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Introduction

The challenges posed by climate change and population aging are among the most pressing concerns in public debate and policymaking. These phenomena not only threaten long-term sustainability but also raise complex questions about efficiency and intergenerational equity. The economic analysis of policies that address these challenges, like any other, requires a clear description of the environment in which economic agents operate. However, despite these phenomena having begun to receive attention a few decades ago, many relevant aspects remain underexplored in existing economic models. This can lead to incomplete or even misleading policy recommendations and significant gaps in our understanding of potential trade-offs.

Environmental and public policies, particularly those related to social security, have attracted the attention of economists since at least the early twentieth century. Economic analysis of environmental policies dates back to Arthur Pigou's work on externalities, which laid the foundation for using taxes to correct market failures, such as pollution. At the same time, the concepts of public goods and market failure began to justify state-provided benefits, such as pensions. After World War II, interest in social security grew, leading economists like Paul Samuelson to study transfers from young to old from a social welfare perspective.

Today, economic research in these areas has shifted from simple theoretical models to complex quantitative models aiming to replicate and forecast real-world behavior (e.g., integrated assessment models or Bewley-Ayagari life-cycle models). Although quantitative models offer detailed predictions and policy guidance, stylized theoretical models remain indispensable. They help identify core mechanisms, clarify trade-offs, and generate results broadly applicable across contexts. Such models are particularly valuable for uncovering unintended consequences of government interventions, providing new *qualitative* insights for policymakers.

With this general motivation, background, and objectives, the present dissertation explores how environmental and public policies and/or their effectiveness can be affected by various factors, such as individual motivations, political considerations, and social interactions. It consists of three chapters (papers), each focusing on one of these factors and developing a distinct theoretical model. Using the tools of general equilibrium and game theory, these models constitute rigorous but simple analytical frameworks. Furthermore,

their fully dynamic structure enables them to capture interactions and potential conflicts between short- and long-run effects.

The first chapter is motivated by the observation that the personal involvement of individuals in environmental protection is increasing in many countries. However, it is unclear whether their private contributions will improve the long-run quality of the environment when these contributions can influence government environmental efforts. Using a growth model with environmental quality as a public good, I examine the interplay between individuals' voluntary contributions to environmental protection, driven by a 'warm glow' of giving, and public environmental policy. I ask how these interactions affect the joint evolution of economic output and environmental quality, showing that the long-run impact of warm-glow motivation on both variables is negative if the government intervenes optimally.

Related to environmental policy as well, the second chapter, co-authored with Gerhard Sorger, investigates how political pressure influences optimal climate policy and the energy transition. A key feature of our analytical framework is its ability to capture dynamic inconsistency in climate policies, which naturally arises from their long time horizon and susceptibility to modifications due to political pressure. Among other results, we show, somewhat counterintuitively, that political pressure from the polluting sector can lead to a higher rate of clean energy investment chosen by the government. On the other hand, political pressure to increase investment in clean technology leads to a higher rate of emissions, with its negative effect on the long-run stock of greenhouse gases.

The third chapter, co-authored with Kirill Borissov, examines aggregate and distributional effects of public pensions in an economy with positional externalities. More specifically, pensions are assumed to be financed according to a pay-as-you-go scheme, while individuals' decisions are subject to social comparisons. The latter can lead to overspending on personal consumption, which, in turn, can result in zero bequests left within poor dynasties. Relying on this idea, we develop an overlapping-generations model with bequests and demonstrate that the pay-as-you-go pension system exacerbates wealth inequality in an inegalitarian regime. However, it also encourages capital accumulation, which contradicts most of the previous results in the literature.

The dissertation offers new insights for policymakers. Chapter 1 emphasizes the need to consider the potential long-term consequences of relying on or advocating for private efforts to achieve sustainability goals. The results of Chapter 2 suggest that governments facing political pressure that leads to higher current emissions should invest in clean technologies, enabling future policymakers to rely less on polluting alternatives. Chapter 3 highlights the importance of accounting for heterogeneity in bequest behavior when assessing the aggregate long-run effects of old-age social security. Overall, the findings of this dissertation can inform the design of forward-looking policies that address sustainability challenges.

Chapter 1

Protecting the Environment: On the Interplay Between Voluntary Contributions and Public Policy

1.1 Introduction

As climate change intensifies and environmental degradation persists, the responsibility for environmental protection is no longer seen as falling solely on governments. While policymakers implement environmental taxes and invest in large-scale environmental maintenance programs, individual citizens are increasingly stepping up with their own voluntary efforts. This shift is especially evident in developed countries. Between the years 2010 and 2020, for example, the share of individuals donating to environmental organizations increased from 11.4% to 13.5% in France, from 15.3% to 20.3% in Germany, and from 18.2% to 23.6% in the United States. Similarly, the share of those willing to incur costs — in terms of money or time — while "doing right for the environment" increased from 40.3% to 47.2% in Japan, from 51.4% to 59.6% in Germany, and from 59.8% to 64.8% in Australia (ISSP [20, 21]).

Interestingly, though individuals are willing to exert voluntary efforts to protect the environment, they are not necessarily willing to support governments' efforts. According to a recent survey conducted in nine OECD countries (OECD [35]), about 40% of households in these countries are willing to compromise their lifestyle for the benefit of the environment but not to pay for a stricter public environmental policy. This naturally raises questions about the interplay between private and public environmental actions. Understanding their relationship is essential for designing effective measures that combine individual and government efforts to promote sustainability. Thus, it is important to examine how increasing individual involvement affects public environmental policy and when it leads to an overall long-run improvement in environmental quality.

In the present paper, we address these questions by developing a growth model with environmental quality in which both the government and individuals can contribute to environmental maintenance. To capture the observed discrepancy between the willingness of individuals to pay for private versus public environmental protection, we assume that environmental quality is a public good and, therefore, is taken as given by each individual. Yet, each individual can improve it through her voluntary contribution driven by a 'warm glow' of giving. Here, we follow the approach of Andreoni [4, 5], which suggests that in a large economy, individuals tend to view their private contributions to the public good as a source of personal fulfillment rather than a means to enhance social welfare.[1] The government, on the other hand, chooses environmental policy so as to maximize social welfare, thereby internalizing the environmental externality.

Our main interest lies in the interplay between individuals' voluntary contributions to environmental protection and public environmental policy. More specifically, we ask how the motivation for voluntary contributions — i.e., the strength of warm glow — affects government abatement expenditures financed by environmental taxes and what implications this has for the long-run levels of output and environmental quality. To uncover these implications, we separately examine two scenarios: the laissez-faire scenario without any government intervention and the scenario where the government intervenes and decides on the environmental policy in the interest of individuals.

We find that if the government does not intervene, aggregate output decreases with the strength of warm glow in the steady state, whereas environmental quality increases with it, which is intuitive since environmental quality improves solely through private warm-glow contributions. In contrast, if the government intervenes and optimally sets the environmental policy under consideration, not only aggregate output but also environmental quality declines in the long run as the warm-glow strength increases. The intuition underlying this result can be explained as follows. Private efforts to improve the environment divert resources from capital accumulation, implying lower output in the economy in the long run. This reduces the amount of resources available in the long run for government actions. Moreover, private contributions lower the need for government intervention. As a consequence, public abatement expenditures fall with an increase in the warm-glow strength to the point that total abatement expenditures, private plus public, decrease in the steady state, leading to lower steady-state environmental quality.

We also discuss the robustness of our findings to several extensions of the model and to alternative assumptions. In particular, we further demonstrate that a stronger warm-glow

[1] Taufik et al. [41] have found that acting environmentally friendly is intrinsically rewarding, while Bolderdijk et al. [9] have argued that appealing to moral responsibility rather than economic concerns is more effective in promoting pro-environmental actions. Furthermore, several experimental studies have shown that when individuals make private contributions to an environmental public good, they are primarily motivated by the warm-glow effect (see, e.g., Menges et al. [32] and Ma and Burton [29]). In a more general context, the warm-glow motivation for private contributions to a public good has been documented by Ribar and Wilhelm [39] and Crumpler and Grossman [12].

motive can lead to a higher long-run environmental quality when private efforts to improve it are more efficient than public ones. In this case, however, the government would find it optimal to allocate all tax revenue to subsidize private contributions, provided that such subsidies are available. Then, the negative long-run effect of the warm-glow motive on environmental quality persists. In other words, increasing individuals' motivation for private environmental protection lowers long-run environmental quality despite such activity receiving the maximum support from the government. Alternatively, by subsidizing savings rather than private contributions, the government can reverse the sign of the long-run environmental impact of warm glow, but this would again require higher efficiency of private environmental protection efforts over public ones. If the government is more efficient than individuals in environmental protection, the negative effect would still hold.

A number of studies have examined the relationship between private environmental protection and public environmental policy using static partial-equilibrium models, which are not able to capture the fact that environmental quality and efforts to improve it (private and/or public) may endogenously change over time as the economy develops.[2] Perino [37] has proposed a general-equilibrium, yet also static, model with two sectors and investigated how campaigns that stimulate voluntary contributions to environmental improvements by intrinsically motivated consumers affect aggregate emissions for different mandatory climate policies. Ballet et al. [6] have studied the crowding-out effect of the government's abatement effort on the voluntary abatement effort of individuals in a dynamic general-equilibrium setting with overlapping generations. They, however, assume that the voluntary effort is determined at the aggregate rather than the individual level, disregarding the free-rider problem. Dam and Heijdra [13] have asked whether public environmental policy is effective in the presence of the households' warm-glow motive. They have developed a dynamic general-equilibrium framework in which households, unlike in Ballet et al. [6], do not fully internalize the environmental externality but experience a warm glow from contributing to the public good. This is similar to our approach. Nevertheless, none of these papers allow for endogenous public environmental policy.

Our paper also contributes to the ongoing discussion in the literature on whether pro-environmental preferences are good for the environment. Ambec and De Donder [3] have considered consumers experiencing a warm glow from buying a greener product (green consumerism) and shown that the warm-glow effect reduces environmental quality with

[2]Within these models, it has been discussed, for example, how green taxes and subsidies can encourage morally motivated individuals to purchase green goods (Nyborg et al. [34]) and how a decrease in consumption of brown goods by such individuals affects the optimal tax on these goods (Daube and Ulph [14]). Similarly, Chan [11] has analyzed how optimal environmental policy changes in the presence of different behavioral motives, including green consumerism, warm glow, and moral norms. Herweg and Schmidt [17] have studied how the intrinsic motivation of climate-conscious consumers is affected by the chosen policy instrument to reduce emissions and how it affects the efficiency of regulation. Taking a mechanism design approach, Wichman [43] has examined how a regulator can achieve the socially optimal provision of an environmental public good when consumers' preferences feature the warm-glow benefit from private contribution and are not fully known to the regulator.

emission standards but not with emission taxes. In the above-mentioned work of Perino [37], the author concludes that increasing the intrinsic motivation of consumers to reduce their carbon footprint can result in higher aggregate emissions if emissions of some industries are not capped. Marini et al. [31] found that environmentalism, defined by psychic benefits (resp. costs) associated with a green (resp. brown) product purchase, is detrimental to the environment when the green product is of better quality than the brown one. In a broader context of goods generating externalities, including pollution, Kaufmann et al. [26] demonstrated that the social responsibility of consumers may lead to overconsumption of these goods due to the violation of price-taking behavior. Complementing these studies, we provide a novel rationale for why pro-environmental preferences may harm the environment, which relies on the negative long-run impact of warm-glow motivation on total (private and public) abatement expenditures.

Finally, from a more policy-oriented perspective, our paper contributes to the debate on the role of private actions in ensuring a better environment in the future. Contrary to popular belief (see, e.g., IPCC [19] and Capstick et al. [10]), we argue that advocating for private actions may not necessarily lead to improved environmental outcomes. While governments may find themselves less pressured by individuals to intervene, advocating for private environmental actions can also diminish the resources available for these interventions in the long run, which could result in the environment being worse than without private actions at all. Policymakers, thus, need to carefully consider the potential long-term consequences of relying on and/or advocating for private efforts to achieve sustainability goals.

The remainder of the paper is organized as follows. Section 1.2 describes the model. In Section 1.3, we define an equilibrium for a given environmental policy and examine the effects of the warm-glow strength in the laissez-faire case. In Section 1.4, we endogenize the policy and present our main results. Section 1.5 discusses their robustness. The last section concludes and suggests some directions for future research.

1.2 The Model

Our model has an overlapping-generations (OLG) structure and is similar to the standard OLG model with environmental quality developed by John and Pecchenino [22]. A key difference is that we allow individuals to privately contribute to environmental protection, which is otherwise carried out solely by the government.[3]

[3]John and Pecchenino [22] assume that individuals alive are represented by a short-lived government whose responsibility is to maintain environmental quality for their benefit. We will make the same assumption when considering endogenous public environmental policy in Section 1.4.

1.2.1 Individuals

Time is discrete and indexed by $t \in \{0, 1, \dots\}$. In every period t, a new generation of identical individuals is born. The size of each generation is normalized to 1, i.e., there is no population growth. Each individual lives for two periods: young and old. In the first period, each individual of generation t inelastically supplies one unit of labor, earning a wage w_t, which can be used for consumption c_t, savings s_t, or making a contribution to environmental protection m_t. In the second period, individuals retire and consume the proceeds of their savings. Therefore, the individual's first and second-period budget constraints read

$$c_t + s_t + m_t = w_t, \tag{1.1}$$

$$d_{t+1} = R_{t+1} s_t, \tag{1.2}$$

where d_{t+1} is the old-age consumption level and R_{t+1} is the gross interest rate in the economy in period $t+1$.

Individuals derive utility from consumption, the quality of the environment they live in (e.g., air or water quality), and the contribution to environmental protection they voluntarily make. Formally, we assume that the preferences of each individual born at time t are represented by the following lifetime utility function:

$$u(c_t, E_t) + \alpha v(m_t) + \delta u(d_{t+1}, E_{t+1}),$$

where E_t denotes the measure of environmental quality in period t. The parameters $\alpha \geq 0$ and $\delta > 0$ are the strength of warm glow derived from voluntary contribution and the time-preference factor, respectively. The function u is assumed to satisfy $u_1'(\cdot) > 0$, $u_2'(\cdot) > 0$, $u_{11}''(\cdot) < 0$, $u_{22}''(\cdot) < 0$, $u_{12}''(\cdot) \geq 0$, $\lim_{x \to 0} u_1'(x, E) = \infty$, while the function v is assumed to satisfy $v'(\cdot) > 0$ and $v''(\cdot) < 0$.

Concerning the interpretation of m_t, one can think of private donations to environmental organizations or, more broadly, expenditures associated with individual pro-environmental behavior (to the extent that they represent a private contribution to the public good), such as installing solar panels for residences or switching to an electric car. The key idea here is that individuals are willing to incur these expenditures because they are intrinsically motivated to protect the environment and not because they internalize the effect of their pro-environmental expenditures on environmental quality per se. As mentioned in the Introduction, we treat environmental quality as a public good, assuming that each (infinitely small) individual takes the level of this public good as given when choosing her optimal behavior. Other economy-wide variables, such as the wage rate and the gross interest rate, are also taken as given by each individual. That said, all individuals of generation t are assumed to correctly anticipate R_{t+1} and E_{t+1} when young.

The problem of each individual born at time t is to maximize her lifetime utility subject to her budget constraints by choosing the vector (m_t, c_t, d_{t+1}). Taking into account all the above assumptions, this yields the following first-order conditions for given values of w_t, E_t, R_{t+1}, and E_{t+1}:

$$\alpha v'(m_t) = u_1'(c_t, E_t), \tag{1.3}$$

$$u_1'(c_t, E_t) = \delta R_{t+1} u_1'(d_{t+1}, E_{t+1}). \tag{1.4}$$

The second condition reflects the usual intertemporal trade-off between consumption when young and when old. The first condition, on the other hand, reflects the willingness of individuals to sacrifice their consumption when young for the internal satisfaction derived from voluntarily contributing to the environmental public good.[4] This willingness increases as the strength of warm glow, $\alpha \geq 0$, increases, with the boundary case $\alpha = 0$ corresponding to the situation where individuals do not voluntarily contribute at all. Therefore, one can interpret α as a measure of individual pro-environmental behavior in the economy. Our ultimate goal is to understand how α influences the long-run level of environmental quality.

1.2.2 Environmental Quality

We assume that for a given initial value $E_0 > 0$, the environmental quality E_t evolves according to the following equation:

$$E_{t+1} = b\bar{E} + (1-b)E_t - \beta Y_t + \gamma(m_t + g_t). \tag{1.5}$$

Considering first the last two terms on the right-hand side, the term βY_t represents environmental degradation or pollution as a by-product of production activities in period t, with the parameter $\beta > 0$ measuring how much each unit of output produced Y_t degrades the environment. This degradation can be overcome by increasing individuals' voluntary contributions to environmental protection, m_t, or increasing government expenditures, which we denote by g_t. Following our examples for voluntary contributions mentioned, one could think of g_t as government expenditures on supporting solar power plants or developing public transportation. Notice that private and public pollution abatement

[4] An interesting alternative approach to modeling private contributions to environmental protection has been proposed in Bezin [7], where individuals make them in order to socialize others into environmental attitudes. Similarly to our study, this paper assumes that private contributions are determined at the individual level, i.e., individuals ignore the effect of their own contribution on environmental quality. In our model, the motivation for private contributions is purely intrinsic. This aligns with the approach taken in the recent paper by Aghion et al. [2], where individuals are assumed to be intrinsically motivated and incur costs to act in an environmentally friendly way, despite having a negligible individual impact on emissions.

efforts have the same efficiency level, denoted by $\gamma > 0$. We will relax this assumption in Section 1.5.[5]

The first two terms on the right-hand side of (1.5) capture the internal dynamics of environmental quality. The autonomous level of environmental quality is given by $\bar{E} \geq 0$, while $b \in (0, 1)$ measures how fast environmental quality converges to this level in the absence of human activities. Given that b is less than one, current production and abatement activities have a greater influence on the environment in the near future than in the distant future.

1.2.3 Firms

In every period, the economy produces Y_t according to a neoclassical production function $F(K, L)$:

$$Y_t = F(K_t, L_t), \tag{1.6}$$

where K_t is the capital stock and L_t is the amount of labor used in period t. We assume that the function F is homogeneous of degree 1 in both variables and satisfies $F_1'(\cdot) > 0$, $F_2'(\cdot) > 0$, $F_{11}''(\cdot) < 0$, $F_{22}''(\cdot) < 0$, and $F_{12}''(\cdot) > 0$. For the sake of simplicity, and because one period lasts half a generation, capital is assumed to depreciate fully each period.

There exists a representative firm that chooses the levels of K_t and L_t every period to maximize its profit π_t, which is defined as follows:

$$\pi_t = (1 - \tau_t)Y_t - R_t K_t - w_t L_t.$$

Since production generates pollution, we assume that the government taxes firms, following the polluter-pays principle. Specifically, each unit of firms' production is taxed at a rate of $\tau_t \in [0, 1)$. Then, under the conventional assumption of perfectly competitive markets, the imposition of this environmental tax (hereafter referred to as the pollution tax) translates into a reduction in the gross interest rate, R_t, and the wage rate, w_t, which are given by

$$R_t = (1 - \tau_t)f'(k_t), \tag{1.7}$$

$$w_t = (1 - \tau_t)[f(k_t) - k_t f'(k_t)]. \tag{1.8}$$

[5]It is also worth noting that a linear relationship among polluting activity, abatement, and the quality of the environment, which is a standard simplifying assumption in the literature, makes private and public abatement perfect substitutes for environmental protection. This could be seen as a limitation of our approach. However, if we consider the opposite extreme, where they act as perfect complements, the model's main implications when private and public abatement coexist would remain unchanged. Therefore, we do not expect the presence of complementarity between private and public abatement to impact our main results.

where $k_t := K_t/L_t$ is the capital stock per worker in period t and $f(k) := F(k, 1)$ is the intensive-form production function. Because we normalize the size of each generation to 1 and assume that the young supply labor inelastically, we have $k_t = K_t$ for every $t \geq 0$.

In equilibrium, the capital stock used for production in period $t + 1$ is equal to the aggregate savings in period t, i.e.,

$$K_{t+1} = s_t. \tag{1.9}$$

The initial capital stock $K_0 > 0$ is given exogenously and is equally distributed among all old individuals in period 0.

1.2.4 Environmental Policy

We have introduced two environmental policy measures, namely, government pollution abatement expenditures, g_t, and the pollution tax, τ_t. This environmental policy is assumed to be budget-balanced every period, i.e., pollution taxes collected from firms in period t are used to finance public abatement in the same period.[6] Thus, the government's budget constraint is given by

$$g_t = \tau_t Y_t = \tau_t f(K_t), \tag{1.10}$$

where the second equality follows from $L_t = 1$. It is clear that the variable τ_t fully determines the environmental policy under consideration and can equivalently be viewed as the share of public abatement expenditures in GDP. In the next section, we consider exogenous (budget-balanced) environmental policy and treat τ_t as given. Later, in Section 1.4, we will consider endogenous τ_t, which is optimally chosen by the government.

1.3 Equilibrium for a Given Policy

In this section, we define the equilibrium of the economy described above for a given sequence of pollution tax rates $\{\tau_t\}_{t=0}^{\infty}$ and discuss a laissez-faire scenario without any public policy. In equilibrium, individuals solve their utility maximization problem, firms solve their profit maximization problem, all markets clear every period, and the law of motion for environmental quality is given by (1.5). More formally, we have the following definition.

Definition 1.1. *For a given sequence of pollution tax rates $\{\tau_t\}_{t=0}^{\infty}$ and a given initial state of the economy (K_0, E_0), an equilibrium (or an equilibrium path) is a sequence $\{m_t, c_t, d_t, K_{t+1}, E_{t+1}\}_{t=0}^{\infty}$ such that for every period $t \geq 0$, equations* (1.1)-(1.10) *hold.*

[6] The absence of public debt is justified by the fact that when endogenizing environmental policy, as you will see in Section 1.4, we focus on a short-lived government. With public debt, such a government would have incentives to default on previously issued debt and issue an infinite amount of new debt to finance current public abatement, which is not plausible. On the other hand, if the public debt-to-capital ratio is kept constant over time, our main results would remain qualitatively the same as with no public debt.

Throughout the paper, we use a log-linear specification for the lifetime utility function and a Cobb-Douglas specification for the production function to keep the analysis tractable.[7] In particular, we will assume

$$v(m) = \ln m,\ u(x, E) = \ln x + \lambda \ln E,\ f(k) = Ak^{\nu}, \tag{1.11}$$

where $\nu \in (0, 1)$ is the share of capital income in total output, $A > 0$ is a fixed productivity parameter, and $\lambda > 0$ is the degree of environmental concerns among individuals, which is unrelated to the warm-glow parameter α. Under these specifications, the equilibrium (for a given policy) is characterized by the following dynamics of the state variables:[8]

$$K_{t+1} = \frac{\delta(1-\tau_t)(1-\nu)AK_t^{\nu}}{1+\alpha+\delta}, \tag{1.12}$$

$$E_{t+1} = b\bar{E} + (1-b)E_t + \left\{\gamma\left[\frac{\alpha(1-\tau_t)(1-\nu)}{1+\alpha+\delta} + \tau_t\right] - \beta\right\} AK_t^{\nu}. \tag{1.13}$$

It follows from the first equation that an increase in the pollution tax rate τ_t discourages capital accumulation, which is due to a reduction in the wage rate. As for the second equation, we see that for a given $K_t > 0$, a higher τ_t leads to a higher environmental quality in period $t+1$. This is because public pollution abatement, $g_t = \tau_t AK_t^{\nu}$, directly increases with τ_t, while the private one, m_t, decreases with τ_t due to a reduction in the wage rate, but not so much as to exceed the increase in g_t. To put it differently, one can infer from (1.13) that public abatement today increases environmental quality next period while partially crowding out private abatement.[9]

Let us now consider the laissez-faire situation, where $\tau_t = 0$ holds for all $t \geq 0$. The following result follows directly from equations (1.12) and (1.13).

Proposition 1.1. *In the absence of public environmental policy (i.e., $\tau_t = 0$ holds for all $t \geq 0$), the capital stock K_t and the environmental quality E_t converge to their steady-state values K^* and E^*, respectively. These steady-state values are given by*

$$K^* = \left[\frac{\delta A(1-\nu)}{1+\alpha+\delta}\right]^{\frac{1}{1-\nu}},$$

$$E^* = \bar{E} + \frac{1}{b}\left[\frac{\gamma\alpha(1-\nu)}{1+\alpha+\delta} - \beta\right] A(K^*)^{\nu}.$$

[7] In the context of an OLG model with environmental externalities, a log-linear lifetime utility function is also used in John et al. [23] and in subsequent studies such as Mariani et al. [30]. A Cobb-Douglas production function is used, for example, in Prieur [38] and Bezin [7].

[8] The omitted derivations and proofs can be found in Appendix 1.A.

[9] In contrast to the related work by Ballet et al. [6] mentioned in the Introduction, public abatement necessarily (but partially) crowds out private abatement efforts in our general-equilibrium setting. Similarly to Dam and Heijdra [13], however, also mentioned in the Introduction, pro-environmental behavior by individuals (i.e., $\alpha > 0$) partially offsets the positive effect of public abatement on environmental quality.

Note from Proposition 1.1 that the steady state of the laissez-faire economy is characterized by the negative dependence of K^* on the parameter α, that is, a higher intensity of warm glow from making voluntary contributions to the environmental public good leads to a lower steady-state capital stock. This finding is not surprising, given that private pollution abatement diverts resources from consumption but also savings and, consequently, capital formation. On the other hand, because of this effect, the amount of resources available in the long run for private abatement falls, too, and the steady-state level of environmental quality can potentially decrease with α. The value of α for this to happen, though, must already be quite high. More precisely, we have $\partial E^*/\partial\alpha > 0$ if $\gamma(1-\nu) \leq \beta$; otherwise, we have

$$\frac{\partial E^*}{\partial \alpha} \gtreqless 0 \;\Leftrightarrow\; \alpha \lesseqgtr \frac{1+\delta}{\nu}\left[1-\nu+\frac{\beta}{\gamma(1-\nu)-\beta}\right].$$

Since, in the real world, individuals are more likely to prioritize their consumption over their private contribution to the environment, the value of α is realistically below 1. At the same time, the capital share ν is realistically below $1/2$, which implies that the critical threshold for α in the above relationship is likely to exceed 1. Thus, under plausible restrictions, our model predicts a positive effect of the warm-glow strength on steady-state environmental quality in the absence of public environmental policy. In the next section, we will show that if public policy is present and endogenously chosen by the government, a higher warm-glow intensity always leads to a lower steady-state environmental quality.

1.4 Endogenous Environmental Policy

So far, pollution tax rates have been assumed to be exogenously determined. Let us now consider a one-period lived government, as suggested by John and Pecchenino [22], and endogenize the sequence $\{\tau_t\}_{t=0}^{\infty}$.[10] With endogenous environmental policy, our main interest lies in its dependence on the warm-glow strength as a measure of individual pro-environmental behavior.

The one-period government chooses environmental policy in the interest of all individuals alive during its period of office, i.e., both the young and the old. In particular, we assume that the government that is in office in period t, or for short, the time-t government, seeks to maximize a weighted utilitarian social welfare function, i.e.,

$$\mu u(d_t, E_t) + u(c_t, E_t) + \delta u(d_{t+1}, E_{t+1}), \tag{1.14}$$

[10] It is worth mentioning that John and Pecchenino [22] assume the existence of a one-period lived government but do not model it explicitly. This is done in John et al. [23], which studies a similar model with a different focus. We rely on this second paper, too.

where $\mu \geq 0$ is the weight the government attaches to the welfare of the current old relative to that of the current young. Since the private contribution m_t provides warm-glow utility and increases the utility from the public good (environmental quality), we exclude the warm-glow component from the social welfare function to avoid double counting. This is in line with Diamond [16], who argued against including warm glow in social welfare.[11] Nevertheless, if we include it, our main result, namely, the negative dependence of steady-state environmental quality on α continues to hold for all $\alpha \geq 0$ as long as its dependence on the degree of environmental concerns, λ, is positive (see Appendix 1.A for details).

The time-t government's problem is to choose τ_t that maximizes (1.14) subject to all the equilibrium conditions (1.1)-(1.10). The fact that the government takes the equilibrium conditions into account implies, in particular, that the government, unlike individuals, internalizes the effect of voluntary contributions m_t on the future environmental quality E_{t+1}. Similarly, it internalizes the effect of savings s_t on the future interest rate R_{t+1}. At the same time, we assume that the time-t government correctly anticipates the next-period pollution tax rate τ_{t+1}, which also influences R_{t+1}, but takes this tax rate as given. The last assumption reflects the idea that the current government ignores the potential impact of its chosen policy on that determined by the future government.[12] Therefore, the current policy τ_t is set for a given τ_{t+1} and, in general, depends on it. Furthermore, since we consider a dynamic setting, the chosen tax τ_t also depends in general on the values of two state variables K_t and E_t.

We stick to our specification assumption (1.11). Then, the social welfare function (1.14) becomes

$$\mu(\ln d_t + \lambda \ln E_t) + \ln c_t + \lambda \ln E_t + \delta(\ln d_{t+1} + \lambda \ln E_{t+1}),$$

with c_t and d_t being proportional to $(1-\tau_t)K_t^\nu$ in equilibrium, d_{t+1} being proportional to $(1-\tau_{t+1})K_{t+1}^\nu$, and (K_{t+1}, E_{t+1}) being given by (1.12)-(1.13). Note that because the time-t government takes τ_{t+1} as given, it can disregard the additive term involving τ_{t+1} when maximizing this objective function, evaluated at the equilibrium, with respect to τ_t. Thus, under (1.11), the current policy turns out to be independent of τ_{t+1}. Moreover, because the current government inherits the state (K_t, E_t) from the previous government, the time-t government's problem can further disregard additive terms involving K_t and

[11] Considering environmental policy in an economy with intrinsically motivated individuals, Herweg and Schmidt [17] also assumed that the government (regulator) wants to maximize *material* social welfare. In Ambec and De Donder [3], the social welfare function similarly excludes the warm-glow component.

[12] The same assumption is made in Ono [36]. Alternatively, one can assume, as in Karp and Rezai [25], that all time-t governments choose the same policy rule that maps the state of the economy into the pollution tax. We do not explore this alternative assumption in the present paper. However, in Subsection 1.5.6, we consider a long-lived government, which can also be viewed as the case where the current government that cares about currently living as well as yet unborn generations realizes its impact on the policy chosen by the future government.

E_t, reducing eventually to

$$\max_{\tau_t \in [0,1)} (1 + \mu + \delta\nu) \ln(1 - \tau_t) + \delta\lambda \ln E_{t+1}, \tag{1.15}$$

where E_{t+1} is given by (1.13). It is clear that the only reason for the government to intervene and choose $\tau_t > 0$ is to improve the future quality of the environment for the current young, E_{t+1}, which, as pointed out in the previous section, increases with τ_t for a given K_t. Since we already examined the situation without policy interventions, we focus on an interior solution to the government's problem, implying that private and public pollution abatement coexist. The next proposition characterizes this solution.

Proposition 1.2. *The interior solution to the time-t government's problem is given by*

$$\tau_t = 1 - \frac{b\bar{E} + (1-b)E_t + (\gamma - \beta)AK_t^\nu}{\gamma(1 + \delta\hat{\lambda})[1 - \frac{\alpha(1-\nu)}{1+\alpha+\delta}]AK_t^\nu},$$

where $\hat{\lambda} := \lambda/(1 + \mu + \delta\nu)$.

Proposition 1.2 shows that while the optimal current pollution tax rate τ_t does not depend on τ_{t+1}, it evolves with the state of the economy (K_t, E_t). Specifically, it increases as the current capital stock K_t and, hence, the output $Y_t = AK_t^\nu$ increases or as the current environmental quality E_t decreases. That is, the government imposes a higher pollution tax and thus raises tax revenue to finance public abatement when the economy produces more output or when the environment is in worse condition. Intuitively, these findings reflect a higher demand for environmental protection when environmental quality is low and pollution is high. At the same time, a larger amount of resources is available for this protection when more (polluting) output is produced. Proposition 1.2 also shows that τ_t is low for a given (K_t, E_t) when the warm-glow strength α is high. This result is also intuitive: the demand for government intervention in environmental protection declines as individuals are more motivated to protect the environment privately.[13]

We now turn to the dynamics. Applying the expression for τ_t from Proposition 1.2 to the state equations (1.12) and (1.13) derived for a given policy, we obtain that K_t and E_t evolve as

$$K_{t+1} = \frac{\delta(1-\nu)[b\bar{E} + (1-b)E_t + (\gamma - \beta)AK_t^\nu]}{\gamma(1 + \delta\hat{\lambda})(1 + \alpha\nu + \delta)}, \tag{1.16}$$

$$E_{t+1} = \frac{\delta\hat{\lambda}[b\bar{E} + (1-b)E_t + (\gamma - \beta)AK_t^\nu]}{1 + \delta\hat{\lambda}}. \tag{1.17}$$

[13] Note that this effect relies on private and public abatement being modeled as perfect substitutes. If, alternatively, they were perfect complements, the effect would disappear.

These equations highlight the role of savings in environmental protection, as the environmental quality in period $t+1$ is proportional to the capital stock in that period and, consequently, to the aggregate savings in period t. Proposition 1.3 shows that this property, together with our assumption of a linear natural regeneration of environmental quality, leads to simple expressions for the long-run values of K_t and E_t.

Proposition 1.3. *Suppose that $\gamma > \beta$ holds. When public environmental policy is optimally chosen by one-period-lived governments, the capital stock K_t and the environmental quality E_t converge to their steady-state values K^{**} and E^{**}, respectively. These steady-state values are given by*

$$K^{**} = \frac{\delta(1-\nu)[b\bar{E} + (\gamma-\beta)A(K^{**})^{\nu}]}{\gamma(1+\delta\hat{\lambda}b)(1+\alpha\nu+\delta)},$$

$$E^{**} = \frac{\delta\hat{\lambda}[b\bar{E} + (\gamma-\beta)A(K^{**})^{\nu}]}{1+\delta\hat{\lambda}b}.$$

The condition $\gamma > \beta$ in Proposition 1.3 means that if *all* output is invested in pollution abatement, the resulting environmental improvement must be able to exceed the environmental degradation caused by the production of this output. Similar conditions are used in other OLG models with environmental quality (see, e.g., Schumacher [40]), and it can be seen as a condition that rules out inefficient pollution abatement technologies. Moreover, without this condition, the economy may not converge to a steady state.[14]

Equipped with Proposition 1.3, we can now examine how individual efforts to protect the environment affect environmental quality in the long run when they also influence the choice of environmental policy by the short-lived government. To do so, we consider the comparative statics with respect to the warm-glow strength, $\alpha \geq 0$. Let us first note that the relationship between α and the steady-state capital stock K^{**} is negative, i.e.,

$$\frac{\partial K^{**}}{\partial \alpha} < 0. \tag{1.18}$$

Similarly to the case where public environmental policy was absent, this result is driven by the negative effect of α on the share of wage income saved by the young, reflecting that private abatement diverts resources from savings. On the other hand, with public policy, savings are also positively influenced by α via a reduced pollution tax rate for a given state (K_t, E_t) and, hence, an increased wage rate. Yet, this positive indirect effect is weaker than the former negative direct effect in the long run.

[14] More precisely, if $\gamma < \beta$ and the difference $\beta - \gamma$ is sufficiently large, the model dynamics exhibit explosive oscillations around the steady state. The critical threshold for $\beta - \gamma$ decreases as the autonomous level of environmental quality, $\bar{E}$, becomes closer to 0 (most of the previous literature assumes $\bar{E}$ to be 0).

Next, due to the negative dependence of steady-state capital stock on α, the steady-state level of output produced by firms decreases with the warm-glow strength, too. Furthermore, as α increases, the pollution tax rate that firms face decreases not only in the short run but also in the long run. Denoting the steady-state pollution tax rate by τ^{**}, it is straightforward to check that it is given by

$$\tau^{**} = 1 - \frac{b\bar{E} + (\gamma - \beta)A(K^{**})^{\nu}}{\gamma(1 + \delta\hat{\lambda}b)[1 - \frac{\alpha(1-\nu)}{1+\alpha+\delta}]A(K^{**})^{\nu}},$$

which, taking into account (1.18), indeed implies $\partial\tau^{**}/\partial\alpha < 0$ for any value of α. Thus, in the long run, the government collects fewer taxes from firms and therefore spends less on public pollution abatement (equal to $\tau^{**}A(K^{**})^{\nu}$). The question then is whether the negative effect of the warm-glow strength on public pollution abatement is sufficiently strong for environmental quality to fall. It follows from the expression for E^{**} and (1.18) that the following result is true:

$$\frac{\partial E^{**}}{\partial \alpha} < 0. \tag{1.19}$$

Note that this result holds despite production, which worsens environmental quality through pollution, decreasing with α in the long run. As a corollary, the sum of private and public abatement must also fall with the warm-glow strength in the steady state.

In sum, a higher degree of warm glow leads to a lower capital stock and a lower pollution tax revenue in the long run. The latter reduces abatement expenditures by the government, leading to a decrease in total (private and public) abatement expenditures, which, in turn, reduces environmental quality.

In concluding the present section, we would like to stress that while a stronger motivation to contribute to environmental protection privately causes steady-state environmental quality to fall, it does not have to fall as individuals become more concerned about the environment. In fact, by studying the effect of the degree of environmental concerns λ on E^{**}, we obtain

$$\frac{\partial E^{**}}{\partial \lambda} > 0 \quad \text{if} \quad \lambda < \frac{(1-\nu)(1+\mu+\delta\nu)}{\nu\delta b}, \tag{1.20}$$

which indicates that greater individuals' concerns for environmental quality are associated with higher levels of environmental quality in the long run, provided that these concerns are not too strong.[15] This finding is explained by a more stringent environmental policy adopted by the government, whose intervention is necessary to account for the public good nature of environmental quality. Indeed, as one can see from Proposition 1.1, the parameter λ does not appear in the expressions for K^* and E^*, in contrast to those for

[15] For $\nu < 1/2$, for example, it is sufficient that λ does not exceed $1/\delta$, meaning that environmental quality when old is less important for the current young than their young-age consumption.

K^{**} and E^{**}, as if individuals did not care about the level of the environmental public good at all.

1.5 Robustness

In this section, we examine whether the negative long-run effect of the warm-glow strength on environmental quality is robust to several extensions and alternative assumptions.

1.5.1 Different Efficiencies of Private and Public Abatement

We begin with an extension in which private and public pollution abatement efforts may have different efficiency levels, denoted by $\gamma_m > 0$ and $\gamma_g > 0$, respectively. The special case where $\gamma_m = \gamma_g = \gamma$ corresponds to our original model. The law of motion for environmental quality (1.5) is modified as

$$E_{t+1} = b\bar{E} + (1-b)E_t - \beta Y_t + \gamma_m m_t + \gamma_g g_t. \tag{1.21}$$

Since individuals are atomistic in our economy, one can reasonably assume that the government is more efficient in protecting the environment than individuals, i.e., $\gamma_g > \gamma_m$. In practice, governments, unlike individuals, can implement large-scale pollution abatement programs that benefit from economies of scale, providing a more cost-effective way to improve the environment. In addition, governments can use recent technological advancements that might be inaccessible to individual citizens, further increasing the relative efficiency of public abatement. Nonetheless, no restrictions on the relationship between γ_g and γ_m are required to derive the results below, so they also hold for the case $\gamma_g \leq \gamma_m$.

While all other equations in the model remain unchanged, the law of motion for the capital stock is modified, too. The latter depends on the environmental policy chosen by the government, which now has to account for the difference in the efficiency of private and public pollution abatement. In particular, proceeding analogously to the baseline model, we obtain the following proposition.

Proposition 1.4. *When private and public pollution abatement may differ in their efficiency levels (i.e., $\gamma_m \neq \gamma_g$), the interior solution to the time-t government's problem is given by*

$$\tau_t = 1 - \frac{b\bar{E} + (1-b)E_t + (\gamma_g - \beta)AK_t^{\nu}}{(1+\delta\hat{\lambda})[\gamma_g - \frac{\gamma_m \alpha(1-\nu)}{1+\alpha+\delta}]AK_t^{\nu}}.$$

Note that the higher the efficiency of private abatement, γ_m, the lower the optimal pollution tax rate for a given state (K_t, E_t). In other words, it is optimal for the government to rely less on public abatement as each unit invested in abatement activities privately causes higher environmental improvement.

Then, under this optimal policy, the joint dynamics of the capital stock K_t and the environmental quality E_t are given by

$$K_{t+1} = \frac{\delta(1-\nu)[b\bar{E} + (1-b)E_t + (\gamma_g - \beta)AK_t^{\nu}]}{(1+\delta\hat{\lambda})[\gamma_g(1+\alpha+\delta) - \gamma_m\alpha(1-\nu)]}, \tag{1.22}$$

$$E_{t+1} = \frac{\delta\hat{\lambda}[b\bar{E} + (1-b)E_t + (\gamma_g - \beta)AK_t^{\nu}]}{1+\delta\hat{\lambda}}, \tag{1.23}$$

which gives equations (1.16)-(1.17) for identical efficiency between private and public abatement. The next proposition generalizes Proposition 1.3 to heterogeneous efficiency.

Proposition 1.5. *Suppose that $\gamma_g > \beta$ holds. When public environmental policy is optimally chosen by one-period-lived governments, the capital stock K_t and the environmental quality E_t converge to their steady-state values K^{**} and E^{**}, respectively. These steady-state values are given by*

$$K^{**} = \frac{\delta(1-\nu)[b\bar{E} + (\gamma_g - \beta)A(K^{**})^{\nu}]}{(1+\delta\hat{\lambda}b)[\gamma_g(1+\alpha+\delta) - \gamma_m\alpha(1-\nu)]},$$

$$E^{**} = \frac{\delta\hat{\lambda}[b\bar{E} + (\gamma_g - \beta)A(K^{**})^{\nu}]}{1+\delta\hat{\lambda}b}.$$

It is easy to deduce from this proposition that if, realistically, the government is more efficient in protecting the environment than individuals (i.e., $\gamma_g > \gamma_m$), the negative effect of the warm-glow strength α on the steady-state environmental quality E^{**} still holds. If, on the contrary, individuals are more efficient in protecting the environment (i.e., $\gamma_g < \gamma_m$), the strength of their motivation to do so still affects E^{**} negatively as long as the efficiency γ_m remains smaller than $\gamma_g/(1-\nu)$ but positively when it is greater.[16] That said, as we will show in the following subsection, for $\gamma_g < \gamma_m$, the government would find it optimal to subsidize private abatement rather than to finance public abatement if such subsidies are available. The negative effect of α would then persist even in the case $\gamma_m \geq \gamma_g/(1-\nu)$.

1.5.2 Subsidies for Voluntary Contributions

Next, we allow the government to allocate revenue from the pollution tax not only to public abatement but also to subsidies for voluntary contributions. Let $\theta_t^m \in [0,1)$ denote the subsidy rate that the time-t government applies to each unit contributed to environmental protection privately, $m_t > 0$. Then, the government's budget constraint (1.10) becomes

$$g_t + \theta_t^m m_t = \tau_t Y_t = \tau_t f(K_t),$$

[16] In any case, γ_g must be greater than $\gamma_m\alpha(1-\nu)/(1+\alpha+\delta)$ for the equilibrium to exist and greater than β to ensure its convergence to a steady state.

while the individual's first-period budget constraint (1.1) becomes

$$c_t + s_t + (1 - \theta_t^m)m_t = w_t.$$

Let us also denote by σ_t the share of tax revenues used to finance subsidies, i.e., $\sigma_t := \theta_t^m m_t/[\tau_t f(K_t)]$. The pair (τ_t, σ_t) thus characterizes all feasible combinations of the three environmental policy measures g_t, τ_t, and θ_t^m.

To make the analysis interesting, we maintain the assumption from the previous subsection that private and public pollution abatement may differ in efficiency. Therefore, equation (1.21) holds. With the functional forms given in (1.11), this equation can be expressed in equilibrium as

$$E_{t+1} = b\bar{E} + (1-b)E_t + \left\{ \frac{\gamma_m \alpha(1-\tau_t)(1-\nu)}{1+\alpha+\delta} + \gamma_g \tau_t - \beta + (\gamma_m - \gamma_g)\sigma_t \tau_t \right\} A K_t^{\nu}, \quad (1.24)$$

which coincides with (1.13) for $\gamma_m = \gamma_g = \gamma$. On the other hand, we can show (see Appendix 3.A) that the objective function of the time-t government's problem with subsidies remains the same as in (1.15). Then, if $\gamma_m = \gamma_g = \gamma$, any value of σ_t from 0 to 1 can be chosen by the time-t government without affecting our initial results. Similarly, if $\gamma_g > \gamma_m$, the time-t government chooses not to subsidize voluntary contributions, i.e., $\sigma_t = 0$, and we return to the model discussed in the previous subsection.

Finally, consider the case $\gamma_g < \gamma_m$. If individuals are more efficient in environmental protection than the government, the government finds it optimal to choose $\sigma_t = 1$, and therefore public abatement disappears from the model. Then, it is straightforward to see that the law of motion for environmental quality (1.24) coincides with (1.13) holding with $\gamma = \gamma_m$. As a consequence, with $\gamma = \gamma_m$, Propositions 1.2 and 1.3 remain valid for the case where the entire tax revenue is used to finance subsidies for private abatement, and so do the effects of an increase in the strength of warm glow, $\alpha > 0$. The latter finding means, in particular, that a stronger motivation of individuals to voluntarily contribute to the environmental public good still leads to a lower level of this public good in the long run (E^{**}), despite contributions receiving the maximum support from the government. As before, this long-run effect works through reduced pollution taxes collected from firms. However, their reduction now does not translate into lower public pollution abatement, which is absent. Instead, it reduces the government's support for individuals' voluntary contributions. A direct positive effect of α on these contributions becomes relatively weak, so private environmental protection eventually decreases with the degree of motivation for such protection (provided that the degree is initially positive).

1.5.3 Savings Subsidies

Alternatively to subsidies for voluntary contributions, the government can subsidize savings (or tax them if the subsidy rate is negative) to mitigate the negative effect of voluntary contributions on capital accumulation.[17] While savings subsidies are received by individuals, one can also consider subsidies received by firms for their capital use, leading to the same equilibrium dynamics of the capital stock and environmental quality. Denoting by θ_t^s the subsidy rate on the returns to savings received in period t, we rewrite the individuals' second-period budget constraint (1.2) as

$$d_{t+1} = (1 + \theta_{t+1}^s) R_{t+1} s_t$$

and the government's budget constraint (1.10) as

$$g_t + \theta_t^s R_t K_t = \tau_t Y_t = \tau_t f(K_t).$$

With these new conditions, the previous specifications in (1.11), and possibly different efficiency between private and public abatement, we obtain that not only does the law of motion for environmental quality change, but so does the objective function of the time-t government. Specifically, the time-t government's problem (1.15) becomes to choose τ_t and θ_t^s to maximize

$$\mu \ln(1 + \theta_t^s) + (1 + \mu + \delta\nu) \ln(1 - \tau_t) + \delta\lambda \ln E_{t+1}, \tag{1.25}$$

where E_{t+1} is given by

$$E_{t+1} = b\bar{E} + (1 - b)E_t + \left\{ \frac{\gamma_m \alpha (1 - \tau_t)(1 - \nu)}{1 + \alpha + \delta} + \gamma_g [\tau_t - \theta_t^s \nu (1 - \tau_t)] - \beta \right\} A K_t^{\nu}. \tag{1.26}$$

As the parameter μ represents the relative weight that the government attaches to the current old in the social welfare function, one can see that there exists an intergenerational conflict due to savings subsidies: they increase consumption of the current old but reduce public abatement and, hence, the future environmental quality that will be experienced by the current young. The optimal policy is given in the following proposition.

[17] Another possible extension is the one where the young receive lump-sum transfers (or pay lump-sum taxes if the transfers are negative). In this case, the main results of the model would be similar to those obtained in the present subsection.

Proposition 1.6. *When savings subsidies are available, the interior solution to the time-t government's problem is given by*

$$\theta_t^s = \frac{\hat{\mu}(1-\nu)[\gamma_g(1+\alpha+\delta)-\gamma_m\alpha]}{\gamma_g\nu(1+\alpha+\delta)} - 1,$$

$$\tau_t = 1 - \frac{b\bar{E} + (1-b)E_t + (\gamma_g-\beta)AK_t^\nu}{(1+\hat{\mu})(1+\delta\hat{\lambda})(\gamma_g - \frac{\gamma_m\alpha}{1+\alpha+\delta})(1-\nu)AK_t^\nu},$$

where $\hat{\mu} := \mu/(1+\delta\nu)$.

Note that an increase in the warm-glow strength α reduces both the savings subsidy rate and the pollution tax rate for a given state (K_t, E_t). The latter effect was already discussed. However, it can now have important consequences for capital accumulation. Substituting the expression for τ_t into (1.12), which still holds, and the expressions for both θ_t^s and τ^s into (1.26) yields

$$K_{t+1} = \frac{\delta[b\bar{E} + (1-b)E_t + (\gamma_g-\beta)AK_t^\nu]}{(1+\hat{\mu})(1+\delta\hat{\lambda})[\gamma_g(1+\alpha+\delta)-\gamma_m\alpha]}, \tag{1.27}$$

$$E_{t+1} = \frac{\delta\hat{\lambda}[b\bar{E} + (1-b)E_t + (\gamma_g-\beta)AK_t^\nu]}{1+\delta\hat{\lambda}}. \tag{1.28}$$

Thus, we obtain:

Proposition 1.7. *Suppose that* $\gamma_g > \beta$ *holds. When public environmental policy is optimally chosen by one-period-lived governments, the capital stock* K_t *and the environmental quality* E_t *converge to their steady-state values* K^{**} *and* E^{**}*, respectively. These steady-state values are given by*

$$K^{**} = \frac{\delta[b\bar{E} + (\gamma_g-\beta)A(K^{**})^\nu]}{(1+\hat{\mu})(1+\delta\hat{\lambda}b)[\gamma_g(1+\alpha+\delta)-\gamma_m\alpha]},$$

$$E^{**} = \frac{\delta\hat{\lambda}[b\bar{E} + (\gamma_g-\beta)A(K^{**})^\nu]}{1+\delta\hat{\lambda}b}.$$

An immediate corollary from this proposition is that the long-run effect of warm glow on environmental quality remains negative if the government is more efficient, at least slightly, in environmental protection than individuals. If the reverse holds (i.e., $\gamma_g < \gamma_m$), the direct negative effect of α on capital accumulation becomes weaker than its positive effect on capital accumulation through the reduced pollution tax rate. As a result, steady-state capital increases with the warm-glow strength, which further leads to higher steady-state environmental quality. Similarly, in the special case where $\gamma_g = \gamma_m$, the model with saving subsidies predicts neutral, rather than negative, effects of α on K^{**} and E^{**}, with total abatement expenditures being independent of α.

1.5.4 Contributing When Retired

Our fourth extension deals with individual pro-environmental behavior among the old. Since individuals in our economy are intrinsically motivated, they could, in principle, make voluntary contributions to environmental protection not only when working but also during retirement. Under our timing assumptions, contributions by the old would not be able to change the level of the environmental public good from which the old benefit themselves. Therefore, their intrinsic motivation should come from altruistic feelings toward the young. While a rigorous examination of intergenerational altruism is beyond the scope of this paper, it is still reasonable to allow individuals to derive warm-glow utility (for whatever reasons) from their voluntary contributions in both periods of life.

Let us denote by m_t^y and m_{t+1}^o the voluntary contributions made by each individual of generation t when young and when old, respectively. Then, we can rewrite the individual budget constraints (1.1) and (1.2) as

$$c_t + s_t + m_t^y = w_t,$$

$$d_{t+1} + m_{t+1}^o = R_{t+1} s_t.$$

As for the lifetime utility function, it is modified as follows:

$$u(c_t, E_t) + \alpha^y v(m_t^y) + \delta[u(d_{t+1}, E_{t+1}) + \alpha^o v(m_{t+1}^o)],$$

where $\alpha^y \geq 0$ and $\alpha^o \geq 0$ measure the level of intrinsic motivation for making a voluntary contribution (the strength of warm glow) when young and when old, respectively. Consistent with empirical studies showing that younger individuals are more willing to contribute to environmental protection (see, e.g., Torgler and Garcia-Valinas [42], Wiser [44], Adaman et al. [1]), we suppose that $\alpha^y = \alpha$ and $\alpha^o = \varepsilon\alpha$, with the proportionality factor ε being strictly smaller than 1 (but non-negative). The case $\varepsilon = 0$ corresponds to our baseline model. The last modification to mention here is that since the current old can now make voluntary contributions simultaneously with the current young, the law of motion for environmental quality (1.5) must hold with

$$m_t = m_t^y + m_t^o, \tag{1.29}$$

i.e., m_t in (1.5) is now given by the sum of all voluntary contributions made in period t.

With these modifications, we first derive the state equations for a given environmental policy, analogous to (1.12) and (1.13):

$$K_{t+1} = \frac{\delta(1+\varepsilon\alpha)(1-\tau_t)(1-\nu)AK_t^{\nu}}{1+\alpha+\delta(1+\varepsilon\alpha)}, \tag{1.30}$$

$$E_{t+1} = b\bar{E} + (1-b)E_t + \left\{\gamma\left[\frac{\alpha(1-\tau_t)(1-\nu)}{1+\alpha+\delta(1+\varepsilon\alpha)} + \frac{\varepsilon\alpha\nu(1-\tau_t)}{1+\varepsilon\alpha} + \tau_t\right] - \beta\right\} AK_t^{\nu}. \tag{1.31}$$

Comparing the first equation with (1.12) and noting that $\varepsilon < 1$, it is evident that incorporating voluntary contributions by the old does not change the fact that an increase in the strength of warm glow α discourages capital accumulation for a given pollution tax rate τ_t. At the same time, comparing the second equation with (1.13) reveals that for given τ_t and K_t, an increase in α still leads to an improvement in environmental quality. Because of this, the optimal tax rate chosen by the time-t government remains negatively dependent on α when $\varepsilon > 0$, as demonstrated in the following proposition.[18]

Proposition 1.8. *When the old can contribute to environmental protection privately, similar to the young (i.e., $\varepsilon > 0$), the interior solution to the time-t government's problem is given by*

$$\tau_t = 1 - \frac{b\bar{E} + (1-b)E_t + (\gamma-\beta)AK_t^{\nu}}{\gamma(1+\delta\hat{\lambda})[1 - \frac{\alpha(1-\nu)}{1+\alpha+\delta(1+\varepsilon\alpha)} - \frac{\varepsilon\alpha\nu}{1+\varepsilon\alpha}]AK_t^{\nu}}.$$

Next, by substituting this expression for τ_t into our new state equations (1.30)-(1.31), we obtain

$$K_{t+1} = \frac{\delta(1-\nu)[b\bar{E} + (1-b)E_t + (\gamma-\beta)AK_t^{\nu}]}{\gamma(1+\delta\hat{\lambda})[\delta(1-\nu) + \frac{1-\nu+\nu\delta}{1+\varepsilon\alpha} + \frac{\nu(1+\alpha)}{(1+\varepsilon\alpha)^2}]}, \tag{1.32}$$

$$E_{t+1} = \frac{\delta\hat{\lambda}[b\bar{E} + (1-b)E_t + (\gamma-\beta)AK_t^{\nu}]}{1+\delta\hat{\lambda}}, \tag{1.33}$$

where (1.33) is identical to (1.17), while (1.32) differs from (1.16) by a constant factor. Hence, the long-run behavior of the economy changes only quantitatively.

Proposition 1.9. *Suppose that $\gamma > \beta$ holds. When public environmental policy is optimally chosen by one-period-lived governments, the capital stock K_t and the environmental quality E_t converge to their steady-state values K^{**} and E^{**}, respectively. These steady-*

[18] As before, we assume that warm-glow utility (now of the young and the old) is excluded from the policy objective function.

state values are given by

$$K^{**} = \frac{\delta(1-\nu)[b\bar{E} + (\gamma-\beta)A(K^{**})^{\nu}]}{\gamma(1+\delta\hat{\lambda}b)[\delta(1-\nu) + \frac{1-\nu+\nu\delta}{1+\varepsilon\alpha} + \frac{\nu(1+\alpha)}{(1+\varepsilon\alpha)^2}]},$$

$$E^{**} = \frac{\delta\hat{\lambda}[b\bar{E} + (\gamma-\beta)A(K^{**})^{\nu}]}{1+\delta\hat{\lambda}b}.$$

As in Section 1.4, the effect of increasing the warm-glow strength α on the steady-state value K^{**} has the same sign as its effect on E^{**}. However, this sign is not necessarily negative now. Intuitively, as the old become intrinsically motivated to make voluntary contributions alongside the young, the negative overall impact of voluntary contributions on capital formation decreases since contributions in old age represent an additional saving motive. Furthermore, because contributions by the old directly improve environmental quality, the need for public abatement also decreases, putting downward pressure on the optimal pollution tax rate. Both of these effects work in favor of a higher long-run capital stock and, thus, a higher long-run environmental quality. However, if individuals are sufficiently more motivated to protect the environment when young than when old, that is, the factor ε is sufficiently low, the negative effects of α on K^{**} and E^{**} survive. Specifically, if ε is smaller than $\nu/(1+\nu+\delta\nu)$, the following relationships hold:

$$\frac{\partial E^{**}}{\partial \alpha} < 0 \;\Leftrightarrow\; \frac{\partial K^{**}}{\partial \alpha} < 0 \;\Leftrightarrow\; \alpha < \frac{\nu(1-\varepsilon) - \varepsilon(1+\delta\nu)}{\varepsilon[\nu(1-\varepsilon) + \varepsilon(1+\delta\nu)]}.$$

From this, we can conclude that our main results remain robust to the inclusion of voluntary contributions in old age as long as the strength of warm glow (α) is not too high and the relative importance of voluntary contributions by the old (ε) is sufficiently smaller than 1.

1.5.5 Time Contributions

In the last two subsections, we briefly discuss the robustness of our results to two alternative modeling assumptions: first, that individuals voluntarily allocate part of their time, rather than their income, to protecting the environment, and second, that the government is long-lived rather than short-lived.[19]

We start by discussing the model with time contributions. Here, our approach is similar to that of Ballet et al. [6], while we treat environmental quality as a public good. Suppose each young individual is endowed with one unit of time that she can allocate between working and protecting the environment (for example, sorting domestic waste or volunteering in local environmental organizations). Let $m_t \in [0,1)$ denote the share of time spent by the individual of generation t on environmental protection. Then, her

[19]Both alternative frameworks are studied in detail in Supplementary Appendix 1.B.

first-period budget constraint is given by

$$c_t + s_t = (1 - m_t)w_t, \tag{1.34}$$

where the term $1 - m_t$ represents individual but also aggregate labor supply, which must always be equal to aggregate labor demand, L_t, in equilibrium. Furthermore, we modify the law of motion for environmental quality (1.5) to express individuals' voluntary contributions in terms of their opportunity costs, namely unpaid labor income, given by $m_t w_t$:

$$E_{t+1} = b\bar{E} + (1 - b)E_t - \beta Y_t + \gamma(m_t w_t + g_t), \tag{1.35}$$

where $Y_t = L_t f(k_t) = (1 - m_t)f(K_t/(1 - m_t))$. On the other hand, as before, we assume that individuals derive warm-glow utility directly from m_t, with the parameter $\alpha \geq 0$ measuring the strength of such warm glow.[20]

Solving this alternative model under the specifications in (1.11) used previously, three important features can be noted. First, the share of time spent by individuals of different generations on abatement activities is the same and independent of public environmental policy. Specifically, we obtain $m_t = \alpha/(1 + \alpha + \delta)$ for all $t \geq 0$. Second, for a given policy, the aggregate saving rate (i.e., K_{t+1}/Y_t) does not depend on the presence of voluntary contributions (i.e., does not depend on the parameter α). Due to this effect, in the laissez-faire scenario, an increase in the warm-glow strength α does not affect output per worker in the steady state and unambiguously leads to a higher steady-state environmental quality, even if α is already high. These findings are in contrast to the original model, where the aggregate saving rate was a decreasing function of α (for a given τ_t), and steady-state environmental quality in the laissez-faire case decreased with α if α becomes too high.

Third, and most importantly, under the optimally chosen environmental policy by short-lived governments, steady-state environmental quality is positively related to steady-state output, as in the original model. Moreover, their relationship is described by the same equation as in Proposition 1.3. However, steady-state output is influenced now by a reduced labor supply rather than reduced savings due to voluntary contributions. More precisely, we show in Supplementary Appendix 1.B that the steady-state capital stock is an increasing function of α, implying that aggregate savings are higher in the long run in the presence of voluntary contributions (i.e., when $\alpha > 0$). At the same time, the steady-state levels of output and environmental quality first decline and then increase with α, both reaching their minimum at $\alpha = (1 + \delta)(1 - \nu)$. Thus, if the strength of warm glow remains sufficiently low, its negative effect on long-run environmental quality shown

[20]Equivalently, one can think of the alternative setting in this subsection as the one with endogenous labor supply where individuals derive utility from leisure time, which is spent entirely on abatement activities.

in Section 1.4 remains robust to the alternative assumption that individuals voluntarily contribute their time to environmental protection.[21]

1.5.6 Long-Lived Government

Up to this point, we have assumed that the government is short-lived and cares only about generations that are currently alive. But what if the government also cares about generations that have not yet been born? In this case, the government can be considered long-lived, and the government's problem becomes to choose a sequence of pollution tax rates that maximizes the welfare of current (young and old) generations as well as all future generations. For the solution of this problem to be time-consistent, that is, for the sequence of tax rates chosen in period 0 to continue to be optimal in any subsequent period, one needs to define the objective function properly. Following the existing OLG literature (see, e.g., De la Croix and Michel [15]), we assume that the government in period $t \geq 0$ maximizes

$$\frac{\delta}{\delta_G} u(d_t, E_t) + \sum_{s=0}^{\infty} \delta_G^s [u(c_{t+s}, E_{t+s}) + \delta u(d_{t+s+1}, E_{t+s+1})]$$

or, equivalently,

$$\sum_{s=0}^{\infty} \delta_G^s \left[u(c_{t+s}, E_{t+s}) + \frac{\delta}{\delta_G} u(d_{t+s}, E_{t+s}) \right], \tag{1.36}$$

where $\delta_G \in (0,1)$ is the government's discount factor, which may differ from the individual discount factor δ and reflects how the government values different generations relative to the current young. Note that this objective function is not a more general version of the objective function (1.14) in the baseline model (even if $\mu = \delta/\delta_G$). Therefore, they yield different optimal policies. However, the equilibrium behavior of the economy for a given policy remains unchanged.[22]

Although showing the convergence of the economy to a steady state would be considerably more complicated with (1.36) than with (1.14), the steady state itself is still not difficult to characterize. Once again, using the functional forms in (1.11), we obtain (see Supplementary Appendix 1.B) that if the government maximizes (1.36), the steady-state

[21] Concerning the threshold for α, if $\alpha > (1+\delta)(1-\nu)$, individuals would spend $m_t > (1-\nu)/(2-\nu)$ of their unit of time on (unpaid) environmental protection and the remainder on (paid) work. For $\nu = 1/3$, this implies that an individual working 40 hours per week, for example, would spend more than 26 hours per week on recycling or environmental volunteering, which is unrealistically high. It is therefore unlikely that the level of α exceeds the threshold.

[22] Note also that because the government decides on an intertemporal policy plan, it does not now take future policy as given, contrary to the government in the baseline model.

values of the capital stock K_t and the environmental quality E_t are given by

$$K^{**} = \frac{\delta(1-\nu)[b\bar{E} + (1+\nu\delta_G^2\tilde{\lambda}b)(\gamma-\beta)A(K^{**})^{\nu}]}{\gamma(1+\delta_G\tilde{\lambda}b)(1+\alpha\nu+\delta)}, \tag{1.37}$$

$$E^{**} = \frac{\delta_G\tilde{\lambda}[b\bar{E} + (1-\delta_G\nu)(\gamma-\beta)A(K^{**})^{\nu}]}{1+\delta_G\tilde{\lambda}b}, \tag{1.38}$$

where $\tilde{\lambda} := \lambda/[1-\delta_G(1-b)]$. It follows then that the signs of $\partial K^{**}/\partial\alpha$ and $\partial E^{**}/\partial\alpha$ are the same as we had with (1.14), namely negative ones. The explanation for these negative long-run effects is similar, too, and relies on the negative effect of the warm-glow strength on the aggregate saving rate for a given policy and its negative effect on public abatement. Hence, we conclude that the main results of this paper do not depend on whether the government is short- or long-lived, or put differently, whether the government cares only about the currently living generations or all generations, including those not yet born.

1.6 Conclusion

In the present paper, we develop a two-period OLG model with an environmental public good — environmental quality — where individuals derive warm-glow utility from their voluntary contributions to this public good while the government chooses environmental policy. We are primarily interested in the role of these contributions in improving environmental quality in the long run when they can also influence public policy. In the basic version of the model, we abstract away from possibly different efficiencies between private and public abatement efforts, the possibility that the government can subsidize voluntary contributions or savings, and the possibility that individuals can contribute not only during working age but also during retirement. We examine these extensions successively, along with two alternative assumptions concerning the form of voluntary contributions and the government's planning horizon.

The main result of this paper can be summarized as follows. When public environmental policy is endogenous, i.e., depends on private contributions to environmental protection both directly and indirectly through the state of the economy, an increase in individuals' motivation to contribute privately (the strength of warm glow) will, in the long run, lead to poorer environmental quality. This result remains robust in most extensions, particularly if the government is more efficient in protecting the environment than individuals. Furthermore, the result survives if voluntary contributions are concentrated among the working population rather than retirees.

There are several directions in which future research might proceed. First, since we model private environmental protection as individuals' voluntary contributions to a pure public good, it would be natural to allow for a situation where an environmental public

good is impure (see Kotchen [27] and Kotchen and Moore [28]). This would require considering an economy with two sectors, clean and dirty, similar to that in Nakabayashi [33] or Bezin [8]. Second, one could model abatement in a more sophisticated way than we do in the current paper by allowing for two types of capital, productive and abatement capital (see Xepapadeas [45]). Third, it would be interesting to consider a setting where individuals derive utility from social approval for their voluntary contributions, as in Holländer [18], rather than directly from contributions. Finally, following Jouvet et al. [24], it is important to incorporate intergenerational altruism into the analysis, capturing the trade-off between contributing to an environmental public good and transferring private wealth to children.

1.A Appendix

To prove the results in the main text, we begin by examining a general model that integrates all considered extensions of the basic model and then focus on several special cases.

Consider the equilibrium for a given environmental policy as defined in Section 1.3 and replace conditions (1.1)-(1.2) with

$$c_t + s_t + (1-\theta_t^m)m_t^y = w_t,$$
$$d_{t+1} + (1-\theta_{t+1}^m)m_{t+1}^o = (1+\theta_{t+1}^s)R_{t+1}s_t,$$

conditions (1.3)-(1.4) with

$$\alpha^y v'(m_t^y) = (1-\theta_t^m)u_1'(c_t, E_t),$$
$$\alpha^o v'(m_{t+1}^o) = (1-\theta_{t+1}^m)u_1'(d_{t+1}, E_{t+1}),$$
$$u_1'(c_t, E_t) = \delta(1-\theta_{t+1}^s)R_{t+1}u_1'(d_{t+1}, E_{t+1}),$$

condition (1.5) with (1.21), and condition (1.10) with

$$g_t + \theta_t^m m_t + \theta_t^s R_t K_t = \tau_t Y_t = \tau_t f(K_t)$$

such that $\{\theta_t^m, \theta_t^s\}_{t=0}^{\infty}$ is given and m_t is determined by (1.29). Applying (1.11), we obtain that in this equilibrium, the following relationships hold for all $t \geq 0$:

$$m_t^y = \frac{\alpha^y c_t}{1-\theta_t^m},\ m_t^o = \frac{\alpha^o d_t}{1-\theta_t^m},\ c_t = \frac{(1-\tau_t)(1-\nu)AK_t^\nu}{1+\alpha^y+\delta(1+\alpha^o)},\ d_t = \frac{(1-\theta_t^s)(1-\tau_t)\nu AK_t^\nu}{1+\alpha^o},$$

along with

$$K_{t+1} = s_t = \frac{\delta(1+\alpha^o)(1-\tau_t)(1-\nu)AK_t^\nu}{1+\alpha^y+\delta(1+\alpha^o)} \tag{1.39}$$

and

$$E_{t+1} = b\bar{E}+(1-b)E_t-\beta AK_t^\nu+\gamma_m(m_t^y+m_t^o)+\gamma_g[\tau_t AK_t^\nu-\theta_t^m(m_t^y+m_t^o)-\theta_t^s(1-\tau_t)\nu AK_t^\nu].$$

Next, we generalize the social welfare function (1.14) as

$$\mu u(d_t, E_t) + u(c_t, E_t) + \delta u(d_{t+1}, E_{t+1}) + \phi[\mu\alpha^o v(m_t^o) + \alpha^y v(m_t^y) + \delta\alpha^o v(m_{t+1}^o)],$$

where $\phi \in \{0,1\}$ is an indicator variable for whether the warm-glow utility of (currently living) individuals is included. Then, applying (1.11) once again and using our equilibrium conditions given above, we obtain that social welfare can be expressed as

$$\begin{aligned}
&\mu(1+\phi\alpha^o)\ln(1-\theta_t^s) + [1+\phi\alpha^y+(\mu+\delta\nu)(1+\phi\alpha^o)]\ln(1-\tau_t) \\
&- \phi(\alpha^y+\mu\alpha^o)\ln(1-\theta_t^m) + \delta\lambda\ln E_{t+1} + \Theta(K_t, E_t) \\
&+ \delta[(1+\phi\alpha^o)\ln(1-\tau_{t+1}) - \phi\alpha^o\ln(1-\theta_{t+1}^m) + (1+\phi\alpha^o)\ln(1-\theta_{t+1}^s)],
\end{aligned} \tag{1.40}$$

where E_{t+1} is given by

$$\begin{aligned}
E_{t+1} = b\bar{E} + (1-b)E_t + [\gamma_g\tau_t - \gamma_g\theta_t^s\nu(1-\tau_t) - \beta]AK_t^\nu + (1-\tau_t) \\
\times \left[\gamma_m + \frac{\theta_t^m(\gamma_m-\gamma_g)}{1-\theta_t^m}\right]\left[\frac{\alpha^y(1-\nu)}{1+\alpha^y+\delta(1+\alpha^o)} + \frac{\alpha^o\nu(1-\theta_t^s)}{1+\alpha^o}\right]AK_t^\nu
\end{aligned} \tag{1.41}$$

and the function Θ includes all constant terms and terms involving K_t and E_t, which the time-t government takes as given, along with the policy $(\tau_{t+1}, \theta_{t+1}^m, \theta_{t+1}^s)$ determined by the next-period government.

Case 1: $\alpha^y = \alpha$, $\alpha^o = 0$, $\gamma_m = \gamma_g = \gamma$, and $\theta_t^m = \theta_t^s = 0$ for $t \geq 0$

Using (1.39)-(1.41) and letting $\alpha^y = \alpha$, $\alpha^o = 0$, $\gamma_m = \gamma_g = \gamma$, and $\theta_t^m = \theta_t^s = 0$ for all $t \geq 0$, we now prove the results from Section 1.4. Note that in this case, equations (1.39) and (1.41) reduce to (1.12) and (1.13), respectively. As far the time-t government's problem is concerned, the tax rate τ_t maximizing (1.40) for given τ_{t+1}, K_t, and E_t solves

$$\max_{\tau_t\in[0,1)} (1+\mu+\delta\nu+\phi\alpha)\ln(1-\tau_t) + \delta\lambda\ln E_{t+1},$$

where E_{t+1} is given by (1.13). If we set $\phi = 0$, this problem coincides with (1.15), leading to Proposition 1.2. If ϕ is not necessarily 0, its interior solution is given by

$$\tau_t = 1 - \frac{b\bar{E} + (1-b)E_t + (\gamma - \beta)AK_t^\nu}{\gamma(1 + \frac{\delta\lambda}{1+\mu+\delta\nu+\phi\alpha})[1 - \frac{\alpha(1-\nu)}{1+\alpha+\delta}]AK_t^\nu},$$

which generalizes Proposition 1.2 to the case where the warm-glow utility is included in the social welfare function. Substituting this expression for τ_t into (1.12) and (1.13) further yields

$$K_{t+1} = \frac{\delta(1-\nu)[b\bar{E} + (1-b)E_t + (\gamma - \beta)AK_t^\nu]}{\gamma(1 + \frac{\delta\lambda}{1+\mu+\delta\nu+\phi\alpha})(1 + \alpha\nu + \delta)},$$

$$E_{t+1} = \frac{\delta\lambda[b\bar{E} + (1-b)E_t + (\gamma - \beta)AK_t^\nu]}{1 + \mu + \delta\nu + \phi\alpha + \delta\lambda}.$$

Then, it holds that for all $t \geq 1$,

$$K_{t+1} = \frac{\delta\lambda(1-b)K_t}{1 + \mu + \delta\nu + \phi\alpha + \delta\lambda} + \frac{\delta(1-\nu)[b\bar{E} + (\gamma - \beta)AK_t^\nu]}{\gamma(1 + \frac{\delta\lambda}{1+\mu+\delta\nu+\phi\alpha})(1 + \alpha\nu + \delta)},$$

$$E_t = \frac{\lambda\gamma(1 + \alpha\nu + \delta)K_t}{(1-\nu)(1 + \mu + \delta\nu + \phi\alpha)}.$$

Provided that $\gamma > \beta$ is satisfied, dK_{t+1}/dK_t decreases with K_t and goes to the value that is less than 1 as K_t goes to infinity. Thus, we conclude that for any given initial levels K_0 and E_0, K_t and E_t converge to their steady-state values. To find these values, one needs to solve the pair of equations above for $K^{**} = K_t = K_{t+1}$ and $E^{**} = E_t$. By doing so, we obtain

$$K^{**} = \frac{\delta(1-\nu)[b\bar{E} + (\gamma - \beta)A(K^{**})^\nu]}{\gamma(1 + \frac{\delta\lambda b}{1+\mu+\delta\nu+\phi\alpha})(1 + \alpha\nu + \delta)},$$

$$E^{**} = \frac{\delta\lambda[b\bar{E} + (\gamma - \beta)A(K^{**})^\nu]}{1 + \mu + \delta\nu + \phi\alpha + \delta\lambda b},$$

which proves Proposition 1.3 when $\phi = 0$. The comparative statics of K^{**} and E^{**} with respect to α are

$$\frac{\partial K^{**}}{\partial \alpha} \gtreqless 0 \;\Leftrightarrow\; \frac{\nu(1 + \mu + \delta\nu + \phi\alpha + \delta\lambda b)}{1 + \alpha\nu + \delta} \lesseqgtr \frac{\phi\delta\lambda b}{1 + \mu + \delta\nu + \phi\alpha},$$

$$\frac{\partial E^{**}}{\partial \alpha} \gtreqless 0 \;\Leftrightarrow\; \frac{\nu(1 + \mu + \delta\nu + \phi\alpha + \delta\lambda b)}{1 + \alpha\nu + \delta} \lesseqgtr \phi\left[\frac{\delta\lambda b}{1 + \mu + \delta\nu + \phi\alpha} - \frac{1-\nu}{\nu} - \frac{b\bar{E}}{(\gamma - \beta)\nu A(K^{**})^\nu}\right],$$

leading to equations (1.18) and (1.19) when $\phi = 0$. When $\phi = 1$, the last inequality holds with ">" for any $\alpha \geq 0$ as long as $\partial E^{**}/\partial\lambda > 0$ holds. Indeed, taking the partial derivative of E^{**} with respect to λ, we obtain

$$\frac{\partial E^{**}}{\partial \lambda} \gtreqless 0 \;\Leftrightarrow\; \frac{\delta\lambda b}{1+\mu+\delta\nu+\phi\alpha} \lesseqgtr \frac{1-\nu}{\nu} + \frac{b\bar{E}}{(\gamma-\beta)\nu A(K^{**})^{\nu}},$$

which proves that the inclusion of the warm-glow utility in the social welfare function (i.e., $\phi = 1$) does not change the sign of $\partial E^{**}/\partial\alpha$ if the sign of $\partial E^{**}/\partial\lambda$ is positive. This relationship also proves (1.20) for $\phi = 0$.

Case 2: $\alpha^y = \alpha$, $\alpha^o = 0$, $\phi = 0$, and $\theta_t^m = \theta_t^s = 0$ for $t \geq 0$

Now suppose that $\alpha^y = \alpha$, $\alpha^o = 0$, $\phi = 0$, and $\theta_t^m = \theta_t^s = 0$ for all $t \geq 0$, and show the results from Subsection 1.5.1. Using (1.39)-(1.41) and proceeding as in the previous case, we obtain that equation (1.12) still holds. Furthermore, the time-t government's problem remains as in (1.15), with E_{t+1} being now given by

$$E_{t+1} = b\bar{E} + (1-b)E_t + \left[\frac{\gamma_m\alpha(1-\tau_t)(1-\nu)}{1+\alpha+\delta} + \gamma_g\tau_t - \beta\right] AK_t^{\nu}, \tag{1.42}$$

instead of (1.13). The interior solution to this problem is given by

$$\tau_t = 1 - \frac{b\bar{E} + (1-b)E_t + (\gamma_g-\beta)AK_t^{\nu}}{(1+\frac{\delta\lambda}{1+\mu+\delta\nu})[\gamma_g - \frac{\gamma_m\alpha(1-\nu)}{1+\alpha+\delta}]AK_t^{\nu}},$$

which proves Proposition 1.4. Applying this solution to (1.12) and (1.42) yields (1.22) and (1.23), respectively. It follows then that for all $t \geq 1$,

$$K_{t+1} = \frac{\delta\lambda(1-b)K_t}{1+\mu+\delta\nu+\delta\lambda} + \frac{\delta(1-\nu)[b\bar{E} + (\gamma_g-\beta)AK_t^{\nu}]}{(1+\frac{\delta\lambda}{1+\mu+\delta\nu})[\gamma_g(1+\alpha+\delta) - \gamma_m\alpha(1-\nu)]},$$

$$E_t = \frac{\lambda[\gamma_g(1+\alpha+\delta) - \gamma_m\alpha(1-\nu)]K_t}{(1-\nu)(1+\mu+\delta\nu)},$$

which, by the same argument leading to Proposition 1.3, proves Proposition 1.5.

Case 3: $\alpha^y = \alpha$, $\alpha^o = 0$, $\phi = 0$, and $\theta_t^s = 0$ for $t \geq 0$

Next, suppose that $\alpha^y = \alpha$, $\alpha^o = 0$, $\phi = 0$, and $\theta_t^s = 0$ for all $t \geq 0$, and show the results from Subsection 1.5.2. Using (1.39)-(1.41) and proceeding as before, we obtain that equation (1.12) still holds, while the time-t government's problem becomes to choose $\tau_t \in [0,1)$ and $\theta_t^m \in [0,1)$ that maximize the same objective as in (1.15), with E_{t+1} being

now given by

$$E_{t+1} = b\bar{E} + (1-b)E_t + \left\{ \left[\gamma_m + \frac{\theta_t^m(\gamma_m - \gamma_g)}{1-\theta_t^m} \right] \frac{\alpha(1-\tau_t)(1-\nu)}{1+\alpha+\delta} + \gamma_g \tau_t - \beta \right\} AK_t^\nu.$$

We also impose an additional constraint

$$\theta_t^m \leq \frac{\tau_t(1+\alpha+\delta)}{\alpha(1-\tau_t)(1-\nu) + \tau_t(1+\alpha+\delta)},$$

which is equivalent to $\theta_t^m m_t \leq \tau_t A K_t^\nu$ in equilibrium. Then, by

$$\sigma_t := \frac{\theta_t^m m_t}{\tau_t A K_t^\nu} = \frac{\theta_t^m \alpha(1-\tau_t)(1-\nu)}{\tau_t(1-\theta_t^m)(1+\alpha+\delta)},$$

the above problem can be conveniently expressed as

$$\max_{\tau_t \in [0,1), \sigma_t \in [0,1]} (1+\mu+\delta\nu)\ln(1-\tau_t) + \delta\lambda \ln E_{t+1},$$

where E_{t+1} is given by (1.24). The discussion following equation (1.24) in Subsection 1.5.2 is thus true.

Case 4: $\alpha^y = \alpha$, $\alpha^o = 0$, $\phi = 0$, and $\theta_t^m = 0$ for $t \geq 0$

Suppose that $\alpha^y = \alpha$, $\alpha^o = 0$, $\phi = 0$, and $\theta_t^m = 0$ for all $t \geq 0$, and show the results from Subsection 1.5.3. Using (1.39)-(1.41) and proceeding as before, we obtain that equation (1.12) still holds, while the time-t government's problem becomes to choose $\tau_t \in [0,1)$ and θ_t^s that maximize (1.25), with E_{t+1} being given by (1.26). The interior solution to this problem is given by

$$\theta_t^s = \frac{\mu(1-\nu)[\gamma_g(1+\alpha+\delta) - \gamma_m\alpha]}{\gamma_g\nu(1+\delta\nu)(1+\alpha+\delta)} - 1,$$

$$\tau_t = 1 - \frac{(1+\delta\nu)[b\bar{E} + (1-b)E_t + (\gamma_g - \beta)AK_t^\nu]}{(1+\mu+\delta\nu+\delta\lambda)(\gamma_g - \frac{\gamma_m\alpha}{1+\alpha+\delta})(1-\nu)AK_t^\nu},$$

which proves Proposition 1.6. Applying this solution to (1.12) and (1.26) yields (1.27) and (1.28), respectively. It follows then that for all $t \geq 1$,

$$K_{t+1} = \frac{\delta\lambda(1-b)K_t}{1+\mu+\delta\nu+\delta\lambda} + \frac{\delta(1+\delta\nu)[b\bar{E} + (\gamma_g - \beta)AK_t^\nu]}{(1+\mu+\delta\nu+\delta\lambda)[\gamma_g(1+\alpha+\delta) - \gamma_m\alpha]},$$

$$E_t = \frac{\lambda[\gamma_g(1+\alpha+\delta) - \gamma_m\alpha]K_t}{1+\delta\nu},$$

which, by the same argument leading to Proposition 1.3, proves Proposition 1.7.

Case 5: $\alpha^y = \alpha$, $\alpha^o = \varepsilon\alpha$, $\gamma_m = \gamma_g = \gamma$, $\phi = 0$, and $\theta_t^m = \theta_t^s = 0$ for $t \geq 0$

Finally, suppose that $\alpha^y = \alpha$, $\alpha^o = \varepsilon\alpha$, $\gamma_m = \gamma_g = \gamma$, $\phi = 0$, and $\theta_t^m = \theta_t^s = 0$ for all $t \geq 0$, and show the results from Subsection 1.5.4. Using (1.39)-(1.41) and proceeding as before, we obtain that equation (1.39) becomes (1.30), while the time-t government's problem remains as in (1.15), with E_{t+1} being now given by (1.31). The interior solution to this problem is given by

$$\tau_t = 1 - \frac{b\bar{E} + (1-b)E_t + (\gamma - \beta)AK_t^\nu}{\gamma(1 + \frac{\delta\lambda}{1+\mu+\delta\nu})[1 - \frac{\alpha(1-\nu)}{1+\alpha+\delta(1+\varepsilon\alpha)} - \frac{\varepsilon\alpha\nu}{1+\varepsilon\alpha}]AK_t^\nu},$$

which proves Proposition 1.6. Applying this solution to (1.30) and (1.31) yields (1.32) and (1.33), respectively. It follows then that for all $t \geq 1$,

$$K_{t+1} = \frac{\delta\lambda(1-b)K_t}{1+\mu+\delta\nu+\delta\lambda} + \frac{\delta(1-\nu)[b\bar{E} + (\gamma-\beta)AK_t^\nu]}{\gamma(1 + \frac{\delta\lambda}{1+\mu+\delta\nu})[\delta(1-\nu) + \frac{1-\nu+\nu\delta}{1+\varepsilon\alpha} + \frac{\nu(1+\alpha)}{(1+\varepsilon\alpha)^2}]},$$

$$E_t = \frac{\lambda\gamma K_t}{(1-\nu)(1+\mu+\delta\nu)}\left[\delta(1-\nu) + \frac{1-\nu+\nu\delta}{1+\varepsilon\alpha} + \frac{\nu(1+\alpha)}{(1+\varepsilon\alpha)^2}\right],$$

which, by the same argument leading to Proposition 1.3, proves Proposition 1.9.

1.B Supplementary Appendix

Analysis of the model with time contributions

Suppose that each individual born in period t chooses (m_t, c_t, d_{t+1}) so as to maximize the original lifetime utility function, i.e.,

$$u(c_t, E_t) + \alpha v(m_t) + \delta u(d_{t+1}, E_{t+1}),$$

subject to the budget constraints (1.34) and (1.2). The first-order conditions of this problem are given by (1.4) and

$$\alpha v'(m_t) = w_t u_1'(c_t, E_t).$$

The production side of the economy remains unchanged, while the labor market clearing condition becomes $L_t = 1 - m_t$ (instead of $L_t = 1$), implying that $k_t = K_t/(1-m_t)$ holds for every t.

Taking into account the modified law of motion for environmental quality (1.35) and applying (1.11), the equilibrium for a given policy is now characterized by

$$m_t = \frac{\alpha}{1+\alpha+\delta},\ c_t = \frac{(1-\tau_t)(1-\nu)Ak_t^{\nu}}{1+\alpha+\delta},\ d_t = (1-\tau_t)\nu Ak_t^{\nu},$$

along with

$$K_{t+1} = s_t = \frac{\delta(1-\tau_t)(1-\nu)Ak_t^{\nu}}{1+\alpha+\delta} = \frac{\delta(1-\tau_t)(1-\nu)Y_t}{1+\delta},$$
$$E_{t+1} = b\bar{E} + (1-b)E_t - \beta Y_t + \gamma\left[\frac{\alpha(1-\tau_t)(1-\nu)}{1+\delta} + \tau_t\right]Y_t,$$

where $Y_t = (1-m_t)Ak_t^{\nu} = A(K_t)^{\nu}(1-m_t)^{1-\nu}$. Then, in the laissez-faire scenario (i.e., $\tau_t = 0$ for all $t \geq 0$), the steady-state values of the capital stock and environmental quality are given by

$$K^* = \frac{1+\delta}{1+\alpha+\delta}\left[\frac{\delta A(1-\nu)}{1+\delta}\right]^{\frac{1}{1-\nu}},$$
$$E^* = \bar{E} + \frac{1}{b}\left[\frac{\gamma\alpha(1-\nu)}{1+\delta} - \beta\right]\frac{A(1+\delta)}{1+\alpha+\delta}\left[\frac{\delta A(1-\nu)}{1+\delta}\right]^{\frac{\nu}{1-\nu}}.$$

Note that K^* depends on the warm-glow strength α negatively, whereas E^* depends on it positively.

Next, keeping the assumptions that the time-t government maximizes (1.14) and that it correctly anticipates next-period's values of all variables, including m_{t+1}, but takes the value of τ_{t+1} as given, the time-t government's problem can be expressed exactly as (1.15) under the specifications in (1.11).[23] This leads to the following interior solution:

$$\tau_t = \frac{1}{\beta} - \frac{b\bar{E} + (1-b)E_t + (\gamma-\beta)Y_t}{\beta\gamma(1+\frac{\delta\lambda}{1+\mu+\delta\nu})[1-\frac{\alpha(1-\nu)}{1+\delta}]Y_t}.$$

Substituting this expression for τ_t into the above expressions for K_{t+1} and E_{t+1} yields

$$K_{t+1} = \frac{\delta(1-\nu)[b\bar{E} + (1-b)E_t + (\gamma-\beta)Y_t]}{\gamma(1+\frac{\delta\lambda}{1+\mu+\delta\nu})[1+\delta-\alpha(1-\nu)]},$$
$$E_{t+1} = \frac{\delta\lambda[b\bar{E} + (1-b)E_t + (\gamma-\beta)Y_t]}{1+\mu+\delta\nu+\delta\lambda}.$$

[23] In fact, one could also include the warm-glow utility in (1.14), without any effects on our results.

Thus, in the steady state, where $K_{t+1} = K_t = K^{**}$, $E_{t+1} = E_t = E^{**}$, and $Y_{t+1} = Y_t = Y^{**}$, we have

$$K^{**} = \frac{\delta(1-\nu)[b\bar{E} + (\gamma-\beta)Y^{**}]}{\gamma(1+\frac{\delta\lambda b}{1+\mu+\delta\nu})[1+\delta-\alpha(1-\nu)]},$$

$$E^{**} = \frac{\delta\lambda[b\bar{E} + (\gamma-\beta)Y^{**}]}{1+\mu+\delta\nu+\delta\lambda b}.$$

with

$$Y^{**} = A(K^{**})^{\nu}\left(\frac{1+\delta}{1+\alpha+\delta}\right)^{1-\nu}.$$

Finally, we note that as long as $\gamma > \beta$ and $1+\delta > \alpha(1-\nu)$ hold, the partial derivatives of K^{**}, E^{**}, and Y^{**} with respect to α are such that

$$\frac{\partial K^{**}}{\partial \alpha} > 0,$$

i.e., the capital stock and, hence, aggregate savings increase with the warm-glow strength in the steady state, whereas

$$\frac{\partial E^{**}}{\partial \alpha} \gtreqless 0 \;\Leftrightarrow\; \frac{\partial Y^{**}}{\partial \alpha} \gtreqless 0 \;\Leftrightarrow\; \alpha \gtreqless (1+\delta)(1-\nu),$$

i.e., steady-state environmental quality and steady-state aggregate output first decrease and then increase with the warm-glow strength.

Analysis of the model with a long-lived government

Suppose that the objective function of the government is given by equation (1.36). The government's problem is then to choose a sequence of tax rates $\{\tau_t\}_{t=0}^{\infty}$ that, given an initial state (K_0, E_0), maximizes

$$\sum_{t=0}^{\infty} \delta_G^t \left[u(c_t, E_t) + \frac{\delta}{\delta_G} u(d_t, E_t)\right]$$

subject to the equilibrium conditions (1.1)-(1.10) holding for every $t \geq 0$. The solution to this problem leads to the same equilibrium as if the government chooses the allocation

$\{m_t, c_t, d_t, K_{t+1}, E_{t+1}\}_{t=0}^{\infty}$ so as to maximize the above objective subject to

$$\begin{aligned} f(K_t) &= c_t + d_t + m_t + g_t + K_{t+1}, \\ E_{t+1} &= b\bar{E} + (1-b)E_t - \beta f(K_t) + \gamma(m_t + g_t), \\ \alpha v'(m_t) &= u_1'(c_t, E_t), \\ u_1'(c_t, E_t)K_{t+1} &= \delta u_1'(d_{t+1}, E_{t+1})d_{t+1}, \end{aligned}$$

where g_t is given by

$$g_t = f(K_t) - \frac{d_t f(K_t)}{f'(K_t)K_t}.$$

The optimal pollution tax rate in period t then can be computed using

$$\tau_t = 1 - \frac{d_t}{f'(K_t)K_t}.$$

Under (1.11), this latter problem simplifies to[24]

$$\max_{\{K_{t+1}, E_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \delta_G^t \left(\ln K_{t+1} + \delta_G \lambda \ln E_{t+1}\right)$$

subject to

$$E_{t+1} = b\bar{E} + (1-b)E_t + (\gamma - \beta)AK_t^{\nu} - \frac{\gamma(1 + \alpha\nu + \delta)}{\delta(1-\nu)}K_{t+1}.$$

Denoting the Lagrange multiplier associated with the constraint by ε_t, we obtain the following first-order conditions:

$$\begin{aligned} \frac{\delta_G^t}{K_{t+1}} &= \frac{\gamma(1 + \alpha\nu + \delta)\varepsilon_t}{\delta(1-\nu)} - \nu(\gamma - \beta)AK_{t+1}^{\nu-1}\varepsilon_{t+1}, \\ \frac{\delta_G^{t+1}\lambda}{E_{t+1}} &= \varepsilon_t - (1-b)\varepsilon_{t+1}. \end{aligned}$$

[24]Note that for the short-lived government, which takes the future tax rates as given, the analogous problem would differ only in the objective function. This function would be written as

$$(1 + \mu + \delta\nu)\ln K_{t+1} + \delta\lambda \ln E_{t+1}.$$

In the steady state, where $K_{t+1} = K_t = K^*$ and $E_{t+1} = E_t = E^{**}$, we must have $\varepsilon_t \delta_G = \varepsilon_{t+1}$ and therefore

$$\frac{1}{K^{**}} = \frac{\varepsilon_t}{\delta_G^t}\left[\frac{\gamma(1+\alpha\nu+\delta)}{\delta(1-\nu)} - \nu(\gamma-\beta)A(K^{**})^{\nu-1}\delta_G\right],$$
$$\frac{\delta_G\lambda}{E^{**}} = \frac{\varepsilon_t}{\delta_G^t}[1-\delta_G(1-b)].$$

Taking into account the constraint, this leads to (1.37)-(1.38).

Bibliography

[1] Adaman, F., Karalı, N., Kumbaroğlu, G., Or, I., Özkaynak, B., Zenginobuz, Ü., 2011. What determines urban households' willingness to pay for CO_2 emission reductions in Turkey: A contingent valuation survey. Energy Policy 39, 689-698.

[2] Aghion, P., Bénabou, R., Martin, R., Roulet, A., 2023. Environmental preferences and technological choices: Is market competition clean or dirty? American Economic Review: Insights 5, 1-19.

[3] Ambec, S., De Donder, P., 2022. Environmental policy with green consumerism. Journal of Environmental Economics and Management 111, 102584.

[4] Andreoni, J., 1989. Giving with impure altruism: Applications to charity and Ricardian equivalence. Journal Political Economy 97, 1447–1458.

[5] Andreoni, J., 1990. Impure altruism and donations to public goods: A theory of warm-glow giving. Economic Journal 100, 464-477.

[6] Ballet, J., Bazin, D., Lioui, A., Touahri, D., 2007. Green taxation and individual responsibility. Ecological Economics 63, 732-739.

[7] Bezin, E., 2015. A cultural model of private provision and the environment. Journal of Environmental Economics and Management 71, 109-124.

[8] Bezin, E., 2019. The economics of green consumption, cultural transmission and sustainable technological change. Journal of Economic Theory 181, 497-546.

[9] Bolderdijk, J.W., Steg, L., Geller, E.S., Lehman, P.K., Postmes, T., 2013. Comparing the effectiveness of monetary versus moral motives in environmental campaigning. Nature Climate Change 3, 413-416.

[10] Capstick, S., Khosla, R., Wang, S., van den Berg, N., Ivanova, D., Otto, I.M., Gore, T., Corner, A., Akenji, L., Hoolohan, C., Power, K., 2020. Bridging the gap — the role of equitable low-carbon lifestyles. UNEP Emissions Gap Report 2020, 62-75.

[11] Chan, N.W., 2024. Pigouvian policies under behavioral motives. Journal of the Association of Environmental and Resource Economists 11, 97-135.

[12] Crumpler, H., Grossman, P.J., 2008. An experimental test of warm glow giving. Journal of Public Economics 92, 1011-1021.

[13] Dam, L., Heijdra, B.J., 2011. The environmental and macroeconomic effects of socially responsible investment. Journal of Economic Dynamics and Control 35, 1424-1434.

[14] Daube, M., Ulph, D., 2016. Moral behaviour, altruism and environmental policy. Environmental and Resource Economics 63, 505-522.

[15] De la Croix, D., Michel, P., 2002. A Theory of Economic Growth: Dynamics and Policy in Overlapping Generations. Cambridge University Press.

[16] Diamond, P., 2006. Optimal tax treatment of private contributions for public goods with and without warm-glow preferences. Journal of Public Economics 90, 897–919.

[17] Herweg, F., Schmidt, K.M., 2022. How to regulate carbon emissions with climate-conscious consumers? Economic Journal 132, 2992-3019.

[18] Holländer, H., 1990. A social exchange approach to voluntary cooperation. American Economic Review 80, 1157-1167.

[19] IPCC, 2018. Global Warming of 1.5°C: An IPCC Special Report. Cambridge University Press.

[20] ISSP, 2019. International Social Survey Programme: Environment III — ISSP 2010. GESIS Data Archive, Cologne.

[21] ISSP, 2023. International Social Survey Programme: Environment IV — ISSP 2020. GESIS, Cologne.

[22] John, A., Pecchenino, R., 1994. An overlapping generations model of growth and the environment. Economic Journal 104, 1393-1410.

[23] John, A., Pecchenino, R., Schimmelpfennig, D., Schreft, S., 1995. Short-lived agents and the long-lived environment. Journal of Public Economics 58, 127-141.

[24] Jouvet, P.A., Michel, P., Vidal, J.P., 2000. Intergenerational altruism and the environment. Scandinavian Journal of Economics 102, 135-150.

[25] Karp, L., Rezai, A., 2014. The political economy of environmental policy with overlapping generations. International Economic Review 55, 711-733.

[26] Kaufmann, M., Andre, P., Kőszegi, B., 2024. Understanding markets with socially responsible consumers. Quarterly Journal of Economics 139, 1989–2035.

[27] Kotchen, M.J., 2006. Green markets and private provision of public goods. Journal of Political Economy 114, 816-834.

[28] Kotchen, M.J., Moore, M. R., 2007. Private provision of environmental public goods: Household participation in green-electricity programs. Journal of Environmental Economics and Management 53, 1-16.

[29] Ma, C., Burton, M., 2016. Warm glow from green power: Evidence from Australian electricity consumers. Journal of Environmental Economics and Management 78, 106-120.

[30] Mariani, F., Pérez-Barahona, A., Raffin, N., 2010. Life expectancy and the environment. Journal of Economic Dynamics and Control 34, 798-815.

[31] Marini, M.A., Tarola, O., Thisse, J.F., 2022. When is environmentalism good for the environment? Environmental and Resource Economics 82, 1-28.

[32] Menges, R., Schroeder, C., Traub, S., 2005. Altruism, warm glow and the willingness-to-donate for green electricity: An artefactual field experiment. Environmental and Resource Economics 31, 431-458.

[33] Nakabayashi, M., 2010. Optimal tax rules and public sector efficiency with an externality in an overlapping generations model. Journal of Public Economics 94, 1028-1040.

[34] Nyborg, K., Howarth, R.B., Brekke, K.A., 2006. Green consumers and public policy: On socially contingent moral motivation. Resource and Energy Economics 28, 351-366.

[35] OECD, 2023. How Green is Household Behaviour? OECD Studies on Environmental Policy and Household Behaviour, OECD Publishing, Paris.

[36] Ono, T., 2005. The political economy of environmental taxes with an aging population. Environmental and Resource Economics 30, 165-194.

[37] Perino, G., 2015. Climate campaigns, cap and trade, and carbon leakage: Why trying to reduce your carbon footprint can harm the climate. Journal of the Association of Environmental and Resource Economists 2, 469-495.

[38] Prieur, F., 2009. The environmental Kuznets curve in a world of irreversibility. Economic Theory 40, 57-90.

[39] Ribar, D.C., Wilhelm, M.O., 2002. Altruistic and joy-of-giving motivations in charitable behavior. Journal of Political Economy 110, 425-457.

[40] Schumacher, I., 2015. The endogenous formation of an environmental culture. European Economic Review 76, 200–221.

[41] Taufik, D., Bolderdijk, J.W. and Steg, L., 2015. Acting green elicits a literal warm glow. Nature Climate Change 5, 37-40.

[42] Torgler, B., Garcia-Valiñas, M.A., 2007. The determinants of individuals' attitudes towards preventing environmental damage. Ecological Economics 63, 536-552.

[43] Wichman, C.J., 2016. Incentives, green preferences, and private provision of impure public goods. Journal of Environmental Economics and Management 79, 208-220.

[44] Wiser, R.H., 2007. Using contingent valuation to explore willingness to pay for renewable energy: A comparison of collective and voluntary payment vehicles. Ecological Economics 62, 419-432.

[45] Xepapadeas, A., 2005. Economic growth and the environment. In: Mäler, K.-G., Vincent, J.R. (Eds.). Handbook of Environmental Economics. Elsevier, Ch. 23.

Chapter 2

Climate Policy Under Political Pressure[1]

2.1 Introduction

The protection of the environment is a high priority for many governments these days. Being concerned about severe damages induced by climate change in the near future, they are actively seeking to design policies that can reduce greenhouse gas emissions. However, despite increased awareness and intensive discussions, the environmental policy implemented by governments tends to be less ambitious than their promised policy and far from sufficient to reach their long-term climate targets. According to the recent UNEP report (UNEP [58]), with the currently implemented policies, half of the G20 countries, including the United States, are unlikely to meet their original commitments under the Paris Agreement.

Available evidence suggests that one of the factors playing a significant role in the implementation of environmental policies is political pressure. Political pressure takes different forms and may have different objectives. The last UN Climate Change Conference COP27 saw an unprecedented number of corporate lobbyists from the oil and gas industry, who tried to prevent binding agreements on the ban of fossil fuel technologies. Indeed, corporate lobbying has risen considerably over the past two decades and is now the dominant force in political interest intermediation (Hanegraaf and Poletti [34]). To protect their industries, lobby groups successfully exert pressure to reduce the stringency of environmental regulations.[2] Furthermore, the behavior of policymakers is subject to

[1] Joint work with Gerhard Sorger. The chapter has been published in the Journal of Environmental Economics and Management.

[2] For example, Anger et al. [6] have shown, using data on energy taxes in Germany, that industries that are more exposed to regulation and have more powerful lobbies face lower taxes. Similarly, based on US data, Pacca et al. [48] have indicated that lobbying pressure from the polluting industries causes governors to deviate from their own political preferences and allocate fewer resources to environmental preservation. See also Damania et al. [19], Fredriksson and Svensson [26], and Fredriksson and Vollebergh [27], among

political pressure from the public. Governments nowadays face growing public concerns over climate change and yet important climate policies are often opposed by the public.[3] Taking the form of mass demonstrations (e.g., the Yellow Vests protests in France), such opposition has to be addressed by governors if they want to maintain their legitimacy.

Motivated by these facts, we propose a theoretical framework where the government is subject to political pressure when controlling emissions and investment in clean technology. We interpret political pressure broadly to include interest groups or lobbies, but also the power of the street and other possible means of influence. A key feature of our model is that as a result of this pressure, the preferences of the government become dynamically inconsistent.

Dynamic inconsistency is a particularly relevant and plausible characteristic of climate policies because they involve a very long time horizon. Casual observations reveal many instances where long-term climate policies were not implemented as initially planned but were either modified or canceled altogether due to political influence of various sorts. Both at the national and international levels, policymakers give in to political pressure by taking the concerns of protesters and industry lobbyists into account, at least to a certain extent. At the same time, governments continue to emphasize the importance of their long-run goals such as effective limitation of global warming and future economic prosperity. But when the future becomes the present, new political pressure makes governments bend again and deviate from previously announced policies.[4]

Our paper relates to a large literature on environmental regulation and dynamic inconsistency. One of the most well-studied sources of dynamic inconsistency in this literature is non-constant discounting. Starting with Cropper and Laibson [18], who discussed the application of hyperbolic discounting to environmental projects, researchers have used a non-constant discount rate to examine its role in incentive-based and command-and-control environmental regulations (Heutel [40]), resource management (Hepburn et al. [39]), and, most importantly, in climate change policy (Karp [44], Fujii and Karp [28],

others, for empirical evidence on the negative dependence of the stringency of environmental policy on corruption, which is usually considered to enhance the influence of lobby groups.

[3]See the work of Dechezlepretre et al. [20] who have conducted a survey covering twenty countries and found that such policies as a carbon tax with cash transfers or a tax on fossil fuels/flying/cattle products are opposed by a large number of people. For further evidence of people's opposition to environmental policies, see Sælen and Kallbekken [55], Cherry et al. [14], and Douenne and Fabre [22].

[4]A good illustration of the situation described is climate policy in Australia. After the drought in the early 2000s, the Australian government promised to implement an emissions trading scheme by 2010. In 2009, the implementation was postponed until after 2012. In 2010, however, the government announced a new initiative, namely a carbon pricing scheme, which was implemented in 2011 but repealed already in 2014. In replacement, the government introduced another climate policy that paid businesses and communities to reduce their emissions. In 2015, Australia signed up for the Paris Agreement, and the same year the government proposed a plan for integrating climate and energy policy, but it was not realized. Following the bushfire season in 2019, the government announced that climate policy could change again.

Karp and Tsur [45]).[5] Another strand of this literature views dynamic inconsistency as a problem arising from interactions between an environmental regulator and regulated entities such as firms (Biglaiser et al. [11], Gersbach and Glazer [30], Abrego and Perroni [1], Petrakis and Xepapadeas [51], Helm et al. [38], Acemoglu and Rafey [3], Chiappinelli and May [15]), consumers (Marsiliani and Renstrom [47]), or individual countries (Pichler and Sorger [53]). It is also worth mentioning here the work of Ulph and Ulph [57], where the source of dynamic inconsistency is uncertainty about environmental concerns of the future government, and the works of Harstad [37] and Battaglini [9], where dynamic inconsistency in environmental policy is generated by a stochastic rotation of political power.

In the present paper, we consider an alternative source of dynamic inconsistency, which relies on the presence of political pressure. Our approach is nevertheless sufficiently flexible to capture the effects of non-constant (in particular, quasi-hyperbolic) discounting. Indeed, our model implies that the pressure from myopic individuals on the government has the same policy implications as the assumption that the government itself is present-biased. Moreover, when slightly extended, our model can capture uncertainty about the current government's survival in power along the lines suggested in Harstad [37] and Battaglini [9].

The only study we are aware of that deals with dynamic inconsistency in environmental policy caused by the presence of lobbying or other political pressure is Kalkuhl et al. [43]. Similar to our framework, the model of Kalkuhl et al. [43] assumes that lobbying makes the government's objective different from social welfare and more in line with the interests of lobby groups.[6] However, their model cannot capture that the preferences of the government evolve over time, nor can it distinguish between short- and long-run effects of lobbying, in contrast to our model. In addition, whereas Kalkuhl et al. [43] assume that lobbying power is exogenous throughout their analysis, our model also allows for endogenous lobbying power (intensity).

Our analysis lends itself to both positive and normative interpretations.[7] On the one hand, we assume that the government cannot commit to future policies, capturing the realistic feature that the term of office of the incumbent government is limited. Yet, the

[5] As in our study, the analysis in all these papers abstracts from general equilibrium effects and employs a simplified description of environmental processes. In Gerlagh and Liski [29], the socially optimal climate policies are analyzed in a general-equilibrium growth model with quasi-hyperbolic discounting and detailed climate dynamics. Iverson and Karp [42] consider an even more general setting allowing for general discounting and general functional forms.

[6] As noted by Kalkuhl et al. [43], this assumption is justified by the result of the standard static lobbying model proposed by Grossman and Helpman [31]. It says that the political equilibrium with lobbying can be characterized as a solution to maximizing social welfare plus the utility of lobby groups (with given weights between the two terms). A more direct application of the Grossman and Helpman [31] model to environmental politics has been developed in Schopf and Voss [56] and Voss and Schopf [59], where the authors examine a resource extraction problem.

[7] See Karp [44] for the same feature.

government still cares about the policies implemented after leaving office, which can also be a reasonable reflection of the actual preferences of climate policymakers. On the other hand, we examine how the government optimally responds to various political pressures when determining its policies to increase current and future social welfare. In this sense, our analysis is normative, and it can serve as a basis for policy recommendations made by an environmental agency for politicians when they face a specific type of pressure.

Examining several examples of political pressure in an economy with two energy sectors (clean and dirty), we show that political pressure from the dirty energy lobby increases the emission rate chosen by the government. Interestingly, the government chooses a higher level of investment in clean technology, too, in an effort to counteract the increased environmental damage. This policy leads to a rise in the steady-state amount of clean energy produced and can even enhance the steady-state share of the total energy produced by clean technology. However, it cannot prevent an increase in steady-state emissions due to such pressure. The same positive long-run impact on clean energy production arises when the government is subject to pressure for higher energy consumption. Moreover, we show that this pressure, if strong enough, reduces the production of dirty energy in the steady state and the steady-state stock of greenhouse gases. Furthermore, our analysis demonstrates that facing pressure to increase investment in clean technology, the government chooses a higher rate not only of investment but also of emissions. In fact, this type of political pressure reinforces the effects of lobbying by the dirty energy sector, except for the negative effect of the former on the long-run stock of greenhouse gases.

These results contribute to the literature on the influence of lobbying on environmental regulation. Our contribution here is twofold. First, we offer a new rationale for why green lobbying can make pollution regulation less stringent. Such a possibility has been shown by Conconi [17] and Aidt [4], using static two-country models. In contrast to them, we use a single-country setting and provide a dynamic argument. Second, to the best of our knowledge, we are the first paper that examines the effect of lobbying on the regulation of investments in clean technology.[8] We also contribute to the literature on the transition to clean technologies (see, e.g., Acemoglu et al. [2] and Rozenberg et al. [54]) by arguing that the energy mix can be shifted towards renewables in the long run due to political pressure from the dirty sector or from members of the public who care only about immediate consumption. The recent works of Grey [32] similarly shows that lobbying from polluting firms can induce a switch to an economy with cleaner production. His reasoning, however, focuses on the competition for market share between firms, while we highlight the role of the dynamically inconsistent preferences of the government.

From a more policy-oriented perspective, our results are related to the approach to climate policy suggested by Pahle et al. [49], according to which the government facing

[8]Note, though, that the question of how investment in clean technology or pollution abatement affects lobbying has been addressed in the literature (see Fredriksson [25] and Grey [32]).

political barriers should choose policies that lay the foundations for a shift to more efficient policies in the future. Consistent with this line of reasoning, our model suggests that the government facing political pressure that leads to higher emissions should promote investment in clean technologies that allows future policymakers to rely less on polluting ones for economic activity. To the extent that real-world governments are forward-looking and respond optimally to political pressure, our model can thus explain why countries with powerful business associations, such as the US, extend tax credits for renewable energy instead of restricting fossil fuel industries (Downie [23]), or why countries with strong social movements, such as France, build infrastructure for electric vehicles instead of increasing fuel taxes (Haidar and Rojas [33]).

The rest of the paper is organized as follows. The next section presents our basic framework and defines the optimal climate policy selected by the government under political pressure. In Section 2.3, we illustrate our approach by applying this framework to several examples of political pressure. In Section 2.4, we further explore three extensions of the basic model. The last section concludes the paper and outlines directions for future research.

2.2 Model Formulation

The current section describes the basic model that we use to illustrate the implications of political pressure for climate policy. We adopt the framework of Harstad [35, 36]. Whereas the focus of these papers is on the structure of climate agreements between several countries, we want to highlight a new potential source of dynamic inconsistency that would arise in every single state. We therefore restrict our attention to a single-country version of Harstad's model.[9]

2.2.1 Energy Production

Time is discrete and indexed by $t \in \{0, 1, \dots\}$. In each period t, the economy produces energy, which cannot be stored and hence is entirely consumed within that period. Energy can be produced by two technologies: one that emits greenhouse gases as by-products and one that does not cause greenhouse gas emissions. We will refer to the first technology and energy produced by this technology as "dirty" and to the second technology and energy produced by it as "clean". One can think of dirty technology as being based on the combustion of fossil fuels, while clean technology involves the conversion of solar, nuclear, water, or wind power into electricity.

[9] Extending the model to a multi-country setting goes beyond the scope of this paper and is left for future research.

We assume that the relationship between emissions and energy generated by dirty technology is linear and choose the units of measurement so that one unit of greenhouse gases is emitted per unit of energy that is produced. The amount of energy produced by dirty technology in period t is denoted by g_t. The stock of greenhouse gases then evolves according to

$$G_{t+1} = \gamma G_t + g_t, \tag{2.1}$$

where G_t denotes the stock of greenhouse gases in the atmosphere at the beginning of period t and $1 - \gamma \in (0, 1)$ is the rate of their natural decay.

It is assumed that the infrastructure for dirty technology is already in place, that it does not depreciate over time, and that there is no investment in this infrastructure.[10] In contrast, clean energy production requires new investment (e.g., the construction of windmills). The stock of technology for clean energy production that is available at the beginning of period t is denoted by R_t and the investment in this stock during period t is denoted by r_t. Consequently, it holds that

$$R_{t+1} = \rho R_t + r_t, \tag{2.2}$$

where $1 - \rho \in (0, 1)$ captures depreciation of the technology stock. For clean technology, we also assume linearity. More specifically, one unit of energy can be produced per period by each unit of clean technology. Thus, the total energy production in the economy in period t is equal to $g_t + R_t$. Both initial stocks G_0 and R_0 are exogenously given.

Alternatively, as pointed out by Harstad [35], the variable R_t can be interpreted as the stock of abatement technology available in period t. According to this interpretation, there is only one technology available for energy production, which allows the generation of y_t units of energy with $g_t = y_t - R_t$ units of emissions. We shall therefore occasionally refer to the technology for clean energy production as the abatement technology.

2.2.2 Social Welfare

Social welfare in period t depends positively on energy consumption $g_t + R_t$ and negatively on the investment rate r_t and the greenhouse gas concentration G_t. Following Harstad [35, 36] and subsequent papers such as Pichler and Sorger [53], we use an additively separable welfare function of the form

$$u(g_t + R_t, r_t, G_t) = B(g_t + R_t) - K(r_t) - D(G_t), \tag{2.3}$$

[10] This assumption is also made in Harstad [35, 36] and in the literature building on these papers, but it is not innocuous. Kollenbach [46], for example, has introduced a "black capacity" necessary to extract and burn fossil fuels in the related setting of Battaglini and Harstad [10] and has shown that this affects some of the results. For the sake of analytical tractability, however, we stick to the simpler setting of Harstad [35, 36] in the present paper.

where B is a concave benefit function and K and D are convex cost functions. Total welfare from period t onwards is the discounted sum of per-period welfare, i.e.,

$$u(g_t + R_t, r_t, G_t) + \sum_{s=1}^{+\infty} \delta^s u(g_{t+s} + R_{t+s}, r_{t+s}, G_{t+s}), \tag{2.4}$$

where $\delta \in (0, 1)$ is the time-preference factor applied by the government.[11]

Under the assumptions made so far, the government would maximize (2.4) subject to the state equations (2.1) and (2.2). Since the discount rate is constant, the preferences represented by (2.4) are dynamically consistent and the government solves a stationary discounted dynamic programming problem. Its solution can be conveniently described by means of policy functions, that is, in the form $g_t = \phi(G_t, R_t)$ and $r_t = \psi(G_t, R_t)$. In the next subsection, we shall introduce a key element of our model, namely political pressure, that makes the government's preferences dynamically inconsistent.

2.2.3 Political Pressure

As argued in the Introduction, climate policymakers give in to political pressure today (at least to some extent) while promising stringent rules and measures for the future. We take this observation as an indication of dynamically inconsistent behavior and model it by assuming that in period t, the government chooses the control sequence $(g_{t+s}, r_{t+s})_{s=0}^{+\infty}$ so as to maximize

$$v(g_t, r_t, G_t, R_t) + \sum_{s=1}^{+\infty} \delta^s u(g_{t+s} + R_{t+s}, r_{t+s}, G_{t+s}), \tag{2.5}$$

where

$$v(g, r, G, R) = u(g + R, r, G) + pw(g, r, G, R). \tag{2.6}$$

Here, the function w reflects the preferences of various interest groups (protesters, industry lobbyists, etc.) and $p \geq 0$ measures how effective these groups are in aligning the government's preferences with their own ones. In other words, the difference between the instantaneous welfare function u and its biased version v reflects the influence of interest groups on the government.[12] Note that the preferences of interest groups are myopic. This reflects the real-world situation in which political pressure is typically exerted with

[11] In fact, there is a small difference between our model formulation and the one used by Harstad [35, 36]. Harstad also postulates a welfare function of the form $u(g_t + R_t, r_t, G_t)$ but, instead of equations (2.1) and (2.2), assumes $G_t = \gamma G_{t-1} + g_t$ and $R_t = \rho R_{t-1} + r_t$. This means, in particular, that in Harstad's model, investment decisions affect clean energy production immediately, i.e., during the period of investment. Which timing assumption is more realistic depends on the period length and the time needed to construct new infrastructure. See Eichner and Kollenbach [24] for a discussion of time lags of emissions and real-world clean investments.

[12] In our basic model, we consider only a single interest group and assume that its influence is exogenous and constant over time. Later in Section 2.4, we allow for a state-dependent (and hence time-varying)

certain short-term objectives (to block a particular policy reform, for instance), even if interest groups may be long-lived. Furthermore, this is equivalent to assuming that the future actions of the government are not taken into account by interest groups because they know that the government cannot commit to those actions, as we specify below.[13]

The presence of political pressure makes the government's preferences non-stationary, i.e., dynamically inconsistent. In period t, the government evaluates its actions in period $t+s$ according to the welfare function $u(g_{t+s} + R_{t+s}, r_{t+s}, G_{t+s})$, but when period $t+s$ arrives, it evaluates these actions according to $v(g_{t+s}, r_{t+s}, G_{t+s}, R_{t+s})$. That said, the government's *time* preferences are stationary since they are represented by the constant discount factor δ. It is also worth stressing that though the government may anticipate that the interest groups will exert pressure in future periods, its period-t preferences described by (2.5) are independent of future pressure. One natural interpretation of this assumption is that the term of office of the incumbent government is limited and, for the time after their tenure, its members care only about social welfare as they are no longer the target of political pressure. Thus, the government in our model is purely benevolent to *future generations*, while it has biased preferences over the current period.[14]

Because the preferences of the government exhibit dynamic inconsistency, one has to be precise about (i) whether it can commit to its future actions and (ii) how it behaves in case such commitment is not possible. Throughout, we assume that there is no commitment and we follow the so-called "sophisticated approach" in modeling behavior without commitment. This approach goes back to the work of Phelps and Pollak [52] and Peleg and Yaari [50] and envisions the decision maker as a collection of different "selves", one for each period, who play a dynamic intrapersonal game. Every self uses a stationary Markov strategy by choosing its actions depending on the current state of the economy only and correctly anticipates its descendants to act according to their own (biased) preferences. The current self of the government takes its future incarnations' strategies as given and chooses its actions by best-responding to those strategies. Formally, the solution to the government's problem is defined as follows.

Definition 2.1. *A pair of policy functions (ϕ, ψ) is a solution to the government's problem if there exists a continuation value function U such that for every state (G, R), it holds that*

$$(\phi(G,R), \psi(G,R)) = \underset{(g,r)}{\operatorname{argmax}}\ v(g, r, G, R) + \delta U(\gamma G + g, \rho R + r) \tag{2.7}$$

intensity of pressure and treat it as a decision variable of the interest group. We will also consider the case of multiple interest groups.

[13] In the environmental context, the assumption of a myopic lobby can be also found in Barbier [7].

[14] In Subsection 2.4.2, we drop this assumption and allow for possible bias in the government's preferences over future periods as well.

and

$$U(G,R) = u(\phi(G,R) + R, \psi(G,R), G) + \delta U(\gamma G + \phi(G,R), \rho R + \psi(G,R)). \tag{2.8}$$

Condition (2.7) states that the current (self of the) government chooses emissions $g = \phi(G,R)$ and investment $r = \psi(G,R)$ optimally under the assumption that welfare from the next period onwards is given by the continuation value function U. Condition (2.8), on the other hand, ensures that the continuation value function U is consistent with the correctly anticipated optimal strategies ϕ and ψ.

For the interpretation of the results in the following sections, it will be helpful to keep in mind that the first-best solution of the model arises as a special case of Definition 2.1 when p equals 0, i.e., when the lobby does not exert any pressure. In this case, the government's preferences are dynamically consistent, and the government's problem coincides with the maximization of social welfare.

2.3 Specific Examples

In this section, we provide explicit examples of political pressure and investigate its consequences for government policy and the evolution of the economy. For the sake of analytical tractability, we restrict our analysis to the same linear-quadratic specification as that used by Harstad [36] and Pichler and Sorger [53]:

$$B(g+R) = -\frac{1}{2}(g + R - \bar{y})^2, \;\; K(r) = kr, \;\; D(G) = \frac{d}{2}G^2. \tag{2.9}$$

Here, $\bar{y}$ is a bliss point for total energy consumption, k is the unit cost of investment in clean technology, and d is the slope of the marginal environmental damage curve.

Both choice variables of the government, g and r, must be non-negative. However, we will ignore these constraints as they would complicate the analysis considerably without providing additional insights. Instead, we interpret the linear-quadratic environment under consideration as a second-order approximation around the steady state of the model and assume that the initial stocks G_0 and R_0 are sufficiently close to the steady-state values. In that case, all variables g, r, G, and R remain non-negative even without explicitly restricting them.

The linear-quadratic structure of the examples discussed in the remainder of the paper allows us to focus on linear policy functions of the form

$$\phi(G,R) = \phi_0 + \phi_G G + \phi_R R \;\text{ and }\; \psi(G,R) = \psi_0 + \psi_G G + \psi_R R, \tag{2.10}$$

where ϕ_0, ϕ_G, ϕ_R, ψ_0, ψ_G, and ψ_R are real numbers.

2.3.1 Traditional Energy Lobby

We start with the case in which political pressure is exerted by the traditional energy sector. The traditional energy sector cares about the amount of energy generated by dirty technology and so we assume $w(g, r, G, R) = g$. The following proposition summarizes our findings for this scenario.[15]

Proposition 2.1. *Suppose that $w(g, r, G, R) = g$ holds.*
(a) *There exists a solution to the government's problem in which the policy functions are given by*

$$\phi(G, R) = \frac{\bar{y} + p - \gamma k(1 - \delta\rho)}{1 + \delta d} - \frac{\gamma\delta dG}{1 + \delta d} - \frac{R}{1 + \delta d}, \tag{2.11}$$

$$\psi(G, R) = \bar{y} - \frac{1 - \delta\rho}{\delta}\left(1 + \frac{1 - \delta\gamma}{\delta d}\right)k + \gamma^2 G - \rho R + \gamma\phi(G, R). \tag{2.12}$$

(b) *In this solution, the stock of greenhouse gases and the stock of clean technology reach their steady-state values G^* and R^*, respectively, after two periods. These steady-state values are given by*

$$G^* = \frac{(1 - \delta\rho)(1 - \delta\gamma)k}{\delta^2 d} + \frac{p}{1 + \delta d}, \tag{2.13}$$

$$R^* = \bar{y} + p - \gamma k(1 - \delta\rho) - (1 - \gamma + \delta d)G^*. \tag{2.14}$$

As mentioned before, the policy functions in (2.11)-(2.12) take the linear form stated in (2.10). In the situation described by Proposition 2.1, the response coefficients ϕ_0, ϕ_G, and ϕ_R can be read off directly from (2.11), while ψ_0, ψ_G, and ψ_R are given by

$$\psi_0 = \bar{y} - \frac{1 - \delta\rho}{\delta}\left(1 + \frac{1 - \delta\gamma}{\delta d}\right)k + \frac{\gamma[\bar{y} + p - \gamma k(1 - \delta\rho)]}{1 + \delta d},$$

$$\psi_G = \frac{\gamma^2}{1 + \delta d}, \quad \text{and} \quad \psi_R = -\rho - \frac{\gamma}{1 + \delta d}.$$

One can see that ϕ_G, ϕ_R, and ψ_R are negative, whereas ψ_G is positive. This means that the optimal level of greenhouse gas emissions decreases both with respect to the current stock of greenhouse gases G and the current stock of clean technology R. As for the optimal investment rate, it increases with the stock of greenhouse gases but decreases with respect to installed clean capital. The negative feedback of the two stock variables G and R on their own input rates g and r, respectively, reflects an inherent stability of the equilibrium dynamics, which is common in linear-quadratic systems of the sort considered here. The negative influence of R on the emission rate g indicates that a higher stock of clean technology allows the government to rely less on dirty technology for

[15] All proofs can be found in Appendix 2.A.

energy production, implying a reduction of emissions. An alternative interpretation for $\phi_R < 0$ is that investment in clean technology serves as a commitment device for future emission reductions. On the other hand, the positive effect of G on the investment rate r demonstrates that a higher concentration of greenhouse gases in the atmosphere urges the government to accelerate the transition to clean energy production.

Next, we note that none of the coefficients ϕ_G, ϕ_R, ψ_G, and ψ_R depends on the intensity of political pressure p. The sensitivity of the optimal policy to changes in the two state variables is thus not affected at all by the activity of the lobby. In contrast, the coefficients ϕ_0 and ψ_0 do depend on p and this dependence is positive. Not surprisingly, the more pressure the lobby of the traditional energy sector exerts, the higher the rate of emissions chosen by the government for any given state (G, R). However, since the government is forward-looking and correctly foresees that higher current emissions will lead to a permanently higher stock of greenhouse gases, it finds it optimal to compensate for the higher emission rate with increased investment in clean technology. Such an investment will help to cut down on emissions in the future and reduce long-run environmental damage. Consistent with this explanation, equation (2.12) shows that $\psi(G, R)$ is independent of p in the boundary case $\gamma = 0$, which corresponds to the full decay of greenhouse gases in the atmosphere within one period. To summarize, investment in clean energy can be seen as a government's effort to substitute for the natural decay of greenhouse gases or to abate future emissions, as was already pointed out in Subsection 2.2.1, and traditional energy lobbying encourages that effort.

Concerning the dynamics of the economy, we find that as in Harstad [36], the steady state is reached after two periods, which is due to the linearity of the investment cost function. Furthermore, one can obtain directly from equations (2.13)-(2.14) that the steady-state values G^* and R^* increase with p:[16]

$$\frac{\partial G^*}{\partial p} > 0 \quad \text{and} \quad \frac{\partial R^*}{\partial p} > 0.$$

Hence, both dirty energy generation and clean energy generation are higher in the long run under lobbying by the traditional energy sector. It is interesting to ask then whether this lobbying increases or decreases the steady-state share of clean energy in total energy production or, more formally, whether the value of $R^*/[(1-\gamma)G^* + R^*]$ increases or decreases with p. We have

$$\frac{\partial}{\partial p}\left[\frac{R^*}{(1-\gamma)G^* + R^*}\right] \gtreqless 0 \;\Leftrightarrow\; \bar{y} \lesseqgtr \frac{1-\delta\rho}{\delta}\left(1 + \frac{1-\delta\gamma}{\delta d}\right)k.$$

[16] Since the steady-state values of emission and investment rates are proportional to G^* and R^*, respectively, the signs of the effects of p on the steady-state controls are identical to those on G^* and R^*.

Because we can view an increase in $R^*/[(1-\gamma)G^* + R^*]$ as the long-run shift in the energy mix from fossil fuels to renewables and $\bar{y}$ as the target for total energy production, we conclude that political pressure from the dirty sector shifts the energy mix towards renewable sources in the long run if the overall production target is not very high.

2.3.2 Quasi-Hyperbolic Discounting

Next, we examine the case of quasi-hyperbolic discounting, which is nested in our model and arises when the utility function of the lobby coincides with the instantaneous social welfare function, that is, if $w(g, r, G, R) = u(g + R, r, G)$ holds.[17] This situation can be interpreted as one where part of the population does not care about the future (i.e., they exhibit complete myopia) and the government faces pressure from that group for higher welfare in the current period. The government's preferences are then given by the objective function $(1+p)u(g+R, r, G) + \delta U(\gamma G + g, \rho R + r)$ or, equivalently,

$$u(g + R, r, G) + \beta\delta U(\gamma G + g, \rho R + r)$$

with $\beta = 1/(1+p) \leq 1$. Obviously, this corresponds to the familiar β-δ model of quasi-hyperbolic discounting. As a result, political pressure from myopic individuals on the government with a constant discount factor has the same implications as the assumption that the government itself is present-biased. These implications can be drawn from the following proposition.

Proposition 2.2. *Suppose that $w(g, r, G, R) = u(g + R, r, G)$ holds.*
(a) *There exists a solution to the government's problem in which the policy functions are given by*

$$\phi(G, R) = \frac{(1+p)\bar{y} - \gamma k(1 + p - \delta\rho)}{1 + p + \delta d} - \frac{\gamma\delta d G}{1 + p + \delta d} - \frac{(1+p)R}{1 + p + \delta d}, \tag{2.15}$$

$$\begin{aligned}\psi(G, R) = \bar{y} - \frac{1-\delta\rho}{\delta}\left(1 + \frac{1-\delta\gamma}{\delta d}\right)k + \gamma^2 G - \rho R + \gamma\phi(G, R) \\ - \frac{pk}{\delta}\left[\frac{1}{\delta d} + \frac{1 + \delta d + p[1 + \delta(\gamma + \rho)]}{(1+p)^2 + \delta d}\right].\end{aligned} \tag{2.16}$$

(b) *In this solution, the stock of greenhouse gases and the stock of clean technology reach their steady-state values G^* and R^*, respectively, after two periods. These steady-state*

[17] There is a substantial body of literature on dynamic inconsistencies in the context of climate policy, which are caused by hyperbolic discounting. For a brief discussion of selected contributions to this strand of the literature, we refer the reader to the Introduction.

values are given by

$$G^* = \left[\frac{1+p-\delta(\gamma+\rho)}{\delta^2}\left[1+\frac{p\delta d}{(1+p)^2+\delta d}\right]+\gamma\rho\right]\frac{k}{d}, \tag{2.17}$$

$$R^* = \bar{y} - \gamma\left(1-\frac{\delta\rho}{1+p}\right)k - \left(1-\gamma+\frac{\delta d}{1+p}\right)G^*. \tag{2.18}$$

Whereas in the situation of the traditional energy lobby studied in the previous subsection, the coefficients ϕ_G, ϕ_R, ψ_G, and ψ_R were independent of the intensity of political pressure p, this is no longer the case in the present situation. In particular, the absolute value of the response coefficient

$$\phi_G = -\frac{\gamma\delta d}{1+p+\delta d}$$

is decreasing with respect to p, while the opposite is true for the absolute values of

$$\phi_R = -\frac{1+p}{1+p+\delta d},\ \psi_G = \frac{\gamma^2(1+p)}{1+p+\delta d},\ \psi_R = -\rho - \frac{\gamma(1+p)}{1+p+\delta d}.$$

This means that higher political pressure from myopic individuals makes the government's emission policy less sensitive to changes in the stock of greenhouse gases and more sensitive to changes in the clean technology stock. The investment policy, on the other hand, becomes more sensitive to changes in both state variables. All of these results can be interpreted simply as a consequence of a decrease in the government's time-preference factor from δ to $\delta/(1+p) = \beta\delta$.

To assess how changes in the lobbying intensity p affect the emission and investment rates given by (2.15)-(2.16), we first note that

$$\frac{\partial\phi(G,R)}{\partial p} = \frac{\delta[d(\bar{y}-\gamma k+\gamma G-R)-\gamma\rho k]}{(1+p+\delta d)^2}.$$

The sign of this derivative depends on the state variables G and R. Since we treat our linear-quadratic model as a second-order approximation around its steady state, we evaluate the above expression at (G^*, R^*) given by (2.17)-(2.18). It follows that near the steady state, the emission rate depends positively on p if and only if $p > \delta(\gamma+\rho) - 1$. Realistically, the parameters γ, ρ, and δ are smaller than but close to 1 so that the right-hand side of the inequality is likely to be positive. Thus, the rate of emissions should first decrease and then increase with p. Proceeding analogously, one can also show that the partial derivative $\partial\psi(G,R)/\partial p$ is negative provided that (G,R) is sufficiently close to (G^*, R^*) and p is sufficiently close to 0. This implies that an increase in political pressure (from a sufficiently low initial level) exerted by myopic individuals reduces the rate of investment in clean technology, which is consistent with the intuition about a present-

biased government that undervalues future environmental damage and therefore invests less in clean energy.[18]

Let us now turn to the effects of political pressure on the steady-state values G^* and R^*. Equations (2.17) and (2.18) imply that (see Appendix 2.A)

$$\frac{\partial G^*}{\partial p} > 0; \quad \frac{\partial R^*}{\partial p} < 0 \quad \text{if} \quad p \leq \sqrt{1+\delta d}. \tag{2.19}$$

Consequently, at least when the government does not attach a very high weight to the utility of the interest group, the pressure from myopic individuals results in a higher steady-state stock of emissions and a lower steady-state stock of clean technology.[19] These findings naturally arise from the above result that the government subject to political pressure invests less in clean technology, thereby contributing to a higher long-run accumulation of greenhouse gases in the atmosphere.

2.3.3 Pressure for Immediate Consumption

In the previous subsection, we analyzed the case where the government is influenced by individuals who are myopic and so disregard future environmental damage, but who correctly evaluate the current costs of investment in clean energy production. Following the existing literature, one can further assume that individuals do not take those costs into account and care only about immediate energy consumption. Douenne and Fabre [22] have shown that people overestimate their losses from higher energy prices, which leads to carbon tax aversion, and those beliefs are persistent. In our paper, we do not model energy prices explicitly. However, since an increase in energy production should, in principle, reduce energy prices, one could interpret public pressure for energy consumption (equal to production) as pressure for energy affordability. The social costs of clean energy investments are likely to be much less salient to individuals because they may be unaware of them or may support additional public spending in general, even paying higher income taxes (Bremer and Bürgisser [12]). Using the terminology of behavioral economics (see, e.g., DellaVigna [21] and Allcott et al. [5]), we can say that energy consumption represents a visible or salient component of public pressure, while investment costs represent a (completely) opaque component. The results for this scenario are provided in the next proposition.

[18] It is noteworthy that equation (2.16) is identical to (2.12), except for an additional negative term given by the second line of (2.16). From this, one can deduce that if the emission channel for clean investments identified in the previous subsection is absent (which would be the case for $\gamma = 0$), the investment rate is always lower with quasi-hyperbolic discounting.

[19] From (2.19) and the fact that the sign of $\partial[R^*/((1-\gamma)G^* + R^*)]/\partial p$ is the same as that of $[G^* \times \partial R^*/(\partial p) - R^* \times \partial G^*/(\partial p)]$, we also know that the steady-state share of the total amount of energy that is produced by clean technology decreases in the presence of the pressure.

Proposition 2.3. *Suppose that $w(g, r, G, R) = B(g + R)$ holds.*
(a) *There exists a solution to the government's problem in which the policy functions are given by*

$$\phi(G, R) = \frac{(1+p)\bar{y} - \gamma k(1-\delta\rho)}{1+p+\delta d} - \frac{\gamma\delta d G}{1+p+\delta d} - \frac{(1+p)R}{1+p+\delta d}, \tag{2.20}$$

$$\psi(G, R) = \bar{y} - \frac{1-\delta\rho}{\delta}\left[1 + \frac{1-\delta\gamma}{\delta d} - \frac{p^2}{(1+p)^2+\delta d}\right] k + \gamma^2 G - \rho R + \gamma\phi(G, R). \tag{2.21}$$

(b) *In this solution, the stock of greenhouse gases and the stock of clean technology reach their steady-state values G^* and R^*, respectively, after two periods. These steady-state values are given by*

$$G^* = \frac{(1-\delta\rho)(1+p)k}{\delta^2 d}\left[\frac{1+p+\delta d}{(1+p)^2+\delta d} - \frac{\delta\gamma}{1+p}\right], \tag{2.22}$$

$$R^* = \bar{y} - \frac{\gamma k(1-\delta\rho)}{1+p} - \left(1 - \gamma + \frac{\delta d}{1+p}\right) G^*. \tag{2.23}$$

Comparing (2.20) with (2.15) and (2.21) with (2.16), we see that the response coefficients ϕ_G, ϕ_R, ψ_G, and ψ_R are identical in Propositions 2.2 and 2.3. Only the constant terms ϕ_0 and ψ_0 differ in the two scenarios. Indeed, both ϕ_0 and ψ_0 are higher in the situation studied in the present subsection than in the case of quasi-hyperbolic discounting. This implies, of course, that for all given values of G and R, both the emission and investment rates are higher if investment costs are ignored by the interest group and are therefore not distorted from the government's perspective.

Furthermore, compared to the quasi-hyperbolic discounting case, the present case is characterized by different comparative statics of the steady state with respect to p. Specifically, we have

$$\frac{\partial G^*}{\partial p} \gtreqless 0 \;\Leftrightarrow\; p \lesseqgtr \sqrt{1+\delta d}; \quad \frac{\partial R^*}{\partial p} > 0,$$

that is, the increased importance given to energy consumption in the current period leads to a higher stock of clean technology in the steady state. On the other hand, the dependence of the steady-state stock of greenhouse gases on p has an inverted U-shape with a peak at $\sqrt{1+\delta d}$. The latter relationship is quite remarkable as it suggests that the pressure to increase immediate energy consumption, which is usually considered detrimental to the environment in the long run, may in fact be beneficial to it provided that the pressure is strong enough. The intuition behind this result is as follows. Political pressure for higher immediate energy consumption encourages the production of energy by dirty technology because it does not require building new infrastructure, but, at the same time, encourages investment in clean technology because the government is aware that future energy production will be subject to the same pressure. Then, if the pressure

is sufficiently high, the government invests so much in the production of clean energy that the stock of greenhouse gases starts to decline in the long run. Note that together with this decline, the steady-state share of dirty energy in the total amount of energy produced also decreases, demonstrating that the pressure for immediate consumption (when strong) causes a long-run shift in the energy mix towards renewables.

2.4 Extensions

Introducing the model in Section 2.2, we made several assumptions to focus on the simplest description of political pressure as a source of the dynamically inconsistent preferences of the government. In this section, we illustrate how some of them can be relaxed. In particular, we deal with versions of the model in which the strength of political pressure is determined endogenously and in which political pressure distorts not only the government's current but also future utility. In addition, we examine the case of multiple (potentially competing) interest groups.

2.4.1 Endogenous Pressure

So far, we have modeled the weight given by the government to the concerns of the interest group as an exogenous parameter p. This weight, however, is likely to depend on the effort exerted by the interest group. For example, industries can decide on how many lobbyists to employ and whether to gain more access to policymakers.[20] Similarly, protesters can choose between less costly activities that do not attract much public attention and more elaborate ones that require a big team and special equipment but have a higher potential for policy change. In this subsection, we therefore treat p as a choice variable of the interest group and assume that more intensive pressure involves higher costs.

The biased welfare function applied by the government in period t can be written as

$$v(p_t, g_t, r_t, G_t, R_t) = u(g_t + R_t, r_t, G_t) + p_t w(g_t, r_t, G_t, R_t),$$

where p_t is the pressure exerted by the interest group in period t. As before, the government in period t seeks to maximize the (biased) total intertemporal welfare:

$$v(p_t, g_t, r_t, G_t, R_t) + \sum_{s=1}^{+\infty} \delta^s u(g_{t+s} + R_{t+s}, r_{t+s}, G_{t+s}).$$

The interest group is assumed to maximize its own utility minus the cost of exerting pressure. Denoting this cost by $C(p_t)$, we can define the interest group's objective in

[20] See Holyoke [41] and Coen and Richardson [16] for an in-depth description of lobbying activities in the US and the EU.

period t as $w(g_t, r_t, G_t, R_t) - C(p_t)$. The interest group can only benefit from exerting pressure if the government responds. An equilibrium concept that captures this feature must therefore be a hierarchical one such that the interest group moves first and the government moves second. Since we have assumed the interest group to be myopic, and since the government is split into multiple selves due to its dynamic inconsistency problem, the hierarchical game is played separately in every period. More precisely, in every period, the interest group and the current self of the government play a Stackelberg game where first the interest group determines the intensity of its political pressure and then the government determines emission and investment rates. Both players correctly anticipate the equilibrium strategies of all future selves of the government.[21]

To describe the situation under consideration formally, we employ the stagewise Stackelberg equilibrium concept (see, e.g., Basar and Olsder [8, p. 140]). For better readability, we split the equilibrium definition into three parts. We start by specifying what we mean by a stage game.

Definition 2.2.1. *Let a state (G, R) and a continuation value function U be given. The subgame or stage game $\Gamma(G, R, U)$ is defined by the government's objective function*

$$v(p, g, r, G, R) + \delta U(\gamma G + g, \rho R + r)$$

and the interest group's objective function

$$w(g, r, G, R) - C(p).$$

In this stage game, the interest group chooses p, while the government chooses (g, r).

The Stackelberg equilibrium of the stage game can be further defined as follows.

Definition 2.2.2. *Let a state (G, R) and a continuation value function U be given and consider the stage game $\Gamma(G, R, U)$. The best response of the government to the interest group's action p is*

$$(\mathrm{BR}^g[p], \mathrm{BR}^r[p]) = \operatorname*{argmax}_{(g,r)} v(p, g, r, G, R) + \delta U(\gamma G + g, \rho R + r).$$

A triple of actions (p, g, r) is a Stackelberg equilibrium of the stage game $\Gamma(G, R, U)$ if $(g, r) = (\mathrm{BR}^g[p], \mathrm{BR}^r[p])$ and if for all feasible actions $\tilde{p}$ of the lobby, it holds that

$$w(\mathrm{BR}^g[p], \mathrm{BR}^r[p], G, R) - C(p) \geq w(\mathrm{BR}^g[\tilde{p}], \mathrm{BR}^r[\tilde{p}], G, R) - C(\tilde{p}).$$

[21] Yet, it is worth emphasizing that the interest group in period t tries to influence only the actions of the government's self that is in charge in that period.

That is, in an equilibrium of the stage game, the current self of the government chooses its best response to the interest group's pressure p. The interest group takes the best-response functions as given and chooses p so as to maximize its objective. We are now ready to define the stagewise Stackelberg equilibrium of the full dynamic game. We continue to denote the Markov strategies of the government determining emissions and investment by ϕ and ψ, respectively. As for the interest group, we denote its strategy to determine the intensity of pressure by μ.

Definition 2.2.3. *A triple of stationary Markov strategies (μ, ϕ, ψ) is a stagewise Stackelberg equilibrium if there exists a continuation value function U such that for every state (G, R), the equation*

$$U(G,R) = u(\phi(G,R) + R, \psi(G,R), G) + \delta U(\gamma G + \phi(G,R), \rho R + \psi(G,R)) \tag{2.24}$$

holds and the triple $(\mu(G,R), \phi(G,R), \psi(G,R))$ is a Stackelberg equilibrium of the stage game $\Gamma(G, R, U)$.

With equation (2.24), we again impose consistency of the continuation value function with the equilibrium strategies.[22] This time, however, the strategies describe not merely the optimal behavior of the multiple selves of the government (given U), but also the hierarchical interaction between the government and the interest group (given U) at every stage of the game.

We illustrate the above definition using the example from Subsection 2.3.1. Recall that this was the case of a traditional energy lobby with the utility function $w(g, r, G, R) = g$. We maintain the linear-quadratic specification (2.9) and augment it with the quadratic cost function $C(p) = (c/2)p^2$. Under these assumptions, one obtains the following result.

Proposition 2.4. *There exists a stagewise Stackelberg equilibrium in which the lobby's strategy is given by*

$$\mu(G,R) = \hat{p} := \frac{1}{c(1+\delta d)}. \tag{2.25}$$

The emission and investment strategies of the government and the steady-state stocks of greenhouse gases and clean technology are as described in Proposition 2.1 with $p = \hat{p}$.

Proposition 2.4 shows that the optimal intensity of political pressure by the traditional energy lobby is independent of the states so that the economy behaves as if the pressure was exogenous. The result can be understood by noting that in the case of exogenous pressure, the latter does not affect how the production of dirty energy reacts to the state of the economy.[23] As a consequence, even if the pressure is endogenous, its marginal

[22] Note that (2.24) is formally identical to (2.8) in Definition 2.1.

[23] This property is reflected by the fact that the coefficients ϕ_G, ϕ_R, ψ_G, and ψ_R in Proposition 2.1 do not depend on p.

benefit for the lobby is independent of the state too. By equating this marginal benefit to the marginal cost, the lobby thus exerts pressure with the same intensity for all G and R. In line with our expectations, the intensity decreases in the slope of the cost function and goes to zero as c goes to infinity. In addition, the pressure depends on the slope of the marginal environmental damage curve and the discount factor of the government. The larger δ or d, the less willing the government is to increase dirty energy production, and the less intensive the lobby's pressure.

2.4.2 Future Pressure

In this subsection, we allow for the possibility that the current government may care about future political pressure. Specifically, we consider the setting where instead of (2.5), the government in period t maximizes

$$v(g_t, r_t, G_t, R_t) + \sum_{s=1}^{+\infty} \delta^s \hat{v}(g_{t+s}, r_{t+s}, G_{t+s}, R_{t+s}), \tag{2.26}$$

with

$$\hat{v}(g, r, G, R) = u(g + R, r, G) + \alpha p w(g, r, G, R). \tag{2.27}$$

Here, we introduce a new parameter $\alpha \in [0, 1]$ to capture the degree to which future political pressure matters for the current government. The extreme case $\alpha = 0$ corresponds to our original model where the current government does not care about future pressure, while the case $\alpha = 1$ corresponds to the situation where it cares about current and future pressure equally (i.e., the function $\hat{v}$ coincides with v). Following the interpretation given earlier in Subsection 2.2.3, we can interpret α as the probability of extending the incumbent government's term of office, and we suppose that the preferences of the incumbent government depend on political pressure it will face in future periods of its tenure.

With the government's period-t preferences described by (2.26), the definition of the solution to the government's problem is modified to

Definition 2.3. *A pair of policy functions (ϕ, ψ) is a solution to the government's problem if there exists a continuation value function U such that for every state (G, R), it holds that*

$$(\phi(G, R), \psi(G, R)) = \underset{(g,r)}{\operatorname{argmax}}\ v(g, r, G, R) + \delta U(\gamma G + g, \rho R + r) \tag{2.28}$$

and

$$U(G, R) = \hat{v}(\phi(G, R), \psi(G, R), G, R) + \delta U(\gamma G + \phi(G, R), \rho R + \psi(G, R)). \tag{2.29}$$

Note that the only difference between this definition and Definition 2.1 is that the former requires that the continuation value function takes into account the bias in the government's preferences due to political pressure.

Similar to the previous subsection, we illustrate our results with the example of a dirty energy lobby considered in Subsection 2.3.1. We obtain:

Proposition 2.5. *Suppose that $w(g, r, G, R) = g$ holds.*
(a) *There exists a solution to the government's problem in which the policy functions are given by*

$$\phi(G, R) = \frac{\bar{y} + p(1 - \alpha\delta\gamma) - \gamma k(1 - \delta\rho)}{1 + \delta d} - \frac{\delta\gamma dG}{1 + \delta d} - \frac{R}{1 + \delta d}, \tag{2.30}$$

$$\psi(G, R) = \bar{y} - \frac{1 - \delta\gamma}{\delta d}\alpha p - \frac{1 - \delta\rho}{\delta}\left(1 + \frac{1 - \delta\gamma}{\delta d}\right)k + \gamma^2 G - \rho R + \gamma\phi(G, R). \tag{2.31}$$

(b) *In this solution, the stock of greenhouse gases and the stock of clean technology reach their steady-state values G^* and R^*, respectively, after two periods. These steady-state values are given by*

$$G^* = \frac{(1 - \delta\rho)(1 - \delta\gamma)k}{\delta^2 d} + \frac{p}{1 + \delta d}\left[1 + \alpha\left(\frac{1 - \delta\gamma}{\delta d} - \delta\gamma\right)\right], \tag{2.32}$$

$$R^* = \bar{y} + p(1 - \alpha\delta\gamma) - \gamma k(1 - \delta\rho) - (1 - \gamma + \delta d)G^*. \tag{2.33}$$

We see from Proposition 2.5 that the positive effect of the pressure intensity p on the emission rate $\phi(G, R)$ found in Subsection 2.3.1 still holds for any given state (G, R) and any degree of the government's concerns for future pressure α. Its positive effect on the investment rate $\psi(G, R)$, also found in Subsection 2.3.1, can fail to hold, however. One can easily check that $\psi(G, R)$ decreases with p for all G and R if and only if

$$\alpha > \frac{\gamma\delta d}{d(\delta\gamma)^2 + (1 + \delta d)(1 - \delta\gamma)}, \tag{2.34}$$

where the right-hand side is always smaller than 1. We have then that the effect of dirty energy lobbying on the investment rate is positive for $\alpha = 0$ and negative for $\alpha = 1$, which highlights the importance of dynamic inconsistency in the preferences of the government. Indeed, the preferences do not exhibit dynamic inconsistency only when $\alpha = 1$. In this case, accounting for the interests of the dirty energy sector in the current and future periods, the government chooses a lower investment rate because a higher investment rate applied today would reduce the dirty energy lobby's utility in the next period.

Consistent with these results, the intensity of political pressure p has the following long-run effects on the stocks of greenhouse gases and clean technology:

$$\frac{\partial G^*}{\partial p} > 0; \quad \frac{\partial R^*}{\partial p} \gtreqless 0 \;\Leftrightarrow\; \alpha \lesseqgtr \frac{\gamma\delta d}{d(\delta\gamma)^2 + (1-\gamma+\delta d)(1-\delta\gamma)}, \tag{2.35}$$

implying in particular that the effect of dirty energy lobbying on the steady-state technology stock is positive for $\alpha = 0$ and negative for $\alpha = 1$. It is noteworthy that the critical threshold for α in (2.35) is greater than that in (2.34) (but still less than 1) and therefore there is a case where p reduces the investment rate in the short run but increases it in the long run. Yet, with plausible values of γ and δ close to 1, both critical thresholds are close to 1, which suggests that our earlier results are robust to the inclusion of future political pressure in the government's preferences (unless $\alpha = 1$, in which case the preferences become dynamically consistent).

2.4.3 Multiple Lobbies

Finally, we consider the case where the government is influenced by multiple lobbies. We will focus on two lobbies pursuing the interests of the dirty and clean energy sectors. As in Subsection 2.3.1, the dirty energy lobby derives utility from dirty energy generation g. The clean lobby, in contrast, derives utility not from the production of clean energy itself (which would not affect our earlier results) but from building new infrastructure for the clean sector. Thus, the clean lobby always prefers to have as much investment r as possible. Assuming also that the intensity of political pressure of each lobby is exogenously given, we can write the biased welfare function of the government as

$$v(g, r, G, R) = u(g + R, r, G) + p^g g + p^r r, \tag{2.36}$$

where $p^g \geq 0$ and $p^r \geq 0$ measure how strong the pressure is from the dirty and clean energy lobbies, respectively.[24] One can subsume this case under the general case considered in Section 2.2 (see equation (2.6)) by defining $w(g, r, G, R) = (p^g g + p^r r)/p$. Moreover, one can endogenize p^g and p^r in essentially the same way as we did in Subsection 2.4.1, without affecting either the government's strategies or the steady state.[25] Our findings are summarized in the proposition below.

[24] In practice, the pressure from the clean energy lobby on the government is comparatively weak. See, for example, the work of Brulle [13], who estimated lobbying expenditures of the fossil fuel sector in the US to be almost five times higher than those of the renewable energy sector.

[25] The requirement in Definition 2.2 that p maximizes $w(\mathrm{BR}^g[p], \mathrm{BR}^r[p], G, R) - C(p)$ has to be replaced by the requirement that (p^g, p^r) is a Nash equilibrium in the stage game played by the two lobbies, given the best response of the government. Then, under our assumptions on v and u, this Nash equilibrium turns out to be in dominant strategies which are independent of the state variables G and R. As in Subsection 2.4.1, we could further assume that the cost of exerting pressure is quadratic (possibly with different coefficients for each lobby) and derive explicit expressions for the lobbies' strategies.

Proposition 2.6. *Suppose that* $w(g, r, G, R) = (p^g g + p^r r)/p$ *holds.*
(a) *There exists a solution to the government's problem in which the policy functions are given by*

$$\phi(G, R) = \frac{\bar{y} + p^g - \gamma[(1 - \delta\rho)k - p^r]}{1 + \delta d} - \frac{\delta\gamma d G}{1 + \delta d} - \frac{R}{1 + \delta d}, \tag{2.37}$$

$$\psi(G, R) = \bar{y} - \frac{1 - \delta\rho}{\delta}\left(1 + \frac{1 - \delta\gamma}{\delta d}\right)k + \frac{p^r(1 + \delta d)}{\delta^2 d} + \gamma^2 G - \rho R + \gamma\phi(G, R). \tag{2.38}$$

(b) *In this solution, the stock of greenhouse gases and the stock of clean technology reach their steady-state values* G^* *and* R^**, respectively, after two periods. These steady-state values are given by*

$$G^* = \frac{(1 - \delta\gamma)(1 - \delta\rho)k - p^r}{\delta^2 d} + \frac{p^g + \gamma p^r}{1 + \delta d}, \tag{2.39}$$

$$R^* = \bar{y} + p^g + \gamma p^r - \gamma k(1 - \delta\rho) - (1 - \gamma + \delta d)G^*. \tag{2.40}$$

The policy functions (2.37) and (2.38) imply that the pressure from the dirty and clean energy lobbies on the government have the same qualitative effects in the short run, even though the two lobbies seem to be antagonistic. Both p^g and p^r increase current emissions as well as current investments in clean technology for any given pair of stocks (G, R). The positive impact of p^r on current emissions is particularly noteworthy and can be explained as follows. From the discussion in Subsection 2.3.1, we know that the production of clean energy substitutes for the natural decay of the stock of greenhouse gases. The pressure for higher investment today promotes this production. As a result, for larger values of p^r, the government can "afford" more current emissions, while keeping the long-term environmental damage unchanged.

Because of the positive effect on emission and investment rates, it is tempting to conjecture that lobbying from the clean energy sector increases the steady-state greenhouse gas concentration G^* and the steady-state clean capital stock R^*. However, this reasoning neglects the interaction between the two state variables. Based on the results of Proposition 2.6, we obtain the following comparative statics results:

$$\frac{\partial G^*}{\partial p^r} < 0 \quad \text{and} \quad \frac{\partial R^*}{\partial p^r} > 0.$$

The presence of the clean energy lobby thus positively influences the installation of clean technology also in the long run. As far as the implication for the greenhouse gas concentration is concerned, we have to distinguish two effects. The direct effect mentioned above would lead to a higher steady-state stock G^*. The indirect effect resulting from the increase in R^* and the negative dependence of g on R works in the opposite direction. It

is then possible that the indirect effect dominates the direct one and this is indeed the case for all values of the parameters.

Finally, we point out that the comparative statics with respect to p^g must be identical to those found in Proposition 2.1, where only the traditional energy lobby was active, i.e.,

$$\frac{\partial G^*}{\partial p^g} > 0 \quad \text{and} \quad \frac{\partial R^*}{\partial p^g} > 0.$$

Hence, the dirty and clean energy lobbies reinforce each other's positive impact on the long-run stock of clean technology. On the other hand, the long-run stock of greenhouse gases can be either lower or higher than without any pressure, which depends on the relative size between p^g and p^r. We therefore conclude that lobbying from the dirty and clean energy sectors are competing but only in their influence on greenhouse gas emissions and only in the long run.

In addition to the formulation of the model with multiple lobbies discussed above, one can think of several alternatives. One possibility would be to replace (2.36) by

$$v(g, r, G, R) = u(g + R, r, G) + \frac{p^g g + p^r r}{p^g + p^r},$$

i.e., the pressure exerted by one lobby not only increases its own influence on the government's objective function but also reduces the influence of the other lobby. This version of the model can be analyzed in much the same way as the one above but has somewhat different properties.[26] Another alternative formulation can be built on a "winner-takes-it-all" assumption, meaning that only the strongest interest group has an influence on the government's objective. In this case, however, due to the implied discontinuity of the government's objective function at $p^g = p^r$, the equilibrium with endogenous pressure may fail to exist. But even if it exists, there is no reason to believe that in the simple linear-quadratic environment used in all our examples, this will change the implications of pressure from the "winning" lobby for the policy functions and the long-run states.[27]

2.5 Conclusion

The present paper uses a framework developed by Harstad [35, 36] to analyze the influence of political pressure on a government's climate policy. Our main assumption is that the government gives in to such pressure (at least to a certain extent) without changing its long-term goals. This assumption makes the government's preferences dynamically

[26] For example, whereas an increase of p^g in Proposition 2.6 results in an increase of $\psi_0 = \psi(0, 0)$, the coefficient ψ_0 is decreasing with respect to p^g in the alternative formulation. This reflects the assumption that the pressure exerted by the dirty lobby reduces the weight that the government attaches to investment r when responding to pressure from the clean lobby.

[27] This is because the lobbies' strategies would still be independent of the state variables.

inconsistent. We study the optimal response of the government to single or multiple pressure groups and illustrate how the intensity of political pressure can be endogenized. Furthermore, we illustrate the importance of dynamic inconsistency in our analysis by allowing the government to care about future political pressure.

We show that political pressure from the dirty and clean energy sectors each increases the emission and investment rates chosen by the government for any given state of the economy. On the other hand, while both kinds of political pressure further increase the steady-state stock of clean technology, the steady-state stock of greenhouse gases decreases with "clean" and increases with "dirty" lobbying. Yet, as long as the overall production target is not very high, dirty energy lobbying negatively affects the steady-state share of the total energy produced by dirty technology. Similarly, facing pressure for higher energy consumption from myopic individuals, the government can shift the energy mix towards renewables in the long run. Indeed, this pressure (if strong enough) reduces the absolute amount of dirty energy produced in the steady state, thereby reducing the concentration of greenhouse gases in the steady state, as well as its relative share in total energy production.

There are a number of directions in which future research can extend the analysis of the present paper. First of all, one can try to modify or relax some of the assumptions that have been made primarily for analytical tractability. This includes the introduction of "black capacity" or a more careful modeling of investment lags in clean energy production, as it is done in Kollenbach [46] or Eichner and Kollenbach [24], respectively. Second, one can extend the proposed model to the case where lobbies or members of the public have a long-term objective in addition to a short-term objective. Third, it would be interesting to consider a multi-country setting with foreign political pressure or a setting with decentralized decisions, where emissions and investments in clean technology are respectively taxed and subsidized by the government. Last but not least, one could try to provide more detailed microfoundations for political pressure by including political contributions from lobby groups and electoral incentives of policymakers.

2.A Appendix

Preliminaries

Since the solution concept presented in Definition 2.1 arises as a special case (for $\alpha = 0$) of the concept specified in Definition 2.3, we start by dealing with the more general case and will later consider the special case of $\alpha = 0$.

Let U_1 and U_2 denote the partial derivatives of the continuation value function $U(G, R)$ with respect to its first and second arguments, respectively. Let also $w_1'(G, R)$, $w_2'(G, R)$, $w_3'(G, R)$, and $w_4'(G, R)$ denote the partial derivatives of the interest group's utility func-

tion $w(g, r, G, R)$ with respect to its first, second, third, and fourth arguments, respectively, each evaluated at $(\phi(G, R), \psi(G, R), G, R)$.

Due to the functional forms stated in (2.3) and (2.9) and the specifications of the function w used in Propositions 2.1-2.6, the problems under consideration have a linear-quadratic structure. We therefore conjecture that the continuation value function U takes the quadratic form

$$U(G, R) = U_0 + U_G G + U_R R + \frac{U_{GG}}{2} G^2 + U_{GR} GR + \frac{U_{RR}}{2} R^2, \tag{2.41}$$

and that the policy functions take the linear form (2.10), where U_0, U_G, U_R, U_{GG}, U_{GR}, U_{RR}, ϕ_0, ϕ_G, ϕ_R, ψ_0, ψ_G, and ψ_R are coefficients to be determined.

Using (2.3), (2.6), (2.9), and (2.10), the first-order optimality conditions for the maximization problem in (2.28), which is identical to (2.7), can be written as

$$\begin{aligned} &\phi_0 + \phi_G G + (1 + \phi_R) R - \bar{y} - p w_1'(G, R) \\ &\qquad = \delta U_1(\phi_0 + (\gamma + \phi_G) G + \phi_R R, \psi_0 + \psi_G G + (\rho + \psi_R) R), \end{aligned} \tag{2.42}$$

$$k - p w_2'(G, R) = \delta U_2(\phi_0 + (\gamma + \phi_G) G + \phi_R R, \psi_0 + \psi_G G + (\rho + \psi_R) R). \tag{2.43}$$

Furthermore, differentiating both sides of (2.29) with respect to G and R and using (2.3), (2.27), (2.9), (2.10), and (2.42)-(2.43) yields

$$\begin{aligned} U_1(G, R) = {} & \gamma[\phi_0 + \phi_G G + (1 + \phi_R) R - \bar{y}] - dG \\ & - p[w_1'(G, R)[\gamma + (1 - \alpha)\phi_G] + (1 - \alpha) w_2'(G, R)\psi_G - \alpha w_3'(G, R)]. \end{aligned} \tag{2.44}$$

Analogously, if one differentiates (2.29) with respect to R and uses (2.3), (2.27), (2.9), (2.10), and (2.42)-(2.43), the result is

$$\begin{aligned} U_2(G, R) = {} & -\phi_0 - \phi_G G - (1 + \phi_R) R + \bar{y} + \rho k \\ & - p[(1 - \alpha) w_1'(G, R)\phi_R + w_2'(G, R)[\rho + (1 - \alpha)\psi_R] - \alpha w_4'(G, R)]. \end{aligned} \tag{2.45}$$

Proof of Proposition 2.1

(a) Under the assumptions of the proposition, it holds that $\alpha = 0$ and $w(g, r, G, R) = g$, which implies $w_1'(G, R) = 1$ and $w_2'(G, R) = 0$. Equations (2.44)-(2.45) boil down to

$$U_1(G, R) = \gamma[\phi_0 + \phi_G G + (1 + \phi_R) R - \bar{y}] - dG - p(\gamma + \phi_G), \tag{2.46}$$

$$U_2(G, R) = -\phi_0 - \phi_G G - (1 + \phi_R) R + \bar{y} + \rho k - p\phi_R. \tag{2.47}$$

Differentiating the first of these equations with respect to R and the second one with respect to G, it follows from (2.41) that

$$U_{GR} = \gamma(1+\phi_R) = -\phi_G. \tag{2.48}$$

Applying $w_1'(G,R) = 1$, $w_2'(G,R) = 0$, and (2.46)-(2.47) to the first-order conditions (2.42)-(2.43) and then equating the coefficients of G and R on both sides of the two resulting equations, we obtain

$$\phi_0 - \bar{y} - p = \delta[\gamma[\phi_0 + \phi_G\phi_0 + (1+\phi_R)\psi_0 - \bar{y}] - d\phi_0 - p(\gamma+\phi_G)], \tag{2.49}$$

$$\phi_G = \delta[\gamma[\phi_G(\gamma+\phi_G) + (1+\phi_R)\psi_G] - d(\gamma+\phi_G)], \tag{2.50}$$

$$1+\phi_R = \delta[\gamma[\phi_G\phi_R + (1+\phi_R)(\rho+\psi_R)] - d\phi_R], \tag{2.51}$$

$$-k = \delta[\phi_0 + \phi_G\phi_0 + (1+\phi_R)\psi_0 - \bar{y} - \rho k + p\phi_R], \tag{2.52}$$

$$0 = \delta[\phi_G(\gamma+\phi_G) + (1+\phi_R)\psi_G], \tag{2.53}$$

$$0 = \delta[\phi_G\phi_R + (1+\phi_R)(\rho+\psi_R)]. \tag{2.54}$$

If $\phi_G = 0$, then (2.48) implies $1+\phi_R = 0$, contradicting (2.50). Hence, ϕ_G must be non-zero. By (2.48), we can rewrite (2.53)-(2.54) as $\phi_G[\gamma(\gamma+\phi_G)-\psi_G] = \phi_G(\gamma\phi_R-\rho-\psi_R) = 0$. Since $\phi_G \neq 0$, we have $\psi_G = \gamma(\gamma+\phi_G)$ and $\phi_R = (\rho+\psi_R)/\gamma$. Using these results and (2.48) once more, it follows from (2.50)-(2.51) that

$$\phi_G = -\frac{\gamma\delta d}{1+\delta d},\ \phi_R = -\frac{1}{1+\delta d},\ \psi_G = \gamma(\gamma+\phi_G),\ \psi_R = -\gamma + \rho\phi_R.$$

By substituting these values into (2.49) and (2.52), one can find

$$\phi_0 = \frac{\bar{y} + p - \gamma k(1-\delta\rho)}{1+\delta d},$$

$$\psi_0 = \bar{y} - \frac{1-\delta\rho}{\delta}\left(1 + \frac{1-\delta\gamma}{\delta d}\right)k + \gamma\phi_0.$$

Finally, taking (2.41) into account, we obtain from (2.46)-(2.47) that $U_{GG} = \gamma\phi_G - d$, $U_{GR} = \gamma(1+\phi_R)$, and $U_{RR} = -(1+\phi_R)$. Using the above expressions for ϕ_G and ϕ_R leads to

$$U_{GG} = -\frac{\gamma^2\delta d}{1+\delta d} - d,\ U_{GR} = \frac{\gamma\delta d}{1+\delta d},\ U_{RR} = -\frac{\delta d}{1+\delta d}.$$

Since $U_{GG} < 0$ and $U_{GG}U_{RR} - U_{GR}^2 = \delta d^2/(1+\delta d) > 0$, the function U is strictly concave so that the first-order conditions that we have used are necessary and sufficient for optimality. This completes the proof of part (a) of the proposition.

(b) Because of (2.1) and (2.2), we have

$$G_2 = \gamma[\gamma G_0 + \phi(G_0, R_0)] + \phi(\gamma G_0 + \phi(G_0, R_0), \rho R_0 + \psi(G_0, R_0)),$$
$$R_2 = \rho[\rho R_0 + \psi(G_0, R_0)] + \psi(\gamma G_0 + \phi(G_0, R_0), \rho R_0 + \psi(G_0, R_0)).$$

It is straightforward to verify that independently of the values G_0 and R_0, the right-hand sides of these equations are equal to the right-hand sides of (2.13) and (2.14), respectively. This proves that the state (G^*, R^*) is reached within two periods. It is also easy to check that the equations $(1-\gamma)G^* = \phi(G^*, R^*)$ and $(1-\rho)R^* = \psi(G^*, R^*)$ hold. Hence, (G^*, R^*) is indeed a steady state. □

Proof of Proposition 2.2

The proof applies the same reasoning as the proof of Proposition 2.1, so we will omit some of the details.

(a) The assumption $w(g, r, G, R) = u(g+r, G, R)$ implies $w_1'(G, R) = -\phi(G, R) - R + \bar{y} = -\phi_0 - \phi_G G - (1 + \phi_R)R + \bar{y}$ and $w_2'(G, R) = -k$. Setting also $\alpha = 0$, we can rewrite equations (2.44)-(2.45) as

$$U_1(G, R) = [\gamma + p(\gamma + \phi_G)][\phi_0 + \phi_G G + (1 + \phi_R)R - \bar{y}] - dG + pk\psi_G,$$
$$U_2(G, R) = -(1 - p\phi_R)[\phi_0 + \phi_G G + (1 + \phi_R)R - \bar{y}] + [\rho + p(\rho + \psi_R)]k.$$

Then, by (2.41),

$$U_{GR} = [\gamma + p(\gamma + \phi_G)](1 + \phi_R) = -(1 - p\phi_R)\phi_G. \tag{2.55}$$

Applying these results to the first-order conditions (2.42)-(2.43) and equating the coefficients of G and R on both sides of the two equations, we obtain the following six equations for the coefficients ϕ_0, ϕ_G, ϕ_R, ψ_0, ψ_G, and ψ_R:

$$(1 + p)(\phi_0 - \bar{y}) = \delta[[\gamma + p(\gamma + \phi_G)][\phi_0 + \phi_G\phi_0 + (1 + \phi_R)\psi_0 - \bar{y}] - d\phi_0 + pk\psi_G], \tag{2.56}$$
$$(1 + p)\phi_G = \delta[[\gamma + p(\gamma + \phi_G)][\phi_G(\gamma + \phi_G) + (1 + \phi_R)\psi_G] - d(\gamma + \phi_G)], \tag{2.57}$$
$$(1 + p)(1 + \phi_R) = \delta[[\gamma + p(\gamma + \phi_G)][\phi_G\phi_R + (1 + \phi_R)(\rho + \psi_R)] - d\phi_R], \tag{2.58}$$
$$-(1 + p)k = \delta[(1 - p\phi_R)[\phi_0 + \phi_G\phi_0 + (1 + \phi_R)\psi_0 - \bar{y}] - [\rho + p(\rho + \psi_R)]k], \tag{2.59}$$
$$0 = \delta(1 - p\phi_R)[\phi_G(\gamma + \phi_G) + (1 + \phi_R)\psi_G], \tag{2.60}$$
$$0 = \delta(1 - p\phi_R)[\phi_G\phi_R + (1 + \phi_R)(\rho + \psi_R)]. \tag{2.61}$$

This system has a unique solution satisfying (2.55), which is given by

$$\phi_G = -\frac{\gamma\delta d}{1+p+\delta d},\ \phi_R = -\frac{1+p}{1+p+\delta d},\ \psi_G = \gamma(\gamma+\phi_G),\ \psi_R = -\gamma+\rho\phi_R$$

and

$$\phi_0 = \frac{(1+p)\bar{y} - \gamma k(1+p-\delta\rho)}{1+p+\delta d},$$
$$\psi_0 = \bar{y} - \frac{1-\delta\rho}{\delta}\left(1+\frac{1-\delta\gamma}{\delta d}\right)k - \frac{pk}{\delta}\left[\frac{1}{\delta d} + \frac{1+\delta d+p[1+\delta(\gamma+\rho)]}{(1+p)^2+\delta d}\right] + \gamma\phi_0.$$

Analogously to the proof of Proposition 2.1, one can further show that $U_{GG} = -\gamma^2\delta d[(1+p)^2+\delta d]/(1+p+\delta d)^2 - d < 0$ and $U_{GG}U_{RR} - U_{GR}^2 = \delta d^2[(1+p)^2+\delta d]/(1+p+\delta d)^2 > 0$, which completes the proof of part (a).

(b) The proof of part (b) is completely analogous to the proof of part (b) of Proposition 2.1 and is therefore omitted. □

Proof of Proposition 2.3

The proof applies the same reasoning as the proofs of Propositions 2.1 and 2.2, so we will omit some of the details.

(a) The assumption $w(g,r,G,R) = B(g+R)$ implies $w_1'(G,R) = -\phi(G,R) - R + \bar{y} = -\phi_0 - \phi_G G - (1+\phi_R)R + \bar{y}$ and $w_2'(G,R) = 0$. Given $\alpha = 0$, equations (2.44)-(2.45) become

$$U_1(G,R) = [\gamma + p(\gamma+\phi_G)][\phi_0 + \phi_G G + (1+\phi_R)R - \bar{y}] - dG,$$
$$U_2(G,R) = -(1-p\phi_R)[\phi_0 + \phi_G G + (1+\phi_R)R - \bar{y}] + \rho k,$$

so that (2.55) holds in this case, too. Applying these results to the first-order conditions (2.42)-(2.43) and equating the coefficients of G and R in the two resulting equations, one obtains (2.57)-(2.58), (2.60)-(2.61), and

$$(1+p)(\phi_0 - \bar{y}) = \delta[[\gamma + p(\gamma+\phi_G)][\phi_0 + \phi_G\phi_0 + (1+\phi_R)\psi_0 - \bar{y}] - d\phi_0],$$
$$-k = \delta[(1-p\phi_R)[\phi_0 + \phi_G\phi_0 + (1+\phi_R)\psi_0 - \bar{y}] - \rho k].$$

We thus conclude that ϕ_G, ϕ_R, ψ_G, and ψ_R take the same values as in Proposition 2.2, while the values of ϕ_0 and ψ_0 are obtained from the last two equations and are given by

$$\phi_0 = \frac{(1+p)\bar{y} - \gamma k(1-\delta\rho)}{1+p+\delta d},$$
$$\psi_0 = \bar{y} - \frac{1-\delta\rho}{\delta}\left[1 + \frac{1-\delta\gamma}{\delta d} - \frac{p^2}{(1+p)^2+\delta d}\right]k + \gamma\phi_0.$$

The rest of the proof, including the proof of part (b), is identical to that of Proposition 2.2. □

Proof of Proposition 2.4

We conjecture that there exists a stagewise Stackelberg equilibrium with constant lobby's strategy, i.e., there exists a pressure intensity p such that $\mu(G,R) = p$ holds for all G and R. We will show that this conjecture is consistent with the equilibrium conditions if p is appropriately chosen.

For a given value of p, let us define the strategies ϕ and ψ as in Proposition 2.1 and let U be the corresponding continuation value function. By this construction, ϕ, ψ, and U satisfy condition (2.8), which is identical to condition (2.24) in Proposition 2.4. Consequently, we only need to show that one can choose p in such a way that $(\mu(G,R), \phi(G,R), \psi(G,R))$ is a Stackelberg equilibrium of the stage game $\Gamma(G,R,U)$.

According to the construction and Proposition 2.1, it holds that $(\phi(G,R), \psi(G,R))$ maximizes $u(g+R, r, G) + pg + \delta U(\gamma G + g, \rho R + r)$ with respect to g and r, respectively. This means that $\phi(G,R) = \mathrm{BR}^g[p]$ and $\psi(G,R) = \mathrm{BR}^r[p]$ hold for all (G,R). Finally, consider the problem of the lobby, which is to maximize

$$\mathrm{BR}^g[p] - C(p) = \frac{\bar{y} + p - \gamma k(1-\delta\rho)}{1+\delta d} + \phi_G G + \phi_R R - \frac{c}{2}p^2.$$

Because the coefficients ϕ_G and ϕ_R are independent of p (see Proposition 2.1), it is clear that the lobby's objective is maximized by choosing $p = \hat{p} = 1/[c(1+\delta d)]$. This completes the proof. □

Proof of Proposition 2.5

We proceed in the same way as in the proof of Propositions 2.1-2.3, but this time α can be non-zero. From $w(g,r,G,R) = g$, it follows that $w_1'(G,R) = 1$ and $w_2'(G,R) =$

$w_3'(G,R) = w_4'(G,R) = 0$. Equations (2.44)-(2.45) become

$$U_1(G,R) = \gamma[\phi_0 + \phi_G G + (1+\phi_R)R - \bar{y}] - dG - p[\gamma + (1-\alpha)\phi_G],$$
$$U_2(G,R) = -\phi_0 - \phi_G G - (1+\phi_R)R + \bar{y} + \rho k - p(1-\alpha)\phi_R.$$

Consequently, condition (2.48) must hold. Applying this to the first-order conditions (2.42)-(2.43) and equating the coefficients of G and R in the two equations, we get (2.50)-(2.51), (2.53)-(2.54), and

$$\phi_0 - \bar{y} - p = \delta[\gamma[\phi_0 + \phi_G\phi_0 + (1+\phi_R)\psi_0 - \bar{y}] - d\phi_0 - p[\gamma + (1-\alpha)\phi_G]],$$
$$-k = \delta[\phi_0 + \phi_G\phi_0 + (1+\phi_R)\psi_0 - \bar{y} - \rho k + p(1-\alpha)\phi_R].$$

Hence, the coefficients ϕ_G, ϕ_R, ψ_G, and ψ_R turn out to be the same as in Proposition 2.1. As for ϕ_0 and ψ_0, however, they are given by

$$\phi_0 = \frac{\bar{y} + p(1-\alpha\delta\gamma) - \gamma k(1-\delta\rho)}{1+\delta d}$$
$$\psi_0 = \bar{y} - \frac{1-\delta\gamma}{\delta d}\alpha p - \frac{1-\delta\rho}{\delta}\left(1 + \frac{1-\delta\gamma}{\delta d}\right)k + \gamma\phi_0.$$

The remainder of the proof is identical to that of Proposition 2.1. □

Proof of Proposition 2.6

We proceed as in the proof of Propositions 2.1-2.3. Since $w(g,r,G,R) = (p^g g + p^r r)/p$, we have $pw_1'(G,R) = p^g$ and $pw_2'(G,R) = p^r$. With this and $\alpha = 0$, equations (2.44)-(2.45) reduce to

$$U_1(G,R) = \gamma[\phi_0 + \phi_G G + (1+\phi_R)R - \bar{y}] - dG - p^g(\gamma + \phi_G) - p^r\psi_G,$$
$$U_2(G,R) = -\phi_0 - \phi_G G - (1+\phi_R)R + \bar{y} + \rho k - p^g\phi_R - p^r(\rho + \psi_R).$$

Condition (2.48) then holds. Applying these results to the first-order conditions (2.42)-(2.43) and equating the coefficients of G and R in the two equations, one obtains (2.50)-(2.51), (2.53)-(2.54), and

$$\phi_0 - \bar{y} - p^g = \delta[\gamma[\phi_0 + \phi_G\phi_0 + (1+\phi_R)\psi_0 - \bar{y}] - d\phi_0 - p^g(\gamma + \phi_G) - p^r\psi_G],$$
$$-k + p^r = \delta[\phi_0 + \phi_G\phi_0 + (1+\phi_R)\psi_0 - \bar{y} - \rho k + p^g\phi_R + p^r(\rho + \psi_R)].$$

Thus, the values of ϕ_G, ϕ_R, ψ_G, and ψ_R remain the same as in Proposition 2.1. The values of ϕ_0 and ψ_0 are modified as

$$\phi_0 = \frac{\bar{y} + p^g - \gamma[(1-\delta\rho)k - p^r]}{1+\delta d},$$
$$\psi_0 = \bar{y} - \frac{1-\delta\rho}{\delta}\left(1 + \frac{1-\delta\gamma}{\delta d}\right)k + \frac{p^r(1+\delta d)}{\delta^2 d} + \gamma\phi_0.$$

From here, the proof is identical to the proof of Proposition 2.1. □

Proof of Equation (2.19)

It follows directly from (2.17) and (2.18) that

$$\frac{\partial G^*}{\partial p} = \left[\frac{1+p+\delta d}{(1+p)^2+\delta d}\frac{2(1+p)-\delta(\gamma+\rho)}{\delta(1+p)} + \frac{1+p}{\delta^2 d} + \frac{p(\gamma+\rho)}{(1+p)^2+\delta d}\right]\frac{(1+p)k}{(1+p)^2+\delta d},$$
$$\frac{\partial R^*}{\partial p} = -(1-\gamma)\frac{\partial G^*}{\partial p} - \left[\frac{(1-p^2+\delta d)d}{(1+p)^2+\delta d} + (\gamma+\rho)\left[1 + \frac{2p\delta d}{(1+p)^2+\delta d}\right]\right]\frac{k}{(1+p)^2+\delta d}.$$

The first expression is obviously positive, whereas the second one is negative whenever p is smaller than $\sqrt{1+\delta d}$.

Bibliography

[1] Abrego, L., Perroni, C., 2002. Investment subsidies and time-consistent environmental policy. Oxford Economic Papers 54, 617-635.

[2] Acemoglu, D., Akcigit, U., Hanley, D., Kerr, W., 2016. Transition to clean technology. Journal of Political Economy 124, 52-104.

[3] Acemoglu, D., Rafey, W., 2023. Mirage on the horizon: geoengineering and carbon taxation without commitment. Journal of Public Economics 219, 104802.

[4] Aidt, T.S., 2005. The rise of environmentalism, pollution taxes and intra-industry trade. Economics of Governance 6, 1–12.

[5] Allcott, H., Mullainathan, S., Taubinsky, D., 2014. Externalities, internalities, and the targeting of energy policy. Journal of Public Economics 112, 72-88.

[6] Anger, N., Böhringer, C., Lange, A., 2015. The political economy of energy tax differentiation across industries: theory and empirical evidence. Journal of Regulatory Economics 47, 78–98.

[7] Barbier, E.B., 2010. Corruption and the political economy of resource-based development: a comparison of Asia and Sub-Saharan Africa. Environmental and Resource Economics 46, 511-537.

[8] Başar, T., Olsder, G.J., 1995. Dynamic Noncooperative Game Theory, 2$^{\text{nd}}$ ed., Academic Press.

[9] Battaglini, M., 2021. Chaos and unpredictability in dynamic social problems, NBER Working Paper No. 28347, `http://www.nber.org/papers/w28347`.

[10] Battaglini, M., Harstad, B., 2016. Participation and duration of environmental agreements. Journal of Political Economy 124, 160-204.

[11] Biglaiser, G., Horowitz, J.K., Quiggin, J., 1995. Dynamic pollution regulation. Journal of Regulatory Economics 8, 33-44.

[12] Bremer, B., Bürgisser, R., 2023. Do citizens care about government debt? Evidence from survey experiments on budgetary priorities. European Journal of Political Research 62, 239-263.

[13] Brulle, R.J., 2018. The climate lobby: a sectoral analysis of lobbying spending on climate change in the USA, 2000 to 2016. Climatic Change 149, 289-303.

[14] Cherry, T.L., Kallbekken, S., Kroll, S., 2017. Accepting market failure: cultural worldviews and the opposition to corrective environmental policies. Journal of Environmental Economics and Management 85, 193-204.

[15] Chiappinelli, G., May, N., 2022. Too good to be true? Time-inconsistent renewable energy policies. Energy Economics 112, 106102.

[16] Coen, D., Richardson, J., 2009. Lobbying in the European Union: Institutions, Actors and Issues. Oxford University Press.

[17] Conconi, P., 2003. Green lobbies and transboundary pollution in large open economies. Journal of International Economics 59, 399-422.

[18] Cropper, M., Laibson, D., 1999. The implications of hyperbolic discounting for project evaluation. In: Portney, P.R., Weyant, J. (Eds.). Discounting and Intergenerational Equity. Resources for the Future. Taylor & Francis, Ch. 16.

[19] Damania, R., Fredriksson, P.G., List, J.A., 2003. Trade liberalization, corruption, and environmental policy formation: theory and evidence. Journal of Environmental Economics and Management 46, 490-512.

[20] Dechezleprêtre, A., Fabre, A., Kruse, T., Planterose, B., Chico, A.S., Stantcheva, S., 2022. Fighting climate change: international attitudes toward climate policies, NBER Working Paper No. 30265, `http://www.nber.org/papers/w30265`.

[21] DellaVigna, S., 2009. Psychology and economics: Evidence from the field. Journal of Economic Literature 47, 315-372.

[22] Douenne, T., Fabre, A., 2022. Yellow vests, pessimistic beliefs, and carbon tax aversion. American Economic Journal: Economic Policy 14, 81-110.

[23] Downie, C., 2017. Business actors, political resistance, and strategies for policymakers. Energy Policy 108, 583-592.

[24] Eichner, T., Kollenbach, G., 2023. Participation and duration of environmental agreements: investment lags matter. Journal of the Association of Environmental and Resource Economists 10, 33-53.

[25] Fredriksson, P.G., 1997. The political economy of pollution taxes in a small open economy. Journal of Environmental Economics and Management 33, 44-58.

[26] Fredriksson, P.G., Svensson, J., 2003. Political instability, corruption and policy formation: the case of environmental policy. Journal of Public Economics 87, 1383-1405.

[27] Fredriksson, P.G., Vollebergh, H.R., 2009. Corruption, federalism, and policy formation in the OECD: the case of energy policy. Public Choice 140, 205-221.

[28] Fujii, T., Karp, L., 2008. Numerical analysis of non-constant pure rate of time preference: a model of climate policy. Journal of Environmental Economics and Management 56, 83-101.

[29] Gerlagh, R., Liski, M., 2018. Consistent climate policies. Journal of the European Economic Association 16, 1-44.

[30] Gersbach, H., Glazer, A., 1999. Markets and regulatory hold-up problems. Journal of Environmental Economics and Management 37, 151-164.

[31] Grossman, G.M., Helpman, E., 1994. Protection for sale. American Economic Review 84, 833-850.

[32] Grey, F., 2018. Corporate lobbying for environmental protection. Journal of Environmental Economics and Management 90, 23-40.

[33] Haidar, B., Rojas, M.T.A., 2022. The relationship between public charging infrastructure deployment and other socio-economic factors and electric vehicle adoption in France. Research in Transportation Economics 95, 101208.

[34] Hanegraaff, M., Poletti, A., 2021. The rise of corporate lobbying in the European Union: an agenda for future research. Journal of Common Market Studies 59, 839–855.

[35] Harstad, B., 2012. Climate contracts: a game of emissions, investments, negotiations, and renegotiations. Review of Economic Studies 79, 1527-1557.

[36] Harstad, B., 2016. The dynamics of climate agreements. Journal of the European Economic Association 14, 719-752.

[37] Harstad, B., 2020. Technology and time inconsistency. Journal of Political Economy 128, 2653-2689.

[38] Helm, D., Hepburn, C., Mash, R., 2003. Credible carbon policy. Oxford Review of Economic Policy 19, 438-450.

[39] Hepburn, C., Duncan, S., Papachristodoulou, A., 2010. Behavioural economics, hyperbolic discounting and environmental policy. Environmental and Resource Economics 46, 189-206.

[40] Heutel, G., 2015. Optimal policy instruments for externality-producing durable goods under present bias. Journal of Environmental Economics and Management 72, 54-70.

[41] Holyoke, T., 2020. Interest Groups and Lobbying. Taylor & Francis.

[42] Iverson, T., Karp, L., 2021. Carbon taxes and climate commitment with non-constant time preference. Review of Economic Studies 88, 764-799.

[43] Kalkuhl, M., Steckel, J.C., Edenhofer, O., 2020. All or nothing: climate policy when assets can become stranded. Journal of Environmental Economics and Management 100, 102214.

[44] Karp, L., 2005. Global warming and hyperbolic discounting. Journal of Public Economics 89, 261-282.

[45] Karp, L., Tsur, Y., 2011. Time perspective and climate change policy. Journal of Environmental Economics and Management 62, 1-14.

[46] Kollenbach, G., 2022. International environmental agreements and black technology. Environmental and Resource Economics 82, 601-624.

[47] Marsiliani, L., Renström, T.I., 2000. Time inconsistency in environmental policy: tax earmarking as a commitment solution. Economic Journal 110, 123-138.

[48] Pacca, L., Curzi, D., Rausser, G., Olper, A., 2021. The role of party affiliation, lobbying, and electoral incentives in decentralized US state support of the environment. Journal of the Association of Environmental and Resource Economists 8, 617-653.

[49] Pahle, M., Burtraw, D., Flachsland, C., Kelsey, N., Biber, E., Meckling, J., Edenhofer, O., Zysman, J., 2018. Sequencing to ratchet up climate policy stringency. Nature Climate Change 8, 861-867.

[50] Peleg, B., Yaari, M.E., 1973. On the existence of a consistent course of action when tastes are changing. Review of Economic Studies 40, 391–401.

[51] Petrakis, E., Xepapadeas, A., 2003. Location decisions of a polluting firm and the time consistency of environmental policy. Resource and Energy Economics 25, 197-214.

[52] Phelps, E., Pollak, R.A., 1968. On second-best national saving and game-equilibrium growth. Review of Economic Studies 35, 185–199.

[53] Pichler, P., Sorger, G., 2018. Delegating climate policy to a supranational authority: a theoretical assessment. European Economic Review 101, 418-440.

[54] Rozenberg, J., Vogt-Schilb, A., Hallegatte, S., 2020. Instrument choice and stranded assets in the transition to clean capital. Journal of Environmental Economics and Management 100, 102183.

[55] Sælen, H., Kallbekken, S., 2011. A choice experiment on fuel taxation and earmarking in Norway. Ecological Economics 70, 2181-2190.

[56] Schopf, M., Voss, A., 2019. Bargaining over natural resources: governments between environmental organizations and extraction firms. Journal of Environmental Economics and Management 97, 208-240.

[57] Ulph, A., Ulph, D., 2013. Optimal climate change policies when governments cannot commit. Environmental and Resource Economics 56, 161-176.

[58] UNEP, 2021. The Emissions Gap Report 2021, United Nations Environment Programme (UNEP), Nairobi, Kenya.

[59] Voss, A., Schopf, M., 2021. Lobbying over exhaustible-resource extraction. European Economic Review 135, 103740.

Chapter 3

Social Security, Bequests, and Social Comparisons[1]

3.1 Introduction

It is well known that when agents do not have a bequest motive, unfunded social security discourages capital accumulation, whereas it is neutral to capital accumulation when agents are linked with their descendants through positive bequests. These are the implications of two standard overlapping generations (OLG) models developed by Diamond [17] and Barro [9], respectively. Though both models are useful for the analysis of public intergenerational transfers, they have both been criticized for ignoring consumer heterogeneity (see, among others, Mankiw [32]), as is apparent in the data. In particular, existing empirical evidence (e.g., Altonji et al. [4], Laitner and Ohlsson [26], Altonji and Villanueva [5], Adermon et al. [1]) suggests that a lot of individuals do not receive inheritances or transfers from their parents, while the amount received by individuals varies significantly and positively with their parents' income or wealth. This naturally raises the question of how unfunded social security affects capital accumulation when bequests are left only within relatively rich families. Should the results lie between those found in Diamond's and Barro's models, and so the long-run capital stock declines due to social security?

The importance of addressing the issue of unfunded social security is highlighted by ongoing discussions in many countries about reforming such systems. In these discussions, however, bequests have received little attention. Unfunded social security systems are primarily designed to support individuals with limited financial resources. At the same time, as bequests become more concentrated among the rich, unfunded social security can potentially increase wealth inequality in the long run. This is because poorer individuals who do not benefit from inherited wealth become more likely to save only for

[1] Joint work with Kirill Borissov. The chapter has been published in the Journal of Public Economic Theory.

their retirement, and these savings are likely to be reduced due to public pensions. Thus, when discussing the role of unfunded social security systems, it is relevant to take the distribution of bequests into account.

This paper investigates the long-run effects of unfunded social security on capital stock and wealth inequality in an OLG model where individuals differ in the bequests they receive from parents. With this heterogeneity, parents who do not inherit enough wealth themselves (if any) may leave no bequest if they overspend on personal consumption. Overspending on personal consumption may arise from various sources in reality. As noted by Stiglitz [40], one important source in developed countries is social comparisons: "Individuals' concerns with their consumption relative to that of others — the problem of "keeping up with the Joneses" — helps explain why so many Americans live beyond their means" (p. 105). Duesenberry [19] was the first to suggest that relative considerations cause poorer families to have a higher propensity to consume. More recently, several papers (Moav and Neeman [36, 37], Alvarez-Cuadrado and Long [7, 8], Borissov [13], Genicot and Ray [21], Dioikitopoulos et al. [18]) have argued that concerns for social status or socially determined aspirations have important implications for inequality and poverty. Following this literature, we assume that individuals' decisions are subject to social comparisons, and we characterize different long-run states of the bequest distribution for various values of the parameters reflecting social comparisons.[2]

We examine three long-run states of the bequest distribution that correspond to three long-run equilibrium regimes of the economy: (i) the egalitarian regime where all individuals leave bequests; (ii) the egalitarian regime where nobody leaves bequests; and (iii) the inegalitarian regime where some individuals leave bequests and some do not. In the first regime, the economy always converges to full equality, and unfunded social security does not affect capital accumulation. Similar to the model of Barro [9], our model predicts that a reduction in savings due to social security is exactly offset by an increase in savings due to parental bequests. On the other hand, we show that in the egalitarian regime with no bequests, as in the model of Diamond [17], unfunded social security discourages capital accumulation and, hence, leads to a lower capital stock in the long run. Finally and most interestingly, we consider the inegalitarian regime, which is characterized by the division of the population into two homogeneous groups of families from some time on, with only individuals from the wealthiest families continuing to leave bequests. In this regime, unfunded social security increases long-run wealth inequality by redistributing wealth from the non-wealthiest families to the wealthiest ones. However, at the same time, it also increases the long-run capital stock.

[2]There is considerable empirical and experimental evidence that people have relative concerns (see, e.g., Heffetz and Frank [22]). Furthermore, there is a large number of studies dealing with the effects of social comparisons on public policies. In the context of social security, social comparisons have been discussed by Knell [23] and Bilancinia and D'Antoni [11]. Their models, contrary to ours, ignore bequests and any heterogeneity across individuals of the same generation.

Our results contribute to the literature analyzing unfunded social security in the presence of heterogeneous (intended) bequests. Under the assumption that some fraction of agents does not have a bequest motive and therefore does not bequeath, Michel and Pestieau [33] and Laitner [25] have shown that in the long run, the neutrality of unfunded social security to capital accumulation observed in Barro's model still holds at the aggregate level. One can reach the same conclusion by considering agents with low and high intergenerational discount factors such that only the latter leave positive bequests asymptotically (see Smetters [39]). Moreover, the neutrality holds in models where bequests are left in the long run by only the most altruistic agents while being additionally heterogeneous in productivity or by the most altruistic agents together with agents having some preference for wealth (see Michel and Pestieau [34] or Pestieau and Thibault [38], respectively). Unlike all these studies and similar to our paper, Caballé and Fuster [16] have assumed that young individuals have identical preferences and differ only in their inheritance. These authors, however, have further assumed that when old, individuals learn whether they are of the altruistic type or not. Using such uncertainty in preferences, the authors have numerically demonstrated that if altruistic agents leave positive bequests in the long run, the introduction of an unfunded social security system crowds out capital.

To the best of our knowledge, our paper is the first to show that unfunded social security increases the long-run capital stock when some individuals leave bequests and some do not. That said, our finding that social security increases long-run wealth inequality too is consistent with the results obtained by Michel and Pestieau [33] and Laitner [25]. Furthermore, consistent with Caballé and Fuster [16], we show that an increase in the social security contribution rate can serve as a source of permanent wealth heterogeneity because it can shift the economy from the egalitarian regime with no bequests to the inegalitarian regime.

More broadly, we contribute to the literature arguing that agent heterogeneity can challenge the implications of government policies from standard representative agent models (see, e.g., Galí et al. [20], Werning [41], and Bilbiie [12]). In our study, we challenge the implications of unfunded public pensions from two such models, the one proposed by Diamond [17] and the one proposed by Barro [9], by focusing on heterogeneity in bequest behavior.

The rest of the paper is organized as follows. We present the model in Section 3.2 and characterize three equilibrium regimes in Section 3.3. Section 3.4 examines the long-run aggregate and distributional effects of social security. In the final section, we conclude the paper and suggest potential avenues for future research.

3.2 The Model

The framework described below is the OLG extension of the successive generations model with intergenerational altruism and social comparisons developed by Borissov [13]. Similar models have been used to analyze public debt and tax policies (Borissov and Kalk [15], Borissov and Hashimzade [14]). Here, our interest lies in the analysis of (unfunded) social security.

3.2.1 Demographics

Consider an OLG economy in discrete time indexed by $t \in \{0, 1, \dots\}$. At any date t, a new generation of individuals is born. They live for two periods: young and old. At the beginning of old age, each individual gives birth to one offspring belonging to the same dynasty. There exist N such dynasties, which we index by $j \in \{1, \dots, N\}$, and hence every generation consists of N individuals.[3] Dynasties differ only in their initial endowment of capital, denoted by s_{-1}^j for dynasty j.

3.2.2 Firms

The economy produces a single good that is either consumed or invested. The production function takes the Cobb-Douglas form with a constant capital share $\alpha \in (0, 1)$:

$$Y_t = K_t^\alpha L_t^{1-\alpha},$$

where Y_t is total output, K_t is the capital stock, and L_t is labor input, all at date t. We assume that capital fully depreciates after one period of use.

All markets are perfectly competitive. Therefore, the rate of return to capital, R_t, and the wage rate, w_t, satisfy

$$R_t = \alpha k_t^{\alpha-1} \quad \text{and} \quad w_t = (1-\alpha)k_t^\alpha, \tag{3.1}$$

where $k_t := K_t/L_t$ is the capital stock per worker at date t.

3.2.3 Individuals

Each individual inelastically supplies one unit of labor when young and retires when old, as in Diamond [17]. We then have $L_t = N$ for every $t \geq 0$ in equilibrium. During working age, individual j born at date t receives a bequest b_t^j from her parent and earns the wage w_t on which she pays a payroll tax at rate $\tau \in [0, 1)$. Denoted by ω_t^j, the resulting income

[3] Throughout the paper, we refer to an individual belonging to dynasty j as individual j.

is allocated between consumption c_t^j and savings s_t^j:

$$c_t^j + s_t^j = \omega_t^j := (1-\tau)w_t + b_t^j. \tag{3.2}$$

After retirement, the individual receives social security benefits θ_{t+1} from the government and the proceeds of her savings $R_{t+1}s_t^j$ (i.e., R_{t+1} corresponds to the gross interest rate in the economy at date $t+1$). These resources can be used by the individual to consume d_{t+1}^j or to leave a bequest b_{t+1}^j to her offspring. The second-period budget constraint therefore reads

$$d_{t+1}^j + b_{t+1}^j = R_{t+1}s_t^j + \theta_{t+1}. \tag{3.3}$$

Bequests are assumed to be non-negative, i.e.,

$$b_{t+1}^j \geq 0. \tag{3.4}$$

Preferences of individual j born at date t are represented by the following lifetime utility function:

$$U_t^j(\hat{c}_t^j, \hat{d}_{t+1}^j, \hat{\omega}_{t+1}^j) = (1-\delta)\ln \hat{c}_t^j + \delta \ln \hat{d}_{t+1}^j + \beta \ln \hat{\omega}_{t+1}^j,$$

where $\delta \in (0,1)$ is the degree of patience and $\beta > 0$ is the degree of parental altruism. When modeling altruism, we follow the approach of Becker and Tomes [10] and assume that individuals care about the disposable income of their offspring, in particular about the disposable income during her working life (similar to Lambrecht et al. [27, 28], Michel et al. [35], Kunze [24], and Maebayashi [31]).

The variables $\hat{c}_t^j$ and $\hat{d}_{t+1}^j$ represent effective young- and old-age consumption, and $\hat{\omega}_{t+1}^j$ represents the effective level of the offspring's disposable income.[4] As mentioned in the Introduction, the behavior of individuals is subject to social comparisons in our economy. Accordingly, we assume that individuals born at t derive utility by comparing their $(c_t^j, d_{t+1}^j, \omega_{t+1}^j)$ with respective values $(\bar{c}_t, \bar{d}_{t+1}, \bar{\omega}_{t+1})$ of some reference group. Following Ljungqvist and Uhlig [29] and subsequent papers such as Alvarez-Cuadrado and Long [7], we use an additive specification for these comparisons, namely

$$\begin{aligned}
\hat{c}_t^j &= c_t^j - \gamma_c \bar{c}_t, \\
\hat{d}_{t+1}^j &= d_{t+1}^j - \gamma_d \bar{d}_{t+1}, \\
\hat{\omega}_{t+1}^j &= \omega_{t+1}^j - \gamma_\omega \bar{\omega}_{t+1},
\end{aligned}$$

with parameters $\gamma_c \in [0,1)$, $\gamma_d \in [0,1)$, and $\gamma_\omega \in [0,1)$ measuring the intensity of social comparisons applied to each argument of the utility function U_t^j. Note that we allow γ_ω to

[4] Our use of the term "effective" follows Alonso-Carrera et al. [2, 3].

be smaller than γ_c or γ_d, so that we can capture that individuals may be less affected by social comparisons when they leave bequests than when they consume. Alvarez-Cuadrado and Long [8] have argued that this may be the case due to the limited observability of bequests and their concentration in the upper tail of the wealth distribution. Similarly, we allow γ_c to be larger than γ_d or γ_ω, which, in turn, can capture that the importance of social comparisons may decrease with age; see Alvarez-Cuadrado and Long [8] and Alvarez-Cuadrado and Japaridze [6] for a discussion of why this is also plausible. In contrast, models with pure altruism of Barro [9] (e.g., Alonso-Carrera et al. [3]) assume that bequests and young-age consumption share the same intensity of social comparisons, thereby failing to accommodate either one or both possibilities.[5]

To sum up, the problem of individual j born at date t is to choose $(c_t^j, s_t^j, d_{t+1}^j, b_{t+1}^j)$ so as to maximize the lifetime utility $U_t^j(c_t^j - \gamma_c \bar{c}_t, d_{t+1}^j - \gamma_d \bar{d}_{t+1}, \omega_{t+1}^j - \gamma_\omega \bar{\omega}_{t+1})$ subject to the budget constraints (3.2)-(3.3), and the non-negativity constraint on bequests (3.4). Provided that the individual takes the reference levels $(\bar{c}_t, \bar{d}_{t+1}, \bar{\omega}_{t+1})$ as given, the first-order conditions of this problem can be written as

$$(1-\delta)(d_{t+1}^j - \gamma_d \bar{d}_{t+1}) = \delta R_{t+1}(c_t^j - \gamma_c \bar{c}_t), \tag{3.5}$$

$$\delta(\omega_{t+1}^j - \gamma_\omega \bar{\omega}_{t+1}) \geq \beta(d_{t+1}^j - \gamma_d \bar{d}_{t+1}) \quad (= \text{if } b_{t+1}^j > 0). \tag{3.6}$$

It is important to stress that in each generation, individuals differ only in bequests transferred by their parents at date t. All preference parameters and reference levels are common across individuals. In what follows, we assume that the reference group for individuals born at date t is the one associated with the average individual of their generation, that is,

$$\bar{c}_t = \frac{1}{N}\sum_{j=1}^{N} c_t^j, \;\; \bar{d}_{t+1} = \frac{1}{N}\sum_{j=1}^{N} d_{t+1}^j, \;\; \bar{\omega}_{t+1} = \frac{1}{N}\sum_{j=1}^{N} \omega_{t+1}^j. \tag{3.7}$$

3.2.4 Government

The government runs a balanced-budget unfunded social security system, collecting taxes from young individuals at date t and transferring those funds to the current old. With constant population, the government's budget constraint is simply given by

$$\theta_t = \tau w_t. \tag{3.8}$$

Our primary objective is to examine the aggregate and distributional effects of changes in the social security contribution rate τ.

[5] In a more general context, the type of altruism considered in the present paper (often referred to in the literature as "family" altruism) seems more realistic compared to pure altruism because the former does not require individuals to forecast the behavior of all their descendants and hence all future prices to make decisions today.

3.2.5 Equilibrium

An equilibrium of the economy described above is defined in a standard way. Individuals and firms solve their maximization problems, the government balances its budget, and all markets clear. Formally, suppose we are given an initial state $\{(s_{-1}^j)_{j=1}^N, K_0\}$, where $(s_{-1}^j)_{j=1}^N$ is the distribution of initial capital endowments among dynasties and $K_0 = \sum_{j=1}^N s_{-1}^j = Nk_0$ is the initial capital stock. An intertemporal equilibrium starting from the initial state $\{(s_{-1}^j)_{j=1}^N, K_0\}$ is a sequence $\{(c_t^j, s_t^j, d_t^j, b_t^j)_{j=1}^N, k_{t+1}\}_{t=0}^\infty$ such that for all $t \geq 0$:

(a) $(c_t^j, s_t^j, d_{t+1}^j, b_{t+1}^j)$ is the optimal choice of individual j born at date t, which is determined by (3.2)-(3.6) with $(\bar{c}_t, \bar{d}_{t+1}, \bar{\omega}_{t+1})$ given by (3.7), θ_{t+1} given by (3.8), and (w_t, w_{t+1}, R_{t+1}) given by (3.1);

(b) (d_0^j, b_0^j) is the optimal choice of individual j who is old at $t = 0$, which is determined by

$$d_0^j + b_0^j = R_0 s_{-1}^j + \theta_0,$$

$$\delta(\omega_0^j - \gamma_\omega \bar{\omega}_0) \geq \beta(d_0^j - \gamma_d \bar{d}_0) \quad (= \text{if } b_0^j > 0)$$

with $b_0^j \geq 0$, $\bar{d}_0 = \sum_{j=1}^N d_0^j/N$, $\bar{\omega}_0 = \sum_{j=1}^N \omega_0^j/N$, θ_0 given by (3.8), and (w_0, R_0) given by (3.1);

(c) $k_{t+1} = \sum_{j=1}^N s_t^j/N$.

3.3 Three Equilibrium Regimes

In the present section, we characterize three equilibrium regimes that differ in whether bequests are left in the long run by all, none, or some of the N dynasties. We will call these regimes an "egalitarian regime with bequests", an "egalitarian regime with no bequests", and "an inegalitarian regime", respectively, and we will provide conditions on the parameters of the model that lead to each of them.

All the results in this section are presented without making additional simplifying assumptions. Yet, it is helpful to begin with a brief informal discussion of the simple case where

$$\gamma_c > \gamma_d = \gamma_\omega = 0, \tag{3.9}$$

meaning that individuals are influenced by social comparisons exclusively during their working life.[6] In this case, conditions for each equilibrium regime to occur can be expressed solely in terms of the intensity of social comparisons when young, γ_c, and the

[6]Note that this case is consistent with the two plausible properties of social comparisons mentioned in Subsection 3.2.3.

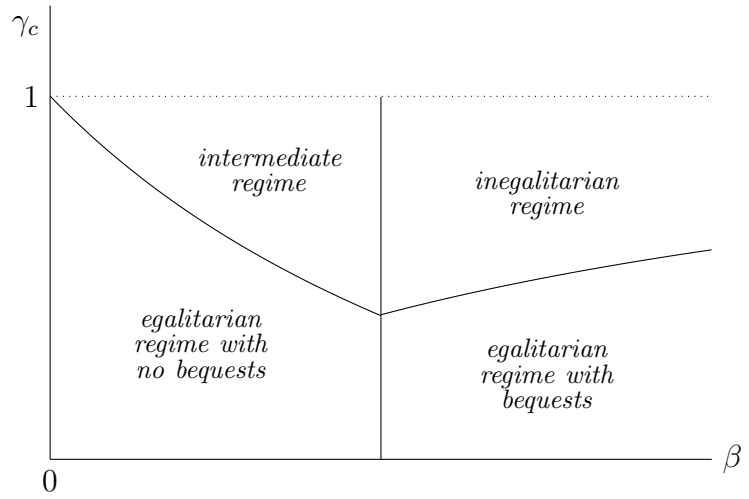

Figure 3.1: Equilibrium regimes for various values of γ_c and β

degree of altruism, β. As illustrated in Figure 3.1, the inegalitarian regime, for example, occurs if both γ_c and β are sufficiently high. Intuitively, social comparisons over young-age consumption discourage individuals from saving, while altruism toward children encourages individuals to bequeath. Furthermore, the first effect is particularly pronounced for individuals belonging to less wealthy dynasties. Then, when both γ_c and β are high, rich individuals are motivated and find it possible to leave bequests, whereas poor individuals, even though equally motivated, find themselves constrained in their bequest decision. When γ_c is low, in contrast, and β is high, all individuals save enough to bequeath, leading to the egalitarian regime with bequests. On the other hand, when both γ_c and β are low, all individuals are motivated to save primarily for their old-age consumption, leading to the egalitarian regime with no bequests. Finally, in Figure 3.1, we indicate one more possible regime, which can be called an "intermediate regime" and which occurs when γ_c is high, whereas β is low. We omit the analysis of this regime in the paper, however, because it involves a complex equilibrium characterization without providing new insights into the role of social security.[7]

We now turn to a more formal analysis of the model. Let us consider an intertemporal equilibrium $\{(c_t^j, s_t^j, d_t^j, b_t^j)_{j=1}^N, k_{t+1}\}_{t=0}^\infty$ starting from $\{(s_{-1}^j)_{j=1}^N, K_0\}$. Without loss of generality, we assume

$$s_{-1}^1 = \cdots = s_{-1}^n > s_{-1}^{n+1} \geq \cdots \geq s_{-1}^N,$$

[7]The intermediate regime excludes the possibility that all dynasties leave bequests in the long run, but not that a fraction of dynasties do. In the latter situation, the policy implications of the model remain the same as in the inegalitarian regime. On the other hand, if none of the dynasties leave bequests in the long run in the intermediate regime, the policy implications of the model remain the same as in the egalitarian regime with no bequests.

where $n \in \{1, \ldots, N\}$ is the exogenous number of dynasties having the highest initial capital endowment. Then, it is not difficult to show that for any pair of dynasties (j, h), $s_{-1}^j = s_{-1}^h$ implies $b_0^j = b_0^h$, while $s_{-1}^j > s_{-1}^h$ implies $b_0^j \geq b_0^h$. Moreover, at any date $t \geq 0$, $b_t^j = b_t^h$ implies $b_{t+1}^j = b_{t+1}^h$, while $b_t^j > b_t^h$ implies $b_{t+1}^j \geq b_{t+1}^h$. As a result, the following relationship holds for all $t \geq 0$:

$$b_t^1 = \cdots = b_t^n \geq b_t^{n+1} \geq \cdots \geq b_t^N. \tag{3.10}$$

That is, individuals belonging to the initially richest dynasties always leave the largest bequests. We use $M_t \in \{1, \ldots, N\}$ to denote the (endogenous) number of individuals leaving positive bequests at date t, i.e.,

$$b_t^j > 0,\ j \leq M_t;\ b_t^j = 0,\ j > M_t,$$

and $m_t := M_t/N$ to denote their share in the population. Let us also denote the average bequest at t by $\bar{b}_t := \sum_{j=1}^N b_t^j/N$.

With these preliminaries, we can state our first result. All proofs can be found in Appendix 3.A.

Proposition 3.1. *Suppose that the following conditions are satisfied:*

$$\frac{1-\gamma_d}{1-\gamma_\omega} > \frac{\delta(1-\tau)(1-\alpha)}{\beta[\alpha+\tau(1-\alpha)]} \quad \textit{and} \quad \frac{1-\gamma_d}{1-\gamma_c} < \frac{1+\beta(1-\alpha)}{1-\delta}\frac{1-\gamma_d}{1-\gamma_\omega} - \frac{\delta\alpha}{1-\delta}. \tag{3.11}$$

There exists a date T_1 such that for all $t \geq T_1$, it holds that $M_t = N$ (or $m_t = 1$) and

$$\frac{\bar{b}_t}{k_t^\alpha} = \frac{\beta(1-\gamma_d)[\alpha+\tau(1-\alpha)] - \delta(1-\gamma_\omega)(1-\tau)(1-\alpha)}{\beta(1-\gamma_d)+\delta(1-\gamma_\omega)}, \tag{3.12}$$

$$\frac{k_{t+1}}{k_t^\alpha} = \frac{\beta\alpha(1-\gamma_c)(1-\gamma_d)}{\beta\alpha(1-\gamma_c)(1-\gamma_d) + (1-\gamma_\omega)[\delta\alpha(1-\gamma_c)+(1-\delta)(1-\gamma_d)]}. \tag{3.13}$$

Furthermore, it holds that

$$\lim_{t\to\infty} \frac{b_t^j}{\bar{b}_t} = \lim_{t\to\infty} \frac{s_t^j}{k_{t+1}} = 1,\ j = 1, \ldots, N.$$

Thus, under parameter restriction (3.11), the bequest motive turns out to be operative for all dynasties from date T_1 onward, and their bequests converge to the same value as t goes to infinity. Moreover, starting from date T_1, the aggregate saving rate k_{t+1}/k_t^α is constant and independent of the distribution of initial capital endowments (i.e., it does not depend on n). Since newborn individuals differ only in their bequests received, which are shown to be identical in the long run, all dynasties also own the same share of long-run

capital stock and of long-run national income. We will therefore refer to this case as the egalitarian regime with (positive) bequests.

Next, we turn to another extreme case where the bequest motive is inoperative for all dynasties. Proposition 3.2 provides sufficient conditions for this case to occur and characterize the associated dynamics of capital accumulation.

Proposition 3.2. *Suppose that the following conditions are satisfied:*

$$\frac{1-\gamma_d}{1-\gamma_\omega} < \frac{\delta(1-\tau)(1-\alpha)}{\beta[\alpha+\tau(1-\alpha)]} \quad \textit{and} \quad \frac{1-\gamma_d}{1-\gamma_c} < \frac{\delta[(1+\beta)(1-\tau)(1-\alpha)-\beta\alpha]}{\beta(1-\delta)[\alpha+\tau(1-\alpha)]}. \tag{3.14}$$

There exists a date T_2 such that for all $t \geq T_2$, it holds that $M_t = 0$ (or $m_t = 0$), $s_t^j = k_{t+1}$ for every $j = 1 \ldots, N$, and

$$\frac{k_{t+1}}{k_t^\alpha} = \frac{\delta\alpha(1-\gamma_c)(1-\tau)(1-\alpha)}{\delta\alpha(1-\gamma_c)+(1-\delta)(1-\gamma_d)[\alpha+\tau(1-\alpha)]}. \tag{3.15}$$

Similar to Proposition 3.1, the distribution of initial capital endowments has no long-run effect on the aggregate saving rate. However, in contrast to Proposition 3.1, we note that dynasties become homogeneous starting from some date (namely, $t = T_2$) rather than asymptotically. This reflects the model feature that inheritance is the sole source of heterogeneity across individuals when they are born. Hence, if none of the dynasties leave bequests, individuals within each generation become identical. We will refer to the case described in Proposition 3.2 as the egalitarian regime with no bequests.

After presenting the results where the economy behaves in the long run as if there was a single representative dynasty, let us now focus on the scenario where inequality between different dynasties persists over time.[8] To state the corresponding proposition, we first need to look at the dynamics of the bequest-output ratio $\bar{b}_t/k_t^\alpha$ when the bequest motive is operative for at least some dynasties. As we show in Lemma 3.2 in Appendix 3.A, if the number of dynasties with an operative bequest motive is positive and does not decrease between dates t and $t+1$, i.e., $M_{t+1} \geq M_t > 0$ (or $m_{t+1} \geq m_t > 0$), then $\bar{b}_t/k_t^\alpha$ evolves according to

$$\frac{\bar{b}_{t+1}}{k_{t+1}^\alpha} = \frac{A(m_{t+1}) + B(m_{t+1})\frac{\bar{b}_t}{k_t^\alpha}}{C(m_{t+1}) + D(m_{t+1})\frac{\bar{b}_t}{k_t^\alpha}}.$$

Defined in Appendix 3.A, the functions A, B, C, and D are such that if

$$\frac{1-\gamma_d}{1-\gamma_\omega} > \frac{\delta(1-\tau)(1-\alpha)}{\beta[\alpha+\tau(1-\alpha)]},$$

[8] The persistence of wealth inequality across multiple generations of the same families has been empirically documented by Adermon et al. [1], who have also found that bequests and inter vivos gifts play a central role in this persistence.

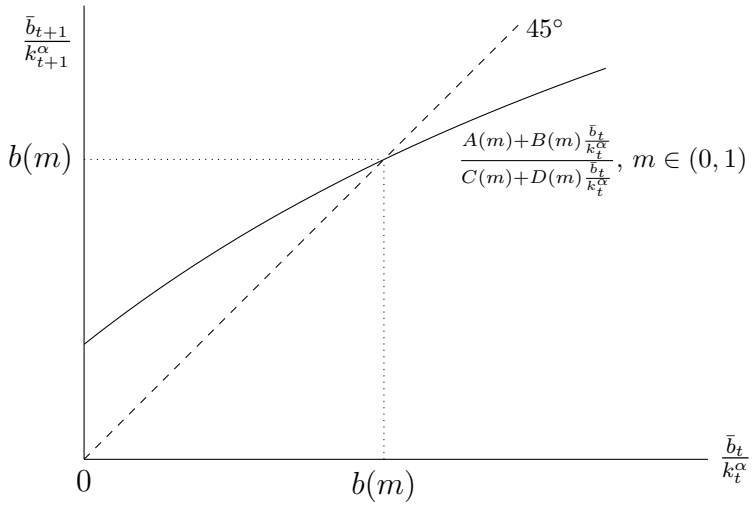

Figure 3.2: Determination of the steady-state value of the bequest-output ratio

then the relationship between $\bar{b}_{t+1}/k^{\alpha}_{t+1}$ and $\bar{b}_t/k^{\alpha}_t$ for any given $m_{t+1} \in (0,1)$ takes the form depicted by the solid curve in Figure 3.2. Provided that m_{t+1} remains constant at a level m, the intersection of this curve with the 45-degree line gives the steady-state value of the bequest-output ratio, which is defined by

$$b(m) := \frac{B(m) - C(m) + \sqrt{[B(m) - C(m)]^2 + 4A(m)D(m)}}{2D(m)}.$$

Denoting further the steady-state saving rate associated with $b(m)$ by

$$s(m) := \frac{\delta\alpha(1-\gamma_c)[(1-\tau)(1-\alpha) + b(m)]}{\delta\alpha(1-\gamma_c) + (1-\delta)(1-\gamma_d)[\alpha + \tau(1-\alpha) - b(m)]},$$

we are now able to formulate the following proposition.

Proposition 3.3. *Suppose that the following conditions are satisfied:*

$$\frac{1-\gamma_d}{1-\gamma_\omega} > \frac{\delta(1-\tau)(1-\alpha)}{\beta[\alpha + \tau(1-\alpha)]} \quad \textit{and} \quad \frac{1-\gamma_d}{1-\gamma_c} > \frac{1+\beta(1-\alpha)}{1-\delta}\frac{1-\gamma_d}{1-\gamma_\omega} - \frac{\delta\alpha}{1-\delta}. \tag{3.16}$$

There exists a date T_3 such that for all $t \geq T_3$, it holds that $M_t = n$ (or $m_t = n/N$) and

$$\frac{b^j_t}{\bar{b}_t} = \frac{N}{n} \quad \textit{and} \quad \frac{s^j_t}{k_{t+1}} = 1 + \frac{N-n}{n}\left(\frac{\delta k^{\alpha}_t}{k_{t+1}}\frac{\bar{b}_t}{k^{\alpha}_t} + \frac{1-\delta}{\alpha}\frac{\bar{b}_{t+1}}{k^{\alpha}_{t+1}}\right), \; j \leq n; \tag{3.17}$$

$$b^j_t = 0 \quad \textit{and} \quad \frac{s^j_t}{k_{t+1}} = 1 - \frac{\delta k^{\alpha}_t}{k_{t+1}}\frac{\bar{b}_t}{k^{\alpha}_t} - \frac{1-\delta}{\alpha}\frac{\bar{b}_{t+1}}{k^{\alpha}_{t+1}}, \; j > n. \tag{3.18}$$

Furthermore, it holds that

$$\lim_{t\to\infty} \frac{\bar{b}_t}{k_t^\alpha} = b\left(\frac{n}{N}\right) \quad \textit{and} \quad \lim_{t\to\infty} \frac{k_{t+1}}{k_t^\alpha} = s\left(\frac{n}{N}\right).$$

This proposition says that as long as the parameters of the model satisfy (3.16), the economy is characterized by a partition of the set of dynasties into two homogeneous groups from date T_3 onward. In one group, dynasties leave the same positive bequest, and those dynasties had the highest capital endowment at $t = 0$. In the other group, dynasties leave no bequests. Therefore, individuals belonging to dynasties of this group consume their entire lifetime resources, saving solely to finance their old-age consumption. While the latter group of dynasties always owns a lower share of capital stock (wealth) than the former group, these shares continue to evolve after $t = T_3$, and so do the bequest-output ratio $\bar{b}_t/k_t^\alpha$ and the aggregate saving rate k_{t+1}/k_t^α. As time goes on, they all converge to the long-run levels that depend on n, which implies that the distribution of initial endowments has a persistent effect not only on individual variables, such as the dynasty's share of capital stock, but also on aggregate variables, such as the aggregate saving rate.[9]

Note that since n can take any integer value from 1 to N, Proposition 3.3 in particular allows for the set of dynasties leaving bequests in the long run to be of size N. Then, it is easy to verify that $b(1)$ and $s(1)$ are equal to the bequest-output ratio and the saving rate, respectively, found in the egalitarian regime with bequests. Here, however, the scenario with $M_t = N$ is possible in the long run only if capital endowments are evenly distributed across the founding members of each dynasty, or in other words, if dynasties are never heterogeneous. In what follows, we will refer to the general case described in Proposition 3.3 as the inegalitarian regime.[10]

3.4 Social Security

The current section aims to examine how the social security contribution rate τ affects capital accumulation in each of the three equilibrium regimes and how it affects inequality in the inegalitarian regime. We are interested in the long run, where, as we have seen in the previous section, the fraction of dynasties leaving positive bequests m_t is constant and equal to either 1, 0, or n/N.

Consider first the economy in the egalitarian regime with bequests. It is apparent that τ does not appear in equation (3.13), implying that social security does not affect

[9] To put it differently, both individual and aggregate variables in the long run depend on the fraction of individuals leaving bequests, which is equal to n/N from date T_3 onward. This stands in contrast to the two papers mentioned in the Introduction, namely Michel and Pestieau [33] and Laitner [25], where the steady-state capital stock (per worker) cannot depend on that fraction and is the same as if all individuals left bequests.

[10] Alternatively, one can call this regime a "two-class" regime.

capital accumulation. This finding is not surprising in view of Ricardian equivalence and is driven by the fact that a reduction in savings due to social security is exactly offset by an increase in savings due to bequests. As a consequence, consumption and investment are unaffected by social security. Indeed, it follows from (3.12) that the value of $\bar{b}_t - \tau(1-\alpha)k_t^\alpha$ or, equivalently, $\bar{b}_t - \tau w_t$ remains constant, which means that a unit increase in the social security contribution leads to a unit increase in the average bequest.

Turning to another egalitarian regime with no bequests, the standard result known in the literature also remains valid. Specifically, by looking at the effect of τ in (3.15), we conclude that in the absence of private intergenerational transfers, social security reduces the aggregate saving rate. As in the canonical OLG model without altruism, this result relies on the fact that social security discourages savings used to finance old-age consumption.

Given that the aggregate saving rate is independent of and negatively dependent on τ when bequests are left by all and none of the dynasties, respectively, it is tempting to conjecture that the presence of social security cannot increase the aggregate saving rate when bequests are left by only a fraction of dynasties. However, such reasoning neglects the distributional effect of social security. Note that under conditions of Proposition 3.3, the following relationships hold (see Appendix 3.A):

$$\frac{\partial s(n/N)}{\partial \tau} \geq 0 \;\Leftrightarrow\; \frac{\partial b(n/N)}{\partial \tau} \geq 1-\alpha \;\Leftrightarrow\; n \leq N. \tag{3.19}$$

That is, as long as initial capital endowments are unevenly distributed across dynasties, the long-run saving rate in the inegalitarian regime necessarily increases with τ. From the second inequality in (3.19), we can deduce that a higher saving rate is required since a unit increase in the social security contribution causes the average bequest to increase by more than one unit. The latter observation, in turn, can be explained as follows. Because social security discourages savings for dynasties that do not leave bequests, it redistributes wealth from these dynasties to those leaving positive bequests, which are the wealthiest ones. As the wealthiest dynasties become even wealthier, their elderly members bequeath a larger fraction of the capitalized value of inheritance they received during their youth. This makes the average bequest overreact to changes in the social security contribution.

Consistent with the explanation given above, social security not only increases the long-run aggregate saving rate but also increases long-run wealth inequality in the inegalitarian regime. Recall that dynasties fall into two homogeneous groups starting from $t = T_3$. This division allows us to easily calculate the Gini coefficient based on the wealth distribution across dynasties for $t \geq T_3$. In particular, the coefficient must be equal to the difference between the share of the total capital stock owned by the wealthiest dynasties, $\sum_{j=1}^{n} s_t^j / K_{t+1}$, and their share in the total number of dynasties, n/N. Applying

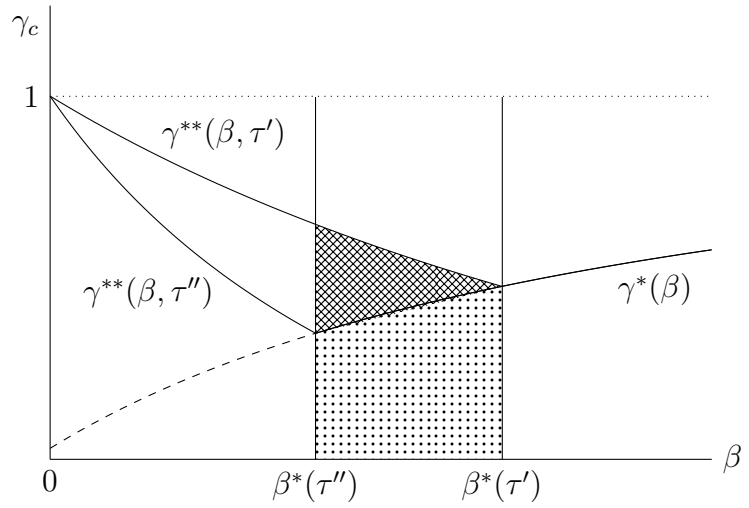

Figure 3.3: Possible regime shifts due to an increase in τ (from τ' to τ'')

Proposition 3.3, we obtain

$$\lim_{t\to\infty} \frac{\sum_{j=1}^{n} s_t^j}{N k_{t+1}} - \frac{n}{N} = \frac{N-n}{N}\left[\frac{\delta}{s(n/N)} + \frac{1-\delta}{\alpha}\right] b\left(\frac{n}{N}\right), \tag{3.20}$$

which is zero for $n = N$ and which increases with τ if and only if[11]

$$\left[\frac{\partial b(n/N)}{\partial \tau} - 1 + \alpha\right] \frac{\beta\left(1-\gamma_d \frac{n}{N}\right) + \delta\left(1-\gamma_\omega \frac{n}{N}\right)}{(1-\alpha)(\beta+\delta)} + \frac{N-n}{N} > 0. \tag{3.21}$$

It directly follows from (3.19) for $n < N$ that this inequality holds. Thus, in the inegalitarian regime, the degree of long-run wealth inequality is always positively dependent on the social security contribution rate.

Finally, it is worth stressing that the comparative statics results that have been discussed so far assume that changes in τ are marginal and do not lead to a change in the equilibrium regime. That being said, both conditions of Proposition 3.2, for example, depend on τ and provided that $(1+\beta)(1-\alpha) > \beta\alpha$, any increase in τ make them less likely to be met. In other words, an increase in the social security contribution rate reduces the set of possible parameter values that correspond to the egalitarian regime with no bequests. This can also be seen graphically. Suppose that simplifying assumption (3.9)

[11] See Appendix 3.A for details.

holds and let

$$\beta^*(\tau) := \frac{\delta(1-\tau)(1-\alpha)}{\alpha+\tau(1-\alpha)},$$

$$\gamma^*(\beta) := \frac{(\beta+\delta)(1-\alpha)}{\beta(1-\alpha)+1-\delta\alpha},$$

$$\gamma^{**}(\beta,\tau) := \frac{\beta\delta(1-\alpha)+[\beta^*(\tau)-\beta][\alpha+\tau(1-\alpha)]}{\beta\delta(1-\alpha)+[\beta^*(\tau)-\beta\delta][\alpha+\tau(1-\alpha)]}.$$

These three thresholds define three curves plotted in Figure 3.1. Figure 3.3 then illustrates how a change in τ from $\tau' \geq 0$ to $\tau'' > \tau'$ would change Figure 3.1, showing that the area corresponding to the egalitarian regime with no bequests shrinks. Two regions are of particular interest in Figure 3.3: the dotted one and the cross-hatched one. The dotted region contains all values of γ_c and β under which an increase in the contribution rate from τ' to τ'' shifts the economy from one egalitarian regime with no bequests to another egalitarian regime with bequests. The cross-hatched region, on the other hand, contains all values of γ_c and β under which an increase from τ' to τ'' shifts the economy from the egalitarian regime with no bequests to the inegalitarian regime. This last result reinforces our conclusion that social security increases wealth inequality in the long run by suggesting that social security may even be a source of inequality.

To sum up, we have:

Proposition 3.4. *Unfunded social security does not affect capital accumulation in the egalitarian regime with bequests and discourages capital accumulation in the egalitarian regime with no bequests. However, if the economy results in the inegalitarian regime, unfunded social security tends to encourage capital accumulation. Furthermore, an increase in the social security contribution rate may cause the economy to shift from the egalitarian regime with no bequests to the inegalitarian regime, where the degree of long-run wealth inequality always increases with the contribution rate.*

3.5 Conclusion

We provide an analysis of unfunded social security in an OLG model where individuals receive different bequests from their parents and are influenced by social comparisons. In this model, individuals belong to infinitely-lived dynasties that are heterogeneous solely in their initial wealth. We characterize three equilibrium regimes in which bequests are left in the long run by either all, none, or only the wealthiest dynasties. Our findings demonstrate that unfunded social security has no effect on capital accumulation in the first case and has a negative effect on capital accumulation in the second case. However, in the regime where only the wealthiest dynasties leave positive bequests in the long run,

unfunded social security tends to encourage capital accumulation. Yet, it also increases long-run wealth inequality.

Further research can be carried out in various directions. First of all, it would be interesting to consider an extended model where labor supply is endogenous, or parents additionally invest in children's human capital, as in Liu and Thøgersen [30] or Maebayashi [31], respectively. In both cases, individuals belonging to different dynasties would pay different amounts of payroll taxes, which could potentially cause the economy to shift from the inegalitarian regime to the egalitarian regime with bequests, or vice versa. Second, one can introduce heterogeneity in reference points to better capture the phenomenon of social comparisons. For example, rich individuals may be more likely to compare themselves with other rich individuals than with the average person (see Alvarez-Cuadrado and Japaridze [6]). Last but not least, a fully-funded element of the social security system could be incorporated into the model to investigate the distributional effects of social security privatization.

3.A Appendix

Three preliminary lemmas

Lemma 3.1. *In the intertemporal equilibrium* $\{(c_t^j, s_t^j, d_t^j, b_t^j)_{j=1}^N, k_{t+1}\}_{t=0}^{+\infty}$ *starting from* $\{(s_{-1}^j)_{j=1}^N, K_0\}$*, the aggregate saving rate* k_{t+1}/k_t^α *evolves according to*

$$\frac{k_{t+1}}{k_t^\alpha} = \frac{\delta\alpha(1-\gamma_c)[(1-\tau)(1-\alpha)+\frac{\bar{b}_t}{k_t^\alpha}]}{\delta\alpha(1-\gamma_c)+(1-\delta)(1-\gamma_d)[\alpha+\tau(1-\alpha)-\frac{\bar{b}_{t+1}}{k_{t+1}^\alpha}]}, \tag{3.22}$$

where

$$\begin{aligned}\bar{b}_t := \frac{1}{N}\sum_{j=1}^N b_t^j = \frac{1}{N}\sum_{j=1}^N \max\Bigg\{0, \frac{\beta}{\delta+\beta}\left[\alpha\frac{s_{t-1}^j}{k_t}+\tau(1-\alpha)\right] \\ -\frac{\delta}{\delta+\beta}(1-\tau)(1-\alpha)-\frac{\gamma_d\beta}{\delta+\beta}\left[\alpha+\tau(1-\alpha)-\frac{\bar{b}_t}{k_t^\alpha}\right] \\ +\frac{\gamma_\omega\delta}{\delta+\beta}\left[(1-\tau)(1-\alpha)+\frac{\bar{b}_t}{k_t^\alpha}\right]\Bigg\}k_t^\alpha\end{aligned} \tag{3.23}$$

and

$$\begin{aligned}\bar{b}_{t+1} &:= \frac{1}{N}\sum_{j=1}^{N} b_{t+1}^{j} \\ &= \frac{1}{N}\sum_{j=1}^{N}\max\Bigg\{0, \frac{\beta\alpha}{1+\beta}\left[(1-\tau)(1-\alpha)+\frac{b_t^j}{k_t^\alpha}\right]\frac{k_t^\alpha}{k_{t+1}} - (1-\alpha)\left(\frac{1}{1+\beta}-\tau\right) \\ &\quad - \frac{\gamma_c\beta\alpha}{1+\beta}\left[(1-\tau)(1-\alpha)+\frac{\bar{b}_t}{k_t^\alpha}-\frac{k_{t+1}}{k_t^\alpha}\right]\frac{k_t^\alpha}{k_{t+1}} \\ &\quad - \frac{\gamma_d\beta}{1+\beta}\left[\alpha+\tau(1-\alpha)-\frac{\bar{b}_{t+1}}{k_{t+1}^\alpha}\right]+\frac{\gamma_\omega}{1+\beta}\left[(1-\tau)(1-\alpha)+\frac{\bar{b}_{t+1}}{k_{t+1}^\alpha}\right]\Bigg\}k_{t+1}^\alpha.\end{aligned} \tag{3.24}$$

Furthermore, for all $t \geq 0$, it holds that

$$\frac{s_t^j}{k_{t+1}} = 1+\delta\left(\frac{b_t^j}{k_t^\alpha}-\frac{\bar{b}_t}{k_t^\alpha}\right)\frac{k_t^\alpha}{k_{t+1}} - \frac{1-\delta}{\alpha}\left(\frac{\bar{b}_{t+1}}{k_{t+1}^\alpha}-\frac{b_{t+1}^j}{k_{t+1}^\alpha}\right),\ j=1,\ldots,N \tag{3.25}$$

and

$$\frac{\bar{b}_t}{k_t^\alpha} \geq \frac{\beta(1-\gamma_d)[\alpha+\tau(1-\alpha)]-\delta(1-\gamma_\omega)(1-\tau)(1-\alpha)}{\beta(1-\gamma_d)+\delta(1-\gamma_\omega)}. \tag{3.26}$$

Proof. Consider the problem of individual j born at date t. Using (3.2)-(3.6), we can write the solution to this problem as

$$\begin{aligned} c_t^j &= (1-\tau)w_t + b_t^j - s_t^j, \\ s_t^j &= \delta[(1-\tau)w_t + b_t^j - \gamma_c\bar{c}_t] - \frac{1-\delta}{R_{t+1}}(\theta_{t+1} - b_{t+1}^j - \gamma_d\bar{d}_{t+1}), \end{aligned} \tag{3.27}$$

$$\begin{aligned} d_{t+1}^j &= R_{t+1}s_t^j + \theta_{t+1} - b_{t+1}^j, \\ b_{t+1}^j &= \max\left\{0, \frac{\beta}{\delta+\beta}(R_{t+1}s_t^j + \theta_{t+1} - \gamma_d\bar{d}_{t+1}) - \frac{\delta}{\delta+\beta}[(1-\tau)w_{t+1} - \gamma_\omega\bar{\omega}_{t+1}]\right\}. \end{aligned} \tag{3.28}$$

At the same time, it is clear that d_t^j and b_t^j chosen by individual j born at date $t-1$ are given by

$$d_t^j = R_t s_{t-1}^j + \theta_t - b_t^j$$

and

$$b_t^j = \max\left\{0, \frac{\beta}{\delta+\beta}(R_t s_{t-1}^j + \theta_t - \gamma_d\bar{d}_t) - \frac{\delta}{\delta+\beta}[(1-\tau)w_t - \gamma_\omega\bar{\omega}_t]\right\}. \tag{3.29}$$

Using the first equation in each of these two systems together with $k_t = \sum_{j=1}^{N} s_{t-1}^j/N$, $k_{t+1} = \sum_{j=1}^{N} s_t^j/N$, and (3.7), we obtain that for all $t \geq 0$,

$$\bar{c}_t = (1-\tau)w_t + \bar{b}_t - k_{t+1}, \tag{3.30}$$

$$\bar{d}_t = R_t k_t + \theta_t - \bar{b}_t, \tag{3.31}$$

$$\bar{\omega}_t = (1-\tau)w_t + \bar{b}_t. \tag{3.32}$$

Applying (3.31) and (3.32) to (3.29) and taking (3.1) and (3.8) into account yields (3.23). Similarly, applying (3.30) and (3.31) to (3.27) and taking (3.1) and (3.8) into account yields for all j and t,

$$\begin{aligned} s_t^j = \Bigg\{ & \delta \left[(1-\tau)(1-\alpha) + \frac{b_t^j}{k_t^\alpha} \right] - \gamma_c \delta \left[(1-\tau)(1-\alpha) + \frac{\bar{b}_t}{k_t^\alpha} - \frac{k_{t+1}}{k_t^\alpha} \right] \\ & + \frac{\gamma_d(1-\delta)}{\alpha} \left[\alpha + \tau(1-\alpha) - \frac{\bar{b}_{t+1}}{k_{t+1}^\alpha} \right] \frac{k_{t+1}}{k_t^\alpha} \Bigg\} k_t^\alpha. \end{aligned} \tag{3.33}$$

Using condition $k_{t+1} = \sum_{j=1}^{N} s_t^j/N$ and rearranging terms, we obtain (3.22). Furthermore, dividing both sides of (3.33) by k_{t+1} and using (3.22), we obtain (3.25). Combining (3.27) and (3.28) along with (3.30)-(3.32), (3.1), and (3.8) yields (3.24). To obtain (3.26), note that (3.23) implies

$$\begin{aligned} \bar{b}_t \geq \frac{1}{N} \sum_{j=1}^{N} \Bigg\{ & \frac{\beta}{\delta+\beta} \left[\alpha \frac{s_{t-1}^j}{k_t} + \tau(1-\alpha) \right] - \frac{\delta}{\delta+\beta}(1-\tau)(1-\alpha) \\ & - \frac{\gamma_d \beta}{\delta+\beta} \left[\alpha + \tau(1-\alpha) - \frac{\bar{b}_t}{k_t^\alpha} \right] + \frac{\gamma_\omega \delta}{\delta+\beta} \left[(1-\tau)(1-\alpha) + \frac{\bar{b}_t}{k_t^\alpha} \right] \Bigg\} k_t^\alpha. \end{aligned}$$

Applying $k_t = \sum_{j=1}^{N} s_{t-1}^j/N$ and rearranging yields (3.26). □

Lemma 3.2. *Let*

$$A(m) := m(1-\gamma_c)(1-\tau)(1-\alpha)\left\{\beta(1-\gamma_d)[\alpha+\tau(1-\alpha)] - \delta(1-\gamma_\omega)(1-\tau)(1-\alpha)\right\},$$

$$\begin{aligned} B(m) := {} & m(1-\gamma_c)\left\{\beta(1-\gamma_d)[\alpha+\tau(1-\alpha)] - \delta(1-\gamma_\omega)(1-\tau)(1-\alpha)\right\} \\ & + \beta(1-m)\left\{\delta\alpha(1-\gamma_c) + (1-\delta)(1-\gamma_d)[\alpha+\tau(1-\alpha)]\right\}, \end{aligned}$$

$$C(m) := (1-\gamma_c)(1-\tau)(1-\alpha)\left\{\delta[1-m\gamma_\omega+\beta(1-m\gamma_d)] + m\beta(1-\delta)(1-\gamma_d)\right\},$$

$$D(m) := \beta(1-m\gamma_c)(1-\delta)(1-\gamma_d) + \delta(1-\gamma_c)[1-m\gamma_\omega+\beta(1-m\gamma_d)].$$

If $M_{t+1} \geq M_t > 0$ for some t, then

$$\frac{\bar{b}_{t+1}}{k_{t+1}^{\alpha}} = \frac{A(m_{t+1}) + B(m_{t+1})\frac{\bar{b}_t}{k_t^{\alpha}}}{C(m_{t+1}) + D(m_{t+1})\frac{\bar{b}_t}{k_t^{\alpha}}}. \tag{3.34}$$

Proof. If $M_{t+1} \geq M_t > 0$ for some t, then one has $\sum_{j=1}^{M_{t+1}} b_t^j/N = \sum_{j=1}^{M_t} b_t^j/N = \bar{b}_t$ for this t. Therefore, by (3.24),

$$\begin{aligned}
\frac{\bar{b}_{t+1}}{k_{t+1}^{\alpha}} = {} & \frac{\beta\alpha}{1+\beta}\left[(1-\tau)(1-\alpha)\frac{M_{t+1}}{N} + \frac{\bar{b}_t}{k_t^{\alpha}}\right]\frac{k_t^{\alpha}}{k_{t+1}} - (1-\alpha)\left(\frac{1}{1+\beta} - \tau\right)\frac{M_{t+1}}{N} \\
& - \frac{\gamma_c\beta\alpha}{1+\beta}\left[(1-\tau)(1-\alpha) + \frac{\bar{b}_t}{k_t^{\alpha}} - \frac{k_{t+1}}{k_t^{\alpha}}\right]\frac{k_t^{\alpha}}{k_{t+1}}\frac{M_{t+1}}{N} \\
& - \frac{\gamma_d\beta}{1+\beta}\left[\alpha + \tau(1-\alpha) - \frac{\bar{b}_{t+1}}{k_{t+1}^{\alpha}}\right]\frac{M_{t+1}}{N} + \frac{\gamma_\omega}{1+\beta}\left[(1-\tau)(1-\alpha) + \frac{\bar{b}_{t+1}}{k_{t+1}^{\alpha}}\right]\frac{M_{t+1}}{N}.
\end{aligned}$$

Let us substitute the expression for k_{t+1}/k_t^{α} from (3.22) into this equation and use $m_{t+1} = M_{t+1}/N$. After rearranging terms, we obtain (3.34). □

Lemma 3.3. *Suppose that $m \in (0, 1]$ and let*

$$\Theta(x) := \frac{A(m) + B(m)x}{C(m) + D(m)x} \quad \textit{and} \quad \Phi(x) := x - \Theta(x).$$

(a) *If*

$$\frac{1-\gamma_d}{1-\gamma_\omega} > \frac{\delta(1-\tau)(1-\alpha)}{\beta[\alpha + \tau(1-\alpha)]}, \tag{3.35}$$

there exists a positive solution to $\Phi(x) = 0$. This solution is unique and given by

$$x_1 = \frac{B(m) - C(m) + \sqrt{[B(m) - C(m)]^2 + 4A(m)D(m)}}{2D(m)}.$$

Furthermore, we have $0 < \Theta'(x_1) < 1$.

(b) *If*

$$\frac{1-\gamma_d}{1-\gamma_\omega} < \frac{\delta(1-\tau)(1-\alpha)}{\beta[\alpha + \tau(1-\alpha)]} \quad \textit{and} \quad \frac{1-\gamma_d}{1-\gamma_c} < \frac{\delta[(1+\beta)(1-\tau)(1-\alpha) - \beta\alpha]}{\beta(1-\delta)[\alpha + \tau(1-\alpha)]}, \tag{3.36}$$

a positive solution to $\Phi(x) = 0$ does not exist.

Proof. Let us first note that $C(m) > 0$ and $D(m) > 0$ for any $m \in (0, 1]$. Hence, equation $\Phi(x) = 0$ is equivalent to $D(m)x^2 - [B(m) - C(m)]x - A(m) = 0$. The two roots of

$D(m)x^2 - [B(m) - C(m)]x - A(m) = 0$ are

$$x_1 = \frac{B(m) - C(m) + \sqrt{[B(m) - C(m)]^2 + 4A(m)D(m)}}{2D(m)},$$

$$x_2 = \frac{B(m) - C(m) - \sqrt{[B(m) - C(m)]^2 + 4A(m)D(m)}}{2D(m)}.$$

To prove part (a), we note that $A(m)$ is positive for any $m \in (0, 1]$ if and only if (3.35) is satisfied. Given $A(m)D(m) > 0$, we then have $x_1 > 0$ and $x_2 < 0$. This proves that under condition (3.35), x_1 constitutes a unique positive solution to $\Phi(x) = 0$. To show $0 < \Theta'(x_1) < 1$, we use

$$\begin{aligned}
&B(m)C(m) - A(m)D(m) \\
&= \beta\delta(1 - m)(1 - \gamma_c)(1 - \tau)(1 - \alpha) \\
&\times \{\alpha(1 - \gamma_c)[\delta(1 - m\omega + \beta(1 - m\gamma_d)) + m\beta(1 - \delta)(1 - \gamma_d)] \\
&+ (1 - \delta)(1 - \gamma_d)[m(1 - \omega)(1 - \tau)(1 - \alpha) + (\alpha + \tau(1 - \alpha))(1 - m\omega + \beta(1 - m))]\} > 0.
\end{aligned}$$

Then, we have

$$\Theta'(x) = \frac{B(m)C(m) - A(m)D(m)}{[C(m) + D(m)x]^2} > 0.$$

Evaluating this at $x = x_1$, we further obtain that $\Theta'(x_1)$ is equal to the inverse of

$$1 + \frac{[B(m) - C(m)]^2 + 4A(m)D(m) + [B(m) + C(m)]\sqrt{[B(m) - C(m)]^2 + 4A(m)D(m)}}{2[B(m)C(m) - A(m)D(m)]}.$$

Since $C(m)$ and $B(m)C(m) - A(m)D(m)$ are positive for any $m \in (0, 1]$ and (3.35) implies $B(m) > 0$ for any $m \in (0, 1]$, the expression in curly brackets is greater than 1, which leads to $\Theta'(x_1) < 1$.

To prove part (b), suppose that the first condition in (3.36) holds. Then, we have $A(m) < 0$ for any $m \in (0, 1]$ and so a positive solution to $D(m)x^2 - [B(m) - C(m)]x - A(m) = 0$ can exist only if $B(m) - C(m) > 0$. Since the first condition in (3.36) is supposed to hold, $B(m) - C(m) > 0$ requires

$$\begin{aligned}
&\beta(1 - m)\{\delta\alpha(1 - \gamma_c) + (1 - \delta)(1 - \gamma_d)[\alpha + \tau(1 - \alpha)]\} \\
&\quad - (1 - \gamma_c)(1 - \tau)(1 - \alpha)\{\delta[1 - m\gamma_\omega + \beta(1 - m\gamma_d)] + m\beta(1 - \delta)(1 - \gamma_d)\} > 0
\end{aligned}$$

or, equivalently,

$$\left\{\frac{1-\gamma_d}{1-\gamma_c}-\frac{\delta[(1+\beta)(1-\tau)(1-\alpha)-\beta\alpha]}{\beta(1-\delta)[\alpha+\tau(1-\alpha)]}\right\}\frac{1-m}{m}$$
$$>\frac{(1-\tau)(1-\alpha)[\beta(1-\gamma_d)+\delta(1-\gamma_\omega)]}{\beta(1-\delta)[\alpha+\tau(1-\alpha)]}.$$

For $m=1$, this condition is necessarily violated, while for $m\in(0,1)$, this condition is violated if the second condition in (3.36) holds. □

Proof of Proposition 3.1

Suppose that both conditions in (3.11) are satisfied. To prove that there exists T_1 such that $M_t=N$ for $t\geq T_1$, it suffices to show that (i) $M_{t+1}=N$ holds for at least some t, and (ii) $M_t=N$ implies $M_{t+1}=N$ for all $t\geq 0$.

To prove that $M_{t+1}=N$ for some t, assume the opposite, i.e., $M_{t+1}<N$ for all $t\geq 0$. By (3.24), we have

$$\frac{\beta\alpha(1-\tau)(1-\alpha)}{1+\beta}\frac{k_t^\alpha}{k_{t+1}}-(1-\alpha)\left(\frac{1}{1+\beta}-\tau\right)$$
$$-\frac{\gamma_c\beta\alpha}{1+\beta}\left[(1-\tau)(1-\alpha)+\frac{\bar{b}_t}{k_t^\alpha}-\frac{k_{t+1}}{k_t^\alpha}\right]\frac{k_t^\alpha}{k_{t+1}}$$
$$-\frac{\gamma_d\beta}{1+\beta}\left[\alpha+\tau(1-\alpha)-\frac{\bar{b}_{t+1}}{k_{t+1}^\alpha}\right]+\frac{\gamma_\omega}{1+\beta}\left[(1-\tau)(1-\alpha)+\frac{\bar{b}_{t+1}}{k_{t+1}^\alpha}\right]\leq 0,$$

because otherwise $M_{t+1}=N$, given $b_t^j\geq 0$ for all $j=1,\ldots,N$. Hence, equation (3.24) implies $b_{t+1}^j/k_{t+1}^\alpha\leq[\beta\alpha/(1+\beta)](b_t^j/k_{t+1})$ for all j and t, which in turn implies

$$\frac{\bar{b}_{t+1}}{k_{t+1}^\alpha}\frac{k_{t+1}}{k_t^\alpha}\leq\frac{\beta\alpha}{1+\beta}\frac{\bar{b}_t}{k_t^\alpha}.$$

Note that since the first condition in (3.11) is satisfied, the right-hand side of (3.26) is positive and therefore $\bar{b}_t/k_t^\alpha>0$ holds for all $t\geq 0$. Furthermore, applying (3.26) to (3.22) gives

$$\frac{k_{t+1}}{k_t^\alpha}\geq\frac{\beta\alpha(1-\gamma_c)(1-\gamma_d)}{\beta\alpha(1-\gamma_c)(1-\gamma_d)+(1-\gamma_\omega)[\delta\alpha(1-\gamma_c)+(1-\delta)(1-\gamma_d)]}.$$

Then, for all $t\geq 0$,

$$\frac{\bar{b}_{t+1}}{k_{t+1}^\alpha}\frac{\beta\alpha(1-\gamma_c)(1-\gamma_d)}{\beta\alpha(1-\gamma_c)(1-\gamma_d)+(1-\gamma_\omega)[\delta\alpha(1-\gamma_c)+(1-\delta)(1-\gamma_d)]}\leq\frac{\beta\alpha}{1+\beta}\frac{\bar{b}_t}{k_t^\alpha}. \quad (3.37)$$

Using also the fact that the second inequality in (3.11) is equivalent to

$$\frac{\beta\alpha(1-\gamma_c)(1-\gamma_d)}{\beta\alpha(1-\gamma_c)(1-\gamma_d)+(1-\gamma_\omega)[\delta\alpha(1-\gamma_c)+(1-\delta)(1-\gamma_d)]} > \frac{\beta\alpha}{1+\beta}, \tag{3.38}$$

we obtain $\bar{b}_{t+1}/k_{t+1}^{\alpha} < \bar{b}_t/k_t^{\alpha}$ for $t \geq 0$. Taking into account that $\bar{b}_t/k_t^{\alpha}$ satisfies (3.26) for all $t \geq 0$, the sequence $\{\bar{b}_t/k_t^{\alpha}\}_{t=0}^{\infty}$ can be monotonically decreasing only if $\bar{b}_t/k_t^{\alpha}$ converges to a value that is greater than the right-hand side of (3.26). However, this value cannot exist because condition (3.37) holding with $\bar{b}_{t+1}/k_{t+1}^{\alpha} = \bar{b}_t/k_t^{\alpha}$ contradicts (3.38).

Let us now prove that $M_t = N$ implies $M_{t+1} = N$ for all $t \geq 0$. Assume the opposite, that is, $M_t = N$ implies $M_{t+1} < N$ for some t. It follows from $M_t = N$ and equation (3.23) that

$$\frac{\bar{b}_t}{k_t^{\alpha}} = \frac{\beta(1-\gamma_d)[\alpha+\tau(1-\alpha)]-\delta(1-\gamma_\omega)(1-\tau)(1-\alpha)}{\beta(1-\gamma_d)+\delta(1-\gamma_\omega)}. \tag{3.39}$$

We know that condition $M_{t+1} < N$ requires (3.37) to hold. Applying (3.39) to (3.37) yields

$$\begin{aligned}&\frac{\bar{b}_{t+1}}{k_{t+1}^{\alpha}}\frac{\beta\alpha(1-\gamma_c)(1-\gamma_d)}{\beta\alpha(1-\gamma_c)(1-\gamma_d)+(1-\gamma_\omega)[\delta\alpha(1-\gamma_c)+(1-\delta)(1-\gamma_d)]}\\ &\leq \frac{\beta\alpha}{1+\beta}\frac{\beta(1-\gamma_d)[\alpha+\tau(1-\alpha)]-\delta(1-\gamma_\omega)(1-\tau)(1-\alpha)}{\beta(1-\gamma_d)+\delta(1-\gamma_\omega)}.\end{aligned}$$

Since (3.26) must be true for all $t \geq 0$, the last inequality together with (3.26) imply

$$\frac{\beta\alpha(1-\gamma_c)(1-\gamma_d)}{\beta\alpha(1-\gamma_c)(1-\gamma_d)+(1-\gamma_\omega)[\delta\alpha(1-\gamma_c)+(1-\delta)(1-\gamma_d)]} \leq \frac{\beta\alpha}{1+\beta}.$$

Again, this contradicts condition (3.38), which is equivalent to the second condition in (3.11).

We are left to prove (3.12), (3.13), $\lim_{t\to\infty} s_t^j/k_{t+1} = 1$, and $\lim_{t\to\infty} b_t^j/\bar{b}_t = 1$. From $M_t = N$ for all $t \geq T_1$ and (3.23), it holds that

$$\forall t \geq T_1: \ \frac{\bar{b}_t}{k_t^{\alpha}} = \frac{\beta(1-\gamma_d)[\alpha+\tau(1-\alpha)]-\delta(1-\gamma_\omega)(1-\tau)(1-\alpha)}{\beta(1-\gamma_d)+\delta(1-\gamma_\omega)},$$

which proves (3.12). Substituting this expression for $\bar{b}_t/k_t^\alpha$ and $\bar{b}_{t+1}/k_{t+1}^\alpha$ into equation (3.22), we obtain (3.13). Furthermore, by (3.24), we have

$$\begin{aligned}\forall t \geq T_1: \ \frac{b_{t+1}^j}{\bar{b}_{t+1}} = {} & \frac{\beta\alpha}{1+\beta}\frac{\bar{b}_t}{k_t^\alpha}\frac{k_{t+1}^\alpha}{\bar{b}_{t+1}}\frac{k_t^\alpha}{k_{t+1}}\frac{b_t^j}{\bar{b}_t} \\ & + \left\{\frac{\beta\alpha}{1+\beta}\left[(1-\tau)(1-\alpha)\right]\frac{k_t^\alpha}{k_{t+1}} - (1-\alpha)\left(\frac{1}{1+\beta}-\tau\right)\right. \\ & -\frac{\gamma_c\beta\alpha}{1+\beta}\left[(1-\tau)(1-\alpha)+\frac{\bar{b}_t}{k_t^\alpha}-\frac{k_{t+1}}{k_t^\alpha}\right]\frac{k_t^\alpha}{k_{t+1}} \\ & -\frac{\gamma_d\beta}{1+\beta}\left[\alpha+\tau(1-\alpha)-\frac{\bar{b}_{t+1}}{k_{t+1}^\alpha}\right] \\ & \left.+\frac{\gamma_\omega}{1+\beta}\left[(1-\tau)(1-\alpha)+\frac{\bar{b}_{t+1}}{k_{t+1}^\alpha}\right]\right\}\frac{k_{t+1}^\alpha}{\bar{b}_{t+1}}, \ j=1,\ldots,N.\end{aligned}$$

Taking (3.12) and (3.13) into account leads to

$$\begin{aligned}\forall t \geq T_1: \ \frac{b_{t+1}^j}{\bar{b}_{t+1}} = {} & \left\{1-\frac{1-\delta}{1+\beta}\frac{1-\gamma_\omega}{1-\gamma_d}\left[\frac{1+\beta(1-\alpha)}{1-\delta}\frac{1-\gamma_d}{1-\gamma_\omega}-\frac{\delta\alpha}{1-\delta}-\frac{1-\gamma_d}{1-\gamma_c}\right]\right\}\frac{b_t^j}{\bar{b}_t} \\ & +\frac{1-\delta}{1+\beta}\frac{1-\gamma_\omega}{1-\gamma_d}\left[\frac{1+\beta(1-\alpha)}{1-\delta}\frac{1-\gamma_d}{1-\gamma_\omega}-\frac{\delta\alpha}{1-\delta}-\frac{1-\gamma_d}{1-\gamma_c}\right], \ j=1,\ldots,N.\end{aligned}$$

Applying the second condition in (3.11), we prove $\lim_{t\to\infty} b_t^j/\bar{b}_t = 1$. Finally, combining (3.25) and $\lim_{t\to\infty} b_t^j/\bar{b}_t = 1$, we prove $\lim_{t\to\infty} s_t^j/k_{t+1} = 1$. □

Proof of Proposition 3.2

Suppose that the first condition in (3.14) holds. To prove that there exists a date T_2 such that $M_t = 0$ for all $t \geq T_2$, we first show that $M_{t+1} \leq M_t$ for all $t \geq 0$. Assume the opposite, i.e., $M_{t+1} > M_t$ for some t. Then, there must exist h such that $b_t^h = 0$ and $b_{t+1}^h > 0$. By (3.24), this implies

$$\begin{aligned}& \frac{\beta\alpha(1-\tau)(1-\alpha)}{1+\beta}\frac{k_t^\alpha}{k_{t+1}} - (1-\alpha)\left(\frac{1}{1+\beta}-\tau\right) \\ & -\frac{\gamma_c\beta\alpha}{1+\beta}\left[(1-\tau)(1-\alpha)+\frac{\bar{b}_t}{k_t^\alpha}-\frac{k_{t+1}}{k_t^\alpha}\right]\frac{k_t^\alpha}{k_{t+1}} \\ & -\frac{\gamma_d\beta}{1+\beta}\left[\alpha+\tau(1-\alpha)-\frac{\bar{b}_{t+1}}{k_{t+1}^\alpha}\right]+\frac{\gamma_\omega}{1+\beta}\left[(1-\tau)(1-\alpha)+\frac{\bar{b}_{t+1}}{k_{t+1}^\alpha}\right] > 0,\end{aligned} \tag{3.40}$$

so that $b_{t+1}^j > 0$ for all $j = 1, \ldots, N$, given $b_t^j \geq 0$. Consequently, by (3.23),

$$\frac{\bar{b}_{t+1}}{k_{t+1}^\alpha} = \frac{\beta(1-\gamma_d)[\alpha+\tau(1-\alpha)] - \delta(1-\gamma_\omega)(1-\tau)(1-\alpha)}{\beta(1-\gamma_d)+\delta(1-\gamma_\omega)}, \tag{3.41}$$

where the right-hand side is negative when the first condition in (3.14) is satisfied. This contradicts $b_{t+1}^j > 0$ for all $j = 1, \ldots, N$.

Similarly, let us show that $M_t < N$ for all $t \geq 0$. If $M_t = N$ for some t, equation (3.23) results in

$$\frac{\bar{b}_t}{k_t^\alpha} = \frac{\beta(1-\gamma_d)[\alpha+\tau(1-\alpha)] - \delta(1-\gamma_\omega)(1-\tau)(1-\alpha)}{\beta(1-\gamma_d)+\delta(1-\gamma_\omega)},$$

where, again, the right-hand side is negative when the first condition in (3.14) is satisfied. This is not possible because $b_t^j \geq 0$ for all j and t.

Thus, we deduce from $M_{t+1} \leq M_t < N$ for all $t \geq 0$ that there exist M and T_2 such that $M_t = M < N$ for all $t \geq T_2$.

Suppose now that the second condition in (3.14) also holds. Let us show that $M = 0$. If $M > 0$, we can apply Lemma 3.2 and write

$$\forall t \geq T_2 : \ \frac{\bar{b}_{t+1}}{k_{t+1}^\alpha} = \frac{A(m) + B(m)\frac{\bar{b}_t}{k_t^\alpha}}{C(m) + D(m)\frac{\bar{b}_t}{k_t^\alpha}},$$

where $m = M/N$. Then, for $M > 0$, this difference equation must have a positive stationary solution. However, from part (ii) of Lemma 3.3, we know that for given $m \in (0,1)$, this solution does not exist when the second condition in (3.14) is satisfied. Therefore, $M = 0$ is the only possibility.

To complete the proof, we note that equation (3.15) is obtained by simply applying $\bar{b}_t = \bar{b}_{t+1} = 0$ to (3.22). Furthermore, using (3.25) and setting $b_t^j = b_{t+1}^j = \bar{b}_t = \bar{b}_{t+1} = 0$, we get $s_t^j = k_{t+1}$ for $t \geq T_2$ and $j = 1, \ldots, N$. □

Proof of Proposition 3.3

Suppose that both conditions in (3.16) hold. We first show that $M_{t+1} \leq M_t$ for all $t \geq 0$. Assume the opposite, i.e., $M_{t+1} > M_t$ for some t. From the proof of Proposition 3.2, we know that (3.40) and (3.41) must be satisfied. Substituting for $\bar{b}_{t+1}/k_{t+1}^\alpha$ from (3.41) into (3.22) yields

$$\frac{k_{t+1}}{k_t^\alpha} = \left[1 + \frac{(1-\delta)(1-\gamma_\omega)}{\beta\alpha(1-\gamma_d)+\delta\alpha(1-\gamma_\omega)}\frac{1-\gamma_d}{1-\gamma_c}\right]^{-1}\left[(1-\tau)(1-\alpha) + \frac{\bar{b}_t}{k_t^\alpha}\right]$$

Substituting this expression for k_{t+1}/k_t^α and (3.41) for $\bar{b}_{t+1}/k_{t+1}^\alpha$ into (3.40), we obtain

$$\begin{aligned}&\frac{\beta(1-\gamma_d)[\alpha+\tau(1-\alpha)]-\delta(1-\gamma_\omega)(1-\tau)(1-\alpha)}{\beta(1-\gamma_d)+\delta(1-\gamma_\omega)}\\&>\frac{\bar{b}_t}{k_t^\alpha}\Bigg\{1+\frac{\beta(1-\delta)(1-\gamma_\omega)}{(1+\beta)(1-\tau)(1-\alpha)[\beta(1-\gamma_d)+\delta(1-\gamma_\omega)]}\\&\quad\times\left[\frac{1-\gamma_d}{1-\gamma_c}-\frac{1+\beta(1-\alpha)}{1-\delta}\frac{1-\gamma_d}{1-\gamma_\omega}+\frac{\delta\alpha}{1-\delta}\right]\Bigg\}.\end{aligned}$$

It follows from the second condition in (3.16) that the expression in curly brackets is positive. Using (3.26) together with the first condition in (3.16), we further obtain

$$\frac{1-\gamma_d}{1-\gamma_c}-\frac{1+\beta(1-\alpha)}{1-\delta}\frac{1-\gamma_d}{1-\gamma_\omega}+\frac{\delta\alpha}{1-\delta}<0,$$

which contradicts the second inequality in (3.16).

Let us also show that $M_t > 0$ for all $t \geq 0$. Assume the opposite, i.e., $M_t = 0$ for some t. Then, we have $\bar{b}_t = 0$ for this t and equation (3.23) implies

$$\beta\left[\alpha\frac{s_{t-1}^j}{k_t}+\tau(1-\alpha)\right]-\gamma_d\beta\left[\alpha+\tau(1-\alpha)\right]-\delta(1-\gamma_\omega)(1-\tau)(1-\alpha)\leq 0,\ j=1,\ldots,N.$$

Consequently, using $k_t = \sum_{j=1}^N s_{t-1}^j/N$, we obtain

$$\frac{1-\gamma_d}{1-\gamma_\omega}\leq\frac{\delta(1-\tau)(1-\alpha)}{\beta[\alpha+\tau(1-\alpha)]},$$

which contradicts the first inequality in (3.16).

Summing up, we have shown $M_{t+1} \leq M_t$ and $M_t > 0$ for all $t \geq 0$. Hence, there exist M and S such that $M_t = M > 0$ for all $t \geq S$.

Next, we prove that there exists a date $T_3 \geq S$ such that $b^j_{t+1}/\bar{b}_{t+1} = b^j_t/\bar{b}_{t+1}$ for $t \geq T_3$ and $j \leq M$. Using equation (3.24), we can write

$$\begin{aligned}\forall t \geq S: \ \frac{b^j_{t+1}}{\bar{b}_{t+1}} = {} & \frac{\beta\alpha}{1+\beta}\frac{\bar{b}_t}{k^\alpha_t}\frac{k^\alpha_{t+1}}{\bar{b}_{t+1}}\frac{k^\alpha_t}{k_{t+1}}\frac{b^j_t}{\bar{b}_t} \\ & + \left\{\frac{\beta\alpha}{1+\beta}(1-\tau)(1-\alpha)\frac{k^\alpha_t}{k_{t+1}} - (1-\alpha)\left(\frac{1}{1+\beta}-\tau\right)\right. \\ & -\frac{\gamma_c\beta\alpha}{1+\beta}\left[(1-\tau)(1-\alpha)+\frac{\bar{b}_t}{k^\alpha_t}-\frac{k_{t+1}}{k^\alpha_t}\right]\frac{k^\alpha_t}{k_{t+1}} \\ & -\frac{\gamma_d\beta}{1+\beta}\left[\alpha+\tau(1-\alpha)-\frac{\bar{b}_{t+1}}{k^\alpha_{t+1}}\right] \\ & \left.+\frac{\gamma_\omega}{1+\beta}\left[(1-\tau)(1-\alpha)+\frac{\bar{b}_{t+1}}{k^\alpha_{t+1}}\right]\right\}\frac{k^\alpha_{t+1}}{\bar{b}_{t+1}}, \ j=1,\ldots,M.\end{aligned} \tag{3.42}$$

At the same time, Lemma 3.2 implies

$$\forall t \geq S: \ \frac{\bar{b}_{t+1}}{k^\alpha_{t+1}} = \frac{A(m)+B(m)\frac{\bar{b}_t}{k^\alpha_t}}{C(m)+D(m)\frac{\bar{b}_t}{k^\alpha_t}},$$

where $m = M/N$, and part (i) of Lemma 3.3 further implies $\lim_{t\to\infty}\bar{b}_t/k^\alpha_t = b(m)$. By (3.22), we therefore have $\lim_{t\to\infty} k_{t+1}/k^\alpha_t = s(m)$. Tedious calculations show that if the first condition in (3.16) is satisfied, the following relationship holds for any $m \in (0,1]$:

$$\begin{aligned}& \frac{\beta\alpha}{1+\beta}\frac{1}{s(m)} > 1 \\ \Leftrightarrow \ & \frac{\beta\alpha}{1+\beta}(1-\tau)(1-\alpha)\frac{1}{s(m)} - (1-\alpha)\left(\frac{1}{1+\beta}-\tau\right) \\ & -\frac{\gamma_c\beta\alpha}{1+\beta}\left[(1-\tau)(1-\alpha)+b(m)-s(m)\right]\frac{1}{s(m)} - \frac{\gamma_d\beta}{1+\beta}\left[\alpha+\tau(1-\alpha)-b(m)\right] \\ & +\frac{\gamma_\omega}{1+\beta}[(1-\tau)(1-\alpha)+b(m)] < 0 \\ \Leftrightarrow \ & \frac{1-\gamma_d}{1-\gamma_c} > \frac{1+\beta(1-\alpha)}{1-\delta}\frac{1-\gamma_d}{1-\gamma_\omega} - \frac{\delta\alpha}{1-\delta}.\end{aligned}$$

Then, there indeed exists a date $T_3 \geq S$ such that $b^j_{t+1}/\bar{b}_{t+1} = b^j_t/\bar{b}_{t+1}$ for $t \geq T_3$ and $j \leq M$, because otherwise $\lim_{t\to\infty} b^j_t/\bar{b}_t = \pm\infty$ for $j \leq M$. As a consequence, we obtain

$$\forall t \geq T_3: \ b^j_t/\bar{b}_t = 1/m, \ j \leq M; \ b^j_t = 0, \ j > M, \tag{3.43}$$

and therefore, by (3.25),

$$\forall t \geq T_3 : \ \frac{s_t^j}{k_{t+1}} = 1 + \frac{1-m}{m}\left(\frac{\delta k_t^\alpha}{k_{t+1}}\frac{\bar{b}_t}{k_t^\alpha} + \frac{1-\delta}{\alpha}\frac{\bar{b}_{t+1}}{k_{t+1}^\alpha}\right), \ j \leq M \tag{3.44}$$

and

$$\forall t \geq T_3 : \ \frac{s_t^j}{k_{t+1}} = 1 - \frac{\delta k_t^\alpha}{k_{t+1}}\frac{\bar{b}_t}{k_t^\alpha} - \frac{1-\delta}{\alpha}\frac{\bar{b}_{t+1}}{k_{t+1}^\alpha}, \ j > M, \tag{3.45}$$

which proves (3.17) and (3.18) if $M = n$.

Finally, let us show that $M = n$ or, equivalently, $m = n/N$. By (3.10), if $b_t^n = 0$ for some t, we have $b_t^j = 0$ for all j and this t, which is impossible since $M_t > 0$ for $t \geq 0$. We thus have $b_t^n > 0$ for $t \geq 0$. Then, $s_{-1}^n > s_{-1}^{n+1}$ together with $b_0^n > 0$ implies, by (3.23), that $b_0^n > b_0^{n+1}$. Similarly, $b_t^n > b_t^{n+1}$ together with $b_{t+1}^n > 0$ implies, by (3.24), that $b_{t+1}^n > b_{t+1}^{n+1}$ for all $t \geq 0$, so we can write

$$\forall t \geq 0 : \ b_t^1 = \cdots = b_t^n > b_t^{n+1} \geq \cdots \geq b_t^N.$$

On the other hand, we know from (3.43) that

$$\forall t \geq T_3 : \ b_t^1 = \cdots = b_t^M > b_t^{M+1} = \cdots = b_t^N,$$

and therefore $M = n$ is the only possibility. This completes the proof of Proposition 3.3. □

Proof of Equation (3.19)

It follows directly from the definition of function s that

$$\frac{\partial s(n/N)}{\partial \tau} = \frac{\delta\alpha(1-\gamma_c)[\delta\alpha(1-\gamma_c) + (1-\delta)(1-\gamma_d)][\partial b(n/N)/\partial\tau - 1 + \alpha]}{\{\delta\alpha(1-\gamma_c) + (1-\delta)(1-\gamma_d)[\alpha + \tau(1-\alpha) - b(n/N)]\}^2}.$$

Furthermore, it follows from the definition of function b and $\partial D(n/N)/\partial\tau = 0$ that

$$\begin{aligned}
\frac{\partial b(n/N)}{\partial \tau} &= \frac{b(n/N)\left[\partial B(n/N)/\partial\tau - \partial C(n/N)/\partial\tau\right] + \partial A(n/N)/\partial\tau}{\sqrt{[B(n/N) - C(n/N)]^2 + 4A(n/N)D(n/N)}} \\
&= 1 - \alpha + (1-\alpha)\frac{N-n}{N}\frac{\beta(1-\gamma_d)(1-\delta) + \delta(1-\gamma_c)(1+\beta)}{\sqrt{[B(n/N) - C(n/N)]^2 + 4A(n/N)D(n/N)}} \\
&\times \left\{\frac{\beta(1-\gamma_d)(1-\delta)[\alpha + \tau(1-\alpha)] + \delta(1-\gamma_c)[\beta\alpha - (1+\beta)(1-\tau)(1-\alpha)]}{\beta(1-\gamma_d)(1-\delta) + \delta(1-\gamma_c)(1+\beta)} - b\left(\frac{n}{N}\right)\right\}.
\end{aligned}$$

Using the relationship between $s(n/N)$ and $b(n/N)$, it is straightforward to verify that the expression in curly brackets is positive if and only if $s(n/N) < \beta\alpha/(1+\beta)$, and we know from the proof of Proposition 3.3 that this condition is satisfied for any $n \in \{1, \ldots, N\}$ if

both conditions in (3.16) hold. Thus, we obtain that (3.19) is true whenever conditions of Proposition 3.3 are satisfied.

Proof of Equation (3.21)

Let us first show that

$$\begin{aligned}
&\lim_{t\to\infty} \frac{\sum_{j=1}^{n} s_t^j}{N k_{t+1}} - \frac{n}{N} \\
&\quad = \left[b\left(\frac{n}{N}\right) - \tau(1-\alpha)\right] \frac{\beta(1-\gamma_d \frac{n}{N}) + \delta(1-\gamma_\omega \frac{n}{N})}{\beta\alpha} \\
&\qquad + \tau(1-\alpha)\frac{N-n}{N}\frac{\beta+\delta}{\beta\alpha} + \frac{n}{N}\frac{\gamma_d\beta\alpha + \delta(1-\alpha)(1-\gamma_\omega)}{\beta\alpha} - \frac{n}{N}.
\end{aligned} \tag{3.46}$$

Using (3.23), we can write

$$\begin{aligned}
\bar{b}_{t+1} = \frac{1}{N}\sum_{j=1}^{M_{t+1}} \Bigg\{ &\frac{\beta}{\delta+\beta}\left[\alpha\frac{s_t^j}{k_{t+1}} + \tau(1-\alpha)\right] - \frac{\delta}{\delta+\beta}(1-\tau)(1-\alpha) \\
&- \frac{\gamma_d\beta}{\delta+\beta}\left[\alpha + \tau(1-\alpha) - \frac{\bar{b}_{t+1}}{k_{t+1}^\alpha}\right] + \frac{\gamma_\omega\delta}{\delta+\beta}\left[(1-\tau)(1-\alpha) + \frac{\bar{b}_{t+1}}{k_{t+1}^\alpha}\right]\Bigg\} k_{t+1}^\alpha,
\end{aligned}$$

which, after rearranging, becomes

$$\begin{aligned}
\frac{\sum_{j=1}^{M_{t+1}} s_t^j}{N k_{t+1}} = &\left[\frac{\bar{b}_{t+1}}{k_{t+1}^\alpha} - \tau(1-\alpha)\right] \frac{\beta(1-\gamma_d m_{t+1}) + \delta(1-\gamma_\omega m_{t+1})}{\beta\alpha} \\
&+ \tau(1-\alpha)(1-m_{t+1})\frac{\beta+\delta}{\beta\alpha} + m_{t+1}\frac{\gamma_d\beta\alpha + \delta(1-\alpha)(1-\gamma_\omega)}{\beta\alpha}.
\end{aligned}$$

From Proposition 3.3, we know that $M_t = n$ (or $m_t = n/N$) for all $t \geq T_3$ and $\lim_{t\to\infty} \bar{b}_t/k_t^\alpha = b(n/N)$, leading to (3.46).

Since the right-hand sides of (3.46) and (3.20) must be equal to each other, the right-hand side of (3.20) increases with τ if and only if

$$\left[\frac{\partial b(n/N)}{\partial\tau} - 1 + \alpha\right] \frac{\beta(1-\gamma_d\frac{n}{N}) + \delta(1-\gamma_\omega\frac{n}{N})}{\beta\alpha} + (1-\alpha)\frac{N-n}{N}\frac{\beta+\delta}{\beta\alpha} > 0,$$

which is equivalent to (3.21).

Bibliography

[1] Adermon, A., Lindahl, M., Waldenström, D., 2018. Intergenerational wealth mobility and the role of inheritance: Evidence from multiple generations. Economic Journal 128, 482-513.

[2] Alonso-Carrera, J., Caballé, J., Raurich, X., 2007. Aspirations, habit formation, and bequest motive. Economic Journal 117, 813-836.

[3] Alonso-Carrera, J., Caballé, J., Raurich, X., 2008. Estate taxes, consumption externalities, and altruism. Journal of Public Economics 92, 1751-1764.

[4] Altonji, J.G., Hayashi, F., Kotlikoff, L.J., 1997. Parental altruism and inter vivos transfers: Theory and evidence. Journal of Political Economy 105, 1121-1166.

[5] Altonji, J.G, Villanueva, E., 2007. The marginal propensity to spend on adult children. The B.E. Journal of Economic Analysis and Policy 7(1) (Advances).

[6] Alvarez-Cuadrado, F., Japaridze, I., 2017. Trickle-down consumption, financial deregulation, inequality, and indebtedness. Journal of Economic Behavior & Organization 134, 1-26.

[7] Alvarez-Cuadrado, F., Long, N.V., 2011. The relative income hypothesis. Journal of Economic Dynamics and Control 35, 1489-1501.

[8] Alvarez-Cuadrado, F., Long, N.V., 2012. Envy and inequality. Scandinavian Journal of Economics 114, 949-973.

[9] Barro, R.J., 1974. Are government bonds net wealth? Journal of Political Economy 82, 1095-1117.

[10] Becker, G.S., Tomes, N., 1979. An equilibrium theory of the distribution of income and intergenerational mobility. Journal of Political Economy 87, 1153-1189.

[11] Bilancini, E., D'Antoni, M., 2012. The desirability of pay-as-you-go pensions when relative consumption matters and returns are stochastic. Economics Letters 117, 418-422.

[12] Bilbiie, F.O., 2008. Limited asset markets participation, monetary policy and (inverted) aggregate demand logic. Journal of Economic Theory 140, 162-196.

[13] Borissov, K., 2016. The rich and the poor in a simple model of growth and distribution. Macroeconomic Dynamics 20, 1934-1952.

[14] Borissov, K., Hashimzade, N., 2022. Fiscal policy and inequality in a model with endogenous positional concerns. Journal of Mathematical Economics 103, 102765.

[15] Borissov, K., Kalk, A., 2020. Public debt, positional concerns, and wealth inequality. Journal of Economic Behavior & Organization 170, 96-111.

[16] Caballé, J., Fuster, L., 2003. Pay-as-you-go social security and the distribution of altruistic transfers. Review of Economic Studies 70, 541-567.

[17] Diamond, P.A., 1965. National debt in a neoclassical growth model. American Economic Review 55, 1126-1150.

[18] Dioikitopoulos, E.V., Turnovsky, S.J., Wendner, R., 2020. Dynamic status effect, savings, and income inequality. International Economic Review 61, 351-382.

[19] Duesenberry, J.S., 1949. Income, Saving and the Theory of Consumer Behavior. Harvard University Press.

[20] Galí, J., López-Salido, J. D., Vallés, J., 2007. Understanding the effects of government spending on consumption. Journal of the European Economic Association 5, 227-270.

[21] Genicot, G., Ray, D., 2017. Aspirations and inequality. Econometrica 85, 489–519.

[22] Heffetz, O., Frank, R.H., 2011. Preferences for status: Evidence and economic implications. In: Benhabib, J., Bisin, A., Jackson, M.O. (Eds.), Handbook of Social Economics, vol. 1. Elsevier, pp. 69-91.

[23] Knell, M., 2010. The optimal mix between funded and unfunded pension systems when people care about relative consumption. Economica 77, 710-733.

[24] Kunze, L., 2012. Funded social security and economic growth. Economics Letters 115, 180-183.

[25] Laitner, J., 2001. Secular changes in wealth inequality and inheritance. Economic Journal 111, 691–721.

[26] Laitner, J., Ohlsson H., 2001. Bequest motives: A comparison of Sweden and the United States. Journal of Public Economics 79, 205–236.

[27] Lambrecht, S., Michel, P., Vidal, J.-P., 2005. Public pensions and growth. European Economic Review 49, 1261-1281.

[28] Lambrecht, S., Michel, P., Thibault, E., 2005. Capital accumulation and fiscal policy in an OLG model with family altruism. Journal of Public Economic Theory 8, 465-486.

[29] Ljungqvist, L., Uhlig, H., 2000. Tax policy and aggregate demand management under catching up with the Joneses. American Economic Review 90, 356-366.

[30] Liu, P., Thøgersen, J., 2020. Pay-as-you-go pensions and endogenous retirement. Macroeconomic Dynamics 24, 1700-1719.

[31] Maebayashi, N., 2020. Is an unfunded social security system good or bad for growth? A theoretical analysis of social security systems financed by VAT. Journal of Public Economic Theory 22, 1069-1104.

[32] Mankiw, N.G., 2000. The savers-spenders theory of fiscal policy. American Economic Review 90, 120-125.

[33] Michel, P., Pestieau, P., 1998. Fiscal policy in a growth model with both altruistic and nonaltruistic agents. Southern Economic Journal 64, 682-697.

[34] Michel, P., Pestieau, P., 2005. Fiscal policy with agents differing in altruism and ability. Economica 72, 121-135.

[35] Michel, P., Thibault, E., Vidal, J.-P., 2006. Intergenerational altruism and neoclassical growth models. In: Kolm, S.-C., Mercier Ythier, J. (Eds.). Handbook of the Economics of Giving, Altruism and Reciprocity: Applications, vol. 2. Elsevier, pp. 1055-1106.

[36] Moav, O., Neeman, Z., 2010. Status and poverty. Journal of the European Economic Association 8, 413-420.

[37] Moav, O., Neeman, Z., 2012. Saving rates and poverty: The role of conspicuous consumption and human capital. Economic Journal 122, 933-956.

[38] Pestieau, P., Thibault, E., 2012. Love thy children or money: Reflections on debt neutrality and estate taxation, Economic Theory 50, 31-57.

[39] Smetters, K., 1999. Ricardian equivalence: Long-run Leviathan. Journal of Public Economics 73, 395-421.

[40] Stiglitz, J., 2012. The Price of Inequality. Penguin UK.

[41] Werning, I., 2007. Optimal fiscal policy with redistribution. Quarterly Journal of Economics 122, 925-967.

Abstract

This dissertation examines how environmental and public (social security) policies and/or their effectiveness can be affected by individual motivations, political considerations, and social interactions.

The first chapter studies the interplay between individuals' warm-glow-driven environmental efforts and public environmental policy. Using a growth model with environmental quality as a public good, I show that without government intervention, the strength of warm glow reduces output but improves environmental quality in the long run. When the government intervenes, however, the warm-glow strength has a negative long-run effect on both variables. This suggests that advocating for private actions may be counterproductive for future sustainability when public policies are in place.

The second chapter, co-authored with Gerhard Sorger, studies the implications of political pressure for optimal climate policy. A key feature of the proposed model is its ability to capture dynamic inconsistency in policy implementation. Our findings offer a new rationale for why green lobbying can make pollution regulation less stringent. We also show that political pressure from the polluting sector leads to a higher rate of clean energy investment chosen by the government and can shift the energy mix towards renewables in the long run.

The third chapter, co-authored with Kirill Borissov, examines the aggregate and distributional effects of unfunded social security. In our model, individuals' decisions are subject to social comparisons, which can result in zero bequests left within poor families. In this case, unfunded social security increases long-run wealth inequality. However, it also increases the long-run capital stock, contrary to the cases where none or all families leave bequests. This highlights the importance of accounting for heterogeneity in bequest behavior when analyzing social security.

Overall, the findings of this dissertation can inform the design of forward-looking policies that address sustainability challenges due to climate change and population aging.

Zusammenfassung

Diese Dissertation untersucht, wie Umwelt- und Sozialpolitik (insbesondere die Sozialversicherung) und/oder deren Wirksamkeit durch individuelle Motivationen, politische Erwägungen und soziale Interaktionen beeinflusst werden können.

Das erste Kapitel analysiert das Zusammenspiel zwischen den durch ein "Warm-Glow"-Motiv getriebenen Umweltanstrengungen von Individuen und der öffentlichen Umweltpolitik. Anhand eines Wachstumsmodells, in dem Umweltqualität ein öffentliches Gut ist, zeige ich, dass in Abwesenheit staatlicher Eingriffe durch den Warm Glow das Produktionsniveau sinkt und die Umweltqualität steigt. Greift jedoch der Staat ein, hat eine stärkere Warm-Glow-Motivation langfristig negative Auswirkungen auf beide Variablen. Dies deutet darauf hin, dass die Förderung privater Maßnahmen kontraproduktiv für die zukünftige Nachhaltigkeit sein kann, wenn öffentliche Politiken bereits existieren.

Das zweite Kapitel, gemeinsam mit Gerhard Sorger verfasst, untersucht die Auswirkungen politischen Drucks auf die optimale Klimapolitik. Ein zentrales Merkmal des vorgeschlagenen Modells ist seine Fähigkeit, dynamische Inkonsistenz in der politischen Umsetzung abzubilden. Unsere Ergebnisse liefern eine neue Begründung dafür, warum grünes Lobbying zu einer weniger strengen Regulierung von Emissionen führen kann. Wir zeigen außerdem, dass politischer Druck aus dem verschmutzenden Sektor zu einer höheren staatlich gewählten Investitionsrate in saubere Energie führt und langfristig den Energiemix zugunsten erneuerbarer Energien verschieben kann.

Das dritte Kapitel, gemeinsam mit Kirill Borissov verfasst, untersucht die aggregierten und verteilungspolitischen Auswirkungen einer umlagefinanzierten Rentenversicherung. In unserem Modell unterliegen individuelle Entscheidungen sozialen Vergleichsprozessen, was dazu führen kann, dass arme Familien keine Erbschaften hinterlassen. In diesem Fall erhöht die umlagefinanzierte Rentenversicherung die langfristige Vermögensungleichheit. Gleichzeitig steigt jedoch auch der langfristige Kapitalstock — im Gegensatz zu den Fällen, in denen entweder keine oder alle Familien Erbschaften hinterlassen. Dies unterstreicht die Wichtigkeit, bei der Analyse von Rentensystemen die Heterogenität im Erbschaftsverhalten zu berücksichtigen.

Insgesamt können die Erkenntnisse dieser Dissertation zur Gestaltung zukunftsorientierter Politiken beitragen, die den Herausforderungen der Nachhaltigkeit durch den Klimawandel und den demografischen Wandel begegnen.