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„Intraday Dynamics of Option-Implied Variance“

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Abstract

This thesis aims to examine the intraday patterns of implied volatility as measured by the VIX (Volatility Index). The focus is on the behavior of the VIX at a one-minute interval, with the goal of closing the gap in the literature on the intraday dynamics of the VIX. The research will include an investigation of the problems associated with the VIX, the influence of liquidity, and the contribution of different data records. Existing literature reveals a notable focus on overall market volatility in financial research, with comparatively less attention directed towards intraday implied volatility and its determining factors. To address this research challenge, this thesis will utilize high-frequency data and apply econometric techniques to explore the complexities of intraday implied volatility patterns. In the methodology section, we will outline the VIX calculation method as detailed in the CBOE white paper. Our empirical analysis will involve a comparison between our own VIX calculation and the CBOE published VIX, an investigation into irregularities and anomalies, an examination of correlations with the underlying asset, a concentration on short time intervals, and an exploration of distinctions among various option exchanges. This study seeks to enhance our comprehension of the VIX and its intraday behavior.

Zusammenfassung

Ziel dieser Arbeit ist es, die intraday Muster der impliziten Volatilität zu untersuchen, die durch den VIX (Volatilitätsindex) gemessen wird. Der Fokus liegt auf dem Verhalten des VIX im einminütigen Intervall, mit dem Ziel, die Lücke in der Literatur zur intraday Dynamik des VIX zu schließen. Im Rahmen dieser Arbeit werden die mit dem VIX verbundenen Probleme, der Einfluss der Liquidität und der Beitrag verschiedener Datensätze von verschiedenen Börsen untersucht. Die bestehende Literatur zeigt, dass der Schwerpunkt in der Finanzforschung auf der Gesamtmarktvolatilität liegt, während der impliziten Intraday Volatilität und den sie bestimmenden Faktoren vergleichsweise wenig Aufmerksamkeit geschenkt wird. Um diese Forschungsherausforderung anzugehen, werden in dieser Arbeit Hochfrequenzdaten verwendet und ökonometrische Techniken angewandt, um die Komplexität der impliziten Intraday Volatilitätsstrukturen zu untersuchen. Im Abschnitt über die Methodik wird die VIX-Berechnungsmethode gemäß dem CBOE White Paper erläutert. Unsere empirische Analyse umfasst einen Vergleich zwischen unserer eigenen VIX-Berechnung und dem von der CBOE veröffentlichten VIX, eine Untersuchung von Unregelmäßigkeiten und Anomalien, eine Ermittlung der Korrelationen mit dem Basiswert, ein genauer Blick auf kurze Zeitintervalle und eine Untersuchung des VIX unter Verwendung verschiedener Optionsbörsen. Diese Arbeit zielt darauf ab, unser Verständnis des VIX und seines intraday Verhaltens zu vertiefen.

Contents

1 Introduction

The VIX (Volatility Index) was introduced by the Chicago Board Exchange and since its debut, it has garnered significant attention and usage as a gauge of market volatility. Not only the financial markets, but also academia has taken a keen interest in this topic. The VIX attracts considerable market interest due to multiple factors. Primarily, the VIX provides valuable insights into investor sentiment, earning its reputation as the "fear index". Monitoring fluctuations in the VIX enables investors to assess the prevailing levels of risk, uncertainty, and market sentiment. As a result, it serves as a crucial instrument for effectively managing portfolio risk and making well-informed investment choices. Additionally, the VIX is perceived to mirror the degree of fear and uncertainty prevailing among market participants. During periods of heightened stock market volatility, the VIX typically exhibits an upward trend, signaling elevated levels of uncertainty among investors. Conversely, a low VIX reading may indicate a greater sense of confidence and optimism among market participants regarding future market conditions. Furthermore, the VIX holds historical significance as it has been in existence for over two decades and has become a widely recognized benchmark in numerous financial studies. Its extensive usage and recognition in both practical and scholarly contexts attest to its enduring importance and relevance in the financial industry. Through the examination of historical VIX data, researchers can deepen their understanding of the factors influencing market volatility and unravel how market participants navigate different market conditions. Despite the extensive body of literature on the VIX, there exists a notable dearth of research focusing specifically on its intraday behavior. This knowledge deficiency offers a valuable chance to enhance the current comprehension of the VIX. Thus, the primary objective of this thesis is to bridge this gap by conducting an in-depth exploration of the intraday dynamics and patterns exhibited by the VIX. By shedding light on the intraday behavior of the VIX, this study aims to enhance our comprehension of this influential volatility index and contribute to the existing body of knowledge in the field. This research endeavor entails a thorough examination of the VIX performance throughout the trading day, with a specific focus on the challenges associated with reproducing the VIX at an intraday level. To accomplish this objective, intraday data sourced from the OPRA market will be utilized. Departing from the conventional fifteen-second intervals, the VIX will be analyzed at both one-minute and one-second intervals, allowing for a more comprehensive exploration of its behavioral patterns. By adopting these varying time intervals, this study seeks to uncover valuable insights into the dynamics of the VIX and advance our understanding of its behavior in different intraday settings. Our objective is to comprehensively investigate and quantify the disruptions that occur within the VIX throughout the trading day. Additionally, we aim to delve into the underlying reasons behind these disruptions and explore why the microstructure of the VIX can potentially lead to misleading conclusions. Furthermore, our research will encompass an examination of the consistency of the VIX across various time frames and exchanges. We will explore the complexities related to the VIX, including the influence of liquidity and the importance of diverse data sources. Of particular significance is the distinction that the official VIX is exclusively released by the CBOE based on SPX data, whereas we predominantly utilize option data associated with SPY. Given that not all data possesses equal quality, we will also assess the influence of data quality on the VIX, further contributing to the understanding of potential biases and limitations in VIX calculations. Through this multifaceted exploration, we aim to shed light on the complex issues surrounding the VIX and provide valuable insights into its behavior under varying conditions and data sources. The methodology for computing the VIX is outlined in detail in the CBOE white paper (Cboe Exchange, 2023). The plan for this research is organized as follows: Firstly, we will provide a comprehensive overview of the essential theoretical foundations. This will encompass the theory of options, futures, volatility, as well as an explanation of the different methods for calculating volatility. The methodology, including a comprehensive description of the VIX calculation based on the official CBOE white paper, will also be covered. Following the theoretical groundwork, we will proceed with presenting the data employed in this study. In the initial segment of this chapter, we will explore the data utilized. Given the significance and extensive volume of the data, a detailed explanation of its characteristics will be provided. Afterwards, we are going to look at the empirical analysis. The chapter will start with a comparison between our own VIX calculation and the officially published VIX by the CBOE, aiming to identify any differences or discrepancies. Furthermore, an examination of irregularities or anomalies arising in the VIX will be conducted, followed by an analysis of the underlying reasons behind such occurrences. A crucial section dedicated to the correlation of the VIX with the underlying asset, as well as other relevant correlations, will also be included, providing valuable insights for investors. Additionally, a thorough investigation of brief intervals will be conducted, offering insights into the behavior of the VIX during these shorter time frames. Finally, we will analyze the impact of various options exchanges on the VIX, uncovering any notable influences or patterns.

2 Theoretical Framework

2.1 Options

This section is intended to be an introduction to derivatives and volatility models. In the world of finance, derivatives refer to instruments whose value is derived from an underlying asset. Essentially, the value of a financial derivative is solely based on the price movement of the underlying asset. As a result, these financial instruments are used by both investors and traders for risk management and speculative purposes. An option is such a financial instrument that provides the holder with the option, but not the obligation, to buy or sell the underlying asset at a predetermined price. A call option is an option contract that gives the holder the right to purchase the underlying asset at a specified price. Conversely, a put option is an option contract that grants the holder the right to sell the underlying asset at a specified price. The process S_t represents the price movement of the underlying instrument. The predetermined price, which is referred to as strike price, is represented by K. Additionally, the expiration date of an option, also referred to as its maturity, is denoted by the variable T. The options necessary for modeling volatility are known as plain vanilla options, reportedly named for their simplicity and "basic" nature, similar to the flavor of vanilla ice cream. Let $\Pi(S,T)$ denote the payoff at maturity respectively the intrinsic value of the option at time T. These payoff function can then be represented by

$$\begin{aligned}\Pi(S,T)^C &= \max(S_T - K, 0) = (S_T - K)^+ \\ \Pi(S,T)^P &= \max(K - S_T, 0) = (K - S_T)^+.\end{aligned}$$

The subscript indicates the option type, where C represents a call option and P symbolizes a put option. Another important concept is moneyness, which refers to the relationship between the current price of the underlying asset and the strike price of the option. Three different states of intrinsic value are recognized, where intrinsic value refers to the market value of the financial derivative, disregarding any potential fluctuations or volatility in the underlying price until the expiration date. In simpler terms, intrinsic value represents the current market value of an option that is unaffected by the passage of time. The first state is referred to as "In-The-Money" (ITM), encompassing all options with a positive intrinsic value. For a call option, it indicates that the strike price of the option is lower than the current price of the underlying. Whereas for the put option the strike price has to be higher than the current spot price to be called ITM. By "At-The-Money" (ATM) options the strike price equals the present price of the underlying, consequently the intrinsic value would be zero. The central emphasis of this study revolves around "Out-of-The-Money" (OTM) options, which are characterized by having a negative intrinsic value. Graphically, this concept is represented by 2.1. Another financial derivative that will be relevant is futures contracts. Futures are agreements between two parties to either buy or sell an underlying

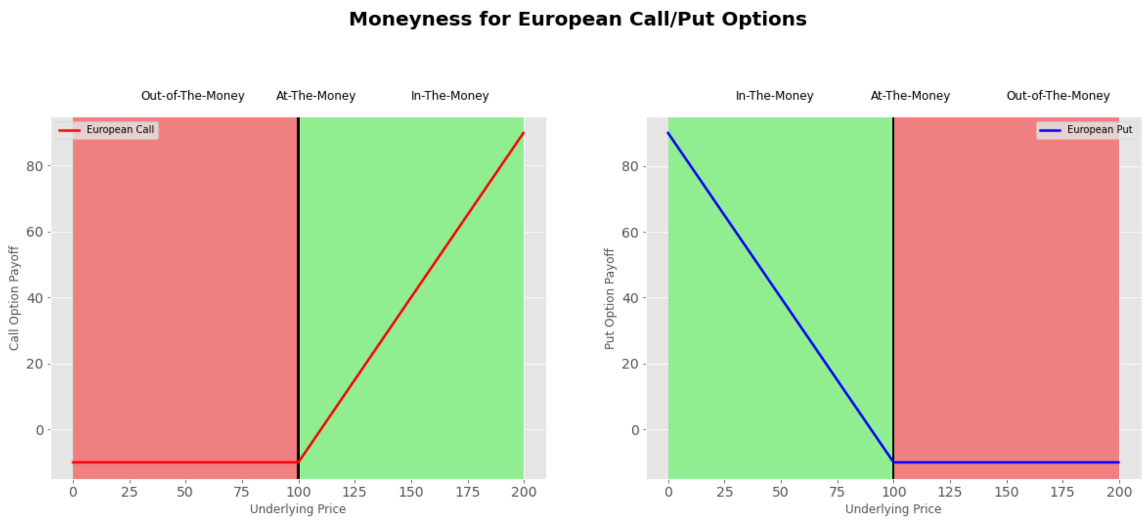


Figure 2.1: Moneyness of Options: Red indicates the OTM part of the option, the thick black line in the middle indicates the moment where the options would be ATM and green signals when an option would be ITM. Options have a strike price of 100 and the premium (price) is set to 10.

asset at a predetermined price and expiration date. Unlike options, the holder of a futures contract is obligated to either buy or sell the underlying asset at the agreed price, regardless of the market price of the asset at the expiration date. Due to the fact that one is obligated to sell or buy in a future deal can result in negative payoffs. The mathematical expression for the profit or loss of the contract holder is represented by $S_T - K$, while the position of the seller is expressed as $K - S_T$. Futures are commonly used for hedging purposes and are particularly popular in the commodities market.

2.2 Volatility

The following section will explore the process of calculating volatility from option prices. Volatility is a statistical measure that quantifies the degree of variation for returns of a given security or market index. It is commonly viewed as a representation of the level of uncertainty regarding the magnitude of fluctuations in the value of an underlying asset. Volatility, which is the measurement of uncertainty about the future price of an asset, is expressed as the standard deviation or variance of the price of the underlying. A higher volatility implies that the price of the asset is more likely to experience larger and more frequent changes in the near future. It is considered more risky as there is an equal chance for the price to increase or decrease, without a direction. Volatility is denoted by the Greek letter σ in the financial literature. The initial differentiation in terms of volatility arises from the method employed to compute it. First, there is the so-called historical volatility also known as statistical volatility. It is based on historic price data, typically over a specified period of time. Some studies thoroughly examining historical volatility include "Realized Volatility: A Review" by Michael McAleer and Marcelo C. Medeiro (McAleer and

Medeiros, 2008), as well as "Forecasting Realized Volatility: A Review" by Dong Wan Shin (Shin, 2018). It is typically calculated using time-series models. The primary limitation of this approach is the dependence on historical data. Hence, this data may not accurately represent the present market situation and conditions.
Another important measure of volatility is known as implied volatility. It is calculated using present option prices. This metric is crucial because it provides insight into the markets expectations for future volatility. It can be calculated by using option pricing models such as the Black-Scholes model or by using model-free methods. While implied volatility is commonly considered to have stronger predictive power for future volatility compared to historical volatility, this assumption does not always hold true (Samsudin and Mohamad, 2016). Nevertheless, implied volatility is a forward-looking measure.

2.3 Model-Based Implied Volatility

The focus of the discussion now shifts to model-based implied volatility models. These models are based on mathematical and economic assumptions that are used to calculate implied volatility. This is the reason why these models are referred to as "model-based implied volatility models". Many model-based volatility estimation models for example assume a logarithmic return of the underlying. Others, however use auto-regressive conditional heteroskedasticity models to estimate the changes in the volatility of asset returns (Degiannakis and Xekalaki, 2004). The most famous model-based estimation for volatility is the Black-Scholes formula. It was introduced in the revolutionary paper 'The Pricing of Options and Corporate Liabilities' by Black and Scholes (Black and Scholes, 1973). The original purpose of the formula was to calculate the price of options, but it is also possible to estimate volatility from this formula. The Black-Scholes model relies on several assumptions, including constant volatility, no dividends, efficient markets and many more. These assumptions may not hold in the real world, leading to inaccurate results.

2.4 Model-Free Implied Volatility

In this section, we focus on model-free implied volatility (MFIV) models, with particular attention to the CBOE VIX measure. The VIX is a financial index that is extensively employed to measure the anticipated volatility of the stock market in the upcoming 30 days. It is derived from the prices of options on the S&P 500 index and is commonly known as the "fear index" or the "fear gauge" of the stock market. The VIX is mathematically expressed as the annualized percentage of the anticipated one standard deviation move in the returns of the S&P 500 index. When VIX values are higher, it indicates a rise in uncertainty or volatility in the market. Through the use of detailed time-stamped SPY and SPX option quote data, we are able to replicate the VIX following the exact CBOE prescription. This allows us to evaluate various types of interference that may not be present in the officially released series. Throughout the trading day, the CBOE calculates and releases the index value almost continuously (every 15 seconds), which results in the VIX being observable nearly in real-time. To calculate the VIX index, the CBOE uses a group of European style OTM options written on the asset price with identical tenor, T, and strike prices K_j with

$j = 1, \cdots, N$ where $0 < K_1 < \cdots < K_f \leq F < K_{f+1}1 \cdots < K_N$. In this context, F represents the forward price of the asset at $t = 0$ for time T, while K_f indicates the first strike price that is below the forward price ($K_f \coloneqq \max_{f \in \{1,\ldots,N\}} K_f < F$). The strike price is selected based on a reasonable cross-section of available options, such that $f \in \mathbb{N}$ and $2 < f < N - 1$. Additionally, it is useful to define $\mathcal{Q}_t(K)$ as the minimum price between the put and call options with strike K, i.e., $\mathcal{Q}_t(K) = \min(P_t(K), C_t(K))$. This implies that $\mathcal{Q}_t(K)$ represents the price of the option that is currently out-of-the-money, between the two vanilla options with strike K. The price of an option with strike price K_j which is currently out of the money is denoted by $\mathcal{Q}(K_j)$. The CBOE calculates the model-free implied volatility in an (annualized) manner as shown below

$$\hat{\sigma}_T^2 = \underbrace{\frac{2\exp(rT)}{T} \sum_{j=2}^{N} \frac{\Delta K_j}{K_j^2 \mathcal{Q}(K_j)}}_{\text{Discrete Approximation}} - \underbrace{\frac{1}{T}\left[\frac{F}{K_f} - 1\right]^2}_{\text{Correction Term}},$$

where r represents the (annualized) risk-free rate for $[0, T]$, which is estimated using the corresponding U.S. Treasury bill rate and additionally, $\Delta K_j = (K_{j+1} - K_{j-1})/2$. By setting r to zero (or the value of the options $\mathcal{Q}(K_j)$ in time T dollars), the first term of the equation yields a discrete estimation of the theoretical MFIV with the exception of the annualization factor $1/T$. The second term in the equation compensates for the absence of a clear distinction between the OTM put and call options, which is caused by the difference between K_f and the forward price. Another complication is that only a limited number of option maturity dates are quoted at any given time, while the VIX is defined for a fixed maturity of $T_M = \frac{30}{365}$, or thirty calendar days. The CBOE overcomes this challenge by combining linearly $\hat{\sigma}_1^2$ and $\hat{\sigma}_2^2$, which are the estimated variances for the two options with tenors T_1 and T_2 closest to thirty days, respectively. However, options with less than 23 calendar days until expiration and those with more than 37 calendar days are excluded from the calculation. The resulting interpolated value is presented as a volatility measure and quoted as an annualized percentage,

$$\text{VIX} = 100 \times \sqrt{\left[w_1(\hat{\sigma}_1^2 T_1) w_2(\hat{\sigma}_2^2 T_2)\right] \times \frac{N_{365}}{N_{30}}},$$

where $w_1 = \frac{T_2 - T_M}{T_2 - T_1}$ and $w_2 = \frac{T_M - T_1}{T_2 - T_1}$ so we can see that $w_1 + w_2 = 1$. The exchange cannot directly observe the forward price F, so it is estimated using put-call parity by selecting the pair of put and call options with identical strikes and the smallest price discrepancy. Therefore, all inputs for the VIX calculation, except for r, are obtained directly from the options price data.

3 Data

The data utilized in this thesis research originates from the Options Price Reporting Authority (OPRA). OPRA is a consortium comprising thirteen securities exchanges that supply last quotations and option-related data. Further details on the different exchanges are presented in the corresponding subchapter. However, it consolidates the most favorable bid and ask price information for each strike price from all participating exchanges. Our primary data source is tick-by-tick SPY option quotes from CBOE, which is part of the OPRA. The sample period encompasses the week spanning from January 26, 2015, to January 30, 2015. The choice of SPY data for the majority of this thesis is driven by the need for comparability across exchanges and the desire to explore one-second intervals. SPY is an exchange traded fund (ETF), which is designed to replicate the performance of the S&P 500 Index. It is the ETF with the highest trading volume, averaging approximately 82.45 million shares per day over a 30-day period. The substantial trading volume of SPY across exchanges facilitates obtaining an adequate number of data points from each exchange, simplifying the estimation of the VIX. Furthermore, we employ tick-by-tick SPX data to assess the accuracy and dependability of the VIX published by the CBOE. Considering that SPX represents the S&P 500 and SPY is an ETF designed to track it, the disparity in data between the two is expected to be minimal (Tsalikis and Papadopoulos, 2019; Johnson, 2008). This aspect will aid in identifying and analyzing the distinctions arising from utilizing different option quotes for nearly identical underlying assets. Usually, volatility analysis involving option data relies on end-of-day option matrices, encompassing only the last quote recorded for the day. In contrast, our study revolves around quotes registered at millisecond intervals, providing a more granular level of data. Moreover, while options matrices typically offer a single cross-section per day, the OPRA data cross-section can change within milliseconds, providing a more dynamic and detailed representation of market conditions. Three different expiration dates are considered in our investigation. The first expiration date for the options falls on 20.02.2015 and follows the regular pattern of expiring on the third Friday of the month. The length of the first expiration date varies during the week, ranging from 25 days to 21 days before its expiration. Furthermore, we are examining the second expiration date, which falls on 28.02.2015, and the third one set for 06.03.2015. These maturities average approximately 62,370 OTM put and 57,510 OTM call options daily. We primarily create one-minute intervals for all available options during the trading day, from 08:30 to 15:15 Central Time, employing the "previous tick" method. Our data adheres to a specific format (3.1). In this format, the first column denotes the timestamp, starting with the first two digits for hours, followed by minutes, seconds, and the final three digits representing milliseconds (HH:MM:SS:SSS). The second column consistently shows the record type code, which remains "Q" for our analysis since the data exclusively comprises quotations. The subsequent column indicates the options exchange code, revealing the source of the assets quotes.

10:05:42.471	Q	X		47.91	202.67	202.68
10:05:46.719	Q	W		47.91	202.69	202.7
10:05:46.927	Q	X		47.89	202.69	202.7
10:05:47.723	Q	X		47.89	202.68	202.69
10:05:49.733	Q	X		47.9	202.67	202.68
10:05:52.986	Q	X		47.92	202.68	202.69
10:05:55.977	Q	W		47.91	202.7	202.71
10:05:57.513	Q	W		47.91	202.71	202.72
10:06:03.852	Q	X		47.94	202.72	202.73
10:06:04.853	Q	X		47.91	202.71	202.72
10:06:06.649	Q	X		47.91	202.7	202.71
10:06:06.676	Q	W		47.91	202.7	202.71
10:06:07.157	Q	X		47.91	202.7	202.71
10:06:07.861	Q	X		47.89	202.7	202.71
10:06:07.861	Q	X		47.89	202.7	202.71

Raw OPRA Data

	80.0	85.0	90.0	95.0	100.0	105.0	110.0	115.0	120.0	125.0
08:30:00	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.01
08:31:00	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.01
08:32:00	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
08:33:00	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
08:34:00	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
08:35:00	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
08:36:00	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
08:37:00	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
08:38:00	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
08:39:00	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
08:40:00	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
08:41:00	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
08:42:00	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
08:43:00	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
08:44:00	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
08:45:00	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
08:46:00	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
08:47:00	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
08:48:00	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
08:49:00	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
08:50:00	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
08:51:00	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
08:52:00	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005

Cleaned OPRA Data

Figure 3.1: The left side displays the data as published by OPRA in milliseconds, while the right side exhibits the cleaned data.

This information is crucial as we will be comparing the VIX across multiple exchanges. If the column neighboring to the options exchange code is empty, it signifies that the transaction was a regular sale reported without any specific conditions. However, if the column contains the letter "A," it indicates the cancellation of a previously reported quotation. Following this is the best bid price for the option, accompanied by the quantity of options available at that price. Subsequently, the next column displays the best ask price, along with the number of bids at that price. The last two entries pertain to the underlying asset, representing the best bid and best ask price, as previously mentioned. Our analysis will solely rely on the mid-price. Subsequently, cleaned data can be seen in the snippet (3.1) on the right side. The provided data showcases the quotations of SPY at CBOE at one-minute intervals, with corresponding strike prices listed in the top row of the CSV file.

4 Empirical Analysis

In this chapter, we present the empirical results of our analysis on the VIX. To begin, we compare our calculated VIX measure with the publicly available VIX data provided by the CBOE. This comparison allows us to assess the accuracy and reliability of our methodology in capturing market volatility. Afterwards, we delve into the intraday variations of the VIX. By examining how the index fluctuates throughout the trading day, we can gain a deeper understanding of the intraday volatility patterns and potential drivers behind them. Building upon this, we explore the reasons behind the VIX reputation as the "fear index". This leads us to investigate the correlation between the underlying asset and the VIX. Expanding further, we scrutinize the VIX on higher-frequency. Lastly, we shift our focus towards the influence of different exchanges on the VIX.

4.1 Comparison to the CBOE VIX

To assess the accuracy, we conduct a comparison between the official VIX values published by the CBOE and the values derived from our own calculations using the SPY and the SPX data from OPRA. This comparative analysis allows us to identify any disparities between the two sets of values and gain insights into the precision of our methodology. In addition, maybe first conclusions can be made regarding the difference of the values due to the different data used. As CBOE VIX presented values in this comparison are sourced from the official publications of the CBOE and were collected from Bloomberg. Bloomberg does not provide historical data with intervals shorter than one minute, hence we limit the comparison to one-minute intervals. Through this comparison, we aim to provide a comprehensive understanding of the alignment between our calculated values and the widely accepted VIX values reported by the CBOE. Such insights contribute to the robustness and credibility of our analysis. Prior to delving into the discussion of the results, it is important to acknowledge the variation in the number of strike prices between SPX and SPY options. For the first expiration date there are 214 different strikes available for SPX options whereas only 159 different strikes are to choose from for SPY options. A similar picture emerges when we take a look at the second expiation date where 169 different strike prices exist for SPX and just 63 strikes for SPY options. When considering the third expiration date, there is a selection of 172 available strikes for SPX options and 59 strikes to choose from for SPY options. Unlike the CBOE VIX methodology, our analysis includes all out-of-the-money strike options. This is in contrast to the CBOE, which excludes options prices after two consecutive zero bids. Including all strike prices in our analysis offers significant value. By doing so, we can thoroughly investigate the dynamics occurring at the extreme ends of the strike prices. Furthermore, incorporating all strike prices should provide us with a wealth of information about the overall market conditions. This approach should enable us to identify the drivers of the VIX more effectively. The figure (4.1)

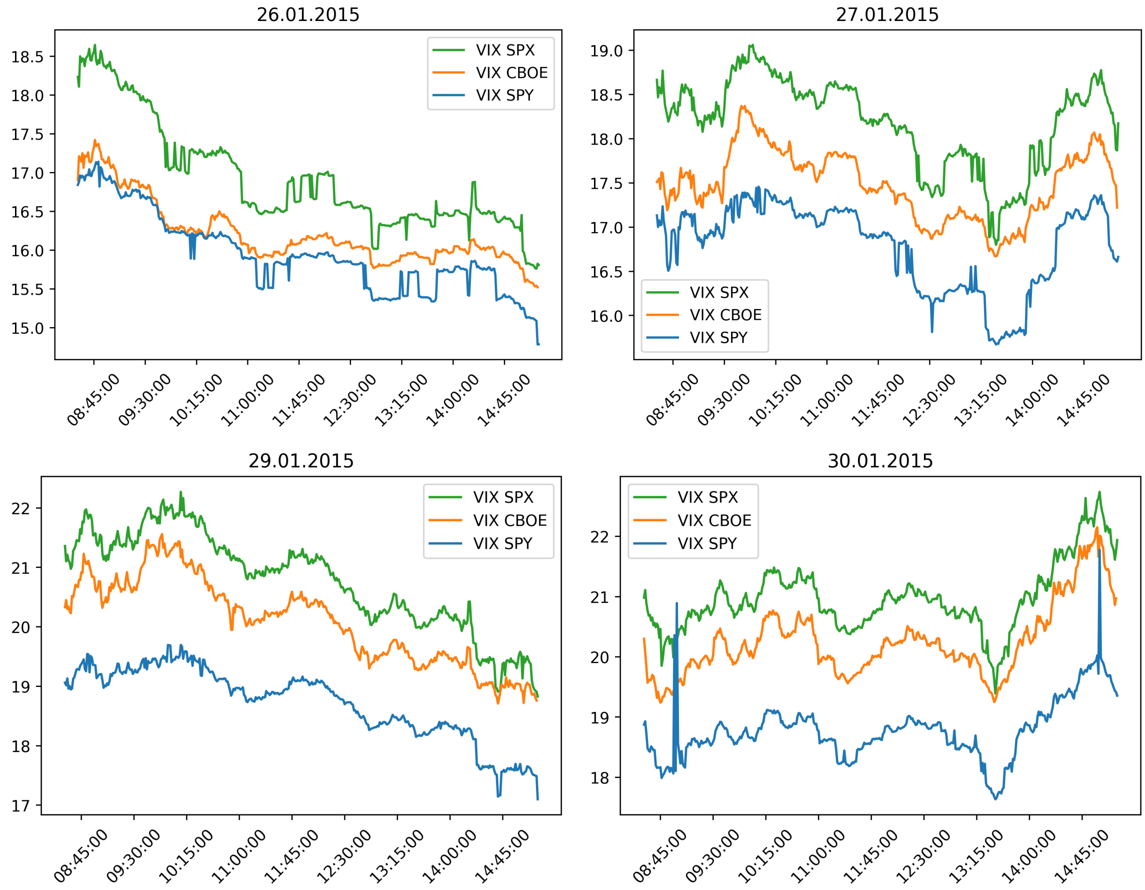


Figure 4.1: Comparison figure between the different VIX values. The green line is the VIX using SPX data, the blue line is the VIX official published by the CBOE and the orange line is the VIX calculated using SPY data. The plot shows four different days, namely Monday, Tuesday, Thursday and Friday.

depicts the three distinct VIX variations. These plots already highlight pretty well that the more strike prices included, the higher the values of the VIX become. Therefore, the VIX calculated using all available SPX option data (green line) consistently ranks the highest, followed by the VIX published by the CBOE (orange line), and the lowest is the one using SPY option data (blue line) with the fewest strike prices. The approximation, therefore, yields satisfactory results. What is striking when looking at the figure is the turbulent pattern throughout Monday. ”Turbulent” in this context refers to the angular movement of the two modeled VIX, characterized by rapid and non-smooth changes in trajectory. This could be explained to some extent by the Monday effect (Singleton and Wingender, 2003; Pettengill, 2003; Berument and Dogan, 2012). Which states that on Monday stock returns tend to be different from those on other trading days. One of the key drivers that particularly intrigues us among the various factors contributing to the Monday Effect is the influence of weekend news and events. Over the weekend many different news can influence the market and those get priced in on Monday. On this particular Monday the S&P credit rating for Russia has been downgraded to junk status for the first time in a decade, which created uncertainty in the market. This new information is then reflected in the option prices and leads subsequently to this pattern. However, there are minor distinctions in the VIX values on Monday. On Tuesdays, the disparity between the values amplifies. Additionally, during the early hours of the trading day, there is a greater discrepancy between the SPX VIX and the CBOE VIX, which diminishes as the day progresses. In contrast, Thursday exhibits a steady and moderate trajectory. Friday was relatively moderate as well while using SPX data. The interesting part here are the spikes in the VIX which is calculated using SPY data. These jumps will be investigated in more detail in the next chapter. When examining Figure 4.2, an intriguing pattern

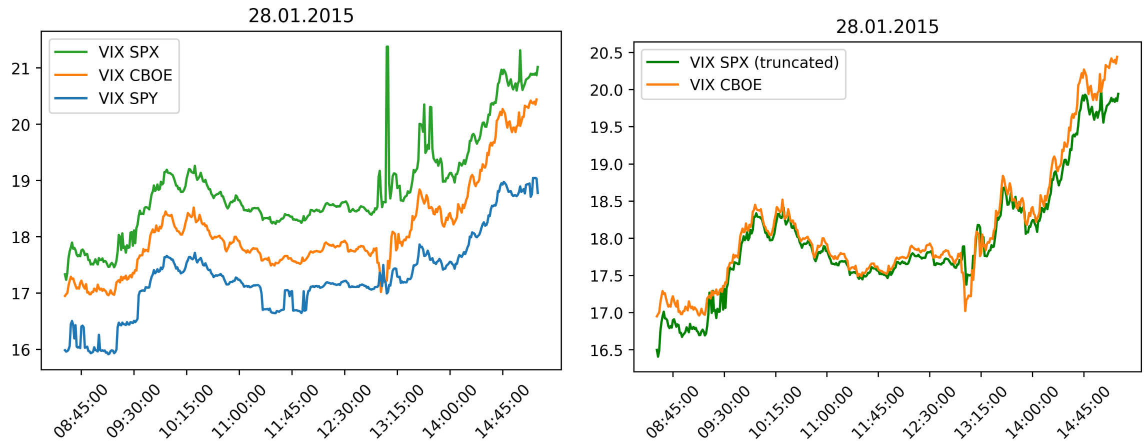


Figure 4.2: Comparison between VIX values for the 28.01.2015. The green line is the VIX calculated using SPX data, the blue line is the VIX official published by the CBOE and the orange line is the VIX using SPY data. The far out-of-the-money options where driving the peak around midday. Particularly, when examining the right plot, the far OTM options from SPX are truncated.

emerges around midday. Notably, during this time frame, the CBOE VIX demonstrates a contrasting behavior compared to the other two plots. They move in a divergent direction temporarily, but then realign, albeit not perfectly, along a similar trajectory. Therefore, the two modeled VIX have better predictive power becomes evident when observing the direction they all continue following the deviation. This discrepancy becomes particularly apparent when comparing the CBOE VIX with the plot that employs SPX data. During that Wednesday in January, there was a FED meeting around that time, introducing new information to the market. This is simultaneously reflected in the prices of the options. Especially in the far out-of-the-money options. This implies that the market was more unsettled at that moment, as it is not shown in the official VIX published by the CBOE. This is precisely why this work is crucial - the CBOE intraday approximation has been proven to not be completely reliable. Upon isolating the far out-of-the-money options from the SPX data, the two lines exhibit a high degree of similarity, as illustrated in Figure 4.2 on the right side. This suggests that the market has extensively protected itself against severe or extreme events by implementing hedging strategies. Thus, the market is very interested in those options at far ends. Observing the cross-section cross-section of the first expiration date, it appears that a significant investor, possibly institutional, strategically acquired numerous put options at almost every second strike price as a hedge, along with some call options. This suggests a coherent strategy behind this cross-section configuration. Such institutional investors often lean towards SPX options due to their higher liquidity compared to SPY options. Nevertheless, this strategic activity does not influence the VIX calculation since the nearest expiration date is within 24 days to maturity and therefore has no impact on the computation. For the other two relevant dates, abnormal behavior is evident in the cross-sections, persisting for about three minutes, from 13:06 to 13:09, before reverting to normal patterns. These deviations in behavior, particularly for the expiration dates that influence VIX calculation, underline the role of ignored abnormal strike prices in rendering the truncated VIX smoother. Examination of the cross-section of SPY options during the same interval strengthens the notion of an institutional investors influence on the market. Plot 4.4 displays a consistent cross-section behavior during this time frame. Only the third expiration date exhibits momentary disturbances. This indicates maybe the absence of any significant new market-relevant information during this period. Importantly, the VIX derived from SPY options becomes a more reliable market signal in this context, as it seems to avoid arbitrage violations. However, that SPX options are exclusively traded on the CBOE, resulting in a concentrated market with substantial market maker influence. This concentrated power might lead to less favorable prices for other market participants.

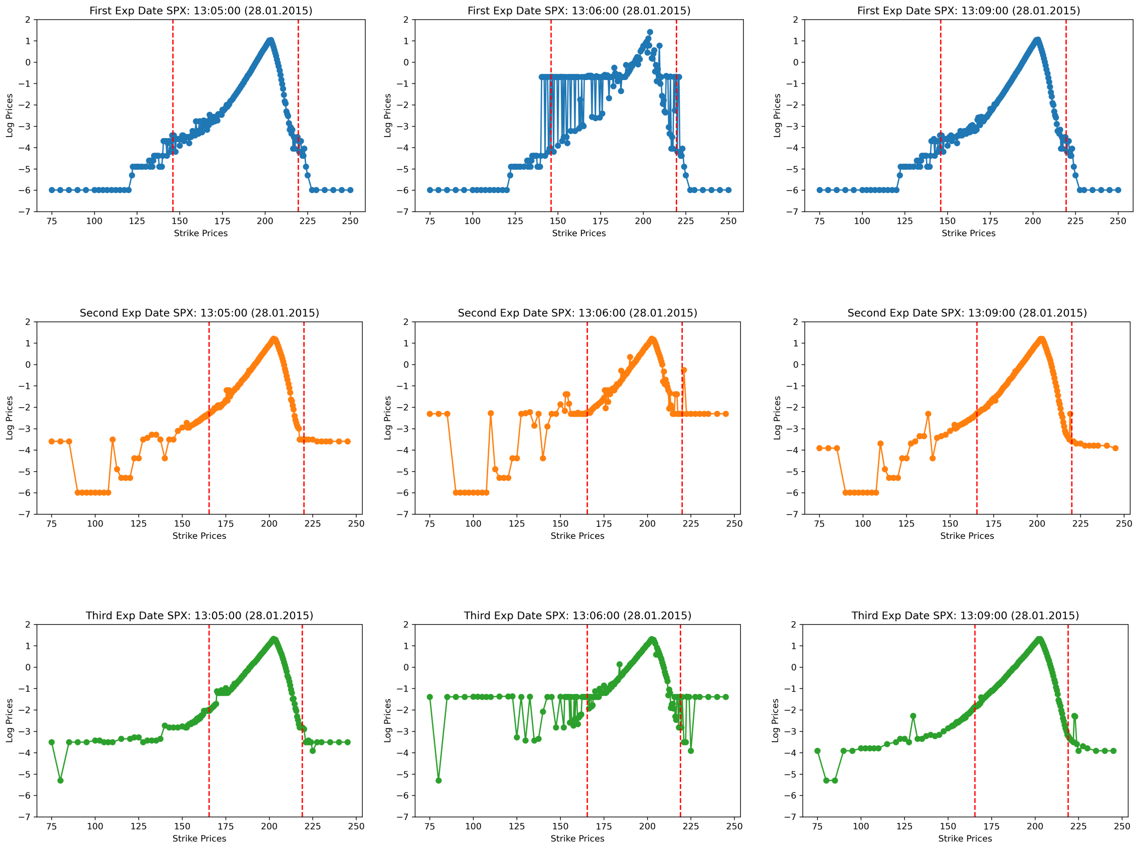


Figure 4.3: The plots illustrate the changes in option prices across different strike prices at around 1 PM for the various expiration dates using SPX data. The blue line in the first three plots represent the cross-section of the nearest expiration date. The orange below depicts the cross-section of the middle expiration date, and the green lines represent the cross-section of the third expiration date, which is furthest from expiration. The x-axis represents the strike prices, while the y-axis displays the logarithmic prices of the options. It shows the behavior of the far out-of-the-money options which are thriving the spikes in the VIX. The red dashed line indicates the point at which the strike prices were truncated for the abbreviated SPX VIX. The interval from 13:06 to 13:09, spanning three minutes, illustrates the duration within which the cross-section reverted to a typical pattern.

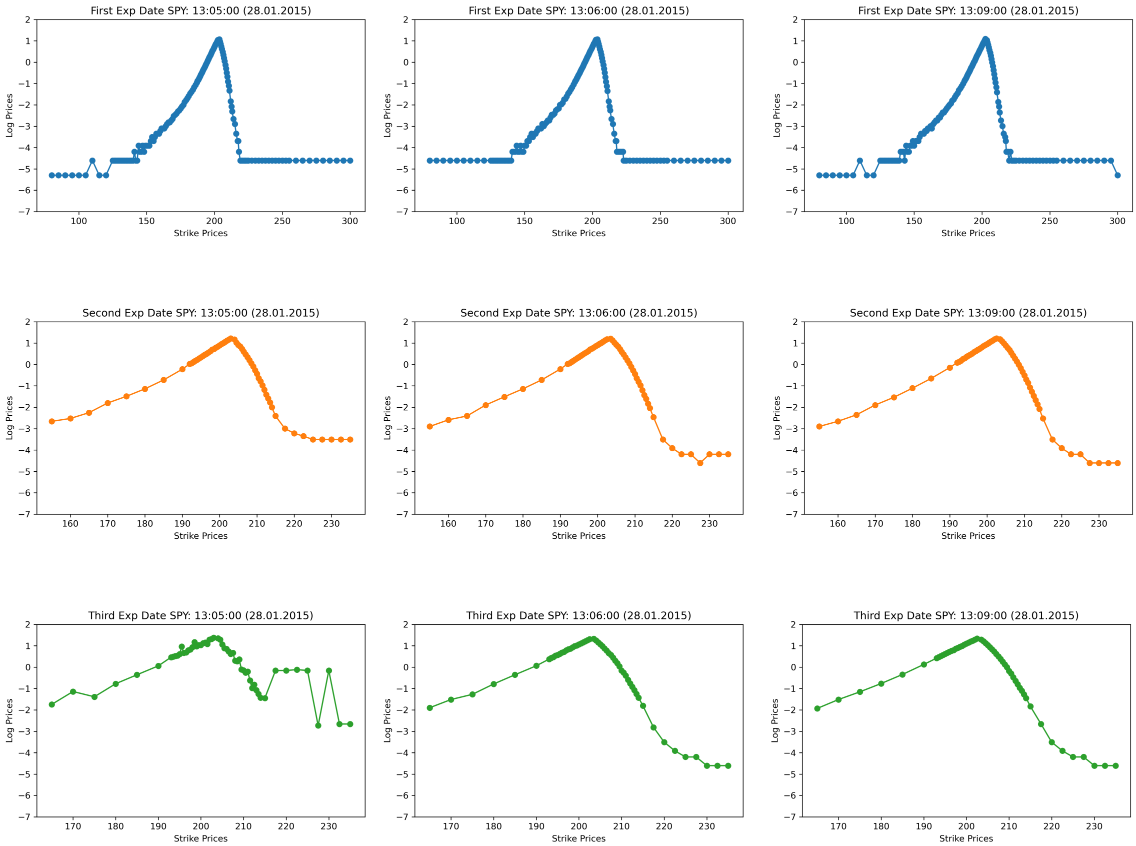


Figure 4.4: The diagrams depict variations in option prices across distinct strike prices, at the same time as the plot 4.3, but using SPY cross-section data. In the initial three graphs, the blue line signifies the cross-section of the nearest expiration date, followed by the orange line for the middle expiration date, and the green lines for the third expiration date, furthest from expiration. The x-axis corresponds to the strike prices, while the y-axis shows the logarithmic option prices.

4.2 Irregularities

The objective of this chapter is to enhance the identification of anomalous patterns or deviations observed within the VIX. By implementing various techniques and methodologies, we aim to improve the detection and understanding of these unusual occurrences. As demonstrated in previous plots, the VIX can undergo rapid changes. The umbrella term "irregularities" encompasses both significant outliers and anomalies observed in the VIX. Moving forward, these abrupt fluctuations in the process will be referred to as "jumps". In general, a jump can be defined as a substantial decrease or sharp increase occurring within a brief time period. The plot displayed in Figure 4.5 graphically illustrates the clear visibility of jumps. These distinctive fluctuations serve as the focal point of our investigation, where we aim to understand the underlying drivers and market factors that contribute to these jumps. Before proceeding, it is important to establish a precise mathematical definition of a "jumps" in this particular context. To identify jumps, we utilize the logarithmic differences of the VIX and calculate their standard deviation. An outlier or anomaly, known as a jump, is identified when the logarithmic difference exceeds two times the standard deviation. In our specific cases, selecting two times the standard deviation has consistently yielded the best results. Henceforth, we will utilize this measure for our analysis, foregoing the more commonly used method of taking three times the standard deviation found in the literature. Utilizing two standard deviations generally encompasses a significant portion of the data (about 95%), which is often deemed adequate for identifying outliers or unusual occurrences. By employing this methodology, we can effectively detect and quantify jumps in the data. The mathematical formula used to identify a jump is as follows:

$$J_t := \Bigg(\log(\mathrm{VIX}_t) - \log(\mathrm{VIX}_{t-1}) \Bigg) > sd_{\mathrm{VIX}} \times 2$$

where J_t denotes the jump at time t, VIX_t is the value of the VIX at time t, log represents the natural logarithm and sd_{VIX} represents the standard deviation of the logarithmic differences of the VIX.

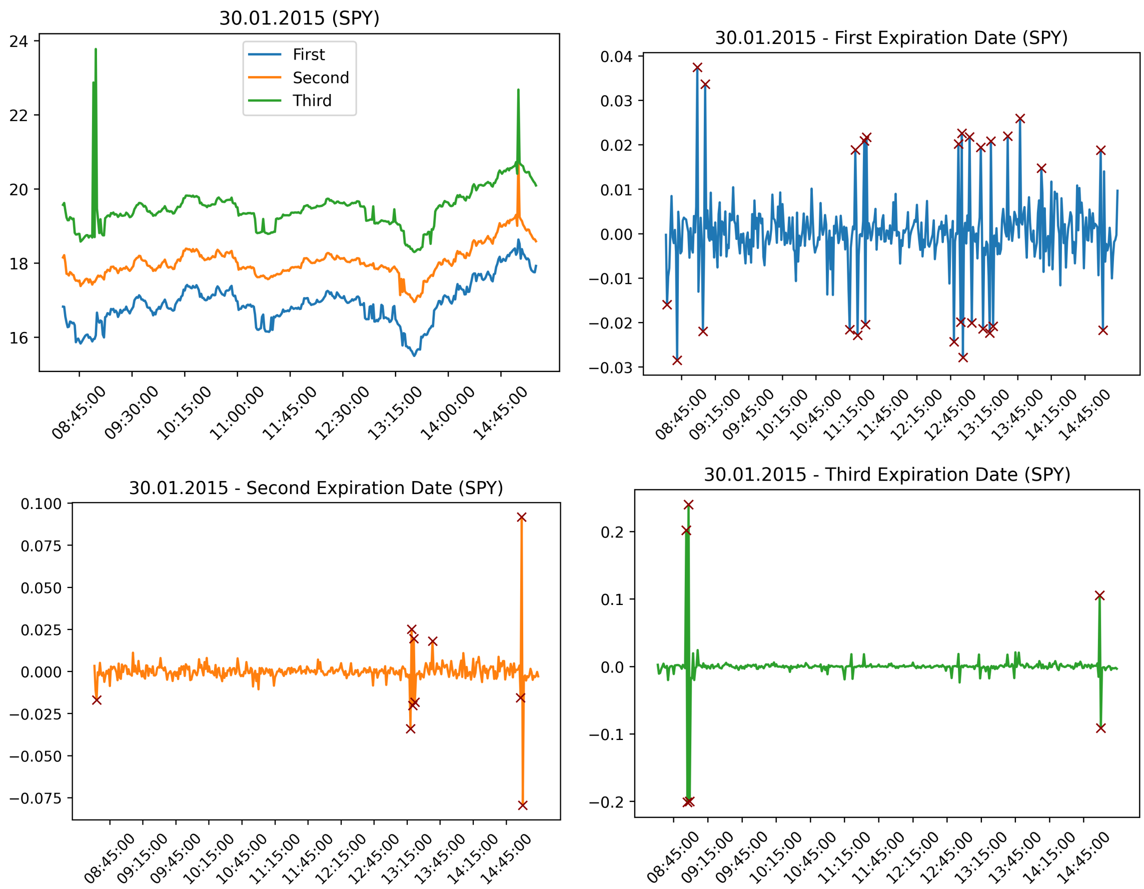


Figure 4.5: The plot in the upper left corner illustrates the price process of the VIX calculated for three different expiration dates. The other three plots showcase the logarithmic differences of the VIX on the y-axis over the course of the day. The crosses indicate different occurrences of VIX jumps. The plot effectively demonstrates the success of the jump detector, accurately capturing and identifying VIX jumps throughout the observed period.

This formula works perfectly for our purposes, as Figure 4.5 demonstrates. To examine the impact of the underlying asset on the occurrence of jumps, we will analyze the logarithmic returns of the SPY index for the corresponding day. When the SPY index experiences a downward movement, people tend to buy protection against potential market downturns. This explains why there are more points indicating jumps when the SPY index declines. Conversely, in bullish markets, participants are less inclined to purchase insurance, resulting in fewer spikes and less impact on the VIX values. Figure 4.6 presents a graph, which combines the logarithmic returns of the underlying asset with the crosses indicating the occurrence of jumps in the VIX for the different data sources. The plot clearly illustrates the connection between jumps in the VIX and alterations in the underlying price process. Throughout the week, significant deviations in the underlying returns are consistently accompanied by at least one cross in one of the three VIX, demonstrating a strong connection between rapid price changes in the underlying asset and jumps in the VIX.

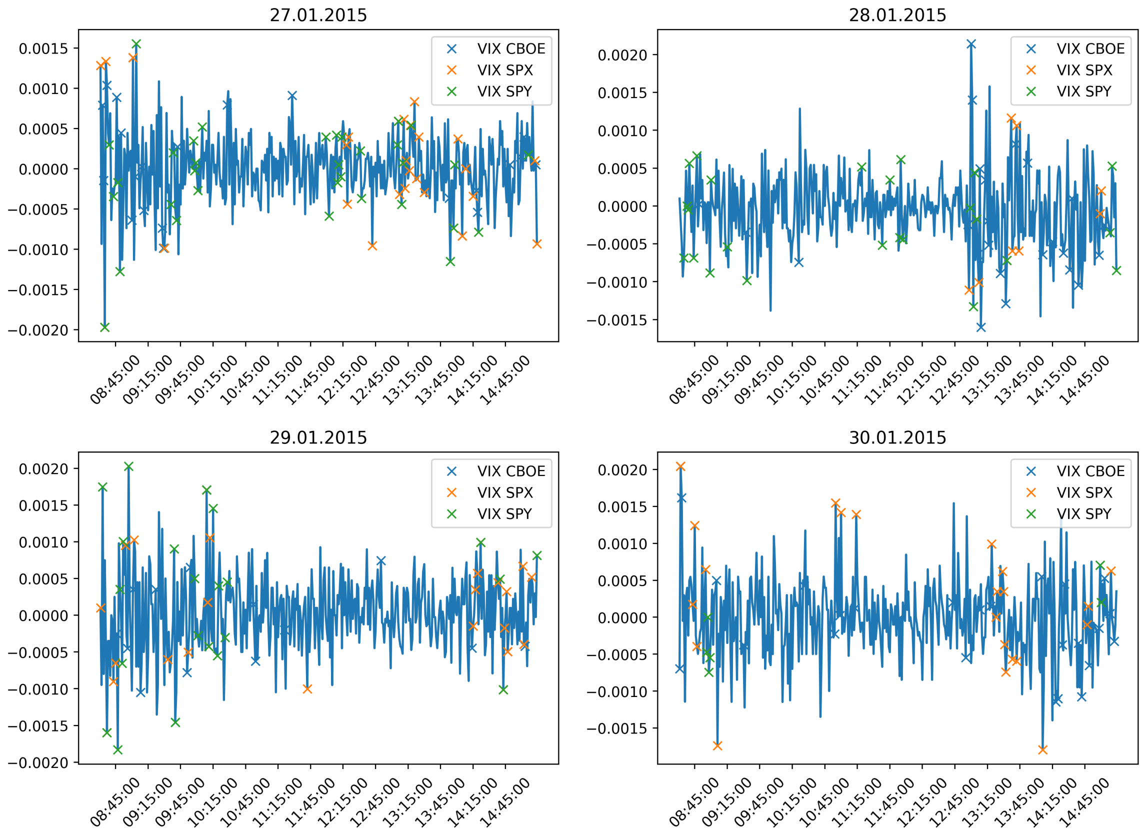


Figure 4.6: The plot displays the logarithmic returns of the SPY ETF on the y-axis throughout the day. The crosses on the blue line represents various occurrences of VIX jumps for different data types. The plot illustrates that changes in the underlying asset are a significant driver of these jumps. On Tuesday, the SPY VIX experienced 33 jumps, followed by 24 on Wednesday, 22 on Thursday, and 6 on Friday. In contrast, the VIX derived from SPX data encountered 27 jumps on Tuesday, 10 on Wednesday, 25 on Thursday, and a notable 23 on Friday. Notably, the officially published VIX exhibited 24 jumps on Tuesday, 25 on Wednesday, 20 on Thursday, and 27 on Friday.

An intriguing observation is the presence of a peak in SPY options on Friday. Particularly, there is a prominent peak occurring around 3:00 PM. Upon closer examination, the second and third expiration dates exhibit a more pronounced peak compared to the first expiration date when analyzing SPY data. This phenomenon warrants further investigation to gain a deeper understanding of its underlying causes. The peak for the second and third expiration dates is significantly higher than that of the nearest expiration date. In contrast, the first expiration date exhibits a gradual build-up compared to the other two, suggesting that there may be several factors contributing to the peaks of the different expiration dates. Additionally, the third expiration date had two significant outliers in the morning at 9 AM, but the simultaneous afternoon jumps for all expiration dates make it even more thrilling to analyze. Mainly because of the comparison of the different cross-section behavior between the different expiration dates. The cross-section of option prices can be seen as a snapshot of market conditions at a certain point in time. This allows us to analyze market trends and pricing behaviors, providing valuable insights for gaining a comprehensive understanding of the prevailing situation. By examining the market for options at a given moment, one can gain information that aids in assessing market dynamics and pricing patterns. When analyzing the cross-section of option prices during jumps, a common observation is the emergence of a consistent pattern in price changes. This pattern is often characterized by a prominent peak, as illustrated in the blue graph. The presence of such a peak suggests that the overall market tends to move in unison, exhibiting a collective directional shift during these periods. Based on the information presented in Plot 4.3, the cross-section for the initial expiration date adheres to the expected standard pattern. On the contrary, the situation diverges for the remaining two expiration dates, as their respective cross-sections exhibit distinct dissimilarities. Upon examining the price changes between 2:59 PM and 3:00 PM for these expiration dates, the market does not exhibit uniform movement. Rather, there are substantial price jumps observed in option prices during this time frame. The price fluctuations observed for the second and third expiration dates deviate from the expected pattern. Options prices for strike prices near or at-the-money undergo rapid and significant increases, which endure for a mere minute before reverting back to their initial levels. The market is witnessing the suppression of inefficiencies, potentially attributed to the exploitation of arbitrage opportunities. Efforts to establish consistency by fixing specific option prices proved ineffective, as the entire cross-section experienced movement. As depicted in Figure 4.7, the entire market underwent a shift, rendering the corridor VIX (Andersen, Bondarenko, and Gonzalez-Perez, 2015) less effective or applicable in this scenario.

The plot labeled as Figure 4.8 illustrates the irregularities discussed in the previous chapter, which occurred on Wednesday. It presents the linear relationship between the VIX calculated using SPY and SPX data. On Wednesday, there are significantly more outliers when using SPX data as opposed to SPY data. This disparity is also noticeable in the linear regression plot, indicating a notable difference in the relationship between the VIX and the two underlying assets on that particular day. Upon analyzing the pairplot, the SPX data exhibits more and stronger outliers compared to the SPY data. This is reflected in the higher values on the y-axis, indicating a notable difference in the distribution of data points between the two sets. When comparing the right figures, the outliers in the other plots are primarily caused by the far out-of-the-money options. Moreover, the linear regression exhibits a more negative trend as expected, with fewer outliers. Therefore, the irregularities are driven by the far out-of-the-money options in combination with changes in the underlying asset.

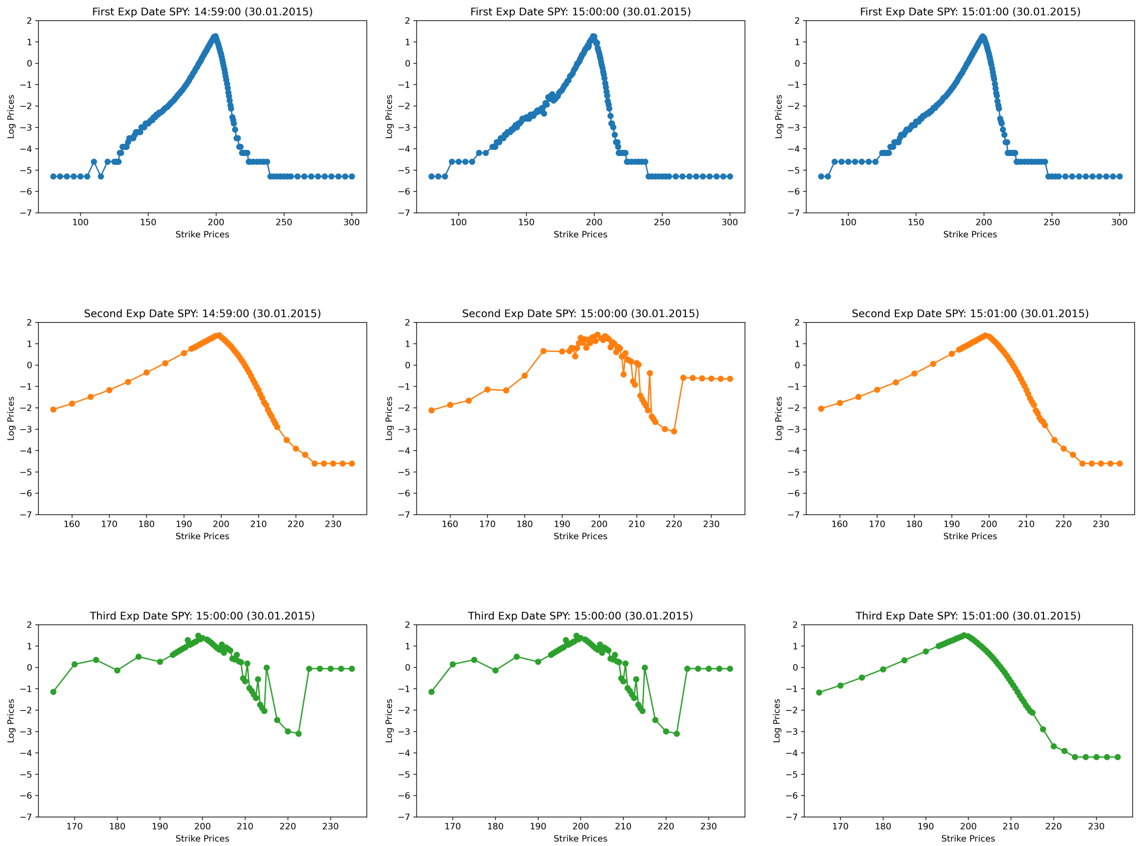


Figure 4.7: The plots illustrate the changes in option prices across different strike prices at 3 PM for the various expiration dates. The blue line in the first three plots represent the cross-section of the nearest expiration date. The orange below depicts the cross-section of the middle expiration date, and the green lines represent the cross-section of the third expiration date, which is furthest from expiration. The x-axis represents the strike prices, while the y-axis displays the logarithmic prices of the options.

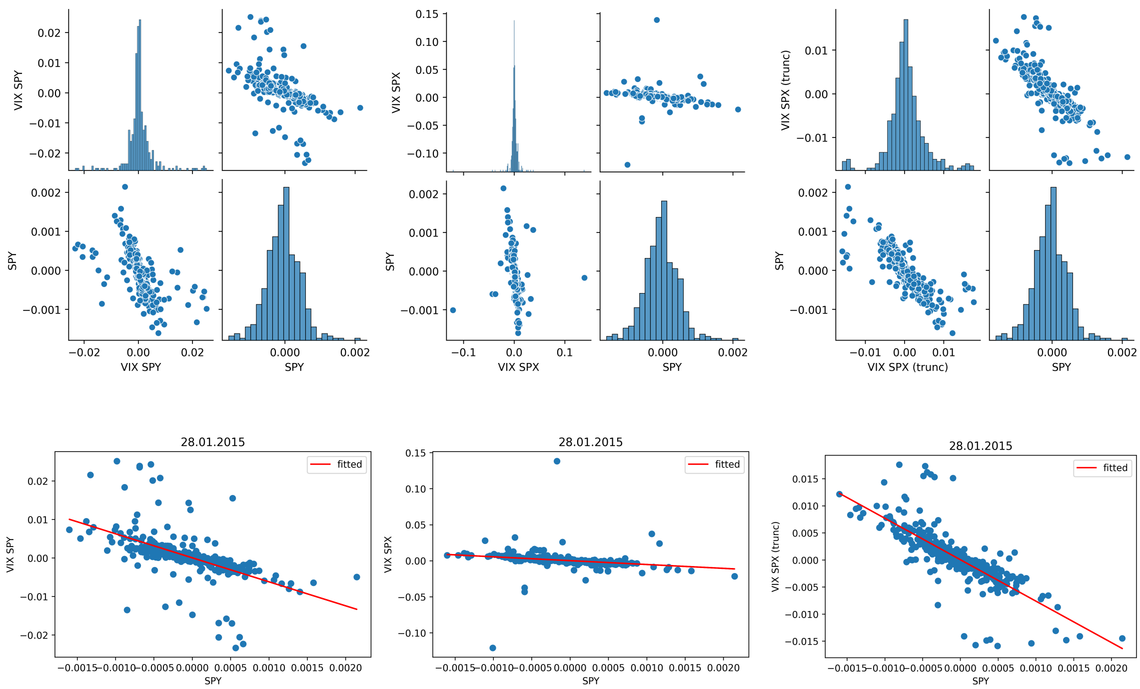


Figure 4.8: The trio of top plots presents a pairplot, showcasing the interrelationships between the VIX and the SPY ETF across different pairs. Directly beneath, the subsequent trio of plots illustrates the findings of a linear regression examination. In these visualizations, the SPY ETF is employed as the independent variable, whereas the VIX serves as the dependent variable. Notably, logarithmic differences were consistently applied to both the underlying SPY ETF and the three distinct VIX values depicted in this visual representation.

4.3 Leverage Effect

The VIX popularity among investors is primarily due to its distinctive behavior of increasing during bearish stock market conditions and either declining or remaining stable during bullish conditions. This unique characteristic has earned it the nickname "fear gauge". The reason behind this behavior can be attributed to the strong demand for options when the S&P 500 prices decline. Market participants tend to purchase protective put options as a means of hedging their portfolios and acquiring insurance, so to speak. This relationship between the S&P 500 and the VIX is a crucial aspect that has contributed to the VIX fame among investors. The negative correlation between the S&P 500 and the VIX has played a significant role in the VIX success for the CBOE. We are going to investigate different kinds of correlation between VIX values calculated from different data types and also the correlation with the SPY ETF. For our investigation we are going to use the Spearman rank correlation coefficient. The formula for this measure is:

$$r_s = \frac{cov(R(X), R(Y))}{\sigma_{R(X)}\sigma_{R(Y)}}.$$

The expression $R(X)$ represents the ranked or sorted set of data points for variable X, while $cov(X, Y)$ denotes the covariance, and σ represents their standard deviation. The first correlation we are going to examine is the one, which made the VIX so attractive among investors, namely the relationship between the VIX and the S&P 500. The correlation coefficient is in the investigated week on average -0.78145, whereas there is just a minimal difference while using the VIX is based on the SPX data where the coefficient is -0.78004. These findings align with previous research conducted in this field, as documented in references (Liu and Dash, 2011) and (Vodenska and Chambers, 2013). One significant discovery of this research is the considerable variation in the correlation between the SPY and the VIX published by the CBOE. This correlation is unexpectedly lower than anticipated (-0.34277), even when compared to the correlations of VIX values estimated using SPY as the underlying asset (0.28045) or SPX (0.26625). The observed low correlation between the different VIX values can be attributed to the fact that the VIX published by the CBOE exhibits a relatively stable and unchanging pattern in comparison to the others. This stability, where values remain consistently at or around the same levels, significantly reduces the degree of linear association or covariance between the different time series. As a result, the correlation coefficient, which measures the strength and direction of their relationship, tends to be low. The first expiration date consistently exhibits the highest correlation with the SPY ETF, with coefficients of -0.85279 for SPY option data and -0.79465 for SPX options. The correlation decreases for the second expiration date to -0.80177 and -0.790214, respectively. Finally, the last expiration date shows a correlation of -0.73538 for SPY data and -0.71150 for SPX data. Similar results with the same pattern emerge when considering the correlation between expiration dates. Therefore, the nearest expiration date shows the strongest relationship with the SPY index, while the correlation weakens as the expiration date moves further into the future. The decreasing correlation as we move further into the future expiration dates can be attributed to the concept of "time decay" or "theta decay" in options. While the expiration date of options approaches, the value of the option tends to decline due to the diminishing time left for the price of the underlying asset to move in a favorable direction. For the nearest expiration date, the option value is highly sensitive to any movement in the price of the underlying asset because there is less time for the market conditions to change significantly. As we move to further expiration dates, the options have more time until expiration, and thus, their value is less affected by short-term movements in the price of the underlying asset. Therefore, the correlation between the VIX and the underlying asset weakens as we move into the future expiration dates. The correlation between the various VIX estimates in this study is as anticipated, as shown in Table 4.1. The correlation among the estimates is generally strong, except for the VIX published by the CBOE, which exhibits weaker correlations with the other VIX estimates. As the interval length increases, the correlation between the VIX estimates becomes weaker. The variation in correlation between VIX estimates at different interval lengths can be attributed to diverse factors. For instance, longer interval lengths can introduce more data gaps and missing information, impacting the accuracy and reliability of VIX estimates and consequently affecting their correlation with other estimates and the underlying. Moreover, high-frequency trading and market microstructure play a role in the correlation as well. Shorter intervals capture more detailed market movements, while longer intervals tend to smooth out some of these microstructure effects, leading to differences in correlation. Additionally, the dynamics of the market may not be fully represented with longer interval lengths. Intraday movements in the VIX can be influenced by various market events, news, and investor sentiments. As the interval length increases, some of these short-term influences may be diluted or averaged out, resulting in weaker correlations between VIX estimates. Overall, with larger interval lengths, the data is aggregated over a longer time period, leading to more volatility smoothing and noise reduction. This can obscure short-term fluctuations and make the VIX estimates less responsive to rapid changes in the price of the underlying asset, thereby affecting the correlation with other estimates, as evident in the data presented in Table 4.1.

One-minute Interval

	SPY	SPY VIX	SPX VIX	SPX VIX (trunc)	CBOE VIX
SPY	*1*	*-0.781*	*-0.780*	*-0.832*	*-0.343*
SPY VIX	*-0.781*	1	0.725	0.759	0.280
SPX VIX	*-0.780*	0.725	1	0.935	0.266
SPX VIX (trunc)	*-0.832*	0.759	0.935	1	0.208
CBOE VIX	*-0.343*	0.280	0.266	0.280	1

Five-minute Interval

	SPY	SPY VIX	SPX VIX	SPX VIX (trunc)	CBOE VIX
SPY	*1*	*-0.782*	*-0.737*	*-0.815*	*-0.329*
SPY VIX	*-0.782*	1	0.683	0.743	0.292
SPX VIX	*-0.737*	0.683	1	0.926	0.232
SPX VIX (trunc)	*-0.815*	0.743	0.926	1	0.275
CBOE VIX	*-0.329*	0.292	0.232	0.275	1

Ten-minute Interval

	SPY	SPY VIX	SPX VIX	SPX VIX (trunc)	CBOE VIX
SPY	*1*	*-0.773*	*-0.705*	*-0.799*	*-0.267*
SPY VIX	*-0.773*	1	0.639	0.707	0.262
SPX VIX	*-0.705*	0.639	1	0.904	0.120
SPX VIX (trunc)	*-0.799*	0.707	0.904	1	0.186
CBOE VIX	*-0.267*	0.262	0.120	0.186	1

Thirty-minute Interval

	SPY	SPY VIX	SPX VIX	SPX VIX (trunc)	CBOE VIX
SPY	*1*	*-0.751*	*-0.745*	*-0.829*	*-0.216*
SPY VIX	*-0.751*	1	0.630	0.714	0.023
SPX VIX	*-0.745*	0.630	1	0.921	0.033
SPX VIX (trunc)	*-0.829*	0.714	0.921	1	0.102
CBOE VIX	*-0.216*	0.023	0.033	0.102	1

Table 4.1: The table presented displays the correlations between the underlying asset and the VIX values at various intervals. The first table showcases the average correlations throughout the week at the one-minute interval. Subsequently, correlations at five-minute intervals, ten-minute intervals, and thirty-minute intervals are provided.

4.4 Higher Frequency

This chapter is dedicated to examining the SPY as the underlying asset, with a specific emphasis on analyzing data at one-second intervals. In the previous analysis conducted using one-minute intervals, a total of 405 data points were available for the trading day, commencing from 08:30 when the CBOE opened and concluding at 15:15 when it closed. Nevertheless, the adoption of one-second intervals considerably increases the number of

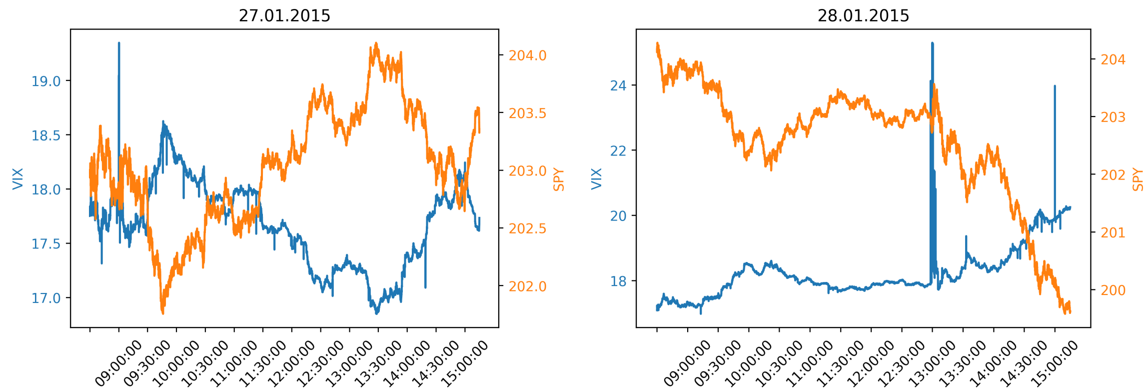
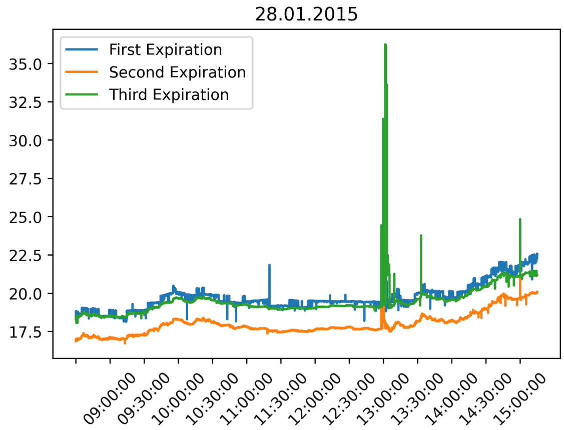


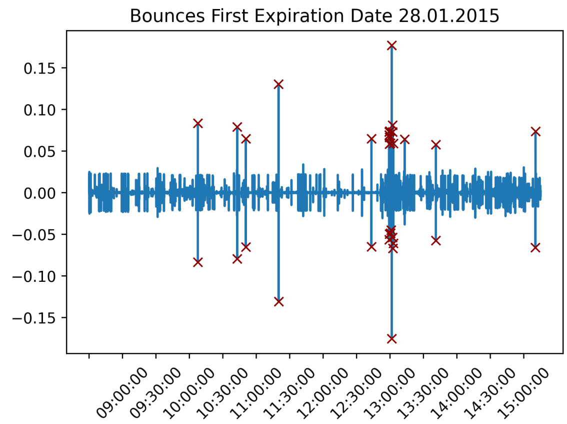
Figure 4.9: The figure displays the VIX calculated in one-second intervals along with the corresponding SPY ETF as the underlying asset. Notably, the spikes in the VIX are more frequent and higher compared to the previous sections.

observations to 24,300 data points for each trading day, offering a higher level of detail for analysis. This investigation seeks to dig into distinct characteristics and aspects that were not observed in the previous analysis conducted using one-minute intervals. The primary focus will be on examining the variations in correlation behavior when employing shorter time intervals, potentially offering insights into the influence of high-frequency trading. Additionally, we will analyze the emergence of the VIX at higher frequencies, aiming to uncover any noteworthy patterns or dynamics at these finer time resolutions. An interesting discovery is the decline in correlation between the underlying asset and the VIX when using this high-frequency data. The average correlation between the SPY ETF and the VIX for the examined week is merely -0.0774, suggesting a weak or almost negligible correlation between these two processes. This contrasts with the previous finding where shorter time intervals exhibited stronger correlations. In addition, the correlation between the different expiration dates has also decreased. The use of high-frequency data introduces more noise and short-term fluctuations, which can obscure the underlying relationship between the VIX and the underlying asset. As a result, the correlation tends to be weaker compared to correlations calculated using longer time intervals, where longer-term trends and market dynamics have a more pronounced impact. While using high-frequency data can be more susceptible to noise, which can obscure the relationship between the underlying asset and the VIX. Additionally, high-frequency data might capture intraday volatility spikes that may not necessarily be reflective of longer-term market dynamics. In situations where investors use the VIX as a hedge against market downturns, relying solely on high-frequency

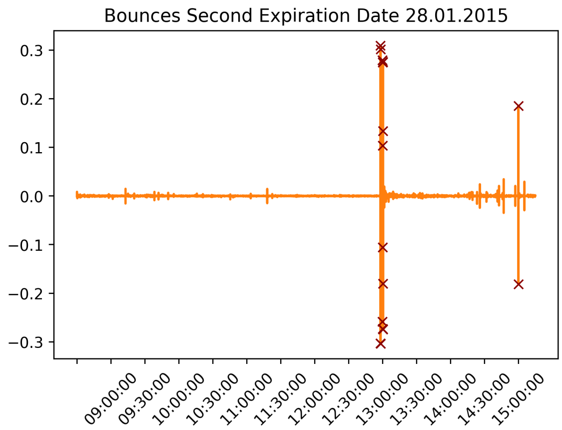
VIX data may result in less reliable protection. the VIX exhibits significantly more jumps in the one-second interval compared to the one-minute interval. On average, the one-second interval VIX experiences 162 jumps each day, while the one-minute interval VIX only has around 22 jumps per day. Figure 4.10 depicts the number of jumps that occurred on January



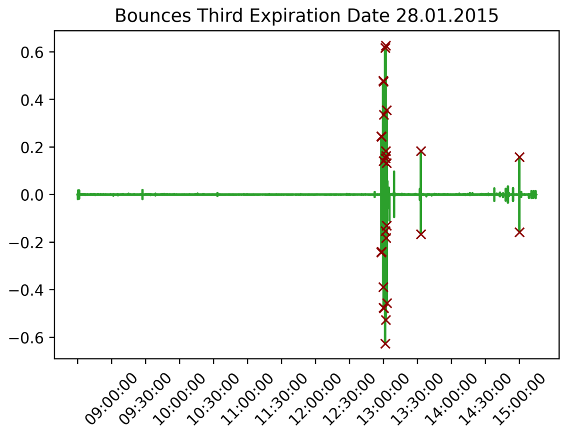
Three different expiration dates



First expiration date



Second expiration date



Third expiration date

Figure 4.10: The plot illustrates the jumps of the VIX at high-frequency on the same Wednesday as previously discussed.

28, 2015, during the FED conference. Clearly, new information entered the market during this event, as reflected in the price changes of the options. The plot also reveals a significant increase in outliers when compared to the plot generated using one-minute intervals. It is crucial to investigate the events that unfolded on Wednesday, as they were not observable in the one-minute interval analysis. These events could potentially offer insights into the discrepancy between the CBOE VIX and the modeled VIX using SPY respectively SPX data. The thickness of the line in figure 4.10 signifies multiple abrupt changes in the VIX within a short time frame, distinguishing it from other jumps. These fluctuations are visually evident as distinct spikes in the graph. Additionally, the occurrence of red crosses in Figure 4.10 highlights several consecutive peaks observed in the data. The sequence of these peaks indicates the potential presence of an arbitrage opportunity or the exploitation of a market inefficiency during that specific period. All three expiration dates experience a spike at the same time, suggesting a synchronized occurrence. This prompts inquiries about a potential shared catalyst or occurrence, which might have influenced the market during that specific period, such as perhaps the already mentioned FED meeting. Further investigation is warranted to explore the underlying factors behind this simultaneous surge in VIX values across different expiration dates. To analyze these jumps, an examination of the cross-section is necessary. Figure 4.11 presents the cross-section of the different expiration dates around that time. It showcases a particular second, which is representative of the time period with fluctuations.

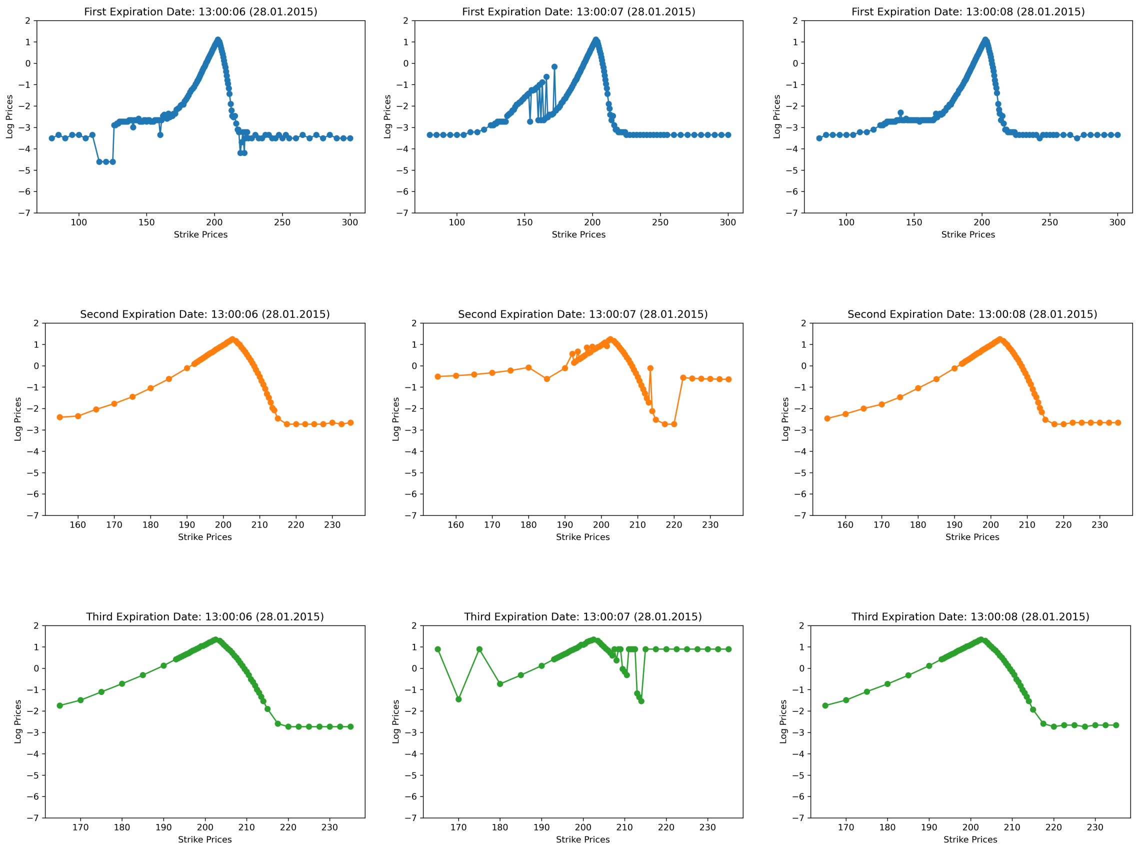


Figure 4.11: Cross-section of the options taken at seconds interval. The blue graph represents the cross-section of the first expiration date, the orange represents the second expiration date and the green line represents the third expiration date. In this time frame, we can observe how the market quickly incorporates new information, as indicated by the actions of high-frequency traders.

Despite the detailed analysis, the available data does not offer sufficient clarity to pinpoint the exact cause of the market inefficiency observed. The figure highlights the distinct behavior exhibited by the cross-sectional prices of the options. Interestingly, the prices of these options revert back to their previous levels in some instances, only to change again in the following seconds. The market response to new information is evident in the sudden and significant price movements observed in the option prices (4.11). Potentially, in the immediate term, institutional investors might have anticipated a market decline, prompting their substantial purchase of put options for the nearest expiration date. However, their strategy seems geared towards a bullish outlook in the long run, evident in their acquisition of call options for the more extended expiration dates. Once more, the far out-of-the-money options emerge as the primary drivers behind these price changes. These findings indicate that market participants are responding robustly to the new information, and these specific options are reflecting the impact of that information most prominently. The VIX experienced an increase during the Wednesday jump, as observed not only in the one-minute interval SPY data, but also in the SPX data and the one-second interval SPY data. This intriguing finding contradicts the information provided in the CBOE VIX report, which indicated a decrease in the VIX during the same period. Our analysis of the one-second interval data suggests that our model predicts an upward movement of the VIX during the Wednesday jump, which diverges from the information reported by the CBOE. This discrepancy underscores the necessity for further investigation and evaluation to understand the underlying factors contributing to these contrasting observations.

4.5 Effect of Exchange Platforms

This chapter aims to analyze the behavior of the VIX by incorporating option quotes from multiple exchanges. By utilizing option price quotes from different exchanges for the SPY, we can examine and evaluate the impact of these exchanges on VIX values. Since the SPX is exclusively traded on the CBOE, the comparison will focus on utilizing SPY data. Below is a summary of the option exchanges previously discussed in the Methodology chapter. The exchanges under investigation in this study include NYSE Amex Equities (AMEX), BATS Options (BATS), Boston Stock Exchange (BSE), Chicago Board Options Exchange (CBOE), BOX Options Exchange, GEMX (ISE Gemini), International Securities Exchange (ISE), MIAX Options Exchange, NYSE Arca Options, OMX Options Market (NOM), OMX BX Options (NOBO), C2 Options Exchange, OMX PHLX, and BATS BZX Options. Certain exchanges may have limited data available for the one-minute interval due to lower liquidity levels. This aspect will be further discussed later on to provide a comprehensive understanding of the data limitations associated with these exchanges. In order to streamline the terminology, this study will utilize the symbols designated by the Options Price Reporting Authority (OPRA). The table presented provides the association between each exchange, its corresponding symbol, and the group to which the options exchange belongs. For example, if a 'B' is depicted on the graph, it represents the BOX Options Exchange of the TMX Group, which can be cross-referenced in the accompanying table 4.5. Additionally, the table highlights the fact that certain larger exchanges have acquired smaller option exchanges, indicating consolidation within the industry.

Option Exchanges		
Group	Option exchange operator	OPRA symbol
Intercontinental Exchange (ICE)	NYSE AMEX Options	A
	NYSE Arca Options	N
TMX Group	BOX Options Exchange	B
CBOE	Chicago Board Options Exchange	C
	C2 Options Exchange	W
	BATS BZX Options	Z
Nasdaq	GEMX (ISE Gemini)	H
	International Securities Exchange (ISE)	I
	OMX Options Market (NOM)	Q
	OMX BX Options (NOBO)	T
	OMX PHLX	X
Miami International Holdings (MIH)	MIAX Options Exchange	M

The examination of liquidity across various exchanges forms a key focus of exploration in this chapter. This analysis holds the potential to unveil insights into the impact of liquidity on VIX behavior, particularly by investigating whether exchanges with lower liquidity exhibit greater fluctuations, potentially attributable to wider spreads. The trajectory of the SPY price process, chosen by the most liquid exchange for each specific day, can be considered a dependable indicator. While the price process is of stochastic nature, sporadic irregularities can occur. Nonetheless, it serves as a valuable reference for understanding the characteristics of a typical or normal process. Figure 4.12 illustrates the calculated VIX

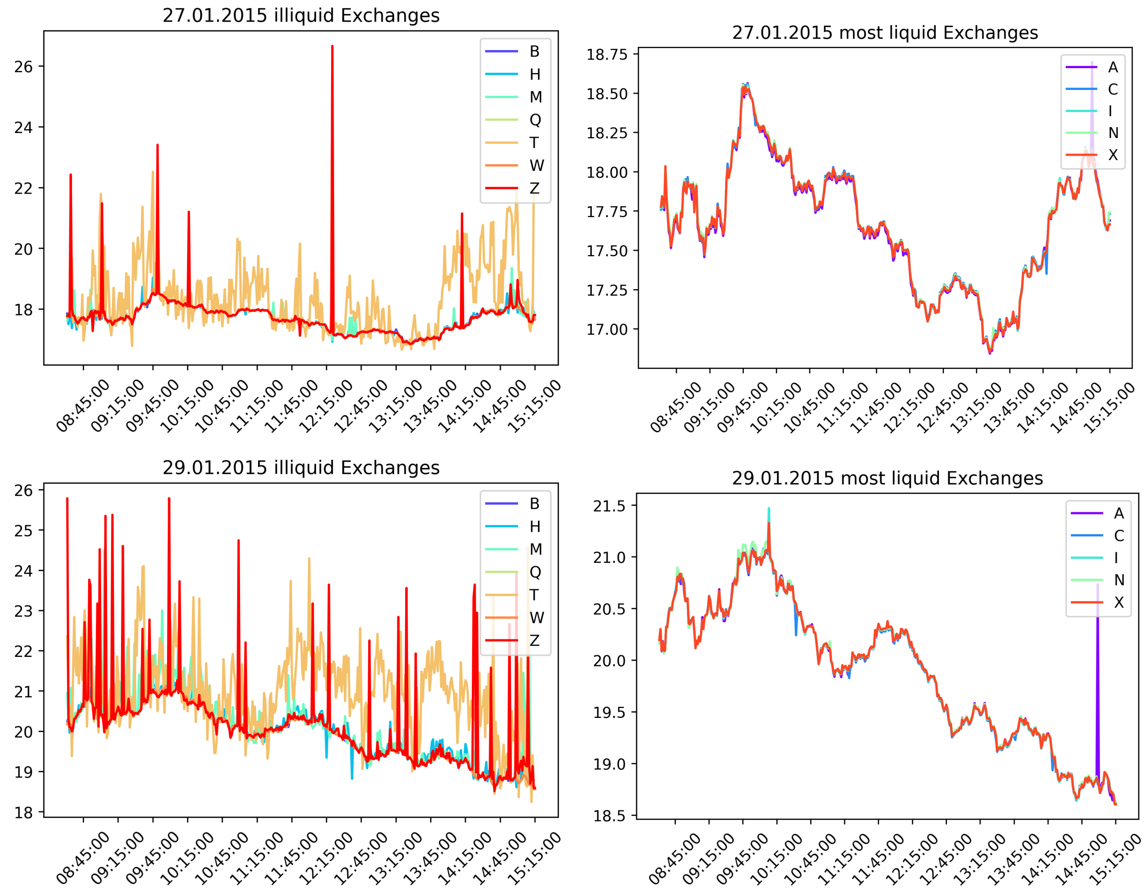


Figure 4.12: The figure illustrates the contrast between exchanges with different trading volumes. On the left side, we have the less liquid exchanges, while the right side showcases the more liquid exchanges, as per (Andersen, Archakov, Grund, Hautsch, Li, Nasekin, Nolte, Pham, Taylor, and Todorov, 2021).

values obtained from different exchanges, from Tuesday and Thursday. The plot notably showcases the presence of unusual and extreme behavior in certain exchanges, particularly exchange Z, which exhibits significant outliers. Apart from these anomalies, the VIX process derived from option quotes on BATS BZX Options (Z) demonstrates relatively moderate behavior. Significantly, the light orange line representing the VIX from OMX BX Options (T) exhibits heightened fluctuations on Tuesday. Furthermore, throughout the week, this exchange displays more pronounced fluctuations compared to the others, making it an intriguing feature of the graph. The observed phenomenon can potentially be explained by the presence of fewer transactions or quotes, leading to larger and more pronounced changes in the VIX values. Another possible explanation for the rapid fluctuations

observed in the red line (representing BATS BZX Options VIX (Z)) could be attributed to infrequent but significant quotes entering the market. The spikes in the VIX values may be a consequence of hedging strategies implemented by major market participants. These participants may be engaging in large-scale hedging activities, resulting in notable impacts on the VIX measurements. The plot clearly reveals that less liquid exchanges tend to exhibit greater volatility and more frequent spikes in the VIX process. Interestingly, the five largest exchanges in terms of trading volume in 2015 (CBOE, AMEX, ARCA, PHLX, and ISE), as mentioned in the study "A Descriptive Study of High-Frequency Trade and Quote Option Data" (Andersen, Archakov, Grund, Hautsch, Li, Nasekin, Nolte, Pham, Taylor, and Todorov, 2021), demonstrate fewer spikes and a more stable VIX process in the plot. This observation suggests a potential correlation between liquidity levels and the stability of the VIX process. Exchanges categorized as "less liquid" still maintain a considerable level of trading volume. However, their volume is relatively lower compared to larger exchanges like CBOE and AMEX. This lower volume translates to fewer reported quotes and reduced market activity, resulting in a decrease in the number of buyers and sellers participating in option contracts. Therefore, this has repercussions on trade execution ease and overall market activity. Consequently, exchanges with higher liquidity typically display enhanced market efficiency, which manifests in quicker price adjustments to new information. In simpler terms, exchanges with greater liquidity tend to have more stable prices. This relationship between liquidity and price stability is evident in the plot (4.12) depicting the VIX calculated from various exchanges. More liquid exchanges display a smoother VIX process, reflecting the influence of liquidity on price dynamics. The presence of numerous market participants in more liquid exchanges enables them to accommodate large trades without causing substantial price impacts on the underlying asset. The VIX, derived from different option quotes, serves as a reflection of the liquidity level of an exchange.
On Friday afternoon, there appears to be a widespread occurrence of unusual behavior across nearly all exchanges, suggesting the possibility of an arbitrage opportunity in the market. As for example the arbitrage opportunity that traders can exploit price discrepancies by simultaneously buying and selling option contracts on the same underlying asset across different exchanges. Although we already analyzed the outliers on Friday in detail in chapter 4.2. Referring back to the plot in Figure 4.12, on Tuesday, the less liquid options market witnesses VIX values surpassing 26, as evidenced by the highest peak on exchange Z. In contrast, the highest VIX value recorded in the liquid options market is slightly above 18.50, indicating a substantial disparity in volatility levels between the exchanges. This observation emphasizes not only the contrasting trajectories of the VIX, but also the varying levels reached by the index across different exchanges. Throughout the week, the calculated VIX levels from different exchanges consistently display variations, which adds to the fascination surrounding the divergence of VIX values. The discrepancy in VIX values among exchanges raises intriguing questions that warrant further exploration. That illustration shows how important liquidity is for efficient markets. The VIX calculated from quotes on one exchange should align closely with the VIX calculated from quotes on other exchanges. This is because factors influencing the VIX, such as economic uncertainty, market corrections, and significant events, should be accessible and known to all market participants. However, as depicted in the plot presented in Figure 4.12, this ideal scenario is not always observed, with varying values of the VIX observed across different exchanges.

This discrepancy invites further investigation into the underlying factors contributing to the differences in VIX values among exchanges, potentially shedding light on market dynamics and the impact of liquidity variations on the index. Prompting the question of why these differences in VIX values exist among exchanges. One contributing factor to this disparity can be attributed to the liquidity issues present in certain exchanges. When orders in large volumes cannot be efficiently executed on these exchanges, it can lead to an increase in the VIX values.
Figure 4.13 illustrates VIX values from various exchanges alongside their corresponding underlying price processes. This data includes three highly liquid option exchanges and three less liquid ones. The left side of the plot exhibits three graphs representing the more liquid option exchanges, while the right side depicts three graphs representing the less liquid exchanges. Particularly, the more liquid exchanges demonstrate a greater degree of similarity in their behavior, as evidenced by the uniformity of their y-axis values, all falling within the same range. In contrast, the less liquid option exchanges exhibit a wider range of y-axis values, spanning from 17 to 32.5. This discrepancy suggests that the less liquid exchanges experience greater variability and potential divergence in their VIX values, highlighting the influence of liquidity on the stability and consistency of VIX measurements across different exchanges. Nevertheless, a common characteristic among all exchanges is the significant spike observed around midday, indicating the pricing in of new information. However, this spike was not adequately reflected in the VIX published by the CBOE. As a result, it raises doubts about the full reliability of the CBOE VIX. Upon closer observation, the VIX calculated from less liquid option exchanges consistently exhibits a higher frequency of spikes, as indicated by the quotes and prices of options. This observation underscores the importance of utilizing high-quality data when calculating the VIX, as its accuracy and reliability heavily depend on the input data. When fed with data from less liquid option exchanges, the VIX may become less dependable as an approximation of market volatility. To gain a comprehensive understanding of these phenomena and to provide a more conclusive explanation, further investigation is warranted. Additional research and analysis can shed more light on the factors contributing to the observed differences in VIX behavior across different option exchanges.

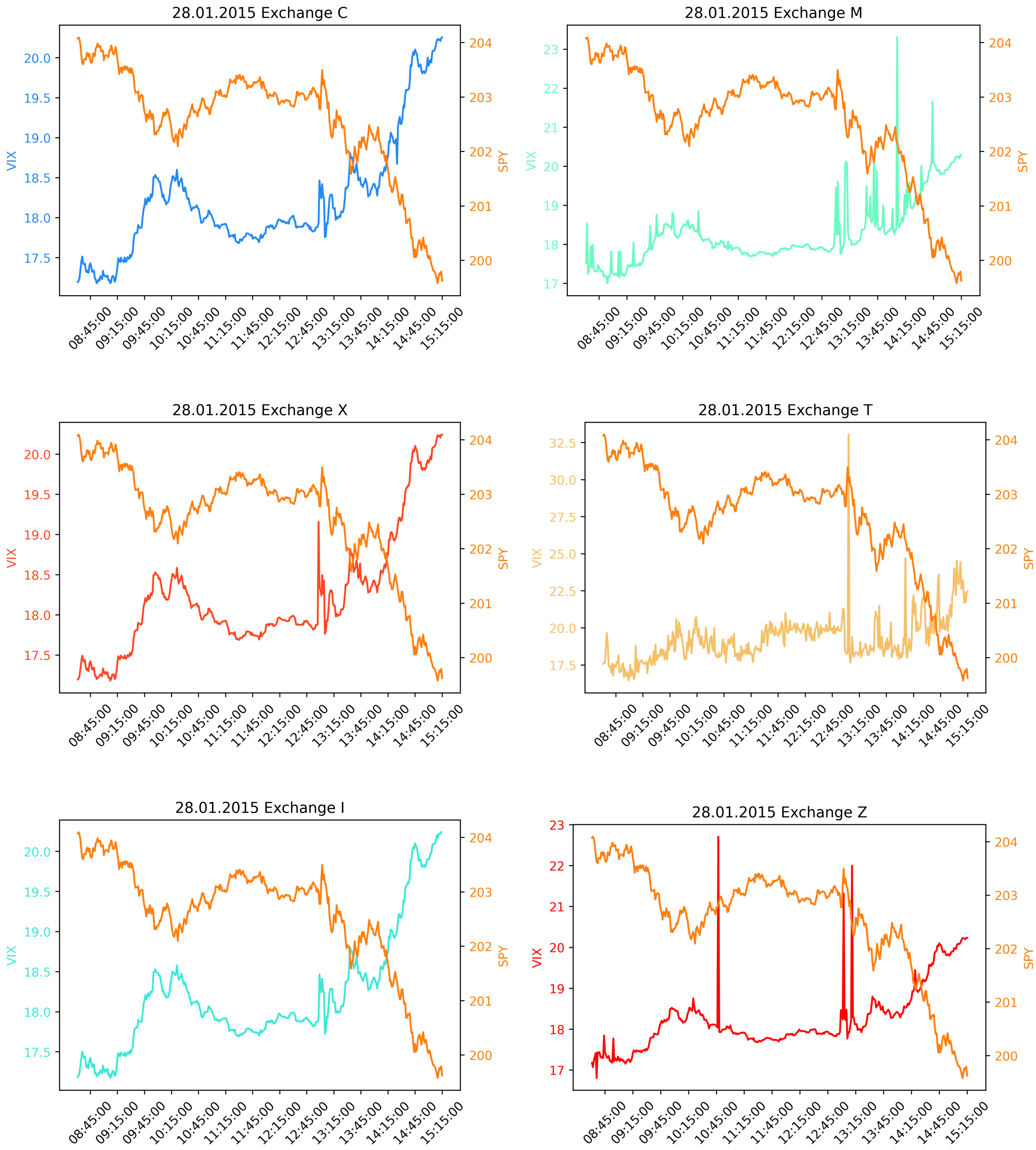


Figure 4.13: The plot depicts the VIX calculated from various exchanges on Wednesday, the 28th, along with the corresponding underlying asset. Nearly all exchanges displayed a synchronized reaction around midday.

5 Conclusion

The study of intraday patterns of the implied volatility VIX is crucial for understanding the dynamics of financial markets and their behavior. Through this research, we have discovered various factors that impact the VIX on a minute-to-minute basis, such as liquidity, correlations with the underlying, and the influence of different option exchanges. Additionally, this thesis effectively illustrates how the VIX is frequently affected by far-out-of-the-money options. However, these options are not incorporated into the CBOE VIX, potentially depriving market participants of valuable information. Our findings suggest that the VIX can exhibit significant deviations from its expected behavior in short time intervals, which highlights the importance of careful analysis when using the VIX as a market indicator. In conclusion, the examination of intraday patterns of the VIX provides valuable insights into the volatility dynamics of financial markets and is an important area for further research. As our thesis suggests, the VIX performs well when applied to high-quality data, meaning that there is little difference in VIX levels for liquid exchanges. However, if the exchange lacks liquidity, the reliability of the VIX decreases significantly. Especially the results of our comparison with the officially published VIX by the CBOE indicated that there may be discrepancies in the calculation, which highlights the importance of using high-quality data. Another issue is that the reliability of the VIX decreases even further when using a higher-frequency. At high-frequencies, the behavior of the VIX becomes even more abnormal, leading to an increase in outliers. Our findings provide valuable insights for investors, as well as for researchers looking to further study the dynamics of implied volatility. Ultimately, this thesis underscores the importance of understanding the intraday patterns of the VIX, as they have implications for risk management and investment decisions. For future research, the exploration of different underlyings across various sectors and diverse exchanges, within even more compressed time frames, holds a captivating potential. The study of intraday volatility remains a rich domain with numerous discoveries yet to be unearthed.

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