universität wien

MASTERARBEIT / MASTER'S THESIS

Titel der Masterarbeit / Title of the Master's Thesis

„The Natural Rate of Unemployment

- Validity and Estimation Methods - "

verfasst von / submitted by

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angestrebter akademischer Grad / in partial fulfilment of the requirements for the degree of

Master of Science (MSc)

Wien, / Vienna 2023

Studienkennzahl lt. Studienblatt / degree programme code as it appears on the student record sheet:	UA 066 913
Studienrichtung lt. Studienblatt / degree programme as it appears on the student record sheet:	Masterstudium Volkswirtschaftslehre
Betreut von / Supervisor:	Univ.- Prof. i.R. Dipl.- Ing. Dr. Robert Kunst

Abstract

The natural rate of unemployment hypothesis was a groundbreaking concept (Friedman & Phelps, 1967, 1968); however, with the introduction of the hysteresis hypothesis in unemployment rate (Blanchard & Summers, 1986) the existence of the natural rate is severely questioned by large number of economists. In this work we present further evidence against the hysteresis hypothesis by finding level- or/and trend stationarity in most of the chosen 21 OECD countries` unemployment rates in the period of 1969-2022 with performing the KPSS- (1992) and Kruse (2011) test. Further, we attempt to reproduce the natural rate of unemployment of France and the United States represented by the OECD NAIRU estimates with the trend component of measured unemployment rate (Rogerson, 1997), through various smoothing and decomposition methods applied for the actual unemployment rates of the two countries in the period of 1970-2022. Although the Hodrick-Prescott- (with ʎ equal to 100) curve and the Baxter-King filtered line came closest statistically to the OECD benchmark in each case, neither these curves nor their homoscedastic (but autocorrelated) regression curves were able to reproduce the respective NAIRU estimates or their homoscedastic (but autocorrelated) regression curves.

Die Hypothese der natürlichen Arbeitslosenquote war ein bahnbrechendes Konzept (Friedman & Phelps, 1967, 1968); jedoch mit der Einführung der Hysterese-Hypothese in der Arbeitslosenquote (Blanchard & Summers, 1986) wird die Existenz der natürlichen Rate von einer großen Anzahl von Ökonomen ernsthaft in Frage gestellt. In dieser Arbeit präsentieren wir weitere Evidenz gegen die Hysterese-Hypothese, indem sie die Niveau- und/oder Trendstationarität in den meisten der 21 ausgewählten OECD-Länder in den Arbeitslosenquoten im Zeitraum von 1969-2022 mit der Durchführung des KPSS- (1992) und Kruse (2011) Tests feststellt. Darüber hinaus versuchen wir, die natürliche Arbeitslosenquote Frankreichs und der Vereinigten Staaten, die durch die OECD-NAIRU-Schätzungen repräsentiert wird, mit der Trendkomponente der gemessenen Arbeitslosenquote (Rogerson, 1997) zu reproduzieren, indem wir verschiedene Glättungs- und Zerlegungsmethoden anwenden, die für die tatsächlichen Arbeitslosenquoten der beiden Länder im Zeitraum von 1970-2022 angewendet werden. Obwohl die Hodrick-Prescott-Kurve (mit ʎ gleich 100) und die Baxter-King-gefilterte Linie dem OECD-Benchmark jeweils statistisch am nächsten kamen, waren weder diese Kurven noch ihre homoskedastischen (aber autokorrelierten) Regressionskurven in der Lage, die jeweiligen NAIRU-Schätzungen oder ihre homoskedastischen (aber autokorrelierten) Regressionskurven zu reproduzieren.

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1. Introduction

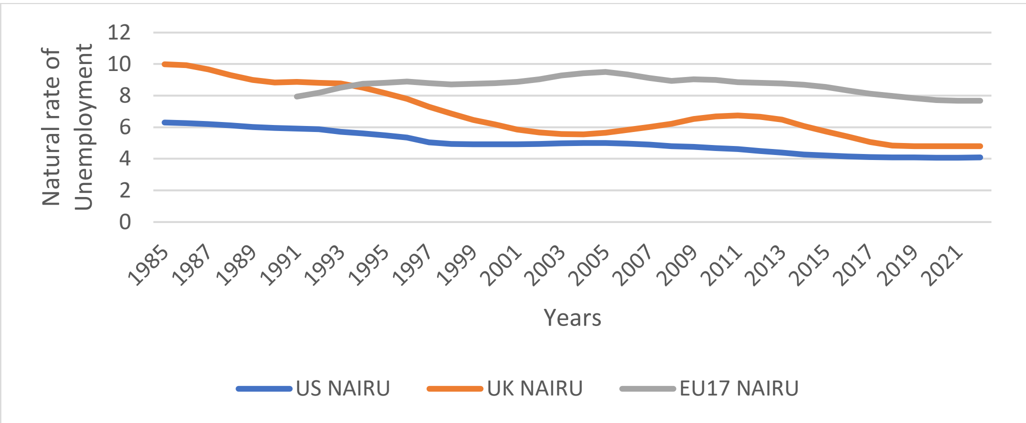
Unemployment has been a major concern of governments of advanced economies since the two great oil crises in the 1970s, leading to a high degree of persistence in unemployment rates, especially in European countries. A high and persistent unemployment rate has serious negative impact on economic growth and living society materializing in significant socio-economic and socio-political consequences (i.e., decreasing productivity, rising economic inequality, expanding individual costs in form of mental health issues, loss of human capital, civil unrest etc.).

Friedman and Phelps (1968) more than 50 years ago developed a joint hypothesis as the natural rate of unemployment[1] which is independent of monetary policy (independence hypothesis) and as monetary policy - defined by Friedman -, offers only a temporary trade-off between inflation and unemployment, therefore it could not maintain any unemployment rates below the natural rate without leading to higher inflation (acceleration hypothesis) (Blanchard, 2018). Economists often call the natural rate of unemployment for a more general macroeconomic equilibrium (King et al., 2005).

While the natural rate hypothesis was disputed at the time, its popularity increased overnight and has become a dominant paradigm in macroeconomics. The natural rate of unemployment (determined by labor market equilibrium) is the long-run equilibrium level of unemployment characterized by a mean-reverting estimated time series around which the actual rate of unemployment fluctuates and returns after any economic shocks, as economic shocks have a transitory effect on the actual rate of unemployment (Cheng, 2022).

While the Great Moderation (Stock & Watson, 2002; Bernanke 2004) took place in the US from the mid-1980s up to approximately 2007, represented by less volatility in business cycle fluctuations in contrast to previous decades, most European economies were plagued by periods of recessions with high unemployment rates from the 1980s leading to the popular proposition of rising natural rate of unemployment.

Table 1. [2]



Source: OECD

[1] In the further text the joint hypothesis of natural rate of unemployment will be referred as the natural rate hypothesis or the natural rate of unemployment.
[2] EU17: Euro area: AUT, BEL, EST, FIN, FRA, GER, GRE, IRE, ITA, LUX, LATV, LITH, NED, POR, SLOV, SLOVENIA, SPA.

The hysteresis hypothesis of the natural rate was first formulated by Blanchard and Summers (1986) arguing that the natural rate of unemployment can follow a path-dependent process, which can be the explanation for its increasing level. As due to the financial crisis (2008-2009) further adverse fluctuation happened in the actual rate of unemployment as well, Summers (2014) interpreted it as the confirmation of the doctrine of hysteresis in the natural rate of unemployment.

Many analyses and studies have been performed examining the accuracy of the natural rate of unemployment since the introduction of the hypothesis by Friedman and Phelps. A large number of economists agreed with the presence of the hysteresis hypothesis in unemployment rate, thus rejecting the existence of the natural rate (Cross et al., 1990; Mitchell, 1993; Roed, 1996; Furuoka, 2014 etc.), but numerous studies rejected hysteresis (Bianchi & Zoega, 1998; Papell et al., 2000; Camarero et al., 2004; Chang et al., 2005; Lee et al., 2010; Khraief et al., 2020 etc.) and reported important evidence in support of the natural rate hypothesis observing that unemployment rates in general have a high degree of persistence (Song et al., 1998).

The natural rate of unemployment is an essential constituent of economic thinking and models used by central banks (Blanchard, 2018). It is the basis of inflation targeting framework applied by the European Union, OECD, Bank of England, and the FED as well. The natural rate of unemployment is calculated as the NAIRU estimates (non-accelerating inflation rate of unemployment) from the Phillips curve relationship applying the Kalman filter and used as a benchmark for policy applications.

Blanchard (2018) stated: "I see the macro- and microeconomic evidence as suggestive but not conclusive evidence against the natural rate hypothesis. Policy makers (central banks) should keep the natural rate hypothesis as their null hypothesis, but also keep an open mind and put some weight on the alternatives". Therefore, more empirical corroborations are needed to understand the dynamics of the development of the unemployment rate.

This study aims to empirically reconsider the dynamics of the unemployment rate for selected 21 OECD countries (Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Iceland, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, South Korea, Spain, Sweden, Turkey, UK, and US) in the period of 1969 – 2022 by applying conventional and nonlinear unit root tests. In the second part of this work following the argument of Rogerson (1997) that the natural rate of unemployment is the rate of unemployment that would exist after seasonal and cyclical components are taken out, smoothing and filtering statistical methods such as the Hodrick-Prescott-, the Baxter-King, the Christiano-Fitzgerald filters and the Beveridge-Nelson decomposition will be applied for the unemployment rate time series of France and the US for the period of 1970 – 2022 to extract the trend component and examining and comparing to the NAIRU estimates as benchmark, calculated by the OECD for the identical period.

This work`s novelty is threefold: (i) The examined period encompasses the financial crisis (2008-2009) and the COVID time (2019-2022) as well, rarely assessed in the literature so far. (ii) The Beveridge-Nelson decomposition mostly applied for the GDP analysis in the literature not for the unemployment rate time series. Also, there are not many attempts to estimate the natural rate of unemployment by the Hodrick-Prescott Filter (Claar, 2000). (iii) The results of the analysis will be compared not only with the NAIRU estimates calculated by the OECD but also with the NAIRU derivative curves obtained by regression analysis.

The study is built up as follows: after the introduction and the research questions the literature review is performed in the third chapter, assessing relevant documents about the development of the notion of the natural rate of unemployment and the NAIRU estimates, continuing with the greatest criticism of the concept by the hysteresis hypothesis and closing with the evaluation of the main estimation methods of the natural rate of unemployment. Following the methodology description of the assessment in chapter 4, the analysis is performed in chapter 5 in two steps: in the first part the author is looking for new evidence about the stationarity properties of the unemployment rates in the chosen time series with different testing methods; in the second part the natural rate of unemployment calculated by the NAIRU estimates of the OECD is examined in two countries, namely France and the United States, through various smoothing and decomposition methods applied for the actual unemployment rates. The author also checks whether these methods can reproduce the NAIRU estimates or their regression derivations. Chapter 6 concludes the assessment of the validity and the estimation methods of the natural rate of unemployment performed in the previous section.

2. Research Questions

- Does the unemployment rate time series indicate a stationary or a non-stationary process?
- Can the natural rate of unemployment in the form of NAIRU estimates or of their regression derivations (curves obtained by regression analysis) be estimated with a smoothing or filtering approach such as the Hodrick-Prescott-, the Baxter-King-, the Christiano-Fitzgerald filters or the Beveridge-Nelson decomposition method?

3. Literature Review

3.1. Natural Rate of Unemployment

In the late 1960s Friedman (1968) and Phelps (1967) established the “natural rate” of unemployment hypothesis. It was “born” from the criticism of the two main paradigms of Keynesian macroeconomics, as boosting of aggregate demand did not work anymore in the 1960s to reduce the unemployment rate. Friedman blamed the “short-run” Keynesian model as it did not differentiate temporary from permanent changes in monetary policy in terms of their effects on unemployment. Respectively, he dispraised the “long-run” growth model, as it included no assumption at all about the unemployment rate (Rogerson, 1997). The standard definition of the natural rate is that it depends on “the actual structural characteristics of the labour and commodity markets, including market imperfections, stochastic variability in demands and supplies, the cost of gathering information about job vacancies and labour availabilities, the costs of mobility, and so on“ (Friedman, 1968). The vagueness of the definition was intentional, as Friedman never meant to present a formal notion but to declare his strong belief that the real side of the economy assumes a unique (long-run) equilibrium (Dixon, 1995). Phelps (1968) described the same concept more clearly as an equilibrium unemployment that was invariant to monetary policy (Rogerson, 1997).

The missing micro-foundations of the natural rate concept were developed and formalized by Lucas (1973). He relied partially on imperfect information or forecast errors – the presence of uncertainty regarding to the future in the Lucas-Rapping model (1969); imperfect information pertaining to aggregates such as the price level and money stock in the Island-model (1979) -. However, an underlying assumption in his conception of the significance of intertemporal substitution was that despite possessing full information and perfect foresight, economic activity would still fluctuate as rational households vary labor supply in response to alterations in present and future prices and wages. Hence, instead of the static concept of the natural rate the dynamics of short-run deviations became important around the long-run static equilibrium (Dixon, 1995).

Real business cycle theorists welcomed the idea of competitive equilibrium and expanded it to a fully dynamic equilibrium: there exists an intertemporal equilibrium that extends through time, thus the equilibrium level of employment is itself fluctuating. For them, the natural rate of unemployment is equal to the actual rate whatever it is today (Tobin, 1995), fluctuating as it reflects changes in the willingness of people to work.

Rogerson (1997) proposed that the natural rate of unemployment could be clearly defined in dynamic economic theory as the deterministic steady state equilibrium unemployment rate. He argued as well that it could be interpreted as an empirical concept equal to the trend component of the observed, actual time series for measured unemployment and that was the rate of unemployment that would exist after seasonal and cyclical components were taken out.

Despite many criticisms of the notion of the natural rate of unemployment (i.e., Hahn 1995, Dixon 1995, Rogerson 1997), one of the concept's innovations was the application of the doctrine of monetary neutrality to unemployment. The terminology "natural rate" indicated that, contrary to pursued "full employment", in equilibrium there might be unemployment (frictional), which is natural and acceptable. Hence, a change in policy was required to see unemployment as a primarily microeconomic concern. The way to decrease it was not through macroeconomic policy, but through policy towards the operation of markets – especially the labor market – in order to remove "imperfections" and "frictions" (Dixon, 1995). Thus, the notion of the natural rate of unemployment encouraged significant research work on micro labour markets, both at theoretical and empirical level, such as contract-, bargaining-, and search theories as well as careful studies of the characteristics of the unemployed (Hahn, 1995). It led to a strong focus on reforming the labour market that marked the Thatcher years (1979-91) (Dixon, 1995).

3.2. The NAIRU

In sharp contrast to the Lucas developments, Layard and Nickell (1985, 1986) introduced an empirical model of the UK labour market that placed imperfect competition at the very heart of the natural rate. Two papers provided the basis for this approach (Layard & Nickell, 1985, 1986) and a new name was created: "non-accelerating inflation rate of unemployment" or NAIRU. The NAIRU approach had the significant advantage of enabling the natural rate to be empirically modelled. Nickell and Layard were able to classify factors such as union power, labour mismatch, unemployment benefits etc., which affected the wage determination curve (WW curve), and those factors which affected the labor demand curve (DD curve) like world energy, commodity prices, capital stock etc., to monitor the evolution of the NAIRU over time (Dixon, 1995).

With the central claim that there exists an unobserved rate of unemployment where inflation stays constant for any economy and at any point in time, the NAIRU became a key concept in contemporary macroeconomics. Over time the NAIRU has been linked to the concept of the natural rate of unemployment (Ball & Mankiw 2002), which would occur in the absence of any cyclical fluctuation and therefore constitutes the structural unemployment rate, existing independently of all seasonal and temporary fluctuations (Friedman, 1968; Phelps, 1967). The European Commission and the OECD utilize the NAIRU estimates as an equivalent measure for structural unemployment, which has significant direct consequences on the scope and evaluation of fiscal policy.

Consequently, from the late 1970s to the 1990s research concentrated on the cross-country determinants of structural unemployment as the dominant theoretical interpretation of Europe`s increasing unemployment was the rigidity of labor market organizations. Yet, several empirical studies followed and showed that the empirical evidence for this view is limited (the underlying correlation lacks robustness in estimation techniques, variations in control variables of selected countries and time periods). Other studies had found no considerable correlation between the OECD measure of labor market deregulation and alterations in the NAIRU (Baker et al., 2005), hence researchers started including additional control variables representing alternative explanations for the evolution of structural unemployment. Blanchard and Wolfers (2000) emphasized the link between macroeconomic shocks (i.e., changes in the long-term interest rate, shifts in labor demand, deviation from the trend in total factor productivity growth) and labor market institutions. Stockhammer and Klär (2011) have taken capital accumulation (investments) in their regression analysis as the most significant variable in explaining unemployment (Heimberg et al., 2017).

In summary, most economists agree that the NAIRU is a specific concept that stands for the non-accelerating inflation rate of unemployment (Claar, 2000), i.e., the unemployment rate consistent with stable inflation (OECD, 2015). However, its theoretical- and empirical assessments are disputable. The author will apply the NAIRU in terms of the interpretation by the OECD as the level of the natural rate of unemployment which should be complied by the actual rate of unemployment, otherwise necessary corrective policy measures should be implemented (i.e., EU fiscal surveillance framework, OECD, 2015).

3.3. The Hysteresis Hypothesis

The greatest criticism on the concept of natural rate of unemployment was presented by Blanchard and Summers (1986) with the hysteresis hypothesis implying that shock effects or cyclical variations in unemployment rate would continue even after the economy recovered due to the rigidity of the labor market (Lindbeck & Snower, 1988). These permanent effects imply that the equilibrium unemployment rate will not be reinstated in the long run (Smyth 2003; Bolat et al., 2014). Thus, the current unemployment rate is assumed to be a dynamic process, it is path-dependent (Phelps, 1972; Carlin et al., 1990), and depends on its past values with the sum of their coefficients equal or very similar to unity.

The validity of the unemployment hysteresis hypothesis is very important as it has considerable policy implications: when unemployment rate development is defined as a non-stationary process, labor market policies should be concentrated on structural reforms to ease the effects of unfavorable shocks. Contrary, when the process is stationary, the focus would be on avoidance of short-term disequilibria (Akay et al., 2020). Since the late 1980s, unit root tests have been widely applied to investigate the accuracy of the hysteresis effect in the development of unemployment rate time series.

Unit root tests were first presented by Dickey and Fuller (1979, 1981) and became the standard procedure to test for stationarity in univariate time series (Gray, 2004) under the linear framework. The augmented Dickey-Fuller test (ADF) (Said & Dickey, 1984) allows for error autocorrelations and can include a drift and/or trend variable besides the lag polynomials and gives more accuracy in testing than the Dickey-Fuller (DF) test. Pesaran & Smith (1995) and Cross (1995) emphasize that the unemployment rate is a bounded variable within the closed interval of [0 - 100%], as technically it cannot fall below 0% or increase over 100%. Therefore, they would expect it to be stationary (or being integrated of order 0 as I(0)) in the very long run. However, for a short period of time an upward trend can happen - i.e., the post 1970s in the UK - but in contrast to a permanent rise, which is the main criterion for a trend, the unemployment rate will return to its equilibrium level (UK unemployment rate decreased in the mid-80s). Thus, Gray (2004) suggests considering the UK unemployment time series as being integrated of order one (I(1)) without a drift or trend in the post 1974 era.

Kwiatkowski et al. (1992) note that the power of the ADF test to reject the unit root is low, as the way conventional hypothesis testing is performed ensures that the null hypothesis is maintained unless there is convincing evidence against it. Thus, relevant alternatives such as stable autoregressive processes with roots near unity (DeJong et al., 1989) or fractionally integrated time series (Diebold et al., 1991) would most probably not be rejected by the ADF test. Kwiatkowski et al. propose a new statistical test (KPSS test) for nonlinear time series[3], complementing the conventional unit root tests, using Bartlett window of 8 with the null hypothesis of stationarity, either around a level or around a linear or nonlinear trend. The nonlinear time series are expressed as the sum of a deterministic trend, a random walk, and a stationary error and in the H_0 hypothesis the random walk has zero variance. Examining US annual data covering 62 to 111 years ending with 1970 with time series including the unemployment rate, the authors concluded that the unit root test was not supported, and the hypothesis of level and trend stationarity were supported at usual critical levels for the US unemployment rate time series.

A change in general testing concept was introduced by Perron (1989) who argued that the traditional unit root tests tend to be not valid in case of structural breaks (shocks). Similar criticism was formulated by Enders & Granger (1998), stating that standard tests for unit root (and cointegration) have low power when the presence of dynamics in a time series is not specified correctly. Conventional univariate unit root tests have been criticized as well, when the time series are short (Campbell & Perron, 1991; DeJong et al., 1992). To control the size problem, panel unit root tests under the assumption of cross-section independence were developed which led to mixed results (Song & Wu, 1997; Leon-Ledesma; 2002).

Later the assumption of cross-section independence was weakened to cross-section dependence and structural breaks were included in the unit root tests (Philips & Sul, 2003; Jewell et al., 2003;

[3] Nonlinear time series are any stochastic process that is not linear, thus their features cannot be modelled by linear processes (i.e., time-changing variance, asymmetric cycles, thresholds & breaks) (Kunst, 2009).

Camarero et al., 2006). Then, significant evidence was found against the hysteresis hypothesis (Cheng, 2022). Phelps (1994) indicated as well that economic shocks may lead to a shift in the natural rate of unemployment.

Becker et al. (2006) introduced the flexible Fourier transformation which is frequently used to model nonlinear time series with structural breaks. Holl and Kunst (2011) used simulation to successfully reproduce the Canadian unemployment rate time series with a two-part nonlinear transformation function of a serially correlated I(1) process and concluded that the Range and the Forward-Backward Range Unit Root tests (Leybourne, 1995) support the hysteresis hypothesis. Various types of further models have been established in unit root testing for nonlinear time series in recent years where the most popular are KSS from Kapetanios et al. (2003), Sollis (2004, 2009) and Kruse (2011).

Kruse (2011) proposed a new unit root test for nonlinear time series, considering a time series y_t that behaves like a random walk if y_{t-1} is not far from some location parameter *c* and is mean reverting as y_{t-1} moves too far from *c.* The economic intuition behind mean-reversion is that economic time series (i.e., exchange rates, unemployment rates, interest rates) behave like a non-stationary random walk if they are close enough to their long run equilibrium value in the last period and there are driving forces (i.e. arbitrage, central equilibrating policies etc.) leading to mean-reversion when high deviation from the long run equilibrium level occur (Kruse, 2011). In these time series, modeled as exponential smooth transition autoregressive processes (ESTAR), the degree of mean reversion is dependent on the squared difference between y_{t-1} and *c*.

The debate on the validity of the hysteresis hypothesis in unemployment rate time series is still ongoing, many recent studies have been presenting evidence for and against the notion (i.e., Omay et al., 2021, Ball et al., 2021 etc.).

3.4. Estimation Methods of the Natural Rate of Unemployment

3.4.1. Econometric Models

Adams and Coe (1990) allocate the methods of estimating the natural rate of unemployment into two categories: econometric- and smoothing methods (Claar, 2000).

The first approach includes the four basic single-equation econometric methods where the most popular technique is to induce the natural rate from the parameters of an estimated Philips curve (Gordon, 1997; Staiger et al., 1997; Tootell, 1998) or the estimation is based upon an unemployment rate equation using the output gap, a set of structural variables and a set of other relevant variables (i.e., dummies for certain periods) (Barro, 1977; Adams et al., 1987; Coe, 1990) focusing explicitly upon the natural rates' structural determinants (Claar, 2000). The two other single-equation economic methods make use of Okun's law explicitly and implicitly and estimate the level of potential output as a proxy for the natural rate (1998; Masson et al., 1988; Torres et al., 1990). Adam and Coe argue that the best approach is to estimate the natural rate in a system of four equations, similar to those mentioned above.

Since the 1990s substantial empirical econometric models have been born in order to specify the determinants of the natural rate of unemployment. These econometric models either explained the fluctuation in the natural rate of unemployment with the change in (labor market) institutions (Belot and Van Ours, 2000 & 2001; Nickell et al., 2002 & 2005) or with shocks and baseline components shifting over time and usually interacting with average level of institutional factors (i.e., country dummies).

Good examples for the latter models are the panel data econometric model from Layard et al. (1991) using a dynamic model with baseline variables (wage pressure, benefit replacement ratio, real import price changes, and monetary shocks); Blanchard and Wolfers (2000) applied five-year averages to evaluate the long-run interaction of baseline variables (level of TFP[4], real interest rate, change in inflation, and labor demand shift) with time-invariant institutional variables. In a later analysis Blanchard and Wolfers replaced the observed shock variables with unobserved common shocks represented by time dummies and the explanatory power of the model increased substantially. Bertola et al. (2001) extended the basic Blanchard and Wolfers model with an additional baseline variable the share of young people (age 15-24) in the population, resulting in a more successful explanation by his model. Fitoussi et al. (2000) adopted a slightly different way, first examining the interaction of baseline variables (nonwage support relative to labor productivity, real price of oil, real rate of interest, and productivity growth) with country dummies and then analyzing the cross-sectional relationship between these and labor market institutions. The result demonstrated that changes depend on long-run shifts in the set of baseline variables with the impact of these being more severe and longer-lasting in some countries than others because of stable institutional differences.

While Blanchard and Katz (1997) note that economists are still a long way from having a good quantitative interpretation of the determinants of the natural rate as empirical knowledge still lags behind, Werding (2006) concludes that variations in institutions alone may offer a reasonable explanation for the long-run fluctuations in unemployment.

Further econometric procedure for estimating the natural rate of unemployment assumes that it is constant with a few structural breaks in its level (Papell et al., 2000 etc.). Other methods include the structural vector autoregression (VAR) model of aggregate output, inflation, and the unemployment rate to estimate the natural rate as the time-varying steady state of the unemployment rate defined by Phelps (King et al., 2005).

3.4.2. Statistical Approach

Following the second category's approach by Adams and Coe (1990) the natural rate is a time-varying realization of a certain smoothed time series by applying techniques such as calibrated unobserved-components models (Gordon, 1997), deterministic polynomial trends (Staiger et al., 1997), low-pass filtering (Staiger et al., 2001) or the Hodrick-Prescott filter (Ball and Mankiw, 2002).

The Hodrick-Prescott (1997) filter and other filtering methods (i.e. Hamilton filter, Baxter-King filter, Christiano-Fitzgerald, Beveridge-Nelson decomposition) are based on a standard macroeconomic principles decomposition of the macroeconomic time series (i.e., the actual rate of unemployment) into its cyclical (noise) and noncyclical (trend) elements, where the noncyclical part is the resulting estimate of the natural rate of unemployment (Claar, 2000).

[4] Total Factor Productivity

Business cycle theory examines the identification of different shocks and fluctuations in macroeconomic time series which deviate from trend; however, there is a fundamental disputation among economists about the connection between the trend and the cycle (Canova, 1998). Some scientists assumed that short- and long run fluctuations had different influence on the economy, i.e., Blanchard & Quah (1989) describe two kinds of shocks where one has a permanent impact[5] on the current rate of unemployment and one has not[6]. Soon was this theory disputed by the findings of Nelson & Posser (1982) identifying unit roots in several historical US time series, implying that economic shock will not only influence the cyclical element but will have permanent changes on the trend component as well (Bjornland, 2000). It has become a complex task to isolate the two components which resulted in different filter methods but none of them leads to a superior approach working optimally for every macroeconomic time series (Larsson et al., 2012). Burnside (1997) explained the different outcomes with the claim that there was no standard agreement among academics about the definition of a business cycle, thus there would be always different ways to extract the cyclical elements. According to relevant literature (van Aarle et al., 2008; de Haan et al., 2008), the most widely applied filters are the Hodrick-Prescott-, Baxter-King- and the Christiano-Fitzgerald filters.

The Hodrick-Prescott (1997) filter (HP) is a popular smoothing method which can estimate the smooth natural rate time series (low-frequency trend components) within the actual unemployment rate series by minimizing the variation of the actual rate around the natural rate subject to a penalty parameter Λ (penalizing rapid changes in growth component). A high value of Λ gives a more linear (smoother) trend and allows for more variation in the cyclical elements. The convention is to set Λ equal to 100 for annual data to avoid the cycles with frequencies equal or greater than eight years (Burke et al., 1998); however, Hassler et al. (1994), Baxter and King (1999) suggest Λ equal to 10, whereas Ravn & Uhlig (2002) propose 6.25 for annual data.

The views related to robustness of results across diverse filtering methods are mixed: Jaeger (1994), Cogley et al. (1995), Park (1996) demonstrate that through the HP filter completely artificial cyclical movements may be abstracted when the underlying time series is a random walk or the order of integration increases. Spurious indications of cyclical behavior may result from the HP filter near the ends of the sample confirmed by empirical examples from King et al. and Harvey et al. (1993). However, analysis from Blackburn et al. (1991), Claar (2000) show contrary conclusion.

While the Hodrick-Prescott filter focuses on the identification of slowly moving trend components of macroeconomic time series, the Baxter-King- (1999) and the Christiano-Fitzgerald (2003) filters concentrate on the different frequency specifications of the cycle. The latter two methods use the theory of spectral analysis of time series for the decomposition relying on the Spectral Representation Theorem stating that any time series within a broad class cyclical elements can be decomposed into different frequency components. As a tool the band-pass filter is applied, and an ideal filter requires that the sample is infinite and some sort of approximation.

The Baxter-King (BK) linear filter assumes a symmetric finite moving average series (resulting in stationarity data if the original time series is integrated of order one or two), where the cycles are run through predefined lower and upper bound (i.e., through the period of 6- and 32 quarters or of

[5] Supply shocks i.e., changes in technology, price changes in raw materials, mid- and long-term labour migration, natural catastrophes etc.
[6] Demand shocks i.e., changes in investment- and consumption level, changes in foreign income, in monetary- and fiscal policies which do not change the long-run aggregate supply curve and the full employment level of output.

2- and 8 years). Their business cycle components do not only exclude the very slow-moving trend parts (like the HP filter) but additionally the very high-frequency irregular components (noise) as well, retaining the intermediate elements which are the major features of business cycles (Baxter & King, 1999).

The Christiano-Fitzgerald (CF) filter has basic similarities with the BK filter; however, it assumes that the time series follows a random walk without a drift. The cycle components are considered like in the BK filter, as outside of the range of 6- and 32 quarters for quarterly data. The CF filter is not symmetric, as it puts different weights to every observation contrary to the BK filter where the weights are set regardless of the number of observations (Christiano & Fitzgerald, 2003). The CF filter is consistent compared to the BK, as it converges to an ideal band pass filter (the approximation error of the weights decreases) as the sample size grows (Haug & Dewald, 2004).

The Beveridge – Nelson (BN) decomposition (1981) developed from the observation that economic growth is not predictable and has an intrinsic stochastic component, thus an integrated economic time series can be broken down into the sum of unobservable trend (permanent) and cyclical (transitory) components (Morley, 2001). The estimation of the BN trend is derived by forecasting the integrated economic time series with the application of a univariate AR(1) model for its first differences and considering the implicated Wold form from the AR(1) model with the assumption of normality, the minimum squared error (MSE) of j-period ahead first difference forecasts can be determined. Therefore, the BN trend is defined as the minimum MSE forecast of the long-run level of the economic time series (minus the deterministic drift), while the cycle is described in the traditional way as the deviation from the trend (Morley, 2001).

Further methods include moving average estimates to model the natural rate of unemployment: centered, equally weighted moving averages (long used by business cycle researchers) (Baxter & King 1999): i.e., Mankiw (2000) applies the 21-year-centered moving average, while Claar (2000) considers the 11-year-centered moving average of the actual unemployment rate.

4. Methodology

Firstly, a descriptive statistical analysis will be given about the development of the unemployment time series in the 21 OECD countries for the specified period of 1969 – 2022. Besides the depicted graphs, a linear and nonlinear trend line will be indicated for each graph to give a hint about the existence of any trend behavior in the examined time series (with R-squared values based on F-total test performed by MS Excel).

Secondly, the standard Dickey-Fuller test incorporating a constant and a linear trend[7] and the augmented Dickey Fuller tests with three specifications (Enders, 2014) will be performed for benchmarking. Then the nonlinear KPSS univariate unit root test (Kwiatkowski et al., 1992) and the nonlinear Kruse unit root test (Kruse, 2011) will be completed. The Kruse test is performed in two steps: at first the data set is examined whether it depicts a (i) mean-reverting, (ii) a trend or (iii) none of these processes following with demeaning, detrending or neither of them (taking the raw data). In

[7] The time series are examined in the form of $\Delta y_t = \alpha + \beta_t + \theta(y_{t-1}) + e_t$ where α is a constant; β is a linear trend coefficient; $\theta = \rho-1$; $\rho[0, 1]$; $\{e_t\}$ is a martingale difference sequence w.r.t. $[y_{t-1}, y_{1-2}, \ldots.]$, assumed to be i.i.d. with $\mu = 0$, and is independent from y_0. The H_0 hypothesis is $H_0 : \theta = 0$ (or $\rho = 1$) existence of unit root, and $H_1 : \theta < 0$ (or $\rho < 1$) stationarity (Woodridge, 2013).

the second step the test regression will be estimated with a lag length chosen according to the information criterion AIC (all calculation carried out in RStudio).

Finally, following the suggestion of Rogerson (1997) that the natural rate of unemployment can be interpreted as an empirical concept equal to the trend component of the observed or actual time series for measured unemployment, that is the rate of unemployment that would exist after seasonal and cyclical components are taken out, the Hodrick-Prescott- (ʎ equal to 100 and 10 and additional calculation with drift as well), the Baxter-King-, the Christiano-Fitzgerald filters and the Beveridge-Nelson decomposition statistical method will be applied to extract the trend components from the unemployment time series of France and the US in the period of 1970-2022. The two countries were chosen for different reasons: The unemployment rate of France shows a relatively strong trend line and non-stationarity for the examined period, documented in the following chapter. Contrary, the US unemployment time series revealed no trend line and exhibited a more stationary behavior through most of the performed tests detailed later in this document.

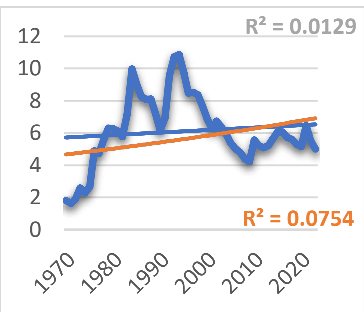
The extracted seven trend curves of the unemployment rates for each country will be examined for their statistical properties and for stationarity. Statistical comparison will be undertaken with the NAIRU estimates calculated by the OECD which are based on the Kalman filter approach. For those trend curves with the closest values to the NAIRU estimates a regression analysis will be performed in order to compare the growth rate (β slopes) of the fitted lines to that of the NAIRU estimates and to those values provided by the regression on the NAIRU estimates. Conclusion will be provided how close the examined filtering-, the decomposition methods and the regressions perform to the NAIRU estimates and to their regression curves in case of the two countries in the analyzed period. All calculations are carried out in RStudio, data are provided by OECD statistics.

5. Analysis

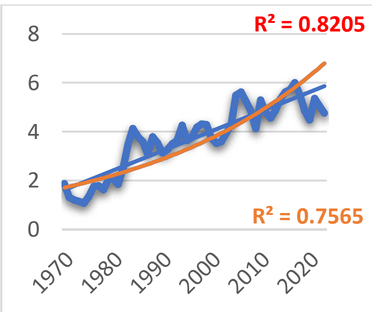
5.1. Linear or Nonlinear Trend or any Trends at all?

Examining the potential trends in the development of the unemployment time series in the period of 1969 – 2022, the analysis delivered the results summarized in Table 2:

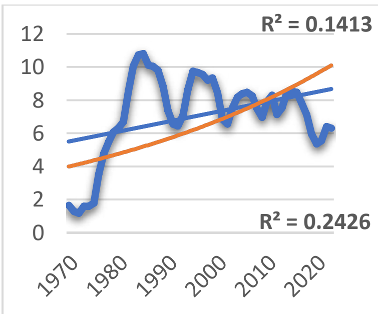
Table 2



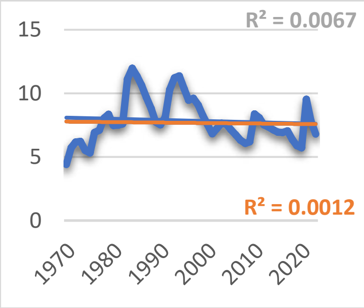
Australia



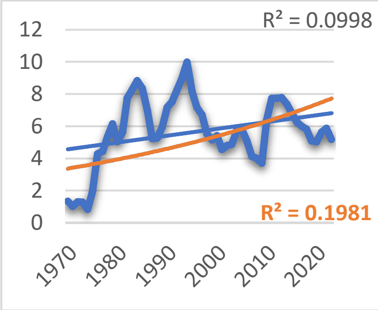
Austria



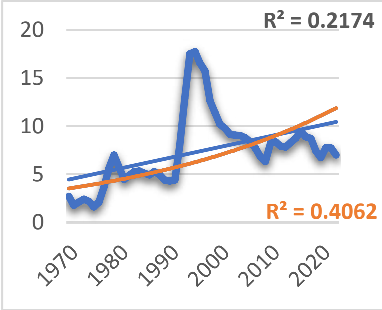
Belgium



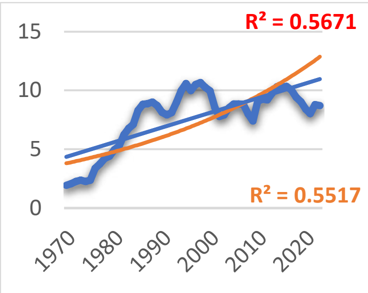
Canada



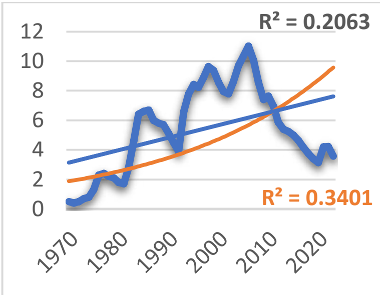
Denmark



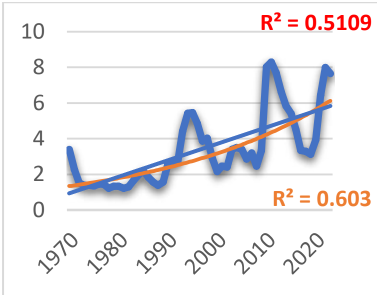
Finland



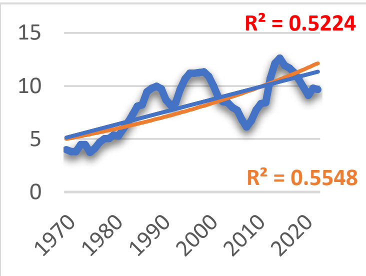
France



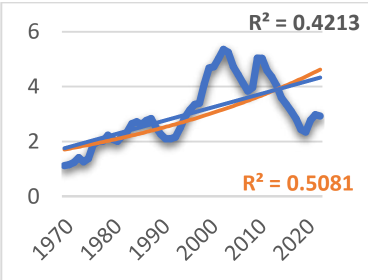
Germany



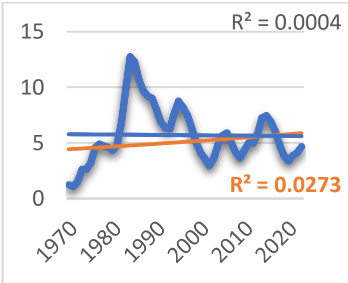
Iceland



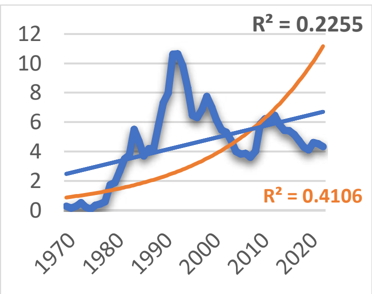
Italy



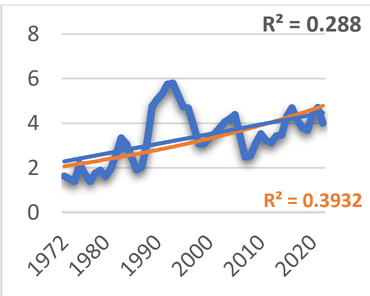
Japan



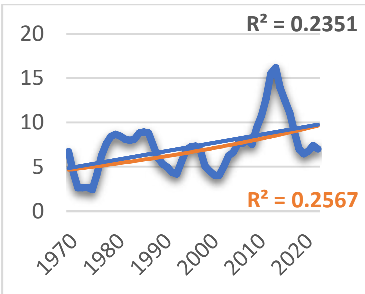
The Netherlands



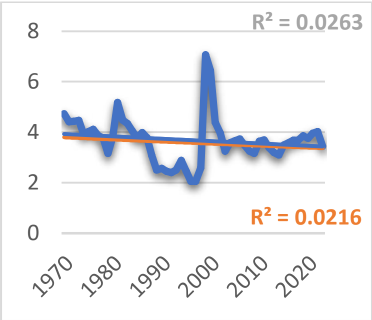
New Zealand



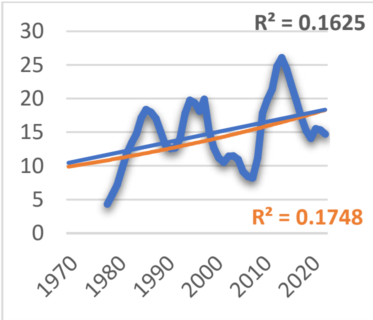
Norway



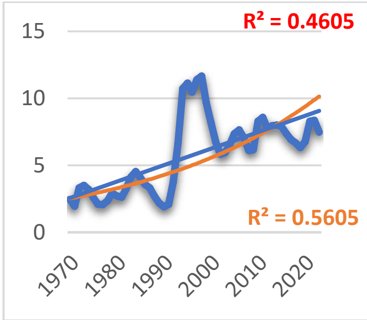
Portugal



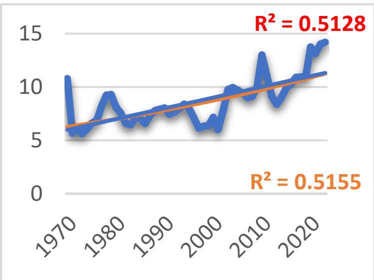
South-Korea



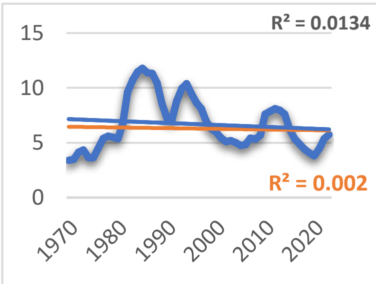
Spain



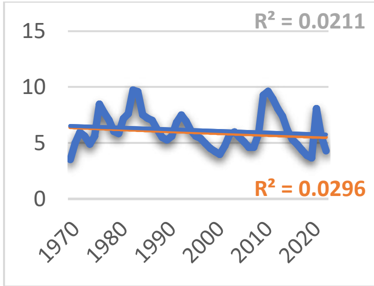
Sweden



Turkey



UK



USA

In case of six countries (Austria, France, Iceland, Italy, Sweden, and Turkey) the fitted linear trends showed significant R-squared values (in the range of 0.46 – 0.83) (exponential trend was a slightly better fit only in two cases).

In case of eight countries (Belgium, Spain, Germany, Finland, New Zealand, Portugal, Norway, and Japan) the R-squared values were at a lower level in the rage of 0.14 – 0.42 for the fitted linear trend (exponential trend was a similar fit for Portugal but inefficient for the rest of the countries).

For 7 countries (Australia, Canada, Denmark, Netherlands, South-Korea, UK, and US) neither a linear- nor an exponential trend could be fitted, thus R-squared values remained negligible (0.0004 – 0.0998) and F-total tests indicated insignificant regression models.

In summary, the author was able to fit a linear trend only in case of six countries out of the 21 with good F-total tests and large R-squared values (> ~ 0.50) (the exponential trend revealed a slightly better fit only in case of two countries). Therefore, it cannot be stated that the development of the unemployment rates suggests a certain trend in general in the examined time series.

5.2. Behavior of the examined Time Series

Examining the behavior of the unemployment rate time series in the period of 1969 – 2022, we can allocate the countries to four different groups (Féve et al., 2003):

- The first group is built by Belgium, Denmark, Germany, Japan, New Zealand, Portugal, and Spain, where approximately an upward trend can be recognized. Each trend is stronger after the 1st and 2nd oil shocks in the 1980s and 1990s and after the financial crisis in 2007 – 2008; however, it shows significant decreases by 2020 (Germany`s longer lasting increasing trend in the 90s was due to the German unification in 1990).
- The second group comprises the Scandinavian countries Finland, Norway, and Sweden where fairly an increasing trend can be detected (especially for Sweden), but the strong upward trend happened after the 90s followed by consolidation in 2010 and onwards.
- The third group consists of the countries Austria, France, Italy, Iceland, and Turkey where a strong, continuously increasing trend of unemployment rate can be located in the whole examined period.
- The countries in group four such as Australia, Canada, the Netherlands, South-Korea, UK, and US do not display any trend.

Comparing the result with Féve et al. (2003), who examined the unemployment trend for a shorter period (1966 – 1999), most of the countries remained in the same categorization except Austria (deterioration, moved to the third group), Canada, the Netherland, UK (improvement, moved to the fourth group), Spain and Portugal (both moved to the first group; however, improvement for Spain, deterioration for Portugal).

5.3. Descriptive Statistics

The descriptive statistics show that only the time series of Australia and New Zealand depict a normal distribution for the examined period, and the data from Spain is very close to it. The null hypothesis of the Jarque-Bera test (H_0 = normal distribution) is rejected in the rest of the time series. There are three countries - Denmark, Japan, and Norway - where the values of skewness and kurtosis are very close to the normal distribution, but still not close enough to change the rejection of the Jarque-Bera test.

The colors in Table 3 indicate the lowest and the highest values of descriptive statistic figures of the countries, where the lowest rates of unemployment are rewarded (see in green) and the highest rates are penalized (red color). The colors demonstrate as well which countries are the closest to (green) or the furthest (red) from the normal distribution. The most numbers in red are indicated in the time series of Spain, Portugal and South-Korea revealing high unemployment rates and in case of the two latter countries significantly deviating distribution from the normal distribution which also applies for the time series of France. The most favorable figures are shown for the time series of Japan and Norway with one of the lowest unemployment rate and standard deviation with a distribution close to the normal distribution.

Table 3

Country	Mean	Median	Min.	Max.	Range	Standard dev.	Skewness	Kurtosis	Jarque-Bera test statistic	p_value	Jarque-Bera test result
Australia	6.13	5.99	1.64	10.87	9.23	2.144	0.044	3.031	0.020	0.990	✓
Austria	3.74	3.79	1.08	6.01	4.93	1.388	-0.423	2.160	3.199	0.202	×
Belgium	7.09	7.51	1.18	10.82	9.64	2.508	-0.977	3.371	8.902	0.012	×
Canada	7.86	7.54	4.42	11.99	7.57	1.741	0.631	2.768	3.700	0.157	×
Denmark	5.69	5.70	0.85	9.99	9.14	2.101	-0.548	3.160	2.758	0.252	×
Finland	7.44	7.57	1.62	17.76	16.14	3.799	0.896	3.866	8.907	0.012	×
France	7.65	8.63	1.92	10.65	8.73	2.604	-1.052	2.854	10.006	0.007	×
Germany	5.39	5.31	0.40	11.02	10.62	2.932	-0.011	1.978	2.353	0.308	×
Iceland	3.39	2.88	1.22	8.29	7.07	2.034	1.007	3.028	9.133	0.010	×
Italy	8.24	8.42	3.74	12.62	8.88	2.546	-0.299	1.979	3.151	0.207	×
Japan	3.04	2.80	1.12	5.36	4.24	1.179	0.351	2.119	2.854	0.240	×
Netherland	5.69	4.99	1.08	12.74	11.66	2.568	0.705	3.337	4.734	0.094	×
New Zealand	4.58	4.65	0.08	10.65	10.57	2.640	0.039	2.838	0.073	0.964	✓
Norway	3.38	3.40	1.39	5.79	4.40	1.202	0.053	2.131	1.630	0.443	×
Portugal	7.28	7.12	2.42	16.18	13.76	3.000	0.919	4.136	10.502	0.005	×
S. Korea	3.68	3.70	2.06	7.07	5.01	0.917	1.150	6.205	35.012	0.000	×
Spain	14.97	14.91	4.32	26.09	21.77	4.956	0.127	2.666	0.337	0.845	☺
Sweden	5.81	6.26	1.92	11.67	9.75	2.837	0.251	2.006	2.793	0.248	×
Turkey	8.64	8.06	5.61	14.18	8.57	2.202	0.879	3.170	7.024	0.030	×
UK	6.67	5.72	3.40	11.77	8.37	2.382	0.649	2.295	4.914	0.086	×
USA	6.14	5.79	3.51	9.72	6.21	1.601	0.590	2.607	3.477	0.176	×

Jarque-Bera test H_0: data is normally distributed
Jarque-Bera test critical values for a sample size 50 at 1%, 2.5%, 5% and 10%: 0.0015, 0.02, 0.079, 0.201 respectively (source Matlab).

✓ Data is normally distributed
× Data is not normally distributed
☺ Data is close to normal distirbution

5.4. Conventional and Non-Conventional Unit Root Tests

The standard Dickey-Fuller tests with a constant and/or a linear trend revealed non-stationarity for most of the time series, only in case of South-Korea, Turkey, and the US, the author was able to reject the H_0 hypothesis (Appendix 2).

The augmented Dickey-Fuller tests (ADF) were performed with three extra specifications (Enders, 2014). The first regression for each country included the autoregressive time series. The second regression with drift besides the autoregressive time series comprised a constant; and the third regression equation with trend contained the said autoregressive time series, a constant and the trend variable. For each regression the presence of the unit root was examined as the H_0 hypothesis (τ_1, τ_2, τ_3); for regressions with drift additionally the presence of the unit root and the absence of the intercept were jointly tested as the H_0 hypothesis (Φ_1); for regressions with both intercept and trend variables the presence of both the unit root and the trend and the absence of intercept (Φ_3) then the presence of the unit root and the absence of both the intercept and the trend variables (Φ_2) were jointly investigated as the H_0 hypothesis (Appendix 1). The optimal lag order was determined with the AIC information criterion.

The author was not able to reject the H_0 hypothesis in any of the time series for the first specification indicating non-stationarity in the autoregressive time series. In case of the second specification, the author was able to reject the unit root in case of seven countries (Australia, Belgium, Canada, Denmark, Finland, Norway, and the UK). In case of further six countries, the H_0 hypothesis was rejected not only with the second specification but with the third one as well (Netherland, Portugal, South-Korea, Spain, Sweden, and the US). Only with the third specification the author was able to reject the unit root for the time series of Iceland as well. In case of seven countries the examined time series included the unit root (Austria, France, Germany, Italy, Japan, New Zealand and Turkey); however, in case of the time series of France, Germany, Italy, Japan, and New Zealand it would be possible to reject to H_0 hypothesis a bit over the 10% significance level with the presence of a drift (second specification) and in case of the data from Austria with the presence of both a drift and a trend variables (third specification) (Appendix 2).

By performing the nonlinear KPSS (1992) univariate unit root tests most of the countries showed level- or trend stationarity or both (H_0 hypothesis), while only in case of France was the H_0 hypothesis rejected (Appendix 3).

The Kruse (2011) tests showed similar results, in the majority of the countries the H_0 hypothesis – the existence of a unit root – were rejected at the defined significance levels (1% or 5% or 10%) except for the countries Canada and Turkey (Appendix 3).

5.5. Stationary or Non-Stationary Time Series?

On the basis of the performed tests the author can conclude that only in case of six countries (Austria, France, Iceland, Italy, Sweden, and Turkey) it was possible to fit a linear/nonlinear trend with relatively high R-squared values (based on F-total test). The nonlinear unit root tests (KPSS, Kruse) showed that most of the time series are level- or trend stationary. In case of three countries (Canada, France, and Turkey) one of the two tests were not successful, thus these time series revealed non-stationarity and these countries with Germany performed poorly with the other tests as well. Three countries (Australia, South-Korea, and US) performed successfully in most of the tests confirming a strong level- or trend stationarity. In summary, the analysis provides strong evidence in favour of stationarity for 18 OECD countries' unemployment rates in the examined period.

Table 4.

	Proposed Linear or Nonlinear Trend (R^2, F-total test)	Jarque-Bera Test	DF Test	ADF Test	KPSS Test	Kruse Test	✓
Australia	✗	✓	✗	✓	✓	✓	4
Austria	✓	✗	✗	✗	✓	✓	3
Belgium	😐	✗	✗	✓	✓	✓	3.5
Canada	✗	✗	✗	✓	✓	✗	2
Denmark	✗	✗	✗	✓	✓	✓	3
Finland	😐	✗	✗	✓	✓	✓	3.5
France	✓	✗	✗	✗	✗	✓	2
Germany	😐	✗	✗	✗	✓	✓	2.5
Iceland	✓	✗	✗	✓	✓	✓	4
Italy	✓	✗	✗	✗	✓	✓	3
Japan	✓	✗	✗	✗	✓	✓	3
Netherland	✗	✗	✗	✓	✓	✓	3
New Zealand	😐	✓	✗	✗	✓	✓	3.5
Norway	😐	✗	✗	✓	✓	✓	3.5
Portugal	😐	✗	✗	✓	✓	✓	3.5
S.Korea	✗	✗	✓	✓	✓	✓	4
Spain	😐	✗	✗	✓	✓	✓	3.5
Sweden	✓	✗	✗	✓	✓	✓	4
Turkey	✓	✗	✓	✗	✓	✗	3
UK	✗	✗	✗	✓	✓	✓	3
US	✗	✗	✓	✓	✓	✓	4

✓	good test performance for standard distribution and level or trend stationarity (rated as 1)
✗	bad test performance for standard distribution and level or trend stationarity (rated as 0)
😐	some trend observed but with low R-squared (F-total test) value (rated as 0.5)

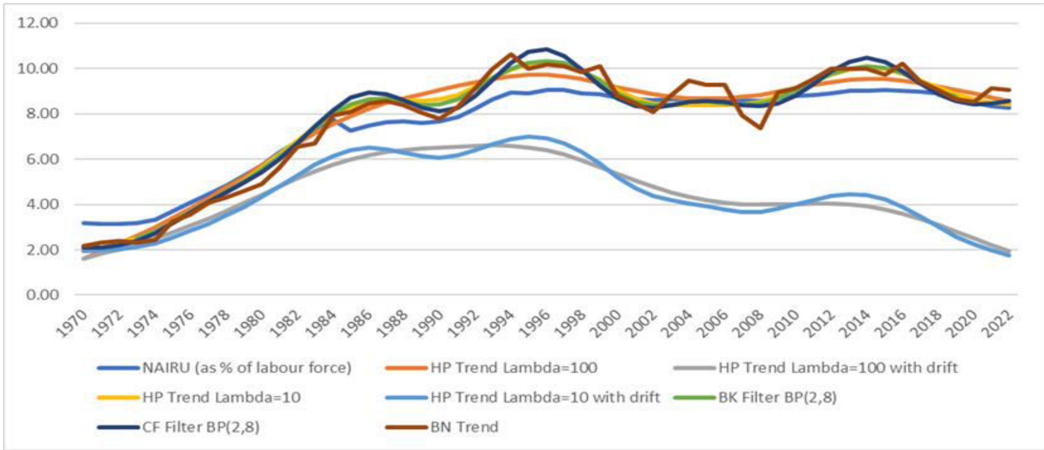
5.6. Estimation of the Natural Rate of Unemployment

5.6.1. France

Depicting the trend components from the three filtering methods and from the BN decomposition, Table 5 presents the seven calculated curves with comparison to the NAIRU estimates.

The line of the NAIRU estimates (middle blue) pictures three slight peaks of the French unemployment rate for the examined period as of 1984, 1996-1997 and 2015. All of the calculated curves follow approximately that shape with higher values around the three high spots (except the HP curves with drifts in light blue and grey); however, the BN trend curve (dark brown) exhibits an additional peak in 2004 and a trough in 2009 as well.

Table 5



It stands out that the two HP trends with a drift (ʎ equal to 100 in grey and to 10 in light blue) show the least similarities to the NAIRU estimates (middle blue) and the rest of the curves, especially after 1982 with lower values.

The HP trend with ʎ equals to 100 (orange) seems to follow the closest the NAIRU estimates; however, with a slightly higher volatility in the period of 1985 - 1999 and 2012 - 2018. Next closest trends are the HP trend with ʎ equals to 10 (yellow) and the BK filter line (green) developing almost identically and overlapping each other in most of the examined time; nevertheless, their exhibited volatilities are different than that of the HP trend with ʎ equals to 100 (orange): lower between 1989 and 1993 and larger in the period of 1993 – 1999, 2012 – 2017. The CF filtered trend (dark blue) and the BN trend curve (dark brown) progress the most distinguishably, representing more bumpiness and the highest volatility around the three peaks and an additional high (in 2004) and a low area (in 2009).

The descriptive statistics confirm the visual analysis: the HP trend with ʎ equals to 100 (orange) and the BK filtered line (green) come the closest to the NAIRU values in terms of range, standard deviation, and variance. The mean and the median are very similar to the values of the NAIRU for all of the filtered and decomposed curves. The values of all the curves and the NAIRU do not follow the standard normal distribution (Jarque-Bera test rejects in each case); although the kurtosis for the NAIRU estimates-, the HP filter ʎ equals to 10 -, and the CF curve are close to that of the normal distribution. The skewness values with around -1 for all of the curves and the NAIRU estimates show a negative skew, where the mass of the distribution is concentrated on the right of a right leaning bell curve with the mean smaller than (to the left of) the median.

Table 6.

Statistical method	Mean	Median	Min.	Max.	Range	Standard dev.	Varriance	Skewness	Kurtosis	Jarque Bera test statistics	p-values	Jarque Bera test result
HP Trend Lambda=100	*7.76*	8.84	1.61	*9.71*	*8.11*	*2.380*	*5.662*	-1.393	3.526	17.759	0.00014	✘
HP Trend Lambda=10	*7.76*	*8.56*	1.94	10.25	8.30	2.434	5.926	*-1.265*	*3.268*	14.299	0.0008	✘
BK Filter BP(2,8)	8.06	*8.56*	*2.52*	10.35	*7.83*	*2.115*	*4.474*	*-1.339*	3.717	15.048	0.0005	✘
CF Filter BP(2,8)	*7.76*	*8.52*	2.08	10.86	8.78	2.486	6.179	-1.135	*3.138*	11.413	0.0033	✘
BN Trend	*7.72*	*8.56*	*2.16*	10.61	8.45	2.520	6.351	-1.091	2.852	10.56	0.0051	✘
NAIRU (as % of labour force)	7.48	8.58	3.13	9.06	5.93	1.945	3.784	-1.278	3.157	14.477	0.000719	✘

Jarque-Bera test H_0 hypothesis = Normal distribution

Jarque-Bera test critical values for a sample size 50 at 1%, 2.5%, 5% and 10%: 0.0015, 0.02, 0.079, 0.201 respectively (source Matlab).

✘	Jarque-Bera test rejected
italic style	closest value to the NAIRU

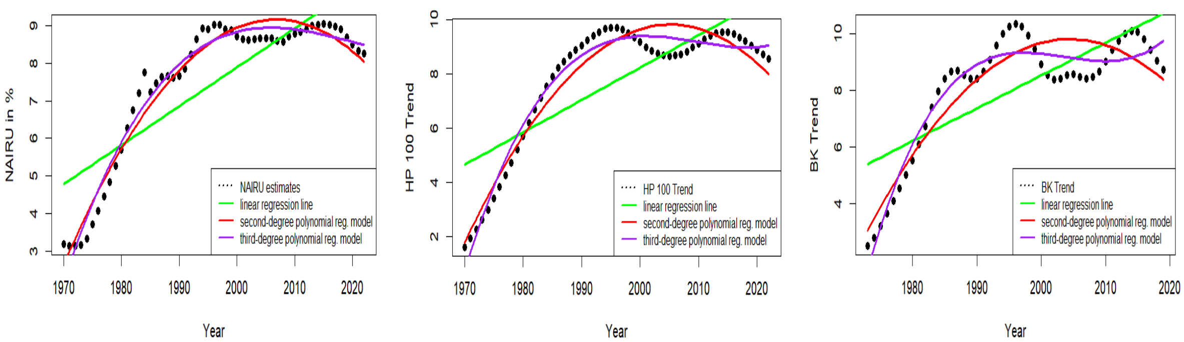
In order to fit an appropriate regression line or curve to the trends and the NAIRU estimates the stationarity and the behavior of the error terms and the residuals will be examined. Macroeconomic time series, as the unemployment rates as well, can show high autocorrelation as the observation values may depend on their past observation values (no random sampling, second assumption of Gauss-Markow theorem), especially in case of trending components. Therefore, error terms would not behave as white noise (i.i.d.) in a regression and their expected values may exhibit dependence on the explanatory variables violating the fourth assumption of the Gauss-Markow theorem (zero conditional mean). These violations can lead to biased estimators in regression analysis. Furthermore, when the variance of the error term is not constant (heteroscedasticity), certain efficiency property of the regression is damaged.

The regression analysis is carried out besides the NAIRU curve for the HP trend with ʎ equals to 100 and the BK curve, as these two curves showed the closest statistical similarities to the NAIRU estimates for the examined period.

Although the decomposed trend elements are non-stationary in the HP-, BK methods, it can happen that finite derived curves reveal stationarity for the examined period, which represents a significant information when performing econometric analysis. The augmented Dickey-Fuller test (ADF) showed stationarity for the two trend curves and the NAIRU estimates, while the standard DF test, the Kruse test (2011) and the KPSS test (1992) have not provided any stationarity evidence (Appendix 4).

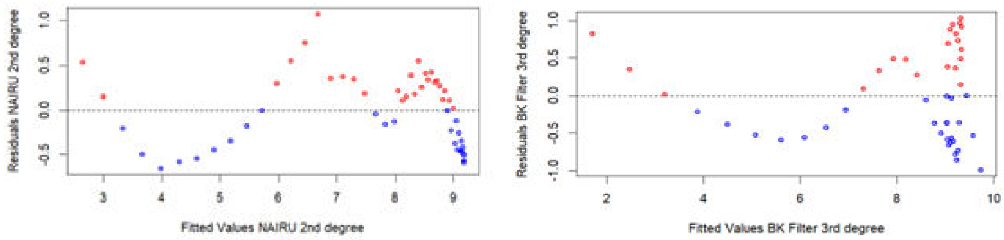
The author made several experiments with linear-, log- and polynomial regressions and concluded that the polynomial regression curves showed a good visual fit (Table 7, red- & purple curves) for the NAIRU estimates (Table 7, black dots, first graph), for the HP 100 trend curve (Table 7, black dots, second graph) and for the BK trend line (Table 7, black dots, third graph) presented in Table 7. The green line pictures a lineal regression with the best fit in each cases as a benchmark and the mostly applied regression method during the author`s scientific studies.

Table 7



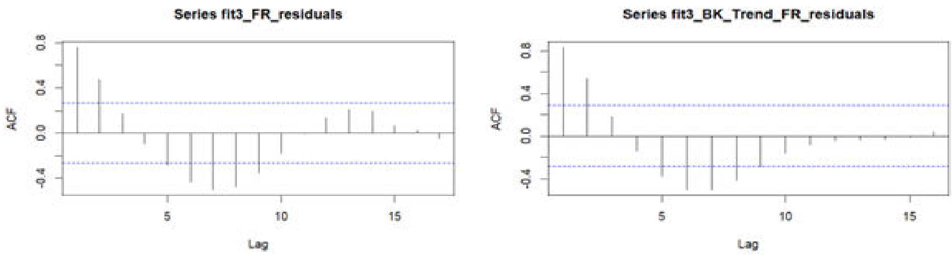
The regression analysis shows that although the values of the F-total tests indicate a good prediction model in each case (three regressions per black dotted curves), there are only two regression models revealing no evidence of heteroscedasticity in the residuals at the 10% significance level (see Appendix 5), namely the second-degree polynomial regression for the NAIRU estimates (red curve, first graph) and the third-degree polynomial regression for the BK trend curve (purple curve, third graph) on the basis of the Breusch-Pagan test. In Table 8 the graphs of the residuals from these two regression models are presented.

Table 8



Examining the autocorrelation graphs of the residuals in each regression model (Appendix 6), they exhibit the existence of serial correlation. Only two models show an autocorrelation with less and less values from the 10th lag (Table 9); however, the null hypothesis of the Breusch-Godfrey test (no serial correlation of any lags up to "p" order in the errors) can be rejected only up to the 30th lag in the residuals of the third-degree polynomial regression models of the NAIRU estimates (Table 7, purple curve, first graph) and in the same polynomial regression of the BK trend curve (Table 7, purple curve, third graph).

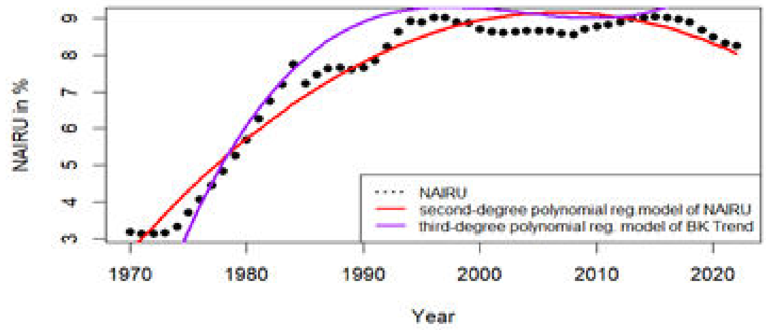
Table 9



5.6.1.1. Results

The author concludes that on the basis of the regression analysis only two models, the second-degree polynomial regression for the NAIRU estimates (red curve) and the third-degree polynomial regression for the BK trend curve (purple) created regression curves with homoscedastic error term (constant variance) but with the presence of autocorrelation in the residuals (Table 10).

Table 10



According to the descriptive statistics (Table 11), although the range of the BK filtered curve third degree polynomial regression's values (purple) became a bit greater than that of the original BK curve, the standard deviation and variance reduced slightly getting closer to the values of the original NAIRU curve (black dotts). Negative skewness decreased but kurtosis increased significantly compared to the values of the original BK curve and the NAIRU estimates, indicating more outliers (leptokurtic) in the BK filtered curve third-degree polynomial regression. The NAIRU second-degree polynomial regression's values volatility increased slightly with higher range value; however, other statistical behaviour remained very similar to the original estimates of the NAIRU curve.

Table 11

Statistical method	Mean	Median	Min.	Max.	Range	Standard dev.	Varriance	Skewness	Kurtosis	Jarque Bera test statistics	p-values	Jarque Bera test result
BK Filter BP(2,8)	8.06	8.56	2.52	10.35	7.83	2.115	4.474	-1.339	3.717	15.048	0.00054	x
BK Filter third-degree polynomial regression	8.06	9.05	*1.69*	*9.73*	*8.04*	*2.033*	*4.132*	-1.761	*5.008*	32.198	1.02E-07	x
NAIRU second-degree polynomial regression	7.48	8.32	*2.64*	9.17	*6.53*	1.903	3.623	-1.124	3.020	11.152	0.003787	x
NAIRU (as % of labour force)	7.48	8.58	3.13	9.06	5.93	1.945	3.784	-1.278	3.157	14.477	0.000719	x

Jarque-Bera test H_0 hypothesis = Normal distribution

Jarque-Bera test critical values for a sample size 50 at 1%, 2.5%, 5% and 10%: 0.0015, 0.02, 0.079, 0.201 respectively (source Matlab).

x Jarque-Bera test rejected

italic style interesting changes compared to the original curve

The additional comparison of the coefficients from the two regression equations revealed significantly different slopes for both of the x squared and the x values with neglectable intercepts and a tiny x cubed value for the third-degree polynomial regression of the BK curve as well (Table 12).

Table 12

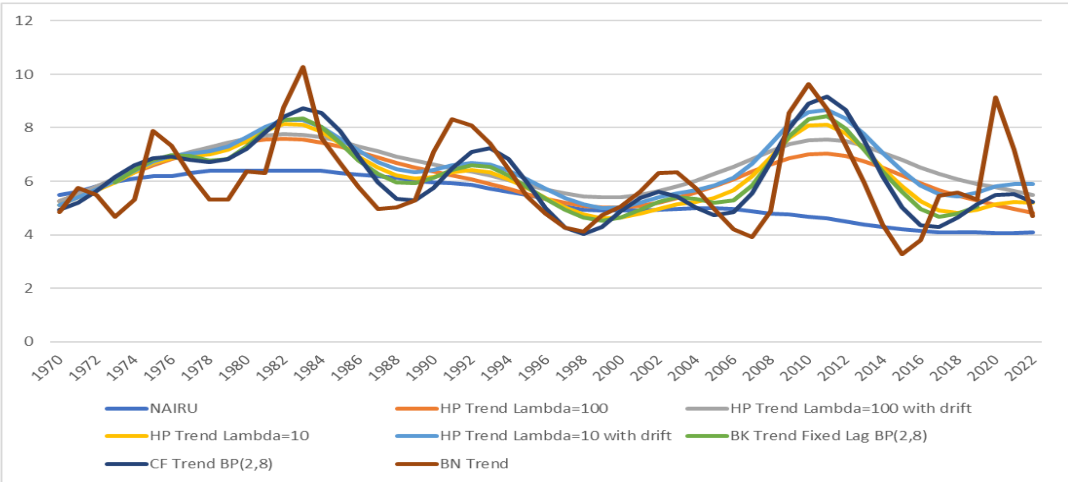
Regression	Equation
NAIRU Polyinomial regression second-degree	$y = -19490 - 0.00484x^2 + 19.44x + \varepsilon_t$
BK Filter Polynomial regression , third-degree	$y = -2415000 + 0.0003003x^3 - 1.805x^2 + 3616x + \varepsilon_t$

Therefore, together with the visual and descriptive statistical analysis, the author was not able to find any evidence that the NAIRU estimates or the NAIRU second-degree polynomial values can be reproduced by the BK filter third-degree polynomial regression.

5.6.2. The United States

Depicting the trend components from the three filtering methods and from the BN decomposition Table 13 presents the seven calculated curves with comparison to the NAIRU estimates and reveals that all the derived trend time series present a higher volatility than the NAIRU estimates.

Table 13



The NAIRU estimates (middle blue) are very close to a straight line with low volatility between the values of 4% and a bit over 6% in the examined period; only one peak can be observed in the period of 1978 – 1984 (the two oil crises).

The closest curves following the NAIRU estimates are the HP curves with ʎ equals to 100 (orange) and the same curve with a drift (light grey); however, with higher volatility in the peak period. The curves also exhibit a second high spot period between 2007 – 2015 (financial crisis).

The next curves closest to the NAIRU estimates are the HP curve with ʎ equals to 10 (yellow) and the BK trend line (green) with high overlapping of each other in most of the examined period, showing similarities in behavior to the previously examined French unemployment rates. These two curves picture higher volatility in the two peak periods of 1981 – 1985 and 2008 – 2013. The HP curve with ʎ equals to 10 with a drift (light blue) has the exact same shape as that of the previous two curves except with a slightly higher volatility in peak periods.

The CF - and the BN trend curve are the most distinguishable among the derived curves and the most opposite to the NAIRU estimates line, presenting more bumpiness with relative high volatility with more peak periods, respectively some lower values to the NAIRU estimates as well . The CF trend curve (dark blue) provides a similar curvature to the BK trend line; however, with higher values in peak areas and with lower ones in troughs: besides the two mentioned peak periods extra high spots appear between 1975 – 77 and 1992 – 94 and lower values in the period of 1987 – 90, 1997 – 99. The BN trend line (dark brown) pictures the highest volatility and a different shape line compared to the other filtered trend lines presenting further two extra peaks as of 2001 – 04 and 2020 – 21 and several more trough times (1973-74, 1978-79, 2006-07 and 2014-2015).

The descriptive statistical analysis reveals that the mean is similar in all of the derived curves with around 6% while that of the NAIRU estimates is slightly smaller (5.3%). The median and the minimum values of the CF and BN curves are the closest to those of the NAIRU; however, all the rest of the statistical descriptive indicators such as maximum values, range, standard deviation, variance, skewness, and kurtosis of the HP curves with ʎ equals to 100 and of the same curve with a drift provide the closest similarity to the respective figures of the NAIRU estimates. The skewness values are close to zero like for the standard normal distribution - except for the BN trend curve - with positive skewness; however, the values of the kurtosis are well below the required value for the normal distribution – except for the BN trend curve -. Therefore, the Jarque-Bera test null hypothesis for the existence of standard normal distribution of the data is rejected for each of the derived curves and for the NAIRU estimates (Table 14).

Table 14

Statistical method	Mean	Median	Min.	Max.	Range	Standard dev.	Varriance	Skewness	Kurtois	Jarque Bera test statistics	p-values	Jarque Bera test result
HP Trend Lambda=100	*6.19*	6.21	4.81	*7.60*	*2.79*	*0.834*	*0.695*	*0.074*	*1.748*	3.5071	0.1732	✘
HP Trend Lambda=100 with a drift	6.53	6.53	5.24	*7.76*	*2.52*	*0.776*	*0.602*	*0.027*	*1.660*	3.9746	0.1371	✘
HP Trend Lambda=10	*6.19*	6.17	*4.63*	8.14	3.52	1.079	1.164	0.319	1.911	3.5166	0.1723	✘
HP Trend Lambda=10 with a drift	6.53	6.40	5.00	8.66	3.66	1.035	1.072	0.445	2.147	3.3584	0.1865	✘
BK Filter BP(2,8)	6.29	6.26	*4.54*	8.43	3.89	1.145	1.310	0.271	2.042	2.3693	0.3058	✘
CF Filter BP(2,8)	*6.19*	*5.96*	*4.03*	9.18	5.15	1.366	1.865	0.478	2.273	3.1842	0.2035	✘
BN Trend	*6.08*	*5.60*	*3.27*	10.27	7.00	1.604	2.574	0.700	2.809	4.4075	0.1104	✘
NAIRU (as % of labour force)	5.33	5.34	4.07	6.40	2.33	0.816	0.666	-0.130	1.618	4.3671	0.1126	✘

Jarque-Bera test H_0 hypothesis = Normal distribution

Jarque-Bera test critical values for a sample size 50 at 0.01, 0.025, 0.05 and 0.10: 0.0015****, 0.02***, 0.079**, 0.201* respectively (source Matlab).

✘ Jarque-Bera test rejected

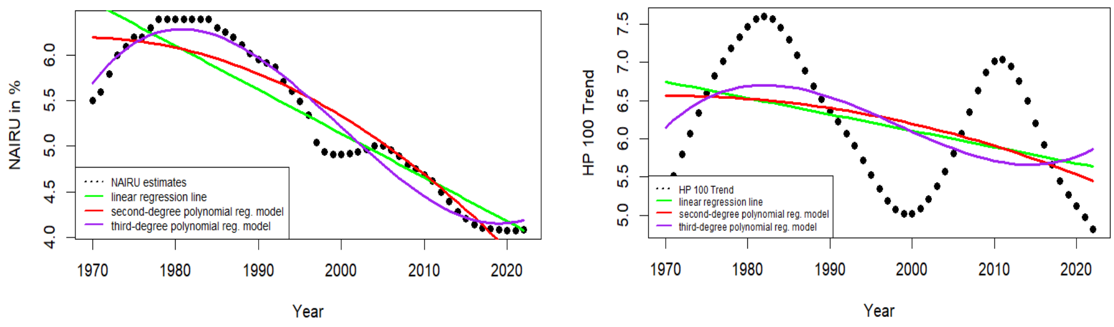
italic style closest value to the NAIRU

Examining the stationarity for the derived time series and the NAIRU estimates showed that the Dickey-Fuller test (DF) and the nonlinear Kruse test (2011) have not provided any evidence for stationarity except for the BN trend curve (DF, Kruse test) and the NAIRU estimates (DF). Contrary, the augmented Dickey-Fuller test (ADF) and the nonlinear KPSS test (1992) showed trend- or level (or both) stationarity for all of the time series and the NAIRU estimates (Appendix 7).

The HP curve with ʎ equals to 100 and of the same curve with a drift showed the closest values to the NAIRU estimates. As these HP curves are very similar to each other statistically, the regression analysis involves only the HP curve with ʎ equals to 100 besides the NAIRU estimates.

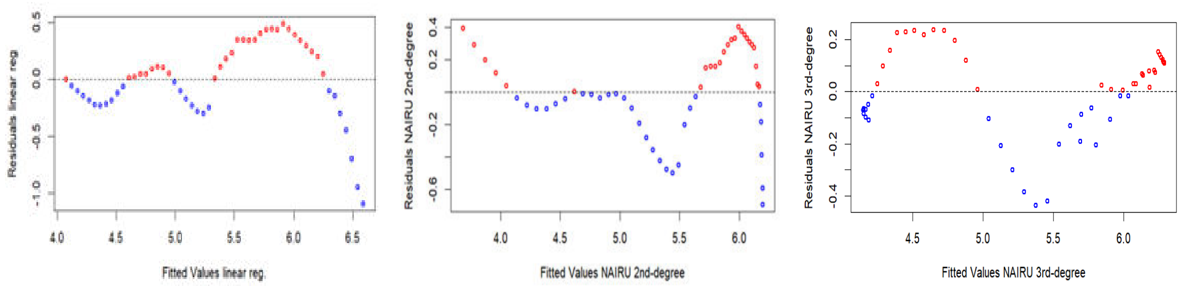
The previously applied linear and polynomial regressions exhibited in these cases a good visual fit for the NAIRU estimates; however, they were not successful for the HP trend curve with ʎ equals to 100 (HP 100) as shown in Table 15:

Table 15



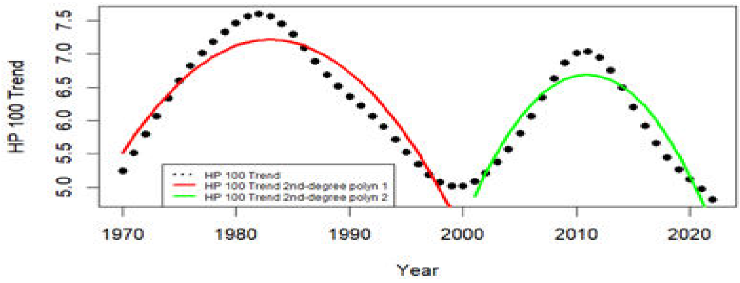
The regression models of the NAIRU estimates provided good values for the F-total test with low p-values; the residual analysis showed no evidence of heteroscedasticity only for the third-degree polynomial regression (purple line, first graph in Table 15) at 1% significance level on the basis of the Breusch-Pagan test (see Table 16 and Appendix 8). High autocorrelation was revealed in the residuals as well in all of the three NAIRU regressions by the performed Breusch-Godfrey tests (Appendix 8) and by the graphical presentation (Appendix 9).

Table 16



The regression analysis in case of the HP 100 trend confirmed the poor visual results with low F-total test values, autocorrelation in the residuals of the regression models with the presence of heteroskedasticity (Appendix 8 and 10). However, the author was able to fit two polynomial regressions to the HP 100 trend curves by dividing the examined period into two parts and apply a second-degree polynomial regression for each curve per period (Table 17):

Table 17



The regression analysis of the two second-degree polynomial curves showed that the models are a good fit for the underlying and divided HP 100 trend curve with high F-statistic- and low p-values and with homoscedastic but autocorrelated residuals (Appendix 11-12).

5.6.2.1. Results

In summary, the NAIRU estimates curve (Table 18, blue line) was predicted with constant variance of the error term (homoscedasticity) by a third-degree polynomial regression (Table 18, purple curve); however, with the presence of autocorrelation in the regression residuals. Among the derived curves the HP 100 trend line (Table 18, black dotts) came statistically the closest to the NAIRU estimates and two second-degree polynomial regression models (Table 18, red and green curves) were able to predict with homoscedastic error term each half of the HP 100 trend line but with serial correlation in the models` residuals.

Table 18

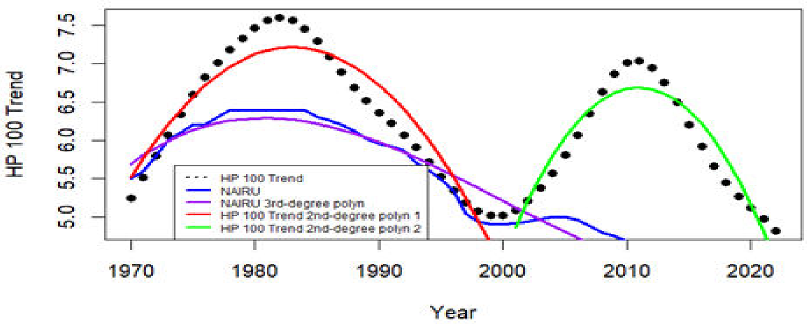


Table 18 and the descriptive statistical analysis (Table 19) reveal that the estimated values of the two second-degree regression models of the HP 100 curve have slightly smaller deviation and variance than the original HP 100 curve; however, the skewness turned into negative around -1 and the kurtosis of the two single second-degree regression curves became closer to the value of the standard normal distribution but still not close enough to provide evidence for the null hypothesis of the Jarque-Bera test. The NAIRU third-degree polynomial regression's values volatility decreased slightly with lower range values; however, other statistical behaviour remained very similar to the original estimates of the NAIRU curve.

Table 19

Statistical method	Mean	Median	Min.	Max.	Range	Standard dev.	Varriance	Skewness	Kurtosis	Jarque Bera test statistics	p-values	Jarque Bera test result
HP Trend 100	6.19	6.21	4.81	7.60	2.79	0.834	0.695	0.074	1.748	3.5071	0.1732	✘
HP Trend 100_1 second-degree polynomial regression	6.37	6.57	4.31	7.22	2.90	0.815	0.665	*-0.928*	*2.921*	4.4569	0.1077	✘
HP Trend 100_2 second-degree polynomial regression	5.94	6.13	4.42	6.69	2.27	0.695	0.483	*-0.689*	*2.287*	2.206	0.3319	✘
Estimated values of HP Trend_1 and HP Trend_2 second-degree poly. reg. as one time series (1970-2022)	6.19	6.40	*4.31*	*7.22*	2.90	*0.790*	0.625	n/a	n/a	n/a	n/a	n/a
NAIRU third-degree polynomial regression	5.33	5.54	4.15	6.29	*2.14*	*0.799*	*0.639*	-0.276	1.487	5.7274	0.0571	✘
NAIRU (as % of labour force)	5.33	5.34	4.07	6.40	2.33	0.816	0.666	-0.130	1.618	4.3671	0.1126	✘

Jarque-Bera test H_0 hypothesis = Normal distribution

Jarque-Bera test critical values for a sample size 50 at 1%, 2.5%, 5% and 10%: 0.0015, 0.02, 0.079, 0.201 respectively (source Matlab).

✘ Jarque-Bera test rejected

italic style interesting changes compared to the original curve

The additional comparison of the coefficients from the three polynomial regressions shows significantly different slopes with negligible and negative intercepts especially in case of the NAIRU third-degree polynomial (Table 20). Therefore, together with the visual and descriptive statistical analysis the author was not able to find any evidence that the NAIRU estimates or the NAIRU third-degree polynomial regression values can be reproduced by the two HP 100 trend second-degree polynomial regressions.

Table 20.

Regression	Equation
NAIRU Polyinomial regression , third-degree	$y = -498600 + 0.00006250x^3 - 0.3747x^2 + 748.6x + \varepsilon_t$
HP Trend 100_2 second-degree polynomial regression 2001-2022)	$y = -39520 - 0.01005x^2 + 39.87x + \varepsilon_t$
HP Trend 100_2 second-degree polynomial regression 2001-2022)	$y = -1608 - 0.0004160x^2 + 1.639x + \varepsilon_t$

6. Conclusion

The natural rate of unemployment hypothesis was a groundbreaking concept from Friedman (1968) and Phelps (1967) 55 years ago, claiming that there exists a macroeconomic equilibrium with a level of unemployment where inflation is stable. The notion was as important following the Keynes-era as it is nowadays in inflationary times.

High and persistent unemployment rates – especially in Europe after the 1980s– supported the hysteresis hypothesis (Blanchard & Summers, 1986), finding that the dynamics of the unemployment time series followed a non-stationary process. However, with the development of the unit root testing and econometric modelling the theory was often questioned, and significant evidence was presented for the level- or trend stationarity in unemployment time series, strengthening the natural rate hypothesis.

The author was looking for further evidence whether stationarity was present in the chosen 21 OECD countries unemployment rates in the period of 1969-2022 with the help of conventional- and nonlinear unit root tests. The performed analysis showed that the conventional DF and ADF tests revealed mixed results; however, most of the time series were level- or trend stationary according to the nonlinear KPSS (1992) and the Kruse (2011) test.

Following the proposal of Rogerson (1997) to establish the natural rate of unemployment as the trend component of the measured unemployment rate, the author applied the Hodrick-Prescott-, the Baxter-King-, the Christiano-Fitzgerald filters, and the Beveridge-Nelson decomposition on the unemployment time series of France and the United States in the period of 1970-2022 and compared the results with the NAIRU estimates as benchmark calculated by the OECD for the identical period. The analysis showed that in both cases the HP 100 curves and the BK filtered lines came closest statistically to the NAIRU estimates and the CF and BN trends revealed the least similar behavior to the OECD benchmark, with more bumpiness and the highest volatility.

The regression analysis for the closest trend lines in case of France showed a regression model with autocorrelation in the residuals but constant variance of the error term for the second-degree polynomial NAIRU estimates and the third-degree polynomial of the BK curve; however, neither the regression coefficients nor the statistical and visual analysis confirmed that the BK regression curve can reproduce any of the NAIRU curves. The author received a similar result in case of the US time series: the regression analysis showed a regression model with autocorrelation in the residuals but homoscedastic error term for the third-degree polynomial NAIRU estimates and the second degree polynomials of the HP 100 curve (the HP 100 curve's regression provided two polynomial regressions for each half of the original HP 100 curve); however, neither the regression coefficients nor the statistical and visual analysis confirmed that the two new HP 100 regression curves can reproduce any of the NAIRU curves.

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8. APPENDIX

1. The ADF tests with the three specifications defined by Enders (2014):

$$\Delta y_t = \gamma y_{t-1} + \underbrace{\sum_{i=2}^{p} \beta_i \Delta y_{t-i+1}}_{\text{control of serial correlation}} + \epsilon_t$$

$$\rightarrow (\tau 1) \quad H_0 : \gamma = 0$$

$$\Delta y_t = \gamma y_{t-1} + \underbrace{a_0}_{\text{constant}} + \sum_{i=2}^{p} \beta_i \Delta y_{t-i+1} + \epsilon_t$$

$$\rightarrow (\phi 1) \quad H_0 : \gamma = 0 \quad \& \quad a_0 = 0$$

$$\rightarrow (\tau 2) \quad H_0 : \gamma = 0$$

$$\Delta y_t = \gamma y_{t-1} + \underbrace{a_2 t}_{trend} + a_0 + \sum_{i=2}^{p} \beta_i \Delta y_{t-i+1} + \epsilon_t$$

$$\rightarrow (\phi 2) \quad H_0 : \gamma = 0 \quad \& \quad a_2 = 0 \quad \& \quad a_0 = 0$$

$$\rightarrow (\phi 3) \quad H_0 : \gamma = 0 \quad \& \quad a_2 = 0$$

$$\rightarrow (\tau 3) \quad H_0 : \gamma = 0$$

2. Results of DF- and ADF tests:

	Dickey-Fuller Test (DF)		Augmented Dickey-Fuller Test (ADF)									
	DF with drift & a linear trend		ADF τ1			ADF τ2, φ1			ADF τ3, φ2, φ3			
Country	DF	p value	Test statistic	Critical value at 0.1 sign. level	p-value	Test statistic	critical value	p-value	Test statistic	critical value	p-value	**ADF Result**
Australia	-2.013	0.569	-0.408	-1.61	0.231	**-2.639*/ 3.572**	**-2.58 / 3.86**	**0.0105**	-2.524 / 2.850 / 4.186	-3.15 / 4.16 / 5.47	0.0157	**reject the unit root with the precence of a constant at 10% sign.lev.**
Austria	-2.774	0.263	0.297	-1.61	0.899	-1.838 / 2.190	-2.58 / 3.86	0.1904	*-3.118/ 3.836 / 5.195*	-3.15 / 4.16 / 5.47	0.0228	**non-stationary**
Belgium	-1.585	0.742	-0.467	-1.61	0.000	**-3.196**/ 5.254***	**-2.89 / 4.71**	**0.0000**	-2.854 / 3.454 / 5.037	-3.15 / 4.16 / 5.47	0.0000	**reject the unit root with the precence of a constant at 5% sign.lev.**
Canada	-2.845	0.234	-0.456	-1.61	0.587	**-2.853*/ 4.073 ***	**-2.58 / 3.86**	**0.0153**	-2.96 / 3.089 / 4.63	-3.15 / 4.16 / 5.47	0.0247	**reject the unit root with the precence of a constant at 10% sign.lev.**
Denmark	-2.003	0.573	-0.59	-1.61	0.037	**-3.192**/ 5.194***	**-2.89 / 4.71**	**0.0005**	-2.942 / 3.408 / 5.015	-3.15 / 4.16 / 5.47	0.0018	**reject the unit root with the precence of a constant at 5% sign.lev.**
Finland	-1.318	0.849	-1.063	-1.61	0.000	**-3.158**/ 5.033***	**-2.89 / 4.71**	**0.0000**	*-3.117 / 3.52 / 5.230*	-3.15 / 4.16 / 5.47	0.0000	**reject the unit root with the precence of a constant at 5% sign.lev.**
France	-1.229	0.885	0.31	-1.61	0.023	*-2.334 / 3.337*	*-2.58 / 3.86*	*0.0037*	-1.762 / 2.216 / 2.722	-3.15 / 4.16 / 5.47	0.0109	**non-stationary**
Germany	-0.709	0.964	-0.714	-1.61	0.001	*-2.112 / 2.268*	*-2.58 / 3.86*	*0.0002*	-1.581 / 1.576 / 2.327	-3.15 / 4.16 / 5.47	0.0007	**non-stationary**
Iceland	-2.74	0.277	-0.599	-1.61	0.034	*-2.330 / 2.898*	*-2.58 / 3.86*	*0.0038*	**-4.127*** / 5.897** / 8.626****	**-4.04 / 4.88 / 6.49**	**0.0001**	**reject H_0 with unit root but with drift & trend at 1%; only with trend at 5%**
Italy	-1.303	0.855	-0.025	-1.61	0.000	*-2.328 / 2.942*	*-2.58 / 3.87*	*0.0000*	-2.580 / 2.582 / 3.637	-3.15 / 4.16 / 5.47	0.0001	non-stationary
Japan	-0.788	0.957	-0.272	-1.61	0.003	*-2.027 / 2.166*	*-2.58 / 3.86*	*0.0005*	-1.763 / 1.48 / 2.11	-3.15 / 4.16 / 5.47	0.0016	non-stationary
Netherland	-1.997	0.576	-1.07	-1.61	0.000	**-3.802*** / 7.285*****	**-3.51 / 6.70**	**0.0000**	**-3.825** / 5.076** / 7.55****	**-3.45 / 4.88 / 6.49**	**0.0000**	**reject H_0 unit root with presence of a drift at 1%. Reject H_0 with unit root but with drift & trend at 5%; only with trend at 5%**
New Zealand	-1.224	0.887	-0.636	-1.61	0.020	*-2.206 / 2.548*	*-2.58 / 3.86*	*0.0029*	-1.91 / 1.664 / 2.384	-3.15 / 4.16 / 5.47	0.0090	non-stationary
Norway	-1.887	0.62	-0.536	-1.61	0.018	**-2.907** / 4.304***	**-2.89 / 3.86**	**0.0005**	*-3.035 / 3.257 / 4.805*	-3.15 / 4.16 / 5.47	0.0011	**reject the unit root with the precence of a constant at 5% sign.lev.**
Portugal	-1.524	0.766	-0.881	-1.61	0.000	**-3.540***/ 6.345****	**-3.51 / 4.71**	**0.0000**	**-3.595** / 4.643* / 6.887****	**-3.45 / 4.16 / 6.49**	**0.0000**	**reject H_0 unit root with presence of a drift at 1%. Reject H_0 with unit root but with drift & trend at 5%; only with trend at 5%**
S. Korea	**-3.452**	**0.057****	-0.964	-1.61	0.563	**-4.161*** / 8.675*****	**-3.51 / 6.70**	**0.0005**	**-4.162*** / 5.793** / 8.673****	**-4.04 / 4.88 / 6.49**	**0.0017**	**reject H_0 unit root with presence of a drift with 1%. Reject H_0 with unit root but with drift & trend at 1%; only with trend at 5%**
Spain	-1.886	0.619	-0.685	-1.61	0.000	**-3.323**/ 5.550***	**-2.93 / 4.86**	**0.0000**	**-3.297* / 3.7999 / 5.67 ***	-3.18 / 4.31 / 5.61	0.0000	**reject H_0 with presence of a drift at 5%. Reject H_0 with unit root but with drift & trend at 10%; only with trend at 10.%**
Sweden	-1.711	0.691	-0.762	-1.61	0.001	**-2.712* / 3.764**	**-2.58 / 3.86**	**0.0000**	**-3.277* / 3.745 / 5.528***	-3.15 / 4.16 / 5.47	0.0000	**reject H_0 unit root with presence of a drift at 10%. Reject H_0 with unit root but with drift &trend at 10%; only with trend at 10.%**
Turkey	**-3.836**	**0.023*****	0.923	-1.61	0.584	-1.08 / 1.298	-2.58 / 3.86	0.5148	-2.418 / 2.593 / 3.116	-3.15 / 4.16 / 5.47	0.1071	non-stationary
UK	-1.809	0.651	-0.699	-1.61	0.000	**-2.936**/ 4.335***	**-2.89 / 3.86**	**0.0000**	*-3.041 / 3.208 / 4.786*	-3.15 / 4.16 / 5.47	0.0000	**reject the unit root with the precence of a constant at 5% sign.lev.**
USA	**-3.281**	**0.084***	-0.836	-1.61	0.500	**-3.546***/ 6.293****	**-3.51 / 4.71**	**0.0026**	**-3.701** / 4.695* / 7.036****	**-3.45 / 4.16 / 6.49**	**0.0043**	**reject H_0 unit root with presence of a drift at 1%. Reject H_0 with unit root but with drift & trend at 5%; only with trend at 5%**

DF test incorporating a constant and/or a linear trend H_0 hypothesis = non-stationarity (presence of unit root)
crtical values (for sample size 50) 0.01, 0.025, 0.05, 0.1: - 3.58****, -3.22***, -2.93**, -2.60* respectively (without trend); Source Brooks et al., Introductory econometrics for finance (2019) pp.641
Stationarity for South-Korea, Turkey, and US.

AFD test H_0 hypothesis = non-stationarity (presence of unit root)
critical values for τ1 (presence of unit root) 0.01, 0.05, 0.1: -2.6***, -1.95**, -1.61*
critical values for τ2 (presence of unit root but drift) 0.01, 0.05, 0.1: -3.51*, -2.89**, -2.58***
critical values for τ2 (presence of unit root but drift) for SPAIN 0.01, 0.05, 0.1: -3.58*, -2.93**, -2.60***
critical values for φ1 (presence of unit root but absence of drift) 0.01, 0.05, 0.1: 6.70***, 4.71**, 3.86*
critical values for φ1 (presence of unit root but absence of drift) for SPAIN 0.01, 0.05, 0.1: 7.06***, 4.86**, 3.94*

critical values for τ3 (presence of unit root but drift & trend) 0.01, 0.05, 0.1: -4.04***, -3.45**, -3.15*
critical values for τ3 (presence of unit root but drift & trend) for SPAIN 0.01, 0.05, 0.1: -4.15***, -3.50**, -3.18*
critical values for φ2 (presence of unit root and absence of drift & trend) 0.01, 0.05, 0.1: 6.50***, 4.88**, 4.16*
critical values for φ2 (presence of unit root and absence of drift & trend) for SPAIN 0.01, 0.05, 0.1: 7.02***, 5.13**, 4.31*
critical values for φ3 (presence of unit root but only trend) 0.01, 0.05, 0.1: 8.73*, 6.49**, 5.47***
critical values for φ3 (presence of unit root but only trend) for SPAIN 0.01, 0.05, 0.1: 9.31*, 6.73**, 5.61***

(yellow cell)	reject the H_0 hypothesis
italic numbers	reject the H_0 hypothesis a bit over 10% sign. level

3. Results of KPSS (1992)- and Kruse (2011) test:

	KPSS Test				KRUSE Test						
Country	KPSS Test Statistic	Truncation lag parameter	p-value	**Result**	raw/dem/detr.	selected lag (AIC)	Kruse Test Statistic	F Statistic	p-value	R^2	**Result**
Australia	**0.2695*** / 0.271	3	0.1 / 0.01	level stationarity	detrended	10	**2.868**	2.472	**0.021**	**0.291**	stationary (reject the H_0 at 5%)
Austria	1.264 / **0.154***	3	0.043	trend stationary	raw	0	**5.762**	2.881	**0.065**	**0.066**	stationary (reject the H_0 at 10%)
Belgium	**0.394*** / 0.245	3	0.08	level stationary	raw	10	**8.653**	3.726	**0.002**	**0.432**	stationary (reject the H_0 at 1%)
Canada	**0.244*** / 0.218	3	0.1	level stationarity (nearly trend stationary)	demeaned	5	5.876	1.594	0.165	0.080	non-stationary (high p-value and low F-total test)
Denmark	**0.288* / 0.190***	3	0.1 / 0.0199	level- and ternd stationary	demeaned	10	**16.509**	3.645	**0.002**	**0.425**	stationary (reject the H_0 at 1%)
Finland	**0.501* / 0.205***	3	0.042 / 0.014	level- and ternd stationary	detrended	10	**9.684**	3.818	**0.001**	**0.440**	stationary (reject the H_0 at 10%)
France	0.936 / 0.294	3	0.01 / 0.01	non-stationary	raw	1	**13.538**	7.871	**0.000**	**0.284**	stationary (reject the H_0 at 1%
Germany	**0.582*** / 0.315	3	0.024	level stationary	demeaned	10	**8.703**	3.145	**0.005**	**0.375**	stationary (reject the H_0 at 5%)
Iceland	0.936 / **0.045***	3	0.1	trend stationarity	detrended	4	**9.456**	3.731	**0.005**	**0.251**	stationary (reject the H_0 at 10%)
Italy	0.828 / **0.178***	3	0.024	trend stationary	raw	6	**7.691**	4.969	**0.000**	**0.403**	stationary (reject the H_0 at 10%
Japan	0.817 / **0.216***	3	0.01	trend stationary	detrended	3	**5.597**	5.290	**0.001**	**0.300**	stationary (reject the H_0 at 1%)
Netherland	**0.189* / 0.184***	3	0.1 / 0.022	level- and trend stationary	detrended	10	**12.755**	9.106	**0.000**	**0.693**	stationary (reject the H_0 at 1%
New Zealand	**0.526*** / 0.271	3	0.0359	level stationary	detrended	5	**5.348**	3.532	**0.005**	**0.270**	stationary (reject the H_0 at 1%)
Norway	**0.526* / 0.181***	3	0.036 / 0.023	level- and trend stationary	demeaned	10	**4.289**	2.795	**0.012**	**0.350**	stationary (reject the H_0 at 5%)
Portugal	**0.434* / 0.081***	3	0.0627	level- and trend stationarity	detrended	8	**17.576**	7.918	**0.000**	**0.606**	stationary (reject the H_0 at 1%)
S. Korea	**0.146* / 0.096***	3	0.1	level- and rend stationarity at sign.level 10%	raw	5	**7.341**	2.876	**0.015**	**0.215**	stationary (reject the H_0 at 5%)
Spain	**0.272* / 0.070***	3	0.1	level- and trend stationarity	detrended	2	**10.104**	8.677	**0.000**	**0.417**	stationary (reject the H_0 at 10%)
Sweden	0.826 / **0.132***	3	0.0762	trend stationarity	detrended	10	**4.084**	3.507	**0.002**	**0.412**	stationary (reject the H_0 at 10%)
Turkey	0.964 / **0.209***	3	0.0126	trend stationarity	detrended	10	3.160	1.350	0.242	0.089	non-stationary (high p-value and low F-total test)
UK	**0.230* / 0.195***	3	0.1 / 0.018	level- and trend stationary	raw	10	**5.052**	3.354	**0.003**	**0.396**	stationary (reject the H_0 at 10%)
USA	**0.130* / 0.072***	3	0.1	level- and trend stationarity	detrended	1	**9.698**	3.508	**0.022**	**0.126**	stationary (reject the H_0 at 5%)

KPSS test H_0 hypothesis = level or trend stationarity
critical values of level stationarity: 0.01***, 0.05**, 0.1*: 0.739; 0.463; 0.347.
critical values of trend stationarity: 0.01***, 0.05**, 0.1*: 0.119; 0.146; 0.216.

(yellow) not enough evidence to reject the H_0 hypothesis for KPSS Test
H_0 hypothesis (unit root) rejected for Kruse Test

4. Stationarity tests for the NAIRU estimates and the derived curves of the French unemployment rate in the period of 1970 – 2022:

	Dickey-Fuller Test (DF)		Augmented Dickey-Fuller Test (ADF)									
	DF with a drift & lin. Trend		ADF τ1			ADF τ2, φ1			ADF τ3, φ2, φ3			
Statistical method	DF	p value	Test statistic	Critical value at 0.1 sign. level	p-value	Test statistic	critical value	p-value	Test statistic	critical value	p-value	**ADF Result**
HP Trend Lambda=100	-1.969	0.587	-2.56**	-1.95	2.20E-16	-4.7*** / 13.1***	-3.51 / 6.70	2.20E-16	-4.5*** / 8.96*** / 11.4***	-4.04 / 6.50 / 8.73	2.20E-16	stationary
HP Trend Lambda=10	-0.794	0.957	-0.843	-1.61	2.20E-16	-3.9*** / 7.5***	-3.51 / 6.70	2.20E-16	-3.9** / 5.9*** / 8.8***	-3.45 / 4.88 / 8.73	2.20E-16	stationary
BK Filter BP(2,8)	-1.641	0.717	-0.876	-1.61	2.20E-16	-3.6*** / 6.4**	-3.51 / 4.71	2.20E-16	-3.4* / 4.7* / 7.1**	-3.15 / 4.16 / 6.49	2.20E-16	stationary
CF Filter BP(2,8)	-0.944	0.939	-0.347	-1.61	2.20E-16	-3.6*** / 6.8***	-3.51 / 6.70	2.20E-16	-4.2*** / 6.5** / 9.4***	-4.04 / 4.88 / 8.73	2.20E-16	stationary
BN Trend	-1.329	0.845	0.564	-1.61	0.1944	-2.32 / 3.61	-2.58 / 3.86	0.02969	-1.63 / 2.38 / 2.67	-3.15 / 4.16 / 5.47	0.07157	non-stationary
NAIRU (as % of labour force)	-0.194	0.99	0.747	-1.61	1.30E-06	-3.08** / 6.002**	-2.89 / 4.71	8.89E-07	-1.12 / 4.2* / 5.1	-3.15 / 4.16 / 5.47	3.07E-06	stationary

DF test H_0 hypothesis = non-stationarity (presence of unit root)

crtical values (for sample size 50) 0.01, 0.025, 0.05, 0.1: - 3.58****, -3.22***, -2.93**, -2.60* respectively (without trend); Source: Brooks et al., Introductory econometrics for finance (2019) pp.641

AFD test H_0 hypothesis = non-stationarity

critical values for τ1 (presence of unit root) 0.01, 0.05, 0.1: -2.6*, -1.95**, -1.61***

critical values for τ2 (presence of unit root but drift) 0.01, 0.05, 0.1: -3.51*, -2.89**, -2.58***

critical values for φ1 (presence of unit root but absence of drift) 0.01, 0.05, 0.1: 6.70***, 4.71**, 3.86*

critical values for τ3 (presence of unit root but drift & trend) 0.01, 0.05, 0.1: -4.04*, -3.45**, -3.15***

critical values for φ2 (presence of unit root and absence of drift & trend) 0.01, 0.05, 0.1: 6.50***, 4.88**, 4.16*

critical values for φ3 (presence of unit root but only trend) 0.01, 0.05, 0.1: 8.73*, 6.49**, 5.47***

H_0 hypothesis rejected; stationary time series

	KPSS Test				KRUSE Test						
Statistical method	KPSS Test Statistic	Truncation lag parameter	p-value	**Result**	raw/dem/detr.	selected lag (AIC)	Kruse Test Statistic	F Statistic at 5% sign.	p-value	R^2 (adjusted)	**Result**
HP Trend Lambda=100	0.960 / 0.328	3	0.01	rejected both at 1% non-stationary	raw/dem/detr.	N/I	N/I	very high	negligable	nearly 1	non-stationary
HP Trend Lambda=10	0.917 / 0.304	3	0.01	rejected both at 1% non-stationary	raw/dem/detr.	N/I	N/I	very high	negligable	nearly 1	non-stationary
BK Filter BP(2,8)	0.799 / 0.259	3	0.01	rejected both at 1% non-stationary	raw/dem/detr.	N/I	N/I	very high	negligable	nearly 1	non-stationary
CF Filter BP(2,8)	0.888 / 0.286	3	0.01	rejected both at 1% non-stationary	raw/dem/detr.	N/I	N/I	very high	negligable	nearly 1	non-stationary
BN Trend	0.973 / 0.303	3	0.01	rejected both at 1% non-stationary	raw	10	8.150	2.726	0.013	0.33	stationary
NAIRU (as % of labour force)	1.055 / 0.328	3	0.01	rejected both at 1% non-stationary	raw	1	19.083	22.45	3.17E-09	0.56	non-stationary

KPSS test H_0 hypothesis = level or trend stationarity

critical values of level stationarity: 0.01***, 0.05**, 0.1*: 0.739; 0.463; 0.347.

critical values of trend stationarity: 0.01***, 0.05**, 0.1*: 0.216; 0.146; 0.119.

Kruse test H_0 hypothesis = existence of unit root

N/I = not interpretable

H_0 hypothesis rejected; stationary time series

5. Results of the regression analysis for the NAIRU estimates, HP 100 trend- and the BK trend curve for the unemployment time series of France in the period of 1970 – 2022:

NAIRU		Model parameters			Breusch-Pagan Test		Durbin-Watson Test		Breusch-Godfrey Test		
Regression	Equation	F-Statistic	p-values	R^2 (adjusted)	Test Statistic	p-values	Test Statistic	p-values	Lags (DF)	Test Statistic	p-values
Linear Regression	$y = -200 + 0.104x + \varepsilon_t$	108.9	2.96E-14	0.675	6.3805	0.01154	0.0407	0	10	49.57	3.20E-07
Polyinomial regression , second-degree	$y = -19490 - 0.00484x^2 + 19.44x + \varepsilon_t$	561.7	2.20E-16	0.956	4.271***	0.1182	0.2201	0	10	42.54	6.02E-06
Polynomial regression , third-degree	$y = -564900 + 0.00006859x^3 - 0.416x^2 + 839.2x + \varepsilon_t$	484.2	2.20E-16	0.965	16.537	0.00088	0.3155	0	10	35.33	0.00011

HP 100 Trend		Model parameters			Breusch-Pagan Test		Durbin-Watson Test		Breusch-Godfrey Test		
Regression	Equation	F-Statistic	p-values	R^2 (adjusted)	Test Statistic	p-values	Test Statistic	p-values	Lags (DF)	Test Statistic	p-values
Linear Regression	$y = -230.46 + 0.119x + \varepsilon_t$	76.5	1.01E-11	0.592	10.381	0.00127	0.0216	0	10	48.25	5.59E-07
Polyinomial regression , second-degree	$y = -26140 - 0.006504x^2 + 26.08x + \varepsilon_t$	349.4	2.20E-16	0.931	11.369	0.0034	0.0425	0	10	52.89	7.82E-08
Polynomial regression , third-degree	$y = -1291000 + 0.000159x^3 - 0.9588x^2 + 1927x + \varepsilon_t$	512.1	2.20E-16	0.967	9.3797	0.02465	0.1094	0	10	46.40	1.22E-06

BK Filter		Model parameters			Breusch-Pagan Test		Durbin-Watson Test		Breusch-Godfrey Test		
Regression	Equation	F-Statistic	p-values	R^2 (adjusted)	Test Statistic	p-values	Test Statistic	p-values	Lags (DF)	Test Statistic	p-values
Linear Regression	$y = -222.027 + 0.1153x + \varepsilon_t$	56.9	1.61E-09	0.549	11.027	0.0009	0.0527	0	10	41.33	9.87E-06
Polyinomial regression , second-degree	$y = -27350 - 0.006809x^2 + 27.30x + \varepsilon_t$	119.7	2.20E-16	0.838	8.1099	0.01734	0.0994	0	10	46.02	1.42E-06
Polynomial regression , third-degree	$y = -2415000 + 0.0003003x^3 - 1.805x^2 + 3616x + \varepsilon_t$	173.1	2.20E-16	0.918	6.3914**	0.09405	0.2245	0	10	42.27	6.71E-06

F-statistic H_0 = bad model fit

critical values with DF 1 and 51; DF 2 and 50, DF 3 and 49 for 0.01, 0.05, 01: 7.18, 4, 2.68; 5.32, 3.18, 2.46; 4.98, 2.82, 2.18.

good model fit

Breuch-Pagan Test H_0 = no evidence of heteroskedasticity

critical values of χ^2 distribution with DF=1 : 0.01***, 0.05**, 0.1*: 6.635, 3.841, 2.706; with DF=2: 9.21, 5.99, 4.61; with DF=3: 11.345, 7.815, 6.251.

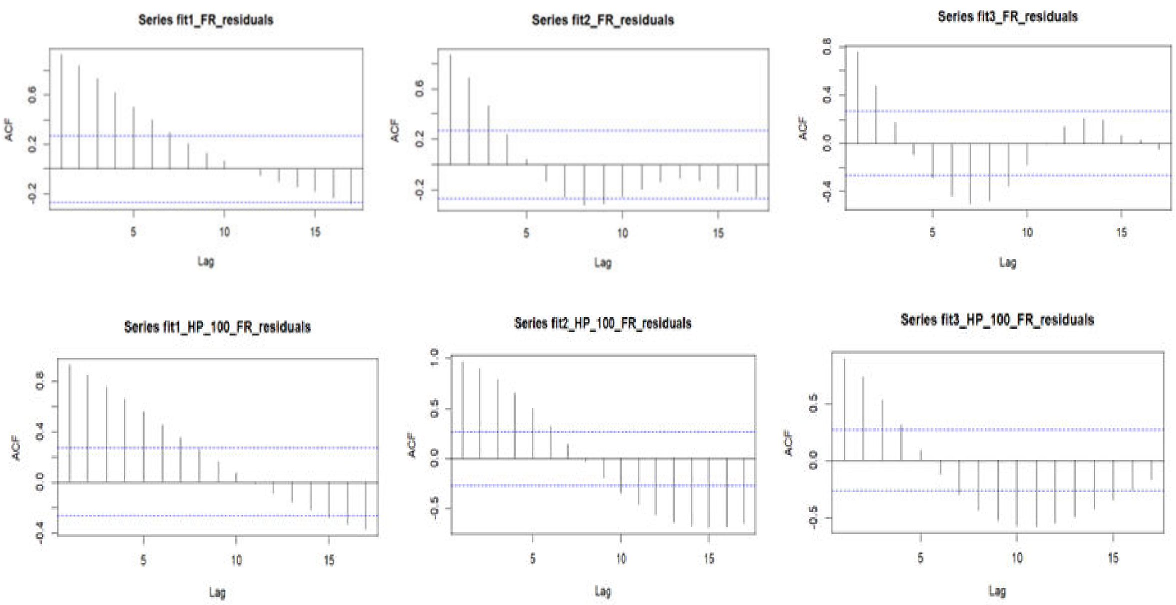
no evidence of heteroskedasticity

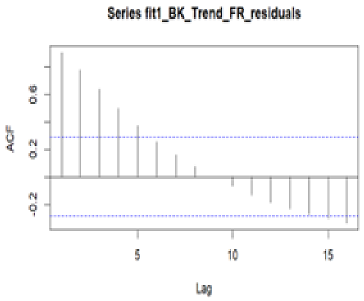
Durbin-Watson Test: Test statistic = 2 no autocorrelation; < 2 positive serial correlation; > 2 negative serial correlation

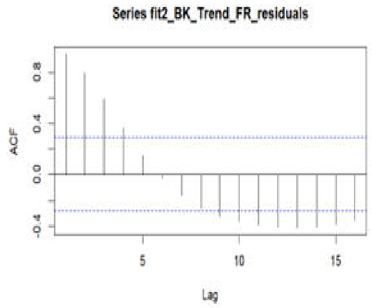
Breusch-Godfrey Test H_0 = no seriel correlation in the residuals

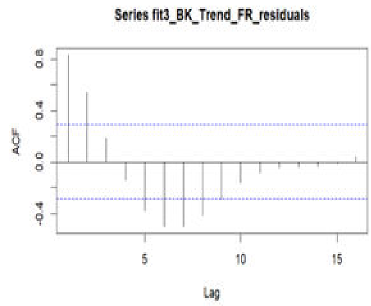
critical values at sign. level 0.01, 0.05, 0.1 with DF=5: 15.09, 11.07, 9.24; with DF=10: 23.21, 18.31, 15.99; with DF=15: 30.58, 24.99, 22.31; with DF=25: 44.31, 37.65, 34.38; DF=30: 50.89, 43.77, 40.26.

6. The autocorrelation of the residuals in the regressions of the NAIRU estimates, the HP 100 trend- and the BK trend curve for the unemployment time series of France in the period of 1970 – 2022:









7. Stationarity tests for the NAIRU estimates and the derived curves of the US unemployment rate in the period of 1970 – 2022:

	Dickey-Fuller Test (DF)		Augmented Dickey-Fuller Test (ADF)									
	DF with a drift & lin. Trend		ADF τ1			ADF τ2, φ1			ADF τ3, φ2, φ3			
Statistical method	DF	p value	Test statistic	Critical value at 0.1 sign. level	p-value	Test statistic	critical value	p-value	Test statistic	critical value	p-value	ADF Result
HP Trend Lambda=100	-1.73	0.683	-2.088**	-1.95	2.20E-16	-7.9*** / 33.5***	-3.51 / 6.70	2.20E-16	-9.3*** / 30.6*** / 43.3***	-4.04 / 6.50 / 8.73	2.20E-16	Stationary
HP Trend Lambda=100 with a drift	-1.73	0.683	-1.953**	-1.95	2.20E-16	-9.1*** / 43.98***	-3.51 / 6.70	2.20E-16	-9.3*** / 30.4*** / 43.3***	-4.04 / 6.50 / 8.73	2.20E-16	Stationary
HP Trend Lambda=10	-1.704	0.694	-0.999	-1.61	2.20E-16	-5.6*** / 15.9***	-3.51 / 6.70	2.20E-16	-6.1*** / 12.2*** / 18.3***	-4.04 / 6.50 / 8.73	2.20E-16	Stationary
HP Trend Lambda=10 with a drift	-1.704	0.694	-0.892	-1.61	2.20E-16	-6.02*** / 18.2***	-3.51 / 6.70	2.20E-16	-6.1*** / 12.2*** / 18.3***	-4.04 / 6.50 / 8.73	2.20E-16	Stationary
BK Filter BP(2,8)	-1.470	0.785	-1.006	-1.61	3.53E-12	-5.8*** / 16.6***	-3.58 / 7.06	2.20E-16	-5.9*** / 11.8*** / 17.7***	-4.15 / 7.02 / 9.31	2.85E-16	Stationary
CF Filter BP(2,8)	-1.941	0.598	-1.239	-1.61	5.20E-13	-7.1*** / 25.2***	-3.51 / 6.70	2.20E-16	-7.8*** / 20.2*** / 30.3***	-4.04 / 6.50 / 8.73	2.20E-16	Stationary
BN Trend	-3.26*	0.088	-1.126	-1.61	0.1071	-4.9*** / 12.1***	-3.51 / 6.70	1.15E-05	-4.97*** / 8.*** / 12.4***	-4.04 / 6.50 / 8.73	3.57E-05	Stationary
NAIRU (as % of labour force)	-5.22****	0.01	-1.146	-1.61	5.71E-12	-0.69 / 0.79	-2.58 / 3.86	1.39E-10	-3.95** / 5.8** / 7.99**	-3.45 / 4.88 / 6.49	1.36E-12	Stationary

DF test H_0 hypothesis = non-stationarity (presence of unit root)

crtical values (for sample size 50) 0.01, 0.025, 0.05, 0.1: - 3.58****, -3.22***, -2.93**, -2.60* respectively (without trend); Source: Brooks et al., Introductory econometrics for finance (2019) pp.641

AFD test H_0 hypothesis = non-stationarity

critical values for τ1 (presence of unit root) 0.01, 0.05, 0.1: -2.6*, -1.95**, -1.61***

critical values for τ2 (presence of unit root but drift) 0.01, 0.05, 0.1: -3.51*, -2.89**, -2.58***

critical values for φ1 (presence of unit root but absence of drift) 0.01, 0.05, 0.1: 6.70***, 4.71**, 3.86*

critical values for τ3 (presence of unit root but drift & trend) 0.01, 0.05, 0.1: -4.04*, -3.45**, -3.15***

critical values for φ2 (presence of unit root and absence of drift & trend) 0.01, 0.05, 0.1: 6.50***, 4.88**, 4.16*

critical values for φ3 (presence of unit root but only trend) 0.01, 0.05, 0.1: 8.73*, 6.49**, 5.47***

H_0 hypothesis rejected; stationary time series

	KPSS Test				KRUSE Test						
Statistical method	KPSS Test Statistic	Truncation lag parameter	p-value	Result	raw/dem/detr.	selected lag (AIC)	Kruse Test Statistic	F Statistic	p-value	R^2 (adjusted)	Result
HP Trend Lambda=100	0.344* / 0.103*	3	0.1	Level and Trend Stationary	raw/dem/detr.	N/I	N/I	very high	negligable	nearly 1	Non-Stationary
HP Trend Lambda=100 with a drift	0.153* / 0.103*	3	0.1	Level and Trend Stationary	raw/dem/detr.	N/I	N/I	very high	negligable	nearly 1	Non-Stationary
HP Trend Lambda=10	0.237* / 0.087*	3	0.1	Level and Trend Stationary	raw/dem/detr.	N/I	N/I	very high	negligable	nearly 1	Non-Stationary
HP Trend Lambda=10 with a drift	0.116* / 0.087*	3	0.1	Level and Trend Stationary	raw/dem/detr.	N/I	N/I	very high	negligable	nearly 1	Non-Stationary
BK Filter BP(2,8)	0.269* / 0.100*	3	0.1	Level and Trend Stationary	raw/dem/detr.	N/I	N/I	very high	negligable	nearly 1	Non-Stationary
CF Filter BP(2,8)	0.176* / 0.069*	3	0.1	Level and Trend Stationary	raw/dem/detr.	N/I	N/I	very high	negligable	nearly 1	Non-Stationary
BN Trend	0.078* / 0.047*	3	0.1	Level and Trend Stationary	raw	10	5.4973	4.643	0.0003	0.51	Stationary
NAIRU (as % of labour force)	1.257 / 0.191***	3	0.01	Trend Stationary	detrended	1	11.7937	37.08	1.50E-12	0.6797	Non-Stationary

KPSS test H_0 hypothesis = level or trend stationarity

critical values of level stationarity: 0.01*, 0.05**, 0.1*: 0.739; 0.463; 0.347.**

critical values of trend stationarity: 0.01*, 0.05**, 0.1*: 0.216; 0.146; 0.119.**

H_0 hypothesis not rejected; Level or Trend stationary time series

Kruse test H_0 hypothesis = existence of unit root

N/I = not interpretable

H_0 hypothesis rejected; stationary time series

8. Regression analysis of NAIRU and HP 100 curve for the US unemployment rate in the period of 1970-2022:

NAIRU		Model parameters			Breusch-Pagan Test		Durbin-Watson Test		Breusch-Godfrey Test		
Regression	Equation	F-Statistic	p-values	R^2 (adjusted)	Test Statistic	p-values	Test Statistic	p-values	Lags (DF)	Test Statistic	p-values
Linear Regression	$y = 101.77 - 0.0483x + \varepsilon_t$	259.9	2.20E-16	0.833	14.614	0.0001	0.0714	0	10	40.15	1.59E-05
Polyinomial regression , second-degree	$y = -3441 - 0.000889x^2 + 3.5x + \varepsilon_t$	200.1	2.20E-16	0.885	9.977	0.0068	0.0884	0	10	43.56	3.95E-06
Polynomial regression , third-degree	$y = -609700 + 0.00007625x^3 - 0.4575x^2 + 914.8x + \varepsilon_t$	382.2	2.20E-16	0.957	11.177***	0.0108	0.1333	0	10	48.57	4.88E-07

HP 100 Trend		Model parameters			Breusch-Pagan Test		Durbin-Watson Test		Breusch-Godfrey Test		
Regression	Equation	F-Statistic	p-values	R^2 (adjusted)	Test Statistic	p-values	Test Statistic	p-values	Lags (DF)	Test Statistic	p-values
Linear Regression	$y = 48.86 - 0.021376x + \varepsilon_t$	9.483	0.00334	0.140	1.465	0.2262	0.063	0	10	48.18	5.75E-07
Polyinomial regression , second-degree	$y = -1608 - 0.0004160x^2 + 1.639x + \varepsilon_t$	5.044	0.0101	0.135	1.574	0.4553	0.060	0	10	49.03	4.03E-07
Polynomial regression , third-degree	$y = -498600 + 0.00006250x^3 - 0.3747x^2 + 748.6x + \varepsilon_t$	4.421	0.0079	0.165	7.884	0.04846	0.062	0	10	51.10	1.68E-07

F-statistic H_0 = bad model fit

critical values with DF 1 and 51; DF 2 and 50, DF 3 and 49 for 0.01, 0.05, 01: 7.18, 4, 2.68; 5.32, 3.18, 2.46; 4.98, 2.82, 2.18.

good model fit

Breuch-Pagan Test H_0 = no evidence of heteroskedasticity

critical values of χ^2 distribution with DF=1 : 0.01***, 0.05**, 0.1*: 6.635, 3.841, 2.706; with DF=2: 9.21, 5.99, 4.61; with DF=3: 11.345, 7.815, 6.251.

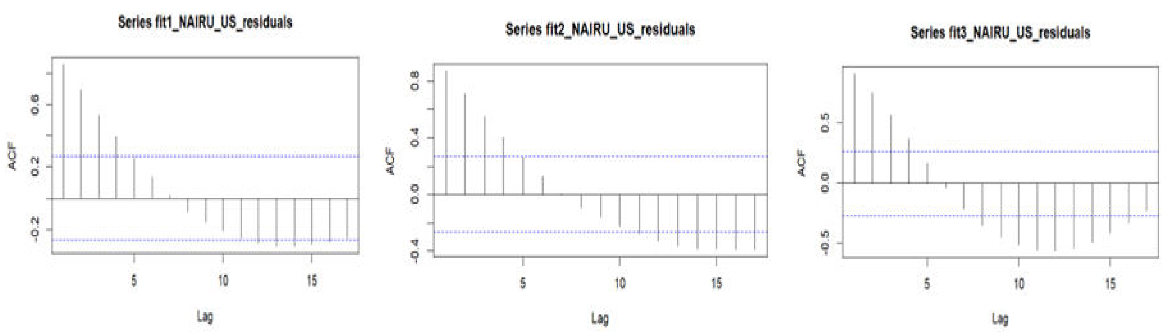
no evidence of heteroskedasticity

Durbin-Watson Test: Test statistic = 2 no autocorrelation; < 2 positive serial correlation; > 2 negative serial correlation

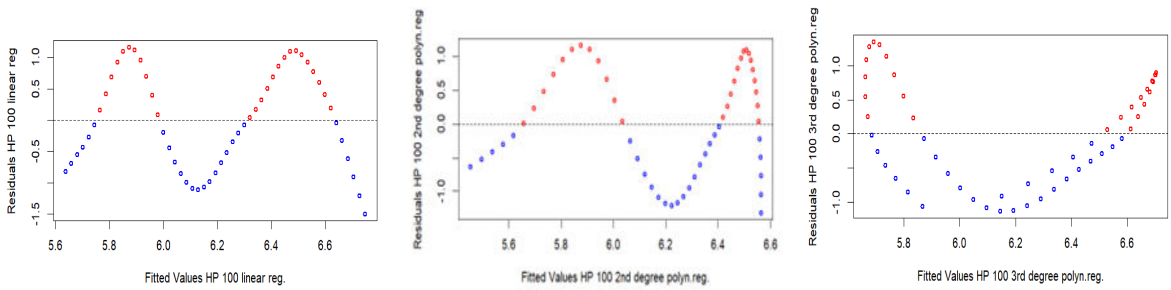
Breusch-Godfrey Test H_0 = no seriel correlation in the residuals

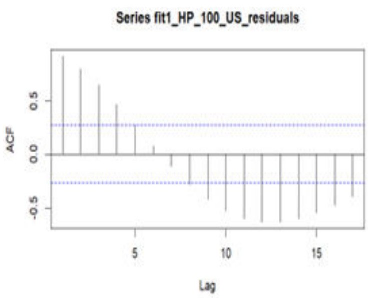
critical values at sign. level 0.01, 0.05, 0.1 with DF=5: 15.09, 11.07, 9.24; with DF=10: 23.21, 18.31, 15.99; with DF=15: 30.58, 24.99, 22.31; with DF=25: 44.31, 37.65, 34.38; DF=30: 50.89, 43.77, 40.26.

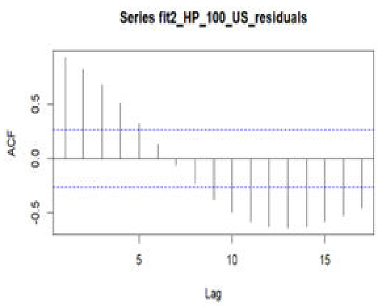
9. Graphical presentation of the autocorrelation function in the residuals of the NAIRU US regression models: linear-, second-degree-, and third-degree polynomial regression:

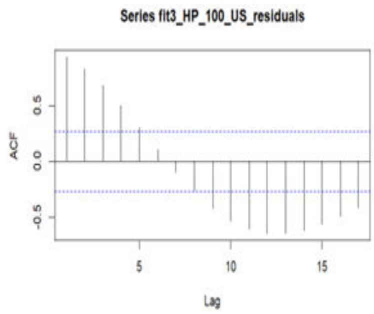


10. Graphical presentation of heteroscedasticity and the autocorrelation function in the residuals of the HP 100 regression models: linear-, second-degree-, and third-degree polynomial regression:









11. Regression analysis of the HP 100 curve for the US unemployment rate when the original curve is divided into two periods and a second-degree polynomial is fitted for each period:

HP 100 Trend		Model parameters			Breusch-Pagan Test		Durbin-Watson Test		Breusch-Godfrey Test		
Regression	Equation	F-Statistic	p-values	R^2 (adjusted)	Test Statistic	p-values	Test Statistic	p-values	Lags (DF)	Test Statistic	p-values
Polyinomial regression_1, 1970-2000 , second-degree	$y = -39520 - 0.01005x^2 + 39.87x + \varepsilon_t$	112.5	4.13E-14	0.881	4.916**	0.086	0.144	0	10	29.66	0.001
Polyinomial regression_2 , 2001-2022, second-degree	$y = -1608 - 0.0004160x^2 + 1.639x + \varepsilon_t$	81.8	4.61E-10	0.885	2.414***	0.299	0.320	0	*10*	*20.12*	*0.028*

F-statistic H_0 = bad model fit

critical values with DF 1 and 51; DF 2 and 50, DF 3 and 49 for 0.01, 0.05, 01: 7.18, 4, 2.68; 5.32, 3.18, 2.46; 4.98, 2.82, 2.18.

good model fit

Breuch-Pagan Test H_0 = no evidence of heteroskedasticity

critical values of χ^2 distribution with DF=1 : 0.01***, 0.05**, 0.1*: 6.635, 3.841, 2.706; with DF=2: 9.21, 5.99, 4.61; with DF=3: 11.345, 7.815, 6.251.

no evidence of heteroskedasticity

Durbin-Watson Test: Test statistic = 2 no autocorrelation; < 2 positive serial correlation; > 2 negative serial correlation

Breusch-Godfrey Test H_0 = no seriel correlation in the residuals

critical values at sign. level 0.01, 0.05, 0.1 with DF=5: 15.09, 11.07, 9.24; with DF=10: 23.21, 18.31, 15.99; with DF=15: 30.58, 24.99, 22.31; with DF=25: 44.31, 37.65, 34.38; DF=30: 50.89, 43.77, 40.26.

12. Residual behavior (homoskedasticity and autocorrelation) of the two new second-degree polynomial regressions of the HP 100 curve for the US unemployment rate: