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1 Introduction

The purchasing power parity (PPP) is an economic concept which postulates that international goods arbitrage will, in the long run, equate purchasing power across countries. The variable of adjustment is the nominal exchange rate: If the national price level in country A is lower than in country B, international goods arbitrage will cause the currency of A to appreciate with respect to the currency of B, such that the price levels are equalized if expressed in one currency and no arbitrage opportunities remain. From the definition of the real exchange rate as the nominal exchange rate adjusted for relative price levels it follows that the PPP—the concept that the nominal exchange rate will adjust to the relative price levels—is equivalent to the condition that the real exchange rate is constant. In a statistical sense, the PPP is an empirically relevant equilibrium concept as long as the real exchange rate fluctuates closely around its mean, keeping the nominal exchange rate and the relative price levels together in the long run; i.e. as long as the real exchange rate is stationary. This theoretical concept lacks, however, significant empirical corroboration.

Rather than searching for stationarity of the real exchange rate, newer studies follow the co-integration approach: denoting the nominal exchange rate by s_t and the price differential by $p_t - p_t^*$, it holds that s_t and $p_t - p_t^*$ are co-integrated as long as a linear combination of the form $z_t = s_t - \lambda(p_t - p_t^*)$ is stationary. The main difference between testing for a unit root in the real exchange rate q_t and in the co-integration relation z_t is that the latter approach is less restrictive, allowing for a co-integration vector $(1, -\lambda)$ different from $(1, -1)$ (Sarno and Taylor 2009). Further, numerous studies have further tried to approach the problem through panel studies or very long time series. Although some of these studies have accomplished to find some evidence for mean-reversion, the speed of adjustment is still surprisingly low. *‘The purchasing power parity puzzle then is this’*, writes Kenneth Rogoff, *‘how can one reconcile the enormous short-term volatility of real exchange rates with the extremely slow rate at which shocks appear to damp out?’* (Rogoff 1996, as quoted by Sarno and Taylor 2009).

Alan Taylor (2001) proposes that two pitfalls—namely sampling (temporal aggregation) and model specification (linear specification)—could cause an underestimation of mean-reversion and thus explain much of the persistence of real exchange rates. In this thesis, we will concentrate on tackling the second pitfall, by formulating and estimating a non-linear model for the PPP.

The following overview of theoretical reasons for non-linear mean reversion of real exchange rates draws largely on Sarno and Taylor's 'The Economics of Exchange Rates', which provides an extensive overview over the various approaches towards cracking the purchasing power parity puzzle. According to Sarno and Taylor, the idea that the real exchange rate might exhibit non-linear mean-reversion dates back as far as 1916, when Heckscher proposed that trading costs could create a 'band of inaction' in which no international goods arbitrage occurs, even though purchasing power differs between two countries. Only as the purchasing power differential reaches some 'commodity points', at which the arbitrage opportunities outweigh the trading costs, international goods arbitrage takes place and mean-reversion occurs. Newer approaches consider variable costs of trade in addition to fixed costs: While there is mean-reversion outside some band of inaction, real-exchange rates will rarely cross their mean because of the proportional trading costs. Further, the thresholds might reflect not only shipping costs and trade barriers but also the idea that sunk costs from international trade lead risk-averse traders to wait for sufficiently large arbitrage opportunities before entering the international market. An alternative analysis was developed by Kilian and Taylor (2001), who focus on the determination of exchange rates on international financial rather than on international goods markets. In their model, traders take the advice of economic fundamentalists on the one hand, who might differ in their opinions about the exchange rate misalignments with respect to the equilibrium exchange rate; and technical analysts on the other hand, who focus on trend-following forecasts. The more uncertain the economic fundamentalists are about the equilibrium exchange rate, the more weight is given to the technical analyst's forecast, which transmits a unit root to the exchange rate and causes it to deviate from its equilibrium value. As the deviations become greater, however, the degree of agreement between economic fundamentalists on the misalignment increases, and more weight is given to their evermore conclusive consensus forecasts, such that mean-reversion sets in.

Following these theoretical arguments for non-linear mean reversion of the real exchange rate, this thesis will formulate an econometric model for the PPP between the Euro Area and the United States. Focusing on the original 'band of inaction'-theory, the model will be specified in a threshold non-linear way. More precisely, the real exchange rate will be modeled as a unit-root process within certain thresholds, and as mean-reverting outside these thresholds. The interest of this thesis lies in analyzing whether the 'band of inaction'-theory can explain the persistence of real exchange rates (i.e. whether there is a threshold effect) and whether the underestimation of the adjustment speed can be traced down to these rigidities (i.e. whether the speed of mean-reversion of real exchange rates found outside the thresholds is significantly faster than the speed determined over the whole sample).

Among the related empirical studies, Obstfeld and Taylor (1997) find non-linear mean-reversion of the real exchange rate in a threshold autoregressive (TAR) model, in which real exchange rates are modeled as unit-root autoregressive processes within two thresholds and switch to stationary autoregressive processes after transgressing them. Taylor, Peel and Sarno (2001) allow for smooth transition between the regimes through an exponential smooth transition autoregressive (ESTAR) model, in which the real exchange rate becomes increasingly mean-reverting rather than switching abruptly between diverting and mean-reverting regimes. This thesis differs from these papers in two respects: First, we use a multivariate rather than a univariate model setup, i.e. we formulate a Threshold-VAR rather than a Threshold-AR model. The multivariate model setup allows for a comparison between the out-of sample properties of the non-linear model and those of the conventional linear model. Second, we employ the model to the recent EUR/USD exchange rate from 1990 to 2009. While these data might present some challenges stemming from potential structural breaks through the process of European monetary integration on the one hand and the broad usage of the US-Dollar in international trade on the other hand, it also provides interesting insights about the history of over- and undervaluation of the Euro since the beginning of European monetary integration.

The thesis is structured as follows: Section 2 gives a short theoretical overview of the law of one price and the purchasing power parity. Section 3 recalls important econometric preliminaries and Section 4 presents our analytical framework, the co-integrated VAR model. Section 5 provides a description of the data set and a discussion of the data properties. Section 6 contains the statistical model, including the long-run identification and estimation of the linear model as well as the estimation of the thresholds and the non-linear model. All estimations are carried out with the statistical software package ‘E-Views’. Section 7 provides a test for the presence of a threshold effect. Since critical values for the distribution of the Threshold-VEC under the null of no threshold effect are unknown, they are generated through a bootstrap procedure such that inference on the threshold estimates can be drawn. Section 8 compares the out-of-sample forecasting properties of the linear and the non-linear model. Section 9 summarizes the main findings, concludes and provides an outlook for further research.

2 Theoretical Framework

Consider two countries (domestic and foreign) with one currency each.¹ The *law of one price (LOOP)* postulates that – under the assumption of frictionless international good arbitrage – the same good should have the same price across countries if expressed in terms of the same currency. Formally, let S_t denote the spot exchange rate in price notation (the price of the foreign currency in terms of domestic currency), and let $P_{i,t}$ and $P_{i,t}^*$ denote the domestic and foreign price of good i respectively. Then, the LOOP requires that $P_{i,t} = S_t P_{i,t}^*$ or, after taking natural logarithms,

$$p_{i,t} = s_t + p_{i,t}^*. \quad (1)$$

Following from the LOOP, the *purchasing power parity (PPP)* states that not only the prices of individual goods but also the national price levels should be the same across countries if expressed in the same currency. Under two assumptions – namely that all goods $i = 1, \dots, n$ are tradable and that their weights γ_i in the price index are identical in each country – the national price levels can be expressed as $p_t = \sum_{i=1}^n \gamma_i p_{i,t}$ and $p_t^* = \sum_{i=1}^n \gamma_i p_{i,t}^*$ where the weights satisfy $\sum_{i=1}^n \gamma_i = 1$. Then, the PPP condition can be derived from a weighted average over the n LOOP conditions:

$$\sum_{i=1}^n \gamma_i p_{i,t} = s_t + \sum_{i=1}^n \gamma_i p_{i,t}^* \Rightarrow p_t = s_t + p_t^*. \quad (2)$$

The PPP exchange rate is the exchange rate between two countries at which this condition holds, i.e. the exchange rate at which the purchasing power of one unit of currency is equalized between the countries:

$$s_t = p_t - p_t^*. \quad (3)$$

From the definition of the real exchange rate as the nominal exchange rate adjusted for relative price levels, $q_t = s_t - p_t + p_t^*$, it follows that the PPP holds as long as the logarithm of the real exchange rate is zero. In a statistical sense, the PPP is an empirically relevant equilibrium concept as long as the real exchange rates fluctuates closely around its zero mean, keeping the nominal exchange rate and the relative price levels together in the long run – i.e. if the real exchange rate is stationary.

¹The section on the purchasing power parity draws largely on Sarno and Taylor (2009).

3 Excursion: Econometric Preliminaries

Note: The econometric preliminaries draw largely on Juselius (2006) and Luetkepohl and Kraetzig (2008), as long as no other references are mentioned.

3.1 Stationarity and co-integration

3.1.1 Stationarity

A stochastic vector process $\{x_t\}$ is said to be *weakly stationary* if

$$\begin{aligned} E(x_t) &= -\infty < \mu < \infty \quad \text{for all } t, \\ \text{Var}(x_t) &= \Sigma_0 < \infty \quad \text{for all } t, \\ \text{Cov}(x_t, x_{t+h}) &= \Sigma_h < \infty \quad \text{for all } t \text{ and } h = 1, 2, \dots \end{aligned}$$

In words, a weakly stationary process has a time invariant, finite mean and variance as well as time invariant, finite covariances (i.e. the covariance of two realizations x_t and x_{t+h} depends solely on h , not on time). If the distribution of $(x_{t_1}, \dots, x_{t_n})$ equals the distribution of $(x_{t_1+h}, \dots, x_{t_n+h})$ for $h = \dots, -1, 0, 1, \dots$, the process is said to be strictly stationary. Henceforth, stationarity will refer to the concept of weak stationarity.

In contrast, a stochastic vector process is said to be integrated of order d —or $I(d)$ —if it becomes stationary only after taking the d^{th} difference, i.e. if $\Delta^d x_t$ is $I(0)$ whereas $\Delta^{d-1} x_t$ is still $I(1)$. From the $I(d)$ -notation it follows that a stationary vector process can be said to be $I(0)$. Economic aggregates are typically $I(1)$, i.e. Δx_t is typically stationary while x_t is not; where x_t denotes a vector of economic variables.

3.1.2 Co-integration

While macroeconomic and financial variables are typically $I(1)$, linear combinations of these variables are often found to be $I(0)$. Following Engle and Granger (1987), a linear combination of integrated variables is stationary if the variables exhibit a common stochastic trend: If, for example, subtracting one $I(1)$ -series from another yields an $I(0)$ -series, one would assume that both series grow at roughly the same rate, deviations of which are only preliminary. More formally, let x_t denote a vector of variables, $x_t = (x_{1,t}, x_{2,t}, \dots, x_{k,t})'$. The components x_t are said to be co-integrated of order (d, b) , denoted $x_t \sim CI(d, b)$, if (1) the vector x_t is $I(d)$, and if (2) there exists a vector $\beta \neq 0$ such that $z_t = \beta' x_t \sim I(d - b)$. The vector β is then called the co-integrating vector. In a system of k integrated variables there are up to $k - 1$ linearly independent co-integrating vectors β_i ($i = 1, \dots, k - 1$).

The concept of co-integration has an important economic interpretation: A theoretical equilibrium condition between the components of a vector of economic variables x_t can be expressed through a linear combination of the form $\beta'x_t = 0$. In reality however, economic variables will fluctuate around their equilibrium such that $\beta'x_t = z_t$, where z_t denotes a zero-mean equilibrium error. Then, the equilibrium concept has empirical relevance as long as z_t rarely drifts too far away from zero, i.e. as long as z_t is stationary – which is true as long as the $I(1)$ components of x_t are co-integrated: *‘Interpreting $\beta'x_t$ as the long-run equilibrium, co-integration implies that equilibrium holds except for a stationary, finite variance disturbance even though the series themselves are non-stationary and have infinite variance’* (Engle and Granger, 1987).

3.2 VAR and VEC Models

3.2.1 VAR Models

Let x_t denote a $(k \times 1)$ vector of variables, $x_t = (x_{1,t}, x_{2,t}, \dots, x_{k,t})'$. The VAR representation with p lags, or VAR(p), has the form

$$x_t = \Gamma_0 + \Pi_1 x_{t-1} + \Pi_2 x_{t-2} + \dots + \Pi_p x_{t-p} + \epsilon_t \quad (4)$$

where Π_i 's are $(k \times k)$ coefficient matrices, Γ_0 is a $(k \times 1)$ vector of constants and ϵ_t is an unobservable error term (an extensive derivation of the VAR model can be found in Juselius 2006). The VAR representation for x_t is *stable* (i.e. there exists a stationary solution for the VAR model) if the roots of the characteristic polynomial $\Pi(z) = I - \Pi_1 z - \dots - \Pi_p z^p$ all lie outside the unit circle, i.e. if $|\Pi(z)| = 0$ only if $z > 1$.

If the vector x_t is not stationary, the characteristic polynomial has a unit root (i.e. $|\Pi(z)| = 0$ if $z = 1$) and the VAR representation for x_t is not stable. A stable VAR representation then exists for $\Delta^d x_t$ only as long as the integrated variables are not co-integrated – in the presence of co-integrating relations, the VAR formulation is no longer the most convenient model setup for $\Delta^d x_t$. While the parameterization of a VAR is general enough to accommodate integrated variables, the co-integration relations between these variables do not appear explicitly. A more appropriate model setup for analyzing the co-integration structure results in the VEC model, which allows to preserve information about the long-run relationship between the variables as well as about their short-run dynamics.

3.2.2 VEC Models

The Granger representation theorem states that the property of co-integration implies, and is implied by, the existence of an error correction formulation.² Let x_t denote a $(k \times 1)$ vector of variables, $x_t = (x_{1,t}, x_{2,t}, \dots, x_{k,t})$. For convenience, assume that the variables are at most $I(1)$. Then the general VAR(p) model can be written as

$$x_t = \Gamma_0 + \sum_{s=1}^p \Pi_s x_{t-s} + \epsilon_t \quad (5)$$

Subtracting x_{t-1} from both sides and rearranging terms yields the general VEC($p-1$) representation,

$$\Delta x_t = \Gamma_0 + \sum_{s=1}^{p-1} \Gamma_s \Delta x_{t-s} + \Pi x_{t-1} + \epsilon_t, \quad (6)$$

where $\Gamma_s = -(\Pi_{s+1} + \dots + \Pi_p)$ and $\Pi = -(I_K - \Pi_1 - \dots - \Pi_p)$ are $(k \times k)$ coefficient matrices and Γ_0 is a $(k \times 1)$ vector of constants. Since the components of x_t were assumed to be at most $I(1)$, the components of Δx_t are all $I(0)$. It follows that Πx_{t-1} must be $I(0)$, thus that Πx_{t-1} must contain the co-integrating relations $\beta' x_{t-1}$.

Write Π as $\Pi = \alpha \beta'$, where β is the $(k \times r)$ co-integrating matrix that contains the $r \in \{1, \dots, k-1\}$ linearly independent co-integrating vectors and α is a $(k \times r)$ loading matrix.³ By construction, $rk(\beta) = rk(\alpha) = rk(\Pi) = r < k$. The rank of Π indicates the number of linearly independent co-integrating vectors and is referred to as the co-integrating rank of the system. Under the reduced-rank restriction on Π , the reduced-rank VEC($p-1$) model can be written as

$$\Delta x_t = \Gamma_0 + \sum_{s=1}^{p-1} \Gamma_s \Delta x_{t-s} + \alpha \beta' x_{t-1} + \epsilon_t, \quad (7)$$

where $\beta' x_{t-1}$ contains the co-integration relations. Denoting this vector as the error correction term EC_{t-1} we can write the VEC($p-1$) model as

$$\Delta x_t = \Gamma_0 + \sum_{s=1}^{p-1} \Gamma_s \Delta x_{t-s} + \alpha EC_{t-1} + \epsilon_t. \quad (8)$$

²A proof is provided in Engle and Granger (1987). A proof that this theorem holds for the multivariate, vector error-correction models can be found in Johansen (1996) or in Juselius (2006).

³Note that while Π is unique, β and α are not: using any $(r \times r)$ matrix B with $rk(B) = r$, we obtain a new co-integrating matrix $\beta B'^{-1}$ and a new loading matrix αB for which $(\alpha B)(\beta B'^{-1}) = \Pi$ holds.

While the coefficient matrices Γ_s are often referred to as the short-run parameter matrices, Π contains the information about the co-integration relationship and may be called the long-run parameter matrix. The co-integration relations contained in $\beta'x_{t-1}$, which are interpreted as long-run steady-state relationships, are of particular economic interest. *'Error-correction models allow the long-run components of variables to obey equilibrium constraints while short-run components have a flexible dynamic specification.'* (Engle and Granger 1987)

Two cases should be remarked, in which the reduced-rank condition is not fulfilled:

- If $rk(\Pi) = 0$, the vector x_t is non-stationary but its components are not co-integrated. Then, the term Πx_{t-1} vanishes from the VEC.
- If $rk(\Pi) = k$, the vector x_t is already stationary.

Then, a VAR specification for $\Delta^d x_t$ and x_t , respectively, is the appropriate model setup.

3.3 Threshold co-integration

If a vector time series process contains one or many structural breaks, the coefficients of its VEC representation will not be constant over the whole sample range. The idea of threshold autoregression is to decompose such complex stochastic systems into simpler subsystems by cutting the globally non-linear relationship into smaller locally linear regimes, i.e. regions of the state space within which coefficients are constant (Tong, 1993). The variable which causes the regime to switch between those regimes, whether it is endogenous or exogenous, is called the threshold variable.

The special case of threshold co-integration was introduced by Balke and Fomby (1997). The essential idea is that the error correction term, which captures the deviations from the long-run equilibrium in an Engle and Granger error correction equation, may display threshold non-linear behavior. Regime switching in threshold error-correction models is thus determined by the realizations of the error correction term in the previous period. In other words, EC_{t-1} is the threshold variable. Multivariate extensions to the Balke and Fomby approach were developed by Lo and Zivot (2001) and Hansen and Seo (2002).

The general T-VEC(p-1) model has the form

$$\Delta X_t = \Gamma(0)^{(j)} + \sum_{s=1}^{p-1} \Gamma(s)^{(j)} \Delta X_{t-s} + \alpha^{(j)} EC_{t-1} + \epsilon_t \quad (9)$$

$$\theta^{(j-1)} < EC_{t-1} \leq \theta^{(j)}$$

where $\Gamma(s)^{(j)}$ are the regime-dependent coefficient matrices and $\alpha^{(j)}$ is the regime-dependent $(k \times r)$ loading matrix associated to the $(r \times 1)$ error correction vector $EC_{t-1} = \beta' X_{t-1}$. The thresholds $\theta^{(j)}$ satisfy $-\infty = \theta^{(0)} < \theta^{(1)} < \dots < \theta^{(l)} < \theta^{(l+1)} = \infty$ and are assumed to be time-invariant. Threshold VEC models rest upon on the idea that the relevant regime in period t , $J_t \in \{1, \dots, l\}$, is determined by the error correction term EC_{t-1} in the previous period $t-1$. Whenever this term crosses one of the thresholds, the regime-dependent parameters of the model change in the following period.

In the general formulation (9), all parameters except for the co-integrating vector β are modeled as potentially regime-dependent. However, we are more interested in holding the short-run coefficients Γ fixed, while allowing only the adjustment coefficient associated to the error correction term, α , to switch across the regimes. This T-VEC(p-1) model can then be written as

$$\Delta X_t = \Gamma(0) + \sum_{s=1}^{p-1} \Gamma(s) \Delta X_{t-s} + \alpha^{(j)} EC_{t-1} + \epsilon_t \quad (10)$$

$$\theta^{(j-1)} < EC_{t-1} \leq \theta^{(j)}$$

Consider the three-regime case, where error correction is absent within a certain ‘band of inaction’ around some equilibrium value while present outside this band. The resulting model is a so-called Equilibrium T-VEC model.

$$\Delta X_t = \Gamma(0) + \sum_{s=1}^{p-1} \Gamma(s) \Delta X_{t-s} \begin{cases} + \alpha^{(1)} EC_{t-1} + \epsilon_t & \text{for } -\infty < EC_{t-1} \leq \theta^{(1)} \\ + \epsilon_t & \text{for } \theta^{(1)} < EC_{t-1} \leq \theta^{(2)} \\ + \alpha^{(3)} EC_{t-1} + \epsilon_t & \text{for } \theta^{(2)} < EC_{t-1} \leq \infty \end{cases}$$

By introducing regime-specific dummy variables $\Theta_t^{(j)}$ indicating the regime state ($\Theta_t^{(j)} = 1$ if $\theta^{(j-1)} < EC_{t-1} \leq \theta^{(j)}$ and zero otherwise) we can rewrite the T-VEC(p-1) as

$$\Delta x_{t-1} = \Gamma(0) + \sum_{s=1}^{p-1} \Gamma(s) \Delta X_{t-s} + \sum_{j=0}^l \alpha^{(j)} \Theta_t^{(j)} EC_{t-1} + \epsilon_t. \quad (11)$$

The model has the form of a linear VEC model with exogenous components, $\Theta_t^{(j)} EC_{t-1}$ or $\Theta_t^{(j)} \beta' x_{t-1}$. The reason for treating β as exogenous roots in the problem of identification. Remember that while Π is unique, β and α are not: Using any $(r \times r)$ matrix B with $rk(B) = r$, we obtain a new co-integrating matrix $\beta B'^{-1}$ and a new loading matrix αB for which $(\alpha B)(\beta B'^{-1}) = \Pi$ holds. In order to obtain identified estimates, we must hold either α or β fixed in the estimation. While linear adjustment towards a non-linear co-integrating relation has no proper economic interpretation, non-linear adjustment towards a fixed long-run equilibrium relation – i.e. a fixed co-integrating vector β – is justified by economic theory.

Consequently, in a first step, the co-integrating vector β is estimated within the linear model and the EC -term $\beta' x_{t-1}$ is calculated. For the moment, assume that the threshold values $\theta^{(1)}$ and $\theta^{(3)}$ are known, such that the regime specific EC -terms $\Theta_t^{(j)} \beta' x_{t-1}$ can be calculated. Then, the adjustment coefficients $\alpha^{(j)}$ can be estimated through the standard OLS procedure, where the EC -terms are treated as two ordinary exogenous components. While the thresholds are, in practice, estimated through a search algorithm rather than assumed as known, the estimation procedure remains unchanged – essentially, it is just repeated for every possible combination of thresholds.

Our search algorithm differs from the algorithm proposed by Hansen and Seo primarily in that these authors perform a joint grid-search over one threshold and one co-integrating vector rather than treating the co-integrating relation as exogenous. The joint grid-search over the thresholds and the co-integrating vectors becomes, however, overly complicated if there is more than one co-integrating vector or more than one threshold.

4 Analytical Framework

The VEC representation for the PPP model can be written as:

$$\begin{aligned}\Delta x_t &= \Gamma(0) + \sum_{s=1}^{p-1} \Gamma(s) \Delta x_{t-s} + \Pi x_{t-1} + \epsilon_t \\ x_t &= [s_t, p_t, p_t^*]' \\ \epsilon_t &= [\epsilon_t^1, \epsilon_t^2, \epsilon_t^3]' \sim NID(0, \Sigma)\end{aligned}\tag{12}$$

where $\Gamma(0)$ is a (3×1) vector of constants, $\Gamma(s)$ ($s = 1, \dots, p-1$) are (3×3) matrices holding the short-run coefficients and $\Pi = \alpha\beta'$ is a (3×3) matrix holding the long-run parameters:

$$\begin{aligned}\begin{pmatrix} \Delta s_t \\ \Delta p_t \\ \Delta p_t^* \end{pmatrix} &= \begin{pmatrix} \gamma(0)_1 \\ \gamma(0)_2 \\ \gamma(0)_3 \end{pmatrix} + \begin{bmatrix} \gamma(1)_{1,1} & \gamma(1)_{1,2} & \gamma(1)_{1,3} \\ \gamma(1)_{2,1} & \gamma(1)_{2,2} & \gamma(1)_{2,3} \\ \gamma(1)_{3,1} & \gamma(1)_{3,2} & \gamma(1)_{3,3} \end{bmatrix} \begin{pmatrix} \Delta s_t \\ \Delta p_t \\ \Delta p_t^* \end{pmatrix} \\ &+ \begin{bmatrix} \alpha_{1,1} & \dots & \alpha_{1,r} \\ \alpha_{2,1} & \dots & \alpha_{2,r} \\ \alpha_{3,1} & \dots & \alpha_{3,r} \end{bmatrix} \begin{bmatrix} \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \cdot & \cdot & \cdot \\ \beta_{r,1} & \beta_{r,2} & \beta_{r,3} \end{bmatrix} \begin{pmatrix} s_{t-1} \\ p_{t-1} \\ p_{t-1}^* \end{pmatrix} + \begin{pmatrix} \epsilon_t^1 \\ \epsilon_t^2 \\ \epsilon_t^3 \end{pmatrix}\end{aligned}\tag{13}$$

The VEC formulation is an appropriate model setup as long as $\Pi = \alpha\beta'$ is of reduced rank $0 < r < 3$. Recall that $rk(\Pi) = 3$ if each variable is stationary and that $rk(\Pi) = 0$ if there is no co-integrating relation even though the variables are integrated. Assume that all variables are $I(1)$ and that there is at least one co-integrating relation between them. In particular, we expect to find a co-integration relation between s_t , p_t and p_t^* , i.e. we expect the PPP to hold, pulling the nominal exchange rate towards a level which equates purchasing power in both countries. Further, we expect that the coefficient associated to p_t is equal in size and opposite in sign to the coefficient associated to p_t^* . Since there are no other theoretically justified co-integrating relationships between the variables, we expect that β is a vector of the form $\beta_1' = (1 \quad -\beta_{1,2} \quad \beta_{1,3})$, where $-\beta_{1,2} = \beta_{1,3}$. A finding of $\beta_{1,2} = -1$ and $\beta_{1,3} = 1$ would be of particular interest, as it supports the absolute rather than only the relative PPP theory. Under these assumptions, the model reduces to

$$\begin{aligned}\begin{pmatrix} \Delta s_t \\ \Delta p_t \\ \Delta p_t^* \end{pmatrix} &= \begin{pmatrix} \gamma(0)_1 \\ \gamma(0)_2 \\ \gamma(0)_3 \end{pmatrix} + \begin{bmatrix} \gamma(1)_{1,1} & \gamma(1)_{1,2} & \gamma(1)_{1,3} \\ \gamma(1)_{2,1} & \gamma(1)_{2,2} & \gamma(1)_{2,3} \\ \gamma(1)_{3,1} & \gamma(1)_{3,2} & \gamma(1)_{3,3} \end{bmatrix} \begin{pmatrix} \Delta s_t \\ \Delta p_t \\ \Delta p_t^* \end{pmatrix} \\ &+ \begin{bmatrix} \alpha_{1,1} \\ \alpha_{2,1} \\ \alpha_{3,1} \end{bmatrix} \begin{bmatrix} \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \end{bmatrix} \begin{pmatrix} s_t \\ p_t \\ p_t^* \end{pmatrix}\end{aligned}\tag{14}$$

where the first line can be expressed through

$$\Delta s_t = \gamma(0)_1 + \gamma(1)_{1,1} \Delta s_t + \gamma(1)_{1,2} \Delta p_t + \gamma(1)_{1,3} \Delta p_t^* + \alpha_{1,1} (\beta_{1,1} s_t + \beta_{1,2} p_t + \beta_{1,3} p_t^*).$$

Error-correcting behavior of the nominal exchange rate towards the PPP rate implies that the adjustment parameter associated to the exchange rate, $\alpha_{1,1}$, is negative. There is, however, no theoretical reason why p_t or p_t^* should respond to deviations of the nominal exchange rate from the PPP. Hence, we expect that the loading vector α_1 has the form $\alpha_1' = (-\alpha_{1,1} \ 0 \ 0)$. The zero rows in α imply that domestic and foreign price levels are weakly exogenous to the co-integrating vector β .

5 Data and Data Properties

5.1 Data

The data cover the USD/EUR exchange rate as well as the US- and European price levels from January 1990 – the beginning of European monetary integration – until October 2008 – the outburst of the world financial crisis – on a monthly basis. The exchange rate was specified as the nominal exchange rate in price notation, i.e. the price of one USD expressed in EUR. An increase in the exchange rate is thus interpreted as a nominal depreciation of the Euro, and vice-versa.⁴ Price levels were specified as producer price levels rather than consumer price levels. Note that the PPP is derived under the assumption that all goods are tradable and traded. While both producer and consumer price indices contain non-traded goods, their proportion is lower in the former than in the latter.⁵ The time series are graphically presented in Figure 1.

The choice of 1990 as the beginning of the sample period results from a trade-off between the number of observations available for estimation and the number and magnitude of structural breaks for the Euro Area data. The year 1990 coincides with the beginning of the first stage of the European Monetary Union (EMU), which was marked by the complete liberalization of capital movements in the European Economic Community. In 1992, the Treaty on the European Union was signed in Maastricht. The treaty, which entered into force the following year, established the completion of the EMU as a formal objective and stated a number of economic convergence criteria concerning inflation, long term interest rates, exchange rates as well as fiscal debt and deficit. Five years later, all large European economies were considered to fulfill the conditions to adopt the Euro as a single currency. Note that the fulfillment of the convergence criteria on inflation might have led to a structural break in the producer price series.

In 1994, one year after the Treaty on the European Union entered to force, the establishment of the European Monetary Institute (EMI) marked the second stage of the EMU. While monetary policy remained within the responsibility of national central banks, the EMI searched to strengthen cooperation between the central banks and their monetary policy operations as well as to carry out the preparatory work required for the establishment of the European System of Central Banks and the European Central Bank in 1998, the conduct of the single monetary policy and the creation of a com-

⁴The monthly average nominal exchange rate was provided by Thomson Reuters (ECU/USD until December 1998 and EUR/USD from January 1999 onwards).

⁵Seasonally adjusted monthly producer price indices were provided by the Federal Reserve for the United States and by the OECD for the Euro Area in its respective composition.

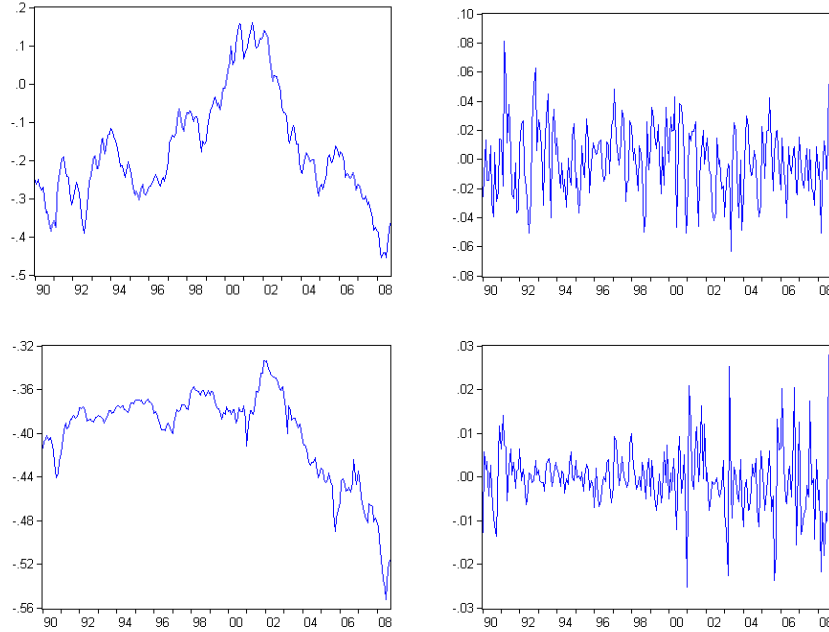


Figure 1: Time series, 1990-2008. Upper left: EUR/USD exchange rate in levels (s_t). Upper right: EUR/USD exchange rate in first differences (Δs_t). Bottom left: producer price differential in levels ($p_t - p_t^*$). Bottom right: producer price differential in first differences ($p_t - p_t^*$).

mon currency in the third stage of the EMU. In January 1999, the beginning of the third stage, the conversion rates against the Euro were irrevocably fixed, the Euro was introduced as a common book currency and the single monetary policy entered to force. Again, the handover of monetary policy responsibility from partly rather ‘dovish’ national central banks to the rather ‘hawkish’ ECB might be noted as a structural break in the producer price series.

Three years later, on January 1st 2002, the Euro was physically introduced in the member states and by the end of February 2002 banknotes and coins became the sole legal tender in the Euro Area. While the physical introduction of the Euro should have had a lesser effect on the exchange rates than the fixation of conversion rates in 1999, the cash changeover might have had a stronger effect on the inflation rates. Beginning in 1999 or 2002, no policy measures are expected to cause important structural breaks to our sample, as the Euro adoption of Slovenia is not expected to affect exchange rates or inflation rates to a significant extent. For an extensive overview of the phases of European monetary integration see Mongelli (2004).

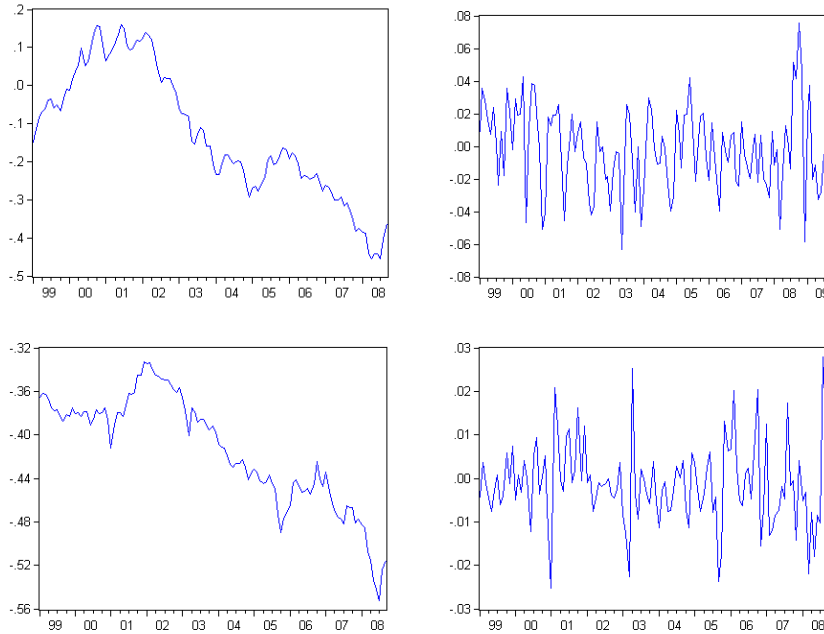


Figure 2: Subsample of Figure 1, 1999-2008.

5.2 Tests on the Suitability of the Data

The exchange rate, price levels and the price differential are all found to be $I(1)$, as the Augmented Dickey-Fuller test on the null hypothesis of a unit root rejects the null only when first differences are taken. Thus, we can conclude that all our variables are $I(1)$ over the whole period and hence fulfill the precondition for further co-integration analysis.

However, as Figure 1 shows, the variance of the differenced producer price differential appears not to be time-invariant but substantially higher from around the year 2000 onwards than before. In the undifferenced series, the producer price differential (which was until then relatively constant) started to fall strongly from 2000 onwards. This process was accompanied by a strong appreciation of the Euro. There is thus a potential structural break in January 1999, when the Euro was introduced in the Euro Area, and similarly in 2002, when the cash changeover took place. In the following chapter it will be shown that policy dummies for the periods 1999 to 2001 and 2002 to the end of our sample are indeed highly significant in the estimated co-integrating relationships.

6 The Statistical Model

6.1 Long-run Identification

It was previously discussed that the relative PPP is a relevant concept as long as the real exchange rate $s_t - (p_t - p_t^*)$ is stationary or, more generally, as long as s_t and $(p_t - p_t^*)$ are co-integrated. For the Engle and Granger test of co-integration we perform a simple regression of s_t on p_t , p_t^* and a constant, treat the residuals of the regression as the potential co-integrating relation and test them on the null hypothesis of a unit root. Since the null can be rejected on the 5% level, we conclude that the variables are co-integrated. While the coefficient associated to p_t^* seems equal in size and opposite in sign to the coefficient associated to p_t , this hypothesis must formally be rejected on any standard confidence level. Also, the equilibrium relationship between the exchange rate and the price differential is not equiproportionate: the coefficients associated to p_t and p_t^* are significantly different from one in absolute value.

To proceed from the univariate to the multivariate specification, we first take a look at the characteristic polynomial of the VAR(2) model. The characteristic polynomial is found to have one near-unit root, which—taking the value 1.01—will be considered to equal one, which suggests that the levels specification is not the appropriate model setup. Whether we can proceed to the VEC setup for x_t or whether we have to proceed to the VAR setup for Δx_t depends on whether or not there exists a co-integrating relation between the components of x_t . As expected, the Johansen test finds exactly one co-integrating relationship between price levels and the exchange rate. This result is robust to all deterministic terms included in the co-integrating equation.

At this point, however, there is no empirical evidence that the PPP is indeed the co-integrating relation between our variables. Remember that α and β are not unique – in order to identify the long-run structure, identifying restrictions must be imposed on the co-integrating relation. In the case of $r = 1$, a convenient identifying restriction is the normalization of the first coefficient, i.e. $\beta_{1,1} = 1$. This restriction is empirically and economically identified: The coefficients are statistically significant and economically meaningful, i.e. consistent in sign, while not in magnitude, with the PPP theory. Since there is a risk of normalizing an insignificant coefficient (i.e. restricting a zero coefficient to take the value one), we cross-check our estimates through normalizing each of the price levels. Regardless of the identifying restriction imposed, all three variables are significant in the co-integrating equation.

Since unrestricted VAR and VEC models involve a relatively large number of parameters—namely $k(d + kp)$, where d is the number of deterministic variables—it is often useful to impose restrictions that reduce the number of coefficients in order to improve the estimation precision (see for example Luetkepohl and Kraetzig, 2008). In the identified co-integrating relation, the restriction that the coefficient associated to p_t is opposite in sign and equal in magnitude to the one associated to p_t^* seems plausible. Henceforth, $x_t = (s_t \ p_t - p_t^*)'$ rather than $x_t = (s_t \ p_t \ p_t^*)'$.⁶

The co-integrating relation determined through the Johansen test is virtually identical to the co-integrating relation determined through the Engle-Granger procedure. Both co-integrating relations present a clear structural break in the form of a mean-shift at the beginning of the third stage of the EMU: While the deviations of the nominal exchange rates from its monetary fundamentals are permanently negative until 1999—indicating an overvaluation of the Euro with respect to the PPP—the deviations are largely positive thereafter. We account for the structural break through the introduction of two mean-shift dummies into the co-integrating relation: The variable $D_1 = (0, \dots, 0, 1, \dots, 1, 0, \dots, 0)$ takes the value one between January 1999 and December 2001; another variable $D_2 = (0, \dots, 0, 1, \dots, 1)$ takes the value one from January 2002 onwards.⁷ Both dummies are highly significant and positive, implying a strong upward shift of the constant after 1999 and a smaller, still significant upward shift after the cash changeover in the Euro Area. Thus, although the common currency underwent a sustained appreciation since 2002, we conclude that it was still undervalued with respect to its PPP fundamentals: The steadily decreasing inflation differential would have implied an even stronger appreciation of the Euro. After the introduction of the dummy variables to the VEC, the likelihood increases substantially, the sum of squared residuals is reduced by half. Rather than only on the 5% level, the null of a unit root in the co-integrating equation can now be rejected at a 1% level. Overall, the structural break which was visible in the baseline model is absent in the Structural Break specification.

⁶Considering the economic and statistic arguments in favor of the homogeneity restriction, we will impose the restriction that the coefficient associated to p_t is opposite in sign and equal in magnitude to the one associated to p_t^* even though this hypothesis cannot formally be rejected on any standard level of significance.

⁷We are aware that the asymptotic distributions of the model might be affected through the introduction of deterministic components other than a constant and a trend (see Juselius (2006) for a discussion).

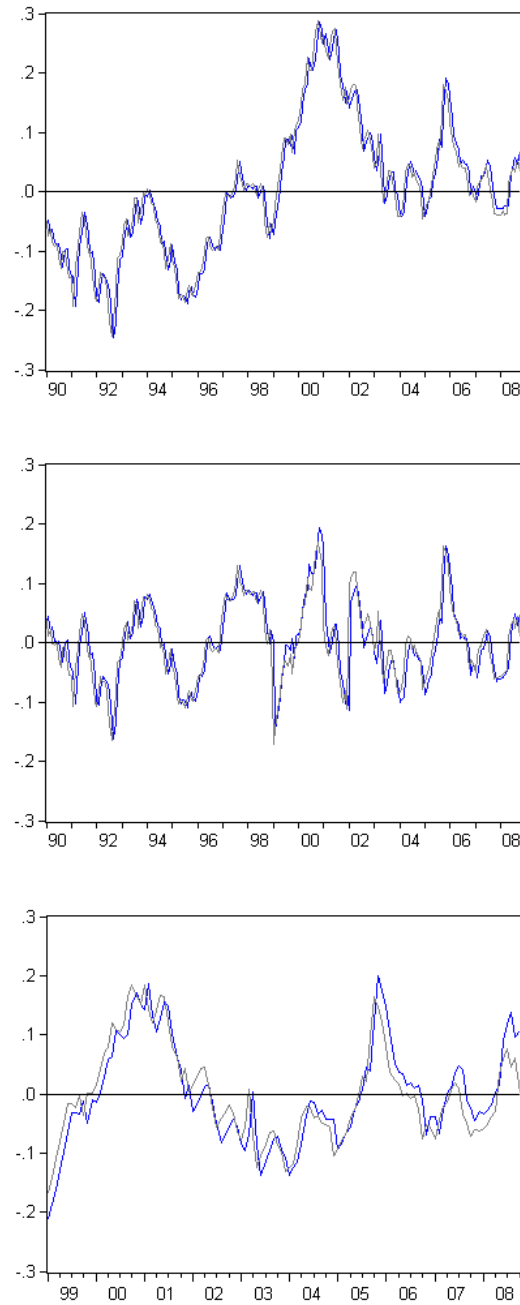


Figure 3: Co-integrating relations for the structural model as identified by the Engle and Granger procedure (grey) and the Johansen procedure (blue). Upper: Baseline-1990 specification. Center: Structural Break specification. Bottom: Baseline-1999 specification.

6.2 Model Selection

For the model selection we use both the Akaike (AIC) and the Schwarz Information Criteria (SIC). The AIC is an asymptotically efficient criterion, that overestimates the lag length with positive probability; the SIC is an asymptotically consistent criterion that favors a more parsimonious specification. It turns out, however, that both information criteria propose a VAR(2) / VEC(1) setup for our PPP model.

To report any potential mis-specification, we test the residuals from our linear model non-autocorrelation and residual normality. First, the portmanteau test finds some significant autocorrelation in the residuals from the VAR(2) / VEC(1) models. The remaining residual autocorrelation, which can be traced back to autocorrelation in the price differential, seems to stem from a structural break rather than an inadequate choice of lag length: If mean-shift dummies for the first and second phases of the Euro introduction are included, the portmanteau test cannot reject the null hypothesis of non-autocorrelation, which indicates that our lag-order is adequate. Finally, we run the multivariate Hansen and Doornik test on the null hypothesis of residual normality. However, joint residual normality must be strongly rejected. While the null of individual normality is not rejected for the residuals of the exchange rate equation it is strongly rejected for price levels and the price differential. Even in the Structural Break specification, residuals are not found to be normal.

6.3 The Linear Model

Summing up the remarks on model selection, the initial specification will not have to be modified: The lag length of two for the levels and one for the difference specification seems appropriate, the existence of exactly one co-integrating relationship was confirmed and the normalization of the first coefficient in the co-integrating relation is a valid identifying restriction. As for the over-identifying restrictions, we will impose the homogeneity constraint even though the homogeneity hypothesis was rejected. The results will henceforth be reported for the baseline specification using the full data set ('Baseline-1990'), the Structural Break specification ('Structural Break') and, further, for the baseline specification on the post-1999 data set only ('Baseline-1999'). As in the Baseline-1990 specification, the existence of exactly one co-integrating relation is confirmed for the Baseline-1999 specification. However, different than in the Baseline-1999 specification, the homogeneity restriction cannot be rejected at the 1% level. Also, while the information criteria again propose a lag length of two and one for the levels and difference specifications respectively, there seems to be no autocorrelation problem in the Baseline-1999 specification.

The estimation of the well-specified linear models reveals that the adjustment coefficients associated to the exchange rate equation are either insignificant or relatively small, which indicates that there is little or no equilibrium correction of the nominal exchange rate towards its PPP fundamentals. The lowest overall speed of adjustment is found over the short sample. The fastest speed is, unsurprisingly, detected in the Structural Break specification. The co-integrating vectors and the adjustment coefficients are displayed in Figure 4. Overall, the low speed of adjustment—the purchasing power parity puzzle—motivates the threshold formulation of the PPP model. Following the argumentation of Taylor (2001), we expect that the segmentation of the sample space to mean-reverting and non mean-reverting regimes will significantly reduce the downward bias of the overall adjustment speed towards the price fundamentals.

	Baseline-1990	Structural Break	Baseline-1999
s_t	+1.0000	+1.0000	+1.0000
$(p_t - p_t^*)$	-2.2320*	-2.5673***	-3.0217***
constant	-0.7196	-0.7653	-1.1232
D1		-0.1734***	
D2		-0.1474***	
α_s	-0.0195*	-0.0479**	-0.0021

Figure 4: Results for the co-integrating vectors (lines 1-5) and the adjustment coefficients for the exchange rate equation (line 6), for the selected specifications of the linear VEC model.

6.4 The Threshold Model

In order to capture the non-linear properties of the long-run equilibrium, we introduce the general T-VEC model as

$$\Delta x_t = \Gamma(0) + \sum_{s=1}^{p-1} \Gamma(s) \Delta x_{t-s} + \alpha^{(j)} EC_{t-1} + \epsilon_t \quad (15)$$

$$\theta^{(j-1)} < EC_{t-1} \leq \theta^{(j)}$$

$$x_t = [s_t, p_t, p_t^*]'$$

$$\epsilon_t = [\epsilon_t^1, \epsilon_t^2, \epsilon_t^3]' \sim NID(0, \Sigma)$$

where the regime-specific correction coefficients α^j ($j \in \{1, \dots, l\}$) are determined by the state of the error term in the previous period. We restrict the T-VEC to a three-regime equilibrium T-VEC, with $-\infty < \theta^1 < \theta^2 < \infty$.

Thus, there are three regimes—one inner and two outer regimes—spanning from $-\infty$ to θ^1 , from θ^1 to θ^2 and from θ^2 to $+\infty$ respectively.

Let Θ^1 denote a vector with $\Theta_t^1 = 1$ if $EC_t \in (\theta^0, \theta^1]$ and $\Theta_t^1 = 0$ otherwise. Analogously, let Θ^2 denote a vector where $\Theta_t^2 = 1$ if $EC_t \in (\theta^2, \theta^3]$ and $\Theta_t^2 = 0$ otherwise. Replacing the case-differentiation notation with these regime-specific dummy variables, the T-VEC can be formulated and estimated like a linear VEC model:

$$\Delta x_t = \Gamma(0) + \sum_{s=1}^{p-1} \Gamma(s) \Delta x_{t-s} + \alpha^1 \Theta_{t-1}^1 EC_{t-1} + \alpha^2 \Theta_{t-1}^2 EC_{t-1} + \epsilon_t \quad (16)$$

First, we estimate the linear model and store the co-integrating relationship $EC_{t-1} = \beta x_{t-1}$. In a second step, we estimate the T-VEC model as a linear VAR in first differences, augmented by two deterministic components, namely $\alpha^1 \Theta^1 EC_{t-1}$ and $\alpha^2 \Theta^2 EC_{t-1}$. Since there is no theoretically justified threshold-value after which real exchange rates should become mean-reverting, we have to estimate the thresholds θ^1 and θ^2 from our sample. This is performed through a search algorithm.

6.4.1 Estimation of the Thresholds

In a first step, we search for upper and lower boundaries for the thresholds, constrained by the condition that at least 10% of observations must remain within each regime, where the 10% condition was chosen deliberately. For example, the upper threshold for the structural model including policy dummies is constrained by +0.085, the lower threshold by -0.088. Then, we estimate the T-VEC model with an upper threshold ranging from zero to +0.085 (the respective upper threshold) in 0.001-increments. For every upper threshold within this range, we estimate the model with a lower threshold ranging from -0.088 (the respective lower threshold) to zero, again in 0.001-increments. To prevent the finding of a local rather than a global optimum, the algorithm runs over all possible combinations of upper and lower thresholds. For each of the thousands of resulting T-VEC models, we select the model in which the likelihood of the exchange rate equation is maximized (or an information criterion is minimized) with respect to the linear formulation. We chose to optimize the exchange rate equation rather than the whole model since we are interested in explaining and forecasting exchange rates, not price levels. A graphical representation of the search algorithm is presented in Figure 5, in which the AIC is shown for each combination of upper and lower thresholds (in 0.005 increments, for purpose of clarity).

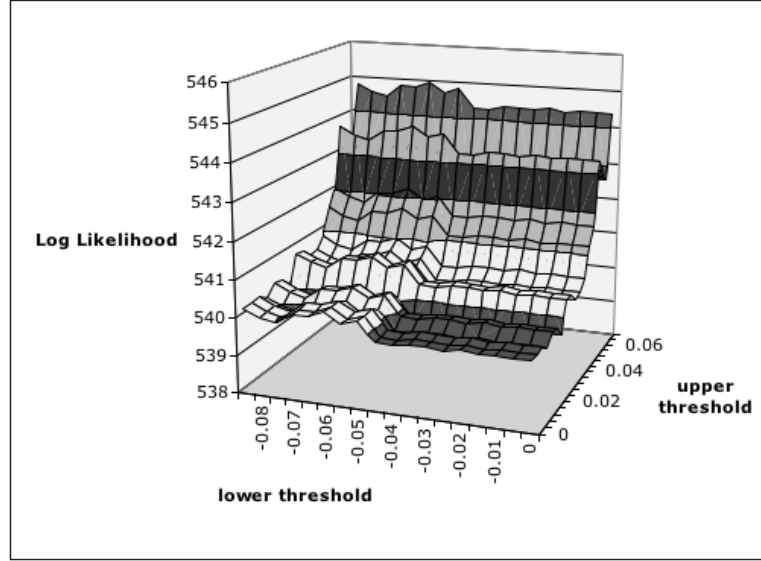


Figure 5: Search algorithm for thresholds in the Structural Break specification. The x - and y - axes show the lower and upper thresholds respectively, the z -axis shows the associated log likelihood. For every upper threshold, the likelihood function is maximized through a lower threshold of around -0.06 and vice versa.

6.4.2 Findings from the Threshold Model

While the specific findings differ somehow among the selected specifications (see Figure 6 for summarized boundaries, thresholds and adjustment parameters) there are some interesting general results. Note, however, that these findings have not yet been tested on statistical significance.

First, for all specifications of the linear VEC model there exists a T-VEC model which minimizes the AIC and, consequently, maximizes the likelihood of the exchange rate equation. Since the AIC is a convenient model selection criterion for forecasting purposes, the T-VEC model will probably outperform the linear VEC model at long-run out-of-sample exchange rate forecasts. This proposition will be tested in Section 9. In contrast, the SIC is hardly ever minimized through a threshold specification, as the improvement in the likelihood function is more than offset by the additional parameters included in the models.

Second, there is evidence for asymmetric adjustment of the nominal exchange rate with respect to its PPP fundamentals: Within the lower regime, equilibrium correction occurs at a faster pace than on the whole sample; however the reverse is true for the upper regime ($\alpha_1 < \alpha < \alpha_3$). A single-threshold model with $\theta = 0$ confirms that the adjustment speed associated to the lower regime (i.e. all negative deviations from the PPP fundamentals)

is more than twice as fast as the adjustment speed associated to the upper regime (-0.090 versus -0.035). Further evidence for asymmetric adjustment of the nominal exchange rate is provided by a graphical representation of the regime-specific dummy variables, as upward deviations are less frequent and more persistent than downward deviations across all models. Since a positive error correction term corresponds to an undervaluation of the Euro relative to its PPP fundamentals in the previous period, the asymmetry implies that an overvaluation of the Euro is corrected at a substantially higher rate than an undervaluation. This finding is consistent with our previous remark that the Euro seems to have been an undervalued currency relative to its PPP fundamentals; especially since its physical adoption in 2002.

Third, there is evidence for smooth rather than abrupt transition between the regimes, since the speed of adjustment increases as the number of observations in the outer regimes is reduced. Starting with the most restrictive constraint, somewhat more than 20% of observations fall into the mean-reverting outer regimes – where adjustment indeed occurs at a faster pace than in the linear model. As the constraint is gradually reduced to 15%, 10% and 5%, the adjustment speed associated to the outer regimes increases even further (see Figure 7 for a comparison of the results for the Structural Break specification). The observation that the exchange rate becomes increasingly mean-reverting partly explains the finding that the thresholds are always close to the outer boundaries, regardless of the proportion of observations required to remain within each regime.

Finally, the proportions of observations in the outer regimes are slightly greater on the reduced sample than on the sample beginning in 1990. The Euro introduction provides a potential economic explanation for this observation: For example, the common currency might have reduced exchange rate uncertainty and thus the costs of hedging against exchange rate risks, which constitute an important part of transaction costs associated to international trade (especially for commercial relationships with the former Southern-European soft currency countries). This tightens the band of inaction, within which there are no arbitrage opportunities, and a greater proportion of observations fall into the mean-reverting regimes.

If the adjustment coefficients associated to the error-correction terms are significantly more negative than the adjustment coefficients found in the linear model, we could conclude in support of non-linear mean-reversion. The finding would then corroborate the hypothesis that non-linear error correction could in part explain the purchasing power parity puzzle. However, the adjustment coefficients from the T-VEC remain relatively low, and it remains to test whether they are statistically significant.

15%	Baseline-1990	Structural Break	Baseline-1999
Min Θ^1	-0.117	-0.066	-0.084
Max Θ^2	+0.116	+0.075	+0.105
Θ^1	-0.115	-0.065	-0.076
Θ^2	0.069	+0.059	+0.052
Proportion Regime 1	15.49%	15.93%	16.95%
Proportion Regime 2	22.57%	19.91%	23.73%
$\Theta^1 \alpha_s$	-0.0503	-0.1150	-0.0565
$\Theta^2 \alpha_s$	-0.0117	-0.0412	+0.0285
(α_s)	(-0.0195)	(-0.0479)	-0.0021

Figure 6: Comparison of the results for three specifications of the T-VEC model (optimal thresholds and associated parameters, where a minimum of 15% of observations was required to remain within each regime.) Lines 1-2: Boundaries for lower and upper thresholds respectively, such that 15% of observations remain within the outer regimes. Lines 3-4: Optimal thresholds. Lines 5-6: Proportion of observations that remain within the outer regimes. Lines 7-8: Adjustment coefficients associated to the exchange rates for the outer regimes. Line 9: Adjustment coefficient associated to the exchange rate in the linear VEC.

	10%	15%	20%
Min Θ^1	-0.088	-0.066	-0.057
Max Θ^2	+0.085	+0.075	+0.058
Θ^1	-0.083	-0.065	-0.033
Θ^2	+0.081	+0.059	+0.050
Proportion Regime 1	11.06%	15.93%	16.95%
Proportion Regime 2	12.39%	19.91%	22.12%
$\Theta^1 \alpha_s$	-0.1726	-0.1150	-0.0989
$\Theta^2 \alpha_s$	-0.0514	-0.0412	-0.0480
(α_s)	(-0.0479)	(-0.0479)	(-0.0479)

Figure 7: Comparison of results for the Structural Break T-VEC, where 10%, 15% and 20% of observations were required to remain within each regime.

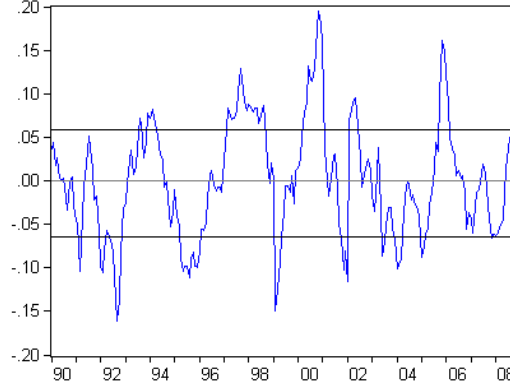


Figure 8: Result for the co-integration relation and the optimal thresholds for the Structural Break T-VEC, where a minimum of 15% of observations was required to remain within each regime.

7 Test for the Presence of a Threshold Effect

We are now interested in testing for the presence of a threshold effect, i.e. we test the null hypothesis of linear co-integration against the alternative hypothesis of threshold co-integration. Consider first the case of one threshold and two regimes. In this case, the class of linear VEC models clearly results from a restriction on the class of T-VEC models, where the restriction is that $\alpha_1 = \alpha_2$. Thus, the null hypothesis can be formulated as $H_0 : \alpha_1 = \alpha_2$. This null could, normally, be tested through a likelihood ratio test, in which the likelihood of the unrestricted model (the T-VEC) is compared to the likelihood of the restricted model (the linear VEC). If their ratio exceeds a certain critical value $k_c > 1$, the null could be rejected. However, conventional tests have non-standard distributions, because the adjustment coefficients α_1 and α_2 , are not identified under the null of linear co-integration.

For our two threshold / three regimes case, the test is complicated once more by the fact, that the class of linear VEC models cannot be formulated through a restriction on the class of T-VEC models. Consider the restriction that $\alpha_1 = \alpha_2 = \alpha_3$: Since α_2 was set to zero, the restriction is equivalent to the hypothesis of no-co-integration rather than the hypothesis of linear co-integration. To account for these limitations of conventional test statistics, we propose to approximate the asymptotic distribution of the test statistic under the null of linear co-integration through the application of a simulation method. The parametric bootstrap procedure contains the following steps:

- 1) Solve the linear model stochastically by bootstrapping the errors ϵ_t for each period. Store the resulting series for the nominal exchange rate and the price differential, s_t^1 and $(p - p^*)_t^1$.
- 2) Calculate the error correction term from the simulated data: $EC_t^1 = s_t^1 - \beta' [(p - p^*)_t^1]$.⁸
- 3) Run the search algorithm on the simulated data s_t^1 , $(p - p^*)_t^1$ and EC_t^1 and estimate the optimal threshold model.⁹
- 4) Store the log likelihood of the simulated linear and the simulated threshold model ($\log l_{lin}^1$ and $\log l_{thr}^1$, respectively). Then calculate the likelihood ratio $LR^1 = (\log l_{thr}^1 - \log l_{lin}^1)$.
- 5) Repeat the simulation 5000 times.
- 6) Sort the likelihood ratios LR^1 to LR^{5000} in an ascending order. Then, the 4500th likelihood ratio in the list approximates the critical value for the 10% level, the 4750th for the 5% level and the 4950th for the 1% level of significance. By construction, the 4500th likelihood ratio in the list— $k_c(0.90)$ —has the value, which is exceeded by only 10% of repetitions. The level of significance α thus indicates the probability that we reject the null of linear co-integration even though the data is generated by a linear model.

For the Structural Break specification, we find the critical values $k_c(0.90) = 5.2200$, $k_c(0.95) = 6.3920$, $k_c(0.99) = 8.4586$. Since the likelihood ratio of the original Structural Break specification takes the value 7.0306, we can reject the null hypothesis of no threshold co-integration on the 5% level of significance.¹⁰

⁸Or, for the model including mean-shift dummies and a constant:

$$EC_t^1 = s_t^1 - \beta' [(p - p^*)_t^1 \quad D_1 \quad D_2 \quad 1]'$$

⁹For the simulation, the model selection criterion is maximized within the class of threshold models rather than within both classes (including the linear model). Hence, for each simulation an optimal threshold is determined, even if the T-VEC doesn't outperform the linear VEC with respect to the selection criterion.

¹⁰If 15% rather than 10% of observations are required to remain within each regime, the null hypothesis can be rejected only at the 10% level.

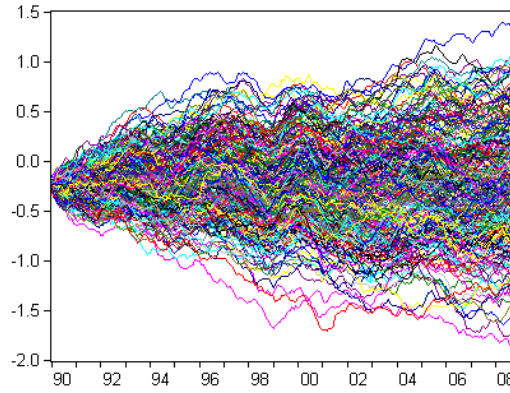


Figure 9: 1000 repetitions of the nominal exchange rate, as generated through a bootstrapping simulation.

8 Forecasting the Exchange Rate

We have found, for each of the selected model specifications, a threshold model that outperforms the linear model in terms of AIC, which is a convenient model selection criterion for forecasting purposes. To test whether the T-VEC model really has a forecasting advantage with respect to the VEC model, the out-of-sample forecasting accuracy of the models is compared for different forecasting horizons h up to two years ahead. The task is performed for the Structural Break specification (with the thresholds determined by minimization of AIC under the restriction that at least 15% of observations remain within each regime). For the out-of-sample forecasts, a window of two years is cut out from the sample. The model is then estimated with the observations before the window and a forecast is made for the 24 months within the window. Finally, the window is shifted one period to the right and the task is repeated (see Figure 10 for a graphical representation of the rolling forecast). This procedure is executed for the T-VEC and the linear VEC model, starting in January 1996 (which leaves at least five full years before the window for the estimation of the model).

For the forecast evaluation, the mean squared error (MSE) is calculated for each horizon $h = 1, \dots, 24$. The MSE is the average forecasting error for a given horizon over each repetition of the rolling regression procedure. Figure 11 shows the MSE-ratio, i.e. the MSE of the T-VEC divided by the MSE of the linear VEC. For all forecasting horizons exceeding three months ahead, the MSE-ratio is smaller than one, implying that the forecasting accuracy of the T-VEC model is higher than the forecasting accuracy of the linear VEC model.

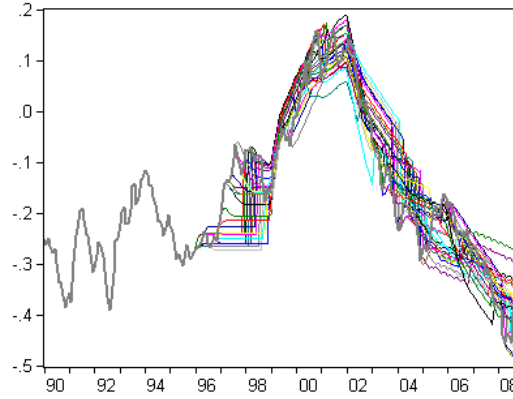


Figure 10: Rolling forecast (for windows with two years length) for the Structural Break T-VEC model.

Moreover, the MSE-ratio decreases with the length of the forecasting horizon, implying that the forecasting advantage of the T-VEC model increases further for longer term forecasts. Eventually, the MSE-ratio converges to around 0.70, implying that the forecasting accuracy of the T-VEC model improves by 30% with respect to the linear model. According to the Diebold-Mariano test on the null hypothesis of equal forecasting accuracy, these findings are highly significant. For the forecasting horizons up to one year ahead, the forecasting accuracy of both models do not differ significantly.

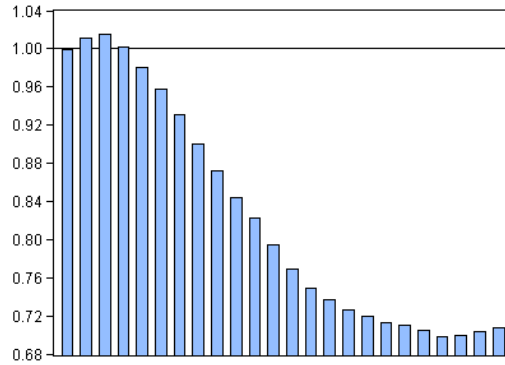


Figure 11: Results for the mean squared error ratios ($MSE_{thr.}/MSE_{lin.}$) for the Structural Break specification, for one to 24 months ahead forecasts.

9 Conclusions and Outlook

Consistent with the theoretical literature on transaction costs in international goods arbitrage as well as empirical findings of non-linear mean-reversion of the nominal exchange rate towards its price fundamentals, this thesis finds significant support for threshold co-integration in the purchasing power parity between the Euro Area and the United States. Using an equilibrium Threshold-VEC framework, we find that—allowing for a band of inaction in which no goods arbitrage occurs and the exchange rate diverts from its price fundamentals—the adjustment parameters associated to the mean-reverting outer regimes imply substantially faster adjustment than the parameters from the linear VEC model. Hence, the speed of adjustment of the nominal exchange rate towards the price fundamentals is underestimated if transaction costs in international goods arbitrage are not taken into account. Based on the empirical test statistics calculated through a bootstrapping procedure, the null hypothesis of no-threshold effects could be rejected on standard levels of significance. In particular, three interesting findings about the EUR/USD exchange rate fundamentals deserve consideration for further research:

- First, the transition between mean-reverting and diverting regimes appears smooth rather than abrupt (the speed of adjustment increases as the thresholds are shifted outwards). For future research, it would be interesting to allow for smooth transition between the T-VEC regimes.
- Second, the identified adjustment parameters are asymmetric, implying that an overvaluation of the Euro with respect to its price fundamentals is corrected faster than an overvaluation of the Dollar. Further research would be needed to conclude whether this is attributable to asymmetric transaction costs between for transatlantic goods arbitrage or to other factors, probably related to financial markets.
- Third, the band of inaction was found to tighten slightly after the introduction of the Euro in 1999, which might reflect a reduction of transaction costs, namely currency risks. For future research, it would be interesting to allow for time-varying thresholds in the T-VEC model.

Using the purchasing power parity model for forecasting the exchange rate, this thesis finds that the T-VEC model clearly outperforms the linear VEC model in terms of forecasting accuracy for medium- to long-run forecasting horizons. Overall, we can conclude that our T-VEC model is a very promising framework for the analysis of long-run exchange rate behavior. The purchasing power parity puzzle, however, will probably continue to keep economists busy for some more time to come.

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11 Appendix A: Abstract

Abstract

Consistent with the theoretical literature on transaction costs in international goods arbitrage, this thesis finds significant support for threshold co-integration in the purchasing power parity between the Euro Area and the United States. Estimating a threshold co-integrated vector autoregressive model, we find that—allowing for a band of inaction in which no goods arbitrage occurs and the exchange rate diverts from its price fundamentals—the adjustment parameters from the regimes outside the band imply substantially faster adjustment than the adjustment parameters from the linear model. In particular, the results indicate asymmetric adjustment, with an overvaluation of the Euro with respect to the purchasing power parity being corrected at a higher rate than an overvaluation of the Dollar. The presence of a threshold effect is statistically significant according to the empirical critical values generated through a bootstrapping procedure. Finally, the out-of-sample forecasting accuracy of the Threshold-VEC model is significantly and substantially higher than the accuracy of the linear VEC model for longer-term forecasting horizons.

Keywords: *Exchange Rates, Purchasing Power Parity, Nonlinearity, Threshold-Models, Bootstrap*

Abstract in German / Zusammenfassung

Diese Diplomarbeit versucht, die langsame Geschwindigkeit, mit welcher Wechselkurse zur Kaufkraftparität zurückkehren, durch eine Nichtlinearität im Anpassungsprozess zu erklären. Im nichtlinearen Modell setzt die Rückkehr des EUR/ USD Wechselkurses zur Kaufkraftparität erst dann ein, wenn die Kaufkraftdiskrepanz gewisse Schwellwerte überschreitet. Diese Schwellen spiegeln etwa Transaktionskosten wider. Tatsächlich implizieren die Parameter, welche in dem nichtlinear-kointegrierten vektorautoregressiven Modell geschätzt wurden, eine wesentlich schnellere Anpassung des Wechselkurses als die Parameter aus dem linearen Modell. Weiters deuten die Resultate auf einen asymmetrischen Anpassungsprozess hin, da eine Überbewertung des Euro in Bezug auf die Kaufkraftparität schneller korrigiert wird als eine Überbewertung des Dollars. Der Schwellwert-Effekt ist statistisch signifikant, wobei die empirischen kritischen Werte durch eine Bootstrapping-Simulation generiert wurden. Abschließend wird gezeigt, dass die längerfristige Prognosegüte im nichtlinearen Modell gegenüber dem konventionellen Modell signifikant verbessert werden kann.

Schlagworte: *Wechselkurse, Kaufkraftparität, Nichtlinearität, Threshold-Modelle, Bootstrap*

12 Appendix B: Curriculum Vitae

PERSONAL

ANNA FRANZISKA ORTHOFER

Born 1987 in Vienna, Austrian citizen

EDUCATION

- | | |
|-----------------|--|
| Oct 05 - Jan 10 | University of Vienna, Austria
Master program in Economics
<i>Specialization: econometrics and monetary economics</i>
Average grade: 1.0 (1= excellent, 5=fail) |
| Oct 07 - Jun 08 | University Paris I - Panthéon Sorbonne, France
EU Erasmus fellowship
<i>Master level courses in international economics</i> |

EMPLOYMENT

- | | |
|------------------|--|
| Oct 09 - present | Oesterreichische Nationalbank, Vienna
Foreign Research Division
<i>Economist</i> |
| Jul 09 - Aug 09 | Oesterreichische Nationalbank, Vienna
Foreign Research Division
<i>Research Assistant</i> |
| Oct 08 - Jun 09 | University of Vienna
Department of Economics
<i>Teaching Assistant to Professor Manfred Nermuth</i> |
| Oct 08 - Jun 09 | RZB Bank Group, Raiffeisen Research, Vienna
Economics and Currency Analysis
<i>Student Trainee</i> |
| Jul - Sep 08 | RZB Bank Group, Raiffeisen Research, Vienna
Quantitative Research and Emerging Markets
<i>Summer Intern</i> |
| Jul - Sep 07 | Vienna Institute for Development Cooperation
Development Policy Research
<i>Summer Intern</i> |