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Lu HAN

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Master Thesis

Unveiling Links between Inflation and The Shadow Economy: A VAR Approach

LU HAN

ON MY HONOR AS A STUDENT OF THE DIPLOMATISCHE AKADEMIE WIEN, I SUBMIT THIS WORK IN GOOD FAITH AND PLEDGE THAT I HAVE NEITHER GIVEN NOR RECEIVED UNAUTHORIZED ASSISTANCE ON IT.

Abstract

This research investigates the relationship between inflation and the size of the shadow economy using both static and dynamic econometric models. The study examines the cross-sectional correlation between inflation rates and the size of the shadow economy for the years 2007, 2008, 2009, and 2019. The characteristics of this correlation, including direction changes, are analyzed when control variables of political stability and trade openness are included, and when global economies are sub-grouped according to income levels and different time periods. In general, this cross-sectional correlation is found to be more pronounced in low-income and high-income countries. A panel fixed effects model is constructed to assess the coefficients and their significance.

Time-series data from individual countries in each subgroup are tested using a Vector Autoregressive (VAR) Model to determine the Granger Causality between the two variables. Furthermore, panel data from global economies and subgroups are analyzed using the Dumitrescu-Hurlin Panel Granger Causality Test.

The study finds strong evidence of bidirectional panel Granger Causality between inflation and the size of the shadow economy globally, and within high-income and middle-income countries. However, in low-income countries, this causality is unidirectional, running from the size of the shadow economy to inflation.

A Panel VAR (PVAR) model is subsequently developed to examine the impulse-response function(IRF) between the two variables, including the first difference of the shadow economy size (its growth rate) and the control variables. The PVAR model becomes stable, and the responses become significant when the growth rate of the shadow economy is applied. The findings indicate that a shock in inflation leads to an increase in the growth rate of the shadow economy, which then returns to normal. Similarly, a shock in the growth rate of the shadow economy elicits a significant response from inflation.

Keywords: VAR Model, Granger Causality, Shadow Economy, Inflation, Impulse-Response Function, Panel Data Analysis, Time Series Analysis

Zusammenfassung

Diese Forschung untersucht die Beziehung zwischen Inflation und der Größe der Schattenwirtschaft unter Verwendung sowohl statischer als auch dynamischer ökonometrischer Modelle. Die Studie untersucht die Querschnittskorrelation zwischen Inflationsraten und der Größe der Schattenwirtschaft für die Jahre 2007, 2008, 2009 und 2019. Die Eigenschaften dieser Korrelation, einschließlich Richtungsänderungen, werden analysiert, wenn Kontrollvariablen wie politische Stabilität und Handelsfreiheit einbezogen werden und wenn globale Volkswirtschaften nach Einkommensniveaus und verschiedenen Zeiträumen untergliedert werden.

Die Korrelation zwischen Inflation und der Größe der Schattenwirtschaft ist in Ländern mit niedrigem und hohem Einkommen ausgeprägter. Ein Panel-Fixed-Effects-Modell wird konstruiert, um die Koeffizienten und deren Signifikanz zu bewerten. Zusätzlich werden Zeitreihendaten einzelner Länder in jeder Untergruppe unter Verwendung eines Vektorautoregressiven (VAR) Modells getestet, um die Granger-Kausalität zwischen den beiden Variablen zu bestimmen. Darüber hinaus werden Paneldaten globaler Volkswirtschaften und Untergruppen mit dem Dumitrescu-Hurlin Panel Granger Kausalitätstest analysiert.

Die Studie findet starke Belege für eine bidirektionale Panel-Granger-Kausalität zwischen Inflation und der Größe der Schattenwirtschaft weltweit und in Ländern mit hohem und mittlerem Einkommen. In Ländern mit niedrigem Einkommen ist diese Kausalität jedoch unidirektional und verläuft von der Größe der Schattenwirtschaft zur Inflation. Ein Panel-VAR (PVAR) Modell wird anschließend entwickelt, um die Impuls-Antwort-Funktion zwischen den beiden Variablen zu untersuchen, einschließlich der ersten Differenz der Größe der Schattenwirtschaft (deren Wachstumsrate) und der Kontrollvariablen. Das PVAR-Modell wird stabil und die Reaktionen werden signifikant, wenn die Wachstumsrate der Schattenwirtschaft berücksichtigt wird. Die Ergebnisse zeigen, dass ein Inflationsschock zu einem Anstieg der Wachstumsrate der Schattenwirtschaft führt, die dann wieder normalisiert. Ein ähnliches Muster zeigt sich bei der Reaktion der Inflation auf einen Schock in der Wachstumsrate der Schattenwirtschaft.

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Introduction

Examining the relationship between inflation and the shadow economy holds academic significance as it offers insights into the complex interdependencies within economic systems. While the assumed correlation between these variables exists, uncovering the nuanced nature, strength, as well as the causal direction of this association remains to be elucidated. Understanding the intricacies of this relationship is important for advancing economic theory and informing policy formulation.

The shadow economy

The shadow economy's emergence and dynamics are shaped by a multitude of macroeconomic and microeconomic factors. These include national and federal monetary and fiscal policies, international economic patterns like currency exchange rates and trade openness, levels of corruption, demographic composition, international migration, institutional effectiveness, and the rule of law. Additionally, consumer preferences, motivations, and socio-cultural influences play a role. The shadow economy, in turn, impacts economic growth and development levels, as well as the effectiveness of policies and institutions, especially in developing countries. It also affects residents' welfare and living standards both in the short and long term.

To study the shadow economy is of great importance because it is a component that cannot be ignored of the world economies. According to the most recent available World Bank data online by the time of writing, the size of the informal economy world-wide (or the shadow economy, the different names and definition of the shadow economy will be discussed in the next section) is estimated to consist of 32-33 percent of global GDP and the informal employment makes up 31 percent of the total employment during the time period between 1990-2018 (Elgin et al., 2021). It is harmful to the global economy by causing the loss of tax revenue, decreasing productivity, weakening the effectiveness of employee protection regulations, defunctionalizing the official economic indicators, and more (Medina and Schneider, 2018).

It is also an insightful factor to understand the development issues all over the world because there is huge discrepancy of the share of the shadow economy between the advanced economies and the less-developed ones. For countries, investigating the situation and dynamics of the “shadow part” of their economies enables them to better design monetary and financial policies and to have more precise expectations of the effects of such policies. In terms of international organizations, such as the World Bank and the International Monetary Fund, and non-governmental organizations who are engaged with international aid, the importance of understanding the shadow market lies in the better implementation of their projects and optimization of their effectiveness (Malpass, 2023). Transnational corporations and trade companies need to get information on the shadow market conditions of their counterparts' countries when engaging in transactions. This knowledge is crucial for reducing transcation risks and preventing potential investment issues.

Inflation

Inflation is an important indicator of the health of economies, serving as a gauge for policymakers, businesses, investors, and consumers to evaluate economic conditions and make well-informed decisions. The inflation rate shows the level of stability of price levels in the economy and the change in the purchasing power of its currency over time. In response to fluctuations in the inflation rate, central banks adjust interest rates, which influence individuals' and investors' decisions on spending or saving. Governments and central banks also heavily rely on inflation data to formulate monetary and fiscal policies, such as inflation targets and the level of money supply. Furthermore, the mechanism used to measure the inflation rate is mature and has stood the test of time. As a result, inflation rate data could be considered reliable and valuable for conducting research on other variables in macroeconomics.

The relationship between inflation and the size of the shadow economy

It is intuitive to imagine a relationship between inflation and the size of the shadow economy, given their interconnected features and potential reciprocal influence. The dynamics of inflation, as a fundamental macroeconomic indicator, are intricately intertwined with the operation and expansion of informal economic activities, characteristic of the shadow economy. Conversely, the size and activities within the shadow economy have a notable impact on inflationary pressures, reflecting a symbiotic relationship where changes in one variable influences the other. It is likely that double-sided causality exists between the two variables. And since they both evolve over time, it is also possible that there is Granger causality between them, meaning the development of one variable can forecast the other's potential change.

Furthermore, studies have shown that macroeconomic variables display diverse patterns when modeled statically or dynamically. Incorporating them into a dynamic system can provide a better understanding of their interactions. Utilizing a Vector Autoregressive (VAR) Model to analyze the impulse-response function can offer valuable insights in this regard.

Overall, gaining a better understanding of Granger causality and its directions between inflation and the size of the shadow economy is crucial for policymakers to formulate effective strategies. By exerting control over one factor, they can also mitigate the impact of the other. Moreover, this understanding can also help investors in making informed decisions by enhancing their ability to forecast future phenomena.

The literature review

The definition, size and data sources of the shadow economy

The shadow economy is difficult to study because of its clandestine attribute. Many aspects of the topic are also controversial, such as the definition, the measurements of its size, and the data sources. In the vast literature, the shadow economy is also used to be called as the black economy[1], the gray economy, the hidden economy, or more neutrally, the informal economy. There are debates on whether illegal activities should be included in the definition of the shadow economy. One concept was defined that the informal economy consists of economic activities that bypass costs and are not covered by the benefits and rights outlined in laws and administrative regulations governing property

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[1] sometimes the black market is also used to call the shadow economy, but it is more often to be used to refer to the illegal trade of goods and activities such as smuggling.

relationships, commercial licensing, labor contracts, financial credit, and social systems (Feige, 1979). Schneider and Medina developed this idea and gave similar definition to the shadow or the informal economy as all the economic activities concealed from official authorities due to monetary, regulatory, and institutional reasons (Medina and Schneider, 2019). This concept is adopted in this study.

The measurement of the size of the shadow economy is difficult by the hidden nature of such economic activities. There are several methods widely used in the literature and more are being developed. There are several categories of them: the direct methods, the indirect methods, and the model-based methods.

The direct approaches mainly utilize surveys, samples derived from voluntary responses, tax audits, and other compliance techniques (Medina and Schneider, 2019). The challenge lies in the fact that the outcomes are directly influenced by how the survey questions are constructed and the willingness of the respondents to give real information, and surveys vary significantly. Moreover, employing the same parameters to gauge and compare the informal economy across different countries is difficult and usually not very accurate (Restrepo-Echavarría, 2015). Many of the surveys are arbitrary because they tend to use simple definitions and few parameters.

The indirect approaches are also called “indicator” approaches, and they are macroeconomic methods which try to use one indicator of the shadow economy to estimate the entire size of it. Some of the most used methods are discrepancy between the national expenditure and income statistics; discrepancy between official and actual labor force; the transactions approach; the currency demand approach; the physical input (electricity consumption) method (Restrepo-Echavarría, 2015).

Discrepancy between the National Expenditure and Income Statistics

In theory, GDP's income and expenditure measures should be equal, but informal activities may affect the expenditure measure more than the income measure. The expenditure side relies on formal sector value added, while the income side involves some self-reporting estimates (Restrepo-Echavarría, 2015). If those in the informal economy hide income but not expenditure, the difference between national income and expenditure could estimate the informal economy's size, assuming accurate and statistically independent expenditure components (Medina and Schneider, 2019).

Discrepancy between Official and Actual Labor Force

Assuming a constant total labor force participation, a decline in official labor force participation is suggested to indicate a rise in informal economic activity. However, this method has limitations, as changes in participation may result from various factors, such as economic downturns. Additionally, individuals often engage in both informal and formal work, making it an unreliable estimator. While interpreting a decrease in official labor force participation as an increase in the informal economy's significance, the method lacks precision due to potential confounding variables like the business cycle, job availability, education choices, and retirement decisions (Medina and Schneider, 2019).

The Transactions Approach

This approach involves using Fischer's quantity equation MV = pT, where M is money, V is velocity, p is prices, and T is total transactions. Assuming the relationship "between the money flows related to transactions and the total value added is constant", which means Prices*Transactions = k (official GDP + informal economy). Combing these two functions, Money*Velocity = k (official GDP + informal economy) is derived. The amount of money in circulation and official GDP figures are available, and it's possible to estimate money velocity. Therefore, if the proportion of the informal economy relative to the formal economy is known for a reference year, it becomes possible to calculate the informal economy for the remaining period of analysis. While theoretically appealing, this method has weaknesses, such as the arbitrary assumption of constant values over time and the influence of factors like the development of checks and credit cards on cash holdings and velocity (Medina and Schneider, 2019).

The Currency Demand Approach

This approach gauges the informal economy by correlating currency demand with tax pressure, assuming that informal activities predominantly involve cash transactions. An increase in the demand for money, triggered by a rise in the tax burden, is considered indicative of an expanding informal economy. To quantify excess money demand, economists employ econometric methods, factoring in income development, payment habits, interest rates, and related variables, along with government regulation and tax system complexity. Criticisms include the limited consideration of factors like "tax morality" and the assumption that both formal and informal economies have the same money velocity (Restrepo-Echavarría, 2015). Additionally, relying on cash payments to measure informal transactions may underestimate the informal economy's size, and increased currency demand deposits could be attributed to a slowdown in demand deposits rather than informal economic activity growth. The assumption of equal money velocity in both economies and the absence of an informal economy in a base year are also contentious points in this method (Medina and Schneider, 2019).

The Physical Input (Electricity Consumption) Method

This method relies on electricity consumption as a physical indicator of both formal and informal economic activity, assuming an elasticity close to 1 between electricity and GDP. By using electricity as a proxy for overall economic activity and subtracting official GDP estimates, an indicator of informal economic activity is obtained. The growth difference between electricity consumption and official GDP is attributed to informal economy growth. Criticisms highlight that not all informal activities require significant electricity, and efficiency improvements in electricity use can impact results. Additionally, variations in the electricity/GDP elasticity across countries and over time are considerations (Medina and Schneider, 2019). Different from using only electric power consumption, some authors developed a new physical input approach relying on the total final energy consumption, which covers inclusive forms of energy used in the economy (Postea, Achim, and Noja, 2024).

The choice of measurement method depends on the research question, with indirect approaches being suitable for macroeconomic studies, whereas direct approaches are preferred in microeconomic studies. Additionally, more recent and technically advanced model-based methods are emerging to improve the estimation of informal economy size. (Restrepo-Echavarría, 2015).

Multiple Indicators, Multiple Causes Method (MIMIC) Model

The Multiple Indicators, Multiple Causes (MIMIC) approach explicitly addresses various causes and effects of the informal economy. This methodology utilizes associations between observable causes and indicators (effects) of the unobserved variable—the informal economy—to estimate its size. It offers a comprehensive perspective, acknowledging the complexity of factors influencing the informal economy and providing a framework to estimate its size (Medina and Schneider, 2019). The MIMIC method also faces criticism from scholars. One major concern is the method's sensitivity to the selection of causative and indicative variables. The validity and reliability of estimates heavily depend on the careful choice of these variables. The diverse choices of variables contribute to different estimates, hindering the establishment of a consistent and dependable measure for the shadow economy (Postea, Achim, and Noja, 2024). Other limitations include: 1) using GDP (both GDP per capita and the growth rates) for both of the cause side and indicator variables side, 2) it needs an independent study for the base-year's estimates to calibrate the size of the informal economy as a percentage of GDP (Elgin et al., 2021).

The Dynamic General Equilibrium (DGE) Model

The DGE model, short for Dynamic General Equilibrium model, analyzes how households optimize labor allocation between formal and informal economies over time. It accounts for changes in this allocation across periods. DGE modelling stands out due to its broad coverage across countries and years, clear theoretical foundation, and its suitability for conducting policy experiments and making projections. It shares the limitations of over reliance on assumptions and the need of an independent study for calibration with other model-based measurement methods. Moreover, a computable DGE model only encompasses a portion of the characteristic features of the informal sector. The availability of data, particularly in Emerging Markets and Developing Economies (EMDEs), poses a challenge in aligning DGE models with all facets of informality (Elgin et al., 2021).

Research Potential of the Shadow Economy

There are numerous research possibilities regarding the shadow economy, covering a wide range of economic sub-fields. Consumer choices and motivations for entering the shadow economy or informal job market can be analyzed using theories of behavioral economics. How individuals assess the risks and profits of such behaviors reveals patterns related to game theory. The causes of the rise of shadow economies can be studied to evaluate the implementation of taxation policies, the impact of financial and market rigidities, and to forecast the outcomes of economic shocks. With the development of digital payment platforms and digital money, their role in curbing the growth of the shadow economy and the design of mechanisms to avert the risks posed by the black market are among the emerging topics of interest.

The interaction between the shadow economy and the formal economy is also widely studied. Some investigated the relation in terms of the effect of shadow economy and GDP growth (International Monetary Fund, 2021). Some studies are similar to this but exploring the shadow economy's influence on sustainable development (Mara, Achim, and Clement, 2023). Results of such studies show that in most of the cases, the shadow economy has a negative influence on economic growth. However, sometimes the correlation is positive, indicating that the shadow economy might provide more resilience to the formal market (Medina and Schneider, 2019).

Another popular research question is about the parallel market exchange rate premium, which refers to the difference between the official exchange rate and the black-market exchange rate. This research question is particularly important for international monetary institutions because financing projects with local currency at inflated official rates reduces available resources, limiting the reach and benefits of initiatives like cash transfers to the poor. Concerns arise about potential diversions of World Bank loan proceeds for unrelated expenditures, posing corruption risks. Increased foreign-currency debt for a given local-currency spending level adds to future debt service burdens (Malpass, 2023). Policy research on how countries can narrow the gap between the official economy and the parallel one—in other words, how to formalize the informal economy—is a key focus for many international organizations.

The relationship between the inflation rate, official exchange rate, and the parallel market exchange rate premium has been explored in many previous studies. This relationship is intricate because, in countries with high inflation rates and low inflation rates, the fluctuation of exchange rates affects price levels differently. Price levels, in turn, further affect inflation rates (Sirag, 2021). Of the top 20 countries experiencing the highest inflation rates in 2021, over 75% have parallel markets (Yacoub and Hamadeh, 2022). However, since the foreign currency black market is only a part of the overall shadow market, and the number of countries with a parallel market for foreign currencies is insufficient to infer a universal conclusion on the relationship between the inflation rate and the dynamics of the shadow economy, broader research on these variables is necessary.

More specifically, researchers have explored causal relationships between the shadow economy and various economic indicators. For example, Davidescu (2015) investigated the connection between the shadow economy and the unemployment rate, while Nikopour et al. (2009) examined its relationship with foreign direct investment. Back to the research topic, there are also many artciles about understanding how the shadow economy interacts with inflation in a broader level rather than only focusing on the parallel exchange rate premium.

The study "Inflation and Endogenous Growth in Underground Economies" researched the impact of inflation on economic growth in economies with underground sectors. By integrating a non-market good into an endogenous growth cash-in-advance economy with human capital, the study considers taxes that influence substitution into the non-market sector. Through dynamic general equilibrium analysis and extensive econometric testing on data from transition countries, the study finds that inflation affects growth in economies with shadow sectors in different levels (Cziraky and Gillman, 2004).

Similarly, "The Economic Growth–Inflation–Shadow Economy Trilogy: Developed Versus Developing Countries" explores the interaction between economic growth, inflation, and the shadow economy, controlling for political stability. Analyzing data from 33 developed and 14 developing countries between 2005 and 2016, the study finds out bidirectional links between economic growth and the shadow economy in OECD countries. In MENA countries, a bidirectional relationship is observed between inflation and the underground economy (Baklouti and Boujelbene, 2019). However, it is noted that the sample size in this study is relatively small.

Furthermore, the journal article "Taxing the Unobservable: The Impact of the Shadow Economy on Inflation and Taxation" observes a positive relationship between inflation and the size of the shadow economy, alongside a negative association between the tax burden and the shadow economy's size. These relationships depend on the independence of central banks and the chosen exchange rate regime (Mazhar and Méon, 2017). Nonetheless, although this study has a larger sample size, causality is not explicitly tested.

Methodology

The research question: the hypothesized double-sided causality between inflation and the shadow economy

The hypothesis posits that there is a double-sided causality between inflation rates and the growth of the shadow economy. This relationship is examined from both directions: how inflation influences the shadow economy and how the shadow economy impacts inflation.

Inflation Rates Cause the Growth of the Shadow Economy

Inflation erodes their own currency's purchasing power, individuals and businesses then seek immediate payments and transactions outside formal channels to maintain their economic stability. This shift is particularly pronounced when official avenues for currency exchange are restricted, increasing the demand for foreign currencies through unofficial means.

High inflation is often associated with elevated unemployment rates. Individuals who lose their jobs may turn to the shadow labor market for opportunities, where they can avoid formal regulations and taxes. The pressure on incomes due to inflation also incentivizes people to engage in the shadow economy, where they can evade taxes and potentially increase their purchasing power.

The Growth of the Shadow Economy Increases Inflation Rates

Conversely, the expansion of the shadow economy can contribute to rising inflation rates. An increase in cash transactions and unregulated money circulation within the shadow economy induces higher demand and exerts upward pressure on prices. The government's reduced revenue from tax-avoidance activities necessitates increasing the money supply to meet fiscal demands, further increasing inflation.

Shortages of goods and services in the official market, driven by the diversion of economic activities to the shadow economy, also lead to higher prices. Additionally, the distortion of economic indicators due to the unrecorded transactions in the shadow economy hampers the effectiveness of inflation-curbing policies. Policymakers rely on accurate data to formulate these policies, and the presence of a substantial shadow economy can render these measures less effective.

The research methods employing static and dynamic econometric models

This study utilizes the most recent panel data of the estimated size of the shadow economy from the Informal Database of the World Bank (Elgin et al., 2021) and the data of annual inflation rate of the same years from the research *One-Stop Source: A Global Database of Inflation* (Ha, Kose, and Ohnsorge 2023) to explore the correlation, Granger causality, and impulse-response relationship between inflation rate and the size of the shadow economy. Countries will be further subdivided based on their income levels. This approach aims to investigate the intricate relationship between the two variables on different scales.

The study proceeds in two main stages: preliminary tests and panel data empirical analysis. For the preliminary tests, examining contemporaneous correlation between the two variables across selected individual years is conducted using static cross-sectional regression models. Additionally, Granger Causality between these variables in specific countries is tested through time series analysis based on VAR (Vector Autoregression) or VECM (Vector Error Correction Model). These initial analyses aim to identify patterns, anomalies, and significant relationships that inform subsequent panel data analysis in the formal empirical study.

Data from individual countries or single time points (cross-sections) present significant limitations and fallacies that can undermine the robustness and reliability of empirical analyses. These data points often lack representativeness, as they may not adequately reflect the features of the broader scale of observations or time span, resulting in misleading generalizations. Additionally, cross-sectional data fail to capture dynamic changes over time, providing an incomplete understanding of long-term effects. They are also prone to short-term fluctuations, which can distort the results and undermine the robustness of the conclusions. Furthermore, such data cannot control for individual heterogeneity, which refers to the variability and differences in characteristics, behaviors, and conditions across the countries observed, making the result not comparable between countries and leading to potential biases. This limitation hampers the identification of general causal relationships. By using panel data, which combines both cross-sectional and time series dimensions, such issues can be addressed, unobserved heterogeneity can be controlled for, dynamic changes can be captured, and the accuracy and reliability of empirical analyses can be improved, leading to more robust and generalizable conclusions.

Therefore, for the second stage, the analysis is conducted using panel data. Firstly, as a preliminary step common to both panel and time-series data analysis, a stationarity check is performed. Once the data has been tested or processed, it becomes suitable for analyses. General correlations, Granger causality and its directions, as well as the propagation of effects throughout the system are tested using models designed for panel data. Concerning correlation, the Fixed Effects estimation model is employed. This model is particularly advantageous for addressing individual heterogeneity issues, as it eliminates the unobserved effects of variables that remain constant over time. Given that the observations in this study pertain to countries or economies worldwide, many variations among them stem from these "fixed effects." However, constructing such a regression model with an adequate number of control variables is challenging. Granger Causality between the two variables and its directions are assessed using a Dumiterscu-Hurlin Panel Granger Causality Test. Understanding Granger Causality provides insight into the ability of one variable to forecast the other, crucial for policymaking. A Panel Vector Autoregression (PVAR) model is utilized to examine the impulse-response function within the system, encompassing both two-variable and multi-variable systems.

It is equally important to bear in mind that the sequence of early and later steps in this research is not rigidly fixed, as it is possible for later stages of analysis to have statistically significant results even if the initial stages do not reveal any discernible patterns. This flexibility in the analytical process allows for the possibility that significant findings can emerge at any point, regardless of the outcomes of preceding steps.

This research employs EViews 13 as the software for data analysis.

Variables and data sources

In total, there are 5 variables selected for this research:

Informality[2]: the size of the shadow economy

Inflation: annual average inflation rate of Headline Consumer Price Inflation

Trade Openness: the share of imports and exports of total GDP

Political Stability: perceptions of the likelihood of political instability and/or politically motivated violence, including terrorism (Kaufmann, Kraay, and Mastruzzi, 2010)

Corruption Perception Index: perceived corruption and transparency in public sectors

Measurement selection and data sources for the size of the shadow economy

The dataset used in this paper for the estimation of size of the shadow economy is taken from the Informal Economy Database of the World Bank including maximum 196 economies during the time span of 1990-2020 and covering 11 most commonly used measures of informal economy of both direct and indirect, model-based approaches such as DGE and MIMIC (Elgin et al., 2021).

The selection of measurement directly impacts the outcome of empirical research on the given question. Survey-based measures concerning perceptions have typically exhibited high stability, making them more suitable for cross-country comparisons rather than longitudinal studies. However, they also have other drawbacks, such as limited data availability. In particular, in Emerging Markets and Developing Economies (EMDEs), data for certain years or countries are missing, complicating the use of such measures for the research question.

Given that the research question aims to explore the relationship between inflation and the size of the shadow economy rather than the absolute value of the size of the shadow economy, and considering that both variables evolve over time, a measurement capable of

[2] In a later stage, the first difference of the size of informality is used as the growth rate of the shadow economy

effectively capturing the dynamic evolution of the shadow economy and encompassing various aspects is required and the result of the extra study for calibration matters less. Therefore, model-based measurement methods, MIMIC or DGE are deemed more appropriate for this research.

The consistency between the two models as well as between them and the survey-based measurement methods is tested by the authors of the Informal Economy Database by examining the correlations among them. Spearman rank correlation is applied to check whether different measures align with one another in terms of capturing the level of informality within an economy, or at least, its ranking among countries. They also explore whether different measures exhibit similar trends over time. This involves calculating the proportion of country pairs where the first differences in two measures share the same direction. The results indicate that the different measurement methods for informality show a positive correlation with each other. The cross-country rank correlation between the two model-based estimates of informal output is nearly 1, and statistically significant at the 1 percent level (Elgin et al., 2021).

Apart from the calibration issue, other limitations of the MIMIC model have been examined. The concern regarding the use of GDP for both the cause side and the indicator side in the MIMIC model was tested by replacing GDP figures with night light density from GPS photos and by removing the GDP variable altogether. The results remained statistically significant (International Monetary Fund, 2021). As inflation rate has not been used to generate the model, and it contains more aspects of the shadow economy, the MIMIC model is to be used as the measurement of the size of the shadow economy in this study.

The six causal factors utilized in building the MIMIC model are: 1) size of government (comprising general government final consumption expenditure as a percentage of GDP, derived from UN data combined with WDI); 2) the proportion of direct taxation (direct taxes as a percentage of total taxation, sourced from WDI); 3) the fiscal freedom index provided by the Heritage Foundation; 4) the business freedom index, also from the Heritage Foundation; 5) unemployment rate and GDP per capita, serving to gauge economic conditions (sourced from WDI, with the latter spliced with WEO data); and 6) government effectiveness, measured using the Worldwide Governance Indicators (Elgin et al., 2021).

The three indicator variables encompass: 1) the growth rate of GDP per capita (sourced from WDI, spliced with WEO data); 2) the labor force participation rate (representing individuals over 15 who are economically active as a percentage of the population, sourced from WDI, with splicing conducted using Haver Analytics); and 3) the currency ratio of M0 (currency in circulation outside of banks) over M1 (sourced from IMF IFS and Haver Analytics) (Elgin et al., 2021).

The indicator of inflation rate and data sources

The indicator of inflation rate in this research is the value of annual average inflation rate of Headline Consumer Price Inflation taken from the research *One-Stop Source: A Global Database of Inflation* (Ha, Kose, and Ohnsorge 2023).

In this dataset, several indicators of the inflation rate are calculated using different methods and are available in both monthly and annual formats. Annual average data is utilized because the size of the shadow economy is measured annually, ensuring consistency in frequency.

The official CPI data is not employed due to its absence or unreliability in certain countries of special research interest, such as Argentina. Among the available options—HCPI (headline consumer price inflation, annual), FCPI(food price index, annual), POI (producer price index), and DEF (GDP deflator growth rate, annual)—a selection must be made based on data availability (particularly for countries of special interest), the number of observations, and the common usage of these indicators in existing studies.

Both CPI and GDP deflators are viable options, but CPI is preferred for the following reasons:

1. CPI includes goods consumed by consumers that are both domestically produced and imported. Imported goods constitute a significant part of the shadow economy.

2. The growth rate of GDP per capita (WDI, spliced with WEO) is employed to estimate the size of the shadow economy. Therefore, to minimize tautology, DEF should not be used.

The control variables

The data for Trade Openness is taken from Penn World Table database. (Feenstra, Inklaar, and Timmer, 2015). The indicator gauging political stability is sourced from the World Bank's Data Bank on Political Stability and Absence of Violence/Terrorism (Kaufmann, Kraay, and Mastruzzi, 2010). The corruption perception index is taken from @Transparency International.

It is worth noting that the Corruption Perception Index before 2012 cannot be used for analyzing trends over time. Therefore, the time span of the multi-variable panel is from 2012 to 2019.

The preliminary study

The cross-sectional model

The first step of this research is to build a cross-sectional model to examine the relationship between inflation and the size of the shadow economy in specific individual years worldwide. This step provides a straightforward overview of the relationship between these two variables across countries. However, it is necessary to further analyze panel data, as these variables are hardly ever constant over time.

Employing such a static model presents clear limitations when exploring the ceteris paribus relationship between variables like the inflation rate and the size of the shadow economy. Firstly, when the units of observation are countries, the sample size is considerably smaller compared to random sampling from the broader population, typically employed in studying income, education, or similar social science features. The second challenge of building such a regression model is finding an adequate number of control variables to address issues such as omitted variable bias and endogeneity. Previous studies on the shadow economy have used variables such as tax burden, labor market restrictions (Enste and Schneider, 2000), political stability, unemployment rate (Davidescu, 2015), foreign direct investment rate, economic growth rate, economic freedom, institutional quality (Nikopour et al., 2009), and others as control variables. However, in this research, the selection of control variables needs to meet a special requirement because the estimation of the size of the shadow economy is done using the MIMIC model, which has already selected some economic variables as the causes and indicators of the shadow economy. These variables include size of government, share of direct taxation, fiscal freedom index, business freedom index, unemployment rate, GDP per capita, government effectiveness, growth rate of GDP per capita, labor force participation rate, and currency as a ratio of M0 over M1. These should be excluded from being selected as control variables to avoid problems such as tautology (Mazhar, Ummad, and Pierre-Guillaume Méon, 2017).

Therefore, the control variables for this research are trade openness, political stability, and corruption level, according to the statistical significance derived from previous research similar to this one.

It is particularly important to examine the years marked by significant global economic events, such as the 2008 global financial crisis, as well as the most recent situation. Therefore, 2007, 2008, 2009, and 2019 are chosen to build the cross-sectional regression models. Analyzing the differences in the relationship before and after such an event can provide valuable insights for event studies on the impact of these economic shocks on the same relationship.

The models are:

$$Informality_t{}^{3} = c_0 + c_1 Inflation_t + c_2 Trade\ Openness_t + c_3 Political\ Stability_t + c_4 Corruption\ Perception\ Index_t + \varepsilon_t$$

Data stratification

For relevant research inquiries in both cross-sectional regression research and panel data analyses, data stratification is essential as the patterns of the relationship between the shadow economy and other variables can vary significantly among countries with different economic characteristics. Some of the control variables show different correlations with the dependent variable when countries are categorized by their level of development. For instance, previous research finds that the marginal effect of GDP per capita on the size of the shadow economy is statistically significant in both developed and developing countries, but it exhibits a positive correlation in developing countries and a negative correlation in developed ones. This reveals non-linearity in the relationship, and

[3] Since causality cannot be tested within cross-sectional models. Therefore, informality is only put at the left-hand side and inflation is always at the right-hand side.

it could further explain why there is no significance when analyzing the entire group of countries (Mazhar, Ummad, and Pierre-Guillaume Méon, 2017). Another study shows that the positive impact of the shadow economy is more pronounced in developed nations (Nguyen and Liu, 2023). The relationship between corruption and the shadow economy is discovered to vary between developed and developing countries (Dreher and Schneider, 2010).

There are two methods for stratifying the data: creating several subgroups based on specific criteria or adding dummy variables to the original models. Each method has its own advantages and disadvantages.

Dividing the observations into subgroups reduces the sample size, which can limit the statistical power and reliability of the estimations (Mazhar, Ummad, and Pierre-Guillaume Méon. 2017.). Additionally, multiple models need to be built for each subgroup, and these models may not be directly comparable due to differing parameters across subgroups. Adding dummy variables maintains the sample size, preserving generalizability, but it may complicate the interpretation of results. Moreover, when several parameters vary simultaneously among different groups, adding dummy variables is not a good option because the prerequisite for adding a dummy variable is "holding everything else constant."

Additionally, although adding dummy variables can be convenient for creating cross-sectional regression models, in the analysis of panel data, many differences captured by dummy variables remain constant over time. When using fixed effects estimations, such variations are eliminated.

To maintain consistency over analyses with different models in this research, the stratification method of this study is subgrouping. The countries are to be categorized into high-income, (upper and lower) middle-income, and low-income countries based on the classification of the World Bank.

Analyzing Time Series Data for Individual Countries

As mentioned, the size of the shadow economy and the inflation rate both fluctuate over time. Cross-sectional models do not effectively capture their dynamics or their interactions over time. In the second stage of the preliminary study, the dimension of time is included to study selected individual countries. Time series analysis offers an examination of country-specific dynamics, focusing on temporal evolution. However, findings may lack generalizability beyond the studied countries due to the difficulty in cross-country comparisons. Therefore, panel data will be used in the formal empirical study of this research.

The past value of one variable could influence its future value. This phenomenon, known as autocorrelation or lagged effects, is often observed in time series data. It is also important to understand how different variables evolve over time.

Stationarity

In time series analysis, the concept of stationarity is very important in understanding and modeling the behavior of a series over time. A stationary time series is characterized by constant statistical properties, including mean, variance, and autocorrelation, which remain consistent across different time intervals (Nau, 2020). The assumption of stationarity is crucial for various statistical forecasting methods including building VAR(Vector Autoregressive) model and Granger Causality Test, as it simplifies prediction by assuming that the series' future statistical properties will mirror its past behavior. Additionally, stationarity enables the calculation of meaningful sample statistics, which are essential for accurate forecasting and understanding relationships between variables (Wooldridge, 2012).

However, achieving stationarity in real-world time series data can be challenging, as many business and economic series exhibit non-stationary behavior such as trends, cycles, and random-walking, sometimes even after adjustment for stationarity. Nonetheless, techniques such as differencing and detrending can be employed to transform non-stationary series into stationary or covariance stationary ones. Unlike strict stationarity, which requires that the entire probability distribution of the time series remains unchanged over time, covariance stationarity requires that the mean, variance, and autocovariance of the time series remain constant over time, while the individual values of the time series vary. These techniques facilitate the application of regression analysis and other statistical methods for inference and prediction.

The Augmented Dickey-Fuller (ADF) Test

There are several ways to test stationarity of time series. One of the most commonly used ones is The Augmented Dickey-Fuller (ADF) Test.

The Augmented Dickey-Fuller (ADF) test is an extension of the Dickey-Fuller (DF) test, designed to address limitations related to autocorrelation in time series data. The DF test assumes a first-order autoregressive (AR) model, capturing only first-order autocorrelation. The ADF model extends the DF test by including additional lagged terms in the regression equation to capture higher-order autocorrelation in the time series data. Specifically, it augments the original DF equation with lagged differences of the series. This augmentation allows the ADF model to address potential autocorrelation issues more effectively, as it can account for higher-order autocorrelation beyond the first lag. However, determining the appropriate number of lags is a practical challenge, as including too many lags can lead to loss of degrees of freedom (Jalil and Rao, 2019).

The ADF model for testing the stationarity of the two variables in this model is:

$$\begin{aligned}\Delta Informality &= \alpha_1 + \beta_1 t + \gamma_1 Informality_{t-1} + \delta_1 \Delta Informality_{t-1} \\ &\quad + \delta_2 \Delta Informality_{t-2} + \cdots + \delta_k \Delta Informality_{t-k} + \varepsilon_t\end{aligned}$$

$$\begin{aligned}\Delta IInflation &= \alpha_2 + \beta_2 t + \gamma_2 inflation_{t-1} + \theta_1 \Delta inflation_{t-1} + \theta_2 \Delta inflation_{t-2} \\ &\quad + \cdots + \theta_k \Delta inflation_{t-k} + \mu_t\end{aligned}$$

where:

- $\Delta Informality,\ \Delta Inflation$ represent the differenced series of the size of the shadow economy and inflation rate at time t, respectively.
- α_1, α_2 are the constant terms.
- $\beta_{1,}\ \beta_2$ represent the coefficient on time trend.
- γ_1, γ_2 are the coefficients on the lagged level of the series.
- $\delta_1, \delta_2, \dots, \delta_k; \theta_1, \theta_2, \dots, \theta_k$ are the coefficients on the lagged differences of the series up to lag k.
- ε_t, μ_t are the error terms.

The null hypothesis of the ADF test is that the time series has a unit root[4], indicating non-stationarity. The alternative hypothesis is that the series is stationary. The test statistic (f-value) is compared to critical values from the Dickey-Fuller distribution to determine whether to reject the null hypothesis. Rejection of the null hypothesis by the ADF test suggests that the time series can be utilized directly for time-series data analyzing models including Granger causality testing. This type of time series is said to be integrated of order zero, denoted as I(0).

When conducting the ADF test with data analysis software, it involves testing three models: first, testing a model with both intercept and trend terms, followed by a test of a model with only the intercept term, and finally, a test of a model with no intercept or trend terms. Non-stationarity is concluded only when all three models fail to reject the null hypothesis. Conversely, if the null hypothesis is rejected in any one of the models, the time series is deemed to be stationary.

First differencing

When the null hypothesis of the ADF test cannot be rejected while testing the original time series data, a common method to process the data is to conduct first differencing. The first difference of a time series represents the changes from one period to the next. If Y_t denotes the value of the time series Y at period t, then the first difference at period t is given by Y_t - Y_{t-1}. This process often helps to remove trends and stabilize the mean of the series, making it more likely to be stationary. If the time series becomes stationary after first differencing, it is said to be integrated of order one, denoted as I(1). This means that the series itself is non-stationary, but the first difference of the series is stationary. When interpreting the relationship between first-differenced variables, it is important to bear in mind that the statistical and economic meaning has changed to reflect the relationship between the variation of the two variables.

Cointegration:

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[4] In time series analysis, a unit root refers to a characteristic of a stochastic process where the root of the characteristic equation is equal to 1. Mathematically, a unit root exists when the autoregressive coefficient in a time series model equals 1.

Even if both the time series representing the size of the shadow economy and the inflation rate are initially non-stationary but become stationary after first differencing (indicating they are both I(1)), there is a possibility of a unique relationship known as cointegration between them. Cointegration occurs when two or more non-stationary time series, which are integrated of the same order (typically I(1)), have a linear combination that is stationary (I(0)). This is crucial because it addresses the issue of spurious correlation, where two non-stationary series might appear related due to shared trends rather than a genuine relationship. Identifying cointegration confirms a legitimate long-term equilibrium relationship between the variables, thus avoiding misleading results from spurious regressions (IHSEViews, 2017).

The Engle-Granger two-step cointegration test involves performing a unit root test on the residuals of a regression equation. From the perspective of cointegration, if the dependent variable can be explained by a linear combination of the independent variables, it indicates a stable equilibrium relationship between them. The part of the dependent variable that cannot be explained by the independent variables forms a residual series, which should not exhibit serial correlation—in other words, the residuals should be stationary. The model of such test in this research can be written as:

$$inflation_t - informality_t = \mu_t$$

where μ_t represents the residual.

If two or more variables exhibit cointegration, a VAR (Vector Autoregression) model is not appropriate. Instead, a VECM (Vector Error Correction Model) is more suitable for constructing a dynamic model. Additionally, an Autoregressive Distributed Lag (ARDL) model can be used to capture both long-term and short-term equilibrium relationships (Wooldridge, 2012).

The Granger Causality Test:

The Granger Causality Test is a statistical hypothesis test employed to assess the predictive utility of one time series for another. It evaluates whether one variable can be considered a driver of changes in another variable. Essentially, it aids in establishing the existence of a causal relationship between two variables, where changes in one variable leads to changes in another. This test is valuable across various fields, including economics, finance, and social sciences, as it facilitates the comprehension of relationships between different variables and their predictive capabilities (ScienceDirect Topics, 2013).

Granger causality is based on the idea that if the past values of one variable *y* can improve the prediction of another variable *x*, then *y* is said to Granger-cause *x*. This does not necessarily imply a direct causal relationship but indicates that there is some predictive power in the historical values of *y* for forecasting *x* (Shojaie and Fox, 2022). Specifically, Granger causality does not address contemporaneous causality between *y* and *x*, meaning it cannot determine whether *x* is an exogenous or endogenous variable in an equation involving y_t and x_t. This limitation is also why Granger causality is not applicable in purely cross-sectional studies (Wooldridge, 2012).

Granger's original argument was based on identifying a unique linear model where the variables at a given time are expressed in terms of their past values (indicating autoregression). This linear modeling framework involves creating a system of equations where current values of a set of variables are explained by their own lagged values. The innovation or error terms in these models capture the new information that cannot be predicted from past values. If including the past values of *y* in this information set leads to better predictions of *x* (i.e., reduces the prediction error variance), then *y* is considered to Granger-cause *x*. This does not directly point to a cause-and-effect relationship in the traditional sense but suggests that *y* contains useful information for predicting *x* (Shojaie and Fox, 2022).

The model of such test in this research can be written as[5]:

$$inflation_t = \alpha_0 + \alpha_1 inflation_{t-1} + \alpha_2 inflation_{t-2} + \cdots + \alpha_m inflation_{t-m} + error_t$$

and comparing with the same model including lagged value of informality:

[5] if inflation and informality are stationary series I(0), or if I(1), then "inflation" and "informality" should be replaced by the growth rate of inflation and the growth rate of informality.

$$Inflation_t = \alpha_0 + \alpha_1 inflation_{t-1} + \alpha_2 inflation_{t-2} + \cdots + \alpha_m inflation_{t-m} + \beta_p informality_{t-p} + \cdots + \beta_q informality_{t-q} + error_t$$

And the opposite direction is:

$$informality_t = \gamma_0 + \gamma_1 informality_{t-1} + \gamma_2 informality_{t-2} + \cdots + \gamma_m informality_{t-n} + error_t$$

and comparing with the same model including lagged value of inflation:

$$Informality_t = \gamma_0 + \gamma_1 informality_{t-1} + \gamma_2 informality_{t-2} + \cdots + \gamma_n informality_{t-n} + \delta_j inflation_{t-j} + \cdots + \delta_k inflation_{t-k} + error_t$$

In the preliminary study stage, Granger causality between the size of the shadow economy and the inflation rate is tested within selected individual countries using their time series data. The countries are randomly chosen from each income level subgroup.

In the formal empirical study stage, the test is then applied to panel data encompassing global economies and each subgroup. Instead of using the traditional pairwise Granger Causality Test, which is more suitable for time series analysis, the Dumitrescu-Hurlin Panel Granger (Non) Causality Test is employed.

For conducting the Granger Causality Test, only stationary time series are suitable. If the original series are not stationary, differencing the series to the same order can be used to achieve stationarity. This approach was common before the development of cointegration test models, as differencing ensures stationarity and helps mitigate the risk of spurious results.

The Vector Autoregressive (VAR) Model and The Vector Error Correction Model (VECM)

There are several models for testing Granger causality, and the Vector Autoregressive (VAR) model is both traditional and among the most commonly used.

A Vector Autoregressive model is a statistical model comprising multiple equations and variables. In this model, each variable is influenced not only by its own past values but also by the past and current values of other variables. VARs offer a structured method for understanding the intricate interactions within multiple time series data, making them valuable for forecasting purposes compared to simpler univariate autoregressive models (Rossi and Wang 2019).

By simultaneously modeling multiple variables and their interdependencies, the VAR model captures complex dynamic relationships, thus mitigating the risk of endogeneity. Its ability to account for contemporaneous correlations between variables enables the identification of causal relationships without reliance on predetermined causal structures, characteristic of traditional regression models.

In a VAR model, Granger causality testing assesses whether one variable provides useful information in predicting another variable within the system. This involves estimating multiple equations simultaneously and revealing the direction of causality between variables without building another group of equations.

The model can be expressed as:

$$\begin{aligned} Inflation_t = {} & \alpha_0 + \alpha_1 inflation_{t-1} + \alpha_2 inflation_{t-2} + \cdots + \alpha_m inflation_{t-m} \\ & + \beta_1 informality_{t-1} + \cdots + \beta_p informality_{t-p} + \mu_t \end{aligned}$$

$$\begin{aligned} Informality_t = {} & \gamma_0 + \gamma_1 informality_{t-1} + \gamma_2 informality_{t-2} + \cdots \\ & + \gamma_m informality_{t-n} + \delta_1 inflation_{t-1} + \cdots + \delta_k inflation_{t-k} + \varepsilon_t \end{aligned}$$

The Vector Error Correction Model (VECM) is a VAR model with cointegration constraints, meaning it exhibits long-term stable relationships. It is often used for modeling non-stationary time series with cointegration relationships.

When building these models, choosing the right lag order is sometimes challenging. Researchers often rely on criteria like the Akaike Information Criterion (AIC), which penalizes models based on the number of parameters and selects the lag order that minimizes the AIC value. In practical data analysis scenarios, researchers may compare outcomes from different criteria and opt for a lag order deemed appropriate for their study.

If the variables are not stationary but their differenced series become stationary at the same order, cointegration tests should be performed on the original series. If cointegration is found, a VECM is suitable to test Granger causality. In this research, if two series cointegrate, both a VECM with the original series and a VAR with the same order differenced series are used for testing Granger causality. If no cointegration is found, a VAR model with the same level differenced series is employed for testing Granger causality. If two series do not become stationary at the same order, continue differencing them until the order they both achieve stationarity, and use the differenced series at that order to conduct VAR model.

Granger causality is important also for the models themselves because if causal relationships are not met, any variables failing to meet these criteria can be considered as exogenous variables. Therefore, it becomes necessary to revise the VAR model, integrating these exogenous variables into the updated model.

The formal empirical study

Panel data for inflation-informality

Panel data offers several advantages over cross-sectional or time-series data due to its unique blend of inter-individual differences and intra-individual dynamics. Firstly, panel data typically provide more accurate inference of model parameters, benefiting from increased degrees of freedom and sample variability compared to single cross-sectional or time-series data. This enhances the efficiency of econometric estimates. Secondly, panel data allows for better control of omitted variables' impact by incorporating intertemporal dynamics and individuality, thus improving the accuracy of predictions and providing micro foundations for aggregate data analysis. Lastly, panel data simplifies computation and statistical inference in certain cases, particularly in analyzing nonstationary time series, handling measurement errors, and modeling dynamic Tobit models (Hsiao, 2007).

Before applying any models, it's crucial to check the stationarity of panel data. Various methods exist for conducting panel data unit root tests, such as the LLC Test, Breitung Test, IPS Test, Fisher Type Test using ADF and PP Test, and Hadri Test. These methods differ in their assumptions and null hypotheses. For instance, the LLC and Breitung Tests assume a common unit root process, with the LLC Test testing for a unit root presence and the Breitung Test for its absence. On the other hand, the IPS and Fisher Type Test assume individual unit root processes, with the former testing for a unit root and the latter for its absence. The Hadri Test, meanwhile, hypothesizes that the panel data is free from a unit root. In practical research, combining and comparing results from multiple methods is necessary to determine whether the panel data is stationary.

Fixed Effects Estimation in Panel Data Analysis

Fixed effects estimation and random effects estimation are two common approaches employed in panel data analysis, particularly when examining observations across different countries.

The fixed effects estimator adjusts by eliminating the unobserved effect before estimation, effectively removing time-constant explanatory variables. Conversely, the random effects estimator assumes that the unobserved effect is unrelated to all explanatory variables and is random across observations. It is typically employed when the unobserved effect varies randomly across observations, as its name suggests. However, fixed effects estimation cannot be used when the explanatory variable of interest remains constant over time (Wooldridge, 2012).

When analyzing panel data with countries as observations and variables changing over time, fixed effects estimation often provides significant advantages and is generally more convincing than random effects estimation for policy analysis on aggregated data. This holds particularly true when each country is treated as a distinct unit of observation, and the sample cannot be treated as a random sample from a large population. In this study, while fixed effects estimation is anticipated to be more effective, a Hausman Test is conducted to determine whether fixed effects estimation outperforms random effects estimation.

Fixed effects estimation allows for the estimation of a unique intercept for each cross-sectional unit, thereby offering a more robust approach to controlling for unobserved heterogeneity and ensuring the reliability of estimates (Wooldridge, 2012).

General regression models with fixed effects (cross-sectional) estimation are built with the panels of this research.

The models:

$$Informality_{it} = a_{i0} + a_1 Inflation_{it} + \omega_i + \varepsilon_{it}$$

$$Informality_{it} = c_{i0} + c_1 Inflation_{it} + c_2 Trade\ Openness_{it} + c_3 Political\ Stability_{it} + c_4 Corruption\ Perception\ Index_{it} + \omega_i + \varepsilon_{it}$$

Where ω_i is the unobserved time-invariant individual effect.

The Panel Vector Autoregressive (PVAR) Model

Built on the same principles as standard VARs but with an added cross-sectional dimension, panel VARs offer a more powerful tool for tackling relevant policy questions. Panel VARs are particularly prominent in addressing contemporary issues in academia and policy-making. They can capture both static and dynamic interdependencies, handle links across units without restrictions, easily incorporate time variation in coefficients and shock variances, and account for cross-sectional dynamic heterogeneities. The current surge in empirical analyses using panel VARs in fields such as macroeconomics, banking, finance, and international economics highlights their utility (Canova and Ciccarelli, 2013). But in most of the cases, PVAR model is for I(1) panels, and if cointegration is found, PVECM is used. However, since the time period (T) for the multi-variables model is short(T=8), PVAR model is still built to study for Impulse-Response Function without testing stationarity.

Dumiterscu-Hurlin Panel Granger Causality Test

When the panel data is I(0), the Dumiterscu-Hurlin Panel Granger Causality Test can be directly ultilised for testing panel Granger causality. The Dumitrescu-Hurlin Panel Granger Causality Test is designed to determine whether one variable (x) can be used to predict another variable (y) across a panel dataset that includes multiple cross-sectional units (e.g., different countries). The test accounts for heterogeneity among these units and operates under two main hypotheses. The null hypothesis, called Homogeneous Non-Causality (HNC), posits that there is no causal relationship from x to y for any cross-sectional unit in the panel. The alternative hypothesis allows for the existence of two subgroups: one where a causal relationship from x to y exists, and another where it does not (Dumiterscu and Hurlin, 2012). When $p > 0.05$, it is said that homogeneous causality does not exist between the variables, or the presence of heterogeneous causality. Conversely, when $p < 0.05$, it says that homogeneous causality exists among the variables.

To perform the test, individual Granger causality tests are conducted for each unit, and the resulting Wald statistics are averaged across the panel. This average is then compared to a standard normal distribution under the assumption of cross-sectional independence. The test can handle unbalanced panels and different lag orders, and a block-bootstrap procedure is employed to address cross-sectional dependence. This method offers a straightforward and effective way to assess causality in panel data, enhancing the power of Granger causality tests even for small samples. However, it does not indicate the specific units where causality is present, and its asymptotic properties assume cross-sectional independence (Dumiterscu and Hurlin, 2012).

Impulse-Response Function (IRF)

Impulse-Response Functions (IRFs) offer insights into how variables react to external shocks and evolve over time within a dynamic system studied using a VAR model when Granger causality between the variables is identified.

In dynamic systems, variables are not isolated; they are all interconnected, with each variable's behavior influenced by the historical evolution of others within the system. IRFs allow to quantify how a sudden shock to one variable ripples through the system, affecting the behavior of other variables over time, capturing not just the immediate impact but also how the shock persists and spreads throughout the system.

The PVARs are further tested with the IRF function.

Analysis

The preliminary study

Cross-sectional regression models:

$$
\begin{aligned}
Informality_t &= c_0 + c_1 Inflation_t + c_2 Trade\ Openness_t + c_3 Political\ Stability_t \\
&\quad + c_4 Corruption\ Perception\ Index_t + \varepsilon_t
\end{aligned}
$$

Where t=2007, 2008, 2009, 2019.

Multicollinearity is identified after adding the variable of corruption level. Therefore, in this stage, this variable is removed from the model.

When t = 2007, the regressions are run with all countries (149 observations), high-income countries (48 observations), middle-income countries (68 observations), and low-income countries (41) countries, respectively. The results are shown in the figures and tables below.

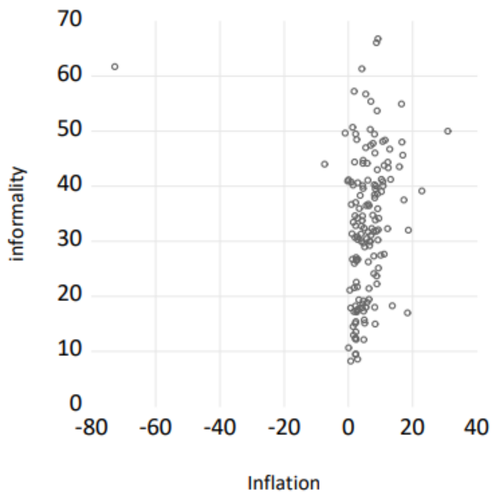

Figure 1- scatter plot between informality-inflation (for all countries 2007)

Table 1 - correlation between informality and inflation 2007 (all countries)

Dependent variable	Informality	
Inflation	0.0586	-0.0784
	(0.1320)	(0.1122)
Trade openness		-0.0617***
		(0.0230)
Political stability		-5.7041***
		(1.1438)
Corruption perception index		
Constant	32.4914***	36.2967***
	(1.3024)	(1.9270)
Observations	149	149
R-Squared	0.0013	0.3094

Number of countries	149	149

standard errors in parentheses. ***p<0.01, **p<0.05, *p<0.1.

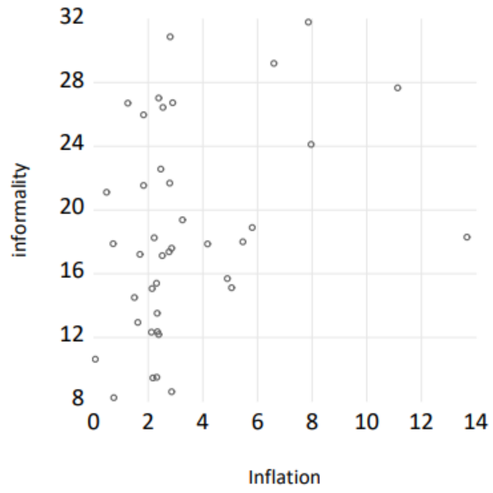

Figure 2 - scatter plot between informality-inflation within High-income countries (2007)

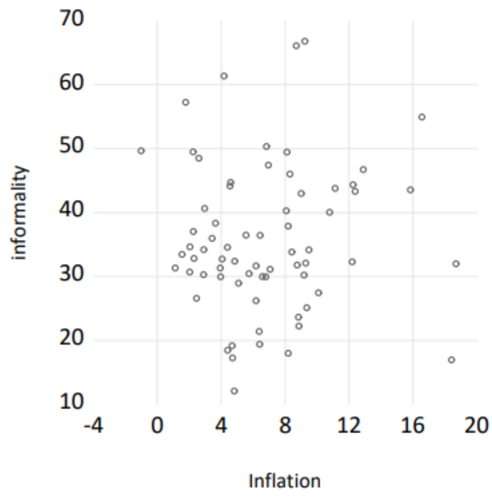

Figure 3 - scatter plot between informality-inflation within Middle-income countries (2007)

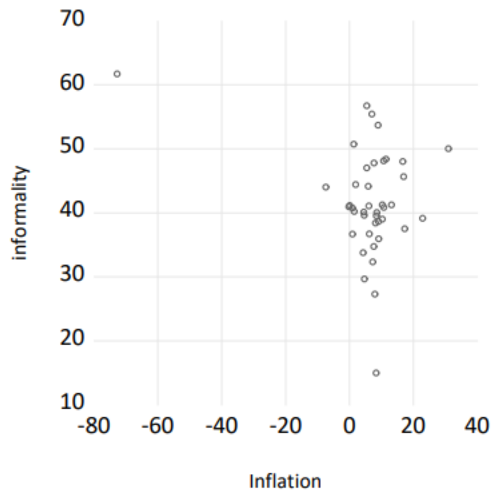

Figure 4 - scatter plot between informality-inflation within Low-income countries (2007)

Table 2 - correlation between informality and inflation 2007 (within subgroups)

Dependent variable	High-income Informality		Middle-income Informality		Low-income Informality	
Inflation	0.7968**	0.7958**	0.1152	0.1265	-0.1757*	-0.2108**
	(0.3490)	(0.3243)	(0.3470)	(0.3497)	(0.0893)	(0.0977)
Trade openness		0.0141		0.0531		-0.0959
		(0.0172)		(0.0501)		(0.1054)
Political stability		-4.6168***		-3.2879		-0.7255
		(1.6215)		(2.0456)		(1.4948)
Corruption perception index						
Constant	15.9918***	17.9275***	35.0368***	31.2565***	42.6888***	44.9641***
	(1.5169)	(2.4083)	(2.7458)	(4.2284)	(1.3583)	(3.6759)
Observations	40	40	68	68	41	41
R-Squared	0.1206	0.2823	0.0013	0.0460	0.0904	0.3094
Number of countries	40	40	68	68	41	41

standard errors in parentheses. ***p<0.01, **p<0.05, *p<0.1.

From the scatter plots, it is plausible that a positive correlation between inflation and informality exists within each group. However, the statistics show that in 2007:

1. Across all countries, whether control variables are added or not, the correlation is not significant, and the explanatory power remains notably weak. However, there is a change in direction.

2. The correlation is significant at the 5% level within high-income countries when no variables are controlled. When trade openness and political stability are controlled, the correlation is still significant at the 5% level. This indicates that within high-income countries, every one percent increase in the inflation rate corresponds to a 0.7968 percent increase in the size of the shadow economy when no variables are controlled, or a 0.7958 percent increase when controlled for trade openness and political stability.

3. Within middle-income countries, whether control variables are added or not, the correlation is not significant, and the explanatory power is also notably weak.

4. The correlation is significant at the 10% level within low-income countries when no variables are controlled. When trade openness and political stability are controlled, the correlation is significant at the 5% level. This indicates that within low-income countries, every one percent increase in the inflation rate corresponds to a 0.1757 percent decrease in the size of the shadow economy when no variables are controlled, or a 0.2108 percent decrease when controlled for trade openness and political stability. Overall, the findings indicate that in 2017, the correlation is more pronounced in both high-income and low-income countries, with effects observed in differing directions. This discrepancy could potentially explain why the correlation across all countries is not significant, as the effects may cancel each other out.

When $t = 2008$, the regressions are run with all countries (149 observations), high-income countries (41 observations), middle-income countries (70 observations), and low-income countries (38 observations), respectively. It is noticeable that the size of the subgroups changed. Some countries moved from one group to another. Therefore, such countries are removed when time series and panel data analyses are applied. The results are shown in the figures and tables below.

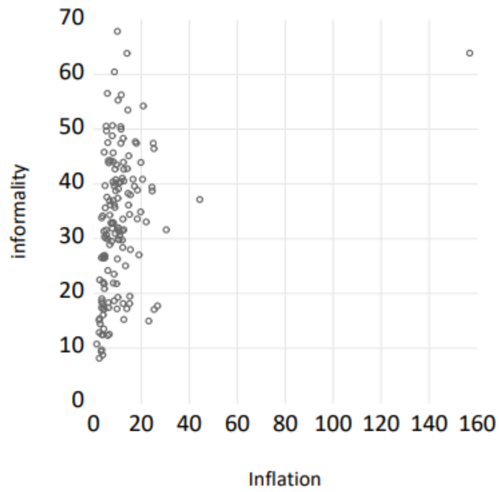

Figure 5 - scatter plot between informality-inflation (for all countries 2008)

Table 3 - correlation between informality and inflation 2008 (all countries)

Dependent variable	Informality	
Inflation	0.2933***	0.1722**
	(0.0739)	(0.0666)
Trade openness		-0.0422**
		(0.0207)
Political stability		-5.3510***
		(1.1453)
Corruption perception index		
Constant	29.4231***	32.7073***
	(1.3013)	(1.8538)
Observations	149	149
R-Squared	0.0968	0.3254
Number of countries	149	149

standard errors in parentheses. ***p<0.01, **p<0.05, *p<0.1.

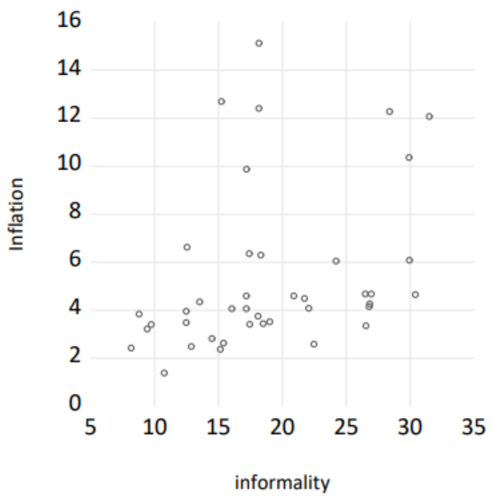

Figure 6 - scatter plot between informality-inflation within High-income countries (2008)

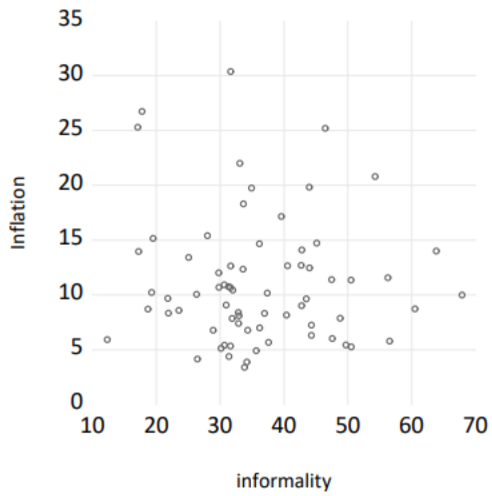

Figure 7 - scatter plot between informality-inflation within Middle-income countries (2008)

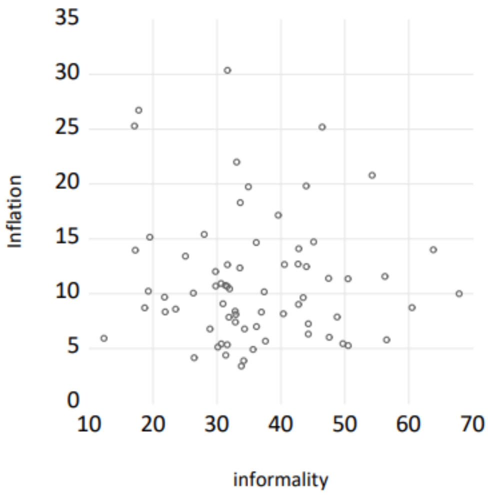

Figure 8 - scatter plot between informality-inflation within Low-income countries (2008)

Table 4 - correlation between informality and inflation 2008 (within subgroups)

Dependent variable	High income informality		Middle income informality		Low income informality	
Inflation	0.6538**	0.5763**	-0.1239	-0.1183	0.1304**	0.1722***
	(0.2951)	(0.2777)	(0.2416)	(0.2347)	(0.0521)	(0.0542)
Trade openness		0.0073		0.0881*		-0.2252**
		(0.0152)		(0.0490)		(0.1060)
Political stability		-4.5792***		-4.1544**		1.2393
		(1.6852)		(1.8289)		(1.6079)
Corruption perception index						
Constant	15.4657***	18.4153***	37.5023***	31.0161***	38.8693***	45.8486***
	(1.8665)	(2.5451)	(3.0168)	(4.2994)	(1.5508)	(3.6061)
Observations	41	41	70	70	38	38
R-Squared	0.1118	0.2617	0.0039	0.0902	0.1483	0.2505
Number of countries	41	41	70	70	38	38

standard errors in parentheses. ***p<0.01, **p<0.05, *p<0.1.

The scatter plots also show some positive relationship between the two variables. Comparing with 2007, there are no countries (as observations) with negative inflation rate anymore. The statistics show that in 2007:

1. Across all countries, the correlation appears positive and statistically significant. The absence of control variables results in a significant correlation at the 1% level, while controlling for trade openness and political stability maintains significance at the 5% level. This suggests that globally, a one percent rise in the inflation rate corresponds to a 0.2933 percent increase in the shadow economy when no variables are controlled, or a 0.1722 percent increase when controlling for trade openness and political stability.

2. Significant correlation is observed at the 5% level among high-income countries without control variables. Even with the inclusion of trade openness and political stability as controls, the correlation remains significant at the 5% level. Within high-income countries, each one percent increase in the inflation rate corresponds to a 0.6538 percent increase in the shadow economy without controls, or a 0.5763 percent increase when controlling for trade openness and political stability.

3. In middle-income countries, regardless of the addition of control variables, the correlation does not reach significance, and the explanatory power remains notably weak.

4. Within low-income countries, a significant correlation is found at the 5% level without control variables. When controlling for trade openness and political stability, significance drops to the 10% level. This implies that within low-income countries, a one percent increase in the inflation rate corresponds to a 0.1304 percent increase in the shadow economy without controls, or a 0.1722 percent increase when controlling for trade openness and political stability. The direction of correlation in low-income economies mirrors that of high-income countries and the global trend.

Overall, the findings suggest that in 2017, the correlation remains more pronounced in both high-income and low-income countries, with effects observed consistently. This disparity may stem from the shock of the global financial crisis this year.

When t = 2009, the regressions are run with all countries (150 observations), high-income countries (44 observations), middle-income countries (71 observations), and low-income countries (35 observations), respectively. The results are shown in the figures and tables below.

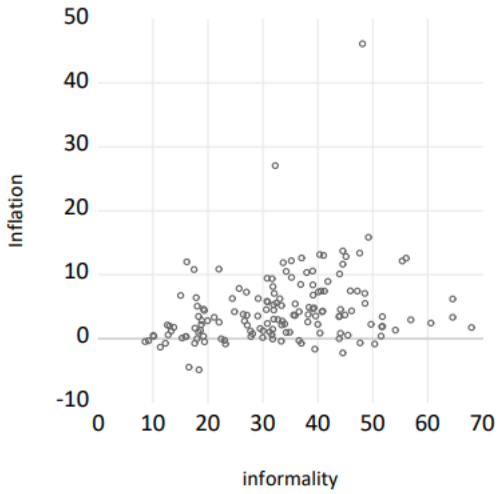

Figure 9 - scatter plot between informality-inflation (for all countries 2009)

Table 5 - correlation between informality and inflation 2008 (all countries)

Dependent variable	informality	
Inflation	0.6247***	0.2235
	(0.1779)	(0.1727)
Trade openness		-0.0573**
		(0.0262)
Political stability		-4.7039***
		(1.2147)
Corruption perception index		
Constant	30.2446***	34.4752***
	1.2918	(1.9835)
Observations	150	150
R-Squared	0.0769	0.2667
Number of countries	150	150

standard errors in parentheses. ***p<0.01, **p<0.05, *p<0.1.

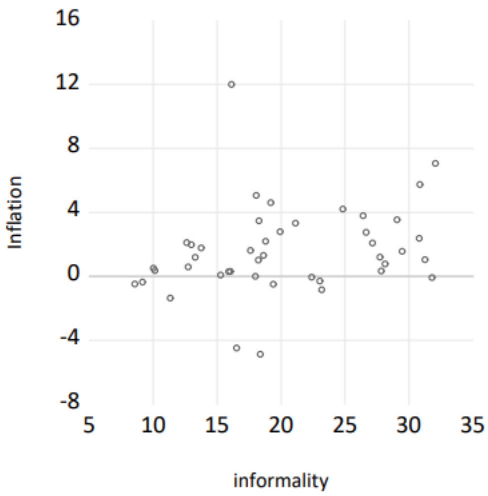

Figure 10 - scatter plot between informality-inflation within High-income countries (2009)

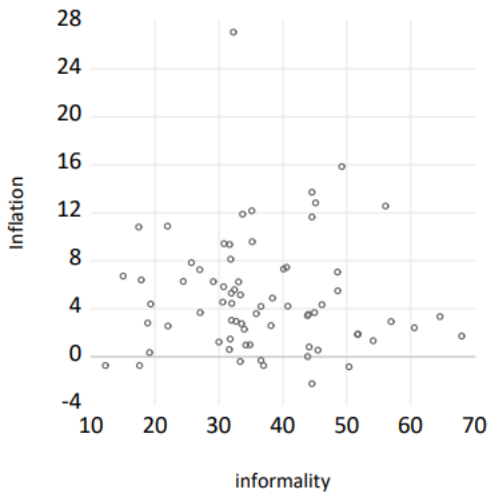

Figure 11 - scatter plot between informality-inflation within Middle-income countries (2009)

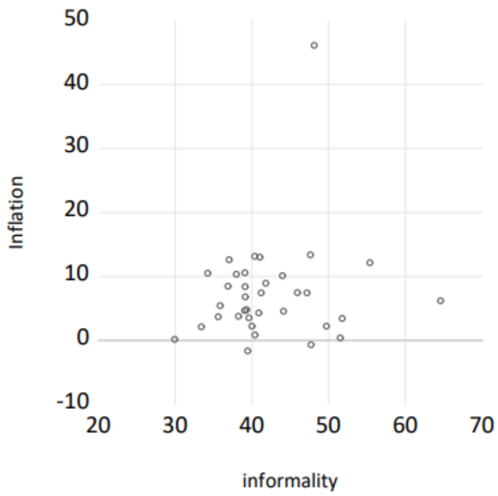

Figure 12 - scatter plot between informality-inflation within Low-income countries (2009)

Table 6 - correlation between informality and inflation 2009 (within subgroups)

Dependent variable	High income informality		Middle income informality		Low income informality	
Inflation	0.6342*	0.4611	-0.1033	-0.1349	0.1284	0.1726
	(0.3709)	(0.3835)	(0.2935)	(0.2945)	(0.1478)	(0.1599)
Trade openness		0.0020		0.0691		0.0521
		(0.0217)		(0.0574)		(0.1219)
Political stability		-2.8567		-2.9435		1.1589
		(1.8302)		(1.8942)		(1.6710)
Corruption perception index						
Constant	19.2810***	21.3183***	37.0472***	32.9084***	41.2698***	40.4497***
	(1.1779)	(2.3824)	(2.0381)	(3.6202)	(1.5885)	(3.9151)
Observations	44	44	71	71	35	35
R-Squared	0.0651	0.1226	0.0018	0.0427	0.0224	0.0437
Number of countries	44	44	71	71	35	35

standard errors in parentheses. ***p<0.01, **p<0.05, *p<0.1.

The pattern of correlation appears notably different in 2009. While scatter plots still suggest a positive relationship, significant correlation is observed only when no variables are controlled, with overall and high-income groups showing significance at the 5% and 10% levels, respectively. Though direct comparisons of cross-sectional regression models across years are challenging, it is plausible that this difference reflects the enduring effects of the shock experienced one year prior.

When t = 2019, the regressions are run with all countries (148 observations), high-income countries (50 observations), middle-income countries (76 observations), and low-income countries (22 observations), respectively. In this year, Venezuela experiences an unusually high inflation rate due to its presidential crisis. Similarly, Zimbabwe faces significant inflationary pressure stemming from past crises. For the sake of consistency and generalizability in the research, these observations are removed. The results are shown in the figures and tables below.

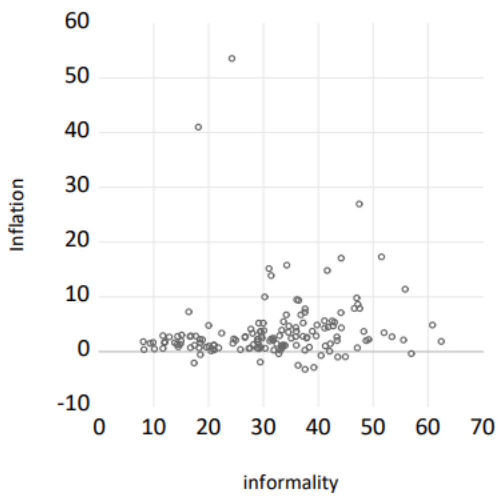

Figure 13 - scatter plot between informality-inflation (for all countries 2009)

Table 7 - correlation between informality and inflation 2019 (all countries)

Dependent variable	informality	
Inflation	0.2224	-0.0336
	(0.1484)	(0.1323)
Trade openness		-0.0580**
		(0.0246)
Political stability		-5.3832***
		(1.2141)
Corruption perception index		
Constant	30.5674***	34.2514***

	(1.1342)	(1.8555)
Observations	148	148
R-Squared	0.0151	0.2833
Number of countries	148	148

standard errors in parentheses. ***p<0.01, **p<0.05, *p<0.1.

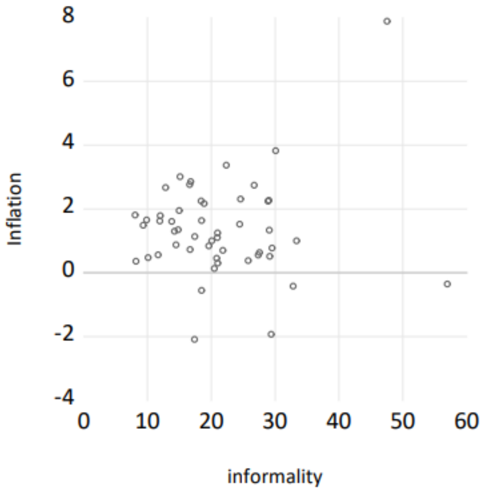

Figure 14 - scatter plot between informality-inflation within High-income countries (2019)

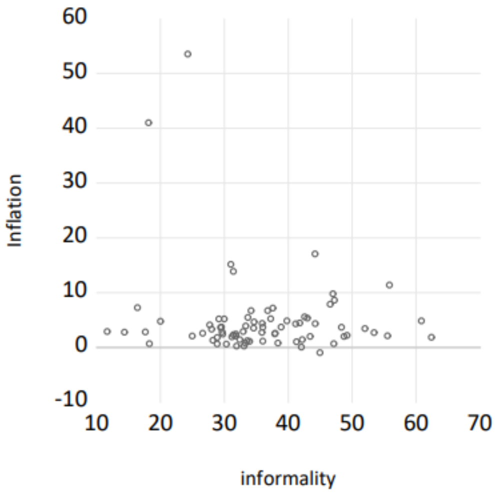

Figure 15 - scatter plot between informality-inflation within Middle-income countries (2019)

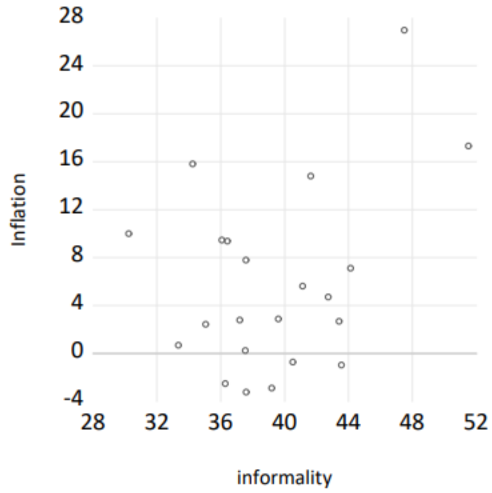

Figure 16 - scatter plot between informality-inflation within Low-income countries (2019)

Table 8 - correlation between informality and inflation 2019 (within subgroups)

Dependent variable	High income informality		Middle income informality		Low income informality	
Inflation	0.7047	1.0463	-0.2056	-0.2897*	0.2246	0.1796
	(0.8931)	(0.9105)	(0.1518)	(0.1560)	(0.1337)	(0.1457)
Trade openness		-0.0170		-0.0215		0.0579
		(0.0285)		(0.0531)		(0.0622)
Political stability		-3.8963		-3.4539*		0.4576
		(2.8613)		(2.0005)		(1.4399)
Corruption perception index						
Constant	20.2555***	23.9189***	36.8738***	36.8583***	38.0530***	36.9692***
	(1.8028)	(3.1238)	(1.3980)	(3.1149)	(1.2737)	(2.9499)
Observations	50	50	76	76	22	22
R-Squared	0.0128	0.0733	0.0242	0.0803	0.1236	0.1753
Number of countries	50	50	76	76	22	22

standard errors in parentheses. ***p<0.01, **p<0.05, *p<0.1.

In 2019, the correlation is only significant within middle-income countries when controlling for trade openness and political stability, at a 10% significance level. This finding stands in stark contrast to the patterns observed in previous years.

It may be concluded that cross-sectional models cannot depict stable, general relationships between variables that evolve over time. Such models fail to capture dynamic changes and lack predictive power for the future.

Time Series Analyses:

Countries from each income level subgroup are randomly selected for time series analyses spanning from 1998 to 2019, focusing on variables of inflation and informality.

These countries are Angola (AGO), Argentina (ARG), Armenia (ARM), Austria (AUT), Brazil (BRZ), Ghana (GHA), New Zealand (NZL), Nigeria (NGA), Norway (NOR), Russia (RUS), and Venezuela (VEN).

The graphs of the two variables are illustrated for Armenia as an exemple:

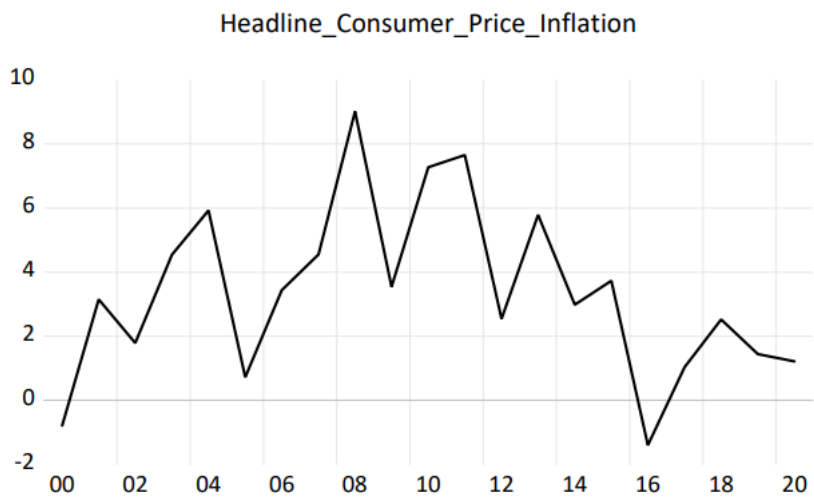

Figure 17 - Armenia, inflation

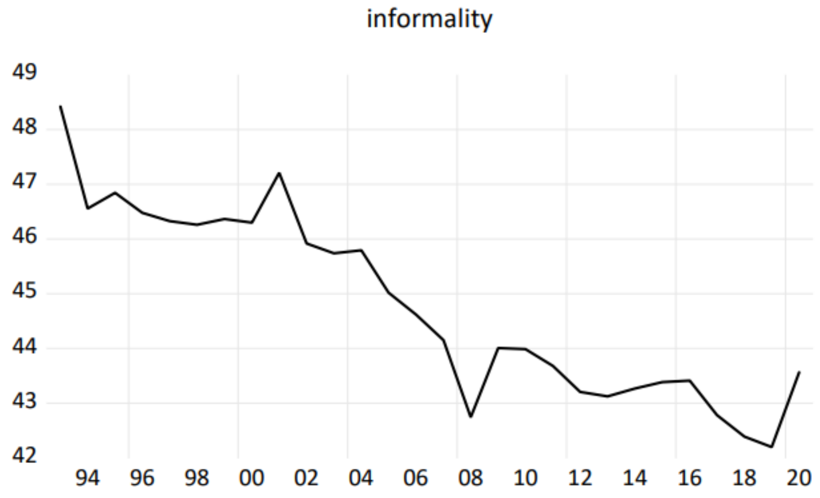

Figure 18 - Armenia, informality

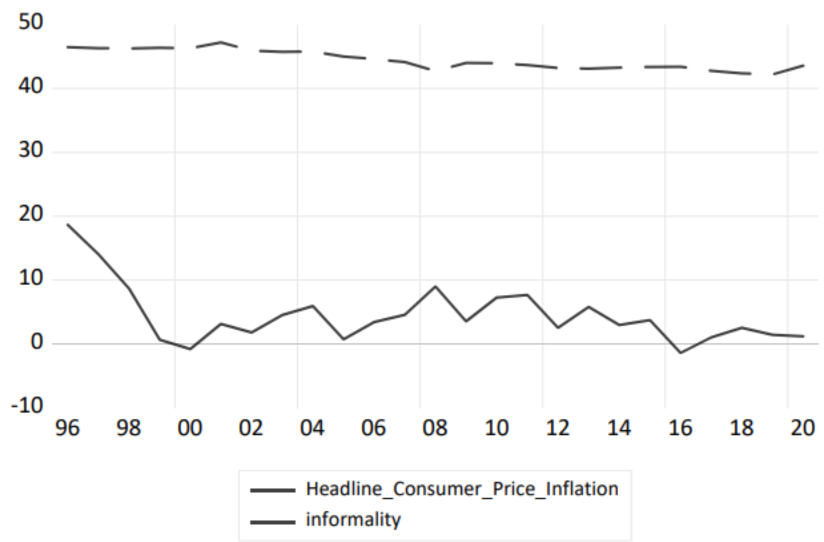

Figure 19 - Armenia, combined

The graphs roughly indicate that the magnitude of fluctuations between the two variables differs, with inflation exhibiting much larger fluctuations[6]. Additionally, the variation in informality suggests a potential trend over time.

The outcomes of stationarity checks, cointegration assessments, models examining Granger causality, and the results of their corresponding Granger Causality Tests of the selected countries are presented in the two tables below. The significance thresholds for the p-values are set at 0.05 and 0.10, respectively.

Table 9 - time series analyses result (threshold for p value: 0.05)

Country Code	Stationarity of inflation rate	Stationarity of informality	Cointegration between inflation and informality	VAR or VECM for Granger Causality Test	Optimal lag interval	Granger Causality between inflation and informality
ARG	I(1), 1% significance level	I(1), 5% significance level	No	VAR with first-differenced series	1	the variation of inflation does not Granger cause the variation of informality; the variation of informality does not Granger cause the variation of inflation
NZL	I(0), 5% significance level	I(2), 1% significance level	N/A different order	N/A, second difference of informality does not have economic significance	N/A	N/A
NOR	I(0), 1% significance level	I(1), 5% significance level	N/A different order	VAR with first-differenced series	1	the variation of inflation does not Granger cause the variation of informality; the variation of informality does not Granger cause the variation of inflation
AUT	I(0), 1% significance level	I(1), 5% significance level	N/A different order	VAR with first-differenced series	2	the variation of inflation does not Granger cause the variation of informality; the variation of informality does not Granger cause the variation of inflation
RUS	I(0), 1% significance level	I(1), 5% significance level	N/A different order	VAR with first-differenced series	0	N/A

[6] They can be compared directly because both are expressed as percentages.

BRZ	I(0), 5% significance level	I(1), 5% significance level	N/A different order	VAR with first-differenced series	1	the variation of inflation does not Granger cause the variation of informality; the variation of informality does not Granger cause the variation of inflation
VEN	Near singular matrix, it needs to be removed from the panel	I(1), 5% significance level	N/A	N/A	N/A	N/A
GHA	I(2), 5%	I(1), 5% significance level	N/A different order	N/A, second difference of informality does not have economic significance	N/A	N/A
NGA	I(1), 5% significance level	I(2), 1% significance level	N/A different order	N/A, second difference of informality does not have economic significance	N/A	N/A
AGO	I(0), 1% significance level	I(2), 1% significance level	N/A different order	N/A, second difference of informality does not have economic significance	N/A	N/A
ARM	I(1), 1% significance level	I(1), 1% significance level	Yes	VECM, with original series, and VAR with first-differenced series	VECM, original 2; VAR first difference 1	VECM: inflation does not Granger cause informality, informality does not Granger cause inflation; VAR: the variation of inflation does not Granger cause the variation of informality; the variation of informality Granger causes the variation of inflation, at 5% significance level

Table 10 - time series analyses result (threshold for p value: 0.10)

Country Code	Stationarity of inflation rate	Stationarity of informality	Cointegration between inflation and informality	VAR or VECM for Granger Causality Test	Optimal lag interval	Granger Causality between inflation and informality

ARG	I I(0), 10% significance level	I(1), 5% significance level	N/A different order	VAR with first-differenced series	1	the variation of inflation does not Granger cause the variation of informality; the variation of informality does not Granger cause the variation of inflation
NZL	I(0), 5% significance level	I(1), 10% significance level	N/A different order	VAR with first-differenced series	1	The variation of informality Granger causes the variation of inflation, at 5% significance level; the variation of inflation does not Granger cause the variation of informality
NOR	I(0), 1% significance level	I(1), 5% significance level	N/A different order	VAR with first-differenced series	1	the variation of inflation does not Granger cause the variation of informality; the variation of informality Granger causes the variation of inflation at 10% significance level
AUT	I(0), 1% significance level	I(1), 5% significance level	N/A different order	VAR with first-differenced series	2	the variation of inflation does not Granger cause the variation of informality; the variation of informality does not Granger cause the variation of inflation
RUS	I(0), 1% significance level	I(0), 10% significance level	N/A	VAR with original series	1	inflation Granger causes informality at 1% significance level, informality Granger causes inflation at 1% significance level
BRZ	I(0), 5% significance level	I(1), 5% significance level	N/A different order	VAR with first-differenced series	1	the variation of inflation does not Granger cause the variation of informality; the variation of informality Granger causes the variation of inflation at 10% significance level
VEN	Near singular matrix, it needs to be removed from the panel	I(1), 5% significance level	N/A	N/A	N/A	N/A

GHA	I(1), 10%	I(1), 5% significance level	Yes	VECM, with original series, and VAR with first-differenced series	VECM:2 VAR:0	VECM: inflation does not Granger cause informality, informality does not Granger cause inflation; VAR: N/A
NGA	I(1), 5% significance level	I(1), 10% significance level	No	VAR with first-differenced series	0	N/A
AGO	I(0), 1% significance level	I(2), 1% significance level	N/A different order	N/A, second difference of informality does not have economic significance	N/A	N/A
ARM	I(0), 10% significance level	I(0), 1% significance level	N/A	VAR with original series	1	inflation does not Granger cause informality, informality does not Granger cause inflation

The results show that stationarity, cointegration, and Granger causality between the two variables vary significantly across these countries. Therefore, panel data analyses are needed to generalize the relationship between the two variables. Additionally, Granger causality is not always observed within these countries.

The formal empirical study

Graphically illustration of the relationship

Overall

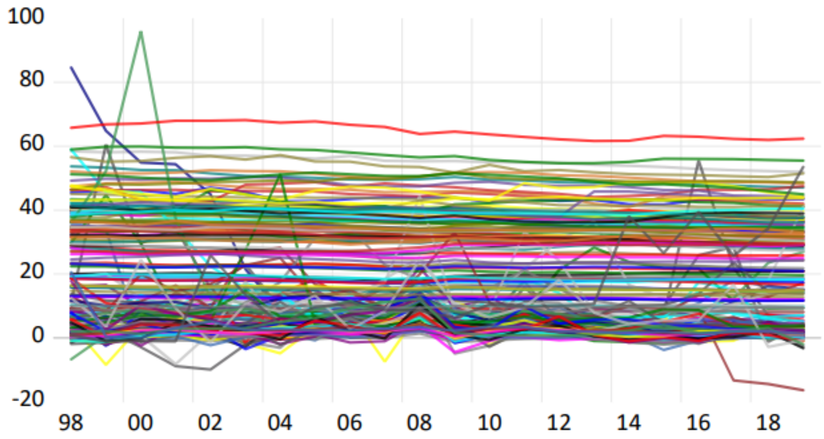

Figure 20 - all countries, combined

High-income countries:

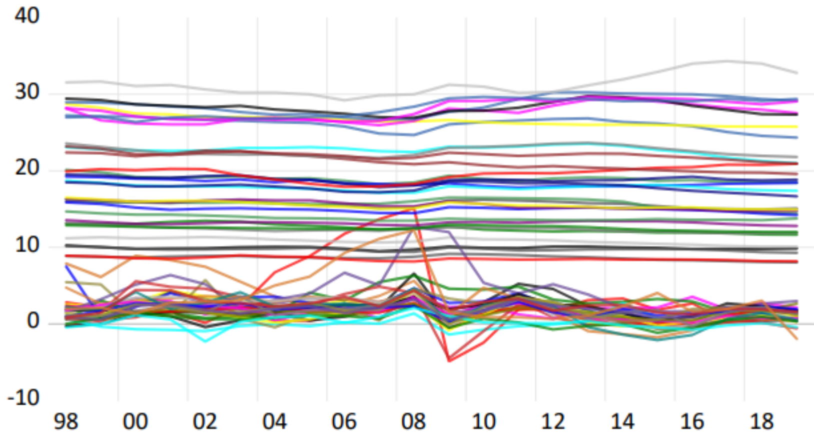

Figure 21 - high-income, combined

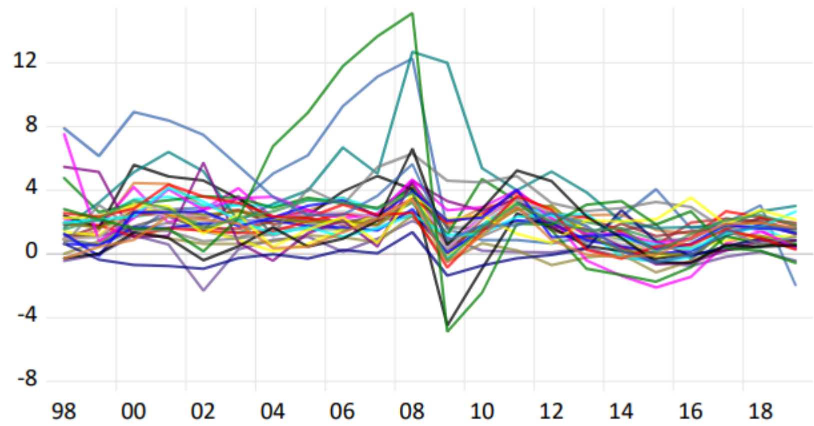

Figure 22 - high-income, inflation

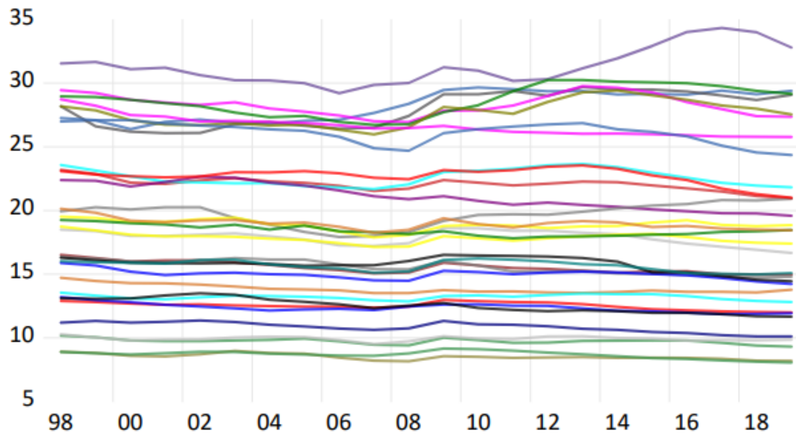

Figure 23 - high-income, informality

Middle-income countries:

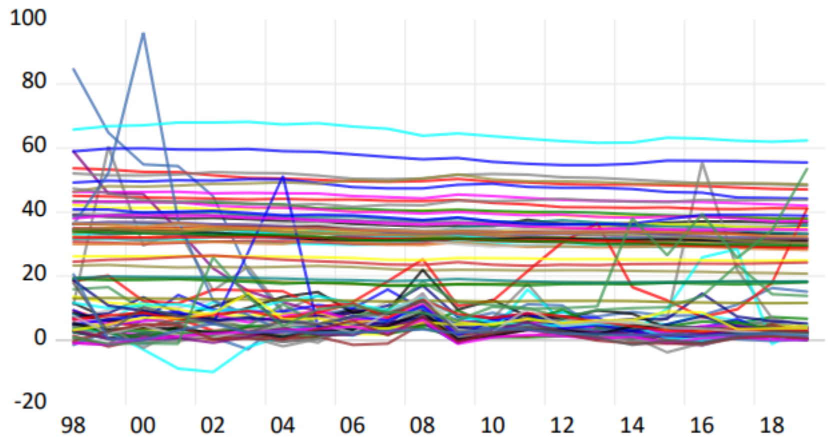

Figure 24 - middle-income, combined

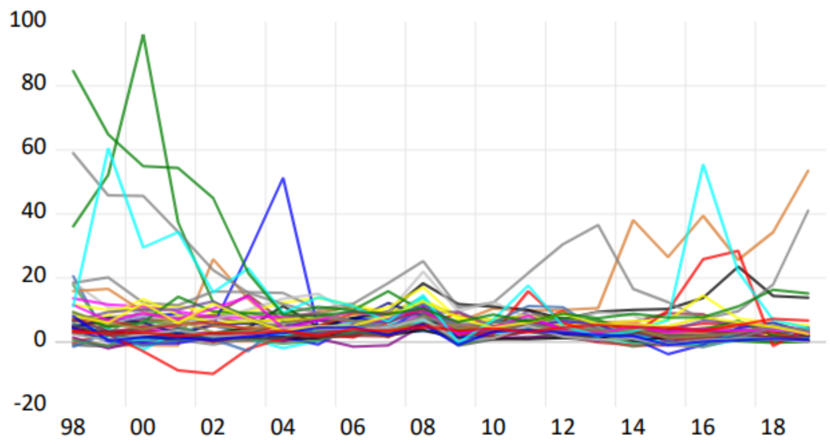

Figure 25 - middle-income, inflation

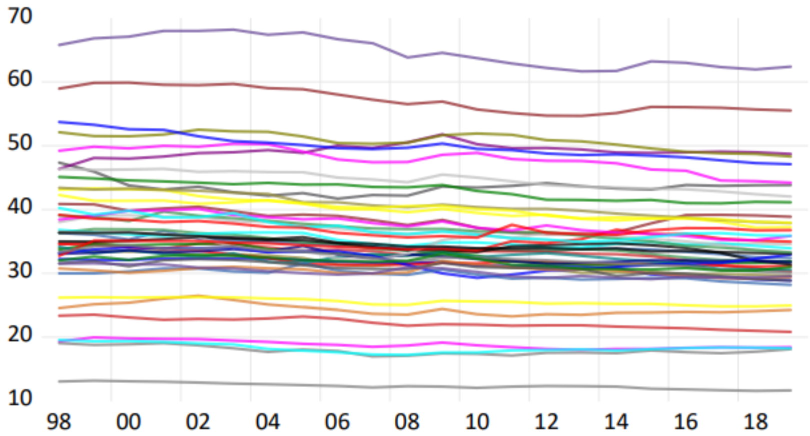

Figure 26 - middle-income, informality

Low-income countries:

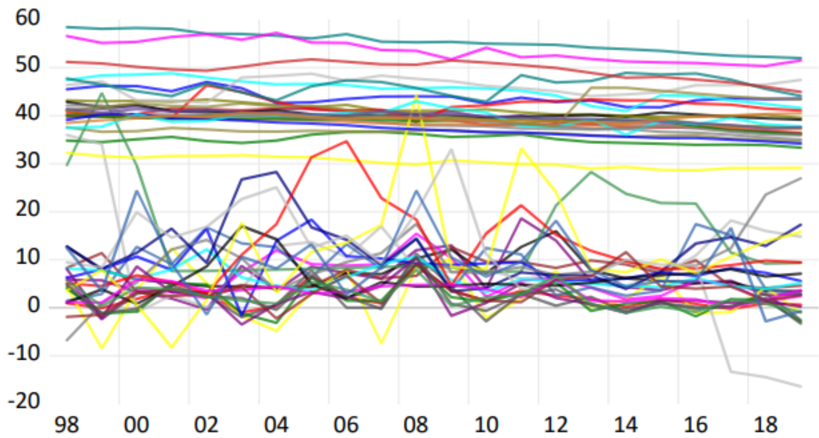

Figure 27 - low-income, combined

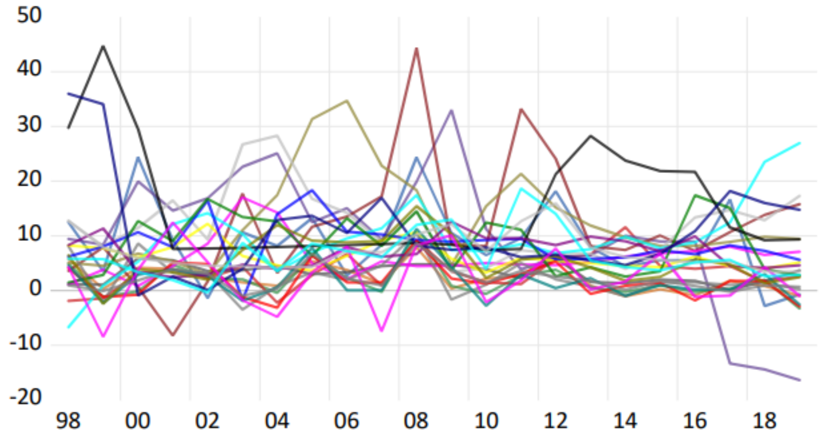

Figure 28 - low-income, inflation

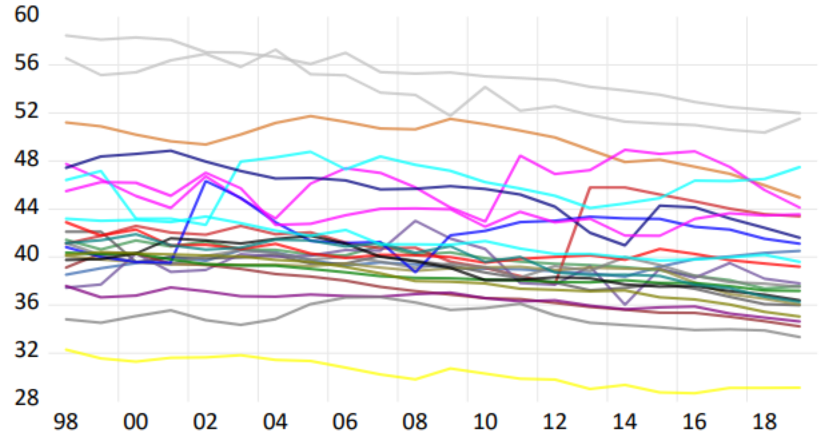

Figure 29 - low-income, informality

The magnitude of fluctuations between the two variables differs greatly, with inflation fluctuating much more. It is difficult to observe any straightforward dynamic relationship between the two variables.

Panel data fixed-effects estimation model

Fixed effects estimator is selected after conducting Hausman Test. When all the countries are used to build the model, the estimated coefficients and their significance are shown in the table below[7]:

Table 11 - fixed effects panel model (all countries)

Dependent variable	informality	
Inflation	0.0266***	-0.0002
	(0.0045)	(0.0001)
Trade openness		-0.0176**
		(0.0078)
Political stability		-0.2812
		(0.5066)
Corruption perception index		-0.4002
		(0.0222)
Observations	2288	1079
Number of countries	104	136

standard errors in parentheses. ***p<0.01, **p<0.05, *p<0.1.

Table 12 - fixed effects panel model (within subgroups)

Dependent variable	High income informality		Middle income informality		Low income informality	
Inflation	-0.0457***	-0.1069***	0.0317***	4.26^{e-05}***	0.0266***	0.0036
	(0.0151)	(0.0233)	(0.0057)	(1.27^{e-05})	(0.0045)	(0.0042)
Trade openness		0.0081***		0.0102**		0.0124
		(0.0022)		(0.0044)		(0.0125)
Political stability		0.2206		-0.5547***		-0.2976
		(0.1916)		(0.1752)		(0.3278)
Corruption perception index		-0.0064		-0.0272*		-0.1407***
		(0.0101)		(0.0141)		(0.0344)
Observations	726	352	1012	519	550	208
Number of countries	33	44	46	66	25	26

standard errors in parentheses. ***p<0.01, **p<0.05, *p<0.1.

The results indicate that if a fixed effects estimator is utilized in the panel, the correlation between inflation and informality is significant at the 1% level when no control variables

[7] Corruption perception index is kept as a variable because in Panel models, multicollinearity is not as problematic as in cross-sectional models.

are included. However, the direction of the correlation is negative within high-income countries, while in other cases, it is positive. When controlling for trade openness, political stability, and the corruption perception index, the correlation is significant only in the high-income and middle-income subgroups, at the 1% level. In high-income countries, the effect is negative. And in middle-income countries, it is positive but is noticeably small.

However, dynamics are not considered in fixed effects estimated panel data models. Therefore, the results can only reveal relationships between variables in a static and average manner. The dimension of time is still needed to understand the true relationship.

Dumiterscu-Hurlin Panel Granger Causality Test

The panel data from 104 countries spanning 1998 to 2019 are used to conduct the Dumitrescu-Hurlin Panel Granger Causality Test. Panel unit root test results indicate that both inflation and informality are stationary at the original level (both I(0)), which contrasts with the time series stationarity results for individual countries that have been tested. Therefore, these variables can be directly used for the Dumitrescu-Hurlin Panel Granger Causality Test.

The results of the panel unit root test and the Dumitrescu-Hurlin Panel Granger Causality Test are shown in the following two tables:

Table 13 - Panel unit root test results ($p<0.05$)

panel (all 104 countries)		panel high-income		Panel middle-income		panel low-income	
inflation	informality	inflation	informality	inflation	informality	inflation	informality
stationary	stationary	stationary	stationary	stationary	stationary	stationary	stationary

Null hypothesis of Dumitrescu-Hurlin Panel Granger Causality Test:

Informality does not homogeneously cause inflation

Inflation does not homogeneously cause informality

Table 14 - Dumiterscu-Hurlin Panel Granger Causality Test Results

	All 104 countries	High-income	Middle-income	Low-income
lag length	2	2	2	2
W-Stat.	5.200269416	4.079260974	5.320486537	6.458801059
	5.05103951	7.496442245	4.765814941	2.34792111
Zbar-Stat.	10.94647842	3.77648506	7.582653265	7.702060509
	10.38174259	11.06095236	6.186646081	0.074641424
Granger causality	Informality homogeneously causes inflation	Informality homogeneously causes inflation	Informality homogeneously causes inflation	Informality homogeneously causes inflation
	Inflation homogeneously causes informality	Inflation homogeneously causes informality	Inflation homogeneously causes informality	Inflation does not homogeneously cause Informality; or inflation heterogeneously causes informality

The results of Dumiterscu-Hurlin Panel Granger Causality Test indicate that worldwide, as well as in high-income and mid-income countries, there is a consistent Granger causal relationship from informality to inflation across all countries in each panel. This suggests that an increase in informality Granger causes an increase in inflation in a similar way for each country in each panel. There is also a consistent Granger causal relationship from inflation to informality across all countries in each panel. In other words, an increase in inflation Granger causes an increase in informality in a similar way for each country in each panel. This double-sided Granger causality is conceptually different from the hypotheses of the research question because Granger causality only reveals forecasting power.

The Granger causality between these two variables in low-income countries shows a single-directional manner, where an increase in informality Granger causes an increase in inflation similarly for each country in this panel. However, there is no uniform causal effect of inflation on informality across the countries in the panel. This maybe because the informal economy in these countries are very persistent and in a significant scale that they are not sensitive to the variation of inflation anymore.

Impulse-Response Function Analyses

The final stage of this research involves examining the system's response to external shocks or changes in variables over time using impulse-response functions (IRFs). Both the duo-variable panel comprising 104 countries from 1998 to 2019 and the multi-variable panel encompassing 136 countries from 2012 to 2019, along with the respective subgroups of the multi-variable panel, are subjected to testing.

The PVAR using informality as a variable is not stable. Because of this, the IRF results are also unstable and non-significant. However, when the first difference of informality (which indicates the growth rate of the size of the shadow economy) is used, the system becomes stable and the IRF results become meaningful, except in the low-income subgroup of the multi-variable panel.

Multi-variable panel:

All countries

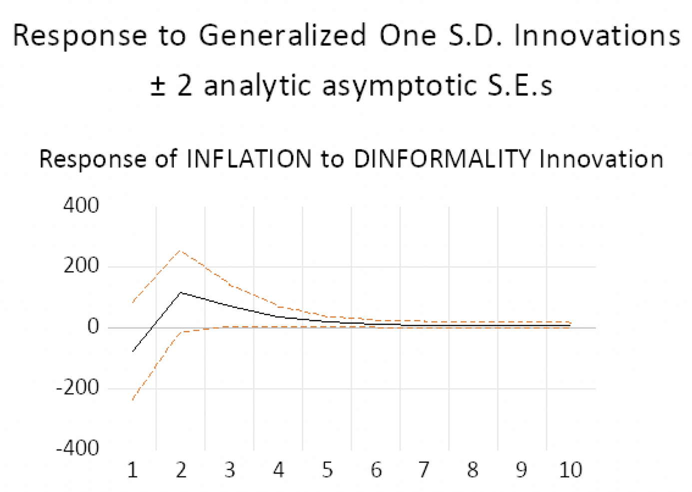

Figure 30 - IRF, multi-variable, all countries, 1

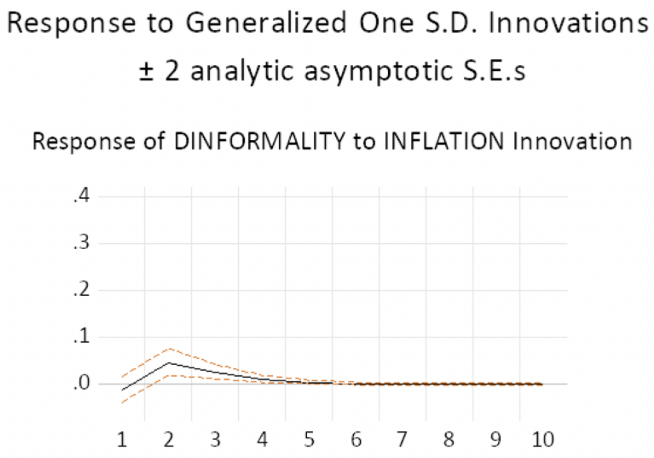

Figure 31 - IRF, multi-variable, all countries,2

These two graphs show that in this system (including all countries, controlling for the corruption perception index, trade openness, and political stability): (1) when there is an external shock on the growth rate of the size of the shadow economy, inflation experiences an increase from a lower-than-initial value, reaches the highest point at period 2, and returns to normal after around 5 periods; (2) when there is an external shock on the inflation rate, the growth rate of the shadow economy also increases, reaches its highest value at period 2, and returns to normal after around 5 periods.

High-income:

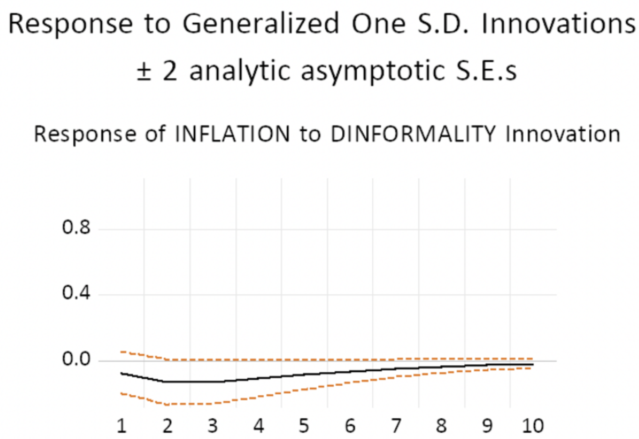

Figure 32 - IRF, multi-variable, high-income, 1

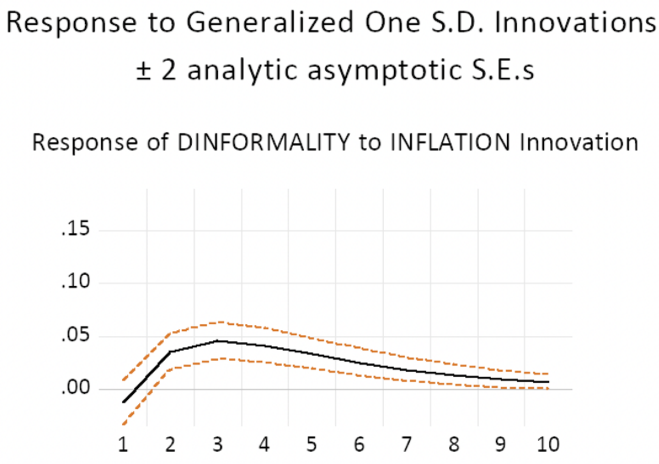

Figure 33 - IRF, multi-variable, high-income, 2

These two graphs indicate that in this system (within high-income countries, controlling for the corruption perception index, trade openness, and political stability): (1) when there is an external shock on the growth rate of the size of the shadow economy, inflation decreases, reaches its lowest point at period 2, and returns to normal after around 10 periods; (2) when there is an external shock on the inflation rate, the growth rate of the shadow economy increases, reaches its highest point at period 3, and returns to normal after around 10 periods.

Middle-income

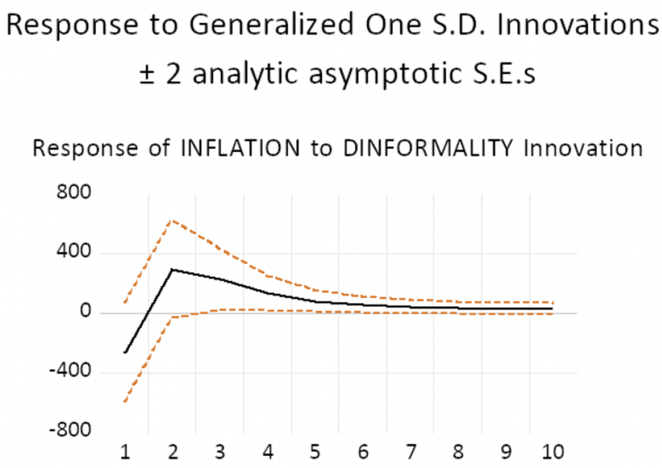

Figure 33 - IRF, multi-variable, middle-income, 1

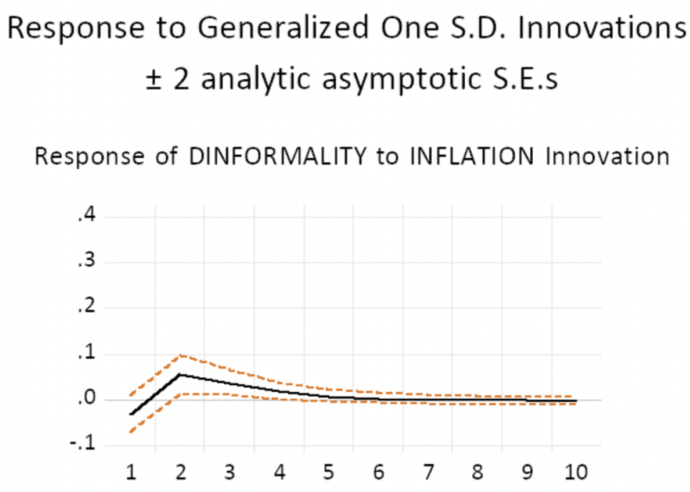

Figure 34 - IRF, multi-variable, middle-income, 2

These two graphs illustrate that in this system (within middle-income countries, controlling for the corruption perception index, trade openness, and political stability): (1) when there is an external shock on the growth rate of the size of the shadow economy, inflation experiences an increase from a lower-than-initial value, reaches the highest point at period 2, and returns to normal after around 7 periods; (2) when there is an external shock on the inflation rate, the growth rate of the shadow economy increases, reaches its highest point at period 2, and returns to normal after around 5 periods.

Duo-variable panel

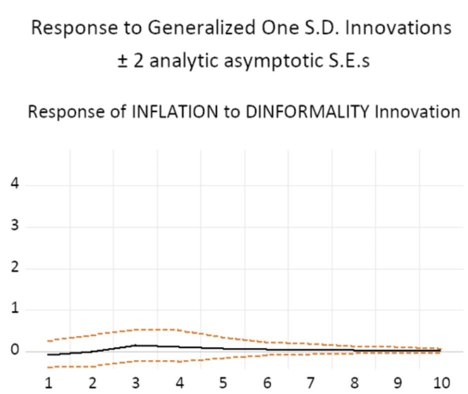

Figure 35 - IRF, duo-variable,1

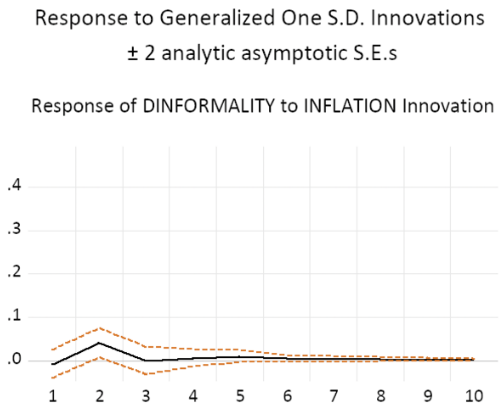

Figure 36 - IRF, duo-variable,2

These two graphs suggest that in this system (within all countries, no control variables): (1) when there is an external shock on the growth rate of the size of the shadow economy, inflation increases, reaches the highest point at period 3, and returns to normal after around 8 periods; (2) when there is an external shock on the inflation rate, the growth rate of the shadow economy increases, reaches its highest point at period 2, experiences one fluctuation at period 3, and returns to normal after around 6 periods.

These results further unveil the intricated endogenous relationship between inflation and another level of informality, its growth rate.

Conclusion

This paper thoroughly investigated the relationship between the inflation rate and the size of the shadow economy using both static and dynamic models. Cross-sectional regression models show different directions and significance of correlations during different times. Therefore, such a model could not unveil the general relationship between these two variables as they evolve over time. Fixed effect estimation models also show the relationship in a static way but control for heterogeneity. The correlation between the two variables in panel data is mostly significant and positive, except for high-income countries where the correlation is always negative. This indicates different patterns of the relationship among different groups.

When time series are tested for individual countries, most of them are not stationary at the original level. Granger causality of the series or the differenced series is only witnessed in a few of the selected countries.

However, the Dumitrescu-Hurlin Panel Granger Causality Test shows that in panel data, double-sided Granger causality exists in overall observations, high-income, and middle-income countries, while only uni-directional homogeneous causality is observed in low-income countries. This result suggests proved the symbiotic relationship between the two variables in general, and in high-income and middle-income countries, while the shadow economy might be too persistent to reaction the variation of inflation.

Finally, PVAR models are constructed to test the system's reaction to external shocks on its variables using Impulse-Response Functions. The results show that within systems including inflation and the growth rate of the shadow economy, in general, one variable increases in response to a sudden increase in the other variable. However, in the high-income subgroup with control variables, inflation decreases in response to an increase in the growth rate of the shadow economy.

The outcomes of this research demonstrate the necessity of examining macroeconomic indicators within groups of countries at different levels of development or income. When formulating economic policies, policymakers should consider that controlling inflation or the shadow economy will influence the other variable in the future.

Limitation

This study has several limitations that must be acknowledged.

Estimated Values

One primary limitation is that the size of the shadow economy is an estimated value rather than a directly observed one. This reliance on estimates can introduce errors and uncertainties that may affect the robustness and accuracy of the findings.

Granger Causality

The use of Granger causality in this study only indicates forecasting power rather than true causality. Granger causality can show whether one variable can predict another, but it does not confirm a direct causal relationship in the real world. To establish real causality, more sophisticated models, such as simultaneous equations models with appropriate instrumental variables, are necessary.

Long-run and Short-run Relationships

This study does not differentiate between long-run and short-run relationships. Understanding how inflation and the shadow economy interact over different time horizons is crucial, and this gap needs to be addressed in future research to provide a more comprehensive view of the dynamics.

Varying Frequency of Variables

The inflation rate tends to vary more frequently than the size of the shadow economy. Using yearly data for both variables may not adequately capture their true dynamics and interactions. This mismatch in frequency could lead to an incomplete understanding of how these variables influence each other. Future research should consider using data with higher frequency for inflation to better capture its variations and interactions with the shadow economy.

Model Assumptions

Certain assumptions of the models used may not have been thoroughly tested, partly due to the author's current level of expertise in econometrics. This oversight could affect the validity of the models and, consequently, the study's findings.

These limitations highlight areas for future research and underscore the need for cautious interpretation of the study's findings. Addressing these issues could lead to more robust and reliable insights into the relationship between inflation and the shadow economy.

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