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Optimizing Last-Mile Delivery for Individual Customer Service

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Abstract (German)

Die vorliegende Dissertation befasst sich mit der Optimierung der Lieferung auf der letzten Meile für den individuellen Kundenservice und fokussiert auf die Evaluierung des Potentials von mobilen Packstationen. Unter Last-Mile-Delivery versteht man den Transport von Paketen vom letzten Verteilzentrum bis zum Endkunden. Die Berücksichtigung des Kunden bei der Zustellung wird immer wichtiger, um einen profitablen, aber auch wettbewerbsfähigen Service anbieten zu können. Mobile Packstation, bzw. Mobile Parcel Locker (MPL), ist ein neues Konzept, das traditionelle Dienstleistungen kombiniert und auf individuelle Kundenwünsche zugeschnitten werden kann. Um das Potenzial von MPLs zu bewerten, werden sie zunächst mit den traditionellen Konzepten der anwesenden Haustürbelieferung (Attended Home Delivery, AHD) und der stationären Packstation verglichen. Anschließend wird ein neues Konzept vorgestellt, das AHDs und MPLs kombiniert, genannt Mobile Home Delivery Parcel Locker (MHDPL). Aufgrund der zusätzlichen Komplexität wird ein heuristischer Ansatz zur Lösung des MHDPL Problems entwickelt, basierend auf dem Iterated Local Search Verfahren. Abschließend wird der erste Ansatz einer dynamischen Anwendung untersucht, bei der die Kunden nacheinander eintreffen. Wie ein Kunde bedient werden soll wird individuell vereinbart, bevor die zukünftigen Kunden bekannt sind. Es werden verschiedene Strategien vorgestellt, um zu bestimmen welche Serviceoptionen jedem Kunden während der dynamischen Kundenannahme angeboten werden sollen. Diese Strategien reichen von einfachen, intuitiven Ansätzen bis hin zu anspruchsvolleren Ansätzen, die versuchen realistisches Problemwissen über die dynamisch eintreffenden Kunden zu nutzen. Experimente zeigen, dass die Kombination von MPL und AHD vielversprechend ist. Der MHDPL Service kann sich an jede Kundensituation anpassen und bietet großes Potenzial in Szenarien, in denen sowohl AHDs als auch MPLs gut funktionieren. Allerdings ist die dynamische Kundenannahme bei MPL Services besonders schwierig, aber mit geeigneten Strategien können deutliche Verbesserungen erzielt werden.

Abstract (English)

This thesis deals with the optimization of the last-mile delivery for individual customer service and focuses on the evaluation of the potential of mobile parcel lockers. Last-mile delivery refers to the transportation of parcels from the last distribution center to the end customer. Taking the preferences of customer into account during delivery is becoming increasingly important in order to be able to offer a profitable but also competitive service. Mobile Parcel Locker (MPL) is a new concept that combines traditional services and can be tailored for individual customer preferences. To evaluate the potential of MPLs, they are first compared with the traditional concepts of Attended Home Delivery (AHD) and stationary parcel locker. Subsequently, a new concept is presented that combines AHDs and MPLs, called Mobile Home Delivery Parcel Locker (MHDPL). Due to additional complexity, a heuristic approach to solving the problem is developed, using an iterated local search. Finally, the first approach of a dynamic application is investigated, where customers arrive successively. How a customer is to be served is determined individually before the future customers are known. Different strategies are presented to determine which service options should be offered to each customer during the dynamic customer acceptance. These strategies range from simple, intuitive approaches to more sophisticated approaches that attempt to utilize realistic problem knowledge about dynamic customer arrivals. Experiments show that the combination of MPL and AHD is promising. The MHDPL service can adapt to any customer situation and offers great potential in scenarios where both AHDs and MPLs work well. However, dynamic customer acceptance is particularly difficult for MPL services, but significant improvements can be achieved with appropriate strategies.

Preface

This thesis contains research that has been carried out and has been published over the course of the previous years. Two papers have been published, and one is submitted. The first publication in the Networks journal called *Mobile Parcel Lockers with Individual Customer Service* is discussed in Chapter 2. Chapter 3 refers to the article *Mobile Home Delivery Parcel Lockers* which has been published in Transportation Research Part E. *Dynamic Customer Acceptance for MHDPL Services* has been submitted and addressed in more detail in Chapter 4. In addition, the results and intermediate progress of this research has been presented and discussed in many workshops and conferences over the past several years.

Contents

Chapter 1

Introduction

In the introduction to this thesis, first the relevance of last-mile delivery within the parcel shipping process is highlighted and existing as well as upcoming concepts for handling the last mile are described. This is followed by a discussion on last-mile delivery in terms of customer perspective and expectations, with a focus on parcel locker services to enable different levels of service. In this way, the unique potential of Mobile Parcel Lockers (MPLs) to accommodate individual customer preferences is highlighted. Finally, the research questions and further structure of the thesis are presented.

1.1 Last-Mile Delivery

The global parcel shipping volume has been rising steadily in recent years, and forecasts predict that the market size will further increase with a compound annual growth rate of around 12% until 2030 (Future Data Stats, 2023; Mali, 2025). In 2014, 43 billion parcels were shipped worldwide, in 2022 this had risen to 161 billion parcels and 259 billion parcels are forecast for 2027 (Statista, 2023b, 2023c). Given these figures and the associated economic value, the transportation research community is constantly looking for new concepts and improved optimization methods to cope with larger shipment volumes and exploit the potential available. The parcel delivery process is divided into three parts: first-mile delivery, middle-mile delivery and last-mile delivery. First-mile delivery deals with the first stage of delivery, that is, collecting the deliveries from the retailer or manufacturer and transporting them to the first warehouse or distribution center (Giménez-Palacios et al., 2022; Goodman, 2021). Middle-mile delivery deals with the onward transportation of the consolidated goods over longer distances to the final distribution center (Greening et al., 2023; Onfleet, 2023). Finally, last-mile delivery deals with the distribution from the last distribution center to the individual customers (Chopra & Meindl, 2015), which is the most challenging factor in terms of costs and planning (Joerss et al., 2016). Each part has its own unique challenges to be addressed: Inventory management, packaging and timely pickups on the first mile; Long-distance coordination, route optimization, and multiple modes of transport on the middle mile; Tight delivery windows, urban congestion, managing returns and failed deliveries, as well as offering flexible delivery options on the last mile (Onro, 2025). The last mile is of particular importance, as it accounts for 53% of total shipping costs (Statista, 2023a) and involves direct contact with the end customer. For this thesis, the focus is on individual customer service, which is why the emphasis is on last-mile delivery and its challenges and developments.

The expected increase in shipment volumes creates several challenges, particularly for last-mile delivery (Ha et al., 2023). In general, more vehicles, more congestion and more emissions are expected as more customers must be served. Ha et al., 2023 identify 18 different types of challenges and categorize them into five domains: operations, infrastructure, delivery, logistics and environment. The challenges include, for example, loading and unloading times, the search for parking spaces, different customer expectations, unoptimized routes and noise pollution. Some research focuses on the environmental challenges associated with last-mile deliveries (Comi & Savchenko, 2021; Mucowska, 2021; Qi et al., 2018), although this is also studied in the field of green vehicle routing problems (Asghari, Al-e, et al., 2021; Lin et al., 2014; Moghdani et al., 2021; Sabet & Farooq, 2022). Effective collaboration between service providers is one approach that could address several of these challenges, as it would reduce fleet size and traffic volumes (Savelsbergh & Van Woensel, 2016). But collaborations create completely new economic and competitive challenges (Cleophas et al., 2019; Elting et al., 2024; Gansterer & Hartl, 2018; Gansterer et al., 2019). Beyond this, there are plenty innovative last-mile delivery concepts that Boysen et al., 2021 summarize and categorize together with the more traditional concepts. They present a classification scheme in which all aspects to be considered are divided into storage, transportation and handover. Storage includes, for example, the depot, intermediate hubs or parcel lockers. The handover takes place either via self-service by customers or attended or unattended at home. The greatest variety can be seen in the possible transportation options, which can be, for example, bikes, vans, ships or airplanes. Delivery concepts already available today include home delivery with vans or cargo bikes as well as self-service pickup from parcel stores or parcel lockers. Soon, the authors expect deliveries by drones and robots, but also crowd shipping and parcels being stored in private car trunks. In the future, however, they see autonomous vans, drones taking off from trains or airplanes and Mobile Parcel Lockers (MPL). While there are literature overviews of articles considering drones (Garg et al., 2023) or robots (Alverhed et al., 2024) for last-mile deliveries, research on MPLs is still in the early stages.

MPLs are similar to the well-known stationary parcel lockers and were first introduced by Schwerdfeger and Boysen, 2020. Customers visit the MPL and pick up their delivery. However, the MPL is only available for a certain period of time, referred to as the service provision period. This can vary from a few hours to several days. In this way, a single MPL can serve customers in different areas by moving and positioning it to different locations. Another difference to stationary parcel lockers is the ability to be filled at the depot, eliminating the need for a filling tour. Furthermore, no strategic location planning is required, as MPLs can be deployed based on operational needs. For example, Wang, Bi, and Chen, 2020 use MPLs in addition to stationary parcel lockers to be able to react to peaks in demand. Schwerdfeger and Boysen, 2022 describe how an MPL can be realized in practice. They consider the implementation as an independent vehicle, as a trailer or as a station that can be loaded onto vehicles. In addition, MPLs can be positioned by humans or they may operate autonomously.

1.2 Individual Customer Service

The previous chapter discussed traditional and innovative concepts for optimizing last-mile delivery. Beyond these concepts, the customer's perspective is becoming increasingly important (Esper et al., 2020; Olsson et al., 2022). This is because better customer service promises higher profits for the service provider (Puri, 2024). There are various aspects that facilitate good customer experience, such as timely delivery, same-day or next-day options, appropriate delivery windows, connectivity and automation, route optimization, proof of delivery, or friendly delivery drivers (Reinblatt, 2025). Customers also want more control and flexibility to decide when and how they want to receive their deliveries (Puri, 2024; Tam, 2025). For 72% of customers, a favorable time window for delivery is even more important than fast delivery (Codept, 2023; FlashBox, 2023).

From the customer's perspective, most of the infrastructure of last mile delivery, such as transit hubs, crowd delivery or consolidation, are generally less noticeable. For consumers, the delivery conditions are decisive, which determine both the time and the mode of delivery. The time includes aspects such as next and same day delivery, but more importantly the agreement on a time slot during which the delivery will take place. There are two modes of delivery to be distinguished: home delivery, which can be attended or unattended, and self-service delivery, which takes place at a specific collection point.

Self-service delivery is relatively new and the incentives for customers to switch from home delivery to parcel lockers have not been fully investigated. Vakulenko et al., 2018 investigate the customers' view on the self-service concept with parcel lockers. They consider functional, social, emotional and financial values for customers. Functionally, parcel lockers enable fast and flexible collection. Emotionally, for some customers it feels easy and familiar, just like opening the mailbox every day. Socially it is more mixed, but some customers appreciate the service without direct contact. Financially, parcel lockers are attractive if they can be offered at a lower cost. Yuen et al., 2018 and Yusoff et al., 2025 investigate consumers' intentions to use parcel locker services. They point out that it is individual and depends on the customer's own needs whether they find parcel locker services advantageous or not. However, it is evident that the location is crucial and how easily it can be integrated into everyday life, e.g. at shopping centers or transport networks. The size of the goods also matters in terms of how far customers are willing to walk. Choo, 2016 suggests that the customers attitude towards parcel locker services is more important than optimizing the network of available parcel lockers. To increase consumer interest in using parcel lockers as an alternative to home delivery, the service needs to become more attractive.

Research has shown that customers opt for useful, effective, and environmentally friendly delivery options if they satisfy their needs for flexibility, savings, and efficiency (Kolasińska-Morawska et al., 2022). To provide a flexible and highly competitive service, a sufficiently large network of parcel lockers is required. Furthermore, Vakulenko et al., 2019 argue that a competitive service provider should consider offering multiple delivery options to its customers. This is because multiple delivery options allow customers to develop a sense of control, choose the most convenient delivery option and have better overall experience. They further describe that customers are inherently loyal to traditional services and tend to distrust new services. A parcel locker service just around the corner can speed up the transition and acceptance.

MPLs combine these requirements and advantages. An extensive network of parcel lockers is not necessary, as customers can be offered various service options at different times and locations with the same vehicle. MPLs are therefore a promising innovative concept for last-mile delivery to offer customers a cost-effective yet individually personalized service in terms of time and location.

1.3 Research Questions and Structure

AHDs and Fixed Parcel Lockers (FPLs) are established concepts for last-mile delivery. In contrast, MPLs are a new concept and offer a service that can be tailored to individual customer preferences. This thesis examines the potential of MPLs to optimize last-mile delivery for individual customer service. The following three research questions are addressed.

- RQ1 – Comparison of MPLs with AHDs and FPLs
- RQ2 – Integrated MPL and AHD services (MHDPL services)
- RQ3 – Dynamic customer acceptance mechanism for MHDPL services

The first research question aims to investigate how MPL vehicles perform in comparison to FPL and AHD vehicles. To investigate the impact of individual customer preferences, two distinctively different customer types are introduced, which differ in the requested time window length and the maximum travel distance to the parcel locker. To enable a fair comparison, the fleet sizes and vehicle capacities are fixed. The aim is to maximize the number of customers accepted with the given resources. This enables an assessment of how well the different services can adapt to customers and their individual preferences. In addition, combined fleets are considered, as AHDs and FPLs are frequently investigated in combination in the literature. Managerial insights include the impact of structural demand differences and different fleet sizes as well as operational fleet utilization and individual customer experience.

For the second research question, the concept of MPLs is expanded to include the option of offering AHD services. This new service is called Mobile Home Delivery Parcel Locker (MHDPL) service and combines the advantages of MPLs and AHDs. Since this is a complex problem and MPLs show their greatest potential with a larger number of available customers, a heuristic solution approach is developed. The iterated local search has proven to be promising for conceptually related problems.

However, it needs to be adapted and requires new operators to consider the characteristics of MPLs and the ability to handle both types of services. The managerial insights analyze the novel MHDPL service in terms of various instance and customer characteristics and compare it to dedicated MPL and AHD services.

The third research question deals with the dynamic customer acceptance mechanism for MHDPL services. The focus is on the decision whether a customer should be offered an MPL or an AHD service. In dynamic customer acceptance, this decision must be made before the subsequent customers are known. Each decision has a significant impact and must therefore be made carefully. Various strategies are being developed to determine which service should be offered to each customer. The strategies range from simple, intuitive approaches to more sophisticated approaches that attempt to utilize realistic problem knowledge about dynamic customer arrivals. The managerial insights focus on the challenges of dynamic customer acceptance mechanisms for MHDPL services and on the potential of the proposed strategies.

The thesis is structured as follows. Chapter 2 deals with the comparison of MPL with traditional modes (RQ1). Chapter 3 deals with the combined MHDPL service, where MPL and AHD services can be performed by a single vehicle (RQ2). Chapter 4 analyzes dynamic customer acceptance mechanisms for MHDPL services (RQ3). Chapter 5 then summarizes the findings and provides an overview of possible starting points for further research questions.

Chapter 2

Mobile Parcel Lockers with Individual Customer Service

Kötschau, R., Soeffker, N., & Ehmke, J. F. (2023).
Networks, 82(4), 506-526.

Abstract The ongoing growth of e-commerce deliveries has led to a significant increase in last-mile delivery volumes. New technologies are being investigated to provide these deliveries efficiently and in a customer-friendly manner. A common practice is to use fixed parcel lockers (FPLs) to make deliveries independent from the presence of the customer as is the case in attended home deliveries (AHDs). FPLs are usually installed at key locations in cities, and customers can collect their package at any time once it has been delivered to this particular location. Mobile parcel lockers (MPLs) represent a new idea: they can be parked for temporary collection of items at different locations, keeping the pickup distance to the customer short and avoiding high infrastructure costs. However, customers need to collect their parcels within a restricted time window. This service is supposed to become especially efficient through autonomously operating vehicles that can move MPLs at low costs. In this article, we introduce the heterogeneous locker location problem to study the effects of fleets combining two of the services—FPLs, MPLs, AHDs—within one framework. In our comparison, we consider that customers may have different expectations regarding their maximum pickup distance as well as their temporal flexibility in accepting deliveries. A fixed fleet is applied to maximize the number of customers served, respecting individual customer preferences in terms of pickup distances and time windows. We evaluate the different delivery services regarding managerial insights on service quality and efficiency. In the experiments, we analyze the impact of structural demand differences and different fleet sizes, as well as the operational fleet utilization and the individual customer experience. Results show the potential to increase the number of customers served by about 14%–19% through the use of MPLs while considering individual customer preferences.

2.1 Introduction

Last-mile operations are the most challenging factor in terms of costs and planning when delivering goods to private customers (Joerss et al., 2016). Traditionally, customers have received deliveries directly at home, which is very convenient but involves uncertain arrival times and relatively high costs for logistics service providers. Consequently, the customer may not be present for accepting a delivery, which can result in failed deliveries and in further delivery attempts. Therefore, customers and logistics service providers are increasingly planning *attended home deliveries* (AHDs) through delivery time windows. This reduces the risk of failed deliveries and enables customers to plan their daily schedules more flexibly (Hübner et al., 2016). Besides these advantages, providers are restricted in their tour planning. For a survey on AHD related problems, the interested reader is referred to Dixit et al. (2019) or Kumar and Panneerselvam (2012).

To avoid the high cost of deliveries and failed deliveries, another common way of last-mile services is *fixed parcel lockers* (FPLs), which are filled by the logistics provider regularly, e.g. once every morning. FPLs promise a flexible pickup time for the customer, lower costs for logistics service providers (Seghezzi et al., 2022), a positive impact on ecological sustainability (Eliyan et al., 2021), and the possibility to manage the constantly increasing transportation volume better (Zurel et al., 2018). From a customer perspective, on the one hand, the customer has the advantage of being able to plan his or her daily schedule more flexibly; on the other hand, he or she needs to become active and travel to the parcel locker for pickup. A study by McKinsey showed that 70% of customers prefer low-cost delivery, but prefer home delivery to parcel lockers if the costs are the same (Joerss et al., 2016). For the provider, detours and failed deliveries are minimized. However, the decisive factors for the efficiency of FPLs are their location and capacity, which are determined by strategic planning (Deutsch & Golany, 2018; Faugère & Montreuil, 2020; Grabenschweiger et al., 2021).

A new and innovative service is provided by *mobile parcel lockers* (MPLs), which can change their location with or without a driver (Schwerdfeger & Boysen, 2022). In research and everyday life, this service has not received much attention yet. Some practical trials can be observed, e.g. by the postal service in Poland (InPost S.A., 2022) or by a start-up in Berlin, Germany (Krisch, 2019). Compared to the established FPLs, MPLs do not require an extra filling tour but can be filled directly at the depot. In addition, while the long-term planning of FPLs is associated with high initial investments, MPLs can be repositioned as often as required and therefore adapt to the current customer demand. However, for MPLs, the customers are given a certain time window to pick up their parcels. In contrast to ADHs, where short time windows are often preferred, it can be assumed that a longer time window is better for the customers when being serviced via MPLs, as it allows greater self-determination and flexibility. While MPLs are not prevalent neither in the real world nor in the literature yet, they are an attractive last-mile transportation mode as they combine the consolidation advantage of FPLs with the flexibility of AHDs.

As MPLs, FPLs, and AHDs show different characteristics, in this paper, we present the heterogeneous locker location problem (HLLP) to study the effects of fleets combining two of the services. Similar to Schwerdfeger and Boysen (2020), we also consider the services separately, but in contrast we study the operational problem of serving customers with a given fleet and analyze the effects of different fleets on service quality and delivery efficiency.

We compare last-mile deliveries with FPLs to a fleet of MPLs and to a fleet of AHD vehicles. We also investigate all combinations of two of these services to accommodate individual customer preferences. The combinations seem promising, as AHDs and FPLs represent two very different services: for AHDs, no pickup efforts are required, but delivery time windows can be very restrictive. FPLs reduce costs and increase flexibility, but require some pickup efforts by customers. MPLs as a new variant offer a blend of both features and can operationally adapt for a more diverse and efficient service.

We assume that customers have individual requirements regarding flexibility, and we try to accommodate these requirements as logistics service providers. We consider two types of customers with different levels of willingness to pick up their item (reflected by the allowed maximal distance to the pickup location) and with varying pickup flexibility (reflected by the length of pickup time windows). We evaluate the efficiency of the different services by means of the number of customers served and by how well we can accommodate the different customers' expectations.

Our contributions to the literature are as follows. First, we introduce the HLLP that maximizes the number of customers that can be served with a given fleet of service vehicles, where the fleet may consist of different types of service vehicles and where customers have individual preferences. Second, we analyze the effects of different service fleets on the achieved service level. While we cannot assess the investment costs of the new MPL technology, we particularly focus on managerial insights for a service provider as we analyze the service level for different customer types and fleet mixtures.

Chapter 2.2 summarizes the literature related to the considered delivery services. The HLLP is described in Chapter 2.3. Then, a mathematical model is presented and implementation details are discussed in Chapter 2.4. The design of the experiments and the results are presented in Chapter 2.5. We conclude in Chapter 2.6.

2.2 Related Literature

While the problem considered in this work is related to several other problem settings, this chapter primarily focuses on literature dealing with parcel lockers in an operational setting and the integration of customer preferences. The most relevant papers are listed in Table 2.1. Which types of delivery “services” were included in each study is marked with [+]. In “locations per customer”, it is indicated how many delivery locations a customer can specify. In the case of MPL, the pickup distance is used to determine whether a customer can be served at a particular location. Whether the pickup efforts are individual ([yes]) or the same for all customers ([no]) is shown in the “individ. MPL pickup effort” column. Customers specify either “no” time window at all or “any” number of time windows, which is given in the column “windows per customer”. If time windows are considered, the lengths are individual for different customers or the same for all customers, as indicated in the column “individ. window lengths”. The last column “solution” indicates whether the applied solution methods are of exact “[E]”, heuristic “[H]”, or matheuristic “[(M)H]” nature.

Orenstein et al., 2019 investigate FPLs in the context of customers selecting multiple locations and assigning priorities for delivery options. Several papers address the integration of FPLs and AHDs. In the medical field, Veenstra et al., 2018 let the logistics service provider choose the mode of delivery, i.e. whether to deliver via AHD or by FPL, which implies opening costs. Enthoven et al., 2020 investigate a case where customers can choose between home delivery or FPL in a two-echelon system. Mancini and Gansterer, 2021 add time windows and the third option that the customer would accept deliveries both via FPL and AHD. Grabenschweiger et al., 2021 always allow AHDs while the customers can specify several FPL locations as alternatives. The next extension is introduced by Dumez et al., 2021 and Tilk et al., 2021, where multiple locations can be provided with time windows and individual delivery location priorities.

research studies	services						options				solution
	single			combination							
	AHD	FPL	MPL	FPL &MPL	AHD &FPL	AHD &MPL	locations per customer	individ. MPL pickup effort	windows per customer	individ. window lengths	
Orenstein et al., 2019		+					any	–	none	–	H
Veenstra et al., 2018					+		one	–	none	–	E/H
Enthoven et al., 2020	+				+		one	–	none	–	H
Mancini and Gansterer, 2021					+		one / two	–	one	no	(M)H
Grabenschweiger et al., 2021					+		any	–	one	yes	H
Dumez et al., 2021	+				+		any	–	any	yes	H
Tilk et al., 2021					+		any	–	any	yes	E
Schwerdfeger and Boysen, 2020		+	+				any	no	any	yes	E
Schwerdfeger and Boysen, 2022		+	+				any	yes	any	yes	H
Wang, Bi, and Chen, 2020				+			one	no	one	yes	(M)H
Wang, Bi, Lai, and Chen, 2020				+			one	no	none	–	E
This work	+	+	+	+	+	+	one	yes	one	yes	E

TABLE 2.1: Literature comparison

Schwerdfeger and Boysen present the idea of MPLs for the first time in Schwerdfeger and Boysen, 2020 and address various realization concepts in Schwerdfeger and Boysen, 2022. Schwerdfeger and Boysen, 2020 allow customers to specify multiple locations with time windows. They evaluate the required fleets to serve all customers with only FPL or MPL, respectively, for different pickup efforts. In Schwerdfeger and Boysen, 2022, driver assignments and incurred costs are incorporated to investigate different MPL concepts, such as mounted or loaded and fixed drivers, swap drivers, or autonomous driving. Wang, Bi, and Chen, 2020 consider a pickup and delivery problem, where customers are served using either FPLs or MPLs. They consider single customer locations and add pickup services to the delivery problem. This work has been further extended to incorporate stochastic demand in Wang, Bi, Lai, and Chen, 2020.

Recently, individual customer preferences have become more important in the context of last-mile deliveries. For instance, customers can now define several delivery options (Grabenschweiger et al., 2021; Mancini & Gansterer, 2021; Schwerdfeger & Boysen, 2020; Tilk et al., 2021) and even assign individual priorities to these options (Dumez et al., 2021; Orenstein et al., 2019). In this paper, however, we assume that a customer does not want to provide his or her entire daily schedule to the provider. Instead, the customers have precise but individual preferences as to when they want to be served, over what length of time, and with how much pickup effort.

We introduce the HLLP to integrate FPL, MPL, and AHD services into a single problem. We maximize the number of customers served, for a given fleet, respecting individual customer preferences. This enables a fair comparison of how the single and combined services perform in different situations regarding structural demand and customer preferences. In order to efficiently address our customers' preferences, we consider six variations of services, namely the three single services FPL, MPL, and AHD, as well as the combinations FPL&MPL, AHD&FPL and AHD&MPL. With this work, we therefore contribute to the literature by considering not only single services or the AHD and FPL combination, but additionally the MPL combinations. Further, we consider individual customer preferences regarding the individual pickup effort.

2.3 Heterogeneous Locker Location Problem

In this chapter, we present the HLLP that incorporates not only different types of delivery services but also considers individual customer preferences. The aim is to serve as many customers as possible with a given set of delivery resources considering the technological constraints and individual customer preferences. Customer preferences are represented by spatio-temporal restrictions. For spatial restrictions, each customer has a maximum pickup distance from the customer's home to a pickup location, reflecting the maximum individual effort the customer is willing to invest for a pickup. For temporal restrictions, a customer specifies when and over what length of time the pickup or delivery should occur. We assume the service provider knows the customer preferences perfectly.

The services we investigate are AHDs, FPLs, MPLs, and any combination of two of these. The first is found in the majority of the literature and corresponds to established industry practice. Here, a vehicle with a driver handles the delivery to the customer within a specified time window. The second are FPLs, which do not move and have exactly one location where customers can pick up their delivery as soon as it is available. Contrasting FPLs, MPLs are parcel lockers which can change their location during the day. The MPLs stop within reach of the customers so that they can pick up their delivery at any time within their desired pickup time window. An MPL service can be carried out in a variety of ways. Schwerdfeger and Boysen, 2022 compare the costs when the MPLs are realized with a driver, autonomously or as a trailer.

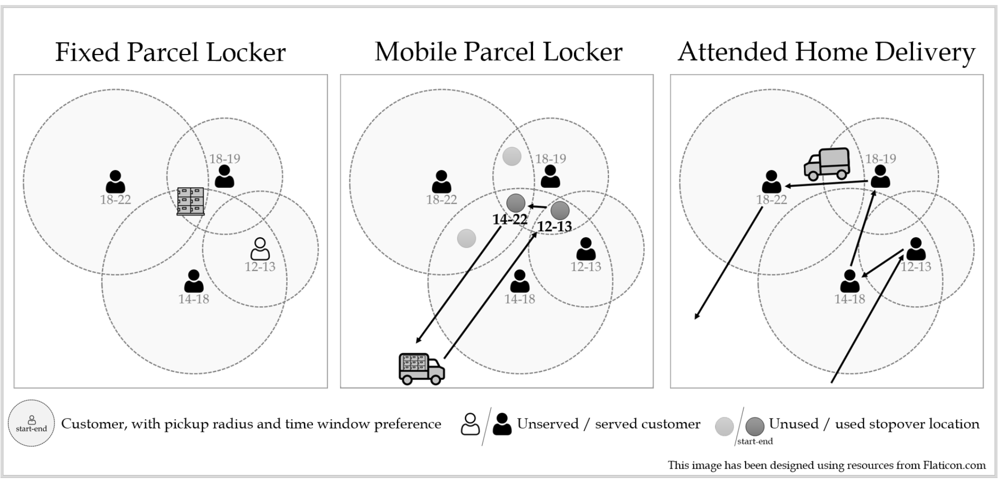


FIGURE 2.1: Example for FPL, MPL, and AHD

Figure 2.1 shows examples of problem realizations for FPLs, MPLs, and AHDs with one single (fixed) locker or truck. The four customers have an individual maximum willingness of pickup effort (indicated by the pickup radius) and expect deliveries in individual customer time windows (indicated by the time window). The FPL has one dedicated location for pickup, which is not reachable by one customer due to the restricted pickup distance. The MPL has four predefined parking locations to potentially perform stopovers. In the solution, two stopovers are chosen to reach all customers and fulfill their time window constraints. These stopovers are defined by a parking location and a period of time. AHD simply visits each of the customer locations in their respective time windows. In the end, AHD and MPL can serve all customers – AHD with four stopovers and MPL with two stopovers –, and FPL can serve three customers. To illustrate one possible combination of services: If FPL and AHD operate together, AHD can, for example, serve customers who could not be reached via FPL, as shown in Figure 2.2.

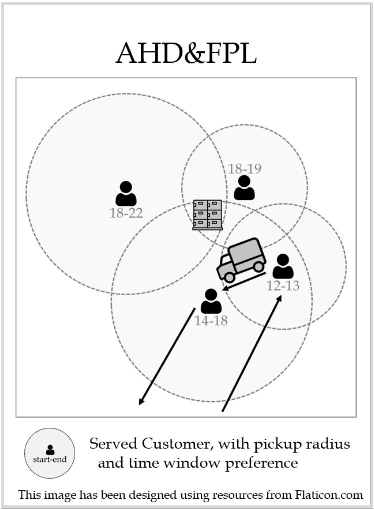


Figure 2.2: Example for the combination of AHD and FPL

To solve the HLLP, we exploit that each service can be modeled as a vehicle that stops at different locations for a different amount of time, following a feasible routing schedule. In this way, we can represent all services simultaneously in the model with only minor changes, i.e. we refer to all vehicles as lockers that operate in a specified "mode". For each locker, the solution includes a series of stopovers to be made in order to maximize the number of customers served. In the case of combinations, the optimal solution will determine which customers should be served with which of the available service modes.

In the simplest case, FPLs “stop” at one location for the whole day. MPLs can stop at several locations multiple times for an indefinite period of time. AHDs stop only once at each location for exactly the same relatively short period of time. FPLs and MPLs can serve many customers at the same time from one location, whereas AHDs serve each customer individually. However, AHDs have a rather short service time to hand over the delivery, whereas MPLs have to be available for a longer period of time in order to allow the customers to pick up the deliveries.

Schwerdfeger and Boysen (2020) discuss conceptually similar problem settings such as the location routing problem and present the time discretization-based mobile locker location problem. The discrete-time approach is closely related to the HLLP and helps to find a representative solution in a suitable time. However, their problem focuses on the optimal fleet size from a strategic perspective. We need to adapt it to the HLLP, which analyzes the operational perspective with predefined fleets. We consider customers with individual preferences, which includes having the parcel lockers available to the customer for the entire pickup time window to allow the desired flexibility, which is different from the current literature. Furthermore, we add the functionality to handle lockers of multiple modes and let the model choose the optimal allocation of customers by efficiently using the available resources.

2.4 Mathematical Model

The problem described in Chapter 2.3 can be formalized as a Mixed Integer Program (MIP). The most important concepts are the modes, the stopovers, and the drives. Each locker is assigned to a mode, such as FPL, MPL, or AHD. Stopovers and drives are time-discrete characteristics that describe all possibilities of the lockers to perform actions. If a locker is assigned to a stopover, it is specified that the locker remains at the location of the stopover from the start time of the stopover until the end time of the stopover. In addition, the assigned drives describe the movements between the stopovers. In the HLLP, lockers are only allowed to use stopovers and drives that match the locker mode. In addition, the locker fleet is fixed and the number of customers accepted is to be maximized.

2.4.1 General Model

In the following, we provide a detailed explanation of the MIP and then address its parameterization for different locker modes. The required sets and parameters are summarized in Table 2.2. All locations p which are available for a stopover independent of the temporal restrictions are contained in the set P. In addition, γ represents another location at which each route of a locker must start and end. This could be a depot location, but in the following we consider a pseudo location to initialize the locker without travel time.

The spatial and temporal assignment of the locations is realized by the stopovers. The set S contains all relevant possibilities to perform a stopover. Each stopover $s \in S$ is represented by a specific stopover location p_s^S, a stopover start time $\underline{t}_s^S$, a stopover end time $\overline{t}_s^S$, and a stopover mode m_s^S. The mode represents the restriction that a stopover can only be used by a locker type of the corresponding service, i.e. FPL, MPL, or AHD.

The possibility to move from one stopover to another stopover is expressed by the drives. A drive d of the set of drives D is represented by a drive mode m_d^D, a drive start location $\underline{p}_d^D$, a drive end location $\overline{p}_d^D$, and a drive time t_d^D, which specifies when the locker departs from the start location. Besides drives between stopovers, additional drives to and from the location γ have to be defined to enable a complete tour. For each $p \in P$, drives from γ with arrival corresponding to $\underline{T}_p^{S\gamma}$ and drives to γ with departure corresponding to $\overline{T}_p^{S\gamma}$ are created. In addition, the parameter $\sigma_{\underline{p},\overline{p},t}$ indicates the departure time of a particular drive which starts from $\underline{p}$ and arrives on time t at $\overline{p}$.

The complete time horizon is given in the set T, which contains all discrete points in time. For example, with a time horizon of 1:00 to 5:00, discretized in 1 hour intervals, T contains {1:00, 2:00, 3:00, 4:00, 5:00}. For each available location $p \in P$, one subset of T represents the times at which at least one stopover starts $\underline{T}_p^S$ and at least one stopover starts that does not have a prior stopover at the same location $\underline{T}_p^{S\gamma}$. In the same way, $\overline{T}_p^S$ represents points in time at which at least one stopover ends and $\overline{T}_p^{S\gamma}$ when at least one stopover ends that does not have a subsequent stopover at the same location. The customers c are given in set C. The binary parameter $b_{c,p,\underline{t},\overline{t}}$ indicates whether it is possible to serve a customer c by a stopover at location p in the time interval $\underline{t}$ to $\overline{t}$. Finally, the lockers are defined by the set L and each locker l has a mode m_l^L and a capacity κ_l.

$p \in P$	$\rightarrow$ Set of available parking locations
γ	$\rightarrow$ Pseudo (depot) location, representing start and end of a tour
$s \in S$	$\rightarrow$ Set of stopovers, for each exists: mode (m^S), parking location (p^S), start time ($\underline{t}^S$), and end time ($\bar{t}^S$)
m_s^S	$\rightarrow$ Mode of stopover s
p_s^S	$\rightarrow$ Parking location of stopover s
$\underline{t}_s^S$	$\rightarrow$ Start time of stopover s
$\bar{t}_s^S$	$\rightarrow$ End time of stopover s
$d \in D$	$\rightarrow$ Set of drives, for each exists: mode (m^D), starting location ($\underline{p}^D$), final location ($\overline{p}^D$), and departure time (t^D)
m_d^D	$\rightarrow$ Mode of drive d
$\underline{p}_d^D$	$\rightarrow$ Parking location where drive d starts
$\overline{p}_d^D$	$\rightarrow$ Parking location where drive d ends
t_d^D	$\rightarrow$ Departure time of drive d
$\sigma_{\underline{p},\overline{p},t}$	$\rightarrow \sigma \in \mathbb{Z}$ is the departure time at $\underline{p}$ to arrive in $\overline{p}$ at time t
$t \in T$	$\rightarrow$ Set of discrete points in time
$\underline{T}_p^S$	$\rightarrow$ Set of subsets of T where a stopover-start exists at parking location p
$\overline{T}_p^S$	$\rightarrow$ Set of subsets of T where a stopover-end exists at parking location p
$\underline{T}_p^{S\gamma}$	$\rightarrow$ Set of subsets of T where a stopover-start exists at parking location p, where no prior stopover exists at p - enables a possible start drive, from γ
$\overline{T}_p^{S\gamma}$	$\rightarrow$ Set of subsets of T where a stopover-end exists at parking location p, where no successive stopover exists at p - enables a possible end drive, to γ
$c \in C$	$\rightarrow$ Set of customers
$b_{c,p,\underline{t},\bar{t}}$	$\rightarrow b \in \{0,1\}$ indicates if a customer c can be served by a stopover with: $p, \underline{t}, \bar{t}$
$l \in L$	$\rightarrow$ Set of lockers, for each exists: mode (m^L) and capacity (κ)
m_l^L	$\rightarrow$ Mode of locker l
κ_l	$\rightarrow$ Capacity of locker l

Table 2.2: Sets and Parameters

Based on the given parameters, three decisions are made. The variable $y_{l,c}$ decides if customer $c \in C$ should be served by locker $l \in L$. The two remaining decisions are which stopovers $x^S_{l,p,\underline{t},\bar{t}}$ and which drives $x^D_{l,\underline{p},\bar{p},t}$ are used by the respective lockers $l \in L$. The mathematical model is as follows.

$$
\begin{array}{lll}
(1) & \max \sum\limits_{l \in L} \sum\limits_{c \in C} y_{lc} & \\
s.t. & & \\
(2) & \sum\limits_{l \in L} y_{l,c} \leq 1 & \forall c \in C \\
(3) & \sum\limits_{c \in C} y_{l,c} \leq \kappa_l & \forall l \in L \\
(4) & \sum\limits_{\substack{s \in S \\ m^S_s = m^L_l}} b_{c,p^S_s,\underline{t}^S_s,\bar{t}^S_s} * x^S_{l,p^S_s,\underline{t}^S_s,\bar{t}^S_s} \geq y_{lc} & \forall l \in L, c \in C \\
(5) & \sum\limits_{\substack{s \in S \\ m^S_s = m^L_l \\ p^S_s = p \\ \underline{t}^S_s = \underline{t}}} x^S_{l,p,\underline{t},\bar{t}^S_s} = \sum\limits_{\substack{d \in D \\ m^D_d = m^L_l \\ \bar{p}^D_d = p \\ t^D_d = \sigma_{\underline{p}^D_d,p,\underline{t}}}} x^D_{l,\underline{p}^D_d,p,t^D_d} & \forall l \in L, p \in P, \underline{t} \in \underline{T}^S_p \\
(6) & \sum\limits_{\substack{s \in S \\ m^S_s = m^L_l \\ p^S_s = p \\ \bar{t}^S_s = \bar{t}}} x^S_{l,p,\underline{t}^S_s,\bar{t}} = \sum\limits_{\substack{d \in D \\ m^D_d = m^L_l \\ \underline{p}^D_d = p \\ t^D_d = \bar{t}}} x^D_{l,p,\bar{p}^D_d,\bar{t}} & \forall l \in L, p \in P, \bar{t} \in \overline{T}^S_p \\
(7) & \sum\limits_{p \in P} \sum\limits_{t \in \overline{T}^{S\gamma}_p} x^D_{l,p,\gamma,t} \leq 1 & \forall l \in L \\
(8) & y_{l,c} \in \{0,1\} & \forall l \in L, c \in C \\
(9) & x^S_{l,p,\underline{t},\bar{t}} \in \{0,1\} & \forall l \in L, p \in P, \underline{t} \in T, \bar{t} \in T \\
(10) & x^D_{l,\underline{p},\bar{p},t} \in \{0,1\} & \forall l \in L, \underline{p} \in P \cup \{\gamma\}, \bar{p} \in P \cup \{\gamma\}, t \in T
\end{array}
$$

The objective function (1) maximizes the number of customers that are served by the available locker capacity. Constraint (2) ensures that each customer is served at most once. Each locker can serve a maximum of customers according to its capacity (3). For a customer to be served by a locker, the locker must stop at a compatible stopover according to the customer's location and time window preferences. Whether a customer and a location are compatible in a certain time window is identified by the parameter b. In addition, the locker must also be compatible with the stopover according to its delivery mode. Constraint (4) checks these compatibilities. Constraints (5) and (6) represent the flow conditions, i.e., if a locker uses a stopover, it must also drive to and from this stopover. To ensure that each locker is used only once, (7) limits the number of drives of a locker to the pseudo location γ to a maximum of 1. The binary decision variables are defined in the equations (8–10). Constraint (8) defines which locker $l \in L$ serves which customer $c \in C$; (9) defines at which stopover of S a locker l dwells; and Constraint (10) represents which drives of D a locker l uses. In Constraints (9) and (10), stopovers and drives are associated with each other following their unique properties regarding location and time window.

2.4.2 Model Adaptations for FPL, MPL, AHD

In the HLLP, each locker is operated in one of the three modes FPL, MPL, or AHD. The stopovers as well as the associated drives belong to a certain mode, which expresses that they can be used by lockers of the same mode. In the following, we discuss how the available stopovers and drives are defined for the different modes and which customers can be served by a specific stopover.

For FPL, one stopover is created at each FPL location $p \in P_{FPL} \subseteq P$. These stopovers start at the earliest possible working time $\underline{w}$ and end at the latest possible working time $\overline{w}$ according to the considered time horizon. Hence, for each FPL location $p \in P_{FPL}$, we create a stopover $(\text{FPL}, p, \underline{w}, \overline{w})$, which means a stopover usable by lockers in FPL mode, at location p from $\underline{w}$ to $\overline{w}$. Since these lockers are permanently located in one place, no drives are needed, except for the start and end at the pseudo location γ for all available locations. Moreover, to define the parameter b, the maximum pickup distances need to be checked for each customer and stopover combination. As each FPL stopover persists the entire time horizon, the customer time windows do not need to be verified.

For MPL, at each MPL location $p \in P_{MPL} \subseteq P$, multiple stopovers are created in the considered working time horizon ($\underline{w}$ until $\overline{w}$) according to the discrete time interval z. This means that we create a stopover $(\text{MPL}, p, start, end)$ for each $p \in P_{MPL}$ with $start$ as an element from the set of $\underline{w}$ to $\overline{w} - z$ with z interval steps and end as an element from the set of $start + z$ to $\overline{w}$ with z interval steps. For example, with $\underline{w}$ = 1:00, $\overline{w}$ = 4:00 and z = 1 hour, the time windows at any given location would be {1:00–2:00, 1:00–3:00, 1:00–4:00, 2:00–3:00, 2:00–4:00, 3:00–4:00}. In addition to the drives from and to the pseudo location γ, further drives will be created between two stopovers whenever they can be reached in time. Travel times are rounded up to the next discrete point in time. In order for a customer to be served by an MPL stopover, the location must be accessible within the allowed pickup distance, and the customer time window must be within the stopover time window.

For AHD, the stopovers are defined differently than those for MPL. In this context, the potential AHD locations are the customer locations. Hence, at each AHD location $p \in P_{AHD} \subseteq P$, the earliest possible time $\underline{w}_{c(p)}$ and latest possible time $\overline{w}_{c(p)}$ are now determined by the associated customer time window. According to the discrete-time interval z, the stopovers are defined exclusively within this customer time window ($\underline{w}_c$ until $\overline{w}_c$). Thus, stopovers $(\text{AHD}, p, start, end)$ are created for each $p \in P_{AHD}$ with $start$ as an element from the set of $\underline{w}_{c(p)}$ to $\overline{w}_{c(p)} - z$ with z interval steps and end equals $start + z$. For example, a discrete time interval of 10 min and a one-hour customer time window from 1:00 until 2:00 results in six possible stopover time windows {1:00-1:10, 1:10-1:20, 1:20-1:30, 1:30-1:40, 1:40-1:50, 1:50-2:00}. The drives are set identically as in the case of the MPL mode. In order for a customer to be served during an AHD stopover, the stopover must occur at the customer's location and fit within the customer's time window.

Table 2.3 provides examples of stopover calculations. To keep the example simple, no travel times are involved. Therefore, the arrival time is equal to the departure time for all generated σ. An example with one hour of travel time between two points is σ(MPL, p_1, p_2, 3:00) = 2:00.

The predefined stopovers and drives are essential parts of the presented model. The number of stopovers and drives, along with the available lockers, are decisive for the computational complexity of the problem. To reduce the problem size without losing optimality, we apply preprocessing as presented by Schwerdfeger and Boysen, 2020. This includes reducing stopovers when they are no option for the current customers as well as eliminating non-essential drives. For instance, in our example in Table 2.3, we could remove all AHD drives that do not include γ, as we only need to address each customer once. Additionally, in the given example with only one MPL location and no possibility to move to any other location, we could remove all MPL stopovers except the stopover (MPL, p, 1:00, 4:00).

	FPL-Location p	MPL-Location p	AHD-Location p
available from	$\underline{w} = 1{:}00$	$\underline{w} = 1{:}00$	$\underline{w}_{c(p)} = 1{:}00$
available until	$\overline{w} = 4{:}00$	$\overline{w} = 4{:}00$	$\overline{w}_{c(p)} = 2{:}00$
discretization	-	$z = 1$hr	$z = 10$min
nr of stopovers	1	6	6
generated stopovers	(FPL, p, 1:00, 4:00)	(MPL, p, 1:00, 2:00) (MPL, p, 1:00, 3:00) (MPL, p, 1:00, 4:00) (MPL, p, 2:00, 3:00) (MPL, p, 2:00, 4:00) (MPL, p, 3:00, 4:00)	(AHD, p, 1:00, 1:10) (AHD, p, 1:10, 1:20) (AHD, p, 1:20, 1:30) (AHD, p, 1:30, 1:40) (AHD, p, 1:40, 1:50) (AHD, p, 1:50, 2:00)
nr of drives	2	4	7
generated drives	(FPL, γ, p, 1:00) (FPL, p, γ, 4:00)	(MPL, γ, p, 1:00) (MPL, p, p, 2:00) (MPL, p, p, 3:00) (MPL, p, γ, 4:00)	(AHD, γ, p, 1:00) (AHD, p, p, 1:10) (AHD, p, p, 1:20) (AHD, p, p, 1:30) (AHD, p, p, 1:40) (AHD, p, p, 1:50) (AHD, p, γ, 2:00)
nr of σ	2	4	7
derived σ	(FPL, γ, p, 1:00) = 1:00 (FPL, p, γ, 4:00) = 4:00	(MPL, γ, p, 1:00) = 1:00 (MPL, p, p, 2:00) = 2:00 (MPL, p, p, 3:00) = 3:00 (MPL, p, γ, 4:00) = 4:00	(AHD, γ, p, 1:00) = 1:00 (AHD, p, p, 1:10) = 1:10 (AHD, p, p, 1:20) = 1:20 (AHD, p, p, 1:30) = 1:30 (AHD, p, p, 1:40) = 1:40 (AHD, p, p, 1:50) = 1:50 (AHD, p, γ, 2:00) = 2:00

Table 2.3: FPL/MPL/AHD example with only one location

2.5 Computational Study

We investigate the performance of the individual delivery services and the service combinations through a computational study. First, the experimental setup is described. This is followed by a detailed presentation and discussion of the experimental results, differentiated according to instance, locker, and service characteristics.

2.5.1 Experimental Setup

We consider 100, 200, and 400 customers distributed in an area of 10 km in length and width. Customer locations stem from the VRPTW instances by Solomon, 1987 and Gehring and Homberger, 1999. They are tightly clustered in C1, less tightly clustered in C2, randomly distributed in R1, and a mixture of R1 and C1 locations is chosen in RC1. The distribution of 200 customer locations for these instance types are shown in the appendix, see Figure A.1.

We define two customer types with different preferences: *restrictive* and *flexible* customers. Based on the above instances, these are randomly assigned with equal probability. Restrictive customers have a restrictive willingness for pickup efforts – i.e. only accept closer stopovers – and have shorter pickup time windows. Flexible customers are willing to travel a bit further to their pickup location in exchange for more flexibility in terms of a longer pickup time window. To clearly distinguish between these types of customers, we have made the following assumptions: Restrictive customers are willing to travel a maximum of 0.5 km and are offered pickup time windows of 1 hour length. Flexible customers have 4 hours to pick up their items and are willing to travel up to 2.5 km to the pickup location. The restrictive customers have a very limited pickup range, whereas the flexible customers demand a considerable amount of time for pickup. These two customers with their own unique characteristics allow us to observe how the different services behave under certain circumstances.

The temporal distribution of time windows is based on the real-world data by Köhler and Haferkamp, 2019 for unequal demand in metropolitan areas. The available time windows span 12 hours (10:00 to 22:00). Since Köhler & Haferkamp considered only two-hour time windows, we distribute/aggregate the demand probabilities equally for one-hour time windows and four-hour time windows. The combined probabilities for customer type and time window preferences are shown in Figure 2.3. As an example, the probability for a time window from 10:00 to 12:00 according to Köhler & Haferkamp is 25%. Half of these contribute to flexible 10-14 customers and the remaining 12.5% are divided equally between 10-11 and 11-12 restrictive customers (6.25% and 6.25%). Based on these distributions, 10 instances were randomly generated for each of the four instance types in the time period from 10:00 to 22:00.

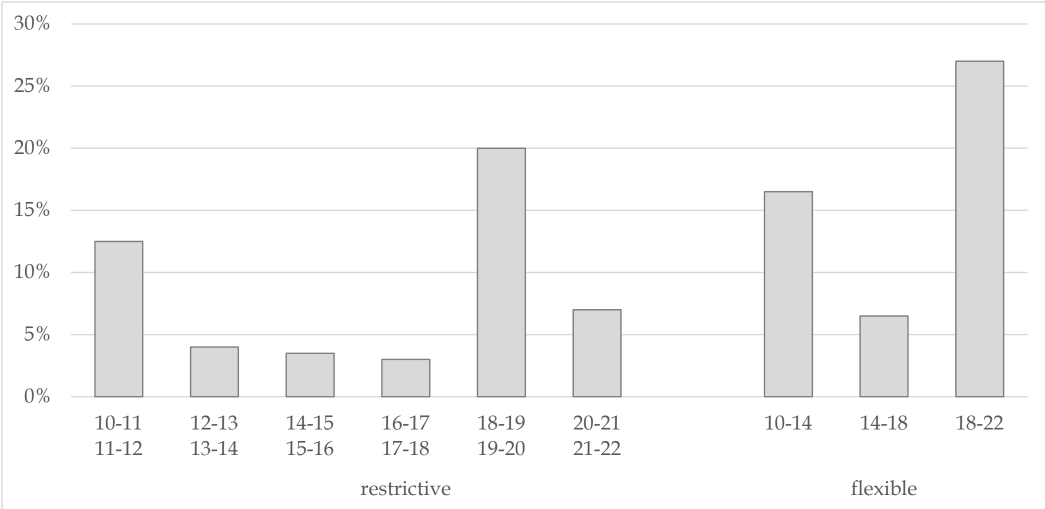


Figure 2.3: Probabilities for customer type and time window preferences

The above combinations of four instance types and ten customer scenarios are examined for each delivery service, namely FPL, MPL, AHD, and the combinations FPL&MPL, AHD&FPL, and AHD&MPL. The number of available lockers is varied and set to four, eight, or ten in each service. The total capacity of the delivery resources corresponds to the number of customers and is distributed equally among the locker-based services. Given a total capacity of 100/200/400 compartments, this results in four big lockers with 25/50/100 compartments, eight medium lockers with 13/25/50 compartments, and ten small lockers with 10/20/40 compartments, respectively. For the combined services, resources are split 50/50, i.e. two fixed and two mobile lockers, four fixed and four mobile lockers, etc.

We assume that customer locations can serve as potential stopover locations for each service. Inherently, AHD stopovers are permitted at every customer location. For FPL and MPL, we use a k-medoid clustering algorithm to identify smaller sets of locker locations. To determine appropriate FPL locations, customer locations were clustered for each instance type separately (C1, C2, R1, RC1) according to the number of available lockers (2, 4, 5, 8, 10). As a result, cluster centers (here: locker locations) have been selected such that the overall customer travel distance is minimal for each instance type and number of available lockers. To identify reasonable locations for MPL, 50 clusters were generated. Thus, it is possible to have a stopover at 25% of the customer locations, and each stopover will cover an average of 2 km^2.

The duration of a stopover varies according to the delivery service. Since FPLs have a fixed location, they offer pickup services across the whole planning horizon (12 hours). MPLs can stay at a stopover for 1 to 12 hours. For AHD, the time required for stopovers is set to 12 minutes as various service times ranging from 5 to 20 minutes can be found in the literature (Campbell & Savelsbergh, 2006; Tilk et al., 2021). The travel times for repositioning for AHD and MPL services are based on a travel speed of 30 km per hour and are discretized in 12-min intervals. The discretization is based on experiments conducted with 200 customers to achieve an acceptable calculation time of below one hour, see Table A.1 in the appendix for details. For a customer to be served by MPL, the locker must cover the entire pickup time window, i.e. a minimum stopover of one hour is required. Hence, if a MPL changes its location, it is not available for service in the corresponding one-hour time slot. Therefore, observed times for repositioning may be higher than the actual times, especially for the MPL service.

All experiments were performed on a computational cluster with Xeon-G 6226R processors at 2,9 GHz, using up to five cores and 384 GB memory. For 68 of 4320 calculations, no solutions could be found within the maximum runtime of 1 hour, for details see Table A.2; these 68 cases were excluded from the reported results.

2.5.2 Results and Discussion

With our computational results, we focus on managerial insights including service quality and efficiency of the individual delivery services, in particular the impact of restrictive and flexible customers. The *number of served customers* (objective function) will be the main indicator for efficiency. In addition, efforts for repositioning of the lockers (repositioning time), the utilization of the locker capacities, as well as the efforts required for pickup (distances) and acceptance probabilities for different time windows will be examined. First, Chapter 2.5.2.1 provides an overview of the results by generally comparing the performance of the investigated services with 100, 200, and 400 customers. This is followed by a more detailed discussion of the impact of the instance type characteristics (Chapter 2.5.2.2), the utilized lockers (Chapter 2.5.2.3), and the service characteristics for the different customer types (Chapter 2.5.2.4) with 200 customers. Finally, we briefly discuss a more conservative scenario in Chapter 2.5.2.5.

2.5.2.1 Overview

To assess the potential of each of the separate services and service combinations, Figure 2.4 presents the proportion of served customers aggregated across all instances and locker variations. There are clear differences despite the same number of lockers being used. For example, FPL can handle only 56% of customers on average, whereas the combination of AHD&MPL can achieve 81%. Among single services, AHD performs best with 71% accepted customers, followed by MPL with 66%, and FPL with 56% served customers. FPL&MPL is exactly in the middle of the separate counterparts. Since the lockers in MPL can also remain in one place all day and have more locations to choose from, the combination with FPL leads to a “downgrade” of certain lockers. In contrast, AHD&FPL and AHD&MPL can handle more customers than the separate services. Here again, the combination with MPL is superior to the FPL combination. In sum, we see that FPL and MPL services are inferior to AHD in terms of customer acceptance. The combination of different services leads to a higher number of customers to be accepted.

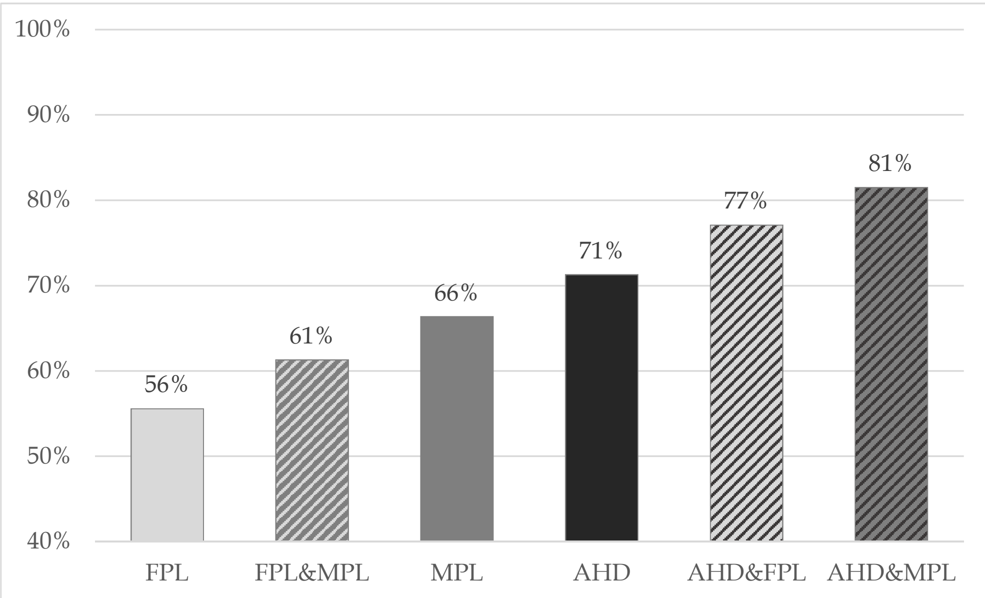


FIGURE 2.4: Accepted customers for different services

Figure 2.5 compares how the services perform for 100, 200, or 400 customers. For each service, the proportion of customers served is highest for 100 customers and lowest for 400 customers. The differences are rather small for the FPL and MPL services. FPL can serve from 54% of the 400 customers to 57% of the 100 customers, whereas MPL achieves between 60% and 74%. In contrast, AHD&FPL and AHD&MPL show a significant performance difference with 63% and 90% as well as 66% and 95%. However, the largest spread is seen for AHD, where 95% of the 100 customers can be served and only 45% of the 400 customers. With AHDs, a higher number of customers is more difficult to serve because, unlike with parcel lockers, there is no possibility of serving several customers at the same time.

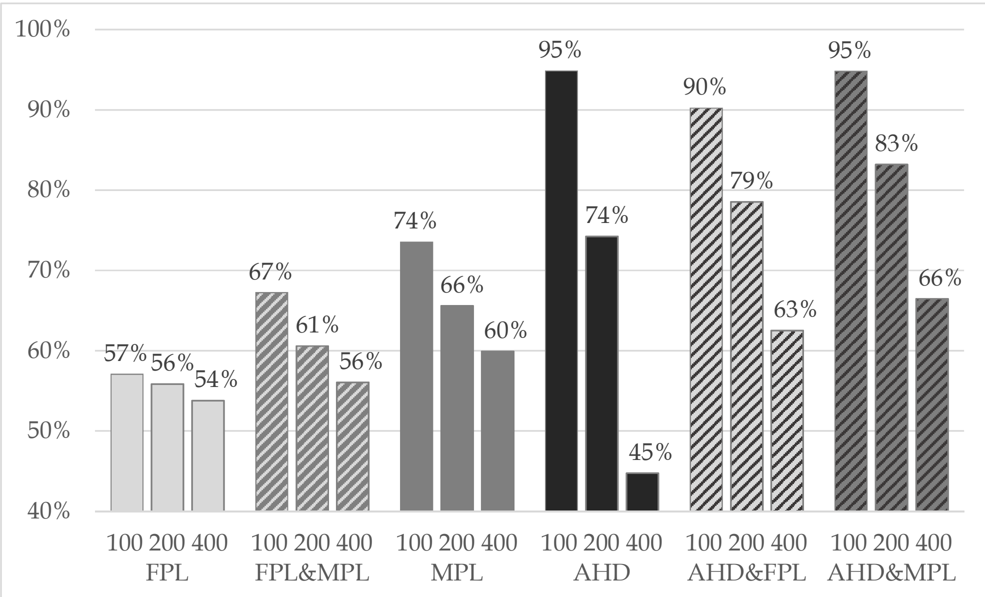


Figure 2.5: Accepted customers for different services and number of customers

When there are 100 customers, in many cases all customers are served. When there are 400 customers, for 38 instances no solutions could be found. While 33 of these instances involve AHDs, it can be observed that the upper bound of maximum number of customers served for all AHD instances with 400 customers is 48%. For these reasons, in the following, we will focus on the details of instances with 200 customers.

2.5.2.2 Instance Characteristics

We now assess the results of the instance types for 200 customers in detail. Figure 2.6 compares the proportion of customers served for the different services across the four different instance types.

Generally, customer acceptance follows the same ranking across all instances as observed before (see Figure 2.4), with FPL showing the smallest acceptance rates (51–59%) and AHD&MPL showing the highest acceptance rates (81–85%). However, FPL and MPL work much better for clustered instance types than for random customer locations, as represented by R1. AHD-based variants can adapt best to varying spatial characteristics and accommodate at least 74% of customers in every instance type. Overall, the impact of spatial instance characteristics can be seen clearly for MPL: the more clustered the customer locations, the higher the potential of customer acceptance. Combinations of AHD and MPL can adapt to any of these characteristics.

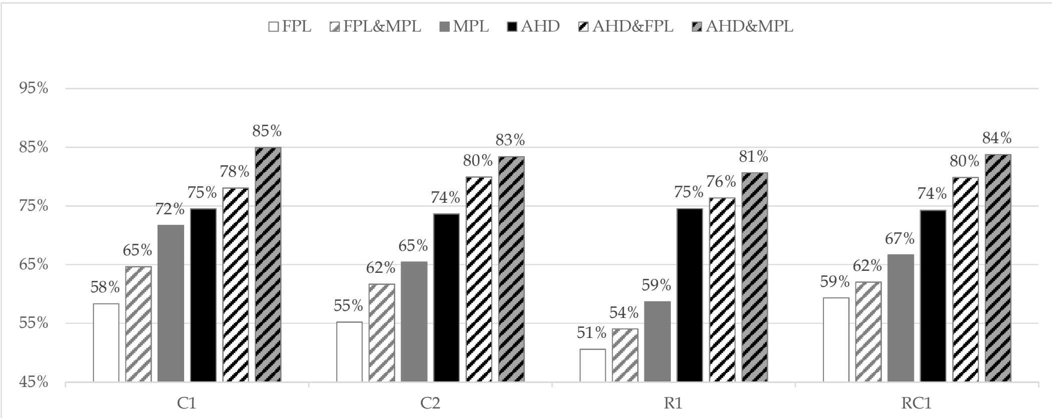


FIGURE 2.6: Accepted customers for different services and instance types

A detailed look into why customers could not be accepted is provided by Figure 2.7 for separate services and Figure 2.8 for service combinations. “Restrictive” and “flexible” represent the type of accepted customers, while “time/capacity” and “distance” represent the reasons for rejection. The rejection due to time window or capacity is based on resource constraints, while the other category highlights the customers that have been impossible to be reached due to their limited willingness of pickup efforts.

For FPL, we see a large chunk of rejections related to infeasible pickup distances, while there are barely rejections due to capacity issues. Across all instance types, the unserved customers are therefore predetermined by their pickup distances. The large majority of accepted customers are flexible customers. For both FPL and MPL, the rejected customers are determined by the customer's pickup distance. A higher number of accepted customers arises from the more flexible MPLs. No differences are apparent in AHD, where rejections only happen due to time or capacity issues, but not due to limited pickup distances.

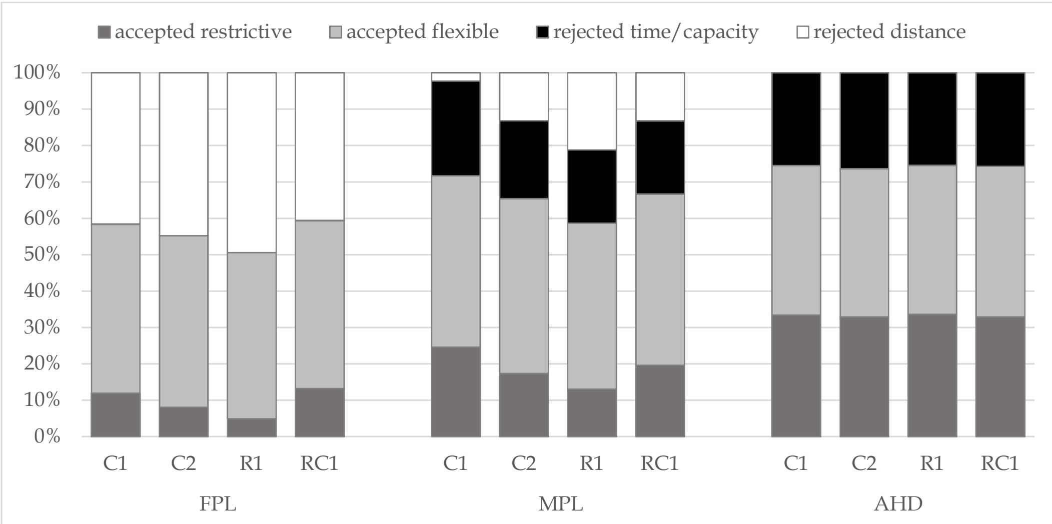


Figure 2.7: Types of accepted/rejected customers for single services/instance types

As can be seen in Figure 2.8, FPL&MPL reduces the number of customers that have to be rejected by FPL due to insufficient distance. Adding the AHD option to FPL or MPL helps transforming customers rejected due to pickup distance into accepted customers; again, only time/capacity-based rejections occur for AHD&FPL and AHD&MPL with almost no variations across the instance types.

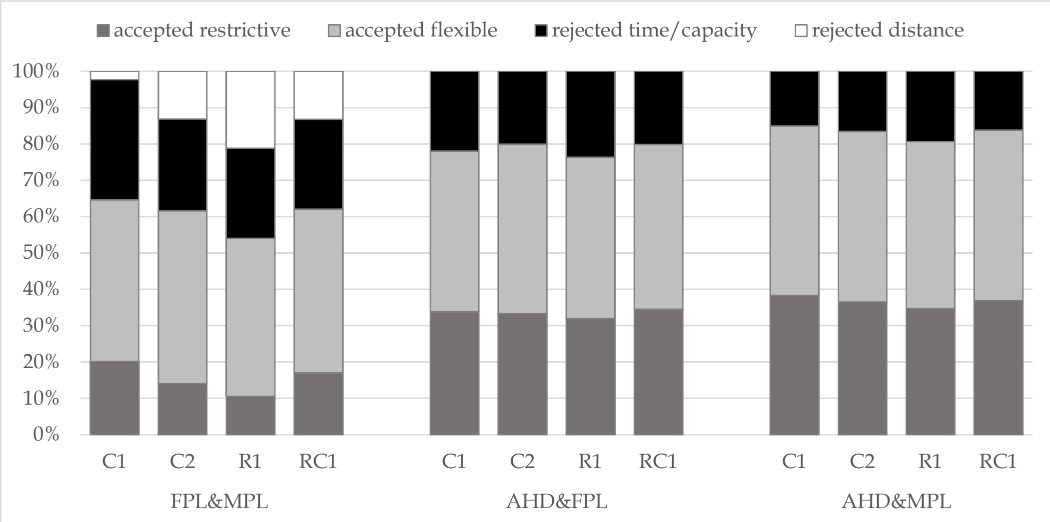


FIGURE 2.8: Types of accepted/rejected customers for combined services/instance types

2.5.2.3 Locker Characteristics

We now investigate the impact of different numbers of lockers and their compartments for 200 customers. To this end, we consider four big lockers with 50 compartments, eight medium lockers with 25 compartments, and ten small lockers with 20 compartments, as described in Chapter 2.5.1. First, we show how this changes the number of served customers. Furthermore, we investigate how operational efficiency varies in terms of repositioning times of MPL and AHD as well as utilization of FPL, MPL, AHD, and their combinations.

Customer Acceptance. Figure 2.9 shows the proportion of accepted customers for the separate and combined services with four big, eight medium, and ten small lockers, respectively. Overall, despite the total capacity remaining the same, we see an increasing yet saturating trend in all cases: More and smaller lockers allow for finer spatial dispersion of logistics resources and can hence meet customer demand better. While a significant proportion of flexible customers can already be served with four big lockers, regardless of the actual service, eight medium-sized lockers allow customer acceptance of over 90% for services combined with AHD. Restrictive customers benefit most from additional lockers.

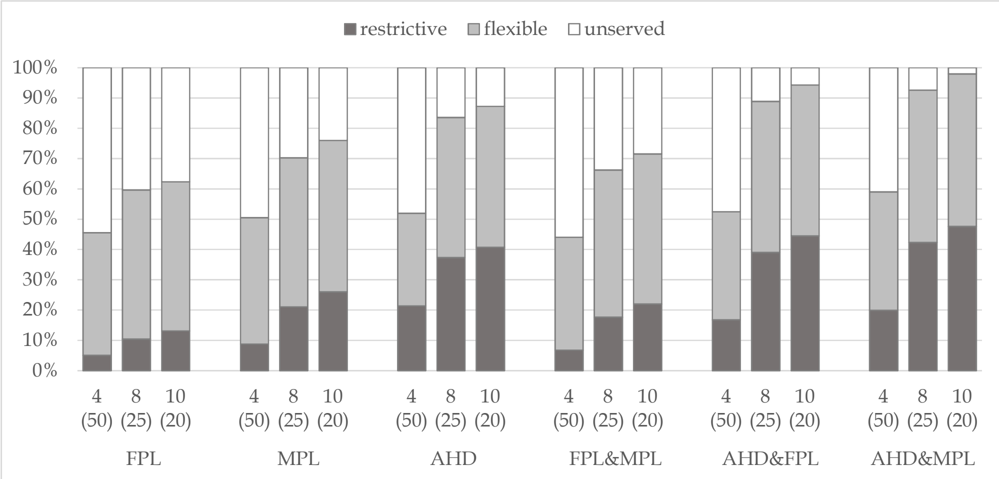


FIGURE 2.9: Customer types for different services and number of lockers (compartments)

As discussed before, combining services can increase the acceptance of restrictive customers significantly. Figure 2.10 shows how restrictive and flexible customers distribute across the individual services of combined services with varying locker sizes. Generally, being the most flexible service, AHD has the largest chunk of restrictive customers for AHD&FPL and AHD&MPL. For FPL&MPL, the more flexible service MPL has the larger chunk of restrictive customers. FPL and AHD seem to be particularly good at exploiting their individual advantages. Since MPL is capable of integrating restrictive customers when necessary and AHD is able to focus especially on remote customers, these two services complement each other particularly well.

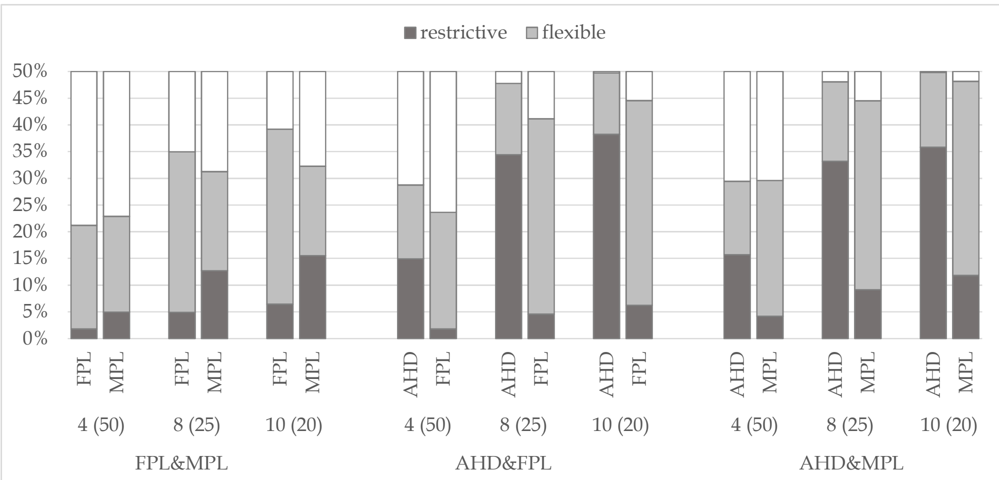


FIGURE 2.10: Customer types for combined services and number of lockers (compartments)

With more lockers (and fewer compartments) at hand, MPL can adjust more flexibly to customer locations. Hence, they reposition more often, which is reported in Figure 2.11 showing how often MPL stop in total and how often they stop for only one customer. Both the number of total stopovers per locker and the number of single-customer stopovers per locker increase as the number of available lockers increases for separate MPL and when combined with FPL. The latter result in the highest number of single-customer stopovers. For AHD&MPL, with increasing number of lockers, both the total number of MPL stopovers and single-customer stopovers remain quite low and rather stable, indicating a reasonable allocation of customers between AHD and MPL.

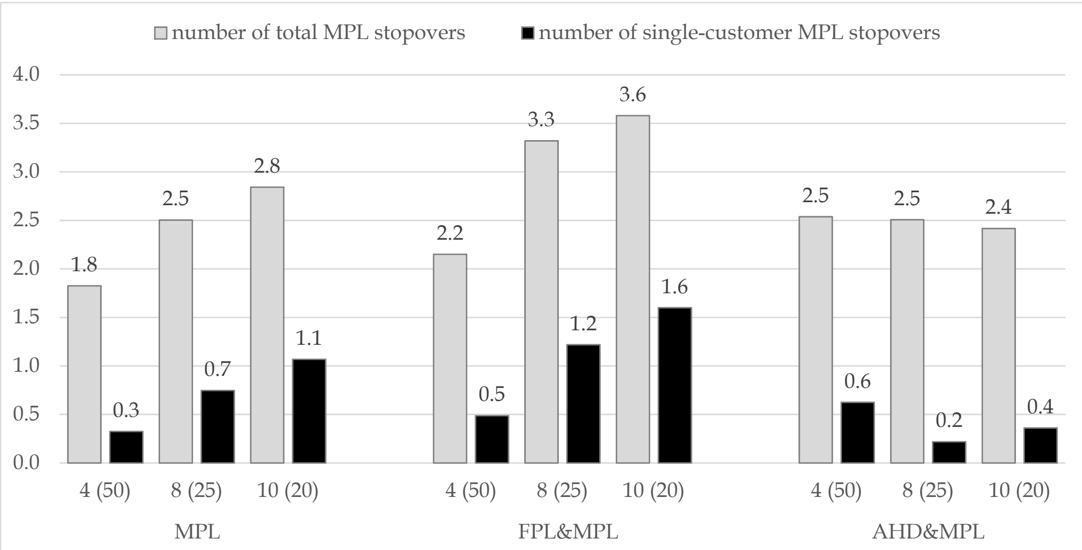


Figure 2.11: Number of MPL stopovers per locker, for different services and number of lockers (compartments)

Impact on Traffic. Table 2.4 shows the percentage of the operating time one locker spends with repositioning relative to the total operating time (FPLs do not move at all). MPLs reposition 6.9–15.4% of the total operating time. This increases as their number increases and helps accessing more remote customers. AHD drive the most with about 55% of the operational time being travel time. For FPL&MPL, repositioning times for MPLs increase up to 21.5%. AHD&MPL keep the MPL repositioning times around 12.5% of the total operating time. For AHD, the travel time remains similar, but in the combined services the travel time increases with a higher number of lockers. In summary, MPL repositioning times are significantly lower compared to AHD. For AHD&MPL, differences become more pronounced with a higher number of lockers, the repositioning time increases for AHD and decreases for MPL.

lockers number (compartments)	FPL	MPL	AHD	FPL&MPL		AHD&FPL		AHD&MPL	
				FPL	MPL	AHD	FPL	AHD	MPL
4 (50)	0%	6.9%	54.5%	0%	9.6%	51.4%	0%	50.9%	12.8%
8 (25)	0%	12.5%	56.3%	0%	19.3%	55.3%	0%	54.5%	12.6%
10 (20)	0%	15.4%	55.9%	0%	21.5%	56.9%	0%	57.0%	11.8%

TABLE 2.4: Repositioning times per locker, for different services and lockers

Table 2.5 shows the results from an absolute perspective, i.e. it reports how many hours all lockers are actually in transit, based on the time-discrete implementation. Using ten lockers, MPL are on the road for 18 hours in total and AHD are creating 67 hours of travel time. It can be observed that the repositioning time increases with an increasing number of lockers. AHD combinations cause significantly less total repositioning time than separate AHD. The combination of AHD with MPL not only serves more customers, but also reduces repositioning time up to 39%.

lockers number (compartments)	FPL	MPL	AHD	FPL&MPL	AHD&FPL	AHD&MPL
4 (50)	0 hrs	3 hrs	26 hrs	2 hrs	12 hrs	15 hrs
8 (25)	0 hrs	12 hrs	54 hrs	9 hrs	27 hrs	32 hrs
10 (20)	0 hrs	18 hrs	67 hrs	13 hrs	34 hrs	41 hrs

TABLE 2.5: Total repositioning time comparison, for different services and lockers

Locker Utilization. From an economic perspective, it is interesting how well the available resources – different number of lockers with different capacities, i.e. number of compartments – are used. As seen before, the total capacity utilization increases with combined services, as an increasing number of customers served aligns with how the logistics capacity is used overall. Now we narrow this down and investigate in which cases capacity limits are the reason for rejection. Figure 2.12 shows the percentage of lockers that have reached the capacity limit for all services. With four lockers and a capacity of 50 compartments per locker, there is never a capacity problem, which is why this has been omitted in the figure. With an increasing number of lockers and smaller capacity per locker, for AHD combinations, lockers reach capacity significantly more often than all other services. In particular, while for most services there are between 10% to 23% of lockers at capacity, this increases to 38 or 68% for the AHD combinations. Again, this aligns with the high ratio of customers served.

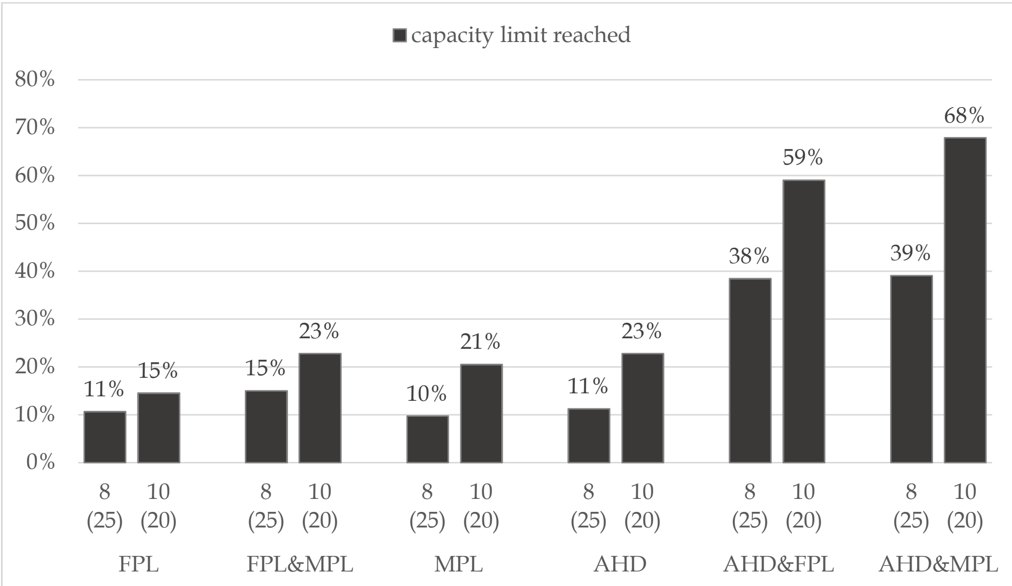


Figure 2.12: Percentage of lockers at capacity limit, for different services and number of lockers (compartments)

Figure 2.13 presents the proportion of lockers at their capacity limit for the combined services. Generally, the more flexible the service and the higher the number of lockers, the higher the risk of limited capacity. For FPL&MPL, FPLs are more likely to be fully utilized, and this increases with an increasing number of (smaller) lockers. For AHD&FPL, the use of both modes increases with the number of lockers. FPLs are less often at the limit than AHDs, with a gap of about 20%. For AHD&MPL, there is no clear pattern. Limited AHD capacities are more often at their capacity limit when eight medium-sized lockers are in operation, but this shifts to MPLs when operating ten small lockers. Therefore, with ten lockers, more MPLs reach the capacity limit than AHDs. Overall for AHD&MPL, the capacity utilization seems to be the most evenly distributed, suggesting that there is further potential to serve additional customers, even though the majority of existing customers have already been served.

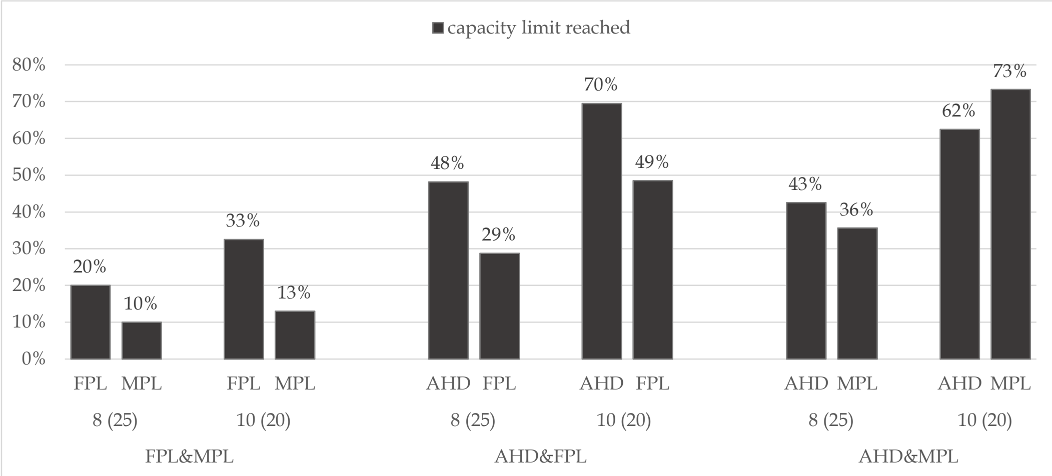


FIGURE 2.13: Percentage of lockers at capacity limit, for combined services and number of lockers (compartments)

2.5.2.4 Service Characteristics

In the following, we analyze the services from the customer's perspective. This involves the impact of the pickup distances, and the probability of acceptance of a specific customer.

Figure 2.14 shows the percentage of the maximum pickup distance, the average restrictive or flexible customer might expect, across the different delivery services. The average customer has to travel about half the maximum pickup distance unless AHD is involved (which does not require traveling by definition). In favor of their preferences, the restrictive customers need to travel less than the flexible customers. The restrictive customers consume their maximum pickup distance by 45%, whereas the flexible customers have to travel up to 57% on average. For the AHD combinations, on average, customers benefit from shorter distances compared to the other services. Matching their preferences, the restrictive customers benefit to a greater extent, as they are supplied mainly via AHD.

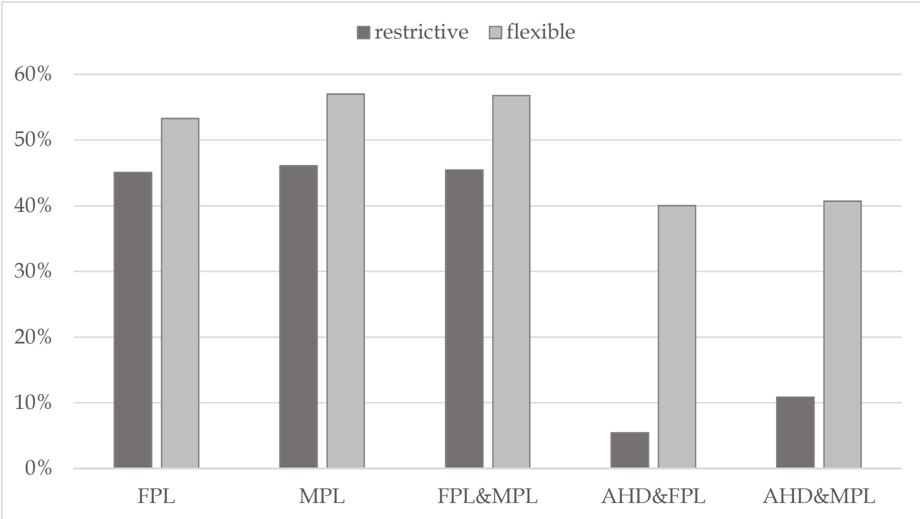


FIGURE 2.14: Average pickup distance for different services

Some customers cannot be served by FPL or MPL at all due to their restricted willingness to travel. Figure 2.15 shows the percentage of restrictive and flexible customers that could not be served by any of the available FPL and MPL locations due to pickup restrictions. The number of customers that can be served varies with the number of FPL locations, whereas the available MPL locations remain the same in each case.

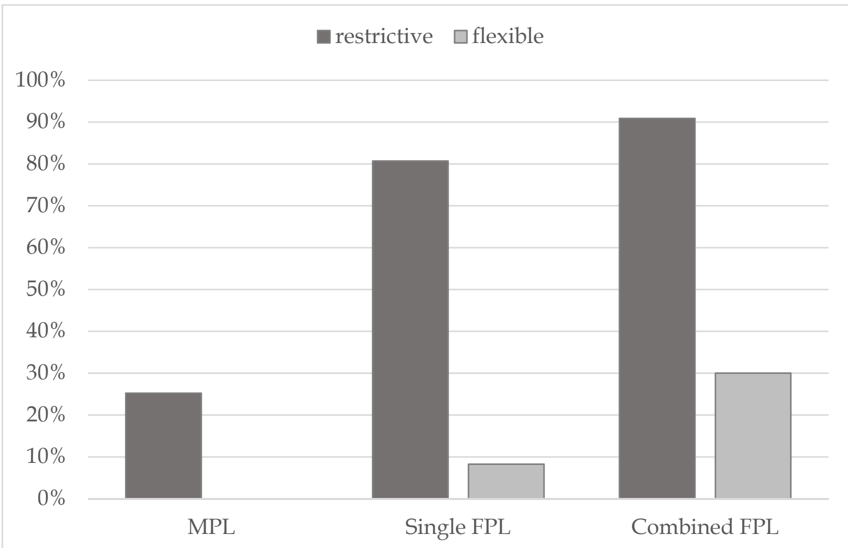


FIGURE 2.15: Proportion of customers who cannot be served per service/service combination

The limited pickup range of the restrictive customers is a clear disadvantage, especially for the FPL service. Overall, FPL is not able to serve 81% of restrictive customers individually and 91% when combined with MPL or AHD, respectively. Only 8% and 30% of flexible customers are affected. Based on the available MPL locations, none of the flexible customers and 25% of the restrictive customers cannot be served due to their pickup distance. A small number of FPLs cannot be positioned well enough and therefore leads to significantly lower customer acceptance rates. The possibility of repositioning through MPL helps alleviating that.

We now consider the acceptance probabilities of customer types by time window for separate services (Figure 2.16). Note that the one-hour time windows belong to restrictive customers and the four-hour time windows belong to flexible customers. Results for the combined services can be found in the appendix, see Figure A.2. Overall, restrictive customers have a probability of 56% of being accepted across all constellations and services, whereas flexible customers can expect an acceptance rate of around 91%. For restrictive customers (left side), we see for FPL that it is very balanced, always just below 20%, regardless of the time window. For MPL and AHD, the acceptance probability depends on the number of customers in the respective time window. However, this is much more pronounced for AHDs with up to 99% acceptance probability than for MPLs with up to 51%.

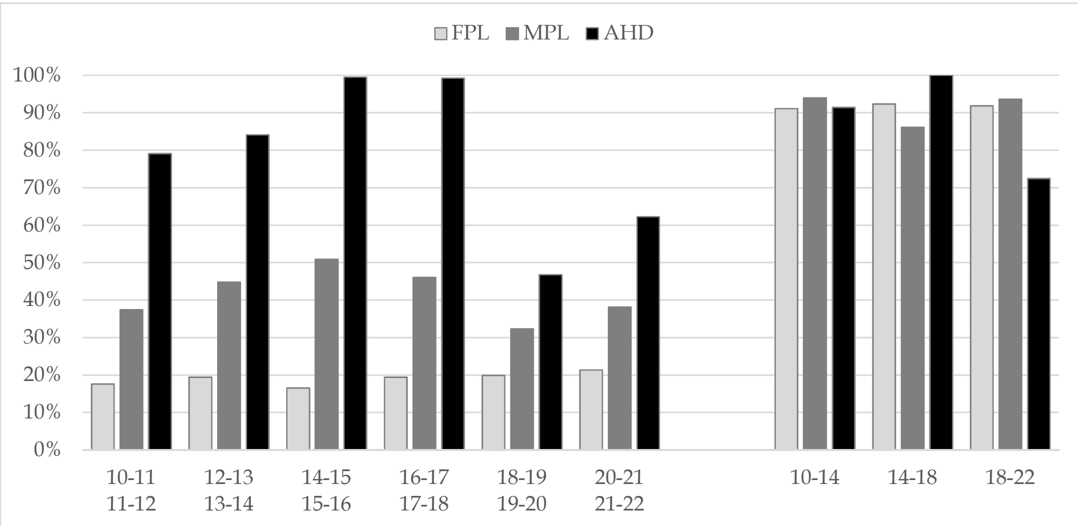


Figure 2.16: Acceptance probability of customers for single services by time window

Second, we take a look at the flexible customers on the right side of the Figure 2.16. In the case of FPLs, the time windows again show no influence, and the probability of acceptance is about 92% for each of them. With AHD, this increases up to 100% in the time window 14-18. For MPLs, however, no obvious trend can be identified. In each of the time windows 10-14 and 18-22, 94% of the customers can be accepted, although the customer demand is clearly different. In time window 14-18, the probability of acceptance is the lowest, although demand has also been the lowest.

Third, if we compare the restrictive customers with the flexible customers, we see a clear difference between the customer types, with acceptance probabilities of up to 20% and 92% if served by FPLs. This is similar but less significant in the case of MPLs. In contrast, for AHDs this trend is barely discernible, but the flexible customers are also favored. For example, customer demand in the 20-21 and 21-22 and 14-18 time windows is roughly identical, yet the flexible 14-18 customers are more likely to be served. This could be because there is more time in the longer flexible time window or because there are many flexible customers in the 18-22 time window and they are preferred over the restrictive customers.

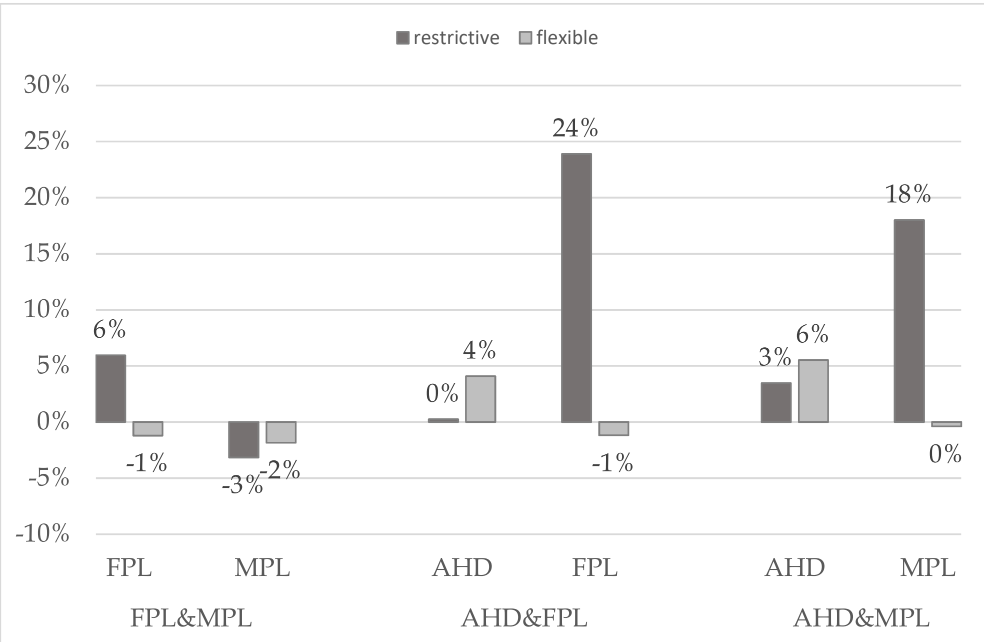


FIGURE 2.17: Comparison of served customer types, through combined services instead of single services

Finally, we examine how the acceptance probabilities of the average restrictive and flexible customer are affected by combined services. For this purpose, Figure 2.17 shows the three service combinations and in each case the corresponding change in the acceptance rate of each customer type based on the underlying separate service type. Relative to FPLs, FPL&MPL can serve 6% more restrictive customers, whereas MPLs lose a small number of both restrictive and flexible customers. Note that the acceptance of flexible customers even decreases slightly in comparison to separate FPLs.

In summary, in most cases, both customer types benefit from the AHD combinations. Only for AHD&FPL, the served flexible customers decrease by 1% in comparison to separate FPLs. By combining AHDs with FPLs or MPLs, it is possible to serve 4% or 6% more flexible customers without losing restrictive customers. In contrast, the combination of FPLs or MPLs with AHDs particularly benefits restrictive customers, whose acceptance probability increases by 24% or 18%, respectively. Both customer types benefit most from the combination with MPL.

2.5.2.5 Conservative Scenario

In practice, it is quite unlikely that the customer types are equally distributed, as it was assumed in our previous experiments to better highlight differences between customer types. Since we do not have real data on customer behavior, the following analysis aims to provide additional insights using a more "conservative" scenario. For this purpose, we consider a distribution of 70% restrictive customers and 30% flexible customers for the example of C1 and R1 instances with 200 customers.

As Figure 2.18 shows, the 30% flexible customers can be served almost completely by all services or service combinations. However, due to the high proportion of restrictive customers, the gap between instances with different spatial characteristics increases for both FPL and MPL services. This is particularly evident for MPLs, where 30% of the customers in R1 instances cannot be reached due to the limited pickup distances. For C1 instances, AHD&MPL is on par with single AHDs in terms of acceptance rates, but for R1 instances, it is 4.8 percentage points below AHD. Furthermore, the AHD combinations can no longer achieve higher acceptance rates than single AHD, which highlights that a minimum number of more flexible customers is required to make the combinations advantageous regarding the number of accepted customers. Nevertheless, the effects explained in the previous chapters, such as less traffic or undesired service, are still in place. Thus, even in the more conservative scenario, the AHD&MPL combination gives 66% of the flexible customers the desired flexibility. Furthermore, in Figure A.3, we depict the accepted customers for the conservative scenario with 100, 200, and 400 customers. The results are similar to the ones depicted in Figure 2.5, so not only the ratio of customer types is relevant for the efficiency of the combined services, but also the ratio of available lockers to the number of customers.

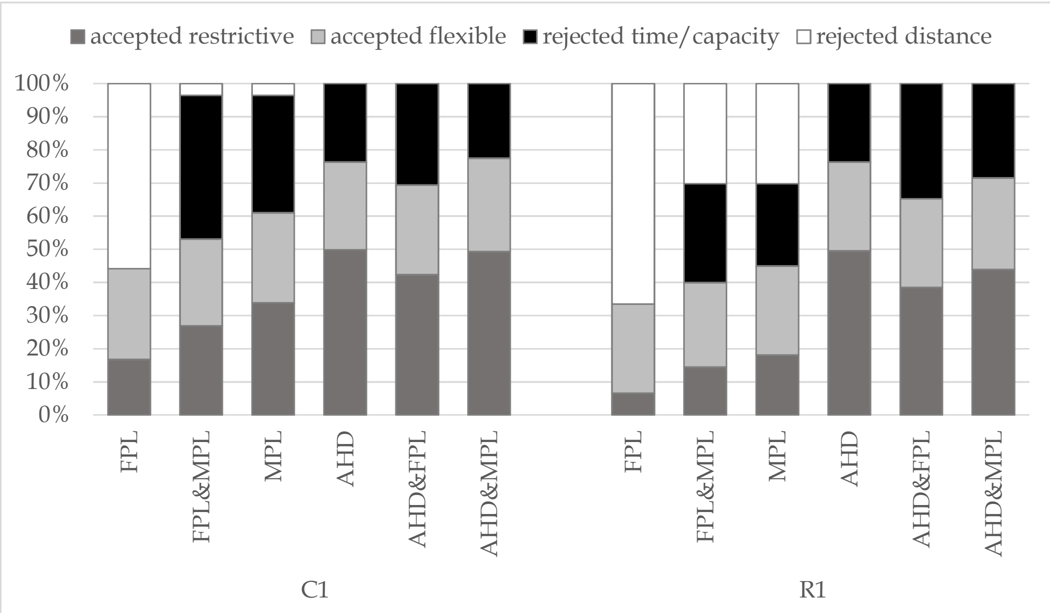


FIGURE 2.18: Types of accepted/rejected customers for 200 customers with 70% restrictive and 30% flexible customers

2.6 Conclusion

We have presented the HLLP to evaluate different innovative delivery services for last-mile deliveries: FPLs, MPLs, AHDs, and their combinations. In the experiments, we have considered instance types that differ in spatial characteristics. Two distinctly different types of customers have been designed. Restrictive customers have a limited willingness to travel and shorter pickup time windows, while flexible customers are willing to travel further in exchange for more flexibility.

The results show that AHDs dominate FPLs and MPLs. However, combining delivery services can improve acceptance rates for both types of customers and produces significantly less traffic compared to a single AHD service. FPLs and MPLs serve many customers in a short period of time, and AHDs serve customers efficiently regardless of their location and willingness to travel. The AHD combinations achieve good results, but AHD&MPL is slightly better and seems to have more spare capacity for additional customers. The main weakness of a small number of FPLs is accessibility, hence they are mainly used by flexible customers. For MPLs, spatially clustered customers showed clear advantages, whereas spatial characteristics do not affect AHDs. From the customers' point of view, the acceptance probabilities result from the selected customer type with FPLs, from the selected time window with AHDs, and from a blend of both features with MPLs.

We see some directions for future research. In the literature, well advanced solution approaches for AHDs are available, but we used a discrete-time based MIP structure to solve it for the sake of simpler comparability. Continuous and metaheuristic approaches can help to increase the degree of realism and to find good solutions for larger problems. In addition, stochastic customer demand might be considered in order to evaluate how well the different services can adapt to uncertainties. Furthermore, considering multiple delivery locations and time windows for each customer as well as more diverse customer types would be of relevance. This includes sensitivity analyzes on varying willingness to travel and spatio-temporal flexibility. This could help gaining a deeper understanding of the relationship between customer preferences and service offerings. To provide strategic guidance to service providers, investment and operating costs should be considered and evaluated in relation to the level of customer service achieved.

Chapter 3

Mobile Home Delivery Parcel Lockers

Kötschau, R., Scherr, N., Tilk, C., & Ehmke, J. F. (2025).
Transportation Research Part E: Logistics and Transportation Review, 193, 103867.

Abstract The demand for last-mile parcel delivery services is expected to increase. Great efforts are therefore being made to develop new ideas for more efficient last-mile delivery. One of the more recent concepts are Mobile Parcel Lockers (MPL), which can be easily repositioned to meet the actual daily demand for deliveries. In this work, we introduce the Mobile Home Delivery Parcel Locker (MHDPL) problem. Therein, a single vehicle can provide both, MPL service and Attended Home Deliveries (AHD), the current industry standard. Hence, the MHDPL problem integrates two characteristics: the simultaneous delivery of many customers as with MPLs, and the delivery of particularly remote customers in a short time as with AHDs. We present a mathematical formulation and propose an Iterated Local Search (ILS) as a solution approach. Since our ILS works on a time-space network, we introduce new problem and network-specific operators. The performance of the ILS is evaluated and the efficiency of the new service is analyzed in terms of various instance and customer characteristics, comparing it to dedicated MPL and AHD services. Our computational analysis reveals that the combination of services has economic potential and satisfies customer preferences more effectively. According to our study, an average MHDPL vehicle can serve 21% more customers than a comparable AHD vehicle.

3.1 Introduction

The demand for last-mile parcel delivery services is predicted to soar in the coming years (Statista, 2023d). Consequently, a great deal of engineering and research is going into developing novel ideas to handle last-mile deliveries more efficiently. A range of service models are being investigated in addition to disruptive technology like robotics and drones (Boysen et al., 2021). These models aim to utilize resources more efficiently. To guarantee the customers' availability, for instance, delivery time windows are communicated. Moreover, traditional attended home delivery (AHD) services are supported by stationary fixed parcel lockers (FPLs). FPLs are typically found in areas that are convenient for a large number of customers, or close to the customer's residence. FPLs must, however, be placed strategically in key locations and come with significant investment costs.

One intriguing alternative last-mile delivery approach is the use of mobile parcel lockers (MPLs) (Schwerdfeger & Boysen, 2022). MPLs are easily repositionable to accommodate the actual need for package deliveries throughout daily operations. After being filled at the depot, MPLs can shift their location as needed multiple times during the day. When there is a large number of customers, it may be more effective for both, logistics service providers and customers, that the delivery vehicles are positioned efficiently rather than the delivery person moving from customer to customer. This service will become especially cost-effective once vehicles can operate autonomously. However, MPL services might be less cost-effective in areas with low customer density or incompatible time windows. Thus, if a delivery person is driving a MPL vehicle, the vehicle could be used to provide both, MPL services and AHD services. Such a service model can easily adapt to customer density as well as increase customer satisfaction and planning flexibility. Moreover, customer time windows can be better utilized since previous studies have shown that with AHD, longer time windows are less favorable for customers but better for planning (Köhler et al., 2020), whereas with MPLs, it is assumed to be the opposite.

In this paper, we introduce the Mobile Home Delivery Parcel Locker (MHDPL) problem. Given a fixed fleet of delivery vehicles that can perform both service types, AHD and MPL, the number of customers served within a period is to be maximized, taking into account their preferences. The MHDPL problem integrates two features, (i) serving many customers at the same time like MPLs, and (ii) serving particularly secluded customers in a short time with AHD services while utilizing the same vehicle.

To adapt the service according to customer requirements, we assume that each customer defines a spatial and temporal scope in which they are available. The MHDPL problem can be modelled as a generalization of the team orienteering problem (TOP). However, it features significant novelties: it works on a time-space network, the profit is correlated, and there is a mixture of fixed and variable profits.

Recently, Kötschau et al., 2023 introduced the Heterogeneous Locker Location Problem (HLLP) and showed that mixed fleets of dedicated delivery vehicles which can only perform either MPL or AHD service can benefit from the specific advantages of the individual service types. However, this requires the adaptation of the fleet composition. For the MHDPL problem, in contrast, it is a question of operations management to use the fleet effectively. To the best of our knowledge, the MHDPL service has not yet been considered in the literature.

The contributions of this work are as follows. First, we introduce the Problem of the MHDPL service and adapt the mathematical formulation of the HLLP. Second, we design an iterated local search (ILS) working on a time-space network introduced by Schwerdfeger and Boysen (2020). While defining the first heuristic solution approach for the MHDPL, we introduce new problem and network-specific operators. Third, we conduct an extensive computational study in which we evaluate i) our ILS compared to solving the mathematical model directly with the Gurobi solver, and ii) the efficiency of the MHDPL service. For this purpose, characteristics of instances and customers are varied to determine when the use of a MHDPL service is particularly rewarding and under which circumstances an AHD or MPL service would be sufficient. Here, the fixed fleet size allows a better assessment of the potential of MHDPLs in different circumstances and facilitates a fair comparison with AHD and MPL services.

The remainder of this work is structured as follows: The existing literature relevant to the MHDPL service is discussed in Chapter 3.2, and a detailed description of the problem including the mathematical formulation is given in Chapter 3.3. The solution approach with the ILS structure and all operators applied is described in Chapter 3.4. This is evaluated through a computational study, and managerial insights are provided in Chapter 3.5. Finally, a conclusion is given in Chapter 3.6.

3.2 Related Literature

In the search for suitable mathematical models and heuristics, we will first address problems already considered in the literature related to the MHDPL service and describe their limitations. Then, to create the foundations for our solution approach, we will focus on the literature in the field of the TOP and explain the similarities with the MHDPL problem.

Table 3.1 summarizes the existing literature that addresses problems with MPLs. MPLs were first introduced by Schwerdfeger and Boysen, 2020 and Wang, Bi, and Chen, 2020. Schwerdfeger and Boysen, 2020 observe the minimum fleet needed to serve all customers using MPL services. Several mixed integer problems (MIPs) are presented to solve the problem exactly. Wang, Bi, and Chen, 2020 consider a service where MPLs are used in addition to FPLs to compensate for peaks in demand. The presented solution approach is based on a non-linear integer programming (NLP) model with an embedded genetic algorithm (GA). In Wang, Bi, Lai, and Chen, 2020, the problem is extended to include stochastic demand, and an exact solution approach based on an integer linear programming (ILP) model is proposed. Schwerdfeger and Boysen, 2022 present various concepts of how MPLs can be realized with drivers or even autonomously. In the introduced multi-stage heuristic, the individual selection and assignment problems are solved one after the other to minimize the total costs. Liu et al., 2023 apply a reinforcement-learning-based approach with global and local search methods, minimizing fleet size, travel distance, and service delay with separate weights. They benchmark their results with an additionally developed GA. Wang et al., 2024 discuss recourse strategies in the event of uncertainties such as demand volume and pickup time. Their solution approach is based on a modified tabu search algorithm (TS).

Rolf et al., 2022 are the first to combine MPLs and home deliveries. For that purpose, they try to utilize MPLs efficiently and serve the remaining customers at home. For the deployment of MPLs, a greedy algorithm is used to solve the maximum coverage problem. Afterward, the nearest neighbor heuristic and local search procedures are used to serve the remaining customers. Kötschau et al., 2023 investigate MPLs, FPLs, AHDs and mixed fleets. They compare the efficiency of different service fleets by optimising the potential to serve as many customers as possible. They present the HLLP and solve the problem exactly. Peppel et al., 2024 develop a mathematical model to determine near-optimal locations for MPL stops. For that purpose, they present a network design problem that includes MPLs alongside FPLs and home deliveries to assess the economic and environmental impact. Customers' decisions are integrated using a multinomial logit model.

In summary, research on MPLs is rather new and exact solution methods are predominant. In contrast to the MHDPL, the few heuristic solution approaches consider problems with different objective functions and, more importantly, they focus on deliveries exclusively using MPL services.

contribution	**delivery options**	**objective**	**solution approach**
Schwerdfeger and Boysen, 2020	MPL	min fleet size	MIP
Schwerdfeger and Boysen, 2022	MPL	min costs	multi-stage, TS
Liu et al., 2023	MPL	min fleet size, travel distance and service delay	Q-Learning mix (& GA)
Wang et al., 2024	MPL	min costs	TS algorithm
Wang, Bi, and Chen, 2020	MPL/FPL	min costs	NLP with GA
Wang, Bi, Lai, and Chen, 2020	MPL/FPL	min costs	Robust ILP
Rolf et al., 2022	MPL/HD	min fleet size and travel distance	two stages, greedy MPL assignment and NN with LS
Kötschau et al., 2023	MPL/FPL/AHD	max served customers	MIP
Peppel et al., 2024	MPL/FPL/HD	min costs	MIP

TABLE 3.1: Literature on mobile parcel lockers

There is further research on combining AHD with FPL services. In Veenstra et al., 2018 and Enthoven et al., 2020, customers can choose between home delivery or FPL. In Mancini and Gansterer, 2021, a third option is added, for customers without a preferred mode of service. If the service is not specified by the customer, this increases the complexity of the problem by allowing the provider to choose the best option. In Grabenschweiger et al., 2021, all customers accept AHD services by default, but additionally, a selection of FPLs can be specified as alternatives. Tilk et al., 2021 and Dumez et al., 2021 consider the closest approaches to the MHDPL problem. Here, each customer can define several FPL and AHD options for where and when they want to receive their delivery. However, FPL services are conceptually quite different from MPL services. For an FPL service, the provider has to visit the location for a short time only, whereas a MPL must be available for an entire service provision period, i.e., the time window of the customer. Additionally, customers typically select specific FPLs, whereas, with MPL services, the provider attempts to match customers smartly.

There is a similarity in problems where multiple customers are served from a single parking location, but the delivery person is walking instead of the customers. For a recent overview of such problems, we refer to Le Colleter et al., 2023. Particularly relevant are articles where the most suitable parking locations have to be selected from a set of possible parking locations. Reed et al., 2022 present the capacitated delivery problem with parking. This problem has a strong emphasis on the operational times of delivery: Travel times, parking times, walking times, handover times, and unloading times are considered. The objective is to minimize the total tour duration for a single vehicle. Besides providing exact solutions, they further present a two-echelon location-routing heuristic. Le Colleter et al., 2023 consider the park-and-loop routing problem with parking selection. In their case, each delivery person has a maximum carrying capacity and maximum total travel distance. First, the fleet is minimized and subsequently, the total walking, driving, and parking time is minimized. The problem is solved using a small and large neighborhood search.

Senarclens de Grancy and Reimann, 2015 consider a vehicle routing problem with time windows and multiple workers per vehicle, but each worker can only carry one delivery at a time. They minimize the costs of the fleet, staff, and driving distances and solve the problem by applying multiple clustering first and routing second approaches, in which customers are clustered sequentially or in parallel. The parking problems share the characteristic that suitable locations have to be found to serve the customers. However, they focus more on the delivery process connected with the parking rather than on the most suitable location for the customers. Commonly, this only involves consideration of a single vehicle and no customer time windows. In addition, a suitable parking location must be found even for individual customers and it is not considered that customers can pick up the delivery at the appropriate time in a self-determined manner. Therefore, modeling that parking time is used for a delivery person to visit each customer is significantly different from modeling that parking time is used for customers to flexibly collect their delivery during their time window.

The MHDPL problem is a generalization of the TOP with time windows. The TOP has been introduced by Chao et al., 1996 and deals with finding multiple routes through a set of nodes, each with different profits, maximizing the total profit with a limited fleet. For surveys on current literature on the TOP, we refer to Archetti et al., 2007, Vansteenwegen et al., 2011, Gunawan et al., 2016 and Vansteenwegen and Gunawan, 2019. Recent research deals with time-varying profits, depending on arrival and service times (Yu et al., 2022). In the MHDPL problem, profits differ depending on the number of customers served with a MPL service which, in turn, depends on the start time (i.e., arrival time) and duration (i.e., length of the service provision period) of the MPL service. In addition, we consider the new notion of correlated profits, as profit is represented by customers, and one customer can be served at different locations. If a customer is served at one location , the potential profit at another location changes, since this customer cannot be served more than once. The most relevant literature is summarized in Table 3.2.

contribution	vehicle fleet	customer time windows	arrival-time-dependent profit	service-time-dependent profit	correlated profit	solution approach
Vansteenwegen et al., 2009	x	x				ILS
Gunawan et al., 2017	x	x				SA-ILS
Gunawan et al., 2018	x			x		ILS
Yu et al., 2019				x		TS + NLP
Yu et al., 2022	x		x	x		AILS
This work	x	x	x	x	x	ILS

TABLE 3.2: Literature on related team orienteering problems

Vansteenwegen et al., 2009 propose a simple and effective ILS meta-heuristic for the TOP with time windows. The ILS is based on insertion as a local search procedure and a shake as a perturbation to escape from local optima. Gunawan et al., 2017 extend the ILS with algorithmic modifications, a greedy construction, and additional local search and perturbation operators. In addition, they propose a hybridization of simulated annealing and ILS. In Gunawan et al., 2018, the TOP is considered without time windows but with variable profits depending on the service time. This approach accepts deviations in the objective and is based on two phases with decreasing thresholds. Furthermore, they implement the most efficient criterion: Based on the score a new node is inserted, or the service time of an existing node is extended. A similar problem is studied by Yu et al., 2019 for a single vehicle, but in this case, it is solved using a matheuristic based on TS and nonlinear programming. Yu et al., 2022 extend the problem by adding arrival time-based profits and multiple vehicles. They solve the problem using Bender's branch-and-cut algorithm and an ILS heuristic with integrated modified coordinate search. In the problem setting of search and rescue operations, the authors exploit the properties of a nonconcave profit function, which is not possible with the MHDPL problem. In contrast, we model the arrival-time and service-time-dependent profits as separate nodes in a time-space network, which are called stopovers. We adapt the ILS framework presented by Vansteenwegen et al., 2009 such that it can be used with this time-space network. Moreover, ideas for nodes with variable and time-dependent profits presented in Gunawan et al., 2018 and Yu et al., 2019 are further developed to introduce new problem and network-specific operators. The most efficient criterion presented in Gunawan et al., 2018 inspired the stopover-based operators presented in Chapter 3.4.2.2, which insert or extend stopovers based on the ratio of profit per duration. Yu et al., 2019 reorganise/remove and especially balance nodes so that the available time budget can be allocated to the more profitable nodes; this idea formed the basis for the customer movement operators presented in Chapter 3.4.2.1.

3.3 Problem Description

The narrative for the MHDPL problem is given in Chapter 3.3.1, where we explain the approach involving stopovers and discuss the characteristics of MPL and AHD stopovers. Afterward, the nomenclature, listing all relevant sets and notations, and the mathematical formulation are presented in Chapter 3.3.2. Last, Chapter 3.3.3 presents an illustrative example that demonstrates potential solutions based on the utilisation of the various services. The purpose of this is not only to understand the problem under consideration but in particular to provide a fundamental understanding of the details of the ILS, which is described in Chapter 3.4.

3.3.1 Problem Narrative

The MHDPL problem deals with the service of providing packages to customers in last-mile deliveries. The objective is to maximize the number of customers served. The customer locations and a selection of possible parking locations are given as well as the travel time between all locations and the distance between customers and parking locations. Each customer has preferences regarding his or her availability (time window start), flexibility (time window length), and maximum pickup effort (maximum travel distance to a parking location). We assume that all customers accept delivery by both AHD and MPL services. The number of available vehicles with limited capacity is given, and each vehicle can perform AHD and MPL services. An AHD service performs a short stop at the customer's location within the customer's time window. This enables the provider to quickly serve customers who are not particularly compatible with the others in terms of location and/or time. For a MPL service, a vehicle is parked at a parking location for a *service provision period*. Customers are willing to pick up their parcels only if the service provision period covers their entire time window. This enables the customers to pick up their packages flexibly, and the service provider can serve several customers at the same time.

Depending on the considered problem instance, there are various options for performing AHD and MPL services at each available location. Hence, there are various delivery options for each customer at different locations and at different times. Therefore, we model the MHDPL with the help of a time-space network, following the ideas of Schwerdfeger and Boysen, 2020 for MPL services. The nodes in this network are called *stopovers* which answer the question of where and for how long the delivery vehicles should be positioned. The set of stopovers describes all relevant spatial-temporal combinations of location and time in which a vehicle can perform a service. We precompute which customers can potentially be served at which stopover, based on their time preferences and distances to the stopover location. To ensure that a stopover can be scheduled following another stopover, based on the travel time between these two stopovers, all arcs of the time-space network are defined in the set of *drives*.

For each customer, several stopovers emphasizing AHD services are defined at the corresponding customer location within their time window, serving only this particular customer. MPL stopovers are defined at the predefined parking locations and multiple customers might be served if their entire time window is covered by the service provision period and their maximum travel distance is not exceeded. Consequently, AHD stopovers are typically significantly shorter than MPL stopovers. Moreover, this definition implies that longer customer time windows increase the flexibility of AHD services but decrease the flexibility of MPL services. As the number of customers located within the range of a parking location increases, the number of reasonable stopovers increases significantly, too. Schwerdfeger and Boysen, 2020 further describe how to compute and reduce the number of stopovers and drives without losing optimality of the solution.

An example with six locations to perform AHD and MPL stopovers is illustrated in Figure 3.1. Three customer locations for possible AHD services (C1, C2, and C3) and three predefined parking locations for MPL services (P1, P2, and P3) are depicted. Therein, we assume that AHD services are completed within a 15-minute service time. The customer at location C1 has specified a time window from 15:00 to 16:00, which results in four possible AHD stopovers at this location (15:00-15:15, 15:15-15:30, 15:30-15:45, 15:45-16:00). For MPL services, more than one customer can be served from each parking location. Customer C3 is reachable from location P1, customers C1 and C2 are reachable from location P2, and all three customers are reachable from location P3. Therefore, at parking location P1, only a single stopover from 14:00 to 16:00 serving customer C3 is defined. However, at location P2, there are three possible stopovers: 15:00-16:00 (to serve customer C1), 12:00-14:00 (to serve customer C2), and 12:00 to 16:00 (to serve both customers). At location P3, stopovers for the same periods exist, however, during the service provision period from 12:00 to 16:00, all three customers could be served. In addition, it is possible to serve customers C1 and C3 at location P3 with a stopover from 14:00 to 16:00. The entire set of relevant stopovers for the given example can be found in the appendix, see Table B.1.

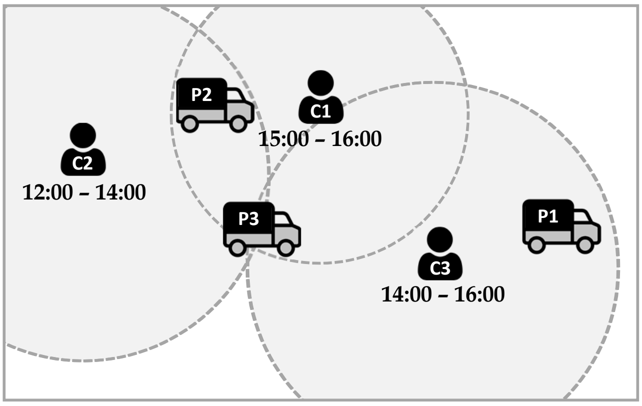


FIGURE 3.1: Example for AHD and MPL stopover locations

3.3.2 Mathematical Formulation

In the MHDPL problem, the sets of locations L, discrete points in time T, customers C, and vehicles V are given. The set of locations L includes all customer locations and parking locations, but separately a pseudo depot location $\gamma \notin L$ is specified. Travel times t_{ij} for each pair of locations $i, j \in L$ are given. In addition, distances between customers and parking locations are used to determine which customer can be served at which parking location. The overall time horizon is defined as a set of discrete points in time $T = \{T_0 := 0, 1, \dots T_n\}$. Each customer $c \in C$ is defined by a location $l_c \in L$, a time window $[\underline{t}_c, \bar{t}_c]$ where $\underline{t}_c, \bar{t}_c \in T$, and a maximum travel distance to a parking location d_c^{max}. For each vehicle $v \in V$, a capacity κ_v of how many customers can be served is specified. In a pre-processing step, all reasonable stopovers and drives are determined based on the existing customers, their preferences, and the available parking locations. We refer to Schwerdfeger and Boysen, 2020 for details.

Let S be the set of all stopovers and D the set of all drives. Each stopover $s \in S$ is defined by a location $l_s \in L$, start time $\underline{t}_s \in T$, end time $\bar{t}_s \in T$, and a mode of service $m_s \in \{AHD, MPL\}$. The parameter $b_{c,l,\underline{t},\bar{t}}$ describes whether customer $c \in C$ can be served through the corresponding stopover. In addition, for each stopover $s \in S$, let $dur_s = \bar{t}_s - \underline{t}_s$ be the duration of the service provision period of this stopover and let δ_s^c be the realized profit by this stopover, corresponding to the number of customers served. Each drive $d \in D$ is defined by a departure location $\underline{l}_d \in L$, arrival location $\bar{l}_d \in L$, and departure time $t_d \in T$. Additionally, the parameter $\sigma_{\underline{l},\bar{l},t}$ specifies the latest available departure time based on the arrival time $t \in T$ for a drive from departure location $\underline{l} \in L$ to arrival location $\bar{l} \in L$. For each location $l \in L$, the following subsets of the time horizon are derived: a) $\underline{T}_l \subset T$: points in time where at least one stopover starts, b) $\overline{T}_l \subset T$: points in time where at least one stopover ends, and c) $\overline{T}_l^\gamma \subset T$: points in time where at least one stopover ends and no following stopover exists at the same location l. A comprehensive summary of all sets and notations can be found in Table 3.3.

$l \in L$	Set of available locations
γ	Additional (pseudo) depot location $\gamma \notin L$
t_{ij}	Travel time for each pair of locations $i, j \in L$
$t \in T$	Set of discrete points in time, with T_0 being the first point in time and T_n being the last point in time
$c \in C$	Set of customers, each defined by: location (l_c), time window $[\underline{t}_c, \bar{t}_c]$, and maximum travel distance d_c^{max}
$v \in V$	Set of vehicles, each with a capacity κ_v
$s \in S$	Set of stopovers, each defined by: location (l_s), start time ($\underline{t}_s$), end time ($\bar{t}_s$), and mode of service (m_s)
$b_{c,l,\underline{t},\bar{t}} \in \{0,1\}$	Indicates if a customer c can be served by a stopover with: $l, \underline{t}, \bar{t}$
dur_s	Duration of stopover s, calculated as $\bar{t}_s - \underline{t}_s$
δ_s^c	Realized profit of stopover s (corresponds to number of customers served), depends on the current solution and customers who are not served elsewhere
$d \in D$	Set of drives, each defined by: departure location ($\underline{l}_d$), arrival location ($\bar{l}_d$), and departure time (t_d)
$\sigma_{\underline{l},\bar{l},t} \in \mathbb{Z}$	Departure time at $\underline{l}$ to arrive in $\bar{l}$ at time t
$\underline{T}_l$	Set of subsets of T where a stopover-start exists at location l
$\overline{T}_l$	Set of subsets of T where a stopover-end exists at location l
$\overline{T}_l^\gamma$	Subsets of $\overline{T}_l$ where no successive stopover exists at location l

TABLE 3.3: Nomenclature

A solution to the problem is described by three types of decision variables. For each vehicle $v \in V$, the decisions are: $y_{v,c}$ if it serves customer $c \in C$, $x^S_{v,l,\underline{t},\bar{t}}$ if it performs the corresponding stopover, and $x^D_{v,\underline{l},\bar{l},t}$ if it requires the corresponding drive. The mathematical model is as follows.

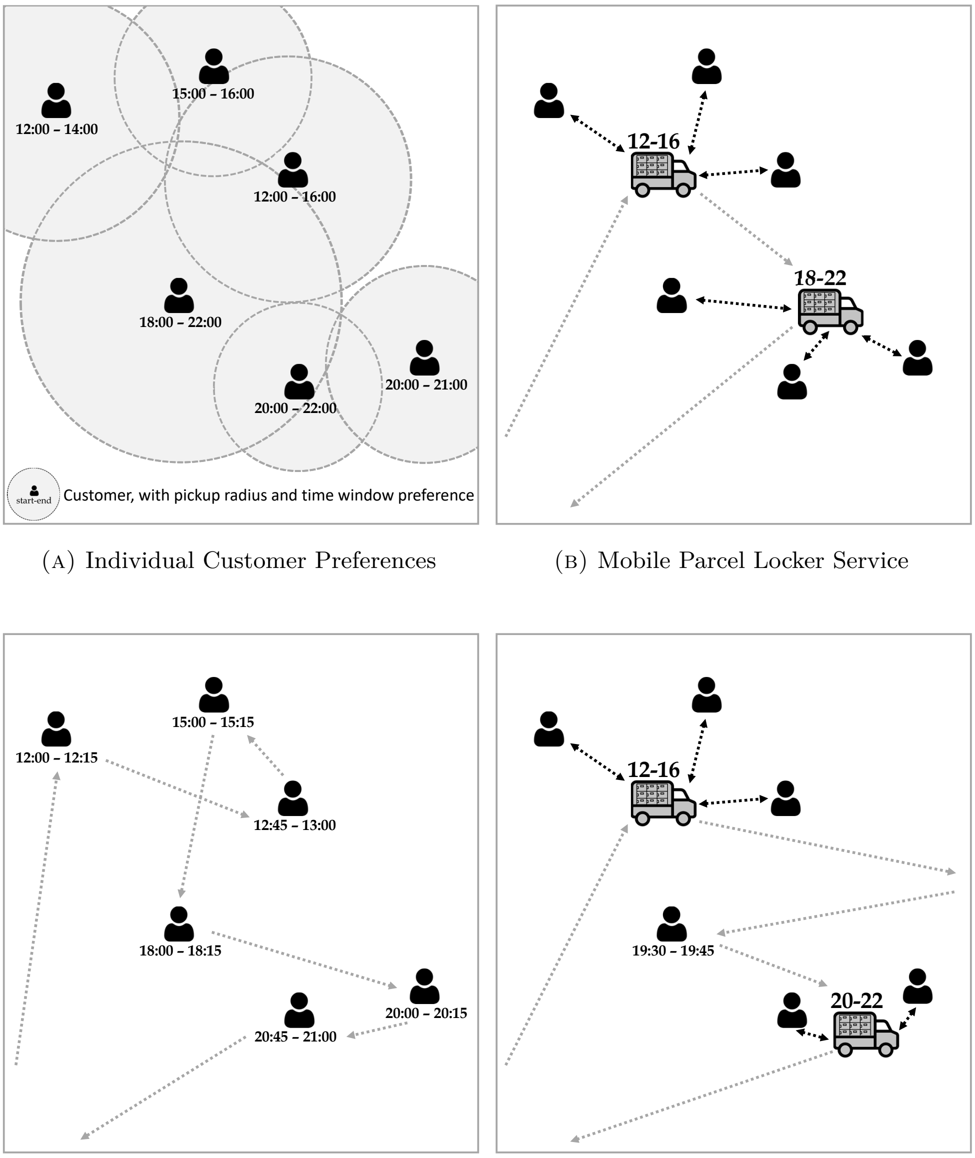
The objective (1) is to maximize the number of customers served by a given fleet, under the following constraints. (2) No customer will be served more than once. (3) No vehicle may serve more customers than it has capacity. (4) When a vehicle serves a customer it must perform a stopover at which the customer can be served. The flow conservation is ensured by (5) and (6): For each stopover performed, there must be a drive to (5) and from (6) that stopover. Finally, (7) each vehicle may have a maximum of one drive to a pseudo depot to ensure that it is not used more than once. The binary decision variables are specified in the constraints (8)–(10). This mathematical formulation adds the combined application of AHD and MPL stopovers to the HLLP presented by Kötschau et al., 2023. Furthermore, we relax the assumption that only homogeneous vehicles are used.

$$
\begin{array}{lll}
(1) & max \sum_{v \in V} \sum_{c \in C} y_{v,c} & \\
(2) & \sum_{v \in V} y_{v,c} \leq 1 & \forall c \in C \\
(3) & \sum_{c \in C} y_{v,c} \leq \kappa_v & \forall v \in V \\
(4) & \sum_{s \in S} b_{c,l_s,\underline{t}_s,\bar{t}_s} * x^S_{v,l_s,\underline{t}_s,\bar{t}_s} \geq y_{v,c} & \forall v \in V, c \in C \\
(5) & \sum_{\substack{s \in S \\ l_s = l \\ \underline{t}_s = \underline{t}}} x^S_{v,l,\underline{t},\bar{t}_s} = \sum_{\substack{d \in D \\ \bar{l}_d = l \\ t_d = \sigma_{\underline{l}_d,l,\underline{t}}}} x^D_{v,\underline{l}_d,l,t_d} & \forall v \in V, l \in L, \underline{t} \in \underline{T}_l \\
(6) & \sum_{\substack{s \in S \\ l_s = l \\ \bar{t}_s = \bar{t}}} x^S_{v,l,\underline{t}_s,\bar{t}} = \sum_{\substack{d \in D \\ \underline{l}_d = l \\ t_d = \bar{t}}} x^D_{v,l,\bar{l}_d,\bar{t}} & \forall v \in V, l \in L, \bar{t} \in \overline{T}_l \\
(7) & \sum_{l \in L} \sum_{t \in \overline{T}^{\gamma}_l} x^D_{v,l,\gamma,t} \leq 1 & \forall v \in V \\
(8) & y_{v,c} \in \{0,1\} & \forall v \in V, c \in C \\
(9) & x^S_{v,l,\underline{t},\bar{t}} \in \{0,1\} & \forall v \in V, l \in L, \underline{t} \in T, \bar{t} \in T \\
(10) & x^D_{v,\underline{l},\bar{l},t} \in \{0,1\} & \forall v \in V, \underline{l} \in L \cup \{\gamma\}, \bar{l} \in L \cup \{\gamma\}, t \in T
\end{array}
$$

3.3.3 Exemplary Comparison of MPL, AHD, and MHDPL Services

Figure 3.2a shows an example with six customers and their individual time windows and willingness to travel, expressed as a radius surrounding each customer. Figure 3.2b shows a possible solution with two MPL stopovers serving all customers for the given example. For a MPL service, a vehicle is available within customers' reach during their entire time window so that these customers can flexibly collect their delivery. In contrast, for an AHD service, the vehicle is positioned directly at the customers' location for a shorter period of time so that individual customers can be served in a time-saving manner. A corresponding AHD tour is shown in

Figure 3.2c. The time spans depicted here refer to the service time at the customer's location. A delivery person can carry out the delivery as usual. Note that in order to reduce the total driving distance, the second customer could be served after the third customer, but this does not impact the problems' objective function.



(A) Individual Customer Preferences

(B) Mobile Parcel Locker Service

(C) Attended Home Delivery Service

(D) Mobile Home Delivery Parcel Locker Service

FIGURE 3.2: Example for MPL, AHD and MHDPL service with individual customer preferences

Applying a given fleet of MHDPL vehicles, it is the providers' responsibility to decide how to serve the available customers most efficiently with MPL or AHD services. A possible solution to the problem with MHDPLs is shown in Figure 3.2d. In this case, a MPL stopover is performed at the beginning and at the end of the tour, to accommodate most customers' preferences. The customer located in the middle has a large maximum travel distance but an unfavorable time window concerning the other customers. Therefore, this customer could be served by an AHD service to create additional availability for other potentially occurring delivery services in between.

3.4 Iterated Local Search

This chapter outlines the adapted ILS and its problem and network-specific operators. In Chapter 3.4.1, an overview of the ILS is given in order to gain a basic understanding, explain the structure and the time-based adaptation. Afterward, the operators designed for the time-space network with stopovers and drives are described in detail, in Chapter 3.4.2 for the local search, and in Chapter 3.4.3 for the perturbation procedure.

3.4.1 Overview

ILS is a metaheuristic procedure that alternates between local search and perturbation. During local search, the current solution is improved step by step until no further improvement is possible with the applied operators. To escape from local optima, the improved solution is then partially destroyed by perturbation and used as the initial solution in the next iteration.

The following ILS approach utilizes the time-space network introduced by Schwerdfeger and Boysen (2020). To develop an efficient ILS, we introduce various new problem and network-specific operators. First, during the local search procedure, single customers are moved within the solution or included in existing stopovers; existing stopovers are extended, or new stopovers are inserted. Then, during perturbation, stopovers are removed by one of two methods: Stopovers are removed randomly until a random threshold of customers has been removed, or all activities within a varying time interval are removed. The latter is inspired by Vansteenwegen et al., 2009, who propose two values q and r that are used to indicate a varying interval of customers that are removed from the current solution for each vehicle. We adapt this such that q and r now define a time interval in which all activities are removed from each vehicle.

These parameters are gradually modified every time the time-based perturbation is performed. Furthermore, for better comparability, a fixed maximum runtime of the algorithm is defined as a stopping criterion. The outline of the implemented ILS is shown in Algorithm 1.

Algorithm 1 Iterated Local Search

```
Used Parameters:
  _AlgRuntime_; _PerturbationRatio_; _RemovalTimeStep_; _RemovalTimeMax_
q ← T0
r ← _RemovalTimeStep_
Solution ← create empty solution with two pseudo depot stopovers for each vehicle
BestFound ← Solution ← LocalSearch(Solution, random = false)
while _AlgRuntime_ is not passed do
    Solution ← LocalSearch(Solution, random = true)
    if Solution better than BestFound then
        BestFound ← Solution
        r ← _RemovalTimeStep_
    end if
    if random number between 0 and 1 ≤ _PerturbationRatio_ then
        Solution ← RandomPerturbation(Solution)
    else
        Solution ← TimeBasedPerturbation(Solution, q, r)
        q ← q + r
        r ← r + _RemovalTimeStep_
        if q ≥ Tn then
            q ← q − Tn + T0
        end if
        if r > _RemovalTimeMax_ then
            r ← _RemovalTimeStep_
        end if
    end if
end while
return BestFound
```

Predefined ILS parameters and problem-specific input parameters are used to guide the ILS. The maximum runtime for the algorithm is specified in *_AlgRuntime_*. The *_PerturbationRatio_* indicates the probability for the two different perturbation procedures and is 0.5 by default. For the time-based perturbation, the maximum length of the time interval for removal is specified in *_RemovalTimeMax_*, which is 75% of the time horizon T by default. The length of the time interval for removal r varies during the process. By default, r will be varied in time increments of *_RemovalTimeSteps_*, which is set to 5 by default. All default values demonstrated promising results during development and extensive testing.

The algorithm starts by initializing the values q and r, which are used for the time-based perturbation. An initially empty solution is then created, consisting of two pseudo stopovers for each vehicle. These pseudo stopovers represent the beginning and the end of the tour and thus determine the vehicle's availability. The first greedy solution is obtained by performing the local search procedure without randomization. Within code lines 5 to 24, new solutions are explored iteratively until the maximum runtime is reached. In each iteration, the local search procedure is applied in a randomized variant. The best solution found in the course of the algorithm is stored in $BestFound$. A perturbation is then carried out to destroy parts of the solution preparing for a new iteration. Whether random or time-based perturbation will be performed is randomly chosen in line 11. If the time-based perturbation has been applied, the values q and r will be updated. Value q is set to the current ending time of the removal interval, i.e., $q = q + r$, and r is increased by exactly one removal time step. The corresponding value is reset when q exceeds the end of the time horizon T_n or r exceeds the maximum length of the removal time interval.

3.4.2 Local Search

The local search procedure gradually improves the current solution. For this purpose, various operators have been developed that move customers or add stopovers. The move operators presented in Chapter 3.4.2.1 perform the insertion or relocation of a single customer. All move operators are executed in a nested way at the beginning of the local search procedure. The stopover operators, which are described in more detail in Chapter 3.4.2.2, are applied afterward. In contrast to the move operators, they perform insertion or extension of a stopover and, hence, consider several customers simultaneously. Stopovers are iteratively extended or added to the solution until no more improvement can be found. The stopover to be added is selected either by choosing the best score or randomly by a score-based roulette wheel.

3.4.2.1 Customer Movement Operators

This section describes the four operators that decide how customers are moved within the solution and how not yet served customers are added to stopovers already in the solution. The aim of moving customers within the solution is manifold: a) to create potential to add further customers directly; b) to be able to add further stopovers, and c) to further explore different modes of service for each customer. Each operator is checked for each stopover in the solution, and each time an improvement can be found, it is applied. Then, the next operator is applied in the order in which they are introduced.

(1) AHD to AHD: For each AHD stopover, it is checked if the corresponding customer can be served by another vehicle using any compatible AHD stopover while respecting all time constraints. If a feasible movement can be found, it will only be implemented if the new vehicle has more remaining capacity than the current vehicle. In addition, after checking all AHD stopovers of a single vehicle, if at least one move has been performed, it is determined whether a not-served customer can be included in an existing MPL stopover planned for the current vehicle.

(2) NotServed to MPL: Each MPL stopover is checked to determine whether a customer not currently in the solution can be added to that stopover.

(3) MPL to MPL: For each customer currently served via a MPL stopover, it is determined whether this customer can be served by another MPL stopover. If it is possible and the vehicle currently serving the customer has less remaining capacity than the vehicle performing the new MPL stopover, then the customer will be moved. Should a movement result in the initial vehicle now having a remaining capacity of exactly one, it is checked whether a not-served customer could be served during the initial MPL stopover.

(4) AHD to MPL: All MPL stopovers performed by vehicles with remaining capacity are analyzed. If there is a customer currently served through an AHD stopover in the solution which can be served at any MPL stopover considered here, the customer will be moved accordingly. The customer is now served via the existing MPL stopover and the AHD stopover is no longer required. This provides more options for adding further stopovers or extending the existing stopovers as the local search procedure continues.

Figure 3.3 shows an example solution consisting of two tours that are carried out simultaneously by two different vehicles. Starting from the depot, it is illustrated how individual customers are served by AHD stopovers and several grouped customers by MPL stopovers. The lines below each stopover indicate the duration of the stopover. The presented operators are highlighted in black. (1) demonstrates how a customer served by an AHD stopover is moved to another tour. (2) shows a customer who has not been served yet and who is added to a MPL stopover. (3) moves a customer between two MPL stopovers, and (4) moves a customer from an AHD stopover to a MPL stopover.

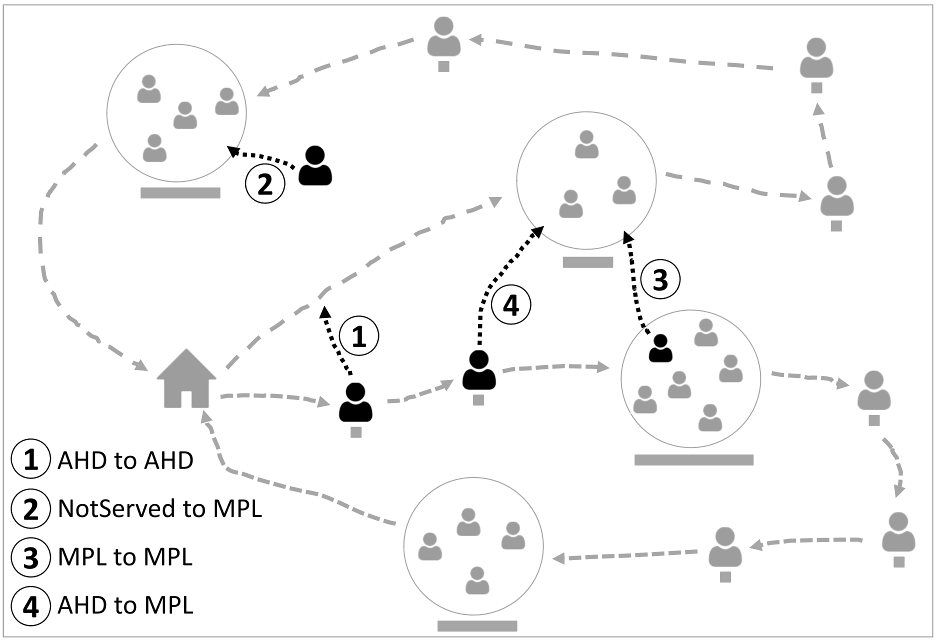


FIGURE 3.3: Example for ILS move operators

The figure illustrates a solution with two tours. Each tour consists of AHD stopovers (single customers) and MPL stopovers (grouped customers). The lines below the stopovers represent the duration of the stopover. The customers to be moved are highlighted in black and can be moved from one tour to another, following the applied move operator.

3.4.2.2 Stopover-based Operators

The two operators based on stopovers are referred to as *extension* and *insertion*. Instead of considering individual customers, this approach focuses on including stopovers and thus considers several customers simultaneously. Both operators explore the complete set of stopovers S. It is crucial to ensure the feasibility of incorporating a stopover into the existing solution for each insertion or extension. To insert or extend a stopover, it is necessary that the vehicle in question has at least one free capacity, and that the resulting route meets all time constraints. For this purpose, it is allowed to move AHD stopovers in the current solution within the customer's time window. A score is calculated for each possible insertion or extension of a stopover. The score serves to identify the most suitable candidates, intending to optimise the utilisation of available time. Following this, for the most suitable candidates, the costs of insertion are calculated for all feasible insertion positions. Based on the insertion costs, the combination of stopover and insertion position with the lowest cost is then implemented. This process of inserting or extending stopovers is repeated until no further customer can be inserted feasibly.

Stopover Insertions: When an insertion is performed, a stopover is selected and added to the solution; this can be either an AHD or a MPL stopover. Which stopover is selected depends not only on the respective score and the insertion costs but also on whether the local search procedure is randomized or not. When inserting an AHD stopover s, we calculate the $score_s^{AHD}$ hierarchically based on a) the number of customers served ($\delta_s^c = 1$) divided by the duration (dur_s) of this stopover plus 1 and b) the maximum travel distance (d_c^{max}) of the corresponding customer c in relation to the maximum of the maximum travel distance of all customers (d^{max}). The hierarchical order is realised by multiplying a sufficiently large number M. Formally, this leads to $score_s^{AHD} = M * (\frac{1}{dur_s+1}) + (d^{max} - d_c^{max})$. The former aims at maximizing the number of customers served in the required time. The latter prioritizes AHD stopovers corresponding to customers who are hard to serve with MPL stopovers. For the insertion of a MPL stopover s, the $score_s^{MPL}$ is calculated as the number of customers served (δ_s^c) divided by the duration (dur_s) of that stopover plus 1, i.e., $score_s^{MPL} = M * (\frac{\delta_s^c}{dur_s+1})$. Note that for both, the insertion of AHD and MPL stopovers, we add $+1$ in the denominator to prefer stopovers serving more customers, especially to prefer MPL over AHD.

Stopover Extensions: An extension means that a MPL stopover currently in the solution is extended, i.e., the location is visited earlier and/or longer. This operator replaces a shorter stopover with a longer stopover at the same location. Practically, this is realised by removing a stopover that is currently in the solution and inserting a replacement stopover. When extending an existing MPL stopover $s_j \in S$ to a replacement MPL stopover $s_i \in S$ at the same location, the $score_{s_i,s_j}^{ext}$ is calculated by dividing the number of additional customers served ($\delta_{s_i}^c - \delta_{s_j}^c$) by the additional duration ($dur_{s_i} - dur_{s_j}$). As with the stopover insertions, the result is multiplied by a sufficiently large number M for the purpose of ensuring comparability. This results in the following formula: $score_{s_i,s_j}^{ext} = M * (\frac{\delta_{s_i}^c - \delta_{s_j}^c}{dur_{s_i} - dur_{s_j}})$.

Score Calculation Example: An example for score calculations is given in Table 3.4 for different stopover *extensions* and stopover *insertions*. The table contains the values: a) profit ($\delta_{s_i}^c$) and duration (dur_{s_i}) of the new selected stopover s_i,

b) profit ($\delta^c_{s_j}$) and duration (dur_{s_j}) of the existing extended stopover s_j, and c) customers' maximum travel distance (d^{max}_c) for AHD stopovers; values that are not required are indicated with "-". $M = 1000$ and $d^{max} = 2$ are set for the computations. Highlighted in black, the first extension shows the vehicle leaving later, the second shows the vehicle arriving earlier, and the third shows earlier arrival and later departure together. In the first case, for example, a score of 333.33 results from the additional duration of 3 $(11 - 4)$ and an additional customer $(5 - 4)$ that can be served. Two MPL and two AHD stopovers are shown as exemplary insertions. A MPL stopover lasting four time units where four customers can be served achieves a score of 800.00 $(1000 * (\frac{4}{4+1}))$. In addition, AHD stopovers differ in particular by the maximum travel distance of the respective customer, whereby the last stopover in the example achieves a score of 500.50 $(1000 * (\frac{1}{1+1}) + (2 - 1.5))$.

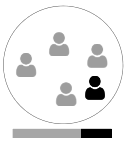
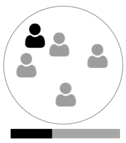
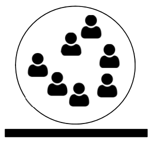
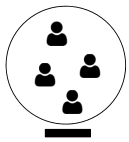
type	extension			insertion			
visualization							
$\delta^c_{s_i}$	5	5	6	7	4	1	1
dur_{s_i}	10	11	14	14	4	1	1
$\delta^c_{s_j}$	4	4	4	-	-	-	-
dur_{s_j}	7	7	7	-	-	-	-
d^{max}_c	-	-	-	-	-	0.5	1.5
score	**333.33**	**250.00**	**285.71**	**466.67**	**800.00**	**501.50**	**500.50**

TABLE 3.4: Example for the score calculation in the local search

The visualisations show stopovers which can either be extensions of already planned stopovers (extension type) or stopovers that are completely new to the current solution (insertion type). The additional customers and the additional time span of the service provision period compared to the current solution are highlighted in black.

Insertion Costs Calculation: If the local search procedure is greedy, we select all insertion or extension options with the highest score as candidates and compute the insertion costs for all candidates. In the random local search variant, only a single stopover is selected using a score-based roulette wheel with the ten best insertion options and the best extension option. The insertion costs are then calculated exclusively for this stopover. Ties in the score are broken arbitrarily.

For all feasible insertion positions, we compute the insertion costs that hierarchically include 1) the sum of the required travel times to and from the new stopover, and 2) the minimum of the elapsed time before and after the new stopover. Let s_i and s_{i+1} be two successively served stopovers in the current solution and let s be the stopover to insert: the insertion cost is defined as $cost_{s,s_i,s_{i+1}} = M * (t_{l_{s_i},l_s} + t_{l_s,l_{s_{i+1}}}) + \min\{\underline{t}_s - \bar{t}_{s_i}, \underline{t}_{s_{i+1}} - \bar{t}_s\}$ for a sufficiently large number M. We then conduct the combination of insertion position and stopover that has the lowest insertion cost. Ties are broken arbitrarily. There is an exception when determining the lowest insertion costs: if a valid insertion position is found for one vehicle, the other vehicles are considered only if their remaining capacity is equal or larger.

3.4.3 Perturbation

If no further improvements can be found in the local search procedure, the current solution is partially destroyed by applying either *random* (see Chapter 3.4.3.1) or *time-based* (see Chapter 3.4.3.2) perturbation selected randomly based on the probability defined in the perturbation ratio parameter.

3.4.3.1 Random Perturbation

The random perturbation randomly determines a threshold for the number of customers to be removed from the current solution. It is possible to remove any number of customers currently included in the solution. Iteratively, a stopover is randomly selected and removed, including all customers served by this stopover. This is repeated until the previously defined threshold has been reached or exceeded. Note that only complete stopovers are removed.

3.4.3.2 Time-Based Perturbation

Applying time-based perturbation removes all activities from all vehicles within a given time interval. The values q and r mentioned above describe the start time and the length of the interval to be removed. It is important to note that if the sum of q

and r exceeds the considered time horizon, the excess amount of time is removed at the beginning of the time horizon. All stopovers occurring within the time interval for removal will be completely removed. If a stopover intersects the removal interval, the stopover is not removed completely but truncated, and all customers who can still be served remain in the solution.

Figure 3.4 illustrates time-based perturbation using the already familiar example with two tours. The removed customers and the reduced service provision period of completely or partially removed stopovers are highlighted in black. In the tour on the bottom half of the figure, one AHD and one MPL stopover are completely removed, as highlighted in black. In the other tour, two AHD stopovers are removed, and one MPL stopover is only partially removed. All deleted stopovers are in the same removal interval near the end of the time horizon.

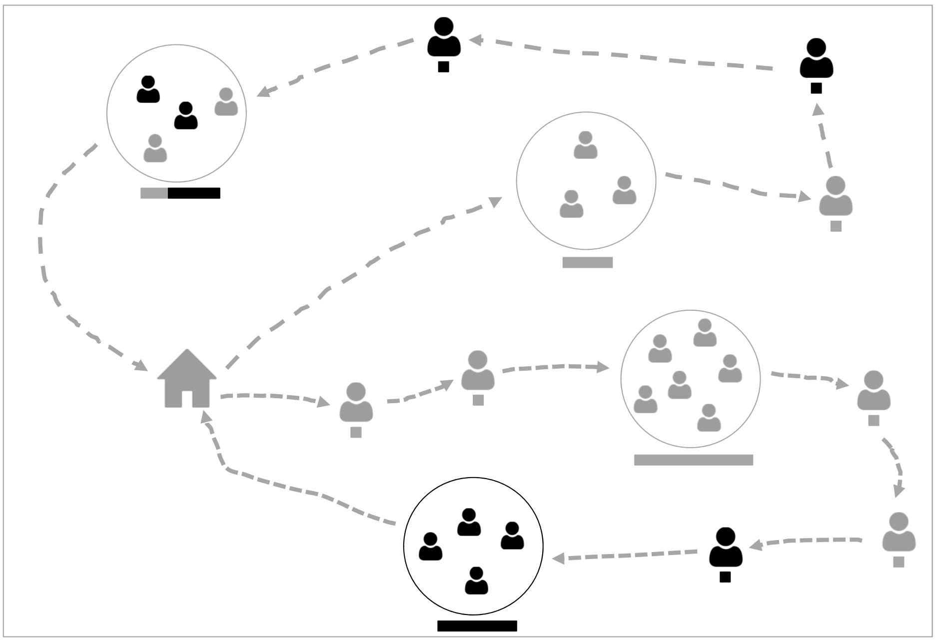


FIGURE 3.4: Example for time-based perturbation

The figure illustrates the familiar solution with two tours, where the stopover times and customers to be removed are highlighted in black. The highlighted customers were scheduled to be served during the removal interval.

The special case where the removal interval occurs within a stopover is shown in Figure 3.5. The time interval and customers to be removed are highlighted in black. Initially, in the upper part of the figure, there is a single stopover where many customers are served over a relatively long service provision period. Since a time interval in the middle of the service provision period of the stopover is being removed, this stopover will be divided into two significantly shorter stopovers, as shown in the lower part of the figure. Although these two stopovers can still serve some of the initially contained customers, the customers with time windows within or overlapping the removed time interval can no longer be served.

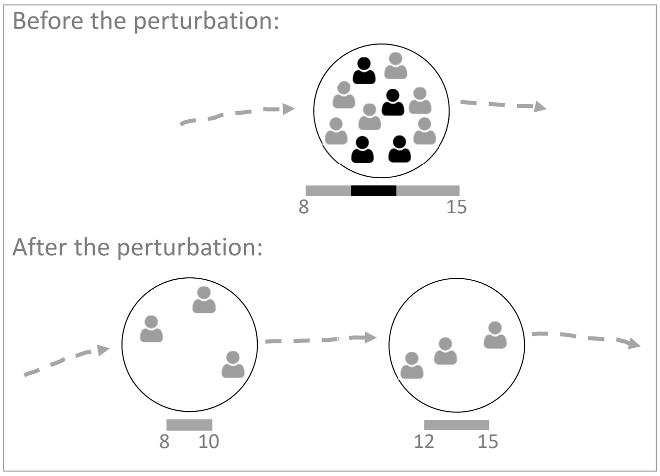


Figure 3.5: Example of a time-based perturbation dividing an existing MPL stopover into two shorter ones

The upper part of the figure illustrates a MPL stopover from which the time interval and customers highlighted in black are to be removed. The lower part of the figure shows the two new MPL stopovers created through the removal.

3.5 Computational Study

The aim of this study is to analyze the distinguishing factors for the proposed MHDPL service and to evaluate the performance of the ILS. To determine under which conditions the MHDPL service is particularly beneficial or whether an AHD or MPL service is sufficient, various instance and customer characteristics are investigated. In addition, it is analyzed how often MHDPL vehicles perform which service and whether it would be sufficient if only dedicated vehicles were used. The study concludes with an evaluation of the MHDPL service from the customer's perspective.

The experimental setup is described in Chapter 3.5.1. This is followed by evaluating the performance in Chapter 3.5.2. Sensitivity analyses and discussions of the instance- and customer-specific characteristics are carried out in Chapter 3.5.3 and Chapter 3.5.4, providing managerial insights on the AHD, MPL, and MHDPL services. The utilization of AHD and MPL services is explored in more detail in Chapter 3.5.5. Finally, the acceptance probabilities and expected service quality from a customer's perspective are discussed in Chapter 3.5.6.

3.5.1 Experimental Setup

For each result in the following experiments, 100 instances are evaluated. These instances consist in equal parts of four differently clustered customer locations derived from Solomon, 1987 and Gehring and Homberger, 1999 instances for the vehicle routing problem with time windows. They define customer locations as densely clustered (C1), as less densely clustered (C2), as randomly distributed (R1), and as a combination of randomly distributed and densely clustered (RC1). However, the original distances have been scaled to reflect a variety of customer populations. Vehicles can perform stopovers at any customer location to execute AHD services. For MPL services, we identify 50 specific parking locations based on 50 clusters of customer locations. For each of the four different spatial distributions, the locations of 400 customers and 50 MPL parking options are shown in the appendix Figure B.1. The travel distances between each location are calculated using Euclidean distances, and an average speed of 30 km/h is assumed to determine travel times.

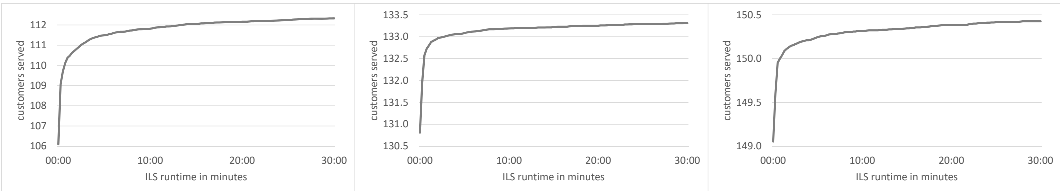
Another adjustment to the original instances is that we consider a 12-hour time horizon from 10 am to 10 pm, and time window distributions derived from real-world data provided by Köhler and Haferkamp, 2019. The probabilities for each time window are given in the appendix Table B.2. Moreover, one time unit corresponds to 12 minutes which, in turn, corresponds to the AHD service time. However, the minimum duration of a MPL service is set to one hour (5 time units) due to the real-world application of the problem: Customers need to specify a certain time slot for delivery and we decided to provide time slots on an hourly basis.

For AHD, MPL, and MHDPL services, vehicles with a total capacity according to the number of available customers, are utilized. All experiments were carried out on a computing cluster with 2.9 GHz Xeon-G 6226R processors and 384 GB memory with a maximum runtime of 30 minutes.

3.5.2 ILS Performance

To analyze the performance of the proposed ILS, we compare it to solving the MIP presented in Chapter 3.3.2 directly with Gurobi 9.1.2. Moreover, we evaluate three different services: using the MHDPL service and using only AHD or MPL services. For that purpose, we use three sets of instances with increasing problem complexity, each containing 100 instances. In the small instances, 100 customers distributed over 25 km^2 are served by only two vehicles. The complexity has been doubled in the medium-sized instances, with 200 customers in a service area of 49 km^2, and four vehicles. The large instances consist of 400 customers, 100 km^2, and 8 vehicles. Customer preferences are randomly assigned time windows of 1, 2, or 3 hours and maximum travel distances of 0.5, 1.0, or 1.5 km.

On average across all 300 instances, the ILS can find good solutions quickly. As shown in Figure 3.6, the solution quality improves continuously with increasing computing time, for (a) MPL, (b) AHD, as well as (c) MHDPL services. The average results for the differently sized instances are shown separately in the appendix, Figure B.2.



(A) MPL (B) AHD (C) MHDPL

Figure 3.6: Best found solution by ILS over time, aggregated over all instances

Figure 3.7 shows the average number of customers served by the MIP and ILS solutions operating a MPL, AHD, or MHDPL service for the 100-customer instances. 100% of these instances are solved to optimality for MPL, compared to 96% for AHD and 54% for MHDPL. On average, 50.9 of the 100 customers can be served by MPL, 56.3 by AHD, and 65.9 by MHDPL. The performance of the ILS in these small instances is excellent. The average ILS solutions are only 0.1 customers lower than the MIP solutions for MPL and MHDPL services. For AHD, both methods are equally successful. For MPL, the ILS achieves the optimal solution in 95% of the instances. When using MHDPL service, the ILS achieves the MIP solution in 88% of cases. However, the ILS can also achieve a better solution than the MIP within the specified maximum runtime in one instance. While the average runtime for the ILS is always 30 minutes, the MIP runtimes are on average 2 seconds for MPL, 8 minutes for AHD, and 22 minutes for MHDPL.

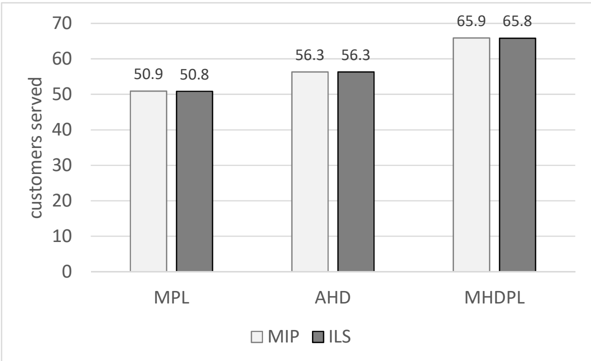


FIGURE 3.7: ILS performance for small instances

Results for medium-sized instances are shown in Figure 3.8. For these instances, optimal solutions can only be found when the MPL service is used, for more details see Table B.3 in the appendix. The best results indicate that 95.4 of the 200 customers can be served by MPL, at least 113.6 by AHD, and 126.7 by MHDPL. For MPL, the average ILS solution is 1.3 customers below the MIP solution, whose runtime is just six seconds. In all other cases, the average runtime is 30 minutes. The ILS was able to find better average objective values for both AHD and MHDPL than the MIP in the same time. The difference is 0.5 customers for AHD and 21.2 customers for MHDPL. For MPL, the ILS found the optimal solution in 22% of instances. For AHD, the ILS was on par with the MIP in 65% of instances and better in the remaining 35%. In terms of MHDPL, the ILS performed better than the MIP in every instance.

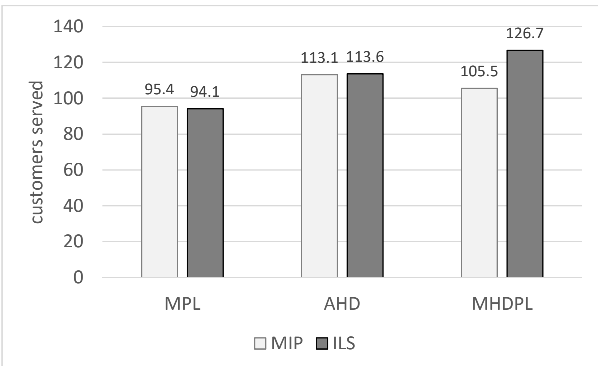


FIGURE 3.8: ILS performance for medium-sized instances

The results for the large instances in Figure 3.9 show the same trend as the medium-sized instances. The MIP indicates that 197 of the 400 customers can be served by MPL, while according to the ILS 230 by AHD, and 258.8 by MHDPL. The MIP achieves the optimal solution within 22 seconds for MPL, whereas the average ILS solution is 6.3 customers below the optimum. For some AHD and MHDPL instances, no solution is ever found with the MIP, therefore their objective value is included with 0. The average MIP solution is 172.3 or 193.4 customers lower than the ILS solution. In summary, for large instances, the MIP is better for MPL, while the ILS always achieves better results for AHD and MHDPL.

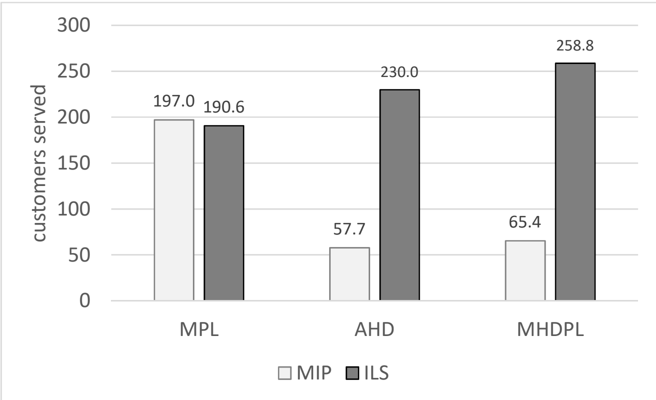


FIGURE 3.9: ILS performance for large instances

It can be concluded that the ILS achieves excellent to optimal results with small instances. With increasing instance size, the ILS continues to achieve good results, whereas the MIP struggles with AHD and MHDPL services. In the given setting, the MIP can find the optimal solutions for MPL quickly, even for large instances. Therefore, the MIP is used as the solution method for MPLs while the ILS is used for AHDs and MHDPLs in the following analyses.

3.5.3 Instance Characteristics

In this section, the instance-specific characteristics will be examined to determine when the use of a MHDPL service is particularly rewarding and under which circumstances an AHD or MPL service would be sufficient. To this end, the distribution of customers in the service area, the customer density (i.e., customers per square kilometer), and the number of vehicles available for service are varied (one at a time). The baseline setting corresponds to the previously presented large instances, consisting of

400 customers, 100 km^2, eight delivery vehicles, and random customer preferences, with 1, 2, or 3-hour time windows and 0.5, 1.0, or 1.5 km maximum travel distances.

Figure 3.10 shows the number of customers served for the four different spatial distributions of customer locations. The results are given for AHD, MPL, and MHDPL, based on 25 instances for each spatial distribution. MPLs are most affected by the different spatial distributions. The largest difference is 56 customers, with an average number of 167 for R1 instances and 223 for C2 instances. In contrast, AHD can constantly serve around 230 customers in each case. MHDPLs can compensate for the weaknesses of MPLs in R1 instances and serve 244 customers. Nevertheless, MHDPL benefits the most in C2 instances, where 272 customers can be served. This is where the difference towards the individual services is largest, with 41 (49) additional customers compared to AHD (MPL). In all further experiments, we provide averages across the different spatial distributions, i.e., over 100 instances.

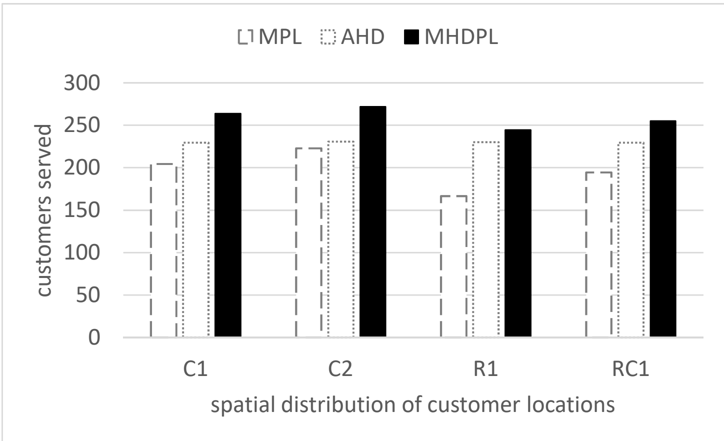


FIGURE 3.10: Experimental results, varying spatial distributions

The impact of the service area on the three services under consideration is analyzed in Figure 3.11. Here, we vary the size of the service area between 4 and 324 km^2. The highest number of customers can be served when there is a very high customer density, i.e., when the 400 customers are distributed over 4 km^2. As the size of the service area increases, the performance of MPL declines. In contrast, the number of accepted customers with AHD decreases only very little. If the density is high, all 400 customers can be served by MPL and only 231 by AHD. For a service area size of 100 km^2, the 197 customers served by MPL fall below the 230 customers served by

AHD. With a very low customer density at 324 km^2, AHDs have a clear advantage with 222 customers, as MPLs only serve 118 customers.

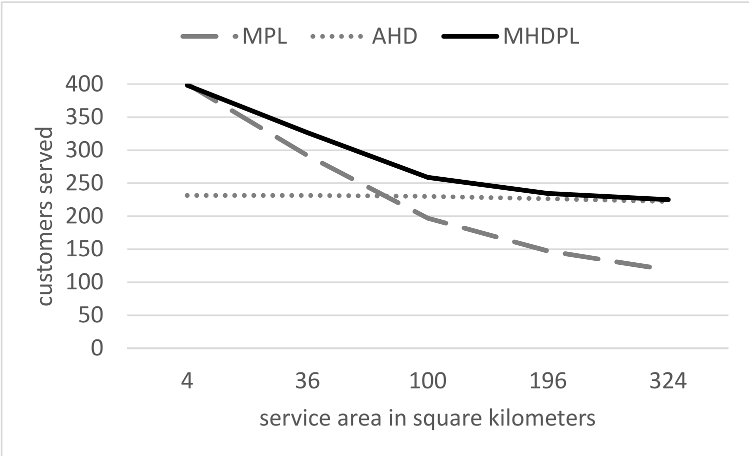


FIGURE 3.11: Experimental results, varying service area size

Summarizing, the MHDPL service is capable of adapting to different circumstances and achieves the same results as MPL when customer density is high and the same results as AHD when customer density is very low. Between these two extremes, MHDPL outperforms the individual service operations. If the individual services perform similarly well, which is the case at the intersection at around 70 km^2, the advantage of combining the two services is the highest.

To finalize the analysis concerning the instance characteristics, Figure 3.12 illustrates the potential of the services according to different fleet sizes. For the instances of the baseline settings, the number of vehicles is varied between four and 12. For each service, more customers can be served as the number of vehicles increases. However, while AHD and MPL are on par using four vehicles with about 118 served customers, numbers drift apart when the number of vehicles increases. Operating 12 vehicles, 253 customers can be served by MPL and 308 by AHD. MHDPL consistently serves around 28 more customers than AHD, regardless of the number of vehicles. This shows that the number of vehicles is less decisive for the performance of MHDPL services than the distribution and density of customers.

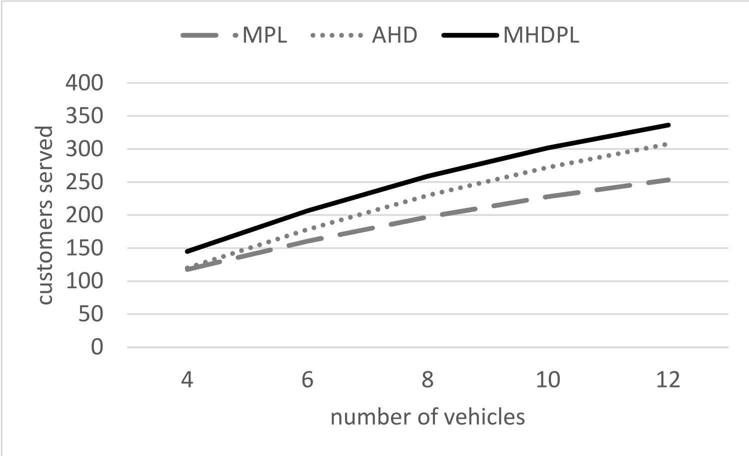


Figure 3.12: Experimental results, varying number of vehicles

3.5.4 Customer Characteristics

Alongside the previously discussed characteristics, the customers' individual preferences are fairly important for the successful application of a customer-friendly delivery service. This section therefore analyzes the implications of the different time window and travel distance preferences. For this purpose, the baseline settings are used as before, with 400 customers distributed across 100 km^2 and eight service vehicles while either the length of the time window or the maximum travel distance of customers are varied.

In Figure 3.13, the time window for each customer is equally set to 1, 2, 3, or 4 hours. At first glance, it appears that the time windows in these instances do not have a significant impact. AHD services are more flexible the longer the time window, allowing slightly more customers to be served. MPLs, on the other hand, are less flexible with longer time windows, resulting in a moderate decrease in the number of customers served by MPLs and MHDPLs. There is a recognizable saturation at 4-hour time windows for AHD and MHDPL. It can therefore be concluded that with longer time windows, the positive effect on AHDs and the negative effect on MPLs cause MHDPLs to slightly lose their advantage over AHDs.

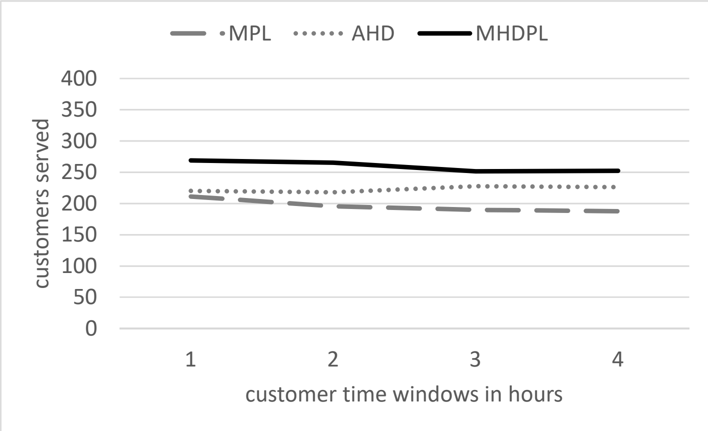


FIGURE 3.13: Experimental results, varying customer time windows

Figure 3.14 concludes this analysis by setting the maximum travel distance of all customers to 0.5, 1, 1.5, and 2 km. Varying the maximum travel distance has a similar effect to varying the customer density as described in the previous section. For a very short distance of 0.5 km, only 110 customers can be served using MPLs, compared to around 230 customers with AHDs and MHDPLs. For longer distances of 2 km, however, MPLs and MHDPLs can cover up to 374 customers, while AHDs remain at 230 customers. As the AHDs are not influenced by the customers' willingness to travel, a horizontal line is shown which intersects at approximately 1.2 kms with the relatively linearly ascending line of the MPLs. Again, it can be seen that the MHDPLs have the largest advantage when both individual services perform equally well, i.e., where their curves intersect.

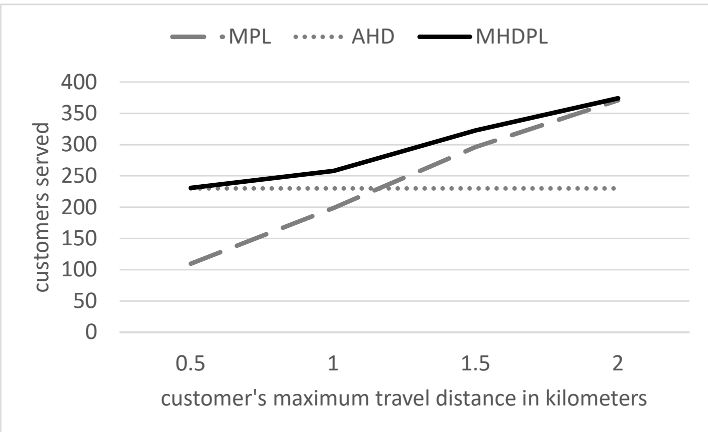


FIGURE 3.14: Experimental results, varying maximum travel distances

3.5.5 Service Utilization

Following the evaluation of instance parameters and customer preferences, the utilization of the services will be examined in this section. In particular, we analyze how the vehicles are used in the MHDPL service, how often and for how long AHDs and MPLs are carried out, and, finally, the frequency of using particular MPL parking locations. The following results are based on all 1700 instances previously considered in Chapter 3.5.3 and Chapter 3.5.4.

Since any MHDPL vehicle can perform any type of service, it is interesting to observe how often the vehicles perform which service type. In 870 of the 1700 instances, at least one vehicle specializes in AHD or MPL services. More precisely, in 712 (169) instances, at least one vehicle is specialized in AHD (MPL). Therefore, in 11 instances, both AHD and MPL specialized vehicles are operated. Exclusively AHD vehicles are used in 40 instances. In contrast, MPL vehicles are never used exclusively in any instance. Therefore, the benefits of the mixed services are evident, as in 98% of the instances MHDPL vehicles operate in both service types.

Figure 3.15 shows how many of the 13600 vehicles in the instances under consideration exclusively represent MPLs, AHDs or use both services. While just 2% of the vehicles operate exclusively as MPLs, 21% of them operate as AHDs. The majority of vehicles (77%) operate as MHDPLs. Details on the occurrence of vehicles specializing in MPL or AHD in the various instances can be found in the appendix, Figure B.4 and Figure B.5. Focusing on the AHD specialized vehicles, 14% (8%) occur in the instances where 12 (10) vehicles are used and 15% in the instances with larger time windows of 3 or 4 hours. 21% of AHD specialized vehicles occur when the maximum travel distance is short at 0.5km and 22% (14%) when the service area is large at 324km^2 (196km^2). In contrast, vehicles specializing in MPL services are more likely to occur with 20% (10%) in instances with longer maximum travel distances of 2km (1.5km). Moreover, the majority of MPL specialized vehicles occur for a very small service area of 4km^2 (36km^2), which is 58% (7%). In summary, the most important criteria for the occurrence of MPL or AHD specialized vehicles are extreme characteristics in service area sizes or maximum travel distances.

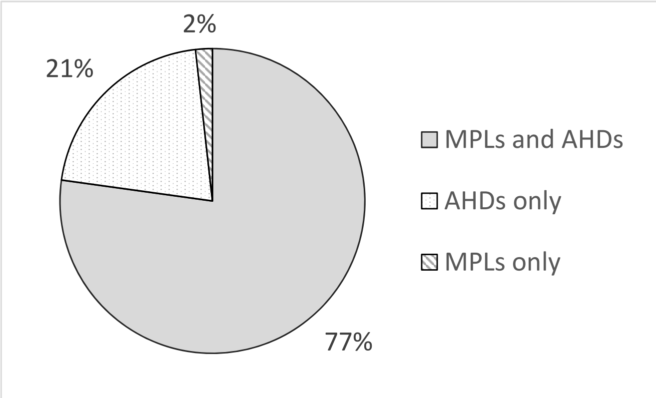


FIGURE 3.15: MHDPL service vehicle types

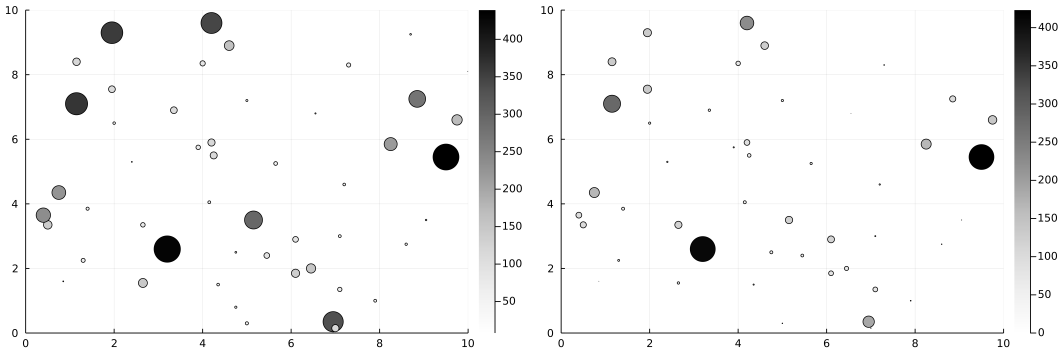
Next, we analyse the average vehicle behavior across all instances per service type. Table 3.5 shows in more detail how often and how long AHDs and MPLs are performed, excluding the travel times between the stops. For AHD, the average vehicle performs 28.2 stops and spends 5.6 hours serving 28.2 customers. For MHDPL, the average vehicle only serves 18.3 customers using AHDs and spends 3.7 hours on their service time. Simultaneously, this vehicle also carries out 1.3 MPL stops, which take 3.6 hours and serve 16 customers on average. In contrast, on average, the MPL service vehicle serves 27 customers and performs 2 stops with a total service provision period of 11 hours. MHDPL vehicles spend only 34% of the time with MPLs than MPL-only vehicles, resulting in 59% of customers served. It can therefore be concluded that, on average, a MHDPL vehicle can serve 34.2 customers through the combination of services, whereas the dedicated AHD/MPL service vehicle is only able to serve 28.2/27 customers.

service	performed	number of stops	total service duration	customers served
AHD	AHDs	28.2	5.6 h	28.2
MPL	MPLs	2.0	11.0 h	27.0
MHDPL	AHDs	18.3	3.7 h	18.3
	MPLs	1.3	3.6 h	16.0

TABLE 3.5: Comparison of the average vehicle performing AHDs/MPLs

Finally, the use of the available MPL parking locations is shown in Figure 3.16. We focus on RC1 instances here, i.e., the mixture of clustered and random customer locations. Visualizations of the remaining instances can be found in the appendix, see

Figure B.3. How often the parking locations were used in the MPL/MHDPL services is shown in Figure 3.16a/Figure 3.16b. Larger and darker circles represent more frequent use. Some locations are used in every instance for both the MPL and the MHDPL services considering the 425 RC1 instances. However, there is a difference in the number of locations used very frequently, which are significantly more locations for the MPL service. In addition, the average duration of the stop is significantly longer for the MPL service (4.83 hours) than for the MHDPL service (2.69 hours). With the MHDPL service, two locations are used particularly frequently, two locations are used moderately frequently, and two locations are not used at all. It can therefore be stated that promising locations for the MHDPL service could be identified and reserved in advance for a real service scenario.



(A) MPL service

(B) MHDPL service

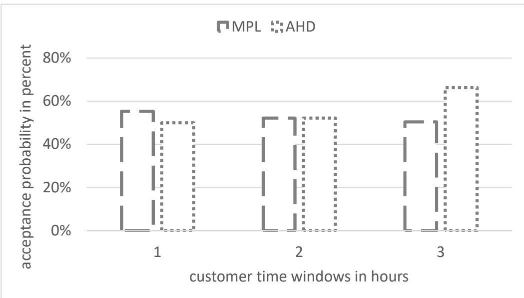
FIGURE 3.16: Number of visits at MPL parking locations, RC1 instances

3.5.6 Customer Service

This section is dedicated to analyzing which customer service can be achieved with the MHDPL service compared to individual AHD or MPL services. Firstly, the time window and travel distance preferences with which customers are most likely to be accepted or rejected are discussed. Secondly, the actual travel distances that customers can expect when collecting deliveries from a mobile parcel locker are analyzed. The evaluations consist of all instances of the previous experiments. The customers have preferences of 1, 2, or 3-hour time windows and 0.5, 1, or 1.5 km maximum travel distance. The special cases where customers have 4-hour time windows or 2 km travel distances are not analyzed in detail, as the data basis is not directly comparable.

Nevertheless, the acceptance probabilities for all customer preference combinations are given in appendix Figure B.6.

Figure 3.17 shows the probabilities that a customer with a certain time window preference will be accepted for the different services. Figure 3.17a contains the services where the customers were served exclusively via MPLs or AHDs, whereas in Figure 3.17b, the MHDPL service is shown but the probability is split into whether they would be served with MPLs or AHDs. Customers with a 3-hour time window are most likely to be accepted by the MHDPL service with a probability of 71%. This is closely followed by customers with 1 or 2-hour time windows with 65% in each case. In terms of how likely it is to be served by the MHDPLs using MPL/AHD services, however, the proportion decreases/increases steadily as the time window length becomes longer, from 36% to 23% via MPL and from 29% to 48% via AHD. The same trend of decreasing/increasing acceptance probability can be observed when customers are served exclusively with MPLs/AHDs, as shown in Figure 3.17a. The acceptance probabilities range from 55% to 50% for MPL services and from 50% to 66% for AHD services. From the customer's point of view, a longer time window indicates a better acceptance of the service, but if the customer prefers to be served with the MPL service, it would be recommended to choose a shorter time window.



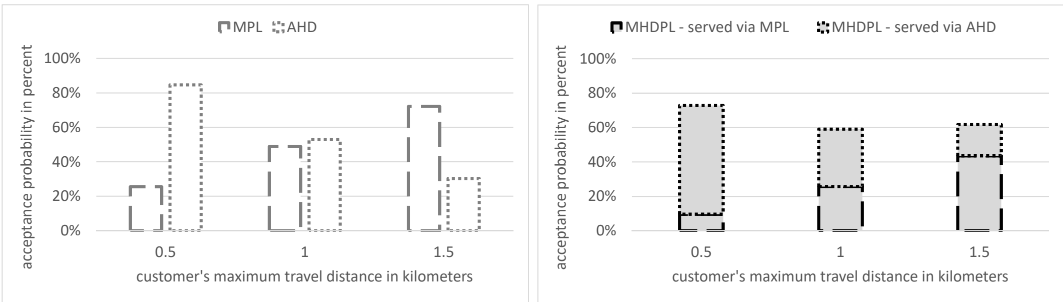
(A) MPL service and AHD service

(B) MHDPL service

FIGURE 3.17: Customer acceptance probabilities based on time window preference

Figure 3.18 shows the probabilities that a customer with a certain maximum travel distance preference will be accepted for the different services. Like the previous figure, Figure 3.18a contains the services where the customers were served exclusively via MPLs or AHDs. For the individual services, the acceptance probabilities increase/decrease significantly with increasing maximum travel distance for MPLs/AHDs; for MPL services from 26% to 72% and for AHD service from 85%

to 30%. The maximum travel distance should not affect AHD services, but this can be attributed to the fact that the ILS prioritizes customers with a shorter maximum travel distance in order to support MPL services. In Figure 3.18b, the MHDPL service is shown, with the probability split into whether they are served with MPL or AHD services. Customers with a very short maximum travel distance have the highest probability of being accepted for the MHDPL service (73%). With increasing maximum travel distance, the probability of acceptance first decreases (59% at 1 km) and then slightly increases again (62% at 1.5 km). The decrease is clearly due to fewer customers served by AHD and the subsequent increase due to the increase in customers served by MPL. In summary, For MPL services, the probability of being served increases with a longer maximum travel distance.



(A) MPL service and AHD service

(B) MHDPL service

FIGURE 3.18: Customer acceptance probabilities based on maximum travel distance preference

After considering the acceptance probabilities for customers with certain service preferences, the actual travel distances compared to the maximum travel distance preference will be discussed in more detail. Figure 3.19 shows for all customers and for customers grouped by their maximum travel distance how far they would actually have to travel on average. The average maximum travel distance of all customers served by MPL services is 1.18 km, of which the average customer has to travel 0.68 km to collect their delivery. Customers who are inclined to travel up to 0.5 km must on average travel almost exactly half this distance, i.e. 0.25 km. The customer group with the highest maximum travel distance (2 km) has to travel 1.21 km on average. As the maximum travel distance increases, the relative actual travel distance also increases, from 50% in the 0.5 km group to 61% in the 2 km group. It can be concluded that the more inclined customers are to travel, the more likely it is that this will be exploited.

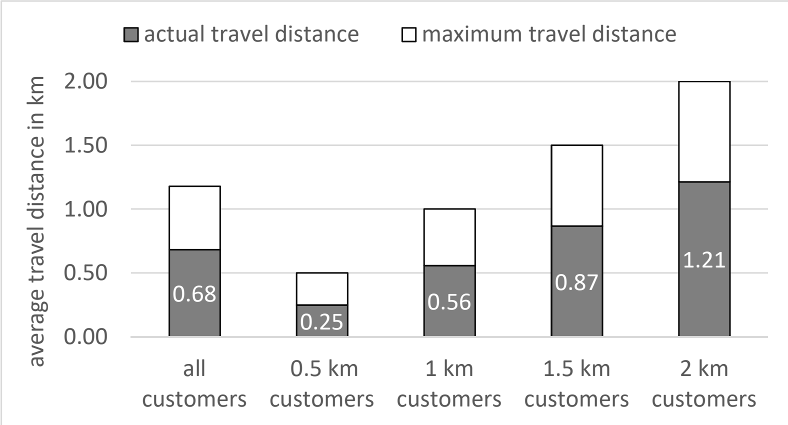


FIGURE 3.19: Average actual travel distances for different customer groups

Another interesting aspect is the consideration of how the actual travel distances react with different fleet sizes. With additional vehicles, more customers can be served, so it could be assumed that more difficult customers with greater distances might be accepted. However, this is not the case, as the actual distances decrease with additional vehicles, as can be seen in Figure 3.20. The resulting average travel distances to the pickup locations with varying fleet sizes are shown separately for MPL only services and for MHDPL services. The actual travel distances decrease with increasing fleet size, but this trend is relatively small for the MHDPLs, from 0.69 km with 4 vehicles to 0.67 km with 12 vehicles. For the MPLs, on the other hand, this is more pronounced, from 0.69 km to 0.62 km. It can be deduced that the additional vehicles are used for better coverage and therefore the travel distances of all customers are reduced. Additional vehicles can therefore not only accept more customers, but also reduce average travel distances and thereby improve customer service.

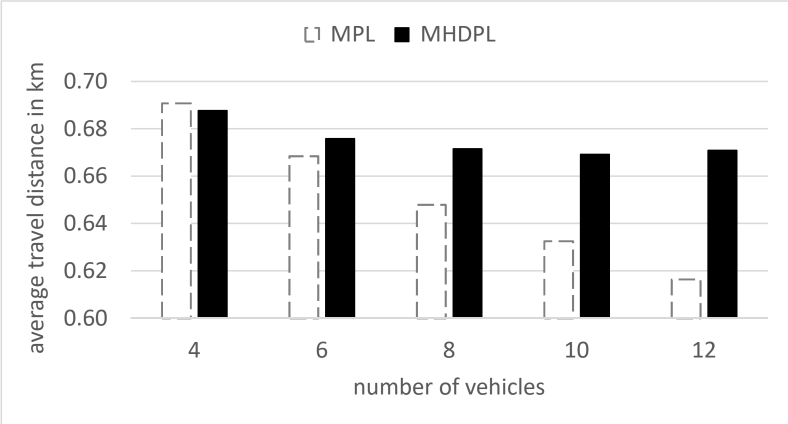


FIGURE 3.20: Average actual travel distances for different fleet sizes

3.6 Conclusion

We have introduced the MHDPL service, where both AHD and MPL services can be performed by each vehicle allowing to adapt to the existing customer situation. Our problem relies on a fixed homogeneous fleet of vehicles to maximize the number of customers served. To find good solutions for larger problem instances, we have proposed an ILS and have developed a variant specially designed for the problem at hand. For the local search procedure, we have designed operators that move individual customers. We have also introduced operators that consider customer groups via stopovers. During the perturbation procedure, parts of the solution are destroyed randomly or specifically all activities within a certain time interval.

We have conducted a study exploring different numbers of customers, spatial distributions, service area sizes, fleet sizes, and customer preferences. The solutions obtained by the MIP and the ILS for the AHD, MPL, and MHDPL services have been presented and discussed. We have been able to show that the ILS performs well for MHDPL services. For small instances, 89% of the solutions found were equal to or better than the MIP solutions, while for larger instances, the MIP was outperformed in every instance.

The managerial insights indicate that customer density and customer willingness to travel are the key characteristics to decide whether AHD or MPL services are the most efficient. We have observed that individual home deliveries would utilize the available time more efficiently if the customer density is lower. Peppel et al. (2024) similarly conclude that there is limited potential for MPLs in areas with only a few customers and that home deliveries may be more efficient from a sustainability perspective. They suggest cities with at least 20,000 inhabitants to ensure a sufficient population density for MPL services. In contrast, Rolf et al. (2022) focus on the fleet costs (time, distance, emissions) and conclude that from their point of view MPLs could be used even in small-scale areas where FPLs would not be appropriate.

However, we have demonstrated that the MHDPL service is able to adapt to any given scenario and has the greatest advantage when both services work similarly well. Although Liu et al. (2023) do not consider home deliveries, they recommend the combination of MPLs with other mainstream delivery methods (e.g. home deliveries), especially in the introductory phase of MPLs. Kötschau et al. (2023) showed the potential of combining AHDs with MPLs using two distinct fleets of vehicles with only a single delivery mode. In our experiments, the benefits of the mixed service vehicles are evident, as in 98% of the instances, MHDPL vehicles operate in both service types. The combination of services has economic potential and satisfies customer preferences more effectively. We could show that an average MHDPL vehicle can serve 21% more customers than a comparable AHD vehicle. Furthermore, additional vehicles not only allow more customers to be accepted but also improve the service quality through shorter travel distances for customers.

To identify MPL locations, Liu et al. (2023) recommend focusing on strategic locations (e.g., near commercial buildings or schools) and time intervals in order to utilize MPL services most efficiently. However, our study and Rolf et al. (2022) were able to show that some MPL locations appear to be particularly useful and dominant in retrospect, regardless of strategic considerations. It would therefore be beneficial for the following studies to identify and utilize these particularly suitable MPL locations in advance.

For future studies, it would additionally be interesting to investigate how the various services can react to dynamic and stochastic demand. This goes hand in hand with the possibility of developing a customer acceptance mechanism in which customers do not show up with fixed preferences. Still, the preferences are negotiated depending on current and forecasted demand. Furthermore, a cost analysis of the various services, e.g., with a guarantee of a certain service level, would be particularly important for enhancing these services.

Chapter 4

Dynamic Customer Acceptance for MHDPL Services

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submitted, June 2025

Abstract The two traditional modes for parcel deliveries to customers are attended home deliveries (AHD) and stationary parcel lockers. A very innovative concept for last-mile delivery service is the Mobile Parcel Locker (MPL). Recent studies have shown that the combination of AHD and MPL services is promising, and a service called Mobile Home Delivery Parcel Lockers (MHDPL) has been introduced. For an efficient service, the customer acceptance decision must consider whether a customer should be served via AHD or MPL. Moreover, this decision must be made for each individual customer without knowing future customers. Therefore, this study deals with dynamic customer acceptance mechanisms for MHDPL problems. The focus is on what type of service (AHD or MPL) should be offered to each customer in order to maximize the number of accepted customers. We provide fundamental components from which effective offer strategies for MHDPL services can be developed. A comprehensive computational study provides managerial insights into the challenges of dynamic customer acceptance mechanisms for MHDPL services and the potential of the developed offer strategies. Additionally, different offer strategies are compared, ranging from simple, intuitive approaches to more sophisticated approaches that attempt to utilize realistic problem knowledge about dynamic customer arrivals. An average of 16% improvement can be achieved with a strategy that exploits realistic problem knowledge compared to the dynamic benchmark.

4.1 Introduction

Last-mile deliveries have been on the rise in recent years, and predictions indicate that this trend will continue (Statista, 2023d). The two traditional modes for parcel deliveries to customers are home deliveries and parcel lockers. From a provider's point of view, home deliveries are challenging, as they are associated with uncertainties (attendance of the customer) and linearly increasing working hours (handover time for each customer). This uncertainty can be avoided through effective communication with the customer regarding their availability, a service known as Attended Home Deliveries (AHD) (Campbell & Savelsbergh, 2006). Linearly increasing working hours can be avoided with parcel locker services. In this case, customers collect the parcels themselves from a previously specified location. Bundling customers at predefined service locations enables economies of scale as parcel volumes increase. However, stationary parcel lockers raise a new strategic problem as to where the parcel lockers or similar service locations should be implemented and what capacity is required to accommodate fluctuations. Additionally, initial investment costs are high, especially if the service provider strives for a high level of service with short travel distances for each customer. This creates a trade-off between a high service level and lower working hours since a higher service level requires a higher density of parcel lockers. If the parcel lockers are closer to the customers, then fewer customers are bundled and more parcel lockers need to be visited, requiring additional working time.

A very innovative concept for last-mile delivery service is the Mobile Parcel Locker (MPL). MPLs are parcel lockers that can be moved and positioned at different locations. They can be realized with dedicated vehicles, trailers, or stations that are loaded onto vehicles (Schwerdfeger & Boysen, 2022). These are filled at the depot beforehand and then placed and relocated based on the actual demand. An MPL offers the flexibility to address diverse customer requirements, from short to longer service provision periods with different maximum travel distances. For example, MPLs can be positioned for a short period of time to serve a few customers who are less willing to walk, or they could be positioned to serve more customers who are willing to walk longer distances. Although AHD and MPL services can be conceptually modeled in a similar way, they have important differences. AHD services take a short period of time within a specific time window, while MPL services are available to customers during the entire service provision period. In addition, AHD services take place directly at the customer's location, while for MPL services, a vehicle is positioned within the previously defined maximum travel distance.

Recent studies have shown that the combination of AHD and MPL services is promising. Kötschau et al., 2025 introduce a service called Mobile Home Delivery Parcel Lockers (MHDPL), where each vehicle can provide both types of service. While they consider a static problem where all customers are known in advance, in this paper, we consider a dynamic problem where customers arrive successively, and a suitable service option for each customer must be agreed on without knowing the future customers. Assuming that all customers contribute the same, the service provider naturally aims to serve as many customers as possible with its available fleet. To achieve this, the service provider must assess which service options to offer to a customer under the given circumstances. This decision has to be made while the customer is inquiring and before subsequent customers are known. As service options, various forms of MPL services (with different service provision periods and maximum travel distances) and AHD services (with different time windows) can be considered. For example, a customer requesting an all-day MPL service provision period at their doorstep would be very expensive for the service provider.

Dynamic customer acceptance mechanisms have been studied frequently for AHD services (Waßmuth et al., 2023). However, to the best of our knowledge, they have not yet been considered for MPL services. Compared to AHDs, MPL services have the potential to bundle several customers efficiently, but this is difficult if future customers are unknown. Furthermore, an MPL service occupies the available resources for a longer period of time, which means that shortsighted planning will inevitably reduce the potential to accept more customers in the future.

The contributions of this paper include customer acceptance mechanisms for MHDPL problems. The focus of the study is on what type of service (AHD or MPL) should be offered to each customer in order to maximize the number of accepted customers. In this way, MPLs are considered for the first time in customer acceptance mechanisms. To this end, we present a dynamic feasibility check for MHDPL service options. The feasibility check considers the requirements (service options) with which each customer must be served. The actual tour plans (exact order of AHD and MPL stopovers as well as the final locations of MPL stopovers) can be redefined with each customer request. For this purpose, the iterated local search for the static MHDPL problem by Kötschau et al., 2025 is adapted. In the form of continuous re-optimization, the previous best solution is reused every time a new customer arrives.

Moreover, we provide fundamental components from which effective offer strategies for MHDPL services can be developed. These components use set and max covering formulations to derive decisions on whether a specific customer should be served with either AHD or MPL service. We consider the value of each customer to be equal and do not explicitly choose to reject customers beforehand. We conduct a comprehensive computational study to create managerial insights. On the one hand, the challenges of the dynamic customer acceptance mechanisms with MHDPL services are discussed in direct comparison to the static problem with hindsight information. On the other hand, the potential of the developed offer strategies is evaluated by applying hindsight knowledge. Finally, different service offer strategies are compared, ranging from simple, intuitive to more sophisticated approaches that attempt to utilize realistic problem knowledge about dynamic customer arrivals.

In the following, Chapter 4.2 describes the narrative for the static and dynamic MHDPL customer acceptance problem. Chapter 4.3 situates the addressed issue in the existing literature. Customer acceptance mechanisms involving feasibility checks and components for suitable offer strategies are described in Chapter 4.4. Chapter 4.5 deals with the computational study, in which problem characteristics and hindsight solutions, as well as different dynamic offer strategies, are analyzed and compared. Finally, the conclusion is given in Chapter 4.6.

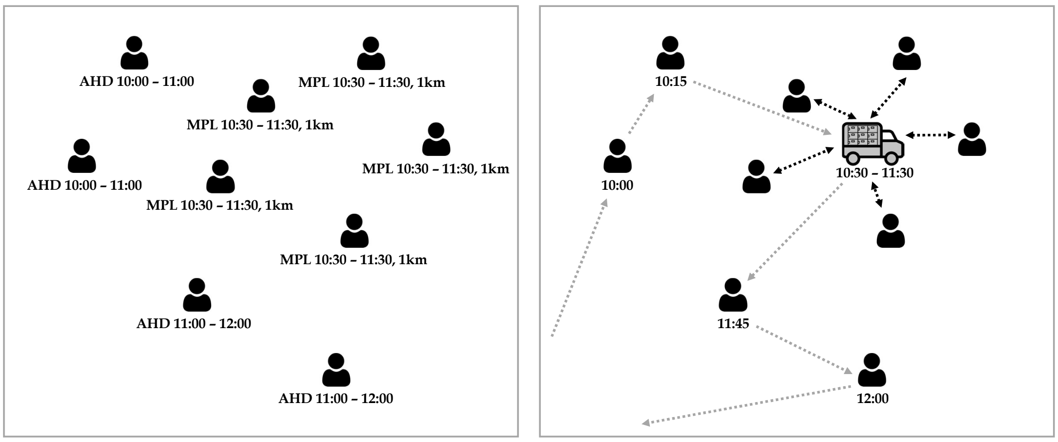
4.2 Problem Narrative

In the dynamic MHDPL problem, customers arrive successively and are served with either AHD or MPL service. In the following, we first describe the static problem assuming that all information is already known at the beginning of the planning horizon. This serves to better understand the following dynamic problem and is also used as a benchmark for the computational study. Afterwards, the dynamic MHDPL problem is presented, which illustrates the characteristics of the dynamic customer acceptance mechanisms.

In the static problem, a fixed fleet of MHDPLs is used to maximize the number of served customers using AHD and MPL services. Each customer is defined by a location, the time period during which they are available for service, and a service preference for either AHD or MPL services. A set of *service options* is predefined and consists of one or more AHD service options and one or more MPL service options. AHD service options are defined by different *time windows*, e.g., 10:00-11:00 and

11:00-12:00. The AHD service will take place at the customer's location for a short service time within the designated time window. MPL service options are defined by a *service provision period* and a *maximum travel distance*, e.g., 10:30-11:30 within 1 km from the customer's location. Hence, during MPL service, the mobile parcel locker is positioned close to the customer (e.g., within 1 km) and the customer has the flexibility to collect their delivery within the service provision period (e.g., 10:30-11:30).

The problem now is to select a service option for each customer within their availability (or reject the customer) and construct a tour plan such that the number of customers served is maximized. A tour plan consists of the routes of all vehicles of the fleet, which can be described by sequences of so-called *stopovers*. A stopover is defined by a location, the time period the vehicle remains there, and the set of customers served. While an AHD stopover serves a single customer at their location, several customers served by MPL service option can be bundled to a single MPL stopover if i) the stopover location is within the maximum travel distance from the customer's location and ii) the service provision period of the MPL service is a subset of the time period the vehicle remains at the location. Travel times between all pairs of possible stopover locations are given.



(A) Assigned service options (B) Finalized tour plan

FIGURE 4.1: Example solution for the static MHDPL problem

Figure 4.1 shows an example of the assigned service options and a feasible tour plan for the static MHDPL problem. In the example, we have nine customers who are available between 10:00 and 12:00. Travel and service times have been streamlined for easier traceability; It is assumed that a new service can begin every 15 minutes. At the time of planning, all customer locations and their availability periods are known. In Figure 4.1a, each customer is assigned a service option so that all customers can be served. The assigned service option is the binding commitment towards the customer regarding the service to be provided. However, the tour plan in Figure 4.1b is just one feasible solution for serving customers while adhering to the given service options. All customers can be served with a single vehicle performing one MPL and four AHD stopovers. Two customers are served with AHD stopovers at 10:00 and 10:15, then the vehicle performs an MPL stopover from 10:30 to 11:30 to allow five customers to collect their deliveries, and the last two customers are served with AHD stopovers at 11:45 and 12:00. This example highlights the dependency of the final tour plan from the assigned service options and that service options must be selected in a way that ensures a feasible tour plan.

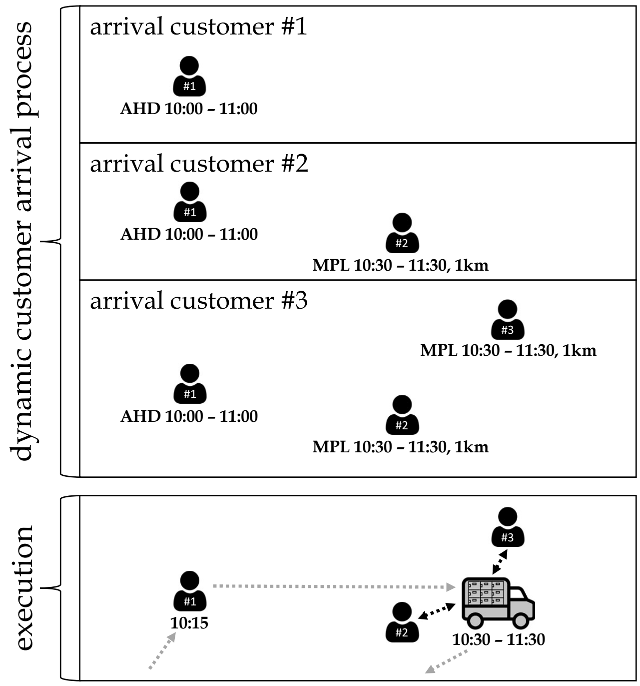
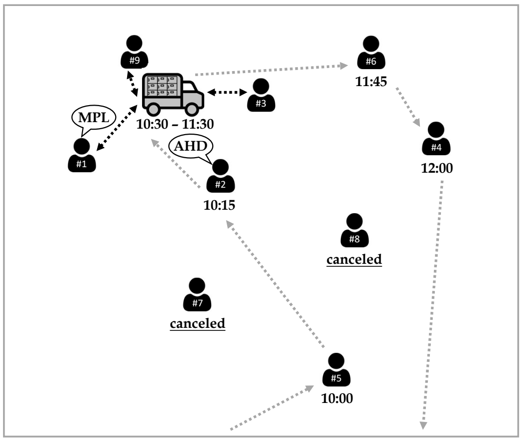


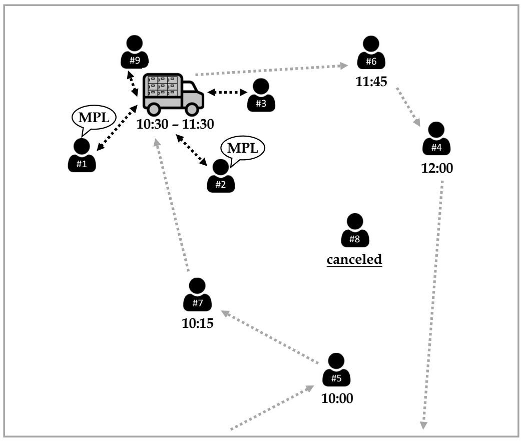
FIGURE 4.2: Example dynamic customer arrivals

In the dynamic MHDPL problem, the customers arrive successively, and a dynamic customer acceptance mechanism is carried out before the resulting tour plan is executed. For instance, customers may arrive during the course of a day, and the vehicle executes the plan the next day. An illustrative example with three customers arriving one after the other is given in Figure 4.2. For each customer, the commitment to a service option must be made before the subsequent customers are known. With every new customer that arrives, each service option of the predefined set must be evaluated, and the service provider and customer must agree on a suitable service option. Thereby, it is possible to constantly revise the temporary tour plan. The service provider may choose to offer the entire set of feasible service options to each arriving customer, or only some of the service options can be offered to each customer individually. Based on the customer's availability and personal preferences, the customer selects the best possible service from the options offered. If the customer cannot find an acceptable service option, the customer withdraws the request. From the provider's point of view, it is important that the options offered are not too expensive and do not restrict flexibility too much, as the available resources will be blocked, and subsequent customers will increasingly have to be turned away. At the same time, the service provider wants to offer more time-consuming MPL services in order to serve subsequent customers more efficiently during the same stopover.

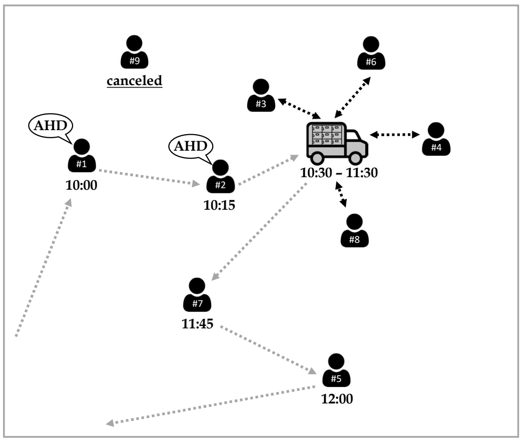
The example in Figure 4.3 illustrates the dynamic effects of different assigned service options. The focus is on the differences in offering AHD or MPL service. Different time window lengths, service provision periods, or maximum travel distances are not incorporated, and all nine customers are available between 10:00 and 12:00. The sub-figures show the effects of different selected services for the first two customers arriving. If the first customer selects MPL service and the second customer AHD service, as shown in Figure 4.3a, the customers #7 and #8 cannot be served. These customers are too far away from the MPL location, and AHD service is not possible due to time constraints. Although customer #9 arrives after the declined customers, this customer can be integrated into the MPL stopover together with customers #1 and #3. If the second customer would also opt for MPL service, as shown in Figure 4.3b, customer #2 could be included in the MPL stopover, and customer #7 could be served with AHD service instead. Customer #8 is still not served. If the first two customers have opted for the AHD service, as shown in Figure 4.3c, customers #3, #4, #6 and #8 can be served with an MPL service and #5 and #7 are served with AHD services. Although it would also be possible to serve the MPL customers with separate AHD services, customer #9 cannot be served. Figure 4.3d demonstrates that if AHD service was agreed upon with the first customer and MPL service with the second customer, all nine customers can be served. In summary, the acceptance of a single customer and, in particular, the selected service option, have a substantial impact on the acceptance of future customers. This highlights the importance for service providers to carefully decide on the offered service options.



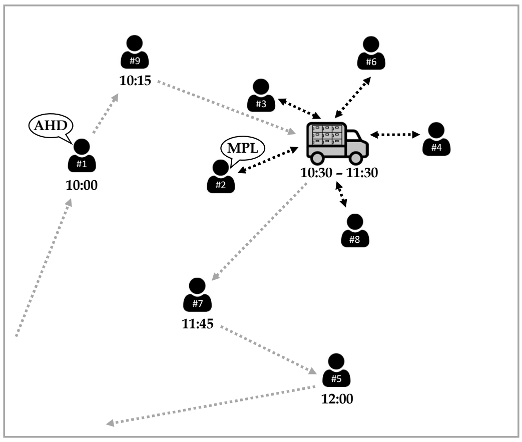
(A) #1 MPL and #2 AHD



(B) #1 MPL and #2 MPL



(C) #1 AHD and #2 AHD



(D) #1 AHD and #2 MPL

Figure 4.3: Example solutions for the dynamic MHDPL problem, based on different service options for the first two customers.

The service provider uses one vehicle to serve nine customers between 10:00 and 12:00. The sub-figures show the effects if AHD or MPL service was agreed separately for the first two customers. For the first customer, there are not enough direct neighbors, which is why AHD service would be more efficient. For the second customer, MPL service would be better, as there are many customers in the surrounding area. However, customers arrive successively, which is why these decisions are difficult due to a lack of information about future customers.

4.3 Related Literature

The challenge in the dynamic MHDPL problem is to accept as many customers as possible with dynamically arriving information. In the following, we briefly discuss the relevant literature on the static problem variant, outline the related dynamic literature on customer acceptance mechanisms, and situate our contribution regarding the dynamic MHDPL problem.

For the static MHDPL problem, all information is known at the planning stage, and vehicles perform MPL and AHD services sequentially. The existing literature covers MPLs used on their own (Liu et al., 2023; Schwerdfeger & Boysen, 2020, 2022; Wang et al., 2024), MPLs combined with stationary parcel lockers (Wang, Bi, & Chen, 2020; Wang, Bi, Lai, & Chen, 2020), and MPLs in combination with home deliveries (Kötschau et al., 2023, 2025; Peppel et al., 2024; Rolf et al., 2022). Kötschau et al., 2025 consider the problem with MHDPLs for the first time. Here, all customers are known at the beginning of the planning horizon, and the aim is to serve as many customers as possible with the given fleet. The authors model the MHDPL problem as a generalization of the team orienteering problem (TOP). The TOP was first introduced by Chao et al., 1996 and aims to find a defined number of routes through a set of nodes, each with different profits, to maximize the total profit. For more detailed reviews on the TOP, we refer to Archetti et al., 2007, Vansteenwegen et al., 2011, Gunawan et al., 2016 and Vansteenwegen and Gunawan, 2019. Kötschau et al., 2025 model the problem on a time-space network with correlated profits and propose an iterated local search (ILS) as solution approach. This approach can, therefore, be used to solve the static problem as a benchmark.

To the best of our knowledge, there is no research on dynamic problems that take MPLs into account. A TOP in the dynamic context, called Online Capacitated TOP, is considered by Shiri et al., 2024. However, in contrast to our problem, all request locations are known at the beginning, and only the possible profits are dynamic. The profits are not revealed until the location of the corresponding request is reached, whereas in the MHDPL problem, the request locations (i.e., the locations of the customers) are dynamic and are successively revealed over time. Closely related to the TOP is the vehicle routing problem (VRP), which has been studied under dynamic aspects for several decades. The VRP, first introduced by Dantzig and Ramser, 1959, aims to find a number of routes through a set of nodes minimizing the associated routing costs. In the dynamic VRP, some static information is available at the time of planning, but unknown dynamic events will realize in the future. There are various aspects to be considered dynamically, such as vehicle locations, travel times, and new customer requests (Ghiani et al., 2003). Examples for such problems include cab services with a focus on relocating temporarily unused vehicles or dial-a-ride systems where static customers book in advance, and dynamic customers request a ride at short notice (Jaillet & Wagner, 2008). Therefore, the entire solution needs to be re-evaluated as more information becomes available. Dynamic VRPs are characterized by fast response times and the ability to deny or defer orders, for example, to avoid excessive delays or inappropriate costs (Ghiani et al., 2003). In general, deterministic dynamic problems are re-optimized when new events occur, either periodically or continuously (Pillac et al., 2013). Periodically, a static variant of the problem is solved whenever new information is available. This allows already existing algorithms to be used, although the processing times may be rather long. With continuous re-optimization, information about good solutions is stored and reused for the next planning epoch.

The stream of dynamic VRPs focusing on dynamically arriving customer requests aims to decide whether a customer should be accepted or rejected in the prevailing situation. For the service provider, each agreement with every single customer is a dynamic event that can take place even before the vehicle fleet is put into operation. This customer acceptance process and the delivery process can therefore be treated as two independent problems. Campbell and Savelsbergh, 2005 point out that the integration of these two problems promises substantial potential to improve the profitability of last-mile deliveries, especially for services where the customer must be present. Customers must be present for security reasons, when goods are perishable, or where goods are collected or exchanged. This also applies to the MHDPL services, as attendance is not only required for home deliveries, but also for the temporarily available mobile parcel locker services, where customers have to collect the delivery themselves.

Research combining the customer acceptance and the delivery process has been summarized by Waßmuth et al., 2023 as demand management. Possible objectives of demand management are to prioritize higher-value customers, to serve more customers, or to reduce costs through more efficient delivery. During customer acceptance, the service provider offers different options (e.g., time slots) for the customer to be served; This set is referred to as service options. The emphasis is on offering and/or pricing of service options. In pricing, the service options are priced differently to create incentives but still leave the customer with a choice. When the focus is on offering, the question is which service options are offered to a particular customer. In our paper, we focus on the offering of service options, with emphasis on the different characteristics of MPLs and AHDs. The decisions about service options can be divided into three different planning levels: The strategic level, where service regions, pricing models, design of service options, and the identification of service segments are addressed; The tactical level, where the service options and prices for different service segments are defined based on demand forecasts; the operational level, where individual customers are accepted, and the availability and prices of service options offered are adjusted. We focus on the operational level, where smart decisions are required to be able to accept as many customers as possible. The providers' operational decisions on the service options are made individually for each service option. However, before decisions on offerings can be made, the feasibility of service options must be assessed. Related literature on AHD services applies different approaches for checking feasibility, ranging from functional approximation (Köhler & Haferkamp, 2019; Van Der Hagen et al., 2024), over insertion heuristics (Campbell & Savelsbergh, 2005; Ehmke & Campbell, 2014; Köhler et al., 2020) to more advanced routing methods (Casazza et al., 2016; Hungerländer et al., 2017; Truden et al., 2022; Visser et al., 2019). We use an advanced routing method: the ILS for the static MHDPL problem is adapted to be applied in continuous re-optimization. Although demand management has not yet been studied for MPL services, it has been studied for stationary parcel lockers by Sailer et al., 2024. However, this differs significantly from the MHDPL problem as it focuses on utilization and does not consider routing. With a focus on different compartment sizes and priority customers, they carefully consider which lockers should be offered to maximize the expected number of accepted customers, weighted by their priority.

4.4 Customer Acceptance Mechanisms

Dynamic customer acceptance involves a series of essential steps, which are illustrated in Figure 4.4: 0) The service provider defines the initial set of available service options and maintains a list of already accepted requests, including the chosen service options, as well as a preliminary solution for the routing problem. 1) Customer arrival and input of delivery location and availability in terms of time. 2) The service provider checks which service options are feasible with respect to the requests already accepted as well as the new customer's location and availability (see Chapter 4.4.2). 3) The service provider decides which of the feasible service options are offered to the customer (see Chapter 4.4.3). 4) The customer either accepts one of the offered service options and is added to the set of already accepted requests or rejects. If the customer is offered AHD and MPL services, they choose the service that corresponds to their personal preference. In the case that more than one service option is offered for one of the two types of services, the customer chooses randomly between these options. Steps 1 to 4 are repeated for each newly arriving customer.

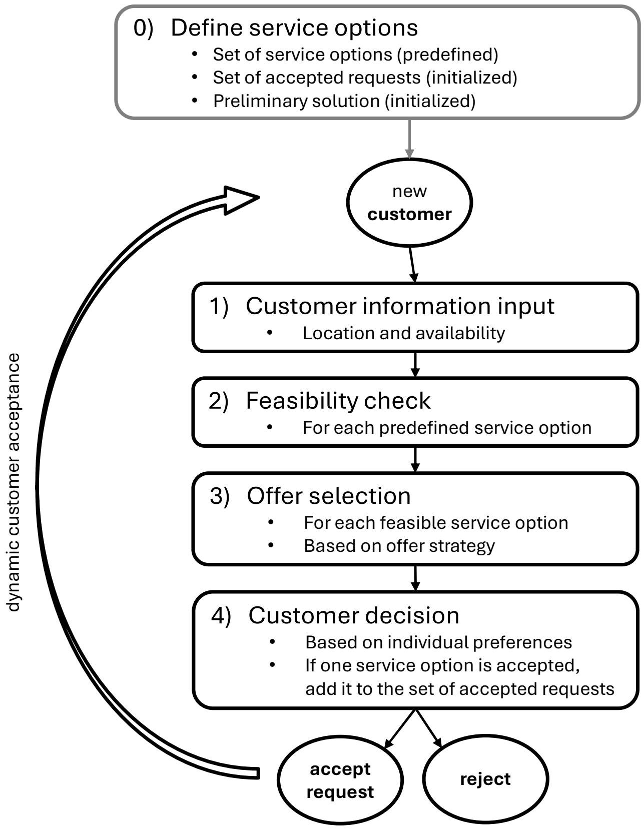


FIGURE 4.4: Customer acceptance mechanism

4.4.1 Formal Problem Description

The input parameters defined before the planning horizon are given in Table 4.1. Each service option o in the set of predefined service options O consists of a service mode $m_o \in \{\text{AHD, MPL}\}$, a time period t_o represented through a start and end time (e.g., 10:00 to 12:00), and a maximum travel distance d_o, which might vary for different MPL service options and is always 0 for AHD services. Note that the concrete location where the service is performed is not part of the definition and depends on the customer(s) to whom the service option is offered. For the AHD service, the location is always the customer's location, while for the MPL service, the location can be chosen flexibly within the maximum distance d_o from the customer's location. The ordered set C lists all customers in chronological order in which they will arrive in dynamic customer acceptance. Each customer c is specified by a location $l_c \in L$, availability period a_c and service preference $p_c \in \{\text{AHD, MPL}\}$. We assume that l_c and a_c are communicated upon arrival to the service provider, but p_c is not. For example, a customer c might be available around noon from 10:00 to 14:00 (a_c) and prefer (p_c) MPL services. In addition, a fleet of vehicles V is given, where for each vehicle $v \in V$, a capacity κ_v denotes the maximum number of customers that can be served. The available parking locations for MPL services are defined in the set P. Travel times $t_{i,j}$ between all pairs of locations $(i, j) \in L \cup P \times L \cup P$ are known.

$o \in O$	Service option o in set of predefined service options O
m_o	Service mode of service option o
t_o	Time period of service option o
d_o	Maximum travel distance of service option o
$c \in C$	Customer c in set of customers C, in arrival order
$l_c \in L$	Location l_c of customer c in set of locations L
a_c	Availability period of customer c
p_c	Service preference of customer c
$v \in V$	Vehicle v in set of vehicles V
κ_v	Capacity of vehicle v
$p \in P$	Parking location p in set of available MPL parking locations P
$t_{i,j}$	Travel time between locations i and j

Table 4.1: Input parameters

Table 4.2 shows the variables required during dynamic customer acceptance. All accepted requests are stored in the set R. For each request $r \in R$, the parameter i_r^C denotes the index of the corresponding customer $c \in C$ and parameter i_r^O the selected service option $o \in O$. For each arriving customer c, the set of feasible service options $O^F \subseteq O$ is determined with respect to the requests already accepted. The set of offered service options O^O is a subset of the feasible service options O^F for the new customer and is determined by the applied offer strategy.

$r \in R$	Request r in the set of accepted requests R
i_r^C	Index of customer c associated with request r
i_r^O	Index of service option o associated with request r
$O^F \subseteq O$	Subset of feasible service options for the current customer for each $o \in O^F$ exists a feasible solution T^F
$O^O \subseteq O^F$	Subset of offered service options to the current customer

TABLE 4.2: Planning variables

A final solution for the particular delivery problem is described through the variables in Table 4.3. Most important for the definition of a solution are the performed stopovers. Let S be the set of all stopovers used so serve the current request set R. A stopover $s \in S$ is defined by a location l_s for the time period $t_s = [\bar{t}_s, \underline{t}_s]$ and serves the customers specified in C_s with the service mode defined in m_s. For example, an AHD stopover at customer location l_c could serve customer c in the time period from 10:00 to 10:10 or an MPL service could be provided to serve the customers specified in the set C_s at the MPL parking location p in the time period from 11:00 to 13:00. A solution T is a set of routes $S_v, v \in V$, and each route S_v consists of a list of stopovers. A route S_v is feasible if i) it serves at most κ_v customers ($\sum_{s \in S_v} |C_s| \leq \kappa_v$) and (ii) all stopovers can be conducted in a time-feasible manner, i.e., $\underline{t}_s + t_{l_s, l_{s+1}} \leq \bar{t}_{s+1}$ for all consecutive stopovers $(s, s+1) \in S_v$. A solution T is feasible if all routes are feasible and no customer is served more than once. The objective value is the number of customers served ($\sum_{v \in V} \sum_{s \in S_v} |C_s|$).

$s \in S$	Stopover s in the set of stopovers S used for a given request set R
l_s	Location of stopover s
$t_s = [\bar{t}_s, \underline{t}_s]$	Time period of stopover s
m_s	Service mode of stopover s
$C_s \subseteq C$	Set of customers served during stopover s
S_v	List of stopovers performed by vehicle v (a route)
T	A solution consisting of routes $S_v, v \in V$

TABLE 4.3: Solution description

4.4.2 Feasibility Check

The feasibility check is an essential part of dynamic customer acceptance. After a customer c_{new} has arrived, the predefined set of service options O must be checked. For each $o \in O$, it has to be evaluated whether it can be offered to the customer or not. For this purpose, a heuristic is used to quickly check whether the customer can be served. We use the ILS heuristic by Kötschau et al. (2025), which is suitable for the MHDPL problem due to the specifically developed local search, permutation, and insertion operators. Since this heuristic maximizes the number of accepted customers, this ILS can be used directly to determine hindsight solutions where all incoming customers, as well as their locations and availabilities, are known in advance. For the dynamic problem, however, a few adjustments are necessary. Here, the ILS is initialized with a start solution that corresponds to the last solution found, containing all previously accepted requests R.

In addition, the termination criterion has been changed. On the one hand, the search terminates immediately if a better solution than the starting solution has been found, and on the other hand, the search is limited to a restricted number of iterations. As the ILS is applied for each new customer, an improvement of the starting solution means that a solution could be found with the new customer included. If no improvement is found, the search is terminated after a predefined number of iterations. This allows reasonable runtime, and previous experiments have shown that good solutions can be achieved with just a small number of iterations. The feasibility check is performed for every service option $o \in O$ with a time period t_o within the customer's availability $a_{c_{new}}$. Each time the feasibility check is successful, this service option o is included in the set of feasible service options O^F.

An overview of the feasibility check is presented in Algorithm 2. The algorithm uses the predefined set of service options O, the already accepted requests R, the most recently determined solution T, and the newly arrived customer c_{new}. R and T are empty when the first customer appears. First, empty lists are initialized in which all feasible service options O^F and the corresponding solutions T^F will be stored. For each service option $o \in O$, it is checked whether its time period t_o is within the customer's availability period $a_{c_{new}}$. If this is the case, a temporary set of requests R_{tmp} is created in which R is extended by a new request of customer c_{new} and the corresponding service option o. The emerging static MHDPL problem with request set R_{tmp} is then solved using the ILS. The maximum number of iterations for the ILS is predetermined and will be handed over, and the current solution T is used as the starting solution. The solution of the ILS is stored in T^o. If more customers were served in T^o than in T, then the used service option o and the found solution T^o are stored in O^F and T^F. After all service options have been evaluated, the two sets with the feasible options and solutions are passed along. Based on the option selected in the following steps, the corresponding solution is set as the new current solution.

Algorithm 2 Feasibility Check

Used Parameters:
O; R; T; c_{new}; $\_MaxIterations\_$

$O^F \leftarrow \emptyset$
$T^F \leftarrow \emptyset$
for $o \in O$ **do**
  if t_o is within $a_{c_{new}}$ **then**
    $R_{tmp} \leftarrow R \cup \{newRequest(i^C = c_{new}, i^O = o)\}$
    $T^o \leftarrow runILS(R_{tmp}, \_MaxIterations\_, StartSolution \leftarrow T)$
    if $countCustomers(T^o) > countCustomers(T)$ **then**
      $O^F \leftarrow O^F \cup \{o\}$
      $T^F \leftarrow T^F \cup \{T^o\}$
    end if
  end if
end for
return O^F, T^F

4.4.3 Offer Selection

The most important aspect of dynamic customer acceptance is deciding which service options to offer a newly arriving customer. The provider attempts to offer services to customers in a way that maximizes the number of accepted customers. In general, the provider faces the challenge that no information about future customers is available at the time of decision-making. Without offer selection, the provider makes all feasible options available and allows the customers to choose freely from the set of feasible service options O^F. With this approach, however, there is a high probability that customers will make less favorable choices, resulting in fewer customers being accepted. While some customers may receive their preferred service, this is detrimental to the overall situation for all customers and the service provider. The provider, therefore, needs to carefully select the service options offered.

One particular important decision is which service options should be considered for the initial set of service options O. For the MHDPL problem, we assume that at least one service option $o \in O$ with $m_o =$ AHD and at least one service option $o \in O$ with $m_o =$ MPL have been defined. Now, the aim is to investigate the differences between these two services. AHD services offer more flexibility compared to MPL services, which can facilitate the acceptance of future customers. In contrast, MPL services have the potential to bundle customers, although initial experiments have shown that this is particularly difficult under dynamic circumstances. The key challenge is to assess how beneficial a specific MPL service might be for the acceptance of customers arriving in the future.

The following sub-chapters introduce three fundamental components that contribute to a successful offer strategy. Chapter 4.4.3.1 deals with limiting the number of simultaneously occurring MPL services, Chapter 4.4.3.2 focuses on encouraging particularly desirable MPL services, and Chapter 4.4.3.3 restricts the geographical areas in which MPL services are offered.

4.4.3.1 Limiting the Number of MPL Services

Considering the number of simultaneous MPL services and limiting them is significant, as it allows the provider to determine the ratio of flexibility (AHD service) and bundling (MPL service). If all available vehicles perform MPL stopovers at the same time, no AHD services can be offered flexibly during that time. It is, therefore, necessary to evaluate whether bundling of customers or flexibility should be preferred in a particular situation.

Hence, when deciding whether or not to offer an MPL service option to the current customer, we propose to solve a set covering problem. This problem determines the minimum number of vehicles required to serve the new customer with the MPL service option and all accepted requests with MPL service option in the time interval under consideration. The considered time interval depends on the MPL service option currently in question to be offered to the new customer. If the minimum number of vehicles with MPL services is larger than the predefined maximum number of simultaneous MPL services, then the current customer would exceed the limit, and no MPL service is offered. However, if possible, the customer might still be offered other service options, e.g., AHD service.

In particular, for the current customer c_{new}, the set covering problem is solved for each feasible service option $o \in O^F$ where $m_o = \text{MPL}$. Let o be the service option under consideration, then the objects to be covered are $R_{tmp} = \{r \in R : m_{i^O_r} = \text{MPL and } t_{i^O_r} \subseteq t_o\} \cup newRequest(i^C = c_{new}, i^O = o)$, i.e., the set of accepted requests with MPL service option that needs to be performed in the time interval of service option o and additionally the newly arrived request of c_{new} with service option o. The minimum set is selected from the available MPL parking locations $p \in P$. Hence, there are binary decision variables x_p for all $p \in P$ indicating whether a parking location is used or not. The parameter $b_{r,p}$ indicates whether request $r \in R_{tmp}$ can be served at parking location $p \in P$. This is determined by the travel distance of the corresponding customers' location $l_{i_r^C}$ to the parking location p and the maximum travel distance associated with the agreed service option $d_{i_r^O}$. The objective function $(S.1)$ minimizes the number of parking locations used. Constraints $(S.2)$ ensure for each request $r \in R_{tmp}$ that at least one parking location at which this customer can be served is used. The domain of the decision variables is given in Constraints $(S.3)$.

$$(S.1) \qquad min \sum_{p \in P} x_p$$

$$(S.2) \qquad \sum_{p \in P} b_{r,p} * x_p \geq 1 \qquad \forall r \in R_{tmp}$$

$$(S.3) \qquad x_p \in \{0, 1\} \qquad \forall p \in P$$

4.4.3.2 Focusing on MPL Services

When deciding whether or not to offer an MPL service option to a customer, it seems intuitive to check whether an MPL service is already provided in the temporal and spatial vicinity of the current customer. If other requests have been accepted with an MPL service that is compatible with the new request and the current MPL service option, the new customer can be integrated cost-efficiently. This means that this new request can be bundled with the existing requests in a single MPL stopover. However, it should be noted that this might further restrict the final MPL parking location, which has not been determined at this stage. If one or more service options are found this way, no further service options (e.g., AHD service) will be offered to this customer. In this way, the customer is encouraged to decide in favor of this (for the service provider) convenient MPL service. Whether a new MPL request can be combined with existing MPL requests is checked using the set covering problem already described in Chapter 4.4.3.1. The problem is solved twice, once without and once with the new request, and if the minimum number of MPL parking locations required is identical, the customer can be bundled and conveniently integrated using the considered service option.

4.4.3.3 Limiting the Area of MPL Services

Based on typically available geographical information, it seems reasonable to offer MPL services only in selected areas. This includes locations which are accessible to a particularly large number of customers or, conversely, if customers are located very remotely, only offering them AHD service. This approach improves the potential to bundle customers and tries to prevent unfavorable MPL stopovers. Consequently, MPL services will only be offered to or initialized by those customers who are within the distance of the predefined locations. This can be implemented by predefining a set of promising MPL parking locations.

Furthermore, service providers might know the general geographical structures of the delivery area and maintain a list of all known customers, but they do not know the locations of the customers who require delivery on a particular day. Therefore, it would be reasonable to adjust the predefined locations constantly when additional information becomes available. We propose a max covering problem to implement this adaptive approach. It allows us to determine whether the current customer should be served by the MPL service or not, based on the requests that have already been accepted and the customers that could potentially show up in the future.

For the current customer c_{new}, the max covering problem is solved for each feasible service option $o \in O^F$ where $m_o =$ MPL. Three sets of customers are considered: (a) the current customer c_{new}; (b) all accepted MPL customers within the considered time period C_{acc}; (c) a new set of potential customers who have not yet arrived C_{pot}.

The set C_{acc} consists of all customers with relevant accepted requests, i.e., $C_{acc} = \{i_r^C : r \in R_{tmp}, m_{i_r^O} =$ MPL and $t_{i_r^O} \subseteq t_o\}$. The potential customer set C_{pot} contains all customers known to the service provider based on the geographical structures and not only those who will actually arrive as defined in C. We remove all customers who have already arrived, i.e., all customers $c \in C$ whose index is less than or equal to c_{new}, from C_{pot}. Parameters $b_{c,p}$ indicate for each customer $c \in C_{pot} \cup C_{acc} \cup \{c_{new}\}$ whether it can be served at a certain MPL parking location $p \in P$ or not. Additionally, the new parameters k and w define the maximum number of MPL stopovers and the weight ratio between the current customer and the potential future customers.

Binary decision variables x_p for all $p \in P$ indicate whether a parking location is used or not. Similarly, the binary decision variables y_c indicate if a customer $c \in C_{acc} \cup$

$C_{pot} \cup \{c_{new}\}$ is covered by the chosen parking locations. The objective function (M.1) maximizes the potential customers plus the new customer in the solution, whereby the new customer is weighted relative to a single potential customer. Constraints (M.2) ensure that all already accepted customers must be served in the solution. Constraint (M.3) defines that no more MPL parking locations may be used than there are vehicles available for MPL services. Constraints (M.4) couple parking and customer variables. The domains of the decision variables are defined in (M.5) and (M.6).

$$
\begin{array}{lll}
(M.1) & max \sum_{c \in C_{pot}} y_c + y_{c_{new}} * w & \\
(M.2) & y_c = 1 & \forall c \in C_{acc} \\
(M.3) & \sum_{p \in P} x_p \leq k & \\
(M.4) & y_c \leq \sum_{p \in P} b_{c,p} * x_p & \forall c \in C_{acc} \cup C_{pot} \cup \{c_{new}\} \\
(M.5) & x_p \in \{0,1\} & \forall p \in P \\
(M.6) & y_c \in \{0,1\} & \forall c \in C_{acc} \cup C_{pot} \cup \{c_{new}\}
\end{array}
$$

4.5 Computational Study

The aim of this study is to understand the determinants of successful offer strategies for the dynamic customer acceptance MHDPL problem. The focus is on the decision whether to offer MPL or AHD services to the individual customer. For this purpose, instances and the initial set of service options are defined and described in Chapter 4.5.1. Chapter 4.5.2 compares the dynamic and the static problem with hindsight information. This comparison illustrates the relevance of effective offer strategies for dynamic problems with MPL services and analyzes the critical obstacles. Then, Chapter 4.5.3 shows the potential of solving the dynamic problem by exploiting hindsight knowledge of the MPL stopovers used. This also underlines the effectiveness of the offer strategies presented in this paper. Finally, Chapter 4.5.4 presents several offer strategies that complement each other in terms of complexity and available realistic problem knowledge. These strategies are further analyzed with respect to MPL and AHD services to better understand the critical differences and provide practical implications for the implementation of such a service.

4.5.1 Experimental Setup

In the following experiments, four different clustered areas of 5 km in length and width with 1000 customer locations are considered. The spatial distribution of customer locations is derived from the instances of Gehring and Homberger, 1999: random (R1); clusters (C2); dense clusters (C1); mixture of random and dense clusters (RC1). A number of 50 possible MPL parking locations were identified by clustering the 1000 customer locations for each area. The four areas with 1000 customer locations and 50 MPL parking locations are shown in the appendix in Figure C.1. To determine the accessibility of MPL parking locations for each customer, the travel distance is calculated as Euclidean distance. The travel times of vehicles between all locations are considered in 12-minute steps and calculated based on the Euclidean distance and a travel speed of 30 km/h.

100 random instances were generated for each area, i.e., 400 in total. Each instance considers 100 randomly sampled customer locations (out of the 1000 available). These 100 customers arrive dynamically one after the other and have an equally random preference for AHD or MPL services. The availability periods of the customers have been drawn with the following probabilities: 33% at noon (10:00-14:00); 13% in the afternoon (14:00-18:00); 54% in the evening (18:00-22:00). These probabilities are derived from real-world data presented in Köhler and Haferkamp, 2019.

Focusing on the differences between offering AHD or MPL service options, the initial set of service options includes only a limited number of options. There are AHD service options of two hours every two hours (i.e., 10:00-12:00, 12:00-14:00, ..., 20:00-22:00). For MPL service, a maximum travel distance of 1 km is assumed and the service provision periods are likewise two hours for comparability, which are offered at 11:00, 13:00 and 15:00. This results in two AHD and one MPL service option for each availability period of the customers.

The maximum number of iterations for the ILS to check feasibility during dynamic customer acceptance is based on the number of customers already included in the solution, i.e., the maximum number of iterations is *number of customers times 2* per arriving customer. For example, if 10 customers have been accepted, the ILS would have 20 iterations, and if 40 customers have already been accepted, the ILS would have 80 iterations.

Four MHDPL vehicles are used, each with a maximum capacity of 25 customers. All experiments were conducted on a computing cluster with 2.9 GHz Xeon-G 6226R processors and 384 GB RAM using Julia 1.11 and Gurobi 9.12. Each experiment was solved four times. Average results are presented to consider randomness in the heuristic solution approach and the customer selection of service options.

4.5.2 Comparison of Dynamic and Hindsight

In this chapter, we compare the dynamic problem with the static problem, where hindsight information is available. A general comparison of the customers accepted is given, including the differences for MPL-only, AHD-only, and MHDPL services. This is followed by a more detailed analysis of the AHD and MPL services provided for the four different spatial distributions and the three different time periods. In addition, the utilization of MPL stopovers is discussed. Finally, the difficulty of the decisions in the dynamic problem is emphasized using an example.

First, we want to assess the effects of the dynamic arrival of customers and the successive selection of service options. In this *`dynamic`* variant, 80.3 customers can be accepted on average if no particular offer strategy is applied, and customers select their preferred service options from the available feasible service options. To classify this number, we compare it with how many customers could be accepted if they did not arrive successively but were all known from the beginning. For this purpose, the static problem is solved using the original ILS with a runtime of 10 minutes, considering hindsight customer knowledge. In this ***`hindsight`*** variant, an average of 97.6 customers could be accepted. Therefore, 17.2 customers could not be accepted due to dynamic customer arrival. Further experiments in which only AHD services or only MPL services were offered show that the potential, respectively, the loss due to dynamic customer acceptance, is greatest for the combined MHDPL services (see Figure 4.5). If only MPL services are offered, the difference between `hindsight` and `dynamic` is 11.8, with 75.5 and 60.7 accepted customers on average. In the case of only AHD services, the same number of customers can be accepted dynamically as with hindsight customer information (84.0). The dynamic problem with AHDs is relevant when considered at a higher and more complex level of detail, but with the availability times given and simplified travel times, they are not as difficult as MPL services.

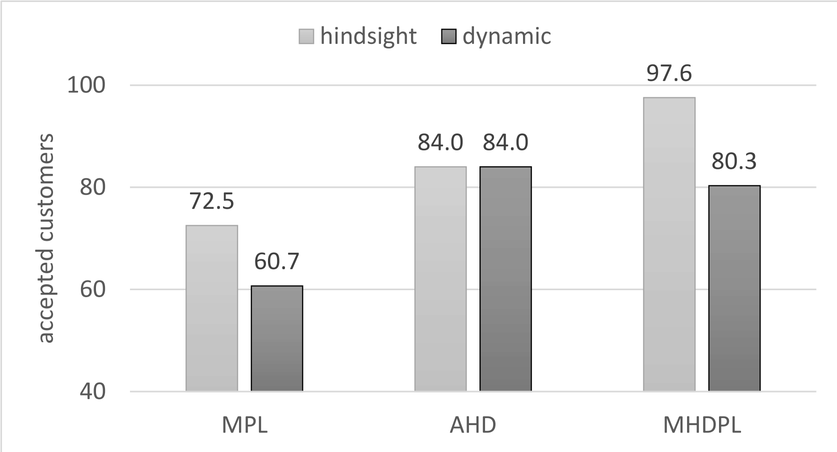


FIGURE 4.5: Accepted customers, average of `dynamic` and `hindsight` solutions

Considering MHDPL services, on average, the number of customers served with MPL services barely differs from those served with AHD services. In `hindsight`, 48.6 (49.0) customers are served using MPL (AHD) services. In `dynamic`, 40.0 (40.3) customers are served using MPL (AHD) services. On average, most customers are accepted in the clustered C2 instances (99.5 in `hindsight` and 83.5 in `dynamic`) and the fewest in the random R1 instances (95.4 in `hindsight` and 77.8 in `dynamic`). Further details on the differences based on spatial distributions are given in the appendix in Figure C.2 for `hindsight` and Figure C.3 for `dynamic`. The actual differences (`hindsight` minus `dynamic`) are shown in Figure 4.6. Although a similar number of customers is served with both MPL and AHD services in total, there are noticeable variations based on spatial distributions. Dynamically, in the clustered instances (C1/C2), it is slightly more difficult to accept customers with MPL services (the differences are 10.8/8.7 for MPL and 6.8/7.2 for AHD), whereas in the randomly distributed instances (R1/RC1), it encounters more difficulties with the customers served by AHD services (the differences are 7.2/7.6 for MPL and 10.4/10.2 for AHD). This suggests that in areas with clustered customer locations, it is more important to find advantageous MPL stopovers, whereas in areas with more random customer locations, it is more important to avoid unfavorable MPL stopovers.

Significant service differences in customer acceptance can be seen when the different time periods are analyzed in detail. The greatest potential to serve customers with MPL services is in the evening, where, on average, 23.1 (36.9) customers are accepted in `dynamic` (`hindsight`), in contrast to 16.1 (16.6) customers with AHD services. At

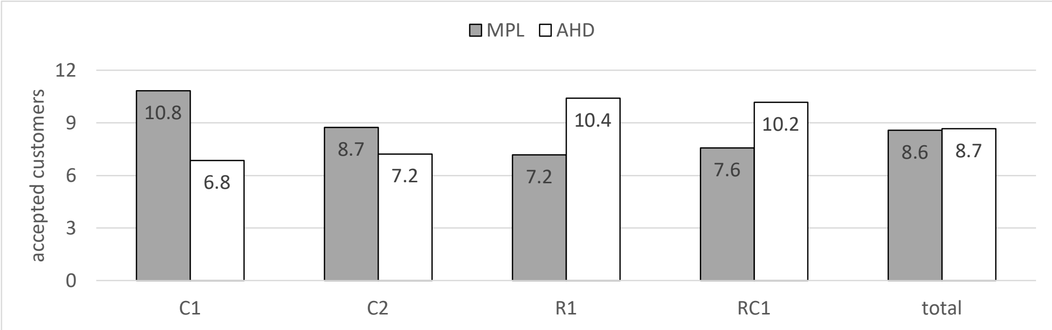


FIGURE 4.6: Difference `hindsight` minus `dynamic` solutions, average accepted customers, for each area

noon and in the afternoon, more customers are served with AHD than with MPL services. Further details are shown in the appendix in Figure C.5 for `hindsight` and Figure C.4 for `dynamic`. The differences between `hindsight` and `dynamic` are shown in Figure 4.7. At noon, 3.6 fewer customers are served with AHD services, but 0.8 more with MPL services. In the afternoon, the situation is nearly balanced, as 4.5 fewer customers are served with AHD services, but 4.4 more with MPL services. The largest difference between `hindsight` and `dynamic` is in the evening, where 13.8 fewer customers are served with MPL services, although only 0.6 fewer with AHD services. This shows the relevance of planning MPL services in the evening. However, appropriate strategies must consider the different characteristics of the different times of the day.

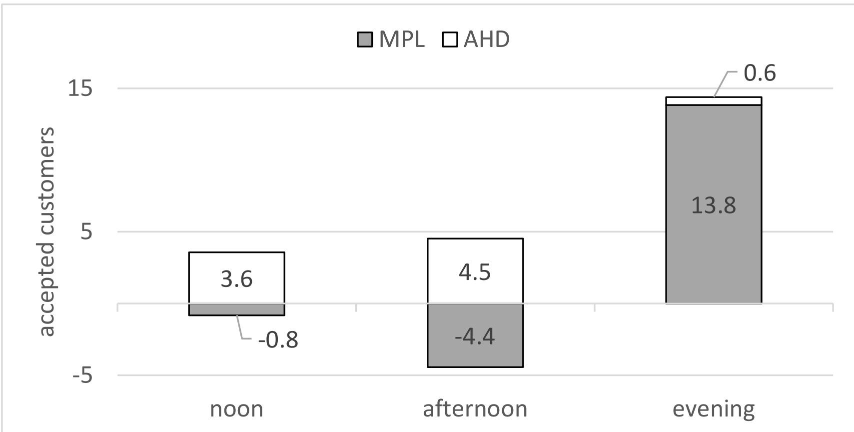


FIGURE 4.7: Difference between `hindsight` and `dynamic` solutions, average number of accepted customers, for each time period

Another useful performance measure for utilization is the average number of customers served during a single MPL stopover. This is shown in the appendix in Figure C.6 for `hindsight` and in Figure C.7 for `dynamic`. In addition, the differences are highlighted in Figure 4.8 as a percentage relative to the `hindsight` solutions. On average, 3.6 (8.0) customers are served per MPL stopover in `dynamic` (`hindsight`). The most important stopovers are in the evening, with an average of 5.9 (9.7) customers. In absolute terms, the difference is greatest in the evening with 3.9 customers. In relative terms, however, the performance of MPL stopovers decreases the most at noon, falling by 50%, although it was previously shown that 0.8 more customers are served with MPL services at noon. Furthermore, the performance of MPL stopovers decreases overall by 55%; in the afternoon by 30%, and in the evening by 40%. This implies that the selection of appropriate MPL stopovers is most difficult at noon.

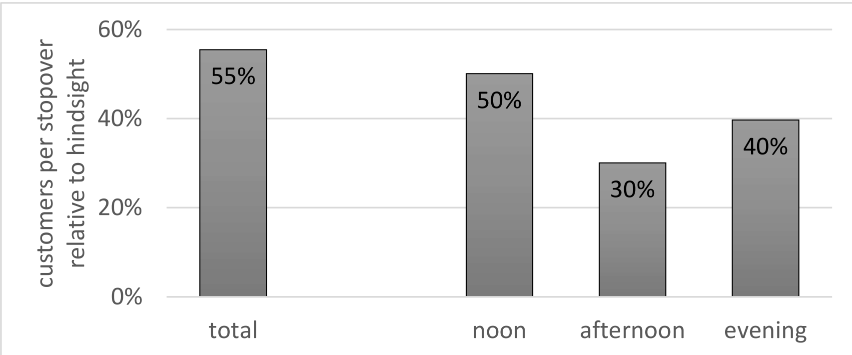
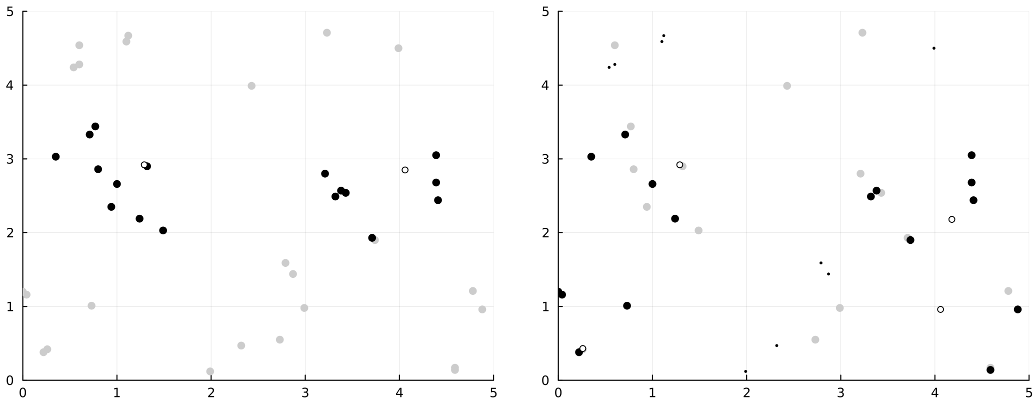


FIGURE 4.8: `Dynamic` solutions, number of customers served per MPL stopover on average relative to `hindsight`, for each time period

The following example of two solutions for a C1 instance helps to further illustrate the differences between `hindsight` and `dynamic`. For this purpose, all customers available at noon are visualized in Figure 4.9. Appendix Figure C.8 also shows these visualizations for customers in the afternoon, in the evening and in total. Figure 4.9a shows the `hindsight` solution in which 24 customers are served with AHD services and 17 customers with MPL services by two MPL stopovers. Figure 4.9b shows the `dynamic` solution in which 9 customers are rejected, 16 customers are served with AHD services, and the remaining 16 customers with MPL services by four MPL stopovers. It can be observed that with dynamically arriving customers, too many MPL stopovers are required at unfavorable locations in order to serve the customers accepted with MPL services. Furthermore, it is shown that in the area (horizontal

1 and vertical 3) all customers are served with MPL services in `hindsight`, while in `dynamic` some customers chose the AHD service and thus subsequent customers must be rejected.



(A) `hindsight` (B) `dynamic`

FIGURE 4.9: Example, comparing the time period from 10:00 to 14:00 (noon) of one `hindsight` and one `dynamic` solution for a C1 instance

The locations of customers served with AHD service (gray) or MPL service (large black) and the rejected customers (small black), as well as the MPL parking locations used (white with black border) are shown for the customers at noon.

Lastly, we want to consider the customer acceptance decisions made over time for the same instance as in the previous example. Figure 4.10 shows the preferences and the customer acceptance decisions for all customers in detail. In `hindsight`, all customers could be accepted, while in the `dynamic` solution, 22 customers had to be rejected. Although `hindsight` is not designed to ensure that every customer receives their preferred service, the distribution between customers who received the preferred service and customers who received the alternative service is fairly balanced. Namely, 52 customers prefer the AHD service, and in `dynamic` (`hindsight`), 34 (26) have received the preferred AHD service and 6 (26) have received the MPL service instead. A number of 48 customers prefer the MPL service, in `dynamic` (`hindsight`), 34 (23) have received the preferred MPL service, and 4 (25) have received the AHD service instead. In `dynamic`, therefore, more customers received the preferred service, but some customers could not be served at all. The `hindsight` progression bar shows that even at the beginning, some customers did not receive the preferred service. In contrast, in the `dynamic` progression bar, 49 of the first 50 customers received their preferred service, while 22 of the last 50 customers had to be rejected. This shows the importance of the early decisions of customer acceptance, which have a decisive impact on all subsequent customer acceptance decisions, especially in the case of MPL services.

customer preference	AHD			MPL		
accepted service	AHD	MPL	reject	MPL	AHD	reject
symbol	•	•	•	○	○	○
hindsight	26	26	0	23	25	0
dynamic	34	6	12	34	4	10

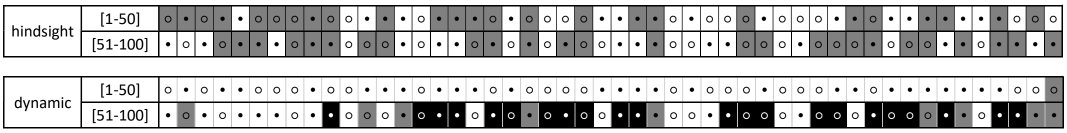


FIGURE 4.10: Comparison different accepted services, for one example `hindsight` and one `dynamic` solution

The preferences and decisions for all customers are listed in order of arrival. A small filled circle indicates a preference for an AHD service, a larger unfilled circle indicates a preference for MPL service. A field with a white background indicates that the customer has been accepted with the preferred service; a field with a gray background indicates that the customer has been accepted with the non-preferred service; a field with a black background indicates that the customer has been rejected. Below the table are progression bars for `hindsight` and `dynamic` in which the decisions for customers 1 to 50 and 51 to 100 are shown in two rows in chronological order.

4.5.3 Potential of Dynamic with Hindsight Knowledge

In this chapter, the theoretical potential of the presented offer strategies for customer acceptance is evaluated. The dynamic problem is solved by applying an offer strategy where hindsight knowledge about the best MPL stopovers is available. This is put into context with the findings above. Finally, the value of hindsight knowledge is investigated, whereby parts of the hindsight knowledge are not known or only known in aggregated form. The importance of the duration of feasibility checks for the effectiveness of advanced offer strategies is further discussed in Chapter C.2.

The previous chapter showed that in the `dynamic` variant, a total of 80.3, and in the `hindsight` variant, a total of 97.6 customers can be accepted on average. Hence, for the dynamic problem, 80.3 represents a lower bound if no specific offer strategies are applied. Next, we want to estimate the potential upper bound using an offer strategy based on the components introduced in Chapter 4.4.3. For this, we assume perfect knowledge of the used MPL parking locations provided by the `hindsight` solutions. Thereby, the number of available MPL services is limited (see Chapter 4.4.3.1) to the number in the corresponding `hindsight` solution for each instance and time period. In addition, MPL services are only offered to customers located within the range of the MPL parking locations used in `hindsight` (see Chapter 4.4.3.3). The focus on MPL services (see Chapter 4.4.3.2) is considered so that suitable customers are not offered AHD services. With this variant, which is called ***`dynamic with hindsight knowledge`***, on average, 96.7 customers can be accepted, as shown in Figure 4.11. Hence, incorporating some hindsight knowledge, it is possible to get excellent results even in the dynamic problem variant: The difference to `hindsight` is only 0.9 customers.

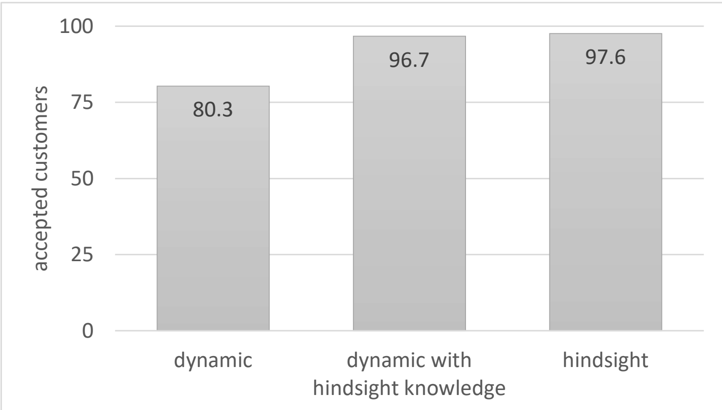


FIGURE 4.11: Accepted customers, average of `dynamic`, `dynamic with hindsight knowledge` and `hindsight` solutions

Next, the value of the information about the number of MPL stopovers available at the same time and the exact locations of the MPL stopovers will be evaluated. Figure 4.12 shows the number of not accepted customers for different degrees of hindsight knowledge compared to `hindsight`. The first bar shows the difference between `hindsight` and `dynamic with hindsight knowledge`. The next two bars show the solutions when the knowledge of the number of stopovers that should be made is reduced, but the best MPL parking locations are still known. If more stopovers are accepted than parking locations are known, customers will be accepted in other areas as well. For the second bar, the number of stopovers is aggregated separately for each time period and each spatial distribution. The aggregation takes 10% of the `hindsight` solutions into account and is calculated as the average number of stopovers for the respective time period and spatial distribution. With the known MPL parking locations and the aggregated number of MPL stopovers, on average, 1.3 customers are lost. For the third bar, the number of stopovers is not known, and a maximum of four stopovers is assumed based on the number of available vehicles. This results in an average of 3.1 customers being lost. If only the number of MPL stopovers is known, but not the exact MPL parking locations, the number of accepted customers is reduced by about 8. In these cases, it makes little difference whether no MPL parking locations are known (-8.2) or whether the best parking locations of 10 percent of the `hindsight` solutions are known (-7.9), separately for each time period and spatial distribution. As already mentioned above, 17.2 fewer customers are accepted if no information is available, and customers choose freely.

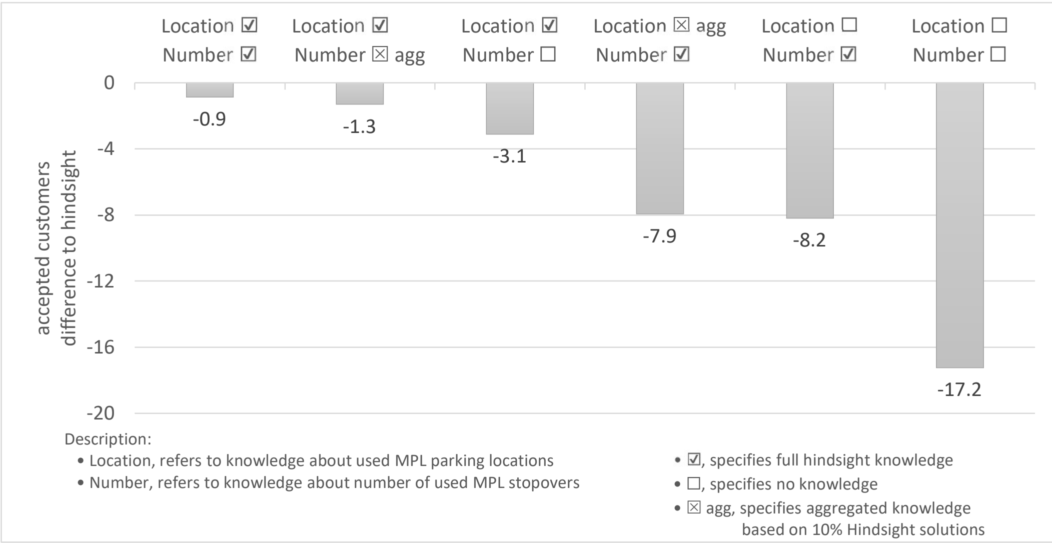


FIGURE 4.12: Effects of reduced hindsight knowledge

4.5.4 Evaluation of Dynamic Offer Strategies

This chapter introduces promising offer strategies that are composed of the components developed in this work. These offer strategies complement each other in terms of complexity and the available realistic problem knowledge. The performance of the different offer strategies is discussed and compared with the variants presented above. Furthermore, it is analyzed in detail which services are offered to customers in the different strategies and how this differs in the spatial distributions and time periods. An explicit example is used to evaluate the customer acceptance decisions for the best-performing offer strategy. Finally, the acceptance of preferred services or alternative services, as well as rejections for the different offer strategies, are discussed.

Chapter 4.5.2 discussed the challenges of the dynamic MHDPL problem and defined the ***base*** of what can be achieved without further consideration and information. Afterwards, Chapter 4.5.3 discussed the opposite, i.e., what is possible with hindsight knowledge applying the components presented in Chapter 4.4.3. It represents the maximum expected outcome for the dynamic problem in this analysis and is referred to as ***hs_knowledge*** in the following. In this chapter, we will consider more realistic offer strategies that require little or no knowledge about the instances. All strategies are categorized between `base` and `hs_knowledge` in terms of the components involved and the assumed problem knowledge in Table 4.4.

	applied components			problem knowledge	
ID	Focus on MPL (Chapter 4.4.3.2)	Number of MPL (Chapter 4.4.3.1)	Area of MPL (Chapter 4.4.3.3)	number of stopovers	location of stopovers
`base`					
`F`	yes				
`FN_one`	yes	3		single estimate	
`FN`	yes	2/1/4		estimate for each time period	
`FNA_pre`	yes	2/1/4	predefined	estimate for each time period	potential customer locations
`FNA_lin`	yes	2/1/4	adapts with (1x) linear weights	estimate for each time period	potential customer locations
`FNA_exp`	yes	2/1/4	adapts with (12.5x) exponential weights	estimate for each time period	potential customer locations
`hs_knowledge`	yes	varies for each instance	varies for each instance	perfect information	perfect information

Table 4.4: Overview offer strategies

These strategies consist of focusing on MPL services (***F***), limiting the number of MPL services (***N***), and limiting the area of MPL services (***A***). Component (`F`) is used in every strategy (except `base`) and ensures that customers are not offered AHD services if it is possible to combine them cost-effectively with other MPL service customers. Component (`N`) is considered in one case with a rough estimate of a maximum of three MPL stopovers for all simultaneous MPL services. This variant is denoted with the suffix ***_one***. In all other cases, the number of maximum simultaneous MPL stopovers is estimated for each time period. As previous experiments have shown, the number of required simultaneous stopovers can be predicted based on the expected customer volume. It is assumed that the approximate customer demand for the different times of the day is known; in our case, it is about 33 customers at noon, 13 in the afternoon, and 54 in the evening. Considering this and based on trial runs, it has been shown that 2/1/4 stops at noon/afternoon/evening are appropriate for the given instances.

Component (`A`) defines restrictions for the area in which MPL services are offered, and three variants are considered. This is done by clustering the 1000 potential customer locations separately for each time period according to the number of MPL stopovers. These clusters are described by the MPL parking locations from which most customers could be served. For *A_pre*, only customers within range of these MPL parking locations are offered MPL services. For *A_lin* and *A_exp*, the selected MPL parking locations are adjusted during dynamic customer acceptance based on the customers who have already arrived, as described in Chapter 4.4.3.3. In the case of `FNA_lin`, the new customer is valued with a linearly increasing function ($w = 1 * x$). In `FNA_exp`, the new customer is valued with an exponentially increasing function ($w = 12.5 * 1.04713^x$) with x as the number of already accepted requests.

The number of accepted customers for the different offer strategies is shown in Figure 4.13. The line at the bottom indicates the `base` with 80.3 accepted customers on average. This is the result when no specific strategy is applied and all feasible options are offered, while customers can decide what they prefer. And the line at the top indicates the `hs_knowledge` with 96.7 accepted customers on average. This is the highest achievable result with the proposed components, i.e., assuming that the MPL stopovers are known. Strategy `F`, not offering AHD services in the appropriate situations, leads to an increase of about 8 customers over the `base` and achieves 88.1 accepted customers on average. Despite this being a strong increase, some customers are still offered MPL services that are not efficient. Strategy `FN_one` reduces the chance of unprofitable MPL services and can achieve 89.1, meaning about 1 additional accepted customer on average. With a more precise estimate for each time period, only minor improvements (+0.3 customers on average) can be achieved with the `FN` strategy.

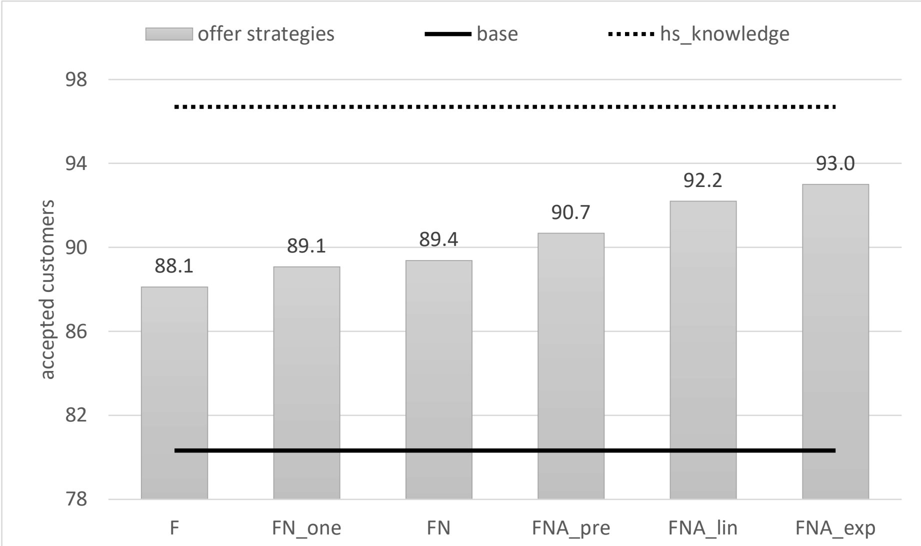


FIGURE 4.13: Overview offer strategies, accepted customers

Furthermore, it can be assumed that the service provider knows the spatial distribution of customers in the service area. If the best areas for MPL stopovers are identified and MPL services are only offered to customers within the range of these areas, as with strategy `FNA_pre`, 90.7 customers can be accepted on average. This is another improvement of 1.3 customers. Further improvements are possible by progressively adjusting the areas where MPL stopovers are allowed with each new customer using the max covering model defined in Chapter 4.4.3.3. With `FNA_lin`, an average of 92.2 customers can be accepted, and with `FNA_exp`, it is an average of 93.0 accepted customers. Using the proposed components, an improvement of approximately 16% compared to the `base` variant can be achieved with only limited but realistic problem knowledge. This means that the gap compared to the `hindsight` solution (97.6) could be reduced from around 17% to 4%.

An important issue with MHDPLs and especially with the dynamic customer acceptance is whether customers should be served with AHD or MPL services. Therefore, the following figures show how many customers were served with the respective service in the different offer strategies. Figure 4.14 shows how many customers were served with AHD and MPL. In Figure 4.15, this is differentiated for the four different spatial distributions and in Figure 4.16 for the three time periods. Although the individual offer strategies are not directly comparable with each other, they build on each other and represent a continuous increase in the number of accepted customers.

For this reason and for better interpretability, the results obtained are linked by lines.

Figure 4.14 shows, for `base` as a starting point, that the number of customers served by AHD and MPL services is equivalent with about 40 customers on average. The increase in the average number of customers accepted with strategy `F` is due to the increase of 14.3 customers served by MPL services, while the number of customers served with AHD decreases by 6.5. The restriction of stopovers with `FN_one` causes MPL services to decrease by 8.7 customers and AHD services to increase by 9.7 customers. The difference in accepted customers between `FN_one` and `FN` is only 0.3, but interestingly, the ratio of the number of customers served with each service is reversed, as 2.5 fewer are served with MPL and 2.8 more with AHD services. For `FNA_pre`, it remains that more customers are served by AHD, but the gap is slightly larger. With adaptive strategies, the ratio is reversed again, and 46.7 customers are served by MPL services and 45.5 customers by AHD services with `FNA_lin`. With `FN_exp`, 1.4 more customers can then be served with MPL, causing AHD services to decrease by 0.6 accepted customers on average. This means that `FNA_exp` serves 0.5 fewer customers with MPL and 3.2 fewer customers with AHD than `hs_knowledge`.

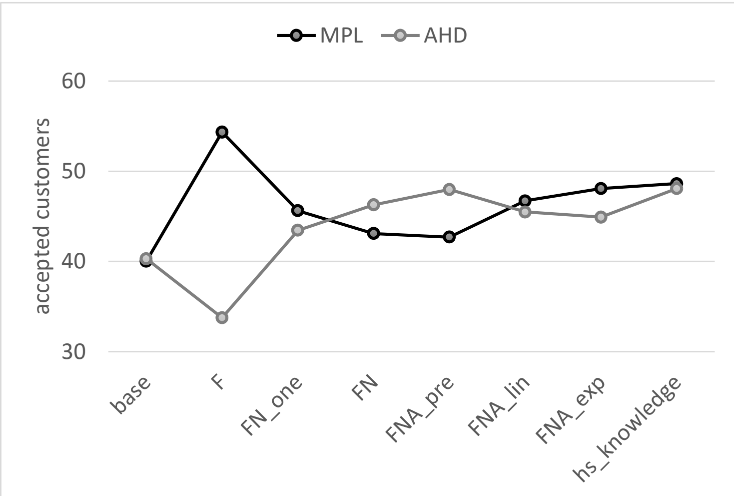
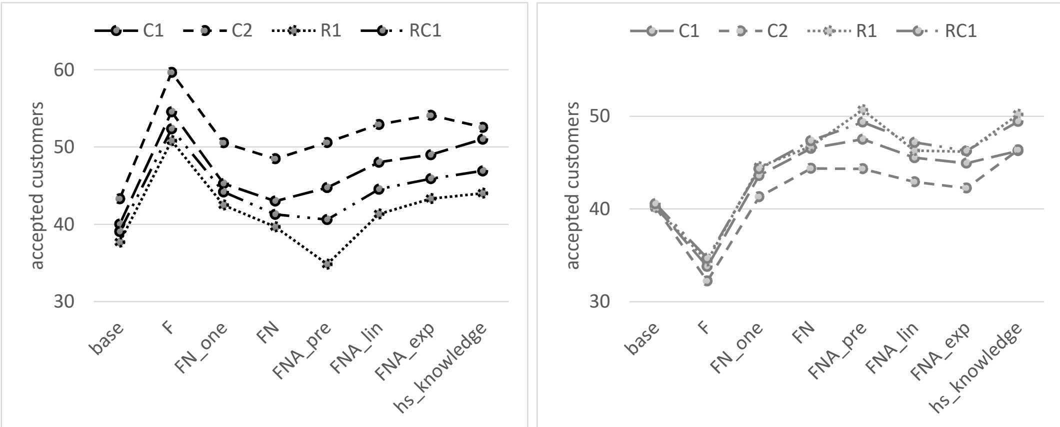


FIGURE 4.14: Accepted customers, served with MPL / AHD

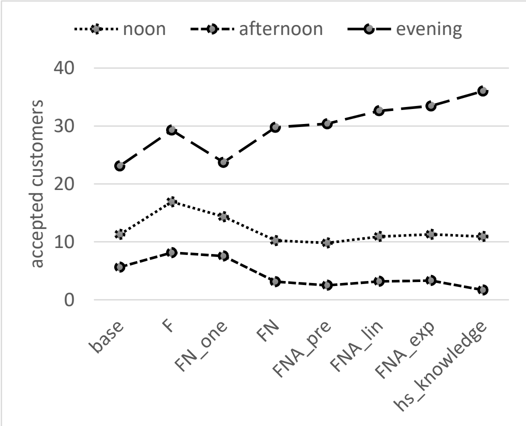
Looking at the different spatial distributions in terms of customers served with MPL services (Figure 4.15a) and customers served with AHD services (Figure 4.15b), we can see very similar trends to the overall trend of the number of customers served with MPL and AHD services. However, in instances with less dense clusters (C2), consistently more customers are served with MPL services than in the other types of instances. The fewest customers are served with MPL services in the random (R1) instances. Mixed RC1 instances are situated in the middle between R1 and the denser clusters (C1). With the AHD services, the situation is exactly the opposite, although the differences in the spatial distributions are less pronounced. The only noticeable observation is that the `FNA_pre` strategy accepts significantly fewer MPL services for the R1 instances, however, this is not the case with the adaptive strategies, `FNA_lin` and `FNA_exp`.



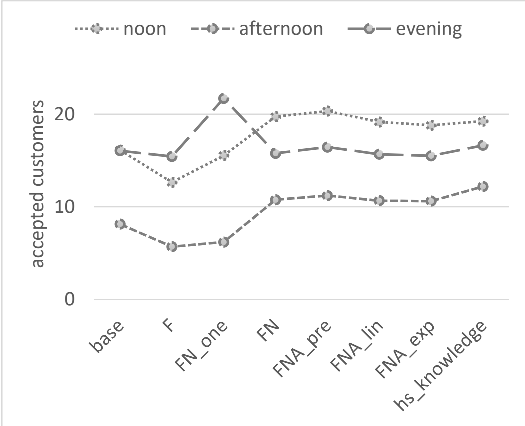
(A) MPL (B) AHD

FIGURE 4.15: Accepted customers for different spatial distributions

For the different time periods, Figure 4.16a shows the number of customers served with MPL services and Figure 4.16b the number of customers served with AHD services. As the number of accepted customers increases with the applied strategy, in the afternoon, the number of MPL services decreases and the number of AHD services increases, except for the outliers `F` and `FN_one`. At noon, the number of MPL services remains consistent, except for the same outliers. For AHD services at noon, there is a decrease from `base` to `F`, then an increase via `FN_one`, and for the other strategies, it remains roughly the same level. In the evening, there is a significant increase in the number of MPL services, while the number of AHD services remains the same. An outlier is only evident in `FN_one`, where fewer MPL and more AHD services are performed. To summarize, the increase in the number of accepted customers results from better MPL stopovers in the evening and increased AHD services at noon.



(A) MPL



(B) AHD

FIGURE 4.16: Accepted customers during each time period

Figure 4.17 illustrates the explicit decisions of the `base` and `FNA_exp` strategy using the same example as in Chapter 4.5.2. With `FNA_exp`, it is more likely that the first customers will not receive the preferred service, allowing most later customers to be accepted and many to be served with their preferred services. In total, 29 (34) of the AHD-preferring customers and 23 (34) of the MPL-preferring customers receive the desired service with `FNA_exp` (`base`) and 3 (22) customers are rejected.

customer preference	AHD			MPL		
accepted service	AHD	MPL	reject	MPL	AHD	reject
symbol	•	•	•	○	○	○
FNA_exp	29	22	1	23	23	2
base	34	6	12	34	4	10

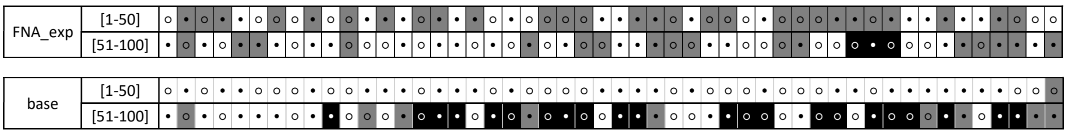


FIGURE 4.17: Comparison different accepted services, for one example `FNA_exp` and one `base` solution, for explanations see Figure 4.10.

Figure 4.18 shows how often the preferences were fulfilled on average with the different strategies. Customers receive the preferred service most frequently if no specific strategy is applied and they are allowed to decide freely, but in this case, the number of customers rejected is the highest. With `base`, 66.0 customers receive the preferred service, and 19.7 customers are rejected. Both numbers decrease until the `FNA_pre` strategy, where, on average, 46.5 customers receive the preferred service and 9.3 customers are rejected. While the number of rejected customers continues to decrease, the proportion of customers who receive their preferred service increases to 47.8 with `FNA_exp` and 48.9 with `hs_knowledge`. Note that the objective is to maximize the number of accepted customers regardless of the preferred services.

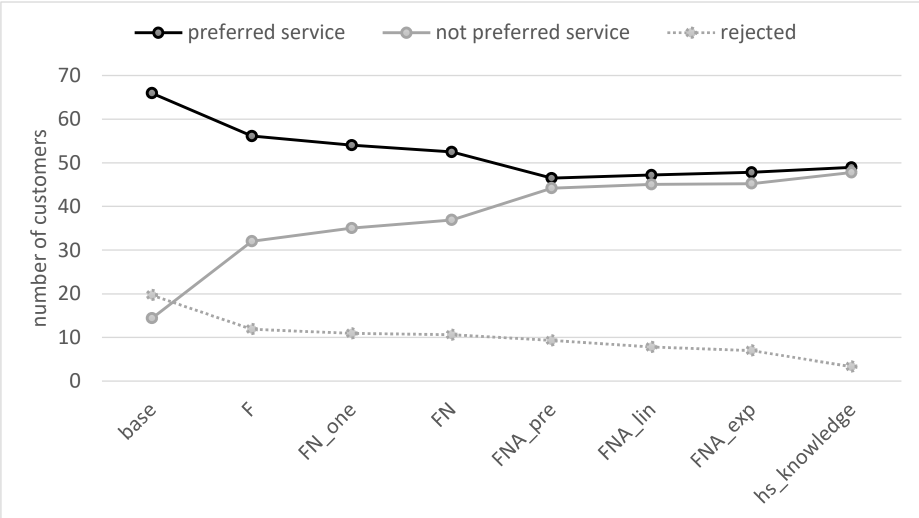


FIGURE 4.18: Number of customers, served with preferred service, other service, or rejected

4.6 Conclusion

In this paper, we analyzed dynamic customer acceptance mechanisms for MHDPL services. On the one hand, we explained the adaptation and integration of an already established ILS. This ILS is used to check the feasibility of possible service options for each arriving customer. On the other hand, new effective offer strategies were developed with an emphasis on set and max covering formulations. These components are limiting the number of simultaneously available MPL stopovers, focusing on MPL services when appropriate, and limiting the area where MPL services are offered. The area of MPL services is also dynamically adapted as more information becomes available.

We conducted experiments to investigate the implications of the dynamic customer acceptance mechanisms and evaluate different strategies. To this end, the characteristics that distinguish the solutions of the dynamic problem from those of the static problem with hindsight information were examined. Although too many inefficient MPL stopovers occurred, more customers in the clustered instances and fewer customers in the randomized instances were served with MPL services in direct comparison to the `hindsight` solutions. Furthermore, it could be shown that different problems occur with different numbers of customers. At noon (33% of the customers), it is important to avoid too many MPL stopovers, in the afternoon (13% of the customers), it seems less relevant, and in the evening (54% of the customers), it is important to ensure that MPL services are offered in the appropriate areas. In addition, unrestricted customer decisions lead to resources being exhausted too quickly and subsequent customers having to be rejected.

We discussed the importance of the different types and degrees of information. It has been found that an approximate estimate of the number of MPL stopovers required is sufficient, but the exact locations of the MPL stopovers are very valuable information. Strategies that do not take any knowledge into account can already accept around eight additional customers on average (10% improvement). However, on average, about 13 additional customers can be accepted (16% improvement) if we know the estimated number of MPL stopovers for each time period and potential customer locations (from which only 10% will arrive in a single instance).

The limitations and possible further developments include the initial set of service options. We chose to include only a few service options, as the focus of this study is on the offer of MPL vs AHD services. Further studies may analyze other variants of AHD and MPL services, but, in particular, MPL with different maximum travel distances. For practical applicability, further solution approaches could be developed to enable faster feasibility checks. The specific integration and consideration of customer preferences in the offer strategies could be part of a study that focuses on customer satisfaction in more detail. It could be assumed that customers are likely to decline if the service offered is not satisfactory. In general, stochastic elements could be included, such as order volume or customers failing to collect their delivery. An exciting analysis would include practical historical data that indicates the probability of customer occurrences.

Chapter 5

Conclusion

The conclusion of this thesis summarizes the work carried out and the findings obtained from answering the research questions in Chapter 5.1. Subsequently, Chapter 5.2 discusses possibilities for further research that would complement the presented findings.

5.1 Summary

This thesis deals with the optimization of last-mile delivery for individual customer service and focuses on the evaluation of the potential of MPLs. First, the potential of MPL was evaluated in comparison to the traditional modes (FPL and AHD). Secondly, the combined service, called MHDPL, was analyzed. Third, the MHDPL service was considered in the context with dynamically arriving customers. The experiments show that the combination of MPL and AHD is very promising. The MHDPL service can adapt to any customer situation and offers great potential in scenarios where both AHDs and MPLs work well. However, the dynamic customer acceptance for MPL services is particularly difficult, but significant improvements can be achieved if strategies based on some knowledge about expected customer volumes and their potential locations is considered. Summaries of the work and the results for answering the three research questions investigated are given below.

To answer the first research question, the literature dealing with parcel lockers in an operational setting and considering different customer preferences was analyzed. The heterogeneous parcel locker location problem was introduced, where the number of accepted customers is to be maximized using a fixed heterogeneous fleet consisting of FPLs, MPLs, and AHDs. The mathematical formulation is derived from a MIP for MPLs known from the literature and solved exactly. Experiments are conducted with two distinctly different customer types for different spatial distributions. It is found that AHDs (71% accepted customers) dominate FPLs (56% accepted customers) and MPLs (66% accepted customers). However, the combination of delivery services can improve acceptance rates (81%) and produce significantly less traffic compared to only AHD services. MPLs serve many customers in a short period of time, and AHDs serve individual customers efficiently regardless of their location and willingness to travel. From the customer's perspective, the acceptance probabilities for FPLs result from the selected customer type, for AHDs from the selected time window, and for MPLs from a blend of both features.

The new MHDPL service was introduced to investigate the second research question. Both AHD and MPL services can be provided by any vehicle, allowing it to adapt to the existing customer situation. AHD and MPL services perform well together, and they can be combined in practice due to the high flexibility of MPLs. The literature was examined for related problems and their heuristic solution approaches. Due to similarities with the team orienting problem the ILS structure was identified as a promising solution approach. In particular, the MHDPL problem is modeled with arrival-time and service-time-dependent profits as separate nodes in a time-space network. The mathematical model for the MHDPL problem was described and the ILS was developed using an adapted perturbation and newly developed local search operators. Experiments have shown that the ILS works well for MHDPL services. For small instances, 89% of the solutions found were equal to or better than the MIP solutions, while for larger instances, the MIP was outperformed in every instance. The managerial insights indicate that AHDs utilize the available time more efficiently if the customer density and customer willingness to travel are lower. However, the MHDPL service can adapt to any given scenario and has the greatest advantage when both services work similarly well. The combination of services has economic potential and satisfies customer preferences more effectively. It is shown that an average MHDPL vehicle can serve 21% more customers than a comparable AHD vehicle.

To answer the third research question, the dynamic customer acceptance of MHDPL services was presented and explored in the literature. Essential components for effective service offer strategies were developed, with an emphasis on set and max covering formulations. The ILS developed in the previous research question was used to test the feasibility of each service option. The experiments show the impact of dynamic customer acceptance, as unrestricted customer decisions lead to resources being exhausted too quickly and subsequent customers having to be rejected. For MHDPL services 17.3 fewer customers could be accepted, for MPL services 11.8 fewer customers could be accepted, and for AHD services the same number of customers could be accepted. Furthermore, it has been shown that with a medium number of customers, it is important to avoid too many MPL stopovers, with a low number of customers, it seems less relevant, and with a high number of customers, it is important to ensure that MPL services are offered in the appropriate areas. Strategies that do not take any problem knowledge into account can accept around eight additional customers on average (10% improvement). A strategy that exploits realistic problem knowledge can achieve an average improvement of 16% compared to the naive strategy with unrestricted customer decisions.

In summary, it can be said that MPL services dominate AHD services under certain conditions. However, an MPL service can be adapted so that it operates like an AHD service. This enables not only the efficient optimization of last-mile delivery, but also that individual customer service can be offered. MPLs can adapt to different customer preferences, which promotes acceptance during the introductory phase. Overall, MPLs are particularly suitable in areas with a high but varying customer density.

5.2 Outlook

This final chapter provides an outlook on further opportunities that build on the findings obtained in this thesis. First, all experiments and implementations were performed with a discrete-time structure. This enabled suitable solutions for qualified conclusions to be obtained even for larger instances. For further realistic investigations, experiments and solution approaches with more fine-grained discretization should be considered. Existing advanced solution approaches for AHDs could be used as a starting point. Approaches that consider stochastic input data could be developed. For example, the impact of uncollected deliveries could be analyzed. In addition, the number of customer requests might be uncertain, especially in the case of dynamic customer acceptance. Stochastic information could also be available on the probability of a customer arriving in a particular area. For dynamic and practical use, it is worthwhile to further improve the heuristic solution approach used for the feasibility check to enable runtime improvements and fast response times for the requesting customers.

Focusing on the last-mile delivery service with MPLs, some research potential can be identified. A key extension for the MHDPL service is to consider what can be accomplished while the vehicle is parked. For example, agents such as drivers, robots or drones can swarm out to serve other customers and/or enable further customized service configurations. Synchronization would be possible, with agents swarming out at one location and returning at a different location. A significant issue with MPLs is the assessment of potential parking locations. Previously, it was assumed that some locations are simply available, but it needs to be investigated what is possible in real application areas and perhaps parking is only possible with time restrictions. Furthermore, it was shown that some parking locations appear to be more dominant than others in the experimental data. It would be interesting to analyze this in more detail and whether this can be confirmed with more realistic data. The concept of parking slot management, e.g., where the city offers parking slots that can be reserved by service providers, seems particularly exciting. The combination of dynamic customer acceptance and parking slot management would be an interesting research question. Furthermore, to give service providers a strategic orientation, the investment and operating costs as well as the environmental impact should be considered and evaluated in relation to the level of customer service achieved. It would be interesting to see for which type of services and at which customer volume MPLs would be realistically profitable. This is exciting for dynamic customer acceptance in order to be able to consider the costs of a customer for different service options.

When considering individual customer service, there are further limitations and promising opportunities to build on the initial research questions. For example, further research should address more realistic scenarios. For the initial experiments and to ensure comparability, only two distinctly different customer types and random customer types were considered. Further studies could focus on modeling realistic customer types and spatial distributions of customers based on realistic city structures. Experiments in which customers are guaranteed a certain level of service or customer satisfaction is maximized would be conceivable. In addition, it is assumed that customers have a preferred availability and service option and do not wish to communicate further details, but customers could specify several alternatives. In the dynamic customer acceptance process, further increments of service options with different time windows, service provision periods and maximum travel distances should be analyzed. In addition, the impact of intentionally rejecting customers or using incentives (pricing, green labels) to motivate customers to choose certain service options should be investigated.

Appendix A

Mobile Parcel Lockers with Individual Customer Service

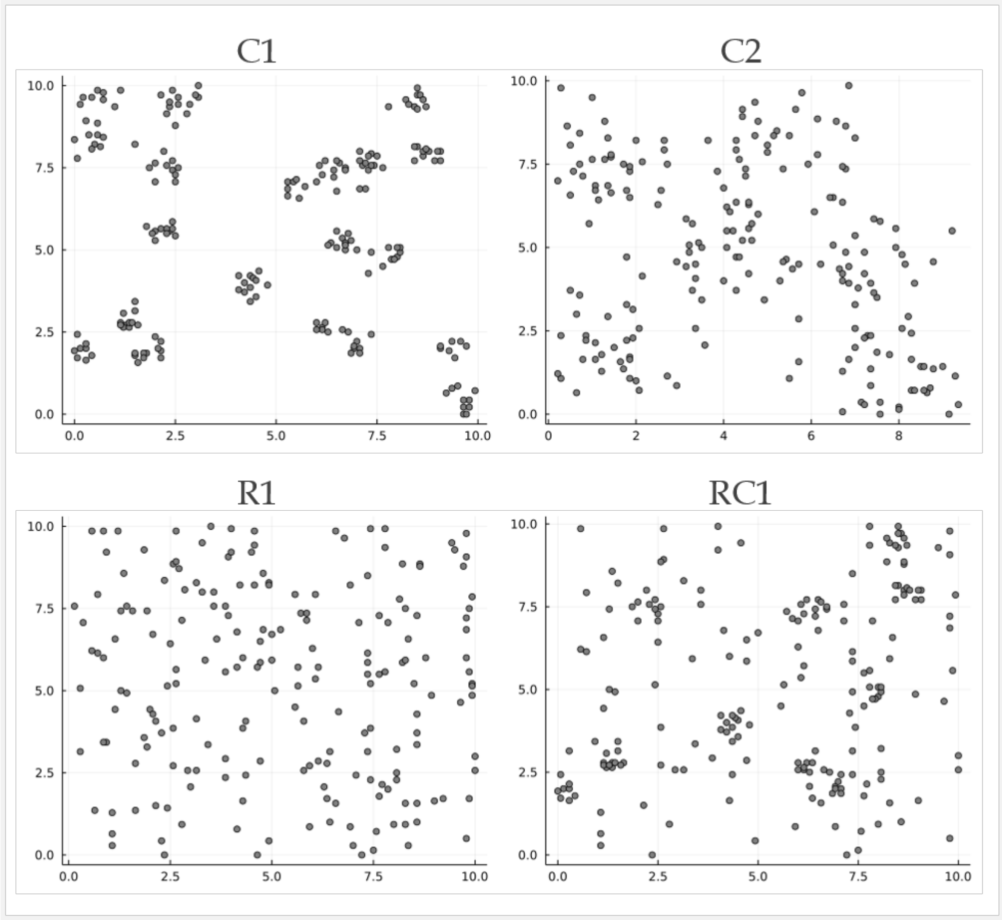


FIGURE A.1: 200 customer location distributions following Gehring and Homberger, 1999

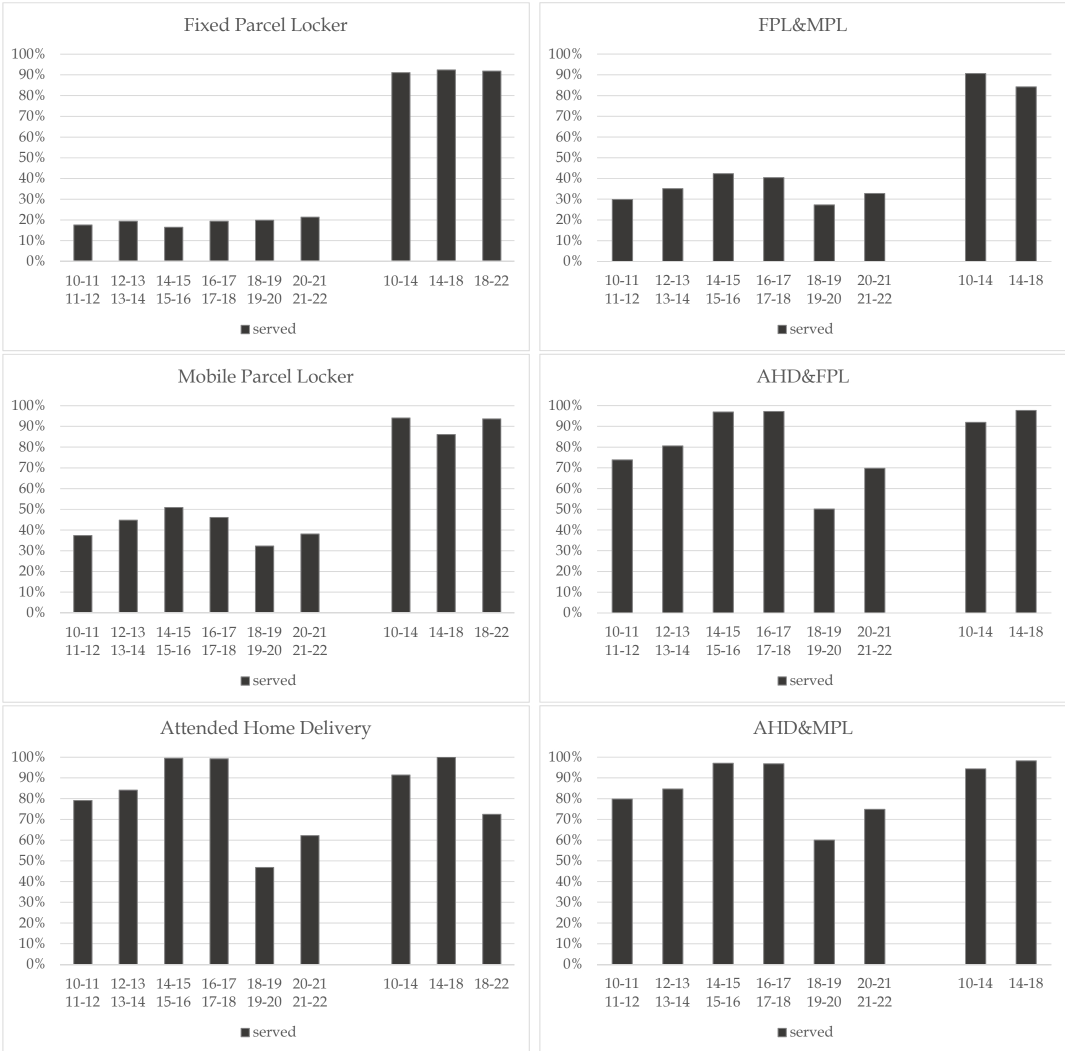


Figure A.2: Acceptance probability of customers for single services by time window

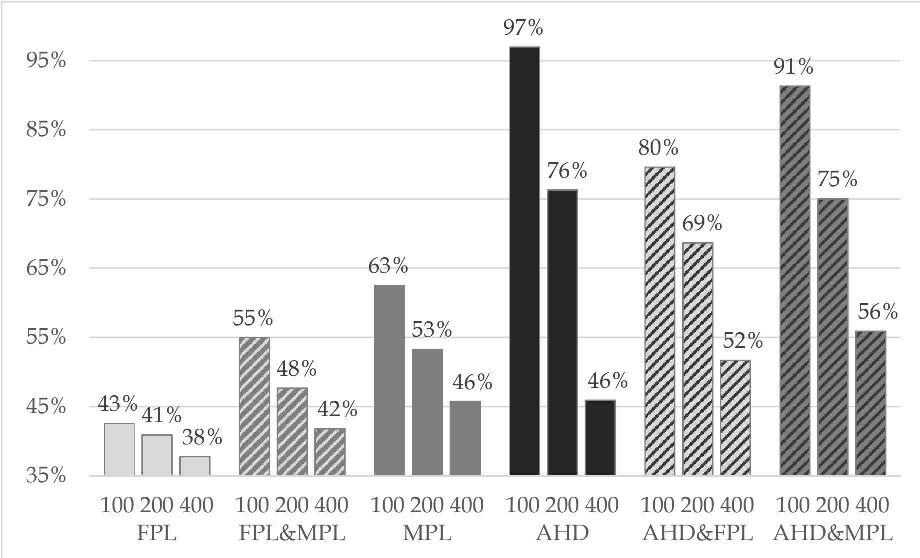


Figure A.3: Accepted customers for different services and number of customers for 70% restrictive and 30% flexible customers

discretization	15 min	12 min	10 min
AHD with 8 lockers	5 min	27 min	103 min
AHD with 10 lockers	9 min	41 min	169 min

Table A.1: Average runtime for different discretization levels (200 customers, three C1 instances and 180 min time limit)

customers	service	runtime	gap	instances with no solution
100	FPL	0.1s	0.00%	-
50% restrictive	FPL&MPL	9.5s	0.00%	-
50% flexible	MPL	74.8s	0.01%	-
	AHD	45.8s	0.00%	-
	AHD&FPL	35.9s	0.00%	-
	AHD&MPL	58.3s	0.00%	-
...	...	...	...	...

customers	service	runtime	gap	instances with no solution
...	...	...	...	...
200	FPL	0.2s	0.00%	-
50% restrictive	FPL&MPL	12.5s	0.00%	-
50% flexible	MPL	104.7s	0.01%	-
	AHD	1613.1s	2.62%	-
	AHD&FPL	920.8s	0.19%	-
	AHD&MPL	1180.5s	0.05%	1xC1
400	FPL	0.3s	0.00%	-
50% restrictive	FPL&MPL	11.4s	0.00%	-
50% flexible	MPL	95.2s	0.00%	-
	AHD	2625.9s	5.80%	10xC1, 6xC2, 9xR1, 8xRC1
	AHD&FPL	1716.0s	1.15%	-
	AHD&MPL	1864.7s	0.84%	1xC1, 3xC2, 1xRC1
100	FPL	0.2s	0.00%	-
70% restrictive	FPL&MPL	7.8s	0.00%	-
30% flexible	MPL	23.7s	0.00%	-
	AHD	31.6s	0.00%	-
	AHD&FPL	20.9s	0.00%	-
	AHD&MPL	113.9s	0.01%	-
200	FPL	0.1s	0.00%	-
70% restrictive	FPL&MPL	7.4s	0.00%	-
30% flexible	MPL	23.2s	0.00%	-
	AHD	326.5s	0.00%	-
	AHD&FPL	113.8s	0.00%	-
	AHD&MPL	390.9s	0.00%	-
400	FPL	0.2s	0.00%	-
70% restrictive	FPL&MPL	9.5s	0.00%	-
30% flexible	MPL	35.8s	0.00%	-
	AHD	2141.5s	4.13%	7xC1, 6xC2, 7xR1, 9xRC1
	AHD&FPL	431.6s	0.03%	-
	AHD&MPL	924.7s	0.15%	-

Table A.2: Average runtime and gap for each service
(excluding instances with no solution)

Appendix B

Mobile Home Delivery Parcel Lockers

service mode	location	arrival time	departure time	reachable customers
MPL	P1	14:00	16:00	C3
MPL	P2	15:00	16:00	C1
MPL	P2	12:00	14:00	C2
MPL	P2	12:00	16:00	C1, C2
MPL	P3	15:00	16:00	C1
MPL	P3	12:00	14:00	C2
MPL	P3	14:00	16:00	C1, C3
MPL	P3	12:00	16:00	C1, C2, C3
AHD	C1	15:00	15:15	C1
AHD	C1	15:15	15:30	C1
AHD	C1	15:30	15:45	C1
AHD	C1	15:45	16:00	C1
AHD	C2	12:00	12:15	C2
AHD	C2	12:15	12:30	C2
AHD	C2	12:30	12:45	C2
AHD	C2	12:45	13:00	C2
AHD	C2	13:00	13:15	C2
AHD	C2	13:15	13:30	C2
AHD	C2	13:30	13:45	C2
AHD	C2	13:45	14:00	C2
AHD	C3	14:00	14:15	C3
AHD	C3	14:15	14:30	C3
AHD	C3	14:30	14:45	C3
AHD	C3	14:45	15:00	C3
AHD	C3	15:00	15:15	C3
AHD	C3	15:15	15:30	C3
AHD	C3	15:30	15:45	C3
AHD	C3	15:45	16:00	C3

TABLE B.1: Example set of stopovers based on Figure 3.1

length \ start time	10	11	12	13	14	15	16	17	18	19	20	21
1 hour	12,5%	12,5%	4%	4%	3,5%	3,5%	3%	3%	20%	20%	7%	7%
2 hours	25%		8%		7%		6%		40%		14%	
3 hours	29%			11%			26%			34%		
4 hours	33%				13%				54%			

TABLE B.2: Time window probabilities

	small instances			medium instances			large instances		
	MPL	AHD	MHDPL	MPL	AHD	MHDPL	MPL	AHD	MHDPL
MIP found optimum	100%	96%	54%	100%	0%	0%	100%	0%	0%
ILS found optimum	95%	96%	47%	22%	0%	0%	0%	0%	0%

Table B.3: Optimal solutions found

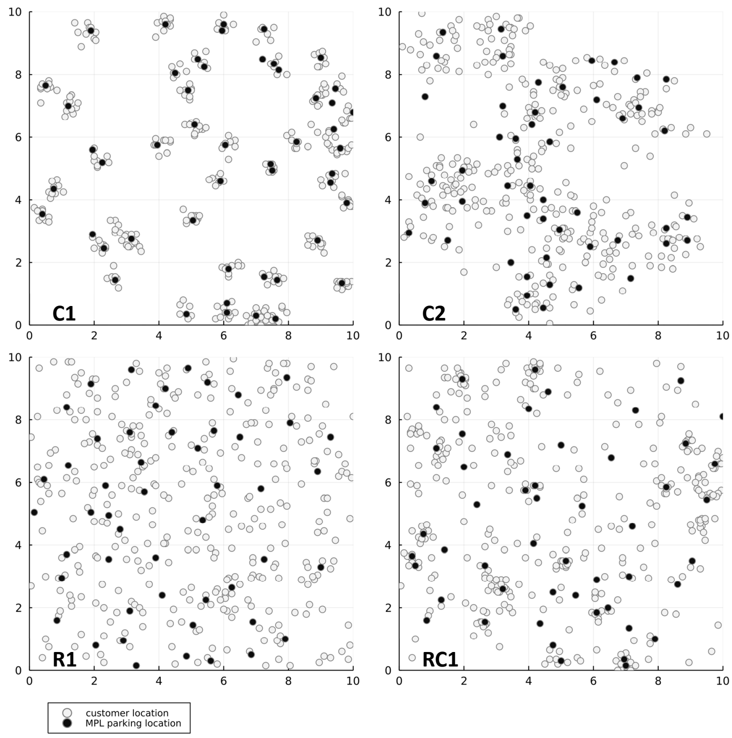
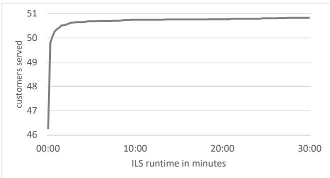
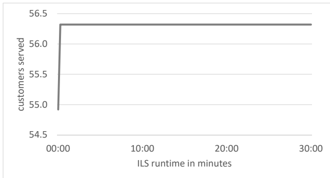


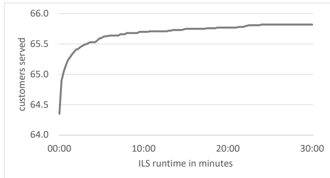
Figure B.1: Different spatial distributions of 400 customer and 50 MPL parking locations



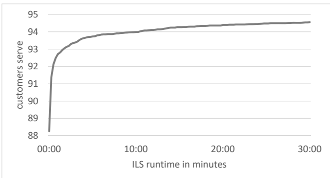
(A) MPL, small instances



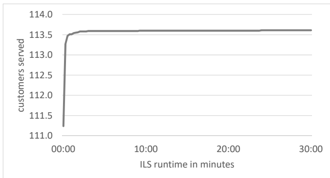
(B) AHD, small instances



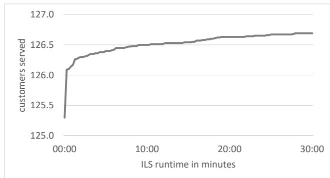
(C) MHDPL, small instances



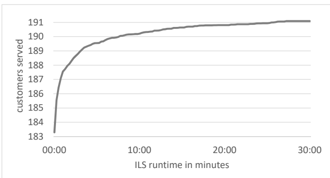
(D) MPL, medium instances



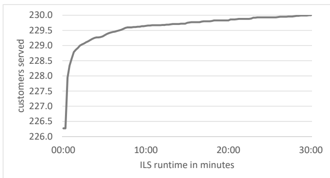
(E) AHD, medium instances



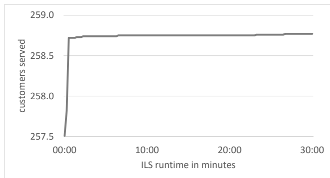
(F) MHDPL, medium instances



(G) MPL, large instances

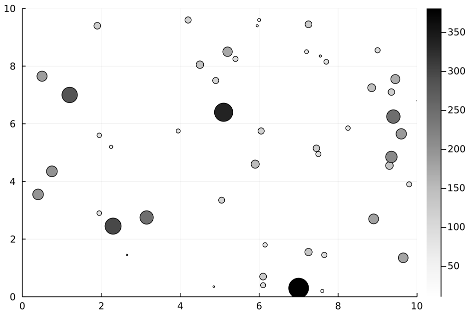


(H) AHD, large instances

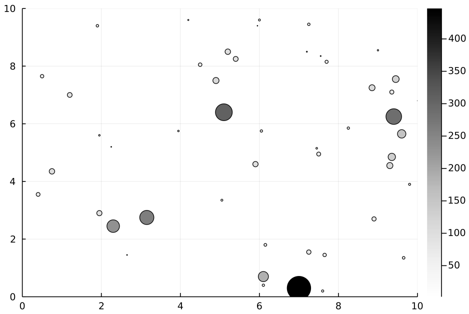


(I) MHDPL, large instances

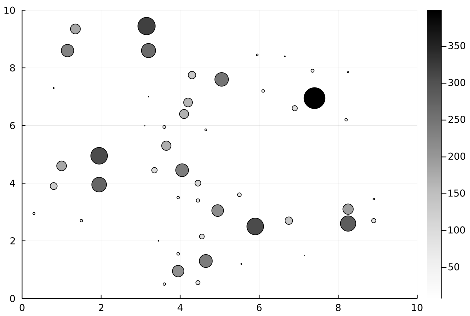
FIGURE B.2: ILS average best found solution over time



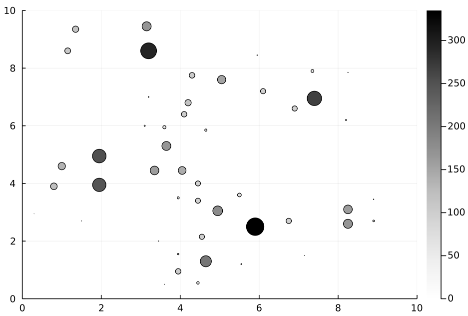
(A) MPL service (C1)



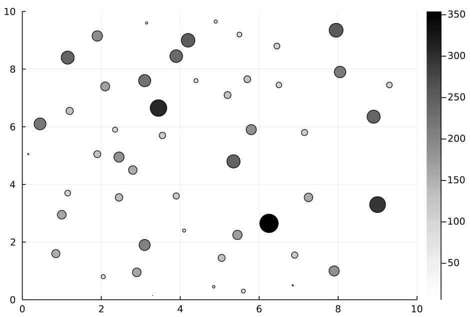
(B) MHDPL service (C1)



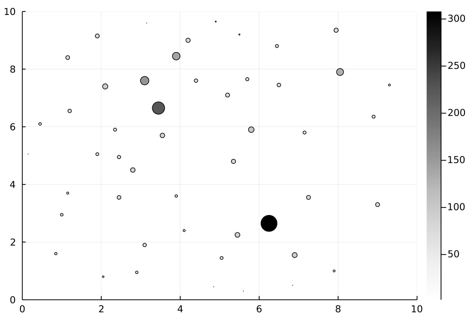
(C) MPL service (C2)



(D) MHDPL service (C2)



(E) MPL service (R1)



(F) MHDPL service (R1)

FIGURE B.3: Number of visits at MPL parking locations, C1/C2/R1 instances

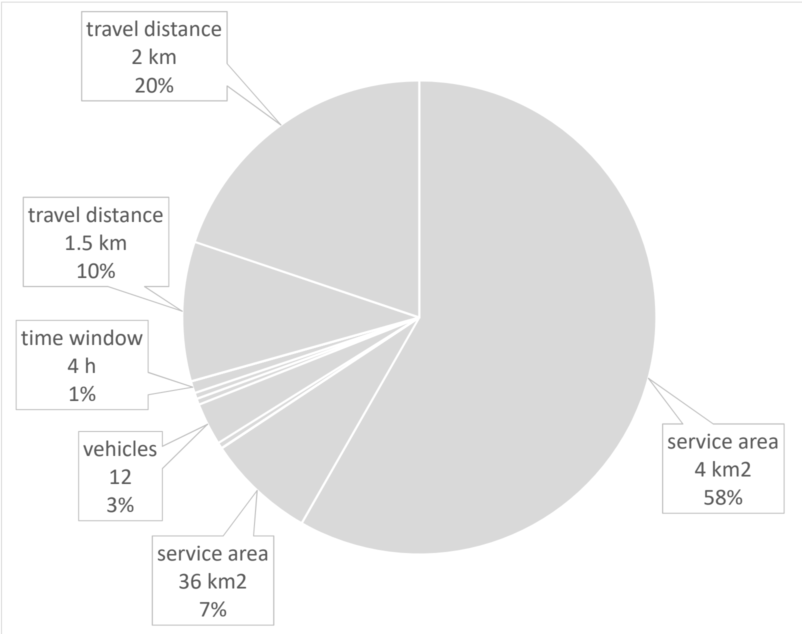


Figure B.4: Occurrence of MPL specialized vehicles

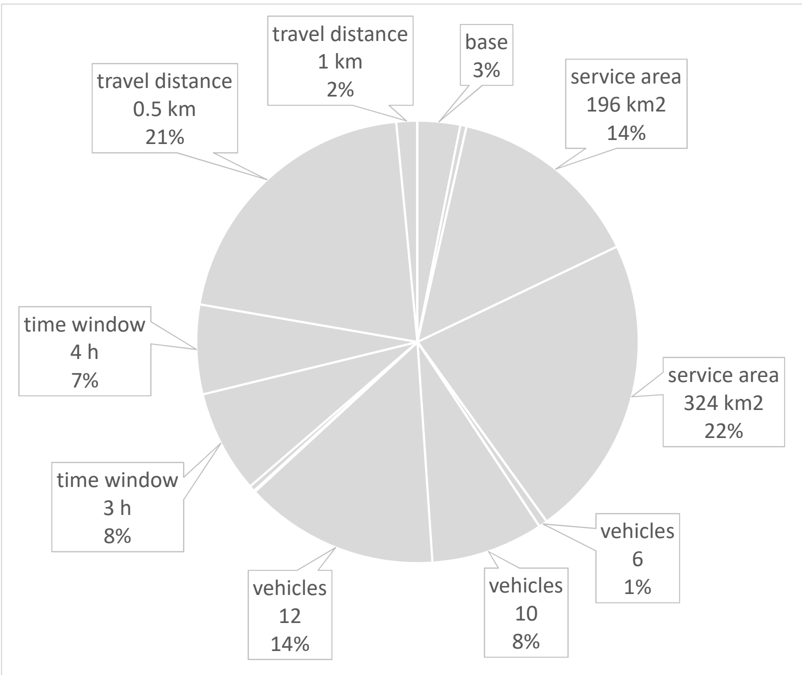
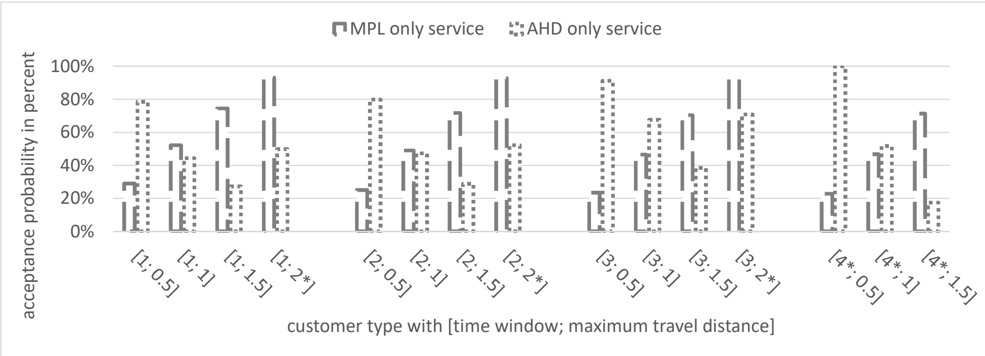
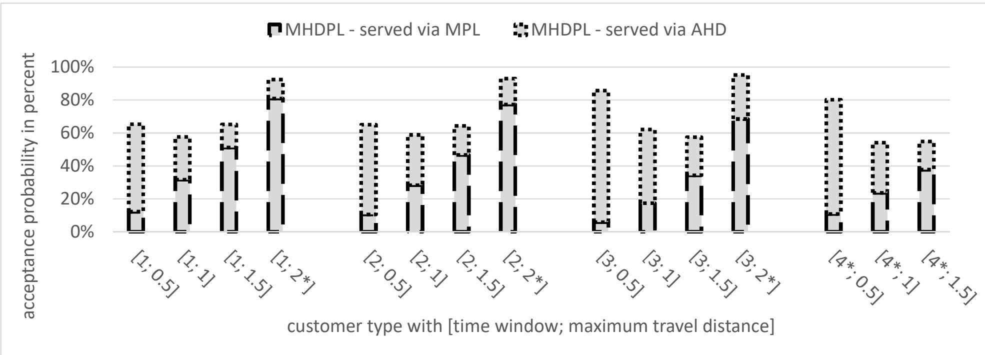


Figure B.5: Occurrence of AHD specialized vehicles



(A) MPL service and AHD service



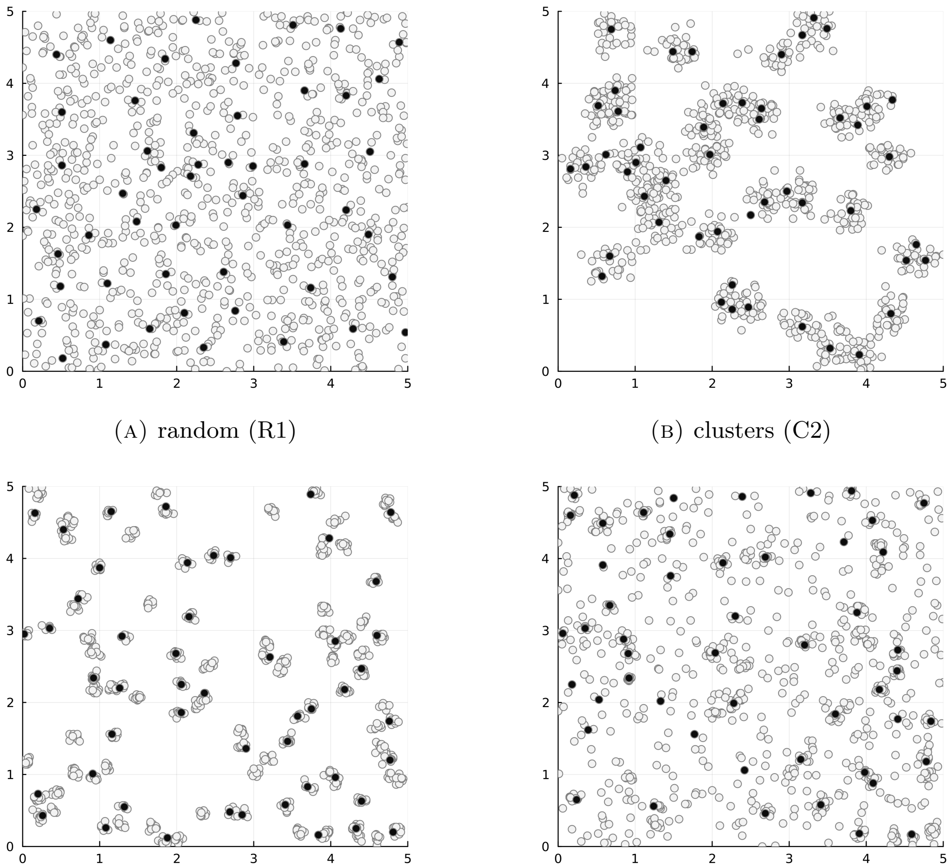
(B) MHDPL service

FIGURE B.6: Customer acceptance probabilities based on preference combinations

Appendix C

Dynamic Customer Acceptance for MHDPL Services

C.1 Additional Figures



(A) random (R1)

(B) clusters (C2)

(C) dense clusters (C1)

(D) mixture (RC1)

FIGURE C.1: 1000 customer locations (white) and 50 MPL parking locations (black)

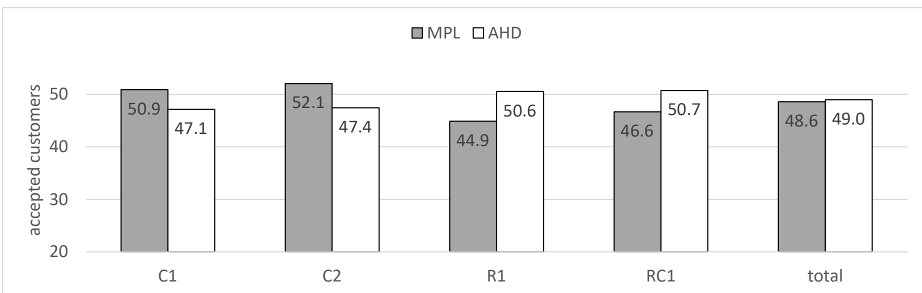


FIGURE C.2: Hindsight solutions, average accepted customers, for each area

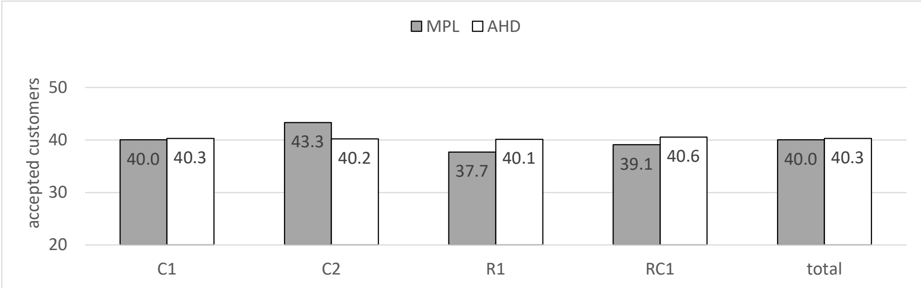


FIGURE C.3: Dynamic solutions, average accepted customers, for each area

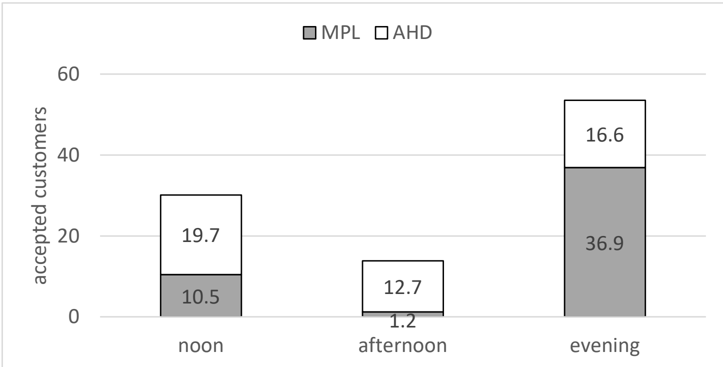


FIGURE C.4: Hindsight solutions, average accepted customers, for each time period

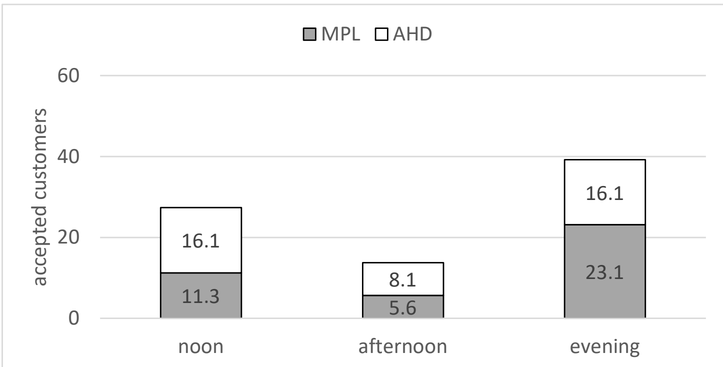


FIGURE C.5: Dynamic solutions, average accepted customers, for each time period

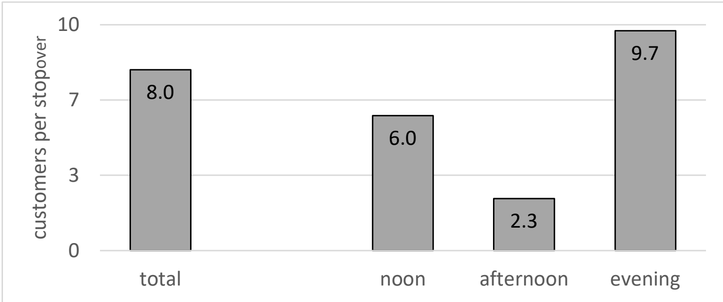


FIGURE C.6: Hindsight solutions, average number of customers served per MPL stopover, for each time period

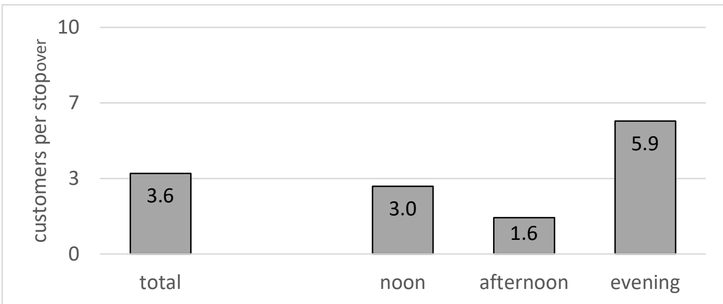
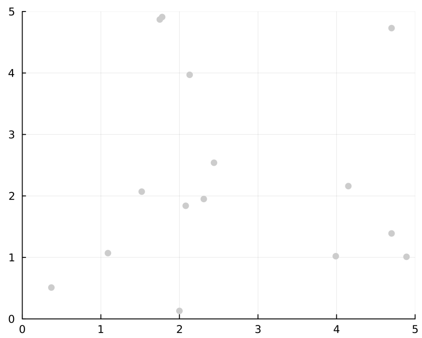
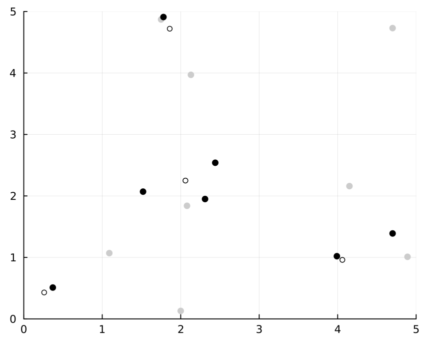


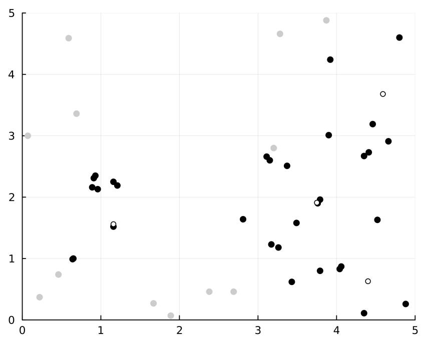
FIGURE C.7: Dynamic solutions, average number of customers served per MPL stopover, for each time period



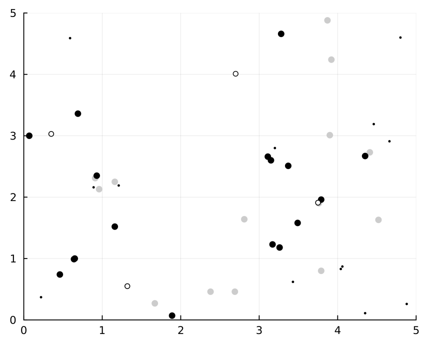
(A) Hindsight, afternoon customers



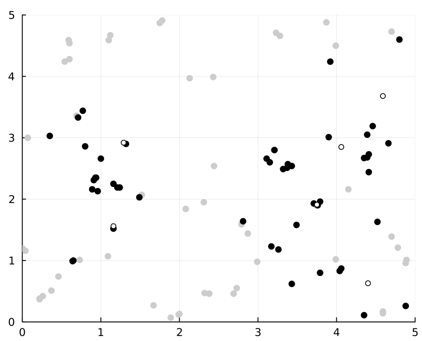
(B) Dynamic, afternoon customers



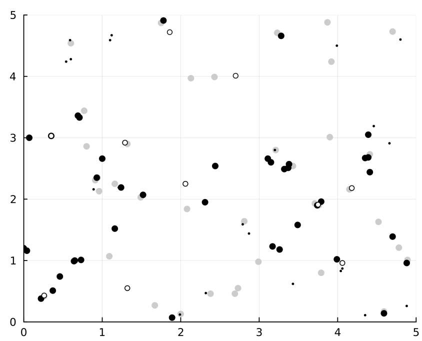
(C) Hindsight, evening customers



(D) Dynamic, evening customers



(E) Hindsight, all customers



(F) Dynamic, all customers

FIGURE C.8: Example, comparing one hindsight and one dynamic solution for a C1 instance

The locations of customers served with AHD service (gray) or MPL service (large black) and the rejected customers (small black), as well as the MPL parking locations used (white with black border) are shown for the respective periods afternoon, evening and for all customers throughout the day.

C.2 Feasibility Check Duration

Another crucial aspect for the performance of an offer strategy is the approach used to determine the feasible service options. The runtime for determining whether a service option can be feasibly offered to a customer is the most time-intensive factor in our calculations. Especially in more complex situations, i.e., when many MPL services are considered, it could be observed that the ILS needs a few more iterations to validate feasibility. In `dynamic with hindsight knowledge`, where perfect information on the stopovers is available, the average runtime for a feasibility check is 3.2 seconds. The maximum runtime for a feasibility check is 43.3 seconds. This is the time that a customer might have to wait in practice before being offered all possible service options. This would be too long for a real-time application, although the implementation of the experiments is not optimized for runtime, as the focus is on the effectiveness of the strategies and the effects of the dynamic process. Nevertheless, the effects of shorter runtimes are shown in Figure C.9, in which the number of maximum iterations of the ILS to check feasibility was reduced. The maximum number of iterations is determined as a function based on the customers already included in the solution (defined in x). The maximum number of iterations is varied from x times 2, to x, x divided by 2, and x divided by 4. As a result, the average duration for a feasibility check is reduced from 3.2 to 1.6, 0.9, and 0.5 seconds, while the maximum duration is reduced from 43.3 to 21.9, 11.6, and 6.4 seconds. However, shorter runtimes increase the chance that a feasible service option cannot be found and therefore cannot be offered to the customer. While the runtimes appear to decrease linearly, the accepted customers decrease exponentially from 97.6 to 96.6, 95.1, and 93.5. Therefore, it can be concluded that significant runtime reduction is possible by reducing the number of iterations the ILS searches for feasible solutions, but at the expense of customers that could potentially be accepted.The strategic decision on acceptable runtime respectively waiting time for a customer is highly relevant for a practical application. Moreover, the runtime has a decisive influence on the best possible solution, and therefore on the offer strategies discussed below. This is why we accept longer runtimes for the scope of this study.

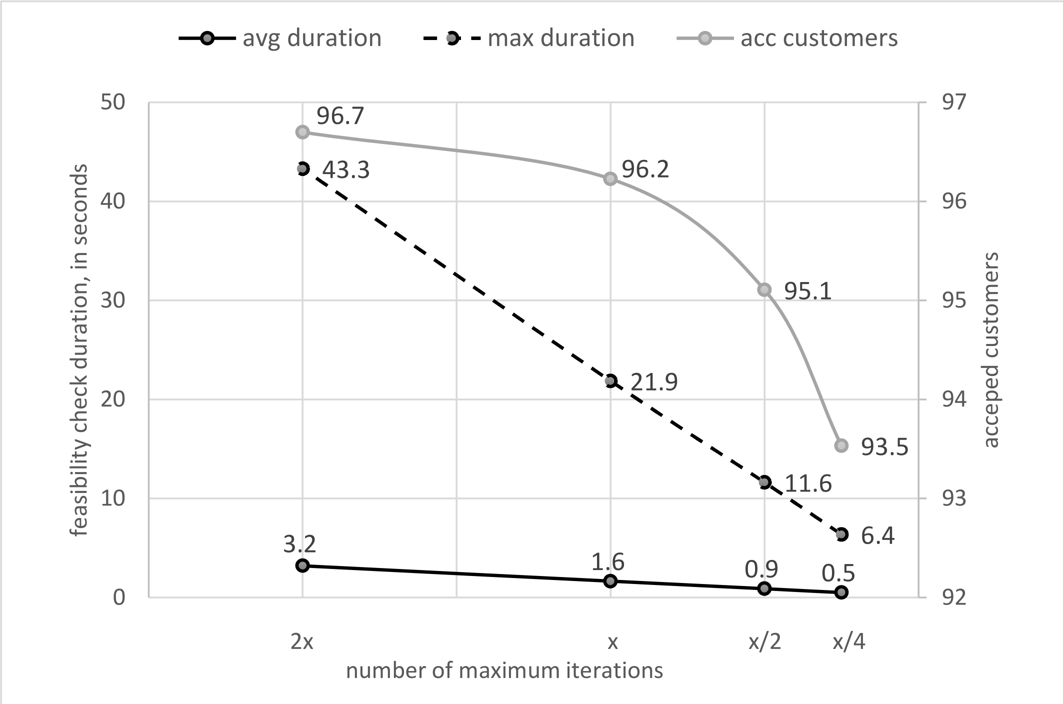


FIGURE C.9: Effects of reduced number of maximum iterations for ILS

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