



universität  
wien

# DISSERTATION / DOCTORAL THESIS

Titel der Dissertation /Title of the Doctoral Thesis

Essays on Intertemporal Decision Making in Industrial-, Energy-  
and Environmental Management

verfasst von / submitted by

Markus Eigruber

angestrebter akademischer Grad / in partial fulfilment of the requirements for the degree of

Doctor of Philosophy (PhD)

Wien, 2019 / Vienna, 2019

Studienkennzahl lt. Studienblatt /

Degree programme code as it appears on the student  
record sheet:

UA 794 370 146

Dissertationsgebiet lt. Studienblatt /

Field of study as it appears on the student record sheet:

Wirtschaftswissenschaften - Management

Betreut von / Supervisor:

Univ.-Prof. Franz Wirl



### **Acknowledgements**

I want to thank my supervisor, Prof. Wirl, not only for his guidance and support, but also for providing an open space from which many stimulating ideas and discussions were created. It is not a coincidence that a large part of this thesis has been produced in concert with my supervisor. Needless to say, I learned a lot from his endless passion for scientific work. A true perpetuum mobile.

Of course, I also want to thank some of my other colleagues and the University of Vienna in general, for providing the infrastructure as well as the funding.

I also take this opportunity to formally thank myself for everything I have done and accomplished so far.

Finally, and most importantly, I want to thank the anchor of my life, my perfect complement, Marianne, for her unconditional support at all times and in all aspects.



### **Abstract**

This cumulative thesis consists of three articles and two notes, covering dynamic issues in industry-, energy- and environmental management. In all but one note we analyse constrained problems stemming from strategic interaction of decision makers over time.

The first paper deals with a set of firms competing not only in output but also in exaggerated claims made to customers, i.e., in cheating efforts. We set up the differential game as a reversed advertising for goodwill model a la Nerlove-Arrow and highlight the importance of information structure and how it can be made use of in order to deter corporate cheating.

In the second paper we analyse how firms, subject to internal learning by doing as well as positive external effects, set up their production path. This situation resembles markets composed of manufacturers competing in the production of solar-photovoltaic panels. We analyse and compare the outcome under differing industrial organizations and also identify a technical anomaly, in a sense that we find multiple stable linear Markov perfect equilibria in the corresponding LQ differential game.

The third chapter addresses possible incentive issues arising from modern climate engineering techniques, in particular, Solar Radiation Management. We propose that effects on trade could be part of the set of all, as of yet, unforeseen side effects of climate-interfering techniques and should therefore also be taken into consideration.

Finally, we conclude by putting forward two novel uses of a particular dynamic in ecological economics, namely the shallow lake game. Both, the societal response to uncovered corporate fraud as well as the existence of tipping points in the global climate system can be modelled by the dynamics of the shallow lake. These notes are topically related to the first and third chapter of this thesis, respectively.



# Contents

|                                                                                   |           |
|-----------------------------------------------------------------------------------|-----------|
| <b>Acknowledgements</b>                                                           | <b>3</b>  |
| <b>Abstract</b>                                                                   | <b>5</b>  |
| <b>Introduction</b>                                                               | <b>10</b> |
| <b>1 Cheating as a dynamic marketing strategy in monopoly, cartel and duopoly</b> | <b>15</b> |
| 1.1 Introduction . . . . .                                                        | 16        |
| 1.2 Model . . . . .                                                               | 19        |
| 1.3 Solution of the static game . . . . .                                         | 24        |
| 1.4 Intertemporal equilibria . . . . .                                            | 26        |
| 1.4.1 Monopoly and cartel . . . . .                                               | 26        |
| 1.4.2 Duopoly . . . . .                                                           | 28        |
| 1.5 Comparison . . . . .                                                          | 30        |
| 1.6 Final remarks . . . . .                                                       | 35        |
| <b>2 Market equilibrium strategies under learning by doing spillovers</b>         | <b>37</b> |
| 2.1 Introduction . . . . .                                                        | 38        |
| 2.2 Framework . . . . .                                                           | 41        |
| 2.2.1 Power cost function . . . . .                                               | 42        |
| 2.2.2 Linear cost function . . . . .                                              | 43        |
| 2.2.3 Objectives under different market structures . . . . .                      | 43        |
| 2.3 Power cost function . . . . .                                                 | 45        |
| 2.3.1 Monopoly . . . . .                                                          | 45        |
| 2.3.2 Cartel . . . . .                                                            | 49        |
| 2.3.3 Oligopoly . . . . .                                                         | 50        |
| 2.3.4 Competition . . . . .                                                       | 54        |

---

|          |                                                                             |            |
|----------|-----------------------------------------------------------------------------|------------|
| 2.3.5    | Comparison . . . . .                                                        | 55         |
| 2.4      | Linear cost function . . . . .                                              | 57         |
| 2.4.1    | Cartel and monopoly . . . . .                                               | 58         |
| 2.4.2    | Oligopoly - Markov perfect equilibrium . . . . .                            | 60         |
| 2.4.3    | Oligopoly - open loop equilibrium . . . . .                                 | 65         |
| 2.4.4    | Comparison . . . . .                                                        | 67         |
| 2.5      | Final remarks . . . . .                                                     | 71         |
| <b>3</b> | <b>Climate engineering in an interconnected world - the role of tariffs</b> | <b>75</b>  |
| 3.1      | Introduction . . . . .                                                      | 76         |
| 3.2      | Framework . . . . .                                                         | 79         |
| 3.3      | Monopoly . . . . .                                                          | 80         |
| 3.3.1    | $F$ disregards temperature changes . . . . .                                | 81         |
| 3.3.2    | $F$ accounts for temperature changes . . . . .                              | 86         |
| 3.4      | Oligopoly . . . . .                                                         | 88         |
| 3.4.1    | $F$ disregards temperature changes . . . . .                                | 89         |
| 3.4.2    | $F$ accounts for temperature changes . . . . .                              | 92         |
| 3.5      | Conclusions . . . . .                                                       | 94         |
| <b>4</b> | <b>Two applications of the shallow lake system</b>                          | <b>97</b>  |
| 4.1      | Corporate cheating under nonlinear reputation dynamics . . . . .            | 99         |
| 4.1.1    | Introduction and literature review . . . . .                                | 100        |
| 4.1.2    | The model . . . . .                                                         | 103        |
| 4.1.3    | Graphical analyses . . . . .                                                | 108        |
| 4.1.4    | Conclusion and discussion . . . . .                                         | 113        |
| 4.2      | (Non)Cooperative climate engineering with a regime shift . . . . .          | 115        |
| 4.2.1    | Introduction . . . . .                                                      | 116        |
| 4.2.2    | The model . . . . .                                                         | 119        |
| 4.2.3    | Numerical analysis . . . . .                                                | 121        |
| 4.2.4    | Conclusion and discussion . . . . .                                         | 132        |
|          | <b>Bibliography</b>                                                         | <b>135</b> |

---

|                                                          |            |
|----------------------------------------------------------|------------|
| <b>List of figures</b>                                   | <b>144</b> |
| <b>List of tables</b>                                    | <b>148</b> |
| <b>German abstract</b>                                   | <b>150</b> |
| <b>Appendices</b>                                        | <b>152</b> |
| 2A Ad Chapter 2.3.4: Competition . . . . .               | 152        |
| 2B Ad Chapter 2.4.1: Cartel solution . . . . .           | 156        |
| 2C Ad Chapter 2.4.2: Stability of fixed points . . . . . | 157        |
| 2D Ad Chapter 2.4.3: Feedback representation . . . . .   | 159        |
| 3A Ad Chapter 3.0.0: Online appendix . . . . .           | 160        |
| 4A Ad Chapter 4.1.1: Cheating examples . . . . .         | 166        |
| Curriculum Vitae . . . . .                               | 169        |



# Introduction

This cumulative thesis contains three articles and two notes in reverse chronological order. The main theme of all articles lies in the exploration of dynamic decision making problems in industrial-, energy- and environmental management and represents the output while employed at the same-named chair of Prof. Wirl. Although each chapter is devoted accordingly (industry, energy and environment), special attention has been paid to the analysis of dynamic interaction under differing industrial organizations, in fact, all but the note in chapter 4.1, is comprised of applied differential games, studying the impacts of certain policies to particular IO's.

The (chronologically)<sup>0</sup> first research question addresses the role of strategic behaviour in response to geoengineering, a newly proposed technology to tackle the problems resulting from climate change. At this point in time, the most promising proposition includes the injection of sulfur particles into the stratosphere in order to reduce Earth's albedo, i.e., to marginally increase the amount of sunlight reflected back to Space. This undertaking falls under the category of Solar Radiation Management and is, of course, no panacea and brings about many other issues, including ones resulting from strategic interaction of sovereign countries. The topic is highly polarizing and thus we propose tariffs, i.e., an economic sanction mechanism, as one avenue which might be made use of as a result of employed geoengineering (although others consider much more vigorous efforts, including potential warfare). Hence, in chapter 3 we examine the incentive issues arising in a two-country trade model, where actors also have access to the global thermostat, i.e., they have the ability to change the, dynamically evolving, temperature by the use of geoengineering. We find that in a world where some are decidedly averse to such technologies, i.e., where negative externalities exist (perceived or real), tariffs might be put into place in order to reduce the incentive to engage with geoengineering in the first place. Therefore, we think it is important to include potential effects on trade relationships to a comprehensive discussion on the impacts of climate engineering. A copy of this chapter has been published in Eigruber & Wirl (2018): Climate Engineering

---

<sup>0</sup>The actual structure of the thesis is laid out afterwards.

in an Interconnected World: The Role of Tariffs in *Dynamic Games and Applications*, 8, 573-587. Of course, as time goes on and progress on the climate front continues to be weak, these issues only increase in importance. However, after having observed some of the recent decisions of world leaders, even we came to question whether increased tariffs are indeed (perceived to be) a punishment. Tendencies towards nationalism and protectionism can be observed around the world and it will be interesting which role suboptimal technical solutions have, as temperature continues to increase. Indeed, as many researchers warn, temperatures might not increase steadily, i.e., evidence of tipping points in Earth's climate system exists. How may one deal with this issue? The note in chapter 4.2 sheds some light on how decision makers might interact in a system which is prone to tipping. We contribute to the literature by considering strategic interactions (in a weak sense, i.e., open loop) and find that, even small asymmetries among countries, e.g., in the availability of geoengineering, might induce preferences for drastically different long run outcomes. Of course, it would be interesting to also identify feedback strategies, however, we were not able to make progress in that direction. These two articles (chapter 3 and 4.2) are dedicated to the environmental part of this thesis.

Next, we wanted to understand and model some of the real world corporate cheating scandals we have been observing in the marketplace. Chapter 1, as well as the note in chapter 4.1, explore how firms may choose to trade off some of their reputational capital for increased profits, by making claims that are hard to disprove, but nonetheless incorrect. Imperfect credence goods and some experience goods fall into this category, where it is hard and costly to find out whether the seller has been cheating. Of course, the recent scandal of Volkswagen comes to mind, but we also report other examples in the respective chapters. The traditional literature in this field has primarily (and mostly correctly) focused on the efforts made by firms to better their image, i.e., to invest in advertising in order to increase brand image or goodwill. In chapter 1 we turn these dynamics upside down and study how firms operate in an environment where they can cheat – which yields short term profits but long term reputational damages, as corporate wrongdoing will be uncovered over time. While chapter 4.1 deals with a control problem of a monopoly facing nonlinear loss in reputation after disclosed cheating,<sup>0</sup> chapter 1 targets the multi-player case, i.e., the corresponding (linear quadratic)

---

<sup>0</sup>... using the same dynamic as the note in 4.2, hence chapter 4: notes on applications of the shallow lake dynamic.

differential game. We compare different market (monopoly, cartel, oligopoly) and information (open loop vs. feedback strategies) structures and find that while cheating is largest in an open loop setting, the contrary is the case for the Markov perfect equilibrium. That is, lacking the knowledge about the evolution of the state (in this case reputation) leads to overshooting. Hence, revealing the state, e.g., by more regular inspections, naturally decreases the amount of cheating being done – even if these inspections are economically inconsequential. A copy of this chapter has been published in Eigruber & Wirl (2019): Cheating as a dynamic Marketing Strategy in Monopoly, Cartel and Duopoly in *Central European Journal of Operations Research*, 1-18. Although all articles are somewhat focusing on dynamic industrial issues we devote chapter 1 and note 4.1 to the industrial part of the thesis.

And finally, chapter 2 concludes by addressing the question how firms, in particular solar photovoltaic manufacturers, set up their production paths when they not only learn from own output (i.e., classic learning by doing) but also benefit from industry spillovers (positive externalities). We again analyze the outcome under different IO's and find that small markets, composed of 'not too many' competing firms are optimal from the standpoint of social welfare. Such constellations outperform non-competitive markets, like a monopoly or cartel, on the basis of increased output and lower prices while also being superior to markets composed of 'too many' actors. The reason lies in the duplication of costs per added firm and, indeed, solar-PV industries in the real world are neither monopolistic, nor competitive and have been subject to many bankruptcies over time (e.g., in Germany). Additionally, in this paper we also identify a technical anomaly, in a sense that we find multiple stable linear Markov perfect strategies in the LQ solar-PV game (one too many). This article is dedicated to the energy part and therefore closes this thesis thematically.

This thesis is structured as follows. The first chapter deals with a group of firms competing not only in production, but also in exaggerated claims made to customers. Afterwards, in chapter 2 we study firms – subject to internal as well as external sources of learning – competing in the solar-PV industry. Chapter 3 examines some not so far explored issues of climate engineering, namely the option to impose a tariff on goods produced by firms located in a country which is engaging in climate-interfering techniques. These chapters are devoted to the industrial-, energy-, and environmental management parts, respectively. Finally, chapter

4 concludes this thesis by putting forward two novel uses of the shallow lake game.

All figures/tables/equations/assumptions/remarks and results are numbered according to their respective section number.

### Scientific state of each article

- Chapter 1**     Eigruber, M., & Wirl, F. (2019), Cheating as a dynamic Marketing Strategy in Monopoly, Cartel and Duopoly. *Central European Journal of Operations Research*, 1-18.
- Chapter 2**     Eigruber, M., & Wirl, F. (2019). *Market Equilibrium Strategies under Learning by Doing Spillovers*. Advanced Working Paper.
- Chapter 3**     Eigruber, M., & Wirl, F. (2018), Climate Engineering in an Interconnected World: The Role of Tariffs. *Dynamic Games and Applications*, 8, 573-587.
- Chapter 4.1**   Eigruber, M. (2018), *Corporate Cheating under Nonlinear Reputation Dynamics*. Contribution to ORCOS 2018, Vienna. Working paper.
- Chapter 4.2**   Eigruber, M. (2019), *(Non)Cooperative Climate Engineering with a Regime Shift*. Advanced working paper.

# Chapter 1

## Cheating as a dynamic marketing strategy in monopoly, cartel and duopoly

### Abstract

In 2015 it was discovered that Volkswagen had manipulated the exhaust emissions of its (diesel) cars. Since then, numerous other automotive car manufacturers were strongly suspected to violate against the same emission standards. This paper investigates how and why firms (monopoly, cartel and duopoly) engage in cheating, more precisely, promising attributes that are actually not part of the product. Firms make claims in order to better market their product but risk damaging their future reputation. The upshot of the paper is the stark difference between open loop and Markov perfect oligopolistic equilibrium outcomes. More precisely, the latter mitigates cheating substantially, even below the levels attained by monopolies and cartels (unless consumers have a very short memory), which is contrary to the outcome in the limiting static version of the game. Therefore, revealing the true state (e.g., by mandating strict inspections) could force firms to use this information and play in Markov instead of open loop strategies.

**Keywords:** Cheating, Differential Game, Competition, Dieselgate.

**JEL index:** C71, C72, D25, D43, L10, L14, M30.

## 1.1 Introduction

Economic agents often face strong incentives to engage in immoral/non-conforming/deceptive or fraudulent behaviour in the marketplace. Such decisions to cheat may be made at small-scale bilateral trade relationships but can also be made at the highest corporate level, as the recent Volkswagen emissions scandal has shown. In this case, one of the leading automotive manufacturing companies in the world, the Volkswagen Group, misled customers by manipulating emissions, in particular of nitrogen oxides (NOx), of their cars during testing. Dubbed, *inter alia*, ‘diesel dupe’ (Hotten 2015), ‘diesel fraud’ (Smith & Parloff 2016), or ‘Dieselgate’ (Gorzelany 2015) by the media, and referred to as ‘diesel issue’ by the company (Volkswagen AG 2015) this cheating scandal had substantial effects not only on the perpetrator itself, but also on industry standards as well as other competitors in the marketplace (who, after the scandal was made public at the end of September 2015 also became the subject of investigations due to suspected cheating). Fig. 1.1.1 compares the evolution of cumulative fractional changes in the share prices for the Volkswagen AG with its main German competitors (BMW AG, Daimler AG) during 2012 - 2018.

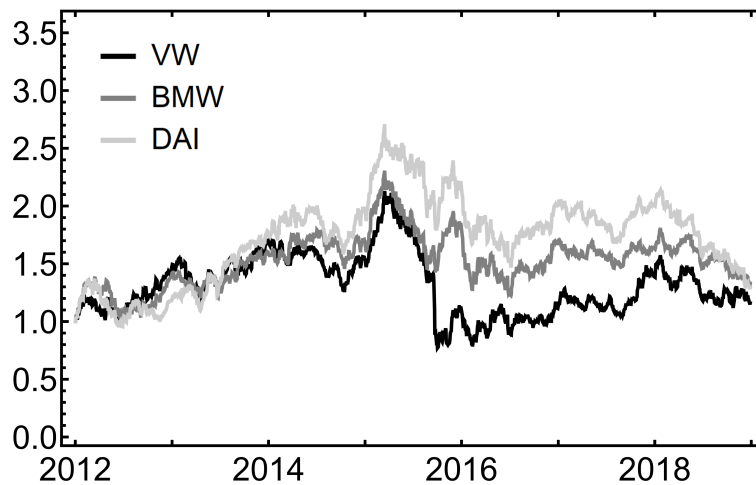


**Figure 1.1.1:** Share prices of the three main German carmakers.

The diesel scandal had a substantial (at least short term) impact on the whole industry. Of course, this is not surprising since the current stock price approximates the present value

of future cash flows, which are influenced by information on the company and its assets, including consumers' goodwill and trust. The combination of negative publicity due to the disclosed scandal and legal consequences, with their inherent uncertain outcomes, affected the stock price substantially. However, despite this major event the industry has rebounded surprisingly strongly, as recent performance indicators show (2017 was Volkswagen's best year in terms of sales, only to be topped by the numbers of 2018, Volkswagen AG 2017, Volkswagen AG 2018). Still, since firms knowingly engaged in inappropriate or illegal activities towards their customers by promising non-existent product features, their so far excellent brand's reputation is being tarred over time.

So why do companies occasionally trade off reputational capital for increased profits? Corporate reputation can be defined as "... a stakeholder's overall evaluation of a company over time. This evaluation is based on the stakeholder's direct experiences with the company, any other form of communication and symbolism that provides information about the firm's actions and/or a comparison with the actions of other leading rivals" (Gotsi & Wilson 2001, p. 29). It is therefore a critical measure of trust for agents as well as a valuable intangible asset for corporations (Coombs 2010, 2015; Haskel & Westlake 2017). Reputation is also rivalry, since it can be represented as a continuum, where each subject can be compared and ultimately ranked to another one. Generally, firms recognize this fact and focus on taking actions that increase their respective image, which is well researched in the literature on advertising for goodwill (Nerlove & Arrow 1962, Fershtman 1984, Cellini & Lambertini 2003 and many others). However, just like most other resources, reputational capital can not only be built up, but can also be consumed and restored. Stakeholders have certain expectations with regard to the organization's actions, which goes hand in hand with their perception of the firm. Clearly, organizational reputation suffers when such expectations are not fulfilled (Coombs 2015). This link is mediated by how much the organization is perceived to be responsible for a scandal. According to attribution theory, intentional corporate misbehavior leading to preventable scandals with substantial damages evoke some of the strongest perceptions of blame and responsibility, which results in a high potential for reputational damage (Coombs & Holladay 1996, Weiner 2012). Furthermore, one company's crisis may spill over to the entire industry (compare Fig. 1.1.1). Such externalities may be positive or negative in nature. It is possible that exposed cheating weakens the cheater's market position and hence

favors his competitors. On the other hand, detected cheating could lead deceived victims to put whole industries under general suspicion, or at least increased scrutiny and, hence, to negative externalities. The case of Volkswagen highlights the Glauconian dilemma in business and here in particular with reference to the issue of Corporate Social Responsibility. Plato reports in *The Republic* how Glaucon challenges Socrates' view by the thought experiment in which a ring (Gyges) makes the person invisible (the following is quoted from Haidt 2012), "... no one, it seems, would be so incorruptible that he would stay on the path of justice or away of other people's property .... Rather his actions would be in no way different from those of an unjust person and both would follow the same path." In short, having the choice between doing good even at the risk of being considered a scoundrel, or appearing virtuous but actually being a scoundrel, most economic agents, and firms in particular, would presumably have to opt for the second. The contribution of this paper is to investigate the incentives for firms to cheat and in particular how competition induces or mitigates cheating. Of course, intensity of competition matters, however, the evidence in which direction this effect holds is mixed. Cartwright & Menezes (2014) report that intermediate levels of competition induce the highest level of cheating (misreporting rate in their study), whereas Kopel & Lamantia (2018) roughly report the opposite.<sup>1</sup> Schwieren & Weichselbaumer (2010) argue that competitive pressures increase nonethical conduct (also compare Shleifer 2004). In this paper we emphasize the dynamics of firm cheating, accounting for the trust or mistrust consumers have about a particular firm and its relation to its competitors. The states of trust depend on past actions, deceits or honesty, of the firms. In short, we propose a differential game setup to study and compare outcomes under different market and information structures.

A surprising finding of this paper is that dynamic strategic oligopolistic interactions (i.e., a Markov perfect equilibrium) mitigate cheating and in fact lead to a ranking opposite to the one obtained from the corresponding static game. Particularly open loop outcomes lead to excessive cheating, which could be somewhat mitigated by regular (even inconsequential) inspections, such that firms can adjust their policies and use the more honest Markov strategies.

The paper is organized as follows: the next section introduces the model in all its versions for the respective market structures (monopoly, cartel and duopoly). In section 1.3 we derive

---

<sup>1</sup>In terms of long run equilibrium share (they use an evolutionary setup) of companies which also include consumer surplus in their objective.

the solutions for the static game. Afterwards, section 1.4 is devoted to the dynamics, considering differing information structures. Section 1.5 compares the derived results. Finally, we conclude and offer directions for future research.

## 1.2 Model

What is crucial if a firm  $i = 1, 2$  cheats, greenwashes or, in general, exaggerates the quality of its product? Actual quality,  $q_0$ , is normalized to zero, while  $q_i > q_0 = 0$  denotes the extent to which firm  $i$  exaggerates the quality of its product. This generates a benefit on its own for the firm. However, this immediate benefit requires that cheating is not observable, at least not easily and directly, which holds in particular for experience (Nelson 1974) and credence goods (Darby & Karni 1973, Dulleck & Kerschbamer 2006). Although consumers can judge the quality of an experience good once bought (e.g., the food served at an unknown restaurant or a movie in the cinema), this is not necessarily the case with a credence good. Here, consumers may also be unable to identify the properties of the good ex post, i.e., after purchase. Hence, for these types of goods it is often impossible, or at least very costly, to uncover cheating. This applies to many goods like the claims of fluoride in toothpaste, organic food served in restaurants (the horse meat scandal of 2013 also comes to mind), to Volkswagen claiming to meet all emission standards (even by saving the costs for the more expensive SCR-catalytic converter, ‘BLUE’, that other companies installed) and to corporate social (ir)responsibility (Langevoort 2002, Lin-Hi & Müller 2013) in general.<sup>2</sup> Indeed, even claims that are known to be exaggerated matter. For example, it is common knowledge that the officially stated vehicle fuel consumption numbers will be exceeded by far in practice. Nevertheless, the relative position of claims still plays a role if car A is claimed to be more efficient than the competitor B, even though on paper the absolute numbers are implausible. However, cheating is not for free. Stated exaggerated claims can be costly because of the necessity of a camouflage (e.g., via software) or negligible if restricted to purely unverifiable technical claims. Misbehaviour may also cause moral costs for decision makers, including risk of fines and even of imprisonment. For instance, the stakes in the Dieselgate scandal are enormous. On the one hand, additional revenue streams from selling products with

---

<sup>2</sup>Also compare BP’s claims and its disasters, first in a refinery in Texas and then also ‘Deepwater Horizon’.

exaggerated quality (lemons) have been generated for many years. On the other hand, the costs range from allowing/ordering a software to fool inspectors, fines (recently, one billion dollars which Volkswagen accepted), full compensation for the cars sold in the US, costly upgrades and still pending law suits in Europe, to terms in jail (that Volkswagen's managers face in the United States).

While cheating enhances (*ceteris paribus*) current sales and revenues this comes at the expense that a firm's future reputation is tarred. We let negative reputation, a firm's 'badwill' so to speak ( $B_i$ ), evolve dynamically,

$$\dot{B}_i = q_i - \delta B_i, \quad i = 1, 2, \quad (1.2.1)$$

i.e., badwill  $B$  is an exponentially weighted (by the parameter  $\delta$ ,  $1/\delta$  being the time constant) average of past cheating. Higher values of  $B_i$  lower firm  $i$ 's profit ( $\pi^i$ ) but exaggerated claims ( $q_i$ ) increase it. Our setup, in which truth is uncovered only slowly, seems to be particularly applicable to (imperfect) credence goods, so that the firms can hide truth only temporarily but not forever ('you can fool all of the people some of the time, you can fool some of the people all of the time, but you can't fool all of the people all of the time', Abraham Lincoln). Not only the firm's own actions and standing matter but also those of its competitors. Presumably, the relative position to the competitor  $j$  matters in both cases – the immediate gain from cheating as well as the reputational damage. Yet, instead of setting up firm  $i$ 's,

$$\text{payoff} = \text{benefit (cheating)} - \text{loss (due to badwill)} - \text{costs (cheating)},$$

in a more or less ad hoc manner, we motivate the firms' payoffs from an underlying (linear) demand system. As a consequence, firms compete in cheating and production. We consider two market segments of identical size and their inverse demands, or respectively, their marginal willingness to pay,  $p_i$ ,  $i = 1, 2$ ,

$$p_1 = 1 + aq_1 - bq_2 - \alpha B_1 + \beta B_2 - x_1 - x_2, \quad (1.2.2)$$

$$p_2 = 1 + aq_2 - bq_1 - \alpha B_2 + \beta B_1 - x_1 - x_2, \quad (1.2.3)$$

where  $x_i$  denotes the quantity of firm  $i$ 's product and  $a, b, \alpha$  and  $\beta$  are positive constants.

Thus, willingness to pay for an additional unit decreases with respect to total supply ( $x_1 + x_2$ ). Absent cheating, the maximum willingness to pay is normalized to unity. Technically, the two goods are homogeneous, however, from the consumers' point of view they differ according to their respective perceived quality. More precisely, the difference between the two segments is that consumers from segment 1 are more susceptible to the claims and the reputation of firm 1 while consumers from segment 2 have the same symmetric feelings towards firm 2. Exaggerated (false) claims of the 'own' good are beneficial, yet similar claims of the competitor are harmful. It seems plausible to assume that the own effect dominates,

$$a > b. \quad (1.2.4)$$

A firm's negative reputation,  $B_i > 0$ , lowers consumers' willingness to pay as the firm's claims are mistrusted, while a negative reputation of the competitor is beneficial; again own effects dominate,  $\alpha > \beta$ . Of course, one may allow for  $\beta < 0$ , as increased badwill of a firm can have a negative spillover on the competing firm since the entire industry faces increasing distrust. In order to render cheating demand enhancing and thus at least potentially profitable, the short run gain must exceed the associated (marginal) long run damage. More precisely, since cheating by one (infinitesimal) unit creates the benefit  $a$  but implies the (stationary) damage  $\alpha/\delta$ , or respectively after accounting for discounting ( $r$ ), we require

$$a > \frac{\alpha}{r + \delta}, \quad (1.2.5)$$

and similarly for the cross effects. In particular, for a monopoly (or cartel) selling two brands we need,

$$b > \frac{\beta}{r + \delta} \wedge a - b > \frac{\alpha - \beta}{r + \delta} > 0. \quad (1.2.6)$$

These inequalities must hold strictly as the costs of cheating, for simplicity quadratic,

$$\frac{c}{2} q_i^2, \quad c > 0, \quad (1.2.7)$$

must be covered too.

Summarizing, if firms maximize their net present value of profits over an infinite horizon,

$$\max_{q_i, x_i} \int_0^\infty e^{-rt} \pi^i dt, \quad i = 1, 2, \quad (1.2.8)$$

with

$$\pi^i = p_i x_i - \frac{c}{2} q_i^2, \quad (1.2.9)$$

then they must solve the above optimizations subject to the evolution of the (two) states,  $B_i$ ,  $i = 1, 2$ , accounting for competitor's strategies; production costs are normalized to zero.

**Remark 1.2.1:** *In this setup production and cheating are chosen simultaneously by both players and both move simultaneously in each period  $t$ . An alternative and also plausible assumption is that production ( $x_i$ ) takes place after reputations (the states  $B_i$ ) and the degrees of cheating (actions, controls,  $q_i$ ) are determined. Computing the Nash equilibrium of the intraperiod levels of production for given cheating and badwill would result in,*

$$x_i = \frac{1 + a(2q_i - q_j) - b(2q_j - q_i) - \alpha(2B_i - B_j) - \beta(B_i - 2B_j)}{3},$$

which implies the reduced form profit,

$$\pi^i = \frac{(1 + (2a + b)q_i - (a + 2b)q_j - (2\alpha + \beta)B_i + (\alpha + 2\beta)B_j)^2}{9} - \frac{c}{2} q_i^2,$$

of firm  $i$ , with cheating as then the only control. This objective includes all linear, mixed and quadratic terms between firms' actions and states. Of course, in this case a monopoly would keep the flexibility over both controls, cheating and production, since a sequential optimization would be suboptimal.

Returning to our simultaneous setup, we consider two different monopolistic solutions: a single-brand monopoly and a monopoly with different brands, or equivalently, a cartel of different firms. Selling different brands allows to benefit from the cross effects, even if symmetric strategies, to which the analysis is limited, are applied. Therefore, a monopoly or

cartel can be modelled in two ways, even if assuming a symmetric allocation, i.e.,

$$q = q_1 = q_2, \quad x = x_1 = x_2,$$

and thus,

$$B = B_1 = B_2.$$

1. A single-brand monopoly cannot benefit/lose from cross effects, thus,

$$b = \beta = 0.$$

2. A multi-brand monopoly or cartel, such that the cross effects matter, i.e.,  $b \neq 0$  and  $\beta \neq 0$  and both are presumably positive.

Therefore, a higher willingness to pay results if the monopoly commits to a single brand if the current loss induced on the other brand ( $b > 0$ ), is larger than the marginal gain due to the implied stationary badwill ( $\beta/\delta$ ), and again ( $\beta/(r + \delta)$ ) on a net present value basis; this is analogue to assumption (1.2.6). Combining this with the assumption that own effects dominate, inequality (1.2.4), introducing different brands will lower the willingness to pay and will lower cheating due to the overall negative cross effects. Therefore, more brands are, if at all, only preferable for the opposite inequalities, and will, surprisingly, result in less cheating.

**Remark 1.2.2:** *The model could also be interpreted in a second way: while (almost) all models take product differentiation as given, one may see ‘cheating’ as an investment in current product differentiation and then the state as the reputation of this particular property (after reversing the sign), e.g., the carmaker Volvo produces(d) comparably safe cars and thereby acquired a reputation of building cars with protective properties.*

### 1.3 Solution of the static game

Unfortunately, an analytical solution of the dynamic setup is either too cumbersome to draw analytical conclusions (in the case of a monopoly and a cartel) or not existing (at least, we could not obtain a closed-form solution). Therefore, we first consider the outcome using the steady state relationships and then use carefully calibrated relations to draw some inferences how competition, actual or inter-brand, (as in the case of different brands sold by a monopolist) affects the honesty of the firms, transiently and in the long run. Computing the symmetric equilibrium choices of  $x_i$  and  $B_i$ , substituting for  $q_i = \delta B_i$ , i.e., the steady states of (1.2.1), yields the following equilibria.

**Result 1.3.1.** *For the single-brand monopoly (superscript  $m$ ),*

$$B^m = \frac{a\delta - \alpha}{4c\delta^2 - (a\delta - \alpha)^2}, \quad (1.3.1)$$

$$x^m = \frac{c\delta^2}{4c\delta^2 - (a\delta - \alpha)^2}, \quad (1.3.2)$$

*for the cartel, or equivalently a two-brands monopoly (superscript  $c$ ),*

$$B^c = \frac{(a\delta - \alpha) - (b\delta - \beta)}{4c\delta^2 - ((a\delta - \alpha) - (b\delta - \beta))^2}, \quad (1.3.3)$$

$$x^c = \frac{c\delta^2}{4c\delta^2 - ((a\delta - \alpha) - (b\delta - \beta))^2}, \quad (1.3.4)$$

*and for a duopoly (superscript  $d$ ),*

$$B^d = \frac{a\delta - \alpha}{3c\delta^2 - (a\delta - \alpha)((a\delta - \alpha) - (b\delta - \beta))^2}, \quad (1.3.5)$$

$$x^d = \frac{c\delta^2}{3c\delta^2 - (a\delta - \alpha)((a\delta - \alpha) - (b\delta - \beta))^2}. \quad (1.3.6)$$

*A single-brand monopoly engages in more cheating than a monopoly selling two brands,*

$$B^m > B^c, \quad (1.3.7)$$

and a duopoly cheats even more than a single-brand monopoly,

$$B^d > B^m \iff a < \frac{c\delta^2 + \alpha(b\delta - \beta)}{\delta(b\delta - \beta)}, \quad (1.3.8)$$

i.e., except high immediate gains from cheating (large  $a$ ).

**Proof:** The first-order-conditions (FOC's) of the cartel (assuming symmetry) result from equating the marginal costs of cheating to the induced marginal revenue, and production results from equating marginal revenues to the marginal production costs (assumed to be zero),

$$\begin{aligned} c\delta^2 B &= ((a\delta - \alpha) - (b\delta - \beta))x, \\ 1 + ((a\delta - \alpha) - (b\delta - \beta))B - 2x &= 0. \end{aligned}$$

Solving these two equations for  $(B, x)$  yields the claim  $(B^c, x^c)$  in (1.3.3) and (1.3.4). These conditions simplify for the monopoly,  $b = \beta = 0$ , to

$$\begin{aligned} c\delta^2 B &= (a\delta - \alpha)x, \\ 1 + (a\delta - \alpha)B - 2x &= 0, \end{aligned}$$

and thus imply  $(B^m, x^m)$  as reported in (1.3.1) and (1.3.2). The FOC's of the duopoly (again after imposing symmetry),

$$\begin{aligned} c\delta^2 B &= (a\delta - \alpha)x, \\ 1 + ((a\delta - \alpha) - (b\delta - \beta))B - 3x &= 0, \end{aligned}$$

imply the same *marginal revenue equals marginal costs condition* for cheating as the monopoly. Solving those two equations verifies the claim  $(B^d, x^d)$  in (1.3.5) and (1.3.6).

Compared to the cartel, the monopoly equates its marginal costs to higher marginal revenues from cheating (keeping output the same) and, similarly, from production (now keeping cheating constant). Directly, the numerator in the monopoly's state, (1.3.1), is larger than the one of the cartel, (1.3.3), yet the denominator of (1.3.1) is smaller since a larger number is

subtracted from the term  $4c\delta^2$ . This is the case under (1.2.6), hence,  $B^m > B^c$ , as reported in (1.3.7). A comparison of the numerators of (1.3.1) and (1.3.5) yields

$$a\delta - \alpha > (a - b)\delta - (\alpha - \beta) \iff b\delta > \beta,$$

i.e., if the immediate effect is dominating the intertemporal one, which is in accordance to our assumptions in (1.2.6). Now, comparing the denominators of (1.3.1) and (1.3.5) yields,

$$\begin{aligned} 4c\delta^2 - (\alpha - a\delta)^2 &< 4c\delta^2 - (\alpha - \beta - (a - b)\delta)^2, \\ \iff (\beta - b\delta)(\beta - b\delta + 2a\delta - 2\alpha) &< 0, \end{aligned}$$

and the claim is equivalent to that this difference is negative, as  $B^d$  must have the smaller denominator. Solving this inequality for  $a$  verifies that  $B^d > B^m$  and yields the critical value reported in (1.3.8). QED

Cheating,  $B > 0$ , results for a duopoly and a single-brand monopoly only if the intuitively derived condition in assumption (1.2.5) holds. For a monopoly selling two brands, the analogous assumption (1.2.6) is necessary for cheating. The monopoly cheats more than the cartel (or a multi-brand monopoly), because the associated marginal revenue from cheating is diminished for a cartel due the cannibalization by the different brands (the cross effect). The duopoly's high rate of cheating is a consequence of the larger production, because cheating is determined by a condition identical to the monopoly's and the marginal revenues are scaled by the output.

## 1.4 Intertemporal equilibria

### 1.4.1 Monopoly and cartel

A symmetric cooperative equilibrium eliminates the gain from cheating that results from relative positions. This mitigates but does not eliminate the incentive for cheating. In spite of the two cases, we can restrict the analysis to the more general case of the cartel by solving

the Hamilton-Jacobi-Bellman (HJB) equation for the value function  $J$ ,

$$rJ(B) = \max_{q_i, x_i} \left\{ \sum_{i=1}^2 \left[ p_i x_i - \frac{c}{2} q_i^2 + J_{B_i}(q_i - \delta B_i) \right] \right\}, \quad (1.4.1)$$

and obtain the monopoly solution by setting  $\beta = b = 0$ . The first-order-conditions of the maximization on the right side of (1.4.1),

$$\begin{aligned} ax_1 - bx_2 + J_{B_1} - cq_1 &= 0, \\ 1 + aq_1 - bq_2 - 2(x_1 + x_2) - \alpha B_1 + \beta B_2 &= 0, \end{aligned}$$

imply for the strategies of the monopoly (symmetry allows to confine the analysis to  $q = q_1 = q_2$  and  $x = x_1 = x_2$ ),

$$q^m = \frac{a(\alpha B - 1) - 4J'}{a^2 - 4c}, \quad (1.4.2)$$

$$x^m = \frac{c(\alpha B - 1) - aJ'}{a^2 - 4c}. \quad (1.4.3)$$

as well as the cartel,

$$q^c = \frac{(a - b)[(\alpha - \beta)B - 1] - 4J'}{(a - b)^2 - 4c}, \quad (1.4.4)$$

$$x^c = \frac{(\alpha - \beta)cB - c - (a - b)J'}{(a - b)^2 - 4c}. \quad (1.4.5)$$

i.e., optimal cheating from a myopic perspective is adjusted for the intertemporal effect ( $J'$ ) of how cheating affects the future badwill. Although a closed-form solution exists, after all only a quadratic polynomial must be solved after proper substitutions and rearrangements when comparing coefficients of the Hamilton-Jacobi-Bellman equation for a quadratic guess of  $J$ , the result is very cumbersome and thus suppressed.

### 1.4.2 Duopoly

#### Open loop

The open loop equilibrium results if both firms solve their intertemporal optimization of (1.2.8) subject to (1.2.1), where the strategy  $\{q_i(t), t \in [0, \infty)\}$  is a best response to the strategy of the opponent  $\{q_j(t), t \in [0, \infty)\}$  and vice versa. Moreover, due to symmetry we have  $q_i = q_j$  in any Nash equilibrium. This setup ignores how the state, in particular of the competitor, evolves and is thus not subgame perfect if the opponent deviates, for whatever reason, from his optimal policy. However, the open loop strategy can make sense in this case, at least temporarily, since the badwill as well as cheating of the competitor may not be observable – or only occasionally, such as at the beginning of the game. Indeed, Volkswagen and presumably others, were able to hide their policy of cheating for quite some time. Setting up the Hamiltonian of player  $i$ ,

$$H^i = \pi^i + \lambda^i (q_i - \delta B_i) + \mu^i (q_j - \delta B_j), \quad i \neq j,$$

the first-order-conditions for interior solutions are,

$$\begin{aligned} ax_i - cq_i + \lambda^i &= 0, \\ 1 + aq_i - bq_j - 2x_i - x_j - \alpha B_i + \beta B_j &= 0, \\ \dot{\lambda}^i &= (r + \delta) \lambda^i + \alpha x_i, \quad \lim_{t \rightarrow \infty} e^{-rt} \lambda^i B^i = 0, \\ \dot{\mu}^i &= (r + \delta) \mu^i - \beta x_i, \quad \lim_{t \rightarrow \infty} e^{-rt} \mu^i B^j = 0. \end{aligned}$$

The maximization of the Hamiltonian is independent of the competitor's action and state and moreover, the second costate vanishes,  $\mu^i = 0$  for  $i = 1, 2$  due to the corresponding transversality condition. Assuming a symmetric equilibrium (thus dropping the index), the open loop strategies are given by,

$$q^o = \frac{(\alpha - \beta) aB - a - 3\lambda}{a^2 - ab - 3c}, \quad (1.4.6)$$

$$x^o = \frac{(\alpha - \beta) cB - c - (a + b) \lambda}{a^2 - ab - 3c}. \quad (1.4.7)$$

The symmetric, intertemporal open loop Nash equilibrium is characterized by the following pair of differential equations,

$$\begin{aligned}\dot{B} &= \frac{(\alpha - \beta) a B - a - 3\lambda}{a^2 - ab - 3c} - \delta B, \\ \dot{\lambda} &= (r + \delta) \lambda + \frac{(\alpha - \beta) \alpha c B - \alpha c - (a + b) \alpha \lambda}{a^2 - ab - 3c},\end{aligned}$$

which implies the steady state,

$$B_\infty^o = \frac{\alpha - a(r + \delta)}{A}, \quad \lambda_\infty^o = \frac{\alpha c \delta}{A}, \quad (1.4.8)$$

with

$$A = (\alpha - \beta + b\delta)(\alpha - a(r + \delta)) - \alpha a \delta + \delta(r + \delta)(a^2 - 3c).$$

### Markov perfection

Finally, we consider the possibility of state-contingent strategies as well. In contrast to the time-dependent strategies in the open loop setup, the duopolistic outcome is now characterized by linear Markov strategies. The resulting equilibrium is strongly time-consistent, more precisely, subgame perfect such that the strategies remain optimal even if a player took an erroneous move. The Markov strategies are the natural outcome if applying (continuous time) dynamic programming. Therefore, we set up the Hamilton-Jacobi-Bellman equation (see Dockner et al. 2000) for the value function of player  $i = 1$ ,

$$rV^1(B_1, B_2) = \max_{q_1, x_1} \{ \pi^i + V_{B_1}^1(q_1 - \delta B_1) + V_{B_2}^1(q_2 - \delta B_2) \}, \quad (1.4.9)$$

and analogously for player  $i = 2$ . The first-order-condition of the maximization of the right hand side for the instrument of cheating is,

$$cq_i = ax_i + V_{B_i}^i, \quad i = 1, 2, \quad (1.4.10)$$

i.e., the marginal cost of cheating must equal the marginal benefit, which consists of the induced additional revenues ( $ax_i$ ) minus the intertemporal loss from a damaged reputation,

captured by  $V_{B_i}^i < 0$ . The production choice follows from the familiar condition that marginal revenue must equal marginal cost (zero in our case),

$$1 + aq_i - bq_j - \alpha B_i + \beta B_j - 2x_i - x_j = 0.$$

As usual for LQ dynamic games, we guess a quadratic solution of the value functions,

$$V^1 = v_0 + v_1 B_1 + v_2 B_2 + \frac{1}{2} (v_{11} B_1^2 + 2v_{12} B_1 B_2 + v_{22} B_2^2), \quad (1.4.11)$$

$$V^2 = w_0 + w_1 B_1 + w_2 B_2 + \frac{1}{2} (w_{11} B_1^2 + 2w_{12} B_1 B_2 + w_{22} B_2^2) \quad (1.4.12)$$

with coefficients  $v_i, w_i \in \mathbb{R}$  and apply symmetry,

$$w_0 = v_0, w_1 = v_2, w_2 = v_1, w_{11} = v_{22}, w_{12} = v_{12}, w_{22} = v_{11},$$

such that ultimately only the coefficients of  $V^1$  must be determined. Yet, even exploiting all these conditions does not allow for a meaningful analytical representation of the Markov perfect equilibrium. Therefore, the following numerical examples have to substitute for this shortcoming.

## 1.5 Comparison

We base our analysis on the following reference case:

$$a = 0.2, b = 0.1, \alpha = 0.02, \beta = 0.01, c = 0.1, r = 0.05, \delta = 0.2. \quad (1.5.1)$$

We let firms discount at 5% per annum and the consumers' rate of forgetting/forgiving be given by 20%, which assumes a rather weak memory. Turning to the demand parameters, cheating by one unit increases a consumer's willingness to pay by  $a = 0.2$  (for the preferred good/brand), i.e., by 20% relative to the otherwise maximum willingness to pay (which has been normalized to unity). This increase in willingness to pay is reduced by  $b = 0.1$  if also the competitor exaggerates his claims by one unit, which is half of the own effect and in line with assumption (1.2.4). The long run effect due to the firm's bad reputation amounts to  $\alpha = 0.02$ ,

which satisfies assumption (1.2.5), and the effect from the competitor's badwill as half of that,  $\beta = 0.01$ . More precisely, the immediate effects of cheating are twice the long run effects, own and cross: cheating by one incremental unit triggers the gain  $a = 0.2$  (and respectively the loss  $b = 0.1$  from the competitor's cheating) but also the long run reputational loss  $\alpha/\delta = 0.1$  and  $\beta/\delta = 0.05$  if ignoring discounting and transient behavior. That is, substantial incentives for cheating are assumed. The parameters also satisfy inequality (1.2.6). Finally, the cost parameter implies for hypothetical cheating levels of  $1/2$ ,  $1$  and  $2$ , marginal cheating costs of  $0.05$ ,  $0.1$ , and  $0.2$ .

Starting with the monopoly and the reference case, Fig. 1.5.1a and b compare the monopolistic strategies of a single-brand with a multi-brand monopoly.

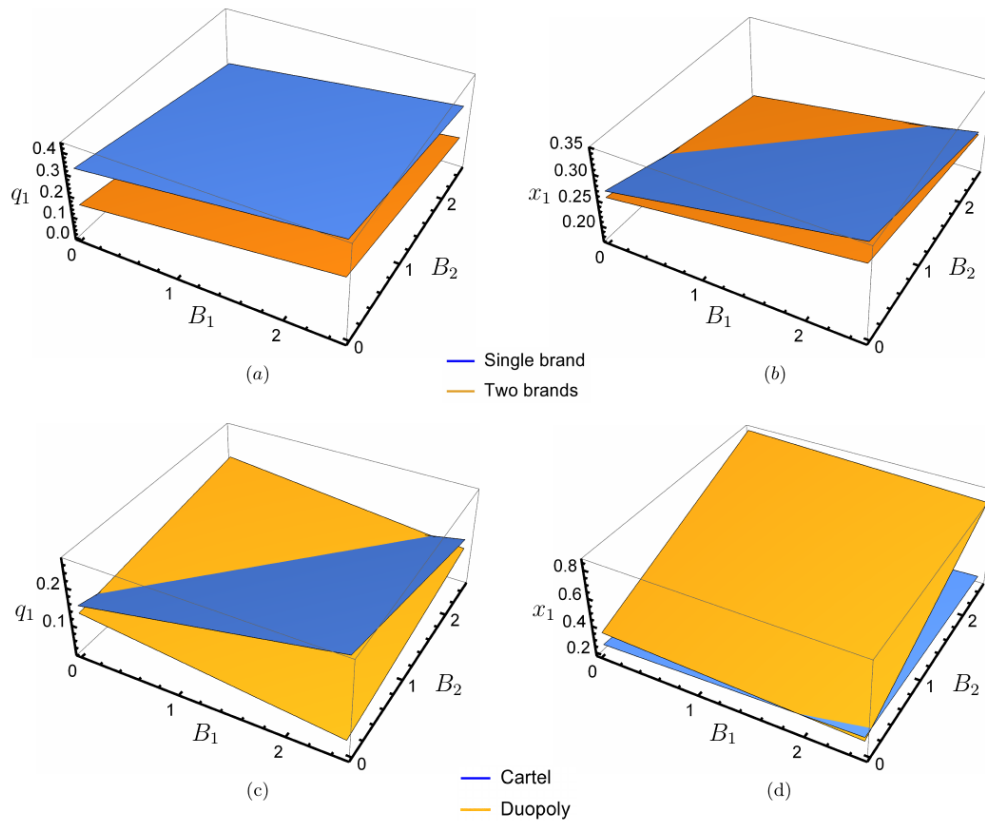


**Figure 1.5.1:** Strategies, cheating in (a) and (c) and production in (b) and (d), in state space.

This figure highlights two things: first, how much more cheating is employed by the single-brand monopoly because countereffects are missing. Second, both monopolistic strategies are

increasing in their own badwill, i.e., the worse the reputation the more the monopoly tries to compensate by exaggerating claims, but declining with respect to the opponent's state of badwill. This is clearly counterintuitive and we may just quote as an explanation a proverb (which rhymes in German) that one can proceed unabashedly once one's reputation is ruined. The same signs apply to the production strategy, and we note that both strategies (cheating and production) are fairly insensitive to changes in the state. Fig. 1.5.1c and d compare the cartel (multi-brand) and the oligopolistic strategies also in state space. In contrast to the monopoly, the oligopolistic strategy has the expected sign: cheating decreases with respect to own badwill but increases with respect to the opponent's. Table 1.5.1 compares the steady states of the reference case across the different scenarios, including the outcomes of the static game.

|            | Monopoly |      | Cartel |      | Duopoly |      |      |
|------------|----------|------|--------|------|---------|------|------|
|            | stat.    | dyn. | stat.  | dyn. | stat.   | dyn. |      |
|            |          |      |        |      |         | OL   | MP   |
| $q_\infty$ | 0.26     | 0.31 | 0.13   | 0.15 | 0.34    | 0.41 | 0.13 |
| $x_\infty$ | 0.26     | 0.26 | 0.25   | 0.25 | 0.34    | 0.34 | 0.34 |
| $p_\infty$ | 0.51     | 0.52 | 0.50   | 0.50 | 0.34    | 0.34 | 0.34 |
| $B_\infty$ | 1.28     | 1.55 | 0.63   | 0.76 | 1.69    | 2.04 | 0.64 |
| $NPV_i$    | 0.13     | 5.19 | 0.13   | 5.05 | 0.11    | 2.16 | 2.24 |

**Table 1.5.1:** Steady state comparison of the controls and states including total discounted utility.

The insensitivity of all strategies with respect to the actual reputation is confirmed by how little production changes due to the dynamic analysis. However, compared to the cartel, the monopoly's badwill increases and almost doubles, which rather shamefully exploits its monopoly position – not only with respect to output but also with respect to cheating. In contrast, the dynamic analysis of reputational effects in duopolistic competition shows a dramatic reduction – stationary badwill is cut to below one half of the static game, which is the opposite for a single-brand monopoly. In fact, the duopoly ends up in less cheating

than the monopoly (at least stationary and weighted over the evolution as the lowest level of badwill results). However, this positive characterization of competition is due to strong intertemporal strategic interactions (i.e., the linear Markov perfect equilibrium), as the open loop equilibrium of the dynamic game ends up at the highest level of badwill. The reason is that playing in open loop strategies ignores the evolution of the opponent's badwill, which in this case leads to excessive cheating.

**Result 1.5.1.** *Accounting for dynamics results in an ambiguous effect on the long run behaviour of the firm, depending on the market and information structure. For the monopoly as well as the cartel (marginally) more cheating results, whereas a more nuanced picture emerges for actual competition. To summarize:*

$$q_{dyn}^m > q_{stat}^m, \quad q_{dyn}^c > q_{stat}^c \quad \text{and} \quad q_{OL}^d > q_{stat}^d > q_{MP}^d.$$

That is, a dynamic consideration can turn the static results upside down if the firms play in the usually more aggressive Markov strategies. These findings are robust as the sensitivity analyses in Fig. 1.5.2, which compares stationary levels with respect to the model parameters around the reference case, document. Fig. 1.5.2a displays the evolution of the long run state for all market structures with respect to  $\delta$  and shows that the relation is non-monotonic. However, increasing  $\delta$  (i.e., consumers' memories become shorter and shorter), the dynamic solutions, including the Markov perfect one, approach the static ones. Therefore, the relation between monopoly and duopoly can be reversed at very high values of  $\delta$ .<sup>3</sup> While long run cheating is declining in the marginal effect of one's own state in all models (Fig. 1.5.2b), this is not true for the opponent's state (Fig. 1.5.2c). In the Markov perfect equilibrium cheating (as well as the state) is declining in  $\beta$ , which then also results in slightly less production due to a smaller inflated market size (Fig. 1.5.2d). These figures corroborate the conclusions from Fig. 1.5.1 and the previous results and further emphasize that Markovian strategies mitigate cheating. We note further that the monopoly's cheating (and also of the cartel over certain domains of low values of  $\delta$  and over significant spillovers  $\beta$ ) exceeds that of Markov perfect

<sup>3</sup>The rankings described in Result 1.3.1 and 1.5.1 hold not only for changes in  $\delta$  but also for large intervals around the reference case for all other parameters not shown here due to brevity.

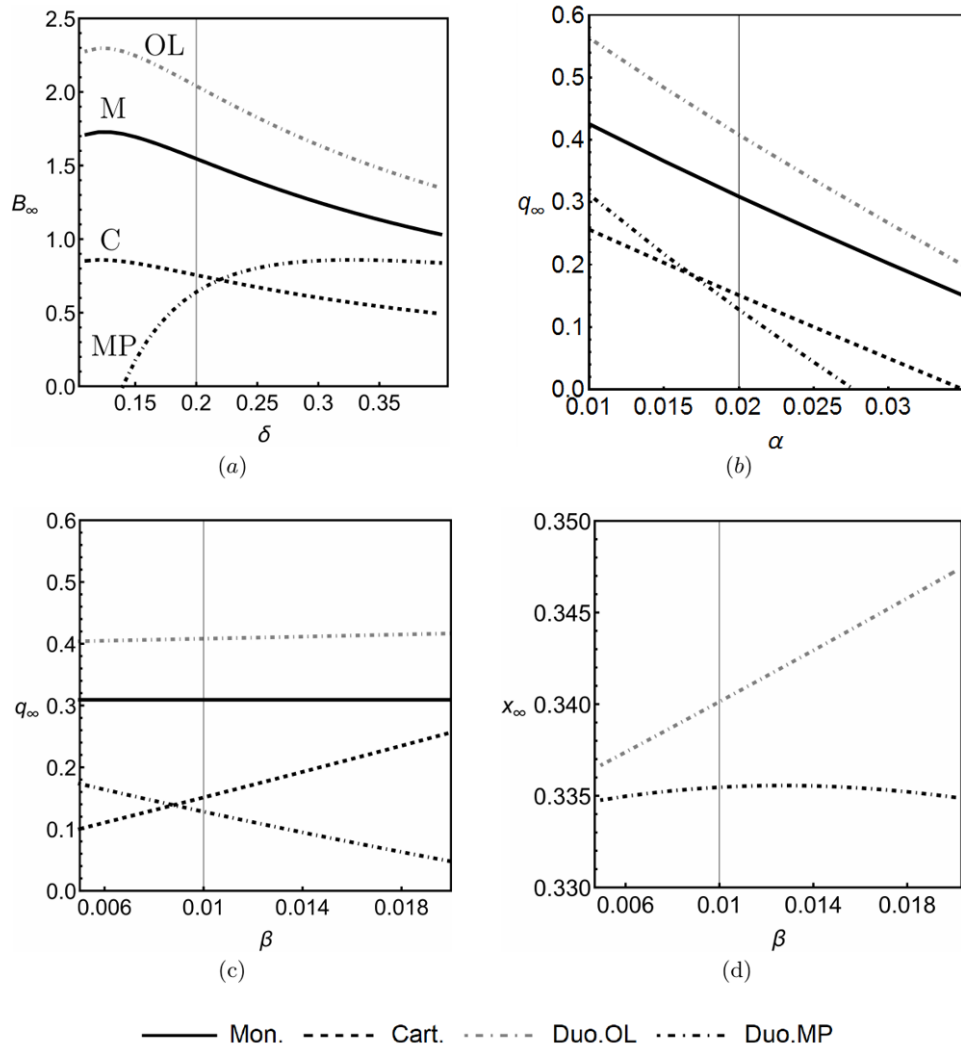


**Figure 1.5.2:** Sensitivity of stationary values with respect to model parameters around the reference case (vertical line) in (1.5.1). Production of the monopoly (constant at 0.24) and cartel (increasing but rather flat, from 0.25-0.26) are much lower and excluded in Fig. 1.5.2d in order to highlight the difference between the open loop and the Markov perfect equilibrium.

(but not open loop) competition.<sup>4</sup> This result - the cheating of German carmakers is due to their implicit cartelization - seems to fit hypotheses expressed in various media, including Domen & Hawranek (2017). As a consequence, further investigations by the EU commission

<sup>4</sup>That is, for most pairs of parameters governing firm internal, as well as external benefits.  $(\alpha, \beta)$ .

in the direction of suspected long-lasting collusion among German carmakers have also been initiated recently.

Hence, information structure is crucial, i.e., the ability to monitor the state of the opponent (and the fact that this also holds for the opponent) leads to a dramatic reduction in cheating and therefore also to less reputational loss. On the other hand, lacking this knowledge or lacking the ability to condition one's strategy on the evolution of the states leads to overshooting and therefore to the worst outcome with respect to consumers (i.e., by how much they are cheated, see Table 1.5.1). A trivial way of revealing the true underlying state is by inspections. These would make information public and thus would enable firms to credibly adjust their strategies in time. We note that even inconsequential audits, that is, audits devoid of any material consequences like a punishment fine, are sufficient in mitigating misbehaviour of firms. Indeed, this tightening of standards and regulations has followed after Dieselgate, but probably did not go far enough.

## 1.6 Final remarks

In this paper we introduced a simple dynamic game theoretic representation of cheating. Both aspects – dynamics as well as strategic interactions – are crucial. Indeed, a major result is that the analysis of the dynamic game turns the counterpart from the static game upside down. More precisely, a Markov perfect oligopolistic equilibrium leads to the lowest level of badwill while a duopoly leads to the highest level of cheating (and thus also of badwill) in the static game and in the dynamic game in open loop strategies. Not only the consideration of dynamics is important but also the degree of strategic interaction. It is the particular interaction in the Markov perfect equilibrium that lowers cheating, which is in contrast to the resulting outcome if the duopolists compete in open loop strategies. The reason seems to be the countervailing effect of the opponent's strategy, such that the Markov perfect strategies are fairly flat in the symmetric equilibrium. Our model is general enough so that it may be applied to many examples. In regards to the game the German carmakers played, one may conjecture that those firms have not acted completely non-cooperatively. Or they colluded tacitly which was enabled by the reliance on common suppliers (e.g., Bosch). We observe that a cartel but even more so, a game played in commitment strategies, increases the incentives

to cheat (i.e., cheating need not be due to cartelization but due to the commitment, e.g., to certain technical solutions). This suggests that establishing competition (as is the case in most markets anyway) but coupled with sufficient public verification of claims should have the highest priority. These observations are based on numerical examples and a theoretical and more robust verification of this claim could be part of future research. Of course, this is a tough task given a problem with multiple states and their interaction in the objectives. Also, the analysis of possible multiple nonlinear equilibria is ignored, because in this setting, a partial differential equation implied by the Hamilton-Jacobi-Bellman equation rules out graphical phase plane techniques in order to derive and characterize nonlinear strategies. Therefore, this is also left for future research (and as the one above, hard to solve too). Other possible extensions would be to include spillovers in the dynamics of the competitor's badwill and to include stochastic processes. Considering the discrete event of 'Dieselgate', Poisson jumps could capture the uncovering of cheating and its subsequent effect on consumers' perception (see Hirschmann 2014).

## Chapter 2

# Market equilibrium strategies under learning by doing spillovers

### Abstract

This paper investigates intertemporal strategic interactions if monopolies, cartels or oligopolies benefit from firm internal as well as external *learning by doing*. Our analysis addresses two issues: first, do the empirical estimates, which are often based on cumulative production aggregated over firms, indeed measure learning by doing? Or are these estimates biased due to simultaneously ongoing intertemporal strategic interactions between competing firms? Second, are asymmetric outcomes conceivable, given a symmetric setup, if firms start from different levels of initial costs/learning? The analysis is carried out for the standard power function representation of learning cost curves and also for a linear cost degression. The latter assumption implies a corresponding linear-quadratic differential game with the surprising feature that multiple symmetric linear Markov perfect equilibria exist – or none at all. The major implication is that learning by doing restricts not only the feasibility of perfect but also of oligopolistic competition while the existence of multiple equilibria highlights some flexibility.

**Keywords:** Differential Game, Competition, Learning by doing, Solar PV.

**JEL index:** C71, C72, C73, D25, D43, L13, P48.

## 2.1 Introduction

Learning by doing is a phenomenon that is known at least since Ford produced its Model T (and presumably much longer) and features prominently in the production from airplanes to microchips (related to Moore's law of doubling memory capacity every two years, and this now for decades with no end in sight, compare Brynjolfsson & McAfee 2017). Nevertheless, Arrow (1962) provided the first theoretical analysis, and in addition learning by doing is also used as a source to generate endogenous growth. The theoretical literature is complemented by many empirical verifications. In particular, even in the narrow field of renewable energy sources many studies highlight the reduction of the costs of photovoltaic (PV) panels, wind energy, batteries and earlier also for nuclear power (Joskow & Rozanski 1979). Indeed, the learning curve estimation of producing PV panels in Reichelstein & Sahoo (2018) motivated this analysis. The upshot of Reichelstein & Sahoo (2018) is that the learning curve factor has not been reduced during the recent years (contrary to the results of many others).

Such claims are established by linking the reduction of world market prices (as a proxy of costs in a competitive industry) to cumulative production. Although this linkage is common in the empirical literature, and particularly in the literature on renewable energy, this assumption raises a few questions: how to differentiate learning from autonomous technological improvement? Or from the effects of competition resulting from a reduction of Ricardian rents. As new firms enter or existing firms expand production, the least efficient (= marginal) producers are eliminated. This reduces the future market price even absent any cost degression at the technical level. Indeed, the PV industry is littered with bankruptcies (recently in Germany despite strong local demand). How could the, now insolvent, German panel producer reduce its cost due to large volumes produced in China? Similarly for the American panel producers facing transatlantic as well as transpacific production. If this were the case, then spillover effects from production must be 100%. However, such large spillovers could lead in turn to a tragedy of the commons as each producer wants the competitors to undertake the costly learning, at least at the beginning. Of course, spillovers stemming from production technologies are crucial and they are indeed observed. For example, Levitt et al. (2013) report that new shifts entering production of a new car start already at fairly efficient levels – for some almost miraculous reason, as they have no experience of their own (in a

narrow sense, this cannot be learning by doing).

An analysis of this interplay between learning from own production and from those of others and interaction in the good's market requires to account for dynamics as well as strategic interactions. Therefore, we consider in this paper intertemporal (equilibrium) production strategies that account for learning in different markets: monopoly, cartel (contrasting a single firm with a set of firms coordinating their production), oligopoly and perfect competition. Among others, we address the following questions:

1. Does learning by doing allow for oligopolistic equilibria? Under what conditions and if so, is the outcome unique?
2. Does learning by doing allow for competition in the long run, in particular if some, possibly only weak, asymmetries exist (like one firm having a head start)?
3. Is learning at all possible in (perfectly) competitive markets?

We study two setups. The first one uses a power function to model the cost degression due to learning, which represents the standard choice in many theoretical and empirical papers. This nonlinear setup can lead to different outcomes depending on the initial conditions – even for a monopoly. More precisely, a threshold determines whether the process of learning will start or not. The analysis of oligopolistic competition is confined to open loop strategies because of the technical difficulties arising due to the nonlinearity.

In the second setup we assume that costs decline linearly with respect to learning. The resulting linear-quadratic game allows to compute the Markov perfect equilibrium. This and the comparison with the open loop equilibrium allows to investigate how the more intense intertemporal interaction between firms affects the market outcome and whether it can compensate for less individual learning compared with the monopoly. We also address the bias if learning curves are estimated from prices observed in non-competitive markets.

The analysis of the linear-quadratic game of oligopolistic competition under learning by doing produces two surprising and interesting theoretical results of its own: first, more than one linear (or singular in the terminology of Rowat 2007) Markov perfect equilibrium may exist. This multiplicity is very rare and unlike the issue of multiple equilibria in nonlinear strategies following Tsutsui and Mino (1990), e.g., Dockner and van Long (1993), Rowat

(2007) and Wirl (2007). It seems that nonuniqueness of linear Markov perfect equilibria is entirely ignored in the literature: it is not mentioned in Dockner et al. (2000) and many differential game experts we contacted were also unaware of this possibility. To the best of our knowledge only Engwerda (2007, 2016) investigates this possibility but in entirely different and somewhat constructed setups (by analyzing the implied Riccati-equations). In contrast, the multiplicity of stable strategies in this paper arises in a natural and completely untwisted setup.

Second, no Markov perfect equilibrium need exist, in particular for little or no spillovers, although the model is familiar to the many dynamic models of oligopolistic competition (learning is the ‘capital’ stock in our model) that started with Reynolds (1987, 1991). Furthermore, the other solutions of the dynamic setup (monopoly, cartel, open loop) as well as of the corresponding static game all exist. The reason is that time-consistency in the strong sense (i.e., subgame perfection) requires convergence to a steady state also for initial conditions off the equilibrium path. Hence, any head start of one firm, an erroneous move or simply a production shortfall renders duopolistic competition in Markov strategies infeasible unless the spillovers are sufficiently large such that they compensate for an any arising cost disadvantage.

This paper is organized as follows. We introduce the framework in section 2.2. In section 2.3, we analyze the nonlinear setup and compare the results (restricted to open loop) including an asymmetry in initial conditions. In section 2.4, we use the assumption of a linear cost function (which allows to solve for feedback strategies) that leads to the surprising theoretical results of nonuniqueness and nonexistence of a linear Markov perfect equilibrium. We also investigate the role of information structure as well as industrial organization on market outcomes. Finally, we conclude in section 2.5.

## 2.2 Framework

Market demand is determined by the consumers' marginal willingness to pay such that the price (for reasons of simplicity linear and normalized),

$$p(t) = 1 - U(t), \quad (2.2.1)$$

is determined by the aggregate supply of  $n$  firms,

$$U(t) := \sum_{i=1}^n u_i(t). \quad (2.2.2)$$

In the terminology of Arrow we define learning as the time-dependent result from producing a good. An individual firm's knowledge ( $l_i$ ) increases with respect to its own production ( $u_i$ ) and the (aggregate) production of its competitors ( $u_j$ ),

$$\dot{l}_i(t) = au_i(t) + b \sum_{j \neq i}^n u_j(t) - \delta l_i(t), \quad l_i(0) = l_{i0}, \quad \delta > 0, \quad (2.2.3)$$

where  $l_i(t)$  denotes the learning in period  $t$ ,  $\delta$  is the rate at which knowledge deteriorates in the absence of production and  $a$  is the marginal learning effect from own production, i.e., increasing production by one (infinitesimal) unit increases the firm's knowledge by  $a$ . The coefficient  $b$  describes how much firm  $i$  learns from the aggregate volume produced by its competitors. The forgetting rate establishes diminishing returns taking place when cost cuts approach the technically feasible lower bound. We assume a symmetric setup throughout the paper, however, we make one exception: firms are allowed to start from different initial conditions ( $l_{i0}$ ). Usually, learning from own production exceeds the learning from others', i.e., we assume  $b < a$ . However, what about the aggregate effect? Although one cannot rule out that the sum of spillovers exceeds the learning from own production (e.g., if own production is very small compared to the aggregate,  $u_i \ll U$ , which generally holds in markets characterized by many participants  $n$ ), we make

**Assumption 2.2.1:** *Learning from own production exceeds what can be learned from the*

production of all others. For different numbers of firms and symmetric outcomes, we assume a stricter version,

$$b < \frac{a}{n-1}. \quad (2.2.4)$$

After all, it is called learning by doing and not learning by copying. Therefore, it is necessary to vary parameter  $b$ , whenever one varies the number of firms or plants, i.e.,  $b := b(n)$ . Normalizing, we set,

$$b(n) := \frac{\bar{b}}{n-1}, \quad \text{and } \bar{b} < a, \quad (2.2.5)$$

which implies that the spillover effect depends only on the aggregate volume produced by the competitors – whether by many or by few. However, Assumption 2.2.1 does not rule out that firm  $i$  learns more from its competitors than from its own production in an asymmetric setup in which  $i$ 's own production is small relative to the sum of its competitors, e.g., due to  $l_i(0) < l_{-i}(0)$ .

### 2.2.1 Power cost function

The first setup is nonlinear. We determine the effect of knowledge ( $l_i$ ) on the unit production costs ( $c$ ) by the power function (which is applied in almost all empirical applications),

$$c_i(t) = k l_i(t)^{-\beta}, \quad (2.2.6)$$

in which  $\beta$  is the so-called learning curve slope and  $k$  a constant scalar. It is typically estimated from how much the costs are reduced at each doubling of output (thus ignoring the depreciation of knowledge) since the power function is scale invariant. As a rule of thumb,<sup>5</sup> a reduction of 20% implies  $\beta = -\ln 0.8 / \ln 2 \approx 0.322$ ; the larger  $\beta$  the higher is the

---

<sup>5</sup>See Swanson (2011) and also Reichelstein & Sahoo (2018) who estimates a beta of 0.6, i.e., an even steeper cost decay. It should also be mentioned that Reichelstein & Sahoo (2018) work with an increasing demand function over time, which contrasts the static demand setup here.

rate of learning.<sup>6</sup> This specification rules out  $l_0 = 0$  and suggests to constrain  $l_0$ , because otherwise initial costs exceed the choke price of 1, given the demand (2.2.1). Furthermore, costs must be scaled (with  $k < 1$ ) in order to allow for equilibria across the different cases. Still, production may start with losses in anticipation of the future cost decline, particularly for a monopoly.

### 2.2.2 Linear cost function

In order to solve for the Markov perfect equilibria, we also consider the case that learning lowers costs linearly, i.e.,

$$c_i(t) = k - \beta l_i(t). \quad (2.2.7)$$

In contrast to (2.2.6) we may allow for  $l_0 = 0$  and can thus analyze whether initial costs above the choke price,  $k > 1$ , allow to start production and subsequent learning, and if it does, in which of the different market setups this is feasible.

### 2.2.3 Objectives under different market structures

As mentioned above, we consider different market structures: monopoly, cartel, oligopoly and perfect competition, all using production as a control (i.e., a Cournot setting). By definition, a monopoly cannot learn from competitors and thus chooses its production strategy that maximizes its net present value of profits. Assuming that future profits are discounted at rate  $r > 0$ , it solves,

$$\max_{U(t) > 0} \int_0^\infty e^{-rt} (p(t) - c(t)) U(t) dt, \quad (2.2.8)$$

$$s.t. \quad \dot{l}(t) = aU(t) - \delta l(t), \quad l(0) = l_0. \quad (2.2.9)$$

A cartel coordinates the decisions of  $n$  firms maximizing the joint profit (from now on omitting

---

<sup>6</sup>Assuming  $l_i > 1$ , otherwise the opposite holds.

time arguments),

$$\max_{\{u_i > 0, i=1, \dots, n\}} \int_0^\infty e^{-rt} \left( 1 - \left( \sum_{i=1}^n u_i \right) - c_i \right) \sum_{i=1}^n u_i dt, \quad (2.2.10)$$

subject to intertemporal learning (2.2.3) in each firm. The difference compared to the monopoly is that the firms, as members of the cartel, can enjoy spillovers but experience less direct learning due to individually lower production levels ( $u_i$ ). *Maximizing a cartel's profit of  $n$  firms (and all producing) makes only sense in a completely symmetric case, i.e., with identical parameters and initial conditions.* The reason is that the slightest head start of one cartel member renders it optimal that it becomes the sole supplier because of its lower costs today and then also in all future periods. After all, maximizing the cartel's profits implies cost minimization. Of course, real world cartels may not (and presumably do not) maximize profits in this strict way and their production arrangements can be the result of cooperative agreements.

The  $n$  players of an oligopoly maximize their individual net present value of profits having control only over their own production,

$$\max_{u_i > 0} \int_0^\infty e^{-rt} \left( 1 - \left( \sum_{i=1}^n u_i \right) - c_i \right) u_i dt, \quad (2.2.11)$$

subject to the same  $n$  dynamic constraints (2.2.3) as the cartel. In contrast to the cartel, asymmetry restricted to initial conditions may allow for a stable oligopolistic outcome in the long run. On the other hand, if a firm starts with the by far lowest costs, it can outproduce its competitors and will therefore run down its learning cost curve ahead of the competitors. This includes the possibility of driving one or all of them out of the market in order to win the prize of becoming a monopoly. Since we focus on interior outcomes, we make

**Assumption 2.2.2:** *All firms produce at all times,  $u_i(t) > 0$  for all  $i = 1, \dots, n$  and all  $t \geq 0$ .*

This assumption excludes the trivial cases in which an interior solution (stationary or in the long run) implies negative profits so that immediately stopping production would be optimal

(for small  $l_{i0}$ ).<sup>7</sup> Nevertheless, the corresponding domains in which learning is absent will be characterized. Assumption 2.2.2, excludes possible outcomes due to an asymmetry in initial conditions: laggards quit or decide to enter later after having sufficiently learned from the production of the incumbents, or an incumbent adjusts its production to keep potential entrants out of the market.<sup>8</sup> The analysis of those aspects is left for future extensions.

## 2.3 Power cost function

### 2.3.1 Monopoly

The optimization problem of the monopoly is given by (2.2.8)-(2.2.9). We define the Hamiltonian,

$$H = (1 - U)U - kUl^{-\beta} + \lambda(aU - \delta l),$$

in which  $\lambda$  denotes the costate (= shadow price) associated with the state of learning. The Hamiltonian-maximizing-condition determines the optimal quantity (if positive),

$$H_U = 1 - 2U - kl^{-\beta} + a\lambda \stackrel{!}{=} 0 \implies U = \frac{1 + a\lambda - kl^{-\beta}}{2}. \quad (2.3.1)$$

Since a bigger stock of knowledge enters the objective positively, the relevant domain of the costate is positive. Hence, (2.3.1) shows that the monopolist produces more compared to the static case due to enjoying lower costs in the future by choosing a higher output today. The

---

<sup>7</sup>A necessary condition can be obtained from the condition that there must exist a level of knowledge ( $l > 0$ ) such that the resulting stationary market price exceeds the costs,

$$1 - \frac{n\delta l}{a + (n-1)b} > c(l) \text{ for at least some } l.$$

<sup>8</sup>Entry deterrence may not be easy, at least for large spillovers: if the incumbent produces little to keep the competitor's learning from spillovers small, then this raises the price that can render the competitor's production profitable; producing high volumes lowers the price and thus provides an entry barrier but reduces the future costs of the potential entrant.

dynamic costate equation is given by,

$$\dot{\lambda} = r\lambda - H_l = (r + \delta)\lambda - k\beta U l^{-\beta-1}, \quad (2.3.2)$$

and implies, combined with (2.2.9), (2.3.1) and an appropriate transversality condition, the canonical system,

$$\begin{aligned} \dot{l} &= \frac{a}{2} (1 + a\lambda - kl^{-\beta}) - \delta l, \quad l(0) = l_0, \\ \dot{\lambda} &= (r + \delta)\lambda - \frac{k\beta}{2} (1 + a\lambda - kl^{-\beta}) l^{-\beta-1}, \quad \lim_{t \rightarrow \infty} e^{-rt} \lambda l = 0. \end{aligned} \quad (2.3.3)$$

Differentiating (2.3.1) with respect to time, substituting (2.3.2) and using (2.3.1) to replace  $\lambda$  yields a dynamic system in state and control,

$$\begin{aligned} \dot{U} &= \frac{r + \delta}{2} (2U - 1) + \frac{r + \delta - \beta\delta}{2} (kl^{-\beta}), \\ \dot{l} &= aU - \delta l. \end{aligned} \quad (2.3.4)$$

Since the maximized Hamiltonian is convex at low and concave at high levels of the state  $l$ , multiple steady states are possible. This in turn suggests the existence of multiple intertemporal long run outcomes that depend on the initial conditions (compare Skiba 1978, Sethi 1977/1979 and Dechert & Nishimura 1983). By analyzing the nullclines of the system (2.3.4), more precisely, substituting  $U = \delta l/a$  into  $\dot{U} = 0$ , yields the following characterization of a steady state,

$$\begin{aligned} \phi_1 l^{-\beta} &= 1 - \phi_2 l, \\ \text{with } \phi_1 &= k \left( \frac{r + \delta(1 - \beta)}{r + \delta} \right) > 0 \quad \text{and} \quad \phi_2 = \frac{2\delta}{a} > 0. \end{aligned} \quad (2.3.5)$$

The left-hand side of (2.3.5) is an exponentially decreasing function that crosses the right-hand side (a linearly decreasing function with intercept at 1) no more than twice, yielding two steady states (generically). The Jacobian of the system (2.3.4),

$$J = \begin{pmatrix} r + \delta & -\frac{1}{2}k\beta(r - \beta\delta + \delta)l^{-\beta-1} \\ a & -\delta \end{pmatrix} \quad (2.3.6)$$

allows to characterize the local dynamics off the steady states as shown in the phase diagram in Fig. 2.3.1.<sup>9</sup>

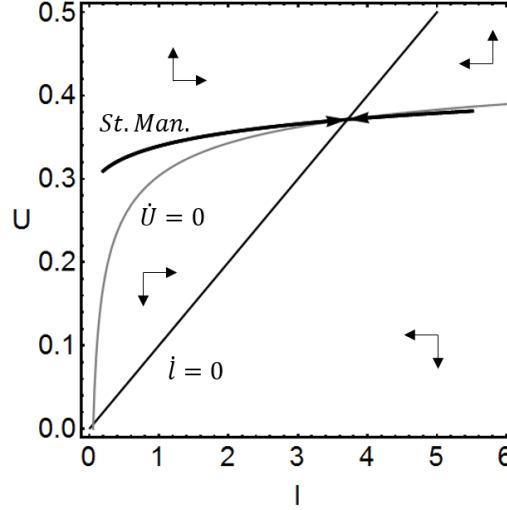


**Figure 2.3.1:** Phase diagram of system (2.3.4) with (2.3.18).

The high steady state is a saddle point (two real eigenvalues with opposing sign) whereas the one at a low level of learning is unstable. It is worth noting that the eigenvalues of (2.3.6) can become complex for sufficiently small levels of  $l$ ,

$$EV_{12} = \frac{1}{2} \left( r \pm l^{-\beta-1} \sqrt{l^{\beta+1} (2a\beta k(\beta\delta - \delta - r) + l^{\beta+1}(2\delta + r)^2)} \right).$$

As the optimal path has to be monotonic in the state (Hartl 1987), the lower steady state (which is an unstable focus) disqualifies as a long run solution to the intertemporal decision making problem of the monopoly.<sup>10</sup> Instead, a threshold can (but need not, compare Hartl et al. 2004 for capital accumulation and Wagener (2003) for the shallow lake model) exist that separates the basins of attractions – either quitting or continuing to run down the learning curve.

Fig. 2.3.2a and b vary the cost ( $k$ ) and the learning parameter ( $\beta$ ) and plot the corresponding

<sup>9</sup>This figure as well as all the following figures in this section draw on the parameter set given in (2.3.18).

<sup>10</sup>If the unstable steady state is a node, two different outcomes are in principle possible (compare Hartl et al. 2004): as above for the unstable spiral, or a continuous policy function passing through the unstable steady state, which then serves simultaneously as the critical threshold.

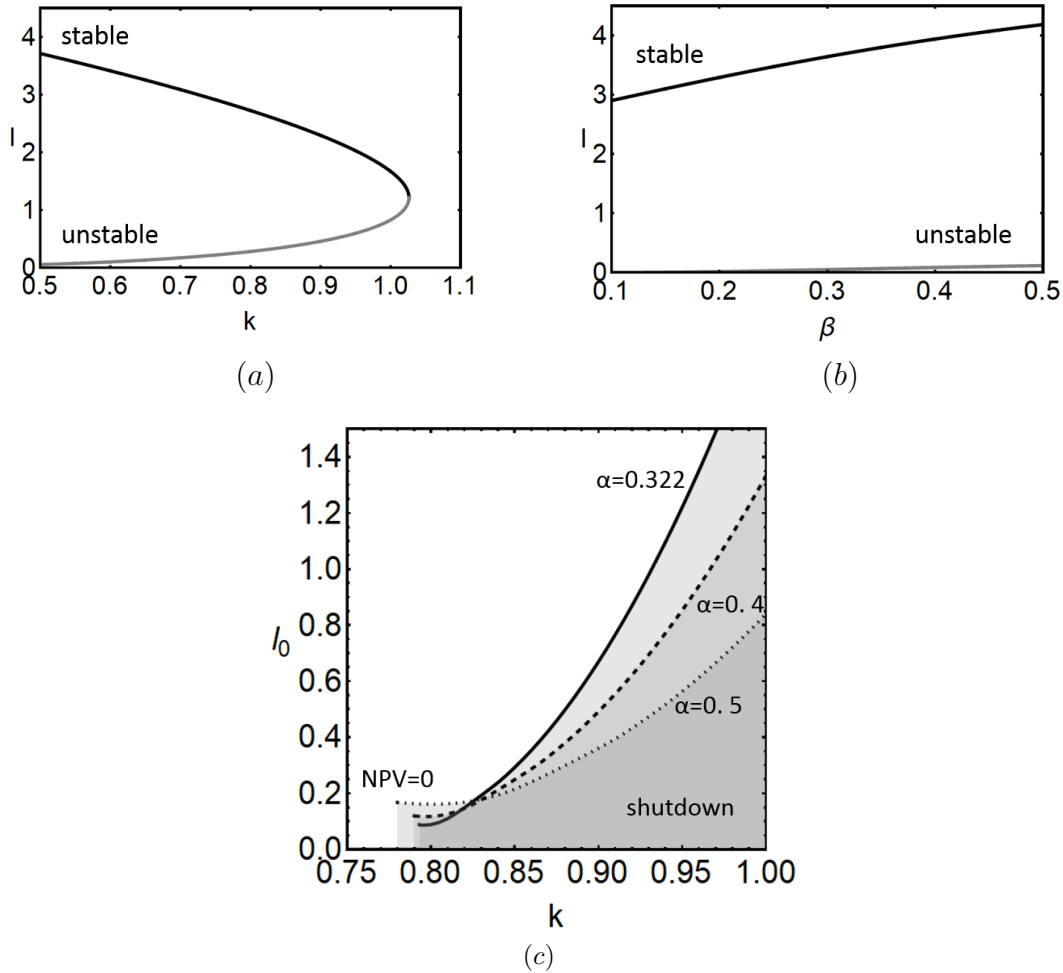


**Figure 2.3.2:** Bifurcation diagrams of (2.3.4) in  $k$ , shown in (a) and in  $\beta$ , shown in (b). (c) shows the Skiba contours in  $(k, l_0)$  space for different learning speeds.

two steady states. High initial costs, more precisely, scaled costs such that  $k > k^* \approx 1.064$  do not allow for steady states and thus for the accumulation of knowledge. In such a case the firm will shut down immediately if the losses from non-profitable production cannot be recovered by profits in the future.<sup>11</sup> As expected, higher learning curve slopes raise the stable steady state and thus the knowledge in the long run but also, and less intuitive, the unstable

<sup>11</sup>This does not exclude the possibility for early instantaneous losses which can later be recuperated by larger turnover with lower unit costs, i.e., when the consumers' maximum willingness to pay finally exceeds unit production costs.

one (but only very marginally, see Fig. 2.3.2b). The lowest feasible initial condition (and thus the highest cost) for which the stable steady state can be reached depends on the cost parameter  $k$ , since the marginal willingness to pay is bounded from above (by 1). We find that arbitrarily small but positive  $l_0$ 's lead to learning by doing, i.e., the higher steady state will be approached, as long as  $k \leq k_c \approx 0.795$ . Fig. 2.3.2c shows the zero-profit contour line in  $(k, l_0)$  space and the steep and convex increase of the necessary level of initial knowledge in order that the firm chooses production instead of a shutdown. It is important to stress that the threshold is, as expected, declining in  $\beta$ , which emphasizes the difference between the behavior of the threshold one the one hand and the unstable steady state on the other (see Fig. 2.3.2b and c).

The qualitative structure of the solutions, in particular the existence of two steady states, is similar in the other industrial organizations studied in this paper. Therefore, we keep the following analyses short but compare the results thereafter.

### 2.3.2 Cartel

We assume full cooperation, i.e., cartel members delegate their individual production decision to the organization which maximizes the joint profit of all its members (2.2.10) subject to (2.2.3). For the reason already given, we impose symmetry not only on the parameters but also on the initial conditions, i.e., we set  $l_i(0) = l(0)$ . This implies  $u_i = u$ ,  $l_i = l$ , and  $\lambda_i = \lambda$  for all  $i$  and thus leads to the Hamiltonian,

$$H = nu(1 - nu - kl^{-\beta}) + \lambda(au + b(n-1)u - \delta l).$$

The choice of the optimal quantity (from  $\arg \max_u H$ ),

$$u = \frac{U}{n} = \frac{n(1 - kl^{-\beta}) + \lambda(a + b(n-1))}{2n^2}, \quad (2.3.7)$$

must account for the learning spillovers and is thus different from the monopoly (2.3.1).

Using the costate differential equation,

$$\dot{\lambda} = (r + \delta) \lambda - nk\beta ul^{-\beta-1},$$

the following canonical equations result,

$$\begin{aligned} \dot{l} &= (a + b(n - 1)) \frac{n(1 - kl^{-\beta}) + \lambda(a + b(n - 1))}{2n^2} - \delta l, \quad l(0) = l_0, \\ \dot{\lambda} &= (r + \delta) \lambda - \frac{k\beta}{2n} (n - nkl^{-\beta} + \lambda(a + b(n - 1))) l^{-\beta-1}, \quad \lim_{t \rightarrow \infty} e^{-rt} \lambda l = 0. \end{aligned} \quad (2.3.8)$$

As before, we transform them into equations in state and control,

$$\begin{aligned} \dot{u} &= \frac{r + \delta}{2n} (2nu - 1) + \frac{r + \delta - \beta\delta}{2n} (kl^{-\beta}), \\ \dot{l} &= (a + b(n - 1)) u - \delta l. \end{aligned} \quad (2.3.9)$$

Of course, by setting  $n = 1$  one obtains the dynamic system for the monopoly, (2.3.3) and (2.3.4).

### 2.3.3 Oligopoly

Due to the nonlinearity in the state and the irreducibility of the state space to one dimension, we restrict ourselves to the open loop solution of the oligopolistic game and compare them to the other market structures afterwards. The analysis of the open loop equilibrium is often criticized because it assumes a single revelation of the state of learning but none in the future. In contrast, the subgame perfect equilibrium of Markov strategies assumes perfect observation of the state and this in each competing firm at each point in time.<sup>12</sup> However, player  $i$  may not be able to observe how its competitors  $j \neq i$  run down their learning curves at each point in time. At best, sporadic and imprecise signals from the evolution of the market price may allow to infer how the competitors' costs decrease. Therefore, the open loop solutions are not entirely unreasonable and the truth is presumably in between the two information structures assumed in this paper.

We define the corresponding Hamiltonian of player  $i$  (now substituting the assumed functional

<sup>12</sup>A good comparison are ballistic vs. guided missiles in military affairs. Whereas the former cannot be influenced after propulsion, the path of the latter can be adjusted during flight.

forms),

$$\begin{aligned} H_i &= \left(1 - \sum_{j=1}^n u_j - kl_i^{-\beta}\right) u_i + \mu_i^i \left(au_i + b \sum_{j \neq i} u_j - \delta l_i\right) \\ &+ \sum_{j \neq i} \mu_j^i \left(au_j + b \sum_{f \neq j} u_f - \delta l_j\right), \end{aligned}$$

where  $\mu_j^i$  represents the costate that player  $i$  associates with state  $l_j$  for  $j \neq i$ . The first-order-conditions for interior solutions of each firm are given by,

$$\begin{aligned} H_{u_i}^i &= 1 - \sum_{j \neq i} u_j - kl_i^{-\beta} - 2u_i + a\mu_i^i + b \sum_{j \neq i} \mu_j^i \stackrel{!}{=} 0, \\ \implies u_i &= \frac{1 - \sum_{j \neq i} u_j - kl_i^{-\beta} + a\mu_i^i + b \sum_{j \neq i} \mu_j^i}{2}, \text{ for } i = 1, \dots, n, \end{aligned} \quad (2.3.10)$$

$$\dot{\mu}_i^i = (r + \delta) \mu_i^i - \beta kl_i^{-\beta-1} u_i, \quad \lim_{t \rightarrow \infty} e^{-rt} \mu_i^i l_i = 0, \text{ for } i = 1, \dots, n, \quad (2.3.11)$$

$$\dot{\mu}_j^i = (r + \delta) \mu_j^i, \quad \lim_{t \rightarrow \infty} e^{-rt} \mu_j^i l_j = 0, \text{ for } i = 1, \dots, n, \quad j \neq i, \quad (2.3.12)$$

and must hold simultaneously in a Nash equilibrium. The  $n(n-1)$  costates  $\mu_j^i$ ,  $i = 1, \dots, n$ ,  $j \neq i$  must vanish for all  $t$  due to the limiting transversality condition, which captures the open loop spirit that the impact on the opponents' learning rates is ignored.

### Symmetry

After imposing symmetry we obtain

$$u = \frac{1 - kl^{-\beta} + a\mu}{n + 1}, \quad (2.3.13)$$

which represents the usual Cournot solution except that the production costs are reduced by the intertemporal gain  $a\mu$ . The canonical system is given by

$$\begin{aligned} \dot{l} &= (a + (n-1)b) \frac{1 - kl^{-\beta} + a\mu}{n+1} - \delta l, \quad l(0) = l_0, \\ \dot{\mu} &= (r + \delta)\mu - k\beta \left( \frac{1 - kl^{-\beta} + a\mu}{n+1} \right) l^{-\beta-1}, \quad \lim_{t \rightarrow \infty} e^{-rt} \mu l = 0, \end{aligned} \quad (2.3.14)$$

and can be transformed into

$$\begin{aligned} \dot{u} &= \frac{1}{n} [(u + nu - 1)(r + \delta) + k(r + \delta - \beta\delta)l^{-\beta} + b\beta k(n-1)ul^{-1-\beta}], \\ \dot{l} &= (a + b(n-1))u - \delta l. \end{aligned} \quad (2.3.15)$$

The open loop oligopolistic equilibrium also includes the possibility of multiple steady states (which cannot be analytically given) and thus the possibility of thresholds.

### Asymmetric initial knowledge

Before we compare the different market forms we briefly address the question whether asymmetric initial conditions can lead to an asymmetric long run outcome. For this purpose, we consider two firms,  $i = 1, 2$  which are symmetric except that one player is disadvantaged at the start, i.e.  $l_1(0) \geq l_2(0)$ . The question is whether this difference leads to the same symmetric long run outcome or whether such initial differences persist leading to an asymmetric steady state in which both produce (Assumption 2.2.2). Therefore, assuming interior solutions,  $u_i > 0$ , the interdependent first-order-conditions of (2.3.10) simplify to

$$u_i = \frac{1 - 2kl_i^{-\beta} + l_j^{-\beta} + 2a\mu_i^i - a\mu_j^j}{3}, \quad i, j = 1, 2, \quad j \neq i$$

and imply

$$\begin{aligned} \dot{l}_i &= \frac{1}{3} \left( a + b + (2a - b)(a\mu_i^i - kl_i^{-\beta}) + (2b - a)(a\mu_j^j - kl_j^{-\beta}) - 3\delta l_i \right), \quad l_i(0) = l_{i0}, \\ \dot{\mu}_i^i &= (r + \delta)\mu_i^i - \frac{k\beta}{3} \left( 1 - 2kl_i^{-\beta} + kl_j^{-\beta} + 2a\mu_i^i - a\mu_j^j \right) l_i^{-\beta-1}, \quad \lim_{t \rightarrow \infty} e^{-rt} \mu_i^i l_i = 0, \end{aligned}$$

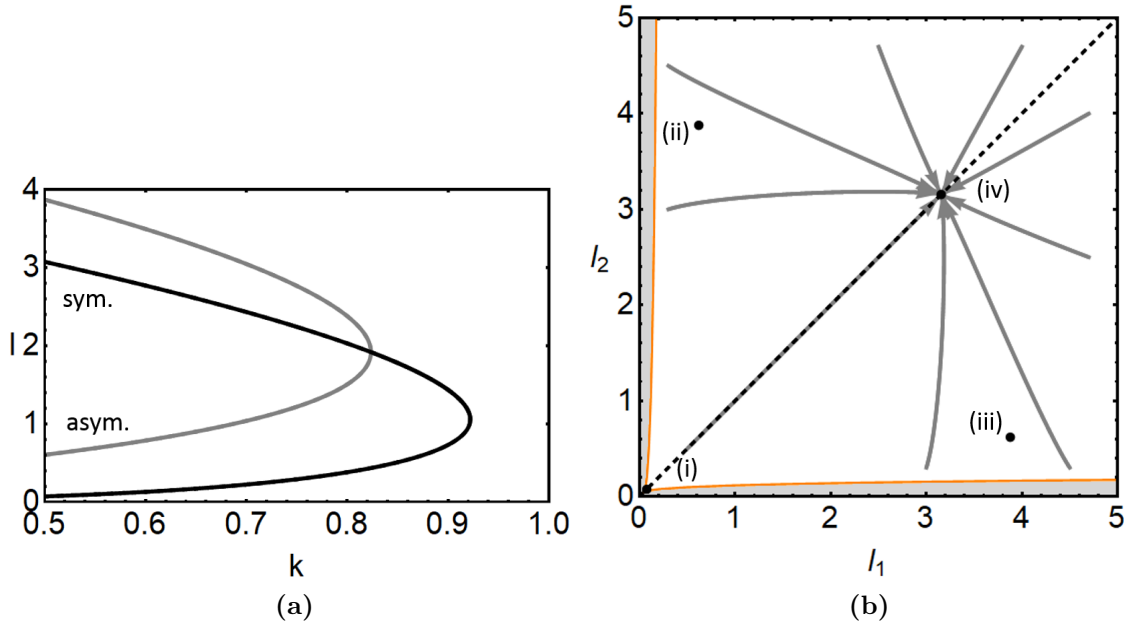


**Figure 2.3.3:** Equilibria for varying levels of  $k$  are plotted in (a). In (b) fixed points, along with some converging paths, are shown in state space.

for  $i, j = 1, 2, j \neq i$ . Although the two state equations can be solved for the costates, the whole four-dimensional system is not analytically solvable.<sup>13</sup> Hence, we have to continue numerically and find at most four steady states in our examples: (i) a symmetric one at very low and economically meaningless levels (where the costs are larger than the choke price), (ii) an asymmetric equilibrium, (iii) the counterpart of the asymmetric equilibrium in (ii), and (iv) a symmetric equilibrium at higher levels of learning. Fig. 2.3.3a plots the four steady states in the form of a bifurcation diagram in the parameter  $k$ .

While (i) is an unstable focus and (iv) is a saddle point, both asymmetric equilibria (ii) and (iii) are unstable as can also be seen in Fig. 2.3.3b, where the solution trajectories for several asymmetric initial conditions are plotted in state space. Additionally, the border of the basin of attraction for (iv) is plotted (less the three other steady states of course). This means that any transient cost differential will ultimately be eliminated, as the disadvantaged firm's cost

<sup>13</sup>After plugging in the solutions for the costates in the respective equations, we are able to find a closed-form solution for one of the costates. However, inserting this result into the last equation yields a nonlinear function which cannot be solved for the remaining costate.

catches up and eventually converges with the other one to the symmetric steady state. This catching-up takes place because lower initial costs allows for more production early, leading to relatively large spillovers that benefit the cost-disadvantaged firm. Thus, entry deterrence is an incredible threat that cannot be supported in the open loop equilibrium, at least for the considered parameters. Hence, unless initial conditions are highly asymmetric (the small gray shaded areas in Fig. 2.3.3b), learning will take place in both firms.

### 2.3.4 Competition

Taking the limit,  $n \rightarrow \infty$ , of the above canonical equations (2.3.15) characterizing the symmetric Nash-Cournot oligopolistic outcome yields,

$$\begin{aligned} \dot{u} &= u(r + \delta + b\beta kl^{-1-\beta}), \\ \dot{l} &= bu - \delta l. \end{aligned}$$

In any steady state,

$$u(r + \delta + b\beta kl^{-1-\beta}) = 0, \quad (2.3.16)$$

$$bu - \delta l = 0, \quad (2.3.17)$$

must hold and thus after simplifying also,

$$\underbrace{lr}_{\geq 0} + \underbrace{l\delta}_{\geq 0} + \underbrace{b\beta kl^{-\beta}}_{> 0} = 0,$$

which, given our assumptions, has no solution. Intuitively, no learning can take place at infinitesimally small quantities. Therefore,

**Result 2.3.1.** *The limit of oligopolistic competition does not allow for learning and thus for competitive and infinitesimally small firms. This limit result extends to the linear cost function analysed below as well as to the approach that models competition directly (instead of as the limit of a particular Nash equilibrium) subject to aggregate spillovers following Krugman (1991) and the endogenous growth literature (see Appendix 2A).*

### 2.3.5 Comparison

In order to compare the outcomes under different market structures, we assume symmetry (the possibility of asymmetric outcomes for cartels and oligopolies was already addressed above) and the parameter values,

$$a = 1, \beta = 0.322, \delta = 0.1, b = \bar{b}/(n-1), \bar{b} = 0.3, k = 0.5, r = 0.05. \quad (2.3.18)$$

The choice of  $\beta$  roughly equates to a learning coefficient of 0.8, i.e., a 20% reduction in unit costs with a doubling of accumulated learning via cumulative production. Table 2.3.1 depicts steady state- and net present values for all market forms with  $l_{i0} = 0.5$  and (2.3.18) for the duopoly as well as for markets with more participants. This highlights that a monopoly attains the highest level of learning (and of course, the highest profit), yet the oligopoly does better in terms of learning than the cartel, which performs worst. However, whereas the cartel decreases total output as more members join, oligopolists are forced to do exactly the opposite, as market power decreases the more firms enter. This additional production is, however, not enough to counterbalance the loss in learning from one's own, now reduced, production. This leads to a smaller knowledge stock and thus to larger costs. Consumers clearly prefer an oligopoly of any size over a cartel and a monopoly over a cartel because of larger production and thus lower prices by the monopoly.

On the supply side, the monopoly's profit ( $PS$  denotes the producers' surplus subscripted for oligopoly, monopoly and cartel) is way beyond that of a cartel and oligopoly. Hence,

$$CS_O > CS_M > CS_C \quad \text{and} \quad PS_M > PS_C > PS_O,$$

in which  $CS$  (subscripted as  $PS$ ) denotes the consumer surplus. Thus, what is best for society depends on the trade-off between the increase in consumer surplus versus lower producer surplus as competition in the market increases. Standard economic theory tells us that total welfare is increasing in the number of firms and that it is maximal under perfect competition. However, this is not the case under learning by doing<sup>14</sup> as can be seen in Fig. 2.3.4 that plots social welfare against the number of market participants.

---

<sup>14</sup>This may also not be the case with the assumption of fixed costs (from which we abstract from) and thus markets characterized by economies of scale.

|            | n | Monopoly | Cartel  | Oligopoly |
|------------|---|----------|---------|-----------|
| $u_\infty$ | 2 |          | 0.175   | 0.237     |
|            | 4 | 0.371    | (0.080) | (0.132)   |
|            | 6 |          | [0.049] | [0.088]   |
| $U_\infty$ | 2 |          | 0.351   | 0.474     |
|            | 4 | 0.371    | (0.318) | (0.528)   |
|            | 6 |          | [0.292] | [0.529]   |
| $l_\infty$ | 2 |          | 2.338   | 3.157     |
|            | 4 | 3.713    | (1.273) | (2.111)   |
|            | 6 |          | [0.834] | [1.511]   |
| $c_\infty$ | 2 |          | 0.380   | 0.345     |
|            | 4 | 0.328    | (0.463) | (0.393)   |
|            | 6 |          | [0.530] | [0.438]   |
| $\pi$      | 2 |          | 0.764   | 0.661     |
|            | 4 | 1.837    | (0.283) | (0.134)   |
|            | 6 |          | [0.147] | [0.022]   |

**Table 2.3.1:** Steady state values for output, learning, costs and total discounted profits ( $\pi$  for  $l_{i0} = 0.5$ ). In addition to the duopoly, values for  $n = 4$  (in parentheses) as well as for  $n = 6$  (in brackets) are shown.

In our example, the increase in consumer surplus is more than outweighed by the loss in firms' profits beyond markets composed of 3 firms. This is due to the fact that each additional firm reduces the speed of cost reduction due to Assumption 2.2.1 of imperfect spillovers. The resulting stock of knowledge is therefore smaller<sup>15</sup> and the monopoly becomes optimal

<sup>15</sup>Learning from one's own production takes place at a slower rate compared to the rate knowledge is acquired

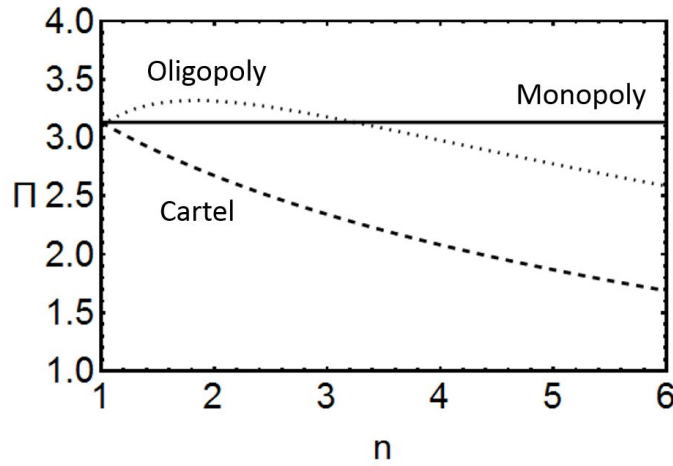


**Figure 2.3.4:** Social welfare depending on the number of firms,  $n$ , under the usual parameter assumptions.

compared to markets with many participants. Thus, we summarize,

**Result 2.3.2.** *Social welfare is generally maximized by an intermediate level of competition (i.e., usually a low number of competing firms) and not by any of the extrema (i.e., a monopoly or perfect competition). For markets composed of many competing firms oligopolistic competition is not welfare maximizing since a monopoly, lacking the duplication of costs in the oligopoly, does better. In all cases, a cartel performs worst, not only for consumers but also for social welfare.*

## 2.4 Linear cost function

The major objective of this section is to solve the learning by doing game also for a linear Markov perfect equilibrium (MPE) between oligopolistic suppliers in order to address the consequences of different information structures on intertemporal competition.

---

from others. Additionally, each firm has a natural decay in its knowledge generation, i.e., the forgetting rate.

### 2.4.1 Cartel and monopoly

It suffices to analyze the (symmetric) cartel, because the monopolistic outcome results for the special case of  $n = 1$  and  $b = 0$ . Therefore, we maximize (2.2.10) subject to the linear cost dynamics (2.2.7), and  $l_{i0} = l_0, \forall i$ . Let  $W(l)$  denote the value function of the cartel. Under the above assumptions – symmetry and all cartel members produce – the following functional equation must hold,

$$rW(l) = \max_{u>0} \{nu(1 - nu - k + \beta l) + W'(l)(au + (n - 1)bu - \delta l)\}, \quad (2.4.1)$$

for the cartel's value function  $W(l)$ . The maximization on the right hand side of (2.4.1) implies,

$$u = \frac{(1 - k + \beta l)}{2n} + \frac{(a + (n - 1)b)}{2n^2} W'. \quad (2.4.2)$$

Since the intercept of (2.4.2) represents the maximization of static profits, intertemporal output will be larger, as  $W' > 0$ , according to economic intuition and also arithmetically as shown below. Substitution of (2.4.2) into (2.4.1) yields an ordinary differential equation for the functional equation (2.4.1). We guess a linear quadratic value function  $W$ ,

$$W = w_0 + w_1 l + \frac{w_2}{2} l^2,$$

with  $w_0, w_1, w_2 \in \mathbb{R}$ , substitute this guess into the left and right hand side of (2.4.1) and obtain the solution by comparing coefficients (see the Appendix 2B). This procedure leads to,

**Result 2.4.1.** *The cartel policy is given by the feedback law  $u^c = A^c + B^c l$  with*

$$A^c = \frac{(1 - k)(r + \delta) \left( r - (r + 2\delta) \sqrt{\cdot} \right)}{2((r + 2\delta)\beta(a + (n - 1)b) - 2\delta n(r + \delta))}, \quad (2.4.3)$$

$$B^c = \frac{(r + 2\delta) \left( 1 - \sqrt{\cdot} \right)}{2(a + (n - 1)b)}, \quad (2.4.4)$$

$$\sqrt{\quad} := \sqrt{1 - \frac{2\beta(a + (n-1)b)}{n(r+2\delta)}}, \quad (2.4.5)$$

which converges to the steady state,

$$l_{\infty}^c = \frac{(1-k)(r+\delta)(a+(n-1)b)}{2\delta n(r+\delta) - \beta(r+2\delta)(a+(n-1)b)},$$

that increases with respect to own learning (i.e., a larger  $a$ ), spillovers (i.e., a larger  $b$ ) and decreases with respect to initial costs ( $k$ ), discounting ( $r$ ), depreciation ( $\delta$ ) and the number of firms ( $n$ ) if  $b = b(n)$  according to (2.2.5). In particular, the process of learning cannot start for initial costs exceeding the choke price,  $k > 1$ , no matter how effective learning is.

Setting  $n = 1$  immediately yields the monopoly case,

**Result 2.4.2.** *The production strategy of a monopoly, accounting for intertemporal learning by doing, exceeds the one of maximizing its static profit (based on current costs) and is given by the linear feedback rule,*

$$u^m = \frac{(1-k)(r+\delta)\left(r - (r+2\delta)\sqrt{1 - \frac{2a\beta}{r+2\delta}}\right)}{2a\beta(r+2\delta) - 4\delta(r+\delta)} + \frac{(r+2\delta)\left(1 - \sqrt{1 - \frac{2a\beta}{r+2\delta}}\right)}{2a}l,$$

that converges to the steady state,

$$l_{\infty}^m = \frac{a(1-k)}{(2\delta - a\beta) - \frac{a\beta\delta}{r+\delta}},$$

which exceeds the one of the cartel,

$$l_{\infty}^m = \frac{(1-k)(r+\delta)a}{2\delta(r+\delta) - a\beta(r+2\delta)} > \frac{(1-k)(r+\delta)(a+(n-1)b)}{2\delta n(r+\delta) - \beta(r+2\delta)(a+(n-1)b)} = l_{\infty}^c.$$

### 2.4.2 Oligopoly - Markov perfect equilibrium

Turning to the analysis of the oligopoly, each profit maximizing firm  $i$  has to solve the following HJB equation for its value function  $V^i$ ,

$$rV^i(l_1, \dots, l_n) = \max_{u_i > 0} \left\{ \left[ \left( 1 - \sum_{j=1}^n u_j \right) - k + \beta l_i \right] u_i + V_{l_i}^i \left( au_i + b \sum_{j \neq i} u_j - \delta l_i \right) + \sum_{j \neq i} V_{l_j}^i \left( au_j + b \sum_{k \neq j} u_k - \delta l_j \right) \right\}, i = 1, \dots, n. \quad (2.4.6)$$

which depends on all players' states. The first-order-condition for the maximization on the right hand side of (2.4.6) is given by,

$$1 - \sum_{j \neq i} u_j - k + \beta l_i - 2u_i + aV_{l_i}^i + b \sum_{j \neq i} V_{l_j}^i = 0, \quad i = 1, \dots, n.$$

Although all actions (and all derivatives of the value functions) of  $i$ 's competitors enter only additively, i.e., as a sum, even imposing symmetry,

$$V_{l_i}^i = V_l, \quad i = 1, \dots, n,$$

and

$$V_{l_j}^i = V_m, \quad \text{for all } i = 1, \dots, n, \text{ and } j \neq i,$$

does not allow to reduce the dimension to below  $n$ . Thus, solving for the Markov perfect equilibria turned out to be challenging for large  $n$  as each player adds an additional state, which itself is interdependent with the controls of all players.<sup>16</sup> In general, the linear quadratic

---

<sup>16</sup>More precisely, although one may (and we did) conjecture that it is sufficient to reduce the system to two value functions for two representative players, with the own state  $l$  and state  $m$  for the others, this is not possible, unless  $n = 2$ . Therefore, examples for  $n > 2$  must rely on numerical calculations. To make matters worse, no closed-form solutions for the high-order polynomial equations could be determined in reasonable time, e.g., not in 24h and not even for  $n = 3$ . Therefore, Newton's Method was used instead, which requires different initial values and trials in order to approximate all roots, the number being determined by the fundamental theorem of algebra.

form of the value function of player  $i$ ,

$$V_i = \theta_{i0} + \sum_{j=1}^n \phi_{ij} l_j + \frac{1}{2} \sum_{m,k=1}^n \psi_{i,mk} l_m l_k, \text{ with } \psi_{i,mk} = \psi_{i,km} \quad (2.4.7)$$

solves (2.4.6) for a set of linear feedback strategies for each player's choice of current output. The  $i$ -th player adds  $i + 1$  additional coefficients per player to the system,<sup>17</sup> which yields

$$\Lambda := 1 + 2n + \binom{n}{2} = \frac{2 + 3n + n^2}{2},$$

equations per player or

$$n\Lambda = \frac{n}{2}(1 + n)(2 + n),$$

equations in total to solve for the same number of coefficients. Using symmetry allows to reduce the complexity down to  $\Lambda$ . For example, the 6-player game consists of 168 simultaneous equations, which can be reduced to 28 by imposing symmetry on the coefficients of the value functions in (2.4.7). In case of an oligopoly and in particular of a duopoly – and in contrast to the cartel – a long run equilibrium with both firms producing exists even if one firm starts with a cost disadvantage. Not only that, the same long run outcome results as if both started from identical initial conditions. To see this assume for simplicity two firms such that the optimal policy in feedback form is given by,

$$u_1 = a_0 + a_1 l_1 + a_2 l_2,$$

and consequently by symmetry,

$$u_2 = a_0 + a_2 l_1 + a_1 l_2,$$

for real valued coefficients  $a_i$  such that the implied system of the state equations (2.2.3) is stable. As a consequence, both will converge to the same symmetric equilibrium even if starting from different initial conditions, e.g., without loss in generality firm 1 has a head

---

<sup>17</sup>One linear term, one quadratic term and all interactions with existing states,  $n - 1$ .

start, i.e.,  $l_1(0) > l_2(0)$ .

Having settled the issue of asymmetric starting conditions we make the following observations.

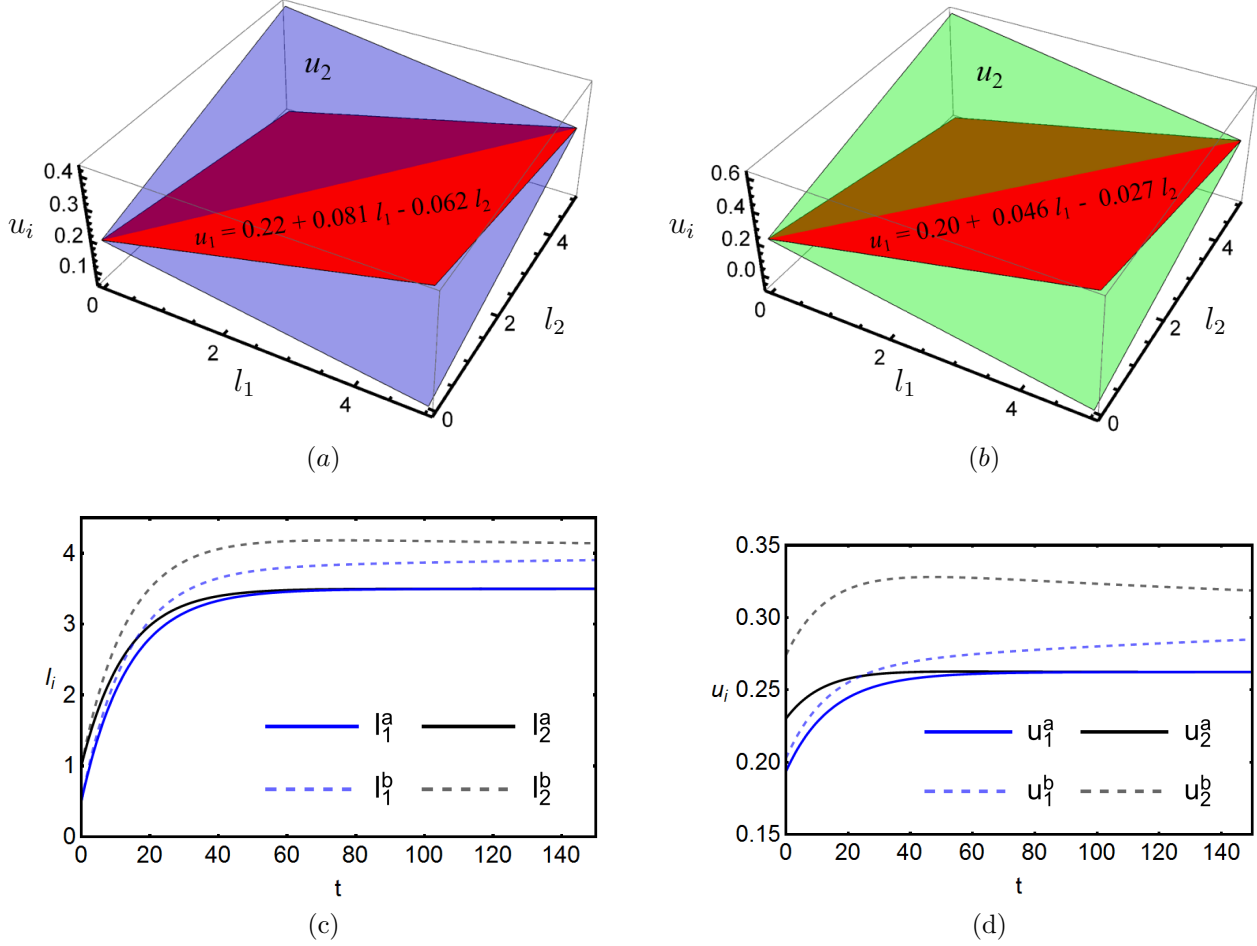


**Figure 2.4.1:** Different Markov perfect symmetric equilibria (a and b) for the duopoly are shown in (a) and (b). The other two graphs in (c) and (d) plot paths of learning and output, departing from different initial conditions,  $l_1(0) = 1$  and  $l_2(0) = 0.5$ .

First, and quite surprisingly, we find that the equilibrium in linear Markov strategies need not be unique. Of course, the possibility of non-singular = nonlinear strategies is well known since Tsutsui & Mino (1990). However, the nonuniqueness in linear strategies puzzled us (we find two) because it is not mentioned in the differential games literature, including Dockner et al. (2000). Furthermore, non-uniqueness originates here in a model with no obvious

distinct feature, except for the impossibility to reduce the dimension of the state space by imposing symmetry. We also checked for asymmetric equilibria but did not find any. Fig. 2.4.1 compares the two different linear Markov perfect (and also symmetric) equilibria and plots the corresponding time paths for different initial conditions. This highlights that the long run outcome does not depend on initial conditions whereas the choice among the two equilibrium strategies, either a or b, matters. More precisely, strategy b, the one with the larger intercept and more sensitivity with respect to learning from own (thus slower convergence) and spillovers, leads to a larger steady state and thus to a higher stationary output than the strategy a. However, strategy b converges much more slowly, in particular for the asymmetric initial conditions shown in Fig. 2.4.1, and this in spite of very similar adjustment speeds for  $l_1 = l_2$ . The convergence to the same steady states from asymmetric initial conditions confirms the intuition from the analysis of the open loop equilibrium for the power function in section 2.3.3.

In order to investigate the issue of existence and uniqueness of equilibria, we perform a bifurcation analysis with respect to the spillover parameter,  $b$  (see Fig. 2.4.2).

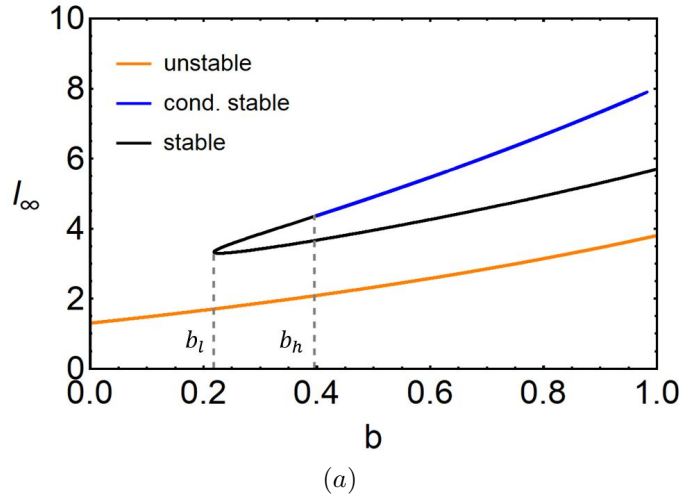


**Figure 2.4.2:** Bifurcation diagram in  $b$  with indicated stability of the given fixed points.

A globally stable Markov perfect equilibrium requires that the corresponding strategies imply stability (i.e., two negative eigenvalues of the Jacobian) of the dynamic system  $(\dot{l}_1, \dot{l}_2)$ . We find that multiple ( $=2$ ) equilibria exist around the reference case (due to  $b = \bar{b}/(n-1) = 0.3$  and we note that, serendipitously, the reference case is included in the parameter region im-

plying multiple MPEs) and that the MPE is unique for stronger spillovers, i.e.,  $b \geq b_h \approx 0.4$ . Economically intuitive is the observation that higher spillovers increase learning and output and this for all MPEs. Next to a locus of unstable saddle points (orange) the figure also includes solutions (blue) that are stable but only along perfectly symmetric paths,  $l_1 = l_2$  for all  $t$ . These solutions cannot resemble subgame perfect equilibria as any unilateral deviation necessarily leads players astray. Hence,

**Result 2.4.3.** *The Markov perfect equilibrium need not be unique. Asymmetry with respect to initial values is eliminated over time, i.e., convergence to the same steady states (respectively) results if starting from different initial conditions.*

The second interesting feature shown in Fig. 2.4.2 is that no linear Markov perfect equilibrium exists if the spillovers are too small, i.e.,  $b \leq b_l \approx 0.2$ . Therefore,

**Result 2.4.4.** *Spillovers of a sufficient magnitude, i.e.,  $b > b_l$ , are necessary in order to support a linear Markov perfect equilibrium.*

This seems puzzling. First, because the equilibrium exists for the limiting static game based on the stationary relation between production and learning. Second, all other solutions including the open loop one (derived in the next section) exist. Third, most (if not all) related games of oligopolistic competition allow for an equilibrium. In particular, the case of no spillovers at all ( $b = 0$ ) leads to a game that is very similar to the many games of oligopolistic competition originating since Reynolds (1987, 1991) in which learning just replaces the firms' capital stocks. How can one explain this puzzle?<sup>18</sup> The conditionally stable solutions (Fig. 2.4.2) offer a clue to solve this puzzle. Along those solutions, the paths still converge to the steady state but only along a perfect symmetric evolution,  $l_1 = l_2$ . If the spillover is too weak, the firm with a disadvantage, no matter how small – either at the beginning or by an error along the evolution – will quit, rendering the other firm the position of a monopoly. The implication for this scenario is that oligopolistic competition must end

<sup>18</sup>In contrast, it is economically intuitive that we lack sensible equilibria for  $a, b \gg 0$ , as costs will eventually be driven down into the negative domain, therefore contributing to the objective in a fictitiously positive way ( $\beta l_i$ ). See also the Appendix 2C.

up as a monopoly. This happens also for increasing the effectiveness of own learning beyond a certain threshold, i.e., for sufficiently large  $a$  s.t.  $a > a_h$ , where  $a_h$  is defined by the saddle-point bifurcation in  $a$  (see Appendix 2C). The reason is similar to the one given above, i.e., as learning from own experience gets larger, relative to the spillovers, any deviation from the equilibrium path cannot be matched by the laggard in any future periods. Hence, no Markov perfect equilibrium exists for such cases. Furthermore, any increase in the number of competitors in the marketplace decreases prices and therefore profits, and by extension, the feasibility of an equilibrium. For this reason, no Markov perfect equilibrium exists for  $n > 2$  (see also Table 2.4.1 in section 2.4.4, although one can construct examples where they do exist).

### 2.4.3 Oligopoly - open loop equilibrium

The open loop equilibrium requires to solve simultaneously the first-order-conditions of the  $n$  firms intertemporal optimization problem. Thus, defining the Hamiltonian of player  $i$  as in the case of the power function, carrying out the calculations as outlined in section 2.3.3, substituting the corresponding linear learning cost function and assuming symmetry yields the rule,

$$u = \frac{1 - k + \beta l + a\lambda}{n + 1},$$

and consequently the following canonical equations,

$$\dot{l} = \frac{(1 - k + \beta l + a\lambda)(a + (n - 1)b)}{n + 1} - \delta l, \quad l(0) = l_0, \quad (2.4.8)$$

$$\dot{\lambda} = (r + \delta)\lambda - \frac{\beta(1 - k + a\lambda + \beta l)}{n + 1}, \quad \lim_{t \rightarrow \infty} e^{-rt}\lambda = 0. \quad (2.4.9)$$

This dynamic system converges along a saddle-point path to the following unique steady state,

$$l_\infty^o = \frac{(1 - k)(a + (n - 1)b)}{(n + 1)\delta - (n - 1)b\beta - a\beta \frac{2\delta + r}{r + \delta}}, \quad (2.4.10)$$

and considering the limit  $n \rightarrow \infty$  for normalized  $b = \bar{b}/(n-1)$ ,

$$\lim_{n \rightarrow \infty} l_\infty^o = 0.$$

Furthermore, the open loop policy can be rewritten as a linear feedback law (see Appendix 2D).

**Result 2.4.5.** *The symmetric open loop equilibrium,  $u = A^o + B^o l$  written in feedback form for a better comparison with the MPE, is given by*

$$\begin{aligned} A^o &= \frac{(1-k)(r+\delta) \left( (n+1)r + \beta b(n-1) - \sqrt{\phantom{x}} \right)}{2(n+1) \left( (\beta b(n-1) - (n+1)\delta)(r+\delta) + a\beta(r+2\delta) \right)}, \\ B^o &= \frac{(n+1)(r+2\delta) + \beta b(n-1) - \sqrt{\phantom{x}}}{2(n+1)(a + (n-1)b)} > 0, \\ \sqrt{\phantom{x}} &:= \sqrt{[(n+1)(r+2\delta) - \beta(2a + (n-1)b)]^2 - 4a\beta^2(a + (n-1)b)}, \end{aligned} \tag{2.4.11}$$

Therefore, it exists, is unique and is increasing with respect to own and independent of others' learning. It converges to the steady state  $l_\infty^o$  in (2.4.10). Again, assuming  $k \geq 1$  rules out long run supply (i.e., starting at a sufficiently small  $l_0 > 0$ , the system converges to  $l = 0$ ) and cannot escape the origin.

Comparative statics are, as expected, similar to Result 2.4.1 (most follows from simple differentiations of (2.4.10)): learning increases in  $a$ ,  $b$ , and decreases with respect to all other model parameters  $k$ ,  $r$ ,  $\delta$  and  $n$  (applying the normalization (2.2.5)).

An interesting question is how the industrial organization affects learning and consequently lowers costs and prices. The explicit solution (2.4.10) allows to address this issue. First, suppose that spillovers are small ( $\bar{b} = \bar{b}_l$  small) and that we increase competition by adding a new supplier. As a consequence of the above result, learning will unambiguously decrease. However, a larger spillover parameter, say  $\tilde{b} > \bar{b}_l$  which can be explicitly computed (also see Appendix 2D), allows to compensate for that. Now consider large spillovers,  $\bar{b}_h$  close to  $a$ . The additional firm will again lower each firm's production but will increase aggregate output due to lower prices and the costs due to the spillover. Yet, an increased spillover

cannot counter the negative effect of  $n$  on learning because the necessary larger spillover (as in the above example of  $\bar{b}_l$ ) would then violate assumption (2.2.4).

The puzzling outcome, the lack of a Markov perfect equilibrium, requires to address the issue of stability in detail and off symmetric initial conditions also for the open loop equilibrium. That is, whether the long run outcome  $l_\infty^o$  is reachable for different initial conditions or only for identical ones. Since this property, or lack thereof, is hidden by assuming symmetry in the derivation above, we consider a duopoly, as in section 2.3.3 for the power function and MPE, in detail. Substituting the corresponding open loop strategies,

$$\begin{aligned} u_1 &= \frac{1 - k + 2\beta l_1 - \beta l_2 + 2a\lambda_1 - a\lambda_2}{3}, \\ u_2 &= \frac{1 - k + 2\beta l_2 - \beta l_1 + 2a\lambda_2 - a\lambda_1}{3}, \end{aligned}$$

into the canonical equations and computing the eigenvalues of the Jacobian (evaluated at the steady states identical to above) yields

$$\begin{aligned} e_{12} &= \frac{r - b\beta \pm \sqrt{b^2\beta^2 + (r + 2\delta)(2b\beta + (r + 2\delta) - 4a\beta)}}{2}, \\ e_{34} &= \frac{3r + b\beta \pm \sqrt{b^2\beta^2 - 3(r + 2\delta)(4a\beta - 3(r + 2\delta))}}{6}. \end{aligned}$$

Now, substituting the usual parameters, yields a pair of negative eigenvalues and thus stability. The stability is lost, or more precisely, restricted to initial conditions along the  $45^\circ$  line if the second eigenvalue,  $e_2 = r - b\beta - \sqrt{\quad}$ , turns positive, which happens, e.g., roughly for  $a \geq 1.4$ . Therefore, the open loop equilibrium need not exist (except for the non-generic case of  $l_i(0) = l(0)$  for all  $i$ ).

#### 2.4.4 Comparison

We use the same parameters as in (2.3.18), i.e., a significant but not too large spillover, learning depreciates at 10% (i.e., a time constant of ten periods) at twice the level of discounting, 5% per period. We account for the linear cost depression by setting  $\beta = 0.05$ . This configuration is compatible with the constraint that initial costs must be below the choke

price, i.e.,  $k < 1$ , no matter how effective learning is.<sup>19</sup>

Table 2.4.1 provides an overview of the results for the analyzed markets and for the same set of players as in Table 2.3.1.

Besides the already mentioned properties with respect to existence and uniqueness, we note that the general structure of the linear model is similar to the nonlinear case: the monopoly runs down the learning curve further than the cartel does and therefore produces more. Again, in the oligopoly aggregate production increases for more players and the opposite is true for a cartel. Since in any symmetric steady state,

$$u_\infty = \frac{\delta l_\infty}{a + b(n-1)} = \frac{\delta l_\infty}{a + \bar{b}}, \quad (2.4.12)$$

the fixed points of all market forms, with their respective size, are located on the same straight line through the origin and with the slope in (2.4.12). This also means that  $u_\infty^C/x_\infty^C = u_\infty^{OL}/x_\infty^{OL} = u_\infty^{FB}/x_\infty^{FB}$  and thus similar induced transient dynamics as well.

Even though total production is largest in an oligopolistic market, the existence of the forgetting rate outbalances individual outputs, such that costs ultimately increase with competition. Compared to the open loop outcome, the increased intensity of the feedback solutions lead to more learning, therefore more production but lower profits.

Fig. 2.4.3a compares all computed strategies, assuming symmetric states ( $l_1 = l_2 = l$ ) and aggregating production. The intersection of the strategies with the steady state condition ( $\dot{l} = 0$ , different for the monopoly) determines stationary learning. As expected, oligopolistic competition induces more learning and both of the more aggressive linear Markov strategies exceed their (unique) open loop counterpart. The monopoly produces less than an oligopoly, but attains the highest level of learning and thus the lowest costs of production. A cartel produces even less and, due to imperfect spillovers, ends up at the far lowest level of learning but with high prices.

Fig. 2.4.3b returns to the familiar approach of estimating the learning coefficient ( $\beta$ ) from price data. This chart highlights that this common procedure can be very problematic in non-competitive markets, as the ‘observed’ price depression falls short of the cost depression. More precisely, while cost depression is assumed at  $\beta = 0.05$ , the (absolute) slopes of the

---

<sup>19</sup>In this setting only a state of learning at  $l = 10$  reduces the costs to 0.

|            | n | Monopoly | Cartel | Oligopoly |       |       |
|------------|---|----------|--------|-----------|-------|-------|
|            |   |          |        | OL        | FB    |       |
| $u_\infty$ | 2 |          | 0.173  | 0.250     | 0.262 | 0.302 |
|            | 4 | 0.429    | 0.075  | (0.129)   | —     | — — — |
|            | 6 |          | 0.047  | [0.086]   | —     | — — — |
| $U_\infty$ | 2 |          | 0.346  | 0.500     | 0.525 | 0.604 |
|            | 4 | 0.429    | 0.300  | (0.517)   | —     | — — — |
|            | 6 |          | 0.284  | [0.516]   | —     | — — — |
| $l_\infty$ | 2 |          | 2.308  | 3.333     | 3.499 | 4.024 |
|            | 4 | 4.286    | 1.200  | (2.069)   | —     | — — — |
|            | 6 |          | 0.811  | [1.475]   | —     | — — — |
| $c_\infty$ | 2 |          | 0.385  | 0.333     | 0.325 | 0.299 |
|            | 4 | 0.286    | 0.440  | (0.397)   | —     | — — — |
|            | 6 |          | 0.459  | [0.426]   | —     | — — — |
| $\pi$      | 2 |          | 1.370  | 1.758     | 1.812 | 1.966 |
|            | 4 | 1.953    | 0.480  | (0.505)   | —     | — — — |
|            | 6 |          | 0.282  | [0.232]   | —     | — — — |

**Table 2.4.1:** Total discounted profits (for  $l_{i0} = 0.5$ ) and steady state values for production, costs, learning and differing number of players.

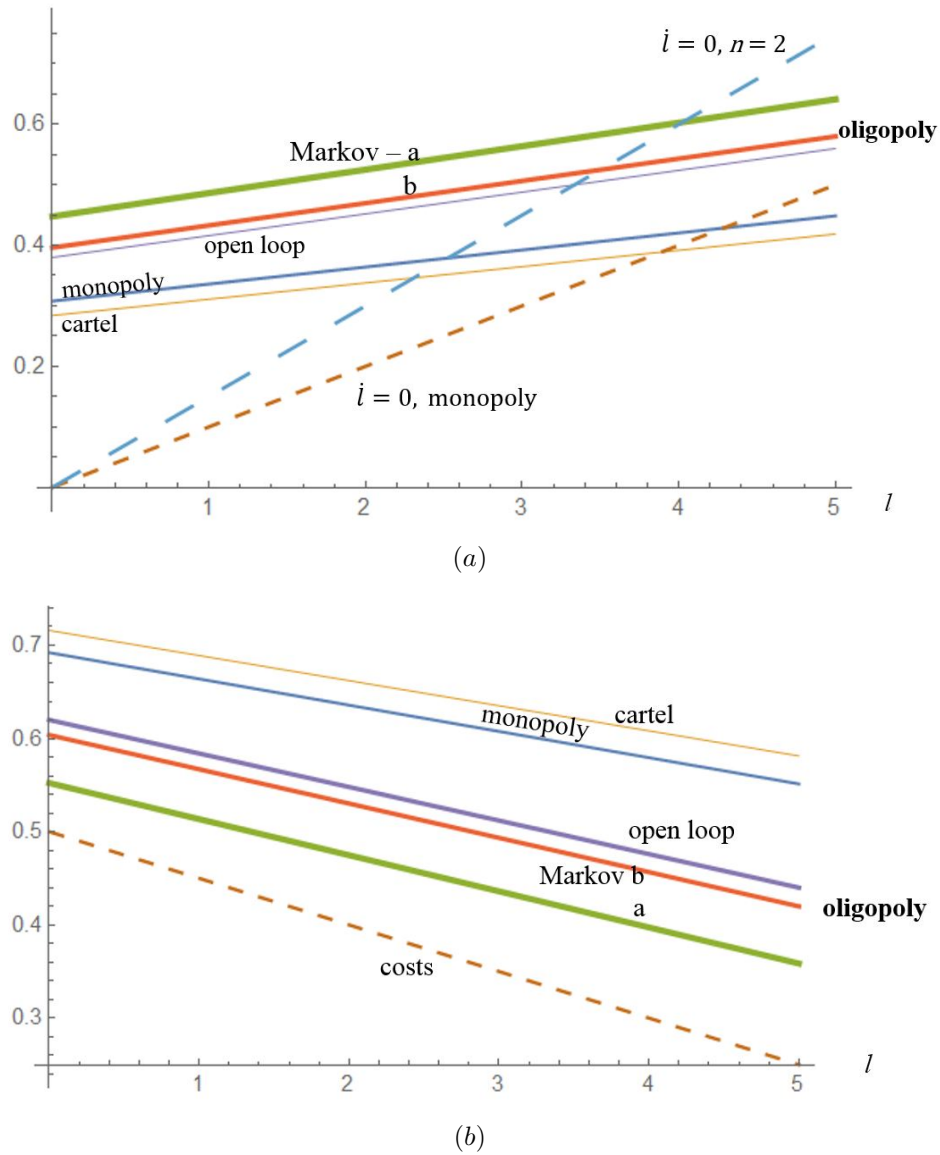


**Figure 2.4.3:** (a) Comparison of all (aggregated) strategies under all market forms. (b) compares prices versus learning with costs and shows that the price degression is much less than the cost degression.

resulting prices are roughly only half of that for the monopoly and cartel and between 0.027 (cartel) and 0.039 for the duopoly (Markov perfect equilibrium a).

Returning to the multiplicity of equilibria we note that the game has no subgame perfect

solution for  $n > 2$  under the usual conditions.<sup>20</sup> The reason for this is that the additional firm will lead to more competition, thus lower outputs, hence less learning and consequently higher unit costs. Coupled with the effect that the consumer price decreases (as total outputs increase), this very quickly leads to a sudden reduction in profits. Hence, quantity choosing firms will not begin the learning process in order to surpass the initial burden of costs,  $k$ , and – every competing firm having the same beliefs – therefore no equilibrium results. As mentioned above, in addition to the locus of stable strategies there also exist(s) a manifold(s) of unstable strategies that repel close orbits as well as conditionally stable fixed points that only attract identical ICs.

Finally, Fig. 2.4.4a compares the net present value of social surplus for the three market forms and different  $n$ . Fig. 2.4.4b and 2.4.4c decompose social surplus into consumers' and producers' surplus. Not surprisingly, consumers prefer the oligopolistic outcome due to the lowest price charged and dislike the formation of a cartel. The Result 2.3.2 in section 2.3.5 holds true for the linear case as well, i.e., a market composed of only a few and not too many oligopolists is the best outcome in terms of total welfare. Again, the intuition holds that every added oligopolist induces decreased individual production and coupled with the depreciation of knowledge this leads to less learning and thus larger costs. Whether the monopoly outperforms the oligopolistic market for large  $n$  is an open question (it is not the case for  $n = 6$ , contrasting the OL outcome in the nonlinear example). Producers' welfare is almost identical in all market forms (except of course for the monopoly), which means that the cartel should be avoided, as it charges the largest price and is ranked last by consumers.

## 2.5 Final remarks

We have analysed intertemporal market equilibria under learning by doing, using first the familiar power function representation of costs affected by learning and then a linear cost function in order to solve for the Markov perfect equilibrium. The major results are:

1. While it is well known since Tsutsui & Mino (1990) and Dockner & van Long (1993) that Markov strategies can include a family of uncountable many nonlinear equilibria, the

---

<sup>20</sup>This also holds for  $n = 3$ , which is not shown here due to brevity.

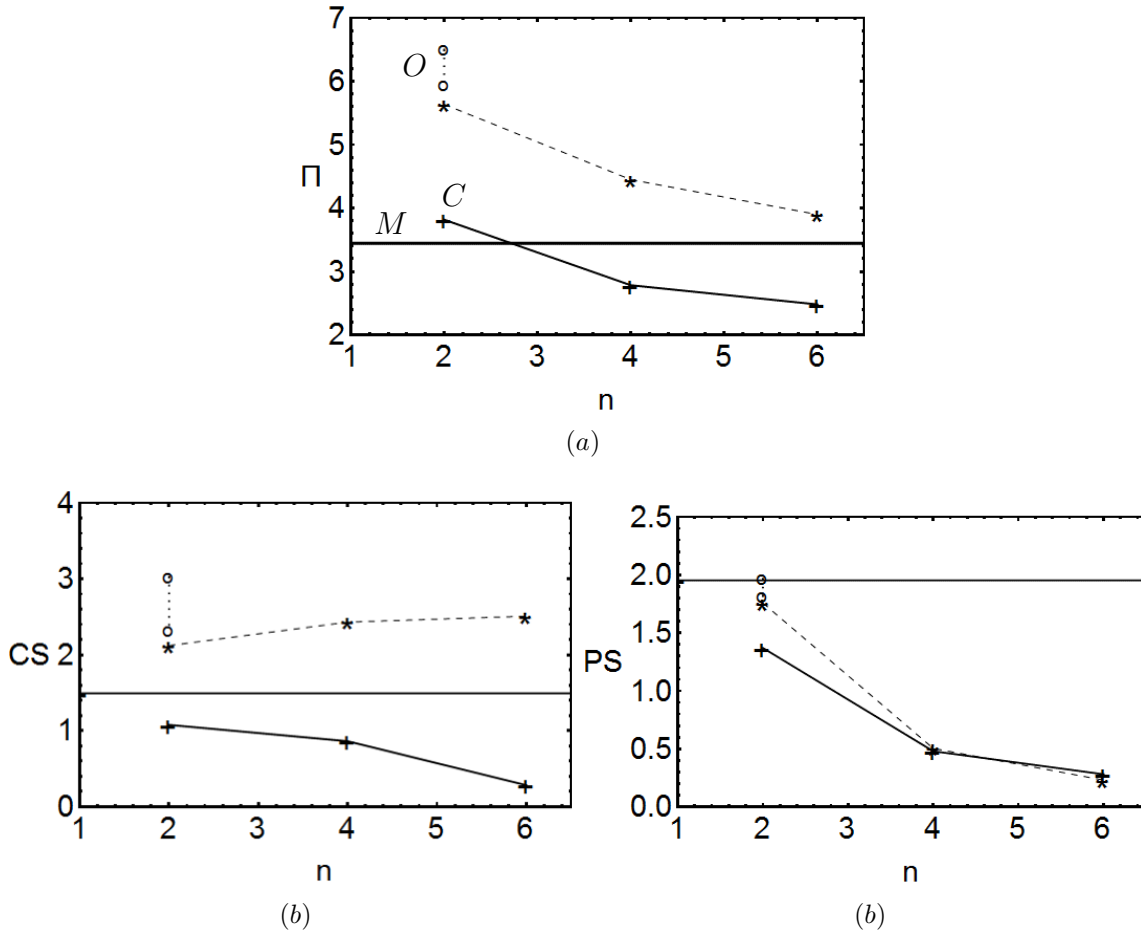


**Figure 2.4.4:** Net present value of social welfare (a), decomposed in consumer (b) and producer surplus (c) for  $n = 2, 4, 6$ . The horizontal line indicates the monopoly, the dashed line the cartel and the dotted line the oligopoly.

restriction to linear strategies (or more general to singular strategies, compare Rowat 2007 and Wirl 2007) usually renders uniqueness. Not here, and to our big surprise, multiple equilibria can and do exist in linear Markov strategies.

2. Even more puzzling is the observation that no MPE need exist in this at first sight very simple game in which all other solutions exist. This is a consequence of competition driven by learning by doing effects, which severely limit the viability of all firms staying in the race, in particular if spillovers are small and a firm starts with a disadvantage or makes erroneous moves at some future date (e.g., of producing too little).

3. A direct consequence of this non-existence is that a Markov perfect oligopolistic equilibrium in industries characterized by learning by doing can only exist if the spillovers from learning are sufficiently large (and the number of firms is small).
4. Perfect competition cannot support intertemporal learning. This follows if increasing the number of firms to infinity within a framework of oligopolistic competition but holds also for a direct setup of a competitive equilibrium a la Krugman (1991).
5. (Oligopolistic) Competition may, but need not always, be beneficial. However, a cartel results in the worst outcome from the consumers' perspective, as it falls short on learning but charges the monopolistic mark up.
6. The long run open loop outcome remains unique even for an asymmetry in initial conditions (assuming that all firms produce).
7. Strategic intertemporal interaction, i.e., a Markov perfect equilibrium, results in more learning compared with a game in open loop strategies – at least under the caveat that all firms remain in the market.
8. A Markov perfect equilibrium, if existing, allows per definitionem to start from different initial conditions, which nevertheless lead to convergence to the same steady state.

One major implication of our analysis is that the pair of competition and learning by doing does not match well. Of course, not under perfect competition, but also for cartels and oligopolies. In particular, a MPE exists only over a fairly restricted parameter domain, and if, it may lead to multiple equilibria. Open loop strategies, e.g., due to the impossibility to observe the competitor's degree of learning, allow for learning by doing as an oligopolistic outcome over a larger domain of weak spillovers.

There are a number of extensions to our analysis and this in many directions. Starting with demand, one may consider unit demand by each consumer accounting for saturation effects. We model the spillover depending on aggregate output of the competitors. Therefore, one extension is to include neighbourhood structures determining the spillover. We assume that all firms produce, which excludes the possibility of entry deterrence in duopoly (and oligopoly) and the possibility to drive one of the competitors out of the market. Entry deterrence by the incumbent suggests a Stackelberg setup. However, such equilibria will not be time consistent in the open loop case and the analysis of Stackelberg equilibria in feedback strategies (with

either short run or global commitment) is very tedious if not impossible for the nonlinear cost function; the linear-quadratic setup may therefore be a more suitable model for this kind of analysis. The puzzling feature of multiple linear Markov perfect equilibria suggests to explore the underlying arithmetical reasons and conditions for such an outcome in other models too and Engwerda (2007, 2009, 2016) may be helpful for such an undertaking.

On the empirical side, it would be interesting to quantify the magnitude of spillovers and how that affects the outcomes across different industries and industrial organizations. In particular, how far can larger spillovers counter the negative effect of increased competition on learning?

## Chapter 3

# Climate engineering in an interconnected world - the role of tariffs

### Abstract

This paper investigates strategic trade policies as a response to negative externalities linked to climate engineering. Parties negatively affected, or which only perceive damages, may react to geoengineering by deploying trade sanctions, i.e., the imposition of tariffs. By introducing a dynamic trade model we show that geoengineering-averse countries have an incentive to implement or increase existing tariffs when the other country uses geoengineering. Our contribution is to highlight that potential consequences on trade should be taken into account before climate engineering techniques are applied. This is particularly crucial in our globalized world since a successful climate policy demands large scale if not global cooperation.

**Keywords:** Global warming, Externalities, Tariffs, Asymmetric differential game, Climate engineering.

**JEL index:** C72, C73, F18, H23, Q54, Q55, Q56.

### 3.1 Introduction

The extraction and use of fossil fuels arising from the industrial revolution has brought tremendous and so far unseen growth in populations and incomes.<sup>21</sup> However, these gains and the modern way of life have been bought by future damages due to anthropogenic change in the composition of Earth's atmosphere and the consequent increases in temperature. The concentration of atmospheric  $CO_2$  has increased from roughly 280ppm at pre-industrial times to well over 400ppm at present. Attempts to limit future damages have been notoriously slow due to a number of different reasons. Well recognized and analysed is the global tragedy of the commons involving more than seven billion people or roughly 200 governments. Moreover, the issue is inherently dynamic as global warming is caused by the stock of greenhouse gases accumulated since the Industrial Revolution (see Dockner & Long 1993 for their seminal contribution). If this were the only problem, it could be solved, as has been proven by the Montreal protocol concerning the global agreement of banning CFC's. Therefore, the major obstacles must be somewhere else. The primary issue is the lack of available substitutes, i.e., of renewable energy, at the necessary scale, cost and time. Further problems arise from the issue of proper time discounting (compare the debate in Nordhaus 2007), economic and meteorological uncertainties as well as geographic asymmetry (some latitudes will gain from global warming). If these problems were not enough, one must add that governments cannot commit which renders R&D into alternatives problematic and at least will slow them down (Wirl 2014). Not to mention the naive policies of the past and present that ignore elementary system properties, e.g., in any electricity supply where the marginal unit is fossil, and this holds all over the world (even for Norway if one includes all NordPool countries), any addition of an electric car is fossil fuelled at 100%. Nevertheless, electric cars are currently heavily subsidized not only directly at the purchase (e.g., buying an electric car in Austria warrants up to €6.000 in subsidies for reducing GHG emissions if at all at a negligible amount) but also indirectly by being exempted from contributing to the infrastructure (roads, garages, police, hospitals, etc.), which is covered by petrol taxes. These resources would be better utilized somewhere else, e.g., in R&D. Furthermore, constraints from asymmetric information (e.g.,

---

<sup>21</sup>Both, anyway linked by Malthusian dynamics, show negligible growth till around 1700 since the agricultural revolution 10.000 BC (Clark 2008).

Joint Implementation, Clean Development Mechanism, energy conservation, currently in the form of white certificates imposed on energy utilities, which make the ‘butcher selling fish’ and induce cheating, Ruff 1988) are entirely ignored, even the rebound effect is ignored (e.g., Bertoldi 2011) that already Jevons observed when analyzing British coal in the 19th century. The COP21 meeting in Paris generated some optimism in form of a new agreement, but the lack of sanction mechanisms and insufficient early contributions together with recent experimental evidence of strategic behaviour in pledge-and-review processes curb enthusiasm (Barrett & Dannenberg 2016). The problems listed above will not go away by wishful thinking. Indeed, recent trends towards more nationalism and protectionism may make it even more difficult to establish the necessary level of global cooperation to address climate change. Not to mention political changes like the election of Trump who promised to bring back coal. Given the dire situation of reducing greenhouse gas emissions, alternative, technical solutions including climate- or geoengineering enter the debate. Defined as the “... deliberate large-scale intervention in the Earth’s climate system, in order to moderate global warming” (The Royal Society 2009, p. ix) these methods can be broadly categorized into two classes: Carbon Dioxide Removal (CDR) and Solar Radiation Management (SRM). CDR methods directly remove  $CO_2$  from the atmosphere and are generally considered to be more safe but also more expensive and act on relatively long time-scales. No large-scale CDR technology exists today that could remove  $CO_2$  from the atmosphere at the required rate and cost. With respect to SRM, the most promising technique is the injection of sulfate aerosols into the stratosphere, which aims at increasing Earth’s albedo - the proportion of reflected sunlight. These aerosols increase reflectivity and therefore directly influence Earth’s radiation balance. This method could effectively and timely lower global average temperatures at manageable projected costs. However, many considerable drawbacks associated with SRM should not be neglected (Robock 2008). It is unclear whether the modern climate engineering techniques would be captured by any of the already existing international treaties (Victor 2008). Many governance issues related to geoengineering exist. Who should decide and how should the optimal level of climate engineering be decided (Barrett 2008, Weitzman 2015)? Climate modellers expect highly asymmetric effects due to changing precipitation patterns and, in general, warn of many unintended consequences not foreseen at present (Schmidt et al. 2012, Bala et al. 2008). Another crucial argument against SRM is that once deployed, it is hard

to stop, even if evidence of substantial harm shows up, because terminating would lead to a sudden jump in temperatures.

The suggested technologies are at best imperfect substitutes for abatement. Therefore, more research is needed on technical as well as economic aspects of geoengineering, not only because of the continuing lack of sufficient abatement, but also because of the increasing threat that many of Earth's systems may pass a tipping point relatively soon (Lenton et al. 2008). Some of the effects associated with catastrophic regime shifts can potentially be damped or at least delayed by appropriate technological intervention.

We develop a dynamic model of strategic interaction between two countries diverging in their perception of geoengineering and production possibilities. The model structure is similar to the asymmetric transboundary pollution game with a geoengineering option in Manoussi & Xepapadeas (2017). The difference is that we study a fully asymmetric trade game, in the sense of entirely different objectives for each country, instead of different parameter values only. In particular, we let only one player apply geoengineering, as opposed to Manoussi & Xepapadeas (2017) or Heyen et al. (2017), where the prospects of a geoengineering-war are studied. Given the interconnected channels of production via trade in a globalized world, production and hence abatement of others can be strategically influenced, especially in a non-cooperative setting. Our aim is to account for trade by investigating the potential role of tariffs as an instrument of a country negatively affected by climate intervention of others. Economic modelling of negative external effects due to geoengineering are not new in the literature. Among others see Moreno-Cruz & Keith (2013), Urpelainen (2012), Millard-Ball (2012) and Manoussi & Xepapadeas (2017). How can one counter such climate intervention of others? Since the corresponding SRM techniques discussed above do not require global reach – once aerosols have been injected locally, stratospheric winds will distribute them globally – directly interfering with geoengineering practices seems unlikely. Due to the projected costs and uncertainty whether SRM is covered by existing treaties, it can be implemented without universal agreement, rendering veto power and voting mechanisms ineffective. While the potential for military conflict over the thermometer exists (Schelling 1996), we focus on an economic instrument often used in the past and present, namely tariffs. It is well known that from a general welfare perspective tariffs are inefficient as they distort prices. Nevertheless, they are often used to protect domestic industries or to sanction others. In the

environmental area tariffs have traditionally played a minor role. Recently, tariff issues have been extensively discussed in two directions (but independent of geoengineering). First, the introduction of carbon border tax adjustments, i.e., carbon is also taxed if it enters as an input of imported goods. Second, the use of trade sanctions (e.g., exclusion of the WTO) has been proposed in the area of international environmental agreements in order to foster the necessary international cooperation in climate change mitigation policies (Nordhaus 2015).

The paper is structured as follows. Section 3.2 introduces the framework of the bilateral trade game. In section 3.3 we investigate a setup where one country has monopoly power in a foreign market. We consider both cases where the trade partner, which is averse to geoengineering, accounts for temperature change and where it does not. Afterwards, in section 3.4 we apply the same analysis to the case of duopolistic competition in production. Finally, section 3.5 concludes.

## 3.2 Framework

We consider a home country ( $H$ ), which exports its produced goods to a foreign country ( $F$ ), which might face negative spillovers from  $H$ 's geoengineering activities. In the following, we explore two different foreign market structures – one where  $F$  is not engaging in production (monopoly) and one where it does (Cournot). Global warming results from increasing the  $CO_2$  concentration in the atmosphere, which arises here solely due to total production in both countries,

$$\dot{P}(t) = q_H(t) + q_F(t) - \delta P(t), \quad P(0) = P_0, \quad (3.2.1)$$

where  $q_H$  and  $q_F$  denote the quantities produced by industries in  $H$  and  $F$ , respectively. Since depreciation is very small (Nordhaus 1991 estimates a time constant of 200 years but many natural scientists put this figure even much higher), we set  $\delta = 0$ , which does not alter our main findings and simplifies some of the anyway cumbersome formulas.

We allow country  $H$  the use of geoengineering,  $g$ , which has the capacity to reduce temperature levels (to  $P - g$ ) at quadratic costs. Given that the speed of temperature decrease is

effectively instantaneous we let geoengineering reduce it immediately.<sup>22</sup> The above mentioned negative external costs of SRM may arise from changing precipitation patterns or unwanted changes in average temperature, but may also be of perceived nature. Artificially changing major geophysical properties of Earth's systems is a controversial topic with little awareness and knowledge of the general public (see Corner et al. 2012 for an overview). However, as attention devoted to this topic increases, public attitudes will form more clearly. The range of public's perception will be wide, from complete acceptance on one side to outright rejection on the other and there will likely be more weight attached to the latter part of the spectrum (The Royal Society 2009, Heyen et al. 2015). Scepticism like 'humans should never interfere with Earth's fundamental systems' does not seem entirely unjustified, especially given past experiences with originally very promising technologies such as nuclear power. Indeed, currently we observe this in Europe in the context of genetically modified food. Therefore, we assume in our baseline models that country  $F$  neglects the increasing pollution stock but perceives geoengineering negatively. We propose a linear negative externality term (instead of the standard quadratic one) in order to account for the fact that the marginal damage from geoengineering is already large at low levels. Afterwards, we relax the assumption of  $F$ 's ignorance in the two respective models and explore the effects of both damages on optimal trade policies.

### 3.3 Monopoly

The exports of the firm in  $H$  are the only source of income, hence the firm's and the government's objective coincide.<sup>23</sup> In addition,  $H$ 's firm has market power in  $F$ . If  $p(t)$  denotes the price charged per unit of the good in period  $t$ , foreign normalized and linear demand is given by  $1 - p(t)$ . The final consumer price consists of  $H$ 's export price  $p$  and  $F$ 's imposition of tariff  $\tau$ . Therefore, the profit of  $H$ 's firm is,

$$\pi(p, \tau) = (1 - p(t) - \tau(t)) p(t)$$

<sup>22</sup>Similar, but simpler compared to the way it is modelled in Manoussi & Xepapadeas 2017.

<sup>23</sup>In this sense we assume that the firm or the industry which is producing is either owned by  $H$  or that there exists a competitive market in  $H$  for which the government charges export taxes (which affect prices).

if we ignore costs (or respectively normalize for constant unit costs). The stock of pollution evolves according to (3.2.1), which implies,

$$\dot{P}(t) = 1 - p(t) - \tau(t) \quad , \quad P(0) = P_0. \quad (3.3.1)$$

The objective of country  $H$  is to maximize the net present value (using the constant discount rate  $r > 0$ ) of profits minus expenditures for geoengineering and convex damage due to increased temperature,

$$\max_{\substack{p(t) \geq 0 \\ g(t) \geq 0}} \int_0^\infty e^{-rt} \left[ (p(t) - p(t)^2 - \tau(t)p(t)) - \frac{g(t)^2}{2} - \frac{\alpha_H}{2} (P(t) - g(t))^2 \right] dt. \quad (3.3.2)$$

The benevolent government of country  $F$  maximizes its net present value of welfare consisting of consumer surplus from imports and tariff revenues less damages caused by geoengineering and changes in temperature,

$$\max_{\tau(t)} \int_0^\infty e^{-rt} \left[ \frac{1}{2} (1 - 2p(t) + p(t)^2 - \tau(t)^2) - \beta g(t) - \frac{\alpha_F}{2} (P(t) - g(t))^2 \right] dt. \quad (3.3.3)$$

### 3.3.1 $F$ disregards temperature changes

First, we analyze the case that country  $F$  does not care about global warming, i.e., for  $\alpha_F = 0$ . We are interested in a subgame perfect and hence a strongly time-consistent equilibrium (Dockner et al. 2000) and thus compute the (linear) Markov perfect strategies for each player. Therefore, we set up the Hamilton-Jacobi-Bellman equations (now and subsequently suppressing the argument of time),

$$\begin{aligned} rV_H &= \max_{p \geq 0, g \geq 0} \left\{ (p - p^2 - \tau p) - \frac{g^2}{2} - \frac{\alpha_H}{2} (P - g)^2 + V'_H(1 - p - \tau) \right\}, \\ rV_F &= \max_{\tau} \left\{ \frac{1}{2} (1 - 2p + p^2 - \tau^2) - \beta g + V'_F(1 - p - \tau) \right\}, \end{aligned} \quad (3.3.4)$$

where  $V_i$  represents the value function of country  $i = H, F$ . The intra-period Nash equilibrium results from solving the first-order optimality conditions on the right hand sides of (3.3.4)

for both players,

$$\begin{aligned} p &= \frac{1 - \tau - V'_H}{2}, \\ g &= \frac{\alpha_H}{1 + \alpha_H} P, \\ \tau &= -V'_F. \end{aligned} \tag{3.3.5}$$

The exporting country  $H$  chooses a price above the monopoly level given the tax. The reasons are that  $H$  is aware of future damages associated with today's emissions ( $V'_H < 0$ ) and therefore has an incentive to pre-empt  $F$ 's future taxes. The choice of geoengineering is given right away by a feedback rule independent of the value function. Country  $F$ 's tax looks Pigouvian, because it is not explicitly correcting for the monopoly distortion, as the tax equals the net present value of the marginal damage from higher temperatures. Nevertheless, it appropriates parts of  $H$ 's monopoly rent.

Linear quadratic games allow for quadratic solutions of the value functions, which imply linear strategies. The analysis of equilibria in nonlinear strategies (Dockner & Long 1993, Tsutsui & Mino 1990) is ignored due to the complexity of our asymmetric framework. Therefore, we guess,

$$\begin{aligned} V_H &:= \theta_1 + \theta_2 P + \frac{\theta_3}{2} P^2 & \implies V'_H &= \theta_2 + \theta_3 P, \\ V_F &:= \psi_1 + \psi_2 P + \frac{\psi_3}{2} P^2 & \implies V'_F &= \psi_2 + \psi_3 P. \end{aligned} \tag{3.3.6}$$

Plugging (3.3.5) and (3.3.6) into (3.3.4) and rearranging terms yields a system of six simultaneous equations in the six coefficients of the two value functions  $V_H$  and  $V_F$  in (3.3.6). Since optimal strategies do not depend on the intercepts we can drop them and solve the resulting system of four equations in  $\theta_2, \theta_3, \psi_2$  and  $\psi_3$ .

**Result 3.3.1.** *The linear Markov perfect strategies for the asymmetric differential game are given by (3.3.5) and (3.3.6) with the coefficients,*

$$\begin{aligned} \theta_2 &= 2A [(2B - C - D)(r + r\alpha_H - \alpha_H\beta)], \\ \theta_3 &= \alpha_H A [2D - C - 4B], \end{aligned}$$

$$\begin{aligned}\psi_2 &= \alpha_H \beta A [D - 2B - 2C] - rA(1 + \alpha_H) [C + D - 2B], \\ \psi_3 &= \alpha_H A [C + D - 2B]\end{aligned}$$

and

$$\begin{aligned}A &= \frac{1}{9\alpha_H r^2 (1 + \alpha_H)^2} \quad , & B &= \sqrt{r^4 (1 + \alpha_H)^3 (3\alpha_H + r^2 (1 + \alpha_H))}, \\ C &= 3r\alpha_H (1 + \alpha_H) \quad , & D &= 2r^3 (1 + \alpha_H)^2.\end{aligned}$$

It holds that  $\psi_2 < 0$  for all  $r, \beta, \alpha_H > 0$  and that  $\theta_2 < 0$  for  $r, \alpha_H > 0$  and

$$\alpha_H < \frac{r}{\beta - r} \iff \beta < \frac{r(1 + \alpha_H)}{\alpha_H}, \quad (3.3.7)$$

which is assumed to hold throughout the paper. In terms of the coefficients that determine the slopes we find that  $\psi_3 > 0$  and  $\theta_3 < 0$  for all  $r, \beta, \alpha_H > 0$ , meaning that the optimal price strategy is always upwards-sloping while the optimal tariff is downwards-sloping. Sensitivity analysis yields,

$$\begin{aligned}\partial\theta_2/\partial\beta &= -2\alpha_H A (2B - C - D) > 0, \\ \partial\psi_2/\partial\beta &= \alpha_H A (D - 2B - 2C) < 0, \\ \partial\theta_3/\partial\beta &= \partial\psi_3/\partial\beta = 0,\end{aligned}$$

which implies that a larger externality due to geoengineering (i.e., a larger  $\beta$ ) decreases the intercept of the optimal price strategy of the exporter and increases the intercept of the optimal tariff strategy but does not affect the slopes of the strategies due to linear external costs in  $g$ . As a consequence, lower prices and larger tariffs are charged for all pollution stock levels if the externality of geoengineering is larger.

Clearly,  $\lim_{\alpha_H \rightarrow 0} \theta_i|_{\beta=0} = \lim_{\alpha_H \rightarrow 0} \psi_i|_{\beta=0} = 0$  for  $i = 2, 3$ , imply the static monopoly price and an absence of tariffs. A tax would generate revenues at the expense of consumer surplus leading to a triangular dead-weight loss. However, due to  $\psi_2|_{\alpha_H > 0, \beta=0} < 0$  and  $\psi_3|_{\alpha_H > 0, \beta=0} > 0$  country  $F$  finds it optimal to implement a tax transiently even if it does not suffer from  $H$ 's geoengineering activity. The reason is that for larger values of  $P$ , country  $H$  (suffering

from increased temperatures) has an incentive to charge a higher price, which induces less production and hence lower pollution stocks over time. This deviation to larger price levels of  $H$  is anticipated and can be exploited by  $F$  through the introduction of a tariff. On these grounds  $F$  accrues rents through its tax policy (part of the price increase that  $H$  plans) which would otherwise go to  $H$ . Hence, despite its Pigouvian look the optimal tariff includes a rent shifting component. The Pigouvian component of the tax is given by,

$$\begin{aligned}\tau_{Pig} &= \tau - \tau|_{\beta=0} = -\psi_2 - \psi_3 P - \left(-\psi_2|_{\beta=0} - \psi_3|_{\beta=0} P\right) \\ &= -\psi_2 + \psi_2|_{\beta=0} \\ &= \alpha_H \beta A (2B + 2C - D),\end{aligned}$$

and is thus constant with respect to time and the pollution stock, which rests at

$$P_\infty = r \left(1 + \frac{1}{\alpha_H}\right) - \beta. \quad (3.3.8)$$

Fig. 3.3.1 plots the time paths implied by the above Markov perfect equilibrium. The six paths correspond to 1/5 increments in the external costs from geoengineering,  $\beta$ . A larger externality leads  $F$  to charge higher tariffs since this is its only option of sanctioning  $H$ . Surprisingly, the tax is declining over time although external costs are increasing due to more geoengineering, countering the increase in  $CO_2$  concentration. The reason is that, as mentioned above, the tax includes a component that appropriates monopoly rents. The environmental Pigouvian component,  $\tau_{Pig}$ , is increasing in  $\beta$  but constant over time (as computed above).

Facing higher tariffs, country  $H$  has an incentive to lower its export prices. Since in steady state there cannot be positive production, the consumer price must equal the choke price,  $\tau_\infty + p_\infty = 1$ , which together with the previous arguments implies that the quantity imported declines faster for more geoengineering-averse countries. This in turn decreases the stock of pollution too, which reduces  $H$ 's incentive to use geoengineering in the first place. This is what country  $F$  wants to achieve when introducing and increasing tariff levels. Fig. 3.3.2 compares total payoffs for each country under a tariff-free regime (solid lines) vs. the regime in which the tariff option is introduced (dashed lines) for  $P_0 = 0$ . The payoffs under the

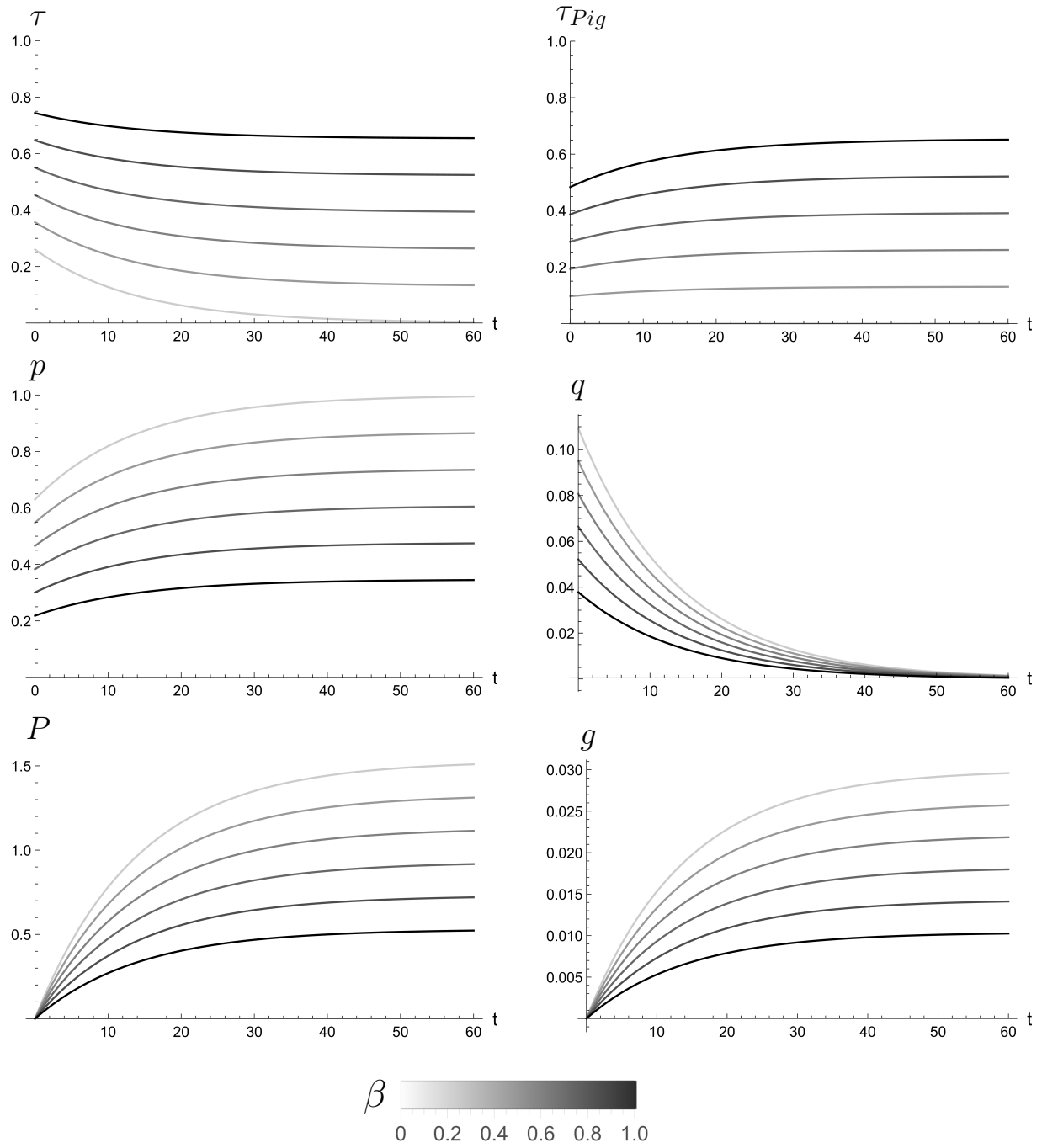


**Figure 3.3.1:** Trajectories of controls and state over time with  $\alpha = 0.02$  and  $r = 0.03$ .

tariff-free regime are derived from the trivial problem defined by setting  $\tau(t) = 0$  for all  $t$ .

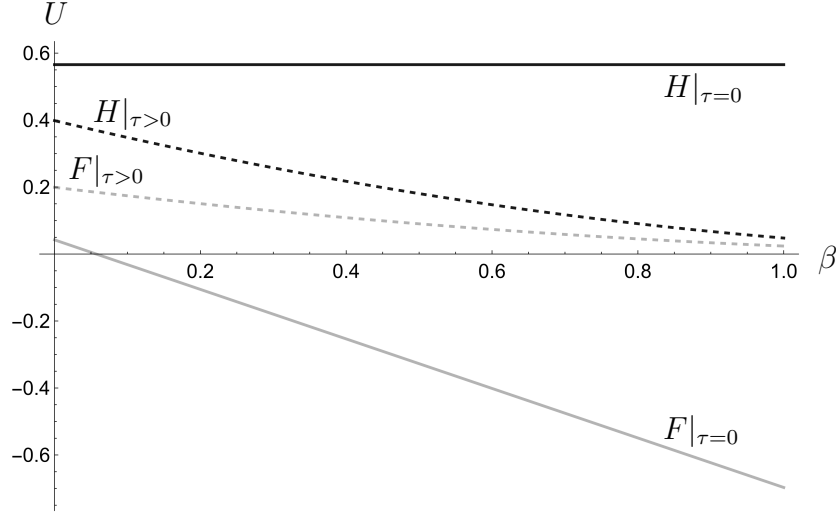


**Figure 3.3.2:** Total payoffs for both countries under a tariff-free (solid) and a tariff regime (dashed).

Under this scenario the exporter does not face any reaction to geoengineering. Therefore, it can produce the optimal quantity and choose its subjectively optimal geoengineering level in each period, yielding a payoff invariant to changes in  $\beta$ . If the importing country introduces a tariff,  $H$ 's payoffs decline the higher the external costs experienced by  $F$  are, due to the fact that from its point of view tariffs sub-optimally distort prices upwards.

Of course, the payoffs of country  $F$  are always higher in the tariff regime. A geoengineering-averse importer without the option of a tariff would face substantial losses, since the exporter would heavily produce and geoengineer and in turn offload this externality on  $F$ . Hence, the deployment of tariffs, increasing in  $\beta$ , forces the exporting country to internalize parts of this externality.

### 3.3.2 $F$ accounts for temperature changes

How will the results change if both countries face damages from the pollution stock? We obtain closed-form solutions for Markov perfect strategies but do not report them due to their length.<sup>24</sup> Compared to (3.3.8) the limiting value of the stock is now also influenced by

<sup>24</sup>Please see Appendix 3A.

$\alpha_F$  and given by

$$P_\infty = \frac{(\alpha_H + 1)(\alpha_H(r - \beta) + r)}{\alpha_F + \alpha_H(1 + \alpha_H)}.$$

The qualitative result of the previous subsection still holds, i.e.,  $\partial\tau/\partial\beta > 0$ ,  $\forall P$  but now due to  $\alpha_F \neq 0$  there exists the possibility for a qualitative difference in the slopes of the strategies.

**Result 3.3.2.** *The slopes of the optimal price and tariff strategies change sign at*

$$\begin{aligned}\hat{\alpha}_F &:= \frac{1}{8}(1 + \alpha_H) \left[ \alpha_H(4 - r^2) - r^2 + r\sqrt{(1 + \alpha_H)(r^2 + 8\alpha_H + \alpha_H r^2)} \right], \\ \tilde{\alpha}_F &:= \frac{1}{2}(1 + \alpha_H) \left[ \alpha_H(1 + r^2) + r^2 - r\sqrt{(1 + \alpha_H)(r^2 + 2\alpha_H + \alpha_H r^2)} \right],\end{aligned}$$

respectively. Specifically,

$$\frac{\bar{\psi}_3 - \bar{\theta}_3}{2} \begin{cases} > 0, & \text{for } \alpha_F < \hat{\alpha}_F \\ = 0, & \text{for } \alpha_F = \hat{\alpha}_F \\ < 0, & \text{for } \alpha_F > \hat{\alpha}_F \end{cases}, \quad \bar{\psi}_3 \begin{cases} > 0, & \text{for } \alpha_F < \tilde{\alpha}_F \\ = 0, & \text{for } \alpha_F = \tilde{\alpha}_F \\ < 0, & \text{for } \alpha_F > \tilde{\alpha}_F \end{cases},$$

where we have  $p = \frac{1 + \bar{\psi}_2 - \bar{\theta}_2}{2} + \frac{\bar{\psi}_3 - \bar{\theta}_3}{2}P$ ,  $\tau = -\bar{\psi}_2 - \bar{\psi}_3P$  and the bar denotes coefficients of the extended model.

Due to  $\hat{\alpha}_F \neq \tilde{\alpha}_F$  there exist three qualitatively different price/tariff strategies. Fig. 3.3.3a shows the slopes of the optimal price and tariff strategies with respect to  $\alpha_F$  for  $r = 0.03$ ,  $\beta = 0.3$  and  $\alpha_H = 0.02$ . Fig. 3.3.3b plots the strategies in state space for the same parameter values and where  $\alpha_F$  is varied in three steps from 0.001, 0.01, to 0.02. At a low stock externality in  $F$ , the export price is increasing and the tax is decreasing as the IO element – acquiring parts of the monopoly rent – dominates. At high external costs, the Pigouvian element dominates the IO element, such that the tariff increases (with respect to state and time) and prices have to decrease. In the intermediate case, both tax and export price are increasing. Hence,  $F$  chooses decreasing (increasing) tariffs over time if it suffers relatively little (much) of increased temperature. In the case where  $F$  gains from global warming

( $\alpha_F < 0$ ) the tariff may become negative at later stages. This induces  $H$  to produce more (e.g., by offering tax incentives) than it would find optimal otherwise.

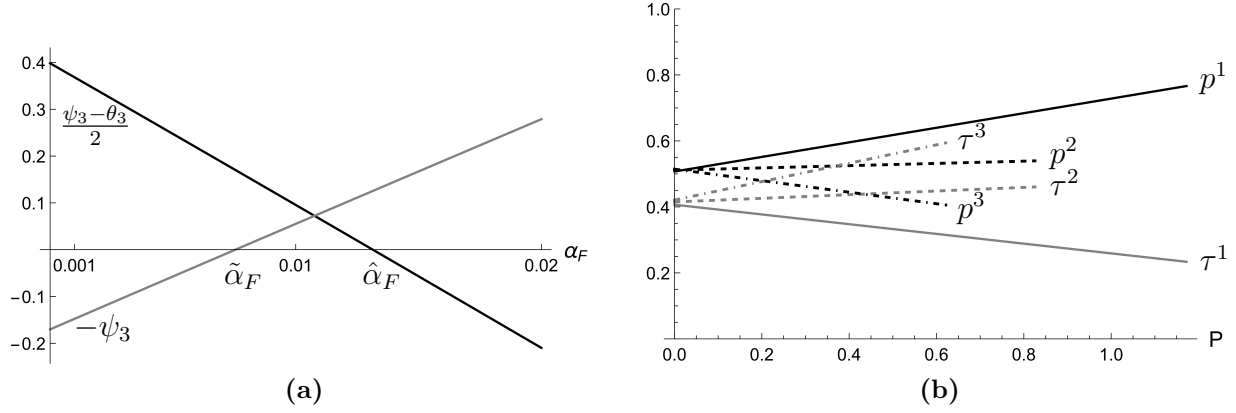


**Figure 3.3.3:** (a) Slopes of optimal price and tax. (b) Price and tax strategies for three different scenarios up to the respective steady state.

### 3.4 Oligopoly

We extend our framework for international competition. More precisely, two firms (or industries), one located in  $H$  the other in  $F$ , compete a la Cournot in  $F$  and  $F$ 's government accounts for the profits of its domestic firm. Total inverse demand for the homogeneous good in  $F$  is given by the normalized and linear relation  $1 - q_H - q_F$ , in line with the single supplier case. We now allow  $F$ 's government to impose a tax/subsidy scheme  $(\tau_H, \tau_F)$  on both industries. Ignoring costs, the firms' profits are

$$\begin{aligned}\Pi_H &= (1 - q_H - q_F - \tau_H)q_H, \\ \Pi_F &= (1 - q_H - q_F - \tau_F)q_F,\end{aligned}\tag{3.4.1}$$

which imply the Cournot-Nash outputs,

$$\begin{aligned}q_H &= \frac{1 - 2\tau_H + \tau_F}{3}, \\ q_F &= \frac{1 - 2\tau_F + \tau_H}{3}.\end{aligned}\tag{3.4.2}$$

We are interested in how the government of  $F$  chooses  $\tau_H$  and  $\tau_F$  under climate intervention of  $H$  with the assumption that the two firms choose the equilibrium Cournot quantities (3.4.2) in each period. That is, while the governments act intertemporally, the firms in both countries act intratemporally only. Hence, country  $H$ 's objective is given by

$$\max_{g \geq 0} \int_0^\infty e^{-rt} \left[ (1 - q_H - q_F - \tau_H) q_H - \frac{g^2}{2} - \frac{\alpha_H}{2} (P - g)^2 \right] dt, \quad (3.4.3)$$

whereas  $F$  maximizes consumer surplus, producer surplus and tax revenues minus damages caused by  $H$ 's geoengineering and increased temperature (note that the taxes or subsidies, which  $F$  imposes on its local firm, cancel),

$$\begin{aligned} \max_{\tau_H, \tau_F} \int_0^\infty e^{-rt} & \left[ \frac{(q_H + q_F)^2}{2} + (1 - q_H - q_F)q_F + \tau_H q_H - \beta g \right. \\ & \left. - \frac{\alpha_F}{2} (P - g)^2 \right] dt, \end{aligned} \quad (3.4.4)$$

subject to

$$\begin{aligned} \dot{P} &= q_H + q_F = \frac{2 - \tau_H - \tau_F}{3}, \quad P(0) = P_0, \\ q_H &= \frac{1 - 2\tau_H + \tau_F}{3}, \quad q_F = \frac{1 - 2\tau_F + \tau_H}{3}. \end{aligned} \quad (3.4.5)$$

### 3.4.1 $F$ disregards temperature changes

We employ the same solution procedure as before. The HJB equations corresponding to this scenario are

$$\begin{aligned}
rV_H &= \max_{g \geq 0} \left\{ \frac{1}{9} (\tau_F - 2\tau_H + 1)^2 - \frac{g^2}{2} - \frac{\alpha_H}{2} (P - g)^2 \right. \\
&\quad \left. + V'_H \left( \frac{2 - \tau_H - \tau_F}{3} \right) \right\}, \\
rV_F &= \max_{\tau_H, \tau_F} \left\{ \frac{1}{6} (2 + 2\tau_H - 2\tau_F + 2\tau_H\tau_F - 3\tau_H^2 - \tau_F^2) - \beta g \right. \\
&\quad \left. + V'_F \left( \frac{2 - \tau_H - \tau_F}{3} \right) \right\}.
\end{aligned} \tag{3.4.6}$$

We guess

$$\begin{aligned}
V_H &:= \lambda_1 + \lambda_2 P + \frac{\lambda_3}{2} P^2, \\
V_F &:= \mu_1 + \mu_2 P + \frac{\mu_3}{2} P^2
\end{aligned} \tag{3.4.7}$$

and derive the following FOC's

$$\begin{aligned}
g &= \frac{\alpha_H}{1 + \alpha_H} P, \\
\tau_H &= -V'_F = -\mu_2 - \mu_3 P, \\
\tau_F &= -1 - 2V'_F = -1 - 2\mu_2 - 2\mu_3 P.
\end{aligned} \tag{3.4.8}$$

As in the previous section we determine the strategy relevant coefficients  $(\lambda_2, \lambda_3, \mu_2, \mu_3)$  by comparing coefficients in (3.4.6) using (3.4.7) and (3.4.8). Since  $\mu_3 = 0$  the value function of  $F$  is linear in the state, thus rendering the following constant tariffs.

**Result 3.4.1.** *The Linear Markov perfect strategies are given by*

$$g = \frac{\alpha_H}{1 + \alpha_H} P, \quad \tau_H = \frac{\beta \alpha_H}{r + r \alpha_H}, \quad \tau_F = -1 + \frac{2\beta \alpha_H}{r + r \alpha_H}. \tag{3.4.9}$$

*Both taxes levied on the industry in  $H$  as well as  $F$  (the latter one may be negative and thus a subsidy) are constant with respect to time and are increasing in  $\beta$ . These policies shut down*

*the imports from  $H$  and imply a subsidy/tax for the industry located in  $F$ ,*

$$q_H = 0, \quad q_F = 1 - \frac{\beta\alpha_H}{r + r\alpha_H}.$$

Note that

$$q_F > 0 \iff \beta < \frac{r(1 + \alpha_H)}{\alpha_H},$$

which is assured due to assumption (3.3.7). Since  $F$  does not suffer from increased temperatures it continues to produce forever (only correcting for the negative externality of  $H$ 's geoengineering), resulting in boundless accumulation of the stock of pollution, i.e.,  $P_\infty = \infty$ . This obvious lack of a steady state sets this case apart from all the others in which  $P$  converges.

We also observe that  $F$  changes its domestic tariff policy from a subsidy to a tax. In fact,  $F$  subsidizes its own firm for

$$\beta \in [0, r(1 + \alpha_H)/(2\alpha_H)) \tag{3.4.10}$$

and taxes it for  $\beta \in (r(1 + \alpha_H)/(2\alpha_H), r(1 + \alpha_H)/(\alpha_H))$ , where the upper bound is again due to (3.3.7). This argument that non-competitive polluting firms may voluntarily internalize and even over-internalize the external costs dates back to Buchanan (1969).

The economic rationale behind this specific tariff policy can be explained as follows. Suppose for the moment that  $\beta = 0$ . In this case the government in  $F$  chooses to subsidize its industry in order to incentivize it to produce at the welfare optimal level ( $q_F = 1$ ) since the subsidies – as pure transfers – play no role. No tariff for the industry in  $H$  is necessary. Now, as  $\beta$  increases  $F$ 's government has an incentive to reduce total production, in order to lower geoengineering efforts by  $H$ . To do this it needs to cut emissions of its own industry too. However, as it lowers production, entry of the firm located in  $H$  becomes profitable. This is not in the interest of  $F$ 's government, since one unit provided by  $F$ 's firm is worth more than one unit provided by  $H$ 's firm. Hence, it imposes taxes on exports of  $H$  in order to deter entry.

### 3.4.2 $F$ accounts for temperature changes

Finally, we study the case of  $\alpha_F > 0$ . The FOC's are still given by (3.4.8) with a different set of coefficients.

**Result 3.4.2.** *The linear Markov perfect strategies for the game of international competition and concern about global warming are given by*

$$\begin{aligned} g &= \frac{\alpha_H}{1 + \alpha_H} P, \\ \tau_H &= 1 + M + \left(N - \frac{1}{2}r\right) P, \\ \tau_F &= 1 + 2M + (2N - r) P, \end{aligned} \tag{3.4.11}$$

with

$$\begin{aligned} M &= \frac{(r + \alpha_H r - \alpha_H \beta) \left(r + \alpha_H r - \sqrt{4\alpha_F + r^2(1 + \alpha_H)^2}\right)}{2\alpha_F}, \\ N &= \frac{\sqrt{4\alpha_F + r^2(1 + \alpha_H)^2}}{2(1 + \alpha_H)}. \end{aligned}$$

Since

$$\begin{aligned} \frac{\partial \tau_H}{\partial \beta} &= \frac{\alpha_H \left(-r(1 + \alpha_H) + \sqrt{4\alpha_F + r^2(1 + \alpha_H)^2}\right)}{2\alpha_F} > 0, \\ \frac{\partial \tau_F}{\partial \beta} &= 2 \frac{\partial \tau_H}{\partial \beta}, \end{aligned}$$

for  $\alpha_H, \alpha_F, r > 0$ , we find that both tariffs are increasing in  $\beta$ . This is due to the fact that  $F$  now also suffers from pollution and hence has to reduce its own production over time. It does so by increasingly taxing its own industry – no subsidy is given in the later and warmer stages. Note that for  $\lim_{\alpha_F \rightarrow 0}$  the results of the previous model apply, i.e., initial subsidies are possible. Intuitively, a subsidy is initially feasible only if  $F$  is able to cope with both the damages from geoengineering and increased temperature, i.e., if in addition to (3.4.10)  $\alpha_F$  is

sufficiently small,

$$\alpha_F < \check{\alpha}_F, \text{ with } \check{\alpha}_F = 2[r(1 + \alpha_H) - 2\alpha_H\beta][r(1 + \alpha_H) - \alpha_H\beta].$$

Since  $M < 0$  and  $N > 0$ , we find that the intercept of the optimal tariff for the home country is larger and that the slope is smaller compared to that for the foreign country. Our assumption of  $\delta = 0$  ensures that  $\lim_{t \rightarrow \infty} \tau_H = \lim_{t \rightarrow \infty} \tau_F = 1$ , implying  $\tau_H > \tau_F$  for all  $t$ . This, in turn means that the quantity produced by  $F$ 's firm is larger than that for  $H$ 's. In fact,  $F$  again designs its tax scheme in such a way that  $H$ 's firm is deterred to export to  $F$ ,

$$q_H = 0, \quad q_F = -M + \left(\frac{r}{2} - N\right) P,$$

which means that the market is characterized by potential but not actual competition (similar but different to contestable markets a la Baumol et al. 1982). In this sense geoengineering introduces a new damage term that country  $F$  has to account for. This threat by  $H$  forces  $F$  to lower the production of its own industry over time. A typical set of optimal controls, the state and production quantities are depicted in Fig. 3.4.1, where we used  $r = 0.03$ ,  $\alpha_H = 0.02$ ,  $\alpha_F = 0.015$  and  $\beta = 0.3$ . Note that since  $\alpha_F > 0$  an explosion in the state similar

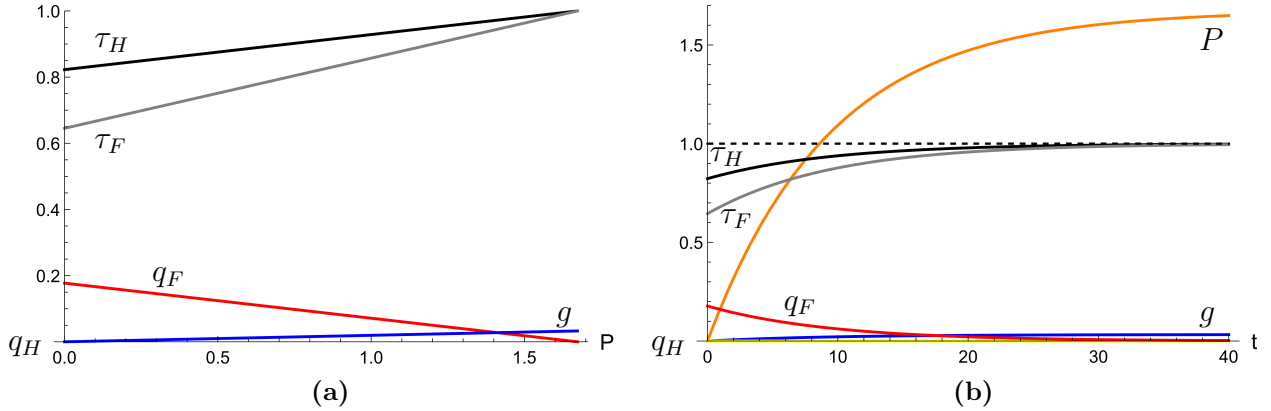


**Figure 3.4.1:** Markov perfect strategies in state space (a) and optimal trajectories over time (b).

to the previous subsection is not optimal.  $F$  needs to correct its production not only for the threat of geoengineering from  $H$  but also for increased temperatures stemming from said

production. We find

$$P_{\infty} = \frac{M}{r/2 - N}.$$

### 3.5 Conclusions

First experiments of SRM are already underway. However, more research is needed not only on technical but also on economic aspects which may arise before, during and after implementing climate engineering techniques. This paper extends the economic literature by investigating bilateral trade relationships between a geoengineering country and its geoengineering-averse trade partner for different industrial organizations (monopoly, duopoly) and trade policies. We find that geoengineering may induce countries with no or negligible production to increase import tariffs (depending on the level of their aversion to geoengineering) and that it could induce producing nations to enforce a trade restriction even if they do not suffer or care about increasing temperature levels. This analysis highlights possible future international tensions due to geoengineering which can induce tariffs and thereby lead to a loss in global welfare. Of course, this tariff policy cannot serve as a mean to sufficiently mitigate global warming, since the reduction in trade cannot be large enough in order to solve the problem. In fact, lowering trade by imposing tariffs gives geoengineering countries more ‘room’ for their domestic emissions, which we have not explicitly analysed in this paper due to the focus on trade. Our warning is that unilateral applications of geoengineering can lead to protectionist policies and in turn to international frictions among key-players that render a global cooperative climate policy even harder to achieve or to maintain. While research on technical and meteorological consequences of climate engineering is critical, it is also necessary to study and to account for social preferences and attitudes before geoengineering is implemented in order to assess the scope of possible ramifications for each party. This means that a comprehensive discussion on the impacts of climate engineering should include potential effects on global trade flows as well as all other potential side effects. Of course, this framework is just among the first steps for further investigations. Extensions and alternatives are to include costly abatement, carbon capture and storage or other  $CO_2$  removal techniques (already built in most of the climate models run by the IPCC) and R&D.

---

Other potential extensions are in the directions of uncertainty (e.g., treating temperature increases as a stochastic process) or nonlinearities including the possibilities of tipping points and the threat of catastrophes.



## Chapter 4

### Two applications of the shallow lake system



## 4.1 Corporate cheating under nonlinear reputation dynamics

### Abstract

Reputation is a critical measure of trust for agents as well as a valuable asset for corporations. Generally, firms recognize this fact and focus on taking actions that increase their respective image, which is well researched in the literature on advertising for goodwill. However, some real world cases point to the fact that firms are sometimes also willing to trade off their reputation for increased profits through deceptive behavior. In this note, we model these decisions by exploring the situation of a monopoly facing this tradeoff over an infinite horizon. The social effects of cheating are depicted via a well known nonlinear dynamic of ecological economics, the shallow lake equation. By the means of a phase diagram as well as bifurcation analysis we find that the enfolding dynamics are similar to the problems of optimal resource extraction with a tipping point, i.e., the existence of multiple steady states and the possibility for a Skiba point.

**Keywords:** Cheating, Monopoly, Reputation, Tipping point, Optimal control.

**JEL index:** D25, D42, L12, L14, M30, Q21, Q29.

### 4.1.1 Introduction and literature review

Roth (1985) defines reputation as "... a characteristic or attribute ascribed to one person (firm, industry, etc.) by another". In this way we define the concept as any stakeholder's evaluation of a company over time, which is clearly valuable to firms; many potential benefits, as well as potential damages, can be extracted from such (individual) assessments. Reputation, as an intangible asset,<sup>25</sup> is inherently a dynamic concept, since it can be accumulated, consumed and restored over time – always subject to change in either direction. Operationally, the reputation of a firm/or individual usually represents a prediction about likely future behaviour, hence, it links past histories with future opportunities in repeated interactions (Roth 1985). Due to its importance, strategic actors in general, and firms in particular, try to influence their respective image. It is tough to think about any modern multinational corporation without a CSR department. There exists a vast and diverse literature on the latter, which is recently also complemented by a small but growing literature on CSI, i.e., Corporate Social Irresponsibility (Vallaster et al. 2012, Lin-Hi & Müller 2013). It is useful to have an overview of how corporate reputation is formed and by what means it is modified over time. Hillenbrand & Money (2007) identify such channels by which stakeholders' evaluations of firms are influenced (see Fig. 4.1.1).

In general, it is critical how a given company relates to stakeholders themselves, to third parties, and lastly also to itself. The reasons and channels by which perceptions are formed are straightforward, e.g., how well a firm communicates and how easy (or hard) it is for stakeholders to communicate with the firm; what kind of benefits the firm offers stakeholders, e.g., whether it provides competitive products with a reasonable price. However, even though most often companies aim to influence their perception positively, the channels listed in Fig. 4.1.1 are a two-way street, i.e., the direction may also be reversed if it is in the interest of the firm. For instance, communication can be withheld or be carried out deceptively. Actively shirking with respect to product or service quality decreases the benefits stakeholders draw from the firm. Lying, deceiving, misrepresenting and unethical behaviour in general, will reduce a firm's reputation once uncovered. Thus, feelings of customers and stakeholders are often strategically influenced and exploited, e.g., the practice of greenwashing, deceiving

---

<sup>25</sup>See Roberts & Dowling (2002). We may also define the concept as an immaterial good, which cannot be easily reproduced by others.

| <i>Three themes of responsibility</i>                                                   | <i>Eight clusters of responsibility</i>                                                                                                                                                                                 |
|-----------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>A business is responsible for...</i>                                                 |                                                                                                                                                                                                                         |
| ...how it relates to <i>ME</i>                                                          | through <i>communication</i><br>through the <i>kind of benefits it offers to me</i><br>through the way it behaves with <i>integrity, transparency</i><br>and <i>accountability</i><br>and how that makes me <i>feel</i> |
| ...how it relates to <i>OTHERS</i><br>(that includes stakeholders and society in large) | The <i>local community</i><br>The <i>wider society</i><br>Towards other direct <i>exchange stakeholders</i> (ie employees, customers, suppliers and shareholders)                                                       |
| ...how it relates to <i>ITSELF</i>                                                      | <i>Long-term business success</i>                                                                                                                                                                                       |

**Figure 4.1.1:** Identified channels by which corporate reputation is being determined. From Hiltenbrand & Money (2007), p.268.

customers and exploiting green preferences, comes to mind.

In this note we want to understand and explore the other side of the coin. How, when and why do firms trade off some of their reputation for increased profits? For the purpose of this note we simply define cheating as any act which is fraudulent/deceptive/non-conforming or simply unethical. Critically, such actions need not be illegal, in fact most of the above mentioned ways a firm cheats its stakeholders are perfectly legal, or at least in a legal gray area. What influences such decisions made by corporations? The literature has identified many of them, such as intensity of competition,<sup>26</sup> the belief that others are performing the same actions (Langevoort 2002), inattention to moral standards (Mazar & Ariely 2006), temporal preferences (Ainslie & Haslam 1992) and the degree of expected benefits, i.e., the probability of being detected and the magnitude of potential penalties (Gneezy 2005, Gilpatric 2011).

The literature most related to this note are the classic works in the dynamic advertising literature. Since reputation is not static and always subject to change, a dynamic analysis

<sup>26</sup>The evidence on competition effects on cheating is mixed, some say a more competitive environment induces cheating, others say it deters cheating because the benefits drawn from it are smaller. See Cartwright & Menezes, 2014 as well as chapter 1, where we explore a similar setting to study the effects of competition.

is required. We draw inspiration from the Lanchester models (originally analyses of military combat, which later were being used to understand ‘warfare for market share’), sales response models (Vidale & Wolfe 1957), product diffusion models (Bass 1969) and most importantly, the models developed in the advertising for goodwill literature (a la Nerlove & Arrow 1962).<sup>27</sup> The latter in particular considers a standard capital accumulation equation,

$$\dot{G}(t) = a(t) - \delta G(t), \quad G(0) \geq 0, \quad (4.1.1)$$

where companies invest in advertising effort,  $a(t)$ , which increases the goodwill,  $G(t)$  of the firm and where customers have the tendency to forget, ( $\delta > 0$ ). For the purpose of this note, we understand reputation as being equivalent to the concept of goodwill, i.e., brand image or advertising capital. While the focus of the above mentioned models mostly lies on increasing goodwill, we aim to model the inverse, i.e., we let firms invest in cheating efforts (in its most general definition, see above), which have some inherent benefit but, when uncovered, reduce a company’s reputation in the long run.<sup>28</sup> Before we introduce the setup we briefly mention some real-world examples where firms have opted to cheat.<sup>29</sup> First, we mention the almost endless examples where firms have greenwashed their products in order to exploit green attitudes of consumers. We define this practice as deception, as the underlying product does not have the advertised properties, i.e., is in most cases not healthy or environmentally friendly (Coca Cola, BP and Green oil). Next, smartphone and other electronics manufacturers are sometimes using the practice of planned obsolescence (which is illegal in France), to deliberately make old products look worse in order to push sales of a newly introduced product (Apple vs. Samsung). Many examples can be found in the video gaming industry where firms have invented a completely new way to increase profits, namely through so-called lootboxes – a package containing virtual content such as a cosmetic or playable character. These boxes act as highly addictive random devices (the output of a given lootbox is random), where consumers are ‘forced to’ spend large amounts of money. The practice of introducing this mechanism into a product is perceived very badly by consumers (firms ‘milking’ a popular

<sup>27</sup>Of course, many other articles can be mentioned here, including issues of strategic interaction (Sorger 1989, Lambertini & Zaccour 2015, ...).

<sup>28</sup>See also the literature on ‘reputation gaps’, i.e, the difference between a firm’s claims and how it is being perceived by stakeholders.

<sup>29</sup>See Appendix 4A for illustrations of the following examples mentioned in this section.

trademark for increased profits, e.g., Electronic Arts, Blizzard Entertainment and many others). Lastly, we also mention the automotive industry, where large-scale cheating has been observed many times (from Hyundai and Kia to Takata and Volkswagen).

As mentioned above, the goal of this note is to establish the dynamic connection between cheating and reputation (the inverse of an advertising for goodwill model in the spirit of Nerlove-Arrow) to understand and model the kinds of behaviours we observe in the marketplace.

### 4.1.2 The model

In many cases, uncovered cheating will not steadily decrease a company's reputation. We propose that the effects stemming from a firm's cheating decisions on its reputation are nonlinear. Thus, cheating may have a different effect on a company's reputation, depending on the actual point on the reputational spectrum. Consider the internet and the modern communication technologies, which make it possible to transfer information and opinions much faster than before. In particular, all kinds of stakeholders, who perceive a company's actions and form their beliefs about these actions, are connected via such technologies and may change or form their opinion through these channels. Any unethical behaviour may be observed by one or a few stakeholders, who can then directly and quickly influence others as well, which one can observe in a number of so-called online 'firestorms', or shitstorms. Basically, negative word-of-mouth dynamics formed over the internet, which may cause rapid and substantial loss of corporate reputation.<sup>30</sup>

**Remark 4.1.1:** *While the advertising models have the capacity for normative implications (how, when and how much one should advertise), once we move to the domain of cheating we propose (at this point) a purely positive or descriptive analysis – to model and understand the evidence listed above.*

---

<sup>30</sup>There are obviously many reasons for a shitstorm, so it is important to distinguish pure cheating or unethical behaviour from simple mistakes in a firm's communication strategy, where the company meant to actually do good, but through some error the message did not come through as such to stakeholders. We also exclude engineering mistakes in the production of a good, which may also lead to a loss in the perceived value of stakeholders towards the company (e.g., Samsung Galaxy Note 8 issues).

We consider a monopoly facing demand,

$$D(p(t)) = a - p(t), \quad (4.1.2)$$

where  $a$  is the market potential and  $p(t)$  is the price charged per unit of the good at time  $t$ . Let,  $k(t)$  be time-dependent investments into cheating (in its most general form, defined above), which is demand enhancing (now omitting time notation),

$$D(p, k) = a + \alpha k - p, \quad (4.1.3)$$

where  $\alpha \in \mathbb{R}_+$ . Hence, more cheating implies more sales, which is in accordance with most of the examples of firm cheating above. As a consequence, cheating is done towards the customer, i.e., it increases willingness to pay. For instance, by claiming higher-than-actual product quality. It is important to differentiate this kind of cheating from other types of misbehaviour like tax evasion or financial misrepresentation. In our model there is no conflict inside the firm or within the organization of the firm. This policy is costly on two fronts: direct implementation costs,

$$c(k(t)) = \frac{\gamma}{2}k(t)^2, \quad \gamma > 0, \quad (4.1.4)$$

e.g., investments into rebranding (ad-campaign) or R&D. In addition, direct costs may also be interpreted as emotional costs of the decision maker, meaning that if the same results (increased profits) could be obtained by ethical means, most would opt to do so.<sup>31</sup> Next to the instantaneous implementation costs, we let cheating affect a firm's Reputation intertemporally through,

$$\dot{R}(t) = -\delta k(t) - \frac{R(t)^2}{1 + R(t)^2} - \epsilon R(t), \quad R(0) = R_0 = 0, \quad (4.1.5)$$

which also influences demand (scaled by  $\beta \in \mathbb{R}_+$ ),

$$D(p, k) = a + \alpha k - p + \beta R. \quad (4.1.6)$$

---

<sup>31</sup>We don't need the convex implementation costs to get rid of a linear Hamiltonian, i.e., also  $\gamma = 0$  does not imply a Bang-Bang solution.

**Remark 4.1.2:** *Instead of letting cheating reduce a firm's reputation, i.e., its goodwill one may also equivalently set up a fictitious badwill state and define  $B = -G$ . In fact, in chapter 1 we use such a specification.*

In the following we motivate the chosen dynamics (4.1.5).

### The shallow lake

We model the social impact of cheating via the so-called shallow lake dynamic (Carpenter et al. 1999, Mäler et al. 2003). Originally used in ecological economics it describes the internal dynamics of plant growth in a shallow lake. Consider,

$$\dot{x}(t) = l(t) - bx(t) + \frac{x(t)^2}{1 + x(t)^2}, \quad x(0) = x_0 \geq 0, \quad (4.1.7)$$

where  $x(t)$  is the amount of phosphorus in the lake,  $l(t)$  is the loading (the input of phosphorus) and  $b$  the rate of loss of phosphorus. In its original version farmers' fertilization efforts spill over and lead to the accumulation of phosphorus in the lake. The equilibrium of the lake can be very sensitive to small changes in the state, i.e., ecologists have found that the internal processes of shallow lakes<sup>32</sup> make them subject to regime shifts. What this means is that, depending on the load  $l(t)$ , these ecosystems have a tendency to tip from a healthy to a highly polluted eutrophic state.

The main idea is to use these dynamics in (4.1.7) (but turned upside-down) to describe the dynamic processes and properties of uncovered cheating among customers. Fig. 4.1.2a depicts the static equilibria of (4.1.5) and indicates how small changes in the load, i.e., in cheating efforts, may lead to dramatic changes in the state. Suppose a firm already built up some reputation capital,<sup>33</sup> then, initially, cheating leads to an almost linear reputational loss. However, at some point 'enough is enough' and there is a collapse, akin to a social shitstorm – a sharp drop in reputation once a critical level of cheating has been observed.

<sup>32</sup>Phosphorus used for fertilization gets washed into the lake and this increases plant growth, like algae, and therefore decreases the amount of light that can penetrate the surface of the lake. This eventually leads to eutrophication, a state where there is too much algae and nutrients in the water. The lake tips from a healthy, clear state to a highly fertilized polluted state.

<sup>33</sup>Otherwise it is unable to trade off its reputation for increased profits. Note, that this is not the case under the badwill specification in Fig. 4.1.2b.

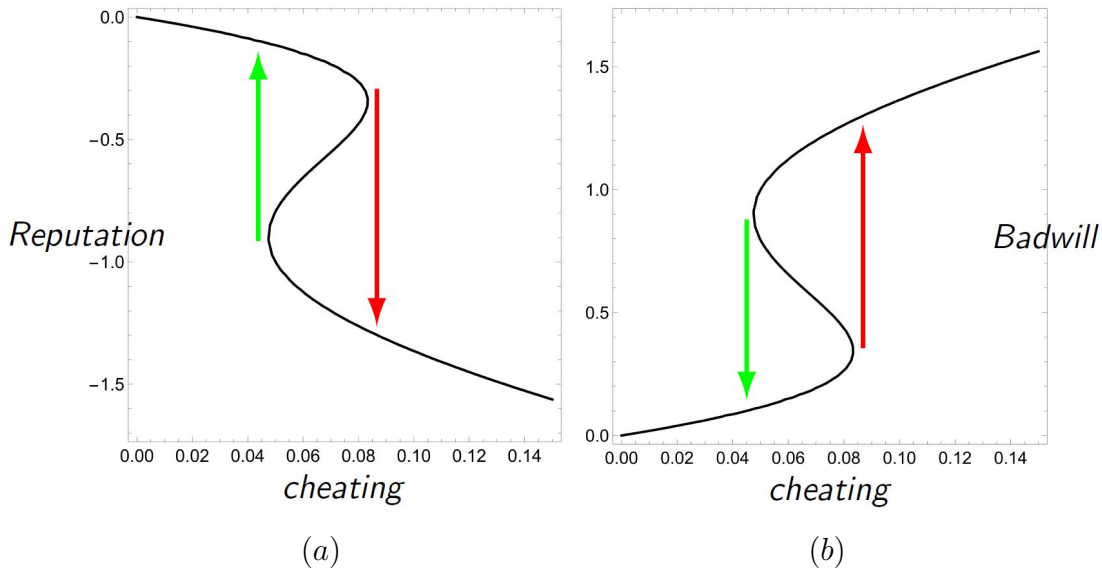


**Figure 4.1.2:** Static equilibria of the shallow lake dynamic for  $b = 0.55$  and two possible settings: (a) exploits an existing resource, i.e., reputation capital. (b) builds up a fictitious stock of bad will.

Additionally, the system exhibits hysteresis, i.e., after the shitstorm the firm needs to reduce cheating efforts by much more in order to get back to the old equilibrium again.

Hence, we let reputation be a (renewable) resource that can be exploited. One interpretation is that, as the investments into cheating increase, more and more customers will be aware of the firm's behaviour, which will ultimately lead to a wave of social interaction (negative word-of-mouth interactions) and consequently to a sudden drop in reputation. Alternatively, consider a sole customer, who after observing different levels of cheating, reduces the quantity of the good bought from the firm a la Fig. 4.1.2, i.e., the customer is willing to be cheated a small amount, but not too much. For the firm this creates a trade-off between making exaggerated claims, which inflate demand, on the one hand and managing both types of costs, instantaneous and accumulating through time, (which might rise sharply when not managed accordingly) on the other hand.

Thus, we are interested in solving the firm's problem,

$$\max_{\substack{p(t) \geq 0, \\ k(t) \geq 0}} \int_0^\infty e^{-\rho t} \left[ \underbrace{p(t)(a + \alpha k(t) - p(t) + \beta R(t))}_{\text{revenue}} - \underbrace{\frac{\gamma}{2} k(t)^2}_{\text{direct costs}} \right] dt, \quad (4.1.8)$$

subject to,

$$\dot{R}(t) = -\delta k(t) - \frac{R(t)^2}{1 + R(t)^2} - \epsilon R(t), \quad R(0) = R_0 = 0, \quad (4.1.9)$$

where the price,  $p$ , as well as the cheating efforts,  $k$ , can be controlled in order to maximize profits (or revenues, as we abstract from production costs) and  $\alpha, \beta, \gamma, \delta, \epsilon, \rho$  are positive parameters.

The specific reputation dynamics in (4.1.9) unfortunately do not allow for a closed-form solution due to the nonlinearity (the fraction in (4.1.9)) but we can still use Pontryagin's maximum principle to set up the canonical equations and analyze them graphically/numerically. Hence, the current value Hamiltonian is given by

$$H = p(a + \alpha k - p + \beta R) - \frac{\gamma}{2} k^2 + \lambda \left( -\delta k - \frac{R^2}{1 + R^2} - \epsilon R \right), \quad (4.1.10)$$

where  $\lambda$  is the dynamic costate variable. The first-order-conditions of (4.1.10) imply,

$$\begin{aligned} H_p &= a + \alpha k - 2p + \beta R \stackrel{!}{=} 0, \\ H_k &= \alpha p - \gamma k + \delta \lambda \stackrel{!}{=} 0. \end{aligned}$$

and thus,

$$\implies p = \frac{a\gamma - \alpha\delta\lambda + \beta\gamma R}{2\gamma w - \alpha^2}, \quad k = \frac{a\alpha + \alpha\beta R - 2\delta\lambda w}{2\gamma w - \alpha^2}. \quad (4.1.11)$$

Finally, the adjoint equation,

$$\dot{\lambda} = \rho\lambda - H_R = \lambda \left( \rho + \epsilon + \frac{2R}{(R^2 + 1)^2} \right) - \beta p, \quad (4.1.12)$$

induces the following canonical system,

$$\dot{R} = -\frac{a\alpha\delta + \alpha\beta\delta R - 2\delta^2\lambda w}{\alpha^2 - 2\gamma w} - \frac{R^2}{1 + R^2} - \epsilon R, \quad (4.1.13)$$

$$\dot{\lambda} = \lambda \left( \rho + \epsilon - \frac{2R}{(1 + R^2)^2} \right) + \frac{a\beta\gamma - \alpha\beta\delta\lambda + \beta^2\gamma R}{\alpha^2 - 2\gamma w}. \quad (4.1.14)$$

with transversality condition,

$$\lim_{t \rightarrow \infty} e^{-\rho t} \lambda(t) R(t) = 0. \quad (4.1.15)$$

### 4.1.3 Graphical analyses

First of all, since the firm favours higher reputation levels, the costate  $\lambda$  will evolve in the positive domain. In other words, the firm is willing to pay  $\lambda$  in order to increase  $R$  by one unit (shadow price interpretation). In terms of the parameter set we will use,

$$a = 1, \rho = 0.03, \beta = 0.1, \gamma = 1, \delta = 0.5, \epsilon = 0.55, \quad (4.1.16)$$

where  $a$  corresponds to the market size,  $\rho$  the discount rate,  $\beta$  the effect of the indirect costs,  $\gamma$  the effect of the direct costs,  $\delta$  the customer sensitivity and  $\epsilon$  the decay or forgetting rate. We first show how the effectiveness, or the marginal utility of cheating,  $\alpha$ , affects the long run outcome of the system. Fig. 4.1.3a and b display phase planes for  $\alpha = 0.5$ , i.e., for a rather low effectiveness (how strongly one unit of cheating inflates demand).

It is well known, that (4.1.7) may induce a problem of multiple steady states (see also Fig. 4.1.2). However, for  $\alpha = 0.5$ , the isoclines intersect only once, leading to a uniquely determined outcome along the saddle-point stable path indicated in Fig. 4.1.3b. Intuitively, if investments in cheating do not pay off sufficiently the firm is willing to trade off very little of its reputation. Increasing the effectiveness of cheating may lead to a system of multiple steady states (see Fig. 4.1.3c and d where we set  $\alpha = 1$ ).

In terms of stability, we note that the lowest and highest fixed points are saddles whereas the one in the center is an unstable spiral (see also Wagener 2003). In this case, demand can be inflated much more effectively, i.e., the marginal short run benefit of cheating is larger compared to the long run damages, so that the company is willing to take the large drop in its

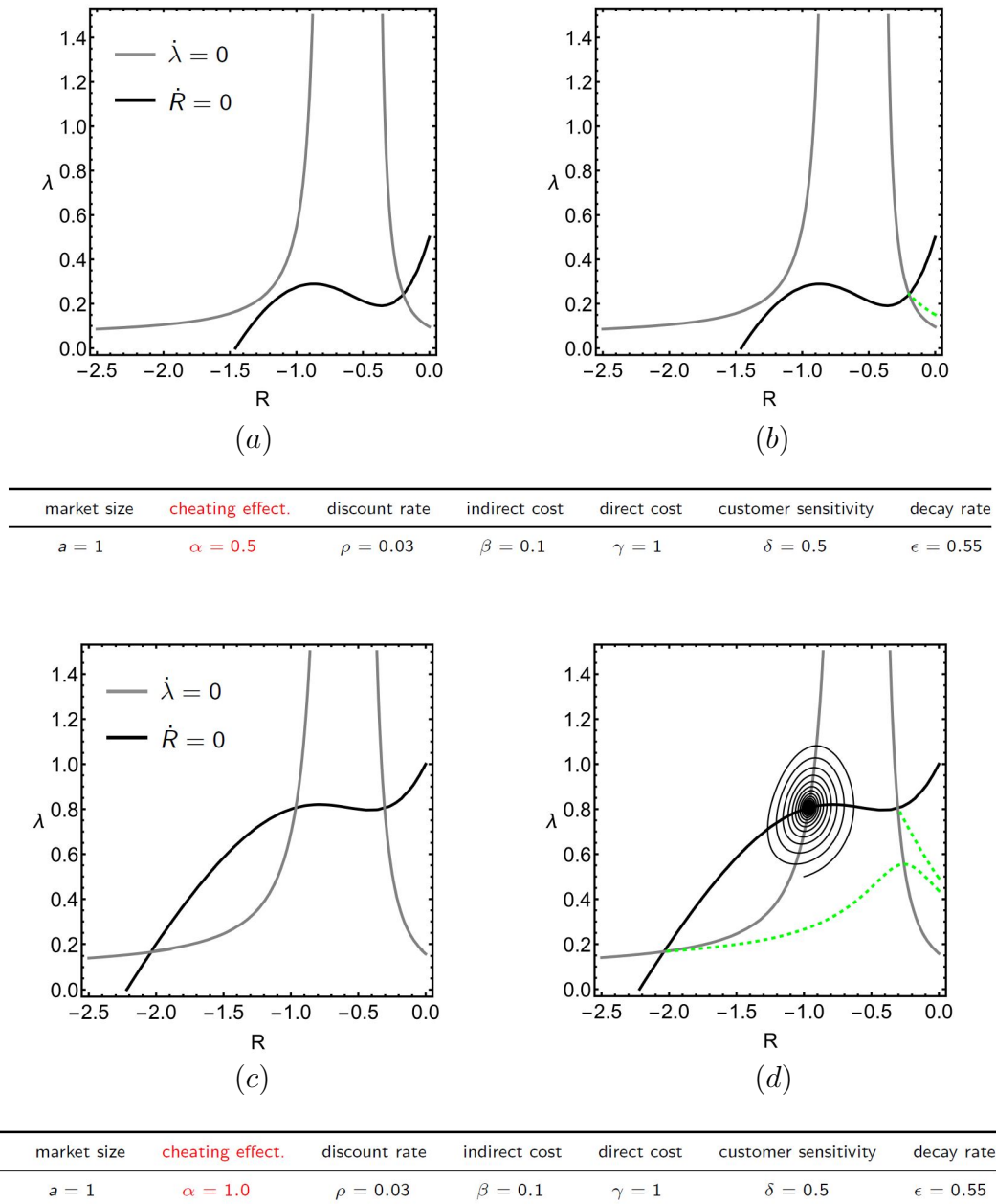


**Figure 4.1.3:** Phase portraits of system (4.1.14) and varying effectiveness levels of  $\alpha$ .

reputation (to roughly  $R = -2.1$ ). A shitstorm follows (the bottom trajectory is optimal in Fig. 4.1.3d). In these types of problems other scholars are often interested in a Skiba point,

i.e., a point of indifference. And indeed, for many examples such a point exists (see Wagener 2003, Kiseleva & Wagener 2010). Fig. 4.1.4a and b show such a point in state space, for which the total discounted utility is exactly identical for both paths to the respective equilibria.

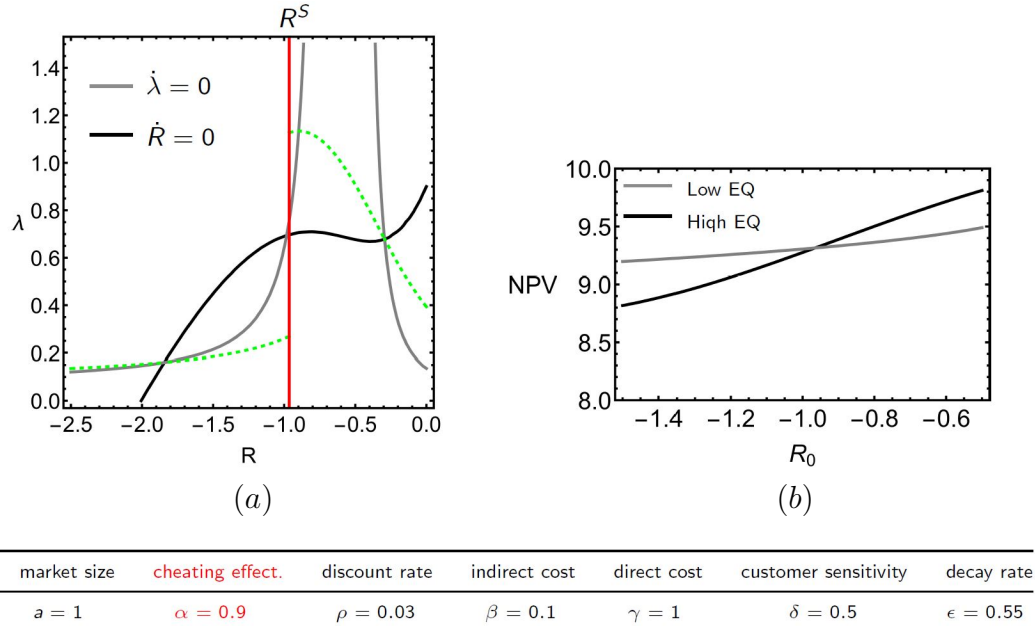


**Figure 4.1.4:** A skiba point in state space (a). (b) compares NPV's for both trajectories.

However, given the story of a firm considering whether to trade off reputation for increased demand, we analyse the converse, i.e., fixing the state we check for Skiba sets in parameter space. Fig. 4.1.5a displays the bifurcation diagram in  $\alpha$ . This shows the cut-off parameter value,  $\alpha^*$ , such that the firm is first indifferent and then strictly prefers the low equilibrium on a net present value basis (see Fig. 4.1.5b). Of course, we need sufficient effectiveness,  $\alpha > 0$ , in order for the firm to even consider cheating its customer base, i.e., otherwise the firm sets  $k = 0$ , as the instantaneous costs (4.1.4) would have to be covered also. Fig. 4.1.5a also shows the parameter interval for which there exists a Skiba point in state space (bounded by the two dashed red vertical lines).

To solve the exercise of computing the set of Skiba points in parameter space (in contrast to the singleton shown in Fig. 4.1.5a), we plot the bifurcation diagrams also in  $(\alpha, \beta)$  and  $(\alpha, a)$  space.

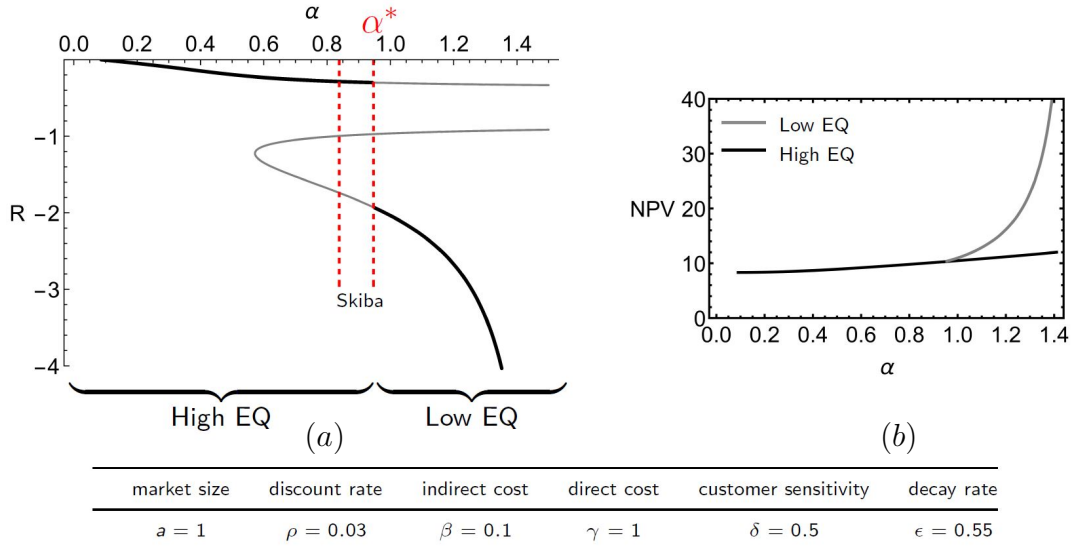


**Figure 4.1.5:** Bifurcation diagram in  $\alpha$  on the left and NPV's on the right.

Fig. 4.1.6 shows the areas where there is either no equilibrium (0 EQ), a uniquely determined one (1 EQ), or three steady states (3 EQ).<sup>34</sup> For the areas implying multiple fixed points Fig. 4.1.6b and Fig. 4.1.6d show the Skiba set partitioning the area of multiple steady states into sub-areas where the low and high equilibrium is optimal, respectively. Clearly, if the cost-benefit ratio is low (i.e., where the indirect costs,  $\beta$ , are small and the effectiveness,  $\alpha$ , is relatively large) a shitstorm is likely to follow as the firm's decision will be to cheat (a lot, because it pays). In addition, Fig. 4.1.6c and d also indicate that the market needs to be of a sufficient size ( $a \gg 0$ ) or otherwise very little cheating is being performed, even if it is effective in inflating demand. The reason seems to be the direct convex costs that have to be recovered (which grow quadratically with  $k$ ).

What can be learned from this brief exercise? The main point is that not only small changes in the initial condition can induce drastically different long run outcomes (compare Fig. 4.1.4) but, of course, also small changes in a given parameter set.

To see this, consider Fig. 4.1.7ab and c where trajectories of controls and the state are plotted over time for two very close parameter configurations and identical initial conditions. The black solid orbit, where the higher equilibrium is optimal is quantitatively and qualitatively

<sup>34</sup>Up until  $\alpha = \sqrt{2\gamma}$ , see (4.1.11).

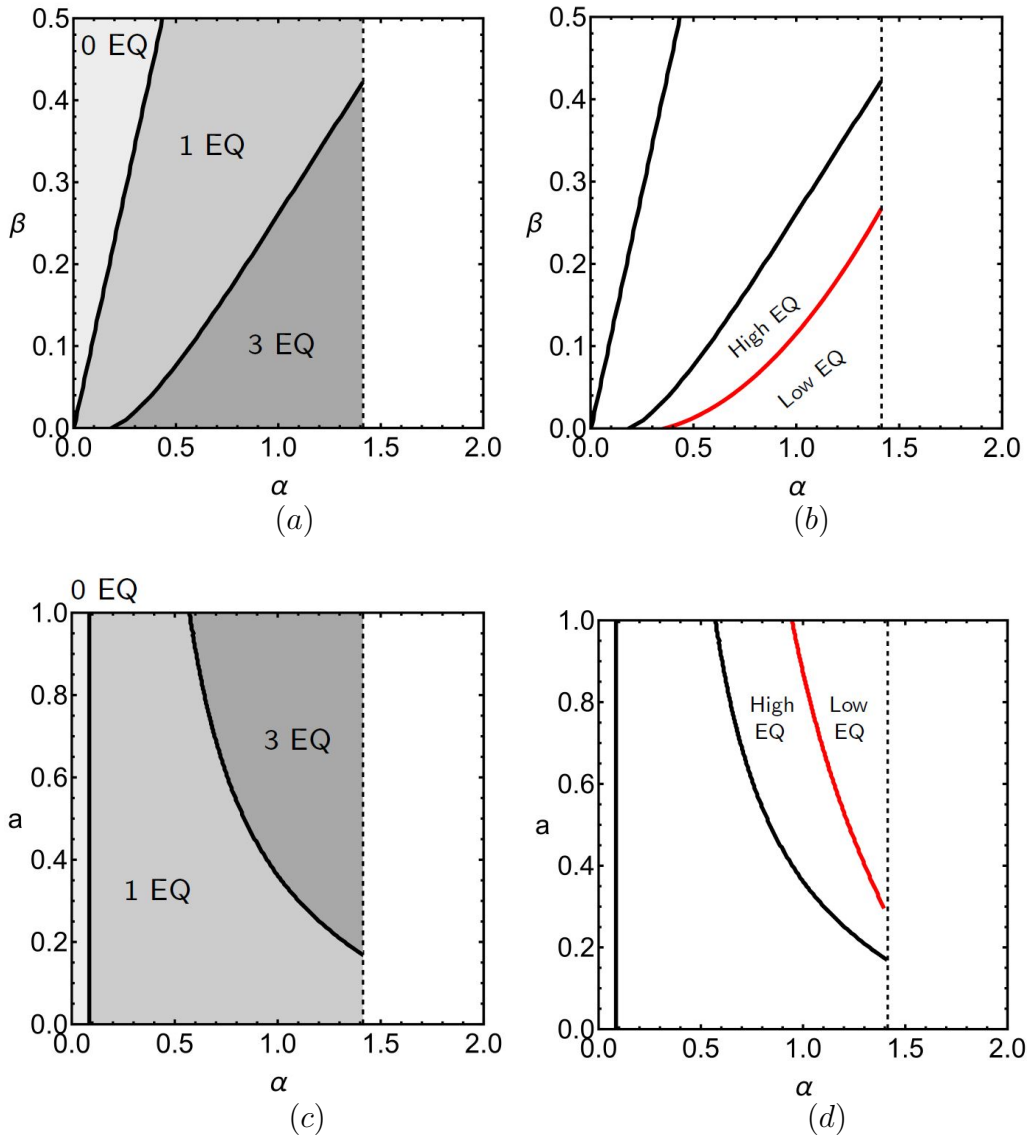
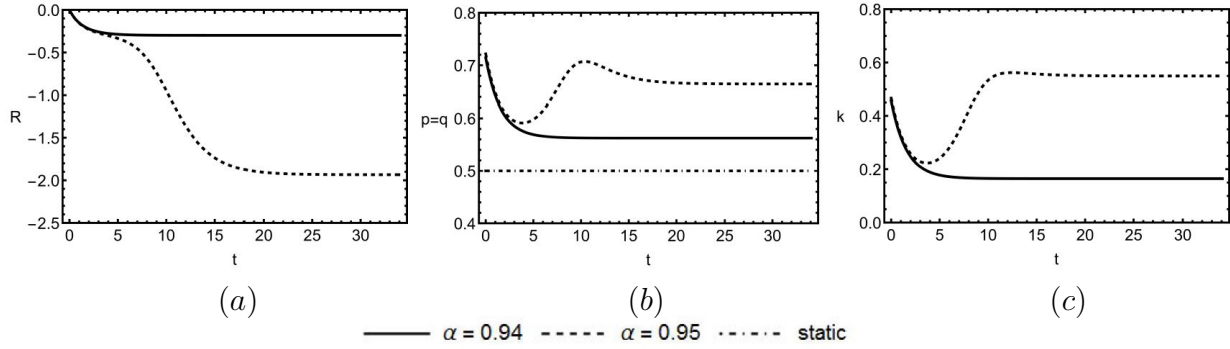


**Figure 4.1.6:** Bifurcation diagrams in  $\alpha$  and  $\beta$  in (a) and (c). Chart (b) and (d) indicate optimality in areas where multiple steady states exist.

different from the dashed one where  $\alpha$  is only marginally larger (from 0.94 to 0.95).<sup>35</sup> This figure also plots the static solution devoid of cheating opportunities ( $p = q = 0.5$ ) and shows

<sup>35</sup>Of course, the depicted qualitative characteristics can be established for any arbitrarily small difference around  $\alpha^*$ .



| indirect costs | market size | consumer sensitivity | direct cost  | decay             | discount rate rate |
|----------------|-------------|----------------------|--------------|-------------------|--------------------|
| $\beta = 0.1$  | $a = 1$     | $\delta = 0.5$       | $\gamma = 1$ | $\epsilon = 0.55$ | $\rho = 0.03$      |

**Figure 4.1.7:** Two close parameter sets inducing very different transient (and long run) behaviors.

how, by proper control of  $k$  (Fig. 4.1.7c), the market is being inflated (Fig. 4.1.7b). This allows the monopoly to charge its respective optimal markup over the static case.

#### 4.1.4 Conclusion and discussion

The goal of this short note is to model the connection between corporate cheating and the consecutive reputation loss to offer an explanation why we sometimes observe this process in reality. Next to the analysis performed, one may want to compare these results to a similar linear model. Of course, the linear model does not feature multiple steady states (and therefore also no Skiba points), but rather a uniquely determined saddle-point stable steady state. In contrast to the model presented here, a firm would be gradually decreasing its reputation, the more effective/less expensive investing in cheating becomes. Unfortunately, a limitation of the nonlinear version (that the linear version does not have) is that solving for closed-form solutions is impossible and that the analysis has to be carried out numerically. In contrast to the study of ecosystems, i.e., shallow lakes, we emphasize again the fact that not only small changes in the state, but also small changes in parameters may lead to drastically different optimal long run outcomes. Of course, most often one is not able to choose the

initial value (one may have a given ecosystem in a specific, measurable, state) but this holds true for the exogenous parameters also.

Next to the obvious candidates of adding uncertainty (stochastic processes) and a second state (sticky price) one could also extend to a game (see chapter 4.2 for a different topic but similar system dynamics). Both, settings with a shared reputation as well as a setup with individual reputations could make sense. Further research in this area may study problems with a fixed endpoint. Finite horizon models with little concern about the scrap value come to mind, which could potentially explain ‘exit-scams’ performed by some firms.

## 4.2 (Non)Cooperative climate engineering with a regime shift

### Abstract

We apply the shallow lake system of Mäler et al. (2003) to study an asymmetric trans-boundary pollution game with a tipping point and an option to geoengineer the climate. Non-cooperative outcomes as well as outcomes under full cooperation are analyzed. We identify areas in parameter space for which countries' preferences towards the long run outcome diverge. Both the cooperative as well as the non-cooperative systems are likely to exhibit multiple steady states in the relevant parameter region. The non-cooperative setup resembles a dynamic *battle of the sexes game*, where coordination, or pre-game commitments, have to be followed. Once climate engineering technologies are sufficiently effective, the temptation to endure a catastrophe increases. But lock in at high temperature levels is especially problematic in case of irreversible damage due to global warming, which also implies alarming consequences in regards to the termination effect.

**Keywords:** Global warming, Externalities, Regime Shift, Asymmetric differential game, Climate engineering, Natural Disasters and Their Management.

**JEL index:** C71, C72, C73, Q34, Q52, Q54, Q55.

### 4.2.1 Introduction

The existence of regime shifts or tipping points in the global climate system is a real and alarming possibility (Lenton et al. 2008, Schellnhuber 2009, Steffen et al. 2018, Stoerk et al. 2018). Such a regime shift would not only trigger direct economic damages in the form of decreased crop yields, relocation expenses and devaluation/destruction of property and infrastructure due to more frequent extreme weather events, but could also initiate cascades of co-extinction among animal and plant species (Bascrompte et al. 2019). Such self-reinforcing loops could dramatically reduce biodiversity, and by extension whole ecosystems, and therefore a valuable pool of potential resources. To limit damages caused by climate change the Copenhagen Accord announced the ceiling of a 2°C temperature increase over pre-industrial levels, which would ‘reduce risks of large and possibly irreversible future damage due to global warming.’ This agreement also provides a more strict goal of 1.5° which, however, could either be impossible to reach, given the current emission path, or simply be uneconomic (Dietz et al. 2018) at this point in time. Even though almost all countries of the world emphasized how important it is to limit further temperature increases (officially, at the COP21 meeting, resulting in the Paris agreement), these declarations are, as of yet, not reflective in the real world data: both in terms of global emissions and therefore also in global average temperature (see Fig. 4.2.1a and b).

Some international agreements turned out to be extremely successful, like the Montreal protocol in 1987 which led to a substantial reduction in the use of CFC’s<sup>36</sup> and the International Convention for the Regulation of Whaling agreement in 1946. However, it is doubtful whether the Paris agreement will be similarly effective in sufficiently reducing the risks of climate change. Trends in global population growth and energy consumption for the coming decades (Dudley, BP, 2018) further challenge the declaration and its outcome. Even if the flow of emissions could be reduced, the stock of carbon dioxide, which has been accumulated through centuries, is still existing and actively changes the radiation balance of Earth going forward. With some exceptions, most countries’ pledges and commitments put humanity on a trajectory to catastrophe (See Fig. 4.2.1a for some IPCC emission pathways, compare Solomon et al. 2007). Further, we note that without a binding agreement, the global atmosphere always acts as a common good and is thus subject to the decisions of strategically acting countries

---

<sup>36</sup>Even though quite recently it is suspected that China started to emit much more than expected.

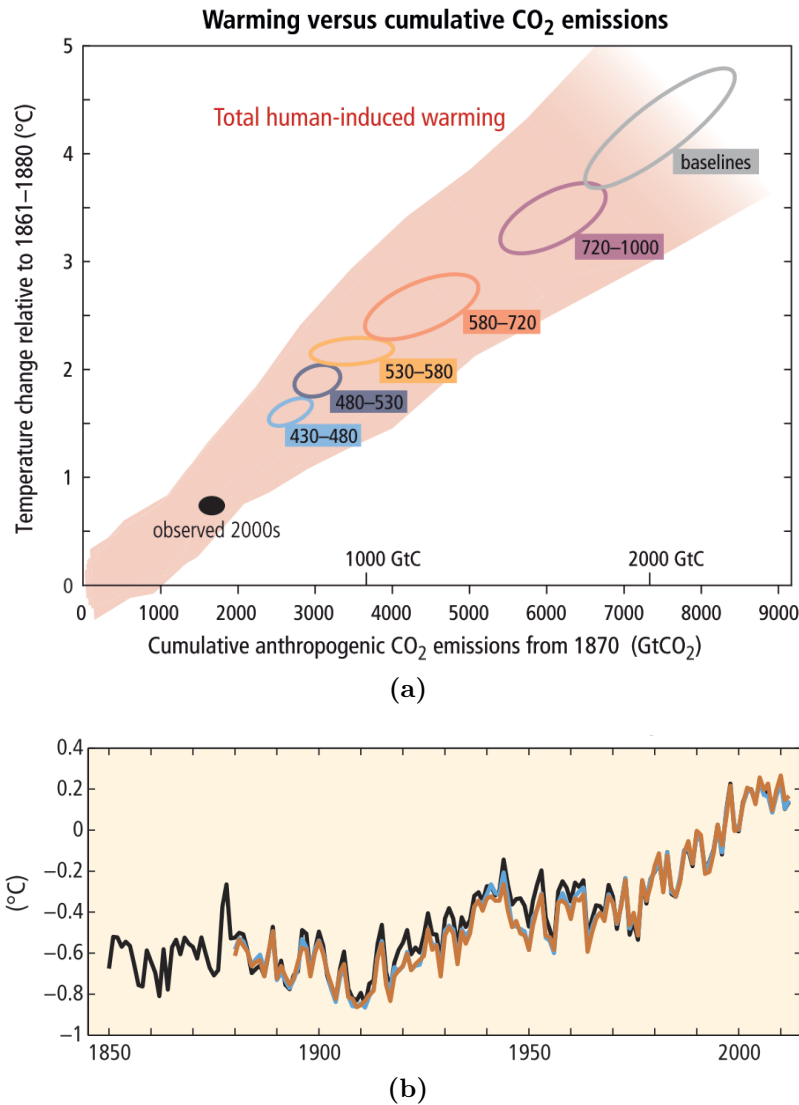


**Figure 4.2.1:** Projected emission pathways (a) and average relative temperature increase (b). Taken from IPCC, 2014.

and, hence, to the tragedy of the commons. Although the folk theorems tell us that many equilibria in dynamic endless public goods games exist, countries were so far unsuccessful in dealing with this issue.<sup>37</sup> The slow progress on the climate front has led researchers and policymakers to suggest novel technical tools, such as climate- or geoengineering techniques.

<sup>37</sup>In addition, some countries in the northern hemisphere would even benefit from a warmer world.

Many creative inventions fall under this definition: from installing a Space mirror at the Lagrange point to ocean fertilization, covering the deserts with a reflective material,<sup>38</sup> afforestation and carbon capture and storage.<sup>39</sup> Many techniques exist with differing cost-benefit ratios, however, one method is projected to compare very favourably (The Royal Society 2009): injecting sulfate aerosols in the stratosphere would increase global albedo and therefore reflect a small portion of light back into Space. This specific method falls under the category of *Solar Radiation Management*, SRM. Why does this method stack up so well? In contrast to all other technical proposals, SRM requires only a very small quantity of materials and could be deployed very quickly, leading to a timely reduction in global average temperature (within 1-2 years, effectively instantaneous in geological time). All other techniques either act on longer time-scales or are simply too expensive.<sup>40</sup> Even though SRM is supposed to be relatively safe, it is far from a panacea, as researchers point out that some estimated critical side-effects should be taken into account. Most importantly, SRM would modify regional weather and hydrological cycles (The Royal Society 2009, Schmidt et al. 2012, Bala et al. 2008) and would, depending on the region, lead to highly asymmetric outcomes. Changed precipitation patterns resulting from SRM could potentially be more costly to affected countries than the burden of increased temperature.

The goal of this note is to study the role of climate-interfering technologies in a system where tipping is possible and initiate further research in this direction. To the best of our knowledge, this issue has not yet been addressed in a multi-player setting (see Heutel et al. 2016 or Bickel 2013 for integrating tipping points in IAM models). Geoengineering, especially the use of SRM, is seen to be a very polarizing topic, i.e., the approval and the use of such a technology is heterogeneously distributed among a given set of countries in the real world. In this paper we therefore analyze which role such a technology plays and what that would entail for asymmetric countries.

This paper is structured as follows. In the next section we set up the model and derive the

---

<sup>38</sup>A surface made of polyethylene-aluminium could increase the local albedo and therefore mitigate, marginally, temperature increase (Gaskill 2004).

<sup>39</sup>Which most of the optimistic emission pathway scenarios already include in the form of negative emissions.

<sup>40</sup>Yet, costs have been decreasing, compare Tollefson 2018.

necessary conditions for the non-cooperative solution. Afterwards we numerically analyse these outcomes and specifically study the effects of asymmetry. The cooperative solution is also investigated in order to highlight the differences to the case when countries do not cooperate. Finally, we conclude, discuss the results and offer extensions for further research.

### 4.2.2 The model

There are several ways of modelling catastrophic environmental damages in a dynamic system. Bickel (2013) works with a discontinuous damage function in an IAM model. One can also introduce a hazard rate function with appropriate event distribution density and study uncertain shifts in state space via a control problem. This approach due to Kamien & Schwarz (1971) has been applied by de Zeeuw & Zemel (2012) and many others. However, none of them focus on strategic decision makers in a tipping environment. The model developed here is an extension of the shallow lake game presented in Mäler et. al (2003) – a nonlinear dynamic game originating in ecological economics, which has recently also been used to model the climate system (Kiseleva 2016). We let temperature,  $T$ , evolve according to

$$\dot{T} = l_i + l_j - bT + \frac{T^2}{1 + T^2}, \quad T_0 = T(0) = 0, \quad (4.2.1)$$

where  $l_i$ ,  $l_j$  are the players' loads, i.e., their emission choices. This specification represents a standard transboundary emission problem (larger emissions will increase temperature) with the added nonlinear term used in the shallow lake system (it is this nonlinearity in particular, which induces the interesting dynamics of a regime shift, including hysteresis). This setup allows us to account for strategic interaction and therefore explicitly check for non-cooperative behaviour of heterogeneous countries (Manoussi & Xepapadeas 2017, Heyen et al. 2015). To summarize, we let countries choose their emissions,  $l_i$ , and SRM levels,  $s_i$ , in order to maximize the net present value of their associated logarithmic benefits from emitting greenhouse gases, i.e., their production, less quadratic costs due to deploying this technology. Additionally, countries suffer quadratically from temperature change and have the ability to mitigate some of that damage via the use of SRM. Specifically, we study a two-player

asymmetric differential game (with omitted time-subscripts) of the following form:

$$\max_{l_i, s_i} \int_0^\infty e^{-\rho t} \left[ \alpha_i \ln l_i - \frac{c_i}{2} s_i^2 - \frac{\delta_i^T}{2} (T - e_i s_i - e_j s_j)^2 - \frac{\delta_i^E}{2} (s_i^2 + s_j^2) \right] dt, \quad (4.2.2)$$

s.t. the evolution of temperature, (4.2.1), where we use the following notation for  $i, j = 1, 2$  and  $i \neq j$ :

$l_i$  = emissions,  $s_i$  = SRM,  $\alpha_i$  = marginal benefit of emissions,

$c_i$  = implementation costs of SRM,  $e_i$  = efficacy coefficient of SRM,

$\delta_i^T, \delta_i^E$  = damage coefficients of temperature change and external effects of SRM,

$T$  = Temperature,  $b$  = recharge rate,  $\rho$  = discount rate.

As many scholars point out, the use of SRM bears many side effects (still ongoing ocean acidification, local air pollution) including external effects on the other player, e.g., in the form of changing precipitation patterns (Bala et al. 2008). We also mention psychological costs that some decision makers may have to pay by interfering with nature in this way (for most individuals there exists a natural personal deterrence to overcome in order to engage in such an activity; for some this deterrence is quite large). In the following, we study and analyze the non-cooperative open loop solutions of the game.

#### 4.2.2.1 The non-cooperative solution

Following Pontryagin's maximum principle, the non-cooperative solution can be derived by setting up the following current-value Hamiltonians,

$$H_i = \alpha_i \ln l_i - \frac{c_i}{2} s_i^2 - \frac{\delta_i^T}{2} (T - e_i s_i - e_j s_j)^2 - \frac{\delta_i^E}{2} (s_i^2 + s_j^2) + \lambda_i \left( l_i + l_j - bT + \frac{T^2}{1 + T^2} \right),$$

where  $\lambda_i, \lambda_j$  denote the dynamic costate variables for the two countries. Deriving the FOC's and solving the resulting interdependent equations for the respective controls yields,

$$l_i = -\frac{\alpha_i}{\lambda_i}, \quad (4.2.3)$$

$$s_i = \frac{\delta_i^T e_i (c_j + \delta_j^E) T}{(c_i + \delta_i^E) \delta_j^T e_j^2 + (c_j + \delta_j^E) (\delta_i^T e_i^2 + c_i + \delta_i^E)}, \quad (4.2.4)$$

for  $i, j = 1, 2$  and  $i \neq j$ . Since larger temperatures (the state) enter the objectives negatively, the relevant domain of the costate is also negative. Hence, emissions, (4.2.3), are increasing in their respective marginal gain (intuitively). Due to our model specification, the geoengineering control is linear in state space (similar as in Manoussi & Xepapadeas 2017). A larger stock of pollution is combated by more geoengineering, i.e.,

$$\frac{\partial s_i}{\partial T} > 0. \quad (4.2.5)$$

Similarly we find,

$$\frac{\partial s_i}{\partial \delta_i^T} > 0, \quad \frac{\partial s_i}{\partial \delta_j^T} < 0, \quad \frac{\partial s_i}{\partial \delta_i^E} < 0, \quad \frac{\partial s_i}{\partial \delta_j^E} > 0.$$

If own costs from increased temperature increase, countries opt for more SRM, however, if the costs for the opponent's increase the opposite is the case. Hence, taking into account the opponent's action, countries counterbalance  $s_i$ , respectively. The evolution of the costates as well as the limiting transversality conditions are given by

$$\begin{aligned} \dot{\lambda}_i &= \rho \lambda_i - \frac{\partial H_i}{\partial T} = \lambda_i \left( \rho + b - \frac{2T}{(1 + T^2)^2} \right) + \delta_i^T (T - e_i s_i - e_j s_j), \\ \lim_{t \rightarrow \infty} e^{-\rho t} \lambda_i T &= 0, \end{aligned} \quad (4.2.6)$$

for  $i, j = 1, 2$  and  $i \neq j$  and form together with (4.2.1) and the optimal rules for the controls (4.2.3)-(4.2.4) the canonical system for the non-cooperative game.

## 4.2.3 Numerical analysis

### 4.2.3.1 The effect of SRM

Given that the climate system has the potential for a tipping point but, as far as we know, has not tipped yet (see Lenton 2011) we focus on sufficiently small initial conditions of the state, i.e., on scenarios prior to any possible regime shift. In terms of the reference parameter

values we will use,

$$\alpha_i = 0.5, \quad c_i = 0.1, \quad \rho = 0.03, \quad b = 0.6, \quad \delta_i^T = 4, \quad (4.2.7)$$

for  $i = 1, 2$  throughout the paper. Note that for the special case of  $e_i = e_j = 0$  geoengineering technologies are effectively not available and that this specification implies symmetry. Hence, for this case the dynamics of the non-cooperative game enfold in two-dimensional phase space instead of the normally three-dimensional one implied by (4.2.1) and (4.2.6).<sup>41</sup> For this reference case we find that the non-cooperative long run solution is uniquely determined by a single saddle-point stable steady state (Fig. 4.2.2a) and that stationary temperature rests at a moderate level.

Since SRM mitigates damages caused by higher temperature levels, its availability makes any occurring regime shift less costly. Hence, increasing the effectiveness of it may qualitatively change the system to one with multiple steady states. Analytically, this is due to the fact that increasing  $e_i$  for  $i = 1, 2$  shifts an arc of the  $\lambda$ -nullcline upwards, eventually leading via a saddle-node bifurcation to a system of three steady states (Fig. 4.2.2b, Fig. 4.2.3a). In this case, the outer two equilibria (we will denote these outcomes as ‘low-temperature’ and ‘high-temperature’, respectively)<sup>42</sup> are saddles and are separated by an unstable focus in the middle. As is well researched in Wagener (2003) and Kiseleva & Wagener (2010), the shallow lake system allows for the existence of a Skiba point – an indifference point, which separates the state space into two regions from which convergence to the outer two respective equilibria is optimal. Often the Skiba point is located between the two saddles, where initial conditions to the left result in long run optimality of the low-temperature equilibrium and initial conditions to the right result in long run optimality of the high-temperature equilibrium. However, since we assume a situation prior to any regime shift, i.e., a climate system that has not tipped yet, the focus lies on sufficiently small initial conditions in the state ( $T_0 = 0$ ). Thus, the issue of finding indifference points in state space is not critical to this analysis. We analyse the converse, meaning we fix an initial condition – we assume an initially pristine environment but

<sup>41</sup>Note also that due to the fact that each decision maker chooses emissions and geoengineering it is not feasible to transform the dynamics into state-control space.

<sup>42</sup>In applications to the ecosystem of the shallow lake the respective outcomes are termed ‘oligotrophic’ and ‘eutrophic’, indicating a clean or polluted state of the lake.

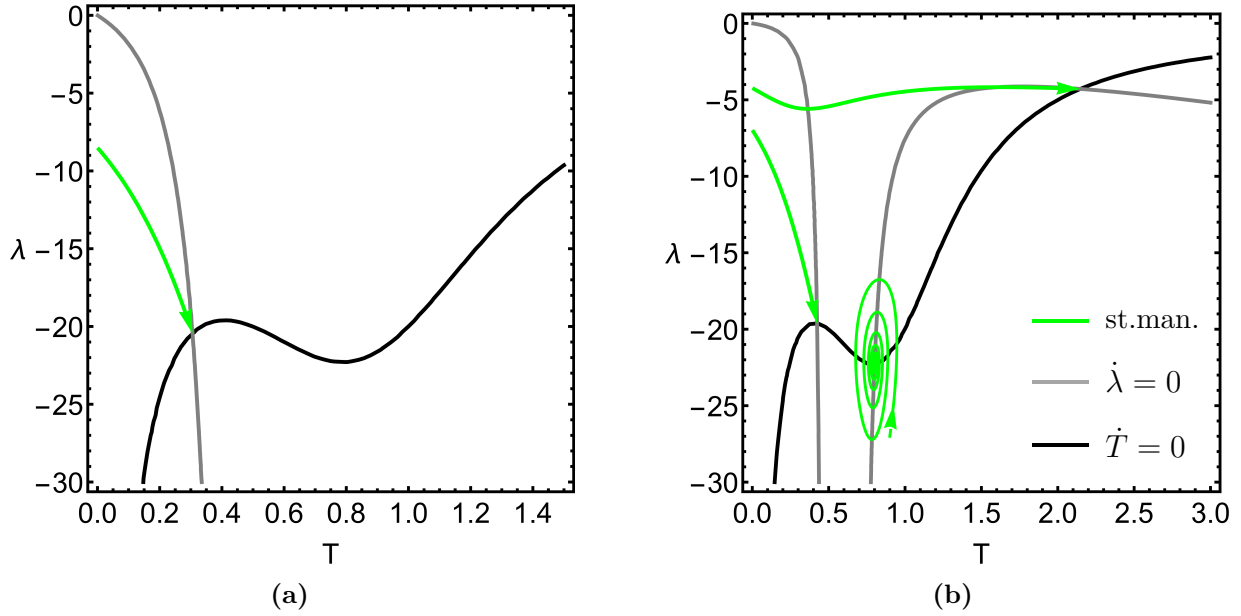


**Figure 4.2.2:** Phase diagrams in state-costate space for the symmetric case. (a) SRM technology is not available, i.e.  $e_i = e_j = 0$ . (b) SRM technology is available and effective, i.e.  $e_i = e_j = 0.7$  and  $\delta_i^E = \delta_j^E = 0.5$ .

the analysis is analogous for other sufficiently small starting values<sup>43</sup> – and vary parameters to identify indifference manifolds in parameter space. Specifically, we are interested in whether countries choose to accept the regime shift, or not, i.e., whether they tend to the low or high-temperature equilibrium, and which distributional consequences these decisions have on total welfare for both countries. Distributional concerns and issues of equity play a major role in climate negotiations since the polluters and those that suffer from increased pollution are often different countries. It is the low-income countries in central Africa and Asia which will suffer most from increased temperature and rising sea levels. Many rich countries are either located in geographically more preferably areas or have the tools and resources to prepare and adapt to a changing climate.

One crucial difference to the study of tipping points in ecological systems is that due to the possibility of geoengineering, the high-temperature equilibrium does not necessarily yield low stationary payoffs to each country. Since larger temperature levels are countered by more

<sup>43</sup>Starting values below the low temperature equilibrium.

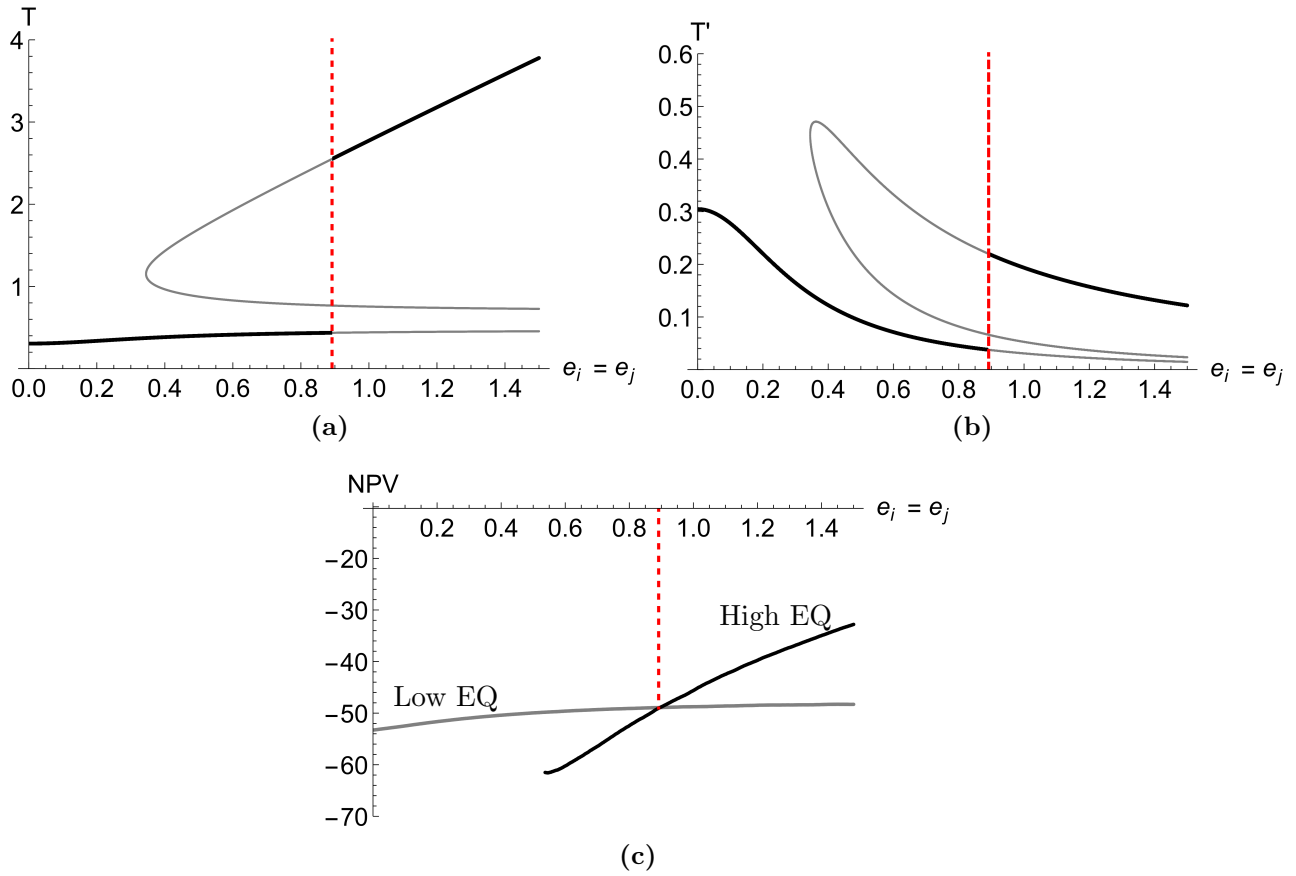


**Figure 4.2.3:** Bifurcation diagrams (a), (b) and total welfare per country (c) for symmetric technology efficacies and  $\delta_i^E = \delta_j^E = 0.5$ .

geoengineering, (4.2.5), these damages can be mitigated. Hence, one needs to differentiate between the true underlying temperature without geoengineering,  $T$ , which evolves according to (4.2.1) and the technically modified ex post temperature,

$$T' = T - e_i s_i - e_j s_j. \quad (4.2.8)$$

Fig. 4.2.3a and b display bifurcation diagrams for identical geoengineering efficacies and show that, once sufficiently effective, these technologies may induce countries to accept the jump in temperatures and converge to the high-temperature equilibrium (black outline indicates long run optimality). Note, that due to heavy use of geoengineering  $T$  and  $T'$  differ substantially

in the high-temperature equilibrium. Depending on  $\delta_i^E$ , this leads to larger side effects, including external costs for the other country. In Fig. 4.2.3c total welfare per country is plotted for the same efficacy continuum. Naturally, net present values for each country coincide for this reference case due to symmetry. At the intersection of the respective curves (roughly at  $e_i = e_j \approx 0.891$ ) each country is exactly indifferent, i.e., for this specific parameter configuration the origin becomes a Skiba point. Using the equivalent terminology above, we may say that all efficacy levels to the right of this particular point lead countries to opt for the high-temperature equilibrium and all efficacy levels to the left (including non-availability) lead to the low-temperature equilibrium. Of course, due to the combination of  $e_i = e_j$  and (4.2.7) countries opt for the same long run outcome.

### Asymmetry

What are the consequences for the asymmetric case? Interestingly, although the state space is the three-dimensional one implied by (4.2.1) and (4.2.6) the relevant dynamics for a fixed set of parameters enfold in a two-dimensional invariant subspace – a plane orthogonal to the axes of the costates, that contains the set of steady states.

To see this we need to find  $\tilde{c}_1, \tilde{c}_2 \in \mathbb{R}$  such that

$$\tilde{c}_1 \lambda_i + \tilde{c}_2 \lambda_j = 0.$$

Hence, by using (4.2.3), (4.2.4) and (4.2.6) we have

$$\begin{aligned} \tilde{c}_1 \dot{\lambda}_i + \tilde{c}_2 \dot{\lambda}_j &= \rho(\tilde{c}_1 \lambda_i + \tilde{c}_2 \lambda_j) - \tilde{c}_1 \frac{\partial H_i}{\partial T} - \tilde{c}_2 \frac{\partial H_j}{\partial T} \\ &= -\tilde{c}_1 \delta_i^T (T - e_i s_i - e_j s_j) - \tilde{c}_2 \delta_j^T (T - e_i s_i - e_j s_j) \\ &= -\tilde{c}_1 \delta_i^T - \tilde{c}_2 \delta_j^T \stackrel{!}{=} 0, \end{aligned}$$

and the combination  $(\tilde{c}_1, \tilde{c}_2)$  can thus be chosen accordingly. Typical cases are depicted in Fig. 4.2.4a and b. We note that the number of possible equilibria as well as their stability properties do not change in the asymmetric case, i.e., there is either a unique steady state being a saddle or three steady states where two are saddles and one is an unstable focus.<sup>44</sup>

<sup>44</sup>For the saddle we find that all EW's are real, two of them positive and one negative. At the unstable focus

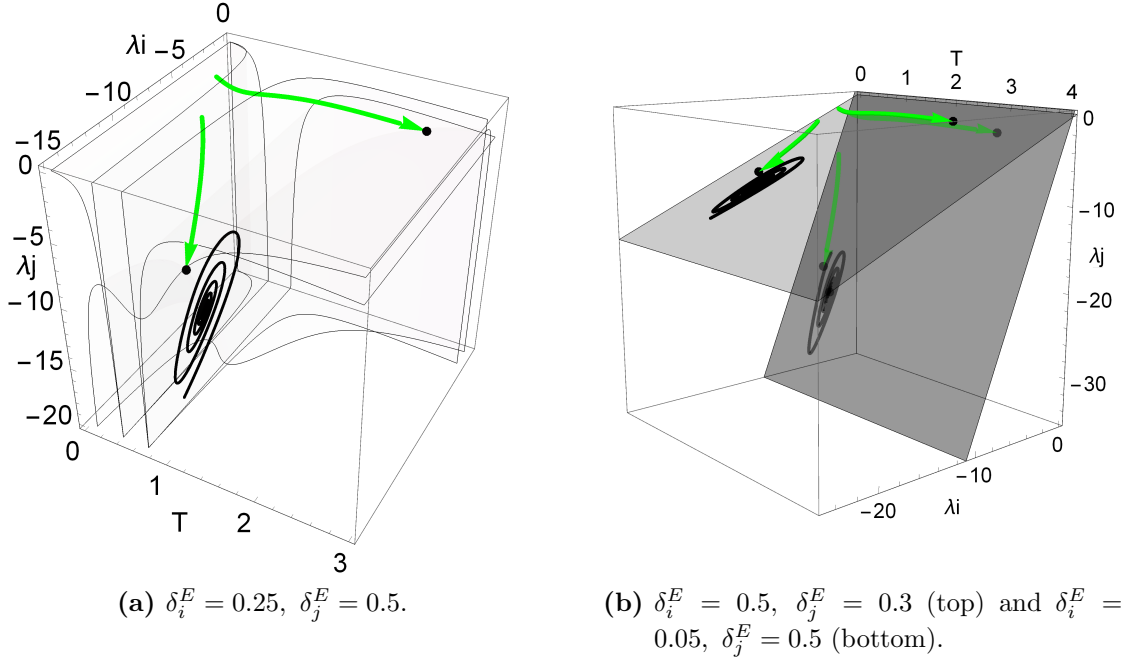


**Figure 4.2.4:** Typical phase diagrams for the general asymmetric case ( $e_i = 1, e_j = 0.1$ ).

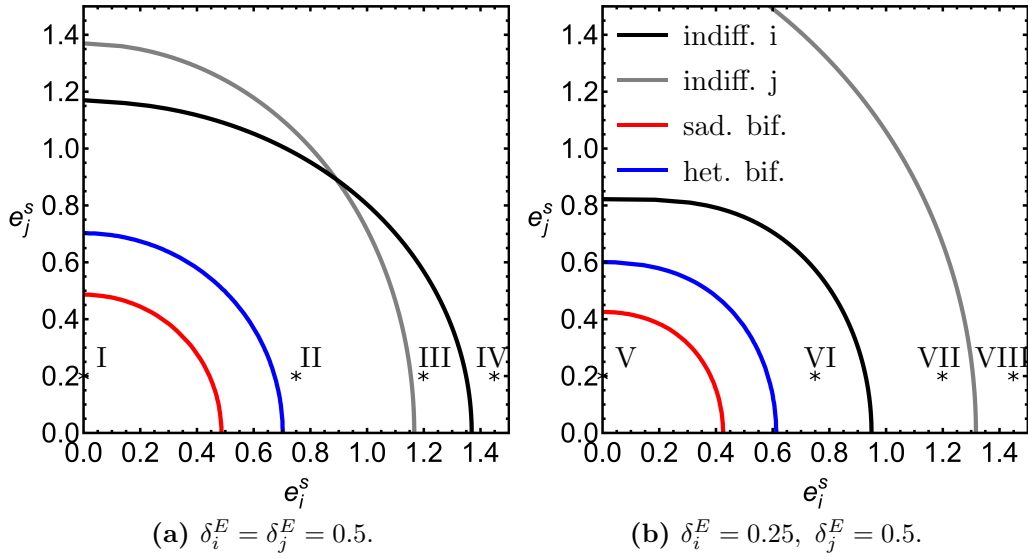


**Figure 4.2.5:** Two-dimensional bifurcations diagram including Skiba contours for each player.

---

we have one real and two complex EW's, all with positive real part.

As mentioned above, it is plausible to assume that countries have differing preferences towards using climate-interfering technologies as well as the costs due to their unwanted side effects. This is reflected in the differing side effects ( $\delta_i^E$ ) and efficacies ( $e_i$ ) of geoengineering technologies. In Fig. 4.2.5a the bifurcation diagram for symmetric side effects ( $\delta_1^E = \delta_2^E$ ) but asymmetric efficacies of SRM ( $e_1 \neq e_2$ ) is plotted. The saddle-node bifurcation curve (red) splits the region with a unique steady state from the region with three. However, even for the case of multiple steady states it is not guaranteed that the high-temperature equilibrium is technically reachable from low initial conditions. This particular threshold is determined by the heteroclinic bifurcation curve (blue), which marks when the unstable manifold of the low-temperature equilibrium merges with the stable manifold of the high-temperature equilibrium. Additionally, this diagram also shows each player's indifference curve (the Skiba contour in parameter space) for the two saddles in case of multiple steady states. For pairs of efficacy parameters to the left of the respective indifference curve the focal country prefers the low-temperature equilibrium and to the right the high-temperature equilibrium is optimal (optimality in a sense of largest discounted welfare, i.e., net present value). As can be seen, these curves do not overlap, i.e., there exists regions in  $(e_i, e_j)$  space where one country prefers the high-temperature equilibrium over the low one and vice versa. The 45-degree line goes through the intersection of each player's indifference curve and depicts the special case of symmetry, which is shown in Fig. 4.2.2. Interestingly, if the efficacy of a country's geoengineering technology is very high – relative to the opponent's – then this focal country may actually prefer staying at the low temperature equilibrium, whereas the other country is willing to take the regime shift, e.g., at the boundary for  $e_j = 0$  the high-temperature equilibrium is optimal for country  $j$  for a larger interval of  $e_i$  compared to country  $i$ . The reason is that due to the, per assumption small but strictly positive, implementation costs (here  $c_i = c_j = 0.1$ ) involved in applying SRM, country  $j$  has an incentive to *freeride* on these expenditures. Knowing the availability of  $i$ 's technology and given the symmetric side effects, country  $j$  chooses to emit more while  $i$  heavily implements geoengineering in order to counteract the warming effect of the emissions stemming from both parties (Table 4.2.1). In fact, for cost-less implementation ( $c_i = c_j = 0$ ) the indifference curves for both players overlap, as any asymmetry in the efficacy has indistinguishable consequences on total welfare.

|              | I (V)                          |          | II (VI)                        |                  | III (VII)                      |                         | IV (VIII)        |                                |
|--------------|--------------------------------|----------|--------------------------------|------------------|--------------------------------|-------------------------|------------------|--------------------------------|
|              | L                              | H        | L                              | H                | L                              | H                       | L                | H                              |
| $l_i^\infty$ | 0.051<br>(0.053)               | -<br>(-) | 0.048<br>(0.059)               | 0.151<br>(0.231) | 0.047<br>(0.060)               | 0.292<br>(0.450)        | 0.047<br>(0.060) | 0.377<br>(0.581)               |
| $l_j^\infty$ | 0.049<br>(0.047)               | -<br>(-) | 0.054<br>(0.043)               | 0.171<br>(0.167) | 0.055<br>(0.042)               | 0.341<br>(0.319)        | 0.055<br>(0.042) | 0.444<br>(0.410)               |
| $s_i^\infty$ | 0.000<br>(0.000)               | -<br>(-) | 0.406<br>(0.464)               | 1.809<br>(2.221) | 0.320<br>(0.341)               | 1.834<br>(2.132)        | 0.280<br>(0.294) | 1.804<br>(2.075)               |
| $s_j^\infty$ | 0.340<br>(0.344)               | -<br>(-) | 0.108<br>(0.072)               | 0.482<br>(0.346) | 0.053<br>(0.033)               | 0.306<br>(0.207)        | 0.039<br>(0.024) | 0.249<br>(0.167)               |
| $T^\infty$   | 0.323<br>(0.327)               | -<br>(-) | 0.407<br>(0.416)               | 1.815<br>(1.994) | 0.435<br>(0.441)               | 2.492<br>(2.756)        | 0.443<br>(0.448) | 2.853<br>(3.167)               |
| $T'^\infty$  | 0.255<br>(0.258)               | -<br>(-) | 0.081<br>(0.054)               | 0.362<br>(0.259) | 0.040<br>(0.025)               | 0.229<br>(0.155)        | 0.029<br>(0.018) | 0.187<br>(0.125)               |
| $NPV_i$      | <b>-53.8</b><br><b>(-52.6)</b> | -<br>(-) | <b>-51.5</b><br><b>(-47.2)</b> | -65.8<br>(-52.9) | <b>-50.7</b><br><b>(-46.4)</b> | -54.3<br><b>(-38.8)</b> | -50.5<br>(-46.2) | <b>-48.3</b><br><b>(-33.0)</b> |
| $NPV_j$      | <b>-54.6</b><br><b>(-55.4)</b> | -<br>(-) | <b>-49.2</b><br><b>(-53.2)</b> | -59.9<br>(-69.1) | -48.0<br><b>(-52.4)</b>        | <b>-46.8</b><br>(-55.1) | -47.7<br>(-52.2) | <b>-40.8</b><br><b>(-48.9)</b> |

**Table 4.2.1:** Total welfare and steady state values for controls and the state.

$(e_i, e_j) = \{(0, 0.2), (0.75, 0.2), (1.2, 0.2), (1.45, 0.2)\}$  was used for respective cases I-IV and V-VIII in Fig. 4.2.5.

The situation changes when we additionally introduce asymmetry in the side effects of SRM, i.e., in  $\delta_i^E$ . Fig. 4.2.5b displays such a case where country  $i$  suffers relatively little from the side effects compared to the competitor  $j$ . In this case, the combined costs of implementing SRM are much smaller, such that country  $i$  opts to *freedrive* (Wagner & Weitzmann 2012)

on this technology, using it even more and therefore incurring an even larger externality on  $j$  (compare Table 4.2.1). As a consequence, country  $i$  prefers to jump to the high-temperature equilibrium much sooner for any efficacy value of  $j$ 's technology ( $i$ 's indifference curve is located to the left of  $j$ 's). In Table 4.2.1 we also report total discounted welfare for both countries. We note that the structure of the non-cooperative game in areas of diverging interests resembles a dynamic version of the *battle of the sexes game*, where a couple has differing preferences towards two leisure activities and need to, somehow, coordinate their decisions. The same holds true here in this global climate game played in commitment strategies. Asymmetry therefore may imply differing preferences towards the long run outcome. Thus, we not only have multiple fixed points but also multiple non-rankable (open loop) Nash equilibria (e.g., in cases III and VII in Table 4.2.1/ Fig. 4.2.5). On the other hand, in areas of identical interests the long run outcome is settled, i.e., both countries either prefer the low temperature equilibrium (e.g., II and VI) or the high temperature equilibrium (e.g., IV and VIII).

#### 4.2.3.2 Cooperation

We now very briefly mention the case where both countries fully cooperate with each other. The necessary conditions for the cooperative solution can be obtained by setting up the following joint current-value Hamiltonian,

$$H = \sum_{i=1}^2 \left( \alpha_i \ln l_i - \frac{c_i}{2} s_i^2 - \frac{\delta_i^T}{2} (T - e_1 s_1 - e_2 s_2)^2 - \frac{\delta_i^E}{2} (s_1^2 + s_2^2) \right) + \lambda \left( l_1 + l_2 - bT + \frac{T^2}{1 + T^2} \right),$$

where  $\lambda$  denotes the dynamic costate variable. Deriving the FOC's and solving the resulting interdependent equations yields,

$$l_i = -\frac{\alpha_i}{\lambda}, \quad (4.2.9)$$

$$s_i = \frac{e_i (\delta_i^E + \delta_j^E + c_j) (\delta_i^T + \delta_j^T) T}{e_j^2 (c_i + \delta_i^E + \delta_j^E) (\delta_i^T + \delta_j^T) + (\delta_i^E + c_j + \delta_j^E) (e_i^2 (\delta_i^T + \delta_j^T) + c_i + \delta_i^E + \delta_j^E)}, \quad (4.2.10)$$

for  $i, j = 1, 2$  and  $i \neq j$ . The evolution of the costate as well as the limiting transversality condition are given by

$$\begin{aligned} \dot{\lambda} &= \rho\lambda - \frac{\partial H}{\partial T} = \lambda \left( \rho + b - \frac{2T}{(1+T^2)^2} \right) + (\delta_i^T + \delta_j^T) (T - e_i s_i - e_j s_j), \\ \lim_{t \rightarrow \infty} e^{-\rho t} \lambda T &= 0 \end{aligned} \quad (4.2.11)$$

and form together with (4.2.1) and the optimal rules for the controls (4.2.9), (4.2.10) the canonical system for the cooperative ‘game’. Since this control model satisfies all the necessary properties in the analysis given in Wagener (2003), the results obtained therein hold. Specifically, tending to the high-temperature equilibrium is optimal if and only if there exists a stable manifold from the origin leading to it. This is not true for the non-cooperative game studied above where there may exist an orbit leading to the high-temperature equilibrium but it is still optimal to converge to the low-temperature steady state (see Fig. 4.2.2b for an example). In technical terms, a heteroclinic bifurcation – the event where the stable manifold of one saddle merges with the unstable one of the other saddle – marks the specific point of optimality from small initial conditions.<sup>45</sup> Fig. 4.2.6a displays the respective bifurcation diagram for the cooperative solution and shows that the saddle-node bifurcation takes place at higher SRM efficacies relative to those under non-cooperation (compare Fig. 4.2.5).

However, a joint welfare maximizer accepts the regime shift sooner in comparison to the non-cooperative case where we additionally distinguished each country’s preferences. The black line – indicating indifference between both equilibria – is at the same time the aforementioned heteroclinic bifurcation curve. For the area between the two bifurcations (e.g. case VI) multiple steady states exist but the high-temperature equilibrium is not reachable from small initial conditions. Under full cooperation countries prefer the same equilibrium (compare Table 4.2.2), which contrasts the results from non-cooperative behavior. Fig. 4.2.7a and b report welfare differences between the two countries for the specific paths implied by the points of interest in Fig. 4.2.5 and Fig. 4.2.6 and highlight the potential for substantial equity issues arising from the availability of SRM. These distributional issues alone could

<sup>45</sup>Such an event may happen twice as we increase the effectiveness of geoengineering technologies. For our purposes the relevant bifurcation happens where the unstable manifold of the low-temperature equilibrium is merging with the stable manifold of the high-temperature one.

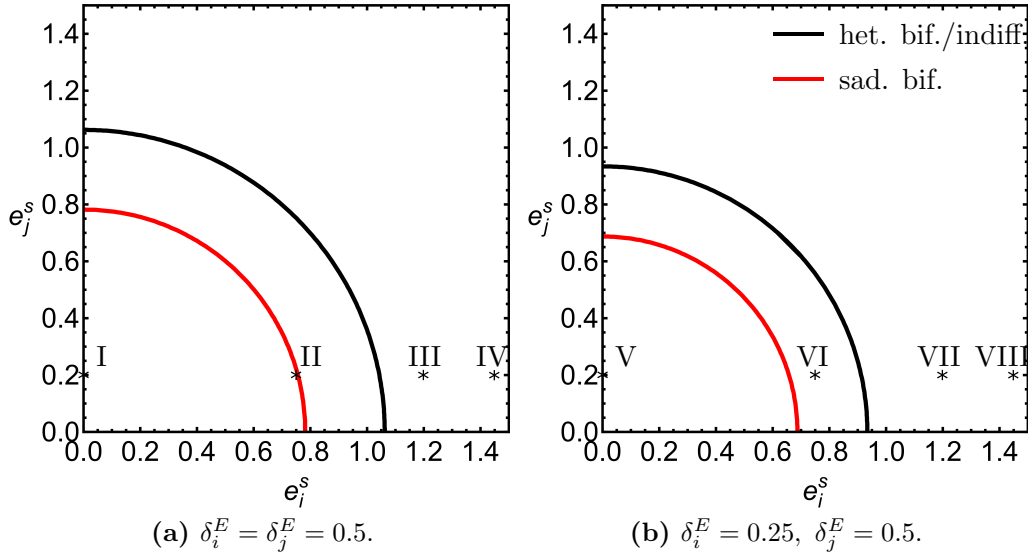


**Figure 4.2.6:** Two-dimensional bifurcations diagrams including the Skiba contours/heteroclinic bifurcation curves for the cooperative game.

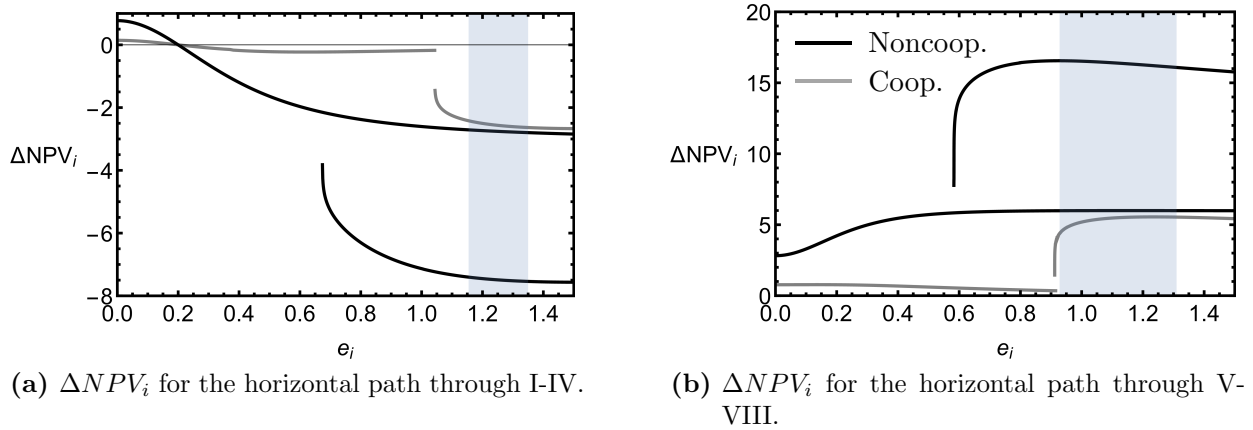


**Figure 4.2.7:** Welfare differentials for the corresponding Nash equilibria, i.e.,  $\Delta NPV_i = NPV_i - NPV_j$  for the implied path in Fig. 4.2.5/4.2.6 (a) and (b). The shaded areas display the regions where there is indeterminacy in the long run outcome of the game. To the left of the areas both countries prefer the low temperature equilibrium whereas areas to the right imply a regime shift.

trigger further sub-optimal welfare-reducing actions and should be therefore be taken into account as well.

Before we conclude we have to note the weak stability properties of the open loop solutions – they are not subgame perfect, i.e., they break down under deviation. These policies can therefore only be seen as an ex ante commitment to the game. Of course, one would like to derive the truly intertemporal strategically relevant Markov perfect Nash equilibria. However, this is a hard task that we were not able to accomplish. It requires solving the coupled nonlinear Bellman equations,

$$rV_i = \max_{l_i \geq 0, s_i \geq 0} \left\{ \alpha_i \ln l_i - \frac{c_i}{2} s_i^2 - \frac{\delta_i^T}{2} (T - e_i s_i - e_j s_j)^2 - \frac{\delta_i^E}{2} (s_i^2 + s_j^2) + V_i' \left( l_i + l_j - bT + \frac{T^2}{1 + T^2} \right) \right\}, \quad (4.2.12)$$

for  $i = 1, 2$  and where  $V_i$  represents country  $i$ 's value function. Naturally, the difficulty arises from the nonlinearity in the state which is also well acknowledged in the literature (Karp & Seguin 2014, Mäler et al. 2003, Kossioris et al. 2008 and Dockner & Wagener 2014). Whether Markovian equilibria simply lead to the same qualitative structure and differ solely quantitatively is an open question and should be on the research agenda for more sophisticated studies.

#### 4.2.4 Conclusion and discussion

Despite all the efforts by countries to cut CO2 emissions, there remains a strictly positive probability (ever-increasing under business-as-usual) of causing a regime shift in the global climate system, possibly triggering catastrophic damages for many nations. This is especially true when a global cooperative abatement strategy with broad participation remains unobtainable. We have shown that asymmetry of countries (a very realistic assumption) plays an even more important role when studying equilibrium behaviour in a tipping environment. Although a cheap carbon dioxide removal technique would be safe and could avoid a catastrophe if sufficiently effective, they remain mostly irrelevant at this point due to their technical and cost constraints. The purported affordability of SRM makes it an attractive option but its side effects could potentially be more costly to other countries than the shift

|         | I (V)                          |          | II (VI)                        |          | III (VII)        |                                | IV (VIII)        |                                |
|---------|--------------------------------|----------|--------------------------------|----------|------------------|--------------------------------|------------------|--------------------------------|
|         | L                              | H        | L                              | H        | L                | H                              | L                | H                              |
| $l_i$   | 0.047<br>(0.047)               | -<br>(-) | 0.051<br>(0.051)               | -<br>(-) | 0.051<br>(0.051) | 0.175<br>(0.217)               | 0.051<br>(0.051) | 0.237<br>(0.290)               |
| $l_j$   | 0.047<br>(0.047)               | -<br>(-) | 0.051<br>(0.051)               | -<br>(-) | 0.051<br>(0.051) | 0.175<br>(0.217)               | 0.051<br>(0.051) | 0.237<br>(0.290)               |
| $s_i$   | 0.000<br>(0.000)               | -<br>(-) | 0.377<br>(0.406)               | -<br>(-) | 0.305<br>(0.318) | 1.397<br>(1.570)               | 0.270<br>(0.278) | 1.377<br>(1.538)               |
| $s_j$   | 0.306<br>(0.378)               | -<br>(-) | 0.100<br>(0.108)               | -<br>(-) | 0.051<br>(0.053) | 0.233<br>(0.262)               | 0.037<br>(0.038) | 0.190<br>(0.212)               |
| $T$     | 0.272<br>(0.277)               | -<br>(-) | 0.372<br>(0.384)               | -<br>(-) | 0.411<br>(0.420) | 1.883<br>(2.076)               | 0.424<br>(0.432) | 2.165<br>(2.385)               |
| $T'$    | 0.210<br>(0.201)               | -<br>(-) | 0.069<br>(0.058)               | -<br>(-) | 0.035<br>(0.028) | 0.160<br>(0.139)               | 0.026<br>(0.020) | 0.131<br>(0.113)               |
| $NPV_i$ | <b>-53.7</b><br><b>(-53.1)</b> | -<br>(-) | <b>-50.3</b><br><b>(-49.7)</b> | -<br>(-) | -49.4<br>(-49.0) | <b>-47.2</b><br><b>(-39.3)</b> | -49.1<br>(-48.8) | <b>-41.7</b><br><b>(-33.7)</b> |
| $NPV_j$ | <b>-53.8</b><br><b>(-53.9)</b> | -<br>(-) | <b>-50.1</b><br><b>(-50.1)</b> | -<br>(-) | -49.2<br>(-49.2) | <b>-44.7</b><br><b>(-44.9)</b> | -49.0<br>(-49.0) | <b>-39.0</b><br><b>(-39.2)</b> |

**Table 4.2.2:** Total welfare and steady state values for controls and the state under full cooperation.  $(e_i, e_j) = \{(0, 0.2), (0.75, 0.2), (1.2, 0.2), (1.45, 0.2)\}$  was used for respective cases I-IV and V-VIII in Fig. 4.2.6.

in temperature itself. In this paper we focused on the intersection of asymmetry and tipping point dynamics. In our model, the existence of multiple non-rankable Nash equilibria essentially generates an issue of equilibrium selection. It is reasonable to assume that coordination takes place when a pareto-dominating Nash equilibrium exists, i.e., when both parties strictly prefer one path over the other (areas to the left/right of both indifference curves). Which of the two possible Nash equilibria will eventually be realized in the case of diverging preferences is hard to predict. It is possible that the country with the more effective geoengineering technology is (perceived to be) in a dominating position. As discussed above, in such a case a credible announcement of  $i$  to either deter the freeriding of  $j$  or to actively freedrive on its own and thereby to force the regime shift on the other country is conceivable. The consequence is significant welfare differentials for both parties. This difference in welfare is increasing in the asymmetry in the total costs of higher temperature levels and SRM among the two players.

A similar situation arises in case of irreversible climate damage, i.e., when the original state cannot be restored and the shift in temperature is permanent.<sup>46</sup> Clearly, the termination effect – the abrupt increase in temperature after suspension of climate engineering – becomes even more of an issue in case geoengineering drives temperature above a critical threshold from which it is impossible to revert. However, coordination on a specific equilibrium path could be made possible by a legally binding international environmental agreement, which is signed by both parties. Such a contract could also include side payments and would eliminate uncertainty in transient behaviour for the countries.

We conclude that further research is needed on (non)cooperative emission games with a potential regime shift and the role of climate-interfering technologies therein. As long as countries are effectively disunited in fighting climate change (the status quo; many sovereign countries and their leaders, constrained by conflicts of interest), merely solving for the globally optimal abatement strategy reflects an utopian benchmark – unattainable, given countries' present actions and declarations. Consequently, we emphasize the importance of strategic interaction, i.e., on games, specifically in settings where temperatures do not increase linearly. Moving forward, studying the consequences stemming from strategic efforts of competing

---

<sup>46</sup>While at  $b = 0.6$  there are hysteresis effects, the low pollution equilibrium can theoretically be restored by lowering the emission rate. This is not the case once  $b$  is sufficiently low, e.g.,  $b = 0.45$ .

---

players with differing endowments and beliefs should therefore provide valuable insights.

## Bibliography

- [1] Ainslie, G., & Haslam, N. (1992). *Hyperbolic discounting*. In G. Loewenstein & J. Elster (Eds.), *Choice over time* (pp. 57-92). New York. Russell Sage Foundation.
- [2] Arrow, K. (1962). The economic implications of learning by doing. *The Review of Economic Studies*, 29(3), 155-173.
- [3] Bala, G., Duffy, P. B., & Taylor, K. E. (2008). Impact of geoengineering schemes on the global hydrological cycle. *Proceedings of the National Academy of Sciences*, 105(22), 7664-7669.
- [4] Barrett, S. (2008). The incredible economics of geoengineering. *Environmental and Resource Economics*, 39(1), 45-54.
- [5] Barrett, S., & Dannenberg, A. (2016). An experimental investigation into ‘pledge and review’ in climate negotiations. *Climatic Change*, 138(1-2), 339-351.
- [6] Bascompte, J., García, M. B., Ortega, R., Rezende, E. L., & Pironon, S. (2019). Mutualistic interactions reshuffle the effects of climate change on plants across the tree of life. *Science advances*, 5(5).
- [7] Bass, F. M. (1969). A new product growth for model consumer durables. *Management science*, 15(5), 215-227.
- [8] Baumol, W. J. and Panzar, J. C. and Willig, R. D. (1982). *Contestable markets and the theory of industry structure*. San Diego: Harcourt Brace Jovanovich.
- [9] Bertoldi, P. (2011). *Assessment and experience of white certificate schemes in the european union*. European Commission, Directorate General JRC.
- [10] Bickel, J. E. (2013). Climate engineering and climate tipping-point scenarios. *Environment Systems & Decisions*, 33(1), 152-167.
- [11] Brynjolfsson, E., & McAfee (2017). The business of artificial intelligence. *Harvard Business Review*.
- [12] Buchanan, J. M. (1969). External diseconomies, corrective taxes, and market structure. *The American Economic Review*, 59(1), 174-177.
- [13] Carpenter, S. R., Ludwig, D., & Brock, W. A. (1999). Management of eutrophication

- for lakes subject to potentially irreversible change. *Ecological applications*, 9(3), 751-771.
- [14] Cartwright, E., & Menezes, M. L. (2014). Cheating to win: Dishonesty and the intensity of competition. *Economics Letters*, 122(1), 55-58.
- [15] Cellini, R., Lambertini L. (2003). Advertising with spillover effects in a differential oligopoly game with differentiated goods. *Central European Journal of Operations Research*, 11, 409-423.
- [16] Clark, G. (2008). *A farewell to alms: a brief economic history of the world (Vol. 25)*. Princeton University Press.
- [17] Coombs, W. T., & Holladay, S. J. (1996). Communication and attributions in a crisis: an experimental study in crisis communication. *Journal of Public Relations Research*, 8(4), 279-295.
- [18] Coombs, W. T. (2010). Conceptualizing crisis communication. In *Handbook of Risk and Crisis Communication (pp. 111-130)*. Routledge.
- [19] Coombs, W. T. (2015). *Ongoing crisis communication: planning, managing, and responding*. Sage Publications.
- [20] Corner, A., Pidgeon, N., & Parkhill, K. (2012). Perceptions of geoengineering: public attitudes, stakeholder perspectives, and the challenge of 'upstream' engagement. *Wiley Interdisciplinary Reviews: Climate Change*, 3(5), 451-466.
- [21] Darby, M. R., & Karni, E. (1973). Free competition and the optimal amount of fraud. *The Journal of law and economics*, 16(1), 67-88.
- [22] Dechert, W. D., & Nishimura, K. (1983). A complete characterization of optimal growth paths in an aggregated model with a non-concave production function. *Journal of Economic Theory*, 31(2), 332-354.
- [23] De Zeeuw, A., & Zemel, A. (2012). Regime shifts and uncertainty in pollution control. *Journal of Economic Dynamics and Control*, 36(7), 939-950.
- [24] Dietz, S., Bowen, A., Doda, B., Gambhir, A., & Warren, R. (2018). The economics of 1.5 C climate change. *Annual Review of Environment and Resources*, 43, 455-480.
- [25] Dockner, E. J., & Van Long, N. (1993). International pollution control: cooperative versus noncooperative strategies. *Journal of Environmental Economics and Management*, 25(1), 13-29.
- [26] Dockner, E. J., Jorgensen, S., Van Long, N., & Sorger, G. (2000). *Differential games in*

- economics and management science*. Cambridge University Press.
- [27] Dockner, E., & Wagener, F. (2014). Markov perfect Nash equilibria in models with a single capital stock. *Economic Theory*, 56(3), 585-625.
  - [28] Domen, F., & Hawranek, D. (2017). *Das geheime Kartell der deutschen Autobauer*. Der Spiegel, 21.
  - [29] Dudley, B. (2018). BP statistical review of world energy. *BP Statistical Review*, London, UK, accessed Aug, 6, 2018.
  - [30] Dulleck, U., & Kerschbamer, R. (2006). On doctors, mechanics, and computer specialists: the economics of credence goods. *Journal of Economic literature*, 44(1), 5-42.
  - [31] Eigruher, M., & Wirl, F. (2018). Climate Engineering in an Interconnected World: The Role of Tariffs. *Dynamic Games and Applications*, 8, 573-587.
  - [32] Eigruher, M., & Wirl, F. (2019). Cheating as a dynamic Marketing Strategy in Monopoly, Cartel and Duopoly. *Central European Journal of Operations Research*, 1-18.
  - [33] Engwerda, J. (2007). Algorithms for computing Nash equilibria in deterministic LQ games. *Computational Management Science*, 4(2), 113-140.
  - [34] Engwerda, J. (2009). Linear quadratic differential games: an overview. In *Advances in dynamic games and their applications* (pp. 1-34). Birkhäuser Boston.
  - [35] Engwerda, J. (2016). Properties of feedback Nash equilibria in scalar LQ differential games. *Automatica*, 69, 364-374.
  - [36] Fershtman, C. (1984). Goodwill and market shares in oligopoly. *Economica*, 51(203), 271-281.
  - [37] Gaskill, A. (2004). Desert Area Coverage: Global Albedo Enhancement Project.
  - [38] Gilpatric, S. M. (2011). Cheating in contests. *Economic Inquiry*, 49(4), 1042-1053.
  - [39] Gneezy, U. (2005). Deception: The role of consequences. *American Economic Review*, 95(1), 384-394.
  - [40] Gorzelany, J. (2015). *Dieselgate: what VW TDI owners should know*. Forbes. Retrieved August, 26, 2019 from <https://www.forbes.com/sites/jimgorzelany/2015/09/23/dieselgate-what-vw-tdi-owners-should-know/#1de07e1b443b>.
  - [41] Gotsi, M., & Wilson, A. M. (2001). Corporate reputation: seeking a definition corporate communications. *An International Journal*, 6(1), 24-30.

- 
- [42] Haidt, J. (2012). *The righteous mind: why good people are divided by politics and religion*. Vintage.
- [43] Hartl, R. F. (1987). A simple proof of the monotonicity of the state trajectories in autonomous control problems. *Journal of Economic Theory*, 41(1), 211-215.
- [44] Hartl, R. F., Kort, P. M., Feichtinger, G., & Wirl, F. (2004). Multiple equilibria and thresholds due to relative investment costs. *Journal of Optimization Theory and Applications*, 123(1), 49-82.
- [45] Haskel, J., & Westlake, S. (2018). *Capitalism without capital: the rise of the intangible economy*. Princeton University Press.
- [46] Heutel, G., Moreno-Cruz, J., & Shayegh, S. (2016). Climate tipping points and solar geoengineering. *Journal of Economic Behavior & Organization*, 132, 19-45.
- [47] Heyen, D., Wiertz, T., & Irvine, P. J. (2015). Regional disparities in SRM impacts: the challenge of diverging preferences. *Climatic Change*, 133(4), 557-563.
- [48] Heyen, D. and Horton, J. and Moreno-Cruz, J. B. (2017). Strategic implications of counter-geoengineering: escalation, cooperation, or nonuse? *Working Paper*.
- [49] Hillenbrand, C., & Money, K. (2007). Corporate responsibility and corporate reputation: two separate concepts or two sides of the same coin?. *Corporate Reputation Review*, 10(4), 261-277.
- [50] Hirschmann, D. (2014). Optimal quality provision when reputation is subject to random inspections. *Operations Research Letters*, 42(1), 64-69.
- [51] Hotten, R. (2015). *Volkswagen: the scandal explained*. BBC. Retrieved August, 26, 2019 from <http://www.bbc.com/news/business-34324772>.
- [52] IPCC, (2014): Climate Change 2014: Synthesis Report. Contribution of Working Groups I, II and III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change.
- [53] Joskow, P., & Rozanski, G. A. (1979). The effects of learning by doing on nuclear plant operating reliability. *The Review of Economics and Statistics*, 61(2), 161-68.
- [54] Kamien, M. I., & Schwartz, N. L. (1971). Optimal maintenance and sale age for a machine subject to failure. *Management Science*, 17(8), B-495.
- [55] Karp, L., & Séguin, C. (2014). Nonlinear differential games with Skiba points. *Working paper*.

- 
- [56] Kiseleva, T., & Wagener, F. O. (2010). Bifurcations of optimal vector fields in the shallow lake model. *Journal of Economic Dynamics and Control*, 34(5), 825-843.
- [57] Kiseleva, T. (2016). Heterogeneous beliefs and climate catastrophes. *Environmental and Resource Economics*, 65(3), 599-622.
- [58] Kopel, M., & Lamantia, F. (2018). The persistence of social strategies under increasing competitive pressure. *Journal of Economic Dynamics and Control*, 91, 71-83.
- [59] Kossioris, G., Plexousakis, M., Xepapadeas, A., de Zeeuw, A., & Mäler, K. G. (2008). Feedback Nash equilibria for non-linear differential games in pollution control. *Journal of Economic Dynamics and Control*, 32(4), 1312-1331.
- [60] Krugman, P. (1991). History versus expectations. *The Quarterly Journal of Economics*, 106(2), 651-667.
- [61] Lambertini, L., & Zaccour, G. (2015). Inverted-U aggregate investment curves in a dynamic game of advertising. *Economics Letters*, 132, 34-38.
- [62] Langevoort, D. C. (2002). The Organizational Psychology of Hyper-Competition: Corporate Irresponsibility and the Lessons of Enron. *Geo. Wash. L. Rev.*, 70, 968.
- [63] Lenton, T. M., Held, H., Kriegler, E., Hall, J. W., Lucht, W., Rahmstorf, S., & Schellhuber, H. J. (2008). Tipping elements in the Earth's climate system. *Proceedings of the national Academy of Sciences*, 105(6), 1786-1793.
- [64] Lenton, T. M. (2011). Early warning of climate tipping points. *Nature climate change*, 1(4), 201.
- [65] Levitt, S. D., List, J. A., & Syverson, C. (2013). Toward an understanding of learning by doing: evidence from an automobile assembly plant. *Journal of political Economy*, 121(4), 643-681.
- [66] Lin-Hi, N., & Müller, K. (2013). The CSR bottom line: Preventing corporate social irresponsibility. *Journal of Business Research*, 66(10), 1928-1936.
- [67] Mäler, K. G., Xepapadeas, A., & De Zeeuw, A. (2003). The economics of shallow lakes. *Environmental and resource Economics*, 26(4), 603-624.
- [68] Manoussi, V., & Xepapadeas, A. (2017). Cooperation and competition in climate change policies: mitigation and climate engineering when countries are asymmetric. *Environmental and Resource Economics*, 66(4), 605-627.
- [69] Mazar, N., & Ariely, D. (2006). Dishonesty in everyday life and its policy implications.

- Journal of public policy & Marketing*, 25(1), 117-126.
- [70] Millard-Ball, A. (2012). The tuvalu syndrome. *Climatic Change*, 110(3-4), 1047-1066.
- [71] Moreno-Cruz, J. B., & Keith, D. W. (2013). Climate policy under uncertainty: a case for solar geoengineering. *Climatic Change*, 121(3), 431-444.
- [72] Nelson, P. (1974). Advertising as information. *Journal of political economy*, 82(4), 729-754.
- [73] Nerlove, M., & Arrow, K. J. (1962). Optimal advertising policy under dynamic conditions. *Economica*, 29(114), 129-142.
- [74] Nordhaus, W. D. (1991). To slow or not to slow: the economics of the greenhouse effect. *The Economic Journal*, 101(407), 920-937.
- [75] Nordhaus, W. D. (2007). A review of the Stern review on the economics of climate change. *Journal of Economic Literature*, 45(3), 686-702.
- [76] Nordhaus, W. D. (2015). Climate clubs: overcoming free-riding in international climate policy. *American Economic Review*, 105(4), 1339-70.
- [77] Reichelstein, S., & Sahoo, A. (2018). Relating product prices to long-run marginal cost: Evidence from solar photovoltaic modules. *Contemporary Accounting Research*, 35(3), 1464-1498.
- [78] Reynolds, S. S. (1987). Capacity investment, preemption and commitment in an infinite horizon model. *International Economic Review*, 69-88.
- [79] Reynolds, S. S. (1991). Dynamic oligopoly with capacity adjustment costs. *Journal of Economic Dynamics and Control*, 15(3), 491-514.
- [80] Roberts, P. W., & Dowling, G. R. (2002). Corporate reputation and sustained superior financial performance. *Strategic management journal*, 23(12), 1077-1093.
- [81] Robock, A. (2008). 20 reasons why geoengineering may be a bad idea. *Bulletin of the Atomic Scientists*, 64(2), 14-18.
- [82] Roth, A. E. (Ed.). (1985). Game-theoretic models of bargaining. *Cambridge University Press*.
- [83] Rowat, C. (2007). Non-linear strategies in a linear quadratic differential game. *Journal of Economic Dynamics and Control*, 31(10), 3179-3202.
- [84] The Royal Society (2009), *Geoengineering the climate: science, governance and uncertainty*. The Royal Society.

- [85] Ruff, L. E. (1988). Least-cost planning and demand-side management: six common fallacies and one simple truth. *Public Utilities Fortnightly*, 121(9), 19-26.
- [86] Schelling, T. C. (1996). The economic diplomacy of geoengineering. *Climatic Change*, 33(3), 303-307.
- [87] Schellnhuber, H. J. (2009). Tipping elements in the Earth System. *Proceedings of the National Academy of Sciences*, 106(49), 20561-20563.
- [88] Schmidt, H., Alterskjær, K., Bou Karam, D., Boucher, O., Jones, A., Kristjánsson, J. E., ... & Lawrence, M. (2012). Solar irradiance reduction to counteract radiative forcing from a quadrupling of CO<sub>2</sub>: climate responses simulated by four earth system models. *Earth System Dynamics*, 3, 63-78.
- [89] Schwieren, C., & Weichselbaumer, D. (2010). Does competition enhance performance or cheating? A laboratory experiment. *Journal of Economic Psychology*, 31(3), 241-253.
- [90] Sethi, S. P. (1977). Nearest feasible paths in optimal control problems: theory, examples, and counterexamples. *Journal of Optimization Theory and Applications*, 23(4), 563-579.
- [91] Sethi, S. P. (1979). Optimal advertising policy with the contagion model. *Journal of Optimization theory and applications*, 29(4), 615-627.
- [92] Shleifer, A. (2004). Does competition destroy ethical behavior?. *American Economic Review*, 94(2), 414-418.
- [93] Skiba, A. K. (1978). Optimal growth with a convex-concave production function. *Econometrica*, 46(3), 527-539.
- [94] Smith, G., & Parloff, R. (2016). *Hoaxwagen*. Fortune. Retrieved August, 26, 2019 from <http://fortune.com/inside-volkswagen-emissions-scandal/>.
- [95] Solomon, S., Qin, D., Manning, M., Averyt, K., Marquis, M., & Tignor, M. M. (Eds.). (2007). Climate change 2007-the physical science basis: Working group I contribution to the fourth assessment report of the IPCC (Vol. 4). *Cambridge university press*.
- [96] Sorger, G. (1989). Competitive dynamic advertising: A modification of the Case game. *Journal of Economic Dynamics and Control*, 13(1), 55-80.
- [97] Steffen, W., Rockström, J., Richardson, K., Lenton, T. M., Folke, C., Liverman, D., ... & Donges, J. F. (2018). Trajectories of the Earth System in the Anthropocene. *Proceedings of the National Academy of Sciences*, 115(33), 8252-8259.
- [98] Stoerk, T., Wagner, G., & Ward, R. E. (2018). Policy Brief—Recommendations for Im-

- proving the Treatment of Risk and Uncertainty in Economic Estimates of Climate Impacts in the Sixth Intergovernmental Panel on Climate Change Assessment Report. *Review of Environmental Economics and Policy*, 12(2), 371-376.
- [99] Swanson, R. (2011). The silicon photovoltaic roadmap. *Stanford Energy Seminar*, Vol. 14, p. 2011.
- [100] Trucco, L.C., & Mullainathan, S. (2014). *Hold the Phone: A Big-Data Conundrum*. The New York Times, Retrieved August, 26, 2019 from [https://www.nytimes.com/2014/07/27/upshot/hold-the-phone-a-big-data-conundrum.html?\\_r=2](https://www.nytimes.com/2014/07/27/upshot/hold-the-phone-a-big-data-conundrum.html?_r=2).
- [101] Tollefson, J. (2018). Sucking carbon dioxide from air is cheaper than scientists thought. *Nature*, 558(7709), 173.
- [102] Tsutsui, S., & Mino, K. (1990). Nonlinear strategies in dynamic duopolistic competition with sticky prices. *Journal of Economic Theory*, 52(1), 136-161.
- [103] Urpelainen, J. (2012). Geoengineering and global warming: a strategic perspective. *International Environmental Agreements: Politics, Law and Economics*, 12(4), 375-389.
- [104] Vallaster, C., Lindgreen, A., & Maon, F. (2012). Strategically leveraging corporate social responsibility: A corporate branding perspective. *California management review*, 54(3), 34-60.
- [105] Victor, D. G. (2008). On the regulation of geoengineering. *Oxford Review of Economic Policy*, 24(2), 322-336.
- [106] Vidale, M. L., & Wolfe, H. (1957). An operations-research study of sales response to advertising. *Operations research*, 5(3), 370-381.
- [107] Volkswagen AG (2015). *Clarification moving forward: internal investigations at Volkswagen identify irregularities in CO2 levels*. Retrieved August, 26, 2019 from [https://www.volkswagen-media-services.com/en/detailpage/-/detail/Clarification-moving-forward-internal-investigations-at-Volkswagen-identify-irregularities-in-CO2-levels/view/2857367/18108d8d101b6284fe7b29bcf415eda5?p\\_p\\_auth=7WdHl2wa](https://www.volkswagen-media-services.com/en/detailpage/-/detail/Clarification-moving-forward-internal-investigations-at-Volkswagen-identify-irregularities-in-CO2-levels/view/2857367/18108d8d101b6284fe7b29bcf415eda5?p_p_auth=7WdHl2wa).
- [108] Volkswagen AG (2017). *Annual report 2017*. Wolfsburg: Volkswagen AG.
- [109] Volkswagen AG (2018). *Annual report 2018*. Wolfsburg: Volkswagen AG.
- [110] Wagener, F. O. (2003). Skiba points and heteroclinic bifurcations, with applications to the shallow lake system. *Journal of Economic Dynamics and Control*, 27(9), 1533-1561.

- [111] Wagner G., Weitzman M.L. (2012) Playing God. *Foreign Policy* [Internet].
- [112] Weiner, B. (2012). *Human motivation*. Springer Science & Business Media.
- [113] Weitzman, M. L. (2015). A voting architecture for the governance of free-driver externalities, with application to geoengineering. *The Scandinavian Journal of Economics*, 117(4), 1049-1068.
- [114] Wirl, F. (2016). Indeterminacy and history dependence of strategically interacting players. *Economics Letters*, 145, 19-24.
- [115] Wirl, F. (2007). Do multiple Nash equilibria in Markov strategies mitigate the tragedy of the commons? *Journal of Economic Dynamics and Control*, 31(11), 3723-3740.
- [116] Wirl, F. (2014). Taxes versus permits as incentive for the intertemporal supply of a clean technology by a monopoly. *Resource and Energy Economics*, 36(1), 248-269.

## List of Figures

|                                                                                                                                                                                                                                                                                                                                                                                       |    |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| 1.1.1 Share prices of the three main German carmakers. . . . .                                                                                                                                                                                                                                                                                                                        | 16 |
| 1.5.1 Strategies, cheating in (a) and (c) and production in (b) and (d), in state space. . . . .                                                                                                                                                                                                                                                                                      | 31 |
| 1.5.2 Sensitivity of stationary values with respect to model parameters around the reference case (vertical line) in (1.5.1). Production of the monopoly (constant at 0.24) and cartel (increasing but rather flat, from 0.25-0.26) are much lower and excluded in Fig. 1.5.2d in order to highlight the difference between the open loop and the Markov perfect equilibrium. . . . . | 34 |
| 2.3.1 Phase diagram of system (2.3.4) with (2.3.18). . . . .                                                                                                                                                                                                                                                                                                                          | 47 |
| 2.3.2 Bifurcation diagrams of (2.3.4) in $k$ , shown in (a) and in $\beta$ , shown in (b). (c) shows the Skiba countours in $(k, l_0)$ space for different learning speeds. . . . .                                                                                                                                                                                                   | 48 |
| 2.3.3 Equilibria for varying levels of $k$ are plotted in (a). In (b) fixed points, along with some converging paths, are shown in state space. . . . .                                                                                                                                                                                                                               | 53 |
| 2.3.4 Social welfare depending on the number of firms, $n$ , under the usual parameter assumptions. . . . .                                                                                                                                                                                                                                                                           | 57 |
| 2.4.1 Different Markov perfect symmetric equilibria (a and b) for the duopoly are shown in (a) and (b). The other two graphs in (c) and (d) plot paths of learning and output, departing from different initial conditions, $l_1(0) = 1$ and $l_2(0) = 0.5$ . . . . .                                                                                                                 | 62 |
| 2.4.2 Bifurcation diagram in $b$ with indicated stability of the given fixed points. . . . .                                                                                                                                                                                                                                                                                          | 63 |
| 2.4.3 (a) Comparison of all (aggregated) strategies under all market forms. (b) compares prices versus learning with costs and shows that the price degression is much less than the cost degression. . . . .                                                                                                                                                                         | 70 |
| 2.4.4 Net present value of social welfare (a), decomposed in consumer (b) and producer surplus (c) for $n = 2, 4, 6$ . The horizontal line indicates the monopoly, the dashed line the cartel and the dotted line the oligopoly. . . . .                                                                                                                                              | 72 |
| 3.3.1 Trajectories of controls and state over time with $\alpha = 0.02$ and $r = 0.03$ . . . . .                                                                                                                                                                                                                                                                                      | 85 |

---

|                                                                                                                                                                                                                                                     |     |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| 3.3.2 Total payoffs for both countries under a tariff-free (solid) and a tariff regime (dashed). . . . .                                                                                                                                            | 86  |
| 3.3.3 (a) Slopes of optimal price and tax. (b) Price and tax strategies for three different scenarios up to the respective steady state. . . . .                                                                                                    | 88  |
| 3.4.1 Markov perfect strategies in state space (a) and optimal trajectories over time (b). . . . .                                                                                                                                                  | 93  |
| 4.1.1 Identified channels by which corporate reputation is being determined. From Hillenbrand & Money (2007), p.268. . . . .                                                                                                                        | 101 |
| 4.1.2 Static equilibria of the shallow lake dynamic for $b = 0.55$ and two possible settings: (a) exploits an existing resource, i.e., reputation capital. (b) builds up a fictitious stock of bad will. . . . .                                    | 106 |
| 4.1.3 Phase portraits of system (4.1.14) and varying effectiveness levels of $\alpha$ . . . .                                                                                                                                                       | 109 |
| 4.1.4 A skiba point in state space (a). (b) compares NPV's for both trajectories. . .                                                                                                                                                               | 110 |
| 4.1.5 Bifurcation diagram in $\alpha$ on the left and NPV's on the right. . . . .                                                                                                                                                                   | 111 |
| 4.1.6 Bifurcation diagrams in $\alpha$ and $\beta$ in (a) and (c). Chart (b) and (d) indicate optimality in areas where multiple steady states exist. . . . .                                                                                       | 112 |
| 4.1.7 Two close parameter sets inducing very different transient (and long run) behaviors. . . . .                                                                                                                                                  | 113 |
| 4.2.1 Projected emission pathways (a) and average relative temperature increase (b). Taken from IPCC, 2014. . . . .                                                                                                                                 | 117 |
| 4.2.2 Phase diagrams in state-costate space for the symmetric case. (a) SRM technology is not available, i.e. $e_i = e_j = 0$ . (b) SRM technology is available and effective, i.e. $e_i = e_j = 0.7$ and $\delta_i^E = \delta_j^E = 0.5$ . . . . . | 123 |
| 4.2.3 Bifurcation diagrams (a), (b) and total welfare per country (c) for symmetric technology efficacies and $\delta_i^E = \delta_j^E = 0.5$ . . . . .                                                                                             | 124 |
| 4.2.4 Typical phase diagrams for the general asymmetric case ( $e_i = 1, e_j = 0.1$ ). . .                                                                                                                                                          | 126 |
| 4.2.5 Two-dimensional bifurcations diagram including Skiba contours for each player.                                                                                                                                                                | 126 |
| 4.2.6 Two-dimensional bifurcations diagrams including the Skiba contours/heteroclinic bifurcation curves for the cooperative game. . . . .                                                                                                          | 131 |

---

|       |                                                                                                                                                                                                                                                                                                                                                                                                    |     |
|-------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| 4.2.7 | Welfare differentials for the corresponding Nash equilibria, i.e., $\Delta NPV_i = NPV_i - NPV_j$ for the implied path in Fig. 4.2.5/4.2.6 (a) and (b). The shaded areas display the regions where there is indeterminacy in the long run outcome of the game. To the left of the areas both countries prefer the low temperature equilibrium whereas areas to the right imply a regime shift. . . | 131 |
| B.1   | Comparison of fixed point stability with the threshold in (2C.0.1). . . . .                                                                                                                                                                                                                                                                                                                        | 158 |
| B.2   | Bif. in $(a, b)$ including the infeasible region where costs become negative. Numbers indicate multiplicity of Markov perfect equilibria. . . . .                                                                                                                                                                                                                                                  | 158 |
| A.1   | Coca Cola, BP and Shell. . . . .                                                                                                                                                                                                                                                                                                                                                                   | 166 |
| A.2   | Comparison of Apple's trend in (a) and Samsung's trend in (b), as shown in Trucco, 2014. . . . .                                                                                                                                                                                                                                                                                                   | 167 |
| A.3   | Example in the video game industry. . . . .                                                                                                                                                                                                                                                                                                                                                        | 168 |
| A.4   | Examples in the automotive industry. . . . .                                                                                                                                                                                                                                                                                                                                                       | 168 |

## List of Tables

|                                                                                                                                                                                                                                          |     |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| 1.5.1 Steady state comparison of the controls and states including total discounted utility. . . . .                                                                                                                                     | 32  |
| 2.3.1 Steady state values for output, learning, costs and total discounted profits ( $\pi$ for $l_{i0} = 0.5$ ). In addition to the duopoly, values for $n = 4$ (in parentheses) as well as for $n = 6$ (in brackets) are shown. . . . . | 56  |
| 2.4.1 Total discounted profits (for $l_{i0} = 0.5$ ) and steady state values for production, costs, learning and differing number of players. . . . .                                                                                    | 69  |
| 4.2.1 Total welfare and steady state values for controls and the state. $(e_i, e_j) = \{(0, 0.2), (0.75, 0.2) (1.2, 0.2), (1.45, 0.2)\}$ was used for respective cases I-IV and V-VIII in Fig. 4.2.5. . . . .                            | 128 |
| 4.2.2 Total welfare and steady state values for controls and the state under full cooperation. $(e_i, e_j) = \{(0, 0.2), (0.75, 0.2) (1.2, 0.2), (1.45, 0.2)\}$ was used for respective cases I-IV and V-VIII in Fig. 4.2.6. . . . .     | 133 |



### German abstract

Diese kumulative Arbeit besteht aus drei Artikeln und zwei kleineren Arbeiten, die sich mit dynamischen Problemen des Industrie-, Energie- und Umweltmanagements befassen. In allen Essays (bis auf Kapitel 4.1) analysieren wir Probleme, die sich aus der strategischen Interaktion von Entscheidungsträgern über die Zeit ergeben.

Der erste Aufsatz befasst sich mit einer Reihe von Unternehmen, die nicht nur im Output, sondern auch in übertriebenen Behauptungen gegenüber Kunden konkurrieren, d. h. im Schummeln. Wir setzen das Differentialspiel als umgekehrte Version vom bekannten Goodwill-Modell a la Nerlove-Arrow an und weisen auf, wie wichtig die konkrete Informationsstruktur ist – und wie sie genutzt werden kann um Betrug durch Unternehmen zu verhindern.

Im zweiten Artikel analysieren wir, wie Unternehmen, die sowohl internen Lernprozessen als auch positiven externen Effekten ausgesetzt sind, ihren Produktionspfad einrichten. Diese Situation ähnelt Märkten, in denen Hersteller im Wettbewerb um die Herstellung von Solar-Photovoltaik-Modulen stehen. Wir analysieren und vergleichen das Ergebnis unter verschiedenen Industrieorganisationen und identifizieren dabei auch eine technische Anomalie, in dem Sinne, dass wir im entsprechenden LQ-Differentialspiel mehrere stabile lineare Markov perfekte Gleichgewichte finden.

Das dritte Kapitel befasst sich mit möglichen Anreizproblemen, die sich aus neuartigen Geoengineering Techniken ergeben, insbesondere dem *Solar Radiation Management*. Wir zeigen auf, dass der Einsatz solcher (polarisierender) Techniken mögliche Auswirkungen auf den internationalen Handel haben könnte und weisen darauf hin, dass man diesen Effekt daher vor/während und nach dem Einsatz berücksichtigen sollte.

Abschließend möchten wir zwei alternative Anwendungen einer bestimmten Dynamik in der ökologischen Ökonomie vorschlagen, nämlich des *shallow lake Spiels*. Sowohl die gesellschaftliche Reaktion auf aufgefliegenen Betrug durch Unternehmen, als auch die Existenz von Kipppunkten im Weltklimasystem lassen sich durch die Dynamiken des shallow lakes modellieren. Diese Anmerkungen beziehen sich thematisch auf das erste bzw. dritte Kapitel und schließen die gesamte Arbeit ab.



## Appendix 2A

### Ad Chapter 2.3.4: Competition

Since the limit of oligopolistic competition does not allow for learning of competitive and thus infinitesimal small firms, the approach applied in Krugman (1991) and in the endogenous growth literature is used below (see Wirl (2016) for an extension of this model to strategically acting agents that, nevertheless, leads to the same history dependent outcomes as competition). Small letters, refer to firm specific and thus also controllable variables like  $u$  for the production of a competitive firm and capitals to market aggregates, e.g.,  $U$ . In this setting, each competitive firm,

$$\max_u \int_0^\infty e^{-rt} [(1 - U) u - uc(l)] dt,$$

where  $c(l)$  denotes the unit production costs conditional on learning, either the power or the linear function in our paper.

$$\max_u \int_0^\infty e^{-rt} [(1 - U) u - ukl^{-\beta}] dt,$$

subject to

$$\dot{l} = au + \bar{b}U - \delta l, \quad l(0) = l_0,$$

where  $\bar{b} < a$  accounts for the spillover to an individual firm from aggregate production. Assuming a symmetric competitive equilibrium of identical firms with their aggregate mass normalized to 1, then individual and market aggregates are identical,

$$U = u, \text{ and } L = l.$$

It is important to note that the capitalized market aggregate variables are exogenous to the individual firm's optimization, yet their evolution can be predicted perfectly in a rational expectation equilibrium.

**Remark:** *The industry is competitive but not subject to free entry, because a later entering firm lacks the learning (at least the own learning but presumably also the spillover if this is tied to producing) and thus must start at higher costs rendering entry unprofitable. In short, this allows us to ignore the zero profit condition, yet (the net present value of) profits must be non-negative to get the business going.*

The Hamiltonian of the competitive firm's intertemporal optimization problem,

$$H = (1 - U)u - uc(l) + \lambda(au + bU - \delta l),$$

$$H = (1 - U)u - ukl^{-\beta} + \lambda(au + bU - \delta l),$$

is linear in the control

$$H_u = 1 - U - c + a\lambda,$$

$$H_u = 1 - U - kl^{-\beta} + a\lambda,$$

and

$$\dot{\lambda} = r\lambda - H_l = (r + \delta)\lambda + uc',$$

$$\dot{\lambda} = r\lambda - H_l = (r + \delta)\lambda - u\beta kl^{-\beta-1}.$$

A competitive equilibrium is only possible along the singular arc,  $H_u = 0$ , because otherwise nothing or very large volumes were produced by each firm. Using in addition from now on

the condition of symmetry yields for the singular arc,

$$H_u = 0 \implies u = 1 + a\lambda - c,$$

and thus the canonical equations,

$$\begin{aligned} \dot{l} &= (a+b)(1+a\lambda-c) - \delta l, \quad l(0) = l_0, \\ \dot{\lambda} &= (r+\delta)\lambda + (1+a\lambda-kl^{-\beta})c'. \end{aligned}$$

$$\begin{aligned} \dot{l} &= (a+b)(1+a\lambda-kl^{-\beta}) - \delta l, \quad l(0) = l_0, \\ \dot{\lambda} &= (r+\delta)\lambda - (1+a\lambda-kl^{-\beta})\beta kl^{-\beta-1}. \end{aligned}$$

A long run competitive equilibrium is only possible if prices exceed cost and this inequality can be strict and thus allow for long run profits due to the assumption of many competitive firms but no entry. Since,

$$\begin{aligned} \dot{l} &= 0 \implies u = \frac{\delta l}{a+b}, \\ \dot{\lambda} &= 0 \implies \lambda = -\frac{uc'}{r+\delta}, \end{aligned}$$

combining

$$\begin{aligned} u &= 1 - kl^{-\beta} + a\lambda = 1 - c - a\frac{uc'}{r+\delta} \implies (r+\delta)u = (r+\delta)(1-c) - auc' \\ \implies u &= \frac{(1-c)(r+\delta)}{r+\delta+ac'}. \end{aligned}$$

Therefore, positive production requires for costs below the choke price,  $c < 1$ , that

$$r + \delta + ac' > 0.$$

Since the market price cannot fall below the costs in a viable competitive equilibrium, at

least in the long run (but not necessarily transiently),

$$p = 1 - u = \frac{r + \delta + ac' - (1 - c)(r + \delta)}{r + \delta + ac'} \geq c \iff \frac{(r + \delta)c + ac'}{r + \delta + ac'} \geq c$$

Therefore,

$$p \geq c \iff \frac{r + \delta + a\frac{c'}{c}}{r + \delta + ac'} \geq 1 \iff \frac{c'}{c} > c' \iff c > 1,$$

and the costs must exceed the choke price (equal to 1).

## Appendix 2B

### Ad Chapter 2.4.1: Cartel solution

This leads to the following system of equations,

$$\begin{aligned}rw_1 &= \frac{(aw_1 + bnw_1 - bw_1 - kn + n)(aw_2 + b(n-1)w_2 + \beta n) - 2\delta n^2 w_1}{2n^2} \\r\frac{w_2}{2} &= \frac{(aw_2 + b(n-1)w_2 + \beta n)^2}{4n^2} - \delta w_2.\end{aligned}$$

the equation for  $w_0$  is dropped since the intercept is irrelevant for the policy (2.4.2). It turns out (as we know) that we guessed correctly and the coefficients are given by,

$$\begin{aligned}w_1 &= \frac{\beta\psi_1(k-1)n}{\psi_3 - \psi_1(\psi_2 - \delta n)}, \\w_2 &= \frac{\psi_2(\psi_1 - \delta(k-1)n) + \psi_3}{(k-1)(\delta + r)(a + b(n-1))^2}\end{aligned}$$

and

$$\begin{aligned}\psi_1 &= (k-1)n(2\delta + r), \\ \psi_2 &= \beta(b-a) + n(-b\beta + 2\delta + r), \\ \psi_3 &= \sqrt{(k-1)^2 n^3 (\delta + r)^2 (2\delta + r) (2\beta(b-a) + n(-2b\beta + 2\delta + r))}.\end{aligned}$$

Substituting and rearranging yields Result 2.4.1.

## Appendix 2C

### Ad Chapter 2.4.2: Stability of fixed points

As usual, stability has been determined by computing the magnitude and sign of the eigenvalues of the fixed points. Clearly, both EWs being negative and real establishes stability. However, this is only the case for the set of fixed points coloured black in Fig. 2.4.2. To determine conditional stability we also computed the sign restricted to the 45 degree line and find, that given

$$\begin{aligned} \dot{l} &= au_i + bu_j - \delta l, \\ \text{symm.} \implies \dot{l} &= au + bu - \delta l, \\ \text{subst.opt.} \implies \dot{l} &= (a + b)u^{opt} - \delta l, \\ \text{linear struct.} \implies \dot{l} &= (a + b)(\alpha + \beta l) - \delta l, \\ \text{simplify to} \implies \dot{l} &= (a + b)(\alpha) + l(\beta(a + b - \delta)). \end{aligned}$$

Thus the system converges along symmetric states if

$$\begin{aligned} \beta(a + b) - \delta &< 0, \\ \iff \beta &< \frac{\delta}{a + b}, \end{aligned} \tag{2C.0.1}$$

i.e., if the state coefficient is sufficiently small, determined by the threshold in (2C.0.1). Computing all fixed points, evaluating their respective eigenvalues and comparing coefficients to the threshold above leads to the following picture

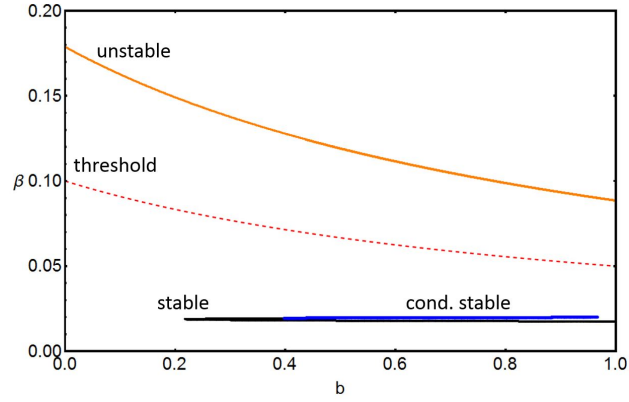


**Figure B.1:** Comparison of fixed point stability with the threshold in (2C.0.1).

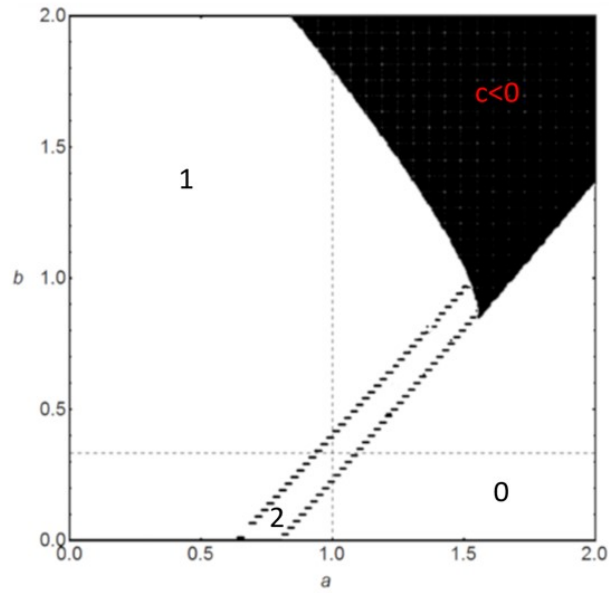


**Figure B.2:** Bif. in  $(a, b)$  including the infeasible region where costs become negative. Numbers indicate multiplicity of Markov perfect equilibria.

## Appendix 2D

### Ad Chapter 2.4.3: Feedback representation

Departing from the canonical equations, we 'guess'

$$\lambda = \alpha + \kappa l$$

and substitute this guess into (2.4.9) on the left hand and on the right hand side. Then equating the coefficients (after substituting (2.4.8)) and solving the two equations yields,

$$\begin{aligned}\kappa &= \frac{(n+1)(r+2\delta) - \beta(2a + (n-1)b) - \sqrt{\dots}}{2a(a + (n-1)b)}, \\ \alpha &= \frac{(1-k)(\beta + (a + (n-1)b)\kappa)}{(n+1)(r+\delta) - a^2\kappa - a(\beta + (n-1)b\kappa)} = \frac{(1-k)(\beta + (a + (n-1)b)\kappa)}{(n+1)(r+\delta) - a\kappa(a + (n-1)b) - a\beta}\end{aligned}$$

with the root reported in (2.4.11).

Similarly considering the long run learning of  $n$  firms with normalized spillover  $\bar{b}$ , one can compute the necessarily higher spillover parameter,  $\tilde{b} > \bar{b}$ , that compensates for the decrease of  $l_\infty^o$  due to an additional firm,

$$(b0(-2 + nl + nl^2)(d + r) + a(b0(c - cnl) + (1 + nl)(d + r)))$$

$$nl(1 + nl)(-ac + (1 + nl)(d + r))$$

$$\tilde{b} = \frac{(n+2)(\bar{b}(n(n+1) - 2)(r+\delta) + a((n+1)(r+\delta) - \kappa\bar{b}(n-1)))}{n(n+1)((n+1)(r+\delta) - a\kappa)}$$

and of course only if  $\tilde{b} < a$  according to assumption (2.2.4).

## Appendix 3A

### Ad Chapter 3.0.0: Online appendix

#### The model of chapter 3.3.1

The final system of HJB's for the monopoly-model is given by

$$r\theta_1 + r\theta_2 P + r\frac{\theta_3}{2}P^2 = \frac{1}{4}(\theta_2 + \psi_2 + 1)^2 + \frac{1}{2}P(\theta_2 + \psi_2 + 1)(\theta_3 + \psi_3) + \frac{P^2((\alpha_H + 1)(\theta_3 + \psi_3)^2 - 2\alpha_H)}{4(\alpha_H + 1)},$$

$$r\psi_1 + r\psi_2 P + r\frac{\psi_3}{2}P^2 = \frac{1}{8}(\theta_2 + \psi_2 + 1)^2 + \frac{P(-4\alpha_H\beta + (\alpha_H + 1)\theta_3(\theta_2 + \psi_2 + 1) + (\alpha_H + 1)\psi_3(\theta_2 + \psi_2 + 1))}{4(\alpha_H + 1)} + \frac{1}{8}P^2(\theta_3 + \psi_3)^2,$$

and after comparing coefficients imply the following equations to be solved

$$\begin{aligned} r\theta_2 &= \frac{1}{2}(\theta_2 + \psi_2 + 1)(\theta_3 + \psi_3), \\ r\frac{\theta_3}{2} &= \frac{((\alpha_H + 1)(\theta_3 + \psi_3)^2 - 2\alpha_H)}{4(\alpha_H + 1)}, \\ r\psi_2 &= \frac{(-4\alpha_H\beta + (\alpha_H + 1)\theta_3(\theta_2 + \psi_2 + 1) + (\alpha_H + 1)\psi_3(\theta_2 + \psi_2 + 1))}{4(\alpha_H + 1)}, \\ r\frac{\psi_3}{2} &= \frac{1}{8}P^2(\theta_3 + \psi_3)^2, \end{aligned}$$

and finally the coefficients of the value functions which ensure convergence as described in Result 3.3.1.

$$\begin{aligned}\theta_2 &= \frac{2 \left( -2(\alpha_H + 1)^2 r^3 + 2\sqrt{(\alpha_H + 1)^3 r^4 (3\alpha_H + (\alpha_H + 1)r^2)} - 3\alpha_H(\alpha_H + 1)r \right) (-\alpha_H \beta + \alpha_H r + r)}{9\alpha_H(\alpha_H + 1)^2 r^2}, \\ \theta_3 &= \frac{4(\alpha_H + 1)^2 r^3 - 4\sqrt{(\alpha_H + 1)^3 r^4 (3\alpha_H + (\alpha_H + 1)r^2)} - 3\alpha_H(\alpha_H + 1)r}{9(\alpha_H + 1)^2 r^2}, \\ \psi_2 &= \frac{1}{9\alpha_H(\alpha_H + 1)^2 r^2} (-2(\alpha_H + 1)^3 r^4 + 2\alpha_H(\alpha_H + 1)^2 \beta r^3 - 3\alpha_H(\alpha_H + 1)^2 r^2 \\ &\quad + 2(\alpha_H + 1)r \left( \sqrt{(\alpha_H + 1)^3 r^4 (3\alpha_H + (\alpha_H + 1)r^2)} - 3\alpha_H^2 \beta \right) \\ &\quad - 2\alpha_H \beta \sqrt{(\alpha_H + 1)^3 r^4 (3\alpha_H + (\alpha_H + 1)r^2)}), \\ \psi_3 &= \frac{2(\alpha_H + 1)^2 r^3 + 2\sqrt{(\alpha_H + 1)^3 r^4 (3\alpha_H + (\alpha_H + 1)r^2)} + 3\alpha_H(\alpha_H + 1)r}{9(\alpha_H + 1)^2 r^2}.\end{aligned}$$

### The model of chapter 3.3.2

The final system of HJB's for the extended monopoly-model is given by

$$\begin{aligned}r\bar{\theta}_1 + r\bar{\theta}_2 P + r\frac{\bar{\theta}_3}{2} P^2 &= \frac{1}{4} (\bar{\theta}_2 + \bar{\psi}_2 + 1)^2 + \frac{1}{2} P (\bar{\theta}_2 + \bar{\psi}_2 + 1) (\bar{\theta}_3 + \bar{\psi}_3) \\ &\quad + \frac{P^2 ((\bar{\theta}_3 + \bar{\psi}_3)^2 + ((\bar{\theta}_3 + \bar{\psi}_3)^2 - 2) \alpha_H)}{4(\alpha_H + 1)}, \\ r\bar{\psi}_1 + r\bar{\psi}_2 P + r\frac{\bar{\psi}_3}{2} P^2 &= \frac{1}{8} (\bar{\theta}_2 + \bar{\psi}_2 + 1)^2 \\ &\quad + \frac{P ((\bar{\theta}_2 + \bar{\psi}_2 + 1) (\bar{\theta}_3 + \bar{\psi}_3) + \alpha_H (-4\beta + \bar{\theta}_3 (\bar{\theta}_2 + \bar{\psi}_2 + 1) + \bar{\psi}_3 (\bar{\theta}_2 + \bar{\psi}_2 + 1)))}{4(\alpha_H + 1)} \\ &\quad + \frac{P^2 ((\bar{\theta}_3 + \bar{\psi}_3)^2 (\alpha_H + 1)^2 - 4\alpha_F)}{8(\alpha_H + 1)^2},\end{aligned}$$

and after comparing coefficients imply the following equations to be solved

$$r\bar{\theta}_2 = \frac{1}{2} (\bar{\theta}_2 + \bar{\psi}_2 + 1) (\bar{\theta}_3 + \bar{\psi}_3),$$

$$\begin{aligned}
r \frac{\bar{\theta}_3}{2} &= \frac{((\bar{\theta}_3 + \bar{\psi}_3)^2 + ((\bar{\theta}_3 + \bar{\psi}_3)^2 - 2) \alpha_H)}{4(\alpha_H + 1)}, \\
r \bar{\psi}_2 &= \frac{((\bar{\theta}_2 + \bar{\psi}_2 + 1)(\bar{\theta}_3 + \bar{\psi}_3) + \alpha_H(-4\beta + \bar{\theta}_3(\bar{\theta}_2 + \bar{\psi}_2 + 1) + \bar{\psi}_3(\bar{\theta}_2 + \bar{\psi}_2 + 1)))}{4(\alpha_H + 1)}, \\
r \frac{\bar{\psi}_3}{2} &= \frac{((\bar{\theta}_3 + \bar{\psi}_3)^2(\alpha_H + 1)^2 - 4\alpha_F)}{8(\alpha_H + 1)^2},
\end{aligned}$$

and finally the coefficients of the value functions which ensure convergence

$$\begin{aligned}
\bar{\theta}_2 &= -\frac{1}{9r^2(\alpha_H + 1)^3(\alpha_F + \alpha_H^2 + \alpha_H)}(2(-2\sqrt{r^4(\alpha_H + 1)^6(3\alpha_F + (\alpha_H + 1)((r^2 + 3)\alpha_H + r^2)} \\
&\quad + 3r(\alpha_H + 1)^2(\alpha_F + \alpha_H^2 + \alpha_H) + 2r^3(\alpha_H + 1)^4(\alpha_H(r - \beta) + r)), \\
\bar{\theta}_3 &= \frac{-4\sqrt{r^4(\alpha_H + 1)^6(3\alpha_F + (\alpha_H + 1)((r^2 + 3)\alpha_H + r^2))}}{9r^2(\alpha_H + 1)^4} \\
&\quad - \frac{-3r(\alpha_H + 1)^2(-2\alpha_F + \alpha_H^2 + \alpha_H) + 4r^3(\alpha_H + 1)^4}{9r^2(\alpha_H + 1)^4}, \\
\bar{\psi}_2 &= \frac{1}{9r^2(\alpha_H + 1)^3(\alpha_F + \alpha_H^2 + \alpha_H)}(-3r^2(\alpha_H + 1)^3(\alpha_F + \alpha_H^2 + \alpha_H) \\
&\quad - 2\beta\alpha_H\sqrt{r^4(\alpha_H + 1)^6(3\alpha_F + (\alpha_H + 1)((r^2 + 3)\alpha_H + r^2))} \\
&\quad + 2r\sqrt{r^4(\alpha_H + 1)^6(3\alpha_F + (\alpha_H + 1)((r^2 + 3)\alpha_H + r^2))} \\
&\quad + 2r\alpha_H\sqrt{r^4(\alpha_H + 1)^6(3\alpha_F + (\alpha_H + 1)((r^2 + 3)\alpha_H + r^2))} \\
&\quad - 6\beta r\alpha_H(\alpha_H + 1)^2(\alpha_F + \alpha_H^2 + \alpha_H) - 2r^4(\alpha_H + 1)^5 + 2\beta r^3\alpha_H(\alpha_H + 1)^4), \\
\bar{\psi}_3 &= \frac{-\frac{2\sqrt{r^4(\alpha_H + 1)^6(3\alpha_F + (\alpha_H + 1)((r^2 + 3)\alpha_H + r^2))}}{(\alpha_H + 1)^4} + \frac{3r(-2\alpha_F + \alpha_H^2 + \alpha_H)}{(\alpha_H + 1)^2} + 2r^3}{9r^2}.
\end{aligned}$$

The slope of the optimal price strategy is given by  $\frac{\bar{\psi}_3 - \bar{\theta}_3}{2}$  and by plugging in from above yields

$$\frac{\frac{\sqrt{r^4(\alpha_H + 1)^6(3\alpha_F + (\alpha_H + 1)((r^2 + 3)\alpha_H + r^2))}}{(\alpha_H + 1)^4} + \frac{3r(-2\alpha_F + \alpha_H^2 + \alpha_H)}{(\alpha_H + 1)^2} - r^3}{9r^2},$$

which has its route at

$$\hat{\alpha}_F = -\frac{1}{8}(\alpha_H + 1)\left((r^2 - 4)\alpha_H + r\left(r - \sqrt{\alpha_H + 1}\sqrt{(r^2 + 8)\alpha_H + r^2}\right)\right).$$

The slope of the optimal tariff strategy is given by  $-\bar{\psi}_3$  and is equal to zero at

$$\tilde{\alpha}_F = \frac{1}{2} \left( \alpha_H (\alpha_H + 1) + r^2 (\alpha_H + 1)^2 - \sqrt{r^2 (\alpha_H + 1)^3 ((r^2 + 2) \alpha_H + r^2)} \right),$$

which, after rearranging, constitutes Result 3.3.2.

### The model of chapter 3.4.1

The final system of HJB's for the Cournot-model is given by

$$r\lambda_1 + r\lambda_2 P + r\frac{\lambda_3}{2}P^2 = \lambda_2\mu_2 + \lambda_2 + P(\lambda_3(\mu_2 + 1) + \lambda_2\mu_3) + \frac{P^2(2\lambda_3\mu_3 + (2\lambda_3\mu_3 - 1)\alpha_H)}{2(\alpha_H + 1)},$$

$$r\mu_1 + r\mu_2 P + r\frac{\mu_3}{2}P^2 = \frac{1}{2}(\mu_2 + 1)^2 + P\left((\mu_2 + 1)\mu_3 - \frac{\beta\alpha_H}{\alpha_H + 1}\right) + \frac{1}{2}P^2\mu_3^2,$$

and after comparing coefficients imply the following equations to be solved

$$\begin{aligned} r\lambda_2 &= \lambda_3(\mu_2 + 1) + \lambda_2\mu_3, \\ r\frac{\lambda_3}{2} &= \frac{2\lambda_3\mu_3 + (2\lambda_3\mu_3 - 1)\alpha_H}{2(\alpha_H + 1)}, \\ r\mu_2 &= (\mu_2 + 1)\mu_3 - \frac{\beta\alpha_H}{\alpha_H + 1}, \\ r\frac{\mu_3}{2} &= \frac{1}{2}\mu_3^2, \end{aligned}$$

and finally the coefficients of the value functions

$$\begin{aligned} \lambda_2 &= -\frac{\alpha_H(\alpha_H(r - \beta) + r)}{r^3(\alpha_H + 1)^2}, \\ \lambda_3 &= -\frac{\alpha_H}{r\alpha_H + r}, \\ \mu_2 &= -\frac{\beta\alpha_H}{r\alpha_H + r}, \\ \mu_3 &= 0. \end{aligned}$$

Hence, F's value function is linear in  $P$ , rendering optimal tariffs to be constant. This directly gives Result 3.4.1.

## The model of chapter 3.4.2

The final system of HJB's for the extended Cournot-model is given by

$$r\bar{\lambda}_1 + r\bar{\lambda}_2 P + r\frac{\bar{\lambda}_3}{2}P^2 = \bar{\lambda}_2\bar{\mu}_2 + \bar{\lambda}_2 + P(\bar{\lambda}_3(\bar{\mu}_2 + 1) + \bar{\lambda}_2\bar{\mu}_3) + \frac{P^2(2\bar{\lambda}_3\bar{\mu}_3 + (2\bar{\lambda}_3\bar{\mu}_3 - 1)\alpha_H)}{2(\alpha_H + 1)},$$

$$r\bar{\mu}_1 + r\bar{\mu}_2 P + r\frac{\bar{\mu}_3}{2}P^2 = \frac{1}{2}(\bar{\mu}_2 + 1)^2 + P\left((\bar{\mu}_2 + 1)\bar{\mu}_3 - \frac{\beta\alpha_H}{\alpha_H + 1}\right) + \frac{P^2(\bar{\mu}_3^2(\alpha_H + 1)^2 - \alpha_F)}{2(\alpha_H + 1)^2},$$

and after comparing coefficients imply the following equations to be solved

$$\begin{aligned} r\bar{\lambda}_2 &= \bar{\lambda}_3(\bar{\mu}_2 + 1) + \bar{\lambda}_2\bar{\mu}_3, \\ r\frac{\bar{\lambda}_3}{2} &= \frac{2\bar{\lambda}_3\bar{\mu}_3 + (2\bar{\lambda}_3\bar{\mu}_3 - 1)\alpha_H}{2(\alpha_H + 1)}, \\ r\bar{\mu}_2 &= (\bar{\mu}_2 + 1)\bar{\mu}_3 - \frac{\beta\alpha_H}{\alpha_H + 1}, \\ r\frac{\bar{\mu}_3}{2} &= \frac{\bar{\mu}_3^2(\alpha_H + 1)^2 - \alpha_F}{2(\alpha_H + 1)^2}, \end{aligned}$$

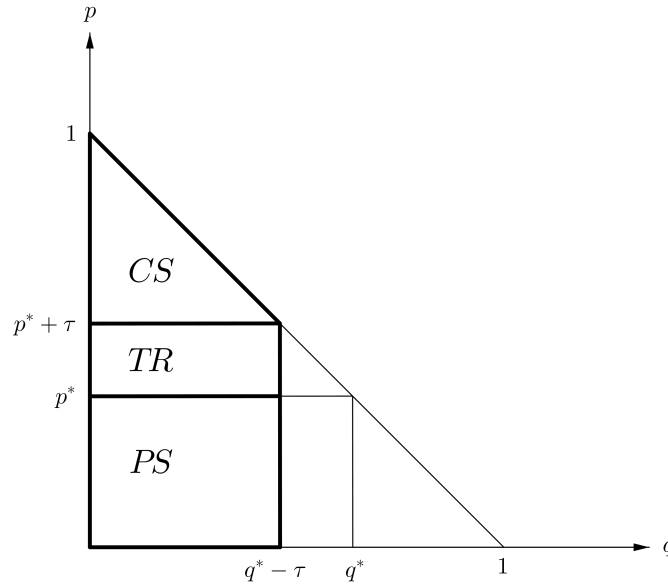
and finally the coefficients of the value functions which ensure convergence

$$\begin{aligned} \bar{\lambda}_2 &= \frac{1}{2\alpha_F^2(4\alpha_F + r^2(\alpha_H + 1)^2)}(\alpha_H(\alpha_H + 1)(r^2(\alpha_H + 1)^2(-\sqrt{4\alpha_F + r^2(\alpha_H + 1)^2} + r\alpha_H + r) \\ &\quad + \alpha_F(4r(\alpha_H + 1) - 2\sqrt{4\alpha_F + r^2(\alpha_H + 1)^2}))(\alpha_H(r - \beta) + r)), \\ \bar{\lambda}_3 &= -\frac{\alpha_H}{\sqrt{4\alpha_F + r^2(\alpha_H + 1)^2}}, \\ \bar{\mu}_2 &= \frac{\left(\sqrt{4\alpha_F + r^2(\alpha_H + 1)^2} - r(\alpha_H + 1)\right)(\alpha_H(r - \beta) + r) - 2\alpha_F}{2\alpha_F}, \\ \bar{\mu}_3 &= \frac{1}{2}\left(r - \frac{\sqrt{4\alpha_F + r^2(\alpha_H + 1)^2}}{\alpha_H + 1}\right). \end{aligned}$$

Plugging this into the rules for the optimal strategies and rearranging yields Result 3.4.2.

## Objectives

In the following we depict how the terms in both objectives of the two countries were calculated. Given demand of  $p = 1 - q$  we can easily calculate all relevant utilities in case of a tariff. To do this consider the following diagram.



It directly follows, that

$$\begin{aligned}
 CS &= \frac{(1 - p - \tau)^2}{2} = \frac{1 - 2p + p^2 + \tau^2 - 2\tau + 2\tau p}{2}, \\
 TR &= \tau(1 - p - \tau) = \tau - \tau p - \tau^2, \\
 CS + TR &= \frac{1 - 2p + p^2 - \tau^2}{2}, \\
 PS &= p(1 - \tau - p) = p - p^2 - \tau p.
 \end{aligned}$$

## Appendix 4A

### Ad Chapter 4.1.1: Cheating examples

Here, we briefly show and explain some real world examples where the model could apply. First, green washing, deceptive advertising – manipulating the perception of consumers in order to exploit green attitudes, even though the underlying product is in these cases not green/healthy or environmentally friendly at all.



**Figure A.1:** Coca Cola, BP and Shell.

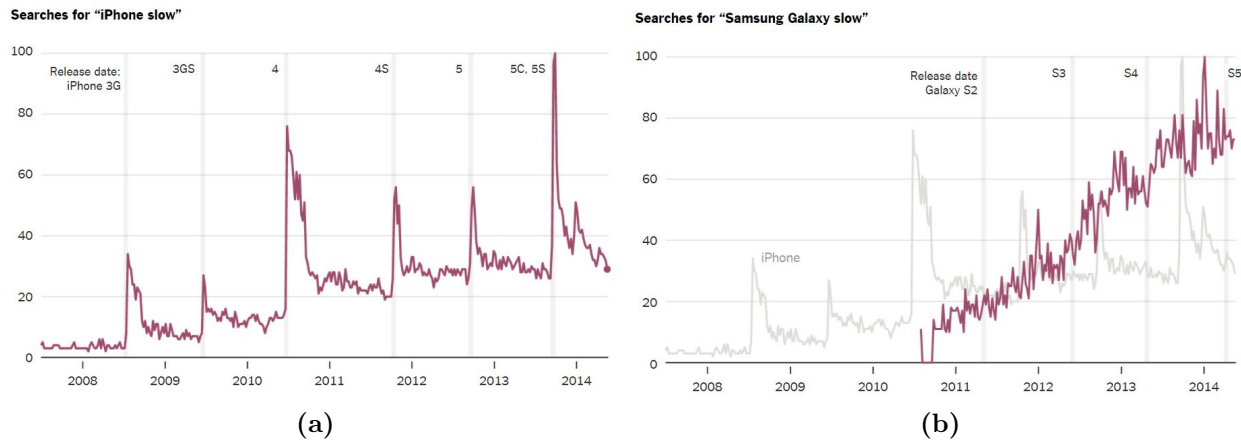


**Figure A.2:** Comparison of Apple's trend in (a) and Samsung's trend in (b), as shown in Trucco, 2014.

Google trend analyses of Apple's and Samsung's (perceived) smartphones performance (see Trucco & Mullainathan 2014). The figure shows peaks that take place exactly when a new Iphone is being released implying that Apple deliberately slows down old Iphone models in order to sell more of the new ones (for comparison, Samsung's graph does not have these peaks (it has a trend)).

The next example is from the video gaming industry. The screenshot below shows data for a recent video game (Star Wars Battlefront 2) where the publisher (Electronic Arts) chose to include the above mentioned 'lootboxes' as a way to increase profits (i.e., letting the consumer pay twice, once for the product itself and then again for trying to acquire the necessary (in this case Darth Vader) characters). Consumers perceive this strategy very negatively, as the "user score" of 0.9 (out of 10) shows.

The game itself has not been reviewing this badly (68 out of 100, average reviews out of 60 critics), which indicates that the unethical practice of including lootboxes into the game. This is an example of a firm milking a popular trademark (in this case Star Wars) for profits and at the same time being aware of the following reputational loss.

The last examples stem from the automotive industry, where large scale cheating has been observed several times. Hyundai and Kia (one company) have misstated fuel economy figures. Takata, (a japanese producer of airbags) got caught producing and selling (knowingly) defect airbags. And recently also Volkswagen which got involved in probably the largest scandal

**STAR WARS BATTLEFRONT II** PlayStation 4

Electronic Arts | Release Date: Nov 14, 2017 | Also On: PC, Xbox One

Summary Critic Reviews User Reviews Details & Credits Trailers & Videos

 **68** **Metascore**  
Mixed or average reviews  
based on **60 Critics**

**0.9** **User Score**  
Overwhelming dislike  
based on **7283 Ratings**

**What's this?**

**Summary:** Star Wars Battlefront II allows players to experience the untold story of an Imperial elite special forces soldier in an all-new single player campaign. In epic multiplayer battles, players pilot a First Order TIE fighter through intense dogfights in space, and play as ground troopers or... [Expand ▼](#)

**Developer:** EA DICE  
**Genre(s):** Action, Shooter, First-Person, Tactical  
**Cheats:** [On GameFAQs](#)  
**Rating:** T  
[More Details and Credits »](#)

Your Score:  0

**Figure A.3:** Example in the video game industry.

in the automotive industry. The company developed a special software in order to deceive regulators, specifically, in the US where the standards for allowed nitrogen-oxide emissions (NOx) are much tougher compared to the EU. In the end, they got caught and the scandal went public in late September 2015.



**Figure A.4:** Examples in the automotive industry.



# Curriculum Vitae

|                       |                                                                                                                                                                                                                                                                                                                                                                                                                         |
|-----------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Name</b>           | Markus Eigruber                                                                                                                                                                                                                                                                                                                                                                                                         |
| <b>Birth</b>          | Steyr, Austria                                                                                                                                                                                                                                                                                                                                                                                                          |
| <b>Education</b>      | <p>PhD Program in Management (since 10/2015)<br/>University of Vienna</p> <p>Bsc Mathematics (since 10/2014)<br/>University of Vienna</p> <p>Master Program in Economics (10/2012-09/2014)<br/>Vienna University of Economics and Business</p>                                                                                                                                                                          |
| <b>Publications</b>   | <p>Eigruber, M., &amp; Wirl, F. (2019). <i>Cheating as a dynamic Marketing Strategy in Monopoly, Cartel and Duopoly</i>. Contribution to ÖFG Symposium 2018 and resubmitted at CEJOR in 09/2019, Vienna. SI Feichtinger.</p> <p>Eigruber, M., &amp; Wirl, F. (2018). Climate Engineering in an Interconnected World: The Role of Tariffs. <i>Dynamic Games and Applications</i>, 8, 573-587. Published. SI Dockner.</p> |
| <b>Working Papers</b> | <p>Eigruber, M. (2019). <i>(Non)Cooperative Climate Engineering with a Regime Shift</i>.</p> <p>Eigruber, M., &amp; Wirl, F. (2018). <i>Market Equilibrium Strategies under Learning by Doing Spillovers</i>.</p> <p>Eigruber, M. (2018). <i>Corporate Cheating under Nonlinear Reputation Dynamics</i>. Contribution to ORCOS 2018, Vienna.</p>                                                                        |