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Geometric Aspects of the Standard Model Effective Field Theory

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Zusammenfassung

Die Suche nach Physik jenseits des Standardmodells geht weiter. Diese Arbeit untersucht den Ansatz der effektiven Feldtheorien. Wir präsentieren die geometrische universelle Beschreibung der effektiven Feldtheorie des Standardmodells und überprüfen Ward-Identitäten bis zur Ein-Schleifen-Ordnung, wobei die Theorie im Hintergrundfeld-Eichfixierungsschema quantisiert wird. Im Rahmen dieses Formalismus entspricht Physik jenseits des Standardmodells einer nicht verschwindenden Krümmung, während das Standardmodell dem krümmungsfreien Fall entspricht.

Abstract

The search for Beyond Standard Model physics continues. This thesis explores the Effective Field Theory approach. We present the Geometric Universal Development of the Standard Model Effective Field Theory and verify Ward identities to one-loop order when background field gauge is used to quantize the theory. The geometric SMEFT organizes the theory and is the physics of curved field spaces. Beyond the Standard Model corresponds to non-zero curvature, whereas the Standard Model corresponds to zero curvature.

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1 Introduction

The search for Beyond the Standard Model (BSM) physics continues. After the discovery of the Higgs particle at CERN[1], the quantum field theoretic description of elementary particles and their interactions, summarized by the Standard Model (SM), can be regarded as complete. Despite its successes[2], the SM fails to account for several key observations, such as missing neutrino masses, the baryon asymmetry of the universe, and the apparent abundance of dark matter. In general, fundamental physics currently lacks direct observational evidence for new physics. Promising tabletop experiments that search for quantum features of gravity[3, 4], might shed some new light towards phenomenological physics.

The recent Large Hadron Collider’s (LHC) discovery of the Higgs boson in 2012 was a major success. However, at the current energies or even with a moderate increase in the center of mass energy, a new detection of any BSM particle is unlikely. Searches for supersymmetric particles, dark matter candidates, and other exotic states at the LHC have so far yielded no positive results. While higher-energy colliders, like the planned Future Circular Collider (FCC), extend the reach of experimental particle physics, it remains uncertain whether such a facility will uncover new phenomena beyond the SM. One thing that is certain is that with the high luminosity phase at the LHC, a new level of experimental precision will be reached for many channels. Thus, this precision improves the sensitivity to potential deviations from SM predictions.

Effective field theories (EFTs) offer a systematic and model-independent framework for analyzing such new precision data, particularly in the absence of direct evidence for new physics. EFTs reveal high-energy physics by precise measurements at low energy. To search for this new physics, both experimental and theoretical progress is necessary. On the theory side, progress has been made by developing and applying EFTs[5]. An EFT is a quantum field theory, which is not regarded as “fundamental”. That means, the effective theory is only valid under specific circumstances, i.e. in a given energy range. In this regard, the SM may serve as an effective low-energy approximation of a more fundamental UV-complete theory at the energy scales currently accessible at the LHC. One such effective theory of the SM is referred to as the *Standard Model Effective Field Theory* (SMEFT).

The basic idea of the SMEFT is to extend the SM in a model-independent way by systematically including the effects of possible new physics at higher energy scales using the framework of effective field theories. The SMEFT extends the SM Lagrangian by higher dimensional operators with *Wilson* coefficients, also called low energy constants, which are suppressed by the scale of new physics and obey the SM symmetries. These additional operators may not only describe new physics phenomena, but also shift masses and coupling constants[6]. Fits to LHC data then give constraints on the coefficients and provide a tool to analyze the SMEFT[7]. As an illustrative example of SMEFT-induced effects, we consider the process $h \rightarrow \gamma\gamma$. This process happens in the SM only through a loop, but in the SMEFT already at tree level.

One caveat of the SMEFT is its complexity. At dimension six, there are already 2499[6] independent Wilson coefficients. However, there is a method to efficiently organize the physics of the SMEFT which uses aspects of geometry and is hence called *the geometric SMEFT* (geoSMEFT). The geoSMEFT uses the fact that after spontaneous symmetry breaking the Higgs field takes on a vacuum expectation value (vev). Conceptually, the Higgs fields can be seen as a background field with curvature as in general relativity. New effects and deviations from the SM appear through the metric defined on this curved space, with non-vanishing Riemann curvature tensor[8]. The SM is flat and has zero curvature[9]. The geoSMEFT organizes the theory and also helps to understand the SMEFT differently. This geometric approach uses scalar field-space geometry and connections – which depend on the scalar field coordinates – to write down the SMEFT Lagrangian in terms of these metrics and composite operator forms. The geometric ideas can be seen as a result of the general field redefinition invariance.

Moreover, one can use the so-called background field gauge together with the geoSMEFT formalism, which is advantageous since this background field gauge invariance leads to the “naive” *Ward identities*. In what follows Ward Identities are derived using the *Background Field method* (BFM). The BFM Ward identities constrain n-point functions as well as masses, gauge couplings, and mixing angles in the SMEFT[10]. Since the higher-dimensional operators of the SMEFT respect the gauge symmetry of the SM, one-loop Ward identities formulated within the BFM are respected operators in the SMEFT. In this work, we verify that this is indeed the case using the formalism of the geoSMEFT. The geometric formulation provides a framework for performing calculations to all orders within the vev expansion of the SMEFT[11]. This provides a clearer understanding of how neglected higher-order contributions affect the interpretation of global SMEFT data. Fitting and constraining the parameters that encode deviations from the SM should be guided as much as possible by these results, ensuring a robust bottom-up SMEFT analysis that forms a key part of the LHC’s future program. The goal is to develop strategies to identify new physics (NP) at short distances, i.e. at high energies[12]. We will conclude this introduction with the following phrase: *SMEFT and Higgs physics is the physics of curved field space(s)*.

The structure of the thesis is as follows. In Sec. 2 we give a short review on aspects of the SM and its construction. We outline the success of the SM and also what the SM *cannot* explain. In Sec. 3 we will go into some aspects of EFTs and will give the introduction to the SMEFT and geoSMEFT formalism. In Sec. 4 we briefly review the concept of two-point functions and as the first example we will give the calculation of the vacuum polarization of the photon in quantum electrodynamics (QED). We also introduce the method of background fields, gauge fixing and Ward Identities – they constrain two-point functions – and will calculate those in the electroweak theory at one-loop in perturbation theory. In Sec. 5 we present some preliminary material to be able to understand how we proceed in the calculation of the Ward Identities in the electroweak geoSMEFT. This is illustrated within the QCD sector of the geoSMEFT, since it is easier to see the principles. The same principles also hold in the electroweak geoSMEFT. In Sec. 6 we incorporate the complete electroweak geoSMEFT and derive the corresponding Lagrangian. The associated Feynman rules are then obtained in Sec. 7. In Sec. 8 we present the results for the Ward identities in the electroweak sector and Sec. 9 is our conclusion.

2 The Standard Model

In nature, four fundamental forces or interactions are known to exist. These are the gravitational, electromagnetic, strong, and weak nuclear forces. Gravity is described by GR, developed by Einstein. The resulting paper and the field equations were published in 1915[13], ten years after the special theory of relativity[14]. The other three forces describe the other non-gravitational effects and interactions. They describe the structure of matter and the corresponding interactions at a fundamental level. These three forces are collectively combined in a theoretical model, which is called the *Standard Model of Particle Physics*, or *Standard Model* (SM) for short. Many physicists spent decades working on the SM resulting in many Nobel prizes. Here we will not give a detailed review of the SM but rather outline the key features and ideas behind the model. Some texts that discuss the phenomenology of the SM include[15, 16, 17, 18].

The theoretical framework of the SM is relativistic quantum field theory (QFT), which describes the fundamental elementary particles and interactions. The basis of the model is symmetry. It is a gauge invariant QFT with the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. $SU(3)$ is the “color” group, describing the strong nuclear interactions of quarks and gluons. $SU(2)_L \times U(1)_Y$ is the electroweak sector of the SM with W and Z bosons, and the photon. The electroweak sector is the framework describing electromagnetism – the interaction between charged particles and the photon – and the weak nuclear force – the theory describing for example nuclear decay processes – in a unified way. Local gauge invariance plays an essential role in this formulation. Inspired by the success of quantum electrodynamics with the precise agreement between theory and experiment of the anomalous magnetic moment of the electron[19], the principle of local gauge invariance was used as well in Yang-Mills theories. Yang-Mills theories correspond to a local $SU(N)$ symmetry.

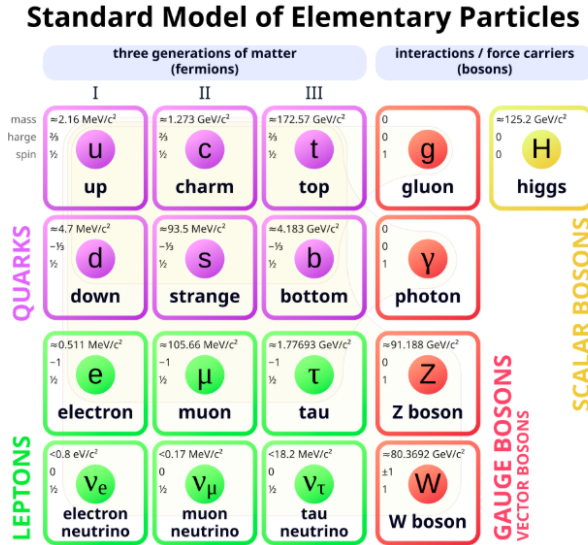


Figure 1: Standard Model of Particle Physics[20].

One feature of the SM is that the weak interaction only couples to left chiral fields which was experimentally seen in the experiment by Chien-Shiung Wu[21]. In this experiment a parity violation in β -decay was observed. This is also encoded in the description of the SM. There are three generations of left handed leptons L^i and quarks Q^i . Here $i=1,2,3$ indexes the three observed generations. Then there are the right handed leptons e_R^i and quarks u_R^i, d_R^i . This is also known as the fermionic sector of the SM. Then there is

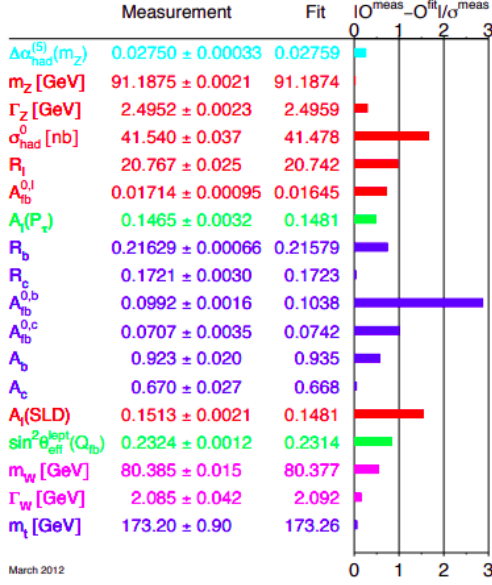


Figure 2: Agreement of SM observables on the Z-pole; LEP Electroweak Working Group[2]

the bosonic sector with the gauge bosons mediating the interactions. The Higgs boson, which generates the masses of the SM particles via the Higgs mechanism, completes the SM. This last part – the discovery of the Higgs boson at CERN[1] – completes the SM as a model of the fundamental particles and their interactions. The data is in good agreement with the Standard Model predictions[22, 16, 2], see Fig. 2. Recent analysis showed a p-value of 0.53, corresponding to a 0.6 σ in a gaussian distribution[23]. The SM is a model with 19 parameters, which have to be experimentally determined. From then on, one can make predictions. A schematic picture of the SM can be seen in Fig. 1. Open questions remain, as elaborated in Sec. 2.3.

2.1 Groups, Fields and the SM Lagrangian

We discuss the symmetry group and field content of the SM. The symmetry group is $SU(3)_C \times SU(2)_L \times U(1)_Y$. $SU(3)$ with color charge C belongs to the strong interaction and $SU(2)_L \times U(1)_Y$ belongs to the electroweak interactions with weak $SU(2)$ isospin and $U(1)$ hypercharge Y . The L indicates that only left-chiral fields couple to the weak interaction. After electroweak symmetry breaking, we obtain massive W , Z bosons and the massless photon of quantum electrodynamics (QED) corresponding to the unbroken subgroup written as $U(1)_{\text{QED}}$. Following electroweak symmetry breaking (EWSB) we obtain massive gauge bosons and QED. The SM is constructed with fields having different charges under the SM gauge group. In what follows the notation is:

$$(\text{color charge, weak isospin})_{\text{hypercharge}}. \quad (1)$$

The SM scalar and fermion fields and charges are given by[16]:

$$\begin{aligned} H &\sim (1, 2)_{\frac{1}{2}} & e &\sim (1, 1)_{-1} \\ L &\sim (1, 2)_{-\frac{1}{2}} & d &\sim (3, 1)_{-\frac{1}{3}} \\ Q &\sim (3, 2)_{\frac{1}{6}} & u &\sim (3, 1)_{\frac{2}{3}} \end{aligned}$$

A “1” indicates that the field is in a singlet representation of a given symmetry, a “2” corresponds to a doublet and a “3” to a triplet. The Higgs field has no color charge and does not interact with the strong force, but is a doublet under $SU(2)$ and has hypercharge of $1/2$. The SM Lagrangian is organized by the following terms:

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{Gauge}} + \mathcal{L}_{\text{Fermions}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}. \quad (2)$$

The gauge term describes pure gauge kinetic and self-interaction terms. The fermionic part includes fermion-kinetic and fermion-gauge interactions. The Higgs part covers the mass generation of the gauge bosons, the Higgs kinetic term and the potential. Finally, the Yukawa term is needed for the generation of fermion masses. All the terms are constructed to be local and gauge invariant. Gauge invariance is a core feature of the SM. The SM is then given by:

$$\begin{aligned} \mathcal{L}_{\text{Gauge}} &= -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W_{\mu\nu}^I W^{I\mu\nu} - \frac{1}{4}G_{\mu\nu}^A G^{A\mu\nu}, \\ \mathcal{L}_{\text{Fermions}} &= i\bar{L}\not{D}L + i\bar{Q}\not{D}Q + i\bar{e}\not{D}e + i\bar{u}\not{D}u + i\bar{d}\not{D}d, \\ \mathcal{L}_{\text{Higgs}} &= (D_\mu H)^\dagger (D^\mu H) + \mu^2 (H^\dagger H) - \lambda (H^\dagger H)^2, \\ \mathcal{L}_{\text{Yukawa}} &= -\left(Y_e \bar{L}H e + Y_d \bar{Q}H d + Y_u \bar{Q}\tilde{H}u + \text{h.c.}\right), \\ \mathcal{L}_{\text{SM}} &= \mathcal{L}_{\text{Gauge}} + \mathcal{L}_{\text{Fermions}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}. \end{aligned} \quad (3)$$

The B, W are the gauge fields in the weak eigenstate basis corresponding to $U(1)_Y, SU(2)_L$ and G is the gauge field of $SU(3)_C$. The equations of motion (EOMs) for the bosons follow from the Euler-Lagrange equation and are given by:

$$D^\rho G_{\mu\rho}^A = -\frac{g_3}{2}[\bar{Q}\gamma_\mu \lambda^A Q + \bar{u}\gamma_\mu \lambda^A u + \bar{d}\gamma_\mu \lambda^A d] \equiv J_{\mu,G}^A, \quad (4)$$

$$D^\rho W_{\mu\rho}^I = -\frac{g_2}{2}[H^\dagger i\overleftrightarrow{D}_\mu H + \bar{L}\gamma_\mu \sigma^I L + \bar{Q}\gamma_\mu \sigma^I Q] \equiv J_{\mu,W}^I, \quad (5)$$

$$D^\rho B_{\mu\rho} = -g_1 Y_H H^\dagger i\overleftrightarrow{D}_\mu H - g_1 Y_\psi \bar{\psi}\gamma_\mu \psi \equiv J_{\mu,B}, \quad (6)$$

$$D^2 H = \mu^2 H - 2\lambda (H^\dagger H) H - Y_e \bar{e}L - Y_d \bar{d}Q. \quad (7)$$

The definition of the field strength tensors

$$G_{\mu\nu}^A = \partial_\mu G_\nu^A - \partial_\nu G_\mu^A - g_3 f^{ABC} G_\mu^B G_\nu^C, \quad (8)$$

$$W_{\mu\nu}^I = \partial_\mu W_\nu^I - \partial_\nu W_\mu^I - g_2 \epsilon^{IJK} W_\mu^J W_\nu^K, \quad (9)$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \quad (10)$$

2.2 Failings

As we have discussed, the SM is a highly successful framework describing a wide range of phenomena, yet it does not explain everything. Some open questions include:

- Neutrino masses

- Baryon Asymmetry
- Dark Matter
- Dark Energy
- Quantum Gravity
- The Hierarchy Problem
- The Strong CP-Problem and Naturalness

Neutrinos are strictly massless in the SM. However, the fact that neutrinos can oscillate implies that neutrinos are massive and their mass range is constrained by experimental results such as those from KATRIN experiment[24]. An effective field theory may provide a characterization of the missing mass description of the model.

Cosmological observations suggest that the SM is incomplete. *Dark matter* (DM), as postulated by Fritz Zwicky after he noticed that the velocity dispersion in galaxies cannot entirely be explained by visible matter alone[25], is not within the framework of the SM yet. The discovery of the *cosmic microwave background* (CMB)[26] opened a door to precise cosmology and supports the big bang theory. This discovery led to the conclusion that DM apparently dominates the matter density in the universe[27]. Therefore, the most compelling evidence for DM arises from observations of the CMB and the large-scale structures of the universe. Another cosmological issue is the baryon asymmetry, meaning that the universe is mostly composed of matter and not antimatter. To explain this asymmetry, Andrei Sakharov outlined three necessary conditions[28]—known as the Sakharov conditions—that must be satisfied to generate a baryon asymmetry from an initially symmetric state: C and CP violations, baryon number violation and a departure from thermal equilibrium. Furthermore, the discrepancy between the theoretical prediction and the experimentally measured value of the cosmological constant Λ remains a problem[29, 30, 31]. Thus, the calculation of the vacuum energy using quantum field theory leads to unresolved inconsistencies.

A peculiarity of the SM is the strong CP problem in quantum chromodynamics (QCD). It concerns the question of why the strong interaction does not appear to violate CP symmetry. Through the totalitarian principle of Gell-Mann[32] – all terms in a Lagrangian that are not forbidden by symmetry must be included – an additional term in the QCD Lagrangian exists,

$$\mathcal{L}_{QCD} \supset \frac{\theta g_s^2}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu}, \quad (11)$$

which violates CP if $\theta \neq 0$ [16]. G is the field strength, $\tilde{G}$ the dual field strength, g_s the strong coupling constant and θ , a new parameter that is related to the degree of the CP violation. The θ -term in QCD gives rise to an *electric dipole moment* (EDM) for the neutron that is directly proportional to θ , making the EDM a sensitive probe of CP violation in the strong interaction. Since CP is not a symmetry of the SM and is already known to be violated by the weak interaction, there is no reason within the SM to expect θ to be particularly small – it could naturally be of order one. However, experimental measurements constrain θ to be less than about 10^{-10} [16], which implies an unexpectedly small value. This extreme suppression, without an apparent theoretical reason, is known as the *strong CP problem*. A way to resolve this issue is by introducing a mechanism that forces θ to be zero. One such mechanism is the Peccei-Quinn mechanism[33, 34]. This approach also results in a possible dark matter candidate, the *axion*[35]. However, despite the appearance of the CP violating term in the Lagrangian, there are still arguments for CP conservation[36]. The strong CP problem can be regarded as part of the *naturalness problem*[37]. It is the question of why certain parameters take their specific value. The best example is the Higgs mass. Why is the Higgs mass so low, when it should receive a large quantum correction through the interaction with other particles of the SM[16]? This *hierarchy problem*[38] – though its significance is debated between physicists – can also be seen as an issue, which could be resolved by some new physics.

The SM describes three out of the four fundamental forces of nature. Gravity is not included. The question whether gravity has quantum properties still needs to be answered. Tabletop experiments may shed some light onto this issue[4, 3]. Theoretical models of quantum gravity do exist. GR can be regarded as the low-energy theory of quantum gravity. Quantum gravity effects become important at 10^{-35} m (the *Planck length*)[16]. To describe any physics beyond the Planck scale – for instance, in describing an evaporating black hole in the final stage of its lifetime – a new model, like *string theory*, is needed.

2.3 Physics Beyond the SM

In the last chapter, we discussed established problems of the SM. We now consider aspects of *beyond SM* (BSM) physics that are of particular interest, yet still somewhat speculative. One such topic is *charged lepton flavor violation* (CLFV). This refers to a process – for example, the interaction of $\mu^+ \rightarrow e^+ \gamma$ – through which a lepton changes its flavor. Such processes are heavily suppressed in the SM. Thus, if one observed such phenomena, it would indicate new BSM physics[39, 40]. Processes involving bound muons can be of great importance. When a low-energy muon travels through matter, it gradually loses energy and eventually comes to rest in a target material. At this point, it becomes bound to a nucleus N with atomic number Z , forming a muonic atom μ_H . After reaching the ground state, it can undergo one of three possible processes: *muon-to-electron conversion*, *decay-in-orbit (DIO)*, or *nuclear capture*. Muon conversion is described by the branching corresponding to the rate of muon conversion normalized to the rate of muon capture:

$$R_{\mu e} = \frac{\Gamma(\mu_H \rightarrow eN)}{\Gamma(\mu_H \rightarrow \nu_\mu N')} \quad (12)$$

N' corresponds to the nucleus with atomic number $Z - 1$. The best measurement of $R_{\mu e}$ with the upper limit of $R_{\mu e} < 7 \times 10^{-13}$, was obtained by the SINDRUM-II[41]. Muon conversion, the process through which a muon converts into an electron in the presence of a nucleus, provides a sensitive probe for CLFV and is thus a promising probe of new physics.

Another topic is semileptonic B-meson decays. Using the branching ratio of $B^+ \rightarrow K^+ \mu^+ \mu^-$ and $B^+ \rightarrow K^+ e^+ e^-$, one can look for possible lepton universality violation. Lepton universality is the principle that all charged leptons — the electron, muon, and tau — couple to interactions with equal strength, meaning that any differences in their behavior arise solely from their differing masses, not from any fundamental difference. Such a difference, if measured, could indicate that new heavy particles interact differently with different lepton flavors. Measurements at CERN indicate that the result is compatible with the SM prediction within 2.6σ (standard deviations)[42]. A result with a 2.6σ deviation from the SM prediction is not considered statistically significant in particle physics. To claim a discovery, a deviation of 5σ is required. A similar case is the anomalous magnetic moment of the muon, which in recent years seemed to indicate new physics. However, since June 2025 with the latest measurements from Fermilab, the result appears to be in agreement with the SM prediction[43, 44].

2.4 Summary

To summarize, the SM is a well-tested model of elementary particles and their interactions. The SM of particle physics is a relativistic quantum field theory using local symmetry principles for its construction. It is a gauge theory based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. Despite the successes, physics beyond the SM is needed: Gravity is not included in the SM, it does not explain neutrino masses or conceptual issues like the hierarchy and the naturalness problem, it offers no explanation of the observed baryon asymmetry, neither does it explain the origin of dark matter and dark energy. Thus, further theoretical work is required.

3 Effective Field Theories

One method to search for BSM physics is through EFTs. The hope or idea of having one fundamental theory describing everything is still alive. Such a theory would describe all physics across all energy scales, even beyond the Planck scale. The SM would then be described as a low-energy approximation[45]. This approach of finding such a complete theory may be overly ambitious and recently has not been particularly successful. A newer approach is to develop theories, valid in specific “domains”, i.e. energy ranges. The idea is to reduce the complexity and the degrees of freedom of the model. This brings us to the concepts of EFTs. EFTs provide a powerful framework to describe physical phenomena at a given energy scale without requiring detailed knowledge of the underlying high-energy theory. The central idea is based on the separation of energy scales: physics at low energies can often be described accurately by focusing only on the relevant degrees of freedom at that scale, while the effects of heavier particles or high-energy processes are encoded in effective interactions. Consider a QFT with a heavy scalar S coupled to light fields denoted by J . The Lagrangian is then given by

$$\mathcal{L}(S, J) = \frac{1}{2}(\partial^\mu S)(\partial_\mu S) - M_S^2 S^2 - JS. \quad (13)$$

If one is interested in the effective Lagrangian – in an energy regime below the S – “without” the S , one obtains

$$\mathcal{L}_{\text{eff}} = \frac{1}{2M_S^2} J^2. \quad (14)$$

A detailed description of the used procedure of *integrating out* a heavy field is given below in Sec. 3.1.

EFTs allow physicists to do calculations model-independently and enable a method for *beyond SM physics*.

We now discuss several examples. If one only operates in the energy range of $E < m_W \simeq 80$ GeV, then one can integrate out the W, Z bosons, the top quark and the Higgs. One is then left with a lighter, effective version of the SM called weak effective field theory (WEFT). It consists of photons, the gluon octet, the three lepton flavours and five quark flavours. In this theory, the underlying UV-complete theory is known and the Wilson coefficients – we will discuss them later in more detail – are determined by SM parameters. Since this theory contains no W and Z bosons, the symmetry is no longer the full SM symmetry. It is only invariant under $SU(3)_C \times U(1)_{\text{QED}}$. Specifically, weak interactions, which are mediated by the exchange of W and Z bosons in the SM, appear in the WEFT as four-fermion interactions between quarks and leptons. Many of these effective operators break the approximate flavor symmetry, enabling transitions between different quark flavours. As such, WEFT provides the theoretical framework for a broad range of phenomena in flavor physics, which involves studying transitions among mesons and baryons composed of different types of quarks[45].

A historical example is the effective theory of beta decay. Although at the time of development and prior to the development of the SM, it was not understood that beta decay can be seen as an EFT. This EFT is the low energy description of an interaction without the exchange of the W -boson in beta decay. The interaction is instead described by a four-fermion interaction vertex with an effective operator including Fermi’s constant G_F . This constant is determined by *matching*, i.e. the SM and the EFT should give the same result at a specific energy scale. This determines G_F . This is only possible if the UV-complete theory is known. Otherwise, the constants remain undetermined and are free parameters. Fig. 3 shows diagrammatically the difference between the full and effective theory.

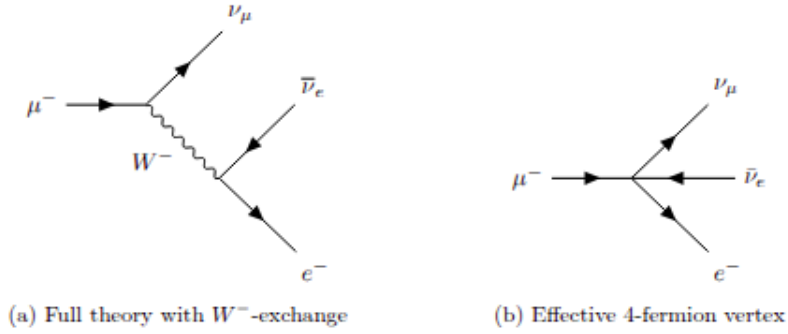


Figure 3: Muon decay: comparison between full and effective theory

Another EFT is *chiral perturbation theory* (ChPT). It is a method to describe light mesons – particles consisting of a quark and an antiquark – such as pions and kaons. Since QCD is strongly coupling at $\simeq 2\text{GeV}$, the effective Lagrangian cannot be calculated analytically. But the chiral symmetry of QCD, if the quark masses are set to zero, provides an organizing principle for the construction of an EFT[16].

For completeness, we briefly note that there also exists an effective field theory description of gravity involving a massless spin-2 particle, the *graviton*[45, 16].

Another example is *heavy quark effective field theory* (HQEFT). We have seen that light mesons, at lower energies can be studied using ChPT. In physics at center of mass energies $\sqrt{s} \gg \Lambda_{QCD}$, one can use perturbation theory in α_s . One can also use perturbation theory for charm and bottom quarks by expanding in $\frac{\Lambda_{QCD}}{m_b}$ or $\frac{\Lambda_{QCD}}{m_c}$ [16]. HQEFT can be used to study bottom and charmed hadrons.

We now discuss two general approaches to EFTs and then look at how one can integrate out heavy fields – as in the example above – using the path integral.

3.1 Top-Down Approach

The first approach being discussed is the top-down approach. Here, the UV-complete model is known – valid up to a given energy scale – and systematically derives a lower-energy description that captures only the relevant physics at that scale. This can be done by *integrating out* the heavy-degrees of freedom in the path integral. The symmetry is embedded into the low energy or infrared regime. An example is the four-Fermi theory as mentioned above.

Now let us see how one can lower the degrees of freedom by integrating out fields[46, 47]. We start with the Lagrangian of eq. (13)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu S)(\partial^\mu S) - \frac{1}{2}M_S^2 S^2 - JS = -\frac{1}{2}S(\square + M_s^2)S - JS. \quad (15)$$

We obtain the EFT by integrating out the heavy scalar by

$$e^{iZ_{\text{eff}}[J]} = \frac{\int \mathcal{D}S e^{i \int d^4x \mathcal{L}(S, \mathcal{J})}}{\int \mathcal{D}S e^{i \int d^4x \mathcal{L}(S, \mathcal{I})}}. \quad (16)$$

Before evaluating eq. (16), some definitions are useful:

$$D = \square + M_S^2, \quad (17)$$

$$D^{-1}J = - \int d^4y \Delta_F(x-y)J(y), \quad (18)$$

$$D\Delta_F(x-y) = -\delta^{(4)}(x-y). \quad (19)$$

Starting from the action as

$$\int d^4x L(S, J) = - \int d^4x \frac{1}{2} [SDS - SJ - SJ + JD^{-1}J - JD^{-1}J], \quad (20)$$

$$= - \int d^4x \frac{1}{2} \left[SDS - SDD^{-1}J - D^{-1}JDS + D^{-1}JDD^{-1}J - JD^{-1}J \right], \quad (21)$$

$$= - \int d^4x \frac{1}{2} [(S - D^{-1}J)D(S - D^{-1}J) - JD^{-1}J], \quad (22)$$

$$= - \int d^4x \frac{1}{2} [S'DS' - JD^{-1}J]. \quad (23)$$

In the second line we used integration by parts twice to arrive at

$$\int d^4x JS = \int d^4x D^{-1}DJS = \int d^4x (D^{-1}J)(DS), \quad (24)$$

and used the field redefinition of $S' = S - D^{-1}J$.

From there we conclude that

$$\exp(iZ_{\text{eff}}) = \frac{\int [dS'] \exp\left\{-\frac{i}{2} \int d^4x (S' D S' - J D^{-1} J)\right\}}{\int [dS] \exp\left\{-\frac{i}{2} \int d^4x (S D S)\right\}}. \quad (25)$$

Since the integration is over all the field content, we can take $[dS'] = [dS]$ and we obtain

$$\exp(iZ_{\text{eff}}) = \exp\left(-\frac{i}{2} \int d^4x J D^{-1} J\right) = \exp\left(-\frac{i}{2} \int d^4x d^4y J(x) \Delta_F(x-y) J(y)\right). \quad (26)$$

We can expand J at small distances

$$J(y) = J(x) + (\partial_\mu J(y))(x-y)^\mu \quad (27)$$

and use the propagator

$$\int d^4y \Delta_F(x-y) = \int d^4y \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - M^2} e^{ip(x-y)} \quad (28)$$

to arrive at

$$Z_{\text{eff}} = \int d^4x \frac{J(x)J(x)}{M_S^2} \longrightarrow \mathcal{L}_{\text{eff}} = 1/M_S^2 J^2. \quad (29)$$

Thus, after integrating out the heavy degrees of freedom, the effective Lagrangian depends only on the light fields, with the influence of the heavy sector entering solely through the heavy mass parameter M_S and the current-current interaction.

A more general approach, which allows for loop improvement of this procedure, can be found in [48]. To summarize, the logic of the top-down approach is as follows:

1. Having a UV model
2. Match onto the IR model
3. Make prediction of the low energy behaviour
4. Low energy constants are determined by matching
5. Symmetries of the UV are embedded in the IR

3.2 Bottom-up approach

In the bottom up approach, the UV model is not known, only the IR one. The idea is to construct all possible operators and use experimental data to constrain the UV model. Since the number of operators is infinite, they are modeled and organized in a power counting. Symmetries in the UV region will constrain the low energy constants (Wilson coefficients) in the IR regime. The method of calculating in both regimes – the UV and the IR – and requiring that they agree is called matching. For example, Fermi’s constant G_F can be determined by writing down the amplitude of muon decay using the full electroweak theory with W and Z bosons and using Fermi’s interaction rule. By requiring that the amplitudes “match”, G_F can be determined. The logic for the bottom-up approach is as follows:

1. UV model is unknown, only the IR
2. Low energy degrees of freedom are arbitrary
3. All possible operators are constructed and experiments are used for constraints
4. Symmetries are reflected in the experimental constraints

3.3 Renormalizability

We also want to discuss *renormalizability* of EFTs. EFTs are usually non-renormalizable in the standard sense, but they still can be used to make predictions at a quantum level[16]. Renormalizable theories are of central importance in quantum field theory. Only a finite number of counter-terms are needed to make the theory finite. As such, the SM is an example of a renormalizable theory. But sometimes they can be very difficult and cumbersome to work with or it is even impossible to do calculations. That is the advantage of EFTs. One can make the theory “simpler” by reducing the degrees of freedom and restrict the domain of applicability. Despite the non-renormalizability, EFTs are predictive[16].

3.4 Standard Model Effective Field Theory

As already stated, this thesis will focus on the SMEFT. In this section, the basics of the SMEFT formalism will be discussed. The SMEFT is an example of a bottom-up approach EFT. The SMEFT Lagrangian contains all the field content of the SM and also an infinite set of higher-dimensional gauge-invariant operators. For clarity, the SM has dimension four.

Let us now see this effective theory on a larger scale. It is an effective extension to the SM to a yet unknown theory, which might be itself an EFT of some larger theory. The UV theory is valid at energy scales $\Lambda \gg m_W$ [45]. At lower energies the SMEFT reduces to the SM and at even lower energies to the WEFT, as mentioned before. Fig. 4 shows the different applicability of the specific models[45]. The SMEFT formalism is a fully consistent framework and extension to the SM degrees of freedom. That does not mean, however, that it is the “correct” description of nature since three assumption go into this consideration[45].

1. **QFT**: That the model is described manifestly as a Poincaré-invariant local QFT
2. **Mass Gap**: That the scale Λ is for above the electroweak scale, i.e. $\Lambda \gg m_w$

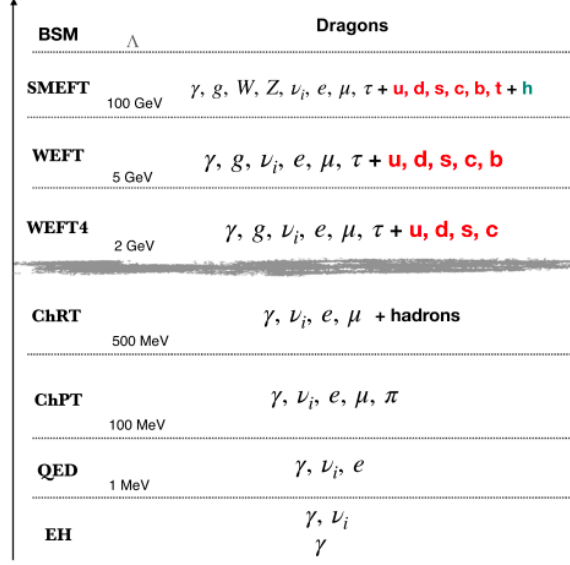


Figure 4: The ladder of EFTs describing nature, assuming SMEFT is a valid description of nature in the regime $100 \text{ GeV} \ll E \ll \Lambda$, with $\Lambda \gg 100 \text{ GeV}$ [45].

3. **Gauge Symmetry:** That the model describing the higher-energy effects is symmetric under the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ and the Higgs is a doublet

Here, we take the first two assumptions for granted, and briefly discuss the third assumption. The principle of gauge symmetry has proven to be extremely successful. It is a foundational concept in QFT. The question now is, why the SMEFT formalism should also respect this symmetry. In the end it should describe new heavy particle physics *beyond* the SM. Also, gauge symmetry can be seen not as an actual symmetry of the physical system, but merely as a redundancy in its description[45].

Now we want to discuss, how the SMEFT is constructed. The SMEFT Lagrangian is defined as

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \mathcal{L}^{(d)}, \quad \mathcal{L}^{(d)} = \sum_i \frac{C_i^{(d)}}{\Lambda^{d-4}} Q_i^{(d)} \quad \text{for } d > 4. \quad (30)$$

Q_i^d are d-dimensional operators and C_i^d are the corresponding dimensionless Wilson-coefficients. Λ is the scale of the “new heavy physics” regime. The dominant extension of the SMEFT to the SM are the dimension six operators, see App. B. There are no gauge invariant operators at dim=0,1 and 3. So the first relevant operators start at dimension five. In dimension five there is only one operator known as the Weinberg operator.

$$\mathcal{O}^W = \frac{c^{(5)}}{\Lambda} (\tilde{H}^\dagger L)^T C (\tilde{H}^\dagger L), \quad (31)$$

which creates a majorana mass term after spontaneous symmetry breaking for the neutrinos[45]. C is the charge conjugation matrix. One can then make an estimate on the scale of this physics by $c^{(5)} \approx 1$ and $m_\nu \approx 1 \text{ eV}$ and then

$$\Lambda \approx 10^{13} \text{ GeV}. \quad (32)$$

As such we neglect this dimension five operator in this thesis. For scattering at larger energy E than the

particle masses, one finds[45] that

$$\mathcal{M} \sim \left(\frac{E}{\Lambda}\right)^{d-4}. \quad (33)$$

Thus, effects from four-dimensional operators are unsuppressed, effects in five dimensions are suppressed by E/Λ , effects in six dimensions are suppressed by $(E/\Lambda)^2$ and so on. At dimension six, the SMEFT formalism can be written in terms of 59 independent operators, which conserve baryon number[49].

Additional SMEFT operators at dimension six shift masses and gauge coupling constants[6], i.e.

$$\bar{g}_3 = g_3 (1 + C_{\text{HG}} v_T^2), \quad \bar{g}_2 = g_2 (1 + C_{\text{HW}} v_T^2), \quad \bar{g}_1 = g_1 (1 + C_{\text{HB}} v_T^2) \quad (34)$$

$$m_W^2 = \frac{\bar{g}_2 v_T^2}{4}, \quad m_Z^2 = \frac{v_T^2 (\bar{g}_1^2 + \bar{g}_2^2)}{4} + \frac{v_T^4}{8} C_{\text{HD}} (\bar{g}_1^2 + \bar{g}_2^2) + \frac{v_T^4}{2} \bar{g}_1^2 \bar{g}_2^2 C_{\text{HWB}}, \quad (35)$$

where v_T is the new minimum due to the operator $C_{\text{H}}(H^\dagger H)^3$ yielding

$$\langle H^\dagger H \rangle = \frac{v^2}{2} \left(1 + \frac{3C_{\text{H}} v^2}{4\lambda} \right) \equiv \frac{1}{2} v_T^2. \quad (36)$$

The renormalized mixing angles are:

$$\sin \bar{\theta} = \frac{\bar{g}_1}{\sqrt{\bar{g}_1^2 + \bar{g}_2^2}} \left[1 + \frac{v_T^2}{2} \frac{\bar{g}_2}{\bar{g}_1} \frac{\bar{g}_2^2 - \bar{g}_1^2}{\bar{g}_2^2 + \bar{g}_1^2} C_{\text{HWB}} \right] \quad (37)$$

$$\cos \bar{\theta} = \frac{\bar{g}_2}{\sqrt{\bar{g}_1^2 + \bar{g}_2^2}} \left[1 - \frac{v_T^2}{2} \frac{\bar{g}_1}{\bar{g}_2} \frac{\bar{g}_2^2 - \bar{g}_1^2}{\bar{g}_2^2 + \bar{g}_1^2} C_{\text{HWB}} \right] \quad (38)$$

As already mentioned, the SMEFT is a tool to indirectly probe new heavy physics effects. One way to search for such effects, might be through CP sensitive observables[45]. CP is the combined transformation of Charge Conjugation (C) and Parity (P). CP is violated by the electroweak theory by Kaon decay processes[16]. CP symmetry physics is covered mainly by Kaon and B-meson decays. Also there is the strong CP problem. There may be beyond-the-SM physics in CP violation just around the corner[16, 45].

3.5 Geometric Standard Model Effective Field Theory

In this section we introduce the concept of the Geometric SMEFT, or “geoSMEFT” formalism. It is a way to organize the SMEFT and provides a different way of understanding QFT and the SMEFT using geometric aspects. The following geometric ideas can be seen as a result of the field redefinition invariance of QFT and the SMEFT. GeoSMEFT organizes the theory (SMEFT) space in terms of field-space connections G_i multiplying composite operator forms f_i . This decomposition is schematically represented by[9]

$$\mathcal{L}_{\text{SMEFT}} = \sum_i f_i G_i(A, I, \phi, \dots), \quad (39)$$

where G depends on group indices A, J of the (non-spacetime) symmetry groups. Thus, the SMEFT can be written in terms of a scalar field expansion and a derivative expansion. The fact that the theory can be written in this form is a result of the completeness relation of the symmetry generators. The Higgs doublet is written in terms of scalar field coordinates $\phi_I = \{\phi_1, \phi_2, \phi_3, \phi_4\}$ with normalization

$$H(\phi_I) = \frac{1}{\sqrt{2}} \begin{bmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{bmatrix}. \quad (40)$$

The gauge boson field coordinates are combined into $\mathcal{W}^A = \{W^1, W^2, W^3, B\}$ with $A = 1, 2, 3, 4$. The geometric expressions that appear in the Ward Identities are related to the field space connections. Most important are h_{IJ} and g_{AB} , which are defined by[9];

$$h_{IJ}(\phi) = \frac{g^{\mu\nu}}{d} \frac{\delta^2 \mathcal{L}_{\text{SMEFT}}}{\delta(D_\mu \phi)^I \delta(D_\nu \phi)^J} \bigg|_{\mathcal{L}(\alpha, \beta) \rightarrow 0}, \quad (41)$$

$$g_{AB}(\phi) = -\frac{2g^{\mu\nu}g^{\sigma\rho}}{d^2} \frac{\delta^2 \mathcal{L}_{\text{SMEFT}}}{\delta \mathcal{W}_{\mu\sigma}^A \delta \mathcal{W}_{\nu\rho}^B} \bigg|_{\mathcal{L}(\alpha, \beta) \rightarrow 0, \text{CP-even}}. \quad (42)$$

The variable d is the space time dimension and the matrix square root of these field-space connections are defined as $\sqrt{g_{AB}} = \langle g_{AB} \rangle^{1/2}$. The explicit form of the above field space metrics is basis dependent. Using the conventional Warsaw basis at $\mathcal{L}^{(6)}$ and some operators at $\mathcal{L}^8$ as defined in[50] h_{IJ} and g_{AB} are given by

$$\begin{aligned} h_{IJ} = & \left[1 + \phi^2 C_{H\Box}^{(6)} + \sum_{n=0}^{\infty} \left(\frac{\phi^2}{2} \right)^{n+2} \left(C_{HD}^{(8+2n)} - C_{H,D2}^{(8+2n)} \right) \right] \delta_{IJ} \\ & + \frac{\Gamma_{A,J}^I \phi^K \Gamma_{A,L}^K \phi^L}{2} \left[\frac{C_{HD}^{(6)}}{2} + \sum_{n=0}^{\infty} \left(\frac{\phi^2}{2} \right)^{n+1} C_{H,D2}^{(8+2n)} \right], \end{aligned} \quad (43)$$

$$\begin{aligned} g_{AB}(\phi_I) = & \left[1 - 4 \sum_{n=0}^{\infty} \left(C_{HW}^{(6+2n)} (1 - \delta_{A4}) + C_{HB}^{(6+2n)} \delta_{A4} \right) \left(\frac{\phi^2}{2} \right)^{n+1} \right] \delta_{AB} \\ & - \sum_{n=0}^{\infty} C_{HW,2}^{(8+2n)} \left(\frac{\phi^2}{2} \right)^n (\phi^I \Gamma_{A,J}^I \phi^J) (\phi^L \Gamma_{B,K}^L \phi^K) (1 - \delta_{A4})(1 - \delta_{B4}) \\ & + \left[\sum_{n=0}^{\infty} C_{HWB}^{(6+2n)} \left(\frac{\phi^2}{2} \right)^n \right] [(\phi^I \Gamma_{A,J}^I \phi^J) (1 - \delta_{A4}) \delta_{B4} + (A \leftrightarrow B)]. \end{aligned} \quad (44)$$

The rotation that transforms the weak eigenstate basis to the mass eigenstate basis is[9]

$$W_\nu^A = \sqrt{g}^{AB} U_{BC} A_\nu^C, \quad (45)$$

$$\alpha^A = \sqrt{g}^{AB} U_{BC} \beta^C, \quad (46)$$

$$\phi^J = \sqrt{h}^{JK} V_{KL} \Phi^L, \quad (47)$$

where $\Phi_L = \{\Phi^-, \Phi^+, \chi, h\}$ and in the SM limit

$$\begin{aligned} \alpha^A &= \{g_2, g_2, g_2, g_1\}, & W^A &= \{W_1, W_2, W_3, B\}, \\ \beta^C &= \left(\frac{g_2(1-i)}{\sqrt{2}}, \frac{g_2(1+i)}{\sqrt{2}}, \sqrt{g_1^2 + g_2^2} (\cos^2 \bar{\theta} - \sin^2 \bar{\theta}), \sqrt{2} \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}} \right), & A^C &= \{W^+, W^-, Z, A\}. \end{aligned}$$

The matrices U and V are unitary and given by

$$U^{BC} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{i}{\sqrt{2}} & -\frac{i}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & c_{\bar{\theta}} & s_{\bar{\theta}} \\ 0 & 0 & -s_{\bar{\theta}} & c_{\bar{\theta}} \end{pmatrix}, \quad V^{JK} = \begin{pmatrix} -\frac{i}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

All classified field space connections for two- and three-point function can be found in[9]. It should be noted that such explicit and finite metrics can be written down only for two- and three-point functions[9]. For higher-point functions, the number of field space connections is infinite. This infinite set of metrics is related to the increasing number of operators at higher dimensions.

The geoSMEFT is a method for organizing the physics of the SMEFT using geometric aspects – such as field space connections and defining metrics – as a result of field redefinition invariance of QFT. Effects of higher-dimensional operators are encoded in the curvature of the field space geometry and for two- and three-point functions there is a finite amount of these metrics and they can be written down explicitly.

Since the effects of new physics (NP) are described geometrically, the redefined masses and gauge couplings are expressed in terms of geometric quantities. The redefined masses are given by[9]

$$\bar{m}_W^2 = \frac{\bar{g}_2^2}{4} \sqrt{h_{11}}^{-2} \bar{v}_T^2, \quad \bar{m}_Z^2 = \frac{\bar{g}_Z^2}{4} \sqrt{h_{33}}^{-2} \bar{v}_T^2, \quad \bar{m}_A^2 = 0, \quad (48)$$

and the relation of the gauge couplings to those of the SM are given by

$$\bar{g}_2 = g_2 \sqrt{g}^{11} = g_2 \sqrt{g}^{22}, \quad (49)$$

$$\bar{e} = g_2 (s_{\bar{\theta}} \sqrt{g}^{33} + c_{\bar{\theta}} \sqrt{g}^{34}) = g_1 (c_{\bar{\theta}} \sqrt{g}^{44} + s_{\bar{\theta}} \sqrt{g}^{34}), \quad (50)$$

$$\bar{g}_Z = \frac{g_2}{c_Z^2} (\bar{c} \sqrt{g}^{33} - \bar{s} \sqrt{g}^{34}) = \frac{g_1}{s_Z^2} (\bar{s} \sqrt{g}^{44} - \bar{c} \sqrt{g}^{34}). \quad (51)$$

For convenience, we have defined $s_{\bar{\theta}}, c_{\bar{\theta}} \equiv \bar{s}, \bar{c}$. The new minimum vacuum $\bar{v}_T$ is given by

$$\langle H^\dagger H \rangle = \frac{v^2}{2} \left(1 + \frac{3 C_H^{(6)} v^2}{4\lambda} + \frac{v^4}{8\lambda^2} \left[9 \left(C_H^{(6)} \right)^2 + 4\lambda C_H^{(8)} \right] + \dots \right) \equiv \frac{\bar{v}_T^2}{2}. \quad (52)$$

As an example that the geometric formulation yields indeed the same expression as the ordinary SMEFT, we verify it for the W-boson mass to mass dimension six. In SMEFT we have from Eq. (34,35)

$$m_W^2 = \frac{g_2^2 (1 + c_{HW} v_T^2)^2}{4} v_T^2 \quad (53)$$

and the geoSMEFT yields from Eq. (48) and using the explicit form ¹ of $\sqrt{h}^{ij}$ and $\sqrt{g}^{ij}$ from[51]

$$\frac{g_2^2}{4} (1 + \tilde{c}_{HW})^2 v_T^2, \quad (54)$$

with $\tilde{c}_{HW} = v_T^2 c_{HW}$. Thus, both expressions agree, as expected. In addition to the SM mixing angle $\bar{s}^2$, we have in the geoSMEFT formulation a second mixing angle s_Z^2 given by:

$$s_Z^2 = \frac{g_1 (\sqrt{g}^{44} \bar{s} - \sqrt{g}^{34} \bar{c})}{g_2 (\sqrt{g}^{33} \bar{c} - \sqrt{g}^{34} \bar{s}) + g_1 (\sqrt{g}^{44} \bar{s} - \sqrt{g}^{34} \bar{c})}, \quad (55)$$

$$\bar{s}^2 = \frac{(g_1 \sqrt{g}^{44} - g_2 \sqrt{g}^{34})^2}{g_1^2 [(\sqrt{g}^{34})^2 + (\sqrt{g}^{44})^2] + g_2^2 [(\sqrt{g}^{33})^2 + (\sqrt{g}^{34})^2] - 2g_1 g_2 \sqrt{g}^{34} (\sqrt{g}^{33} + \sqrt{g}^{44})}. \quad (56)$$

The covariant derivative acting on fermion fields ψ is then expressed as

$$D_\mu \psi = \left[\partial_\mu + i \bar{g}_3 G_\mu^A T^A + i \frac{\bar{g}_2}{\sqrt{2}} (W_\mu^+ T^+ + W_\mu^- T^-) + i \bar{g}_Z (T_3 - s_Z^2 Q_\psi) Z_\mu + i Q_\psi \bar{e} A_\mu \right] \psi. \quad (57)$$

The mixing angle denoted by the subscript Z is introduced in order to express the covariant derivative, as shown above, in a form analogous to that of the SM. The second angle diagonalizes the mass mixing in the gauge boson sector.

¹Note that $\sqrt{g}^{IJ} \sqrt{g}_{JK} \equiv \delta_K^I$ and $\sqrt{h}^{IJ} \sqrt{h}_{JK} \equiv \delta_K^I$.

4 Two-Point Functions in the Electroweak Standard Model

4.1 Two-Point Functions

In this section, we give a short review of two-point functions. Two-point functions are functions that describe the motion of a particle from one point in space-time to another and are therefore related to the propagator.

First, we will have a look at the definition of a two-point function in canonical quantization and then in the path integral approach.

4.1.1 Canonical two-point functions

For simplicity, let us consider the free field operator:

$$\phi_0(x, t) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2E_k}} \left(a_k e^{-ikx} + a_k^\dagger e^{ikx} \right), \quad (58)$$

where $a_k, a_k^\dagger$ are the time-independent creation and annihilation operators. Then the two point function is defined as

$$\langle \phi_1(x_1) \phi_2(x_2) \rangle = \langle 0 | \phi_1(x_1) \phi_2(x_2) | 0 \rangle, \quad (59)$$

where two field insertions are present and $x_1^0 > x_2^0$. Particularly important are time-ordered products of two fields:

$$\langle 0 | T \{ \phi_1(x_1) \phi_2(x_2) \} | 0 \rangle, \quad (60)$$

which accounts for both cases, i.e. $x_1^0 > x_2^0$ and $x_2^0 > x_1^0$. These play an essential role in the calculation of S-matrix elements for scattering processes.

The Feynman propagator D_F is defined as the time-ordered correlation function defined as in Eq. (60):

$$D_F(x - y) = \langle 0 | T \{ \phi(x) \phi(y) \} | 0 \rangle. \quad (61)$$

In position space, the propagator takes the form,

$$D_F(x - y) = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m^2 + i\epsilon} e^{-ip(x-y)}, \quad (62)$$

and in momentum space,

$$D_F(p) = \frac{i}{p^2 - m^2 + i\epsilon}. \quad (63)$$

In an interacting theory, the propagator gets a correction from the self-energy Σ , i.e.

$$D_F(p) = \frac{i}{p^2 - m^2 - \Sigma(p^2) + i\epsilon}. \quad (64)$$

4.1.2 Path integral two-point functions

Another method to calculate two-point functions is using the path integral and generating functionals[16, 52]. We will discuss the generating functional in greater detail in Sec. 5.2. The path integral is a more modern approach to quantum field theory and has some advantages over the canonical formulation[16]. For example, it allows for a systematic way to derive Feynman rules in perturbation theory. Two-point functions in the free theory can be calculated by taking variations of the generating functional Z :

$$\langle 0 | T \{ \phi_1(x_1) \phi_2(x_2) \} | 0 \rangle = (-i)^2 \frac{1}{Z_0[0]} \frac{\partial^2 Z_0[J]}{\partial J(x_1) \partial J(x_2)}. \quad (65)$$

4.1.3 Two-point functions from the effective action

A useful application of the effective action is that it facilitates the derivation of Feynman rules and two-point functions[16, 15]. To obtain two-point functions, one identifies the effective action with the action, i.e. $\Gamma[\phi] = iS[\phi]$ – where ϕ is a generic field. N -point functions and vertices are then $\{\phi(p_1), \dots, \phi(p_n)\}$ related by variations with respect to the fields. These variations generate 1PI Green's functions (n -point correlation functions) and two derivatives yield the inverse of the 1PI two-point function (i.e. the propagator)[16], i.e.

$$\langle \phi_1(x_1) \phi_2(x_2) \rangle = \left\{ \frac{\partial}{\partial \phi(x_1)} \frac{\partial}{\partial \phi(x_2)} (-i) \Gamma[\phi] \right\}_{\phi=0}^{-1}. \quad (66)$$

As an example, we calculate the interaction vertex for the interaction term

$$\mathcal{L}_{\text{int}} = c\phi(\partial_\mu\phi)(\partial^\mu\phi), \quad (67)$$

with ϕ being a scalar field and c a constant. We convert the interaction term into momentum space:

$$\begin{aligned} iS &= ic \int d^4x_1 d^4x_2 d^4x_3 \delta(x_2 - x_1) \delta(x_3 - x_1) \phi(x_1) (\partial_{x_2,\mu} \phi(x_2)) (\partial^{x_3,\mu} \phi(x_3)), \\ &= ic \int d^4x_1 d^4x_2 d^4x_3 \frac{d^4p_1 d^4p_2 d^4p_3}{(2\pi)^{4 \cdot 3}} \delta(x_2 - x_1) \delta(x_3 - x_1) e^{-ip_1 x_1} \phi(p_1) (-ip_2) e^{-ip_2 x_2} \phi(p_{2,\mu}) (-ip_3) e^{-ip_3 x_3} \phi(p_3^\mu), \\ &= ic \int \frac{d^4p_1 d^4p_2 d^4p_3}{(2\pi)^{4 \cdot 3}} (2\pi)^4 \delta(p_1 + p_2 + p_3) (-p_2 \cdot p_3) \phi(p_1) \phi(p_2) \phi(p_3). \end{aligned} \quad (68)$$

We take the following variation

$$\frac{\delta^3 i\mathcal{S}}{\delta\phi(k_1)\delta\phi(k_2)\delta\phi(k_3)} = ic \int \frac{d^4 p_1 d^4 p_2 d^4 p_3}{(2\pi)^{4 \cdot 3}} (2\pi)^4 \delta(p_1 + p_2 + p_3) (-p_2 \cdot p_3) \quad (69)$$

$$\times (\delta_{11}\delta_{22}\delta_{33} + \delta_{11}\delta_{23}\delta_{32} + \delta_{12}\delta_{21}\delta_{33} \quad (70)$$

$$\delta_{12}\delta_{23}\delta_{31} + \delta_{13}\delta_{21}\delta_{32} + \delta_{13}\delta_{22}\delta_{31}) \quad (71)$$

$$= \frac{-ic}{(2\pi)^{4 \cdot 3}} (2\pi)^4 \delta(k_1 + k_2 + k_3) \cdot 2[k_1 \cdot k_2 + k_2 \cdot k_3 + k_1 \cdot k_3], \quad (72)$$

where $\delta_{ij} = \frac{\delta\phi(p_i)}{\delta\phi(k_j)}$.

The triple variation applied to the effective action is

$$\frac{\delta^3 \Gamma[\phi]}{\delta\phi(k_1)\delta\phi(k_2)\delta\phi(k_3)} = \frac{i}{(2\pi)^{3 \cdot 4}} (2\pi)^4 \delta^{(4)}(k_1 + k_2 + k_3) \Gamma^{(3)}(k_1, k_2, k_3). \quad (73)$$

Thus, we identify the Feynman rule for the interaction vertex of Eq. (67) as:

$$\{\phi(k_1), \phi(k_2), \phi(k_3)\} = \Gamma^{(3)}(k_1, k_2, k_3) = -2ic [k_1 \cdot k_2 + k_2 \cdot k_3 + k_1 \cdot k_3]. \quad (74)$$

4.2 Functional and Background Field Method, Gauge Fixing and Ward Identities

In this section, functional techniques and the *background field method* (BFM) will be discussed. We will have a look at the process of gauge fixing and will show how Ward identities can be derived using background fields, which will be calculated in the following section at one-loop in perturbation theory. The BFM is a technique to quantize gauge theories while maintaining explicit gauge invariance. The BFM method can help to understand gauge theories differently and can also make calculations simpler[52].

Cross sections and decay rates are among the most important quantities calculated in quantum field theory, as they provide direct predictions for experiments. An important object in the calculation of cross sections and decay rates is the S-matrix. The S-matrix is obtained via the LSZ reduction[16] formula using Green's functions. These Green's functions can be determined by taking functional derivatives of the generating functional with respect to the source

$$Z[J] = \int \mathcal{D}\phi \exp \left\{ iS[\phi] + i \int d^4 x J(x) \phi(x) \right\}. \quad (75)$$

The generating functional generates disconnected Feynman diagrams and can be seen as the QFT analog of the partition function in statistical physics. Having a closed-form of $Z[J]$ means that one has, in principle, solved the theory[16].

The generator of connected diagrams is[52]

$$W[J] = -i \ln Z[J]. \quad (76)$$

The effective action Γ is an important quantity in a quantum field theory. It is the generating functional of 1PI (one-particle irreducible) diagrams[52]. The effective action is related to the generating functional of connected diagrams W via[52]

$$\Gamma[\bar{Q}] = W[\bar{Q}] - \int d^4x J_\phi(x) \bar{Q}(x), \quad (77)$$

where J is implicitly defined as the solution to

$$\frac{\partial W[J]}{\partial J(x)} = \bar{Q}(x). \quad (78)$$

Thus Γ is a Legendre transform of W .

The basis of every gauge theory is the gauge invariant Lagrangian. In order to quantize the theory and to derive Feynman rules to do calculations, one has to fix the gauge. However, the process of gauge fixing leads to the appearance of two additional terms –the ghost term and the gauge-fixing term– that explicitly break gauge invariance. Quantizing the theory can explicitly break gauge invariance. Dimensional regularization can also violate gauge invariance[53]. In the end, any physical quantity must be gauge independent. Quantities with no direct physical interpretation, like off-shell Green's functions, do not reflect the gauge invariance of the theory. As such, they need not be gauge invariant[52]. In the BFM approach, background gauge invariance is still present after gauge fixing and ghost terms have been added. In this formalism, Green's functions obey *Ward identities* as we will see below. These Ward identities arise naturally from gauge invariance. To introduce a propagator in an abelian (QED), as well as in a non-abelian (SM) model, one has to fix the gauge[16]. The path integral approach gives a way to gauge fix any model[17]. In the abelian case the procedure is quite straight forward. In the non-abelian case however, we get new fields, called ghost-fields. Ghosts are not physical particles, because their purpose is to cancel unphysical degrees of freedom. They also violate the spin-statistics theorem as Grassmannian bosonic fields. Thus they cannot be physical particles. So now we have additional terms in the Lagrangian $\mathcal{L}$:

$$\mathcal{L} \longrightarrow \mathcal{L} + \mathcal{L}_{\text{ghosts}} + \mathcal{L}_{\text{gauge}}, \quad (79)$$

where $\mathcal{L}$ depends on the model you consider. $\mathcal{L}_{\text{gauge}}$ is the gauge fixing term. In QED one only has the gauge term and no ghosts.

The BFM Ward identities can be derived from the invariance of the effective action with respect to the background field gauge[10]. An example from QED illustrates how one arrives at Ward identities. Without the gauge fixing term, the effective action is gauge invariant,

$$\delta\Gamma = 0. \quad (80)$$

Thus:

$$\delta\Gamma = \frac{\delta\Gamma}{\delta A_\mu} \delta A_\mu + \frac{\delta\Gamma}{\delta \psi_a} \delta \psi_a + \frac{\delta\Gamma}{\delta \bar{\psi}_a} \delta \bar{\psi} \quad (81)$$

$$= \left[-\partial_\mu \frac{\delta\Gamma}{\delta A_\mu} - ie \left(\frac{\delta\Gamma}{\delta \psi_a} \psi_a - \bar{\psi}_a \frac{\delta\Gamma}{\delta \bar{\psi}_a} \right) \right] \alpha(x) = 0, \quad (82)$$

where α comes from the gauge term $A_\mu \longrightarrow A_\mu + \partial_\mu \alpha(x)$ and from $\psi \longrightarrow e^{-i\alpha} \psi$. From that we conclude that

$$\left[-\partial_\mu \frac{\delta\Gamma}{\delta A_\mu} - ie \left(\frac{\delta\Gamma}{\delta \psi_a} \psi_a - \bar{\psi}_a \frac{\delta\Gamma}{\delta \bar{\psi}_a} \right) \right] = 0, \quad (83)$$

which is known as the *Master Ward identity*. Further identities are derived via variations with respect to the fields.

If we perform another variation with respect to A and respecting Lorentz invariance we get

$$\partial_\mu \frac{\delta^2 \Gamma}{\delta A_\mu \delta A_\nu} = 0 \xrightarrow{\text{momentum space}} i p_\mu \frac{\delta^2 \Gamma}{\delta A_\mu \delta A_\nu} = 0. \quad (84)$$

This reflects that the photon is transverse. Also the photon mass does not receive corrections from higher order terms and stays massless due to gauge symmetry. Furthermore, even though the gauge fixing term explicitly breaks gauge invariance, the Ward identities still hold due to current conservation and the abelian structure of the theory. In non-abelian models one needs background fields to obtain gauge invariance and to arrive at the Ward Identities.

The background field method (BFM) is useful in non-abelian gauge theories to maintain gauge invariance after gauge fixing. In this method the fields are split into background fields (denoted with $\tilde{A}_\mu^a$) and quantum fields, i.e.

$$A_\mu^a \longrightarrow \tilde{A}_\mu^a + A_\mu^a \quad (85)$$

in the Yang-Mills field strength. The quantum fields A_μ^a are integrated out with fixed background fields $\tilde{A}_\mu^a$. Since only the quantum fields propagate by construction, only they need to be gauge fixed[16]. The background fields are fixed by construction and do not propagate. The advantage of this method is that one can choose a gauge fixing functional associated with the background field.

The Yang-Mills field strength, after applying Eq. (85), turns into[16]

$$F_{\mu\nu} \longrightarrow \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \partial_\mu \tilde{A}_\nu^a - \partial_\nu \tilde{A}_\mu^a + g f^{abc} \left(A_\mu^b A_\nu^c + \tilde{A}_\mu^b A_\nu^c + A_\mu^b \tilde{A}_\nu^c + \tilde{A}_\mu^b \tilde{A}_\nu^c \right) \quad (86)$$

$$= \tilde{F}_{\mu\nu}^a + \tilde{D}_\mu A_\nu^a - \tilde{D}_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c, \quad (87)$$

where $\tilde{F}_{\mu\nu}^a = \partial_\mu \tilde{A}_\nu^a - \partial_\nu \tilde{A}_\mu^a + g f^{abc} \tilde{A}_\mu^b \tilde{A}_\nu^c$ is the field strength for the background field and

$$\tilde{D}_\mu A_\nu^a = \partial_\mu A_\nu^a + g f^{abc} \tilde{A}_\mu^b A_\nu^c \quad (88)$$

is the covariant derivative.

The Lagrangian

$$\mathcal{L}_{\text{BF}} = -\frac{1}{4} \left(\tilde{F}_{\mu\nu}^a + \tilde{D}_\mu A_\nu^a - \tilde{D}_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c \right)^2 \quad (89)$$

is invariant under

$$\tilde{A}_\mu^a \longrightarrow \tilde{A}_\mu^a + \frac{1}{g} \partial_\mu \alpha^a - f^{abc} \alpha^b \tilde{A}_\mu^c, \quad A_\mu^a \longrightarrow A_\mu^a - f^{abc} \alpha^b A_\mu^c. \quad (90)$$

Since $\tilde{A}_\mu$ is not dynamical, we do not need its propagator and do not need to gauge fix the background field. In this way we are able to keep the background field gauge invariance manifest throughout the

calculation. We gauge fix A_μ through the Fadeev-Popov procedure using $\tilde{D}_\mu A_\mu = 0$ (instead of the usual $\partial_\mu A_\mu = 0$) as the gauge condition. This gauge condition breaks the gauge invariance for the propagating quantum A_μ , but retains the background gauge invariance for the $\tilde{A}_\mu$.

The resulting Lagrangian is[16]

$$\mathcal{L}_{\text{BFG}} = \mathcal{L}_{\text{BF}} - \frac{1}{2\xi}(\tilde{D}_\mu A_\mu^a)^2 + (\tilde{D}_\mu \bar{c}^a)(\tilde{D}_\mu c^a) + gf^{abc}(\tilde{D}_\mu \bar{c}^a)A_\mu^b c^c. \quad (91)$$

This Lagrangian is gauge invariant with respect to $\tilde{A}_\mu$ and this type of gauge fixing is called *background field gauge*.

This background gauge is parametrized by the background gauge parameter $\tilde{\xi}$. The effective action is not guaranteed to be gauge invariant. However, it can be made gauge independent using the background field method. It will be gauge invariant with respect to the background field gauge. This concept brings us to Ward identities. As mentioned earlier in this section, Ward identities are a consequence of gauge invariance of the effective action:

$$\delta\Gamma = 0. \quad (92)$$

4.2.1 Ordinary Gauge fixing vs BFM Gauge fixing

Ordinary gauge fixing can be achieved by using the path integral and a gauge function G [17]. For example, in Yang-Mills theory the generating functional takes the form

$$Z \propto \int \mathcal{D}A \det\left(\frac{\delta G}{\delta\theta}\right) \prod_{x,a} \delta(G) e^{iS_{\text{YM}}}. \quad (93)$$

$\theta^a(x)$ is an arbitrary gauge transformations, $G^a(x)$ is the gauge function. The final form of Eq. (93) is then given by

$$Z \propto \int \mathcal{D}A \mathcal{D}\bar{c} \mathcal{D}c \exp(iS_{\text{YM}} + iS_{\text{gh}} + iS_{\text{gf}}), \quad (94)$$

where c and $\bar{c}$ stand for the ghost fields, S_{gh} is the ghost action and S_{gf} is the gauge fixing term. The latter two Lagrangians are given by

$$\mathcal{L}_{\text{gh}} = -\partial_\mu \bar{c}^a \partial^\mu c^a + gf^{abc} A_\mu^c \partial^\mu \bar{c}^a c^b, \quad (95)$$

$$\mathcal{L}_{\text{gf}} = -\frac{1}{2\xi} \partial^\mu A_\mu^a \partial^\nu A_\nu^a. \quad (96)$$

What is not obvious is that both approaches, with and without background fields, yield the same physics. We will give now the proof, following Abbot[52], that the BFM approach yields indeed the same physics.

We start by defining the generating functional for a non-abelian gauge theory

$$Z[J] = \int \mathcal{D}A \left(\frac{\partial G}{\partial \theta} \right) \exp \left\{ iS[A] - \int d^4x \frac{i}{2\xi} G^a(x) \cdot G^a(x) + i \int d^4x J(x)A(x) \right\}. \quad (97)$$

Incidentally, the background field method for abelian models, is the ordinary “fields shifting” method. A is the gauge field with field strength $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c$.

The background field generating functional for the gauge theory is

$$\tilde{Z}[J, \phi] = \int \mathcal{D}A \left(\frac{\partial \tilde{G}}{\partial \theta} \right) \exp \left\{ iS[A + \phi] - \int d^4x \frac{i}{2\xi} \tilde{G}^a(x) \cdot \tilde{G}^a(x) + i \int d^4x J(x)A(x) \right\}, \quad (98)$$

where ϕ is the additional background field. For the following, we also introduce the following abbreviations:

$$\int d^4x \tilde{G}^a(x) \tilde{G}^a(x) \equiv \tilde{G}\tilde{G}, \quad (99)$$

$$\int d^4x J(x)A(x) \equiv JA. \quad (100)$$

After a field redefinition $A \rightarrow A - \phi$ in Eq.(98) one finds

$$\tilde{Z}[J] = \int \mathcal{D}A \left(\frac{\partial \tilde{G}}{\partial \theta} \right) \exp \left\{ iS[A] - \frac{i}{2\xi} \tilde{G}^a(x) \cdot \tilde{G}^a(x) + iJ(x)A(x) - iJ(x)\phi(x) \right\} \quad (101)$$

$$= Z[J] \exp\{-iJ\phi\}. \quad (102)$$

Taking the logarithm and multiply with (-i) gives

$$\tilde{W}[J, \phi] = W[J] - J\phi. \quad (103)$$

After differentiating with respect to J and using that $\frac{\delta W}{\delta J} = \bar{Q}$ and $\frac{\delta \tilde{W}}{\delta J} = \tilde{Q}$ we obtain

$$\tilde{Q} = \bar{Q} - \phi. \quad (104)$$

Now we use the definition of the effective action $\tilde{\Gamma}$ and Eq. (103,104), which gives

$$\tilde{\Gamma}[\tilde{Q}, \phi] = \tilde{W}[J, \phi] - J\tilde{Q} \quad (105)$$

$$= W[J] - J\phi - J\tilde{Q} = W[J] - J\phi - J\tilde{Q} + J\phi. \quad (106)$$

Thus, we finally obtain

$$\tilde{\Gamma}[\tilde{Q}, \phi] = W[J] - J\bar{Q} = \Gamma[\bar{Q}] = \Gamma[\tilde{Q} + \phi]. \quad (107)$$

And as a specific case, take $\tilde{Q} = 0$ so that

$$\tilde{\Gamma}[0, \phi] = \Gamma[\phi]. \quad (108)$$

Therefore, we see that the background field effective action is related to the ordinary effective action in the presence of the background field ϕ . Eq. (108) demonstrates that one can obtain the background field effective action by vacuum graphs (graphs with no external legs) and it ensures that one gets the same result as in the conventional approach. The gauge fixing term differs between the two approaches, yet both approaches produce the same S-matrix elements. Thus, the background field method provides a consistent framework for performing gauge theory calculations. Background fields can also simplify some calculations, such as the calculation of the Yang-Mills β -function[52, 16].

4.2.2 Summary

The BFM is a method that keeps gauge symmetry explicit after gauge fixing. In ordinary gauge fixing, gauge invariance is lost. Maintaining gauge invariance after quantization implies that the naive Ward identities follow. The BFM yields the same S-matrix and is therefore an equivalent description of the theory. Using background fields can help reduce the complexity of the calculations. Maintaining explicit gauge invariance, as achieved in the background field gauge, is extremely useful.

4.3 Vacuum Polarization Tensor in QED

As a first two-point function we will calculate the vacuum polarisation tensor $i\Pi^{\mu\nu}$ in QED. As we will see, this loop diagram, see Figure 5, as a correction to the photon propagator, has a physical effect.

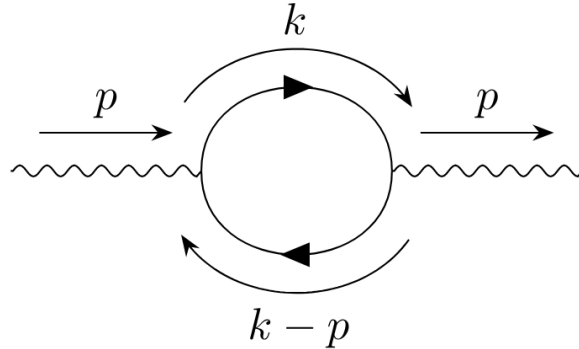


Figure 5: Feynman diagram of vacuum polarisation

The matrix element corresponding to this loop is

$$i\mathcal{M} = -(ie)^2 \int \frac{d^4 k}{(2\pi)^4} \cdot \frac{\text{Tr}[\gamma^\mu(\not{k} - \not{p} + m)\gamma^\nu(\not{k} + m)]}{((k-p)^2 - m^2 + i\epsilon)(k^2 - m^2 + i\epsilon)}, \quad (109)$$

where the additional -1 and the trace come from the closed fermion loop.

Using the trace formulas gives

$$\text{Tr}[\gamma^\mu(\not{k} - \not{p} + m)\gamma^\nu(\not{k} + m)] = 4[g^{\mu\nu}(-k^2 + k \cdot p + m^2) + 2k^\mu k^\nu - k^\nu p^\mu - k^\mu p^\nu]. \quad (110)$$

After using the Feynman parameters in the denominator and completing the square and going to $d = 4 - 2\epsilon$ dimensions in the integral we obtain

$$i\Pi^{\mu\nu} = -e^2 \tilde{\mu}^{4-d} \int_0^1 dx \int \frac{d^d k}{(2\pi)^d} \frac{4[g^{\mu\nu}(-k^2 + k \cdot p + m^2) + 2k^\mu k^\nu - k^\nu p^\nu - k^\nu p^\nu]}{[(k - p(1-x))^2 + p^2 x(1-x) - m^2 + i\epsilon]^2}, \quad (111)$$

where $\tilde{\mu}$ is used to keep the same mass dimension of the integral as in $d = 4$.

We can make the change $k^\mu \rightarrow k^\mu + p^\mu(1-x)$ and after omitting terms linear in k^μ , since they are odd and vanish, we obtain

$$i\Pi^{\mu\nu} = -4e^2 \tilde{\mu}^{4-d} \int_0^1 dx \int \frac{d^d k}{(2\pi)^d} \frac{2k^\mu k^\nu - g^{\mu\nu}(k^2 - p^2 x(1-x) - m^2) - 2p^\mu p^\nu x(1-x)}{(k^2 - \Delta + i\epsilon)^2}, \quad (112)$$

where $\Delta = -p^2 x(1-x) + m^2$.

Eq. (112) can be expressed in terms of the Master-integrals of dimensional regularization

$$i\Pi^{\mu\nu} = -4e^2 \tilde{\mu}^{4-d} \int_0^1 dx \{2I_2^{\mu\nu} - g^{\mu\nu} I^{(2)} + g^{\mu\nu}(p^2 x(1-x) + m^2) I_2 - 2p^\mu p^\nu x(1-x) I_2\}. \quad (113)$$

Since we make the replacement $k^\mu k^\nu \rightarrow \frac{1}{d} g^{\mu\nu} k^2$ we can combine the first two integrals

$$2I_2^{\mu\nu} - g^{\mu\nu} I^{(2)} = \int \frac{d^d k}{(2\pi)^d} \frac{(\frac{2}{d} - 1)g^{\mu\nu} k^2}{(k^2 - \Delta + i\epsilon)^2} = -(\frac{2}{d} - 1) \frac{g^{\mu\nu}}{2} \frac{id}{(4\pi)^{\frac{d}{2}}} \Gamma(1 - \frac{d}{2}) \Delta^{\frac{d}{2}-1} \quad (114)$$

$$= \frac{-ig^{\mu\nu} \Delta}{(4\pi)^{\frac{d}{2}}} \Gamma(2 - \frac{d}{2}) \Delta^{\frac{d}{2}-2}. \quad (115)$$

In the last step we used that $\Gamma(2 - \frac{d}{2}) = (1 - \frac{d}{2})\Gamma(1 - \frac{d}{2})$.

Combining this result with the two remaining integrals we obtain

$$\begin{aligned} & \left\{ 2I_2^{\mu\nu} - g^{\mu\nu} I^{(2)} + g^{\mu\nu}(p^2 x(1-x) + m^2) I_2 - 2p^\mu p^\nu x(1-x) I_2 \right\} \\ &= \frac{i}{(4\pi)^{d/2}} \Gamma(2 - \frac{d}{2}) \Delta^{\frac{d}{2}-2} \\ & \quad \times \left[g^{\mu\nu}(p^2 x(1-x) + m^2) - 2p^\mu p^\nu x(1-x) - g^{\mu\nu} \Delta \right] \\ &= \frac{i}{(4\pi)^{d/2}} \Gamma(2 - \frac{d}{2}) \Delta^{\frac{d}{2}-2} \\ & \quad \times \left[2g^{\mu\nu} p^2 x(1-x) - 2p^\mu p^\nu x(1-x) \right] \\ &= \frac{2i}{(4\pi)^{d/2}} \Gamma(2 - \frac{d}{2}) \Delta^{\frac{d}{2}-2} x(1-x) \left[g^{\mu\nu} p^2 - p^\mu p^\nu \right]. \end{aligned} \quad (116)$$

Thus

$$i\Pi^{\mu\nu} = -\frac{8ie^2 \tilde{\mu}^{4-d}}{(4\pi)^{\frac{d}{2}}} (p^2 g^{\mu\nu} - p^\mu p^\nu) \Gamma(2 - \frac{d}{2}) \int_0^1 dx x(1-x) \Delta^{\frac{d}{2}-2}. \quad (117)$$

Setting $d = 4 - 2\epsilon$ and expanding around $\epsilon = 0$ we get

$$i\Pi^{\mu\nu} = -\frac{ie^2}{2\pi^2}(p^2 g^{\mu\nu} - p^\mu p^\nu) \int_0^1 dx x(1-x) \left[\frac{1}{\epsilon} - \gamma_E + \ln \frac{\tilde{\mu}^2 4\pi}{\Delta} \right], \quad (118)$$

where γ_E is the Euler-Mascheroni constant. Setting $\tilde{\mu}^2 = \frac{\mu^2 e^{-\gamma_E}}{4\pi}$ we obtain the final result

$$i\Pi_{1-loop}^{\mu\nu} = -\frac{ie^2}{2\pi^2}(p^2 g^{\mu\nu} - p^\mu p^\nu) \int_0^1 dx x(1-x) \left[\frac{1}{\epsilon} + \ln \frac{\mu^2}{m^2 - p^2 x(1-x)} \right]. \quad (119)$$

This loop diagram has an effect on the Coulomb potential. It corrects the potential by the Uehling term[16] and thus has an effect on electromagnetic interactions and on the splitting effect of atomic energy levels, which can be measured, such as the lamb shift.

The loop can also be calculated using Mathematica with package X[54, 55] to again verify the general structure, i.e. the transverse structure multiplied by a function dependent on p^2 . This structure is due to Lorentz and gauge invariance. The result as calculated using Package X is

$$i\Pi^{\mu\nu} = \frac{i \left(12m^2 + 3 \text{DiscB}[p^2, m, m] (2m^2 + p^2) + p^2 \left(5 + \frac{3}{\epsilon} + 3 \log \left(\frac{\mu^2}{m^2} \right) \right) \right) (-p_\mu p_\nu + p^2 g_{\mu\nu})}{36\pi^2 p^2}, \quad (120)$$

where DiscB gives the part of the Passarino-Veltman B_0 function containing the s plane branch cut. For example, written out explicitly we have

$$\text{Disc}[s, m, m] = \frac{\sqrt{s(-4m^2 + s)} \log \left(\frac{2m^2 - s + \sqrt{s(-4m^2 + s)}}{2m^2} \right)}{s}. \quad (121)$$

4.4 Ward identities and definitions

In this section, we calculate two-point functions in the electroweak sector of the SM at one loop and verify some Ward identities. These Ward Identities are exact and hold to all orders in perturbation theory.

For this, some definitions and notation are required[10]:

$$-i\Gamma_{\mu\nu}^{\hat{V}\hat{V}'} = \left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2} \right) \Sigma_T^{\hat{V}\hat{V}'} - \frac{k_\mu k_\nu}{k^2} \Sigma_L^{\hat{V}\hat{V}'}. \quad (122)$$

This refers to the two-point function of the background field bosons in terms of transverse and longitudinal components. The first two Ward identities of interest to us are:

$$\frac{\delta^2 \Gamma}{\delta \hat{A}_\mu \delta \hat{A}_\nu} = 0 \quad (123)$$

$$\frac{\delta^2 \Gamma}{\delta \hat{A}_\mu \delta \hat{Z}_\nu} = 0. \quad (124)$$

Eq. (123) refers to the fact that the photon is transverse. The $\hat{A}/\hat{Z}$ mixing, from Eq. (124), vanishes for zero momentum transfer. For the neutral sector we find the following Ward identities:

$$0 = \Sigma_L^{\hat{Z}\hat{Z}}(k^2) - im_Z \Sigma^{\hat{Z}\hat{\chi}}(k^2), \quad (125)$$

$$0 = k^2 \Sigma^{\hat{Z}\hat{\chi}}(k^2) - im_Z \Sigma^{\hat{\chi}\hat{\chi}}(k^2) + i \frac{g_z}{2} T^H. \quad (126)$$

The first identity relates the longitudinal part of $\hat{Z}\hat{Z}$ with the $\hat{Z}\hat{\chi}$ mixing. Latter one relates $\hat{Z}/\hat{\chi}$, $\hat{\chi}/\hat{\chi}$ with the tadpole T^H .

The analogous identities corresponding to the charged sector are:

$$0 = \Sigma_L^{\hat{W}^\pm \hat{W}^\mp}(k^2) \pm m_W \Sigma^{\hat{\Phi}^\pm \hat{W}^\mp}(k^2), \quad (127)$$

$$0 = k^2 \Sigma^{\hat{W}^\pm \hat{\Phi}^\mp}(k^2) \pm m_W \Sigma^{\hat{\Phi}^\pm \hat{\Phi}^\mp}(k^2) \mp \frac{g_2^2}{2} T^H. \quad (128)$$

$\hat{\phi}^\pm$ are charged background goldstone bosons and $\hat{\chi}$ is the neutral background goldstone boson.

The tree-level masses, in the SM, are related by

$$m_W = \frac{g_2 v}{2}, \quad (129)$$

$$m_Z = \frac{v}{2} \sqrt{g_1^2 + g_2^2}, \quad (130)$$

$$m_h = \sqrt{2\lambda} v, \quad (131)$$

where v is the vacuum expectation value (vev), λ the higgs coupling, $g_z = \sqrt{g_1^2 + g_2^2}$ and g_1, g_2 the U(1), SU(2) couplings. The right hand side of Eq. (125-128) are related by

$$\frac{\delta^2 \Gamma}{\delta \hat{\chi} \delta \hat{Z}^\nu} = ik^\nu \Sigma^{\hat{Z}\hat{\chi}}(k^2) = -\frac{\delta^2 \Gamma}{\delta \hat{Z}^\nu \delta \hat{\chi}}, \quad (132)$$

$$\frac{\delta^2 \Gamma}{\delta \hat{\chi} \delta \hat{\chi}} = i \Sigma^{\hat{\chi}\hat{\chi}}(k), \quad (133)$$

$$-i \frac{\delta \Gamma}{\delta \hat{H}} = T^H. \quad (134)$$

The different two-point functions (Sigmas) are calculated using Package X[55], FeynArts[56] and FormCalc[57]. All the different Feynman diagrams contributing to the various two-point functions, can be seen in the App C.

4.5 Results

This is the section of the results of the calculation of the ‘‘Sigmas’’ and the Ward identities in the SM of the previous section. We present the UV-divergent parts at one loop. They are for the Ward identities of Eqs. (123, 124):

$$\left[\Sigma_T^{\hat{A}\hat{A}}(k^2)\right]_{\text{div}}^{\text{SM}} = \frac{g_1^2 g_2^2}{g_1^2 + g_2^2} k^2 \left(-\frac{7}{16\pi^2\epsilon}\right), \quad (135)$$

$$\left[\Sigma_L^{\hat{A}\hat{A}}(k^2)\right]_{\text{div}}^{\text{SM}} = 0, \quad (136)$$

$$\left[\Sigma_T^{\hat{A}\hat{Z}}(k^2)\right]_{\text{div}}^{\text{SM}} = -\frac{g_1 g_2}{g_1^2 + g_2^2} k^2 \left(\frac{43g_2^2 + g_1^2}{96\pi^2\epsilon}\right), \quad (137)$$

$$\left[\Sigma_L^{\hat{A}\hat{Z}}(k^2)\right]_{\text{div}}^{\text{SM}} = 0. \quad (138)$$

and for the neutral sector:

$$\left[\Sigma_T^{\hat{Z}\hat{Z}}(k^2)\right]_{\text{div}}^{\text{SM}} = \frac{8k^2(g_1^4 - 43g_2^4) + 3(\xi + 3)v^2(g_1^2 + g_2^2)^2(g_1^2 + 3g_2^2)}{768\pi^2\epsilon(g_1^2 + g_2^2)}, \quad (139)$$

$$\left[\Sigma_L^{\hat{Z}\hat{Z}}(k^2)\right]_{\text{div}}^{\text{SM}} = \frac{(\xi + 3)v^2(g_1^2 + g_2^2)(g_1^2 + 3g_2^2)}{256\pi^2\epsilon}, \quad (140)$$

$$\left[\Sigma_T^{\hat{Z}\hat{\chi}}(k^2)\right]_{\text{div}}^{\text{SM}} = -i \frac{(\xi + 3)v\sqrt{g_1^2 + g_2^2}(g_1^2 + 3g_2^2)}{128\pi^2\epsilon}, \quad (141)$$

$$(142)$$

With the relevant charged Greens functions given by:

$$\left[\Sigma_T^{\hat{W}^+\hat{W}^-}(k^2)\right]_{\text{div}}^{\text{SM}} = \frac{g_2^2 [3(\xi + 3)v^2(g_1^2 + 3g_2^2) - 344k^2]}{768\pi^2\epsilon}, \quad (143)$$

$$\left[\Sigma^{\hat{W}^+\hat{\phi}^-}\right]_{\text{div}}^{\text{SM}} = -\left[\Sigma^{\hat{W}^-\hat{\phi}^+}\right]_{\text{div}}^{\text{SM}} = \frac{g_2(\xi + 3)v(g_1^2 + 3g_2^2)}{128\pi^2\epsilon}, \quad (144)$$

$$\left[\Sigma^{\hat{\chi}\hat{\chi}}(k^2)\right]_{\text{div}}^{\text{SM}} = \left[\Sigma^{\hat{\phi}^+\hat{\phi}^-}(k^2)\right]_{\text{div}}^{\text{SM}}, \quad (145)$$

$$= \frac{1}{256\pi^2\epsilon} \left[v^2 (3g_1^4 + 9g_2^4 + 96\lambda^2 + 12g_2^2\lambda\xi + g_1^2(6g_2^2 + 4\lambda\xi)) - 4(g_1^2 + 3g_2^2)(3 + \xi)k^2 \right], \quad (146)$$

$$= \frac{1}{v} [T^H]_{\text{div}}^{\text{SM}} - \frac{(\xi + 3)k^2(g_1^2 + 3g_2^2)}{64\pi^2\epsilon}, \quad (147)$$

$$(148)$$

And finally the tadpole is given by:

$$[T^H]_{\text{div}}^{\text{SM}} = \frac{v^3}{256\pi^2\epsilon} (3g_1^4 + 9g_2^4 + 96\lambda^2 + 12g_2^2\lambda\xi + g_1^2(6g_2^2 + 4\lambda\xi)). \quad (149)$$

The goal of the following sections is the calculation of one-loop Ward-Identities in the SMEFT also employing the background field method and the geoSMEFT formalism. We require the Lagrangian and Feynman rules to perform the computation of the identities. In order to achieve this, a number of preparatory considerations are necessary. These will be demonstrated within the QCD sector and subsequently apply equally to the electroweak sector of the SMEFT.

5 GeoSMEFT, Universality and the QCD sector as an Example

We begin with the QCD sector to illustrate the formal structure. Using the geometric formulation of the SMEFT, we include metrics to organize the theory. We include $\mathcal{K}$ as the metric for the operator, that contains two derivatives and two field strengths and is typically treated as redundant in non-universal theories. The *universal* theory[58] captures the leading BSM physics through dimension-six operators, which include only SM bosons ("bosonic operators"). Possible UV completions of the effective theory couple to the SM via the SM currents, see Sec. 2.1. These operators involving these SM currents can be eliminated in favor of bosonic operators by field redefinitions or, equivalently, by applying the SM EOMs. The geoSMEFT formulation classifies all two- and three-point functions of the SMEFT. This relies on the use of the EOM to reduce the operator structure[9]. Universality does not account for this EOM redundancy. Our aim is to demonstrate consistency with the geoSMEFT formulation by using field redefinitions up to and including dimension eight. In other words, we show that the reduction of basis for universal theories also holds in the geometric formulation. In this section we eliminate higher-derivative operators, such as those involving the $\mathcal{K}$ metric. Operators of the form $\mathcal{K}(DG)(DG)$ introduce additional unphysical poles in the propagators, which we seek to avoid. We therefore restrict our analysis to theories with physical propagators.

Our effective Lagrangian takes the form:

$$\mathcal{L} = -\frac{1}{4}\kappa_{AB}G_{\mu\nu}^AG^{\mu\nu,B} + i\bar{\psi}\not{D}\psi + \mathcal{K}_{AB}(D^\mu G_{\nu\mu}^A)(D^\rho G_{\nu\rho}^B) + F^{ABC}G_{\mu\nu}^AG_{\nu\rho}^BG_{\rho\mu}^C, \quad (150)$$

with field space connections[9]

$$\kappa\delta_{AB} = \kappa_{AB} = \delta_{AB} - 4\delta_{AB}\sum_{n=0}^{\infty}c_{\text{HG}}^{(6+2n)}\left(\frac{\phi^2}{2}\right)^{n+1}, \quad (151)$$

$$\mathcal{K}\delta_{AB} = \mathcal{K}_{AB} = \delta_{AB}\sum_{n=0}^{\infty}c_{\text{DG}}^{(6+2n)}\left(\frac{\phi^2}{2}\right)^n, \quad (152)$$

$$Ff^{ABC} = F^{ABC} = f^{ABC}\sum_{n=0}^{\infty}c_{\text{G}}^{(6+2n)}\left(\frac{\phi^2}{2}\right)^n. \quad (153)$$

For the remainder of the thesis, for a better reading experience we sometimes relax the raised and lowered index convention for the Lorenz indices. This is possible as we work in flat space time. For example, in Eq. (150) we ignored this rule in the last operator form (FGGG).

5.1 Field Redefinitions

Here we use the field redefinition invariance of QFT. In the path integral approach field redefinition invariance follows directly from invariance of the measure. We will not show that invariance here, but we give a small example: Suppose that we have the Lagrangian of the free scalar theory

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}m^2\phi^2. \quad (154)$$

We proceed by performing a field redefinition of $\phi \longrightarrow \phi + \lambda\phi^3$. This changes the Lagrangian, but the matrix element for $2 \longrightarrow 2$ scattering stays the same, i.e.

$$i\mathcal{M} = 0, \quad (155)$$

as should be the case in a free theory without interactions.

We will use a field redefinition to remove the operator form $(DG)(DG)$. Returning to Eq. (150) we perform a field redefinition of the following form:

$$G_\mu^A \longrightarrow G_\mu^A + \delta G_\mu^A = G_\mu^A + M_B^A (D^\rho G_{\mu\rho}^B). \quad (156)$$

We find the following relations:

$$\delta G_{\mu\nu}^A = D_\mu \delta G_\nu^A - D_\nu \delta G_\mu^A - f^{ABC} \delta G_\mu^B \delta G_\nu^C \quad (157)$$

$$\delta D^\mu G_{\nu\mu}^A = D^\mu \delta G_{\nu\mu}^A - f^{ABC} G_\mu^B \delta G_{\nu\mu}^C - f^{ABC} \delta G_\mu^B \delta G_{\nu\mu}^C \quad (158)$$

$$= D^2 \delta G_\nu^A - D_\mu D_\nu \delta G_\mu^A - f^{ABC} D_\mu (\delta G_\mu^B \delta G_\nu^C) - f^{ABC} \delta G_\mu^B G_{\nu\mu}^C \quad (159)$$

$$- f^{ABC} \delta G_\mu^B (D_\nu \delta G_\mu^C) + f^{ABC} \delta G_\mu^B (D_\mu \delta G_\nu^C) + f^{ABC} f^{CIJ} \delta G_\mu^B \delta G_\nu^I \delta G_\mu^J \quad (160)$$

Applying the field redefinition yields

$$\begin{aligned} \mathcal{L} \longrightarrow & -\frac{1}{4} \kappa_{AB} G_{\mu\nu}^A G^{\mu\nu,B} + i\bar{\psi} \not{D} \psi + F^{ABC} G_{\mu\nu}^A G_{\nu\rho}^B G_{\rho\mu}^C - J_A^\mu M_D^A D^\gamma G_{\mu\gamma}^D \\ & + [\mathcal{K}_{CB} - \kappa_{AB} M_C^A] (D^\gamma G_{\nu\gamma}^C) (D^\mu G_{\nu\mu}^B) + \frac{\delta\kappa_{AB}}{\delta\phi^2} \phi^I (D_\mu \phi^I) M_C^A (D^\sigma G_{\nu\sigma}^C) G^{\mu\nu,B} \\ & + \frac{1}{2} \kappa_{AB} f^{ACD} M_E^C M_F^D G^{\mu\nu,B} D^\sigma G_{\mu\sigma}^E D^\gamma G_{\nu\gamma}^F + 2\mathcal{K}_{AB} M_C^A (D^\rho G_{\nu\rho}^B) D^\mu D^\nu D^\sigma G_{\mu\sigma}^C \\ & - 2\mathcal{K}_{AB} M_C^A (D^\rho G_{\nu\rho}^B) (D^2 D^\sigma G_{\nu\sigma}^C) - 2\mathcal{K}_{AB} f^{ACD} M_F^C (D^\sigma G_{\mu\sigma}^F) G_{\nu\mu}^D (D^\gamma G_{\nu\gamma}^B) \\ & + 3F^{ABC} M_D^A [D^\mu D^\sigma G_{\nu\sigma}^D - D^\nu D^\sigma G_{\mu\sigma}^D] G_{\nu\rho}^B G_{\rho\mu}^C \\ & - \frac{1}{2} \kappa_{AB} M_C^A M_D^B (D^\mu D^\sigma G_{\nu\sigma}^C) [D_\mu D^\gamma G_{\nu\gamma}^D - D_\nu D^\gamma G_{\mu\gamma}^D] \end{aligned} \quad (161)$$

Choosing $M_D^B = \kappa^{BC} \mathcal{K}_{CD}$, which removes the operator form $D^2 G^2$, gives

$$\begin{aligned}
\mathcal{L} \longrightarrow & -\frac{1}{4}\kappa_{AB}G_{\mu\nu}^AG^{\mu\nu,B} + i\bar{\psi}\not{D}\psi + F^{ABC}G_{\mu\nu}^AG_{\nu\rho}^BG_{\rho\mu}^C - J_A^\mu\kappa^{AC}\mathcal{K}_{CD}D^\gamma G_{\mu\gamma}^D \\
& + \frac{\delta\kappa_{AB}}{\delta\phi^2}\phi^I(D_\mu\phi^I)\kappa^{AD}\mathcal{K}_{DC}(D^\sigma G_{\nu\sigma}^C)G^{\mu\nu,B} \\
& + \frac{1}{2}\kappa_{AB}f^{ACD}\kappa^{CI}\mathcal{K}_{IE}\kappa^{DJ}\mathcal{K}_{JF}G^{\mu\nu,B}(D^\sigma G_{\mu\sigma}^E)(D^\gamma G_{\nu\gamma}^F) \\
& + 6F^{ABC}\kappa^{AC}\mathcal{K}_{CD}D^\mu D^\sigma G_{\nu\sigma}^DG_{\nu\rho}^BG_{\rho\mu}^C + 2\mathcal{K}_{AB}\kappa^{AI}\mathcal{K}_{IC}(D^\rho G_{\nu\rho}^B)[(D^\mu D^\nu - D^2)D^\sigma G_{\nu\sigma}^C] \\
& - 2\mathcal{K}_{AB}f^{ACD}\kappa^{CI}\mathcal{K}_{IF}(D^\sigma G_{\mu\sigma}^F)G_{\nu\mu}^D(D^\gamma G_{\nu\gamma}^B) \\
& - \frac{1}{2}\kappa_{AB}\kappa^{AI}\mathcal{K}_{IC}\kappa^{BJ}\mathcal{K}_{JD}(D^\mu D^\sigma G_{\nu\sigma}^C)[D_\mu D^\gamma G_{\nu\gamma}^D - D_\nu D^\gamma G_{\mu\gamma}^D] + \dots
\end{aligned} \tag{162}$$

We again used that $\kappa^{AB} = (\kappa_{AB})^{-1}$ and $\kappa^{AB}\kappa_{BC} = \delta_C^A$. The $+\dots$ indicates terms beyond dimension eight.

The first line of Eq. (162) corresponds to the dimension-six operators plus metric coefficients. The rest are dimension-eight terms still including two- or three-point functions, but are neglected, since they correspond to higher orders in the derivative (momentum) expansion, which are neglected in the geoSMEFT approach, which only seeks to sum the vev expansion. We are then left with:

$$\mathcal{L} = -\frac{1}{4}\kappa_{AB}G_{\mu\nu}^AG^{\mu\nu,B} + i\bar{\psi}\not{D}\psi + F^{ABC}G_{\mu\nu}^AG_{\nu\rho}^BG_{\rho\mu}^C - J_A^\mu\kappa^{AC}\mathcal{K}_{CD}D^\gamma G_{\mu\gamma}^D + \dots \tag{163}$$

The last operator of Eq.(163) $(\bar{\psi}\psi)DG$ contains a derivative and can be reduced for consistency with other conventional SMEFT treatments. We do a second field redefinition of the form,

$$G_\mu^A \longrightarrow G_\mu^A + B^{AD}J_{\mu,D}, \tag{164}$$

and obtain:

$$\begin{aligned}
\mathcal{L} \longrightarrow & -\frac{1}{4}\kappa_{AB}G_{\mu\nu}^AG^{\mu\nu,B} + i\bar{\psi}\not{D}\psi + F^{ABC}G_{\mu\nu}^AG_{\nu\rho}^BG_{\rho\mu}^C - J_A^\mu B^{AD}J_{\mu,D} \\
& - J_A^\mu [\kappa^{AC}\mathcal{K}_{CD} + \kappa_{BD}B^{BA}] D^\gamma G_{\mu\gamma}^D + \frac{\delta\kappa_{AB}}{\delta\phi^2}\phi^I(D_\mu\phi^I)B^{AD}J_{\nu,D}G^{\mu\nu,B} \\
& + f^{DIJ}\kappa^{AC}\mathcal{K}_{CD}B^{IF}J_{\gamma,F}J_{\mu,A}G_{\mu\gamma}^J + 6F^{ABC}B^{AD}(D_\mu J_{\nu,D})G_{\nu\rho}^BG_{\rho\mu}^C \\
& - \frac{1}{2}\kappa_{AB}B^{AD}B^{BF}D_\mu J_{\nu,D}[D_\mu J_{\nu,F} - D_\nu J_{\mu,F}] + \frac{1}{2}\kappa_{AB}f^{ACD}B^{CI}B^{DJ}J_{\mu,I}J_{\nu,J}G^{\mu\nu,B} \\
& - \kappa^{AC}\mathcal{K}_{CD}B^{DF}J_A^\mu D^\nu [D_\mu J_\nu^F - D_\nu J_\mu^F] + f^{DIJ}B^{IF}\kappa^{AC}\mathcal{K}_{CD}J_A^\mu J_{\gamma,F}G_{\mu\gamma}^F.
\end{aligned} \tag{165}$$

We eliminate the DG operator form by using $\bar{\mathcal{K}}^{AD} = \kappa^{AC}\mathcal{K}_{CF}\kappa^{FD}$. Again, the dimension-eight terms belong to the derivative expansion for four- and higher-point functions and we can neglect them. Finally we obtain:

$$\mathcal{L} = -\frac{1}{4}\kappa_{AB}G_{\mu\nu}^AG^{\mu\nu,B} + i\bar{\psi}\not{D}\psi + F^{ABC}G_{\mu\nu}^AG_{\nu\rho}^BG_{\rho\mu}^C + \bar{\mathcal{K}}^{AD}J_A^\mu J_{\mu,D} + \dots \quad (166)$$

where $\bar{\mathcal{K}}^{AF} = \kappa^{AC}\mathcal{K}_{CD}\kappa^{DF} = -B^{AF}$.

At each order, we can redefine Wilson coefficients such that $\bar{\mathcal{K}}$ has the same form as $\mathcal{K}$. Consider, we expand $\bar{\mathcal{K}}$ to dimension eight, obtaining

$$\kappa^{AB}\mathcal{K}_{BC}\kappa^{CD} = c_{DG}^{(6)}\delta^{AD} + \left(c_{DG}^{(8)} + 8c_{HG}^{(6)}c_{DG}^{(6)}\right)\left(\frac{\phi^2}{2}\right)\delta^{AD}. \quad (167)$$

Taking $c_{DG}^{(8)} \longrightarrow c_{DG}^{(8)} - 8c_{HG}^{(6)}c_{DG}^{(6)}$ we obtain

$$\bar{\mathcal{K}}^{AB} = \sum_{n=0}^{\infty} c_{DG}^{(6+2n)} \left(\frac{\phi^2}{2}\right)^n \delta^{AB}. \quad (168)$$

We see that we can achieve the same form of $\kappa^{AB}K_{BC}\kappa^{CD}$ as K_{AD} , this holds to all orders.

We have thus shown that we can make the universal SMEFT consistent with the standard geoSMEFT classification of two-and three-point functions. We eliminate the $(DG)^2$ operator – removed by the standard geoSMEFT classification – in favor of a four-fermion type operator.

5.2 Equation of Motion

The same result as in Eq. (163) can be obtained by neglecting dimension-eight and higher operators and applying the equations of motion:

$$D_\mu G^{\nu\mu,A} = \kappa^{AC} \frac{\delta\kappa_{CB}}{\delta\phi^2} \phi^i (D_\mu \phi^I) G^{\mu\nu,B} - \kappa^{AC} J_C^A \quad (169)$$

From there we obtain the relations:

$$\mathcal{K}_{AB}(D_\mu G^{\nu\mu,A})(D_\gamma G^{\nu\gamma,B}) = -\mathcal{K}_{AB}\kappa^{AC} J_{\nu,C}(D_\gamma G^{\nu\gamma,B}) \quad (170)$$

$$-\mathcal{K}_{AB}\kappa^{AC} J_{\nu,C}(D_\gamma G^{\nu\gamma,B}) = \kappa^{DB}\mathcal{K}_{BA}\kappa^{AC} J_{\nu,C}J_{\nu,D} \quad (171)$$

We have neglected dimension-eight and higher operator forms. With these relations we can replace the higher derivative operator of Eq. (150) with the four fermion operator $(\bar{\mathcal{K}}JJ)$ of Eq. (166).

We also obtain the same result by adding the equation of motion explicitly, i.e.

$$0 = \frac{\delta \mathcal{L}}{\delta G_\rho^D} X_\rho^D + \frac{\delta \mathcal{L}}{\delta \partial_\sigma G_\rho^D} \partial_\sigma X_\rho^D. \quad (172)$$

From the evaluation of Eq. (172) we obtain

$$0 = -\kappa_{AB} G_{\mu\nu}^A D_\mu X_\nu^B - \bar{\psi} \gamma^\rho \tilde{T}^D \psi X_\rho^D = -\kappa_{AB} G_{\mu\nu}^A D_\mu X_\nu^B - J_A^\mu X_\mu^A \quad (173)$$

Adding Eq. (173), using dimension-six terms only, to the Lagrangian and choosing $X_\rho^D = C_E^D D^\mu G_{\rho\mu}^E$ gives

$$\mathcal{L} = -\frac{1}{4} \kappa_{AB} G_{\mu\nu}^A G^{\mu\nu,B} + \mathcal{K}_{AB} D_\mu G_\nu^A D_\rho G_{\nu\rho}^B + i\bar{\psi} \not{D} \psi - \kappa_{AB} G_{\mu\nu}^A D_\mu X_\nu^B - \bar{\psi} \gamma^\rho \tilde{T}^D \psi X_\rho^D \quad (174)$$

$$= -\frac{1}{4} \kappa_{AB} G_{\mu\nu}^A G^{\mu\nu,B} + i\bar{\psi} \not{D} \psi + (\mathcal{K}_{AB} \delta_{EB} - \kappa_{AB} C_E^B) (D^\mu G_{\rho\mu}^A) (D^\sigma G_{\rho\sigma}^E) - J_A^\mu X_\mu^A. \quad (175)$$

The choice $C_E^D = \kappa^{DA} \mathcal{K}_{AE}$ removes the operator of the form $(DG)^2$, yielding

$$\mathcal{L} = -\frac{1}{4} \kappa_{AB} G_{\mu\nu}^A G^{\mu\nu,B} + i\bar{\psi} \not{D} \psi - J_A^\mu \kappa^{AG} \mathcal{K}_{GE} (D^\sigma G_{\mu\sigma}^E). \quad (176)$$

We have used the notation that $\kappa^{AB} = (\kappa_{AB})^{-1}$ and $\kappa^{AB} \kappa_{BC} = \delta_C^A$. Applying the EOM again, choosing $X_\mu^A = B^{AD} J_{\mu,D}$ yields:

$$\mathcal{L} = -\frac{1}{4} \kappa_{AB} G_{\mu\nu}^A G^{\mu\nu,B} + i\bar{\psi} \not{D} \psi - J_A^\mu (\kappa^{AG} \mathcal{K}_{GE} + B^{BA} \kappa_{BE}) (D^\sigma G_{\mu\sigma}^E) - J_A^\mu B^{AD} J_{\mu,D}. \quad (177)$$

Choosing $B^{AF} = -\kappa^{AG} \mathcal{K}_{GE} \kappa^{EF}$ removes the $(\bar{\psi}\psi)(DG)$ operator and we finally obtain

$$\mathcal{L} = -\frac{1}{4} G_{\mu\nu}^A G^{\mu\nu,B} + i\bar{\psi} \not{D} \psi - J_A^\mu \kappa^{AC} \mathcal{K}_{CF} \kappa^{FD} J_{\mu,D} \quad (178)$$

$$= -\frac{1}{4} G_{\mu\nu}^A G^{\mu\nu,B} + i\bar{\psi} \not{D} \psi + \bar{\mathcal{K}}^{AD} J_A^\mu J_{\mu,D}, \quad (179)$$

$$(180)$$

where we again have defined $\bar{\mathcal{K}}^{AD} = \kappa^{AC} \mathcal{K}_{CF} \kappa^{FD}$. This shows that, for our purposes, the EOM are sufficient.

In Sec. 6 we will use exclusively the EOM approach for the full universal electroweak geoSMEFT, as we have demonstrated the EOM are sufficient.

That the EOM are sufficient. This can also be seen due to the of the following:

Suppose we take

$$\phi \longrightarrow \phi + \lambda G(\phi) \equiv F(\phi). \quad (181)$$

Taylor expanding in λ yields

$$S[\phi] \longrightarrow S[F(\phi)]|_{\lambda \rightarrow 0} + \frac{\delta S[F(\phi)]}{\delta F} \frac{\delta F}{\delta \lambda} \Big|_{\lambda \rightarrow 0} \lambda + \frac{\delta^2 S[F(\phi)]}{\delta F^2} \left(\frac{\delta F}{\delta \lambda} \right)^2 \Big|_{\lambda \rightarrow 0} \frac{\lambda^2}{2} + \dots, \quad (182)$$

where these expressions hold:

$$F|_{\lambda \rightarrow 0} = \phi \quad (183)$$

$$S[F(\phi)]|_{\lambda \rightarrow 0} = S[\phi] \quad (184)$$

$$\frac{\delta F}{\delta \lambda} = G(\phi). \quad (185)$$

From there we obtain:

$$S[\phi] \longrightarrow S[\phi] + \lambda \frac{\delta S[\phi]}{\delta \phi} G + \frac{\lambda^2}{2} \frac{\delta^2 S[\phi]}{\delta \phi^2} G^2 + \dots \quad (186)$$

Treating ϕ and $D_\mu \phi$ as separate variables yields (to order λ):

$$S[\phi, D_\mu \phi] \longrightarrow S[\phi, D_\mu \phi] + \lambda \frac{\delta S[\phi, D_\mu \phi]}{\delta \phi} G + \lambda \frac{\delta S[\phi, D_\mu \phi]}{\delta(D_\mu \phi)} D_\mu G + \dots \quad (187)$$

$$= S[\phi, D_\mu \phi] + \lambda \left(\frac{\delta S[\phi, D_\mu \phi]}{\delta \phi} - D_\mu \frac{\delta S[\phi, D_\mu \phi]}{\delta(D_\mu \phi)} \right) G + \dots \quad (188)$$

Where in the last equivalence we have used integration by parts to move the covariant derivative onto the variation of the action. At leading order in the expansion in λ , the use of the EOM is justified. If $\lambda \approx \frac{1}{\Lambda^2}$, then the EOM relates dimension-six operators. Going further in the expansion, i.e. λ^2 , one gets the EOM equation *and* additional terms. The EOM is no longer applicable. These extra terms correspond precisely to higher derivatives, which we neglect in the geoSMEFT.

6 Inclusion of the Full Universal Electroweak geoSMEFT

The full effective Lagrangian of the electroweak universal geoSMEFT at dimension-six is given by

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}g_{AB}W_{\mu\nu}^AW^{\mu\nu,B} + \frac{1}{2}h_{IJ}(D_\mu\phi)^I(D^\mu\phi)^J + i\bar{\psi}\not{D}\psi - V - J_{H,J}\phi^J \\ & + Y_{AB}(D^\mu W_{\nu\mu}^A)(D^\rho W_{\nu\rho}^B) + H_{IJ}(D^2\phi)^I(D^2\phi)^J + \kappa_{IJ}^A(D^\mu\phi)^I(D^\nu\phi)^J W_{\mu\nu,A} + F^{ABC}W_{\mu\nu}^AW_{\nu\rho}^BW_{\rho\mu}^C. \end{aligned} \quad (189)$$

We will now remove the Y and H operator forms, just as in our QCD example, such that the propagators are again not modified. The potential V is defined as

$$V = \lambda \left(\frac{\phi^2}{2} - \frac{v^2}{2} \right)^2 + \sum_{n=0}^{\infty} c_H^{6+2n} \left(\frac{\phi^2}{2} \right)^{3+n}, \quad (190)$$

the Higgs current as

$$J_{H,J} = \bar{L}^i Y_e e M_{iJ} + \bar{Q}^i Y_d d M_{iJ} + \bar{Q}^i Y_u u \tilde{M}_{iJ} + h.c. \quad (191)$$

and the following matrices are employed in order to write the Yukawa couplings in terms of ϕ :

$$M_{iJ} = \frac{1}{\sqrt{2}} \begin{pmatrix} i & 1 & 0 & 0 \\ 0 & 0 & i & 1 \end{pmatrix} \quad \tilde{M}_{iJ} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & i & 1 \\ i & 1 & 0 & 0 \end{pmatrix} \quad (192)$$

The definitions of the various metrics employed in the geoSMEFT[9] can be found in App. D.

We need the following definitions and relations:

$$(D^\mu\phi^I) = \left(\partial^\mu\phi^I - \frac{1}{2}W^{\mu,A}\tilde{\gamma}_{A,J}^I\phi^J \right) \quad (193)$$

$$\frac{\delta D^\mu\phi^I}{\delta W_\rho^D} = -\frac{1}{2}\tilde{\gamma}_{D,J}^I\phi^J \quad (194)$$

The W EOM is:

$$D_\mu W^{\nu\mu,A} = -\frac{1}{2}g^{AB}h_{IJ}(D^\nu\phi^I)\tilde{\gamma}_{BK}^J\phi^K - g^{AB}J_B^\nu, \quad (195)$$

with current J defined as:

$$(J_W^\mu)_A = (J_W^\mu)^A = \begin{cases} \frac{g_2}{2} \bar{\psi} \gamma^\mu \sigma^A \psi, & A = 1, 2, 3 \\ g_1 Y_\psi \bar{\psi} \gamma^\mu \psi, & A = 4. \end{cases} \quad (196)$$

With these definitions we obtain a relation for the $Y(DW)^2$ term of Eq. (189)

$$\begin{aligned} Y_{AB}(D^\mu W_{\nu\mu}^A)(D^\rho W_{\nu\rho}^B) &= [gYg]^{CM} [h(D^\nu \phi) \tilde{\gamma} \phi]_C J_{\nu,M} + [gYg]^{MC} J_M^\nu J_{\nu,C} \\ &+ \frac{1}{4} [gYg]^{CD} [h(D^\nu \phi) \tilde{\gamma} \phi]_C [h(D_\nu \phi) \tilde{\gamma} \phi]_D, \end{aligned} \quad (197)$$

where we used a shorthand notation:

$$[gYg]^{CD} \equiv g^{CA} Y_{AB} g^{BD}, \quad (198)$$

$$[h(D^\nu \phi^I) \tilde{\gamma} \phi]_C \equiv [h_{IJ}(D^\nu \phi^I) \tilde{\gamma}_C^J \phi^K]. \quad (199)$$

Now we absorb the g dependence into Y and take $gYg \rightarrow Y$ in complete analogy with $\mathcal{K}_{AB}$. With that we can eliminate the operator form $Y(DW)^2$ yielding

$$\mathcal{L} = -\frac{1}{4} g_{AB} W_{\mu\nu}^A W^{\mu\nu,B} + \frac{1}{2} h_{IJ} (D_\mu \phi)^I (D^\mu \phi)^J + i \bar{\psi} \not{D} \psi - V - J_{H,K} \phi^K \quad (200)$$

$$+ H_{IJ} (D^2 \phi)^I (D^2 \phi)^J + \kappa_{IJ}^A (D^\mu \phi)^I (D^\nu \phi)^J W_{\mu\nu}^A + F^{ABC} W_{\mu\nu}^A W_{\nu\rho}^B W_{\rho\mu}^C \quad (201)$$

$$+ Y^{AB} [J_A^\nu + [h(D^\nu \phi) \tilde{\gamma} \phi]_A] J_{\nu,B} \quad (202)$$

Next we will remove the H operator form using the Higgs EOM. The Higgs EOM is:

$$(D^2 \phi)^I = h^{IJ} \left[\frac{1}{2} \phi_J \frac{\delta h_{KM}}{\delta \phi^2} (D_\mu \phi)^K (D_\mu \phi)^M - 2 \phi_J \frac{\delta V}{\delta \phi^2} - J_{H,J} \right] \quad (203)$$

From there we find

$$H_{MN} (D^2 \phi)^M (D^2 \phi)^N = \left[\frac{\delta V}{\delta \phi^2} \phi^C - J_{H,C} \right] (h^{CM} H_{MN} h^{NK}) \left[\frac{\delta V}{\delta \phi^2} \phi^K - J_{H,K} \right] \quad (204)$$

$$= H^{CK} \left[\left(\frac{\delta V}{\delta \phi^2} \right)^2 \phi^C \phi^K + 4 \frac{\delta V}{\delta \phi^2} \phi^C J_{H,K} + J_{H,C} J_{H,K} \right]. \quad (205)$$

In Eq. (205) we have taken $hHh \rightarrow H$. The expression

$$\mathcal{V} \equiv H^{CK} \left(\frac{\delta V}{\delta \phi^2} \right)^2 \phi^C \phi^K \quad (206)$$

can be absorbed into the definition of V . The redefinitions of Y and H are discussed in App. E. As in the QCD example this amounts to a redefinition of the Wilson coefficients.

Thus, we finally obtain the effective Lagrangian

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} g_{AB} W_{\mu\nu}^A W^{\mu\nu,B} + \frac{1}{2} h_{IJ} (D_\mu \phi)^I (D^\mu \phi)^J + i \bar{\psi} \not{D} \psi - V - J_{H,K} \phi^K \\ & + \kappa_{IJ}^A (D_\mu \phi)^I (D_\nu \phi)^J G_{\mu\nu}^A + F^{ABC} G_{\mu\nu}^A G_{\nu\rho}^B G_{\rho\mu}^C \\ & + Y^{MC} [J_M^\nu + [h(D^\nu \phi) \tilde{\gamma} \phi]_M] J_{\nu,C} + H^{CK} \left[4 \frac{\delta V}{\delta \phi^2} \phi^C + J_{H,C} \right] J_{H,K}. \end{aligned} \quad (207)$$

This Lagrangian of Eq. (207), together with the Lagrangian of the QCD sector of Eq. (150) defines the geometric universal SMEFT (geoUSMEFT). The next step is to work out the Feynman rules in terms of the geometric quantities, such as the metrics.

We can relate the result of Eq. (207) with the purely dimension six results found in [58]. We define:

$$\mathcal{O}_6 = \lambda |H|^6 \quad (208)$$

$$\mathcal{O}_H = \frac{1}{2} (\partial_\mu |H|^2)^2 \quad (209)$$

$$\mathcal{O}_y = |H|^2 (H_\alpha J_H^\alpha + \text{h.c.}) \quad (210)$$

$$\mathcal{O}_{2y} = J_{H\alpha}^\dagger J_H^\alpha \quad (211)$$

$$\mathcal{O}_{HJW} = \frac{ig}{2} (H^\dagger \sigma^a D_\mu H) J_a^\mu \quad (212)$$

$$\mathcal{O}_{2JW} = J_{W,\mu}^a J_W^{\mu,a} \quad (213)$$

$$\mathcal{O}_W = (H^\dagger \sigma^a \overleftrightarrow{D}_\mu H) D^\nu W_{\mu\nu}^a \quad (214)$$

$$Q_{H\Box} = |H|^2 \Box |H|^2 \quad (215)$$

And:

$$\frac{1}{2} (H^\dagger \overleftrightarrow{D}_\mu H)^2 \leftrightarrow 2Q_{HD} - \frac{1}{2} Q_{H\Box} \quad (216)$$

We see:

$$\mathcal{O}_{K4} = |D^2 H|^2 \xrightarrow{\text{EoM}} \left(\frac{\lambda^2}{4} v^4 |H|^2 - 4\lambda^2 v^2 |H|^4 + 4\lambda \mathcal{O}_6 \right) - (\lambda v^2 (H_\alpha J_y^\alpha + \text{h.c.}) - 2\lambda \mathcal{O}_y) + \mathcal{O}_{2y}. \quad (217)$$

We obtained the following relation:

$$H_{IJ}(D^2\phi)^I(D^2\phi)^J = V + 4H^{CK}\frac{\delta V}{\delta\phi^2}\phi^C + J_C H^{CK} J_K. \quad (218)$$

Eq. (217) can be compared with Eq. (218). The first parenthesis corresponds to V . The second parenthesis corresponds to the shifted Yukawa couplings, and the last term corresponds to the four-fermion operator going as J_H^2 .

We also have:

$$\mathcal{O}_{2W} = \left(\frac{1}{2}g^2\lambda v^2|H|^4 + g^2\mathcal{O}_6\right) + \left(-\frac{3}{4}g^2\mathcal{O}_H - \frac{3}{4}g^2Q_{H\Box} - 2g^2\lambda Q_H\right) - \left(\frac{1}{4}g^2\mathcal{O}_y\right) - Q_{HJW} - \left(\frac{1}{2}\mathcal{O}_{2JW}\right) \quad (219)$$

We can relate Eq. (219) with our result:

$$Y_{AB}(D^\mu W_{\nu\mu}^A)(D^\rho W_{\nu\rho}^B) = [gYg]^{CM}[h(D^\nu\phi)\tilde{\gamma}\phi]_C J_{\nu,M} + \frac{1}{4}[gYg]^{CD}[h(D^\nu\phi)\tilde{\gamma}\phi]_C[h(D_\nu + \phi)\tilde{\gamma}\phi]_D + [gYg]^{MC}J_M^\nu J_{\nu,C}. \quad (220)$$

The second parenthesis corresponds to the two-derivative operator form and Q_{HJW} to our first term, which can be thought of as interactions coupled to the SM via the current $J_{\nu,M}^W$. $\mathcal{O}_{2JW}$ corresponds to the four-fermion operator. The first and third parentheses do not appear in our result. These terms contribute to potential and Yukawa terms. They are missed, since we retain the third $H^4 D^2$ operator, whereas most bases replace this via the EOM with $\mathcal{O}_y$ and shifts to the potential.

7 Feynman Rules

We gauge fix the geoUSMEFT using the background field method following [8, 59] and Sec. 4.2. We illustrate the determination of the Feynman rules with an example. Our goal is to derive the rules in terms of the geometric expressions (=metrics). We will expand in orders of field insertions by expanding the metric to classify all possible Feynman rules contributing to an n -point function. This can be done in the following way:

1. Operator forms result differently in the number of field insertions. For example, the operator form $(D_\mu \phi)^I (D^\mu \phi)^J$ will result in at least two fields, while the operator form $G_{\mu\nu}^A G_{\nu\rho}^B G_{\rho\mu}^C$ results in at least three.
2. For a two-point function, one needs at most four fields. Thus, for the first operator form in (1) we expand h_{ij} to include at most two fields. For the second operator of (1) we expand F_{ABC} to include at most one field.
3. We can then derive the Feynman rules in terms of geometric quantities such as the metric.

In the next section we provide an explicit example. We explore the gluon two-point function at one-loop. This calculation shows the development of Feynman rules and the results. The same principles are then applied to the full electroweak theory.

7.1 The Transverse Gluon

Due to the unbroken structure of the $SU(3)_C$ symmetry, the gluon remains transverse and we have

$$p^\mu \frac{\delta^2 \Gamma}{\delta \hat{G}_\mu^A(p) \delta \hat{G}_\nu^B(-p)} = 0. \quad (221)$$

Thus the longitudinal part, as defined in Sec. 4.4 vanishes. Moreover the transverse part also vanishes at zero momentum transfer, i.e. $\Sigma_T^{\hat{G}\hat{G}}(0) = 0$.

The Lagrangian contributing to the gluon two-point function is

$$\mathcal{L} = -\frac{1}{4} \kappa_{AB} G_{\mu\nu}^A G^{\mu\nu,B} + i \bar{\psi} \not{D} \psi + F^{ABC} G_{\mu\nu}^A G_{\nu\rho}^B G_{\rho\mu}^C + \mathcal{K}^{AB} J^{\mu,A} J_\mu^B. \quad (222)$$

The first three term terms contribute to the two-point function at one-loop order. So we expand κ , see App. F.1 for additional details:

$$\begin{aligned} \kappa = & \langle \kappa \rangle + \langle \kappa_R \rangle \sqrt{h^{44}} H + \langle \kappa_{RR} \rangle \langle h^{44} \rangle H^2 \\ & + \langle \kappa_{RA} \rangle [2 \langle h^{11} \rangle \omega^+ \omega^- + \langle h^{33} \rangle \omega_0^2 + \langle h^{44} \rangle H^2] + \dots \end{aligned} \quad (223)$$

The subscript R (A) stands for the radial (angular) component of the metric. The radial component includes interactions with the goldstone bosons. Notice that there are no $\omega^+\omega^+$ or $\omega^-\omega^-$ terms as they would violate QED. Both terms appear, each with opposite signs, and thus they cancel each other. Now we are ready to determine the Feynman rules:

$$\begin{pmatrix} \hat{G}_1 \\ \hat{G}_2 \\ H \end{pmatrix} = -i \langle \kappa_R \rangle \langle \kappa \rangle^{-1} \sqrt{h^{44}} (p_1^{\mu_2} p_2^{\mu_1} - p_1 \cdot p_2 \eta^{\mu_1 \mu_2}), \quad (224)$$

$$\begin{pmatrix} G_1 \\ \hat{G}_2 \\ \omega^+ \\ \omega^- \end{pmatrix} = -2i \langle \kappa_{RA} \rangle \langle h^{11} \rangle \langle \kappa \rangle^{-1} (p_1^{\mu_2} p_2^{\mu_1} - p_1 \cdot p_2 \eta^{\mu_1 \mu_2}), \quad (225)$$

$$\begin{pmatrix} \hat{G}_1 \\ \hat{G}_2 \\ \omega^0 \\ \omega^0 \end{pmatrix} = -2i \langle \kappa_{RA} \rangle \langle h^{33} \rangle \langle \kappa \rangle^{-1} (p_1^{\mu_2} p_2^{\mu_1} - p_1 \cdot p_2 \eta^{\mu_1 \mu_2}), \quad (226)$$

$$\begin{pmatrix} \hat{G}_1 \\ \hat{G}_2 \\ H \\ H \end{pmatrix} = -2i [\langle \kappa_{RA} \rangle + \langle \kappa_{RR} \rangle] \langle h^{44} \rangle \langle \kappa \rangle^{-1} \langle \kappa \rangle^{-2} (p_1^{\mu_2} p_2^{\mu_1} - p_1 \cdot p_2 \eta^{\mu_1 \mu_2}). \quad (227)$$

The relevant Feynman diagrams contributing to the gluon two-point function are shown in Fig. 6.

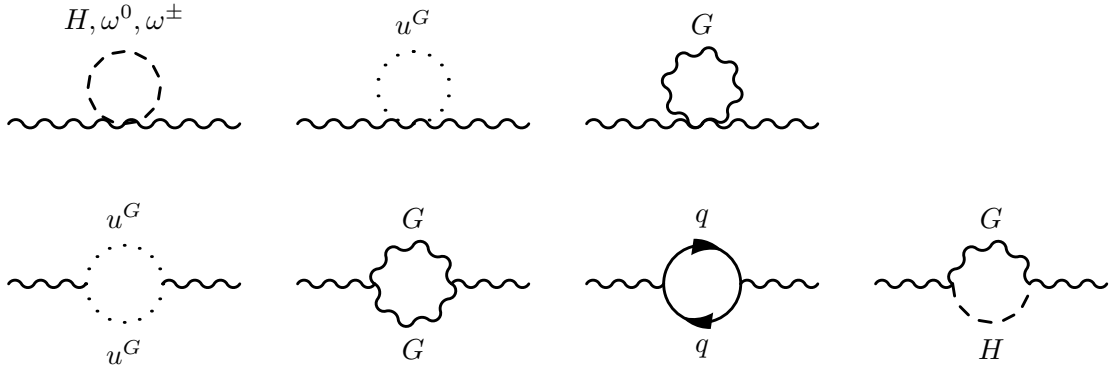


Figure 6: Diagrams contributing to the gluon two-point function

Where in Fig. 6 ω are the goldstone bosons and u^G are the ghosts. The above rules result purely from the metric expansion. The rules for gauge fixing and ghosts are not included here. The rules for gauge fixing and ghost terms can be found in [8, 59].

The gauge fixing parameter in the QCD sector is denoted by ξ_G and in the electroweak sector by ξ_W . We use again FeynRules[60], FeynArts[56], FormCalc[57] and Package X[55].

We work in $d = 4 - 2\epsilon$ dimensions and the pole is written compactly as

$$\frac{1}{\tilde{\epsilon}} = \frac{1}{\epsilon} - \gamma_E + \ln(4\pi), \quad (228)$$

where γ_E is the Euler-Mascheroni constant.

The first three bubble diagrams of Fig. 6 are transverse on their own and yield

$$\Sigma_{\tilde{G}\tilde{G}}^S = -\frac{p^2 N_C}{3 \cdot 16\pi^2} \langle \kappa^{-1} \rangle \left[\langle h^{33} \rangle \langle \kappa_{RA} \rangle \left(\frac{\xi_W \bar{m}_Z^2}{\tilde{\epsilon}} + A_0^F [\xi_W \bar{m}_Z^2] \right) + 2 \langle h^{11} \rangle \langle \kappa_{RA} \rangle \left(\frac{\xi_W \bar{m}_W^2}{\tilde{\epsilon}} + A_0^F [\xi_W \bar{m}_W^2] \right) \right. \quad (229)$$

$$\left. + \langle h^{44} \rangle (\langle \kappa_{RR} \rangle + \langle \kappa_{RA} \rangle) \left(\frac{\bar{m}_H^2}{\tilde{\epsilon}} + A_0^F [\bar{m}_H^2] \right) \right]. \quad (230)$$

Where the finite part of the scalar A_0 is defined as:

$$A_0^F(\bar{m}^2) = \bar{m}^2 \left(1 + \log \left[\frac{\mu^2}{\bar{m}^2} \right] \right). \quad (231)$$

As we work in dimensional regularization, the scaleless ghost and gluon tadpole-like diagrams vanish exactly. The sum of the ghost and gluon loop diagrams gives:

$$\begin{aligned} \Sigma_{u,G}^{\tilde{G}\tilde{G}} = & -\frac{\bar{g}_3^2 N_c}{16\pi^2} p^2 \left[\frac{11}{3\tilde{\epsilon}} - \frac{59 - 54\xi_G + 9\xi_G^2}{36} + \frac{11}{3} B_0^F(p^2, 0, 0) \right] \\ & + \frac{\langle \kappa \rangle^{-3/2} N_c}{16\pi^2} \langle F \rangle \left[2\bar{g}^3 p^4 + \langle \kappa \rangle^{-3/2} \langle F \rangle p^6 \right] \left[\frac{3}{\tilde{\epsilon}} - 1 + 3B_0^F(p^2, 0, 0) \right]. \end{aligned} \quad (232)$$

The finite part of the scalar B function is given by:

$$B_0^F(p^2, \bar{m}_1^2, \bar{m}_2^2) = 2 + \text{Disc}[p^2, \bar{m}_1^2, \bar{m}_2^2] + \frac{p^2 + \bar{m}_1^2 - \bar{m}_2^2}{2p^2} \log\left(\frac{\mu^2}{\bar{m}_1^2}\right) + \frac{p^2 + \bar{m}_2^2 - \bar{m}_1^2}{2p^2} \log\left(\frac{\mu^2}{\bar{m}_2^2}\right), \quad (233)$$

$$\text{Disc}[p^2, m_1^2, m_2^2] = \frac{\sqrt{\lambda(m_1, m_2, p^2)}}{p^2} \log\left(\frac{m_1^2 + m_2^2 - p^2 + \sqrt{\lambda(m_1, m_2, p^2)}}{2m_1 m_2}\right), \quad (234)$$

$$\lambda(m_1, m_2, p^2) = (p^2 - (m_1 + m_2)^2) (p^2 - (m_1 - m_2)^2). \quad (235)$$

The ghost and gluon loops are transverse and therefore their sum is also transverse.

The result for the fermionic loop is given by:

$$\Sigma_q^{\hat{G}\hat{G}} = \frac{\bar{g}_3^2 N_c}{16\pi^2} \left[\frac{2p^2}{9\tilde{\epsilon}} - \frac{2p^2 - 12\bar{m}_q^2}{27} - \frac{12}{27} A_0^F(\bar{m}_q^2) + \frac{2}{9} (p^2 + 2\bar{m}_q^2) B_0^F(p^2, \bar{m}_q^2, \bar{m}_q^2) \right] \quad (236)$$

$$= \frac{\bar{g}_3^2 N_c}{16\pi^2} \left[\frac{2p^2}{9} \left(\frac{1}{\tilde{\epsilon}} + \log \frac{\mu^2}{\bar{m}_q^2} \right) + \frac{10p^2 + 24\bar{m}_q^2}{27} + \frac{2p^2 + 4\bar{m}_q^2}{9} \text{Disc}[p^2, \bar{m}_q^2, \bar{m}_q^2] \right]. \quad (237)$$

The last diagram of Fig. 6, the Higgs-gluon loop diagram, is purely transverse and yields:

$$\begin{aligned} \Sigma_{HG}^{\hat{G}\hat{G}} &= \langle \kappa_4 \rangle^2 \langle h^{44} \rangle \langle \kappa \rangle^{-2} \frac{N_c}{16\pi^2} \left[\frac{p^2 + 3\bar{m}_H^2}{18\tilde{\epsilon}} - \frac{p^2 - 3\bar{m}_H^2}{54} + \frac{5p^2 - \bar{m}_H^2}{18} A_0^F(\bar{m}_H^2) \right. \\ &\quad \left. + \frac{(p^2 - \bar{m}_H^2)^2}{18} B_0^F(p^2, 0, \bar{m}_H^2) \right]. \end{aligned} \quad (238)$$

After expanding out the scalar functions we obtain:

$$\begin{aligned} \Sigma_{HG}^{\hat{G}\hat{G}} &= \langle \kappa_4 \rangle^2 \langle h^{44} \rangle \langle \kappa \rangle^{-2} \frac{N_c}{16\pi^2} \left[\frac{p^2 + 3\bar{m}_H^2}{18} p^2 \left(\frac{1}{\tilde{\epsilon}} + \log \frac{\mu^2}{\bar{m}_H^2} \right) \right. \\ &\quad \left. + \frac{5p^4 + 6\bar{m}_H^2 p^2 + 3\bar{m}_H^4}{54} + \frac{(p^2 - \bar{m}_H^2)^3}{18p^2} \log \left(\frac{\bar{m}_H^2}{\bar{m}_H^2 - p^2} \right) \right]. \end{aligned} \quad (239)$$

7.2 Organizing the electroweak sector

The expansion of the metrics – needed to generate Feynman rules – in the electroweak sector is much more evolved. In the following discussion, we only consider the $c_{H\Box}^{(6+2n)}$ component of the metric h_{ij} and ignore the other components. The metric h then has the form

$$h_{IJ} = -2 \sum_{n=0}^{\infty} \left(\frac{\phi^2}{2} \right)^n \phi_I \phi_J c_{H\Box}^{(6+2n)} + \dots \quad (240)$$

After taking a variation with respect to Φ_k and taking the expectation value we obtain

$$\left\langle \frac{\delta h_{IJ}}{\delta \Phi_K} \right\rangle = -2 \sum_{n=0}^{\infty} c_{H\Box}^{(6+2n)} \left(\frac{v^2}{2} \right)^n \left[2nv \delta_{I4} \delta_{J4} \frac{\delta \phi_4}{\delta \Phi_K} + v \left(\delta_{I4} \frac{\delta \phi_J}{\delta \Phi_K} + \delta_{J4} \frac{\delta \phi_I}{\delta \Phi_K} \right) \right] + \dots \quad (241)$$

As in the κ case, the first term in Eq. (241) probes the radial component of ϕ , whereas the second term probes the angular component. This in mind, we rewrite h as

$$\left\langle \frac{\delta h_{IJ}}{\delta \Phi^K} \right\rangle \Phi^K = \langle h_{R2} \rangle (T_{R2})_{IJ} + \langle h_{A2} \rangle (T_{A2})_{IJ} + \dots. \quad (242)$$

Again, R (A) stands for the radial (angular) component. The latter includes structure with goldstone boson.

The matrices T then contain the field dependence including the rescaling to the weak eigenstates in the from of $\sqrt{h}^{IJ}$. In this example we have:

$$\langle h_{R2} \rangle = - \sum_{n=0}^{\infty} 4n v c_{H\Box}^{(6+2n)} \left(\frac{v^2}{2} \right)^n, \quad (243)$$

$$\langle h_{A2} \rangle = - \sum_{n=0}^{\infty} 2v c_{H\Box}^{(6+2n)} \left(\frac{v^2}{2} \right)^n, \quad (244)$$

$$T_{R2} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{h^{44}} H \end{pmatrix}, \quad (245)$$

$$T_{A2} = \begin{pmatrix} 0 & 0 & 0 & -i\sqrt{h^{11}} \frac{\omega^+ - \omega^-}{\sqrt{2}} \\ * & 0 & 0 & \sqrt{h^{11}} \frac{\omega^+ + \omega^-}{\sqrt{2}} \\ * & * & 0 & -\sqrt{h^{33}} \omega^0 \\ * & * & * & 2\sqrt{h^{44}} H \end{pmatrix}. \quad (246)$$

T_{A2} is symmetric and we only display the upper triangular part.

Following this procedure for the full h metric we have

$$\left\langle \frac{\delta h_{IJ}}{\delta \Phi^K} \right\rangle \cdot \Phi^K = \langle h_{R1} \rangle (T_{R1})_{IJ} + \langle h_{R2} \rangle (T_{R2})_{IJ} + \langle h_{A2} \rangle (T_{A2})_{IJ} + \langle h_{R3} \rangle (T_{R3})_{IJ} + \langle h_{A3} \rangle (T_{A3})_{IJ}. \quad (247)$$

The additional expectation structures are given by,

$$\langle h_{R1} \rangle = \sum_{n=0}^{\infty} (n+1) v \left(\frac{v^2}{2} \right)^n c_{HD,2}^{(6+2n)}, \quad (248)$$

$$\langle h_{A1} \rangle = 0, \quad (249)$$

$$\langle h_{R3} \rangle = \sum_{n=0}^{\infty} n v \left(\frac{v^2}{2} \right)^n c_{HD}^{(6+2n)}, \quad (250)$$

$$\langle h_{A3} \rangle = \frac{1}{2} \sum_{n=0}^{\infty} v \left(\frac{v^2}{2} \right)^n c_{HD}^{(6+2n)}, \quad (251)$$

and the matrix structures by

$$T_{R1} = \mathbf{1}\sqrt{h^{44}} H, \quad (252)$$

$$T_{R3} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 1 & 0 \\ * & * & * & 1 \end{pmatrix} \sqrt{h^{44}} H, \quad (253)$$

$$T_{A3} = \begin{pmatrix} 0 & 0 & -\sqrt{h^{11}} \frac{\omega^+ + \omega^-}{\sqrt{2}} & -i\sqrt{h^{11}} \frac{\omega^+ - \omega^-}{\sqrt{2}} \\ * & 0 & -i\sqrt{h^{11}} \frac{\omega^+ - \omega^-}{\sqrt{2}} & \sqrt{h^{11}} \frac{\omega^+ + \omega^-}{\sqrt{2}} \\ * & * & 2\sqrt{h^{44}} H & 0 \\ * & * & * & 2\sqrt{h^{44}} H \end{pmatrix}. \quad (254)$$

We also require the double variations of the metric and we define:

$$h_{IJ} = \langle h_{IJ} \rangle + \sum_{n=1}^3 \left[\sum_{r \in \{R, A\}} \langle h_{r,n} \rangle (T_{r,n}) + \frac{1}{2} \sum_{r,p \in \{R, A\}} \langle h_{rp,n} \rangle (T_{rp,n}) \right] \quad (255)$$

in this case the expressions quickly become cumbersome.

For the term independent of fields, we have

$$\langle h_{IJ} \rangle = \begin{pmatrix} \langle h_{11} \rangle & 0 & 0 & 0 \\ * & \langle h_{11} \rangle & 0 & 0 \\ * & * & \langle h_{33} \rangle & 0 \\ * & * & * & \langle h_{44} \rangle \end{pmatrix} \quad (256)$$

$$= \begin{pmatrix} 1 + \langle h_1 \rangle & 0 & 0 & 0 \\ * & 1 + \langle h_1 \rangle & 0 & 0 \\ * & * & 1 + \langle h_1 \rangle + \langle h_3 \rangle & 0 \\ * & * & * & 1 + \langle h_1 \rangle + \langle h_2 \rangle + \langle h_3 \rangle \end{pmatrix}. \quad (257)$$

The expectations are expressed as

$$\langle h_1 \rangle = \sum_{n=0}^{\infty} \left(\frac{v^2}{2} \right)^{n+1} c_{HD,2}^{(6+2n)}, \quad (258)$$

$$\langle h_2 \rangle = -4 \sum_{n=0}^{\infty} \left(\frac{v^2}{2} \right)^{n+1} c_{H\Box}^{(6+2n)} = v \langle h_{A2} \rangle = v^2 \langle h_{AA2} \rangle, \quad (259)$$

$$\langle h_3 \rangle = \sum_{n=0}^{\infty} \left(\frac{v^2}{2} \right)^{n+1} c_{HD}^{(6+2n)} = v \langle h_{A3} \rangle = 2v^2 \langle h_{AA3} \rangle. \quad (260)$$

For the terms quadratic in fields, the relevant expectations are given by

$$\langle h_{RR1} \rangle = \sum_{n=0}^{\infty} 2n(n+1) \left(\frac{v^2}{2} \right)^n c_{HD,2}^{(6+2n)}, \quad (261)$$

$$\langle h_{RR2} \rangle = -8 \sum_{n=0}^{\infty} n(n-1) \left(\frac{v^2}{2} \right)^n c_{H\Box}^{(6+2n)}, \quad (262)$$

$$\langle h_{RR3} \rangle = \frac{1}{4} \sum_{n=0}^{\infty} 4n(n-1) \left(\frac{v^2}{2} \right)^n c_{HD}^{(6+2n)}, \quad (263)$$

$$\langle h_{RA1} \rangle = \langle h_{AR1} \rangle = \frac{1}{v} \langle h_{R1} \rangle, \quad (264)$$

$$\langle h_{RA2} \rangle = \langle h_{AR2} \rangle = \frac{1}{v} \langle h_{R2} \rangle, \quad (265)$$

$$\langle h_{RA3} \rangle = \langle h_{AR3} \rangle = \frac{1}{2v} \langle h_{R3} \rangle, \quad (266)$$

$$\langle h_{AA1} \rangle = 0, \quad (267)$$

$$\langle h_{AA2} \rangle = \frac{1}{v^2} (\langle h_{44} \rangle - \langle h_{33} \rangle), \quad (268)$$

$$\langle h_{AA3} \rangle = \frac{1}{2v^2} (\langle h_{33} \rangle - \langle h_{11} \rangle). \quad (269)$$

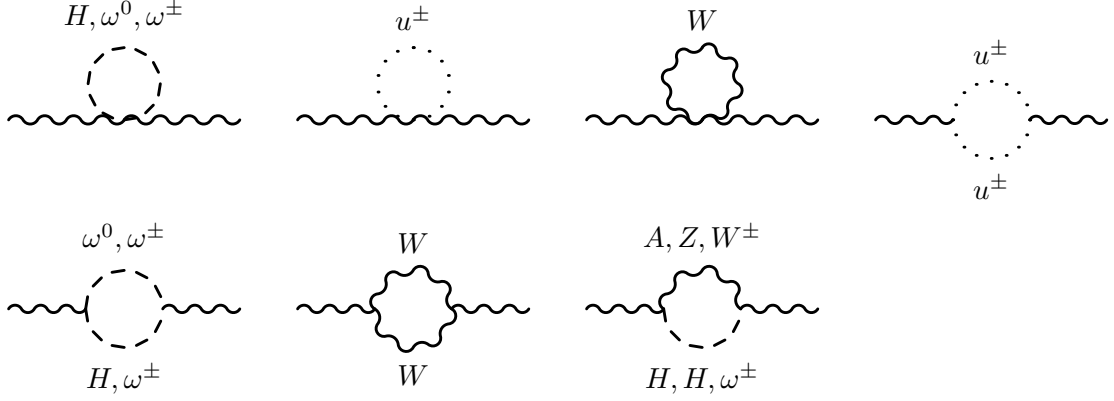


Figure 7: Diagrams contributing to the photon two-point function. When multiple labels are present in bubble diagrams, they should be understood as pairing upper and lower labels. For example, $\omega^0, \omega^\pm$ and $H, \omega^\pm$ should be interpreted as two separate diagrams: one with ω^0 and H running in the loop, and the other with charged Goldstone bosons. The remaining combinations violate charge conservation and therefore do not exist.

8 The Ward Identities in the Electroweak Sector

In this section we consider the Ward identities in the electroweak sector and analyze their dependence on geometric quantities. That the Ward identities are indeed fulfilled at one-loop and leading order in the SMEFT expansion has been shown in [10].

8.1 The Transverse Photon

As in the gluon case in Sec. 7.1, the photon is transverse due to the residual $U(1)_{\text{EM}}$ gauge symmetry. Therefore the longitudinal part vanishes

$$\Sigma_L^{\hat{A}\hat{A}}(p^2) = 0. \quad (270)$$

The relevant diagrams contributing can be found in Fig. 7. The full transverse part, however, is prohibitively large to write for this thesis. As such we give some meaningful expressions coming from the geometric perspective. Table 1 shows which combinations of the expectations of metric variations contribute.

Starting from the expectations of the metrics, i.e. with no variations applied, the UV-divergent part of these contributions take the form:

$$\begin{aligned}
\Sigma_{\hat{A}\hat{A}}^T \Big|_{\text{exp}} = & -\frac{p^2}{16\pi^2\tilde{\epsilon}} \left[\bar{e}^2 (2 - \xi_W + \langle g_{33} \rangle \langle g^{11} \rangle [4 + (1 + \xi_W) \langle g_{33} \rangle \langle g^{11} \rangle]) \right. \\
& + 2\bar{e}\bar{g}_2 \sqrt{g^{11}} \bar{c} \langle g_{34} \rangle \sqrt{g^{44}} (2 + \langle g_{33} \rangle \langle g^{11} \rangle [1 + \xi_W]) \\
& + 2\bar{e}\bar{g}_2 \sqrt{g^{11}} \bar{s} (2\sqrt{g_{44}} - 2\langle g_{44} \rangle \sqrt{g^{44}} + \langle g_{34} \rangle \langle g^{11} \rangle \sqrt{g_{34}} [1 + \xi_W]) \\
& \left. + \bar{g}_2^2 \langle g_{34} \rangle^2 \langle g^{11} \rangle (1 + \xi_W) \left(\bar{c} [\sqrt{g^{44}}]^2 + \bar{s} [(\sqrt{g^{34}})^2 - 2\sqrt{g^{33}} \sqrt{g^{44}}] \right) \right]. \quad (271)
\end{aligned}$$

In the SM limit $g_{ij} \rightarrow \delta_{ij}$ we recover the SM result

$$\Sigma_T^{\hat{A}\hat{A}} \Big|_{\text{exp}} \rightarrow -\frac{7e^2}{16\pi^2\tilde{\epsilon}} k^2 = \frac{g_1^2 g_2^2}{g_1^2 + g_2^2} \left(\frac{-7k^2}{16\pi^2\tilde{\epsilon}} \right). \quad (272)$$

In Eq. (271) we used the expressions from electroweak symmetry breaking:

$$e = g_1 \cos \theta_W, \quad (273)$$

$$e = g_2 \sin \theta_W, \quad (274)$$

$$\tan \theta_W = \frac{g_1}{g_2}, \quad (275)$$

$$e = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}. \quad (276)$$

The simplest additional contributions come from the $\langle g_{RRi} \rangle$ contributions. The UV-divergent part of these expressions is given by:

$$\Sigma_T^{AA} \Big|_{RR} = -\frac{\bar{e}^2}{16\pi^2} \frac{\bar{m}_H^2 \langle h \rangle^{44} p^2}{2g_1^2 g_2^2} [g_2^2 \langle g_{RR2} \rangle + g_1^2 (\langle g_{RR1} \rangle + \langle g_{RR3} \rangle) + 2g_1 g_2 \langle g_{RR4} \rangle].$$

The other contributions from the metric g 's quadratic dependence on fields gets contributions from the Higgs and goldstone bosons. The result is given by:

$$\Sigma_T^{\hat{A}\hat{A}} \Big|_{RA,AA} = -\frac{\bar{e}^2}{16\pi^2\tilde{\epsilon}} \frac{p^2}{g_1^2 g_2^2} \left(c_H^{\hat{A}\hat{A}} \bar{m}_H^2 \langle h^{44} \rangle + c_W^{\hat{A}\hat{A}} \xi_W \bar{m}_W^2 \langle h^{11} \rangle + c_Z^{\hat{A}\hat{A}} \xi_W \bar{m}_Z^2 \langle h^{33} \rangle \right). \quad (277)$$

The coefficients are given by:

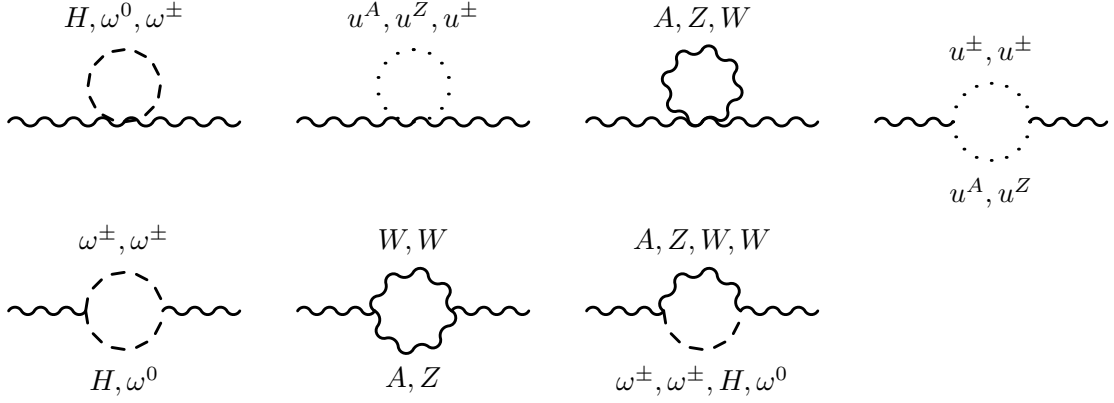


Figure 8: Diagrams contributing to the WW two-point function. The labeling is explained in Fig. 7

$$c_H^{\hat{A}\hat{A}} = \frac{1}{2} [g_2^2 \langle g_{RA2} \rangle + g_1^2 (12 \langle g_{AA3} \rangle + \langle g_{RA1} \rangle + 9 \langle g_{RA3} \rangle) + 2g_1 g_2 (2 \langle g_{AA4} \rangle + 5 \langle g_{RA4} \rangle)] , \quad (278)$$

$$c_W^{\hat{A}\hat{A}} = g_2^2 \langle g_{RA2} \rangle + g_1^2 (\langle g_{RA1} \rangle + \langle g_{RA3} \rangle - 4 \langle g_{AA3} \rangle) + 2g_1 g_2 (\langle g_{RA4} \rangle - 2 \langle g_{AA4} \rangle) , \quad (279)$$

$$c_Z^{\hat{A}\hat{A}} = \frac{1}{2} [g_2^2 \langle g_{RA2} \rangle + g_1^2 (4 \langle g_{AA3} \rangle + \langle g_{RA1} \rangle + \langle g_{RA3} \rangle) + 2g_1 g_2 (2 \langle g_{AA4} \rangle + \langle g_{RA4} \rangle)] . \quad (280)$$

Terms quadratic in the metric are not included in the current analysis. These quadratic terms induce higher momentum dependence in the poles, as in the gluon case (see Eq. (239)).

An example is the p^4 dependence of the $\langle g_{R1} \rangle^2$ contribution to the UV-divergent part:

$$\Sigma_T^{\hat{A}\hat{A}} \Big|_{(R1)^2} = (\text{long expression}) p^2 + \frac{\bar{e}^2}{16\pi^2 \bar{\epsilon}} \frac{p^4}{6g_2^2} \langle g_{R1} \rangle^2 \langle h_{44} \rangle \langle g_{33} \rangle . \quad (281)$$

8.2 W Bosons

The Ward identities for the charged bosons are more involved. The first relation connects the longitudinal W gauge-boson two-point function with the $W^\pm - \omega^\pm$ mixing. The relation is expressed as:

$$\Sigma_L^{\hat{W}^\pm \hat{W}^\mp}(p^2) \pm \bar{m}_W \Sigma^{\hat{\omega}^\pm \hat{W}^\mp}(p^2) = 0, \quad (282)$$

where we relate the mixed Σ to the effective action as:

$$\frac{\delta^2 \Gamma}{\delta \hat{\omega}^\pm \delta \hat{W}_\nu^\mp} = -i p_\nu \left[\pm \bar{m}_W + \Sigma^{\hat{\omega}^\pm \hat{W}^\mp}(p^2) \right] . \quad (283)$$

The relevant Feynman diagrams for the WW two-point function can be found in Fig. 8 and those for $W\omega^\pm$ can be found in Fig. 9.

The second identity

linear variations	quadratic variations	squares of linear contributions
$\langle h_{R1} \rangle \quad \Sigma_L^{\hat{W}\hat{W}}, \Sigma_T^{\hat{W}\hat{W}}, \Sigma^{\hat{\omega}^+\hat{\omega}^-}, T^H$	$\langle g_{RR1} \rangle \quad \Sigma_T^{\hat{A}\hat{A}}, \Sigma_T^{\hat{W}\hat{W}}$	$\langle g_{R1} \rangle^2 \quad \Sigma_T^{\hat{A}\hat{A}}$
$\langle h_{R2} \rangle \quad T^H$	$\langle g_{RA1} \rangle \quad \Sigma_T^{\hat{A}\hat{A}}, \Sigma_T^{\hat{W}\hat{W}}, \Sigma^{\hat{\omega}^+\hat{\omega}^-}$	$\langle g_{R1} \rangle \langle g_{R2} \rangle \quad \Sigma_T^{\hat{A}\hat{A}}$
$\langle h_{A2} \rangle \quad \Sigma_L^{\hat{W}\hat{W}}, \Sigma_T^{\hat{W}\hat{W}}, \Sigma^{\hat{\omega}^+\hat{\omega}^-}, T^H$	$\langle g_{RR2} \rangle \quad \Sigma_T^{\hat{A}\hat{A}}$	$\langle g_{R1} \rangle \langle g_{R3} \rangle \quad \Sigma_T^{\hat{A}\hat{A}}$
$\langle h_{R3} \rangle \quad T^H$	$\langle g_{RA2} \rangle \quad \Sigma_T^{\hat{A}\hat{A}}, \Sigma^{\hat{\omega}^+\hat{\omega}^-}$	$\langle g_{R1} \rangle \langle g_{A3} \rangle \quad \Sigma_T^{\hat{A}\hat{A}}$
$\langle h_{A3} \rangle \quad \Sigma_L^{\hat{W}\hat{W}}, \Sigma_T^{\hat{W}\hat{W}}, \Sigma^{\hat{\omega}^+\hat{\omega}^-}, T^H$	$\langle g_{RR3} \rangle \quad \Sigma_T^{\hat{A}\hat{A}}$	$\langle g_{R1} \rangle \langle g_{R4} \rangle \quad \Sigma_T^{\hat{A}\hat{A}}$
$\langle g_{R1} \rangle \quad \Sigma_T^{\hat{W}\hat{W}}, T^H$	$\langle g_{RA3} \rangle \quad \Sigma_T^{\hat{A}\hat{A}}, \Sigma^{\hat{\omega}^+\hat{\omega}^-}$	$\langle g_{R1} \rangle \langle g_{A4} \rangle \quad \Sigma_T^{\hat{A}\hat{A}}$
$\langle g_{R2} \rangle \quad T^H$	$\langle g_{AA3} \rangle \quad \Sigma_T^{\hat{A}\hat{A}}, \Sigma_T^{\hat{W}\hat{W}}, \Sigma^{\hat{\omega}^+\hat{\omega}^-}$	$\langle g_{R2} \rangle^2 \quad \Sigma_T^{\hat{A}\hat{A}}$
$\langle g_{R3} \rangle \quad T^H$	$\langle g_{RR4} \rangle \quad \Sigma_T^{\hat{A}\hat{A}}$	$\langle g_{R2} \rangle \langle g_{R3} \rangle \quad \Sigma_T^{\hat{A}\hat{A}}$
$\langle g_{A3} \rangle \quad \Sigma_T^{\hat{W}\hat{W}}, \Sigma^{\hat{\omega}^+\hat{\omega}^-}, T^H$	$\langle g_{RA4} \rangle \quad \Sigma_T^{\hat{A}\hat{A}}, \Sigma^{\hat{\omega}^+\hat{\omega}^-}$	$\langle g_{R2} \rangle \langle g_{A3} \rangle \quad \Sigma_T^{\hat{A}\hat{A}}$
$\langle g_{R4} \rangle \quad T^H$	$\langle g_{AA4} \rangle \quad \Sigma_T^{\hat{A}\hat{A}}, \Sigma^{\hat{\omega}^+\hat{\omega}^-}$	$\langle g_{R2} \rangle \langle g_{R4} \rangle \quad \Sigma_T^{\hat{A}\hat{A}}$
$\langle g_{A4} \rangle \quad \Sigma_T^{\hat{W}\hat{W}}, \Sigma^{\hat{\omega}^+\hat{\omega}^-}, T^H$	$\langle h_{RR1} \rangle \quad \Sigma_L^{\hat{W}\hat{W}}, \Sigma_T^{\hat{W}\hat{W}}, \Sigma^{\hat{\omega}^+\hat{\omega}^-}$	$\langle g_{R2} \rangle \langle g_{A4} \rangle \quad \Sigma_T^{\hat{A}\hat{A}}$
	$\langle h_{RA1} \rangle \quad \Sigma_L^{\hat{W}\hat{W}}, \Sigma_T^{\hat{W}\hat{W}}, \Sigma^{\hat{\omega}^+\hat{\omega}^-}$	$\langle g_{R3} \rangle^2 \quad \Sigma_T^{\hat{A}\hat{A}}$
	$\langle h_{RA2} \rangle \quad \Sigma^{\hat{\omega}^+\hat{\omega}^-}$	$\langle g_{R3} \rangle \langle g_{A3} \rangle \quad \Sigma_T^{\hat{A}\hat{A}}$
	$\langle h_{AA2} \rangle \quad \Sigma_L^{\hat{W}\hat{W}}, \Sigma_T^{\hat{W}\hat{W}}, \Sigma^{\hat{\omega}^+\hat{\omega}^-}$	$\langle g_{R3} \rangle \langle g_{R4} \rangle \quad \Sigma_T^{\hat{A}\hat{A}}$
	$\langle h_{RA3} \rangle \quad \Sigma^{\hat{\omega}^+\hat{\omega}^-}$	$\langle g_{R3} \rangle \langle g_{A4} \rangle \quad \Sigma_T^{\hat{A}\hat{A}}$
	$\langle h_{AA3} \rangle \quad \Sigma_L^{\hat{W}\hat{W}}, \Sigma_T^{\hat{W}\hat{W}}, \Sigma^{\hat{\omega}^+\hat{\omega}^-}$	$\langle g_{A3} \rangle^2 \quad \Sigma_T^{\hat{A}\hat{A}}, \Sigma_T^{\hat{W}\hat{W}}, \Sigma^{\hat{\omega}^+\hat{\omega}^-}$
	$\langle g_{R1} \rangle \langle h_{R1} \rangle \quad \Sigma_T^{\hat{W}\hat{W}}$	$\langle g_{A3} \rangle \langle g_{R4} \rangle \quad \Sigma_T^{\hat{A}\hat{A}}$
	$\langle g_{A4} \rangle \langle h_{A3} \rangle \quad \Sigma_T^{\hat{A}\hat{A}}, \Sigma^{\hat{\omega}^+\hat{\omega}^-}$	$\langle g_{A3} \rangle \langle g_{A4} \rangle \quad \Sigma_T^{\hat{A}\hat{A}}, \Sigma_T^{\hat{W}\hat{W}}, \Sigma^{\hat{\omega}^+\hat{\omega}^-}$
	$\langle g_{A4} \rangle \langle h_{A3} \rangle \quad \Sigma_T^{\hat{A}\hat{A}}, \Sigma^{\hat{\omega}^+\hat{\omega}^-}$	$\langle g_{R4} \rangle^2 \quad \Sigma_T^{\hat{A}\hat{A}}$
		$\langle g_{R4} \rangle \langle g_{A4} \rangle \quad \Sigma_T^{\hat{A}\hat{A}}$
		$\langle g_{A4} \rangle^2 \quad \Sigma_T^{\hat{A}\hat{A}}, \Sigma_T^{\hat{W}\hat{W}}, \Sigma^{\hat{\omega}^+\hat{\omega}^-}$
		$\langle h_{R1} \rangle^2 \quad \Sigma_L^{\hat{W}\hat{W}}, \Sigma_T^{\hat{W}\hat{W}}, \Sigma^{\hat{\omega}^+\hat{\omega}^-}$
		$\langle h_{R1} \rangle \langle h_{A2} \rangle \quad \Sigma_L^{\hat{W}\hat{W}}, \Sigma_T^{\hat{W}\hat{W}}, \Sigma^{\hat{\omega}^+\hat{\omega}^-}$
		$\langle h_{R1} \rangle \langle h_{A3} \rangle \quad \Sigma_L^{\hat{W}\hat{W}}, \Sigma_T^{\hat{W}\hat{W}}, \Sigma^{\hat{\omega}^+\hat{\omega}^-}$
		$\langle h_{A2} \rangle^2 \quad \Sigma_L^{\hat{W}\hat{W}}, \Sigma_T^{\hat{W}\hat{W}}, \Sigma^{\hat{\omega}^+\hat{\omega}^-}$
		$\langle h_{A2} \rangle \langle h_{A3} \rangle \quad \Sigma_L^{\hat{W}\hat{W}}, \Sigma_T^{\hat{W}\hat{W}}, \Sigma^{\hat{\omega}^+\hat{\omega}^-}$
		$\langle h_{A3} \rangle^2 \quad \Sigma_L^{\hat{W}\hat{W}}, \Sigma_T^{\hat{W}\hat{W}}, \Sigma^{\hat{\omega}^+\hat{\omega}^-}$

Table 1: Table of contributions from various terms in the expanded metrics. Contributions coming purely from metric expectations are neglected, see the discussion of SM-like parts in the main text.

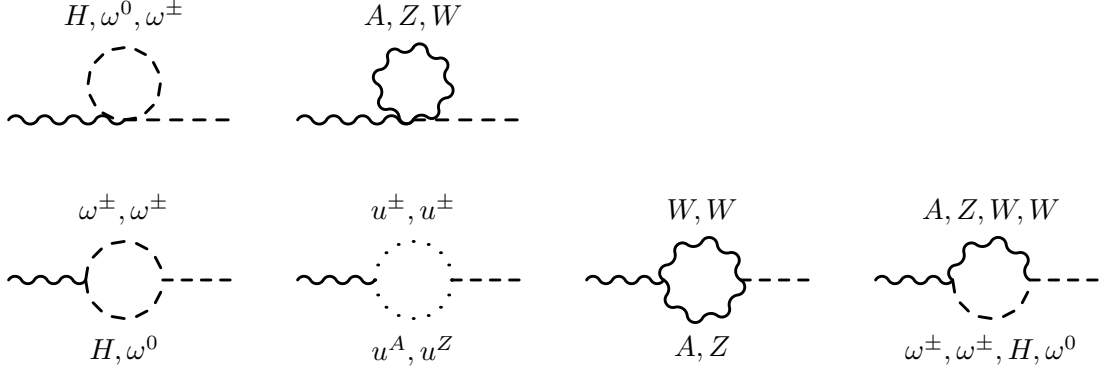


Figure 9: Diagrams contributing to the $W^\mp \omega^\pm$ mixing

$$0 = p^2 \Sigma^{\hat{W}^\pm \hat{\omega}^\mp}(p^2) \pm \bar{m}_W \Sigma^{\hat{\omega}^\pm \hat{\omega}^\mp}(p^2) \mp \frac{\bar{g}_2^2}{2} \sqrt{h_{44}} \sqrt{h^{11}} T^H \quad (284)$$

explicitly involves h_{IJ} . This relation relates the mixing between $W^\pm$ and $\omega^\mp$ with the $\omega^\pm \omega^\mp$ two-point function and the tadpole T^H .

The two-point function for the charged Goldstones and the tadpole T_H are related to the effective action by:

$$\frac{\delta^2 \Gamma}{\delta \hat{\omega}^+ \delta \hat{\omega}^-} = ip^2 + i \Sigma^{\hat{\omega}^+ \hat{\omega}^-}(p^2), \quad (285)$$

$$\frac{\delta \Gamma}{\delta \hat{H}} = iT_H. \quad (286)$$

Table 1 lists which variations contribute to the transverse and longitudinal parts separately, we discuss a subset here.

Consider first the contributions from double variations of the h metric (e.g. $\langle h_{RR1} \rangle$):

$$\begin{aligned} \Sigma_L^{\hat{W}^+ \hat{W}^-} \Big|_{\delta^2 h} &= \frac{\bar{g}_2^2 v^2}{16\pi^2 \tilde{\epsilon}} \frac{1}{16} \left[\bar{m}_H^2 \langle h^{44} \rangle \langle h_{RR1} \rangle + 4\xi_W \bar{m}_W^2 \langle h^{11} \rangle (\langle h_{AA2} \rangle + 4\langle h_{AA3} \rangle) \right] \\ &+ \frac{\bar{g}_2^2 v^2}{16\pi^2 \tilde{\epsilon}} \frac{\langle h_{RA1} \rangle}{16} \left[\bar{m}_H^2 \langle h^{44} \rangle + 2\xi_W \bar{m}_W^2 \bar{m}_H^2 \langle h^{11} \rangle + \xi_W \bar{m}_Z^2 \langle h^{33} \rangle \right]. \end{aligned} \quad (287)$$

We have that the longitudinal part is independent of double variations of g, but the transverse part has a single contribution from $\langle g_{AA3} \rangle$:

$$\Sigma_T^{\hat{W}^+ \hat{W}^-} \Big|_{\delta^2 g} = -\frac{1}{16\pi^2 \tilde{\epsilon}} \frac{\langle g_{11} \rangle}{2} p^2 \left[\langle g_{RR1} \rangle \bar{m}_H^2 \langle h^{44} \rangle + 8 \langle g_{AA3} \rangle \xi_W \bar{m}_W^2 \langle h^{11} \rangle \right]. \quad (288)$$

We also have contributions from geometric expressions, that is in terms of expectations of the metrics and geometric relations such as those in Eqs. (48-51).

$$\begin{aligned} \Sigma_T^{\hat{W}^+ \hat{W}^-} \Big|_{\text{mixed}} &= -\frac{p^2}{16\pi^2\tilde{\epsilon}} \frac{\bar{g}_2^2 v^2 \langle g^{11} \rangle \langle h^{44} \rangle}{4} \langle g_{R1} \rangle \langle h_{R1} \rangle + \frac{p^2}{16\pi^2\tilde{\epsilon}} \frac{\bar{g}_z^2 c_Z^2 v^2 \langle g^{11} \rangle \langle h^{11} \rangle}{4} \langle g_{A3} \rangle \langle h_{A3} \rangle \\ &+ \frac{p^2}{16\pi^2\tilde{\epsilon}} \frac{\bar{g}_2 \bar{g}_Z^2 s_Z^2 v^2 \sqrt{g^{11}} \langle h^{11} \rangle}{4g_1^2} \langle g_{A4} \rangle \langle h_{A3} \rangle. \end{aligned} \quad (289)$$

As was the case for the photon we also obtain higher momentum contributions. The longitudinal part has contributions up to strictly p^2 , whereas the transverse part can receive higher momentum contributions. Considering only contributions from g to the transverse two-point function, which scales as p^4 , we obtain:

$$\begin{aligned} \Sigma_T^{\hat{W}^+ \hat{W}^-} \Big|_{p^4} &= \frac{p^4}{16\pi^2\tilde{\epsilon}} \frac{\langle g_{11} \rangle \langle h_{11} \rangle}{6} \left(\langle g_{33} \rangle \langle g_{A3} \rangle^2 + 2\langle g_{34} \rangle \langle g_{A3} \rangle \langle g_{A4} \rangle + \langle g_{44} \rangle \langle g_{A4} \rangle^2 \right) \\ &+ \frac{p^4}{16\pi^2\tilde{\epsilon}} \frac{\langle g_{11} \rangle \langle h_{44} \rangle}{6} \langle g_{11} \rangle \langle g_{R1} \rangle^2. \end{aligned} \quad (290)$$

The reason higher momentum structures only occur in the transverse part can be understood from the renormalizability of the theory. The only counter term that can be used to cancel such higher momentum terms come from operators like

$$\phi^n W_{\mu\nu} D^m W_{\mu\nu} \longrightarrow v^n p^{m+2} \left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right), \quad (291)$$

where the arrow indicates the counter term structure arising from this operator.

We have confirmed the Ward identity of Eq. (271) in all cases except that proportional to $\bar{m}_H$ (see below). An explicit example is the dependence on double variations of the h metric

$$\Sigma^{\hat{\omega}^+ W^-} = -\frac{\bar{g}_2 v}{16\pi^2\tilde{\epsilon}} \frac{\sqrt{h}^{11}}{8} \left[\bar{m}_H^2 \langle h^{44} \rangle \langle h_{RR1} \rangle + 4\xi_W \bar{m}_W^2 \langle h^{11} \rangle (\langle h_{AA2} \rangle + 4\langle h_{AA3} \rangle) \right]. \quad (292)$$

When multiplied by $\bar{m}_W = \bar{g}_2 v \sqrt{h_{11}}/2$, this term cancels against Eq. (287) and the Ward identity is satisfied.

As this thesis only deals with contributions from g and h , and neglects the additional contributions from variations of the scalar potential V , there remains a piece proportional to $\bar{m}_H^2$. The mass of the Higgs is generated (at tree-level) from $\delta^2 V$. Absent the full potential contributions this part is not expected to cancel properly. This will be addressed in our upcoming publication, which includes the dependence of all metrics appearing in Eq. (207).

Considering the relation of Eq. (271) and comparing with Table 1 there appears to be an issue: Tadpole structures exist without compensatory other structures as required by the Ward identities. For example,

for the $\langle g_{R2} \rangle$ dependence there exists no $\Sigma^{\hat{\omega}^+ \hat{\omega}^-}$ or $\Sigma^{\hat{\omega}^+ \hat{W}^-}$. This can be resolved by recalling that *our parametrization of the metrics is redundant*, for example:

$$\langle g_{RA1} \rangle = \frac{1}{v} \langle g_{R1} \rangle, \quad (293)$$

and we see the compensatory structures are those coming from $\langle g_{RA1} \rangle$ in our parametrization.

Taking the $\langle g_{RA1} \rangle$ dependence for the two-point function for $\hat{\omega}^+ \hat{\omega}^-$,

$$\Sigma^{\hat{\omega}^+ \hat{\omega}^-} \Big|_{g_{RA1}} = -\frac{\langle g_{RA1} \rangle}{16\pi^2 \tilde{\epsilon}} \frac{3\langle h^{11} \rangle v^4}{32g_2^2} \left[2g_2^6 \langle g^{11} \rangle \langle h_{11} \rangle^2 + c_Z^4 \bar{g}_Z^6 \langle h_{33} \rangle^2 \right]. \quad (294)$$

and the tadpole's $\langle g_{R1} \rangle$ dependence is given by:

$$T^H \Big|_{g_{R1}} = -\frac{\langle g_{R1} \rangle}{16\pi^2 \tilde{\epsilon}} \frac{3v^4 \sqrt{h}^{44}}{32g_2^2} \left(2g_2^6 \langle g^{11} \rangle \langle h^{11} \rangle^2 + c_Z^4 \bar{g}_Z^6 \langle h_{33} \rangle^2 \right). \quad (295)$$

There is no related contributions from $\Sigma^{\hat{\omega}^+ \hat{W}^-}$. The relative factor between these two is dictated by the Ward identity in Eq. (273) and is given by:

$$-\frac{1}{\bar{m}_W} \frac{\bar{g}_2^2}{2} \sqrt{h_{44}} \sqrt{h}^{11} = -\frac{1}{v\sqrt{h_{11}}} \sqrt{h_{44}} \sqrt{h}^{11}. \quad (296)$$

With this relative factor the sum of the two diagrams vanishes in agreement with the Ward identity, that is:

$$\Sigma^{\hat{\omega}^+ \hat{\omega}^-} \Big|_{g_{RA1}} - \frac{1}{v\sqrt{h_{11}}} \sqrt{h_{44}} \sqrt{h}^{11} \cdot T_H \Big|_{g_{R1}} \propto (\langle g_{R1} \rangle - v\langle g_{RA1} \rangle) = 0. \quad (297)$$

The last equivalence follows from Eq. (293).

As the metric term $\langle h_{RR1} \rangle$ is linearly independent from the others considered and it necessarily corresponds to a four-point Feynman rule², it cannot contribute to the Tadpole. As such its contribution to the W-goldstone mixing must cancel against the pure charged goldstone two-point function. We find for the latter:

²As it corresponds to sourcing two scalars and multiplies $(D\phi)^2$ it must source at least 4 fields.

$$\Sigma^{\hat{\omega}^+\hat{\omega}^-}\Big|_{h_{RR1}} = -\frac{p^2}{16\pi^2\tilde{\epsilon}} \frac{\bar{m}_H^2 \langle h^{11} \rangle \langle h^{44} \rangle}{8}. \quad (298)$$

Taking into account Eq. (271) and (273), as well as the $\langle h_{RR1} \rangle$ dependence of Eq. (287), we see that the Ward identity *is satisfied*.

Combining all dependence we find that the second charged boson Ward identity is satisfied in all cases except precisely where the scalar potential dependence we have neglected becomes necessary for the cancellation to occur. This extra dependence will be covered in our upcoming publication.

9 Conclusion and Outlook

In this thesis, we investigated the structure of the Standard Model Effective Field Theory (SMEFT) with particular emphasis on its geometric formulation and the description of one-loop two-point functions appearing in Ward Identities. Starting from the Standard Model (SM) and the foundations of a bottom-up effective field theory, we reviewed the construction of the SMEFT operator basis and discussed the role of dimension-six operators in parametrizing possible effects of physics beyond the SM in a model-independent framework.

The geometric SMEFT (geoSMEFT) is a consequence of the field redefinition invariance defining the SMEFT[11]. The geoSMEFT organizes the theory geometrically using field space connections. The flat field space corresponds to the SM, whereas curvature refers to beyond SM physics. For two- and three-point functions the field space connections can be written down compactly and explicitly to all orders in mass dimension. This provides a better understanding of the impact of neglected higher-order terms. Corrections to SM predictions for future LHC analyses should be guided by these results[11].

We first calculated two-point functions and Ward identities in the SM. We used the Background Field Method (BFM) approach to gauge fixing. This BFM approach keeps background gauge invariance manifest. Background gauge invariance implies Ward identities, which were evaluated at one-loop. Ward Identities provide an excellent opportunity to cross check loop-calculations within the SMEFT.

We defined the concept of universality and demonstrated the consistency of the *universal* SMEFT with the standard geoSMEFT classification of two- and three-point functions. We arrived at the universal electroweak geoSMEFT Lagrangian.

From there we determined the corresponding Feynman rules, calculated the relevant Feynman diagrams and checked the Ward identities. Again, we used the BFM to gauge fixing. We checked the identity that corresponds to the transversality of the photon as well as the identity that relates the longitudinal W gauge-boson two-point function with the $W^\pm\omega^\mp$ mixing. Furthermore, we investigated the identity that relates the mixing between $W^\pm$ and $\omega^\mp$ with the $\omega^\pm\omega^\mp$ two-point function and the tadpole T^H .

We have confirmed the Ward identities in all considered cases. We noted that in order to satisfy the Ward identities in cases of Higgs mass dependence we need to extend our analysis to include variations of the potential as $\bar{m}_H \sim \delta^2 V$. This will be covered in our forthcoming paper.

A Dimensional Regularization

Dimensional regularization is nowadays a standard mathematical procedure in dealing with UV-divergent integrals, which appear frequently in particle physics calculations. This regularization scheme was developed by Gerard 't Hooft and Martinus Veltman[61]. The key idea is to calculate the integral not in the usual four dimensions but in d dimensions - where d is defined usually as $d = 4 - 2\epsilon$ - and the ϵ parameter serves as a regulator. The UV-divergent part of the integral appears as a $\frac{1}{\epsilon}$ term. We will show an example of such a divergent integral in a scalar field theory. One has to evaluate the integral by using Feynman-parameters[16] and then use “Master-integrals” from[16, 15] to obtain a result.

Let us consider the Lagrangian $\mathcal{L}$ of a scalar field ϕ with mass a term and a 3-point vertex interaction ϕ , where

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2 \phi^2 - \frac{\lambda}{3!} \phi^3. \quad (\text{A.1})$$

Now consider the one-loop self-energy diagram, see Figure 10.

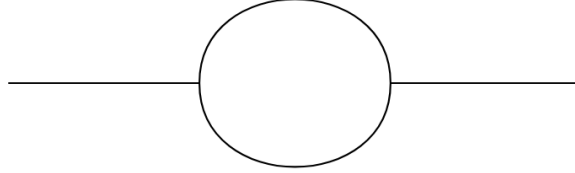


Figure 10: Self-energy loop-diagram in ϕ^3 theory

If we evaluate the diagram using Feynman-rules we find the matrix-element

$$i\mathcal{M} = -i\frac{\lambda}{2} \int \frac{d^4 k}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\epsilon} \frac{i}{(k-p)^2 - m^2 + i\epsilon}. \quad (\text{A.2})$$

Rewriting the integral in d dimensions we obtain

$$i\mathcal{M} = -i\frac{\lambda}{2} \tilde{\mu}^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{i}{k^2 - m^2 + i\epsilon} \frac{i}{(k-p)^2 - m^2 + i\epsilon}, \quad (\text{A.3})$$

where $\tilde{\mu}$ is introduced to keep the same mass dimension as in $d=4$. This integral diverges in $d = 4$ but converges in $d < 4$ dimensions. We need a way to solve the integral in d dimensions and after that we can substitute back the $d = 4 - 2\epsilon$ dimensions. After introducing Feynman parameters and shifting $k \rightarrow k+xp$ we get

$$i\mathcal{M} = i\frac{\lambda}{2} \tilde{\mu}^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{1}{[k^2 - \Delta + i\epsilon]^2}, \quad (\text{A.4})$$

where $\Delta = m^2 - p^2 x(1-x)$.

Using the formulas for dimensional regularization one arrives at the final result

$$i\mathcal{M} = \frac{-\lambda}{2 \cdot 16\pi^2} \left(\frac{1}{\epsilon} + \ln\left(\frac{\tilde{\mu}^2}{\Delta}\right) \right), \quad (\text{A.5})$$

where $\tilde{\mu}^2 = 4\pi e^{-\gamma_E} \mu^2$ and γ_E the Euler-Mascheroni constant.

Dimensional regularization isolates out the divergences into singularities. Quadratically, as well as logarithmically divergent integrals have $\frac{1}{\epsilon}$ poles in 4 dimensions[16].

Master Integrals:

$$I_2 = \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - \Delta + i\epsilon)^2} = \frac{i}{(4\pi)^{d/2}} \frac{1}{\Delta^{2-d/2}} \Gamma\left(\frac{4-d}{2}\right) \quad (\text{A.6})$$

$$I_2^{\mu\nu} = \int \frac{d^d k}{(2\pi)^d} \frac{k^\mu k^\nu}{(k^2 - \Delta + i\epsilon)^2} = \frac{g_{\mu\nu}}{d} \int \frac{d^d k}{(2\pi)^d} \frac{k^2}{(k^2 - \Delta + i\epsilon)^2} \quad (\text{A.7})$$

$$I_2^2 = \int \frac{d^d k}{(2\pi)^d} \frac{k^2}{(k^2 - \Delta + i\epsilon)^2} = \frac{-d}{2} \frac{i}{(4\pi)^{d/2}} \frac{1}{\Delta^{1-d/2}} \Gamma\left(\frac{2-d}{2}\right) \quad (\text{A.8})$$

B SMEFT operators at dimension six

In Table 2 we have the 59 independent, baryon number conserving dimension-six operators built from SM fields[62]. They are categorized into eight classes. EOMs are used to eliminate a few redundant operators. This elimination is a choice of basis and is not unique.

1 : X^3		2 : H^6		3 : $H^4 D^2$		5 : $\psi^2 H^3 + \text{h.c.}$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_H	$(H^\dagger H)^3$	$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$	Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$
$Q_{\tilde{G}}$	$f^{ABC} \tilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$			Q_{HD}	$(H^\dagger D_\mu H)^* (H^\dagger D_\mu H)$	Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \tilde{H})$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$					Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$
$Q_{\tilde{W}}$	$\epsilon^{IJK} \tilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$						
4 : $X^2 H^2$		6 : $\psi^2 XH + \text{h.c.}$		7 : $\psi^2 H^2 D$			
Q_{HG}	$H^\dagger H G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I H W_{\mu\nu}^I$	$Q_{Hl}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{l}_p \gamma^\mu l_r)$		
$Q_{H\tilde{G}}$	$H^\dagger H \tilde{G}_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$	$Q_{Hl}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{l}_p \tau^I \gamma^\mu l_r)$		
Q_{HW}	$H^\dagger H W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \tilde{H} G_{\mu\nu}^A$	Q_{He}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$		
$Q_{H\tilde{W}}$	$H^\dagger H \tilde{W}_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \tilde{H} W_{\mu\nu}^I$	$Q_{Hq}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$		
Q_{HB}	$H^\dagger H B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tilde{H} B_{\mu\nu}$	$Q_{Hq}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{q}_p \tau^I \gamma^\mu q_r)$		
$Q_{H\tilde{B}}$	$H^\dagger H \tilde{B}_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H G_{\mu\nu}^A$	Q_{Hu}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$		
Q_{HWB}	$H^\dagger \tau^I H W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I H W_{\mu\nu}^I$	Q_{Hd}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r)$		
$Q_{H\tilde{W}B}$	$H^\dagger \tau^I H \tilde{W}_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) H B_{\mu\nu}$	$Q_{Hud} + \text{h.c.}$	$i(\tilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$		
8 : $(\bar{L}L)(\bar{L}L)$		8 : $(\bar{R}R)(\bar{R}R)$		8 : $(\bar{L}L)(\bar{R}R)$			
Q_{ll}	$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$		
$Q_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$		
$Q_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar{d}_p \gamma_\mu d_r)(\bar{d}_s \gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t)$		
$Q_{lq}^{(1)}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$		
$Q_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{u}_s \gamma^\mu u_t)$		
		$Q_{ud}^{(1)}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{u}_s \gamma^\mu T^A u_t)$		
		$Q_{ud}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_r)(\bar{d}_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{d}_s \gamma^\mu d_t)$		
				$Q_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{d}_s \gamma^\mu T^A d_t)$		
8 : $(\bar{L}R)(\bar{R}L) + \text{h.c.}$		8 : $(\bar{L}R)(\bar{L}R) + \text{h.c.}$					
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_{tj})$	$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r) \epsilon_{jk} (\bar{q}_s^k d_t)$				
		$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r) \epsilon_{jk} (\bar{q}_s^k T^A d_t)$				
		$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r) \epsilon_{jk} (\bar{q}_s^k u_t)$				
		$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r) \epsilon_{jk} (\bar{q}_s^k \sigma^{\mu\nu} u_t)$				

Table 2: The 59 independent dimension-six operators built from SM fields, which conserve baryon number

C SM Feynman Diagrams

In this section we present the relevant Feynman diagrams for the calculation of the Ward Identities in the SM, as discussed in Sec. 4.5. The notation is the following: GP denotes Goldstone bosons, G0 the neutral goldstone boson and ghWp (ghWm) to the ghosts corresponding to W^+ (W^-). The suffix BG means that it refers to the background field. Hence A is the ordinary and ABG is the background field photon. The diagrams are generated using the FeynArts package[56].

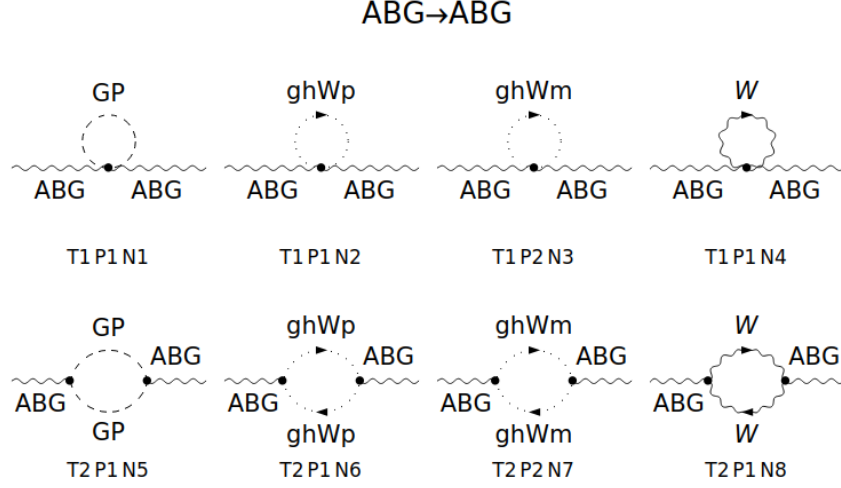


Figure 11: ABG $\rightarrow$ ABG two point function

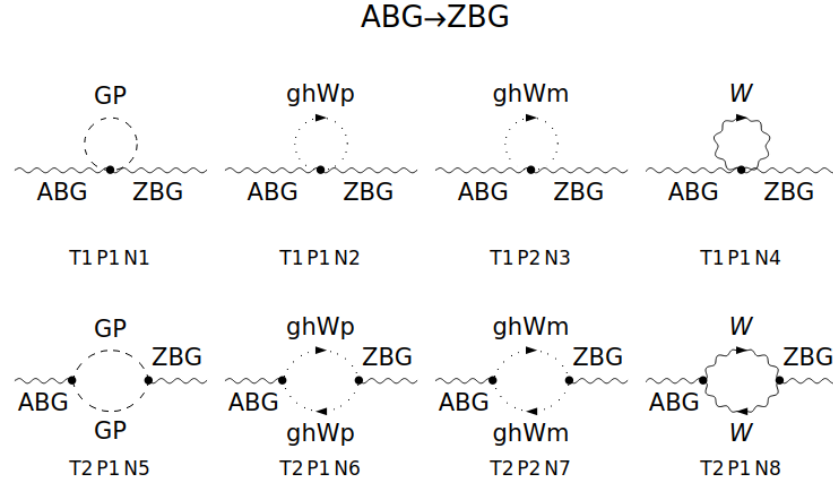


Figure 12: ABG $\rightarrow$ ZBG two point function

ZBG→ZBG

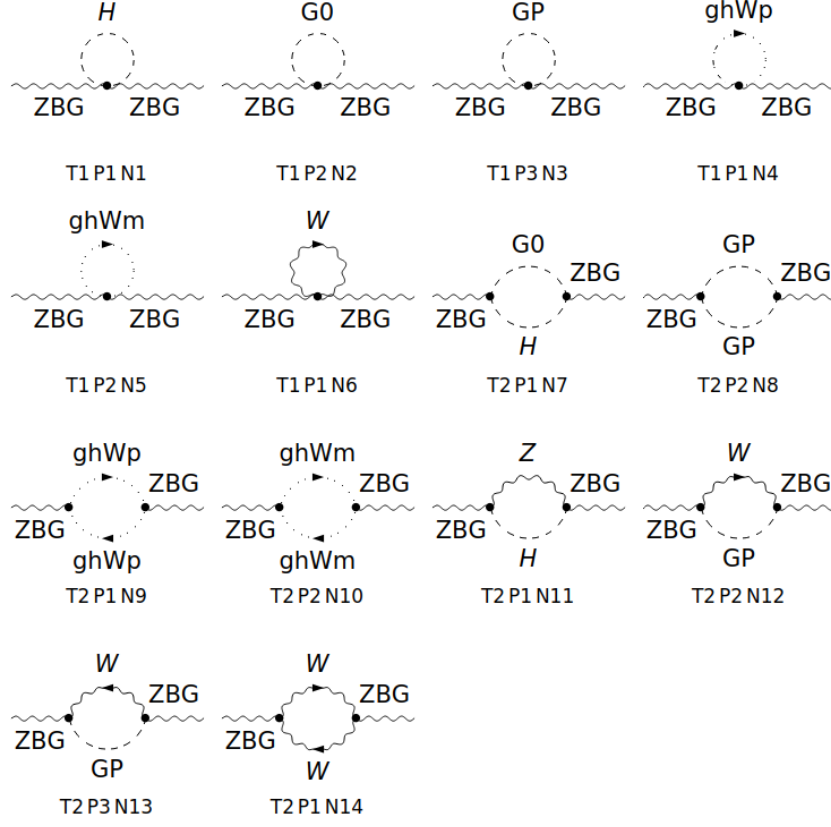


Figure 13: $ZBG \rightarrow ZBG$

ZBG→G0BG

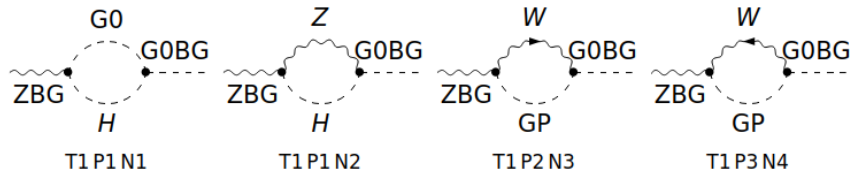


Figure 14: $ZBG \rightarrow G0BG$

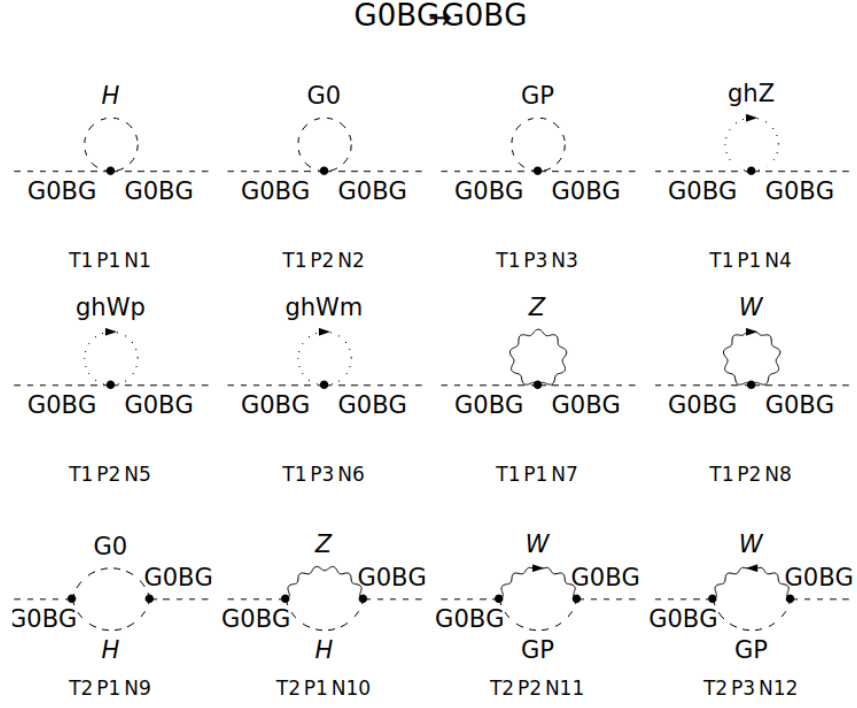


Figure 15: $G0BG \rightarrow G0BG$

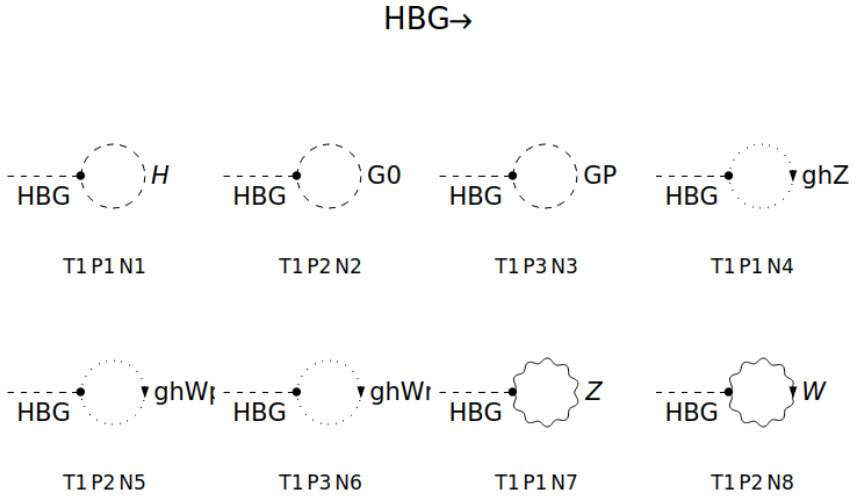


Figure 16: Background Higgs tadpole

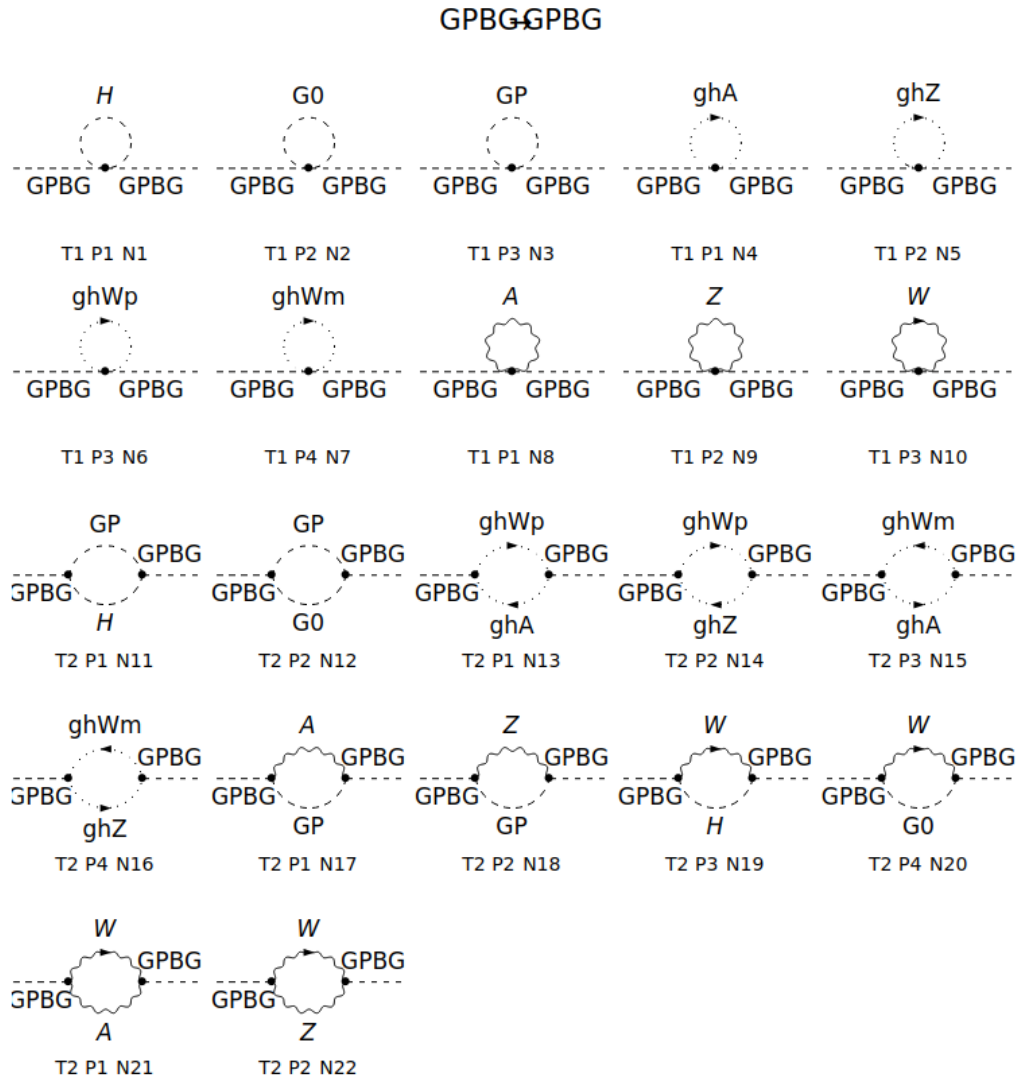


Figure 17: GPBG → GPBG

WBG → WBG

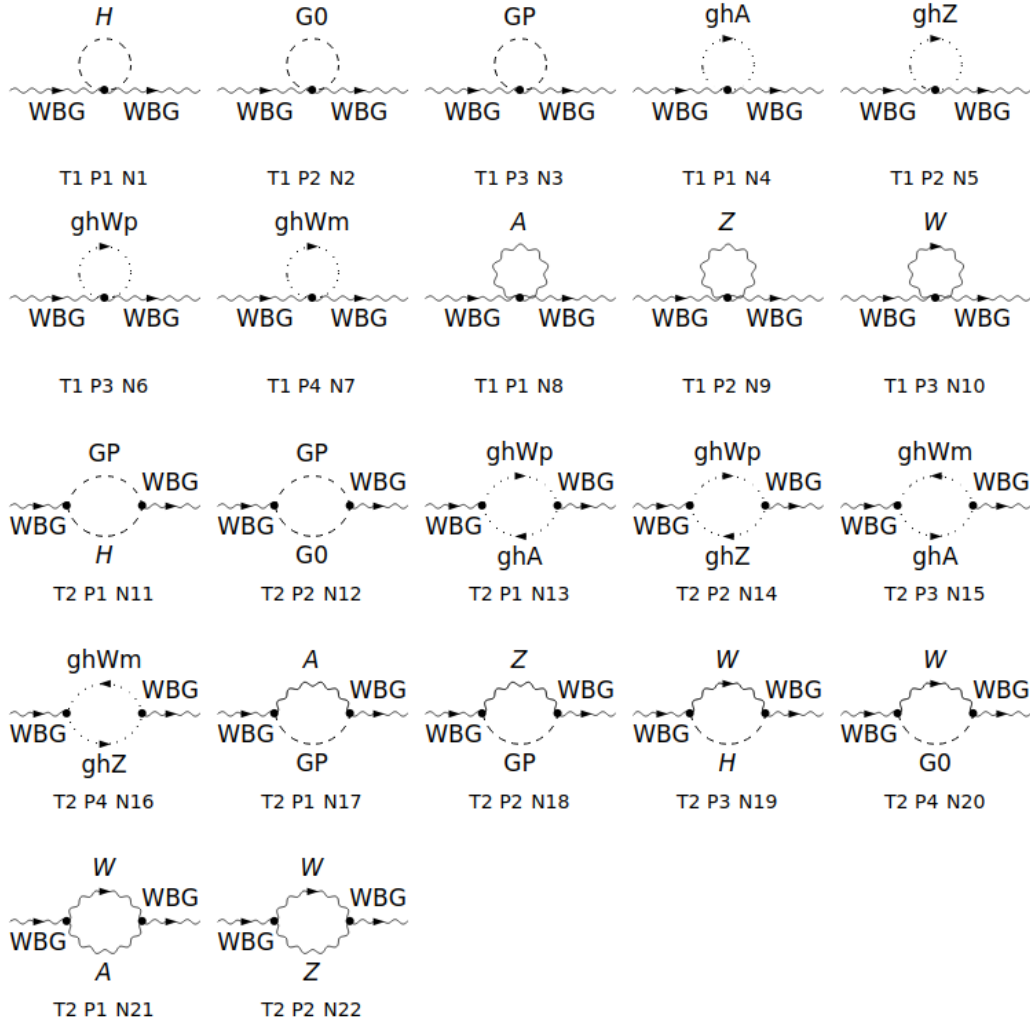


Figure 18: WBG → WBG

WBG $\rightarrow$ GPBG

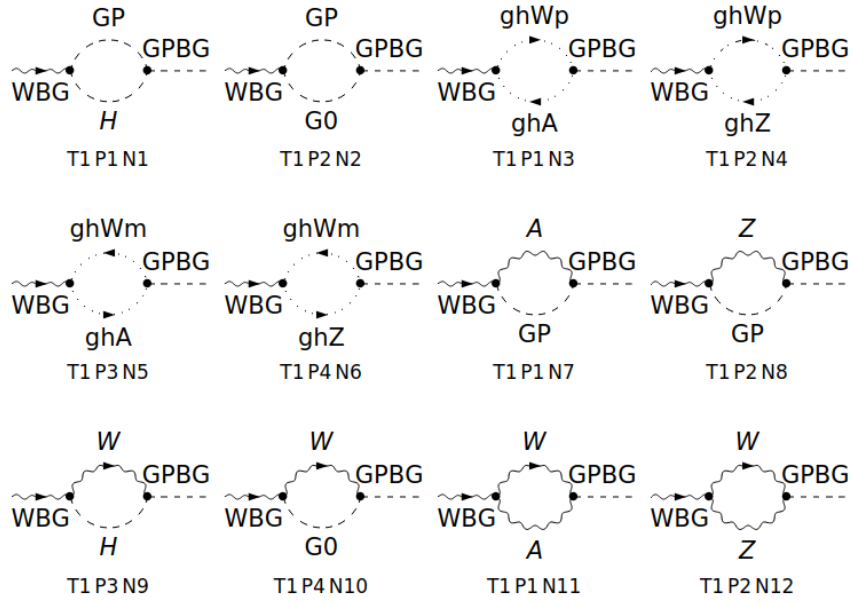


Figure 19: $\text{WBG} \rightarrow \text{GPBG}$

D Electroweak geoSMEFT metrics

The metrics ³ as defined in Sec. 5.[9]:

$$V(\phi) = \lambda \left(\frac{\phi^2}{2} - \frac{v^2}{2} \right)^2 - \sum_{n=3}^{\infty} c_H^{2n} \left(\frac{\phi^2}{2} \right)^n \quad (D.1)$$

$$h_{IJ} = \delta_{IJ} + \sum_{n=0}^{\infty} \left(\frac{\phi^2}{2} \right)^n \left[\frac{\phi^2}{2} \delta_{IJ} c_{HD,2}^{(6+2n)} - 2 \phi_I \phi_J c_{H\Box}^{(6+2n)} + \frac{1}{4} \Gamma_{AJ}^I \phi_K \Gamma_{AL}^K \phi^L c_{HD}^{(6+2n)} \right] \quad (D.2)$$

$$H_{IJ} = \sum_{n=0}^{\infty} \left(\frac{\phi^2}{2} \right)^n \left[\delta_{IJ} c_{DH,1}^{(6+2n)} - 2 \phi_I \phi_J c_{DH,2}^{(8+2n)} + \frac{1}{2} \Gamma_{AJ}^I \phi_K \Gamma_{AL}^K \phi^L c_{DH,3}^{(8+2n)} \right] \quad (D.3)$$

$$Y_{AB} = 4 \sum_{n=0}^{\infty} \left(\frac{\phi^2}{2} \right)^n \left[c_{DW}^{(6+2n)} \delta_{A4} + c_{DB}^{(6+2n)} \delta_{A4} \right] \delta_{AB} \quad (D.4)$$

$$\begin{aligned} & + \sum_{n=0}^{\infty} c_{DW,2}^{(8+2n)} \left(\frac{\phi^2}{2} \right)^n (\phi_I \Gamma_{AJ}^I \phi^J) (\phi_L \Gamma_{BK}^L \phi^K) (1 - \delta_{A4})(1 - \delta_{B4}) \\ & + \sum_{n=0}^{\infty} c_{DW,B}^{(8+2n)} \left(\frac{\phi^2}{2} \right)^n \left[(\Gamma_{AJ}^I \phi^J) (1 - \delta_{A4}) B_4 + (A \leftrightarrow B) \right] \\ g_{AB} = & \left[1 - 4 \sum_{n=0}^{\infty} \left(C_{HW}^{(6+2n)} (1 - \delta_{A4}) + C_{HB}^{(6+2n)} \delta_{A4} \right) \left(\frac{\phi^2}{2} \right)^{n+1} \right] \delta_{AB} \end{aligned} \quad (D.5)$$

$$\begin{aligned} & - \sum_{n=0}^{\infty} C_{HW,2}^{(8+2n)} \left(\frac{\phi^2}{2} \right)^n (\phi_I \Gamma_{A,J}^I \phi^J) (\phi_L \Gamma_{B,K}^L \phi^K) (1 - \delta_{A4})(1 - \delta_{B4}) \\ & + \left[\sum_{n=0}^{\infty} C_{HW,B}^{(6+2n)} \left(\frac{\phi^2}{2} \right)^n \right] [\phi_I \Gamma_{A,J}^I \phi^J (1 - \delta_{A4}) \delta_{B4} + (A \leftrightarrow B)] \\ f_{ABC} = & \epsilon_{ABC} \sum_{n=0}^{\infty} c_W^{(6+2n)} \left(\frac{\phi^2}{2} \right)^n + \frac{1}{2} \delta_{A4} \epsilon_{BCD} (\phi_K \Gamma_{DL}^K \phi^L) \sum_{n=0}^{\infty} c_{W,2}^{(8+2n)} \left(\frac{\phi^2}{2} \right)^n. \end{aligned} \quad (D.6)$$

$$\begin{aligned} \kappa_{IJ}^A = & -\frac{1}{2} \gamma_{4J}^I \sum_{n=0}^{\infty} c_{DHDHB}^{(6+2n)} \left(\frac{\phi^2}{2} \right)^n - \frac{1}{2} \gamma_{AJ}^I (1 - \delta_{A4}) \sum_{n=0}^{\infty} c_{DHDHW}^{(6+2n)} \left(\frac{\phi^2}{2} \right)^n \\ & + \frac{1}{4} \epsilon_{ABC} (\phi_K \Gamma_{BL}^K \phi^L) \gamma_{CJ}^I \sum_{n=0}^{\infty} c_{DHDHW,2}^{(8+2n)} \left(\frac{\phi^2}{2} \right)^n \\ & - \frac{1}{8} (1 - \delta_{A4}) (\phi_K \Gamma_{AL}^K \phi^L) (\phi_B \Gamma_{LN}^B \phi^N) \gamma_{BJ}^I \sum_{n=0}^{\infty} c_{DHDHW,3}^{(10+2n)} \left(\frac{\phi^2}{2} \right)^n \end{aligned} \quad (D.7)$$

³ h_{IJ} is written slightly differently than in[9], see App. E for an explanation

E Redefining Y and H

In this section we demonstrate the redefinition of different metric forms as we did in Sec. 5. We first look at h_{IJ} , which has the form

$$h_{IJ} = \delta_{IJ} + \sum_{n=0}^{\infty} \left(\frac{\phi^2}{2} \right)^n \left[\frac{\phi^2}{2} \delta_{IJ} c_{HD,2}^{(6+2n)} - 2\phi_I \phi_J c_{H\Box}^{(6+2n)} + \frac{1}{4} \Gamma_{AJ}^I \phi_K \Gamma_{AL}^K \phi^L c_{HD}^{(6+2n)} \right]. \quad (\text{E.1})$$

Note that h_{IJ} takes a slightly different form than in App. D. We show now how one arrives at Eq. (E.1) starting from Eq. (3.4) of [9]. Our basis of operators relevant for h_{IJ} are:

$$\mathcal{O}_1 = (H^\dagger H) \Box (H^\dagger H) \quad (\text{E.2})$$

$$\mathcal{O}_2 = |H^\dagger DH|^2 \quad (\text{E.3})$$

$$\mathcal{O}_3 = (H^\dagger H) |DH|^2 \quad (\text{E.4})$$

To construct h_{IJ} as in (E.1) we need $\mathcal{O}_1, \mathcal{O}_2, (H^\dagger H)\mathcal{O}_3$, which are labeled in [9] as $Q_{H\Box}^{(6)}, Q_{HD}^{(6)}, Q_{HD}^{(8)}$. At each order we factorize out the factor $(\frac{\phi^2}{2})^n$ – first appearing at dimension-eight ($n=1$) – to arrive at the result as given in Eq. (E.1).

Thus, the set of all operator forms $(D\phi)^2$ is spanned by the basis vectors

$$v_1 = \delta_{IJ}, \quad (\text{E.5})$$

$$v_2 = \frac{1}{\phi^2} \phi_I \phi_J, \quad (\text{E.6})$$

$$v_3 = \frac{1}{2\phi^2} \Gamma_{AJ}^I \phi_K \Gamma_{AL}^K \phi^L. \quad (\text{E.7})$$

The Gram matrix G is given by

$$G_{ij} = \langle v_i | v_j \rangle = \begin{pmatrix} 4 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 2 \end{pmatrix}. \quad (\text{E.8})$$

G is invertible – i.e. has a non-zero determinant – and therefore the above basis indeed spans the space.

In near-complete analogy, the metric g_{AB} takes the form

$$g_{AB} = \left[1 - 4 \sum_{n=0}^{\infty} \left(c_{HW}^{(6+2n)} (1 - \delta_{A4}) + c_{HB}^{(6+2n)} \delta_{A4} \right) \left(\frac{\phi^2}{2} \right)^{n+1} \right] \delta_{AB} \quad (\text{E.9})$$

$$- \sum_{n=0}^{\infty} c_{HW,2}^{(8+2n)} \left(\frac{\phi^2}{2} \right)^n (\phi_I \Gamma_{AJ}^I \phi^J) (\phi_L \Gamma_{BK}^L \phi^K) (1 - \delta_{A4})(1 - \delta_{B4}) \quad (\text{E.10})$$

$$+ \sum_{n=0}^{\infty} c_{HWB}^{(6+2n)} \left(\frac{\phi^2}{2} \right)^n \left[(\phi_I \Gamma_{AJ}^I \phi^J) (1 - \delta_{A4}) \delta_{B4} + (A \leftrightarrow B) \right]. \quad (\text{E.11})$$

The corresponding basis vectors are

$$w_1 = (1 - \delta_{A4}) \delta_{AB}, \quad (\text{E.12})$$

$$w_2 = \delta_{A4} \delta_{AB}, \quad (\text{E.13})$$

$$w_3 = \frac{1}{4} (\phi_I \Gamma_{AJ}^I \phi^J) (\phi_L \Gamma_{BK}^L \phi^K) (1 - \delta_{A4})(1 - \delta_{B4}), \quad (\text{E.14})$$

$$w_4 = \frac{1}{2} (\phi_I \Gamma_{AJ}^I \phi^J) (1 - \delta_{A4}) \delta_{B4}, \quad (\text{E.15})$$

$$w_5 = \frac{1}{2} (\phi_I \Gamma_{BJ}^I \phi^J) (1 - \delta_{B4}) \delta_{A4}. \quad (\text{E.16})$$

The operator structure corresponding to $c_{HWB}^{(6+2n)}$ is given by the sum

$$w_+ \equiv w_4 + w_5,$$

and for later convenience we also define

$$w_- \equiv w_4 - w_5.$$

Replacing w_4 and w_5 with $w_{\pm}$, the Gram matrix takes the form

$$G_{ij} = \langle w_i | w_j \rangle = \begin{pmatrix} 3 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix}. \quad (\text{E.17})$$

Since this matrix is invertible, the basis vectors $\{w_i\}$ form a complete basis. However, unlike the case of h_{IJ} , the following products project onto the individual components of $w_{\pm}$:

$$w_1 w_+ = w_4, \quad (\text{E.18})$$

$$w_2 w_+ = w_5. \quad (\text{E.19})$$

This appears to suggest that g_{AB} is insufficient to describe all scattering amplitudes, since it is spanned only by $\{w_1, w_2, w_3, w_+\}$, which do not themselves form a basis.

Let $\{w_i\}$ be a basis and define the index-swapping operation

$$\sigma(M)_{AB} \equiv M_{BA}.$$

It follows that

$$\sigma(w_1) = w_1, \tag{E.20}$$

$$\sigma(w_2) = w_2, \tag{E.21}$$

$$\sigma(w_3) = w_3, \tag{E.22}$$

$$\sigma(w_+) = w_+, \tag{E.23}$$

$$\sigma(w_-) = -w_-. \tag{E.24}$$

Since g , g^{-1} , and Y are symmetric, they are even under this swapping operation. Thus, any product of these will remain even under this operation and does not leave the subspace spanned by w_1, w_2, w_3, w_+ . the subspace spanned by w_4 is not relevant for g .

In particular, $gYg = g^{-1}Yg^{-1}$ can be taken to have the same operator structure as Y without loss of generality. For definiteness, consider the explicit example $M_{AB} = (gYg)_{AB}$. For a general matrix M in this space, its components may be projected using the Gram matrix,

$$c_i = (G^{-1})_{ij} \langle w_j | M \rangle. \tag{E.25}$$

Applying this to $M = gYg$, we find that the dependence on the Wilson coefficients in g_{AB} may be absorbed into a redefinition of the coefficients appearing in Y_{AB} (up to dimension eight), as explicitly done in Sec. 5.1.

F Variations of Metrics

F.1 Expansion of κ

We will expand κ written as

$$\kappa = 1 - 4 \sum_{n=1}^{\infty} c_{\text{HG}}^{(4+2n)} \left(\frac{\phi^2}{2} \right)^n. \quad (\text{F.1})$$

We take a variation with respect to $\Phi_i = \{\omega^+, \omega^-, \omega^-, h\}$, yielding

$$\frac{\delta \kappa}{\delta \Phi_i} = \frac{\delta \kappa}{\delta \phi_k} \frac{\delta \phi_k}{\delta \Phi_i} \quad (\text{F.2})$$

$$= \left[-4 \sum_{n=1}^{\infty} c_{\text{HG}}^{(4+2n)} n \left(\frac{\phi^2}{2} \right)^{n-1} \phi_k \right] \frac{\delta \phi_k}{\delta \Phi_i}. \quad (\text{F.3})$$

After taking the expectation value we obtain

$$\left\langle \frac{\delta \kappa}{\delta \Phi_i} \right\rangle = \left[-4 \sum_{n=1}^{\infty} c_{\text{HG}}^{(4+2n)} \left(\frac{v^2}{2} \right)^{n-1} v \delta_{k4} \right] \frac{\delta \phi_k}{\delta \Phi_i} \quad (\text{F.4})$$

$$= \left\langle \frac{\delta \kappa}{\delta \phi_4} \right\rangle \sqrt{h^{44}} \quad (\text{F.5})$$

$$\equiv \langle \kappa_R \rangle \sqrt{h^{44}}. \quad (\text{F.6})$$

Since only the higgs takes on a vev, only the fourth component does contribute.

Continuing with the second derivative we have:

$$\frac{\delta^2 \kappa}{\delta \Phi_i \delta \Phi_j} = \frac{\delta^2 \kappa}{\delta \Phi_k \delta \Phi_l} \frac{\delta \phi_k}{\delta \Phi_i} \frac{\delta \phi_l}{\delta \Phi_j} \quad (\text{F.7})$$

$$= -4 \sum_{n=1}^{\infty} c_{\text{HG}}^{(4+2n)} \left[n(n-1) \left(\frac{\phi^2}{2} \right)^{n-2} \phi_k \phi_l + n \left(\frac{\phi^2}{2} \right)^{n-1} \delta_{kl} \right] \frac{\delta \phi_k}{\delta \Phi_i} \frac{\delta \phi_l}{\delta \Phi_j}. \quad (\text{F.8})$$

Taking the expectation value:

$$\left\langle \frac{\delta^2 \kappa}{\delta \Phi_i \delta \Phi_j} \right\rangle = -4 \sum_{n=1}^{\infty} c_{\text{HG}}^{(4+2n)} \left(\frac{v^2}{2} \right)^{n-1} [2n(n-1)\delta_{k4}\delta_{l4} + n\delta_{kl}] \frac{\delta \phi_k}{\delta \Phi_i} \frac{\delta \phi_l}{\delta \Phi_j} \quad (\text{F.9})$$

$$= \left\langle \frac{\delta^2 \kappa}{\delta \phi_k \delta \phi_l} \right\rangle \frac{\delta \phi_k}{\delta \Phi_i} \frac{\delta \phi_l}{\delta \Phi_j} \quad (\text{F.10})$$

$$\equiv \langle \kappa_{RR} \rangle \frac{\delta \phi_4}{\delta \Phi_i} \frac{\delta \phi_4}{\delta \Phi_j} + \langle \kappa_{RA} \rangle \frac{\delta \phi_k}{\delta \Phi_i} \frac{\delta \phi_k}{\delta \Phi_j}. \quad (\text{F.11})$$

Where we have defined:

$$\langle \kappa_{RR} \rangle = -4 \sum_{n=1}^{\infty} 2n(n-1)c_{\text{HG}}^{(4+2n)} \quad (\text{F.12})$$

$$\langle \kappa_{RA} \rangle = -4 \sum_{n=1}^{\infty} n c_{\text{HG}}^{(4+2n)} \quad (\text{F.13})$$

The expanded κ has the following form:

$$\begin{aligned} \kappa = & \langle \kappa \rangle + \langle \kappa_R \rangle \sqrt{h^{44}} H + \langle \kappa_{RR} \rangle \langle h^{44} \rangle H^2 \\ & + \langle \kappa_{RA} \rangle [2\langle h^{11} \rangle \omega^+ \omega^- + \langle h^{33} \rangle \omega_0^2 + \langle h^{44} \rangle H^2] + \dots \end{aligned} \quad (\text{F.14})$$

Requiring $\kappa^{-1}\kappa = 1$ we find:

$$\begin{aligned} \kappa^{-1} = & \langle \kappa \rangle^{-1} - \langle \kappa \rangle^{-1} \langle \kappa_R \rangle \langle \kappa \rangle^{-1} \sqrt{h^{44}} H \\ & - \langle \kappa \rangle^{-1} [\langle \kappa_{RR} \rangle - \langle \kappa_R \rangle \langle \kappa \rangle^{-1} \langle \kappa_R \rangle] \langle \kappa \rangle^{-1} \langle h^{44} \rangle H^2 \\ & - \langle \kappa \rangle^{-1} \langle \kappa_{RA} \rangle \langle \kappa \rangle^{-1} [2\langle h^{11} \rangle \omega^+ \omega^- + \langle h^{33} \rangle \omega_0^2 + \langle h^{44} \rangle H^2] + \dots \end{aligned} \quad (\text{F.15})$$

F.2 Variation of F

G^3 combined with a scalar from the metric does not contribute to our Green's functions. Thus, we take only the expectation value and we have

$$F f^{abc} G_{\mu\nu}^A G_{\nu\rho}^B G_{\rho\mu}^c \longrightarrow \langle F \rangle f G_{\mu\nu}^A G_{\nu\rho}^B G_{\rho\mu}^C, \quad (\text{F.16})$$

$$\langle F \rangle = \sum_{n=0}^{\infty} c_G^{(6+2n)} \left(\frac{v^2}{2} \right)^n. \quad (\text{F.17})$$

F.3 Variations of h and H

As discussed in the main text, we decompose the metric h_{IJ} as

$$h_{IJ} = \langle h_{IJ} \rangle + \sum_{n=1}^3 \left[\sum_{r \in \{R,A\}} \langle h_{r,n} \rangle (T_{r,n}) + \frac{1}{2} \sum_{r,p \in \{R,A\}} \langle h_{rp,n} \rangle (T_{rp,n}) + \dots \right], \quad (\text{F.18})$$

with the terms linear in fields given by:

$$\langle h_{R1} \rangle = \sum_{n=0}^{\infty} (n+1)v \left(\frac{v^2}{2} \right)^n c_{HD,2}^{(6+2n)}, \quad (\text{F.19})$$

$$\langle h_{R2} \rangle = - \sum_{n=0}^{\infty} 4nv \left(\frac{v^2}{2} \right)^n c_{H\Box}^{(6+2n)}, \quad (\text{F.20})$$

$$\langle h_{R3} \rangle = \sum_{n=0}^{\infty} nv \left(\frac{v^2}{2} \right)^n c_{HD}^{(6+2n)}, \quad (\text{F.21})$$

$$\langle h_{A1} \rangle = 0, \quad (\text{F.22})$$

$$\langle h_{A2} \rangle = \frac{1}{v} (\langle h_{44} \rangle - \langle h_{33} \rangle), \quad (\text{F.23})$$

$$\langle h_{A3} \rangle = \frac{1}{v} (\langle h_{33} \rangle - \langle h_{11} \rangle). \quad (\text{F.24})$$

and matrix parts given by:

$$T_{R1} = \sqrt{h^{44}} H, \quad (\text{F.25})$$

$$T_{R2} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \sqrt{h^{44}} H, \quad (\text{F.26})$$

$$T_{R3} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 1 & 0 \\ * & * & * & 1 \end{pmatrix} \sqrt{h^{44}} H, \quad (\text{F.27})$$

$$T_{A2} = \begin{pmatrix} 0 & 0 & 0 & -i\sqrt{h^{11}} \frac{\omega^+ - \omega^-}{\sqrt{2}} \\ * & 0 & 0 & \sqrt{h^{11}} \frac{\omega^+ + \omega^-}{\sqrt{2}} \\ * & * & 0 & -\sqrt{h^{33}} \omega^0 \\ * & * & * & 2\sqrt{h^{44}} H \end{pmatrix}, \quad (\text{F.28})$$

$$T_{A3} = \begin{pmatrix} 0 & 0 & -\sqrt{h^{11}} \frac{\omega^+ + \omega^-}{\sqrt{2}} & -i\sqrt{h^{11}} \frac{\omega^+ - \omega^-}{\sqrt{2}} \\ * & 0 & -i\sqrt{h^{11}} \frac{\omega^+ - \omega^-}{\sqrt{2}} & \sqrt{h^{11}} \frac{\omega^+ + \omega^-}{\sqrt{2}} \\ * & * & 2\sqrt{h^{44}} H & 0 \\ * & * & * & 2\sqrt{h^{44}} H \end{pmatrix}. \quad (\text{F.29})$$

For the quadratic dependence on fields we find for the Wilson coefficient dependence:

$$\langle h_{RR1} \rangle = \sum_{n=0}^{\infty} 2n(n+1) \left(\frac{v^2}{2} \right)^n c_{HD,2}^{(6+2n)}, \quad (\text{F.30})$$

$$\langle h_{RR2} \rangle = -8 \sum_{n=0}^{\infty} n(n-1) \left(\frac{v^2}{2} \right)^n c_{H\Box}^{(6+2n)}, \quad (\text{F.31})$$

$$\langle h_{RR3} \rangle = \frac{1}{4} \sum_{n=0}^{\infty} 4n(n-1) \left(\frac{v^2}{2} \right)^n c_{HD}^{(6+2n)}, \quad (\text{F.32})$$

$$\langle h_{RA1} \rangle = \langle h_{AR1} \rangle = \frac{1}{v} \langle h_{R1} \rangle, \quad (\text{F.33})$$

$$\langle h_{RA2} \rangle = \langle h_{AR2} \rangle = \frac{1}{v} \langle h_{R2} \rangle, \quad (\text{F.34})$$

$$\langle h_{RA3} \rangle = \langle h_{AR3} \rangle = \frac{1}{2v} \langle h_{R3} \rangle, \quad (\text{F.35})$$

$$\langle h_{AA1} \rangle = 0, \quad (\text{F.36})$$

$$\langle h_{AA2} \rangle = \frac{1}{v^2} (\langle h_{44} \rangle - \langle h_{33} \rangle), \quad (\text{F.37})$$

$$\langle h_{AA3} \rangle = \frac{1}{2v^2} (\langle h_{33} \rangle - \langle h_{11} \rangle). \quad (\text{F.38})$$

The matrix dependence becomes difficult to express in publication form. In what follows we take,

$$\bar{\omega}^{\pm} \equiv \sqrt{h^{11}} \omega^{\pm}, \quad (\text{D.39})$$

$$\bar{\omega}^0 \equiv \sqrt{h^{33}} \omega^0, \quad (\text{D.40})$$

$$\bar{H} \equiv \sqrt{h^{44}} H, \quad (\text{D.41})$$

which makes our expressions slightly more compact. With these definitions, the matrix dependence is given by:

$$T_{RR1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ * & 1 & 0 & 0 \\ * & * & 1 & 0 \\ * & * & * & 1 \end{pmatrix} \bar{H}^2, \quad (\text{F.39})$$

$$T_{RA1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ * & 1 & 0 & 0 \\ * & * & 1 & 0 \\ * & * & * & 1 \end{pmatrix} (2\bar{\omega}^+ \bar{\omega}^- + \bar{\omega}_0^2 + \bar{H}^2), \quad (\text{F.40})$$

$$T_{RR2} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 1 \end{pmatrix} \bar{H}^2, \quad (\text{F.41})$$

$$T_{RA2} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 1 \end{pmatrix} (2\bar{\omega}^+ \bar{\omega}^- + \bar{\omega}_0 \bar{\omega}_0 + \bar{H}^2), \quad (\text{F.42})$$

$$T_{AR2} = \begin{pmatrix} 0 & 0 & 0 & -i\sqrt{2}(\bar{\omega}^+ - \bar{\omega}^-)\bar{H} \\ * & 0 & 0 & \sqrt{2}(\bar{\omega}^+ + \bar{\omega}^-)\bar{H} \\ * & * & 0 & -2\bar{\omega}_0\bar{H} \\ * & * & * & 4\bar{H}^2 \end{pmatrix}, \quad (\text{F.43})$$

$$T_{AA2} = \begin{pmatrix} -(\bar{\omega}^+ - \bar{\omega}^-)^2 & -i[(\bar{\omega}^+)^2 - (\bar{\omega}^-)^2] & i\sqrt{2}(\bar{\omega}^+ - \bar{\omega}^-)\bar{\omega}_0 & \sqrt{2}(\bar{\omega}^+ - \bar{\omega}^-)\bar{H} \\ * & (\bar{\omega}^+ + \bar{\omega}^-)^2 & -\sqrt{2}(\bar{\omega}^+ + \bar{\omega}^-)\bar{\omega}_0 & \sqrt{2}(\bar{\omega}^+ + \bar{\omega}^-)\bar{H} \\ * & * & 2\bar{\omega}_0^2 & -2\bar{\omega}_0\bar{H} \\ * & * & * & 2\bar{H}^2 \end{pmatrix}, \quad (\text{F.44})$$

$$T_{RR3} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 1 & 0 \\ * & * & * & 1 \end{pmatrix} \bar{H}, \quad (\text{F.45})$$

$$T_{RA3} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 1 & 0 \\ * & * & * & 1 \end{pmatrix} (2\bar{\omega}^+ \bar{\omega}^- + \bar{\omega}_0^2 + \bar{H}^2), \quad (\text{F.46})$$

$$T_{AR3} = \begin{pmatrix} 0 & 0 & -2\sqrt{2}(\bar{\omega}^+ + \bar{\omega}^-)\bar{H} & -2i\sqrt{2}(\bar{\omega}^+ - \bar{\omega}^-)\bar{H} \\ * & 0 & -2i\sqrt{2}(\bar{\omega}^+ - \bar{\omega}^-)\bar{H} & 2\sqrt{2}(\bar{\omega}^+ + \bar{\omega}^-)\bar{H} \\ * & * & 8\bar{H}^2 & 0 \\ * & * & * & 8\bar{H}^2 \end{pmatrix}, \quad (\text{F.47})$$

$$T_{AA3} = \begin{pmatrix} 8\bar{\omega}^+ \bar{\omega}^- & 0 & 2i\sqrt{2}[\bar{\omega}_0(\bar{\omega}^+ - \bar{\omega}^-) + i\bar{H}(\bar{\omega}^+ + \bar{\omega}^-)] & -2\sqrt{2}[\bar{\omega}_0(\bar{\omega}^+ + \bar{\omega}^-) + i\bar{H}(\bar{\omega}^+ - \bar{\omega}^-)] \\ * & 8\bar{\omega}^+ \bar{\omega}^- & -2\sqrt{2}[\bar{\omega}_0(\bar{\omega}^+ + \bar{\omega}^-) + i\bar{H}(\bar{\omega}^+ - \bar{\omega}^-)] & -2i\sqrt{2}[\bar{\omega}_0(\bar{\omega}^+ - \bar{\omega}^-) + i\bar{H}(\bar{\omega}^+ + \bar{\omega}^-)] \\ * & * & 4(\bar{\omega}_0^2 + \bar{H}^2) & 0 \\ * & * & * & 4(\bar{\omega}_0^2 + \bar{H}^2) \end{pmatrix}. \quad (\text{F.48})$$

We can re-express the H_{IJ} metric immediately using the above relations and appropriate renaming of

Wilson coefficients, e.g. $c_{HD,2}^{(6+2n)} \rightarrow c_{DH,1}^{(6+2n)}$. Our expanded expression for H_{IJ} then simply takes the form:

$$H_{IJ} = \langle H_{IJ} \rangle + \sum_{n=1}^3 \left[\sum_{r \in \{R,A\}} \langle H_{r,n} \rangle (T_{r,n}) + \frac{1}{2} \sum_{r,p \in \{R,A\}} \langle H_{rp,n} \rangle (T_{rp,n}) + \dots \right]. \quad (\text{F.49})$$

F.4 Variation of g

Following the previous parts of this section, we expand g_{AB} as:

$$g_{AB} = \langle g_{AB} \rangle + \sum_{n=1}^4 \left[\sum_{r \in \{R,A\}} \langle g_{r,n} \rangle (T_{r,n}^g) + \frac{1}{2} \sum_{r,p \in \{R,A\}} \langle g_{rp,n} \rangle (T_{rp,n}^g) + \dots \right], \quad (\text{F.50})$$

The Wilson coefficient dependence is given by:

$$\langle g_{11} \rangle = \langle g_{22} \rangle = -4 \sum_{n=0}^{\infty} \left(\frac{v^2}{2} \right)^{n+1} c_{HW}^{(6+2n)}, \quad (\text{F.51})$$

$$\langle g_{33} \rangle = -4 \sum_{n=0}^{\infty} \left(\frac{v^2}{2} \right)^{n+1} \left[c_{HW}^{(6+2n)} + \frac{v^2}{2} c_{HW,2}^{(8+2n)} \right], \quad (\text{F.52})$$

$$\langle g_{34} \rangle = 2 \sum_{n=0}^{\infty} \left(\frac{v^2}{2} \right)^{n+1} c_{HWB}^{(6+2n)}, \quad (\text{F.53})$$

$$\langle g_{44} \rangle = -4 \sum_{n=0}^{\infty} \left(\frac{v^2}{2} \right)^{n+1} c_{HB}^{(6+2n)}. \quad (\text{F.54})$$

For the single variation dependence we have:

$$\langle g_{R1} \rangle = -4 \sum_{n=0}^{\infty} (n+1) v \left(\frac{v^2}{2} \right)^n c_{HW}^{(6+2n)}, \quad (\text{F.55})$$

$$\langle g_{R2} \rangle = -4 \sum_{n=0}^{\infty} (n+1) v \left(\frac{v^2}{2} \right)^n c_{HB}^{(6+2n)}, \quad (\text{F.56})$$

$$\langle g_{R3} \rangle = - \sum_{n=0}^{\infty} 2n v^3 \left(\frac{v^2}{2} \right)^n c_{HW,2}^{(8+2n)}, \quad (\text{F.57})$$

$$\langle g_{R4} \rangle = \sum_{n=0}^{\infty} 2n v \left(\frac{v^2}{2} \right)^n c_{HWB}^{(6+2n)}, \quad (\text{F.58})$$

$$\langle g_{A1} \rangle = 0, \quad (\text{F.59})$$

$$\langle g_{A2} \rangle = 0, \quad (\text{F.60})$$

$$\langle g_{A3} \rangle = \frac{2}{v} (\langle g_{33} \rangle - \langle g_{11} \rangle), \quad (\text{F.61})$$

$$\langle g_{A4} \rangle = \frac{2}{v} \langle g_{34} \rangle. \quad (\text{F.62})$$

The corresponding matrix dependence is given by:

$$T_{R1}^g = \begin{pmatrix} 1 & 0 & 0 & 0 \\ * & 1 & 0 & 0 \\ * & * & 1 & 0 \\ * & * & * & 0 \end{pmatrix} \bar{H}, \quad (\text{F.63})$$

$$T_{R2}^g = \begin{pmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 1 \end{pmatrix} \bar{H}, \quad (\text{F.64})$$

$$T_{R3}^g = \begin{pmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 1 & 0 \\ * & * & * & 0 \end{pmatrix} \bar{H}, \quad (\text{F.65})$$

$$T_{A3}^g = \begin{pmatrix} 0 & 0 & -\frac{1}{\sqrt{2}}(\bar{\omega}^+ + \bar{\omega}^-) & -\frac{i}{\sqrt{2}}(\bar{\omega}^+ - \bar{\omega}^-) \\ * & 0 & -\frac{i}{\sqrt{2}}(\bar{\omega}^+ - \bar{\omega}^-) & \frac{1}{\sqrt{2}}(\bar{\omega}^+ + \bar{\omega}^-) \\ * & * & 2\bar{H} & 0 \\ * & * & * & 0 \end{pmatrix}, \quad (\text{F.66})$$

$$T_{R4}^g = \begin{pmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 0 & 1 \\ * & * & * & 0 \end{pmatrix} \bar{H}, \quad (\text{F.67})$$

$$T_{A4}^g = \begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}}(\bar{\omega}^+ + \bar{\omega}^-) \\ * & 0 & 0 & -\frac{i}{\sqrt{2}}(\bar{\omega}^+ - \bar{\omega}^-) \\ * & * & 0 & \bar{H} \\ * & * & * & 0 \end{pmatrix}. \quad (\text{F.68})$$

For terms quadratic in the scalar fields we have:

$$\langle g_{RR1} \rangle = -4 \sum_{n=0}^{\infty} 2n(n+1) \left(\frac{v^2}{2} \right)^n c_{HW}^{(6+2n)}, \quad (\text{F.69})$$

$$\langle g_{RR2} \rangle = -4 \sum_{n=0}^{\infty} 2n(n+1) \left(\frac{v^2}{2} \right)^n c_{HB}^{(6+2n)}, \quad (\text{F.70})$$

$$\langle g_{RR3} \rangle = - \sum_{n=0}^{\infty} 4n(n-1)v^2 \left(\frac{v^2}{2} \right)^n c_{HW,2}^{(8+2n)}, \quad (\text{F.71})$$

$$\langle g_{RR4} \rangle = \sum_{n=0}^{\infty} 4n(n-1) \left(\frac{v^2}{2} \right)^n c_{HWB}^{(6+2n)}, \quad (\text{F.72})$$

$$\langle g_{RA1} \rangle = \langle g_{AR1} \rangle = \frac{1}{v} \langle g_{R1} \rangle, \quad (\text{F.73})$$

$$\langle g_{RA2} \rangle = \langle g_{AR2} \rangle = \frac{1}{v} \langle g_{R2} \rangle, \quad (\text{F.74})$$

$$\langle g_{RA3} \rangle = \langle g_{AR3} \rangle = \frac{1}{v} \langle g_{R3} \rangle, \quad (\text{F.75})$$

$$\langle g_{RA4} \rangle = \langle g_{AR4} \rangle = \frac{1}{v} \langle g_{R4} \rangle, \quad (\text{F.76})$$

$$\langle g_{AA1} \rangle = 0, \quad (\text{F.77})$$

$$\langle g_{AA2} \rangle = 0, \quad (\text{F.78})$$

$$\langle g_{AA3} \rangle = \frac{1}{v^4} (\langle g_{33} \rangle - \langle g_{11} \rangle), \quad (\text{F.79})$$

$$\langle g_{AA4} \rangle = \frac{2}{v^2} \langle g_{34} \rangle. \quad (\text{F.80})$$

The relevant matrix structures are then given by:

$$T_{RR1}^g = \begin{pmatrix} 1 & 0 & 0 & 0 \\ * & 1 & 0 & 0 \\ * & * & 1 & 0 \\ * & * & * & 0 \end{pmatrix} \bar{H}^2, \quad (\text{F.81})$$

$$T_{RA1}^g = \begin{pmatrix} 1 & 0 & 0 & 0 \\ * & 1 & 0 & 0 \\ * & * & 1 & 0 \\ * & * & * & 0 \end{pmatrix} (2\bar{\omega}^+ \bar{\omega}^- + \bar{\omega}_0^2 + \bar{H}^2), \quad (\text{F.82})$$

$$T_{RR2}^g = \begin{pmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 1 \end{pmatrix} \bar{H}^2, \quad (\text{F.83})$$

$$T_{RA2}^g = \begin{pmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 1 \end{pmatrix} (2\bar{\omega}^+ \bar{\omega}^- + \bar{\omega}_0^2 + \bar{H}^2), \quad (\text{F.84})$$

$$T_{RR3}^g = \begin{pmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 1 & 0 \\ * & * & * & 0 \end{pmatrix} \bar{H}^2, \quad (\text{F.85})$$

$$T_{RA3}^g = \begin{pmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 1 & 0 \\ * & * & * & 0 \end{pmatrix} (2\bar{\omega}^+ \bar{\omega}^- + \bar{\omega}_0^2 + \bar{H}^2), \quad (\text{F.86})$$

$$T_{AR3}^g = \begin{pmatrix} 0 & 0 & -2\sqrt{2}(\bar{\omega}^+ + \bar{\omega}^-)\bar{H} & 0 \\ * & 0 & -2i\sqrt{2}(\bar{\omega}^+ - \bar{\omega}^-)\bar{H} & 0 \\ * & * & 8\bar{H}^2 & 0 \\ * & * & * & 0 \end{pmatrix}, \quad (\text{F.87})$$

$$T_{AA3}^g = \begin{pmatrix} 4(\bar{\omega}^+ + \bar{\omega}^-)^2 & 4i[(\bar{\omega}^+)^2 - (\bar{\omega}^-)^2] & 2i\sqrt{2}[\bar{\omega}_0(\bar{\omega}^+ - \bar{\omega}^-) + 3i(\bar{\omega}^+ + \bar{\omega}^-)\bar{H}] & * \\ * & -4(\bar{\omega}^+ - \bar{\omega}^-)^2 & -2\sqrt{2}[\bar{\omega}_0(\bar{\omega}^+ + \bar{\omega}^-) + 3i(\bar{\omega}^+ - \bar{\omega}^-)\bar{H}] & * \\ * & * & 4(\bar{\omega}_0^2 - 2\bar{\omega}^+ \bar{\omega}^- + 3\bar{H}^2) & 0 \\ * & * & * & 0 \end{pmatrix}, \quad (\text{F.88})$$

$$T_{RR4}^g = \begin{pmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 0 & 1 \\ * & * & * & 0 \end{pmatrix} \bar{H}^2, \quad (\text{F.89})$$

$$T_{RA4}^g = \begin{pmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 0 & 1 \\ * & * & * & 0 \end{pmatrix} (2\bar{\omega}^+ \bar{\omega}^- + \bar{\omega}_0^2 + \bar{H}^2), \quad (\text{F.90})$$

$$T_{AR4}^g = \begin{pmatrix} 0 & 0 & 0 & -2\sqrt{2}(\bar{\omega}^+ + \bar{\omega}^-)\bar{H} \\ * & 0 & 0 & -2i\sqrt{2}(\bar{\omega}^+ - \bar{\omega}^-)\bar{H} \\ * & * & 0 & 4\bar{H}^2 \\ * & * & * & 0 \end{pmatrix}, \quad (\text{F.91})$$

$$T_{AA4}^g = \begin{pmatrix} 0 & 0 & 0 & 2i\sqrt{2}[\bar{\omega}_0(\bar{\omega}^+ - \bar{\omega}^-) + i(\bar{\omega}^+ + \bar{\omega}^-)\bar{H}] \\ * & 0 & 0 & -2\sqrt{2}[\bar{\omega}_0(\bar{\omega}^+ + \bar{\omega}^-) + i(\bar{\omega}^+ - \bar{\omega}^-)\bar{H}] \\ * & * & 0 & 2(\bar{\omega}_0^2 - 2\bar{\omega}^+ \bar{\omega}^- + \bar{H}^2) \\ * & * & * & 0 \end{pmatrix}. \quad (\text{F.92})$$

As in the case of h_{IJ} and H_{IJ} we can directly re-express Y_{AB} in terms of the forms for g_{AB} derived above. Our expanded form of Y is then given by:

$$Y_{AB} = \langle Y_{IJ} \rangle + \sum_{n=1}^4 \left[\sum_{r \in \{R,A\}} \langle Y_{r,n} \rangle (T_{r,n}^g) + \frac{1}{2} \sum_{r,p \in \{R,A\}} \langle Y_{rp,n} \rangle (T_{rp,n}^g) + \dots \right]. \quad (\text{F.93})$$

As it plays a part in the gauge fixing, we also need the general inverse of the metric g_{AB} . This can be found by requiring that $g_{AB}^{-1} \equiv g^{AB}$ vanishes order by order in fields. Defining,

$$(g_1)_{AB} = \sum_{n=1}^4 \sum_{r \in \{R,A\}} \langle g_{r,n} \rangle (T_{r,n}^g), \quad (\text{F.94})$$

$$(g_2)_{AB} = \frac{1}{2} \sum_{n=1}^4 \sum_{r,p \in \{R,A\}} \langle g_{rp,n} \rangle (T_{rp,n}^g), \quad (\text{F.95})$$

we find for the inverse:

$$g^{AB} = \langle g^{AB} \rangle - \langle g^{AC} \rangle (g_1)_{CD} \langle g^{DB} \rangle + \langle g^{AC} \rangle (g_1)_{CD} \langle g^{DE} \rangle (g_1)_{EF} \langle g^{FB} \rangle - \langle g^{AC} \rangle (g_2)_{CD} \langle g^{DB} \rangle + \dots. \quad (\text{F.96})$$

Where the Wilson coefficient dependence of the field independent part is given by:

$$\langle g^{11} \rangle = \langle g_{11} \rangle^{-1}, \quad (\text{F.97})$$

$$\langle g^{33} \rangle = \frac{\langle g_{44} \rangle}{\langle g_{33} \rangle \langle g_{44} \rangle - \langle g_{34} \rangle^2}, \quad (\text{F.98})$$

$$\langle g^{34} \rangle = \frac{\langle g_{34} \rangle}{\langle g_{34} \rangle^2 - \langle g_{33} \rangle \langle g_{44} \rangle}, \quad (\text{F.99})$$

$$\langle g^{44} \rangle = \frac{\langle g_{33} \rangle}{\langle g_{33} \rangle \langle g_{44} \rangle - \langle g_{34} \rangle^2}. \quad (\text{F.100})$$

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