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# Contents




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# An Evolutionary Model of the Hold-up Problem

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## 1. Introduction

The hold-up problem describes how inefficiency might emerge in situations that feature relation-specific investment. The inefficiency arises if it is impossible to write a contract on the investment level or on the compensations of the investor. Because the investment is relation-specific, the concerned parties will bargain over the terms of the transaction after the investment costs are sunk. Traditionally it is assumed that sunk costs can not influence bargaining outcomes (Klein, Crawford, and Alchian, 1978). This implies that the bargaining relates to the gross surplus available and ignores the net surplus that takes into account the investment costs. If this is the case, then Grout (1984), Grossman and Hart (1986), and Tirole (1986) have shown that there is necessarily underinvestment, because the marginal return on investment will not be obtained by the investing party.

To see this consider two players $\alpha$ and $\beta$ where player $\alpha$ chooses an investment level $I$. This investment level is associated with a private cost $C(I) \equiv cI$ and leads to a potential surplus $V(I)$ where $V$ fulfils the Inada conditions. Clearly the optimal investment level is given by $C'(I) = V'(I)$. However, to realise the surplus $V$ player $\alpha$ has to reach an agreement with player $\beta$ about the distribution of the surplus among them. If their negotiation leads to the Nash bargaining solution, then the private return on investment for player $\alpha$ will be $V/2$. Thus the optimal investment level that player $\alpha$ can choose for herself is given by $C'(I) = V'(I)/2$, which leads to a suboptimally low investment level.

The argument that sunk costs do not influence bargaining outcomes is most convincing, when the bargaining model under consideration has a unique solution. Examples for this are the Nash bargaining solution (Nash, 1950) and the Rubinstein-Ståhl bargaining game, which has a unique subgame perfect Nash equilibrium (Ståhl, 1972; Rubinstein, 1982). But there exist many non-cooperative games with multiple solutions. For example in the Nash demand game (Nash, 1953) every division is a subgame perfect Nash equilibrium. Van Damme, Selten, and Winter (1990) have shown that the Rubinstein-Ståhl bargaining game has multiple subgame perfect equilibria under reasonable assumptions. It is obvious that if multiple bargaining solutions are possible then efficient investment will be sustained in some equilibria.

In recent years evolutionary models by Tröger (2002) and Ellingsen and Robles (2002) have shown that under certain circumstances investment decisions indeed influence the behaviour of agents in subsequent bargaining. These models reject the paradigm of sunk costs. Players do form their beliefs about their opponents bargaining behaviour conditional on the investment decisions that are taken by players. In combination with a bargaining model that exhibits multiple Nash equilibria, this allows the selection of an equilibrium that can be justified by a forward induction argument. This might permit to sustain the efficient investment level and thus solve the hold-up problem for one sided investment.

While the traditional literature on the hold-up problem is mostly concerned with situation where investment is one-sided, there exist interesting situations where the investment is two-sided. Imagine an employee who acquires some firm specific skills. This might be associated with costs for the employee, e.g., effort, and costs for the firm, e.g., training costs. Dawid and MacLeod (2001, 2008) have shown that the optimistic results of Tröger (2002) and Ellingsen and Robles (2002) do not generalise to the case of two-sided investment. Their model predicts that the hold-up problem might be worse in an evolutionary setting than in the traditional strategic models

This paper extends the literature on the application of evolutionary game theory to the hold-up problem. In contrast to the existing work I consider players who in line with traditional strategic models do not condition their beliefs on the investment decisions. But both players consider themselves to be the sole residual claimants in case of a change in the investment level. This provides an incentive for efficient investment, but will lead to conflicting demands when the investment level changes since it is impossible that both players are the sole residual claimant.

But because players base their beliefs on incomplete samples of past plays, it is possible for them to coordinate on a stable convention with efficient investment. This is not only true in the case of one sided investment, but also in the case of two-sided investment for a considerable range of parameters. The distribution of the surplus will guarantee both players a positive net payoff. But in contrast to Tröger (2002) and Ellingsen and Robles (2002), the distribution of the surplus will not favour the investing party.

The remainder of the paper is organised as follows. The next section discusses the existing literature on evolutionary models of the hold-up problem. Section 3 discusses the set up of the present model in detail. Section 4 derives the results for the case of one-sided investment and Section 5 does so for two-sided investment. Section 6 concludes.

# 2. Related Literature

## 2.1. Bargaining

Evolutionary analysis has been used as an equilibrium selection criterion in the Nash demand game by Young (1993b). He considers a two population model to allow for asymmetric bargaining conventions between the two populations. Every round two players, one from each population, are randomly matched to play a Nash demand game. The players have only very bounded rationality. Every player observes a sample of other agents' past plays that is drawn from the last $m$ periods of plays. Then the strategies of players are best responses to the observed distribution of the play of their opponents population. The strategic considerations exclude every reasoning concerning preferences, rationality, or the information sets of their opponents. If they would play against a natural stochastic process, then their behaviour would not be different. In addition they are assumed to make a mistake (or experiment) from time to time in which case they play a random strategy.

Young shows that for any initial history of plays, players will coordinate on some bargaining conventions in which everybody always plays a certain demand that is compatible with her opponents' demand. These conventions can be displaced by a number of subsequent errors though. Note that a sufficiently large number of such errors has to occur within $m$ periods (the length of history) because players do only samples from the last $m$ periods of play. Young shows that if such errors occur, then in the very long run the convention that corresponds to the (generalised) Nash bargaining solution is most likely to prevail. This result is driven by the fact that the number of errors that are necessary to displace the Nash bargaining solution is larger than for any other bargaining convention. Thus Young provides an evolutionary model that supports the (generalised) Nash bargaining solution.

## 2.2. The Hold-up Problem

The framework of Young has been extended by Tröger (2002) to study a version of the hold-up problem. Ellingsen and Robles (2002) consider a very similar model that yields the same results. In Tröger's model players play a two-stage investment-bargaining game. In the first stage one of the two players chooses an investment $I$ from a discrete set at some cost $C_I$. This investment determines the size of the potential gross surplus $V_I$ that can be realised. In the second stage both players bargain over the gross surplus by playing a Nash demand game. Note that the costs of investment are sunk at the time of bargaining.

For this case of one sided investment Tröger shows that the unique stochastically stable equilibrium is an investment bargaining convention where the efficient investment level is chosen.

The models of Tröger and Ellingson and Robles use more sophisticated decision rules than Young's original model. Players choose their demands in the bargaining stage contingent on the investment level that has been chosen. Players believe that the demand from their opponents following different investment levels are independent from each other, but depend on the investment level chosen.

Thus players form separate beliefs about their opponents play for each bargaining subgame. The beliefs about bargaining behaviour that follows a specific investment level $I$ are based exclusively on past bargaining behaviour that has followed upon this specific investment level. Given these beliefs about the distribution of their opponents' demands, players choose a best response to this distribution.

These beliefs imply that the investing players believe that their investment decision will influence their opponents behaviour in a very specific way, i.e. they can choose among several demand distributions of their opponents by setting their investment level. They will then decide upon their investment level taking into account their expectations about the following bargaining behaviour.

If given a history of play, a specific investment can not be a best response, then this investment level would only be chosen if the investing player makes an error. Both papers by Tröger and Ellingson an Robles assume that in this case the player who makes an error at the investment stage also makes an error at the bargaining stage i.e. makes a random demand. This particular error structure allows that the history of bargaining play following an investment level that is not a best response can take on a random form.

Because of the way players form their beliefs, it follows that the beliefs about the bargaining behaviour following such an investment level may vary randomly. Thus a convention with investment level $I$ can only be stochastically stable if for all beliefs that a player can hold about the bargaining behaviour of their opponents following all other investment levels, choosing $I$ is his best response at the investment stage. The only conventions for which this is true are those where the most efficient investment level is chosen and the investing player gets a surplus that is higher than the maximum potential payoff from any other investment level.

### 2.2.1. Signalling

The choice of the investment level can be interpreted as a signal. Suppose that $I$ and $I'$ would just be two different signals without any influence on the available surplus that one player can send prior to the Nash bargaining game. If this signal would be interpreted by both players as a credible signal concerning the play in the Nash bargaining game, then this would not alter the argument presented above. In Tröger's model the structure of the players' beliefs assures that the choice of the investment level is regarded as a credible signal by the players.

This implies that the choice of the investment level does not alter the bargaining behaviour by changing the size of the available surplus, but by sending a credible signal. Thus the investing player can claim almost the entire surplus to himself because he is able to send a credible signal.

This point can be illustrated by the Battle of the Sexes game. It is commonly believed that if only one player is allowed to send a signal prior to playing the game, then this has two effects. The coordination problem will be solved and the player who sends the signal is able to determine the outcome of this game.

## 2.3. Two-sided Investment

Now consider a Battle of Sexes game where both players can send a signal. In this case coordination can not be guarantied anymore. This would indicate that the results of Tröger and Ellingson do not generalise to the case of two-sided investment.

This intuition is confirmed by Dawid and MacLeod (2001). Using a very similar framework they show that if both players can choose between different investment levels no stable investment-bargaining convention can be assured. The reason for this is that an investment-bargaining convention in the case of two-sided investment requires both players to choose a specific investment level. As in the models of Tröger and Ellingson the out of equilibrium beliefs about the bargaining behaviour can take a random form. But in general there does not exist an investment bargaining convention where both players are better of than in every other bargaining convention following different investment levels. Consequently every convention can be displaced by some out of equilibrium beliefs. This implies that no investment-bargaining convention will be stochastically stable.

# 3. The Model

I consider a model with two populations $\mathbb{A}$ and $\mathbb{B}$. Every period $t = 1, 2, ...$ a player $\alpha$ is randomly selected from $\mathbb{A}$ and a another player $\beta$ is randomly selected from $\mathbb{B}$. These two players are matched to play a two stage investment-bargaining game.

## 3.1. The Game

At the investment stage $\alpha$ chooses an investment level $I$ which is associated with a private cost $C_I$. Simultaneously $\beta$ chooses an investment level $J$ at cost $C_J$. The investments of the players creates a gross surplus of the size $V_{IJ}$.

At the bargaining stage players divide the gross surplus by playing a Nash demand game. Players $\alpha$ and $\beta$ simultaneously make demands $x$ and $y$. Players receive their respective demand if, and only if, the sum of their demands is smaller or equal the gross surplus that is available. The surplus of $\alpha$ and $\beta$ in the investment bargaining game is then given by

$$(\pi_\alpha(I, xJ, y), \pi_\beta(I, xJ, y)) = \begin{cases} (x - C_I, y - C_J) & \text{if } x + y \leq V_{IJ} \\ (-C_I, -C_J) & \text{otherwise} \end{cases}.$$

Two demands $x$ and $y$ are said to be compatible if given $I$ and $J$, $x + y \leq V_{IJ}$.

Because every division is a Nash equilibrium of the Nash demand game (Nash, 1953), every combination of investment levels that yields a positive net surplus $V_{IJ} - C_I - C_J$ can be supported in a subgame perfect Nash equilibrium. To see this consider a strategy combination $(I, x, J, y)$ where $x + y = V_{IJ}$ and and $x - C_I$ and $y - C_J$ are positive. Now suppose that for all $I' \in \mathbb{I} \setminus I$ the strategies $(x', y')$ in the subgame following the investment levels $I'$ and $J$ are $x' = 0$ and $y' = V_{I'J}$. Analogously suppose that for all investment levels $J' \in \mathbb{J} \setminus J$ the strategies $(x'', y'')$ in the subgame following the investment levels $I$ and $J'$ are $x'' = V_{IJ'}$ and $y'' = 0$. It is obvious that given these strategies no player has an incentive to deviate from their investment level because this would yield a negative payoff of $-C_{I'}$ or $-C_{J'}$ respectively and thus $(I, x, J, y)$ is a Nash equilibrium. Because every division of the surplus that satisfies $x + y = V$ is a Nash equilibrium, it is subgame perfect.

While there can in general be other Nash equilibria, the above equilibria are clearly the relevant ones for the analysis of the hold-up problem. In particular this implies that inefficient low investment as well as efficient high investment can be a subgame perfect Nash equilibrium. The aim of the following evolutionary analysis is to identify those equilibria which are most likely to be chosen.

For technical reasons it is assumed that the action sets of players are finite. The investment levels of players $I$ and $J$ are chosen from the finite sets of feasible demands $\mathbb{I} = \{I_0, I_1, \ldots, I_N\}$ and $\mathbb{J} = \{J_0, J_1, \ldots, J_N\}$ respectively. Let $\delta$ be a base unit that is chosen such that for all possible investment levels $\frac{V_{IJ}}{\delta}$, $\frac{C_I}{\delta}$, $\frac{C_J}{\delta}$, and $\frac{max_{IJ}V_{IJ}}{2\delta}$ are integers. The demands of the players are assumed to be elements of the set of possible demands $X = \{\delta, 2\delta, \ldots max_{IJ}(V_{IJ}) - \delta\}$.

The lower bound of this set is motivated by the fact the a demand of zero is weakly dominated by any other demand that a player could make. The upper bound of this set is justified by the fact that given the lower bound of $X$ all demands that are larger than $max_{IJ}(V_{IJ}) - \delta$ are weakly dominated by all other demands in $X$.

The action sets of players from a $\mathbb{A}$ and $\mathbb{B}$ are then given by the finite sets $\mathbb{I} \times X$ and $\mathbb{J} \times X$ respectively.

## 3.2. Players Behaviour

The beliefs of the players about their opponents strategies are based on partial observations of the history of plays. Every period a player $\alpha$ draws a sample $A$ which consists of $a$ entries from the last $m$ records of play. Similarly, $\beta$ draws a sample $B$ consisting of $b$ entries from the last $m$ records of play.

The beliefs of player $\alpha$ about the investment behaviour of $\beta$ are given by the observed distribution of investment choices of $\beta$ which is denoted by $q(J|A)$. Similarly the beliefs of $\alpha$ about the bargaining behaviour of $\beta$ are given by the observed cumulative distribution of *absolute demands* which is denoted by $F(y|A)$. Analogously the beliefs of $\beta$ can be characterised by $q(I|B)$ and $F(x|B)$.

If players interpret their opponents' demands as absolute values which are independent of the available surplus, this implies that every player considers himself to be the sole residual claimant. This obviously provides an incentive to choose the most efficient investment level, which is crucial in driving the results in this paper. But it evidently is impossible that both players are the sole residual claimant. Thus this beliefs will lead to a situation of conflict where the players make incompatible demands. The evolutionary analysis will then serve to answer two questions: Whether given this conflict an efficient investment-bargaining norm is likely to emerge and how the surplus will be divided among the two players.

The occurrence of a conflict at the bargaining stage contrasts with the interpretation of demands as relative shares. In this case there would be no reason why players should make incompatible demands after a change of the investment level. The present model thus provides an explicit description of the conflict that will arise about the distribution of the additional surplus after a change in the investment level.

On a more fundamental level this assumption deals with the question of how agents interpret past events in a similar but different situation. This question arises because the players beliefs about their opponents behaviour at the bargaining stage following the investment levels $I'$ and $J'$ are based on past bargaining behaviour that followed upon any investment levels $(I, J) \in \mathbb{I} \times \mathbb{J}$. This is expressed by $F(y|A)$ and $F(x|B)$ being independent of $I$ and $J$. This is the main difference between this model and the models by Tröger (2002), Ellingsen and Robles (2002) and Dawid and MacLeod (2001, 2008).

In their models players do not draw on information about bargaining situations that have followed different investment levels to form their beliefs about the bargaining that follows an specific investment level. In Dawid and MacLeod (2001) the players beliefs consists of different cumulative probability distributions of their opponents demands $\hat{F}_{IJ}(.)$ for all $(I, J) \in \mathbb{I} \times \mathbb{J}$ and a distribution $\hat{q}(.)$ of the investment levels[1]. These beliefs about the bargaining behaviour following a specific investment level are updated exclusively by bargaining that takes place after this investment level has been chosen. Since no demand is considered independently of an investment level, the question of interpreting them as absolute or relative demands does not arise.

These assumptions, however, imply that even if a player beliefs that the probability $\hat{q}(I')$ that the investment level $I'$ will be played is zero, he still holds very specific beliefs $\hat{F}_{I'J}$ about what will happen when this investment level will be played[2]. Thus players that observe an investment level $I'$ will not use any information about bargaining behaviour that has followed different investment levels. This is even the case if $I'$ is a very unusual investment level that has not been observed for a very long time.

The separation of beliefs along the lines of different investment levels also assumes away the possibility that players actually regard prior investment as irrelevant for the bargaining situation. In these models the investment decision serves as a credible and unambiguous signal that coordinates the expectations of players. However, the interesting difference of these models and the concept of forward induction is that the content of the signal that is sent by a specific investment level is endogenously determined by the evolutionary process. But if the goal of these papers is to explain why people seem to be able to overcome the hold-up problem without relying on explicit forward induction, then the assumption that investment serves as a credible signal does not seem to be a very powerful test of this hypothesis.

While there certainly does not exist a well developed theory about how players use

[1] They also assume that players have the constant belief that the surplus will be shared according to the Nash bargaining solution if both players choose the same investment level.

[2] Evidently beliefs that contain a distribution of bargaining demands after an investment level that is believed to be chosen with probability zero can not be based on a single sample of past play. This is due to the fact that no sample of play can contain zero occurrences of a specific investment level $I'$ and at the same time occurrences of demands that followed upon $I'$. This problem only occurs in the case of two-sided investment since in models of one sided investment the probabilities of investment levels $\hat{q}(.)$ are not modelled explicitly.

Dawid and MacLeod (2001) circumvents this problem by associating the state of the model with the beliefs of all players which are updated according to a specific algorithm. This implies however that a population has to consists of a set of players that must not change. In contrast, if as in the present model the state space is associated with a truncated history of past play, then the players in each population can be replaced at all time. The new player will simply draw a sample of the history of play and forms beliefs based on the observed past plays.

information obtained in similar but different situation I believe that they certainly do so. Including this feature therefore constitutes a major advantage of the present model over the existing literature.

The interpretation of demands as absolute values seems to be sensible for the given context. As an example of this consider two firms where firm $\alpha$ produces a relation specific good that is used in the production process of firm $\beta$. Suppose that $\alpha$ invests into a more efficient production technology. Firm $\alpha$ might very well expect to be able to charge the same price to $\beta$ as before. On the contrary firm $\beta$ might believe that changes in production costs should be passed through to output prices.

Given their beliefs the players choose their strategies as follows. At the bargaining stage players simultaneously choose their best response given their beliefs and the investment decisions taken by both players. The payoff expected by $\alpha$ playing $x$ is then given by

$$E(\pi_\alpha(x)|I, J, A) = xF(V_{IJ} - x|A) - C_I \tag{1}$$

and defined analogously for $\beta$. Note that both players know both investment decisions $I$ and $J$ at the bargaining stage. Thus $\alpha$ and $\beta$ will demand $\arg\max_x E(\pi_\alpha(x)|I, J, A)$ and $\max_y E(\pi_\beta(y)|I, J, B)$ respectively.

If for any player different demands yield the highest expected payoff at the bargaining stage, then this player plays a probability distribution over these demands. The support of this distribution is the set of demands that yield the maximum expected payoff for and which there exists a compatible demand of their opponent given the investment levels $I$ and $J$. This is motivated by the fact that any other demand is weakly dominated by the demands for which there exists a compatible demand. This implies that given the investment level $I$ and $J$ a player chooses a demand from the set $X_{IJ} = X \cap [\delta, V_{IJ} - \delta]$.

Since $x$ and $y$ are understood as absolute value it can happen that some or all of the expected demands are larger than the surplus available after a specific choice of investment levels. This case can be interpreted as insecurity about the distribution of opponents demands in which case the player assumes that the worst case will happen, i.e. no coordination will take place. If all demands with positive probability in $A$ or $B$ are larger than the total surplus, then the expected payoff of all demands is zero and consequently the player will play a probability distribution with support $X_{IJ}$.

At the investment stage players simultaneously decide upon their investment levels. They choose their best response given their sample of play and their anticipations about the payoff in the bargaining games. These anticipations are based on backwards induction which associates the payoffs $\max_x E(\pi_\alpha(x)|I, J, A)$ and $\max_y E(\pi_\beta(y)|I, J, B)$ with the subgame following the investment levels $I$ and $J$.

Therefore the payoff expected by $\alpha$ when choosing an investment level is given by

$$E(\pi_\alpha(I)|A) = \sum_{J \in \mathbb{J}} q(J|A) \max_x (E(\pi_\alpha(x)|I, J, A)) \tag{2}$$

and defined analogously for $\beta$. Thus at the investment stage $\alpha$ and $\beta$ will choose $\arg\max_I E(\pi_\alpha(I)|A)$ and $\arg\max_J E(\pi_\beta(J)|B)$ respectively.

Again if different actions yield the highest expected payoff for any player, then this player plays a probability distribution over these actions. At the investment stage this probability distribution has full support over all investment levels that yield the maximum expected payoff.

In addition players make occasional mistakes at both stages with probability $\epsilon$ in which case they choose a random action either at the investment or the bargaining stage. The probabilities of making an error at the investment and bargaining stage are independent. It is thus possible that no, one, or both players make a mistake at the investment stage and that in the same period no, one, or both players make a mistake at the bargaining stage

This constitutes the second major difference between this paper and Tröger (2002), Ellingsen and Robles (2002) and Dawid and MacLeod (2001). In these papers if a player makes an error at the bargaining stage, then he will also make an error at the bargaining stage i.e. make a random demand. While this assumption is crucial for the results of these papers no particular motivation is provided for this specific assumption. The results of the present model would not change if this assumption would be adopted instead of independent probabilities.

## 3.3. Equilibrium Concepts

The play in a period $t$ is denoted by $(I_t, x_t, J_t, y_t)$ which is abbreviated by $(I, x, J, y)_t$. The complete history of play in period $t$ is given by

$$((I, x, J, y)_1, (I, x, J, y)_2, ...(I, x, J, y)_t).$$

The behaviour of players determines a Markov chain $P^\epsilon$. This Markov chain is called the perturbed investment-bargaining process. The state space $\mathbb{S}$ of $P^\epsilon$ consists of sequences $s$ of length $m$ that consist of records of play $(I, x, J, y) \in \mathbb{I} \times X \times \mathbb{J} \times X$. Because the strategy spaces are finite, the state space is finite as well.

A state at time $t$ is denoted by

$$s_t = ((I, x, J, y)_{t-m+1}, (I, x, J, y)_{t-m-+2}, ...(I, x, J, y)_t).$$

The Markov chain $P^\epsilon$ is irreducible and aperiodic. To see this suppose that $P^\epsilon$ is in state $s_t$ at time $t$. Suppose further that both players make errors at the investment and the bargaining stage for $m$ consecutive periods. After this $m$ periods the state $s_{t+m}$ consists entirely of random demands. Consequently the state $s_{t+m}$ can be every state in $\mathbb{S}$ which implies irreducibility. Clearly the same state can also be reached after $m+1$ periods of consecutive errors which implies aperiodicity.

From the theory of Markov Processes it follows that there exists a unique stationary distribution $\mu^\epsilon$ of the Markov process $P^\epsilon$ where $\mu^\epsilon(s)$ is the cumulative relative frequency with which a state $s$ will occur if the process $P^\epsilon$ runs for a very long time.

In the following analysis of the long run behaviour of the investment bargaining process I use the equilibrium concept of stochastic stability introduced by Foster and Young (1990).

**Definition 1.** A state $s \in \mathbb{S}$ is stochastically stable if $\lim_{\epsilon \to 0} \mu^\epsilon(s) > 0$.

Let $S^*$ denote the set of stochastically stable states. For very small error probabilities $\epsilon$ the process $P^\epsilon$ will be in $S^*$ most of the time. This equilibrium notion allows for constant perturbations of the best response dynamic.

To compute the stochastically stable states the techniques pioneered by Freidlin and Wentzel (1984) and adapted by Young (1993a,b) will be applied.

The probability of a one period transition from a state $s$ to a different state $s'$ under $P^\epsilon$ is denoted by $P^\epsilon_{ss'}$. A state $s'$ is called a successor of $s$ if $P^\epsilon_{ss'} > 0$.

Let $s'$ be a successor of $s$ under $P^\epsilon$. The resistance between $s$ and $s'$, $r(s, s')$ is given by the number of errors that are necessary for a transition from $s$ to $s'$. Let a path $\rho$ from $s^1$ to $s^2$ be a sequence of one period transitions that leads from $s^1$ to $s^2$. The resistance of a path $\rho$ is the sum of the resistances of the one-period transitions in $\rho$. For any two states $s^1$ and $s^2$, let $r(s^1, s^2)$ be the least total number of mistakes in any path that leads from $s^1$ to $s^2$.

It has been shown in Young (1993a) that the set of stochastically stable states with respect to $P^\epsilon$ is a subset of the so called recurrent communication classes $S_i$ of $P^\epsilon$.

**Definition 2.** The set $S$ of recurrent communication classes $S_i$ of $P^\epsilon$ is defined by the following properties.

1. From every state $s \in \mathbb{S}$ there exists a path with zero resistance which leads to at least one state that lies in a recurrent communication class.
2. For every two states $s^1$ and $s^2$ in any recurrent communication class $S_i$ there exists a path from $s^1$ to $s^2$ with zero resistance .
3. Every path that leads from a state $s^1$ that lies in a recurrent communication class $S_i$ to any state $s^2$ that lies outside this recurrent communication has resistance strictly greater than zero.

Let $P^0$ denote the Markov chain that is determined exclusively by the best response behaviour of players. This Markov chain is called the unperturbed investment bargaining process. A transition that has positive probability under $P^0$ has zero resistance under $P^\epsilon$. This allows to compute the recurrent communication classes of $P^\epsilon$ using the properties of $P^0$.

Define a graph $G$ as follows. There is a vertex for every recurrent communication class $S_i$ of $P^\epsilon$ and a directed edge from every vertex to every other vertex. The "weight" or resistance of any directed edge $S_i \to S_j$ is the least resistance of all paths that start in $S_i$ and end in $S_j$.

Let an i-Tree be a collection of edges in $G$ such that from every Vertex $S_{j \neq i}$ there is a unique directed path to $S_i$, and there are no cycles. Let $\mathbb{T}_i$ be the set of all i-Trees

**Definition 3.** The stochastic potential of a recurrent communication class $S_i$ is the least resistance among all i-Trees:

$$\gamma_i = \min_{T \in \mathbb{T}_i} \sum_{(s,s') \in T} r(s, s').$$

The elements of a recurrent communication class $S_i$ are stochastically stable if $S_i$ has the least stochastic potential among all $S_j \in S$.

The further analysis will also use the concept of conventions. Write $(I, x, J, y)^l$ for a sequence of length $l$ that consists of identical records $(I, x, J, y)$

**Definition 4.** A state $s \in \mathbb{S}$ is a *convention* $\sigma = (I, \xi, J, V_{IJ} - \xi)^m$ with $I \in \mathbb{I}$, $J \in \mathbb{J}$, and $\xi \in X_{IJ}$ if it consists of $m$ identical records and the demands of the players are compatible. A convention is stable if it is an absorbing state of the Markov process $P^0$.

Stable conventions are the only states where all possible beliefs of players coincide with the best response behaviour of their opponents. Thus as long as the process is in a stable convention beliefs of players will constantly be confirmed.

# 4. One-sided Investment

This section deals with the case of one-sided investment. This case has been the centre of the discussions concerning the hold-up problem.The present model describes a situation of one-sided investment if players from one population have only one feasible investment level.

For simplicity assume that there only exist two investment levels, high investment, $H$ and low investment, $L$. The number of investment levels is not crucial, but must be finite. Assume further without loss of generality that players from $\mathbb{B}$ can only choose low investment ($\mathbb{J} = \{L\}$) while players from $\mathbb{A}$ can choose between high and low investment ($\mathbb{I} = \{H, L\}$).

In the remainder of this section I concentrate on the case where high investment is efficient. This clearly is the relevant case for the discussion of the hold-up problem. In the present model high investment is said to be efficient if

$$V_H - C_H > V_L - C_L. \tag{3}$$

As a point of reference briefly consider the traditional case where the surplus $V_I$ is split among both players according to the Nash-bargaining rule which attributes $V_I/2$ to each player. Clearly in this case a player $\alpha$ will choose high investment if $\frac{V_H}{2} - C_H > \frac{V_L}{2} - C_L$. It follows that the hold-up problem would lead to an inefficient, low level of investment if

$$V_H - V_L \in (C_H - C_L, 2C_H - 2C_L).$$

The following section will present an evolutionary analysis of the case where $V_H > V_L$ and $C_H > C_L$ which obviously is a necessary condition for the hold-up problem to arise. For simplicity the cost associated with low investment $C_L$ is assumed to be zero for both players. To shorten the notation, $J$ and $C_L$ will be suppressed.

## 4.1. Stable High Investment

In this part it will be shown that for the case of one-sided efficient investment all recurrent communication classes of $P^\epsilon$ consist of a stable convention with high investment. This implies that all stochastically stable states must be associated with high investment. In the next part on surplus distribution it is then shown that there exists a unique stochastically stable convention.

Consequently the present model predicts that the evolutionary dynamics lead to a situation where the efficient investment level is chosen and consequently the hold-up problem is solved in the sense that the efficient investment level is chosen. This is consistent with the Results of Tröger (2002) and Ellingsen and Robles (2002). It will become clear, however, that their results rest on quite different arguments than the results that are presented in this paper. While in their models efficiency can only be guaranteed when the concept of stochastic stability is evoked, in this model the unperturbed process converges to efficient outcomes directly. This is formulated in the following Theorem:

**Theorem 5.** *Let investment be efficient and let the sample lengths of both players be smaller or equal to $m/2$. Then from any initial state $s$ the unperturbed bargaining process $P^0$ converges almost surely to a stable convention where investment is high.*

The expression "almost surely" is used as in Young (1993b) and describes the fact that the probability that $P^0$ does not converge to a stable convention with high investment goes to zero as the number of periods goes to infinity. The Proof of the Theorem will use several Lemmata. The first Lemma describes the investment behaviour of players $\alpha$:

**Lemma 6.** *If the sample $A$ that is drawn by player $\alpha$ consists of identical records $(I, \xi, y)$, then $\alpha$ will choose high investment if, and only if, $y < V_H - C_H$.*

*Proof.* Consider the best response of $\alpha$ drawing $A$. Since all records in $A$ are identical, $\alpha$ expects $\beta$ to play $y$ with probability 1 at the bargaining stage i.e. $F(y|A) = 1$. Consequently the best response of $\alpha$ at the bargaining stage is to make a compatible demand $x = V_I - y$ if this demand can possibly yield a positive payoff, i.e. $V_I - y \geq \delta$. Since by assumption no player can obtain a negative share of the gross surplus, it follows from equation 2 that for the case of one-sided investment the expected payoff of choosing an investment level is given by

$$\max_x (E(\pi_\alpha(x)|I, A)) = \max\{V_I - y, 0\} - C_I.$$

Because investment is efficient, it follows from equation 3 that $V_H - y - C_H > V_L - y - C_L$ for all $y$. Consequently $\max\{V_H - y, 0\} - C_H > \max\{V_L - y, 0\} - 0$ holds if, and only if, $y < V_H - C_H$ in which case $\alpha$ will choose high investment. $\square$

The condition that $A$ consists of identical records may seem limiting. It will be shown later, however, that this special case allows to characterise the set of stable conventions. This clearly is the case because every sample that is drawn from a convention by definition consists of identical records.

This proof rests on the assumptions about how the players form their beliefs. Since the beliefs of players $\alpha$ are unconditional on their own investment decision, they believe that their opponents $\beta$ will make their demands according to the same distribution irrespective of the investment that $\alpha$ chooses. Because in this sense the demands of $\beta$ are given, players $\alpha$ expect to maximise their payoff by maximising the surplus. This of course only holds conditional on the fact that $\alpha$ expects to get a positive payoff at all, which is a sufficient condition for high investment.

The next step is to characterise the set of stable equilibria with high investment:

**Lemma 7.** *Let $\Sigma_H$ denote the set of stable conventions where investment is high. If investment is efficient, then $\Sigma_H$ is the set of conventions $\sigma = (H, \xi, V_H - \xi)$ with $\xi \in X \cap [C_H + \delta, V_H - \delta]$.*

*Proof.* Consider a convention $\sigma$ with high investment and some division $\xi$.

From Lemma 6 it follows that $\alpha$ will choose high investment if, and only if, $V_H - \xi < V_H - C_H$. This condition also implies that the best response of $\alpha$ at the bargaining stage is $\xi$.

For $\beta$ to play $V_H - \xi$ with probability one it must be the case that $V_H - \xi > 0$.

Together this shows that if $\sigma$ is a convention with high investment, then the unique best response of both players is $(H, \xi, V_H - \xi)$ if, and only if, $\xi \in X \cap [C_H + \delta, V_H - \delta]$. It follows that $\Sigma_H$ is the set of absorbing states with high investment of the unperturbed investment bargaining process $P^0$ . □

Lemma 7 expresses that every convention with high investment is stable if both players get a strictly positive net gain. The proof is a simple application of Lemma 6 and the fact that $\alpha$ and $\beta$ will only make a demand with probability one if their expected payoff from this demand is strictly greater than the payoff of every other demand.

In the following proofs of Lemma 8 and Theorem 5 it will be assumed that both players draw samples of the same length, i.e. $a = b = k$. This assumption is made for convenience and is not crucial. The assumption that $2\max\{a, b\} \leq m$ would be crucial to the argument used. References to probabilities always refer to probabilities of the unperturbed investment bargaining process $P^0$.

**Lemma 8.** *Suppose that both players draw samples of length $k$ and that $2k \leq m$. Let $\mathbb{S}'$ be the set of states where the last $k$ records $(I, x, y)$ are identical and the best response of $\alpha$ drawing a sample that consists of these last $k$ records is $(H, V_H - y)$. For any initial state $s \in \mathbb{S}$ there is a positive probability under $P^0$ of transition within finite time to a state in $\mathbb{S}'$.*

*Proof.* Suppose that the process is in state $s_t = [(I, x, y)_{t-m+1}, \ldots, (I, x, y)_t]$ at time $t$. Let $A_t$ denote the sample of the records that are the last $k$ records in $s_t$. Distinguish two cases:

Case 1: Suppose that the best response of $\alpha$ to $A_t$ implies low investment.

Suppose that $\alpha$ and $\beta$ both sample $A_t$ in period the periods from $t+1$ to $t+k$ inclusive, which has a positive probability. This is possible because by assumption $2k \leq m$. Then $\alpha$ will play its best response $(L, x)$ and $\beta$ will play its best response $y$. At the bargaining stage $\alpha$ and $\beta$ will by assumption play a demand that is compatible with some demand of their respective opponent. Since $\alpha$ will play low investment, $\alpha$ and $\beta$ will choose demands that are lower than the potential surplus associated with low investment i.e. $x, y \in X \cap [\delta, V_L - \delta]$.

Let $A_{t+k}$ denote the last last $k$ records in $s_{t+k}$. Observe that from the argument above it follows that $A_{t+k}$ consists entirely of records $(L, x, y)$ where $\beta$ made demands $y$ that were smaller than $V_L$. Consider the best response of $\alpha$ drawing $A_{t+k}$. Because investment is efficient and $y < V_L$, it follows from equation 3 that $y < V_H - C_H$. Then it follows from Lemma 6 that $\alpha$ will choose high investment. Because $y < V_H$, it is clear that $\alpha$ will demand $V_H - y$ at the bargaining stage.

Case 2: Suppose that the best response of $\alpha$ to $A_t$ implies high investment.

Suppose that $\alpha$ and $\beta$ both sample $A_t$ in period $t+1$ to $t+k$ inclusive, which again has a positive probability. Then $\alpha$ and $\beta$ will play their best response $(H, x, y)$ in periods $t+1$ to $t+k$ inclusive. Let $A_{t+k}$ denote the last $k$ records in $s_{t+k}$ that consist entirely of records $(H, x, y)$. If the best response of $\alpha$ to $A_{t+k}$ implies low investment, then $s_{t+k}$ is a state to which case 1 applies.

If the best response of $\alpha$ to $A_{t+k}$ implies high investment, then it follows from Lemma 6 that $y < V_H - C_H$, which implies that the best response of $\alpha$ at the bargaining stage must be $V_H - y$.

Together the Cases 1 and 2 prove the Lemma. □

This proof of Lemma 8 shows that if investment was low for sufficiently many periods, then players $\alpha$ will choose high investment with positive probability. This implies that there do not exist stable conventions that are not elements of $\Sigma_H$. The proof of this rests on the fact that if investment is low, then by assumption players $\beta$ will make demands $y$ that are lower than the total gross surplus $V_L$. After enough periods players $\alpha$ will come to believe that the demands of $\beta$ will be indeed smaller than $V_L$. If this is the case, however, then it has been shown in Lemma 6 that $\alpha$ will find it profitable to choose high investment.

Lemma 8 also shows that from any initial state there exists a positive probability that play in the next $k$ periods is identical. This implies that every recurrent communication classes of $P^\epsilon$ must include such a state. This follows from the fact that the same sample can be drawn for $k$ periods because by assumption $2k \leq m$.

This technical result will be used to complete the proof of Theorem 5. Starting from a state that fulfils the properties derived in Lemma 8, a sequence of states is constructed where each transition has positive probability and that leads to a stable convention with high investment. The fact that the set of states is finite completes the argument from any initial state the process $P^0$ converges almost surely to an absorbing state with high investment.

*Proof of Theorem 5.* Consider a state $s_t$ in period $t$ where the last $k$ records $(I, x, y)$ are identical and the the best response of $\alpha$ drawing a sample of that consists of the last $k$ records is $(H, V_H - y)$. Let $A_t$ denote the sample of the last $k$ records in $s_t$.

Let $\alpha$ and $\beta$ sample $A_t$ in the periods from $t+1$ to $t+k$ inclusive. The demand by $\alpha$ will then be $V_H - y > C_H$. Thus with positive probability $\alpha$ and $\beta$ will choose

$(H, V_H - y, V_H - x)$ in the periods $t+1$ to $t+k$ inclusive. Let $B_{t+k}$ denote the sample of the last $k$ records in $s_{t+k}$.

In period $t+k+1$, let $\alpha$ sample $A_t$ and let $\beta$ sample $B_{t+k}$. Note that this is possible since $2k \leq m$ and $A_t$ and $B_{t+k}$ together consist of the last $2k$ records in $s_{t+k}$. The best response of $\alpha$ to $A_t$ must still be $(H, V_H)$. Since all demands from $\alpha$ in $B_{t+k}$ are $V_H - y$ and $\alpha$ plays high investment, $\beta$'s best reply is $y$. Thus $(H, V_H) - y, y)$ is played in period $t+k+1$ with positive probability.

In period $t+k+2$ the first record of $A_t$ may have disappeared from the state $s_{t+k+2}$ (if $m = 2k$). But since $\beta$ has demanded $y$ in period $t+k+1$, there still exists a sample of length $k$ in $s_{t+k+2}$ in which $\beta$ always demanded $y$. Let this be the sample of $\alpha$ in period $t+2k+2$ and let the sample of $\beta$ continue to be $B_{t+2k}$. Thus in period $t+k+2$, $(H, V_H - y, y)$ will be played with positive probability. By an analogous argument $(H, V_H - y, y)$ will be played with positive probability in period $t+k+3$ to $t+2k$ inclusive.

Thus in period $t+2k$ the last $k$ records in $s_{t+2k}$ consist entirely of records $(H, V_H - y, y)$. Let $A_{t+2k}$ denote the last $k$ records in $s_{t+2k}$. Let $\alpha$ and $\beta$ sample $A_{t+2k}$ in period $t+2k+1$ to $t+2k+(m-k)$ inclusive. Clearly the best response of $\alpha$ and $\beta$ will be $(H, V_H - y, y)$. Thus there is a positive probability that in period $t+2k+(m-k)$, $s'_{t+2k+(m-k)}$ will consist exclusively of records $(H, V_H - y, y)$. Since by construction $\alpha$ gets a positive net gain i.e. $V_H - y > C_H$, Lemma 7 implies that $s'_{t+2k+(m-k)}$ is a stable convention with high investment.

Together with Lemma 8 this shows that for any given state $s$ there exists a positive probability of reaching a stable convention with high investment within finite time. The fact that the state space $\mathbb{S}$ is finite implies that as the number of periods goes to infinity the probability of reaching a stable convention with high investment goes to one. Thus the Proof is complete. □

Theorem 5 implies that the set $S$ of recurrent communication classes of the perturbed investment bargaining process $P^\epsilon$ consists entirely of stable conventions. From Theorem 2 of Young (1993a) it follows that the set of stochastically stable states consists exclusively of stable conventions with high investment.

**Corollary 9.** *The set of stochastically stable equilibria $S^*$ must be a subset of the set of stable conventions $\Sigma_H$.*

*Proof.* From Theorem 5 and Lemma 7 it follows that if investment is efficient, then from any state $s \in \mathbb{S}$ there exists a path with positive probability under $P^0$ to a stable convention with high investment. This implies that every recurrent communication class of $P^\epsilon$ must contain a stable convention because of property 3 in definition 2. Because all stable conventions are absorbing states of $P^0$, it follows from property 2 of definition 2 that all recurrent communication classes are stable conventions.

Theorem 2 in Young (1993a) states that the set of stochastically stable equilibria $S^*$ must be a subset of set recurrent communication classes of the perturbed Markov process $S$ which completes the proof. □

## 4.2. Surplus distribution

Corollary 9 confirms the results of Tröger (2002) and Ellingsen and Robles (2002) that the stochastically stable equilibrium is a convention with high, efficient investment. But the proofs of the corresponding Proposition 4.1 in Tröger (2002) and Proposition 4 in Ellingsen and Robles (2002) also imply that as $\delta \to 0$ the share of the investing player converges to

$$\xi = \max\left\{ \max_{I \in \mathbb{I} \setminus I^*} (V_I - C_I) + C_{I^*}, \frac{V_{I^*}}{2} \right\}, \tag{4}$$

where $I^* \equiv \arg\max_{I \in \mathbb{I}}(V_I - C_I)$ is the most efficient investment level. Thus the investing player either gets the symmetric Nash bargaining solution $V_{I^*}/2$ or the maximum potential net surplus $V_I - C_I$ of the next efficient investment level. Thus if the set of possible investment levels represents a sufficiently fine grid, then almost all the surplus will be attributed to the investing player.

To understand this observe that in these models a stable convention can be associated with almost any investment level $I \in \mathbb{I}$. This is due to the fact that the expected payoffs of the investment levels $I_0 \dots I_N$ depend on the independent distributions $\hat{F}_I(y)$. Thus a convention $\sigma_I = (I, x, V_I - x)$ with $I \neq I^*$ is stable if

$$\max_x E(\pi_\alpha(x)|I, \hat{F}_I) = \max_x (x\hat{F}_I(V_I - x|A) - C_I) > \max_x E(\pi_\alpha(x)|I', \hat{F}_{I'})$$

for all $I' \in \mathbb{I} \setminus I$.

If this is the case, then an investment level $I'$ will only be chosen if a player $\alpha$ makes an error at the investment stage. Because of the specific assumption made in Ellingsen and Robles (2002) and Tröger (2002), this also implies that $\alpha$ will also make an error at the bargaining stage. Thus if a player $\alpha$ chooses $I'$ by error, she will make a random demand at the bargaining stage. This implies that the beliefs $\hat{F}_{I'}$ of players $\beta$ will change by some random demand. Because $I'$ is only chosen by error, this change of $\hat{F}_{I'}$ will be permanent in the absence of future errors.

Thus the absorbing state $s$ which is associated with the stable convention $\sigma_I$ will move to another absorbing state $s'$ that is also associated with $\sigma_I$ but differs in terms of $\hat{F}_{I'}$. Moreover a number of such errors can change the beliefs of players $\hat{F}_{I'}$ in such a way that $\max_x E(\pi_\alpha(x)|I', \hat{F}_{I'}) > \max_x E(\pi_\alpha(x)|I, \hat{F}_I)$ which will lead to a displacement of the convention $\sigma_I$ and will possibly lead to a state that is associated with another stable convention $\sigma_{I'}$. Consequently $\sigma_I$ can not be the unique stochastically stable convention because it can be displaced by a sequence of states that are connected by single errors.

Now consider a convention $\sigma^* = (I^*, x, V_{I^*} - x)$ where $x > \max_{I \in \mathbb{I} \setminus I^*}(V_I - C_I) + C_{I^*}$. Evidently this implies that $\max_x E(\pi_\alpha(x)|I^*, \hat{F}_{I^*}) > \max_x E(\pi_\alpha(x)|I, \hat{F}_I)$ for all possible beliefs $\hat{F}_I$ and investment levels $I \in \mathbb{I} \setminus I^*$. Thus such a convention can not be displaced by any number of errors a the investment stage. It can of course be displaced by errors that occur at the bargaining stage after the choice of the investment level $I^*$. But given that the beliefs $\hat{F}_I$ are based on sufficiently long samples of play, more than one error is necessary to displace the convention $\sigma^*$ in this way. Also as long as the convention $\sigma^*$ is in place a single error at the bargaining stage will not change the beliefs $\hat{F}_{I^*}$ permanently because $I^*$ is chosen every period. Thus the unique stochastically stable convention will fulfil the condition of equation 4 where the dependence on the Nash bargaining solution follows from Young (1993b).

In the present model the net gain $x - C_I$ that the investing player obtains in a stochastically stable convention does not depend on the net surplus of other investment levels. Thus the nature of the grid that is represented by $\mathbb{I}$ does not influence the equilibrium distribution of the surplus. The reason for this is that the expectations about bargaining behaviour are the same for all investment levels. This is why the assumption of only two investment levels is not crucial in the present paper.

The distribution of the gross surplus $V_I$ will be determined by the asymmetric Nash bargaining solution as long as this does result in a positive net surplus for the investing player. The weights of the asymmetric Nash bargaining solution depend on the relative length of the sample sizes $a$ and $b$ of the players $\alpha$ and $\beta$ respectively. Hence the predictions about the evolution of surplus distribution in bargaining situations by Young (1993b) are replicated in the present model.

**Theorem 10.** *Let investment be efficient, let $a$ and $b$ be sufficiently large, and let $a \leq b \leq \frac{m}{2}$ hold. There exists at least one and at most two generically stochastically stable conventions. As $\delta \to 0$ they converge to*

$$\xi = \max \left\{ \arg\max_x ((x)^{a/m}(V_H - x)^{b/m}), C_H \right\}.$$

Clearly because of the assumption $a \leq b$ this Theorem does not describe all relevant cases. It encompasses, however, the case considered by Ellingsen and Robles (2002) and Tröger (2002) who assume that $a = b$. The characterisation of the general case remains for further research.

The Proof of this Theorem will largely rely on the arguments of the Proof of Theorem 3 in Young (1993b). To establish that these arguments can be used in the present case the following Lemma will be used.

**Lemma 11.** *If investment is one-sided, then the behaviour of players at time $t$ depends only on the distribution of demands $x$ and $y$ in $s_t$.*

*Proof.* From equation 2 it follows that the expected payoff of a player $\alpha$ choosing an investment level $E(\pi_\alpha(I)|A)$ reduces to $\max_x E(\pi_\alpha(x)|I, A)$. The bargaining behaviour of $\alpha$ and $\beta$ is given by equation 1 and the analogon for $\beta$. Because $F(y|A)$ and $F(x|B)$ are independent the investments taken in $A$ and $B$ respectively the play of $\alpha$ and $\beta$ depends exclusively on the history of demands that have been made by players $\alpha$ and $\beta$. □

Note that this implies that for one-sided investment past investment decisions do not influence the behaviour of players.

*Proof of Theorem 10.* From Theorem 2 in Young (1993a) it follows that the set of stochastically stable states must be a subset of $S$. From Lemma 11 it follows that the transition from one state to another depends only on the history of bargaining demands. Consequently only errors at the bargaining stage have to be considered to determine the minimal number of errors that is necessary to displace a stable convention and move into the basin of attraction of another stable convention. This allows us to use Lemma 1 in Young (1993b): The minimum number of error that is necessary to displace a convention $(H, \xi, V_H - \xi)^m$ in $S$ is given by the smallest integer that is greater or equal than

$$r_\delta(\xi) = \min \left\{ a(1 - \frac{\xi - \delta}{\xi}), b(1 - \frac{V_H - \xi - \delta}{V_H - \xi}), a\frac{\xi}{V_H - \delta} \right\},$$

which is denoted by $\lceil r_\delta \rceil$.

If players $\beta$ make mistakes $\xi'$ that are *higher* than the conventional demands $\xi$, then the minimal number of mistakes that is necessary to make a player $\alpha$ choose $\xi'$ is given by $a\frac{\xi}{\xi'}$. This expression is minimised if $\xi' = V_H - \delta$ which yields the third term of $r_\delta$.

If players $\alpha$ make mistakes $\xi'$ that are *higher* than the conventional demands $\xi$, then the minimal number of mistakes that is necessary to make a player $\beta$ choose $\xi'$ is given by $b(1 - \frac{V_H - \xi'}{V_H - \xi})$. This expression is minimised by $\xi' = \xi + \delta$ which yields the second term of $r_\delta$.

If players $\beta$ make mistakes $\xi'$ that are *lower* than the conventional demands $\xi$, then the minimal number of mistakes that is necessary to make a player $\alpha$ choose $\xi'$ is given by $a(1 - \frac{\xi'}{\xi})$. This expression is minimised by $\xi' = \xi - \delta$ which yields the first term of $r_\delta$.

Of course also players $\alpha$ make mistakes $\xi'$ that are *lower* than the conventional demands $\xi$. But for $b \geq a$ the minimal number of such mistakes that is necessary to make a player $\beta$ choose $\xi'$, given by $b\frac{V_H - \xi}{\xi'}$, is never strictly smaller than $a(1 - \frac{\xi - \delta}{\xi})$. Hence these mistakes are irrelevant for the discussion of stochastic stability.

A formal derivation of $r_\delta(x)$ would be analogous to the proof of Lemma 1 of Young (1993b). Note, however, that the expressions by Young differ from those above because he assumes $V_H = 1$ and $a \geq b$ instead of $b \geq a$.

Because $b\left(1 - \frac{V_H - \xi - \delta}{V_H - \xi}\right)$ and $a\frac{\xi}{V_H - \delta}$ are strictly increasing in $\xi$ and $a(1 - \frac{\xi - \delta}{\xi})$ is strictly decreasing in $\xi$, $r_\delta$ is maximised by a unique value $\xi^*$or two adjunct values $\xi^*$ and $\xi^* + \delta$. To construct a minimal i-Tree on $S$ distinguish three cases:

Case 1: Suppose that $C_H < \xi^*$ and suppose that $\xi^* = \arg\max_{\xi \in X} r_\delta$ is unique. Now follow the Proof of Lemma 2 in Young (1993b). Construct an i-tree $T_\delta$ with a vertex for every stable convention and with root $(H, \xi^*, V_H - \xi^*)^m \in \Sigma_H$ as follows:

1. For every $(H, \xi, V_H - \xi)^m \in S$ with $\xi > \xi^*$ put the directed edge $((H, \xi, V_H - \xi)^m, (H, \xi - \delta, V_H - \xi + \delta)^m)$ in $T_\delta$ and let its weight be $\left\lceil a(1 - \frac{\xi - \delta}{\xi}) \right\rceil$.

2. For every $(H, \xi, V_H - \xi)^m \in S$ with $\xi < \xi^*$ and $b\left(1 - \frac{V_H - \xi - \delta}{V_H - \xi}\right) \leq a\frac{\xi}{V_H - \delta}$ put the directed edge $((H, \xi, V_H - \xi)^m, (H, \xi + \delta, V_H - \xi + \delta)^m)$ in $T_\delta$ and let its weight be $\left\lceil b(1 - \frac{V_H - \xi - \delta}{V_H - \xi}) \right\rceil$.

3. For every $(H, \xi, V_H - \xi)^m \in S$ with $\xi < \xi^*$ and $b\left(1 - \frac{V_H - \xi - \delta}{V_H - \xi}\right) > a\frac{\xi}{V_H - \delta}$ put

the directed edge $((H, \xi, V_H - \xi)^m, (H, V_H - \delta, \delta)^m)$ in $T_\delta$ and let its weight be $\left\lceil a(\frac{\xi}{V_H-\delta}) \right\rceil$.

This tree is constructed such that it has the minimal stochastic potential among all i-Trees on $S$. Because its root is chosen such that it maximises $r_\delta$, the number of errors that is necessary to displace this convention is higher than for any other stable convention if $a$ and $b$ are sufficiently large. For a more detailed exposition compare the proof of Lemma 2 in Young (1993b)[3].

Case 2: Suppose that $C_H \leq \xi^*$ and that $r_\delta(\xi)$ is maximised by two neighbouring values in $X$, $\xi^*$ and $\xi^* + \delta$. Simply construct a i-Tree as above for each of them. By similar arguments as above these i-Trees have minimum stochastic potential.

Case 3: Suppose now that $C_H \geq \xi^*$. Construct an i-Tree with a vertex for every stable convention and root $(H, C_H + \delta, V_H - C_H - \delta)^m$ as follows:

For every $(H, x, V_H - x)^m \in S$ put the directed $((H, x, V_H - x)^m, (H, x - \delta, V_H - x + \delta)^m)$ in $T_\delta$ and let its weight be $\left\lceil a(1 - \frac{\xi-\delta}{\xi}) \right\rceil$. By construction this i-Tree has minimum stochastic potential among all i-Trees on $S$. Because $r_\delta$ is decreasing in $\xi$ for $\xi \geq \xi^*$, it follows that the number of errors that is necessary to displace $(H, C_H + \delta, V_H - C_H - \delta)^m$ is higher than for any other stable convention.

Together with Theorem 2 in Young (1993a) this implies that for $C_H \geq \xi^*$, $(H, C_H + \delta, V_H - C_H - \delta)^m$ and for $C_H < \xi^*$, $(H, \xi^*, V_H - \xi^*)$ and possibly $(H, \xi^* + \delta, V_H - \xi^* - \delta)$ are the unique generically stochastically stable conventions. Together with Lemma 3 in Young (1993b) that shows that $\lim_{\delta \to \infty} \arg\max_{\xi \in X} r_\delta(\xi)$ equals the asymmetric Nash bargaining solution, this completes the proof of the Theorem. $\square$

This Theorem shows that in the present model the distribution of the surplus does not favour the investing player. In contrast for $a \leq b$ the net gain $\xi^* - C_H$ of the investing player is strictly smaller than the net gain $V_H - \xi^*$ of the non-investing player. This confirms the traditional results of the literature on the hold-up problem that emphasises the possibility that the investing player is "ripped off" by the other player.

Indeed Theorem 10 might be seen as describing an intermediate case. On the one hand, the evolutionary learning process ensures that investment will be high and that the investing player will not get a negative net surplus. This confirms the insights that evolution can overcome the underinvestment in the hold-up problem and thereby solve the efficiency problem. On the other hand, it allows for the non-investing party to ensure more than half of the net surplus for himself. Consequently the investing party still has an incentive to search for possibilities that allow a different distribution of the surplus.

## 5. Two-sided Investment

While one-sided investment features prominently in the literature there is no reason to believe that in general relation specific investment is one-sided. As an example one

[3]Note that the tree described in Young 1993 is a mirror image of the x-Tree constructed above. The reason is that $b \geq a$ is assumed instead of $a \geq b$. Also note that in the paper by Young the sample length of players $\alpha$ and $\beta$ is defined by $am$ and $bm$ respectively.

| | L | H |
|---|---|---|
| L | $\frac{V_{LL}}{2} - C_L$ , $\frac{V_{LL}}{2} - C_L$ | $\frac{V_{HL}}{2} - C_L$ , $\frac{V_{HL}}{2} - C_H$ |
| H | $\frac{V_{LH}}{2} - C_H$ , $\frac{V_{LH}}{2} - C_L$ | $\frac{V_{HH}}{2} - C_H$ , $\frac{V_{HH}}{2} - C_H$ |

Figure 1: Strategic Model of the Two-sided Hold-up Problem.

might think about an employee who invests effort to acquire a firm specific skill and uses costly resources of the firm to do so.

This section will concentrate on the case where high investment by both players is efficient, which clearly is the appropriate frame of reference for discussing the hold-up problem. It is again assumed that there only exist two investment levels, $H$ and $L$. It is also assumed that the investment by players from $\mathbb{A}$ and $\mathbb{B}$ is symmetric in the sense that $V_{HL} = V_{LH}$. This is in line with the assumptions made in Dawid and MacLeod (2001, 2008). Investment is said to be efficient if

$$V_{HH} - 2C_H > \max\{V_{HL} - C_H - C_L, V_{LL} - 2C_{LL}\}. \quad (5)$$

The focus on one-sided investment might be due to the fact that for many traditional bargaining models the nature of the hold-up problem is the same for one- and two-sided investment. As a point of reference consider again that the surplus $V_{IJ}$ is split among the two player according to the Nash bargaining rule. If the players from both populations are identical, then the game can be represented by the matrix in Figure 1.

High investment by both players will be a Nash equilibrium if, and only if, $\frac{V_{HH}}{2} - C_H > \frac{V_{HL}}{2} - C_L$. Thus if $V_{HH} - 2C_H > V_{LL} - 2C_L$ is satisfied, it follows from equation 5 that the hold-up problem would lead to inefficient, low investment if

$$V_{HH} - V_{HL} \in (C_H - C_L, 2C_H - 2C_L). \quad (6)$$

Evidently this is very similar to the condition for one-sided investment (equation 3). Figure 2 depicts the parameter values of the hold-up problem for $C_L = 0$ which occurs in the areas B and C.

This similarity of the result does not carry over to the evolutionary models in the spirit of Tröger (2002) and Ellingsen and Robles (2002). Dawid and MacLeod (2001) show that if investment is two-sided, no unique stochastically stable convention with high investment might exist. The intuition behind their result is relatively straightforward. Consider a stable convention with high investment of both players, $\sigma = (H, x, H, V_{HH} - x)$. If $\sigma$ is a stable convention, then the beliefs of the players must be such that choosing high investment yields the highest expected payoff. Because the the fact that $\sigma$ is a stable

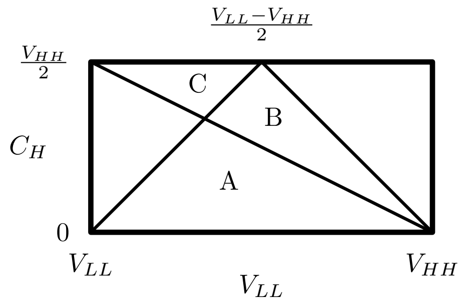


Figure 2: Parameters of the hold-up problem.
The concept of this Figure is due to Dawid and MacLeod (2008).

convention implies that both players believe that $q(L) = 0$, this is equal to

$$\begin{aligned} \max_x E(\pi_\alpha(x)|H, H, \hat{F}_{HH}(y)) &\equiv \max_x(x\hat{F}_{HH}(V_{HH} - x) - C_H) \\ > \max_x E(\pi_\alpha(x)|L, H, \hat{F}_{LH}(y)) &\equiv \max_x(x\hat{F}_{LH}(V_{LH} - x) - C_L), \end{aligned}$$

and the analogous condition for $\beta$.

As explained in the discussion of Corollary 9, $\hat{F}_{LH}(.)$ can take a random form for both players due to errors at the investment stage, which imply errors at the bargaining stage. Thus a convention is the unique stochastically stable convention only if it is stable for all possible $\hat{F}_{LH}(.)$.

Clearly the maximum expected payoff from low investment that is compatible with some beliefs $\hat{F}_{LH}(.)$ and $q(H) = 0$ is $V_{HL} - C_L$. Consequently a convention can only be stable for all possible $\hat{F}_{LH}(.)$ if $x - C_H > V_{HL} - C_L$ and $V_{HH} - x - C_H > V_{HL} - C_L$. This can only be the case if

$$V_{HH} - V_{HL} > 2C_H - 2C_L + V_{HL},$$

which because of equation 6 implies that there would be no hold-up problem in the traditional strategic model. This implies that there does not exist a unique stochastically stable convention for the relevant range of parameters and hence no coordination in the long run.

Thus Dawid and MacLeod (2001) show that in a situation with two-sided investment the hold-up problem might turn out to be more severe in an evolutionary framework. This point also made in Dawid and MacLeod (2008). In this paper the probabilities of making an error at the investment and the bargaining stage are independent. Moreover the probability of making an error at the investment stage is assumed not to converge towards zero while the probability of making an error at the bargaining stage converges to zero. In this setup a unique stochastically stable bargaining norm always exist. But the set of parameters for which this stochastically stable norm is associated with efficient high investment is strictly smaller than in the strategic model.

Different to this in the present model all stochastically stable conventions are associated with high efficient investment by both players for a relevant range of parameters. It will be shown that this is the case for what I shall call unilaterally efficient investment.

Investment is said to be unilaterally efficient if, high investment by only one player is more efficient, then low investment by both players. Formally investment is unilaterally efficient if, it is efficient and

$$V_{HL} - C_H > V_{LL} - C_L. \tag{7}$$

The parameters for which investment is unilaterally efficient correspond to the area A and B in Figure 2.

**Theorem 12.** *Let investment be unilaterally efficient and let the sample lengths of both players be $k \leq m/3$. Then from any initial state $s$ the unperturbed bargaining process $P^0$ converges almost surely to a stable convention where investment is high.*

The proof of this Theorem is similar to the proof of the one-sided case. For simplicity $C_L = 0$ is assumed again which is not crucial, however. The following Lemma is the equivalent of Lemma 6.

**Lemma 13.** *Suppose that investment is unilaterally efficient and that the sample $A$ consists of $k$ identical records $(I, x, J, y)$. If player $\alpha$ draws $A$, then $\alpha$ will play high investment if, and only if, $y < V_{HJ} - C_H$. If player $\beta$ draws $A$, then $\beta$ will play high investment if, and only if, $x < V_{IH} - C_H$.*

*Proof.* Consider the best response of $\alpha$ drawing $A$. Since all records in $A$ are identical, $\alpha$ expects $\beta$ to play $J$ and $y$ with probability one. Consequently the best response of $\alpha$ at the bargaining stage is to play $x = V_{IJ} - y$ if $V_{IJ} - y \geq \delta$.

From equation 2 it follows that the expected payoff for $\alpha$ of playing an investment level $I'$ is

$$\max_x (E(\pi_\alpha(x)|I', J, A)) = \max\{V_{IJ} - y, 0\} - C_I.$$

If $J_A = H$, then because investment is unilaterally efficient which implies efficiency it follows from equation 5 that $\max\{V_{HH} - y, 0\} - C_H > \max\{V_{LH} - y, 0\} - 0$ holds if, and only if, $y < V_{HH} - C_H$ in which case $\alpha$ will choose high investment.

If conversely $J_A = L$, then because investment is unilaterally efficient equation 7 implies that $\max\{V_{HL} - y, 0\} - C_H > \max\{V_{LL} - y, 0\} - 0$ if, and only if, $y < V_{HL} - C_H$ in which case $\alpha$ will choose high investment.

Together this implies that $\alpha$ will chooses high investment if, and only if, $y < V_{HJ} - C_H$.

Because the players from $\mathbb{A}$ and $\mathbb{B}$ are symmetric, $\beta$ chooses high investment if, and only if, $x < V_{IH} - C_H$. $\square$

In the remainder of this section use the following definition to shorten notation:

**Definition 14.** Let $A_t$ denote the last $k$ records in a state $s_t$.

The following three Lemmata will show that from any initial state there exists a positive probability of reaching a state in $\Sigma_{HH}$ within finite time. The condition that both players sample $k \leq m/3$ records is not referred explicitly. It is, however, used implicitly and is sufficient to ensure that the proofs of the following Lemmata are valid.

**Lemma 15.** *For any state $s_t$ where $A_t$ consists of identical records $(L, x, L, y)$ there exists a positive probability of transition to a stable convention with high investment within finite time.*

*Proof.* Distinguish 2 cases:

Case 1: Assume that $x$ or $y$ in $A_t$ are larger than $C_H$. Then without loss of generality assume that $x > C_H$.

Suppose that $\alpha$ and $\beta$ sample $A_t = (L, x, L, y)^k$ in period $t+1$ to $t+k$ inclusive. Because $x$ and $y$ are smaller than $V_{LL}$ and investment is unilaterally efficient, $x, y < V_{HL} - C_H$. Thus Lemma 13 implies that both players will play high investment. Consequently at the bargaining stage $\alpha$ will choose $V_{HH} - y$ and $\beta$ will choose $V_{HH} - x$.

Suppose that in period $t + k + 1$ to $t + 2k$ inclusive $\alpha$ samples $A_{t+k} = (H, V_{HH} - y, H.V_{HH} - x)^k$ and $\beta$ samples $A_t$. Because of $x > C_H$, Lemma 13 implies high investment for player $\alpha$. Player $\beta$ will also choose high investment again. Thus at the bargaining stage $\beta$ will choose $V_{HH} - x$ again and $\alpha$ will choose $V_{HH} - (V_{HH} - x) = x$.

Suppose that in period $t + 2k + 1$ to $t + 3k$ inclusive $\alpha$ and $\beta$ sample $A_{t+2k} = (H, x, H, V_{HH} - x)^k$. Because $x < V_{LL}$, player $\beta$ will choose high investment. Player $\alpha$ will also choose high investment again since $V_{HH} - x < V_{HH} - C_H$. At the bargaining stage $\alpha$ demands $x$ and $\beta$ demands $V_{HH} - x$.

Thus $A_{t+3k}$ equals $A_{t+2k}$ after both players have sampled $A_{t+2k}$ . Consequently if both player sample the last $k$ records in every period, then $s_{t+2k}$ will converge to a stable convention in $m - 2k$ periods.

Case 2: Assume that $x$ and $y$ in $A_t$ are smaller or equal than $C_H$.

Suppose that $\alpha$ and $\beta$ sample $A_t = (L, x, L, y)^k$ in period $t+1$ to $t+k$ inclusive. Then because $x$ and $y$ are smaller than $V_{LL}$ and investment is unilaterally efficient, Lemma 13 implies that both players will play high investment. Consequently at the bargaining stage $\alpha$ will choose $V_{HH} - y$ and $\beta$ will choose $V_{HH} - x$.

Suppose that in period $t + k + 1$ to $t + 2k$ inclusive $\alpha$ samples $A_{t+k} = (H, V_{HH} - y, H.V_{HH} - x)^k$ and $\beta$ samples $A_t$. Because from $x \leq C_H$, it follows that $V_{HH} - x \geq V_{HH} - C_H$, Lemma 13 implies low investment for player $\alpha$. Player $\beta$ will choose high investment again. Thus at the bargaining stage $\beta$ chooses $V_{LH} - x$. Because investment is efficient, $V_{HH} - x > V_{HL}$ which implies that $\alpha$ makes a random demand $y'$.

Suppose that in period $t+2k+1$ to $t+3k$ inclusive $\alpha$ samples $A_{t+2k} = (L, y', H.V_{LH} - x)^k$ and $\beta$ samples $A_t$. Because investment is efficient, $V_{LH} - x < V_{HH} - C_H$ and Lemma 13 implies that $\alpha$ will choose high investment. Player $\beta$ will choose high investment again. Thus at the bargaining stage $\alpha$ demands $V_{HH} - (V_{LH} - x)$ and $\beta$ demands $V_{HH} - x$.

Suppose that in period $t + 3k + 1$ to $t + 4k$ inclusive $\alpha$ samples $A_{t+2k}$ and $\beta$ samples $A_{t+3k} = (H, V_{HH} - (V_{LH} - x), H, V_{HH} - x)^k$. Because investment is unilaterally efficient, $V_{LH} - V_{LL} > C_H$ and since $x < V_{LL}$, it follows that $V_{HH} - (V_{LH} - x) < V_{HH} - C_H$. Thus Lemma 13 implies that player $\beta$ will choose high investment. Player $\alpha$ will again choose high investment. At the bargaining stage $\alpha$ will again demand $V_{HH} - (V_{LH} - x)$ and $\beta$ will demand $V_{HH} - (V_{HH} - (V_{LH} - x)) = V_{LH} - x$.

Suppose that in period $t + 4k + 1$ to $t + 5k$ inclusive $\alpha$ and $\beta$ sample $A_{t+4k} = (H, V_{HH} - (V_{LH} - x), H, V_{LH} - x)^k$. Since $\alpha$ and $\beta$ both face the same investment and demand distributions as in period $t+3k+1$ to $t+4k$, both will choose high investment again and make the same demands at the bargaining stage. Thus if both players sample the last $k$ records in every period, then $s_{t+5k}$ will converge to a stable convention with high investment in $m-2k$ periods. □

**Lemma 16.** *For any state $s_t$ where $A_t$ consists of identical records $(H, x, L, y)$ or $(L, xH, y)$ there exists a positive probability of transition to a stable convention with high investment within finite time.*

*Proof.* Because $\mathbb{A}$ and $\mathbb{B}$ are symmetric, assume without loss of generality that $A_t = (H, x, L, y)^k$. Distinguish 4 cases:

Case 1: Assume that $y < V_{HL} - C_H$ and $x > C_H$. Suppose that $\alpha$ and $\beta$ sample $A_t$ in period $t+1$ to $t+k$ inclusive. Because of Lemma 13, $y < V_{HL} - C_H$ implies that player $\alpha$ will choose high investment. Because investment is efficient and $x < V_{HL}$ which implies $x < V_{HH} - C_H$, it follows from Lemma 13 that $\beta$ will choose high investment. At the bargaining stage $\alpha$ will demand $V_{HH} - y$ and $\beta$ will demand $V_{HH} - x$.

Suppose that in period $t+k+1$ to $t+2k$ inclusive $\alpha$ samples $A_{t+k} = (H, V_{HH} - y, H, V_{HH} - x)^k$ and $\beta$ samples $A_t$. Because $x > C_H$, Lemma 13 implies that $\alpha$ will choose high investment. Player $\beta$ will again choose high investment. At the bargaining stage $\alpha$ will demand $V_{HH} - (V_{HH} - x) = x$ and $\beta$ will demand $V_{HH} - x$.

Suppose that $\alpha$ and $\beta$ sample $A_{t+2k} = (H, x, H, V_{HH} - x)^k$. Because $x > C_H$ and investment is efficient, $V_{HH} - x < V_{HH} - C_H$ and because $x < V_{HL}$ and investment is efficient, $x < V_{HH} - C_H$. Thus Lemma 13 implies that $\alpha$ and $\beta$ will choose high investment. At the bargaining stage $\alpha$ will demand $x$ and $\beta$ will demand $V_{HH} - x$. Since this is identical to the records in $A_{t+2k}$, the state $s_{t+2k}$ will converge to a stable convention with high investment if both players sample the last $k$ records in the next $m-k$ periods.

Case 2: Assume that $y \geq V_{HL} - C_H$ and $x \leq C_H$. Suppose that $\alpha$ and $\beta$ sample $A_t = (H, x, L, y)^k$ in period $t+1$ to $t+k$ inclusive. Because of Lemma 13, $y \geq V_{HL} - C_H$ implies that player $\alpha$ will choose low investment. Because investment is efficient and $x < V_{HL}$ which implies $x < V_{HH} - C_H$, it follows from Lemma 13 that $\beta$ will choose high investment. At the bargaining stage $\alpha$ will demand $V_{HL} - y$ and $\beta$ will choose $V_{HL} - x$.

Suppose that in period $t+k+1$ to $t+2k$ inclusive $\alpha$ samples $A_t$ and $\beta$ samples $A_{t+k} = (L, V_{HL} - y, H, V_{HL} - x)^k$. Because $x \leq C_H$, $V_{HL} - x \geq V_{HL} - C_H$ and Lemma 13 imply that $\beta$ chooses low investment. Player $\alpha$ will choose low investment as before.

Thus $A_{t+2k} = (L, x', L, y')$ and Lemma 15 implies that $s_{t+2k}$ converges to a stable convention with high investment with positive probability.

Case 3: Assume that $y < V_{HL} - C_H$ and $x \leq C_H$. Suppose that $\alpha$ and $\beta$ sample $A_t = (H, x, L, y)^k$ in period $t+1$ to $t+k$ inclusive. Because of Lemma 13 $y < V_{HL} - C_H$ implies that player $\alpha$ will choose high investment. Because investment is efficient and $x < V_{HL}$ which implies $x < V_{HH} - C_H$, it follows from Lemma 13 that $\beta$ will choose high investment. At the bargaining stage $\alpha$ will demand $V_{HH} - y$ and $\beta$ will demand $V_{HH} - x$.

Suppose that in period $t+k+1$ to $t+2k$ inclusive $\alpha$ samples $A_{t+k} = (H, V_{HH} - y, H, V_{HH} - x)^k$ and $\beta$ samples $A_{t+k}$. Because $x \leq C_H$, $V_{HH} - x \geq V_{HH} - C_H$ and Lemma 13 implies low investment by player $\alpha$. Player $\beta$ will choose high investment as before. Thus at the bargaining stage $\beta$ will demand $V_{LH} - x$. Because $x \leq C_H$ and investment is efficient, $V_{HH} - x > V_{HL}$. Thus player $\alpha$ will make any demand $x' \in [\delta, V_{LH} - \delta]$ with positive probability.

Thus $A_{t+2k} = (L, x', H, V_{LH} - x)^k$. If $V_{LH} - x > C_H$, then suppose that $x' < V_{HL} - C_H$ in period $t+k+1$ to $t+2k$. Because players in $\mathbb{A}$ and $\mathbb{B}$ are symmetric, it then follows from Case 1 that $s_{t+2k}$ will converge to a stable convention with high investment with positive probability.

If $V_{LH} - x \leq C_H$, then suppose that $x' \geq V_{HL} - C_H$ in period $t+k+1$ to $t+2k$. Because players in $\mathbb{A}$ and $\mathbb{B}$ are symmetric, it then follows from Case 2 that $s_{t+2k}$ will converge to a stable convention with high investment with positive probability.

Case 4: Assume that $y \geq V_{HL} - C_H$ and $x > C_H$. Suppose that $\alpha$ and $\beta$ sample $A_t = (H, x, L, y)^k$ in period $t+1$ to $t+k$ inclusive. Because of Lemma 13, $y \geq V_{HL} - C_H$ implies that player $\alpha$ will choose low investment. Because investment is efficient and $x < V_{HL}$, $\beta$ will choose high investment. At the bargaining stage $\alpha$ will demand $V_{HL} - y$ and $\beta$ will choose $V_{HL} - x$.

Suppose that in period $t+k+1$ to $t+2k$ inclusive $\alpha$ samples $A_{t+k} = (L, V_{HL} - y, H, V_{HL} - x)^k$ and $\beta$ samples $A_t$. Because investment is efficient $V_{HL} - x < V_{HH} - C_H$ and thus $\alpha$ will choose high investment. Player $\beta$ will choose high investment again. At the bargaining stage $\alpha$ will demand $V_{HH} - (V_{HL} - x)$ and $\beta$ will demand $(V_{HH} - x)$.

Suppose that in period $t+2k+1$ to $t+3k$ inclusive $\alpha$ samples $A_{t+2k} = (H, V_{HH} - (V_{HL} - x), H, V_{HH} - x)^k$ and $\beta$ samples $A_t$. Because $x > C_H$, $V_{HH} - x < V_{HH} - C_H$ and thus $\alpha$ will choose high investment. Player $\beta$ will choose high investment again. At the bargaining stage $\alpha$ will demand $x$ and $\beta$ will demand $V_{HH} - x$.

Suppose that $\alpha$ and $\beta$ sample $A_{t+3k} = (H, x, H, V_{HH} - x)^k$. Because $x < V_{HL}$ and investment is efficient, $\beta$ will choose high investment. Because $x > C_H$, $\alpha$ will also choose high investment again. At the bargaining stage $\alpha$ will demand $x$ and $\beta$ will demand $V_{HH} - x$. Thus $A_{t+3k+1} = A_{t+3k}$ and consequently if $\alpha$ and $\beta$ sample the last $k$ records in period $t+3k+1$ to $t+2k+m$ the state $s_{t+3k}$ will converge to a stable convention with high investment.

Together Cases 1 to 4 imply that from every state $s_t$ with $A_t = (H, x, L, y)^k$ or $A_t = (L, x, H, y)^k$ there exists a positive probability of transition to a stable convention with high investment within finite time. □

**Lemma 17.** *For any state where $A_t$ consists of identical records $(H, x, H, y)$ there exists a positive probability of transition to a stable convention with high investment within finite time.*

*Proof.* Distinguish two cases:

Case 1: Assume that if $\alpha$ and $\beta$ sample $A_t = (H, x, H, y)^k$ this does not imply high investment for both players. Thus if both players sample $A_t$ in period $t+1$ to $t+k$ inclusive, then $A_{t+k}$ will have one of the following forms: $(L, x'L, y')$, $(L, x', H, y')$ or

$(H, x', L, y')$. Then Lemma 15 and 16 together imply that $s_{t+k}$ will converge to a stable convention with high investment with positive probability.

Case 2: Assume that if $\alpha$ and $\beta$ sample $A_t = (H, x, H, y)^k$, this implies high investment for both players. Suppose that both players sample $A_t$ in period $t + 1$ to $t + k$ inclusive. At the bargaining stage $\alpha$ will demand $V_{HH} - y$ and $\beta$ will demand $V_{HH} - x$.

Suppose that both players sample $A_{t+k} = (H, V_{HH} - y, H, V_{HH} - x)^k$ in period $t + k + 1$ to $t + 2k$ inclusive. If this does not imply high investment for both players, then Lemma 15 and 16 again ensure that $s_{t+2k}$ will converge to a stable convention with high investment with positive probability.

If this is not the case, suppose that $\alpha$ samples $A_{t+k}$ and $\beta$ samples $A_t$ in period $t + k + 1$ to $t + 2k$ inclusive. By construction this implies high investment for both players. At the bargaining stage $\alpha$ will demand $x$ and $\beta$ will demand $V_{HH} - x$.

Suppose that in period $t + 2k + 1$ both players sample $A_{t+2k} = (H, x, H, V_{HH} - x)$. By construction this implies high investment for both players. At the bargaining stage $\alpha$ will demand $x$ and $\beta$ will demand $V_{HH} - x$. Thus $A_{t+2k+1} = A_{t+2k}$ and consequently if $\alpha$ and $\beta$ sample the last $k$ records in period $t + 2k + 1$ to $t + k + m$ the state $s_{t+2k}$ will converge to a stable convention with high investment. □

The next Lemma claims that a state where $A_t$ consists of identical records can be reached from any possible state.

**Lemma 18.** *For every initial state $s$ the investment bargaining process $P^0$ moves to a state $s'$ where the last $k$ records of the state consist of identical records with positive probability.*

*Proof.* Suppose that the process is in state $s_t$ at time $t$ .

Suppose that $\alpha$ and $\beta$ sample $A_t$ in period $t + 1$ to $t + k$ inclusive. This implies that there play is identical in the periods $t + 1$ to $t + k$ inclusive with positive probability. Thus the last $k$ records in state $s_{t+k}$ are identical with positive probability. □

Together these Lemmata can be used to prove Theorem 12:

*Proof of Theorem 12.* Lemmata 15 to 18 together imply that for any given state $s$ there exist a positive probability of reaching a stable convention with high investment by both players within finite time. The fact that the state space $\mathbb{S}$ is finite completes the proof. □

The next Lemma characterises the set of stable conventions with high investment.

**Lemma 19.** *Let $\Sigma_{HH}$ denote the set of stable conventions where investment by both players is high. If investment is efficient, then $\Sigma_{HH}$ is the set of conventions $\sigma = (H, \xi, H, V_{HH} - \xi)$ with $\xi \in X \cap [C_H + \delta, V_{HH} - C_H - \delta]$.*

*Proof.* Consider a convention $\sigma$ with high investment by both players and some division $\xi$.

From Lemma 13 it follows that $\alpha$ will choose high investment if, and only if, $V_{HH} - \xi < V_{HH} - C_H$. This condition also implies that the best response of $\alpha$ at the bargaining stage is $\xi$.

Analogously it follows from Lemma 13 that $\beta$ will choose high investment if, and only if, $\xi < V_{HH} - C_H$. This condition also implies that the best response of $\beta$ at the bargaining stage is $V_{HH} - \xi$.

Together this shows that if $\sigma$ is a convention with high investment, then the unique best response of both players is $(H, \xi, H, V_{HH} - \xi)$ if, and only if, $\xi \in X \cap [C_H + \delta, V_{HH} - C_H - \delta]$. It follows that $\Sigma_{HH}$ is the set of absorbing states with high investment of the unperturbed investment bargaining process $P^0$ . □

Thus if investment is unilaterally efficient, then every convention is stable if both players choose high investment and results in a positive payoff for both players.

The fact that the unperturbed investment bargaining process $P^0$ converges to a stable convention with high investment allows to state the following Corollary.

**Corollary 20.** *If investment is unilaterally efficient, then the set of stochastically stable equilibria $S^*$ is a subset of the set of stable conventions with high investment $\Sigma_{HH}$.*

*Proof.* Analogous to the Proof of Corollary 9. □

It has thus been shown that as long as for both players choosing high investment is more efficient than low investment regardless of the investment decision of their opponent, the evolutionary learning process will lead to a set of states where both players choose high investment. The distribution of the surplus will be such that no player receives a negative net surplus.

Consequently the fact that relation specific investment is two-sided does not necessarily imply that the hold-up problem will occur in an evolutionary setting. A characterisation of the set of stochastically stable equilibria if investment is not unilaterally efficient remains for further research though.

# 6. Conclusion

In this paper I have shown that in situations associated with the hold-up problem an evolutionary learning process will lead to a stable convention where the efficient investment levels are chosen. This result is obtained, although the players hold beliefs that lead to conflicting demands at the bargaining stage. The evolutionary learning process is shown to overcome this conflict. This efficiency result carries over at least partially to the case of two-sided investment which contradicts the findings of Dawid and MacLeod (2001, 2008).

But while efficiency is achieved by the evolutionary learning process, the investing player does not necessarily profit from his own investment: In the case of one-sided investment the gross surplus is distributed according to the Nash bargaining solution under the condition that the investing player receives a positive surplus given his investment. Since the investing player bears the investment costs, his net surplus will be smaller than the net surplus of the non-investing player. Thus at the bargaining stage the costs of investment are partially treated as sunk costs. This contrasts with the findings of Ellingsen and Robles (2002) and Tröger (2002), where evolution leads to a surplus distribution that would follow from a forward induction argument.

From a methodological point of view the present paper shows that the exact specification of a myopic best response is non-trivial in a changing environment. If the set of feasible actions of the players does not necessarily remain the same, then it becomes difficult to determine how the actions that have been taken given different action sets should be interpreted for determining the best response.

Moreover the results of the model are sensitive to the rules of interpretation as demonstrated by comparing the present model to those of Ellingsen and Robles (2002), Tröger (2002) and Dawid and MacLeod (2001, 2008). In their models it is simply assumed that only actions that have been made given the same action set are considered. However, since in reality two situations are never exactly the same, this does not seem to be a very sensible assumption.

In contrast to this, the present model does specify an interpretation of actions made given different action sets. While I believe that the assumption that demands are interpreted as absolute shares of the surplus is reasonable, it definitely is not perfect. How people in general use past experiences in similar but different situation remains an open question for research though.

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# A. Abstract


## English

This paper explores an evolutionary model of the hold-up problem. Contrary to the existing literature, I assume that players use information about past actions in similar bargaining situations that differ with respect to the available surplus. Based on beliefs that are derived from this information players choose their myopic best responses, which lead to conflicting demands after changes of the investment levels.

In this setup I show that the evolutionary learning process will coordinate the players actions on the efficient investment levels not only in the case of one-sided investment, but also in the case of two-sided investment for a relevant range of parameters. It is also shown that the distribution of the surplus yields a positive payoff for both players but does not favour the investing party in the one-sided case.

## Deutsch

Diese Diplomarbeit untersucht ein evolutionäres Modell des sogennanten Hold-up Problems. Im Gegensatz zur bestehenden Literatur nehme ich an, dass die Spieler Informationen über vergangenes Verhalten nutzen, das in ähnlichen Verhandlungsitutationen mit unterschiedlich hohen Investitionsüberschüssen stattfand. Basierend auf diesen Informationen wählen die Spieler ihre "myopic best responses", was bei Änderungen der Investitionsniveaus zu inkompatiblen Forderungen führt.

In diesem Modell zeige ich, dass der evolutionäre Lernprozess das Verhalten der Spieler so koordiniert, dass diese das effiziente Investitionsniveau nicht nur im Falle einseitiger, sondern für einen relevanten Parameterbereich auch im Falle beidseitiger Investitionen wählen. Es wird auch gezeigt, dass die Verteilung des Investitionsüberschusses zu positiven Gewinnen für beide Spieler führt, jedoch bei einseitigen Investitionen die investierende Partei nicht begünstigt wird.

# B. Curriculum Vitae

## General Information

Nationality: Austria

Date of Birth: 18.09.1986

## Education

since 10/05: Studies in economics at the University of Vienna. Graduate level courses in Game Theory, Evolutionary Game Theory, Macroeconomics, International Macroeconomics, Economics of Development, Labour Economics, Econometrics, Topology, Dynamic Optimisation, Law and Economics, and Theory of Taxation.

10/07–06/08: Studies in economics at the University Paris 1 - Sorbonne Panthéon (Erasmus).

## Experience

08/08: Internship at the Austrian Ministry of Finance. My work consisted of the preparation of economic background information for the Austrian Minister of Finance.

06/08–07/08: Consultant at the Organisation of Economic Co-Operation and Development, Economics Dept. Country Studies Desk for Germany and Slovakia: My work was about Slovak house prices in preparation for the OECD Economic Survey on the Slovak Republic 2009. It focused on the estimation of house price equations to assess potential overvaluations in the real estate market.

07/07: Internship at the Federal Chancellery of Austria (Bundeskanzleramt Österreich), Dept. of Economics and OECD relations: Mainly working on the effects of the openness of the austrian economy on the wage share.

Summer term 07: Tutor for an undergraduate course in Mathematics at the Dept. of Statistics, University of Vienna.

07/06: Internship at the Vienna Chamber of Workers (Arbeiterkammer Wien), Dept. of Economics and Statistics: Working on a literature overview about income distributions in long run growth models.

10/04–09/05: Compulsory social service, Volkshilfe Österreich, Vienna: Assistance work in a refugee dormitory.

## Skills

Languages: English, German, French,

Statistical Packages: R (using JGR), EViews

## Scholarships

Erasmus scholarship for the academic year 07/08 at the University Paris 1.

Performance scholarship of the University of Vienna for the academic year 05/06.

## Extracurricular Activities

10/07–09/09: Elected student representative at the Dept. of Economics, University of Vienna. My activities include the participation in several university commission (an appointment commission for a full professor, two curricular commissions, and the faculty commission), inscription advisor and tutor for first year students.