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„A multi-objective and temperature-dependent steel slab transportation problem"

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Abstract

In this master´s thesis we introduce a multi-objective steel slab transportation problem that stems from an industrial partner's steel plant. Steel slabs are heavy and rectangularly shaped casting products that can measure high temperatures. They are transported by heavy-duty vehicles among the many facilities and storage locations within the steel plant's network. During the transportation of steel slabs, heavy-duty vehicles are subjected to abrasion due to the heat that the steel slabs emit. The overall goal of this particular steel slab transportation problem is two-fold: one the one hand, the distance travelled by the vehicles' must be minimized, on the other hand, the abrasion must also be minimized by keeping the heat-exposure of the vehicles low. In order to consider the latter aspect, we first must know how to measure and quantify said heat exposure. The latter aspect constitutes the thesis' main emphasis. The problem is novel in so far that the scientific literature on vehicle routing problems has not dealt with travel distances and vehicle heat-exposure as a multi-objective problem. We propose multiple vehicle heat-exposure measures and thoroughly investigate them using an insertion-based Large Neighbourhood Search (LNS) metaheuristic. First, we conduct extensive computational experiments. We use a small fictional problem instance and analyse the particular traits of the resulting vehicle routes when applying one of the heat-exposure measures in combination with the distance travelled, i.e., representing the two objective functions. Subsequently, we apply the objective functions on real-world data sets provided by the industrial partner and compare the solutions obtained through the LNS with real-world solutions. Second, we discuss the proposed vehicle heat-exposure measures with representatives of the industrial partner. This provides detailed insights on the multi-objective steel slab transportation problem overall and allows for an evaluation of this master´s thesis' findings by technical experts.

Zusammenfassung

In dieser Masterarbeit stellen wir ein Mehrziel-Stahlbrammentransportproblem vor, das aus dem Stahlwerk eines Industriepartners stammt. Stahlbrammen sind schwere und rechteckig-geformte Stahlfabrikate, die hohe Temperaturen messen können. Sie werden mit Schwerlastfahrzeugen zwischen den vielen Produktions- und Bearbeitungsanlagen sowie innerhalb des Betriebsgeländes des Stahlwerks transportiert. Beim Transport von Stahlbrammen werden Schwerlastfahrzeuge durch die von den Stahlbrammen abgegebene Hitze einem Verschleiß ausgesetzt. Das übergeordnete Ziel dieses speziellen Transportproblems von Stahlbrammen ist zweierlei: Einerseits muss die von den Fahrzeugen zurückgelegte Strecke minimiert werden, andererseits muss auch der Verschleiß minimiert werden, indem die Hitzebelastung der Fahrzeuge minimiert wird. Um den letztgenannten Aspekt zu berücksichtigen, müssen wir zunächst wissen, wie diese Hitzebelastung gemessen und quantifiziert werden kann. Der letztere Aspekt ist der Schwerpunkt dieser Masterarbeit. Das Problem ist insofern neu, als dass die wissenschaftliche Literatur zu Tourenplanungsproblemen die Fahrstrecken und die Hitzebelastung von Fahrzeugen noch nicht als Mehrzielproblem behandelt hat. Wir schlagen mehrere Maße zur Hitzebelastung von Fahrzeugen vor und untersuchen diese umfassend mit einer Einfüge-basierten Large Neighborhood Search (LNS) Metaheuristik. Zunächst führen wir umfangreiche Computerexperimente durch. Wir verwenden eine kleine fiktive Probleminstanz und analysieren die besonderen Eigenschaften der resultierenden Fahrzeugrouten, wenn wir eines der Maße zur Hitzebelastung in Kombination mit der zurückgelegten Strecke anwenden, d. h. die beiden Zielfunktionen darstellen. Anschließend wenden wir die Zielfunktionen auf reale Datensätze des Industriepartners an und vergleichen die mittels der LNS erhaltenen Lösungen mit realen Lösungen. Zweitens diskutieren wir mit Vertretern des Industriepartners die vorgeschlagenen Maße zur Hitzebelastung von Fahrzeugen. Dies ermöglicht detaillierte Einblicke in die Mehrzielproblematik des Stahlbrammentransports im Allgemeinen und ermöglicht eine Bewertung der Erkenntnisse dieser Masterarbeit durch Fachexperten.

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1 Introduction

Studies done by Oxford Economics on behalf of “Worldsteel”, an industry association for the iron and steel industry, show that the steel industry contributes to an estimated 3.8% of global GDP. Within these 3.8%, less than 20% are generated in the steel industry itself. The remaining 80% are either suppliers or customers of steel manufacturers. Globally, the steel industry enables 96 million jobs, or about 3% of all globally employed persons according to the International Labour Organisation (ILO). 6 Million persons work directly in the steel industry, while the remaining 90 million work either for customers or suppliers of the steel industry. (Basson, 2020b)

The European Union (28), is the largest steel importer in 2019 with 40.2 Mt (megatonnes), followed by the United States with 27.1Mt. Within the European Union (28) Germany is the largest steel importer with 23.1Mt. The largest steel exporter is China with 63.9 Mt of steel exports in 2019. (Basson, 2020a)

This master thesis deals with the transportation of steel slabs in the industrial partner´s steelplant. We call the problem Steel Slab Transportation Problem (TDSSTP) henceforth. The objective is the minimization of the travel distance and the minimization of the vehicle heat exposures. The higher the temperature of the steel slabs the vehicle transports and the longer the vehicle transports those steel slabs the greater is the vehicle´s heat exposure. Vehicle heat exposure leads to abrasion of vehicle parts and hence costs. Our goal is to mitigate extremes. In particular we want to minimize the peak vehicle temperature across the routes and achieve an even distribution of the batches containing items with high temperatures across the routes. Additionally, we want to avoid transporting items longer than necessary (especially items that are considered warm).

This master thesis is organized as follows: We start by describing the underlying problem of the TDSSTP in chapter 2. Subsequently, we classify the problem according to the existing literature and describe existing multi-objective Vehicle Routing Problems (VRP) as well as steel-related logistics problems in the literature in chapter 3. We develop an algorithm to solve a problem with multiple objectives. Hence, in section 4 we explain the concept of Pareto-dominance, the solution techniques for multi-objective problems and how to compare those solutions with each other. The algorithm developed in this thesis is a Large Neighbourhood Search (LNS) metaheuristic based on (Shaw, 1997), which we describe in section 5.2. The repair operator of the LNS is adopted from (Madsen, Ravn and Rygaard, 1995) which we illustrate in section 5.1. The measure of the vehicle heat exposure is a central topic of this master thesis. We illustrate different measures of vehicle heat exposure in section 6. To the best of our knowledge there is no existing literature that deals with the temperature of items as a second objective beside travel distance minimization. In section 7.1 we create a small network of 10 nodes to illustrate the routes generated by the different measures of vehicle heat exposure of section 6. We apply the algorithm on problem instances inspired by a real-world data set provided by the industrial partner in section 7.2 and compare the routes generated with the routes by the industrial partner. In section 7.3 we investigate the trade-off between a distance related measure and the different vehicle heat exposure measures. In chapter 8 we provide an interview conducted with the industrial partner regarding the vehicle heat exposure measures. Finally, in section 9 we provide a conclusion of our most important findings and give a brief outlook on the topic.

The **research questions** of this thesis are**:**

1. How can the temperature of a transportation batch consisting of steel slabs of different temperatures be quantified and how can its impact on the vehicle be measured, i.e., how can we measure a vehicle´s heat exposure?

2. How do different ways of measuring a vehicle´s heat exposure impact the solution structure and characteristics by means of different key performance indicators?

3. What are current state of the art solution methods for solving similar Vehicle Routing Problems and what multi-objective solution approach is suitable for solving the *Temperature-dependent Steel Slab Transportation Problem* (TDSSTP) at hand?

4. How do the results of our proposed solution approach compare to the results of real-life practice?

2 Problem description

The problem considered in this thesis, which we call *Temperature-dependent Steel Slab Transportation Problem* (TDSSTP) henceforth, can be described as follows: A set of requests R is given. Each request $r \in R$ contains a steel slab, which is a heavy, rectangular shaped steel product. Hereinafter we refer to a steel slab as an item. Every item must be transported from a pickup location p^r to a delivery location d^r. Every item has a weight denoted by w^r, a height denoted by h^r and a temperature denoted by $temperature^r$. We neglect the cooling of items during the entire process and assume temperatures to remain constant at start temperatures once the route(s) begin. The item temperatures range between 0°C and 450°C. In the following, we will refer to items as being *warm*. In this case an item´s temperature is greater or equal to a parameter set by the decision maker[1]. Every request must be served exactly once. The pickup and delivery services of a request must be carried out by the same vehicle (pairing constraint). Every item must be picked up before it is delivered (precedence constraint).

Two types of special heavy-duty vehicles are used for steel slab transportation by the industrial partner, straddle carriers and trucks with trailers. In this thesis we only consider straddle carriers. Hereinafter we refer to a straddle carrier as a vehicle. Multiple items are piled up and transported together in a vehicle. We refer to a pile of items transported together in a vehicle as a batch hereinafter. Batches are transported by the vehicle by establishing physical contact with the lowermost item in those batches. Hence, all items in the batch rest on the lowermost item. Each vehicle has capacity restrictions w.r.t. the batch height and batch weight. A batch cannot exceed a maximum height of 880mm (h^{max}) and a maximum weight of 105 tons (w^{max}) due to the vehicle's technical restrictions. The set of homogenous vehicles is denoted as *K*. Each vehicle $k \in K$ performs exactly one route. Hence, every k also corresponds to a route. Vehicle k is restricted to begin its route at a start location s^k, but no end location is defined. We introduce a dummy depot with distance 0 to all other nodes. We assume the dummy depot to be the start and end node of every vehicle/route k.

The transport network in the TDSSTP is a complete, directed graph $G = (V, A)$, where V is the set of vertices and A is the set of arcs. A vertex in the network represents either a storage location or a facility in the plant´s network, while the arcs represent shortest distances between

[1] In our case this parameter is set to 300°C

the storage locations and/or facilities. Each arc (i, j) is associated with a distance t_{ij} travelled between i and j. Since a batch consists of the composition of the unique items it contains, and the composition and hence the batch changes at every node, since other items can be picked up and/or delivered at those nodes, each arc of a vehicle's route is associated with a batch. This batch either contains one or more items or it is empty, constituting an idle ride. We disregard item loading and unloading operations for simplicity. Any item can be retrieved from the stack at any location instantly without complex reshuffling operations.

The TDSSTP considers a time horizon of 3 hours. Within that time horizon all requests must be fulfilled. There are no time windows with respect to the requests. Beside the objective of minimizing travelling distance, our second objective is to minimize the vehicle heat exposure, because the higher the temperature of the items in the batch the greater is the vehicles' exposure to heat and thus abrasion. The measure of the vehicle heat exposure is a central topic of this master thesis and one of the research questions. Chapter 6 describes the different measures of the vehicle heat exposure.

We distinguish between evaluation function and objective function. While both capture distance-related and/or temperature-related measures, the evaluation function records the change of a solution due to request insertions or removals while the objective function records the total value of the measures of a solution.

3 Literature review and classification

In this chapter we classify the underlying problem of this thesis according to the existing literature. Subsequently, we review the most relevant literature on multi-objective VRPs and give a brief insight into the logistics literature regarding the steel industry.

3.1 The family of general pickup and delivery problems (GPDP)

The *General Pickup and Delivery Problem* consists of a fleet of vehicles and multiple requests. Each vehicle has a given capacity, a start and an end location. Each request specifies a given amount of load, a location where the load has to be picked up (source) and a location where it has to be delivered to (destination). Every load has to be transported from the source to the destination without any transshipments in-between. The objective is to construct a set of routes, where each route corresponds to a vehicle, in order to satisfy the transportation requests. (Savelsbergh and Sol, 1995) Parragh, Doerner and Hartl (2008) distinguish the *General Pickup and Delivery Problem* into two subclasses: The *Vehicle Routing Problem with Backhaul* (VRPB) and the *Vehicle Routing Problem with Pickup and Delivery* (VRPPD). In the VRPB goods are transported from a depot to linehaul customers and from backhaul customers to a depot. In the VRPPD goods are transported between pickup and delivery locations. The VRPPD itself can be divided into subclasses, paired and unpaired VRPPDs. In unpaired VRPPDs a homogenous good is transported, which can be picked up at any pickup node and can be used to fulfil the demand at any delivery node. Paired VRPPDs deal with goods that have a specific pickup node and a specific delivery node. The *Dial a Ride Problem* (DARP) and the classical *Pickup and Delivery Problem* (PDP) belong to this subclass of problems. While the DARP deals with the transportation of people, the PDP deals with the transportation of goods. The differences are additional constraints or objectives with regards to passenger (in)convenience in case of the DARP. The steel slab transportation problem (TDSSTP) is a variant of the classical *Pickup and Delivery Problem* (PDP).

3.2 Multi-objective VRP review

Despite many vehicle routing problems encountered in the industries being multi-objective by nature, they are often limited by modelling them with the objective of minimizing a single cost, where costs can be expressed in terms of distance travelled, time required or customers visited.

In real life the objectives might not be limited to a single cost. Numerous aspects can be taken into account by adding new objectives. When multiple objectives are considered, the different objectives are often in conflict. For example, in the Travelling Salesman Problem with Profits (TSPP) every customer is associated with a profit, but not every customer has to be visited. There are two conflicting objectives: The profit is maximized by visiting more customers. But the more customers are visited the longer is the tour and hence the costs. The cost is minimized by visiting less customers, but so is the profit. (Jozefowiez, Semet and Talbi, 2008) Another example is the bi-objective covering tour problem (BOCTP). In the BOCTP the length of a tour must be minimized, on which a subset of nodes is visited. At the same time the greatest distance between the nodes of another set and the nearest visited node has to be minimized. (Jozefowiez, Semet and Talbi, 2007) Lastly, the DARP can incorporate objectives to minimize the user inconvenience, such as the mean user ride time, beside the objective of minimizing the travelling distance. However, minimizing the mean user ride time might lead to longer routes.

In what follows, we provide a brief overview on some literature relevant to multi-objective VRPs. A study of multi-objective VRPs can be found in (Jozefowiez, Semet and Talbi, 2008). They examine VRPs in terms of their definitions, objectives and multi-objective algorithms proposed for solving them. Evolutionary algorithms constitute a popular approach in solving multi-objective VRPs. Evolutionary algorithms approximate the entire Pareto-Front in a single run due to their population-based nature. (Demir, Bektaş and Laporte, 2014) A survey on multi-objective evolutionary algorithms can be found in (Zitzler, Deb and Thiele, 2000). More recently, Parragh *et al.* (2009) use a two-phase solution procedure to solve a bi-objective dial-a-ride problem. The two objectives are total distance travelled and mean user ride time. In the first phase a variable neighbourhood search-based heuristic generates approximate weighted sum solutions. In the second phase additional efficient solutions are calculated by using a path-relinking module and seeding it with the partial Pareto-Front computed in phase one. Comparisons with solutions for smaller instances generated by the epsilon constraint method show high-quality approximations to the true Pareto-efficient Front. Paquette *et al.* (2013) use a tabu search metaheuristic to solve the multi-objective DARP. In addition to cost, three objectives related to quality of service are considered. Extensive tests on real-life data show that the algorithm generates a rich set of non-dominated solutions and can be employed to find good trade-offs between cost and quality of service. Demir, Bektaş and Laporte (2014) use an Adaptive Large Neighbourhood Search (ALNS) combined with a speed optimization procedure to solve the bi-objective Pollution-Routing Problem. The two objective functions are fuel consumption and driving time are conflicting and are thus considered separately. They test 4 a-posteriori methods, namely the weighted method, the weighted method with normalization, the epsilon constraint method and a hybrid method combining adaptive weighting with the epsilon constraint method, using the ALNS as a search engine.

Beside the classical objective of minimizing the travelling distance, we consider different forms of temperature related objectives in this thesis. The objective of temperature is novel to literature. To the best of our knowledge, the topic of item/vehicle temperature has not been considered as an objective in VRP literature so far.

3.3 Steel review

Logistical optimization problems w.r.t. job scheduling that stem from steel plant problems are common in literature. Özgür, Uygun and Hütt (2021) give a comprehensive review about steel warm rolling mill scheduling from an operation research point of view. A moderate amount of literature exists on transportation problems that take place in steel plants. The reader is referred to (Vonolfen, 2014) and to (Kofler, 2014) for information regarding steel slab transportation problems.

4 Basics of Multi-objective Optimization

4.1 Multi-objective optimization problems and Pareto dominance

A general optimization problem is commonly described as a quadruple (X, Z, f, rel), where X is the search space, Z is the objective space and f the function that maps to each decision vector $x \in X$ from the search space a corresponding objective vector $z = f(x) \in Z$ from the objective space. *rel* represents a binary relation that defines a partial order[2] over the objective space Z. This induces a preorder[3] on the search space. The objective is to find a solution $x^* \in X$ that is mapped to a minimal $z^* = f(x^*)$ of $f(X) = \{z \in Z | \exists x \in X : z = f(x)\}$[4] (Knowles, Thiele and Zitzler, 2006). In case of a single objective function (n=1), the relation 'less than or equal to' is used to define a minimization problem $(X, \mathbb{R}, f_1, \leq)$. A unique optimal minimum value $z^* \in f(X)$ exists, since $\leq$ is a total order[5] over $Z \in \mathbb{R}$ and hence any two values in Z are comparable. However, there might be several different decision vectors that are mapped to z^*, which is why the relation $\leq$ is a preorder but not a partial order on the decision space with $x^1 \leq x^2 \leftrightarrow f(x^1) \leq f(x^2)$. In the case of multiple objective functions (n>1), the relation $\leq$ is extended to $\mathbb{R}^n$ and the relation $\preccurlyeq$ is used to relate the elements of the objective space. The relation $\preccurlyeq$ is called Weak Pareto dominance. A solution z^1 weakly Pareto dominates a solution z^2 if and only if z^1 is not worse than z^2 in all objectives. A solution z^1 dominates a solution z^2 if and only if z^1 is not worse than z^2 in all objectives and z^1 is better than z^2 in at least one objective. A solution z^1 strictly Pareto-dominates a solution z^2 (denoted $\prec\prec$) if and only if z^1 is better than z^2 in all objectives. Finally, two solution z^1 and z^2 are incomparable (denoted ||) if neither $z^1 \preccurlyeq z^2$ nor $z^2 \preccurlyeq z^1$. (Knowles, Thiele and Zitzler, 2006) We use the concept of Pareto dominance to compare solutions of a multi-objective optimization problem.

Figure 1 shows three solutions (A,B,C) in a multi-objective solution plane. The x-axis represents the objective value 1, while the y-axis represents the objective value 2. Both objectives have to

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[2] The axioms for a relation that are necessary for partial order are reflexivity, antisymmetry and transitivity. (For more information on axioms see Appendix A)

[3] The axioms for a relation that are necessary for preorder are reflexivity and transitivity.

[4] There exists a decision vector x that is mapped to the objective vector z (meaning the minimal z* can only be found if the corresponding decision vector x* exists)

[5] The axioms for a relation that are necessary for total order are reflexivity, antisymmetry, transitivity and comparability. (For more information on axioms see Appendix A)

be minimized. Solution B dominates solution C, because B is not worse than C in f_1 or f_2 (B and C are equal in f_1) but B is strictly better than C in f_2. Solution A strictly Pareto-dominates solution C, because it is strictly better than solution C in f_1 and f_2. Solution A and solution B are not comparable, because solution A is strictly better than solution B in f_1 but solution B is strictly better than solution A in f_2.

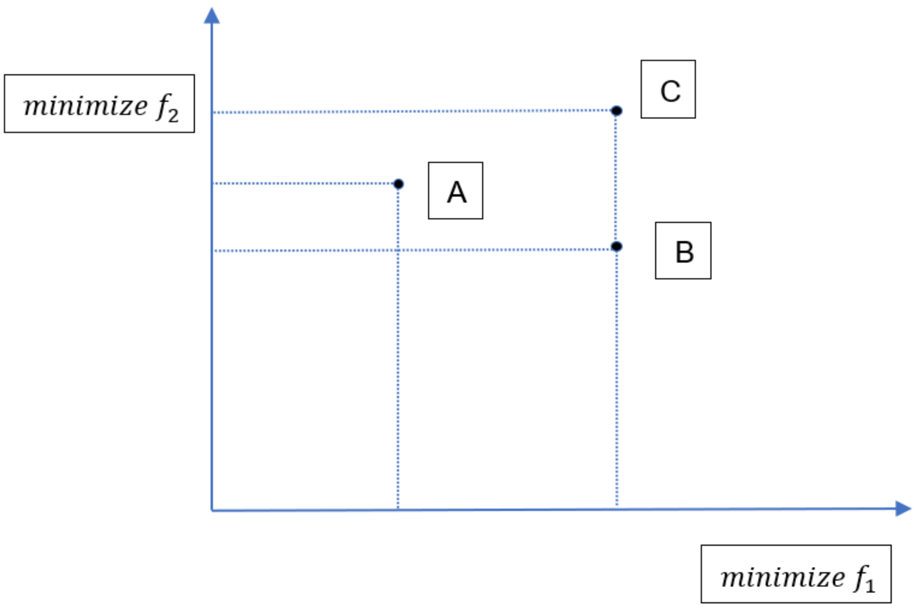

Figure 1 Pareto dominance

A solution is called *pareto-optimal* or *pareto-efficient* if this solution is not weakly dominated by any other solution. The solution to a multi-objective problem is the set of *non-dominated decision-vectors* called *Pareto-Set*. (Knowles, Thiele and Zitzler, 2006; Jozefowiez, Semet and Talbi, 2008) The set of non-dominated objective vectors are called Pareto-Front and each objective vector represents a different trade-off in the solutions. The solutions of a Pareto-Front are incomparable. (Knowles, Thiele and Zitzler, 2006)

4.2 Solution approaches to multi-objective optimization problems

Pareto efficient sets can be further distinguished into *supported* and *non-supported* solutions. Supported, efficient solutions lie on the convex hull of the solution space. For a bi-objective minimization problem the entire set of supported efficient solution can be found as optimal solutions by varying ω in a weighted sum objective function:

$$\min[\omega\, z_1(x) + (1-\omega) z_2(x)]$$

$$0 \leq \omega \leq 1$$

Non-supported, efficient solutions, which are a major part of the Pareto-set cannot be found as optimal solutions by means of the weighted sum objective function with any combination of ω. Non-supported, efficient solutions lie in the interior of the convex hull of the solution space. (Parragh *et al.*, 2009) They can only be found accidentally as intermediate solutions when searching for supported solutions. (Laumanns, Thiele and Zitzler, 2006)

Three main solution approaches can be distinguished in multi-objective problems. They differ in terms of the order in which the decision maker provides preferences for the different objectives: *A priori* approaches consider the decision maker's preference as known before solving the problem. *A posteriori* approaches generate a set of solutions that are potentially non-dominated first. Then the decision maker chooses according to her preferences. *Interactive* approaches consider the decision maker´s preferences during the solution process. (Jozefowiez, Semet and Talbi, 2008)

We use the weighted aggregation method as an a posteriori approach. The objective functions are scalarized, meaning the problem is formulated such that only a single objective function has to be solved. This is done by means of weights attributed to each objective function, reflecting their relative importance. This approach is simple to implement, however setting weights according to the importance of the objectives can be difficult. (Jozefowiez, Semet and Talbi, 2008)

5 Solution approach

The PDP is a NP-hard problem (Garey and Johnson, 1979), making exact solution methods unfit to find solutions for large instances. NP-hard problems are those problems that are not solvable in polynomial time. We make use of a Large Neighbourhood Search (LNS) metaheuristic (Shaw, 1997) to solve the TDSSTP. We use the insertion-algorithm for the *DARP with time windows* in (Madsen, Ravn and Rygaard, 1995) to generate an initial solution and as the repair operator for the LNS.

5.1 A sequential cheapest insertion algorithm for the Steel Slab Transportation Problem

(Madsen, Ravn and Rygaard, 1995) developed their algorithm REBUS as a solution system for a dial-a-ride problem with time windows (DARPTW). We omit the time windows constraints and use the algorithm in the TDSSTP, which is a PDP and hence belongs to the same class of paired VRPPDs. The algorithm is a sequential cheapest insertion heuristic. We use the algorithm to construct an initial solution and subsequently we use it as the repair operator in our metaheuristic solution approach in section 5.2. Depending on the vehicle heat exposure measure used, we sort the request to be inserted either by their item temperature or randomly before the insertion process and then insert them one by one into the routes. The algorithm is described by the following pseudocode:

```
function INSERTION_ALGORITHM (a set of routes K, a set of requests R)
    for every route k in K do
        insert start and end depot node 0 in k
    end for
    for every request r in set of requests R do
        least cost insertion = big M (a very large number)
        for every route k in set of routes K do
            insert p^r after first depot node
            insert d^r after p^r
            compute cost of insertion
            if cost of insertion < least cost insertion then
                least cost insertion = cost of insertion
                save insertion position
                insertion route = k
            end if
            else-if cost of insertion == least cost insertion and size of route k < size of insertion route then
                save insertion position
                insertion route = k
            end else-if
            repeat
                move d^r one stop to the right
                for all nodes between p^r and d^r do
                    check feasibility
                    if feasible then
                        compute cost of insertion
                        if cost of insertion < least cost insertion then
                            least cost insertion = cost of insertion
                            save insertion position
                            insertion route = k
                        end if
```

else-if $cost\ of\ insertion == least\ cost\ insertion$ **and** size of route k < size of $insertion\ route$ **then**
save $insertion\ position$
$insertion\ route = k$
end else-if
else
goto infeasible1
end else
end for
until d^r is last position before end depot 0
infeasible1:
repeat
move p^r one position to the right
place d^r right after p^r
check feasibility
if feasible **then**
compute $cost\ of\ insertion$
if $cost\ of\ insertion < least\ cost\ insertion$ **then**
$least\ cost\ insertion = cost\ of\ insertion$
save $insertion\ position$
$insertion\ route = k$
end if
else-if $cost\ of\ insertion == least\ cost\ insertion$ **and** size of route k < size of $insertion\ route$ **then**
save $insertion\ position$
$insertion\ route = k$
end else-if
repeat
move d^r one stop to the right
for all nodes between p^r and d^r **do**
check feasibility
if feasible **then**

```
                                        compute cost of insertion
                                        if cost of insertion < least cost insertion then
                                            least cost insertion = cost of insertion
                                            save insertion position
                                            insertion route = k
                                        end if
                                        else-if cost of insertion == least cost insertion and size of route k < size of
                                        insertion route  then
                                                save insertion position
                                                insertion route = k
                                        end else-if
                                    else
                                        go to infeasible2
                                    end else
                                end for
                            until d^r is last position before end depot 0
                        end if
                            infeasible2
                    until p^r is the last position before end depot 0
                end for
            end for
end function
```

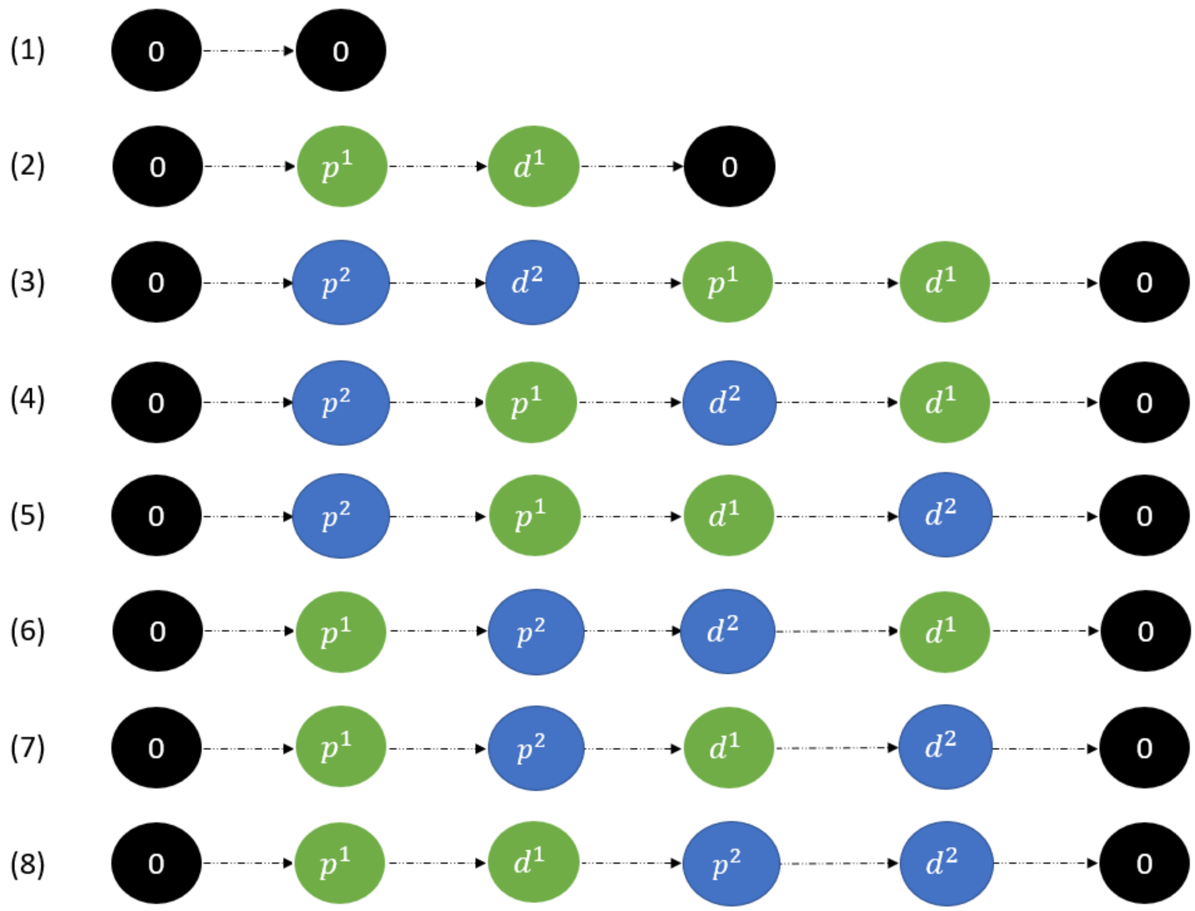

Figure 2 Insertion-based construction algorithm

Figure 2 illustrates an example of the insertion-algorithm in (Madsen, Ravn and Rygaard, 1995). The numbers in brackets indicate the respective route in Figure 2. We use an auxiliary dummy node 0 as depot node in the algorithm to initialize routes (1). It´s distance to all the other nodes is zero. The figure shows two requests 1 and 2 inserted into the route. First request 1 is inserted into the route at its only possible position as shown in (2). Then, all feasible insertion positions of request 2 are explored as shown in (3)-(8). In what follows, we describe all steps in detail. The precedence constraint requires all routes to have the pickup node followed by the delivery node for every request inserted. Hence, d^1 follows p^1 for the first request inserted (2). Subsequently request 2 is inserted right after the start depot (3). Delivery node d^2 is moved one stop to the right at a time until it is the last stop before the end depot (4-5). Then pickup node p^2 is moved one stop to the right and d^2 is placed right after it (6). Again d^2 is moved one stop to the right until it is the last stop before the end depot (7). Finally, p^2 and d^2 are both the last stops before the end depot (8).

Following every move of an item's pickup and/or delivery node the route is checked for feasibility. The feasibility checks ensure that the restrictions of the vehicle regarding the batch heights and weights are not violated. Once a route is infeasible, moving the delivery node further to the right still leads to infeasible routes, hence the pickup node must be moved one stop to the right and the delivery node placed right after it. In addition, the $cost\ of\ insertion$ is computed in every feasible insertion position and the insertion position with the least cost is stored for the request at hand. Since insertions across different routes can lead to the same insertion costs, we insert the request into the shortest route in order to balance route lengths.

At the time of the insertion the algorithm only checks if the adjacent nodes in the insertion positions are identical to the nodes to insert. However, it could happen that the delivery node already occurs between the pickup and delivery positions. Hence, after the request has been inserted into the position that incurs the least cost, a post processing step is necessary to ensure that the delivery node inserted is necessary. All nodes between the pickup and delivery position are checked for the delivery node. If the delivery node is found prior to its insertion position, then the first occurrence is kept and the other one deleted. This is because the delivery node inserted in an earlier stage might contain multiple items, while the delivery node inserted recently does not. Keeping the former delivery node also gives us spare capacity which we can use for other items.

The cost of an insertion is calculated by means of an evaluation function. The evaluation function is a weighted aggregation function as described in section 4.2, containing evaluation criteria as functions. Evaluation criteria capture the change in the measures of a route due to request insertions or removals[6]. Every evaluation criterion is normalized and is multiplied with a weight. We normalize the evaluation criteria by dividing them by a value that is at least as high as the highest value found for this evaluation criterion given the set of requests R. These normalization values are found experimentally before we run the algorithm to find solutions. We therefore simply run the algorithm without normalization and save the largest values for the respective objectives. Each normalized evaluation criterion can take a value between 0 and 1. This transforms the evaluation criteria to similar sizes and makes them comparable. Weights are multiplied with the evaluation criteria, representing their relative importance. Evaluation criteria with a higher weight have a higher priority. The weights are varied from 0 to 1 in incremental steps and are complementary, meaning the increase in one weight has to be

[6] We provide a more detailed description of the evaluation criteria in chapter 6.

decreased from the other weight/weights. E.g. for 2 evaluation criteria, we start with weight 1 for the first evaluation criterion and 0 for the second evaluation criterion. Hence, only the first evaluation criterion is considered in the solution. Then, if we decrease the weight for evaluation criterion 1 by 0.1 we would have to increase the weight of evaluation criterion 2 by the same amount (0.1). We repeat this step until the weight for evaluation criterion 2 is 1 and the weight for evaluation criterion 1 is 0. For every weight vector we run the LNS $number\ of\ iteration$ times and subsequently pick the solution yielding the lowest cost ($best\ solution$). We use the objective function to compute the $best\ solution$, in contrast to the cost calculation due to the insertions/removal of requests where we use the evaluation function. Objective functions capture the total measures of a solution (e.g. *Total distance* of routes, *Average temperature of the warmest batch*, etc). We sort the set of solutions obtained for the weight vectors such that only non-comparable solutions remain.

5.2 Large neighbourhood search (LNS)

We use a LNS metaheuristic as introduced by (Shaw, 1997). The LNS starts by assuming that an initial solution is provided. The LNS then repeatedly destroys and rebuilds parts of the solution by removing and reinserting certain items into different positions in the route. After each iteration a $new\ solution$ is found and it´s corresponding objective value is calculated.

We apply a $deterioration\ threshold$ as the acceptance criterion. The $deterioration\ threshold$ is a parameter that describes the percentage of deterioration accepted compared to the best solution (Danloup, Allaoui and Goncalves, 2018). The $deterioration\ threshold$ together with the $best\ solution$ determine whether this $new\ solution$ replaces the $incumbent\ solution$. The $new\ solution$ becomes the $incumbent\ solution$ if it is not worse than $deterioration\ threshold\ *\ best\ solution$. This does not mean that the $new\ solution$ has to be better than the $incumbent\ solution$ however, and as a consequence the $incumbent\ solution$ can deteriorate over time. The aim of this approach is to escape from local optima. Should the $new\ solution$ be even better than the $best\ solution$, then it replaces the $best\ solution$. The LNS stops as soon as the stopping criterion is met.

```
function LNS (requests, number of iterations, deterioration threshold)
    incumbent solution = INSERTION_HEURISTIC
    best solution = incumbent solution
    request bank = ∅
    for every iteration in number of iterations do
        new solution = incumbent solution
        WORST_REMOVAL (new solution, request bank)
        INSERTION_HEURISTIC (new solution, request bank)
        if f(new solution) ≤ f(best solution) ∗ deterioration threshold then
            incumbent solution = new solution
        end if
        if f(new solution) ≤ f(best solution) then
            best solution = new solution
        end if
    end for
    return best solution
end function
```

Our initial solution is the solution found by the sequential cheapest insertion heuristic in section 5.1. We use an acceptance criterion of 5%. The LNS stops after a predefined $number\ of\ iterations$, which we choose experimentally depending on the evaluation criterion and instance at hand. The amount of removed items at each iteration and hence the size of the neighbourhood is determined by the $destroy\ rate$. The larger it is, the larger the explored solution space, but the slower is the insertion procedure. Additionally, choosing a $destroy\ rate$ that is too high might give bad results when one uses a heuristic for inserting requests. (Ropke and Pisinger, 2006) We start with a $destroy\ rate$ of 80 percent and uniformly decrease it by 2 percent in the course of the following iterations. The performance and robustness of the overall heuristic is dependent on the removal and insertion procedures used (Ropke and Pisinger, 2006). There are several removal and insertion procedures in the literature. For simplicity, we use the insertion procedure described in Figure 2. We use an adapted version of the worst removal procedure from (Ropke and Pisinger, 2006):

function WORST_REMOVAL ($new\ solution, request\ bank$)
    $counter = 0$
    while ($counter < destroy\ rate * |requests|$)
        L = array of all requests i
        randomly shuffle L
        sort L by descending $cost(i, s)$
        request: $r = L[counter\ ]$
        remove r from solution s
        store r in $request\ bank$
        $counter = counter + 1$
    end while
end function

At first the array L, containing all requests, is shuffled randomly. Then the removal cost of every request i is calculated. The cost of every request i is its detour.

$$detour_i = distance_i - distance_{pickup_i\,,\,delivery_i}$$

The requests are sorted such that those requests with the highest cost are removed first. Not all requests that are removed have a cost. Those requests removed without a cost are randomly picked out, due to the shuffling operation before. We do not consider the request temperature to keep the method simple.

6 Measuring distance and temperature

The temperature of a transportation batch b is dependent on the temperatures of the items that are contained in this batch. Every batch has a temperature denoted $temperature_b$, where $b \in B$ and B denotes the set of all batches in a solution. The heat impact of the transportation batch on the vehicle can be assumed in many ways. Therefore, we investigate multiple vehicle heat exposure measures that allow us to quantify the temperature of a transportation batch and its impact on the vehicle. Note that there is no specification of which evaluation criteria to combine. Any combination of evaluation criteria is possible. In the following we present the evaluation criteria used in this thesis:

a) *Increase of distance*: This evaluation criterion simply minimizes the *Total distance*, which is the sum of the distances travelled by all vehicles. Requests are inserted such that the *Total distance* is increased by as little as possible. The increase of the *Total distance* by request r is obtained by subtracting the *Total distance* when request r is not included from the *Total distance* when request r is included. For each request $r \in R$, we insert r´s pickup and delivery node into positions in a route such that the resulting *Increase of distance* is as minimal as possible. If the nodes p^r and d^r are already part of a route, then the item of request r is added into all batches between p^r and d^r, provided that the vehicle capacity restrictions for those batches are not violated. In this case the *Increase in distance* is 0. If p^r and d^r are not part of any route then by inserting them an *Increase in distance* greater than 0 would incur.

b) *Temperature dependent item detour*: This evaluation criterion considers the difference between the actual distance that is travelled with the item of request r by a vehicle k, i.e., t_r^k and the item´s minimum distance to be travelled, which is the distance between the item´s pickup and delivery node, i.e., t_{p^r,d^r}. This difference is called detour.

$$detour_r = t_r^k - t_{p^r,d^r}$$

The goal is to choose an insertion position for every request $r \in R$ such that $detour_r$ is minimal. The detour should be smaller the warmer an item´s temperature is. Hence the item detours are multiplied with the item temperature. Consequently, the algorithm is inclined to insert items in positions with less detours the warmer the items are. All requests that are to be inserted are sorted with respect to increasing temperature before the process takes place. This ensures that paths of warm items (which are inserted at last) are less likely to be broken up by subsequent insertions.

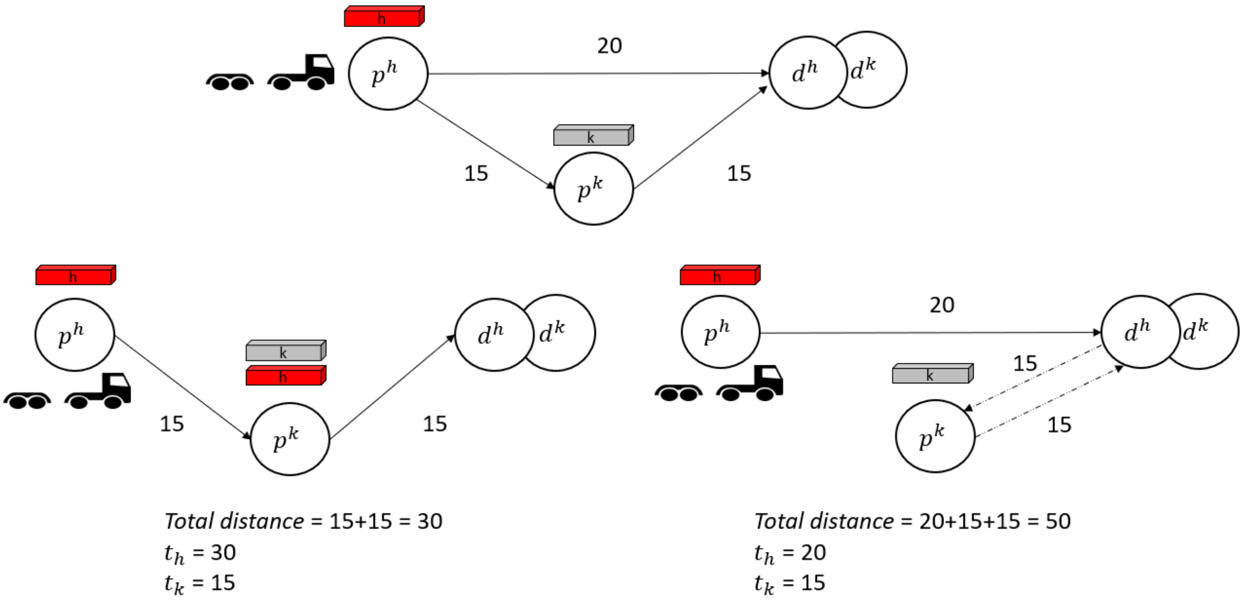

Figure 3 Item detour

Figure 3 illustrates the application of the evaluation criterion *Temperature dependent item detour*, where we minimize the detour for all requests contained in a vehicle´s route. We can see the two requests h and k. Item h is warm, while item k is not. Additionally, we can see the pickup and delivery nodes for item h or k, respectively. Nodes d^h and d^k are at the same location since items h and k have the same delivery location and hence drawn overlapped. The distance p^h to d^h amounts to 20, while the distance p^k to d^k amounts to 15. The route on the bottom left shows the route that results if the *Total distance* is minimized. The *Total distance* is smaller when items h and k are transported in the same batch from p^k to d^h= d^k. In this case the *Total distance* amounts to 30. However, the warmer the items, the more are they preferred to be transported as quickly as possible. The route on the bottom right shows the route that results if items h and k are transported separately. The vehicle transports item h from p^h to d^h first, then travels to p^k, picks up item k and then delivers it at d^h= d^k. This results in a *Total distance* of 50, however the items are transported directly from their pickup to their delivery nodes.

c) *Average temperature by weight*: In this evaluation criterion we assume a vehicle´s temperature to be the average temperature of the items it carries. The evaluation criterion considers the temperature of every item in a batch, i.e., $\forall r \in b$, in relation to an item´s percentual weight contribution to the transportation batch weight. The heavier an item, the more its temperature affects the average batch temperature:

$$temperature_b = \frac{w^r}{\sum_{r \in b} w^r} * temperature^r$$

The expectation of this evaluation criterion is to purposefully put warm items into batches together with cold items in order to reduce the average batch temperature.

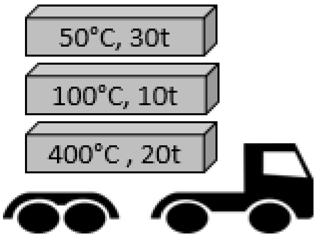

Figure 4 Average temperature by weight

Figure 4 shows a batch where the temperatures of all three items are considered. We divide the item weights by the batch weight (the sum of the item weights). The topmost item has a weight of 30t, divided by the sum of item weights (60t), yields 0.5, thus it contributes 50% to the batch weight with a temperature of 50°C. The item at the bottom contributes only 20/60= 1/3 to the batch temperature, despite being the warmest item in the batch. The item in the middle contributes 10/60 = 1/6 to the batch temperature with 100°C. Thus, the batch weight:

$$temperature_b = \frac{30t}{60t} * 50°C + \frac{10t}{60t} * 100°C + \frac{20t}{60t} * 400°C = 175°C$$

d) *Temperature dependent load*: This evaluation criterion considers the temperature of the warmest item of the batch and the total batch weight:

$$temperature_b = \max_{r \in b}(temperature^r) * \sum_{r \in b} w^r$$

This evaluation criterion models the thermodynamic concept of heat transfer between two solid state bodies by conduction. The heat transfer between two solid state bodies is, among other things, dependent on the pressure with which the two bodies are pressed against each other. (Cengel, 1976). Since the vehicle picks up and transports a batch of items by physical contact with the lowermost item, the temperature of this item along with the pressure it is subjected to from the top items is crucial for the heat exposure to the vehicle. In the worst case the warmest item lies at the bottom of the batch. Since the item at the bottom is the one the vehicle establishes physical contact with and the weight of the entire batch lies on this bottom item, heat exposure will be greater the warmer the bottom item is and the more weight lies on top of it.

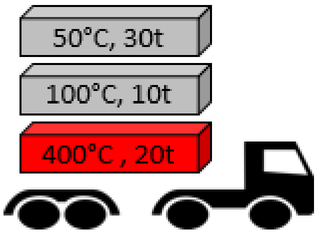

Figure 5 Temperature dependent load

The same batch as in Figure 4 is considered in Figure 5. Yet here only the temperature of the warmest item is considered, since we assume the worst case, where the warmest item of a transportation batch is the lowermost item, such that all other items are piled on top of it (as depicted). The temperature of the warmest item (400°C) is multiplied with the sum of item weights that rest on top of it (60t)

$$temperature_b = 400°C * (30t + 10t + 20t) = 24000$$

f) *Distribution penalty*: This evaluation criterion adds a penalty to the insertion cost in case a *warm* item is inserted into a route at a position where other *warm* items are nearby, i.e., before or after said position. The penalty is especially high when a *warm* item is added into a batch that already contains other *warm* items. As discussed in chapter 2 the decision maker chooses what item temperature is considered warm. The penalty consists of a constant *fee* that decreases the higher the distance between warm items is. For a warm item that is considered for insertion h and the closest warm item i that is already part of the route the penalty for inserting h is computed as follows:

$$penalty_h = \frac{fee}{t_{d^h,p^i}}$$

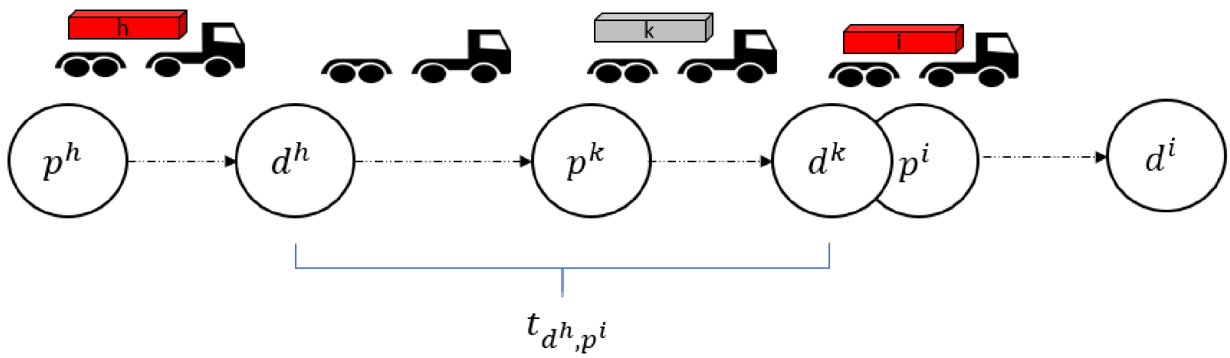

Figure 6 Distribution penalty

Figure 6 displays the application of the distribution penalty for item h. Items k and i are already part of the route and item h is about to be inserted, hence it´s penalty is calculated. Items h and i are warm, while item k is not. When item h is inserted the algorithm checks all batches where h is included if there are other *warm* items in the respective batches. If so, the value for the denominator is set to 1. Hence the full *fee* is incurred. This penalises having multiple warm items in a batch at the same time. The value for the denominator is also set to 1 in case a warm item is delivered right when item h is picked up, i.e. location p^h is also the delivery location d for a *warm* item that is already part of the route, or in case a warm item is picked up right when item h is delivered, i.e. location d^h is also the pickup node p for a *warm* item that is already part of the route. If there is at least one arc between item h and the closest batch with a batch containing no *warm* items, the fee is divided by the distance from pickup/delivery node of item h to the pickup/delivery node of next closest *warm* item. In Figure 6 item h is inserted and the closest *warm* item that is already part of the route is item i. Hence, we divide the *fee* by the distance between the location when h is delivered and the next *warm* item i is picked up (t_{d^h,p^i}).

7 Experimental results and comparison to current practices

7.1 Experimental results

We show how potential goals can be reached by applying the evaluation criteria introduced in chapter 6 on an artificial instance we created. While we optimize some evaluation criteria separately (*Increase of distance*, *Average temperature by weight*) in an evaluation function, we combine other evaluation criteria (*Temperature dependent load*, *Temperature dependent item detour* and *Distribution penalty*) with the evaluation criterion *Increase of distance* into an evaluation function, because optimizing them separately would lead to route lengths that are in our perception impractically high. The artificial instance consists of 10 requests, where the pickup and delivery locations are distributed such that they model real-world historic data provided by our industrial partner. We choose 10 requests and consider a single-vehicle scenario, because it creates an instance small enough to have a clear visual representation but large enough to demonstrate our purposes. Table 1 shows the solutions obtained. In the top row the evaluation functions are denoted. They are numbered from 1 to 5. The remaining rows show different KPIs (Key Performance Indicators) used to analyse the solution structure and characteristics of routes. Each column represents the KPIs of one evaluation function.

We use the following KPIs: Travel distance, Dead headings, Average utilization, Average batch temperature, Average temperature of the warmest batch, Average temperature of the second warmest batch, and Total detour. We illustrate the computation of the KPIs for evaluation function *Increase of distance* in Table 2: *Total distance* denotes the sum of the distances of all arcs travelled by the vehicle(s). In total, those 9 arcs amount to a distance of 1115. 2 of the 9 arcs are travelled without any load (*Dead headings*), they amount to 279. *Average utilization* is calculated as follows: First the percentage of the vehicle weight capacity that is utilized is calculated and multiplied with the arc distance for every arc. For example, for the first arc of the route the percentual weight utilization is 90/105, since the batch weight on that arc is 90 tons and the vehicle capacity is 105 tons. The products of all arcs are summed up and divided by the *Total distance* of the route, which is 1115. This yields 45%, meaning that on average 45% of the vehicle(s) weight capacity is utilized on the route(s). We calculate the *Average batch temperature* as follows: First we build the products of the batch temperatures and the respective distance travelled with the batch. For example, the vehicle at hand travels a distance of 121 ($t_{22,3}$) with batch 1, which has a batch temperature of 250°C. It travels a distance of 50($t_{3,0}$) with batch 2, which has a batch temperature of 325°C, etc. Then we sum up these products and divide them by the *Total distance.* The *Average batch temperature* is the average temperature of the batches while the vehicle travels through the route. KPIs *Average batch temperature of the warmest batch* and *Average batch temperature of the second warmest batch* denote the two warmest batch temperatures in the route. The temperatures of the two warmest batches amount to 325°C, which is batch 2 and 250°C, which is batch 1. Finally, *Total detour* is calculated as the sum of all detours of the requests in the route. For example, item 1 is transported between nodes 2 and 17 in arc 4 and then between nodes 17 and 8 in arc 5. The shortest distance for item 1 would be from node 2 to node 8 however. Thus, the detour for item 1 is computed as $t_{2,7} + t_{17,8} - t_{2,8}$. We do the same computation for all items in the route. The sum of these computations for all items yields the *Total detour*.

Additionally, we want to give the reader a visual representation of the solutions obtained. The routes to the solutions in Table 1 are depicted from Figure 7 to Figure 11. The nodes represent locations (pickup and/or delivery), while the arcs represent the shortest distances between the locations. The rectangles display the items that belong to the batch that the rectangle is pointing at. The first number in the rectangle is the arc and batch number. It displays the sequence in which the vehicle traverses the arcs, but also denotes the batch that is transported in this arc. The numbers in the brackets correspond to the items in the respective batch. The subscript stands for the order of occurrence of the item in the route while the superscript stands for item temperature. Empty denotes a dead heading. Finally, the last number in the rectangle corresponds to the average batch temperature. For example, $1\ (2_{1/3}^{300°C}, 4_{1/2}^{350°C}, 7_{1/1}^{100°C}), 250°C$, denotes the first batch in the route. Items 2, 4 and 7 belong to this batch. Item 2 has a temperature of 300°C and is contained in three consecutive batches, starting with batch 1. Item 4 has a temperature of 350°C and appears first in batch 1. It is contained in 2 consecutive batches, meaning that batch 2 also contains item 4. Item 7 is contained only in batch 1. It has a temperature of 100°C. The average temperature of all items in batch 1 amounts to $\frac{(300+350+100)}{3}°C = 250°C$. The textboxes indicate the start and end locations of the route.

	1) Increase of distance	2) Average temperature by weight	3) 0.1*Increase of distance+ 0.9*Temperature-dependent load	4) 0.5*Increase of distance+ 0.5*Temperature dependent item detour	5) 0.5*Increase of distance+ 0.5*Distribution penalty
Total distance	1115 (9 arcs)	1580 (12 arcs)	1606 (11 arcs)	1385 (12 arcs)	1465 (12 arcs)
Dead headings	279 (2 arcs)	329 (3 arcs)	408 (3 arcs)	365 (4 arcs)	434 (3 arcs)
Average utilization	45 %	42 %	25%	35%	30%
Average batch temperature	122 °C	72°C	123°C	84°C	113°C
Average temperature of the warmest batch	325 °C	167°C	400°C	300°C	400°C
Average temperature of the second warmest batch	250 °C	150°C	350°C	225°C	200°C
Total detour	395	976	74	353	200

Table 1 Comparison of evaluation functions

Total distance	$t_{22,3} + t_{3,0} + t_{0,2} + t_{2,17} + t_{17,8} + \mathrm{t}_{8,34} + \mathrm{t}_{34,8} + \mathrm{t}_{8,14} + \mathrm{t}_{14,12}$ = 1115 (9 arcs)
Dead headings	$t_{8,34} + t_{8,14}$ = 279 (2 arcs)
Average utilization	$$\frac{\sum_{(i,j)\in A}\left(\left(\frac{\sum_{r\in b} w^r}{w^{max}}\right) * t_{ij}\right)}{Total\ distance}$$ 1: (90/105)*121=103,7 2: (60/105)*50=28,6 3: (60/105)*160=91,4 4: (60/105)*77=44 5: (90/105)*192=164,6 6: (0/105)*120=0 7: (30/105)*120=34,3 8: (0/105)*159=0 9: (30/105)*116=33,1 $$\frac{103{,}7 + 28{,}6 + 91{,}4 + 44 + 164{,}6 + 0 + 34{,}3 + 0 + 33{,}1}{121 + 50 + 160 + 77 + 192 + 120 + 120 + 159 + 116} = 45\%$$
Average batch temperature	$$\frac{\sum_{b\in B}\frac{\sum_{r\in b} temperature^r}{\lvert r \in b \rvert} * t_{ij}}{Total\ distance}$$ $$\frac{250 * 121 + 325 * 50 + 175 * 160 + \cdots + 50 * 120 + 0 * 159 + 50 * 116}{1115} = 122°C$$
Average temperature of the warmest batch	325°C ($t_{3,0}$)

Average temperature of the second warmest batch	250°C ($t_{22,3}$)
Total detour	$$\sum_r t_r^k - t_{p^r,d^r}$$ Item 1: $t_{2,17} + t_{17,8} - t_{2,8} = 77 + 192 - 225 = 44$ Item 2: $t_{22,3} + t_{3,0} + t_{0,2} - t_{22,2} = 121 + 50 + 160 - 36 = 295$ Item 3: $t_{17,8} - t_{17,8} = 0$ Item 4: $t_{22,3} + t_{3,0} - t_{22,0} = 121 + 50 - 160 = 11$ Item 5: $t_{0,2} + t_{2,17} + t_{17,8} - t_{0,8} = 160 + 77 + 192 - 384 = 45$ Item 6: $t_{14,12} - t_{14,12} = 0$ Item 7: $t_{22,3} - t_{22,3} = 0$ Item 8: $t_{34,8} - t_{34,8} = 0$ 44+295+11+45=395

Table 2 KPI calculation Total distance

1) Increase of distance: We obtain the first solution by optimizing the evaluation criterion *Increase of distance*. The route obtained has a *Travel distance* of 1115 on 9 arcs. 279 of those 1115 are *Dead headings*. It is the shortest route among the 5 solutions in Table 1. The *Average temperature of the warmest item* amounts to 325°C and the *Average temperature of the second warmest item* amounts to 250°C. The *Average batch temperature* amounts to 122°C, which is the second highest *Average batch temperature* among the 5 solutions, because the temperature of items is not considered at all in this solution. The solution has the second highest *Total detour* with 395, because loading up many items at once and subsequently visiting one delivery node after the other leads to a lower *Total distance* then having to visit every pickup node in-between. This also results in the highest *Average utilization* among the 5 solutions with 45%.

Figure 7 shows the route corresponding to solution 1 (*Total distance*) in Table 1. The warmest items are transported at the beginning of the route with considerably less heat exposure on the subsequent parts of the route. The first 5 arcs correspond to the batches with the highest thermal load, while the last 4 arcs contain two empty trips (dead headings). We prefer a more even distribution of thermal load across the route to avoid extremes. Batches 1 and 2 include items 2 (300°C) and 4 (350°C). We prefer to distribute warm items across batches such that the average temperature of the warmest batches is a low as possible. The disadvantage of this evaluation criterion is that it can lead to high detours (the *Total distance* of the route is low, however, the distances travelled by individual items can be long). The fact that the temperature is not considered means that every possible disadvantage in connection with the temperature can occur.

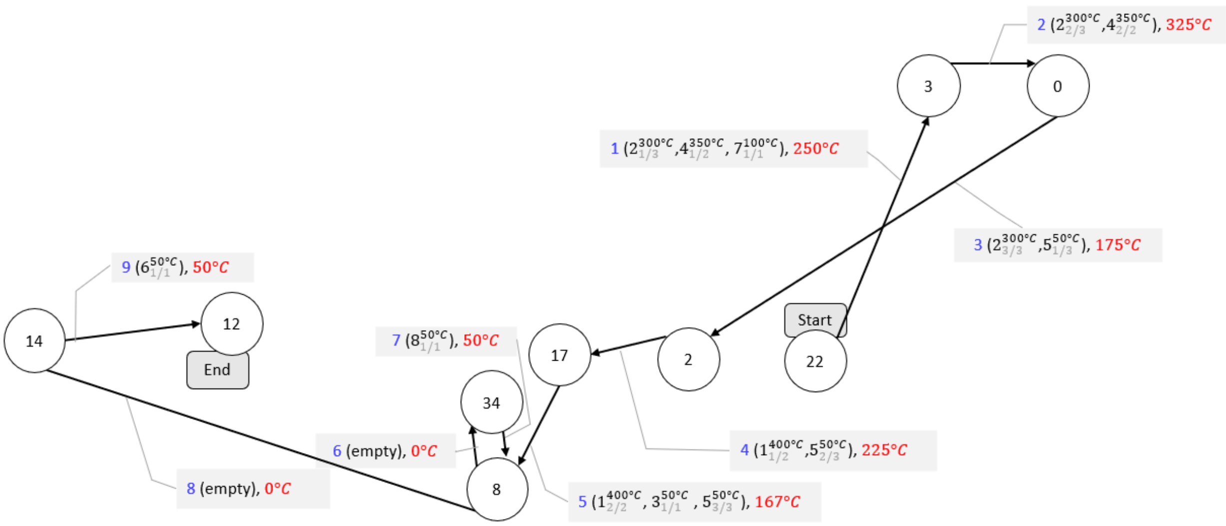

Figure 7 Total distance

2) Average temperature by weight: The second solution is obtained by optimizing with respect to the *Average temperature by weight*. The route obtained has a *Total distance* of 1580, an increase by 42% in comparison to solution 1, while *Dead headings* increase by 18% to 329. The *Average batch temperature* is the lowest among the 5 solutions with 72°C. It is about 50°C lower than the highest *Average batch temperature*s (solutions 1 and 3). The *Average temperature of the warmest batch* (167°C) as well as the *Average temperature of the second warmest batch* (150°C) are the lowest of the 5 solutions as well. They were reduced by 158°C and 100°C, respectively, in comparison to solution 1. The *Average utilization* is 42% and only slightly lower than the highest Average utilization (solution 1 with 45%). However, the *Total detour* amounts to 976, which is about 1.5 times the *Total detour* of solution 1.

Figure 8 displays the route obtained by solution 2. Overall, warm items are more evenly distributed along the route. The algorithm deliberately transports cold items together with warm items in order to reduce the average temperature of the batch, when this objective is considered. This can lead to undesirable features such as high detours however as it can be seen for the transportation of item 5. Item 5 is picked up and transported along arcs 3 to 8 (6 arcs) for the sole purpose of reducing the average temperature of batches 5, 7 and 8, because those batches contain warm items (item 4 in batch 5 with 350°C, item 2 in batch 7 with 300°C and item 1 in batch 8 with 400°C). Similarly, item 3 is transported along arcs 4 to 8 for the same purpose.

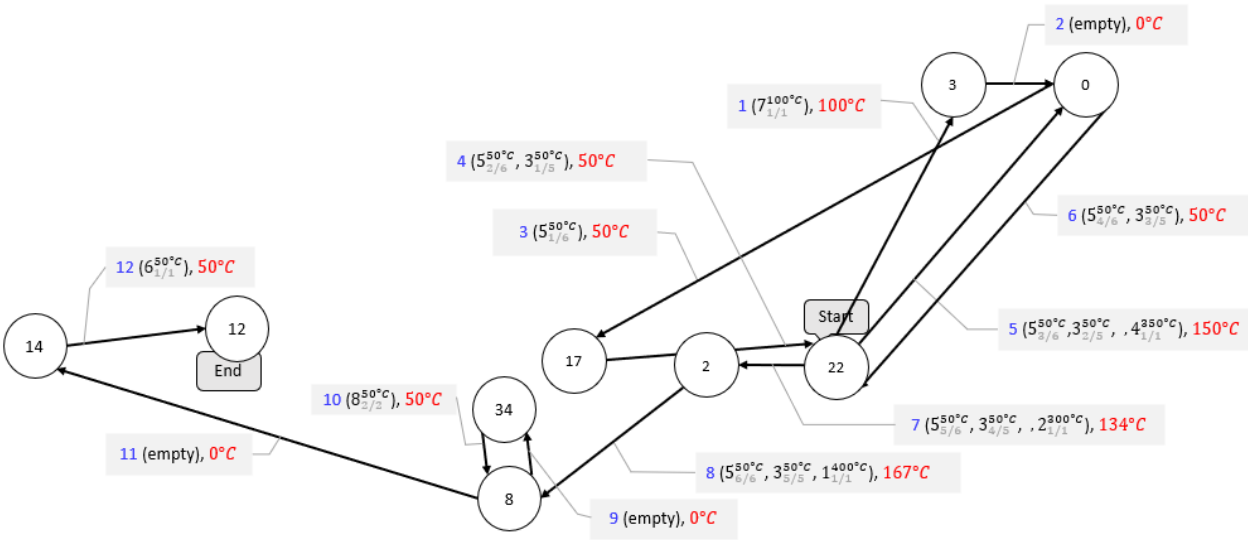

Figure 8 Average temperature by weight

3) Temperature-dependent load + Total distance: The evaluation function of the third solution in Table 1 consists of the two evaluation criteria *Temperature-dependent load* (weighted with 0.9) and Total distance (weighted with 0.1). The solution yields a *Total distance* of 1606, which is the highest in Table 1 and an increase by 44% compared to the lowest *Total distance* (solution 1 with 1115). *Dead headings* increase in comparison to solution 1 by 46% to 408. The *Average batch temperature* is the highest in Table 1 amounting 123°C. It is only slightly higher than solution 1 (122°C). The *Average batch temperature of the warmest batch* and the *Average batch temperature of the second warmest batch* are 400°C and 350°C, respectively. Solution 3 has the highest values for these 3 KPIs. The *Average utilization* amounts to 25%, which is significantly lower than in the other solutions.

Figure 9 shows the route corresponding to solution 3 in Table 1. Items 1 (400°C), 2 (300°C), 4 (350°C) and 7 (100°C) are all transported separately, such that the weight resting on top of them is as low as possible. This also has the effect that they are transported directly. Items 3, 5, and 8 (all 50°C) are consolidated into batches such that the *Total distance* is reduced. The fact warm items are transported separately explains the low *Average utilization. Temperature-dependent detour* features a high amount of *Dead headings*, which is good for cooling the vehicle down. It also results in low detours. The few items that are subjected to detours are cold. The separate transportation of warm items leads to high values for the batch temperatures.

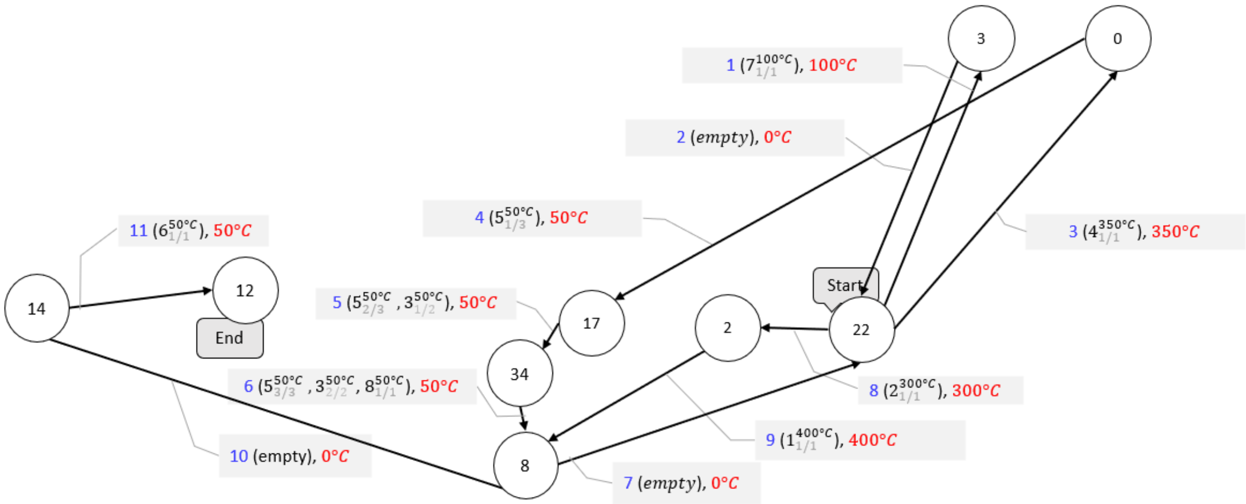

Figure 9 Temperature dependent Load + Total distance

4) Temperature dependent item detour + Total distance: We include the evaluation criterion *Temperature dependent Item detour* to obtain solution 4 in Table 1. Both, *Temperature dependent Item detour* and *Total distance* are both weighted with 0.5. Since both objectives are distance-related, this solution yields the second lowest *Total distance* among the 5 solutions in Table 1 with 1385. It is only 24% higher than the lowest value (solution 1 with 1115). The *Average utilization* amounts to 35%, which is ordinary. The *Average batch temperature* is the second lowest in Table 1 with 84°C. Only solution 2 has a lower *Average batch temperature* with 72°C. This is a coincidence, because the warmest item (item 1 with 400°C) shares its way along arc 8 with two cooler items 3 and 5 (both 50°C) as it can be seen in Figure 10. The *Average temperature of the warmest batch* is 300°C and the *Average temperature of the second warmest batch* is 225°C. The *Total detour* amounts to 353. This is a high value considering the fact that *Temperature dependent item detour* is part of the evaluation function.

However, detours only occur in connection with items 3 (50°C), 5 (50°C) and 7 (100°C), none of the warm items are affected.

When weighted accordingly the evaluation criterion *Temperature dependent item detour* guarantees that no detours occur w.r.t. warm items. Detours w.r.t. cooler items can still occur. *Temperature dependent item detour* also does not consider the composition of batches. Multiple warm items could be transported in a common batch.

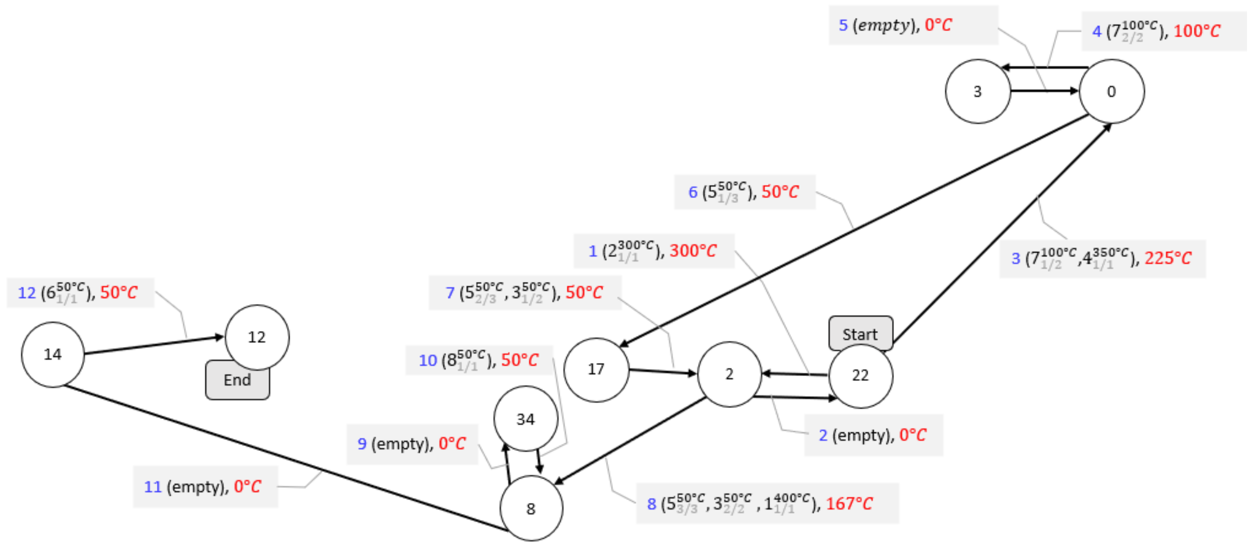

Figure 10 Item detour + Total distance

5) Distribution penalty + Total distance: Solution 5 in Table 1 is obtained by combining the evaluation criteria *Distribution penalty* and *Total distance* and weighting each with 0.5. The *Total distance* amounts to 1465, an increase of 32% in comparison to solution 1, while *Dead headings* increase by 56% compared to solution 1 to 434. The *Average utilization* amounts to 30%, which is the second lowest value (*Average utilization* of solution 3: 25%). The *Average batch temperature* amounts to 113°C, which is ordinary compared to the other values. *Average temperature of the warmest batch* amounts to 400°C and *Average temperature of the second warmest batch* amounts to 200°C. The *Total detour* amounts to 200, however, just as in solution 4 detours do not occur in connection with warm items.

The route corresponding to solution 5 can be seen in Figure 11. With this evaluation function we achieve several important characteristics for a route. Warm items are distributed across the entire route. Item 4 (350°C) is transported in batch 1, while item 2 (300°C) is transported in batch 5 and item 1 (400°C) is transported in batch 10. The arcs in-between serve to cool down the vehicle. Warm items are transported either separately or along with cold items, but never with other warm items. Item 1 (400°C) is transported separately, while item 2 (300°C) is transported together with item 5 (50°C) and item 4 (350°C) is transported together with item 7 (100°C). Warm items are transported directly from their pickup node to their delivery node.

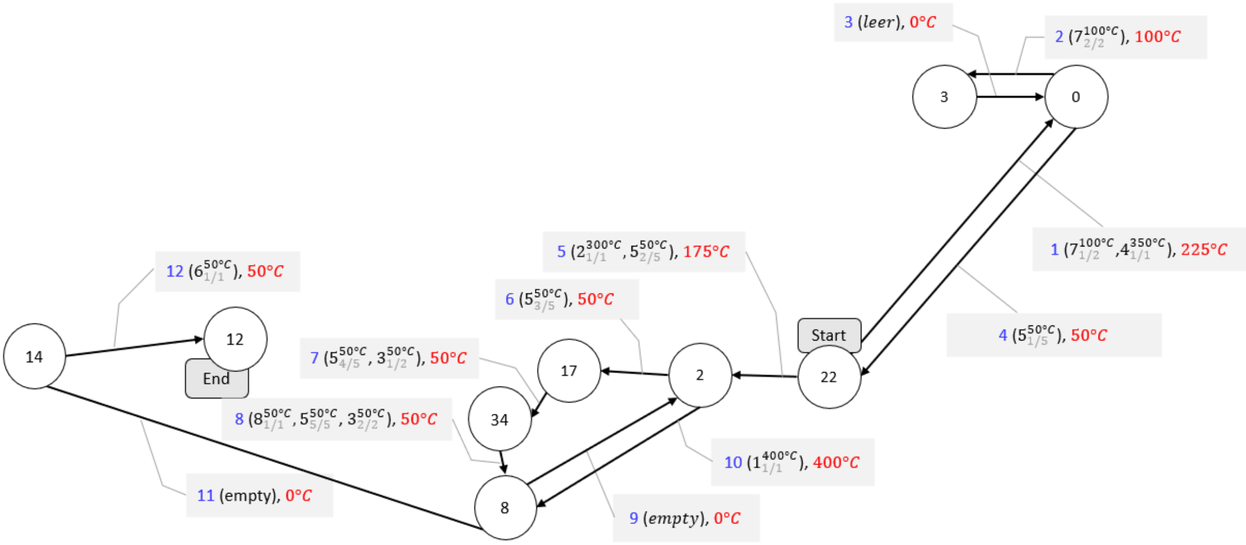

Figure 11 Distribution penalty + Total distance

7.2 Comparison to current practices

Our test instance originates from real world data. We picked a sample data set, where we consider a time span of three hours. The data set contains 162 requests, 30 of those requests or 18,5% have a temperature of at least 300°C. Figure 12 shows the distribution of pickup-and-delivery-pairs (p^r, d^r) among requests for the data set. The most frequently occurring requests in the data set are transported from node 17 to node 8 (distance 192), from node 0 to node 8 (distance 384) and from node 14 to node 2 (distance 263). Unlike in section 7.1 we consider a multiple vehicle case.

Figure 12 pickup-delivery locations of requests for data set

In the following we generate solutions for the data set described above, using the LNS in combination with the same evaluation functions used in section 7.1. Subsequently we analyse these solutions and compare them with the solutions obtained by the industry by means of different KPIs.

Beside the 7 KPIs used in section 7.1 we use 4 new KPIs to measure qualities that are specific to the evaluation criteria *Heat dependent load*, *Temperature dependent item detour* and *Distribution penalty*:

- *Maximum (item temperature * item detour)* captures the greatest product of temperature and detour of all items $r \in R$: $\max_{\mathrm{r \in R}}(temperature_r * detour_r)$, while
- *Total (item temperature * item detour)* captures the sum of the products of temperature and detour of all items $r \in R$: $\sum_{r \in R}(temperature^r * detour_r)$.

- *Maximum conduction (Warmest Item per batch * batch weight)* captures the greatest product of the highest item temperature per batch and the respective batch weight of all batches in the routes: $\max_{b \in B}(\max_{r \in b}(temperature^r) * \sum_{r \in b} weight^r)$.
- *Total penalty* captures the sum of the penalties incurred during the insertion of the requests: $\sum_{r \in R} penalty_r$.

Note that we only consider warm items for the computation of *Maximum (item temperature * item detour)* and *Total (item temperature * item detour).*

Table 3 shows the KPIs obtained for the data set described in the beginning of this section. We used the same evaluation functions as in section 7.1, but changed the weights for the evaluation criteria to 0.5 for better comparability, the only exception being *Increase of distance*, which we optimized separately. The rows display the different KPIs. The first columns (*Industrial practice*) display the KPIs obtained for the solutions by the industrial partner. The remaining columns show the KPIs obtained for the solutions for the evaluation functions. The solutions are numbered from 1 to 6. Solution 1 constitutes the Industrial practice, while solutions 2 to 6 constitute the solutions obtained by applying the respective evaluation criteria. Note that the scheduling is dependent on the schedule planner. Scheduling can be done in different ways. Hence, this instance is an example and does not represent the general practice at the industrial partner.

Appendix B to Appendix G show the routes generated by the evaluation criteria/*Industrial practice* for the data set in Table 3. Numbers in front of the brackets represent the various locations in the plant, while numbers inside the brackets represent the items that are on the vehicle as it approaches the next location. Empty brackets indicate all items previously on the vehicle have been unloaded and the vehicle drives to its next pickup location without load. Red numbers inside two $ signs indicate warm items (items with a temperature greater or equal to 300°C).

Note that in our algorithm we do not consider time windows and other practically relevant constraints that stem from business policies. Also, we simplify the problem by neglecting stochastic information, which is considered by the industrial partner. These are, besides our optimization efforts, contributing factors why our results improve upon the real-world results provided by the industrial partner.

1) The current practice can be seen in column ***Industrial practice***. The *Total distance* amounts to 21661 and the *Dead headings* amount 9958, which is about half of *Total distance. Total distance* and *Dead heading* are the highest among the 6 solutions in Table 3. The *Average utilization* is the lowest among the 6 solutions with 38%. The Average batch temperature is also the lowest among the 6 solutions with 56°C. However, *Average batch temperature of the warmest batch* amounts to 406°C and is relatively high in comparison with the other solutions. All detour-related KPIs amount 0. Judging from the solution structure it seems like, in this instance, the routing practice involves the consolidation of items with the same destination into a batch, transporting this batch to its destination and then travelling back empty to repeat the same procedure for the next batch. Hence, there are no detours for this solution. The *Maximum conduction* amounts to 38381 and is approximately the same for all solutions except for solution 4. The *Total penalty* amounts to 28024, which is rather high. Appendix B shows the routes for solution 1. On several occasions multiple warm items are transported in the same batch, hence the high *Total penalty.* We can also see that batches containing warm items are concentrated in routes 1 and 3, while in routes 2 and 4 warm items are not transported at all. Despite this, there is always an empty arc between the transportation of batches containing warm items, which provides time to cool down the vehicle.

2) Evaluation criterion ***Increase of distance*** provides the second solution. This solution provides the lowest *Total distance* with 15279 (a decrease of nearly 30% compared to solution 1) and also low Dead headings amounting 3995 (a decrease of nearly 60% compared to solution 1). It provides the highest *Average utilization* with 63%. This is a big difference compared to solution 1. *Dead headings* decrease stronger than *Total distance* compared to solution 1 and as a consequence the average vehicle load is higher (together with the *Average utilization*). Additionally, the *Average batch temperature* increases by 28°C in comparison to solution 1 as a consequence of less Dead headings in solution 2. The *Average batch temperature of the warmest batch* and the *Average batch temperature of the second warmest batch* both decrease compared to solution 1 by 25°C or 28°C, respectively. Since *Increase of distance* does not consider item temperatures we assume this to simply be a consequence of the request pickup and delivery nodes. In contrast to *Industrial practice* all the other solutions lead to detours. Appendix C shows that solution 2 leads to the detour of multiple hot items, namely items 36,44,73,145 and 148. The detour is rather low amounting 2311. However, because the detours concern multiple, warm items that are transported over 3 consecutive arcs each, *Maximum (item temperature * item detour)* amounts to 111384 and *Total (item temperature * item detour)* amounts to 227570. *Maximum conduction* takes an ordinary

magnitude with 39329 among the solutions, where conduction is not considered. The *Total penalty* is high with 33610. Just like in solution 1 multiple warm items are transported in the same batch.

3) We weight the evaluation criteria ***Increase of distance*** **and** ***Average temperature by weight*** and obtain solution 3. Total distance amounts to 16904 (a decrease of approximately 22% compared to solution 1) and also low Dead headings amounting 3954 (a decrease of nearly 60% compared to solution 1). The *Average utilization* amounts to 58%. The *Average batch temperature* for solution 3 amounts to 89°C, which is 33°C higher than solution 1. However, the *Average batch temperature of the warmest batch* amounts to 330°C (a decrease of 76°C compared to solution 1) and the *Average batch temperature of the second warmest batch* amounts to 324°C (a decrease of 31°C compared to solution 1). These are the lowest numbers for the two KPIs among the 6 solutions. The *Total detour* is very high with 7984. If we compare solution 3 with solution 2, which had similar KPIs up to this point, we can see that, despite *Total detour* being much higher for solution 3, *Maximum (item temperature * item detour)* and *Total (item temperature * item detour)* are higher for solution 2. This indicates that most of the items subjected to detours are colder items. In fact, Appendix D shows that a few cold items (items 112,122,123) are subjected to very long detours causing a *Maximum (item temperature * item detour)* of 15752 and a *Total (item temperature * item detour)* of 47388. The *Total penalty* decreases by about 12% compared to solution 1. It is similar among the 6 solutions except for solutions 4 and 6, where it is lower.

4) Solution 4 consists of the evaluation criteria ***Increase of distance*** **and** ***Temperature dependent load***. Both evaluation criteria are weighted with 0.5. *Total distance* decreases by approximately 19% to 17643 compared to solution 1. *Dead headings* are higher for solutions 4 to 6 compared to solutions 2 and 3. *Dead headings* decrease by 53% compared to solution 1 to 4713. The *Average utilization* is characteristically low for this solution amounting 49%. *Average batch temperature of the warmest batch* amounts to 409°C, which is only 3°C warmer than in solution 1 and the *Average batch temperature of the second warmest batch* amounts to 408°C, which is 53°C warmer than in solution 1. The *Total detour* amounts to 1740, which is low. The few items that are subjected to detours are cold items which is why *Maximum (item temperature * item detour)* and *Total (item temperature * item detour)* are 0. Since conduction is only considered in this solution *Maximum conduction* amounts to 19195, which 50% lower than for solution 1. *Maximum conduction* is about the same for all other solutions. *Total penalty* is

unusually low among the solutions 1 to 5. Appendix E shows that at most 2 warm items are consolidated into a batch, while multiple cold items are consolidated into batches of 4 items.

5) Solution 5 is a combination of the evaluation criteria ***Increase of distance and Temperature dependent item detour***. The solution yields a Total distance of 15790, which is a decrease of 27% compared to solution 1. The *Dead headings* amount 4574 (a decrease of 54% compared to solution 1). The Average utilization is very high with 60%. The *Average batch temperature* amounts to 70°C, which is the lowest *Average batch temperature* among the solutions 2 to 6. The *Average batch temperature* is still 14°C higher than in solution 1. The *Average batch temperature of the warmest batch* amounts to 382°C (24°C lower than in solution 1), while the *Average batch temperature of the second warmest batch* amounts to 348°C (7°C lower than in solution 1). The *Total detour* is the lowest among the solutions 2 to 6. It amounts to 1416. In addition, the little detours that occur concern cold items only, hence *Maximum (item temperature * item detour)* and *Total (item temperature * item detour)* are 0. The *Maximum conduction* amounts to 40807. *Total penalty* is similarly high compared to solution 1 amounting 28114. In Appendix F we can see multiple batches containing 3 warm items as well as one batch even containing 4 warm items.

6) Solution 6 is a combination of the evaluation criteria **Increase of distance and Distribution penalty**. It has the highest *Total distance* among solutions 2 to 6. The *Total distance* amounts to 20577 and only decreases by 0.5% compared to solution 1. The *Dead headings* are also the highest among solutions 2 to 6 with 4944 (a decrease of 50% compared to solution 1). The Average utilization amounts to 53%. The Average batch temperature amounts to 89°C, which is 33°C higher than in solution 1. The *Average batch temperature of the second warmest batch* amounts to 349°C, which is 6°C lower than in solution 1. Total detour is the highest among all 6 solution, amounting to 8283. There is one warm item subjected to a detour. That is why *Maximum (item temperature * item detour)* and *Total (item temperature * item detour)* both amount 32085. The *Maximum conduction* is 40395. The *Total penalty* amounts to only 693, which is a 98% decrease compared to solution 1. Appendix G shows an almost uniform distribution of warm items across the routes. Solution 6 shows similarities to solution 3 w.r.t some KPIs, namely *Average batch temperature*, *Total detour*, *Maximum (item temperature * item detour)* and *Total (item temperature * item detour)*. Both evaluation criteria seem to optimize particular objectives at the expense of *Total detour*. In case of solution 3 these objectives are the *Average batch temperature of the warmest batch* and *Average batch temperature of the second warmest batch*. In case of solution 6 the objective is *Total penalty*.

Data set	1) Industrial practice	2) Increase of distance	3) 0.5*Increase of distance + 0.5*Average temperature by weight	4) 0.5*Increase of distance + 0.5*Temperature dependent load	5) 0.5*Increase of distance + 0.5*Temperature dependent item detour	6) 0.5*Increase of distance + 0.5*Distribution penalty
Total distance	21661	15279	16904	17643	15790	20577
Dead headings	9958	3995	3954	4713	4574	4944
Average utilization	38%	63%	58%	49%	60%	53%
Average batch temperature	56°C	84°C	89°C	104°C	70°C	89°C
Average batch temperature of the warmest batch	406°C	381°C	330°C	409°C	382°C	361°C
Average batch temperature of the second warmest batch	355°C	327°C	324°C	408°C	348°C	349°C
Total detour	0	2311	7984	1740	1416	8283
Maximum (item temperature * item detour)	0	111384	15752	0	0	32085
Total (item temperature * item detour)	0	227570	47388	0	0	32085
Maximum conduction (Warmest Item per batch * batch weight)	38381	39329	42432	19195	40807	40395
Total penalty	28024	33610	24697	14671	28114	693

Table 3 Comparison of industrial practice and evaluation criteria

7.3 Trade-off between distance and temperature

Figure 13 shows a Pareto-approximation of an evaluation function consisting of the evaluation criteria *Increase of distance* and *Average temperature by weight*. We use the KPIs *Total distance* to evaluate the solution quality of *Increase of distance* and *Average batch temperature of the warmest batch* to evaluate the solution quality of *Average temperature by weight*. We choose the KPI *Average batch temperature of the warmest batch*, because we are interested in the worst-case scenario, i.e., the warmest batch.

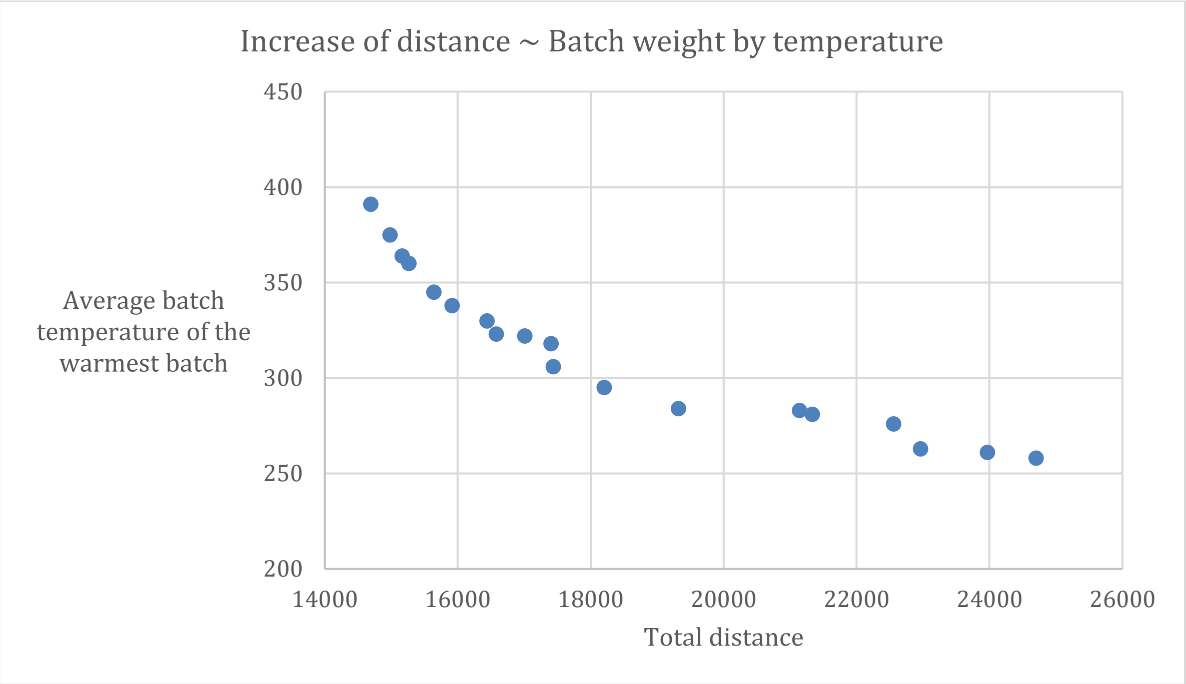

Figure 13 Relationship between Increase of distance and batch temperature

We notice an inverse relationship between *Average batch temperature of the warmest batch* and *Total distance*. The lower the *Average batch temperature of the warmest batch* the higher the *Total distance* of the routes and vice versa. The highest *Average batch temperature of the warmest batch* is around 390°C while the lowest amounts to around 260°C. The *Total distance* ranges from around 14500 to around 25000.

Figure 14 shows a Pareto-approximation that is obtained with an evaluation function consisting of the evaluation criteria *Increase of distance* and *Temperature dependent load*. The KPIs *Total distance* and *Maximum conduction* measure the solution quality of those evaluation criteria. We choose the KPI *Maximum conduction*, because it is the only KPI accounting for the evaluation criterion *Temperature dependent load.*

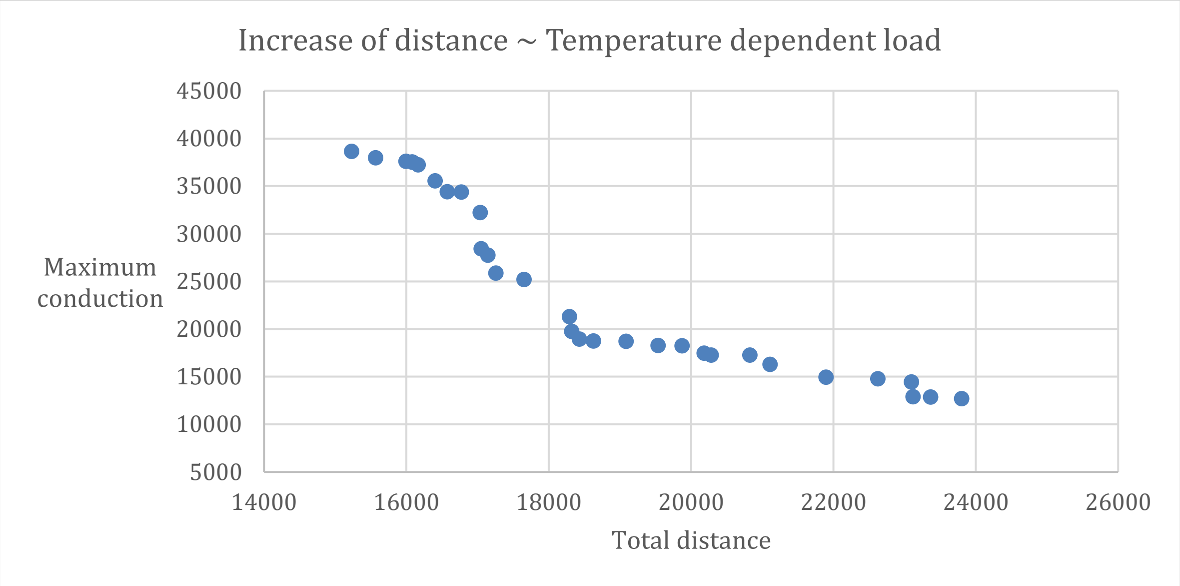

Figure 14 Relationship between Increase of distance and Temperature dependent load

Again, we see an inverse relationship between the two evaluation criteria. This time between *Total distance* and *Maximum conduction*. The highest *Maximum conduction* amounts to about 39000 while the lowest amounts to approximately 12500. The *Total distance* ranges from around 15000 to around 24000. We can also see two knicks in the graph appearing at a *Total distance* of approximately 17000 and 18000. The graph is generally steeper than in Figure 13.

Figure 15 shows a Pareto-approximation of the evaluation criteria *Increase of distance* and *Distribution penalty*, measured by the KPIs *Total distance* and *Total penalty*.

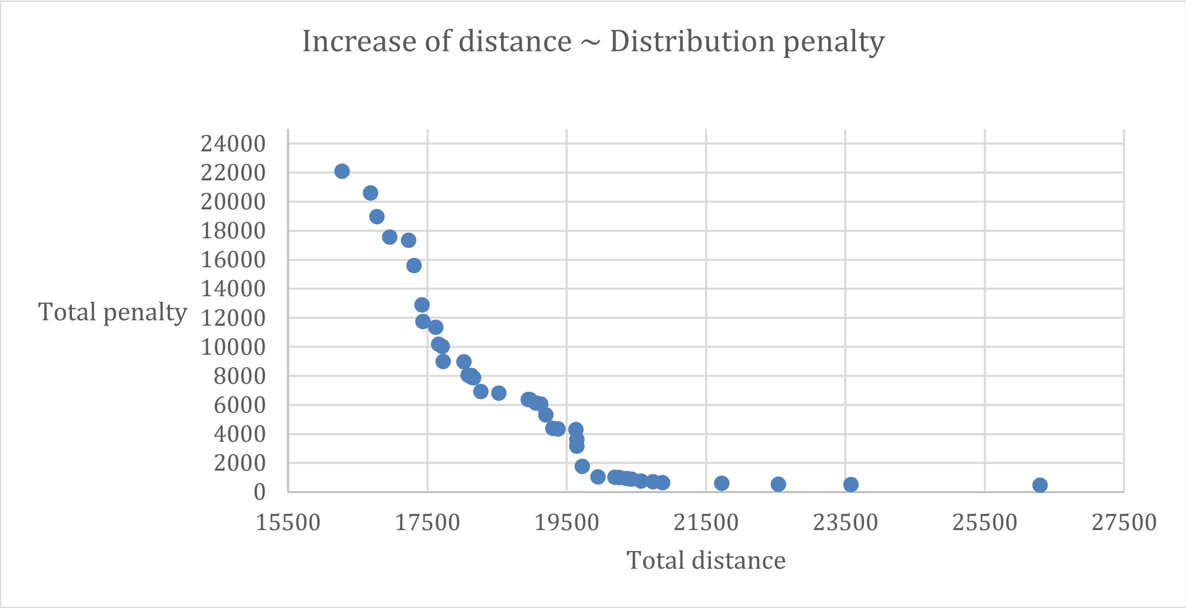

Figure 15 Relationship between Increase of distance and Distribution penalty

Total distance and *Total penalty* have an inverse relationship. The smaller the *Total distance* the larger the *Total penalty* and vice versa. The *Total distance* ranges from around 16000 to around 26000. The *Total penalty* ranges from around 500 to around 22000. The graph is steeper than the graphs in Figure 13 and Figure 15. There are two knicks at *Total distance* = 19000 and 19600.

Temperature dependent item detour and *Total distance* do not show a Pareto-approximation. We assume the relationship of those two evaluation criteria not to be conflicting.

8 Feedback of the industrial partner

An interview was conducted with technical employees of the industrial partner where the evaluation criteria were reviewed and suggestions concerning further adjustments were discussed.

The industrial partner considers peak item temperature as especially problematic. Item temperatures around 450°C are considered as temperature peaks. It is more detrimental to the vehicle in terms of abrasion to transport peak item temperatures for a short duration than to transport items with moderate temperatures over a long duration, e.g. the industrial partner estimates it would be preferrable to transport items with 300°C for 20 minutes than to transport items with 500°C for 10 minutes.

The industrial partner distinguishes between vehicle parts that are sensitive to high temperatures and vehicle parts that are not. For vehicle parts that are sensitive, high temperatures lead to abrasion and/or other damages. Temperatures of temperature-sensitive vehicle parts ought to be measured and decisions on whether a critical vehicle temperature is reached should be made on the basis of the temperatures of those temperature-sensitive vehicle parts. Vehicle parts that are sensitive to high temperatures are hydraulic hoses, angular gears and the vehicles' tires:

Hydraulic oil fires were the original reason why temperatures of the transported items were considered in the schedules of the industrial partner in the first place. Two material-specific temperatures are relevant in this context. The flashpoint is the lowest temperature under which a flammable vapor forms, which in the contact with an external ignition source, immediately creates a flame. The ignition point is the temperature at which the material, in the presence of air exclusively due to its own temperature self-ignites (kolb Cleaning Technology GmbH, 2021). On the bases of the flashpoint and the ignition point of the hydraulic oil, items can be categorized into 4 groups. This helps finding the right parameter for warms request items:

- critical items (item temperatures greater than 450°C)
- very hot items (item temperatures 316°C – 450°C, the ignition point of the hydraulic oil lies at 315°C) and
- hot items (item temperatures 280°C – 315°C, the flashpoint of the hydraulic oil lies at 279°C)

- According to the driver item temperatures below 200°C are harmless

Cracks in the **hydraulic hoses** can lead to dripping of the hydraulic oil on the items and result in hydraulic oil fire.

Angular gears contain lubricant oil. The lifespan for lubricant oils is provided by the manufacturer and measured in hours of operation. The more heat the lubricant oil is subjected to, the more it loses its viscosity and the lifespan is shortened drastically. One way to deal with abrasion of lubricant oils would be to increase the rate of oil change intervals. However, measuring the lubricant oil temperatures inside the angular gears and hence determining the rate of oil change intervals is difficult. Another problem is water condensation in the lubricant oil, which is also a by-product of high temperatures. Peak temperatures can wear out the lubricant oil in only a few hours.

Beside item temperatures the weight of batches also contributes to heat generation in the **vehicle´s tires** due to a higher tire pressure. Combining heavy batch weights with warm items regularly is not recommended. Instead, the industrial partner recommends transporting cooler items between batches with warm items, and additionally, at times, transporting batches with a low batch weight.

The vehicle part establishing physical contact between the vehicle and the lowermost item, which is called **jaw** is subjected to scaling the warmer the transported items are. Jaws contain precision milled gears that wear out quicker or even break out when overstressed.

The industrial partner indicated that the most critical item w.r.t. temperature is the top item while the second most critical item is the lowermost item. From the perspective of preserving the vehicle it would be the most beneficial to transport items in a “sandwich-structure”, meaning that the top item and the lowermost item are cooler, while the warm items are in-between them. However, from a logistical point of view (if shuffling is considered as well) it does not pay off to engage in shuffling operations to achieve this “sandwich-structure”. The industrial partner rather changes vehicles than to aim for specific batch compositions.

The industrial partner confirmed that request scheduling depends on the drivers’ experience. Contrary to our knowledge, at times, warm items are requested if the driver notices that items on the vehicle are slipping. The industrial partner also pointed out a difference in the time of the year. In the winter season items cool down quicker and the topic temperature is a less critical one in scheduling.

9 Conclusion

We developed five different evaluation criteria (one distance-related and four temperature related evaluation criteria), each of which captures a different objective of the TDSSTP. We used a weighted aggregation function to consider multiple objectives in the optimization. We varied the weights and generated Pareto Front approximations. The evaluation criterion *Increase of distance* was always combined with one other evaluation criterion. Distance-related and temperature-related objectives turned out to be conflicting. In addition to the trade-off between distance-related and temperature-related objectives, the recommended solution structures among the temperature-related objectives were opposing. Sometimes they were diametrically opposed to each other, for example, while *Average temperature by weight* recommended mixing warm items with many cooler items to reduce the average batch temperatures, *Temperature dependent load* recommended transporting warm items separately, to reduce the pressure on the lowermost item in the batch.

We investigated all evaluation criteria in our result section by comparing the solution qualities and solution structures by means of different KPIs. The solution structures differed between the different evaluation criteria. Ultimately, the industrial partner examined our findings and determined that our suggested evaluation criteria *Temperature dependent item detour* and *Distribution penalty* were most suitable and representable of the TDSSTP.

With regards to further research, we would like to highlight that the industrial partner mentioned ex post that the vehicles' tires are exposed to a high degree of heat radiation at some storage locations. Hence, a possible future adaptation of the TDSSTP would be to introduce site dependencies, meaning that certain vehicle heat exposure measures should be used only in case items are transported between specific nodes, where a high degree of heat radiation is found.

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Appendix A Axioms

1. reflexiveness: $\forall a \in A: a \sim a$
2. antisymmetry: $\forall\ a, b \in A:\ a \sim b\ \wedge\ b \sim a \Rightarrow a = b$
3. transitivity: $\forall a, b, c \in A: a \sim b\ \wedge\ b \sim c \Rightarrow a \sim c$
4. comparability: $\forall a, b \in A: a \sim b\ \vee b \sim a$

<u>Reflexiveness</u> describes the property of a relation where every item **a** of a set **A** relates to itself.
<u>Antisymmetry</u> describes the property of a relation between two items **a** and **b** of a set **A**, where if **a** relates to **b** and **b** relates to **a** then it follows that **a** equals **b**.
<u>Transitivity</u> describes the property of a relation between three items **a**, **b** and **c** of a set **A**, where if **a** relates to **b** AND **b** relates to **c** then it follows that **a** relates to **c**.
<u>Comparability</u> describes the property of a relation between two items **a** and **b** of a set A, where **a** relates to **b** OR **b** relates to **a**, but not both. (Spannagel, 2010a, 2010b)

Appendix B solution industrial partner

14 (45, 46, 47,) - 2 ( ) - 14 (54, 55, 56,) - 2 ( ) - 14 (71, 72, 73,) - 2 ( ) - 14 (88, 89, 90,) - 2 ( ) - 14 (98, 99, 100,) - 17 ( ) - 14 (109,) - 2 ( ) - 14 (114, 115, 116,) - 2 ( ) - 14 (117, 118,) - 12 ( ) - 14 (122, 123,) - 12 ( ) - 17 (125, 126, 127, 128, 129,) - 8 ( ) - 49 (134, 135, 136, 137, 138,) - 8 ( ) - 49 (139, 140, 141, 142,) - 8 ( ) - 2 (153, 154, 155, 156, 157,) - 8 ( ) –

12 (1, 2,) - 8 ( ) - 49 (18,) - 8 ( ) - 2 (28, 29,) - 8 ( ) - 3 (51,) - 0 ( ) - 0 (59, 60, 61,) - 8 ( ) - 0 (77, 78, 79,) - 8 ( ) - 2 (95, 96, 97,) - 8 ( ) - 3 (104, 105,) - 0 ( ) - 0 (110, 111, 112, 113,) - 8 ( ) - 11 (124,) - 8 ( ) - 0 (130,) - 8 ( ) - 3 (143, 144,) - 0 ( ) - 0 (149, 150, 151, 152,) - 8 ( ) –

22 (3, 4, 5, 6,) - 2 ( ) - 2 (7,) - 17 ( ) - 22 (10, 11, 12, 13, 14,) - 17 ( ) - 22 (19, 20, 21,) - 49 ( ) - 22 (25, 26, 27,) - 49 ( ) - 22 (30, 31, 32,) - 49 ( ) - 22 (36, 37, 38, 39,) - 0 ( ) - 22 (42, 43, 44,) - 0 ( ) - 22 (52, 53,) - 0 ( ) - 22 (62, 63, 64, 65, 66,) - 2 ( ) - 22 (67, 68, 69, 70,) - 2 ( ) - 12 (84, 85, 86,) - 17 ( ) - 17 (87,) - 8 ( ) - 12 (92, 93, 94,) - 17 ( ) - 17 (101, 102,) - 8 ( ) - 2 (103,) - 8 ( ) - 14 (145, 146, 147, 148,) - 49 ( ) - 11 (158, 159, 160, 161, 162,) - 5 ( ) –

0 (8, 9,) - 3 ( ) - 3 (15, 16, 17,) - 8 ( ) - 17 (22, 23, 24,) - 8 ( ) - 17 (33, 34, 35,) - 8 ( ) - 17 (40, 41,) - 8 ( ) - 17 (48, 49, 50,) - 8 ( ) - 17 (57, 58,) - 8 ( ) - 0 (74, 75, 76,) - 11 ( ) - 11 (80, 81, 82, 83,) - 8 ( ) - 3 (91,) - 0 ( ) - 0 (106, 107, 108,) - 8 ( ) - 11 (119, 120, 121,) - 8 ( ) - 3 (131, 132, 133,) - 8 ( ) -

Appendix C solution Increase of distance

54 () - 3 (104, 51, 91,) - 0 (61, 150, 78, 111,) - 8 () - 14 (46, 45, 72,) - 2 () - 22 (26, 20, 19,) - 49 (141, 138, 137, 135,) - 8 () - 14 (123, 122, 117, 118,) - 12 (92, 84, 94, 93,) - 17 (40, 127, 126, 24,) - 8 () - 14 (71, 109, 148,) - 2 (148,) - 22 (21, 31, 148,) - 49 () - 3 (16, 131, 17,) - 8 () - 22 (69,) - 2 (156, 95, 153, 96,) - 8 () - 54 () –

54 () - 0 (76, 77, 60, 149,) - 11 (81, 77, 60, 149,) - 8 () - 22 (44, 36,) - 11 (158, 44, 36,) - 5 (44, 36,) - 0 (112, 113, 108, 151,) - 8 () - 17 (101, 50, 34,) - 8 () - 22 (27, 32, 52,) - 49 (142, 140, 52,) - 0 (59, 107, 142, 140,) - 8 () - 14 (47, 89, 88, 147,) - 2 (147,) - 49 () - 11 (121, 119, 120,) - 8 () - 14 (73, 99, 146,) - 49 (73, 99,) - 0 (73, 130, 99,) - 2 (130, 7, 99,) - 17 (41, 130, 57,) - 17 (41, 130, 57,) - 17 (41, 130, 57,) - 8 () - 22 (53, 42, 43,) - 0 (152,) - 22 (11, 152, 12,) - 17 (58, 102, 152,) - 8 () - 14 (100, 98, 145,) - 17 (145,) - 22 (145, 30, 25,) - 49 (18, 136, 134, 139,) - 2 (18, 136, 134, 139,) - 8 () - 54 () –

54 () - 3 (105, 144, 133, 143,) - 0 (8, 133, 9,) - 3 (132, 133, 15,) - 8 () - 12 (85, 86,) - 17 (23, 22, 35,) - 17 (23, 22, 35,) - 17 (23, 22, 35,) - 8 () - 14 (114, 54, 56,) - 2 (28, 103, 29,) - 8

() - 17 (48,) - 12 (2, 1, 48,) - 8 () - 22 (37, 38, 39,) - 0 (110, 79, 106,) - 8 () - 11 (124, 82, 83,) - 8 () - 17 (49, 33, 87,) - 8 () - 54 () –

54 () - 22 (64, 65, 3, 4,) - 2 (97, 155, 157, 154,) - 8 () - 14 (90, 55, 116, 115,) - 2 () - 22 (5, 6, 62, 67,) - 2 () - 22 (68, 63, 70, 66,) - 2 () - 0 (75, 74,) - 11 (160, 159, 161, 162,) - 5 () - 11 (80,) - 22 (14, 10, 13, 80,) - 17 (129, 128, 80, 125,) - 8 () - 54 () –

Appendix D solution Average temperature by weight

54 () - 14 (89, 72, 45, 115,) - 2 () - 12 (85,) - 14 (90, 46, 116, 85,) - 17 (90, 46, 116,) - 2 () - 22 (38, 52, 37, 39,) - 0 (107, 108, 152, 78,) - 8 () - 14 (71, 88, 147, 55,) - 2 (157, 147,) - 49 (142, 140, 136, 157,) - 22 (142, 140, 136, 157,) - 8 () - 11 (160, 158, 159, 162,) - 5 () - 3 (133, 132, 17,) - 8 () - 54 () –

54 () - 22 (70, 62, 4, 63,) - 2 (7,) - 17 (49, 40, 125, 87,) - 8 () - 22 (3,) - 49 (18, 138, 139, 3,) - 2 (18, 96, 138, 139,) - 8 () - 22 (19, 27, 14, 67,) - 49 (135, 14, 67,) - 2 (135, 29, 97, 14,) - 8 (14,) - 14 (100, 99, 98, 14,) - 17 () - 0 (149, 60, 150, 151,) - 8 () - 2 (155,) - 11 (120, 82, 119, 155,) - 8 () - 17 (50, 128, 22, 126,) - 8 () - 54 () –

54 () - 22 (12, 10, 13, 11,) - 17 () - 14 (73, 148, 145, 123,) - 2 (148, 145, 123,) - 49 (123, 134,) - 22 (25, 32, 123, 134,) - 49 (123, 134, 141,) - 11 (83, 123, 134, 141,) - 8 (123,) - 14 (146, 123, 122, 56,) - 2 (146, 123, 122, 103,) - 49 (123, 122, 103,) - 0 (123, 122, 103, 113,) - 8 (123, 122,) - 12 (93, 86, 94,) - 14 (93, 86, 94, 54,) - 17 (23, 54,) - 2 (23,) - 22 (20, 30, 23,) - 49 (23,) - 11 (121, 124, 23,) - 8 () - 17 (129, 24, 35, 48,) - 8 () - 22 (42,) - 3 (51, 143, 42, 105,) - 0 (110, 61, 111,) - 11 (110, 61, 111, 80,) - 8 () - 2 (95,) - 17 (57, 101, 127, 95,) - 8 () - 22 (21, 66, 6, 65,) - 49 (137, 66, 6, 65,) - 2 (28, 137, 156, 154,) - 8 () - 54 () –

54 () - 22 (64, 5, 69, 68,) - 2 (153,) - 0 (79, 153, 77,) - 11 (81, 79, 153, 77,) - 8 () - 14 (118, 117,) - 12 (2, 1, 92, 84,) - 8 (92, 84,) - 17 () - 22 (43, 36,) - 0 (76, 74, 75, 112,) - 11 (161, 112,) - 5 (112,) - 22 (31, 112,) - 49 (112,) - 22 (26, 112,) - 49 (112,) - 17 (102, 34, 112,) - 8 () - 17 (41, 33, 58,) - 8 () - 22 (44, 53,) - 0 (9,) - 0 (8, 9,) - 3 (131, 16, 15,) - 8 () - 14 (114, 47, 109,) - 2 () - 3 (104, 144, 91,) - 0 (106, 130, 59,) - 8 () - 54 () –

Appendix E solution Temperature dependent load

54 () - 22 (30,) - 49 () - 22 (32,) - 49 () - 22 (69, 66, 67, 4,) - 2 () - 22 (31,) - 49 () - 3 (91, 51, 105,) - 0 (79, 151, 111, 61,) - 8 () - 14 (100, 99,) - 17 () - 22 (37, 36,) - 0 () - 3

(131, 133, 16,) - 8 () - 14 (46, 145,) - 2 (145,) - 49 ()[7] - 22 (10, 11, 12,) - 17 (126, 50, 33, 48,) - 8 () - 12 (85, 93,) - 17 () - 2 (153, 97, 103, 29,) - 8 () - 22 (20,) - 49 () - 22 (53, 43,) - 0 (106, 108, 152,) - 49 (136, 106, 108, 152,) - 8 () - 14 (146, 88,) - 2 (146,) - 49 () - 22 (25,) - 49 () - 11 (159, 162, 158,) - 5 () - 0 (77, 59, 149, 130,) - 8 () - 14 (114, 116,) - 2 () - 2 (155, 95, 156, 28,) - 8 () - 17 (23, 24, 49, 129,) - 8 () - 17 (101, 41, 58,) - 8 () - 54 () –

54 () - 3 (144, 143,) - 0 (75, 76, 74, 78,) - 11 (78, 80,) - 22 (13, 78, 80,) - 17 (128, 87, 78, 80,) - 8 () - 14 (118, 98,) - 12 (94, 84, 98,) - 17 () - 22 (19,) - 49 () - 22 (65, 63, 3,) - 2 () - 22 (27,) - 49 () - 11 (124, 83, 120,) - 8 () - 14 (147, 148,) - 49 (134,) - 22 (6, 5, 70, 134,) - 2 (134, 157, 7,) - 22 (14, 134, 157, 7,) - 17 (125, 35, 134, 157,) - 8 () - 14 (89, 90,) - 2 () - 22 (42, 52,) - 0 (150, 110, 60, 113,) - 8 () - 54 () –

54 () - 49 (137, 138, 141, 139,) - 2 (137, 138, 141, 139,) - 8 () - 14 (72, 56,) - 2 () - 22 (38, 39,) - 0 () - 3 (104, 15,) - 0 (15,) - 49 (135, 18, 142, 15,) - 8 () - 14 (55, 71,) - 2 () - 17 (57, 22, 34,) - 8 () - 12 (86, 92,) - 17 () - 0 (107,) - 3 (132, 17, 107,) - 8 () - 54 () –

54 () - 22 (64, 68,) - 2 () - 22 (62,) - 2 () - 22 (44,) - 0 (9, 112,) - 3 (112,) - 11 (161, 160, 119, 112,) - 5 (119, 112,) - 0 (8, 119, 112,) - 3 (119, 112,) - 49 (140, 119, 112,) - 2 (154, 140, 119, 112,) - 8 () - 14 (45, 109,) - 2 (96,) - 22 (21, 96,) - 49 (96,) - 11 (121, 81, 82, 96,) - 8 () - 14 (122, 117, 123,) - 12 (1, 2,) - 8 () - 14 (73, 47,) - 2 () - 17 (40, 127, 102,) - 8 () - 14 (115, 54,) - 2 () - 22 (26,) - 49 () - 54 () -

Appendix F solution Temperature dependent item detour

54 () - 17 (49, 127, 23, 48,) - 8 () - 22 (63,) - 2 () - 22 (25,) - 49 (139, 136, 141,) - 2 (139, 136, 141, 95,) - 8 () - 22 (43, 38, 39,) - 0 (9,) - 3 (16, 15, 132,) - 8 () - 22 (12, 14,) - 17 (35, 24, 41,) - 8 () - 14 (146, 72, 73,) - 2 (146,) - 49 (140, 134, 135, 18,) - 2 (140, 134, 135, 18,) - 8 () - 17 (22, 40, 58,) - 8 () - 14 (55, 47, 56,) - 2 () - 0 (60, 107, 108,) - 8 () - 54 () –

54 () - 22 (4, 64, 3, 65,) - 2 () - 3 (51, 91, 105,) - 0 (74, 76, 75,) - 11 (160, 161, 159,) - 5 () - 0 (111, 110, 151, 113,) - 8 () - 14 (147,) - 22 (21, 31, 147,) - 49 ()[8] - 0 (106, 79, 61,) - 8 () - 14 (118, 122, 117, 123,) - 12 (94, 85, 84, 92,) - 17 (102, 125, 34, 87,) - 8 () - 22 (6, 62, 5, 67,) - 2 (7,) - 17 (33, 126, 128, 129,) - 8 () - 12 (1, 2,) - 8 () - 54 () –

54 () - 11 (83, 81, 82,) - 8 () - 12 (93, 86,) - 17 () - 22 (66, 70, 68, 69,) - 2 () - 22 (13, 10, 11,) - 17 () - 2 (28, 155, 157, 156,) - 8 () - 22 (52, 53, 42,) - 0 (8,) - 3 (133, 17, 131,) - 8 () - 14

[7] The distance from node 14 to node 2 plus from node 2 to node 49 equals the distance from node 14 to node 49, i.e., $t_{14,2} + t_{2,49} = t_{14,49}$

[8] The distance from node 14 to node 22 plus from node 22 to node 49 equals the distance from node 14 to node 49, i.e., $t_{14,22} + t_{22,49} = t_{14,49}$

(71, 116, 115,) - 2 (97, 154, 153,) - 2 (96, 97, 154, 153,) - 8 () - 14 (90, 114, 109,) - 2 () - 22 (20, 32, 30,) - 49 () - 2 (29,) - 17 (29, 101, 50,) - 8 () - 54 () –

54 () - 3 (104, 144, 143,) - 0 () - 11 (158, 162,) - 5 () - 0 (130, 59, 112,) - 8 () - 14 (100, 98, 99,) - 17 () - 22 (19, 27, 26,) - 49 (138, 137, 142,) - 2 (103, 138, 137, 142,) - 8 () - 22 (37, 44, 36,) - 0 (78, 150,) - 0 (149, 78, 150, 77,) - 8 () - 14 (148, 145, 54, 89,) - 2 (148, 145,) - 49 () - 11 (121, 124, 120,) - 8 () - 14 (46, 45, 88,) - 2 () - 11 (80, 119,) - 17 (80, 119, 57,) - 0 (80, 119, 57, 152,) - 8 () - 54 () –

Appendix G solution Distribution penalty

54 () - 22 (37,) - 0 (9, 110, 149,) - 3 (131, 110, 149,) - 2 (97, 131, 110, 149,) - 8 () - 14 (89, 115, 109, 114,) - 2 (155, 156, 157, 95,) - 8 () - 14 (88,) - 2 () - 3 (144, 143, 105,) - 0 (75, 78, 76,) - 11 (121, 83, 78,) - 8 () - 14 (47,) - 12 (47, 86,) - 2 (7, 96, 86,) - 17 (127, 128, 96, 126,) - 8 () - 14 (54,) - 2 () - 3 (51, 91, 132,) - 0 (74, 132, 60,) - 11 (119, 132, 60,) - 8 () - 14 (118, 117,) - 12 () - 54 () –

54 () - 14 (56,) - 2 () - 22 (5, 65, 63,) - 2 () - 11 (162, 160, 159, 161,) - 5 () - 22 (64,) - 2 () - 22 (42, 39, 52,) - 0 () - 3 (104, 133, 16,) - 0 (8, 133, 16,) - 3 (15, 133, 16,) - 8 () - 14 (116, 71, 72,) - 2 (154,) - 22 (154, 38,) - 0 (154, 130,) - 49 (136, 154, 130,) - 2 (136, 154, 130,) - 17 (136, 154, 130, 101,) - 8 () - 14 (122, 123, 46,) - 2 (122, 123,) - 12 (92, 85, 93,) - 17 () - 2 (153,) - 22 (153, 20,) - 49 (153,) - 11 (158, 153, 81, 124,) - 5 (153, 81, 124,) - 0 (153, 81, 124, 150,) - 8 () - 14 (73,) - 2 () - 49 (18,) - 17 (40, 129, 18,) - 8 () - 14 (147,) - 49 () - 54 () –

54 () - 14 (45,) - 2 () - 22 (32, 70,) - 49 (138, 137, 135, 70,) - 2 (28, 138, 137, 135,) - 8 () - 22 (44, 53,) - 0 (113, 79, 111, 151,) - 8 () - 22 (31,) - 49 () - 17 (58, 33, 41,) - 8 () - 22 (30, 12,) - 49 (12,) - 11 (120, 82, 80, 12,) - 8 (12,) - 22 (19, 12, 11,) - 49 (12, 11,) - 0 (106, 12, 11,) - 22 (106, 12, 11, 27,) - 49 (106, 12, 11,) - 17 (24, 34, 106,) - 8 () - 22 (43, 69,) - 0 (59, 152, 69,) - 49 (139, 59, 152, 69,) - 2 (139, 103, 59, 152,) - 8 () - 22 (26,) - 49 () - 17 (50, 35, 57,) - 8 () - 14 (146,) - 49 () - 54 () –

54 () - 14 (98, 99, 100, 148,) - 49 (98, 99, 100,) - 17 () - 22 (3, 21,) - 49 (140, 142, 141, 3,) - 2 (29, 140, 142, 141,) - 8 () - 12 (2, 84, 1, 94,) - 8 (84, 94,) - 14 (145, 84, 94,) - 22 (145, 84, 94, 14,) - 49 (84, 94, 14,)[9] - 22 (10, 84, 94, 14,) - 17 (102, 49, 48, 87,) - 8 () - 22 (68, 4, 66, 67,) - 2 () - 22 (36,) - 0 (107, 77, 61,) - 8 () - 22 (25, 13, 6, 62,) - 49 (134, 13, 6, 62,) - 2 (134, 13,) - 17 (23, 125, 22, 134,) - 8 () - 14 (90,) - 2 () - 3 (17,) - 0 (108, 112, 17,) - 0 (108, 112, 17,) - 8 () - 14 (55,) - 2 () - 54 () -

[9] The distance from node 14 to node 22 plus from node 22 to node 49 equals the distance from node 14 to node 49, i.e., $t_{14,22} + t_{22,49} = t_{14,49}$