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Raluca Fonoage

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O.Univ.Prof. Dipl.-Ing. Dr. Richard F. Hartl

For my parents, Corina and Vasile Fonoage

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1. Introduction

Nowadays, companies are aiming to reach high revenues by delivering goods to customers all over the world. Therefore Logistics and Supply Chain Management are playing an important role in the economic environment. Among many challenges one of the most important ones regarding globalization is to keep the costs of the distribution network under control. High costs can be avoided by smart and effective strategic decisions, such as favorable location and full capacity of hubs. In order to generate an optimal solution tactical decisions such as service level and mode of transport as well as operational decisions such as order size and lead time have to be taken into consideration.¹

The problem of locating facilities and allocating customers covers the core components of the distribution system design.² The Facility Location Problem has its origin in the paper of Weber (1909) who considered that by allocating only one location, the total distance between the site and customers will be minimized. The objective of the work is to minimize the whole distribution network costs by considering fixed and variable hub costs, transportation costs and inventory costs. Croxton and Zinn (2005) have already solved this problem in their work “Inventory considerations in network design”. They presented the problem set ups and the results without providing a detailed algorithm regarding the programming part. The challenge of my work is to reach with the help of Xpress on basis of another data base the same results as Croxton and Zinn. They demonstrated that the inclusion of inventory costs will lead to important changes in the structure of the optimal network design.

The study case consists of two parts: the traditional model and the new model. Each of the models are presented theoretically and then exemplified on the data base. The challenge consists in the calculation and incorporation of the inventory costs in the

¹ cf. R. G. Askin, I. Baffo, M. Xia. 2013.

² cf. A. Klose, A. Drexl. 2004.

linear program by using the square root law. Several scenarios were run in order to demonstrate that the configuration of optimal network design is looking different in the traditional model compared to the new model. Moreover, a check of the safety stock calculations with the SRL was made in order to demonstrate the reliability of this approach.

2. Literature Review

The Facility Location Problem (FLP) was analyzed throughout time by several scientists. For example Comuejols, Sridharan and Thizy (1991) compared Lagrange heuristics with the normal heuristics. Their conclusion was that the Lagrange heuristic is more efficient than the normal heuristic both with regard to calculation time as well as final results. Sridharan (1995) has analyzed the Lagrangean relaxations of the Capacitated Plant Location Problem with single source constraint. He also demonstrated that by using this approach other combinatorial problems can be solved.

Revelle, Eiselt and Daskin (2008) have done some research concerning the Facility Location Problem (FLC) and Capacitated Facility Location Problem (CFLP). Both Lagrange heuristics and the concept of variable splitting were analyzed. At the same time, authors like Geoffrion and Powers (1995) were focusing on models regarding the distribution network by optimally locating hubs and deciding from which hub should each customer be supplied. The traditional CFLP is only considering transportation and warehousing costs. Heskett (1996) and Ballou (2001) recognized that the inventory cost have to be included in the network design models. Croxton and Zinn (2005) included the above mentioned issues in their work. They first solved the traditional model by considering hub costs and transportation costs and then calculated the inventory costs by using the square root law. In the end they introduced all these variables into the linear model. All these steps will be explained in the next chapters.

The recent literature is presenting improvements in this direction. Daskin, Coullard, and Shen (2002) and Ozsen, Coullard and Daskin (2008) also considered inventory control in CFLP.³ Sourirajan, Ozsen, and Uzsoy (2007, 2009) included the service level, lead time and inventory in their works. The most recent paper on this subject is written by Askin, Baffo and Xia (2013). They developed a genetic algorithm and a specific problem

³ cf. R. G. Askin, I. Baffo, M. Xia. 2013.

heuristic in order to solve a multi-commodity hub location and distribution planning with inventory consideration problem.⁴

The Square Root Law (SRL) is an estimate of safety stock savings. Brown (1967), Heskett, Glaskowsky, and Ivie (1974) considered the SRL as an estimate of safety stock savings but did not mentioned anything about the cycle stock. Maister (1976) was the first author that directed his research in the cycle stock direction. He considered only the situation when the cycle stock is determined through economic order quantity.

The SRL can be applied under certain assumptions. Zinn, Levy and Bowersox (1989) have eliminated two of these assumptions: the first one assumes that the demand variance is the same at all inventory locations and the second one considers that orders concerning the same item are independent in all inventory locations. The portfolio effect was further extended to include procurement and transportation costs (Mahmoud, 1992), multiple consolidation points (Evers and Beier 1993), variable lead times (Tallon, 1993) and transshipments (Evers, 1997).

Continuous Facility Location Problems were described for the first time by Ballou in 1968. These models can be used in different areas for example to choose the location of pollution sensors in order to measure certain environments or to determine the location of video cameras. By Single Facility Location Problem, a new facility has to be located so that the distance between facilities is minimized (Wesolowsky, 1973). In case of Multiple Facilities Location Problem, the optimal location of several facilities has to be determined.(Akyuz, Oncan and Altnel, 2009). The Facility Location-allocation Problem searches not only for the optimal locations, but also assigns the customers to locations in order to minimize the transportation costs (ReVelle and Swain, 1970).

Discrete Facility Location Problems consist of two sets of demands and candidate locations. Koopmans and Beckmann (1975) proposed the Integer Quadratic Assignment Problem for the first time. The Plant Location Problem (PLP) was in attention of several authors. Plant Location with Procurement Planning (Lim and Kim,

⁴ cf. R. G. Askin, I. Baffo, M. Xia. 2013.

2001) and a Multi-Commodity PLP with technology selections (Dasci and Verter, 2001) are the two main directions of the PLP research. The first formulation of median problem was solved by ReVelle in 2008. An improvement of the median problem was offered by Nicekl and Puetro (2005) and Jamshidi (2009). Each customer can be served from each facility if the distance between these locations is not bigger than a certain coverage distance (Toregas, Swain, ReVelle and Bergman, 1971). In case of the center problem, the goal is to find sites that permit the 100% fulfillment of customers demand (Daskin 1995, ReVelle 2008). The hub location problem was initially proposed by O'Kelly (1987). The hubs have to be located in such a way in order to minimize the transportation costs (Daskin, 1995). One of the initial formulations of Hierarchical Location Problem is the problem with a single-flow two-level system (Klose and Drexl, 2005).

The Discrete and Continuous Models and the SQL Models are formulated so that the Supply Chain related costs become minimal. In comparison to the SQL Models, the Discrete and Continuous Models are taking into consideration only the transportation and warehousing costs, without calculating and including the inventory costs. The inclusion of inventory related costs is important because it can lead to different optimal network design.

The network design optimization problem is considered an extended version of the facility location and demand allocation problem. The multi-source facility location-allocation problem integrates inventory problem in a mixed integer nonlinear programming model (Yao, Lee, Jaruphongsa, Tan and Hui, 2010). The number of locations of warehouses, allocation of customers to warehouses and inventory levels of warehouses will be established with an iterative heuristic method utilizing approximation and transformation techniques. No specific information about safety stock calculation is provided.

A continuous approximation modelling technique was developed to solve a facility location-allocation problem and inventory management problem (Tsao, Mangotra, Lu, Dong, 2012). On the other hand, Croxton and Zinn (2005) have calculated the safety stock costs with the Square Root Law and then introduce it discrete into the linear

program, avoiding the mixed integer linear programming and the heuristic. In this case, no inventory sharing was taken into consideration.

3. Network Design Optimization

3.1 Network Design Decisions

The problem of the optimization of the network design was mentioned in literature for the first time in 1970. Geoffrion and Powers are explaining the importance of the network design optimization and tools that can be used to reach the goal: evolution of logistics and the improving of computer programs.⁵ The first models analyze one decision problem: which customer has to be delivered from which location.

In the standard network design problem, the focus is on minimizing the transportation and fixed warehousing costs by taking into consideration demand and capacity constraints. It is usually modeled as a mixed integer linear programming and it is solved with a different algorithm.⁶ The most common algorithm is the branch and bounds algorithm.

Two types of decisions are known in supply chain management: operational and strategic decisions. The strategic decisions influence the company for a long term perspective, are related to corporate strategy and are bringing input on a design basis. On the other hand, operational decisions are made for daily business and for a short time success. The literature presents four big areas concerning the decision management:⁷

⁵ cf. A. M. Geoffrion , R. F. Powers, 1195.

⁶ cf. K. L. Croxton , .W. Zinn, 2005.

⁷ cf. R. Ganeshan, T., P. Harrison. 1995

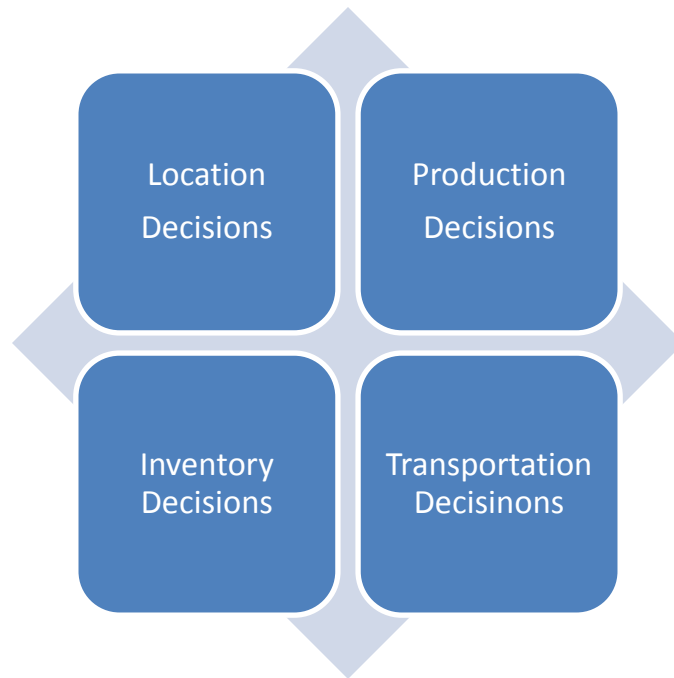


Figure 1 Network Design Decisions

Network Design is including production, stocking and sourcing facilities taking also into consideration customers' locations and demand. In order to establish a potential network design, two main questions have to be answered: which type of location, how many locations and where? These questions are very important because the setting of the network design is influencing the corporate strategy of the company and has a long term perspective. A big part of a company costs are direct influenced by the decision regarding the network design. Even if we tend to say that the location decision is the only decision that influences the network design, each of the four decision areas contribute to the network design optimization. A complex network design looks like in Figure 2.

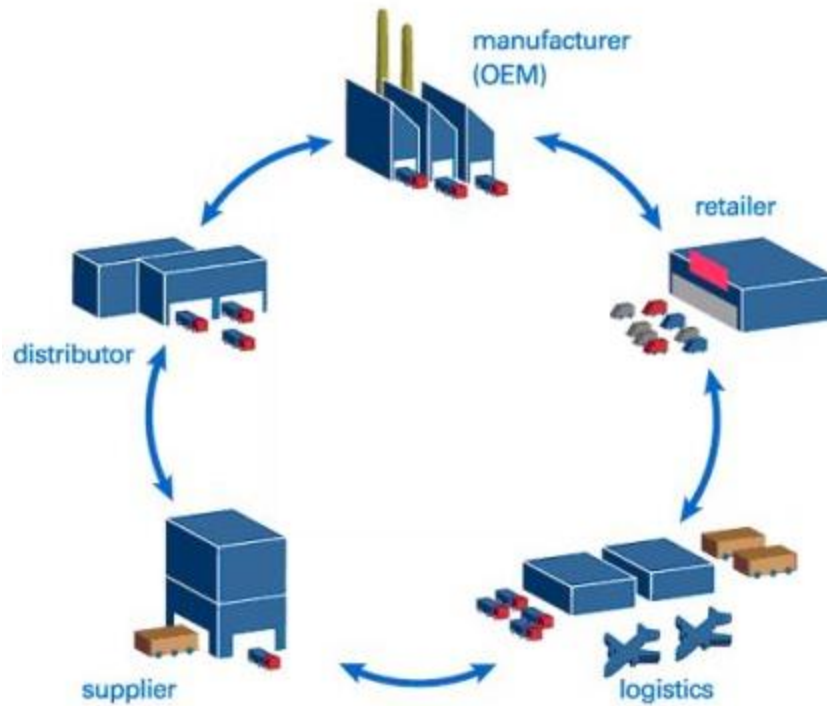


Figure 2 Network Design

3.1.1 Location Decisions

The geographic placement of plants, stocking places and sourcing places is the first step in creating a supply chain⁸. All location decisions are creating the network design and are influencing the costs and process matrix for a long time. Therefore, an optimal location decision will lead to an effective network design and will have a big impact of the daily cost structure of the company. The best approaches is to take into consideration several locations from each category (plant, storage location, supplier), to research the costs for each of them and then to take a decision which of these location will be opened and which will be closed.

⁸ cf. R. Ganeshan, T., P. Harrison 1995

Location decision can be taken by the startup of a company or can be implemented as change management. In both situations a detailed cost analysis has to be taken into consideration in order to get minimum cost further on a daily business level.

3.1.2 Production Decisions

These decisions assume the existence of plants but decide the exact path through which a product reaches the destination. These decisions include also which product has to be produced in which plant, which suppliers have to supply each individual plant and which customers have to be delivered from each plant. These decisions are influencing the lot sizing model, production scheduling on machines.⁹

3.1.3 Inventory Decisions

Inventory management can decrease or increase the cost of a product with 40%. Storage costs are high and safety stock costs are highly influenced the inventory policy. Not optimal acquisitions can lead to long storage periods and a significant increase in the cost of the product. There is no standard optimal inventory policy. It depends on the kind of product, the type of demand posted by customers and the level of costs for each cost type.

3.1.4 Transportation Decisions

Products logistic can be done through water, air or roads. Depending on the placement of the network design elements, it can be that not all these three types of transport are available. Depending on the kind of products, it is important the cheapest and effective mean of transportation to be used.

⁹ cf. Hasuike (2012)

3.2 Designing the Supply Chain Network

The goal of designing a supply chain network is to increase the profitability of the company but in the same time to satisfy customer needs regarding demand, services and responsiveness.¹⁰ There are several steps that have to be followed in order to reach the optimal Supply Chain Network.

Phase 1: The Definition of Regional Facility Configuration

Customers play an important role for each business. A forecast of demand has to be run by considering following factors: size of the demand, ABC and XYZ analyze to determine several characteristics of demand. These factors influence the location decision in terms of position and size. Economies of scale and economies of scope can also play an important role by defining the potential facilities of the network design.

Phase 2: The Selection of Potential Sites

In this phase, a set of several potential sites for facilities are analyzed. Both hard infrastructure requirements and soft infrastructure requirements have to be taken into consideration.

Phase 3: The Choice of Locations

The locations for each facility will be chosen from the set of potential sites from Phase 2. This decision influences the cost structure of the network design: transportation and warehousing costs both between plants and hubs and hubs and customers.

¹⁰ cf. Chopra, S., Meindl, P. 2010

3.3 Facility Location Models ¹¹

Companies are dealing with Facility Location Problems (FLP) whenever they want or are required to operate changes in their network design. Some companies are willing to start and cover other geographical areas while others just want to optimize their network costs: New companies that want to start any economic activity are also dealing with FLPs. FLP is a Supply Chain Management decision problem and works in a similar way independent from the type of facility that has to be chosen: distribution centers, plants, hubs, maritime ports. The complexity of the problem is given by the amount of different restrictions that have to be considered.

In case of a Discrete Facility Location Problem (DIFLP) demand occurs at specific geographical points. Here two discrete sets of demand as well as the possible locations are given. The quadratic assignment problem was introduced by Koopmans und Beckmann (1957) and describes an assignment problem: a set of customers has to be assigned to a set of facilities in order to generate customer's satisfaction with minimal costs.

$$\text{Minimize } Z = \sum_{i=1}^D \sum_{j=1}^D \sum_{k=1}^D \sum_{l=1}^D c_{ij} d_{kl} x_{ik} x_{jl} \quad (1)$$

Subject to:

$$\sum_{i=1}^m x_{ij} = 1 \quad j = 1, \dots, D \quad (2)$$

$$\sum_{j=1}^m x_{ij} = 1 \quad i = 1, \dots, D \quad (3)$$

$$x_{ij} = \begin{cases} 1 & \text{if facility } i \text{ is assigned to customers } j \\ 0 & \text{otherwise} \end{cases} \quad i, j = 1, \dots, D \quad (4)$$

¹¹ cf .A. B. Arabani, R. Z. 2012

D represents the total number of facilities and customers. C_{ij} symbolizes the costs of assigning facility i to customer j. d_{kl} denotes the distance between facility k and customer l. x_{ij} is an integer variable and shows if facility i is assign to customer j. All these elements are formulated in the minimization problem under (1). (2) and (3) describe the main constraints of the linear formulation: each customer has to be assigned to only one facility. This solution is applied in the plant location formulation and there is no know algorithm to solve this problem in polynomial time.

PLP deals with the assignment of a single facility from a subset of sources to a set of customers with a given demand. No plant capacity constraint is assumed. PLP is formulated as follows:

$$\text{Minimize } Z = \sum_{j \in F} C_j x_j + \alpha \sum_{j \in F} w_j d_{ij} y_{ij} \quad (5)$$

As in the previous model, the objective function has the purpose to minimize the total cost. C_j represents the cost incurred by facility j, w_j the transportation cost per unit of distance, d_{ij} the distance between plant i and facility j and α converse the demand total distance to cost unit. The constraints of this model are similar to (2)-(4). The PLP finds its application in computational geometry as geographical information systems or computer aided design (CAD).

In case of Network Facility Location Problem, demand occurs typically on nodes. In some cases, demand can occur on links and nodes on the same time. Taking these issues into consideration, the Network Facility Location Problem (NFLP) can be classified into five categories.

First Network Facility Location Problem is the median problem and it has its origin in Weber's Problem (1909) which is the first formulized Facility Location Problem in the literature. The p median model identifies locations for p new facilities in a certain space to supply n demand points so that the total distance is minimized. It is applied to find the optimal facility location of public or industrial facilities.

The Covering Problem was mentioned for the first time by Toregas, Swain, ReVelle and Bergman (1971). Customers can be served by a particular facility if they are located at a specific distance from each other, so called coverage distance.

The difference between the Covering Problem and the Center Problem is that the Center Problem searches for the facilities that fulfill all demands. At the same time, the distance between the facility and the customer should be minimal.

In a classical Hub Location Problem, a set of locations and a quantity of products that have to be transported from a location to another are given. The most important decision that needs to be taken is to decide which of these locations will become the hub.

In comparison with the above mentioned problem, the Hierarchical Location Problem consists of a distribution system with hierarchical facilities. The facilities on a higher level will select their locations independent from the facilities on a lower level. It is also assumed that the first ones have enough capacity in order to fulfill the demand. At the same time they have the handling and the transshipments costs proportional with the amount of products they are operating.

In comparison with the Discrete Problem, the facilities of a Continuous Problem can be located anywhere in a planning area. The performance of such models depends on the distance between facilities and customers and the continuous solution space in which facilities are allowed to be seated.

In the case of a SFLP the new facility has to be located in such a way that the distance between nodes is minimal. A model from this category was formulated by Wesolowsky (1973):

$$\text{Minimize } Z = \sum_{i \in F} w_i d_{X, P_i} \quad (6)$$

F represents the set of existing facilities; w_i comes out by transforming distance in costs, X and P denote the position of the existing facility and d_{XP_i} describes the distance between these two facilities.

The Multi Facility Location Problem is similar to the Single Facility Location Problem. In this case, several locations have to find their position in order to get the minimum distance in the network design. Each SFLP can be transformed in a MFLP:

$$\text{Minimize } Z = \sum_{i \in F} \sum_{j \in F} w_{ij} d_{X_j P_j} \quad (7)$$

Equation (7) is an extension of equation (6). w_{ij} represents the weight between facility i and j , X_j and P_i denote the position of the new facility j and the existing facility i .

The aim of Facility Location-Allocation Problem is not only to position optimally the facilities, but also to assign these facilities to different customers in order to get minimum costs and high demand satisfaction. One model was proposed by ReVelle and Swain (1970):

$$\text{Minimize } Z = \sum_{i \in D} \sum_{j \in F} C_{ij} y_{ij} \quad (8)$$

Subject to:

$$\sum_{j \in F} x_j = p \quad (9)$$

$$\sum_{j \in F} y_{ij} = 1 \quad \forall i \in D \quad (10)$$

$$x_j \geq y_{ij} \quad \forall i \in D, j \in F \quad (11)$$

The objective function minimizes the total network design costs, which are determined by the allocation of customers to facilities. Exactly p facilities are opened (9), each demand node is used (10) and all opened facilities are assigned to demand nodes (11).

3.4 Network Design Optimization – Traditional Model

Two network design optimization models will be presented in this chapter. The traditional model integrates warehousing and transportation costs. The new model takes also the inventory costs into consideration. The calculation of safety stock costs will be described later in this chapter.

The model is based on the mixed integer linear problem solved by Croxton and Zinn (2005). The formulation suffered some adaptations. No transshipments or cross docking are allowed and all deliveries are performed just in time. All potential factories will work and therefore, no factory costs are taken into consideration. An important role is played by warehousing costs because a hub that is not satisfying any customer demand will not generate any costs. The fix hub costs are constant, not depending on the amount of stored goods. The proposed network design structure is presented in Figure 3. Each plant can deliver products to any hub and each hub can serve any customer.

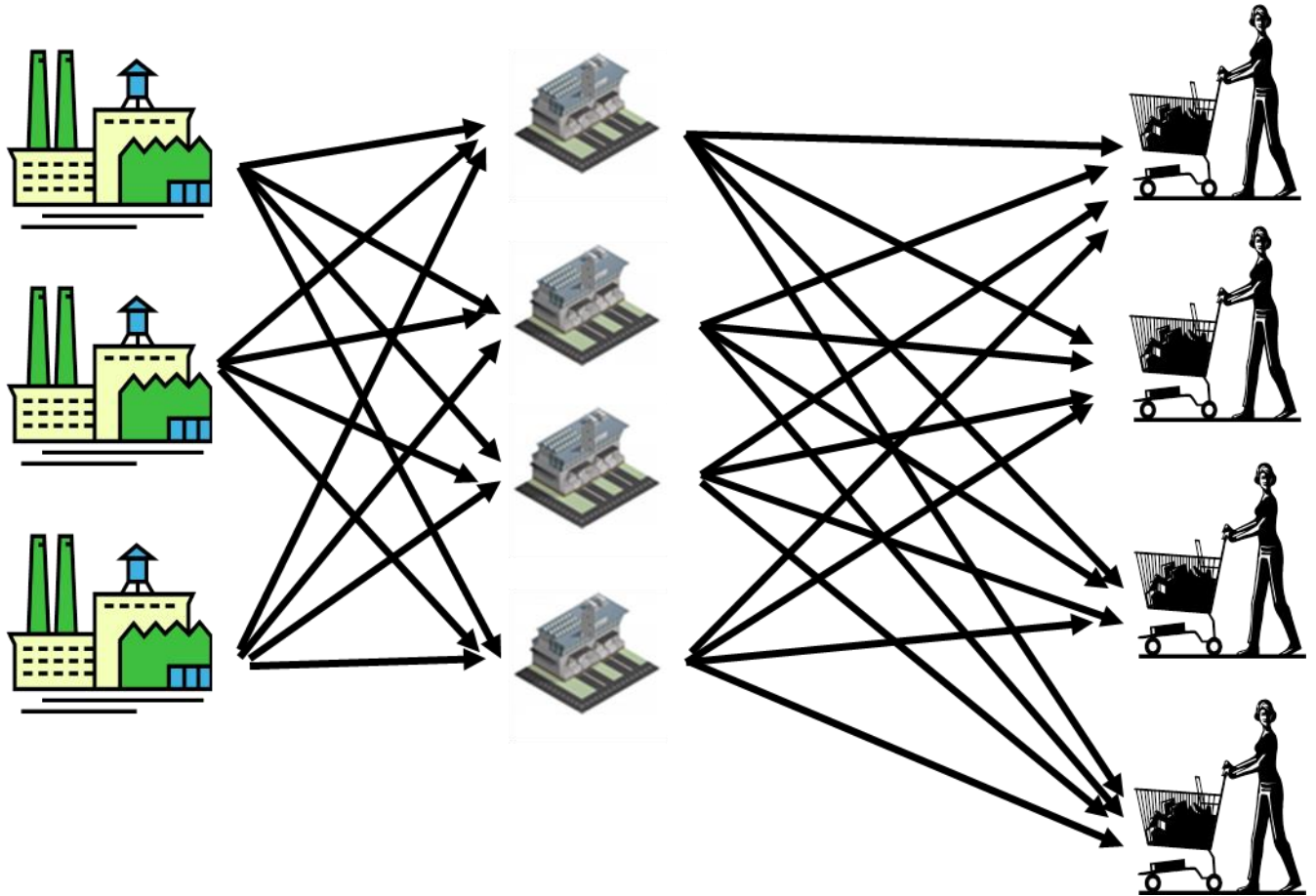


Figure 3 Network Design Elements and Flow

The needed data for the model and the variables are presented below.

Data:

T_{ij} = The transportation costs per unit between factory i and hub j

T_{jk} = The transportation costs per unit between hub j and customer k

F_j = Fixed warehousing costs

V_j = Variable warehousing costs per unit

D_{kp} = Demand from customer k in period p

P_j = Capacity of hub j

Variables:

X_{jk} = transportation units between hub j and customer k

V_{ij} = transportation units between factory i and hub j

Y_j = binary variable that shows if the hub j is opened or not

$j \in J$	hubs
$k \in K$	customers
$i \in I$	factories
$p \in P$	plants

The model was formulated considering the transportation amount each chain of the network. At the same time the decision if a hub is opened or not is declared to be the decision variables. One hub can deliver products to a customer only if it receives the necessary products from a factory.

The standard model is looking as follows:

$$\min \sum_{j \in J} F_j Y_j + \sum_{p \in P} \sum_{j \in J} \sum_{i \in I} T_{ijk} X_{jk} + \sum_{p \in P} \sum_{j \in J} \sum_{k \in K} T_{jpk} X_{jk} + \sum_{p \in P} \sum_{j \in J} \sum_{k \in K} C_j T_{ijk} \quad (12)$$

$$\sum_{j \in J} T_{jpk} = D_{kp}, \quad \forall k \in K, p \in P \quad (13)$$

$$\sum_{i \in I} T_{ijp} = \sum_{k \in K} T_{jkp}, \quad \forall j \in J, p \in P \quad (14)$$

$$\sum_{k \in K} \sum_{p \in P} T_{ijp} \leq P_j O_j \quad \forall j \in J \quad (15)$$

$$O_j \text{ is binary}, \quad \forall j \in J \quad (16)$$

The objective function (12) minimizes the fixed warehousing costs, the transportation costs between plants and hubs and between hubs and customers and the variable hub costs.

The aim of the objective function (13) is to minimize the fixed warehousing costs. It also has the role to reduce the transportation costs between plants and hubs as well as between hubs and customers. Not to be neglected are the variable hub costs, which should also be reduced.

All three decision variables can be found in this function.

The entire demand of the customer has to be satisfied in each period (14) and the products transported from plant i to hub j have to be transported from hub j to customer k (14). It is not allowed to overcome the capacity of a hub (15). The case if a hub is opened or closed is expressed in constraint (16). These constraints make sure that the customer service level remains high and that the hubs have on stock the products that should be delivered to customers.

Until now, no safety stock costs were considered. The next section will present relevant data and calculation of inventory costs.

3.5 Inventory Models in the Network Design

Apart of the warehousing and transportation costs, inventory costs have also to be taken into consideration. The safety stock costs can be simply calculated for an already existent supply chain design. In case of a Hub Location Problem, safety stock costs have to be estimated. Moreover, because of the square root, it is impossible to include these costs as a variable in the linear model. The only solution is to calculate separately the safety stock costs and to include them on a scenario basis in the linear program.

3.5.1 The Traditional SRL¹²

A diminishment of inventory levels will decrease costs and therefore, increase profit. The number of stock-keeping facilities influences the level of inventory among the distribution system. The first evidence regarding this subject can be found in a paper written by Smykay from 1973. The author states that the safety stock held at a single centralized location is equal to the aggregate safety stock held at multiple locations divided by the square root of the number of locations. Several economists adopted this issue by analyzing the above mentioned theory. The cost reduction reached by diminishing the number of facilities was also mentioned in further research papers and different square root laws were developed. Maister (1976) derived the first authentically version of the square root law (17).

$$\frac{\sqrt{m}}{\sqrt{n}}, \quad (17)$$

where m is number of locations after consolidation

and n number of locations before consolidation

¹² cf. Smykay, E. W. 1973

The square root law always influences the safety stock level, whereas cycle stocks only in case of use of EOQ policy. If cycle stocks are only depending on the mean of demand, their level will stay the same, no matter of the number of opened facilities.

Ballou proposed an empirical approach of Maister's work and his conclusion was that the square root law is too optimistic regarding the inventory savings. Zinn, Levy and Bowersox (1989) showed that Smykay square root law is a special case of their portfolio effect model. They took into consideration only the case when the number of locations is reduced to 1. Furthermore, Mahmoud, Tallon and other brought improvements to the square root law and in 1993 Everes and Bieier generalized the portfolio effect by considering also multiple locations and lead time variability. Therefore, the ratio of centralized safety stock to decentralized safety stock is equal to (18).

$$\frac{1}{\sqrt{n}} \quad (18)$$

Putting together all the above mentioned ideas expressed by scientists in the last years, it can be stated that the square root law estimates the safety stock savings from inventory centralization. These savings are proportional to the square root of the ratio of the new number of stocking locations over the original number of stocking locations. For example, if the original number of locations is 4 and the current number of location is 2, the safety stock is increased by a factor of square root of 0.5. Even if the majority of papers regarding the SRL are treating the centralization phenomena, this formula can be also used for calculating the increasing ratio of the safety stock as a result of decentralization. Maybe it sounds strange, but the general optimization does not always mean centralization, it can be also reached through decentralization. One case when decentralization is better than centralization is when the transportation costs are very high, representing an important percentage of the total cost of the product. In this case a possible decentralization should be considered. A more detailed explanation of these

phenomena will be given later in this thesis, by analyzing the network design and the main considerations that influences it.

3.5.2 Alternative Models¹³

The Square Root Law is very useful under certain assumptions. The most relevant ones are: the average total system demand remains the same, no transshipments between facilities are allowed, lead time are independent and normal distributed and last but not least all facilities use the safety factor and the standard deviation. These assumptions were all developed throughout time in order to improve the Square Root Law.

The extension of the Square Root Law with regard to both cycle and safety stocks leads to the consolidation effect which is the percent reduction in average total inventory made possible by the consolidation of inventory from multiple locations. The first formula of this concept is presented in (19).

$$CE = 1 - \frac{\sum_{j=1}^m \left(\frac{1}{2} CS_{a_j} + SS_{a_j} \right)}{\sum_{i=1}^n \left(\frac{1}{2} CS_{b_i} + SS_{b_i} \right)}, \quad (19)$$

CE = consolidation Effect

CS_{b_i} = cycle stock (order quatity)at decentralized location i

CS_{a_j} = cycle stock (order quatity)at centralized location j

SS_{a_j} = safety stock at centralized location j

SS_{b_i} = safety stock at decentralized location i

The consolidation effect has two parts: order quantity effect and portfolio effect.

¹³ cf. P. T., Evers, F. J. Beier 1993.

As defined by Evers (1995), the order quantity effect is quantified by the percent reduction in average total cycle stock due to consolidation. This rule can be applied only by using the EOQ system. The total cycle stock cost in a supply chain is higher in a decentralized setting than in a centralized one. In case of very low level of fixed ordering costs, the EOQ method is not useful anymore and it can be replaced by the replacement principle: always order the quantity that was delivered last period. In this case, we cannot talk about order quantity effect any more.

The portfolio effect is defined as the percent reduction in total safety stock due to consolidation. It provides a more precise estimation of safety stocks savings from inventory centralization by eliminating two of the assumptions of the square root law: the demand variance for an item is the same at all inventory location and sales for the same item in all inventory locations are independent. The numerical factor that makes this diminishment possible is the sum of the standard deviations that has a relevant role in the case of computing safety stocks. The sum of standard deviations of demand is greater or equal than the standard deviation of the sum of demand, when safety stock is centralized, the portfolio effect is observable.

3.5.3 An expanded SRL¹⁴

In order to maximize the consolidation effect the next linear program presented by (12) to (20) is used.

$$MAX: CE = 1 - \frac{\sum_{j=1}^m \sqrt{\frac{AD_b}{2C}} \sqrt{\sum_{i=1}^n W_{ij}} + k\theta_{D_k} \sqrt{L} \sqrt{\sum_{i=1}^n W_{ij}}}{n \left(\sqrt{\frac{AD_b}{2C}} + k\theta_{D_k} \sqrt{L} \right)} \quad (20)$$

$$ST: \sum_{i=1}^n (W_{ij}) = \frac{n}{m} \text{ for all } j \quad (21)$$

¹⁴ cf. P. T., Evers, F. J. Beier 1993.

$$\sum_{j=1}^m (W_{ij}) = 1 \text{ for all } i \quad (22)$$

$$W_{ij} \geq 0 \text{ for all } i \text{ and } j \quad (23)$$

The main function is the consolidation effect (20). Concerning constraints, (21) expresses that the ratio of the mean demand for each centralized facility is the same. The second one (22) assures that the ratio of mean demand transferred from each centralized location to decentralized location is one: all products have to be transferred to the central location. Concerning the third constraint (23), the mean demand transferred from decentralized locations to centralized location is greater than zero. This is actually the non-negativity constraint that is important in order to maintain the basic assumptions and set-up of the model.

After implementing and solving this linear program, the consolidation effect is optimized when all W_{ij} are equal to $1/m$ which demonstrate the findings of Evers and Beier when considering the consolidation of safety-stocks only. Substituting W_{ij} with $1/m$ in the main equation the following will result (24).

$$CE = 1 - \frac{m(\sqrt{\frac{AD_b}{2C}} \sqrt{\sum_{i=1}^n \frac{1}{m}} + k\theta_{D_k} \sqrt{L} \sqrt{\sum_{i=1}^n \frac{1}{m^2}})}{n(\sqrt{\frac{AD_k}{2C}} + k\theta_{D_k} \sqrt{L})} \quad (24)$$

The square root law can be applied for a network design only when all assumptions are fulfilled. Important to note is the Square Root Law can be applied only once. No further optimization is possible because the correlation of demands on the remaining locations is equal to 1.

The square root law will be applied in the next section to get a realistic value for the inventory costs in each possible scenario.

3.6 The new model

Apart from transportation and fix warehousing costs, hubs are facing inventory costs. An important part of the inventory costs are the safety stock costs. They are higher with each additional hub that is opened. In order to prove the effectiveness of the above described statement, the safety stock cost was introduced into the objective function.

The new model takes over all components and setups from the traditional model but it also includes the safety stock cost in the linear program (25).

$$\min \sum_{j \in J} F_j Y_j + \sum_{p \in P} \sum_{j \in J} \sum_{i \in I} T_{ijk} X_{jk} + \sum_{p \in P} \sum_{j \in J} \sum_{k \in K} T_{jkp} X_{jk} + \sum_{p \in P} \sum_{j \in J} \sum_{k \in K} C_j T_{ijk} + \mathbf{SST} \quad (25)$$

The safety stock cost was calculated with the SRL which implies a nonlinear component. The next step that has to be done is to calculate the square root for the SS for each scenario and then introduce it in the objective function as a number.

As mentioned in the part that includes the Square Root Law, the safety stock for n opened hubs is the square root of the opened hubs times the safety stock needed when only 1 hub is opened (26).

$$SS_n = SS_1 \sqrt{n} \quad (26)$$

Where:

SS_n is the total network wide safety stock required if inventory is decentralized in n

wahrehouses

SS₁ is the total safety stock required if inventory is centralized in a single wahrehouse

The safety stock in case only one hub has been opened, was calculated as the k factor times the standard deviation of the whole demand (27).

$$SS_1 = k\sigma \quad (27)$$

Where:

k is the safety factor for a 99,9% service level, the value is taken from the k table

σ is the satandard deviation of demand

A new constraint was introduced in order to respect the number of opened hubs taken as true in each scenario (28).

$$\sum_{j \in J} Y_j = n \quad (28)^{15}$$

where: Y_j is a binary variable that shows if the warehouse j is opened or not

n is the number of opened warehouses

The scenario with the smallest total cost will give the optimal network design structure. If the number of opened warehouses from traditional model will differ from the number of opened warehouses from the new model, the relevance of safety stock costs in the network design will be demonstrated.

¹⁵ cf. P. T., Evers 1995.

4. Study Case

The problem of network design optimization was also researched by Croxton and Zinn (2005). They solved a complex Facility Location Problem by determining firstly the hubs that have to be opened, secondly the distribution route from each plant to its optimal hub and from each hub to all customer and last but not least the quantity of products stocked at each hub.¹⁶ In the standard network design problem, the objective is to minimize the transportation and fixed warehousing costs. It should be considered that the incorporation of inventory costs often brings changes in the network design. The Square Root Law was applied in order to calculate the safety stock costs in each possible scenario. These costs cannot be included in the main function because the square root does not have a place in a linear program. A mixed-integer linear program is used to get the network design configuration. This one delivers minimal total costs. These optimal results will be generated with the help of Xpress.

The current case study deals with the medium term optimization of an existing network structure.¹⁷ It is assumed that the plants remain unchanged, whereas there are no constraints for hubs. We can choose to work with any of these hubs but we have to be able to satisfy the whole customer demand.

4.1 The Data

It was a challenge to get the whole needed data for a general optimization model. The transportation costs between factory and hub and between hub and customer were given explicitly, whereas the warehousing costs and the capacity of each hub were

¹⁶ cf. R. G. Askin, I. Baffo, M. Xia. 2013.

¹⁷ cf. M. Preusser, 2008

estimated in relation to the given data. The transportation costs were calculated by multiplying the distance in km between facilities with the cost per km per product. All hubs have the same capacity because we want to simulate similar situations for all hubs.

As mentioned in the paper of Croxton and Zinn (2005), the model includes several decision variables: firstly the hubs that will be opened, secondly which customer will be delivered from which hub and finally which hub will be delivered from which plant.

The given network structure consists of a set P of European plants with elements going from 1 to 4 which deliver one product to customers situated also in Europe.¹⁸ All deliveries have to be fulfilled through a set H of 14 possible hubs. No direct delivery between plant and customer is allowed. No hub has sufficient capacity to satisfy the entire demand. The demand generated by a set C of customers from 1 to 167 was provided on a daily basis for a period of one year. The plants and customers are fixed, whereas any combination of the fourteen hubs is possible. The Supply Chain structure is shown in Figure 4.

¹⁸ cf. M. Preusser, 2008

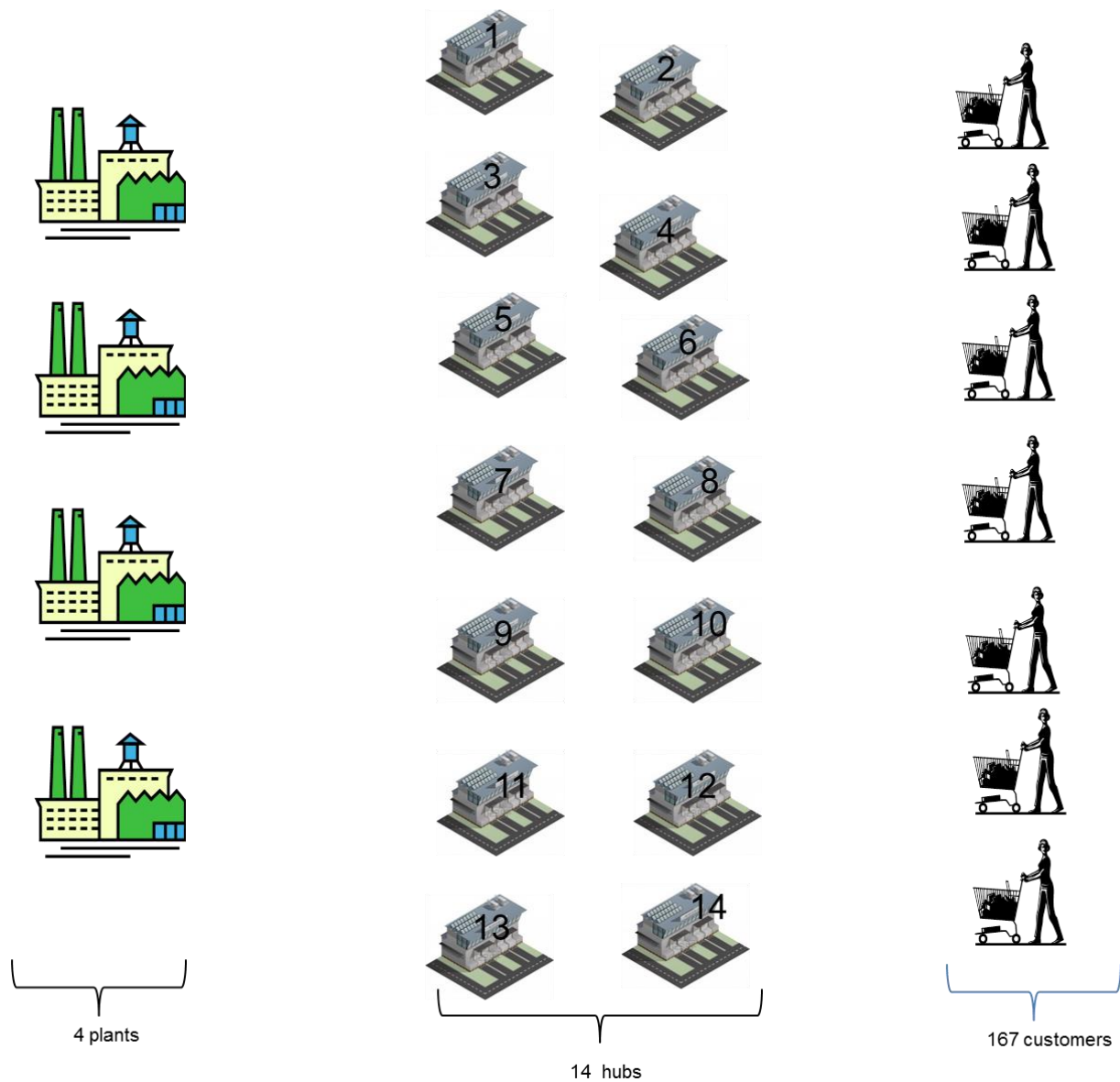


Figure 4: Supply Chain Design

Following data is given: the transportation costs between plants and hubs, the transportation costs between hubs and customers, fixed and variable warehousing costs, the capacity of each hub and the demand of each customer. No plant capacity is taken into consideration. Table 1 is an excerpt of the transportation cost between plant and hub.

Hub\Plant	P1	P2	P3	P4
H1	24,61	100,31	50,29	43,09
H2	42,80	63,15	63,23	83,75
H3	11,50	40,09	27,96	37,90
H4	32,08	95,06	41,18	35,50
H5	38,60	89,76	39,93	30,00
H6	76,55	125,83	62,79	72,50

Table 1 Excerpt from Transportation Costs between Plants and Hubs

The optimization problem of this model was implemented as described in section 3.3.

4.2 Traditional Model - Solutions and Interpretation

Xpress was run by integrating the above mentioned conditions and the first step was to check the feasibility of the provided solutions. Table 2 shows the values for two decision variables: transported units between plants and hubs and between hubs and customers. For each position the following informations are given: plant, hub, customer and period. The solutions integrate zero and non-zero values. In the case of zero values, no product delivery occurred. The capacity of each hub was also taken into consideration in generating the solution.

Transported units Factory-Hubs				Transported units Factory-Hubs			
<i>Factory</i>	<i>Hubs</i>	<i>Period</i>	<i>Units</i>	<i>Hubs</i>	<i>Customers</i>	<i>Period</i>	<i>Units</i>
1	3	2	48	3	1	2	48
1	3	5	21	3	1	5	21
1	3	6	105	3	1	6	105
1	3	7	49	3	1	7	49
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...

Table 2 Optimization results transported units

The deliveries from plants to hubs were generated by the demand of the customers. Both the transportation costs plants-hubs, hubs-customers and hub costs of the hub are taken into consideration by assigning customers to hubs and hubs to plants. The 100% demand satisfaction of customers was also reached.

The results of the most important decision variables are presented in Table 3. The main question of the model is the number of hubs that have to operate in order to minimize the overall costs.¹⁹ This question will be answered by taking into consideration all costs generated by all operations. Only the hubs number 3, 4, 8 and 10 are functioning. Hub 8 has the smallest fixed cost. The fixed costs of the other three hubs find themselves in the upper segment while the variable warehousing costs are placed in the medium segment. This means that the transportation costs play also an important role in this decision making process.

The goal of the model is to get the minimum possible total cost by respecting the model setups. The total cost of operating the most efficient network design is 9.15056E+07.

¹⁹ cf. K. L. Croxton , .W. Zinn, 2005.

Opened Hubs		Solution Xpress
Hubs	Open/Closed	9,15056E+12
1	0	
2	0	
3	1	
4	1	
5	0	
6	0	
7	0	
8	1	
9	0	
10	1	
11	0	
12	0	
13	0	
14	0	

Table 3 Optimization results total cost

The evolution of total costs and network structure will be analyzed in the next part of this thesis. Compared to the above presented traditional model, the updated model will include one additional constraint (29), namely the variable n which shows how many hubs have to be opened. In order to go through all possibilities, 12 scenarios have been implemented. The scenario setups differ through the number of operating hubs. All other program setups mentioned above stay the same.

$$\sum_{j \in J} o_j = n, \quad n \text{ is the number of opened warehouses} \quad (29)$$

Table 4 is presenting the results of the 12 scenarios. The column *Opened W* is actually representing n opened hubs depicted from the equation (29). For example, Scenario 4

includes 4 values in the additional constraint; this means that a number of 4 hubs will be opened.

Opened H	Which Hubs are opened	Costs without Inventory costs
1	∞	∞
2	3,4	9,33832E+07
3	3,4,10	9,19782E+07
4	3,4,8,10	9,15056E+07
5	3,4,8,10,12	9,15079E+07
6	2,3,4,8,10,12	9,15104E+07
7	2,3,4,7,8,10,12	9,15131E+07
8	2,3,4,5,7,8,10,12	9,15158E+07
9	1,2,3,4,5,7,8,10,12	9,15188E+07
10	1,2,3,4,5,7,8,9,10,12	9,15218E+07
11	1,2,3,4,5,7,8,9,10,11,12	9,15251E+07
12	1,2,3,4,5,6,7,8,9,10,11,12	9,15286E+07
13	1,2,3,4,5,6,7,8,9,10,11,12,14	9,15324E+07
14	1,2,3,4,5,6,7,8,9,10,11,12,13,14	9,15364E+07

Table 4 Results without Safety Stock Costs

The first scenario is not able to provide a feasible solution because no hub has enough capacity to deliver all the necessary goods in order to satisfy in proportion of 100% the whole demand. The network structure from scenario 2 provides the highest total cost. In this case the transportation costs are far too high.

The costs are decreasing from scenario 2 to Scenario 4. As already observed in Table 3, scenario number 4 is providing the best solution resulting in the smallest overall cost. From scenario 5 to scenario 14, total cost is increasing slowly. This phenomenon is a consequence of the fix hub costs. This experiment can also be considered a check of the optimal solution.

By modeling the supply chain as showed in the optimal solution, its representation will look as in Figure 5.

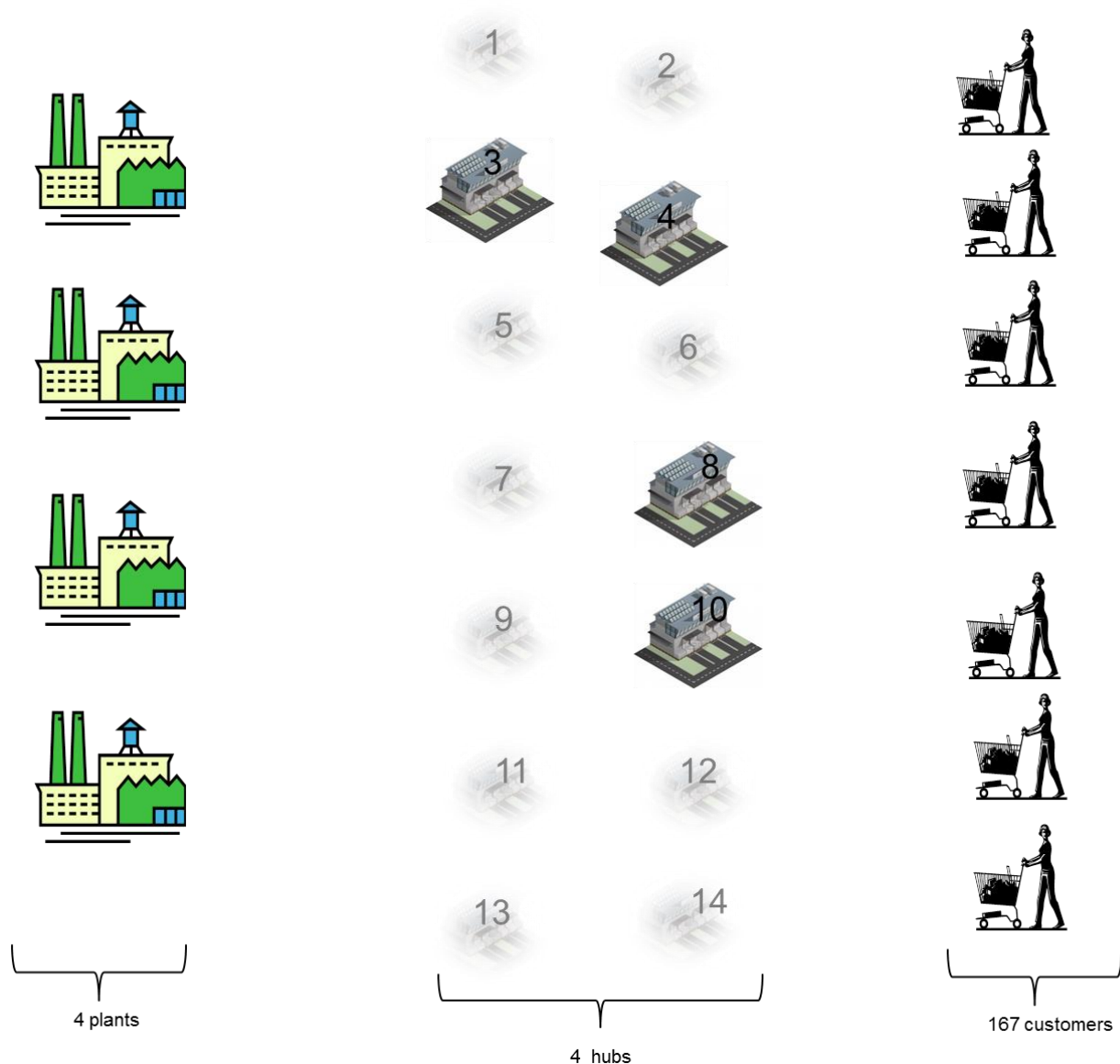


Figure 5: Optimal network design - traditional model

All hubs apart from 3, 4, 8 and 10 will not exist anymore in the network structure. As already mentioned in the model setup, all four plants are active and all customer demands will be satisfied.

4.3 The New Model

The linear problem for the study case was formulated as in section 3.5. The safety stock costs will be calculated as shown in section 3.4 and the new results will be generated from Xpress as for the traditional model.

4.3.1 Safety Stock Calculation

The safety stock was included in the program as a discrete value. Because of the 14 given hubs, 14 scenarios were taken into consideration. The number of scenarios is giving the number of opened hubs: Scenario 1: 1 Hub was opened, Scenario 2: 2 hubs were opened, and so on until 14. For each scenario the safety stock was calculated separately.

Equation (16) was applied in order to calculate the safety stock costs for one global location. The results are presented in Table 5.

Standard Deviation	1431809,49
Service Level 99,9%	3,09
Safety Stock	4424291,32

Table 5 Safety Stock calculation

Standard deviation was calculated for the entire range of demand from all customers in all periods. The model is supposed to satisfy the entire demand of all customers. Therefore, a service level of 99,9% was taken into consideration. The k factor was taken from the k Table (Appendix 1: Table of k values for calculating the safety stock).

4.3.2 Solutions and Interpretations

Xpress was run 14 times with a different objective function. The safety stock cost was added as an absolute value to the objective function and a new constraint was introduced in order to respect the number of opened hubs taken as true in each scenario (30).

$$\sum_{j=1}^{14} Y_j = n \quad (30)^{20}$$

where: Y_j is a binary variable that shows if the warehouse j is opened or not

n is the number of opened warehouses

²⁰ P. T., Evers, 1995.

	Additional value to the objective Function	Additional Constraint
1	4424291,32	$\sum_{j=1}^{14} Y_j = 1$
2	6256892.786	$\sum_{j=1}^{14} Y_j = 2$
3	7663097.35	$\sum_{j=1}^{14} Y_j = 3$
4	8848582.636	$\sum_{j=1}^{14} Y_j = 4$
5	9893016.139	$\sum_{j=1}^{14} Y_j = 5$
6	10837256.2	$\sum_{j=1}^{14} Y_j = 6$
7	11705574.55	$\sum_{j=1}^{14} Y_j = 7$
8	12513785.57	$\sum_{j=1}^{14} Y_j = 8$
9	13272873.95	$\sum_{j=1}^{14} Y_j = 9$
10	13990837.6	$\sum_{j=1}^{14} Y_j = 10$
11	14673714.26	$\sum_{j=1}^{14} Y_j = 11$
12	15326194.7	$\sum_{j=1}^{14} Y_j = 12$
13	15952009.2	$\sum_{j=1}^{14} Y_j = 13$
14	16554182.29	$\sum_{j=1}^{14} Y_j = 14$

Table 6 Scenarios for the SRL

The different elements of the new model in comparison with the traditional one are structured in Table 6.

Table 7 presents the results according to the simulation of the 14 scenarios generated by Xpress.

The values in the first scenario are ∞ . This implies that it is impossible to fulfill the entire demand when only one hub is opened because of the hub capacity constraint.

The optimal number of hubs that should be opened is two because it generates the minimum costs for the entire network design. In the traditional model, the optimal solution was delivered by Scenario 4: which consisted in four opened hubs (Table 7).

Opened H	Which Hubs are opened	Total Cost taking the inventory costs (SRL) into consideration
1	∞	∞
2	3,4	9,96401E+07
3	3,4,10	9,96413E+07
4	3,4,8,10	1,00354E+08
5	3,4,8,10,12	1,01401E+08
6	2,3,4,8,10,12	1,02348E+08
7	2,3,4,7,8,10,12	1,03219E+08
8	2,3,4,5,7,8,10,12	1,04030E+08
9	1,2,3,4,5,7,8,10,12	1,04792E+08
10	1,2,3,4,5,7,8,9,10,12	1,05513E+08
11	1,2,3,4,5,7,8,9,10,11,12	1,06199E+08
12	1,2,3,4,5,6,7,8,9,10,11,12	1,06855E+08
13	1,2,3,4,5,6,7,8,9,10,11,12,14	1,07484E+08
14	1,2,3,4,5,6,7,8,9,10,11,12,13,14	1,08091E+08

Table 7 Results with SRL

By taking the safety stock costs into consideration, the structure of the supply chain will receive a new shape. Hubs 3 and 4 stays the same, but no other additional hub was chosen to be opened. Decentralization brings in this case an increase in safety stock costs and as consequence, an increase in the total cost.

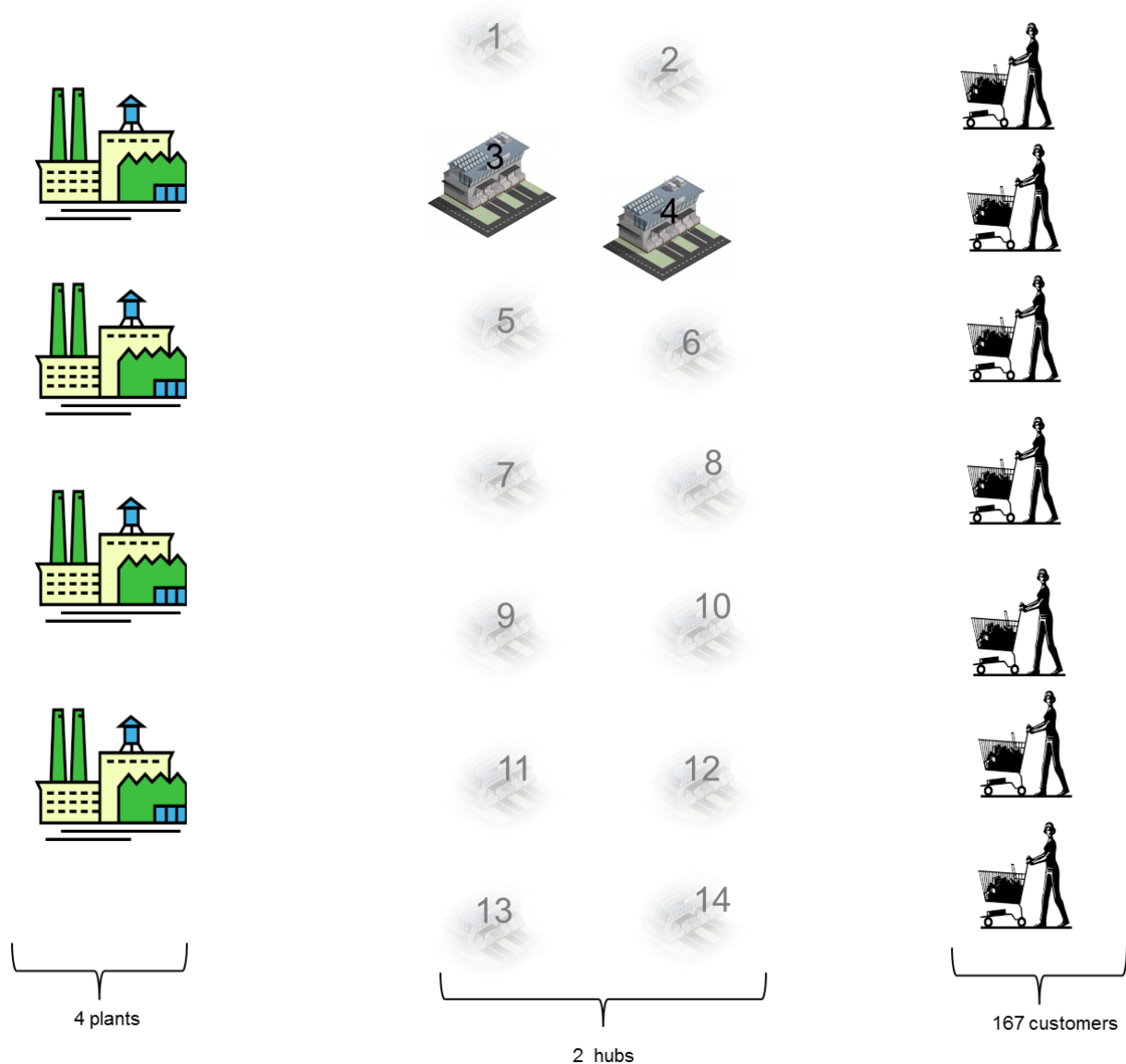


Figure 6: Optimal network design - the new model

Even if the new configuration leads to higher transportation costs, this increase in costs is lower than the cost generated by opening any additional hub. The optimal network structure is presented in Figure 6.

4.3.3 Safety Stock Check

The next step is to check if the previously calculated safety stock is reliable in case of using the square root law. Each of the two optimal hubs was associated individually with other possible hub and 26 scenarios were simulated. The safety stock for each scenario was calculated by using Equation 18.

Delivered quantities from each hub were taken into consideration when calculating the standard deviation.

The standard deviation was then multiplied with the safety factor k (3.09). The individual safety stock costs for each of the two hubs are represented in Table 8.

Hub 3		Hub 4	
Standard Deviation	182533.6057	Standard Deviation	144424.5976
Service Level 99%	3.09	Service Level 99%	3.09
Safety Stock Costs	5640288.416	Safety Stock Costs	4462720.067

Table 8 Safety Stock Results

Summing up the values of the two safety stocks delivers a total cost of 10103008.48.

Comparing the safety stock costs calculated by taking into consideration the Square Root Law with the safety stock costs calculated without taking the Square Root Law comes to a difference of 38%. A more detailed view of these costs is presented in Table 9.

Safety Stock total cost (without SRL):	10103008.48
Safety Stock total cost (with SRL):	6256892.786
Difference (Percentage)	0.380690138
Total cost (without SRL):	1.03486E+08
Total cost (with SRL):	9.96401E+07
Difference (Percentage)	0.037163481

Table 9 Optimal Results new model detailed

Moreover, the total cost of the network structure including the safety stock calculated with the SRL is only 3.7% lower than the total cost of the network by calculating the safety stock discrete. The difference of 38% between the safety stock values does not affect the total cost of the network design. Therefore it can be said that the safety stock cost approximation is reasonable.

In order to check the relevance of these results, all combinations between hub 3 and all other hubs have to be taken into consideration. Because the combination hub 3 and 3 does not exist, 13 scenarios remain for discussion.

The program was run two times for each scenario with different value settings: the first case takes into consideration the safety stock calculated with the SRL. In the second case, the safety stock was calculated discrete: the first case of the program was run and the safety stock of each individual hub was calculated using the formula (28).

Scenario	Hubs	Total cost (without SRL):	Total cost (with SRL):	Difference (Percentage)
1	3,1	1.15854E+08	1.11714E+08	0.036
2	3,2	1.67136E+08	1.63691E+08	0.021
3	3,4	1.03486E+08	9.96401E+07	0.037
4	3,5	1.10529E+08	1.06490E+08	0.037
5	3,6	2.29881E+08	2.26501E+08	0.015
6	3,7	1.98409E+08	1.94232E+08	0.021
7	3,8	1.02954E+08	9.97354E+07	0.031
8	3,9	1.94807E+08	1.90931E+08	0.020
9	3,10	1.03522E+08	1.00714E+08	0.027
10	3,11	1.95683E+08	1.92250E+08	0.018
11	3,12	1.54489E+08	1.50150E+08	0.028
12	3,13	1.33864E+08	1.29801E+08	0.030
13	3,14	1.52198E+08	1.48301E+08	0.026

Table 10 Scenarios Hub 3

Looking at the above mentioned results in Table 10, the total cost without SRL between scenario 7 and scenario 3 (the optimal solution) differs with just 0.51%. This result is predictable because of the small difference between the total cost with the SRL in

scenario 3 and 7. All other scenarios generate a higher cost compared to the optimal scenario.

Scenario	Hubs	Total cost (without SRL):	Total cost (with SRL):	Difference (Percentage)
1	4,1	1.52198E+08	1.48301E+08	0.026
2	4,2	2.23609E+08	2.21531E+08	0.009
3	4,3	1.03486E+08	9.96401E+07	0.037
4	4,5	1.71663E+08	1.68070E+08	0.021
5	4,6	2.91354E+08	2.87843E+08	0.012
6	4,7	2.53413E+08	2.51416E+08	0.008
7	4,8	1.61369E+08	1.57354E+08	0.025
8	4,9	2.50518E+08	2.48494E+08	0.008
9	4,10	1.60563E+08	1.56234E+08	0.027
10	4,11	2.52703E+08	2.49853E+08	0.011
11	4,12	2.15378E+08	2.11293E+08	0.019
12	4,13	1.89879E+08	1.86858E+08	0.016
13	4,14	2.07677E+08	2.04720E+08	0.014

Table 11 Scenarios Hub 4

In case of hub 4, all total costs without SRL are higher than the optimal solution, scenario 3 (Table 11).

The inclusion of the safety stock costs (as an active cost) in the network design will change in most of the cases the configuration of the network. The result of the study case by applying the traditional model consists of four opened hubs, whereas in the new model two hubs have to be opened. The Square Root Law approximates the costs of the safety stocks and makes the inclusion of these costs in the linear programming possible. The last information from the last two tables was generated in order to check the reliability of the Square Root Law.

Analyzing both cases and considering all scenarios, the difference of 5% in total costs is considered to be reasonable.

5. Conclusion

Croxton and Zinn (2005) proposed a linear model in order to solve a hub location problem for a company with a complex network design. The data that I used in my work was different to the one used by the above mentioned authors, but the main structure of the network design remains similar: facilities, hubs and customers. Plants and customers are considered fix and several potential locations for hubs and their specific costs are given. The program has to decide which of these facilities will be opened in order to minimize the costs.

Transportation costs, fix and variable hub costs can be easily calculated. Apart from all these costs, inventory costs have also to be taken into consideration and therefore a new model was developed. In the case of safety stock costs, the calculation is more complex because each different combination of opened hubs will derive different safety stock costs. The square root law has been used in order to calculate a general safety stock factor for a certain number of opened hubs. Because of the nonlinear component of the square root, we cannot introduce the safety stock costs in the objective function in form of decision variables. The only solution is to introduce them as numbers and to generate these safety stock costs for each possible scenario.

The reliability of the safety stock costs was checked for the optimal solution by calculating the effective safety stock costs for all combinations between the optimal solution hubs and all other hubs. The result was differing with a few percent as absolute value, but the optimal hubs stay the same.

The traditional model was simulated and the optimal solution consists of four opened hubs. The new model delivered only two opened hubs as optimal. Therefore, the inclusion of safety stock costs in the optimization model has a big influence in the structure of the network design and in the approximation of the total optimal costs. An incomplete data input in such a strategic decision can lead to high supply chain management costs.

6. Appendix

F(k)	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900

Appendix 1: Table of k values for calculating the safety stock

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8. Abstract

Supply Chain Management and Logistics are becoming more and more important due to the diversification of the needs of the consumers and the explosion of economical processes. A large number of companies are producing and distributing goods all over the world. Globalization and competition lead to a strong need of optimization.

Costs optimization is the goal of each company. It can be achieved on a daily basis through intelligent acquisition, lean production and economies of scales or scopes but high costs can also be avoided by strategic decisions that have a long term influence on the success of a company. Tactical decisions such as service level and mode of transportation and operational decisions as order size and lead time have to be taken into consideration by establishing facilities location and customer allocation.

The objective of this paper is to demonstrate that inventory costs can change the structure of the optimal network design because of the significant influence brought to the cost structure of the entire supply chain. The traditional approach considers only warehousing and transportation cost. The new model takes also safety stock costs into consideration. Croxton and Zinn (2005) solved this problem but they present only the model setups and not the method of safety stock calculation. These optimization problems are solved with help of Xpress and are formulated as linear program.

Transportation costs and warehousing costs were estimated and easily integrated in the model. Safety stock costs can take different values for each combination of opened warehouses. The square root law was used to calculate an approximation of safety stock cost for each possible number of opened hubs. Because of the square root, it is impossible to integrate this cost factor in the objective function as a decision variable. Therefore, its value was added as a number that was calculated individually for each scenario.

The correctness of safety stock approximation was successfully checked by calculating the effective safety stock costs for all combinations of opened hubs as in the optimal

solution and all other hubs. The traditional model suggests four warehouses to be opened and the new model indicates two warehouses. Therefore, the inclusions of the safety stock costs offers a realistic image of supply chain costs and influence the whole network design structure.

9. Zusammenfassung

Aufgrund von steigenden Bedürfnissen der Kunden und anspruchsvolleren Anforderungen im Bereich betriebswirtschaftlicher Prozesse werden Lieferkettenmanagement und Logistik zunehmend wichtiger für Unternehmen. Immer mehr Unternehmen produzieren und vertreiben ihre Güter auf dem globalen Markt. Globalisierung und Wettbewerb zwingen die Unternehmen die Geschäftsprozesse besser aufeinander abzustimmen.

Kostenoptimierung ist das große Ziel eines jeden Unternehmens. Dieses Ziel kann im Bereich der täglichen Einkaufspolitik und Massenproduktion erreicht werden. Um einen optimalen Erfolg zu erzielen und eine langfristige Senkung der Kosten zu erreichen bedarf es einer strategischen Planung im Hinblick auf die Entscheidungsprozesse. Taktische und operative Entscheidungen in Bezug auf die Produktverfügbarkeit und den Transportwesen müssen bei der Standortwahl und der Kundenzuordnung berücksichtigt werden.

Das Ziel dieser Abhandlung ist es zu erörtern, dass Lagerkosten und alle in der Beschaffungskette eines Betriebes fallenden Kosten die Struktur des Optimal Network Design stark beeinflussen. Die traditionelle Geschäftspolitik berücksichtigt lediglich Lager – und Transportkosten. Das neue Modell kalkuliert aber auch den Sicherheitsbestand im Lager ein. Croxton und Zinn (2005) lösten dieses Problem, doch sie präsentierten nur den Aufbau des neuen Modells aber nicht die Methode der Kalkulation des Sicherheitsbestandes im Lager eines Betriebes. Diese Probleme der Optimierung kann mit Hilfe des linear formulierten Programmes Xpress gelöst werden.

Kosten für Transport und Lagerung werden somit in dem Modell integriert. Kosten für den Mindestbestand (Sicherheitsbestand) können abhängig von der Art der Lagerung verschiedene Werte aufweisen. Unter Verwendung der Quadratwurzel Regel wird die Näherungsrechnung der Kosten für den Mindestbestand durchgeführt. Die Quadratwurzel erlaubt es nicht diesen Kostenfaktor in der objektiven Funktion als eine

Entscheidungsvariable zu integrieren. Deshalb ist der Wert des Mindestbestandes individuell für jedes mögliche Szenario zu kalkulieren.

Die Korrektur des Näherungswertes für den Mindestbestand (Sicherheitsbestand) wurde erfolgreich durch die Berechnung der effektiven Kosten für den Mindestbestand aller Güterverteilzentren durchgeführt. Im Gegensatz zum traditionellen Modell, das 4 offene Verteilzentren berücksichtigt, empfiehlt das neue Modell nur zwei Lagerhäuser. Deshalb bietet die Einbeziehung der Kosten für die Menge des Sicherheitsbestandes in die Kalkulation eine realistischere Aufzeichnung der Kosten für die gesamte Lieferkette und beeinflusst somit die gesamte Netzwerk-Design Struktur.

10. Lebenslauf

Ausbildung

Seit 09/2010	Universität Wien, BWZ Magisterstudium, Studienrichtung Betriebswirtschaft Spezialisierung: Supply Chain Management und Industrial Management
10/2009-04/2010	Katholische Universität Eichstätt Ingolstadt , Ingolstadt, Deutschland Erasmus Stipendium während des Bachelor Studiums
10/2006-07/2010	Universität von Oradea , Oradea, Rumänien Bachelor der Sozialwissenschaften in Finanzen Studienrichtung: Wirtschaftswissenschaften Spezialisierungen: Finanzen und Banken
10/2002-07/2006	Gymnasium „Emanuil Gojdu“ , Oradea, Rumänien Matura Generelle Grundschulausbildung mit Gymnasiumabschluss Spezialisierungen: Mathematik und Informatik

Berufserfahrung

Seit 08/2012	AIDA Produktionsges.mbH & Co KG , Wien, Österreich (Lebensmittel Branche, FMCG) Projekt Manager – Supply Chain Management
07/2011-08/2011	Atlas GmbH , Lodz, Polen (Baumaterialien Branche) Praktikum im Einkauf/Logistik, Controlling, Vertrieb und Kundenservice
07/2010-10/2010	Romcolor AG , Oradea, Rumänien (Baumaterialien Branche) Praktikum im Logistikbereich
05/2009-06/2009	BCR Erste Bank , Oradea, Rumänien (Bankwesen) Praktikum

Computerkenntnisse

Microsoft Office (Word, Excel, Power Point), **C++**, **Anylogic** (Managerial simulation),
Xpress (Linear Programming), **SAP Beraterzertifizierung (TERP 10)**

Sprachkenntnisse

Rumänisch	Muttersprache
Deutsch	fließend in Wort und Schrift (ÖSD Zertifikat)
Englisch	fließend in Wort und Schrift (Cambridge Zertifikat)