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„Sovereign Default Incentives in Emerging Market Economies"

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Zusammenfassung

Die vorliegende Magisterarbeit präsentiert und analysiert theoretische Modelle mit endogenem Ausfallrisiko für Schwellenländer. Es werden Gemeinsamkeiten und Unterschiede der Studien von Eaton und Gersovitz (1981), Arellano (2008) und Cuadra, Sanchez und Sapriza (2010) ausgearbeitet und aufgezeigt. Diese Studien basieren auf denselben Hauptannahmen, sie unterscheiden sich jedoch in der vorherrschenden Wirtschaftsart und der Art der Kreditaufnahme. Das endogene Ausfallrisiko wird in den Modellen durch denselben Mechanismus modelliert. Kreditobergrenzen dienen der Kreditrationierung und sollen einen Kreditausfall im Gleichgewicht verhindern. Die Einführung von stochastischem Einkommen kann jedoch zu Situationen führen, in denen eine Nichtzahlung vorteilhaft ist. Es sind verschiedene Arten vom rekursiven Gleichgewicht möglich. Ein zentrales Resultat dieser Modelle sind die antizyklischen Default-Anreize. Endogenes Ausfallrisiko in Schwellenländern fördert zudem prozyklische Fiskalpolitik. Die Ergebnisse dieser Studien werden durch die Möglichkeit der privaten Kreditaufnahme nicht verändert.

Abstract

The master thesis presents the endogenous default risk models of Eaton and Gersovitz (1981), Arellano (2008) and Cuadra, Sanchez and Sapriza (2010), and discusses the similarities and differences. The studies are based on the same main assumptions. The most obvious distinctions concern the type of economy and borrowing. Endogenous default risk models rely on the same basic mechanism to model sovereign default risk. Credit rationing in the form of credit ceilings should prevent default in equilibrium but stochastic income can generate situations in which a decision to default is preferable. Several types of the recursive competitive borrowing equilibrium are possible. The studies establish different theoretical results, which are supported by other studies. The finding of countercyclical default incentives is a central result of the investigated papers. Endogenous default risk leads to a procyclical fiscal policy in these economies, which is also supported by the data. The quantitative predictions of Arellano and Cuadra, Sanchez and Sapriza match well the business cycle statistics of Argentina and Mexico, respectively. Allowing for private borrowing does not modify the main message of endogenous default risk models with endogenous fiscal policy.

Contents

1. Introduction

The recent years were marked by sovereign debt crises in the European Union and several other countries. Many sovereigns were not able to satisfy their repayment obligations and defaulted on their debt. To analyse this issue, it is necessary to define the key terms. The rating agency Moody's Investors Service (1999, p.1) defines default in the following way:

> Moody's defines default as any missed or delayed disbursement of interest and/or principal. We include as defaults distressed exchanges where: (1) the issuer offers bondholders or depositors a new security or package of securities that amount to a diminished financial obligation (such as preferred or common stock, debt with a lower coupon or par amount, or a less liquid deposit either because of a change in maturity or currency of denomination, or required credit maintenance facilities) and (2) the exchange has the apparent purpose of helping the borrower avoid default.

A rational agent will only default, if he is (highly) indebted. Typically, sovereign debt is presented at its face value and it is "defined as the undiscounted sum of future principal repayments" (Tomz, Wright 2013; p.2). Therefore, it does not include interest payments and it does not discount the principal.

According to Eichengreen et al. (2005), most developing countries borrow abroad but they are not able to do this in their own currency. More than 93 % of all developing countries issue debt in a foreign currency. This percentage is much lower for developed countries. The reason why most poor countries borrow in a foreign currency is that the country can not default by increasing the inflation rate and therefore, the default risk and the interest rates are lower. On the other hand, the exchange rate risk increases. (Tomz, Wright 2013)

However, the World Bank (2012) reports that the developing countries' total external outstanding debt rose from \$ 2,514.1 billion in 2005 to \$ 4,076.3 billion at the end of 2010. This amount corresponds to 21 % of GNI in 2010. From this \$ 4 trillion, the major part was attributable to the middle income countries with an amount of \$ 3,959.7 billions. The country with the highest external debt stock in 2010 was China with \$ 548.6 billions. Also among the top ten borrowers were Mexico with an external outstanding debt of \$ 200.1 billions (19.5 % of GNI) and Argentina with \$ 127.9 billions (36.1 % of GNI). Furthermore, Roxburgh et al. (2011) report that total sovereign debt[1] rose significantly during the last decade. Public outstanding debt reached an amount of \$ 41.1 trillion at the end of 2010, which corresponds to 69 % of global GDP or 19 % of global financial stock. These numbers

[1] including developed economies and internal debt

demonstrate the importance of this topic. If the trend of increasing sovereign debt rates holds on, maybe we see decades of sovereign debt crises all over the world, like the 1970s, 1980s and 1990s which were characterized by numerous debt crises in emerging economies.

The rating agency Standard & Poor's reports about 84 default events between 1975 and 2008. (Yue 2010) During this time, the second largest default in history (by present values) occurred when Argentina defaulted on $ 81.8 billion in December 2001 (its total debt was $ 144.5 billion). (Damill et al. 2006) Up to now, the largest default in history was the Greek restructuring in March 2012, which covered about $ 265 billion. At the same time, it was the first default of an advanced country with an annual income per capita over $ 25,000. (Porzecanski 2012) Although the Greek government had not missed any payments, it demanded new terms. This was the reason why the rating agencies conclude that a default had occurred, it is in line with Moody's definition of default. (Tomz, Wright 2013)

Sovereign debt crises are not a new phenomenon, countries have borrowed money for hundreds of years. Sovereign debt represents a large fraction of total global financial assets. (Tomz, Wright 2013)

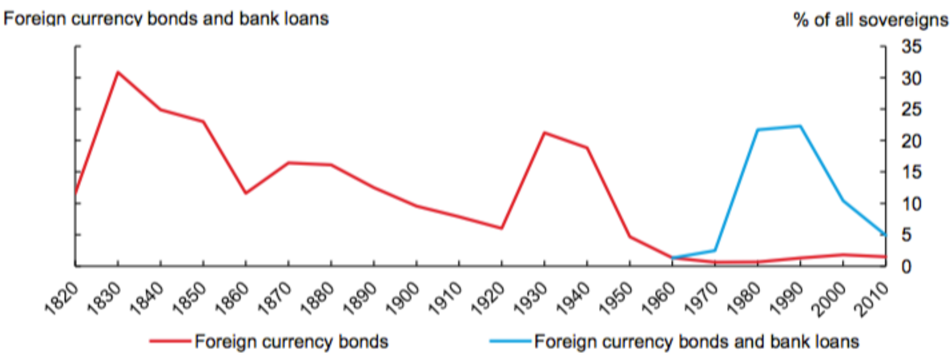

Figure 1.1.: Sovereign default rates as a percentage of all sovereigns, Source: Beers and Nadeau (2014)

Beers and Nadeau (2014) provide some trends in defaults. Figure 1.1 presents the sovereign default rate of foreign borrowing as a percentage of all sovereigns in the last two centuries. It shows three main default episodes since 1820, when default rates exceeded 20 %. More than each fifth country defaulted in the 1830s and 1840s, in the 1930s and in the 1980s. Furthermore, the figure shows a sharp decline in default rates of foreign currency bonds after the Second World War. A resolution of prewar defaults and the foundation of the IMF, World Bank and other institutions, like development agencies, were the main reason for the drop in default rates of foreign currency bonds and the increasing importance of bank loans.

As Beers and Nadeau report, typically low income and emerging market economies are vulnerable to debt crises. The consequences of a decision to default are many and varied. An immediate result is the loss of reputation, which is normally attached to higher interest rates or even to a loss in access to capital markets. Usually, the domestic economy is affected by a negative growth rate of income and by a decrease of bilateral trade. According to Borenzstein and Panizza (2009), a sovereign default increases the probability of a domestic banking crisis by 11 %. Furthermore, also the policymakers are negatively affected. It is likely that the head of the executive, the minister of finance and/or the head of the central bank will be replaced after a default. These political costs may increase social welfare since it generates incentives for repayment.

The first model which has addressed the issue of international borrowing with endogenous default risk was developed by Eaton and Gersovitz (1981, from now on EG). In the literature, it is called the classical framework. EG show that a level of debt can be sustainable on the threat of a permanent exclusion of international capital markets after a default. Many papers follow and extend this seminal approach. Recent papers in the quantitative sovereign debt literature additionally concentrate on business cycle statistics, like Arellano (2008) and Cuadra, Sanchez and Sapriza (2010, from now on CSS). The main characteristic of these models is the endogenous decision on default. The master thesis investigates this key issue and compares the three mentioned papers. The aim is to present the sovereign default incentives and to model the basic mechanism of endogenous default risk.

A sovereign default is different from the bankruptcy of an individual economic agent since no international bankruptcy law for countries exist. (Yue 2010) No explicit mechanism deters a government from defaulting, but nevertheless private loans to foreign governments exist. To model endogenous default risk, it is necessary to understand the lending and the repayment incentives. In the classical framework of EG, the repayment incentives rely on the loss of reputation after a repudiation. Such self-enforcing mechanisms can be sufficient to sustain some level of debt. However, an investment in a foreign country must be worthwhile for lenders. On the assumption of risk neutrality, they make only loans if the expected rate of return is at least as high as in the safe case. (Martins, Vailakis 2014)

The master thesis is organized as follows: section 2 presents the classical approach by EG with a quite general model of borrowing, a deterministic model and a stochastic model. Section 3 shows Arellano's model and section 4 the extension by CSS. The models are ordered with increasing complexity, beginning with the general model of borrowing and ending with a stochastic model in which the fiscal policy is endogenous and the private sector has access to international capital markets. Section 5 provides a discussion of the studies while section 6 concludes. In the following, the terms *default* and *repudiation* are used as synonyms as well as *government/public spending*, *government/public expenditures* and *public consumption*.

2. The Classical Model

Based on three models, Eaton and Gersovitz (1981, EG) provide a theoretical analysis of borrowing by sovereign nations in private foreign capital markets. The bankruptcy of an individual agent is not part of the analysis, only borrowing and default by a government is of interest in the models. There exists no explicit mechanism which prevents a repudiation of the governments' external debt. Only the coercion of the governments of private creditors could enforce a repayment. However, creditors can penalize defaulting debtors by various actions. One of the most important penalties is the permanent exclusion from the capital market and therefore from future borrowing.

A credit ceiling, which does not automatically imply credit rationing, arises due to the scarcity of the resources. The price mechanism fails in capital markets since the debtor will not pay the price after a default and hence a sort of credit rationing, may occur. EG emphasize the endogenously determined penalty of default and the endogenously determined credit ceiling. Both of them are important aspects of the models. No exogenous cost occurs after a repudiation.

The first model presents a quite general specification of competitive equilibrium, with a random net output but without an explicit solution. The government decides on the repayment and the new borrowing in each period. The equilibrium borrowing is determined by the minimum of the desired amount and the credit ceiling. Credit rationing prevents a default.

The second model describes a deterministic model of borrowing, with alternating periods of high income relative to trend and periods of low income relative to trend. If the debtor has not previously defaulted, he can borrow in bad periods and save in good periods. Debts and savings mature in one period. Both of them can be used to smooth consumption. Normally countries specialize either on borrowing or on saving. EG use a constant relative risk aversion utility function to get an explicit solution of the desired amount of borrowing. The set of sustainable debt levels is characterized by a finite interval. In the deterministic case, no default will occur in equilibrium.

The third model provides a stochastic model of borrowing with uncertain income in the period in which loans are due. Regardless of income in the previous period, bad and good periods occur with equal probability. The analysis implements Taylor-series approximations of a general utility function. Various types of equilibria are possible. The desired amount of borrowing is determined based on the assumption of repayment. A debt-service payment in periods with low income reduces utility more than in periods with high income. Thus, the safe credit ceiling is characterized by the optimality of repayment in bad periods. Extended credits with higher interest rates are risky but possible. For instance, if the debtor pays only in good periods, the probability of default is at least $\frac{1}{2}$ and the interest rate has to be at least twice as high as in the safe case.

2.1. Assumptions

The theoretical models characterize international borrowing of a small open economy in international private capital markets. The net output in period t is a random variable and it is denoted by y_t. Output is not storable and can be used for consumption c_t or for debt-service payments p_t.

The small open economy possesses a benevolent government which optimizes its objective function over an infinite horizon. It can borrow or save to smooth the consumption. If the country has previously defaulted, borrowing is not possible. Borrowing matures in one period and has to be repaid. The actual borrowing is the minimum of the amount the country wishes to borrow and its credit ceiling. A property of all debtors is that they are dishonest if a default is better than repayment.

Domestic households and firms are not considered. The only agents that are part of the theoretical analyses, with exception of the domestic government, are foreign lenders. They are rational and know all relevant characteristics of individual borrowers. Lenders are also assumed to know the probability of default in each period. Private lenders are competitive and risk neutral. Competition among them maximizes the borrowers' utility. Loans to alternative borrowers at the safe interest rate are also possible. The interest rates are given, lenders and borrowers are price takers. The total loanable funds are bounded by the finiteness of the resources. Foreign creditors have no legal possibility to enforce repayment.

A default leads to an endogenous penalty. This could be a cutoff of aid or some restrictions on commodity trade. A further consequence is the permanent embargo on future borrowing. The costs of default are determined endogenously by several characteristics of the country and its variability and growth rate of income. The higher its future demand for debt, the more the country suffers from an exclusion of international credit markets. There are no other exogenous penalties in the models. The benefits of a default grow with the size of debt.

2.2. The General Model of Borrowing

This section contains a very general model for borrowing in international capital markets. The government decides on the size of debt $b_t \in L_t$ to smooth the consumption c_t in each period t. L_t denotes the set of loan amounts available in period t, which are bounded by $sup L_t \leq \overline{L}_t < \infty$ for all t $< \infty$. This means that the country is not able to borrow more than the total loanable funds available. $\overline{L}_t$ denotes the upper bound of the total available loanable funds. In this model, saving is not possible.

The consumption in period t is given by

$$c_t = y_t + b_t - p_t \tag{2.1}$$

It depends positively on output y_t and on borrowing b_t. The debt-service payments p_t reduce the consumption.

Debt matures in one period. The repayment function satisfies

$$R(b_t) = d_{t+1} \tag{2.2}$$

where d_{t+1} denotes the debt-service obligations in period t+1. It is defined by

$$d_{t+1} = (1 + r_t)b_t \tag{2.3}$$

The country has to pay an interest rate of r_t for the borrowing b_t.

The borrower decides on the size of the debt-service payments p_t in each period. If the debt-service payments p_t fall short of debt-service obligations d_t in period t, the country faces an embargo on future loans. It follows that $L_\tau = 0$ for all $\tau \geq$ t and the government is constrained to $b_\tau = 0$ for all $\tau \geq$ t. If, however, the borrower satisfies his debt obligations so that $p_t = d_t$, the set of loan amounts available in period t, is given by $L_t = L(y_t, d_t)$.

The benevolent government maximizes the expected discounted utility over an infinite horizon. The utility depends positively on consumption and negatively on a possible penalty P_t after a default. The government's objective function is

$$E[\sum_{t=0}^{\infty} \beta^t U(c_t - P_t)] \tag{2.4}$$

where $U' > 0$ and $U'' < 0$. The discount factor β is strictly positive and smaller than 1.

If the borrower decides to repudiate his liabilities in t, the value of the objective function V^D is defined as the expected discounted sum of the current and all future levels of utility under autarchy. The consumption equals the output since no borrowing is possible.

$$V^D(y_t) \equiv E[\sum_{t=0}^{\infty} \beta^{\tau-t} U(y_\tau - P_\tau)] \tag{2.5}$$

The value of the objective function given a decision to satisfy the debt-service obligations $p_t = d_t$ in period t is denoted by V^R. It depends positively on consumption. Since the borrower does not default, there is no penalty. A decision against default in period t does not mean that default is never optimal. The government decides on default in each period, it could be optimal to wait one period and then to repudiate the liabilities.

$$V^R(y_t, d_t) \equiv sup_{b_t \in L_t}\{U(y_t + b_t - d_t) + \beta E max[V^R(y_{t+1}, d_{t+1}), V^D(y_{t+1})]\} \tag{2.6}$$

An explicit solution of the contraction equation requires a very simple utility function. Provided that V^R exists and is unique, default is optimal if and only if the value function given a decision to default in t is larger than the value function given a decision not to default in t.

$$V^D(y_t) > V^R(y_t, d_t) \tag{2.7}$$

The probability of default λ_t in t is given by

$$\lambda(d_t) = Pr[V_t^D > V_t^R] \tag{2.8}$$

It increases monotonically with debt-service obligations d_t in t because V^R decreases monotonically with d_t and V^D is independent of d_t. The proof appears in Appendix B.1.1.

2.2.1. Competitive Borrowing Equilibrium

A competitive borrowing equilibrium satisfies all mentioned assumptions about the borrower, borrowing and lending. It is characterized by the functions $V(y_t, d_t)$, $L(y_t, d_t)$ and $R(b_t)$. The set of loan amounts available and the repayment function are defined for $b_t \in L(y_t, d_t)$ which maximize $V(y_t, d_t)$ subject to the zero expected profit condition

$$\{1 - \lambda[R(b_t)]\}R(b_t) = (1 + \overline{r})b_t \qquad \forall b_t \in L(y_t, d_t) \tag{2.9}$$

This constraint means that competitive lenders will not make loans which have a lower expected rate of return as in the safe case with $\overline{r}$. Governments with a higher probability of default have to pay a higher interest rate to make loans worthwhile to lenders.

The set of available loan amounts $L(y_t, d_t)$ is bounded by $[0, \overline{b}_t]$ where $\overline{b}_t < \infty$. This follows directly from the finite wealth of the world. The repayment function $R(b_t)$ is increasing in b_t and convex over $[0, \overline{b}_t]$. The proof of the convexity appears in Appendix B.1.2.

2.2.2. Demand for Credit

An unconstrained borrower, who could choose any positive amount of borrowing, would borrow $b_t^* \in R^+$ such that his concave value function V^* is maximized.

$$V^*(y_t, d_t) \equiv sup_{b_t \in R^+}\{U(y_t + b_t - d_t) + \beta E max[V^R(y_{t+1}, R(b_t)), V^D(y_{t+1})]\} \tag{2.10}$$

The right-hand side is increasing in b_t for $b_t < b_t^*$ and reaches its maximum at b_t^*. If the desired amount of borrowing exceeds the credit ceiling in equilibrium, $b_t^* > \overline{b}_t$, the country is rationed and borrows the largest amount available, $\overline{b}_t$. The ceiling is not binding if $b_t^* < \overline{b}_t$ and the government can borrow its desired amount.

2.2.3. Credit Rationing

Actual borrowing b_t in period t is the minimum of the desired amount of borrowing b_t^* and the credit ceiling $\overline{b}_t$.

$$b_t = min(b_t^*, \overline{b}_t) \tag{2.11}$$

Rationing can occur because the wealth of the world is finite and the price mechanism fails in credit markets. Repayment is not possible if

$$d_{t+1} > y_{t+1}^{max} + \overline{L}_{t+1} \tag{2.12}$$

where y_{t+1}^{max} denotes the maximum output, which is smaller than infinity. This condition means that the debt-service obligations d_{t+1} in t+1 exceed the available resources and therefore, the probability of default $\lambda(d_{t+1})$ is 1. Clearly, the greatest possible debt-service obligations $\overline{d}_{t+1}$ have to be smaller than infinity. If λ is differentiable, then $\overline{d}_{t+1}$ is defined by

$$\overline{d}_{t+1} \equiv inf[d : 1 - \lambda(d_{t+1}) - \lambda'(d_{t+1})d_{t+1} = 0] \tag{2.13}$$

This condition comes from the expected rate of return, which corresponds to the left-hand side of the zero expected profit condition 2.9. Given $R(b_t) = d_{t+1}$ and the differentiability of the default probability, the derivative with respect to d_{t+1} leads then to the greatest possible debt-service obligations. The zero expected profit condition 2.9 implies also the maximum expected debt-service payment $\overline{p}_{t+1}$ in period t+1

$$\overline{p}_{t+1} = [1 - \lambda(\overline{d}_{t+1})]\overline{d}_{t+1} < \infty \tag{2.14}$$

and the credit ceiling $\overline{b}_t$ in t.

$$\overline{b}_t = \frac{[1 - \lambda(\overline{d}_{t+1})]\overline{d}_{t+1}}{1 + \overline{r}} \tag{2.15}$$

If $b_t > \overline{b}_t$, the debt-service obligations exceed $\overline{d}_{t+1}$ and therefore $p_{t+1} < d_{t+1}$ rational lenders anticipate this behaviour, no loans with $b_t > \overline{b}_t$ will be made. The maximum amount which can be borrowed corresponds either to b_t^*, or if rationing occurs to $\overline{b}_t$.

2.3. The Deterministic Model of Borrowing

The biggest difference to the previous model is the alternating income. Periods with high income relative to trend and periods with low income relative to trend alternate with one another. Provided that the country has not defaulted, it can borrow in bad periods with low income. The debt becomes due in the following period of relatively high income. Alternatively, governments can also save via lending in good periods. If default is no option for the foreign borrower, the amount will be repaid in any case in the succeeding period. It is assumed that borrowing is only possible in bad periods, while saving can only occur in good periods. Otherwise, smoothing the consumption would not be possible and the country suffers more if the income is low. A country which has access to international capital markets will specialize in either borrowing or lending since both of them can be used for smoothing purposes.

To get an explicit solution of the analysis, EG use a constant relative risk aversion utility function.

$$U(c) = \begin{cases} \frac{c^{1-\gamma}}{1-\gamma} & \text{if } \gamma > 0,\ \gamma \neq 1 \\ ln(c) & \text{if } \gamma = 1 \end{cases} \tag{2.16}$$

The utility depends on the consumption and on the elasticity of marginal utility γ.

The existence of a growth rate g of trend income and consumption is a further difference to the general model. The initial level of trend income is denoted by y_0 and for reasons of simplification it is assumed to be 1. Income deviates from trend by the percent σ in period t. The income is therefore either $y_t^l = (1-\sigma)G^t$ in bad periods or $y_t^h = (1+\sigma)G^t$ in good periods, where $G \equiv 1+g$. If the country has not defaulted previously, it can borrow a percent $b \geq 0$ of σ at a given gross rate of interest R in bad periods. Alternatively, it can save a percent $s \geq 0$ of σ at a gross rate of interest R' in good periods. Borrowing and saving must ensure the non-negativity of consumption in each period. The consumption c^l in period t with relatively low income y_t^l depends positively on possible borrowing and on a possible liquidation of lending from the previous period.

$$c^l(s,b,t) = [1 - \sigma(1 - b - \frac{R's}{G})]G^t \qquad \text{t} = 0, 2, 4, ... \tag{2.17}$$

The consumption c^h in period t with relatively high income y_t^h depends negatively on possible saving and on a possible repayment of borrowing from the previous period.

$$c^h(s,b,t) = [1 + \sigma(1 - \frac{Rb}{G} - s)]G^t \qquad \text{t} = 1, 3, 5, ... \tag{2.18}$$

If the country has defaulted and is therefore constrained to $b = 0$, it faces a permanent exclusion from international capital markets. The consumption under autarchy is given by $c^l(t) = y_t^l$ in bad periods and $c^h(t) = y_t^h$ in good periods. Smoothing the consumption via borrowing is not possible.

For the further analysis, EG define the two-period discounted utility W^l on the assumption of repayment. The first period of W^l is given by y^l and the second by y^h. The discount factor β is strictly positive and smaller than 1.

$$W_t^l(s,b) \equiv U[c^l(s,b,t)] + \beta U[c^h(s,b,t+1)] \qquad \text{t} = 0, 2, 4, ... \tag{2.19}$$

In periods of relatively low income $y = y^l$ where $c = c^l$, the borrower maximizes W^l with respect to b. s is given since the government can save only in good periods.

W^h is defined as the two-period discounted utility from a perspective of high income on the assumption of repayment.

$$W_t^h(s,b) \equiv U[c^h(s,b,t)] + \beta U[c^l(s,b,t+1)] \qquad \text{t} = 1,3,5,... \tag{2.20}$$

In periods of relatively high income $y = y^h$ where $c = c^h$, the government maximizes W^h with respect to s. b is given since it can borrow only in bad periods.

2.3.1. Demand for Credit

If the income is low relative to trend, it is assumed that the country has the option of borrowing. The two-period discounted utility determines the optimal value of b since it contains the utility of the current period and the discounted utility of the repayment period. The government decides on the amount it wishes to borrow, given saving from the previous period. For a maximum, the first-order condition of the two-period discounted utility W^l (2.19) with respect to b is given by

$$W_t^l = \frac{1}{1-\gamma}\{[1-\sigma(1-b-\frac{R's}{G})]G^t\}^{1-\gamma} + \frac{1}{1-\gamma}\beta\{[1+\sigma(1-\frac{Rb}{G}-s)]G^{t+1}\}^{1-\gamma}$$

$$\frac{\partial W_t^l}{\partial b} = \sigma G^{t(1-\gamma)}[1-\sigma(1-b-\frac{R's}{G})]^{-\gamma} - \sigma\beta R G^{t(1-\gamma)-\gamma}[1+\sigma(1-\frac{Rb}{G}-s)]^{-\gamma} = 0 \tag{2.21}$$

On the other hand, saving is only possible if the income is high relative to trend. The government decides on the amount it wishes to lend, given borrowing from the previous period. The two-period discounted utility determines the optimal value of s since it contains the utility of the current period and the discounted utility of t+1. For a maximum, the first-order condition of W^h (2.20) with respect to s is given by

$$W_t^h = \frac{1}{1-\gamma}\{[1+\sigma(1-\frac{Rb}{G}-s)]G^t\}^{1-\gamma} + \frac{1}{1-\gamma}\beta\{[1-\sigma(1-b-\frac{R's}{G})]G^{t+1}\}^{1-\gamma}$$

$$\frac{\partial W_t^h}{\partial s} = -\sigma G^{t(1-\gamma)}[1+\sigma(1-\frac{Rb}{G}-s)]^{-\gamma} + \sigma\beta R' G^{t(1-\gamma)-\gamma}[1-\sigma(1-b-\frac{R's}{G})]^{-\gamma} = 0 \tag{2.22}$$

To smooth the consumption, a government with access to international capital markets will focus on either borrowing or saving. What is better for the country depends on different variables. Desired borrowing σb^* is positive if the first-order condition of borrowing (2.21) is satisfied. This means that the first-order condition of saving (2.22) is negative and the desired saving σs^* is zero. It holds that $0 = \frac{\partial W_t^l}{\partial b} > \frac{\partial W_t^h}{\partial s}$ and therefore

$$[1-\sigma(1-b-\frac{R's}{G})]^{-\gamma}-\frac{\beta R}{G^\gamma}[1+\sigma(1-\frac{Rb}{G}-s)]^{-\gamma} > [1-\sigma(1-b-\frac{R's}{G})]^{-\gamma}-\frac{G^\gamma}{\beta R'}[1+\sigma(1-\frac{Rb}{G}-s)]^{-\gamma}$$

$$-\frac{\beta R}{G^\gamma}[1+\sigma(1-\frac{Rb}{G}-s)]^{-\gamma} > -\frac{G^\gamma}{\beta R'}[1+\sigma(1-\frac{Rb}{G}-s)]^{-\gamma}$$

$$G^{2\gamma} > \beta^2(R'R)$$

$$G^\gamma > \beta(R'R)^{\frac{1}{2}} \tag{2.23}$$

If in equilibrium $b^* > 0$ and $s^* = 0$, the growth rate is large relative to the geometric average of the interest rates and the discount factor. In this case, rearranging (2.21) and solving for σb result in the desired amount of borrowing relative to income. Since the country specializes in borrowing, it will not save in good periods.

$$[1-\sigma(1-b)] = [1+\sigma(1-\frac{Rb}{G})]G(\beta R)^{-\frac{1}{\gamma}}$$

$$\sigma b + \sigma b R(\beta R)^{-\frac{1}{\gamma}} = G(\beta R)^{-\frac{1}{\gamma}} + \sigma G(\beta R)^{-\frac{1}{\gamma}} - 1 + \sigma$$

$$\sigma b^* = \frac{[G(\beta R)^{-\frac{1}{\gamma}} - 1] + \sigma[G(\beta R)^{-\frac{1}{\gamma}} + 1]}{R(\beta R)^{-\frac{1}{\gamma}} + 1} \tag{2.24}$$

The desired amount is independent of t. The higher the growth rate g and the standard deviation σ, the more the borrower wishes to borrow. A higher σ means that less income is available in periods with $y = y^l$ and a higher borrowing is needed to smoothen the consumption. On the other hand, the income is higher in the repayment period and the country can repay a higher amount. The situation is similar for a higher growth rate g. It causes more resources in $y = y^h$ and therefore a higher repayment is possible. To smooth the consumption, the government wishes to borrow more in a bad period.

The government specializes on saving if the first-order condition of saving (2.22) is satisfied. This means that the first-order condition of borrowing (2.21) is negative and the desired borrowing σb^* is zero. It holds that $\frac{\partial W_t^l}{\partial b} < \frac{\partial W_t^h}{\partial s} = 0$ and therefore

$$G^\gamma < \beta(R'R)^{\frac{1}{2}} \tag{2.25}$$

Based on the assumption of a positive growth rate, an equilibrium with $s^* > 0$ and $b^* = 0$ is more likely if the elasticity of marginal utility γ is low. Rearranging (2.22) and solving for σs result in the desired amount of saving relative to income. Since the country specializes in saving, it will not borrow in bad periods.

$$[1-\sigma(1-\frac{R's}{G})]G(\beta R')^{-\frac{1}{\gamma}} = [1+\sigma(1-s)]$$

$$\sigma s + \sigma s R'(\beta R')^{-\frac{1}{\gamma}} = \sigma G(\beta R')^{-\frac{1}{\gamma}} - G(\beta R')^{-\frac{1}{\gamma}} + 1 + \sigma$$

$$\sigma s^* = \frac{[1-G(\beta R')^{-\frac{1}{\gamma}}] + \sigma[1+G(\beta R')^{-\frac{1}{\gamma}}]}{1+R'(\beta R')^{-\frac{1}{\gamma}}} \tag{2.26}$$

The desired amount of saving is independent of t. The higher the standard deviation σ, the more the country wishes to save. A higher σ means that more income is available in periods with $y = y^h$, while less income is available in periods with $y = y^l$. The government wishes to save more in good periods to smooth the consumption. An increase in g has a negative effect on the desired amount since $\sigma - 1 < 0$. A high growth rate causes more resources in $y = y^l$ and therefore the country wishes to save less in the good period.

If both first-order conditions are satisfied, then $\frac{\partial W_t^l}{\partial b} = \frac{\partial W_t^h}{\partial s} = 0$ and therefore

$$G^\gamma = \beta(R'R)^{\frac{1}{2}} \tag{2.27}$$

The government is indifferent between borrowing and saving.

For the rest of the model, we consider only the case $b^* > 0$ and $s^* = 0$.

2.3.2. Sustainable Debt Level

A debt level is sustainable if the borrower may borrow an amount in bad periods, which he will fully repay in the succeeding period with high income. The corresponding discounted utility given repayment *Rb* in good periods is given by

$$V^R(b) = \sum_{t=0}^{\infty} \beta^{2t} max_{s\geq 0}[W_{2t}^h(s,b)] \qquad \text{t} = 1,3,5,... \tag{2.28}$$

or equivalently,

$$V^R(b) = [max_{s\geq 0} W^h(s,b)]\phi^{-1} \tag{2.29}$$

where $\phi \equiv 1-(\beta G^{1-\gamma})^2 > 0$. The computation appears in Appendix B.1.3.

A sustainable debt level satisfies the conditions

$$V^R(b) \geq V^R(0) \tag{2.30}$$

$$c^h \geq 0 \tag{2.31}$$

$$s \geq 0 \tag{2.32}$$

The first condition means that a value function with borrowing b must be at least as high as the discounted utility without borrowing. If this condition should fail, the country would repudiate its debt. The non-negativity condition of consumption is also valid in the period of repayment. To satisfy the second condition, it must hold that $\sigma(\frac{Rb}{G} + s - 1) \leq 1$ where $s \geq 0$. There exists a non-negativity condition for saving s in $y = y^h$ otherwise debt would be repaid with new debt (no-Ponzi game condition).

The set of sustainable debt levels is characterized by a finite interval $[0, \bar{b}]$.

If $V^{R\prime}(0) < 0$, the value function decreases with borrowing b and $V^R(b) < V^R(0)$ for all $b > 0$. No debt level is sustainable and the credit ceiling corresponds to $\bar{b} = 0$.

If $V^{R\prime}(0) > 0$, the value function increases with borrowing b. The credit ceiling $\bar{b}$ is the minimum of two amounts.

$$\bar{b} = min(\tilde{b}, \hat{b}) \tag{2.33}$$

$\tilde{b}$ is strictly positive and defined by

$$\tilde{b} \equiv b : V^R(b) = V^R(0) \tag{2.34}$$

Given a decision not to default, the value function of $\tilde{b}$ equals the value function without borrowing. At most one such debt level exists. The proof appears in Appendix B.1.4.

$\hat{b}$ denotes the highest value b such that $c^h \geq 0$ and $s \geq 0$. It is defined by

$$\hat{b} \equiv \frac{G(1+\sigma)}{\sigma R} \tag{2.35}$$

The conditions (2.30) to (2.32) are satisfied for $b \leq \tilde{b}$ and $b \leq \hat{b}$. A debt level with $b > \bar{b}$ is not sustainable, the government will default in each case. If $b > \bar{b} = \tilde{b}$, condition (2.30) is violated since $V^R(b)$ is a strictly concave function of b (see Appendix B.1.4). This means that the expected discounted utility given a decision to default is higher than the value function of repayment. If $b > \bar{b} = \hat{b}$, either the consumption is negative or $s < 0$ in the repayment period. The borrower will default. Rational lenders know all relevant information, they anticipate this behaviour and will not lend $b > \bar{b}$.

$V^{R\prime}(b) > 0$ for $b < b^*$, the value function increases with borrowing b until the desired level of borrowing b^* is reached. Rational lenders do not offer more than $\bar{b}$, loans are restricted by $\bar{b}\sigma G^t$ in period t. Actual borrowing is therefore

$$b_t = min(b^* \sigma G^t, \bar{b}\sigma G^t) \tag{2.36}$$

The borrower is rationed if $\bar{b}\sigma G^t < b^* \sigma G^t$, what prevents default. The equilibrium is characterized by the full repayment of the outstanding debt.

2.4. The Stochastic Model of Borrowing

The biggest difference to the deterministic model is the stochastic income. Periods of relatively high income $y = y^h = 1 + \sigma$ and periods of relatively low income $y = y^l = 1 - \sigma$ alternate indefinitely. Regardless of income in the previous period, they occur with the same probability. It is possible that two or more good periods follow each other, the same could happen for bad periods.

For reasons of simplification, there is no saving $R' = 0$. The only remaining option is smoothing via borrowing. It is assumed that this is possible in periods with $y_t = y_t^l$ and under the conditions that the country has not defaulted previously and that there was no outstanding debt at the beginning of the current period, i.e. $b_{t-1} = 0$. Debt matures in one period and must be repaid in t+1 regardless of the income y_{t+1}. A further simplification is the assumption of no growth, that is, $g = 0$ and therefore $G = 1$. The initial level of trend income y_0 is still 1.

The stochastic income complicates the model, derivations of exact analytic expressions from a constant relative risk aversion utility function are not possible. This is why EG use Taylor-series approximations of a general utility function. The equilibrium is not unique, several types are possible.

2.4.1. Demand for Credit

Borrowing can occur in periods with $y_t = y_t^l$ if the condition $b_{t-1} = 0$ is satisfied and the country has not defaulted. Based on the assumption of repayment in t+1, the borrower will select b to maximize his two-period discounted utility

$$W_t^l = U[1 - \sigma(1 - b)] + \frac{1}{2}\beta\{U[1 + \sigma(1 - Rb)] + U[1 - \sigma(1 + Rb)]\} \tag{2.37}$$

The income in the period of repayment is either $y_{t+1} = y_{t+1}^h$ or $y_{t+1} = y_{t+1}^l$ with equal probability. W_t^l therefore contains the discounted average utility of repayment in the second period. The growth rate g and the interest rate of saving R' are zero.

The desired amount of borrowing relative to income is determined by a second-order Taylor series approximation about $c = 1$. The derivation appears in Appendix B.1.5.

$$\sigma b^* = \frac{\frac{1-\beta R}{\tilde{A}} + \sigma}{1 + \beta R^2} \tag{2.38}$$

where $\tilde{A} \equiv -\frac{U''}{U'}$ denotes the degree of absolute risk aversion. The higher the standard deviation σ, the more the borrower wishes to borrow. The interpretation is the same as in the deterministic case.

2.4.2. Sustainable Debt Level

The maximum amount b_t that can be contracted at R is determined by the willingness to pay of the borrower in any foreseeable situation. Debt-service payments p_{t+1} reduce utility more in periods with $y_{t+1} = y^l_{t+1}$ than if the income is $y_{t+1} = y^h_{t+1}$. The probability of default is higher in bad periods.

The expected discounted utility of borrowing in periods with low income $y_t = y^l_t$, when $b_{t-1} = 0$, and under the assumption of full repayment is given by

$$V^R(b) = \frac{1}{2a}\{U[1-\sigma(1-b)]+U[1+\sigma]\}+\frac{1}{4a}\beta\{U[1-\sigma(1+Rb)]+U[1+\sigma(1-Rb)]\} \quad (2.39)$$

where $a \equiv 1 - \frac{\beta}{2} - \frac{\beta^2}{2}$. Appendix B.1.6 contains the derivation. Borrowing occurs in periods with low income. It is only possible if the country has access to international capital markets. The income in each period is either $y^l = 1 - \sigma$ or $y^h = 1 + \sigma$ with equal probability. The utility of the first period is the average of both a bad period with borrowing and a good period. The second period contains the expected discounted utility given a decision to repay Rb.

A country that faces a credit embargo is constrained to $b_t = 0$. In this case, the expected discounted utility under autarchy is given by

$$V^D(b) = \frac{U[1+\sigma]+U[1-\sigma]}{2(1-\beta)} \quad (2.40)$$

The complete derivation appears in Appendix B.1.7. The government does not have the possibility of smoothing via borrowing, the expected utility is the average of a good and bad period.

Default is not tempting in good periods. If the country does not settle its debt in good periods, it loses access to capital markets and it will suffer more in hard times. Default will be optimal if and only if the income in the repayment period is low and the expected discounted utility of repayment is lower than the expected discounted utility without payment. Conversely, the optimality of payment in bad periods requires the satisfaction of the condition

$$U[1-\sigma(1+Rb)] + \beta V^R(b) \geq U[1-\sigma] + \beta V^D \quad (2.41)$$

The credit ceiling relative to income is determined by a second-order Taylor series approximation about $c = 1$ since exact analytic expressions are not possible because of the stochastic income. The derivation is shown in Appendix B.1.8.

$$\sigma\bar{b} = \frac{\frac{2\beta(1+R)-4R}{A} + \sigma(2\beta - 4Ra)}{\beta(1+\beta R^2) + 2R^2 a} \quad (2.42)$$

Payment is optimal if $0 \leq b \leq \bar{b}$. Lenders will not make loans in excess of $\bar{b}$ for a given interest rate R. At an interest rate R and if $b > \bar{b}$, the expected discounted utility under autarchy exceeds the expected discounted utility of repayment in a bad period. In this case, condition (2.41) is violated and the probability of default λ is 1.

2.4.3. Demand for Credit in the Risky Case

Provided that the interest rate is larger than R, borrowing with $b > \bar{b}$ can occur despite a positive probability of default. Rational lenders will make loans in excess of the safe credit ceiling $\bar{b}$ only if the expected rate of return of $b > \bar{b}$ is at least as high as in the safe case when $b \leq \bar{b}$. A situation may arise in which repayment for a credit with $b > \bar{b}$ is optimal if and only if the income in the repayment period is $y = y^h$. The probability of default is then $\lambda = \frac{1}{2}$, the borrower defaults in a bad period. Lenders will make such loans if and only if the gross interest rate is at least $2R$. Based on the assumption of repayment if $y = y^h$, the borrower will select b to maximize his two-period discounted utility.

$$W_t^l = U[1 - \sigma(1 - b)] + \frac{1}{2}\beta U[1 + \sigma(1 - 2Rb)] \tag{2.43}$$

Payment occurs with a probability of $\frac{1}{2}$ in the second period.

The desired amount of borrowing relative to income for $b > \bar{b}$ and $2R$ is determined by a second-order Taylor series approximation about $c = 1$. The derivation appears in Appendix B.1.9.

$$\sigma b^* = \frac{\frac{1-\beta R}{\tilde{A}} + \sigma(1 + \beta R)}{1 + 2\beta R^2} \tag{2.44}$$

The desired amount of borrowing depends positively on the standard deviation σ. The lower the income in periods with $y = y^l$, the more the country wishes to borrow for smoothing purposes.

2.4.4. Sustainable Debt Level in the Risky Case

The maximum amount b_t that can be contracted depends on the willingness to pay of the borrower in any foreseeable situation for $y = y^h$. The expected discounted utility of borrowing b in periods with low income $y_t = y_t^l$, when $b_{t-1} = 0$, and under the assumption of full repayment only in the event $y_{t+1} = y_{t+1}^h$ is given by

$$V^R(b) = \frac{1}{2e}\{U[1-\sigma(1-b)]+U[1+\sigma]\}+\frac{1}{4e}\beta\{U[1+\sigma(1-2Rb)]+U[1-\sigma]+\beta V^D\} \tag{2.45}$$

where $e \equiv 1 - \frac{\beta}{2} - \frac{\beta^2}{4}$. Appendix B.1.10 contains the derivation. The income in each period is either high or low. The utility of the first period is the average of these two possibilities, taking into account that borrowing occurs in the bad period. The second period describes the expected discounted utility given a decision to repay Rb in a good period and to default in a period with low income. The probability of repayment $1 - \lambda$ is at most $\frac{1}{2}$. Default leads to autarchy, the expected discounted utility of borrowing therefore contains the expected discounted utility under autarchy V^D, which is given by (2.40).

Payment in t+1 when $y_{t+1} = y_{t+1}^h$ will be optimal if and only if the expected discounted utility of repayment is at least as high as the expected discounted utility without payment.

$$U[1 + \sigma(1 - 2Rb)] + \beta V^R(b) \geq U[1 + \sigma] + \beta V^D \tag{2.46}$$

The credit ceiling relative to income for an interest rate $2R$ has to satisfy this condition. It is determined by a second-order Taylor series approximation about $c = 1$. The derivation is shown in Appendix B.1.11.

$$\sigma \bar{b}_{2R} = \frac{\frac{2f(1-\beta R)-4R}{\bar{A}} + \sigma[2f(1+\beta R) + 4R]}{f(1 + 2\beta R^2) + 4R^2} \tag{2.47}$$

where $f \equiv \frac{\beta}{2e} = \frac{\beta}{2-\beta-\frac{\beta^2}{2}}$. Payment is optimal if $0 \leq b \leq \bar{b}_{2R}$. Lenders will not make loans in excess of $\bar{b}_{2R}$ for a given interest rate $2R$. At an interest rate $2R$ and if $b > \bar{b}_{2R}$, the expected discounted utility under autarchy exceeds the expected discounted utility of repayment in a bad period. Condition (2.46) is violated and the borrower will default.

This stochastic model can have several types of equilibria. If $\bar{b}_{2R} > \bar{b}$, loans can be either safe and constrained by $\bar{b}$ or risky with a credit ceiling $\bar{b}_{2R}$. Furthermore, other types of risky lending are also possible. For instance, a probability of default of $\frac{2}{3}$ would require an interest rate of at least $3R$ to satisfy the zero expected profit condition (2.9). In this case, actual borrowing depends on the desired borrowing b^*. If $\bar{b}_{2R} \leq \bar{b}$, no risky loan will be made. Whenever $b^* \geq \bar{b} \geq \bar{b}_{2R}$ the borrower is rationed to $\bar{b}$ and only safe loans will be made. If, however, $b^* < \bar{b}$, loans are determined by the desired amount of borrowing b^*, which is also safe.

2.5. Econometric Analysis

Actual borrowing b_t in period t is the minimum of the credit ceiling $\bar{b}_t$ and the desired amount of borrowing b_t^*. In this section, an econometric analysis shows the impact of different variables on the level of debt and the probability of a supply regime and a demand regime. The supply regime is characterized by the credit ceiling, whereas actual borrowing corresponds to the desired amount of borrowing in the demand regime. Appendix C.1.1 presents the used model and the variables. The model contains two regression analyses. The first regression analysis shows the relation between the credit ceiling and a set of variables, whereas the second analysis shows the relation between the demand for credit and the same set of variables. Only the amount of borrowing can be observed. It is not clear whether b_t corresponds to the credit ceiling $\bar{b}_t$ or to the desired debt b_t^*. The used data is a cross section of poor countries, including Argentina and Mexico. The observed years are 1970 and 1974.

In this econometric analysis, the public debt to private creditors serves as dependent variable. The level of debt is a function of the growth rate of income, the export variability, income, the ratio of imports to income, population, the public debt to public institutions and a dummy for the year in both regimes. The two regimes use the same country characteristics, they can be identified by the theory and the results from the theoretical models.

The regression of the desired debt is identified because of a positive relation between the debt and the growth rate. As mentioned in 2.3.1, a higher growth rate causes more resources in the repayment period and therefore a higher repayment is possible. Some future income is desired in the current period. On the other hand, the relation between the credit ceiling and the growth rate is ambiguous. One regression shows a negative relation, therefore it is identified as the supply regime. Other variables reinforce this identification. The negative relation suggests that a high growth rate causes fewer negative effects of a future credit embargo, which increases default incentives and decreases the credit ceiling.

The export variability is used instead of the income variability because the documentation of exports is better than the documentation of income for many poor countries. The coefficients are both positive, which means that the export variability increases the credit ceiling and the desired debt. These findings are also consistent with the theoretical part.

In the case of the supply regime, the coefficient of income is positive. This means that the credit ceiling increases in income. In good periods, the ceiling is higher than in bad periods. On the other hand, income and the demand for credit are not related. This result is not suggested by the theory. It means that the government wishes to borrow the same amount in good and bad periods.

The ratio of imports to income serves as a proxy for the penalty. If imports are important for the poor country, an embargo on trade is more harmful. The coefficient of the supply regime is positive. This is consistent with the theory. A penalty has a positive effect on the credit ceiling since it decreases the value function given a decision to default V^D (2.5). As a consequence, the probability of default λ decreases. Surprisingly, the ratio of imports to income has a negative effect on the demand regime. The demand for credit decreases in the penalty.

The population of a country has a negative effect on the credit ceiling. The more inhabitants a country has, the lower its ceiling is. This result is also surprising. On the other hand, the size of the population has no effect on the demand for credit.

The public debt to public institutions is taken as predetermined and exogenous. It has a positive effect on the credit ceiling, which means that the public debt to private creditors increases with the public debt to public institutions. A high level of debt to public creditors could mean that the country is stable and reliable. The negative effect on the credit demand is insignificant. It would suggest that the debt to public institutions is a substitute for the debt to private creditors.

The dummy allows for a year-specific error term. The negative coefficient in the supply regime indicates a lower credit ceiling in 1974 than in 1970. On the other hand, the dummy has a positive effect on the demand. The desired debt was insignificantly lower in 1970.

EG estimate the probability of a supply constrained regime for the poor countries. 16 of 81 observations are more likely to be demand determined and 65 observations are more likely to be supply determined. For instance, Turkey is one of the countries which had a probability of a supply constrained regime of 100 % in 1970 and 1974. On the other end of the scale is Trinidad and Tobago, it had a probability of a demand regime of 99 % in 1970 and 100 % in 1974. Argentina's probability of a supply constrained regime was 68 % in 1970, which increased to 72 % in 1974. Mexico's probability decreased from 89 % in 1970 to 69 % in 1974.

3. Arellano's Model

Arellano (2008) studies endogenous default risk in a stochastic general equilibrium model of a small open economy. The incompleteness of asset markets and the stochastic stream of income are important characteristics of the analysis. The equilibrium is determined by the policy functions for the government and the consumers, and the price function for bonds. Default in equilibrium is possible because of the incompleteness of asset markets. The probability of default is countercyclical and therefore higher in periods with low income. To prevent Ponzi schemes, debt is bounded.

In each period, the country trades discount bonds with foreign creditors on international financial markets to maximize its objective function. Bonds can be either positive or negative. A positive value means that the government saves an amount. The government sells bonds if it wants to borrow. These non-contingent assets mature in one period, but debt contracts are not enforceable. The benevolent government can choose to default at any time. However, creditors can penalize defaulting debtors by various actions. One of the most important penalties is the temporary or permanent exclusion from financial markets and therefore from future borrowing and saving. A lower output during financial autarchy is another exogenous consequence of a default.

In this study, the notation is slightly different to the one used in the seminal approach by Eaton and Gersovitz. The reason is that Eaton and Gersovitz explicitly distinguish between saving and borrowing. They show that it is better for a country to specialize in either borrowing or saving and exclude the saving option in the stochastic model. Therefore, it is helpful to use a different notation for borrowing and saving. On the other hand, Arellano treats borrowing and saving equally. In each period, the country has the option to borrow or to save regardless of the sign of initial assets. The current study uses a smooth transition from borrowing to saving. Equilibrium interest rates for a low level of debt can be equal to the interest rate for saving which corresponds to the risk free interest rate. Therefore, it is helpful to use the same notation for borrowing and saving in this study.

The theoretical analysis treats two different shocks. The first model is a simpler version, it contains i.i.d. income shocks and characterizes analytically the equilibrium properties of borrowing. Persistent shocks can be observed in the second model. Both models characterize the default decision and the equilibrium bond price function. Bond prices are determined endogenously by the incentives for repayment.

With i.i.d. shocks, the income shock of the repayment period is independent of the shock from the previous period. Given initial assets, a default boundary determines the required income for the satisfaction of debt obligations. A lower output shock leads to default and the government faces an exclusion from international financial markets.

Therefore, the probability of default is a function of the default boundary. It is decreasing in assets. An endogenous borrowing limit defines the range of risky borrowing. No rational borrower will enter into a contract with a higher level of debt because of the ‚Laffer Curve‘ of total resources borrowed.

With persistent shocks, it is likely that the income shock in the repayment period can be predicted by the shock from the previous period. In this setting, the probability of default depends on initial assets and on the income shock. A constant relative risk aversion utility function is used for the analysis. The parameters are calibrated for Argentina. This model shows a numerical solution for the sustainable level of debt, equilibrium interest rates and the bond price schedule for a recession and a boom. The country defaults earlier if the income is low. With a higher income, the bond prices are higher and the equilibrium interest rates are lower. Assets are reported as ratio of mean output.

3.1. Assumptions

The theoretical models analyse the borrowing equilibrium of a small open economy that experiences a stochastic stream of income. Output is not storable and can be used for consumption or for debt-service payments. The output shocks follow a Markov process with a transition function $f(y_{t+1}, y_t)$ and have a compact support. International asset markets are incomplete because of the endogenous probability of default and the set of assets available that pays only a non-contingent face value. Countries can not insure themselves against the income uncertainty. Loans are non-negotiable.

The small open economy possesses a benevolent government, which optimizes its objective function over an infinite horizon. It trades bonds to smooth the consumption of the households. If the country has previously defaulted, neither borrowing nor saving is possible. The risk averse government refunds to households all the operations from international credit markets in a lump sum fashion. It does not have commitment and decides on default after observing the income shock in each period.

Defaulting only makes sense when the country is indebted. A permanent or a temporary exclusion for a stochastic period of time from international financial markets is a consequence of a repudiation. During this time, no borrowing and no saving is possible. The re-entry takes place with an exogenous probability. Direct output costs during financial autarchy are further consequences. The probability of default is endogenous to the incentives of repayment.

Foreign lenders trade bonds on international financial markets to maximize their expected profits. They have perfect information about the state of the domestic economy and the income every period, but they have no legal possibility to enforce repayment. Alternatively, trade between foreign agents at the safe interest rate $\overline{r}$ is possible. Since governments can buy and sell bonds, the foreign agents also buy and sell bonds. Lenders are price takers and risk neutral. Competition among them maximizes the borrowers' utility.

Domestic households are assumed to be identical and risk averse. In each period, they receive a stochastic stream of tradable goods and transfers from the government. Since output is not storable, households consume their entire endowment and the transfers.

The government, lenders and the households act sequentially in this model. First, the government observes the income shock and decides on default given negative initial assets. If the country satisfies its debt obligations, it takes the bond price schedule as given and chooses an amount of bonds subject to the resource constraint. Lenders take also price schedules as given and choose the amount of bonds subject to their profit maximization problem. After the signing of contracts and the lump sum transfers took place, households consume their endowment.

3.2. The Stochastic Model of Borrowing

This section presents the stochastic model of borrowing. If the government has access to international capital markets, it can trade bonds B_t to smooth the consumption c_t in each period. The discount bond has a positive value, $B_t > 0$, if the government wishes to save. However, borrowing occurs with a negative value in period t, $B_t < 0$. The price of each bond q_t depends on the stochastic income shock y_t and on B_t, and is smaller than 1. This means that a purchase of $q_t B_t$ units of bonds in period t leads to debt-service obligations B_t or to savings B_t in period t+1, depending on the sign of B_t. The corresponding notation in the study by Eaton and Gersovitz is s for the positive value and b for the negative value of $q_t B_t$. A lower bound on debt, $B_t \geq -\underline{Z}$, prevents Ponzi schemes.

The consumption of the households in period t is given by

$$c_t = y_t - q_t B_t + B_{t-1} \tag{3.1}$$

It depends positively on the output shock y_t. Savings from period t-1 and borrowing in t increase the current consumption, whereas debt-service payments and savings in t decrease c_t. If the government decides to default, the country faces financial autarchy for a stochastic number of periods. Under this condition, the consumption satisfies

$$c_t = y_t^D \tag{3.2}$$

where $y_t^D = h(y_t) < y_t$ and $h' > 0$. It denotes the lower income under financial autarchy. The government does not repay its debt after a default and can not trade bonds during its exclusion of international financial markets.

The zero expected profit condition of the competitive and risk neutral lenders requires that

$$(1-\lambda_{t+1})B_t = (1+\overline{r})q_t B_t \tag{3.3}$$

where λ_{t+1} denotes the probability of default in the repayment period. This condition means that the expected rate of return has to be at least as high as in the safe case. Condition (3.3) implies the bond price in period t.

$$q_t = \frac{1-\lambda_{t+1}}{1+\overline{r}} \tag{3.4}$$

The probability of default is determined endogenously by the incentives of the government to satisfy its debt obligations. With initial assets $B_{t-1} \geq 0$, a default makes no sense. After a repudiation, the economy can neither borrow nor save during financial autarchy, it is constrained to $B_t = 0$. Therefore, $\lambda_t = 0$ when the country is not indebted. In the case of $B_{t-1} < 0$, the probability of default lies in the closed interval [0,1]. This implies that the bond prices lie in the closed interval

$$q_t \in [0, \frac{1}{1+\overline{r}}] \tag{3.5}$$

and are smaller than 1. A bond price of zero means that the default probability is equal to 1. The inverse of the bond price defines the country gross interest rate in period t.

$$R_t \equiv \frac{1}{q_t} \tag{3.6}$$

Each period starts with initial assets B_{t-1}. After observing the income shock y_t, the government decides on default. If it satisfies the debt obligations, the benevolent government takes the bond price schedule as given and decides on the size of B_t subject to the resource constraint (3.1). The government's objective function is

$$E[\sum_{t=0}^{\infty} \beta^t U(c_t)] \tag{3.7}$$

where $U' > 0$ and $U'' < 0$. The discount factor β is strictly positive and smaller than 1.

If the borrower decides to repudiate his liabilities in t, the value of the objective function V^D is defined as the expected discounted sum of the current and all future levels of utility with a temporary financial autarchy. The consumption equals the output under autarchy since no borrowing and no saving via bonds is possible.

$$V^D(y_t) \equiv U(y_t^D) + \beta \int_{y_{t+1}} \{\theta max[V^R(y_{t+1}, 0), V^D(y_{t+1})] + (1-\theta)V^D(y_{t+1})\} f(y_{t+1}, y_t) dy_{t+1} \tag{3.8}$$

where θ denotes the exogenous probability of regaining access to international financial markets after default.

The value of the objective function given a decision on not defaulting in period t is denoted by V^R. It depends positively on consumption. A decision against default in period t does not mean that default is never optimal. The government decides on default in each period, it could be optimal to wait one period and then to repudiate the liabilities.

$$V^R(y_t, B_{t-1}) \equiv max_{B_t}\{U(y_t - q_t B_t + B_{t-1}) + \beta \int\limits_{y_{t+1}} max[V^R(y_{t+1}, B_t), V^D(y_{t+1})] f(y_{t+1}, y_t) dy_{t+1}\} \tag{3.9}$$

Provided that V^R exists and is unique, default is optimal if and only if the value function given a decision to default in t is larger than the value function given a decision not to default in t.

$$V^D(y_t) > V^R(y_t, B_{t-1}) \tag{3.10}$$

When period t begins with initial assets B_{t-1}, the set of output shocks y_t for which default is optimal is given by

$$D(B_{t-1}) \equiv \{y_t \in Y : V^D(y_t) > V^R(y_t, B_{t-1})\} \tag{3.11}$$

The set of output shocks y_t for which repayment B_{t-1} is optimal is defined by

$$P(B_{t-1}) \equiv \{y_t \in Y : V^D(y_t) \leq V^R(y_t, B_{t-1})\} \tag{3.12}$$

The probability of default λ_{t+1} satisfies

$$\lambda(y_t, B_t) = Pr[V^D_{t+1} > V^R_{t+1}] = \int\limits_{D(B_t)} f(y_{t+1}, y_t) dy_{t+1} \tag{3.13}$$

In period t, it decreases monotonically in assets B_{t-1} because V^R increases monotonically with B_{t-1} and V^D is independent of initial assets. Therefore, default sets $D(B_{t-1})$ decrease also in assets. Consider for this the assets B^1_{t-1} and B^2_{t-1} where $B^1_{t-1} < B^2_{t-1}$. If a country defaults on B^2_{t-1} at a given y_t, it defaults also on B^1_{t-1} since B^1_{t-1} represents a higher level of debt. The proof appears in Appendix B.2.1.

Empty default sets, $D(B_{t-1}) = \emptyset$, mean that the default probability is zero, $\lambda_t = 0$. When the period starts with B_{t-1}, a decision to default is not optimal for any output shock $y_t \in Y$ and $P(B_{t-1}) = Y$. This is true at least for all $B_{t-1} \geq 0$. Furthermore, there exists a threshold value $\overline{B}_{t-1}$ such that $D(B_{t-1}) = \emptyset$ holds for all $B_{t-1} \geq \overline{B}_{t-1}$.

$$\overline{B}_{t-1} \equiv inf[B_{t-1} : D(B_{t-1}) = \emptyset] \tag{3.14}$$

There exists also a threshold value $\underline{B}_{t-1}$ such that $P(B_{t-1}) = \emptyset$ holds for all $B_{t-1} \leq \underline{B}_{t-1}$. This level of debt is not sustainable. A decision to default is optimal for any output shock and therefore $D(B_{t-1}) = Y$. The bounded support of the stochastic shock is a necessary condition for the existence of such a level of debt.

$$\underline{B}_{t-1} \equiv sup[B_{t-1} : D(B_{t-1}) = Y] \tag{3.15}$$

It holds that $\underline{B}_{t-1} \leq \overline{B}_{t-1} \leq 0$ since default can only be optimal for negative assets and default sets are decreasing in assets.

3.3. Competitive Borrowing Equilibrium

A recursive competitive borrowing equilibrium satisfies all mentioned assumptions about the borrower, the households, borrowing and lending. It is characterized by the functions q_t, B_t and c_t, given initial assets B_{t-1} and given the stochastic income shock y_t. In equilibrium, the bond price is a function of $\lambda(y_t, B_t)$. The probability of default depends on y_t since the probability distribution of the stochastic income shock in the repayment period is a function of today's income shock. It depends also on the level of debt and therefore on the policy functions for the government B_t. Thus, the equilibrium bond price is given by $q_t = q(y_t, B_t)$.

$$q(y_t, B_t) = \frac{1 - \lambda(y_t, B_t)}{1 + \overline{r}} \tag{3.16}$$

It is increasing in B_t for a given y_t because $\lambda(y_t, B_t)$ decreases in bonds. Low bond prices and high interest rates are consequences of a high level of debt. Since $q(y_t, B_t)$ determines the government's budget set, the set of contracts available in period t is characterized by

$$q(y_t, B_t)B_t = \frac{[1 - \lambda(y_t, B_t)]B_t}{1 + \overline{r}} \tag{3.17}$$

over the space B_t. In the case of borrowing, each contract increases consumption by $q(y_t, B_t)B_t$ in period t.

The borrowing equilibrium is defined as a set of policy functions for the government, foreign lenders and households. The price function for bonds $q(y_t, B_t)$, asset holdings $B(y_t, B_{t-1})$, default sets $D(B_{t-1})$, repayment sets $P(B_{t-1})$ and the consumption $c(y_t, B_{t-1})$ must satisfy the following conditions:

- Bond prices $q(y_t, B_t)$ satisfy the zero expected profit condition (3.3) and reflect the government's default probability $\lambda(y_t, B_t)$
- Given y_t, B_{t-1} and $q(y_t, B_t)$, the government's policy functions $B(y_t, B_{t-1})$, together with the repayment sets $P(B_{t-1})$ and the default sets $D(B_{t-1})$, satisfy the government's optimization problem
- Given y_t, B_{t-1} and $B(y_t, B_{t-1})$, the consumption $c(y_t, B_{t-1})$ satisfies the resource constraint (3.1).

The lower bound on debt $-\underline{Z}$ is not binding in equilibrium.

3.4. Equilibrium with i.i.d. Shocks

This section characterizes the borrowing equilibrium with i.i.d. endowment shocks. The income shock in period t gives no information on y_{t+1}. This means that the default probability and the equilibrium bond prices are independent of y_t and therefore $q_t = q(B_t)$. For reasons of simplification, it is assumed that the economy does not experience an output loss during autarchy, $h(y) = y$. The exclusion from international capital markets is permanent after a decision to default, $\theta = 0$. Smoothing the consumption via bonds is never possible afterwards.

With a positive probability of default, the highest possible consumption corresponds to the endowment y_t. Possible capital outflows, defined by $B_{t-1} < q(B_t)B_t$ for $B < 0$, reduce the consumption for a given B_{t-1}. Capital inflows, $B_{t-1} > q(B_t)B_t$, are possible only if $D(B_{t-1}) = \emptyset$, given initial assets B_{t-1}. The proof is shown in Appendix B.2.2.

Default happens in situations with a non-empty default set $D(B_{t-1})$ for a given B_{t-1} when no roll over of the current debt is possible. In such a situation, the economy experiences capital outflows which means that the consumption under autarchy is higher. Therefore, the expected discounted utility under autarchy is larger than the expected discounted utility under the contract and the country defaults.

Capital outflows prevent smoothing the consumption in a period with low income, the consumption can not be increased relative to income. Outflows reduce the concave utility function more when the income is low. A decision to default is more likely in bad periods than in good periods. The value of default dominates the value of repayment because of the i.i.d. shocks and the incompleteness of asset markets. The proof of the countercyclical default incentives appears in Appendix B.2.3.

The equality of the value of default and the value of repayment defines the default boundary $\overline{y}(B_{t-1})$. It determines the required income to satisfy the given debt obligations B_{t-1}.

$$\overline{y}(B_{t-1}) \equiv y_t \in Y : V^D(y_t) = V^R(y_t, B_{t-1}) \tag{3.18}$$

where $B_{t-1} \in (\underline{B}_{t-1}, \overline{B}_{t-1})$. If $y_t < \overline{y}(B_{t-1})$, the income is too low for a repayment and the government will default. However, a higher income, $y_t \geq \overline{y}(B_{t-1})$, implies that $V^D(y_t) \leq V^R(y_t, B_{t-1})$ and the government satisfies its debt obligations. The default boundary $\overline{y}(B_{t-1})$ is decreasing in assets since V^R increases monotonically with B_{t-1}, while V^D is independent of initial assets. To satisfy the equality, the default boundary must decrease to reduce the value of repayment. The economy experiences capital outflows because $D(B_{t-1}) \neq \emptyset$ (see Appendix B.2.2) and the utility function is strictly concave. Therefore, the lower income reduces V^R more than V^D. When $B_{t-1} \in (\underline{B}_{t-1}, \overline{B}_{t-1})$, default happens in the range of $[y_t^{min}, \overline{y}(B_{t-1}))$ where y_t^{min} denotes the minimum output in period t. On the other hand, the repayment set is characterized by $[\overline{y}(B_{t-1}), y_t^{max}]$ where y_t^{max} denotes the maximum output in period t.

The default boundary is not defined if $B_{t-1} \notin (\underline{B}_{t-1}, \overline{B}_{t-1})$. For lower initial assets,

$B_{t-1} < \underline{B}_{t-1}$, the borrower defaults in each case regardless of the income shock. The default boundary is not defined since $V^D(y_t) > V^R(y_t, B_{t-1})$ for all $y_t \in Y$. If $B_{t-1} > \overline{B}_{t-1}$, the repayment set constitutes the entire set and $V^D(y_t) < V^R(y_t, B_{t-1})$ for all $y_t \in Y$.

With i.i.d. shocks, the probability of default λ_t in period t is characterized by the default boundary $\overline{y}(B_{t-1})$ and the cumulative probability distribution of shocks $F[\overline{y}(B_{t-1})]$. Since equilibrium bond prices depend on the default probability, also $q(B_t)$ is a function of the default boundary and the cumulative probability distribution of shocks.

$$q(B_t) = \frac{1 - F[\overline{y}(B_t)]}{1 + \overline{r}} \tag{3.19}$$

If $B_t > \overline{B}_t$, the default set is empty which means that the contract can be concluded at the risk free rate $\overline{r}$. Equilibrium bond prices correspond to $\frac{1}{1+\overline{r}}$. If $B_t < \underline{B}_t$, the default set constitutes the entire set and the probability of default is 1. Equilibrium bond prices are equal to zero and no contract will be concluded. The country could not increase its consumption with such a contract but should repay B_t in the next period. Otherwise, it would lose forever access to international financial markets. No rational borrower signs a contract with $q(B_t) = 0$. Bond prices are increasing in $B_t \in (\underline{B}_t, \overline{B}_t)$ because in this range, the default boundary is decreasing in assets.

3.4.1. Set of Contracts Available

The entire set of contracts available in period t is characterized by

$$q(B_t)B_t = \frac{\{1 - F[\overline{y}(B_t)]\}B_t}{1 + \overline{r}} \tag{3.20}$$

over the space B_t. Since the shocks are independent and identically distributed, the entire set of contracts available is the same in each period.

Figure 3.1 illustrates the total resources borrowed or saved under various asset choices B_t. In this parameterized scenario, the shocks are i.i.d. Gaussian shocks. Borrowing and lending B_t are plotted on the horizontal axis, and the additional consumption $q(B_t)B_t$ on the vertical axis. A negative value of $q(B_t)B_t$ means that the country borrows an amount and the consumption increases. If the value is positive, then the country saves an amount and the consumption decreases in the current period.

For lendings $B_t \geq 0$, the default set is empty and contracts are concluded at the risk free rate $\overline{r}$. The equilibrium bond prices $q(B_t)$ determine the positive slope of the graph for savings, that is $\frac{1}{1+\overline{r}}$. Given positive assets, the consumption decreases in B_t.

In the case of borrowing, four intervals can be distinguished.

If $B_t > \overline{B}_t$, the default set is empty and contracts can be concluded at the risk free interest rate $\overline{r}$. The slope of the graph is positive and given by $\frac{1}{1+\overline{r}}$. The total resources

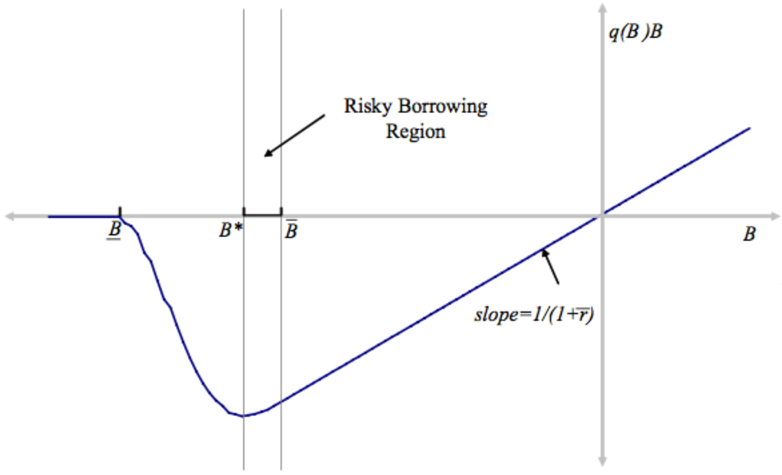

Figure 3.1.: Total resources borrowed/saved under various asset choices, Source: Arellano (2008)

borrowed are decreasing in assets. That is, the higher the level of debt is, the more consumption increases. The equilibrium bond prices are constant in this interval. The absolute value of assets is decreasing and therefore the absolute value of $q(B_t)B_t$ is decreasing. The economy can experience capital inflows.

If $B_t \in [B_t^*, \overline{B}_t)$, risky borrowing occurs. Equilibrium bond prices are lower than in the risk free case, the slope is smaller. The increase in $q(B_t)$ is smaller than the decrease of the absolute value of assets. In equilibrium, the only signed contracts with a positive default premium correspond to this interval. B_t^* serves as an endogenous borrowing limit since it generates the highest possible additional consumption. No higher level of utility can be realized with contracts available, the borrower is constrained. Therefore, the consumption can not be increased further by additional bonds.

If $B_t \in (\underline{B}_t, B_t^*)$, no contract will be concluded. No rational borrower will enter into a contract because alternative contracts are available so that the borrower can increase his consumption by the same amount, while repayment obligations are lower in the next period. The slope of the total resources borrowed is negative in this interval. This means that consumption increases in assets. Equilibrium bond prices are increasing, whereas the absolute value of assets is decreasing. Thus, bond prices must increase faster than the assets to increase consumption by $q(B_t)B_t$ units in the current period. However, uncertainty in income smooths out $q(B_t)$.

If $B_t < \underline{B}_t$, the probability of default is equal to 1 and $q(B_t) = 0$. The household's consumption can not be increased by additional assets with contracts available.

With a positive probability of default, the default boundary $\overline{y}(B_{t-1})$ decreases in assets and therefore equilibrium bond prices increase. The borrowing limit $B_t^* \in (\underline{B}_t, \overline{B}_t)$ characterizes the endogenous Laffer Curve for borrowing. $q(B_t^*)B_t^*$ generates the highest possible additional consumption in period t. The endogenous borrowing limit determines the range in which borrowing occurs. Only contracts with $B_t \geq B_t^*$ will be concluded since they increase the consumption by the same amount as $B_t < B_t^*$ but the repayment obligations are lower for these contracts.

The risky borrowing region $B_t \in [B_t^*, \overline{B}_t)$ is non-empty if a higher borrowing is associated with larger capital inflows. Given an empty default set for initial assets B_{t-1}, capital inflows are defined by $B_{t-1} > q(B_t)B_t$. To satisfy this condition, the increase in $q(B_t)$ has to be smaller than the decrease of the absolute value of assets. Then, $q(B_t^*)B_t^* \leq q(B_t)B_t$ for all $B_t \in [B_t^*, \overline{B}_t)$ and the economy experiences larger capital inflows for a higher level of debt. Risky borrowing contains a default premium since the probability of default in t+1 is positive. This, in turn, means that the default set is non-empty in the repayment period and the economy can not experience capital inflows (see Appendix B.2.2) in t+1.

However, the risky borrowing region $B_t \in [B_t^*, \overline{B}_t)$ can disappear in examples with differently chosen parameters. In this case, the endogenous borrowing limit is determined by $B_t^* = \overline{B}_t$. Borrowing is only possible if it leads to an empty default set in t+1. It does not occur with $B_t < \overline{B}_t$ because alternative contracts are available so that the borrower can increase his consumption by the same amount while repayment obligations are lower in the next period.

In a deterministic case with perfectly forecastable income shocks, figure 3.1 looks different. Negative assets are only sustainable if $D(B_{t-1}) = \emptyset$. For a positive default probability, the bond price function is equal to zero. The graph of the total resources borrowed jumps from $\frac{1}{1+\overline{r}}$ to zero at $\overline{B}_t$. Default does not happen in the equilibrium of the deterministic case.

3.5. Equilibrium with Persistent Shocks

The biggest difference to the previous equilibrium with i.i.d. shocks is the discretized persistent income process. It is more likely that the shock in the repayment period is low if today's shock is low. Therefore, a default is more likely during a recession even with small liabilities (see Appendix B.2.3). After a default, the economy experiences some direct output costs in the following way

$$h(y_t) = min(\hat{y}_t, y_t) \tag{3.21}$$

where $\hat{y}_t$ is defined as the default costs threshold in period t. It represents the highest possible output under financial autarchy. As a consequence of these asymmetric output costs, the expected discounted utility under autarchy is less sensitive to the shocks. This, in turn, increases the risky borrowing interval $[B_t^*, \overline{B}_t)$. The autarchy is temporary, the exogenous probability of re-entering international capital markets is given by θ. Equilibrium bond prices in period t depend on the income shock y_t and on the assets B_t, $q_t = q(y_t, B_t)$.

A constant relative risk aversion utility function is used for the numerical analysis.

$$U(c) = \frac{c^{1-\gamma}}{1-\gamma} \qquad \text{for } \gamma > 0, \gamma \neq 1 \tag{3.22}$$

The utility depends on the consumption and on the risk aversion coefficient γ.

3.5.1. Graphical and Numerical Analysis

The parameterized example is calibrated to match the Argentinean economy. Details are shown in Appendix C.2.1. Each period starts with initial assets B_{t-1}. The government observes the income shock in period t and decides on default. The persistent shocks are either high or low with a deviation of $\sigma = \pm 5$ % from trend. If the country satisfies its debt obligations, it takes as given the bond price schedule and chooses an amount of bonds subject to the resource constraint. The graphical analysis contains four figures. Figure 3.2 shows the decision on default, given initial assets B_{t-1}. Figure 3.3 presents the bond price schedule for the policy functions, conditional on not defaulting, while the relation between initial assets and policy functions in period t is described by figure 3.4. Figure 3.5 contains the equilibrium interest rate, which the country pays along the equilibrium path if it satisfies its debt obligations. Assets are reported as ratio of mean output.

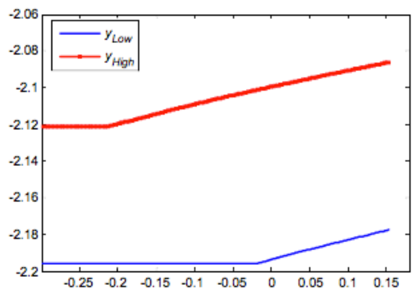

Figure 3.2.: Decision on default, Source: Arellano (2008)

Figure 3.2 shows the relation between the value function and initial assets B_{t-1} for a high income shock y_t^h and a low income shock y_t^l. The value function is defined by $max[V^R(y_t, B_{t-1}), V^D(y_t)]$. The values are negative because the risk aversion coefficient of the constant relative risk aversion utility function is set to 2. The value function is strictly increasing in initial assets for both y_t^h and y_t^l after a threshold. This means that default is preferred for a lower B_{t-1}. The utility function is increasing in consumption, which in turn depends positively on income. Therefore, the value function is higher during a boom than during a recession regardless of initial assets. During a boom, the government defaults if $B_{t-1} < -0.21$. It holds that $V^D(y_t^h) > V^R(y_t^h, B_{t-1})$ for a higher level of debt. For all $B_{t-1} \geq -0.21$, repayment is preferred and the value function increases in assets. During a recession, the government defaults already on $B_{t-1} < -0.02$.

The lower the income, the earlier the government defaults.

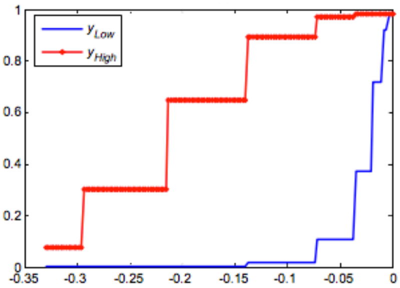

Figure 3.3.: Bond price schedule, Source: Arellano (2008)

Figure 3.3 describes the bond price schedule $q(y_t, B_t)$ for borrowing B_t. It is increasing in assets since the default probability is decreasing in B_t. Equilibrium bond prices are lower during recessions, $q(y_t^h, B_t) \geq q(y_t^l, B_t)$. This reflects the countercyclical default probability (see Appendix B.2.3). $q(y_t, B_t)$ determines the government's budget set, which is larger in good periods. The country is often at its constraint during recessions, it faces a high endogenous borrowing limit, $B^*(y_t^h) \leq B^*(y_t^l)$. A decision to default is more likely when the income is low (see figure 3.2) and therefore the prices are lower. This means that the country pays lower interest rates in good periods. Given y_t^l, equilibrium prices for $B_t < -0.14$ are zero. No contract will be concluded since a contract would give zero resources to the borrower. The price schedule is very steep during recessions.

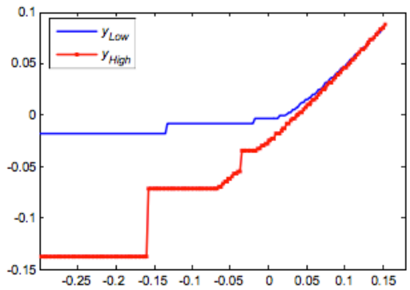

Figure 3.4.: Policy functions, Source: Arellano (2008)

Figure 3.4 presents the policy functions $B(y_t, B_{t-1})$ as a function of initial assets B_{t-1}, conditional on not defaulting. Given the income shock y_t, B_t increases in B_{t-1}. The economy saves less in recessions than in booms if $B_{t-1} > 0.1$. At all lower levels of wealth, the government saves more in hard times. An indebted economy can borrow only small amounts during recessions, it borrows more during good shocks because of the countercyclical interest rate (see figure 3.5) and the lower endogenous borrowing limit.

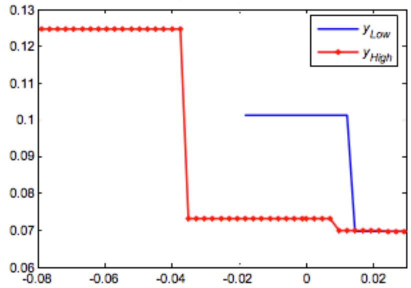

Figure 3.5.: Equilibrium interest rate, Source: Arellano (2008)

Figure 3.5 plots the equilibrium interest rate r_t as a function of initial assets B_{t-1}, where $r_t = \frac{1}{q(y_t, B_t)} - 1$. It is decreasing in B_{t-1} since B_t is increasing in initial assets and therefore also $q(y_t, B_t)$ increases. Equilibrium interest rates are lower in booms than in recessions for a given B_{t-1} because bond prices are higher for y_t^h (see figure 3.3). During a recession, the government defaults if $B_{t-1} < -0.02$ (see figure 3.2). Afterwards, it faces an exclusion of international financial markets. The country can not trade bonds during autarchy, no borrowing is possible. Contracts are only available if $B_{t-1} \geq -0.02$. The highest equilibrium interest rate is then 0.1 in this parameterized scenario. During booms, the country can choose higher amounts because of risky borrowing. The interest rate increases in default risk.

In this numerical example, we can distinguish between a recession and a boom. The discretized income shocks are persistent and either 5 % below or 5 % above trend. During a recession, the government defaults if the initial debt is higher than 2 % of mean output. In this case, it faces an exclusion of international financial markets and is not able to trade bonds for an exogenous period of time. On the other hand, if the country satisfies its debt obligations, it can smooth the household's consumption by trading bonds taking the bond price schedule as given. Equilibrium bond prices depend on the level of assets B_t, which in turn depends on the initial assets. Rational lenders anticipate the behaviour of the borrower, they lend at most 2 % of mean output to a country. Otherwise, the government would default in the repayment period. Given the bond price schedule, a higher borrowing would be possible. The bond prices are positive for $B_t > -0.14$ but contracts will not be signed because equilibrium interest rates are too high. In periods with an income shock of 5 % below trend, contracts will be concluded only if the equilibrium interest rate is not higher than 10 %, which corresponds to a bond price of 0.91. Given the bond price schedule, assets with a lower bond price leads to a level of debt that is not sustainable.

During booms, the government defaults if $B_{t-1} < -0.21$. In the case of risky borrowing, equilibrium interest rates can be much higher than in bad periods. This means, in turn, that the country can choose a lower level of assets with a lower price and a higher repayment obligation.

Equilibrium bond prices are higher in good periods than in bad periods, which correspond to lower interest rates during booms. The endogenous borrowing limit is lower for y_t^h and the sustainable level of debt is higher. The economy suffers more during recessions.

3.6. Quantitative Analysis

This section presents a quantitative analysis in terms of matching the data for the Argentinean default in 2001. The country was indebted by 144.5 billion U.S. dollars when it defaulted on over 80 billion U.S. dollars in December. (Damill et al. 2006) In present values, this was the second largest default in history after the Greek restructuring in 2012. (Tomz; Wright 2013) A severe economic crisis was one of the consequences.

Appendix C.2.2 contains details of the used data and the business cycle statistics. The used model corresponds to the model with persistent shocks from the previous chapter.

The quantitative predictions contain the interest rate spread, the trade balance, consumption and output for the default period which corresponds to the first quarter of 2002.

The interest rate spread is defined as the difference between the Argentinean interest rate and the U.S. interest rate[1]. As shown in the previous chapter, interest rates are countercyclical. This means that the spread is high if the income is low (see figure 3.5). In the case of the Argentinean default, the interest rate spread was 28.60 % in the first quarter of 2002. The model underestimates the spread and generates a value of 24.32 %. It matches well the volatility. The main anomaly of this model is the generation of a small mean annual spread prior the default. It corresponds to 10.25 % for Argentina, while the model generates a value of 3.58 %. In good periods, the interest rate spread is lower than in bad periods.

The trade balance was positive in the first quarter of 2002. However, the model predicts a slightly negative trade balance but it approximately matches the volatility.

Consumption decreased by 16 % during the default episode. The model generates a lower reduction, the deviation from trend corresponds to 9.47 % in the benchmark model.

The situation is very similar for output. The reduction was larger than the model predicts. In the default episode, the Argentinean output decreased by 14.21 %. Thus, the consumption decreased more than output but the model generates a slightly larger reduction of output. As in the data, it delivers a higher volatility of consumption than output. The correlation between consumption and income is very high during recessions. The borrower is constrained in bad times. Borrowing is expensive because of countercyclical interest rates. On the other hand, debt is cheap during booms and the borrower can consume more. This causes the higher volatility of consumption relative to income, given persistent income shocks.

The model generates a mean deviation of output during financial autarchy of -8.13 %, it approximately matches the mean reduction of 7.3 % of the Argentinean output during

[1] 5 year U.S. treasury bond

its exclusion of international financial markets. The prediction of the ratio of mean debt to output prior the default event corresponds to 5.95 %. This matches closely the debt-service to GDP ratio of 5.53 %.

Argentina experienced a significant reduction in output and consumption after its decision to default, while the interest rate spread increased sharply. The model predicts well these facts. The predictions for the trade balance are less well. Furthermore, it can predict the default event in 2001. Using the time series starting in 1993, the model predicts the Argentinean default event in the last quarter of 2001.

4. Cuadra, Sanchez and Sapriza's Model

Cuadra, Sanchez and Sapriza (2010, CSS) study endogenous default risk in a dynamic stochastic general equilibrium model of a small open economy with endogenous fiscal policy. The study follows the seminal approach of Eaton and Gersovitz and the approach of Arellano. One big difference to Arellano is the production economy. The output is determined endogenously, domestic households have to decide on labour effort. They derive direct utility not only from private consumption and public spending but also from leisure. The economy experiences a persistent productivity shock. The incompleteness of asset markets is a common feature of both studies. CSS extend the approach of Arellano by focussing on the effect of endogenous default risk on fiscal policy. The same notation is used as in the previous study.

The equilibrium in the theoretical model is characterized by the value and the policy functions for the government, the policy functions for the households, and the price function for bonds. Default in equilibrium is possible because of the incompleteness of asset markets. As shown in Arellano, the probability of default is countercyclical in emerging economies and therefore higher in periods with low income. In each period, the country observes the technology shock and decides on default. Given repayment, it trades one-period discount bonds with foreign creditors on international financial markets and collects consumption taxes to finance its expenditures. If the government defaults, it faces a temporary exclusion of international financial markets and the economy experiences an economic loss. Then, the output is lower for the same labour effort.

The theoretical analysis contains the optimal behaviour of the private sector and the government. The intertemporal problem of the government corresponds to a recursive dynamic programming form. The representative household takes as given the fiscal policy and decides on its private consumption and labour effort. Then, the benevolent government takes into account the optimal reaction of the households and chooses its fiscal policy and borrowing to maximize the private sector's utility. Since the default risk is countercyclical, the resulting optimal fiscal policy is procyclical. During recessions, interest rates are higher and foreign borrowing is lower. The consumption tax rate increases to finance the government expenditures. On the other hand, borrowing is cheap in good periods and the tax rate decreases. In a graphical and numerical analysis, the benchmark model is calibrated to match the Mexican economy.

CSS analyse also the effect of private borrowing on public borrowing. Therefore, the benchmark model is modified by various aspects. It is extended by the possibility of private borrowing, the production economy is substituted by an endowment economy and the model consists of only two periods. The interest rate paid by the public sector serves as a lower bound for the interest rates of the private sector. This concept is known as the sovereign credit ceiling. Both agents can default. The penalty of a repudiation depends on the defaulting agent. The result of this modification is that the possibility of private borrowing does not change the main results of the benchmark model. Fiscal policy and borrowing is still procyclical.

4.1. Assumptions

The theoretical model analyses the borrowing equilibrium of a small open economy. Labour services and a production technology are needed for the production. The productivity factor follows a Markov process with a transition function $f(A_{t+1}, A_t)$, shocks are persistent. The production function has constant returns to scale. Output is not storable and can be used for private or public consumption. Public expenditures can be financed by trading bonds or by consumption taxes. A labour tax and a flat tax are no options for the government. International asset markets are incomplete because of the endogenous probability of default and the set of assets available that pays only a non-contingent face value. The model consists of infinitely lived agents: the domestic government, households and foreign lenders.

The small open economy possesses a benevolent government, which optimizes its objective function over an infinite horizon. Given the bond price schedule, it trades one-period discount bonds on international capital markets and taxes households to finance its public expenditures. If the country has previously defaulted, it faces a temporary exclusion of financial markets. During this time, borrowing and saving is not possible. The fully rational government does not have commitment and decides on default after observing the productivity shock in each period. The fiscal policy is endogenous in this model. The government sets a consumption tax rate and determines the amount of public spending.

Foreign lenders are assumed to be risk neutral. They trade bonds on international financial markets to maximize their expected profits. A large number of identical creditors guarantees a perfectly competitive market. In the case of borrowing, it is assumed that foreign lenders will always repay their debt. On the other hand, they have no legal possibility to enforce repayment of the domestic government. Alternatively, trade between foreign agents at the safe interest rate $\overline{r}$ is possible.

Domestic households are assumed to be identical. The representative household takes as given the actions of the government and chooses its labour effort and its consumption. Direct utility is derived by consumption, public spending and leisure. The private sector has no access to international capital markets, it pays taxes on consumption. Households prefer a smooth path of government expenditures.

Each period starts with initial assets. The government observes the productivity shock and decides on default. If the economy is not indebted, the rational country will never default. Otherwise, it will lose access to international credit markets for a temporary period of time and experience an economic loss. The re-entry takes place with an exogenous probability. If the government satisfies its debt-service obligations, it chooses an amount of borrowing and its fiscal policy, that is, the level of public spending and the consumption tax rate. The aim of the government is to maximize the utility of the private sector, taking into account the reaction of the households.

4.2. The Stochastic Model of Borrowing

This section presents the stochastic model of borrowing of the small open economy. If the government has not previously defaulted, it can trade non-contingent bonds B_t in each period. Bonds mature in one period and have to be repaid. A positive value $B_t > 0$ means that the country buys bonds abroad because it wishes to save. On the other hand, it borrows an amount in period t if $B_t < 0$. The price of each discount bond q_t depends on the stochastic productivity shock A_t and on B_t, and is smaller than 1. Therefore, a purchase of $q_t B_t$ units of bonds in period t leads to savings B_t in period t+1. If the country borrows $q_t B_t$ units of bonds in period t, it has to repay the debt-service obligations B_t in t+1. Otherwise, it defaults and loses access to international financial markets.

Households consume c_t and work in period t, l_t denotes the labour effort of the representative household. The output of the economy depends on labour and the production technology A_t. A positive productivity shock increases output, whereas a negative shock decreases y_t. Shocks are assumed to be persistent, the production function has constant returns to scale.

$$y_t = A_t l_t \tag{4.1}$$

It is assumed that the productivity factor follows a Markov process. The transition function is given by $f(A_{t+1}, A_t)$, the values of A are defined over the set Υ. The consumption of the households in period t is given by

$$(1 + T_t)c_t = A_t l_t \tag{4.2}$$

where T_t denotes the tax rate on private consumption. c_t depends positively on output and negatively on the consumption tax rate. Households derive direct utility from consumption, government spending g_t and leisure $1 - l_t$. Utility decreases in labour effort. The expected discounted utility is given by

$$E[\sum_{t=0}^{\infty} \beta^t U(c_t, g_t, 1 - l_t)] \tag{4.3}$$

The utility function is twice differentiable, it is strictly increasing in each argument and strictly concave. The discount factor β is strictly positive and smaller than 1. The representative household maximizes its discounted utility subject to its budget constraint (4.2). It behaves like a Stackelberg follower and takes as given the actions of the government, which are the decision on default, the public spending g_t and the tax rate T_t. If the government has previously defaulted, the country faces financial autarchy for a temporary period of time. During this time, the output is lower for the same labour effort.

$$y_t^D = h(A_t)l_t \tag{4.4}$$

where $h(A_t) < A_t$ and therefore $y_t^D < y_t$. The corresponding budget constraint is given by

$$(1 + T_t^D)c_t^D = h(A_t)l_t \tag{4.5}$$

The superscript D denotes variables under financial autarchy.

The risk neutral lenders maximize their expected profits. Since international financial markets are perfectly competitive, the zero expected profit condition requires that

$$(1 - \lambda_{t+1})B_t = (1 + \overline{r})q_t B_t \tag{4.6}$$

where λ_{t+1} denotes the probability of default in the repayment period. This condition means that the expected rate of return has to be at least as high as in the safe case with a risk free interest rate $\overline{r}$. The zero expected profit condition implies the bond price in period t.

$$q_t = \frac{1 - \lambda_{t+1}}{1 + \overline{r}} \tag{4.7}$$

Bond prices lie in the closed interval

$$q_t \in [0, \frac{1}{1 + \overline{r}}] \tag{4.8}$$

and are smaller than 1. A bond price of zero means that the default probability is equal to 1. On the other hand, the highest possible price $\frac{1}{1+\overline{r}}$ corresponds to the safe case, when the default probability is zero. The inverse of the bond price defines the country gross interest rate in period t.

$$R_t \equiv \frac{1}{q_t} \tag{4.9}$$

Each period starts with initial assets B_{t-1}. The government observes the productivity shock A_t and decides on default. If it satisfies its debt obligations, the benevolent government takes as given the bond price schedule and decides on the size of B_t. Additionally, it decides on its fiscal policy. The government's budget constraint is then given by

$$g_t = T_t c_t - q_t B_t + B_{t-1} \tag{4.10}$$

Government expenditures are increasing in the tax rate and in private consumption. Furthermore, savings from period t-1 and borrowing in t increase the current public spending, whereas debt-service payments and savings in t decrease g_t. The endogenous fiscal policy determines the amount of public expenditures and the level of the consumption tax rate T_t.

The government's objective function is given by condition (4.3). The benevolent government cares about the domestic households and maximizes their utility function, which depends positively on private consumption, government expenditures and leisure. The value of the objective function given a decision in favour of repayment in period t is denoted by V^R. A decision against default in period t does not mean that default is never optimal. The government decides on default in each period, it could be optimal to wait one period and then to repudiate the liabilities.

$$V^R(A_t, B_{t-1}) \equiv max_{T_t, g_t, B_t}\{U(c_t^*, g_t, 1-l_t^*)+\beta \sum_{A_{t+1}\in\Upsilon} max[V^R(A_{t+1}, B_t), V^D(A_{t+1})]f(A_{t+1}, A_t)\} \tag{4.11}$$

The government acts like a Stackelberg leader. It maximizes its objective function by choosing its fiscal policy and the foreign assets, taking into account the optimal reaction of the private sector, c_t^* and l_t^*.

If the government has previously defaulted, the country can not trade bonds during the financial autarchy. The government's budget constraint is then given by

$$g_t^D = T_t^D c_t^D \tag{4.12}$$

The value of the objective function given a decision in favour of default is denoted by V^D. It is defined as the expected discounted sum of the current and all future levels of utility with a temporary financial autarchy.

$$\begin{aligned} V^D(A_t) \equiv max_{T_t^D, g_t^D}\{U(c_t^{D*}, g_t^D, 1-l_t^{D*}) + \beta \sum_{A_{t+1}\in\Upsilon} [\theta max[V^R(A_{t+1}, 0), V^D(A_{t+1})] \\ + (1-\theta)V^D(A_{t+1})]f(A_{t+1}, A_t)\} \end{aligned} \tag{4.13}$$

where θ denotes the exogenous probability of regaining access to international financial markets after a default. The government maximizes its objective function by choosing T_t^D and g_t^D, taking into account the optimal reaction of the private sector.

Provided that V^R exists and is unique, default is optimal if and only if the value function given a decision to default in t is larger than the value function given a decision not to default in t.

$$V^D(A_t) > V^R(A_t, B_{t-1}) \tag{4.14}$$

When period t begins with initial assets B_{t-1}, the set of productivity shocks A_t for which default is optimal is given by

$$D(B_{t-1}) \equiv \{A_t \in \Upsilon : V^D(A_t) > V^R(A_t, B_{t-1})\} \tag{4.15}$$

The set of productivity shocks A_t for which repayment B_{t-1} is optimal is defined by

$$P(B_{t-1}) \equiv \{A_t \in \Upsilon : V^D(A_t) \leq V^R(A_t, B_{t-1})\} \tag{4.16}$$

The probability of default λ_t satisfies

$$\lambda(A_{t-1}, B_{t-1}) = Pr[V_t^D > V_t^R] = \sum_{A_t \in D(B_{t-1})} f(A_t, A_{t-1}) \tag{4.17}$$

Empty default sets, $D(B_{t-1}) = \emptyset$, mean that the default probability is zero, $\lambda_t = 0$. When period t starts with B_{t-1}, a decision to default is not optimal for any productivity shock $A_t \in \Upsilon$ and $P(B_{t-1}) = \Upsilon$. The probability of default decreases monotonically in assets B_{t-1} because V^R increases monotonically with B_{t-1} and V^D is independent of initial assets. Therefore, default sets $D(B_{t-1})$ decrease also in assets. The proof is similar to Appendix B.2.1.

4.3. Competitive Borrowing Equilibrium

A recursive competitive borrowing equilibrium satisfies all mentioned assumptions about the borrower, the households, borrowing and lending. In equilibrium, the bond price is a function of $\lambda(A_t, B_t)$. The probability of default depends on A_t since the probability distribution of the persistent productivity shock in the repayment period is a function of today's productivity shock. It depends also on the level of debt and therefore on the policy functions for the government B_t. Thus, the equilibrium bond price is given by $q_t = q(A_t, B_t)$. It is increasing in B_t for a given A_t because $\lambda(A_t, B_t)$ decreases in bonds. Low bond prices and high interest rates are consequences of a high level of debt. In the case of borrowing, each contract increases government spending by $q(A_t, B_t)B_t$ in period t.

The recursive borrowing equilibrium is defined as a set of policy functions for the government, foreign lenders and households. The price function for bonds $q(A_t, B_t)$, the value functions for the government $V^R(A_t, B_{t-1})$ and $V^D(A_t)$, default sets $D(B_{t-1})$, repayment sets $P(B_{t-1})$, asset holdings $B(A_t, B_{t-1})$, fiscal policies g_t, g_t^D and T_t, T_t^D, the optimal consumption c_t^*, c_t^{D*} and the optimal labour effort l_t^*, l_t^{D*} must satisfy the following conditions:

- Bond prices $q(A_t, B_t)$ satisfy the zero expected profit condition (4.6) and reflect the government's default probability $\lambda(A_t, B_t)$
- Given A_t, B_{t-1}, $q(A_t, B_t)$ and the actions of the government, the consumption c_t^*, c_t^{D*} and the labour effort l_t^*, l_t^{D*} solve the household's problem
- Given A_t, B_{t-1}, $q(A_t, B_t)$ and the optimal behaviour of the households, the value functions $V^R(A_t, B_{t-1})$ and $V^D(A_t)$, the government's policy functions $B(A_t, B_{t-1})$, the repayment sets $P(B_{t-1})$, the default sets $D(B_{t-1})$ and the fiscal policies g_t, g_t^D and T_t, T_t^D satisfy the government's optimization problem.

4.3.1. Optimal Behaviour

If the government has not previously defaulted, the representative household maximizes its discounted utility (4.3) subject to its budget constraint (4.2). It behaves like a Stackelberg follower and takes as given the decision of the government on default, the public spending g_t and the tax rate T_t. A combination of the first order conditions results in the optimal behaviour of the households.

$$\frac{A_t}{1+T_t} = \frac{U_l(c_t^*, g_t, 1-l_t^*)}{U_c(c_t^*, g_t, 1-l_t^*)} \tag{4.18}$$

The derivation appears in Appendix B.3.1. The distortionary consumption tax affects the labour-leisure tradeoff. The optimal choice of consumption and leisure of the private sector is denoted by c_t^* and $1-l_t^*$. These solve the households' problem, given the actions of the government. The right-hand side of this intratemporal condition describes the marginal rate of substitution of leisure for consumption. The left-hand side shows the ratio between the marginal product of labour and the consumption tax.

The government behaves like a Stackelberg leader. It maximizes its objective function by choosing its fiscal policy and the foreign assets, taking into account the optimal reaction of the private sector. Given access to capital markets and a decision in favour of repayment, the government maximizes $V^R(A_t, B_{t-1})$ subject to the budget constraint of the private (4.2) and public sector (4.10), and the optimal reaction of the households (4.18). The government's problem is then given as

$$V^R(A_t, B_{t-1}) \equiv max_{T_t, g_t, B_t}\{U(c_t^*, g_t, 1-l_t^*) + \beta \sum_{A_{t+1}\in\Upsilon} max[V^R(A_{t+1}, B_t), V^D(A_{t+1})]f(A_{t+1}, A_t)\}$$

subject to

$$(1+T_t)c_t = A_t l_t^*$$

$$g_t = T_t c_t - q_t B_t + B_{t-1}$$

$$\frac{A_t}{1+T_t} = \frac{U_l(c_t^*, g_t, 1-l_t^*)}{U_c(c_t^*, g_t, 1-l_t^*)}$$

Given a decision in favour of repayment, the benevolent government chooses a consumption tax rate T_t, government spending g_t and the borrowing B_t. For a maximum, the first order condition for T_t can be written as

$$U_c(c_t^*, g_t, 1-l_t^*)\frac{A_t l_t^*}{(1+T_t)^2} = U_g(c_t^*, g_t, 1-l_t^*)[\frac{A_t l_t^*}{(1+T_t)^2} + \frac{T_t A_t}{1+T_t}\frac{\partial l_t}{\partial T_t}] \tag{4.19}$$

The derivation is shown in Appendix B.3.2. The distortionary consumption tax affects the government spending, private consumption and the labour supply of the households. Private consumption is negatively related to the tax rate. The left-hand side of the condition shows the reduction of private consumption when the government increases the tax rate. An increase of the tax rate reduces private consumption by $\frac{A_t l_t^*}{(1+T_t)^2}$ units. On the other hand, government spending depends positively on the tax rate. Public spending increases by the same amount $\frac{A_t l_t^*}{(1+T_t)^2}$ as private consumption decreases. The resources are reallocated from private to public consumption. The possible advantages depend on the preferences of the households. The consumption tax rate also affects the labour effort. The term $\frac{T_t A_t}{1+T_t}\frac{\partial l_t}{\partial T_t}$ characterizes the reaction of the labour supply on changes in the tax rate that depends on the sign of $\frac{\partial l_t}{\partial T_t}$. If the labour effort decreases in the consumption tax rate, this corresponds to a negative sign, then the public spending decreases since the proceeds from taxing the households are reduced. Condition (4.19) means that the marginal costs of a change in the tax rate equals the marginal benefits.

The government also chooses its optimal borrowing. For a maximum, the first order condition for B_t can be written as

$$U_g(c_t^*, g_t, 1-l_t^*)[q_t + B_t\frac{\partial q_t}{\partial B_t}] = \beta \sum_{A_{t+1}\in P(B_t)} U_g(c_{t+1}^*, g_{t+1}, 1-l_{t+1}^*) f(A_{t+1}, A_t) \qquad (4.20)$$

Condition (4.20) presents the Euler equation, the derivation appears in Appendix B.3.2. An additional unit of borrowing changes the government spending by $[q_t + B_t\frac{\partial q_t}{\partial B_t}]$ units in period t. The first term corresponds to a situation in which the bond price is constant for all levels of assets. In this case, an additional unit of borrowing increases government expenditures in q_t units. However, the essential feature of the model is default risk. The probability of default increases in debt, a higher borrowing causes lower bond prices and higher interest rates. The second term $B_t\frac{\partial q_t}{\partial B_t}$ takes this effect into account. Decreasing bond prices reduce the government spending and the utility of the households. Given a positive default probability, the benefits of borrowing decrease in interest rates. Borrowing could also affect the supply of labour. Households can work more or less if the country sells additional bonds. A negative effect would arise if additional borrowing reduces labour supply, given the consumption tax rate. Then, the proceeds from the tax would be lower and the government spending is reduced. The effect of the labour supply on government spending is given by the term $\frac{T_t A_t}{1+T_t}\frac{\partial l_t}{\partial B_t}$ and depends on the sign of $\frac{\partial l_t}{\partial B_t}$. In this model, $\frac{\partial l_t}{\partial B_t} = 0$ which means that the labour supply does not depend on the government spending. The behaviour of the households is fully described by their budget constraint and the first order condition, which are both characterized by the absence of borrowing. However, additional borrowing can only be optimal if $[q_t + B_t\frac{\partial l_t}{\partial B_t}] > 0$ holds. Otherwise, an additional borrowing decreases government spending and the utility of the representative household. An additional unit of borrowing affects not only the current utility but also the utility in the repayment period. Debt matures in one period, the borrowing has to be repaid in t+1. This reduces the government spending and the utility in the repayment period. If the government refuses the payment, it loses access to international capital markets. The right-hand side of the Euler equation describes the discounted marginal utility of government spending in t+1. Therefore, the current marginal utility of government spending equals the discounted marginal utility of public consumption in the next period. This equality guarantees that the private sector is indifferent between government spending in the current period and in t+1.

If the government has previously defaulted, the representative household maximizes its discounted utility (4.3) subject to its budget constraint (4.5) taking as given the public spending g_t^D and the tax rate T_t^D. Under financial autarchy, the optimal behaviour of the private sector is given by

$$\frac{h(A_t)}{1+T_t^D} = \frac{U_l(c_t^{D*}, g_t^D, 1-l_t^{D*})}{U_c(c_t^{D*}, g_t^D, 1-l_t^{D*})} \tag{4.21}$$

The optimal choice of consumption and labour effort of the private sector is denoted by c_t^{D*} and l_t^{D*}. The interpretation of this condition is the same as for condition (4.18).

Given a decision to default, the government maximizes $V^D(A_t)$ subject to the budget constraint of the private (4.5) and the public sector (4.12), and the optimal reaction of households (4.21). The government's problem is then given as

$$\begin{aligned} V^D(A_t) \equiv max_{T_t^D, g_t^D} \{ U(c_t^{D*}, g_t^D, 1-l_t^{D*}) + \beta \sum_{A_{t+1} \in \Upsilon} [\theta max[V^R(A_{t+1}, 0), V^D(A_{t+1})] \\ + (1-\theta) V^D(A_{t+1})] f(A_{t+1}, A_t) \} \end{aligned}$$

subject to

$$(1+T_t^D)c_t^D = h(A_t) l_t^{D*}$$

$$g_t^D = T_t^D c_t^D$$

$$\frac{h(A_t)}{1+T_t^D} = \frac{U_l(c_t^{D*}, g_t^D, 1-l_t^{D*})}{U_c(c_t^{D*}, g_t^D, 1-l_t^{D*})}$$

The first order condition for T_t^D can be written as

$$U_c(c_t^{D*}, g_t^D, 1-l_t^{D*}) \frac{h(A_t) l_t^{D*}}{(1+T_t^D)^2} = U_g(c_t^{D*}, g_t^D, 1-l_t^{D*}) [\frac{h(A_t) l_t^{D*}}{(1+T_t^D)^2} + \frac{T_t^D h(A_t)}{1+T_t^D} \frac{\partial l_t^D}{\partial T_t^D}] \tag{4.22}$$

This intratemporal condition can be interpreted in the same manner as condition (4.19). Since the economy faces financial autarchy, the government can not trade bonds and the Euler equation disappears.

4.4. Equilibrium with Persistent Shocks

This section presents a numerical solution for the benchmark model. The production function is linear and given by equation (4.2).

Labour is the only input factor, the production function has constant returns to scale. The productivity shock A_t is assumed to be persistent. It is more likely that the shock in the repayment period is low if today's shock is low. Therefore, a default is more likely during a recession even with small liabilities. The previous study of Arellano proves the countercyclical default incentives in a simpler version of an endowment economy with incomplete asset markets and i.i.d. income shocks (see Appendix B.2.3). This result holds also for the current model with the production economy and persistent shocks. Given a non-empty default set, the economy can not experience capital inflows (see Appendix B.2.2). Capital outflows reduce the concave utility function more when the income is low. This makes a decision to default more likely during a recession. If the government decides to default, the economy experiences some direct output costs in the following way

$$h(A_t) = min(\hat{A}_t, A_t) \tag{4.23}$$

where $\hat{A}_t$ represents the highest possible productivity factor under financial autarchy. As a consequence of these asymmetric output costs, the expected discounted utility under autarchy is less sensitive to the shocks. The autarchy is temporary, the exogenous probability of re-entering international capital markets is given by θ. Equilibrium bond prices in period t depend on the productivity shock A_t and on the assets B_t, $q_t = q(A_t, B_t)$. The following utility function is used for the numerical analysis

$$U(c_t, g_t, 1 - l_t) = \pi[\frac{g_t^{1-\gamma}}{1-\gamma}] + (1-\pi)[\frac{(c_t - \frac{l_t^{1+\psi}}{1+\psi})^{1-\gamma}}{1-\gamma}] \tag{4.24}$$

The utility depends on private consumption, public spending and leisure. Public spending and the variables of the households are additively separable, the utility of government spending does not directly affect the utility of leisure and consumption. The optimal behaviour of the households is independent of the government spending. $\pi \in (0, 1)$ denotes the weight of g_t. A high π means that government spending is preferred, whereas a lower value means that private consumption and leisure are preferred. The risk aversion coefficient is denoted by γ and the labour elasticity is given by $\frac{1}{\psi}$. The first term of (4.24) represents the utility of the government spending which follows a constant relative risk aversion utility function. The second term corresponds to Greenwood-Hercowitz-Huffman preferences for private consumption and leisure. The private sector variables are not additively separable, the marginal rate of substitution of leisure for consumption is given by $\frac{U_l(c_t, g_t, 1-l_t)}{U_c(c_t, g_t, 1-l_t)} = l_t^{\psi}$. The labour supply is independent of private consumption. Given the production function and this utility function, the optimal labour supply of the representative household satisfies

$$l_t^* = (\frac{A_t}{1+T_t})^{\frac{1}{\psi}} \tag{4.25}$$

The derivation appears in Appendix B.3.3. Households work more during booms since the labour supply depends positively on the productivity factor. On the other hand, the tax rate has a negative effect on the optimal labour supply. If the government increases the tax rate, the households respond with taking more leisure. The first order conditions of the government satisfies

$$(1-\pi)[\frac{\psi}{1+\psi}(\frac{A_t}{1+T_t})^{\frac{1+\psi}{\psi}}]^{-\gamma} = \pi g_t^{-\gamma}(1-\frac{T_t}{\psi}) \tag{4.26}$$

and

$$g_t^{-\gamma}[q_t + B_t\frac{\partial q_t}{\partial B_t}] = \beta \sum_{A_{t+1}\in P(B_t)} g_{t+1}^{-\gamma} f(A_{t+1}, A_t) \tag{4.27}$$

If the government has previously defaulted, the optimal labour supply is given by

$$l_t^{D*} = [\frac{h(A_t)}{1+T_t^D}]^{\frac{1}{\psi}} \tag{4.28}$$

The government chooses its fiscal policy given the reaction of the households. Then, the first order condition for the tax rate is given by

$$(1-\pi)\{\frac{\psi}{1+\psi}[\frac{h(A_t)}{1+T_t^D}]^{\frac{1+\psi}{\psi}}\}^{-\gamma} = \pi g_t^{D^{-\gamma}}(1-\frac{T_t^D}{\psi}) \tag{4.29}$$

4.4.1. Graphical and Numerical Analysis

The parameterized example is calibrated to match the Mexican economy. Details are shown in Appendix C.3.1. Each period starts with initial assets B_{t-1}. The government observes the productivity shock in period t and decides on default. The persistent shocks are either high or low. If the country satisfies its debt obligations, it takes as given the bond price schedule and chooses an amount of bonds and its fiscal policy. Lenders also take as given the bond price schedule and choose an amount of bonds subject to their profit maximization problem.

The graphical analysis contains three figures for a high productivity shock A_t^h and a low productivity shock A_t^l. Given A_t, figure 4.1 presents the bond price schedule for the policy functions, figure 4.2 shows the relation between initial assets and borrowing and figure 4.3 contains the consumption tax rate. Assets are reported as ratio of mean output.

Figure 4.1 plots the bond price schedule $q(A_t, B_t)$ for borrowing B_t in period t for a high and a low productivity shock. In both cases, bond prices are increasing in B_t because the probability of default decreases monotonically in assets. As mentioned in chapter 4.2, bond prices lie in the closed interval $[0, \frac{1}{1+\bar{r}}]$ and are smaller than 1. If the country is highly indebted and the probability of default is 1, the bond prices are equal

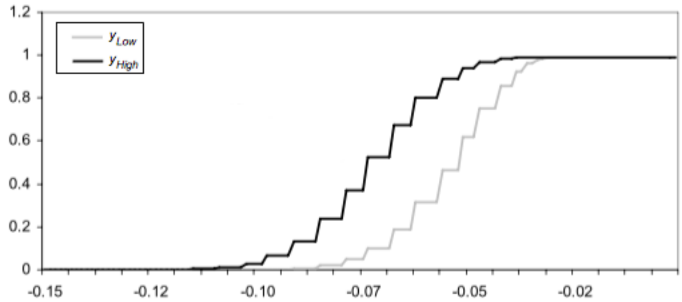

Figure 4.1.: Bond price schedule, Source: Cuadra et al. (2010)

to zero. Then, the borrower can not increase its consumption by additional borrowing because $q(A_t, B_t)B_t = 0$. A contract with $q(A_t, B_t) = 0$ gives zero resources to the country but leads to repayment obligations of B_t in t+1. Given A_t^h, equilibrium prices for $B_t < -0.115$ are zero. The default incentives are countercyclical and therefore higher if the income is low. Bond prices are already zero if $B_t < -0.091$ given A_t^l. For all higher levels of assets, the default probability is decreasing and the bond price schedule is increasing. Since the default probability is higher during recessions, the bond prices are lower when the productivity is low. That is, $q(A_t^h, B_t) \geq q(A_t^l, B_t)$ for all B_t. The upper bound of the bond price schedule corresponds to a low level of debt when the probability of default is zero. In this case, contracts can be concluded at the risk free interest rate $\overline{r}$. The equilibrium bond prices are the inverse of the gross risk free interest rate. In this scenario, the government will always repay its debt if $B_t > -0.039$ given a positive productivity shock. Given a recession, the government can borrow at the risk free rate if $B_t > -0.034$. For a higher debt level, the default incentives are positive and increasing.

This figure reflects the countercyclical default probability of the domestic government. During recessions, the country is often at its constraint. The price schedule is then very steep, the lower prices lead to higher interest rates. The borrower suffers more during recessions.

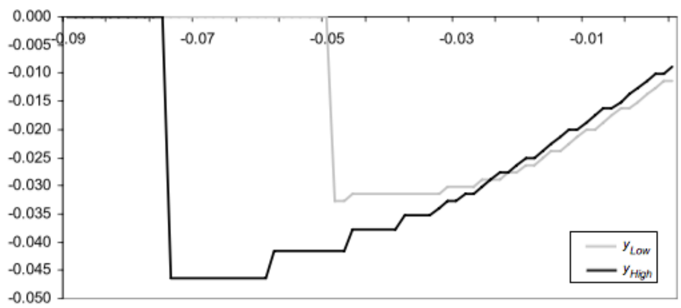

Figure 4.2.: Policy functions, Source: Cuadra et al. (2010)

Figure 4.2 shows the policy functions $B(A_t, B_{t-1})$ as a function of initial assets B_{t-1}, given repayment. Initial assets B_{t-1} are plotted on the horizontal axis, and the borrowing in period t, $B(A_t, B_{t-1})$, on the vertical axis. If the initial debt is very high and the default set constitutes almost the entire set but the government does not default, the country is constrained and can borrow only a small amount. Given the persistent productivity shock A_t, B_t is increasing in B_{t-1} for all lower levels of initial debt. A highly indebted country borrows more during booms because of the lower interest rates and the higher equilibrium bond prices in good periods (see figure 4.1). On the other hand, the country borrows more during recessions if the initial debt is low. The corresponding value is given by $B_{t-1} > -0.024$ in this parameterized example. In this case, the government has no incentives to default and the interest rates contain no risk premium. The borrowing in period t is then given by $B_t > -0.028$ for A_t^l and A_t^h. The government can borrow at most $B_t = 0.046$ if $A_t = A_t^h$ and $B_t = 0.034$ during recessions. Otherwise, the value of default surpasses the value of repayment in t+1 and the country will default. A higher borrowing leads to a level of debt that is not sustainable since the interest rates are too high.

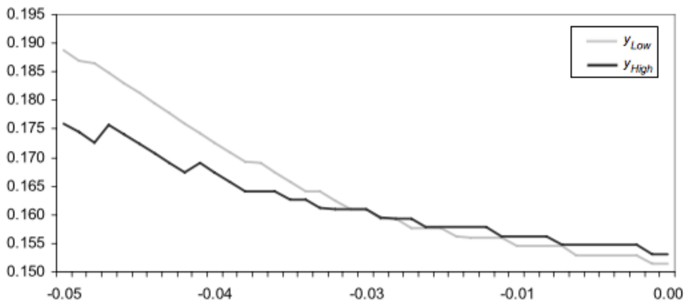

Figure 4.3.: Tax rate, Source: Cuadra et al. (2010)

Conditional on not defaulting, the government decides on the borrowing B_t and its fiscal policy. Figure 4.3 presents the consumption tax rate for the parameterized example. Given the productivity shock, the tax rate is procyclical. It decreases in assets and can potentially reinforce booms or recessions. Given a low income period, the bond prices are low and the interest rates are high. Financing of government spending by debt is expensive, the country is often at its constraint. Therefore, the tax rate increases during recessions. A highly indebted country taxes consumption more if the persistent productivity shock is low. Given the productivity shock and the bond price schedule, the government taxes consumption less in bad periods if the borrowing is $B_t > -0.028$. Then, the tax rate during booms is at least as high as the tax rate during recessions.

In this numerical example, a low and a high productivity shock hits the production economy. Conditional on not defaulting, the bond price schedule is steeper if the shock is negative. A positive shock causes a lower probability of default and higher bond prices for each level of assets. Given these productivity shocks, the government has no incentives to default in t+1 if $B_t > -0.028$, it will always repay its debt. In this case, contracts can be concluded at the risk free interest rate. The corresponding bond prices are the inverse of the gross risk free interest rate. For all higher levels of assets, the economy borrows more during recessions. Since borrowing is cheap, the tax rate decreases and is lower than in booms.

Given an initial debt $B_{t-1} < -0.024$, the country borrows more if the income is high because of the countercyclical default probability. Then, the government finances its expenditures mostly by borrowing and therefore the consumption tax rate is lower than during recessions.

4.5. Quantitative Analysis

This section presents a quantitative analysis in terms of matching the data for the Mexican economy. Appendix C.3.2 contains details of the used data and the business cycle statistics. The used model corresponds to the model from chapter 4.4. It matches well the stylized facts of the Mexican economy.

The quantitative predictions contain the correlations between output and the tax rate, private consumption, public consumption, the interest rate spread and the trade balance.

The tax rate and the output are negatively correlated. The benchmark model predicts well this result. During booms, the probability of default is low, bond prices are high and interest rates are low. The government uses mainly cheap foreign credits to finance its expenditures, it reduces consumption tax rates. In low income states, interest rates are high and the government is often constrained. It increases tax rates to finance the public consumption. Thus, default risk causes procyclical tax rates. In a model without default, tax rates are countercyclical. A government with commitment faces constant bond prices and constant interest rates. In this case, foreign credits are also cheap during recessions and the government borrows more on international capital markets while it taxes less consumption. The correlation between the tax rate and income is positive in a model without default risk. As reported in Ilzetzki and Vegh (2008), a countercyclical fiscal policy is counterfactual. The procyclicality of tax rates is predominantly in emerging economies such as Mexico. A model with labour tax delivers the same results as the model with consumption tax.

Private and public consumption are highly positively correlated with output. The procyclical government spending can be explained by the countercyclical interest rates. In good periods, the government borrows more and taxes consumption less, this in turn increases private consumption. The correlation between output and private consumption is higher than the correlation between government spending and output. The model predicts also this result but it overestimates the procyclicality of the public spending.

The interest rate spread and the trade balance are negatively correlated with output. In both cases, the benchmark model underestimates the correlations. The default probability is lower during booms and the interest rates are lower. Thus, the interest rate spread, defined as the difference between the country interest rate and the risk free interest rate, decreases in output. The countercyclical interest rates make smoothing the consumption more difficult.

The model also predicts the volatility of the private consumption, output, the tax rate and the trade balance.

The data suggests a higher volatility of private consumption than output. The benchmark model delivers the same result. During booms, the probability of default is lower and bond prices are higher. This causes lower interest rates and a higher borrowing. Tax rates decrease and the private consumption increases relative to output. On the other hand, the government is often constrained during recessions. It increases the tax rate to finance its expenditures. This causes a higher reduction in private consumption and therefore, private consumption has a higher volatility than output. In a model without default risk, the volatility of output is higher than the volatility of private consumption. A higher income directly increases consumption of the households, but at the same time it decreases consumption because the correlation of output and the tax rate is positive. On the other hand, a lower income decreases the tax rate and the reduction of private consumption is lower than the reduction of income.

The model underestimates the volatility of the tax rate and of the trade balance.

4.6. Private Borrowing

In one third of the developing countries, the private sector was responsible for more than 10 % of total external borrowing in the late 1990s. (Reinhart et al. 2003) Therefore, the assumption that only the government can borrow abroad does not reflect the reality. This section contains a variation of the model with additional private foreign borrowing to examine the role of public borrowing. By the exclusion of households from capital markets, public borrowing is potentially overestimated. Private debt could serve as a substitute for public debt. In this model, private borrowing is restricted to international financial markets. Interactions between domestic households are ruled out. The model consists of two periods and the income is given exogenously. The agents can borrow in the first period, while the repayment takes place in the second period of the model, conditional on not defaulting. Furthermore, the first period starts with initial assets and the agents can already default in period 1. In this endowment economy, the government and the private sector have the option to default. The penalty consists of an exclusion from international capital markets and direct output costs. The level of penalty depends on the agent that defaults. After a repudiation, both domestic agents are excluded from international capital markets. The output loss is at its highest if both the private and the public sector do not satisfy their debt obligations. If only the representative household defaults, the economic loss is lower than if the public sector defaults. This model uses the concept of sovereign credit ceiling, the interest rate paid by the public sector serves as a lower bound for the interest rates of the private sector.

In this endowment economy, the income $\tilde{y}_t$ depends positively on output. Furthermore, it depends negatively on the default decision D_t^P of the households and on the default decision D_t of the government in the current period. These indicator functions take the value of 0 if the corresponding agent has not defaulted and 1 if it has defaulted. $\tilde{y}_t$ is defined by

$$\begin{aligned}\tilde{y}_t(y_t, D_t^P, D_t) \equiv\; & I_{(D_t=1)} I_{(D_t^P=1)} \kappa^B y_t + I_{(D_t=1)} I_{(D_t^P=0)} \kappa^G y_t + I_{(D_t=0)} I_{(D_t^P=1)} \kappa^P y_t \\ & + I_{(D_t=0)} I_{(D_t^P=0)} y_t\end{aligned} \tag{4.30}$$

$I_{(D_t=1)}$ denotes the indicator function of default for the government and $I_{(D_t^P=1)}$ denotes the indicator function of default for the private sector in period t. It is 1 if the domestic agent decides to default and 0 if the repayment takes place. That is, $I_{(D_t=1)} = 1$ if $D_t = 1$ and $I_{(D_t=1)} = 0$ if $D_t = 0$ for the government. On the other hand, $I_{(D_t=0)}$ denotes the indicator function of repayment. It is 1 if the government satisfies its debt obligations and 0 if the government defaults. The same holds for the indicator functions of the private sector. After a default, the country experiences some direct output costs. The economic loss is denoted by $y_t - \kappa y_t$, where $\kappa \in (0, 1)$ represents the punishment. It is at its highest if both default at the same time, the corresponding punishment is then κ^B. If only the government defaults, the economic loss is higher than if the private sector defaults. It holds that $\kappa^B \leq \kappa^G \leq \kappa^P \leq 1$. Condition (4.30) distinguishes between these three cases. If both agents repudiate their liabilities, $I_{(D_t=1)} I_{(D_t^P=1)} = 1$, the income in period t is given by $\kappa^B y_t$. The income is $\kappa^G y_t$ if only the government defaults and it is $\kappa^P y_t$ if only the representative household defaults. Given repayments, the economy does not experience an output loss and the income is y_t.

The representative household trades bonds B_t^P and consumes c_t in period t. It does not decide on labour effort since the model describes an endowment economy. The utility function depends on private and public consumption, it is twice differentiable, strictly increasing in each argument and strictly concave. It is assumed that all domestic agents have the same preferences.

Given access to international capital markets, the private consumption in period t is

$$(1 + T_t)c_t = \tilde{y}_t(y_t, D_t^P, D_t) - q_t^P(y_t, B_t^P, B_t)B_t^P + B_{t-1}^P \tag{4.31}$$

The superscript P denotes the variables of the private sector. The bond prices of the representative household depend on output, private borrowing and public borrowing because of the concept of sovereign credit ceiling. Therefore, the private sector faces lower bond prices of the discount bonds than the government. In the second period, the term $q_t^P(y_t, B_t^P, B_t)B_t^P$ disappears because the agents can borrow only in the first period. Given access to international capital markets, the public consumption satisfies

$$g_t = T_t c_t - q_t(y_t, B_t^P, B_t)B_t + B_{t-1} \tag{4.32}$$

Public borrowing or saving is only possible in the first period. Thus, $q_t(y_t, B_t^P, B_t)B_t$ disappears in the second period. The bond prices of the government depend also on the borrowing of the private sector, on the borrowing of the government and on output in period t because of the persistent output shocks. Both the government and the representative household are price takers. Bond prices of the private sector are determined by the zero expected profit condition

$$[1 - \lambda^P(y_t, B_t^P, B_t)]B_t^P = (1 + \overline{r})q_t^P B_t^P \tag{4.33}$$

and are given by

$$q_t^P = \frac{1 - \lambda^P(y_t, B_t^P, B_t)}{1 + \overline{r}} \tag{4.34}$$

The corresponding gross interest rate of the private sector in period t is

$$R_t^P \equiv \frac{1}{q_t^P} \tag{4.35}$$

Since interest rates of the public sector serves as a lower bound, it holds that $R_t \leq R_t^P$ and therefore $q_t \geq q_t^P$. The probability of default is higher for the representative household.

4.6.1. Model with Private Borrowing

Each period starts with initial assets B_{t-1} and B_{t-1}^P. In the first period, the government decides on default and chooses its consumption tax T_t and its borrowing B_t. Then, the private sector decides also on default and chooses its borrowing B_t^P. These decisions determine the public expenditures g_t and the private consumption c_t. If one agent defaults, both lose access to financial markets and the output decreases. In the second period, they decide only on default if none of them has already defaulted in the first period. Additionally, the government decides also on the tax rate. As in the model without private borrowing, the government takes as given the reaction of the representative household and maximizes the lifetime utility. The private sector responds to the actions of the government and maximizes also its lifetime utility. The model can be solved by backward induction.

Given repayments in the first period and repayment of the government in period 2, the government's budget constraint satisfies

$$g_2 = T_2 c_2 + B_1 \tag{4.36}$$

Then, the household's problem in the last period is given by

$$v_2^R(y_2, B_1^P, B_1, T_2) \equiv max_{D_2^P \in \{0,1\}} U(c_2, g_2) \tag{4.37}$$

subject to the household's budget constraint

$$(1 + T_2)c_2 = \tilde{y}_2(y_2, D_2^P, D_2) + I_{(D_2^P=0)} B_1^P \tag{4.38}$$

$I_{(D_2^P=0)}$ denotes the indicator function of repayment for the private sector in period 2. It is 0 if the household decides to default and 1 if the repayment takes place. That is,

$I_{(D_2^P=0)} = 1$ if $D_2^P = 0$ and $I_{(D_2^P=0)} = 0$ if $D_2^P = 1$. The household maximizes its value function by the decision on default.

Given a default of the public sector in the second period, the government's budget constraint is given by

$$g_2^D = T_2^D c_2^D \tag{4.39}$$

Then, the household's problem is

$$v_2^D(y_2, B_1^P, T_2^D) \equiv max_{D_2^P \in \{0,1\}} U(c_2^D, g_2^D) \tag{4.40}$$

subject to the household's budget constraint

$$(1 + T_2^D)c_2^D = \tilde{y}_2(y_2, D_2^P, D_2) + I_{(D_2^P=0)} B_1^P \tag{4.41}$$

Given the reaction of the representative household, the government's problem is then

$$V_2(y_2, B_1^P, B_1) \equiv max_{D_2 \in \{0,1\}, T_2, T_2^D} \{D_2 v_2^D(y_2, B_1^P, T_2^D) + (1 - D_2) v_2^R(y_2, B_1^P, B_1, T_2)\} \tag{4.42}$$

where D_2 denotes the government's indicator function of default. It is 1 if the public sector defaults and 0 if the repayment takes place. The public sector decides on its consumption tax and on repayment, given the optimal reaction of the households. These decisions determine the government spending in the current period. The government defaults if $v_2^D(y_2, B_1^P, T_2^D) > v_2^R(y_2, B_1^P, B_1, T_2)$, whereas a decision in favour of repayment requires that $v_2^D(y_2, B_1^P, T_2^D) \leq v_2^R(y_2, B_1^P, B_1, T_2)$.

Each period starts with initial assets B_{t-1} and B_{t-1}^P. In the first period, the agents decide on default and on the level of borrowing. Given a decision in favour of repayment in period 1, the government's budget constraint satisfies

$$g_1 = T_1 c_1 - I_{(D_1^P=0)} q_1(y_1, B_1^P, B_1) B_1 + B_0 \tag{4.43}$$

If the private sector defaults, that is, $D_1^P = 1$ and $I_{(D_1^P=0)} = 0$, also the public sector faces financial autarchy and is excluded from capital markets. The government can not trade bonds on international financial markets although it satisfies its debt obligations B_0. As a further consequence, the income decreases.

The household's problem in the first period is given by

$$v_1^R(y_1, B_0^P, B_0, T_1, B_1) \equiv max_{D_1^P \in \{0,1\}, B_1^P} \{U(c_1, g_1) + \beta E V_2(y_2, I_{(D_1^P=0)} B_1^P, I_{(D_1^P=0)} B_1)\} \tag{4.44}$$

subject to the household's budget constraint

$$(1 + T_1)c_1 = \tilde{y}_1(y_1, D_1^P, D_1) - I_{(D_1^P=0)} q_1^P(y_1, B_1^P, B_1) B_1^P + I_{(D_1^P=0)} B_0^P \tag{4.45}$$

and given $q_1^P(y_1, B_1^P, B_1)$ and $q_1(y_1, B_1^P, B_1)$.

If the government defaults in the first period, both sectors are excluded from international borrowing. The government's budget constraint is

$$g_1^D = T_1^D c_1^D \tag{4.46}$$

The problem of the representative household is then

$$v_1^D(y_1, B_0^P, T_1^D) \equiv max_{D_1^P \in \{0,1\}} \{U(c_1^D, g_1^D) + \beta E V_2(y_2, 0, 0)\} \tag{4.47}$$

subject to the household's budget constraint

$$(1 + T_1^D)c_1^D = \tilde{y}_1(y_1, D_1^P, D_1) + I_{(D_1^P=0)} B_0^P \tag{4.48}$$

Given the reaction of the representative household, the government's problem in the first period is given by

$$V_1(y_1, B_0^P, B_0) \equiv max_{D_1 \in \{0,1\}, T_1, T_1^P, B_1} \{D_1 v_1^D(y_1, B_0^P, T_1^D) + (1-D_1) v_1^R(y_1, B_0^P, B_0, T_1, B_1)\} \tag{4.49}$$

where D_1 denotes the government's indicator function of default. The public sector decides on default and its consumption tax. It decides on borrowing only if the private sector does not default. The government does not satisfy its debt obligations if $v_1^D(y_1, B_0^P, T_1^D) > v_1^R(y_1, B_0^P, B_0, T_1, B_1)$.

4.6.2. Quantitative Analysis

The model is also evaluated in a quantitative analysis. Appendix C.3.3 contains details of the used data. Only the parameters of the economic loss differ to the benchmark model. The highest punishment occurs if the government and the representative household default. In this case, the income decreases more than if only one agent does not repay its debt. A decision to default of the government causes a lower income than a repudiation of the private sector.

The simulation of the model implies that private foreign borrowing does not modify the main message of the previous benchmark model. The quantitative predictions contain the correlations between output and the tax rate, public foreign borrowing, private foreign borrowing, private consumption and public consumption in the first period for a high and a low level of initial debt, where $B_0^P = B_0$.

The model predicts the negative correlation between output and the consumption tax. Foreign borrowing is costly during recessions because of high interest rates, the government increases the consumption tax to finance its expenditures. In times of low interest rates, the public sector borrows more abroad and decreases the consumption tax.

If the initial debt is high, the model predicts a strong procyclicality of the tax rate. In this case, the government is often constrained, taxing the households seems to be the better option to finance the public spending.

The model with private borrowing predicts that public borrowing increases in output. Similar to the correlation between output and the consumption tax rate, the relation is much stronger if the initial debt is high. Since the probability of default is countercyclical, interest rates are lower and bond prices are higher during booms. Therefore, borrowing is cheap and additional consumption increases. On the other hand, the lower bond prices during recessions cause a reduction in public borrowing.

In this model, the households have the option of foreign borrowing to increase the private consumption. The model predicts also a positive correlation between output and private borrowing in the first period for a low and a high level of initial debt. The private sector borrows more during booms because of the countercyclical interest rates. In bad periods, it is often constrained. Households can not smoothen their consumption with private borrowing because of the procyclical tax rates. The government finances its expenditures mainly by cheap foreign borrowing during booms and the tax rate decreases. Then, the private sector pays less tax and consumes more. The procyclical private borrowing increases the consumption further. If the income is low, the government increases taxes and the private consumption decreases. The private sector wants to increase its borrowing to reduce the fall in consumption but the concept of sovereign credit ceiling implies that the households pay higher interest rates and are often constrained during recessions.

Private and public consumption are highly correlated with output in the first period. The consumption increases in output.

The model with private foreign borrowing delivers the same results as the benchmark model. Public borrowing is procyclical and the government borrows more during good periods than during bad periods. Since the default probability is countercyclical, foreign borrowing is costly during recessions and the government increases the consumption tax rate to finance its expenditures. The possibility of private borrowing from abroad reduces the costs of an increasing tax rate. Since also the borrowing of the private sector is procyclical, it is not a useful instrument to smooth the consumption.

5. Discussion

The aim of this section is to provide a comparison of assumptions, mechanisms and results of the previously presented models. The similarities and the differences are worked out and the results are listed in a comparative way.

5.1. Assumptions

This section analyses the assumptions of the models and presents the similarities and the differences.

5.1.1. Similarities

All three studies are based on the same main assumptions. The models consist of a small open economy with international capital markets and three infinitely lived agents: a domestic government, a representative domestic household and foreign lenders. The government is assumed to be benevolent, its aim is to maximize the utility of the representative household through actions on the capital market. Furthermore, the government is fully rational and risk averse. Domestic households are also assumed to be risk averse and identical. An important characteristic of the foreign lenders is the risk neutrality. A risk averse lender would only make safe loans and a risk-seeking creditor would prefer loans with a high interest rate (high default risk). Therefore, the zero expected profit condition would not hold without this assumption. Another common assumption is that the government and the creditors take as given the bond price schedule. The agents are price takers, they are not able to affect bond prices and interest rates for a given default risk.

A large number of identical creditors guarantees for a perfectly competitive capital market. The assumptions of rationality and perfect information are common features of the studies. Nevertheless, foreign creditors have no possibility to enforce a repayment obligation. The domestic government lacks commitment whereas foreign agents can commit its payments in the case of borrowing. Thus, the models provide a one-sided commitment problem. Kletzer and Wright (2000) present a model of sovereign default incentives with a two-sided commitment problem, where also the foreign agents can decide to default. They find that the surplus of international borrowing for consumption smoothing purposes is enough to sustain the cooperation between the agents along the equilibrium path and also after a default.

The country faces an endogenous punishment if the government does not repay its debt in full on the due date. An important assumption is that the debt is completely erased after a default. This is not supported by evidence, as Yue (2010) reports in her Paper. Usually, the defaulting country and the creditors renegotiate over a reduction of the debt. Such a renegotiation result on average in a 40 % loss for creditors. Yue (2004, p.30) shows “that the introduction of an endogenous debt recovery schedule[1] leads to a higher default probability and greater interest rate volatility” in models with endogenous default risk.

Ponzi schemes are ruled out in these studies. Bergman (2001, p.27) shows that the no-Ponzi game condition is violated “when government debt is an explosive process.” Such a process leads to a high probability of default (see Appendix B.1.1, the probability of default increases monotonically with debt-service obligations) and therefore to low bond prices and high interest rates. Then, the country defaults when net repayment is required. Since the creditors are assumed to be rational, they make no loans if the government debt is characterized by an explosive process.

5.1.2. Differences

The differences of the assumptions lie in the detail. The most obvious distinctions concern the type of economy and the type of borrowing. The small open economies in the models of EG and Arellano are endowment economies, this means that the income is given exogenously. On the other hand, CSS describe a production economy in which the income is determined endogenously depending on the labour effort of the private households. The used production function has constant returns to scale.

Also the kind of borrowing differs. EG assume that the government can only borrow an amount from private creditors. The reason is that they view public debt to sovereigns and international organizations “as predetermined by political and other considerations not part of the economic decision-making process of a poor country” (Eaton, Gersovitz 1981; p.303). Given access to the capital markets, borrowing is only possible if the income is low relative to trend and if the country is initially not indebted. Except the maturity of one period, they make no further assumptions about the type of borrowing.

However, Arellano and CSS use one-period discount bonds. An important feature of their studies is the incompleteness of asset markets since the only available bonds pay a non-contingent face value. According to CSS, the incompleteness of asset markets characterizes more emerging economies than advanced economies. The reason is that governments in developing countries are not able to issue such a wide range of types of assets like the developed countries. For instance, most low and middle income countries have only developed a short-term market for public debt but not for long-term debt since short-term borrowing is cheaper than long-term borrowing for these economies. (Broner et al. 2013) Arellano and CSS do not distinguish between private and public creditors. De Bolle et al. (2006) report that, on average, the privately held debt contributes for about 50 % of total external public debt prior emerging market crises. Additionally, Arellano and CSS assume that the countries can borrow in each period regardless of the

[1] the debt recovery rate determines the fraction of debt which has to be repaid in the case of default

income and the initial debt, given access to the capital market. In contrast to EG, the government is not able to insure away the idiosyncratic income uncertainty with the set of bonds available.

A further difference concerns savings. EG exclude the possibility of saving in the general and in the stochastic model because of simplicity. In the deterministic model, saving is only allowed in a good period. Arellano and CSS do not use such restrictions. There is a smooth transition from borrowing to saving in their models.

The country loses the ability of saving during financial autarchy. EG do not mention this assumption explicitly but it has to be satisfied in the deterministic model. Otherwise, the specialization on borrowing or saving could fail. For instance, consider a situation in which $G^{\gamma} \leq \beta(R'R)^{\frac{1}{2}}$ holds. The country is either indifferent between borrowing and saving or saving is preferred. In such a situation, the rational government would borrow in the first period and default when repayment is required. Instead of satisfying its debt obligations, it would save the amount. This is known as the Bulow and Rogoff (1989) paradox. According to this paradox, a default happens with probability one in equilibrium if the country can hold foreign assets with the same interest rates after a default.

The classical approach of EG renounce on stochastic shocks in the deterministic and the stochastic model. The income is either deterministic or it alternates indefinitely between a high and a low level. Therefore, the income in the stochastic model of EG is determined by only two possible outputs which occur with the same probability. The income in the general model is a random variable.

On the other hand, Arellano and CSS assume stochastic shocks which are either i.i.d. (Arellano) or persistent (Arellano and CSS). In the case of persistence, the shocks are discretized into a Markov chain using the procedure in Hussey and Tauchen (1991). The state space contains 21 values in the study of Arellano and 25 values in the study of CSS. The number of values differ because the authors approximate the stochastic process of output for different countries. The probability that the process moves from one state to another state is given by the transition function. Arellano uses stochastic income shocks because of the endowment economy, whereas the economy of CSS experiences productivity shocks.

After observing the shocks, the domestic government decides on default in each period and, given repayment, on borrowing. In the model that is characterized by the production economy, the government also decides on its expenditures and the consumption tax rate. Gavin and Perotti (1997) report that indirect taxes are much more important for Latin American countries than for industrialized countries. During the period 1970 to 1995, indirect taxes (including taxes on international trade) were responsible for about 70 % of total tax revenues in Latin America. The largest share of these relates to taxes on goods and services. The share of direct taxes declined during this period, it accounted for around 25 % of total tax revenues.

Domestic households derive direct utility from private consumption. This is a common feature of all three studies. Additionally, CSS assume that the utility also depends on government spending and leisure. In this case, households play a more important role than in the other two models. They decide on consumption and labour effort and therefore, domestic households affect the decisions of the government and the equilibrium borrowing. In the studies of EG and Arellano, the private sector plays only a subordinate role since it does not influence the decisions of the government and the equilibrium. In these studies, the households can only consume their endowment and what they get from the government. There is no need for access to international financial markets. However, CSS provide a variation of their model with additional private foreign borrowing to examine the role of public borrowing.

If the government defaults, the private and public creditors can take some retaliatory actions. In the case of EG, the private foreign creditors will not lend to the defaulting government anymore, the country faces a permanent exclusion of international capital markets. Arellano also assumes this in her simpler version in which she characterizes the properties of the equilibrium. Kletzer and Wright (2000) argue that the maintenance of the threat of permanent exclusion requires that all potential lenders must participate in punishment. This means that some lenders forgo on loans to the defaulting country and therefore on possible gains.

The models with persistent shocks assume that the exclusion of international capital markets is temporary. The country regains access with an exogenous probability. This is a more realistic assumption since Richmond and Dias (2008) report that during the period 1980 to 2005, a country regains full market access[2] after 8.4 years on average after a decision to default (median of 7 years). These authors establish stylized facts regarding the duration of exclusion from international capital markets by an empirical analysis. They find that, on average, the defaulting country regains partial access to international capital markets[3] after 5.7 years. The median is 3 years. Interestingly, the exclusion after a decision to default on foreign currency bonds is much shorter than after a default on foreign currency bank debt. Richmond and Dias also find regional differences in their study. Defaulters from Latin America and the Caribbean face the shortest time of complete market exclusion. These countries regain partial market access after 3.1 years on average. For instance, Argentina was excluded from capital markets for 3.5 years afters its default in 2001. On the other hand, African and Middle Eastern countries are excluded the longest time after a default.

Unfortunately, these findings are not in line with the numerical examples of Arellano and CSS. Both studies use a calibrated exogenous probability of re-entering financial markets that is too low. The reason is that their calibrations are based on findings of Gelos et al. (2002 and 2003, respectively). The results of these early versions are biased downwards since countries which did not regain access to international capital markets after a default during the 1990s were not included. These early versions and the most

[2]according to Richmond and Dias, full market access corresponds to the first year in which the public or the private sector experiences net bond and bank transfers of at least 1 % of GDP

[3]according to Richmond and Dias, partial access corresponds to the first year in which the public or the private sector experiences net bonds and bank transfers after a default

recent version of 2011 of Gelos et al. show significantly different results for the 1990s[4]. However, evidence suggests that an exclusion of capital markets does not last forever.

The papers assume that the output decreases and the small open economy experiences an economic loss after a decision to default. A repudiation diminishes the economic performance through different channels. In addition to the exclusion of international capital markets, empirical observations suggest that a sovereign default on foreign debt also affects negatively the domestic capital market and "diminishes the aggregate credit available in the economy" (Arellano 2008; p.702). Since capital is an important input factor for production, the output decreases. De Paoli et al. (2009) report that sovereign debt crises commonly coincide with other economic crises. If they occur together with currency and/or banking crises, the output losses can be very large. Moreover, Rose (2002) finds that a default is associated with a reduction of bilateral trade by 8 % a year. This decline in trade between the debtor and its creditors persists for approximately 15 years.

In the seminal study, EG assume in their general model that the defaulting country has to pay a penalty after a default. This is in line with the findings of Rose, it could be a cutoff of aid or some restrictions on commodity trade. For reasons of simplification, the deterministic and the stochastic model are characterized by the absence of a penalty. In the numerical examples of Arellano and CSS, the economy experiences asymmetric output costs in the form of a default costs threshold. This approach reflects more the idea of De Paoli et al. since the level of output loss depends on whether the sovereign debt crisis occurs in isolation or whether the economy suffers by a ‚twin' or ‚triple' crisis. By the assumption of an asymmetric loss, the expected discounted utility under autarchy is less sensitive to the shocks which increases the risky borrowing interval.

The benefits of a repudiation grow with the size of debt. The assumption of an endogenous punishment is a crucial characteristic of these models. The endogeneity is based on the fact that the harshness of the punishment is determined by country characteristics such as the variability of income. The higher the future demand for debt, the more the country suffers from an exclusion of capital markets.

An exogenous penalty could be either too severe or too weak and therefore, it could lead to different incentives. For instance, if the (exogenous) punishment is too weak, the country has no incentives to repay its debt. Since creditors are assumed to be rational, no lending occurs under such conditions. On the other hand, no borrowing occurs if the punishment is too severe because the government is assumed to be risk averse.

5.2. Mechanism

There exists neither an international bankruptcy law for countries (Yue 2010) nor an explicit mechanism which deters a government from defaulting on its external debts.

[4]for instance: the median number of years of exclusion from financial markets corresponds to zero in the early version of 2003 and to two in the most recent version of 2011

Therefore, sovereign external debt appears to be a paradox. (Eaton, Gersovitz 1981) It is important to understand the borrower's repayment incentives and the incentives of international lending to sovereigns to analyse sovereign risk. Without a legal framework, repayment incentives should be based on a self-enforcing mechanism. (Martins, Vailakis 2014) In all three studies, the defaulting country loses its reputation and the access to international capital markets. The mechanism of modelling endogenous sovereign default risk is the same for all investigated models.

As economic theory predicts, the target of the foreign lenders is to maximize their utility. A large number of them guarantees for a perfectly competitive international capital market. The zero expected profit condition is the same for EG, Arellano and CSS. The risk neutral lenders have positive incentives to make loans to the sovereigns as long as the expected rate of return is at least as high as the rate of return in the safe case. The only differences arise because of the different assumptions of borrowing. Since EG do not use discount bonds in their seminal study, it can be shown that the repayment obligation in period t is given by

$$R(b_t) = (1 + r_t)b_t = (1 + r_t)q_t B_t = B_t \tag{5.1}$$

The value functions are basically the same in these models since the benevolent government maximizes the expected discounted utility over an infinite horizon. Again, they differ only slightly because of different assumptions. Since EG assume a permanent loss in access to capital markets, their value of default is characterized by the absence of future borrowing. On the other hand, the value of default of Arellano and CSS includes an exogenous probability of regaining access to international capital markets. If this θ is zero, the exclusion is permanent and the value of default collapses to equation 2.5, where the income is given by $y_t^D = y_t - P_t$ and follows a Markov process with a transition function. Then, the value of default is the sum of the current and all future levels of utility under autarchy.

The value of repayment is defined as the sum of the current utility and the expected discounted utility of the future, given a decision not to default. The government will always repay its debt if the value of repayment surpasses the value of default. Then, the sovereign has no incentives to default. However, if the value of default exceeds the value of repayment, the optimal decision is to repudiate the debt obligations. This is the most crucial property of these models since the value functions are determined endogenously. To avoid a repudiation, the repayment incentives, which are given by the expected discounted sum of the current and all future levels of utility given a decision to satisfy the debt obligations, must be higher than the default incentives.

The probability of default is defined endogenously as the probability that the value of default exceeds the value of repayment when loans are due. It depends on debt-service obligations in period t+1 and if the stochastic shock is assumed to be persistent, on the income in period t. This probability of default can also be computed by the concept of default and repayment sets, which is used by Arellano and CSS. Then, λ_{t+1} is the sum of the transition functions over all shocks of the default set. However, the probability of default is based on expectations since it is defined from an ex-ante perspective.

The bond price schedule of Arellano and CSS is driven by these expectations. In the study of EG, the borrowing is traded at its face value which does not depend directly on the probability of default but indirectly because of the credit ceiling which can imply credit rationing. The formula of the gross interest rate of borrowing is the same for all three models. It depends positively on the risk free interest rate and on the probability of default. If the government satisfies its debt obligations in each case, the country faces a risk free and constant interest rate.

$$R_t = (1 + r_t) = \frac{1 + \overline{r}}{1 - \lambda_{t+1}} = \frac{1}{q_t} \tag{5.2}$$

This equation is in line with empirical observations. The terms of borrowing depend on the creditworthiness of the government, which depends, inter alia, on ratings of international credit rating agencies. According to Reinhart (2002), such agencies deliver good results in predicting defaults. As a consequence, the risk premium and the interest rates increase with the probability of default.

5.3. Competitive Borrowing Equilibrium

The recursive competitive borrowing equilibrium is defined as a set of policy functions for the government, foreign lenders and households in all three studies. In these models, the equilibrium need not be unique, several types of equilibria are possible. For instance, the stochastic model of EG is characterized by various types of equilibria, depending on the desired borrowing, the credit ceiling in the risk free case and the credit ceilings in the risky cases.

While several types of equilibria are possible, these models are characterized by the absence of multiple equilibria. A multiple equilibrium is a situation where for a given borrowing two or more values satisfy the zero expected profit condition. (Villemot 2012) De Grauwe (2011) presents a simple theoretical model in which multiple equilibria are possible. The volatility of capital markets and "the self-fulfilling nature of market expectations" (De Grauwe 2011; p.6) can lead to multiple equilibria. Lenders sell government bonds if they expect a decision to default. As a result, the bond prices decrease and the interest rates increase, making a default more likely. On the other hand, if creditors expect a repayment, they do not sell the government bonds and the interest rates remain constant. De Grauwe mentions that a bad equilibrium is more likely for countries "which have no control of the currency in which they issue their debt" (De Grauwe 2011; p.6). This includes members of a monetary union but also emerging countries. In the studies of EG, Arellano and CSS, the domestic government and the foreign lenders are price takers, they take as given the bond price schedule. The borrower chooses a desired amount and the lenders reply to it, given the bond price schedule that is decreasing in desired borrowing. Since borrowing matures in one period and the government immediately decides on default after observing the shock, lenders can not sell bonds prior the maturity. Therefore, several types are possible but not multiple equilibria.

5.4. Results

This section is divided into a theoretical, a numerical and a quantitative part. First, the theoretical results of the studies are presented while the second part discusses the numerical results of the papers. Then, the third part presents the quantitative results. Additionally, it contains some political implications. The theoretical part of CSS is consistent with Arellano's findings and therefore, the next section compares mainly the results of EG and Arellano while the numerical and quantitative part also relies on the results of CSS.

Both the theoretical and the numerical results of these endogenous default risk models rely on a constant relative risk aversion utility function. It is the only utility function with a constant relative risk aversion coefficient, $-\frac{cU''(c)}{U'(c)} = \gamma$. There also exist absolute risk aversion functions and partial relative risk aversion functions. The absolute risk aversion function is used mainly in situations in which the risk remains constant but the wealth varies. On the other hand, the partial relative risk aversion function shows the change of the risk premium when wealth remains fixed but risk varies. However, the relative risk aversion function is used when wealth and risk vary proportionally. Since this is the case in these endogenous default risk models, the authors prefer a relative risk aversion utility function. As Menezes and Hanson (1970, p.485) mention, it "gives information about the behavior of the proportional change in the risk premium when wealth and the risk are changed in the same proportion". The relative risk aversion coefficient of the used utility function is constant. This means that an increase of the borrower's wealth leads to an increase in risky assets such that their fraction remains constant in the portfolio.

In the models with an endowment economy, the direct utility depends only on private consumption. EG and Arellano use the same resource constraint, the government spending corresponds to the lump sum transfers and is defined as the borrowing minus the repayment in the current period. However, private households derive direct utility from consumption, public spending and leisure in the production economy of CSS. Therefore, the authors use a more complex utility function, where government spending and the variables of the households are additively separable. As in the studies of EG and Arellano, the households derive utility from the public expenditures by a relative risk aversion utility function with a constant relative risk aversion coefficient. Since the government collects consumption taxes, the public expenditures depend not only on borrowing and repayment but also on taxes.

5.4.1. Theoretical Results

The studies present different theoretical results. An important result of EG is that the maximum amount which can be borrowed corresponds either to the desired amount or if rationing occurs, to the credit ceiling. EG compute the government's demand for credit, which depends on the standard deviation of income, the discount rate, the interest rate of borrowing and the degree of absolute risk aversion. The credit ceiling also depends on these variables, it relies additionally on the optimality of repayment. The minimum of the desired amount and the credit ceiling serves as an endogenous borrowing limit since it generates the highest possible level of utility.

Arellano uses a similar concept in her simpler model with i.i.d. income shocks. Budget sets are determined by the bond prices, which in turn decrease in borrowing. If the probability of default goes to 1, the bond prices go to zero. Then, budget sets are bounded from above. The upper bound is given by $min_{B_t}[\frac{\{1-F[\overline{y}(B_t)]\}B_t}{1+\overline{r}}]$ and defines the highest possible additional consumption $q(B_t^*)B_t^*$ in period t. Since the utility of the households depends positively on consumption, no higher level of utility can be realized. Therefore, this upper bound also serves as an endogenous borrowing limit.

These endogenous borrowing limits can be either safe or risky. A safe borrowing limit is defined by the optimality of repayment in each case. On the other hand, a risky borrowing limit is characterized by a positive probability of default. As already mentioned earlier[5], risky borrowing regions can disappear. Then, the endogenous borrowing limits are characterized by the absence of default risk.

Nevertheless, there are crucial differences between the concept of EG and the concept of Arellano. In the study of EG, the endogenous borrowing limit is characterized by the minimum of the desired amount, which maximizes the value function, and the highest amount that guarantees the optimality of repayment. On the other hand, Arellano's endogenous borrowing limit maximizes only the additional consumption in the current period but not necessarily the value function. Therefore, this borrowing limit is not equivalent to the desired borrowing. The second important difference is the existence of a credit ceiling in the seminal paper of EG. If the desired borrowing exceeds the credit ceiling, the country is rationed and the endogenous borrowing limit decreases. The more recent studies of Arellano and CSS do not use such an ex-ante borrowing limit. The reason is that the income in the stochastic model of EG is determined by only two possible outputs, which occur with the same probability, whereas the stochastic shocks can have more values in the other two studies. The state space contains 21 values in the study of Arellano and 25 values in the study of CSS in the case of persistent shocks.

However, Arellano defines an ex-post default boundary. If the income in the repayment period falls short of this boundary, the borrower will default with a probability of 1. It does not serve as an endogenous borrowing limit because the default boundary determines the required income, which is given exogenously, to satisfy the debt obligations. This is the crucial difference to the credit ceiling of EG. The default boundary is defined only for risky borrowing with a positive probability of repayment. On the other hand, the

[5] see chapter 2.4.4 and chapter 3.4.1

credit ceilings can exist for riskless and risky borrowing, depending whether the risky credit ceilings exceed or fall short of the riskless credit ceiling.

The existence of a risky credit ceiling or default boundary implies that a default in equilibrium is possible. In the deterministic case, the equilibrium is characterized by the full repayment of the outstanding debt. The introduction of stochastic income leads to an uncertain situation about the borrower's ability of repayment when loans are due. A decision to default happens in equilibrium if the value of default is larger than the value of repayment. The borrowing equilibrium of the stochastic model of EG can be characterized by a repudiation if and only if the risky credit ceilings and the desired amount exceed the risk free credit ceiling, that is, $\bar{b}_{2R} > \bar{b}$ and $b^* > \bar{b}$. Consider the risky situation from chapter 2.4.3, in which repayment can only be optimal if the income is high. This means that the value of default exceeds the value of repayment in a low income period. Since periods of relatively high and relatively low income occur with the same probability, the probability of default is at least $\frac{1}{2}$ in equilibrium. In Arellano's model with i.i.d. income shocks, a default happens in equilibrium if the (exogenous) income falls short of the default boundary. Then, the country experiences an exclusion of international capital markets and the income decreases.

To avoid default in equilibrium, a sustainable debt level has to satisfy three important conditions:

- The value of repayment has to be at least as high as the value function without debt (value of default)
- The consumption can not be negative in the repayment period
- The no-Ponzi game condition has to be satisfied

If only one of these conditions is violated, the debt level is not sustainable. The first condition is the main characteristic of the endogenous default risk models. The stochastic model of the classical framework requires that $\sigma(1+Rb) \leq 1$ holds in the risk free case and that $\sigma(2Rb-1) \leq 1$ holds in the presented risky case to satisfy the positive consumption in the repayment period. In the study of Arellano and CSS, the consumption is positive in t+1 if $y_{t+1} \geq q_{t+1}B_{t+1} - B_t$ is satisfied, the income must be larger than the difference between repayment and borrowing. This, in turn, means that the debt obligations must be smaller than the sum of borrowing and income. It will always be satisfied for Ponzi schemes, which are ruled out by assumption. If only one of these conditions is violated, the debt level is not sustainable.

The risk free credit ceiling satisfies all these conditions in a good and a bad period. It represents the highest level of debt that is sustainable regardless of income in the repayment period. The presented risky credit ceiling does not satisfy the first and/or the second condition in a bad period, which means that the corresponding debt level is not sustainable if the income is low. The demand for credit can be sustainable or unsustainable, depending whether it exceeds or falls short of the risk free credit ceiling.

The same holds for Arellano's endogenous borrowing limit, it can be sustainable or unsustainable. The borrowing limit generates the highest possible additional consumption in the current period, which could lead to a violation of the first and/or second condition if the probability of default is positive. A risky debt level is sustainable if and only if the income surpasses the default boundary and $y_{t+1} \geq q_{t+1}B_{t+1} - B_t$ holds in the repayment period. Then, the value of default falls short of the value of repayment and the consumption is positive. Generally it applies that a level of debt is always sustainable if $B_t \geq \overline{B}_t$ and it can never be sustainable if $B_t \leq \underline{B}_t$.

It is more likely that a level of debt is sustainable when the income is high. The countercyclical default incentives are one of the most, if not even the most important findings of the underlying studies. EG do not explicitly prove this statement since a repayment in bad times reduces the concave utility more than in good times in their models. CSS take as given the countercyclical default incentives because of findings of empirical studies. However, Arellano proves the countercyclical default incentives in her paper, which also holds for the model of CSS.

The countercyclicality is in line with findings of Tomz and Wright (2007) who use a dataset of borrowing to study the sovereign default decisions during the period 1820 to 2004. According to them, there exists "a negative but unexpectedly weak relationship between output and default." (Tomz, Wright 2007; p.358) The empirical work mentions that 36 % of the observed 169 default episodes began in good times, which is not supported by the predictions of the theoretical models. However, nearly two thirds of the default episodes began in low income periods. Furthermore, the authors find that a severe recession does not automatically lead to a default. For instance, the Argentinean government was one of the biggest borrowers a hundred years ago. Despite declines in output of 21 % in 1881 and of 24 % in 1917, it did not default on its liabilities. On the other hand, Argentina defaulted in 1982 when output was only 3 % below trend.

5.4.2. Results of the Numerical Examples

The models of Arellano and CSS are solved numerical. The parameterized examples are calibrated to match the Argentinean (Arellano) and the Mexican (CSS) economy. The used parameter values are similar for the two studies, the productivity shocks follow an AR(1) process $\log(X_t) = \rho \log(X_{t-1}) + \epsilon_t^X$ with $E(\epsilon^X) = 0$ and $E(\epsilon^2) = \eta_X^2$ where X stands for the income shock y (Arellano) or for the productivity shock A (CSS). The observed time periods start in 1980.

	Arellano	CSS
Country	Argentina	Mexico
Time period	1980-2001	1980-2007
Stochastic process	AR(1)	AR(1)
ρ	0.945	0.850
η	0.025	0.006
Markov chain (states)	21	25
Risk aversion coefficient	2	2
Risk free interest rate	0.017	0.010
Discount factor	0.953	0.970
Probability of regaining access to capital markets	0.282	0.100
Default costs threshold	0.969E(y)	0.990E(A)

Table 5.1.: Calibration of the numerical examples, Based on Arellano (2008) and Cuadra et al. (2010)

Table 5.1 provides some information on the calibration of the numerical examples and summarizes the used parameter values. As mentioned earlier, the probability of re-entering financial markets is not in line with the findings of Richmond and Dias (2008).

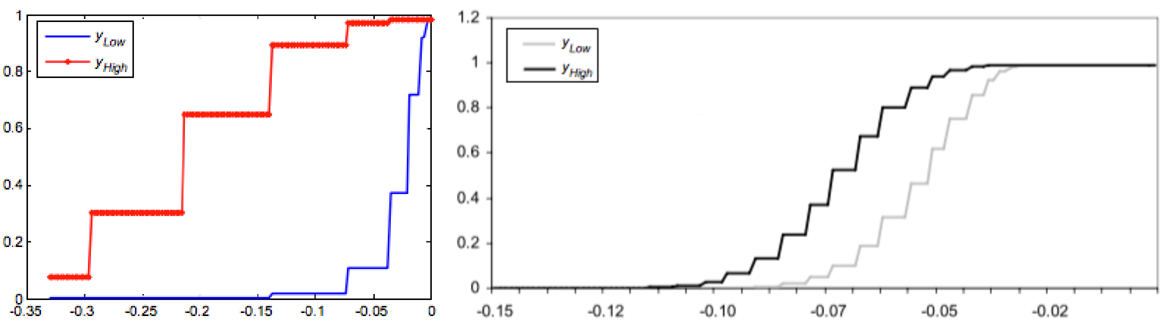

Figure 5.1.: Comparison of the bond price schedules, Sources: Arellano (2008) and Cuadra et al. (2010)

Figure 5.1 plots the bond price schedule for Arellano and CSS. The left panel of the figure shows the calibrated example of Argentina and the right panel the calibrated example of Mexico. The horizontal axes contain the borrowing B_t as a ratio of mean output and the equilibrium prices are presented on the vertical axes. The two figures show two main results of these models: bond prices are decreasing in borrowing and prices are lower during recessions. The comparison of these panels shows that if the economy experiences a high shock, the equilibrium bond prices are higher for the calibrated Argentinean economy for each level of borrowing with a positive probability of default. This has different reasons. On the one hand, Arellano uses a higher risk free interest rate, and on the other hand, the default risks differ. Arellano assumes a default probability of 3 %, which is in line with the number of Argentinean defaults in the past 100 years until 2008. The author tries to get this percentage by calibration. However, the determination of the Mexican default probability is much more complex. Tomz and Wright (2013) report that researchers disagree in the number of Mexican defaults during the 1980s. They count one continuous default while other researchers list four, five or even 23 defaults during this period. CSS use an algorithm to compute the Mexican probability of default.

The government has the possibility to borrow a significant higher amount in Arellano's benchmark model. Equilibrium bond prices for $B_t < -0.3$ are still positive in the endowment economy whereas a contract with $B_t < -0.115$ leads to bond prices of zero in the production economy, given a high shock. In these numerical examples, risk free borrowing is possible if $B_t > -0.035$ and $B_t > -0.039$, respectively. Then, the equilibrium bond prices of Mexico exceed the Argentinean bond prices because of the lower risk free interest rate in the model of CSS. The bond price schedule of CSS's benchmark model is much steeper than Arellano's bond price schedule if the economy experiences a positive shock.

The situation is similar for a low shock. In Arellano's model, borrowing with $B_t < -0.14$ leads to default for sure while $B_t < -0.091$ can never be sustainable in the model of CSS. Also with lower income, the bond price schedule is steeper for the Mexican economy since its upper bound of the bond price schedule is lower. Mexico can already conclude contracts at the risk free interest rate if $B_t > -0.034$ while Argentina has to pay a positive risk premium for all $B_t < -0.002$.

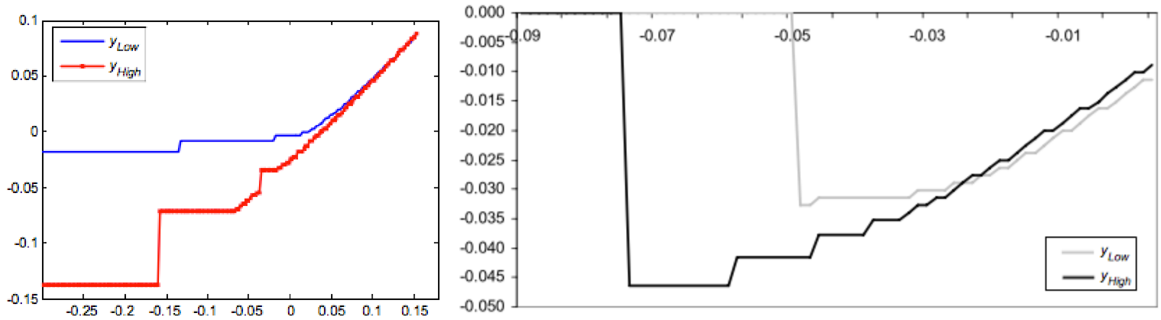

Figure 5.2.: Comparison of the policy functions, Sources: Arellano (2008) and Cuadra et al. (2010)

Figure 5.2 plots the policy functions for Arellano and CSS. The left panel of the figure shows the calibrated example of Argentina and the right panel the calibrated example of Mexico. The horizontal axes contain the initial assets B_{t-1} as a ratio of mean output and the borrowing in period t, B_t, is plotted on the vertical axes. In both cases, the highly indebted country can borrow more during booms because of the countercyclical default risk premium. In the parameterized example of CSS, the government borrows more during recessions than during booms if the initial debt is given by $B_{t-1} > -0.024$. In Arellano's benchmark model, the government is not able to borrow more during recessions than during booms, its borrowing will always be higher in good periods. However, it saves more in recessions than in booms if $B_{t-1} > 0.1$. This can be explained by the bond price schedule. Argentina has to pay a positive risk premium for almost every level of borrowing in bad periods while the Mexican government can borrow a higher amount at the risk free interest rate.

Another finding is that the country from Arellano's approach is able to borrow at a higher level of initial debt while the government of CSS is already constrained at a lower level of debt. Furthermore, the country from the left panel of the figure can borrow a higher maximum amount in good periods than the country from the right panel. The situation is reversed when the economy experiences a bad shock, the highest possible borrowing of the Argentinean economy is lower than the highest possible borrowing of the Mexican economy. This is in line with the bond price schedule and the risk premium.

5.4.3. Quantitative results

According to EG, a credit ceiling can imply credit rationing. Zoli (2004) confirms the existence of credit ceilings for emerging economies by her empirical work. She also finds that the country risk premium on sovereign debt is upward sloping for these countries. The empirical work suggests that the supply of funds curve becomes vertical after a critical level of debt. This defines the maximum amount that can be borrowed, the credit ceiling. EG estimate the probability of a supply constrained regime for poor countries in their econometrical part. They find that 65 of 81 observations are more likely to be supply determined and only 16 observations are more likely to be demand determined. This can be explained by the high probability of default of emerging economies. Peter (2002) estimates these probabilities for 78 emerging markets for the period 1984 to 1997 and finds that they are very high for many poor countries. The main explanatory variables are a bad repayment performance, a high variability of the real GNP per capita growth and high political risk.

However, a government could also benefit from an ex-ante credit ceiling. According to Hatchondo et al. (2012), the introduction of a political motivated credit ceiling can generate expectations of a lower level of debt in the future, given commitment of the government. It is important to mention that not each introduced ex-ante credit ceiling has the same effect. Credit ceilings can be countercyclical, constant or procyclical. A countercyclical debt ceiling is decreasing in income, which means that the government does not necessarily benefit from this fiscal rule. Hence, the government prefers procyclical credit ceilings that are lower during bad times. Then, interest rates decrease because of the creditors' expectations of a lower level of debt in the future and the government has to pay lower interest rates for the same level of borrowing. The risky credit ceiling of the stochastic model from the classical approach is procyclical since it depends positively on the standard deviation. The risk free ceiling also depends positively on the standard deviation, but only if the value of the discount factor is near 1. A discount factor near 0 leads to a countercyclical credit ceiling. The derivation appears in Appendix B.4.1.

Villemot (2012) report that these endogenous default risk studies deliver mixed results in replicating realistic debt levels and default probabilities. Dias et al. (2011, p.2) mention that these models produce "levels of the face value of external sovereign debt that are between five and ten times smaller than the levels reported in traditional sovereign debt statistics." For instance, the numerical example of Arellano suggests a sustainable debt ratio of $B_{t-1} = 0.21$ relative to mean output for a persistent shock of 5 % above trend. During a recession, a debt ratio of $B_{t-1} < 0.02$ already leads to a repudiation. It seems to be too low since the World Bank (2012) reports that the external debt of developing countries was, on average, 21 % of GNI in 2010 and the Argentinean external debt accounted for 36.1 % of its GNI.

The models of Arellano and CSS are more successful in reproducing key business cycle statistics in terms of matching the data. They match well the stylized facts of the Argentinean and the Mexican economy. An important result of the quantitative analysis of CSS is that fiscal policy is procyclical in emerging economies. This finding is confirmed by the use of various econometric methods by Ilzetzki and Vegh (2008). The authors identify "a causal relation from output to government consumption" (Ilzetzki, Vegh 2008; p.26), their analysis shows the procyclicality of fiscal policy in developing countries. Furthermore, Ilzetzki and Vegh also report about an expansionary effect of the government spending on output, which means that fiscal policy reinforces booms and recessions. Therefore, a less procyclical fiscal policy is desirable for consumption smoothing purposes. The target of consumption smoothing could be realized by the introduction of a credit ceiling, as in the model of EG. According to Hatchondo et al. (2012), a constant or procyclical credit ceiling affects positively the creditors' expectations about the future level of debt and therefore also the interest rates. A fiscal rule with lower ceilings makes interest rates less sensitive to shocks when the income is low. This means that the government borrows more abroad and taxes less consumption than it would without such fiscal rule. Thus, fiscal policy becomes less procyclical. As a further consequence, the consumption volatility decreases. This also holds for the analysis with private foreign borrowing.

6. Conclusions

This master thesis shows the evolution of endogenous sovereign debt models. It presents and compares three studies with endogenous default risk of a small open economy. The model of EG was the first that addressed the issue of endogenous sovereign default risk in international capital markets. The seminal study consists of three models with increasing complexity. The general model introduces the basic structure of endogenous default risk models. A deterministic model of borrowing extends this general approach, its equilibrium is characterized by the full repayment of the outstanding debt. The stochastic model of borrowing builds upon the previous findings and models endogenous default risk in a stochastic dynamic framework. EG show that the amount that can be borrowed by a country is either determined by its demand or by its credit ceiling. Credit rationing should prevent default in equilibrium but the introduction of stochastic income can generate situations in which a decision to default is preferable. The stochastic model can have several types of equilibria. This classical study shows that a good reputation can sustain some level of debt. The loss of reputation, which results from a default, leads to a lower output and a permanent exclusion of international capital markets.

The study of Arellano, like many other studies in the (quantitative debt) literature, follows and extends the approach of EG. The analytical characterization of the equilibrium properties is the focus of a simpler version of its model with i.i.d. income shocks, whereas a model with persistent shocks is calibrated for Argentina and solved numerical. The study of CSS can also be assigned to the quantitative debt literature. It is based on the study of EG and extends the approach of Arellano by adding endogenous fiscal policy. It shows that these models are able to match several empirical regularities.

All three investigated studies rely on the same main assumptions. The models consist of a small open economy with an infinite time horizon and the benevolent government is assumed to be fully rational. Its aim is to smooth the households consumption via borrowing on international capital markets. The debt matures in one period and has to be repaid. Otherwise, the country experiences a punishment in the form of an exclusion of international capital markets and a loss in output. A large number of risk neutral foreign lenders guarantees for a perfectly competitive capital market. The incompleteness of the international financial markets is more appropriate for emerging economies. The most obvious distinctions concern the type of economy and the kind of borrowing. EG and Arellano assume an endowment economy, whereas the study of CSS is based on a production economy. In the seminal approach, borrowing is traded at its face value while the quantitative debt studies are characterized by discount bonds.

Since no international bankruptcy law for countries exist and the legal enforcements are rather weak, international lending appears to be a paradox if the default probability is positive. The endogenous default risk models rely on the same basic mechanism to model sovereign default risk. According to economic theory and the classical framework of EG, lenders want to maximize their profits. They only make loans if the expected rate of return is at least as high as in the safe case. Therefore, the zero expected profit condition is one of the most important equations of these models, it represents the lending incentives of the foreign lenders, even in the case of positive default probabilities. However, the most important equation of the endogenous default risk models represents the borrower's repayment incentives, it shows the relation between the value of default and the value of repayment. If the value of default exceeds the value of repayment, the government will default. As a consequence, a level of debt is desirable at which the costs of default just exceed the benefits.

Several types of the recursive competitive borrowing equilibrium are possible in these models. The studies establish different theoretical results, which are supported by other studies. The finding of countercyclical default incentives is a central result of the investigated papers. Endogenous default risk leads to a procyclical fiscal policy in these economies, which is also supported by the data. The quantitative predictions of Arellano and CSS match well the business cycle statistics of Argentina and Mexico, respectively. Allowing for private borrowing does not modify the main message of endogenous default risk models with endogenous fiscal policy.

A. Notation

Notation	Description
a	Defined by $a \equiv 1 - \frac{\beta}{2} - \frac{\beta^2}{2}$
A_t	Production technology in period t
$\tilde{A}$	Degree of absolute risk aversion, defined by $\tilde{A} \equiv -\frac{U''}{U'}$
$\hat{A}_t$	Default costs threshold
b_t	Actual borrowing in period t
b_t^*	Desired amount of borrowing in period t
$\bar{b}_t$	Credit ceiling in period t
$\tilde{b}$	Possible credit ceiling, defined as the lowest value of $b > 0$ such that $V^R(b) = V^R(0)$
$\hat{b}$	Possible credit ceiling, defined as the highest value of $b > 0$ such that $c^h \geq 0$ and $s \geq 0$
B_t	Repayment obligation in period t
B_t^P	Repayment obligation of the private sector in period t
$\overline{B}_t$	Lower bound of assets for which default sets are empty
$\underline{B}_t$	Upper bound of assets for which default sets constitute the entire set
B_t^*	Endogenous borrowing limit
c_t	Consumption in period t
c_t^*	Optimal consumption in period t
c_t^D	Consumption under financial autarchy in period t
c_t^{D*}	Optimal consumption under financial autarchy in period t
c^h	Consumption in a period of high income
c^l	Consumption in a period of low income
d_t	Debt-service obligations in period t
$\overline{d}_t$	Greatest possible debt-service obligations in t
D_t	Indicator function of default of the government in period t
D_t^P	Indicator function of default of the private sector in period t
$D(B_{t-1})$	Default set in period t
e	Defined by $e \equiv 1 - \frac{\beta}{2} - \frac{\beta^2}{4}$
f	Defined by $f \equiv \frac{\beta}{2e} = \frac{\beta}{2-\beta-\frac{\beta^2}{2}}$
$f(A_{t+1}, A_t)$	Transition function of the productivity shock A
$f(y_{t+1}, y_t)$	Transition function of the income shock y
F	Cumulative probability distribution of shocks
g_t	Government spending in period t
g_t^D	Government spending under financial autarchy in period t
g	Growth rate of trend income and consumption
G	Growth factor of trend income and consumption

$h(A_t)$	Production technology under financial autarchy in period t
$h(y_t)$	Output under financial autarchy in period t
l_t	Labour effort of the representative household in period t
l_t^*	Optimal labour effort in period t
l_t^D	Labour effort under financial autarchy in period t
l_t^{D*}	Optimal labour effort under financial autarchy in period t
$1 - l_t$	Leisure in period t
L_t	Set of loan amounts available in period t
$L(y_t, d_t)$	Set of loan amounts available in the competitive borrowing equilibrium
$\overline{L}_t$	Upper bound of the total loanable funds in period t
p_t	Debt-service payments in period t
$\overline{p}_t$	Maximum expected debt-service payments in period t
P_t	Penalty imposed for defaulting in period t
$P(B_{t-1})$	Repayment set in period t
q_t	Price of one discount bond in period t
q_t^P	Price of one discount bond for the private sector in period t
r_t	Country interest rate
$\overline{r}$	Safe (international) interest rate
R	Gross rate of interest on borrowing
R'	Gross rate of interest on saving
R_t^P	Gross rate of interest of the private sector in period t
$R(b_{t_1})$	Repayment function in period t
s	Saving
s^*	Desired amount of saving
T_t	Tax rate on private consumption in period t
T_t^D	Tax rate on private consumption under financial autarchy in period t
U	Utility function
v_t^D	Value function of the private sector, given a decision to default of the gov.
v_t^R	Value function of the private sector, given repayment of the government
V^D	Expected discounted utility given a decision to default in period t
V^R	Expected discounted utility given a decision of repayment in period t
V^*	Expected discounted utility given the desired amount of borrowing
$V(y_t, d_t)$	Expected discounted utility in the competitive borrowing equilibrium
W_t^h	Two-period discounted utility with a perspective from a high period
W_t^l	Two-period discounted utility with a perspective from a low period
y_0	Initial level of trend income
y_t	Output in period t
y_t^{max}	Maximum output in period t
y_t^{min}	Minimum output in period t
y^h	Borrower's income in periods with positive deviation from trend
y^l	Borrower's income in periods with negative deviation from trend
y_t^D	Output under financial autarchy in period t
$\overline{y}(B_{t-1})$	Default boundary in period t

$\hat{y}$	Default costs threshold
$\tilde{y}_t$	Income in the endowment economy with private borrowing
Y	Set of income
$-\underline{Z}$	Lower bound on debt
β	Discount factor
γ	Elasticity of marginal utility / risk aversion coefficient
θ	Probability of regaining access to international capital markets
κ^B	Punishment if the government and the private sector default
κ^G	Punishment if the government defaults
κ^P	Punishment if the private sector defaults
λ_t	Probability of default in period t
λ_t^P	Probability of default of the private sector in period t
π	Weight of government spending in period t
σ	Standard deviation of income
Υ	Set of the production technology
ϕ	Defined by $\phi \equiv 1 - (\beta G^{1-\gamma})^2 > 0$
ψ	Inverse of the labour elasticity

B. Mathematical Appendix

B.1. The Classical Model

B.1.1. The Probability of Default Increases Monotonically with Debt-Service Obligations

The probability of default is increasing if $d_t^1 \leq d_t^2$ implies that $\lambda(d_t^1) \leq \lambda(d_t^2)$ for all d_t^1, d_t^2.

Consider any pair d_t^1 and d_t^2 such that $d_t^1 \leq d_t^2$.
Given a decision not to default, $V^R(y_t, d_t^1)$ depends on $c_t^1 = y_t + b_t^1 - d_t^1$ and $V^R(y_t, d_t^2)$ on $c_t^2 = y_t + b_t^2 - d_t^2$.

$$\begin{aligned} V^R(y_t, d_t^1) &\equiv U(y_t + b_t^1 - d_t^1) + E\beta V[y_{t+1}, R(b_t^1)] \\ &\geq U(y_t + b_t^2 - d_t^1) + E\beta V[y_{t+1}, R(b_t^2)] \\ &\geq U(y_t + b_t^2 - d_t^2) + E\beta V[y_{t+1}, R(b_t^2)] \equiv V^R(y_t, d_t^2) \end{aligned}$$

and therefore $V^R(y_t, d_t^1) \geq V^R(y_t, d_t^2)$. V^R decreases monotonically in d_t.
Note that $V[y_{t+1}, R(b_t^i)] \equiv max\{V^R[y_{t+1}, R(b_t^i)], V^D(y_{t+1})\}$ for $i = 1, 2$.

$V^D(y_t) \equiv E[\sum_{\tau=t}^{\infty} \beta^{\tau-t} U(y_\tau - P_\tau)]$ is independent of d_t. This implies

$$\lambda(d_t^1) = Pr[V^D(y_t) > V^R(y_t, d_t^1)] \leq Pr[V^D(y_t) > V^R(y_t, d_t^2)] = \lambda(d_t^2)$$

Thus, λ increases monotonically with d_t.

B.1.2. Convexity of the Repayment Function

From the zero expected profit condition (2.9) follows

$$R(b_t) = \frac{(1 + \overline{r})b_t}{1 - \lambda[R(b_t)]}$$

Therefore, $R(0) = 0$.

Consider any pair b_t^1, $b_t^2 \in L(y_t, d_t)$ such that $0 \leq b_t^1 \leq b_t^2 \leq \overline{b}_t$.

From the zero expected profit condition (2.9) follows

$$\{1-\lambda[R(b_t^1)]\}R(b_t^1) = (1+\overline{r})b_t^1 \tag{B.1}$$
$$\{1-\lambda[R(b_t^2)]\}R(b_t^2) = (1+\overline{r})b_t^2 \tag{B.2}$$

$b_t^1 \leq b_t^2$ implies

$$\{1-\lambda[R(b_t^1)]\}R(b_t^1) = (1+\overline{r})b_t^1 \leq (1+\overline{r})b_t^2 = \{1-\lambda[R(b_t^2)]\}R(b_t^2)$$

If $R(b_t^2)$ is a repayment function for b_t^1, it provides excess expected profits for the lender so that $R(b_t^1) \leq R(b_t^2)$. Hence, $R(b_t)$ increases with b_t.

From (B.2)

$$\frac{R(b_t^2)}{b_t^2} = \frac{1+\overline{r}}{1-\lambda[R(b_t^2)]} \tag{B.3}$$

Multiply (B.3) by b_t^1 and define

$$x \equiv \frac{R(b_t^2)b_t^1}{b_t^2} = \frac{(1+\overline{r})b_t^1}{1-\lambda[R(b_t^2)]} \tag{B.4}$$

From (B.2), (B.4) and $b_t^1 \leq b_t^2$

$$R(b_t^2) = \frac{(1+\overline{r})b_t^2}{1-\lambda[R(b_t^2)]} \geq \frac{(1+\overline{r})b_t^1}{1-\lambda[R(b_t^2)]} = x \tag{B.5}$$

From Appendix B.1.1, λ increases monotonically with $d_{t+1} = R(b_t)$, and $R(b_t^1) \leq R(b_t^2)$

$$\lambda[R(b_t^1)] \leq \lambda[R(b_t^2)] \tag{B.6}$$

From (B.1), (B.4) and (B.6)

$$R(b_t^1) = \frac{(1+\overline{r})b_t^1}{1-\lambda[R(b_t^1)]} \leq \frac{(1+\overline{r})b_t^1}{1-\lambda[R(b_t^2)]} = x \tag{B.7}$$

Dividing (B.7) by b_t^1 and combining with (B.3)

$$\frac{R(b_t^1)}{b_t^1} \leq \frac{x}{b_t^1} = \frac{1+\overline{r}}{1-\lambda[R(b_t^2)]} = \frac{R(b_t^2)}{b_t^2} \tag{B.8}$$

$R(b_t)$ is convex over $[0, \overline{b}_t]$ because of $R(0) = 0$ and (B.8).

B.1.3. Discounted Utility Given a Decision to Repay Rb

$$V^R(b) = \sum_{t=0}^{\infty} \beta^{2t} max_{s\geq 0}[W^h_{2t}(s,b)] \qquad \text{t} = 1,3,5,...$$

$$\begin{aligned} W^h_{2t}(s,b) &= \frac{1}{1-\gamma}\{[1+\sigma(1-\frac{Rb}{G}-s)]^{1-\gamma}+\beta[1-\sigma(1-b-\frac{R's}{G})]^{1-\gamma}G^{1-\gamma}\}G^{2t(1-\gamma)} \\ &= W^h(s,b)G^{2t(1-\gamma)} \end{aligned}$$

$$\begin{aligned} V^R(b) &= \sum_{t=0}^{\infty} \beta^{2t} max_{s\geq 0}[W^h_{2t}(s,b)] \\ &= \sum_{t=0}^{\infty} \beta^{2t} max_{s\geq 0}[W^h(s,b)G^{2t(1-\gamma)}] \\ &= \sum_{t=0}^{\infty} \beta^{2t} G^{2t(1-\gamma)} max_{s\geq 0}[W^h(s,b)] \\ &= [max_{s\geq 0}W^h(s,b)] \sum_{t=0}^{\infty} \beta^{2t} G^{2t(1-\gamma)} \end{aligned}$$

Using the geometric series

$$\begin{aligned} &= [max_{s\geq 0}W^h(s,b)]\frac{1}{1-(\beta G^{1-\gamma})^2} \\ &= [max_{s\geq 0}W^h(s,b)]\phi^{-1} \end{aligned}$$

where $\phi^{-1} = \frac{1}{1-(\beta G^{1-\gamma})^2}$

B.1.4. $\tilde{b}$ Exists at Most Once

The continuity of $V^R(b)$ implies a local extremum at some point $\tilde{b}^*$, where $0 < \tilde{b}^* < b$. If the condition (2.23) is satisfied at $\tilde{b}^*$, i.e. $G^\gamma > \beta(R'R)^{\frac{1}{2}}$, then $\tilde{b}^* > 0$ and $s^* = 0$ at any extremum.

Optimal savings can be expressed as a function of b, similar to (2.26).

$$
\begin{aligned}
&[1-\sigma(1-b-\frac{R's}{G})]G(\beta R')^{-\frac{1}{\gamma}} = [1+\sigma(1-\frac{Rb}{G}-s)]\\
&\sigma s[1+R'(\beta R')^{-\frac{1}{\gamma}}] = 1+\sigma-\sigma\frac{Rb}{G}-(1-\sigma+\sigma b)G(\beta R')^{-\frac{1}{\gamma}}\\
&\sigma s = \frac{[1-G(\beta R')^{-\frac{1}{\gamma}}]+\sigma[1-\frac{Rb}{G}+G(\beta R')^{-\frac{1}{\gamma}}(1-b)]}{1+R'(\beta R')^{-\frac{1}{\gamma}}}\\
&s^* = max[\frac{[1-G(\beta R')^{-\frac{1}{\gamma}}]+\sigma[1-\frac{Rb}{G}+G(\beta R')^{-\frac{1}{\gamma}}(1-b)]}{\sigma[1+R'(\beta R')^{-\frac{1}{\gamma}}]}, 0]
\end{aligned}
$$

Optimal saving s^* can not be negative, $s^*(\tilde{b}^*) = 0$.

$$
\frac{\partial \sigma s^*}{\partial b} = -\frac{R}{G[1+R'(\beta R')^{-\frac{1}{\gamma}}]} - \frac{G(\beta R')^{-\frac{1}{\gamma}}}{1+R'(\beta R')^{-\frac{1}{\gamma}}} < 0
$$

Thus, $s^*(b) = 0$ for all $b \geq \tilde{b}^*$. Optimal savings decrease monotonically in b.

The corresponding value function is given by $V^R(b) = W^h(0,b)\phi^{-1}$ for all $b \geq \tilde{b}^*$.

$$
\begin{aligned}
&W^h(0,b)\phi^{-1} = \frac{1}{1-\gamma}\{[1+\sigma(1-\frac{Rb}{G})]^{1-\gamma}+\beta[1-\sigma(1-b)]^{1-\gamma}G^{1-\gamma}\}\phi^{-1}\\
&\frac{dV^R(b)}{db} = \{-[1+\sigma(1-\frac{Rb}{G})]^{-\gamma}\frac{R}{G}+\beta[1-\sigma(1-b)]^{-\gamma}G^{1-\gamma}\}\sigma\phi^{-1}\\
&\frac{d^2V^R(b)}{db^2} = -\gamma\sigma^2\{[1+\sigma(1-\frac{Rb}{G})]^{-\gamma-1}\frac{R^2}{G^2}+\beta[1-\sigma(1-b)]^{-\gamma-1}G^{1-\gamma}\}\phi^{-1} < 0
\end{aligned}
$$

Thus, $V^R(b)$ is a strictly concave function of b, it can attain at most one maximum and $V^R(b) = V^R(0)$ for at most one $\tilde{b} > 0$. If $b > \tilde{b}$, then $V^R(b) < V^R(0)$ and condition (2.30) is not satisfied.

B.1.5. Derivation of the Demand for Credit

The two-period discounted utility is given by (2.37)

$$
W_t^l = U[1-\sigma(1-b)] + \frac{1}{2}\beta\{U[1+\sigma(1-Rb)] + U[1-\sigma(1+Rb)]\} \tag{B.9}
$$

Second-order Taylor series approximation about $c = 1$ (M=Multiplier)

$$U[1-\sigma(1-b)] \approx U(1) - U'(1)\sigma(1-b) + \frac{1}{2}U''(1)\sigma^2(1-b)^2 \qquad M = 1$$
$$U[1+\sigma(1-Rb)] \approx U(1) + U'(1)\sigma(1-Rb) + \frac{1}{2}U''(1)\sigma^2(1-Rb)^2 \qquad M = \frac{\beta}{2}$$
$$U[1-\sigma(1+Rb)] \approx U(1) - U'(1)\sigma(1+Rb) + \frac{1}{2}U''(1)\sigma^2(1+Rb)^2 \qquad M = \frac{\beta}{2}$$

Combining the approximations

$$\begin{aligned}(B.9) \approx U(1)[1+\frac{\beta}{2}+\frac{\beta}{2}] + U'(1)[-\sigma(1-b) + \frac{\beta}{2}\sigma(1-Rb) - \frac{\beta}{2}\sigma(1+Rb)] \\ + \frac{1}{2}U''(1)[\sigma^2(1-b)^2 + \frac{\beta}{2}\sigma^2(1-Rb)^2 + \frac{\beta}{2}\sigma^2(1+Rb)^2]\end{aligned} \tag{B.10}$$

For a maximum, the first-order condition with respect to b is given by

$$\begin{aligned}\frac{\partial(B.10)}{\partial b} =& U'(1)\sigma[1-\frac{\beta}{2}R-\frac{\beta}{2}R] + \frac{1}{2}U''(1)\sigma^2[-2(1-b) - \beta R(1-Rb) + \beta R(1+Rb)] = 0 \\ & [1-\beta R] + \frac{1}{2}\frac{U''(1)}{U'(1)}\sigma[-2+2b+2\beta R^2 b] = 0 \\ & 1-\beta R = \tilde{A}[-\sigma + \sigma b(1+\beta R^2)] \\ & \sigma b^* = \frac{\frac{1-\beta R}{\tilde{A}} + \sigma}{1+\beta R^2}\end{aligned}$$

where $\tilde{A} \equiv -\frac{U''}{U'}$.

B.1.6. Derivation of the Value Function Given Repayment in the Safe Case

Given access to international financial markets and $b_{t-1} = 0$, it is assumed that the government borrows in a bad period and it will always repay its debt in t+1. The probability of a good period is $\frac{1}{2}$. In this case, the country can not borrow. The next period is then characterized by the same situation since $b_t = 0$.

$$V^R = \frac{1}{2}\{U[1+\sigma] + \beta V^R\} + \frac{1}{2}\{U[1-\sigma(1-b)] + \beta V^B\} \tag{B.11}$$
$$V^B = \frac{1}{2}\{U[1+\sigma(1-Rb)] + \beta V^R\} + \frac{1}{2}\{U[1-\sigma(1+Rb)] + \beta V^R\} \tag{B.12}$$

Combining (B.11) and (B.12)

$$\begin{aligned}
V^R &= \frac{1}{2}U[1+\sigma] + \frac{1}{2}\beta V^R + \frac{1}{2}U[1-\sigma(1-b)] + \frac{1}{4}\beta U[1+\sigma(1-Rb)] \\
&\quad + \frac{1}{4}\beta U[1-\sigma(1+Rb)] + \frac{1}{2}\beta^2 V^R \\
V^R(1-\frac{\beta}{2}-\frac{\beta^2}{2}) &= \frac{1}{2}\{U[1-\sigma(1-b)] + U[1+\sigma]\} \\
&\quad + \frac{1}{4}\beta\{U[1-\sigma(1+Rb)] + U[1+\sigma(1-Rb)]\} \\
V^R &= \frac{1}{2a}\{U[1-\sigma(1-b)] + U[1+\sigma]\} + \frac{1}{4a}\beta\{U[1-\sigma(1+Rb)] + U[1+\sigma(1-Rb)]\}
\end{aligned}$$

where $a \equiv 1 - \frac{\beta}{2} - \frac{\beta^2}{2}$.

B.1.7. Derivation of the Value Function Given a Decision to Default

If the government has defaulted, it faces a permanent exclusion from the capital market. The income in each period is either high or low with equal probability.

$$\begin{aligned}
V^D &= \frac{1}{2}[U(1+\sigma) + \beta V^D] + \frac{1}{2}[U(1-\sigma) + \beta V^D] \\
V^D &= \frac{1}{2}[U(1+\sigma) + U(1-\sigma)] + \beta V^D \\
V^D &= \frac{1}{2}[U(1+\sigma) + U(1-\sigma)]\sum_{t=0}^{\infty}\beta^t
\end{aligned}$$

Using the geometric series

$$\begin{aligned}
V^D &= \frac{1}{2}[U(1+\sigma) + U(1-\sigma)]\frac{1}{1-\beta} \\
V^D &= \frac{U[1+\sigma] + U[1-\sigma]}{2(1-\beta)}
\end{aligned}$$

B.1.8. Derivation of the Credit Ceiling in the Safe Case

The optimality of payment in bad periods requires the satisfaction of condition (2.41)

$$U[1-\sigma(1+Rb)] + \beta V^R(b) - U[1-\sigma] - \beta V^D \geq 0 \tag{B.13}$$

where

$$V^R = \frac{1}{2a}\{U[1-\sigma(1-b)] + U[1+\sigma]\} + \frac{1}{4a}\beta\{U[1-\sigma(1+Rb)] + U[1+\sigma(1-Rb)]\}$$
$$V^D = \frac{U[1+\sigma] + U[1-\sigma]}{2(1-\beta)}$$

Second-order Taylor series approximation about $c = 1$ (M=Multiplier)

$$U[1-\sigma(1+Rb)] \approx U(1) - U'(1)\sigma(1+Rb) + \frac{1}{2}U''(1)\sigma^2(1+Rb)^2 \qquad M = 1$$
$$U[1-\sigma(1-b)] \approx U(1) - U'(1)\sigma(1-b) + \frac{1}{2}U''(1)\sigma^2(1-b)^2 \qquad M = \frac{\beta}{2a}$$
$$U[1+\sigma] \approx U(1) + U'(1)\sigma + \frac{1}{2}U''(1)\sigma^2 \qquad M = \frac{\beta}{2a}$$
$$U[1-\sigma(1+Rb)] \approx U(1) - U'(1)\sigma(1+Rb) + \frac{1}{2}U''(1)\sigma^2(1+Rb)^2 \qquad M = \frac{\beta^2}{4a}$$
$$U[1+\sigma(1-Rb)] \approx U(1) + U'(1)\sigma(1-Rb) + \frac{1}{2}U''(1)\sigma^2(1-Rb)^2 \qquad M = \frac{\beta^2}{4a}$$
$$U[1-\sigma] \approx U(1) - U'(1)\sigma + \frac{1}{2}U''(1)\sigma^2 \qquad M = -1$$
$$U[1+\sigma] \approx U(1) + U'(1)\sigma + \frac{1}{2}U''(1)\sigma^2 \qquad M = -\frac{\beta}{2(1-\beta)}$$
$$U[1-\sigma] \approx U(1) - U'(1)\sigma + \frac{1}{2}U''(1)\sigma^2 \qquad M = -\frac{\beta}{2(1-\beta)}$$

Combining the approximations

$$\begin{aligned}(B.13) \approx\ & U(1)\{1 + \frac{\beta}{a} + \frac{\beta^2}{2a} - 1 - \frac{\beta}{1-\beta}\} + U'(1)\sigma\{-1 - Rb + \frac{\beta}{2a}[-1+b+1] \\ & + \frac{\beta^2}{4a}[-1-Rb+1-Rb] + 1 - \frac{\beta}{2(1-\beta)}[-1+1]\} + \frac{1}{2}U''(1)\sigma^2\{(1+Rb)^2 \\ & + \frac{\beta}{2a}[(1-b)^2+1] + \frac{\beta^2}{4a}[(1+Rb)^2 + (1-Rb)^2] - 1 - \frac{\beta}{2(1-\beta)}[1+1]\} = 0\end{aligned} \tag{B.14}$$

Simplification of (B.14)

$$U(1)\{1 + \frac{\beta}{a} + \frac{\beta^2}{2a} - 1 - \frac{\beta}{1-\beta}\} = U(1)\{\frac{\beta}{2a}(2+\beta) - \frac{\beta}{1-\beta}\}$$

It holds that

$$\frac{\beta}{2a}(2+\beta) - \frac{\beta}{1-\beta} = 0$$
$$(2+\beta)(1-\beta) = 2a$$
$$1 - \frac{\beta}{2} - \frac{\beta^2}{2} = a$$

and therefore

$$U(1)\{0\} \tag{B.15}$$

$$\begin{aligned} &U'(1)\sigma\{-1 - Rb + \frac{\beta}{2a}[-1+b+1] + \frac{\beta^2}{4a}[-1-Rb+1-Rb] + 1 - \frac{\beta}{2(1-\beta)}[-1+1]\} \\ &\qquad = U'(1)\sigma b\{-R + \frac{\beta}{2a}(1-\beta R)\} \end{aligned} \tag{B.16}$$

$$\begin{aligned} &\frac{1}{2}U''(1)\sigma^2\{(1+Rb)^2 + \frac{\beta}{2a}[(1-b)^2+1] + \frac{\beta^2}{4a}[(1+Rb)^2+(1-Rb)^2] - 1 - \frac{\beta}{2(1-\beta)}2\} \\ &\quad = \frac{1}{2}U''(1)\sigma^2\{1 + 2Rb + R^2b^2 + \frac{\beta}{2a}[2-2b+b^2] + \frac{\beta^2}{4a}[2+2R^2b^2] - 1 - \frac{\beta}{1-\beta}\} \\ &\quad = \frac{1}{2}U''(1)\sigma^2\{2Rb + R^2b^2 + \frac{\beta}{2a}[-2b+b^2] + \frac{\beta^2}{2a}R^2b^2\} + \frac{1}{2}U''(1)\sigma^2\{\frac{\beta}{2a}(2+\beta) - \frac{\beta}{1-\beta}\} \\ &\quad = \frac{1}{2}U''(1)\sigma^2 b\{2R + R^2b + \frac{\beta}{2a}[b-2] + \frac{\beta^2}{2a}R^2b\} + \frac{1}{2}U''(1)\sigma^2\{0\} \end{aligned} \tag{B.17}$$

Combining (B.15), (B.16) and (B.17)

$$U'(1)\sigma b\{-R + \frac{\beta}{2a}(1-\beta R)\} + \frac{1}{2}U''(1)\sigma^2 b\{2R + R^2b + \frac{\beta}{2a}(b-2) + \frac{\beta^2}{2a}R^2b\} = 0$$
$$2U'(1)\{-R + \frac{\beta}{2a}(1-\beta R)\} = -U''(1)\sigma\{2R + R^2b + \frac{\beta}{2a}(b-2) + \frac{\beta^2}{2a}R^2b\}$$
$$2U'(1)\{-R + \frac{\beta}{2a}(1-\beta R)\} = -U''(1)\sigma\{2R - \frac{\beta}{2a}2\} - U''(1)\sigma\{R^2b(1+\frac{\beta^2}{2a}) + \frac{\beta}{2a}b\}$$
$$-2R + \frac{\beta}{2a}(2-2\beta R) = \tilde{A}\sigma\{2R - \frac{\beta}{2a}2\} + \tilde{A}\sigma b\{R^2 + \frac{\beta}{2a}(1+\beta R^2)\}$$
$$\frac{-4Ra + 2\beta(1-\beta R)}{\tilde{A}} - \sigma\{4Ra - 2\beta\} = \sigma b\{2R^2a + \beta(1+\beta R^2)\}$$
$$\sigma\bar{b} = \frac{\frac{2\beta(1-\beta R)-4Ra}{\tilde{A}} + \sigma\{2\beta - 4Ra\}}{\beta(1+\beta R^2) + 2R^2a}$$

It holds that

$$\begin{aligned}2\beta(1-\beta R)-4Ra &= 2\beta(1-\beta R)-4R(1-\frac{\beta}{2}-\frac{\beta^2}{2})\\ &= 2\beta-2\beta^2R-4R+2\beta R+2\beta^2R\\ &= 2\beta(1+R)-4R\end{aligned}$$

and therefore

$$\sigma\bar{b}=\frac{\frac{2\beta(1+R)-4R}{\tilde{A}}+\sigma(2\beta-4Ra)}{\beta(1+\beta R^2)+2R^2a}$$

where $\tilde{A}\equiv-\frac{U''}{U'}$ and $a\equiv 1-\frac{\beta}{2}-\frac{\beta^2}{2}$.

B.1.9. Derivation of the Demand for Credit in the Risky Case

The two-period discounted utility is given by (2.43)

$$W_t^l=U[1-\sigma(1-b)]+\frac{1}{2}\beta U[1+\sigma(1-2Rb)] \tag{B.18}$$

Second-order Taylor series approximation about $c=1$ (M=Multiplier)

$$\begin{aligned}U[1-\sigma(1-b)] &\approx U(1)-U'(1)\sigma(1-b)+\frac{1}{2}U''(1)\sigma^2(1-b)^2 && M=1\\ U[1+\sigma(1-2Rb)] &\approx U(1)+U'(1)\sigma(1-2Rb)+\frac{1}{2}U''(1)\sigma^2(1-2Rb)^2 && M=\frac{\beta}{2}\end{aligned}$$

Combining the approximations

$$\begin{aligned}(B.18)\approx U(1)[1+\frac{\beta}{2}]+U'(1)[-\sigma(1-b)+\frac{\beta}{2}\sigma(1-2Rb)]+\frac{1}{2}U''(1)[\sigma^2(1-b)^2\\ +\frac{\beta}{2}\sigma^2(1-2Rb)^2]\end{aligned} \tag{B.19}$$

For a maximum, the first-order condition with respect to b is given by

$$\begin{aligned}\frac{\partial(B.19)}{\partial b} =&U'(1)\sigma[1-\beta R]+\frac{1}{2}U''(1)\sigma^2[-2(1-b)-2\beta R(1-2Rb)]=0\\ &[1-\beta R]+\frac{U''(1)}{U'(1)}\sigma[-1+b-\beta R(1-2Rb)]=0\\ &1-\beta R=\tilde{A}[-\sigma(1+\beta R)+\sigma b(1+2\beta R^2)]\\ &\sigma b^*=\frac{\frac{1-\beta R}{\tilde{A}}+\sigma(1+\beta R)}{1+2\beta R^2}\end{aligned}$$

where $\tilde{A}\equiv-\frac{U''}{U'}$.

B.1.10. Derivation of the Value Function Given Repayment in the Risky Case

Given access to international financial markets and $b_{t-1} = 0$, it is assumed that the government borrows in a bad period. It will repay its debt if the income is high in t+1, otherwise the country will default. The probability of $y_t = y_t^h$ is $\frac{1}{2}$. In this case, the country can not borrow. The next period is then characterized by the same situation since $b_t = 0$.

$$V^R = \frac{1}{2}\{U[1+\sigma] + \beta V^R\} + \frac{1}{2}\{U[1-\sigma(1-b)] + \beta V^B\} \tag{B.20}$$
$$V^B = \frac{1}{2}\{U[1+\sigma(1-2Rb)] + \beta V^R\} + \frac{1}{2}\{U[1-\sigma] + \beta V^D\} \tag{B.21}$$

Combining (B.20) and (B.21)

$$\begin{aligned}
V^R &= \frac{1}{2}U[1+\sigma] + \frac{1}{2}\beta V^R + \frac{1}{2}U[1-\sigma(1-b)] + \frac{1}{4}\beta U[1+\sigma(1-2Rb)] + \frac{1}{4}\beta^2 V^R \\
&\quad + \frac{1}{4}\beta U[1-\sigma] + \frac{1}{4}\beta^2 V^D \\
V^R(1 - \frac{\beta}{2} - \frac{\beta^2}{4}) &= \frac{1}{2}\{U[1-\sigma(1-b)] + U[1+\sigma]\} \\
&\quad + \frac{1}{4}\beta\{U[1+\sigma(1-2Rb)] + U[1-\sigma] + \beta V^D\} \\
V^R &= \frac{1}{2e}\{U[1-\sigma(1-b)] + U[1+\sigma]\} + \frac{1}{4e}\beta\{U[1+\sigma(1-2Rb)] + U[1-\sigma] + \beta V^D\}
\end{aligned}$$

where $e \equiv 1 - \frac{\beta}{2} - \frac{\beta^2}{4}$.

B.1.11. Derivation of the Credit Ceiling in the Risky Case

The optimality of payment in bad periods requires the satisfaction of condition (2.46)

$$U[1+\sigma(1-2Rb)] + \beta V^R(b) - U[1+\sigma] - \beta V^D \geq 0 \tag{B.22}$$

where

$$\begin{aligned}
V^R &= \frac{1}{2e}\{U[1-\sigma(1-b)] + U[1+\sigma]\} + \frac{1}{4e}\beta\{U[1+\sigma(1-2Rb)] + U[1-\sigma] + \beta V^D\} \\
V^D &= \frac{U[1+\sigma] + U[1-\sigma]}{2(1-\beta)}
\end{aligned}$$

Second-order Taylor series approximation about $c = 1$ (M=Multiplier)

$$U[1+\sigma(1-2Rb)] \approx U(1) + U'(1)\sigma(1-2Rb) + \frac{1}{2}U''(1)\sigma^2(1-2Rb)^2 \quad M = 1$$

$$U[1-\sigma(1-b)] \approx U(1) - U'(1)\sigma(1-b) + \frac{1}{2}U''(1)\sigma^2(1-b)^2 \quad M = \frac{\beta}{2e}$$

$$U[1+\sigma] \approx U(1) + U'(1)\sigma + \frac{1}{2}U''(1)\sigma^2 \quad M = \frac{\beta}{2e}$$

$$U[1+\sigma(1-2Rb)] \approx U(1) + U'(1)\sigma(1-2Rb) + \frac{1}{2}U''(1)\sigma^2(1-2Rb)^2 \quad M = \frac{\beta^2}{4e}$$

$$U[1-\sigma] \approx U(1) - U'(1)\sigma + \frac{1}{2}U''(1)\sigma^2 \quad M = \frac{\beta^2}{4e}$$

$$U[1+\sigma] \approx U(1) + U'(1)\sigma + \frac{1}{2}U''(1)\sigma^2 \quad M = \frac{\beta^3}{8e(1-\beta)}$$

$$U[1-\sigma] \approx U(1) - U'(1)\sigma + \frac{1}{2}U''(1)\sigma^2 \quad M = \frac{\beta^3}{8e(1-\beta)}$$

$$U[1+\sigma] \approx U(1) + U'(1)\sigma + \frac{1}{2}U''(1)\sigma^2 \quad M = -1$$

$$U[1+\sigma] \approx U(1) + U'(1)\sigma + \frac{1}{2}U''(1)\sigma^2 \quad M = -\frac{\beta}{2(1-\beta)}$$

$$U[1-\sigma] \approx U(1) - U'(1)\sigma + \frac{1}{2}U''(1)\sigma^2 \quad M = -\frac{\beta}{2(1-\beta)}$$

Combining the approximations

$$\begin{aligned}(B.22) \approx\ & U(1)\{1 + \frac{\beta}{e} + \frac{\beta^2}{2e} + \frac{\beta^3}{4e(1-\beta)} - 1 - \frac{\beta}{1-\beta}\} + U'(1)\sigma\{1 - 2Rb + \frac{\beta}{2e}[-1+b+1] \\ & + \frac{\beta^2}{4e}[1-2Rb-1] + \frac{\beta^3}{8e(1-\beta)}[1-1] - 1 - \frac{\beta}{2(1-\beta)}[1-1]\} + \frac{1}{2}U''(1)\sigma^2\{(1-2Rb)^2 \\ & + \frac{\beta}{2e}[(1-b)^2+1] + \frac{\beta^2}{4e}[(1-2Rb)^2+1] + \frac{\beta^3}{8e(1-\beta)}[1+1] - 1 - \frac{\beta}{2(1-\beta)}[1+1]\} \\ & = 0 \end{aligned} \tag{B.23}$$

Simplification of (B.23)

$$U(1)\{1 + \frac{\beta}{e} + \frac{\beta^2}{2e} + \frac{\beta^3}{4e(1-\beta)} - 1 - \frac{\beta}{1-\beta}\} = U(1)\{\frac{\beta}{2e}(2+\beta) + \frac{\beta^3}{4e(1-\beta)} - \frac{\beta}{1-\beta}\}$$

It holds that

$$\frac{\beta}{2e}(2+\beta) + \frac{\beta^3}{4e(1-\beta)} - \frac{\beta}{1-\beta} = 0$$
$$(2+\beta)(1-\beta) + \frac{\beta^2}{2} = 2e$$
$$2+\beta-2\beta-\beta^2+\frac{\beta^2}{2} = 2e$$
$$1-\frac{\beta}{2}-\frac{\beta^2}{4} = e$$

and therefore

$$U(1)\{0\} \tag{B.24}$$

$$\begin{aligned} &U'(1)\sigma\{-2Rb + \frac{\beta}{2e}[-1+b+1] + \frac{\beta^2}{4e}[1-2Rb-1] + \frac{\beta^3}{8e(1-\beta)}[1-1] - \frac{\beta}{2(1-\beta)}[1-1]\} \\ &\qquad = U'(1)\sigma b\{-2R + \frac{\beta}{2e}(1-\beta R)\} \end{aligned} \tag{B.25}$$

$$\begin{aligned} &\frac{1}{2}U''(1)\sigma^2\{(1-2Rb)^2 + \frac{\beta}{2e}[(1-b)^2+1] + \frac{\beta^2}{4e}[(1-2Rb)^2+1] + \frac{\beta^3}{8e(1-\beta)}[1+1] - 1 \\ &\quad - \frac{\beta}{2(1-\beta)}2\} \\ &\quad = \frac{1}{2}U''(1)\sigma^2\{1-4Rb+4R^2b^2 + \frac{\beta}{2e}[2-2b+b^2] + \frac{\beta^2}{4e}[2-4Rb+4R^2b^2] + \frac{\beta^3}{4e(1-\beta)} \\ &\quad - 1 - \frac{\beta}{1-\beta}\} \\ &\quad = \frac{1}{2}U''(1)\sigma^2\{-4Rb+4R^2b^2 + \frac{\beta}{2e}[-2b+b^2] + \frac{\beta^2}{2e}(-2Rb+2R^2b^2)\} \\ &\quad + \frac{1}{2}U''(1)\sigma^2\{\frac{\beta}{2e}(2+\beta) + \frac{\beta^3}{4e(1-\beta)} - \frac{\beta}{1-\beta}\} \\ &\quad = \frac{1}{2}U''(1)\sigma^2 b\{-4R+4R^2b + \frac{\beta}{2e}[b-2] + \frac{\beta^2}{2e}(2R^2b-2R)\} + \frac{1}{2}U''(1)\sigma^2[0] \end{aligned} \tag{B.26}$$

Combining (B.24), (B.25) and (B.26)

$$U'(1)\sigma b\{-2R+\frac{\beta}{2e}(1-\beta R)\}+\frac{1}{2}U''(1)\sigma^2 b\{-4R+4R^2b+\frac{\beta}{2e}(b-2)+\frac{\beta^2}{2e}(2R^2b-2R)\}=0$$
$$2U'(1)\{-2R+\frac{\beta}{2e}(1-\beta R)\}=-U''(1)\sigma\{-4R+4R^2b+\frac{\beta}{2e}(b-2)+\frac{\beta^2}{2e}(2R^2b-2R)\}$$
$$2U'(1)\{-2R+\frac{\beta}{2e}(1-\beta R)\}=-U''(1)\sigma\{-4R-\frac{\beta}{2e}(2+2\beta R)\}-U''(1)\sigma\{2R^2b(2+\frac{\beta^2}{2e})+\frac{\beta}{2e}b\}$$
$$-4R+2f(1-\beta R)=\tilde{A}\sigma\{-4R-2f(1+\beta R)\}+\tilde{A}\sigma b\{2R^2(2+f\beta)+f\}$$
$$\frac{-4R+2f(1-\beta R)}{\tilde{A}}+\sigma\{4R+2f(1+\beta R)\}=\sigma b\{4R^2+2R^2f\beta+f\}$$
$$\sigma\bar{b}_{2R}=\frac{\frac{2f(1-\beta R)-4R}{\tilde{A}}+\sigma[2f(1+\beta R)+4R]}{f(1+2\beta R^2)+4R^2}$$

where $\tilde{A}\equiv-\frac{U''}{U'}$, $e\equiv 1-\frac{\beta}{2}-\frac{\beta^2}{4}$ and $f\equiv\frac{\beta}{2e}$.

B.2. Arellano's Model

B.2.1. Default Sets Are Decreasing in Assets

This proof is similar to Appendix B.1.1.
Consider any pair B^1_{t-1}, B^2_{t-1} such that

$$B^1_{t-1}\leq B^2_{t-1} \tag{B.27}$$

Given y_t, if default is optimal for B^2_{t-1}, then default is also optimal for B^1_{t-1} because B^1_{t-1} represents a higher level of debt and therefore $D(B^2_{t-1})\subseteq D(B^1_{t-1})$.

Suppose that $D(B^2_{t-1})\neq\emptyset$. It holds for all $y_t\in D(B^2_{t-1})$

$$U(y_t)+E\beta[\theta V^O(y_{t+1},0)+(1-\theta)V^D(y_{t+1})]>U[y_t-q(y_t,B_t)B_t+B^2_{t-1}]+E\beta V^O(y_{t+1},B_t) \tag{B.28}$$

where $V^O(y_{t+1},B_t)\equiv max[V^R(y_{t+1},B_t),V^D(y_{t+1})]$.
This condition holds for all B_t. (B.28) means that the value of the contract is higher under default.

Given y_t and B_t, consumption in period t is given by

$$y_t-q(y_t,B_t)B_t+B^1_{t-1}\qquad\text{for } B^1_{t-1} \tag{B.29}$$
$$y_t-q(y_t,B_t)B_t+B^2_{t-1}\qquad\text{for } B^2_{t-1} \tag{B.30}$$

From (B.27), (B.29) and (B.30)

$$y_t - q(y_t, B_t)B_t + B^1_{t-1} \leq y_t - q(y_t, B_t)B_t + B^2_{t-1} \tag{B.31}$$

(B.31) implies that

$$U[y_t - q(y_t, B_t)B_t + B^1_{t-1}] + E\beta V^O(y_{t+1}, B_t) \leq U[y_t - q(y_t, B_t)B_t + B^2_{t-1}] + E\beta V^O(y_{t+1}, B_t) \tag{B.32}$$

Given repayment, the value of the contract is increasing in initial assets.

Combining (B.28) and (B.32)

$$U(y_t) + E\beta[\theta V^O(y_{t+1}, 0) + (1-\theta)V^D(y_{t+1})] > U[y_t - q(y_t, B_t)B_t + B^1_{t-1}] + E\beta V^O(y_{t+1}, B_t) \tag{B.33}$$

Hence, $y_t \in D(B^1_{t-1})$ and $D(B^2_{t-1}) \subseteq D(B^1_{t-1})$. Default sets are decreasing in assets.

B.2.2. Capital Inflows Are Possible Only if the Default Set of a Given Initial Asset Is Empty

The exclusion of financial markets after defaulting is permanent, $\theta = 0$.
The country is indebted if $B_{t-1} < 0$.

Proof by contradiction:
Suppose that the economy can experience capital inflows

$$B_{t-1} > q(B_t)B_t \tag{B.34}$$

but the government chooses some lower level of debt B'_t that leads to capital outflows

$$B_{t-1} < q(B'_t)B'_t \tag{B.35}$$

The government is benevolent and wants to maximize the utility. However, under condition (B.35) default is optimal.

Given y_t and B_{t-1}, consumption in period t is given by

$$y_t \qquad \text{under financial autarchy} \tag{B.36}$$

$$y_t - q(B'_t)B'_t + B_{t-1} \qquad \text{given repayment and capital outflows} \tag{B.37}$$

Given capital outflows, consumption is higher during autarchy in each period.

Since the utility is increasing in consumption, it holds that

$$U(y_t) + E\beta V^D(y_{t+1}) > U[y_t - q(B'_t)B'_t + B_{t-1}] + E\beta V^O(y_{t+1}, B'_t) \tag{B.38}$$

where $V^O(y_{t+1}, B'_t) \equiv max[V^R(y_{t+1}, B'_t), V^D(y_{t+1})]$. Defaulting is the optimal decision.

On the other hand, default is not optimal for contracts that deliver capital inflows. It holds that

$$U(y_t) + E\beta V^D(y_{t+1}) \leq U[y_t - q(B_t)B_t + B_{t-1}] + E\beta V^O(y_{t+1}, B_t) \tag{B.39}$$

because of (B.34)

$$U(y_t) \leq U[y_t - q(B_t)B_t + B_{t-1}] \tag{B.40}$$

and by definition

$$E\beta V^D(y_{t+1}) \leq E\beta V^O(y_{t+1}, B_t) \tag{B.41}$$

If the government chooses B'_t to maximize the utility, it results in a contradiction. B'_t is not the maximizing level of bonds.
Therefore, given the benevolent government has chosen B_t to maximize the value of the contract, it must hold that only contracts are available such that the economy experiences capital outflows, $B_{t-1} < q(B_t)B_t$, if $D(B_{t-1}) \neq \emptyset$. No contracts that deliver capital inflows, $B_{t-1} > q(B_t)B_t$, can be available. Otherwise, the country could repay the entire debt and consume more in the current period. In such a situation, a rational borrower would never default. This means that the default probability is zero and $D(B_{t-1}) = \emptyset$.

B.2.3. Default Is More Likely in Bad Periods

The exclusion of financial markets after defaulting is permanent, $\theta = 0$.
The country is indebted if $B_{t-1} < 0$.

Consider two output shocks y_t^1, $y_t^2 \in Y$ with

$$y_t^1 \leq y_t^2 \tag{B.42}$$

Default is countercyclical and therefore more likely the lower the output shock is. This means that if $y_t^2 \in D(B_{t-1})$, then $y_t^1 \in D(B_{t-1})$.

If $y_t^1 \in D(B_{t-1})$, then the government defaults and $D(B_{t-1}) \neq \emptyset$.

$$U(y_t^1) + E\beta V^D(y_{t+1}) > U[y_t^1 - q(B_t^1)B_t^1 + B_{t-1}] + E\beta V^O(y_{t+1}, B_t^1) \tag{B.43}$$

$y_t^2 \in D(B_{t-1})$ implies that

$$U(y_t^2) + E\beta V^D(y_{t+1}) > U[y_t^2 - q(B_t^2)B_t^2 + B_{t-1}] + E\beta V^O(y_{t+1}, B_t^2) \tag{B.44}$$

where $V^O(y_{t+1}, B_t^i) \equiv max[V^R(y_{t+1}, B_t^i), V^D(y_{t+1})]$ for $i = 1, 2$.

Countercyclical default incentives imply

$$\begin{aligned} &U(y_t^1) + E\beta V^D(y_{t+1}) - U[y_t^1 - q(B_t^1)B_t^1 + B_{t-1}] - E\beta V^O(y_{t+1}, B_t^1) \\ &\quad \geq U(y_t^2) + E\beta V^D(y_{t+1}) - U[y_t^2 - q(B_t^2)B_t^2 + B_{t-1}] - E\beta V^O(y_{t+1}, B_t^2) \end{aligned} \tag{B.45}$$

(B.45) implies

$$\begin{aligned} &U[y_t^2 - q(B_t^2)B_t^2 + B_{t-1}] + E\beta V^O(y_{t+1}, B_t^2) - U[y_t^1 - q(B_t^1)B_t^1 + B_{t-1}] - E\beta V^O(y_{t+1}, B_t^1) \\ &\quad \geq U(y_t^2) + E\beta V^D(y_{t+1}) - U(y_t^1) - E\beta V^D(y_{t+1}) \end{aligned} \tag{B.46}$$

If this condition holds, then $y_t^2 \in D(B_{t-1})$ implies that $y_t^1 \in D(B_{t-1})$.

From the right-hand side of (B.46) and because i.i.d. income shocks imply that $E\beta V^D(y_{t+1})$ is equal for y_t^1 and y_t^2

$$U(y_t^2) + E\beta V^D(y_{t+1}) - U(y_t^1) - E\beta V^D(y_{t+1}) = U(y_t^2) - U(y_t^1) \tag{B.47}$$

B_t depends on y_t and on a given B_{t-1}

$$B_t^1 = B(y_t^1, B_{t-1})$$
$$B_t^2 = B(y_t^2, B_{t-1})$$

From (B.42) and because B_t depends negatively on output, that is, the higher the output is, the higher the possible debt

$$B_t^1 \geq B_t^2 \tag{B.48}$$

From (B.48), the total resources borrowed are given by

$$q(B_t^1)B_t^1 \geq q(B_t^2)B_t^2 \tag{B.49}$$

From (B.49) and because utility is increasing in consumption

$$U[y_t^2 - q(B_t^2)B_t^2 + B_{t-1}] + E\beta V^O(y_{t+1}, B_t^2) \geq U[y_t^2 - q(B_t^1)B_t^1 + B_{t-1}] + E\beta V^O(y_{t+1}, B_t^1) \tag{B.50}$$

Combining (B.46), (B.47) and (B.50)

$$U[y_t^2 - q(B_t^1)B_t^1 + B_{t-1}] + E\beta V^O(y_{t+1}, B_t^1) - U[y_t^1 - q(B_t^1)B_t^1 + B_{t-1}] - E\beta V^O(y_{t+1}, B_t^1) \\ \geq U(y_t^2) - U(y_t^1) \tag{B.51}$$

i.i.d. shocks imply

$$U[y_t^2 - q(B_t^1)B_t^1 + B_{t-1}] - U[y_t^1 - q(B_t^1)B_t^1 + B_{t-1}] \geq U(y_t^2) - U(y_t^1) \tag{B.52}$$

Condition (B.52) holds due to Appendix B.2.2. $D(B_{t-1}) \neq \emptyset$ means that $B_{t-1} < q(B_t)B_t$ for all available contracts. The utility function is increasing and strictly concave, which means that capital outflows reduce utility more for $y_t^1 - q(B_t^1)B_t^1 + B_{t-1}$ than for $y_t^2 - q(B_t^1)B_t^1 + B_{t-1}$ and condition (B.52) is satisfied. Hence, condition (B.46) is satisfied because of (B.50) and $y_t^1 \in D(B_{t-1})$. Default incentives are stronger the lower the endowment.

B.3. Cuadra, Sanchez and Sapriza's Model

B.3.1. Derivation of the Optimal Behaviour of Households

The representative household maximizes its discounted utility

$$max_{c_t,l_t} E[\sum_{t=0}^{\infty} \beta^t U(c_t, g_t, 1 - l_t)]$$

subject to its budget constraint

$$(1 + T_t)c_t = A_t l_t$$

It behaves like a Stackelberg follower and takes as given the decision of the government on default, the public spending g_t and the tax rate T_t.

Using Lagrange

$$L = \sum_{t=0}^{\infty} \beta^t \{U(c_t, g_t, 1 - l_t) + \mu[c_t - \frac{A_t l_t}{1 + T_t}]\}$$

First order conditions

$$\frac{\partial L}{\partial c_t} = \beta^t \{U_c(c_t, g_t, 1 - l_t) + \mu\} = 0 \tag{B.53}$$

$$\frac{\partial L}{\partial l_t} = \beta^t \{-U_l(c_t, g_t, 1 - l_t) - \mu \frac{A_t}{1 + T_t}\} = 0 \tag{B.54}$$

From (B.53)

$$-\mu = U_c(c_t, g_t, 1 - l_t) \tag{B.55}$$

Combining (B.54) and (B.55)

$$U_l(c_t, g_t, 1 - l_t) = U_c(c_t, g_t, 1 - l_t) \frac{A_t}{1 + T_t} \tag{B.56}$$

From (B.56)

$$\frac{A_t}{1 + T_t} = \frac{U_l(c_t^*, g_t, 1 - l_t^*)}{U_c(c_t^*, g_t, 1 - l_t^*)}$$

The derivation of the optimal reaction of the representative household under financial autarchy is similar.

B.3.2. Derivation of the Optimal Behaviour of the Government

The government maximizes its objective function

$$V^R(A_t, B_{t-1}) \equiv max_{T_t, g_t, B_t} \{U(c_t^*, g_t, 1 - l_t^*) + \beta \sum_{A_{t+1} \in \Upsilon} max[V^R(A_{t+1}, B_t), V^D(A_{t+1})] f(A_{t+1}, A_t)\}$$

subject to the household's budget constraint,

$$(1 + T_t) c_t = A_t l_t \tag{B.57}$$

to the government's budget constraint

$$g_t = T_t c_t - q_t B_t + B_{t-1} \tag{B.58}$$

and to the optimal reaction of the households

$$\frac{A_t}{1+T_t} = \frac{U_l(c_t^*, g_t, 1-l_t^*)}{U_c(c_t^*, g_t, 1-l_t^*)} \tag{B.59}$$

Assume that $\mathbb{C}(T_t, g_t, A_t)$ and $\mathbb{L}(T_t, g_t, A_t)$ solve the household's problem, such that $c_t^* = \mathbb{C}(T_t, g_t, A_t)$ and $l_t^* = \mathbb{L}(T_t, g_t, A_t)$.

From (B.57)

$$\mathbb{C}(T_t, g_t, A_t) = \frac{A_t}{1+T_t}\mathbb{L}(T_t, g_t, A_t) \tag{B.60}$$

From (B.58)

$$g_t = T_t\mathbb{C}(T_t, g_t, A_t) - q_t B_t + B_{t-1} \tag{B.61}$$

From (B.59)

$$\frac{A_t}{1+T_t} = \frac{U_l[\mathbb{C}(T_t, g_t, A_t), g_t, 1-\mathbb{L}(T_t, g_t, A_t)]}{U_c[\mathbb{C}(T_t, g_t, A_t), g_t, 1-\mathbb{L}(T_t, g_t, A_t)]} \tag{B.62}$$

Using Lagrange

$$\begin{aligned} L = {} & U[\mathbb{C}(T_t, g_t, A_t), g_t, 1-\mathbb{L}(T_t, g_t, A_t)] + \beta \sum_{A_{t+1}\in\Upsilon} max[V^R(A_{t+1}, B_t), V^D(A_{t+1})]f(A_{t+1}, A_t) \\ & + \mu\{T_t\mathbb{C}(T_t, g_t, A_t) - q(A_t, B_t)B_t + B_{t-1} - g_t\} \end{aligned} \tag{B.63}$$

First order conditions

$$\begin{aligned} \frac{\partial L}{\partial T_t} = {} & U_c[\mathbb{C}(T_t, g_t, A_t), g_t, 1-\mathbb{L}(T_t, g_t, A_t)]\mathbb{C}_T(T_t, g_t, A_t) \\ & - U_l[\mathbb{C}(T_t, g_t, A_t), g_t, 1-\mathbb{L}(T_t, g_t, A_t)]\mathbb{L}_T(T_t, g_t, A_t) + \mu[\mathbb{C}(T_t, g_t, A_t) + T_t\mathbb{C}_T(T_t, g_t, A_t)] = 0 \end{aligned} \tag{B.64}$$

$$\begin{aligned} \frac{\partial L}{\partial g_t} = {} & U_c[\mathbb{C}(T_t, g_t, A_t), g_t, 1-\mathbb{L}(T_t, g_t, A_t)]\mathbb{C}_g(T_t, g_t, A_t) + U_g[\mathbb{C}(T_t, g_t, A_t), g_t, 1-\mathbb{L}(T_t, g_t, A_t)] \\ & - U_l[\mathbb{C}(T_t, g_t, A_t), g_t, 1-\mathbb{L}(T_t, g_t, A_t)]\mathbb{L}_g(T_t, g_t, A_t) + \mu[T_t\mathbb{C}_g(T_t, g_t, A_t) - 1] = 0 \end{aligned} \tag{B.65}$$

$$\frac{\partial L}{\partial B_t} = \beta \sum_{A_{t+1}\in P(B_t)} V_{B_t}^R(A_{t+1}, B_t)f(A_{t+1}, A_t) + \mu[-q(A_t, B_t) - q_B(A_t, B_t)B_t] = 0 \tag{B.66}$$

Since the value of default does not depend on borrowing, the index of summation contains only elements of the repayment set.

From (B.60)

$$\mathbb{C}_T(T_t, g_t, A_t) = -\frac{A_t}{(1+T_t)^2}\mathbb{L}(T_t, g_t, A_t) + \frac{A_t}{1+T_t}\mathbb{L}_T(T_t, g_t, A_t) \tag{B.67}$$

From (B.60) and (B.67)

$$\begin{aligned}
&\mathbb{C}(T_t, g_t, A_t) + T_t\mathbb{C}_T(T_t, g_t, A_t) \\
&\quad = \frac{A_t}{1+T_t}\mathbb{L}(T_t, g_t, A_t) - \frac{A_tT_t}{(1+T_t)^2}\mathbb{L}(T_t, g_t, A_t) + \frac{A_tT_t}{1+T_t}\mathbb{L}_T(T_t, g_t, A_t) \\
&\quad = \frac{A_t}{1+T_t}\mathbb{L}(T_t, g_t, A_t)(1 - \frac{T_t}{1+T_t}) + \frac{A_tT_t}{1+T_t}\mathbb{L}_T(T_t, g_t, A_t) \\
&\quad = \frac{A_t}{(1+T_t)^2}\mathbb{L}(T_t, g_t, A_t) + \frac{A_tT_t}{1+T_t}\mathbb{L}_T(T_t, g_t, A_t)
\end{aligned} \tag{B.68}$$

From (B.62)

$$U_c[\mathbb{C}(T_t, g_t, A_t), g_t, 1 - \mathbb{L}(T_t, g_t, A_t)] = U_l[\mathbb{C}(T_t, g_t, A_t), g_t, 1 - \mathbb{L}(T_t, g_t, A_t)]\frac{1+T_t}{A_t} \tag{B.69}$$

$$U_l[\mathbb{C}(T_t, g_t, A_t), g_t, 1 - \mathbb{L}(T_t, g_t, A_t)] = U_c[\mathbb{C}(T_t, g_t, A_t), g_t, 1 - \mathbb{L}(T_t, g_t, A_t)]\frac{A_t}{1+T_t} \tag{B.70}$$

From (B.67), (B.69) and (B.70)

$$\begin{aligned}
&U_c[\mathbb{C}(T_t, g_t, A_t), g_t, 1 - \mathbb{L}(T_t, g_t, A_t)]\mathbb{C}_T(T_t, g_t, A_t) \\
&\quad - U_l[\mathbb{C}(T_t, g_t, A_t), g_t, 1 - \mathbb{L}(T_t, g_t, A_t)]\mathbb{L}_T(T_t, g_t, A_t) \\
&\quad = U_l[\mathbb{C}(T_t, g_t, A_t), g_t, 1 - \mathbb{L}(T_t, g_t, A_t)]\frac{1+T_t}{A_t}[-\frac{A_t}{(1+T_t)^2}\mathbb{L}(T_t, g_t, A_t) \\
&\quad + \frac{A_t}{1+T_t}\mathbb{L}_T(T_t, g_t, A_t)] - U_l[\mathbb{C}(T_t, g_t, A_t), g_t, 1 - \mathbb{L}(T_t, g_t, A_t)]\mathbb{L}_T(T_t, g_t, A_t) \\
&\quad = -\frac{1}{1+T_t}U_l[\mathbb{C}(T_t, g_t, A_t), g_t, 1 - \mathbb{L}(T_t, g_t, A_t)]\mathbb{L}(T_t, g_t, A_t) \\
&\quad + U_l[\mathbb{C}(T_t, g_t, A_t), g_t, 1 - \mathbb{L}(T_t, g_t, A_t)]\mathbb{L}_T(T_t, g_t, A_t) \\
&\quad - U_l[\mathbb{C}(T_t, g_t, A_t), g_t, 1 - \mathbb{L}(T_t, g_t, A_t)]\mathbb{L}_T(T_t, g_t, A_t) \\
&\quad = -\frac{A_t}{(1+T_t)^2}U_c[\mathbb{C}(T_t, g_t, A_t), g_t, 1 - \mathbb{L}(T_t, g_t, A_t)]\mathbb{L}(T_t, g_t, A_t)
\end{aligned} \tag{B.71}$$

Combining (B.64), (B.68) and (B.71)

$$-\frac{A_t}{(1+T_t)^2}U_c[\mathbb{C}(T_t,g_t,A_t),g_t,1-\mathbb{L}(T_t,g_t,A_t)]\mathbb{L}(T_t,g_t,A_t) + \mu[\frac{A_t}{(1+T_t)^2}\mathbb{L}(T_t,g_t,A_t)+\frac{A_tT_t}{1+T_t}\mathbb{L}_T(T_t,g_t,A_t)]=0 \tag{B.72}$$

From (B.60)

$$\mathbb{C}_g(T_t,g_t,A_t)=\frac{A_t}{1+T_t}\mathbb{L}_g(T_t,g_t,A_t) \tag{B.73}$$

From (B.69) and (B.73)

$$\begin{aligned}&U_c[\mathbb{C}(T_t,g_t,A_t),g_t,1-\mathbb{L}(T_t,g_t,A_t)]\mathbb{C}_g(T_t,g_t,A_t)\\&\quad-U_l[\mathbb{C}(T_t,g_t,A_t),g_t,1-\mathbb{L}(T_t,g_t,A_t)]\mathbb{L}_g(T_t,g_t,A_t)\\&\quad=U_l[\mathbb{C}(T_t,g_t,A_t),g_t,1-\mathbb{L}(T_t,g_t,A_t)]\mathbb{L}_g(T_t,g_t,A_t)\\&\quad-U_l[\mathbb{C}(T_t,g_t,A_t),g_t,1-\mathbb{L}(T_t,g_t,A_t)]\mathbb{L}_g(T_t,g_t,A_t)=0\end{aligned} \tag{B.74}$$

Combining (B.65) and (B.74)

$$\begin{aligned}&U_g[\mathbb{C}(T_t,g_t,A_t),g_t,1-\mathbb{L}(T_t,g_t,A_t)]+\mu[T_t\mathbb{C}_g(T_t,g_t,A_t)-1]=0\\&U_g[\mathbb{C}(T_t,g_t,A_t),g_t,1-\mathbb{L}(T_t,g_t,A_t)]=-\mu[T_t\mathbb{C}_g(T_t,g_t,A_t)-1]\\&\mu=U_g[\mathbb{C}(T_t,g_t,A_t),g_t,1-\mathbb{L}(T_t,g_t,A_t)]\frac{1}{1-T_t\mathbb{C}_g(T_t,g_t,A_t)}\end{aligned} \tag{B.75}$$

Combining (B.72) and (B.75)

$$\begin{aligned}&\frac{A_t}{(1+T_t)^2}U_c[\mathbb{C}(T_t,g_t,A_t),g_t,1-\mathbb{L}(T_t,g_t,A_t)]\mathbb{L}(T_t,g_t,A_t)\\&\qquad=U_g[\mathbb{C}(T_t,g_t,A_t),g_t,1-\mathbb{L}(T_t,g_t,A_t)]\frac{1}{1-T_t\mathbb{C}_g(T_t,g_t,A_t)}[\frac{A_t}{(1+T_t)^2}\mathbb{L}(T_t,g_t,A_t)\\&\qquad+\frac{A_tT_t}{1+T_t}\mathbb{L}_T(T_t,g_t,A_t)]\end{aligned}$$

The utility function is assumed to be additively separable, it holds in optimum

$$\mathbb{C}_g(T_t,g_t,A_t)=0$$
$$\mathbb{L}_g(T_t,g_t,A_t)=0$$

It follows

$$\frac{A_t}{(1+T_t)^2} U_c[\mathbb{C}(T_t, g_t, A_t), g_t, 1 - \mathbb{L}(T_t, g_t, A_t)]\mathbb{L}(T_t, g_t, A_t)$$
$$= U_g[\mathbb{C}(T_t, g_t, A_t), g_t, 1 - \mathbb{L}(T_t, g_t, A_t)][\frac{A_t}{(1+T_t)^2}\mathbb{L}(T_t, g_t, A_t)$$
$$+ \frac{A_t T_t}{1+T_t}\mathbb{L}_T(T_t, g_t, A_t)]$$

Hence,

$$U_c(c_t^*, g_t, 1 - l_t^*)\frac{A_t l_t^*}{(1+T_t)^2} = U_g(c_t^*, g_t, 1 - l_t^*)[\frac{A_t l_t^*}{(1+T_t)^2} + \frac{T_t A_t}{1+T_t}\frac{\partial l_t}{\partial T_t}]$$

The envelope condition is given by

$$V^R_{B_{t-1}}(A_t, B_{t-1}) = \mu$$
$$= U_g[\mathbb{C}(T_t, g_t, A_t), g_t, 1 - \mathbb{L}(T_t, g_t, A_t)]\frac{1}{1 - T_t\mathbb{C}_g(T_t, g_t, A_t)}$$
$$= U_g[\mathbb{C}(T_t, g_t, A_t), g_t, 1 - \mathbb{L}(T_t, g_t, A_t)]$$

It follows that

$$V^R_{B_t}(A_{t+1}, B_t) = \mu$$
$$= U_g[\mathbb{C}(T_{t+1}, g_{t+1}, A_{t+1}), g_{t+1}, 1 - \mathbb{L}(T_{t+1}, g_{t+1}, A_{t+1})] \tag{B.76}$$

Combining (B.66) and (B.76)

$$U_g[\mathbb{C}(T_t, g_t, A_t), g_t, 1 - \mathbb{L}(T_t, g_t, A_t)][q(A_t, B_t) + q_B(A_t, B_t)B_t]$$
$$= \beta \sum_{A_{t+1}\in P(B_t)} U_g[\mathbb{C}(T_{t+1}, g_{t+1}, A_{t+1}), g_{t+1}, 1 - \mathbb{L}(T_{t+1}, g_{t+1}, A_{t+1})]f(A_{t+1}, A_t)$$

Hence,

$$U_g(c_t^*, g_t, 1 - l_t^*)[q_t + B_t\frac{\partial q_t}{\partial B_t}] = \beta \sum_{A_{t+1}\in P(B_t)} U_g(c_{t+1}^*, g_{t+1}, 1 - l_{t+1}^*)f(A_{t+1}, A_t)$$

The derivation of the first order condition for T_t^D under financial autarchy is similar.

B.3.3. Derivation of the Optimal Behaviour in the Numerical Example

The production function is given by

$y_t = A_t l_t$

The utility function is given by

$$U(c_t, g_t, 1 - l_t) = \pi[\frac{g_t^{1-\gamma}}{1-\gamma}] + (1-\pi)[\frac{(c_t - \frac{l_t^{1+\psi}}{1+\psi})^{1-\gamma}}{1-\gamma}]$$

From Appendix B.3.1, the optimal behaviour of the households is given by

$$\frac{A_t}{1+T_t} = \frac{U_l(c_t^*, g_t, 1 - l_t^*)}{U_c(c_t^*, g_t, 1 - l_t^*)} \tag{B.77}$$

The marginal rate of substitution of leisure for consumption is given by

$$\frac{U_l(c_t, g_t, 1 - l_t)}{U_c(c_t, g_t, 1 - l_t)} = l_t^{\psi} \tag{B.78}$$

Since

$$U_l(c_t, g_t, 1 - l_t) = -(1-\pi)(c_t - \frac{l_t^{1+\psi}}{1+\psi})^{-\gamma}(-l_t^{\psi})$$
$$U_c(c_t, g_t, 1 - l_t) = (1-\pi)(c_t - \frac{l_t^{1+\psi}}{1+\psi})^{-\gamma}$$

Combining (B.77) and (B.78)

$$l_t^{\psi} = (\frac{A_t}{1+T_t}) \tag{B.79}$$

(B.79) implies the optimal labour effort of the households

$$l_t^* = (\frac{A_t}{1+T_t})^{\frac{1}{\psi}}$$

The first order conditions of the government are given by

$$U_c(c_t^*, g_t, 1-l_t^*)\frac{A_t l_t^*}{(1+T_t)^2} = U_g(c_t^*, g_t, 1-l_t^*)[\frac{A_t l_t^*}{(1+T_t)^2} + \frac{T_t A_t}{1+T_t}\frac{\partial l_t}{\partial T_t}] \tag{B.80}$$

$$U_g(c_t^*, g_t, 1-l_t^*)[q_t + B_t\frac{\partial q_t}{\partial B_t}] = \beta \sum_{A_{t+1}\in P(B_t)} U_g(c_{t+1}^*, g_{t+1}, 1-l_{t+1}^*) f(A_{t+1}, A_t) \tag{B.81}$$

Where

$$U_c(c_t^*, g_t, 1-l_t^*) = (1-\pi)(c_t^* - \frac{l_t^{*1+\psi}}{1+\psi})^{-\gamma} = (1-\pi)[c_t^* - \frac{1}{1+\psi}(\frac{A_t}{1+T_t})^{\frac{1+\psi}{\psi}}]^{-\gamma}$$

$$U_g(c_t^*, g_t, 1-l_t^*) = \pi g_t^{-\gamma}$$

$$A_t l_t^* = A_t(\frac{A_t}{1+T_t})^{\frac{1}{\psi}}$$

$$c_t^* = \frac{A_t l_t^*}{1+T_t} = (\frac{A_t}{1+T_t})^{\frac{1+\psi}{\psi}}$$

$$\frac{\partial l_t^*}{\partial T_t} = -\frac{1}{\psi}(\frac{A_t}{1+T_t})^{\frac{1}{\psi}-1}\frac{A_t}{(1+T_t)^2} = -\frac{1}{\psi(1+T_t)}(\frac{A_t}{1+T_t})^{\frac{1}{\psi}}$$

Combining these equations to (B.80)

$$\begin{aligned}(1-\pi)[(\frac{A_t}{1+T_t})^{\frac{1+\psi}{\psi}} - \frac{1}{1+\psi}(\frac{A_t}{1+T_t})^{\frac{1+\psi}{\psi}}]^{-\gamma}\frac{A_t}{(1+T_t)^2}(\frac{A_t}{1+T_t})^{\frac{1}{\psi}} \\ = \pi g_t^{-\gamma}[\frac{A_t}{(1+T_t)^2}(\frac{A_t}{1+T_t})^{\frac{1}{\psi}} - \frac{T_t A_t}{1+T_t}\frac{1}{\psi(1+T_t)}(\frac{A_t}{1+T_t})^{\frac{1}{\psi}}]\end{aligned}$$

$$\frac{1-\pi}{1+T_t}[(\frac{A_t}{1+T_t})^{\frac{1+\psi}{\psi}}(1-\frac{1}{1+\psi})]^{-\gamma}(\frac{A_t}{1+T_t})^{\frac{1+\psi}{\psi}} = \frac{\pi}{1+T_t}g_t^{-\gamma}(\frac{A_t}{1+T_t})^{\frac{1+\psi}{\psi}}(1-\frac{T_t}{\psi})$$

$$(1-\pi)[\frac{\psi}{1+\psi}(\frac{A_t}{1+T_t})^{\frac{1+\psi}{\psi}}]^{-\gamma} = \pi g_t^{-\gamma}(1-\frac{T_t}{\psi})$$

Combining the equations to (B.81)

$$\pi g_t^{-\gamma}[q_t + B_t\frac{\partial q_t}{\partial B_t}] = \beta \sum_{A_{t+1}\in P(B_t)} \pi g_{t+1}^{-\gamma} f(A_{t+1}, A_t)$$

$$g_t^{-\gamma}[q_t + B_t\frac{\partial q_t}{\partial B_t}] = \beta \sum_{A_{t+1}\in P(B_t)} g_{t+1}^{-\gamma} f(A_{t+1}, A_t)$$

The computations are similar for the financial autarchy.

B.4. Discussion

B.4.1. Comparative Static Effects of the Stochastic Model of EG

The safe credit ceiling is given for $b \leq \bar{b}$.

$$\sigma\bar{b} = \frac{\frac{2\beta(1+R)-4R}{\tilde{A}} + \sigma(2\beta - 4Ra)}{\beta(1+\beta R^2) + 2R^2 a}$$

where $a \equiv 1 - \frac{\beta}{2} - \frac{\beta^2}{2}$

$$\frac{d\sigma\bar{b}}{d\sigma} = \frac{2(\beta - 2Ra)}{\beta(1+\beta R^2) + 2R^2 a} \tag{B.82}$$

$$(B.82) > 0 \text{ if } \frac{\beta}{2-\beta-\beta^2} > R \implies \beta \text{ near } 1$$
$$(B.82) < 0 \text{ if } \frac{\beta}{2-\beta-\beta^2} < R \implies \beta \text{ near } 0$$

The risky credit ceiling is given for $b > \bar{b}$.

$$\sigma\bar{b}_{2R} = \frac{\frac{2f(1-\beta R)-4R}{\tilde{A}} + \sigma[2f(1+\beta R) + 4R]}{f(1+2\beta R^2) + 4R^2}$$

where $f \equiv \frac{\beta}{2e} = \frac{\beta}{2-\beta-\frac{\beta^2}{2}}$

$$\frac{d\sigma\bar{b}_{2R}}{d\sigma} = \frac{2f(1+\beta R) + 4R}{f(1+2\beta R^2) + 4R^2} > 0$$

C. Empirical Appendix

C.1. The Classical Model

C.1.1. Details of the Econometric Analysis

This analysis contains cross section data of 45 poor countries for the years 1970 and 1974. Not all data was available for both years for each country.

Model

The current debt is given by $b = min(\bar{b}, b^*)$, where

$$\bar{b} = f(x) + u \tag{C.1}$$

$$b^* = g(x) + v \tag{C.2}$$

This is the form of the model. b can be observed but it is not known whether it corresponds to $\bar{b}$ or b^*. x defines a vector of country characteristics. The random errors u and v are independent and normally distributed. $\bar{b}$ and b^* depend on the same variables. EG use maximum likelihood methods for the joint estimation of (C.1) and (C.2). The probability of a supply constrained regime or a demand regime is also computed based on the values of the variables and the maximum likelihood methods.

Country Characteristics

Public debt to private creditors: The maturity of this type of public debt is over one year. It is given in U.S. dollars and logged.

Growth rate of income: This variable describes the growth rate of the real GNP (logged) for the periods 1964 to 1970 (for 1970) and 1968 to 1974 (for 1974).

Export variability: The export variability is the standard error of a regression of real exports (logged) on a constant and time for the periods 1964 to 1970 and 1968 to 1974. The export variability generates basically similar results to the income variability. The benefit of the export variability is the better measurement of exports than income.

Income: The income is given by the GNP. This variable is in U.S. dollars and logged.

Ratio of imports to income: This is the ratio of imports to GNP.

Population: It represents the total population of the country in the specific year. This variable is logged.

Public debt to public institutions: This variable is given in U.S. dollars and logged. It contains debt to non-communist governments and international organizations.

Dummy: The dummy is zero in 1970 and one in 1974.

C.2. Arellano's Model

C.2.1. Parameters of the Graphical Analysis

The parameterized example of 3.5.1 is calibrated to match the Argentinean economy. The default probability is assumed to be 3 % because the Argentinean government defaulted 3 times (1956, 1982, 2001) in the last 100 years. The most recent default event of 2014 is not considered since Arellano wrote her paper in 2008. The average debt-service to GDP ratio is reported by the World Bank for the years 1980 to 2001 and corresponds to 5.53 %. The standard deviation of the trade balance is given as a percentage of output for the years 1993 to 2001. The trade balance volatility corresponds to 1.75.

The stochastic income process: The stochastic process for output is assumed to follow a log-normal AR(1) process $\log(y_t) = \rho \log(y_{t-1}) + \epsilon_t^y$ with $E(\epsilon^y) = 0$ and $E(\epsilon^2) = \eta_y^2$. It is estimated using seasonally adjusted quarterly time series data of the Argentinean GDP for 1980 to 2001, which are log and filtered with a linear trend. The values are $\rho = 0.945$ and $\eta = 0.025$. Arellano uses a quadrature based procedure to discretize the shock into a 21 state Markov chain.

The risk free interest rate: The risk free interest rate corresponds to the average quarterly interest rate of a 5 year U.S. treasury bond during 1983 to 2001, $\bar{r} = 0.017$.

The risk aversion coefficient: The risk aversion coefficient is a common value in real business cycle studies, $\gamma = 2$.

The discount factor: The discount factor is calibrated to match the Argentinean economy, $\beta = 0.953$.

The probability of regaining access to international capital markets: The probability of regaining access to international capital markets is calibrated to match the Argentinean economy, $\theta = 0.282$.

The default costs threshold: The default costs threshold is calibrated to match the Argentinean economy, $\hat{y} = 0.969E(y)$.

C.2.2. Details of the Quantitative Analysis

The parameterized model is used for the quantitative analysis in chapter 3.6. Arellano simulated the model and found 100 default events. 74 observations prior to each default, which correspond to the number of periods from Q3 1983 to Q4 2001, were extracted and the mean statistics reported. Table C.1 summarizes the results.

The business cycle statistics are quarterly real series data and seasonally adjusted.

The Argentinean interest rate spread: The Argentinean interest rates are given by the Emerging Markets Bond Index, starting in Q3 1983. The spread corresponds to the difference of the Argentinean interest rate and the yield of a 5 year U.S. treasury bond which represents the safe interest rate.

Trade balance: The trade balance is given as a percentage of output, starting in 1993.

Consumption: The data are log and filtered with a linear trend, starting in 1980.

Output: The data are log and filtered with a linear trend, starting in 1980.

	Default episode		*std(x)*		*corr(x,y)*		*corr(x,r_t)*	
Business Cycle Statistics	Argentina	Model	Argentina	Model	Argentina	Model	Argentina	Model
Interest rate spread	28.60	24.32	5.58	6.36	-0.88	-0.29		
Trade balance	9.90	-0.01	1.75	1.50	-0.64	-0.25	0.70	0.41
Consumption	-16.01	-9.47	8.50	6.38	0.98	0.97	-0.89	-0.36
Output	-14.21	-9.60	7.78	5.81			-0.88	-0.29
Other Statistics								
Mean Debt (% output)	5.53	5.95						
Default Probability	3.00	3.00						
Mean Spread	10.58	3.58						
Output Deviation in Default	-7.30	-8.13						

Table C.1.: Business cycle statistics and other statistics for Argentina and the benchmark model, Based on Arellano (2008)

C.3. Cuadra, Sanchez and Sapriza's Model

C.3.1. Parameters of the Graphical Analysis

The parameterized example of 4.4.1 is calibrated to match the Mexican economy. The used data are reported by the Banco de Mexico. The seasonally adjusted quarterly time series data are from 1980 to 2007. Public spending, consumption of the private sector and the output are in logs. The Hodrick-Prescott filter is used to smooth the data. The authors computed an effective tax rate for the consumption tax. The used interest rate spreads are obtained from the Emerging Markets Bond Index for the time period 1994 to 2007.

The stochastic productivity factor: The stochastic productivity factor is assumed to follow an AR(1) process $\ln(A_t) = \rho \ln(A_{t-1}) + \epsilon_t^A$ with $E(\epsilon^A) = 0$ and $E(\epsilon^2) = \eta_A^2$. It is estimated using seasonally adjusted quarterly time series data of the Mexican GDP for 1980 to 2007, which are log and filtered. The values are $\rho = 0.85$ and $\eta = 0.006$. The authors discretize the shock into a 25 state Markov chain.

The risk free interest rate: The risk free interest rate corresponds to the U.S. annual interest rate, $\overline{r} = 0.01$.

The risk aversion coefficient: The risk aversion coefficient is a common value in real business cycle studies, $\gamma = 2$.

The discount factor: The discount factor is calibrated to match the Mexican economy, $\beta = 0.97$. In this case, β is set to match a volatility of 3 % for the public spending during 1980 to 2007.

The probability of regaining access to international capital markets: The probability of regaining access to international capital markets is calibrated to match the Mexican economy, $\theta = 0.10$. This value means that the country needs about 10 quarters to re-enter financial markets after a decision to default.

The default costs threshold: The default costs threshold is calibrated to match the Mexican economy, $\hat{A} = 0.99E(A)$. In this case, $\hat{A}$ is set to match the Mexican external debt-service to GDP ratio of 4.5 %.

The inverse of the labour elasticity: The inverse of the labour elasticity is given by $\psi = 0.455$. This value implies the labour supply elasticity of $\frac{1}{\psi} = 2.22$.

The weight on government spending: The weight on government spending is calibrated to match the Mexican economy, $\pi = 0.30$. In this case, π is set to match the average value of 16 % for the ratio of public spending to private consumption.

C.3.2. Details of the Quantitative Analysis

The parameterized model is used for the quantitative analysis in chapter 4.5. The authors simulated the model 1,000 times and reported the average values of the business cycle statistics. Table C.2 summarizes these results including a variation of the model without default risk and a variation with labour tax instead of consumption tax.

The Mexican interest rate spread: The Mexican interest rates are given by the Emerging Markets Bond Index for 1994 to 2007. The spread corresponds to the difference of the Mexican interest rate and the U.S. annual interest rate which represents the safe interest rate.

Trade balance: The trade balance is given as a percentage of output for 1980 to 2007.

Private consumption: The data are log and filtered with the Hodrick-Prescott filter for 1980 to 2007.

Public consumption: The data are log and filtered with the Hodrick-Prescott filter for 1980 to 2007.

Output: The data are log and filtered with the Hodrick-Prescott filter for 1980 to 2007.

Consumption tax rate: The authors computed an effective consumption tax rate for 1980 to 2007.

	std(x)				*corr(x,y)*			
Business Cycle Statistics	Mexico	Model	ND	LT	Mexico	Model	ND	LT
Output	2.37	2.37	2.37	2.37				
Consumption	2.90	2.44	2.23	2.43	0.92	0.99	0.99	0.99
Tax rate	0.57	0.20	0.18	0.18	-0.33	-0.35	0.63	-0.34
Trade balance	2.04	0.22			-0.72	-0.36	0.61	-0.35
Government expenditures					0.55	0.97	0.97	0.97
Interest rate spread					-0.63	-0.29		

Table C.2.: Business cycle statistics for Mexico, the benchmark model, a model without default (ND) and a model with labour tax (LT), Based on Cuadra et al. (2010)

C.3.3. Details of the Quantitative Analysis with Private Borrowing

The quantitative analysis of chapter 4.6.2 uses the same values of parameters, functional forms and the same stochastic processes as the quantitative analysis of chapter 4.5. Only the output loss after a default is computed differently. It depends on the defaulting agent and the corresponding punishment. The economy faces the highest penalty if both agents decide to default. If only one agent defaults, the income decreases more when the government does not repay its debt.

The simulation uses the policy functions that solve the first period problem. Given the initial state, the government's policy functions are $T(y_1, B_0^P, B_0)$, $B(y_1, B_0^P, B_0)$ and $D(y_1, B_0^P, B_0)$. The policy functions for the private sector are $B^P(y_1, B_0^P, B_0)$ and $D^P(y_1, B_0^P, B_0)$.

Table C.3 presents the results of the simulation for a high and a low level of initial assets. The correlations in the last column are computed from a simulation of time series. Since no filter was used in this model with private foreign borrowing, the correlations are not comparable with those in table C.2.

	Initial Debt		
Correlations with output	*Low*	*High*	Time Series
Tax rate	-0.238	-0.937	-0.864
Public foreign borrowing	-0.189	-0.927	-0.207
Private foreign borrowing	-0.863	-0.823	-0.716
Government expenditures	0.990	0.994	0.971
Private consumption	0.990	0.994	0.971

Table C.3.: Correlations of the model with private foreign borrowing, Source: Cuadra et al. (2010)

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