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„The Effects of Secondary Shares Sales in IPOs“

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Abstract

This thesis analyses the relation between a stochastic liquidity shock suffered by the entrepreneur and the possibility of secondary share sales in Initial Public Offerings (IPOs). In the model, entrepreneurs will sell secondary shares either due to the liquidity shock or due to negative private information. An equilibrium is characterised in which issues both with only primary shares and with primary and secondary shares are seen. The optimal contracts between the entrepreneur and the investors for each type of issue are derived. The conditions for the existence of such an equilibrium are identified. The probability and the severity of the liquidity shock affect the going-public decision and the IPO price in the market, for both primary only and for primary and secondary issues.

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List of Abbreviations

ICC incentive compability constraint. 31–33

IPO Initial Public Offering. 3, 8, 10–18, 23–25, 35

NPV Net Present Value. 20

PC participation constraint. 30, 31

SEO Seasoned Equity Offer. 10, 19

VC Venture Capital. 23

WPBE Weak Perfect Bayesian Equilibrium. 26

1 Introduction

In the period between 2000 and 2015, Initial Public Offerings (IPOs) in the United States (US) raised an annual average of \$32 billion in proceeds.[1] Due to the amount of money involved, IPOs have been extensively documented and studied. Many different motivations for IPOs are documented in the literature, both on the firm and the insider level. Nonetheless, it is still an active field of both theoretical and empirical research.

When talking about the sale of secondary shares two motivations are particularly worth mentioning: negative information and liquidity or diversification needs. Insiders with better access to (negative) information might want to cash-out before this information is made public. And insiders with a need for higher liquidity or diversification might wish to sell secondary shares independently of any private information about the firm value.

In their seminal paper, Leland and Pyle, 1977 develop a model in which the retention of shares by insider acts as a positive signal of firm value to investors. This became the main theoretical work on secondary share sales. However, the empirical literature is not always supportive of the model. For example, Ang and Brau, 2003 find no relation between secondary share sales and long-term underperformance.

What is the relation between information asymmetries, IPOs and secondary share sales? More specifically, what effect does the liquidity needs of the entrepreneur have on the market price of shares? This thesis characterises how an unobservable liquidity shock suffered by the entrepreneur and the possibility of secondary share sales affect the market prices for both primary only and for mixed primary and secondary issues.

In the model an entrepreneur has the rights to all returns from an existing project and access to a positive NPV reinvestment opportunity. The entrepreneur has no cash at hand and can only finance the reinvestment by issuing equity to outside investors. The entrepreneur might also be hit

[1] Data as reported in Lowry et al., 2017.

by a liquidity shock, in which case she wishes to sell her stake in the returns of the project in exchange for the market price of equity. By assumption, secondary shares can only be sold if the reinvestment is performed and primary shares are issued. The entrepreneur also wishes to sell her shares if she is in possession of negative private information about the project, in a "cash-out" strategy.

The value obtained from the reinvestment and for the entrepreneur's shares depends on the investors' beliefs about the value of the project. This in turn depends on the beliefs about the occurrence of the liquidity shock. The investors observe a partial separation between entrepreneurs with and without liquidity needs which affects the level of pricing in all issues.

This thesis describes a specific type of equilibrium. In this equilibrium an entrepreneur not subject to a liquidity shock will perform the reinvestment and raise funds by selling primary shares if the market price of shares does not create a unacceptable dilution in her stake. And an entrepreneur subject to a liquidity shock will perform the reinvestment and sell her part in the returns if the market value of her equity is satisfactory. Also, an entrepreneur not subject to a liquidity shock will sell her stake in the project returns if the market value of her equity is larger than her share in the returns from the project and the reinvestment, in short: if her shares are overpriced. In this equilibrium the probability and the severity of the liquidity shock will impact the price both of primary only and of primary and secondary issues.

The conditions for the existence of such equilibrium are derived. Both entrepreneurs with and without liquidity shock will forgo positive NPV reinvestment opportunities due to underpricing. However, entrepreneurs subject to a liquidity shock will accept a higher underpricing. Consequently, the conditions for the performance of the reinvestment will be relaxed for all entrepreneurs. For low returns, the entrepreneur will sell secondary shares even in the absence of liquidity shock, in a "cash-out" strategy. The threshold values for each action depend on the probability and on the severity of the liquidity shock.

The basic model in this thesis is adapted from Myers and Majluf, 1984.

The work is also related to that of Aghion et al., 2004 and Habib and Ljungqvist, 1999. A discussion of these works is shown in Section 3. In the context of financial intermediation, Parlour and Plantin, 2008 develop a model in which a bank is subject to a liquidity shock with a certain probability. The liquidity shock changes the conditions for the existence of a secondary market for loans. The model in this thesis borrows the description of the liquidity shock and applies it to a reinvestment problem in the context of corporate finance. While Parlour and Plantin, 2008 model the investment and the secondary sale in two subsequent periods, this work has both reinvestment and secondary sale in the same period. This means that the decision of selling secondary shares directly influences the decision of performing the reinvestment. In Parlour and Plantin this relation is indirect.

Section 2 provides a review of the institutional details related to IPOs. Section 3 discusses some of the relevant empirical literature followed by a description of related theoretical literature. The model is shown in Section 4. Section 5 characterises the specific type of equilibrium and establishes the conditions for its occurrence. Conclusions are in Section 6. All proofs are shown in the Appendix.

2 Initial Public Offerings: institutional details

An IPO is the first time the shares of a company are traded on the stock market and made available for the general public. The firm goes from being a private company to being a public company. The ownership becomes dispersed and the disclose requirements increase significantly.

2.1 Why go public?

There are many reasons why a company would want to go public. The obvious advantage is to raise capital for the expansion of operations, both in the initial offer and in later Seasoned Equity Offers (SEOs). While equity is a more expensive source of capital, the company might already have too much debt and need to issue equity in order to achieve a better capital structure and decrease its cost of capital. Many other advantages can be seen: increasing liquidity for shareholders, increasing publicity and analyst coverage and creating a currency for employees compensation or acquisitions.

An IPO also comes with many costs, that should be weighed against the benefits. The direct costs of an IPO include fees from filing and registration and compensation for the underwriting bank, auditor company and lawyers. Besides the direct costs, the IPO process is associated with indirect costs. Some examples of those are distractions imposed on management and possible competition costs arriving from sensitive information made public due to disclosure requirements. It is also important to say that IPOs tend to be underpriced, which is another cost occurred in the process.

2.2 Characteristics

The percentage of public companies varies a great deal between countries, with IPOs being much more frequent in the US and in the UK than in the rest of the world. Porta et al., 1997 shows that the level of shareholder protection is lower in continental Europe than in the US and the UK. The authors argue that this alters the financial structure of companies and decreases incentives to go public, which could explain the discrepancy in the number of public companies.

In an equity offer the shares can be of two types. New shares whose proceeds go directly to the company in the form of capital are known as primary shares. Existing shares sold by current shareholders are known as secondary shares. The secondary shares do not increase the number of shares outstanding and their sale is merely a transfer of ownership. While the company is required to disclose how many of the shares are primary and how many are secondary in an issue, the shares are indistinguishable, have the same market value and give the new owners the same rights.

An IPO usually involves at least four main participants: the issuing firm (issuer), the current shareholders, the underwriter - or underwriter syndicate - and the investors.

2.3 The Process

The IPO process starts with a preliminary valuation of the company to arrive at a price range for the offer. This price range is then altered based on the information the underwriter receives through the IPO process.

The process also involves creating a registration statement to the regulators which includes a preliminary prospectus. After the share sale is approved by the regulator the company prepares a final prospectus with the IPO details including price and number of shares, both primary and secondary. The information in the prospectus can be altered up to a few days before the IPO in the form of an amendment. Possible alterations are the price range and the number or primary and secondary shares offered.

Large offerings are managed not by one single underwriter but by a group of several underwriting banks. This group is formed by the lead underwriter - the main bank responsible for managing the deal - and the syndicate - a group of banks that participates in the marketing and selling of the issue. Besides selling the shares, the underwriter is responsible for advising the company during the IPO process.

An offering can take many forms, the most common are best-effort IPOs and firm-commitment IPOs. In a best-effort offering the underwriter makes an effort to sell the shares at the best possible price, without, however, committing to a specific price. These types of deal can have a all-or-none clause: either all shares are sold or the deal is called off. Best-effort deals are commonly used in smaller IPOs.

The other option is a firm-commitment IPO. In this case the underwriter guarantees that it will sell all shares at the offer price. In practice this means that the underwriter will acquire all shares at a price slightly lower than the offer price and later resell them in the open market. In the case where demand is smaller than predicted the underwriting bank will bear the loss.

Recently a third IPO process has been tried: the auction IPO. In this process the shares are sold directly to the public through an online auction. Google used this process in its IPO, which happened in 2004. However, the auction process is still not widely adopted.

2.4 Timeline

The IPO process typically starts up to two years before the offering date. The timeline described below is standard for the US, nevertheless the process is similar in European and Asian countries. At first, the company needs to create the accounting and reporting structure required of a public company. The company accounting then needs to be audited for at least two balance sheets and three income statements. Still in the initial steps, the company might look for legal counsel with experience in the IPO process and in counselling public companies.

The underwriter is hired three to six months before the planned offering date. Once this happens, the registration statement starts to be prepared for submission to the regulator. This registration involves accountants and auditors, lawyers and the company team besides the underwriter, who coordinates the process.

The next step of the process is choosing the stock exchange and registering on it. Each exchange has its own requirements for accepting an issue and also charges different fees. Usually the application for the listing at an exchange happens 60 days before the predicted offering date.

At around 20 days before the IPO date a preliminary prospectus is distributed to investors and the *road show* starts. A *road show* is a series of presentations done by the company's management. It usually lasts for two to three weeks and it is an opportunity for marketing the company and the IPO. It also offer the underwriter a chance to collecting information from investors. Using information collected in the road show, the company and the underwriter make the final valuation of the company and set the

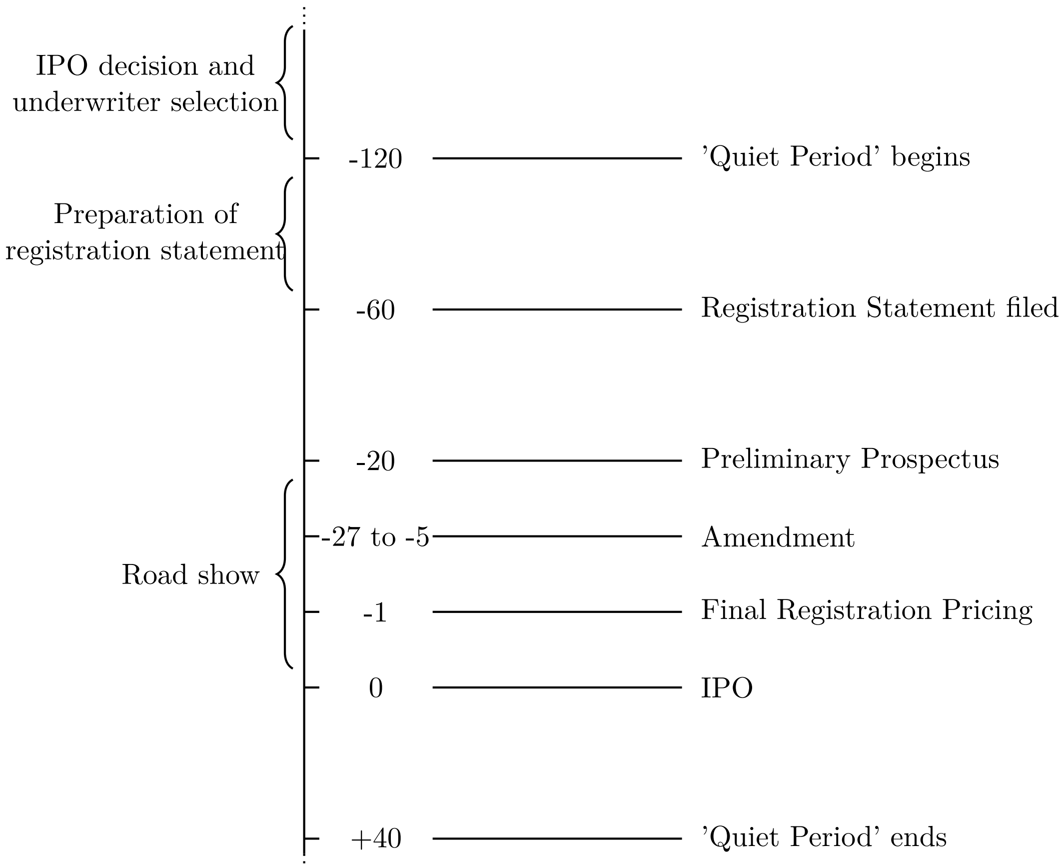

Figure 1: Timeline of events in the IPO process.

final price. The final registration statement - or prospectus amendment - is usually filled with the regulator on the day before the issue.

The quiet period lasts from 120 days before until 40 days after the IPO. During this period the company management is not allowed to reveal any additional information about the company, make forecasts or express opinions about the company's future value.

Figure 1 shows the typical timeline of events for an IPO in the US.

3 Related Literature

3.1 Empirical Literature

The empirical literature on IPOs is extremely rich and diverse, however many empirical puzzles remain, such as hot markets, IPO underpricing and long-run underperformance. The motivations for IPOs are also an open field of research, and are particularly hard to study empirically. Besides that, many studies are dedicated to the effect of secondary shares and insider sale in IPOs, often with contradictory results.

In Downes and Heinkel, 1982, the authors examine the relation between firm value and two possible signals from the entrepreneur to the investors. These signals are the proportion of equity retained by the entrepreneur (as proposed by Leland and Pyle, 1977) and a policy of the firm for higher dividends (as proposed by Bhattacharya, 1979 and Heinkel, 1978). Both are signals of firm quality, as they are costlier to low quality firms. Only high quality firms are able to maintain high dividend policies in the long run and only entrepreneurs with high quality projects will forgo the diversification possibilities of selling the full project to investors. If they serve as signals, there should exist an observable relation between them and firm value.

Using data from 449 American IPOs between 1965 and 1969 the authors examine pure primary and mixed primary and secondary issues. First, they test the hypothesis that retention by the entrepreneur is related to firm value. Their results provide strong support in favour of the hypothesis. However, their regression assumes the risk of the firms in the sample is constant. Secondly, they test for both the retention and the dividend hypotheses. Again, the results support the retention hypothesis. The results, however, reject the dividend hypothesis.

Habib and Ljungqvist, 1999 make a theoretical and empirical exploration of the underpricing in IPOs. In particular, they explore the idea that insiders will engage in costly actions to reduce underpricing when those are consistent with their wealth maximisation strategy. These actions include, but are not limited to, promotion actions, underwriter choice and choice between firm-commitment and best-effort offerings.

They define the participation ratio as the number of secondary shares sold normalised by the total number of original shares and the dilution factor as the number of primary shares normalised by the total number of original shares. The authors reach three testable empirical implications: 1. Wealth losses increase in the participation ratio, in the dilution factor and in uncertainty. 2. Promotion costs increase in the participation ratio, the dilution factor and uncertainty. 3. Underpricing decreases in promotion costs and participation ratio and, when controlling for promotion costs, increases in uncertainty.

The authors use a large sample of American IPOs between 1991 and 1995 to test the empirical implications. The empirical results support all predictions. Particularly interesting is the finding that underpricing decreases with the sale of shares by insiders. This implies that insiders take firm-level decisions while considering the maximisation of their personal wealth. The model in this thesis constructs on this implication by making the decision process for the IPO dependent on the utility maximisation of the entrepreneur.

Both Downes and Heinkel, 1982 and Habib and Ljungqvist, 1999 are consistent with the signalling hypothesis of secondary shares. However, many other studies in the literature address the topic of IPO underpricing and the sale of secondary shares with results that do not always support the signalling hypothesis.

Su, 2004, using data from 587 firm-commitment Chinese IPOs between 1994 and 1999, investigates the relation between underpricing and pre-IPO information asymmetries. The author finds only weak evidence supporting the signalling by underpricing theories as in Allen and Faulhaber, 1989, Grinblatt and Hwang, 1989, Welch, 1989. On the other hand the author finds strong evidence in favour of the adverse selection theories of underpricing as the Winner's Curse as in Rock, 1986.

Ang and Brau, 2003 study the effect of the sale of secondary shares as a negative signal about firm value. In particular they explore possible concealing and confounding strategies used by insiders when selling secondary shares. Those strategies are the increase in prospectus amendments of both the number of secondary shares and the lockup period. The authors also test the implicit assumption that secondary shares represent a negative signal.

They find evidence that insiders indeed adopt certain concealing and confounding strategies when selling secondary shares. Firstly, insiders tend to under-report the number of secondary shares in the prospectus and later increase this number in amendments. This behaviour is observed more frequently when the IPO price is higher than the preliminary price published in the prospectus. Secondly, if the number of secondary shares is increased in the amendment, it is often accompanied by an increase in the lockup period. This relation is even stronger for low-retention insiders. Thirdly, in amendments, the increase in secondary shares is larger than the increase in primary shares or associated with a decrease in primary shares. Finally, a regression model of the first day returns supports the assumption that secondary shares convey negative information.

Brau et al., 2007 explores relation between secondary share sale in IPOs and negative aftermarket performance. By addressing the long-run returns they expect to better understand the motivations for secondary share sales. The authors test for four hypotheses. The first hypothesis predicts that secondary shares are associated with poor aftermarket returns. The second hypothesis is that the sale of secondary shares in general does not reveal negative information, but the sale of shares by insiders - defined as managers and directors - does. If secondary share sales are not motivated by overpricing but rather but other reasons, as liquidity or diversification, they should not be associated with poor aftermarket performance.

Following Ang and Brau, 2003, the authors also explore the topic of secondary share revisions. The third hypothesis is that revisions are used to hide negative information and are connected to poor aftermarket performance. As a fourth hypothesis they test for an alternative explanation for revisions. The authors assert that secondary share increases may be connected to the firm raising a sufficient amount of primary capital and not with negative insider information. In this case, revisions should not be associated with poor long-term performance.

To study aftermarket performance the authors measure post-IPO performance within five time-spans - from 25 days to three years. The results do not support the assumption that secondary sales reveal negative information. However, the results support the second hypothesis: selling by insiders does reveal negative information. The authors, nevertheless, do not explore the issue of whether this is due to negative information at the time of the IPO or due to a subsequent moral hazard problem.

Next, the authors test for the effects of secondary shares revisions in amendments. They find no statistically significant difference in the returns of IPOs with and without secondary share increases. At last, the authors test for the fourth hypothesis: secondary share sales are dependent on whether the company is able to raise enough primary capital and not on negative insider information. If this is true, it would be expected that more secondary shares are sold when the originally planned primary proceeds are reached. The results support this hypothesis.

On the subject of motivations for IPOs, Pagano et al., 1998 test eight hypotheses on why companies go public. The authors use a large database of private and public Italian companies in order to test for ex ante and ex post characteristics of IPOs. They find that the main determinant of the going-public decision is the industry's market-to-book ratio. This implies that companies take advantage of "windows of opportunity" when they perceive shares to be overpriced.

The authors also test the hypothesis that IPOs are motivated by liquidity or diversification needs of the insiders. They find two distinct groups in the sample: companies in which insiders divest significantly and those in which the insiders keep a strong financial position in the company. This supports the hypothesis that the insiders sell secondary shares either in order to diversify and increase consumption or they sell to ”cash out” before negative information becomes public.

3.2 Theoretical Literature

Rock, 1986 develops a model for IPO underpricing and abnormal first day returns. The model uses adverse selection and the concept of the ”Winner’s Curse” to explain the underpricing. The main characteristic of this model is the existence of a group of informed investors whose information is superior to both that of the issuing firm and that of the other investors.

The informed investor would then only move in the cases were the shares are underpriced. On the other side, the uninformed investor will participate in both underpriced and overpriced issues. This creates an adverse selection problem for the uninformed investor. If the shares are distributed ’pro rata’ the uninformed investor will receive more shares of overpriced IPO’s as the informed investor has not invested and demand was low. In other to have uninformed investors participate in the process, companies issue shares underpriced, reducing the effects of adverse selection observed by the investors.

Opposite to Rock, Allen and Faulhaber, 1989, Grinblatt and Hwang, 1989, Welch, 1989 all develop models in which the underpricing is used as a signalling device by companies. High quality companies will accept a higher underpricing in order to signal their high type to investors and later receive the correct value for the shares in SEOs. According to Lowry et al., 2017 this class of models has little empirical support.

The work in this thesis is also related Aghion et al., 2004 and Habib and Ljungqvist, 1999 in modelling the entrepreneur utility maximisation problem in the context of IPO’s and underpricing.

Habib and Ljungqvist, 1999 model the entrepreneur’s going-public decision. In their paper, the entrepreneur can affect the level of underpricing through costly actions. The extent to which these actions are taken relates to the number of secondary shares being sold and the minimisation of the entrepreneur's wealth loss. The main difference between their work and this paper is the fact that here price cannot be affect by the actions of the entrepreneur.

Aghion et al., 2004 introduce a liquidity shock for a large block shareholder and look at its effects on monitoring incentives. This can be interpreted as a venture capitalist exiting the investment, possibly through an IPO. In this case the liquidity shock is modelled by a modification in the monitor's utility function with a certain probability, creating a preference for early consumption.

The authors show that there is not always a trade-off between liquidity and monitoring incentives. The paper differs from the work in this thesis as their work focuses in the trade-off between liquidity and monitoring incentives and not on secondary market pricing.

Myers and Majluf, 1984 consider the decision of issuing stock in an asymmetric information context. The authors consider a firm with a positive Net Present Value (NPV) investment opportunity and not enough assets in place to fund the project. The firm then has to issue equity to raise cash. The managers know more about the firm's value than investors and have no credible way to convey this information.

The investor will value any issues based on his beliefs about the value of the project given that equity was issued. This creates an underpricing in firms with higher values. If the managers act in the interest of old shareholders they will not issue stock when the underpricing is too high, even if this leads to a positive-NPV project being forgone. The authors conclude that even without any other market imperfections besides information asymmetry some positive NPV projects will not be financed by rational managers acting in the interest of incumbent shareholders.

The first paper to explore the effects of secondary shares sale was Leland and Pyle, 1977. In this paper, the authors explore the signalling effect of the retention of a fraction of the project by the original entrepreneur.

The model has two agent types: a risk-averse under-diversified entrepreneur and a large pool of diversified competitive investors. The entrepreneur wishes to invest in a costly project with stochastic returns. She needs to sell a fraction of the project to investors in order to raise funds. As the entrepreneur is risk averse, she would prefer selling all the project in exchange for the expected value of the project's return. The entrepreneur knows the specific value of the project, however she has no means to convey this information to the investors. So the entrepreneur retains a fraction of the project in order to signal the high quality of the project to investors. The investors see the retention fraction as a signal of the value of the project and perceive the value as a function of the retention fraction.

In short, a risk-averse entrepreneur will retain a fraction of the project as a way to signal its quality to investors. In this paper, Leland and Pyle, 1977 argue that, the higher the retention of shares by the original entrepreneur, the higher the value of the project. This means that secondary share sales are a negative signal to the market.

Parlour and Plantin, 2008 create a model of Collateralised Loan Obligations, characterising when the secondary market for loans is liquid and when secondary sales are socially desirable. The paper assumes that banks might sell loans for one of two reasons, either due to (negative) private information or due to a liquidity shock. The liquidity shock is described by a stochastic discount factor in the bank's utility function. If hit by the liquidity shock the bank wishes to sell the loans in the secondary market. The bank also wishes to sell in the secondary market any nonperforming loans.

The loans being sold in the secondary market will be valued as a function of the probability of an loan being nonperforming and the probability of a liquidity shock. The authors arrive at the result that a secondary market only is liquid if the liquidity shock occur with a probability higher than some threshold. If that is not true the secondary loan market breaks down due to adverse selection.

This thesis uses a liquidity shock adapted from Parlour and Plantin, 2008 and also borrows on the idea that an agent can have two motivations for secondary sale, one derived by negative private information and one derived from a liquidity need. However, opposite to their paper, here the investment decision and the secondary sales occur in the same period, and the decision-making process is simultaneous. This leads to a more direct relation between the decision of reinvesting and the decision of selling secondary shares.

4 The Model

This paper develops a model for outside financing and going-public decision and is an adaptation from Myers and Majluf, 1984. The departure from the original model is in allowing the entrepreneur to undergo a liquidity shock, which alters his utility function.

Consider two classes of agents - an entrepreneur and a large pool of competitive investors in an economy with two periods: $t = 0, 1$. Also consider that all agents are protected by limited liability and risk-neutral.

4.1 The project

At $t = 0$, the entrepreneur owns all rights to an existing project which will yield a return R at $t = 1$. The return R is drawn from a continuous uniform distribution $\mathcal{U}(R)$ on the interval $[0, \bar{R}]$. The return R is private information of the entrepreneur at $t = 0$.

Still at $t = 0$, the entrepreneur has access to a reinvestment opportunity. At a cost I the reinvestment will increase by τ the returns at $t = 1$. Assume the reinvestment has positive NPV ($\tau > I$) and is observable and verifiable. As the entrepreneur has no available cash, she must raise funds by selling a fraction r of the total returns $R + \tau$ to the investors.

The investors are competitive, risk-neutral and do not discount future cash-flows, and as such, will fund any project subject to breaking-even in expectation. As the return is the entrepreneur's private information and she has no credible way to reveal this information to the market, the investor will value the project according to their rational homogeneous beliefs about the value of the project R.

The reinvestment process proxies for an IPO, where the entrepreneur sells primary shares in order to raise funds for the growing of the company. Here, as in the case of actual IPOs, the entrepreneur might withstand a dilution of his original shares due to an underpricing of the new shares. In the model this happens if R is larger than the value the investor assigns to the company.

4.2 The liquidity shock

Now assume the entrepreneur is subject to a liquidity shock, which means that she will possibly value consumption at $t = 0$ more than consumption at $t = 1$. This is modelled by a utility function with a stochastic discount factor, similar to the one found in Parlour and Plantin, 2008. At $t = 0$, the utility function of the entrepreneur is

$$U^E(c_0, c_1) = c_0 + \tilde{\delta} c_1,$$

where $\tilde{\delta}$ is a random variable whose realisation at $t = 0$ is

$$\tilde{\delta} = \begin{cases} \delta \in (0,1) & \text{with probability } p \\ 1 & \text{with probability } 1 - p. \end{cases}$$

The realisation of $\tilde{\delta}$ at $t = 0$ is private information of the entrepreneur and independent of the value of the project R.

In this model the liquidity shock $\tilde{\delta}$ proxies not only for an actual liquidity need on the part of the entrepreneur, but it can also proxy for diversification needs or for changes in the opportunity cost of the investment. This is particularly relevant when considering entrepreneurs who are also founders and have the majority of their resources invested in the company or in the case of Venture Capitals (VCs), who might have other lucrative investment opportunities.

After the entrepreneur observes the realisation of $\tilde{\delta}$ and the project return, she can offer a contract to investors also selling her stake in the project. She will then receive E, the market value of her equity in the project, at $t = 0$ and forgo all future returns. Again, the investors will accept the contract subject to breaking even in expectation.

It is important to observe that the entrepreneur decides whether or not to perform the reinvestment and whether or not to sell this stake in the project simultaneously. So, the realisation of the stochastic discount factor $\tilde{\delta}$ affects the entrepreneur's decision of raising funds and reinvesting.

The investors observe whether the issue includes only primary or primary and secondary shares. However, they are not able to differentiate between primary and secondary shares of the same company and therefore both are priced equally. This is consistent with the actual set-up of IPOs, where primary and secondary shares are indistinguishable and subject to the same price.

4.3 Information

All participants know at time $t = 0$ the probability distribution of returns $F(R)$ and the probability p of a liquidity shock. The entrepreneur also knows the return R of her project and the realisation of the stochastic discount factor $\tilde{\delta}$. Investors can form beliefs about R based on the actions of the entrepreneur.

At time $t = 1$ the returns are realised and become public information.

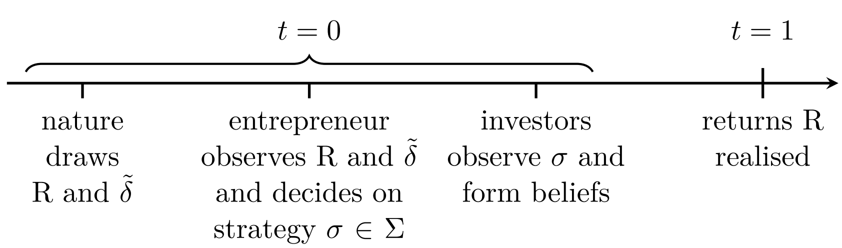

Figure 2: Timeline of the model.

The timeline of the model can be seen in Figure 2.

4.4 Strategies

The entrepreneur then has three possible strategies $\sigma \in \Sigma$:

- to issue primary shares only;
- to issue both primary and secondary shares;
- not to issue shares.

By assumption, it is not possible for the entrepreneur to sell secondary shares without the sale of primary shares. Meaning that if the entrepreneur sells her part in the returns, she will, necessarily, also raise funds and reinvest. This requirement is consistent with a real situation, where insiders cannot public trade secondary shares of a private company. Here the process of making the company public happens through the sale of primary shares, which explains the absence of secondary shares only issues in the model.

Another assumption of the model is that secondary shares sales are all or nothing: either the entrepreneur retains all her shares or she sells all her shares. While in a real IPO this is unlikely to happen, as insiders usually sell only a fraction of their equity, this assumption simplifies the model.

The entrepreneur will choose the strategy which maximises her utility. This is dependent, among other parameters, on the realisation of $\tilde{\delta}$.

4.5 Beliefs

The investors will accept the contract offered by the entrepreneur if they, at least, break even in expectation. After observing the entrepreneur's strategy, they will form homogeneous beliefs about the value of R and the realisation of $\tilde{\delta}$ using Bayesian updating. The posterior beliefs are then a function of the anterior beliefs and the likelihood of the entrepreneurs' actions. The belief can be seen as the probability distribution attached by the investor on project returns given that strategy σ was observed.

More specifically, the beliefs about $\tilde{\delta}$ will affect the beliefs about R, as the investors recognise that entrepreneurs subject to a liquidity shock will accept a higher underpricing for their projects. The investor's pricing strategy is then the optimal strategy for the corresponding belief.

Upon observing an issue of both primary and secondary shares the investors form the beliefs that either the entrepreneur has a liquidity need or that the returns R are sufficiently low to motivate a cash-out. The specific probability assigned to each scenario is a function of the probability distribution of the returns $F(R)$ and of the probability of a liquidity shock p. Furthermore, upon observing an issue of only primary shares the investors form the beliefs that the entrepreneur has no liquidity needs and that the returns are not sufficiently low to motivate a cash-out strategy.

5 Characterisation of the Equilibrium

The analysis in this thesis is restricted to Weak Perfect Bayesian Equilibria (WPBEs) in which the entrepreneur plays a pure strategy and which are described by three threshold values for the returns of the project R: R^{*}, R^{**} and R^{***}. Each threshold value separates between the optimal strategies for the entrepreneur.

The entrepreneur can follow one of the three strategies described in Section 4. The optimal choice will be the one that maximises her utility for each given value of R and realisation of $\tilde{\delta}$ and taking into account the investors believes.

An entrepreneur subject to a liquidity shock will raise equity and sell her part in the returns if the market value of equity E is acceptable. This will happen if the returns of the project R are less than the second threshold value R^{**}.

An entrepreneur not subject to a liquidity shock will raise equity and perform the reinvestment if the fraction r of returns sold to investors does not cause a too severe dilution of her shares. This will happen if the returns of the project R are less than the first threshold value R^*. An entrepreneur not subject to a liquidity shock will sell her stake in the project returns if the market value of equity E is larger than the returns from the project and the reinvestment. This will happen if the returns of the project R are less than the third threshold value R^{***}.

In order to characterise such an equilibrium certain assumptions have to be made. They are explained in detail in the following paragraphs.

Assumption 1. *The value of δ is small enough such that if the entrepreneur performs the reinvestment and is hit by the liquidity shock she will always sell her share in the project at $t = 0$.*

Assumption 1 states that the liquidity shock is severe enough so that every entrepreneur hit by the shock that performs the reinvestment also sells secondary shares. This means that if the underpricing for primary shares is acceptable, the price of equity also will be. This simplifies the analysis of the equilibrium.

Initially, it is also assumed that the threshold values of R follow the ordering $R^{***} < R^* < R^{**}$. Later it will proven to be a characteristic of this type of equilibria. Nonetheless, the ordering is intuitive. It is expected that the entrepreneur subject to the liquidity shock will accept the largest underpricing, so $R^{**} > R^*$ and $R^{**} > R^{***}$. Besides that, from the entrepreneurs without liquidity shock, it is expected that the ones selling secondary shares believe her shares to be overpriced, and the ordering $R^{***} < R^*$ follows from there.

In Figure 3 it is possible to see a scheme of the entrepreneur's optimal action for each combination of $\tilde{\delta}$ and R. The threshold values and the action observed by the investors are depicted in the Figure 4.

More specifically, the entrepreneur's action will be:

- to reinvest and sell her part in the returns if $\tilde{\delta} = \delta$ and $R < R^{**}$ or

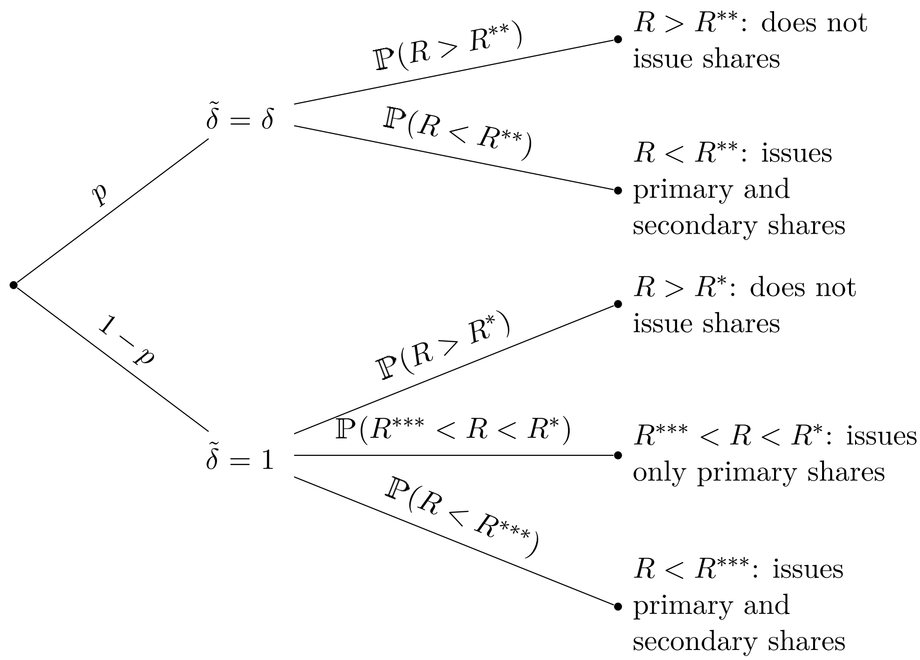

Figure 3: Schematization of the entrepreneur's optimal strategy for each realisation of $\tilde{\delta}$ and R.

$\tilde{\delta} = 1$ and $R < R^{***}$;

- to reinvest and keep her part in the returns if $\tilde{\delta} = 1$ and $R^{***} < R < R^{*}$;
- not to reinvest and keep the full amount of the original return R if $\tilde{\delta} = 1$ and $R > R^{*}$ or if $\tilde{\delta} = \delta$ and $R > R^{**}$.

After observing each of the possible actions the investors will be able to form rational beliefs about the value of the project.

5.1 The investors' beliefs

Upon observing the entrepreneur's strategy the investors form rational homogeneous beliefs about the value of the project.

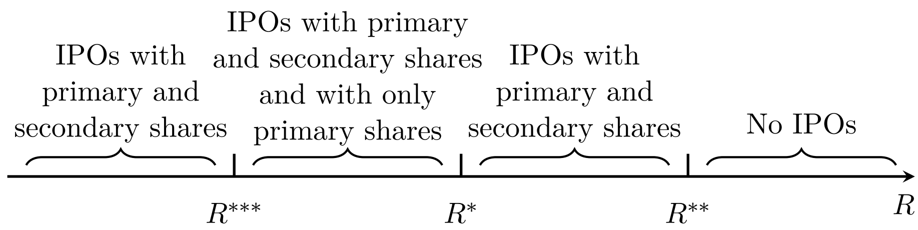

Figure 4: Threshold values and entrepreneur's actions observed by the investors.

After observing an issue of primary shares only, the investors form the belief that the entrepreneur was not hit by a liquidity shock ($\tilde{\delta} = 1$) and that the returns of the project lie between threshold values R^* and R^{***}. The expected return of the project conditional on observing an issue of primary shares only is then

$$\mathbb{E}[R|\sigma = \text{primary issue only}] = \frac{\int_{R^{***}}^{R^*} Rf(R)dR}{F(R^*) - F(R^{***})}.$$

Using the proprieties of the Uniform Distribution $\mathcal{U}(R)$ on the interval $[0, \bar{R}]$ this is equivalent to:

$$\mathbb{E}[R|\sigma = \text{primary issue only}] = \frac{R^* + R^{***}}{2}$$

After observing an issue of primary and secondary shares, the investors infer that either the entrepreneur was hit by a liquidity shock ($\tilde{\delta} = \delta$) and the returns of the project are smaller than R^{**} or that the entrepreneur was not hit by a liquidity shock ($\tilde{\delta} = 1$) and the returns of the project are smaller than R^{***}. The expected returns of the project conditional on observing an issue of both primary and secondary shares is then

$$\mathbb{E}[R|\sigma = \text{primary and secondary}] = \frac{(1-p)\int\limits_0^{R^{***}} Rf(R)dR + p\int\limits_0^{R^{**}} Rf(R)dR}{(1-p)F(R^{***}) + pF(R^{**})}.$$

This can be rewritten as:

$$\mathbb{E}[R|\sigma = \text{primary and secondary}] = \frac{1}{2}\frac{(1-p)(R^{***})^2 + p(R^{**})^2}{(1-p)R^{***} + pR^{**}}$$

Lastly, after observing no issue, the investor forms the beliefs either that the entrepreneur was hit by a liquidity shock ($\tilde{\delta} = \delta$) and the returns of the project are smaller than R^{**} or that the entrepreneur was not hit by an liquidity shock ($\tilde{\delta} = 1$) and the returns are smaller than R^{*}. The expected returns of the project conditional on observing no issue is then

$$\mathbb{E}[R|\sigma = \text{no issue}] = \frac{(1-p)\int\limits_{R^*}^{\bar{R}} Rf(R)dR + p\int\limits_{R^{**}}^{\bar{R}} Rf(R)dR}{(1-p)(1-F(R^*)) + p(1-F(R^{**}))}.$$

Which is equivalent to:

$$\mathbb{E}[R|\sigma = \text{no issue}] = \frac{1}{2}\frac{\bar{R}^2 - (1-p)(R^*)^2 - p(R^{**})^2}{1-(1-p)R^* - pR^{**}}$$

5.2 The threshold values

As the investor can differentiate between issues involving only primary shares and issues involving primary and secondary shares, two different prices will arise. They will accept the contracts subject to their participation constraint (PC) and based on their beliefs about the value of the project. The participation constraint of the investors when acquiring shares of an issue consisting of primary shares only is

$$(\mathbb{E}[R|\sigma = \text{primary issue only}] + \tau)\, r \geq I.$$

In this situation the investors will pay I for a fraction r of the total returns of the project and reinvestment. Using the proprieties of the uniform distribution this becomes

$$\left(\frac{R^{*} + R^{***}}{2} + \tau\right) r \geq I.$$

The participation constraint of the investor when acquiring shares in an issue composed by primary and secondary shares is

$$\mathbb{E}[R|\sigma = \text{primary and secondary}] + \tau \geq E + I.$$

In this case the investors will acquire all shares of the company, both the new issued shares, represented by I, and all existing equity E. The equation above can be rewritten as

$$\frac{1}{2}\frac{(1-p)(R^{***})^2 + p(R^{**})^2}{(1-p)R^{***} + pR^{**}} + \tau \geq E + I$$

As the investors are competitive, they will make zero profits in expectation. This means that both PCs bind.

$$\left(\frac{R^{*} + R^{***}}{2} + \tau\right) r = I \tag{1}$$

$$\frac{1}{2}\frac{(1-p)(R^{***})^2 + p(R^{**})^2}{(1-p)R^{***} + pR^{**}} + \tau = E + I \tag{2}$$

The entrepreneur offers the investors the contract that optimises her utility subject to the realisation of $\tilde{\delta}$ and the returns R of the project. Which contract will be chosen by the entrepreneur is defined by her incentive compability constraint (ICC). In the equilibrium described in this work there are three relevant ICCs.

The entrepreneur not hit by a liquidity shock will only deepen the investment if her returns with reinvestment after paying the investors are larger that the returns without reinvestment. The incentive compatibility constraint of the entrepreneur not subject to a liquidity shock for issuing equity and deepening the investment is then

$$(R + \tau)(1 - r) \geq R.$$

At the threshold value R^* the entrepreneur is indifferent between both strategies, so the ICC above holds with equality.

$$(R^* + \tau)(1 - r) = R^* \tag{3}$$

The entrepreneur subject to a liquidity shock will deepen the investment and sell all her stake at the project if the market value of the equity E is larger than the returns without reinvestment, discounted by δ. So the incentive compatibility constraint of the entrepreneur hit by a liquidity shock for issuing equity, deepening the investment and selling secondary shares is

$$E \geq \delta R.$$

Similar to equation 3, at the threshold value the ICC above holds with equality.

$$E = \delta R^{**} \tag{4}$$

The entrepreneur not subject to a liquidity shock will sell her returns in the deepened project if her equity is overpriced. This happens if the market value of the equity E is larger than her returns with reinvestment after paying the investors. The incentive compatibility constraint of the entrepreneur not hit by a liquidity shock for selling secondary shares is then

$$E \geq (R + \tau)(1 - r).$$

The ICC holds with equality at the threshold R^{***}.

$$E = (R^{***} + \tau)(1 - r) \tag{5}$$

The equilibrium is characterised by the system of equations 1-5.

Lemma 1. *Suppose an equilibrium of the type described above exists. If $p < \frac{\delta^2}{(\delta-1)^2}$ and $\frac{2I}{\tau} - 1 < \delta$, the threshold values R^*, R^{**} and R^{***}, the fraction r of returns required by investors when acquiring shares of a primary issue and the market value of equity E are:*

$$R^* = 2\tau \frac{\tau - I}{2I - \tau} \frac{1 + p\frac{1-\delta}{\delta}}{1 - p(\frac{1-\delta}{\delta})^2} \tag{6a}$$

$$R^{**} = 2\frac{\tau - I}{\delta} \frac{1 + p\frac{1-\delta}{\delta}}{1 - p(\frac{1-\delta}{\delta})^2} \tag{6b}$$

$$R^{***} = 2(\tau - I)\left(\frac{1 + p\frac{1-\delta}{\delta}}{1 - p(\frac{1-\delta}{\delta})^2} - 1\right) \tag{6c}$$

$$r = \frac{(2I - \tau)\left(1 - p(\frac{1-\delta}{\delta})^2\right)}{2(\tau - I)\left(1 + p\frac{1-\delta}{\delta}\right) + (2I - \tau)\left(1 - p(\frac{1-\delta}{\delta})^2\right)} \tag{6d}$$

$$E = 2(\tau - I)\frac{1 + p\frac{1-\delta}{\delta}}{1 - p(\frac{1-\delta}{\delta})^2}. \tag{6e}$$

The proof of Lemma 1 is shown in the Appendix.

Lemma 1 presents the solution for the model in a restricted parameter space:

$$p < \frac{\delta^2}{(\delta - 1)^2}$$

and

$$\frac{2I}{\tau} - 1 < \delta.$$

The first inequality ensures that R^{**} is positive. The second inequality ensures that $R^* < R^{**}$.

Proposition 1. *If $p < \frac{\delta^2}{(\delta-1)^2}$ and $\frac{2I}{\tau} - 1 < \delta$, there exists an equilibrium characterised by the threshold values R^*, R^{**} and R^{***}, the return r of investors when acquiring shares of a primary issue and the market value of equity E and with values described by Lemma 1.*

Under the conditions described in Proposition 1 the equilibrium exists. This means that investors beliefs and the optimal action for the entrepreneur in equilibrium are equivalent.

Proposition 2. *The threshold values R^*, R^{**} and R^{***} follow the ordering $R^{***} < R^* < R^{**}$.*

This means that an entrepreneur subject to a liquidity need will accept a higher degree of underpricing. It also means that an entrepreneur not subject to a liquidity shock will sell secondary shares for lower values of the return of the project R than she will sell only primary shares.

Corollary 1. *As the severity of the liquidity shock decreases (value of δ increases), all threshold values R^*, R^{**} and R^{***} decrease.*

Corollary 1 states that for more severe liquidity shocks (smaller values of δ), the entrepreneurs hit by the liquidity shock will accept a higher underpricing. This in turn will affect the actions of the entrepreneur not hit by a liquidity shock, and increase all threshold values for R.

Corollary 2. *As the probability p of a liquidity shock increases, so increase all threshold values R^*, R^{**} and R^{***}.*

Corollary 2 states that the more probable a liquidity shock is the higher are the values of the thresholds for R: R^*, R^{**} and R^{***}.

Detailed proof of the propositions and corollaries can be seen in Appendix.

6 Conclusion

This thesis develops a model with the aim to explore the relation between the sale of secondary shares and underpricing in IPOs. While secondary share sales can be motivated by a ”cash-out” strategy, they can also be motivated by liquidity (or diversification) needs by the entrepreneur. The fact that secondary shares do not always reveal negative information about the issue is reflected in the pricing of the issues in the market.

This work puts together features from two models, Myers and Majluf, 1984 and Parlour and Plantin, 2008. From the first the model in this thesis borrows the basic formulation of the model and the reinvestment opportunity. From the second it borrows the description of the liquidity shock and its effects in the pricing of shares. The model describes an entrepreneur who has access to a reinvestment opportunity that can only be financed by the issuance of shares. The entrepreneur’s decision of whether to issue shares is further complicated by the possibility of a liquidity shock. If the liquidity shock occurs, she might wish to issue primary shares and sell her secondary shares alongside it.

This thesis proves the existence of an equilibrium in which entrepreneurs can have three strategies: 1. to issue only primary shares, 2. to issue primary and secondary shares and 3. not to issue shares. The optimal strategy is a function of the return of the project and of the realisation of stochastic discount factor. The investors will value the shares based on their beliefs about the value of the project and about the occurrence of a liquidity shock. The modification in the price does not happen only for issues with primary and secondary shares, but also for issues of only primary shares.

The existence of the equilibrium is shown for a particular set of conditions. Also shown is the dependence of this equilibrium on the probability of a liquidity shock and on the severity of the liquidity shock.

The work in this thesis can be further developed by exploring other equilibria or by proving equilibrium unicity. Besides that, a welfare analysis of secondary share sales can be constructed, specially in the form of comparison with equilibria without secondary share sales. A third possible development would be to allow for a fractional sale of secondary shares, when the entrepreneur sells only a part of her equity in the project.

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Appendix

Proof of Lemma 1

As seen in section 5, the equilibrium is characterised by the system of equations 1-5:

$$\left(\frac{R^* + R^{***}}{2} + \tau\right) r = I \tag{1}$$

$$\frac{1}{2}\frac{(1-p)(R^{***})^2 + p(R^{**})^2}{(1-p)R^{***} + pR^{**}} + \tau = E + I \tag{2}$$

$$(R^* + \tau)(1-r) = R^* \tag{3}$$

$$E = \delta R^{**} \tag{4}$$

$$E = (R^{***} + \tau)(1-r) \tag{5}$$

There are five equations and five variables in the system and the system can be solved by simple substitution.

From equation 3:

$$(1-r) = \frac{R^*}{R^* + \tau} \implies r = \frac{\tau}{R^* + \tau}$$

Substituting in equations 1, 2, 4 and 5:

$$\begin{aligned}
\left(\frac{R^* + R^{***}}{2} + \tau\right) \frac{\tau}{R^* + \tau} &= I \\
\frac{1}{2}\frac{(1-p)(R^{***})^2 + p(R^{**})^2}{(1-p)R^{***} + pR^{**}} + \tau &= E + I \\
\delta R^{**} &= E \\
(R^{***} + \tau)\frac{R^*}{R^* + \tau} &= E
\end{aligned}$$

Now substituting $E = \delta R^{**}$ (forth equation in the system above) in the remaining equations:

$$\left(\frac{R^* + R^{***}}{2} + \tau\right)\frac{\tau}{R^* + \tau} = I \tag{6}$$

$$\frac{1}{2}\frac{(1-p)(R^{***})^2 + p(R^{**})^2}{(1-p)R^{***} + pR^{**}} + \tau = \delta R^{**} + I \tag{7}$$

$$(R^{***} + \tau)\frac{R^*}{R^* + \tau} = \delta R^{**} \tag{8}$$

From equation 6 one obtains:

$$R^{***} = \left(\frac{2I}{\tau} - 1\right) R^* + 2(I - \tau) \tag{9}$$

Which can be written as

$$R^{***} + \tau = \left(\frac{2I}{\tau} - 1\right)(R^* + \tau).$$

Substituting in equation 8 one obtains

$$\left(\frac{2I}{\tau} - 1\right)(R^* + \tau)\frac{R^*}{R^* + \tau} = \delta R^{**}$$

which can be rewritten as

$$R^* = \frac{\delta R^{**}}{\frac{2I}{\tau} - 1}. \tag{10}$$

Substituting back into equation 9 one obtains

$$R^{***} = \delta R^{**} + 2(I - \tau) \tag{11}$$

Substituting equation 10 in equation 7 one obtains

$$R^{**} = \frac{2(I-\tau)[(1-p)\delta + p]}{-(1-p)\delta^2 + p - 2q\delta}$$

which can be simplified to

$$\boxed{R^{**} = 2\frac{\tau - I}{\delta}\frac{1 + p\frac{1-\delta}{\delta}}{1 - p(\frac{1-\delta}{\delta})^2}} \tag{12}$$

By replacing equation 11 back into equation 10 one obtains

$$\boxed{R^{***} = 2(\tau - I)\left(\frac{1 + p\frac{1-\delta}{\delta}}{1 - p(\frac{1-\delta}{\delta})^2} - 1\right).} \tag{13}$$

By replacing equation 12 back into equation 9 one obtains

$$\boxed{R^{*} = 2\tau\frac{\tau - I}{2I - \tau}\frac{1 + p\frac{1-\delta}{\delta}}{1 - p(\frac{1-\delta}{\delta})^2}.} \tag{14}$$

The expressions for r and E are obtained by replacing equations 11, 12 and 13 into equations 1 and 4.

$$\boxed{r = \frac{(2I - \tau)\left(1 - p(\frac{1-\delta}{\delta})^2\right)}{2(\tau - I)\left(1 + p\frac{1-\delta}{\delta}\right) + (2I - \tau)\left(1 - p(\frac{1-\delta}{\delta})^2\right)},} \tag{15}$$

and

$$\boxed{E = 2(\tau - I)\frac{1 + p\frac{1-\delta}{\delta}}{1 - p(\frac{1-\delta}{\delta})^2}.} \tag{16}$$

Proof of Proposition 1

Proposition 1 states the existence of an equilibrium as described on Section 5. In order to prove the equilibrium exists it is necessary to prove that the each player's strategy is the best response to the strategy played by other players. As in the text of the thesis it was already shown that the investors' beliefs are rational and are to the best response to the entrepreneur's actions. It remains to proof that the entrepreneur strategy is the best response for the investors' beliefs.

First consider the case where $\tilde{\delta} = 1$ and $R = R^*$. In this case it already know that the entrepreneur is indifferent between issuing primary only shares or not issuing shares. Now it is necessary to show that both these strategies are preferred to issuing primary and secondary shares. This means that:

$$U^E[\text{primary only}] = U^E[\text{no issue}] > U^E[\text{primary and secondary}],$$

$$(R^* + \tau)(1 - r) = R^* > E.$$

As mentioned before, the first two terms are the definitions of R^*, so the equality hold. Its left to prove that the inequality holds:

$$R^* > E.$$

At equilibrium values (Lemma 1):

$$2\tau \frac{I - \tau}{2I - \tau} \frac{1 + p\frac{1-\delta}{\delta}}{1 - p(\frac{1-\delta}{\delta})^2} > 2(I - \tau)\frac{1 + p\frac{1-\delta}{\delta}}{1 - p(\frac{1-\delta}{\delta})^2}$$

$$\frac{1}{\frac{2I}{\tau} - 1} > 1$$

As $\frac{2I}{\tau} - 1 \in (0, 1)$, the inequality holds.

Now consider the case where $= \delta$ and $R = R^{**}$. In this case the entrepreneur is indifferent between not issuing and issuing primary and secondary shares. It is necessary to show that both these strategies are preferred to issuing primary shares only.

$$U^E[\text{primary and secondary}] = U^E[\text{no issue}] > U^E[\text{primary only}],$$

$$E = \delta R^{**} > \delta(R^{**} + \tau)(1 - r).$$

Again, the first two terms are the definition of the threshold R^*, so its left to prove the inequality

$$\delta R^{**} > \delta(R^{**} + \tau)(1 - r).$$

It can be simplified to

$$R^{**} > \tau(1 - r).$$

At equilibrium values:

$$2\frac{I-\tau}{\delta}\frac{1+p\frac{1-\delta}{\delta}}{1-p(\frac{1-\delta}{\delta})^2} >$$
$$\tau\left(1 - \frac{\tau(2I-\tau)\left(1-p(\frac{1-\delta}{\delta})^2\right)}{(2I-2\tau)\left(1+p\frac{1-\delta}{\delta}\right)+\tau(2I-\tau)\left(1-p(\frac{1-\delta}{\delta})^2\right)}\right)$$

which can be simplified to

$$\frac{1}{1-p(\frac{1-\delta}{\delta})^2} > \frac{\tau\delta}{(2I-2\tau)\left(1+p\frac{1-\delta}{\delta}\right)+\tau(2I-\tau)\left(1-p(\frac{1-\delta}{\delta})^2\right)}$$

which holds due to the proprieties of the model.

Now consider the case where $= 1$ and $R = R^{***}$. In this case the entrepreneur is indifferent between primary shares only and issuing primary and secondary shares. It is necessary to show that both these strategies are preferred to not issuing shares.

$$U^E[\text{primary and secondary}] = U^E[\text{primary only}] > U^E[\text{no issue}],$$

$$E = (R^{***} + \tau)(1 - r) > R^{***}.$$

Similar to the cases before, the equality is the definition of R^{***}, so it remains to prove that the inequality holds,

$$E > R^{***}.$$

At equilibrium values:

$$2(I - \tau)\frac{1 + p\frac{1-\delta}{\delta}}{1 - p(\frac{1-\delta}{\delta})^2} > 2(I - \tau)\left(1 + \frac{1 + p\frac{1-\delta}{\delta}}{1 - p(\frac{1-\delta}{\delta})^2}\right).$$

That is equivalent to

$$0 > 2(I - \tau),$$

which holds by definition of the model.

Proof of Proposition 2

Proof of $R^* < R^{**}$

From equation 12 and 14 its possible to see that $R^* < R^{**}$ iff

$$\frac{2\tau(I - \tau)}{2I - \tau}\frac{1 + p\frac{1-\delta}{\delta}}{1 - p(\frac{1-\delta}{\delta})^2} < 2\frac{I - \tau}{\delta}\frac{1 + p\frac{1-\delta}{\delta}}{1 - p(\frac{1-\delta}{\delta})^2}$$

$$\frac{2\tau(I - \tau)}{(2I - \tau)} < 2\frac{I - \tau}{\delta}$$

which is equivalent to:

$$\frac{\tau}{2I - \tau} < \frac{1}{\delta}.$$

From this one obtains

$$\boxed{\delta < \frac{2I}{\tau} - 1,}$$

which is one of the conditions for the existence of the equilibrium.

Proof of $R^{***} < R^{**}$

From equation 10

$$R^{***} = \delta R^{**} + 2(I - \tau),$$

$R^{***} < R^{**}$ iff

$$\delta R^{**} + 2(I - \tau) < R^{**}.$$

As $\delta \in (0, 1)$ and $(I - \tau) < 0$, the inequality always holds.

Proof of $R^{***} < R^{*}$

From equation 9

$$R^{***} = \left(\frac{2I}{\tau} - 1\right) R^{*} + 2(I - \tau),$$

$R^{***} < R^{*}$ iff

$$\left(\frac{2I}{\tau} - 1\right) R^{*} + 2(I - \tau) < R^{*}.$$

Due to the condition $\tau \in (I, 2I)$, $\frac{2I}{\tau} - 1$ is in the interval $(0, 1)$. And as $I - \tau < 0$, the inequality always holds.

Proof of Corollary 1

In order to check the dependency of R^*, R^{**} and R^{***} in δ, one can calculate the derivatives $\frac{dR^*}{d\delta}$, $\frac{dR^{**}}{d\delta}$ and $\frac{dR^{***}}{d\delta}$.

$$\frac{dR^*}{d\delta} = \frac{d}{d\delta}\left(2\tau\frac{\tau - I}{2I - \tau}\frac{1 + p\frac{1-\delta}{\delta}}{1 - p(\frac{1-\delta}{\delta})^2}\right) = 2\tau\frac{\tau - I}{2I - \tau}\frac{d}{d\delta}\left(\frac{1 + p\frac{1-\delta}{\delta}}{1 - p(\frac{1-\delta}{\delta})^2}\right)$$

$$\frac{dR^{**}}{d\delta} = \frac{d}{d\delta}\left(2\frac{\tau - I}{\delta}\frac{1 + p\frac{1-\delta}{\delta}}{1 - p(\frac{1-\delta}{\delta})^2}\right) = 2\frac{\tau - I}{\delta}\frac{d}{d\delta}\left(\frac{1 + p\frac{1-\delta}{\delta}}{1 - p(\frac{1-\delta}{\delta})^2}\right)$$

$$\frac{dR^{***}}{d\delta} = \frac{d}{d\delta}\left(2(\tau - I)\left(\frac{1 + p\frac{1-\delta}{\delta}}{1 - p(\frac{1-\delta}{\delta})^2} - 1\right)\right) = 2(\tau - I)\frac{d}{d\delta}\left(\frac{1 + p\frac{1-\delta}{\delta}}{1 - p(\frac{1-\delta}{\delta})^2}\right)$$

As the first term in each derivative is positive, the sign of the derivative is the sign of $\frac{d}{d\delta}\left(\frac{1+p\frac{1-\delta}{\delta}}{1-p(\frac{1-\delta}{\delta})^2}\right)$.

$$\frac{d}{d\delta}\left(\frac{1 + p\frac{1-\delta}{\delta}}{1 - p(\frac{1-\delta}{\delta})^2}\right) = \frac{-p(\delta^2(p-1) - 2\delta(p-1) + p))}{(\delta^2(p-1) - 2\delta p + p)^2} \tag{17a}$$

$$= \frac{-p((p-1)(\delta^2 - 2\delta) + p)}{(\delta^2(p-1) - 2\delta p + p)^2} \tag{17b}$$

As $(p-1)$ is smaller than zero and so is $(\delta^2 - 2\delta)$, the whole numerator becomes negative. As the expression in the denominator is squared, it is positive. So, the derivative of the expression with relation to δ is negative.

$$\frac{d}{d\delta}\left(\frac{1 + p\frac{1-\delta}{\delta}}{1 - p(\frac{1-\delta}{\delta})^2}\right) < 0$$

And the sign of $\frac{dR^*}{d\delta}$, $\frac{dR^{**}}{d\delta}$ and $\frac{dR^{***}}{d\delta}$ is also negative.

This means that when δ increases (less severe liquidity shock) the thresholds R^*, R^{**} and R^{***} decrease.

Proof of Corollary 2

For the dependence in p, the same argument can be used as for the dependency in δ. This means that the sign of the derivatives of R^*, R^{**} and R^{***} with respect to p only depends on the sign of $\frac{d}{dp}\left(\frac{1+p\frac{1-\delta}{\delta}}{1-p(\frac{1-\delta}{\delta})^2}\right)$

$$\frac{d}{dp}\left(\frac{1+p\frac{1-\delta}{\delta}}{1-p(\frac{1-\delta}{\delta})^2}\right) = \frac{(1-\delta)\delta^2}{(\delta^2(p-1)-2\delta p+p)^2} \tag{18a}$$

As both $(1-\delta)$ and δ^2 are positive, the numerator is positive. As the expression for the denominator is squared it is positive. So, the derivative of the expression with relation to p is positive.

$$\frac{d}{dp}\left(\frac{1+p\frac{1-\delta}{\delta}}{1-p(\frac{1-\delta}{\delta})^2}\right) > 0$$

This means that as the probability of a liquidity shock p increases, the thresholds R^*, R^{**} and R^{***} increase.

German Abstract

Diese Arbeit analysiert den Zusammenhang zwischen einem stochastischen Liquiditätsschock eines Unternehmers und der Möglichkeit von Secondary-Share-Verkäufen bei Börsengang (IPO). Im folgenden Modell verkaufen Unternehmer Sekundäraktien entweder aufgrund eines Liquiditätsschocks oder aufgrund negativer privater Informationen. Es ist ein Gleichgewicht charakterisiert, bei dem IPO sowohl mit nur Primäranteilen als auch mit Primär- und Sekundäranteilen gesehen werden. Die optimalen Verträge zwischen dem Unternehmer und den Investoren wurden für jede Emissionsart hergeleitet. Die Bedingungen für die Existenz eines solchen Gleichgewichts sind bekannt. Die Wahrscheinlichkeit und die Schwere des Liquiditätsschocks wirken sich auf die Entscheidung über den Börsengang und den IPO-Preis auf dem Markt, sowohl für Primär- als auch für Sekundäremissionen, aus.