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Constructing an Inflation-Resistant Portfolio in the Cryptocurrency Universe

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Daria Isakova

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Abstract

This thesis investigates the potential of cryptocurrencies as an inflation hedge, focusing on their ability to track and mitigate the effects of inflation expectations. Using data from December 2019 to April 2024, the research applies a sparse index tracking model to select and assign a weight to cryptocurrencies, aiming to minimize the empirical tracking error against the 5-year break-even inflation rate. The methodology incorporates a rolling window approach and hyperparameter tuning to enhance the model's adaptability to market conditions. Two distinct portfolio compositions are examined: one including all selected cryptocurrencies and another excluding a gold-backed cryptocurrency to assess the impact of traditional commodity-linked assets. Results indicate that cryptocurrencies demonstrate some potential as inflation hedges, particularly when we consider bi-weekly or monthly returns. The portfolio excluding the gold-backed cryptocurrency shows stronger correlations and more significant beta coefficients with inflation expectations. However, high volatility and substantial tracking errors persist, highlighting the challenges of using cryptocurrencies for inflation protection. The thesis also explores practical implementations, including the integration of cash and traditional assets like ETFs to balance risk and return. These findings contribute to the growing body of research on cryptocurrency's role in modern portfolio management and offer insights for investors seeking innovative approaches to inflation hedging in an evolving financial landscape.

Kurzfassung

Die vorliegende Arbeit widmet sich der Untersuchung des Potenzials von Kryptowährungen als Instrument des Inflationsschutzes. Dabei wird ein besonderes Augenmerk auf die Fähigkeit von Kryptowährungen gelegt, Inflationserwartungen zu antizipieren und deren Auswirkungen abzumildern. Die Untersuchung basiert auf Daten aus dem Zeitraum von Dezember 2019 bis April 2024. Mithilfe eines spärlichen Index-Tracking-Modells erfolgt eine Auswahl und Gewichtung von Kryptowährungen mit dem Ziel, den empirischen Tracking-Fehler gegenüber der 5-Jahres-Break-Even-Inflationsrate zu minimieren. Die Methodik umfasst eine rollierende Fensteranalyse sowie ein Hyperparameter-Tuning, um die Anpassungsfähigkeit des Modells an Marktbedingungen zu optimieren. Im Rahmen der Untersuchung werden zwei unterschiedliche Portfolio-Zusammensetzungen analysiert. Dabei umfasst die erste Portfolio-Zusammensetzung alle ausgewählten Kryptowährungen, während die zweite Portfolio-Zusammensetzung eine goldgedeckte Kryptowährung ausschließt. Auf diese Weise lässt sich der Einfluss traditioneller rohstoffgebundener Vermögenswerte auf die Ergebnisse bewerten. Die Resultate demonstrieren, dass Kryptowährungen ein beachtliches Potenzial als Instrument zur Absicherung gegen Inflation aufweisen, insbesondere bei einer Betrachtung von Renditen mit einer Periodizität von zwei Wochen oder Monaten. Das Portfolio, welches die goldgedeckte Kryptowährung ausschließt, zeigt eine stärkere Korrelation sowie signifikant höhere Beta-Koeffizienten in Bezug auf die Inflationserwartungen. Allerdings bestehen weiterhin hohe Volatilität und erhebliche Tracking-Fehler, was die Herausforderungen bei der Verwendung von Kryptowährungen zum Inflationsschutz verdeutlicht. Des Weiteren werden praktische Umsetzungen analysiert, wobei insbesondere die Integration von Bargeld und traditionellen Anlagen wie ETFs zur Risiko- und Renditegleichgewichtung untersucht wird. Diese Erkenntnisse tragen zur wachsenden Forschung über die Rolle von Kryptowährungen im modernen Portfoliomanagement bei und bieten Investoren, die innovative Ansätze zum Inflationsschutz in einer sich entwickelnden Finanzlandschaft suchen, wertvolle Einblicke.

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1. Introduction

During the last few years, the global economic landscape has been tremendously impacted by a series of events that added fuel to inflationary pressures worldwide. These events, including the COVID-19 pandemic, international conflicts, and logistical breakdowns in global trade networks, have created a challenging environment for investors. As a result, individuals are now faced with the task of reassessing their risk tolerance and adjusting their portfolio structures to protect against the erosive effects of inflation.

In the meantime, the significant rise of cryptocurrency adoption allows us to consider cryptocurrencies not only as a store of value, medium of exchange, or a tool for blockchain governance[1], but also has created a possibility to recognize cryptocurrencies as a potential alternative to traditional inflation hedges such as gold, real estate and Treasury Inflation-Protected Securities (TIPS) due to systemic evidence on the relationship between them and inflation expectations Choi and Shin (2022).

By analyzing market data and performing advanced, realistic portfolio simulations, this study comprehensively examines the potential of cryptocurrencies as inflation-hedging assets. Our research focuses on developing optimal strategies for constructing inflation-tracking portfolios, encompassing both asset selection and allocation methodologies. Through this process, we aim to identify which cryptocurrencies, if any, are most effective in creating a robust inflation-hedging portfolio. Unlike previous studies that often focus on individual cryptocurrencies, our approach considers the asset class as a whole, employing dynamic portfolio management and automated trading strategies to mirror inflation expectations.

The structure of this thesis is as follows. Chapter 2 offers a comprehensive overview of inflation measurement, inflation-hedging strategies, and the characteristics of cryptocurrencies as a potential inflation-hedging asset class. Chapter 3 presents the methodology of the research, including data selection and preprocessing, model development, and the design of our tracking strategy. This chapter details the sparse index tracking algorithm used to construct inflation-hedging portfolios and explains our approach to training,

[1] The governance token explained: `https://www.coinbase.com/ru/learn/crypto-basics/what-is-a-governance-token`

validation, and testing data. Chapter 4 presents the empirical results, discussing the outcomes from both the validation and testing periods. It includes analyses of portfolio performance using various metrics such as tracking error, correlation, and OLS beta. Chapter 5 explores possible implementations of our findings, including the integration of cash and traditional assets like ETFs into cryptocurrency portfolios. Chapter 6 concludes the study, summarizing key findings and suggesting directions for future research.

2. Theoretical Background: Inflation, Hedging, and Cryptocurrencies

This Chapter provides a comprehensive overview of inflation measurement techniques, including CPI, PPI, and the 5-year break-even inflation rate. It explores various inflation hedging strategies and their theoretical foundations. The Chapter then delves into cryptocurrencies as an asset class, examining their empirical properties, market efficiency, volatility patterns, and connectedness with traditional financial markets. Finally, it discusses the challenges and potential benefits of integrating cryptocurrencies into inflation hedging strategies, reviewing recent research on the relationship between cryptocurrencies and inflation expectations.

2.1. Inflation measurements

Inflation refers to the general increase in the prices of goods and services over time, resulting in a decrease in the purchasing power of money[1]. The impact of inflation influences various aspects of an individual's life: it lowers the purchasing power of money, erodes the value of cash over time (since no interest is earned), and significantly reduces real returns on fixed income investments and stocks. This can lead to lower returns or even losses when the inflation rate exceeds the nominal return on investments. As a result, investors and average individuals seek assets that can hedge against inflation, preserving their purchasing power and generating real returns that outpace the increase in prices.

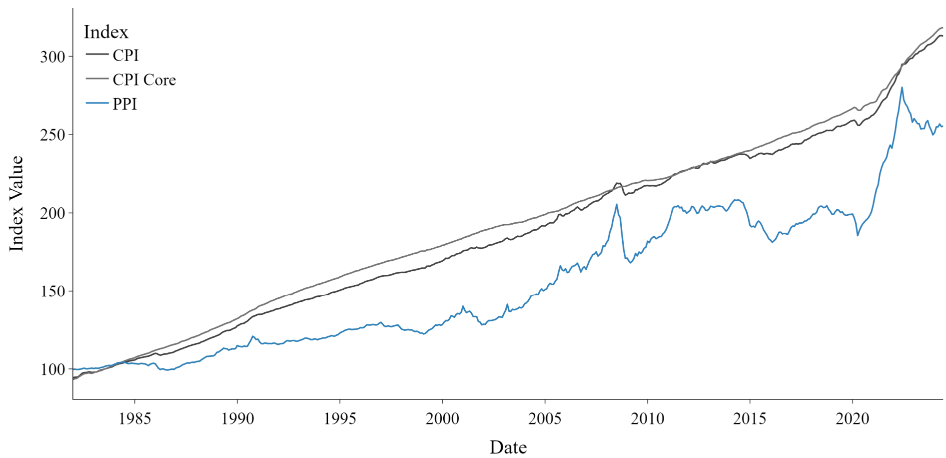
There exist various schools of thought concerning inflation causes and implications. The major economic theories on inflation, for instance, the Monetarist model and Keynesian, - each provides different explanations for its origins and effects on the economy. According to the Monetarist school of thought, inflation is exclusively linked to changes in the money supply. They argue that it occurs when the amount of money in circulation increases more rapidly than the economy's ability to produce goods and services. Keynesians, on

[1] https :// vocal .media / trader / the -impact -of -inflation -on -trading -strategies -for -navigating-volatile-markets

the other hand, highlight demand-side factors, suggesting that when an economy operates near full capacity, increased demand leads to higher prices. In our research, the specific causes behind inflation are not our primary focus, rather, we perceive inflation as a macroeconomic indicator or index that reflects broader economic trends. This perspective allows us to focus more on developing a proper optimized hedging strategy rather than trying to define the causal relationship between inflation and assets.

The following question arises - if we want to use inflation as a macroeconomic index and build our hedging strategy on that, what inflation measurement should we use for this approach? To accurately measure inflation, it is essential to get familiar with established economic indicators such as the Consumer Price Index (CPI)[2] and Producer Price Index (PPI)[3]. The CPI measures the average change over time in the prices (1 year) paid by urban consumers for a market basket of goods and services, making it a direct gauge of the purchasing power that consumers can maintain against the backdrop of rising prices. On the other hand, the PPI measures the average change over time in the selling prices received by domestic producers for their output, a key indicator of cost pressures within the production process that can precede consumer price changes.

Figure 2.1.: CPI, CPI Core and PPI trends



> ”Monthly inflation in the U.S. reached a peak in June 2022 at 9.1 percent. This means that prices were 9.1 percent higher than they were in June of 2021.” (Statista Research Department, 2024)[4]

[2] https://www.bls.gov/cpi/

[3] https://www.bls.gov/ppi/

[4] https://www.statista.com/statistics/191077/inflation-rate-in-the-usa-since-1990/

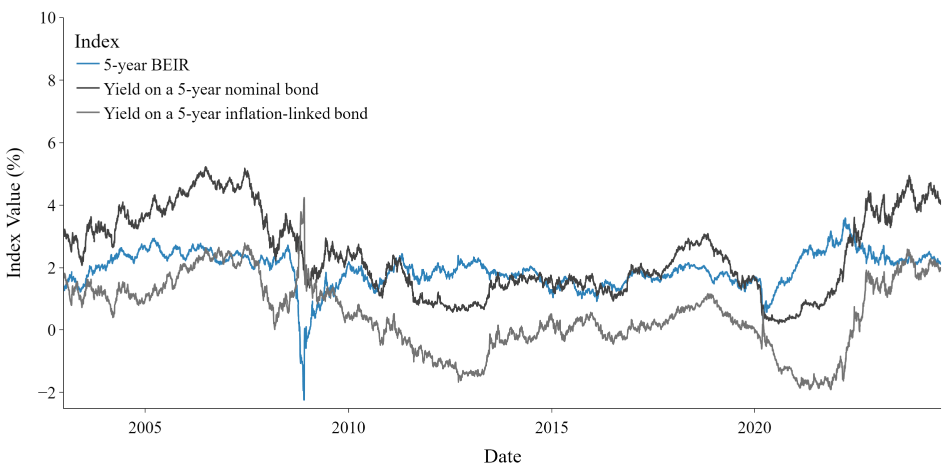
CPI and PPI indices are released by the United States Bureau of Labor Statistics on a monthly basis and have a backward-looking nature according to the calculation formula. They also can be adjusted by seasonality and volatile goods/services inclusion (i.e. core CPI excludes volatile food and oil prices to provide a more reliable measure of inflationary and deflationary periods).

Despite some limitations - like delays in updates, differences in how countries measure inflation, etc., - CPI is still the most commonly used measure for inflation. Fewer studies use other indices such as PPI, Wholesale Price Indices (WPI), Retail Price Indices (RPI), or GDP deflators. The frequency of data used in these studies can vary, with some using monthly, quarterly, bi-annual, or annual changes to track inflation rates (Arnold & Auer, 2015).

Given the need to understand inflation's impact on investment strategies, another crucial aspect that we need to consider - is expected inflation, which plays a vital role in economic forecasting and developing investment strategies to mitigate inflation. Expected inflation reflects the future rate of inflation anticipated by consumers and producers, shaping their financial decisions.

There are different methods exist to gauge expected inflation. For instance, conducting consumer and business surveys to assess the inflation expectations for the upcoming period. Another method - is developing forecasting models using econometric methods based on past data with the help of Bayesian or Frequentist Statistics. These models attempt to predict future inflation by identifying underlying patterns and relationships in economic data.

Figure 2.2.: 5-year BEIR composition



The current research is based on one particular forward-looking indicator - the break-even inflation rate. This rate, derived from the yield difference between fixed-rate bonds and inflation-protected securities (like TIPS in the United States) of the same maturity, provides an estimate of market expectations for inflation over the duration of the bonds.

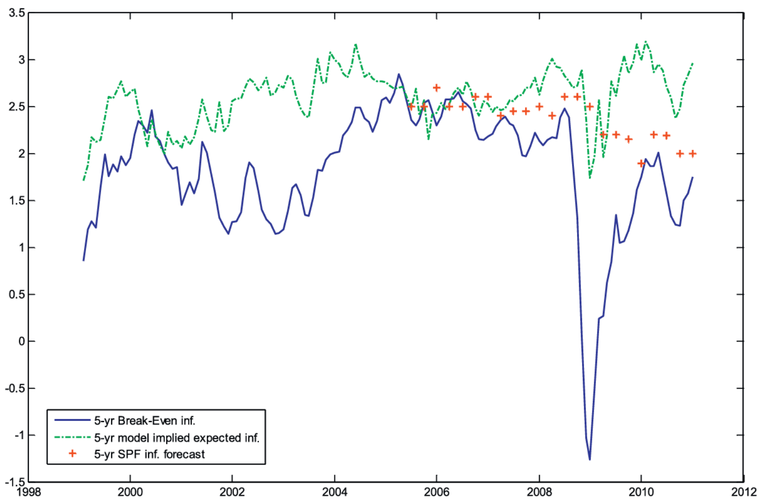
Specifically, this study focuses on the 5-year break-even inflation rate (5-year BEIR, T5YIE), which offers insights into medium-term inflation expectations as perceived by market participants. The 5-year BEIR at time t is expressed by the following formula:

$$BEIR_t^{(5)} = Y_{nominal,t}^{(5)} - Y_{real,t}^{(5)}$$

, where $BEIR_t^5$ represents the 5-year Break-Even Inflation Rate at time t, $Y_{nominal,t}^5$ is the yield on a 5-year nominal bond at time t and $Y_{real,t}^5$ is the yield on a 5-year inflation-linked bond at time t. Figure 2.2 provides a visual representation of this relationship, showing the decomposition of the 5-year BEIR into its components.

In other words, the break-even inflation rate is the point where nominal and inflation-linked bonds become equally attractive to investors. When actual inflation aligns with the break-even rate, there's no clear advantage to choosing one type of bond over the other.

Figure 2.3.: Break-even inflation and model-implied expected inflation (Zeng, 2013, p. 133)



We use the 5-year BEIR as a measure of future inflation expectations for several reasons. First, it is published on a daily basis, making it ideal for short-term strategies. Secondly, given that the underlying assets in our study are cryptocurrencies, which have a relatively short history, the break-even rate enables us to directly relate shifts in inflation expectations to changes in cryptocurrency prices. This approach provides insights into the potential of our future generated portfolios (composed of cryptocurrencies) as a viable hedge against inflation. However, the reader must know, that perceiving the break-even rate as a 1-to-1 representation of expected inflation is misleading due to its limitations such as the inclusion of liquidity and inflation risk premia.

According to the study by Zeng (2013) ”the inflation forecasts implied by the model show that break-even inflation rates underestimate inflation expectations” (almost for the whole sample) due to liquidity risk (Figure 2.3). Similar results were also found by (2011)Kajuth and Watzka (2011).

2.2. Inflation hedge strategies

During periods of financial distress, when inflation rates significantly exceed expectations for an extended period, investors must seriously consider the loss of purchasing power in their investment decisions. Here comes the term “hedge” as the concept of risk mitigation.

Perfectly written by Bodie (1976) and summarized by Arnold and Auer (2015), one of the definitions of inflation hedge states that an asset is an inflation hedge if and only if its real return is independent of the rate of inflation, which means that a positive correlation between the nominal return of the hedging asset and inflation exists. A correlation of 1 is called a perfect hedge since price increases are perfectly compensated by corresponding asset returns. Fisher (1930) initially formulated the theoretical link between asset returns and inflation, observing that the nominal interest rate comprises an expected real return plus an expected inflation rate. Fama and Schwert (1977) elaborated on this, suggesting that expected nominal returns for all forms of assets incorporate market expectations of inflation rates. If the information at time $t-1$ is processed efficiently, any asset j will be priced according to the following formula:

$$E_{t-1}(r^n_{jt}) = E_{t-1}(r^r_{jt}) + E_{t-1}(\pi_t)$$

, where $E_{t-1}(r^n_{jt})$ is the expected nominal return of any asset j at time t, $E_{t-1}(r^r_{jt})$ is the equilibrium expected real return at time t and the best possible assessment of the expected inflation rate $E_{t-1}(\pi_t)$ from $t-1$ to t.

After deriving an empirical measure of the expected inflation rate the following standard linear regression model can be estimated:

$$r^n_{jt} = \alpha_j + \beta_j E_{t-1}(\pi_t) + \varepsilon_{jt}$$

, where the dependent variable r^n_{jt} - the nominal rate of return of asset j at time t is regressed on explanatory variable $E_{t-1}(\pi_t)$ - expected inflation rate estimated at time $t-1$. If the slope/coefficient β_j is not significantly different from one – it goes in line with the hypothesis that the expected nominal rate of return of asset j has a positive synchronized relationship with the expected inflation rate, signifying hedge properties.

Much of the literature and research conducted on inflation hedging employ OLS regression, VAR models, or cointegration techniques to analyse whether specific asset classes - such as common stocks, gold, fixed-income securities, and real estate - possess inflation-hedging properties. These significant studies are comprehensively summarized in the work of Arnold and Auer (2015), which details the main findings and tools used.

Of course, the concept of creating portfolios specifically targeted at hedging against inflation is not new. Several well-known portfolios have been designed with this objective in mind, such as the classic 60/40 stock-to-bond ratio, the famous Ray Dalio's "All Weather Portfolio"[5], and other diversified investment strategies. For instance, according to Ray Dalio's "All Weather Portfolio", the best assets for the regime of growing inflation expectations are Inflation Linked Bonds, Commodities and EM Credit, and when we experience deflation – Equities and Nominal Bonds[6]. However, most of them are based on the properties of the particular asset class and its historical behaviour during different economic environments.

Moreover, the rise of cryptocurrency implementation influenced many investors and investing companies to create portfolios consisting solely from crypto assets with the main purpose - mitigate the eroding effect of inflation. For instance:

- 21Shares Bytetree BOLD Exchange-traded product[7]. Implemented inverse volatility weighting methodology – also known as Naïve Risk Parity (Lee, 2011) – where a larger portfolio weight is assigned to less volatile assets. Portfolio selection includes Bitcoin and Gold.

- Inflation Shield by Coinpanion (now: Altify). Implemented generalized Markowitz

[5] Ray Dalio's All Weather Portfolio, renowned for its approach to portfolio optimization, is designed around asset classes holding the same risk and have shown resilience in various economic conditions: when inflation rises/falls, growth rises/falls relative to expectations.

[6] `https://www.bridgewater.com/research-and-insights/the-all-weather-story`

[7] ISIN: CH1146882308.

(1952) model, where the weights for USDC[8] and PAX Gold[9] were determined by Mean-Risk portfolio optimization method with Minimum Risk objective function and standard deviation as risk parameter. Whereas the fixed 5% of Bitcoin allocation was chosen based on changes in Maximum and Average Drawdown alongside with Sharpe ratio.

- Inflation Shield Bundle by Revix (now: Altify). A monthly rebalanced portfolio that keeps the allocation weights of the proportion of 75% to PAX Gold and 25% to Bitcoin.

Building on the existing body of research, our study adopts a novel approach by focusing on the emerging asset class of cryptocurrencies, independent of the singular hedging properties of individual assets (i.e. Bitcoin). We design a dynamically managed portfolio, optimized through automated trading strategies that aim to mirror inflation expectations. This method will integrate various statistical and portfolio optimization techniques, such as index tracking and mean-variance optimization, as discussed in Chapter 3. By developing a portfolio that adapts to inflation trends, our goal is to assess the potential of cryptocurrencies - not just a single currency but the asset class as a whole - to act as effective safeguards against inflation.

2.3. Cryptocurrencies as an asset class: evaluating their potential for inflation hedging

2.3.1. Overview

The remarkable growth of the cryptocurrency market in recent years, mostly due to Bitcoin phenomenon, attracted considerable attention not only in tech and trading communities but has become a crucial research object in economics. Both users and academics are shedding light on this new technology, with research delving into its potential to integrate into traditional financial systems, its role as an investment asset, and its implications for monetary policy.

While the most famous cryptocurrency Bitcoin pioneered the concept, the cryptocurrency landscape has exploded. Today, thousands of cryptocurrencies exist[10], each with unique features and use cases. Some, like Ethereum, focus on smart contracts and

[8]Stablecoin 1:1 to US Dollar.
[9]Cryptocurrency backed by 1:1 physical gold in London Brinks vaults.
[10]According to Statista Research Department, there are over 9000 cryptocurrencies exist in the beginning of 2024 https://www.statista.com/statistics/863917/number-crypto-coins-tokens/

decentralized applications (dApps). Others, like USDC, are stablecoins pegged to 1:1 to fiat currencies, designed to minimize volatility. This diversity creates both opportunities and challenges for investors and researchers.

The true potential of cryptocurrencies extends far beyond their use as currency. Their underlying blockchain technology offers a distributed, tamper-proof ledger system that could revolutionize industries ranging from supply chain management to identity verification and voting systems. This potential stems from the core feature of decentralization. Unlike fiat currencies, many cryptocurrencies operate without central control, providing advantages like censorship resistance, transparency, and reduced risk of government manipulation. However, decentralization also poses governance and stability challenges[11], illustrating the complex trade-offs inherent in this emerging technology.

The volatile nature of crypto markets fuels ongoing debate. The question remains: Are cryptocurrencies revolutionary assets offering high-growth potential, or merely a bubble driven by speculation? Historical price swings underscore the risk, while the growing interest from institutional investors[12] suggests a shift in perception. Empirical research continues to examine their behaviour as an asset class and their correlation with traditional investments.

2.3.2. Empirical properties of the cryptocurrency time series

The inherent design of cryptocurrencies as digital currencies doesn't preclude them from sharing properties with digital commodities or traditional financial assets, sparking ongoing debate about their classification. Empirical evidence shows that cryptocurrencies share many asset-like qualities, such as volatility, liquidity, and trading volume. According to Phillip, Chan, and Peiris (2018), cryptocurrencies exhibit predictable patterns, mild leverage effects, varied kurtosis, and volatility clustering. In this respect, cryptocurrencies often demonstrate extreme returns, impacting their risk-return characteristics compared to traditional assets. Despite these qualities, cryptocurrencies still show significant inefficiencies due to speculative trading and regulatory uncertainties. Caporale, Gil-Alana, and Plastun (2018) note that while Bitcoin shows improving efficiency over time, smaller cryptocurrencies remain less efficient.

Cryptocurrency market efficiency

[11] See Blockchain Trilemma `https://www.theblock.co/learn/249536/what-is-the-blockchain-trilemma`

[12] In 2021, ProShares introduced the Bitcoin Strategy ETF, the first Bitcoin futures ETF approved by the U.S. Securities and Exchange Commission (SEC). On January 10th, 2024 The SEC approved 11 proposals from issuers including BlackRock, Fidelity and VanEck, among others, to launch spot bitcoin ETFs.

The cryptocurrency market exhibits characteristics of both efficiency and inefficiency, a debate highlighted by extensive research. Some studies suggest signs of efficiency, such as low volatility premiums and improving price discovery over time (Alvarez-Ramirez & Rodriguez, 2021; Burggraf & Rudolf, 2021; Köchling, Müller, & Posch, 2019). Others support the Efficient Markets Hypothesis (EMH), particularly for Bitcoin, due to its unpredictable future values and signs of random walk returns (Wei, 2018; Yaya, Ogbonna, Mudida, & Abu, 2021). Certain market events, like the introduction of Bitcoin futures, seem to have improved market efficiency (Kim, Kim, & Kim, 2020). However, contradictory evidence points to market inefficiencies. These include delayed price reactions to events, the impact of specific large Bitcoin transfers, and the presence of reversal effects driven by both market inefficiency and liquidity compensation (Ante & Fiedler, 2021; Kozlowski, Puleo, & Zhou, 2021; Mohammad Hashemi Joo & Dandapani, 2020). Furthermore, some studies argue that the efficiency of the cryptocurrency market is time-varying, supporting the Adaptive Market Hypothesis (AMH), where periods of efficiency and inefficiency alternate (Chu, Zhang, & Chan, 2019; Tran & Leirvik, 2020).

Volatility and Price Behaviour

Cryptocurrency markets exhibit exceptionally high volatility compared to traditional markets, with distinct volatility patterns evident across different cryptocurrencies. Research highlights characteristics like long memory effects, heavy tails, volatility clustering, and substantial spillover effects between cryptocurrencies (Canh, Wongchoti, Thanh, & Thong, 2019; Phillip et al., 2018; Wei Zhang & Shen, 2018). Bitcoin, while sometimes considered less risky than other cryptocurrencies, still experiences extreme volatility that responds to news events and geopolitical risk (Aysan, Demir, Gozgor, & Lau, 2019; Dyhrberg, Foley, & Svec, 2018; Gkillas & Katsiampa, 2018). Volatility in cryptocurrencies is often driven by public interest and market developments (Chaim & Laurini, 2019).

Additionally, research conducted by Corbet and Katsiampa (2020) reveals that cryptocurrency prices exhibit a positive serial correlation, meaning that an upward trend is likely to continue. However, this trend lessens after negative price movements, indicating an asymmetric pattern. In Bitcoin, specifically, positive returns persist longer than negative ones, further supporting this asymmetric price behaviour.

Interconnectedness

The main ideas regarding cryptocurrency market characteristics were gathered and summarised by Almeida and Gonçalves (2023), one of them is interconnectedness (correlation, cointegration and causality effect of price movements between different cryptocurrencies) and connectedness with financial markets (the relationship between cryptocurrencies and traditional financial assets).

Most of the research focuses on the relationships between cryptocurrencies. There is no firmly established consensus regarding the level of interconnectedness among cryptocurrencies. According to Kostika and Laopodis (2019), correlations between cryptocurrencies are weak and do not present a common long-term path. However, there is also evidence suggesting that interconnectedness, as described by the spillover index (which measures the average percentage of variance explained by shocks transmitted across coins), varies between 25% and 75% among cryptocurrencies. This means that up to three-quarters of the variances can be explained by shocks transmitted across cryptocurrencies (Aslanidis, Bariviera, & Martínez-Ibañez, 2019). Similar results were obtained by Antonakakis, Chatziantoniou, and Gabauer (2019) using a time-varying parameter connectedness methodology (TVP-FAVAR model).

Furthermore, researchers have identified some causality relations among major cryptocurrencies (including Bitcoin, Litecoin, Ripple, Stellar, Monero, Dash, and Bytecoin) with evidence of volatility spillovers (Canh et al., 2019). Notably, the authors suggest that systematic fluctuations in prices spread from smaller cryptocurrencies (in market capitalization) to larger ones. This pattern may be attributed to the possibility that smaller cryptocurrencies are more susceptible to market manipulation. In addition, co-movements within the market are also observed, with Bitcoin leading its relationship with Dash, Monero, and Ripple, while Ethereum appears to lead its relationship with Bitcoin (Mensi, Al-Yahyaee, & Kang, 2019; Mensi, Lee, Al-Yahyaee, Sensoy, & Yoon, 2019; Mensi, Rehman, Al-Yahyaee, Al-Jarrah, & Kang, 2019).

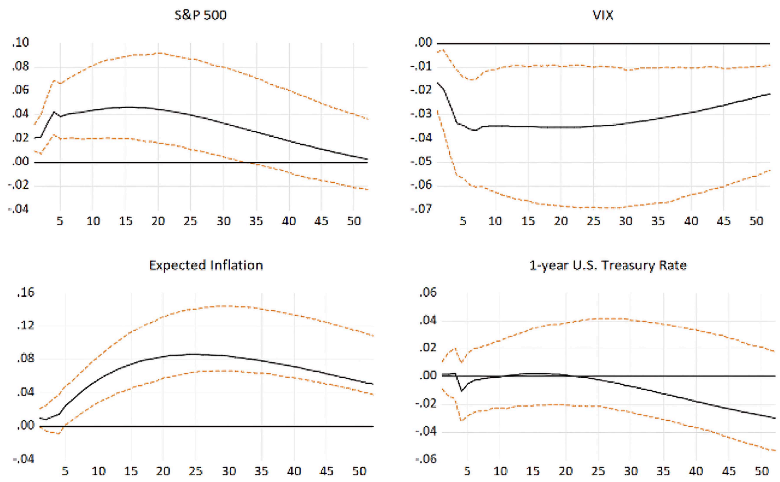
Importantly, research suggests that cryptocurrencies may be decoupled from traditional financial assets. Aslanidis et al. (2019) in their study demonstrate insignificant correlations between cryptocurrencies and assets like bonds, stocks, indices, and gold. Other studies, such as Corbet, Meegan, Larkin, Lucey, and Yarovaya (2018) and Gil-Alana, Abakah, and Rojo (2020), similarly find no cointegration between cryptocurrencies and traditional markets. However, during periods of uncertainty, such as the COVID-19 outbreak, the crypto market experiences a significant increase in connectedness with financial markets (with cryptocurrencies mostly acting as shock receivers), while interconnectedness within the crypto market itself also increases over short-term horizons (Ha & Nham, 2022; Kumar, Iqbal, Mitra, Kristoufek, & Bouri, 2022).

2.3.3. Challenges of integrating cryptocurrencies for inflation hedging

In the wake of the COVID-19 pandemic and the subsequent global inflationary surge, a key question emerged: can cryptocurrencies, given potential similarities to commodities, offer hedging or safe-haven properties?

In the research Choi and Shin (2022) utilize a Vector Autoregression (VAR) model to systematically analyze the relationship among inflation, uncertainty, Bitcoin and gold prices. The authors in Cholesky's decomposition arranged variables in a way that puts Bitcoin last[13], assuming that markets for trading gold are much larger, more established, and more liquid than Bitcoin. With the help of impulse response analysis (Figure 2.4), they find that Bitcoin prices increase significantly after positive inflation shocks, suggesting that Bitcoin could be a useful hedge against inflation. However, Bitcoin prices decline substantially in response to shocks to the VIX index (a measure of financial market volatility), indicating that Bitcoin is not a safe-haven asset. Interestingly, the results are the same for daily and weekly data frequency.

Figure 2.4.: The impulse response of Bitcoin (Choi & Shin, 2022, p. 4)



In the study Blau, Griffith, and Whitby (2021) conduct a variety of multivariate time-series tests to examine the lead-lag relation between Bitcoin and the Federal Reserve Bank of St. Louis 5-year forward inflation expectation rates across the pre and post COVID-19 periods. They find that changes in the price of Bitcoin Granger cause changes in the forward inflation rate without the opposite direction of causation. They then estimate a series of vector autoregressive equations (VAR(1), VAR(3), VAR(3)) and "impose an exogenous shock of the change in Bitcoin and estimate impulse response functions for changes in the forward inflation rate: accumulated impulse response functions become

[13] "Ordering a variable first" means assuming that it is not affected by the other variables contemporaneously and "ordering a variable last" means assuming that it does not affect the other variables contemporaneously.

positive shortly after the exogenous stock to the change in Bitcoin". Similar results were derived by Liu and Valcarcel (2024), but instead of Bitcoin and Ethereum time series, the authors concluded that BTC and ETH futures prices increase following exogenous and unanticipated increases in inflation expectations. However, they find a diametrically opposite dynamic during the turbulent latter part of 2022, a period with high uncertainty in cryptocurrency markets.

Figure 2.5.: Wavelet coherence - inflation expectations and Bitcoin 2010-2021 (Conlon et al., 2021, p. 2)

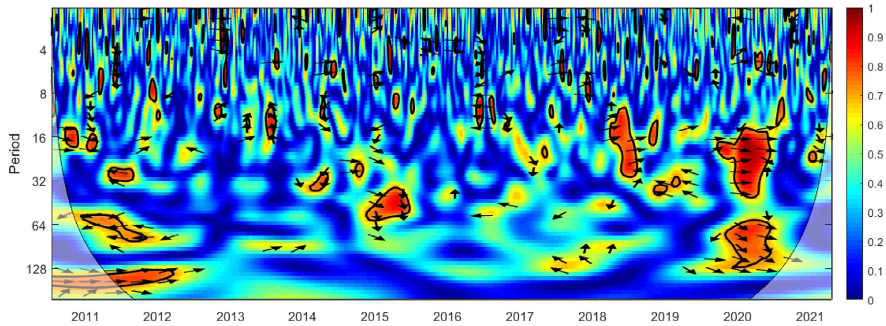
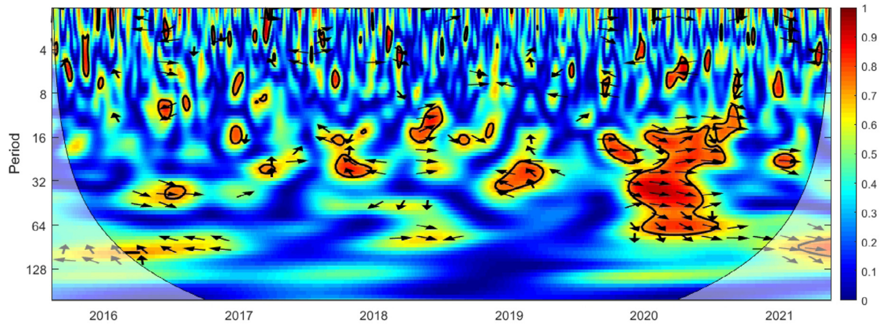


Figure 2.6.: Wavelet coherence - inflation expectations and Ethereum 2015-2021 (Conlon et al., 2021, p. 4)



The research by Conlon, Corbet, and McGee (2021) revisits the analysis of Blau et al. (2021) using wavelet time-scale analysis[14] to establish the dynamic relationship between forward inflation expectations and cryptocurrencies Bitcoin and Ethereum (Figure 2.5, 2.6. Even though Bitcoin has a limited supply (scarcity of supply) compared to unlimited in Ethereum, and therefore allegedly has more similarities with traditional commodities,

[14]Wavelet coherency is a localized measure of the correlation between inflation and the real returns on cryptocurrencies. Positive coherency is indicative of hedging potential (right-facing arrows).

the analysis shows similar outputs for both cryptocurrencies. The study shows the link between both Bitcoin and Ethereum and expected inflation, however, the authors find that any relationships are predominantly associated with a single point in time (hedging potential is limited outside of the crisis period).

The same results can be found in the research by Gaies, Chaâbane, Arfaoui, and Sahut (2024) proving again that Bitcoin and Ethereum provide effective coverage over both short-term (16-32 days) and long-term (64-128 days) and medium-term (16-64 days) investment horizons accordingly, and showing that ”cryptocurrency's effectiveness as an inflation hedge diminishes in bearish market conditions, and, therefore, proves unreliable as a hedge against financial instability”.

The paper by Sakurai and Kurosaki (2023) examines how the co-movement between cryptocurrencies (Bitcoin, Ethereum, and Litecoin) and U.S. inflation expectations has evolved pre and post COVID-19 times. They developed a comparatively new concept - exceedance co-kurtosis - to assess co-movement and asymmetry between cryptocurrencies and inflation expectations. The key findings highlight the fact that after the reopening of the U.S. economy after COVID-19, major cryptocurrencies became slightly better inflation hedges; the downside co-kurtosis (risk of simultaneous decline in cryptocurrencies and inflation expectations) decreased significantly. However, unconditional correlations between cryptocurrencies and inflation expectations remain low.

3. Data and Methods

This Chapter outlines the data selection process and preliminary analyses conducted on a pool of 87 cryptocurrencies. It describes the development of an index tracking model for inflation, introducing a sparsity component to manage portfolio complexity. The Chapter details the implementation of a sparse index tracking algorithm using the Majorization-Minimization (MM) framework to solve the non-convex optimization problem. It then explains the design of the tracking strategy, including the division of data into training, validation, and testing sets, and the creation of two distinct portfolio compositions. Finally, it discusses the execution of the strategy and the performance metrics used for evaluation.

3.1. Data and preliminary analyses

We download the historical data of the 5-year inflation expectation rate (BEIR, T5YIE) from FRED, the website of the Federal Reserve Bank of St. Louis. As stated before, the BEIR is derived as the yield spreads between 5-year US Treasury Constant Maturity Securities and 5-year US Treasury Inflation-Indexed Constant Maturity Securities. We have one sample period: from December 1, 2019 to April 30, 2024. The cryptocurrency price data is sourced from the Santiment API[1] in US Dollars yielding 1614 daily observations for each crypto asset, where the closed price and market capitalization metrics are obtained from different centralized and decentralized exchanges. We use a daily percent change in prices as a return for cryptocurrencies and a daily difference as a change for inflation expectation rates.

3.1.1. Crypto pre-selection

Prior to implementing our inflation hedging strategy, which employs sparsity hyperparameters for cryptocurrency selection as detailed in Section 3.2, we apply several pre-selection criteria to refine the initial pool of potential assets. While the cryptocurrency market

[1] https://api.santiment.net/

boasts over 9,000 options, many lack the liquidity required for practical investment, posing barriers to potential users and investors. To create a realistic portfolio, we narrowed our selection to the top 300 cryptocurrencies based on their market capitalization. This approach ensures that our chosen assets are readily tradable and represent a significant portion of the overall market activity.

To further refine our pool of potential crypto assets for inflation hedging, we apply a second pre-selection criterion: only cryptocurrencies that existed throughout the entire sample period from December 1st, 2019 to April 30th, 2024 are considered. This decision introduces survivorship bias, as it excludes cryptocurrencies that failed or underperformed during this timeframe. While this bias may limit the scope of our analysis, it ensures that the remaining assets have demonstrated a certain degree of resilience which is relevant to the inflation-hedging capabilities. This pre-selection process might also introduce a bias towards established cryptocurrencies, potentially excluding newer or less popular assets with promising inflation-hedging characteristics.

Additionally, we excluded stablecoins pegged 1-to-1 to fiat currencies, as well as wrapped coins designed to replicate existing crypto assets on different blockchains (i.e Wrapped-bitcoin). This decision was made because stablecoins, while relevant to the cryptocurrency ecosystem, do not exhibit the price fluctuations necessary for our inflation hedging analysis. Similarly, wrapped coins, being essentially derivatives of existing assets, do not introduce fundamentally new price dynamics to our dataset.

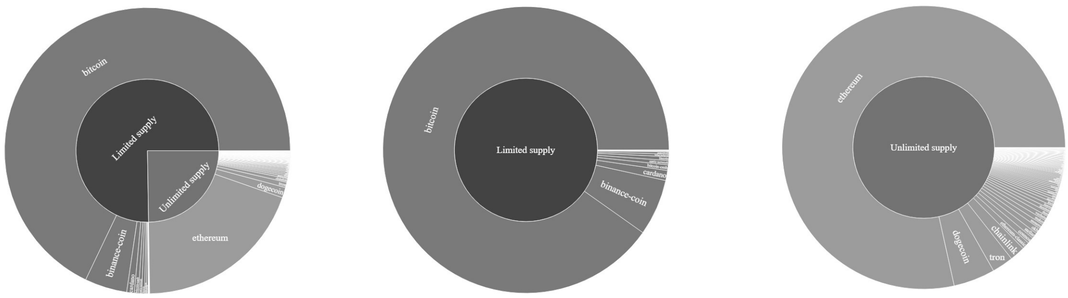
After applying these selection criteria, our final dataset comprises 87 cryptocurrencies. The complete list of these cryptocurrencies can be found in Appendix A.1. The data presented in the table, including market capitalization, volume, and price information, was collected on April 30, 2024, providing a snapshot of the cryptocurrency market at the end of our study period.

3.1.2. Capped vs unlimited supply

Despite utilizing over 80 cryptocurrencies in our pool of potential assets, Bitcoin and Ethereum collectively represent approximately 50-60% of the total cryptocurrency market capitalization. Not only do they dominate in terms of market value, but they are also more developed and exhibit greater market efficiency compared to other cryptocurrencies (Tran & Leirvik, 2020).

Moreover, these two leading cryptocurrencies represent distinct supply tokenomics: Bitcoin's supply is capped at 21 million coins (Nakamoto, 2009), highlighting a deliberate scarcity similar to precious metals like gold, while Ethereum supply is not predetermined and is influenced by factors like block rewards, transaction fees, and programmed mechanisms (Buterin, 2014). This parallel between Bitcoin's limited supply and the inherent scarcity of commodities has fuelled arguments for its potential as a long-term store of value and inflation hedge (Choi & Shin, 2022).

Figure 3.1.: Cryptocurrency supply distribution (USD): limited vs unlimited supply



(a) Total supply (b) Limited Supply (c) Unlimited supply

Therefore, the critical question emerges: do the time series properties of cryptocurrencies depend on the nature of their supply? Does the presence or absence of a maximum supply cap fundamentally alter their price behaviour and volatility patterns? This distinction could have significant implications for future portfolio construction since it is possible to reduce the potential pool of crypto assets to those that only have a fixed supply like Bitcoin. To investigate this question empirically, we examine whether the statistical properties of the time series of cryptocurrencies with limited and unlimited supply are significantly different.

Since Bitcoin and Ethereum are the primary representatives of capped and unlimited supply tokenomics, respectively, we utilize their time series data (returns) to conduct a Mann-Whitney U test. This non-parametric test allows us to assess whether there are statistically significant differences in their time series dynamics, even if the underlying distributions are not normal.

The Mann-Whitney U test evaluates the null hypothesis that the distributions of two independent samples (Bitcoin and Ethereum returns) are equal. The U test statistic is calculated as follows:

$$U_{\mathrm{BTC}} = n_{\mathrm{BTC}} n_{\mathrm{ETH}} + \frac{n_{\mathrm{BTC}}(n_{\mathrm{BTC}}+1)}{2} - R_{\mathrm{BTC}}$$

, where n_{BTC} is the sample size of Bitcoin returns, n_{ETH} is the sample size of Ethereum returns, R_{BTC} is the sum of ranks for Bitcoin returns. A corresponding test statistic

U_{ETH} for Ethereum can be calculated using $U_{\mathrm{ETH}} = n_{\mathrm{BTC}} n_{\mathrm{ETH}} - n_{\mathrm{BTC}}$.

According to Table 3.1, both cryptocurrencies exhibit significant skewness and/or kurtosis, suggesting that their returns do not follow a normal distribution (p-value less than 5% for both Bitcoin and Ethereum rejects the null hypothesis of normality of the data). Therefore, these results justify using a Mann-Whitney U test, which does not require normal distribution.

Table 3.1.: Jarque-Bera test results

Statistic	**Limited supply (Bitcoin)**	**Unlimited supply (Ethereum)**
Jarque-Bera Statistic	84.5693	50.4028
p-value	0.0000	0.0000

Table 3.2.: Mann-Whitney U test results

Statistic	**Value**
U Statistic	1302113.0000
p-value	0.4962

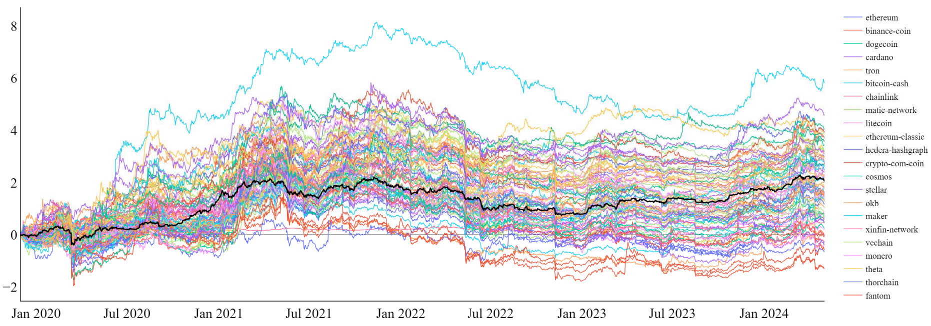
Table 3.2 presents the results of the Mann-Whitney U test comparing the time series of two cryptocurrencies: Bitcoin (limited supply) and Ethereum (unlimited supply). The calculated U statistic is 1302113.0000, with a corresponding p-value of 0.4962. As this p-value exceeds the conventional significance level of 5%, we fail to reject the null hypothesis. This outcome indicates no statistically significant difference between the time series of cryptocurrencies with limited and unlimited supply, suggesting that their distributions are similar in terms of the ranks of their values. Based on these findings, it appears unnecessary to filter cryptocurrencies from the potential pool of assets based on their supply tokenomics

3.1.3. Overall price dynamics

To visually assess the information regarding the latest price dynamics of the cryptocurrencies that are used in our pool of assets (87), we present the line graph (Figure 3.2) with the timeline from December 2019 to April 2024. Prices are normalized to the same starting point for comparative analysis, and a logarithmic scale is used to effectively capture relative changes over time.

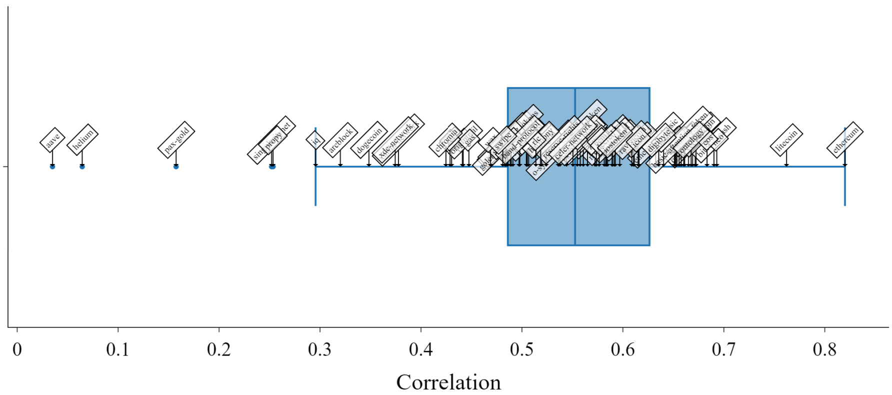
At first glance, a general trend of co-movement is observed among most cryptocurrencies, with noticeable fluctuations in early 2021 and early 2024. Bitcoin's trajectory, highlighted

Figure 3.2.: Overall price dynamics of cryptocurrencies



by a thicker black line, stands out amidst these broader market trends. However, a closer look reveals nuanced differences in the performance of individual cryptocurrencies. Notably, Helium (top line in April 2024) demonstrates unique pattern that diverge from the overall market, suggesting that while cryptocurrencies may be interconnected, their price movements can also be influenced by distinct factors. This graph clearly highlights the importance of considering a wide range of cryptocurrencies, beyond just Bitcoin and Ethereum, to make well-informed investment decisions based on both general trends and specific asset behaviours.

Figure 3.3.: Correlation between Bitcoin and other cryptocurrenicies



We continue the analysis with the following box plot (Figure 3.3) illustrating the distribution of correlation coefficients of daily returns between Bitcoin and the remaining cryptocurrencies in our sample (from December 2019 to April 2024). Each point on the plot represents a specific cryptocurrency, with its position along the x-axis determined by its correlation coefficient with Bitcoin.

While the majority of cryptocurrencies exhibit a positive correlation with Bitcoin, the strength varies significantly. Notably, a few outlier cryptocurrencies display minimal correlation with Bitcoin, showcasing some potential diversification opportunities. Surprisingly, no negative correlation was detected. The median correlation of 0.5531 suggests a moderate positive relationship between most cryptocurrencies and Bitcoin. However, the broad interquartile range (IQR = Q3 - Q1 = 0.6250 - 0.4846 = 0.1404) of 0.1404 indicates substantial low variability in this relationship. But overall, diversifying across different cryptocurrencies may not reduce portfolio risk significantly due to the high positive correlations.

In addition to the general interconnectedness analysis, we conduct a cointegration analysis using the Engle-Granger two-step procedure (Engle & Granger, 1987). This analysis allows us to examine whether two non-stationary time series share a common long-term trend, indicating that their prices tend to move together over time, even if they deviate in the short term. The Engle-Granger procedure evaluates the null hypothesis that there is no cointegration between the two time series. The test involves two main steps:

Step 1: Estimate the long-run equilibrium relationship and rearrange the linear model:

$$\varepsilon_t = \beta_0 + \beta_1 x_t - y_t$$

, where y_t and x_t are the two time series being tested, β_0 is the intercept, β_1 is the coefficient, and ε_t is the error term.

Step 2: Test the residuals (ε_t) for stationarity using an Augmented Dickey-Fuller (ADF) test: if the residuals ε_t are found to be stationary (i.e., integrated of order zero, $I(0)$), then we conclude that the variables y_t and x_t are cointegrated. This implies that the relationship $y_t - \beta_0 - \beta_1 x_t$ is stationary, indicating a long-run equilibrium relationship between y_t and x_t.

Table 3.3.: Cointegration with Bitcoin test results

Cryptocurrency	**Index**	**t_statistic**	**p_value**
maker	1	−3.7022	0.0182
aragon	2	−3.6183	0.0233
aelf	3	−3.4045	0.0418

Continued on next page

Table 3.3 – *Continued from previous page*

Cryptocurrency	**Index**	**t_statistic**	**p_value**
origintrail	4	−3.3945	0.0430
rlc	5	−3.2104	0.0682
tron	6	−2.8293	0.1565
dogecoin	7	−2.7894	0.1691
thorchain	8	−2.6960	0.2010
ethereum-classic	9	−2.6067	0.2341
propy	10	−2.5714	0.2485
	...		
kava	80	−0.7987	0.9349
band-protocol	81	−0.7830	0.9369
tezos	82	−0.7729	0.9382
reserve-rights	83	−0.5053	0.9639
decred	84	−0.4936	0.9648
zilliqa	85	−0.2911	0.9763
icon	86	−0.2440	0.9783

Table 3.3 presents the snapshot of the results of the Engle-Granger cointegration tests between Bitcoin and other cryptocurrencies[2]. Out of the 86 cryptocurrencies analysed, only five exhibit a p-value below 10% significance level, demonstrating a statistically significant long-term relationship with Bitcoin's price movements (Maker, Aelf, Origintrail, Aragon, and Rlc).

To sum up all the above, we can highlight the following points stemming from our multiple hypothesis testing and analysis:

- Initial observation of the line graph (Figure 3.2) reveals that many cryptocurrencies exhibit co-movement with Bitcoin. Furthermore, analysis of daily returns shows positive and moderate correlations between Bitcoin and other cryptocurrencies, with a median correlation of 0.5531. This low variability in correlation coefficients suggests that diversifying across various cryptocurrencies may be problematic.

- While many cryptocurrencies showed a positive correlation with Bitcoin, implying they tend to move in the same direction in the short term, only five cryptocurrencies

[2] A complete list of cointegration results can be found in Appendix A.2.

(Maker, Aelf, Origintrail, Aragon, and Rlc) exhibit cointegration at the 10% significance level. This indicates that fewer cryptocurrencies share a long-term trend with Bitcoin, even though short-term relationships are quite strong for most of them.

These findings warrant further investigation into the specific nature of these relationships, considering both short-term correlations and potential long-term cointegration, as those relationships may significantly influence the design of trading strategies and portfolio construction.

3.1.4. Summary statistics and data transformation

In Appendix A.3 you can find summary statistics for the inflation expectation rate (BREIR, T5YIE) and all 87 cryptocurrencies used in the research. A quick overview of the analysis is presented in Table 3.4. The data reveals substantial volatility in each series, which is relatively unusual compared to more traditional financial assets such as stocks and commodities.

Table 3.4.: Summary statistics

Variable	Index	Count	Mean (%)	Std (%)	Min (%)	Max (%)
Inflation						
T5YIE		1105	2.22	0.61	0.14	3.59
T5YIE (difference)		1104	0.00	0.05	−0.34	0.19
Returns						
bitcoin	1	1613	0.19	3.43	−36.57	18.75
ethereum	2	1613	0.29	4.44	−42.17	25.95
binance-coin	3	1613	0.34	4.94	−42.38	69.76
dogecoin	4	1613	0.61	11.40	−40.26	355.57
cardano	5	1613	0.29	5.28	−39.58	32.24
			...			
wax	80	1613	0.29	7.09	−51.37	112.41
storj	81	1613	0.37	7.88	−44.76	102.38
band-protocol	82	1613	0.40	8.17	−44.18	132.50
celer-network	83	1613	0.37	7.42	−52.20	44.95
swipe	84	1613	0.30	10.18	−69.90	280.22
digibyte	85	1613	0.23	6.51	−41.43	56.81

Continued on next page

Table 3.4 – *Continued from previous page*

Cryptocurrency	Index	Count	Mean (%)	Std (%)	Min (%)	Max (%)
iostoken	86	1613	0.22	6.22	−46.58	69.62
iq	87	1613	0.36	7.68	−46.84	103.03

Applying the Augmented Dickey-Fuller (ADF) test to the cryptocurrency prices and inflation expectation rate suggests the presence of a unit root, indicating nonstationarity, a common characteristic of time series data (Table 3.5)[3]. In our case, performing Pearson correlation analyses, Ordinary Least Squares (OLS) regressions, and tracking error minimization procedures based on nonstationary data (prices) can be incorrect or deliver misleading results. For instance, calculating Pearson correlation coefficients using raw price data instead of returns could lead to inflated correlations. This is because two nonstationary series often appear highly correlated even if they are fundamentally unrelated, simply due to their shared trend components. Furthermore, our tracking error optimization model based on nonstationary time series may violate a crucial assumption of constant mean and variance of time series, leading to suboptimal portfolio construction.

Table 3.5.: ADF test results for cryptocurrencies

Variable	Index	ADF statistics before (p-value)	ADF statistics after (p-value)
Inflation			
T5YIE		-1.71 (0.43)	-29.13 (0.00)
Cryptocurrency			
bitcoin	1	-1.51 (0.53)	-18.76 (0.00)
ethereum	2	-1.70 (0.43)	-12.07 (0.00)
binance-coin	3	-1.51 (0.53)	-10.22 (0.00)
dogecoin	4	-2.78 (0.06)	-6.89 (0.00)
cardano	5	-1.88 (0.34)	-10.34 (0.00)
		...	
wax	80	-2.37 (0.15)	-16.37 (0.00)
storj	81	-2.41 (0.14)	-14.55 (0.00)
band-protocol	82	-2.13 (0.23)	-12.57 (0.00)

Continued on next page

[3]The full version of the ADF test can be found in Appendix A.4.

Table 3.5 – *Continued from previous page*

Cryptocurrency	**Index**	**ADF statistics before (p_value)**	**ADF statistics after (p_value)**
celer-network	83	-2.31 (0.17)	-12.85 (0.00)
swipe	84	-2.53 (0.11)	-9.73 (0.00)
digibyte	85	-2.40 (0.14)	-7.79 (0.00)
iostoken	86	-2.18 (0.21)	-12.96 (0.00)
iq	87	-2.43 (0.13)	-25.09 (0.00)

Additional analysis reveals that taking the first differences of the variables for inflation expectation and calculating returns for cryptocurrencies successfully removes the nonstationarity. Consequently, the following transformations were applied to each series: For cryptocurrency prices:

$$r_t = \frac{P_t}{P_{t-1}} - 1$$

For inflation expectation rate:

$$\Delta T5YIE_t = T5YIE_t - T5YIE_{t-1}$$

, where P_t represents the price of a cryptocurrency at time t, and $T5YIE_t$ denotes the inflation expectation rate at time t. These transformations ensure that the time series are stationary, which is essential for valid statistical analysis and modelling.

3.2. Model development

3.2.1. Minimizing tracking error: a passive approach

Portfolio optimization is a crucial aspect of investment management, aiming to construct a portfolio that maximizes returns while minimizing risk. In this study, our goal is to minimize the discrepancy between the portfolio's returns and changes in inflation expectations, represented by the 5-year break-even inflation rate (BEIR, T5YIE). For this purpose, we use Tracking Error (TE), which is a measure of how closely a portfolio follows its benchmark index. Minimizing TE is essential for investors seeking to replicate the performance of a specific index or asset. In our case, inflation is going to be the benchmark that we aim to replicate by constructing an optimal portfolio consisting of cryptocurrencies. This requires selecting an appropriate measure of tracking error and understanding the implications of active versus passive management strategies. Different approaches to measuring tracking error exist:

Empirical Tracking Error (ETE) or Mean Squared Tracking Error (MSTE). The ETE or MSTE is defined as the mean of the squared tracking errors over the given time periods. Mathematically, it can be expressed as:

$$ETE(w) = \frac{1}{T} \|Xw - r_{idx}\|_2^2$$

, where T is the number of time periods, X is the matrix of the asset returns, w is the vector of the portfolio weights and r_{idx} is the vector of the index returns (benchmark).

Root Mean Square Tracking Error (RMSTE). The RMSTE is essentially the square root of the MSTE. It is defined as:

$$RMSTE(w) = \sqrt{\frac{1}{T} \|Xw - r_{idx}\|_2^2}$$

Tracking Error Variance (TEV). The TEV is defined as the variance of the tracking errors over the given time period. Mathematically, it can be expressed as:

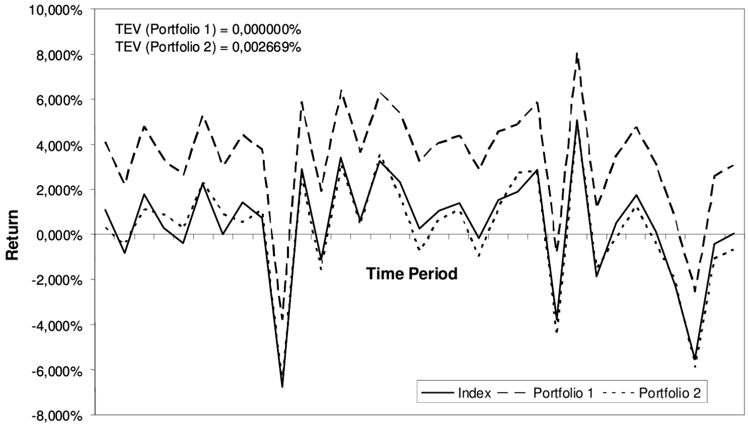
$$TEV(w) = \mathrm{Var}(Xw - r_{idx})$$

While ETE and RMSTE are closely related (RMSTE is a rescaled version of ETE, making it more interpretable, but essentially providing the same information), they are different from TEV in what they measure and how they assess the tracking performance. ETE and RMSTE are about the average size of errors, while TEV is about the consistency or variability of those errors.

Choosing ETE as a main component of the objective function for portfolio optimization purposes, given the context of our study, is preferable due to its simplicity, interpretability, and desirable mathematical properties. Moreover, selecting ETE over TEV aligns with our passive investment strategy, as it focuses on 1-to-1 inflation replication rather than constructing a portfolio that aims to consistently outperform the benchmark. In his paper Roll (1992) notes that optimizing for excess returns leads to the undesirable outcome of the active portfolio having a systematically higher risk than the benchmark.

Additionally, TEV suffers from a significant limitation known as the shift problem, as highlighted by Beasley, Meade, and Chang (2003). This issue occurs when the returns of the tracking portfolio are consistently different from the index returns by a constant factor. In such cases, the tracking error remains constant, and the TEV becomes zero, even though there is a persistent deviation from the index. As a result, TEV fails to accurately capture the true extent of the discrepancy between the portfolio and the benchmark, potentially leading to misleading assessments of tracking performance. This limitation is demonstrated in Figure 3.4, adapted from Roßbach and Karlow (2011), which showcases a scenario where a portfolio with inferior tracking quality has a TEV equal to zero.

Figure 3.4.: Weakness of the TEV (Roßbach and Karlow, 2011, p. 8)



Several other tracking error measures exist alongside the empirical tracking error and the tracking error variance. These include the mean absolute tracking error, downside risk relative to an index, etc. It is crucial to recognize that the choice of tracking error measure significantly impacts the solution to the index tracking optimization problem, resulting in the construction of different index tracking portfolios with distinct characteristics.

Minimizing tracking error as an objective function of portfolio optimization problem is similar to Markowitz (1952) mean-variance model approach, but instead of finding the optimal trade-off between return and risk, the goal is to find an optimal trade-off between tracking error and sparsity (Section 3.2.3). Consequently, minimizing the tracking error does not necessarily lead to an efficient portfolio in the Markowitz sense, as it does not explicitly consider the trade-off between risk and return and does not lie on the efficient frontier, in general. For instance, a portfolio with minimal tracking error might exhibit lower volatility relative to the benchmark, but this does not necessarily imply that it has an optimal combination of risk and return (does not lie on the Markowitz's Efficient Frontier)[4].

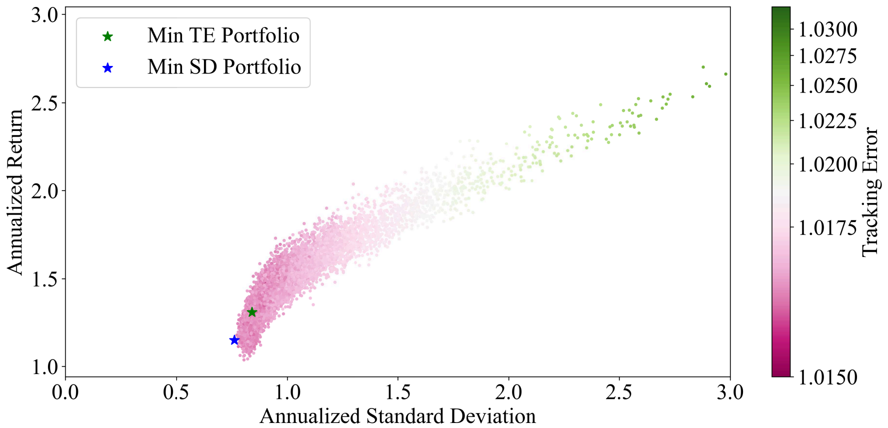
This problem of suboptimality of tracking error portfolio was extensively studied by Roll (1992) and elaborated by Jorion (2004), where they both establish that portfolios

[4]The concept introduced by Markowitz visually depicts a range of investment combinations that maximize potential gains for specific risk levels.

constructed solely to minimize tracking error, without considering overall portfolio risk, are generally not mean-variance efficient. Jorion (2004) further elaborated on Roll's concept, demonstrating that portfolios with a constant level of tracking error volatility lie on an ellipse in the mean-variance space, implying that adding constraints on the absolute portfolio risk can significantly enhance the performance of tracking error-constrained portfolios.

The main difference between the authors' findings and the current research lies in the objective function. While they study the effect of minimizing the tracking error volatility (TEV), we focus on the empirical tracking error (ETE), which can be perceived as a special case of TEV in terms of volatility. Furthermore, the benchmark in Roll's and Jorion's research consists of the same assets as the replication portfolio, whereas our benchmark is inflation, a macroeconomic index that cannot be fully replicated due to its inherent structure. Despite these differences, the fundamental principles and insights regarding the suboptimality of tracking error portfolios remain applicable to our research. The following graph (Figure 3.5) illustrates how a portfolio constructed solely to minimize ETE may not be optimal in terms of risk and return, as there may exist other portfolios that offer better risk-adjusted performance.

Figure 3.5.: Risk-return profile of cryptocurrency portfolios with tracking error colouring



For illustration purposes, we use the daily returns of 87 cryptocurrencies from December 2019 to July 2020 and plot the annualized return and standard deviation of 10,000 randomly generated portfolios with different weights. Additionally, we calculate each portfolio's tracking error against a randomly generated benchmark return time series with a mean of 0 and a standard deviation of 1. The tracking error is represented by the colour of the points, where purple indicates a smaller tracking error and green indicates a comparatively larger tracking error. The portfolio with the lowest standard deviation is marked with a blue star, while the portfolio with the smallest tracking error is marked with a green star. As we can see, the portfolio with the smallest tracking error does not lie on the efficient frontier, demonstrating that, in general, minimizing the tracking error objective is not efficient under the Markowitz mean-variance approach. For our purposes, prioritizing minimal tracking error is justified by the need for a reliable inflation hedge, even if it means sacrificing some elements of mean-variance optimization.

3.2.2. Index tracking model for inflation

Model: Minimizing Empirical Tracking Error (ETE)

In constructing an index tracking model for inflation, the primary objective is to minimize the Empirical Tracking Error (ETE) between the portfolio returns and the returns of an inflation index. This approach ensures that the constructed portfolio replicates the behaviour of the inflation index as closely as possible.

The optimization problem can be written as:

$$\begin{aligned} \min_{w} \quad & \frac{1}{T}\|Xw - r_{inf}\|_2^2 \\ \text{s.t.} \quad & w^T 1 = 1 \\ & w_i \geq 0 \quad \forall i \end{aligned} \tag{3.1}$$

, where T is the number of time periods, X is the matrix of the crypto asset returns, w is the vector of the portfolio weights and r_{inf} is the vector of the inflation expectations returns.

With this optimization approach, a particular challenge arises: how to manage a large number of crypto assets in our potential pool (87). We address this issue by introducing a sparsity component into our index tracking model. This approach aims to create more efficient portfolios by including only the most relevant assets, thereby reducing complexity. The next section explores the implementation of sparsity regularization, examining how it balances tracking error minimization with portfolio efficiency.

3.2.3. Sparse index tracking algorithm and updated index tracking model

Problem formulation

In this section we continue to develop our inflation-tracking strategy. As mentioned in Section 3.2.2, we encounter a significant challenge: managing portfolio complexity while maintaining effectiveness. Not all assets may contribute equally to tracking inflation expectations, and including too many can lead to increased transaction costs and management difficulties. To address this, we incorporate a sparsity element into our index tracking model, following the approach developed by Benidis, Feng, and Palomar (2018):

$$\begin{aligned} \min_{w} \quad & \frac{1}{T}\|Xw - r_{inf}\|_2^2 + \lambda\|w\|_0 \\ \text{s.t.} \quad & w^T 1 = 1 \\ & w_i \geq 0 \quad \forall i \end{aligned} \tag{3.2}$$

, where $\|w\|_0$ is the ℓ_0-norm and denotes card(w)[5] and λ is a hyperparameter that is responsible for portfolio sparsity.

However, the ℓ_0-norm presents significant computational challenges due to its discontinuous, non-differentiable, and non-convex nature. These properties make it difficult to optimize directly using standard mathematical techniques.

ℓ_0-norm approximation

To address this issue, researchers have widely adopted the ℓ_1-norm as a convex approximation. This approach, known as ℓ_1 regularization or LASSO (Least Absolute Shrinkage and Selection Operator), has proven highly effective in various fields, including signal processing, machine learning, and statistics. However, when it comes to portfolio optimization, the ℓ_1-norm approximation encounters significant limitations. The primary issue stems from the nature of portfolio weights: unlike many other optimization problems, portfolio weights are constrained to sum to one ($w^T 1 = 1$), resulting ℓ_1-norm of the weight vector becomes a constant, since

$$\|w\|_1 = \sum_{i=1}^{N} |w_i| = \sum_{i=1}^{N} w_i = 1$$

, regardless of the distribution of weights across assets. As a result, the ℓ_1-norm loses its sparsity-inducing property.

Following the work of Benidis et al. (2018), we use approximation function to the ℓ_1-norm given by

$$\rho_p(w) = \frac{\log\left(1 + \frac{|w|}{p}\right)}{\log\left(1 + \frac{1}{p}\right)}$$

, where parameter p controls the approximation of the function $\rho_p(w)$ $(0 < p \ll 1)$[6].

[5] Measures the number of non-zero elements in a vector.

[6] In our research we use a constant approximation p = 0.3.

With this approximation of ℓ_0-norm, the inflation tracking optimization problem transforms to:

$$\min_{w} \quad \frac{1}{T}\|Xw - r_{inf}\|_2^2 + \lambda 1^T \rho_p(w) \tag{3.3}$$

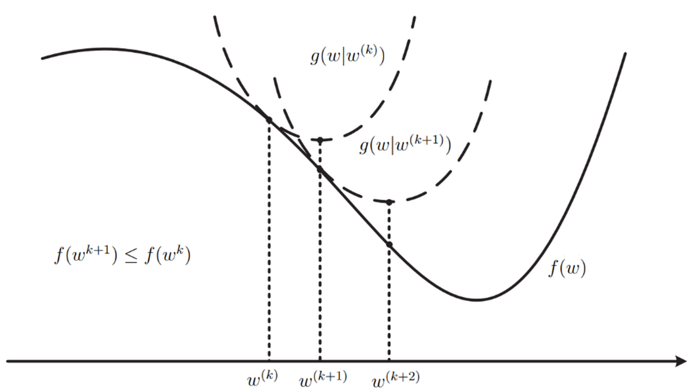
, where $\rho_p(w) = [\rho_p(w_1), \rho_p(w_2), \ldots, \rho_p(w_N)]^T$.

As stated by the authors, this problem remains non-convex because the function $\rho_p(w)$ is concave for $w \geq 0$, but now it is continuous and differentiable.

The Majorization-Minimization (MM) Framework

Despite introducing the ℓ_0-norm approximation function $\rho_p(w)$ into our inflation-tracking optimization problem, the challenge of directly optimizing our index tracking model still persists due to the non-convex nature of the objective function. Specifically, the sparsity term $\lambda 1^T \rho_p(w)$, where $\rho_p(w)$ introduces concavity into the otherwise convex quadratic tracking error term. This results in a non-convex optimization problem, which presents significant computational difficulties. Non-convex functions may have multiple local minima, saddle points, and complex optimization landscapes that standard convex optimization techniques cannot overcome. Consequently, direct optimization methods may converge to suboptimal local minima or fail to converge altogether, necessitating more sophisticated approaches like the Majorization-Minimization (MM) framework (Dempster, Laird, & Rubin, 1977; Hunter & Lange, 2004; Sun, Babu, & Palomar, 2017) to effectively address this complex optimization problem.

Figure 3.6.: The MM procedure



The MM algorithm, illustrated in Figure 3.6, is a method designed to offer an elegant solution by iteratively solving a series of simpler problems. The fundamental idea behind

MM is straightforward: rather than minimizing the original, complex objective function $f(w)$, the MM approach constructs and minimizes a surrogate function $g(w|w^{(k)})$ at each iteration k.
Before: complex optimization problem:

$$\min_{w} f(w) \quad \text{s.t.} \quad w \in W$$

After: create a manageable surrogate function to minimize:

$$w^{(k+1)} = \min_{w \in W} g(w|w^{(k)})$$

, aiming the sequence of minimizers $\{w^{(k+1)}\}$ to converge to the optimal w^*.

This surrogate function has several key properties:

- It is tangent to $f(w)$ at the current point $w^{(k)}$: $g(w^{(k)} \mid w^{(k)}) = f(w^{(k)})$
- It serves as an upper bound for $f(w)$: $g(w \mid w^{(k)}) \geq f(w)$
- The equality of gradients: $\nabla g(w^{(k)} \mid w^{(k)}) = \nabla f(w^{(k)})$

The core principle of the MM algorithm is captured in the following inequality, where the sequence $\{f(w^{(k)})\}$ generated by MM is non-increasing (Benidis et al., 2018):

$$f(w^{(k+1)}) \leq g(w^{(k+1)} \mid w^{(k)}) \leq g(w^{(k)} \mid w^{(k)}) = f(w^{(k)})$$

Finding an appropriate surrogate function $g(w \mid w^{(k)})$ can be challenging since there is no systematic method for constructing them. The derivation of these functions heavily depends on the structure and unique characteristics of each specific problem.

Algorithm LAIT and updated index tracking model

In our specific case, the MM algorithm allows us to transform our non-convex optimization problem into a series of convex subproblems. This approach is particularly beneficial for handling the ℓ_0-norm approximation, as it provides a computationally efficient method to navigate the complex landscape of our objective function while ensuring convergence to a local optimum. We continue optimizing the index tracking model for inflation following the procedure described in "Optimization Methods for Financial Index Tracking: From Theory to Practice" by Benidis et al. (2018) and principles outlined in the previous subsection.

Step 1: Constructing the majorization function.

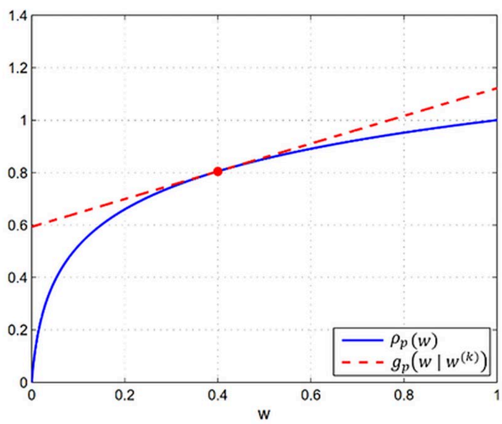
To implement the MM algorithm, we begin by deriving a majorization function for $\rho_p(w)$. This surrogate function $g_p(w \mid w^{(k)})$ serves as an upper bound for $\rho_p(w)$ and is obtained using the first-order Taylor expansion:

$$\begin{aligned}
\rho_p(w) &= \frac{\log(1+w/p)}{\log(1+1/p)} \\
&\leq \frac{1}{\log(1+1/p)}\left[\log\left(1+\frac{w^{(k)}}{p}\right)+\frac{1}{p+w^{(k)}}(w-w^{(k)})\right] \\
&= d_p(w^{(k)})w + c_p(w^{(k)})
\end{aligned}$$

Thus, our ℓ_0-norm approximation function $\rho_p(w)$ with $w \geq 0$ is upperbounded by the surrogate function $g_p(w \mid w^{(k)})$ at the point $w^{(k)}$, where $d_p(w^{(k)})$ and $c_p(w^{(k)})$ are constant:

$$\begin{aligned}
g_p(w \mid w^{(k)}) &= d_p(w^{(k)})w + c_p(w^{(k)}), \\
d_p(w^{(k)}) &= \frac{1}{\log(1+1/p)(p+w^k)} \\
c_p(w^{(k)}) &= \frac{\log(1+w^{(k)}/p)}{\log(1+1/p)} - \frac{w^{(k)}}{\log(1+1/p)(p+w^{(k)})}
\end{aligned}$$

Figure 3.7.: Approximation function to the ℓ_0-norm $\rho_p(w)$. Its majorization function $g_p(w \mid w^{(k)})$ plotted for $w = 0.4$ (Palomar, 2021, p.28)



This process is illustrated in Figure 3.7, where the $\rho_p(w)$ function (blue line) is upper-bounded by its majorization (surrogate) function $g_p(w \mid w^{(k)})$ (red dotted line)(Palomar, 2021).

Step 2: Updating our final convex optimization problem.

After discarding the constant $c_p(w^{(k)})$ in every $(k+1)$-th iteration, we need to solve the final convex optimization problem, which can be solved with the preferred solver[7].

$$\begin{aligned} \min_{w} \quad & \frac{1}{T}\left\|Xw - r_{\text{inf}}\right\|_2^2 + \lambda d_p^{(k)} w \\ \text{s.t.} \quad & w^T 1 = 1 \\ & w_i \geq 0 \quad \forall i \end{aligned} \tag{3.4}$$

, where $d_p^{(k)} = \left[d_p(w_1^{(k)}), \ldots, d_p(w_N^{(k)})\right]$.

The following Algorithm 1 outlines the key steps of the procedure (Linear Approximation for Index Tracking problem):

Algorithm 1 LAIT - Linear Approximation for Index Tracking problem (Benidis et al., 2018, p. 45)

1: Set $k = 0$, initialize $\mathbf{w}^{(0)} \in \mathcal{W}$
2: **repeat**
3: Compute $\mathbf{d}_{p,1}^{(k)}$
4: Solve the Final Convex Optimization Problem and set the solution as $\mathbf{w}^{(k+1)}$
5: $k \leftarrow k + 1$
6: **until** convergence
7: **return** $\mathbf{w}^{(k)}$

This algorithm iteratively refines the portfolio weights w by computing the approximation vector $d_p^{(k)}$ and solving a convex optimization problem at each step. The process continues until convergence, typically when the change in portfolio weights between iterations becomes negligibly small[8]. The final output $w^{(k)}$ represents the optimized sparse portfolio weights that aim to track the index while satisfying the sparsity constraints. This approach balances the trade-off between tracking accuracy and portfolio simplicity, making it particularly useful for practical implementation in financial markets with numerous potential assets.

[7]In this research we use Quadratic Programming Solver.
[8]Conversion threshold in our research is 1e-9.

3.3. Design of tracking strategy

3.3.1. Training, validation and testing

To develop a dynamically adjusted portfolio capable of tracking inflation expectations, we employed a comprehensive approach, dividing our sample into distinct training, validation, and testing sets using a rolling window methodology. This strategic segmentation of data allows for robust model development, validation, and out-of-sample testing, ensuring the reliability and adaptability of our inflation-tracking strategy. The tracking timeline is illustrated in Figure 3.8 for a rolling window of 180 days and Figure 3.9 for a rolling window of 120 days.

Figure 3.8.: Arrangement for training, validation and testing sets during the whole sample period (for a rolling window of 180 days). Part I

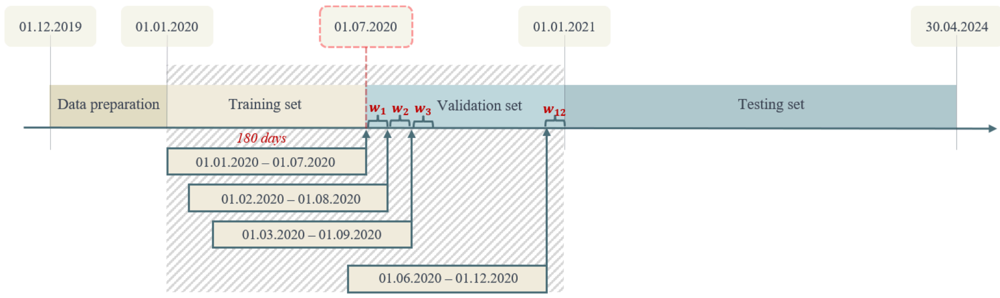
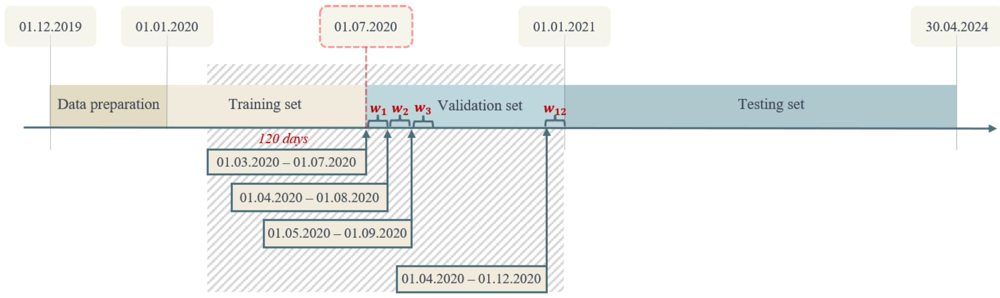


Figure 3.9.: Arrangement for training, validation and testing sets during the whole sample period (for a rolling window of 120 days). Part I



Our Data preparation phase starts on December 1, 2019, and lasts 1 month. While this phase does not significantly impact the design of our inflation tracking strategy, it serves a crucial purpose. This period ensures that all necessary calculations (i.e. monthly data returns), are fully computed and available at the onset of the Training period on January 1, 2020, guaranteeing a smooth transition into the model development phase.

The Training set, spanning from January 1, 2020, to July 1, 2020 (six full months), is reserved for the stable calculation of portfolio weights using different rolling windows (RW): 120-day, 150-day, and 180-day. This approach allows us to assess the impact of varying historical data lengths on our model's performance. The inflation tracking model which takes the returns of the cryptocurrencies as input is then trained on the training set to obtain the assets' weights. It is constructed in such a way that on the first day of the Validation period (July 1, 2020), we generate our first necessary subset of components for the three RWs considered.

We implement a monthly rebalance (30-day) approach within our analysis framework for the entire timeline. Consequently, the Training set used to train the optimization model for selecting asset weights shifts forward every time we perform a 30-day rebalance. As illustrated in Figure 3.9, which examines a 120-day rolling window, the first training period for the model spans from 01.03.2020 to 01.07.2020. This allows us to generate the first model output on July 1, 2020, aligning with our strategy design. Due to the rebalancing approach, subsequent training periods are adjusted accordingly. For instance, the second training period shifts to 01.04.2020 through 01.08.2020 and so on. This technique continuously updates the generated weights as we progress through time, ensuring that our model remains responsive to the most recent market trends.

The Validation set, extending from July 1, 2020, to December 31, 2020, plays a crucial role in fine-tuning our model. Using the weights generated in the Training set for each rolling window and monthly rebalance, we vary our asset sparsity hyperparameter lambda values from 0 to 1e-5 (specifically: 0, 1e-8, 1e-7, 1e-6, and 1e-5) to capture its influence on portfolio performance. We choose the lambda that minimizes the Empirical Tracking Error (ETE) while also taking into consideration transaction costs[9]. The sparsity parameter lambda may directly affect the turnover and transaction costs of a strategy. In general, higher lambda values lead to fewer assets being selected (due to fewer trades) and thus lower turnover, which could lower transaction costs but might also increase the tracking error[10]. Therefore, our aim in the Validation period is to identify the optimal level of sparsity that balances tracking accuracy with practical considerations of transaction costs.

[9]Transaction costs are calculated as 1% of the dollar amount of executed orders relative to the final portfolio value at the end of a period.

[10]This only holds if the majority of assets chosen in one rebalancing period are sequentially chosen in the next rebalancing period.

Figure 3.10.: Arrangement for training, validation and testing sets during the whole sample period (for a rolling window of 180 days). Part II

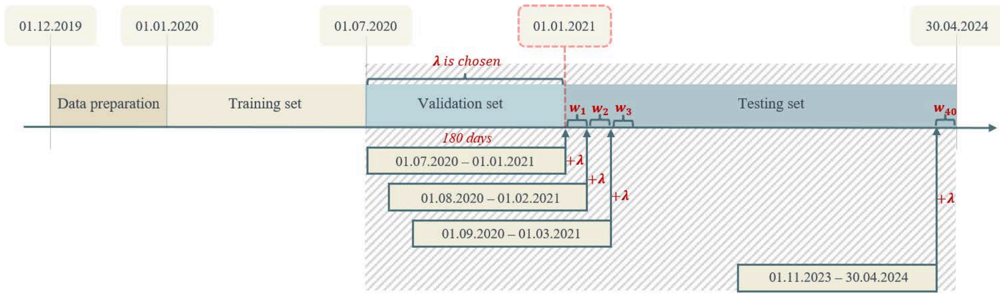
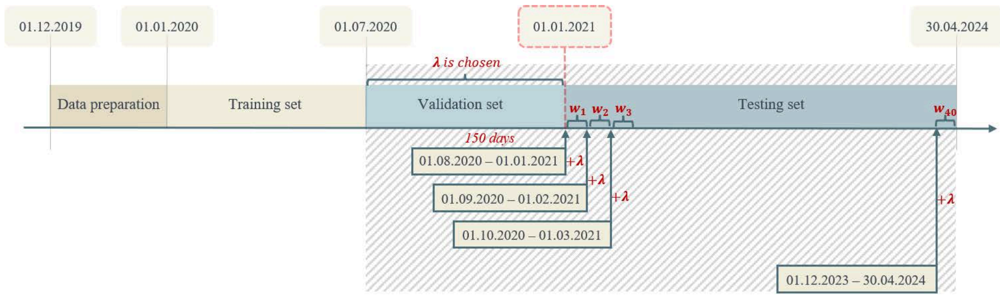


Figure 3.11.: Arrangement for training, validation and testing sets during the whole sample period (for a rolling window of 150 days). Part II



The Testing set, running from January 1, 2021, to April 30, 2024, represents the most critical phase of our analysis. This extended period (more than 3 years) allows us to evaluate the long-term performance and robustness of our inflation-tracking strategy under various market conditions. For each rolling window, we compare the performance of different portfolio compositions based on the weights generated by our optimization model (Equation 3.4) and the sparsity parameter lambda derived in the Validation phase. Taking the 150-day rolling window as an example (Figure 3.11), we generate our first asset weights in the Testing set (January 1, 2021) based on the data from 01.08.2020 to 01.01.2021 and the chosen sparsity parameter lambda obtained in the Validation set (from 01.07.2020 till 01.01.2021). Due to monthly rebalancing, in February 2021, new weights are established based on the data from 01.09.2020 to 01.02.2021 using the same (chosen) sparsity parameter lambda. This approach continues until the end of the sample data, deriving updated weights every 30 days. Eventually, the comparison of strategies is conducted based on the metrics defined in Section 3.3.4.

3.3.2. Portfolio compositions

While exploring different strategies and conducting a preliminary analysis of cryptocurrencies, we noticed one particular crypto asset with characteristics significantly different from others. The cryptocurrency PAX Gold[11] exhibits a lower correlation with Bitcoin and substantially reduced volatility compared to most other cryptocurrencies, which typically show high volatility and a strong positive correlation with Bitcoin. The main reason why PAX Gold stands out lies in the nature of the coin itself. Unlike most cryptocurrencies, PAX Gold is a digital asset backed by physical gold, with each token representing one fine troy ounce of gold stored in secure vaults. This direct link to a tangible, precious metal gives PAX Gold unique characteristics that set it apart from other cryptocurrencies.

During the analysis performed on validation data, it has become noticeable that PAX Gold consistently dominates the portfolio composition, occupying approximately 80% of the total allocation (Section 4.1). To enhance the robustness of our research, the decision was made to create two distinct portfolios with different asset selections:

- Portfolio 1: Including all crypto assets (87 out of 87), providing a comprehensive view of the entire asset universe under consideration.

- Portfolio 2: Excluding PAX Gold (86 out of 87), to assess the strategy's performance without the influence of this unique, gold-backed cryptocurrency.

By differentiating these two portfolios, we aim to uncover valuable insights into the effectiveness of different asset combinations in tracking inflation expectations across various market conditions. Moreover, analysing these two asset selection models separately enables us to identify potential benefits and drawbacks of including or excluding PAX Gold, providing a more comprehensive understanding of its role in our inflation-tracking strategy.

3.3.3. Strategy execution

To fully execute our inflation tracking strategy, we implement a sophisticated backtesting algorithm that closely mimics real-world trading conditions. This algorithm is carried out in Python, leveraging packages such as `vectorbt`[12] and `portfoliolib`[13]. The core of our strategy execution revolves around several key functions, components of each we examine below:

[11] `https://paxos.com/paxgold/`
[12] `https://vectorbt.dev/`
[13] `https://riskfolio-lib.readthedocs.io/en/latest/`

`pre_sim_func_nb`: This function establishes the rebalancing schedule for our strategy. It creates a boolean mask that determines when portfolio rebalancing should occur. The `every_nth` parameter allows us to set the frequency of rebalancing, ensuring our portfolio adapts to market changes at regular intervals.

`pre_segment_func_nb`: This function performs the critical task of assigning optimal weights for our portfolio for every rebalancing period. It incorporates several key steps:

- Selecting the relevant historical asset price data based on either a fixed lookback period or the entire available history.

- Calling the `opt_weights` function to determine the optimal asset allocation.

- Sorting order execution (reorders order sequence such that selling orders come first and buying last).

`order_finc_nb`: This function handles the actual order placement based on the optimized weights. It uses a target percentage approach for sizing orders and incorporates a 1% fee to account for transaction costs.

`opt_weights`: This function is the cornerstone of our strategy, responsible for determining the optimal portfolio weights. It calls the optimization function (index tracking model for inflation with LAIT approach 3.4), which optimizes weights while considering both the asset returns and the inflation expectations returns. The chosen optimization function with the LAIT approach can be easily replaced by any other optimization algorithm (i.e. Markowitz mean-variance optimization) for comparison purposes.

Overall, the execution of our strategy follows a cyclical process at each rebalancing point. This process begins with gathering and preparing historical asset price data. Next, our optimization algorithm generates portfolio weights considering both tracking performance and regularization. Based on these optimized weights, new target allocations are determined, and corresponding orders are generated. Finally, these orders are executed, effectively adjusting the portfolio composition to align with our inflation-tracking objectives.

3.3.4. Performance measurements

When dealing with inflation-tracking strategies, properly defining the accurate measurement of performance is crucial. This section delves into several key metrics that provide a comprehensive assessment of our portfolio's ability to track inflation expectations. The primary metrics considered include the Empirical tracking error, Tracking error variance, Fee metric, Pearson correlation, and Ordinary Least Squares beta (OLS beta), along with the underlying assumptions of the OLS model. These metrics, when analysed collectively, offer valuable insights into the effectiveness and reliability of our inflation-tracking strategy.

Expected Tracking Error and Tracking Error Variance

As previously defined in Section 3.2.1, the Expected Tracking Error (ETE) provides a measure of the average deviation of our portfolio's returns from the inflation expectations and is defined as:

$$ETE(w) = \frac{1}{T} \left\| Xw - r_{inf} \right\|_2^2$$

, where T is the number of time periods, X is the matrix of the asset returns, w is the vector of the portfolio weights and r_{inf} is the vector of the index returns (inflation expectations).

While ETE gives us an average measure of tracking performance, Tracking Error Volatility (TEV) provides insight into the consistency of this tracking. TEV measures the standard deviation of the difference between the portfolio and benchmark returns and is given by:

$$TEV(w) = \mathrm{Var}(Xw - r_{inf})$$

In the context of our inflation-tracking portfolio, a lower value of ETE and TEV indicates better tracking performance, providing a dual perspective on the robustness of our inflation-tracking strategy across various market conditions.

Fee Metric

The fee metric is introduced to quantify the impact of transaction costs on portfolio performance. This metric is calculated as:

$$\text{Fee Metric} = \frac{1\% \times \text{Executed Order Value}}{\text{Final Value of the Portfolio}}$$

, where the executed order value represents the total value of trades executed during the rebalancing process. This metric allows us to incorporate the costs associated with rebalancing the portfolio, providing a more comprehensive view of net performance. Higher transaction costs can significantly erode returns, particularly in portfolios requiring frequent adjustments.

Pearson Correlation

Pearson correlation coefficient measures the strength and direction of the linear relationship between our portfolio returns and the inflation expectation returns. The significance of the Pearson correlation can be assessed using the p-value and t-statistic, where the null hypothesis states that there is no true correlation (i.e., $\rho = 0$). The Pearson correlation coefficient itself is calculated as:

$$\rho = \frac{\text{Cov}(r_{tracking}, r_{inf})}{\sigma_{tracking}\sigma_{inf}}$$

, where $\text{Cov}(r_{tracking}, r_{inf})$ is the covariance between the portfolio returns and the index returns (inflation expectations), $\sigma_{tracking}$ is the standard deviation of the portfolio returns, and σ_{inf} is the standard deviation of the inflation expectations.

In the context of our inflation-tracking strategy, we aim for a positive correlation as close to $+1$ as possible, indicating that our portfolio moves in tandem with inflation expectations. A high positive correlation suggests that our asset allocation effectively captures the dynamics of the inflation expectations.

Ordinary Least Squares Beta (OLS beta)

The OLS beta is a measure derived from the linear regression model, which captures the sensitivity of the portfolio returns relative to changes in the benchmark (inflation expectations) returns. In the context of our inflation-tracking strategy, beta provides insight into how closely our portfolio mimics the movements of the benchmark. The OLS beta is calculated using the following regression model:

$$r_{tracking,t} = \alpha + \beta r_{inf,t} + \epsilon_t$$

, where α is the intercept, β is the slope coefficient representing the OLS beta, and ϵ_t is the error term, $r_{tracking,t}$ is the tracking portfolio return at time t, and $r_{inf,t}$ is the inflation expectations return at time t.

The beta coefficient is estimated as:

$$\beta = \frac{\text{Cov}(r_{tracking,t}, r_{inf,t})}{\text{Var}(r_{inf,t})}$$

For our inflation-tracking strategy, we ideally aim for a beta to be significant and positive (ideally: close to 1), indicating that our portfolio closely tracks the movements of inflation expectations. A beta significantly different from 1 suggests that our portfolio may be over- or under-reacting to changes in the benchmark, which can be also acceptable if the OLS beta is significant and positive. One such case is explored in Chapter 5.

Assumptions of OLS Regression

The validity and reliability of our OLS beta estimate rely on several key assumptions. Understanding and verifying these assumptions is crucial for the proper interpretation of our results:

- Model is linear in the parameter vector β.
- Strict exogeneity.
- Conditional homoscedasticity.
- No autocorrelation.
- Conditional normality of error terms.

Before calculating OLS beta, we conduct various tests such as the Breusch-Pagan test (homoscedasticity), Jarque-Bera test (normality of errors), Ljung-Box test (autocorrelation), and others, to assess whether OLS regression is an appropriate modelling approach[14]. Through these tests, we conclude that we occasionally observe autocorrelation and heteroscedasticity of residuals[15] in the constructed portfolio returns. To address these issues, we impose the following assumption on error terms: Heteroskedasticity and Autocorrelation Consistency (HAC)[16]. When using HAC standard errors, the interpretation of the coefficient estimates (OLS beta) remains the same, but the standard errors, t-statistics, and p-values are adjusted to be robust to heteroskedasticity and autocorrelation.

Ad-hoc Benchmarks

For comparative analysis, while evaluating our strategies using the ETE, TEV, and Fee metrics, we aim to benchmark their performance against classic models that do not incorporate sparsity restrictions. We have selected two such models for comparison:

1. Minimum tracking error optimization with no sparsity constraint ($\lambda = 0$):

$$\begin{aligned} \min_{w} \quad & \frac{1}{T} \|Xw - r_{\text{idx}}\|_2^2 \\ \text{s.t.} \quad & w^T 1 = 1 \\ & w_i \geq 0 \quad \forall i \end{aligned}$$

[14]See examples for daily and monthly data in Appendix.A.5

[15]The significance level is set at 5%.

[16]The Newey-West (1987) estimator for HAC standard error is used. This method accounts for both heteroskedasticity and autocorrelation by using weighted autocovariances up to a specified lag.

2. Minimum variance optimization based on Markowitz's model[17]:

$$\begin{aligned} \min_{w} \quad & \phi(w) \\ \text{s.t.} \quad & \mu w^T \geq \bar{\mu} \\ & w^T 1 = 1 \\ & w_i \geq 0 \quad \forall i \end{aligned}$$

These traditional approaches serve as important reference points, allowing us to assess the effectiveness of our sparsity-constrained strategies relative to well-established portfolio optimization techniques. By including these comparisons, we can better understand the trade-offs between tracking performance and the benefits of introducing sparsity in our inflation-tracking portfolios.

[17] Where $\phi(w)$ is a chosen risk measure: standard deviation.

4. Empirical Results

This Chapter presents the findings from both the validation and testing periods of the study. It analyzes the performance of different sparsity hyperparameters and their impact on asset selection and portfolio composition. The Chapter compares the tracking error, tracking error volatility and transaction costs of the optimized portfolios against ad-hoc benchmark portfolios. It examines the correlation between portfolio returns and inflation expectations across various time horizons. Finally, the Chapter presents the results of OLS regression analyses, providing insights into the sensitivity of the portfolios to changes in inflation expectations.

4.1. Validation period

During the validation period (June 2020 to December 2020 inclusive), we conduct several analyses, focusing on the following tasks:

- **Performance Analysis of Sparsity Hyperparameters**: We examine the performance of different sparsity hyperparameters. Specifically, we control for the number of assets selected by each hyperparameter value to ensure they perform their intended function. A larger hyperparameter value of lambda is expected to result in fewer assets chosen in the portfolio each rebalancing period.

- **Selection of Sparsity Parameter Lambda**: As outlined in Section 3.3.1, we select our sparsity parameter lambda to minimize the ETE (Empirical Tracking Error) of our portfolio and inflation expectations (T5YIE, BEIR) over the entire validation sample. During this process, we also account for transaction fees to evaluate how the sparsity parameter influences turnover costs.

- **Asset Allocation Analysis**: We analyse the asset allocation for Portfolio 1 (including PAX Gold) and Portfolio 2 (excluding PAX Gold) to gain insights into the composition of the portfolios.

All these tasks are performed for each rolling window: 120, 150, and 180 days. By conducting these analyses, we aim to validate the robustness and effectiveness of our portfolio optimization methodology across different time horizons and asset compositions.

Figure 4.1.: Number of assets selected for each rolling window (120, 150, and 180 days) according to different sparsity parameters. Portfolio 1 with PAX Gold.

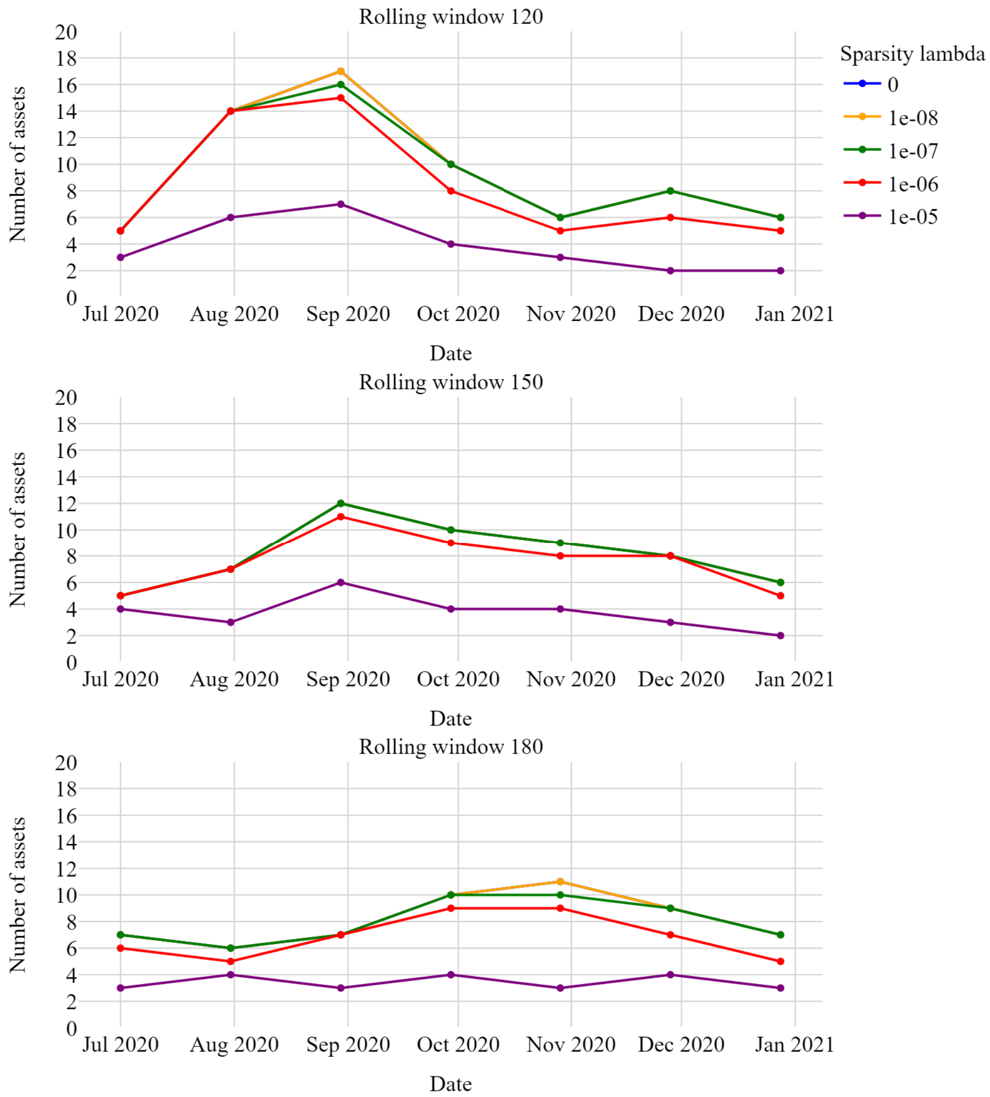


Figure 4.1 represents the number of assets selected for various sparsity lambda values (0, 1e-08, 1e-07, 1e-06, and 1e-05) across three different rolling window sizes (120, 150, and 180 days) over the validation period from July 2020 to December 2020 for Portfolio

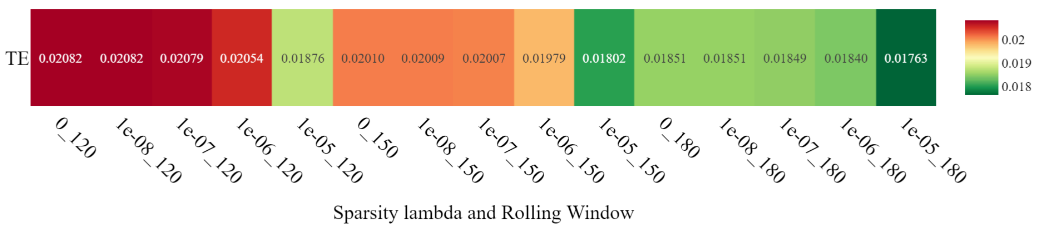
1[1], that includes PAX Gold.

As expected, the number of assets selected decreases as the sparsity lambda value increases, confirming the effectiveness of the sparsity regularization technique. This trend is consistent across all three rolling window sizes. For example, in the 120-day rolling window, the number of assets drops from 14 to 6 when the lambda value increases from 0 to 1e-05 in August 2020[2]. Similarly, in the 180-day rolling window, the number of assets decreases from 7 to 3 when the lambda value changes from 0 to 1e-05 in September 2020. Important to mention that asset selection for lambda equal to zero (no sparsity regularization) often coincides with asset selection with lambda 1e-08 and 1e-07.

Comparing the different rolling window sizes, it is apparent that the number of assets selected varies depending on the window size. The 120-day rolling window generally results in a higher number of assets compared to the 150-day and 180-day windows, especially for lower lambda values. This suggests that shorter rolling windows may lead to more diversified portfolios, while longer rolling windows tend to concentrate on fewer assets. However, the overall trend of decreasing the number of assets with increasing lambda values remains consistent across all rolling window sizes.

The next step in our analysis is the selection of the sparsity parameter lambda. Since our main optimization problem is to create a portfolio that minimizes the tracking error with inflation expectations, choosing ETE as a measure of portfolio performance during the validation period sounds legitimate. As stated in Section 3.3.1, we also account for transaction cost to examine the existence of an inverse relationship between tracking error and fees.

Figure 4.2.: Heatmap of ETE (Validation data). Portfolio 1 with PAX Gold.

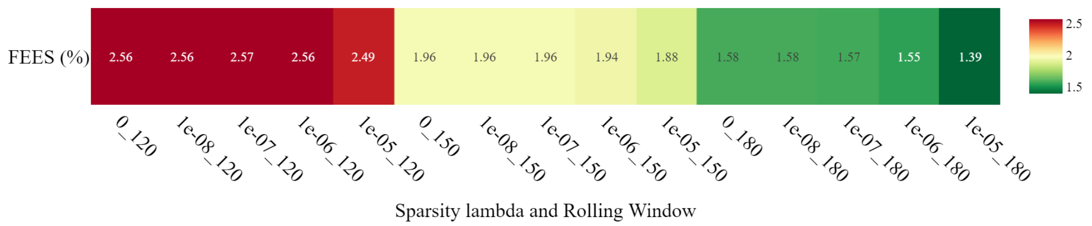


[1] The same analyses for Portfolio 2 (excluding PAX Gold) can be found in AppendixA.6

[2] Assets number for lambda 0, 1e-08, 1e-07, 1e-06 is the same (14) in August 2020.

Figure 4.2 shows the heatmap of ETE for different sparsity lambda values and rolling windows for Portfolio 1[3]. As the sparsity lambda increases, the ETE generally decreases, indicating that higher sparsity leads to better tracking performance, which is a surprising result. This trend is consistent across all rolling window sizes (120, 150, and 180 days). For example, in the 120-day rolling window, the ETE decreases from 0.02082 at $\lambda = 0$ to 0.01876 at $\lambda = 1e - 06$. Similarly, in the 180-day rolling window, the ETE reduces from 0.01851 at $\lambda = 0$ to 0.01763 at $\lambda = 1e - 05$. This suggests that even with a highly sparse portfolio (higher lambda), a reasonable level of tracking accuracy can be maintained.

Figure 4.3.: Heatmap of FEES (Validation data). Portfolio 1 with PAX Gold.



Similarly, Figure 4.3 presents the heatmap of transaction fees (FEES) for the same set of sparsity lambda values and rolling windows. As we can see, transaction fees tend to decrease as the sparsity lambda increases. This observation suggests that higher sparsity results in lower transaction costs, likely due to fewer assets being included in the portfolio. For instance, in the 120-day rolling window, the FEES decrease from 2.56% at $\lambda = 0$ to 2.49% at $\lambda = 1e - 05$. The same pattern is observed in the 150-day and 180-day rolling windows.

To select the optimal sparsity lambda value for each rolling window, we need to balance the tracking error and transaction costs. A common approach is to choose the lambda value that minimizes the ETE while keeping the FEES within an acceptable range. From the validation data for Portfolio 1, lambda = 1e-05 appears to be a suitable choice for all the 120-day, 150-day, and 180-day rolling windows, as it achieves the lowest ETE and lowest transaction fees. From the validation data for Portfolio 2, $\lambda = 1e - 05$ appears to be a suitable choice only for the 120-day rolling window, for 150-day, and 180-day rolling windows the best lambda turns out to be 1e-06. These observations suggest

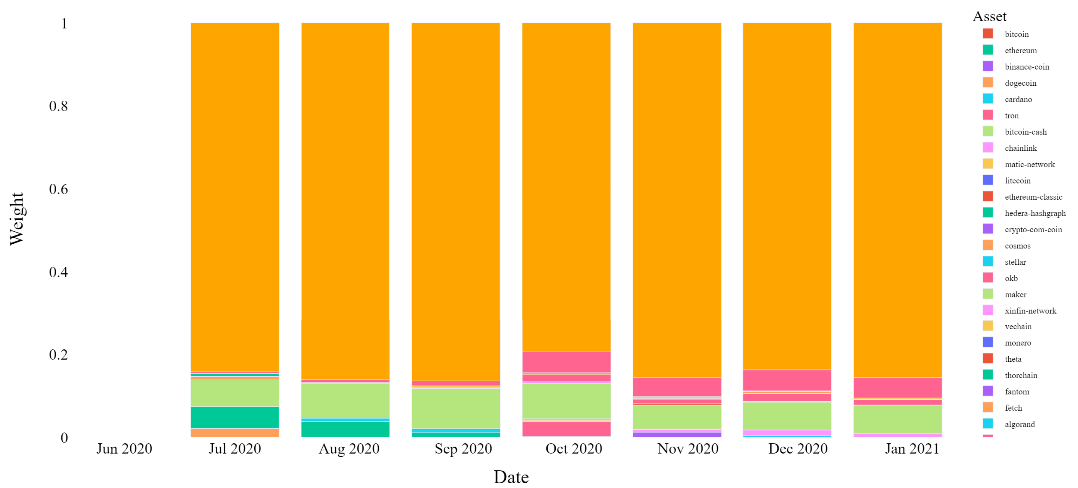
[3]The same analyses for Portfolio 2 (excluding PAX Gold) can be found in Appendix A.7

that, in this particular case, there is no apparent trade-off between tracking performance and transaction costs. The reason for this outcome could be attributed to the specific characteristics of the selected assets (cryptocurrencies) and the market conditions during the validation period.

In conclusion, we perform an asset allocation analysis to examine the behaviour of crypto assets chosen by our optimization algorithm. In Section 3.3.2, we suggest two portfolio asset selections based on the inclusion or exclusion of the cryptocurrency PAX Gold, since the nature of this asset is solely determined by the price of commodity gold and may significantly influence the performance of the portfolio.

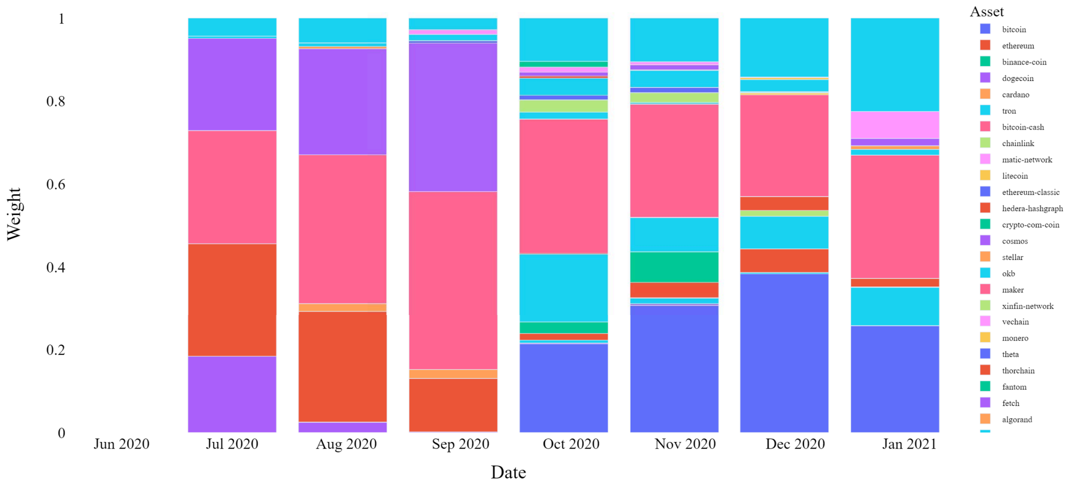
Comparing the asset allocations of Portfolio 1 and Portfolio 2 in Figure 4.4 and Figure 4.4 accordingly, we observe notable differences in the composition of the portfolios due to the inclusion or exclusion of PAX Gold. In Portfolio 1, PAX Gold consistently occupies a significant portion of the allocation for the whole validation period. For instance, in July 2020, PAX Gold accounted for approximately 80% of the portfolio and remained the weight between 79% and 86% throughout the entire validation period. This indicates that the optimization algorithm consistently assigns a high weight to PAX Gold, likely due to its low volatility and return.

Figure 4.4.: Asset allocation for Portfolio 1 with PAX Gold (Validation data).



In contrast, Portfolio 2 exhibits a more diverse allocation across various cryptocurrencies (Figure 4.5). Without the presence of PAX Gold, the optimization algorithm distributes the weights more evenly among the remaining assets. For example, in August 2020, Ftx-token (purple) and Gatechain-token (pink) and Kucoin-shares (red) dominate the portfolio, each holding a share of around 25-35%. As the validation period progresses, the allocation becomes more concentrated, with Bitcoin (blue) and Gatechain-token (pink) emerging as the primary assets in December 2020, together accounting for approximately 65% of the portfolio.

Figure 4.5.: Asset allocation for Portfolio 2 without PAX Gold (Validation data).



The distinct allocation patterns between the two portfolios highlight the impact of including or excluding PAX Gold on the overall composition. The presence of PAX Gold in Portfolio 1 appears to provide stability. On the other hand, the absence of PAX Gold in Portfolio 2 allows for a more dynamic allocation among the remaining assets. Analysing those two asset selection models separately allows us to gain insights into the potential benefits and drawbacks of including or excluding this specific asset for the subsequent testing phase.

4.2. Testing period

4.2.1. ETE/TEV

Based on the analysis conducted during the validation period, we determine the optimal hyperparameter value for sparsity lambda for each rolling window size:

- For Portfolio 1 (which includes PAX Gold), a lambda value of 1e-05 is found to be suitable for all rolling window sizes: 120-day, 150-day, and 180-day.

- For Portfolio 2 (which excludes PAX Gold), a lambda value of 1e-05 is found to be suitable for the 120-day rolling window, while a lambda value of 1e-06 is deemed appropriate for the 150-day and 180-day rolling windows.

In the next stage, we apply the selected sparsity lambda values for Portfolio 1 and Portfolio 2 during the testing period, which spans from January 2021 to April 2024 inclusive. To evaluate the performance of our portfolios, we employ two key metrics: Empirical Tracking Error (ETE) and Tracking Error Volatility (TEV). These metrics allow us to compare the performance of our portfolios against two ad-hoc benchmark portfolios (Section 3.3.4):

- Tracking error minimization with lambda = 0 (which represents no sparsity regularization).

- Minimum volatility optimization by Markowitz (without sparsity regularization).

By using these evaluation metrics and benchmarks, we aim to assess the effectiveness of our sparse portfolio optimization approach and determine whether the selected lambda values contribute to improved performance compared to the traditional methods.

To assess the general time series co-movements between our portfolio performance and the target benchmark, we visualize the asset values of Portfolio 1 and Portfolio 2 against the 5-year BEIR (inflation expectations) for all rolling windows (120, 150, and 180 days) using their respective optimized sparsity lambdas. The asset values are normalized to 100 as a starting point, and a logarithmic scale is employed to better visualize relative changes over the observation period.

According to Figure 4.6 Portfolio 1 across all rolling window sizes (120-day, 150-day, and 180-day) comparatively closely track the inflation expectations throughout the majority of the testing period. Blue areas indicate ideal co-movements, whereas red areas show the opposite directions. The Portfolio 1 values generally move in tandem with the BEIR, indicating that they are effectively capturing the inflation dynamics. However, there are some areas (for instance, September 2022 and the beginning of 2023), where the Portfolio 1 values significantly deviate from the BEIR, showing sometimes even opposite co-movements.

Figure 4.6.: Portfolio 1 (with PAX Gold) asset performance across all rolling windows (Testing data).

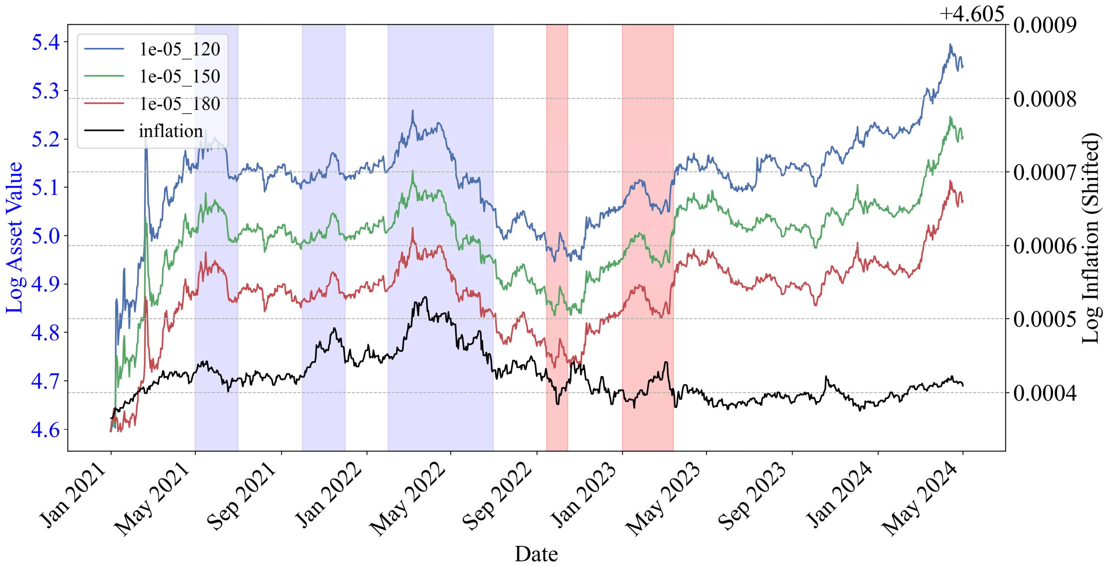
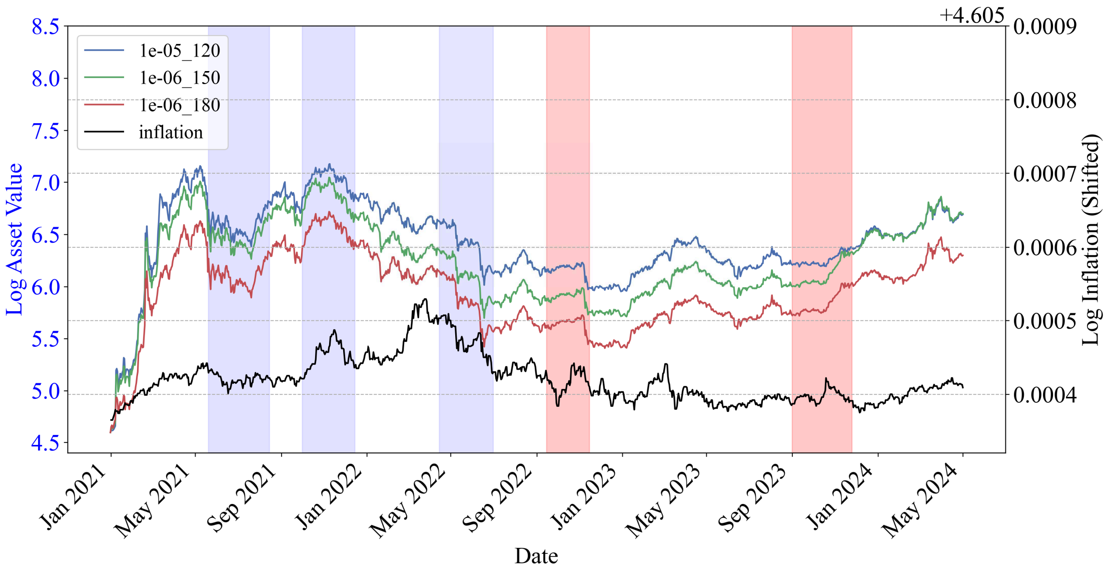


Figure 4.7.: Portfolio 2 (without PAX Gold) asset performance across all rolling windows (Testing data).



Variations of Portfolio 2 (Figure 4.7) seem to have a moderate to small degree of daily co-movement, mostly coinciding when BEIR experiences a negative trend (mid-2021, May 2022). Similarly to Portfolio 1, Portfolio 2 variations have periods when significant deviations exist: September 2022 and September 2023. In addition, it is noticeable that portfolios in Figure 4.7 experience a higher volatility[4] compared to those in Figure 4.6, which is anticipated since excluding the asset with low volatility (PAX Gold).

Next, we will examine the performance of our portfolios using the ETE and TEV metrics. To gain a more comprehensive understanding of the impact of the sparsity parameter, we will also consider transaction fees in our analysis.

According to Table 4.1, only for the rolling window of 180 days does the 1e-5 sparsity parameter yield the lowest TE (0.88×10^{-4}) and TEV (0.93×10^{-2}) values, indicating superior tracking performance compared to both the zero-sparsity and minimum volatility portfolios. In contrast, for rolling windows of 120 and 150 days, the sparsity parameter of 1e-5 underperforms the benchmark portfolios in terms of both TE and TEV. The transaction fees (FEES) for the selected lambda portfolios (1e-5 for all rolling windows), as expected, are lower than the benchmark portfolios (i.e. RW 120: 3.89% vs 4.63% and 4.76%). Lower transaction fees hold for all rolling windows, demonstrating the effectiveness of the chosen sparsity regularization.

Table 4.1.: Comparison of TE and TEV for Portfolio 1 variations with controlling for transaction fees (FEES)

Portfolio	RW 120	RW 150	RW 180
Portfolio 1 with PAX Gold ($\lambda =$)	1e-5	1e-5	1e-5
TE	2.11×10^{-4}	1.27×10^{-4}	0.88×10^{-4}
TEV	1.45×10^{-2}	1.12×10^{-2}	0.93×10^{-2}
FEES (%)	3.89	3.37	2.85
Benchmark Portfolio ($\lambda = 0$)			
TE	1.90×10^{-4}	1.24×10^{-4}	0.92×10^{-4}
TEV	1.37×10^{-2}	1.11×10^{-2}	0.95×10^{-2}
FEES (%)	4.63	3.85	3.39
Benchmark Portfolio (MinVol)			
TE	1.91×10^{-4}	1.25×10^{-4}	0.92×10^{-4}
TEV	1.38×10^{-2}	1.11×10^{-2}	0.96×10^{-2}
FEES (%)	4.67	3.90	3.40

[4] See logarithmic scale on the left.

Table 4.2.: Comparison of TE and TEV for Portfolio 2 variations with controlling for transaction fees (FEES)

Portfolio	RW 120	RW 150	RW 180
Portfolio 2 without PAX Gold ($\lambda =$)	1e-5	1e-6	1e-6
TE	2.05×10^{-3}	1.80×10^{-3}	1.39×10^{-3}
TEV	4.51×10^{-2}	4.24×10^{-2}	3.73×10^{-2}
FEES (%)	20.60	14.95	15.19
Benchmark Portfolio ($\lambda = 0$)			
TE	1.98×10^{-3}	1.79×10^{-3}	1.39×10^{-3}
TEV	4.44×10^{-2}	4.23×10^{-2}	3.72×10^{-2}
FEES (%)	19.92	15.18	14.97
Benchmark Portfolio (MinVol)			
TE	2.00×10^{-3}	1.82×10^{-3}	1.39×10^{-3}
TEV	4.46×10^{-2}	4.25×10^{-2}	3.73×10^{-2}
FEES (%)	20.40	15.06	15.05

Analysing the performance of Portfolio 2 (without PAX Gold), we see a general increase in ETE and TEV for all portfolio types compared to Portfolio 1 (with PAX Gold), which is expected due to the highly volatile nature of the chosen crypto assets. In contrast to previous results, the benchmark portfolio with no sparsity regularization ($\lambda = 0$) shows the lowest ETE and TEV across all rolling windows, without much sacrifice in transaction fees. Only for the rolling window of 150 days does the portfolio with sparsity lambda 1e-6 show the lowest transaction fees (14.95%). While the results from the performance of the portfolio with $\lambda = 0$ are consistent with the theory of an inverse trade-off between transaction costs and tracking error, surprisingly, portfolios with sparsity lambdas different from zero do not deliver lower transaction fees compared to ad-hoc benchmarks. This can be explained by the fact that while the number of assets chosen for our sparsity portfolios may be lower than for ad-hoc benchmark portfolios, the portfolio turnover rate[5] may still be quite high, resulting in an increase in fees.

4.2.2. Correlation

As stated in Section 3.3.4, to assess the relationship between our portfolio returns and the 5-year break-even inflation rate, we employ the Pearson correlation coefficient. In our context, the Pearson correlation allows us to evaluate the degree to which changes in our portfolio returns align with changes in inflation expectations. The test is performed on

[5]Portfolio turnover rate indicates the frequency with which changes are made in the fund's portfolio.

different return frequencies (daily, weekly, bi-weekly and monthly) in order to capture any potential variations in the relationship that might be dependent on the time horizon considered, both short-term and long-term.

Table 4.3.: Pearson correlation coefficients for Portfolio 1 and 5-Year BEIR returns across different timeframes

Portfolio 1	RW 120 (λ=1e-5)	RW 150 (λ=1e-5)	RW 180 (λ=1e-5)
Daily	0.023	0.041	0.052*
Weekly	0.023	0.002	(0.008)
Bi-Weekly	0.202*	0.200*	0.172
Monthly	0.269*	0.196	0.132

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 4.3 reveals a mostly positive correlation between Portfolio 1 variations returns and BEIR across all time horizons examined. The correlation is statistically significant at the 10% level for daily returns of the portfolio with 180 rolling window (0.052), for bi-weekly returns of the portfolio with 120 and 150 rolling windows (0.202 and 0.200), and for monthly returns with 120 rolling window (0.269). The stronger correlation at the bi-weekly and monthly level indicates that this relationship may be more pronounced over longer time horizons, possibly due to the smoothing out of short-term noise in both the portfolio returns and inflation expectations.

Table 4.4.: Pearson correlation coefficients for Portfolio 2 and 5-Year BEIR returns across different timeframes

Portfolio 2	RW 120 (λ=1e-5)	RW 150 (λ=1e-6)	RW 180 (λ=1e-6)
Daily	0.020	0.026	0.033
Weekly	0.148*	0.151**	0.146*
Bi-Weekly	0.304***	0.305***	0.319***
Monthly	0.365**	0.363**	0.379**

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Now let's take a look at the correlation results for Portfolio 2 variations excluding PAX Gold (Table 4.4). The correlations are significant at the 10% level for weekly, at the 1% level for bi-weekly, and at the 5% level for monthly timeframes, with the monthly correlation being the strongest at 0.379 (180 rolling window). The daily correlation, while positive, is not statistically significant at conventional levels. The increasing magnitude of the correlation coefficients as the time horizon lengthens (from daily to monthly) indicates that the relationship between Portfolio 2 returns and inflation expectations becomes stronger over longer periods.

Compared to the results for Portfolio 1, the correlations for Portfolio 2 are consistently higher and more significant across all timeframes, excluding daily frequency. This suggests that the returns of Portfolio 2 are more closely linked to market expectations of inflation than those of Portfolio 1.

The overall positive correlation both for Portfolio 1 and Portfolio 2 implies that as market expectations for inflation rise, the returns of Portfolios tend to increase as well. This could be interpreted as the market viewing cryptocurrencies as a potential hedge against inflation, with investors allocating more capital to this asset class when they anticipate higher inflation in the future. However, it is important to note that correlation does not necessarily imply causation, and other factors may be driving the returns of both the portfolio and the BEIR.

4.2.3. OLS Beta

Having established the correlation between the returns of Portfolio 1 and Portfolio 2 with BEIR, the next step in our analysis is to examine the sensitivity of these portfolios to changes in inflation expectations using the Ordinary Least Squares (OLS) beta. While in Section 3.3.4, the necessary assumptions for the OLS regression, such as linearity, normality, homoscedasticity, autocorrelations of residuals, etc., are addressed to ensure the validity of our results, in this Section we focus solely on the results of the regression analysis, presenting the estimated OLS Betas for Portfolio 1 and Portfolio 2 across different time horizons.

When analysing the results of regressing Portfolio 1 returns on BEIR returns (Table 4.5) one can see that there is no single significant beta coefficient produced by OLS regression, indicating that the relationship between Portfolio 1 returns and changes in inflation expectations is not strong enough to be considered reliable at these time horizons. The lack of significant beta coefficients across all timeframes implies that Portfolio 1, which includes PAX Gold, may not be an effective hedge against inflation as measured by the 5-year BEIR, which could be a concern for investors seeking to protect their investments from the erosive effects of rising prices.

Table 4.5.: OLS Beta coefficients for 5-Year BEIR returns across different timeframes (Portfolio 1)

Portfolio 1	RW 120 (λ=1e-5)	RW 150 (λ=1e-5)	RW 180 (λ=1e-5)
Daily	0.854	1.198	1.269
Weekly	0.665	0.056	(0.175)
Bi-Weekly	5.820	4.398	3.535
Monthly	8.710	5.104	3.009

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 4.6.: OLS beta coefficients for 5-Year BEIR returns across different timeframes (Portfolio 2)

Portfolio 2	RW 120 (λ=1e-5)	RW 150 (λ=1e-6)	RW 180 (λ=1e-6)
Daily	2.371	2.838	3.203
Weekly	18.100*	16.894**	15.105**
Bi-Weekly	40.454*	36.377*	33.553**
Monthly	60.117*	54.220*	51.337**

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 4.5 provides information regarding the OLS beta coefficients for Portfolio 1 returns regressed on BEIR across different timeframes. As one can notice, the beta coefficients are not statistically significant at the conventional levels of 1%, 5%, or 10% for any of the timeframes considered. The lack of statistical significance suggests that the relationship between Portfolio 1 returns and changes in the 5-year BEIR is not strong enough to be considered reliable, regardless of the timeframe examined.

In contrast to Portfolio 1, the OLS beta Coefficients for Portfolio 2 are statistically significant at the 10% and 5% level for all timeframes except daily (Table 4.6). For instance, for a rolling window of 120 days and $\lambda = 1e - 5$, the beta = 18.100 suggests that a one percent increase in inflation expectation leads to an 18 percent increase in returns of Portfolio 2. Those high levels of beta signal that even though Portfolio 2 moves in line with BEIR, the volatility of the former is much higher than inflation expectations.

Overall, based on the OLS beta coefficients presented in both tables, Portfolio 2 appears to be a more suitable choice for investors looking to hedge against inflation risks, as it demonstrates a strong and statistically significant sensitivity to changes in the 5-year BEIR across weekly, bi-weekly, and monthly timeframes. However, the increasing magnitude of the beta coefficients of Portfolio 2 as the timeframe lengthens may undermine the primary aim of inflation hedging, imposing on potential investors significantly more risk than he or she is willing to take. The possible solution for this particular problem is discussed in the following Chapter 5.

5. Possible Implementations

This Chapter explores practical applications of the research findings. It demonstrates how to construct an inflation-hedging portfolio that combines cryptocurrencies with cash to reduce overall volatility while maintaining exposure to potential benefits. The Chapter also examines the incorporation of cryptocurrencies into traditional asset portfolios, using the Vanguard Total Stock Market Index Fund ETF (VTI) as an example. It compares the performance of portfolios with and without cryptocurrency inclusion, considering metrics such as tracking error, correlation with inflation expectations, total returns, Sharpe ratio, etc. The chapter concludes by discussing the potential benefits and drawbacks of including cryptocurrencies in investment portfolios aimed at hedging against inflation.

5.1. Introducing cash

Without a doubt, constructing an inflation hedge portfolio consisting only of volatile crypto assets adds additional risk, which may be unsuitable for investors with low risk tolerance. This investment approach may be unattainable, especially for conservative investors who prioritize stable returns over high-risk opportunities. The high volatility associated with cryptocurrencies can lead to significant fluctuations in portfolio value, potentially causing substantial losses in the short term. However, we cannot deny the increasing role of cryptocurrencies as a financial asset in the modern investment landscape. As the adoption of cryptocurrencies continues to grow and more institutional investors enter the market, the perception of them as a legitimate asset class is strengthening.

Therefore, instead of completely abandoning crypto assets for inflation hedging purposes, we can significantly decrease the volatility that they introduce by allocating a portion of the portfolio to cash or cash equivalents. This approach allows investors to maintain some exposure to the potential benefits of cryptocurrencies as an inflation hedge while mitigating the overall risk of the portfolio. The importance of including cash in the inflation hedge portfolio is widely discussed and revised in the work of Roache and Attie (2009) and Brière and Signori (2011), where cash is represented by a 3-month T-bill rate. The authors imply that both in times of highly volatile and stable economic environments, an investor solely focused on inflation protection should primarily allocate his or her assets into cash for short investment horizons.

To illustrate the practical application of our findings and demonstrate a potential implementation strategy, we construct a portfolio based on the following parameters and assumptions:

- Asset selection: cash (1-to-1 to the US Dollar, instead of T-bills) and 86 cryptocurrencies (excluding PAX Gold).

- Asset allocation: using Minimizing Tracking Error optimization function with a rolling window of 180 days, sparsity lambda = 1e-6 and monthly rebalancing, since this model showed the best results in terms of correlation and OLS beta with 5-year BEIR.

- Sample period: from January 1, 2021 to April 30, 2024.

- Capturing the total weight of all cryptocurrencies in the portfolio to 1 over 51.337 (approx. 2%) in order to gain a one-to-one relationship between portfolio returns and inflation changes (OLS beta = 1).

- Allowing for the look-ahead bias for demonstration purposes and for the sake of simplicity.

Figure 5.1 represents the cryptocurrency allocation within the newly constructed portfolio. The weights dedicated to crypto most of the time do not exceed 2%, which is in line with the stated assumptions.

The results of the OLS regression of the returns of the new portfolio, consisting of cash and crypto assets, on the 5-year Break-Even Inflation Rate show that the OLS beta is significant at the 5% level and equal to 0.985. This finding aligns with the expectations based on the portfolio construction design. The OLS beta coefficient indicates that a one percent increase in inflation expectations leads to an almost one percent (0.985%) increase in the portfolio's returns. These results are accompanied by a positive monthly Pearson correlation of 0.366, which is also significant at the 5% level.

The near-perfect co-movement between the portfolio's returns and the 5-year BEIR can be observed in the following graph (Figure 5.2), which depicts the monthly returns of both the portfolio (1e-06_180_assets_returs) and the 5-year BEIR. The graph visually illustrates the strong positive relationship between the two variables, supporting the findings from the OLS regression and correlation analysis.

Figure 5.1.: Crypto asset allocation under 2% of the total portfolio weights

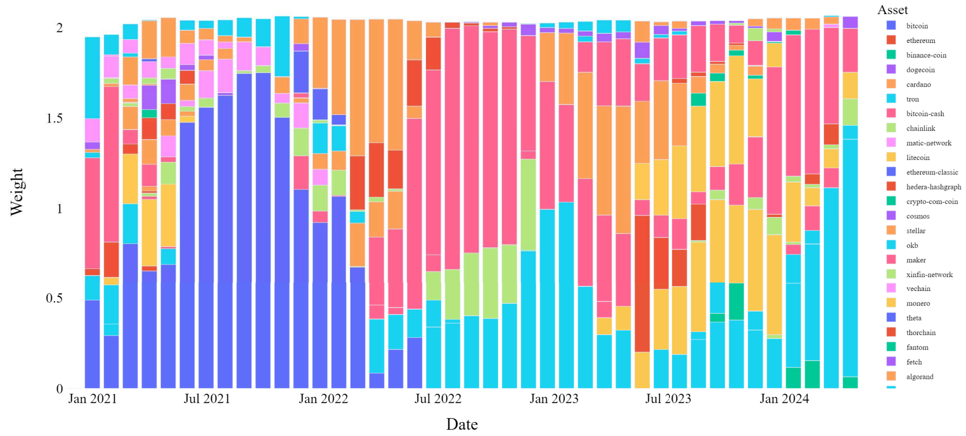
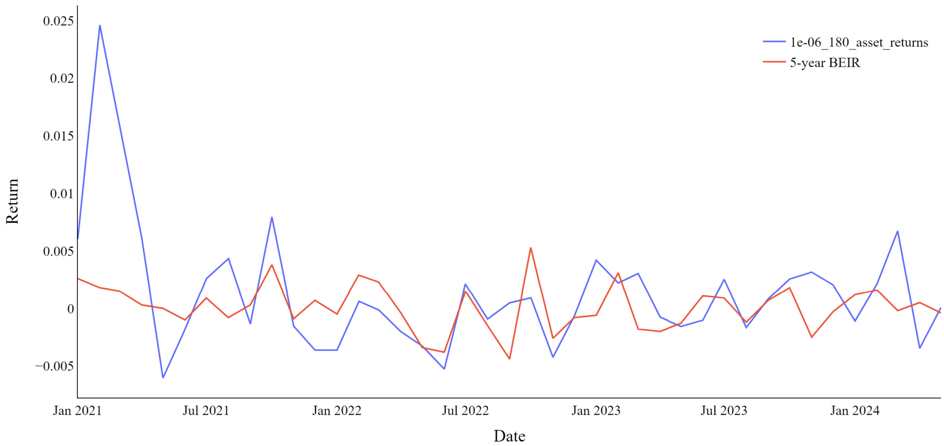


Figure 5.2.: Monthly returns of Portfolio and Inflation Expectations



This portfolio construction serves as a good example of how cryptocurrencies can be incorporated into an inflation hedge portfolio for investors with high risk aversion. By allocating a portion of the portfolio to cash, the overall volatility introduced by cryptocurrencies can be significantly reduced, making the investment more attractive. However, it is crucial to acknowledge that the current analysis is subject to look-ahead bias. The OLS beta coefficient of 51.337, which was used to construct the new portfolio, was derived based on the same sample period as the portfolio itself. This means that the portfolio's performance and its ability to track inflation expectations may be overstated, as the beta coefficient was optimized using data that would not have been available in real time. To build a fully sophisticated and reliable inflation hedge portfolio that incorporates both cash and cryptocurrencies, further analysis is needed, which can be addressed in future research.

5.2. Incorporating cryptocurrencies into traditional asset portfolios

We continue exploring possible implementations of cryptocurrencies in portfolio with traditional assets. In this particular example, we incorporate Vanguard Total Stock Market Index Fund ETF (VTI)[1] that tracks the performance of the CRSP US Total Market Index[2], which measures the performance of the overall US equity market. We construct two portfolios:

- Portfolio A solely includes the VTI.
- Portfolio B uses the same optimization model as in Section 5.1, but instead of cash VTI is used.
- Sample period: from January 1, 2021 to April 30, 2024.

The graph below (Figure 5.3) shows the allocation weights of Portfolio B over time, consisting of VTI (green) and 86 different cryptocurrency assets. Over the time period shown, from January 2021 to April 2024, the portfolio allocation changes notably. For most of the time horizon, the portfolio is heavily allocated to VTI. From January 2023 onwards, the allocation to cryptocurrencies increases compared to the previous period, but it remains relatively small, not exceeding 20% of the total portfolio weights. This increase in cryptocurrency allocation can be explained by the overall cryptocurrency price increase and comparatively low volatility during that time. However, VTI continues to dominate the portfolio throughout the entire period.

We continue our analysis with an evaluation of portfolio performance. According to Table 5.1, Portfolio A, which only includes VTI, demonstrates slightly better tracking performance (lower TE and TEV) and a stronger relationship with the 5-year BEIR (higher correlation and OLS Beta) compared to Portfolio B, which includes VTI and 86 crypto assets. While the difference between the portfolios in tracking performance is not dramatically different, it is essential to consider the higher fees associated with Portfolio

[1] `https://investor.vanguard.com/investment-products/etfs/profile/vti`
[2] `https://www.crsp.org/indexes/crsp-us-total-market-index`

B (3.54%) compared to the zero fees of Portfolio A. These fees can have a significant impact on the cost-effectiveness of Portfolio B and influence its overall performance.

Figure 5.3.: Asset allocation in Portfolio B with cryptocurrencies and VTI

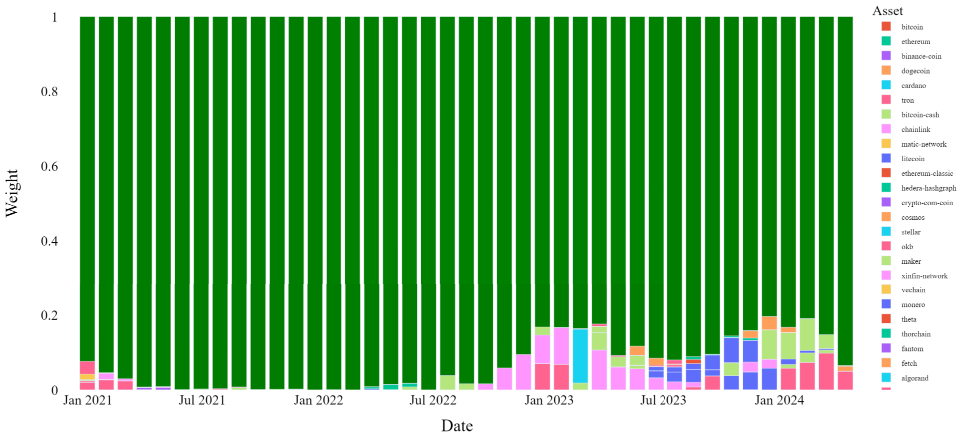


Table 5.1.: Performance evaluation for Portfolio A and Portfolio B

Portfolio	TE	TEV	Corr (daily)	OLS Beta (daily)	FEES [%]
Portfolio A	8.40×10^{-5}	9.15×10^{-3}	0.176^{***}	4.201^{***}	0.00
Portfolio B	9.10×10^{-5}	9.55×10^{-3}	0.165^{***}	4.069^{***}	3.54

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

(A: only VTI, B: VTI with 86 crypto assets)

The following graph (Figure 5.4) shows the evolution of the two portfolios over time. Portfolio A (blue line) outperforms Portfolio B (red line) for an entire timeline. The performance gap between portfolios widens significantly from January 2023 onwards.

Despite the fact, that Portfolio B (VTI with cryptocurrencies) slightly underperforms Portfolio A (VTI) in terms of tracking performance, the situation changes if we consider risk and return metrics. Portfolio B generates a significantly higher total return (61.85%) than Portfolio A (32.75%) (Table 5.2), suggesting that the inclusion of cryptocurrency assets has enhanced the portfolio's returns. When considering risk, both portfolios exhibit similar annualized standard deviations (0.162 for Portfolio B and 0.167 for Portfolio A), indicating comparable levels of volatility. The maximum drawdown percentages are also similar for both portfolios (25.49% for Portfolio B and 25.36% for Portfolio A), suggesting that they experienced similar worst-case scenarios. However, Portfolio B demonstrates a shorter maximum drawdown duration (566 days) compared to Portfolio A (714 days), indicating a quicker recovery from the worst-case scenario. Finally, the Sharpe Ratio, which measures risk-adjusted returns (we assume risk free rate $R_f = 0$ for simplicity), is higher for Portfolio B (0.816) than for Portfolio A (0.526). This suggests that Portfolio B offers better risk-adjusted returns.

Figure 5.4.: Portfolio A (only VTI) and Portfolio B (VTI and cryptocurrencies) performance

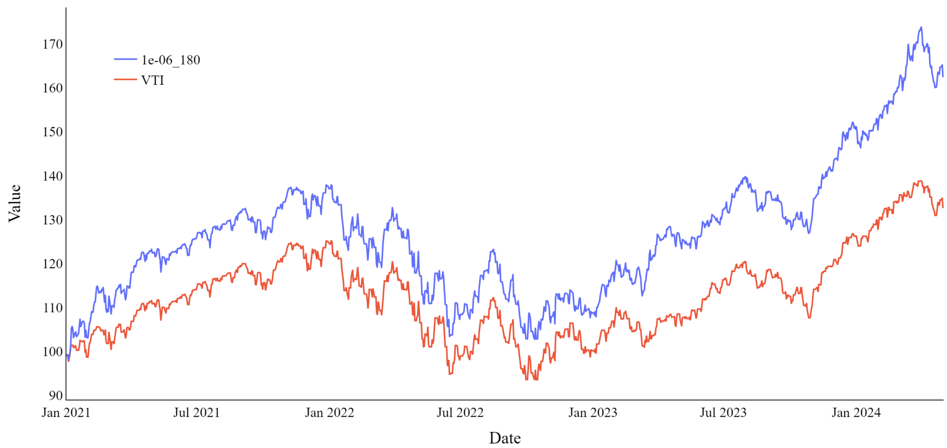


Table 5.2.: Extended performance evaluation for Portfolio A and Portfolio B

Portfolio	End Value	Total Ret. (%)	Ann. Std Dev	Max DD (%)	Max DD Dur.	Sharpe
Portfolio A	132.75	32.75	0.167	25.36	714	0.526
Portfolio B	161.85	61.85	0.162	25.49	566	0.816

(A: only VTI, B: VTI with 86 crypto assets. Sharpe ratio calculated with $R_f = 0$)

Overall, Portfolio B, which includes VTI and 86 cryptocurrency assets, demonstrates superior performance compared to Portfolio A, which only consists of VTI. Portfolio B has a higher end value, total return, and Sharpe ratio, as well as a shorter maximum drawdown duration while maintaining similar volatility to Portfolio A. This suggests that the inclusion of cryptocurrency assets in Portfolio B enhances its overall performance, risk-adjusted returns, and provides a diversification effect. While Portfolio B slightly underperforms Portfolio A in terms of tracking performance and has higher fees (3.54% compared to 0% for Portfolio A), the better risk-return performance of Portfolio B may still be attractive for active investment strategies aiming to outperform the benchmark.

In conclusion, the implementation examples in Sections 5.1 and 5.2 demonstrate the potential value of incorporating cryptocurrencies into inflation-hedging portfolios. The cash-and-crypto portfolio in 5.1 achieves a near one-to-one relationship with inflation expectations, while the VTI-and-crypto portfolio in 5.2 outperforms the VTI-only portfolio in terms of total returns and risk-adjusted performance (while keeping an inflation tracking performance on the same level). These results highlight the potential benefits of including cryptocurrencies in investment portfolios, such as enhanced returns, improved Sharpe ratio, and diversification effects. However, it's crucial to consider the drawbacks, including higher volatility (when crypto allocations are substantial), increased transaction costs, and potential regulatory risks associated with cryptocurrencies. The optimal implementation strategy will depend on individual investor risk tolerance, investment horizons, and market conditions.

6. Conclusion

This thesis has provided a detailed examination of the role that cryptocurrencies can play in constructing an inflation-resistant portfolio. By exploring the theoretical foundations, applying rigorous empirical methods, and analysing the results, this research has contributed valuable insights to the growing body of knowledge on the intersection of cryptocurrencies and inflation-hedging strategies.

The core objective of this thesis was to assess whether cryptocurrencies can serve as effective tools for inflation protection. To this end, we implemented a sparse index tracking model aimed at replicating the performance of the 5-year break-even inflation rate. This approach represents a sophisticated method for constructing portfolios that not only track inflation expectations but also harness the unique characteristics of cryptocurrencies. Our research has yielded several significant findings that contribute to the ongoing discourse on cryptocurrency's role in modern portfolio management and inflation hedging strategies, offering both theoretical contributions and practical implications for investors and financial professionals.

First and foremost, the empirical results demonstrate that cryptocurrencies can indeed act as inflation hedges, but their effectiveness is conditional on several factors, such as portfolio composition and the time frame. One of the most striking findings of our research is the difference in performance between portfolios that included PAX Gold (Portfolio 1) and those that excluded it (Portfolio 2). Portfolio 2, which excludes traditional commodity-linked assets, such as gold-backed cryptocurrencies, demonstrates stronger correlations and more significant beta coefficients with inflation expectations, particularly over bi-weekly and monthly timeframes. This suggests that a more diverse cryptocurrency portfolio, rather than one heavily weighted towards low-volatility, gold-backed digital assets, may offer greater potential for inflation hedging.

However, the high volatility introduced by cryptocurrencies in both portfolio types underscores the inherent risk associated with these investments and highlights the challenges of using these assets for consistent inflation protection. The persistence of high volatility and substantial tracking errors indicates that while cryptocurrency portfolios may move in the same direction as inflation expectations, the magnitude of these movements is often disproportionate. This characteristic potentially exposes investors to greater risk compared to traditional inflation hedges.

The analysis of different time horizons (daily, weekly, bi-weekly, and monthly) revealed that the relationship between cryptocurrency portfolios and inflation expectations strengthens over longer periods. This finding suggests that cryptocurrencies may be more suitable for medium to long-term inflation hedging strategies rather than short-term tactical allocations. It also implies that investors and portfolio managers should consider their investment horizon carefully when incorporating cryptocurrencies into inflation-hedging portfolios.

Another critical finding of this research is the impact of the sparsity parameter lambda on portfolio performance with regard to tracking errors and fees. Our implementation of a sparse index tracking model with the Majorization-Minimization framework proved effective in managing the complexity of portfolio construction. However, the optimal level of sparsity varied across different rolling windows and portfolio compositions, highlighting the need for dynamic and adaptive strategies in cryptocurrency portfolio management. Interestingly, while a larger value of sparsity lambda (higher sparsity) delivered the lowest tracking error and transaction fees across almost all rolling windows during the validation period, in the testing phase, our constructed portfolios outperformed ad-hoc benchmarks in terms of tracking error and fees only in several cases.

Our exploration of practical implementations, including the integration of cash and traditional assets like ETFs, provides valuable insights into how cryptocurrencies might be incorporated into broader investment strategies. The cash-and-crypto portfolio demonstrated the potential to achieve a near one-to-one OLS beta relationship with inflation expectations while significantly reducing overall portfolio volatility. Similarly, the inclusion of cryptocurrencies in a portfolio alongside traditional equity ETFs showed potential for enhancing returns and improving risk-adjusted performance metrics such as the Sharpe ratio, while maintaining the same tracking error as the portfolio without crypto inclusion. For investors with a higher risk tolerance, cryptocurrencies can provide an additional layer of diversification and potential upside, playing a complementary role in a portfolio aimed at protecting against inflation. On the other hand, for more conservative investors, a hybrid approach that combines cryptocurrencies with traditional assets like cash and ETFs may offer a more stable path to inflation protection.

It is important to acknowledge the limitations of this study. The relatively short history of cryptocurrency markets compared to traditional financial assets limits the long-term historical analysis possible. Additionally, the rapidly evolving nature of the cryptocurrency ecosystem, including regulatory changes and technological advancements, may impact the stability of observed relationships over time. Future research could benefit from longer data series and the inclusion of a wider range of cryptocurrencies as the market matures.

Moreover, several potential extensions may be beneficial for future research, such as: incorporating unexpected inflation into the model, exploring the model under different market conditions (bearish and bullish markets) and examining varying inflationary regimes (increasing and decreasing inflation). These extensions could provide a more comprehensive understanding of cryptocurrencies' potential as inflation hedges across various economic scenarios and market conditions.

In conclusion, this thesis contributes to the understanding of how cryptocurrencies can be integrated into inflation-hedging portfolios. While our findings indicate some promise for cryptocurrencies in tracking inflation expectations, they also highlight the complexities and risks involved. The varying performance across different portfolio compositions and time horizons underscores the need for sophisticated, adaptive strategies when incorporating cryptocurrencies into inflation-hedging portfolios. By carefully balancing these factors, investors and financial professionals with different risk preferences can potentially harness the unique properties of cryptocurrencies to protect against inflation in an increasingly complex economic environment. This research adds to the growing body of knowledge on digital assets in modern finance, providing insights that may be valuable for those seeking to combat inflation in today's dynamic financial landscape.

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A. Appendix

A.1. Cryptocurrencies used in the research

Table A.1.1.: Selected cryptocurrencies.

Cryptocurrency	Index	Marketcap (USD)	Volume (USD)	Price (USD)
bitcoin	1	1,193,544,962,108	37,840,840,057	60,609.4979
ethereum	2	367,894,024,585	18,266,377,489	3,014.1315
binance-coin	3	85,374,554,819	1,727,536,120	578.5365
dogecoin	4	19,212,870,615	1,342,359,702	0.1333
cardano	5	15,713,667,150	409,012,811	0.4409
tron	6	10,459,047,155	495,929,321	0.1195
bitcoin-cash	7	8,552,906,177	462,293,011	434.1709
chainlink	8	7,711,163,445	352,077,103	13.1343
matic-network	9	6,604,103,535	300,963,583	0.6671
litecoin	10	5,920,910,168	409,800,293	79.4953
ethereum-classic	11	3,736,072,402	292,051,330	25.4466
hedera-hashgraph	12	3,347,457,720	196,344,120	0.0937
crypto-com-coin	13	3,569,988,369	60,226,116	0.1344
cosmos	14	3,310,159,756	217,872,921	8.4674
stellar	15	3,112,925,671	91,934,242	0.1077
okb	16	3,060,275,328	7,142,494	51.0046
maker	17	2,466,883,852	86,203,403	2,666.5442
xinfin-network	18	486,122,333	6,916,214	0.0349
vechain	19	2,621,176,069	69,561,619	0.0360
monero	20	2,200,103,136	54,513,579	119.3725
theta	21	2,018,427,939	73,061,045	2.0184
thorchain	22	1,622,867,398	263,312,655	4.8381

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Table A.1.1 – *Continued from previous page*

Cryptocurrency	Index	Marketcap (USD)	Volume (USD)	Price (USD)
fantom	23	1,807,558,931	173,251,062	0.6447
fetch	24	1,718,914,021	212,768,715	2.0262
algorand	25	1,448,995,940	86,432,092	0.1780
aave	26	1,234,793,397	89,110,787	83.3719
bitcoin-sv	27	1,191,893,247	46,345,465	60.5170
quant	28	1,200,988,011	22,037,857	99.4793
singularitynet	29	1,053,562,868	90,028,222	0.8217
neo	30	1,199,323,169	171,108,470	17.0023
chiliz	31	957,825,041	88,050,082	0.1078
kucoin-shares	32	925,733,960	1,522,517	9.6611
tezos	33	895,986,302	35,713,931	0.9143
eos	34	854,215,087	185,530,450	0.7596
o-synthetix-network-token	35	868,226,973	33,196,189	2.6489
decentraland	36	795,401,532	64,477,612	0.4168
gatechain-token	37	732,430,822	9,283,478	7.5936
helium	38	846,484,823	39,383,499	5.2617
gnosis-gno	39	812,241,728	15,915,176	313.6567
nexo	40	669,954,140	6,106,459	1.1963
kava	41	699,588,864	22,044,856	0.6461
iota	42	689,143,957	14,466,053	0.2134
bitcoin-gold	43	533,007,165	7,796,073	30.4333
ftx-token	44	446,023,768	11,089,249	1.3561
xdc-network	45	485,576,086	6,919,862	0.0349
golem-network-tokens	46	493,889,941	126,267,049	0.4939
ocean-protocol	47	479,006,587	30,966,812	0.8428
iotex	48	503,861,958	19,308,467	0.0534
ankr	49	471,407,269	42,994,689	0.0471
pax-gold	50	423,743,160	10,159,561	2,299.8058
enjin-coin	51	391,997,975	20,688,875	0.2841
rocket-pool	52	392,998,653	7,072,296	19.3662
arcblock	53	221,154,859	7,412,550	2.2440
zilliqa	54	396,323,652	23,289,589	0.0228

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Table A.1.1 – *Continued from previous page*

Cryptocurrency	**Index**	**Marketcap (USD)**	**Volume (USD)**	**Price (USD)**
ravencoin	55	395,500,105	51,856,176	0.0287
0x	56	392,252,580	28,530,596	0.4628
holo	57	399,745,862	14,829,504	0.0023
aelf	58	398,766,902	64,608,755	0.5487
zcash	59	349,125,803	41,863,988	21.3817
qtum	60	385,875,416	53,555,512	3.6836
basic-attention-token	61	355,376,304	18,584,946	0.2384
loopring	62	322,775,302	14,497,589	0.2362
dash	63	330,694,308	43,134,860	28.0803
nem	64	321,027,810	11,064,405	0.0357
aragon	65	329,747,278	5,636,111	7.6366
origintrail	66	312,189,215	3,165,245	0.7696
gas	67	337,285,565	19,436,774	4.9821
reserve-rights	68	274,546,947	12,444,163	0.0054
decred	69	311,100,555	2,156,440	19.4121
zel	70	297,892,181	8,590,942	0.8609
telcoin	71	225,033,506	1,205,036	0.0026
waves	72	258,670,475	120,832,156	2.2597
harmony	73	249,350,793	10,833,486	0.0179
ontology	74	325,686,196	123,603,946	0.3721
propy	75	251,984,641	9,320,671	2.5198
swissborg	76	185,667,058	1,500,232	0.1886
chromia	77	216,259,328	12,310,162	0.2662
rlc	78	181,119,852	7,733,210	2.5023
icon	79	215,123,379	7,684,392	0.2169
wax	80	215,719,209	8,328,328	0.0627
storj	81	208,723,703	17,026,515	0.5029
band-protocol	82	201,472,798	9,113,266	1.4227
celer-network	83	186,082,112	4,601,908	0.0240
swipe	84	196,074,886	11,471,721	0.3255
digibyte	85	180,749,635	8,767,820	0.0106
iostoken	86	189,239,207	11,389,260	0.0089

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Table A.1.1 – *Continued from previous page*

Cryptocurrency	**Index**	**Marketcap (USD)**	**Volume (USD)**	**Price (USD)**
iq	87	156,052,819	10,719,036	0.0086

A.2. Cointegration test results

Table A.2.1.: Cointegration with Bitcoin test results (full version).

Cryptocurrency	**Index**	**t-statistic**	**p-value**
maker	1	−3.7022	0.0182
aragon	2	−3.6183	0.0233
aelf	3	−3.4045	0.0418
origintrail	4	−3.3945	0.0430
rlc	5	−3.2104	0.0682
tron	6	−2.8293	0.1565
dogecoin	7	−2.7894	0.1691
thorchain	8	−2.6960	0.2010
ethereum-classic	9	−2.6067	0.2341
propy	10	−2.5714	0.2485
ocean-protocol	11	−2.5242	0.2685
gnosis-gno	12	−2.4863	0.2852
gas	13	−2.4535	0.3000
singularitynet	14	−2.3783	0.3352
golem-network-tokens	15	−2.3354	0.3560
fetch	16	−2.3346	0.3564
binance-coin	17	−2.3061	0.3705
telcoin	18	−2.2800	0.3836
ethereum	19	−2.2340	0.4069
gatechain-token	20	−2.2130	0.4177
xinfin-network	21	−2.1662	0.4418
xdc-network	22	−2.1585	0.4458
waves	23	−2.0899	0.4817
iotex	24	−2.0846	0.4844

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Table A.2.1 – *Continued from previous page*

Cryptocurrency	Index	t-statistic	p-value
bitcoin-cash	25	−2.0326	0.5116
iq	26	−1.9517	0.5536
matic-network	27	−1.9095	0.5752
pax-gold	28	−1.8943	0.5829
cardano	29	−1.8895	0.5853
neo	30	−1.8827	0.5888
arcblock	31	−1.8634	0.5985
loopring	32	−1.8406	0.6098
zel	33	−1.8209	0.6195
vechain	34	−1.7942	0.6326
helium	35	−1.7928	0.6332
chromia	36	−1.7612	0.6484
bitcoin-gold	37	−1.7220	0.6668
aave	38	−1.6764	0.6876
decentraland	39	−1.6606	0.6947
celer-network	40	−1.6246	0.7104
ankr	41	−1.5950	0.7230
chainlink	42	−1.5858	0.7268
okb	43	−1.5830	0.7280
storj	44	−1.5326	0.7485
crypto-com-coin	45	−1.5156	0.7552
bitcoin-sv	46	−1.4947	0.7632
fantom	47	−1.4890	0.7654
holo	48	−1.4852	0.7668
quant	49	−1.4747	0.7708
monero	50	−1.4483	0.7805
chiliz	51	−1.3927	0.8000
iostoken	52	−1.3807	0.8041
kucoin-shares	53	−1.3490	0.8145
qtum	54	−1.3350	0.8189
theta	55	−1.3241	0.8223
wax	56	−1.3113	0.8263

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Table A.2.1 – *Continued from previous page*

Cryptocurrency	**Index**	**t-statistic**	**p-value**
eos	57	−1.3082	0.8272
ontology	58	−1.3040	0.8285
cosmos	59	−1.2954	0.8311
nexo	60	−1.2611	0.8412
ravencoin	61	−1.2474	0.8451
algorand	62	−1.2416	0.8467
swipe	63	−1.2404	0.8470
swissborg	64	−1.1823	0.8625
enjin-coin	65	−1.1418	0.8725
iota	66	−1.1361	0.8738
litecoin	67	−1.1272	0.8760
o-synthetix-network-token	68	−1.1130	0.8792
digibyte	69	−1.0995	0.8823
nem	70	−0.9996	0.9029
basic-attention-token	71	−0.9859	0.9055
hedera-hashgraph	72	−0.9521	0.9116
rocket-pool	73	−0.9433	0.9131
zcash	74	−0.9349	0.9145
ftx-token	75	−0.9313	0.9151
dash	76	−0.9309	0.9152
0x	77	−0.9212	0.9168
harmony	78	−0.9203	0.9170
stellar	79	−0.8669	0.9253
kava	80	−0.7987	0.9349
band-protocol	81	−0.7830	0.9369
tezos	82	−0.7729	0.9382
reserve-rights	83	−0.5053	0.9639
decred	84	−0.4936	0.9648
zilliqa	85	−0.2911	0.9763
icon	86	−0.2440	0.9783

A.3. Summary statistics

Table A.3.1.: Summary statistics (full version)

Variable	Index	Count	Mean (%)	Std (%)	Min (%)	Max (%)
Inflation						
T5YIE		1105	2.2206	0.6076	0.1400	3.5900
T5YIE (difference)		1104	0.00	0.05	−0.34	0.19
Returns						
bitcoin	1	1613	0.19	3.43	−36.57	18.75
ethereum	2	1613	0.29	4.44	−42.17	25.95
binance-coin	3	1613	0.34	4.94	−42.38	69.76
dogecoin	4	1613	0.61	11.40	−40.26	355.57
cardano	5	1613	0.29	5.28	−39.58	32.24
tron	6	1613	0.23	4.53	−40.93	39.69
bitcoin-cash	7	1613	0.19	5.50	−42.97	58.51
chainlink	8	1613	0.28	5.75	−45.95	31.76
matic-network	9	1613	0.46	7.21	−51.12	58.05
litecoin	10	1613	0.15	4.82	−36.03	28.20
ethereum-classic	11	1613	0.28	5.85	−40.12	42.26
hedera-hashgraph	12	1613	0.31	7.10	−44.51	102.62
crypto-com-coin	13	1613	0.23	5.14	−38.64	56.66
cosmos	14	1613	0.24	6.07	−44.47	32.44
stellar	15	1613	0.18	5.46	−33.56	74.92
okb	16	1613	0.33	5.64	−40.19	63.07
maker	17	1613	0.27	5.84	−55.52	52.62
xinfin-network	18	1613	0.44	6.70	−40.15	81.60
vechain	19	1613	0.28	5.96	−46.54	34.84
monero	20	1613	0.16	4.63	−41.39	41.19
theta	21	1613	0.40	6.34	−45.23	37.60
thorchain	22	1613	0.54	7.77	−42.50	39.29
fantom	23	1613	0.59	8.45	−52.37	51.44
fetch	24	1613	0.51	7.52	−60.67	43.54
algorand	25	1613	0.15	5.94	−47.81	51.92

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Table A.3.1 – *Continued from previous page*

Cryptocurrency	Index	Count	Mean (%)	Std (%)	Min (%)	Max (%)
aave	26	1613	0.20	5.46	−33.24	31.98
bitcoin-sv	27	1613	0.16	6.79	−42.93	141.32
quant	28	1613	0.36	6.10	−46.98	42.57
singularitynet	29	1613	0.81	15.58	−85.53	531.13
neo	30	1613	0.19	5.60	−37.36	41.27
chiliz	31	1613	0.39	7.20	−51.37	104.48
kucoin-shares	32	1613	0.28	5.40	−39.09	51.65
tezos	33	1613	0.14	5.62	−45.72	35.78
eos	34	1613	0.07	5.48	−40.40	55.21
o-synthetix-network-token	35	1613	0.29	6.95	−42.75	56.93
decentraland	36	1613	0.43	7.71	−47.37	154.74
gatechain-token	37	1613	0.29	4.93	−30.96	100.08
helium	38	1613	1.77	49.70	−45.14	1,962.86
gnosis-gno	39	1613	0.33	5.26	−34.83	39.01
nexo	40	1613	0.29	5.13	−37.36	41.73
kava	41	1613	0.18	6.41	−52.94	37.76
iota	42	1613	0.17	5.72	−42.17	47.51
bitcoin-gold	43	1613	0.28	6.36	−42.08	98.54
ftx-token	44	1613	0.27	7.26	−75.08	94.86
xdc-network	45	1613	0.43	6.50	−38.41	60.85
golem-network-tokens	46	1613	0.36	6.54	−44.96	71.34
ocean-protocol	47	1613	0.49	7.18	−54.31	55.10
iotex	48	1613	0.45	8.29	−42.22	119.97
pax-gold	49	1613	0.03	0.93	−7.48	6.68
ankr	50	1613	0.44	7.04	−47.53	47.33
enjin-coin	51	1613	0.31	6.67	−46.38	55.64
rocket-pool	52	1613	0.52	7.67	−43.34	61.64
arcblock	53	1613	0.50	8.72	−43.50	133.35
zilliqa	54	1613	0.32	6.94	−43.69	94.79
ravencoin	55	1613	0.23	6.63	−49.26	65.58
0x	56	1613	0.24	6.52	−37.27	59.55
holo	57	1613	0.26	6.47	−42.34	58.56

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Table A.3.1 – *Continued from previous page*

Cryptocurrency	**Index**	**Count**	**Mean (%)**	**Std (%)**	**Min (%)**	**Max (%)**
aelf	58	1613	0.34	6.90	−38.43	142.71
zcash	59	1613	0.14	5.57	−41.69	28.76
qtum	60	1613	0.23	6.06	−44.22	49.41
basic-attention-token	61	1613	0.17	5.66	−42.14	45.66
loopring	62	1613	0.40	7.43	−44.32	63.53
dash	63	1613	0.11	5.59	−37.22	57.04
nem	64	1613	0.16	5.70	−34.47	50.07
aragon	65	1613	0.40	6.92	−53.77	49.87
origintrail	66	1613	0.63	8.72	−48.96	135.32
gas	67	1613	0.35	7.55	−52.25	110.21
reserve-rights	68	1613	0.39	7.96	−47.10	52.63
decred	69	1613	0.15	5.75	−39.94	103.33
zel	70	1613	0.46	7.51	−33.15	78.31
telcoin	71	1613	0.47	8.35	−37.54	59.84
waves	72	1613	0.31	6.94	−38.24	70.60
harmony	73	1613	0.38	7.94	−51.74	91.54
ontology	74	1613	0.13	5.72	−45.36	46.75
propy	75	1613	0.74	11.29	−46.31	129.12
swissborg	76	1613	0.37	6.04	−38.13	41.90
chromia	77	1613	0.51	8.39	−49.42	69.97
rlc	78	1613	0.37	7.67	−53.44	106.21
icon	79	1613	0.22	6.14	−43.66	41.96
wax	80	1613	0.29	7.09	−51.37	112.41
storj	81	1613	0.37	7.88	−44.76	102.38
band-protocol	82	1613	0.40	8.17	−44.18	132.50
celer-network	83	1613	0.37	7.42	−52.20	44.95
swipe	84	1613	0.30	10.18	−69.90	280.22
digibyte	85	1613	0.23	6.51	−41.43	56.81
iostoken	86	1613	0.22	6.22	−46.58	69.62
iq	87	1613	0.36	7.68	−46.84	103.03

A.4. Augmented Dickey–Fuller test

Table A.4.1.: ADF test results for cryptocurrencies (full version).

Cryptocurrency	Index	ADF statistics before (p-value)	ADF statistics after (p-value)
Inflation			
T5YIE		-1.71 (0.43)	-29.13 (0.00)
Cryptocurrency			
bitcoin	1	-1.51 (0.53)	-18.76 (0.00)
ethereum	2	-1.70 (0.43)	-12.07 (0.00)
binance-coin	3	-1.51 (0.53)	-10.22 (0.00)
dogecoin	4	-2.78 (0.06)	-6.89 (0.00)
cardano	5	-1.88 (0.34)	-10.34 (0.00)
tron	6	-1.56 (0.50)	-9.46 (0.00)
bitcoin-cash	7	-2.46 (0.13)	-8.70 (0.00)
chainlink	8	-2.16 (0.22)	-18.69 (0.00)
matic-network	9	-1.96 (0.31)	-6.13 (0.00)
litecoin	10	-2.04 (0.27)	-15.32 (0.00)
ethereum-classic	11	-2.60 (0.09)	-7.05 (0.00)
hedera-hashgraph	12	-1.78 (0.39)	-41.56 (0.00)
crypto-com-coin	13	-2.39 (0.15)	-17.77 (0.00)
cosmos	14	-2.19 (0.21)	-18.80 (0.00)
stellar	15	-1.92 (0.32)	-11.65 (0.00)
okb	16	-1.10 (0.72)	-25.23 (0.00)
maker	17	-2.02 (0.28)	-15.77 (0.00)
xinfin-network	18	-2.05 (0.27)	-9.87 (0.00)
vechain	19	-2.70 (0.07)	-13.76 (0.00)
monero	20	-2.45 (0.13)	-19.33 (0.00)
theta	21	-2.20 (0.21)	-14.94 (0.00)
thorchain	22	-2.70 (0.07)	-40.93 (0.00)
fantom	23	-2.26 (0.18)	-16.84 (0.00)
fetch	24	-3.08 (0.03)	-11.26 (0.00)
algorand	25	-1.41 (0.58)	-17.71 (0.00)

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Table A.4.1 – *Continued from previous page*

Cryptocurrency	**Index**	**ADF statistics before (p-value)**	**ADF statistics after (p-value)**
aave	26	-1.86 (0.35)	-40.11 (0.00)
bitcoin-sv	27	-2.17 (0.22)	-19.39 (0.00)
quant	28	-1.98 (0.29)	-44.02 (0.00)
singularitynet	29	-2.32 (0.17)	-45.52 (0.00)
neo	30	-2.56 (0.10)	-15.76 (0.00)
chiliz	31	-2.66 (0.08)	-16.87 (0.00)
kucoin-shares	32	-1.84 (0.36)	-16.85 (0.00)
tezos	33	-1.86 (0.35)	-13.04 (0.00)
eos	34	-1.60 (0.48)	-14.06 (0.00)
o-synthetix-network-token	35	-1.79 (0.39)	-10.58 (0.00)
decentraland	36	-2.47 (0.12)	-19.52 (0.00)
gatechain-token	37	-1.61 (0.48)	-11.92 (0.00)
helium	38	-1.69 (0.44)	-39.72 (0.00)
gnosis-gno	39	-1.59 (0.49)	-7.56 (0.00)
nexo	40	-2.23 (0.20)	-7.76 (0.00)
kava	41	-1.63 (0.47)	-13.00 (0.00)
iota	42	-1.81 (0.38)	-42.74 (0.00)
bitcoin-gold	43	-2.46 (0.13)	-38.74 (0.00)
ftx-token	44	-1.35 (0.61)	-16.46 (0.00)
xdc-network	45	-2.22 (0.20)	-8.26 (0.00)
golem-network-tokens	46	-2.06 (0.26)	-31.66 (0.00)
ocean-protocol	47	-2.65 (0.08)	-18.80 (0.00)
iotex	48	-2.69 (0.08)	-39.26 (0.00)
pax-gold	49	-1.69 (0.44)	-45.82 (0.00)
ankr	50	-2.40 (0.14)	-7.33 (0.00)
enjin-coin	51	-2.35 (0.16)	-6.53 (0.00)
rocket-pool	52	-1.94 (0.32)	-28.84 (0.00)
arcblock	53	2.22 (1.00)	-15.01 (0.00)
zilliqa	54	-1.98 (0.29)	-16.39 (0.00)
ravencoin	55	-2.95 (0.04)	-10.27 (0.00)
0x	56	-2.18 (0.21)	-18.27 (0.00)

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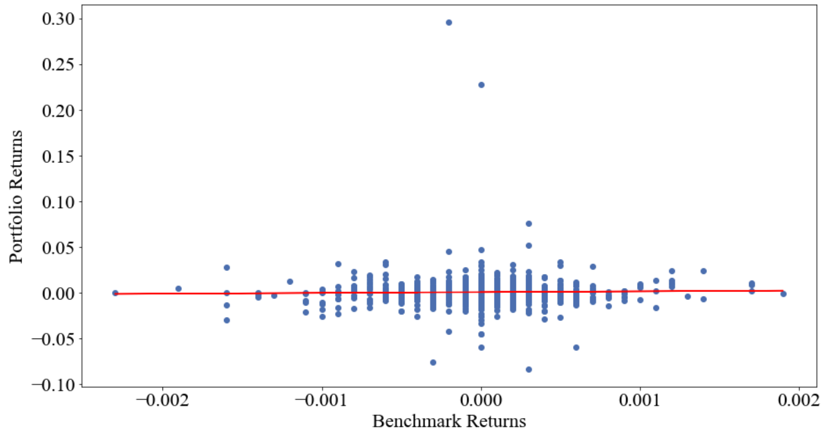
Table A.4.1 – *Continued from previous page*

Cryptocurrency	**Index**	**ADF statistics before (p-value)**	**ADF statistics after (p-value)**
holo	57	-2.76 (0.06)	-6.55 (0.00)
aelf	58	-2.31 (0.17)	-31.22 (0.00)
zcash	59	-2.02 (0.28)	-18.05 (0.00)
qtum	60	-2.37 (0.15)	-8.45 (0.00)
basic-attention-token	61	-1.90 (0.33)	-11.17 (0.00)
loopring	62	-2.99 (0.04)	-11.65 (0.00)
dash	63	-2.06 (0.26)	-18.93 (0.00)
nem	64	-1.94 (0.31)	-12.77 (0.00)
aragon	65	-2.93 (0.04)	-16.29 (0.00)
origintrail	66	-2.92 (0.04)	-19.19 (0.00)
gas	67	-2.45 (0.13)	-18.80 (0.00)
reserve-rights	68	-1.94 (0.31)	-18.14 (0.00)
decred	69	-1.55 (0.51)	-45.36 (0.00)
zel	70	-2.21 (0.20)	-26.37 (0.00)
telcoin	71	-2.21 (0.20)	-12.61 (0.00)
waves	72	-2.58 (0.10)	-12.81 (0.00)
harmony	73	-1.84 (0.36)	-12.10 (0.00)
ontology	74	-2.03 (0.27)	-15.32 (0.00)
propy	75	-2.51 (0.11)	-12.14 (0.00)
swissborg	76	-1.94 (0.32)	-9.24 (0.00)
chromia	77	-2.39 (0.14)	-25.98 (0.00)
rlc	78	-2.75 (0.07)	-10.43 (0.00)
icon	79	-1.84 (0.36)	-18.85 (0.00)
wax	80	-2.37 (0.15)	-16.37 (0.00)
storj	81	-2.41 (0.14)	-14.55 (0.00)
band-protocol	82	-2.13 (0.23)	-12.57 (0.00)
celer-network	83	-2.31 (0.17)	-12.85 (0.00)
swipe	84	-2.53 (0.11)	-9.73 (0.00)
digibyte	85	-2.40 (0.14)	-7.79 (0.00)
iostoken	86	-2.18 (0.21)	-12.96 (0.00)
iq	87	-2.43 (0.13)	-25.09 (0.00)

A.5. OLS assumptions examination. Examples for daily and monthly returns

Table A.5.1.: OLS regression analysis. Portfolio 1 (with PAX Gold) daily returns.

Scatter plot of portfolio returns vs. inflation returns (benchmark)



Autocorrelation (Ljung-Box Test)

lag	statistic	p-value
1	1.0481	0.3059
2	1.7165	0.4239
3	15.6612	0.0013
4	16.3195	0.0026
5	16.7761	0.0049
6	24.8115	0.0004
7	35.5134	0.0000
8	35.5564	0.0000
9	35.5700	0.0000
10	44.3381	0.0000

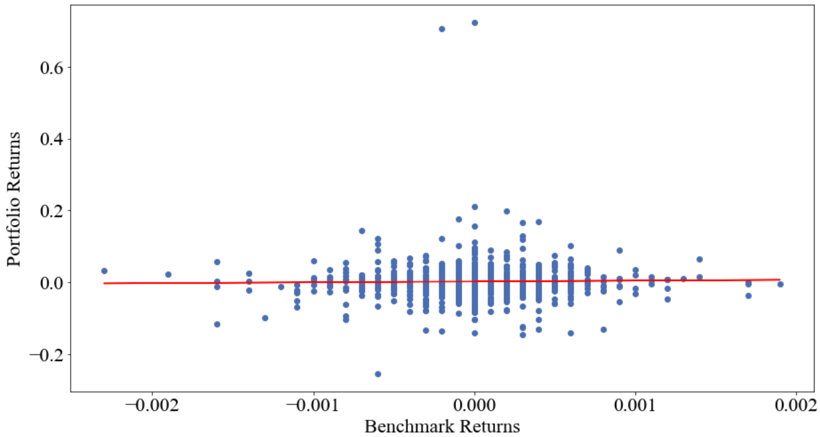
Other Tests

Test	p-value	description
Breusch-Pagan Test	0.6629	residuals are homoscedastic
Jarque-Bera Test	0.0000	residuals are not normal

(Sparsity lambda 1e-5. Rolling window 120 days.)

Table A.5.2.: OLS regression analysis. Portfolio 2 (without PAX Gold) daily returns.

Scatter plot of portfolio returns vs. inflation returns (benchmark)



Autocorrelation (Ljung-Box Test)

lag	statistic	p-value
1	0.0000	0.9983
2	13.5655	0.0011
3	17.4381	0.0006
4	17.4849	0.0016
5	18.4319	0.0025
6	19.1432	0.0039
7	19.8641	0.0059
8	20.6257	0.0082
9	22.0243	0.0088
10	30.4615	0.0007

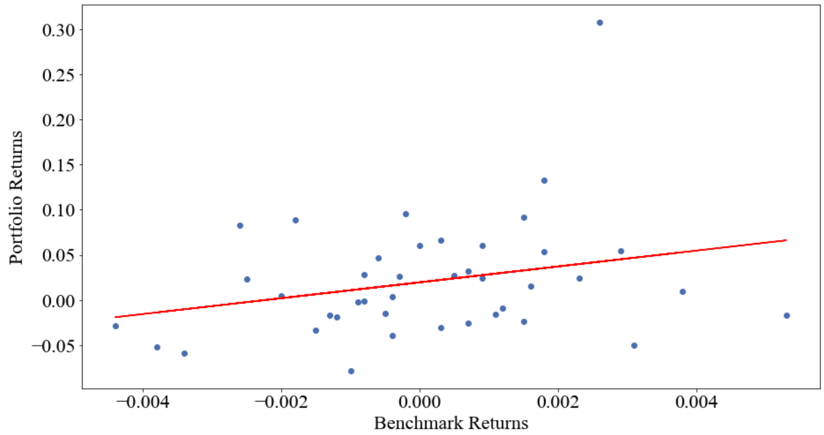
Other Tests

Test	p-value	description
Breusch-Pagan Test	0.6137	residuals are homoscedastic
Jarque-Bera Test	0.0000	residuals are not normal

(Sparsity lambda 1e-5. Rolling window 120 days.)

Table A.5.3.: OLS regression analysis. Portfolio 1 (with PAX Gold) monthly returns.

Scatter plot of portfolio returns vs. inflation returns (benchmark)



Autocorrelation (Ljung-Box Test)

lag	statistic	p-value
1	1.4384	0.2304
2	2.4080	0.3000
3	2.7941	0.4245
4	4.7943	0.3091
5	7.4331	0.1904
6	7.7061	0.2604
7	7.7266	0.3573
8	7.9475	0.4386
9	8.2364	0.5105
10	8.2430	0.6051

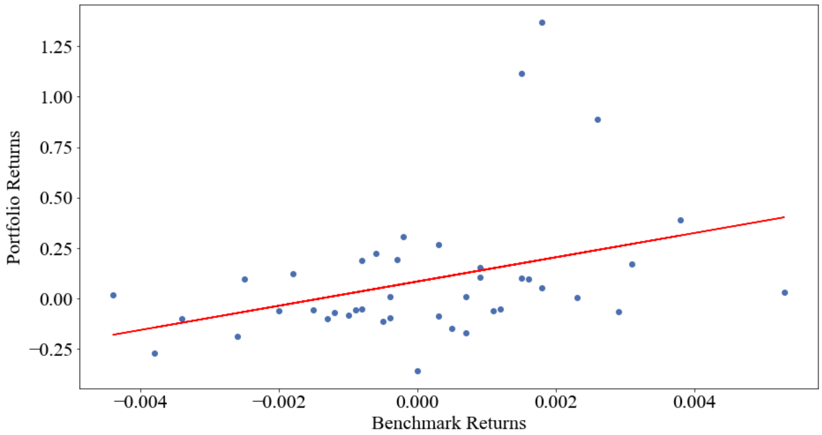
Other Tests

Test	p-value	description
Breusch-Pagan Test	0.1313	residuals are homoscedastic
Jarque-Bera Test	0.0000	residuals are not normal

(Sparsity lambda 1e-5. Rolling window 120 days.)

Table A.5.4.: OLS regression analysis. Portfolio 2 (without PAX Gold) monthly returns.

Scatter plot of portfolio returns vs. inflation returns (benchmark)



Autocorrelation (Ljung-Box Test)

lag	statistic	p-value
1	12.7205	0.0004
2	13.4704	0.0012
3	13.8782	0.0031
4	13.8787	0.0077
5	13.9093	0.0162
6	14.0176	0.0294
7	14.1797	0.0481
8	14.3325	0.0735
9	14.4831	0.1061
10	14.8217	0.1387

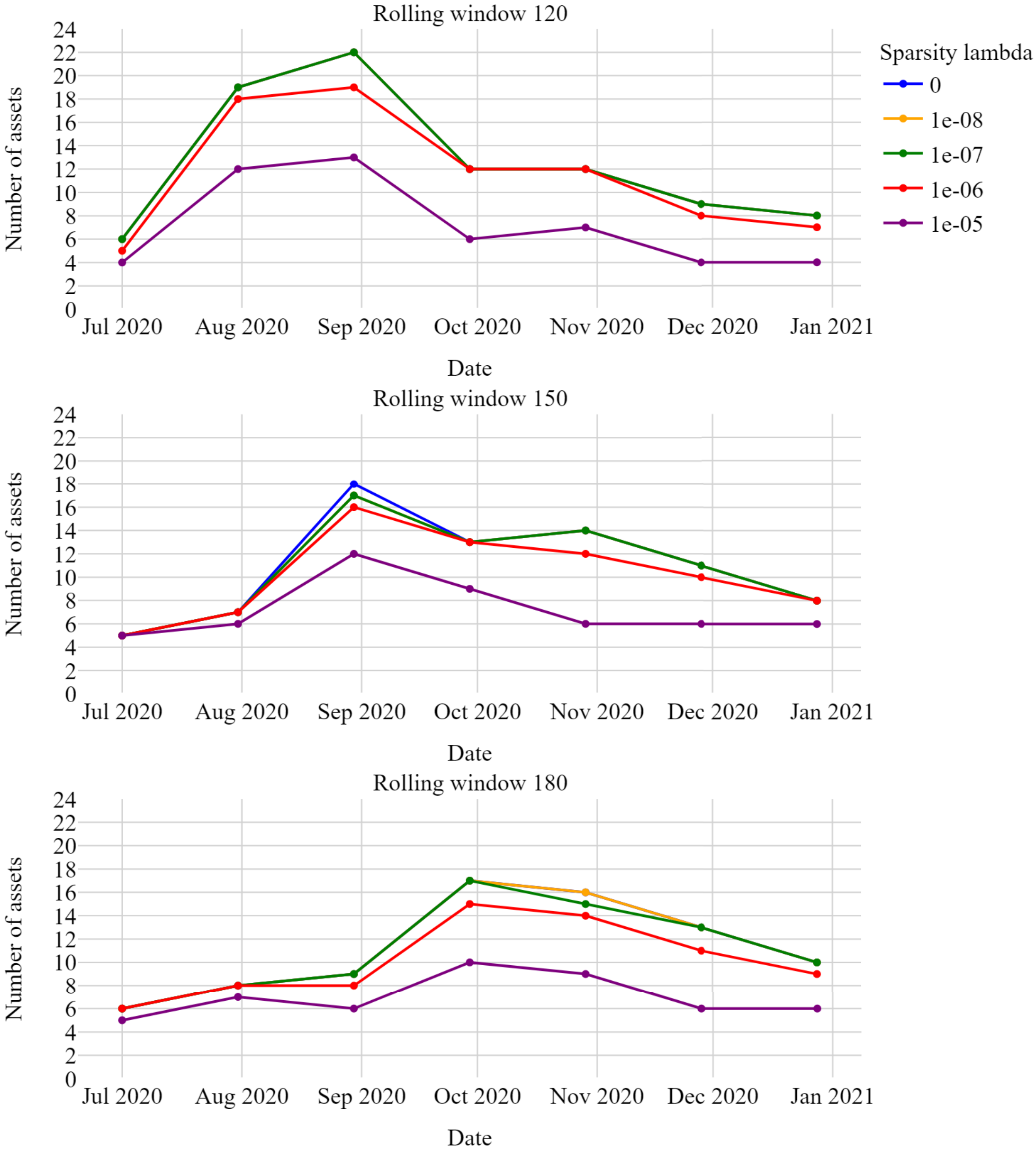
Other Tests

Test	p-value	description
Breusch-Pagan Test	0.0932	residuals are homoscedastic
Jarque-Bera Test	0.0000	residuals are not normal

(Sparsity lambda 1e-5. Rolling window 120 days.)

A.6. Performance analysis of sparsity hyperparameters

Figure A.6.1.: Number of assets selected for each rolling window (120, 150, and 180 days) according to different sparsity parameters. Portfolio 2 without PAX Gold.



A.7. Selection of sparsity parameter lambda

Figure A.7.1.: Heatmap of ETE (Validation data). Portfolio 2 without PAX Gold.

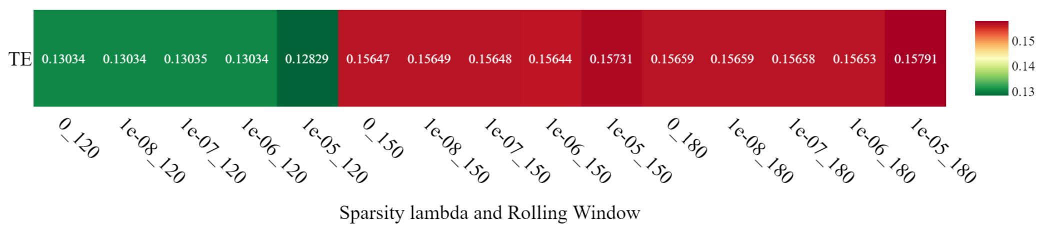


Figure A.7.2.: Heatmap of FEES (Validation data). Portfolio 2 without PAX Gold.