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## **DARK MATTER AXION SEARCH USING NOVEL RF RESONANT CAVITY GEOMETRIES IN THE CAST EXPERIMENT**

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# Abstract

The axion is a hypothetical elementary particle resulting from the Peccei-Quinn mechanism, which was introduced to solve the strong Charge-Parity problem of the Standard model. Its properties also make the axion a perfect cold dark matter candidate. A way to search for axions is by their conversion into a photon in the presence of a strong magnetic field, the power of the photon signal can be enhanced using a cavity resonating at the correct frequency.

The Relic Axions Detector Exploratory Setup (RADES) group is part of the CERN Axion Solar Telescope (CAST) collaboration and employs cavities joined together by irises to search for axions in the 8-9 GHz frequency range. The cavities are placed inside of the 8.8 T magnetic field of the CAST magnet at a temperature of 2 K. Taking advantage of a cavity geometry design and the 9 m long bore of the CAST magnet, the RADES group plans to construct a large volume cavity resonating between 8 and 9 GHz, exploiting the fact that this mass range is poorly explored and the sensitivity to the axion signal is linearly proportional to the cavity volume.

The design, commissioning and data analysis using a prototype consisting of 5 sub-cavities joined by inductive irises are presented. The results of the first data-taking run provide an exclusion limit for the axion-photon coupling constant of  $g_{a\gamma} \gtrsim 4 \times 10^{-13} \text{ GeV}^{-1}$  over an axion mass range of  $34.6738 \mu\text{eV} < m_a < 34.6771 \mu\text{eV}$ , improving the previous best limit at this mass range coming from solar axion searches and yielding one of the most sensitive limits above  $25 \mu\text{eV}$ .

A new cavity design that is suited for the construction of longer cavities and the first results of its characterization are also presented together with the prospects of the RADES experiment.





# Zusammenfassung

Das Axion ist ein hypothetisches Elementarteilchen, das aus dem Peccei-Quinn-Mechanismus entsteht und eingeführt wurde, um das starke CP-Problem des Standardmodells zu lösen. Seine Eigenschaften machen es auch zu einem perfekten Kandidaten für kalte Dunkle Materie. Eine Methode nach Axionen zu suchen, ist die Messung ihrer Umwandlung in ein Photon in Anwesenheit eines starken Magnetfeldes. Die Leistung des Photonensignals kann durch einen Hohlraumresonator, der bei der richtigen Frequenz mitschwingt, verstärkt werden.

Die RADES-Gruppe (Relic Axions Detector Exploratory Setup) ist ein Teil der CERN Axion Solar Telescope (CAST)-Kollaboration und verwendet Hohlraumresonatoren, die durch Iris miteinander verbunden sind. Damit wird es nach Axionen im Frequenzbereich von 8-9 GHz gesucht. Die Hohlraumresonatoren befinden sich innerhalb des Magnetfeldes des CAST-Magneten bei einer Temperatur von 2 K. Unter Verwendung eines neuen Designs der Geometrie des Hohlraumresonators innerhalb der 9 m langen Bohrung des CAST-Magneten plant die RADES-Gruppe einen großvolumigen Hohlraumresonator zu erstellen, der zwischen 8 und 9 GHz mitschwingt. Dabei wird die Tatsache ausgenutzt, dass dieser Massenbereich schlecht erforscht ist und die Empfindlichkeit für das Axion-Signal proportional zum Hohlraumvolumen ist.

Das Design, die Inbetriebnahme und die Datenanalyse mit dem ersten Prototyp, der aus 5 Sub-Hohlraumresonatoren besteht, die durch induktive Iris verbunden sind, wird in dieser Arbeit beschrieben. Die Ergebnisse der ersten Datenaufnahme liefern eine Ausschlussgrenze für die Axion-Photon-Kopplungskonstante von  $g_{a\gamma} \gtrsim 4 \times 10^{-13} \text{ GeV}^{-1}$  über einen Axion-Massenbereich von  $34.6738 \mu\text{eV} < m_a < 34.6771 \mu\text{eV}$ , welche eine Verbesserung der bisherigen Grenzen entsprechen. Die vorherigen Grenzen stammen aus der Suche nach dem solaren Axion und sind eine der empfindlichsten Grenzen oberhalb von  $25 \mu\text{eV}$ .

Ein neues Design, das sich für den Bau längerer Hohlraumresonatoren eignet, und die ersten Ergebnisse seiner Charakterisierung werden genauso wie die Zukunftsaussichten des RADES-Experiments vorgestellt.



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# 1 Introduction

The field of physics has always tried to answer the most fundamental questions of nature. In the last century and with the steady increase in technology, physicists have been able to study the subatomic processes in ways that were not previously possible. The experimental findings together with the theoretical formalism made it possible to construct a model that describes the different forces, matter constituents, and their interactions. This model is known as the Standard Model (SM) of particle physics. The components of the SM are the fermions (quarks and leptons) which make up the known matter and the gauge bosons which transmit the interaction between particles (electromagnetic, strong, and weak). One of the biggest recent successes of the SM was the prediction (the 1960s) and later discovery (2012 at CERN) of the Higgs boson. A boson, which is the result of the needed mechanism to introduce the mass of the particles to the theory. Figure 1 shows all the elementary particles of the SM. The SM is currently the best description of subatomic physics. Nevertheless, there are still many known phenomena that cannot be explained by the SM. For example, gravity is not included in the SM. Also, it does not answer fundamental questions as to why there are three generations of quarks and leptons, or why only matter has been found so far in the universe. Some of the model's parameters have to be "fine-tune", which raises some problems as we will discuss in section 2.2. Since the SM is not complete, extensions to this model are needed.

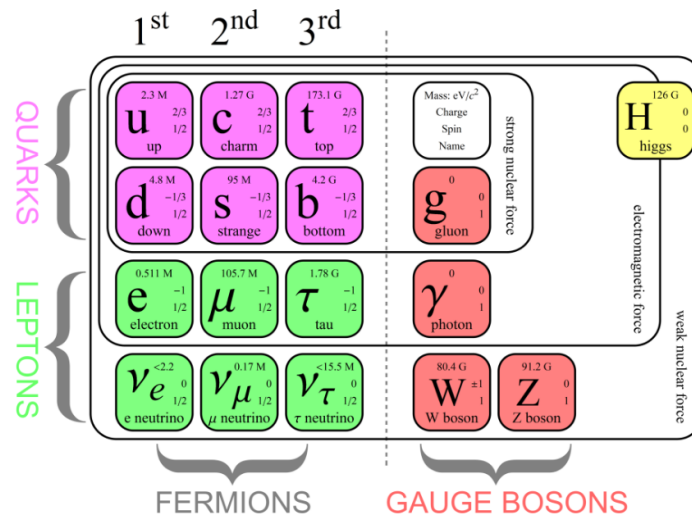


Figure 1: Standard Model of particles physics. Image by Matic Lubej [1]

A field that made significant discoveries in the past decades is cosmology, which studies the beginning, evolution, and development of the universe. Using independent experimental measurements together with the theory of general relativity a model to parameterize the Big Bang cosmological evolution has been developed. This model is known as the  $\Lambda$  Cold Dark Matter model ( $\Lambda$ CDM). The model states that the universe is made of three different components: the cosmological constant  $\Lambda$  that is associated with dark energy, cold dark matter, and ordinary matter. Figure 2 shows the percentage of the three components in the total energy composition of the universe today. Out of these three components, the only one that is currently described by the SM is the so-called ordinary matter. This means that our current physics theories cannot explain the physics behind 95% of the energy composition of today's universe. We do not know what is dark energy and what dark matter is made of. What is known, are some of its properties and where to look for them. Dark energy for example is a type of energy present everywhere in the universe and affects the universe on

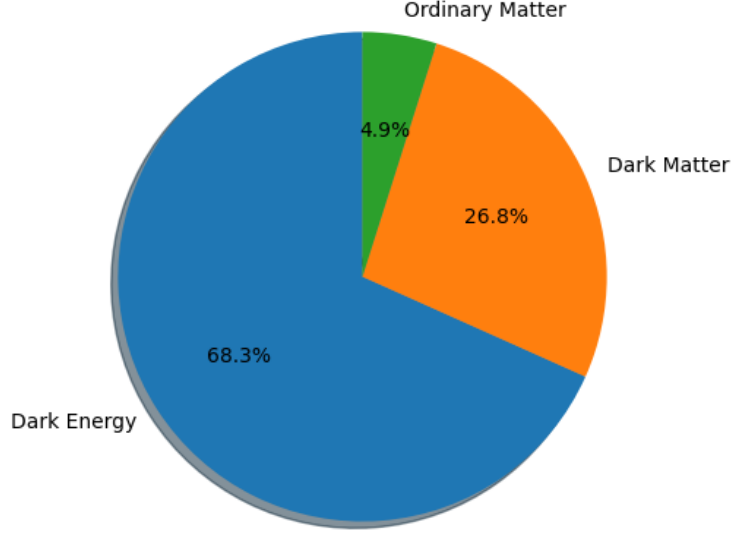


Figure 2: Energy composition of the universe today based on the  $\Lambda$ CDM model.

the largest scales.

Cosmological and astrophysical observations have pointed out the fact that dark matter is concentrated within galaxies and galaxy clusters and dark matter played an important role in the evolution of our universe. Dark matter interacts through the gravitational force with ordinary matter, thus it plays a central part in the formation of the universe structures seen today. However, there has been so far no evidence of a non-gravitational interaction between dark matter and ordinary matter. The name "dark" comes from the fact that this type of matter does not absorb or emit radiation in a significant way in any part of the electromagnetic spectrum. A lot of effort and resources have been dedicated to the study of this type of matter in the last decades. Dark matter opens a portal to new physics beyond the SM. For this reason, many theoreticians have come with different theories to understand the nature of dark matter (see section 3.2). The theories postulate several dark matter candidates. In this work, we will mainly focus on the candidate known as the axion.

The axion is a hypothetical particle first introduced to solve the strong charge conjugation-parity (CP) problem of the standard model (see section 2.2). Later on, it was found that the axion has all the right properties to be one of the most compelling dark matter candidates. In this regard, this new particle "washes out" two problems of the SM. This was the reason why the particle was named after a cleaning product.

There are different types of mechanisms in which the axion can be produced. One of them is the production of axions inside the sun (see section 4.2). This axion flux escapes the sun and travels the space, passing through the Earth. The CERN Axion Solar Telescope (CAST) is a decommissioned LHC dipole magnet with a 9 T magnetic field. The CAST experiment has been searching for solar axions for the last 20 years, yielding the best sensitivity for this type of axion search.

In recent years, CAST has also introduced to its scientific program a second way to look for axions. Axions could also be produced after the Big Bang and later on, they concentrated within the galaxies and galaxy clusters. They are known as relic axions. They are actually passing through the Earth all the time without interacting with ordinary matter. A haloscope-type dark matter experiment (see section 4.5) could potentially see a trace of



the relic axions. The Relic Axion Detector Experimental Setup (RADES) is a sub-group of the CAST experiment which is looking to detect axions in the 8 to 9 GHz range using novel radio frequency (RF) resonant cavity geometries.

## 2 Theoretical motivation for the axion particle

The transformation of an object, which leaves this object unchanged, is known as a symmetry transformation. A set of symmetry transformations is known as a symmetry group. In physics, the understanding of nature's symmetries has been a field of study for many years. Many experiments have been developed in order to measure and test these symmetries. Any conserved symmetry comes with a conserved physical quantity. For example, the momentum is conserved in all systems which are invariant under a translation transformation, the energy under time transformation, and angular momentum under rotational transformation.

In addition to the aforementioned continuous transformations, there are also discrete symmetries, for example, charge conjugation, parity, and time reversal. If for instance, someone reverses all the signs of the electrical charges, the fields and potentials will reverse their sign, but the force will stay the same. This means that classical electrodynamics is invariant under charge conjugation. The parity transformation mirrors the position of the particle. In other words, it swaps the spatial components:  $P(\vec{x}) \rightarrow (-\vec{x})$ . The charge operation transforms the particle to its antiparticle:  $C|\psi\rangle \rightarrow |\bar{\psi}\rangle$ . It reverses the lepton number, baryon number, and charge, but the energy, spin, and mass are unchanged.

It was expected that physical laws obey the combination of charge conjugation and parity symmetry (CP symmetry). In this chapter, it will be shown how the weak interaction violates the CP symmetry and why this was also predicted for the strong interaction. The experimental constraints on the CP symmetry violation in the strong interaction lead to the introduction of new mechanisms to explain the experimental results, which postulated a new particle called the axion.

### 2.1 CP violation in the weak interaction

Prior to 1956, no parity violation was seen in the experiments. However, all of the tests on the parity operator were done for the strong and electromagnetic force. Lee and Yang [2] proposed the idea of testing the parity symmetry in the weak interaction, because at that time the conservation of this symmetry in the weak interaction was an extrapolated hypothesis unsupported by experimental evidence. This test was carried out by Wu [3]. Wu recorded the direction of the emitted electrons for the Cobalt 60 beta decay ( $^{60}\text{Co} \rightarrow ^{60}\text{Ni} + e + \bar{\nu}_e$ ). The cobalt nuclei were aligned in such a way that their spins all pointed in the same direction. She observed that most of the electrons came out in the direction opposite to the nuclear spin. If one looks at the mirror image of this process in which the nuclear spin is now pointing to the opposite direction, one would see that the electrons would yet again be emitted in the same direction as before. In this case, the direction of the electron and the nuclear spin direction is the same. This was the first evidence of parity violation (see figure 3).

In particle physics, helicity is known as the projection of the spin in the direction of the momentum. If both of them are parallel it is called a right-handed particle and if they are opposite it is called a left-handed particle. However, helicity is not Lorentz invariant for massive particles. It is interesting to study the helicity of neutrinos (assuming that the neutrinos are massless). The study of the neutrino and anti-neutrino helicity can be done using the  $\pi^+$  and  $\pi^-$  decay. By measuring the helicity of the produced  $\mu$  (in the  $\pi$  rest frame), one can indirectly measure the helicity of the neutrino. Experiments showed that all neutrinos are left-handed and all anti-neutrinos are right-handed. Applying the charge conjugation operator on a neutrino will give a left-handed anti-neutrino. Once again the weak interaction violates another symmetry, in this case, the charge conjugation symmetry.

The CP symmetry, which produces a right-handed anti-neutrino from a left-handed neu-

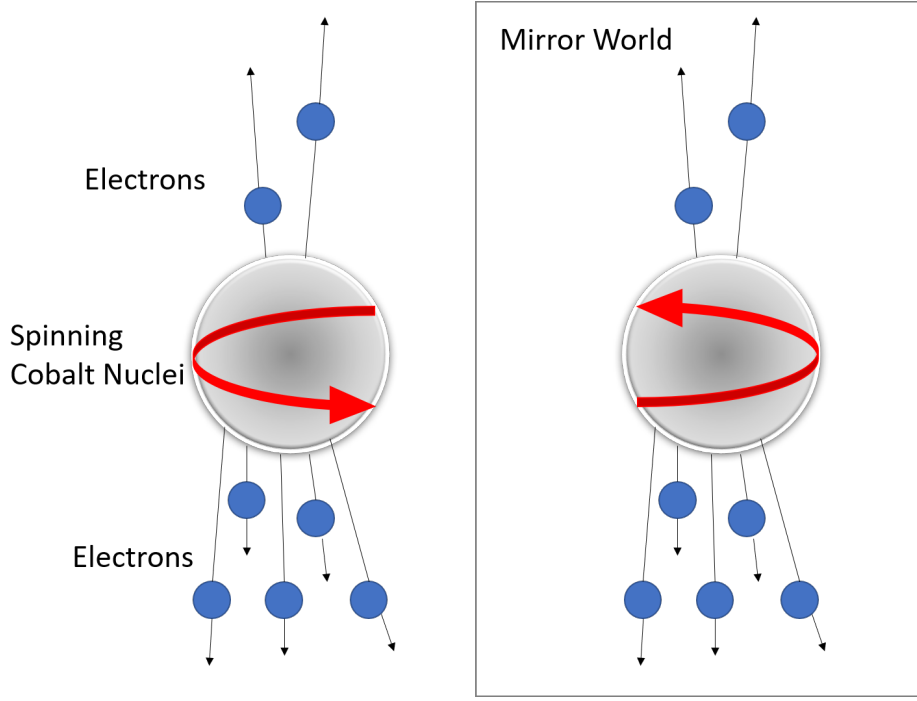


Figure 3: Beta decay of cobalt 60 emits electrons in a preferred direction, violating parity symmetry.

trino, was assumed to be conserved until 1964, when it was found that the decay of the neutral Kaon violates the CP symmetry [4].

CP violation had to be embedded somehow in the theoretical model. This is achieved in the SM with the introduction of a 3x3 unitary matrix known by the name of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The CKM matrix describes the probability of a transition from one quark to another. In other words, it contains the information of the strength of the flavor changing weak interaction. This leaves us with a matrix full of free parameters whose values have to be experimentally determined. However, not all parameters are independent of each other. It was shown that only 4 parameters are necessary to fully define the CKM matrix. The KM parameterization uses three angles and a CP violation phase angle.

The measurement of a CP violation in the weak decay opened the door for more searches of a similar violation in the strong interaction. Based on the SM model there should also be a CP violation term in the strong interaction.

## 2.2 The strong CP problem

In classical field theories, the conserved current  $J^\mu$  of any symmetry of the Lagrangian can be calculated using Noether's theorem. The conserved current is subject to the condition that:

$$\partial_\mu J^\mu = 0 \quad (1)$$

This procedure normally works for quantum field theories, in which the fields are operators. However, in the SM where fermions transform under U(1) symmetry and couple to gauge fields,  $J^\mu$  takes the following form:

$$\partial_\mu J^\mu = \frac{g^2}{32\pi^2} F_a^{\nu\mu} \tilde{F}_{a\nu\mu}, \quad (2)$$

where  $g$  is the gauge coupling and  $F_a^{\nu\mu}$  is the gauge field strength tensor. Equation 2 states the chiral anomaly [5, 6], a more detailed discussion about this can be found in [7]. A term like the right-hand side of equation 2 appears for each coupling between fermions and a gauge group. This effect does not leave the Lagrangian invariant. It adds a term of the form:

$$L_\theta = \frac{g^2}{32\pi^2} \theta F_a^{\nu\mu} \tilde{F}_{a\nu\mu}, \quad (3)$$

where  $\theta$  is the vacuum angle [8, 9] ( $0 \leq \theta \leq 2\pi$ ). For each gauge symmetry there is a corresponding  $\theta$  term ( $\theta_{QCD}, \theta_{EW}$ ). Moreover, the properties of  $F_a^{\nu\mu}$  imply that the  $\theta$  terms will violate the CP symmetry. We have seen in the previous section that the CP violation is a phenomenon widely seen in the weak interaction.

From the theoretical point of view, there is also a  $\theta$  term connected to the strong interaction and there is no particular reason not to expect a CP violation coming from this term. Similar to the experimental observations of CP violation in the weak interaction, the CP violation of the strong interaction will produce effects that can be experimentally measured. One example of this is that a non-zero  $\theta_{QCD}$  would produce an electric dipole moment (EDM) for the neutron.

Using the Wigner–Eckart theorem, it can be shown that the EDM has to be aligned along the spin direction. Moreover, the neutron possesses a nonzero magnetic dipole moment  $\mu$ . Thus, the nEDM ( $d$ ) if it exists will be parallel or anti-parallel to the magnetic moment. The magnetic and electric dipole moments transform differently under time (T) and parity (P) transformations. If the EDM exists the initial system is not symmetric with the final system, violating T and P. This implies an indirect CP violation, for a theory like the SM where CPT is a conserved symmetry. Figure 4 shows how a CP transformation affects a neutron with parallel  $d$  and  $\mu$ . A neutron with anti-parallel  $d$  and  $\mu$  is distinguishable from a neutron with parallel configuration, thus the CP symmetry is violated in this system.

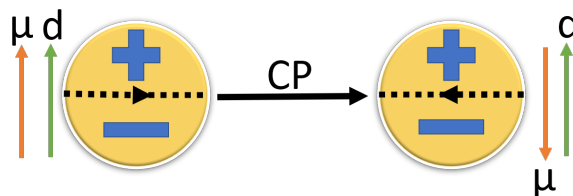


Figure 4: A neutron with an initial parallel  $d$  and  $\mu$  configuration undergoes a CP transformation. The resulting neutron is distinguishable from the initial one. Making this system not symmetric under CP transformation

A full calculation of the nEDM for QCD can be found in [10, 11]. For this work, it is enough to show the mechanism that generates the nEDM (see figure 5).

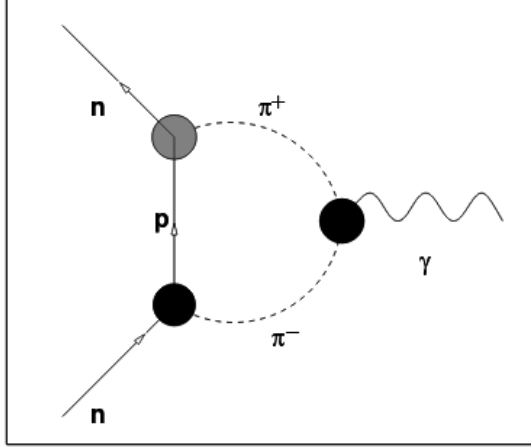


Figure 5: Mechanism responsible for the nEDM. The black circles are CP conserving couplings, while the gray circle is the CP violation vertex. Taken from [11].

A direct computation of this diagram gives:

$$d_n = \frac{g_{\pi NN} \bar{g}_{\pi NN}}{4\pi^2 M_N} \ln \left( \frac{M_N}{m_\pi} \right) \sim 2 \times \theta_{QCD} \times 10^{-16} \text{e-cm}. \quad (4)$$

The experimental upper limit for the nEDM is  $\sim 10^{-26}$  e-cm [12]. From this value, one can constrain the  $\theta_{QCD}$  to roughly less than  $10^{-10}$ . The principle of naturalness introduced by t' Hooft [13] states that any dimensionless free parameters are expected to be of order 1 unless a nonzero value would violate some symmetry of the theory.  $\theta_{QCD}$  is a free parameter of the SM model, a value as small as  $10^{-10}$  is considered unnatural. This is known as the strong CP problem.

So far we have considered just the strong interaction when looking at the strong CP problem and the nEDM. However, quarks also couple to the electroweak gauge group. If we consider the electroweak force, one gets an additional term to  $\theta_{QCD}$  of the form [14]:

$$\theta_Y = \arg(\det[Y_d Y_u]), \quad (5)$$

where  $Y_u$  and  $Y_d$  are the Higgs Yukawa coupling matrices responsible for the quark mass generation. In this case,  $\theta_{QCD}$  in equation 4 has to be replaced by  $\bar{\theta}$  which is defined as:

$$\bar{\theta} = \theta_{QCD} + \theta_Y. \quad (6)$$

Instead of simplifying the problem, adding the electroweak interaction makes it more complicated. Even if one can find a way to argue that  $\theta_{QCD} = 0$ , one would still get a contribution from  $\theta_Y$ . There is a possibility that  $\theta_Y$  is equal to zero, but that is counterintuitive to what is expected from a fundamental theory. It was mentioned in section 2.1 that the CKM matrix has a CP violation phase ( $\delta_{CKM}$ ), which has been measured to be  $1.20 \pm 0.08$  [15]. Both  $\theta_Y$  and  $\delta_{CKM}$  are CP violation terms related to the weak interaction, so it is not expected for one of them to be of order 1 and the other to be zero. Normally, one would expect  $\theta_Y$  to also be of order 1. This assumption would mean that  $\theta_{QCD}$  should have a value of the same order of magnitude. This value has to be equal and with the opposite sign to  $\theta_Y$  with a precision of  $10^{-10}$  to be in agreement with the experimental upper limit of the nEDM. This is considered as a fine-tuning problem of the SM.

## 2.3 The Peccei-Quinn mechanism as a solution to the strong CP problem

Several solutions have been proposed over the years to solve the strong CP problem. Some of them have been so far ruled out by measurements, like the massless up quark solution. Other theories assume that the CP is an exact symmetry of a more fundamental theory [16], but this brings the question of how to explain the CP violation in the weak interaction. Some other theories like the Nelson-Barr mechanism [17–20] are still not completely ruled out, but they introduce more fine-tuning problems.

The most elegant solution to the strong CP problem was introduced by Roberto Peccei and Helen Quinn [21, 22] in 1977. To understand the proposed solution by Peccei and Quinn, we have to mention the concept of spontaneous symmetry breaking (SSB) and how this mechanism was used to give the mass to the leptons and some of the SM bosons. For now, let us consider the following Lagrangian:

$$L = \frac{1}{2}(\partial_\nu \phi^*)(\partial^\nu \phi) - V(|\phi|), \quad (7)$$

where  $\phi(x)$  is a complex scalar field and  $V(|\phi|)$  is a potential of the form:

$$V(|\phi|) = \frac{1}{2}\mu^2|\phi|^2 + \frac{\lambda}{4}|\phi|^4. \quad (8)$$

This theory has a U(1) symmetry because the Lagrangian is invariant under a phase transformation of  $\phi$ . By having a closer look at  $\mu^2$ , one can distinguish between three scenarios:

1.  $\mu^2$  is positive. The theory describes a scalar field with mass  $\mu$ .
2.  $\mu^2$  is equal to zero. The theory describes a massless interacting field.
3.  $\mu^2$  is negative. The minimum of the potential is not located at zero, but at  $|\phi| = v = \mu/\sqrt{\lambda}$ .

Figure 6 shows the potential for a positive and a negative  $\mu^2$  value.

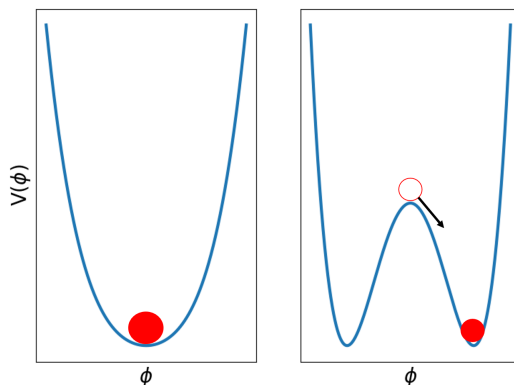


Figure 6:  $V(|\phi|)$  for different  $\mu^2$  values. Left: positive  $\mu^2$ . Right: Negative  $\mu^2$

The SSB is known as the process where there is a phase transition from a positive to a negative  $\mu^2$  value. A good example of SSB is ferromagnetic material, where above a certain

temperature the magnetization is zero and spatially invariant. Below this temperature, the magnetization acquires a non-zero value that points in a certain direction.

If we consider a theory where the universe is currently in the third scenario, one can expand the potential in the following way:

$$\phi(x) = (v + \sigma(x))e^{i\pi(x)}, \quad (9)$$

where  $v$  is the vacuum expectation value,  $\sigma$  and  $\pi$  are the two real scalar fields corresponding to the complex field. Introducing equation 9 into equation 7 one gets:

$$L = \underbrace{\frac{1}{2}(\partial_\nu\sigma)^2 + \frac{v^2}{2}(\partial_\nu\pi)^2}_{\text{kinetic energy terms}} + \underbrace{\left(\frac{1}{2}\sigma^2 + v\sigma\right)(\partial_\nu\pi)^2}_{\text{Interactions between } \sigma \text{ and } \partial_\nu\pi} + V(\sigma(x) + v) \quad (10)$$

Looking at equation 10, one can see that  $\pi$  only appears as a derivative in the Lagrangian, which means there is no mass term for this field. These massless fields are known as Goldstone bosons and are a generic feature of the theory. In a quantum field theory (QFT), the minima are denoted as the vacuum expectation value (VEV) and the mass of a field are small oscillations around the VEV. The  $\sigma$  field represents the potential's radial degree of freedom and the  $\pi$  field the azimuthal degree of freedom, as shown in figure 7.

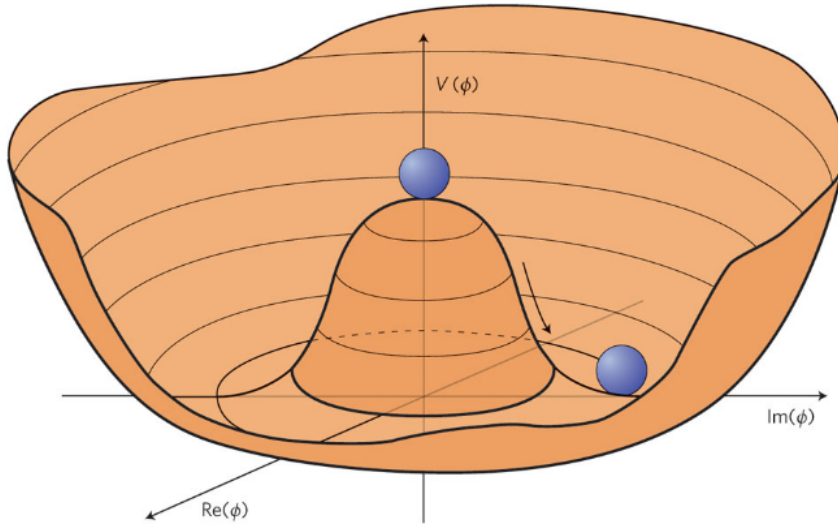


Figure 7: Mexican hat potential with U(1) SBB. The massive boson  $\sigma$  represent the radial degree of freedom and the massless Goldstone boson  $\pi$  the azimuthal degree of freedom.

Figure taken from [23].

These ideas were introduced to the SM, where the universe starts with a very high temperature and then cools down to a point where an SBB occurs. For energies below the electroweak scale  $v = 246$  GeV, the  $SU(2) \times U(1)$  gauge symmetry of the SM is spontaneously broken. In the SM, the complex field is known as the Higgs field and the  $\sigma^2$  term represents the mass term of this field. This term produces a massive boson known as the Higgs boson. The discovery of the Higgs boson in 2012 at CERN has been one of the biggest successes of the SM. Without going into details, the Brout-Englert-Higgs mechanism states that when a gauge symmetry is spontaneously broken, there are no Goldstone bosons and instead the gauge bosons acquire masses. This is the mechanism responsible for giving mass to the  $W^\pm$ , Z bosons, and the fermions.

The Peccei-Quinn mechanism follows a similar idea to the Brout-Englert-Higgs mechanism. They introduced a global U(1) symmetry, designated  $U(1)_{PQ}$  with a complex scalar

field  $\phi_{PQ}$  which has its minimum at  $|\phi|_{PQ} = f_a$  and a fermion type "quark" which is charged under  $SU(3)$ .  $f_a$  is known as the axion scale. A non-zero charge is required to give  $U(1)_{PQ}$  a chiral anomaly with QCD. If the  $U(1)_{PQ}$  symmetry would be an exact spontaneously broken symmetry, then one would have a massless pseudoscalar Goldstone boson analogous to the  $\pi(x)$  field of equation 9. This pseudo-Goldstone boson  $a(x)$  will be denoted as the axion and was introduced by Steven Weinberg [24] and Frank Wilczek [25] shortly after Peccei and Quinn proposed their solution to the strong CP problem. However, the  $U(1)_{PQ}$  is explicitly broken by the chiral anomaly, introducing a term in the Lagrangian of the form:

$$L_{agg} = \frac{g^2}{32\pi^2 f_a} a F_a^{\nu\mu} \tilde{F}_{a\nu\mu}. \quad (11)$$

One can regard this effect as a "misalignment mechanism" in which the Mexican hat potential of the axion field is tilted from an initial random value and "rolls" to its new minimum value. It will be discussed later on how the moment in the evolution of the universe when the  $U(1)_{PQ}$  is broken plays a role in the possible mass of the axion. For now, it is important to remark that the explicit symmetry breaking gives the axion a mass. For the axion to solve the strong CP problem, the axion potential must be minimized at  $\bar{\theta} + a(x)/f_a = 0$ . In other words, the axion must roll to a value where  $a(x)/f_a$  cancels out the initial  $\bar{\theta}$  value. By introducing this extra symmetry, the Peccei-Quinn mechanism "promotes"  $\bar{\theta}$  to a field and the strong CP problem would solve itself dynamically.

## 2.4 Axion models

The interaction between the axion and gluons must exist in order to solve the strong CP problem. This interaction manifests itself in the axion-nucleon interactions and in the axion-meson interactions. In an effective low energy Lagrangian [7, 26], the dominant interaction is the axion- $\pi^0$  mixing, which is responsible for the axion mass. The axion mass can be given by:

$$m_a = \frac{m_\pi^2 f_\pi^2}{f_a^2} \frac{m_u m_d}{(m_u + m_d)^2}, \quad (12)$$

where  $m_u$  and  $m_d$  are the masses of the up and down quark,  $m_\pi$  and  $f_\pi$  are the mass and decay constant of the pion. It is important to notice that the axion mass is inversely proportional to  $f_a$ , so a larger axion scale will lead to a smaller mass. The axion-pion mixing implies that similar to  $\pi^0$ , there is an axion to two-photon interaction. This is a very important feature used to do axion searches as we will see later on. The axion photon interaction is given by:

$$L_{a\gamma\gamma} = g_\gamma \frac{e^2}{16\pi^2 f_a} a F_a^{\nu\mu} \tilde{F}_{a\nu\mu}, \quad (13)$$

where  $g_\gamma$  is a model dependent dimensionless coefficient. The Feynman diagram for this interaction is shown in figure 8.



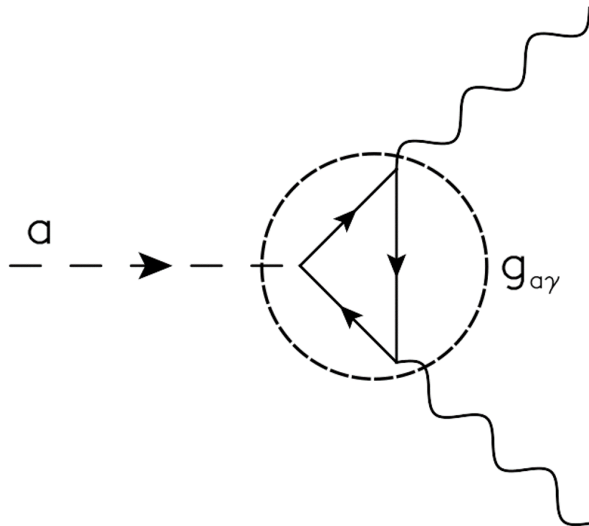


Figure 8: Triangle Feynman diagram for the axion to two photons interactions. The left vertex represents the anomalous current and two right vertices represent the coupling of the gauge fields. Figure taken from [27] and modified.

The first axion model was postulated by Weinberg and Wilczek and was known as the **Peccei-Quinn-Weinberg-Wilczek model (PQWW)**. So far, the  $\phi_{PQ}$  was introduced to solve the strong CP problem, but the properties of this field were not discussed. In the PQWW model it is assumed that the  $\phi_{PQ}$  is a field of the SM. In the SM the Higgs field is a doublet  $(\phi^+, \phi^0)$ , to give mass to the up-type quarks the hermitian conjugate of the Higgs field is used  $(-\phi^{0*}, \phi^-)$ . This was a problem because in this scenario the interaction between the quarks and the  $W^\pm$  bosons would not respect the PQ symmetry. This problem was solved by Peccei and Quinn by extending the SM Higgs field to two doublets. The two Higgs doublets have an VEVs of  $v_1$  and  $v_2$ , while  $f_a = \sqrt{v_1 + v_2}$  and it is around the value for the electroweak scale  $v = 246$  GeV. For this model the axion mass is given by [7, 26]:

$$m_a^{PQWW} = N \left( x + \frac{1}{x} \right) \frac{m_\pi^2 f_\pi^2}{v} \frac{\sqrt{m_u m_d}}{m_u + m_d}, \quad (14)$$

where  $x = v_2/v_1$  is the only free parameter and  $N$  is the number of fermion generations. Plugging the numbers into equation 14, one gets that the axion mass must be larger than 150 keV where the minimum is obtained for  $x = 1$ . There was a lot of excitement among the physicists after this model was presented because these masses could be achieved with the accelerators of that time. However, this model was quickly ruled out by experimental measurements. One of them comes from astrophysical observations. If the axion exists, then it will provide the red giant stars an extra energy loss mechanism (e.g.  $e^- + \gamma \rightarrow e^- + a$  or  $Z, e^- + \gamma \rightarrow Z, e^- + a$ ). This would speed up the stellar evolution of red giants. The calculated evolution results [28] showed that the PQWW axion could not exist. The other evidence came from the studies of the  $J/\Psi \rightarrow a + \gamma$  and the  $\Upsilon \rightarrow a + \gamma$  decays. The product of the branching ratio of these two decays is a parameter-free prediction of the PQWW model. The results from the CUSB collaboration and the Crystal Ball detector were able to rule out the PQWW model [29, 30].

After the downfall of the PQWW model, theoreticians needed to come up with new ideas to save the Peccei-Quinn solution to the strong CP problem. The "invisible" axion model abandoned the idea of  $f_a$  being in the electroweak scale and created new models with  $f_a \gg v$ . From equation 12 one can see that a large  $f_a$  means a small axion mass. This mass was too small to be searched using the detectors of that time and thus the name "invisible" axion models.

The first of these models was pioneered by Kim [31], and Shifman, Vainshtein, and Zakharov [32]. In the **KSVZ model**, the  $\phi_{PQ}$  is not an SM field and does not interact with the SM model quarks or with the Higgs field. So for this model, one would have one Higgs doublet plus one Peccei-Quinn singlet. The  $\phi_{PQ}$  field has to couple to a new heavy non SM quark. Even though the KSVZ model allows for an axion that cannot yet be ruled out by experiments, it introduces the problem of a new heavy quark that is not embedded in most SM extensions.

The other model was introduced by Dine, Fischler, and Srednicki [33] and independently by Zhitnitsky [34]. In the **DFSZ model** the  $\phi_{PQ}$  is not an SM field but it is allowed to interact with the Higgs field. Meaning that the axion has bare coupling with quarks and leptons. For this model, one would need two Higgs doublets plus one Peccei-Quinn singlet.

The axion mass for these models can be also written in the form of equation 12, but they will have different values depending on the electric charge of the new heavy quark for the KSVZ model or the different coupling of the Higgs doublets with the SM fermions for the DFSZ case (see section 3.4).

In this chapter, the theoretical motivation for the axion was presented in the view of the SM and the strong CP problem. The Peccei-Quinn mechanism was introduced to solve this problem and the axion appeared as a product of this new mechanism. The KSVZ and DFSZ models are the most compelling models to include the Peccei-Quinn mechanism in the SM. In the next chapter, it will be seen how the properties of the axion introduced in this chapter also make it a good dark matter candidate.

### 3 Axion as Dark Matter candidate

Cosmology is the science that studies the origin and evolution of the universe. Modern cosmology is based on the Big Bang theory, a theory that brings together particle physics and astronomical observations. The Big Bang theory states that the universe started as a singularity with an extremely high density and temperature and it has then been expanding and cooling down. Within the Big Bang theory, the expansion of the universe is driven by the total energy density. The expansion can be parameterized using the  $\Lambda$ CDM model and the Friedmann equation and is given by:

$$H^2 = H_0^2 \left[ \frac{\Omega_r}{R^4} + \frac{\Omega_m}{R^3} + \Omega_\Lambda + \frac{1 - (\Omega_r + \Omega_m + \Omega_\Lambda)}{R^2} \right], \quad (15)$$

where  $H$  is the Hubble parameter,  $H_0$  is the Hubble constant,  $R$  is the Robertson-Walker scale factor and  $\Omega_r$ ,  $\Omega_m$ , and  $\Omega_\Lambda$  are the normalized present-day energy density for radiation, matter, and dark energy. As we will show later on in this chapter,  $\Omega_{\text{total}} = 1.0 \pm 0.02$ . This means that the last term of the right-hand side part of equation 15 can be neglected. The best current measurements [35] for the energy densities are:  $\Omega_r = 9.10 \times 10^{-5}$ ,  $\Omega_m = 0.309$  and  $\Omega_\Lambda = 0.691$ . Out of this  $\Omega_m = 0.309$ , baryonic matter accounts for just  $\Omega_b = 0.0486 \pm 0.0010$ . The rest is made of a new type of matter known as dark matter. An important remark before we move on is that the energy densities are time-dependent. The universe was not always dominated by the dark energy term, as shown in figure 9.

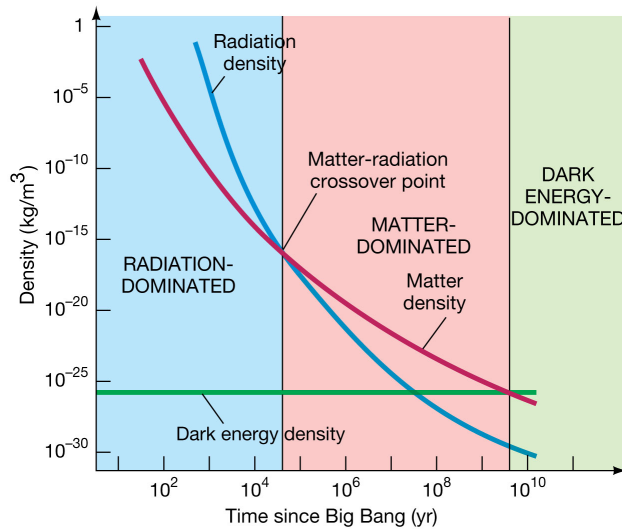


Figure 9: Evolution of the energy density with time in the Big Bang model, with the various eras indicated. Currently, the universe is in the dark energy dominated era. Figure taken from [36].

#### 3.1 Evidence for dark matter

The galaxy rotation curves provided one of the first pieces of evidence of dark matter. Consider a star of mass  $m$  at distance  $r$  from the galactic center, moving tangentially with velocity  $v$ . From the centrifugal and gravitational forces, one gets:

$$\frac{mv^2}{r} = \frac{mM(r)G}{r^2}, \quad (16)$$

where  $M(r)$  is the total mass inside the radius  $r$ . A spiral galaxy like the Milky Way has most of the baryonic matter concentrated in a central hub plus a thin disc. For a star inside

the hub, equation 16 predicts  $v \propto r$  and for a star outside of the hub  $v \propto r^{-1/2}$ . The velocity should decrease at large  $r$ , but the measurements of the galaxy's rotation curves showed a flat behavior at large  $r$  (see figure 10). This leads to the suggestion that most of the mass of the galaxies is in the form of a dark matter halo.

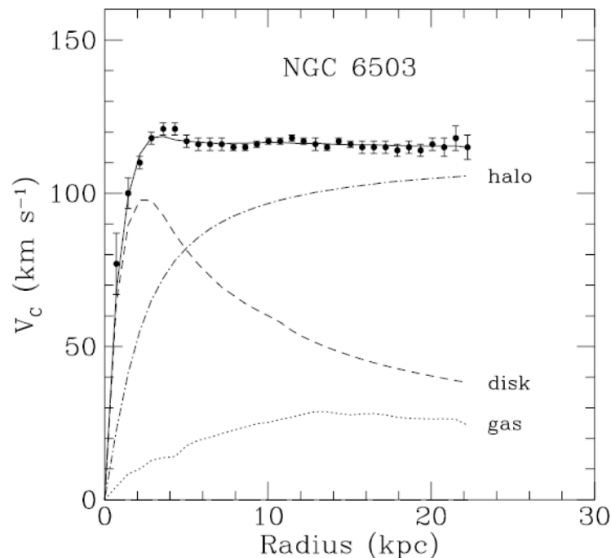


Figure 10: Typical rotation curve for a spiral galaxy (NGC 6503). Figure taken from [37]

Another source of evidence of dark matter is the galaxy clusters velocity dispersion and X-ray emission. The time average of the potential energy  $\langle V \rangle$  and the kinetic energy  $\langle K \rangle$  of a bound state system like a galaxy cluster can be related using the virial theorem. The virial is defined as  $W = \sum \vec{p}_i \cdot \vec{r}_i$ , where  $\vec{p}_i$  is the momenta and  $\vec{r}_i$  is a position vector measured from some arbitrary origin. The time derivative of the virial is defined as:

$$\frac{dW}{dt} = - \sum V_i + 2 \sum K_i. \quad (17)$$

The average for a time interval  $T$  for a bound system results in  $\langle V \rangle = 2\langle K \rangle$ . Using the virial theorem together with estimations of the velocity dispersion of galaxy clusters showed that the velocity dispersion cannot be explained just by the observed baryonic matter distribution.

Most of the visible mass of the galaxy clusters is in the form of very hot X-ray emitting gas. Measurement of these emitted X-rays by the Chandar satellite showed that the velocity of the gas particles exceeds the escape velocity of the galaxy cluster. Dark matter is needed to hold these clusters together.

General relativity was the next one to provide more hints for the existence of dark matter. The gravitational deflection of photons passing by a point with mass  $M$  at a distance of closest approach  $b$  predicted by Einstein's general relativity is given by:

$$\alpha = \frac{4GM}{c^2 b}, \quad (18)$$

This equation implies that massive objects lying between a more distant source may act as gravitational lenses. By measuring the distortion suffered by the light one can estimate the total gravitational mass of the lensing object. In several cases where this approach has been used, the results showed that the object needs to have more mass than the bare baryonic mass. One of the best examples is the galaxy cluster IEO657-558 observation [38]. D. Clowe et.al observed a system of two galaxy clusters having passed through each other.

Using gravitational lensing the experiment was able to determine the total mass distribution. Using the Chandra X-ray satellite the experiment was able to identify the distribution of the hot plasma and finally with optical telescopes the luminous matter in the stars was identified. Figure 11 shows the distribution of the baryonic matter and the total distribution needed for the measured gravitational lensing. The importance of this observation is the location of the X-rays and the dark matter. When these two objects pass through each other, the X-ray clouds will slow down due to electromagnetic interactions. While the dark matter will be unaffected by this. The spatial separation of dark matter and baryonic matter, in this case, cannot be explained by a theory like the modification of the law of Newtonian gravity at large distances, which reinforces the need for dark matter [38].

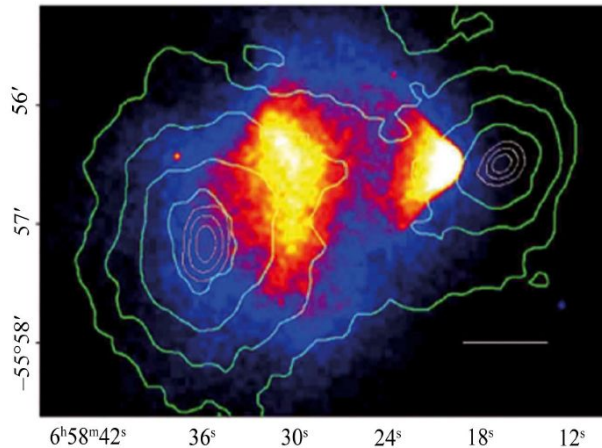


Figure 11: Total mass distribution of the galaxy cluster IEO657-558. The colored areas show the position of the baryonic matter and the green contour lines the distribution of dark matter from the gravitational lensing measurements. Figure taken from [38].

Finally, the Cosmic Microwave Background (CMB) and structure formation can be used to predict the need for dark matter in the universe. When it was hotter and denser at the early stages of the universe, its structure was mainly a high coupled plasma in thermal equilibrium made of protons, electrons, and photons. As the universe expanded and cooled down, the electrons and protons could combine and form neutral bound states. These neutral atoms could not scatter the thermal radiation by Thomson scattering, so matter and radiation were decoupled. This epoch of time is known as recombination. Shortly after the recombination, the photons were able to travel through space freely. These photons have been propagating ever since with less and less energy. Nevertheless, these photons leave a background "noise" better known as the CMB.

Precise measurements of the CMB are critical to cosmology since any proposed model of the universe must explain this radiation. The CMB is a nearly perfect black body radiation but contains very small temperature anisotropies in the order of  $10^{-5}$ . These anisotropies can be quantified precisely by their decomposition into an angular power spectrum<sup>1</sup>, which is observed to contain a series of acoustic peaks. Any proposed universe model needs to explain the CMB and fit the angular power spectrum. The fit parameters of the model give a valuable estimation of the composition of the universe. Figure 12 shows the fit to the angular power spectrum using the  $\Lambda$ CDM model. The accuracy of the fit is robust support for this model, where the amount of matter content in the universe is  $\Omega_m = 0.309$  and just  $\Omega_b = 0.0486 \pm 0.0010$  is baryonic matter.

<sup>1</sup>This power spectrum characterizes the size of the fluctuations as a function of angular scale.

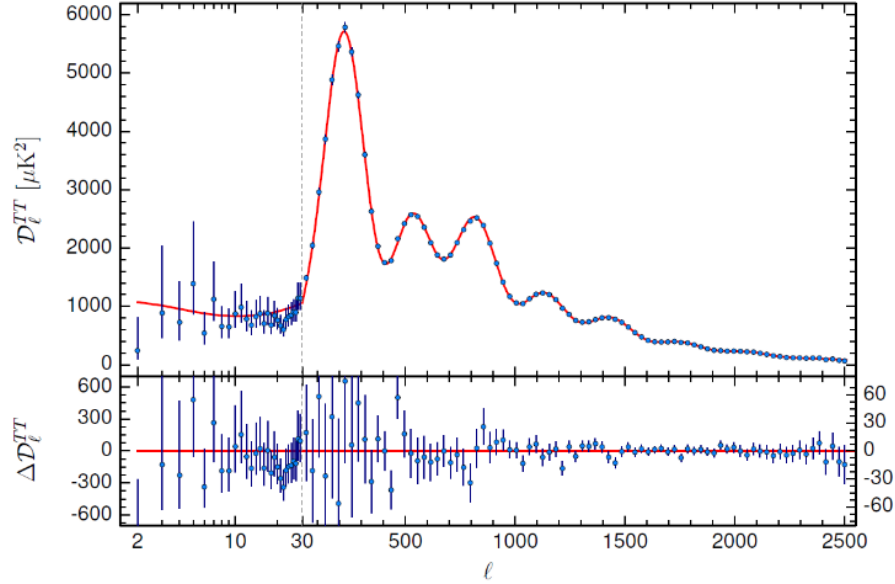


Figure 12: Top: Decomposition of the CMB temperature anisotropies into angular power spectrum (blue points), the red line corresponds to the best fit of these points using the  $\Lambda$ CDM model. Bottom: It shows the residuals. Figure taken from [35]

Moreover, the level of homogeneity of the CMB presents severe issues for the structure formation of the present universe in case there is just baryonic matter. The present-day universe is inhomogeneous. The galaxies and galaxy clusters grew via gravitational interactions from density fluctuations. The anisotropies seen in the CMB are not large enough to have produced today's structures if they were due to baryonic mass only. Also, the baryonic matter is affected by radiation, which is the dominant element of the universe at very early times. As a result, baryonic matter density perturbations are washed out and the baryonic matter was unable to condense into structures. If there were only baryonic matter in the universe, there would not have been enough time for density perturbations to grow into the galaxies and galaxy clusters currently seen.

### 3.2 Dark Matter candidates

In the previous section, several proofs of the existence of dark matter were presented. The specific properties of this new type of matter are unknown. However, it is known that it accounts for  $\Omega_{DM} = \Omega_m - \Omega_b = 0.26$  of the total matter content of the universe and also has to explain the rotational curves of the galaxies. Also, the dark matter candidate has to be a neutral particle, be stable on the cosmological time scale, have mass, and interact weakly. There are different theories, which take into account the existence of dark matter. The various theories propose several dark matter candidates. In principle, dark matter does not need to be made of just one of these candidates. In this section, the most prominent candidates are going to be presented. Also, some initial candidates are now partially excluded because they cannot make up all the dark matter in the universe.

The phenomenon of gravitational lensing explained in section 3.1 cannot be measured for a single star because the resolution of the best optical telescope is not good enough. However, a passing massive single object can work as a lens and amplify the source intensity. This amplification is known as microlensing. **Massive astrophysical compact halo object (MACHOS)** [39, 40] are microlensing objects with masses of the order of stellar masses in our galaxy and were one of the first dark matter candidates. Some of the MACHOs candidates include neutron stars, black holes, and brown dwarfs. Even though experiments

have already seen some MACHOS in the 0.001-0.1 solar mass range, these are not enough to account for the needed dark matter density. If MACHOs were the only constituent of dark matter, many more microlensing events should have been observed by the experiments. The present evidence shows that MACHOS appear to account for only a trivial contribution to the energy density.

The best candidate will, of course, be a particle that is already included in the SM. **Neutrinos** are weakly interacting with SM particles, neutral, stable, and the observation of neutrino oscillations is proof that the neutrino masses are non-zero.

Depending if the candidate was relativistic or non-relativistic at the relevant time of structure formation, it would be classified as Hot Dark Matter (HDM) or Cold Dark Matter (CDM). The production mechanism and evolution of the neutrinos show that at the time of structure formation, the neutrinos were relativistic particles. HDM is responsible for a process known as collisionless damping, a process where the neutrinos are moving around the universe and thus smoothing out the primordial density fluctuations on galactic scales. This phenomenon leads to a top-down structure formation, where the galaxy clusters were formed first, after that subdividing into smaller structures. Numerical simulations have shown that a top-down model would produce a universe quite different from what we live in, making an HDM candidate less likely. Another argument against neutrinos as constituting all of DM is that the amount of neutrino density needed to account for the DM density on the Milky Way will result in neutrinos going faster than the typical escape velocity due to their small mass.

**Weakly Interactive Massive Particle (WIMP)** [41] is a new hypothesized particle that would behave like a neutrino, but it is more massive. WIMPs are regarded as CDM candidates and the lower limit for its mass ( $m_{\text{WIMP}} > 2 \text{ GeV}$ ) is set by the Lee-Weinberg bound [42]. In the most simple WIMP models, it is assumed that in the early universe, the WIMPs are in thermal equilibrium with all the other particles. The thermal equilibrium is possible through their self-annihilation into SM particles. When the expansion of the universe reached a point where the density of the WIMPs is low enough that the self-annihilation process cannot occur anymore, the WIMPs density will freeze out from equilibrium and remain fixed. Under this simplified assumption, one can relate the possible mass of the WIMPs to its cosmic energy density ( $\Omega_{\text{WIMP}}$ ). If the WIMPs account for all the dark matter in the universe ( $\Omega_{\text{WIMP}} \sim \Omega_{\text{DM}}$ ), the expected WIMP mass is  $\sim 100 \text{ GeV}$ , which is the sensible mass range for a particle that interacts via the weak force. This is known as the WIMPs "miracle".

WIMPs come out naturally from theories beyond the SM like Supersymmetry. Supersymmetry is a theory that gives every SM particle a more massive supersymmetric partners. The SM bosons get a fermionic partner, and the SM fermions a bosonic partner. This theory will solve, among other things, the hierarchy problem of the Higgs mass and the unification of the gauge coupling. For this theory to account for the experimental limits of lepton and baryon number conservation, a new parity known as the R-parity is introduced. The SM particle has a  $+1$  R-parity, while the supersymmetric particles have a  $-1$  R-Parity. The conservation of R-parity provides a stable supersymmetric particle. The lightest supersymmetric particle (LSP) is a neutral particle that interacts through the weak and gravitational forces with the SM particles and makes a good DM candidate. Some of the LSP candidates are the neutralino, sneutrino, or gravitino. However, most of the simplest supersymmetric models have already been ruled out by the LHC detectors. More complex models are still possible, but much higher energies are needed to test them.

Another approach to explain dark matter comes from theories which include extra dimensions. If a particle can propagate in extra dimensions, it will have an infinite tower of partner states with identical quantum numbers. This was shown by Kaluza and Klein [43, 44] and

they are known as **Kaluza-Klein particles**. Universal extra dimensions (UED) theories postulate that the SM fields propagate in compact extra dimensions. Similar to supersymmetry, this theory has a KK parity that needs to be conserved, making the lightest Kaluza-Klein particle (LKP) a good dark matter candidate. The KK partners of the electroweak gauge bosons and neutrinos are all possible dark matter candidates.

### 3.3 Axions

In the previous section, we introduced different dark matter candidates. This thesis is focused on the search for axions. Axions are also an excellent dark matter candidate. The axion is a neutral particle and interacts weakly with the SM particles. The lifetime of the axion is dominated by its decay into two photons and is given by [45]:

$$\tau_{a \rightarrow \gamma\gamma} = \frac{64\pi^3 f_a^2}{\alpha^2 g_\gamma^2 m_a^3} = g_\gamma^{-2} H_0^{-1} \left( \frac{26 \text{ eV}}{m_a} \right)^5. \quad (19)$$

For any  $m_a$  below 26 eV the axion is stable on cosmological time scales. One of the arguments used to dismiss the neutrinos as candidates was that they would have a velocity above the galaxy's escape velocity. That is because neutrinos are very light fermions and obey the Pauli exclusion principle. This is not the case for the axions, which are bosons. However, they are still light enough to be considered HDM, and the top-down structure formation problem would also rule them out. So far, we have assumed that the dark matter was initially in thermal equilibrium with the SM model particles, and then at some point in the evolution of the universe, it decoupled and froze out. However, it is possible to find a non-thermal production mechanism for the axion, making them CDM. The mechanism to produce CDM axions is known as the misalignment mechanism. The basic idea is that in the early universe, the axion's VEV starts at an initial random value. At some point, the Peccei-Quinn symmetry (see section 2.3) is spontaneously broken, and the axion field "feels" a tilt (see figure 13) and rolls to a new minimum where it oscillates with a frequency  $m_a$ .

To understand better this mechanism, let us consider the equation of motion of the axion field and try to qualitatively understand the behavior of this equation for different times in the evolution of the universe:

$$\ddot{a} + 3H(t)\dot{a} + m_a^2(t)a = 0. \quad (20)$$

In the standard  $\Lambda$ CDM models  $H(t) \propto 1/t$ , while  $m_a^2(t)$  increases with decreasing temperature until it reaches a constant value at a given temperature ( $T \sim \Lambda_{QCD}$ ). There is a time  $t^a$  where  $H(t) = m_a(t)$ . For  $t \ll t^a$  the last term of equation 20 is negligible and  $a$  is frozen at its initial value. On the other hand at  $t \gg t^a$ , equation 20 depends very strongly on  $m_a$ , the Peccei-Quinn symmetry was already broken and the axion field is already at its new minimum. In this scenario, equation 20 describes a simple harmonic oscillator and the axion field can be written as  $a(t) = a_0(t)\cos(m(t)t)$ . Plugging this expression into equation 20, one obtains:

$$\frac{d(m_a a_0^2)}{dt} = -3H(m_a a_0^2). \quad (21)$$

Without going into details, one can interpret  $n_a = m_a a_0^2/2$  to be the axion number density. Moreover, from the Lagrangian, the total axion energy density is defined as  $\rho_a = m_a n_a$ . For  $t \gg t^a$ ,  $m_a$  is constant, and the energy density looks like the one for a non-relativistic particle. Finally, equation 21 tells that the number of axions per co-moving volume is conserved, which is precisely what is expected for any decoupled particle. The misalignment mechanism provides a non-thermal process in which the axion is CDM.



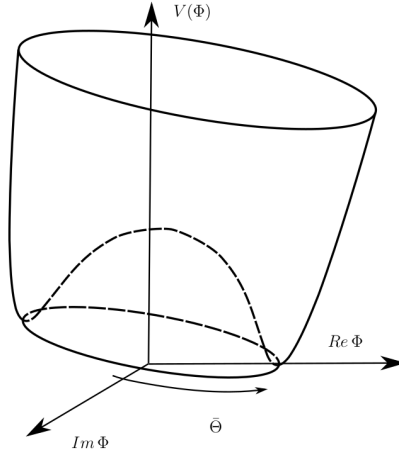


Figure 13: Tilt axion field potential after the Peccei-Quinn spontaneous symmetry breaking. Figure taken from [27].

As it was mentioned in section 3.1, it is essential to estimate the cosmological density of axions ( $\Omega_a$ ). To assess these values, complicated techniques like the dilute instanton gas approximation [46–48] or lattice QCD are used. These techniques give a value of:

$$\Omega_a \sim \left( \frac{f_a}{10^{12} \text{ GeV}} \right)^{\frac{7}{6}}. \quad (22)$$

From this equation, one can compute [16, 49] that for  $\Omega_a \sim \Omega_{DM}$  the value of  $f_a$  has to be around  $3 \times 10^{11} \text{ GeV}$ , this corresponds to  $m_a \sim 20 \mu\text{eV}$ . However, depending on the assumptions of the model and the technique used for the computation, the value of  $\Omega_a$  can change by a factor of 5.

Topological defects on the axion field like cosmic strings and domain walls are other factors that could also contribute to the axion cosmological energy density. Cosmic strings form when a continuous symmetry is spontaneously broken, while domain walls occur when a discrete symmetry is spontaneously broken (see [50] for an introduction to topological defects). Cosmological strings can move around and oscillate, but they cannot evolve into smooth field configurations in the absence of interactions. In case of no interaction, the strings (if they exist) would dominate the universe’s energy density, which goes against the  $\Lambda\text{CDM}$  model. However, studies of cosmic string dynamics have shown [51, 52] that interactions between strings enable them to break apart and eventually decay via the radiation of the field. The strings that could be created by the spontaneous symmetry breaking of the  $U(1)_{PQ}$  symmetry can decay via radiation and constitute another non-thermal mechanism to produce axions. The current knowledge of the string dynamics is not very precise, and the contributions of the cosmic strings to the  $\Omega_a$  may vary by a factor of 100. They can either be negligible compared to the misalignment mechanism or the dominant part of the energy density. The domain walls are more tricky to handle because there is no mechanism for their decay, and any stable domain wall theory goes against the  $\Lambda\text{CDM}$  model. There are different ways to address this problem. Most of them introduce a more elaborated framework to eliminate the discrete symmetry of the Peccei-Quinn mechanism [16].

Finally, it is also important to consider the influence of cosmic inflation in the axion energy density. Inflation is a period of time (from  $10^{-36} \text{ s}$  to between  $10^{-33} \text{ s}$  and  $10^{-32} \text{ s}$  after the Big Bang) in the early universe. During inflation, the universe suffered an exponential expansion. The mechanism responsible for inflation is still unknown, but predictions made by inflation models have already been observed [53]. Inflation could explain things like why the cosmic microwave background radiation is distributed evenly, why the universe appears

to be isotropic and why no magnetic monopoles produced via dilution, assuming they would exist before inflation, have been observed [54]. For this work, inflation's importance relies on whether the Peccei-Quinn symmetry was broken before or after inflation. If it was broken after inflation, one is still in the same situation where topological effects must be taken into account. On the other hand, if it was broken before inflation, all topological defects would be eliminated. This solution does not come for free. In the pre-inflation scenarios, equation 22 get an extra free parameter that once again provokes some fine-tuning problem similar to the strong CP problem. With all of these possible scenarios, it is clear that the theory cannot predict a specific mass for the axion.

Based on the axion idea, other theories introduce a light pseudo-Goldstone boson with similar characteristics and production mechanism. However, these new particles called **Axion Like Particles (ALP) and Weakly Interacting Sub-eV Particles (WISP)** are bosons of an approximate global symmetry broken by something other than the chiral anomaly within QCD and do not solve the strong CP problem.

The hidden sector or dark sector is a theory that introduces a new gauge group that is independent of the SM. New dark sectors quantum fields would produce new unobserved particles. It has been proposed [55] that the misalignment mechanism can be used for the production of **dark photons**, a vector boson that interacts with the SM only through a significantly suppressed kinetic mixing with the SM photon.

ALPs and dark photons are other two examples of dark matter candidates based on similar production mechanisms as the axion but seem less motivated by theory. See [56] for more information.

### 3.4 Axion search space

Even if the specific axion mass is not known, the axion models (introduced in section 2.4) using the dimensionless parameter  $g_\gamma$  can predict a band in which the experiments should look for the axion.  $g_\gamma$  can be written as [57]:

$$g_\gamma = \frac{1}{2} \left( \frac{E}{N} - \frac{24+z}{31-z} \right) = \frac{1}{2} \left( \frac{E}{N} - 1.95 \right), \quad (23)$$

where  $z = m_u/m_d$ ,  $E$  and  $N$  are the electromagnetic and color anomalies of the  $U(1)_{PQ}$ . The best current constraints for  $z$  is  $0.38 < z < 0.58$  [58], the value  $z = 0.56$  was used in equation 23 and it is the standard value used in the axion community for these calculations.  $E/N$  is model dependent and there is no particular reason why it cannot be 1.95.

Let us have a look at the possible  $E/N$  values in the two models introduced in section 2.4. For the KSVZ model, none of the SM fermions interact with the  $\phi_{PQ}$  field, and a new type of quark is introduced. This quark is assumed to be neutral in the simplest versions of the KSVZ model, so  $E = 0$ . Variants of the KSZV model give a non zero charge  $e_q$  to the new quark and define  $E/N = 6e_q^2$ . One of the already known SM charges ( $\pm 1/3, \pm 2/3$  and  $\pm 1$ ) is commonly assigned to the new quark. This leads to  $E/N = 0, 2/3, 8/3$  or  $6$  and  $g_\gamma = -0.97, -0.64, 0.36$  or  $2.03$ .

For the DFSZ model, the PQ charge must be assigned to the SM's quarks and the charged leptons. The symmetries of the SM severely constrain this. At this point, the DFSZ can be divided into two: the type-I model where the leptons and the down type quarks couple to the  $\phi_2$  field of the Higgs doublet. In the type-II model, the leptons and the up-type quarks couple to the  $\phi_1$  of the Higgs doublet. The  $E/N$  values for these models are  $8/3$  and  $2/3$  giving a  $g_\gamma = 0.36$  and  $-0.64$ .

The search space for the axion is quite broad, as shown in figure 14. However, some lower and upper boundaries can be placed. The lower boundary is placed based on equation 22. If

the axion mass is too small ( $m_a < 1 \mu\text{eV}$ ), it would be overproduced resulting in  $\Omega_a/\Omega_{DM} > 1$ . The upper limit is based on astrophysical observations [59]. The neutrino pulse measured from the 1987A Supernovae provides one of the most sensitive upper boundaries for the axion mass. If the axion is too massive ( $m_a > 16 \text{ meV}$ ), then it would provide another energy loss mechanism for the supernovae. The axion would carry away a large amount of energy from the supernovae making the neutrino pulse shorter than the measured value. Some of the other limits shown in figure 14 are going to be discussed in the next chapter. Most of them come from direct experimental exclusion limits.

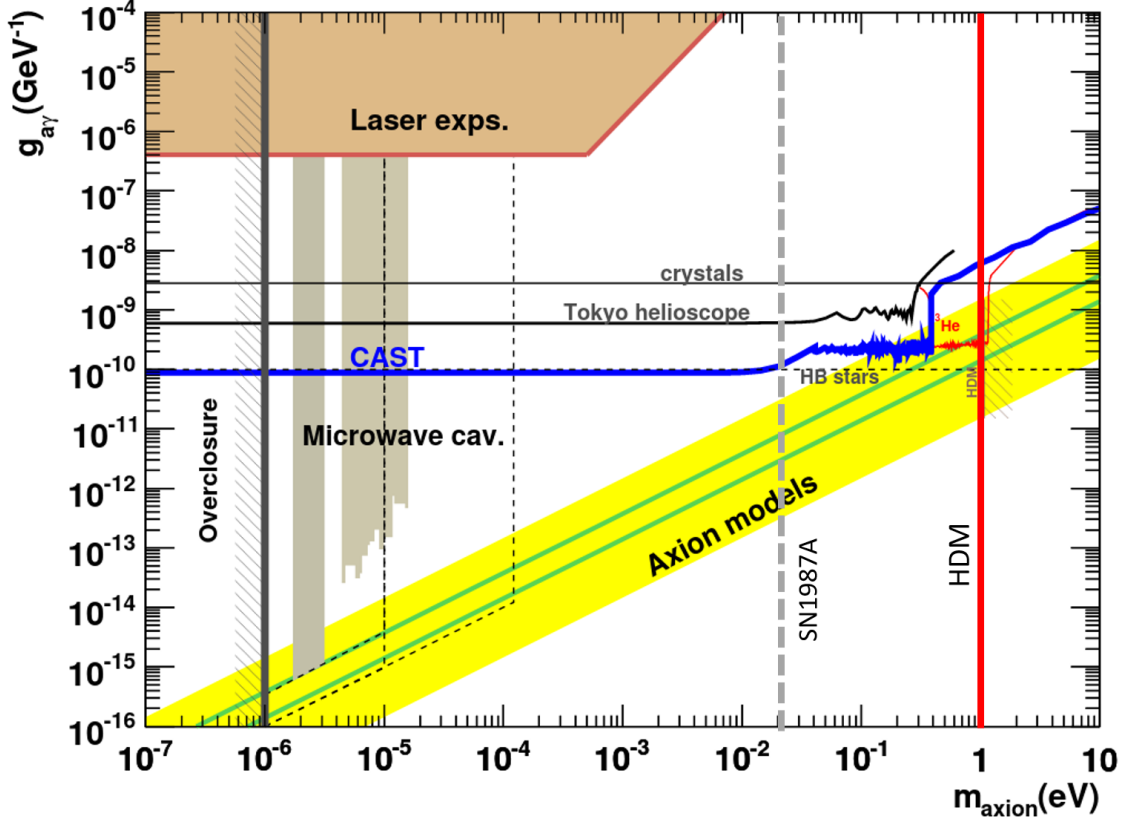


Figure 14: Axion and axion like particles parameter search space. The green lines show the benchmark KSZV and DFSZ models and the yellow region are the QCD model band. The rest of the colored regions are some constraints set by astrophysical observations and direct experimental exclusion limits. Everything above the blue line is already excluded. Also the gray sections labeled microwave cav. are parameter space regions that are already excluded. For details, see text. Figure taken from [60] and modified.

## 4 Experimental axion search methods

In the previous chapters, it was mentioned how some axion models (see section 2.4) were ruled out by observations of red giants and how some boundaries are placed from the neutrino pulse of Supernovae. However, astrophysical observations are not the only way to look for axions. There are different types of methods to look for an axion signature in an Earth-based experiment. For the invisible axion models introduced in section 2.4, the axion has a coupling to two photons of the form:

$$L_{a\gamma} = \frac{1}{4}g_{a\gamma}aF_a^{\nu\mu}\tilde{F}_{a\nu\mu} = -g_{a\gamma}a\mathbf{E} \cdot \mathbf{B}, \quad (24)$$

where  $a$  is the axion field,  $\mathbf{E}$  and  $\mathbf{B}$  are the usual electric and magnetic fields. The model dependent coupling  $g_{a\gamma}$  is defined as:

$$g_{a\gamma} = \frac{\alpha g_\gamma (\hbar c)^{3/2}}{\pi f_a} = \frac{\alpha g_\gamma (\hbar c)^{3/2}}{\pi \sqrt{\chi}} m_a, \quad (25)$$

where  $\alpha$  is the fine structure constant and  $\chi$  is the zero temperature QCD topological susceptibility (77.6 MeV). The interaction in equation 24 can be realized using an external electromagnetic field. The Feynman diagram of this process is shown in figure 15, and it is known as the inverse Primakoff effect.

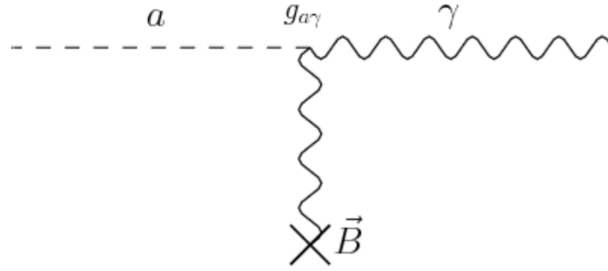


Figure 15: Inverse Primakoff effect: The incoming axion interacts with the virtual photon produced by a magnetic field and transforms into a real photon. Figure taken from [61] and modified.

This process is used by most of the experiments to search for axions. In the following section, the most important axion search methods are going to be briefly discussed.

### 4.1 Light shining through a wall experiment

This type of search was proposed more than 30 years ago by Van Bibber et al. [62]. The idea is to use a laser as a source and shine it into an opaque wall or optical barrier. There is a magnetic field between the source and the wall (see figure 16). The photons coming from the laser can interact with the magnetic field and transform into axions or axion-like particles. The wall will stop the photons that are not converted, but the axions will pass through it. On the other side of the wall, there is another region with a strong magnetic field. The axion can then convert back into a photon, which a single photon detector can measure. The Any Light Particle Search (ALPS) collaboration search for axions and WISPs using this method [63, 64]. One of the advantages of this technique is that the axion production is model-independent. The axion density in the universe and its velocity distribution or the flux associated with the axion's production mechanism inside a star like the Sun do not need

to be known. The main disadvantage is that the axion-photon conversion needs to happen twice.

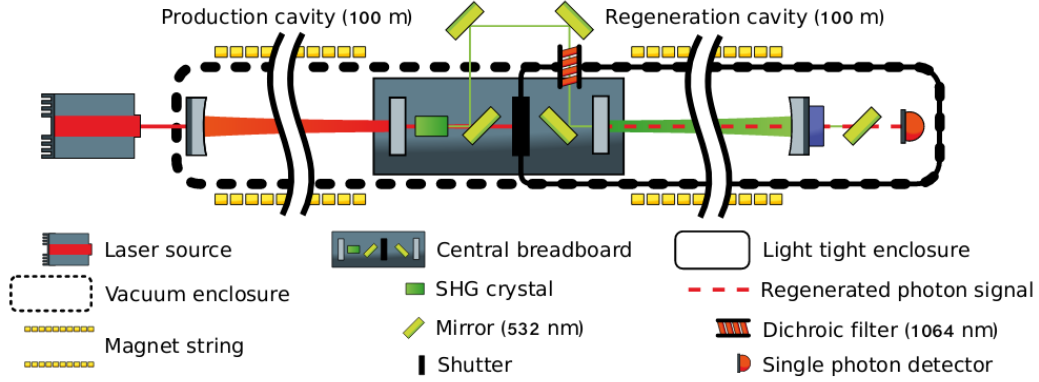


Figure 16: ALPSII simplified design. Figure taken from [65].

## 4.2 Solar axions and helioscope experiments

A near star could provide a source for axions. An excellent candidate to use as a source is our Sun. If axions exist, they can be produced in the Sun's core when the X-rays produced by the nuclear fission processes of the Sun scatter off electrons and protons in the presence of strong electric fields. This is known as the Primakoff effect [66]. These axions can escape the Sun and travel almost unhindered to the Earth due to their weak coupling with ordinary matter. The axion flux coming from the Sun can be converted back into photons using a strong magnetic field. These photons are then measured using X-ray detectors. Figure 17 shows the typical setup of a helioscope experiment.

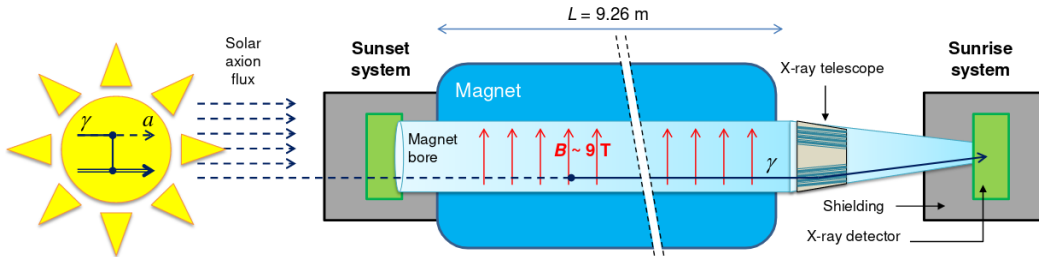


Figure 17: Sketch of a typical helioscope experiment. Figure taken from [67].

For this type of experiments, the probability for the axion-photon conversion inside the magnetic field is given by [68]:

$$P_{a \rightarrow \gamma} = \left( \frac{Bg_{a\gamma}}{2} \right) \frac{1}{q^2 + \Gamma^2/4} (1 + e^{-\Gamma L} - 2e^{-\Gamma L/2} \cos(qL)), \quad (26)$$

where  $B$  is the magnetic field,  $L$  is the length of the magnet,  $\Gamma$  is the inverse absorption length for the photon in the medium inside the magnetic field, and  $q$  is the momentum difference between the axion with mass  $m_a$  and the photon mass  $m_\gamma$  in the medium:

$$q = \frac{m_a - m_\gamma}{2E_a}. \quad (27)$$

The coherent condition  $qL < \pi$  has to be satisfied to convert axions into photons. This condition will set the mass range to which the helioscope will search for an axion. The figure of merit  $F$  for the helioscope experiment is given by [69]:

$$F \propto g_{a\gamma} B^2 L^2 A \epsilon t, \quad (28)$$

where  $A$  is the cross-sectional area,  $\epsilon$  is the efficiency of the detector and optics, and  $t$  is the integration time.

The helioscope experiments were able to set the best  $g_{a\gamma\gamma}$  limit over a large region of axion masses (see the CAST blue line and the Tokyo helioscope line in figure 14). The helioscope experiments have been evolving over the past decades, starting with the work of Lazarus et al. [70], continuing with the SUMICO collaboration [71] and currently with the CAST collaboration (see section 4.3). The International Axion Observatory (IAXO) is a next-generation helioscope that is in the planning phase, and it is expected to surpass the limits set by the CAST collaboration (see section 4.4).

Another type of solar axion experiment uses crystals for axion photon conversion. The typical energies of the solar axions correspond to the wavelength of the lattice spacing in the crystals. This leads to a coherent conversion of axions into X-ray photons as long as the condition for the Bragg reflection is satisfied.

The advantages of the helioscope experiments are that they can probe an extensive mass range, and the axion production is enhanced by the strong electric field of the Sun compared to the magnetic field used in the axion production of the previous method. The main disadvantages are the model dependency of the axion flux inside the Sun and the fact that the helioscope needs to follow the Sun to take data.

### 4.3 The CERN Axion Solar Telescope (CAST)

CAST is a CERN based surface experiment that uses a decommissioned Large Hadron Collider (LHC) dipole magnet with a magnetic field  $\sim 9$  T and a length  $L = 9.26$  m working at a temperature of  $\sim 2$  K using superfluid helium for the cooling. The CAST magnet contains two vacuum pipes inside the magnetic field with a diameter of 43 mm each, known as cold bores. First intended to be solely a helioscope experiment, CAST was proposed back in 1999 [72, 73] and had been taking data since 2003. The CAST magnet is mounted in a movable platform ( $\pm 40^\circ$  horizontally and  $\pm 7^\circ$  vertically) that allows a 90 min solar tracking during the sunrise and sunset. Figure 18 shows a photo of the CAST magnet. CAST is worldwide known for its solar axion searches. It has produced the most sensitive limits to the axion-photon coupling for a large range of masses (see the blue line in figure 14 or [67]).

During its long scientific program, CAST has used different types of X-ray detectors to search for solar axion, for example: a multi-wire time projection chamber [74], a silicon drift detector [67], a  $\gamma$ -ray calorimeter [75], a low-noise charged coupled device attached to a spare x-ray telescope (XRT) from the ABRIXAS x-ray mission [76], and several Micromegas detectors [77]. CAST probed different mass ranges using vacuum,  $^4\text{He}$  [78, 79] and  $^3\text{He}$  [80, 81] inside of the cold bore. In 2013 a new XRT built specifically for axion searches was installed at CAST. The latest CAST limits are presented in [67] and highlight the full potential of the CAST solar axion experiment. Getting an improvement of an order of magnitude to this limit will take decades of data taking. The International Axion Observatory (IAXO) is the next generation helioscope experiment, and it is already under design. A brief description of IAXO is going to be given in section 4.4.

During this work, five detectors (including the RADES detector) were installed and running at CAST. They will be briefly introduced in the next sections to showcase the scientific potential of CAST.



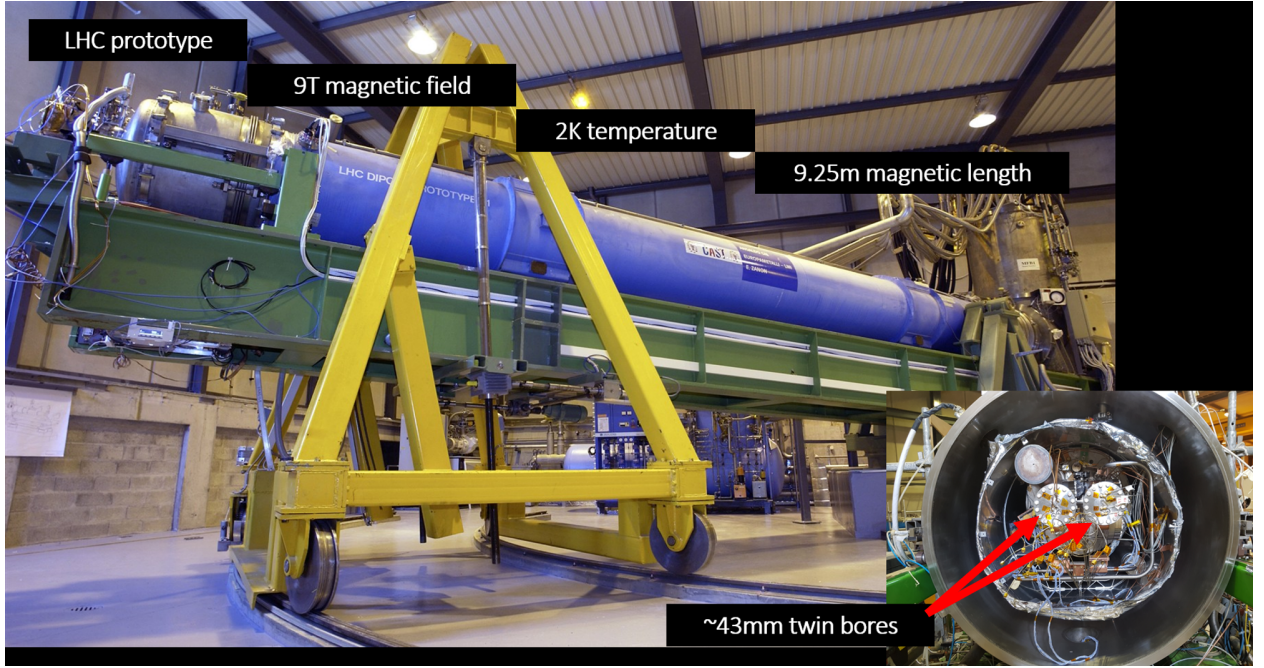


Figure 18: The CAST dipole magnet at CERN (blue) is mounted in a yellow movable structure allowing solar tracking. Recently CAST has also included haloscopes detectors which are installed inside the cold bores. Inset: Front side of the magnet when it is open. The RADES cavity is inserted to the left hand side cold bore.

**Micro-Mesh Gaseous Structures (Micromegas)** detectors were introduced by G. Charpak and Y. Giomataris in 1996 [82]. Micromegas are gas detectors which use fine structure electrodes to create high electrical fields necessary for gas amplification processes. Micromegas measure X-rays photons and other charged particles which ionize the gas. The number of produced electron ion pairs are related to the photon's energy.

The Micromegas technology has been used at CAST for more than ten years. During this period, the detectors have been upgraded, reducing further and further the background level. The latest Micromegas results are presented in [67] and were taken between 2013 and 2015. In 2019 the micro bulk Micromegas detector was installed again at CAST and has been taking data since then. The main goals of this setup are a better understanding of the background limitations of micro bulk detectors, improving the statistics of the previous run, and comparing the background suppression of a Xenon gas mixture inside the detector to the usual Argon mixture. This pathfinder project also provides operational and technical experience for the future IAXO experiment.

One of the major drawbacks of the Micromegas detectors is the readout plane. **The Integrated Grid (InGrid)** technology, also known as GridPix technology, integrate all necessary amplifiers and readout electronics into the readout plane. The Micromegas stage is directly produced on top of the pixel chip to create an almost perfect alignment between the grid holes and the pixel.

An InGrid type detector was built and installed at CAST in 2017 and took data during the 2018 run. This type of technology is still in constant improvement, and CAST was one of the first experiments that used it for data analysis. The InGrid detector used in 2018 is a pathfinder experiment to gain valuable experience and implement this technology in the IAXO magnet.

Apart from the solar axion search performed with the Micromegas and Ingrid detectors, the solar tracking of CAST is also used to search for Chameleons. The Chameleon particle was proposed by Weltman and Koury [83] in 2003, and it is considered a Weakly Interactive

Slim Particle (WISP). Chameleons result from a scalar field introduced to explain the nature of dark energy in the  $\Lambda$ CDM model. Different from other massless scalar fields, the chameleon field acquires a mass that depends on the local matter density (thus the name chameleons).

Chameleons can couple to two photons similar to the axions (see figure 15) for which a magnetic field is needed. However, they can also couple directly to ordinary matter. The **Kinetic WISP (KWISP)** detector installed at CAST is trying to find a chameleon signal using the bounce of the chameleon wave at the boundary between different media [84].

The KWISP detector uses a membrane inside an optical interferometer to measure the net force of an incoming chameleon produced at the Sun. Similar to the helioscope experiments, KWISP has to track the Sun.

#### 4.4 The International AXion Observatory (IAXO)

Despite all the technological improvements on the X-rays detectors, the CAST sensitivity to solar axions is heavily limited by the combination of its volume and magnetic field (see equation 28). The IAXO experimental design consists of a superconductive eight coils toroidal magnet, and each bore will have a 60 cm diameter. The end of each bore will be equipped with X-ray optics and detectors. The intended average magnetic field is 2.5 T and the desired length is 20 m. The IAXO experiment has the potential to be more than one order of magnitude better than the current CAST limit. Figures 19 and 20 show a conceptual design and the prospects for the sensitivity, more details can be found in [69,85]. Currently, an intermediate stage known as BabyIAXO is already under design [86]. Future RADES cavities are envisioned to be installed in the BabyIAXO and IAXO bores (see section 8).

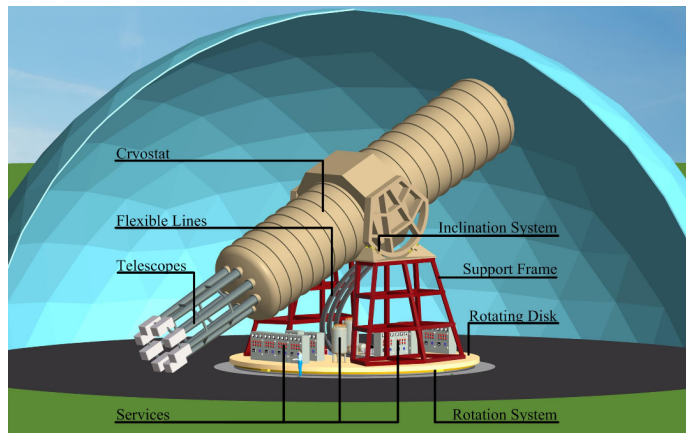


Figure 19: Drawing for the IAXO experimental setup. Figure taken from [69].



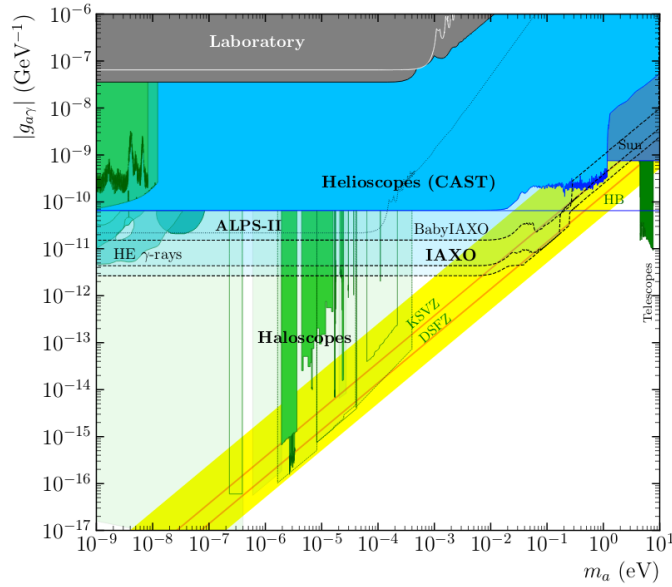


Figure 20: Prospects for the sensitivity of the IAXO experiment for the axion-photon coupling compare to CAST and other experiments. Figure taken from [86].

## 4.5 Haloscope experiment

The axion search done for this work is based on the haloscope type method. This idea was introduced by Pierre Sikivie in 1983 [87] and relies on the assumption that axions were produced in the early universe and therefore are present everywhere. A haloscope experiment uses a high-quality factor cryogenic microwave cavity immersed in a strong magnetic field and connected to a low noise receiver chain to search for relic axions in our local dark matter halo. In the presence of a strong magnetic field, the relic axions transform into photons. The power deposited in the cavity by the axion will be enhanced if the cavity's resonance frequency corresponds to the frequency of the axion mass. Figure 21 shows a typical haloscope experiment.

### 4.5.1 Power deposited in a haloscope

In the presence of a time-varying axion field  $a(t)$  and a spatially constant strong magnetic field along the  $z$ -axis  $B_0\hat{z}$ , the modified Maxwell's equations are given by:

$$\nabla \cdot \mathbf{E} = 0 \quad (29)$$

$$\nabla \times \mathbf{B} - \dot{\mathbf{E}} = -g_{a\gamma}\dot{a}(t)B_0\hat{z} \quad (30)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (31)$$

$$\nabla \times \mathbf{E} + \dot{\mathbf{B}} = 0 \quad (32)$$

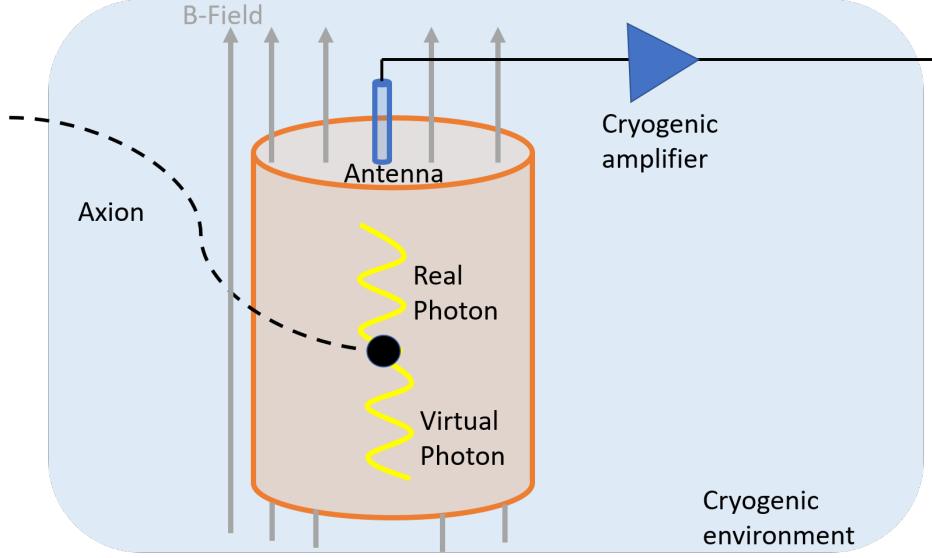


Figure 21: Sketch of a typical haloscope experiment. A resonant cavity (orange) is embedded in a strong magnetic field. A pickup antenna is used to extract the signal from the cavity and a cryogenic amplifier is used to amplify the signal before it arrives to a data acquisition system.

Taking the time derivative of equation 30 gives:

$$\nabla \times \dot{\mathbf{B}} - \ddot{\mathbf{E}} = g_{a\gamma} \ddot{a}(t) B_0 \hat{\mathbf{z}}, \quad (33)$$

plugging equation 32 into equation 33, one obtains:

$$-\nabla \times \nabla \times \mathbf{E} - \ddot{\mathbf{E}} = g_{a\gamma} \ddot{a}(t) B_0 \hat{\mathbf{z}}. \quad (34)$$

Moreover, notice that using equation 29:

$$-\nabla \times \nabla \times \mathbf{E} = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\nabla^2 \mathbf{E}, \quad (35)$$

from this follows that:

$$\nabla^2 \mathbf{E} - \ddot{\mathbf{E}} = g_{a\gamma} \ddot{a}(t) B_0 \hat{\mathbf{z}}. \quad (36)$$

The  $\mathbf{E}$  and  $\mathbf{B}$  can be expanded in a complete set of orthogonal modes  $\mathbf{e}_m$  and  $\mathbf{b}_m$  satisfying the Poisson equation. Here, we are going to focus on  $\mathbf{e}_m$ :

$$\nabla^2 \mathbf{e}_m(\mathbf{x}) + \omega_m^2 \mathbf{e}_m(\mathbf{x}) = 0, \quad (37)$$

where  $\omega_m$  is the characteristic eigenfrequency. The modes can be normalized with:

$$\int_V d^3\mathbf{x} \mathbf{e}_m \cdot \mathbf{e}_n = V \delta_{mn}, \quad (38)$$

where  $V$  is the cavity volume. Writing the electric field in this new basis:

$$\mathbf{E} = \sum_m E_m(t) \mathbf{e}_m(\mathbf{x}) \quad (39)$$

and plugging this in equation 36 gives the time evolution of the amplitude projected in the  $m$ -th mode:

$$\ddot{E}_m + \omega_m^2 E_m + \Gamma_m \dot{E}_m = -g_{a\gamma} \ddot{a}(t) B_0 G_m, \quad (40)$$

where the energy losses are parametrized by  $\Gamma_m$  and the geometric factor  $G_m$  is defined as:

$$G_m = \frac{1}{B_0 V} \int_V d^3 \mathbf{x} B_0 \hat{\mathbf{z}} \cdot \mathbf{e}_m, \quad (41)$$

notice how the magnetic field has to be parallel to the desired electric field mode to be excited by the axion field. Using the Fourier transformation, equation 40 can be expressed in the frequency ( $\omega$ ) domain:

$$\begin{aligned} (\omega_m^2 - \omega^2 + i\omega\Gamma_m)E_m(\omega) &= -g_{a\gamma}B_0G_m\omega^2a(\omega) \\ E_m(\omega) &= -g_{a\gamma}B_0G_m \frac{\omega^2a(\omega)}{(\omega_m^2 - \omega^2 + i\omega\Gamma_m)}. \end{aligned} \quad (42)$$

When excited by an axion field of the form,  $a = a_0 e^{-i\omega t}$ , the E-field amplitude approaches the steady state-solution and the total electromagnetic energy stored in the cavity splits in a sum over the modes:

$$U = \int d^3 \mathbf{x} \frac{1}{2} (|\mathbf{E}|^2 + |\mathbf{B}|^2) = \sum_m \frac{1}{2} |E_m|^2 \left( \frac{\omega^2 + \omega_m^2}{2\omega^2} \right) = \sum_m U_m. \quad (43)$$

The power of a mode can be measured by a coupled antenna. However, this antenna contributes to the total losses, which will be defined as:

$$\Gamma_m = \Gamma_m^c + \Gamma_m^r, \quad (44)$$

where  $\Gamma_m^c$  are the cavity losses and  $\Gamma_m^r$  are the antenna losses. The signal power is then given by:

$$P = \Gamma_m^r U_m = \Gamma_m^r \frac{|g_{a\gamma}B_0G_m a_0|^2 V}{2} \frac{\omega^4}{(\omega_m^2 - \omega^2)^2 + (\omega\Gamma_m)^2}. \quad (45)$$

This is the most general form for the signal power. However, when working with an experiment, this formula can be simplified to parameters commonly measured in radio frequency (RF) experiments. The quality factor of the resonator can be defined as the energy loss per oscillation cycle of the mode  $m$ :

$$Q_m = \frac{\omega_m}{\Gamma_m}. \quad (46)$$

Furthermore, we will concentrate in one particular resonant mode ( $\omega_c$ ), where  $\omega \sim \omega_c \sim m_a$  (see section 5.1) and  $Q_m \rightarrow Q_L$ .  $Q_L$  is known as the loaded Q-value and it is defined as:

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_r}, \quad (47)$$

where  $Q_0$  is the unloaded quality factor, limited by the nonzero resistance of the cavity walls, and  $Q_r$  parameterizes the antenna coupling. The coupling  $\beta$  between the resonant cavity and the antenna (see figure 21) can be defined as:

$$\beta = \frac{Q_0}{Q_r}, \quad (48)$$

from this follows that:

$$Q_L = \frac{Q_0}{1 + \beta}. \quad (49)$$

Finally, the local axion density can be written as:

$$\rho_a = \frac{1}{2} a_0^2 m_a^2. \quad (50)$$

Plugging all these terms in equation 45 gives:

$$P_a = g_{a\gamma}^2 \rho_{\text{DM}} \frac{\beta}{1 + \beta} \frac{1}{m_a} B^2 V Q_L G^2, \quad (51)$$

where it was assumed that axions constitute all the DM density.

#### 4.5.2 Properties of the axion CDM in the Milky Way

Any terrestrial experiment trying to detect relic axions in the Milky Way needs a model that tries to quantify the distribution of axions within the galaxy. The most common model adopted by the axion search experiments is known as the isothermal halo sphere. This model assumes that the galactic halo is virialized, meaning that the system is settled in an equilibrium state where the virial theorem relates its time-averaged kinetic and potential energy. Based on the observations of the velocity of the galaxy rotation curves (see section 3.1) the expected mass distribution  $M(r)$  contained within a sphere of radius  $r$  scales with  $M(r) \sim r$ , which implies that the mass density  $\rho(r) \propto r^{-2}$ . The isothermal halo sphere model is imperfect because it has a nonphysical value at  $r = 0$  and an infinite mass at  $r = \infty$ . The simplest solution is to ignore the behavior at large  $r$  because it does not play a role in the terrestrial experiments and to adopt a pseudo isothermal sphere, where the halo approaches a constant density  $\rho_c$  at radii smaller than a given radius ( $r_c$ ) [88]:

$$\rho(r) = \frac{\rho_c}{1 + (r/r_c)^2}. \quad (52)$$

Applying the pseudo isothermal model to the Milky Way results in  $r_c \approx 3$  kpc. The Earth is located at  $r \approx 8$  kpc from the galactic center, sufficiently away from the core to assume an isothermal halo sphere behavior safely. For an isothermal sphere model, the velocity distribution of the axion DM particles around the local circular velocity  $v_c$  (220 km/s) in the galactic rest frames follows a Maxwell-Boltzmann distribution. The velocity distribution is defined as [89]:

$$f(K) = \frac{2}{\sqrt{\pi}} \sqrt{K} \left( \frac{3}{m_a \langle v^2 \rangle} \right)^{3/2} \exp \left( -\frac{3K}{m_a \langle v^2 \rangle} \right), \quad (53)$$

where  $K$  is the kinetic energy and  $\langle v^2 \rangle = 3v_c^2/2 = (270 \text{ km/s})^2$  is the second moment of the Maxwell-Boltzmann distribution also known as the virial velocity ( $v_r$ ).

There is no consensus in the community for the value of the local DM density  $\rho_{\text{DM}}$ , and some experimental results can be found in [90]. The WIMPs experiments use  $\rho_{\text{DM}} = 0.3 \text{ GeV/cm}^3$  and the axion experiments use  $\rho_{\text{DM}} = 0.45 \text{ GeV/cm}^3$ . To compare the results of this work to other axion experiments, the later value will be used throughout this thesis.

There are other models which describe the distribution of axions within the galaxy, like the Navarro-Frenk-White profile [91], that has a milder  $r$ -dependency compared to the isothermal sphere and fall off faster at larger  $r$  or the SM++ model [92], which takes into account anisotropies in the halo. However, there are not going to be considered in this thesis.

Other axion properties can be derived, assuming a value for the local dark matter density and the virial velocity. The de Broglie wavelength for the axion is defined as:

$$\lambda_a = \frac{h}{m_a v_r} \sim 45 \text{ m}, \quad (54)$$

for  $m_a = 30 \mu\text{eV}$ . The de Broglie length and the coherence length (a region where the axion field can be regarded as constant) are related by a factor of order 1 and will be much larger than the scale of a laboratory experiment. For simplicity,  $\lambda_a$  will be used as the coherence length. The coherence time quantifies the time it takes for a detector (e.g. a resonant cavity) to move into a region of the oscillating axion field where the oscillations are considerably out of phase:

$$\tau_a = \frac{\lambda_a}{v_r} \sim 170 \mu\text{s}. \quad (55)$$

For the design of a resonant cavity, it is helpful to express the coherence time as a quality factor. The "axion quality factor"  $Q_a$  can be defined as:

$$Q_a = \frac{m_a \tau_a c^2}{h} = \frac{c^2}{v_r^2} \sim 10^6, \quad (56)$$

which is independent of the axion mass.  $Q_a$  describes the sharpness of the axion peak in the frequency domain.

#### 4.5.3 Detection of an axion signal above the noise floor

To have any chance of detecting an axion signal, one has first to quantify the noise floor of the receiver chain. The Johnson-Nyquist noise or thermal noise [93] is generated by free electric carriers (usually electrons) in an imperfect conductor due to thermal agitation. In the case of the haloscope experiment, thermal noise will be produced by the resistance of the cavity walls as well as the electronics of the receiver chain (see section 6.8). In general, the noise power coming from thermal noise for a frequency bandwidth ( $\Delta\nu$ ) is given by:

$$P_N = k_b T_{\text{sys}} \Delta\nu, \quad (57)$$

where  $k_b$  is the Boltzmann constant and  $T_{\text{sys}}$  is the sum of the physical temperature of the cavity and the noise temperature of the receiver chain. Let us compare the expected noise power at room temperature with the expected KSVZ axion power. For  $T_{\text{sys}} = 300 \text{ K}$  and

$$\Delta\nu = \Delta\nu_a = \frac{1}{\tau_a} = \frac{\nu_a}{Q_a} \sim 6 \text{ kHz}, \quad (58)$$

the noise power is  $P_N \approx 2 \times 10^{-17} \text{ W}$ . Plugging the typical values of the RADES detector (see table 9 in section 6.9) in equation 51, the expected power is  $P_a^{\text{KSVZ}} \approx 1 \times 10^{-25} \text{ W}$ . The axion power is a factor of  $10^8$  smaller than the noise power! Having the detector in a cryogenic system (e.g., at  $T_{\text{sys}} = 2 \text{ K}$ ) certainly improves the noise power but is not enough to compensate for the huge difference. Nevertheless, it is still possible to detect an axion with a haloscope detector. The signal to noise ratio (SNR) is defined as:

$$\text{SNR} = \frac{P_a}{\sigma_{P_N}}, \quad (59)$$

where  $\sigma_{P_N}$  is the uncertainty of the noise power. The assumption that the noise power is made of purely Johnson-Nyquist noise implies that the noise power follows a Gaussian distribution. From the formula of the standard deviation of a Gaussian distribution,  $\sigma_{P_N}$  can be defined as:

$$\sigma_{P_N} = \sqrt{\frac{2}{n-1}} k_b T_{\text{sys}} \Delta\nu, \quad (60)$$

where  $n$  is the number of independent samples. For a sufficiently large  $n$ ,  $n - 1 \approx n$ . Using the Nyquist sampling theorem, one can express  $n$  as  $n = 2\Delta\nu t$ , where  $t$  is the total integration time. Plugging this into equation 60 gives:

$$\sigma_{P_N} = \frac{k_b T_{sys} \Delta\nu}{\sqrt{\Delta\nu t}}, \quad (61)$$

Inserting 61 into 59 gives the Dicke radiometer equation [94]:

$$\text{SNR} = \frac{P_a \sqrt{t \Delta\nu}}{P_N}. \quad (62)$$

Using equations 62 and 51 one can estimate the sensitivity of the axion search given the external parameters and imposing an SNR of 5. Some experiments [95] are already working in the mK regime. Further developments have also been made in the cryogenic amplifier technology to decrease the noise temperature further and reach the quantum noise limit. However, that is not the only parameter that can be improved to achieve a better sensitivity (see equation 51). The detector can be made, for example, of superconducting material to increase  $Q_L$ , or bigger and stronger magnets can be constructed. The RADES detector will focus on designing a cavity that can in principle be escalated in length, thus increasing its volume and taking advantage of the long bore of the CAST magnet.

## 4.6 Evolution of haloscope detectors

The haloscope technique introduced in section 4.5 is a well-established mechanism that has been used for relic axion searches for more than 30 years. The first haloscope experiments were built shortly after Sikivie's proposal in 1983. The Rochester-Brookhaven-Fermilab collaboration (RBF) and the University of Florida (UF) were the first groups to do a proof of principle experiment, both of them used a cavity resonating at  $\nu_a \sim 1.2 \text{ GHz}$  ( $m_a \sim 5 \mu\text{eV}$ ).

The RBF experiment used the  $\text{TM}_{010}$  mode of a  $V = 101$  cylindrical copper cavity with  $Q_0 \approx 1.9 \times 10^5$  and a solenoid magnet with central field  $B_0 = 6 \text{ T}$ . The tuning was accomplished by the insertion of a dielectric rod. The cavity was immersed in liquid helium, the helium bath was at  $T \approx 4 \text{ K}$  and the amplifier had a noise temperature of  $T_{\text{amp}} \sim 12 \text{ K}$  [96].

The UF experiment worked with the  $\text{TM}_{010}$  mode of a  $V = 71$  cylindrical copper cavity with  $Q_0 \approx 1.5 \times 10^5$  and a solenoid magnet with central field  $B_0 = 7.5 \text{ T}$  and a similar tuning mechanism. The UF experiment worked at a  $T \approx 2 \text{ K}$  and used a high-electron mobility transistor (HEMT) amplifier with a noise temperature of  $T_{\text{amp}} \sim 3 \text{ K}$  [97].

Both experiments achieved a sensitivity to the axion-photon coupling of a factor 10 above the axion models, an outstanding result for the first axion haloscope detectors. However, it was clear that the axion models were out of reach for the technology of those years. The second-generation haloscope experiment in the Lawrence Livermore National Lab is the Axion Dark Matter eXperiment (ADMX) collaboration. The first phase of ADMX [98] used a 7 T magnet with a 60 cm bore diameter. It also used the  $\text{TM}_{010}$  mode of a cylindrical copper cavity with  $Q_0 \approx 1.5 \times 10^5$  immersed in a helium bath at  $T \approx 1 - 2 \text{ K}$ , and a HEMT amplifier noise temperature of  $T_{\text{amp}} \sim 2 \text{ K}$ .

There are two main differences between ADMX phase 1 and the previous detectors. The first one is that the cavity volume is much larger  $V = 2001$  which corresponds to a lower axion frequency (460 MHz). The second difference is that the tuning rod was made of metal instead of dielectric material. The inclusion of the movable metallic rod inside the cavity allowed the search for higher frequencies ( $700 \text{ MHz} < \nu_a < 800 \text{ MHz}$ ) compared to the empty

cavity. Contrary to a dielectric rod that tunes down the frequency. The ADMX collaboration was the first one to reach the KSVZ sensitivity [98].

The next significant technological upgrade of the ADMX experiment included Superconducting QUantum Interference Devices (SQUID) amplifiers [99]. The DC SQUIDs work as a magnetic flux-to-voltage converter using two Josephson junctions. They can work as amplifier if a coil or a tunable microstrip resonator is used to convert the input voltage into a flux. The physical process behind the amplification of the SQUIDs allows the noise temperature to be improved roughly linearly with  $T$  even at temperatures below 1 K, which is not the case for the HEMT amplifiers. The major drawback of the SQUID amplifiers is their sensitivity to magnetic fields. A strong magnetic field is needed to enhance the axion-photon coupling. For that reason, great care has to be taken to shield the SQUID amplifier from this magnetic field. Moreover, to take advantage of the benefits of these amplifiers, a more complex cryogenic system is needed to reduce the temperature to the mK regime.

Between 2008 and 2010 ADMX made a proof of principle run using a SQUID amplifier at  $T \approx 2$  K. They extended their exclusion region with this run to  $460 < \nu_a < 892$  MHz with KSVZ sensitivity. Finally, the last major ADMX upgrade was the inclusion of a dilution refrigerator which allowed the physical temperature of the cavity to go down to 150 mK and the noise temperature of the SQUID amplifier was  $\approx 300$  mK. With these conditions, ADMX was the first experiment to reach the DFSZ sensitivity (in 2018) for a range of  $643 < \nu_a < 790$  MHz [100, 101].

## 4.7 Challenges of haloscope searches at higher frequencies

The next step for the new generation of haloscope detectors is to scan higher axion masses. Naively speaking, to go to higher frequencies (masses) using a cylindrical cavity similar to the ADMX cavity, the radius of the cavity has to be reduced. Since the axion power goes linearly with the volume (see equation 51), a cylindrical cavity resonating at higher frequencies will have low sensitivity compared to the low-frequency cavities.

The next challenge is the application of SQUID amplifiers at higher frequencies. Recently, there has been much development in this field, but there is not much literature or results of SQUID amplifiers working at frequencies above, e.g., 1 GHz. For higher temperatures, this limits the minimum value of the noise power (see equation 57) to the noise temperature of the HEMT amplifiers. A way to solve this problem is to use Josephson Parametric Amplifiers (JPAs), whose operation also relies on physics based on the Josephson effect. However, unlike the SQUID they are designed to work in the 2-12 GHz range. A recent paper [95] shows the results of a squeezed-state receiver JPA amplifier used in a haloscope experiment with a cavity resonating at around 4 GHz.

One more caveat to the higher frequency is the skin depth. Within a conductor, the current density decreases exponentially with greater depth in the conductor. The current is largest near the surface ("skin") of the conductor. Most of the current flows in the region between the surface and the so-called skin depth ( $\delta$ ), which is frequency-dependent and decreases at higher frequencies. A smaller skin depth corresponds to an increment of the unloaded quality factor of the cavity ( $Q_0$ ).

The skin depth is inversely proportional to the conductivity. At low temperatures, the conductivity of the material improves, which also enhances  $Q_0$ . However, at higher frequencies and low temperatures, the value of  $\delta$  will drop below the mean free path of the electron. In this anomalous skin effect regime, the skin depth does not depend on the conductivity and the  $Q_0$  will not improve as we decrease the temperature. This is known as anomalous skin effect [102]. The  $Q_0$  values at higher frequencies (e.g. between 8-9 GHz) are smaller than the ones of the low-frequency regime.

Despite all these challenges, numerous groups have developed and carried out measurements at masses above the ADMX mass limit using new cavity geometries. Some of these groups are: HAYSTAC [95], CAPP [103, 104], ADMX-SIDECAR [105], QUAX [106, 107], ORGAN [108], etc. There also other experiments like MADMAX [109] or BRASS [110] that are under design and development that will use complementary techniques to search at higher frequencies.

In recent years, CAST has added haloscope type detectors to its scientific program. One of them is the CAST-CAPP detector, which consists of 4 tunable rectangular stainless steel cavities of dimensions  $25 \times 24 \times 390 \text{ mm}^3$  electroplated with  $\sim 30 \mu\text{m}$  copper. The cavities are longitudinally split into two identical halves (see top panel of figure 22). To tune the cavity, 2 long parallel sapphire plates are mounted in a frame with a gear that rotates and moves the plates towards or away from each other (see bottom panel of figure 22). The sapphire plates together with the frame are installed inside the empty cavity and then the two halves are screwed together. The gear is moved using a piezoelectric device. Tuning range is  $\sim 400 \text{ MHz}$  (5.1 GHz to 5.5 GHz). The CAPP-CAST group takes advantage of the large volume of the CAST bore to improve their sensitivity. The volume increment is achieved by phase-matching the four cavities.

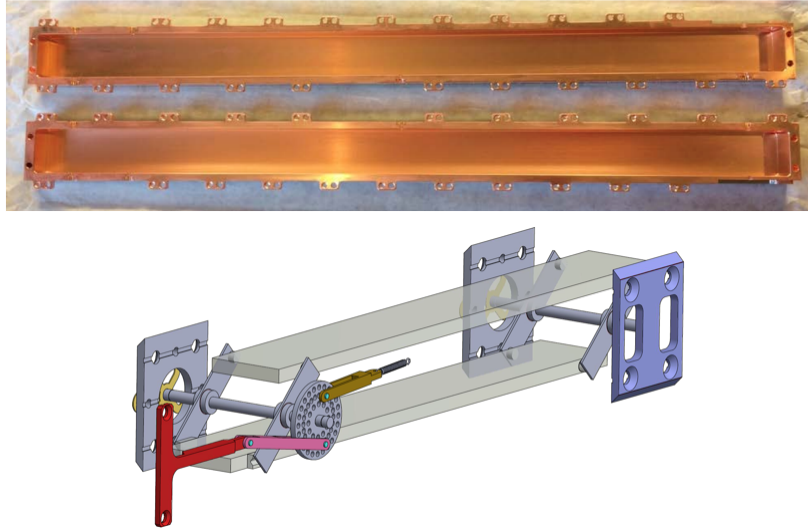


Figure 22: Top: Photo of the two halves of one CAPP-CAST cavity before it was assembled. Bottom: Drawing of the tuning mechanism of the CAST-CAPP cavities, the sapphire plates are represented in gray. They are mounted in the blue frame, which has a gear that rotates and allows the movement of the plates. The gear is attached to two pieces of non-magnetic material (in red) that are moved using a piezo electric device. Figure courtesy of M. Maroudas.



## 5 The RADES haloscope detector

The Relic Axion Detector Exploratory Setup (RADES) group proposed new radio frequency (RF) resonant cavity designs to tackle a mass range in the  $33 \mu\text{eV}$  region (corresponding to 8 GHz). The scaling of the volume and Q factor of conventional haloscope geometries is not favorable for a sensitive axion search at these frequencies. The RADES idea pursued here is to concatenate  $N$  cavities together (where  $N$  denotes the number of cavities) using irises and exploit the large volume of the CAST magnet cold bores. By joining these sub cavities through irises, the RADES detector can increase in volume while at the same time have a resonance frequency in the 8-9 GHz frequency range.

### 5.1 RADES cavity structure general formalism

The general formalism described here is based on the ideas presented in [111]. The modification of Maxwell's equations due to the presence of an axion field was introduced in section 4.5.1, where the amplitude of the mode is given by equation 42. For now, the focus will be on the case where all cavities have one mode close to the common central resonance frequency, and that mode is well separated from other resonance modes. Let us consider the case of  $N$  cavities joined together by irises. Equation 42 is now a system of coupled equations for the amplitude given by:

$$\begin{aligned} (\omega^2 - \Omega_1^2)E_1 &= K_{12}E_2 - g_{a\gamma}B_0G_1\omega^2a(\omega) \\ (\omega^2 - \Omega_2^2)E_2 &= K_{21}E_1 + K_{23}E_3 - g_{a\gamma}B_0G_2\omega^2a(\omega) \\ &\dots \\ (\omega^2 - \Omega_N^2)E_N &= K_{N,N-1}E_{N-1} - g_{a\gamma}B_0G_N\omega^2a(\omega), \end{aligned} \tag{63}$$

where  $\Omega_q = \omega_q^2 + i\omega_q\Gamma_q$  is the complex frequency of the  $q$ -th cavity and  $K_{q,q'}$  is the linear coupling between the cavities. The subindex  $q$  will be used to label the properties of the individual cavities, while the subindex  $i$  will be used for the global solution of the filter array<sup>2</sup>. In the vector notation, this equation can be simplified to:

$$(\omega^2\mathbb{1} - \mathbb{M})\vec{E} = -g_{A\gamma}B_eA_0\omega^2\vec{G} = \vec{J}_a. \tag{64}$$

The  $N \times N$  complex matrix  $\mathbb{M}$  will determine the properties of our detector. By just joining the cavities to their nearest neighbor, the matrix becomes tridiagonal:

$$\mathbb{M} = \begin{pmatrix} \Omega_1^2 & K_{12} & 0 & 0 & 0 & 0 \\ K_{21} & \Omega_2^2 & K_{23} & 0 & 0 & 0 \\ 0 & K_{32} & \Omega_3^2 & K_{34} & 0 & 0 \\ 0 & 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & \ddots & \ddots & \ddots \\ 0 & 0 & 0 & 0 & K_{N,N-1} & \Omega_N^2 \end{pmatrix}. \tag{65}$$

The electromagnetic field modes ( $e_i$ ) of the filter array corresponds to the  $N$  eigenvectors of the  $\mathbb{M}$  and satisfy:

$$\mathbb{M}\vec{e}_i = \lambda_i\vec{e}_i. \tag{66}$$

---

<sup>2</sup>In general we will denote the whole structure (cavities plus irises) the "filter array" to distinguish this from the individual cavities.

Here, the eigenvalue  $\lambda_i$  corresponds to the square of the characteristic resonant frequency of the filter array ( $\lambda_i = \omega_i^2$ ).

Equation 64 can be solved for the electric field using the eigenmodes of the cavity:

$$R^{-1}\mathbb{M}R = \mathbb{D}^2 = \text{diag}(\omega_1^2, \omega_2^2, \dots, \omega_N^2), \quad (67)$$

where  $R$  denotes the matrix that diagonalises  $\mathbb{M}$ . Plugging this in equation 64 gives:

$$\vec{E}_q \simeq \sum_{i \leq N} (\vec{e}_i)_q \left( \frac{\vec{e}_i \cdot \vec{J}_a}{\omega^2 - \omega_i} \right), \quad (68)$$

where  $R_{qi} = (\vec{e}_i)_q$  and  $R^{-1} \simeq R^T$  were used. Equation 68 is the extension of equation 42 for a  $N$  cavity filter. It is important to point out that the fundamental mode has split into  $N$  modes of the filter array. The electric field at each cavity ( $\vec{E}_q$ ) is a linear superposition of the electric fields of the filter array ( $\vec{e}_i$ ). Every mode of the array will be weighted by the resonator response  $(\omega^2 - \omega_i)^{-1}$  and the corresponding geometric factor. Depending on the signs of these factors, the modes can have different phases and cancel each other (e.g., in figure 23).

Let us analyze how the other properties of a cavity are generalized for an  $N$  cavity filter. The geometric factor given in equation 41 is defined for a multi-cavity filter as:

$$G_i = \frac{\sum_q V_q \vec{E}_i \cdot \vec{G}}{V}, \quad (69)$$

where  $V$  is the sum of the cavity volumes. For a filter where the geometric factor of each cavity is similar ( $G_i \approx G_0$ ), the signal will be boosted by the increase in volume, in this case by a factor of  $NV_q$ .

Moreover, the intrinsic losses of each cavity are going to be very similar ( $\Gamma_c^q \approx \Gamma_0^q$ ) in the case where the cavities have similar dimensions. Therefore, the unloaded quality factor of the filter array will be identical to that of each cavity.

With this, similar to equation 45 the power extracted from a port located in the  $q$ -th cavity can be written as:

$$P = \Gamma_q^r \frac{|g_{a\gamma} B_0 G a_0|^2 V_q}{2} \left| \sum_i (\vec{e}_i)_q \frac{\vec{e}_i \cdot \vec{J}_a}{\omega^2 - \omega_i} \right|^2. \quad (70)$$

## 5.2 The 5 sub cavities filter with inductive irises

Based on this general formalism, the RADES group started, in 2017, designing and building a proof of principle detector prototype. For the prototype, it was decided to use five cavities connected with four irises. Using equation 66 this results in:

$$\omega_i^2 \begin{pmatrix} e_{i1} \\ e_{i2} \\ e_{i3} \\ e_{i4} \\ e_{i5} \end{pmatrix} = \mathbb{M} \begin{pmatrix} e_{i1} \\ e_{i2} \\ e_{i3} \\ e_{i4} \\ e_{i5} \end{pmatrix} = \begin{pmatrix} \Omega_1^2 e_{i1} + K_{12} e_{i2} \\ K_{12} e_{i1} + \Omega_2^2 e_{i2} + K_{23} e_{i3} \\ K_{23} e_{i2} + \Omega_3^2 e_{i3} + K_{34} e_{i4} \\ K_{34} e_{i3} + \Omega_4^2 e_{i4} + K_{45} e_{i5} \\ K_{45} e_{i4} + \Omega_5^2 e_{i5} \end{pmatrix}, \quad (71)$$

where  $i$  runs from 1 to 5. From this, a system of linear equations can be constructed and is given by:

$$\begin{aligned}\omega_i^2 &= \frac{\Omega_1^2 e_{i1}}{e_{i1} - \tilde{K}_{12}^i e_{i2}} = \frac{\Omega_2^2 e_{i2}}{e_{i2} - \tilde{K}_{12}^i e_{i1} - \tilde{K}_{23}^i e_{i3}} = \frac{\Omega_3^2 e_{i3}}{e_{i3} - \tilde{K}_{23}^i e_{i2} - \tilde{K}_{34}^i e_{i4}} \\ &= \frac{\Omega_4^2 e_{i4}}{e_{i4} - \tilde{K}_{34}^i e_{i3} - \tilde{K}_{45}^i e_{i5}} = \frac{\Omega_5^2 e_{i5}}{e_{i5} - \tilde{K}_{45}^i e_{i4}},\end{aligned}\quad (72)$$

where  $\tilde{K}_{q-1,1}^i = K_{q-1,1}^i/\omega_i^2$ . It is important to emphasize that the values for the  $\Omega$ 's and  $K$ 's can be chosen by changing the dimensions of the irises and cavities in the design process. For this prototype, the simulations were focused on optimizing the geometric factor for a desired characteristic frequency. To maximize the geometric factor, the electric field of each cavity has to be parallel to the magnetic field.

There are different ways to achieve this. One of the most straightforward solutions is to make all cavities with almost equal dimensions and take all coupling coefficients to be similar. For this case, the  $\mathbb{M}$  is Toeplitz matrix [112] with identical diagonal elements and couplings. This matrix can be immediately diagonalized, and the eigenvectors and frequencies are given by [112]:

$$\vec{e}_i = \frac{1}{\sqrt{(1+N)/2}} \begin{pmatrix} \sin\left(\frac{i\pi}{N+1}1\right) \\ \sin\left(\frac{i\pi}{N+1}2\right) \\ \vdots \\ \sin\left(\frac{i\pi}{N+1}N\right) \end{pmatrix}, \quad \omega_i^2 = \Omega_q^2 + 2K \cos\left(\frac{i\pi}{N+1}\right), \quad (73)$$

where  $i$  goes from 1 to  $N$ , and  $N = 5$ . For a negative  $K$  value, the electric field in all cavities is aligned with the magnetic field for the first mode, i.e.  $\vec{e}_1$  and  $\omega_1^2$ . For this mode and a fixed coupling coefficient of  $\tilde{K}^1 = \tilde{K}_{12}^1 = \tilde{K}_{23}^1 = \tilde{K}_{34}^1 = \tilde{K}_{45}^1$ , equation 72 gives the following solution:

$$\Omega_2^2 = \Omega_3^2 = \Omega_4^2 = \omega_1^2(1 - \tilde{K}^1), \quad \Omega_1^2 = \Omega_5^2 = \omega_1^2(1 - \tilde{K}^1). \quad (74)$$

This equation indicates that all cavities have to resonate at the same frequency, except the first and last cavities that have to resonate at a slightly higher frequency.

The resonance frequency  $\omega_1$  was fixed based on the geometrical constraints of the CAST's cold bore diameter (see figure 18). A WR90 EIA standard rectangular waveguide can easily fit inside the bore. The dimensions of this waveguide were used to constrain the width  $a$  and height  $b$  of all the cavities. Based on these dimensions,  $\omega_1$  was fixed to 8.4 GHz. For a rectangular cavity, the natural frequency  $\Omega_q$  can be determined by its width ( $a$ ) and length ( $L_q$ ):

$$\Omega_q^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{L_q}\right)^2, \quad (75)$$

which holds for an isolated, perfect cavity. The presence of irises, ports, and machining limitations will introduce some perturbations to this relation. In practice, a simulation software that can numerically solve the Maxwell equations for the given geometry is needed to determine the correct value of  $L_q$  for a desired  $\omega_q$ . The CST Microwave Studio electromagnetic commercial software package [113] was used for the design and optimization of the cavity.

The same software was used to determine the dimensions of the irises. Inductive irises were selected for the first prototype because they were easier to machine. The irises were characterized by two dimensions, the width  $W$  and the length  $t$  (see left panel of figure 24). Due to machining constraints,  $t$  had to be at least 2 mm, which was the chosen value. This value also showed satisfactory results in simulations. To determine the coupling  $\tilde{K}^1$  the procedure described in section 14.2 of [114] was used. An inductive iris connected

two cavities. These cavities were "split" into single resonators by applying "Perfect Electric Conductor Wall" (PEC) or "Perfect Magnetic Conductor Wall" (PMC) boundary conditions in the central plane of the iris. The coupling could be calculated using [114]:

$$\tilde{K} = \frac{|f_e^2 - f_o^2|}{f_e^2 + f_o^2}, \quad (76)$$

where  $f_e$  is the resonant frequency for the PEC and  $f_o$  is the resonant frequency for the PMC. Several values of  $\tilde{K}$  were calculated for different values of  $W$  using CST simulations. A value of  $W = 8$  mm was selected. It provided good mode separation and was easy to machine. This  $W$  value corresponded to  $\tilde{K} = -0.0185$ . With the fixed irises dimensions, the geometric factor was optimized. The optimization was done by calculating the electric field for a parameter sweep for the lengths  $L_1$  and  $L_5$  (see left panel of figure 24).

Figure 23 shows the electric field patterns of the five modes calculated using CST, while table 1 shows the signs of the electric field in each cavity, the resonance frequency, and the geometric factor for the five modes of the filter. The signs of the electric field pattern from the CST simulations were in good agreement with the theoretical values given by equation 73.

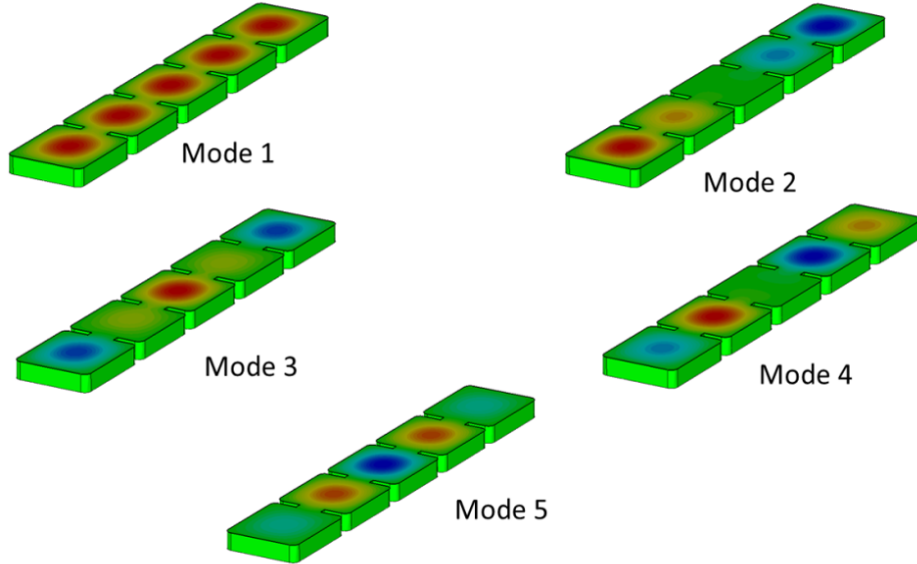


Figure 23: Electric field pattern of the 5 cavity modes, where red and blue indicate opposite directions of the electric field. The first mode, in which the electric field in all sub cavities is parallel to the external magnetic field, couples to the axion. Figure taken from [111].

| Mode | Sign of the E-field at each sub cavity | $\omega_i/2\pi$ [GHz] | $G_i^2$              |
|------|----------------------------------------|-----------------------|----------------------|
| 1    | + + + + +                              | 8.428                 | 0.65                 |
| 2    | + + 0 - -                              | 8.454                 | $3.2 \cdot 10^{-7}$  |
| 3    | - + + + -                              | 8.528                 | $8.1 \cdot 10^{-5}$  |
| 4    | - + 0 - +                              | 8.625                 | $1.6 \cdot 10^{-12}$ |
| 5    | - + - + -                              | 8.710                 | $6.4 \cdot 10^{-6}$  |

Table 1: Sign of the electric field pattern, resonance frequency and geometric factor for every mode of the filter at 2 K. Results produced using CST.

The cavity was manufactured by a standard milling process. Due to the high magnetic field of the CAST magnet and possible quenches, the cavity was manufactured with stainless

| Dimensions [mm]         | $T = 2\text{ K}$ | $T = 298\text{ K}$<br>(including a $30\text{ }\mu\text{m}$ copper coating layer) |
|-------------------------|------------------|----------------------------------------------------------------------------------|
| a                       | 22.86            | 22.99                                                                            |
| b                       | 10.16            | 10.25                                                                            |
| $L_1 = L_5$             | 26.68            | 26.82                                                                            |
| $L_2 = L_3 = L_4$       | 25.00            | 25.14                                                                            |
| $W_1 = W_2 = W_3 = W_4$ | 8.00             | 8.14                                                                             |
| t                       | 2.00             | 1.95                                                                             |

Table 2: Physical dimension at 2 K and 298 K for the 5 sub cavities filter separated by inductive irises. The copper coated layer is included for the room temperature column.

steel. Cavities made out of plain aluminum or copper have been deformed due to quenches. Two prototypes were manufactured, one from stainless steel 316L (which was slightly magnetic<sup>3</sup>) and a second one with stainless steel 316LN (the standard used in the LHC magnets). The second one was used for the data-taking period.

The cavity was coated with  $\sim 30\text{ }\mu\text{m}$  layer of copper with a  $\sim 5\text{ }\mu\text{m}$  nickel layer in between at the CERN coating lab. Even though nickel is magnetic, this small layer was needed for the coating process. Because of the better conductivity of the cavity walls due to the low residual resistance ratio (RRR) of copper, the loaded Q-value ( $Q_L$ ) of the cavity increases compare to pure stainless steel. A silver coating was ruled out because a thicker nickel layer was needed for the coating and the recommendations of the experts were to avoid this if possible.

The dimensions of the cavity were optimized in CST for a temperature of 2 K. The expansion coefficient of stainless steel [115] was used to provide the room temperature dimensions for the machining of the cavity. Table 2 shows the dimensions for the five sub cavities filter with inductive irises.

Two right-angle  $50\text{ }\Omega$  subminiature version A (SMA) coaxial ports were placed at the top center of the first and last sub cavities. One of these ports (label  $P_1$ ) was used to extract the RF signal and was expected to be critically coupled, but ended up being under coupled as explained in section 6.7. The cavity is said to be critically coupled to the receiver chain when the resistance of the cavity walls dissipates half the signal power, and the other half is coupled into the receiver. Critical coupling is needed to maximize the axion power given by equation 51.

The second port ( $P_2$ ) was used to inject an RF signal to characterize the cavity's electromagnetic properties. This port was weakly coupled to minimize power leakage, and it was terminated with a  $50\text{ }\Omega$  load during the data-taking period. The right panel of figure 24 shows the two five sub cavities filters manufactured prototypes. The right-hand side of the figure shows the stainless steel 316LN cavity before it was assembled and copper coated. The cavity has two parts, the main body, where the sub cavities and irises are located, and the cover plate. Once the copper coating is done, the cover plate and the coaxial ports are screwed into the main body, as shown in the right panel of figure 24.

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<sup>3</sup>Due to the selection of the wrong stainless steel, this cavity was used just for a first characterization of the electromagnetic properties of the filter at cryogenic temperatures.

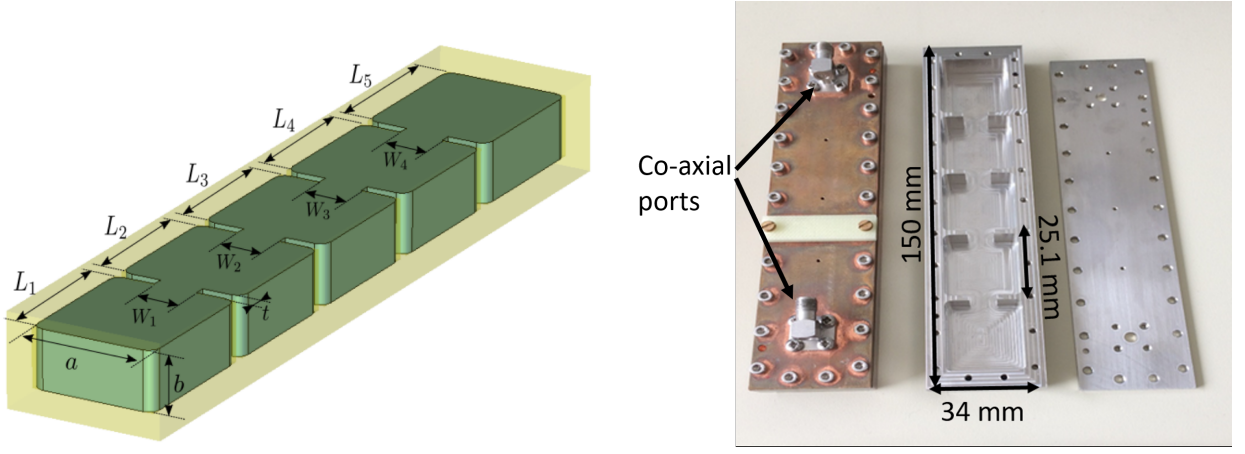


Figure 24: Left: 3D design of the 5 sub cavities filter. Taken from [111]. Right: Manufactured 5 sub cavities filter with inductive irises. On the right is the stainless steel 316LN prototype before it was copper coated. On the left is the 316L cavity after the coating and the assembly were done.

The S-parameters [116] were measured at room temperature. Figure 25 shows the measured transmission parameter ( $S_{12}$ ) in purple and compares it to the expected values given by the theoretical model introduced in section 5.1 (gray line). The red line is the axion DM power normalized to the peak. Not only an excellent agreement is seen between the general formalism and the measurement, but also the axion will couple to the first mode as expected.

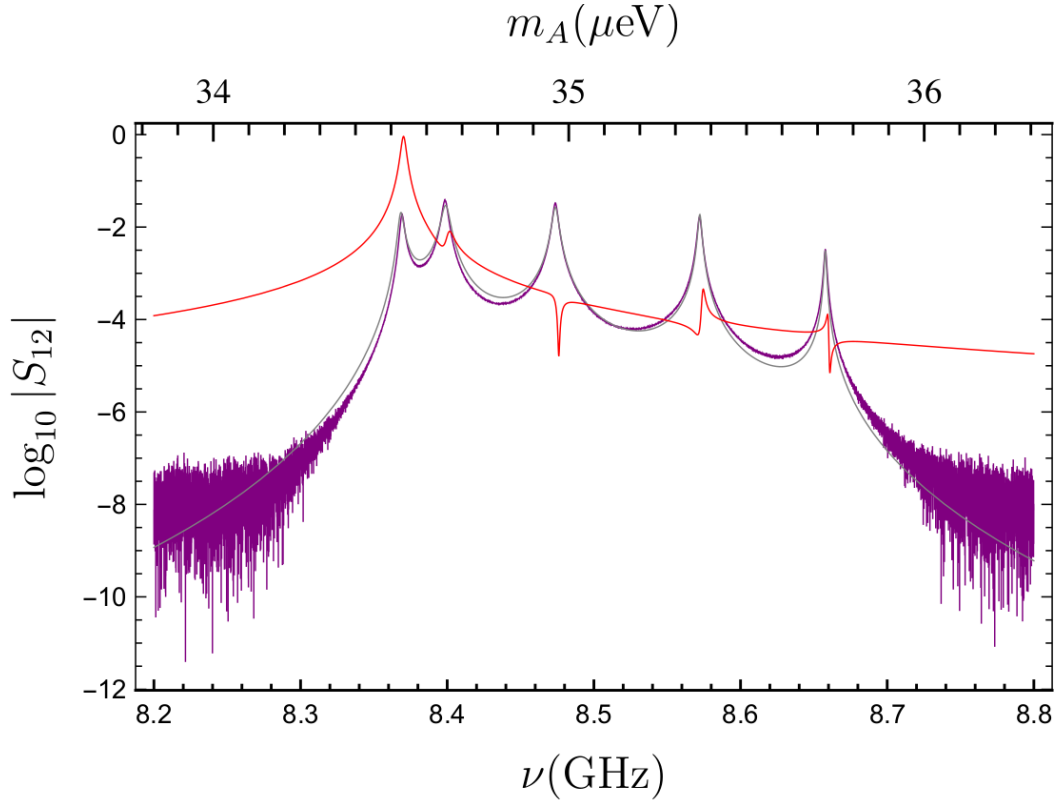


Figure 25: Measured transmission parameter ( $S_{12}$ ) at room temperature (purple line), predicted transmission parameter from the analytical model (gray line) and the axion DM power normalized to the different peaks (red line). Figure taken from [111].

### 5.3 2018 RADES setup at CAST

The CAST-RADES experimental setup was designed to search for a possible axion signal in the  $34\,\mu\text{eV}$  range. The main components of this setup are:

- Decommissioned LHC 8.8 T dipole magnet.
- RF resonant cavity.
- Cryogenic low noise amplifier (LNA).
- Semi-rigid coaxial cables and feedthroughs.
- The data acquisition (DAQ) system made of an analog and a digital stage.

Figure 26 shows the schematic representation of the CAST-RADES setup. The cavity was inserted in one of the bores of the CAST magnet in a region with a magnetic field of approximately 8.8 T and a temperature of 1.8 K. The setup had two main lines. Line 1 connected to  $P_1$  of the cavity was used to extract the signal from the cavity.  $P_1$  was connected to the cryogenic LNA using a  $\sim 1$  m semi-rigid coaxial cable (the specifications are going to be given later on in this section). The LNA had a amplification of 40 decibel (dB) and was located inside of a copper vessel in a region with almost no magnetic field ( $B \ll 0.01$  T) and a temperature of  $\sim 5$  K. The output port of the LNA was connected to one feedthrough of the cold flange via a  $\sim 20$  cm coaxial cable. A 3 m long coaxial cable connected the cold flange's feedthrough with the warm flange's feedthrough. Two copper plates were used for the thermalization of the cables. The DAQ was connected to the feedthrough of line 1 during the data-taking period.

The second line (line 2) was used to inject a signal for characterization purposes. The cabling connections were similar to line 1, except for the fact that for this line, there was no LNA, and a 6 dB attenuator was connected to the cold feedthrough.  $P_2$  was weakly coupled to avoid power leakage during the data-taking period and to reduce the noise introduced by the coaxial cable into the cavity. Figure 26 shows the configuration used for data-taking, where the warm flange feedthrough of line 2 was terminated with a  $50\,\Omega$  load. Other configurations were possible and were used for the characterization of the cavity properties (see sections 6.6, 6.7 and 6.8), e.g. connecting a vector network analyzer (VNA) to line 1 and 2 to measure the S-parameters.

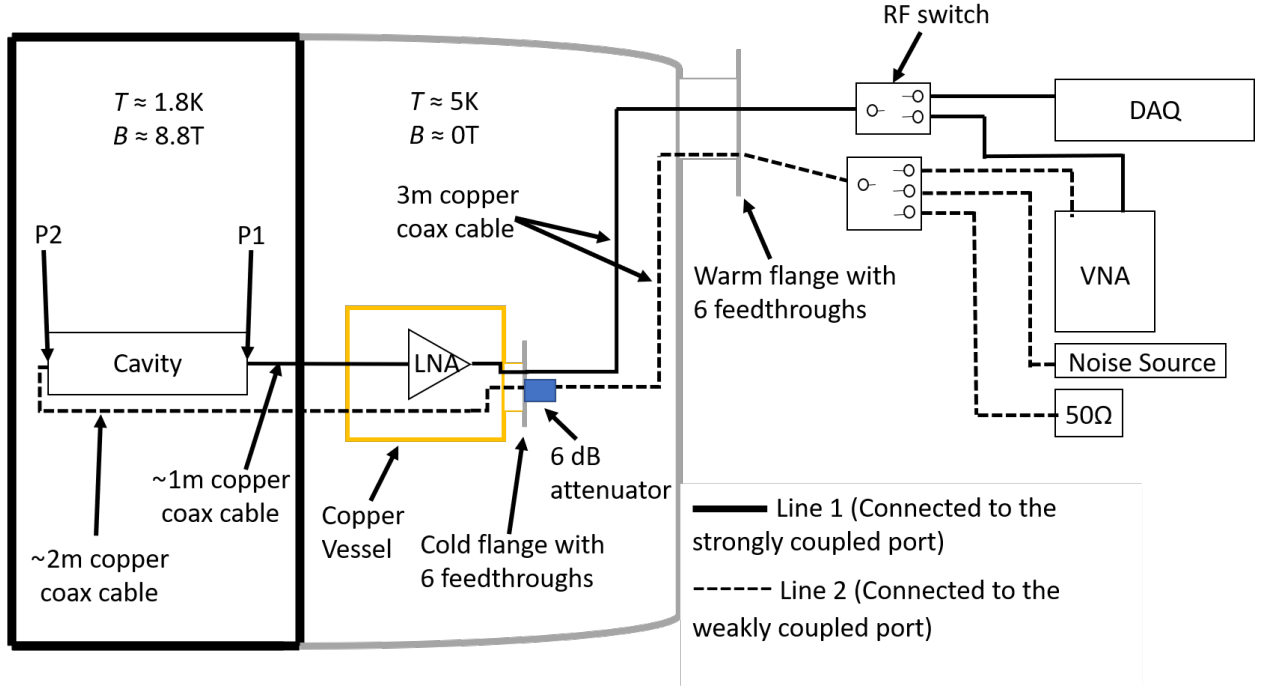


Figure 26: Schematics of the CAST-RADES setup. The cavity was located inside one of the bores of the CAST magnet in a region with  $B \approx 8.8$  T at  $T \approx 1.8$  K. The port from where the signal is extracted ( $P_1$ ) was connected through a coaxial cable to the LNA, which was located in a region with almost no magnetic field ( $B \ll 0.01$  T) at  $\approx 5$  K. The output of the LNA was connected through coaxial cables to a room temperature feedthrough. The weakly coupled port ( $P_2$ ) was located at the rear part of the cavity and was connected through coaxial cables to a room temperature feedthrough. For data taking the DAQ was connected to line 1 and a  $50\ \Omega$  load was connected to line 2. For the noise temperature measurement the noise source was connected to line 2. For coupling and Q-value measurements the VNA was connected to line 1 and 2. The VNA and noise source were available just for a single measurement during the data-taking period, so the RF switches were not needed.

### 5.3.1 Cavity holding pieces and insertion finger

The bore of the CAST magnet is  $\sim 9$  m long and to ensure a region of constant magnetic field of 8.8 T, the cavity was inserted approximately 1 m inside the bore. The cavity was attached to an insertion finger using two holding pieces, one at the rear part and one at the front part of the cavity. The holding pieces also served the purpose of protecting the cavity body during the insertion of the finger. The finger is a 1 m long hollow aluminum cylinder that was inserted together with the cavity inside the CAST magnet. Figure 27 shows the cavity already assembled with the holding pieces and the insertion finger.

Figure 27 shows the end part of the insertion finger moments before it was completely inserted into the bore. To fix the finger to the bore, three screws were used. The top screw also served as an alignment screw because it pointed in the same direction as the electric field lines of the cavity (see figure 23). Note that the electric fields in the cavity have to be aligned with the magnetic field to maximize the geometric factor. This was achieved by positioning the top screw parallel to the magnetic field lines of the CAST's cold bore.



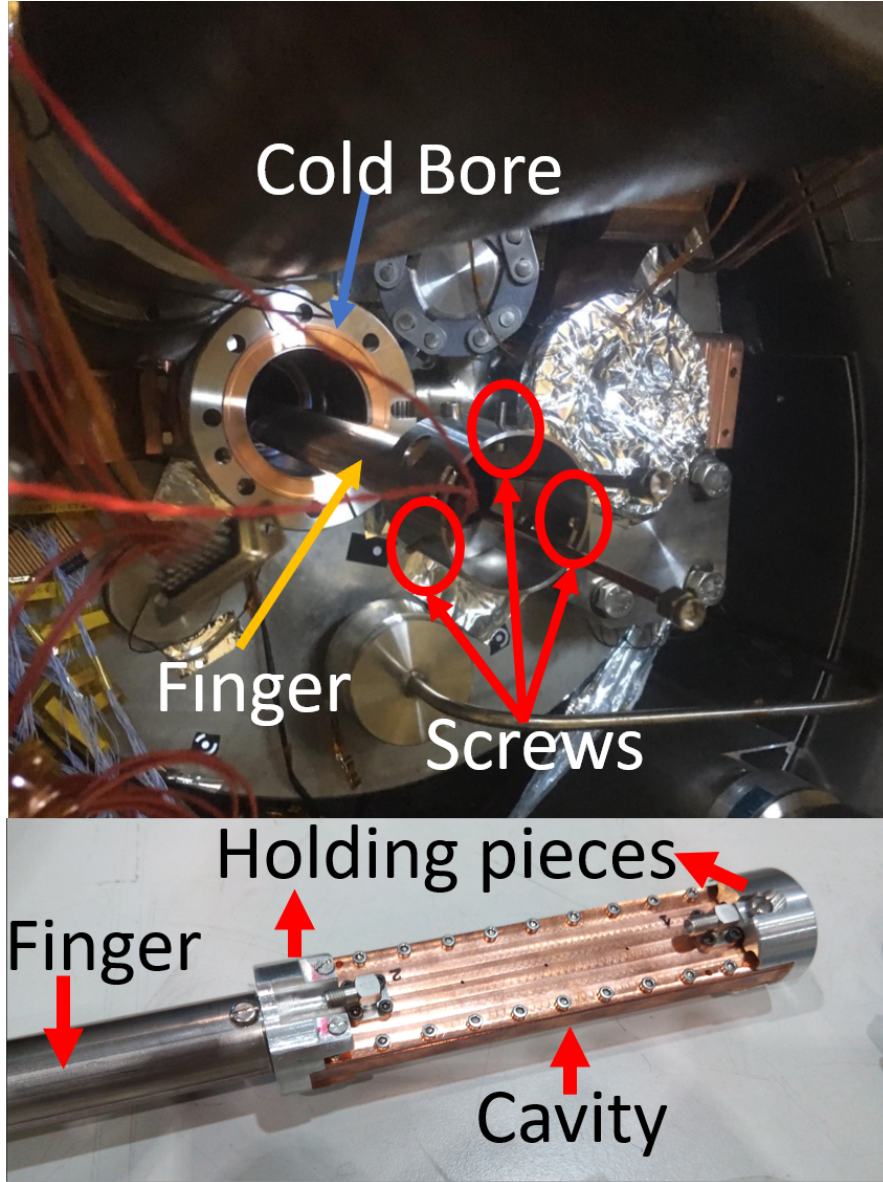


Figure 27: Top: Rear part of the insertion finger before it was completely inserted into the cold bore. Three screws were used to fix, thermalize and align the finger with the bore and the cavity. Bottom: Front part of the insertion finger, with the holding pieces and cavity.

### 5.3.2 Copper vessel and low noise amplifier (LNA)

A copper vessel is connected to the end of the cold bore (see figure 28). This vessel serves as a space where components sensitive to the magnetic can be installed in a cryogenic region ( $\sim 5$  K) with almost no magnetic field ( $B \ll 0.01$  T). Figure 28 shows the copper vessel after it was closed and connected to the cold bore. The bottom right inset of figure 28 shows the inside of the copper vessel. In this figure, two components are visible inside the vessel, the cryogenic LNA and an RF switch. The switch was installed after the 2018 data-taking period was finished. For the 2018 measurement campaign, the LNA was connected directly to  $P_1$  using a semi-rigid coaxial cable as indicated in figure 26.

The setup used a TXA 4028 X-Band LNA produced by TTI Norte [117]. It had a frequency range of 8 to 9 GHz with a gain of around 40 dB and an expected noise temperature of less than 3.7 K at 12 K. A three-stage bias (incorporated inside the DAQ) was used to supply the needed voltages and currents to the amplifier.

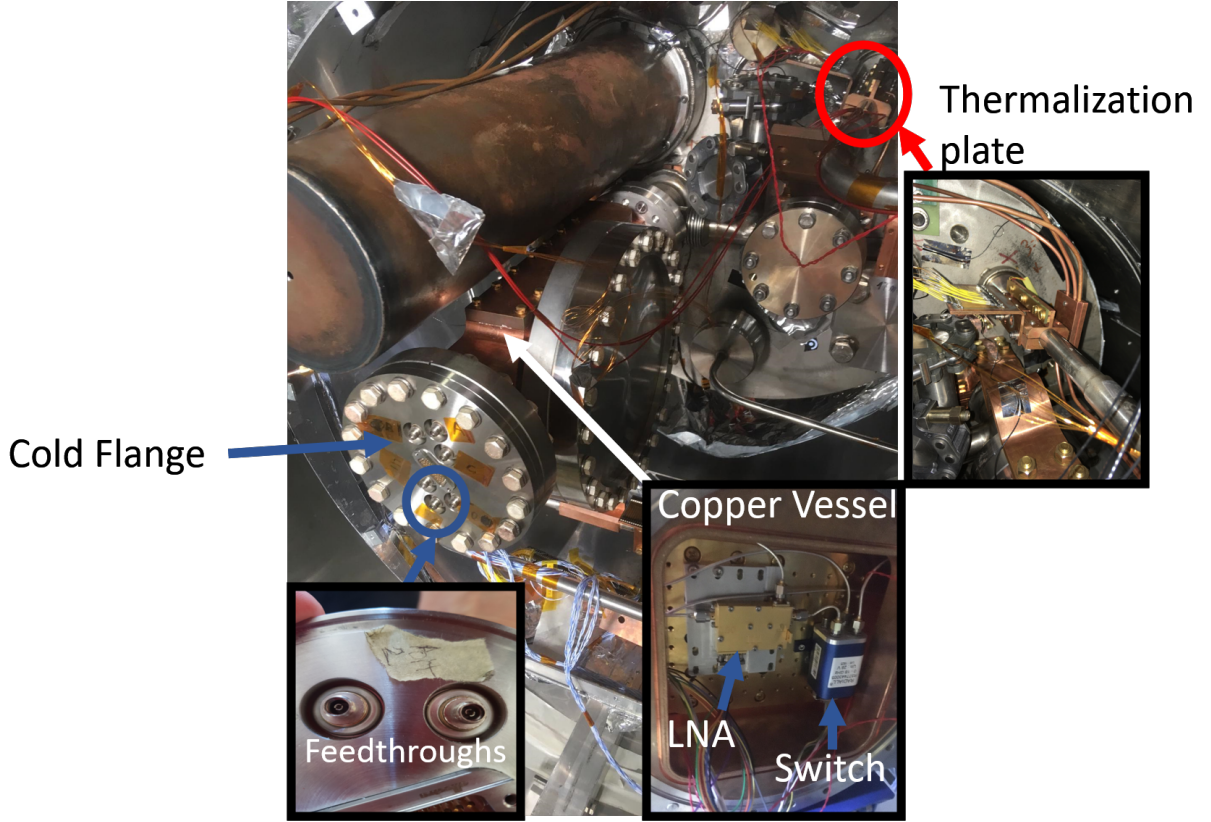


Figure 28: Setup components of the 5 K region and low magnetic field ( $B \ll 0.01$  T).

Bottom left inset: SMAD50 feedthroughs of the cold flange. Bottom right inset: The inside of the copper vessel, the golden rectangle is the cryogenic low noise amplifier and the blue box is the RF switch which was installed after the 2018 data-taking period. Top right inset: One of the thermalization plate

### 5.3.3 Feedthroughs

The setup had two flanges with six feedthroughs, as shown in figure 26. One of them was located at the end of the copper vessel (see figure 28) and was labeled as the cold flange. The second one was located at the outer part of the CAST magnet at room temperature, and it was known as the warm flange. The two flanges were designed and built previous to this work. The design was based on the flanges used by CAST-CAPP (the other haloscope experiment of CAST). The cold flange was a DN100 CF flange with a 26 pin sub-HD connector in the center, surrounded by 6 SMAD50 feedthroughs. The warm flange was a custom-made KS flange with a 25 pin sub-D connector in the center, surrounded by 6 SMAD50 feedthroughs. Allectra produced the flanges and the feedthroughs.

The S-parameters, measured using a VNA, were used to characterize the cavity properties and monitor the setup. A first measurement of the cavity inside the CAST magnet showed an unexpected reflection pattern for the  $S_{11}$  and  $S_{22}$  (see left panel of figure 29). The CAST-RADES setup configuration (cavity, LNA, flanges and cables) shown in figure 26 was moved to a dedicated lab for testing and was labeled the "whole setup" configuration to distinguish it from the test done for each component.

After a dedicated test of each component (see right panel of figure 29 and figure 30), it was found that the reflections were caused by the feedthroughs of the warm and cold flange.

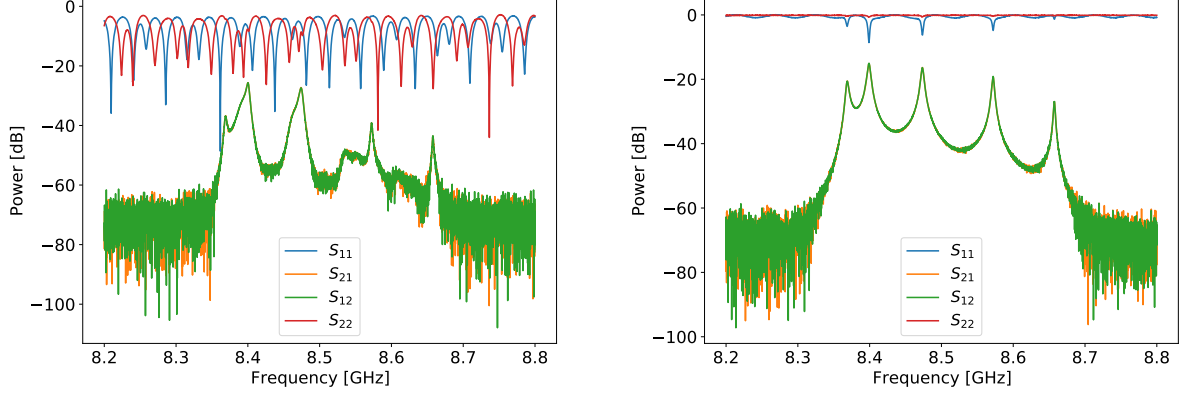


Figure 29: Left: Measurement of the S-parameters for the whole setup in the lab. The reflection parameters show a unexpected peak structure. Right: S-parameters only for the cavity.

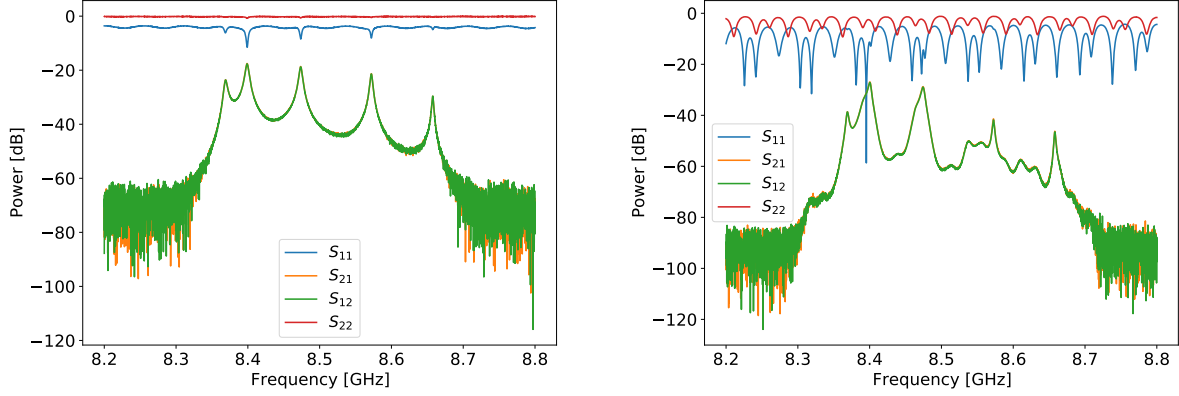


Figure 30: Left: S-parameters for the cavity with a 2m coaxial cable attached to  $P_1$ . Right: S-parameters for the cavity with a 2m coaxial cable attached to  $P_1$  and the other end attached to one feedthrough of the warm flange.

A way to test a connection problem is to measure the S-parameters in the time domain. A bad connection will result in the reflection of the signal, which shows up as a peak in the time domain. Using the time axis, one can calculate the location of this peak and determine which component was producing the problem. The left panel of figure 31 shows the result of  $S_{11}$  and  $S_{22}$  for the whole setup. The measurement showed that there is an important reflection at time zero, which was the location of the warm flange. The second reflection bump at 25 ns is the place where the cold flange was located. Both of these bumps should be of the order of 0.05, for a proper feedthrough connection.

A time-domain measurement was done for all feedthroughs of both flanges. The right panel of figure 31 shows the results for the warm flange. All feedthroughs had a bad connection to the coaxial cables and produced the unwanted reflection structure. Similar results were observed for all feedthroughs of the cold flange. Later, it was found out that these feedthroughs were specified to work up to 8 GHz (below our resonance frequency).

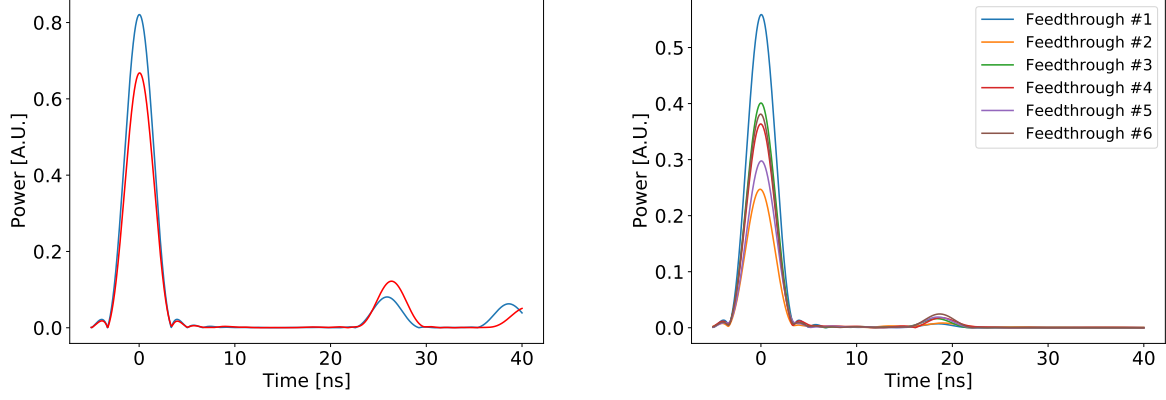


Figure 31: Left: Time domain measurement of  $S_{11}$  (blue) and  $S_{22}$  (red) for the whole setup. The bump at 0 ns corresponds to the feedthrough of the warm flange, the bump at 25 ns to the feedthrough of the cold flange. The amplitude of this bump is smaller because most of the power was reflected by the warm flange feedthrough. Right: Time domain measurement of  $S_{11}$  for the feedthroughs of the warm flange. Similar results were observed for the feedthroughs of the cold flange.

Other sma feedthroughs (type 242-SMAD18G) produced and provided by Allectra were tested. The different behavior can be observed by comparing figure 31 left with figure 32. In figure 32 one of the feedthroughs of the warm flange was changed to the new model. The reflection was reduced from  $\sim 0.6$  (see left panel of figure 31) to  $\sim 0.05^4$ . The 242-SMAD18G feedthrough provided satisfying results. Due to time and budgetary constraints, only the warm flange was redesigned and manufactured with the new feedthroughs.

It will be seen in sections 6.7 and 6.8 how these reflections have an impact on the precision of the coupling measurement.

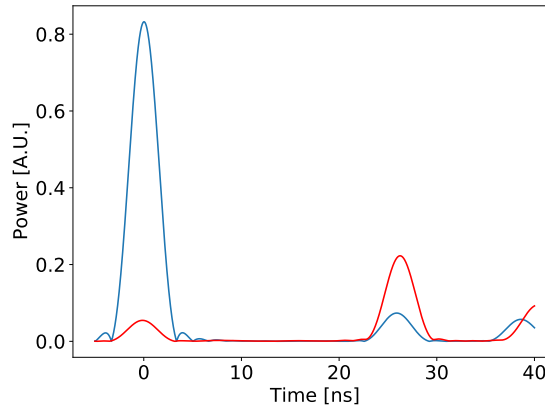


Figure 32: Time domain measurement of  $S_{11}$  (blue) and  $S_{22}$  (red) for the whole setup, where one of the warm flange feedthrough (located in the plot at  $t = 0$ ) was changed with the new model. The reflection coefficient was reduced to about 0.5 for the new feedthrough model.

<sup>4</sup>With the new model more power was transmitted. This means that more power arrived and was reflected by the cold flange. This is the reason why the peak at 25 ns is higher in figure 32



### 5.3.4 Coaxial cables and thermalization plates

RS Times Microwave RG402/U coaxial cables were used to connect the RF components of the setup. The cable was made of silver-coated, copper-plated steel strand, with a PTFE dielectric and copper shielding. It had an outer diameter of 3.58 mm and a characteristic impedance of  $50\ \Omega$ . The cables were terminated with male sma connectors. A  $\sim 1$  m meter and a 1.5 m long cables were used to connect  $P_1$  to the LNA and  $P_2$  to a feedthrough of the cold flange. A small 20 cm cable was used to connect the output of the LNA to a feedthrough of the cold flange. Two 3 m long cables were used to connect the cold flange with the warm flange. These cables were thermalized in two stages using copper plates (see top right inset of figure 28).

### 5.3.5 Data acquisition system

The DAQ was a custom-made product produced by TTI Norte [117]. It was made of two parts, an analog stage that amplified, downconverted in frequency and filtered the signal, and a digital stage that converted the analog signal into a digital signal, computed and stored the Fast Fourier Transformation (FFT) to produce a power spectrum.

Figure 33 shows the schematics of the analog unit. The input signal first went through an LNA with a range of 8 to 9 GHz and a gain of 55 dB. Afterward, a single pole double throw (SPDT) RF switch directed the signal to a test port(T.R.) where the correct functioning of the LNA can be tested. Alternatively, the signal was directed to the treatment chain. An external local oscillator (LO, blue box in figure 33) provided the signal to translate the RF input signal to a down converted 140 MHz signal. An ANAPICO APSIN20G was used as the LO. The down-conversion was based on an image rejection mixer (I.R.)<sup>5</sup> that provided 26 dB of image band rejection. The output frequency after the down-conversion (the intermediate frequency, IF) is defined as:

$$\text{IF}[\text{MHz}] = \text{Input-Signal}[\text{MHz}] - \text{LO-Signal}[\text{MHz}] \quad (77)$$

The IF filtering sub-module filtered the desired IF signal (134MHz to 146MHz). It was made up of two filters in series. A low pass filter (LPF) to reject the LO leakage coming from the down-conversion sub-module and a Surface Acoustic Wave (SAW) band pass filter (BPF) to select the wanted IF frequency band. An adjustable amplification (AA,  $\sim 60$  dB) sub-module transformed the IF signal to the optimum input level for the digital stage.

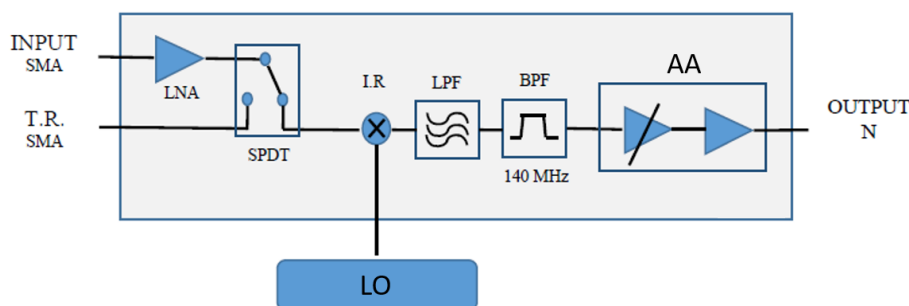


Figure 33: Schematic representation of the DAQ analog stage. The input signal was amplified, down converted and filtered by the analog stage. The original analog stage suffered some modifications due to the presence of noisy bins. Taken from [118].

<sup>5</sup>Unlike standard mixers, image rejection mixers removed one of the two output signals from the mix process by phase cancelling techniques.

The analog stage also comprises power supplies and a multimeter that are used to bias and monitor the cryogenic LNA (see section 5.3.2).

Figure 34 shows the schematics of the digital stage. The IF signal entered the digital stage, where an analog-digital converter board (THDB-ADA from Terasic) digitized the signal. The board integrated a dual 14-bit (16384 levels of dynamic range) 65 megasamples per second analog-digital converter (ADC) model AD9248 from Analog Devices. The sampling rate was 37.5 MHz, resulting in sample taken every 26.6667 ns ( $1/37.5$  MHz). The digital signal went through a de-multiplexer where the single data stream was split into the In-phase and Quadrature signals (I-Q values [119]). The I-Q signals were squared and added before the FFT is calculated in a Field Programmable Gate Array (FPGA), 2048 FFTs were accumulated into the FPGA, and the accumulated FFT (Acc FFT) was sent to a personal computer via Ethernet.

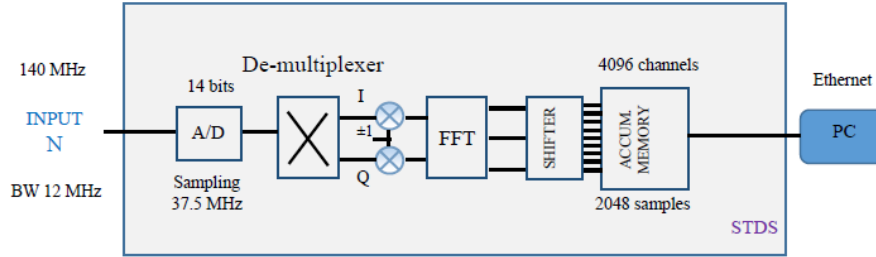


Figure 34: Schematic representation of the original DAQ digital stage. The analog signal was transformed to a digital signal using an ADC, and the accumulated FFT was stored and sent to the PC. The original digital stage suffered some modifications due to the presence of noisy bins. Taken from [118].

The choice of the IF bandpass filter in the analog stage affects the selected sampling rate. The SAW filter has an operative bandwidth of 12 MHz with strong insertion losses at 18.75 MHz. Based on the Nyquist theorem [120], the sampling rate has to be at least twice the maximum bandwidth of the IF filter (37.5 MHz). The design takes advantage of aliasing and under-sampling to work in a higher Nyquist zone and sample a signal of 140 MHz with an ADC sampling rate of just 37.5 MHz.

Aliasing is a phenomenon where the maximum input signal frequency is higher than half the sampling rate. In this case, the frequencies above the value of half the sampling rate will fold from the upper-frequency components into the signal operation band. These aliased signals will add to the original signal, and there will be no way to discriminate the aliased frequency components from the original signal. We can use this effect to our advantage: by sub-sampling, higher-frequency content will be aliased into all of the lower Nyquist zones (see figure 35). However, it is imperative to filter out any noise component in the other Nyquist zones. This was a problem in the original design, as we will discuss later on in this section.

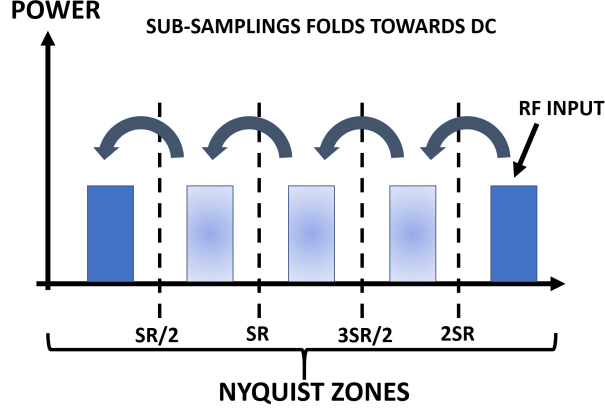


Figure 35: The right blue square represents a RF input signal located at a higher Nyquist zone. Due to aliasing the signal will fold (represented by the light blue squares) to lower Nyquist zones. Once it reaches the lowest Nyquist zone (left blue square) the input signal can be digitize even if its original value is above half the sampling rate.

The digital unit took a new sample every 26.6667 ns ( $1/37.5$  MHz). The FFT was calculated with 8192 samples, so on an accumulated time of  $218.453 \mu\text{s}$  ( $8192 \times 26.6667$  ns) and the resolution bandwidth was 4577.64 Hz ( $1/218.453 \mu\text{s}$ ).

The FPGA made the sum of the magnitude of the complex data (non-coherent integration) of 2048 FFTs. This accumulative FFT was generated every 0.44739 s ( $2048 \times 218.453 \mu\text{s}$ ). Considering the whole bandwidth with insertion losses (18.75 MHz) and the resolution bandwidth (4577.64 Hz), the FPGA generated 4096 ( $18.75 \text{ MHz} / 4577.64 \text{ Hz}$ ) accumulative FFTs every 0.44739 s.

These 4096 FFTs represented the spectra from 131.25 MHz to 150 MHz. However, the system only stored 2622 FFTs, which corresponded to the range from 134 MHz to 146 MHz. This corresponded to the 12 MHz operational bandwidth of the SAW filter.

The DAQ's frequency references determined the uncertainty of the measured frequency. In the presence of an axion signal, the frequency values recorded by the DAQ have to remain stable within the data-taking period. Otherwise, the axion signal could be recorded at different frequency bins and the combination of power spectra over time would be less effective, reducing the sensitivity of the experiment.

There were two frequency references. One at the downmixing module of the analog stage, the uncertainty of the downmixed signal is determined by the stability of the LO frequency. The second frequency reference was the internal reference of the digital stage. The LO oscillator had a reference signal with stability of 0.5 ppm, for a signal of 8.24 GHz this corresponded to a stability of  $\sim 4$  kHz. The latter one used an internal clock with stability of 50 ppm, for an input frequency of 140 MHz this translated to a frequency stability of 7 kHz. However, the datasheet specifies the 50 ppm for a broad temperature range ( $-20$  to  $70^\circ$ ), there is a high probability that the stability is better than that.

The software provided by the company received the accumulated FFT via Ethernet and stored the information on the hard disk of the PC. Additionally, the software provided a graphical user interface (GUI) showing the real-time results on the computer screen (see figure 36). The main tool of this software was the option to select the wanted integration time. The integration time range goes from 0.44739 s to 1832.5 s in steps of 0.44739 s. The power spectra were stored in hexadecimal, and the relation between the stored data in the file and the power is given by:

$$\text{Power [dBm]} = -131 \text{ dB} + 10\log(SD), \quad (78)$$

where  $SD$  is the "stored data". This equation can be understood as the calibration curve of this specific DAQ. The "stored data" is a linear power unit that has a -131 dB offset with respect to the 0 dBm scale<sup>6</sup>. The "stored data" for a given integration time is the power summed over this period of time<sup>7</sup>. This equation was tested using a known input signal (see appendix A).

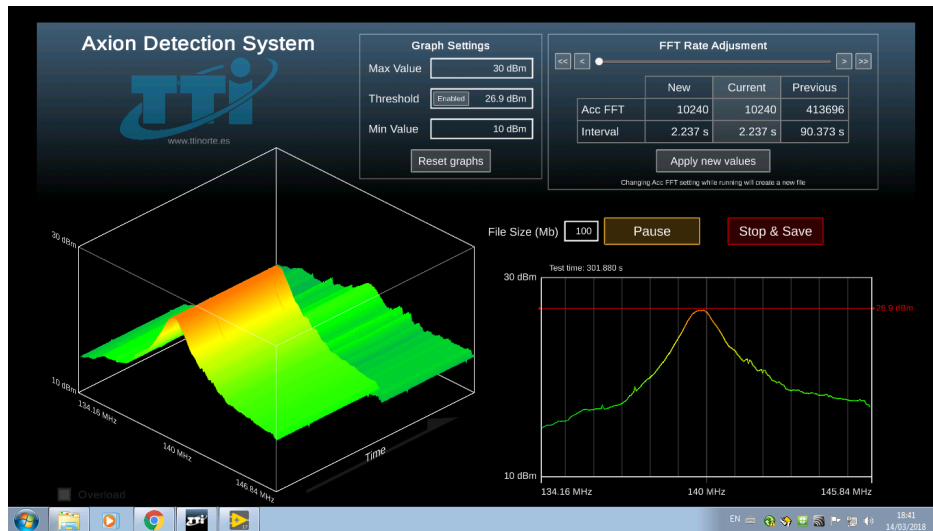


Figure 36: Graphical user interface provided by TTI Norte. The bottom left plot shows the currently acquired power spectrum. The left plot shows the time evolution of the power spectra. The FFT rate adjustment bar allows the user to select the wanted integration time per spectrum.

This software was for online monitoring and data acquisition, but a separate script was used to read out the data for the offline analysis. First tests using the offline analysis script showed a significant amount of noisy bins.

Figure 37 shows the results when the DAQ's input port was terminated with a  $50\ \Omega$  load. The noisy bins were a product of the DAQ electronics and could be classified into two different categories. The first category contained sharply positive and negative peaks with an equal spacing and relative position stability with time. The second was groups of positive peaks with a different spacing which were not stable over time.

<sup>6</sup>The Decibel (dB) expresses the ratio of two power values on a logarithmic scale. The dBm is power level expressed in dB with respect to 1 mW.

<sup>7</sup>For the data-taking period a 90.37278 s integration time was used. The stored data was the sum of 43 spectra taken every 0.44739 s



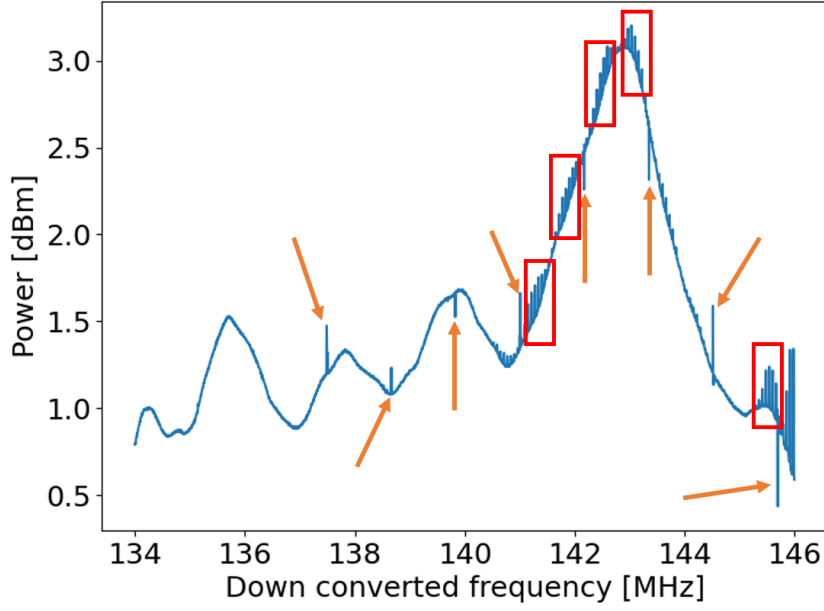


Figure 37: Power spectrum acquired with the original DAQ configuration. The arrows point to the first type of noisy bins which had a separation of around 1.2 MHz. The red squares show examples of the second type of noisy bins.

The first type of noisy bins came from the digital unit, while the second type was coming from the analog stage. Some tests showed that low frequency noise ( $< 20$  MHz) was entering the bandwidth of interest (134-146 MHz). A low pass filter was included after the amplification module of the analog stage to solve this problem. The other higher frequency noise that was entering the desired bandwidth was further suppressed by adding an extra band pass filter at the input of the digital stage. These two filters improved the dynamic range of the DAQ from 25.5 dB (from -14 dBm to 11.5 dBm) to 44.5 dB (from -43.5 dBm to 2 dBm).

The left panel of figure 38 shows the power spectrum after these filters were implemented. A significant improvement can be seen compared to figure 37. This was the DAQ configuration used for the data analysis presented in the following chapter. Even though there were still some noisy bins, the data collected with this configuration was good enough to perform an axion search. For completeness, it will also be mentioned that the problem producing the first type of noisy bins was also repaired after the data-taking period dealt within this thesis. A lack of proper filtering between the ADC and the FPGA let the intrinsic noise of the ADC card enter the FPGA producing the positive and negative noisy bins seen in the power spectrum. The right panel of figure 38 shows the current state of the power spectrum produced by the DAQ after the isolation was included between the ADC and FPGA.

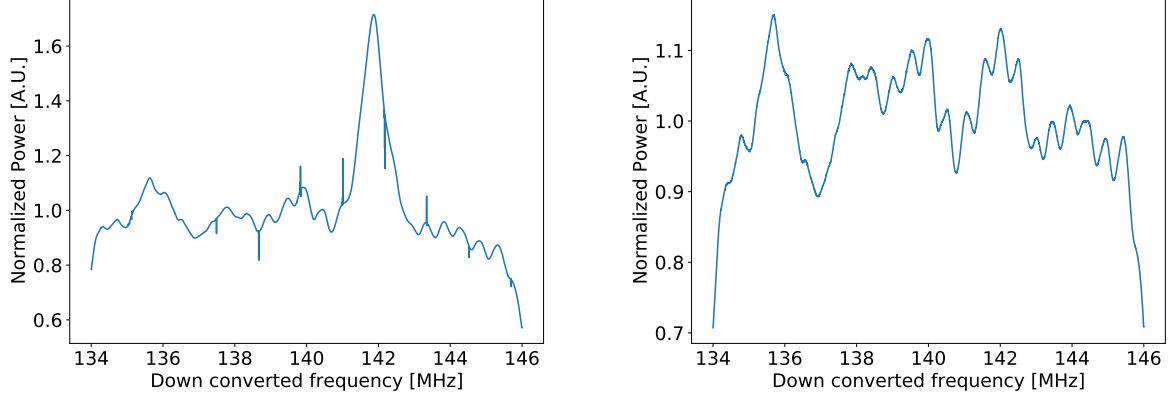


Figure 38: Left: Typical spectrum recorded by the DAQ after the low pass filter and the band pass filter were included. Most of the noisy bins are gone. Right: Typical spectrum recorded by the DAQ used after 2018, when an isolation between the ADC and FPGA was introduced.

Even though the problem with the noisy bins appears to be solved, the DAQ still showed some performance problems. The overall stability in time and frequency of the power spectra is the primary source of issues in the data analysis. From the inner reference clocks of the DAQ, a frequency stability of 7 kHz was expected. The analog and digital stage imprinted a characteristic electronic background structure to the white noise spectrum. Tests described in appendix A showed that this structure is not constant over time and a systematic residual appeared when the spectra were combined over time.

## 6 Data Analysis

In this chapter, the first results of an axion search done by the RADES group will be presented. The search was done based on the haloscope technique (see section 4.5) using the five sub cavities filter with inductive irises (see section 5.2) detector inside of the CAST magnet (see section 4.3). The codes used for analysis were completely developed for the first time during this work. The initial steps were based on the HAYSTAC analysis presented in [89].

### 6.1 2018 CAST data-taking campaign

The data-taking period associated with the axion search presented here started on the 30th of October 2018 and lasted until the 20th of December 2018. The data-taking period began with the ramp-up of the CAST magnet after the quench tests were finalized. As mentioned in section 4.3, there were four detectors (including the RADES detector) taking data at the same time. Two of them needed the solar tracking capability of the CAST magnet. For this reason, the members of the CAST collaboration had to participate in 4 h shifts which started around 45 min before sunrise. During the shift, the magnet was moved from the parking position to the tracking position, followed the sun for approximately 90 min and was moved back to the parking position. The tracking was not needed for the RADES detector, and in fact, the movement of the magnet was detrimental to the data taking. As we will show later, it affected the stability of the power spectra recorded.

At the end of the shift, the magnet was ramped down. The magnetic field stayed off during working hours, and the ramp-up, which lasted around 30 min, was done between 4 pm and 8 pm. The magnetic field stayed on during the entire day on weekends. These were the usual conditions of the data-taking period, but there were always minor issues or interventions that affected the shifts and reduced the total data-taking time.

With this configuration the detector took a total of 432 h of magnet-on and 141 h of magnet-off data at different LO frequencies. The reason for using different LO frequencies will be explained in section 6.2. A spectrum consisted of data integrated for 90.37278 s (from now on, the digits after the comma will not be written down whenever we are referring to these spectra). Table 3 gives a summary of the data sets recorded.

| Magnet-on data |                       | Magnet-off data |                       |
|----------------|-----------------------|-----------------|-----------------------|
| LO freq [GHz]  | # of recorded spectra | LO freq [GHz]   | # of recorded spectra |
| 8.239          | 543                   | 8.239           | 156                   |
| 8.240          | 4093                  | 8.240           | 1203                  |
| 8.241          | 1340                  | 8.241           | 323                   |
| 8.242          | 1649                  | 8.242           | 324                   |
| 8.243          | 1144                  | 8.243           | 0                     |
| 8.244          | 2322                  | 8.244           | 211                   |
| 8.245          | 571                   | 8.245           | 344                   |
| 8.246          | 483                   | 8.246           | 699                   |
| 8.247          | 3145                  | 8.247           | 927                   |
| 8.248          | 581                   | 8.248           | 1047                  |
| 8.249          | 1325                  | 8.249           | 406                   |

Table 3: Number of single spectra taken for magnet-on and magnet-off at different LO frequencies. In gray the sets used for the analysis.

The LO frequency was varied in steps of 1 MHz. Later on, it was realized that choosing a

step that was not a multiple of the resolution bandwidth ( $\Delta\nu = 4577$  Hz) led to the fact that the data taken at different LO frequencies could not be combined straightforwardly because the frequency bins were not aligned.

For the remaining sections, the analysis will mainly focus on the data collected with the LO frequency of 8.240 GHz, labeled  $l_1$ . This LO frequency was selected because it had the largest amount of data for both magnet-on and magnet-off, and the cavity resonance was located in a region without noisy bins. For the removal of the background structure (see section 6.2), a second LO frequency was needed, labeled  $l_2$ .

The 4093  $l_1$  spectra consisting of the magnet-on data set were divided into five sets, while the 1203  $l_1$  spectra of the magnet-off data were divided into four groups. Two LO frequencies were used as  $l_2$ , one for the magnet-data consisting of 3145 spectra divided into three sets and a second one consisting of 1047 spectra divided into three groups. Table 4 summarizes the information of these data sets.

| LO freq [GHz]              | Magnet-on |              |           | Magnet-off |              |           |
|----------------------------|-----------|--------------|-----------|------------|--------------|-----------|
|                            | set #     | # of spectra | Date      | set #      | # of spectra | Date      |
| $l_1 : 8.240$              | 1         | 545          | 14th Nov. | 1          | 355          | 14th Nov. |
|                            | 2         | 592          | 15th Nov. | 2          | 334          | 15th Nov. |
|                            | 3         | 627          | 16th Nov. | 3          | 201          | 16th Nov. |
|                            | 4         | 1745         | 17th Nov. | 4          | 313          | 4th Dec.  |
|                            | 5         | 584          | 4th Dec.  |            |              |           |
| $l_{2\text{-on}} : 8.247$  | 1         | 576          | 10th Dec. |            |              |           |
|                            | 2         | 2446         | 14th Dec. |            |              |           |
|                            | 3         | 124          | 17th Dec. |            |              |           |
| $l_{2\text{-off}} : 8.248$ |           |              |           | 1          | 702          | 6th Nov.  |
|                            |           |              |           | 2          | 310          | 6th Nov.  |
|                            |           |              |           | 3          | 35           | 9th Dec.  |

Table 4: Summary of the LO frequency sets used in the analysis. In blue the sets that were used for the analysis

## 6.2 Removal of the electronic background

Figure 39 shows a typical 90s spectrum which are going to be labeled as single spectra. These spectra consisted of the electronic background (described in appendix A), the cavity resonance peak structure, thermal noise, and any possible axion signal. This part of the analysis aimed to remove the electronic background and the cavity resonance peak structure with minimal impact to an axion signal to produce normalized power excess spectra that can be combined over time.

The LO frequency was changed to shift the cavity peak in the intermediate frequency (IF), while the electronic background stayed qualitatively similar (see figure 40). This allowed the removal of a significant part of the electronic background by dividing the two set at different LO frequencies. The second LO frequency used was 8.247 GHz, labeled  $l_2$ . As shown in Table 4, this LO set contains a total of 3145 spectra accumulated in 3 sets. The largest set was used for the analysis and consists of 2446 spectra<sup>8</sup>.

<sup>8</sup>Each  $l_1$  single spectra was divided by the average spectrum of the  $l_2$  spectra. The uncertainty of this calculation was dominated by the uncertainty of the  $l_1$  single spectra. The combination of data sets from different days increases the chances of systematic residuals. For this reasons, it was justified to neglect the other sets of  $l_2$ .

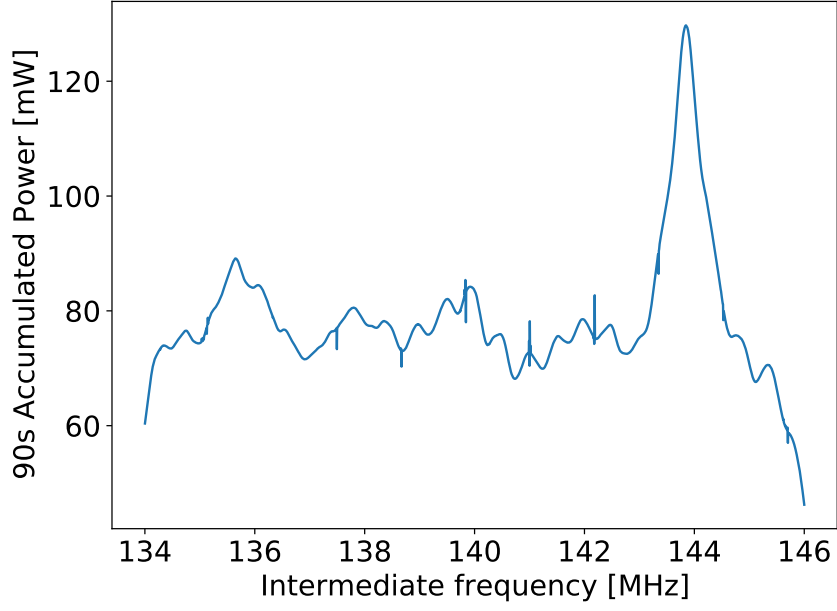


Figure 39: Typical single spectrum recorded by the DAQ when the setup was connected to its input port. The spectrum consisted of the electronic background, thermal noise, the cavity response and any possible axion signal.

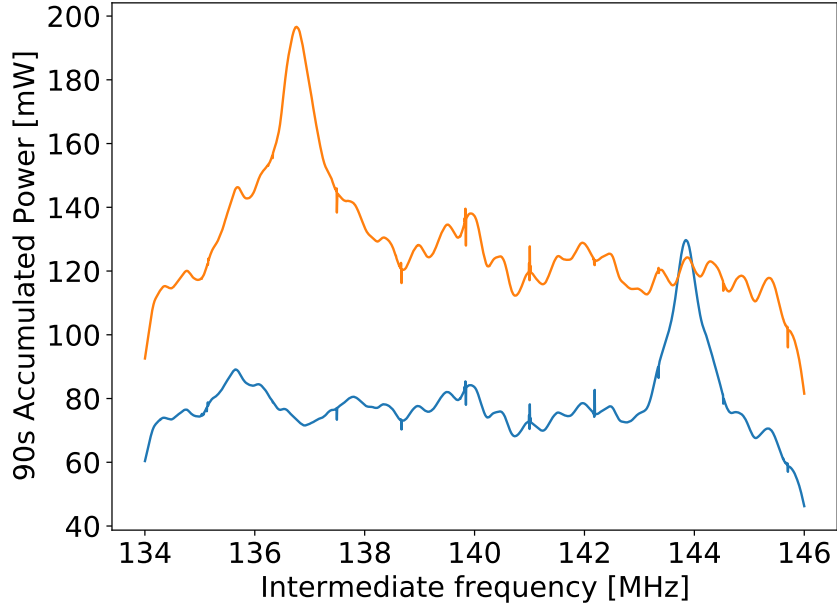


Figure 40: Typical single spectrum recorded by the DAQ for two different LO frequencies ( $l_1 = 8.240$  GHz and  $l_2 = 8.247$  GHz) for the cavity plus LNA as input signal. The cavity resonance appeared at a different IF frequency, but the electronic background remained qualitatively similar.

Each of the five magnet-on sets was treated separately for the removal of the electronic background. Let's denote the data recorded by  $\delta_{ij}$ , where  $j$  represents the value of the IF bin in the  $i$ -th spectrum. For  $i = 1, \dots, M$  and  $j = 1, \dots, 2622$ .  $M$  is the number of spectra for the sets shown in table 4.

The spectra of the two different LO frequencies were denoted as  $\delta_{ij}^{l_1}$  and  $\delta_{ij}^{l_2}$ . A large part of the electronic background was removed by dividing the  $l_1$  spectra by the average spectrum of the  $l_2$  sets:

$$\delta_{ij}^d = \frac{\delta_{ij}^{l_1}}{\frac{1}{M} \sum_{i=1}^M \delta_{ij}^{l_2}}, \quad (79)$$

here  $M = 2446$  was the number of spectra recorded for  $l_2$ . Figure 41 shows a typical  $\delta_{ij}^d$  spectrum. Given the different LO frequencies, the division led to a positive and negative resonance peak. From this point on, the frequency range was limited to a region located at the Lorentzian peak ( $j = 2049, \dots, 2239$ ), which corresponded to a frequency range of  $\sim 0.87$  MHz. This range was selected because it covered more than the full width at half maximum of the Lorentzian peak in a region with no noisy bins. See the vertical lines in figure 41. From then on, the spectra will be displayed in the physical frequency (PF = IF + LO). The index  $k$  is reserved to label bins in the PF space.

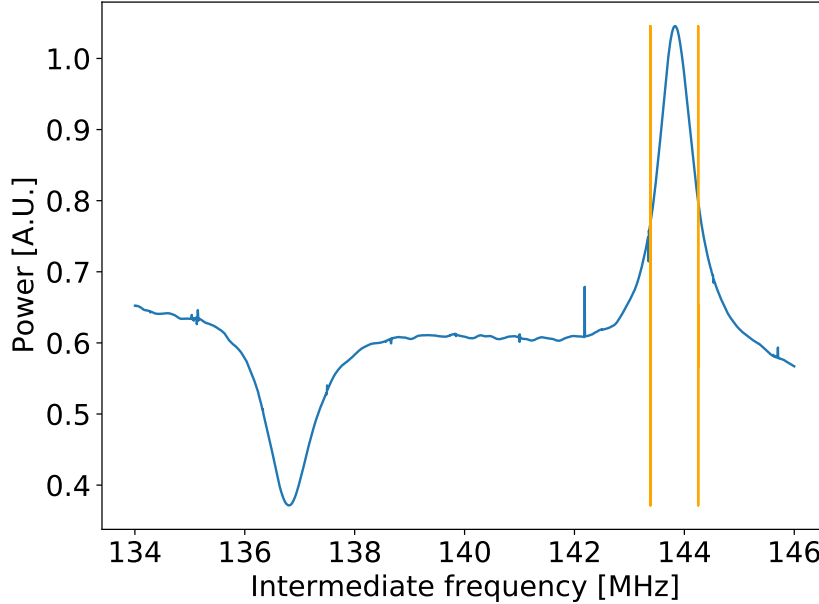


Figure 41: An example of a  $\delta_{ij}^d$  spectrum obtained by the division of the  $l_1$  spectra by the average of the  $l_2$  sets. This procedure removed part of the structures originating from the electronics which were common to both spectra. The orange vertical lines indicate the frequency range considered in the analysis which covered a region without noisy bins.

The removal of the remaining structure was largely based on the HAYSTAC analysis procedure detailed in [89]. A first Savitzky-Golay (SG) filter [121] (with 15 points and a polynomial degree of 3, the reason why these parameter were selected is shown in sections 6.2.1 and 6.2.2) was applied to the average spectrum  $\delta_k^d = (1/M) \sum_{i=1}^M \delta_{ik}^d$ , producing a smoothed spectrum called SG-fit. Figure 42 shows an example of the SG-fit on top of one single spectrum. This first SG filter removed the large structure originating from the electronics. Normalized spectra ( $\delta_{ik}^n$ ) were obtained through dividing  $\delta_{ik}^d$  by the SG-fit:

$$\delta_{ik}^n = \frac{\delta_{ik}^d}{\text{SG}_k}. \quad (80)$$

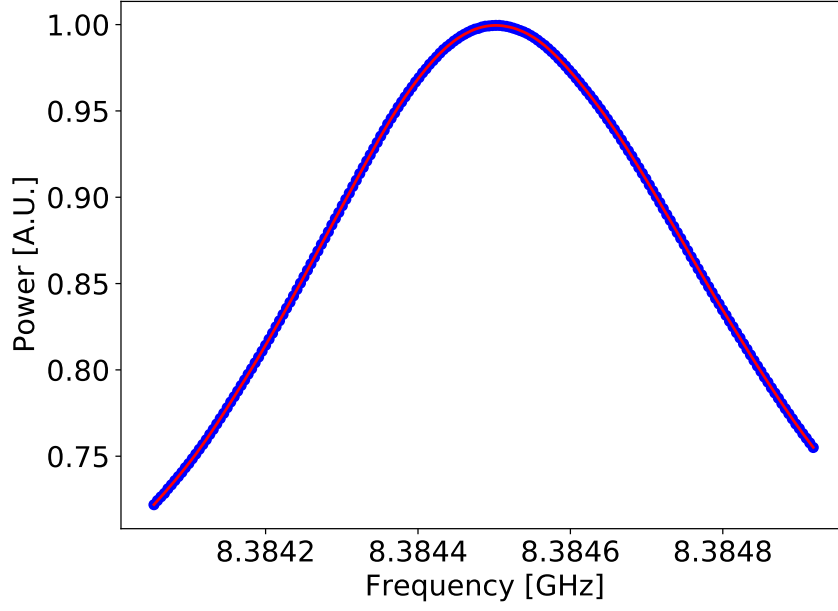


Figure 42: Typical single spectrum after the division procedure. The blue points indicate the data points and the red line the SG-fit produced on the average spectrum.

A second SG-fit (SG\*, produced using a SG-filter with 109 points and a polynomial degree of 3, the reason why these parameter were selected is shown in sections 6.2.1 and 6.2.2) was applied to each  $\delta_{ik}^n$  to remove the effect of gain drifts and the normalized spectra were once again divided by these fits<sup>9</sup> to obtain Unitless Normalized Spectra:

$$\delta_{ik}^u = \frac{\delta_{ik}^n}{\text{SG}_{ik}^*} - 1. \quad (81)$$

The left panel of figure 43 shows an example on a SG\* filter on top of a normalized spectrum ( $\delta_{147,k}^n$ ). The right panel of figure 43 shows a Unitless Normalized Spectrum ( $\delta_{147,k}^u$ ).

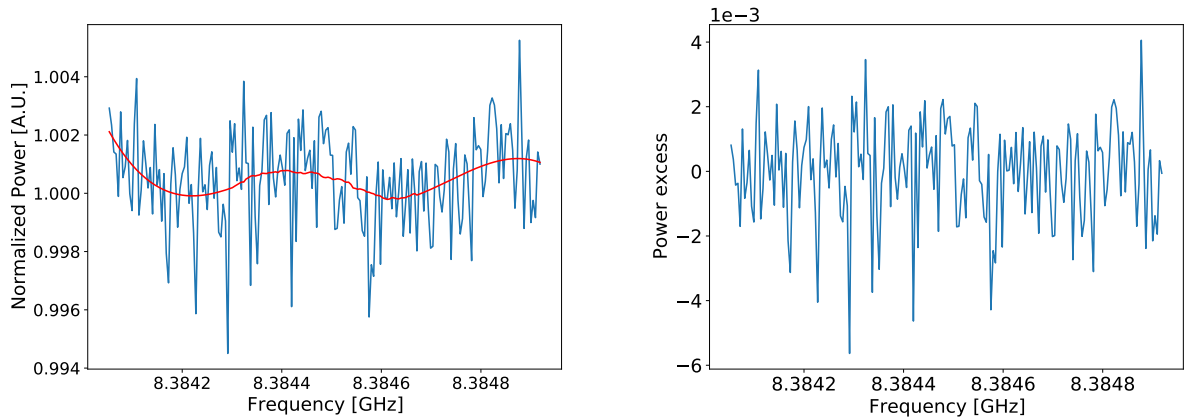


Figure 43: Left: A normalized spectrum ( $\delta_{147,k}^n$ ) and the corresponding SG\*-fit. Right: A Unitless Normalized Spectrum ( $\delta_{147,k}^u$ ).

<sup>9</sup>Each  $\delta_{ik}^n$  got its own SG\* to account for the gain drifts over the data-taking period of 103 hours.

### 6.2.1 The Savitzky-Golay filter

The SG filters are lowpass filters based on the principle of local least-squares fitting of a polynomial. They can be viewed as the polynomial generalization of a running average by two parameters  $N$  and  $W$ , where  $N$  is the polynomial order fit applied to a window of  $2W + 1$  points. For each input point of the sequence, the fit is applied using these parameters, and the output is a smoothed version of the input sequence.

Savitzky and Golay showed that a discrete convolution of a set of input samples with a fixed impulse response is equivalent to fitting a polynomial to this set and then evaluating the resulting polynomial at a single point within the approximation interval [121]. The impulse response is defined as the fixed set of weighting coefficients computed for a given polynomial order within the approximation interval. The impulse response of an SG filter can be computed by fitting the  $N$ th degree polynomial to the unit impulse<sup>10</sup> [122]. Figure 44 shows an example of an impulse response for a SG filter with  $W = 7$  and  $N = 3$ .

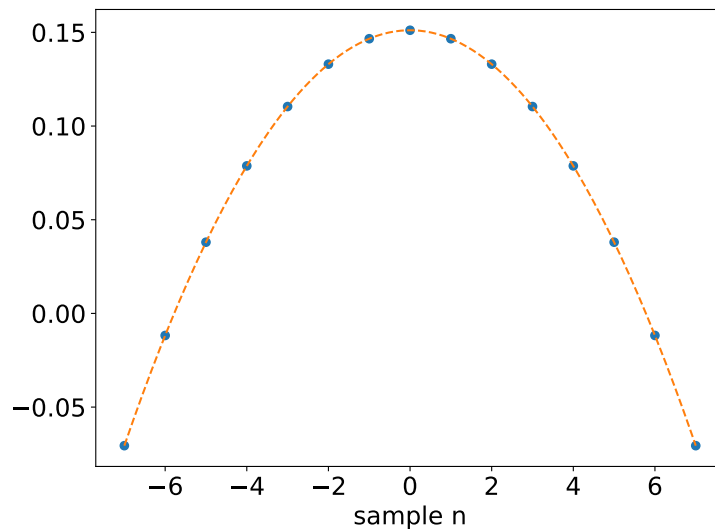


Figure 44: Impulse response for a SG filter with  $N = 3$  and  $W = 7$ . The blue points represent the data points of the impulse response and the orange dotted line is a polynomial fit to these points.

One can learn more about the properties of the SG filters by representing them in the Fourier domain. In the analysis done in this work, the impulse function is convoluted with an input sequence that is itself a power spectrum in the frequency domain. It would be misleading to say that the Fourier Transformation (FT) of the impulse functions is in the frequency domain. The Fourier domain will be labeled as the inverse bin domain, and the representation of the impulse function in this domain will be labeled the transfer function. Figure 45 shows the transfer functions used for the SG-fits of the analysis.

The transfer function highlights two regions in the inverse bin domain, the so-called stop, and pass bands. The transition between the two happens at the 3 dB point, defined by the first dip in the transfer function, located at around  $3 \cdot 10^{-2}$  and  $2.5 \cdot 10^{-1}$  in figure 45. This figure shows that the first SG-filter used in the analysis (orange line in this figure) has a more extended pass band resulting in an SG-fit that preserves large frequency scale features of the power spectrum (like the Lorentzian peak of the cavity). By selecting a larger window

<sup>10</sup>The unit impulse is a sequence with entries equal to zero except the central value. For  $W = 2$ , the unit impulse would be (0,0,1,0,0).



$W$ , the cutoff frequency of the transfer function of the SG\*-filter is shifted to the left. The SG\*-fit produced by this filter removes larger time scale variations like the ones caused by gain drifts and preserves the small statistical noise fluctuations.

The transfer functions had to be optimized to remove the electronic background minimizing the impact on a possible axion signal. The following section will show why the chosen parameters were able to remove the electronic background structure. The impact of the transfer functions and the SG-fit on an axion signal will be discussed in section 6.4.

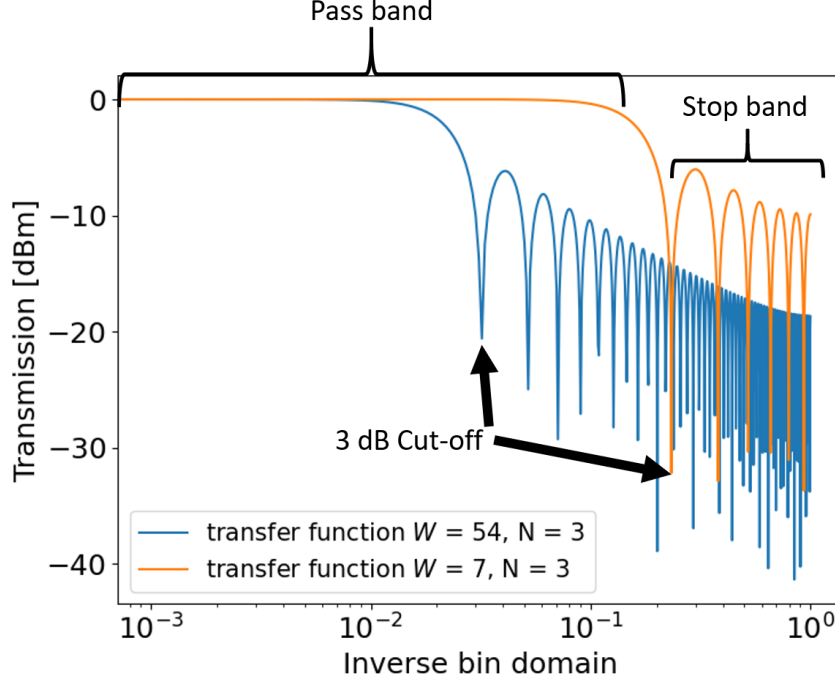


Figure 45: Transfer functions of the two SG filters used for the removal of the electronic background. The 3 dB cut-off frequency is shown for both filters, the corresponding pass band and stop band are shown for the orange transfer function. This transfer function has a longer pass band resulting in a SG-fit that preserves large frequency scale features of the power spectrum. The blue transfer function removes larger time scale variations like caused by gain drifts and preserves the small statistical noise fluctuations

### 6.2.2 Statistics of the Unitless Normalized Spectra

In the absence of axion conversion and if appropriate SG parameters are used, preserving the pure white noise contribution, the  $\delta_{ik}^u$  should be samples of a Gaussian distribution centered at  $\mu = 0$  with a standard deviation:

$$\sigma^\dagger = \frac{1}{\sqrt{\Delta\nu \cdot t}}, \quad (82)$$

where  $t$  is the integration time and  $\Delta\nu$  is the resolution bandwidth. The symbol  $\dagger$  will be used to identify the theoretical standard deviation values computed this way from the measured values computed from the distributions. For  $t = 90$  s and  $\Delta\nu = 4577$  Hz this yields  $\sigma^{u\dagger} = 0.001554$ . The  $\sigma_i^u$  for each  $\delta_{ik}^u$  were computed and compared to  $\sigma^{u\dagger}$  to optimize the SG parameters. The  $\sigma_i^u$  and the histogram of the normalized power excess ( $\delta_{ik}^u/\sigma^{u\dagger}$ ) are shown for each magnet-on data set. The histograms were fitted with a Gaussian distribution, which was expected to have  $\mu = 0$  and  $\sigma = 1$  if the right SG parameter were used.

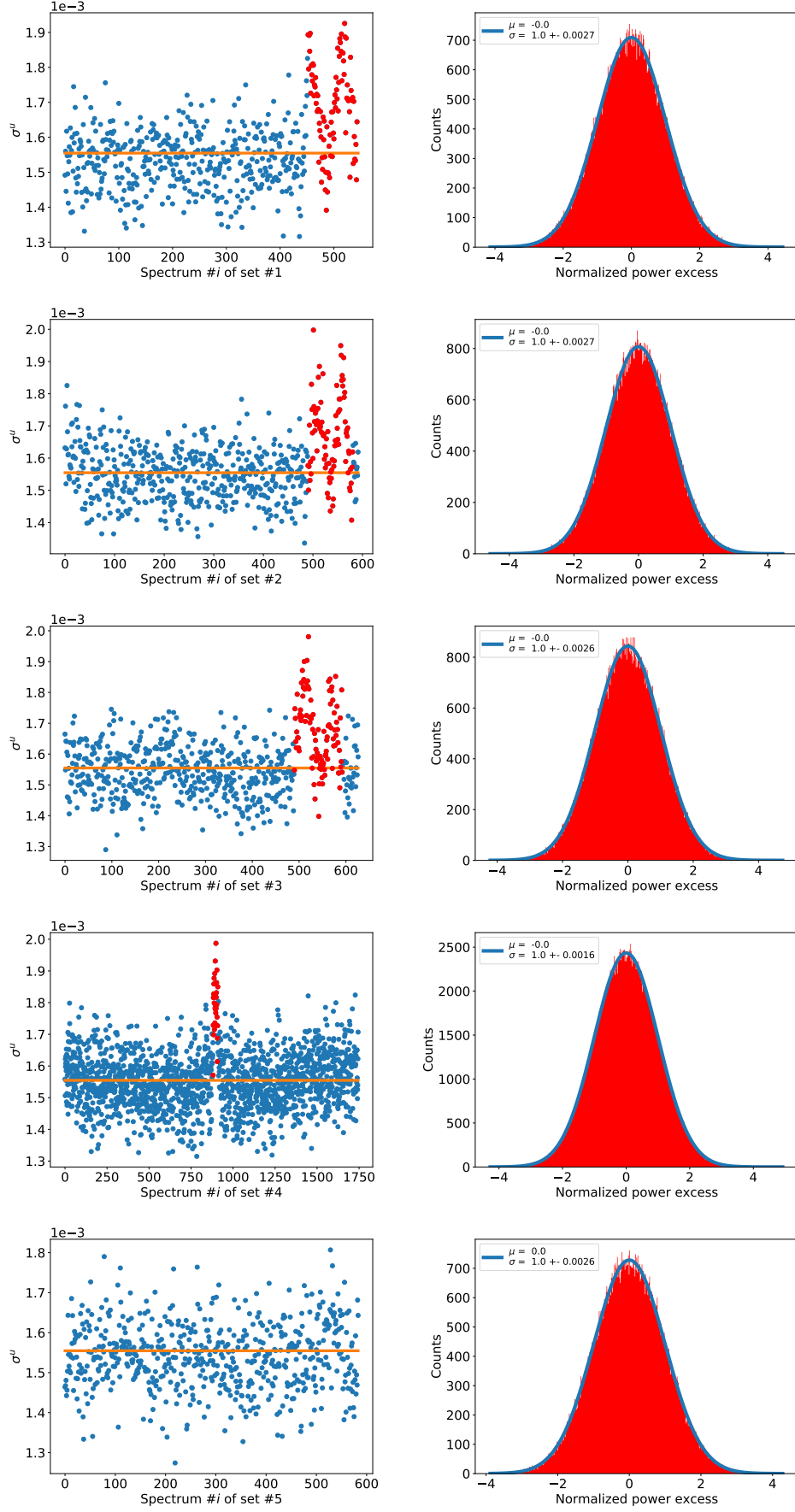


Figure 46: Left column: Standard deviation  $\sigma_i^u$  for all magnet-on sets listed in table 4, The orange line represents the expected theoretical value  $\sigma^{u\dagger}$ . The red points show the regions where the magnet was moving. Right: Histogram and Gaussian fit for the normalized power excess  $(\delta_{ik}^u / \sigma^{u\dagger})$  for the respective magnet-on sets.

The large excursions away from  $\sigma^{u\dagger}$  appearing in set #1, 3-5 were correlated with the movement of the magnet at full speed during the morning shifts<sup>11</sup>. The analysis procedure was done with and without these spectra showing no significant difference in the final result. These spectra were included in the remaining part of the analysis described in this work.

Table 5 summarizes the values of  $\mu$  and  $\sigma$  for all histograms. These results show that the SG fits using the selected parameters removed the electronic background spectrum at this level of integration time.

| Set # | $\mu$ | $\sigma$         |
|-------|-------|------------------|
| 1     | 0.0   | $1.0 \pm 0.0027$ |
| 2     | 0.0   | $1.0 \pm 0.0027$ |
| 3     | 0.0   | $1.0 \pm 0.0026$ |
| 4     | 0.0   | $1.0 \pm 0.0016$ |
| 5     | 0.0   | $1.0 \pm 0.0026$ |

Table 5: Center  $\mu$  and standard deviation  $\sigma$  for the fitted Gaussian distribution to the histograms of the different magnet-on sets.

## 6.3 Grand Unified Spectrum

### 6.3.1 Magnet-on data

The  $\delta_{ik}^u$  are combined into a Grand Unified Spectrum:

$$\delta_k^g = \frac{1}{M} \sum_{i=1}^M \delta_{ik}^u, \quad (83)$$

with the sample variance:

$$(\sigma^g)^2 = \frac{1}{M-1} \sum_{k=1}^M (\delta_k^g - \bar{\delta}^g)^2, \quad (84)$$

here  $M = 190$  and  $\bar{\delta}^g$  is the mean value of  $\delta_k^g$ . The distribution of the  $\delta_k^g$  should also be fully Gaussian if the structures originating from the DAQ and other sources were entirely removed. Equation 82 gives the expected value for the fluctuation of the  $\delta_k^g$  spectrum. In this case the integration time was  $t = 4093 \times 90$  s, yielding  $\sigma^{g\dagger} = 2.43 \times 10^{-5}$ . However, the measured value yielded  $\sigma^g = 5.47 \times 10^{-5}$ .

Figure 47 shows the  $\delta_k^g$  spectrum together with the  $\pm 5\sigma^{g\dagger}$  lines to indicate the level of deviation from the expected theoretical value and the corresponding histogram. The measured  $\sigma$  is different by a factor 1.7 from the expected  $\sigma^\dagger$ . That means that the spectrum was not made of pure white noise and that the combination of data revealed additional structures. The analysis of magnet-off data was able to tell us if these were residual, yet unaccounted for, distortions or an axion signal.

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<sup>11</sup>The magnet moves at full speed around 30 min before and after the solar tracking.

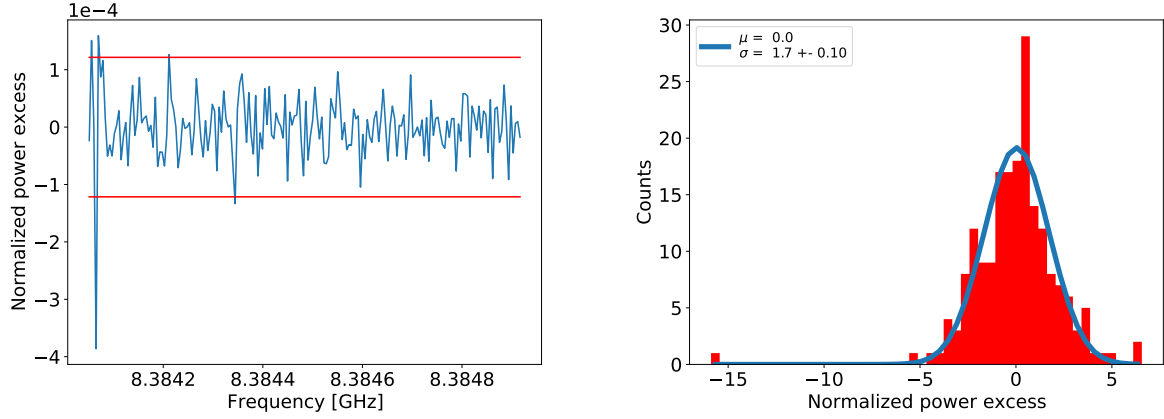


Figure 47: Left: Magnet-on Grand Unified Spectrum  $\delta_k^g$ . The red horizontal lines represent the  $\pm 5\sigma^{g\dagger}$  lines to indicate the level of deviation from the expected theoretical value. Right: Histogram of  $\delta_k^g/\sigma^{g\dagger}$  and the fitted Gaussian function.

### 6.3.2 Magnet-off data

The same procedure as the one explained in section 6.2 using the same SG-filters was done for the magnet-off data. The magnet-off data sets are indicated in table 4. The largest set of the LO frequency 8.248 GHz was used as the second LO frequency  $l_2$ . Table 6 summarizes the values for the means, and standard deviations of the Gaussians fitted to the distributions of the single Unitless Normalized Spectra (similar to section 6.2.2) for each set.

| Set # | $\mu$ | $\sigma$          |
|-------|-------|-------------------|
| 1     | 0.0   | $1.0 \pm 0.0036$  |
| 2     | 0.0   | $1.1 \pm 0.0037$  |
| 3     | 0.0   | $1.0 \pm 0.0034$  |
| 4     | 0.0   | $0.99 \pm 0.0044$ |

Table 6: Mean  $\mu$  and standard deviation  $\sigma$  of the Gaussians fitted to the distributions of the  $\delta_{ik}^u/\sigma^{u\dagger}$  values of the different magnet-off sets.

Figure 48 shows the standard deviations extracted for each single spectra (left) and histogram of  $\delta_{ik}^u/\sigma^{u\dagger}$  (right) for set 3. On the left panel of this figure, one can see that there were two regions in the spectra (between 120 and 140 and between 210 and 240) where the value deviated from the expected noise fluctuation ( $\sigma^{u\dagger}$ ). This translated to a higher than expected  $\sigma$  value for the histogram. The origin of this issue is still unknown. The log files showed that the magnet was not moving during this period of time. It cannot be correlated to this, as was the case for the magnet-on data. No specific interventions on the magnet were noted in the logs either. The remainder of the analysis was done with and without this set, and no significant differences were seen in the Grand Unified Spectrum residual structure. For this work, this set was included. It increased the integration time and, with it, the sensitivity to an axion signal. The other three sets presented the expected distribution.

Using equation 83 the magnet-off Grand Unified Spectrum ( $\delta_k^{g-\text{off}}$ ) was created. In this case  $M = 1203$  and the expected noise fluctuation is given by  $\sigma^{g-\text{off}\dagger} = 4.47 \times 10^{-5}$ . Similarly to the magnet-on data, a significant deviation from a Gaussian distribution appeared once the spectra were combined over time ( $\sigma^{g-\text{off}} = 6.61 \times 10^{-5}$ ). This is shown in figures 47 and 49.

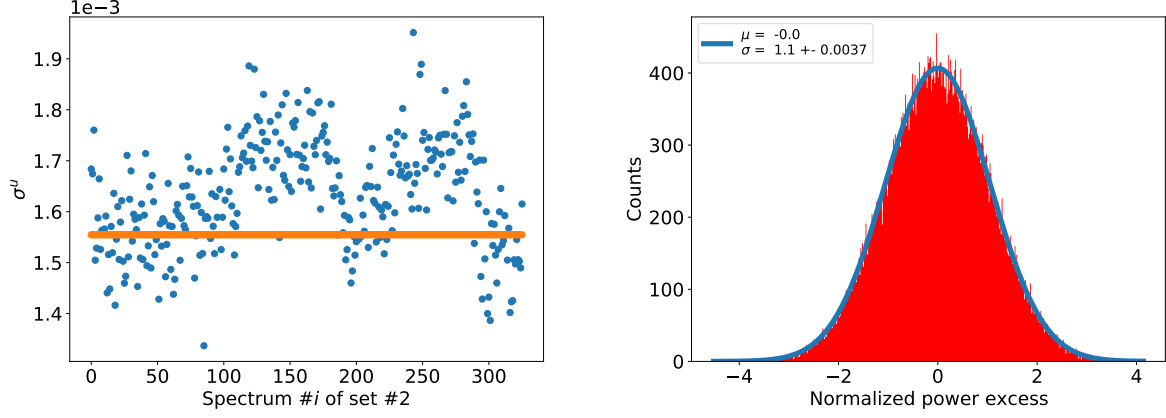


Figure 48: Left: Standard deviation  $\sigma_i^u$  for magnet-off set number 3. The orange line represents the expected theoretical value  $\sigma^{u\dagger}$ . Right: Histogram and Gaussian fit for  $\delta_{ik}^u/\sigma^{u\dagger}$  for magnet-off set number 3.

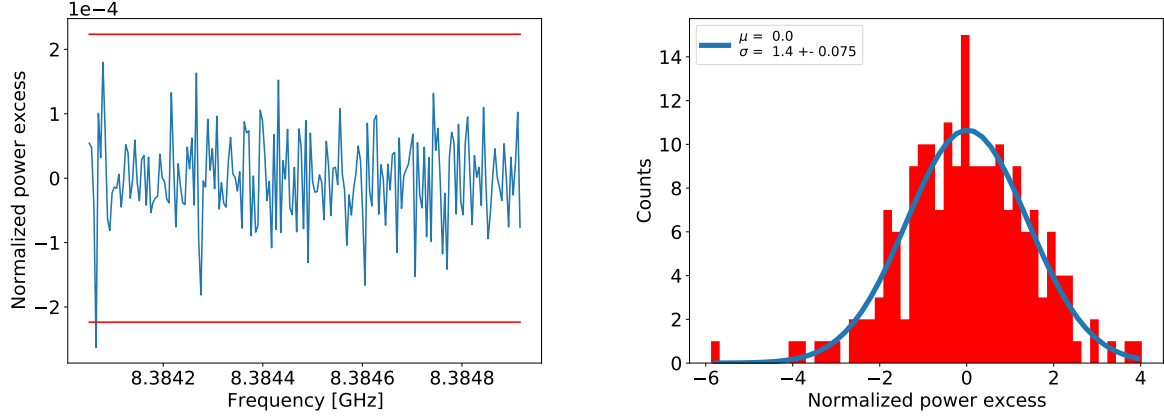


Figure 49: Left: Magnet-off Grand Unified Spectrum  $\delta_k^g$ . The red horizontal lines represent the  $\pm 5\sigma^{g-\text{off}\dagger}$  lines to indicate the level of deviation from the expected theoretical value. Right: Histogram of  $\delta_k^g/\sigma^{g-\text{off}\dagger}$  and the fitted Gaussian function.

### 6.3.3 Removal of the systematic residual

A comparison of the spectra shown in figure 47 and 49 showed that both spectra shared similar structures (e.g., a sharp negative peak at low frequency and another negative peak at  $\sim 8.3843$  GHz). Those shared systematic residual structures were partially removed by subtracting the spectra. The Final Grand Unified Spectrum was computed using the following equation:

$$\delta_k^f = \delta_k^g - \delta_k^{g-\text{off}}. \quad (85)$$

Figure 50 show the Final Grand Unified Spectrum ( $\delta_k^f$ ) and the corresponding histogram.

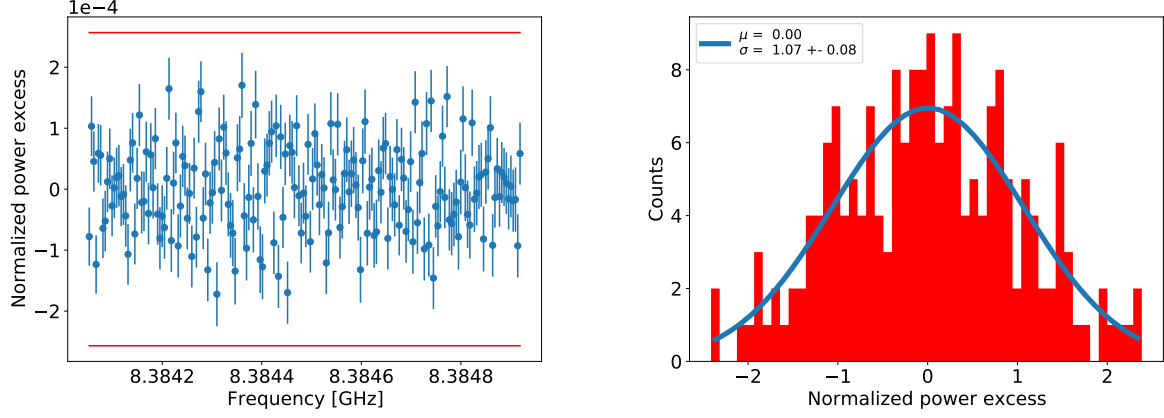


Figure 50: Left: Final Grand Unified Spectrum  $\delta_k^f$ . The red horizontal lines represent the  $\pm 5\sigma^{f\dagger}$  lines to indicate the level of deviation from the expected theoretical value. Right: Histogram of  $\delta_k^g/\sigma^f$  and the fitted Gaussian function.

Since the analysis relied on magnet-off data to obtain the Final Grand Unified Spectrum, the uncertainties on the measurement will be affected by the number of spectra collected in the magnet-off configuration.

The uncertainty of  $\delta_k^g$  and  $\delta_k^{g\text{-off}}$  can be computed with:

$$(\sigma_k^{g*})^2 = \frac{1}{M-1} \sum_{i=1}^M (\delta_{ik}^u - \delta_k^g)^2, \quad (86)$$

where  $M = 4093$  for magnet-on and  $M = 1203$  for magnet-off. The  $\sigma_k^{g*}$  notation indicates that this is the standard deviation for each physical frequency  $k$ , not to be confused with  $\sigma^g$  representing the standard deviation of  $\delta_k^g$  across the frequency range  $k$  given by equation 84. A statistical analysis, similar to the one done in the previous sections, can be done for each  $k$  value. The  $k$ -th frequency of the  $i$ -th spectrum is expected to be a sample taken from a Gaussian distribution with center  $\mu = 0$  and  $\sigma = \sigma^{u\dagger} = 1.554 \times 10^{-3}$ . The expected values for  $\sigma_k^{g*}$  ( $\Delta\nu = 4577$  Hz,  $t \approx 103$  h) and  $\sigma_k^{g*-\text{off}}$  ( $\Delta\nu = 4577$  Hz,  $t \approx 30$  h) are  $\sigma^{g*\dagger} = 2.43 \times 10^{-5}$  and  $\sigma^{g*-\text{off}\dagger} = 4.47 \times 10^{-5}$ . Unlike for  $\sigma^g$ , the measured values agree with the expected value. Figure 51 compares the measured  $\sigma_k^{g*}$  and  $\sigma_k^{g*-\text{off}}$  with the expected values. This shows that the deviation from the Gaussian distribution comes from the residual structure across the frequency range, but the expected reduction of the noise fluctuation with time was preserved at the level of single frequencies.

The error propagation of equation 85 yields:

$$\sigma_k^f = \sqrt{(\sigma_k^{g*})^2 + (\sigma_k^{g*-\text{off}})^2} \quad (87)$$

Given the smaller amount of data collected, the magnet-off uncertainty dominates and thus degrades the sensitivity of the measurement. This could have been avoided if more magnet-off data had been collected.

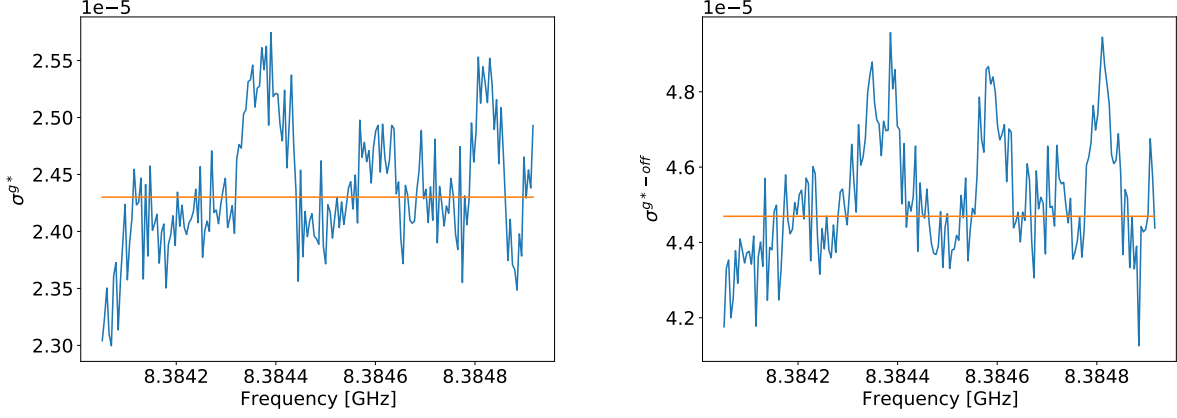


Figure 51: Left: Standard deviation  $\sigma^{g*}$  for magnet-on. Orange horizontal line is the expected theoretical value. Right: Standard deviation  $\sigma^{g*-off}$  for magnet-off. Orange horizontal line is the expected theoretical value.

## 6.4 Axion line shape and fit algorithm

So far, the analysis focused on the removal of the electronic features to obtain a final grand unified spectrum ( $\delta_k^f$ ) on which an axion search could be done. Depending on the strength of the axion-photon coupling  $g_{a\gamma}$ , the signal generated in the cavity would appear as an increment over the background white noise in  $\delta_k^f$ . The relic axions follow a velocity distribution, and their energy is spread over several of the DAQ bins. In the isothermal sphere model (see section 4.5.2), the velocity distribution of the axion follows a Maxwell-Boltzmann distribution (see equation 53). This equation can be rewritten in terms of the measured signal frequency  $\nu$ . In the Earth lab frame the spectrum of the axion signal is defined as:

$$f(\nu) = \frac{2}{\sqrt{\pi}} \left( \sqrt{\frac{3}{2}} \frac{1}{r \nu_a \langle \beta_{MB}^2 \rangle} \right) \sinh \left( 3r \sqrt{\frac{2(\nu - \nu_a)}{\nu_a \langle \beta_{MB}^2 \rangle}} \right) \exp \left( -\frac{3(\nu - \nu_a)}{\nu_a \langle \beta_{MB}^2 \rangle} - \frac{3r^2}{2} \right), \quad (88)$$

where  $\nu_a$  is the Compton frequency of the axion field,  $v_s$ , the velocity of the Sun with respect to the Galaxy, and  $\langle \beta_{MB}^2 \rangle = \langle v^2 \rangle / c^2$  where  $\langle v^2 \rangle$  is the second moment of the Maxwell-Boltzmann distribution defined as  $\langle v^2 \rangle = 3v_c^2/2 = (270 \text{ km/s})^2$ ,  $r = v_s / \sqrt{\langle v^2 \rangle} \approx \sqrt{2/3}$  with  $v_s \approx v_c = 220 \text{ km/s}$ , the velocity of a terrestrial laboratory with respect to the rest frame of the galactic halo.

A discrete version of the axion line shape was obtained as:

$$L_q = \int_{\nu_a + (q-1)\Delta\nu}^{\nu_a + q\Delta\nu} f(\nu) d\nu, \quad (89)$$

where  $\nu_a$  is the axion frequency and  $q$  labels the frequency bin number. In figure 52 a smooth  $L_q$  is plotted in red and the blue points represent the values of  $L_q$  given the DAQ resolution bandwidth  $\Delta\nu = 4577 \text{ Hz}$ . Table 7 shows the value of these points, one can see that 99.9% of the axion power is deposited within 6 bins.

| bin # | $L_q$  |
|-------|--------|
| 1     | 0.4885 |
| 2     | 0.3331 |
| 3     | 0.1256 |
| 4     | 0.0386 |
| 5     | 0.0105 |
| 6     | 0.0026 |

Table 7: Normalized deposited axion signal in each bin.

Ideally, one could fit this line shape to the data points of figure 50. However, one has first to consider the effects of the two SG-fits on the axion line shape. The effect of the SG-fits on the axion signal being independent of the structure of the electronic background, one can study independently how the SG-fits would affect a potential axion signal by applying the fits to a software simulated discrete axion signal.

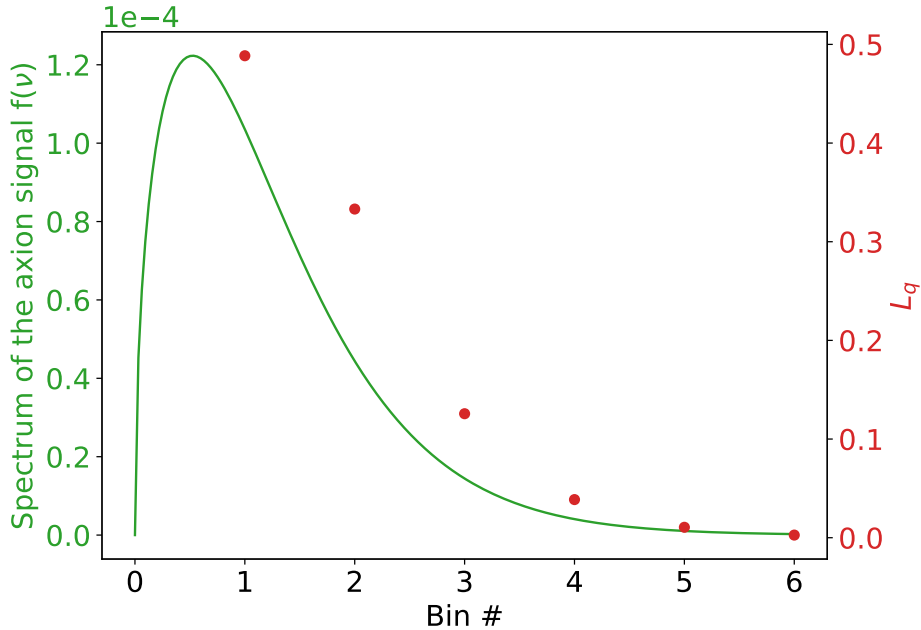


Figure 52: In green the velocity distribution of the axion is shown. In red the  $L_q$  of the discrete axion line shape for the resolution bandwidth of the DAQ ( $\Delta\nu = 4577$  Hz) is plotted. The red points are the result of the integral of the green function for an interval of 1 bin.

To see the expected axion signal, a flat spectrum with the same number of frequency bins as the data was created and the expected 6 bins axion signal was added to the center of the frequency range, the spectrum will be labeled as  $\delta_k^{da}$  (see left panel of figure 53).  $\delta_k^{na}$  (see right panel of figure 53) and  $\delta_k^{ua}$  were created using equations 80 and 81. The SG parameters used were the same as the one used to remove the electronic background (SG:  $W = 7$ ,  $N = 3$ , SG\*:  $W = 54$ ,  $N = 3$ ). The resulting  $\delta_k^{ua}$  spectrum is attenuated and distorted compared to the original axion signal.



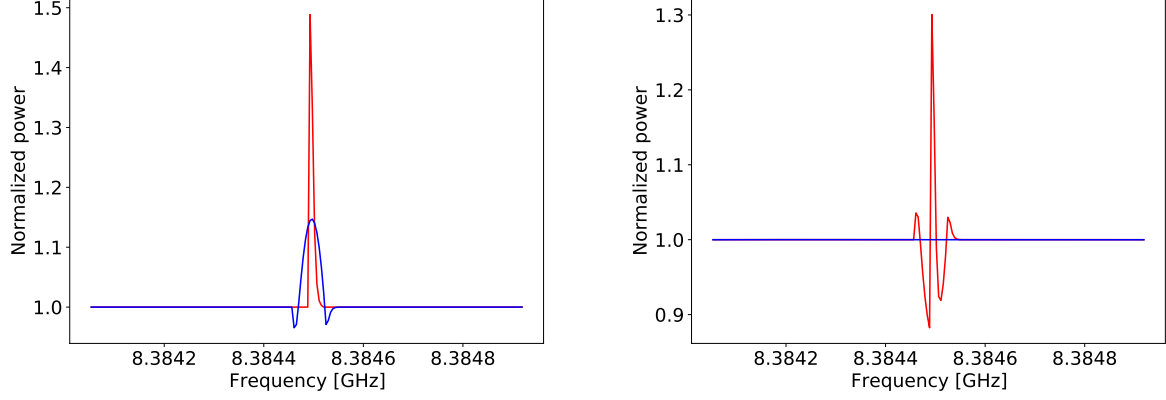


Figure 53: Left: In red, the flat spectrum plus a software simulated axion signal,  $\delta_k^{da}$ . In blue the SG-fit of the red spectrum using the same SG parameters as for the removal of the electronic background ( $W = 7$ ,  $N = 3$ ) is shown. Right: In red,  $\delta_k^{na}$  and in blue the SG\*-fit to  $\delta_k^{na}$  with  $W = 54$ ,  $N = 3$  is plotted.

To estimate the attenuation, the data points (where the axion signal was introduced, see the region between the two biggest dips of the right panel of figure 53) of  $\delta_k^{ua}$  were fitted using a function  $y = A \cdot L_q$ , where  $A$  is the amplitude of the axion signal and compared to the value of the original points ( $A = 1$ ). The distorted axion signal gave an  $A = 0.796$ , which translates to an attenuation of around 20%, reducing the overall sensitivity of the experiment.

A way to check and optimize the SG-fits' parameters was by plotting the Fourier transform of the axion signal and analyzing the effects of the transfer function for the chosen SG-parameters had on the axion signal. In figure 54 the Fourier transform of the axion signal was added to figure 45 as well as a vertical line at  $1/6$  denoted as the axion scale. The so-called axion scale indicates the scale (in inverse bins) of the expected axion signal. To minimize the effect of the SG-filter on the axion, the filter's cut-off should be smaller than the axion scale. The SG\* used in the analysis fulfills this criterion. Its effects on the axion signal were minimal, as shown in the right panel of figure 53. On the contrary, the cut-off frequency of the SG-fit used in equation 80 was larger than the axion scale, attenuating part of the axion signal.

Given the considerable attenuation provided by this filter, other parameters were tried. For example,  $W$  was changed from 7 to 11. The cutoff frequency was then smaller than the axion scale, and the attenuation of the axion signal was reduced from 20% to around 4%. However, for  $W = 11$ , the electronic background was not removed completely, and the values shown in table 5 deviated from the expected values. Eventually, it was found that  $W = 7$  was the parameter that could remove the electronic background the most efficiently while having an acceptable impact on the axion signal.

The SG-fits, apart from attenuating the initially 6-bins-wide signal, also distorted the close-by bins as shown in the left panel of figure 53. A spline interpolation of the modified lineshape (extending over a range of 15 bins) was performed and served as a fit function. To improve the spline interpolation algorithm, a grid of points between the original 15 points was created. These additional points were computed by shifting the axion line shape integral limits (see equation 89) with a misalignment  $(\delta\nu)^{12}$ . These points underwent the same process as the one shown in figure 53. Figure 55 shows the grid of points and the spline interpolation function. This function will be called the **Distorted Axion Fit Function**  $L_q^d$  and will be used for the axion search algorithm.

<sup>12</sup>The process was done for 9 misalignments ( $\delta\nu = \Delta\nu \times 0.1 \times i$ , where  $i$  goes from: 1,...,9)

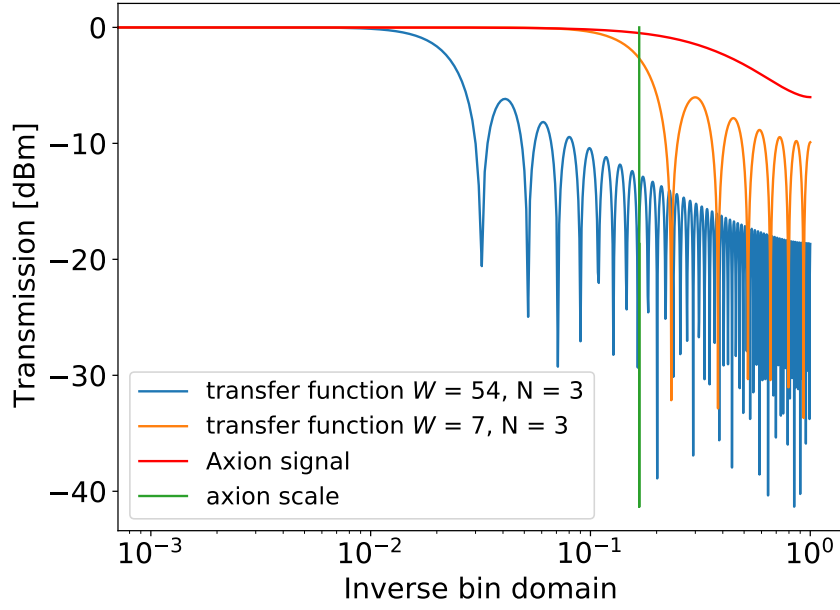


Figure 54: Transfer function of the two SG filters used for the removal of the electronic background and the axion signal. The vertical green line denotes the axion scale determined by the inverse of the number of bins where the axion deposited most of its energy ( $1/6$ ).

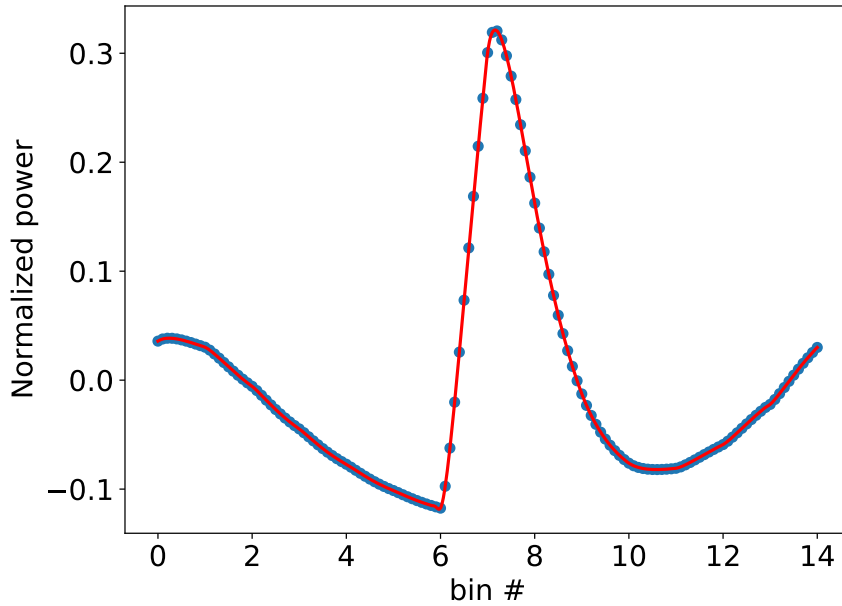


Figure 55: In blue the grid of points resulting of the application of the two SG-fits to the grid of  $L_q$  values. In red the spline interpolation of these points, called the distorted axion fit function  $L_q^d$ .

## 6.5 Axion search and calculation of the upper limit

The fit function was defined as:

$$y(x_k^f) = A \cdot L_q^d(x_k^f), \quad (90)$$

where  $A$  is the amplitude of the axion signal, and  $x_k^f$  are the frequency values of the Final Grand Unified Spectrum ( $\delta_k^f$ ).  $y$  was fitted to a region of  $\pm$  seven bins from the center value (bin 7 in figure 55) in steps of 1 bin. This implied a small reduction (from 190 to 176 frequency points) of the axion search parameter space since the edges (the first and last seven bins) of the  $\delta_k^f$  spectrum could not be used because there were not enough points left or right to perform the fit. Figure 56 shows the amplitude values  $A$  given by the fit for the probed frequencies with the corresponding 3, 4 and 5  $\sigma$  lines, where the standard deviation used was the uncertainty of  $A$ .

Commonly, a signal above 5  $\sigma$  is accepted as an indication for the discovery of a new particle. From figure 56, one can see that there was no signal above this threshold. However, six candidates were above the 3  $\sigma$  line and required further investigation.

Table 8 presents a list with the frequencies, bin #, and the significance level of these candidates. Figure 57 shows an example of the fit function for candidates #3. This table shows that candidates 2 and 3, and 4 and 5 were located at adjacent bins. If an axion signal were present in the data at a given frequency, the amplitude given by the fit for the adjacent bins would be influenced by the axion signal because of the 15-bin-wide fit range, resulting in more than one amplitude above the given threshold at adjacent frequencies. Candidates 2 and 3, and 4 and 5 can be considered two (and not four) different candidates.

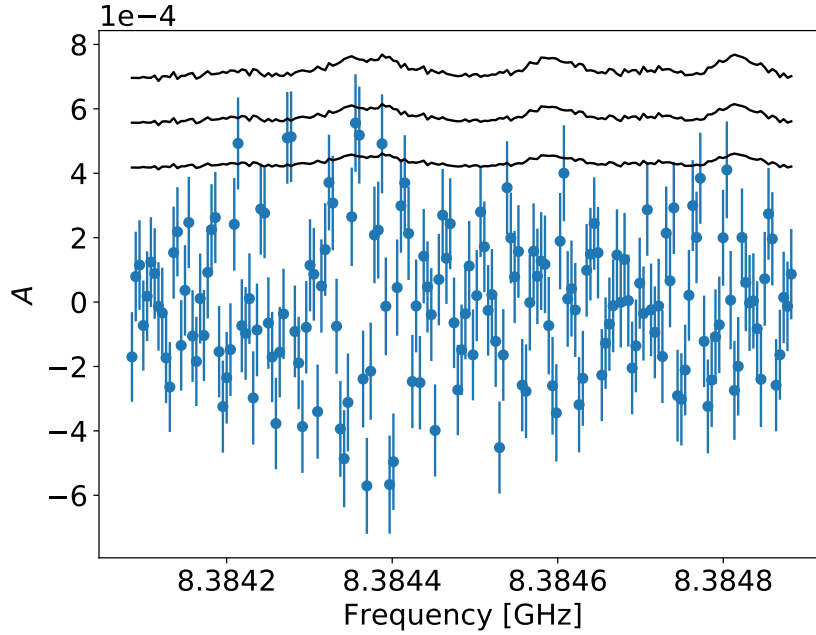


Figure 56: Amplitude  $A$  given by the fit function  $y$  for the probed frequencies. The black horizontal lines are the 3, 4 and 5  $\sigma$  lines computed using the uncertainty of  $A$ . A point above the 5  $\sigma$  line would indicate the discovery of a new particle.

| Candidate # | Frequency [GHz] | bin # | $\sigma$ -value |
|-------------|-----------------|-------|-----------------|
| 1           | 8.384213711     | 35    | 3.47            |
| 2           | 8.384273225     | 48    | 3.49            |
| 3           | 8.384277803     | 49    | 3.59            |
| 4           | 8.384355629     | 66    | 3.49            |
| 5           | 8.384360207     | 67    | 3.34            |
| 6           | 8.384387675     | 72    | 3.09            |

Table 8: Axion signal candidates

One has to analyze the significance of the significance level given in table 8. A distinction between local significance and global significance has to be made. The first one uses the probability value<sup>13</sup> corresponding to the measured test statistic (e.g.  $A$ ) assuming a fixed frequency value for the axion frequency. The global significance is the probability of a background fluctuation at **any** frequency value in the range of interest.

The  $\sigma$ -values given in table 8 represent the local significance of the axion signal candidates. To get the global significance, one has to compute the look-elsewhere effect. The global significance is given by [123]:

$$p^{\text{glob}} = p^{\text{loc}} + \langle N_{u_0} \rangle e^{-(u-u_0)/2}, \quad (91)$$

where  $p^{\text{loc}}$  is the probability value of the highest local excess,  $u$  is the square significance level of the the highest excess,  $u_0$  is a selected significant level and  $N_{u_0}$  are the number of up-crossing at  $u_0$ .

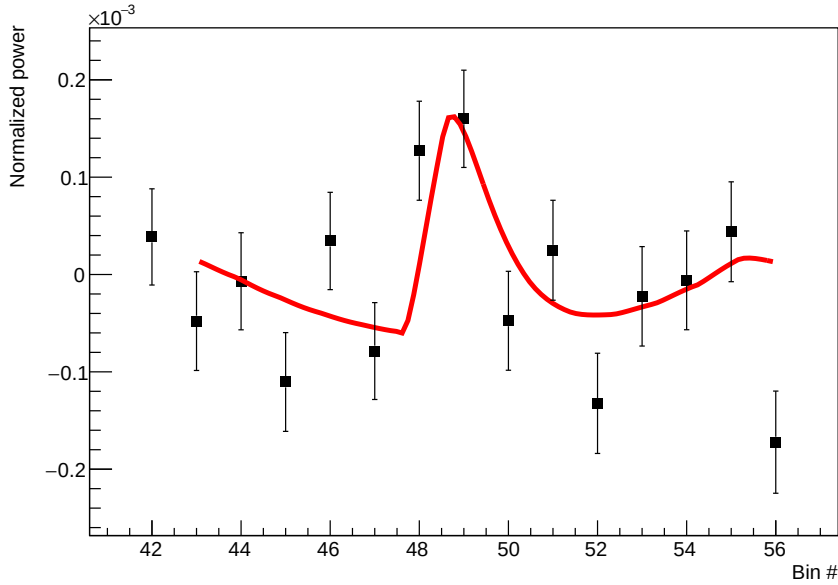


Figure 57: Fit function (red) for the data points of candidate #3.

The up-crossings are defined as the number of times the local test statistic curve (figure 56) crosses a horizontal line at a given level  $u$ . The value of the up-crossing can be minimal (in the order of  $10^{-5}$ ) depending on the selected  $u$  value, in such a case, a huge number of Monte Carlo samples would be required for a precise numerical evaluation. The second

<sup>13</sup>The probability value is the probability of obtaining test results at least as extreme as the observed results, under the assumption that the null hypothesis is correct.

term of the right-hand side of equation 91 is the extrapolation of the up-crossing from a different level  $u_0$  to the desired level  $u$ . The level was chosen to be  $u_0 = 0$ , which allowed the computation of up-crossings (in the other of 10) for the highest excess without relying on Monte Carlo simulations. For  $u_0 = 0$ ,  $N_{u_0} = 36$  (obtained by counting the up-crossings of figure 56). The largest excess in the range of interest had a significant level of  $\sigma = 3.59$ . Plugging all these numbers in equation 91 yields a global  $\sigma$ -value = 1.58 for this candidate, reducing its significance.

So far, it was assumed that the center frequency value (bin 7 in figure 55) and the overall discrete axion lineshape would align perfectly with the frequency binning imposed by the DAQ. However, this did not have to be the case and to allow for the possibility of a misalignment, a search was performed leaving the center frequency value as a free fit parameter within a bin. Figure 58 shows the amplitude  $A$  given by the fit for this search.

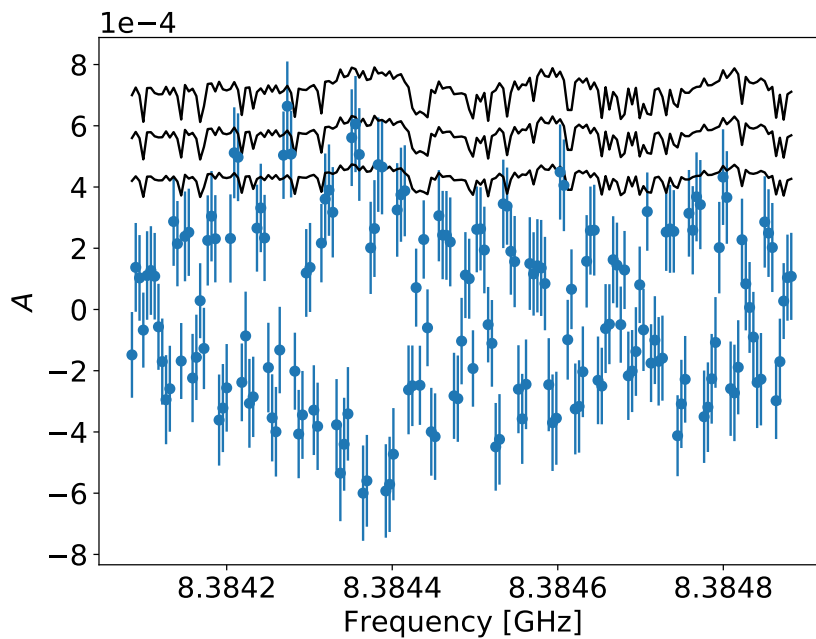


Figure 58: Amplitude  $A$  given by the modified fit function  $y$  where a misalignment parameter is introduced to leave the frequency free within a bin. The black horizontal lines are the 3, 4 and 5  $\sigma$  lines.

In this case, the highest local excess found was 4.54  $\sigma$ , which was translated into a global significance of 3.05  $\sigma$  for this candidate, after the look-elsewhere effect was taken into account. It was concluded that no significant signal above statistical fluctuations was observed.

Finally, a software-produced synthetic axion signal was created and introduced to the data to test the algorithm (see appendix B). The algorithm was able to recover the signal at the correct frequency.

If no signal was found in the data, an upper limit (UL) could be provided. It corresponds to the value at which a signal can be excluded for a given confidence level or credibility interval. The calculation of the upper limit was based on Bayesian statistics [123, 124].

Bayes' theorem states that for a sample of random variables,  $\delta_k^f$ , the posterior probability function  $p(A|\delta_k^f)$  is defined as:

$$p(A|\delta_k^f) = \frac{L(\delta_k^f|A)\pi(A)}{\int L(\delta_k^f|A')dA'}, \quad (92)$$

where  $L(\delta_k^f|A)$  is the likelihood function. Bayesian statistics uses a prior function  $\pi(A)$ , which is the prior degree knowledge of the parameter  $A$ .

The likelihood ratio for an experiment with independent channels can be express as the multiplication of the likelihood functions  $L$  of the individual channels<sup>14</sup>. Furthermore, the analysis is dealing with only Gaussian variables, so the likelihood function is expected to converge to the  $\chi^2$  distribution in the following way:

$$-2\log(L) = \chi^2. \quad (93)$$

From this follows that the posterior function  $p$  can be written as:

$$p(A|\delta_k^f) = \frac{e^{-(S(\delta_k^f, A)/2)}\pi(A)}{N}, \quad (94)$$

where  $A$  is the amplitude given by the axion line shape fit function  $y$  (see equation 90),  $\delta_k^f$  is the normalized power excess of the Final Grand Unified Spectrum. The expected axion signal has to be positive, so a flat prior function  $\pi(A) = 1$  for  $A \geq 0$  and  $\pi(A) = 0$  for  $A < 0$  was used.  $S(\delta_k^f, A)$  is defined as:

$$S(\delta_k^f, A) = \sum_k \left( \frac{(\delta_k^f - y(x_k, A))^2}{\sigma_k^2} \right), \quad (95)$$

where  $\sigma_k$  is the uncertainty on  $\delta_k^f$ .  $N$  is the normalization factor defined as:

$$\int_0^\infty e^{-(\sum_k S(\delta_k^f, A)/2)} dA = N, \quad (96)$$

Finally, the upper limit can be computed with:

$$\frac{1}{N} \int_0^{A_{UL}} e^{-(\sum_k S(\delta_k^f, A)/2)} dA = 1 - \alpha, \quad (97)$$

where  $1 - \alpha$  is the credibility level (CL) of the UL. For this analysis a 95% CL was used.

The solid black line of figure 59 shows the upper limit obtained from the measured data points  $\delta_k^f$ . This result has to be compared to the expected UL given the expected noise fluctuations. A large number of simulated spectra (1000) with the expected white noise fluctuations was created. The expected noise fluctuation ( $5.14 \times 10^{-5}$ ) was calculated using the error propagation formula for the subtraction of the two spectra (see equation 87). The expected uncertainty for the magnet-on and magnet-off data was computed using equation 82 and yielded  $\sigma_{4093t} = 2.43 \times 10^{-5}$  and  $\sigma_{1203t} = 4.47 \times 10^{-5}$ .

The UL was computed for each one of the simulated spectra using equation 97. The dotted red line of figure 59 shows the average expected UL and corresponding uncertainty bands. The bands were calculated by integrating numerically the histogram of the UL distribution of the simulated spectra (see figure 60). Note that the bands are not symmetric because of the choice of the prior.

The unitless  $A_{UL}$  can be converted back to power units using the following equation:

$$P_a = A_{UL} \cdot P_N = A_{UL} k_b T_{sys} \Delta\nu. \quad (98)$$

The exclusion limit to the axion-photon coupling  $g_{a\gamma}$  can be computed by<sup>15</sup>:

$$P_a = g_{a\gamma}^2 \rho_{DM} \frac{\beta}{1 + \beta} \frac{1}{m_a} B^2 V Q_L G^2, \quad (99)$$

<sup>14</sup>In this experiment, the independent channels correspond to the different frequency bins.

<sup>15</sup>This equation was introduced and discussed first in section 4.5.1.

In the following sections, the parameters needed for  $P_a$  are going to be computed. The parameters were inferred from single measurements because of limitations due to hardware availabilities. They were assumed to have been stable over the data-taking period of 103 hours taken within 20 days. Since the cavity did not undergo any mechanical changes<sup>16</sup> during the data-taking period, the assumption of stability of these parameters is believed to be justified.

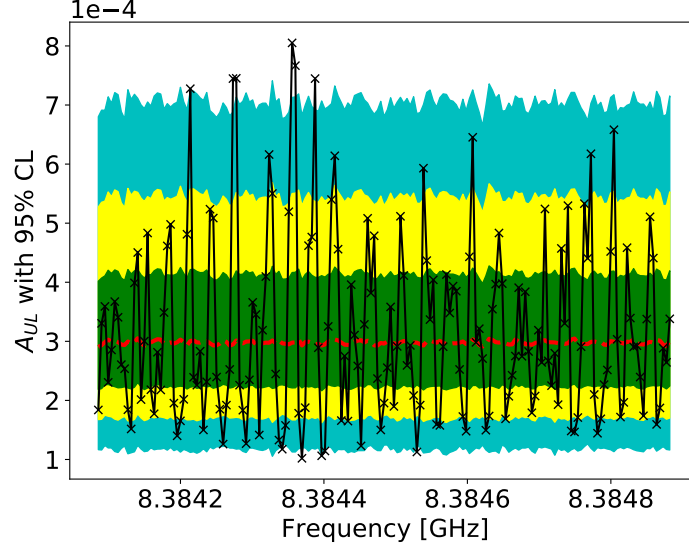


Figure 59

Observed (solid black line) and expected (dotted red line) values of  $A_{UL}$  (in units of normalized power excess) for 95% CL and the 1 (green), 2 (yellow) and 3 (cyan)  $\sigma$  bands for the background-only hypothesis.

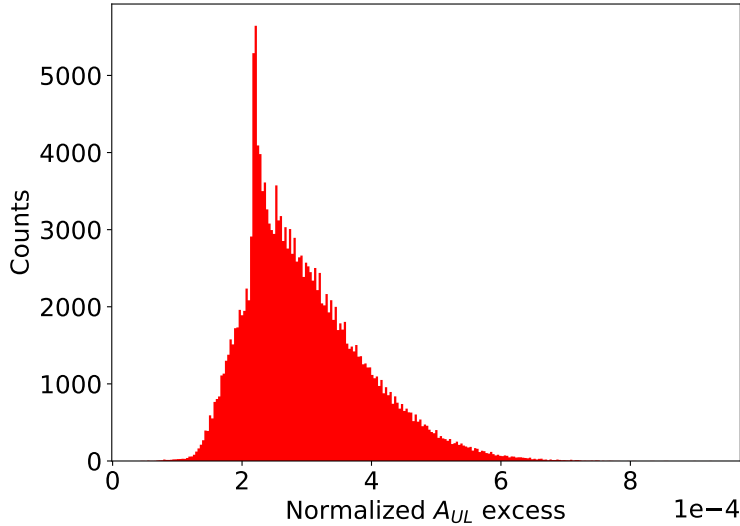


Figure 60

Histogram of the  $A_{UL}$  values for the simulated spectra.

<sup>16</sup>There was not tuning mechanism for the resonance frequency or the coupling of  $P_1$ . The magnet was not open at any point during the 103 hours of data taken for the analysis.

## 6.6 Loaded quality value calculation

In section 5.1 the electromagnetic behavior of the cavity was described showing that the first mode is the one that couples to the axion. At the beginning of the data-taking period, the S-parameters of the cavity were measured using a VNA. The VNA ports were connected to lines 1 (instead of the DAQ) and 2 of figure 26. Using the transmission parameter (see figure 61),  $S_{12}$ , the center frequency (8.384 GHz) of the first mode was qualitatively determined and used to set the LO frequency.

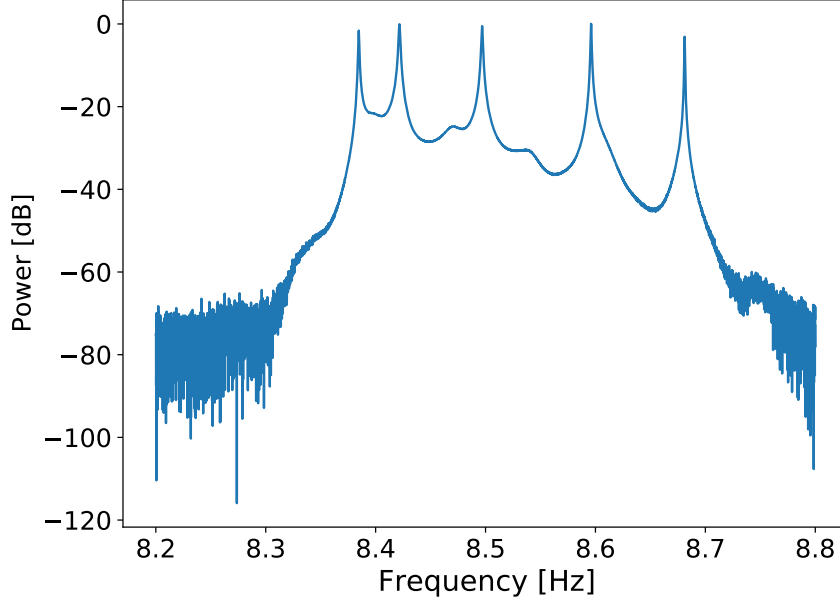


Figure 61

Transmission parameter  $S_{12}$  for the cavity installed in the CAST magnet at cryogenic temperatures.

As was mentioned in section 4.5 the loaded quality factor ( $Q_L$ ) of the cavity impacts the sensitivity of the experiment to an axion signal. The  $Q_L$  is defined as the energy loss per oscillation cycle. There are several ways to compute the Q-value of a cavity [125, 126]. For this work, the  $Q_L$  is going to be calculated using the following equation:

$$Q_L = \frac{f_0}{\Delta f}, \quad (100)$$

where  $f_0$  is the center frequency, and  $\Delta f$  is the resonance width at a value of  $A/\sqrt{2}$ <sup>17</sup>, where  $A$  is the amplitude given by the Lorentzian fit. The spectrum can be fitted using a Lorentzian function (see equation 101) to find  $f_0$  and  $\Delta f$ .

$$L(f) = \frac{A}{\pi} \frac{\frac{1}{2}\Gamma}{(f - f_0)^2 + (\frac{1}{2}\Gamma)^2}, \quad (101)$$

where  $\Gamma$  is the full width at half maximum.

During the data-taking period, the group did not have a VNA to monitor  $Q_L$ . A different, less conventional approach was used to measure  $Q_L$ . The  $\delta_{ik}^d$  magnet-on spectra were divided in groups of approximately 2.5 h. A Lorentzian fit was performed to the average spectrum

<sup>17</sup>This corresponds to a 3 dB drop in the logarithm scale, which is the most common method to measure the Q-value.



of each group. The spectra used for this calculation were the noise response of the system and not a standard VNA  $S_{12}$  measurement, where a signal is injected. Thus, the electronic background, systematic residual, and gain drifts affected the resonance structure. Figure 62 shows an example of one average spectrum and the fit range used to perform the fit. The fit range was selected for a small region where the peak was nearly symmetric, and the influence of systematic believed to be small.  $\Delta f$  was extracted using the fit function.

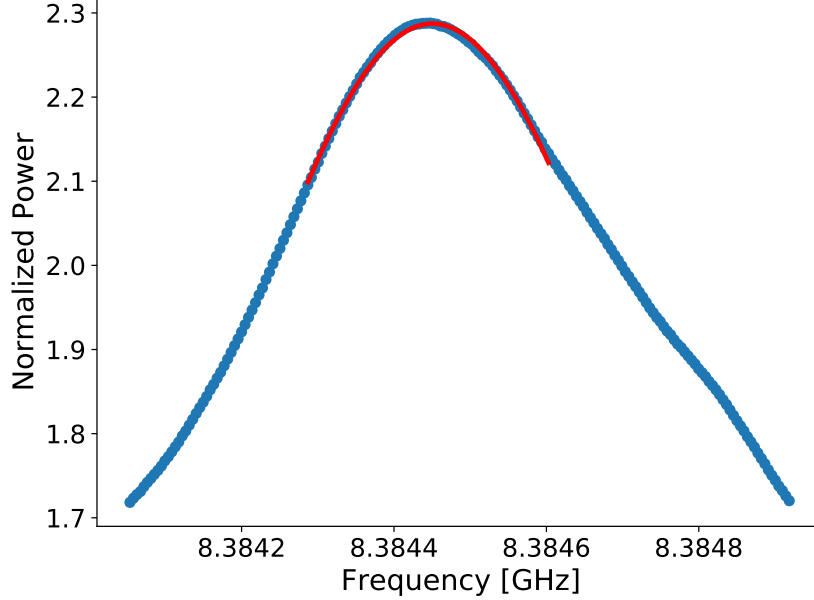


Figure 62

In blue, the average spectrum of one 2.5 h set of data fitted with a Lorentzian function (red line). A region where the peak was nearly symmetric was selected to perform the fit.

Figure 63 shows the  $Q_L$  for all sets. As mentioned in section 6.1 the magnet-on data sets were taken on different days. Thus possible temperature changes affecting the gain of the DAQ amplifiers, gain drifts, magnet movement could explain the difference seen between some days. The final value used for the exclusion limit was the mean value of all these points and yielded  $Q_L = 11009 \pm 483$ .

The value for the center frequency  $f_0$  was 8.384 GHz and the uncertainty of this value ( $< 7$  kHz) was determined by the accuracy of the internal frequency references of the DAQ described in section 5.3.5. It can be assumed that the frequency was known within less than two bins. This could potentially lead to an attenuation of the axion signal, but this effect was not considered in this analysis.

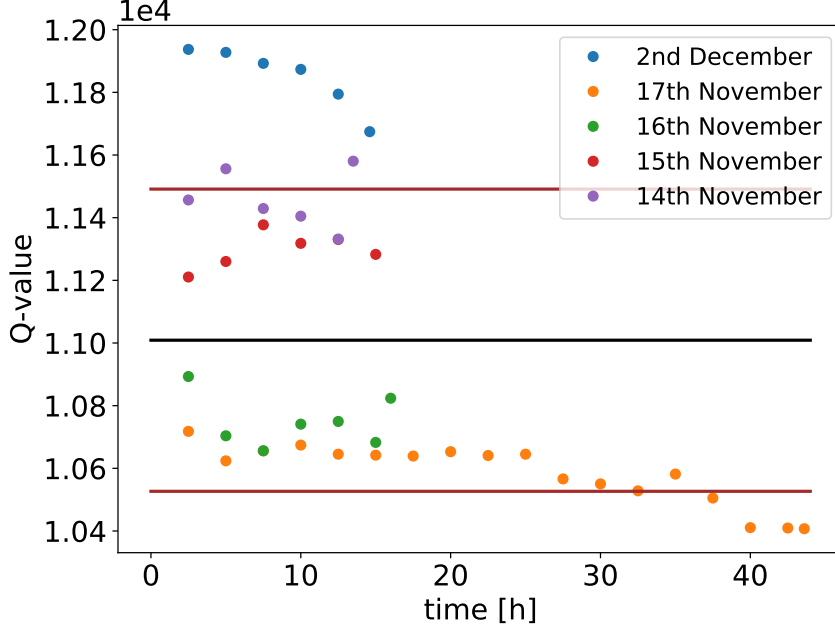


Figure 63

Q-values for all magnet-on sets of 2.5 h. The black line represents the mean value and the brown lines the  $1\sigma$  uncertainty band.

## 6.7 Cavity coupling

Another parameter that goes directly into the axion power (equation 51), is the cavity coupling. The coupling  $\beta$  is defined as the ratio of the power loss in the external circuit to that in the cavity and can be measured using the reflection coefficient,  $S_{11}$ , and the following relation:

$$\beta = \frac{1 \mp |S_{11}|}{1 \pm |S_{11}|}, \quad (102)$$

where  $S_{11}$  is in relative linear units<sup>18</sup>. The minus and plus signs in the nominator and denominator, respectively, correspond to an under-coupled configuration and the opposite to the over-coupled configuration<sup>19</sup>.

The cavity is said to be critically coupled to the receiver chain when the resistance of the cavity walls dissipates half the signal power, and the other half is coupled into the receiver. It is the coupling that maximizes the sensitivity to the axion signal (see appendix of [111]). Furthermore, the critical coupling is seen as a  $50 \, \Omega$  impedance at the LNA input port, minimizing the noise figure of the LNA.

To achieve critical coupling, the  $S_{11}$  was measured at room temperature for different antenna lengths until a minimum was found. The resulting  $S_{11}$  spectrum was compared with the  $S_{11}$  results from CST simulations where a parameter sweep was done to the antenna length. After simulations agreed with the measurements at room temperature, the simulations were done again at cryogenic temperature (2 K) to find the length needed for critical coupling at cryogenic temperatures. The value given by simulations was used for the port where the signal was extracted ( $P_1$ ).

<sup>18</sup>The relative linear units are defined as  $P_L = 10^{(P_{dB})/10}$ , where  $P_{dB}$  is the power given in dB.

<sup>19</sup>In the under-coupled configuration, more power is lost onto the cavity walls than extracted by the antenna, and the opposite for the over-coupled regime.

Simulations relied on available literature values for the RRR of copper at 2 K. Those ranged between 30 and 200<sup>20</sup>. 30 was selected for the calculation of the antenna length. The uncertainty on this value led to the fact that the  $P_1$  port ended up being under-coupled.

The configuration shown in figure 26 did not allow for an in situ measurement of  $\beta$ , because there was no possibility to bypass the amplifier. During a magnet intervention done after the data-taking period during which the cavity was left untouched, a RF switch was installed in the copper vessel between the cavity and the LNA. This switch is shown in the bottom right insert of figure 28. The RF switch allowed us to bypass the LNA and send a signal that reaches  $P_1$ . With this the  $S_{11}$  can be measured and used in equation 102 to calculate the coupling  $\beta$ .

Figure 64 shows the measured  $S_{11}$ . The effects of the incorrect feedthroughs discussed in section 5.3.3 causes the appearance of reflection peaks (like the one around 8.392 GHz), which were not related to the cavity resonances. The axion peak (at around 8.384 GHz) was easily identified because it has the same frequency as the one measured with the  $S_{12}$ . Another effect of the feedthroughs and cables is the introduction of frequency-dependent attenuation structure to the expected flat baseline at 0 dB<sup>21</sup>.

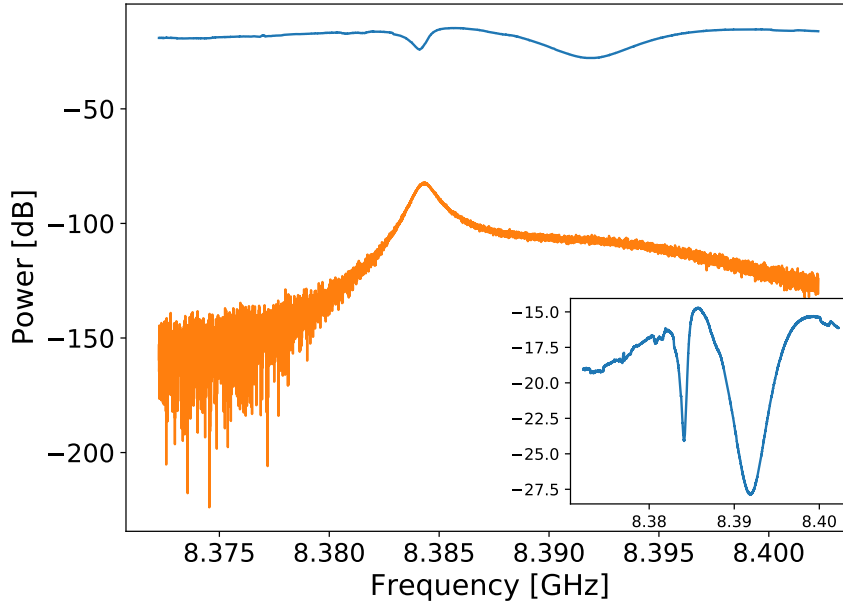


Figure 64

Comparison of in situ measurements of the reflection coefficient  $S_{11}$  (blue) and the transmission coefficient  $S_{12}$  (orange). The peak that appeared in both measurements corresponds to the cavity mode while the other one to the feedthrough reflection. Inset: Highlights the reflection coefficient  $S_{11}$  and shows that the reflection produced by the feedthrough has a deeper minimum ( $\approx -27.5$  dB) than the cavity mode ( $\approx -24$  dB), producing a significant distortion to the baseline.

For this work, a post-processing algorithm was designed to remove part of the offset. A region of points left and right of the resonance peak was selected (see left panel of figure 65).

<sup>20</sup>After the data-taking, tests were done at the CERN's main cryogenic lab to measure the RRR of the cavity. The results yielded an RRR of 10 and showed the antenna length has to provide a coupling of  $\beta = 0.25$  at room temperature to get critical coupling at cryogenic temperatures.

<sup>21</sup>If the VNA was properly calibrated, the  $S_{11}$  would have a flat baseline at zero. The amount of distortion and attenuation produced by the setup would have been improved if the proper feedthroughs were used, simplifying the post-processing analysis.

A linear spline interpolation was done to these points. The resulting function was added to the raw data to remove the offset. The right panel of figure 65 shows the result of this. The value of  $S_{11}$  was measured at the resonance frequency and plugged into equation 102 yielding  $\beta = 0.50 \pm 0.11$ . Thus, the antenna length was in the under-coupled regime and not critically coupled ( $\beta = 1$ ).

Conservatively it was assumed the offset removal procedure led to a 2 dB uncertainty on the determination of the  $S_{11}$  value at resonance. The obtained uncertainty on  $\beta$  was not dominating the systematic error on the axion-photon coupling upper limit (see section 6.9).

Even though the 2 K region where the cavity was located (see figure 26) was not modified during the magnet intervention, we confirmed that the measured coupling was similar to the one during the data taking period by computing the  $Q_L$  value after the installation of the switch. A Lorentzian (see equation 101) was fitted to the linear  $S_{12}$  curve, which yielded,  $f_0 = 8.384$  GHz,  $\Delta f = 0.7440$  MHz and  $Q_L = 11269$ . This  $Q_L$  agrees within uncertainty with the  $Q_L = 11009 \pm 483$  measured during data-taking, justifying the assumption that the coupling  $\beta$  was not significantly different before and after the data-taking period.

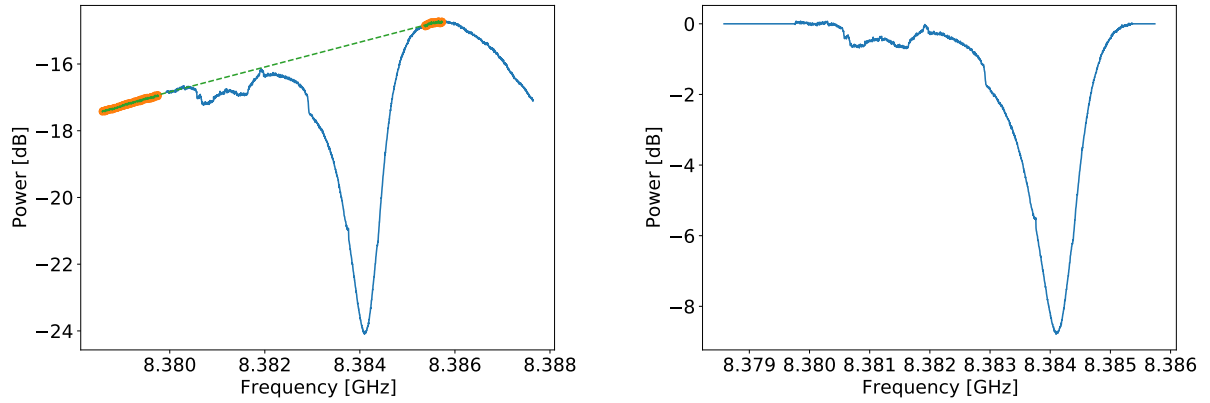


Figure 65: Left: In blue the  $S_{11}$  data points. In orange the range of points selected for the interpolation. In green the result of the linear spline interpolation. Right: Corrected  $S_{11}$  after the offset is removed.

## 6.8 Noise Temperature

The Y method allows the estimation of the noise temperature of a system when it is connected to a noise source. In that case, the Y factor is defined as the ratio of the power measured when the noise source is off ( $P_c$ ) and when it is on ( $P_h$ ). The Y factor together with the temperature when the source is on ( $T_h$ ) and when the source is off ( $T_c$ ) can fully determine the noise temperature of the system.

In our case, a noise source could not be connected directly to the cryogenic LNA. To compute  $T_h$  and  $T_c$ , one needs to estimate how much power was lost between the noise source and the input of the LNA. Knowing the losses of the cable and cavity, this can be done using the following equations:

$$T_{cable} = \frac{T_{amb} + T_{cryo}}{2}, \quad (103)$$

where  $T_{cable}$  is the average physical temperature of the cable,  $T_{amb}$  is the room temperature and  $T_{cryo}$  is the physical temperature of the cavity. The temperature at the point where the cable is connected to the weakly coupled port can be estimated with:

$$T_{cable-end} = \frac{T_{amb}}{L_{cable}} + \left(1 - \frac{1}{L_{cable}}\right) \cdot T_{cable}, \quad (104)$$

where  $L_{cable}$  are the cable losses, this is the estimation of the equivalent noise temperature at the end of the cable. It is composed of the noise coming from the input (attenuated) plus the noise generated by the losses of the cable.

Only a tiny fraction of this noise is coupled to the cavity because the cavity input port is weakly coupled. At the output of the cavity, one has the thermal noise generated by the cavity at  $T_{cryo}$  plus the noise coupled through the weakly coupled port.  $T_c$  can be computed using:

$$T_c = \frac{T_{cable-end}}{L_{cavity}} + \left(1 - \frac{1}{L_{cavity}}\right) \cdot T_{cryo}, \quad (105)$$

and  $T_h$  using:

$$T_h = T_0 \cdot 10^{\frac{ENR_{eff}}{10}} + T_c, \quad (106)$$

where  $ENR_{eff}$  is the effective excess of noise ratio (ENR).

To carry out this measurement an Agilent 346B noise source, which has an ENR of 15 dB was connected to a feedthrough of the warm flange of the line connected to the weakly coupled port of the cavity (see figure 26). This equipment was available just for one measurement at end of the data-taking period. For a  $T_{amb} = 297$  K and  $T_{cryo} = 2$  K, the average physical temperature of the cable was  $T_{cable} = 149.5$  K (determined using equation 103).

The cable losses were estimated using the reflection coefficient  $S_{22}$ . The original signal sent by the VNA traveled to  $P_2$  and back, getting attenuated in the process. Since the  $S_{22}$  is a power ratio, the measurement of this value represented the losses of the line. The left panel of figure 66 shows the results of this measurement. Once again, the effects of the feedthrough can be seen in the spectrum featuring many reflection peaks.

An average value of -18 dB was taken to compute the cable losses. There was a 6 dB attenuator ( $L_{att}$ ) attached to the cold flange of the line connected to the weakly coupled port of the cavity, responsible for a total attenuation of 12 dB (signal went twice through the attenuator). The remaining 6 dB attenuation was produced by the cables when the signal went back and forth. This means that the total cable attenuation is about 3 dB. One can compute the temperature at the end of the cable using equation 104<sup>22</sup>. This gives a value of 223 K.

The cavity cannot be isolated from the rest of the equipment inside the CAST magnet, so it was not possible to evaluate the cavity losses from a  $S_{12}$  measurement. They were instead estimated using CST simulations shown in the right panel of figure 66. A value of 23 dB (corresponding to the value at the resonance frequency) was used for the cavity losses. For an  $ENR_{eff}$  of -17 dB ( $ENR_{eff} = ENR - L_{cavity} - L_{att} - L_{cable}$ ), the final results yielded  $T_c = 3.11$  K and  $T_h = 8.9$  K.

---

<sup>22</sup>Losses have to be plugged in the relative linear unit, not in dBm

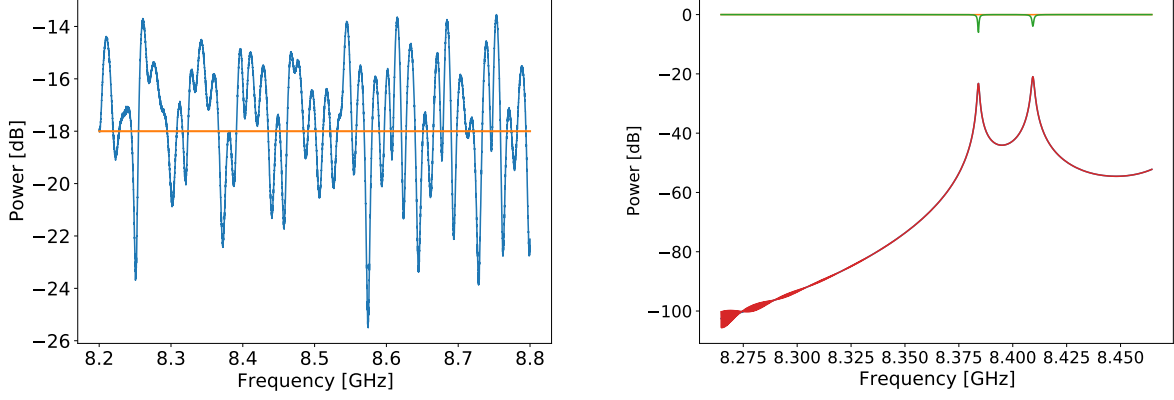


Figure 66: Left: In situ measurement of the reflection coefficient  $S_{22}$  (blue). The orange line represents the  $-18$  dB average value taken as the line losses.. Right: Simulated S-parameters for the cavity. In red the transmission coefficient, in green the reflection coefficient for the under coupled port ( $P_1$ ) and in yellow the reflection coefficient for the weakly coupled port ( $P_2$ ).

The left panel of figure 67 shows the measured power when the noise source was on and off. The right panel of figure 67 shows the Y factor computed using:

$$Y = \frac{P_h}{P_c}. \quad (107)$$

At the time of this measurement, the analysis algorithm was not developed, and a different LO frequency (8.244 GHz) was used. This LO frequency centered the peak in an IF region with noisy bins. These bins had to be removed, and a fit function was implemented to obtain the Y factor for all frequencies (see right panel of figure 67).

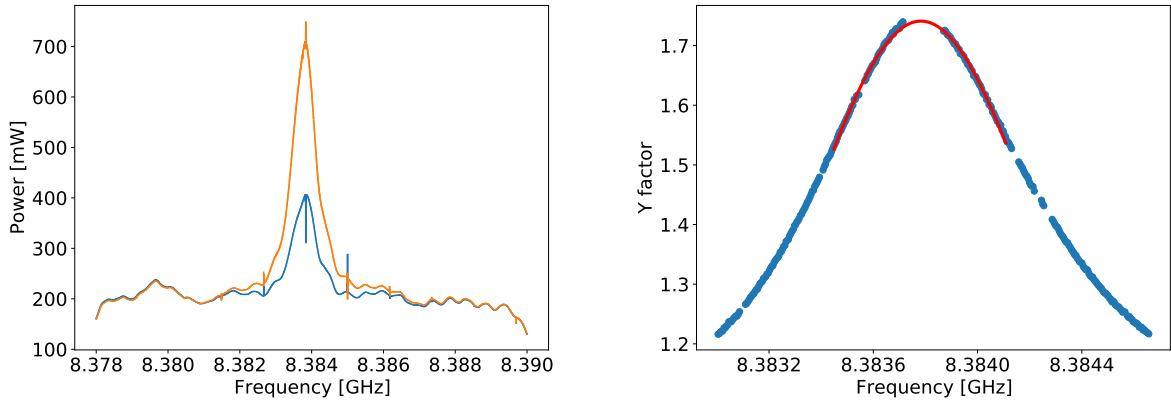


Figure 67: Left: Power measured with the DAQ when the noise source is off (blue) and on (orange). Right: Y factor and corresponding fit function (red line).

Several assumptions were made to estimate the noise temperature of the system. We give here an estimate of the uncertainties associated with the different parameters involved. The uncertainties of the ENR and the attenuator can be extracted directly from the datasheet of the noise source and attenuator, respectively:

- $\Delta ENR = 0.3$  dB

- $\Delta L_{att} = 0.3 \text{ dB}$

The uncertainty for the cable temperature is complicated to compute rigorously. One would need to know the thermal distribution along the five meters of cables and the cable temperature in the different sections. From the results of the rigorous analysis of a cable's thermal distribution [127,128], it can be assumed that the approximation used in this analysis has an uncertainty of about 5%. It is also important to mention that this uncertainty did not have a big influence on the final result. For example, changing the uncertainty from 5% to 50% changes the uncertainty of  $T_c$  from 0.09 to 0.21 K, but its effects on the system temperature were negligible because it was not the dominant factor in the error propagation.

- $\Delta T_{cable} = 5\%$

Given the fluctuations observed in figure 66 it was assumed that the cable losses could be assessed with an uncertainty of 1 dB. However, no additional investigations were performed to have a better estimate of this uncertainty. For future measurements, it is critical to have a good measurement of the cable losses because they have a big influence is the uncertainty of the system temperature. For example, an uncertainty of 0.3 dB instead of 1 dB lowers the uncertainty of the noise temperature from 2.0 to 1.6 K.

- $\Delta \text{Cable-Losses} = 1 \text{ dB}$

The uncertainty of the Y-factor depends critically on the linearity of the system. Some systematic effects could influence the measurement's uncertainty, but the uncertainties given by the fit were used for the calculation.

- $\Delta Y = 0.0084$  (fit uncertainty for the central frequency)

All these uncertainties were used together with the error propagation formula to calculate the uncertainty of the system noise temperature.

The noise temperature of the cryogenic LNA was computed using:

$$T_n = \frac{T_h - Y \cdot T_c}{Y - 1}, \quad (108)$$

The system noise temperature ( $T_{sys}$ ) is the sum of the amplifier plus the cavity temperature. Figure 68 shows the results of this calculation. To compute the noise power ( $P_N = k_B T \Delta \nu$ ), the following temperature was used:

$$T_{sys} = T_c + T_n = (3.1 \pm 0.087) \text{ K} + (4.7 \pm 2.3) \text{ K} = (7.8 \pm 2.0) \text{ K}, \quad (109)$$

where the values given here are at the center frequency of 8.38445 GHz.

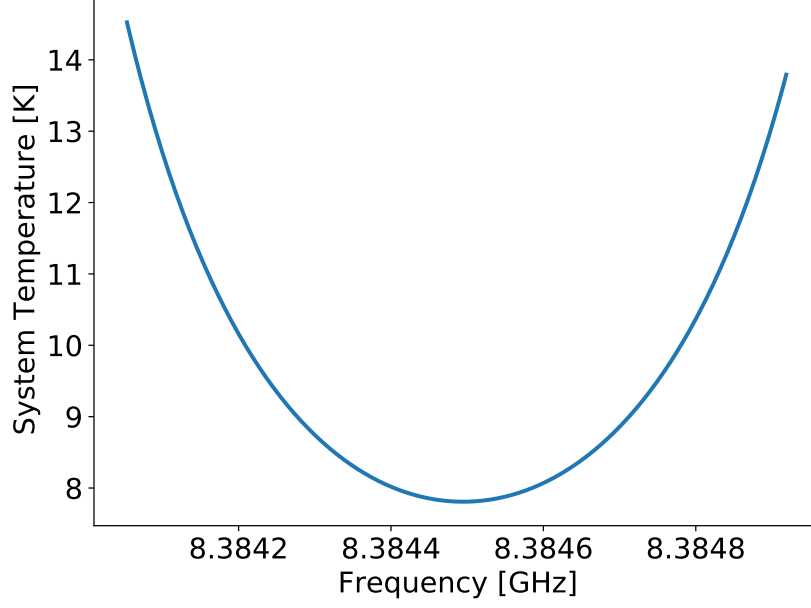


Figure 68  
System temperature  $T_{\text{sys}}$ .

## 6.9 Exclusion limit

The unitless  $A_{UL}$  values shown in figure 59 can be converted back to power units using the following equation:

$$P_a = A_{UL} \cdot P_N = A_{UL} k_b T_{\text{sys}} \Delta\nu. \quad (110)$$

The exclusion limit to the axion-photon coupling  $g_{a\gamma}$  can be computed with<sup>23</sup>:

$$P_a = g_{a\gamma}^2 \rho_{\text{DM}} \frac{\beta}{1 + \beta} \frac{1}{m_a} B^2 V Q_L G^2, \quad (111)$$

The parameters to compute the exclusion limit of the axion-photon coupling are summarized in table 9. The geometric factor and cavity volume were calculated (by J.M. García Barceló) using CST simulations. The  $\eta$  parameter represents the signal attenuation factor through the 1 m cable connecting the cavity output port ( $P_1$ ) and the LNA, which was estimated from the datasheet<sup>24</sup>.

<sup>23</sup>This equation was introduced and discussed first in section 4.5.1

<sup>24</sup>The vendor provides the cables losses at room temperature only, which were 1.24 dB/m at 8.4 GHz, from experience it can be assumed that the losses were half of this value at cryogenic temperatures.



| Parameter        | Value                     |
|------------------|---------------------------|
| $\Delta\nu$      | 4577 Hz                   |
| $T_{\text{sys}}$ | $(7.8 \pm 2.0)$ K         |
| $Q_L$            | $11009 \pm 483$           |
| $\beta$          | $0.50 \pm 0.11$           |
| $B$              | $(8.8 \pm 0.0088)$ T      |
| $\rho_{DM}$      | $0.45 \text{ GeVcm}^{-3}$ |
| $G^2$            | 0.65                      |
| Volume           | 0.03 L                    |
| $\eta$           | 0.83                      |

Table 9: Parameter used to compute the exclusion limit of the axion-photon coupling.

The output power of the cavity is frequency-dependent, determined by the structure of the resonance peak. To account for this effect, the computed  $P_a$  values were divided by a Lorentzian distribution (see equation 101) constructed using the measured center frequency ( $f_0 = 8.384$  GHz) and  $Q_L$ .

An exclusion limit of  $g_{a\gamma} \gtrsim 4 \times 10^{-13} \text{ GeV}^{-1}$  within  $34.6738 \mu\text{eV} < m_a < 34.6771 \mu\text{eV}$  was obtained. Figure 69 compares the exclusion limit achieved by this work with other haloscope experiments and CAST’s solar axion results.

The level of systematic uncertainties is assessed by error propagation of the experimentally determined physical quantities involved in equation 111. Those uncertainties are represented as the green band in the insert of figure 69. They account for less than 10% error, dominated by the knowledge of the noise temperature, and were thus neglected in the estimation of the exclusion limit.

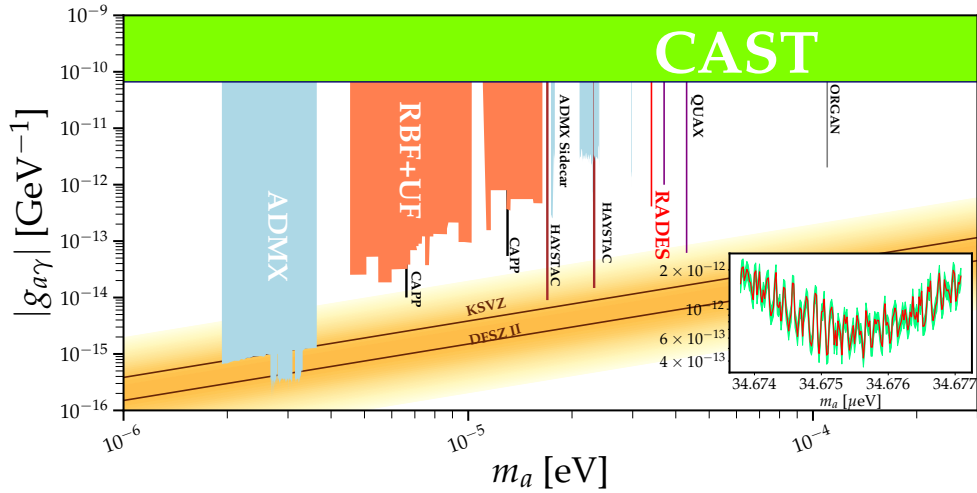


Figure 69: Axion-photon coupling vs axion mass phase-space. In dark red the CAST-RADES axion-photon coupling exclusion limit with 95% credibility level presented in this manuscript. The coupling-mass plane is shown in natural units for consistency with most literature in the field. Other haloscope results: RBF [96], UF [97], ADMX and ADMX-SideCar [100,105,129], CAPP [103,104], HAYSTAC [95,130], QUAX [106,107] and ORGAN [108] and the CAST solar axion results [67] are plotted for comparison, see [131] for a full list of references and the raw source for the plot. Inset: Zoom-in of the parameter range probed in this work ( $34.6738 \mu\text{eV} < m_a < 34.6771 \mu\text{eV}$ ), where the green region represent the uncertainty of the measurement.

## 7 The alternating irises filter

### 7.1 Mode mixing and the alternating irises solution

The RADES group developed a formalism (introduced in 5.1) to guide the design of filter-like cavities to perform direct searches for axions. A five sub cavity prototype was built and took data in the CAST dipole magnet. The sensitivity achieved with this prototype is still far away from the benchmark axion models (see section 2.4). Given that the sensitivity to axion-photon coupling is proportional to  $t^{-1/4}$  [111], an enormous amount of time would be required to reach the axion benchmark models using the same cavity.

The advantage of RADES's geometry compared to the typical cylindrical cavity used for axion searches is that RADES can in principle connect  $N$  cavities, increasing the volume without decreasing the resonance frequency. Moreover, the CAST magnet provides the perfect environment to increase in volume because of its 9 m long bore. Other experiments try to grow in volume by phase matching and combining the signal of different cavities (like the CAST-CAPP experiment described in section 4.5).

The main focus of the RADES R&D project after the 2018 data-taking period was to increase the volume of the cavity. However, it was not as simple as putting more sub cavities together. In section 5.1 it was shown that the resonance frequency splits in  $N$  frequencies when the sub cavities are connected. In the case of the five sub cavities filter, one has five modes, as it is seen in figure 61. The more cavities, the more modes. The cutoff frequency of the rectangular cavity is determined by its width and height, which remain unchanged when adding more cavities. Thus, adding more cavities leads to a smaller spacing between modes which can result in mode-mixing.

The problem of mode mixing resides in the fact that the geometric factor (which is proportional to the scalar product of the magnetic and the electric field, see section 4.5) is almost zero for all the modes except for the one that couples to the axion (see table 1). One needs to identify and separate the axion mode from other modes properly. Let us derive the behavior of the distance between the first and the second mode in the  $N$  number of sub cavities connected with inductive irises. The eigenfrequencies are given by equation 73, the frequency difference between the first two modes is:

$$\Delta\omega_{12} = \omega_1 - \omega_2 \sim \frac{K}{\Omega_q} \left( \cos\left(\frac{\pi}{N+1}\right) - \cos\left(\frac{2\pi}{N+1}\right) \right) \sim \frac{K}{\Omega_q} \frac{3\pi^2}{2(N+1)^2}, \text{ for } N \gg 1. \quad (112)$$

$\Delta\omega_{12}$  is inversely proportional to  $2(N+1)^2$ . For the central mode ( $\omega_c$ ), the behavior is different. The frequency difference with the next mode is:

$$\Delta\omega_{c,c+1} = \omega_{N/2} - \omega_{(N/2)+1} \sim \frac{K}{\Omega_q} \frac{\pi^2}{2(N+1)}, \text{ for } N \gg 1, \quad (113)$$

which is inversely proportional to  $2(N+1)$ .

Thus, the modes are better separated towards the center, which is a key point to consider when scaling in volume and thus in the number of sub cavities. For an odd number of sub cavities the center mode is identified by  $c = (N+1)/2$ , for an even number of cavities the center mode can be either  $c = N/2$  or  $c = N/2 + 1$ .

For the five sub cavities prototype, the same coupling value  $\bar{K}$  is used between all cavities. The eigenvectors of the electric field are independent from  $\bar{K}$ , but the eigenfrequencies are not (see equation 73). When a negative  $\bar{K}$  is used, the first mode couples to the axion. In the case of a positive  $\bar{K}$ , the highest frequency mode couples to the axion.

A new solution has to be introduced to make the center mode couple to the axion. One is looking for a basis transformation of the matrix  $\mathbb{M}$  (see equation 65) that preserves the eigenvalues given in equation 73. Note that if  $\vec{e}_i$  is the eigenvector of  $\mathbb{M}$ , then  $T\vec{e}_i$  is an eigenvector of  $\mathbb{M}_T = T^{-1}\mathbb{M}T$  with the same eigenvalue. The idea is to find a suitable matrix  $T$  that transforms the center mode ( $\vec{e}_c$ ) to the mode that couples to the axion,  $T\vec{e}_c = \vec{e}_1$  [132].

The key observation to find a suitable  $T$  is that for the center mode, the sign of the electric field of the sub cavities changes every two cavities because the factor  $(\pi i)/(N+1)$  is proportional to  $\pi/2$  for the central mode.

The simplest solution to couple  $T\vec{e}_c$  to the axion is to make  $T$  diagonal with entries  $+1, -1$  so that the signs of  $T$  compensate any sign change of  $\vec{e}_c$ .  $T\vec{e}_c$  will have all entries either positive or negative and thus represents a mode with all cavities in phase. For an even number of cavities with the center mode  $c = N/2 + 1$ , the  $T$  matrix is given by:

$$T = \text{diag}\{1, -1, -1, 1, 1, -1, \dots\}. \quad (114)$$

From this follows that:

$$T\mathbb{M}T^{-1} = \begin{pmatrix} \Omega_1^2 & -\bar{K} & 0 & 0 & 0 & 0 \\ -\bar{K} & \Omega_2^2 & \bar{K} & 0 & 0 & 0 \\ 0 & \bar{K} & \Omega_3^2 & -\bar{K} & 0 & 0 \\ 0 & 0 & -\bar{K} & \Omega_4^2 & \bar{K} & 0 \\ 0 & 0 & 0 & \ddots & \ddots & \ddots \\ 0 & 0 & 0 & 0 & \bar{K} & \Omega_N^2 \end{pmatrix} \equiv \mathbb{M}_T. \quad (115)$$

All the eigenvectors of  $\mathbb{M}_T$ ,  $\mathbb{M}_T \vec{t}_i = \lambda_i \vec{t}_i$ , can be calculated using a weighing factor with alternating signs of the  $T$  diagonal [132],  $\vec{t}_i = T\vec{e}_i$ ,

$$(t_i)_q = \frac{(-1)^{s(q)}}{\sqrt{(N+1)/2}} \sin\left(\frac{i\pi}{N+1}q\right) \quad , \quad s(q) = \text{floor}\left(\frac{q}{2}\right), \quad (116)$$

where the floor function gives as output the greatest integer less than or equal to the real number used as input. One can compute the overlap of the axion mode with the central mode [132]:

$$\vec{e}_1 \cdot \vec{t}_c = \frac{1}{\sqrt{N(N+1)/2}} \sum_q (-1)^s \sin\left(\frac{c\pi}{N+1}q\right) \xrightarrow{N \rightarrow \infty} \begin{cases} 1/\sqrt{2} & (N - \text{odd}) \\ \sim 0.900 & (N - \text{even}) \end{cases}. \quad (117)$$

One can see that an even number of sub cavities favors the overlap between the axion mode and the center mode. Also, from the off-diagonal terms of the matrix in equation 115 one can see that the sign of the coupling between the sub cavities is alternating. This can be realized by joining the cavities using alternating capacitive and inductive irises. The capacitive irises provide a positive coupling ( $\bar{K}$ ) between the sub cavities, while the inductive irises provide a negative one. A more detailed calculation can be found in [132]. A similar process as the one described in section 5.2 was followed for the construction of a six sub cavities filter with alternating irises.

## 7.2 The 6 sub cavities filter with alternating irises

The six sub cavities filter with alternating irises was designed using CST simulations. Figure 70 shows a sketch of the irises structure. It was manufactured in Valencia, Spain, using stainless steel 316LN, a  $\sim 30 \mu\text{eV}$  copper coating layer was applied at CERN's coating lab. Figure 71 shows the manufactured filter and table 10 summarizes the dimensions used for

the construction. Figure 72 shows the electric field distribution of the six modes. Note how the center mode (mode 4) is the one that has the electric field pointing in the same direction for all sub cavities and thus is the one that couples to the axion. Table 11 outlines the frequency, geometric factor, and expected  $Q_0$  for the cavity modes.

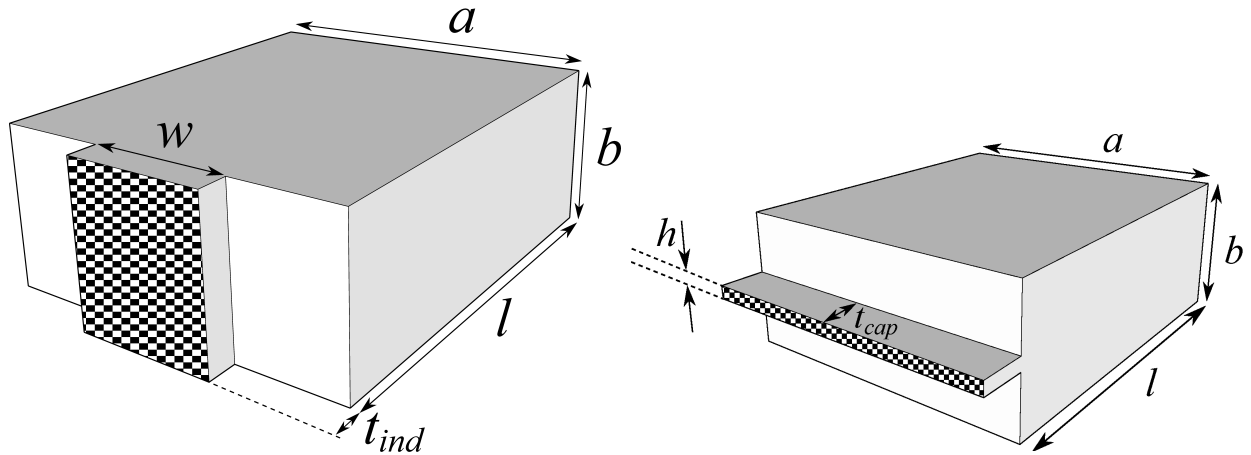


Figure 70: Left: Sketch of the inductive iris. Right: Sketch of the capacitive iris. The hatched zones indicate the connection planes between the irises and the cavity. Figure taken from [132].

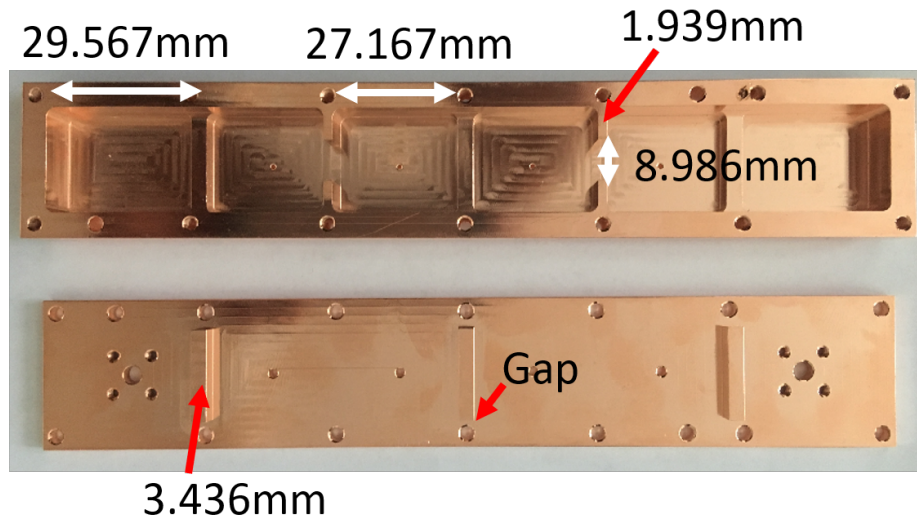


Figure 71: 6 sub cavities filter with alternating irises. Top: The filter's main body consisting of 6 sub cavities, 2 inductive irises and the bottom part of 3 capacitive irises. Bottom: cover plate, the 3 rectangles represent the top part of the capacitive irises.

| Parameter                               | $T = 2$ K<br>(design) | $T = 300$ K<br>(design) | $T = 300$ K<br>(manufactured) |
|-----------------------------------------|-----------------------|-------------------------|-------------------------------|
| cavity width ( $a$ )                    | 22.860                | 22.929                  | 22.860                        |
| cavity height ( $b$ )                   | 10.160                | 10.191                  | 10.272                        |
| length external cavities ( $l_{ext}$ )  | 29.400                | 29.489                  | 29.567                        |
| length internal cavities ( $l_{int}$ )  | 27.000                | 27.081                  | 27.167                        |
| inductive iris width ( $w$ )            | 8.900                 | 8.987                   | 8.986                         |
| inductive iris thickness ( $t_{ind}$ )  | 2.000                 | 2.006                   | 1.939                         |
| capacitive iris height ( $h$ )          | 1.000                 | 1.063                   | 1.019                         |
| capacitive iris thickness ( $t_{cap}$ ) | 3.500                 | 3.511                   | 3.436                         |
| capacitive iris gap ( $g$ )             | 1.100                 | 1.104                   | 1.192                         |

Table 10: Design physical dimensions (in mm) of the filter at 2 K and 300 K (first two columns). The third column indicates the physical dimensions at room temperature of the manufactured cavity measured with an uncertainty of  $3\ \mu\text{m}$ .

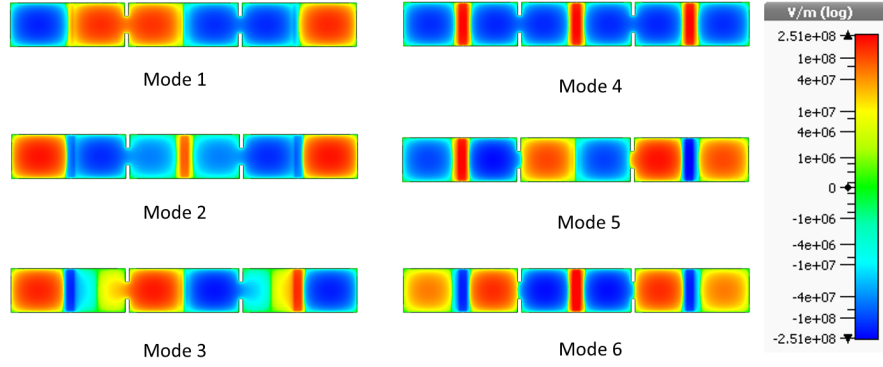


Figure 72: Electric field pattern of the 6 cavity modes. The fourth mode, in which the electric field in all sub cavities is in phase, couples to the axion. Figure taken from [132].

| Mode ( $m$ ) | Resonant frequency (GHz) | $G_i^2$                | $Q_0$ (at 300 K) |
|--------------|--------------------------|------------------------|------------------|
| 1            | 8.298                    | $1.820 \times 10^{-9}$ | 7300             |
| 2            | 8.319                    | $1.257 \times 10^{-3}$ | 7448             |
| 3            | 8.395                    | $1.007 \times 10^{-8}$ | 7426             |
| 4            | 8.508                    | 0.534                  | 6957             |
| 5            | 8.613                    | $1.511 \times 10^{-8}$ | 7563             |
| 6            | 8.698                    | $3.418 \times 10^{-5}$ | 7597             |

Table 11: Electromagnetic properties of the cavity modes given by CST simulations.

The S-parameters were measured at room temperature using a VNA to compute the  $Q_L$  and  $Q_0$  values. Figure 73 shows the result of the  $S_{12}$  measurement. The  $Q_L$  was measured in the same way as explained in section 6.6 and the coupling  $\beta$  as explained in section 6.7.  $Q_0$  can be calculated using equation 49. The initial results yielded  $Q_L = 1125$ ,  $\beta = 0.44$  and  $Q_0 = 1620$ . The measured  $Q_0$  was more than a factor 4 lower than the expected  $Q_0 \approx 7000$  given by simulations.

A measurement of the height across the whole length of the manufactured structure showed a 2 mm difference between the maximum and the minimum values. Some mechanical tests<sup>25</sup> revealed that the contact between the main body and the cover plate was not good. To increase the pressure and improve the contact, the filter was modified with the addition of more screw holes (from 14 to 50 screws). The difference between the two  $S_{12}$  measurements is shown in figure 73. After the screws were added, the contact improved and led to  $Q_0 = 2340$  which, was still almost a factor 3 away from the expected value.

Due to machining constraints, the capacitive irises manufactured at the cover plate (see figure 71) have a 1 mm gap<sup>26</sup> allowing the introduction of the cover plate on top of the main body. Further studies suggest that this gap might allow too much current to flow between the sub cavities and be a possible reason for the low  $Q_0$ .

The CAST schedule dictating the time slot when a cavity can be installed in the magnet, the construction of a 1 m long cavity using alternating irises was started before the complete characterization of the six sub cavities filter was finished. The workshop was aware of the importance of good contact between the cover plate and the main body, but the possible issue with the gap had not been identified before the installation of the 1 m long cavity at CAST.

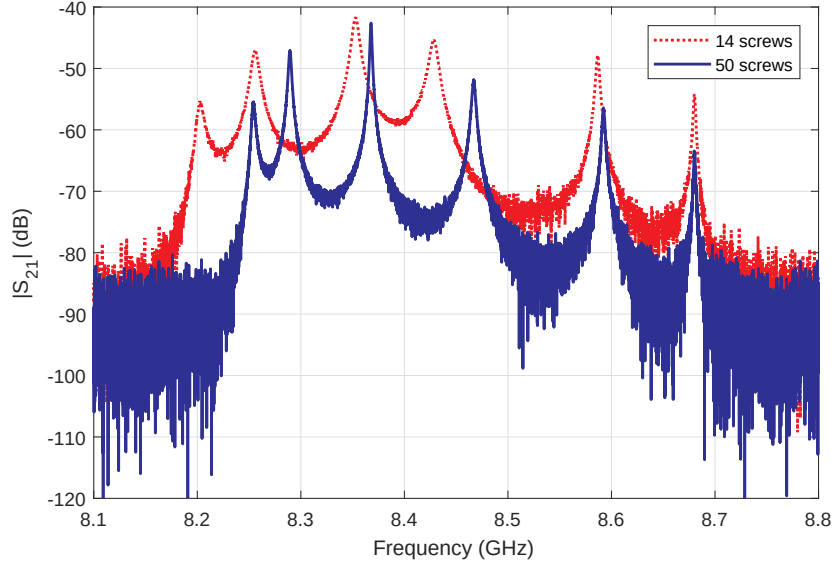


Figure 73: Transmission coefficient  $S_{12}$  of the 6 sub cavities filter for a different number of screws connecting filter and cover. Comparison of the response with 14 screws (dotted line) and 50 screws (solid line). Simulation suggested that the 1 mm gap at the capacitive irises could be a source of problems. Varying the size of this gap changes the coupling at the capacitive irises and the frequencies of the modes. Figure taken from [132].

### 7.3 The 1m long cavity at CAST

Using the dimensions shown in table 10, a 30 sub cavities filter, almost 1 m long, with alternating irises, was constructed in CERN's central workshop. Figure 74 compares this new cavity with the one used in 2018. Due to delays in the production and the tight schedule of data taking of the CAST's collaboration, there was not enough time for the necessary checks before the magnet had to be closed.

<sup>25</sup>Clamps and silver painting were used to put more pressure and improve the contact. These were, however, just temporary solutions.

<sup>26</sup>The gap was between the inner face of the cavity body and the capacitive irises.

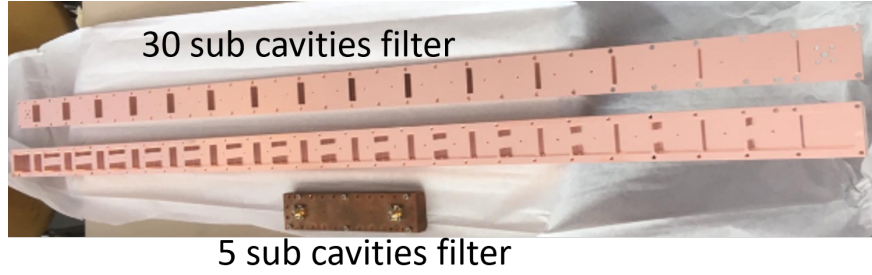


Figure 74: Comparison of the 5 sub cavities filter (bottom) with the  $\sim 1$  meter long cavity (top).

The main goal of this section is to showcase the potential and the challenges of producing a long cavity like this one. The analysis results will be presented elsewhere. The 1 m long cavity was installed in the summer of 2019 following the same arrangements of connections as the one shown in figure 26 with the only difference that an RF switch was installed in the copper vessel between the cavity and the amplifier. The cryogenic LNA was damaged during the quench tests, and the magnet was not opened again until 2020, so it was not possible to take any data in 2019. At the beginning of 2020, the LNA was changed with a Low Noise Factory LNF-LNC6-20B amplifier with a 36 dB gain operating between 6 and 20 GHz<sup>27</sup>.

Figure 75 shows the result for the  $S_{12}$ . As discussed above, 30 modes were expected, but only 25 were observed. There were two possible explanations for this observation. Mode-mixing can appear, particularly at the low and high-frequency values and when the Q values are worse than expected, as discussed above. The second explanation could be linked to the antenna's length, which defines the coupling at the ports. The length of the antenna of one of the ports was selected to ensure a critical coupling to the axion mode. But other modes can be distorted by the antenna to a level where they cannot be measured.

In principle, this would not be a problem because the other modes are not relevant, but it can lead to difficulties finding the center peak for a large number of sub cavities. The first approach to find the center peak was to compare the measurement with simulations. As figure 75 shows, the comparison with the CST simulation made before the production of the cavity did not allow to identify the peak of interest efficiently. This simulation did not take into account imperfections in the contact between the cavity body and the cover plate and the gaps in the conductive irises mentioned above.

Further investigations using simulations were done by J.M. García Barceló using the fact that the set of peaks can be divided in two, the center peak being the last peak of the first set, the furthest away from its next neighbouring peak. This enables the identification of the peak of interest located at a frequency of 8.37 GHz.

Initially, data was taken around another mode located at 8.4 GHz which was wrongly associated with the peak of interest. The antenna length was also determined for critical coupling based on that mode, resulting in a weak coupling for the correct axion mode. Because of the predetermined magnet opening schedule, this could not be rectified.

<sup>27</sup>We would like to thanks our MADMAX colleagues of the MPI in Munich who kindly lend us the amplifier.



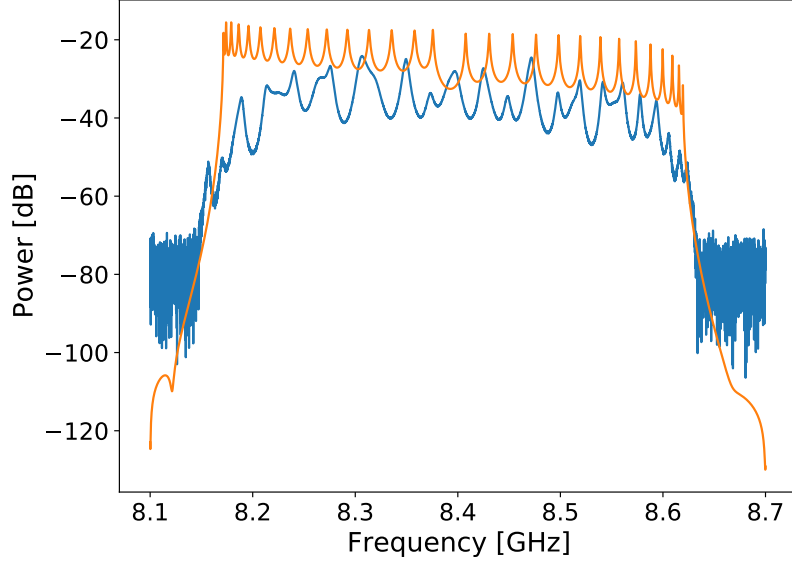


Figure 75: Transmission coefficient  $S_{12}$  of the 30 sub cavities filter (blue) compared to the simulations (orange) done by J.M. García Barceló.

## 7.4 Prospects for the 2020 data-taking period

Approximately 15 days of magnet-on data were recorded in 2020<sup>28</sup>. Table 12 summarizes the preliminary results (described in appendix D) of the 2020 data-taking period. Similar to the previous run, the geometric factor and volume were calculated using CST simulations. At the moment, a very weakly coupled port with  $\beta = 0.1$  was assumed (see appendix D) for the prospects. Using these values, one can estimate the sensitivity of the experiment.

| Parameter        | Value                     |
|------------------|---------------------------|
| $\Delta\nu$      | 4577 Hz                   |
| $T_{\text{sys}}$ | 350 K                     |
| $Q_L$            | 1170                      |
| $\beta$          | 0.1                       |
| $B$              | 8.8 T                     |
| $\rho_{DM}$      | $0.45 \text{ GeVcm}^{-3}$ |
| $G^2$            | 0.56                      |
| Volume           | 0.224 L                   |
| SNR              | 5                         |

Table 12: Parameter used to compute the sensitivity to an axion signal for the 1 m long cavity.

Even with this sub-optimal state and assuming, for an integration time of 15 days the expected sensitivity is better than the current limit at this mass. Moreover, the main issues were already identified and we are confident that both  $\beta$  and  $T_{\text{sys}}$  can be improved to the level of the previous run if not better.

The main challenge of the current setup is the fabrication of the cavity. With this prototype, it was seen that the alternating irises have to be done in another way. New

<sup>28</sup>In 2021 more data could not be taken because of a still unexplained gain loss in the setup.



designs have been studied, where each sub cavity and irises are machined separately and then screwed together. With proper contact, there is no reason to expect a  $Q_L$  lower than the one of the five sub cavities filter.

Figure 76 shows the prospects for the 1 m filter with the measured values and 5 h of data as well as 15 days of data. It also showcase the potential of the filter for a  $\beta = 1$ ,  $Q_L = 10000$  and  $T_{\text{sys}} = 10$  K for a data-taking period of 90 days. With just 3 months of data the 1 m long cavity has the potential of reaching the axion benchmark models.

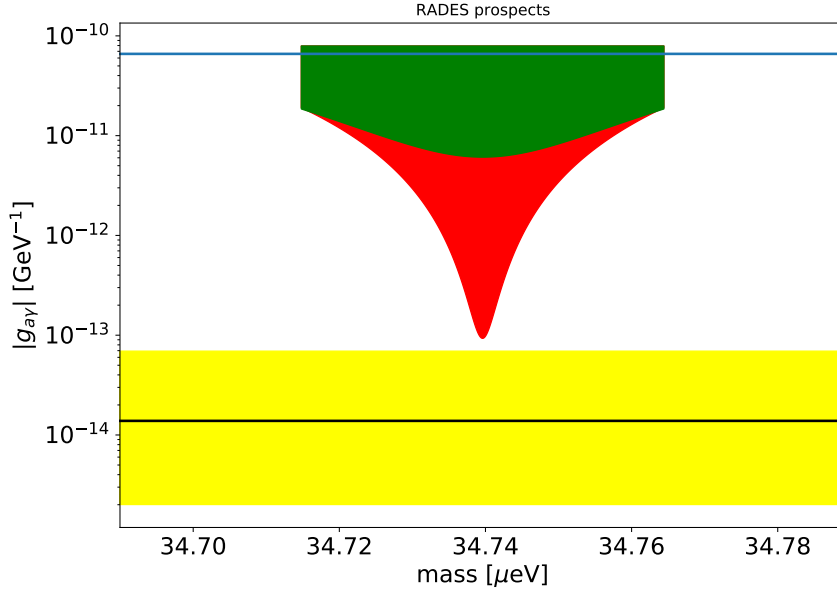


Figure 76: Prospects for the axion-photon coupling sensitivity. In green the prospects for 15 days of integration time. In red the prospects for the case where  $\beta = 1$ ,  $Q_L = 10000$  and  $T_{\text{sys}} = 10$  K and an integration time of 90 days. The blue line is the current best limit for this mass range set by CAST. The black line and yellow region are the KSZV model and band respectively.

## 8 Conclusions and Outlook

The possible existence of a new particle called the axion was a result of the Peccei-Quinn mechanism used as a solution to the strong CP problem. The KSZV and the DFSZ axion models provide characteristic axion-photon coupling relations.

There is plenty of evidence suggesting the existence of a new type of matter called dark matter. The gravitational interaction of dark matter with ordinary matter is expected to be similar to the gravitational interaction between ordinary matter. However, dark matter has a weak (if any) electromagnetic and strong interaction with ordinary matter. To account for some unexplained phenomena in the universe, dark matter must be massive, neutral, and stable on cosmological scales.

A non-thermal production mechanism called the misalignment mechanism produces axion at non-relativistic velocities by spontaneous symmetry breaking of the PQ symmetry, making the axion an excellent dark matter candidate. Depending on the time at which the spontaneous symmetry breaking happens, if axions constitute a part of all of the dark matter, a large parameter space in both mass and couplings to SM particles is still available.

One way of searching for axions is through their conversion into a photon in the presence of a strong magnetic field. The haloscope technique takes advantage of a cavity resonating at the axion frequency to enhance the power of the converted photon signal.

The haloscope technique has been used over the last 20 years to search for axions. However, most of the experiments have searched for masses below  $25\ \mu\text{eV}$ . The RADES group employs N rectangular cavities connected through irises to search for axion above  $25\ \mu\text{eV}$ . By connecting the sub cavities, this new geometry has the advantage of growing in volume while resonating at higher frequencies (between 8 and 9 GHz). The CAST magnet is a de-commissioned LHC dipole magnet, a superconductive magnet cooled to 2 K and producing a  $\sim 9\ \text{T}$  magnetic field. The  $\sim 43\ \text{mm}$  diameter and 9 m long cold bores of the CAST magnet is suited to host a detector resonating between 8-9 GHz, like the one developed by the RADES group.

Based on the solutions of the modified Maxwell's equations for a coupled cavity system and simulations done with CST, a five sub cavities filter with inductive irises was designed and produced. This filter was installed inside of the CAST magnet at the end of 2017.

The main focus of the thesis was to develop an analysis algorithm that could perform an axion search. This was successfully done resulting in an exclusion limit of  $g_{a\gamma} \gtrsim 4 \times 10^{-13}\ \text{GeV}^{-1}$  within  $34.6738\ \mu\text{eV} < m_a < 34.6771\ \mu\text{eV}$ . This is the second-best limit for an axion mass above  $25\ \mu\text{eV}$  showing that the idea of joining N-cavities together worked for five sub cavities.

This work represents the first physics results of the RADES group. The analysis algorithm developed now provides a tool to monitor and control the data in real-time. However, there are still many aspects of the analysis that can be improved. On the data quality side, the removal of part of the electronic background can be done using a calibration curve of the DAQ structure. The calibration curve could be taken by switching the input signal from the cavity plus amplifier to a  $50\ \Omega$  load every given period. This method to remove the electronic background is independent of the cavity properties. It does not need to allocate the peak structure twice within the total bandwidth of the DAQ, which is an advantage compared to the previous method of using two LO frequencies. An algorithm to select the best SG-parameters could be introduced to the code. Other types of filters can be tested to see if they have a better performance in removing the electronic background. If another DAQ is not available, the systematic residual structure and its evolution over time has to be characterized and understood.

So far, the analysis assumed an axion velocity distribution assuming the simplest isother-

mal sphere model. New astrophysical evidence suggests that the standard halo model has to be updated to new models like the SMH++ [92] to account for anisotropies in the halo. These models introduce a slight change to the standard dark matter parameters and velocity distribution. Other experiments use N-body simulations to determine the line-shape utilized to fit the data [133]. The analysis should be able to account for all these types of models and others that might come. Different ways of applying the Bayesian method for axion searches are reported in [134] and could improve the computation of the upper limit.

The outcome of the first data-taking period showcased the potential of the filter geometry. It showed its limitations when in volume due to mode mixing when many cavities are joined using inductive irises. One important milestone was to find a solution to the mode mixing problem. In a rectangular waveguide like our cavity, the central mode has the largest frequency separation with its neighboring peaks. Mode mixing can be avoided by coupling the axion to the central mode. This can be achieved using the alternating irises solution.

The group was able to design and built a 1 m long filter with alternating irises, where the central mode is the one coupling to the axion, improving the cavity volume by a factor of six. However, this structure has a loaded quality factor ( $\sim 1000$ ) ten times lower than the five sub cavities filter due to poor contact between the main body and the cover plate. One option would be to brace the main body with the cover plate to improve the contact. Another option could be to produce each cavity and capacitive iris separately and then put them together using alignment pins and screws. Both of these options are currently under investigation. If the loaded Q-value is improved back to 10 000, the filter could reach the axion models in only three months of data taking. If the manufacturing and assembly process of the cavities is improved by either of these options, there would be no reason why a cavity that fills the whole length of the CAST bore cannot be produced.

Another way to improve the Q-value is by superconducting materials instead of copper. The surface resistance will decrease, allowing higher Q-values. As part of the RADES R&D developments, four test cavities without irises were manufactured. They have been coated with different superconducting materials (REBCO,  $\text{Nb}_3\text{Sn}$  or  $\text{NbN}$ ) and are being tested in the CERN magnet facility SM18.

Since the axion mass is not known, another crucial aspect that needs to be addressed is the tunability of the cavity. A possible way to tune the cavity is by splitting the cavity into two halves in the plane where the electromagnetic wave propagates. The two halves can be separated using a motor, and the increment of the width changes the resonant frequency introducing a tuning mechanism. The first prototype for a five sub cavities filter was already built, and the tuning mechanism was tested at room temperature as reported in [135]. Instead of mechanically tuning the cavity, it can also be tuned using ferroelectric materials (e.g. as irises). By changing the dielectric constant of the material, the resonant frequency can be tuned.

The next-generation helioscope experiment babyIAXO [86] can also be used for haloscope searches, similar to the CAST magnet. Even though it has a lower magnetic field, the bore diameter combined with the bore length gives an overall better figure of merit than the CAST magnet. RADES can, in principle, take advantage of this bore size to install large volume cavities. The group is working on the design of a long rectangular cavity with no irises that could fill the whole bore. This cavity will search for axions at frequencies below the current ADMX frequency ( $\sim 460$  MHz). Also, RADES continues with designs focused on X-Band resonating cavities for babyIAXO. In this case, the idea is to have a short (e.g., five sub cavities) but very tall cavity. Increasing the height of the cavity will not change the resonance frequency, but will increase the overall volume.

Other aspects of the experiment beyond the cavity, can be improved to increase the overall performance of the setup. The current design is still far above the quantum noise

limit. Reducing the noise power increases the sensitivity to an axion signal. Other haloscope experiments have already shown that JPA amplifiers can achieve noise temperatures lower than 1 K and close to the quantum noise limit. In a time frame of 1 or 2 years, there is a possibility for RADES to use a dilution refrigerator in the Canfranc laboratory, located in the Spanish Pyrenees. This opens the door for the introduction of the JPA amplifiers for the first time to the setup.

The axion parameter space is vast, and there are still plenty of mass channels that are currently not explored by any detector. However, every year new groups and collaborations are joining the efforts to search for axions and ALPs. With the construction of the next generation axion search facilities and the development of new technologies, the next decade will bring us a step closer to the possible discovery of dark matter axions.

## Appendix A Characterization of the DAQ electronic background

The components of the DAQ, as well as the noisy bins were described in section 5.3.5. The analysis was done using the power spectra in Watts. A known signal of  $-20$  dBm was used to test the conversion formula given by the company:

$$\text{Power [dBm]} = -131 \text{ dB} + 10\log(SD), \quad (118)$$

where  $SD$  is the "stored data".

The power spectra were recorded for two integration times:  $0.44739$  s (blue spectrum) and  $90.37278$  s (orange spectrum), as shown in the left panel figure 77. The measured value at the peak for the blue spectrum was  $-23$  dBm and corresponded to the expected value for the known input signal for a time  $= 0.44739$  s. This confirmed the fact that equation 118 indeed provided the power in dBm.

The DAQ stored accumulated power when the integration time was changed. There is a factor  $o = 90.37278 \text{ s} / 0.44739 \text{ s}$  between the power of the two spectra. The "stored data" of the blue power spectrum was multiplied by this factor and then plugged in equation 118. The resulting values were compared to the orange power spectrum. The subtraction of the two spectra shown in the right panel of figure 77 show that the two spectra overlapped to a precision of order  $10^{-2}$  dBm, proofing that the DAQ accumulates the power in the linear scale and then stores it.

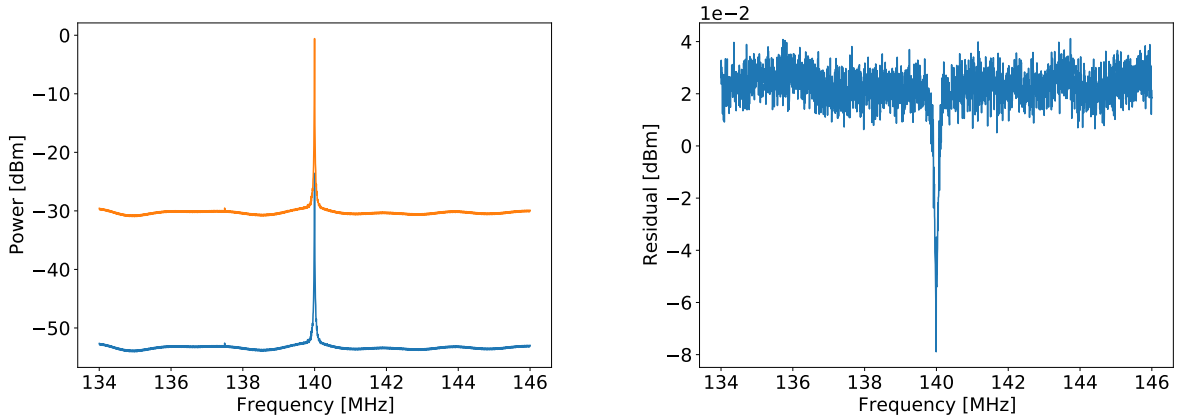


Figure 77: Left: Accumulated power spectrum recorded with the digital stage when a signal generator (sending  $-20$  dBm) was connected to the input port. The blue spectrum is for an integration time of  $0.44739$  s. The orange spectrum is for an integration time of  $90.37278$  s. Right: Subtraction of the two spectra after the stored data of the blue spectrum was multiplied by a factor of  $o = 90.37278 \text{ s} / 0.44739 \text{ s}$ . The difference between the two spectra is in the order of  $10^{-2}$  dBm. The peak appears because the known signal is more sensitive to the stability of the system.

The blue spectrum was transformed to mW using  $\text{Power [dBm]} = 10 \log_{10}(\text{Power [mW]})$ , multiplied by the factor  $o$  and transformed back to dBm. The results were compared to the orange spectrum. The results were similar to the right panel of figure 77. They overlapped to a precision of order  $10^{-2}$  dBm.

With all these tests, it was shown that the "stored data" could be transformed into dBm using equation 118 and  $\text{Power [dBm]} = 10 \log_{10}(\text{Power [mW]})$  can be used to transform the power into mW. The "stored data" is a linear power unit that has a  $-131$  dB offset with respect to the mW scale.

As it was mentioned in section 4.5.3, the power spectra have to be combined over time. To characterize the stability over time of the power spectrum, a  $50\ \Omega$  load was connected to the input of the DAQ.

A spectrum was taken every 90.37278 s (from now on, we will refer to these spectra as the single spectra) for approximately 30 h in this configuration. Figure 78 shows the typical structure of a single spectrum. Both the analog and the digital stage imprint an intrinsic structure (see figure 79) on a purely white noise input signal. The structure seen in figure 78 is a combination of the structures shown in figure 79.

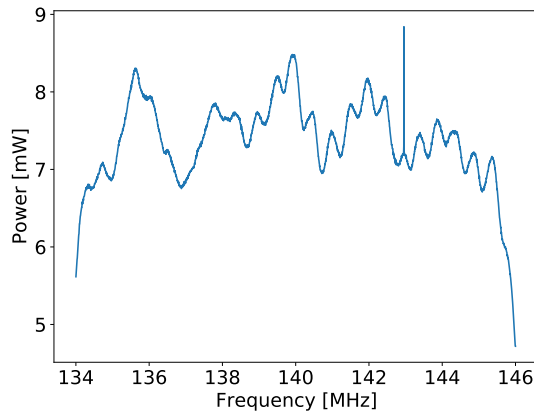


Figure 78: Single power spectrum when the input of the DAQ is terminated with a  $50\ \Omega$  load.

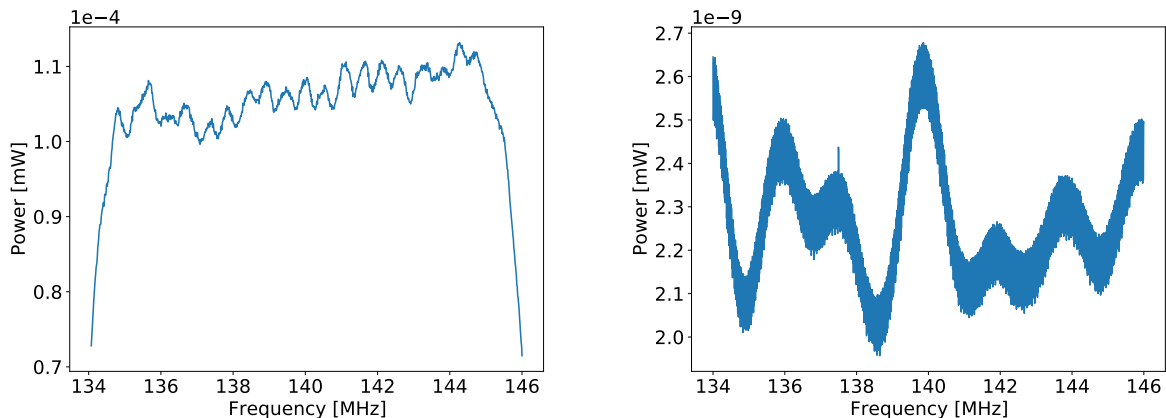


Figure 79: Left: Single spectrum connecting a spectrum analyzer at the output of the analogue stage. The input is terminated with a  $50\ \Omega$  load. Right: Single spectrum at the output of the digital stage. The input is terminated with a  $50\ \Omega$  load.

To test the stability of the spectrum over time and frequency, a subtraction of two spectra was carried out. The left panel of figure 80 shows the result when two consecutively measured spectra were subtracted. The right panel shows the same for two spectra taken with a time difference of  $t = 2.5\ \text{h}$ , a non white noise structure appeared. Moreover, this structure is not constant over time as shown in figure 81, where the subtraction for a time difference of  $t = 12.5\ \text{h}$  is also plotted.

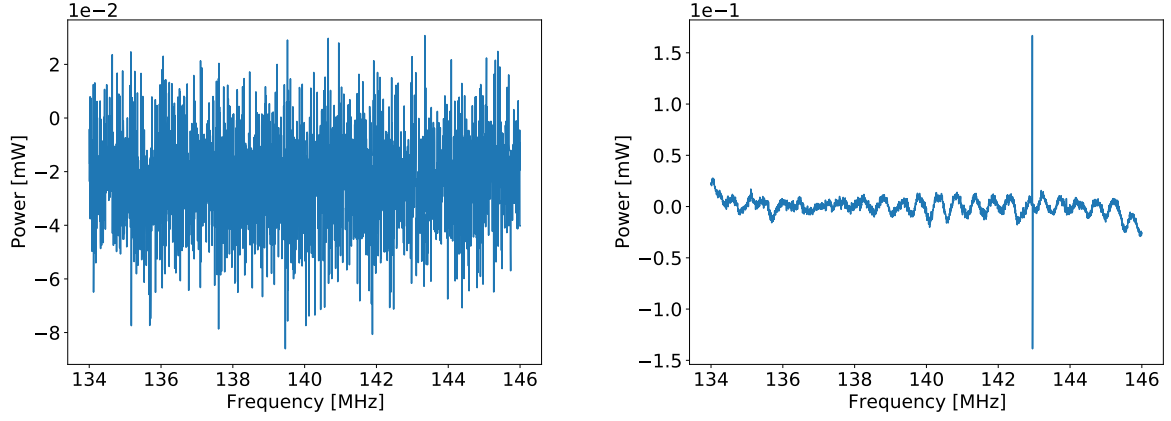


Figure 80: Left: Subtraction of two single spectra taken one after the other. Right: Subtraction of two single spectra taken with a 2.5 h difference.

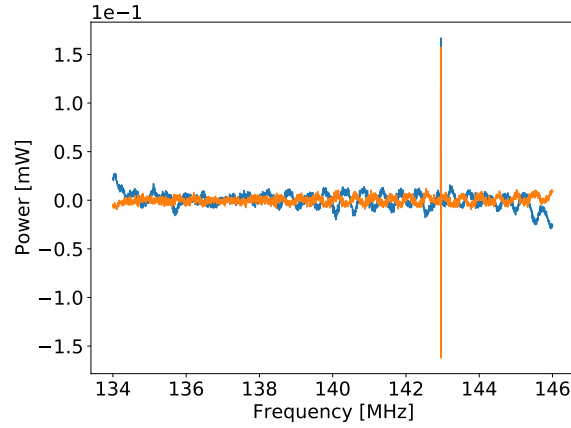


Figure 81: The blue spectrum is the subtraction of two single spectra taken with a 2.5 h difference. The orange spectrum is the subtraction of two single spectra taken with a 12.5 h difference.

The origin of this structure is still unknown. However, when the spectra are combined over time, and the statistical fluctuations are lower than this systematic residual structure, the Grand Unified Spectrum is not Gaussian, and an axion search cannot be done unless this structure is removed. For the results presented in chapter 6 the systematic residual was partially<sup>29</sup> removed by the subtraction of the magnet-on by the magnet-off spectrum.

<sup>29</sup>The measured value of the noise fluctuations was still higher than the theoretical value.

## Appendix B Synthetic axion signal tests

The whole analysis procedure presented in chapter 6 was tested for a software-produced synthetic axion signal. A script developed by C. Cogollos introduced a signal with the line shape of figure 52 to all single spectra ( $\delta_{ij}^{l1}$ ). The axion-photon coupling  $g_{a\gamma}$  used for the power of the axion signal for the initial test created a signal more than  $5\sigma$  away from the mean value of the distribution of the single spectra.

The left panel of figure 82 shows one of these single spectra. The right panel shows the fit to the data points of the axion candidate after the spectra were combined, and the search algorithm using the distorted axion signal was performed. The algorithm found an axion signal with a  $121\sigma$  deviation from the mean value at a specific frequency. The value of the frequency where the signal was inserted was revealed after the analysis was done and compared to the one found by the algorithm. The frequencies matched, giving more confidence that the analysis would find an axion in the data in case there was one.

However, for this initial test, a rigorous analysis of the gain of the receiver chain and how the accumulated power relates to instant power was not done. The conversion factor between the axion power ( $P_a = g_{a\gamma}^2 \rho_{\text{DM}} \frac{\beta}{1+\beta} \frac{1}{m_a} B^2 V Q_L G^2$ ) and the "stored data" was not known. Hence, the exact value of  $g_{a\gamma}$  cannot be extracted from these tests.

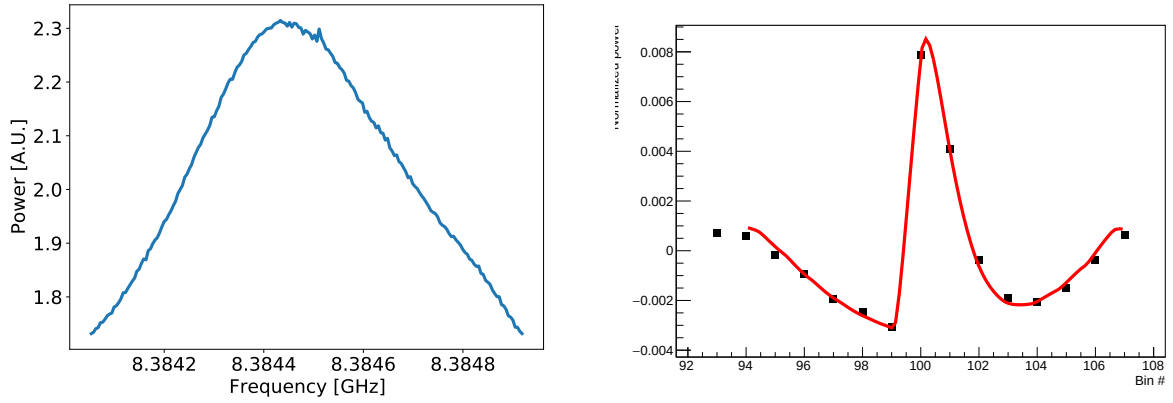


Figure 82: Left: Example of a Normalized single spectrum  $\delta_{ik}^n$  with a synthetic axion signal added to the data. Right: Fit function (red) for the data points for the candidate found by the analysis.



## Appendix C Calibration curve for the electronic background

One of the main lessons learned from the 2018 analysis was that a good understanding of the systematic residual is vital for the quality of the final result. A new method to create a calibration curve to improve the removal of the electronic background structure is under development. Here, the preliminary results of this method are going to be discussed.

The main goal is to remove the structure described in appendix A, especially the systematic structure after the spectra were combined over time. The typical structure seen from the DAQ spectra can be understood as a white noise spectrum multiplied by a frequency-dependent gain curve. Connecting a noise source (which produced white noise with a power 20 dB above the thermal noise power) to the DAQ, one can characterize this gain curve. The resulting curve can be used to remove the electronic background from the spectra of the data-taking period.

The gain curve can be produced using the following equation:

$$G = \frac{P_h - P_c}{T_h - T_c}, \quad (119)$$

where  $P_h$ ,  $P_c$ ,  $T_h$  and  $T_c$  are the measured powers and temperatures when the source is on and off. Figure 83 shows the result of the measured gain curve for a test data set taken on the 12th of January of 2021.

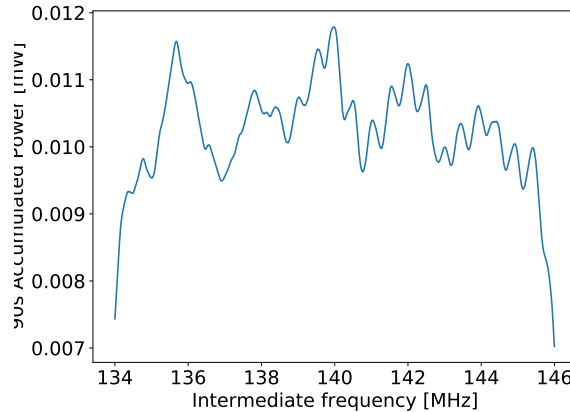


Figure 83: Gain curve of the DAQ. A noise source was used as a input signal.

A second test data set was taken minutes after the first one, with the difference that the noise source was changed and the input signal used was the RADES setup (the 1 m long cavity plus the LNA)<sup>30</sup>. The second set was divided by the gain curve, and the result was compared to the spectrum measured by a spectrum analyzer (SA) for the same input signal as the second set but using a shorter integration time.

Figure 84 shows that the two spectra share the same overall structure across the frequency range. The gain function was able to remove part of the electronic background structure.

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<sup>30</sup>For this test, The CAST magnet was at room temperature and the LNA was not set to the optimum bias for this temperature.

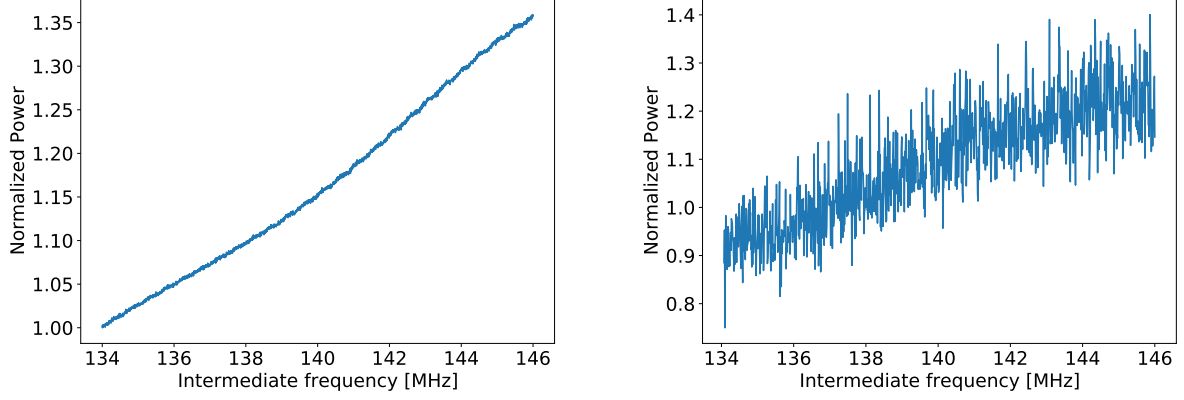


Figure 84: Left: Cavity+LNA spectrum divided by the gain curve for a set taken on the 12th of January of 2021. Right: Cavity+LNA spectrum measured with a spectrum analyzer for a set taken on the 12th of January of 2021.

A similar test was done for a data set taken on the 30th of November of 2020 during the data-taking period. Figure 1 compares the cavity+LNA power spectrum (measured with the DAQ) divided by the gain curve with the same power spectrum recorded with the SA.

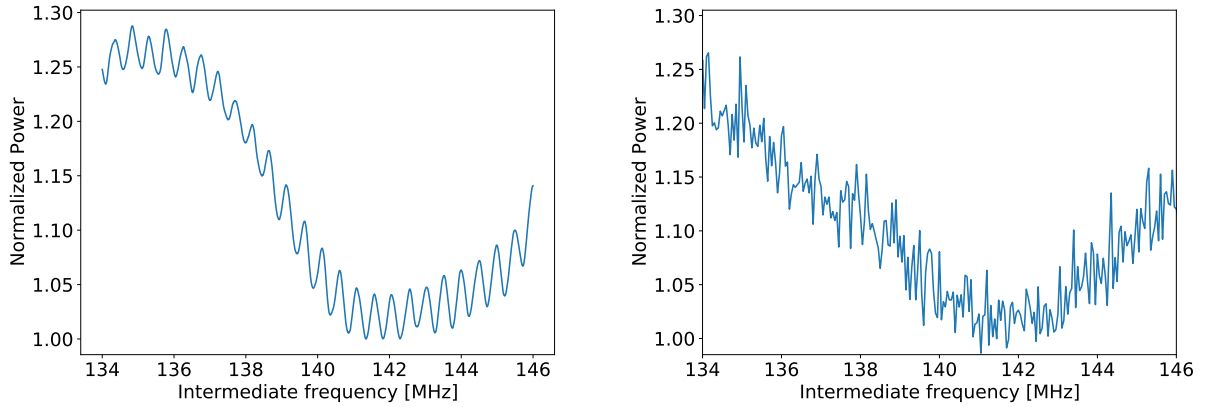


Figure 85: Left: Cavity+LNA spectrum divided by the gain curve for a set taken the 30th of November of 2020. Right: Cavity+LNA spectrum measured with a spectrum analyzer for a set taken the 30th of November of 2020.

Like the previous result, both spectra show the same structure across the frequency range. However, this time the spectrum measured with the DAQ shows a clear sinusoidal structure. This structure could appear because the gain curve was not constant over time, and the one used for the division was taken almost one and half months after the data-taking set used for this test.

Preliminary analysis used the time-domain representation of the spectrum in the left panel of figure 85 to identify the sinusoidal structure. In the time-domain representation this structure appeared as a peak located in one or two bins. These bins were located and removed from the spectra. The resulting spectra were converted back to power spectra. Preliminary results showed that most of the sinusoidal structure was removed using this process. The new power spectra were transformed into structure less power excess spectra using the SG-fits procedure explained in section 6.2. These spectra were combined over time to in principle reduce the noise fluctuation by a factor of  $1/\sqrt{t}$ . However, this procedure

was able to combine 5 hours of data. Afterward, the measured noise fluctuation were not proportional to  $1/\sqrt{t}$  and stayed constant.

At the moment, there are two ideas to improve this analysis. One is to improve the removal of the sinusoidal structure in the time domain. The second one is to determine how often a gain curve must be taken during the data-taking period to remove the electronic background and combine the spectra over time successfully.

## Appendix D 2020 data taking preliminary analysis

As mentioned in section 7.3, scaling in volume implied many more modes appearing closer to each other, affecting the shape and number of the peaks. Additionally, the underperformance of the long cavity in terms of the Q factor had an essential impact on the data taking and subsequent analysis because the full width at half maximum (FWHM) of the peak became larger than half the 12 MHz bandwidth of the DAQ making LO subtraction for electronic background removal unfeasible.

A new method to remove the electronic background (described in appendix C) is under development. Preliminary results showed the expected Gaussian distribution of the Unitless Normalized Spectra, but the noise fluctuation did not follow the  $1/\sqrt{t}$  behavior expected from theory. At the moment, 5 hours of data can be combined using this removal algorithm.

Because of the low  $Q_L$ , the amount of ENR provided by the noise source was not enough to reach the cryogenic LNA. All the extra power was attenuated by the cables and the cavity. The DAQ does not see any difference for the noise source on or off, and the Y method cannot be used for this data.

The direct noise measurement was used to give an estimation of the system temperature. This method is normally used for high noise figure devices. The direct noise measurement method uses  $P_N = k_b T_{sys} \Delta\nu$  (introduced in section 4.5) to calculate the noise temperature. Three parameters are needed for the calculation. The most important one is the gain of the amplifier. The other two are the resolution bandwidth and the cables losses between the output of the LNA and the spectrum analyzer (SA).

To measure the gain of the LNA, a signal generator was connected to the weakly coupled port line of the cavity (line 2 of figure 26), and the SA was connected to the critically coupled line (line 1). A known signal was sent through the line, and the output power of the signal was measured with the SA. The switch was used to bypass the LNA, and the power was measured again with the SA. The difference between the two power measurements gives the gain of the LNA (assuming both lines have the same attenuation). The measured gain was  $\sim 30$  dB.

A  $50\ \Omega$  load was connected to line 2 instead of the signal generator and the noise level of the system ( $P_{TN}$ ) was measured with the SA. Using the following equation, the noise power ( $P_N$ ) at the input of the amplifier can be computed:

$$P_N = P_{TN} - G + L_{cable}, \quad (120)$$

where  $G$  is the gain of the amplifier and  $L_{cable}$  are the cable losses. Plugging  $P_N$  and the resolution bandwidth set at the SA in  $P_N = k_b T_{sys} \Delta\nu$ , the noise temperature was calculated.

The expected noise spectrum  $P_{TN}$  should not have any structure and should have been purely white noise. In the best-case scenario, with an adequate  $Q_L$  and critical coupling, one could see the noise bump produced by the cavity resonance. However, the spectrum showed a non-flat structure where the difference between the maximum and minimum was almost 10 dB (see left panel of figure 86).

The measured power ( $P_{TN}$ ) at the resonant peak was  $-112$  dBm (see right panel of figure 86), with a gain of 30.5 dB and cable losses of 4 dB. This yielded a  $P_T = -138.4$  dBm and the set bandwidth was 3000 kHz, giving a noise temperature of approximately 350 K, which is extremely high compared to the expected 10 K. This measurement was done more than once with different bandwidths and at different weeks of the data-taking period with similar results.

Preliminary studies suggest that two things can be causing this problem. The incorrect antenna coupling produces an impedance mismatch at the input of the LNA. A bad cable

or connection could be responsible of the 10 dB difference seen in the noise power across the frequency range.

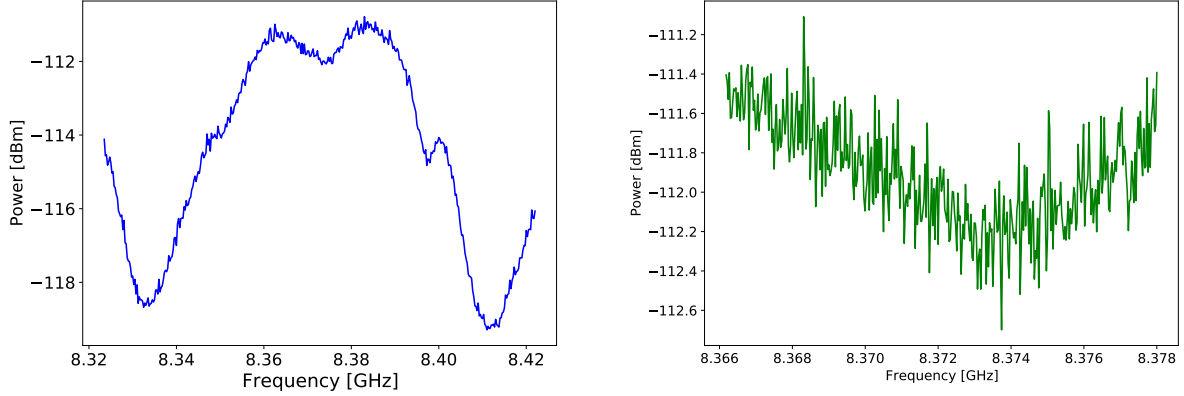


Figure 86: Left: Noise Power ( $P_{TN}$ ) for a 100 MHz span. Right: Noise Power ( $P_{TN}$ ) for a 12 MHz span centered at the resonant peak.

The only parameter that has still not been estimated for the 2020 setup is the cavity coupling  $\beta$ . Figure 87 shows the result of two  $S_{11}$  measurements. The antenna was optimized for the resonance mode at 8.397 GHz. In figure 87 one can clearly see a peak at exactly this frequency, which means that the port is close to critically coupled.  $\beta$  can be calculated in the same way as explained in section 6.7 for this frequency.

However, simulations showed that the mode that couples to the axion was actually the one at 8.37 GHz. Contrary to the peak at 8.397 GHz, at 8.37 GHz one cannot distinguish any peak that deviates too much from the baseline ( $\sim -10$  dB). This means that the coupling for the resonance mode at 8.37 GHz was either too over or too weakly coupled<sup>31</sup>, and the reflection of the cables and feedthroughs dominate the  $S_{11}$  value at this frequency. Post processing algorithms have been developed to try to remove the cable and feedthrough influence, but so far, none of them have successfully done it.

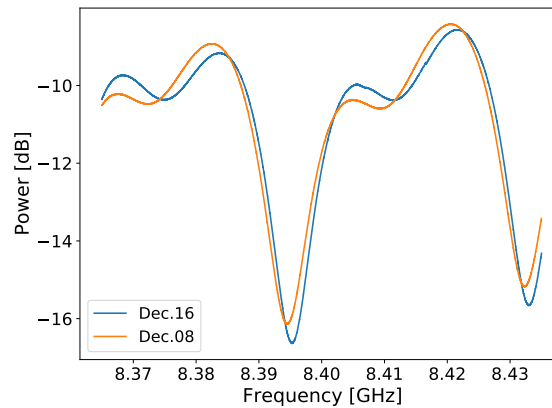


Figure 87: Reflection coefficient  $S_{11}$  measured at two different days during the data-taking period.

The other two solutions to estimate the coupling is to do it through simulations. The coupling at 8.397 GHz can be calculated from the measurements of the reflection coefficient

<sup>31</sup>From the length of the antenna a weakly coupled port ( $\beta = 0.1$ ) was assumed for the prospect of this cavity.

$S_{11}$  (see figure 87). Based on this result, simulations to replicate the cavity structure can be done. The simulations will not suffer from reflections from cables or bad feedthroughs. Thus, the simulations will also show the  $S_{11}$  value of the resonance mode at 8.37 GHz and the coupling  $\beta$  can be estimated. The other solution is to take the 1 m long cavity to a dedicated cryogenic lab, installed it there, and measure the coupling.

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