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„The Effect of Variable Accretion on the Vertical
Structure of Protostellar Disks“

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Abstract

This thesis considers the impact of FUori-like luminosity bursts on the disk vertical structure. In the early stages of star formation the accretion rate onto the star is a highly variable process, where intense bursts of accretion $\geq 10^{-4} M_{\odot}/yr$ can occur. These high accretion rates lead to a sharp increase in luminosity by several orders of magnitude as observed in FU Orionis stars. Based on the two-dimensional, thin-disk, hydrodynamical simulation of Vorobyov & Basu (2006, 2010a), I reconstruct the disk vertical structure at such a luminosity burst and during the subsequent quiescent phase, using coupled equations of local hydrostatic equilibrium and vertical radiation transfer. The results show that the temperature of the disk is significantly increased during the burst, leading to an extended vertical disk structure and lower gas volume densities near the midplane. The requirement of the thin-disk approximation is fulfilled during the burst up to several hundred AU . I find that the disk scale height is significantly reduced at the positions of fragments due to self-gravity, which helps to shield fragments from stellar irradiation. This may increase the likelihood of survival of fragments. Different methods for the reconstruction of the vertical disk structure are tested and the algorithm based on the Henvey method has shown an order of magnitude faster convergence than the shooting and bisection algorithm.

Zusammenfassung

Diese Arbeit befasst sich mit dem Einfluss von FU Orionis ähnlichen Helligkeitsausbrüchen auf die vertikale Struktur einer protostellaren Scheibe. FU Orionis Sterne sind junge Objekte, die einen abrupten Anstieg in ihrer Helligkeit von einigen Größenklassen zeigen. Dieses Phänomen kann durch hohe Massennakkretionsraten von etwa $10^{-4} M_{\odot} / \text{Jahr}$ in der Frühphase der Sternentstehung erklärt werden. Die wesentlichen Scheibenparameter während eines FU Orionis ähnlichen Helligkeitsausbruches werden durch die zwei-dimensionalen, hydrodynamischen Simulation von Vorobyov & Basu (2006, 2010a) berechnet. Dieser Simulation liegt die Annahme einer dünnen Scheibe zu Grunde. Unter der Annahme eines vertikalen thermischen und hydrostatischen Gleichgewichts werden die Gleichungen für den Strahlungstransport gelöst und die vertikale Struktur der Scheibe rekonstruiert. Die Ergebnisse zeigen, dass die vertikale Struktur der Scheibe während eines Helligkeitsausbruches mehr ausgedehnt ist und die Dichten nahe der Mittelebene stark abnimmt. Es wird gezeigt, dass die Annahme einer dünnen Scheibe trotz dieser Ausdehnung gerechtfertigt ist. Die vertikale Dichteverteilung der Scheibe wird basierend auf der Henvey-Methode rekonstruiert, die eine um den Faktor 10 bessere Konvergenz zeigt als der "Shooting-Bisection" Algorithmus.

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1 Introduction

FU Orionis stars (FUors) are characterized by a sharp increase in luminosity by several orders of magnitude (Hartmann & Kenyon, 1996). In 1936 FU Ori, the prototype of this class of stars, showed a steep rise in brightness of about 6 *mag* (Herbig 1966). In the subsequent 70 years this brightening slowly faded by about 1 *mag*. The high Li I abundance and the strong infrared (IR) excess emission in the spectral energy distribution of FUors are indications of the youth of these objects. At the moment, there are more than 10 confirmed FUors.

The phenomenon of FUors made scientists realize that the formerly believed smooth mass accretion onto the protostar may be instead a highly variable process during the early embedded phase of star formation, where the accretion rate can reach $10^{-4} M_{\odot} \text{yr}^{-1}$ and more (Hartmann & Kenyon, 1996). This high accretion rates can be explained by the triggering of disk instabilities. Various sources are discussed in the literature as possible trigger mechanisms, e.g. thermal instability (Bell & Lin 1994), magneto-rotational instability (Armitage et al. 2001, Zhu et al. 2009) and disk fragmentation (Vorobyov & Basu 2006).

The disk fragmentation mechanism for FUori luminosity outbursts can be summarized as follows. Protostellar disks in the early stages of star formation are often gravitational unstable, which leads to the development of a distinct spiral structure. If the Toomre Q-parameter becomes lower than unity and the local cooling time is shorter than the dynamical time, the spiral arms can fragment into gravitationally bound clumps. The infalling matter from the surrounding parental core onto the disk often aids to this process. Due to gravitational exchange of angular momentum with the spiral arms the fragments can be driven onto the protostar, leading to a temporal brightening of the star by several orders of magnitude (Vorobyov & Basu 2006, 2010a).

To study the effects of luminosity bursts of protostars on the vertical structure of a protostellar disk, I have chosen the disk fragmentation model of Vorobyov & Basu (2010a), who used the thin-disk approximation to compute the formation and evolution of protostellar disks. This approximation allows us to follow the disk evolution in a dynamic range of spatial and temporal scales that is much larger than in full three-dimensional simulations. Starting from the gravitational collapse of a starless cloud, the model follows its evolution until the T Tauri phase, when most of the envelope has accreted onto the disk and star system.

The vertical structure of the disk is reconstructed based on a modified de-

scription of Akimkin et al. (2012), where a vertical thermal and hydrostatic equilibrium is assumed. The spectral ranges are split into a IR and ultra-violet (UV) part and described separately, as the disk is illuminated by the star in the UV and radiates in the IR. The Eddington approximation and the two-stream approximation (Mihalas 1987) are used to solve for the radiative transfer equations.

This thesis is structured as follows. An introduction to FUors including historical and observational facts, as well as an outline of possible trigger mechanism, are provided in Chapter 2. The brief description of the two dimensional hydrodynamical model of Vorobyov & Basu (2006) and the justification of the thin-disk approach are given in Chapter 3. Chapter 4 contains a detailed model description of the reconstruction of the disk vertical structure. In Chapter 5, the initial model and the obtained results are given. Chapter 6 gives a summary of the method and the obtained results. The Appendix includes the derivation and a detailed description of the method used for the reconstruction of the vertical density distribution, the derivations of the basic radiative transfer equations and a description of the used opacities and the vertical structure reconstruction code.

2 FU Orionis objects

FU Orionis stars (FUors) are named after the prototype FU Ori. Stars belonging to this small class of pre-main-sequence objects show a sharp increase in their luminosity by several orders of magnitude. The phenomenon of FUors led to the conclusion that accretion rates through disks can be highly variable and do not have to be smooth as formerly assumed. Hartmann & Kenyon (1996) suggested that the usual T Tauri accretion rate of $10^{-7} M_{\odot} \text{yr}^{-1}$ can be interspersed with short FUors outbursts when the accretion rate reaches $10^{-4} M_{\odot} \text{yr}^{-1}$ and more. This enhanced accretion seems to be triggered by some kind of disk instability. There are various mechanism which could be responsible for the mass accretion bursts. In the next sections, I will give an overview over the basic characteristics and some examples of FU Orionis objects and I will discuss possible trigger mechanisms.

2.1 Typical Characteristics

The following characteristics of FUors are summarized from Hartmann & Kenyon (1996), Herbig et al. (2003), Petrov & Herbig (2008) and Semkov et al. (2012):

1. a well-documented rise in brightness of about $4 - 6 \text{ mag}$,
2. high Li I abundance, strong Li I 6707 Å line in absorption,
3. surrounding reflection nebula,
4. association with a molecular cloud,
5. strong infrared (IR) excess emission in the spectral energy distribution,
6. silicate emission or absorption features,
7. optical spectral type of late F to G supergiants,
8. IR spectral type of K to M giants,
9. strong CO absorption at $2.2 \mu\text{m}$,
10. the P Cygni profile of $H\alpha$ and Na ID is dominated by the blueshifted absorption.

Following the argumentation of Reipurth et al. (2002), point 1 is the necessary criterion for an object to be classified as a FUor. Therefore distinctions are made between FUors and "FU Ori like" objects. While "FU Ori like" stars show the basic characteristics of FUors, a luminosity burst has not been observed.

High Li I abundance, a surrounding reflection nebula, as well as an association with a molecular cloud are indications for the youth of FUors. The strong IR excess leads to the conclusion that FUors are most likely surrounded by disks.

Some of the FUors even show a larger IR excess than estimated with simple disk models, leading to the assumption of a circumstellar envelope surrounding the system (Hartmann & Kenyon 1996).

Following the widely accepted model of FUors by Hartmann & Kenyon (1996), the disk outshines the central star by a factor of 100 - 1000. Hence, the optical spectra of FU Ori is thought to be formed on the surface of the disk, whereas the contribution from the star is negligible (Petrov & Herbig 2008). The observed spectral type of late F to G supergiants is therefore produced by the inner disk, while the IR features fitting a K to M giant atmosphere are thought to be produced by the cooler outer disk (Hartmann & Kenyon 1996). This principle of a self-luminous accretion disk has been questioned by Petrov & Herbig (2008). Their observations couldn't confirm the dependency of the line width on wavelength, as well as absorption line doubling as a consequence of a Keplerian disk. These results led them to the conclusion that the concept of a self-luminous accretion disk has to be modified in order to fit observations.

The dominant blueshifted absorption of the P Cygni profile indicates massive outflowing winds (Semkov et al. 2012). These supersonic winds are generated during outbursts and have velocities of several hundred kilometres per second (Hartmann & Kenyon 1996, Semkov et al. 2012).

2.2 Some well studied examples

Currently, there are more than 10 confirmed FU Orionis objects. I summarize several best known examples below.

FU Ori is the first discovered FUor and therefore the eponymous represent of this class (Herbig et al. 2003). The star is located in the dark cloud Barnard 35, in the large λ Orionis HII region and showed a bright flare up of about 6 *mag* in 1936 (Herbig 1966). This brightening was followed by a slow fading of its maximum by about 1 *mag* over the last 70 *yr* (Kenyon et al. 2000). The lightcurves of FU Ori, together with these of V1515 Cyg and V1057 Cyg, during their flare-ups are shown in Figure 2.1, which also demonstrates the differences in timescales and durations.

V1057 Cyg is also one of the best studied FUors. In 1971 it showed a rise in brightness, similar to FU Ori, of about 6 *mag* followed by a slow decline. Nevertheless, it can be seen in Figure 2.1 that the brightness of the star fades faster than that of FU Ori. Before the flare-up, V1057 Cyg was a faint variable star with a T Tauri emission line spectrum, while after it appeared as a high-luminosity

early-type star with only $H\alpha$ in emission (Petrov & Herbig 2008). In 2008 it was still 1 *mag* above its initial B-brightness.

V2493 Cyg is a new FUor star, which has been detected and confirmed only recently through photometric and spectroscopic observations by Semkov et al. (2012). It was the first object observed in all spectral ranges during its outburst in summer 2010 as well as the first year after the maximum was reached. Figure 2.2 taken from Semkov et al. (2012) and references therein, shows the change in position in the V-R/R-I diagram during its outburst. The green diamonds show the position of V2493 before its outburst while the pink triangles indicate the position during the outburst. The shift to bluer colours during the increase of brightness due to heating of the inner disk, which outshines the cooler part of the disk, is clearly visible. Before summer 2010, the star has not been classified as a variable star and earlier photometric observations show its T Tauri like behaviour (Semkov et al. 2012).

BBW 76 is a famous example among the FU Ori like objects. It can be found in the constellation Puppis, within the boundaries of the Gum Nebula and is associated with a small dark cloud (Reipurth et al. 2002). Comparing the spectral characteristics of BBW 76 with FU Ori, Reipurth et al. (2002) showed a very similar behaviour between these stars, indicating the membership of BBW 76 to the class of FUors. Nevertheless, based on the photometric plates from 1900 and 1927, one concludes that the star shows the same brightness during more than a century. Thus the outburst of BBW 76 would be the longest documented FUor at maximum luminosity (Reipurth et al. 2002).

FU Ori outbursts are rare events and slightly more than 10 have been observed in the last 70 years (Green et al. 2006). Given the star formation rate in the solar neighbourhood and the number of observed outbursts, Hartmann (1998) calculated a lower limit of about 4 outbursts during the first Myr of the lifetime of a solar-mass star.

2.3 Mechanisms driving FU Orionis outbursts

There are different trigger mechanisms discussed in the literature, two of which I will briefly review below.

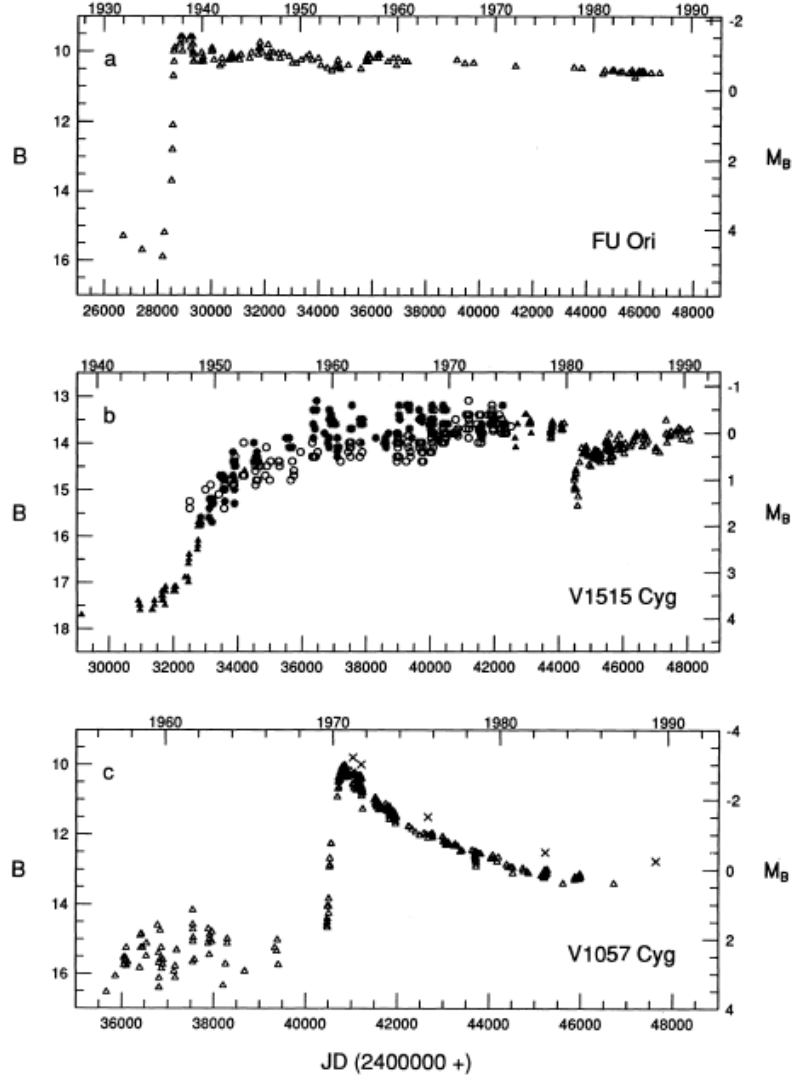


Figure 2.1: Photographic lightcurve of FU Ori, V1515 Cyg and V1057 Cyg, taken from Bell et al. 1995 and references therein. Differences can be seen in the rise times as well as in the decay timescales.

2.3.1 Thermal instability

Thermal instability (TI) is the thermal runaway of the disk due to trapping of heat in the disk. It is discussed as a possible trigger mechanism for FU Ori events in Bell & Lin (1994). The mechanism can be explained by considering the ratio of the radiative losses F_{rad} of the disk to the viscous energy generation F_{visc} (Hartmann 1998):

$$\frac{F_{rad}}{F_{visc}} \propto \frac{T^3}{\kappa_R(T, P)} \Sigma^{-2} \alpha^{-1}, \quad (2.1)$$

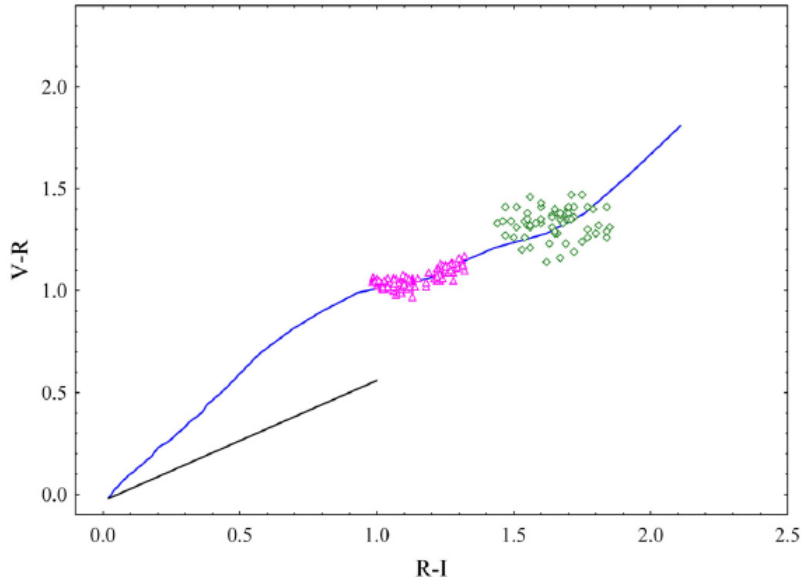


Figure 2.2: V-R vs. R-I color diagram for V2493 Cyg before (open diamonds - green) and during (open triangles - pink) its outburst, taken from Semkov et al. (2012) and references therein.

where κ_R is the Rosseland mean opacity, Σ the gas surface density and α is the Shakura & Sunyaev viscosity parameter. If the disk is thermally perturbed it will regain thermal equilibrium as long as the radiative losses dominate over viscous heating. As Equation 2.1 indicates, this is the case as long as the opacity κ_R does not increase faster with temperature than T^3 .

If κ_R increases faster than T^3 the disk becomes thermally unstable. Radiative cooling is not sufficient anymore and the disk undergoes a thermal runaway. This occurs in regions where the temperatures lies in the range $T = 3\,000\text{ K} - 10\,000\text{ K}$. Here, the ionization of hydrogen leads to a sharp increase in opacity due to H^- (Bell & Lin 1994). The resulting heating causes an increase of the effective local viscous mass flux and the ionization region expands outwards. The radially propagation of the ionization front which separates hot and cold regions, continues until a stabilizing radius is reached. If the surface density behind this ionization front drops, the disk cools below the ionization temperature and the ionization front propagates inward until the whole disk becomes neutral again.

Bell & Lin (1994) stress the self-regulation of this instability, as matter builds up between the outbursts until the temperature is sufficient again to initiate the ionization of hydrogen. TI only operates on extremely small radii $r \leq 0.1\text{ AU}$ and is unlikely to be the main trigger mechanism for FUori events. Armitage et al. (2001) argue that long accretion rates of about $10^{-5} M_{\odot} \text{ yr}^{-1}$ triggered by

magneto rotational instability (MRI) may fall into the thermally unstable regime and cause shorter FUori events as calculated by Bell & Lin (1994). This inner triggering would then be responsible for the detected differences in rise times of FUors (Zhu et al. 2009), see Figure 2.1.

2.3.2 Gravitational instability and fragmentation

Several investigations of disk-fragmentation triggered outbursts are done by Vorobyov & Basu (2006, 2009, 2010a, 2010b). The main idea behind this phenomenon is as follows: In the early stages the disk is still surrounded by an envelope. The infalling matter of this envelope increases the disk mass, which then triggers gravitational instability (GI) and disk fragmentation.

The condition for the disk to become gravitational unstable is given by the Toomre criterion. For a Keplerian disk the Toomre criterion is written as:

$$Q = \frac{c_s \Omega}{\pi G \Sigma}, \quad (2.2)$$

where c_s is the isothermal sound speed, Ω the angular velocity and Σ the mass surface density (Toomre 1964). In order for the disk to become gravitational unstable, Q has to be smaller than some critical value Q_{cr} , which typically is taken to be unity. It has to be noted that this value has an uncertainty, as Q_{cr} may depend on the physical conditions in the disk (Vorobyov & Basu 2010a).

If the disk is gravitational unstable, it can undergo fragmentation (see Figure 2.3). The criterion for the disk to fragment is given by the ratio of the local cooling time t_c to the dynamical time Ω^{-1} , which has to be smaller than a few (Gammie 2001). It is written as:

$$\frac{t_c}{\Omega} \leq C, \quad (2.3)$$

where C may depend on the physical conditions in the disk (e.g. dust content, disk thickness, etc.). Fragments are not likely to build in the inner part of the disk, as Ω is too high to satisfy the fragmentation criterion.

Whether or not the disk will fragment, depends also on the initial conditions in the parent cloud. Vorobyov & Basu (2010a) demonstrated the importance of a sufficiently high initial cloud core rotation. They showed that decreasing the initial rotation of the cloud Ω_0 leads to smaller disk sizes with lower masses, which causes the disk to stabilize against fragmentation. The amount of initial rotation is therefore a necessary condition of disk fragmentation but not a sufficient one,

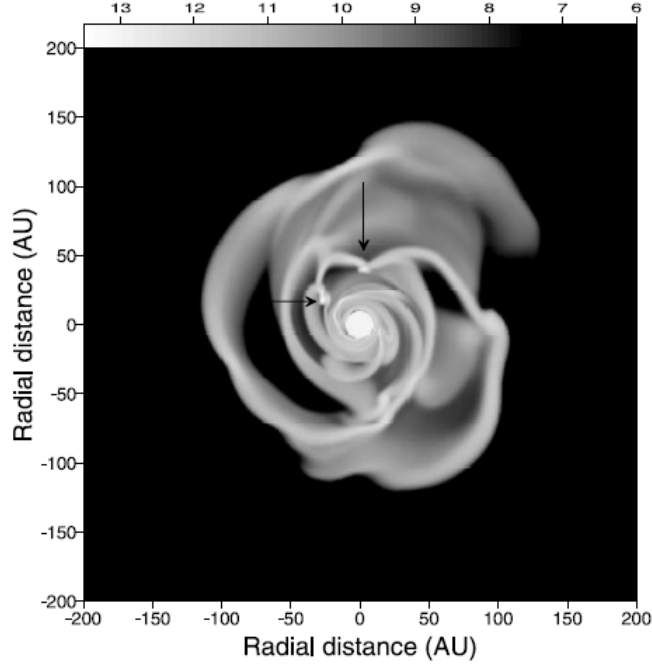


Figure 2.3: Image of the gas volume density distribution in the protostellar disk, taken from Vorobyov & Basu (2006). The arrows indicate the dense ($n \geq 10^{13} \text{cm}^{-3}$) fragments.

as there are still other parameters like turbulent viscosity, external rotation, etc. deciding over the faith of the disk.

If fragments are build, they can be driven onto the protostar by gravitational torques. The release of potential energy then leads to luminosity bursts. Vorobyov & Basu (2010a) calculated accretion rates up to $10^{-4} M_{\odot} \text{yr}^{-1}$ onto the star which are high enough to cause FUori outbursts, see Figure 2.4.

In this work, the fragmentation-driven model of FUors is considered.

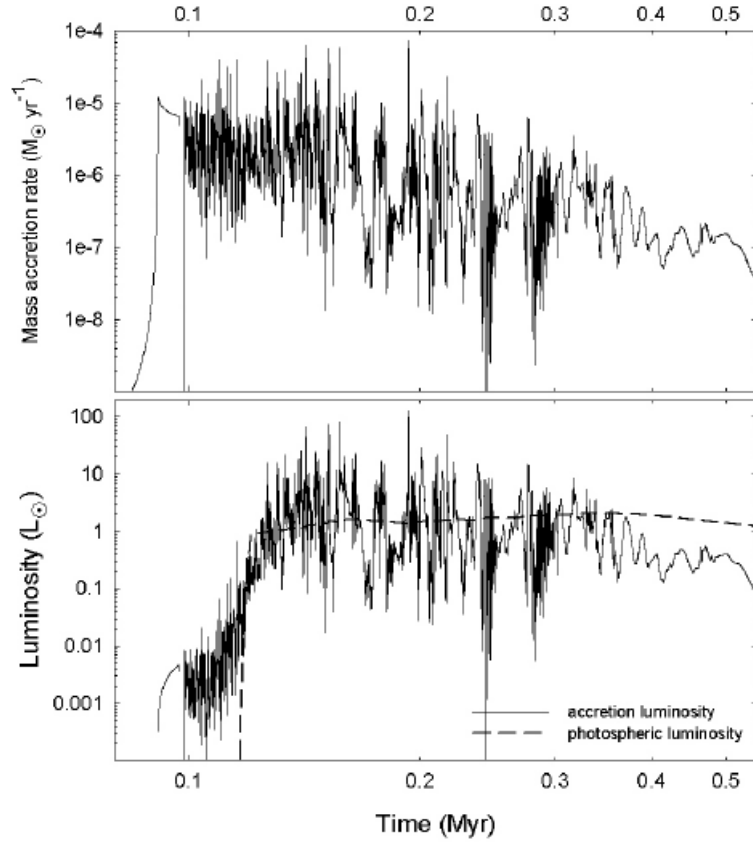


Figure 2.4: Accretion rate and the resulting luminosity due to disk fragmentation in the embedded phase of star formation, taken from Vorobyov & Basu (2010a) [29]. Accretion rates up to $10^{-4} M_{\odot} \text{ yr}^{-1}$ can be noted.

3 Model description

3.1 Thin-disk numerical hydrodynamic simulations

The method for calculating the impact of variable accretion on the vertical disk structure is split into two parts. First the properties of the disk are determined using the two dimensional thin-disk hydrodynamical code developed by Vorobyov & Basu (2010a). Then the vertical disk is reconstructed using coupled equations of local hydrostatic equilibrium and vertical radiation transfer (see Appendix B). This so-called 2 + 1 dimensional treatment makes it possible to study the impact of bursts on the vertical disk structure in a sufficient way. Compared to a full three dimensional treatment this approach has not only advantages in means of simplicity but also better computational speed is gained.

3.2 Hydrodynamical model description

The numerical hydrodynamics model is described in detail in Vorobyov & Basu (2006, 2010a) and is summarized below.

The model computes the gravitational collapse of a starless cloud and follows its evolution through the early embedded phase of star formation until the T Tauri phase, when the bigger part of the cloud has accreted onto the disk and star system. A sink cell is introduced at the inner region of the disk ($r < 6 AU$), a free inflow condition is imposed on the sink cell boundary. As soon as the gas density inside the sink cell exceeds a critical value ($\sim 10^{10} cm^{-3}$), a central point-mass object, a protostar, is introduced. After the formation of the protostar, 90% of the gas falling onto the sink cell is assumed to land on the protostar and its surrounding disk, whereas the remaining 10% are assumed to be carried away by protostellar jets.

The basic equations of mass, momentum and energy transport in a thin disk are given through:

$$\frac{\partial \Sigma}{\partial t} = -\nabla_p \cdot (\Sigma \mathbf{v}_p), \quad (3.1)$$

$$\frac{\partial}{\partial t} (\Sigma \mathbf{v}_p) + [\nabla \cdot (\Sigma \mathbf{v}_p \otimes \mathbf{v}_p)]_p = -\nabla_p P + \Sigma \mathbf{g}_p + (\nabla \cdot \Pi)_p, \quad (3.2)$$

$$\frac{\partial e}{\partial t} + \nabla_p \cdot (e \mathbf{v}_p) = -P(\nabla_p \cdot \mathbf{v}_p) - \Lambda + \Gamma + (\nabla \mathbf{v})_{pp'} : \Pi_{pp'}, \quad (3.3)$$

with Σ being the mass surface density. For this description polar coordinates are

used. Therefore p and p' refer to r and ϕ with the velocity in the disk plane $\mathbf{v}_p = v_r \hat{\mathbf{r}} + v_\phi \hat{\boldsymbol{\phi}}$, the gravitational acceleration in the disk plane $\mathbf{g}_p = g_r \hat{\mathbf{r}} + g_\phi \hat{\boldsymbol{\phi}}$ and the gradient along the planar coordinates of the disk $\nabla_p = \hat{\mathbf{r}} \partial / \partial r + \hat{\boldsymbol{\phi}} r^{-1} \partial / \partial \phi$. P is the vertically integrated pressure, e the internal energy per surface area, Λ the radiative cooling and Γ the heating function due to irradiation.

Viscous heating enters via the term $(\nabla \mathbf{v})_{pp'} : \Pi_{pp'}$ with $\Pi_{pp'}$ being the viscous stress tensor:

$$\Pi = 2\Sigma\nu \left[\nabla \mathbf{v} - \frac{1}{3}(\nabla \cdot \mathbf{v})\mathbf{e}_{ij} \right] \quad (3.4)$$

and $\mathbf{e}_{ij}$ being the unit tensor. Viscosity may be important regarding mass and angular momentum transport. Sources and their magnitudes of viscous transport processes are still not completely understood. Nevertheless, turbulent viscosity induced by magneto-rotational instability is a widely favoured possibility nowadays (Bae et al. 2013, Zhu et al. 2009, Hartmann 1998).

A common way to describe viscosity is the use of the α -description introduced by Shakura & Sunyaev (1973). A modified form of this description used by Vorobyov & Basu (2010a) includes the function $F_\alpha(r) = 2\pi^{-1} \tan^{-1} [(r_d/r)^{10}]$ where r_d is the disk radius. Beyond this radius the viscosity reduces quickly to zero so viscosity is bound to act only in the disk and not in the envelope. The viscosity is then described as:

$$\nu = \alpha c_s Z F_\alpha(r). \quad (3.5)$$

Using this description it is not necessary to specify the sources of turbulence but the magnitude of viscosity can be parametrized. Based on the conclusions of Vorobyov & Basu (2009), where the impact of the viscosity on the development of the disk was tested, α is set to a value of $5 \cdot 10^{-3}$.

3.3 Justification of the thin-disk approach

The thin-disk approach has been used in several simulations (e.g. D'Alessio et al. 1998, Armitage et al. 2001, Vorobyov & Basu 2006, 2010a, 2010b, Zhu et al. 2012, Bae et al. 2013). The condition for its use is that the scaleheight of the disk H must be significantly smaller than the radial extent of the disk R :

$$\frac{H}{R} \ll 1. \quad (3.6)$$

Using Equation (4.29), H can be written in terms of the rotational velocity $\Omega = \sqrt{GM_*/R^3}$:

$$H = \frac{c_s}{\Omega}, \quad (3.7)$$

where c_s is the isothermal sound speed, which can also be expressed as $c_s \leq Q_{cr}\pi G\Sigma/\Omega$. Using that the mass of the disk $M_d(r)$ within the radius r is given through: $M_d(r) = \int \Sigma(r, \phi) dr d\phi$, the ratio $A = H/R$ can be rewritten as:

$$A \leq \frac{Q_{cr}M_d(r)}{CM_*}, \quad (3.8)$$

where Q_{cr} is the critical Toomre parameter and C a constant, dependent on the gas surface density (Vorobyov & Basu 2010a).

To estimate the value of A , typical upper limits for these parameters are adopted from Vorobyov & Basu (2010a): $C = 4$, $Q_r = 2$, $M_d(r)/M_* = 0.5$. Applying these values to Equation (3.8), leads to a ratio of $A \leq 0.25$. Even in the outer disk region, where the disk becomes larger due to a higher ratio of $M_d(r)/M_*$, the criterion is fulfilled.

The justification of the thin-disk approach after the reconstruction of the vertical structure is tested and the results are shown in Chapter 5.3.

4 Reconstruction of the disk vertical structure

The reconstruction of the disk vertical structure follows the description of Akimkin et al. (2012), with significant modifications. These adaptations are necessary in order to couple the vertical reconstruction code with the hydrodynamical code efficiently and self-consistently. The main modifications are:

- the input of the initial parameters obtained by the hydrodynamical code, e.g. disk surface density, midplane temperature, viscous heating, parameters of the forming star, etc.,
- setting the number of vertical grid points consistent with the radial resolution of the hydrodynamical code, thus increasing the computational speed,
- the inclusion of the disk self-gravity into the reconstruction of the disk vertical structure,
- the development of a new method, based on the Henvey method (Bodenheimer et al. 2007), for the reconstruction of the vertical density distribution, which provides a significant increase in the computational speed as compared to the previously used shooting and bisection method,
- the use of density-dependent IR opacities.

The basic assumptions used in the adopted approach, which make the problem tractable, are that the disk is in the vertical thermal and hydrostatic equilibrium.

The dusty and gaseous components of the disk are assumed to be thermally coupled and therefore described with a single temperature. This implies that the collisional and radiative exchange of energy between these species are efficient throughout the vertical extent of the disk (D’Alessio et al. 1998).

4.1 Treating radiative transfer

A detailed derivation of the radiative transfer equations, necessary assumptions, as well as the description of the used opacities can be found in Appendix B. While the disk radiates in the IR, it is illuminated by the star in the UV. Thus the spectral ranges are split and described separately.

Using the Eddington approximation, the equations describing the radiation transfer through the vertical extent of the disk in the IR spectra can be written

as:

$$\frac{dF}{dz} = c\sigma_P(B - E), \quad (4.1)$$

$$F = -\frac{1}{3\sigma_R} \frac{dE}{dz}, \quad (4.2)$$

with σ_P and σ_R are the Planck and Rosseland absorption coefficients, c is the speed of light, F is the integrated over frequency energy flux, E is the integrated energy density and B is the Planck function ($B = aT^4$), where a is given through $a = 4\sigma_B/c$, with σ_B being the Stefan-Boltzmann constant. After introducing column density $\Sigma = \int_0^z \rho(z)dz$, one can write:

$$\frac{dz}{d\Sigma} = \frac{1}{\rho(z)}, \quad (4.3)$$

where $\rho(z)$ is the volume density at the given vertical height z . The radiation transfer equation are thus expressed as:

$$\frac{dF}{d\Sigma} = c\kappa_P(B - E), \quad (4.4)$$

$$F = -\frac{1}{3\kappa_R} \frac{dE}{d\Sigma}. \quad (4.5)$$

The Rosseland κ_R and Planck κ_P mean opacities are given as:

$$\frac{1}{\kappa_R} = \frac{\int_0^\infty \frac{1}{(\kappa_\nu^{abs} + \kappa_\nu^{sca})} \frac{dB_\nu}{dT} d\nu}{\int_0^\infty \frac{dB_\nu}{dT} d\nu}, \quad (4.6)$$

$$\kappa_P = \frac{\int_0^\infty \kappa_\nu^{abs} B_\nu d\nu}{\int_0^\infty B_\nu d\nu}, \quad (4.7)$$

with units of $[cm^2/g]$. Both opacities are calculated directly with the program provided by Semenov et al. (2003) (see Appendix B.1).

For calculating the flux in the UV, the two-stream approximation is used to obtain the mean intensity of the stellar radiation J_{UV} (Mihalas 1987). Since we assume that the disk does not radiate in the UV, the radiation transfer equations

simplify to the following form:

$$\frac{dF_{UV}}{d\Sigma} = -4\pi\kappa_P^{UV} J_{UV}, \quad (4.8)$$

$$F_{UV} = -\frac{4\pi}{4\kappa_F^{UV}} \frac{dJ_{UV}}{d\Sigma}, \quad (4.9)$$

here κ_F^{UV} is the mean flux extinction coefficient and F_{UV} is the UV flux. The intensity of stellar radiation, averaged over the upper hemisphere is defined as:

$$J_{UV}^- = \frac{f}{\pi} \frac{L_*}{4\pi R^2}, \quad (4.10)$$

where L_* is the stellar luminosity and f determines the fraction of the stellar radiation intercepted by the disk, which is dependent on the shape of the disk surface. Therefore, $f L_*$ gives the amount of stellar luminosity penetrating into the disk. Both values are taken from the hydrodynamical code.

With the following boundary conditions at the center and the surface of the disk:

$$F_{UV}(0) = 0, \quad (4.11)$$

$$F_{UV}(\Sigma_0) = 2\pi(J_{UV} - J_{UV}^-), \quad (4.12)$$

where Σ_0 is the gas surface density at the surface of the disk, it is possible to solve system (4.8)-(4.9) analytically if the opacities are not dependent on Σ (Akimkin et al. 2012):

$$J_{UV}(\Sigma) = \frac{J_{UV}^-(1 + e^{-2\lambda\Sigma})}{(1 + q)e^{\lambda(\Sigma_0 - \Sigma)} + (1 - q)e^{-\lambda(\Sigma_0 + \Sigma)}}, \quad (4.13)$$

with

$$\lambda = \sqrt{4\kappa_P^{UV}\kappa_F^{UV}}, \quad (4.14)$$

$$q = \sqrt{\kappa_P^{UV}/\kappa_F^{UV}}. \quad (4.15)$$

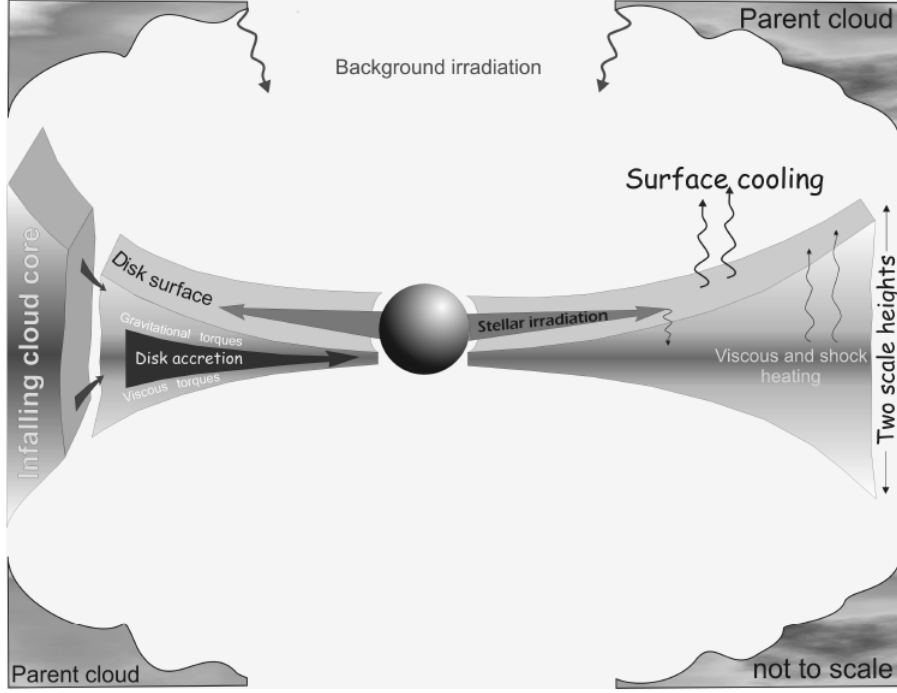


Figure 4.1: Main heating sources in the protostellar disk, taken from Vorobyov & Basu (2010a). While the disk is heated through star radiation and viscosity, cooling occurs due to thermal radiation escaping the disk.

The UV opacities are defined as:

$$\kappa_P^{UV} = \frac{\int_0^\infty \kappa_v^{abs} B_v dv}{\int_0^\infty B_v dv}, \quad (4.16)$$

$$\kappa_F^{UV} = \frac{\int_0^\infty (\kappa_v^{abs} + \kappa_v^{scat}) B_v dv}{\int_0^\infty B_v dv}. \quad (4.17)$$

Here, B_v is the Planck function at the temperature of the star T_* . The latter value is taken from the stellar evolution code of Baraffe & Chabrie (2010), implemented into the hydrodynamical code.

4.2 Heating of the disk

To close the system (4.4)-(4.5), an energy-balance equation needs to be considered. The main heating sources are described in Figure 4.1, taken from Vorobyov & Basu (2010a). The disk is heated by the radiation of the star, by turbulent viscosity and shocks. We assume that viscosity generates energy homogeneously through

the vertical extent of the disk, while stellar radiation penetrates the surface and propagates vertically downwards. The change in the radiation flux with vertical distance z , caused by the heating sources is given as:

$$\frac{dF}{dz} = \Gamma_* + \Gamma_{hydro}, \quad (4.18)$$

where $\Gamma_* [erg\ cm^{-3}\ s^{-1}]$ describes the heating caused by the UV radiation of the star and Γ_{hydro} viscous and shock heating. Using Equation (4.3), the last equation can be rewritten as:

$$\frac{dF}{d\Sigma} = S_* + S_{hydro}. \quad (4.19)$$

Here, S_* and S_{hydro} are the heating functions per unit mass $[erg\ s^{-1}\ g^{-1}]$. The value of the second term is taken directly from the hydrodynamical code (see Chapter 3.2):

$$S_{hydro} = (-P(\nabla_p \cdot \mathbf{v}_p) + (\nabla \mathbf{v})_{pp'} : \Pi_{pp'}) \frac{1}{\Sigma}. \quad (4.20)$$

Heating due to UV radiation is:

$$S_* = 4\pi(\kappa_p^{UV} J_{UV} + \kappa_p^{ISF} J_{UV}^{ISF}), \quad (4.21)$$

where κ_p^{ISF} is the UV opacity at the temperature of the interstellar radiation field $T_{ISF} = 10^4\ K$ and J_{UV}^{ISF} describes the mean intensity of the interstellar radiation field. To calculate $J_{UV}^{ISF}(\Sigma)$, the same assumptions are made as for the calculation of $J_{UV}(\Sigma)$, see Equation (4.13).

4.3 Solution of the radiative transfer equations

To find the vertical temperature distribution, the energy distribution $E(\Sigma)$ needs to be known. As the first step, Equation (4.19) is numerically integrated over Σ , taking into account that F cannot become negative. The resulting flux $F(\Sigma)$ is substituted into Equation (4.5). This procedure provides a value for $dE/d\Sigma$, which can be expressed as:

$$\frac{dE}{d\Sigma} = E(\Sigma) - E(0) = -3\kappa_R F(\Sigma). \quad (4.22)$$

In the next step, the energy at the midplane $E(0)$ needs to be found. We rewrite Equation (4.22) in the following form:

$$E(0) = E(\Sigma_0) + 3\kappa_R F(\Sigma_0) \quad (4.23)$$

and introduce boundary conditions at the disk surface ($\Sigma = \Sigma_0$), where we assume that the IR radiation at the surface is isotropic $F(\Sigma_0) = \frac{1}{2}cE(\Sigma_0)$. We also take into account the energy density of the IR background radiation $E_{bg} = aT_{bg}^4$, with $T_{bg} = 2.73$ K. The energy density at the surface of the disk $E(\Sigma_0)$ is then written as:

$$E(\Sigma_0) = \frac{2F(\Sigma_0)}{c} + E_{bg}. \quad (4.24)$$

To find the energy density distribution at the midplane $E(0)$, this value is substituted back into Equation (4.23). Finally, the temperature structure is obtained by combining Equation (4.4), (4.19) and (4.22):

$$T(\Sigma) = \left[\left(\frac{S_* + S_{hydro}}{c\kappa_P} + E(0) - 3\kappa_R F(\Sigma) \right) \frac{1}{a} \right]^{1/4}. \quad (4.25)$$

This equation is a transcendental equation, as the opacities κ_R and κ_P are dependent on temperature. In order to solve this equation, an iterative process over temperature is introduced. The iterations start with the calculation of the energy density distribution. After each cycle, the opacities are recalculated using the updated temperatures. The iterations are stopped when the absolute error of temperature reaches a value of 0.1 K, which typically is the case within 6 – 12 steps.

4.4 Grid definition and density distribution

Choice of the vertical grid

In order to numerically find the vertical temperature and density distribution, the vertical grid spacing should be set so the actual gradients of the gas volume density ρ are resolved. As these gradients are not known a priori, an isothermal vertical hydrostatic equilibrium is first assumed, for which an analytical solution for ρ can be obtained. Using the equation of vertical hydrostatic equilibrium for

a thin disk:

$$\frac{1}{\rho} \frac{d}{dz} (c_s^2 \rho) = -\frac{GM_*}{R^3} z, \quad (4.26)$$

where R gives the radial distance from the star in the midplane and c_s is the isothermal sound speed, which can be expressed as:

$$c_s^2 = \frac{Tk_b}{\mu m_H}, \quad (4.27)$$

with k_b being the Boltzman constant, μ the mean molecular weight and m_H the mass of hydrogen. Taking into account that c_s does not depend on z and integrating Equation (4.26) over z , the density distribution is found as:

$$\rho(z) = \rho(0) \exp\left(-\frac{z^2}{H^2}\right), \quad (4.28)$$

where H is the vertical scale height of the disk:

$$H = \sqrt{\frac{2c_s^2 R^3}{GM}}. \quad (4.29)$$

Rearranging Equation (4.28), the vertical grid spacing is set as follows:

$$z_i = H \sqrt{\ln \rho(0) - \ln \rho_i}, \quad (4.30)$$

where $i = 2, \dots, N$ and N corresponds to the surface of the disk. Integrating Equation (4.28) over z , the volume density at the midplane of the disk $\rho(0) = \rho_{i=1}$ is derived as:

$$\rho(0) = \frac{2\Sigma_0}{H\sqrt{\pi}}. \quad (4.31)$$

This procedure makes it possible to set the initial density distribution in a way so the density will change by a specific amount from one grid cell to another.

A reasonable choice for the number of vertical grid points N has to be found. This is done in accordance to the radial grid resolution $\Delta x(R)$ at a given radial distance R , in order not to use more points than necessary to resolve the vertical

grid:

$$N = \frac{2 \cdot z_N}{\Delta x(R)}. \quad (4.32)$$

If $N > N_R/4$, where N_R is the number of radial grid points, than N is set to $N_R/4$.

Reconstruction of the density distribution

After the vertical temperature distribution is obtained by solving for the Equation 4.25, the density distribution can be reconstructed numerically. This can, in principle, be done by a combination of the shooting and bisection method (Akimkin et al. 2012).

As this approach has a slow convergence a new method for the reconstruction of the vertical density grid was developed. This method is a combination of the Newton method with the Thomas algorithm. Compared with the shooting and bisection method (SBM), the Newton-Thomas method (NTM) proofs to be faster. Further, the description of a self-gravitating disk can be implemented in this method.

The algorithm for the NTM, also known as *Henvey method*, is described by Bodenheimer et al. (2007) in "Numerical Methods in Astrophysics" and references therein. Bodenheimer et al. (2007) use this approach to solve for the structure of a star and follow its evolution in time. The method described converges fast, only 3 – 5 iterations are needed.

For the reconstruction of the density distribution, the following set of equations has to be solved:

$$\frac{d}{d\Sigma}(c_s^2 \rho) = -\frac{GM_*}{R^3} z, \quad (4.33)$$

$$\frac{dz}{d\Sigma} = \frac{1}{\rho'}, \quad (4.34)$$

where Equation (4.33) is the rewritten equation for the vertical hydrostatic equilibrium Equation (4.26). This modification is necessary as the temperature (and therefore c_s^2) is known as a function of Σ and not of z .

If the self-gravity of the disk is included in the description, Equation (4.33) be-

comes:

$$\frac{d}{d\Sigma}(c_T^2 \rho) = -\frac{GM_*}{R^3}z - 2\pi G\Sigma. \quad (4.35)$$

The derivation, as well as a detailed description of the NTM, are given in the Appendix (A).

4.5 Comparison of different reconstruction methods

To show the advantages of the NTM over the SBM, the computational speed of both approaches are compared in this section. Various cases for the initial conditions are taken into account to find the best initial setup. The iterations for the SBM are stopped when the relative error between the solution at the disk surface and the adopted external density ρ_{ext} becomes smaller than 1 %. The cycle of the NTM is stopped when the absolute difference in density of the iterations becomes smaller than $0.01 \rho_{ext}$.

Isothermal case

The isothermal case was chosen in order to compare the analytical solution for the volume density distribution (Equation 4.28) to the numerical solution obtained with the NTM. The radial distribution of the gas surface density is described with the analytic relation:

$$\Sigma(r) = \Sigma_{in} \left(\frac{r}{r_{in}} \right)^{-1.5}, \quad (4.36)$$

where $\Sigma_{in} = 1\,000 \text{ g cm}^{-2}$ is the gas surface density at the radial distance $r_{in} = 1 \text{ AU}$. The external number density is set to $n_{ext} = 10^3 \text{ cm}^{-3}$, the stellar mass is set to solar mass $M_* = 1.0 M_\odot$, the temperature to $T = 50 \text{ K}$ and the radial distance from the star was set to 15.7 AU . The differences in the volume density δ_ρ of the solutions are calculated with:

$$\delta_\rho = \frac{|\rho_{an} - \rho_{NTM}|}{\rho_{an}}, \quad (4.37)$$

where ρ_{an} indicates the volume density obtained analytically and ρ_{NTM} the volume density calculated with the NTM. The volume density distribution obtained with the analytic solution together with the distribution obtained with the NTM

are shown in Figure 4.2. The highest difference in the volume density δ_ρ between these solutions is found in the midplane, where it reaches a value of about 10%. This difference decreases with vertical distance from the midplane. At a distance of ≥ 0.4 AU δ_ρ is below 5%.

The density distribution obtained with the SBM is in better agreement with the analytically obtained density distribution (see Figure 4.3). The differences are decreasing with vertical distance from the midplane, where at the midplane $\delta_\rho \simeq 6\%$. At a distance of ≥ 0.4 AU δ_ρ is already below 2%.

Choosing different initial conditions for ρ has a significant impact on the convergence speed (see Table 4.1). If the initial conditions are provided by the analytical solution, 4 iterations are needed until the solution converges. 26 iterations are needed if an isotropic distribution for ρ is assumed with $\rho(z) = 100\rho_{ext}$. The slowest convergence is obtained with the SBM, where the solution converges after 42 iterations.

Table 4.1: Comparison of convergence speed for different methods and the impact of initial conditions on the number of iterations.

Case	Method	Initial density distribution	Radial distance	Iterations
Isothermal	NTM	analytic	$r = 15.7$ AU	4
	NTM	$\rho(z) = 100 \rho_{ext}$	$r = 15.7$ AU	26
	SBM	$\rho(0) = \rho(0)_{an}$	$r = 15.7$ AU	42
Non-Isothermal	NTM	analytic	$r = 15.7$ AU	5
	NTM	$\rho(z) = 100 \rho_{ext}$	$r = 15.7$ AU	26
	SBM	$\rho(0) = \rho(0)_{an}$	$r = 15.7$ AU	41
Non-Isothermal with Selfgravity	NTM*	analytic	$r = 15.7$ AU	5
	NTM*	analytic	$r = 250.5$ AU	4

Non-Isothermal case

Assuming a non-isothermal case the vertical temperature distribution is obtained with the numerical model for the reconstruction of the vertical disk structure. The initial conditions are set to: $T_* = 2\,000$ K, $S_{hydro} = 0$, $T_{bg}^{IR} = 2.73$ K, $T_{bg}^{UV} = 10\,000$ K with a dilution of the UV radiation field of $D_{bg}^{UV} = 10^{-14}$.

For this case, the volume density obtained with the NTM converges after 5 iterations if the initial volume density is used from the isothermal analytical solution, whereas 26 iterations are needed if an isotropic density distribution $\rho(z) = 100 \rho_{ext}$ is assumed. The SBM shows a slow convergence with 41 iterations.

Taking into account the better agreement of the SBM obtained density distribution with the analytic solution, the NTM is chosen for the reconstruction of the density distribution as it shows important advantages in convergence speed for the isothermal case and for the non-isothermal case. The importance of the initial guess for the system to converge is shown. Fortunately, the density distribution in the non-isothermal disk shows a similar trend as in an isothermal disk and presents a good initial guess also for the non-isothermal case.

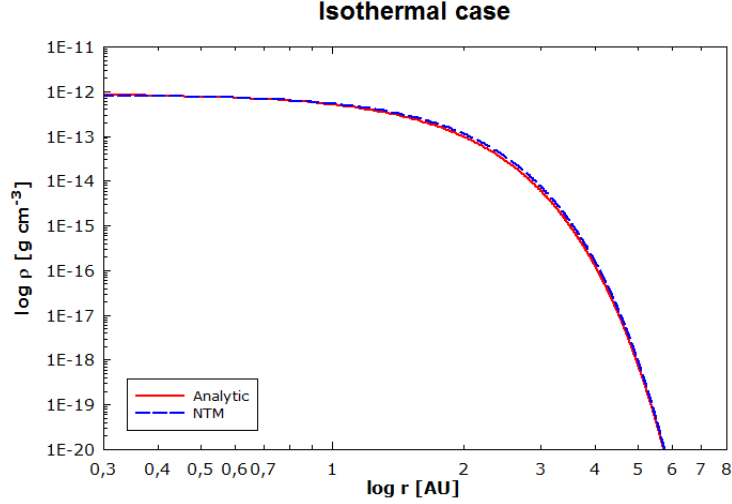


Figure 4.2: The vertical density distribution obtained analytically (red solid line) and obtained with the NTM (blue dashed line) are shown.

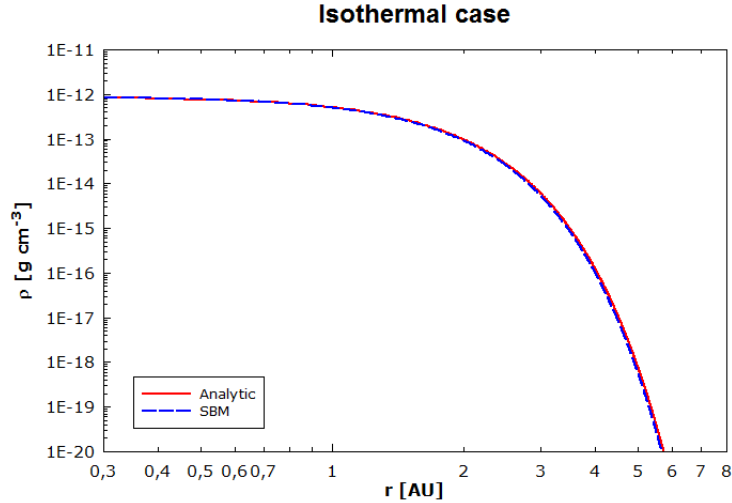


Figure 4.3: The vertical density distribution obtained with the analytic solution (red solid line) and the SBM (blue dashed line) for the isothermal case are shown.

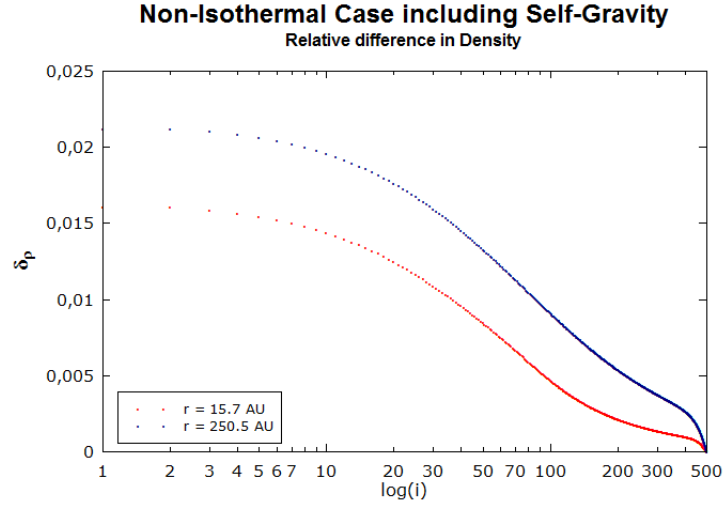


Figure 4.4: Relative difference in volume density $\delta\rho$ obtained with the NTM and the NTM*.

Non-isothermal case including disk self-gravity

NTM and the Newton-Thomas method including disk self-gravity (NTM*) are equally comparing computational speed. Minor differences appear in their resulting density structure. Figure 4.4 shows the relative difference in volume density $\delta\rho$ at two radial distances (15.7 AU, 250.5 AU) from the star. Both differences do not show any significant values but the differences in the outer disk region (blue line) is higher than in the inner disk (red line). The term describing the self-gravity of the disk (see Equation 4.35) is insignificant compared to the gravitational force of the star for the better part of the disk. Equation (4.35) indicates that the second term will be more dominant when the radius and/or the surface density become large. For these cases self-gravity may have a visible impact on the structure.

4.5.1 Stability of the Newton-Thomas method

The stability criterion for the Thomas algorithm implies that the Thomas algorithm is numerically stable if the dominance of the diagonal elements (b_i) is given. Therefore, the magnitude of the diagonal must dominate the sum of the magnitudes of the off diagonal elements (a_i, c_i). The criterion has to be applied on every single line to test the stability of the whole linear system and is expressed as:

$$|b_i| - |a_i| - |c_i| \geq 0 \quad (4.38)$$

This criterion is fulfilled in the isothermal case independent of the chosen initial conditions for ρ . In the non-isothermal case an unstable area appears at a vertical distance from the midplane at $0.15 AU \leq z \leq 0.17 AU$. It is independent of the initial values for ρ as well as on the inclusion of self-gravitation.

In relation to the advantage of computational speed, this relatively small regime of instability will be neglected. Especially as the results gained by the NTM are in good agreement with the analytical solution in the isothermal case as well as with the shooting method in the non-isothermal case.

5 Results

5.1 Initial model

In this section, I reconstruct the vertical structure of a protostellar disk during the FUori-type luminosity outburst and in the subsequent quiescent phase characterized by relatively low stellar luminosity. The required data, shown in Table 5.1, are obtained from the 2D numerical hydrodynamics simulation of a collapsing pre-stellar core with mass $M_c = 1.8 M_\odot$ and the ratio of rotational to gravitational energy $\beta = 0.7\%$. The initial distributions of gas surface density Σ and angular velocity Ω in the prestellar core are:

$$\Sigma = \frac{r_0 \Sigma_0}{\sqrt{r^2 + r_0^2}}, \quad (5.1)$$

$$\Omega = 2\Omega_0 \left(\frac{r_0}{r}\right)^2 \left[\sqrt{1 + \left(\frac{r}{r_0}\right)^2} - 1 \right], \quad (5.2)$$

where $\Omega_0 = 1.2 \text{ km s}^{-1} \text{ pc}^{-1}$ and $\Sigma_0 = 6.2 \cdot 10^{-2} \text{ g cm}^{-3}$ are the angular velocity and gas surface density at the disk center and $r_0 = \sqrt{A} c_s^2 / \pi G \Sigma_0$ is the radius of the central plateau. Here, $c_s = 0.22 \text{ km s}^{-1}$ is the initial sound speed in the core and A the positive density-perturbation amplitude, which is set to 1.2. The simulations of the gravitational collapse of the pre-stellar core continue into the embedded phase of star formation, during which a protostellar disk is formed. In this stage, the disk is subject to intense mass loading from the remnant of the parental cloud. The key feature of this 2D model is the self-consistent disk-core interaction, which allows to observe repetitive episodes of disk fragmentation.

Table 5.1: Data from the hydrodynamical code needed to obtain the vertical structure, where j denotes the radial and k the angular grid points.

$f(j)$	$r(j)$	$\Delta r(j)$	$T_{mp}(j, k)$	$\Sigma_0(j, k)$
fraction of intercepted stellar radiation	radial distance	radial grid spacing	gas midplane temperature	gas surface density
$S_{hydro}(j, k)$	L_*	M_*	T_*	$\dot{M}$
heating due to viscous and shock heating	stellar luminosity	stellar mass	stellar temperature	accretion rate

The evolution of the mass accretion rate $\dot{M}$ in our model as a function of time

elapsed since the beginning of the simulations is shown in Figure 5.1. The black circle indicates the most energetic burst at $\sim 0.186 \text{ Myr}$. This burst was chosen for the reconstruction of the disk vertical structure.

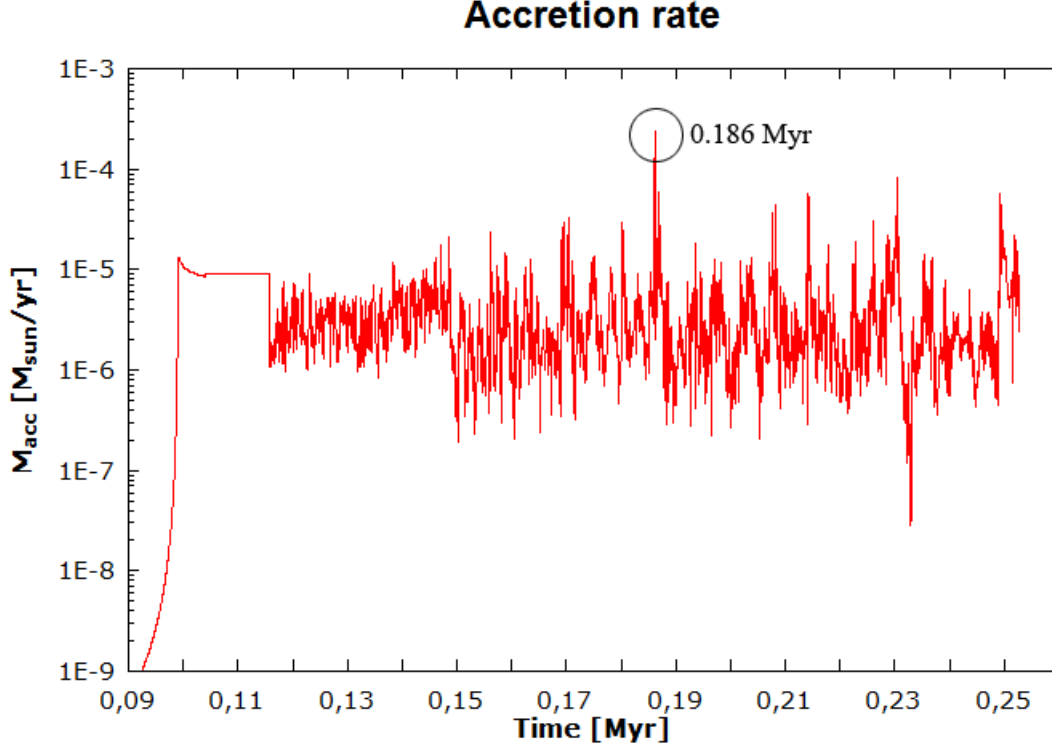


Figure 5.1: Accretion rate onto the star vs. time during disk evolution.

Starting from this moment the necessary data are obtained from the hydrodynamics code every 25 years for about 1 000 years, in order to catch the post-burst phase of the disk. The total luminosity (accretion and photospheric luminosity) of the star L_* during this phase is shown in Figure 5.2. The black circles indicate the points in time, where the disk vertical structure is reconstructed. Snapshot 1 represents the moment of the burst with $L_* \simeq 254 L_\odot$. The luminosity of the star reaches a local minimum with $L_* \simeq 8 L_\odot$ at snapshot 2. This time instance is chosen to represent the post-burst phase. For the reconstruction of the vertical disk, an external number density of $n_{ext} = 10^3 \text{ cm}^{-3}$ was assumed.

5.2 Density structure

Figure 5.3 presents the gas surface density $\log g \text{ cm}^{-2}$ of the protostellar disk at the burst ($t = 0 \text{ yr}$, snapshot 1) and after the burst ($t = 150 \text{ yr}$, snapshot 2). It clearly shows the dense fragments as well as the spiral structure of the disk caused by

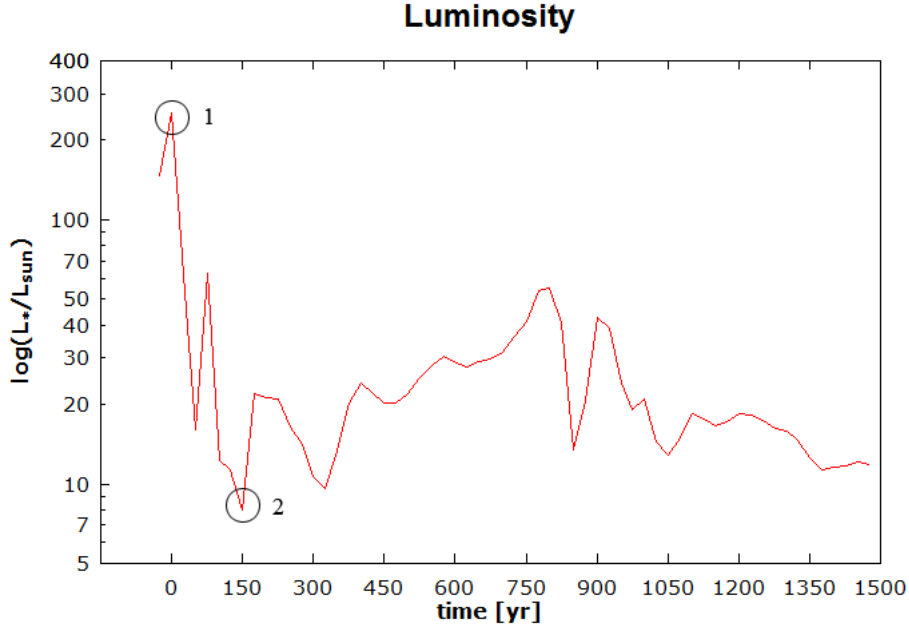


Figure 5.2: Stellar luminosity at and after the burst. The black circles indicate the snapshots taken for the reconstruction of the vertical disk.

gravitational instability.

The black lines indicate the slices, along which the reconstruction of the vertical disk structure is performed. To show the impact of the dense clumps, the first slice contains a fragment at around 192 *AU* (*FG1*), while the second passes through the regular disk. In the post-burst phase, the first cut was taken through the same fragment *FG1*, taking into account the movement of the fragment during the past 150 years. The second cut in the post-burst phase representing the regular disk was chosen at the same position as the cut at the burst.

The vertical distribution of the gas number density n ($\log \text{ cm}^{-3}$) along the chosen radial cuts is shown in Figure 5.4. The black line h_{90} indicates the height, below which 90 % of the total disk mass is contained and marks the dynamically significant region for the hydrodynamical code.

The upper panels show the cuts that include the fragment *FG1* at the burst (left) and after the burst (right). The fragment reaches number densities of almost 10^{13} cm^{-3} . Due to self-gravity, the height of the disk is reduced at these dense regions, the effect that shields the fragment from stellar radiation. As the fragment reaches higher densities at $t = 150 \text{ yr}$, this effect becomes even more pronounced. The shielding of the fragment from radiative heating will lead to a more stable fragment as can be seen from the Toomre criterion (Equation 2.2). The bottom panels in Figure 5.4 show the disk vertical density structure for the

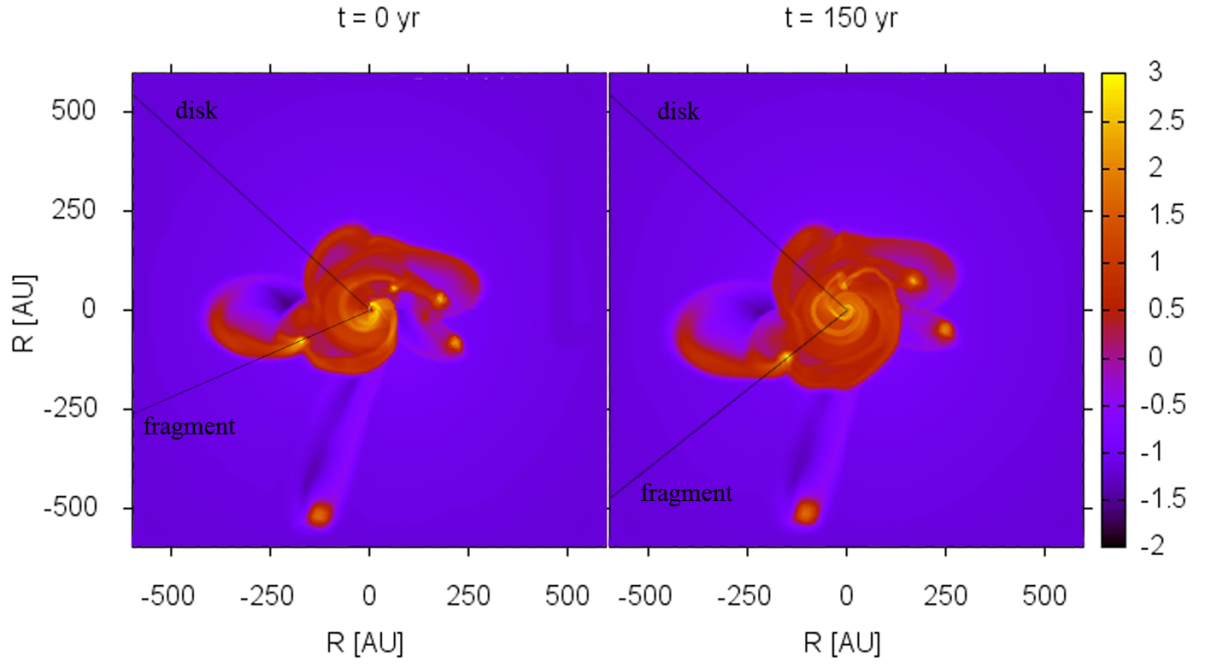


Figure 5.3: Gas surface density of the disk during ($t = 0$ yr) and after ($t = 150$ yr) the burst. The scale bar is in $\log g \text{ cm}^{-2}$.

cut through the regular disk. The extent of the vertical disk height is increased during the burst for both cuts through the disk. Comparing the height of h_{90} at the burst and after the burst, the vertical disk height is in average extended by almost 360 AU during the burst, which is a mean expansion of 62%.

The impact of the burst on the vertical gas volume density distribution at radial distances of $r = 20 \text{ AU}$ (red lines), $r = 40 \text{ AU}$ (blue lines) and $r = 120 \text{ AU}$ (green lines) is shown in Figure 5.5. The gas density profiles at the burst ($t = 0 \text{ yr}$) are presented by the solid lines and the dashed lines give the gas density profiles in the post-burst phase ($t = 150 \text{ yr}$). It is evident that lower gas densities are reached near the midplane of the disk during the burst. This effect is more dominant in the inner disk. At a radial distance of $r = 20 \text{ AU}$ the density during the burst is 3% of the density after the burst, while at $r = 120 \text{ AU}$ the density at the midplane is 35% of the density in the post-burst phase.

In Figure 5.6 the gas volume density distributions of the disk at $r = 20 \text{ AU}$ (red dashed line), $r = 100 \text{ AU}$ (blue dashed line) and at the fragment $r \simeq 192 \text{ AU}$ (green dashed line) are plotted as a function of vertical distance from the midplane together with the analytical solutions for the same radial distances (solid lines). The analytic solution is given by Equation (4.28), where $\rho(0)$ is the volume density at the midplane (Equation 4.31). The sound speed c_s is obtained using

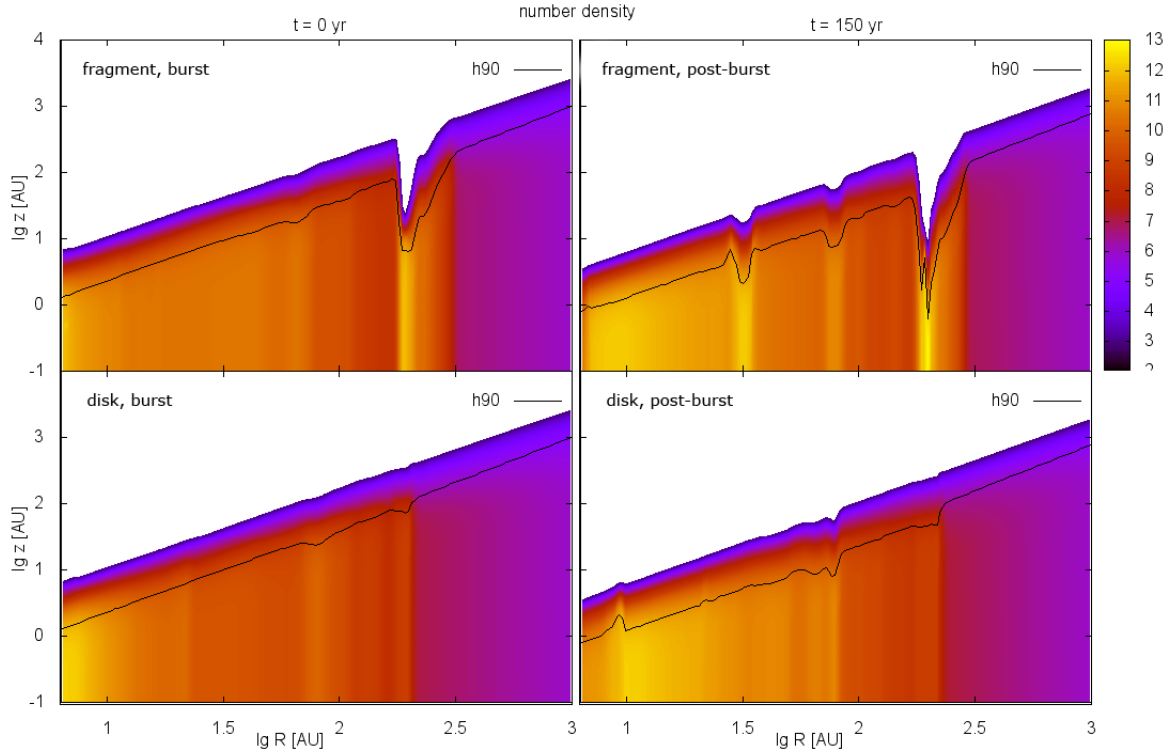


Figure 5.4: Vertical number density distribution of the disk at the burst ($t = 0$ yr) and after the burst ($t = 150$ yr).

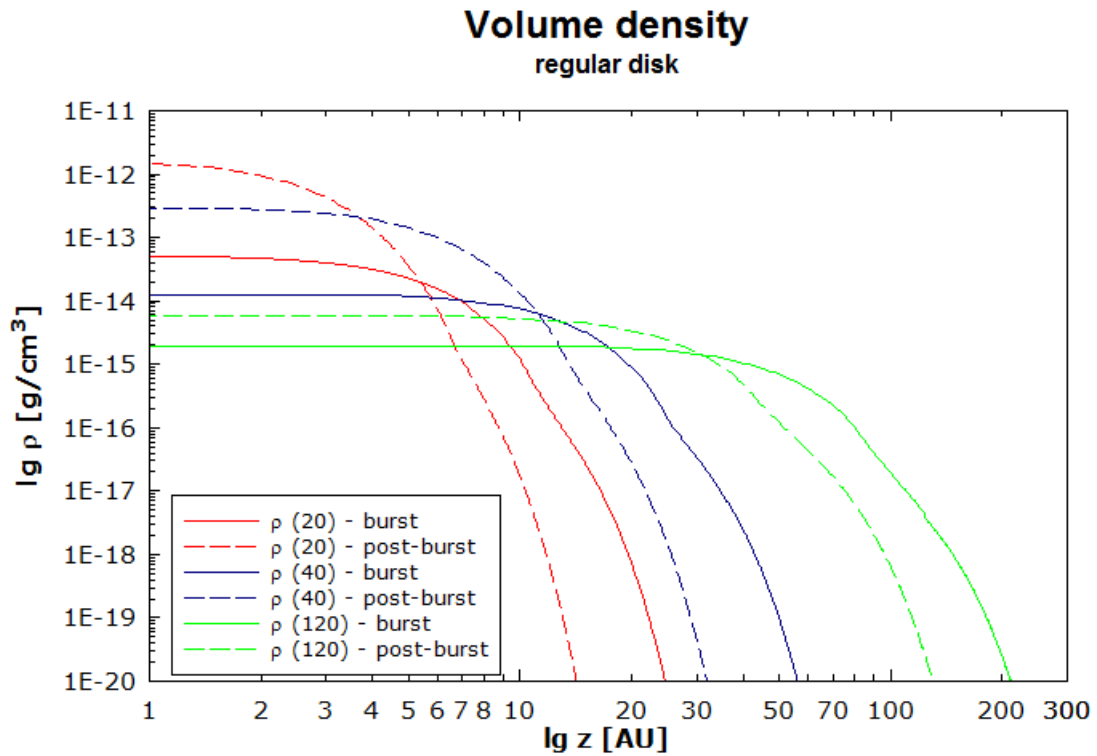


Figure 5.5: Vertical volume density distribution at the burst $t = 0$ yr (solid lines) and after the burst $t = 150$ yr (dashed lines).

the gas midplane temperature taken from the hydrodynamical code. Comparing the accurate numerical reconstruction of the disk to the analytic solution shows the impact of the simplified analytic treatment where an isothermal case was assumed. The numerically obtained density is stronger concentrated to the midplane, where the volume density is about twice as high as that of the analytical solution. The difference is strongest at the fragment where the analytic solution differs from the numerical approach by a factor of almost 60. This significant difference is caused by the impact of the self-gravity of the disk.

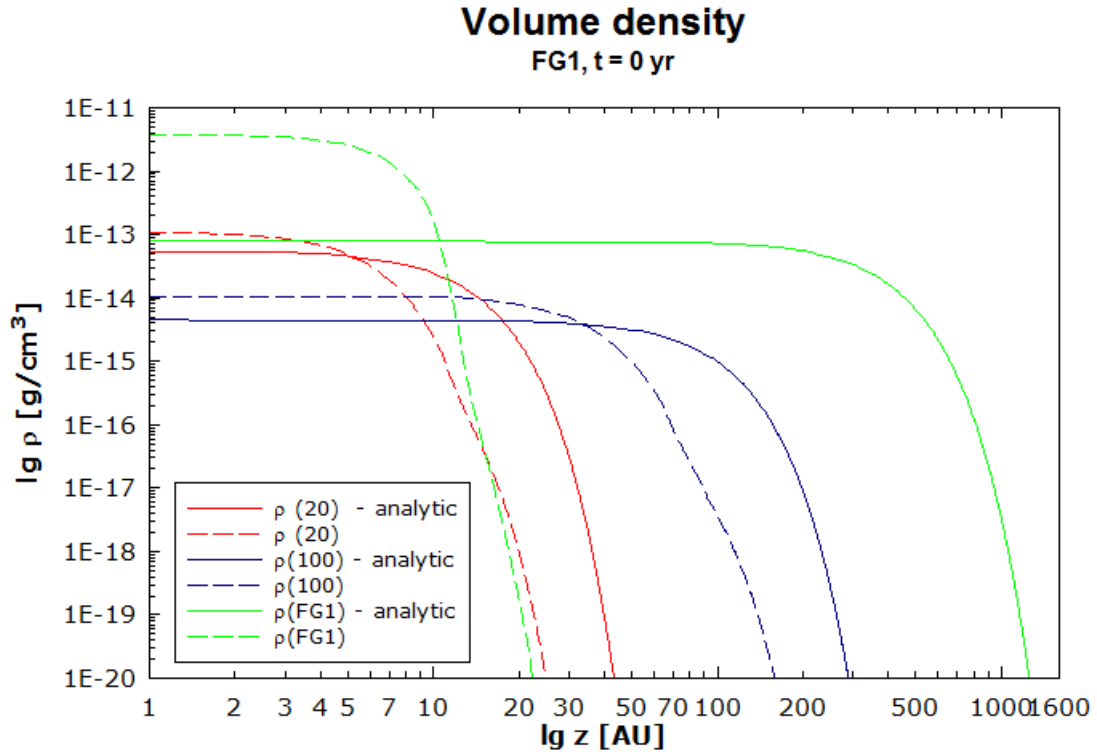


Figure 5.6: Volume density for FG1 at 20 AU, 100 AU and at the fragment, compared with analytic solution.

In Figure 5.7 the number densities of the vertical disk structure are shown for the case where self-gravity is included (left panel) and for the case where the density distribution is reconstructed without taking the self-gravitating disk into account (right panel). If the self-gravitating term is neglected, the vertical height of the disk is more extended at the fragment. The fragment is not shielded from stellar radiation which will act against the stability of the fragment.

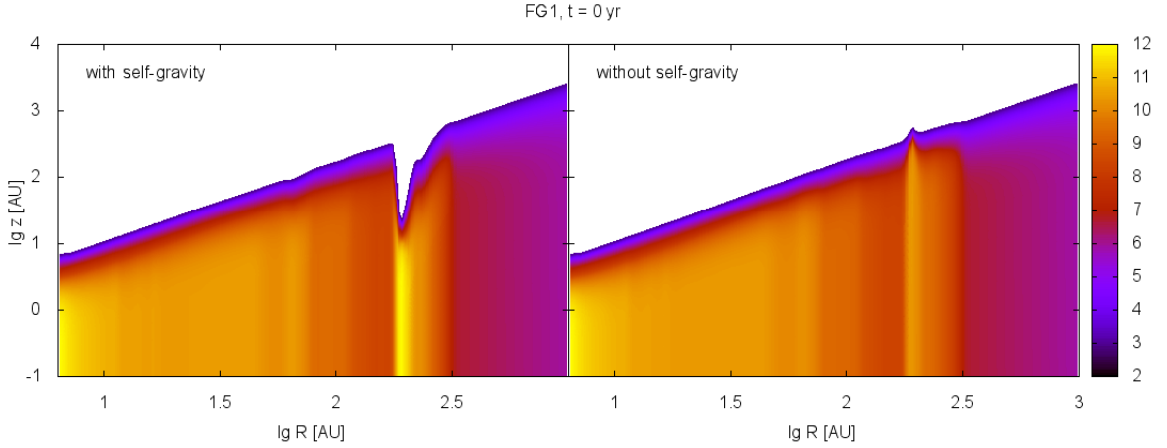


Figure 5.7: Comparison of the vertical density distribution of a disk with self-gravity (left) to a disk without self-gravity (right).

5.3 Characteristic disk heights

The thin-disk approach is valid if the vertical extent of the disk is considerably smaller than the radial extent (see Section 3.3). The applicability of the thin-disk model is tested in this section and the results are given in Figure 5.8. To the As there is no unique definition of the characteristic disk height, different definitions are investigated.

The red solid line indicates the total disk height Z , i.e. the vertical distance from the midplane where the disk merges with the surrounding environment, defined by the external density. It is clear that the condition for the thin-disk approach is not met for this definition. The ratio Z/R is larger than 1 over the whole disk, except for the part where the fragment is located. Here, the height of the disk is significantly smaller.

The extent of the vertical disk is also controlled by the assumed value for the external density, $n_{ext} = 10^3 \text{ cm}^{-3}$. If the external density is set to a higher value $n_{ext} = 10^5 \text{ cm}^{-3}$, the total disk height (black dotted line) becomes smaller than that for $n_{ext} = 10^3 \text{ cm}^{-3}$.

The black solid line (h_{90}) indicates the height of the disk, below which 90 % of the total disk mass is contained. This definition is more appropriate to our purposes, as it marks the dynamically important region for the hydrodynamical calculations and it clearly fulfils the criterion for the thin-disk approach.

The red dashed line indicates the vertical distance where the gas volume density drops by a factor of e as compared to the midplane density ρ_0 . We call this vertical distance the 1σ scale height. Except for the outer part of the disk, where

the 1σ scale height is smaller, it shows a similar trend as h_{90} .

The scale height H derived from a non-gravitating local hydrostatic equilibrium, see Equation (4.29), is shown by the black dashed line. This definition of the scale height fails grossly at the fragment. Contrary to the previous definitions, the disk is even more vertically extended at this region. This effect is caused by the dependence of the height on the thermal sound speed c_s , as fragments also show high temperatures.

The most relevant definitions are h_{90} and the 1σ scale height, because they delineate regions where most of the disk mass is contained. The thin-disk approximation is fulfilled up to several hundred AU , a radial distance beyond which disks rarely grow.

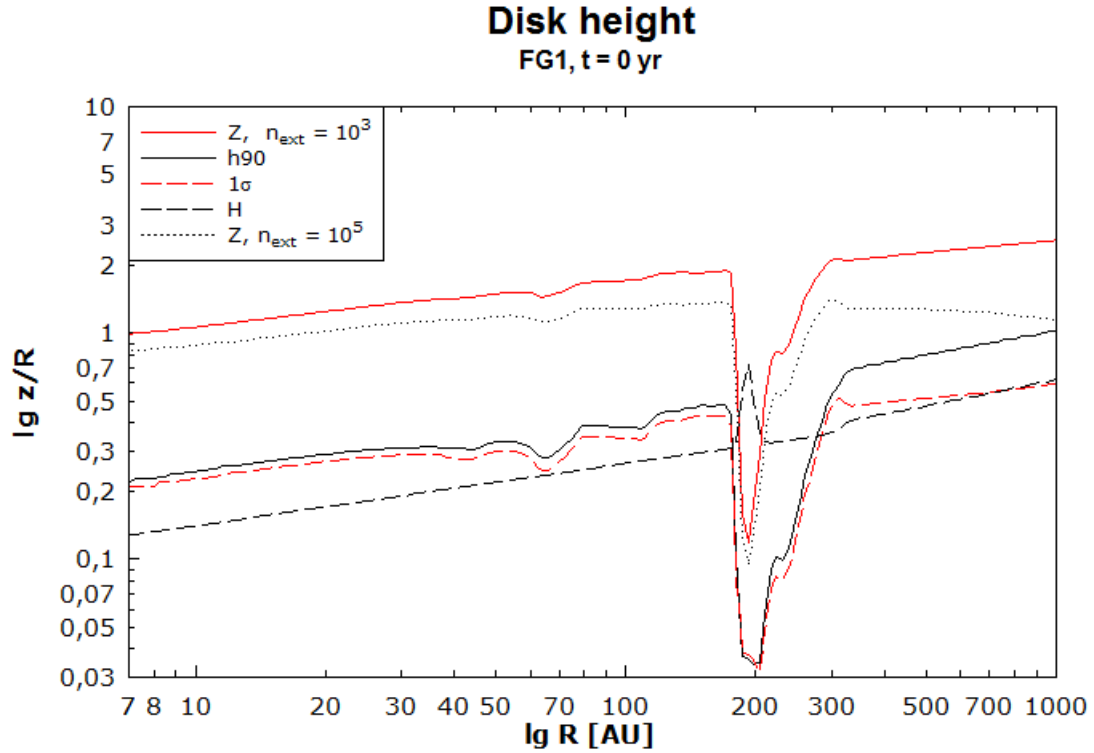


Figure 5.8: Characteristic heights of the disk during the burst.

5.4 Temperature structure

Figure 5.9 presents the vertical distribution of gas temperature in the disk. Because the disk surface is heated by stellar radiation, while viscosity deposits heat homogeneously throughout the vertical extent of the disk (our assumption), the temperature is higher near the disk surface. As the fragment is gravitationally bound the temperature is increased due to compressional heating at the fragment (upper panels).

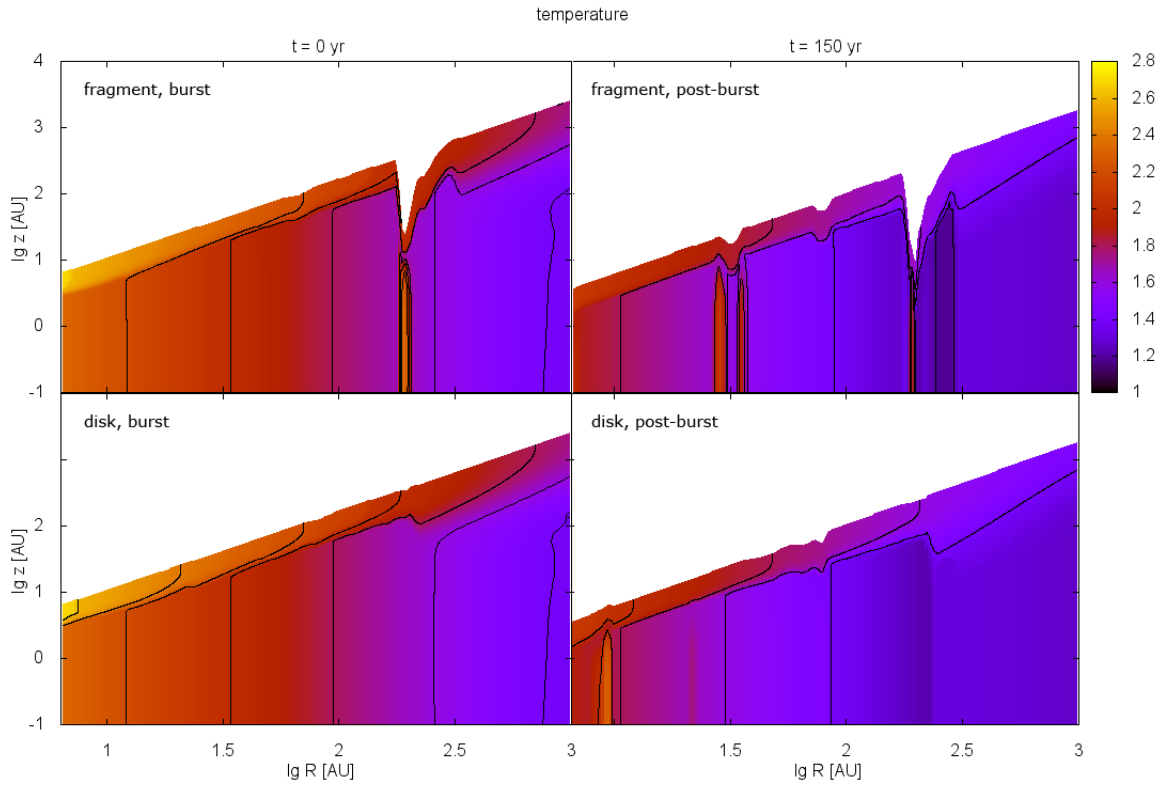


Figure 5.9: Gas temperature structure in the vertical cut of the disk.

Plotting the temperature profiles as a function of the distance z from the midplane at the burst $t = 0$ yr shows these effects in more detail. In Figure 5.10 the profiles at radial distances of $r = 20$ AU (red line), $r = 100$ AU (blue line) and at the fragment $FG1$ $r \simeq 192$ AU (green line) are shown. The temperature profiles at 20 AU and 100 AU show a similar pattern. Heating due to stellar radiation dominates and the hottest regions lie at the disk surface. The opposite behaviour is present at the fragment. Here, compressional heating prevails over radiation and higher gas temperatures can be found near the disk midplane.

The plateau near the surface of the disk arises as the disk is optically thin to the UV radiation and photons can pass through this part of the disk with little

absorption.

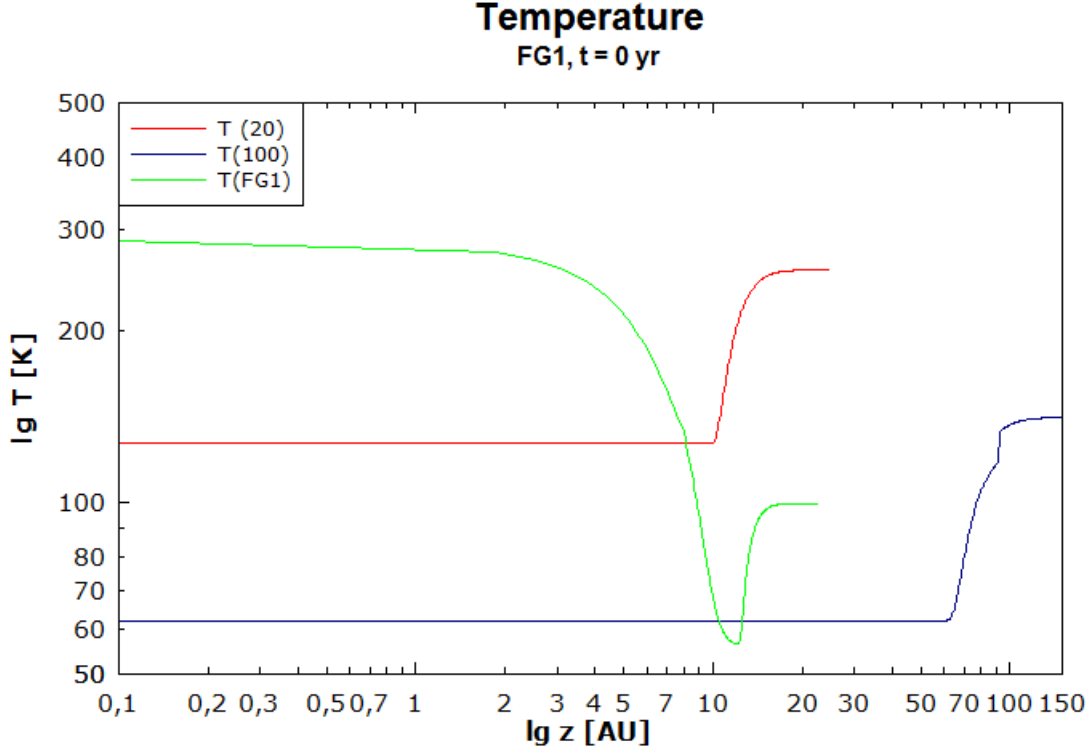


Figure 5.10: Temperature profile at $r = 20$ AU, $r = 100$ AU and at the fragment $r \simeq 192$ AU at the burst $t = 0$ yr.

In Figure 5.11 different definitions for the gas temperature as a function of radius are compared at the moment of the burst $t = 0$ yr, for the cut through the disk where the fragment *FG1* is located. The mass-weighted temperature (red line) is defined as:

$$T(R) = \frac{\sum_i T_i(z) m_i(z)}{\sum_i m_i(z)} \quad (5.3)$$

where i gives the vertical grid cells and $m_i(z)$ is the mass contained in the corresponding cell. The photospheric temperature (green dashed line) is defined by the mass-weighted temperature of the layer, where $\tau \leq \frac{2}{3}$ (D'Alessio et al. 1998). Here, τ is the optical depth. The effective temperature (black dotted line) describes the temperature of the layer where $\tau \leq 1$ and is also mass-weighted.

The midplane temperature of the disk (blue dashed line) coincides with the mass-weighted temperature (red line) almost over the whole disk. The photospheric temperature differs from the effective temperature only in the inner disk region, where higher temperatures are found in the photospheric layer. Compar-

ing these two temperatures with the midplane temperature at the position of the fragment indicates that the temperature near the midplane dominates over the temperatures at the disk surface. Contrary to the vertical temperature structure of the rest of the disk, at the position of the fragment the highest temperatures are found in the midplane.

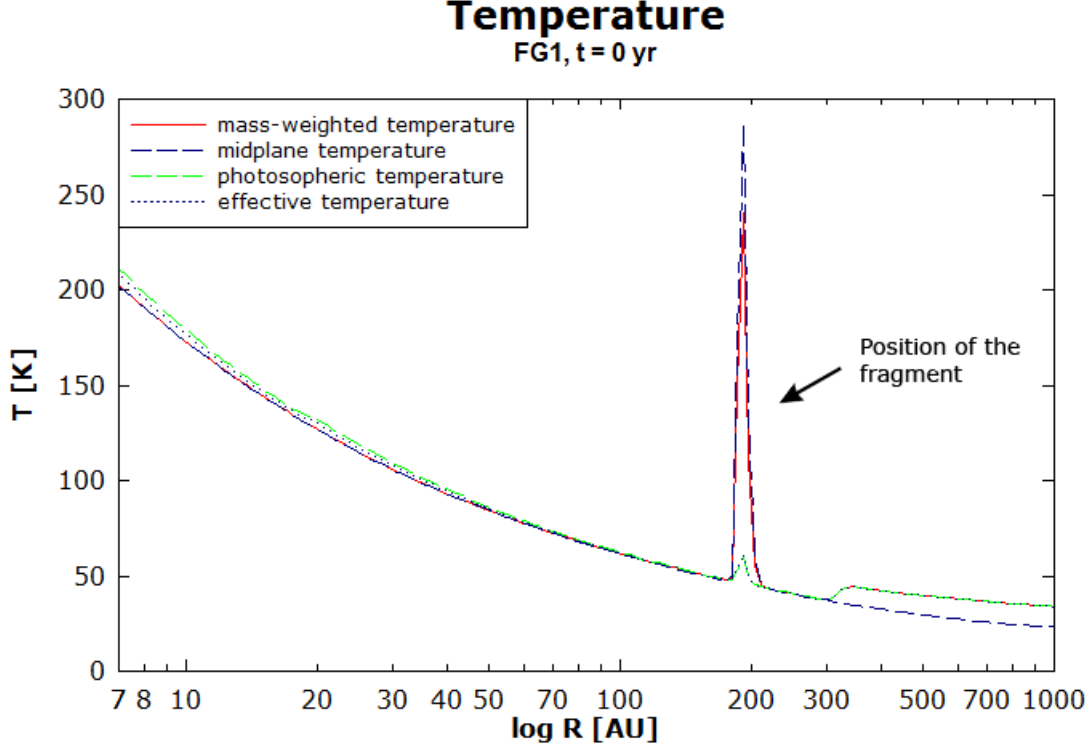


Figure 5.11: Comparison of different disk temperatures at the burst $t = 0$ yr, for the cut through the disk where the fragment *FG1* is located. The arrow indicates the position of the fragment.

The effect of increasing disk temperature during the burst is shown in Figure 5.12. The mass-weighted temperature of the disk is given at the burst (red line) and after the burst (blue line) as function of radius. The left panel shows the temperature of the disk at the fragment *FG1* and the right gives the temperature of the cut through the regular disk. In the post-burst phase the peaks in temperature indicate additional fragments which are present in both cuts through the disk.

As the stellar luminosity L_* during the burst is higher by a factor of ~ 30 than 150 yr later in the post-burst phase, the temperature of the disk during the burst is significantly higher. The difference between these two temperatures is naturally more dominant in the inner disk closer to the star, where the temperature

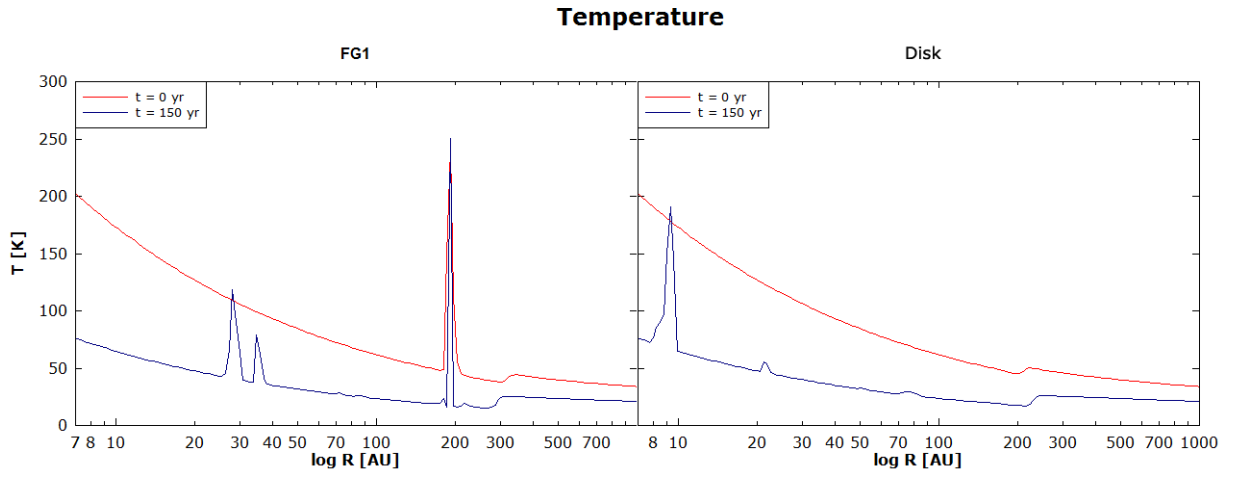


Figure 5.12: The red line shows the mass-weighted temperature of the disk at the burst ($t = 0 \text{ yr}$) and the blue line shows the mass-weighted temperature after the burst ($t = 150 \text{ yr}$).

is increased by a factor of 2.5. This increase in temperature is responsible for the increase of the vertical extent of the disk during the burst.

6 Summary & Conclusion

In this thesis, I study the vertical structure of protostellar disks during luminosity outbursts of young stellar objects and during the subsequent quiescent phase. Stellar and disk parameters, including viscous and shock heating, are obtained from the two-dimensional, thin-disk, hydrodynamical code of Vorobyov & Basu (2006), which computes the formation and evolution of a gravitational unstable disk starting from a starless collapsing cloud core of mass $M_c = 1.8 M_\odot$. The vertical disk structure is reconstructed based on a description of Akimkin et al. (2012), taking into account radiative transfer and the following modifications:

- The use of density- and temperature-dependent IR opacities of Semenov et al. (2003). This is necessary as the temperatures and densities obtained from the hydrodynamical code cover a wide range.
- Development of a new method for the reconstruction of the disk vertical density distribution, based on the Henvey method (Bodenheimer et al. 2007). The method used by Akimkin et al. (2012) is a combination of the shooting and bisection algorithm, which shows a slow convergence. The new method for the reconstruction of the vertical density structure demonstrates a faster convergence by a factor of 8.
- Inclusion of disk self-gravity into the reconstruction of the disk vertical structure. The description of a self-gravitating disk has an important impact on the disk vertical structure if dense fragments are present in the disk.

The vertical disk structure was reconstructed during a FUori-type luminosity burst, where the total stellar luminosity (accretion and photospheric luminosity) reached a value of $L_* \sim 254 L_\odot$. The disk vertical structure at the burst was compared to that of the subsequent quiescent phase, 150 *yr* after the burst, where the total stellar luminosity was $L_* \sim 8 L_\odot$. Two cuts through the disk were considered. The first cut includes a dense fragment at $r \sim 192 AU$ while the second cut passes through the regular disk. Studying the impact of luminosity bursts on the vertical structure of the disk, the following results were obtained:

- During the burst, the gas temperature of the disk is increased on average by a factor of 2.3. In the inner disk region the temperature rises from 81 K to 204 K, whereas at a radial distance of 100 *AU* the temperature rises from 23 K to 62 K.
- The vertical disk height is expanded by 62% during the burst, leading to lower gas volume densities close to the midplane. For example at a radial

distance of $r = 20 \text{ AU}$ the gas volume density at the midplane during the burst is decreased by a factor of 0.03 as compared to that in the quiescent phase.

- Different definitions of the vertical disk height were studied to test the applicability of the thin-disk approach during the luminosity burst. The total disk height, which is defined by the vertical distance from the midplane where the disk merges with the external environment ($n_{ext} = 10^3 \text{ cm}^{-3}$), is comparable to the radial distance, implying the violation of the thin-disk requirement. On the other hand, when vertical scale heights comprising $\sim 90\%$ of the total gas mass and excluding the upper low-density atmosphere are considered, the thin-disk approach turns out to be valid up to a radial extent of $\sim 500 \text{ AU}$. In the quiescent stage the disk is more compact and the thin-disk approach is valid to a higher radial extent.
- While self-gravity has little impact on the vertical scale height of the regular disk, the scale height is significantly reduced at the position of dense fragments. This effect shields the fragment from stellar irradiation, which may increase the likelihood of fragment survival.
- The vertical gas temperature profiles in the regular disk show a maximum temperature at the disk surface, indicating that heating due to stellar radiation dominates over viscous and shock heating. A different behaviour is seen at fragments where the main heating source is compressional heating. Here, the gas temperature near the midplane reaches higher values than at the surface.
- The numerically derived distribution of the gas density is concentrated stronger to the disk midplane than that described by the corresponding analytical solution for the vertically isothermal, non gravitating disk (see Equation 4.28). In particular the densities at the midplane of the disk can be twice higher than those obtained analytically. At the position of the fragment, this difference can amount to a factor of 60 due to strong self-gravity of the fragment.

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A Reconstruction of density distribution

The density distribution of the vertical disk can be obtained by using the Newton method to find the elements of the system described by n equations, where n is given by the number of vertical grid points. The resulting system can be expressed as $\hat{\mathbf{A}}\mathbf{y} = \mathbf{d}$ where $\hat{\mathbf{A}}$ is a tridiagonal matrix and ρ at each grid point is given by $\mathbf{y}$. If the elements are found the solution of this system can be solved using the Thomas algorithm which is a simplified form of the Gaussian elimination.

A.1 Newton method

Two different approaches are compared: The first one, which makes it easier to test and compare the algorithm is a disk where only the gravitational force of the star is taken into account, whereas for the second approach a self-gravitating disk will be assumed.

Star gravitation

To explain this procedure in more detail we start with the following set of equations:

$$\frac{1}{\rho} \frac{dP}{dz} = -\frac{GM_*}{R^3} z \quad (\text{A.1})$$

$$\frac{dz}{d\Sigma} = \frac{1}{\rho} \quad (\text{A.2})$$

After using the equation of state for an ideal gas $P = \rho T k_B / (\mu m_H)$ where k_B is the Boltzmann constant and μ the mean molecular weight. After introducing $\alpha = GM_* \mu m_H / (R^3 k_B)$ the above system is rewritten as:

$$\frac{d(\rho T)}{d\Sigma} = -\alpha z \quad (\text{A.3})$$

$$\frac{dz}{d\Sigma} = \frac{1}{\rho} \quad (\text{A.4})$$

Therefore the equation to solve can be reduced to:

$$\frac{d^2(\rho T)}{d\Sigma^2} = -\frac{\alpha}{\rho} \quad (\text{A.5})$$

which using the Newton method is expressed as:

$$-\frac{\alpha}{\rho^{n+1}} \simeq -\alpha \left[\frac{1}{\rho^n} - \frac{1}{(\rho^n)^2} (\rho^{n+1} - \rho^n) \right] \quad (\text{A.6})$$

For a better overview ρT will be denoted with y in the following description. The discretization of the set of equations for y_i is given by:

$$y_{i+1} = y_i + \frac{dy}{d\Sigma} h_2 + \frac{1}{2} \frac{d^2 y}{d\Sigma^2} h_2^2 \quad (\text{A.7})$$

$$y_{i-1} = y_i - \frac{dy}{d\Sigma} h_1 + \frac{1}{2} \frac{d^2 y}{d\Sigma^2} h_1^2 \quad (\text{A.8})$$

with:

$$h_1 = \Delta\Sigma_{i-1/2} = \Sigma_i - \Sigma_{i-1} \quad (\text{A.9})$$

$$h_2 = \Delta\Sigma_{i+1/2} = \Sigma_{i+1} - \Sigma_i \quad (\text{A.10})$$

Combining the above equations and using $h_{3/2} = \frac{h_1+h_2}{2}$ the following equation for y'' is obtained:

$$y'' = \frac{h_1 y_{i+1} - 2h_{3/2} y_i + h_2 y_{i-1}}{h_1 h_2 h_{3/2}} \quad (\text{A.11})$$

The last expression together with Equation A.6 gives the elements of the matrix:

$$\frac{\rho_{i+1}^{n+1} T_{i+1}}{h_2 h_{3/2}} - \frac{2\rho_i^{n+1} T_i}{h_1 h_2} + \frac{\rho_{i-1}^{n+1} T_{i-1}}{h_1 h_{3/2}} = -\frac{\alpha}{\rho_i^n} + \frac{\alpha}{(\rho_i^n)^2} (\rho^{n+1} - \rho^n) \quad (\text{A.12})$$

The tridiagonal elements of the matrix a_i, b_i, c_i and the solution d_i where $i = 2, \dots, n-1$ are then given by:

$$a_i = \frac{1}{h_1 h_{3/2}} T_{i-1} \quad (\text{A.13})$$

$$b_i = -\frac{2}{h_1 h_2} T_i - \frac{\alpha}{(\rho_i^n)^2} \quad (\text{A.14})$$

$$c_i = \frac{1}{h_2 h_{3/2}} T_{i+1} \quad (\text{A.15})$$

$$d_i = -\frac{2\alpha}{\rho_i^n} \quad (\text{A.16})$$

To find the first and last row of the system, the boundary conditions which are given from the relations:

$$z|_{\Sigma=0} = 0 \quad (\text{A.17})$$

$$\rho|_{\Sigma_0} = \rho_{ext} \quad (\text{A.18})$$

need to be implemented. Where the first one gives the equation:

$$\rho_1^{n+1}T_1 - \rho_2^{n+1}T_2 = 0 \quad (\text{A.19})$$

and the second boundary condition leads to:

$$\rho_n^{n+1} = \rho_{ext} \quad (\text{A.20})$$

which defines the coefficients of the first and last row:

$$b_1 = T_1, c_1 = -T_2, d_1 = 0 \quad (\text{A.21})$$

$$a_n = 0, b_n = 1, d_n = \rho_{ext} \quad (\text{A.22})$$

Self-gravitating disk

Including also the gravitational force of the disk in the description minor changes need to be done in the derivation above. The term describing the self-gravitating disk can be derived from the Poisson Equation:

$$\Delta\phi = 4\pi G\rho \quad (\text{A.23})$$

The gravitational force is given by the first derivative of this equation and is written as:

$$F = -\frac{\partial\phi}{\partial z} \quad (\text{A.24})$$

$$F = -4\pi G\rho z = -2\pi G\Sigma(z) \quad (\text{A.25})$$

where the assumption of an infinitely extended disk is used. This term is now added to Equation A.1 which becomes:

$$\frac{1}{\rho} \frac{dP}{dz} = -\frac{GM_*}{R^3} z - 2\pi G\Sigma \quad (\text{A.26})$$

Applying the same procedure as above this equation is rewritten as:

$$\frac{d(\rho T)}{d\Sigma} = -\alpha z - \beta \quad (\text{A.27})$$

where β is given through: $2\pi G\mu m_H/k_B$. Therefore, Equation A.6 becomes:

$$-\frac{\alpha}{\rho^{n+1}} - \beta \simeq -\alpha \left[\frac{1}{\rho^n} - \frac{1}{(\rho^n)^2} (\rho^{n+1} - \rho^n) \right] - \beta \quad (\text{A.28})$$

All elements of the system are adopted as they stand except d_i for $i = 2, \dots, n-1$ where the new term is added:

$$d_i = -\frac{2\alpha}{\rho_i^n} - \beta \quad (\text{A.29})$$

As β is a constant and relatively small value it is obvious that including a self-gravitating disk in the description will only affect the density distribution if the first term in Equation A.26 is comparable with the second term. This means that either a high density ρ or a big radius R is needed for the self-gravity to have any impact.

A.2 Thomas algorithm

Now that the matrix is composed the tridiagonal system of equations can be solved using the Thomas algorithm.

This algorithm basically consists of two parts, where in the first downward sweep the elements denoted with a_i are eliminated and afterwards the solution is obtained by backward substitution.

For the first part the coefficients need to be modified in a way:

$$c'_1 = \frac{c_1}{b_1} \quad (\text{A.30})$$

$$c'_i = \frac{c_i}{b_i - c'_{i-1}a_i} \quad i = 2, \dots, n-1 \quad (\text{A.31})$$

$$d'_1 = \frac{d_1}{b_1} \quad (\text{A.32})$$

$$d'_i = \frac{d_i - d'_{i-1}a_i}{b_i - c'_{i-1}a_i} \quad i = 2, \dots, n \quad (\text{A.33})$$

where:

$$y_n = d'_n \quad (\text{A.34})$$

and the system can be solved upwards to obtain the final solution with:

$$y_i = d'_i - c'_i y_{i+1} \quad (\text{A.35})$$

Accuracy

The accuracy of the system $\hat{\mathbf{A}}\mathbf{y} = \mathbf{d}$ is tested by comparing the right hand side of the equation $\mathbf{d}$ with $\hat{\mathbf{A}}\mathbf{y}$ and analysing the residual:

$$\hat{\mathbf{A}}\mathbf{y} - \mathbf{b} = \epsilon \quad (\text{A.36})$$

where the euclidean norm of the resulting vector gives a value of $9.0 \cdot 10^{-17}$. Using double precision this result is still smaller than the machine epsilon ($\sim 2.2 \cdot 10^{-16}$) which gives an upper limit of the accuracy due to floating point arithmetic.

Further the accuracy of this algorithm is tested with an already known function. Therefore the equation describing a parabolic function (see Figure A.1) was chosen:

$$\frac{d^2 y}{dx^2} = -1 \quad (\text{A.37})$$

with the boundary conditions:

$$y(0) = 0, y(1) = 0 \quad (\text{A.38})$$

Using these conditions the analytic solution of Equation A.37 is written as:

$$y = \frac{1}{2}x(1 - x) \quad (\text{A.39})$$

Table A.1 shows the resulting set of coefficients, where h is given through:

$$h = \frac{1}{n - 1} \quad (\text{A.40})$$

Comparing the numerical result with the given analytical equation (Equation A.39) the euclidean norm is $6.1 \cdot 10^{-9}$.

Table A.1: Coefficients of the linear system for the Thomas algorithm.

$i = 1$	$y(i) = 0$	$a(i) = 0$	$b(i) = 1$	$c(i) = 0$	$d(i) = 0$
$i = 2, 3 \dots n-1$		$a(i) = 1$	$b(i) = -2$	$c(i) = 1$	$d(i) = -h^2$
$i = n$	$y(i) = 0$	$a(i) = 0$	$b(i) = 1$	$c(i) = 0$	$d(i) = 0$

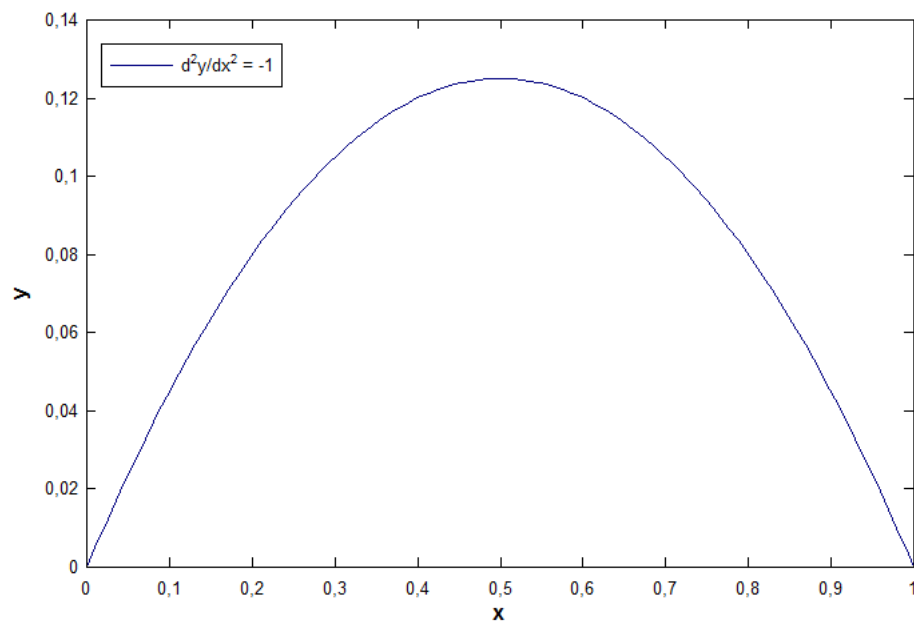


Figure A.1: Numerical and analytical solution of the parabolic function.

B Basic radiative transfer equations

Here, I briefly review the basic radiative transfer equations. I follow the description of Mihalas (1987), Hartmann (1998) and Bodenheimer et al. (2007).

Taking into account the energy changes of the beam with time, the *radiation transfer equation* can be derived from the Boltzmann equation for photons:

$$\frac{1}{c} \frac{\partial I_\nu}{\partial t} + (\hat{\mathbf{s}} \cdot \nabla) I_\nu = -\epsilon_\nu I_\nu + j_\nu \quad (\text{B.1})$$

where I_ν is the radiation intensity with the dimension $\text{erg cm}^{-2} \text{sec}^{-1} \text{Hz}^{-1} \text{sr}^{-1}$. $\hat{\mathbf{s}}$ describes the direction of propagation of the photons. Further j_ν is the mass emission rate which includes thermal emission as well as contributions from energy scattered into the beam. The extinction rate is denoted by ϵ_ν which is defined as the sum of absorption κ_ν and scattering σ_ν . All of these coefficients have the dimension cm^{-1} :

$$\epsilon_\nu = \kappa_\nu + \sigma_\nu \quad (\text{B.2})$$

The dynamic radiation field is described by the three moments of the Boltzmann equation. The mean intensity J_ν , the first moment H_ν^i which can also be expressed in terms of the net flux $F_\nu^i = 4\pi H_\nu^i$ and the second moment K_ν^{ij} :

$$J_\nu = \frac{1}{4\pi} \int_{4\pi} I_\nu d\Omega \quad (\text{B.3})$$

$$H_\nu^i = \frac{1}{4\pi} \int_{4\pi} I_\nu s_i d\Omega \quad (\text{B.4})$$

$$K_\nu^{ij} = \frac{1}{4\pi} \int_{4\pi} I_\nu s_i s_j d\Omega \quad (\text{B.5})$$

After the integration of Equation B.1 over all directions and consideration of a time independent case it can be expressed as:

$$\nabla \cdot \mathbf{H}_\nu + \frac{1}{4\pi} \int_{4\pi} \epsilon_\nu (I_\nu - S_\nu) d\Omega = 0 \quad (\text{B.6})$$

Assuming isotropic scattering as well as absorption and emission of thermal continuum radiation, the source function has the following form:

$$S_\nu = \frac{\kappa_\nu B_\nu + \sigma_\nu J_\nu}{\kappa_\nu + \sigma_\nu} \quad (\text{B.7})$$

and Equation B.6 can be expressed as:

$$\nabla \cdot \mathbf{H}_\nu + \kappa_\nu (J_\nu - B_\nu) = 0 \quad (\text{B.8})$$

With the help of the definition for the radiation energy density E_ν , which gives the radiation energy per unit volume:

$$E_\nu = \frac{1}{c} \int_{4\pi} I_\nu d\Omega = \frac{4\pi}{c} J_\nu \quad (\text{B.9})$$

The time-independent energy equation can be expressed as:

$$\nabla \cdot \mathbf{F}_\nu = 4\pi\kappa_\nu B_\nu - c\kappa_\nu E_\nu \quad (\text{B.10})$$

Finally integrating over all frequencies

$$\int_0^\infty B_\nu d\nu = \frac{\sigma_B T^4}{\pi} = \frac{c}{4\pi} a T^4 = \frac{c}{4\pi} B \quad (\text{B.11})$$

and leads to the *energy radiation equation*:

$$\frac{dF}{dz} = c\sigma_p (B - E) \quad (\text{B.12})$$

with the integrated energy flux F , the integrated energy density E and the integrated density of black-body radiation B . σ_p is the Planck absorption coefficient.

The second radiative equation can be obtained by starting with multiplying Equation B.1 by s_i , integration and using again the assumption of isotropic scattering:

$$\sum_j \frac{\partial K_\nu^{ij}}{\partial x_j} + \epsilon H_\nu^i = 0 \quad (\text{B.13})$$

Together with Equation B.8 these two equations build a set of nine unknowns: J_ν , three components of H_ν^i and five components of K_ν^{ij} . To close this system 5 additional equations are necessary:

$$K_\nu^{ij} = f_\nu^{ij} J_\nu \quad (\text{B.14})$$

where f_ν^{ij} are called the generalized Eddington factors. In an optically thick

medium the Eddington approximation can be used: $f_v^{ij} = \frac{1}{3}$. Therefore Equation B.13 can be simplified as:

$$H_\nu = -\frac{1}{3\epsilon} \nabla J_\nu \quad (\text{B.15})$$

Using again the expression for flux F and the energy density E we arrive at the second radiative transfer equation:

$$F = -\frac{1}{3\sigma_R} \frac{dE}{dz} \quad (\text{B.16})$$

where σ_R is the Rosseland absorption coefficient.

Equation B.16 is the well known *diffusion approximation*. In the code Equation B.12 and Equation B.16 are used to describe the radiative transfer of the IR continuum.

B.1 Opacities

Rosseland mean κ_R and Planck mean κ_P opacities are given as:

$$\frac{1}{\kappa_R} = \frac{\int_0^\infty \frac{1}{(\kappa_\nu^{abs} + \kappa_\nu^{sca})} \frac{dB}{dT} d\nu}{\int_0^\infty \frac{dB}{dT} d\nu} \quad (\text{B.17})$$

$$\kappa_P = \frac{\int_0^\infty \kappa_\nu^{abs} B_\nu d\nu}{\int_0^\infty B_\nu d\nu} \quad (\text{B.18})$$

For calculating the Rosseland mean opacity, the inverse average of the monochromatic opacity which includes absorption as well as scattering processes, is used. The monochromatic opacity used for the Planck mean opacity only accounts for absorption processes.

To calculate Rosseland and Planck mean opacities, the program of Semenov described in Semenov et al. (2003), is used. One of the advantages of this approach is the possibility to obtain opacities in a wide temperature $[5; 10\,000]$ K and density range $[2 \cdot 10^{-18}; 2 \cdot 10^{-7}]$ g/cm³.

The opacities calculated by Bell & Lin (1994) are only Rosseland mean opacities and therefore not adaptable for my needs. Nevertheless, it is interesting to compare them with Semenov opacities, to notice the importance of a complete spectral line data set (see Figure B.1). Alexander & Ferguson (1994) discussed

and compared the opacities obtained by Bell & Lin and pointed out the significant low results, which are an outcome of missing H_2O lines with wavelengths longer than $5 \mu m$ in their calculations. Around $2\,000\,K$ the opacity is primarily dominated by molecular lines, namely H_2O and TiO . Alexander & Ferguson (1994) showed the changing of their opacities by two orders of magnitude two lower values at $2\,000\,K$ if the same lines are excluded.

The primary assumptions made by Semenov et al. (2003) are as follows: In the predominant part of the disk absorption and scattering are caused by dust grains. The destruction of these grains does not set in until the temperature reaches values of about $T \sim 1\,500\,K$. Dust even dominates the absorption properties if there are only 30% of SiO content in the solid phase present. Therefore gas and dust opacities are calculated separately with a breakpoint at $T = 1\,500\,K$. The main components of dust are amorphous pyroxene ($[Fe, Mg]SiO_3$), olivine ($[Fe, Mg]_2SiO_4$), volatile and refractory organics ($CHON_{material}$), amorphous water ice, troilite (FeS) and iron. While for temperatures less than $155\,K$ all of these constituents are present, they are destroyed by and by with higher temperatures. When the temperature lies in the range $685\,K < T \leq 1\,500\,K$, only silicates and iron are present and thus determine the opacity properties.

One problem pointed out by Semenov et al. (2003), is the possibility of changing properties of the dust grains. Due to limitations caused primarily by computational methods the calculations can not account for a more realistic grain structure. In spite of this limitations Semenov et al. (2003) argue that the calculations using simplified grains is still a good and reasonable approach.

With the program provided by Semenov et al. (2003), it is possible to calculate different types of grains. Distinctions are made in the silicate type, the topology of the grains and the shape of the particles. For the calculations of the IR as well as ultra-violet (UV) opacities normal silicates with a relative iron content of $Fe/(Fe + Mg) = 0.3$, homogeneous and spherical particles are used. The UV opacities are calculated under the assumption of being independent of surface density. Therefore, the density is fixed and the UV opacities are obtained as a function of temperature.

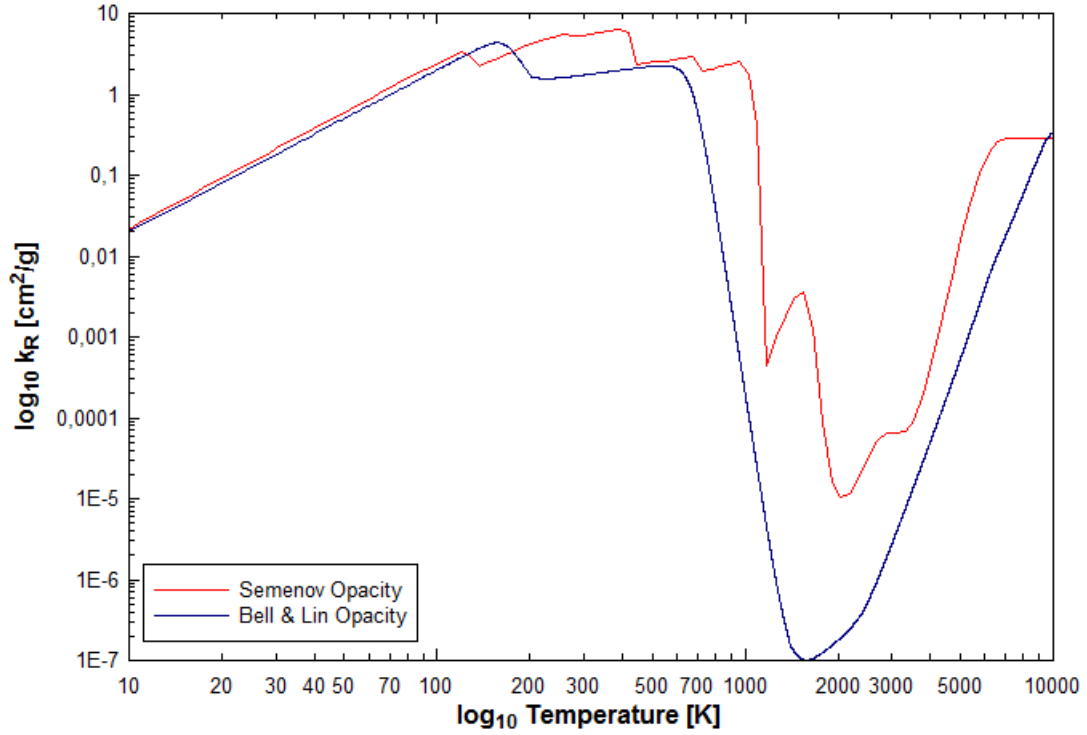


Figure B.1: Semenov et al. (2003) compared with Bell & Lin (1994) calculations for Rosseland mean opacities for densities $n = 10^{-14} \text{ g/cm}^3$. The properties for the dust grains of the Semenov model are normal (in the sense of relative iron content) and we assume homogeneous, spherical particles. The lower values of Bell & Lin opacities due to incomplete set of data for H_2O lines are well notable.

C Source code: Reconstruction of the disk vertical structure

For the calculation of the IR opacities the Fortran77 code of Semenov et al. (2003) is used, which is not reproduced here. The code as well as a description is provided by Semenov et al. (2003) and is freely available in the internet [34].

```
1 MODULE acc                                ! contains accuracy
2   IMPLICIT NONE
3   INTEGER, PARAMETER :: dp = selected_real_kind(15, 307)
4 END MODULE
5 !-----
6 MODULE globalparameters                    ! contains global parameters
7   USE acc
8   IMPLICIT NONE
9
10  INTEGER, PARAMETER :: m = 256
11  ! number of points in radial direction
12  INTEGER, PARAMETER :: endk = 256
13  ! number of angledependent grid points
14  REAL (KIND = dp) :: age_now
15  ! current time
16  REAL (KIND = dp) :: Mstar, Mdot, fact, Lstar, Tstar
17  ! star mass, accretion rate, cos of radiation penetrated in disk,
18  ! star luminosity, star temperature
19  REAL (KIND = dp) :: tkin_r(m), f(m), sacc(m), rad(m), sigma_rad(m)
20  ! midplane temperature, inclination, accretion heating, radius,
21  ! surface density
22 END MODULE
23 !-----
24 MODULE verticalparameters                 ! main variables are defined here
25   USE acc
26   USE globalparameters
27   IMPLICIT NONE
28
29   INTEGER, PARAMETER :: n = m/4
30   ! max. number of vertical points
31   INTEGER :: i, j, iter, itertkin, stat, out, k, gridn(m)
32   ! ----- constants -----
33   REAL (KIND = dp), PARAMETER :: AU = 1.5e13_dp, Grav = 6.67e-8_dp, mH
34   = 1.67e-24_dp, pc=3.086e18_dp
35   REAL (KIND = dp), PARAMETER :: Msun = 2e33_dp, Lsun = 4e33_dp, yr =
36   3.15e7_dp, cl = 3e10_dp
```

```

33  REAL (KIND = dp), PARAMETER :: kBol=1.38e-16_dp, pi=3.141592_dp, ast
    = 7.56e-15_dp, kSBol=5.67e-5_dp
34  REAL (KIND = dp), parameter :: mue = 2.33_dp
35  REAL (KIND = dp), PARAMETER :: rho_ext = (1e3_dp)*(mue*mH)
36  ! -----
37  REAL (KIND = dp) :: kappa_UV_R_star, kappa_UV_P_star
38  ! mean UV opacities of the star
39  REAL (KIND = dp) :: kappa_UV_R_ISF, kappa_UV_P_ISF
40  ! mean UV opacities of the interstellar radiation field
41  REAL (KIND = dp) :: L0, H
42  ! star heating, scaleheight
43  ! ----- helpful parameters -----
44  REAL (KIND = dp) :: r, sigma0, tkin0, rho0, alpha, cs0, delta_log,
    rho_1, rho_2, rho_3
45  REAL (KIND = dp) :: e_0, mdoheat, temptkin, q, dSigma
46
47  REAL (KIND = dp) :: z(n), sigma(n), rho(n), tkin(n), cs(n), data(21),
    lambda, delta_r(m)
48  REAL (KIND = dp) :: sigma_RZ(m,n), rho_RZ(m,n), temp2(m,n),
    max_rel_err, rel_err
49  REAL (KIND = dp) :: z_rad(m,n), density_RZ(m,n), temp(m,n), vphi(m,n)
    , cs_RZ(m,n)
50  REAL (KIND = dp) :: arr_kappa_UV_R_isf(m,n), arr_kappa_UV_P_isf(m,n)
51  REAL (KIND = dp) :: arr_kappa_UV_R_star(m,n), arr_kappa_UV_P_star(m,n)
    , arr_kappa_IR(m,n), arr_kappa_P(m,n)
52  REAL (KIND = dp) :: arr_s_loc(n), flux(n), delta_E(n), erad(n),
    UV_flux_star(n), UV_flux_isf(n)
53
54  REAL (KIND = dp), parameter :: Tcmb=2.73_dp, Ecmb=ast*Tcmb**4
55  REAL (KIND = dp), parameter :: Tbg=1e4_dp, Dbg=1e-13_dp, Ebg=Dbg*
    ast*Tbg**4
56  ! temperature of background UV field, dilution of UV field, radiation
    energy
57  END MODULE
58  ! -----
59  MODULE opacitySem
60  ! Calculation of IR opacities, Program from Semenov et al. 2003 is used
61  ENDMODULE opacitySem
62  ! -----
63  MODULE UVopacity ! Calculation of UV opacities
64
65  CONTAINS
66
67  FUNCTION funkappaP(ttmp) ! Planck opacity

```

```

68  USE acc
69  IMPLICIT NONE
70  INTEGER, PARAMETER :: maxtemp = 10000
71  REAL ( KIND = dp ) :: funkappaP, ttmp1, ttmp, ttmp2, kappa, kappa1,
    kappa2, data(4)
72  REAL ( KIND = dp ), DIMENSION(maxtemp) :: temp, kappaUV_P
73  INTEGER :: stat, j, i
74  SAVE :: stat, temp, kappaUV_P
75
76  IF ( stat.ne.1 ) THEN
77  OPEN(11, file='opacity.dat', status='old')
78      DO j = 1, 10000
79          READ(11,*) ( data(i), i = 1,4)
80          temp(j) = data(1)
81          kappaUV_P(j) = data(2)
82      ENDDO
83      CLOSE(11)
84  stat = 1
85  ENDIF
86
87  IF ( ttmp.lt.temp(1) ) THEN
88      kappa=kappaUV_P(1)
89  ENDIF
90
91  DO j = 1, maxtemp-1
92      ttmp1=temp(j)
93      ttmp2=temp(j+1)
94      IF (( ttmp.ge.ttmp1 ).and.( ttmp.le.ttmp2 )) THEN
95          kappa1=kappaUV_P(j)
96          kappa2=kappaUV_P(j+1)
97          kappa=(ttmp-ttmp1)/(ttmp2-ttmp1)*kappa2+ &
98              (ttmp2-ttmp)/(ttmp2-ttmp1)*kappa1
99      ENDIF
100  ENDDO
101
102  IF ( ttmp.gt.temp(maxtemp) ) THEN
103      kappa=kappaUV_P(maxtemp)
104  ENDIF
105
106      funkappaP = kappa
107  END FUNCTION
108  ! -----
109  FUNCTION funkappaF(ttmp)
110      USE acc

```

```

111  IMPLICIT NONE
112  INTEGER, PARAMETER :: maxtemp = 10000
113  REAL ( KIND = dp ) :: funkappaF, ttmp, ttmp1, ttmp2, kappa, kappa1,
      kappa2, data(4)
114  REAL ( KIND = dp ), DIMENSION(maxtemp) :: temp, kappaUV_F
115  INTEGER :: stat, j, i
116  SAVE :: stat, temp, kappaUV_F
117
118  IF (stat.ne.1) THEN
119  OPEN(11, file='opacity.dat', status='old')
120      DO j = 1, 10000
121          READ(11,*) (data(i), i = 1,4)
122          temp(j) = data(1)
123          kappaUV_F(j) = data(4)
124      ENDDO
125      CLOSE(11)
126  stat = 1
127  ENDIF
128
129  IF (ttmp.lt.temp(1)) THEN
130      kappa=kappaUV_F(1)
131  ENDIF
132
133  DO j = 1, maxtemp-1
134      ttmp1=temp(j)
135      ttmp2=temp(j+1)
136      IF ((ttmp.ge.ttmp1).and.(ttmp.le.ttmp2)) THEN
137          kappa1=kappaUV_F(j)
138          kappa2=kappaUV_F(j+1)
139          kappa=(ttmp-ttmp1)/(ttmp2-ttmp1)*kappa2 &
140              +(ttmp2-ttmp)/(ttmp2-ttmp1)*kappa1
141      ENDIF
142  ENDDO
143
144  IF (ttmp.ge.temp(maxtemp)) THEN
145      kappa=kappaUV_F(maxtemp)
146  ENDIF
147
148  funkappaF = kappa
149  END FUNCTION
150 END MODULE UVopacity
151 ! -----
152 PROGRAM protoplanetary
153     USE acc

```

```

154     USE globalparameters
155     USE verticalparameters
156     IMPLICIT NONE
157
158     CALL initial_output
159     ! Sets initial conditions
160     CALL verticalgrid
161     ! Sets vertical grid initially
162     CALL opacities
163     ! Finds UV and IR opacities
164     CALL main
165     ! Main module for calculation of temperature
166     CALL writeout
167     ! Writes obtained values to external file
168
169 END PROGRAM
170 ! -----
171 SUBROUTINE initial_output
172     USE acc
173     USE globalparameters
174     USE verticalparameters
175     IMPLICIT NONE
176
177     ! Reads in from Hydrodynamical output
178     ! Time, Mass, Luminosity and accretion
179     OPEN(11, file='p01061.dat', status='old')
180     READ(11,*) age_now
181     READ(11,*) Lstar, Mstar, Tstar, Mdot
182
183     ! radial distance, difference between radial points, midplane
184     ! temperature, surface-density, accretion heating, inclination
185     DO j = 1, m
186         DO k = 1, endk
187             READ(11,*) (data(i), i = 1,8)
188             rad(j) = data(3) * pc
189             delta_r(j) = data(4) * pc
190             tkin_r(j) = data(5)
191             sigma_rad(j) = data(6) / 2._dp
192             sacc(j) = data(7)
193             f(j) = data(8)
194         ENDDO
195     ENDDO
196     CLOSE(11)

```

```

197      ! sets number of vertical grid points
198      CALL grid
199
200      ! initial temperature distribution
201      DO j = 1, m
202          DO i = 1, gridn(j)
203              temp(j,i) = tkin_r(j)
204              temp2(j,i) = temp(j,i)
205          ENDDO
206      ENDDO
207  ENDSUBROUTINE
208  ! -----
209  SUBROUTINE verticalgrid
210      USE acc
211      USE verticalparameters
212      IMPLICIT NONE
213
214      DO j = 1, m
215          r = rad(j)
216          sigma0 = sigma_rad(j)
217          tkin0 = tkin_r(j)
218          alpha = Grav * Mstar / (r*r*r)
219          cs0 = kBol * tkin0 / (mue * mH)
220          H = sqrt(2._dp * cs0 / alpha)
221          rho0 = 2._dp * sigma0 / (H*sqrt(pi))
222          CALL grid
223
224          ! disk height
225          delta_log = abs((log(rho0)-log(rho_ext))/(gridn(j)-1._dp))
226          DO i = 1, gridn(j)
227              z(i) = H * sqrt(delta_log*(i-1._dp))
228          ENDDO
229
230          ! density and surface density distribution
231          sigma(1) = 0._dp
232          rho(1) = rho0
233          DO i = 2, gridn(j)
234              rho(i) = rho0 * exp(-(z(i)/H) ** 2)
235              sigma(i) = sigma(i-1) + 0.5_dp*(rho(i-1)+rho(i)) * (z(i)-z(i-1))
236          ENDDO
237
238          DO i = 1, gridn(j)
239              rho_RZ(j,i) = rho(i)
240              sigma_RZ(j,i) = sigma(i)

```

```

241     ENDDO
242
243     DO i = 1, out
244         rho(i) = 0._dp
245         z(i) = 0._dp
246     ENDDO
247 ENDDO
248 END SUBROUTINE verticalgrid
249 ! -----
250 SUBROUTINE opacities
251     USE verticalparameters
252     USE UVopacity
253     USE opacitySem
254     IMPLICIT NONE
255
256     ! Initialization of Semenov opacities for chosen dust model
257     CALL nrm_h_s(eDP,eDR)
258     CALL init_g(eGP,eGR)
259
260     ! Setting UV opacities
261     kappa_UV_P_star = funkappaP(Tstar)
262     kappa_UV_R_star = funkappaF(Tstar)
263     kappa_UV_P_ISF = funkappaP(Tbg)
264     kappa_UV_R_ISF = funkappaF(Tbg)
265
266     ! Setting IR opacities
267     DO j = 1, m
268         DO i = 1, gridn(j)
269             arr_kappa_UV_R_star(j,i) = kappa_UV_R_star
270             arr_kappa_UV_P_star(j,i) = kappa_UV_P_star
271             arr_kappa_UV_R_isf(j,i) = kappa_UV_R_isf
272             arr_kappa_UV_P_isf(j,i) = kappa_UV_P_isf
273         ENDDO
274     ENDDO
275 END SUBROUTINE opacities
276 ! -----
277 SUBROUTINE main
278     USE acc
279     USE globalparameters
280     USE verticalparameters
281     USE UVopacity
282     USE opacitySem
283     IMPLICIT NONE
284

```

```

285 DO j = 1, m
286     fact = f(j)
287     R = rad(j)
288     L0 = fact/pi*Lstar/(4._dp*pi*r*r)
289     sigma0 = sigma_rad(j)
290     tkin0 = tkin_r(j)
291     alpha = Grav * Mstar /(r*r*r)
292     cs0 = kBol * tkin0 /(mue * mH)
293     H = sqrt(2._dp * cs0 / alpha)
294     Rho0 = 2._dp * sigma0/(H*sqrt(pi))
295     mdotheat = sacc(j)
296     out = gridn(j)
297
298 DO i = 1, out
299     sigma(i)=sigma_RZ(j,i)
300 ENDDO
301
302 ! Calculating UV flux from the star and ISM
303     q = sqrt(kappa_UV_P_star/kappa_UV_R_star)
304     lambda = sqrt(4._dp*kappa_UV_P_star*kappa_UV_R_star)
305
306 DO i=1, out
307     UV_flux_star(i)=L0*(1._dp+exp(-2._dp*lambda*sigma(i)))/ &
308         ((1._dp+q)*exp(lambda*(sigma(out)-sigma(i))) &
309         +(1._dp-q)*exp(-lambda*(sigma(out)+sigma(i))))
310 ENDDO
311
312     q = sqrt(kappa_UV_P_isf/kappa_UV_R_isf)
313     lambda = sqrt(4._dp*kappa_UV_P_isf*kappa_UV_R_isf)
314
315 DO i=1, out
316     UV_flux_isf(i)=(0.5_dp*c1*Ebg)*(1._dp+exp(-2._dp*lambda*sigma(i)))/
317         &
318         ((1._dp+q)*exp(lambda*(sigma(out)-sigma(i))) &
319         +(1._dp-q)*exp(-lambda*(sigma(out)+sigma(i))))
320 ENDDO
321 ! heating due to star and accretion
322 DO i = 1, out
323     arr_S_loc(i)=4._dp*pi*(arr_kappa_UV_P_star(j,i)*UV_flux_star(i)+
324         &
325         arr_kappa_UV_P_isf(j,i)*UV_flux_isf(i)) + mdotheat
326 ENDDO

```

```

327 ! Integration of the flux equation
328   Flux(1)=0._dp
329   DO i=2, out
330     dSigma=(sigma(i)-sigma(i-1))
331     Flux(i)=Flux(i-1)+dSigma*arr_S_loc(i)
332     IF (Flux(i).lt.0._dp) Flux(i) = 0.0_dp
333   ENDDO
334
335 ! ——— begin of cycle over temperature ———
336   DO itertkin = 1,20
337
338 ! Calculation of IR opacities
339   DO i = 1, out
340     TempTkin = temp(j,i)
341     ! if density is to low, sets it to minimum value
342     IF (rho0.lt.1e-18_dp) rho0 = 1e-18_dp
343     temptkin = temp(j,i)
344     IF (temptkin.lt.5._dp)temptkin = 5._dp
345     ! if temperature is to low, sets it to minimum value
346     arr_kappa_IR(j,i) = funSem(eDR,eGR,temptkin,rho0)
347     arr_kappa_P(j,i)  = funSem(eDP,eGP,temptkin,rho0)
348   ENDDO
349
350 ! Integration of the energy equation
351   delta_E(1)= 0._dp
352   DO i = 2, out
353     dSigma=sigma(i)-sigma(i-1)
354     delta_E(i)=delta_E(i-1)-3._dp*arr_kappa_IR(j,i)/cl*Flux(i)*dSigma
355   ENDDO
356
357 ! Find E(0) from the boundary condition: F(sigma0)=c*E(sigma0)/2
358   E_0=(2._dp /cl*Flux(out)+Ecmb)-delta_E(out)
359 ! Find the actual energy distribution
360   DO i = 1, out
361     Erad(i)=E_0+delta_E(i)
362   ENDDO
363
364 ! Find temperature and sound speed
365   DO i = 1, out
366     tkin(i)=((Erad(i)+arr_S_loc(i)/(arr_kappa_P(j,i)*cl))/ast)**0.25
367     _dp
368     cs(i)=kBol * tkin(i)/(mue*mH)
369   ENDDO

```

```

370 ! Warning: Check energy-to-flux ratio
371 DO i=1, out
372     IF (Flux(i).gt.cl*Erad(i)) THEN
373         OPEN(20,file='warning.dat',STATUS='unknown',POSITION='append')
374         WRITE(20,*) j,i,'Flux overflow'
375         CLOSE(20)
376     ENDIF
377 ENDDO
378
379 DO i = 1, out
380     temp(j,i) = tkin(i)
381 ENDDO
382
383 ! calculation of the max. error
384 max_rel_err=0._dp
385 DO i=1, out
386     rel_err=abs(temp2(j,i)-temp(j,i))
387     IF (rel_err > max_rel_err) max_rel_err = rel_err
388 ENDDO
389
390 ! update the temperatures
391 DO i=1, out
392     temp2(j,i) = temp(j,i)
393 ENDDO
394 IF (max_rel_err.le.0.1_dp) EXIT
395 ENDDO
396 ! ——— end cycle over temperature ———
397
398 CALL NIM ! reconstruction of the vertical grid
399
400 DO i = 1, out
401     rho_RZ(j,i)=rho(i)
402     density_RZ(j,i)=rho(i)/(mue * mH)
403     temp(j,i)=tkin(i)
404     cs_RZ(j,i) = sqrt(kBol*tkin(i)/(mue * mH))
405     z_rad(j,i) = z(i)
406     IF (z_rad(j,i) == 0) z_rad(j,i)= 0.01_dp*AU
407     Vphi(j,i) = sqrt(Grav*Mstar/rad(j))
408 ENDDO
409
410 ENDDO
411
412 END SUBROUTINE main
413 ! —————

```

```

414 SUBROUTINE NIM
415     USE acc
416     USE globalparameters
417     USE verticalparameters
418     IMPLICIT NONE
419
420     REAL (KIND = dp) :: h_mean, h1, h2, err_rho, rel_err_rho, alphaN
421     REAL (KIND = dp), DIMENSION(out) :: ma, mb, mc, md, rho2
422     REAL (KIND = dp), PARAMETER :: beta = 2._dp*pi*Grav*(mue*mH)/kBol
423     REAL (KIND = dp), PARAMETER :: max_err = 1e-2_dp*rho_ext
424
425     iter = 0
426     alphaN = Grav * Mstar * (mue * mH) / (R**3 * kBol)
427
428     DO i = 1, out
429         rho(i) = rho_RZ(j,i)
430         sigma(i) = sigma_RZ(j,i)
431         rho2(i) = 0._dp
432     ENDDO
433     err_rho = 2._dp
434
435     DO WHILE (err_rho.gt.max_err)
436         ! —— boundary conditions, 1st and last row of matrix ——
437         ma(out) = 0._dp
438         mb(1) = tkin(1)
439         mb(out) = 1._dp
440         mc(1) = -tkin(2)
441         md(1) = 0._dp
442         md(out) = rho_ext
443         ! —— elements of matrix ——
444         DO i = 2, out-1
445             h1 = sigma(i) - sigma(i-1)
446             h2 = sigma(i+1) - sigma(i)
447             h_mean = (h1+h2)/2._dp
448             ma(i) = 1._dp / (h1 * h_mean) * tkin(i-1)
449             mc(i) = 1._dp / (h2 * h_mean) * tkin(i+1)
450             mb(i) = -2._dp / (h1*h2) * tkin(i) - alphaN/rho(i)**2
451             md(i) = -2._dp * alphaN /rho(i) - beta
452         ENDDO
453
454         CALL thomas(out,ma,mb,mc,md,rho) ! calculating rho(i) using Thomas
           algorithm
455
456         err_rho = 0.0_dp

```

```

457     DO i=1, out
458         rel_err_rho=abs(rho2(i)-rho(i))    ! find error of iterations
459     IF (rel_err_rho > err_rho) err_rho = rel_err_rho
460     rho2(i) = rho(i)                      ! save new denstiy distribution
461 ENDDO
462
463     iter = iter+1
464     IF (iter.gt.100) STOP 'NIM does not converge'
465 ENDDO
466
467     z(1) = 0._dp
468     DO i = 2, out
469         z(i)=z(i-1)+(sigma(i)-sigma(i-1))/rho(i-1)
470     ENDDO
471
472 END SUBROUTINE
473 ! -----
474 SUBROUTINE thomas(n,a,b,c,d,y)
475     USE acc
476     IMPLICIT NONE
477     INTEGER :: i
478     INTEGER, INTENT(in) :: n
479     REAL (KIND = dp), DIMENSION(n), INTENT(in) :: a, b, c, d
480     ! matrix elements (a,b,c) and solution (d)
481     REAL (KIND = dp), DIMENSION(n), INTENT(out) :: y
482     ! rho
483     REAL (KIND = dp), DIMENSION(n) :: cn, dn
484     ! new definitions of elements
485     REAL (KIND = dp) :: den, anorm, bnorm, cnorm
486     REAL (KIND = dp) :: ay(n), err(n)
487     ! find error
488
489     ! Definition of new elements – solving downwards
490     cn(1) = c(1)/b(1)
491     dn(1) = d(1)/b(1)
492
493     DO i = 2, n
494         den = b(i) - cn(i-1)*a(i)
495         cn(i) = c(i)/den
496         dn(i) = (d(i) - dn(i-1)*a(i)) / den
497     ENDDO
498
499     ! y(n) can be found – system is solved upwards
500     y(n) = dn(n)

```

```

501
502 DO i = n-1, 1, -1
503     y(i) = dn(i) - cn(i)*y(i+1)
504 ENDDO
505
506 ay(1) = b(1)*y(1)+c(1)*y(2)
507 ay(n) = a(n)*y(n-1)+b(n)*y(n)
508 DO i = 2, n-1
509     ay(i) = a(i)*y(i-1)+b(i)*y(i)+c(i)*y(i+1)
510 ENDDO
511
512 END SUBROUTINE
513 ! -----
514 SUBROUTINE writeout
515     USE acc
516     USE globalparameters
517     USE verticalparameters
518     IMPLICIT NONE
519
520
521     OPEN(unit=19, file='output.dat', status='unknown')
522     WRITE(19, '(2(A6,2X),6(A17,2X))') 'i', 'j', 'R, AU', 'z, AU', 'n(H2),
523         cm-3', 'tkin, K', 'Vphi, km/s', 'Sigma, g/cm**2'
524     WRITE(19, '(A)') ''
525     DO j=1, m
526         DO i=1, gridn(j)
527             WRITE(19, '(2(I6,2X),1p,6(E17.10,2X))') j, i, rad(j)/AU, z_rad(j
528                 , i)/AU, density_RZ(j, i), temp(j, i), &
529                 sigma_RZ(j, i), cs_RZ(j, i)/Vphi(j, i), Vphi(j,
530                     i)/1d5
531         ENDDO
532     ENDDO
533     WRITE(19, *)
534     ENDDO
535     CLOSE(19)
536
537 END SUBROUTINE
538 ! -----
539 SUBROUTINE grid
540     USE acc
541     USE verticalparameters
542     IMPLICIT NONE
543
544     DO j = 1, m
545         alpha = Grav * Mstar / (rad(j)**3)

```

```

542     cs0      = kBol * tkin_r(j) / (mue * mH)
543     H        = sqrt(2._dp * cs0 / alpha)
544     rho0     = 2._dp * sigma_rad(j) / (H*sqrt(pi))
545     delta_log=abs(log(rho0)-log(rho_ext))
546     gridn(j) = Nint(2._dp * H* sqrt(delta_log)/delta_r(j))
547     IF(gridn(j).gt.n) gridn(j) = n
548     ENDDO
549
550 END SUBROUTINE

```

vertcode.f90

Curriculum Vitae

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Studienverlauf

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Titel der Bakkalaureatsarbeit:
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Titel der Masterarbeit:
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- | | |
|--------------|---|
| 05/2012 | Mitarbeit im LOC (local organizing committee) beim Workshop:
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