universität wien

DIPLOMARBEIT / DIPLOMA THESIS

Titel der Diplomarbeit / Title of the Diploma Thesis

„Do the math in English! An analysis of teacher talk in an Austrian mathematics class with English as the language of instruction“

verfasst von / submitted by

Dominik Steinkellner

angestrebter akademischer Grad / in partial fulfilment of the requirements for the degree of

Magister der Philosophie (Mag. phil.)

Wien, 2019 / Vienna, 2019

Studienkennzahl lt. Studienblatt / degree programme code as it appears on the student record sheet:	A 190 344 406
Studienrichtung lt. Studienblatt / degree programme as it appears on the student record sheet:	Lehramtsstudium UF Englisch UF Mathematik
Betreut von / Supervisor:	Univ.-Prof. Mag. Dr. Ute Smit

"The acquisition of mathematical ability is a subtle process, but dialogue between learner and teacher is imperative, and this depends on effective communication."

(UNESCO 1974: 8)

Acknowledgments

I would like to thank my supervisor, Univ.-Prof. Mag. Dr. Ute Smit, for her valuable contributions and expertise, her constructive feedback, and for her continuous support during the entire process of conducting this thesis.

I also want to thank all students and the teacher who I observed for the empirical part of this thesis. Thank you for the great cooperation and your flexibility.

Finally, I am very grateful to my family, especially my wife, for their endless patience and support.

Table of contents

List of abbreviations iv

List of figures v

List of tables vi

1 Introduction 1

2 Language and mathematics 3

2.1 A mathematical register 3

2.1.1 Mathematics lexicon 5

2.1.2 Mathematics syntax 6

2.1.3 Mathematics symbolism 7

2.1.4 Visual display 8

2.2 Mathematics as a universal language? 9

3 Language factors in learning mathematics 12

3.1 Basic notations and “Grundvorstellungen” 12

3.2 Development of math and language skills 15

4 Language in the math classroom 17

4.1 Teacher talk in the math classroom 18

4.2 A communicative approach to math teaching 21

4.3 Communication patterns in math teaching 26

4.4 Making mathematics comprehensible 30

4.4.1 Comprehensible input as a basis for understanding in language teaching 31

4.4.2 Comprehensible input in math 33

5 Teaching math in a foreign language 34

5.1 CLIL 39

6 The study 44

6.1 Introduction 44
6.2 Theoretical framework 45
6.2.1 Comprehensible input strategies 46
6.3 Analysis 52
6.3.1 Focus of analysis and research questions 52
6.3.2 Data 54
6.3.2.1 Context 54
6.3.3 Data collection and analysis 55
6.4 Findings 56
6.4.1 Quantitative data 56
6.4.1.1 Number of teacher-turns 57
6.4.1.2 Time of teacher talk 58
6.4.2 Qualitative data 59
6.4.2.1 Introducing the content of the lesson 59
6.4.2.2 Trigonometry 61
6.4.2.3 Tangent 63
6.4.2.4 Similar triangles 65
6.4.2.5 Complementary and supplementary angles 67
6.4.2.6 Adjacent and opposite side 70
6.4.2.7 Radian 75
6.5 Discussion 78
6.5.1 Limitations and implications for research 83
7 Conclusion 84
8 References 86
9 Appendix 92
A) Transcription Conventions 92

B) Abstract in English.....94

C) Abstract in German.....95

List of abbreviations

CIS	Comprehensible input strategies
CLIL	Content language integrated learning
EAL	English as an additional language
EAL/D	English as additional language or dialect
EFL	English as a foreign language
GVs	“Grundvorstellungen” [basic concepts]
IRE	Initiation-response-evaluation
IRE/F	Initiation-response-evaluation/feedback
L1	First language
L2	Second language
MLU	Mean length of utterance
SLAMS	Second language approach to mathematics skills

List of figures

Figure 1 GVs and Modeling (Blum & Leiß 2007 quoted in Vom Hofe & Blum 2016: 235) 14
Figure 2 SLAMS model (Cuevas 1984: 141) 39
Figure 3 Scores on math tests (Surmont et al. 2016: 327) 42
Figure 4 Relative amount of teacher turns in German 58
Figure 5 Relative amount of teacher turns in English 58
Figure 6 Relative amount of teacher talk 58

List of tables

Table 1 Egyptian symbols (Imhausen 2016:23) 12
Table 2 Components of a math-talk learning community (Hufferd-Ackles, Fuson & Sherin 2004: 90) 25
Table 3 Features and benefits of foreigner talk (from Chaudron 1983:439) 46
Table 4 Modification of input (from Pica, Doughty & Young 1986: 123) 47
Table 5 Elements of explicit instruction (based on Archer & Hughes 2011: 2-3) 49
Table 6 Comprehensible input strategies (from Buri 2012: 2-3, 10-13) 51
Table 7 Number of turns and talking time 56
Table 8 Transcription conventions 92

1 Introduction

Although it is beyond dispute that language and mathematics are inextricably linked, the interconnection between both fields is extremely complex. On the one hand, both areas are received as systems of meaning-making, and mathematical diction and symbolism is often referred to as a distinct language. On the other hand, language is of central importance in the pedagogical context of the math classroom (Schleppegrell 2010: 60). Obviously, mathematical ideas and concepts would be hardly conveyable without language. This is especially true for teaching and learning mathematics, where communication is not solely a matter of imparting mathematical information in the most efficient way, but also concerned with explaining and presenting new concepts comprehensibly for students. In other words, math can hardly be taught or learned without language. Nevertheless, the exact interdependency of both fields is hard to define, but discussed controversially by researchers. While the impact of language on math teaching has been traditionally neglected in math teaching, its essential role for learning mathematics has become clear by now and is increasingly acknowledged in modern contemporary teaching approaches (Schleppegrell 2010: 61). The different aspects of the relations between language and math, however, are multi-faceted and not fully understood.

This thesis aims at examining facets of the interweaving of language and mathematics with specific relevance for math teaching. Hence, it, first of all, elaborates on existing research in this area and scrutinizes and juxtaposes relevant findings in the literature. More precisely, the theory part of this thesis expands on features of mathematical language and the aspects of a mathematical register. In this regard, one section concerns the view of math as a universal language. In the next step, the focus shifts to language aspects which are particularly associated with students' learning process in the math classroom. Therefore, this passage addresses the question in how far language does not only transfer information but also influences and shapes learners' understanding and cognition of the same. Thereupon, typical communication patterns in the math classroom are discussed and possible inferences are drawn. Finally, special attention is paid to issues concerned with teaching math in a second language. Especially the impact on the learning

progress of students is scrutinized in this context. However, it is clearly differentiated between L2 as the language of instruction in a multilingual classroom with language minorities and settings in which the language of instruction is a second language for all learners, such as CLIL.

The latter approach, namely integrating content and language learning, is the setting of interest for the second part of this thesis. More precisely, the empirical part is interested in the language use of a teacher in a CLIL math class. Therefore, a comparative analysis examines differences and similarities in teacher talk between German and English as the language of instruction. For that purpose, four lessons, which were taught by the same teacher, were observed and transcribed. Two lessons were taught in German — the teachers' and learners' L1 — and two lessons were taught in English in a different class. As the newly introduced topic is the same in both sample groups, the analysis provides interesting insights into how mathematical content is presented by the teacher in German and in English. A clear focus is the strategies used to make content comprehensible to students. Obviously, in classrooms specially designed to encourage second language acquisition while teaching non-language-specific content, such as math, history, and so forth, language is not solely a preconditioned medium, but the aim of study at the same time. Thus, it seems natural that CLIL teachers need different strategies to convey content and intended meaning than teachers using students' L1 as the language of instruction. In how far the additional focus on language teaching in a CLIL math class affects a teacher's linguistic complexity of input is the focus of this project. The central aspect of this analysis is the use of comprehensible input strategies (CIS). In other words, how does the teacher modify and simplify mathematical content in order to present it more comprehensibly for students? Clearly, the study also intends to question if such modifications are rather due to the complexity of the content or due to English as the language of instruction. For that purpose, corresponding excerpts from both settings, which are concerned with the introduction of mathematical terms or concepts, are juxtaposed and analyzed, using a qualitative approach. Additionally, statistics about the number and length of teacher turns form a quantitative supplement for the study. Remarkable findings and possible inferences are finally summarized and discussed in the penultimate chapter.

2 Language and mathematics

Obviously, language is crucial in teaching and, consequently, learning mathematics — or any other school subject — as it serves as inevitable medium to convey, explain and scrutinize content. Austin & Howson (1979: 162) agree when they state that "[i]n the teaching and learning of mathematics, language plays a vitally important role". Nevertheless, the interconnection between language and mathematics is extremely complex and "the exact relationship between language and mathematics is not clearly understood" (Dale & Cuevas 1987: 24). The literature and relevant research, however, "certainly point toward close interaction" (Dale & Cuevas 1987: 24). The following section will summarize available observations and inferences in this field as it addresses the question in how far language influences the instruction and comprehension of mathematical content and vice versa.

2.1 A mathematical register

Although it is beyond question that all school subjects are constructed in language, the forms and patterns of the language utilized in the classroom vary among the different disciplines. The language used in math, for example, „tends to be conceptually dense, interpersonally alienating, and highly structured textually in unfamiliar ways" (Schleppegrell 2010: 61). Bearing in mind such distinguishable variations of language use in different subjects, the term register serves to characterize typical features of language used in math. According to Biber & Conrad (2009: 6)

> a register is a variety associated with a particular situation of use (including particular communicative purposes). The description of a register covers three major components: the situational context, the linguistic features, and the functional relationships between the first two components.

More condensed, Cuevas (1984: 136) describes language registers as "meanings that serve a particular function in the language, as well as the words and structures that convey those meanings". Consequently, a mathematics register "can be defined as the meanings belonging to the natural language used in mathematics" (Cuevas 1984: 136). A very prominent function of mathematical language is precision, which is mainly implemented by a narrow scope of meaning and little or no room for interpretation for mathematical

terms. Hence, mathematical terms allow for “an almost totally nonredundant and relatively unambiguous language” (Brunner 1976: 209). In his early work, however, Halliday (1974: 65) emphasizes that a register cannot simply be reduced to words and terms and their meaning in a specific context. Rather, “[i]t is the meanings, including the styles of meaning and modes of argument, that constitute a register” (Halliday 1974: 65). He refines the notion of a mathematics register by stating : “[w]e can refer to a ‘mathematics register’, in the sense of the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is being used for mathematical purposes” (Halliday 1974: 65). Naturally, “[e]very language embodies mathematical meanings in its semantic structure — ways of counting, measuring, classifying and so on” (Halliday 1974: 65). Nevertheless, according to Halliday (1974: 65), these elements are neither sufficient for an academic discipline of mathematics nor a mathematical education in secondary schools or colleges. Thus, he concludes, the development of a mathematical register is a “matter of degree” (Halliday 1974: 65). Clearly, students draw on “everyday” language when learning math, but they need to develop a mathematical register through schooling which “uses language in new ways to serve new functions (Schleppegrell 2010: 63). Schleppegrell (2010: 63-64) further argues that the notion of a mathematical register is, first of all, helpful to differentiate mathematical knowledge from knowledge in other academic subjects by means of language. Secondly, it aids in recognizing a way of language use that is required from students to effectively participate in the knowledge of mathematics. “Learning the language of a new discipline is part of learning the new discipline” (Schleppegrell 2010: 63).

Typically, academic registers and especially a mathematics register features some distinctive characteristics, which are summarized by Snow (2010: 450) as “conciseness, achieved by avoiding redundancy; using a high density of information-bearing words, ensuring precision of expression; and relying on grammatical processes to compress complex ideas into few words“. Yet, it is vital to consider the multi-semiotic nature of mathematics or science in general as highlighted by O'Halloran (2008: 10), who describes constructions in math as “discourses formed through choices from the functional sign systems of language, mathematical symbolism and visual display”. In other words, maths

and science draw from three different resources to construct and convey meaning namely language, subject-specific symbols and visualizations like graphs. Hence, language is — although a very prominent — not the only mode concerned with constructing, conveying and understanding mathematical content. “In both written mathematical texts and classroom discourse, these codes alternate as the primary resource for meaning, and also interact with each other to construct meaning” (O'Halloran 1998: 360).

Nevertheless, words and technical terms are an essential aspect of disciplinary language registers as in math. According to Halliday (1974: 65-66), the development of a register in mathematics inevitably involves the introduction of what he calls new “thing-names” in order to refer to new objects. One way this can be done is the reinterpretation of existing words, for example, $set, point, column, weight$, or $random$. Typical ways of introducing new words are creating new words out of native word stock, borrowing words from another language, creating new words in imitation of another language, inventing totally new words, creating locutions and creating words out of non-native word stock. Moreover, the use of words from other contexts elucidates how learning mathematics necessarily involves a process of learning or at least adapting language appropriate for the discipline’s register and vice versa. This aspect is also highlighted by Sigley & Wilkinson (2015: 76) when they point out that “[o]ne aspect of the mathematics register is that certain mathematical terms may take on different or more precise meanings in the mathematics register in comparison with other usage contexts”. In their research, they focus on the interdependency between acquiring subject-specific knowledge or understanding mathematical concepts and the use of the discipline specific register. Therefore, they provide further essential aspects of a mathematics register in order to understand possible difficulties for students to develop mathematical knowledge. More precisely Sigley & Wilkinson (2015: 78) structure prominent aspects of a mathematics register and elaborate on the following aspects of a mathematics register in the math classroom.

2.1.1 Mathematics lexicon

“The mathematics register requires a highly technical and precise language that is densely structured” (Sigley & Wilkinson 2015: 78). Consequently, there is a need for technical

vocabulary that is uniquely mathematical (e.g., *trigonometric ratios, hypotenuse, sine, cosine, quadrilateral*). As mentioned before, however, there is also an intersection of words which describe mathematical concepts but are also used in everyday language (e.g., *length, prove, greater, value*) (Sigley & Wilkinson 2015: 78). Although these words are familiar to learners, they have to be relearned in a sense as they can have a different or more specified meaning in a mathematical context. In many cases, mathematical vocabulary does not occur in isolation but is combined to create more complex chunks or phrases, such as $least\ common\ multiple, negative\ exponent$ or $highest\ common\ factor$ (Dale & Cuevas 1987: 12). Another complex factor of mathematical vocabulary highlighted by Dale & Cuevas (1987: 12-13) is the fact that the same mathematical operation can be indicated or described in different ways. For example, $add, plus, combine, sum$ or $increased\ by$ all signal addition. Nevertheless, the use of different operation specific words is also likely to require another sentence structure and, consequently, open another perspective on one and the same mathematical operation.

2.1.2 Mathematics syntax

Besides a specific lexicon, the mathematical register is also shaped by special syntactic structures and styles of presentation (Dale & Cuevas 1987: 14). "The grammatical patterns unique to the mathematics register include dense noun phrases, nominalizations, logical connectors, and verbs, which are often employed in mathematical arguments, justifications, and constructions of mathematical ideas" (Sigley & Wilkinson 2015: 78). The importance of nominalization in science in order to " create a discourse that moves forward by logical and coherent steps, each building on what has gone before" is also foregrounded by Halliday (1993: 64). According to him, the experiential content goes into nominal groups to allow backgrounding and foregrounding of information in the representation of processes. Thus, "it sets up the logic relationships of one process to another" (happening a causes happening b rather than a happens, so b happens). Exactly the study of these logical relationships is the core of mathematics. Consequently, comparative structures play a vital role in a math register (Knight & Hargis 1977 quoted in Dale & Cuevas 1987: 14). *Greater/less than,* n *times as much* or *as big/small as* are only a few representatives of such structures. Representing operational relationships in math,

however, requires prepositions which, consequently, form a prominent feature of mathematical language as in *divided by four, added to,* or *increased by.*

2.1.3 Mathematics symbolism

Another special attribute of the math register is its diverse but precisely defined range of symbols. Thus, in written form, a mathematical text is usually distinguishable from more general natural language through the occurrence of subject-specific symbols. Hence, learners of mathematics are not only required to learn new concepts and meanings but also a new form of signifiers, namely the symbols used for describing mathematical relations. Although basic symbols such as numbers, $+$ or $-$ are also used in everyday language, an examination of mathematical content in a written form unavoidably entails the encounter of new and often more complex symbols. To allow for a maximum level of precision and unambiguity, the notation of mathematical symbolism "evolved to encode mathematical relations in the most economical way possible, providing a precise, robust and flexible tool for capturing and rearranging mathematical relations without ambiguity" (O'Halloran 2015: 69). Mathematical symbolism introduced a way of describing rather complex relations in a very condensed but still absolutely precise way. Describing the behavior of a curve, for example, elucidates the efficiency of mathematical symbolism. While described in the form of a mathematical function, the complete curve can be expressed with one single formula. A verbal description of a parabola in natural language, on the other hand, would demand a rather complex description with the need to elaborate on each change of direction separately. At the same time, the compact description of a mathematical function accounts for the description of the entire curve with no room for interpretation. This is hardly possible with natural language.

However, it is this particular strength of efficiency and unambiguity of mathematical symbols which accounts for a prominent characteristic of the syntax of mathematical expressions, namely the "lack of one-to-one correspondence between mathematical symbols and the words they represent" (Dale & Cuevas 1987: 15). Dale & Cuevas (1987: 15) illustrate this discrepancy with the expression, $the\ square\ of\ the\ quotient\ of\ a\ and\ b$. To translate this verbal expression into mathematical symbols, one needs to know that the preceded operation of $the\ square\ of$ needs to be translated last and that

parentheses are required in order to square the entire quotient. Coded in mathematical symbols the expression would be represented correctly as $\left(\frac{a}{b}\right)^2$.

Although such an expression of symbols is sufficient to convey its mathematical relations in a written form, translation skills are needed for a verbal discourse about the very same. For this purpose, it is necessary not only to fully comprehend the correct representation of mathematical relations in the form of mathematical symbols but also to translate the correct sequence of operations into the syntax of natural language. O'Halloran (2015: 68-69) supports this view and states that "the critical issue for mathematical symbolic notation is not the individual sign, but rather the systematic ways in which the signs are organized to create meaning. In other words, the major area of concern is the grammar of mathematical symbolism". Obviously, this challenge suggests a special prominence of mathematical symbolism for learners of math, as it is not simply a matter of learning new words and their corresponding signifiers but also de- and encoding the correct array of mathematical symbols to or from the corresponding verbalization in natural language. Hence, the comparison with learning a foreign language and its specific grammatical system does not seem far-fetched.

2.1.4 Visual display

Finally, it is the presence of visualizations and mathematical images which is typical for a mathematics register. O'Halloran (2015) highlights the relevance of images in connection with mathematical discourse in her multimodal approach to the language of mathematics. Although "visual representations have traditionally been mistrusted as a source of 'mathematical truth' and have accordingly been assigned a secondary status" (O'Halloran 2015: 70), visuals are omnipresent in mathematical discourse. Especially when algebra is linked to geometry, mathematical images play a crucial role "to display mathematical entities and relations and their interactions which were encoded symbolically" (O'Halloran 2015: 70). Modern communication technology has offered new opportunities and "[t]he use of computers has expanded the meaning potential of mathematical image" and "the relations between multiple participants may be visualized, leading to complex, high-density images which today are produced using digital technology" (O'Halloran 2015: 70). Visualizing mathematical relations opens up "a vast potential for viewing the

mathematical representation as a whole and the parts in relation to each other (O'Halloran 2015: 71).

2.2 Mathematics as a universal language?

Although the interconnection between math and language is beyond dispute, approaches to conceive and describe the nexus differ dramatically in their radicality. Ranging from the description of characteristics of a mathematical register, as summarized above, to describing math as a language as such (Pimm 1987), to referring to mathematics as a universal language (Parker Waller & Flood 2016).

Concerning the universality of mathematics, the concept implies that "anybody with mathematical understanding can solve mathematical problems regardless of the language they speak" (Adoniou & Qing 2014: 3). "Universal language can be viewed as a conjectural or antique dialogue that is understood by a great deal, if not all, of the world's population" (Parker Waller & Flood 2016: 297). In this debate, it is argued that mathematical relations always express a universal truth, which can be applied anywhere and, thus, is culture free knowledge. "After all, the argument went, "a negative times a negative gives a positive" wherever you are, and triangles over the world have angles which add up to 180 degrees" (Bishop 1988: 179-180). Based on this approach, Parker Waller & Flood (2016: 297) conclude that "math is a universal language because the principles and foundations of mathematics, previously referred to as the mathematical concepts, are the same everywhere around the world". Nevertheless, Bishop (1988: 180) notes that this seemingly universal truth of math is still based on a development which clearly has a cultural history. According to him, different cultural groups can — similar to individual languages or religious beliefs — generate their own mathematics. Hence, the construction of meaning in mathematics is also culturally influenced. "The thesis is therefore developing that mathematics must now be understood as a kind of cultural knowledge, which all cultures generate but which need not necessarily 'look' the same from one cultural group to another" (Bishop 1988: 180).

However, the concept of math being a universal language is certainly challenged when mathematical discourse exceeds the level of written symbolism and images. "While

arithmetical notations may be mutually understood across some languages — although certainly not all — most mathematical tasks that learners encounter in school are not 'language free'" (Adoniou & Qing 2014: 3). Even if the mathematical symbols and their meaning are seen as a universal language, the context of and discourse about the same is not. Bearing in mind the setting of a classroom, where the meaning and structure of mathematical expression are yet to be understood, math teaching relativizes the influence of universality for the process of learning math. "Although mathematics is based on numbers and symbols, teaching it requires a considerable amount of communication" (Hoffert 2009: 132). For that purpose, the use of natural language is inevitable. "Although mathematics is treated as a visual language of symbols and numbers, in the classroom, it is often expressed and explained through written and spoken words" (Lee & Lee 2017: 355). Consequently, Lee & Lee (2017) claim that — at least — the instruction of math is not universal. Analyzing instructions in Korean and English, they describe typological differences between the two languages and elaborate their relevance for math teaching. A closer look at different approaches to expressing numbers bigger than ten in the decimal system, for example, reveals severe differences between languages. Lee & Lee (2017: 356) argue that verbal representations of numbers between ten and twenty are much more predictable in Korean than in English for example. Compared to seemingly arbitrary labels like *eleven, twelve*, or *thirteen* Korean numerals are systematically structured as they are represented by their cumulative value such as *"ten-one", "ten-two"*, or *"ten-three".* However, since a similar logic applies for numbers bigger than twenty even in English, this is not the case for multiples of ten. Again *twenty, thirty,* and *forty* are — besides a morphological relation — arbitrary labels which are represented by multiplications of ten in Korean, like *"two-ten",* or *"three-ten"*. French, on the other hand, uses an arbitrary representation for multiples of ten from twenty to seventy but then represents eighty and ninety as multiple of twenty. Although all verbalizations represent the same numeral value, the logic to express these values can vary significantly between languages and does not appear universal at all. Consequently, Kaput (2002 quoted in Miura *et al.* 1999: 357) claims that "[m]athematical activities cannot be separated from the cultural contexts in which they are situated, and this is particularly true for the language of mathematics".

Even the universality of symbolic representations can be challenged by considering Roman numerals for example. While Arabic numerals are based on ten different digits which suffice to represent —in different combinations — the infinite set of real numbers, the Roman system only uses three digits for representing values from one to ten. However, new digits are introduced for multiples of ten again. The arbitrariness of our seemingly universal system of numeral symbols becomes even more evident by taking account for Egyptian hieroglyphics. This is clearly evident from a juxtaposition of varying mathematical symbols by Imhausen (2016: 23), as illustrated in table 1 below. Although all occurring numbers —only natural and positive rational ones — are represented in a decimal system, it is not a positional system but uses different symbols for each decimal power. All other numeral values are represented by a concatenation of these symbols. Bearing in mind that this list of variations is by far not exhaustive, it seems obvious that representations of mathematical expressions differ between languages and it is not simply the words that differ but also the systems of mathematical representations.

Nevertheless, it is undisputed that the value of the numerals — even if expressed in different ways — is universal indeed. However, if the represented universal truth is the aim to be reached, there are obviously different ways to reach it. Especially in teaching mathematics, it is, first of all, the way of reaching a learning objective and not the formal expression of the same that matters and what language in the math classroom is all about. Hence, it can be stated that different languages and cultures draw on different strategies and systems in order to express mathematical representations and relations. It seems only natural, however, that the linguistic code or the signifier of mathematical expressions also influences and shapes our cognitive process and understanding of the issues represented — even if they are universal. In other words, communication about mathematics in a foreign language does not only require one to one translation skills but — at least to a certain degree — a change of perspectives and adjustment of one's native system of representations. If someone wants to communicate about mathematics in a foreign language, one necessarily also needs to understand and adopt different systems of representation.

Table 1 Egyptian symbols (Imhausen 2016:23)

hieroglyphic sign	Gardiner number	absolute value	phonetic
	Z1	1	*wˁ*
	V20	10	*mḏ*
	V1	100	*št*
	M12	1000	*ḫ3*
	D50	10.000	*ḏbˁ*
	I8	100.000	*ḥfn*
	C11	1.000.000	*ḥḥ*

3 Language factors in learning mathematics

3.1 Basic notations and “Grundvorstellungen”

As highlighted above the verbal expression of mathematical notions is directly influenced by our understanding and perception of mathematical operations — even on a basic level like numerals. In fact, “[o]ne of the central issues that has long captivated research efforts in mathematical education concerns the question of what mental representations people have of mathematical content” (Vom Hofe & Blum 2016: 225). In other words, what meaning is associated with specific mathematical content or operations and how this is represented mentally by individuals. Addressing such mental representations different categories, such as “intuition”, “use meaning” or “concept image” have been developed. Freudenthal (1983: 33) for example speaks of “mental objects” which need to be constituted and always precede the full attainment of a mathematical concept. He highlights the high effectiveness of mental objects for the comprehension of concepts and, consequently, inverts the established approach of teaching concepts first and applying these concepts afterward. Starting with a problem for application allows a learner to start a process of mathematical modeling and, thus, encourages the

development of mental objects. Language again is the key factor to convey or suggest such basic representations of mathematical concepts. Only with natural language is it possible to translate abstract operations adequately for the resources of learners.

In German math didactics, this field is represented by the concept of "Grundvorstellungen" (GVs) (Vom Hofe & Blum 2016: 225). This concept, which is based on the work of Piaget (1947) is concerned with basic ideas and interpretations of mathematical operations. In other words, they are a description of "fundamental mathematical concepts or methods and its interpretation into real situations. They describe relations between mathematical structures, individual psychological processes and real situations" (Kleine, Kassel & Harvey 2005: 228). As it is the prevailing didactic consensus that math should be taught in the connection with real life problems (Kleine, Kassel & Harvey 2005: 227), finding mathematical abstractions and solutions requires a process of modeling. This process is summarized by Kleine, Kassel & Harvey (2005: 227) with the following phases:

> (1) At first, the complexity of the real situation (RS) has to be focused to the specific problem in hand. You then get a model of reality. (2) This real model (RM) has to be transposed to a mathematical model on a mathematical level. (3) The mathematical model (MM) is solved and you get a mathematical result. (4) Finally the mathematical result (MR) is interpreted with a view to reality.

"Grundvorstellungen" play a crucial role in this process of mathematical modeling. They are the key to a successful transition from concrete real problems to abstract mathematical operations.

> So, *Grundvorstellungen* can be construed as mediating elements or as objects of transition between the world of mathematics and the individual conceptual world of the learner. GVs thus describe relationships between mathematical structures, individual-psychological processes, and subject-related contexts, or, in short: the relationships between mathematics, the individual, and reality. (Vom Hofe & Blum 2016: 231)

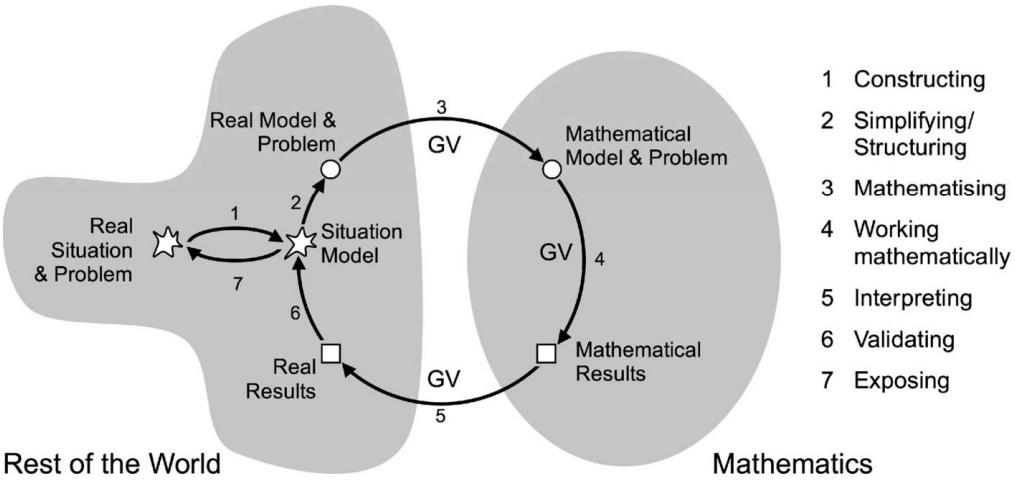

Figure 1 GVs and Modeling (Blum & Leiß 2007 quoted in Vom Hofe & Blum 2016: 235)

One example of the concept of GVs would be the basic representations of dividing. Apart from the clearly defined arithmetic operation and its respective algorithm, which is needed to solve the mathematical problem, a basic mental concept or less abstract image of the operation is needed in order to translate a real problem into a mathematical operation. In the case of dividing two such concepts would be "sharing out" and "splitting up" (Blum & Leiß 2007: 229). "Sharing out" stands for the idea of sharing a quantity among a known number of individuals, for example, share ten apples among five children. "Splitting up", on the other hand, stands for the idea of splitting a quantity into another given quantity. The division now stands for splitting up the original amount and measuring how often the new given amount goes into the original one. An example would be to pour water from a liter bottle into quarter liter glasses. This GV also provides a comprehensible explanation for divisions with divisors smaller than one as the mathematical problem, in this case, would be $1{:}\frac{1}{4}$. In other words, one simply measures how many quarter liter glasses one gets out of a liter of water.

It is not far to seek that in the process of transition between the real world and mathematical abstraction, language, and especially natural language which exceeds a strict mathematical register, is a key factor for comprehension. Hence, regardless of whether math can be considered a universal language or not, making it comprehensible to others always requires natural language and explanations, which are clearly not

universal. On the contrary, as illustrated above, the same mathematical operation can be explained and translated into real problems in many different ways. Based on the concept of mathematical GVs, teaching math clearly demands a targeted use of natural language in order to make more abstract levels of mathematics accessible to students. Thus, the process of learning math is closely intertwined with linguistic development. The language used to communicate about and explain mathematical content shapes and influences our understanding of the same and vice versa.

3.2 Development of math and language skills

The idea that language skills directly affect math performance and vice versa is not new in research and different aspects of this relationship have been addressed in the literature. In his review, Aiken (1972) for example already focuses on the effects of reading abilities on mathematical performance. Given the fact that mathematical problems are often presented as word problems, especially in test situations, it only seems plausible that sufficient reading abilities are indispensable for a positive performance in math. According to Aiken (1972: 359) "[i]t is not difficult to understand how reading ability could affect performance on verbal arithmetic problems, and supporting data are plentiful". However,

> [i]n addition to being related to each other, scores on tests of mathematical and verbal abilities are also correlated with general intelligence. Consequently, the positive correlation between the first two variables may be explicable in terms of their common correlation with the latter variable (Aiken 1972: 362).

Vilenius-Tuohimaa, Aunola & Nurmi (2008: 409) again, support a direct correlation and show that "performance on maths word problems was strongly related to performance in reading comprehension". They further suggest "that both of these skills require overall reasoning abilities" (Vilenius-Tuohimaa, Aunola & Nurmi 2008: 409).

The interplay between reading and math abilities is also reflected in current debates about the validity of math tests which include complex word problems. Walker, Zhang & Surber (2008: 162) address this issue in their study since "[m]any teachers and curriculum specialists claim that the reading demand of many mathematics items is so great that

students do not perform well on mathematics tests, even though they have a good understanding of mathematics". The results of their study support the argument and show an influence of reading abilities on particular math items in tests. However, they also suggest that eliminating such items from math tests would not increase the validity. On the contrary, it "would result in decreasing the construct validity of the test because these items primarily consist of items related to higher order thinking skills such as problem solving and reasoning" (Walker, Zhang & Surber 2008: 179). Reducing a math test to naked number items would only provide information about computation skills. Hence, such tests simply would not cover the demands of math education which also claim reasoning and problem-solving skills. From this perspective, it seems hardly viable to clearly differentiate between reading and math performance in testing. Furthermore, this argumentation implicates a considerable degree of interdependence of mathematical performance and reading skills.

However, it is not only reading skills that closely interfere with mathematical performance. At a much earlier stage of development, when children cannot even read, a number of non-mathematical factors are linked to mathematical development. Purpura & Ganley (2014) for example, focused on the influence of working memory and language on the mathematical performance of four to six-year-old preschool and kindergarten children. The results of their study show that "language was a significant predictor of nearly all mathematics skills and concepts" (Purpura & Ganley 2014: 112). Although language skills were only a marginally significant predictor of both verbal counting and one-to-one counting of sets of objects, language was significantly related to all other mathematical skills assessed, showing the importance of language skills for mathematical development. (Purpura & Ganley 2014: 112-115). Lefevre *et al.* (2010), again, suggest three main pathways for the development of mathematics namely linguistic abilities, spatial attention, and quantitative abilities. Their study, however, prompts that all three factors contribute independently to mathematical outcomes. Although the linguistic pathway affected all mathematical outcomes, the strength of relation clearly depended on the mathematical task. "Thus, as predicted, the relative importance of the pathways depends on the extent to which number system and numerical quantity knowledge is used in the mathematical task" (Lefevre *et al.* 2010: 1763). Purpura & Ganley (2014: 107) also

acknowledge differences in the influence of language skills on performance in math depending on the task. Although they are significantly related to solving story problems, they are less relevant for calculation problems. They ascribe this varying coherence to different linguistic demands. “In story problems, children not only need to be able to complete the mathematical computations but also need to understand that a range of mathematical words can mean the same thing and can be used interchangeably (e.g., ‘‘plus,’’ ‘‘and,’’ ‘‘add,’’ ‘‘together’’)” (Purpura & Ganley 2014: 107). An attempt to specify relations between language skills and mathematical problem-solving skills was made by Vukovic & Lesaux (2013). The findings of their study suggest “that general verbal ability is involved in how children reason numerically whereas phonological skills are involved in executing arithmetic problems” (Vukovic & Lesaux 2013: 90). Again, they approve that word problems acquire a broader set of language skills for successful performance.

All in all, it is undeniable that language skills are directly related to mathematical development and linguistic competence plays a crucial role in the process of learning, comprehending and applying mathematical content.

4 Language in the math classroom

Based on the arguments developed so far, it stands to reason that communication in the classroom can be of considerable influence on the comprehension and learning of mathematical concepts in school. “This sounds like a foreign language to me” or “it’s all Greek to me” are statements that are often used by students to describe particularly, so it seems, their math lessons. No matter if such comparisons of math to foreign languages are drawn for the same reasons as discussed in earlier chapters or simply from the analogy that both a foreign language and math cannot be comprehended by many learners, this metaphor is characteristic of math. At the same time, a description of a good or successful math teacher is often summarized with “She explains it so well”. In both cases, however, teaching and, consequently, learning math is closely tied to the concept of communication. Bennett (2010: 79) agrees and states that “[d]iscourse has long been shown to be influential in supporting students’ learning of mathematics”. However, before the aspects and potential of communication and discourse in math teaching will

be scrutinized, the following section addresses the communicational status quo in the math class. In other words, what is known about characteristic communication features in math teaching and are there any patterns and structures of interaction that typically occur? The main focus, thereby, will be on teacher talk. Not only because it is the matter in question of the empirical part of this thesis, but also because communication in math still is — and many studies in the literature agree (Maier & Schweiger 1999; Franke, Kazemi & Battey 2006; Gregg 1995) — clearly teacher-centered. Thus, the following section addresses crucial aspects of teacher talk and what is known about prevailing communication patterns in the math classroom.

4.1 Teacher talk in the math classroom

The influence of teachers' activities for the quality of teaching and students' achievements in class are beyond question. Brophy (1986: 1069) even claims that "any attempt to improve student achievement must be based on the development of effective teaching behavior". In other words, it is of enormous importance for students' learning process what teachers do and how they behave in class. Unsurprisingly, a substantial part of teaching activities in class is talking and, hence, it is a major concern of researchers to observe and analyze patterns of teacher talk in classroom discourse. In what follows, this section addresses patterns and strategies that are typically observed in teacher talk and how they can influence the learning environment in class — especially with regard to mathematical content.

Although the typical image of a teacher might be associated with explaining subject related content and directly conveying knowledge to the learners, the roles and purposes of teachers' utterances in class are more versatile than that. Christie (2002), therefore, differentiates between different registers of teacher talk depending on the context or aim of what is said by the teacher. What she calls the regulative register is concerned with everything that has "to do with the overall goals, directions, pacing, and sequencing of classroom activity" (Christie 2002: 3). The instructive register, on the other hand, defines utterances that have "to do with the particular 'content' being taught and learned" (Christie 2002: 3). However, even within the respective registers, teacher talk can assume different modes. In more interactional approaches, the teacher can play a less active

communicative role in order to actively engage students in interaction. Examples would be to lead the collectively elaborating class discourse with questions or instructions, to confirm, correct or comment students' utterances or to summarize results of discussions (Maier & Schweiger 1999: 108). In a more traditional, teacher-centered approach teachers are typically concerned with explaining definitions or theorems, illustrating symbols or terms, describing procedures or solution paths or formulating mathematical exercises or problems (Maier & Schweiger 1999: 108). Consequently, teacher talk is not solely based on expertise in the field and content knowledge but is a constant process of decision making in how far to engage learners in classroom discourse or not.

Apart from an active or passive role of the teacher in classroom discourse, it is of particular interest for discourse analysis what the linguistic features that occur in the math classroom are. Although it is easy to say that language is a crucial factor in mathematics, the questions of how and what kinds of language constitute teaching and learning of mathematics content are of considerable complexity (Huang, Normandia & Greer 2005: 4). Driven by a lack of research approaching math classroom discourse from a linguistics perspective Huang, Normandia & Greer (2005: 35) analyzed teacher and student talk in class according to knowledge structures. Based on Mohan (1986: 36-37), they categorized units of discourse as either

> (1) theory aspect knowledge structures or (2) practical aspect knowledge structures. Theory aspect knowledge structures encompassed the subcategories of (a) classification (e.g., definitions, relations among concepts, taxonomic relations), (b) principles (e.g., cause-effect, norms, strategies), and (c) evaluation (e.g., standards, goals). Practical aspect knowledge structures encompassed the subcategories (d) description (e.g., contexts and characteristics), (e) sequence (e.g., process, routines), and (f) choice (e.g., alternatives and dilemmas to be resolved). (Huang, Normandia & Greer 2005: 39)

Their study reflects rich and multiple uses of various knowledge structures (Huang, Normandia & Greer 2005: 39) in teacher talk. The analysis clearly shows involvement of both higher level, theory aspect knowledge structures and lower level, practical aspect knowledge structures. Student discourse, on the other hand, was exclusively constituted by lower-level knowledge structures. The only instances of higher-level structures in

student talk appeared in situations which presented learners in teacher-like positions. This clearly suggests theory aspects of mathematical knowledge to be a unique characteristic of teacher talk in classroom discourse. Mohan (1986: 44) further observed that learners were able to "easily describe an equation or a graph, sequentially tell about procedures they have followed to solve a function and suggest a method or solution". As soon as they were asked to "reference relevant concepts or principles, explain a method used, or justify a decision made for either a method or solution", learners appeared less capable and rather hesitating. Typically, in situations where students failed to articulate theoretical concepts or background, the teacher fell into the trap of taking over the job for them (Huang, Normandia & Greer 2005: 44). Gregg (1995: 459) in his study observes a similar behavior of math teachers. In a qualitative analysis, he highlighted that at each step of a problem the procedure to be used was supplied by the teacher. Although the teacher asked questions, they were not formulated as "how" or "why" questions. Hence, the part of students in classroom discourse was restricted to perform computations which were even identified by the teacher. According to Gregg (1995: 459), "[t]hese emphases contributed to the establishment of a classroom dialogue in which students were not expected to think about when and how to apply procedures". The question is if students can effectively learn and comprehend the concepts and structures which underly these computations without being actively involved in discourse about the same.

Spillane & Zeuli (1999: 14) found that the vast majority of math teachers in their study tend to formulate questions that "required students to do little more than supply the right answer". Although some of the tasks presented in class even directly draw on principled knowledge, students did not get the chance to verbally express their thoughts and knowledge about theoretical aspects as questions posed by the teachers exclusively focussed on procedural knowledge. Hence, teacher questions in math lessons are primarily designed to retrieve facts rather than express conceptual knowledge. However, even if the student's responses were not correct, teachers rarely raised follow-up questions to scrutinize students' mathematical thinking (Spillane & Zeuli 1999: 14). Tasks and discourse of one group of analyzed teachers even "portrayed doing mathematics as a process of memorizing procedures and using these to calculate right answers by

plugging in numbers" (Spillane & Zeuli 1999: 17). Franke, Kazemi & Battey (2006: 231) summarize the prevailing situation in the math class as follows:

> In mathematics classrooms students are typically asked to listen and remember what the teacher said. Usually, little emphasis has been placed on students' explaining their thinking, working publicly through an incorrect idea, making a conjecture, or coming to consensus about a mathematical idea.

Furthermore, Franke, Kazemi & Battey (2006: 229) state that most U.S. mathematics classrooms are dominated by an initiation-response-evaluation (IRE) interaction pattern. Walsh (2011: 17) identifies such an IRE structure as "[o]ne of the most important features of all classroom discourse". This typical pattern comprises three parts, namely a teacher initiation, a student response, and a teacher evaluation or feedback. Evaluation, instead of feedback, however, emphasizes a clear focus on the correctness of student utterances (Walsh 2011: 17). Franke, Kazemi & Battey (2006: 229) also highlight that the evaluation move of math teachers characteristically "focuses on students' answers rather than the strategies they use to arrive at them". The teacher solves the mathematical problem while students' contributions are restricted to providing the next step in a procedure. According to Franke, Kazemi & Battey (2006: 229), "currently many mathematics classrooms do not provide sufficient opportunities for students to develop mathematical understanding".

Although it is consistently controversial, and certainly will remain controversial, which features constitute successful teaching (Franke, Kazemi & Battey 2006: 226), there is more agreement on the assumption that successful learning of mathematical content involves a deeper conceptual understanding of the same. The following chapter will give an overview of strategies and patterns of discourse that support mathematical learning. Furthermore, parallels between math and languages will also be scrutinized from a teaching perspective. In other words, the question at hand is can modern approaches to language teaching also play a role in the math class?

4.2 A communicative approach to math teaching

The fundamental idea of the importance of discourse also in the math class is based on Vygotskij's (1978) sociocultural developmental theory. According to this approach, the development of knowledge and cognition ultimately derives from social interaction. After

the first level of interaction with others, newly encountered concepts are integrated into the individual mental structure on a second level in the process of learning.

> Every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first, between people (interpsychological) and then inside the child (intrapsychological). This applies equally to voluntary attention, to logical memory, and to the formation of concepts. All the higher functions originate as actual relationships between individuals. (Vygotskij 1978: 57)

Given the current development in language teaching, the approach of devoting communication a central role in teaching has become central in modern curricula. However, in contrast to math, active communication is not only a means to an end but the teaching goal at the same time. Hence, it seems reasonable to integrate tasks which represent learning objectives in class activities as well. This concise summary describes the basic idea of the communicative language classroom and it is "premised on the belief that, if the development of communicative language ability is the goal of classroom learning, then communicative practice must be part of the process" (Hedge 2008: 57). What sounds only natural at a first glance, however, is not that self-evident at all. Traditionally, language teaching has often focused on the study of formal systems with a total lack of interactive activities (Hedge 2008: 57). An increasing demand for communicative language abilities clearly challenges this approach and has led to sustainable changes in the language classroom.

In math, on the other hand, communication and interaction abilities are usually not the first objectives that come to one's mind when talking about the goals of mathematical education. Indeed, it is hard to argue the need for a communicative approach in math teaching because of educational aims. Since communicational skills are not a foregrounded learning objective, a product implied learning process does not presuppose interactive communication in the math classroom. Although language competences are separately listed in the Austrian curriculum for AHS-Oberstufe (BMBWF 2018a: 1) as one relevant aspect of math, not all required skills in this field can be directly related to language skills. Thus, the implied parallel to the communicative language classrooms appears far-fetched if based on the same arguments as in language teaching.

However, beyond doubt, communication — particularly speech— is an inevitable factor in the process of teaching math or any other subject as well. “Transfer of knowledge from an experienced other to a novice takes place through the mediation of various tools, particularly speech” (Banse *et al.* 2016: 199). Revisiting the topic of traditionally teacher-centered communication in the math classroom, it is safe to say that students’ active involvement in interaction tends to be neglected. Not least because of this situation, it stands to reason to scrutinize the potential of communicative interaction for the process of learning math.

Going back to Vygotskij’s (1978) theory again, social interaction takes center stage in the process of learning as “[s]peech facilitates social interactions, which in turn produce learning opportunities” (Banse *et al.* 2016: 199). Consequently, communicative interaction, although not the main goal as in the language classroom, is still an absolutely essential means to an end for a successful learning process of mathematical skills. Also, mathematical educators highlight discussion as a key factor for a conceptual understanding of mathematical content. The National Council of Teachers of Mathematics, which operates in the USA and Canada, highlights the importance of learning mathematics with understanding and the positive impact of classroom interaction for conceptual understanding of mathematical content (NCTM 2000: 20-21).

With respect to the influence of language on the development of conceptual understanding, Maier & Schweiger (1999: 17-18) differentiate between two functions of language, namely a communicative and a cognitive function. While the former is concerned with the possibility and process of exchanging thoughts and insights, the latter describes the power to organize individual thinking and to open new opportunities for deeper understanding. Similar categories are introduced by Pimm (1987: 23-24) who speaks about students either “talking for others” or “talking for themselves”. Meyer & Tiedemann (2017: 42), however, highlight the close interweaving between both functions. Given a situation in which the communicative function allows the speaker to impart personal findings and insights, both recipient and speaker can benefit as the exchanged information and eventual feedback can be the basis and trigger for the development of existing new thoughts.

> Articulating aspects of a situation can help the speaker to clarify thoughts and meanings, and hence to achieve a greater understanding. By talking, thoughts are externalized to a considerable extent, which makes them more readily accessible to the speaker's own and other's people's observations (Pimm 1987: 24).

Meyer & Tiedemann (2017: 11) also suggest that the communicative function of language supports the cognitive function. More radically, Vygotskij (1987: 218) claims that verbal expressions never reflect completed thoughts. According to him, speech also structures thoughts and, hence, is an essential part of the process of thinking.

Apart from the undisputed necessity of the communicative function of language for teaching mathematics as a medium to impart knowledge, the consideration of cognitive aspects of language opens an interesting perspective on communication in the math classroom. Drawing on the previous theories and arguments — in the style of communicative language teaching — communicative math teaching would not simply respond to language abilities but also stimulate a deeper understanding of mathematical concepts. In line with this approach, a term recently introduced in math education is math-talk learning communities (Hufferd-Ackles, Fuson & Sherin 2004). "By math-talk learning community, we refer to a classroom community in which the teacher and students use discourse to support the mathematical learning of all participants" (Hufferd-Ackles, Fuson & Sherin 2004: 82). In their framework Hufferd-Ackles, Fuson & Sherin (2004) identify four iterative phases as major components of math-talk learning communities, namely questioning, explaining mathematical thinking, generating mathematical ideas and assuming responsibility for learning.

The focus of questioning in the math-talk learning community clearly is to involve students in classroom interactions and allow their responses to contribute to the classroom discourse. Questioning does not only generate feedback on what students know and understand for the teacher but also "challenges the thinking of the person being questioned by asking for further thinking about his or her work" (Hufferd-Ackles, Fuson & Sherin 2004: 92). However, the cognitive challenge for students and the quality of their responses always depend on the types of questions that are asked. Questions that simply retrieve facts already known by the questioner, for example, contribute differently to

discourse than more open questions which require thorough argumentation or explanation. More details about the role of question types will follow in a later section on teacher talk.

Closely related to questions — as they are the trigger for student responses — are explanations of mathematical thinking. This component of a math-talk learning community aims at providing students with opportunities and abilities to communicate mathematical ideas and concepts. In other words — similar to the communicative language classroom — to let students do the talking about content instead of reducing a math lesson to teacher input only. The role of the teacher is to provide opportunities for communication and support and scaffold students' explanations.

Regarding the source of mathematical ideas, the strategy is again to shift input from the teacher to the students. Rather than anticipating solutions of concepts by explanations of the teacher, the knowledge and especially problem-solving abilities of students should be the main source for developing mathematical concepts.

As the last component of a math-talk learning community, students should also take responsibility for their and others' learning. In the first place, this means a process of co-evaluation in which students listen to each other and initiate clarification for themselves and others. "Students assist each other in understanding and correcting errors" (Hufferd-Ackles, Fuson & Sherin 2004: 90).

Table 2 Components of a math-talk learning community (Hufferd-Ackles, Fuson & Sherin 2004: 90)

Level 3: Teacher as co-teacher and co-learner. Teacher monitors all that occurs, still fully engaged. Teacher is ready to assist, but now in more peripheral and monitoring role (coach and assister).

A. **Questioning**	B. **Explaining mathematical thinking**	C. **Source of mathematical ideas**	D. **Responsibility for learning**
Teacher expects students to ask one another questions about their work. The teacher's questions still may guide the discourse. Student-to-student talk is student-initiated, not dependent on the teacher. Students ask questions and listen to responses. Many questions are "Why?" questions that require justification from the person answering. Students repeat their own or other's questions until satisfied with answers.	*Teacher follows along closely to student descriptions of their thinking, encouraging students to make their explanations more compete; may ask probing questions to make explanations more complete. Teacher stimulates students to think more deeply about strategies.* Students describe more complete strategies; they *defend and justify* their answers with little prompting from the teacher. Students realize that they will be asked questions from other students when they finish, so they are motivated and careful to be thorough. Other students support with active listening.	*Teacher allows for interruptions from students during her explanations; she lets students explain and "own" new strategies. (Teacher is still engaged and deciding what is important to continue exploring.) Teacher uses student ideas and methods as the basis for lessons or miniextensions.* Students interject their ideas as the teacher or other students are teaching, confident that their ideas are valued. Students spontaneously compare and contrast and build on ideas. Student ideas form part of the content of many math lessons.	*The teacher expects students to be responsible for co-evaluation of everyone's work and thinking. She supports students as they help one another sort out misconceptions. She helps and/or follows up when needed.* Students listen to understand, then initiate clarifying other students' work and ideas for themselves and for others during whole-class discussions as well as in small group and pair work. Students assist each other in understanding and correcting errors.

Hufferd-Ackles, Fuson & Sherin (2004) understand their framework as a process during which students do not only develop mathematical skills but also abilities to contribute to classroom discourse and interaction. Based on both teacher and students' behavior their framework describes three levels of a math-talk learning community. As an example, level three is depicted in figure three.

Besides suggestions of experts and comprehensible framework of researchers, there is also empirical evidence for the effects of a communicative approach to math teaching. In a thorough review of the literature, Walshaw & Glenda (2008: 534) come to the conclusion that "[t]hrough students' purposeful involvement in discourse, through listening respectfully to other students' ideas, through arguing and defending their own positions, and through receiving and providing a critique of ideas, students enhance their own knowledge and develop their mathematical identities". Furthermore, they highlight the significant influence of rich cognitive and social experiences in the math classroom on the development of creative thinking and problem-solving skills.

4.3 Communication patterns in math teaching

Bearing in mind the claim for an active integration of students into classroom communication, it is unavoidable to question the role of the teacher in class. Although the arguments elaborated above clearly suggest active communication of students and question a more traditional teacher-centered approach, the math teacher is far from being redundant in a math-talk learning community. No matter if teacher or learner-centered communication, it is the teacher who guides and scaffolds communication. Even if his/her communicative role in the latter setting is of a passive nature, providing and supporting opportunities for communication as well as creating circumstances which enable the development of knowledge and understanding are still in the responsibility of the teacher. Hence, teacher talk — no matter if active or passive — is a central factor for the learning process in the math class. Walshaw & Glenda's (2008: 534) findings also show "that both the cognitive and material decisions that teachers make, in relation to classroom discourse, significantly influence learning". Consequently, teaching mathematics is a constant process of decision making about how to make mathematical content accessible and comprehensible for students, i.e., how to explain the concepts that

are taught. The following section concerns both typical and effective features of teacher talk which aim at conveying knowledge and enable deeper understanding for learners in the math classroom.

According to the Oxford Dictionaries (2019) *to explain* means to "[m]ake (an idea or situation) clear to someone by describing it in more detail or revealing relevant facts". Given this definition, it is safe to say that math teachers —or educators in general —spend ample time of their teaching with explanations. According to Leinhardt (1987: 225), "[t]eaching is the art of transmitting knowledge in a way that ensures the learner receives it". Hence, explanations in a classroom clearly involve special attention on strategies that make content accessible and comprehensible for students. Bearing in mind the unique setting of a classroom, where explanations are aimed at teaching and the explicit dissemination of knowledge among a group of learners, explaining learning content to students is assigned a specific category, namely instructional explanations. More precisely, "[t]hey must coordinate informal colloquial familiar forms of language and understanding with more formal disciplinary ones in the interests of improving learning" (Leinhardt 2010: 3). Thus, the concept of instructional explanation clearly involves "pedagogical moves that engage a group of students and teachers in deep thoughtful activities connected to subject matter learning" (Leinhardt 2010: 2) and can, therefore range from explanations exclusively delivered by the teacher to an interactional process of discussion. In either case, they "are a central deliberate act of teaching" (Leinhardt 2010: 2).

Furthermore, Leinhardt (2010: 4) identifies three significant components of instructional explanations and highlights the importance of a query or problem that needs to be carefully unpacked and is posed with powerful and important questions. Secondly, the completion and interconnection of the launched discussion are crucial. Thus, learners can activate prior knowledge and make connections to previous examples. A third aspect of instructional explanations is the systematic and careful development of examples and representations. Although explanations are definitely not the only category of teaching activities, Leinhardt (2010: 5) stresses the value and validity of the construct: "students learn more when explanations include most of the critical features. In an earlier study,

Leinhardt (1987: 226-227) suggests that — no matter if completed within one lesson or extended over a whole sequence of lessons — an instructional explanation should have the following features:

1. Identification of the goal;
2. Signal monitors indicating progress toward the goal;
3. Examples of the case or instance;
4. Demonstrations that include parallel representations, some level of linkage of these representations, and identification of conditions of use and nonuse;
5. Legitimization of the new concept or procedure in terms of one or more of the following — known principles, cross-checks of representations, and compelling logic;
6. Linkage of new concepts to old through identification of familiar, expanded, and new elements.

For the math classroom, Leinhardt (1987: 226) highlights one core feature within the instruction episode which is "the explanation of new material, including its logical connection to prior knowledge".

Van de Sande & Greeno (2010: 69) again highlight the importance of the role of the learner. Rather than being a neutral recipient of information, it is essential that students are positioned as classroom participants who have a point of view. Hence, it is possible to explain new content with, instead of to, the learner. Only then can the recipient "provide information that contributes to mutual cognizance of the recipient's framing". Consequently, the explainer can draw on the learner's resources and process information in a way which is adequate for the recipient's capabilities, previous knowledge and mental images of the same. A teaching routine that often accompanies the approach of explaining content with, instead of to, the students is the instructional dialogue. "Instructional dialogues are interactive explanatory classroom conversations that serve to both build and transmit knowledge among and to students" (Leinhardt & Steele 2005: 88). According to Lampert *et al.* (2010: 131), this strategy is a "centrepiece of ambitious mathematics teaching". Using instructional dialogues allows for the co-construction of explanations by students and teacher in the math classroom. Leinhardt & Steele (2005: 143-144) found

that this teaching practice often is accompanied by exchange routines in order to integrate students in the instructional explanation process. Typical representations would be the call-on routine, the related revise routine or the clarification routine. While the former is initiated by rather open invitations to discuss a query or problem, the related revise routine is concerned with rethinking assertions and explaining new or different ways of thinking. The clarification routine, on the other hand, intends to clarify confusion that can arise or to scrutinize conjectures or hypotheses constructed by the learners. In such a back-and-forth dialogue, the role of the teacher is to deliberately maintain focus and structure of the explanation process so that mathematical concepts can be co-constructed and are not solely brought in by the teacher (Lampert *et al.* 2010: 131).

In the German "Fachdidaktik" a similar approach is labeled as "fragend-entwickelnder Unterricht" and according to Maier & Schweiger (1999: 134-135), it is the predominant form of math teaching in German classrooms. The motivation for this form of teaching originates from the so-called Socratic method and gained new support during the period of enlightenment. The basic principle of this approach is the attitude that reasonable concepts cannot be just taught by a teacher but also need to be recognized by the learner him/herself through his/her rationality (Maier & Schweiger 1999: 136). However, compared to the concept of instructional dialogues, the term "fragend-entwickelnder Unterricht" appears to be interpreted more broadly. Although the integration of learners in classroom discourse through questions can be easily identified, the didactical quality of classroom discourse also depends on what questions are asked and how interactional patterns are guided by the teacher. Under the lens of structural communication patterns of typical "fragend-entwickelnden Unterricht", Maier & Schweiger (1999: 137) highlight the prominence of what they call "Dreischritt" in classroom discourse. In other words, teacher-student interaction typically occurs in a triadic structure of an impulse by the teacher, a student's response and final comment by the teacher. This is conterminous with the initiation-response-evaluation structure introduced in an earlier chapter and it coincides with the findings of Franke, Kazemi & Battey (2006: 229) for American math classes. They identified such an IRE/F pattern as a typical communication structure in the math classroom.

Bauersfeld (1978 quoted in Maier & Schweiger 1999: 138-139), on the other hand, describes a more complex pattern he observed in math classrooms. What he calls the "Trichter-Muster" basically describes a strategy of leading a learner to an awareness of a mathematical concept which s/he initially did not understand or fully comprehend. Hence, the starting point always is a problem in understanding the content. With specific questions, the teacher tries to elicit the correct answers by the student. In the beginning, these questions are rather broad and require more complex thinking of the student. With every unsuccessful attempt to answer a question they become more focussed and are formulated in a way that direct the learner more and more towards the correct answer. Hence, the model of a funnel. Compared to a straightforward IRF structure, this pattern is not restricted to one student response only. Instead, it stimulates the learner to rethink a problem or misconception and helps to present a concept on a level which is tangible for the student.

Pimm (1987: 52-55), however, highlights that students' responses are often strictly controlled by the teacher's questioning behaviour in practice. Comparable to the cloze procedure in reading, where learners have to fill in gaps of missing words in a text, questions in the math class often only allow or elicit single word answers. Hence, although involved in an interaction process, students do not get the chance to formulate more complex sentences and their utterances are constrained by the teacher to a very high degree. Consequently, the teacher can focus attention on specific items or particularly important aspects of the content being taught but still maintains control of the discourse. Nevertheless, questions interrupt teacher talk and convey a less monotonous style of teaching. Furthermore, asking such display questions (Long & Sato 1983: 271) to which the answer is known already by the questioner allows the teacher to check whether a learner has grasped the content which is being explained.

4.4 Making mathematics comprehensible

Despite different approaches to teaching and distinctive forms of teacher talk, all teaching starts from the same level and encounters the same challenge, namely imparting knowledge in a way which is appropriate for the mental resources of the learner. In other words, teachers can only build on students' prior knowledge and existing capabilities in

order to generate new knowledge. The major challenge thereby is to present new concepts, which usually exceed the intellectual resources of learners, in a way which is still accessible for them and creates a gap between existing and new knowledge that can be cognitively overcome by the students. In other words, it is necessary to find strategies which make input comprehensible for learners.

4.4.1 Comprehensible input as a basis for understanding in language teaching

In language teaching, this challenge is extremely obvious concerning the language of instruction of the language teacher. If the language used by the teacher is the target language itself — which clearly is state of the art in language teaching nowadays and also default by the curriculum in Austria (BMBWF 2018b: 2)— how can learners possibly understand and follow the content of a lesson? The consensus in language teaching is that it is possible if teachers align their language along with the level of their learners and adapt their input accordingly. A prominent theoretical framework for this approach was introduced by Krashen, which is commonly known as the input hypotheses. The basic claim of this approach is that there is only one way for humans to acquire language — "by understanding messages, or by receiving `comprehensible input`" (Krashen 1985: 2). Understanding, however, does not mean that everything of a message needs to be known or internalized by the learner already. The progress of acquisition, in fact, arises from a new or unknown bit of information in the message. The crucial aspect is, however, that new information is embedded in and supported by the level of the learner. Hence, learners progress in acquiring language by "understanding input that contains structures at our next `stage´ — structures that are a bit beyond our current level of competence" (Krashen 1985: 2). If the current level is i, learning can be depicted as moving to the next level which would then be $i + 1$. Reaching the next level $i + 1$ is only possible by understanding input which contains $i + 1$. In other words, language acquisition takes place when we understand something, and we can understand new information only if it is conveyed with what we know already. To understand in this context means that there is a focus on the meaning and not on the form of the message (Krashen 1982: 21). Consequently, language teaching — if there is a focus on fluent communication — can only be effective by providing comprehensible input to students.

Although Krashen's approach appears plausible and is supported by evidence mainly from first language acquisition in children and the input of "caretaker speech" (Krashen 1982: 22), a legitimate question arises — how do we understand new language structures that have not been acquired yet? Krashen (1982: 21) argues that humans use more than just linguistic competence to decode and understand language. We also draw on context, knowledge of the world and extra-linguistic information — meaning all aspects of communication which are not expressed through language — that help us understand. Hence, a language teacher is not only restricted to the linguistic competence of learners but can also draw on a broader base of resources to make input comprehensible. In a second step, however, it is essential to focus on how this approach can be put into practice in the classroom. In other words, how can input be made comprehensible for learners?

Krashen (1982: 64) classifies two basic options to aid comprehension and provide what he calls optimal input, linguistic and non-linguistic support. Typical representatives of the latter in language teaching are the use of imitative body language, objects or pictures to support the understanding of new words or structures. Furthermore, teachers can take advantage of students' knowledge when incorporating topics or situations which are relevant or in the personal interest of students. Generally, optimal input needs to be interesting and relevant for learners, so that acquirers focus on the message and not on the form (Krashen 1982: 66).

With regard to linguistic features that support comprehension, Hatch (1979 quoted in Krashen 1982: 64) summarized linguistic characteristics of simplified input. Among these aspects, which appear to promote comprehension, are:

- Slower rate and clearer articulation, which helps acquirers to identify word boundaries more easily, and allows more processing time;
- More use of high frequency vocabulary, less slang, fewer idioms;
- Syntactic simplification, shorter sentences.

Pica, Doughty & Young (1986: 122) again summarize the following types of modification:

- Repetition and paraphrase of linguistic constituents;
- Restriction of lexis to more common and familiar items;

- Addition of clause boundary markers;
- Reduction in number of embedded and dependent clauses.

More specifically, such modifications often implicate a higher quantity of input, meaning that more words are used in a direction or instruction. More words also automatically result out of redundancy and repetition of input which is typical for simplified input. Furthermore, redundancy is often accompanied by semantic variety and paraphrases. In terms of grammatical structure, modified input tends to be less complex (Pica, Doughty & Young 1986: 123).

According to Krashen (1982: 65), however, this set of typical modification does not represent a set of rules which determines a conscious permanent simplification of speech. Although "[c]onsciously referring to these "rules" might be helpful in occasion, it appears to be the case that we make these adjustments automatically when we focus on trying to make ourselves understood".

What all of these adaption parameters of input have in common, however, is that they are initiated by the instructor or, in the case of a classroom, the teacher. Drawing on Long (1983), Pica, Doughty & Young (1986: 122) referr to this practice as pre-modified input. Interactional modification, on the other hand, "is characterized by the availability of opportunities for non-native speakers to interact with the native speaker, bringing about modifications and restructuring of the interaction by both interlocutors in order to arrive at mutual understanding" (Pica, Doughty & Young 1986: 124). Such modifications are often triggered by requests for input clarification or repetition, comprehension checks and moves which seek input confirmation.

4.4.2 Comprehensible input in math

Although Krashen's input hypothesis is specifically geared towards language acquisition, it stands to reason to scrutinize its viability for other fields as well. Obviously, content in math teaching is constructive and the introduction and successful comprehension of new concepts draw on what learners know already. The crucial factor in effective teaching is to present input that meets learners' needs for a successful learning process. According

to Small & Lin (2010: 2), one approach to meet student needs is providing tasks within students' zone of proximal development. Introduced by Vygotskij (1978: 86) the zone of proximal development describes the "distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers". Related to the math classroom this means that "[i]nstruction within the zone of proximal development allows students, whether with guidance from the teacher or by working with other students, to access new ideas that are beyond what the students know but within their reach" (Small & Lin 2010: 2). This is in line with what Krashen claims as essential for language acquisition and provides a more comprehensive explication of his corresponding formula of $i + 1$ for comprehensible input which is, according to Krashen, the premise for language acquisition.

After this juxtaposition of math and language and the examination of parallels, connections and interdependency in both fields — especially in the classroom — the following chapter will focus on a setting where both math and language teaching go hand in hand explicitly, namely math classes in which the language of instruction is another than the students' first language.

5 Teaching math in a foreign language

The interest in issues of teaching math in a language other than students L1 is evident in the literature and is addressed by researches like Adoniou & Qing (2014), Barton *et al.* (2005), Setati & Adler (2000) or Surmont *et al.* (2016). However, the increasing interest in this field draws from two different backgrounds which will be differentiated in the following review as well. Firstly, the impact of a language of instruction other than the learners' native language — in mathematics as well as in other content subjects — is gaining interest due to demographical reasons. In our globalized world, multilingualism in classrooms is increasing and teachers are often confronted with students who are not proficient or fluent in the language of instruction. Secondly, language teaching has long gone beyond the setting of a pure language classroom. A prevalent concept which facilitates second language acquisition in regular content subjects is the approach of

explicitly integrating language and content learning and is also applied in the math classroom. Referred to as content language integrated learning (CLIL), this approach is also common in Austrian schools. As this is the setting of the empirical study of this thesis, it will be addressed separately in a later section.

Although the two reasons for teaching math in an additional language are not identical in the sense that in CLIL all learners, as well as the teacher, are confronted with an L2 as the language of instruction, there are interesting insights from studies in settings where second language acquisition is not an explicit goal of the math classroom, which will be summarized in the following.

A term commonly used in the literature for this field is bilingual education. Clearly, the term suggests the involvement of two different languages, but a distinct definition of a bilingual speaker raises more issues that need to be considered. Baker (2001: 2), for example, addresses aspects like fluency or the amount of use of both languages as well as the question if bilingualism is an individual characteristic or always bound to a language community. According to Baker, all these factors need to be considered and he demands a careful differentiation of the term bilingualism. “Defining exactly who is or is not bilingual is essentially elusive and ultimately impossible” (Baker 2001: 15). Hence, it comes as no surprise that different interpretations of the term bilingual vary depending on the amount of use and level of proficiency in both languages (Moschkovich 2010: 10). According to Valdés-Fallis (1978: 4), bilingualism is “the product of a specific linguistic community that uses one of its languages for certain functions and the other for other functions or situations”. Considering this definition, it is a misunderstanding that bilinguals are automatically equally fluent in their two languages (Moschkovich 2010: 10). Hence, the term bilingualism is indeed appropriate for learners with a minority L1. Nevertheless, it is crucial to clearly declare the level of proficiency in the L2 for the validity of a study in this field.

Bearing in mind the complexity of language factors in the math classroom which has been elaborated so far, teaching math in an additional language certainly constitutes a further challenge for both teachers and learners.

> Whilst all learners find the shifts in language use in the mathematics classroom challenging, it is particularly problematic for learners who speak English as an additional language or dialect (EAL/D) as they are learning the English language at the same time as they are learning mathematics through that language (Adoniou & Qing 2014: 3).

Accounted for by the increasing number of EAL/D learners in the US, UK, and Australian classrooms, Adoniou & Qing (2014: 3) highlight the need for understanding the language challenges for these students and suggest that teachers must develop skills to help learners overcome such challenges in math. Barton *et al.* (2005: 726) found "that EAL students suffer a disadvantage due to language when studying mathematics". One of their studies showed "that the disadvantage in mathematics achievement due to language suffered by EAL students was at least 10% (Barton *et al.* 2005: 722). A finding which was further supported by succeeding studies of the research team. Particularly critical factors identified in one of their studies were discourse density and logical structure in math classroom communication. Even for EAL students with better language proficiency who, therefore, have fewer disadvantages on vocabulary and syntax items, "the logical complexity of third-year mathematics in English is well beyond the capabilities of many of them" (Barton *et al.* 2005: 726). Neville-Barton & Barton (2005: 13) highlight that "prepositions and word order were key features causing problems at all levels. So also were logical structures such as implication, conditionals, and negation, both at senior secondary and third year university levels".

According to Cummins (1979: 222), an essential factor for successful bilingual learning is proficiency in the L1. In his view, a "cognitively and academically beneficial form of bilingualism can be achieved only on the basis of adequately developed first language (L1) skills". With regard to the math classroom, this would assume sufficient language skills in their L1 for students learning maths in a second language like English. In other words, there is little chance to comprehend mathematical content in a foreign second language if the required linguistic functions are not even sufficient in the L1. Dawe (1983: 349) supports this claim with empirical data from bilingual learners of mathematics showing that "the ability of the child to make effective use of the cognitive functions of his first language is a good predictor of his ability to reason deductively in English as a second language". In his study, he observed bilingual Punjabi, Mirpuri, Italian and Jamaican 11-

13-year-old children growing up in England and basically confirms that competence in the first language is a significant factor in the ability to reason in mathematics in English (Dawe 1983: 325).

Similarly, Clarkson & Galbraith (1992: 34) found support for the thesis that "students' level of competence in their original tongue and in English, the language of their regular schooling, were significant influences on mathematical performance". According to them, the study conducted in five urban community schools in Papua New Guinea reveals evident support for Cummins' threshold theory. Hence, there are also aspects of bilingualism that might positively influence cognitive growth in learners, but they are "unlikely to come into effect until the child has attained a certain minimum or threshold level of competence in a second language" (Cummins 1979: 229).

Van Rinsveld *et al.* (2016: 78) ascertain that arithmetic problems are solved faster and more accurately in the dominant L1 of bilinguals. They further confirmed that such a language effect occurs on all levels of bilingual proficiency. Nevertheless, it decreases with increasing proficiency in the second language. Their study impressively reveals the impact of language on cognitive processes for mathematical problem-solving.

> Further, the present results showed that not only retrieval of learned solutions but also knowledge about the solving procedures required by complex additions seems to be affected by the language of the task in bilinguals, suggesting that some steps of the complex addition solving rely on verbal processes that are difficult to transfer to another language than the language in which they were initially learned. (Van Rinsveld *et al.* 2016: 78).

A positive influence of bilingualism was found by Swanson, Kong & Petcu (2018: 379) who observed that more proficient bilingual children with Spanish as their first and English as their second language with difficulties in math outperformed less proficient bilingual children with similar problems on measures of math calculation. One interpretation of their finding is that increases in bilingualism are also related to increases in working memory and math performance. "However, the results also strongly suggest that a number of other processes were related to the math computation" (Swanson, Kong & Petcu 2018: 390). Hence, it is difficult to deduce direct causalities of different factors involved in math performance in a bilingual setting.

Setati & Adler (2000: 245) also highlight the importance of considering a bigger picture of factors which influence learning progress:

> The argument is that school performance (and by implication, mathematics achievement) is determined by a complex of inter-related factors. The poor performance of bilingual learners thus cannot be attributed to the learner's language proficiencies in isolation of wider social, cultural and political factors that infuse schooling.

Cuevas & Beech (1983: 490), on the other hand, introduced strategies to compensate for disadvantages for bilinguals in the math classroom. The second language approach to mathematics skills (SLAMS) underlies the basic assumption that "second language learners do not possess many or all of the language skills English-speaking students bring with them to the classroom". Consequently, there is a need to differentiate between a content and a language strand when teaching mathematics to students with a low level of proficiency in the language of instruction. In both strands, which are not independent as the math objective determines the content of the language, there is a need to analyze concepts and skills which need to be taught according to the learning objectives. In a second step, learners' skills in both strands need to be diagnosed in order to decide on preventive strategies which are designed to reinforce skills that are prerequisite to the objectives being taught. Based on the insights gained from this process, the teacher develops instructional activities which are suitable for the learners' proficiency level in both fields language and mathematics.

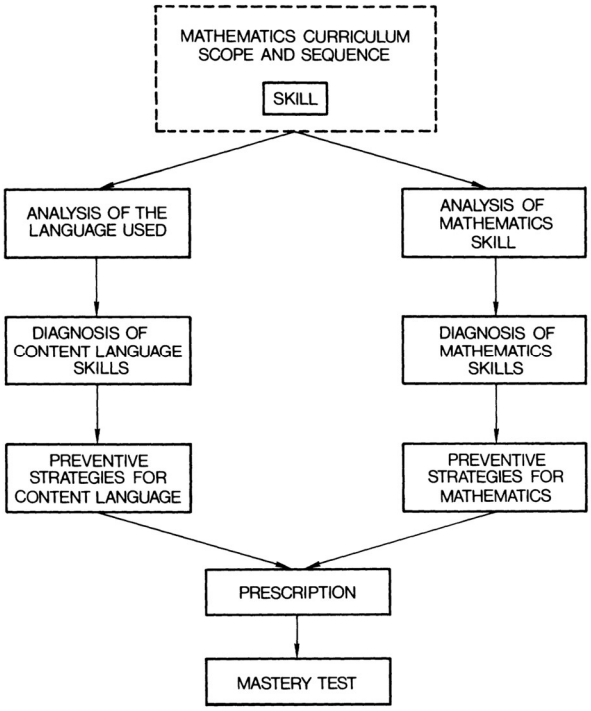

Figure 2 SLAMS model (Cuevas 1984: 141)

5.1 CLIL

The challenges of teaching and learning math in a second language, however, are not restricted to the multilingual classroom with language minorities who are not fully proficient in the language of instruction. Another setting in which content is taught in a language other than the students' L1 is based on the approach of explicitly teaching a second language and subject content at the same time. Hence, the language of instruction in such classrooms is another than the learners' — and in most cases also the teacher's — L1. This approach is referred to as content and language integrated learning and has been gaining relevance in Austrian classrooms over the last decade. "The term *Content-and-Language-Integrated-Learning* (CLIL) refers to educational settings where a language other than the students' mother tongue is used as medium of instruction" (Dalton-Puffer 2007: 1). While this can be the case for any second or foreign language, of course, English is definitely dominant as the language of instruction in CLIL settings (Pérez-Cañado 2012: 320). Although — from a content subject perspective — math is far from being dominant in the Austrian CLIL scene, this educational approach also occurs in math classes. Identical with multilingual classrooms — as described in the preceding section — the interrelation of language and math is of explicit relevance in CLIL. Obviously, language is not merely a

preconditioned medium for communication and instruction but becomes a learning objective as well. Hence, language and content learning are in a special but evident relationship in CLIL and the following section will focus on the expected and actual interplay of both learning strands.

Dalton-Puffer (2007: 5) stresses the conflict and tension between language and content which is often foregrounded in the context of CLIL. According to her, content teachers are often concerned about the negative impact on content learning due to the use of a foreign language for instruction.

> The concern reflects two fears: firstly, that the foreign language may slow down proceedings so that less subject matter can be covered and secondly, that lower language proficiency may result in reduced cognitive complexity of the subject matter presented and/or learned. (Dalton-Puffer 2007: 5)

Indeed, at first glance, it seems plausible that having to concentrate on language and content teaching at the same time automatically entails constraints for the latter compared to an instructional setting in learners' L1. Supported by the findings of studies about the math performance of learners with English as an additional language in the multilingual classroom (Barton *et al.* 2005; Neville-Barton & Barton 2005), the fears of restraints for content learning in CLIL seems reasonable. Nevertheless, just in line with the argumentation in previous chapters of this thesis, Dalton-Puffer (2007: 6) states that content and language cannot be simply separated. "Not only can they not be separated with regard to CLIL, they simply cannot be separated at all". Further, just as highlighted previously, there are aspects of and strategies in language teaching which might, as well, positively influence math teaching. Fading the — all too often — foregrounded tensions of content and language teaching, both fields also complement each other from a different point of view. On the one hand, the focus on language teaching accounts for Vygotskij's (1978) constructivist learning theory and the importance of language and communication for the learning process. On the other hand, teaching content in English can also help to meet the demand for authentic communication in language teaching. The authenticity of language input is a major concern in communicative language teaching and claims "materials which have not been designed especially for language learners and which therefore do not have contrived or simplified language" (Hedge 2008: 67). Beyond

question — especially teacher talk — is also simplified in CLIL classes. However, a focus on content and meaning as well as texts, films, and other materials, can definitely contribute to more authenticity of language input. Hence, there seems to be a lot of potential in CLIL and the setting appears to uniquely join both content and language teaching.

Interestingly, longitudinal studies of the impact of CLIL on performance in math classes are in marked contrast to findings from the settings with learners who speak a minority L1. Jäppinen (2005) conducted a study to learn about the impact of CLIL on cognitional development of students. Her research "focuses on the thinking and content learning processes of Finnish CLIL learners in comparison with learners taught through the mother tongue (Finnish), not on language learning or linguistic issues" (Jäppinen 2005: 152). Comparing CLIL classes with a control group taught in the learners' mother tongue, the findings reveal a resembling cognitive development in both groups (Jäppinen 2005: 162). In some cases, however, "the cognitional development of the experimental group seemed to be even faster than that in the control group" (Jäppinen 2005: 162). Further, the findings suggest a dependence of such an advance for CLIL learners on the age of the student. While in the second age group (10-14-years) the cognitional development was even faster in some cases, younger learners (7-9 years) "had some diffculties [sic] with certain more abstract scientific topics that may not be very well suited for being taught through a foreign language" (Jäppinen 2005: 162). Generally, Jäppinen (2005: 162) concludes that "teaching through a foreign language supports CLIL learners' thinking and content learning".

In accordance with Jäppinen (2005), Surmont *et al.* (2016) found a positive impact of CLIL on math performance. In their longitudinal study, they tested a CLIL group and a control group at the beginning, after three, and after ten months in math. The CLIL group clearly outperformed the non-CLIL group, which was taught in their mother tongue — Dutch — in terms of scores on the second and the third test. " Repeated measures analysis showed that the CLIL group improved significantly better than the non-CLIL group over time" (Surmont *et al.* 2016: 328).

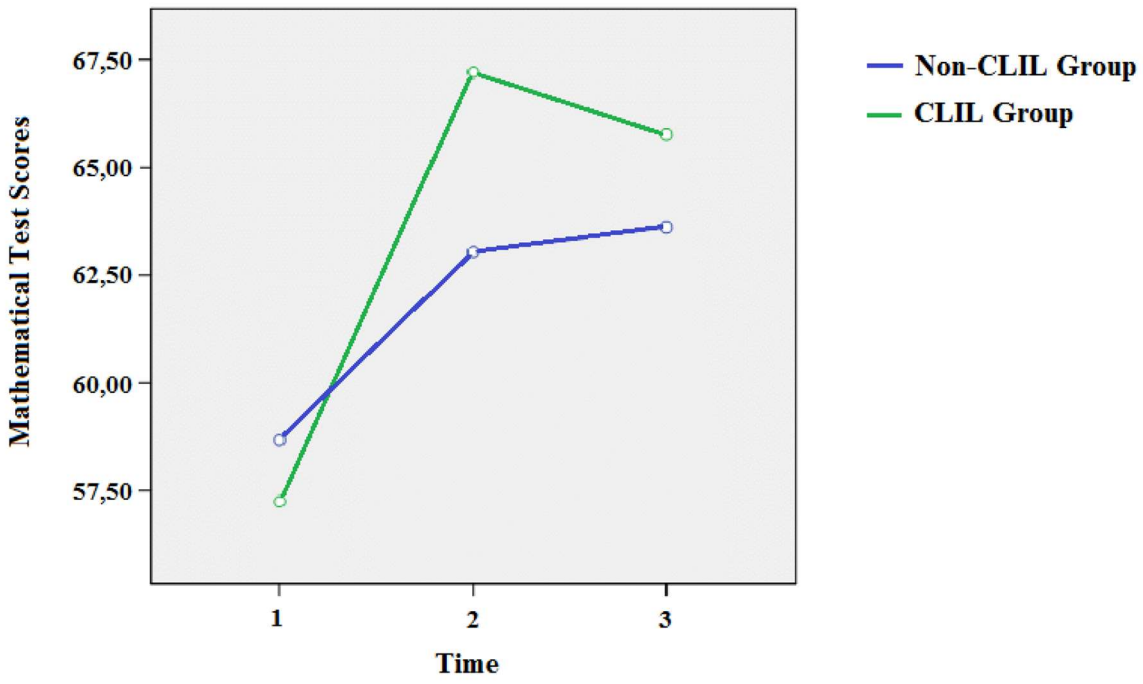

Figure 3 Scores on math tests (Surmont et al. 2016: 327)

Ouazizi (2016), again, focused on the impact of CLIL on both language and content learning. The study suggests that both fields can benefit from CLIL and highlights the high level of language proficiency of learners in CLIL classes (Ouazizi 2016: 129). Additionally, it was observed that CLIL education has a positive influence on learners' motivation to learn new mathematical concepts and terminology. "As far as concerning the subject matter, I concluded that CLIL lead [sic] to better subject matter knowledge than traditional learning" (Ouazizi 2016: 129).

Hence, it can be summarized that teaching math in a foreign language is represented controversially in the literature. While deficits in math performance determine the findings of research in settings with speakers of minority L1 in multilingual classrooms, positive impact on both language and math performance was found in CLIL education. Bilingualism, on the other hand, can have a positive impact on the learning process in both contexts. However, all studies have in common that they clearly underpin the significance of the factor language in math teaching. Furthermore, all teaching contexts quoted share the term language of instructions. Consequently, it is safe to claim that it is, in particular, the quality of instruction that influences students' learning progress. Therefore, it is the teacher's language output that draws special interest. Besides the interesting insights on

CLIL students' development and level of proficiency compared to peers who are taught in their mother tongue, little is known about the impact of CLIL on teachers. The following chapter will address this aspect and intensively scrutinize teacher talk in a CLIL math class and one which is taught in learners L1. After this theoretical part which addressed different language factors in math and especially teaching and learning new mathematical concepts, this thesis turns to this specific aspect of teacher talk now. For this purpose, an empirical study was conducted which will be presented in the following, second part of this thesis.

6 The study

6.1 Introduction

As clearly underpinned in the previous part of this thesis, language and communication are essential factors in teaching and learning mathematics. Undoubtedly, this claim can be generalized and, therefore, also applies to math teaching in learners' L1. However, the interweaving of math and language are of even more interest in settings with a language of instruction other than learners' L1 — such as CLIL. If math is taught in a foreign language, the role of language use takes center stage. Obviously, this approach aims at encouraging second language acquisition and, hence, language is not solely a preconditioned medium but an explicit learning objective at the same time. Consequently, there was a trend of focusing on language use in CLIL classes in order to evaluate learning opportunities and progress from a second language acquisition perspective. Especially in the beginnings of research on CLIL, language learning outcomes were of specific interest (Dalton-Puffer, Nikula & Smit 2010: 9). However, "[w]ith the increase in both CLIL activities and research, awareness has risen about the complexity of factors involved in CLIL" (Dalton-Puffer, Nikula & Smit 2010:9).

One of these aspects are discourse patterns specifically occurring in the CLIL classroom (Smit 2010; Dalton-Puffer 2007), which is also the focus of this study. The detailed interest, however, is in specific features of teacher talk in CLIL lessons compared to content lessons — in this case, math — taught in both teacher's and learners' L1. Assuming a lower level of language proficiency in CLIL classes, teachers also have a narrower scope of language resources to convey content knowledge. Hence, it seems natural that teachers need to adapt their classroom talk accordingly and use different strategies to successfully convey content and intended meaning compared to teaching in L1.

The following study will compare two math lessons taught in learners' and teacher's L1 with two CLIL lessons taught by the same teacher. Thereby, the analysis is focused on the teacher's utterances and the possible effects of English as the language of instruction on teacher talk in an Austrian math class.

6.2 Theoretical framework

Similar to Nikula (2010), who observed a teacher's language use when teaching biology in his L1 and a CLIL setting, this study scrutinizes teacher talk in both settings. In line with her study (2010: 107), the following analysis is grounded in a discourse-pragmatic approach. Hence, the "analytical focus will be on features of classroom interaction rather than on formal aspects of language as a system". Consequently, the interest of this study is comparing "the teacher's ways of 'doing being a teacher'" (Nikula 2010: 107) when teaching math in English and in her L1 — German. In contrast to Nikula (2010), however, it is not the social consequences of language use (e.g. in terms of involvement, politeness, or participation) but the consequences for mathematical content when expressed in the two different languages. More precisely, this study aims at stressing possible differences in depicting mathematical concepts between English and German as the language of instruction.

In this regard, strategies of making mathematical content comprehensible to students will be of special interest. Assuming a lower level of students' language proficiency in English than in German, it stands to reason that a teacher needs to adapt his/her language input accordingly, in order to make him/herself understood by the learners. While such an adaption of language input is thoroughly examined in the field of second language teaching, less is known about its effects on explaining and comprehending content in a second language. As described in section 3.3.4.1, Krashen (1982; 1985) claims in his input hypothesis that language can only be acquired, if learners are exposed to input in the target language which is slightly above their level but can still be understood. Since this approach — as highlighted in section 3.3.4.2 — appears to be applicable for teaching math as well, the CLIL setting lends itself for a scrutinization of overlaps of comprehensible input in language and content teaching. Therefore, a contrastive analysis of teacher talk in a CLIL and L1 math class will focus on the effects of English language instruction on the depiction and explanation of mathematical content.

6.2.1 Comprehensible input strategies

In order to clarify and define a unit of analysis, it is, in a first step, necessary to characterize features of teacher talk which aim at making input comprehensible for learners. An adequate term in this regard is comprehensible input strategies (CIS), which comprises attempts to foster and check comprehension of language input in the classroom. The following section will summarize such strategies with regard to the related literature. Although some of these aspects are mentioned in previous sections already (see 3.3.4.1), relevant characteristics are repeated here, in order to draw a complete picture of the theoretical basis for analysis.

Initially, simplified input was the focus of attention in so-called caretaker speech. Hence, research was mainly concerned with first language acquisition. However, Krashen (1982: 65) draws plausible comparisons to second language acquisitions emphasizing the relevance of caretaker speech or foreigner talk — describing language input from native speakers to non-native speakers of a language — for comprehensible input in second language acquisition. Hatch (1983: 183-184) summarizes key characteristics of foreigner talk. Important aspects and benefits of Hatch's (1983) observation for simplifying input in teacher talk were excerpted by Chaudron (1983: 439).

Table 3 Features and benefits of foreigner talk (from Chaudron 1983:439)

Feature	Benefit
Clear articulation: fewer reduced vowels and fewer contractions	Learner receives the full word form
High frequency vocabulary, less slang, fewer idioms	Learner is more likely to know and/or recognize topic
Fewer pronoun forms of all types Short MLU [mean length of utterance], simple propositional syntax	Reference should be clearer Should be easier to process and analyze
Left-dislocation of topics Repetition and restatement	Should help learner identify topic More processing time and relationship of syntactic forms may be clearer

Pica, Doughty & Young (1986: 122) identify 4 key features which are involved in input modification:

- Repetition and paraphrase of linguistic constituents;
- Restriction of lexis to more common and familiar items;
- Addition of clause boundary markers;
- Reduction in number of embedded and dependent clauses.

More precisely, they provide examples in three main areas of modification — quantity, redundancy, and complexity — as illustrated in table 4 below.

Table 4 Modification of input (from Pica, Doughty & Young 1986: 123)

1	**Quantity**	**Increase in the number of words per direction**
	Baseline:	Moving to the top right corner, place the two mushrooms with the three yellow dots in that grass patch, down toward the road. (23 words)
	Modified:	Move to the top right corner. Take the two mushrooms with the three yellow dots. Put the two mushrooms on the grass. Put the two mushrooms on the grass near the road. (32 words)
2	**Redundancy**	**Increase in repetition**
	Exact/Partial:	
	Baseline:	Place the two mushrooms with the three yellow dots in that grass patch, down towards the road. (0 repetitions)
	Modified:	Take the two mushrooms with the three yellow dots. Put the two mushrooms on the grass. Put the two mushrooms on the grass near the road. (2 repetitions)
	Semantic/Paraphrase:	
	Baseline:	Place the one piece with the two trees right at the edge of the water. (0 repetitions)

	Modified:	Put the two trees at the top of the water. Put the two trees above the water. (1+1 repetitions)
3	**Complexity**	**Reduction in the number of s-nodes** [root or dominating node in a tree graph] **per T-unit** [minimal terminable unit (a main clause with all its subordinate clauses (Hunt 1965: 21-22))]
	Baseline:	In the center of the crossroads, right where the tree meet, put the dog in the — in the carriage. (2 s-nodes per T-unit)
	Modified:	Put the dog in the middle of the three roads. (1 s-node per T-unit)

To sum up, modified input typically features a higher quantity of words for the same amount of information, redundancy, and less complex sentence structures.

Chaudron (1983: 440) recognized a broader field of relevance for simplified input and highlighted its importance for the instructional context, especially in settings and programs in which L2 learners are encouraged to learn subject matter via the medium of the L2. “For their instruction to be effective, teachers, especially lecturers, must anticipate their L2 students' receptive capabilities and provide adequately simplified input for the students to recall or retain the information presented” (Chaudron 1983: 440). In his study, he did not only identify and categorize strategies for simplifying input but rather focused on its effects on learners’ recognition and recall. The specific feature of instructional discourse which was the center of his research is topic reinstatements. With regard to this specific unit of analysis, Chaudron (1983: 440-443) emphasizes the essential role of correctly identifying a given referent for processing new information. For his analysis, he employed five linguistic structures as topic reinstatements and exemplified the devices in the context of the following sentence: *They are selling beer at the picnic*.

a) Simple Noun (simple topic reiteration)
The beer tastes terrific.
b) Synonym
The brew tastes terrific.
c) Repeated Noun
The beer… the beer tastes terrific.
d) Topicalizing Rhetorical question
What about the beer? It tastes terrific.

e) If-Clause (non-conditional)
If you can afford the beer, it tastes terrific.

Besides other interesting insights, the study reveals that positive effects for low-level proficiency groups highly depend on grammatical complexity. Thus, the most effective devices for topic reinstatements are the simple noun and the repeated noun.

Archer & Hughes (2011), on the other hand, focus explicitly on successful instructions in the classroom. Based on the research of several studies in the field, they defined a list of sixteen essential characteristics of what they call explicit instructions. Since the main aim of this approach is achievement for all students, its relatedness to comprehensible input clearly stands to reason. As a matter of fact, the majority of Archer & Hughes' (2011: 2-3) elements of explicit instruction directly concern teacher talk or input in a broader sense, as can be seen in table 5.

Table 5 Elements of explicit instruction (based on Archer & Hughes 2011: 2-3)

Elements of explicit instruction	
Element 1:	Focus instruction on critical content
Element 2:	Sequence skills logically
Element 3:	Break down complex skills and strategies into smaller instructional units
Element 4:	Design organized and focussed lessons
Element 5:	Begin lessons with a clear statement of the lesson's goals
Element 6:	Review prior skills and knowledge before beginning instruction
Element 7:	Provide step-by-step demonstrations
Element 8:	Use clear and concise language
Element 9:	Provide an adequate range of examples and non-examples
Element 10:	Provide guided and supported practice
Element 11:	Require frequent responses
Element 12:	Monitor student performance closely
Element 13:	Provide immediate affirmative and corrective feedback
Element 14:	Deliver the lesson at a brisk pace
Element 15:	Help students organize knowledge
Element 16:	Provide distributed and cumulative practice

More directly concerned with the linguistic aspects of modified teacher talk is the term comprehensible input strategies (CIS), which is applied in a study by Buri (2012). In her study, she observed the use of CIS in Filipino science and math classes with English as the medium of instruction. Initiated by the deficient student achievement in such classes, an overarching goal of the research was to address the effect of language use on comprehension. As Buri (2012: 1) correctly states, "[t]he exchange of ideas between students and teachers is largely done through language as they talk about concepts in science, mathematics and other content areas". Hence, it seems obvious that the adaption of language can positively influence the learning achievements of students in content classes, especially if they are taught in L2. Buris' special interest was the relation of the implementation of CIS with different modes of teacher talk, namely in the operative (giving instructions), interactive (introducing new topics by actively involving students through questions), or informative (extended teacher-centered input) mode. In other words, the aim was to examine how far the purpose of teacher talk influences the use of CIS. For that purpose, ten strategies for making input comprehensible were defined:

1. Translation
2. Paraphrase
3. The use of visual aids
4. Excessive coordination
5. Circumlocution
6. Deviant form
7. Recasting
8. Word coinage
9. Code-switching
10. Repetition

For a better understanding, the following table 6 provides an overview with explanations and examples for the categories introduced and observed by Buri (2012).

Table 6 Comprehensible input strategies (from Buri 2012: 2-3, 10-13)

Translation:	the act of translating words, phrases and sentences until the desired meaning is understood.
	T: What are the foods rich in protein Marvin? What are protein rich foods? 'Yong mayaman sa protina, Marvin?' **M:** Egg. **T:** Egg. Yes, Egg is rich in protein.
Paraphrase:	a restatement of a text usually substituting more commonly used words for the expression that are difficult in the original utterance.
	T: What happens to the balloons inside this bottle? **S:** Becomes bigger. **T:** Okey [sic]. The balloons become bigger or it is inflated. Is it inflated or deflated? **S:** Inflated. **T:** The balloons which represent our lungs expand or it is inflated.
Use of visual aids:	the use of objects as aids to make a concept clear.
	T: Where do you think class is the xylem? **S:** (Students point to the xylem) **T:** How about the phloem? **S:** (Students point to the phloem) **T:** Okey [sic]. That is the phloem. Together class, the xylem and the phloem is known as the vein.
Excessive coordination:	refers to the use of too many conjunctions or linkers resulting in a long, run-on sentence in giving instructions.
	T: Get the oil and then you get the samples of nuts and seeds, then add ethyl alcohol and then you stand it for 12 hours, no, and then you are going to filter the extract and then you fill water and then what will happen to the water?
Circumlocution:	the use of so many words to clarify the term the speaker has in mind. He does not go direct [sic] to the point. Instead, he tries to explain the term in a roundabout manner.
	T: What is the meaning of "translucent"? **S:** Allowing the light to pass through but not transparent. **T:** Yes, allowing the light to pass through but not transparent. Do you understand that? When you see against the light, no. The translucent paper, you can what? The translucent paper you can what? The light allows to pass through but it is not transparent.

Deviant forms:	are non-syntactic forms, usually questioning strategies to secure the expected response.
	T: What is happening when you inhale and then you exhale? Okey. There is a change in, a change in what, brought about by? **S:** Breathing.
Recasting:	refers to syntactic revisions made by the teacher in order to clarify an utterance or an expression.
	T: What about the example of the carbohydrate or the foods that are rich in carbohydrates? Give me examples of the food rich in carbohydrates. **S:** Ma'am, fried rice.
Word coinage:	This is the use of a term familiar to the learners in order to clarify a concept or a science term being taught by the teacher.
	T: What is the major function of a stem? **S:** The path . . . . the path **T:** Okey. All right. The pathway, it is the pathway where the water and mineral goes up from the root to the other parts of the plant. We call that transport carrier.
Code switching:	the shift from the second language to first language to make the meaning clear.
	T: This means that the chlorophyll is in alcohol. Wala na sa dahon, kundi nasa alcohol na. Actually, meron pa „yong dahon, pero konti na lang.
Repetition:	the act of repeating words and phrases until the intended meaning is conveyed.
	T: How about the liquid? What will you do with the liquid sample? What will you do with the liquid sample?

6.3 Analysis

6.3.1 Focus of analysis and research questions

In light of the interweaving of math and language, especially in teaching, the following analysis will single out teacher talk in math lessons and contrast its characteristics by comparing two different settings. More precisely, this study will juxtapose language use and communication patterns of a teacher when teaching math in her and students' L1 — German — and teaching math in a CLIL setting with English as the language of instruction. Obviously, particular interest lies in similarities and differences in introducing

mathematical concepts in the two languages. Hence, the focus of analysis will be in how far the language of instruction influences the patterns of making mathematical input comprehensible to students. Based on the features of comprehensible input introduced above, the use of CIS in both settings will be the major analytical clue. For this purpose, the analysis is of a comparative nature and juxtaposes language use of a math teacher when introducing mathematical concepts in English and in German. The research method is qualitative and aims at revealing possible differences in the use and occurrence of comprehensible input strategies in both settings. More specifically this study seeks to answer the following research questions:

1. How does the language use of a teacher differ between introducing mathematical content in German and in English?

2. What kind of comprehensible input strategies are adopted by the teacher and how do they differ when using students' L1 or English as the language of instruction?

3. To what extent do CIS entail a form of mathematical metalanguage in order to convey mathematical content?

For this purpose, passages of teacher talk that are concerned with introducing and explaining new mathematical terms or concepts were identified and coded accordingly. In a second step, corresponding passages of the German and the English setting were analyzed and contrasted in order to scrutinize differences in language use in both settings.

The identification and categorization of CIS draw on features and relevant aspects of modified speech introduced in the previous section. However, structural analysis of the text samples did not underly a strict predefined categorization. The scheme and relevant categories for analysis rather arose from a close reading and flexible coding process of the data. As a result of this iterative coordination between the literature and the data, the following categories, which were originally introduced by Chaudron (1983) and Buri (2012), were identified and assigned a specific relevance:

- Topic reinstatements
- Repetition

- Paraphrasing
- Recasting
- Use of visual aids

Furthermore, measurable parameters in the data, which can be related to comprehensible input strategies provide a brief quantitative supplement for the study and present some numerical data for the following variables:

- Number of turns by the teacher in each lesson
- Total and relative time of teacher talk in each lesson
- Average time of a teacher turn in each lesson
- Average amount of words in a teacher turn in each lesson

In a further step, a qualitative analysis of representative input situations will then elaborate on the influence of the language of instruction on the use of CIS by the teacher. Furthermore, the analytical focus will enter into the question of how far CIS are directly concerned with math content and how they influence the depiction of mathematical concepts. In other words, it will be examined in how far the use of English as the language of instruction influences the complexity, modality, and formality of content input. In this regard, the use of natural language in comparison to a more formal math register will be foregrounded. This particular aspect aims at analyzing if the abstractness of math content is also reflected in the language proficiency level of students and mathematical input.

6.3.2 Data

6.3.2.1 Context

The complete set of data was collected in a school in Lower Austria. In total, four lessons were observed and recorded in two different 9th-grade math classrooms. As mentioned previously, the major point of interest when choosing both sample groups was the varying language of instruction between the two classes. While one class (sample group A) had been regularly taught in the students' and teacher's L1, namely German, English had been the language of instruction in the second group (sample group B). However, for the sake of meaningful comparability, particular attention was devoted to the content taught in both classes. Consequently, the observation and recording sessions were planned for

lessons with the same mathematical content. Bearing in mind possible differences in temporal progress between the two groups, two lessons in the introductory phase of a new topic seemed to be appropriate for the purpose of this study.

While sample group A was composed of 24 students (seven female and 17 male students), sample group B was slightly smaller with only 15 students (seven female and eight male students). Due to the convenient schedule and flexible cooperation of the teacher, it was possible to record all four sessions within two days. Furthermore, on both days, the German taught lesson took place directly after the CLIL lesson. Both groups were introduced to the topic of trigonometry which was completely new to them. Obviously, all four lessons were taught by the same teacher — a female math teacher, who has been teaching since September 2006. Her second subject is history. Hence, she has no professional background in EFL or language teaching in general. However, her English language competence level is very high. Besides personal interest and English language use in her leisure time, her high proficiency in English also benefited from a year abroad in New York City at the beginning of her teaching career. There she taught math and history in sixth and seventh grade. Consequently, she gained valuable experience in teaching math in English. In Austria, she has taught math lessons with English as the language of instruction since September 2008. However, it needs to be mentioned that most of her teaching in English does not take place in a classical CLIL setting but in an international program — the International Baccalaureate Diploma — integrated into her school. Nevertheless, the sample group analyzed in this study can be considered a typical CLIL class, as all students in the class taught in English are native speakers of German. Hence, the language of instruction is not the students' L1 and, consequently, content and language learning are integrated.

6.3.3 Data collection and analysis

All four lessons were observed and videotaped. Additionally, an audiotape was recorded through an audio recording device which was placed on the teachers' desk in order to maximize intelligibility of the teachers' verbal input. After recording the lessons all of them were transcribed to their full extent using the analytical software MAXQDA 2018. The quantitative information about the temporal extent of teacher talk is based on

synchronizing the audio tape and the transcript according to integrated time stamps at the end of each turn. Hence, it was possible to extract the information needed from the transcript. The qualitative analysis is based on a close reading of the data and a subsequent juxtaposition of relevant passages in relation to differences in teacher talk.

6.4 Findings

6.4.1 Quantitative data

Providing the basis for a quantitative analysis of teacher talk in both settings, the following numbers offer insights into the amount of teacher talk in both German and English. As summarized in table 5, the quantitative focus is on the total of turns — including students and teacher — the total of turns by the teacher, the total time of teacher talk, and the number of words uttered by the teacher in all for lessons.

Before expanding on the numerical data, the analytical procedure needs to be outlined in order to allude to some points of inaccurateness or limitations in the data.

Table 7 Number of turns and talking time

	GER_Thur	GER_FRI	GER_Total	EN_THUR	EN_FRI	EN_Total
∑ Turns	484	481	965	432	408	840
∑ Teacher Turns	203	206	409	181	192	373
Time recorded	43:34.1	47:45.5	01:31:20	43:42.9	44:56.2	01:28:39
Time Teacher talk	36:07.7	37:42.2	01:13:50	37:27.7	38:25.6	01:15:53
Ø time per turn	00:10.7	00:11.0	00:10.8	00:12.4	00:12.0	00:12.2
∑ Words (Teacher)	6768	7513	14281	6633	6910	13543
Ø words per minute	188	203	196	179	182	181

First of all, it needs to be clarified that a turn is one continuous utterance by one of the interlocutors which is, therefore, usually ended by a new turn of another interlocutor, a longer period of silence or other interruptive non-verbal activities like writing something onto the blackboard. In the transcript, each turn is signalled with a new paragraph starting with the abbreviation of the interlocutor. Using the analytical software MAXQDA 2018,

the transcript was synchronized with the audio tape and the end of each turn was assigned with the accurate time stamp. Hence, it was possible to derive the length of each turn from the transcripts. While this approach has no impact on the accuracy of the number of turns, it needs to be stated that pauses and periods of silence might be included, since the end of one turn also indicates the beginning of the succeeding one.

Furthermore, it needs to be mentioned that the word count of the turns also includes comments and notes which are accounted for by the transcription conventions. Hence, these numbers underlie a certain rate of inaccuracy. Nevertheless, a comparative approach still seems legitimate, as such comments occur in all transcripts.

6.4.1.1 Number of teacher-turns

Comparing the total number of teacher-turns, it is noticeable that there are slightly more in the German lessons than in the English ones. While there is a total of 409 teacher turns in the former, there are only 373 in the latter. This, however, does not automatically imply more teacher input in German than in English. A lower number of teacher turns can also result from fewer interruptions or active involvement of learners. Hence, it is essential to consider the number of teacher-turns with regard to the total amount of turns in the lesson. Such a relative perspective, as illustrated in figures 6 and 7, reveals a nearly even share of turns uttered by the teacher compared to the students. Thus, it stands to reason that classroom interaction patterns are rather similar in both languages, meaning that every teacher-turn initiates approximately 1.3 student turns on average. Nevertheless, there is a significantly higher amount of turns in the German lessons. Consequently, in absolute numbers there are more teacher and students turns than in English, which speaks for more student-teacher interaction in the L1 setting. This trend is further underpinned by the average duration of a teacher-turn. With 10. 8 seconds per turn, they are 1.4 seconds shorter in German than in English.

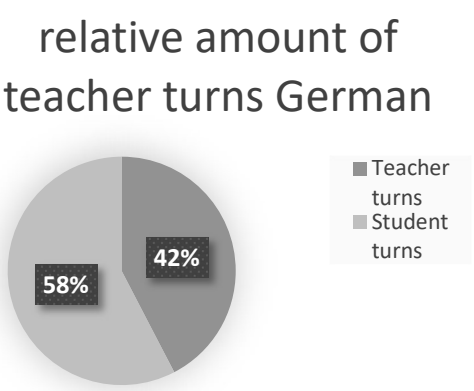

Figure 4 Relative amount of teacher turns in German

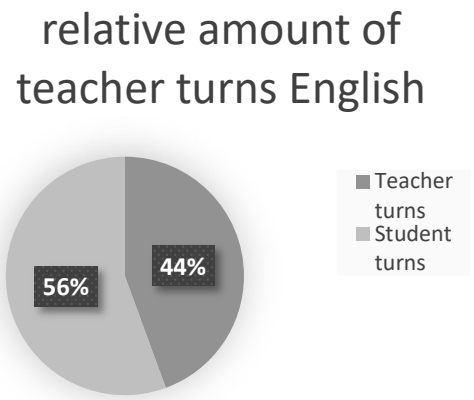

Figure 5 Relative amount of teacher turns in English

6.4.1.2 Time of teacher talk

What both settings have in common, again, is the high amount of teacher talk. As the teacher talks one hour and 13 minutes in German and one hour and 15 minutes in English, it is safe to say that the language has no effect on a typical teacher-centered approach in math teaching. Considering the relative share of teacher talk compared to the complete length of the records, however, the teacher talks slightly more in English. As can be seen in figure 8, teacher talk occupies about 85 % of the lessons in English and about 80 % in German. Hence, there seems to be even more teacher input in L2 — at least in terms of time.

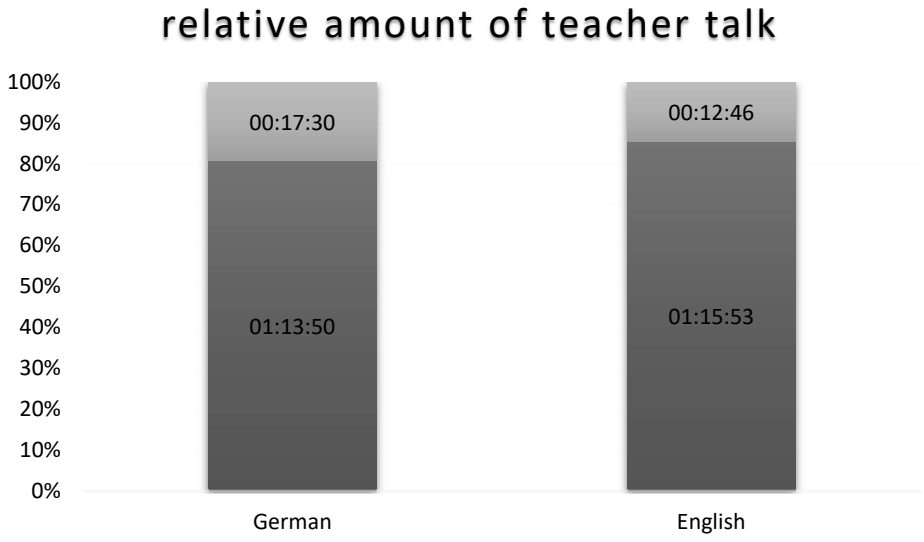

Figure 6 Relative amount of teacher talk

In terms of words, on the other hand, the data reveals more teacher input in German than in English. While the teacher utters 14281 words in the former setting, she clearly needs fewer in the latter, namely 13543. Consequently, the teacher has a faster speech rate in

her L1 than in English. More precisely, she utters 196 words per minute in German and 181 words per minute in English. This finding is also consistent with expectations in teacher talk in a second language for two reasons. First of all, for the sake of better understanding for learners and, secondly, due to a possible lower level of fluency of the teacher in L2.

To summarize, there are no salient differences between the two settings in the numerical data extracted. The high amount of teacher talk especially implicates a clear teacher-centered approach in both languages. Furthermore, there is no quantitative evidence for significant differences in the interactional patterns with and involvement of learners. What is evident from the data, however, is a higher amount of input due to a higher speaking pace in German.

6.4.2 Qualitative data

The following section provides a qualitative analysis of the teacher's language use by contrasting corresponding sequences of the German and English taught lessons. For that purpose, the transcripts were examined for passages which focussed on the introduction of new mathematical content. By juxtaposing the respective excerpts, the analytical approach aims at highlighting similarities and differences in language use in order to convey mathematical content. In this context, special attention is drawn to the use and implementation of strategies to make the content comprehensible for students.

6.4.2.1 Introducing the content of the lesson

In both lessons, the teacher clearly states that she will introduce a new topic and put special emphasis on her impression that it will be something completely new, meaning that it does not intensively build up on indispensable knowledge from previous chapters. However, while this is treated very straightforwardly in group B, the teacher gives much more background information in group A:

Excerpt 1[1] (GER_THUR §1-4)

T: *Gut wir fangen jetzt mit einem neuen Kapitel an. (.) Die gute Nachricht,*

S(m): *Es is neu*

S(f): *Es isn <1> geiles Kapitel </1>*

T: *<1> es ist neu </1> (.) In Mathematik irgendwann ist es schon einmal aufbauend. Also irgendwann werden wir alles verbinden, das kommt aber eher in der siebten Klasse dann (.) wo man wirklich zurückgreift auf andere Sachen, bissl schon in der sechsten (.) Aber eigentlich ist jetzt in der fünften Klasse is etwas immer was Neues. (.) Wo man ganz wenig auf Vorheriges zurückgreift. Das heißt der Vorteil ist, erstens einmal sind wir fertig mit diesen (.) komischen genauen Komma Gleitkomma Sachen achten muss auf diese (.) genauen Definitionen. (1) Wir haben jetzt ein Kapitel mit dem wir auch arbeiten und rechnen können das es fällt vielen Schülern einfach leichter. (.) Und das (.) heißt (.) Trigonometrie*

In group A, the teacher introduces the new topic in the broader context of the curriculum and the organization of mathematical content in this and the following grades. This is a typical example of what was categorized as contextual background information in this study. Hence, the input is definitely related to the content matter in a broader context but not directly relevant on a pure topic specific level. Generally, there is an identifiable trend of more such background information given by the teacher in group A than in group B. Additionally, the repetition of the word *neu [new]* (lines 1, 4, and 8) clearly emphasizes the intended message of a fresh start in the upcoming lesson.

Excerpt 2 (EN_THUR §4-11)

T: *So the GOOD news is (.) that we start with a completely new chapter where the only reassumed knowledge that you should know from before is (1) <pvc> {Pythagorean} </pvc> theorem (1) eh, so that's the only thing you should know and I: (1) hope you know that one*

[…]

T: *Meaning that everything else is pretty much new (.) what of what we are doing (1) ahm <SLOW> and so </SLOW> which means after the difficult chapters from last (.) from the last test we start now now with something a bit easier I think. You like the chapter. Cause you say yes. It is a nice chapter.*

[1]Translations of relevant section in the German excerpts are provided directly in the text.

As can be seen in excerpt 2, the teacher decides against placing the topic in a broader context of the curriculum in order to stress the novelty of the topic. However, similarly to group A, due to the repetition of the words *start* (lines 1 and 8) and *new* (lines 1 and 6) it is made very clear that the upcoming lessons are devoted to a new topic. Interestingly, this is also indicated by stating that the Pythagorean Theorem is everything students need to know from previous lessons. The connection to this particular mathematical concept is not mentioned in excerpt 1 at all. In terms of CIS it is — apart from the repetition already mentioned — noticeable that the teacher automatically and instantly paraphrases the noun phrase *reassumed knowledge* (line 2). The phrase t*he only reassumed knowledge that you should know from before* (line 2) is already redundant which is even intensified by the repetition in form of *that's the only thing you should know* (line 3). Although it does not directly refer to the Pythagorean Theorem, the corresponding phrase in group A would be *Wo man ganz wenig auf Vorheriges zurückgreift [where you rarely draw on previous topics]* (line 8). Hence, in this situation, the teacher's attempt to simplify input is clearly recognizable in the lesson taught in English in order to clarify unknown or newly introduced vocabulary. Nevertheless, it is worth noting that the corresponding phrase in German is restricted to a simpler circumlocution right away and does not even include a more formal and linguistically condensed noun like *Vorwissen [prior knowledge]*.

6.4.2.2 Trigonometry

In both lessons, the introduction of the actual name of the new topic follows the general remarks presented above. In group A as well as in group B, the teacher tries to introduce the key idea of the new topic by deriving it directly from its name. This semantic interpretation of the term *trigonometry*, however, reveals interesting differences between the two settings. As can be seen in excerpt 3, the teacher clearly states that the name of this field is directly related to the mathematical concept behind it. Nevertheless, there is no closer semantic analysis of the term, which would clarify the nexus between the mathematical concept and its formal label in the English version.

Excerpt 3 (EN_THUR §14-24)

***T:** Ähm (2) Ja it's Trigonometry not Geometry, because it's connects angles to: sides of a triangle. […] And it's called right-angled (4) {T writes on board} trigonometry*

***T:** Ehm (.) Why right-angled trigonometry? Because of course there are not only right-angled triangles, (.) but with those formulas we can then find the formulas for the NON-right-angled triangles. (.) which will then be called NON-right-angled trigonometry. […]*

In the German taught lesson, on the other hand, the teacher derives the connection to triangles from prefix tri: *Tri steht für drei [tri stands for three]* (excerpt 4, line 3). Thus, she closely relates the new term to the geometric shape, which is central to this chapter and already familiar to students. Although the same strategy would work in English as well, this correlation is only highlighted in group A. Another distinction between the two settings in this situation is the amount of information given about central aspects of trigonometry. While in group A the essential role of triangles is foregrounded, the main idea of the connectivity between sides and angles of a triangle is — in contrast to group B — not stated in the German class.

Excerpt 4 (GER_THUR §34)

***T:** Also (.) und zwar nicht nur Trigonometrie, sondern wir arbeiten am Anfang mit rechtwinkeliger. […] Also rechtwinkelige Trigonometrie. (.) Trigonometrie (.) Tri steht für drei. Da gehts um DREIecke. (.) Ja? (2) Rechtwinkelige (.) Trigonometrie ist dann klar weil es geht um welche Art von Dreiecken?*

Instead, the key idea of trigonometry is addressed at a much later stage of the lesson in the German class as can be seen in excerpt 5. The phrase *und das machen wir eben jetzt in der Mathematik, dass man das berechnen kann. (.) wie kommt man auf die Winkel [and that's what we do now in math is to calculate that. How we can find the angles]* (lines1-2) is the first attempt in the German lesson to convey the main motivation behind trigonometry.

Excerpt 5 (GER_THUR §134)

1 ***T:*** *[…] und es wär schön und das machen wir jetzt eben in der Mathematik, dass*
2 *man das berechnen kann. (.) wie kommt man auf die Winkel? (.) Und das ist die*
3 *ganze Idee. (.) und dafür brauchen wir eben die Trigonometrie*

Furthermore, in both groups, it is stated right away that the following ratios are only applicable for right-angled triangles. Remarkably, in group B there is a clear differentiation between right-angled and non-right-angled trigonometry. In group A, on the other side, the teacher mentions non-right-angled trigonometry only peripherally as can be seen in excerpt 6: *wir werden nachher auch natürlich mit nicht-rechtwinkwligen Dreieck ah Dreiecken arbeiten (.) aber das it kein Maturastoff [of course we will also deal with non-right-angled triangles later, but that's not relevant for the Matura]* (lines 2-3).

Excerpt 6 (GER_THUR §38)

1 ***T:*** *Ich habs auch in der anderen Klasse gesagt, der Vorteil ist, das ist der Stoff*
2 *der Matura. Wir werden nachher auch natürlich auch mit nicht-rechtwinkeligen*
3 *Dreieck ah Dreiecken arbeiten (.) das ist aber kein Maturastoff.*

In terms of topic iteration, it is noticeable that simple noun repetitions are used in both groups. While trigonometry is used five times in the German introductory turns (line 13 in excerpt 1 and lines 1, 2, and 4 in excerpt 4) it is only mentioned four times in the English setting (lines 1, 3, 4, and 7 in excerpt 3). It needs to be considered, however, that the last time it is mentioned in the context of non-right-angled trigonometry.

6.4.2.3 Tangent

Simple noun repetition also occurs in the introduction of the term *Tangens [tangent]* in the German-speaking class. In this situation, the teacher exposes the learners with the new term right away and offers an explanation subsequently. The focus in the introductory turn, obviously, is to convey that the tangent describes the ratio of two specific sides in a right-angled triangle, and hence, implicates the size of the corresponding angle.

Excerpt 7 (GER_THUR §136)

1 ***T:*** *Ist der Tangens. (.) der heißt Tangens. (.) Die Abkürzung vom Tangens ist (.) t*
2 *a n (2) Das ist das erste Verhältnis. warum sag ich Verhältnis? Verhältnisse sind*
3 *wenn sich etwas zueinander verhält in der Mathematik. (1) Zum Beispiel eine*
4 *Proportion ist mit Gleichheitszeichen von zwei Verhältnissen. (.) und beim*
5 *Tangens ist so definiert, (.) da wenn ich jetzt (.) zum Beispiel mit dem Winkel Mü*
6 *anfange (2) schreib ich auf der Tangens von Mü (1) wenn ich das jetzt in Worten*
7 *ausdrücken oder in länger formuliert würde ich sagen, (1) das Verhältnis der*
8 *Seiten von Mü ist wie? Und das ist eine Definition, das müssts ihr lernen (.) […]*
9 *(1) Das Verhältnis ist definiert als (.) die (.) Seite hier. (2) durch die Seite hier.*

As can be seen in excerpt 7, the term T*angens* (lines 1, 5, and 6) is mentioned five times and *Verhältnis [ratio]* (lines 2, 4, 7, and 9) is mentioned even six times in the initial introduction of the new term. Furthermore, the teacher also elaborates on the term *Verhältnis* which is the basis of the teacher‘s explanation. However, by stating *Verhältnisse sind wenn sich etwas zueinander verhält in der Mathematik [in math it is called ratio if something is related]* (lines 2-3), the teacher only adds another repetition of the term in form of a grammatical variation. Although the teacher intends to explain the concept of mathematical ratio, she simply uses the corresponding verb instead of the noun. Hence, it is rather a matter of repetition than a paraphrase. The particular challenge in this context might be the fact that the term *Verhältnis* is also used in a non-mathematical context and, thus, is already perceived as natural language. Accordingly, it is difficult to find an even more natural or simplified expression for this term. Interestingly, the teacher chooses a contrary strategy by introducing the more formal term of *Proportion [proportion]*. Nevertheless, it needs to be stated that *Proportion* is used as an example and not as a synonym. Consequently, it is implicated that proportion and ratio are not identical concepts.

In group B, the actual term *tangent* is introduced after elaborating on the mathematical concept it defines. Referring to the sketch of two similar right-angled triangles, the teacher draws students‘ attention to the fact that the size of the angles is the same in both examples. Only the length of the three sides differ. The unchanged size of the angles is especially emphasized by the repetition of the word *same,* which is used five times (lines 1, 2, 3, and 4) in the excerpt 8 below. Strikingly, the teacher starts with the observation

that also the ratios of the sides stay the same without reasoning. This observation, however, is not trivial but assumes knowledge of the intercept theorem. Although this should be familiar to learners in both classes, it is only mentioned and revised at a later stage of the lesson in both settings. This detail is of particular interest in the excerpt below from group B. In this situation, the revision of the intercept theorem would lend itself to focus on the fact that not only the size of the angles stay the same but also the ratio of the concerning sides.

Furthermore, in severe contrast to group A, the teacher does not expand on the term ratio in the English class at all. Although the term is repeated numerous times, there is no attempt to paraphrase or clarify the term, which clearly was the case in group A.

Excerpt 8 (EN_THUR §90-92)

T: *that the ratio of the sides stay the SAME for the angle cause you see here (.) this here is still the same angle. I don't need to call this dash, because it's still the same angle. (.) Epsilon is still the same and i still have a right angle here. So all the angles are the same.*

S(m): *<xx>*

T: *So somebody figured out (.) maybe the combination of the ratio of the sides are connected to the actual angles. (.) And that's why people, (.) Somebody defined ratios. (.) And there are three different ratios they defined and the first one (.) call the (2) {T writes on board} <GERMAN> Also auf Deutsch (1) <SLOW> Tangens </SLOW> </GERMAN> (1) it's like (.) and in English it's the?*

6.4.2.4 Similar triangles

As mentioned in the previous excerpt, the development of the mathematical concept behind the term trigonometry was based on the sketch and comparison of two similar right-angled triangles. Since the definition of similar triangles was essential for this phase of the lesson, the term was revised in both classes. The following two excerpts juxtapose the English and the German setting in this particular regard.

Excerpt 9 (EN_THUR §86-88)

1 ***T:*** *[…] Who remembers (.) what are similar triangles? What property is*
2 *important for similar triangles?*
3 ***S(f):*** *They have the same angle?*
4 ***T:*** *The same angle. This is like if I would use my set square (.) and not yours <@>*
5 *because you don't have a set square </@> (.) And your set square they would*
6 *not be congruent of course. They are not the same, but they have the same*
7 *angles. That's why it's called a similar triangle.*

As shown in the excerpt 9, in group B the term is, firstly, introduced by the teacher. Only then, the teacher asks for the definition of the same. The initial question of *what are similar triangles?* (line 1) is instantly followed by a recast, namely *what property is important for similar triangles?* (lines 1-2). After the correct answer from a student, the teacher repeats it and additionally gives an example by comparing the set square of a student to the one of herself. To highlight the essential property of similar triangles, namely that the respective angles have the same size while the sides do not have the same length, the term is confronted with congruent triangles. The term of congruent triangles, again, is instantly paraphrased by stating *They are not the same* (line 6).

In group A, on the other hand, the term *ähnliche Dreiecke [similar triangles]* is not introduced by the teacher right away (excerpt 10). Instead, she tries to elicit the term from the learners by referring to the sketch on the board. Similar to group B, she directs students by contrasting similar triangles to congruent triangles. As opposed to the English setting, however, the teacher refers to the term *deckungsgleich [exactly cover each other]* (line 2) in order to paraphrase this term. In this situation, it seems that both teacher and students simply have more language resources — in terms of vocabulary — to draw from in German than in English. The same argument stands to reason for the higher frequency of recasts when asking questions. Compared to one reformulation of the question in English, in German the teacher uses the strategy of recasting three times in the first turn and another three times in the second turn of the excerpt. Consequently, in contrast to the quantity aspect of simplified input, the amount of words used to transport the same content is much higher in group A (120 words) than in group B (65 words). The higher amount of words for this introduction in German is certainly also influenced by the

eliciting process. Since the teacher intends to prompt learners to come up with the term themselves, she also needs to supply additional support. Hence, she makes use of visual aid — the sketch — and compares the angles of the two triangles. In the course of this process, a number of subject-specific vocabulary appears, for example, *parallel verschoben [moved in parallel]* (line 8) or *Parallelwinkel [corresponding angle]* (line 8) are applied in group A. The English version in group B, on the other hand, does not mention these terms.

Excerpt 10 (GER_THUR §164-169)

T: *[…] aber we wie nennt man die Dreiecke? (.) Da gibts a Aus <x> sie sind nicht kongruent. (.) Kongruent sind zwei Dreiecke wenn sie wirklich deckungsgleich sind wenns euch erinnern könnts. Wie nennt man Dreiecke die nicht kongruent sind, die aber (.) was haben? (.) Was haben die beiden Dreiecke gemeinsam?*

S(m): *Die gleichen Winkel.*

T: *Die haben alle Winkel gleich genau. Weil das ist auch ein rechter Winkel (.) der Winkel ist sowieso drin der hat sich nicht geändert und der Winkel ist ja parallel verschoben nur der Parallelwinkel da. (1) Wie nennt man solche Dreiecke? So was wie mein Geodreick und <first name of S(f)> ah und <first name of S(f)> wars tschuldige <first name of S(f)> Dreieck? (.) Wie nennt man die. Haben gleiche Winkel aber sind sicher nicht gleich groß. (.) Die sind?*

S(f): *<1> Irgendwas irgendwas. </1>*

S(m): *<1> Im Verhältnis </1>*

T: *Haben ma schon ghabt, dass es im. Nein. (.) Ähnlich.*

6.4.2.5 Complementary and supplementary angles

With regard to the terms complementary and supplementary angles, a clear parallel to the excerpts concerning the term similar triangles is recognizable. Although both terms should be familiar to the learners, they are introduced by the teacher in group B. In contrast to excerpt 9, the students are not even asked to recall a definition. In this situation, the complete introduction or repetition is done by the teacher. As apparent from excerpt 11, learners are neither directly involved in the introduction nor the clarification of the two terms.

Excerpt 11 (EN_THUR §63)

T: *Which means vocabulary-wi:se (.) that those two angles are called <SLOW> complementary </SLOW> angles. (1) {T writes on board} <SLOW> complementary (2) {T writes on board} <SLOW> angles. </SLOW> (3) Ehm you should have done that I thi:nk in? second grade when you talked about supplementary angles and complementary angles. Supplementary angles where those angles that add up to one hundred and eighty degrees. (.) to a straight line. (.) and complementary angles where those angles that add up to ninety degrees. (.) the ONLY reason why we mentioned that in second grad actually because now we need it for the whole chapter. just (1) vocabulary-wise (2)*

In group A, on the other hand, the teacher tries to elicit both terms by confronting learners with the definitions first. More precisely, she presents the formal definition as a question when asking *Zwei Winkel die sich auf hundertachzig Grad ergänzen heißen wie? [two angles that complement each other to one hundred and eighty degrees are called?]* (lines 1-2 in excerpt 12). Consequently, in contrast to group B, the students are compelled to actively recall their prior knowledge in the German class. The same is true for the term *complementary angles*. While term and definition are automatically presented by the teacher in English, it is elicited in the German setting, although it is mentioned in both classes that the terms are not new. This is especially emphasized in group A, where the teacher asks twice *Wer kann sich zufällig erinnern? [does someone happen to remember?]* (line 2) and *Könnts euch daran noch erinnern? [Do you still remember?]* (line 5).

Furthermore, the new terms are explicitly introduced as vocabulary. While in English it is mentioned twice that the terms are important *vocabulary-wise* (lines 1 and 9 in excerpt 11), the term *Vokabel [vocabulary*] (lines 1 and 6 in excerpt 12) is used twice in group A. Consequently, in both lessons the introduction of the terms is brought into the focus of language learning rather than acquiring new mathematical knowledge.

Excerpt 12 (GER_THUR §129-134)

T:*[…] (1) Jetzt Vokabeln der Unterstufe (.) Zwei Winkel die sich auf hunderachzig Grad ergänzen heißen wie? Wer kann sich zufällig erinnern? Da gibts ein Wort dafür?*

S(f): *Supplementär?*

T: *Genau, das waren Supplementärwinkel. Könnts euch daran noch erinnern? (.) Das ko bei dem Kapitel genau diese Vokabel brauch ma jetzt bei dem Kapitel und beim nächsten Kapitel. (.) Zwei Winkel die sich auf hunderachzig Grad ergänzen heißen Supplementärwinkel. Wie heißen zwei Winkel die sich auf neunzig Grad ergänzen? Weiß das zufällig jemand?*

S(f): *Ehm.*

S(m): *Komplementärwinkel?*

T: *Sehr gut. Komplementärwinkel. (.) bitte aufschreiben. (.) <SLOW> kom pli mentär (.) Winkel </SLOW> {T writes on board} (2) Das heißt alle Winkel die sich auf neunzig Grad ergänzen. Das heißt in einem rechtwinkeligen Dreieck sind die zwei Winkel die nicht der rechte Winkel sind immer (.) <pvc> {komplementär.} </pvc> […]*

In terms of CIS, it is noticeable that simple topic reiteration is used in both groups. While the term *complimentary* occurs four times in group B (lines 2, 3, 5, and 7), the teacher utters the term *komplementär [complimentary]* three times in the German taught lesson (lines 12, 13, and 15). It is, though, mentioned a fourth time by a student (line 11). Interestingly, the situation is different from the term *supplementary*, which is used only two times by the teacher in the English (line 5) and the German setting (lines 5 and 8).

Different strategies are used in the two lessons regarding the explanation of the terms. While in group A the teacher limits her content relevant remarks to the formal definitions of *Zwei Winkel die sich auf hunderachzig Grad ergänzen heißen Supplementärwinkel [two angles that add up to one hundred eighty degrees are called supplementary angles]* (lines 7-8) and *Das heißt alle Winkel die sich auf neunzig Grad ergänzen [that means all angles that add up to ninety degrees]* (lines 13-14), additional paraphrasing is used in the English version. Here the teacher provides an additional description of supplementary angles with reference to the geometric shape of a straight line: *[…] that add up to one hundred and eighty degrees. (.) to a straight line* (lines 6-7). In the case of the term complementary angles, a geometric interpretation of the same is the basis for introducing the term. The

link that the two smaller angels in a right-angled triangle always add up to ninety degrees directly precedes excerpt 11. In group A, on the other hand, the insight that *in einem rechtwinkeligen Dreieck sind die zwei Winkel die nicht der rechte Winkel sind immer (.) komplementär [in a right-angled triangle the two angles that are not the right angle are always complementary]* (lines 14-15) concludes the introduction of the term. Hence, the potential of using visual aids to make mathematical concepts clear is apparent in both settings. Furthermore, this excerpt clearly reveals that language learning aspects are an important factor in math teaching, no matter if taught in the students' first or a second language.

6.4.2.6 Adjacent and opposite side

This is also true for excerpts 13 and 14, which represent the introduction of the formal terms for the concerning sides in the definition of the tangent. Nevertheless, this situation is not explicitly presented as vocabulary learning, neither in group A nor in group B. This might be due to the fact that the names, in this case, draw from much simpler and everyday language — in English as well as in German — than complementary angle and supplementary angle do. Hence, the notations are easier to deduce from the position in a triangle with respect to the regarding angle. Such a deduction is also the strategy of the teacher in both groups. As can be seen in excerpt 13 she tries to elicit the correct terms by drawing students' attention towards the position of the sides, seen from the perspective of the respective angle. The use of visual aids is of particular relevance in this example, since the teacher intimates which side of the triangle she means by pointing at the sketch on the blackboard. The phrase *I compare (.) this side here (.) with (.) this side here* (lines 1-2) would not make sense at all to the students without this graphical support. Furthermore, this approach offers an alternative to verbally describing or eliciting the name of the term sought. In case of *opposite site* the hints for eliciting the correct notations are also restricted to referring to the sketch which is mirrored in the repeated use of *this side* (lines 1, 2, 5, and 6), *this one* (line 7), *that one* (line 7), or *that side* (line 2). This is, perhaps, down to a lack of alternative vocabulary or synonyms which would accurately describe the position without using the word *opposite* itself.

The term *adjacent*, on the contrary, is elicited by paraphrasing it with the synonym *next to* (line 12 and 14). As indicated by the teacher with the question *What's a fancy word for next to something?* (line 14), the mathematical term sought is more formal and can be easily translated into more natural language. However, the term is obviously already known by students in a different context as they come up with the correct answer themselves (line 21). They even offer another paraphrase in the form of *adjoining* (line 19). Comparing these two instructions, it becomes obvious that successfully explaining mathematical concepts and terms for deeper understanding depend, to a large degree, on natural language sources available.

Excerpt 13 (EN_THUR §107-117)

T: *Tangent of epsilon. And the definition is? I compare (.) this side here (.) with (.) this side here. But I cannot write down that side and that side (.) And that's why I need to NAME them. (.) According with respect to the angles. Which means (.) I wanna know if it's about this angle here (.) What do you think how would? How could I name this sides <x> makes sense? How can I? How could you describe this side so EVERYone knows when you talk about THIS side compared to epsilon when it's here. and not this one (.) and not that one. (.) <1> How could you describe that </1>*

S(x): *<1> <xx> opposite </1> the*

T: *Opposite. That's it. that's fine. Opposite. (.) it's called the (.) <SLOW> opposite side </SLOW> {T writes on board} (5) we write it down in German afterwards. (2) divided by? (2) and now (.) we need (.) this side here that is NEXT to epsilon, but NOT the hypotenuse. (1) So what could be a na:me? (.) How do you call some that is (.) nec What's a fancy word for next to something. If you (.)*

S(x): *<x> ju <x>*

T: *Yeah?*

S(x): *<x>*

T: *S Say it again?*

S(x): *Adjoini:ng maybe?*

T: *Almost, not adjoined a*

S(m): *Adjacent=*

T: *=Adjacent. Very good. Exactly. Divided by <SLOW> adjacent. </SLOW> {T writes on board} (1) <SLOW> adjacent </SLOW> side (.) Always (.) with respect to the <SLOW> given angle. </SLOW> (1) So in our example with epsilon. (.) The opposite side is? (.) p (3) and the adjacent side is? (2)*

Hence, it stands to reason that the scope for paraphrases in explanations increases with a higher level of proficiency in the language of instruction. This assumption is underpinned in excerpt 14 which covers the introduction of the terms *Gegenkathete [opposite side]* and *Ankathete [adjacent side]* in group A. Although the teacher also deploys the sketch on the board as a visual aid for her instruction, she elicits the term via paraphrased wording. With the question *wo liegt die? [where is it located?]* (line 4), the teacher instantly evokes the word *gegenüber [opposite]* (line 6) of one of the students. Consequently, she can reasonably deduce the term *Gegenkathete.* This precise question also allows the teacher to get to the point faster. Even though both excerpts start with pointing at the concerning sides of the triangle, the turn is much shorter in German (excerpt 14, lines 1-4) (53 words) than in English (excerpt 13, lines 1-7) (96 words). Thus, it needed almost twice as many words in L2 than in L1 to elicit the word opposite. This quantitative disparity, however, is not due to a higher amount of information in English but rather caused by a number of recasts of the same question. As can be seen in excerpt 13 (lines 4-8) the question *How could I name this side is?* is reformulated three times. Additionally, there are several incomplete attempts to recast the question and the word *how* is repeated five times. This clearly indicates the teachers' attempt to foster understanding by providing several modified options of the intended question.

The strategy of recasting a question can also be observed in German. However, in excerpt 14 the same question is only reformulated once. The question of *So wie könnte ich die jetzt bezeichen? [So, how could I label it?]* (line 2) is instantly repeated as *Was glaubts ihr, wie könnt ich jetzt sagen? [What do you think? How could I say?]* (lines 2-3) In this situation it is not likely that the recasting of the question aims at a clarification of the same. Rather, repeating a question several times serves to bypass a period of silence between a teacher's question and a student's answer or stressing the relevance of the point.

Excerpt 14 (GER_THUR §136-144)

T: *[…] Das Verhältnis ist definiert als (.) die (.) Seite hier. (2) durch die Seite hier. (.) So wie könnte ich die jetzt bezeichnen? (.) Sind beides Katheten. (.) Was glaubts ihr wie könnt ich jetzt (.) sagen (.) da ist mein Winkel und ich meine diese Kathete hier und nicht diese. Was könnt das für ein Na, oder wo liegt die? (2) Ja?*

S(m): *Irgendwas mit gegenüber?*

T: *Genau und drum heißt sie Gegenkathete. (.) Also es ist Gegenkathete (4) {T writes on board} und das machen wir gleich alles bis aufs g in Klammer weil das ist auch die Abkürzung einfach nur g (.) weil es gibt nur ein g die Gegenkathete. (2) wie könnte ich jetzt DIE da nennen? (.) Die da DRAN ist. (2) Das ist die? (.)*

S(m): *Nebenskathete.*

T: *Fast. (1) Das Wort ist in dran drinnen. <@>*

S(m): *Dranka <1> thete. </1>*

T: *<1> Ja </1>*

S(m): *Ankathete.=*

T: *Ankathete genau. […]*

The elicitation of the term *Ankathete [adjacent side]*, again, is similar in both classes. Just as in group B the teacher tries to elicit the correct term by uttering a synonym, namely *dran [next to]* (line 10). In contrast to the English lesson, the derivation of the correct term is possible on a morphological level as indicated by the teacher with the hint *Das Wort ist in dran drinnen [the word is a part of dran]* (line 12).

Interestingly, both terms are translated into German as well in group B. Although this is the case with other terms in the lesson as well, it is striking that *Ankathete* und G*egenkathete* are not simply translated but elicited from students in their first language as well.

Excerpt 15 (EN_THUR §125-139)

T: *<1> What I want </1> you to do now we write down the German expressions which are? (3) {T writes on board} This is called because in German we have words <GERMAN> Kathete (1) also is das die? (.) was glaubts ihr? die? </GERMAN>*

S(m): *<GERMAN> Gegenüberliegende kathete </GERMAN>*

T: *<GERMAN> Fast </GERMAN>*

S(x): *<GERMAN> Gegen </GERMAN>*

T: *<GERMAN> GEgenkathete einfach. Genau. Die GEGENkathete. (5) {T writes on board} weil Gegenüberliegende Kathete wär zu lang. (.) Und wie nennt man? wo schauts ihr hin? </GERMAN>*

[…]

T: *<GERMAN> und dann die Andere? (2) wie könnte man die nennen? die? mh? </GERMAN>*

S(f): *<GERMAN> Nebenkathete? </GERMAN>*

T: *<GERMAN> Fast (.) So wie </GERMAN> adjacent <GERMAN> is es die An <1> liegende Kathete </1> </GERMAN>*

S(m): *<GERMAN> <1> Anliegende kathete </1> </GERMAN>*

S(f): *<1> Ahh </1>*

T: *<GERMAN> Also Ankathete nennt man das. <SLOW> Ankathete. </SLOW>*

As can be seen in excerpt 15, the German translations are not introduced by the teacher but contributed by the students. Hence, the connection between the name and position of both sides is additionally foregrounded. As a consequence, both labels appear plausible and the mathematical content is directly related to its linguistic representation. In severe contrast to the terms complementary angle and supplementary angle, the opposite and adjacent side are not explicitly labeled as new vocabulary. Although the latter terms are math specific as well, they exploit more natural or everyday language. Hence the labels are more tangible and reasonable for learners. In fact, the terms are not new to them but introduced in a new context and this is clearly mirrored in the two different approaches to introducing new mathematical terms.

6.4.2.7 Radian

A challenging sequence in the session which took place on the following day in both groups was the introduction of the measuring unit radian. Up to that point, all calculations had been computed in degrees. Nevertheless, the learners encountered the new unit earlier, as it is the standard setting on their calculators. Towards the end of the second lesson, radian was introduced and explained in more detail as evident from excerpts 16 and 17.

Clearly, there are striking differences between the approaches chosen to develop the definition of a radian in the English- and the German-taught lesson. However, what both lessons have in common is there starting point of introducing the new unit. In both excerpts 16 and 17, it is noticeable that, first of all, the teacher is questioning the unit of degrees. In group A this is uttered with the phrase *Degrees is pretty much something made up* (lines 2-3). In group A the teacher says *ein Grad ist definiert als ein Neunzigstel von einem rechten Winkel. Das heißt das ist super fiktiv [one degree is defined as a ninetieth of a right angle. This means it is super fictive]* (lines 3-4). Consequently, the focus in the introductory turn is more on *degrees* than on *radian*. This is particularly emphasized through the fourfold repetition of the word *degrees* in group B (lines 2, 3, and 4) and the word *Grad [degree]* (lines 3, 5, and 7) in group A.

More distinguishable is the further development of the new term in the two settings. In group B, the bridge to the new unit is finally forged by the statements that *a circle (.) has the pi included* (line 5) and *so they tried to find a measurement where they can actually work with that pi* (lines 6-7). Without explaining in detail how the number pi and a circle are related, the teacher continues by introducing the definition of a radian.

In group A, on the other hand, the word pi is only mentioned once and in a very different context. It only appears at the very end of excerpt 17 in order to recall the formula for the arc length of a circle sector.

Excerpt 16 (EN_FRI §348-352)

T: *[…] Okay so radian measure (.) this is ehm (.) compared to (.) let's write this down (2) {T writes on board} NOT degrees (2) Degrees is pretty much something made up. (.) One degree is defined as one ninetieth of right angle (1) which hence means that ninety degrees is a right angle. (.) and it's not a very natural (.) measurement compared for a circle? Cause a circle (.) has the pi included which is an irrational number. Right? (1) and so so they tried to find a measurement where they can actually work with that pi. (1) Yeah?*

S(m): *Is it the thing with two pis?*

T: *It's the thing with two pis. A whole circle is two pi. exactly. (.) So the idea is an that's a definition (.) we write down the definition afterwards, I just explain it first. (.) The definition of one radian (1) {T writes on board} so not one degree now (.) it's one radian (2) is ehm (1) {T writes on board} that you have (.) a sector (3) where (.) the length of the arc (2) which is in IB it's called l . Cause it's the LEngth of the arc (.) remember in German what are we using for the arc length.*

S(f): *b.*

T: *b <GERMAN> für Bogenlänge </GERMAN> (1) this is here (.) one (2) {T writes on board} radian. (.) AND with a circle having the same (.) length. that's the definition of one radian. One radian define is defined as the angle created (.) when having the arclength equal to the radius of a circle. (.) That's pretty much the definition of a radian. (1) And we'll derive now how much one radian is and then we'll work with that in the circle. (.) […]*

The connection between circle and radian is brought into effect without mentioning the number pi as a starting point in excerpt 17. In this situation the German introduction of *Und das Bogenmaß ist jetzt ein (.) ein ma: also ein Maß für einen Winkel (.) das ich mit dem Kreis in Zusammenhang setzte sprich das ich über den Kreis definiere [and the radian measure is now a measurement for an angle, which I relate to the circle, that is to say, which I define in terms of the circle]* (lines 9-11) simply gives more details about the mathematical concept behind the radian measure compared to the English version. Furthermore, this principle is repeated as a paraphrase in group A with the phrase *Das heißt ich nehm einen Kreis. Es geht ja um Kreis bei Winkel auch. (2) Ich nehme den Kreis an sich als Definition her (1) um (.) diese Einheit von Winkel zu definieren [That means I take a circle. It is also about a circle when dealing with angles. I take the circle by itself as a definition to define this measurement of angels]* (lines 26-28). In group B, this is narrowed to the aforementioned request to *actually work with that pi* (line 7).

Excerpt 17 (GER_FRI §400-408)

T: *[…] Also wie gesagt das ist einfach eine andere Einheit für Winkel und zwar ist es die natürlichere unter Anführungszeichen (.) Einheit für Winkel. (2) weil ein Grad ist definiert als ein Neunzigstel von einem rechten Winkel. Das heißt das ist super fiktiv. Okay ist ein Neunzigstel, deswegen ist ein rechter Winkel neunzig Grad (.) Weil ein Grad definiert ist als ein Neunzigstel (.) super hat jemand sich so überlegt, ist halt so. (.) deswegen hat ein ganzer Kreis wieviel Grad?*

S(m): *Dreihundersechzig.*

T: *Dreihundertsechzig Grad. Genau. (.) Und das Bogenmaß ist jetzt ein (.) ein ma: also ein Maß für einen Winkel (.) das ich mit dem Kreis in Zusammenhang setzte sprich das ich über den Kreis definiere. (1) und die Definition ist (1) wenn man einen Kreissektor aufzeichnet. Das ist ein Kreis ist so ein Tortenstück. (.) Ein Kreissektor. (1) und ein (1) Grad (.) sozusagen ein (1) Bogenmaß also Radiant also rad R A D ich muss etwas hinschreiben was für eine Einheit ich hab. Ist wie wenn ich ignorier ob ich englisches feet mals Kilometer Centimeter hab. (.) Komplett unterschiedliche Sachen. Ich muss einfach hinschreiben womit ich arbeite. (1) Ist definiert als (.) ein (.) Bogen die Bogenlänge (.) von einem Sektor das kennts noch als b oder? (.) <1> Habts </1> ihr letztes <2> Jahr </2> gmacht beim Kreis.*

S(m): *<1> ja </1>*

S(f): *<2> Ja </2>*

S(m): *Ja.*

T: *Und, dass diese Bogenlänge gleich lang ist wie (.) der Radius. Das ist die Definition von einem (1) Radiant sozusagen.*

S(m): *Ahhh.*

T: *Das heißt ich nehm einen Kreis. Es geht ja um Kreis bei Winkel auch. (2) Ich nehme den Kreis an sich als Definition her (1) um (.) diese Einheit von Winkel zu definieren drum ist es die eigentlich (.) na die (.) beliebtere mathematisch (.) natürlichere Variante um mit Winkeln zu arbeiten. (1) Das ist Definition. (1) Das heißt wenn ich jetzt irgend einen Kreis hab (.) nehm ich an (.) ich hab den Kreis (1) und das ist mein Radius such ich quasi das Bogenmaß das genau so lang wie das ist und damit hab ich den Winkel ein Radiant sozusagen. (2) ich möcht euch jetzt zeigen wie viel Grad das sind cirka damit ihr die Umrechnung sehts. (1)*

Die Formel für (.) eine Bogenlänge (1) bekomm ich in dem ich (2) einen ganzen Kreis nehme (1) ein ganzer Kreis (1) ist die längste Bogenlänge der Umfang vom Kreis. (1) <xxx> größt mögliche (.) Bogenlänge ist der Umfang. (1) und den Umfang von einem Kreis berechne ich mit zwei r pi (.) oder ihr auch. (2) Könnts euch an die Formel erinnern?

Generally, the key role of the circle for the radian measurement is especially emphasized in excerpt 17 as the term *Kreis [circle]* it is repeated 15 times. The word *circle*, on the other hand, is only used five times in the corresponding excerpt 16.

A strategy which is used in both settings, again, is the use of visual aids. In both lessons, the teacher develops her explanation by means of a sketch of a circular sector on the blackboard. In group B, the teacher starts drawing that sketch while uttering *a sector where the length of the arc* (excerpt 16, line 13). Both terms *sector* and *length of the arc* were assigned the corresponding segments of the drawing by pointing at them. Furthermore, the definition of one radian was exemplified by labeling the radius and the arc length with the same letter l. Using an identical sketch, the teacher started to draw the sector in the German group while saying *wenn man einen Kreissektor aufzeichnet [when you sketch a circular sector]* (lines 11-12). Interestingly, there is a tangible comparison for the word *Sektor [sector]* in German. With the phrase *Das ist ein Kreis ist so ein Tortenstück [that is a circle it is like a piece of a cake]* (line 12), the teacher offers a very natural explanation for a formal mathematical term. This is, however, not the case in the English setting.

All in all, both settings are similar in their starting point of questioning the concept of degrees as a measurement for angles and the use of a sketch as a visual aid. Nevertheless, the German version is more detailed and covers the connection between angles and circle on a more complex level.

6.5 Discussion

As evident from the preceding analysis, a different language of instruction in the math classroom does not only imply a one to one translation in a different medium but also variations in the presentation and explanations of mathematical content. In terms of the overall teaching approach, the two different languages have no obvious impact. Clearly, communication is teacher-centered in both settings as described by Maier & Schweiger (1999: 108). This is also in line with findings from Franke, Kazemi & Battey (2006: 231), who observed that "[i]n mathematics classrooms students are typically asked to listen and remember what the teacher said". Nevertheless, a closer scrutinization of the lesson

transcripts revealed interesting insights with regard to research question 1 — How does the language use of a teacher differ between introducing mathematical content in German and in English? From a quantitative perspective, it is striking that the German taught lessons provides a higher amount of teacher input as measured by word count. Hence, the teacher conveys more information in her and the students' L1 than in English. This is also in line with the qualitative analysis of the data, which detects a clear trend to more background information and speaking more verbosely in German. Section 4.4.2.1 is an eminent example in this context.

With respect to student involvement, the findings clearly support previous research by Spillane & Zeuli (1999: 14), Gregg (1995: 459) or Pimm (1987: 52-55), who highlight the dominant role of closed questions to encourage student output. Hence, students are mainly asked to supply short facts rather than to expand on theoretical concepts. Furthermore, it is striking that the teacher tries to elicit more mathematical terms directly from the students in German. As evident from section 4.4.2.4 and 4.4.2.5, students are actively asked to recall terms and previous knowledge in German rather than in English. Although students are also partly involved in developing the corresponding mathematical concepts in English, specialized terminology is primarily introduced by the teacher. Thus, recalling mathematical vocabulary is rather passive for learners in the CLIL setting. In this context, it seems, the second language acts as a deterrent for the teacher for more active student involvement and the teacher tended to take over the students' job as described by Huang, Normandia & Greer (2005: 44). Moreover, a tendency for less student involvement in English can be generalized, as with 556 student turns, there are significantly more in German. In the CLIL setting, 467 student turns were identified. With regard to the explanation of math-specific terminology, it can be said that a higher level of linguistic resources is noticeable in German. This finding also reflects the fears observed by Dalton-Puffer (2007: 5), which concern the negative impact of foreign language use on content teaching. Just as highlighted in sections 4.4.2.2 and 4.4.2.3, the teacher simply seems to have more scope to expand on specific terms and especially the connection between label and concept. In other words, it seems to be easier for the teacher to explain why mathematical concepts are called what they are in German than in English. Besides more linguistic resources for circumscribing technical vocabulary in the L1, this might also

have to do with a greater awareness of the relations between mathematical lexicon and everyday language. Just as described by Sigley & Wilkinson (2015: 78) there is a considerable intersection of words which are used in both fields. Since mathematical terms are often loanwords in both languages, the qualitative comparison has allowed for insights into the interweaving of vocab and content work in math teaching. While this didactical potential is clearly taken up in several instances in the German class, it is more limited in English.

With regard to research question 2 — What kind of comprehensible input strategies are adopted by the teacher and how do they differ when using students' L1 or English as the language of instruction? — a clear differentiation between the two settings is not as obvious. Undoubtedly, attempts to simplify mathematical input are recognizable in both languages and the teacher clearly tries to present the content in an easily accessible way for students. Strategies for simplification and types of modification as introduced by Hatch (1979 quoted in Krashen 1982: 64) and Pica, Doughty & Young (1986: 122) are applied in both settings. The presumption that using English as the language of instruction would foster the occurrence of CIS for the sake of better understanding, however, cannot be confirmed so easily. Rather, the use of CIS also plays a crucial role in the German group as well. First and foremost, this is true for topic reinstatements (Chaudron 1983: 440-443) and repetitions (Buri 2012: 13) in general. In the vast majority of excerpts analyzed, the topic or central message of the sequence is uttered several times. Just in line with findings from Chaudron (1983: 440-443), simple topic reiteration appears to assume a central role in emphasizing the focus of a turn in both languages English and German. Such repetitions, however, often involve syntactic modifications of the initially uttered phrase or sentence. Syntactic revisions of that kind are referred to as recasts (Buri 2012: 13) and also occur in both settings. Nevertheless, recasting is used by the teacher in different situations in each of the two sample groups. While an intensive utilization is observable in section 4.4.2.4 in the German group, it is more prominent in the English class in section 4.4.2.6, for example. Hence, the strategy of recasting is not directly bound to a specific term by the teacher but rather depending on the situation. It is salient, however, that it mainly occurs in the context of asking a question. In both excerpts mentioned above, the teacher reformulates a question immediately, before students can respond. Thus, it stands to reason that

recasting a question is not simply an attempt to clarify the meaning but also to urge a response from the students and to prevent a longer period of silence.

Another comprehensible input strategy that is intensively used in both groups is the use of visual aids (Buri 2012: 11). As highlighted by O'Halloran (2015: 70-71) visualizations and mathematical images play an essential role in mathematical discourse and their potential for teaching was definitely realized by the teacher. In group A as well as in group B this is mainly implemented through sketches on the blackboard. Naturally, a topic related to geometry lends itself for multimodality in the form of graphic representation of the content matter (O'Halloran 2015: 70). Hence, visual representatives are essential for the comprehensibility of the content conveyed. Furthermore, they serve as a vital clue in several elicitation processes in order to trigger the desired response, especially with regard to the label of specific sides of a triangle. All in all, both settings reveal the enormous potential of visual support in teaching mathematics, as a powerful tool to overcome linguistic inaccurateness or unnecessary complicated explanations and circumlocutions.

Finally, the use of paraphrases (Buri 2012: 11) is prevalent in both settings and captured in numerous excerpts. Throughout all four lessons, the teacher tries to offer simplified utterances and expressions for new technical and abstract terms. Thus, the teacher tries to draw on language resources accessible to learners. Consequently, this comprehensible input strategy usually involved the application of more general language. Supporting Schleppegrell's (2010: 63-64) argumentation, the teacher draws on everyday language in order to introduce a more formal mathematical register as defined by Brunner (1976: 209) or Halliday (1974: 65). Hence, the discussion of this analytical focus is clearly relevant for research question 3 — To what extent do CIS entail a form of mathematical metalanguage in order to convey mathematical content? With regard to this question, it can be clearly stated that paraphrases mainly serve to transport mathematical content in a non-mathematical standard. Therefore, they aim at substituting content-specific technical language for more commonly used terms. In other words, the teacher explains mathematical content by speaking about them without using abstract mathematical notions but language which is already familiar to the learners. Although this strategy is

clearly observable in both classes, there are differences in terms of the situations in which they occur between the German and the CLIL lessons. First of all, it needs to be considered that the use of paraphrases concerns a wider scope of the language used by the teacher. In English, there is a higher need for clarifying terms and expressions that are not directly related to mathematics. Since the language of instruction is another than students' L1, there is also a need to elaborate on non-mathematical language. This becomes evident in section 4.4.2.1 for example. Such passages mirror the aspect of second language learning in CLIL, which is supposed to be embedded in the process of content learning. Nevertheless — just as in the German taught lessons — paraphrasing also assumes an important role in simplifying mathematical terms in English. Hence, content and language cannot be simply separated in either of the two settings (Dalton-Puffer 2007: 6). It is striking, however, that instances of this CIS concern different notions in the two settings. While the teacher elaborates on some expressions in German, like in sections 4.4.2.3 or 4.4.2.7, this is not the case in English. This is also true, on the other hand, vice versa, as in section 4.4.2.5. This pattern suggests two conclusions for the practicability of paraphrases in the two different languages. Firstly, the notions used can have a different level of formality or abstractness in the respective language. In other words, while a certain notion might be self-explaining in one language, it might appear meaningful to circumscribe the corresponding term in another one. Such a meaningful explanation, however, heavily depends on the linguistic resources available for both the teacher and the learners, which is the second inference in this context. Consequently, the realization of successful paraphrases might pose a bigger challenge in L2 than in L1. Even though it is a matter of simplification and adopting more commonly used language, finding suitable circumlocutions or synonyms requires a profound repertory of linguistic alternatives. This, of course, accounts for a certain discrepancy when using a second language as the language of instruction. On the one hand, it stands to reason that there is an expanded need for explanations and paraphrases in L2. The resources required for this strategy, on the other hand, are limited as well. Hence, it often proves to be difficult to present the content specific matter in a learner-friendly but still scientifically accurate manner.

All in all, the analysis in the empirical part of this study is a clear illustration of the entanglement of language and mathematics. Although the language is often reduced to

an inevitable medium which is simply a means to an end in content subjects, the insight of this study clearly underpins the view that language is more than just a medium for teaching math. The juxtaposition of two different languages used by the same teacher for teaching the same topic to two different classes unmistakably reveals the influence of the language used on the presentation of mathematical content. Hence, the way teachers use language is a key component in imparting knowledge and enabling comprehension of subject-specific concepts in the math classroom. It is, to a high degree, accounted for by the skillful application of linguistic strategies by the teacher to manage the balancing act of being both mathematically accurate and easily understood at the same time.

6.5.1 Limitations and implications for research

Undoubtedly, the data in this study revealed valid and remarkable insights. Nevertheless, it is also of importance to outline the limitations of the analysis. First of all, it is essential to be distinctly aware of the qualitative nature of this case study. Although the data gained from the observation offered an interesting basis for the analysis of different language use in an L1 and a CLIL setting, it has only investigated the language use of one teacher within a very limited timeframe. Thus, the findings might not be representative of other CLIL teachers and cannot be generalized. Furthermore, it needs to be highlighted that the findings in this study are restricted to the actual procedure in the classroom and are, therefore, descriptive. Despite the fact that all excerpts concern the introduction of mathematical content and attempts to make the very same comprehensible for students, the analysis does not allow for conclusions about the didactic success of the strategies used. Hence, the analytical focus is definitely on how and what the teacher says. The actual didactical implications for the content taught are not part of this thesis. In this regard, the picture is still incomplete. Consequently, further work needs to be done in order to scrutinize the effect of teachers' language use on the learning progress of students in the math classroom. Experimental studies which analyze the impact of teachers' language choice and application of comprehensible input strategies are needed to shed light on this essential key to successful teaching. This is true for both L1 and CLIL content teaching.

7 Conclusion

This thesis is concerned with the interweaving of language and math. Structured in a theoretical and empirical section, it starts with a literature review on language aspects in mathematics. In this regard, the study elaborates on the language of math and in how far math can be considered its own or even a universal language. After focusing on features of a mathematical register, however, the validity of the claim of viewing math as a universal language is seriously challenged — at least in teaching. Hence, the thesis continues with highlighting language factors in learning mathematics and, consequently, in the math classroom. By drawing on the didactical concept of basic notations and Vygotskij's (1987) approach to learning, it is further illustrated how language shapes and fosters learners' learning process. It is further shown how this is true for both listening to the teacher and actively speaking about math. The last sections of the theory part are then concerned with insights and findings from research on teaching math in a foreign language. While such a teaching approach can be quite successful in CLIL settings, it often causes drawbacks for language minorities in multilingual classrooms.

The empirical part of the study is then concerned with strategies of making input comprehensible to learners. Although this is a common approach to second language teaching, it is little investigated in content teaching. Thus, the analytical focus is a teacher's language use in a math classroom that is — in one case — taught in the teacher's and learners' L1, German, and — in the second case — taught in English. As in the latter setting, language is not solely a medium but also a learning objective, the aim of the analysis is to compare the teacher's language use in the two different settings with special attention to the use of comprehensible input strategies. All in all, the analysis shows that CIS frequently occur in both settings and that the overall teaching approach is rather similar. A remarkable finding is that the need for simplifying input is not mainly a matter of language skills but also depends on the complexity of the content taught. Despite the fact that one of the two sample groups was taught in their first language, modification of input for the sake of better understanding occurred throughout all four lessons observed. Hence, the use of comprehensible input strategies was prevalent and sometimes occurred in similar patterns in both languages. While an intensive use of CIS might be expected in

English due to a lower level of language proficiency, it is noticeable that teaching math in learners' L1 also requires modification and simplification of content-specific input.

Nevertheless, situational differences could be observed in the respective language use and attempts of making math content more comprehensible for students. In summary, it can be observed that there was a higher amount of input in German than in English. Hence, the teacher talked less and slower in English. Furthermore, it is apparent from the excerpts that some strategies for making input comprehensible directly depend on pre-existing language resources of both learners and teacher. In other words, one can only paraphrase or express a technical term in more natural language, if one has a suitable alternative in one's linguistic repertoire or if such a synonym even exists. Moreover, the scope of opportunities for finding comprehensible expressions or explanations for a mathematical term directly depends on the abstractness of the label. Interestingly, it might be more difficult to explain a mathematical notion which is also used in everyday language, as simpler varieties for a word are often not that obvious in such situations. This, however, appears to be easier in the L1, since there are more language resources available to the speaker and for the learner. Thus, the study suggests that the language of instruction is not just a matter of different mediums but also affects the presentation and, consequently, the perception of mathematical content in class.

8 References

Adoniou, Misty; Qing, Yi. 2014. “Language mathematics and English language learners”. *Australian Mathematics Teacher* 70(3), 3–13.

Aiken, Lewis R. 1972. “Language factors in learning mathematics”. *Review of Educational Research* 42(3), 359–385.

Archer, Anita L.; Hughes, Charles A. 2011. *Explicit instruction: Effective and efficient teaching*. New York, NY: Guilford Press.

Austin, J. L.; Howson, A. G. 1979. “Language and mathematical education”. *Educational Studies in Mathematics* 10(2), 161–197.

Baker, Colin. 2001. *Foundations of bilingual education and bilingualism*. (3rd ed.). Clevedon England: Multilingual Matters.

Banse, Holland W.; Palacios, Natalia A.; Merritt, Eileen G.; Rimm-Kaufman, Sara E. 2016. “Scaffolding English language learners’ mathematical talk in the context of calendar math”. *The Journal of Educational Research* 110(2), 199–208.

Barton, B.; Chan, R.; King, C.; Neville-Barton, P.; Sneddon, J. 2005. “EAL undergraduates learning mathematics”. *International Journal of Mathematical Education in Science and Technology* 36(7), 721–729.

Bauersfeld, Heinrich. 1978. “Kommunikationsmuster im Mathematikunterricht: Eine Analyse am Beispiel der Handlungsverengung durch Antworterwartung [Communication patterns in the math classroom: An analysis using the example of action narrowing due to expactation of answers]”. In Bauersfeld, Heinrich (ed.). *Fallstudien und Analysen zum Mathematikunterricht [case studies and analyses of math classes]: Festschrift für Walter Breidenbach zum 85. Geburtstag ; zugleich ein Beitrag zur didaktischen Unterrichtsforschung [commemorative publication for the 85th birthday of Walter Breidenbach; likewise a contribution to the didactic classroom research].* Hannover: Schrödel, 158–170.

Bennett, Cory A. 2010. ““It’s hard getting kids to talk about math”: Helping new teachers improve mathematical discourse”. *Action in Teacher Education* 32(3), 79–89.

Biber, Douglas; Conrad, Susan. 2009. *Register, genre, and style*. Cambridge: Cambridge University Press.

Bishop, Alan T. 1988. “Mathematics education in its cultural context”. *Educational Studies in Mathematics* 19(2), 179–191.

Blum, Werner; Leiß, Dominik. 2007. “How do students and teachers deal with modelling problems?”. In Haines, C.; Galbraith, P.; Blum, Werner (eds.). *Mathematical modelling: Education, engineering and economics - ICTMA 12.* San Diego, CA: Elsevier Science, 222–231.

BMBWF. 2018a. “AHS-Lehrpläne Oberstufe neu: Mathematik” https://bildung.bmbwf.gv.at/schulen/unterricht/lp/lp_neu_ahs_07_11859.pdf, (28 Mar. 2019).

BMBWF. 2018b. “Lehrplan AHS Oberstufe: Lebende Fremdsprache (Erste, Zweite)” https://bildung.bmbwf.gv.at/schulen/unterricht/lp/lp_ahs_os_lebende_fs_11854.pdf?61ebyg, (28 Mar. 2019).

Brophy, Jere. 1986. “Teacher influences on student achievement”. *American Psychologist* 41(10), 1069–1077.

Brunner, Regina B. 1976. “Reading mathematical exposition”. *Educational Research* 18(3), 208–213.

Buri, Crisanta C. 2012. “Determinants in the choice of comprehensible input strategies in science classes”. *Journal of International Educational Research* 8(1), 1–18.

Chaudron, Craig. 1983. “Simplification of input: Topic reinstatements and their effects on L2 learners’ recognition and recall”. *TESOL Quarterly* 17(3), 437–458.

Christie, Frances. 2002. *Classroom discourse analysis: A functional perspective*. London: Continuum.

Clarkson, Philip C.; Galbraith, Peter. 1992. “Bilingualism and mathematics learning: Another perspective”. *Journal for Research in Mathematics Education* 23(1), 34–44.

Cuevas, Gilberto J. 1984. “Mathematics learning in English as a second language”. *Journal for Research in Mathematics Education* 15(2), 134–144.

Cuevas, Gilberto J.; Beech, Martha C. 1983. “A second-language approach to mathematics skills: Applications for limited-English proficient students with learning disabilities”. *Learning Disability Quarterly* 6(4), 489–495.

Cummins, James. 1979. “Linguistic interdependence and the educational development of bilingual children”. *Review of Educational Research* 49(2), 222–251.

Dale, Theresa; Cuevas, Gilberto. 1987. “Integrating language and mathematics learning”. In Crandall, Jo A. (ed.). *ESL through content-area instruction: Mathematics, science, social studies.* Englewood Cliffs, NJ: Prentice Hall Regents.

Dalton-Puffer, Christiane. 2007. *Discourse in content and language integrated learning (CLIL) classrooms*. Amsterdam: John Benjamins.

Dalton-Puffer, Christiane; Nikula, Tarja; Smit, Ute. 2010. “Charting policies, premises and research on content and language integrated learning”. In Dalton-Puffer, Christiane; Nikula, Tarja; Smit, Ute (eds.). *Language use and language learning in CLIL classrooms.* Amsterdam: John Benjamins, 1–19.

Dawe, Lloyd. 1983. “Bilingualism and mathematical reasoning in English as a second language”. *Educational Studies in Mathematics* 14(4), 325–353.

Franke, Megan L.; Kazemi, Elham; Battey, Daniel. 2006. “Mathematics teaching and classroom practice”. In Lester, Frank K. (ed.). *Second handbook of research on mathematics teaching and learning: A Project of the national council of teachers of mathematics.* Charlotte, NC: Information Age, 225–256.

Freudenthal, Hans. 1983. *Didactical phenomenology of mathematical structures*. Dordrecht: Reidel.

Gregg, Jeff. 1995. “The tensions and contradictions of the school mathematics tradition”. *Journal for Research in Mathematics Education* 26(5), 442–466.

Halliday, Michael A. 1974. “Some aspects of sociolinguistics”. In UNESCO (ed.). *Interactions between linguistics and mathematical education: Final report of the symposium sponsored by UNESCO, CEDO and ICMI.* Paris: UNESCO, 64–73.

Halliday, Michael A. 1993. “On the language of physical science”. In Halliday, Michael A.; Martin, James R. (eds.). *Writing science: Literacy and discursive power.* Abingdon: Routledge, 54–68.

Hatch, Evelyn. 1979. “Apply with caution”. *Studies in Second Language Acquisition* 2(1), 123–143.

Hatch, Evelyn M. 1983. *Psycholinguistics: A second language perspective*. Rowley, MA: Newbury House Publ.

Hedge, Tricia. 2008. *Teaching and learning in the language classroom*. Oxford: Oxford University Press.

Hoffert, Sharon B. 2009. “Mathematics: The universal language?”. *The Mathematics Teacher* 103(2), 130–139.

Huang, Jingzi; Normandia, Bruce; Greer, Sandra. 2005. “Communicating mathematically: Comparison of knowledge structures in teacher and student discourse in a secondary math classroom”. *Communication Education* 54(1), 34–51.

Hufferd-Ackles, Kimberly; Fuson, Karen C.; Sherin, Miriam G. 2004. “Describing levels and components of a math-talk learning community”. *Journal for Research in Mathematics Education* 35(2), 81–116.

Hunt, Kellogg W. 1965. *Grammatical structures written at three grade levels. NCTE research report no. 3*. Champaign, IL: NCTE.

Imhausen, Annette. 2016. *Mathematics in ancient Egypt: A contextual history*. Princeton, NJ: Princeton University Press.

Jäppinen, Aini-Kristiina. 2005. “Thinking and content learning of mathematics and science as cognitional development in content and language integrated learning (CLIL): Teaching through a foreign language in Finland”. *Language and Education* 19(2), 147–168.

Kaput, James J. 2002. “Notations and representations as mediators of constructive processes”. In Glasersfeld, Ernst (ed.). *Radical constructivism in mathematics education.* Dordrecht: Kluwer, 53–74.

Kleine, Michael; Kassel, Alexander J.; Harvey, Emily. 2005. “With a focus on ´Grundvorstellungen‘ Part 1: A theoretical integration into current concepts”. *ZDM* 37(3), 226–233.

Knight, Lester N.; Hargis, Charles H. 1977. “Math language ability: Its relationship to reading math”. *Language Arts* 54(4), 423–428.

Krashen, Stephen D. 1982. *Principles and practice in second language acquisition*. Oxford: Pergamon.

Krashen, Stephen D. 1985. *The input hypothesis: Issues and implications*. London: Longman.

Lampert, Magdalena; Beasley, Heather; Ghousseini, Hala; Kazemi, Elham; Franke, Megan. 2010. “Using designed instructional activities to enable novices to manage ambitious mathematics teaching”. In Stein, Mary K.; Kucan, Linda (eds.). *Instructional explanations in the disciplines.* New York, NY: Springer, 129–141.

Lee, Wona; Lee, Jin S. 2017. “Math instruction is not universal: Language-specific pedagogical knowledge in Korean/English two-way immersion programs”. *Bilingual Research Journal* 40(4), 353–371.

Lefevre, Jo-Anne; Fast, Lisa; Skwarchuk, Sheri-Lynn; Smith-Chant, Brenda L.; Bisanz, Jeffrey; Kamawar, Deepthi; Penner-Wilger, Marcie. 2010. “Pathways to mathematics: Longitudinal predictors of performance”. *Child Development* 81(6), 1753–1767.

Leinhardt, Gaea. 1987. “Development of an expert explanation: An analysis of a sequence of subtraction lessons”. *Cognition and Instruction* 4(4), 225–282.

Leinhardt, Gaea. 2010. “Introduction: Explaining instructional explanations”. *Instructional explanations in the disciplines*. In Stein, Mary K.; Kucan, Linda (eds.). New York, NY: Springer, 1–5.

Leinhardt, Gaea; Steele, Michael D. 2005. “Seeing the complexity of standing to the side: Instructional dialogues”. *Cognition and Instruction* 23(1), 87–163.

Long, Michael; Sato, Charlene. 1983. “Classroom foreigner talk discourse: Forms and functions of teachers’ questions”. In Seliger, Herbert; Long, Michael (eds.). *Classroom oriented research in second language acquisition.* Rowley, MA: Newbury House, 268–286.

Long, Michael H. 1983. “Linguistic and conversational adjustments to non-native speakers”. *Studies in Second Language Acquisition* 5(2), 177–193.

Maier, Hermann; Schweiger, Fritz. 1999. *Mathematik und Sprache: Zum Verstehen und Verwenden von Fachsprache im Mathematikunterricht [Mathematics and language: About the understanding and use of subject-specific language in the math classroom]*. Wien: öbv & hpt.

Meyer, Michael; Tiedemann, Kerstin. 2017. *Sprache im Fach Mathematik [Language in the subject math]*. Berlin: Springer Berlin Heidelberg.

Miura, Irene T.; Okamoto, Yukari; Vlahovic-Stetic, Vesna; Kim, Chungsoon C.; Han, Jong Hye. 1999. “Language supports for children’s understanding of numerical fractions: Cross-national comparisons”. *Journal of Experimental Child Psychology* 74, 356–365.

Mohan, Bernard A. 1986. *Language and content*. Reading, MA: Addison-Wesley.

Moschkovich, Judit N. 2010. “Language(s) and learning mathematics: Resources, challenges, and issues for research”. In Moschkovich, Judit N. (ed.). *Language and mathematics education: Multiple perspectives and directions for research.* Charlotte, NC: Information Age.

NCTM. 2000. *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Neville-Barton, Pip, and Bill Barton. 2005. “The relationship between English language and mathematics learning for non-native speakers” http://www.tlri.org.nz/pdfs/13909.pdf, (19 Mar. 2019).

Nikula, Tarja. 2010. “Effects of CLIL on a teacher’s classroom language use”. In Dalton-Puffer, Christiane; Nikula, Tarja; Smit, Ute (eds.). *Language use and language learning in CLIL classrooms.* Amsterdam: John Benjamins, 105–123.

O’Halloran, Kay L. 2015. “The language of learning mathematics: A multimodal perspective”. *The Journal of Mathematical Behavior* 40, 63–74.

O’Halloran, Kay L. 1998. “Classroom discourse in mathematics: A multisemiotic analysis”. *Linguistics and Education* 10(3), 359–388.

O'Halloran, Kay L. 2008. *Mathematical discourse: Language, symbolism and visual images*. Oxford: Bloomsbury.
Ouazizi, Khalid. 2016. "The effects of CLIL education on the subject matter (mathematics) and the target language (English)". *Latin American Journal of Content & Language Integrated Learning* 9(1), 110–137.
Oxford Dictionaries. 2019. "Explain | Definition of explain in English by Oxford Dictionaries" https://en.oxforddictionaries.com/definition/explain, (5 Mar. 2019).
Parker Waller, Patrice; Flood, Chena T. 2016. "Mathematics as a universal language: Transcending cultural lines". *Journal for Multicultural Education* 10(3), 294–306.
Pérez-Cañado, María L. 2012. "CLIL research in Europe: Past, present, and future". *International Journal of Bilingual Education and Bilingualism* 15(3), 315–341.
Piaget, Jean. 1947. *La Psychologie de l'Intelligence*. Paris: Libraire Armand Colin.
Pica, Teresa; Doughty, Catherine; Young, Richard. 1986. "Making input comprehensible: Do interactional modifications help?". *Working Papers in Educational Linguistics* 2(1), 121–146.
Pimm, David. 1987. *Speaking mathematically: Communications in the mathematics classrooms*. Abingdon: Routledge.
Purpura, David J.; Ganley, Colleen M. 2014. "Working memory and language: Skill-specific or domain-general relations to mathematics?". *Journal of Experimental Child Psychology* 122, 104–121.
Schleppegrell, Mary J. 2010. "Language in mathematics teaching and learning". In Moschkovich, Judit N. (ed.). *Language and mathematics education: Multiple perspectives and directions for research.* Charlotte, NC: Information Age, 60–84.
Setati, Mamokgethi; Adler, Jill. 2000. "Between languages and discourses: Language practices in primary multilingual mathematics classrooms in South Africa". *Educational Studies in Mathematics* 43(3), 243–269.
Sigley, Robert; Wilkinson, Louise C. 2015. "Ariel's cycles of problem solving: An adolescent acquires the mathematics register". *The Journal of Mathematical Behavior* 40, 75–87.
Small, Marian; Lin, Amy. 2010. *More good questions: Great ways to differentiate secondary mathematics instruction*. New York, NY: Teachers College Press; National Council of Teachers of Mathematics; Nelson Education.
Smit, Ute. 2010. *English as a lingua franca in higher education: A longitudinal study of classroom discourse*. Berlin: Mouton De Gruyter.
Snow, Catherine E. 2010. "Academic language and the challenge of reading for learning about sience". *Science* 328(5977), 450–452.
Spillane, James P.; Zeuli, John S. 1999. "Reform and teaching: Exploring patterns of practice in the context of national and state mathematics Reforms". *Educational Evaluation and Policy Analysis* 21(1), 1–27.
Surmont, Jill; Struys, Esli; Van den Noort, Maurits; Van de Craen, Piet. 2016. "The effects of CLIL on mathematical content learning: A longitudinal study". *Studies in Second Language Learning and Teaching* 6(2), 319–337.
Swanson, H. L.; Kong, Jennifer; Petcu, Stefania. 2018. "Math difficulties and working memory growth in English language learner children: Does bilingual proficiency

play a significant role?". *Language, Speech, and Hearing Services in Schools* 49(3), 379–394.
UNESCO (ed.). 1974. *Interactions between linguistics and mathematical education: Final report of the symposium sponsored by UNESCO, CEDO and ICMI*. Paris: UNESCO.
Valdés-Fallis, Guadalupe. 1978. *Code switching and the classroom teacher*. Arlington VA: Center for Applied Linguistics.
Van de Sande, Carla; Greeno, James G. 2010. "A framing of instructional explanations: Let us explain with you". In Stein, Mary K.; Kucan, Linda (eds.). *Instructional explanations in the disciplines.* New York, NY: Springer, 69–82.
Van Rinsveld, Amandine; Schiltz, Christine; Brunner, Martin; Landerl, Karin; Ugen, Sonja. 2016. "Solving arithmetic problems in first and second language: Does the language context matter?". *Learning and Instruction* 42, 72–82.
Vienna Oxford international Corpus of English. 2007. "Transcription conventions [2.1]: Mark-up conventions" https://www.univie.ac.at/voice/documents/VOICE_mark-up_conventions_v2-1.pdf, (26 Jun. 2019).
Vilenius-Tuohimaa, Piia M.; Aunola, Kaisa; Nurmi, Jari-Erik. 2008. "The association between mathematical word problems and reading comprehension". *Educational Psychology* 28(4), 409–426.
Vom Hofe, Rudolf; Blum, Werner. 2016. ""Grundvorstellungen" as a category of subject-matter didactics". *Journal für Mathematik-Didaktik* 37(S1), 225–254.
Vukovic, Rose K.; Lesaux, Nonie K. 2013. "The relationship between linguistic skills and arithmetic knowledge". *Learning and Individual Differences* 23, 87–91.
Vygotskij, Lev S. 1978. *Mind in society*. Cambridge, MA: Harvard University Press.
Vygotskij, Lev S. 1987. *Thought and language*. (2nd edition). Cambridge, MA: MIT Press.
Walker, Cindy M.; Zhang, Bo; Surber, John. 2008. "Using a multidimensional differential item functioning framework to determine if reading ability affects student performance in mathematics". *Applied Measurement in Education* 21(2), 162–181.
Walsh, Steve. 2011. *Exploring classroom discourse: Language in action*. Abingdon: Routledge.
Walshaw, Margaret; Glenda, Anthony. 2008. "The teacher's role in classroom discourse: A review of recent research into mathematics classrooms". *Review of Educational Research* 78(3), 516–551.

9 Appendix

A) Transcription Conventions

The conventions used in the transcripts are based on the VOICE Transcription Conventions [2.1] (Vienna Oxford international Corpus of English 2007).The following table provides a list of all conventions used in the transcripts consulted in this study.

Table 8 Transcription conventions

T	Teacher
S(m)	Male student
S(f)	Female student
Ss	Several students speaking at the same time
(.)	Pause shorter than a second
(1), (2),…	Timed perceptible pause within a turn (in seconds)
.	Sentence-final falling intonation
,	Phrase-final intonation (more to come)
?	Rising intonation
:	Lengthened vowel sound
=	a turn is immediately followed by another
@	Laughter
Capitals	Stressed words or syllables
<MUMBLING> 2 </MUMBLING>	Unintelligible mumbling
<QUIET> Text </QUIET>	Spoken very quietly
<SLOW> Text </SLOW>	Spoken comparatively slower
<GERMAN> Text </GERMAN>	German words or expressions

<READING> Text </READING>	Text being read aloud
(x)	Unclear speech (one 'x' per syllable)
<1> Text </1>, <2> Text </2>, …	Overlaps, everything that is simultaneous gets the same number
<pvc> text </pvc>	Striking variations on the level of phonology, morphology, and lexis as well as "invented" words
{text}	Additional comments
<first name of S(m)>, <first name of S(f)>	Anonymized name of a student

B) Abstract in English

The entanglement of language and mathematics is a multi-faceted field of research, which has been discussed widely but also controversially in the literature. This thesis is specifically concerned with linguistic aspects of teaching math, especially if the language of instruction is English. For that purpose, the first part of this study elaborates on the language used in the field of mathematics and its implication for the math classroom. Based on a thorough literature review, it is analyzed how language does not solely transfer information but also influences and shapes learners understanding and cognition of the same. In the second — empirical — part, a qualitative analysis of lesson transcripts offers remarkable insights into the effects of CLIL on a teacher's language use in math. The comparative approach refers to data consisting of math lessons observed in two sample groups. While in one group the language of instruction was the teacher's and learners' L1, German, the second group was taught in English. Although the findings reveal similar interaction patterns and strategies to make input comprehensible, they also suggest that in German, the teacher has a wider repertoire of language resources available to modify and comprehensibly explain mathematical content.

C) Abstract in German

Die Verflechtung von Sprache und Mathematik ist ein vielseitiges Forschungsgebiet, welches ausführlich, aber auch kontrovers diskutiert wird. Diese Diplomarbeit beschäftigt sich speziell mit Sprachaspekten im Mathematikunterricht, insbesondere wenn die Unterrichtssprache Englisch ist. Im ersten Teil wird auf den speziellen Sprachgebrauch in der Mathematik und dessen Auswirkungen auf den Unterricht eingegangen. Basierend auf einer ausführlichen Literaturrecherche wird dargelegt, wie Sprache nicht ausschließlich Informationen übermittelt, sondern auch die Wahrnehmung und das Verständnis bei Schülern und Schülerinnen beeinflusst und formt. Im zweiten — empirischen — Teil der Studie, ermöglicht eine qualitative Analyse von Unterrichttranskripten bemerkenswerte Einblicke in die Auswirkungen von CLIL-Unterricht auf den Sprachgebrauch einer Lehrperson. Der vergleichende Zugang bezieht sich auf Daten, welche aus Unterrichtsbeobachtungen in zwei Gruppen bestehen. Während in einer Gruppe Deutsch, die Muttersprache von Lehrperson und Schülern und Schülerinnen, war, wurde die zweite Gruppe in English unterrichtet. Obwohl die Ergebnisse ähnliche Interaktionsmuster und Strategien für besseres Verständnis nahelegen, ist klar ersichtlich, dass die Lehrperson in Deutsch ein größeres Repertoire an sprachlichen Ressourcen zur Verfügung hat, um mathematische Inhalte zu modifizieren und verständlich zu erklären.