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Field deployable Photonic Quantum State Tomography and CHSH Game

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Zusammenfassung

Seit Einstein, Podolsky und Rosen 1935 zum ersten Mal die Gültigkeit und Vollständigkeit der Quantentheorie in Bezug auf die lokale Realität in Frage gestellt haben, hatten viele Wissenschaftler damit zu kämpfen, die Essenz der Theorie, wie es Bell in seiner Veröffentlichung in 1964 gezeigt hatte, zu verstehen und zu akzeptieren. Insbesondere für Menschen ohne Physikkenntnisse kann das Verständnis des kontraintuitiven Bellschen Theorems eine große Schwierigkeit darstellen. In dieser Arbeit bauen wir ein Experiment auf, das vor Ort während einer Veranstaltung eingesetzt wird, und versuchen, das Bellsche Theorem anhand des CHSH-Spiels zu veranschaulichen, um die Menschen dazu zu bringen, die Theorie zu verstehen und sie zu motivieren, tiefer in das Thema einzugehen. Das CHSH-Spiel ist ein Experiment, bei dem zwei Spieler versuchen, ihre Ergebnisse nach bestimmten Regeln miteinander zu korrelieren. Ihre Erfolgsquote ist durch die klassische Physik begrenzt, kann aber durch Quantenverschränkung übertroffen werden, was die Implikationen des Bellschen Theorems auf intuitivere Weise veranschaulicht. Die Gewinnwahrscheinlichkeit von etwa 84% übersteigt das klassische Limit von 75% für das CHSH-Spiel, was den Erfolg des Bellschen Theorems zeigt. Um die Qualität und den Erfolg des Experiments zu sichern, wird zuvor eine Quantentomographie mit demselben Aufbau durchgeführt.

Abstract

Since the very first time the validity and completeness of quantum theory, in regards of local realism, was questioned in 1935 by Einstein, Podolsky and Rosen, many scientists struggled with understanding and accepting the nature of it as shown in Bell's paper in 1964. Especially for people without a physics background, understanding the counterintuitive Bell's theorem may pose a great difficulty. In this work, we set up a field deployable experiment and attempt to show Bell's theorem based on the CHSH game during an outreach event, to get people to understand the theory and motivate them to dig deeper. The CHSH game is an experiment in which two players try to correlate their outputs according to certain rules. Their success rate is limited by classical physics but can be surpassed using quantum entanglement, showing the implications of Bell's theorem in a more intuitive way. The winning probability of around 84% exceeds the classical limit of 75% for the CHSH game, showing the success of Bell's theorem. Also, to secure the quality and success of the experiment, quantum state tomography is performed beforehand using the same setup.

1 Introduction

"God does not play dice with the universe.", is one of the most famous phrases quoted, in one way or another, when talking about quantum mechanics. It declares Albert Einsteins viewpoint on the theory and how he did not want to accept the probabilistic nature of quantum measurements. In their seminal paper of 1935 [1], Albert Einstein, Boris Podolsky, and Nathan Rosen showed that the description of quantum states represented by wave functions cannot be locally real. They concluded that wave functions do not encompass all the information of the quantum system and hence, the theory is incomplete. Their idea suggested that possibly there are other variables, which we are unaware of, that would explain the seeming non-locality of quantum mechanics. These hidden-variable theories, as they are called today, would explain how two quantum systems appear to be communicating faster than the speed of light. However, in 1964 John Bell presented mathematical proof of a theoretically implementable experiment that showcased how no theory of hidden variables could explain the statistical predictions of quantum mechanics [2]. Many experiments applying Bell's theorem and closing loopholes have been performed [3–7] and were successful, but the one experiment that closed all significant loopholes and left no room for doubt, was conducted in 2015 [8] and proved once and for all the incompatibility of quantum theory and local realism.

This work presents an experimental set up which is conducted on a field-deployable breadboard and demonstrated at an outreach event, the Long Night of Research or Lange Nacht der Forschung 2024 in Vienna. The experiment examines a modified version of Bell's theorem, namely the CHSH game. The main goal is to explain Bell's theorem to the average person in a playful way. The CHSH inequality [9], which is regarded in light of this, is a Bell inequality whose violation can be easily implemented in experiments and the game is an altered version, where two players need to come up with a strategy to have the highest possible win probability. To that end, two visitors at the event are encouraged to play the game classically and discuss their strategy. After that, the strategy using quantum mechanics and entangled photons is presented and shown experimentally to surpass the classical limitations.

To check that the quantum state of the entangled photon pair is created as expected, additional measurements are done on the system prior to the event. By doing measurements in different photon polarization states and analyzing the results using maximum likelihood estimation (MLE), the density matrix representing the state of the photons in the ensemble is recreated. This procedure is called quantum state tomography (QST) [10] and is usually done to make sure that the experiment is set up well enough to deliver good results in further measurements.

After a brief introduction to local realism and its clash with quantum mechanics in the first part, section 2, of this thesis, Bell's theorem with special focus on the CHSH inequality and the CHSH game are discussed in 2.2. Section 2.3 briefly explains QST and the theory is finished with section 2.4, where the translation from theory to experiment is described. Section 3 shows the setup and explains in detail how the experiment is conducted for different measurements. The results obtained from these measurements are presented in 4, where two laser sources are compared in regards of quality. Finally, section 5 reviews the results and in section 6 the thesis is concluded by emphasizing the success and possible educational aspect of the experiment.

2 Theory background

2.1 Quantum mechanics and local realism

Local realism is a concept that arises naturally when talking about classical physics. It states following two principles.
Locality: Any physical change on an object happens through an interaction in the immediate surroundings of that object. To interact with something that is separated locally, the interaction must be carried by a particle or a wave.
Realism: Properties of objects exist regardless of whether they are measured or not. Thus, the outcome of an interaction is determined prior to measuring it.

However, this common understanding of our universe does not apply to quantum mechanics. Albert Einstein, Boris Podolsky, and Nathan Rosen challenged the ideas of quantum theory by arguing that any physical theory should be judged by its correctness and completeness [1]. Whether it is correct can be checked through experiments and measurements. But they focused on completeness, which they defined as every element of physical reality having a counterpart in physical theory. The criterion for physical reality was given as follows: "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity." (Einstein et al. 1935, p. 777). Regarding these definitions they analyzed the wave function ψ, which is a full characterization of the state of a quantum system. Now, consider a wave function ψ describing the state of a particle. They show that the momentum of this particle has a certain value and thus a physical reality, but its position can only be obtained by doing a direct measurement. However, this would alter the state and the particle can no longer be described by ψ. Concluding, two non-commuting physical quantities of a system, like the momentum and position of a particle, cannot be measured simultaneously. This means that one of the two arguments must be true: (1) The wave function ψ as a description of physical reality in quantum mechanics is not complete or (2) two non-commuting quantities cannot have simultaneous reality.

They then proceed to argue with two systems that can interact at some point in time, and either one can be measured afterwards without altering the other one. This makes it possible to do two different measurements and thus assign two different wave functions to the same system after the interaction. If these wave functions happen to be eigenfunctions of two non-commuting operators, like position and momentum, one can predict either the position or the momentum with certainty. Depending on which one is done, it must be considered an element of reality. But since both wave functions have the same reality, the argument (2) no longer holds. Thus, the negation of (2) leaves us with the only option that quantum mechanics, as described by the wave function, is an incomplete theory. This means that the wave function is missing information, and a complete description should be given by theories incorporating so-called local hidden variables.

2.2 Bell's Theorem

The idea of local hidden variables was proven to be insufficient by J. S. Bell in 1964 [2], who demonstrated that such a theory could not reproduce all results suggested by quantum theory. Bell's theorem implies the non-locality of quantum mechanics by showing that there must be some sort of interaction between locally separated measurement devices and that this must be instantaneous. In particular, he introduced an inequality that would have to hold for local realistic theories but is violated by quantum mechanics. As this was a theoretical proof, the experimental realization was not possible, which resulted in the derivation of another inequality, the CHSH inequality.

Difficulties in understanding Bell's theorem arise from the fact that measuring in quantum mechanics works differently. In classical mechanics, any object can be fully characterized by doing as many measurements as needed on the same subject. Unfortunately, this is not possible in the quantum case, since a single measurement changes the state of the system. One would need to be able to characterize the state with one measurement, which is not possible due to Heisenberg's uncertainty principle [11], or do many measurements on the same state by creating multiple copies of the unknown system, which is prohibited due to the no-cloning theorem [12]. The process to fully characterize the state of a quantum system is called quantum state tomography and is done by analyzing the measurements done on an ensemble of identical quantum states and will be discussed in section 2.3. Additionally, when doing a measurement on a quantum system, a basis must be defined. For example, if the polarization of a photon is measured, one can choose various bases and will obtain different results for each measurement. A horizontally polarized photon has a 100% chance of being detected when measured in the horizontal basis, while a diagonally polarized photon can be detected in either the horizontal or vertical basis with 50% probability each. This becomes significantly more important for entangled photons, where measurements can be correlated. Two parties sharing entangled photon pairs in a Bell state can each freely choose their basis of measurement and will each have seemingly random outcomes. Nonetheless, when comparing their results, it becomes clear that the outcomes are perfectly correlated or anti-correlated depending on the initial state.

2.2.1 CHSH inequality

Bell's theorem led to derivations of many inequalities that explored the assumptions both theoretically or experimentally. Among the inequalities like the Clauser-Horne inequality [13], the Legett-Garg inequality [14] or Fine's inequalities [15], we are particularly interested in the CHSH inequality by John Clauser, Michael Horne, Abner Shimony, and Richard Holt [9].

Assume, two apparatuses each detect one photon of a correlated pair. The particles then select a channel, either $+1$ or -1 and the results $A(a)$ and $B(b)$, where a and b are adjustable parameters of the apparatuses, is ± 1 depending on the chosen channel. Suppose there are hidden variables, denoted λ, which contain information about the statistical correlation of $A(a)$ and $B(b)$. Since this information is stored in each particle and was created in the past, where communication between them as a pair was possible, the function $A(a,\lambda)$ does not depend on b and $B(b,\lambda)$ does not depend on a. Also, because photons are not influenced by the parameters a and b, the normalized probability distribution $\rho(\lambda)$ is not dependent on them. The correlation function then is

$$P(a,b) = \int A(a,\lambda)B(b,\lambda)\rho(\lambda)d\lambda. \tag{2.1}$$

Suppose there are additional properties a' and b'. Then the following equation can be written:

$$|P(a,b) - P(a,b')| = \int d\lambda[A(a,\lambda]B(b,\lambda) - A(a,\lambda)B(b,\lambda)] \leq 2 \pm [P(a',b') + P(a',b)]. \tag{2.2}$$

After rearranging the terms, we arrive at the inequality

$$|S| = |P(a,b) - P(a,b')| + |P(a',b') + P(a',b)| \leq 2. \tag{2.3}$$

In this experiment, the parameters a, b, a' and b' are the measurement angles regarding the polarization of the photons. Using the angles $(\alpha, \beta, \alpha', \beta')$ for the polarizers, the expectation value can be written as

$$P(\alpha,\beta) = \cos(2(\alpha - \beta)). \tag{2.4}$$

The greatest violation is achieved for the angles $(0°, 22.5°, 45°, 67.5°)$ and reads

$$|S| = \left|\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right| + \left|\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right| = 2\sqrt{2} > 2. \tag{2.5}$$

2.2.2 Polarization measurements

Measurements of photon polarization depend on the basis in which the measurement is performed. The standard bases used in polarization measurements are:

- **Horizontal/Vertical Basis:** $|H\rangle$ and $|V\rangle$
- **Diagonal/Anti-diagonal Basis:** $|+\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$ and $|-\rangle = \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$.
- **Right/Left Circular Basis:** $|R\rangle = \frac{1}{\sqrt{2}}(|H\rangle + i|V\rangle)$ and $|L\rangle = \frac{1}{\sqrt{2}}(|H\rangle - i|V\rangle)$.

Doing a measurement projects the state of the photon with certain probability onto one of the basis states.

When testing the CHSH inequality, we consider an entangled photon pair in one of the Bell states, in our case

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle).$$

Here, the measurement is not done in one of the standard bases but in measurement bases at specific angles, chosen to maximize the violation of the inequality. Experimentally, we measure the number of photon pairs arriving at the detectors simultaneously when projecting according to the angles in 2.7 and calculate the expectation value with

$$P(\alpha,\beta) = \frac{C(\alpha,\beta) + C(\alpha_\perp,\beta_\perp) - C(\alpha_\perp,\beta) - C(\alpha,\beta_\perp)}{C(\alpha,\beta) + C(\alpha_\perp,\beta_\perp) + C(\alpha_\perp,\beta) + C(\alpha,\beta_\perp)}, \tag{2.6}$$

where C is the number of coincidences detected for the given angles. Because half wave plates (HWP) are used for our experiments here, instead of polarizers, the angles must be adjusted to

$$(\alpha,\beta,\alpha',\beta') = (0^\circ, 11.25^\circ, 22.5^\circ, 33.75^\circ). \tag{2.7}$$

The observed coincidence counts might look like:

$$C(\alpha,\beta) = 49414, \quad C(\alpha_\perp,\beta_\perp) = 50326, \quad C(\alpha,\beta_\perp) = 10672, \quad C(\alpha_\perp,\beta) = 10486.$$

Inserting these coincidences into eq. 2.6 yields the probability

$$P(\alpha,\beta) = -0.73.$$

Similarly, the remaining probabilities and the resulting violation can be computed.

When trying to implement Bell's theorem in an experiment and trying to violate it, there are loopholes that need to be considered [16]. Many experiments were conducted that closed some of those loopholes individually or simultaneously [4,5,8,17]. Some significant loopholes are the fair-sampling or detection-efficiency, the locality and the freedom-of-choice loophole. The first one takes into account that the detected photon pairs could in principle be a small subset of the created entangled photons. To be able to assume that the detected ones represent a fair sample of the created ensemble, a decent detection efficiency must be achieved. The second loophole considers the communication between the photon pairs. Closing this requires space-like separation of the measurements ensuring that the choice of setting or the measurement results cannot be communicated in time to the other pair. Lastly, choosing the setting must be done freely or completely randomly. This loophole can be closed assuming human free will [18], by using space-like separated random number generators [6] or using astronomical sources for the choice of setting [7].

2.2.3 CHSH game

The CHSH game [9], as sketched in 2.1 is a game that shows how using quantum mechanics, specifically entanglement, increases one's chance of winning it. The game involves three parties, the players Alice and Bob, and the referee Charlie. At the start of the game, Charlie secretly gives Alice and Bob one bit each. The bits X and Y can have the values 0 or 1. Both players must respond with a bit A and B, without communicating with each other or knowing which bit the other one received. The game is pronounced won by the referee if Alice and Bob respond in a way such that $X \cdot Y = A \oplus B$, where $\cdot$ is the multiplication and $\oplus$ is the XOR logic gate operation. Alice and Bob are prohibited from communicating during the game but can agree on a strategy beforehand. If they choose to always send the same bit, so either both 0

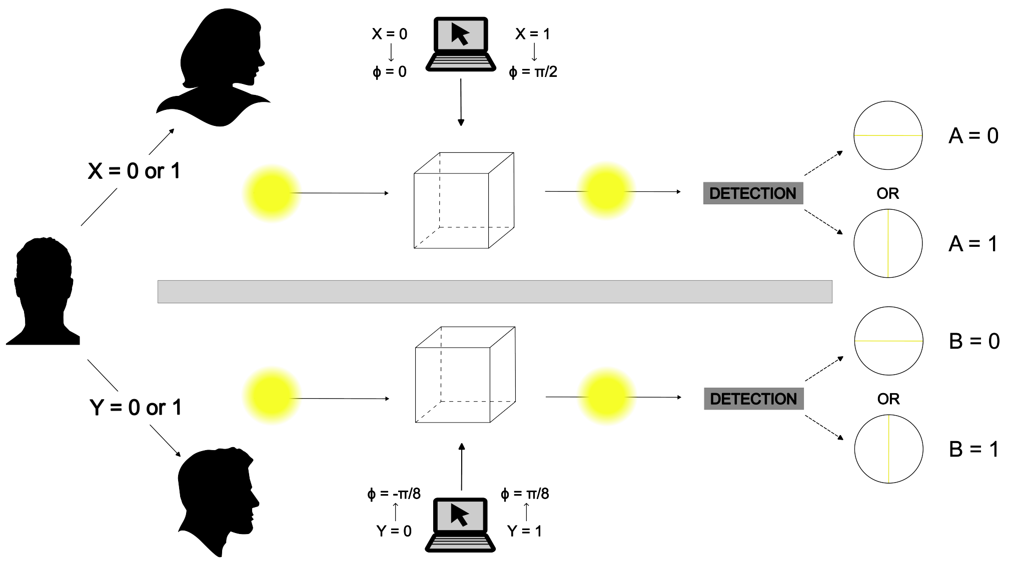


Figure 2.1: Sketch of the CHSH game. Alice and Bob receive a bit from Charlie and perform manipulations on their photons. The photons are detected in either the horizontal $|H\rangle$ or the vertical $|V\rangle$ basis corresponding to $A/B = 0$ or $A/B = 1$, respectively. Depending on the outcome, the game is declared won or lost.

or both 1, they would only lose the game in cases where they both received a 1 from Charlie. The result would be that in 3 out of 4 games they would win, meaning they achieve a winning probability of **P = 75%** which is the best classical strategy. Fig. 2.2 shows all possible outcomes of the game.

In the quantum case the players Alice and Bob share an entangled photon pair in the state $\Phi^+ = \frac{1}{\sqrt{2}}(|H\rangle_A|H\rangle_B + |V\rangle_A|V\rangle_B)$, where H and V are horizontally and vertically polarized photons, and Charlie does measurements in the bases $(|HH\rangle, |HV\rangle|, VH\rangle, |VV\rangle)$. Photons with H and V polarization are equivalent to qubits 0 and 1, respectively. The same rules as for the classical case apply here, with the difference that Alice and Bob apply different strategies to their photons and send it to Charlie. If Alice receives $X = 0$, she does nothing and if she receives $X = 1$, she rotates the polarization of her photon by $\phi = \frac{\pi}{2}$, creating one of the states $|\pm\rangle = \frac{1}{\sqrt{2}}(|H\rangle \pm |V\rangle)$. Similarly, Bob applies $\phi = -\frac{\pi}{8}$ for $X = 0$ and $\phi = \frac{\pi}{8}$ for $Y = 1$. Charlie then detects the photons with certain probabilities in the different bases. The winning condition is measuring coinciding photons in the aforementioned bases because that would mean two same qubits were sent. The bases $(|HH\rangle, |HV\rangle|, VH\rangle, |VV\rangle)$ correspond to $(A = B = 0; A = 0, B = 1; A = 1, B = 0; A = B = 1)$. For example, in the case of $X = Y = 0$ the probability of receiving both qubits in the state $|H\rangle$ would mean $A = B = 0$ and in the state $|V\rangle$ would mean $A = B = 1$. Similarly, if $X = Y = 1$ the winning condition is receiving different qubit values.

Let us regard the case where $X = 1$ and $Y = 0$. Upon receiving their bits, Alice applies a rotation of $\phi = \frac{\pi}{2}$ and Bob a rotation of $\phi = -\frac{\pi}{8}$. When we look at fig. 2.3, we regard the lower left quarter. When Charlie now measures the photons, they have a 42.7% chance of being in the

X	Y	X · Y
0	0	0
1	0	0
0	1	0
1	1	1

A	B	A ⊕ B
0	0	0
1	0	1
0	1	1
1	1	0

Figure 2.2: All possible bit combinations that Charlie can give, and Alice and Bob can respond with, and their outcomes are shown in the tables. To win the game, the colors must match on both sides. For the multiplication, the outcome is 0 (blue), except when $X = Y = 1$. Thus, A and B must be identical to also obtain the same outcome 0 (blue).

state $|HH\rangle$ or $|VV\rangle$, corresponding to $A = B = 0$ and $A = B = 1$, respectively, and a 7.3% chance of being in the state $|HV\rangle$ or $|VH\rangle$ corresponding to $A = 0, B = 1$ and $A = 1, B = 0$, respectively. Therefore, for Charlie to declare the game won, receiving both qubits in either $|HH\rangle$ or $|VV\rangle$ works, so the probabilities add up to $42.7\% + 42.7\% = 85.4\%$. All winning probabilities, as shown in fig. 2.3, in total amount to the sum of the two values and equal **P** $\approx$ **85.4**$\%$.

2.3 Photonic Quantum State Tomography

QST is a method to reconstruct the quantum state of a system by performing measurements on multiple identical copies of it. By doing measurements in different settings, the full description of the state can be derived in the form of a density matrix. The density matrix reveals properties of the quantum system, such as entanglement, and allows us to verify the state of the system before proceeding with further measurements. In order to violate Bell's inequalities or to increase the probability of winning the CHSH game, the created Bell state must be checked to ensure it is as close to maximally entangled as possible.

To understand QST, a more general description of quantum states is needed. The wave function is insufficient, especially in real experiments, since it can only describe pure states and often in reality the systems are in a mixed state, or we are interested in the mixed subsystem of a pure system. The density matrix, generally written as

$$\rho = \sum_i P_i |\psi_i\rangle\langle\psi_i|, \tag{2.8}$$

	Y = 0			Y = 1		
X = 0		B = 0	B = 1		B = 0	B = 1
	A = 0	0.427	0.073	A = 0	0.427	0.073
	A = 1	0.073	0.427	A = 1	0.073	0.427
X = 1		B = 0	B = 1		B = 0	B = 1
	A = 0	0.427	0.073	A = 0	0.073	0.427
	A = 1	0.073	0.427	A = 1	0.427	0.073

Figure 2.3: Winning conditions of the CHSH game using quantum mechanics. For all X and Y from Charlie, Alice and Bob have 4 possible outcomes with different probabilities. In the case of $X = Y = 1$, Alice and Bob have a probability of 0.427 to send different qubits, either $A = 0, B = 1$ or $A = 1, B = 0$, resulting in success. For all other cases coming from Charlie, they have the same probability as before of sending the same qubit, either $A = B = 0$ or $A = B = 1$. There is also a probability of 0.073 of sending the wrong qubits and losing the game.

where P_i is the probability and ψ is the state, is necessary when considering the quantum state of an ensemble which is usually the case during experiments. It allows for a probabilistic description of mixed states and can characterize subsystems of entangled systems.

An alternative representation of the density matrix for one qubit is given in terms of Pauli matrices σ_i and Stokes parameters S_i:

$$\rho = \frac{1}{2}\sum_{i=0}^{3} S_i \sigma_i. \tag{2.9}$$

The Pauli matrices are

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{2.10}$$

and S_i is given by

$$S_i = \mathrm{Tr}\{\sigma_i \rho\}. \tag{2.11}$$

For two-qubit systems, as will be the case in this experiment, the definitions are extended to

$$\rho = \frac{1}{4}\sum_{i=0}^{3}\sum_{j=0}^{3} S_{ij}\sigma_i\sigma_j \quad \text{and} \quad S_{ij} = \mathrm{Tr}\{\sigma_i\sigma_j\rho\}. \tag{2.12}$$

The density matrix can in principle be reconstructed by measuring the outcome probabilities in different bases and calculating the corresponding Stokes parameters. To that end, the experiment is set up to project onto specific basis combinations. The possible bases are $\sigma_1, \sigma_2, \sigma_3$ corresponding to $|+\rangle$ and $|-\rangle$, $|L\rangle$ and $|R\rangle$, and $|H\rangle$ and $|V\rangle$, respectively. The Stokes parameter S_{11}, for example, is calculated with

$$S_{11} = \text{Tr}\{\sigma_1\sigma_1\rho\} = P_{|++\rangle} + P_{|+-\rangle} + P_{|-+\rangle} + P_{|--\rangle}, \tag{2.13}$$

where P is the probability of observing the corresponding outcome. All other parameters can be obtained analogously and inserted into eq. 2.12 to calculate the density matrix. The total number of measurements needed is $4^n - 1$, where n is the number of qubits. Yet, due to statistical errors, there is a possibility of reconstructing a density matrix that is not physical, e.g. it has negative eigenvalues. To circumvent this, the analysis can be done using maximum likelihood estimation (MLE).

This technique finds the density matrix that is most likely to agree with the data obtained from the measurements and constrains it to be physical. First, a correct parametrization of the density matrix is needed. The matrix g of the form

$$g = \frac{T^\dagger T}{\text{Tr}\{T^\dagger T\}} \tag{2.14}$$

is Hermitian, positive semi-definite and has trace one, i.e. all required properties for the density matrix. $T(\vec{t})$ is a tridiagonal matrix with $\vec{t}$ being a vector containing the elements t_i of the matrix g. The density matrix is then given by

$$\rho(\vec{t}) = \frac{T^\dagger(\vec{t})T(\vec{t})}{\text{Tr}\{T^\dagger(\vec{t})T(\vec{t})\}}. \tag{2.15}$$

The probability of ρ giving the expected counts $\overline{n}_{\nu,r}$, where ν is the ν^{th} measurement for detector r, is

$$P(n_{\nu,r}) = \frac{1}{N}\prod_{\nu,r} \exp\Big[\frac{(\overline{n}_{\nu,r)} - n_{\nu,r)})^2}{2\overline{n}_{\nu,r)}}\Big], \tag{2.16}$$

with N being the normalization factor and $n_{\nu,r}$ the experimentally determined coincidence counts. This probability function must be maximized to find the ideal value for $\vec{t}$ and reconstruct ρ.

For two qubit tomography, the number of measurements needed is 16 in total. This means 16 values of $n_{\nu,r}$ must be measured and relate to coincidence counts in the bases

$$\begin{aligned}
&\text{(HH), (HV), (HD), (HR),}\\
&\text{(VH), (VV), (VD), (VR),}\\
&\text{(DH), (DV), (DD), (DR),}\\
&\text{(RH), (RV), (RD), (RR).}
\end{aligned} \tag{2.17}$$

2.4 Collaboration with theorists

The idea of presenting the CHSH game during the LNF came from two PhD students of the Dakić group at the University of Vienna and the experimental realization was done in close collaboration with them. Since they are theoretical physicists, there was some back and forth until the results were satisfactory.

Their original idea to play the game with single photon pairs many times and directly calculating the probabilities from the measurement outcomes. Although this is possible, it is not feasible regarding our goals and options. Single photons are not easy to use for the CHSH game. Each time a photon pair is generated, Alice and Bob would need to do their manipulations, and the photons would be measured. The procedure must then be repeated many times to collect sufficient data to give a representative statistical result of the winning probabilities. Apart from being time-consuming, this method may not align well with the educational goals of providing a practical and engaging demonstration for visitors. A readily available solution is using a spontaneous parametric down-conversion source to generate a high rate of entangled photon pairs. With high enough laser power, the number of created photons exceeds thousands per second and allows for a probabilistic approach to the experiment.

In theory, the measurement of Alice's and Bob's qubits is done in all four basis H, V, D and A simultaneously. This requires the setup to project onto all bases and four detectors to show the coincidences. Our detector, however, only has three detectors, meaning we needed an alternative way to do the measurements. The experimental approach to this is by rotating the HWP by an additional $45°$ after doing the measurement, for example, in the H basis to artificially go to the V basis. From a theorist's point of view, this solution may seem too ambiguous. Nonetheless, their calculations showed that performing a measurement in the H basis with a HWP at $45°$ applied to the photon is equivalent to measuring that photon in the V basis, and the game could indeed be played this way.

3 Experimental implementation

The setup of the experiment is sketched in fig. 3.1. The laser source, as seen in detail in figures 3.2 and 3.3, and the control and readout unit (quCR) are from qutools' quantum entanglement demonstrator [19]. In the source, an entangled photon pair is created by focusing the pump laser of 405 nm wavelength on a β-Barium-Borate (BBO) crystal. Through spontaneous parametric down-conversion a portion of the pump photons hitting the crystal are converted into two photons with 810 nm, conserving energy momentum, thus creating photon pairs in the bell state $|\phi^+\rangle = \frac{1}{\sqrt{2}}(|H\rangle|H\rangle + |V\rangle|V\rangle)$. More information on the laser source and the control and read out unit can be found on the qutools homepage [19]. The photons take two different paths (red dotted lines), are reflected on mirrors and collected in single mode fibers. Since the fibers change the polarization of the photons, polarization controllers are used to compensate for that error. They then leave the fibers to each go through a quarter wave plate (QWP), a half wave plate (HWP) and then a polarizing beam splitter (PBS) and are finally collected and detected on the quCR. The output on the quCR is the total numbers of single photons for both arms coming in during the set time frame and the number of coincidences, meaning the detection of photons from both arms at the same time, i.e. in the coincidence window $\Delta t = 30ns$. One disadvantage of this setup is that the quCR only has two detectors and simultaneous detection of the horizontal and vertical components of both photons after the PBSs is not possible. In the experiments, the part that is reflected at the PBS is ignored and the issue is resolved by adjusting the wave plates accordingly and doing extra measurements.

The measurements are automated to some degree with the use of Python scripts. As I had no prior experience with the components used, I wrote the scripts part by part and only in the end merged the necessary elements for the CHSH game. The first step to automating the experiment was accessing the data from the detector. The quCR can be controlled through an HTTP interface and one is able to show all the information on a browser on a computer. An internet connection is not needed and merely connecting the quCR via an Ethernet cable to the computer works. If wanted, connecting it to a router is also possible. Putting in the IP address shown on the info tab of the quCR into a browser leads to the HTTP interface. My script accesses this page and copies the HTML that contains information such as the single photon counts, the coincidences or measurement time and proceeds depending on the measurement.

The next step is communicating with the wave plates through the Python script. First, the wave plates need to be connected to the laptop and using Thorlabs' software, Elliptec [20], they are correctly defined with their ports and addresses. With that, each wave plate can be manipulated individually.

Two short scripts are used for setting up the experiment. One is used for the calibration of the wave plates and rotates them in small steps over the entire range while saving the single photon counts in an array to be later used for the determination of the fast axis. The second

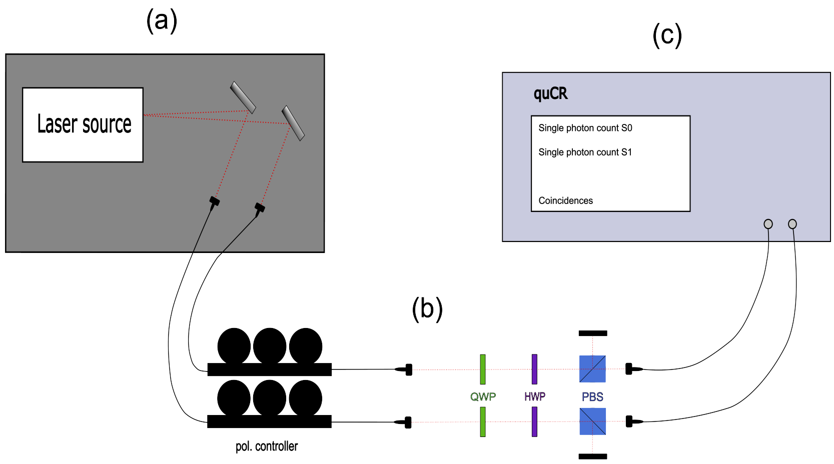


Figure 3.1: Sketch of the setup. **(a)** Entangled photon pairs in the state $|\phi^+\rangle = \frac{1}{\sqrt{2}}(|H\rangle|H\rangle + |V\rangle|V\rangle)$ leave the laser source towards mirrors that reflect them into fibers couplers. The mirrors and fiber couplers are adjusted vertically and horizontally to align the laser and to collect the highest possible amount of the created photons. **(b)** The photons travel through polarization controllers that compensate for the change in polarization caused by the single mode fibers. The fiber is pooled in the three paddles to use the birefringence and realize a retardation of $\pi/4$, $\pi/2$ and then $\pi/4$ again. By changing the angles of the paddles, one can create any polarization. Then, the photons go through quarter wave plates (QWP), half wave plates (HWP) and polarizing beam splitters (PBS) of which only the transmitted light is measured and the reflection is ignored. This combination with different angles for the QWPs and HWPs allows for measurements in all bases. The QWP and HWP are mounted on motorized rotation mounts which are connected to a computer and are manipulated with a Python script. **(c)** The quantum control and entanglement demonstrator (quCR) collects the photons and shows the number of single photons and coincidence counts on the monitor. The quCR is connected to the computer and all, or select values are used for further calculations.

one only rotates one HWP while keeping the other one in either the H or D basis and puts out an array containing the coincidences after subtracting accidental coincidences. This array can then be used to calculate the visibility.

The first main script [21] does quantum state tomography and gives us plots of our density matrix and its properties. First, the script sets the fast axes of the wave plates and then rotates them to project in all 16 measurement bases. After each rotation, the HTTP interface is accessed and the single photon counts and the coincidences are used to calculate the true coincidences in the set basis. These coincidences are used to do a set number of Monte Carlo simulations. After that, the Quantum-Tomography library by the Kwiat Information Group [22] is used to calculate the density matrix. Finally, the properties purity, fidelity and concurrence along with their errors are shown, and the plots are generated.

The script for the CHSH violation [23] works similarly to the one for QST. The HWPs are accessed and rotated according to the angles in 2.7 and the single photon rates and coincidences are read out. Then, the expectation values and the corresponding violation are calculated.

The script for the CHSH game [24] provides two input fields for the players, Alice and Bob, to input their strategy. It asks for an angle to rotate the HWPs. Because the CHSH game is usually defined for polarizers, the given angle is divided by two to correctly apply for HWPs. The wave plates are rotated accordingly, and as for QST, the coincidences are collected. Using the total number of coincidences, the probability matrix is shown and the sum of the diagonal or anti-diagonal, depending on the strategy, is calculated.

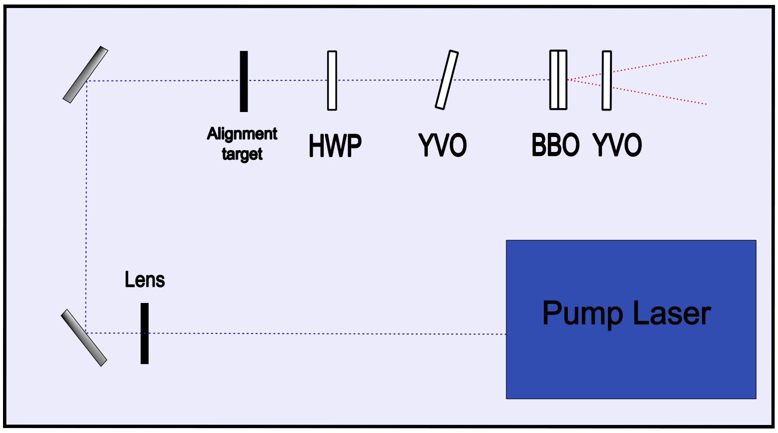


Figure 3.2: Sketch of the laser source. A pump laser at 405 nm wavelength is focused onto β-Barium-Borat crystal. Through spontaneous parametric down-conversion there is a probability of 10^{-11} that the pump photon is converted into two photons at 810 nm. Energy and momentum conservation enables creating entangled photons by compensating temporal shifts using Ytterbium Vanadate (YVO) crystals before and after the BBO crystal. With the half wave plate inserted, the probability of converting the pump photon into $|H\rangle$ or $|V\rangle$ polarization is equal, thus creating the state $|\phi^+\rangle = \frac{1}{\sqrt{2}}(|H\rangle|H\rangle + |V\rangle|V\rangle)$. The alignment target is used to properly align the laser to the fiber couplers.

The most crucial part of the setup is the entanglement created in the laser source and the photons arriving at the fibers. To get the ideal results here, the QWPs and HWPs in both arms are removed and the source is aligned according to the full alignment procedure as described in the quED manual [25]. However, since the fibers are not connected directly to the quCR but go through polarization controllers, the procedure must be adjusted. After maximizing the single photon count rates in both arms, instead of aligning for maximal entanglement, first the polarization must be corrected. To this end, the HWP in the laser source is removed, to create $|HH\rangle$ polarized photons. Now, the paddles of the polarization controllers must be moved to get a minimum in single photon counts in the opposite (V) basis. But, as mentioned before, we can only look at the horizontal components. Thus, HWPs are inserted in both arms and rotated to $45°$ to turn the H polarized light coming from the polarization controller into V. By minimizing this, it is ensured that count rates for horizontal polarized light are maximized.

Furthermore, the fast axes of the wave plates must be determined. This is achieved by doing

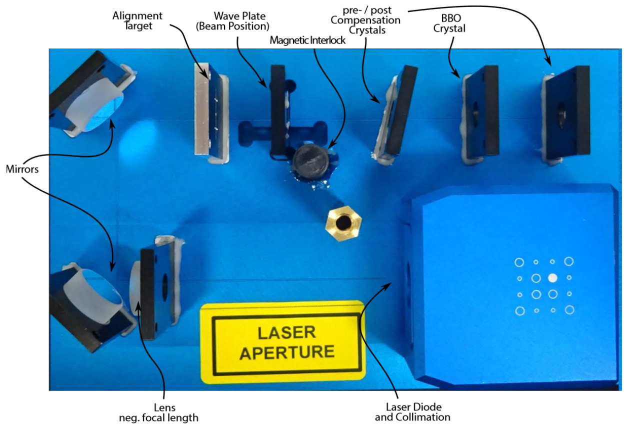


Figure 3.3: Picture of the laser source taken from the quantum entanglement demonstrator manual [25].

a full rotation in small steps and measuring the single photon count rates. Additionally, it must be checked that the fast axes of the QWPs are indeed chosen correctly, as their true effect including the phase shift cannot be seen in the count rates. By putting both QWPs at $45°$ in one arm, the $|H\rangle$ polarization is turned to $|V\rangle$ if they have the same fast axis, because the QWPs act just like a HWP at $45°$. If the axis is wrong, the effects cancel out and the polarization stays the same.

Now that the polarization is correct and the wave plates are adjusted, the wave plates are removed again and the alignment is continued to now increase the entanglement quality to achieve strong correlations. To check the entanglement quality, the visibilities in both the H/V and D/A basis are calculated using

$$V = \frac{N_{max} - N_{min}}{N_{max} + N_{min}}, \tag{3.1}$$

where N_{max}/N_{min} is the highest/lowest coincidence count on the curve. A minimum visibility of $V = 1/\sqrt{2} \approx 0.707$ must be attained to violate the Bell inequality.

3.1 CHSH game application

For the CHSH game, the QWPs are not needed and removed. The only variables that are changed for the experiment are the angles of the HWPs. However, the CHSH game is defined for polarizers and not HWPs, so the angles are halved to mimic the polarizers. The inputs are done by Alice and Bob who manually insert the angles for the chosen strategy into the input fields of the Python script. It is important that they do this subsequently without looking at the chosen angle of the other player to make sure that the game is played according to the rules. Once the angles are chosen, the HWPs rotate accordingly, and a coincidence measurement is done over 10 seconds.

To calculate the probability, measurements in all basis combinations (HH, HV, VH, VV) are required. This cannot be realized in a straightforward way with this setup because of the lack of detectors of the quCR. For measurements in the H basis, nothing changes as per our definition, since the PBS transmits horizontally polarized light. When the V basis is to be measured, the HWP is rotated by $45°$ in addition to the angles chosen by Alice and Bob. From the coincidences, the probabilities are calculated and summed depending on the chosen strategy. The result of $p > 0.75$ is the awaited proof of quantum entanglement.

3.2 QST and Bell violation

For QST and Bell violation the setup is used as depicted in fig. 3.1, with the QWPs inserted this time. The coincidence counts for each projected basis combination as shown in eq. 2.17 are measured over 20 seconds. My Python script then uses the Quantum-Tomography library and calculates the density matrix. Similarly, to violate the CHSH Bell inequality 2.3, the angles for the wave plates are chosen according to 2.7. Again, the coincidences are measured in a time frame of 20 seconds.

3.3 Set up - Lange Nacht der Forschung

The whole experiment consists of a breadboard smaller than one square meter in size, the detection and control unit and a laptop. On the day of the event, after checking the results under ideal conditions in the lab, these parts had to be transported to the venue of the LNF which was approximately 1.5km away. We had to be mindful of the breadboard because it included all the sensitive parts which were prone to misalignment due to vibrations during transport. Minor errors could be corrected relatively quickly, but major issues could only be solved by doing the alignment from scratch, which would pose a major challenge due to time pressure and the bright illumination in the room.

The booth had two tables, of which one was used for the breadboard and the control unit, and the other one for laptops and a monitor, as seen in picture 3.4. Additionally, posters were put up, giving a compact overview of the theory and visual representation of the game.

After setting up the individual parts and connecting them, the control unit was turned on which revealed the first issue. Even though the laser was not yet on, the single photon counts for both detectors were far above the dark counts. Photons coming from the light in the room to the detectors were cut off by shields from all sides and a lid was used to cover the top during measurements. The issue was not the detectors but the fiber cables and the fiber coupling connector on the quCR. Covering them with a black plane reduced the single photon values to acceptable levels, even if not as close to the dark counts in the lab.

The second issue was that no photons were being registered when turning on the laser, hinting at misalignment of the mirrors or fiber couplers. Thankfully, the error was not major and could be resolved quickly. The next step was checking the polarization of the photons. As seen in picture 3.5, the cables were secured with tape, which was not enough to maintain the polarization during transport. The polarization controllers had to be adjusted again.

Figure 3.4: Booth of the experiment with short introduction to entanglement and Bell inequalities on posters and a visualization of the theoretical and experimental results on two monitors

The more time-consuming aspect of these problems was the entanglement quality. After all the small errors and corrections, the number of coincidences was still low. Hence, visibility was calculated in both the H/V and D/A bases and improved over time. Due to the bright lighting and thereby high dark counts, the measurements needed to be done over a longer time period than in the lab to reduce errors.

Overall, once the experiment was set up in the lab, there were no major issues after transport. However, as there are many factors that constitute the quality of the outcome, all parts must be checked individually and corrected accordingly. With the fact that the conditions were not ideal and the dark counts were not negligible, the correction needs time and took me around 1.5 hours.

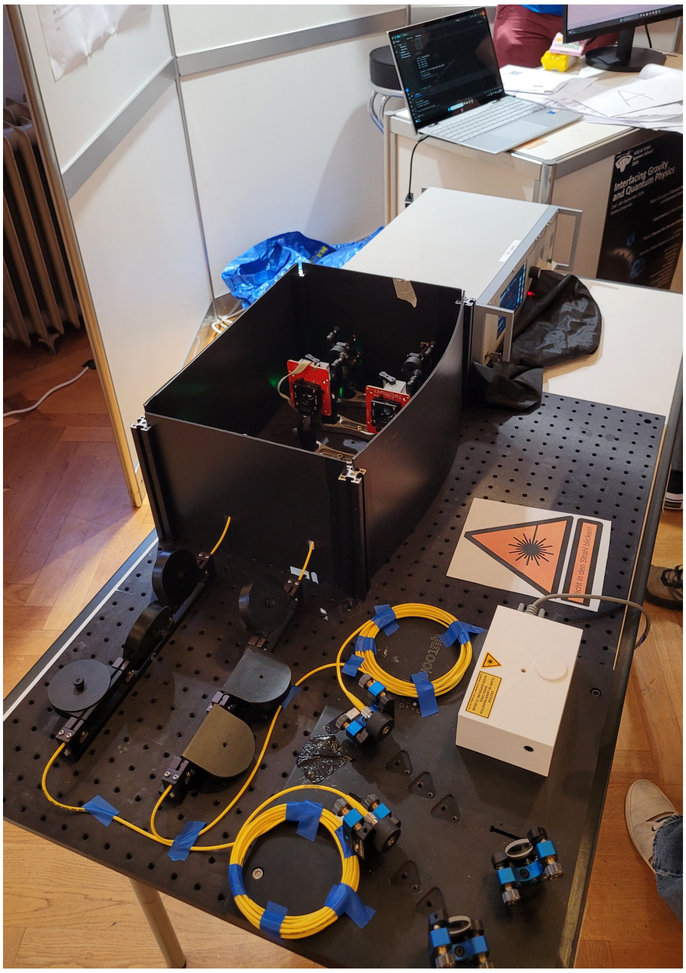


Figure 3.5: Setup as presented during the Lange Nacht der Forschung.

4 Results

The results during the Lange Nacht der Forschung (LNF) were not saved. However, since the setup could not be perfected due to time limitations and the fact that the background noise was very high due to the lighting in the room, the more interesting results that are presented here are from the lab under more ideal conditions and with better entanglement quality. Furthermore, the experiments were done using both available light sources. They will be denoted s_1 and s_2 from now on. One of them has a better quality and it is easier to align or more specifically to increase the entanglement quality. In the following, the results will be presented, and the sources will be compared.

Wave plate calibration

To get the fast axes of the wave plates, they are rotated in $3°$ steps over the whole range of $360°$ and the single photon counts over 3 seconds are measured. The curve is fit with a fit function, and the offset is read out. For the HWPs, the offset values are $40°$ and $-16°$ and can be seen in fig. 7.1. For the QWPs they are $92°$ and $49°$. The QWP in arm 1 shows a maximum at $2°$, but placing the two QWPs in succession in one arm at $45°$ and checking the single counts makes it clear that they are not set to the same fast axis because the counts are at a maximum, even though the effect of the QWPs should be a projection onto the V basis. Adding $90°$ to one QWP corrects this, which is done for the QWP in arm 1.

Visibility

The quality of the entanglement is measured by the visibility, which is determined by having one HWP in the H/D basis and rotating the other HWP in $4°$ steps and measuring the coincidences over 5 seconds. From the fits of the curves, see fig. 7.3, the maximum and minimum coincidences are read out and inserted into eq. 3.1. The visibilities V_1 for the first and V_2 for the second source are

$$
\begin{aligned}
V_{1H} &= 0.919 \pm 0.001, \\
V_{1D} &= 0.929 \pm 0.017; \\
V_{2H} &= 0.916 \pm 0.029, \\
V_{2D} &= 0.954 \pm 0.028.
\end{aligned}
$$

4.1 CHSH game results

The game is played four times with all possible strategies applied. The error is analyzed by doing Monte-Carlo simulations for each strategy and the measured coincidences. The results are shown in fig. 4.1.

	Y = 0			Y = 1		
		B = 0	B = 1		B = 0	B = 1
X = 0	A = 0	0.425	0.074	A = 0	0.417	0.080
	A = 1	0.070	0.410	A = 1	0.083	0.420
		B = 0	B = 1		B = 0	B = 1
X = 1	A = 0	0.405	0.088	A = 0	0.078	0.419
	A = 1	0.090	0.417	A = 1	0.428	0.075

Figure 4.1: Win probabilities of the CHSH game for the first source. The game is won for the anti-diagonal values for $X = Y = 1$ and the diagonal values else. In each case the sum of the corresponding two values exceeds the classical limit.

Summing the diagonal or the anti-diagonal, depending on strategy, the win probabilities for s_1 are

$$\begin{aligned} X = 0,\, Y = 0: &\quad P = 0.837 \pm 0.001, \\ X = 0,\, Y = 1: &\quad P = 0.855 \pm 0.001, \\ X = 1,\, Y = 0: &\quad P = 0.822 \pm 0.001, \\ X = 1,\, Y = 1: &\quad P = 0.847 \pm 0.001. \end{aligned}$$

Similarly, the results for s_2 are presented in fig. 4.2 and the probabilities are

$$\begin{aligned} X = 0,\, Y = 0: &\quad P = 0.864 \pm 0.001, \\ X = 0,\, Y = 1: &\quad P = 0.794 \pm 0.001, \\ X = 1,\, Y = 0: &\quad P = 0.786 \pm 0.001, \\ X = 1,\, Y = 1: &\quad P = 0.852 \pm 0.001. \end{aligned}$$

4.2 Tomography results and CHSH inequality violation

The number of coincidences measured in each wanted basis can be seen in tables 4.1 and 4.2. Doing quantum state tomography on these results yields the density matrices in fig. 4.3. To see

	Y = 0			Y = 1		
X = 0		B = 0	B = 1		B = 0	B = 1
	A = 0	0,428	0,111	A = 0	0,461	0,076
	A = 1	0,094	0,367	A = 1	0,060	0,404
X = 1		B = 0	B = 1		B = 0	B = 1
	A = 0	0,449	0,069	A = 0	0,129	0,386
	A = 1	0,079	0,403	A = 1	0,400	0,085

Figure 4.2: Win probabilities of the CHSH game for the second source. Similar results to the first source can be seen. Even though the values vary more from the theoretical prediction, higher success rate is still guaranteed.

how well the density matrix was recreated by fitting to the data, the root mean square deviation $RMSD$ between the measured probabilities and the probabilities of the reconstructed density matrix is calculated. The errors of the tomography for s_1 and s_2 amount to

$$RMSD_1 = 1.27\%,$$

$$RMSD_2 = 1.10\%.$$

Basis	HH	HV	HD	HR	VH	VV	VD	VR
Coincidences	59604	2071	32093	21197	2472	58228	31694	39175
Basis	DH	DV	DD	DR	RH	RV	RD	RR
Coincidences	27255	32543	58285	27124	21179	40043	28024	4375

Table 4.1: Measured coincidences from s_1 in different basis combinations

Basis	HH	HV	HD	HR	VH	VV	VD	VR
Coincidences	25167	1069	14348	10032	863	21090	10161	14897
Basis	DH	DV	DD	DR	RH	RV	RD	RR
Coincidences	13162	10615	23238	12527	9148	14211	10911	2182

Table 4.2: Measured coincidences from s_2 in different basis combinations

Furthermore, the purity, fidelity and concurrence of the reproduced density matrices ρ_1 and ρ_2 are calculated and the error is determined by simulating 300 values, resulting in

$$purity_{\rho_1} = 0.890 \pm 0.003,$$

(a)

(b)

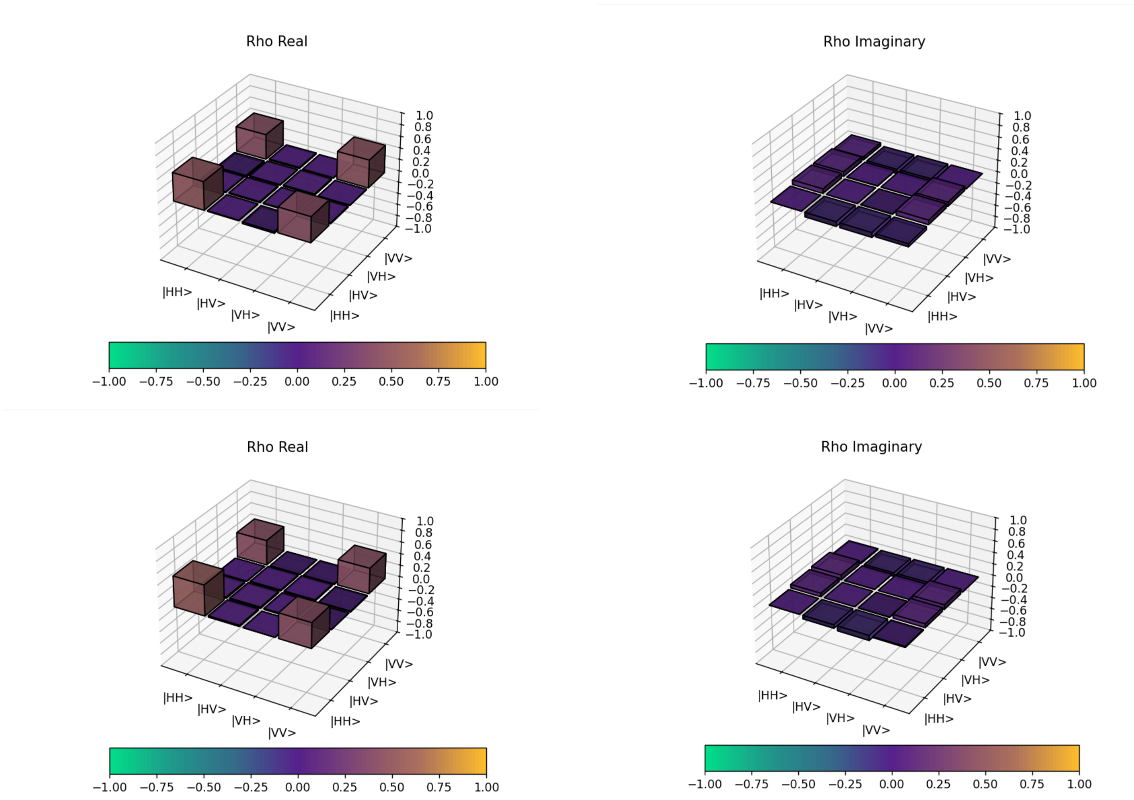


Figure 4.3: Graphical representation of the density matrices ρ_1 in (a) and ρ_2 in (b) of the reproduced states using QST. The figure shows the expected state of $\phi^+ = \frac{1}{\sqrt{2}}(|H\rangle|H\rangle + |V\rangle|V\rangle)$ with almost no imaginary part.

$$fidelity_{\rho_1} = 0.912 \pm 0.002,$$
$$concurrence_{\rho_1} = 0.892 \pm 0.003;$$

$$purity_{\rho_2} = 0.874 \pm 0.004,$$
$$fidelity_{\rho_2} = 0.906 \pm 0.002,$$
$$concurrence_{\rho_2} = 0.865 \pm 0.005.$$

Finally, the Bell violation is calculated according to eq. 2.3 with 1000 simulated values:

$$S_1 = 2.741 \pm 0.002,$$

$$S_2 = 2.594 \pm 0.002.$$

4.3 Die Lange Nacht der Forschung

An important part of this thesis that needs to be addressed is how well the experiment was received and understood by the visitors during the event. Quantum entanglement is a topic that many people have a hard time grasping, even as a physics student. Explaining this theory in 15-20 minutes using an experiment to people with no prior knowledge can be a challenge, but in my experience, we were successful.

The event started at 17:00 and ended at 23:00. From the beginning to the end, the booth was constantly busy with visitors, and as the event was freely accessible to the public, a variety of people attended. Most of them were adults, but also teenagers and children, some at the age of around ten, were very curious and interested. The interactions started with a short description of the topic and an explanation of the classical CHSH game. Visitors were encouraged to play the game themselves and were given envelopes containing sheets of paper with 0 or 1 written on them. The presenters played the role of the referee. Surprisingly, many people were not interested in playing the game themselves. My assumption is that the posters with the explanation were sufficient and that the idea was easy to understand. Another reason might be that the quantum aspect of the experiment was more interesting and appealing, and the guests did not want to waste time on the classical part.

Thanks to the 2022 Nobel Prize and Austrian Nobel Laureate Anton Zeilinger, some people had heard of Bell's theorem in one way or another. For those, the experiment was quite fascinating to see, as many had no idea how experiments in quantum physics looked like. For those who did not know anything about the theory, the original ideas about non-locality and entanglement were discussed before focusing on the experiment.

Understanding the analogy between classical bits and qubits in the sense of photon polarization was not a difficulty for many visitors. Thus, the transition from the classical CHSH game worked out smoothly and it was clear to everyone how Alice and Bob applied their strategy in the quantum case. This time, as the result was not straightforward to see, the game was usually played, albeit not every time. The players could witness their outcome on the laptop and see that the win probability was indeed higher than in the classical game.

All in all, the CHSH game was very successful in explaining quantum entanglement to visitors of the event. On the one hand, the classical version seemed very easy to understand and perfect for catching and keeping people's attention. On the other hand, the quantum version was educational enough for people to get a grasp on entanglement. A lot of interest was shown in the set up itself. Many visitors were curious about the different parts and what their exact purpose was. Having the set up completely open helped catch people's eyes, and having the ability to pick up the lid to show the wave plates during the experiment helped satisfy interested people. Toward the end of the event, some people were even allowed to carefully play around with the screws of the fiber couplers to see how much the alignment was influenced by the smallest rotation. I also showed this to others who were interested in the alignment process. Visitors liked seeing the setup manipulated and adjusted or doing it themselves, as the interactive part of the experiment was not too grand. It seems that this aspect could use some improvement. A possibility for the visitors to be more involved could be the use of non-automated wave plates. By applying the strategy through manual rotation of the HWP the feeling of being more hands-on would be created.

A question that came up quite often was about the practical use of entanglement. Even if people understood the experiment, without any practical advantages, the results might have seemed meaningless. An additional poster explaining applications and the future possibilities or even potential threats posed by the use of quantum entanglement would help to keep people engaged in the topic even after the event.

5 Discussion

The results are all in the expected range with a good error margin. The first laser source outperforms the second one in all areas, but both deliver acceptable results. The second source was initially used when setting up the experiment for the LNF and the results were not promising. After switching sources, the setup was performing as desired, which concludes that s_1 is in a better overall condition. This could either mean that the created entanglement is of higher quality or that the alignment of the mirrors and fiber couplers is easier due to less wear. Looking at the count rates, it is also clear that s_1 has much higher single photon and coincidence rates. In conclusion, source one outperforms s_2 both quantitatively and qualitatively. However, it should be noted that more time was invested in aligning s_1 than s_2 and it could theoretically be possible to reach the same or even higher quality with s_2. But given how comparatively low the count rates are, this seems improbable.

The calibration of the wave plates shows inconsistency, since the count rates should form a sinusoidal curve, but fig. 7.2 shows varying peaks. This should not be the case if the photons during calibration were in a perfect $|HH\rangle$ state. It is likely that the state created after the polarization controllers had some flaws, as HWPs were used during this process and they do not allow for checking the imaginary part of the state. All things considered, this does not seem to be an issue when looking at the results.

The CHSH game was a success both during the outreach event and in the lab. The output matrix almost always showed a win probability of over 0.75 but dipped below that limit in the later hours of the event, as the alignment became out of tune. The lab results show much better values and sometimes exceed the limit of $\cos^2(\pi/8) = 0.854$, even when considering the error. The reason for this is that the entangled state was not perfectly balanced between $|HH\rangle$ and $|VV\rangle$. This changes the probability of detecting coincidences to be higher in some bases and lower in others. Of course, this does not mean that the theoretical limit is wrong or was broken. When we consider all the results and average them, the result is still $P \leq 0.854$.

One thing that was not considered for the CHSH violation and game was the fact that the loopholes were ignored. To be absolutely sure that there are no other variables influencing the results and that we can really accept Bell's theorem, closing the loopholes would be necessary. However, even if some loopholes were to be closed, the locality loophole is unavoidable in the case of a field-deployable setup. The available space on the breadboard does not permit distances between the photon pairs that would secure no faster-than-light communication. Nonetheless, experiments were conducted where these loopholes were closed simultaneously [8], so the results can still be accepted. The goal was not to perform a perfect experiment violating the CHSH inequality, but to show the theory and an actual implementation.

The tomography results show that the reconstruction of the state worked very well when looking at the $RMSD$ and the states are indeed in agreement with the real state $\Phi^+ = \frac{1}{\sqrt{2}}(|HH\rangle +$

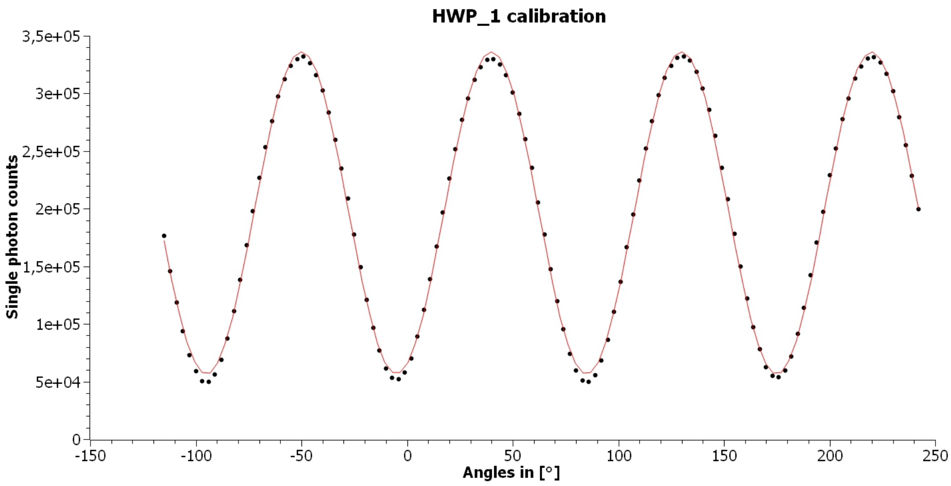
$|VV\rangle$). The purities are slightly below 90% and could have been increased further, but the violation of the CHSH inequality shows that this is not necessary. The violation with the first laser source is relatively close to the maximum of $S = 2\sqrt{2} \approx 2.828$ and S_2 is not as high, but far above the classical limit of $S = 2$.

6 Conclusion

The aim of this work was to set up an experiment that can be deployed outside of the lab and brought to an event like the Lange Nacht der Forschung, where anyone who is interested could convince themselves of the supremacy of quantum mechanics by playing the CHSH game. The visitors, without any prior knowledge, could understand and see within minutes that the results are classically impossible, yet are achieved by the presented setup. The non-intuitive nature of quantum mechanics and entanglement was shown in a playful and easily understandable way to people starting from around 10 years of age. One disadvantage of such a field-deployable setup is that the sensitivity of the laser alignment does not allow for quick setup. It must be readjusted after transport and deteriorates after some time due to external influences.

The loopholes offer room for discussion with the participants but can only be engaged with enough understanding or prior knowledge. For the majority of people this would be too complex and too time-consuming, especially during an event such as the LNF, where hundreds of experiments are presented. The setup could, however, be used as an educational tool to teach students and spark their interest, as an introduction to Bell's theorem.

7 Appendix



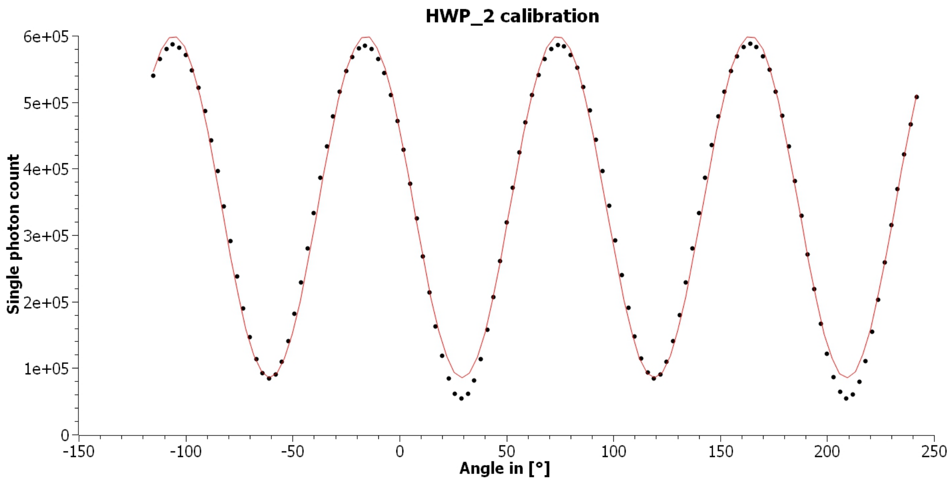
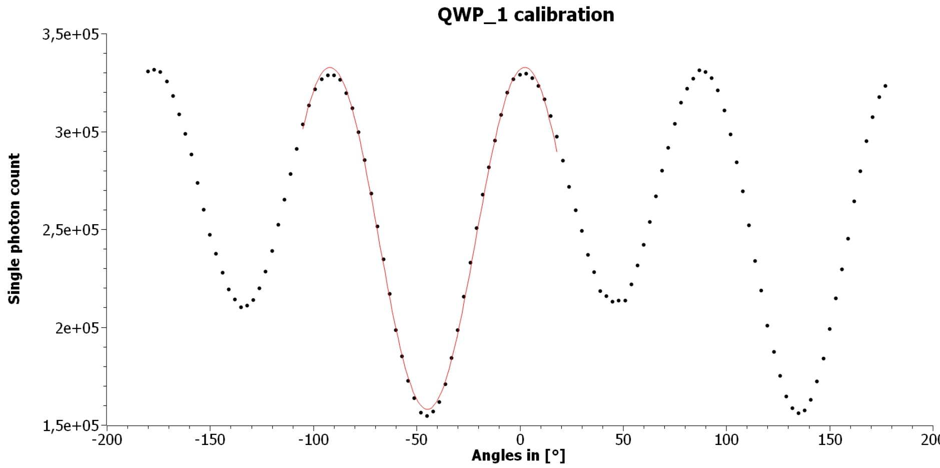


Figure 7.1: Results for the calibration of the HWPs in arm 1 (top) and arm 2 (bottom) of the setup. The data is fit with a sin curve (red line) and the angles corresponding to the fast axis of the HWPs are found out from the maxima, which are at $40°$ and $-16°$, respectively.



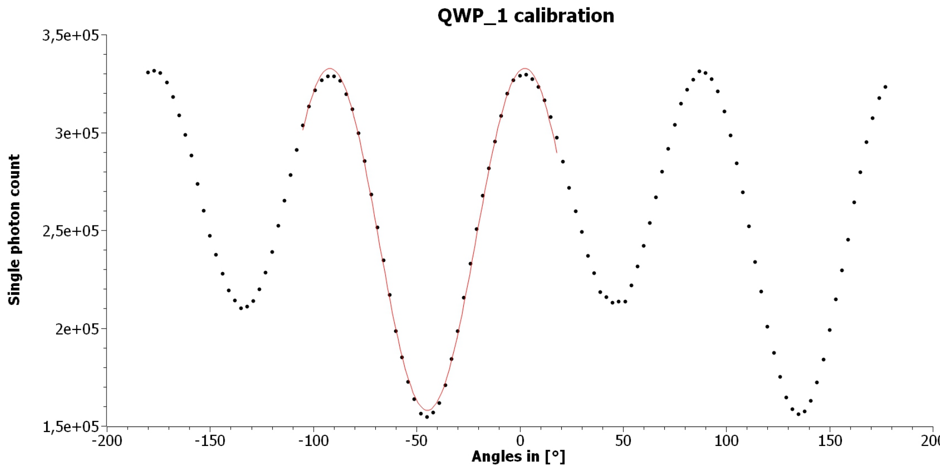
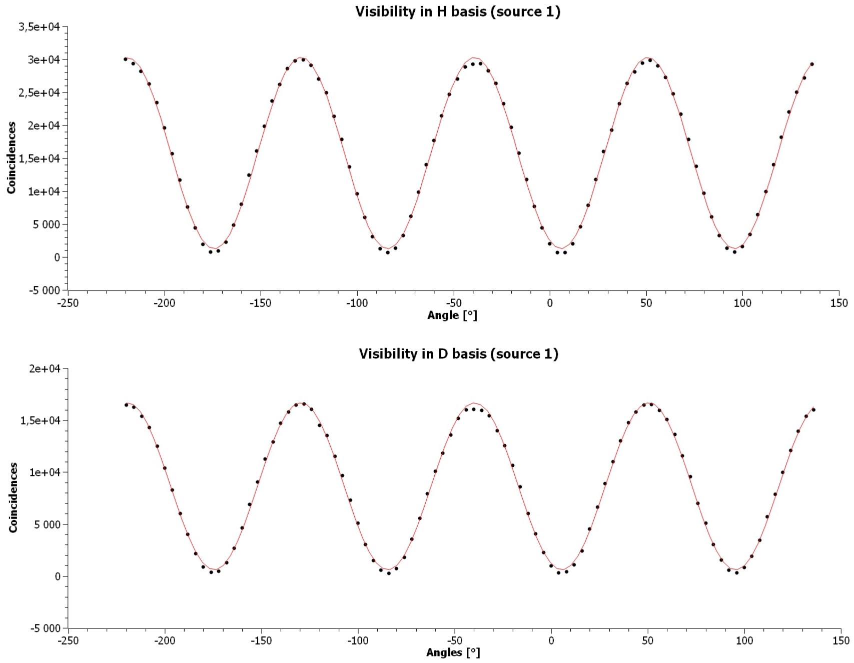


Figure 7.2: Results for the calibration of the QWPs in arm 1 (top) and arm 2 (bottom) of the setup. The curve is fit with a sin function in the region with prominent peaks closest to $0°$. The maxima relating to the fast axis would be at $2°$ and $49°$. Checking the QWPs together shows that $90°$ must be added to one of them, which is done for the QWP of arm 1.

(a)



(b)

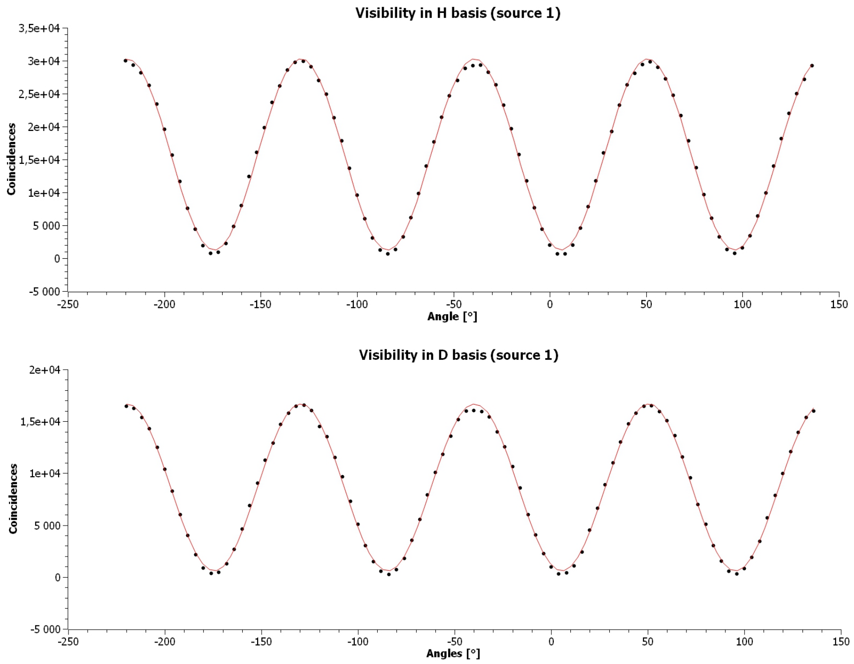


Figure 7.3: Visibility measurements of (a) the first and (b) second source in the H and D basis. The maximum and minimum number of coincidences of the fit curves are taken to calculate the visibility.

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