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The Effect of Adverse Selection on Strategic Shrouding in Add-On Markets

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Disclosure of Usage of Artificial Intelligence

Artificial Intelligence (AI) was used for text revision, specifically spell- and grammar-checking, and for literature research, in order to extend the search for sources that are potentially connected to my work. AI was also used to obtain suggestions to improve readability and clarity in complex passages.

No AI-generated output was directly included. The AI-suggested corrections and improvements were reviewed before incorporating them.

Zusammenfassung

Durch naive Konsument*innen wird Intransparenz in Add-on-Märkten gestützt. Das liegt daran, dass Konsument*innen, die über versteckte Add-on-Attribute aufgeklärt wurden, intransparente Unternehmen bevorzugen, weil dort die Ausbeutung naiver Konsument*innen das Basisprodukt subventioniert. Wenn Konsument*innen hinsichtlich des Risikos heterogen sind, kaufen sophistizierte, kostenintensive Hochrisiko-Konsument*innen das Add-on trotz hoher Preise, was zu einer adversen Selektion führt. Folglich sind im Equilibrium weniger naive Konsument*innen erforderlich, um die Verschleierungspraktiken aufrechtzuerhalten. Die adverse Selektion verändert zudem die Dynamik des Add-on-Marktes. Während eine Aufklärungskampagne bei homogenen Konsument*innen zu Transparenz führen kann, können ähnliche Maßnahmen unter adverser Selektion stattdessen zu einem Zusammenbruch des Add-on-Marktes führen.

Abstract

The existence of naive consumer supports shrouding practices in add-on markets, as educating consumers about the add-on's hidden attributes will drive them to a non-transparent firm where exploitation of naive consumers subsidizes the base good. If consumers are heterogeneous in risk, sophisticated but costly high-risk consumers purchase the add-on even though prices are high, inducing adverse selection. Consequently, fewer naive consumers are required to support shrouding in equilibrium. Adverse selection also changes the dynamics of the add-on market. While an educational campaign can lead to unshrouding when consumers are homogeneous, under adverse selection, similar measures can lead to market breakdown instead.

1 Introduction

Many transactions involve the offer of a complementary good at the point of sale. In cases where one product is essential for the use of the other, but not vice versa, the essential product is referred to as the base good, while the secondary product is called an add-on. Eyewear insurance, protection plans for smartphones, tablets, and laptops, flight cancellation insurance, full-coverage in rental car insurance and credit card lending are some examples of add-ons that share a common feature. All of them are susceptible to adverse selection. For instance, a clumsy or negligent person is disproportionately attracted to eyewear insurance or a protection plan for a smartphone. Similarly, someone prone to illness will attribute more value to flight cancellation insurance and a consumer with high default risk is more likely to accept high overdraft fees. Because these risk attributes are largely unobservable[1] to firms, a classical information asymmetry is created, potentially leading to adverse selection.

Similar to other add-ons, the examples mentioned above are not the primary focus of the consumer. Their prices and attributes are rarely advertised and are difficult to learn beforehand. Many customers do not consider purchasing the add-on until it is offered at the point of sale of the base good. Ellison (2005) suggests that add-on prices are hidden intentionally to exploit boundedly rational consumers. Building upon this premise, the model by Gabaix and Laibson (2006) features an equilibrium in which firms strategically shroud add-on prices, despite competition in the base-good market.

As for Gabaix and Laibson (2006), this thesis is motivated by instances in which firms appear to view informing consumers about product attributes as neither essential nor advantageous, but it departs from their framework by focusing on the interaction between strategic shrouding and adverse selection.

A prominent example of hidden prices in the literature is credit card overdraft fees. This case serves as a primary motivation in the work of Armstrong and Vickers (2012) and Kosfeld and Schüwer (2017), among others. Although differences in default risk can impose an adverse selection problem, this was not considered by the aforementioned authors.

Insurance is the typical example where consumers are heterogeneous in risk. Loss damage waivers for rental cars are supposed to cover losses beyond collision damage and are typically offered as add-ons to the rental agreement. Baker and Siegelman (2013) argue that these waivers are priced well above the actuarially fair price, even when accounting for loss-aversion. Many consumers purchase such insurance because they are unable to evaluate the contract conditions accurately. For instance, the au-

[1] While default risk is observable to some degree for a composite bank, this might not be the case for standalone credit card issuers.

thors report that 24% of consumers purchased the waiver because they were uncertain whether their personal auto insurance already provided coverage.

Prantner and Kollmann (2025) report the results of a market survey on eyewear insurance in Austria, noting that contract terms were unavailable online. Furthermore, mystery shoppers who visited physical stores and requested the terms and conditions when offered insurance were not provided with them in any instance. This makes it impossible for consumers to assess the terms and conditions before purchase, raising the question of how consumers evaluate such products in the absence of essential information.

These examples illustrate how heterogeneity in risk, boundedly rational consumers, and the shrouding or obfuscating of add-on attributes occur simultaneously. While the existing literature covers the interaction of at most two of these features, this thesis focuses on the interaction of all three, addressing the following question: Does the presence of an adverse selection problem in the add-on market increase the possibilities for shrouding add-on attributes?

To answer this, I extend the model developed by Gabaix and Laibson (2006) to incorporate the information asymmetry regarding the heterogeneity in risk, which potentially leads to adverse selection. Using their model as a baseline allows for a direct comparison of the market conditions that facilitate shrouding in equilibrium with and without the presence of risk heterogeneity. Specifically, we compare the shares of naive consumers and maximum add-on prices for which a shrouding and an unshrouding equilibrium exist in each model respectively. The equilibrium analysis is limited to the symmetric, pure-strategy equilibria.

Our results show that when the heterogeneity is relatively small, shrouding is facilitated not because of adverse selection, but because consumers also differ in their valuation, decreasing the appropriable net surplus for transparent firms which unshroud their prices. When heterogeneity is larger, the negative effect of adverse selection is more severe when firms transparently advertise their prices, compared to when they shroud them. This creates more market constellations in which a shrouding equilibrium exists, compared to the baseline model, where consumers do not differ in terms of risk. In the baseline model, if the share of naive consumers or the shrouding markup is decreased sufficiently, the shrouding equilibrium will cease to exist and an unshrouding equilibrium will exist instead. We demonstrate that these dynamics shift fundamentally when low-risk types' valuation is below the add-on's average marginal cost. Specifically, when the share of naive consumers or the shrouding markup is too low to sustain a shrouding equilibrium, no other equilibrium exists. This has important policy implications. Namely, in contrast to add-on markets without risk heterogeneity, when a consumer education campaign disrupts the shrouding equilibrium, an unshrouding equilibrium might not exist, leading to a collapse of the entire add-on market.

Section 2 presents the connection and distinctions to the related literature. Section 3 describes the model with risk heterogeneity. Section 4 reiterates the results of the model by Gabaix and Laibson (2006), which serves as a baseline. Furthermore, it describes how we intend to compare the baseline model to its extension and the assumptions imposed on the model with heterogeneity in risk to maintain comparability. The main results are presented in Section 5, starting with the comparison where the heterogeneity in risk is small, followed by the generalization of these results for any magnitude of heterogeneity in risk. Finally, we present the difference in the dynamics of the models. Policy implications of the results are discussed in Section 6, and Section 7 concludes.

2 Related Literature

This thesis is related to the literature studying how firms interact with boundedly rational consumers, as well as the literature analyzing markets where firms sell highly profitable add-ons alongside a primary base good.

In contrast to Ellison (2005), where firms charge high add-on prices because search costs prevent consumers from observing them prior to the purchase of the base good, we construct our model as an extension of Gabaix and Laibson (2006). In their framework, firms strategically decide whether to shroud or clearly advertise their add-on prices. While shrouding similarly yields high add-on prices, it can be sustained in equilibrium even if advertising is costless. We augment their analysis by incorporating consumers with heterogeneous risk types that are unobservable to firms, allowing us to examine how the threat of adverse selection impacts the strategic shrouding decision.

Structurally, the models presented by Thanassoulis and Vadasz (2023) and Thanassoulis and Vadasz (2025) share very similar features with our framework, as they investigate an add-on market populated by consumers who differ in sophistication and risk. However, two fundamental distinctions separate their setting from ours. First, firms in their models do not face an endogenous, strategic shrouding decision. Second, they allow firms to price discriminate based on a noisy signal regarding the consumers' risk type. In contrast, our model does not allow firms to price discriminate, and must decide whether or not to shroud the add-on price.

The market for extended warranties and its interaction with boundedly rational consumers are examined by Michel (2017) and Parisi and Depoorter (2026). In terms of sales structure and the behavioral mistakes consumers make at the point of sale, extended warranties closely mirror the add-on insurance example. Despite these close parallels, their frameworks fundamentally lack the potential for adverse selection with respect to heterogeneous consumers. Extended warranties cover product breakdowns driven by product failure alone. Because this risk is entirely independent of the consumer type's underlying riskiness, adverse selection is not a concern in the market for extended warranties. Our model departs from this literature by allowing the underlying risk to be consumer-specific.

While Sandroni and Squintani (2007), Herweg and Müller (2016), and Murooka and Yamashita (2025) all incorporate asymmetric information regarding risk types alongside boundedly rational consumers, their models operate in single-good markets. This thesis departs from their line of research by embedding these identical features within a multi-stage add-on market setup.

As in this thesis, the strategic decision to shroud add-on prices plays a central role in Kosfeld and Schüwer (2017), and Ko and Williams (2017). Ko and Williams (2017) focuses primarily on the welfare implications of regulations under the baseline conditions of Gabaix and Laibson (2006) and Kosfeld and Schüwer (2017) demonstrates that consumer education policies can yield, counterintuitively negative welfare effects if firms are able to price discriminate between sophisticated and naive consumer types. Crucially, none of these papers consider an environment in which consumers have heterogeneous risk profiles.

While Armstrong and Vickers (2012) and Armstrong (2015) evaluate the welfare effects of regulatory policy across rational and boundedly rational consumers, they do not feature an endogenous shrouding decision, whereas in our setup, the choice to shroud or transparently advertise add-on prices is central to the analysis.

Heidhues, Kőszegi, and Murooka (2016) and Heidhues, Kőszegi, and Murooka (2017) demonstrate how profitable deception is sustained in competitive markets when the base good has hidden fees. The defining distinction is that their frameworks center on fees that cannot be avoided after purchase. By contrast, we model an add-on market where consumers can choose not to purchase the add-on.

Lastly, this thesis is also related to the literature on obfuscation. While the terms 'shrouding' and 'obfuscating' are sometimes used synonymously, obfuscation is used more broadly. For instance, shrouding is used for actions that make some product information unobservable, as in Gabaix and Laibson (2006) and Heidhues, Kőszegi, and Murooka (2017). In contrast, instead of a binary shrouding decision, Gu and Wenzel (2014) analyze how asymmetry in firms' prominence influences the degree to which firms choose to obfuscate, and in Ellison and Wolitzky (2012), prices are never completely unobservable. Consumers can search to learn prices, but obfuscation influences the search cost. In Spiegler (2006), prices are observable but obfuscated in the sense that consumers have difficulties comparing multi-dimensional goods.

To my knowledge, this thesis presents a novel contribution to this literature by incorporating consumers with heterogeneous risk into a model in which firms strategically shroud add-on prices, with the goal of examining how the incentives in firms' shrouding behavior change when faced with the potential for adverse selection.

3 Model Setup

The model builds upon the framework of Gabaix and Laibson (2006) and a version of their model by Ko and Williams (2017). There are multiple firms selling a product consisting of a base good and an add-on, offered at prices p_1 and p_2, respectively. Consumers have a unit demand for each of the two products. While the base good is sold in a competitive market, the add-on can only be purchased from the firm providing the base good. The price of the add-on is bounded from above by a maximum price $\overline{p}$, such that $p_2 \leq \overline{p}$. In addition to their pricing decisions, firms strategically choose whether to shroud the price of the add-on. Unshrouding add-on prices is costless, but firms prefer shrouding the add-on price over unshrouding when profits are equal.

The unit mass of consumers is divided into sophisticated and naive types, with the share of naive consumers denoted by $\alpha \in [0, 1]$. Naive consumers do not anticipate that the add-on price might exceed their valuation. Consequently, they always purchase the add-on. Sophisticated consumers only remain unaware of the add-on price if the firm chooses to shroud its price. In this case, they form rational beliefs regarding p_2. Conversely, if a firm unshrouds, the add-on price of this firm becomes known to the sophisticated consumers. Furthermore, a fraction $\lambda \in (0, 1]$ of naive consumers becomes educated when at least one firm unshrouds. An educated naive consumer behaves identically to a sophisticated one. Note that the education effect is not cumulative, meaning that the fraction of educated consumers remains λ regardless of how many additional firms choose to unshroud.

These two types of consumers are further divided into low-risk and high-risk consumers. This is the primary point of departure from the model by Gabaix and Laibson (2006), in which consumers are assumed to be homogeneous with respect to risk. We denote the share of high-risk consumers as ρ and assume that the high-risk consumers are evenly distributed across naive and sophisticated consumers. Consequently, the share of naive, high-risk consumers and the share of sophisticated, high-risk consumers are equal to their share in the general population, ρ. Consumers are aware of their risk type from the beginning. While firms remain unaware of an individual's specific type, they are assumed to know the population distributions α and ρ. This splits the consumers into four different categories. The high-risk naives making up $\alpha\rho$, or $\alpha(1-\lambda)\rho$ in case some naives are educated by a firm unshrouding its add-on price. The low-risk naives make up $\alpha(1-\rho)$ et cetera.

Firms face a constant marginal cost for providing the base good. Without loss of generality, we assume the marginal cost for the base good is normalized to zero. The consumers' valuation for the base good is given by $u > 0$ and is uniform across all types. As we normalized the marginal cost to zero this implies that the net surplus of the base good is positive and equal to the valuation.

To account for different risk types, we introduce heterogeneous costs for the add-on. Specifically, serving the add-on to a high-risk type incurs a cost of $c_H > 0$, while serving a low-risk type incurs a lower cost, $c_L < c_H$. We deliberately do not normalize either cost to zero. Retaining this degree of freedom allows us to keep the framework directly comparable to the baseline model without risk types. The baseline model and the specific assumptions made to maintain comparability are discussed in the next section.

Consumers' valuation of the add-on is $v_L \geq c_L$ for the low-risk type and $v_H > c_H$ for the high-risk type. While naive consumers purchase the add-on regardless of its price or their valuation, sophisticated consumers purchase it only if the add-on price (or their belief about the price, if shrouded) is less than or equal to their valuation.

Note that in the original model by Gabaix and Laibson (2006), consumers must purchase the add-on unless they incur a cost of e in advance to substitute away from it. However, as Ko and Williams (2017, p. 47) notes, the analysis is formally equivalent whether one uses the substitution cost e or the utility v of consuming the add-on, as I do.[2]

The model's timeline unfolds as follows:

Period 1: Firms simultaneously set prices for the base good and the add-on. They also choose whether to shroud the add-on price. The decision to shroud or unshroud does not incur any cost.

Period 2: Consumers choose which firm to purchase the base good from. If any

[2]Reiterating the argument from Ko and Williams (2017, p. 47) for a model without risk types, if the utility from purchasing the add-on is $\tilde{v} - p_2$ and the utility from substituting away from the firm's add-on is $\tilde{v} - e$, the net utility is $\tilde{v} - p_2 - (\tilde{v} - e) = e - p_2$. In our version, the net utility between buying and not buying would be $v - p_2$. Consequently, if $v = e$, consumer behavior is identical in both models.

firm previously decided to unshroud, sophisticated consumers and a fraction λ of naive consumers, the educated naive consumers, observe the add-on prices of the firm that unshrouded its price. They do not observe the add-on prices of the firms that shroud the add-on price and form rational beliefs about the unobserved prices. If all firms shroud the add-on price, none of the naive consumers get educated, and sophisticated consumers have to form beliefs about the add-on prices of all firms. While the uneducated naive consumers base their purchase decisions solely on the base good price p_1, the sophisticated and the educated naive consumers also consider their risk-type-specific utility from buying the add-on and its price, or their belief about the price when the price is shrouded.

Period 3: Consumers may purchase the add-on only from the firm that provided them with the base good. Naive consumers purchase the add-on regardless of the price. Sophisticated and educated naive consumers, however, purchase the add-on only if the price (or their rational beliefs about it) is less than or equal to their risk-specific valuation.

Following Gabaix and Laibson (2006), I focus on the symmetric, pure-strategy perfect Bayesian equilibria of the model. An equilibrium is called symmetric in this setting if all firms charge the same price p_1^* for the base good, the same price p_2^* for the add-on, and make the same decision about shrouding or unshrouding the add-on price.

In the model described above, the firm shrouds the add-on price and the uneducated naive consumers ignore it, even if firms choose to unshroud. While it is realistic to say that the cost of overdrafting a credit card is hidden, it is unlikely that consumers buying eyewear insurance or smartphone protection plans do not know the price at the point of sale. Instead, it is more likely that firms shroud the characteristics of the add-on, preventing naive consumers from correctly anticipating the product's quality and value. For instance, Huysentruyt and Read (2010) document that purchasing decisions for extended warranties are highly influenced by emotional considerations. Consumers who focus on the fear of losing their newly acquired base good will often overlook the fact that the insurance does not cover all the insurance cases they expect. Furthermore, Duke et al. (2014) demonstrate in an experiment that offering the add-on insurance at the point of sale leads to more poor purchasing decisions.

Because of these behavioral biases, shrouded characteristics cause some consumers to purchase the add-on even when the price is above their true valuation. Other consumers, however, anticipate that firms shrouding their product characteristics are not selling an item worth buying. Consequently, shrouding characteristics and shrouding prices can be viewed as two sides of the same coin. Moving forward, we will stick to the terminology of shrouding prices, but keep in mind that the framework applies equally to shrouding characteristics. Finally, because failing to anticipate the add-on is not the only reason why naive consumers behave differently from sophisticated consumers in our setting, we prefer the term 'naive consumer' over the term 'myopic consumer' used by Gabaix and Laibson (2006).

The straightforward interpretation of the maximum add-on price, $\overline{p}$, is that it represents an explicit price control imposed by a regulator. Alternatively, $\overline{p}$ can be viewed as the maximum price a naive consumer is willing to pay or the monopoly price with respect to the demand of the behavioral biased consumers. To ensure also in the latter interpretation that all naive consumers purchase the add-on for any $\overline{p}$, as defined in the model, the definition of the naive consumer is every consumer with a strong enough behavioural bias such that they purchase the add-on for this monopoly price.

I acknowledge that when the maximum add-on price $\overline{p}$ is not interpreted as a regulatory constraint, the maximum price $\overline{p}$ and the share of naive consumers α might not be exogenous, as the maximum price itself could influence who is classified as a naive consumer. However, to maintain the analytical tractability of the model, I assume that both $\overline{p}$ and α are exogenously given.

4 The Baseline-Model and Comparability

Moving forward, we refer to the model described in the previous section as the adverse selection model and to the model by Gabaix and Laibson (2006) as the baseline model. Note that our formulation is closer to the version presented by Ko and Williams (2017). However, both versions are equivalent for our purposes, and the latter applies more directly to our specific application examples.

To keep our adverse selection framework comparable to the baseline model, where the marginal cost of the add-on is normalized to zero and consumers have a uniform valuation for the add-on, we introduce two key assumptions.

First, the costs of providing the add-on in the extended model must satisfy

$$c_H\rho + c_L(1-\rho) = 0.$$

This condition ensures that the expected marginal cost for a consumer drawn at random from the general population equals zero, i.e., the marginal cost in the baseline model.

If the expected consumer valuation of the general population matches the consumer valuation in the baseline model, v, such that

$$v_H\rho + v_L(1-\rho) = v,$$

the two models are comparable in the sense that they share the same expected marginal cost and consumer valuation for a random draw from the general population.

Second, we make a parallel restriction to align consumer valuations and costs, setting

$$c_H - c_L = v_H - v_L.$$

With this assumption, we can rewrite the type-specific valuations as $v_H = v + c_H$ and $v_L = v + c_L$. This implies that the individual net surplus for both the low- and high-risk types simplifies to $v_H - c_H = v_L - c_L = v$, making it independent of the cost parameter c_H and the risk type. This ensures that each consumer has the same net surplus independent of their type, as in the baseline model.

In summary, changing c_H has no effect on the average cost, the average valuation, or the individual net surplus. It only affects the spread between high- and low-risk types, shifting costs and valuations in an identical manner.

Given these cost and valuation constraints, setting either $c_H = 0$ or $\rho = 0$ collapses the adverse selection model back into the baseline model. In the following analysis, we use this two-type baseline model as our reference point. This model comprises naive and sophisticated consumers who share identical valuations. We compare it to the adverse selection model, which introduces high- and low-risk consumer types. Because these risk types differ both in their valuation of the add-on and in the marginal costs they impose on the firm, their interaction potentially induces adverse selection, changing the incentives for firms to shroud the add-on price.

4.1 Results on the Baseline-Model

To establish a clear point of comparison, I will first reiterate the result from Gabaix and Laibson (2006), which characterizes the equilibria of the baseline-model. These findings serve as the baseline against which the model with adverse selection is compared.

In case of $c_H = 0$ or $\rho = 0$, the adverse selection model reduces to the baseline model, and we have that:

(i) If the maximum add-on price satisfies $\overline{p} \geq \frac{v}{\alpha}$, there exists a symmetric equilibrium in which firms ***shroud*** *the add-on price. They charge $p_1^* = -\alpha\overline{p}$ for the base good and $p_2^* = \overline{p}$ for the add-on. In this equilibrium, only naive consumers purchase the add-on.*

(ii) If the maximum add-on price satisfies $\overline{p} \leq v$, there exists a symmetric equilibrium in which firms ***shroud*** *the add-on price. They charge $p_1^* = -\overline{p}$ for the base good and $p_2^* = \overline{p}$ for the add-on. In this equilibrium, all consumers purchase the add-on.*

(iii) If the maximum add-on price satisfies $\overline{p} \in \left[v, \frac{v}{\alpha(1-\lambda)}\right]$, there exists a symmetric equilibrium in which firms ***unshroud*** *the add-on price. They charge $p_1^* = -v$ for the base good and $p_2^* = v$ for the add-on. In this equilibrium, all consumers purchase the add-on.*

In each of these equilibria, sophisticated consumers' and educated naives' beliefs about the add-on price are $p_2 = \overline{p}$ when a firm shrouds. These beliefs are the same for any base-good price. Furthermore, there are no other symmetric, pure-strategy equilibria.

In the baseline model, a shrouding equilibrium exists when the maximum price $\overline{p}$ is below the consumers' valuation v. In this case all consumers purchase the add-on, the market is efficient, and firms shroud only because they prefer shrouded prices in the case of equal profits. The more interesting case of shrouding equilibria is when sophisticated consumers are not willing to purchase the add-on and the equilibrium is inefficient. In this case the profit from naive consumer $\alpha\overline{p}$ needs to be large enough to sustain shrouding. This is not the case as long as $\overline{p}$ is close to v. Then, unshrouding is the only equilibrium. The unshrouding equilibrium on the other hand is efficient and has fewer consumers who act like naives, as unshrouding educates some of the previously naive consumers. Fewer naives imply that the maximum price $\overline{p}$ necessary for the unshrouding equilibrium to fail is larger compared to the case when the shrouding equilibrium fails. This is represented by the fact that for some maximum prices both the shrouding and unshrouding equilibria exist in parallel.

It is a central feature of this model that the shrouding is not competed away through the base-good market. In Gabaix and Laibson (2006), they call this mechanism the *curse of debiasing*, describing how educated consumers are driven away from the firm with the transparent prices, as they realize that they will get a better deal at a shrouding firm when they only purchase the base good.

We can think of the magnitude of the *curse of debiasing* as being influenced by two channels. First, how much the naive consumers are ripped off in favor of the sophisticated consumers, who reap the benefits of this exploitation by purchasing only the base good from a shrouding firm. Second, when the net surplus of the add-on is large, a transparent firm earns enough profit on the add-on to compete with the low base-good prices of firms that shroud their add-on prices.

4.2 How to Assess Shrouding and Unshrouding Across Models

Before we present the first results, let us discuss how we can compare shrouding possibilities and the potential for firms to unshroud add-on prices. We will consider both the baseline model and the adverse selection model and compare the respective ranges of exogenous parameters for which a shrouding and unshrouding equilibrium exists.

Consider the equilibria of the baseline model. An unshrouding equilibrium exists only if $\overline{p} \in \left[v, \frac{v}{\alpha(1-\lambda)}\right]$. Should the length of this interval decrease, fewer market configurations exist in which unshrouding prices is an equilibrium. Because the range of the maximum add-on price $\overline{p}$ for which a shrouding equilibrium exists is unbounded, we focus instead on the length of the interval where shrouding fails but an unshrouding equilibrium is sustained. In the baseline model, this interval is defined by $\overline{p} \in \left[v, \frac{v}{\alpha}\right]$. If the length of this interval decreases, it becomes easier to shroud prices because fewer market configurations exist in which shrouding prices is not an equilibrium.

We can also compare the shares of naive consumers α necessary to sustain the shrouding or the unshrouding equilibrium. However, it does not make sense to compare α across models for the same absolute level of $\overline{p}$. Consider the minimum share of naive consumers for which shrouding is still an equilibrium, that is, $\alpha_S = \frac{v}{\overline{p}}$. Hence, for $\frac{\overline{p}}{2} = v$, a shrouding equilibrium only exists if $\alpha \geq \frac{1}{2}$. Now, assume that $p_2^* = \overline{p}$ is the add-on price of a shrouding equilibrium in the adverse selection model, where the low-risk consumers have a valuation $v_L < v$ for the add-on. If we were to use the same level of $\overline{p}$ as in the baseline-model, we can anticipate that it becomes easier to shroud, as $\overline{p} - v_L > \overline{p} - v$. Therefore, when assessing for which α an equilibrium holds, we suggest comparing not at the same absolute level of $\overline{p}$, but at equal relative values $\overline{p} - p_{US}$[3]. Here, p_{US} is the add-on price firms choose in the unshrouding equilibrium when assessing where an unshrouding equilibrium exists, and the add-on price firms would deviate to, when assessing where the shrouding equilibrium fails.

Essentially, we compare the adverse selection model, which features consumers with heterogeneous risk with respect to the add-on, and the baseline model where consumers are identical with respect to riskiness. Across these two models, we compare

[3] Similar to considering the length of the ranges instead of the maximum $\overline{p}$.

whether there are more or fewer market configurations (exogenous parameters) in which shrouding and unshrouding is an equilibrium.

5 Shrouding and Unshrouding in the Adverse Selection Model

Before describing the equilibria of the adverse selection model for the case where it does not coincide with the baseline model (i.e., $c_H > 0$ and $\rho > 0$), some preliminary results can be established.

Lemma 1. *In any symmetric, pure-strategy equilibrium in the adverse selection model,*

(i) firms earn zero profits in equilibrium,

(ii) sophisticated and educated naive consumers' beliefs about shrouded prices are that $p_2 = \overline{p}$ for any p_1,

(iii) the equilibrium price for the add-on satisfies $p_2^* = \overline{p}$ in the shrouding equilibrium and $p_2^* \in \{v_L, v_H\}$ in the unshrouding equilibrium.

Proof. (i): If firms earn positive profits in a symmetric, pure-strategy equilibrium, any firm can marginally undercut the base-good price to attract all customers of its competitors at essentially the same profit margin, increasing its profits.

(ii): If any sophisticated consumers or educated naive consumers believe that $p_2 < \overline{p}$ for some p_1, a firm charging p_1 can increase profits by charging $p_2 = \overline{p}$, since demand does not depend on p_2 but only on the beliefs about p_2. In other words, if firms shroud prices, sequentially rational consumers have to believe that they charge maximum prices.

(iii): From the argument in (ii), we know that $p_2^* = \overline{p}$ in any shrouding equilibrium. In the unshrouded case, demand is constant for $p_2 \leq v_L$, $p_2 \in (v_L, v_H]$ and $p_2 \in (v_H, \overline{p}]$. If demand does not change within these intervals, increasing prices only increases profits. Therefore, the only options for add-on prices in equilibrium are v_L, v_H, and $\overline{p}$. But charging $\overline{p}$ and unshrouding add-on prices cannot be an equilibrium, since shrouding would not change profits and, in the case where profits are equal, firms prefer to shroud the add-on price. □

The features described in Lemma 1 resemble the features of the baseline model. In any symmetric, pure-strategy equilibrium of the baseline model, firms also earn zero profits, and beliefs about shrouded prices match those described in Lemma 1. As in the baseline model, profits from high markups on the add-on are dissipated by competition in the base-good market and sophisticated consumers believe, independent of their risk-type, that a shrouding firm charges the highest possible price. The main difference in these preliminary results lies in point (iii). If firms unshroud add-on prices in the adverse selection model, they must decide whether to sell at a lower price to all consumers or at a higher price to high-risk types and naive consumers only. In the baseline model, however, all consumers share an identical valuation, v, for the add-on, which is the only viable add-on price in an unshrouding equilibrium in the baseline-model.

Having established how we are going to compare unshrouding and shrouding across models in the previous section, we now examine how the range of exogenous parameters for which an unshrouding or shrouding equilibrium exist changes when we introduce consumers with heterogeneous risk types. The following results demonstrate that introducing heterogeneous risk increases the range of parameters in which a shrouding equilibrium exists while simultaneously decreasing the range of parameters in which an unshrouding equilibrium exists. The heterogeneous risk types can be introduced either by raising the cost of high-risk consumers c_H or by raising the share of high risk consumers ρ.

5.1 Introducing Heterogeneous Risk

We begin with a comparative statics analysis where c_H is marginally increased from 0 to $c_H > 0$ for a given ρ. As we will see, the results closely mirror the dynamics of the baseline model, providing an intuitive starting point to illustrate some of the underlying dynamics. We do not conduct a separate comparative statics analysis for ρ, as doing so does not provide a more comprehensible case compared to the full characterization of the equilibria. Instead, we directly move to a general characterization of the equilibria of the adverse selection model for all $c_H > 0$ and $\rho \in (0, 1)$ in the subsequent subsection.

Due to the assumption described in the previous section, we can think of c_H as a measure of the spread between the low- and the high-risk types. If the spread is relatively small, we can characterize the symmetric, pure-strategy equilibria of the adverse selection model as follows.

Proposition 2. *Let $c_H > 0$ be sufficiently small.*

*(i) If the maximum add-on price satisfies $\overline{p} \geq \frac{v_L}{\alpha}$, there exists a symmetric equilibrium in which firms **shroud** the add-on price. They charge $p_1^* = -\alpha\overline{p}$ for the*

base good and $p_2^* = \overline{p}$ for the add-on. In this equilibrium, only naive consumers purchase the add-on.

*(ii) If the maximum add-on price satisfies $\overline{p} \leq v_L$, there exists a symmetric equilibrium in which firms **shroud** the add-on price. They charge $p_1^* = -\overline{p}$ for the base good and $p_2^* = \overline{p}$ for the add-on. In this equilibrium, all consumers purchase the add-on.*

*(iii) If the maximum add-on price satisfies $\overline{p} \in \left[v_L, \frac{v_L}{\alpha(1-\lambda)}\right]$, there exists a symmetric equilibrium in which firms **unshroud** the add-on price. They charge $p_1^* = -v_L$ for the base good and $p_2^* = v_L$ for the add-on. In this equilibrium, all consumers purchase the add-on.*

In each of these equilibria, sophisticated consumers' and educated naives' beliefs about the add-on price are $p_2 = \overline{p}$ when a firm shrouds. These beliefs are the same for any base-good price. Furthermore, there are no other symmetric, pure-strategy equilibria.

A detailed proof of Proposition 2 is provided in the Appendix.

Before we discuss how the introduction of heterogeneous risk types changes the range of exogenous parameters for which an unshrouding or shrouding equilibrium exists in comparison to the baseline model, let us discuss how we can interpret these results that describe the symmetric, pure-strategy equilibria when $c_H > 0$ is sufficiently small.

The results in the above setting are extremely similar to the corresponding equilibrium characterizations of the baseline model (see subsection 4.1), with the subtle but important difference that the valuation for the add-on of the low-risk type, v_L, takes the place of the valuation for the add-on v in the baseline model.

Why is this the case? First of all, in the unshrouded case, firms always prefer charging $p_2 = v_L$ and selling to everyone over charging the higher price $p_2 = v_H$ selling to everyone except the low-risk sophisticates. To acknowledge this, consider a firm going from $p_2 = v_H$ to $p_2 = v_L$. The firm makes additional sales to the low-risk sophisticated consumers at a profit margin $v_L - c_L = v$, which is independent of c_H. On the other hand the price decrease $v_H - v_L = c_H \frac{1}{1-\rho}$ is relatively small when the spread between the low- and the high-risk types is small. Therefore, in the setting of Proposition 2, if firms decide to unshroud add-on prices, they will always lower the price of the add-on to capture additional demand from low-risk, sophisticated consumers.

By doing so, firms avoid lost sales that would occur when firms charge high prices, due to the low-risk sophisticates self-selecting out of the add-on market.

Furthermore, when firms charge $p_2 = v_L$ and unshroud the add-on price, they serve the whole market, and the costs from the high- and low-risk consumers balance out. Additionally, we know from before that charging $p_2 = v_L$ is preferred over charging $p_2 = v_H$, creating the same conditions as in a version of the baseline model where $v = v_L$. Therefore, the same arguments apply. A shrouding equilibrium is sustainable only if the profit from selling the add-on to the share of naive consumers, α, at $\overline{p}$ is higher than profits from selling to everyone at v_L, while an unshrouding equilibrium is sustainable only if the profits from selling the add-on to everyone at v_L are higher than the profit from selling to the share of uneducated naive consumers, $\alpha(1-\lambda)$, at price $\overline{p}$.

We can use the similarity to the baseline model to deduce the welfare analysis of these equilibria. As in the baseline model, the shrouding equilibrium with $\overline{p} > \frac{v_L}{\alpha}$ is inefficient, while the unshrouding equilibrium and the shrouding equilibrium with $\overline{p} \leq v_L$ are efficient. In the inefficient case, the excess profits from naives are transferred to the sophisticated consumers by subsidizing the base good. The inefficiency arises solely because sophisticates do not purchase the add-on.

We can summarize the result in the following way. The introduction of heterogeneous risk types is as if we are considering the baseline model but decreasing the consumers' valuation from v to v_L. Therefore, the change in the range of exogenous parameters for which an unshrouding and shrouding equilibrium exists is similar to the change we would observe when reducing v in the baseline model. This consideration gives us the following result.

Corollary 3. *Let $c_H > 0$ be sufficiently small. In the adverse selection model,*

(i) the range of $\overline{p}$ and α for which a ***shrouding*** *equilibrium exists* ***increases****, and*

(ii) the range of $\overline{p}$ and α for which an ***unshrouding*** *equilibrium exists* ***decreases***

compared to the baseline model.

A detailed proof of Corollary 3 is provided in the Appendix.

This result becomes even more obvious when looking at Figure 1, which displays the range of $\overline{p}$ for which shrouding and unshrouding equilibria exist. Note that the non-shrouding range $\left[v_L, \frac{v_L}{\alpha}\right]$ and the unshrouding range $\left[v_L, \frac{v_L}{\alpha(1-\lambda)}\right]$ not only shift to

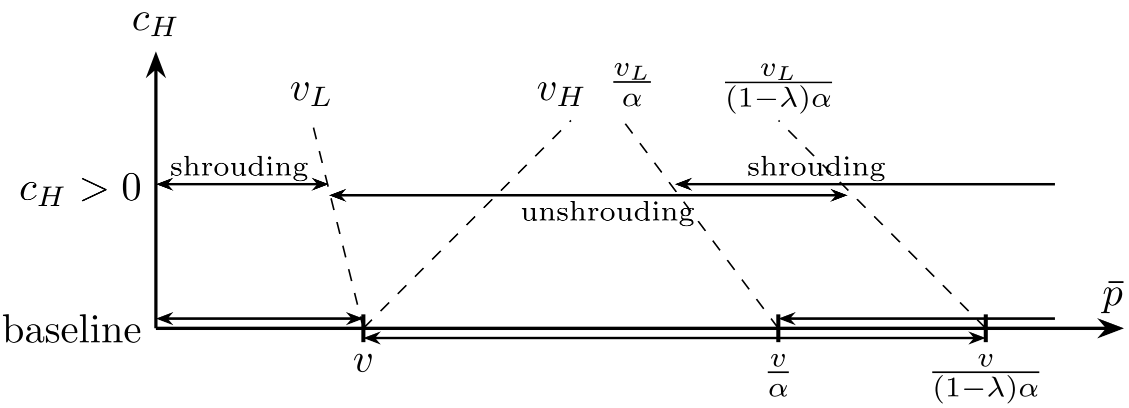

Figure 1: Range of $\overline{p}$ with $c_H > 0$ in which shrouding and unshrouding equilibria exist for $v = \frac{3}{2}, \alpha = \frac{1}{3}, \rho = \frac{1}{5}$, and $\lambda = \frac{1}{4}$.

the left but also become narrower as c_H increases.

Risk heterogeneity in this adverse selection model increases the severity of the curse of debiasing by altering the relative profitability of shrouding versus unshrouding. While risk heterogeneity leaves the add-on profits of shrouding firms unaffected, a transparent firm must lower its add-on price to accommodate the lower-risk type's lower valuation, essentially diminishing its overall profit margin from the add-on when unshrouding. This leaves the transparent firm less capable of competing on the base good, thereby intensifying the curse of debiasing.

5.2 Heterogeneous Risk and Shrouding

Having examined the equilibria for a marginal increase in c_H, which qualitatively resembles the outcome of the baseline model, we now turn to a general characterization of the equilibria for any $c_H > 0$ and $\rho \in (0, 1)$.

Previously, when c_H was sufficiently small (i.e., a small spread between the low- and high-risk types), firms preferred to sell the add-on to all consumers for the lower add-on price $p_2 = v_L$ compared to excluding the low-risk sophisticated consumers by raising prices to v_H. However, this no longer holds when the spread between low- and high-risk type is large, as we will see in the following lemma, which describes the symmetric, pure-strategy equilibria for any $c_H > 0, \rho \in (0, 1)$.

Lemma 4.

(i) If $\overline{p} > v_H$ *and the maximum add-on price satisfies*

$$\overline{p} \geq \max\left\{\frac{v_L}{\alpha}, \frac{v}{\alpha}\left(\rho + \alpha(1-\lambda)(1-\rho)\right) + c_H(1-\lambda)\right\},$$

there exists a symmetric equilibrium in which firms ***shroud*** *the add-on price. They charge* $p_1^* = -\alpha\overline{p}$ *for the base good and* $p_2^* = \overline{p}$ *for the add-on. In this equilibrium, only naive consumers purchase the add-on.*

(ii) If $\overline{p} \in (v_L, v_H]$ *and the maximum add-on price satisfies*

$$\overline{p} \geq \max\left\{\frac{v}{\alpha + (1-\alpha)\rho} + c_L,\ c_H\frac{(1-\alpha)\rho}{\alpha + (1-\alpha)\rho}\right\},$$

there exists a symmetric equilibrium in which firms ***shroud*** *the add-on price. They charge* $p_1^* = -\overline{p}(\alpha + (1-\alpha)\rho) + c_H(1-\alpha)\rho$ *for the base good and* $p_2^* = \overline{p}$ *for the add-on. In this equilibrium, sophisticated high-risk consumers and naive consumers purchase the add-on.*

(iii) If $\overline{p} \in [0, v_L]$*, then there exists a symmetric equilibrium in which firms* ***shroud*** *the add-on price. They charge* $p_1^* = -\overline{p}$ *for the base good and* $p_2^* = \overline{p}$ *for the add-on. In this equilibrium, all consumers purchase the add-on.*

Define $\widehat{c}_H = \frac{(1-\alpha+\alpha\lambda)(1-\rho)^2}{1-(1-\alpha+\alpha\lambda)(1-\rho)}\,v$.

(iv) If the cost of the high-risk consumer satisfies $c_H \leq \widehat{c}_H$ *and the maximum add-on price* $\overline{p} \in \left[v_L, \frac{v_L}{\alpha(1-\lambda)}\right]$*, there exists a symmetric equilibrium in which firms* ***unshroud*** *the add-on price. They charge* $p_1^* = -v_L$ *for the base good and* $p_2^* = v_L$ *for the add-on. In this equilibrium, all consumers purchase the add-on.*

(v) If the cost of the high-risk consumer satisfies $c_H > \widehat{c}_H$ *and the maximum add-on price* $\overline{p} \in \left[v_H, v_H + \frac{v}{\alpha(1-\lambda)}(1-\alpha(1-\lambda))\rho\right]$*, there exists a symmetric equilibrium in which firms* ***unshroud*** *the add-on price. They charge* $p_1^* = -v_H(\alpha(1-\lambda) + (1-\alpha(1-\lambda))\rho) + c_H(1-\alpha(1-\lambda))\rho$ *for the base good and* $p_2^* = v_H$ *for the add-on. In this equilibrium, sophisticated high-risk consumers and naive consumers purchase the add-on.*

(vi) If the cost of the high-risk consumer satisfies $c_H > \widehat{c}_H$*, the valuation of the low-risk consumer* $v_L > 0$ *and the maximum add-on price* $\overline{p} \in \left[v_L, \frac{v}{1-(1-\alpha(1-\lambda))(1-\rho)} + c_L\right]$*, there exists a symmetric equilibrium in which firms* ***unshroud*** *the add-on price.*

They charge $p_1^* = -v_L$ for the base good and $p_2^* = v_L$ for the add-on. In this equilibrium, all consumers purchase the add-on.

In each of these equilibria, sophisticated consumers and educated naives beliefs about the add-on price are $p_2 = \overline{p}$ when a firm shrouds. These beliefs are the same for any base-good price. Furthermore, there are no other symmetric, pure-strategy equilibria.

A detailed proof of Lemma 4 is provided in the Appendix.

First, note that, for c_H small enough Lemma 4 reduces to the results from Proposition 2. For c_H small enough, we have $c_H \leq \widehat{c}_H$ and therefore only need to consider case (iv) for the unshrouding equilibrium corresponding to case (iii) in Proposition 2. Also, for c_H small enough we have that

$$\frac{v_L}{\alpha} > \frac{v}{\alpha}\left(\rho + \alpha(1-\lambda)(1-\rho)\right) + c_H(1-\lambda)$$

and the conditions in (ii) of Lemma 4 are never satisfied. To see this, consider

$$\begin{aligned} & v_H < \frac{v}{\alpha + (1-\alpha)\rho} + c_L \\ \Leftrightarrow \quad & \frac{c_H}{1-\rho} < \frac{v}{\alpha + (1-\alpha)\rho} - v \end{aligned}$$

which is always satisfied for small enough c_H, since the right-hand side is strictly greater than zero. This implies that either $\overline{p} < v_H$ or $\overline{p} \geq \frac{v}{\alpha+(1-\alpha)\rho} + c_L$ but never both.

Why do we get more distinct cases compared to Proposition 2, where the spread between the low- and high-risk consumers was small (i.e., c_H is small)? The restrictions that make the adverse selection model comparable to the baseline model imply a constant profit margin for the high- and low-risk consumers, namely $v_H - c_H = v_L - c_L = v$. On the other hand, the difference between the two risk types in valuation and cost increases with c_H. Selling to everyone becomes less attractive for firms as the difference between v_H and v_L increases. At some point firms will prefer charging $p_2 = v_H$ over $p_2 = v_L$. This is why we have the additional case (v) in which $p_2^* = v_H$ compared to case (iv), where $p_2^* = v_L$. Additionally, if firms prefer charging $p_2 = v_H$ over $p_2 = v_L$ but the maximum price prevents them from charging the higher price, this results in case (vi). Finally, case (ii) is different from (i), because in case (ii) sophisticated, high-risk consumers will purchase the add-on even if firms shroud, because their valuation is above the maximum price, compared to case (i) where only the naive consumers purchase the add-on from a firm that shrouds prices.

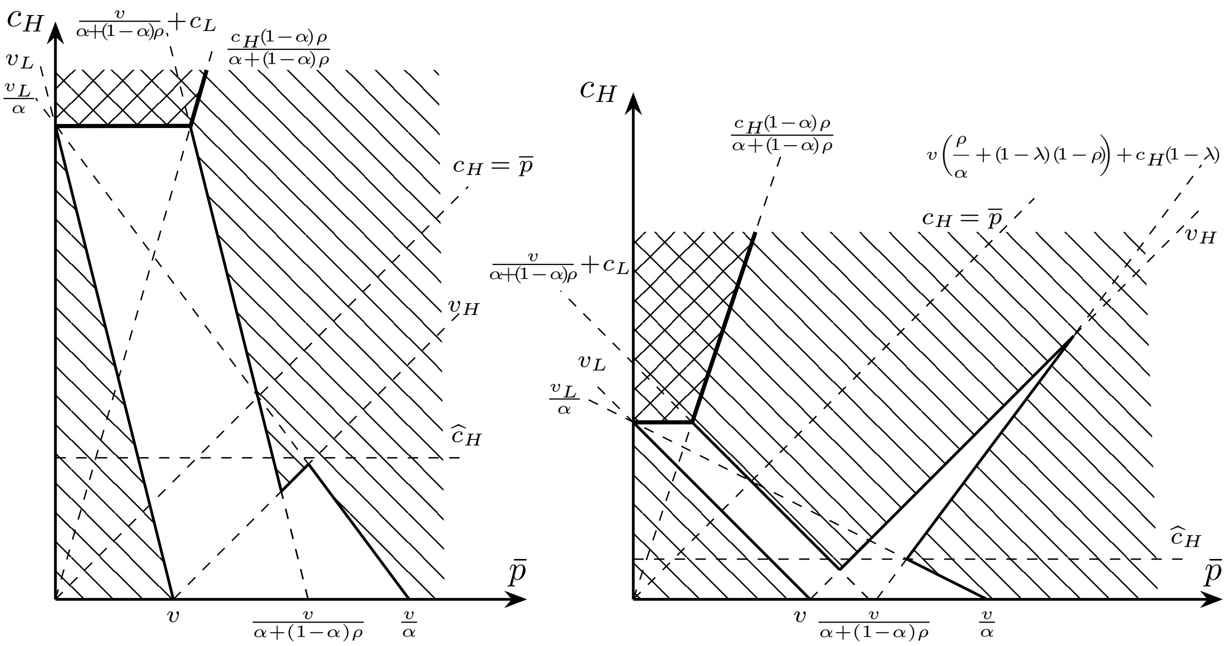

Figure 2: Shrouding Equilibria with, (**left**: $v = 1$, $\rho = \frac{1}{5}$, $\alpha = \frac{1}{3}$, and $\lambda = \frac{1}{4}$), (**right**: $v = \frac{3}{2}$, $\rho = \frac{1}{2}$, $\alpha = \frac{1}{2}$, and $\lambda = \frac{1}{4}$)

Figure 2 illustrates the various cases for shrouding equilibria described in Lemma 4 (i)-(iii) for two different sets of parameters. The regions marked with Northeast diagonal lines indicate the parameter settings where a shrouding equilibrium exists, while the crosshatched area represents the region where the market for add-ons breaks down. Looking at Figure 2 for small c_H, we can recognize the results from the previous subsection 5.1. The left diagram differs from the right one because, for some combinations of α and ρ the second part of the limit in (i) never comes into effect. This tells us that when the share of high-risk consumers is too low, a deviation to unshroud the add-on price at $p_2 = v_H$ and serve only the high-risk consumers and the naives is never profitable.

First, consider case (i). Compared to the previous subsection, we need to consider if firms can profitably deviate to $p_2 = v_H$. If the spread between the low- and the high-risk types is large, the shrouding equilibrium is sustained only if the profits of unshrouding and charging $p_2 = v_H$ attract enough high-risk consumer without driving away too many naive low-risk consumer due to the educational effect of the unshrouded price. In this scenario it helps to sustain the shrouding equilibrium, if the educational effect λ is large, as this leads to more naive consumers realizing that they are low-risk types who should not buy the add-on at this price.

In case (ii) where the high-risk types valuation, v_H, is above the maximum add-on price, $\overline{p}$, the only possible deviation is to unshroud and charge $p_2 = v_L$. Deviating thus attracts demand from the low-risk sophisticated consumers, from whom the full net surplus of the add-on, v, is extracted. On the other hand, firms lose some profit margin on the existing customers, namely $\overline{p} - v_L$. The high-risk consumers already purchase the add-on when firms shroud and cannot be driven away by deviating to unshroud the add-on price. Therefore, for this equilibrium to hold, only the shrouding markup $\overline{p} - v_L$ and the size of the share of sophisticated, low-risk consumers matter.

The second constraint in case (ii) does not arise because firms can profitably unshroud prices, but from the add-on market breaking down. The market for add-ons breaks down if the maximum price for the add-on is too small for the revenue from the naive consumers and the high-risk, sophisticated consumers to make up for the higher cost c_H from the share of high-risk, sophisticated consumers.

The two constraints in case (ii), the constraint necessary for shrouding being preferred over unshrouding, and the constraint for a profitable add-on market when shrouding, intersect where the valuation of the low-risk consumers is zero, $v_L = 0$. Selling the add-on with shrouded prices yields zero profits because the cost from the high-risk, sophisticated consumers dissipates the revenue, a firm would be indifferent to unshrouding, when unshrouding also yields zero profits. This is the case, only if v_L is zero.

Before we analyze the range of exogenous parameters for which a shrouding equilibrium exists, let us consider the unshrouding equilibria of Lemma 4. Figure 3 displays the cases for the unshrouding equilibria (iv)-(vi), where the Northwest diagonal pattern highlights the settings in which an unshrouding equilibrium exists. To allow for direct comparison, both figures are plotted using the exact same set of underlying values for v, ρ, α and λ as in Figure 2. In contrast to the shrouding equilibria, it does not depend on ρ whether some of the conditions restricting the area of the unshrouding equilibria come into effect. That is why the two depictions of the range of the unshrouding equilibrium look qualitatively similar despite the difference in parameters.

For c_H smaller than $\widehat{c}_H$, firms prefer to charge $p_2 = v_L$ over $p_2 = v_H$ for the add-on when they unshroud prices. For the discussion of this case and why firms prefer to charge a lower price $p_2 = v_L$ for small c_H, we refer to the previous subsection 5.1.

When c_H is above $\widehat{c}_H$, firms prefer to charge $p_2 = v_H$ for the add-on when they unshroud prices. If the maximum price for the add-on prevents firms from charging $p_2 = v_H$, their only option is to charge transparent prices $p_2 = v_L$. This is represented by the two arms for values of c_H above $\widehat{c}_H$ in Figure 3.

Case (v) of Lemma 4 describes when the unshrouding equilibrium is sustained despite the possibility to shroud at a higher price $\overline{p} > v_H$. If a firm shrouds, it will lose

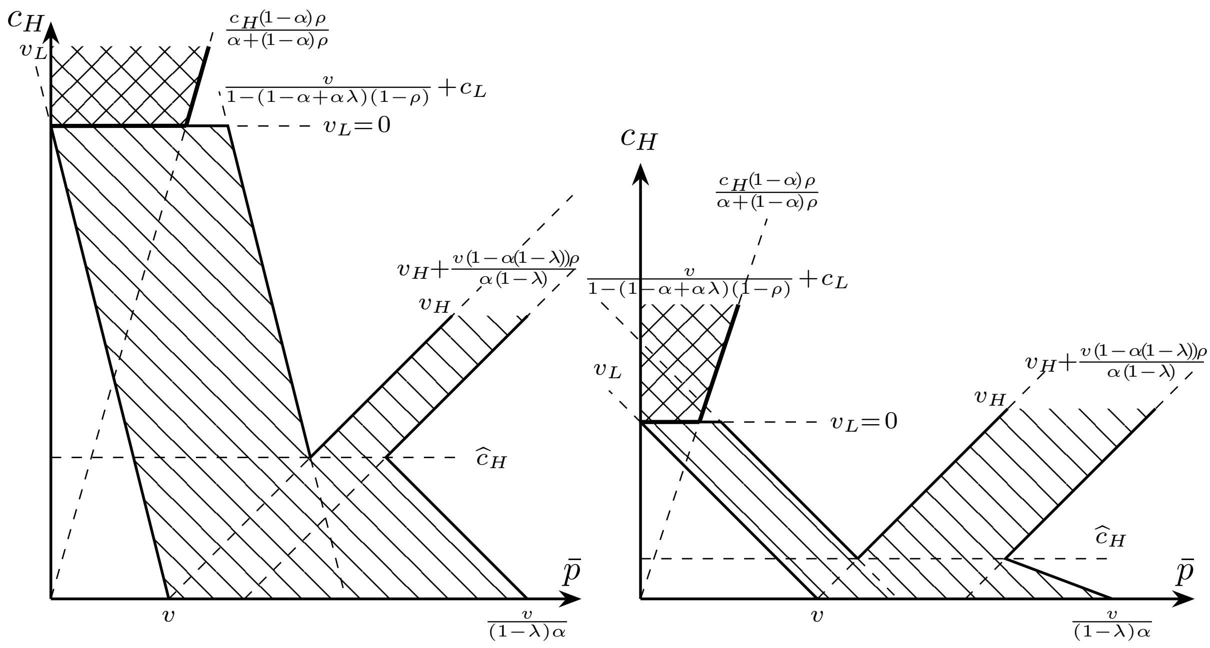

Figure 3: Unshrouding Equilibria with, (**left**: $v = 1$, $\rho = \frac{1}{5}$, $\alpha = \frac{1}{3}$, and $\lambda = \frac{1}{4}$), (**right**: $v = \frac{3}{2}$, $\rho = \frac{1}{2}$, $\alpha = \frac{1}{2}$, and $\lambda = \frac{1}{4}$)

the high-risk sophisticates as customers. Because these customers not only pay high prices but also incur high costs, their profit margin is only v. This is weighed against the markup $\overline{p} - v_H$ charged on the uneducated naives.

The intuition behind case (vi) is the same as for the shrouding equilibrium (ii) constrained by the deviation to unshroud to $p_2 = v_L$. It is the other side of the same coin, only the population shares change due to the educational effect of unshrouding, as we consider the unshrouding equilibrium. Importantly, the unshrouding equilibrium fails once the equilibrium price is below zero, as a firm not selling the add-on would be able to charge lower prices in the base good market.

Using the characterization of the shrouding and unshrouding equilibria from Lemma 4, we can evaluate how the range of exogenous parameters in the adverse selection model changes compared to the baseline model.

Proposition 5. *In the adverse selection model,*

(i) the range of $\overline{p}$ and α for which a ***shrouding*** *equilibrium exists* ***increases****, and*

(ii) the range of $\overline{p}$ and α for which an ***unshrouding*** *equilibrium exists* ***decreases***

compared to the baseline model.

A detailed proof of Proposition 5 is provided in the Appendix.

Behind the technicalities, there are some economically intuitive reasons why the curse of debiasing intensifies compared to the baseline model without risk heterogeneity.

First, if a firm unshrouds prices, it can never attain the full net surplus from all sophisticated consumers. The firm either excludes the low-risk types to get the whole net surplus from the high-risk types, or it lowers the price, losing surplus from the high-risk types in order to attract the low-risk types. This is the main driver of the results from the previous section when the spread between the low- and high-risk types is small.

Second, unshrouding prices goes hand in hand with lowering the add-on price. Otherwise, this only drives away some of the naive consumers who get educated by the firm's transparent pricing. However, lowering prices will only attract more high-risk consumers and never drive them away. Consequently, the negative impact from adverse selection when a firm unshrouds its price is at least as severe as when the price is shrouded.

Lastly, naive, low-risk consumers who were educated by a firm unshrouding its price, might not want to purchase the add-on at the given price. This worsens the adverse selection effect when a firm unshrouds its price.

While adverse selection also affects the profits of the add-on when firms shroud, the effect is at least as severe in the unshrouded setting. Additionally, with unshrouded prices, firms are not able to appropriate the whole net surplus of both the low- and high-risk consumers. Overall, this leads to an intensification of the curse of debiasing.

5.3 Stable Conditions for Shrouding

The previous subsection provides a characterization of the shrouding and unshrouding equilibria of the adverse selection model for any $c_H > 0$ and $\rho \in (0, 1)$. Due to adverse selection, there are constellations in which neither a shrouding nor an unshrouding equilibrium exists, even with positive maximum prices $\overline{p} > 0$. When the low-risk consumers are not willing to purchase at any positive price (i.e., $v_L \leq 0$) and the cost from serving the high-risk sophisticates exceeds the revenue from the add-on at the maximum price, the add-on will incur losses. The firm will avoid to incurring losses in the add-on market by exiting the market, causing a breakdown of the add-on market, as it is not able to compete with a firm that only operates in the base good market. This is in contrast to the baseline model, where a shrouding or unshrouding equilibrium exists for any positive maximum prices $\overline{p} > 0$.

Furthermore, consider a shrouding equilibrium in the baseline model for some maximum price $\overline{p} > v$. If some market intervention would decrease the maximum price and this decrease is sufficiently large, the shrouding equilibrium will fail to exist. In the baseline model, there would still exist an unshrouding equilibrium to which the market could transition, following such a distortion. This is not necessarily the case for the adverse selection model. Instead, when the shrouding equilibrium fails, the add-on market breaks down. The following proposition formally establishes this difference between the two models.

Proposition 6.

(i) For $v_L \leq 0$, $\overline{p} < v_H$, and $\alpha \in (0,1)$, if no shrouding equilibrium exists, then no other equilibrium exists in which the add-on is sold.

(ii) If $v(1-\tilde{\alpha}) < (v_H - c_L)\tilde{\alpha}\tilde{\lambda}(1-\rho)$, and $c_H \geq \widehat{c}_H$, a shrouding equilibrium exists for any $\overline{p} \geq v_H$, $\alpha \in [\tilde{\alpha}, 1)$, and $\lambda \in [\tilde{\lambda}, 1)$.

A detailed proof of Proposition 6 is provided in the Appendix.

Part (i) of Proposition 6 tells us that, when the add-on cannot be sold profitably to the low-risk, sophisticated consumers and the maximum price does not chase away high-risk consumers, the shrouding equilibrium exists unless the add-on market breaks down. Therefore, the shrouding equilibrium is stabilized as long as the add-on market does not break down. This is in contrast to the baseline model, where the shrouding equilibrium is dismantled for $\overline{p} \in (v, \frac{v}{\alpha})$ but the unshrouding equilibrium still exists.

In part (ii) of Proposition 6, irrespective of the maximum price, firms lose more profit by educating some of the highly profitable low-risk naive consumers than they gain from attracting high-risk sophisticated consumers. As in part (i), this suggests that decreasing the maximum price will destabilize the shrouding equilibrium. However, decreasing the share of naive consumers sufficiently would lead to the dismantling of the shrouding equilibrium. Note that this hurdle to unshrouding is driven by not wanting to educate low-risk naive consumers. Therefore, if unshrouding has a large educational effect (large λ), unshrouding becomes even more unattractive. Paradoxically a large educational effect causes both a larger range in which an unshrouding equilibrium exists and a smaller range in which the shrouding equilibrium fails to exist.

These findings imply that measures to decrease the share of naive consumers or lower the maximum add-on price can have substantially different consequences for the add-on market when consumers are heterogeneous in risk.

6 Discussion

Our findings offer several insights for regulatory intervention in markets for add-ons where consumers are heterogeneous in risk, as it is the case, for example, with add-on insurance. First, due to the structural similarities between our framework and the baseline model when the spread between low- and high-risk consumers is small, we conjecture that the results on mandatory disclosure established by Ko and Williams (2017) continue to hold. Specifically, mandatory disclosure does not necessarily improve consumer welfare. It may fail to increase both overall consumer welfare and the welfare of naive consumers.

Second, we expect these shrouding equilibria to persist over time due to a weak consumer learning curve. Unlike general consumer goods where feedback is immediate, add-on insurance only provides feedback about shrouded attributes when an insured event actually occurs. Because these events are relatively rare, the long-term market learning effects envisioned by Gabaix and Laibson (2006) are likely negligible in this context. Consumer organizations could run educational campaigns to support learning and lower the share of naive consumers. For such a campaign to make a difference in shrouding behavior, the impact needs to be larger compared to a market where consumers are identical with respect to risk. We identified a situation where a decrease in naive consumers cannot lead to an unshrouding equilibrium.

Furthermore, our analysis suggests that using price controls to combat strategic shrouding becomes more difficult, especially when the risk spread between the low- and high-risk types is large. The introduction of heterogeneous risk types narrows the range of maximum add-on prices for which firms are forced out of a shrouding equilibrium. If the risk spread is sufficiently large, price controls lose their effectiveness entirely and can disrupt a shrouding equilibrium only by collapsing the entire add-on market.

In order to attain a tractable model that is comparable to the baseline model, we made several restrictive assumptions. For one, we assumed that the low- and high-risk types are uniformly distributed across the naive and sophisticated consumers. We conjecture that our results are stable to slight variations in this distributional assumption, but not to extreme changes.

We also assumed that the spread between the cost and valuation of the low- and high-risk types must move in parallel. This implies that the net surplus is uniform across risk types. We deem this a useful assumption in order to compare the two models, but acknowledge that our results would not hold if net surplus were distributed extremely unequally across risk types.

From our results, we cannot make any conjectures about the existence or properties of non-symmetric or mixed-strategy equilibria, leaving the question open the question of whether there are equilibria in which only some firms shroud and how these compare to non-symmetric equilibria of the baseline model.

7 Conclusion

The model by Gabaix and Laibson (2006) explains why firms shroud attributes of add-ons when some consumers are naive with respect to the firms' shrouding behavior. We extend their model to incorporate consumers who are heterogeneous in the risk they impose on the firm when they purchase the add-on, showing that heterogeneity in risk reinforces shrouding in two ways. First, with heterogeneity in risk, there are fewer market conditions in which shrouding is not an equilibrium. Second, when low-risk consumers are not willing to purchase the product at average marginal cost and high-risk consumers cannot be driven away, the shrouding equilibrium is stable in the sense that there is no level of naive consumers for which the shrouding equilibrium ceases to exist unless the market breaks down. In contrast, when consumers are identical with respect to risk, once the share of naive consumers is low enough, the shrouding equilibrium ceases to exist, instead, an unshrouding equilibrium exists.

Further research might focus on the welfare effects of market interventions or regulatory measures in a setting with adverse selection. As our results only hold when firms are not able to price discriminate among consumers, this suggests further research on how these results change when firms are able to price discriminate between consumers based on some imperfect information.

A Appendix

A.1 Proof of Proposition 2

Proof (Proposition 2). The preliminary results on symmetric equilibria in Lemma 1 tell us that, due to price competition in the base good all profits from the add-on are dissipated. Furthermore, we only need to consider add-on prices $p_2^* \in \{v_L, v_H, \overline{p}\}$, and we know that sophisticated and educated naive consumers believe that $p_2 = \overline{p}$ if a firm shrouds its add-on price.

Since demand for the add-on changes at v_L and v_H, we need to consider the three cases $\overline{p} > v_H$, $\overline{p} \in (v_L, v_H]$, and $\overline{p} \leq v_L$ when checking for profitable deviations in the shrouding and unshrouding equilibria.

Furthermore, when we consider a deviation in which firms shroud the add-on price, we only need to consider $p_2 = \overline{p}$ because when a firm shrouds, demand is not affected by p_2. When considering a deviation in which a firms unshrouds the add-on price, we only need to consider $p_2 = v_L$ and $p_2 = v_H$. This is because for $p_2 > v_H$, shrouding yields at least the same profit, and shrouding is preferred by firms when profits are equal. For $p_2 \in (v_L, v_H)$ or $p_2 < v_L$, increasing p_2 slightly and decreasing p_1 by the same amount keeps the demand of sophisticated consumers unchanged while it potentially increases the demand of naive consumers. Moreover, demand from naive consumers is always at least as profitable as that from sophisticated consumers, since their average cost is zero.

We now check that no firm wants to deviate under the conditions of Proposition 2, starting with the

(i) shrouded equilibria:

We established that firms cannot profitably deviate to any $p_2 < \overline{p}$ and keep prices shrouded, and that the only add-on prices we need to consider when firms deviate to unshroud their add-on prices are $p_2 = v_L$ and $p_2 = v_H$.

- Case $\overline{p} > v_H$:

The equilibrium prices for this setting, given by Proposition 2, are $p_1^* = -\alpha\overline{p}$ and $p_2^* = \overline{p}$.

The deviation to $p_2 \in \{v_L, v_H\}$ is profitable only if the profit from the add-on is enough to undercut the equilibrium price p_1^*. For $p_2 = v_H$, this holds if and only if:

$$-p_1^* < v_H\,(1 - (1 - \alpha + \alpha\lambda)(1 - \rho)) - c_H((1 - \alpha + \alpha\lambda)\rho)$$

The profits from the add-on correspond to purchases by all consumers except the low-risk sophisticated and educated naive consumers, deducting the cost of the high-risk sophisticated and educated naive consumers. Note that the costs of the naive consumers cancel out, since both naive risk types purchase the add-on. Rearranging this expression, we attain the following threshold for $\overline{p}$ at which there is a profitable deviation for $p_2 = v_H$,

$$\overline{p} < \frac{v}{\alpha}\left(\rho + \alpha(1-\lambda)(1-\rho)\right) + c_H(1-\lambda) =: \overline{p}_{US,H}$$

We denote this threshold, for which a deviation to $p_2 = v_H$ is profitable, by $\overline{p}_{US,H}$.

Next, consider the condition for a profitable deviation for $p_2 = v_L$.

$$-p_1^* < v_L \Leftrightarrow \overline{p} < \frac{v_L}{\alpha} =: \overline{p}_{US,L}$$

The profits from the add-on correspond to purchases by all consumers. The costs cancel out, as $c_H\rho + c_L(1-\rho) = 0$. We denote the threshold for which a deviation to $p_2 = v_L$ is profitable by $\overline{p}_{US,L}$.

For the existence of the equilibrium, $\overline{p}$ must be above both thresholds. Rearranging $\overline{p}_{US,L} \geq \overline{p}_{US,H}$, we get:

$$v(1-\alpha+\alpha\lambda)(1-\rho) \geq c_H\frac{1}{1-\rho}\left(1-(1-\alpha+\alpha\lambda)(1-\rho)\right) \tag{1}$$

This condition clearly holds if c_H is sufficiently small, making $\overline{p}_{US,L}$ the binding threshold. In other words, this condition tells us that it is better for firms to sell to the share of low-risk sophisticates, $(1-\alpha+\alpha\lambda)(1-\rho)$, receiving additional profits $v = v_L - c_L$ per customer, than charging higher prices, $c_H\frac{1}{1-\rho} = v_H - v_L$, to everyone except the low-risk sophisticates. Concluding that, for c_H sufficiently small and $\overline{p} > v_H$, there is no profitable deviation from the shrouding equilibrium given in Proposition 2.

- Case $\overline{p} \in (v_L, v_H]$:

If there is a shrouding equilibrium for $\overline{p} \in (v_L, v_H]$, firms charge $p_2^* = \overline{p}$ and charge the lowest possible base good price $p_1^* = -\overline{p}(1-(1-\alpha)(1-\rho)) + c_H(1-\alpha)\rho$. Note that this differs from the former case, since high-risk sophisticated consumers purchase the product.

The only possible deviation we need to check is $p_2 = v_L$, which is profitable if

$$
\begin{aligned}
& -p_1^* < v_L \\
\Leftrightarrow \quad & 0 < -\overline{p}(1-(1-\alpha)(1-\rho)) + c_H(1-\alpha)\rho + v_L \\
\Leftrightarrow \quad & \overline{p} < c_L + \frac{v}{\alpha + (1-\alpha)\rho}, \qquad (2)
\end{aligned}
$$

but since $\overline{p} < v_H$ and

$$
\begin{aligned}
& v_H < c_L + \frac{v}{\alpha + (1-\alpha)\rho} \\
\Leftrightarrow \quad & v + c_H \frac{1}{1-\rho} < \frac{v}{\alpha + (1-\alpha)\rho}
\end{aligned}
$$

is true for c_H small enough, the deviation to $p_2 = v_L$ is always profitable. Hence, shrouding is not an equilibrium when $\overline{p} \in [v_L, v_H)$ and c_H is small enough.

- Case $\overline{p} \leq v_L$:

For c_H small enough, we have that $v_L > 0$. The add-on market does not break down, as firms make positive profits in the add-on. All consumers purchase the add-on, and since shrouding is preferred over unshrouding when profits are equal, we conclude that, for $\overline{p} \leq v_L$, there is no profitable deviation from the shrouding equilibrium given in Proposition 2.

(ii) unshrouded equilibria:

- Case $\overline{p} > v_H$:

The equilibrium prices for this setting, given by Proposition 2, are $p_1^* = -v_L$ and $p_2^* = v_L$.

If we consider the possible profitable deviation to $p_2 = v_H$, we arrive again at condition (1). Therefore, this is not a profitable deviation because we assumed that c_H is small enough for condition (1) to hold.

Consider the possible profitable deviation to $p_2 = \overline{p}$ and shrouding the add-on price. Again, the profits from the add-on at p_2 must be sufficient to undercut p_1^*. Add-on profits come from selling to uneducated naive consumers. This yields the following condition for a profitable deviation:

$$0 < -v_L + \alpha(1-\lambda)\overline{p} \quad \Leftrightarrow \quad \overline{p} > \frac{v_L}{\alpha(1-\lambda)}$$

Consequently, for c_H sufficiently small and $\overline{p} > v_H$, there is no profitable deviation from the unshrouding equilibrium given in Proposition 2.

- Case $\overline{p} \in (v_L, v_H]$:

The equilibrium prices for this setting, given by Proposition 2, are $p_1^* = -v_L$ and $p_2^* = v_L$.

A deviation from unshrouding add-on prices and charging $p_1^* = -v_L$, $p_2^* = v_L$ to $p_2 = \overline{p} < v_H$ cannot be profitable as long as condition (1) holds. Therefore, we conclude that, for c_H sufficiently small and $\overline{p} > v_L$, there is no profitable deviation from the unshrouding equilibrium given in Proposition 2.

- Case $\overline{p} \leq v_L$:

In this case, everyone will purchase the add-on. Consequently, profits for shrouding and unshrouding are equal. But since shrouding is preferred over unshrouding when profits are equal there is no unshrouding equilibrium for $\overline{p} \leq v_L$.

□

A.2 Proof of Corollary 3

Proof (Corollary 3). To verify this, recall that for the baseline model, the range where an unshrouding equilibrium exists is $\left(\frac{1}{\alpha(1-\lambda)} - 1\right) v$ for $\overline{p}$.

The range of the maximum add-on price $\overline{p}$ for which a shrouding equilibrium exists is unbounded, so we consider the length of the interval where shrouding fails but an unshrouding equilibrium is sustained, which is $\left(\frac{1}{\alpha} - 1\right) v$.

By Proposition 2, to get the corresponding parameter ranges for the adverse selection model, we simply need to exchange v with v_L.

Since $v > v_L$, we see that the parameter range of $\overline{p}$ for shrouding increases by looking at:

$$\frac{1-\alpha}{\alpha} v > \frac{1-\alpha}{\alpha} v_L$$

Note that the value for the range of $\overline{p}$ decreases when going from the baseline to the adverse selection model, because for $\overline{p}$, we consider the range where shrouding fails but an unshrouding equilibrium exists. Consequently, the range of $\overline{p}$ for which a shrouding equilibrium exists increases.

We verify that the parameter range of $\overline{p}$ for unshrouding decreases by considering:

$$\frac{1-\alpha(1-\lambda)}{\alpha(1-\lambda)} v > \frac{1-\alpha(1-\lambda)}{\alpha(1-\lambda)} v_L.$$

To show the results on the ranges of α, we will use the following well-established result.

Lemma 7. *Let $f, g : X \subset \mathbb{R} \to Y \subset \mathbb{R}$ be two strictly decreasing, continuous functions with $f(x) < g(x)$, then $f^{-1}(y) < g^{-1}(y)$.*

This essentially tells us that when the range in which a certain equilibrium exists decreases with respect to $\overline{p}$ and the corresponding threshold is decreasing in α, the range also decreases with respect to α.

A shrouding equilibrium fails to exist in the baseline model if $\overline{p} - v > \frac{1-\alpha}{\alpha} v =: g(\alpha)$ and if $\overline{p} - v_L > \frac{1-\alpha}{\alpha} v_L =: f(\alpha)$ in the adverse selection model. Note that because both f and g are decreasing in α and $f(\alpha) < g(\alpha)$, we get $f^{-1}(y) < g^{-1}(y)$. Consequently, if the maximum add-on price is above the reference point by the same amount in both models, i.e., $y = \overline{p} - v = \overline{p} - v_L$, then the α above which the shrouding equilibrium exists is smaller in the adverse selection model. Hence, the range of α in which a shrouding equilibrium exists increases compared to the baseline-model.

Similarly, we can argue that the range of α in which an unshrouding equilibrium exists decreases compared to the baseline model. Consider $\overline{p} - v > \frac{1-\alpha(1-\lambda)}{\alpha(1-\lambda)} v =: g(\alpha)$ for the baseline model and $\overline{p} - v_L > \frac{1-\alpha(1-\lambda)}{\alpha(1-\lambda)} v_L =: f(\alpha)$ for the adverse selection model, and repeat the same argument.

$\square$

A.3 Proof of Lemma 4

Proof (Lemma 4). From Proposition 2, we can conclude that (iii) holds. Because for $c_H = \widehat{c}_H$, we have equality in condition (1), this implies that for $c_H < \widehat{c}_H$, charging $p_2 = v_L$ is preferred over $p_2 = v_H$ when unshrouding. The converse holds for $c_H > \widehat{c}_H$. Therefore, proving case (iv), in which $c_H \leq \widehat{c}_H$, follows by the exact same argument as for the corresponding case in the proof of Proposition 2. Note that we do not need to consider $v_L = 0$ in case (iv) because when c_H is such that $v_L = 0$, we have that $c_H > \widehat{c}_H$.

To show the existence of the remaining equilibria, we use similar arguments as in the proof of Proposition 2. Namely, the only candidates for profitable deviation for the add-on price are $p_2 \in \{v_L, v_H, \overline{p}\}$, and a deviation is profitable only if the profit from the add-on is enough to undercut the equilibrium price p_1^*.

- Proof of (i):

To show that (i) is an equilibrium, consider the proof of Proposition 2 for case (i). There we established that when $\overline{p} > v_H$, the only two viable options to deviate are to

$p_2 \in \{v_L, v_H\}$. Deviation to v_L is profitable if and only if $\overline{p} < \overline{p}_{US,L} = \frac{v_L}{\alpha}$. Deviation to v_H is profitable if and only if $\overline{p} < \overline{p}_{US,H} = \dfrac{v}{\alpha}(\rho + \alpha(1-\lambda)(1-\rho)) + c_H(1-\lambda)$. Consequently, the equilibrium can only hold if $\overline{p} \geq \max\{\overline{p}_{US,L}, \overline{p}_{US,H}\}$.

- Proof of (ii):

  Going back to equation (2) in the proof of Proposition 2, we already established that a shrouding equilibrium only exists if

$$\overline{p} \geq \frac{v}{\alpha + (1-\alpha)\rho} + c_L.$$

Furthermore, an equilibrium can only exist if the profit from the add-on is non-negative. Note that this was not an issue for a small c_H in Proposition 2 for $\overline{p} \geq v_H$. This gives us the second condition in case (ii):

$$\begin{aligned} & 0 \leq \overline{p}(\alpha + (1-\alpha)\rho) - c_H(1-\alpha)\rho \\ \Leftrightarrow \quad & \overline{p} \geq c_H \frac{(1-\alpha)\rho}{\alpha + (1-\alpha)\rho}. \end{aligned}$$

- Proof of (v):

  Since we have $c_H > \widehat{c}_H$, we know that deviation to $p_2 = v_L$ cannot be profitable. Considering a deviation to $p_2 = \overline{p}$ and shrouding, we have a profitable deviation if:

$$\begin{aligned} & -p_1^* < \overline{p}\alpha(1-\lambda) \\ \Leftrightarrow \quad & v_H(\alpha(1-\lambda) + (1-\alpha(1-\lambda))\rho) - c_H(1-\alpha(1-\lambda))\rho < \overline{p}\alpha(1-\lambda) \\ \Leftrightarrow \quad & v_H + \frac{v}{\alpha(1-\lambda)}(1-\alpha(1-\lambda))\rho < \overline{p} \end{aligned}$$

- Proof of (vi):

  Consider a deviation to $p_2 = \overline{p}$ and shrouding, we have a profitable deviation if:

$$\begin{aligned} & -p_1^* < \overline{p}(\alpha(1-\lambda) + (1-\alpha(1-\lambda))\rho) - c_H(1-\alpha(1-\lambda))\rho \\ \Leftrightarrow \quad & v + c_L(1-(1-\alpha(1-\lambda))(1-\rho)) < \overline{p}(1-(1-\alpha(1-\lambda))(1-\rho)) \\ \Leftrightarrow \quad & \frac{v}{1-(1-\alpha(1-\lambda))(1-\rho)} + c_L < \overline{p} \end{aligned}$$

Note that profits from the add-on purchases for $p_2^* = v_L$ are non-negative only if $v_L \geq 0$.

□

A.4 Proof of Proposition 5

Proof (Proposition 5). Let us start with the range of $\overline{p}$ for which the **shrouding** equilibrium fails to exist. Splitting the range into two parts, we get that on one side, a shrouding equilibrium fails to exist when $\overline{p} \in [v_H, \max\{\overline{p}_{US,L}, \overline{p}_{US,H}\}]$, and for the bottom part below v_H, we know the shrouding equilibrium fails to exist when $[v_L, \min\{v_H, c_L + \frac{v}{\alpha+(1-\alpha)\rho}\}]$.

The length of the first part, $\max\{\overline{p}_{US,L}, \overline{p}_{US,H}\} - v_H$, is decreasing in c_H. The length of the second part is increasing only up until $v_H = c_L + \frac{v}{\alpha+(1-\alpha)\rho}$ and is constant thereafter.

Therefore, the candidate for the largest range is the c_H for which $v_H = c_L + \frac{v}{\alpha+(1-\alpha)\rho}$. We know that for $c_H = \widehat{c}_H \Leftrightarrow \overline{p}_{US,L} = \overline{p}_{US,H}$:

$$
\begin{aligned}
c_L + & \frac{v}{\alpha + (1-\alpha)\rho} < v_H \\
\Leftrightarrow \quad & \frac{v}{\alpha + (1-\alpha)\rho} < v + c_H - c_L \\
\overset{c_H = \widehat{c}_H}{\Leftrightarrow} \quad & \frac{v}{\alpha + (1-\alpha)\rho} < v + v\frac{(1-\alpha+\alpha\lambda)(1-\rho)}{\rho + \alpha(1-\lambda)(1-\rho)} \\
\Leftrightarrow \quad & \frac{1}{\alpha + (1-\alpha)\rho} < 1 + \frac{(1-(\rho+\alpha(1-\lambda)(1-\rho))}{\rho + \alpha(1-\lambda)(1-\rho)} = \frac{1}{\rho + \alpha(1-\lambda)(1-\rho)} \\
\Leftrightarrow \quad & \rho + \alpha(1-\lambda)(1-\rho) < \alpha + (1-\alpha)\rho \quad \Leftrightarrow \quad -\alpha\lambda(1-\rho) < 0,
\end{aligned}
$$

and for $c_H = 0$:

$$
c_L + \frac{v}{\alpha + (1-\alpha)\rho} = \frac{v}{\alpha + (1-\alpha)\rho} > v = v_H.
$$

This tells us that v_H and $c_L + \frac{v}{\alpha+(1-\alpha)\rho}$ must intersect at some c_H with $c_H < \widehat{c}_H$. Recall that this intersection is the candidate for the largest range in which no shrouding equilibrium exists.

Consequently, when $v_H = c_L + \frac{v}{\alpha+(1-\alpha)\rho}$, we have that $c_H < \widehat{c}_H$, which implies that $\overline{p}_{US,L} > \overline{p}_{US,H}$. This lets us rewrite the length of the range:

$$
\begin{aligned}
& \max\{\overline{p}_{US,L}, \overline{p}_{US,H}\} - v_H + \min\left\{v_H, c_L + \frac{v}{\alpha + (1-\alpha)\rho}\right\} - v_L \\
= & \overline{p}_{US,L} - v_H + v_H - v_L = \overline{p}_{US,L} - v_L = \frac{v_L}{\alpha} - v_L
\end{aligned}
$$

We already showed that this range is smaller than the corresponding range in the baseline model, $\frac{v}{\alpha} - v$ in the proof of Corollary 3.

Consider the range of $\overline{p}$ for which **unshrouding** equilibria exist. This range consists of two separate sections with lengths

$$\frac{v(1-\alpha+\alpha\lambda)\rho}{\alpha(1-\lambda)}$$

and

$$\frac{v}{1-(1-\alpha+\alpha\lambda)(1-\rho)}+c_L-v_L.$$

Comparing the combined length to the length of the range in the baseline model,

$$\begin{aligned}
&\frac{v(1-\alpha+\alpha\lambda)\rho}{\alpha(1-\lambda)}+\frac{v}{1-(1-\alpha+\alpha\lambda)(1-\rho)}-v<\frac{v}{(1-\lambda)\alpha}-v\\
\Leftrightarrow\quad&\frac{(1-\alpha+\alpha\lambda)\rho}{\alpha(1-\lambda)}+\frac{1}{1-(1-\alpha+\alpha\lambda)(1-\rho)}<\frac{1}{(1-\lambda)\alpha}\\
\Leftrightarrow\quad&\frac{1}{1-(1-\alpha+\alpha\lambda)(1-\rho)}<\frac{1-(1-\alpha+\alpha\lambda)\rho}{(1-\lambda)\alpha}=\frac{(1-\lambda)\alpha+(1-\alpha+\alpha\lambda)(1-\rho)}{(1-\lambda)\alpha}\\
\Leftrightarrow\quad&\frac{(1-\alpha+\alpha\lambda)(1-\rho)}{1-(1-\alpha+\alpha\lambda)(1-\rho)}<\frac{(1-\alpha+\alpha\lambda)(1-\rho)}{(1-\lambda)\alpha}\\
\Leftrightarrow\quad&(1-\lambda)\alpha<1-(1-\alpha+\alpha\lambda)(1-\rho)=(1-\lambda)\alpha+(1-\alpha+\alpha\lambda)\rho\\
\Leftrightarrow\quad&0<(1-\alpha+\alpha\lambda)\rho
\end{aligned}$$

Hence, the range of $\overline{p}$ in which unshrouding is an equilibrium decreases.

Following the same argument as in the proof of Corollary 3, using Lemma 7, the change in the range of α follows the direction of the change in the range of $\overline{p}$, as long as the lengths of the individual sections are decreasing in α. The lengths of these individual sections are

$$\begin{aligned}
&\frac{v}{\alpha}\left(\rho+\alpha(1-\lambda)(1-\rho)\right)+c_H(1-\lambda)-v_H \quad \text{for case (i)},\\
&\frac{v}{\alpha+(1-\alpha)\rho}+c_L-v_L=\frac{v}{\alpha(1-\rho)+\rho}-v \quad \text{for case (ii)},\\
&v_H+\frac{v}{\alpha(1-\lambda)}(1-\alpha(1-\lambda))\rho-v_H=\left(\frac{v}{\alpha(1-\lambda)}-v\right)\rho \quad \text{for case (v) and}\\
&\frac{v}{1-(1-\alpha(1-\lambda))(1-\rho)}+c_L-v_L=\frac{v}{\rho+\alpha(1-\lambda)(1-\rho)}-v \quad \text{for case (vi)},
\end{aligned}$$

which are all decreasing in α. Therefore, the change in the range of α for whichthe shrouded or unshrouded equilibrium exists follows the same direction as the respective range of $\overline{p}$. □

A.5 Proof of Proposition 6

Proof (Proposition 6). To show part (i), recall the relevant equilibria in Lemma 4, namely (ii) and (vi). This tells us that for $v_L \leq 0$ and $\overline{p} < v_H$, no unshrouding equilibrium exists. A shrouding equilibrium exists if

$$\overline{p} \geq \max\left\{\frac{v}{\alpha + (1-\alpha)\rho} + c_L,\, c_H\frac{(1-\alpha)\rho}{\alpha + (1-\alpha)\rho}\right\}$$

But for $v_L \leq 0$, we have that, $\frac{v}{\alpha+(1-\alpha)\rho} + c_L \leq c_H\frac{(1-\alpha)\rho}{\alpha+(1-\alpha)\rho}$. Hence, if no shrouding equilibrium exists, $\overline{p}(\alpha+(1-\alpha)\rho) < c_H(1-\alpha)\rho$, the revenue from the naive consumers and the sophisticated, high-risk consumers does not cover the cost of the high-risk consumers. Under these conditions, no other equilibrium exists in which the add-on is sold, as a firm only selling the base good can undercut any firm that sells the add-on at a loss. This also holds for non-symmetric and mixed-strategy equilibria.

To show part (ii), we show that under the given conditions, there always exists a shrouding equilibrium. Recall from Lemma 4 (i) that for $\overline{p} > v_H$, a shrouding equilibrium only exists if

$$\overline{p} \geq \max\left\{\frac{v_L}{\alpha}, \frac{v}{\alpha}\left(\rho + \alpha(1-\lambda)(1-\rho)\right) + c_H(1-\lambda)\right\}.$$

As $c_H \geq \widehat{c}_H$, we know that

$$\frac{v_L}{\alpha} \leq \frac{v}{\alpha}\left(\rho + \alpha(1-\lambda)(1-\rho)\right) + c_H(1-\lambda).$$

But

$$\begin{aligned} & \frac{v}{\alpha}\left(\rho + \alpha(1-\lambda)(1-\rho)\right) + c_H(1-\lambda) > v_H \\ \Leftrightarrow\quad & v(1-\alpha) > (v_H - c_L)\alpha\lambda(1-\rho) \end{aligned}$$

never holds under the given conditions. Therefore, the shrouding equilibrium always exists. $\square$

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