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Distributional Consequences of Inheritance Taxation

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Diese Arbeit ist euch gewidmet. Alle in der Arbeit enthaltenen Fehler gehen selbstverständlich auf meine Kappe.

Abstract

Inheritance taxation is a controversially debated subject. In his early contribution to the topic, Stiglitz (1978) argues that direct redistribution financed by inheritance taxation might be dominated by the following effect: The reduction in aggregate savings caused by this taxation leads to a decline in the stock of capital. According to standard marginalist theories, this is accompanied by a drop in wages and a rise in interest rates. For sufficiently low values of the elasticity of technical substitution the share of wage incomes will decline. Since working classes are poorer than capitalist classes, this means redistribution from the bottom to the top.

In the following paper I challenge the view of Stiglitz. There is no natural law that inheritance taxation goes ahead with a decline in aggregate savings. Instead of assuming that tax revenues are used to grant transfer incomes, I assume that they are invested into education or public infrastructure which raises the economy's productivity. As a result, aggregate savings might rise. A low value of the elasticity of technical substitution might in fact benefit workers since the rise in the stock of capital is accompanied by a large increase in wages and a sharp decline in interest rates. I proceed by calculating critical values for relevant parameters that grant that the outlined system of inheritance taxation benefits the working class.

Zusammenfassung

Erbschaftssteuern werden bis heute kontrovers diskutiert. Stiglitz (1978) argumentiert in einem frühen Beitrag zum Thema, dass die mittels Erbschaftssteuern finanzierte direkte Umverteilung durch folgenden Effekt konterkariert werden könnte: Der durch die Besteuerung ausgelöste Rückgang an Ersparnissen lässt den in der Ökonomie verfügbaren Kapitalstock sinken. Gemäß üblicher marginalistischer Theorien geht dies mit einem Rückgang der Löhne und einem Anstieg des Zinses einher. Bei hinreichend niedriger technischer Substitutionselastizität wird daher der Anteil der Löhne am Gesamteinkommen sinken. Da ArbeiterInnen ärmer sind als KapitalistInnen können Erbschaftssteuern somit Umverteilung von unten nach oben bewirken.

In der folgenden Arbeit wird der von Stiglitz beschriebene Mechanismus einer kritischen Analyse unterzogen. Der Zusammenhang zwischen Erbschaftssteuern einerseits und einem sinkenden Kapitalstock andererseits ist kein Naturgesetz. Anstatt anzunehmen, dass die Steuererlöse als Transfereinkommen an ArbeiterInnen ausbezahlt werden, wird im hier entwickelten Modell davon ausgegangen, dass die Steuererlöse für öffentliche Investitionen in Infrastruktur oder Bildung verwendet werden. Dies geht folglich mit einem ökonomieweiten Produktivitätsanstieg einher, der von einem Anstieg der Ersparnisse und damit des Kapitalstocks begleitet werden kann. Eine niedrige technische Substitutionselastizität bedeutet in diesem Fall einen starken Anstieg der Löhne und einen starken Rückgang der Zinsen, wodurch der Anteil der Löhne am Gesamteinkommen steigt. In der vorliegenden Arbeit werden schließlich jene Werte für Modellparameter angegeben, die sicherstellen, dass Erbschaftssteuern Umverteilung von oben nach unten bedeuten.

Contents

1 Introduction

Inheritance taxation is a controversially debated political issue. Those who advocate against it typically argue that one should not "punish" those who want to leave something to their dearest. Moreover, they recognize an "unfair" accumulation of taxes, since bequests are formed out of already taxed incomes. [1] Those who are in favor of inheritance taxation often shift the focus of attention from descendants to heirs. They argue that unlike other sorts of income, receiving bequests can hardly be considered as personal "achievements".

Both strands of arguments have in common that they directly judge on the "fairness" of inheritance taxation - without considering its effects on key macroeconomic variables. What are the distributional consequences of inheritance taxation? Is the widespread belief of its supporters, that it has an equalizing effect, correct? These questions are of course not new.

One strand of theoretical literature, based on the works of Becker and Tomes (1976), highlights the potentially equalizing effect not of inheritance taxation but of inheritance itself. In a model economy, where (i) children's earnings are a random variable known by parents and (ii) parents care about their children's income, bequests will be used to compensate for low earnings. Hence, bequests tend to have an equalizing effect and taxation of bequests disturbs this effect. As shown by Davies (1986), a linear tax that is used to finance lump-sum transfers will increase inequality in this setting. For a compact review, see Davies and Shorrocks (2000, chapter 2.3).

Bossmann, Kleiber, and Wälde (2007) change the model in so far that although (i) individual earnings still are a random variable, (ii') parents no longer care about their children's income but about the height of their bequests per se. Bequests no longer target the poor and redistributive taxation of bequests decreases the coefficient of variation of wealth.

In his early contribution on the topic, Stiglitz (1978) similarly assumes that descendants, when saving for bequests, follow simple behavioral rules, ignoring the actual needs of their children. Ceteris paribus, taxation of bequests plus lump-sum redistribution has an equalizing effect. However, according to Stiglitz, ceteris paribus might not be met. More precisely, he accounts for the fact that inheritance taxation might has an effect on the stock of capital and therefore on the distribution of market income.

[1] To consider inheritance taxation as an unparalleled double taxation of income is a very subjective statement. Alternatively, if one considers leaving bequests as a particular way of spending income, it is unparalleled not to tax them since every other form of spending is subject to the value added tax.

To follow his thoughts, imagine an economy that consists of workers and capitalists. Assume that workers' income depends on wage payments to a larger extent than capitalists' income, who make their living out of interest payments. Assume further that workers leave bequests to their children (the future workers) and capitalists leave bequests to their children (the future capitalists) respectively.

The ceteris-paribus-intuitive guess is: If everybody who plans to leave bequests was forced to pay inheritance taxes and tax returns were passed to workers as transfer income, then the combined taxation/subsidy scheme should make workers better off.

However, this guess does not need to hold under all circumstances. The original redistribution of wealth towards workers could be accompanied with a decrease in aggregate savings, even though taxation does not deter people from saving in order to make bequests. They might decline simply because workers are saving a smaller fraction of their income than capitalists per se. Redistribution towards workers then means redistribution towards those who save less. Moreover, aggregate savings might decline because of differential treatment of income and wealth. The propensity to save out of transfer income might be much lower than the respective propensity to save out of inherited wealth. Both arguments would imply that the stock of capital declines. A decline in the capital stock is likely to shift the functional distribution of income. According to standard marginalist theories and for a given labor force, a decline in the capital stock causes a decrease in wages and a rise in interest rates.

More precisely, the total effect on the income share of labor is ambivalent since firm owners receive higher rates of return on less capital. If the income share of labor declines and its decline is large enough, than direct redistribution will be outweighed by the indirect effect that works via the evolution of the capital stock.

This paper changes the setting of Stiglitz' class-model of inheritance in one important aspect. It poses an alternative transfer scheme, as tax returns are not paid out directly, but invested in public infrastructure or the educational system. These investments will be accompanied by a rise in aggregate productivity. Inheritance taxation (plus the proposed transfer scheme) would then lead to a larger output produced from any given stock of (private) capital. Of this output a smaller fraction is saved than would be in the absence of the taxation/transfer scheme. But it is possible that taking both effects together, savings and therefore the stock of (private) capital available in the economy will rise. Wages will rise relative to interest rates. The income share of labor might rise, for which case workers' position improves relative to that of capitalists, even at the absence of direct transfers.

In Stiglitz' original model there exist parameters that specify an economy so that the combined tax-transfer system delivers intense indirect redistribution towards capitalists. In spite of beneficial direct effects, workers are worse off in the end. I show that for some of the same specifications, workers will be better off according to the modified model.

On the other hand, there exist parameters that specify an economy for which the tax-transfer system explained by Stiglitz delivers indirect and direct effects of inheritance taxation that are both beneficial to workers. For some of the same specifications workers will be worse off according to the model developed in this thesis.

In contrast to the model presented by Stiglitz, there no longer exist values for the elasticity of technical substitution that make inheritance taxation generally harmful to workers: Whether or not low levels of the elasticity of technical substitution benefit capitalists relative to workers always depends on other characteristics of the economy.

The remainder of the thesis is structured as follows: In section 2 I summarize the model of inheritance taxation developed by Joseph Stiglitz and replicate how inheritance taxation influences some key distributional measures. A modified model of inheritance taxation is developed in section 3. Again, the effects of inheritance taxation on distributional measures are calculated. In Section 4 these effects are compared between the models. Section 5 presents a conclusion. Rather tedious calculations are outsourced into an Appendix.

2 Stiglitz' Original Model

2.1 Basic Assumptions and the Process of Capital Accumulation

Assume an economy with a standard neoclassical production function $F(K, L)$, where K denotes capital and L denotes labor. Assume further that $F(K, L)$ exhibits constant returns to scale and satisfies INADA-conditions. The factors of production are remunerated by their marginal products. Moreover, the society is split into two classes: workers and capitalists. Children of workers become workers themselves, whereas children of capitalists stay capitalists. Although both classes have capital holdings and therefore receive interest (r) income, only capitalists can make a living from it. In contrast to them, workers have to participate in the labor market in order to receive supplementary wage (w) income. Although wealth ownership may differ between workers and capitalists, there is no difference in wealth ownership within the working class and within the capitalist class.

It is assumed further that workers save a constant fraction s_w of their total income and capitalists save a fraction s_c respectively, with $s_c > s_w$. All savings are used to make bequests, i.e. there are no intragenerational savings. More technically: The economy consists of non-overlapping generations, who each live for one period.

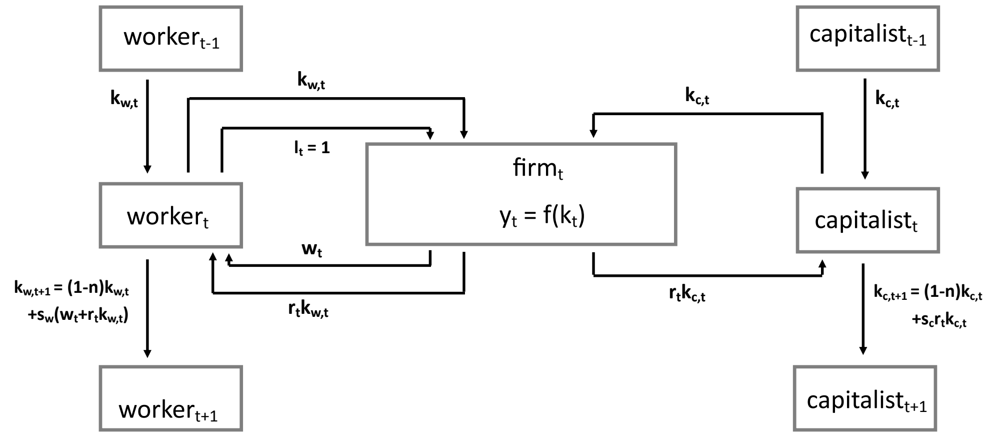

Figure 1: a discrete-time representation of the model economy without taxation

The capital accumulation equations of the economy are therefore given by

$$\dot{k}_w = s_w(w + rk_w) - nk_w \tag{1}$$
$$\dot{k}_c = s_c rk_c - nk_c \tag{2}$$

with n being a measure for the growth rate of population[2], k_c denoting the amount of capital a single capitalist inherits from the past generation and k_w that of a single worker respectively.[3] Dots above variables denote their derivatives with respect to time.

[2]Alternatively, n can be imagined as the rate of capital depreciation.

[3]Specifying capital accumulations this way one assumes that bequests are saved entirely by their receivers.

Stiglitz then introduces a simple tax-transfer system, according to which all inherited wealth is taxed by a constant rate τ. The resulting tax returns are paid out to workers as transfer income.

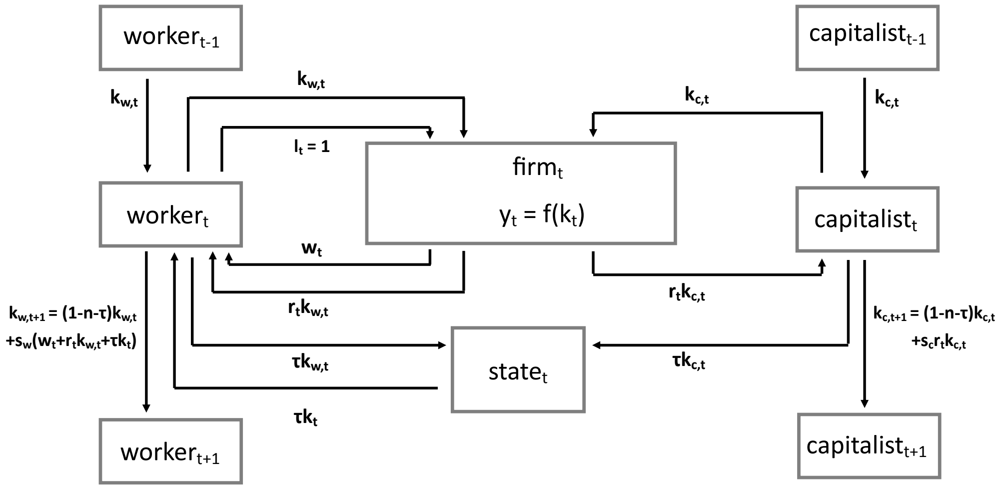

Figure 2: the model economy with the taxation/subsidy scheme outlined by Stiglitz

This gives the following modified capital accumulation equations:

$$\dot{k_w} = s_w(w + rk_w + \tau k) - (n + \tau)k_w \tag{3}$$

$$\dot{k_c} = s_c r k_c - (n + \tau)k_c \tag{4}$$

Capital intensity is denoted by k and consists out of capital owned by workers as well as such owned by capitalists. Hence, it must hold that $k_w \leq k$.[4]

[4] If the number of workers in the economy is given by N_w and the number of capitalists is given by N_c it holds that

$$k = \frac{N_c k_c}{N_w} + k_w.$$

The resulting dynamics of the capital accumulation process guide the economy towards a steady state. Although the behavioral assumptions made on the economic agents are such that their entire capital holdings are handed over to the next generation, there are losses in capital holdings per heir because of population growth n (and because of taxation). These losses rise linearly with wealth of decedents. In contrast to this, additional bequests rise linearly with income (factor s_w, s_c respectively) but not with capital. This is because of the decreasing marginal product of capital incorporated into $F(K, L)$. The determination of the steady state level of capital intensity k therefore works similarly as for the standard Solow model, but the steady state depends now also on the rate of inheritance taxation. Setting $\dot{k}_c = 0$, it follows from equation (4), that

$$r = \frac{n + \tau}{s_c}. \tag{5}$$

Setting r equal to the marginal product of capital $f'(k)$, we obtain by implicit differentiation, that

$$\frac{dk}{d\tau} = \frac{1}{s_c f''(k)} < 0.$$

This result is intuitively reasonable, because according to the model, inheritance taxation is combined with redistribution towards those who save less, i.e. the workers.[5]

2.2 Distributional Consequences of Inheritance Taxation in the Original Model

In order to be able to speak about distributional consequences, a choice on the precise measure of distribution used has to be made. For simplicity, Stiglitz chooses the fraction $\frac{k_w}{k}$ of wealth held by workers. Using equations (4) and (5) Stiglitz calculates $\frac{k_w}{k}$ to be

$$\frac{k_w}{k} = \frac{\frac{s_w s_c \tau}{n+\tau} + \frac{s_w \alpha(\tau)}{1-\alpha(\tau)}}{s_c - s_w}, \tag{6}$$

[5]It should be explicitly noted that according to the model there is no behavioral reaction of decedents to changes in the rate of inheritance taxation: s_w and s_c do not depend on τ. Any changes in the stock of capital due to (changes in) inheritance taxation do not result from changes in behavior but from redistribution towards people with unchanged, but different behavior. For instance towards people, who spend a larger fraction of their income on consumption per se.

where $\alpha(\tau)$ measures the functional distribution of income. The steps in doing so are outlined in Appendix 6.1.

From equation (6) it can be seen that $\frac{k_w}{k}$ depends on the level of inheritance taxation directly and indirectly. The direct effect $\frac{s_w s_c \tau}{n+\tau}$ is caused by granting transfer incomes towards workers. The indirect effect $\frac{s_w \alpha(\tau)}{1-\alpha(\tau)}$ occurs if inheritance taxation shifts the functional distribution of income.

As shown in Subsection 2.1, a rise in inheritance taxation is accompanied by a decline in the stock of capital. For given wages and interest rates, this would increase the labor share of income. However, according to standard marginalist theory, wages will decline whereas interest rates will rise. The overall effect of an increase in τ on α is therefore ambiguous.

This can also be demonstrated formally by deriving $\frac{k_w}{k}$ with respect to τ:

$$\frac{d\frac{k_w}{k}}{d\tau} = \frac{\frac{s_w s_c n}{(n+\tau)^2} + \frac{s_w \frac{d\alpha}{d\tau}}{(1-\alpha)^2}}{s_c - s_w},$$

where the positive left hand side of the numerator represents the direct, the ambiguous right hand side represents the indirect effect. Thus, a substantially negative effect of inheritance taxation on the income share of labor may cause an overall negative effect of inheritance taxation on laborers' share of wealth.

At $\tau = 0$, the effect of inheritance taxation on the distribution of wealth can also be expressed as a function of the elasticity of substitution of the underlying production function, denoted by σ:

$$\frac{d\frac{k_w}{k}}{d\tau}_{|\tau=0} = \frac{s_w}{n(s_c - s_w)}(s_c + \frac{\sigma - 1}{1-\alpha(0)}) \tag{7}$$

For details see Appendix 6.4. Equation (7) shows that for low values of τ high values of σ imply that inheritance taxation causes redistribution towards workers, whereas low values for σ imply that inheritance taxation benefits capitalists.

This can be made intuitively clear, if one recalls that

$$\sigma = \left| \frac{relative\ change\ in\ factor\ input\ ratio}{relative\ change\ in\ marginal\ rate\ of\ technical\ substitution} \right|.$$

Thus, high values for σ mean that a given exogenous change in input ratio (for instance due to a decline in the stock of capital) goes ahead with little change in the marginal rate of technical substitution and therefore has little effect on wages and interest rates. The "indirect effect" of inheritance taxation on the distribution of wealth is small, direct redistribution dominates it.

Instead of looking at the distribution of wealth one can alternatively monitor the effect of inheritance taxation on the distribution of market income between classes. The appropriate variable to observe is $\frac{y_w}{y}$. However, it is not simply identical to the functional distribution of income given by α as workers' savings grant them some income in addition to wages. Hence, it holds that $\frac{y_w}{y} > \alpha$. The precise relation between these two quantities is given by

$$\frac{y_w}{y} = 1 - \left((1-\alpha)\left(1 - \frac{k_w}{k}\right)\right)$$

As $\frac{k_w}{k}$, the above can be derived by τ (for details, see Appendix 6.5). The result is reformulated as a function of σ and, for the case of $\tau = 0$, it reduces to

$$\frac{d\frac{y_w}{y}}{d\tau_{|\tau=0}} = \frac{(1-\alpha)s_c(\sigma - (1-s_w))}{n(s_c - s_w)}. \tag{8}$$

One may argue that wealth and income do matter to individuals only in so far as they determine individuals' consumption. However, Stiglitz does not calculate workers' share in consumption explicitly.

3 Modification of the Model

3.1 Alternative Assumptions and their Effects on Capital Accumulation

In this section I modify the model by respecifying the use of tax returns.

A central feature of Stiglitz' model is that tax returns are used to generate transfer incomes. Alternatively, they can be invested in projects such as infrastructure or public education. This will effect productivity in the economy.

As a starting point, the per capita production function is reformulated as

$$y = af(k),$$

where a is a factor that accounts for individual productivity as a function of public investment per worker[6]. It is assumed that this investment is financed by the returns of inheritance taxation. In order to keep the model simple, we assume that the entire tax returns are used up for this sort of public investment.

[6]It shall be noted that individual productivity a is specified in a Harrod-neutral way: Its rise has no effect on the marginal rate of technical substitution. This means that, roughly speaking, technological progress benefits workers and capitalists equally.

Therefore, a becomes a function of τk and the per capita production function may be reformulated as

$$y = a(\tau k)f(k) \tag{9}$$

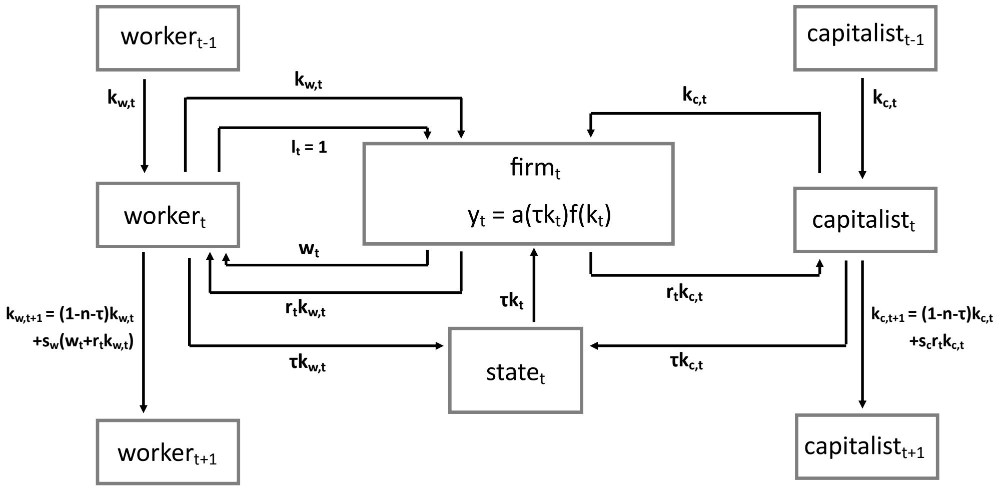

Figure 3: the model economy with public investment into productivity

The intergenerational capital accumulation equation for workers changes, as workers no longer receive transfer income. These equations are now given by

$$\dot{k}_w = s_w(w + rk_w) - (n + \tau)k_w \tag{10}$$
$$\dot{k}_c = s_c rk_c - (n + \tau)k_c. \tag{11}$$

The combined taxation-investment scheme has two opposite effects on capital intensity k:

- By assumption, bequests are saved entirely by their receivers. Therefore, inheritance taxation leads to a decline in private savings. Since the government does not invest the returns into physical capital k the stock of physical capital decreases compared to an economy with no inheritance taxation at all. It even decreases compared to the model used by Stiglitz where a fraction s_w of transfer incomes is saved. However, there are no transfer incomes in the modified model.

- On the other hand, public investment into human capital a raises productivity in the economy, and therefore, compared to an economy without inheritance taxation or compared to Stiglitz' model, the income generated out of a given level of physical capital k increases. As part of this additional income is saved, physical capital k rises.

It remains unclear, which effect dominates. To clarify this issue, one has to look at how changes in the rate of inheritance taxation τ influence the steady-state level of capital intensity k. Formally, the derivative of the steady-state level of k with respect to τ is given by

$$\frac{dk}{d\tau} = \frac{\frac{1}{s_c} - a'f'k}{a'\tau f' + af''}; \tag{12}$$

see Appendix 6.6. However, the sign of this derivative remains unclear. It depends on various parameters. In particular, it may depend on the level of τ as well as on the exact characteristics of the functions a and f. For small levels of τ, capital intensity will rise with taxation if and only if $s_c a' f' k$ is greater than unity. In this case, one additional unit of taxes collected (causing a decline in savings by one unit) is accompanied by a rise in output that is large enough to cause extra savings of more than a unit. This is proven formally in Appendix 6.7.

3.2 Distributional Consequences of Inheritance Taxation in the Modified Model

As shown in Appendix 6.8, the value of $\frac{k_w}{k}$ in the steady state is given by

$$\frac{k_w}{k} = \frac{s_w}{s_c - s_w}\frac{\alpha}{1-\alpha}. \tag{13}$$

So, as tax returns are no longer paid out directly but are invested to increase the economy's productivity, changes in inheritance taxation rate can effect the distribution of wealth between classes only in so far as they shift α, i.e., the functional distribution of income. Deriving $\frac{k_w}{k}$ by τ yields:

$$\frac{d\frac{k_w}{k}}{d\tau} = \frac{s_w}{s_c - s_w}\frac{\frac{d\alpha}{d\tau}}{(1-\alpha)^2}.$$

This means that if the share of wages in total income rises (declines) with inheritance taxation, the share of capital owned by workers rises (declines) as well.

Whether or not the share of wages does in fact increase with inheritance taxation remains highly unclear. In Appendix 6.3 it gets visible that this does not only depend on whether or not k rises in τ but also on the exact characteristics of the underlying production function f.

However, for small levels of τ the issue becomes slightly clearer. In fact, at $\tau = 0$ the direction of the effect of inheritance taxation on workers' position relative to capitalists depends on two parameters: the elasticity of substitution σ and $s_c a' f' k$. Whether or not the stock of capital rises is determined by $s_c a' f' k$. How workers' and capitalists' claims on output are affected by the development of the stock of capital is determined by σ. More precisely, as demonstrated in Appendix 6.9, it holds that

$$\frac{d\frac{k_w}{k}}{d\tau}_{|\tau=0} = \frac{s_w}{n(s_c - s_w)(1-\alpha)}(\sigma - 1)(1 - s_c a' f' k). \tag{14}$$

In contrast to Stiglitz' original model, also the effect of inheritance taxation on workers' share in consumption can be easily calculated in the modified model. In fact, looking at income and consumption, the derivatives are given by

$$\frac{d\frac{y_w}{y}}{d\tau_{|\tau=0}} = \frac{(1-\alpha)s_c}{n(s_c - s_w)}(\sigma - 1)(1 - s_c a' f' k). \tag{15}$$

and

$$\frac{d\frac{c_w}{c}}{d\tau_{|\tau=0}} = \frac{(1-s_w)(1-s_c)(1-\alpha)s_c(\sigma-1)(1-s_c a' f' k)}{n(s_c - s_w)\left((s_c - s_w)\left(1-(1-\alpha)\left(1-\frac{s_w}{s_c - s_w}\frac{\alpha}{1-\alpha}\right)\right) + (1-s_c)\right)^2}. \tag{16}$$

For details, see Appendix 6.10. It becomes visible, that whether σ and $s_c a' f' k$ are below or above unity determines whether inheritance taxation shifts resources towards workers or capitalists. I will explore this systematically in the next section.

4 A Comparison of Policy Effects for the Two Models

As stated above, there is no clear and obvious effect of small levels of inheritance taxation on the distribution of wealth, income and consumption—neither for

Stiglitz' model nor for its modification developed in this paper. Instead, the size and direction of such an effect strongly depend on the exact values of various parameters such as $s_c, s_w, n, \alpha(0), \sigma(0)$ and $a'(0)f'(0)k(0)$.

In this section, it shall be demonstrated that there exist values for these parameters that lead to opposing outcomes for the two different models. For some values for these parameters $\frac{k_w}{k}$ rises if the introduction of inheritance taxation is used to fund transfer payments, but declines if tax returns are invested into the economy's productivity. For other values, combined introduction of inheritance taxation and transfer payments causes $\frac{k_w}{k}$ to rise, whereas using the newly introduced inheritance taxation for investments into productivity causes $\frac{k_w}{k}$ to decline.

For Stiglitz' original model, the critical values of σ at which the positive and negative effects of (low levels of) inheritance taxation on workers' shares offset each other have been calculated by Stiglitz himself:

$$\frac{d\alpha}{d\tau_{|\tau=0}} < 0 \iff \sigma < 1;$$

$$\frac{d\frac{k_w}{k}}{d\tau_{|\tau=0}} < 0 \iff \sigma < 1 - (1 - \alpha(0))\, s_c;$$

$$\frac{d\frac{y_w}{y}}{d\tau_{|\tau=0}} < 0 \iff \sigma < 1 - s_w;$$

One can thus say that if workers' share in capital ownership decreases as a result of (low levels of) inheritance taxation, their share in income decreases as well. [7]Moreover, if their share in income is doomed to decline, the functional share of labor declines as well. Formally:

$$\frac{d\frac{k_w}{k}}{d\tau_{|\tau=0}} < 0 \implies \frac{d\frac{y_w}{y}}{d\tau_{|\tau=0}} < 0 \implies \frac{d\alpha}{d\tau_{|\tau=0}} < 0$$

For the modified model, it holds that

$$\frac{d\frac{k_w}{k}}{d\tau_{|\tau=0}} < 0 \iff \frac{d\frac{c_w}{c}}{d\tau_{|\tau=0}} < 0 \iff \frac{d\frac{y_w}{y}}{d\tau_{|\tau=0}} < 0 \iff \frac{d\alpha}{d\tau_{|\tau=0}} < 0$$
$$\iff (\sigma < 1 \wedge a'f'ks_c < 1) \vee (\sigma > 1 \wedge a'f'ks_c > 1)$$

To express it verbally: Workers' shares decline as a result of (low levels of) inheritance taxation if and only if either

[7]This can be immediately demonstrated if one recalls that $\frac{k_w}{k}_{|\tau=0} < 1$.

- the stock of capital declines while wages decline strongly and interest rates rise a lot

or

- the stock of capital rises while wages rise modestly and interest rates decline only little.

Conversely, workers' shares will rise if and only if

- the stock of capital declines while wages decline modestly and interest rates rise little

or

- the stock of capital rises while wages rise strongly and interest rates decline a lot.

For instance, if the increase in productivity causes an increase in savings going ahead with a sharp increase in wages and a drastic decrease in interest rates, workers will be relatively better off.[8] The position of workers is improved despite of a low level of σ prevalent in the economy. Therefore, one cannot conclude that low levels of the elasticity of substitution make the introduction of inheritance taxation unbeneficial to workers per se. Such a verdict heavily depends on the precise use of tax returns.

5 Concluding Remarks

Mathematical modelling can play a crucial role in criticizing ideologies. In this sense, by modifying a model of Stiglitz, this thesis challenges two claims that are sometimes brought against inheritance taxation.

Claim no. 1: Inheritance taxation is always accompanied with a decline in the stock of capital since (a) *the propensity to save out of inherited wealth is larger than the propensity to save out of transfer income or since* (b) *it goes ahead with redistribution towards those who save less.*

Neither of effects (a) and (b) is neglected in the model developed in this thesis. But when making this claim it is assumed that inheritance taxation lets the economy's aggregate productivity unchanged. This need not be the case. In an environment, where public investment out of tax returns boosts productivity, output, savings and the stock of capital might actually rise.

[8]Since the economy becomes wealthier as a result of the rise in total savings workers are not only relatively but absolutely better off.

Claim no. 2: Inheritance taxation might miss its target of redistributing towards workers, since the combination of a drop in the stock of capital and a low level in the elasticity of substitution between capital and labor will shift the functional distribution of income towards capitalists. That the elasticity of substitution is substantially below unity is shown in numerous studies such as Young (2013), Knoblach, Rößler, and Zwerschke (2016), Chirinko and Mallick (2017) and Gechert, Havranek, Irsova, and Kolcunova (2019).

Claim no. 2 stands and falls with the correctness of claim no. 1. If in contrast to claim no. 1 the stock of capital rises with inheritance taxation, the low elasticity of substitution turns from a weakness into a strength, in the sense that inheritance taxation then causes a sharp rise in wages and a sharp decline in interest rates, shifting the functional distribution of income towards workers.

The main finding of this thesis is that, if designed properly, inheritance taxation can play an equalizing role. However, in obtaining this result some restrictive assumptions have been made. The differential treatment of inherited wealth on the one hand and market/transfer income on the other hand has entered the analysis from the beginning but remains questionable. Moreover, all results were obtained by assuming a closed economy. Checking whether central insights of this paper can be uphold if these assumptions are relaxed is left to future research.

6 Appendix

6.1 Distribution of Wealth in the Original Model, Eq. (6)

In the steady state equation (4) can be rewritten as

$$s_w w + s_w \tau k = \left(n + \tau - s_w r\right) k_w.$$

Dividing by k and r yields

$$s_w \frac{w}{rk} + \frac{s_w \tau}{r} = \left(\frac{n+\tau}{r} - s_w\right) \frac{k_w}{k}.$$

Making use of equation (5) and exploiting the definition of α, one obtains

$$s_w \frac{\alpha}{1-\alpha} + \frac{s_w s_c \tau}{n+\tau} = \left(s_c - s_w\right) \frac{k_w}{k},$$

which can be easily solved for $\frac{k_w}{k}$.

6.2 Reformulation of σ

We will make use of the result obtained in this section in Section 6.3. Formally writing down the definition of σ, one receives

$$\sigma = -\frac{\frac{d\frac{K}{L}}{\frac{K}{L}}}{\frac{d\frac{MP_K}{MP_L}}{\frac{MP_K}{MP_L}}},$$

and thus

$$\frac{1}{\sigma} = -\frac{d\frac{MP_K}{MP_L}}{dk}\frac{k}{\frac{MP_K}{MP_L}} = -\frac{d\frac{f'}{f-f'k}}{dk}\frac{k}{\frac{f'}{f-f'k}}.$$

Deriving by k, one obtains

$$\frac{1}{\sigma} = -\frac{f''(f-f'k) - f'(f' - (f''k + f'))}{(f-f'k)^2}\frac{k(f-f'k)}{f'},$$

which can be simplified to

$$\sigma = -\frac{(f-f'k)f'}{f''fk}.$$

6.3 Derivation of α after τ for both Models

Inserting $w = f - f'k$ into the definition of α, I obtain

$$\alpha = \frac{f - f'k}{f}.$$

Thus, one receives

$$\frac{d\alpha}{d\tau} = \frac{f'kf' - f'f - kf''f}{f^2}\frac{dk}{d\tau}.$$

For the original model it therefore holds that

$$\frac{d\alpha}{d\tau} = \frac{f'kf' - f'f - kf''f}{f^2}\frac{1}{s_c f''} = \frac{f'kf' - f'f - kf''f}{f^2\frac{f'k}{f}}(1-\alpha)\frac{1}{s_c f''} = (\sigma-1)\frac{1-\alpha}{f's_c}.$$

Since at $\tau = 0$ the steady-state rate of interest is given by $\frac{n}{s_c}$, and since in equilibrium $r = f'$, it follows that

$$\frac{d\alpha}{d\tau_{\tau=0}} = (\sigma - 1)\frac{1-\alpha}{n}.$$

For the modified model it holds that

$$
\begin{aligned}
\frac{d\alpha}{d\tau_{\tau=0}} &= \frac{f'kf' - f'f - kf''f}{f^2}\frac{1 - s_c a' f'k}{s_c a f''} \\
&= \frac{f'kf' - f'f - kf''f}{f^2 \frac{f'k}{f}}(1-\alpha)\frac{1 - s_c a' f'k}{s_c a f''} \\
&= (\sigma - 1)\frac{1-\alpha}{s_c a f'}(1 - s_c a' f'k) \\
&= (\sigma - 1)\frac{1-\alpha}{n}(1 - s_c a' f'k).
\end{aligned}
$$

6.4 Reformulation of $\frac{d\frac{k_w}{k}}{d\tau}_{|\tau=0}$ as a Function of σ for the Original Model

Deriving eq. (6) after τ, setting $\tau = 0$ and recalling the result of Section 6.3 yields

$$
\begin{aligned}
\frac{d\frac{k_w}{k}}{d\tau}_{|\tau=0} =& \frac{1}{s_c - s_w}\left(\frac{s_w s_c n}{n^2} + \frac{s_w \frac{d\alpha}{d\tau}_{|\tau=0}}{(1-\alpha)^2}\right) \\
=& \frac{1}{s_c - s_w}\left(\frac{s_w s_c}{n} + \frac{s_w(\sigma-1)(\alpha-1)}{n(1-\alpha)^2}\right) \\
=& \frac{s_w}{n(s_c - s_w)}\left(s_c + \frac{\sigma - 1}{1-\alpha}\right).
\end{aligned}
$$

6.5 Effects of τ on the Distribution of Income between Classes in the Original Model

A worker's market income y_w is given by $y_w = w + rk_w$. Hence, as workers receive some income out of capital holdings, the distribution of income between classes $\frac{y_w}{y}$ is not identical to the functional distribution of income α. In fact, it must hold that $\frac{y_w}{y} > \alpha$. More precisely, it holds that

$$
\frac{y_w}{y} = 1 - (1-\alpha)\left(1 - \frac{k_w}{k}\right).
$$

Deriving by τ yields

$$
\frac{d\frac{y_w}{y}}{d\tau} = \frac{d\alpha}{d\tau}\left(1 - \frac{k_w}{k}\right) + (1-\alpha)\frac{d\frac{k_w}{k}}{d\tau}.
$$

Inserting the results obtained so far, one receives for the special case of $\tau = 0$

$$
\begin{aligned}
\frac{d\frac{y_w}{y}}{d\tau}_{|\tau=0} =& (\sigma-1)\frac{1-\alpha}{n}\left(1-\frac{s_w\alpha}{(1-\alpha)(s_c-s_w)}\right)\\
&+(1-\alpha)\frac{s_w}{n(s_c-s_w)}\left(s_c+\frac{\sigma-1}{1-\alpha}\right)\\
=&(\sigma-1)(1-\alpha)\frac{s_c-s_w-\frac{s_w\alpha}{1-\alpha}}{n(s_c-s_w)}\\
&+(1-\alpha)\frac{s_w}{n(s_c-s_w)}\left(s_c+\frac{\sigma-1}{1-\alpha}\right)\\
=&\frac{1-\alpha}{n(s_c-s_w)}\left(\left(s_c-s_w-\frac{s_w\alpha}{1-\alpha}+\frac{s_w}{1-\alpha}\right)(\sigma-1)+s_ws_c\right)\\
=&\frac{(1-\alpha)s_c(\sigma-(1-s_w))}{n(s_c-s_w)}.
\end{aligned}
$$

6.6 Derivation of k with respect to τ in the Modified Model, Eq. (12)

A single entrepreneur takes a as given and chooses k such that

$$r=\frac{\delta y}{\delta k}=af'$$

holds in equilibrium. From equation (11) we can see that in the steady state $r=\frac{n+\tau}{s_c}$. Substituting this in the expression above, we obtain

$$\frac{n+\tau}{s_c}-a(\tau k)f'=0$$

By implicit differentiation, it follows that

$$\frac{dk}{d\tau}=\frac{\frac{1}{s_c}-a'f'k}{a'\tau f'+af''},$$

assuming the denominator to be non-zero, which holds for small tax rates.

6.7 Meaning of $s_ca'f'k$

If tax returns rise by one unit, total savings increase by

$$
\begin{aligned}
s_w&\left(\frac{dw}{da}a'+a'f'k_w\right)+s_c(a'f'(k-k_w))\\
&=s_w a'\left(\alpha f+(1-\alpha)f\frac{k_w}{k}\right)+s_c a'(1-\alpha)f\left(1-\frac{k_w}{k}\right)\\
&=s_w a'\left(\alpha f+f\frac{s_w}{s_c-s_w}\alpha\right)+s_c a'(1-\alpha)f\frac{(s_c-s_w)(1-\alpha)-s_w\alpha}{(s_c-s_w)(1-\alpha)}\\
&=a'f\alpha\left(s_w+\frac{s_w^2}{s_c-s_w}+\frac{s_c(s_c-s_w-\alpha s_c)}{\alpha(s_c-s_w)}\right)\\
&=a'f\frac{\alpha(s_c-s_w)s_w+\alpha s_w^2+s_c^2-s_w s_c-\alpha s_c^2}{s_c-s_w}\\
&=a'f\frac{(1-\alpha)s_c^2-(1-\alpha)s_w s_c}{s_c-s_w}\\
&=s_c a'(1-\alpha)f\frac{s_c-s_w}{s_c-s_w}\\
&=s_c a'f'k
\end{aligned}
$$

6.8 Distribution of Wealth in the Modified Model, Eq. (13)

As shown above, in the steady-state we have $r=\frac{n+\tau}{s_c}$, and therefore, from (10),

$$\left(n+\tau-s_w\frac{n+\tau}{s_c}\right)k_w=s_w w.$$

Consequently, dividing both sides with the steady-state value of k, we get

$$\frac{k_w}{k}=\frac{s_w}{(n+\tau)\left(1-\frac{s_w}{s_c}\right)}\frac{w}{k}.$$

Because, $\frac{w}{k}=r\frac{\frac{w}{f}}{\frac{rk}{f}}=r\frac{\alpha}{1-\alpha}$ this implies that

$$\frac{k_w}{k}=\frac{s_w s_c}{(n+\tau)(s_c-s_w)}r\frac{\alpha}{1-\alpha}.$$

Using again the fact that $r=\frac{n+\tau}{s_c}$, this can be simplified to

$$\frac{k_w}{k}=\frac{s_w}{s_c-s_w}\frac{\alpha}{1-\alpha}.$$

6.9 The Relationship between $\frac{d\frac{k_w}{k}}{d\tau}$ and σ in the Modified Model at $\tau = 0$

This relationship can be determined as follows:

$$\frac{d\frac{k_w}{k}}{d\tau} = \frac{s_w}{s_c - s_w}\frac{\frac{d\alpha}{d\tau}}{(1-\alpha)^2}$$

Setting $\tau = 0$ and making use of a result obtained in Section 6.3, I obtain

$$\begin{aligned}\frac{d\frac{k_w}{k}}{d\tau}_{|\tau=0} &= \frac{s_w}{s_c - s_w}(\sigma - 1)\frac{(1-\alpha)}{n}(1 - s_s a' f' k)\frac{1}{(1-\alpha)^2} \\ &= \frac{s_w}{n(s_c - s_w)(1-\alpha)}(\sigma - 1)(1 - a' f' k s_c).\end{aligned}$$

6.10 Effects of τ on the Distribution of Income and Consumption between Classes in the Modified Model

Making use of the formula for $\frac{y_w}{y}$ established for the original model we obtain

$$\frac{d\frac{y_w}{y}}{d\tau} = \frac{d\alpha}{d\tau}\left(1 - \frac{k_w}{k}\right) + (1-\alpha)\frac{d\frac{k_w}{k}}{d\tau}.$$

Considering the special case of $\tau = 0$ and making use of the equations obtained in Sections 6.3 and 6.9, this transforms to

$$\begin{aligned}\frac{d\frac{y_w}{y}}{d\tau}_{|\tau=0} =& (\sigma - 1)\frac{1-\alpha}{n}(1 - s_c a' f' k)\left(1 - \frac{s_w \alpha}{(1-\alpha)(s_c - s_w)}\right) \\ &+ (1-\alpha)\frac{s_w}{n(s_c - s_w)(1-\alpha)}(\sigma - 1)(1 - s_c a' f' k) \\ =& \frac{(1-\alpha)}{n}\frac{(1-\alpha)s_c - s_w}{(s_c - s_w)(1-\alpha)}(\sigma - 1)(1 - s_c a' f' k) \\ &+ \frac{s_w}{n(s_c - s_w)}(\sigma - 1)(1 - s_c a' f' k) \\ =& \frac{(1-\alpha)s_c}{n(s_c - s_w)}(\sigma - 1)(1 - s_c a' f' k).\end{aligned}$$

For the modified model, measures of consumption and workers' share in consumption are given by

$$\begin{aligned}c_w &= (1 - s_w)y_w, \\ c &= (1 - s_w)y_w + (1 - s_c)(y - y_w), \\ \frac{c_w}{c} &= \frac{(1 - s_w)\frac{y_w}{y}}{(s_c - s_w)\frac{y_w}{y} + (1 - s_c)}.\end{aligned}$$

Deriving $\frac{c_w}{c}$ after τ yields

$$\frac{d\frac{c_w}{c}}{d\tau} = \frac{(1-s_w)\frac{d\frac{y_w}{y}}{d\tau}\left((s_c-s_w)\frac{y_w}{y}+(1-s_c)\right)-(1-s_w)\frac{y_w}{y}(s_c-s_w)\frac{d\frac{y_w}{y}}{d\tau}}{\left((s_c-s_w)\frac{y_w}{y}+(1-s_c)\right)^2}$$
$$= \frac{(1-s_w)(1-s_c)\frac{d\frac{y_w}{y}}{d\tau}}{\left((s_c-s_w)\frac{y_w}{y}+(1-s_c)\right)^2}.$$

Setting $\tau = 0$ we obtain

$$\frac{d\frac{c_w}{c}}{d\tau_{|\tau=0}} = \frac{(1-s_w)(1-s_c)(1-\alpha)s_c(\sigma-1)(1-s_c a' f' k)}{n(s_c-s_w)\left((s_c-s_w)\left(1-(1-\alpha)\left(1-\frac{s_w}{s_c-s_w}\frac{\alpha}{1-\alpha}\right)\right)+(1-s_c)\right)^2}.$$

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