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Characterizing the fractal dimension of molecular clouds

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Abstract

Understanding the structure of molecular clouds is crucial for unveiling the physical processes that govern star formation. In this thesis, we present a detailed morphological and topological analysis of the Orion A and Orion B molecular clouds, employing tools rooted in fractal geometry and Minkowski functionals. Using column density maps derived from far-infrared dust emission, we apply perimeter–area techniques both globally and locally to quantify the fractal dimension across a wide dynamic range.

Our results reveal a clear structural transition in Orion A at a column density of $N \sim 1.2 \times 10^{22} \text{ cm}^{-2}$, coinciding with the onset of dense core formation and enhanced star formation activity. Orion B, in contrast, maintains a scale-free, more turbulent morphology with no such transition. The Euler characteristic further supports this picture, showing a well-defined topological peak in Orion A, absent in Orion B. We also introduce a local fractal dimension map and mass–size–dimension (MSD) diagrams, enabling a spatially resolved assessment of structural complexity.

By correlating these structural diagnostics with the distribution of young stellar objects (YSOs), we find that regions of higher local fractal dimension tend to host more early-stage protostars, particularly in Orion A. This suggests a connection between morphological complexity and ongoing star formation. Our findings demonstrate the value of fractal and topological frameworks in understanding the complex physics of the interstellar medium and linking cloud structure to star formation processes.

Kurzfassung

Die Struktur von Molekülwolken spielt eine zentrale Rolle für das Verständnis der physikalischen Mechanismen hinter der Sternentstehung. In dieser Arbeit analysieren wir die Molekülwolken Orion A und Orion B mithilfe topologischer Verfahren sowie unter Verwendung von Minkowski-Funktionalen. Die Grundlagen der Analyse sind Säulendichtekarten, die aus fernen Infrarotdaten der Staubemissionen abgeleitet wurden. Daraus bestimmen wir sowohl globale als auch lokale fraktale Dimensionen über einen breiten Dynamikbereich hinweg.

Unsere Ergebnisse zeigen in Orion A einen strukturellen Übergang bei einer Säulendichte von $N \sim 1,2 \times 10^{22} \text{ cm}^{-2}$, der mit dem Auftreten dichter Kerne und erhöhter Sternentstehungsaktivität zusammenfällt. Orion B hingegen weist eine durchgängig skalenfreie, turbulente Morphologie ohne einen solchen Übergang auf. Die Euler-Charakteristik untermauert diese Interpretation durch ein deutliches topologisches Maximum in Orion A, das in Orion B fehlt.

Zusätzlich führen wir eine lokale Karte der fraktalen Dimension sowie sogenannte Masse–Größe–Dimensions–Diagramme (MSD) ein, um die strukturelle Komplexität ortsabhängig zu analysieren. Ein Vergleich mit der Verteilung junger Sternobjekte (YSOs) zeigt, dass Regionen mit höherer fraktaler Komplexität insbesondere in Orion A vermehrt frühe Entwicklungsstadien der Sternentstehung aufweisen. Dies deutet auf eine Verbindung zwischen komplexer Morphologie und aktiver Sternbildung hin.

Unsere Arbeit zeigt, wie Methoden der Fraktal- und Topologieanalyse helfen können, die mehrskalige Struktur des interstellaren Mediums zu erfassen und mit physikalischen Prozessen der Sternentstehung zu verknüpfen.

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1. Introduction

Giant Molecular Clouds (GMCs) are the primary sites of star formation [MO07]. Within these massive regions, stars form from the gravitational collapse of the highest column density structures. Understanding the morphology and internal structure of GMCs is hence essential to understanding the conditions that regulate star formation.

Much of our knowledge of the structures of GMCs comes from the study of dust emission [Dra03]. Dust grains—well-coupled to the gas in these clouds—emit thermal radiation in the far-infrared and submillimeter regimes. These emissions are then used to reconstruct column density maps, offering a window into the distribution of material within the cloud.

To go beyond qualitative interpretation, quantitative tools are needed to describe the spatial complexity of these structures. One compelling approach involves applying methods from fractal geometry and topology. The concept of a fractal dimension, first introduced by Mandelbrot, provides a way to quantify the irregularity or complexity of a structure’s boundary [Can84]. Complementary topological measures, the Minkowski functionals, allow us to describe properties like connectivity and fragmentation.

Historically, various techniques have been employed to estimate the fractal dimension of molecular clouds—from simple box-counting algorithms to power-law scalings of structure size distributions [EF96]. However, these methods can often provide a limited or averaged view of complex, hierarchical structures.

The motivation behind this thesis is hence to develop and apply tools that connect the visual complexity of molecular clouds with physically meaningful, measurable quantities. By refining the analysis of fractal and topological properties, we aim to gain deeper insight into the role of turbulence, structure formation, and star formation in GMCs in a new fashion.

The goals of this thesis are:

- to extract and preprocess observational column density data for selected GMCs,
- to characterize their structure using topological methods such as the Euler characteristic and perimeter–area scaling,
- to explore the fractal dimension framework in both global and local forms, and
- to investigate correlations between structural complexity and star formation.

The thesis is organized as follows:

- **Chapter 2** offers theoretical background on molecular clouds and fractal geometry.
- **Chapter 3** describes the datasets employed, including how column density maps and YSO catalogs are used.

1. *Introduction*

- **Chapter 4** outlines the methods used to calculate fractal and topological descriptors, with a focus on perimeter–area relations and dendrogram-based analysis.
- **Chapter 5** presents the results, including fractal dimension maps, topological behavior, and the relationship between cloud structure and star formation.
- **Chapter 6** discusses the implications of the results and compares them with previous literature and theoretical expectations.
- **Chapter 7** summarizes the main findings and outlines possible directions for future research.

Additional material, including simulation details and visual examples of structures, is provided in the Appendices.

2. Background

2.1. Dust Modeling

Interstellar dust plays a fundamental role in tracing the structure of molecular clouds [MO07]. Though it accounts for only about 1% of the mass of the interstellar medium (ISM), its ability to absorb, scatter, and re-emit radiation makes it an essential tool for indirectly probing the properties of cold, dense gas where stars form.

2.1.1. Emission from Dust Grains

Dust grains absorb ultraviolet (UV), visible, and near-infrared (NIR) radiation from stars and re-emit this energy thermally at longer wavelengths, primarily in the mid-infrared (MIR), far-infrared (FIR), and submillimeter (sub-mm) regimes. This process is governed by the radiative transfer equation (RTE), which couples extinction (absorption and scattering) and emission across the electromagnetic spectrum [RL79]. In molecular clouds, where visual extinction can exceed $A_V > 10$, emission from dust becomes the dominant observable at long wavelengths.

The standard form of the RTE for the specific intensity I_ν at frequency ν along a path s is:

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu \quad (2.1)$$

where:

- I_ν is the *specific intensity* [$\text{erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1}$],
- α_ν is the *absorption coefficient* (also known as the extinction or opacity coefficient),
- j_ν is the *emission coefficient*, representing the power added per unit length, frequency, and solid angle.

This equation describes the change in radiation intensity along a line of sight due to both absorption and emission. In dusty environments such as molecular clouds, thermal dust emission contributes to j_ν , while α_ν accounts for both absorption and scattering by dust grains. Solving the RTE under suitable assumptions enables us to infer critical physical properties, including dust temperature and column density.

Dust emission is commonly modeled as a modified black-body, or "grey-body," spectrum:

$$F_\nu^{\text{obs}} = B_\nu(T_{\text{dust}}) \times (1 - e^{-\tau_\nu}), \quad (2.2)$$

2. Background

where $B_\nu(T_{\text{dust}})$ is the Planck function at the dust temperature T_{dust} , and τ_ν is the optical depth at frequency ν . For optically thin emission at long wavelengths ($\tau_\nu \ll 1$), this simplifies to:

$$F_\nu^{\text{obs}} \approx B_\nu(T_{\text{dust}}) \cdot \tau_\nu. \quad (2.3)$$

The optical depth τ_ν is typically parameterized as:

$$\tau_\nu = \kappa_\nu \cdot \Sigma, \quad (2.4)$$

where κ_ν is the dust opacity and Σ the dust surface density. The dust opacity is often modeled as a power-law:

$$\kappa_\nu \propto \nu^\beta, \quad (2.5)$$

where β is the dust emissivity index, typically between 1.5 and 2 in the ISM [Dra06].

2.1.2. Rayleigh–Jeans Approximation and FIR Observations

To reduce the number of free parameters when inferring dust properties, observations are often carried out in the Rayleigh-Jeans (RJ) regime, where $h\nu \ll k_B T$. This condition is typically satisfied at wavelengths longer than $\sim 250 \mu\text{m}$ for dust temperatures around $T_{\text{dust}} \sim 10\text{--}20 \text{ K}$, which makes submillimeter and millimeter observations (e.g. from *Herschel*) particularly valuable.

In this regime, the flux scales as:

$$F_\nu^{\text{obs}} \propto \nu^{2+\beta}, \quad (2.6)$$

allowing for simplified spectral energy distribution (SED) fitting to extract the dust temperature and column density.

2.1.3. Column Density Estimation

Under the assumption of fixed dust opacity κ_ν and emissivity index β , one can solve for the dust optical depth at a reference frequency (e.g., $850 \mu\text{m}$):

$$\tau_{850} = \frac{I_{850}}{B_{850}(T_{\text{dust}})}. \quad (2.7)$$

Given a calibration between optical depth and column density (e.g., from [Hil83]), the molecular hydrogen column density $N(\text{H}_2)$ can be in principle estimated as:

$$N(\text{H}_2) \propto \tau_{850}. \quad (2.8)$$

This method allows for the derivation of full-resolution column density maps from multi-band FIR/sub-mm observations, providing a key data product for structural and fractal analyses.

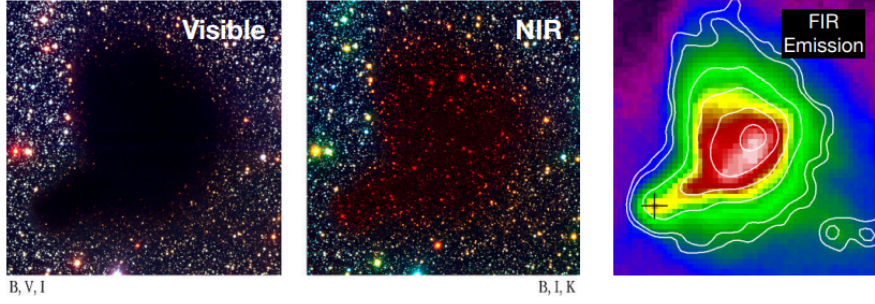


Figure 2.1.: Comparison of dust extinction (optical and NIR) and dust emission (FIR) views of Barnard 68. Credits: ESO, [NLS⁺12].

2.1.4. Comparison with Extinction Methods

Dust column densities can also be inferred from extinction measurements, particularly in the near-infrared (e.g., via NICER/NICEST, [LA01] & [Lom09]). These techniques are independent of dust temperature and often provide high angular resolution in regions with abundant background stars. However, their reliability decreases in high-opacity areas, where few background stars are detectable, and they require robust modeling of the stellar background.

Emission-based methods, by contrast, are more effective in probing dense and cold regions. By modeling the thermal emission from dust at far-infrared and sub-millimeter wavelengths, they can recover column density even where extinction saturates [FR10]. Nevertheless, these methods rely on assumptions about dust temperature, opacity laws, and emissivity index, which can introduce systematic uncertainties.

Each approach has its strengths and limitations, and the choice between them depends on the scientific objectives and the characteristics of the target region. A visual comparison of both techniques is shown in Figure 2.1, highlighting the complementary nature of extinction and emission observations for the dark cloud Barnard 68.

2.1.5. Connection to Star Formation

Dust-based column density maps are crucial for studying the early stages of star formation. Since most dust in molecular clouds is cold ($T_{\text{dust}} \sim 10\text{--}20\text{ K}$), emission in the FIR/sub-mm naturally traces the dense gas where star formation occurs. Thresholds in column density (e.g., $N(\text{H}_2) \sim 1\text{--}2 \times 10^{22}\text{ cm}^{-2}$) have been associated with the onset of core formation and gravitational instability [LLA10].

Thus, dust modeling serves as the foundation for the structural and statistical analyses performed throughout this thesis.

2. Background

2.2. Fractals

Fractals are geometric objects characterized by irregularity and self-similarity over a range of scales. Unlike classical Euclidean shapes, fractals are not usually described by integer dimensions and often exhibit complex, nested structures. Introduced to the scientific community largely through the work of Mandelbrot in the late 20th century [Can84], fractal geometry has since become a powerful framework for describing naturally occurring structures, from coastlines to clouds.

In the context of molecular clouds, fractal behavior arises seemingly naturally everywhere we look. These regions of the interstellar medium exhibit turbulent dynamics and hierarchical fragmentation, producing morphologies that lack a characteristic scale. Theoretical models of interstellar turbulence, particularly those based on compressible, supersonic flows, predict self-similar density structures [Kol62]. Observational studies have consistently found evidence of such fractal-like properties in molecular clouds, supporting the idea that turbulence plays a dominant role in shaping them [EF96, FPW91].

The *fractal dimension* is an important quantitative descriptor of a structure's complexity. Unlike topological dimension, the fractal dimension can take non-integer values. In this work, we explore this quantity through the perimeter-area relation and other related approaches, which are introduced in detail in later chapters.

Understanding fractal properties is not just a matter of geometry. The morphology of molecular clouds is connected to their physical evolution. For instance, more fragmented structures may reflect advanced stages of gravitational collapse or be the signature of driven turbulence. Moreover, star formation tends to occur in regions of low complexity and high density, suggesting a link between the fractal nature of clouds and their star-forming potential.

In summary, fractal geometry offers a quantitative framework to describe the hierarchical, scale-free nature of molecular clouds. Combined with other morphological and topological tools, it provides important insights into the processes that govern the ISM and the onset of star formation. This integrated approach enables the identification of correlations between cloud morphology, physical conditions, and star formation activity, thereby advancing our understanding of the complex interplay between turbulence, gravity, and feedback in the interstellar medium.

3. Data

3.1. *Herschel* column-density maps

3.1.1. Introduction

The main source of data in this work stems from ESA’s *Herschel* Space Observatory [PRP⁺10], which was launched in 2009 with the goal of observing in the far-infrared and submillimetre wavelengths. *Herschel* was equipped with three main instruments: the Photodetector Array Camera and Spectrometer (PACS), the Spectral and Photometric Imaging Receiver (SPIRE), and the Heterodyne Instrument for the Far Infrared (HIFI). These instruments allowed *Herschel* to observe a wide range of astronomical phenomena, including star formation and galaxy evolution. *Herschel*’s mission lasted until 2013, enabling unprecedented studies of the cold and dusty regions of space.

Herschel’s observations of Orion, the target of this work, were part of the *Herschel* Gould Belt Survey (HGBS). This survey aimed to map the nearby star-forming regions in the Gould Belt, a flat structure of a few hundred parsecs inclined by approximately 20° with respect to the galactic plane [AMB⁺10]. The clouds covered by the HGBS span a wide range of physical and environmental conditions.

3.1.2. Dust emission maps

In the ISM, dust typically has temperatures T_{dust} around 20 K and emits thermal radiation that can be approximated by a modified blackbody, peaking at FIR and sub-mm wavelengths. This thermal dust emission represents the dominant source of FIR and mm-continuum radiation in galaxies, as it can be recognized in Figure 3.1

Dust grains absorb UV and NIR photons from stars, reprocessing this stellar radiation and re-emitting it at longer wavelengths. Because dust is highly efficient at radiating energy, it reaches and maintains relatively low equilibrium temperatures [Hi83]. Observations of dust emission hence provide a powerful tool to estimate the column density of dust and gas along the line of sight.

Using mm continuum dust emission to measure column density offers several advantages over traditional extinction-based methods: it can probe much higher column densities where extinction becomes saturated or unreliable, it often provides better angular resolution, and it remains effective even in regions heavily obscured by dust where extinction mapping fails [Dra03]. Moreover, continuum observations allow simultaneous determination of both dust temperature (T_{dust}) and gas column density ($N(\text{H}_2)$), which is essential for characterizing dense molecular clouds. In extragalactic studies, where individual stars

3. Data

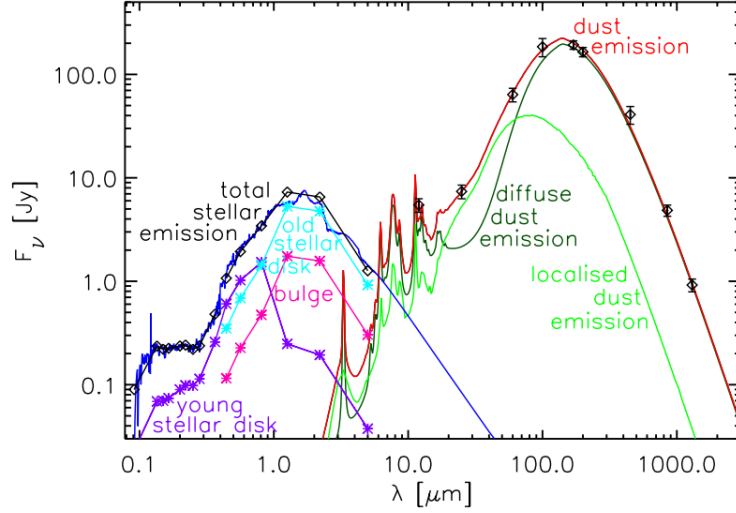


Figure 3.1.: Best fit SED of NGC 891 and observed data in infrared and submillimeter. Also plotted are the main components responsible for each part of the emission spectrum [PTD⁺11]. Dust emission represents the dominant source of FIR and mm-continuum radiation in galaxies.

cannot be resolved, dust emission often represents the only viable method for estimating gas masses.

However, these methods also involve significant assumptions and uncertainties. Different approaches to estimate column density are conceptually equivalent but may yield different results due to methodological degeneracies. One key source of uncertainty arises from the poorly constrained dust opacity, κ , which depends on the composition, size distribution, and physical properties of dust grains [OH94]. Variations in κ directly affect the derived column densities, introducing systematic uncertainties into the analysis.

The data used in this work are *Herschel* column-density maps of the Orion A and B giant molecular clouds. These maps were created by combining data from PACS and SPIRE instruments, which observed at different wavelengths to capture the dust emission across the clouds.

Specifically, PACS provided imaging photometry in the 60-210 μm wavelength range, using two filled silicon bolometer arrays with 16×32 and 32×64 pixels. In photometric mode, PACS simultaneously imaged two bands: either 60-85 μm or 85-125 μm , along with 125-210 μm , covering a field of view of approximately 1.75×3.5 arcminutes with near-Nyquist sampling [PWG⁺10]. These bands are particularly sensitive to warmer dust components.

SPIRE extended the coverage to longer wavelengths, better tracing colder dust. Its photometer operated in three broad bands centered at 250 μm , 350 μm , and 500 μm , well-suited for mapping large-scale dust emission from the cold interstellar medium [GAA⁺10].

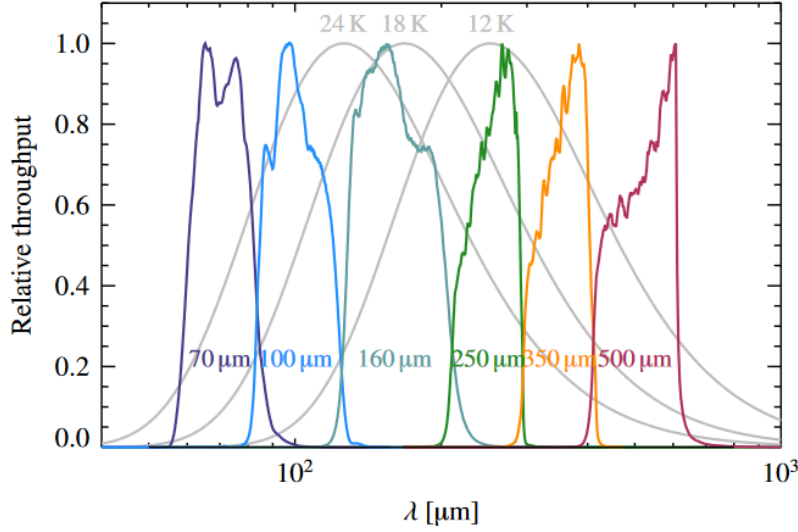


Figure 3.2.: Throughputs in relative units of the PACS and SPIRE bands and three modified blackbodies for typical dust temperatures (12 K, 18 K, 24 K) and with $\beta = 1.8$ [LBAL14].

The combination of PACS and SPIRE photometric data allows for the characterization of the dust SED across a broad range of temperatures. The relative throughputs of the two instruments can be seen in Figure 3.2, on top of black body radiation for typical temperatures of dust. This broad wavelength coverage enables accurate fitting of modified blackbody models to derive dust temperatures, column densities, and emissivity properties.

3.1.3. Derivation of column density maps

To derive the column density maps of the Orion A and B molecular clouds, we used the extinction maps produced by [LBAL14], which are publicly available through the CDS Archive¹. These maps were constructed by fitting the dust emission data from Herschel PACS and SPIRE instruments to a modified blackbody model, providing estimates of the optical depth at 353 GHz (τ_{353}), along with the associated uncertainty maps.

The data products consist of multi-plane FITS files where PLANE1 corresponds to the optical depth map at 353 GHz, and PLANE2 contains the pixel-wise uncertainties. Each map header includes metadata describing projection parameters, coordinate coverage, and the empirical calibration parameters used in the conversion from dust optical depth to extinction, and subsequently to column density. The effective resolution of these maps is 36 arcsec, with pixel dimensions of 3444×2492 , covering both Orion A and B clouds. The given resolution excludes the Planck data, which is at a lower resolution of 5 arcmin.

Following the approach in [LBAL14], we converted the optical depth maps into K -band

¹Orion optical-depth and column-density maps: J/A+A/566/A45 (CDS)

3. Data

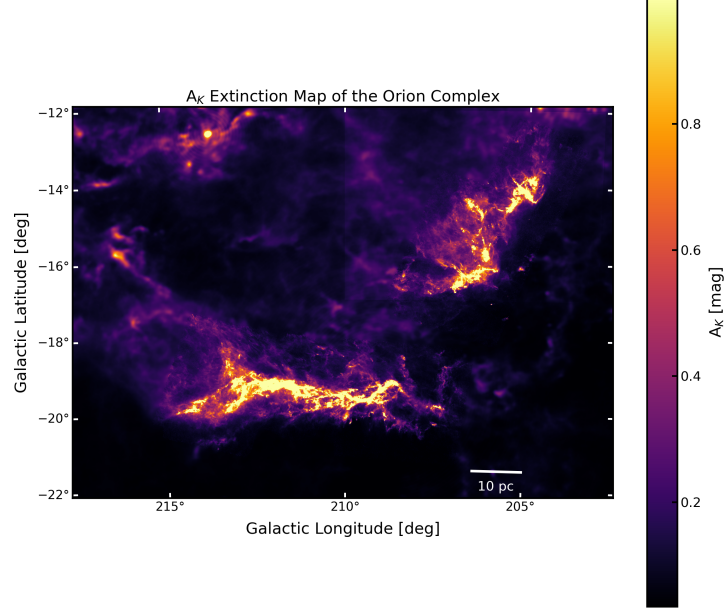


Figure 3.3.: Extinction map of Orion A and B derived from Herschel data, showing the extinction in K-band (A_K) across the clouds. The map is based on the optical depth at 353 GHz, converted to extinction using the relation in Equation 3.1.

extinction maps (A_K) using a linear relation of the form:

$$A_K = \gamma \cdot \tau_{353} + \delta \quad (3.1)$$

The constants γ and δ were calibrated separately for Orion A and B using near-infrared extinction data from the 2MASS/NICEST method [Lom09]. Specifically:

- For Orion A, $\gamma = 2640$ mag and $\delta = 0.012$ mag.
- For Orion B, $\gamma = 3460$ mag and $\delta = -0.001$ mag.

The A_K extinction map can be seen in Figure 3.3. To convert from A_K to visual extinction A_V , we assumed a standard extinction curve with $R_V = 3.1$, following [BSD78] and [RL85], giving:

$$A_V = \frac{A_K}{0.112} \quad (3.2)$$

Finally, assuming all hydrogen is in molecular form (i.e., fully in H_2) and a standard gas-to-dust ratio, the column density of molecular hydrogen was derived using:

$$N(H_2) = 0.93 \times 10^{21} \cdot A_V [cm^{-2}] \quad (3.3)$$

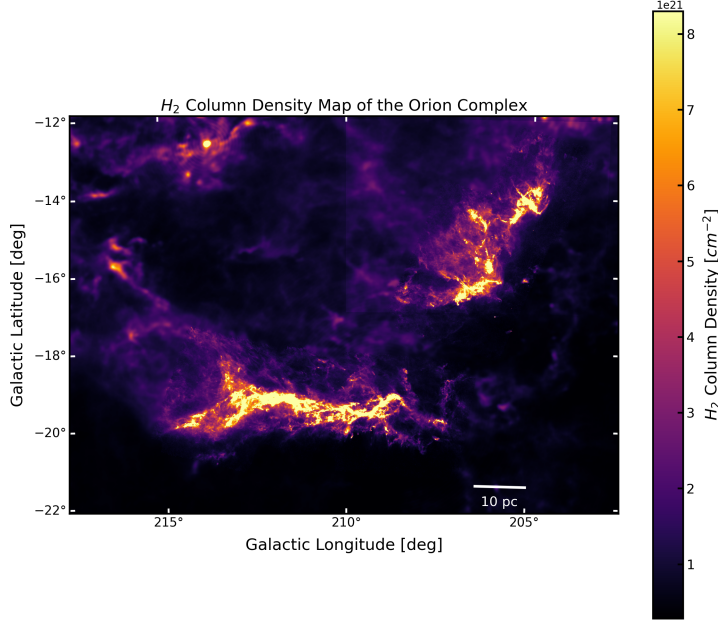


Figure 3.4.: Column density maps of the Orion Complex derived from Herschel data, showing the total column density of molecular hydrogen $N(\text{H}_2)$ across the cloud. The maps are based on the optical depth at 353 GHz.

The derived column density maps of the Orion Complex can be seen in Figure 3.4. The H_2 column densities range from approximately 10^{21} cm^{-2} to a few 10^{23} cm^{-2} , in agreement with values reported in the literature [LBAT14].

This method provides a robust, emission-based estimate of the total column density across Orion A and B, with the advantage of tracing both high and low column density regions uniformly. However, it is important to note that the derivation relies on assumptions about dust properties, uniform opacity, and gas-to-dust ratio, which introduce systematic uncertainties into the absolute calibration of $N(\text{H}_2)$.

Derivation of Mass Maps

Mass maps are a crucial component in this work, allowing us to estimate the total mass of the Orion A and B molecular clouds and their substructures. These maps are computed directly from the H_2 column density maps derived in the previous section, by converting the number of molecules per unit area into a mass per pixel.

The conversion process follows these steps:

1. **Pixel area conversion:**

Each pixel in the column density map subtends a solid angle on the sky, which is converted into a physical area based on the known distance to the Orion complex, assumed to be $D = 412 \text{ pc}$. The pixel scale is $0.00417^\circ/\text{px}$, which is first converted

3. Data

to radians and then used to compute the physical pixel size:

$$\theta_{\text{px}} = \frac{\text{pixel_scale}}{180/\pi}, \quad \text{px_size (pc)} = \sin(\theta_{\text{px}}) \cdot D$$

The area per pixel in cm^2 is then obtained by converting parsecs to centimeters:

$$\text{Area}_{\text{px}} = \text{px_size}^2 \times (3.086 \times 10^{18} \text{ cm/pc})^2$$

2. Column density to mass:

The H_2 column density $N(\text{H}_2)$ in cm^{-2} is multiplied by the pixel area to yield the number of molecules per pixel. This is then converted to mass using the proton mass, $m_p = 1.673 \times 10^{-27} \text{ kg}$, and the mean molecular weight correction factor ($\mu = 2.8$) to include helium and heavier elements.:

$$M_{\text{H}_2} = N(\text{H}_2) \cdot \text{Area}_{\text{px}} \cdot 2.8 \cdot m_p$$

3. Conversion to solar masses:

The result is then converted from kilograms to solar masses using:

$$M_{\text{px}} = \frac{M_{\text{H}_2}}{M_{\odot}}, \quad \text{with } M_{\odot} = 1.9885 \times 10^{30} \text{ kg}$$

The resulting mass map provides the total H_2 mass contained in each pixel of the map, with units of solar masses and can be seen in Figure [3.5](#)

The masses are computed by integrating the mass surface density over the spatial extent of each cloud, defined by fixed Galactic longitude (ℓ) and latitude (b) boundaries:

- Orion A: $206^\circ < \ell < 217^\circ$, $-21^\circ < b < -17^\circ$
- Orion B: $203^\circ < \ell < 210^\circ$, $-17^\circ < b < -12^\circ$

Using these definitions and the mass maps, we estimate the mass of Orion A to be approximately $8.62 \times 10^4 M_{\odot}$, the mass of Orion B to be approximately $7.02 \times 10^4 M_{\odot}$. The total mass of Orion A and B combined is therefore approximately $1.56 \times 10^5 M_{\odot}$. These mass estimates are consistent with previous measurements, such as gamma-ray-based masses for Orion A and B of approximately $8.1 \times 10^4 M_{\odot}$ and $4.0 \times 10^4 M_{\odot}$, respectively ([AAA+12](#)), and with dust-based studies (e.g., [LBAL14](#)), all within observational uncertainties.

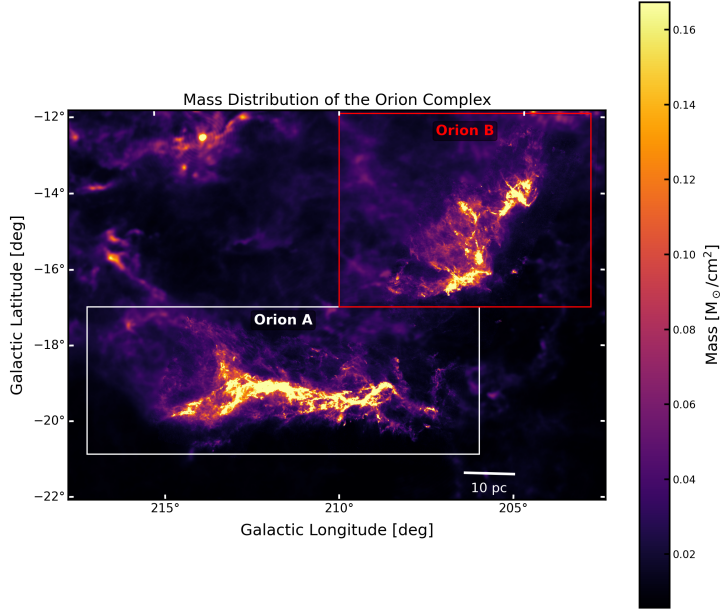


Figure 3.5.: Mass map of the Orion A and B molecular clouds derived from the H_2 column density maps. The map shows the total mass in solar masses contained in each pixel.

3.1.4. Data Pre-processing

To ensure consistency and reliability in the derived column density and mass maps, several pre-processing steps were applied to both the original *Herschel* data and the derived extinction products from [LBAL14]. These steps aimed to isolate high-resolution structure, remove contaminating background emission, and maintain uniform calibration across the dataset.

The original *Herschel* photometric maps used in the derivation of extinction and column density maps were processed using the Herschel Interactive Processing Environment (HIPE; [MMO10]), version 10.0.2843, along with the most up-to-date calibration files available at the time. Standard pipeline procedures were followed, including detector deglitching, baseline removal, and map-making.

Since the raw maps often contain large-scale background emission and lower-resolution contributions from the *Planck* mission, we performed a spatial filtering step to isolate the high-resolution structure traced by *Herschel*’s PACS and SPIRE instruments. Specifically, we masked the lower-resolution *Planck*-dominated regions and retained only those spatial frequencies that correspond to *Herschel*’s native angular resolution. This effectively removed diffuse Galactic foregrounds and background gradients, improving the contrast of compact and filamentary structures within the Orion A and B clouds.

The column density maps from [LBAL14] were obtained by fitting a modified blackbody model to *Herschel* data and calibrated using NIR extinction measurements via the

3. Data

NICEST/2MASS technique. These maps already incorporated several pre-processing steps, including spatial regridding, background subtraction, and convolution to a common beam size.

For our purposes, we applied additional masking to exclude low-signal or edge pixels where the uncertainty in the optical depth was high. These masks ensure that only high-confidence regions are used in the column density and mass derivations.

In summary, the pre-processing workflow ensures that:

- Only high-resolution *Herschel* data contribute to the final maps.
- Large-scale background emission is removed.
- Low signal-to-noise regions and unreliable pixels are masked out.

3.2. Young Stellar Objects (YSOs)

3.2.1. Introduction

Understanding the relationship between the internal structure of molecular clouds and their ability to form stars is a central goal of star formation studies. In this work, we investigate how the morphology and column density distribution of the Orion A and B clouds relate to their young stellar content. To this end, we utilize the Young Stellar Object (YSO) catalogue compiled by [MGM⁺12], which provides a detailed census of the star-forming population in Orion.

The catalogue is based on observations from the *Spitzer Space Telescope*, specifically using data from the Infrared Array Camera (IRAC; [FHA⁺04]) and the Multiband Imaging Photometer for Spitzer (MIPS; [RYE⁺04]), covering approximately 9 deg² in five mid-infrared bands. These wavelengths are sensitive to warm dust and circumstellar material, allowing for the identification of young stars through their infrared excesses. The classification of YSOs is primarily based on the slope of their spectral energy distributions (SEDs) in the mid-infrared, following standard criteria.

Additionally, the catalogue incorporates near-infrared data from the 2MASS survey [SCS⁺06], which helps to identify less embedded sources and complements the mid-infrared sensitivity.

3.2.2. The catalogue

The catalogue² includes a total of 3479 dusty YSOs, classified primarily based on their spectral energy distributions (SEDs). These classifications distinguish between:

- Protostars (P): deeply embedded sources with rising or flat SEDs, indicative of ongoing mass accretion (N=488).
- Disk sources (D): more evolved pre-main sequence stars that still retain circumstellar disks, but with declining mid-infrared emission (N=2991).

²YSO catalogue in the CDS Archive

3.2. Young Stellar Objects (YSOs)

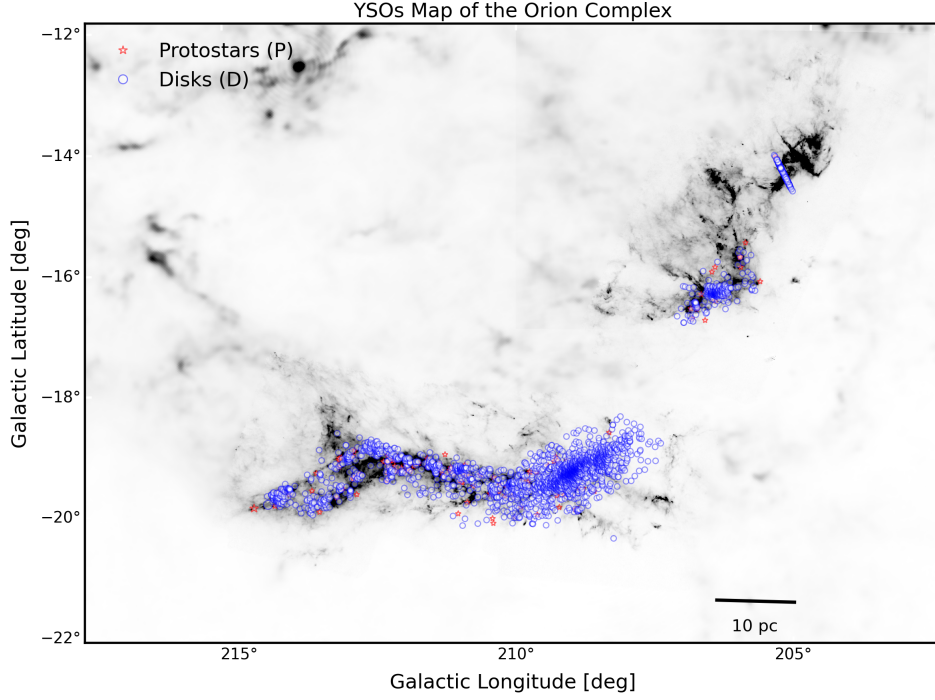


Figure 3.6.: Map of YSOs in the Orion A and B clouds, overlaid on the column density map. YSOs are color-coded by evolutionary class, with protostars (P, red stars) and disk sources (D, blue circles) identified based on their mid-infrared spectral energy distributions.

The YSO dataset provides key information for each object, including spatial coordinates, infrared luminosity, and evolutionary class. This makes it a powerful tracer of the recent and ongoing star formation within the Orion clouds.

A spatial overview of the YSO population is shown in Figure 3.6, where the distribution of protostars and disk sources is overlaid on the column density map. This visualization allows for a direct comparison between the gas structure and the stellar content, enabling a statistical study of the relationship between dense gas and star-forming activity.

4. Methods

4.1. Introduction

The ISM is a complex and dynamic environment, shaped by a broad spectrum of physical processes involving turbulence, gravity, magnetic fields, and stellar feedback. Observationally, the ISM presents a rich variety of morphologies, where the interplay between turbulence and self-gravity leads to intricate patterns. Traditional statistical tools can fall short in fully capturing the spatial complexity of these structures. To address this, the use of topological and morphological descriptors has become increasingly relevant.

In this chapter, we introduce the framework of topological methods used to characterize the structure of the ISM, focusing on the Minkowski functionals and the concept of the fractal dimension. These tools provide a quantitative way to analyze the geometry and topology of cloud structures, offering insight into properties such as size, shape, fragmentation, and connectivity.

In particular, the fractal dimension—derived here through a perimeter-area relation—serves as a proxy for the complexity of a structure’s boundary. It allows us to assess how self-similar or fragmented a region is, providing a direct link between morphology and the physical processes that govern cloud evolution and star formation.

4.2. Minkowski Functionals

The Minkowski functionals are a set of topological measures that can be used to quantify the shape and structures of objects in a space [MBW93]. From these functionals, we can derive various properties of the structures, such as their size, shape, and connectivity. These can be defined for any dimension, but we will focus on the 2D case, which is relevant for the analysis of the column density maps.

First it is convenient to define the concept of threshold, which is important both in understanding the methods but also in the application. We define the *excursion set* S_ν of a given field on a d -dimensional domain $D \subset R^d$ as the set of points where f exceeds a given threshold parameter ν :

$$S_\nu = \{\mathbf{x} \in D : f(\mathbf{x}) > \nu\} \quad (4.1)$$

The boundary of the excursion set is defined as the *level set*, which is the set of points where the field equals the threshold:

$$\partial S_\nu = \{\mathbf{x} \in D : f(\mathbf{x}) = \nu\} \quad (4.2)$$

4. Methods

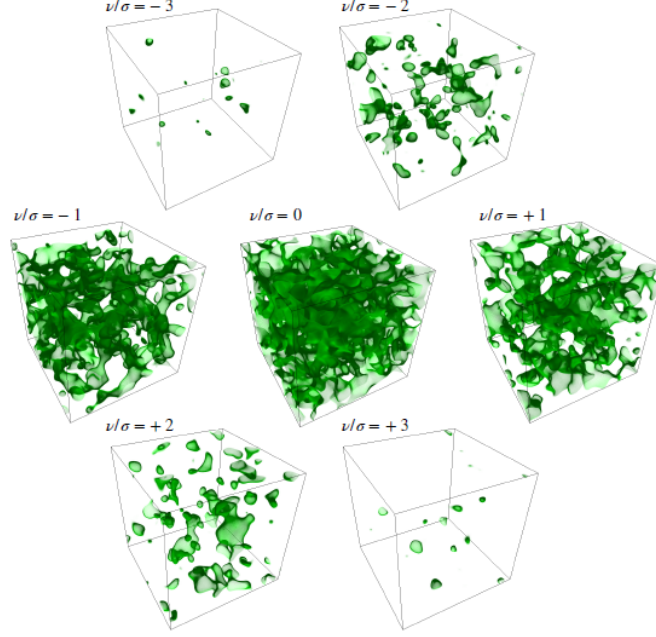


Figure 4.1.: Example of an excursion set and its boundary for a field for a smoothed three-dimensional Gaussian field.

An example of these can be seen in Figure 4.1, adapted from [Hah24].

The motion-invariants are defined as the integrals of the Minkowski functionals over the excursion set S_ν , are $d+1$ and are called the Minkowski functionals. For a 2-dimensional map, these are:

- $M_0(S_\nu)$: the area of the excursion set, which is the integral of the field over the set S_ν .
- $M_1(S_\nu)$: the perimeter of the excursion set.
- $M_2(S_\nu)$: the Euler characteristic of the excursion set, which is a measure of the connectivity of the set.

From these, we can define a lot of properties that can be calculated to characterize the structures in Orion A and B.

The following section is divided into two parts: global and local properties. **Global properties** refer to statistical measures that characterize the cloud as a whole, capturing morphological behaviors across all column density scales. In contrast, **local properties** probe how structural complexity evolves as a function of column density, offering details into spatial variations and scale-dependent behavior within the cloud.

4.3. Global Properties

4.3.1. Global Fractal Dimension

The fractal dimension is a measure of the border complexity of a given structure. This can be calculated using the Minkowski functionals, specifically the perimeter and area [Can84]. The main idea for this approach is that, for a given border length, a simple border will always enclose a larger area than a complex border.

On this idea, the perimeter-area (P-A) relation is defined as:

$$P \propto A^{\frac{D}{2}} \quad (4.3)$$

By measuring the perimeter P and area A of a structure at different thresholds and fitting the logarithm of the perimeter to the logarithm of the area, we can derive the fractal dimension D of the overall structure. Hence this is characterized in this work as a **global property** of the cloud, and it's called global fractal dimension.

The scaling relationship between perimeter and area offers insight into the structural complexity of a shape's boundary. Intuitively, a smoother and more compact boundary can enclose more area with less perimeter compared to a more irregular or fragmented contour. This variation in complexity is quantified through the global fractal dimension, D , which describes how the perimeter scales with area.

For simple geometries—like circles or squares—the perimeter P scales with the square root of the area A , following $P \propto A^{1/2}$, yielding $D = 1$, as expected for a one-dimensional boundary. In contrast, for highly intricate and space-filling contours, the perimeter scales more directly with the area, i.e., $P \propto A$, corresponding to a fractal dimension approaching $D = 2$, the value expected for a boundary that becomes increasingly space-filling [Lov82].

When a structure includes components with distinct spatial scales—for example, large coherent features overlaid with fine-grained substructure—these different regimes may exhibit different perimeter–area scaling behaviors. In such cases, the presence of multiple scaling slopes (or values of D) in different regimes can reveal the existence of characteristic length scales, hinting at underlying physical processes operating differently at those scales.

On the other hand, if the data show a consistent scaling behavior across the full range of column densities—with the perimeter and area following a single power-law in log–log space—this suggests scale-free structure and supports the interpretation that the cloud morphology is fractal in nature. An example of this is illustrated in Figure 4.2 where data from meteorological clouds were used to derive the fractal dimension of the structures. The slope of the linear fit yields the fractal dimension D , providing a quantitative measure of the structural complexity. The lack of a characteristic scale, evidenced by the good fit, suggests that turbulent processes are likely responsible for shaping the observed structures.

The perimeter-area relation is a powerful tool for analyzing the complexity of structures in the ISM. It's simple, yet effective, and can be applied to a wide range of structures, from clouds to galaxies. The fractal dimension derived from the perimeter-area relation can provide insights into the physical processes that shape these structures, such as turbulence, gravity, and magnetic fields. However, it is important to note that the perimeter-area relation is not always applicable to all structures. Furthermore, numerical effects can also

4. Methods

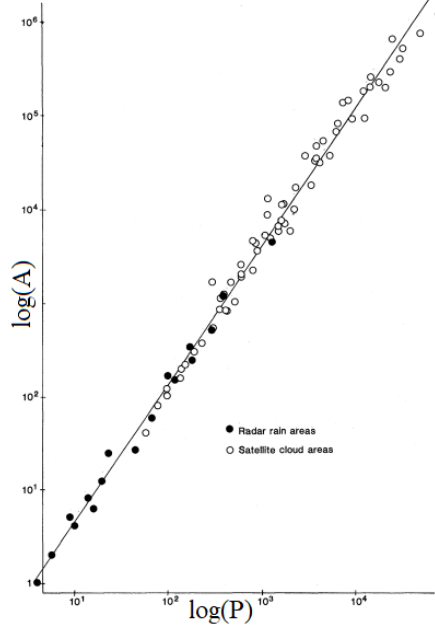


Figure 4.2.: Example of the perimeter-area relation for meteorological clouds [Lov82]. The slope of the linear fit gives the fractal dimension of the structure.

play a role in the calculation of the fractal dimension, as we will see in the simulations section. Artifacts also have an effect on this method [Imr06]. This is to say, that the method has its limitations and simulations play an important role in understanding these effects.

4.4. Local Properties

4.4.1. Local Fractal Dimension

One further advantage of the perimeter-area relation is that it can be inverted to derive the fractal dimension of a structure at a given column density threshold. By inverting Eq. 4.3, we can express the local fractal dimension D as a function of the column density threshold ν :

$$D(\nu) = 2 \times \frac{\log(P(\nu))}{\log(A(\nu))}. \quad (4.4)$$

This approach enables a more detailed characterization of the geometry and scaling behavior of the cloud structures as a function of physical conditions. In particular, by applying a series of increasing thresholds ν , one effectively isolates progressively denser substructures within the molecular cloud. At each threshold, the perimeter $P(\nu)$ and area

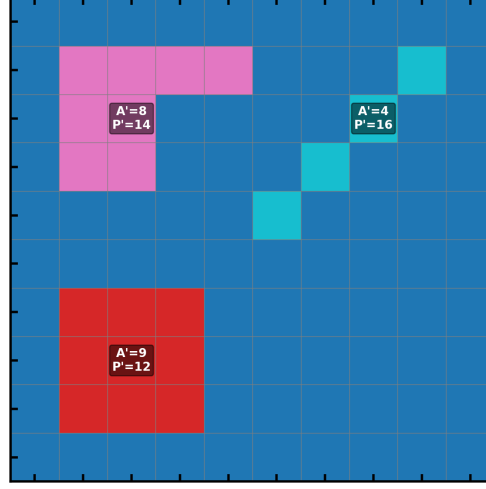


Figure 4.3.: Simplified example of the calculation of perimeters and areas of structures at a given column density threshold. The local fractal dimension can be calculated by summing over all of the structures at that threshold, or for each of the single structures.

$A(\nu)$ can be measured either for all connected structures collectively or for each identified region individually.

In the first case, the perimeters and areas of all structures present in the map at a given threshold are summed, providing a global measurement of the local fractal dimension for that threshold. This yields a single $D(\nu)$ value for each column density level, capturing how the overall morphological complexity evolves as successively denser material is selected. An example of this approach is shown in Figure 4.3, where the perimeters and areas are computed across a range of thresholds, demonstrating the dependence of the fractal dimension on the chosen column density cut.

Conversely, it is also possible to calculate the local fractal dimension separately for each individual structure as a function of threshold. In this case, the evolution of $D(\nu)$ with increasing density can be tracked on a per-object basis, providing insights into whether substructures become more or less fragmented and irregular as higher-density material is isolated. This allows the study of whether certain regions of the cloud exhibit self-similar scaling over a wide range of column densities, or whether there are transitions in the structural properties that may reflect different physical regimes, such as turbulence, gravitational collapse, or feedback from star formation. Because this method analyzes structures as a function of column density, it is considered a **local property**. Moreover, as this approach is relatively novel, its interpretation must be carefully developed based on the results obtained in this work.

Analyzing the local fractal dimension as a function of threshold thus provides a tool to investigate the scale dependence and complexity of molecular clouds, complementing the global measurements and helping to disentangle the contributions from different physical

4. Methods

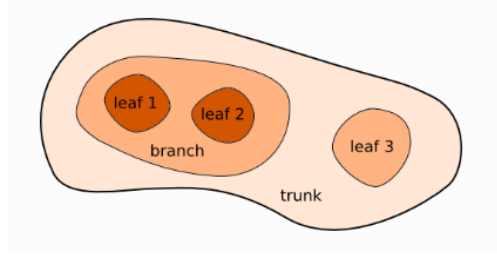


Figure 4.4.: Simplified example of a resulting tree/branch/leave structure from the Astrodendro package.

processes.

4.4.2. Dendrograms

In order to explore the segmentation of structures with changing column density or mass, dendrograms are a useful tool. A dendrogram is a diagram that represents the hierarchical organization of structures within the data [EST10]. Specifically, it shows how large-scale regions can be decomposed into smaller substructures as the threshold for the column density (or mass) increases.

By progressively applying higher thresholds, one can trace how individual dense clumps emerge from the diffuse background and how these clumps further fragment into even denser cores. In this way, dendrograms provide a visual and quantitative framework to analyze the nested, multiscale nature of molecular cloud structure. An example of a resulting tree for an example shape can be seen in Figure 4.4 (from the Astrodendro Package[†]).

In this work, dendrograms were constructed using hierarchical clustering methods applied to the mass maps of Orion A and B. This approach allows identification and characterization of the main branches (large structures) as well as the terminal leaves (smallest resolved substructures). For each identified region, properties such as total mass, size (computed via principal component analysis), and fractal dimension were derived to quantify their physical characteristics and complexity.

4.4.3. Mass-Size-Fractal Dimension (MSD) plane

Since it is possible to calculate the local fractal dimension for single structure, we can connect this property to more physical aspects of the substructures, namely mass and size. This then allows us to include new information on already established mass-size diagrams [HKS⁺25], creating a Mass-Size-Fractal Dimension (MSD) plane. The mass and size of each structure are computed using its binary mask: the mass is obtained by summing the corresponding pixels in the mass map, while the size is estimated by performing a principal component analysis (PCA) on the mask to determine its spatial extent.

[†]Astrodendro Python Package

4.5. Connection to Star Formation

The dendrogram-like segmentation is an important aspect of this process, as it ensures a proper characterization and recognition of the sub-structures.

4.4.4. Euler characteristic

Euler characteristic, often denoted as χ (or $M_2(S_\nu)$ in the context of the Minkowski functionals), and in two dimensions it is given by:

$$\chi = \text{Number of connected regions} - \text{Number of holes} \quad (4.5)$$

For a given excursion set S_ν , the Euler characteristic provides a measure of the topology of the structures, indicating how many isolated regions and holes are present at a specific threshold. In practice, a positive Euler characteristic indicates more connected regions than holes, while a negative value suggests a topology dominated by holes.

The Euler characteristic is closely related to the genus G of the structure, where in two dimensions $G = 1 - \chi$. The genus is commonly used in cosmology and astrophysics to describe the connectivity of structures, such as the filamentary network in the ISM or the cosmic web. By analyzing the Euler characteristic or genus as a function of the threshold, one can gain insights into the morphological transitions and connectivity of the observed structures.

Since this is also a property that can be expressed as a function of the column density thresholds ν , it also counts towards the **local properties** of the cloud.

4.5. Connection to Star Formation

To investigate the link between cloud structure and star formation activity, we employed the catalogue of young stellar objects (YSOs) compiled by [MGM⁺12]. This catalogue provides both positional information and evolutionary classifications of YSOs within Orion A and Orion B, allowing a direct spatial comparison with the structures identified in our analysis.

The degree of association between the local structural properties and the YSO distribution was quantified using the Pearson correlation coefficient [Pea95] and the Spearman rank correlation coefficient [Sed14]. These metrics capture linear and monotonic relationships, respectively, and together provide a robust measure of how variations in the structural parameters relate to the spatial density of YSOs.

We restricted the YSO sample to objects classified as Class I or flat-spectrum sources, based on their infrared slope (**alpha**) and the catalogue classification flag (**C1**). These stages are closely associated with active star-forming regions, while more evolved Class II/III sources may have migrated from their birth sites and were therefore excluded [Lad87]. For comparison, we also computed correlations using the full YSO sample.

4. Methods

4.6. Simulations

4.6.1. Simulations on the Global Properties

The simulations addressing the global properties are primarily designed to verify the interpretation of the values obtained for the global fractal dimension D , as described in the preceding sections. In particular, they aim to confirm the expectation that the perimeter–area relation should yield unreliable or systematically biased results for structures that possess a well-defined characteristic length scale.

In this context, Gaussian Random Fields (GRFs) play an important role as a benchmark for comparison. GRFs allow the generation of synthetic structures with controlled statistical properties and scaling behavior, providing a framework to test whether the methodology is sensitive to the presence or absence of intrinsic scales.

4.6.2. Simulations on the Local Properties

Analogously, simulations are employed to better understand how different values of the local fractal dimension arise in different types of structures. These tests include the analysis of Gaussian Random Fields, purely Gaussian noise, and simple geometric shapes such as lines and circles.

The simulations also allow for quantifying numerical effects and resolution-dependent biases, assessing how limited spatial resolution and discretization influence the measured properties of the identified structures. This helps ensure that the interpretation of the fractal dimension is robust and not merely an artifact of the analysis method or data sampling.

4.6.3. Comparison with Other Methods

In the context of the simulations, a brief comparison with alternative approaches was carried out. This served both to improve understanding of the fractal dimension framework and to identify potential limitations or artifacts of the perimeter–area (PA) method.

The method most frequently used for comparison was the box-counting technique, which relies on covering the shape with a grid of boxes of varying sizes and counting the number of boxes that contain part of the shape. By examining how this count scales with the box size, the fractal dimension can be estimated [Fal13].

4.7. Uncertainties

4.7.1. Perimeter and Area

The measurement of perimeter and area is subject to inherent uncertainties. To estimate these uncertainties, the following procedure was carried out:

- The perimeter (P) and area (A) are measured for a set of circles with known true perimeter and area.

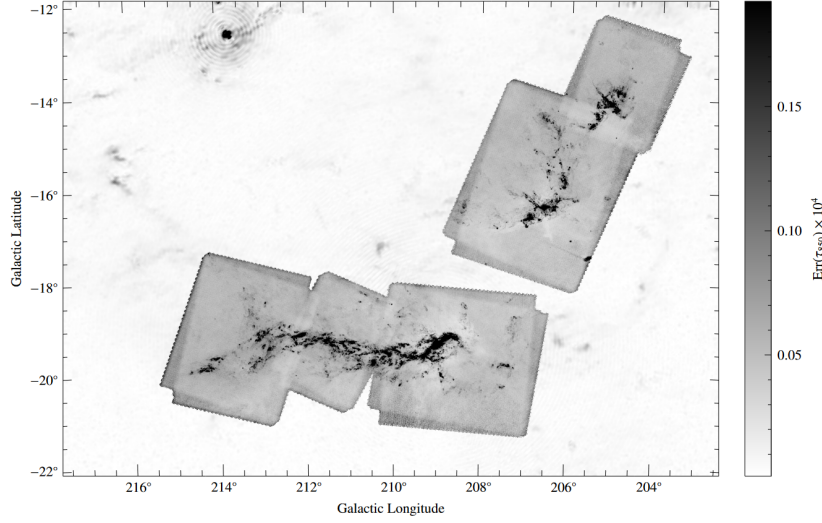


Figure 4.5.: Pixel-wise error map of the τ_{850} data [LBAL14]

- The magnitude of the measurement error are computed by comparing the measured values to the true values.
- The errors are averaged over a sufficiently large number of samples (N) to obtain representative estimates of the typical deviation.

4.7.2. Beam Size

The finite resolution imposed by the telescope beam introduces systematic uncertainties in the measurement of structural properties. The beam convolution smooths small-scale fluctuations and reduces the complexity of the contour at each column density threshold. This effect generally leads to an underestimation of the perimeter and an overestimation of the area, biasing the derived fractal dimension toward lower values.

To quantify this uncertainty, one can simulate synthetic column density maps with known fractal characteristics and convolve them with a Gaussian kernel matching the observational beam. By comparing the perimeter–area relations before and after convolution, the bias introduced by the finite resolution can be estimated. Alternatively, the characteristic beam size provides a natural lower limit to the spatial scales where fractal analysis is reliable. In this work, scales smaller than 2–3 times the beam full-width at half-maximum (FWHM) are treated with caution, and the derived fractal dimensions are interpreted as lower limits to the intrinsic complexity of the cloud morphology.

4.7.3. Uncertainties in the Extinction Map

The dataset also includes a pixel-wise τ_{850} error map, as shown in Figure 4.5.

4. Methods

Although these uncertainties are of a non-negligible magnitude to other sources of error and, in principle, all effects should be taken into account, the dominant contributions to the total uncertainty arise from the measurements of perimeter and area. For this reason, the pixel-wise extinction errors are neglected in the following analysis.

5. Results

5.1. Global Properties

5.1.1. Global Fractal Dimension

To quantify the cloud morphology through the global fractal dimension, we computed perimeter-area measurements over a restricted range of column densities, $N_{\min} \leq N \leq N_{\max}$, with $N_{\min}$ around $2 \times 10^{21} \text{ cm}^{-2}$ and $N_{\max}$ around $8 \times 10^{22} \text{ cm}^{-2}$. This range was chosen to avoid biases caused by the limited angular resolution, which produces artificially smooth contours at low column densities, and by the sparse, irregular features that dominate at the highest column densities. At the same time, this range includes important physical thresholds, such as typical star forming scales and HI to H_2 transition regime.

Figure 5.1 shows the perimeter-area relation for Orion A, together with the best-fit linear regression in log-log space used to derive the global fractal dimension. The residuals of the fit are displayed in the lower panel. From this single-fit model we obtain:

$$D_{\text{OA, Global}} = 1.35 \pm 0.02,$$

with a mean absolute residual of 0.1174 and a correlation coefficient of 0.9893.

However, the Orion A data show systematic deviations from a single power law across the full column-density range. To account for this, we performed a segmented i.e. double linear fit, dividing the data at $N = 1.23 \times 10^{22} \text{ cm}^{-2}$. The two resulting global fractal dimensions from the slopes are:

$$D_{\text{OA},1} = 1.65 \pm 0.01 \quad (\text{high column density}),$$

$$D_{\text{OA},2} = 0.97 \pm 0.3 \quad (\text{low column density}).$$

The corresponding mean absolute residuals are 0.0222 (correlation coefficient = 0.9986) for the high-density regime and 0.0679 (correlation coefficient = 0.9748) for the low-density regime, representing a clear improvement compared to the single-fit model. Figure 5.2 shows the two fitted segments and their residuals. The distinct change in slope suggests a structural transition or the coexistence of different regimes of spatial complexity in Orion A.

In contrast, Orion B exhibits a well-behaved perimeter-area relation that is accurately described by a single linear fit across the entire column-density range:

$$D_{\text{OB, Global}} = 1.40 \pm 0.01.$$

5. Results

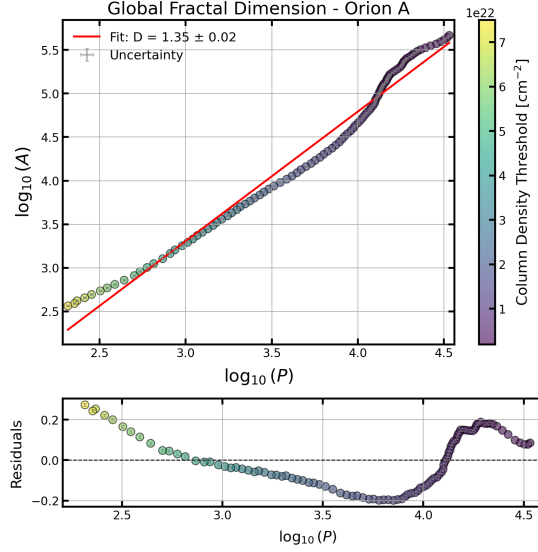


Figure 5.1.: Perimeter-area relation for Orion A. The solid line indicates the best-fit linear regression in log-log space used to estimate the global fractal dimension D_{Global} . Residuals are shown in the lower panel.

The corresponding residuals (mean absolute residual = 0.0493) and correlation coefficient (0.9976) confirm the robustness of this fit. Figure 5.3 shows the fitted relation and residuals. The consistency of the slope across scales indicates a more uniform degree of structural complexity compared to Orion A.

The uncertainties in perimeter and area measurements were estimated at 1.6%, based on simulations demonstrating that this level of error is representative across different reference geometries.

5.2. Local Properties

5.2.1. Euler Characteristic

We evaluated the Euler characteristic, χ , for both Orion A and Orion B as a function of the column-density threshold. This topological descriptor quantifies the connectivity and fragmentation of structures within the clouds: negative values of χ indicate a predominance of holes, while more positive values reflect a higher degree of connectivity.

Figures 5.4 and 5.5 present the results for Orion A and Orion B, respectively. In both cases of Orion A and B, χ was computed over the same range of column densities $N_{\text{min}} \leq N \leq N_{\text{max}}$ used in the global perimeter-area analysis above, for consistency and comparability.

Orion A exhibits a profile that closely resembles the Gaussian-like behavior expected from Gaussian Random Field simulations (Figure A.9), with a pronounced peak at approximately $1.64 \times 10^{22} \text{ cm}^{-2}$. In contrast, Orion B shows a more linear trend. In both

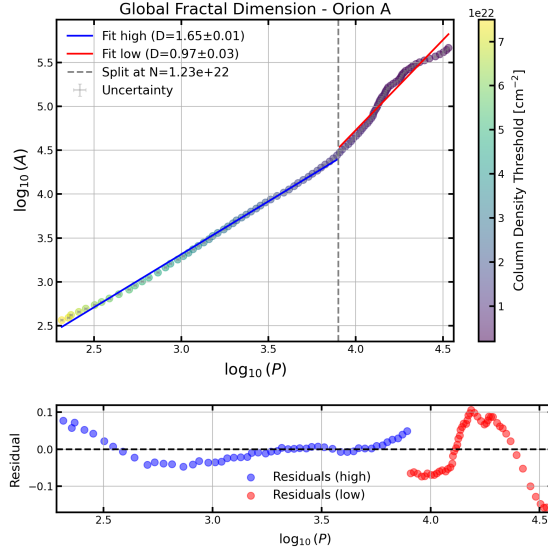


Figure 5.2.: Perimeter-area relation for Orion A with a segmented linear fit. Dashed lines indicate the two fitted regimes, separated at $N = 1.23 \times 10^{22} \text{ cm}^{-2}$. Residuals are shown in the lower panel.

regions, strong numerical fluctuations appear at lower column-density thresholds, leading to noticeable variations in the calculated Euler characteristic.

5.2.2. Local Fractal Dimension

Figure 5.6 shows the variation of the local fractal dimension $D(\nu)$ as a function of column-density threshold for Orion A and Orion B. The column density range $N_{\min} \leq N \leq N_{\max}$ is the same as in the previous sections. In both regions we find a clear overall trend: the local fractal dimension changes systematically as the threshold increases. Superimposed on this trend, several pronounced peaks and deviations are visible. These features likely trace transitions in the morphology of the structures, for instance the emergence of compact cores or the fragmentation of filaments into smaller substructures.

A more detailed interpretation of these peaks and their implications for cloud evolution and turbulence will be addressed in the following section.

The average values and their standard deviations of the local fractal dimension for the two regions are:

$$\bar{D}_{\text{OA, Local}} = 1.73 \pm 0.09,$$

$$\bar{D}_{\text{OB, Local}} = 1.75 \pm 0.08.$$

The uncertainties in perimeter and area measurements were estimated again at approximately 1.6%, based on simulation results.

5. Results

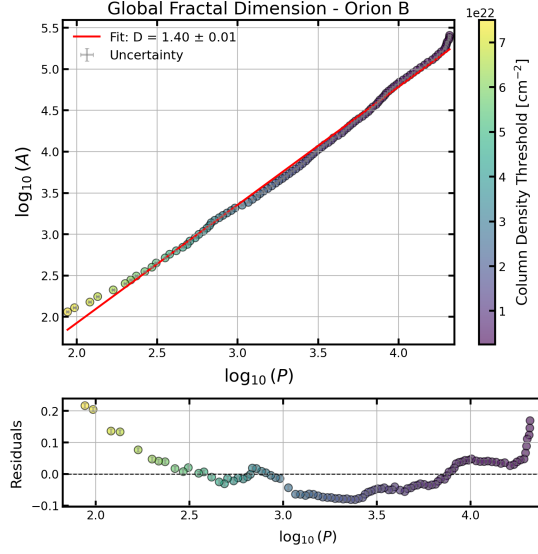


Figure 5.3.: Perimeter-area relation for Orion B with the best-fit linear regression overlaid. Residuals are shown in the lower panel.

5.2.3. Analysis of Individual Structures

To gain deeper insight into the fractal properties of each cloud, we decomposed the emission at every column-density threshold into its set of connected structures. This dendrogram-based approach provides a hierarchical view of how the material is organized into nested substructures, enabling a more detailed examination of their geometrical and topological properties.

Figures 5.7 and 5.8 display the resulting dendrograms for Orion A and Orion B. These diagrams trace how individual regions merge into larger complexes as the threshold decreases, effectively capturing the clustering behaviour of the cloud across spatial scales.

We then applied the same calculation of the local fractal dimension to each of these connected structures. For numerical stability, we first limited the analysis to the largest regions at each threshold, selecting them according to minimum perimeter and area criteria. The resulting measurements are shown in Figures 5.9 and 5.10. Overall, these individual-structure analyses reproduce similar trends to those found in the undifferentiated calculation (Figure 5.6), although with shallower progression at higher column density thresholds. This behaviour has been recorded in other instances, e.g. when the regions got simply divided into N sharp cuts: the calculated local fractal dimensions show similar trends, but with a shallower progression. The average of these then tends to approach the whole region one.

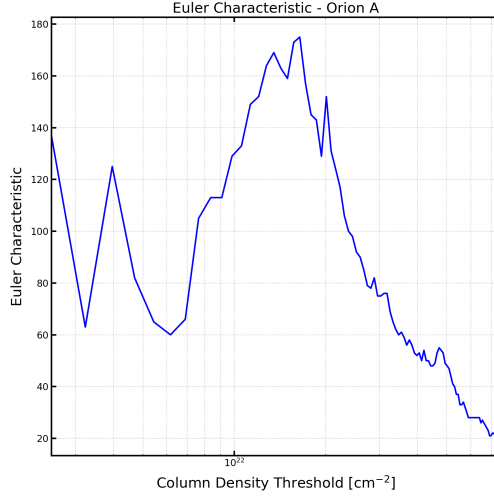


Figure 5.4.: Euler characteristic of Orion A as a function of column-density threshold. Positive values correspond to a predominance of isolated structures, while negative values indicate higher connectivity.

5.2.4. MSD Plane

To investigate how the fractal properties relate to the physical characteristics of the cloud, we computed the mass, size, and local fractal dimension for each connected structure identified at every column-density threshold in the dendrogram hierarchy. These quantities were then plotted against each other, with the local fractal dimension encoded as color.

The resulting Mass-Size-Fractal Dimension (MSD) planes are shown in Figure 5.11 for Orion A, Figure 5.12 for Orion B, and Figure 5.13 for the combined dataset. For reference, the expected scaling relations for idealized filamentary structures ($A = 10$) and spherical structures ($A = 3$) are overplotted, as done in [HKS⁺25].

Using this information, we generated spatial maps of the local fractal dimension by assigning to each pixel the value associated with the structure it belongs to. If multiple structures overlap, we averaged their values. Because of this averaging, the highest values appear slightly reduced compared to the peak values in the previous plots. Nevertheless, the maps in Figures 5.14 and 5.15 reinforce the trends already identified in the MSD analysis.

An interactive 3D version of the MSD plane is available online at: [MSD 3D Visualizer](#).

Finally, the hierarchical relationships extracted from the dendrogram allow us to track how individual structures break down into substructures. This information is visualized in Figures A.1 and A.2 in the Appendix, where lines connect each structure to its parent and child structures in the MSD plane.

5. Results

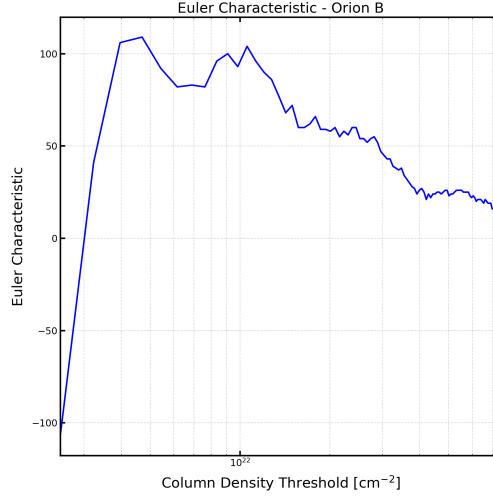


Figure 5.5.: Euler characteristic of Orion B as a function of column-density threshold.

5.3. Connection to Star Formation

Regions of higher gas density are more conducive to star formation. To explore this connection, we compared the spatial distribution of the local fractal dimension with the surface density of young stellar objects (YSOs). The YSO density maps were obtained by smoothing the distribution of identified YSOs across each region; details of the map construction are provided in the Appendix (Figures [A.5](#), [A.6](#) for the full sample, Figures [A.3](#) and [A.4](#)). The spatial coverage is naturally limited by the footprint of the available YSO catalog.

We first restricted the analysis to the subset of YSOs classified as early-stage (Class I and flat-spectrum), which are most likely to be physically associated with ongoing star formation. In Orion A this subset contains 426 objects, while in Orion B it contains 111. Using this subset, the pixel-by-pixel comparison between the YSO density maps and the local fractal dimension yields:

Orion A: Pearson 0.434, Spearman 0.429,

Orion B: Pearson 0.288, Spearman 0.244.

For comparison, we also performed the same analysis using the full YSO sample, without filtering by evolutionary stage. The full catalog contains 2818 YSOs in Orion A and 632 in Orion B. The corresponding correlation coefficients are:

Orion A: Pearson 0.286, Spearman 0.393,

Orion B: Pearson 0.399, Spearman 0.340.

Although the correlations are moderate in both cases, the higher values obtained for the young subset in Orion A indicate a stronger spatial link between regions of high

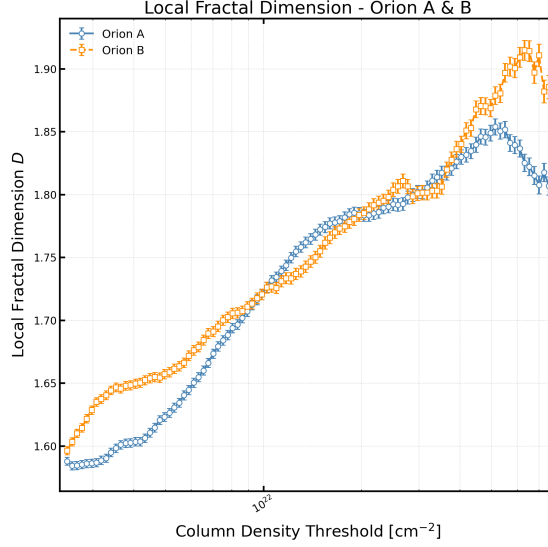


Figure 5.6.: Local fractal dimension as a function of column-density threshold for Orion A and Orion B. Error bars reflect the uncertainties of the fit at each threshold.

local fractal dimension and active star formation. This supports the interpretation that complex, self-similar structures within the molecular cloud preferentially host the earliest stages of star formation.

5.4. Simulations and Uncertainties

5.4.1. Simulations on Global Properties

On the Global Fractal Dimension

To validate the interpretation of the global fractal dimension framework, we performed a series of controlled tests. These tests involved applying the fitting routine to mock structures: some were constructed to exhibit a characteristic scale, while others were explicitly scale-free and therefore self-similar. As outlined earlier, the method is expected to return a robust and consistent fit only in the absence of a characteristic scale, which would indicate self-similarity.

We first carried out tests on simple geometric figures, in order to confirm the intuition that shapes with simple borders have in this framework a global fractal dimension of 1, and this increase more and more the more complex the boundaries of the shape become. These tests confirmed these expectations.

We then carried out tests on Gaussian Random Fields (GRFs), exploring both GRFs with power-law spectra (scale-free) and GRFs with peaked spectra (dominated by a characteristic scale). The simulations consistently confirmed our expectations, showing that the fitting procedure yields more stable and reliable results in the scale-free case,

5. Results

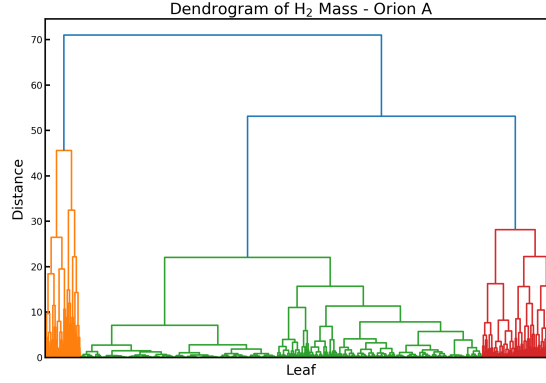


Figure 5.7.: Dendrogram of the Orion A mass distribution, showing the hierarchical merging of structures as the column-density threshold is lowered. The various main branches are coloured differently for highlighting purposes. The y-axis "Distance" represents a degree of similarity between structures.

while deviations appear when a characteristic scale is introduced.

We also tested the impact of resolution effects by smoothing the original data with Gaussian kernels of increasing size. Even under extreme smoothing, the resulting global fractal dimension varied by no more than -20 % relative to the unsmoothed case, indicating that the method is robust within typical observational resolutions. The -20 % is also relative to an extreme smoothing case (see Figure [A.7](#) in the Appendix).

5.4.2. Simulations on Local Properties

On the Euler Characteristic

To better understand the expected behavior of the Euler characteristic when applied to real data, we performed dedicated simulations. As in the previous tests, the main inputs were Gaussian Random Fields (GRFs), chosen for their well-defined statistical properties and the expectation of a symmetric response.

The simulations confirmed this expectation: the Euler characteristic exhibits a symmetric profile as a function of the threshold, with a clear maximum in connectivity at the central peak and a gradual decrease on either side. This behavior closely resembles a Gaussian bell curve, providing a useful reference pattern for interpreting the results from the actual cloud data.

On the Local Fractal Dimension

This section validates the inversion of the perimeter–area (PA) relation and explains how the resulting local fractal dimension should be interpreted in terms of boundary complexity.

We first tested the method on simple, well-understood shapes—straight lines, circles, and Gaussian profiles. In these cases it was sometimes necessary to account for the intercept

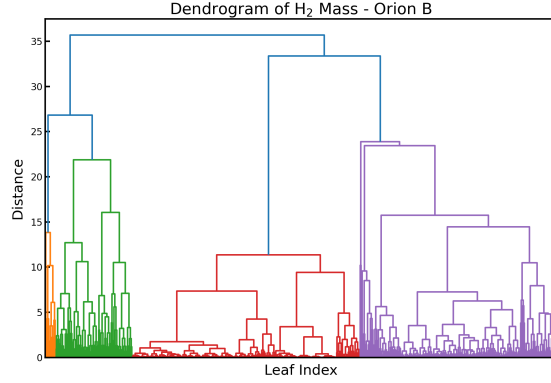


Figure 5.8.: Dendrogram of the Orion B mass distribution, showing the hierarchical merging of structures as the column-density threshold is lowered.

in the original perimeter–area relation (Equation 4.3). The results were consistent with expectations: lines yielded a dimension of $D = 2$, simple boxes and circles gave $D = 1$, and more intricate shapes produced values in between.

We then extended the tests to more complex structures, in particular Gaussian Random Fields (GRFs), to explore how the method behaves in realistic, irregular patterns. These experiments revealed a dichotomy that can be described as “sausage vs. hairy caterpillar”:

- **Smooth, compact structures ("sausages")** tend toward $D(\nu) \approx 2$ in the local PA framework. Their borders are long and coherent with minimal roughness, resulting in a perimeter that scales almost linearly with area.
- **Irregular, space-filling structures ("hairy caterpillars")** tend toward $D(\nu) \approx 1$. Their boundaries are highly convoluted and expand more slowly with area, leading to a sublinear perimeter–area relationship.

A gallery of visual examples illustrating this behavior is provided in the Appendix, and the trends are evident in the observational results (see, for example, Figure 5.13). In practice, small and relatively round structures exhibit local fractal dimensions close to 2, whereas larger, more complex structures trend toward 1.

Finally, we examined the effect of resolution on the local fractal dimension using a procedure similar to that employed for the global dimension. Smoothing the data with a Gaussian kernel generally reduced the local fractal dimension at most scales. At the same time, because the smoothing lowers the highest column-density peaks, the usable column-density range for the analysis becomes narrower. This leads to the earlier appearance of artifacts in the PA relation, as documented in the Appendix in Figure A.8.

5.4.3. Error Estimates

The uncertainties on the perimeter and area measurements, derived from the simulation procedures described above, are on average approximately 1.6% for each quantity. These

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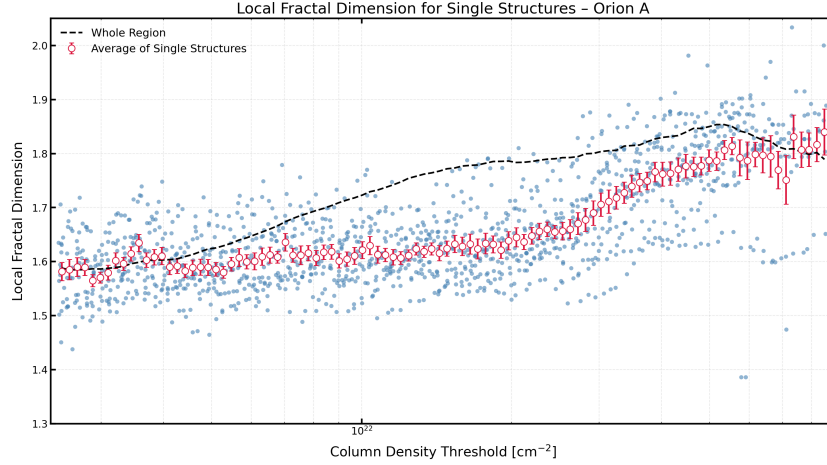


Figure 5.9.: Local fractal dimension for individual structures in Orion A as a function of column–density threshold. Shown are the mean values obtained from the largest structures at each threshold (with uncertainties), overlaid with the corresponding values from the undifferentiated calculation.

uncertainties arise primarily from pixelation effects and the finite resolution of the column–density maps.

An example of the resulting error distributions from a representative simulation is shown in Figure A.10 in the Appendix. The narrow spread around the mean demonstrates that the uncertainty estimates are robust, and that systematic errors remain small compared to the overall dynamic range of the measurements.

5.4.4. Comparison with Alternative Methods

A clear anti-correlation is found when comparing our results with those obtained using the box-counting method, both for the global and the local estimates of the fractal dimension. This systematic offset highlights an important scaling dependence: different techniques for quantifying fractal characteristics do not yield directly comparable numerical values. Such differences should therefore be carefully considered when interpreting results across studies that adopt different methodological frameworks.

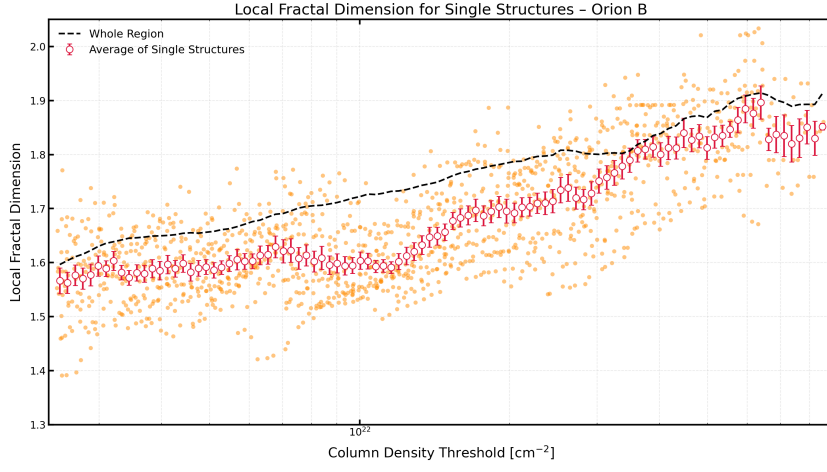


Figure 5.10.: Local fractal dimension for individual structures in Orion B as a function of column-density threshold. As in Orion A, the mean values from the selected structures (with uncertainties) are overlaid with the results from the undifferentiated calculation.

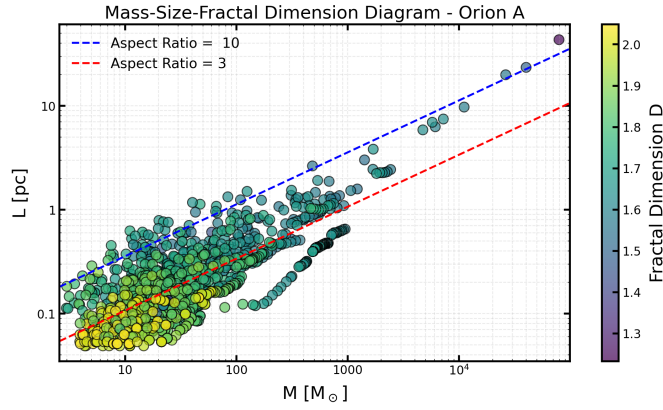


Figure 5.11.: MSD plane for Orion A. The color scale indicates the local fractal dimension, and reference scalings for filamentary and spherical geometries are shown as solid lines.

5. Results

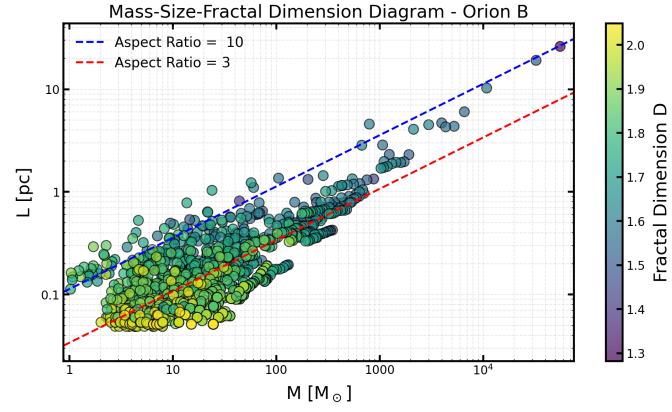


Figure 5.12.: MSD plane for Orion B, with color coding and reference scalings.

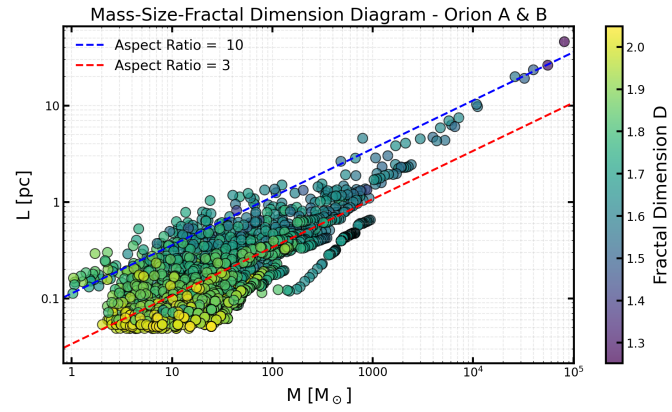


Figure 5.13.: MSD plane for Orion A and Orion B combined. The color bar covers the full range of local fractal dimensions across both clouds.

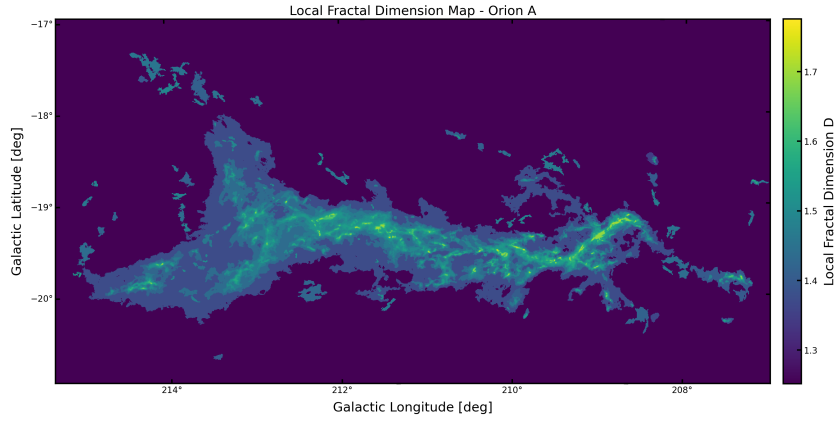


Figure 5.14.: Spatial map of the local fractal dimension for Orion A. Pixels without assigned structures (NaN) are set to the minimum value of the local fractal dimension.

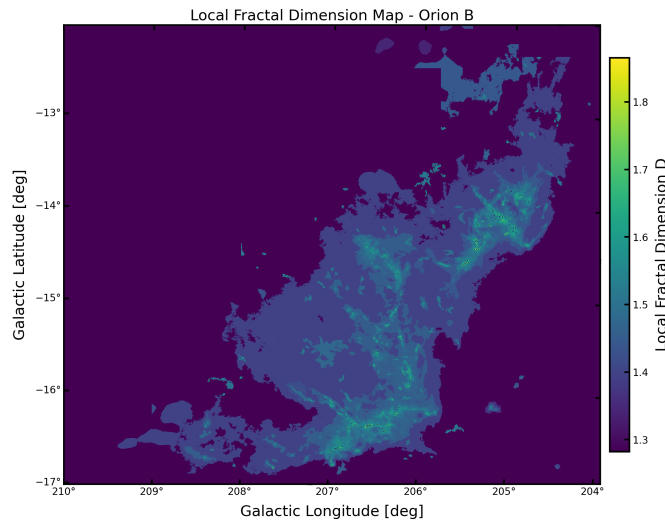


Figure 5.15.: Spatial map of the local fractal dimension for Orion B. Pixels without assigned structures (NaN) are set to the minimum value of the local fractal dimension.

6. Discussion

Our analysis of the Orion A and Orion B molecular clouds reveals a complex interplay between morphology, topology, and star formation activity. By combining perimeter–area scaling, Euler characteristic profiles, and mass–size relations, we obtain a consistent picture of how structural complexity varies across column densities and how these variations relate to the presence of young stellar objects.

The global fractal dimension is based on the idea that if a structure is self-similar—i.e., lacks a characteristic scale—its perimeter and area should follow a power-law scaling, appearing as a straight line in a log–log diagram. Turbulence is known to play a key role in organizing structure within molecular clouds, and the perimeter–area method provides a way to quantify how much this self-similar behavior influences the observed morphology.

Our global perimeter–area analysis reveals distinct structural regimes within Orion A, with a clear change in slope that reflects different levels of complexity at low and high column densities. In contrast, Orion B maintains a more uniform fractal signature. This contrast already begins to suggest that Orion A and Orion B are governed by different physical conditions.

For Orion A, a double fit is required, indicating a change in the underlying physics. The double fit can be seen in Figure 5.2. This transition occurs at a column density of $N = 1.23 \times 10^{22} \text{ cm}^{-2}$, marking a physically significant scale. At this threshold, the structural properties of Orion A shift, coinciding with the onset of active star formation and the emergence of dense cores (LLA10). The concurrent changes in fractal dimension and topology at this scale suggest a direct link between the cloud’s morphology and the physical processes driving star formation.

Without applying the double fit, the global fractal dimension is $D = 1.35 \pm 0.01$. The single fit for Orion A can be seen in Figure 5.1. This value is in good agreement with previously reported fractal dimensions for molecular clouds derived with alternative methods (EF96), where typical values are around $D \sim 1.3$. Numerical simulations based on two-dimensional compressible turbulence in a self-gravitating ISM also reproduce similar values (YZWS94). Note that many studies report the three-dimensional fractal dimension, which is related to the two-dimensional value by adding unity.

When the data are separated into two regimes, we obtain $D = 1.65 \pm 0.01$ at higher column densities and $D = 0.97 \pm 0.03$ at lower column densities. Although a fractal dimension smaller than 1 is not expected in theory, this result is likely influenced by the limited number of data points contributing to the low-density fit. Even so, the trend is informative: a fractal dimension approaching 1 indicates smoother, simpler boundaries, whereas higher column densities exhibit more complex, space-filling structures.

Taken together, these results suggest that the processes driving self-similarity—such as turbulence—become less dominant once dense cores emerge. Beyond this threshold, the

6. Discussion

cloud’s structure deviates from scale-free behavior, reflecting the impact of star formation on its morphology.

With methods such as the perimeter–area analysis, it is important to verify that the conclusions drawn are consistent with what is visually apparent in the cloud. To this end, Figure B.1 shows Orion A with three overlaid contours corresponding to low, intermediate, and high column-density thresholds. The intermediate threshold was chosen close to the transition point identified in the global fractal dimension analysis, where the fit was separated into two regimes.

A visual inspection supports the quantitative results. At low column densities, Orion A appears as large structures with relatively complex boundaries. Around $N \approx 1.2 \times 10^{22} \text{ cm}^{-2}$, the morphology begins to change markedly: structures start to fragment, and the cloud breaks up into smaller, more irregular components. At even higher column densities, the emission is dominated by compact cores and small-scale features.

This visual progression confirms the double trend inferred from the global analysis. Below the transition, the cloud exhibits more complex, interconnected structures, corresponding to the higher global fractal dimension ($D = 1.65$). Above the transition, the morphology is dominated by simpler, more isolated cores, reflected in the lower fitted fractal dimension ($D \approx 1$).

In Orion B, by contrast, no such break is required. This outcome is broadly consistent with expectations, as Orion B is known to be a more turbulent molecular cloud [OPG⁺17]. In such an environment, scale-free behavior dominates across the column-density range analyzed, as reflected in the high quality of the single global fractal dimension fit (Figure 5.3). The emergence of cores does not appear to disrupt the hierarchical structuring. Together, these findings provide the first elements of a coherent picture described by the Minkowski functionals.

As with Orion A, it is important to verify whether the conclusions drawn for Orion B are consistent with its visual appearance. In the corresponding gallery (Figure B.2), Orion B breaks down into smaller components relatively quickly, yet the overall visual complexity remains comparable across the examined thresholds. The structures appear to fragment in a gradual and continuous manner, with each region breaking into substructures at similar rates.

Because this fragmentation proceeds smoothly across the entire column-density range, there is no clear transition point that would require separating the perimeter–area relation into distinct regimes. The morphology gives the impression of a consistent, scale-free process, with no abrupt change in structural organization.

This visual assessment therefore supports the quantitative findings: a single global fractal dimension is sufficient to describe Orion B, consistent with a more turbulent environment in which fragmentation occurs in a self-similar way.

It is also instructive to compare these results with measurements obtained for other molecular clouds. Studies applying perimeter–area methods or similar techniques have typically found global fractal dimensions in the range $D \sim 1.2\text{--}1.4$ for nearby star-forming regions. For example, [FPW91] reported $D \approx 1.36$ for CO maps of Perseus and Ophiuchus, while [SAP05] found values between 1.25 and 1.35 for a sample of Galactic molecular

clouds. These values are broadly consistent with the single-fit results for Orion A ($D = 1.35 \pm 0.01$) and Orion B ($D = 1.40 \pm 0.01$), reinforcing the interpretation that the large-scale morphology of these clouds is comparable to that of other regions in the Milky Way. What is less commonly reported, however, is the clear break we observe in Orion A at $N = 1.23 \times 10^{22} \text{ cm}^{-2}$.

Naturally, this method has limitations. Numerical errors can arise when calculating perimeters and areas for very small structures at high column densities. Straight contours, resulting from limited observational resolution, should also be avoided, as they can distort the scaling relationships on which the method relies.

The local fractal dimension builds on the perimeter–area relation, but in this case the relation is inverted to obtain a proxy for the boundary complexity of individual structures. This measure offers a more nuanced view than the global approach, capturing how structural complexity evolves with column density on a local scale.

The local fractal dimension calculated at each column-density threshold shows a consistent trend toward $D(\nu) \approx 2$ at higher thresholds for both Orion A and Orion B (Figure 5.6). This behavior appears to arise not from the increasing complexity of individual structures, but rather from the growing, interconnected network of cores, fibers, and filaments that emerge as the cloud fragments at higher column densities. The average local fractal dimension for both Orion A and Orion B is approximately 1.7, which remains within the expected range for molecular clouds. This value suggests that, on average, the boundaries of structures in both regions are moderately complex—more intricate than simple geometric shapes, but not fully space-filling. Such averages are consistent with previous studies and support the interpretation that turbulence and hierarchical fragmentation are key drivers of cloud morphology at these scales.

An interesting benchmark for interpreting the local fractal dimension comes from the classical theory of turbulence developed by Kolmogorov [Kol62]. In this framework, the energy spectrum of incompressible turbulence follows a power-law scaling of the form $E(k) \propto k^{-5/3}$, where k is the wavenumber. This spectral slope corresponds to a projected (2D) fractal dimension of approximately $D \approx 1.7$, which is often used as a reference point for the morphology of turbulent structures in astrophysical environments.

In molecular clouds, a local fractal dimension approaching this value is commonly interpreted as evidence for Kolmogorov-like turbulent behavior, particularly in low-compression or subsonic regimes. In our results, we observe that many of the extracted structures cluster around this value, especially in intermediate column density regimes and in regions where extended filamentary features dominate.

At the same time, significant deviations from $D \approx 1.7$ are found—towards both higher and lower values. These shifts likely reflect the breakdown of classical turbulence conditions and the emergence of other physical processes.

The overall distribution of D across the MSD plane thus captures the transition from turbulence-dominated to gravity-dominated regimes. In this sense, the local fractal dimension can be viewed as a sensitive tracer of the internal physics shaping the morphology of the cloud at different scales and densities.

Peaks can be seen in Figures 5.6 for the local fractal dimension at column densities

6. Discussion

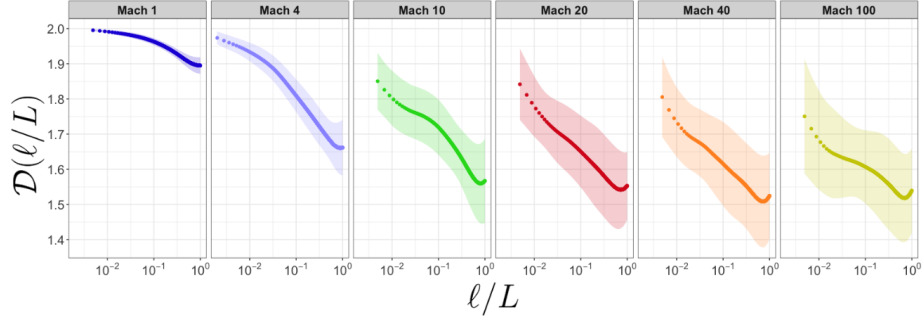


Figure 6.1.: Fractal dimension derived from the mass–size relation as a function of Mach number [BFKS19]. Note that the x-axis is inverted relative to the representation used in our plots above.

of a few times 10^{22} cm^{-2} , roughly corresponding to the regimes where dense cores and filamentary substructures begin to dominate. Visually, it is challenging to determine whether these peaks reflect genuine increases in structural complexity or are influenced by numerical effects—particularly the reduced pixel statistics at high thresholds. However, this behavior could represent a real physical transition, where the cloud morphology becomes increasingly fragmented and irregular as gravitational collapse intensifies. Further investigation using independent tracers or higher-resolution data would be needed to disentangle these effects confidently.

Because this approach is less commonly used in studies of molecular clouds, direct observational comparisons in the literature are limited. Nevertheless, the trends we identify are consistent with the overall picture painted by the global fractal dimension and align well with our physical understanding of these regions.

The closest analytical work on this topic employs a related framework, the mass–size scaling relation [BFKS19]. Since mass can be treated as a proxy for area, and size as a proxy for perimeter, this method is conceptually similar to the perimeter–area scaling relation used here. In their simulations, they report similar trends in the inferred fractal dimension as a function of Mach number (Figure 6.1). This comparison demonstrates that the results from our method are supported by independent simulation-based studies and further emphasizes the sensitivity of fractal measures to the turbulent state of the cloud. The uncertainties associated are also similar in magnitude, proving also that our approach is reasonable.

As with the global method, certain limitations must be considered. Straight edges introduced by limited angular resolution at low column densities, as well as very small structures at the highest column densities, can affect the measurements. For these reasons, the usable column-density range is narrower than the full range provided by the maps.

Moreover, the Euler characteristic highlights clear differences between Orion A and Orion B. Orion A exhibits a profile that closely resembles the behavior expected from Gaussian Random Field simulations, with a well-defined peak at approximately $1.64 \times$

10^{22} cm^{-2} . For comparison, the results for a GRF are included in the Appendix in Figure A.9. This threshold is significant, echoing the transition seen in the global fractal dimension analysis, and marks the emergence of dense cores within the cloud.

In contrast, Orion B shows no such pronounced peak, instead displaying a more gradual, almost linear decrease. If Orion B is indeed more strongly influenced by turbulence, this behavior is consistent with a more scale-free, less clustered structure, where the appearance of dense cores is more gradual and widely distributed. The topological analysis therefore reinforces the notion that turbulence governs the structural evolution of Orion B, while Orion A undergoes distinct transitions closely tied to star-formation thresholds.

Visual examples of the changes in the Euler characteristic for Orion A and Orion B are provided in Figures 6.2 and 6.3. These figures illustrate how the degree of connectivity and fragmentation evolves as a function of column density.

In Orion A, the sharp peak in the Euler characteristic corresponds to a rapid transition from a regime dominated by isolated structures to one in which these structures merge into a more interconnected network. The example cutouts in Figure 6.2 highlight this process as well: at low column densities, the cloud consists of one big connected structure surrounded by many disconnected regions; at mid range that starts to change, whereas at higher thresholds these regions coalesce into larger, more complex entities. This marks the transition from a regime of many disconnected structures to one where connectivity increases rapidly.

In contrast, Figure 6.3 for Orion B shows a more gradual evolution. Here, the Euler characteristic decreases smoothly, indicating that fragmentation and merging occur over a broader range of column densities. The visual examples confirm that Orion B maintains a predominantly filamentary and interconnected morphology across scales, consistent with its interpretation as a more turbulent, scale-free environment.

For visualization, the column density threshold corresponding to the peak in the Euler characteristic for Orion A—along with the same threshold overlaid on Orion B for comparison—is shown in Figures B.3 and B.4 in the Gallery. Together, these visualizations reinforce the quantitative findings and provide intuitive insight into how topology and morphology change across different physical regimes in the two clouds.

The next step in our analysis was to examine individual structures in the context of the local fractal dimension. Using the hierarchical segmentation provided by the dendrogram, we tracked the properties and positions of individual components at each column-density threshold.

As an initial step, we analyzed how the local fractal dimension of these structures varies with column density. We found that the ensemble of structures broadly follows the trend observed for the entire cloud, but with a shallower dependence and greater scatter (Figures 5.9 and 5.10).

These results raise interesting questions. While the ensemble as a whole tends toward $D \approx 2$ (indicating increasing boundary complexity), one might expect that individual structures would appear smoother and simpler at higher column densities, rather than more intricate. This apparent tension motivates a closer look at how structure fragmentation and connectivity influence the derived fractal dimension.

6. Discussion

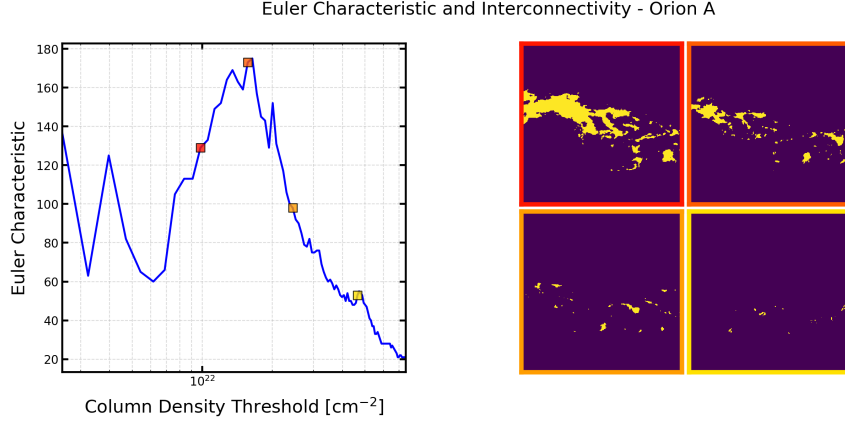


Figure 6.2.: Euler characteristic as a function of column density (left). The right panels show the same zoomed-in area of the cloud at the thresholds indicated by the colored boxes on the graph (Orion A).

Connecting the fractal dimension with mass and size properties further confirmed this puzzling picture: the smaller and lighter (core-like) the structures, the more they tend towards higher local fractal dimension (Figures 5.11, 5.12, 5.13).

The trends discussed in the MSD plane can also be verified visually using the gallery in Appendix B. For Orion A, we begin with large-scale, low-fractal-dimension structures that encompass much of the cloud (see Figure B.5). These correspond to the upper-right region of the MSD plane, representing high mass and size but relatively low structural complexity.

As we move across the MSD diagram—decreasing in size and mass—the local fractal dimension tends to increase. This progression is illustrated in Figures B.6, B.7, and B.8, which show increasingly fragmented structures with more intricate boundaries.

At the extreme end, small and simple shapes dominate, often showing very high local fractal dimensions (approaching 2). An example of such a structure is shown in Figure B.9. The mass, size, and fractal dimension of each example are shown in the figure labels to allow for direct comparison with the corresponding location on the MSD plane.

A similar pattern is observed in Orion B. Figure B.10 corresponds to the top right of the MSD plane—large, coherent structures with lower fractal dimension. Figures B.11, B.12, and B.13 illustrate the gradual transition toward smaller, more fragmented features. Finally, Figure B.14 shows a compact, high-density structure with a high local fractal dimension, representative of the lower-left region of the MSD plot.

Finally, we decided to explore a possible connection between local structural complexity and star formation activity. To do this, we used the YSO catalogue from [MGM⁺12], extracting a sample of young stellar objects (YSOs) from which we computed a YSO density map. This density map was then correlated with our previously derived local fractal dimension maps (Figures 5.14 and 5.15). We first analyzed only early-stage YSOs and then repeated the analysis for the full YSO sample for completeness and comparison.

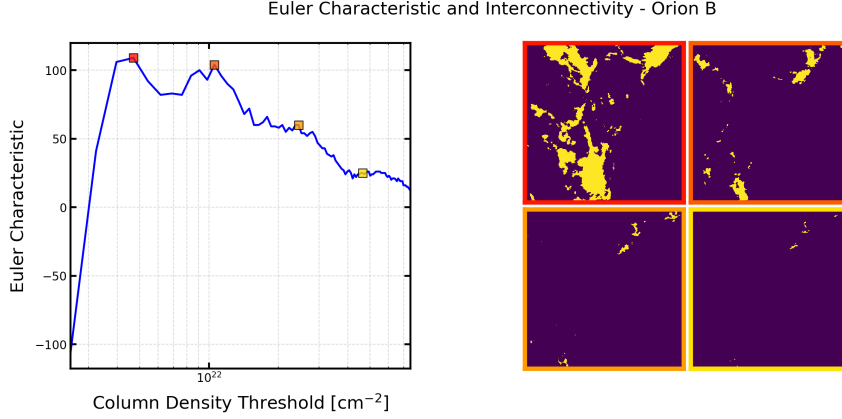


Figure 6.3.: Euler characteristic as a function of column density (left). The right panels show the same zoomed-in area of the cloud at the thresholds indicated by the colored boxes on the graph (Orion B).

The correlation results, expressed in terms of both Pearson and Spearman coefficients, revealed distinct trends for the Orion A and Orion B clouds. In Orion A, we found a stronger correlation when using the early-stage YSO sample (Pearson: 0.434, Spearman: 0.429) compared to the full sample (Pearson: 0.286, Spearman: 0.393). In contrast, Orion B showed the opposite behavior, with a weaker correlation for the early-stage sample (Pearson: 0.288, Spearman: 0.244) and a stronger correlation for the full sample (Pearson: 0.399, Spearman: 0.340).

These results hint again at environmental differences between the two regions. In Orion A, the stronger correlation for early-stage YSOs suggests that the emergence of complex, fragmented structures is closely tied to the onset of star formation. Conversely, the weaker or more distributed trend in Orion B may reflect a more turbulent, less clustered environment where star formation proceeds in a less organized manner, less dependent on localized structural complexity. This interpretation is consistent with long-standing observational evidence: Orion A hosts a significantly higher density of protostars and young stellar objects, including the Orion Nebula Cluster and the Integral Shaped Filament, and is overall far more active in star formation than Orion B [MGM⁺12]. The contrasting correlations therefore align naturally with the known difference in star-forming activity between the two regions.

It is important to note, however, that the method is currently limited by the relatively small number of YSOs in the early-stage catalogue and the restricted spatial coverage of the YSO density maps. These limitations make it difficult to draw definitive conclusions. Nevertheless, if future studies with larger and more complete datasets confirm these patterns, they would provide valuable evidence for fundamental differences in the star formation environments of Orion A and Orion B, and demonstrate the potential of fractal and topological analyses to shed light on the physical conditions governing star formation in molecular clouds.

6. Discussion

In light of these findings, it is instructive to compare our approach with more historically used methods for estimating fractal dimensions in molecular clouds. Classical analyses, such as those presented in [EF96], often relied on a scale-counting relation of the form

$$N(\lambda > L) \propto L^{-D},$$

where N is the number of self-similar structures larger than a given scale L , and D is the inferred fractal dimension. While this method provides a useful first-order characterization of cloud hierarchy, it is inherently limited to counting structures across scales and does not directly incorporate information about their shapes, perimeters, or connectivity.

By contrast, the perimeter–area approach used in this work captures not only the size distribution of structures but also the complexity of their boundaries and how that complexity evolves with column density. When combined with topological diagnostics such as the Euler characteristic and hierarchical segmentation via dendrograms, this method offers a more nuanced view of how morphology and topology change across different physical regimes. Rather than yielding a single, scale-averaged fractal dimension, our approach reveals distinct regimes within the same cloud (e.g., Orion A) and connects those regimes to physically meaningful transitions such as the emergence of dense cores and the onset of star formation.

In summary, while historical methods such as the $N(\lambda > L)$ scaling have been invaluable in establishing the fractal nature of molecular clouds, the combined perimeter–area and topological analysis employed here provides a more detailed, physically interpretable framework that links structural complexity directly to the underlying star-forming processes.

Taken together, our analysis provides a cohesive view of the structural and physical differences between Orion A and Orion B. The combination of global and local fractal dimension measurements, topological diagnostics through the Euler characteristic, and the hierarchical insights from the MSD analysis reveals that both clouds exhibit scale-dependent behavior, but in distinct ways. Orion A shows clear structural transitions—evident in the change of slope in the global perimeter–area relation and the sharp features in the Euler characteristic—marking the emergence of dense cores and a close connection between increasing morphological complexity and active star formation. In contrast, Orion B maintains a more uniform, filamentary character and exhibits weaker correlations with local complexity, consistent with a more turbulent, scale-free environment where star formation proceeds in a more distributed manner.

The accuracy of the derived morphological properties, including the fractal dimension and Euler characteristic, is inherently limited by the resolution of the observational data. At high column density thresholds, the number of pixels above the cut becomes increasingly sparse, which can lead to artificial fluctuations in the measured quantities—particularly in small or isolated structures. This effect may manifest as spurious peaks or enhanced scatter in the local fractal dimension. Conversely, at low column densities, smoothing and beam convolution can oversimplify the intrinsic cloud structure, potentially underestimating its complexity. While efforts were made to exclude clearly unphysical cases (e.g., regions with too few pixels or straight-edged artifacts), care must be taken in interpreting trends

near the resolution limits. Future observations at higher angular resolution could help better resolve small-scale features and test the robustness of the patterns observed here.

These findings reinforce nonetheless the idea that turbulence and self-similarity dominate cloud structure on large scales, but that localized physical processes such as core formation can imprint strong deviations from scale-free behavior. By integrating fractal analysis with topological measures and comparisons to simulations, this study demonstrates how a multi-faceted approach can link the morphology of molecular clouds directly to their star-forming activity, offering a richer and more physically grounded picture than classical methods alone.

7. Conclusion

This thesis set out to revisit and extend the study of structural complexity in molecular clouds through the lens of fractal geometry and topological analysis, originally popularized in the context of the interstellar medium by works such as [EF96]. The aim was to move beyond purely visual or qualitative impressions and provide a quantitative framework that could be applied to real observational data of star-forming regions.

The column density maps used in this work were derived from dust emission measurements from the *Herschel* satellite, as described by [LBAL14], offering high dynamic range and spatial resolution across the Orion complex. This data enabled the application of Minkowski functionals and perimeter–area analysis to characterize the global and local fractal structure of Orion A and Orion B.

A number of techniques were employed to explore the geometry and topology of the molecular clouds, including:

- Global fractal dimension estimation from perimeter–area scaling, as introduced by [Can84];
- Local fractal dimension analysis as a function of column density threshold;
- Euler characteristic computations to trace topological transitions in structure;
- Mass–size diagrams inspired by recent work such as [HKS⁺25], offering a new way to connect local structural properties with complexity and fragmentation;
- A comparison with star formation properties through YSO spatial density and classification.

The global fractal analysis revealed an interesting dichotomy between Orion A and Orion B: while Orion B exhibited a relatively constant fractal dimension across scales, Orion A showed a clear break in its scaling behavior at $N \sim 1.2 \times 10^{22} \text{ cm}^{-2}$. This threshold aligns with the onset of dense core formation and early-stage YSOs, indicating a shift in the dominant physical processes—likely from turbulence-dominated to gravity-driven fragmentation.

The local analysis added further nuance to this picture: as column density increases, the local fractal dimension tends toward 2, reflecting the increased visual and morphological complexity introduced by filamentary networks and core clustering. However, the complexity measured in the mass–size diagrams showed that on the smallest scales, structures tend to simplify again, suggesting self-regulated fragmentation and localized collapse.

7. Conclusion

The Euler characteristic analysis complemented this by showing a distinct peak in Orion A, consistent with a rapid topological transition, whereas Orion B displayed a more gradual evolution, in line with its more turbulent and hierarchical structure.

While the current study provides a comprehensive view of Orion A and Orion B, several directions for improvement and future exploration remain:

- **Improved uncertainty quantification:** The propagation of errors in perimeter–area and fractal dimension estimates could benefit from higher-order error modeling, especially for small or pixel-limited structures.
- **Refined star formation connection:** The analysis of the link between fractal structure and star formation relied on spatial correlations with YSO densities. Future work could involve more rigorous statistical modeling, such as clustering metrics or spatial cross-correlation functions, possibly extended to velocity space.
- **Wider application of methods:** The techniques developed here could be applied to other GMCs in different galactic environments. This would test the generality of the observed trends and potentially uncover environmental dependencies.
- **Simulation comparison:** A natural extension would be to directly compare these observational metrics with synthetic observations of numerical simulations of turbulent, magnetized clouds with varying initial conditions.

Overall, this thesis demonstrates the power of topological and fractal tools to provide insight into the structural and physical processes governing molecular clouds. By combining geometry, topology, and star formation tracers, it offers a coherent framework that can be further developed in both observational and theoretical contexts.

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A. Appendix

This first appendix contains extra graphs and results that mainly compliment the results section.

A.1. Fractal Dimension

This section shows more plots regarding the local fractal dimension and the evolution of structures on the Mass-Size-Fractal Dimension graph.

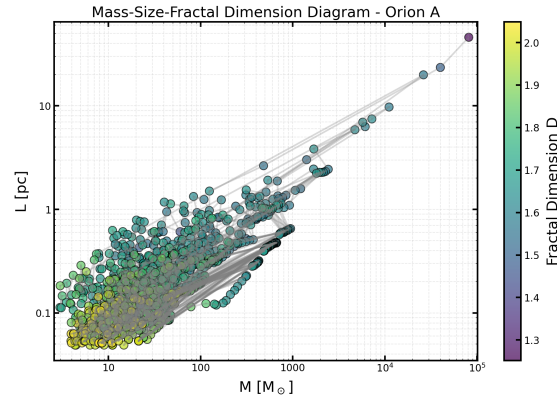


Figure A.1.: MSD plane for Orion A with dendrogram connections overplotted. Lines trace parent-child relationships between structures.

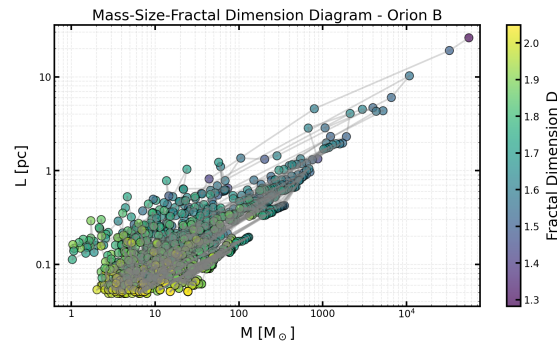


Figure A.2.: MSD plane for Orion B with dendrogram connections overplotted.

A.2. YSO Density Maps

This section of the appendix provides the YSO density maps used in the correlation analysis discussed in the main text.

A.2.1. Class I/flat-spectrum sources

The following maps show the surface density of early-stage YSOs (Class I and flat-spectrum) for Orion A and Orion B. These objects are most closely associated with ongoing star formation and are used in the analysis to trace regions of active star-forming activity.

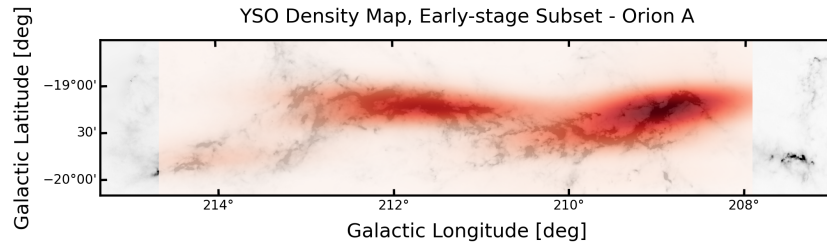


Figure A.3.: YSO density map for Orion A, overlaid on the column-density map. The coverage is limited by the extent of the YSO catalog, so some regions are not sampled.

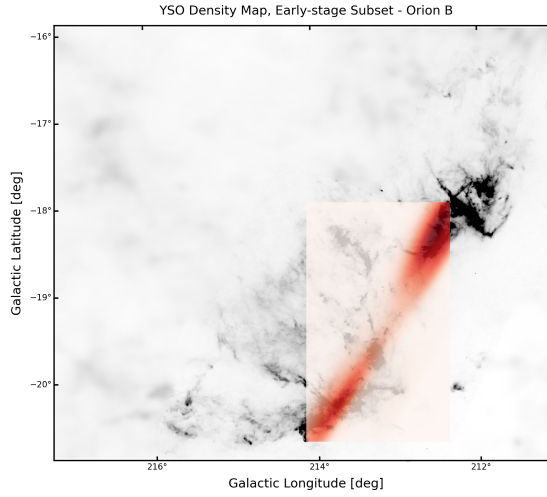


Figure A.4.: YSO density map for Orion A, overlaid on the column-density map. The coverage is limited by the extent of the YSO catalog, so some regions are not sampled.

A.2.2. Full Sample

The next set of maps shows the surface density for the full YSO sample, including more evolved Class II sources. These maps provide a broader view of the stellar content but include stars that may have migrated from their birth sites, resulting in a more diffuse distribution.

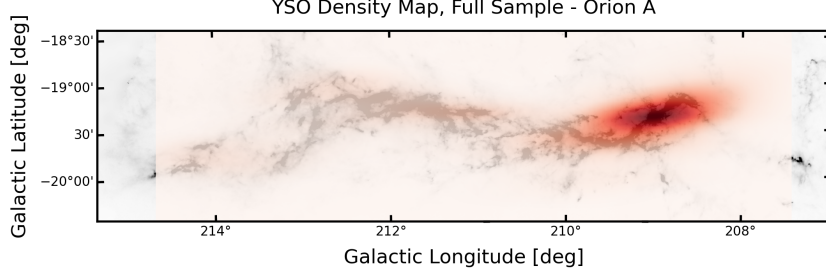


Figure A.5.: YSO density map for Orion A, overlaid on the column-density map. The coverage is limited by the extent of the YSO catalog, so some regions are not sampled.

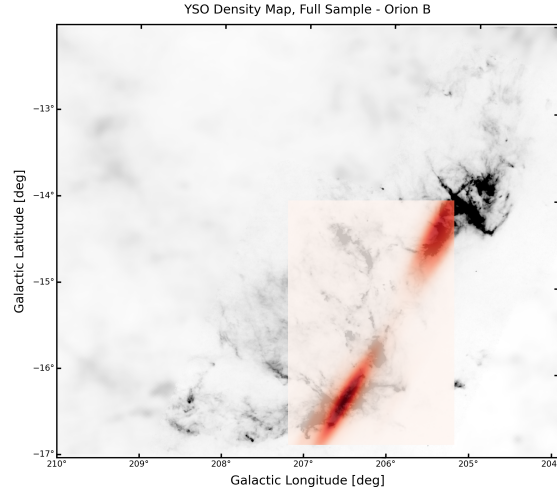


Figure A.6.: YSO density map for Orion A, overlaid on the column-density map. The coverage is limited by the extent of the YSO catalog, so some regions are not sampled.

A.3. Simulations

This section of the appendix provides more context to the results of the simulations and calculation of the uncertainties.

A. Appendix

A.3.1. Global Fractal Dimension

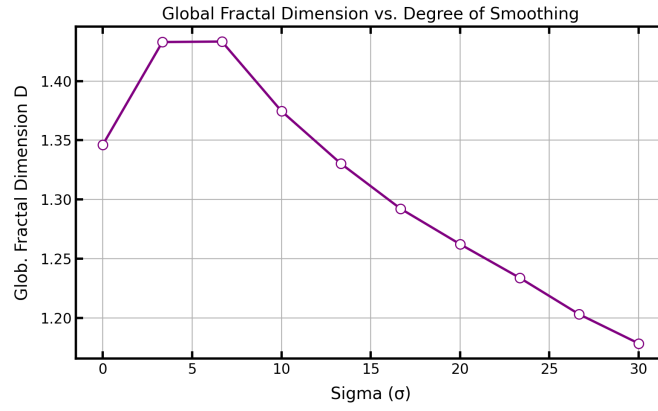


Figure A.7.: Results for the global fractal dimension as a function of the degree of smoothing of the input data.

A.3.2. Local Fractal Dimension

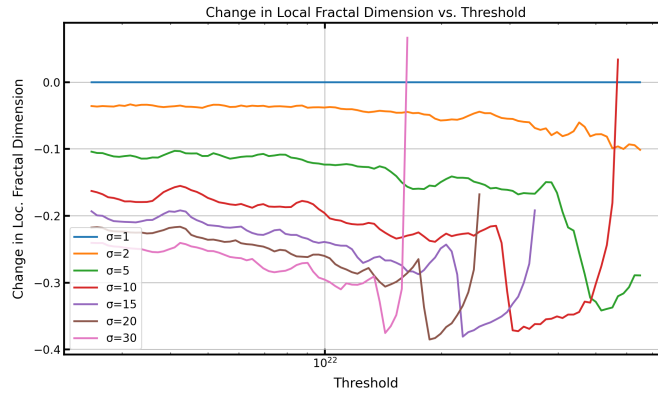


Figure A.8.: Results for the global fractal dimension as a function of the column density thresholds for a number of smoothing kernels of the input data.

A.3.3. Euler Characteristic

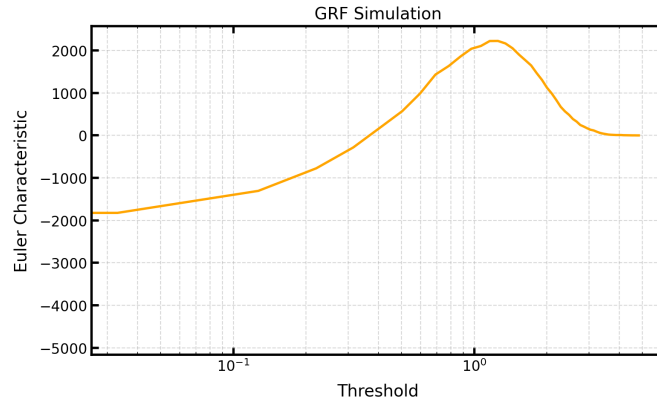


Figure A.9.: Resolution of the Euler Characteristic for a Gaussian Random Field with exponent of the power spectrum $a = -2$.

A.3.4. Uncertainties

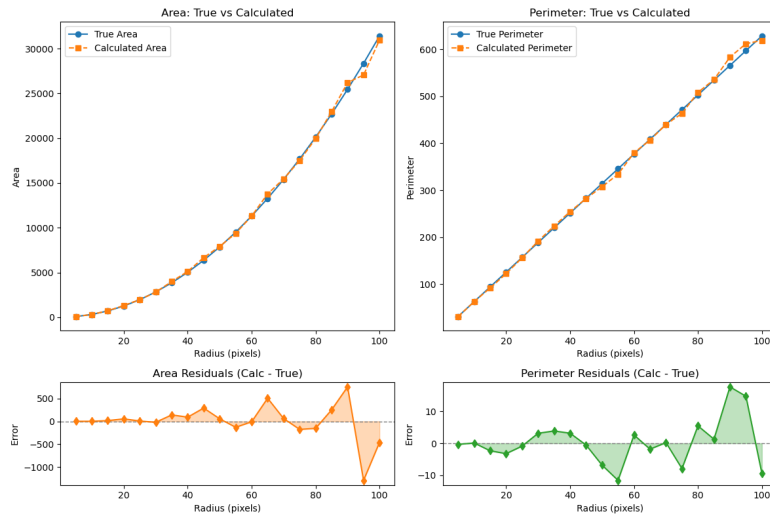


Figure A.10.: Example of the uncertainty distributions in the measurements of perimeter and area from simulated structures (arbitrary units).

B. Gallery of Visual Examples

This second appendix contains visual examples to connect the results with the structures.

B.1. Fractal Structure of Orion

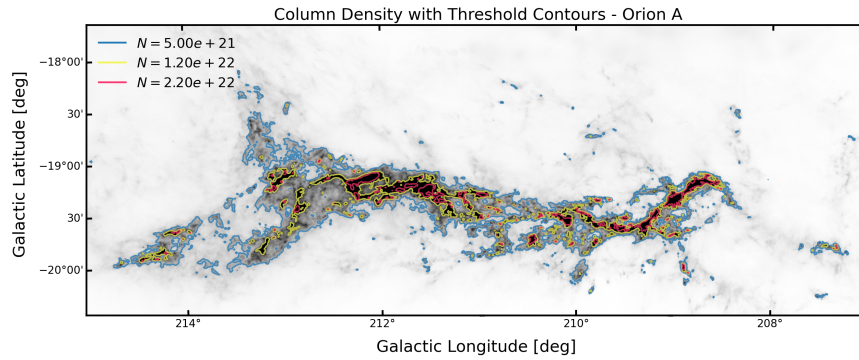


Figure B.1.: Column-density map of Orion A with contours of three characteristic thresholds.

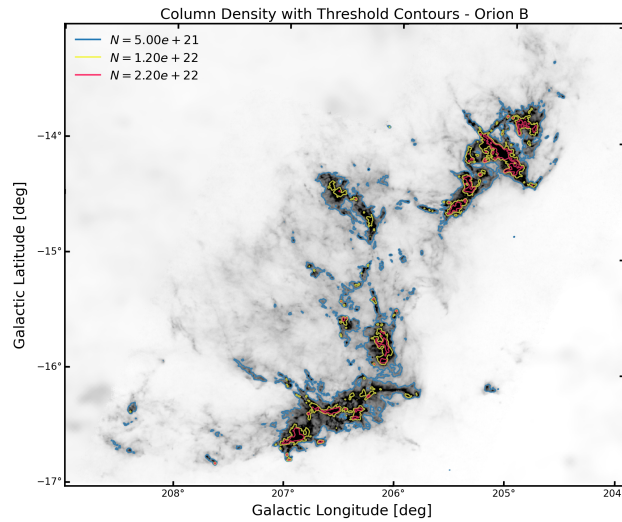


Figure B.2.: Column-density map of Orion B with contours as in Figure [B.1](#).

B.2. Euler Characteristic

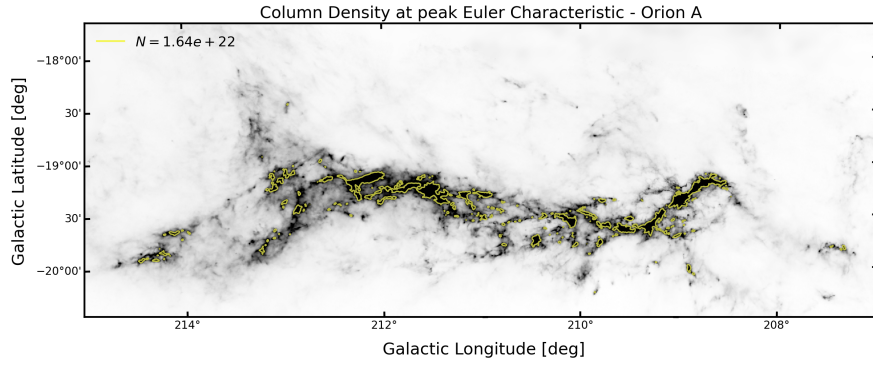


Figure B.3.: Column-density map of Orion A with a contour overlaid at the threshold corresponding to the peak in the Euler characteristic.

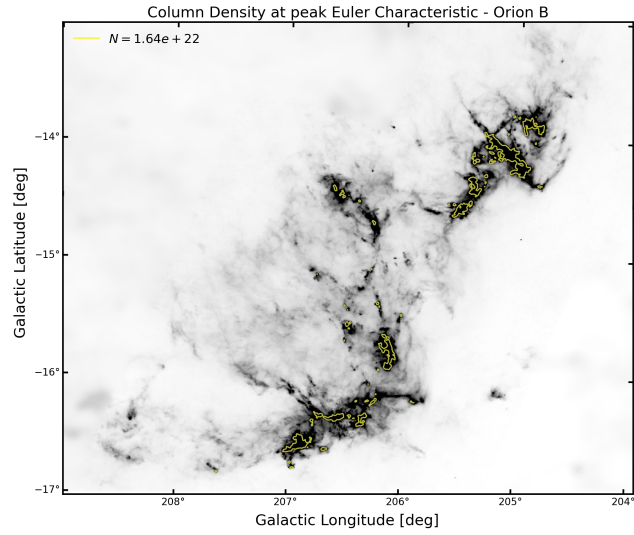


Figure B.4.: Column-density map of Orion B with a contour overlaid at the same threshold used for Orion A in Figure [B.3](#). While this value does not correspond to a peak in Orion B's Euler characteristic, it is shown here for visual comparison.

B.3. MSD Structures

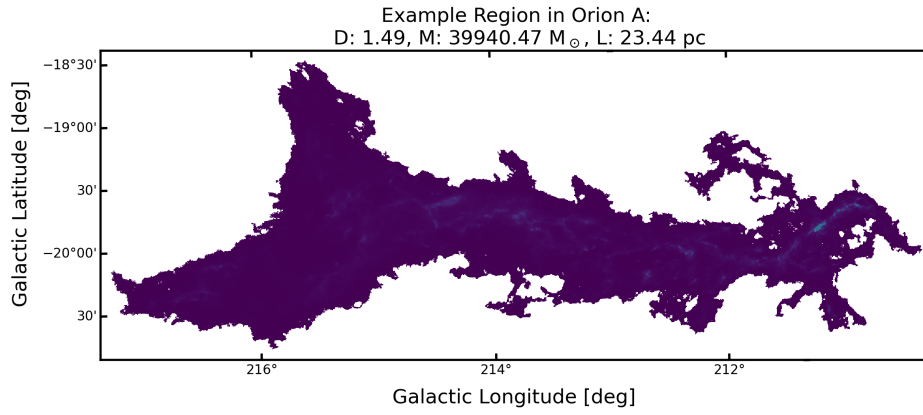


Figure B.5.: Example of the mask of an individual structure extracted from Orion A at a given column density threshold. Key physical properties—mass, size, and local fractal dimension—are indicated in the title of the plot.

B. Gallery of Visual Examples

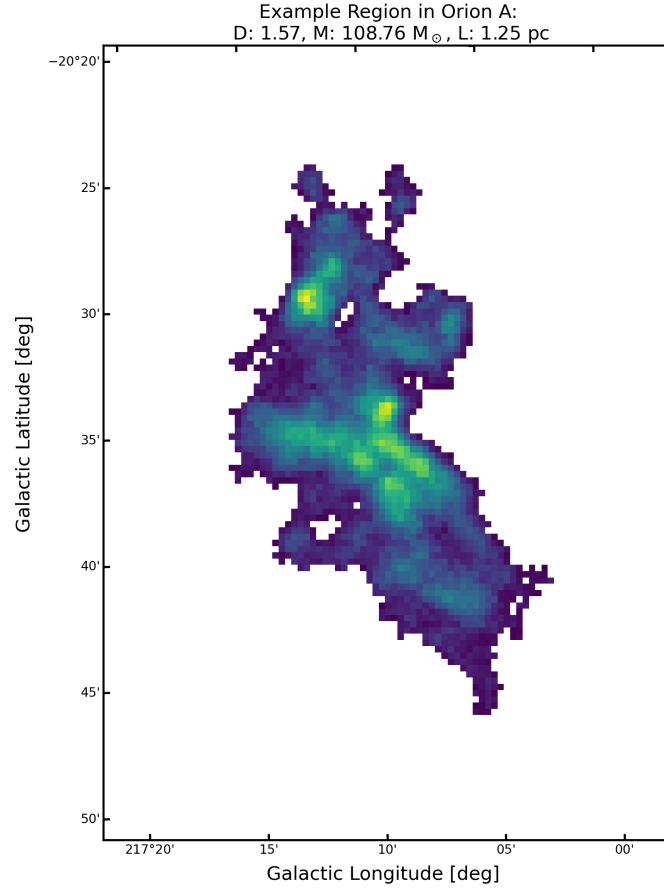


Figure B.6.: Example of the mask of an individual structure extracted from Orion A at a given column density threshold.

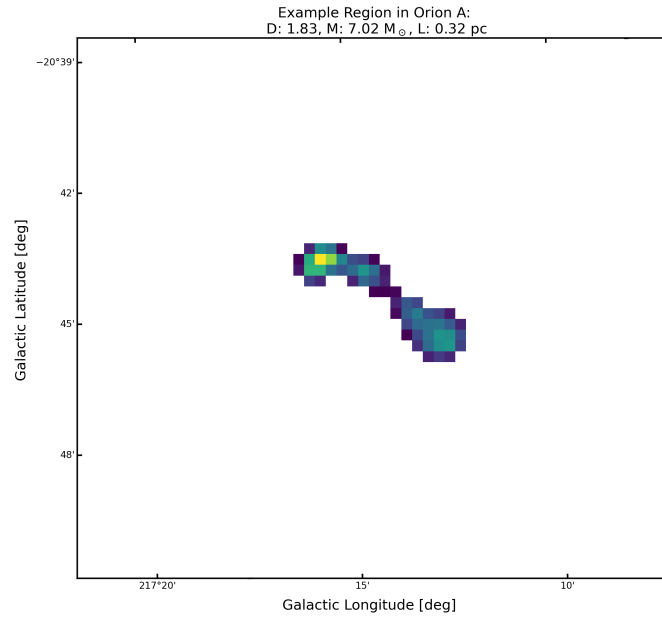


Figure B.7.: Example of the mask of an individual structure extracted from Orion A at a given column density threshold.

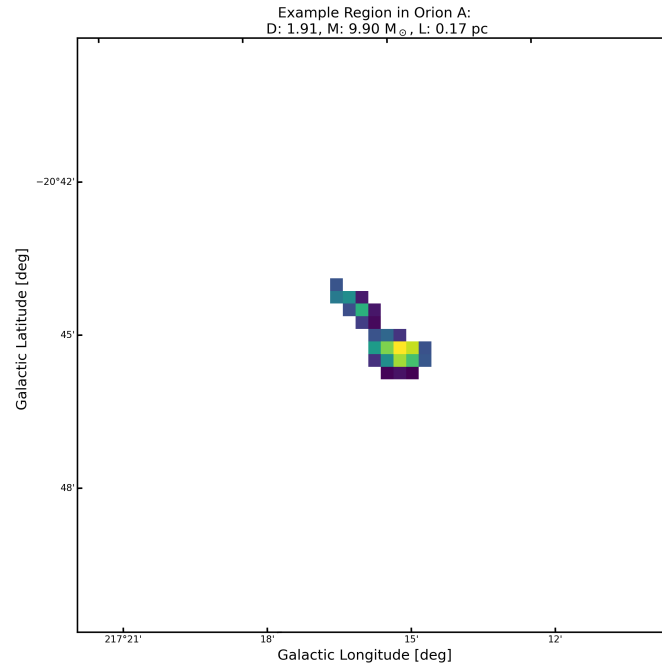


Figure B.8.: Example of the mask of an individual structure extracted from Orion A at a given column density threshold.

B. Gallery of Visual Examples

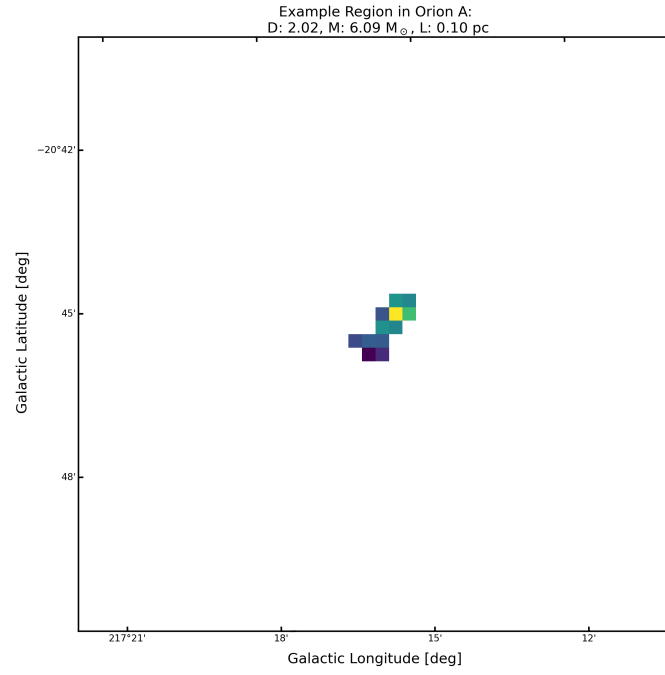


Figure B.9.: Example of the mask of an individual structure extracted from Orion A at a given column density threshold.

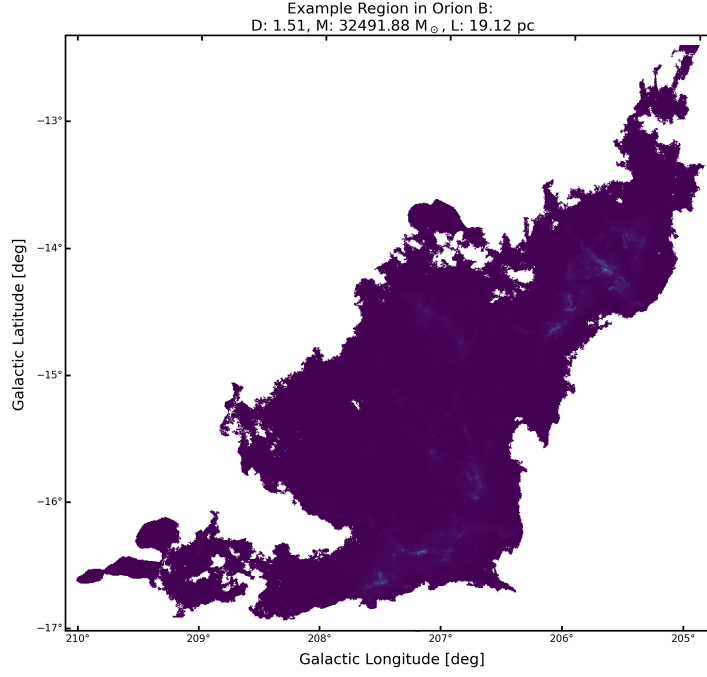


Figure B.10.: Example of the mask of an individual structure extracted from Orion B at a given column density threshold. Key physical properties—mass, size, and local fractal dimension—are indicated in the title of the plot.

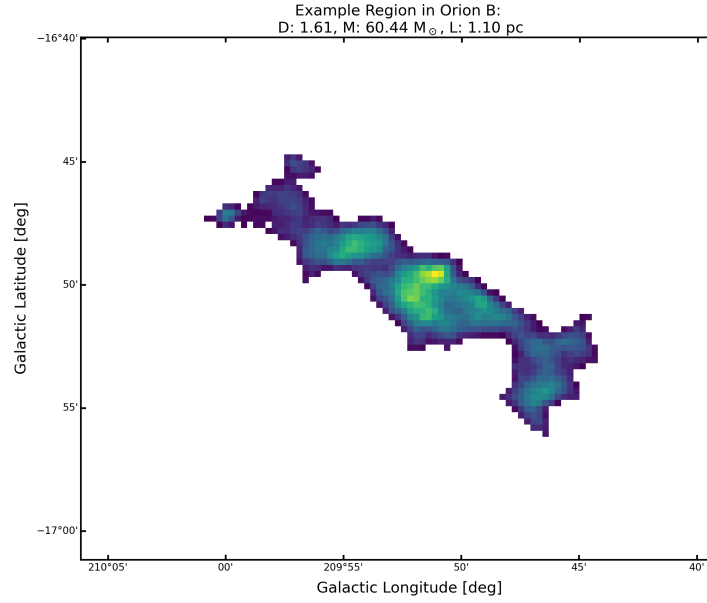


Figure B.11.: Example of the mask of an individual structure extracted from Orion B at a given column density threshold.

B. Gallery of Visual Examples

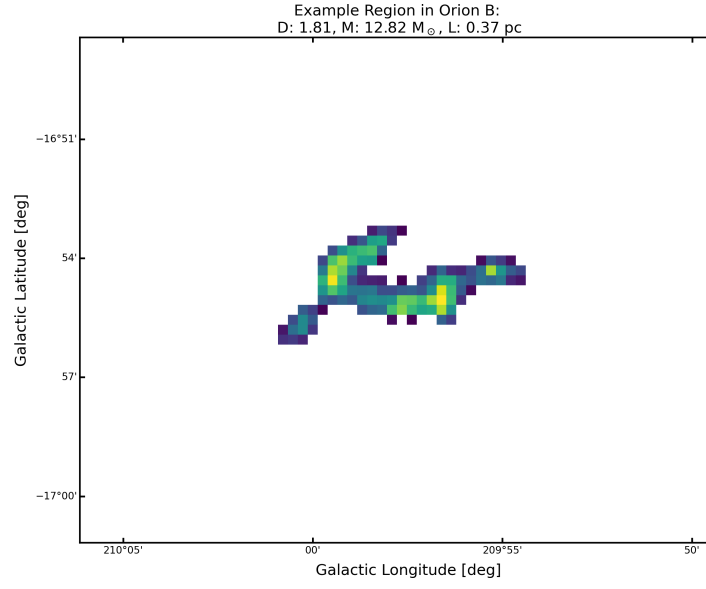


Figure B.12.: Example of the mask of an individual structure extracted from Orion B at a given column density threshold.

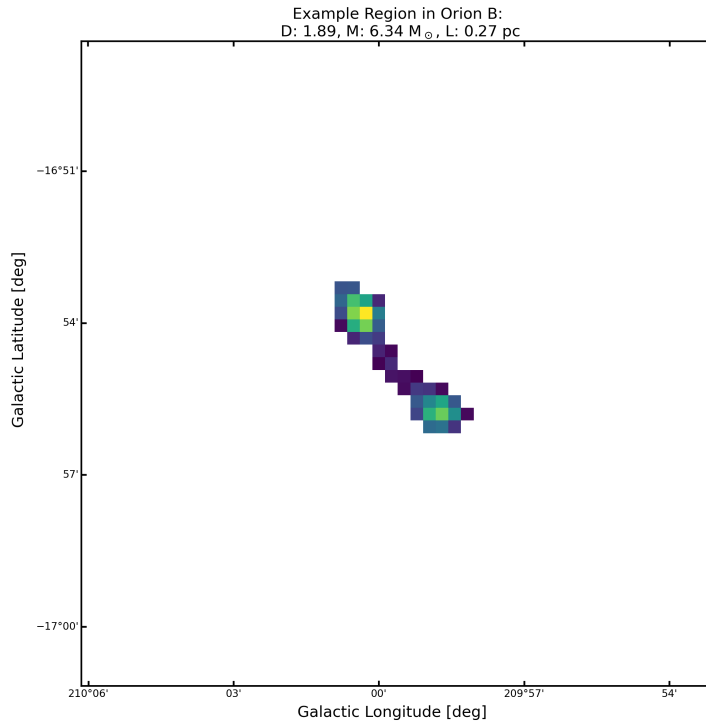


Figure B.13.: Example of the mask of an individual structure extracted from Orion B at a given column density threshold.

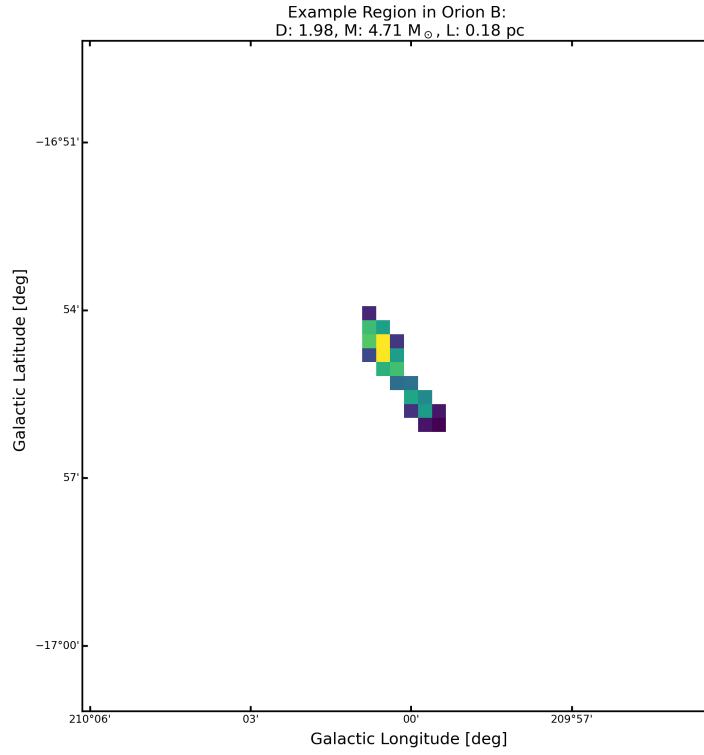


Figure B.14.: Example of the mask of an individual structure extracted from Orion B at a given column density threshold.