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Contents

1 Introduction

Firm heterogeneity is substantial and prevalent along multiple dimensions. Numerous empirical studies, using data from various countries, have emphasized the importance of explicitly accounting for this phenomenon. [1] This thesis takes as its basis empirical evidence on the type and extent of firm heterogeneity, and asks whether aggregate outcomes, such as growth rates of output, investment or employment, can be affected by this variation, and if so, through which economic mechanisms. To address these questions, I develop quantitative equilibrium models of industry dynamics featuring heterogeneous firms. The models explicitly account for empirically-observed differences in the factor adjustment behavior of firms, and focus particularly on the disparity between incumbents and new entrants. Furthermore, the complex structure of the models allows me to quantify how much particular firm traits, such as age or size, contributed to the aggregate outcomes.

The empirical evidence shows substantial degree of firm heterogeneity in patterns of firm dynamics and factor adjustment. Large numbers of plants enter or exit from the market each year, which leads to changes in both the number and the characteristics of active producers. Furthermore, high-entry industries, such as services or the retail sector, are typically also the ones with high exit rates, which is indicative of common determinants relevant for both types of decisions. On the other hand, rates of investment, job creation and job destruction vary substantially for both firms and plants.[2] Firms creating jobs are roughly comparable in numbers to those that destroy jobs, yet businesses with negative investment rates are virtually absent, which stands in sharp contrast to the high proportion of firms with positive investment. This asymmetry suggests there is some form of serious friction present, namely partial investment irreversibility, that hampers the ability of firms to disinvest.

In this thesis, I qualitatively and quantitatively assess how partial investment irreversibility affects firm entry and exit and evaluate the implications of this de-

[1] Examples of empirical papers include Cooper and Haltiwanger (2006), Davis et al. (2010), and Haltiwanger et al. (2010) for the US economy, Asphjell et al. (2014) for manufacturing in Norway, and Hawkins et al. (2015) for Korean manufacturing.

[2] See e.g. Cooper and Haltiwanger (2006) and Davis et al. (2010) for data on the distribution of investment rates and employment growth rates, respectively.

pendence for aggregate output and investment. My analysis links the decision of a firm to enter or exit from the market to its investment choices. The empirical evidence presented in my first chapter indicates such a link, as I document a significant and negative correlation between rates of firm turnover and measures of capital intensity at the industry level. In my model, partial investment irreversibility implies that firms view a constant share of their investment as effectively sunk. This sunk cost of capital acquisition decreases both entry and post-entry investment activity. At the same time, the scrap value of capital is lower, inducing firms to postpone their decision to exit.[3]

My thesis takes as its starting point the empirical observation that entry and exit rates of firms in the US economy widely vary across sectors. I calculate the correlation coefficients between rates of firm turnover and selected measures of capital intensity. These coefficients are negative and statistically significant. Departing from this evidence, I build an equilibrium model of firm entry and exit augmented by partial investment irreversibility. This model augments the framework by Hopenhayn (1992) by adding physical capital as the second production factor and common productivity shocks. I use this model to study in detail the role that partial investment irreversibility at the individual firm level plays in firm entry and exit, and how the responses of the aggregate economy to shocks deviate from the frictionless case.

The thesis consists of three chapters. In the first chapter, I document variation in firm entry and exit rates across sectors in the US economy and provide evidence on the statistical relevance of capital intensity for these differences. In the second chapter, I build on these observations and assess the role of partial investment irreversibility for firm entry and exit, while also studying the implications for the dynamics of aggregate output and investment over the business cycle. In the third chapter, I develop a model featuring firms that adjust labor and physical capital in opposite directions, which is a phenomenon observed in the data, and evaluate the quantitative significance of such plants for the aggregate economy.

[3]Ramey and Shapiro (2001) have documented the discounted prices of equipment sold by exiting plants.

1.1 Sectoral differences in firm dynamics and job flows

In the first chapter, I focus on differences in patterns of firm dynamics across sectors in the US economy.[4] My work is motivated by recent work that emphasizes the importance of market-entering firms for aggregate job creation and job destruction (see e.g. Evans (1987), Dunne et al. (1989), Ibsen and Westergaard-Nielsen (2011) and Sedláček (2011)). Haltiwanger et al. (2010) show for the US private sector that business startups create a large fraction of both gross and net jobs. At the same time, firms in the early stages of their life exhibit relatively high levels of job destruction, as young firms are much more likely to fail and then exit a market. I inspect and document the prevailing empirical patterns.

This chapter provides new empirical evidence on differences between industries in patterns of firm dynamics and on the potential determinants of these differences. I derive this data from publicly-available data sets, the Business Dynamics Statistics (2013b) and the Bureau of Economic Analysis (2012). Firstly, I calculate and compare sector-specific statistics concerning firm entry, post-entry growth, and exit, for industries in the US economy. The sectors can be divided into two groups. The first group, represented by manufacturing, displays low entry and exit rates, and high variation in net job creation rates for age cohorts of firms, especially during the early years of their life-cycles. The second group - containing wholesale, retail and services - is characterized by high entry and exit rates, and smaller changes in net job creation rates over firm age.[5]

Secondly, I empirically assess the role of several factors – sunk entry costs and capital intensity of production – as potential determinants of the observed variation. I calculate statistics representing these characteristics for relevant industries and use them to evaluate the interdependence between these factors and firm entry and exit. My findings suggest that the measures of sunk entry cost and capital intensity of production are significantly negatively correlated with the rate of firm exit. In other words, establishments in an industry characterized by high levels of these capital-related characteristics are on average less likely to quit production. In the following chapter I develop an equilibrium model of firm turnover that features partial irreversibility of physical capital.

[4] I focus on sectors rather than firm-level data, as the former is publicly available.

[5] Net job creation is defined as the difference between the values of job creation and job destruction.

1.2 Firm entry and exit, investment irreversibility, and business cycle dynamics

In the second chapter, I qualitatively and quantitatively assesses the role of plant entry and exit in business cycle dynamics in an environment where investment in physical capital is partially irreversible. Given observed co-movements between the mass of businesses and aggregate economic activity, it is natural to investigate the extent to which firm entry and exit help explain the amplitude and the degree of persistence of macroeconomic fluctuations. I ask whether, and to what extent, variation in the number of firms that are subject to partial investment irreversibility contributes to fluctuations in the aggregate economy when those fluctuations are driven by exogenous shocks. To account for the empirical patterns of firm turnover, I use the equilibrium model of firm turnover by Hopenhayn (1992) and extend this model to consider physical capital and aggregate productivity shocks.

My work contributes to the literature on determinants and implications of firm turnover by identifying and evaluating the entry, exit and post-entry growth of a firm as a potentially important propagation mechanism for business cycles. I find that accounting for endogenous entry and exit of firms in a business cycle model with investment irreversibility increases the persistence of responses of aggregate variables to exogenous disturbances relative to a frictionless model. A shock of one standard deviationto the level of aggregate technology increases the number of entrants by 15%. Higher productivity yields an expected value of entry that outweighs the implicit sunk cost associated with investment irreversibility. As a result, exiting businesses are replaced not only by a greater number of new firms, but also by firms that invest more on average.

Moreover, I show that new firms contribute to aggregate dynamics not only by their initial investment, but also by investment realized in the later stages of their life-cycle. The output level of a new cohort increases steadily in subsequent periods, since their productivity growth exceeds the decay due to the exits of the least productive firms. This built-in selection device enhances the endogenous propagation of the economic activity.

1.3 The joint adjustment of production factors by heterogeneous plants

The last chapter of the thesis is a joint work with Monika Merz. In this chapter, we study the cross-sectional patterns of firms adjusting capital and labor. Recent empirical studies suggest that plants often invest in physical capital while simultaneously laying-off workers.[6] This finding is at odds with earlier empirical literature, which showed that abrupt changes in one production factor (e.g. physical capital) are, *on average*, associated with simultaneous adjustments of the other factor (e.g. labor) in the same direction.[7] We ask what determines the decision of plants to simultaneously invest and lay-off employees, and how much such plants contribute to observed joint occurrences of positive *aggregate* investment and negative growth of *aggregate* employment.

We develop an equilibrium model consistent with the above facts and use it to determine the quantitative importance of aggregate investment undertaken by those plants which simultaneously invest and lay off workers. The prevalence of these plants is not consistent with the standard assumption used in many modern macroeconomic models that output production exhibits constant returns-to-scale, or that the factor substitution elasticity is fixed and equal to one. We depart from the model in Hawkins et al. (2015) featuring idiosyncratic labor productivity shocks and augment it by a shock to the elasticity of factor substitution. These shocks lead to firm heterogeneity that is essential for joint occurrence of plants investing and hiring and those that invest and lay-off workers.

In our quantitative analysis, we focus on US manufacturing. Since the corresponding empirical evidence on the joint distribution of labor and capital adjustment has not been compiled yet, we use empirical moments of the marginal distributions to identify the model parameters.[8] We find that 18% of plants annually undertake positive investment and negative growth of employment, in line with observations for other countries.[9] We also show that these plants provide more than a third of the aggregate investment level.

[6] See Asphjell et al. (2014) for data on Norwegian manufacturing and Hawkins et al. (2015) for statistics regarding Korean manufacturing.

[7] Examples include the studies by Letterie et al. (2004) and Sakellaris (2004).

[8] Focusing on Korean manufacturing, Hawkins et al. (2015) identify model parameters using moments of the joint distribution of capital and labor adjustment.

[9] Moreover, two thirds of investing plants with negative employment growth invest at a rate above 10%.

2 Sectoral differences in firm dynamics and job flows[10]

2.1 Introduction

Many papers have emphasized the importance of entrants firms for the dynamics of aggregate job creation and destruction (see e.g. Evans (1987) and Dunne et al. (1989), or more recently Ibsen and Westergaard-Nielsen (2011) and Sedláček (2011)). In an extensive empirical paper, Haltiwanger et al. (2013) showed that, for the US private sector, business startups create a sizeable fraction of both gross and net jobs. At the same time, however, young firms exhibit relatively high levels of job destruction, as they are also much more likely to exit the market. All in all, the authors find that the empirical evidence "highlight[s] the importance of ... analyses that focus on the startup process - both the entry process and the subsequent post-entry dynamics" ((Haltiwanger et al., 2010, p.3)).

In order to better understand the dynamics of firms' behavior, I shall focus on how those dynamics vary between different sectors. The aim of this chapter is twofold. First, I document sector-specific measures of firm entry, post-entry growth, and exit, and compare their values across industries in the US economy. Second, I analyze several factors – sunk entry costs, capital intensity of production, and the extent of competition in a market – as potential determinants of the observed variation. I calculate statistics representing these factors for the various industries and use them to evaluate the interdependence between these factors and firms' entry and exit activity.

Patterns of firm dynamics – represented by the rates of firm entry and exit, and the net job creation rates – exhibit substantial variation across the sectors in the US economy. Based on the observed data, the sectors fall into two groups. The first group, consisting of the manufacturing sector, displays low entry and exit rates, as measured by the fraction of jobs that were respectively created by entrants or destroyed by exiting firms. In addition, there are relatively high differences in the net job creation rates between different age cohorts of firms, especially during the early years of firms' life-cycles. Businesses in the second group, which

[10] An earlier version of the chapter appeared as a working paper in Majher (2019b).

consists of wholesale, retail and services - are characterized by a large amount of entering and exiting activity, while changes in net job creation rate with age of firm are smaller compared to those for manufacturing.

The evidence on substantial inter-sectoral differences in magnitudes of entry and exit is further reinforced by the variation in the rates of plant turnover observed at higher levels of disaggregation. I document this fact for industries in manufacturing, disaggregated following the 3- and 4-digitlevels of NAICS classification. At the same time, the figures for entry and exit are strongly positively correlated across the sectors.

In the second part of this chapter, I assess interdependencies between the indicators of firm dynamics and the explanatory variables under consideration. One especially important result is that the measures of sunk entry cost and capital intensity of production are significantly negatively correlated with the rate of plant exit. In other words, establishments in an industry characterized by high levels of these capital-related characteristics are on average less likely to quit production. This observation suggests an important role played by accumulated capital stock in the exit decision of a firm, which can be understood in terms of investment irreversibility, that is to say, already-installed capital stock can be sold only at a discounted price, as a result of which businesses find it optimal to postpone exit and remain operational.

Regarding the role of market competition, the statistics indicate a significant positive correlation between plant entry and concentration ratio, calculated as the share of economic activity accounted for by the largest companies. In other words, an industry characterized by a small number of dominant firms is, on average, associated with a higher number of new establishments. This observation suggests an interesting link between the degree of market concentration and new plant openings, although in this work I shall not be able to investigate this further due to data limitations.

I compute values for all empirical statistics using available data sets for the US economy at various levels of industry disaggregation. Patterns of firm dynamics are represented by rates of firm entry, exit, and net job creation. Next, I describe sunk entry costs and capital intensity of production using measures based on an industry's levels of physical capital. The degree of market competition is characterized by values of the concentration ratio and the number of plants. For all these variables I demonstrate graphically the extent of variation between sectors. Ultimately, unconditional correlations are calculated, providing information about the magnitudes and signs of the relationships between variables.

This chapter proceeds as follows. Section 2.2 contains a brief overview of literature relevant to my work. Section 2.3 introduces the data sources and examines the distributions of the values of the examined statistics across sectors in the US economy. Section 2.4 presents the correlations between the indicators of firm dynamics and the explanatory variables under consideration and discusses the observed interdependencies. Finally, this chapter concludes with Section 2.5.

2.2 Review of literature

I start with a brief review of the related literature. My work is part of the field of research that focuses on firm dynamics. Therefore, I firstly present selected papers that incorporate firm entry, growth and exit into their model framework, and secondly, I list several works that documented the relevant empirical patterns.

In the seminal paper of this field, Jovanovic (1982) developed a model of gradual selection and learning about a firm's efficiency. Subsequent literature includes Hopenhayn (1992), who presented a dynamic stochastic model including firms' endogenous entry and exit decisions, Portier (1995), who addresseed and linked the cyclicality of firms' entry and markups, and Campbell (1998), who studied entry and exit of plants in an environment with embodied technological change. Recent relevant contributions in this field are papers by Jaimovich and Floetotto (2008), Bilbiie et al. (2012) and Cheremukhin and Tutino (2012). Jaimovich and Floetotto (2008) focus on the amplification effect on the business cycle of a markup mechanism, which is endogenously determined by the number of firms. Cheremukhin and Tutino (2012) go further in this direction, analyzing the dependence between firm exits, markups and business cycle asymmetry. Finally, Bilbiie et al. (2012) analyze endogenous entry as a propagation mechanism of fluctuations of the business cycle in a model with monopolistic competition and sunk entry costs.

Secondly, a number of contributions have focused on empirical assessment of the observed patterns of firm dynamics and their consequences. Just to name a few, Sutton (1991) and Gschwandtner and Lambson (2002) addressed the effect of sunk costs on entry and exit, while Bartelsman et al. (2009) examined the role of policy and institutions in differences in firm dynamics between countries. Finally, empirical papers by Haltiwanger et al. (2013) and Moscarini and Postel-Vinay (2012) have recently contributed contributions to the body of literature investigating the links between a firm's size, age and growth.

In comparison to the above papers, my work focuses particularly on inter-sectoral differences in patterns of entry, growth and exit. This chapter contributes to the literature by documenting sector-specific patterns of firm dynamics and assessing the role of several factors relevant for variation in these dynamics.

2.3 Empirical Analysis

2.3.1 Underlying Data

Data have been collated from several sources. Firstly, information on firm entry and exit, and job flows of firms over their life-cycles, was collected from the Business Dynamics Statistics (BDS) data set recorded by U.S. Census Bureau (2015a). This source reports statistics on firm and establishment demography (i.e. population, "births", and "deaths" of firms) and associated changes in the employment level, and covers the U.S. private sector annually over the time-period of 1977-2013[11]. The publicly-available data were disaggregated into nine main sectors following the SIC classification. A distinctive advantage of this data set is that it contains data on firm size and age. For my analysis, I have used the data on existing firms, firm births, and deaths, and also the associated levels of employment, job creation and job destruction.

Secondly, in order to observe the magnitudes of production units' entry and exit at a more disaggregated level, I have used the Statistics of U.S. Businesses (SUSB) database published by U.S. Census Bureau (2015c). These administrative records contain total numbers and associated employment levels for the overall population of establishments, new-born plants and establishment deaths. The data are recorded annually over the period 1998-2012, and are available for several levels of disaggregation, down to the 4-digit level of NAICS classification. Compared to the BDS data, SUSB covers a shorter period of time and cntains no information on establishment age. On the other hand, the higher degree of disaggregation allows assessment of sector-specific patterns of plant entry in greater detail. There are two complications: the employment observations contains a substantial amount of noise inserted for purposes of data protection, and the NAICS classification scheme has changed considerably during the time-period under consideration. For these reasons, I consider only the numbers of establishments, existing in, entering, and exiting the market, for industries within the manufac-

[11] The measurement day for every year is March 12^{th}.

turing sector, which was not affected by the classification changes.

Thirdly, I have used the concentration ratios reported by U.S. Census Bureau (2015b) as a proxy for the industry-specific degree of market competition. The values of this statistic represent the fraction of output accounted for by either the four, the eight, the twenty, or the fifty largest companies. This data is available for the years 2002 and 2007 at various level of disaggregation. The most detailed figures are available at the 6-digit level of the NAICS classification.

Finally, data is needed that describe the physical capital stock of firms in a particular sector as well as their output. A suitable database is publicly available at the website of the Bureau of Economic Analysis (2013). The BEA dataset provides the current-value of the net stock of private fixed assets, a variable that covers private equipment and software as well as structures used by businesses in a given industry. Annual data is available over the period 1947-2013, and is available down to the level of 3-digit NAICS industries for the manufacturing sector, with other sectors displaying less detailed disaggregation. This variable is reported at current valuation, but it can be converted into a constant value equivalent using the chain-type quantity indexes for net stock of private fixed assets with a base year of 2009, which are also available from the Bureau of Economic Analysis (2013).

I conclude this subsection by specifying two things: firstly, that my analysis will cover the time span from 1979 to 2013. Although the time period available in the BDS data starts in year 1977, I have avoided using data for the first two years following concerns raised by Moscarini and Postel-Vinay (2012) with respect to apparent measurement error in them. And secondly, in order to make the analysis comprehensive, I shall also focus on comparing the sectors of manufacturing, retail, wholesale and services[12]. The reason for selection of these sectors is that they conveniently display differences in both firm dynamics and the explanatory variables. Specifically, as shall be discussed further bellow, they differ in entry and exit rates, in the net job creation rates for firms in the early years of their existence, and in the sunk entry costs and capital-labor ratios. In addition, they constitute together a significant fraction of the aggregate economy, as their joint share of total employment has steadily remained around 80% during the entire period under discussion. However, in order to achieve consistency in the classification of industries, the data on capital from the BEA have been converted into the SIC classification using the conversion tables available at the North American

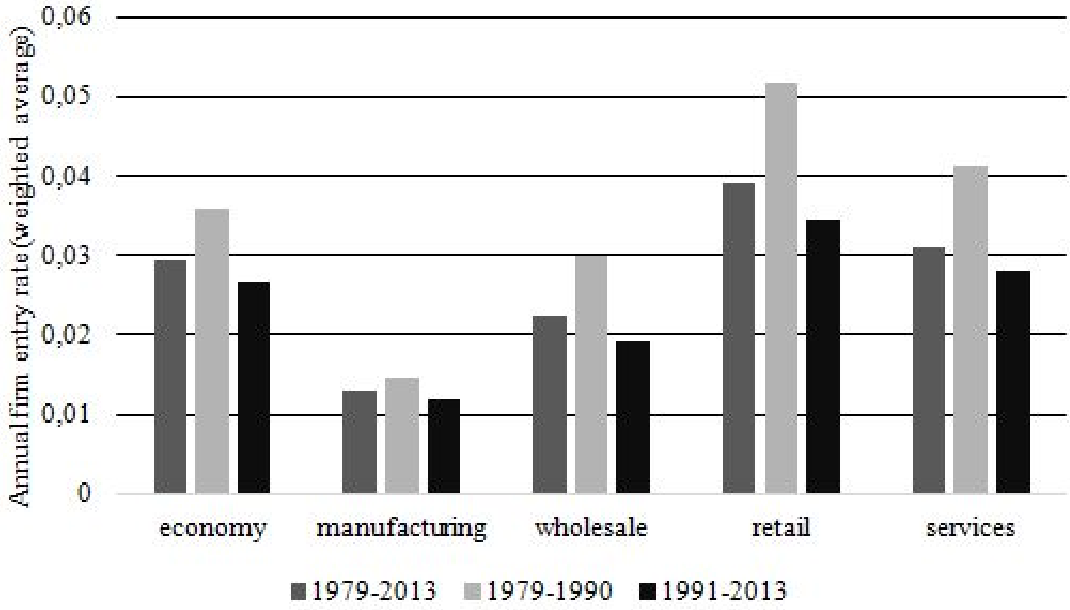
[12]This selection follows the BDS data, i.e. the SIC classification of industries

Industry Classification System (2012) website.

2.3.2 Firm dynamics

Firstly, I shall describe here the variables that characterize firm dynamics. My aim is to find differences that can be observed between the individual sectors. Several such differences have already been described by other works, e.g. in the recent study by Haltiwanger et al. (2013). That work focuses mainly on the role of the age of a firm on aggregate firm dynamics, although the authors also report significant inter-sectoral differences in the employment-weighted entry rates, in age patterns of the net employment growth rates of continuing firms, and in the dynamics of job destruction by exiting businesses. My aim is to examine these facts in more depth.

Figure 2.1: Sector-specific annual firm entry rates (in employment terms)



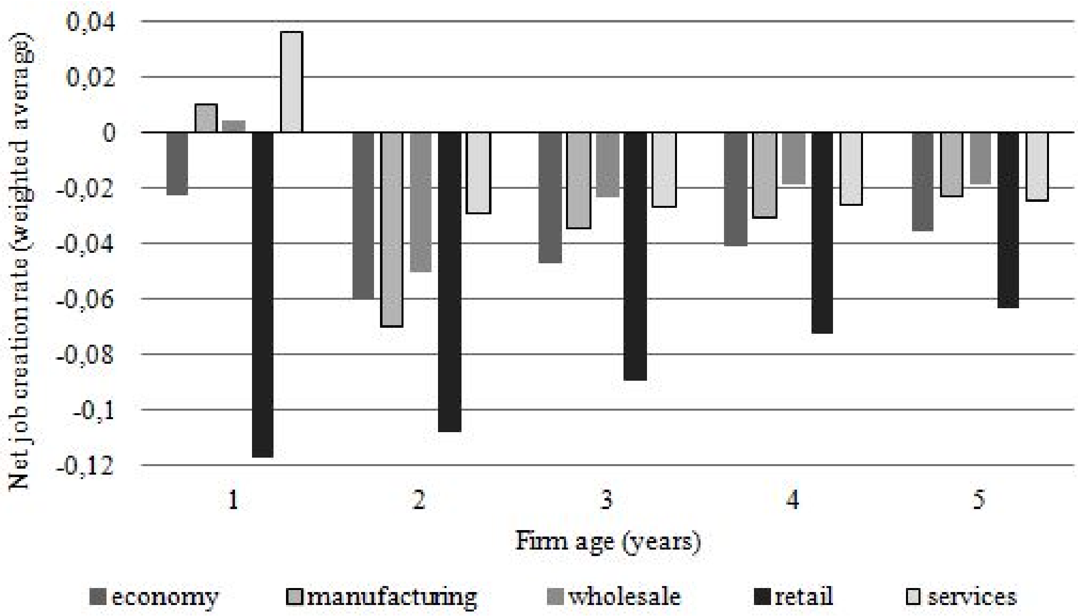
Source: Own calculations using Business Dynamics Statistics of the U.S. Census. Firm entry rate in employment terms is calculated as the fraction of total employment accounted for by newly entered firms. The reported figures are weighted averages over the period 1979-2013.

The first statistics of note is the employment weighted entry rate, calculated as the fraction of jobs in the economy that were created by firms entering the market during the given year. I present the employment-weighted averages of this variable over the entire time-period under analysis, and over the two sub-periods

1979-1990 and 1991-2013, in Figure 2.1. This distinction is made in order to control for a change in the volatility of the aggregate variables, which took place in late 1980s and early 1990s. From the presented Figure, it is obvious that the differences between the individual sectors are substantial for all the time periods considered. Entry rates in the retail or service sectors are several times higher than in manufacturing. The most pronounced difference is between the retail and manufacturing: the average entry rate in the former is about 4% as opposed to 1.3% in the latter, over the period 1979-2013.

Several additional statistics regarding the entry rates across the sectors are presented in Table 2.2 in the Appendix. How the entry rates change over time is also presented in Appendix, Figure 2.6. Entry rates have been declining steadily over time for all sectors, mirroring the pattern in the aggregate economy. In any case, both of these ways of arranging the data also show significant differences between sectors regarding the entry process. Even after dropping significantly in recent years, the entry rate for the retail sector still remains well above that of the manufacturing sector.

Figure 2.2: Sector-specific averages of annual net job creation rates, 1979-2013



Source: Own calculations using Business Dynamics Statistics of the U.S. Census. The net job creation rate is calculated as the fraction of total employment accounted for by total net job creation (i.e. job creation minus job destruction). The reported figures are averages over the period 1979-2013.

As the second indicator of firm dynamics, I have calculated the sector-specific net job creation rates for firms of different age cohorts. This variable is defined as the annual difference between created and destroyed jobs divided by the level of employment in the given year. It is presented separately for the different firm age categories in order to assess the patterns of firms' life-cycle growth in various sectors.[13] Since I need to track the age of firms, my sample is restricted to those firms, whose existence was first registered in the BDS in 1979 or later[14]. Moreover, as the values for firms with age higher than five years are aggregated into five-year age groups, I have not been able to construct a consistent measure of net job flows for any firm age category higher than five years of age. I will therefore report firms' job flow behavior only during the first five years of their existence.

The average net job creation rates for the selected sectors and firm ages are presented in Figure 2.2. The first thing to notice is that the job creation is negative for the majority of sectors and age groups except for firms of age one, where some of the industries exhibit positive levels of net job flow. This observation is consistent with the finding by Haltiwanger et al. (2013) that firms of any age, with the exception of entering businesses and the oldest firms, contribute a negative level of net jobs. Recall that my figure does not include newly entered firms, which by definition have positive net job flows of 100%.

Figure 2.2 also displays another interesting pattern: variation between sectors in the pattern of firms' net job creation in the early stages of their existence. Firms in the service sector exhibit average net job growth of almost 4% in their first year, then this figure falls sharply to a net job loss of 3% in the second year. However, for older firms, the exhibited net job loss remains steady at a level of 3%, so there is not much variation in average labor force adjustments. On the other hand, newly created retail businesses display a high net job loss (about -13%) after their first year of existence. This rate steadily decreases thereafter, but still remains the highest across the observed sectors. In other words, firms in the retail sector on average start shedding jobs at a substantially higher pace than creating them relatively early in their existence. In manufacturing, the net job creation rate falls from an average value of 1% for one-year-old firms to -7% in the second year, followed by -3% in the third and fourth year. Although these rates are not themselves the highest in their respective age categories, their variability with age of firm is relatively high, indicating that labor force adjustments are highly

[13] Following the BDS terminology, the age of an establishment equals the current year minus the year of "birth". Consequently, the age of a firm is given by the age of the oldest establishment of this firm.

[14] This includes all the firms that had their first establishment started after March 12, 1978.

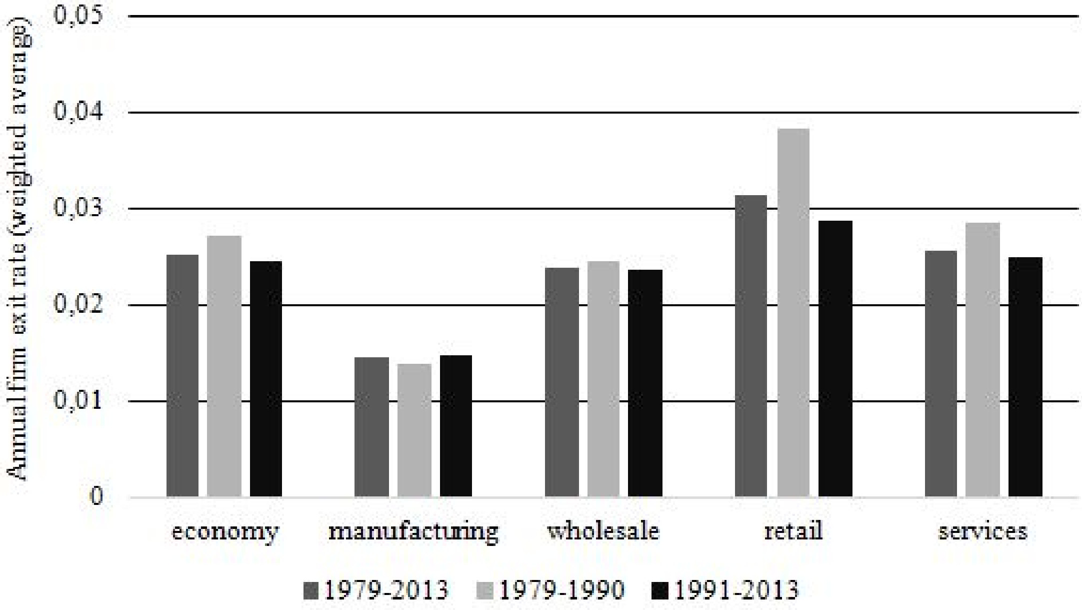
dynamic for businesses in manufacturing in the early stages of their life-cycles.

Finally, I present sector-specific exit rates as the last variable characterizing firm dynamics. I have calculated the value of this variable as the fraction of the total number of jobs that have been destroyed because of firms exiting the market. This provides additional insight into the nature of firm dynamics, as it shows how likely firms from different sectors are to exit the market. As can be seen in Figure 2.3, there are again substantial differences between sectors in the dynamics of firms exit behavior. In the retail sector, more than 3% of jobs (on average) are destroyed every year because of closure of establishments. In contrast, businesses in manufacturing show a significantly lower exit rate of less than 1.5%. This diversity regarding the exit process is also shown by the additional descriptive statistics in table 2.3 and the plot of the exit rates over time, which will be displayed futher down in Figure 2.7. The presented data show that the maximum value of the exit rate for manufacturing is roughly half of that for the retail sector. A similar thing can be said for the minimum values over the observed time period. Even though the exit rates have been steadily declining in all industries, that of manufacturing remains the lowest. All these facts support the conclusion that firms in different industries exhibit contrasting dynamics of exiting from the market.

To sum up, the presented data show significant variability between sectors regarding the entry, growth and exit of firms. In what follows, I shall present the two main patterns of firm dynamics that emerge from analysis of the data. The first of these patterns is seen for the manufacturing sector, and the second for group of other sectors: wholesale, retail and services.

- The manufacturing sector displays a very low rate of entry, indicating that it is more difficult for new businesses to enter the market than for other sectors. In the subsequent stages of firms' life-cycles, there is on average a persistently higher number of jobs destroyed than created. Moreover, the net job creation rates are quite different across the firms' age categories. This signals a more dynamic labor force adjustment during the years following a firm's entry. On the other hand, there is a relatively low rate of job destruction coming from firms exiting the market. TThis suggests that once a firm in manufacturing starts to produce, it is very likely to remain in operation; but, at the same time, such firms exhibit very rapid changes in the labor force adjustments, as reflected in the cross-age volatility of the net job creation rates.

Figure 2.3: Sector-specific annual exit rates of firms (in employment terms)



Source: Own calculations using Business Dynamics Statistics of the U.S. Census. The firm exit rate in employment terms is calculated as the fraction of total employment accounted for by exiting firms. The reported figures are weighted averages over the period 1979-2013.

- Firms in retail, wholesale and services exhibit a relatively higher average rate of entry. I conjecture that firms in these sectors do not face too many barriers during the entry process. In addition, the process of labor force adjustment exhibits less volatility across age cohorts, resulting in either strongly negative net job creation rates (for the retail sector), or remaining at levels around and slightly below zero growth (for services). However, the values of the firms' exit rates are relatively high. In general, it seems that firms operating in these industries are relatively more vulnerable to market conditions, and many of them exit when economic situation turns unfavourable. This, in turn, leads to a somewhat more age-persistent pattern of labor force adjustment.

2.3.3 Entry and exit of plants

One drawback of the BDS database is the low level of disaggregation of the publicly available figures. Due to the small number of sectors, it is not possible to statistically assess thedeterminants of the cross-s ectoral differences in the measures of firm dynamics. For this reason I have employed the statistics of establishment entry and exit activity reported by the SUSB data set at the 3- and 4-digit levels of the NAICS disaggregation. There are two important changes relative to the figures calculated from the BDS data. Firstly, due to several classification adjustments for various industries, I have only used the data on the manufacturing sector, for which the classification remained unaltered throughout the time period under analysis. Secondly, the values of plants' employment levels contain substantial noise. I have therefore calculated rates of plant entry and exit based on the numbers of establishments rather than on the employment levels. Specifically, the rates of plant entry and exit are calculated, respectively, as fractions of the total number of plants accounted for by entering or exiting establishments.

Consistently with the analysis for the broad sectors, I shall assess the variation between industries in the measures of plant turnover. For this purpose I report the distributions of annual plant entry and exit rates across industries in Figures 2.8 and 2.9 in the Appendix. The rates of entry and exit are concentrated around the value of $7-8\%$ for the majority of industries, but in some cases rates go above 14% or below 4%. In addition, the scatter plot in Figure 2.11 depicts the interdependence between the rates of entry and exit. Both values are highly positively correlated across the sector. All in all, the presented evidence suggests substantial variation in the magnitudes of firm entry and exit between industries.

2.3.4 Sunk costs of entry and capital intensity

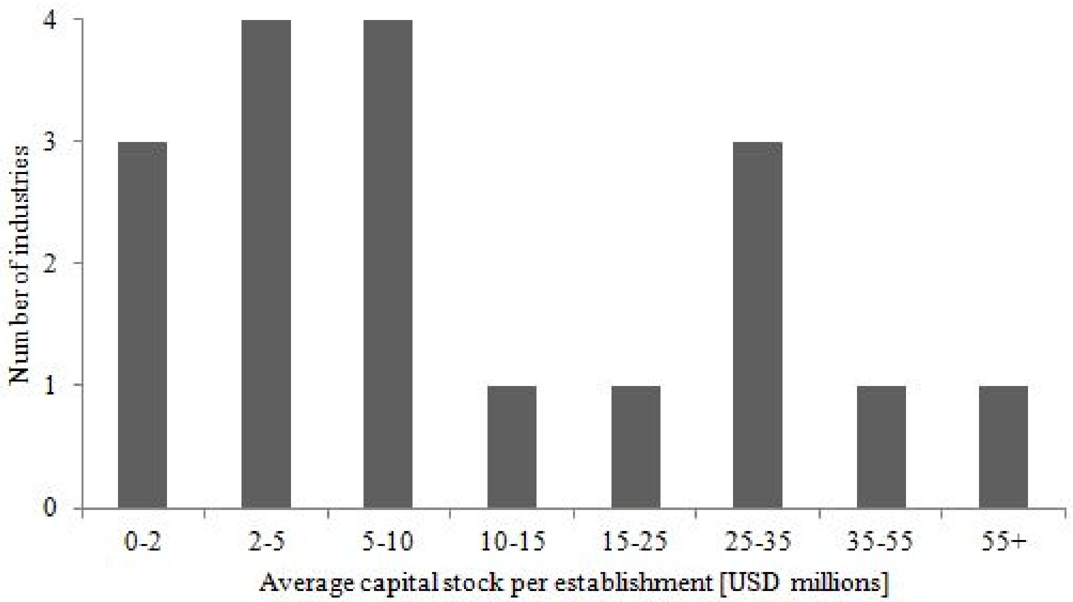
In this section I analyse data characterizing sector-specific sunk costs of entry and capital intensity of production. Particular emphasis is put on how the empirical data differs between individual industries.

I start with an assessment of sunk entry costs. Several authors have pointed out that this variable cannot be observed directly in an economy. As a proxy figure, Lambson and Jensen (1998) use the real gross book value of property, plant and equipment arguing that capital costs are a convenient indicator for sunk (entry) costs. Gschwandtner and Lambson (2002) use two statistics as a proxy for sunk entry costs: the level of cumulative gross fixed capital formation and the ratio of capital to labor.

In my calculations, I have used the average level of capital per firm in an industry, which by construction resembles the proxy vairable used by Lambson and Jensen (1998) or the first proxy variable used by Gschwandtner and Lambson (2002). This statistic is calculated using the industry-specific levels of net stock of fixed assets divided by the numbers of establishments[15]. It covers private equipment, software, and structures used by firms (c.f. Bureau of Economic Analysis (2013)) and thus captures well all the principal investments that a firm has to make when entering a market.

Using the capital stock as a proxy measure for sunk costs has its limitations, as this interpretation depends strongly on the degree of irreversibility. The more reversible the capital stock is, the higher proportion of its value can be recovered upon scrapping. High degree of investment reversibility would imply that the capital costs exceed the sunk costs. Lambson and Jensen (1998) discuss this issue and note that, at the same time, the capital costs may fall short of the sunk costs, as the latter include other expenditures associated with starting up a business. In light of these considerations, I follow their conclusion of the capital costs being a good proxy for the sunk costs if detailed firm-level data is not available.

Figure 2.4: Distribution of industry-specific levels of capital stock per establishment in manufacturing



Source: Own calculations based on BEA and SUSB data. The values of the variable are calculated as sample averages, over the period 1998-2011, of capital stock values divided by numbers of establishments. The figure represents the number of 3-digit NAICS industries in the manufacturing sector with per-establishment capital stock within the specified range. The total number of industries is 18.

In Figure 2.4 I give an overview of the distribution of levels of capital per

[15]Recall that the value of fixed assets is expressed in the price level of the year 2009.

establishment over 3-digit industries in manufacturing. There are pronounced differences observed between individual sectors. A plant in the petroleum and coal products manufacturing industry containts, on average, capital stock worth almost 70 million US dollars. This value is an order of magnitude higher than its counterpart in the furniture and related products manufacturing industry, where a plant is associated, on average, with less than \$1.2 million USD of capital stock value.

Secondly, I analyse data on the sector-specific degree the capital intensity of production, characterized by the values of the capital-labor ratio. This variable is calculated from the source data for every industry as an average ratio between the net stock of private fixed assets and the number of full-time workers.

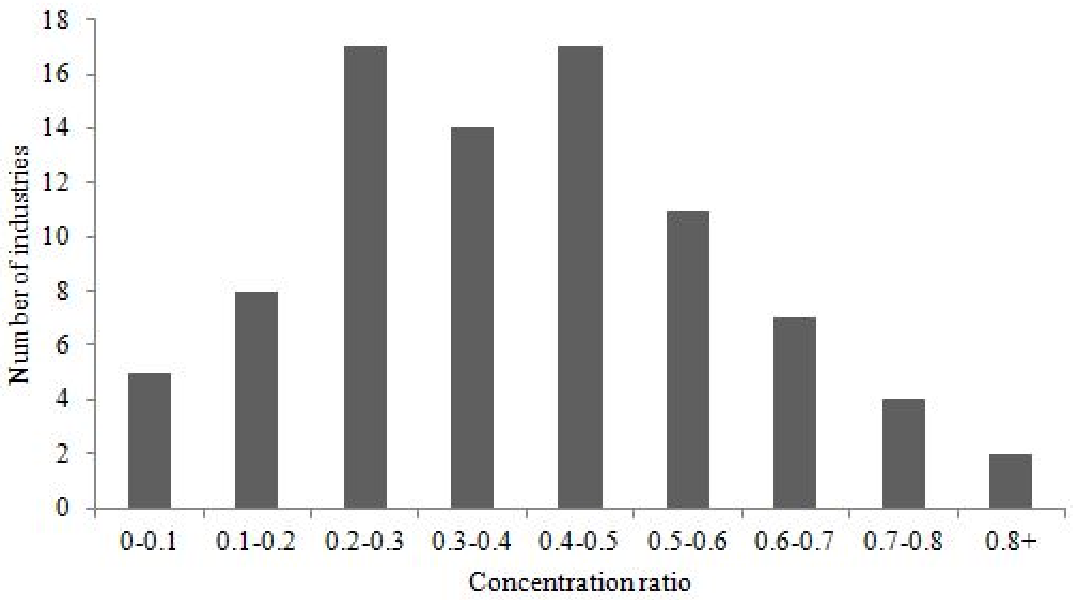
I present the distribution of 3-digit manufacturing industries over the values of the capital-labor ratio in Figure 2.12 in the Appendix. Similarly to the measure of average capital stock discussed above, this statistic exhibits substantial heterogeneity across sectors. The average worth of fixed assets per full-time worker ranges from $\$45,000$ USD (furniture and related products manufacturing) to above \$1.3 million USD (petroleum and coal product manufacturing).

To sum up, both examined indicators – my seleced proxy for sunk entry costs and the capital-labor ratio – exhibit similar patterns of substantial heterogeneity across sectors. The observed variation in the levels of capital-related variables may have significant implications for the firm dynamics (and variations thereof between sectors). My economic intuition is that both entry and exit rate depend negatively on the level of sunk entry costs as well as on the degree of capital intensity of production. High capital requirements make the production process more difficult to initiate. This implies that higher levels of expected profit are necessary to attract entering firms. On the other end of the lifecycle, capital already installed can be sold only at a substantial discount (see the evidence presented by Ramey and Shapiro (2001)). Sale of a larger stock of capital is associated with a higher loss in its value; a firm in a highly capital-intensive industry is therefore more likely to remain in the market.

2.3.5 Competition in the goods market

In this section I report the distributions of the industry-specific values of two statistics, which represent the degree of competition faced by firms in the market. I start with the statistic of the concentration ratio as the most natural proxy measure for the degree of market competition. This statistic expresses the fraction of the total output of an industry that is accounted for by a specific number of the largest firms. In Figure 2.5 I overview the distribution of the concentration ratios across the sample industries. The figures are calculated using statistics based on the value of shipments as a measure of the industry's output, for the sample year 2002. However, the results are robust to changes in this selection, i.e. using value added as an alternative measure of output, and observing values in hte year 2007 instead.

Figure 2.5: Distribution of industry-specific concentration ratios in manufacturing



Source: Own calculations using the concentration ratios database of U.S. Census. The values used here are calculated as the share of the total value of shipments accounted for by the eight largest companies in 2002. The figure represents the number of 4-digit NAICS industries in the manufacturing sector with a concentration ratio within the specified range. The total number of industries is 86.

The graph indicates strong variation across industries in the output shares produced by the eight largest companies. In some industries, the top 8 group accounts for less than 10% of the total value of shipments. On the other hand, there are two 4-digit sectors where this figure is above 80%. The majority of industries have concentration ratios of around $0.4 - 0.5$, meaning that the eight largest firms generate roughly one half of the total value of shipments produced.

I also present one additional measure that can serve as a naive proxy for market competition. The idea is to approximate the degree of competition by the number of firms (or establishments), which compete in a particular market. As stated by Portier (1995), the level of market power is linked to the number of competitors. For each of the 86 industries reported at the 4-digit NAICS level in the manufacturing sector, I have calculated this statistic by taking the average number of plants over the sample period. Figure 2.10 in the Appendix documents the size of different industries, in terms of the number of plants (displayed as a distribution of industries over the numbers of associated plants). Once again, the graph displays substantial variation in the values of the statistic. The number of plants within an industry can be as low as 114 units for tobacco manufacturing, and as high as $33,000$ for the sector of printing and related support activities. This data reinforces the picture already presented of strong variation between sectors in the degree of market competition.

2.4 Correlations

In the previous sections I have examined the patterns of variables characterizing firm and plant dynamics, their potential determining factors, and their variation between sectors. In this section I assess interdependencies between these measures. I calculate unconditional correlations between rates of plant entry and exit and values of variables representing sunk entry costs, capital intensity of production, and competition faced in the goods market. Subsequently I shall comment on the signs and significance of these statistics.

Table 2.1: Correlations between rates of plant turnover and possible determinants

	Turnover rate	Entry rate	Exit rate
log capital per plant[a]	-0.402**	-0.227	-0.527**
log capital-labor ratio[a]	-0.383*	-0.195	-0.520**
concentration ratio - share of top 8 firms[b]	0.283***	0.382***	0.095
concentration ratio - share of top 20 firms[b]	0.271***	0.372***	0.083
log number of plants[b]	-0.238**	-0.285***	-0.125

*** - statistical significance at 1% level, ** - statistical significance at 5% level, * - statistical significance at 10% level,
a - calculated at 3-digit NAICS classification level; *b* - calculated at 4-digit NAICS level.

The statistics are presented in table 2.1. The values of the correlation coefficients between the average capital stock per establishment and the rates of plant entry and exit are, respectively, -0.227 and -0.527. The scatter plots for these correlations are shown in Figure 2.13 in the Appendix. Observe that both coefficient values are negative, indicating that industries characterized by a higher average level of physical capital are associated with lower relative numbers of entering and exiting plants. However, only the correlation coefficient with the exit rate is statistically significant. This suggests that sunk entry costs, as represented by capital costs, are of particular importance for exit decisions of production units. Once a plant starts to produce in a sector requiring higher irreversible costs upon entry, it is on average less likely to exit from the market.

Analogous observations can be made while assessing correlations between the rates of plant turnover and the capital-labor ratios. The respective values of these statistics are -0.195 and -0.520. Once again, for illustrative purposes I represent the respective interdependencies using scatter plots in Figure 2.14 in the Appendix. The results indicate that the degree of capital intensity is inversely proportional to the relative numbers of entrants and exits in an industry. However, only the value of the correlation coefficient involving the exit rate is statistically significant. This points to the exit decision of a plant being particularly affected by the capital intensity of the sector to which it belongs.

Now I shall address the dependence between plant entry and exit and the degree of competition in a market. The first proxy variable representing market competition is the share of value added accounted for by the eight largest companies[16]. The values of the correlation coefficients between this concentration ratio and the rates of plant entry and exit are, respectively, 0.382 and 0.095, where only the first number is statistically significant. Illustrative scatter plots are presented in Figure 2.15 in the Appendix. Note that, by construction, the value of the concentration ratio is inversely related to the degree of market competition. In particular, a high ratio means that a disproportionate share of economic activity is generated by only a handful of firms, implying less severe market competition. The observed coefficients therefore suggest a significant negative correlation between frequency of plant entry and the competition level of an industry.

Let me discuss this last observation in more detail. The negative correlation between frequency of plant entry and the degree of market competition is at odds with the typical assumption that entry of new agents intensifies competition among businesses (cf. Bilbiie et al. (2012)). This discrepancy may be due to a difference in the measurement units, as the concentration ratios are calculated

[16]The results are virtually identical when using the concentration ratio for the twenty largest companies.

for companies as opposed to establishments. It may well be that firms in highly concentrated industries (i.e. with fewer competitors) open relatively more plants, which leads to observed higher rates of plant entry. In any case, my results indicate that the fact that firms open new plants should be given proper attention when considering any relationship between business dynamics and market competition.

The second proxy variable for the degree of competition in a market is the number of plants in a given industry. The correlation coefficients between this number and the rates of plant entry and exit are -0.238 and -0.285, and both values are statistically significant. If a higher number of plants is an indication of more severe competition, then these values suggest a strongly negative linkage between the rates of plant entry and exit and the degree of market competition. This result is consistent with the coefficient values found for the concentration ratio.

To sum up, the presented evidence supports the view that the economic elements mentioned are highly influential on the magnitude of plant entry and exit in an industry. The statistics show that the levels of sunk entry costs and capital intensity of production, as well as the degree of market competition, are negatively correlated with the rates of establishment turnover. These conclusions are reinforced by the fact that most of the calculated correlation coefficients are statistically significant. Overall, the observed dependencies provide insights that are important for development of a model framework aimed at capturing cross-industry differences in plant dynamics.

2.5 Conclusion

In this chapter, I present empirical data on how patterns of firm dynamics differ between sectors. Many more businesses enter the sectors of services and retail than the manufacturing sector, and their labor force adjustments are more volatile. On the other hand, firms in manufacturing are on average less likely to exit from the market. At the more disaggregated level, there is substantial variation in rates of plant entry across industries in the manufacturing sector, even at the 4-digit level of NAICS classification. At the same time, the number of entering and exiting establishments (as fractions of total establishments) are strongly positively correlated with each other.

I then address several potential determinants of this variation: irreversible entry costs, capital intensity of production, and the degree of competition in a market. I introduce variables suitable for representing these elements, and present their values across different industries. Finally, I show that these variables are significantly correlated with the indicators of firm dynamics. Levels of sunk entry costs and capital-labor ratio are negatively correlated with the rates of establishment exit, whereas industry concentration ratios correlate positively with the rates of plant entry.

The presented evidence provides fruitful ideas for future research. Most importantly, the relevance of the respective explanatory variables for patterns of firm dynamics needs to be investigated more thoroughly. It is obvious that these determinants are not mutually independent, but interact through various economic mechanisms, such as the mechanism of markup presented by Jaimovich and Floetotto (2008) or Colciago and Rossi (2011). Thus, a modeling framework that accounts for these interactions can provide novel insights into the consequences of firms' decisions to enter, grow, or exit, at the level of an entire industry or even of an aggregate economy.

2.A Complementary evidence

Table 2.2: Employment weighted entry rates (1979-2013)

	ECON	MAN	WHO	RET	SRV
mean^a	0.0293	0.0129	0.0224	0.0392	0.0309
max	0.0407	0.0174	0.0376	0.0654	0.0528
(year)	(1982)	(1981)	(1982)	(1982)	(1982)
min	0.0198	0.0072	0.0120	0.0229	0.0199
(year)	(2009)	(2013)	(2013)	(2013)	(2013)

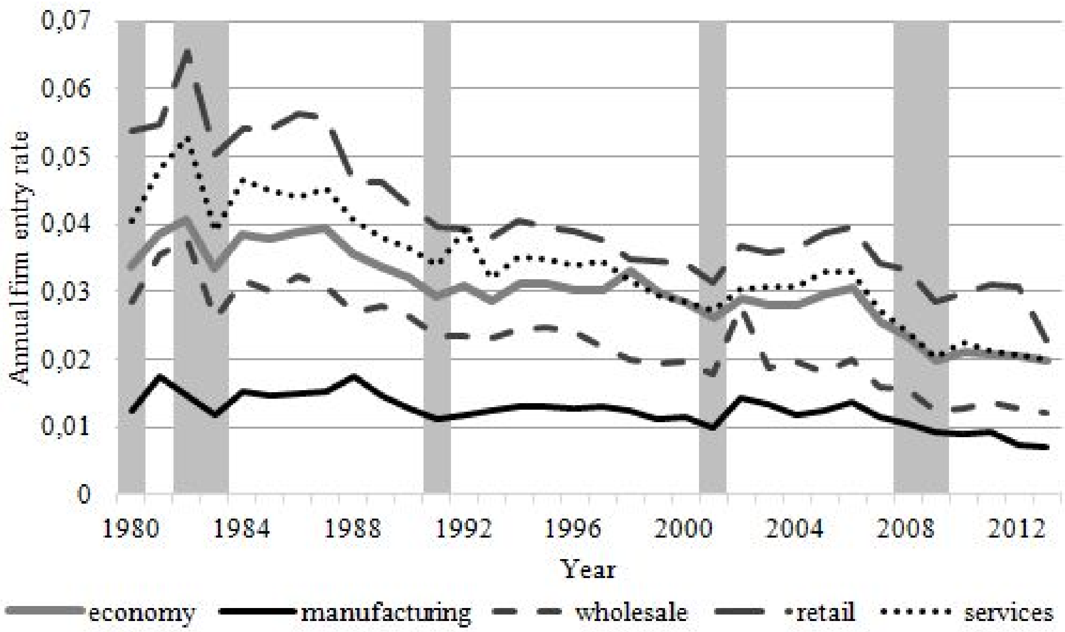
a - weighted averages

Table 2.3: Employment weighted exit rates (1979-2013)

	ECON	MAN	WHO	RET	SRV
mean^a	0.0253	0.0144	0.0238	0.0314	0.0256
max	0.0323	0.0180	0.0291	0.0445	0.346
(year)	(1987)	(1986)	(1987)	(1980)	(1987)
min	0.0204	0.0104	0.0184	0.0235	0.0207
(year)	(2012)	(1984)	(2012)	(2012)	(2012)

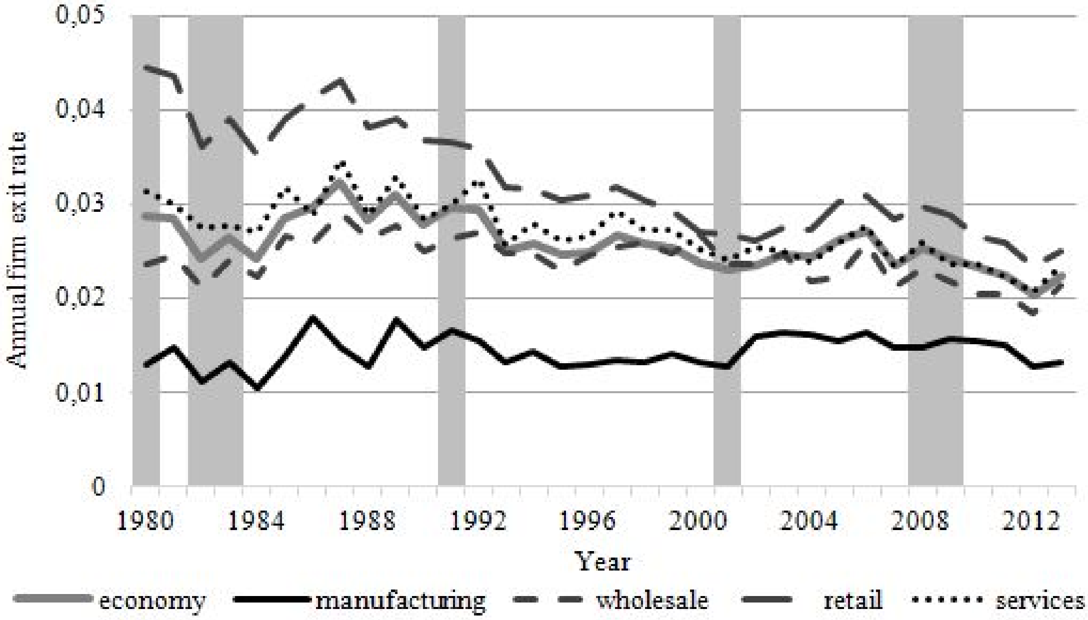
a - weighted averages

Figure 2.6: Sector-specific annual entry rates of firms, 1979-2013



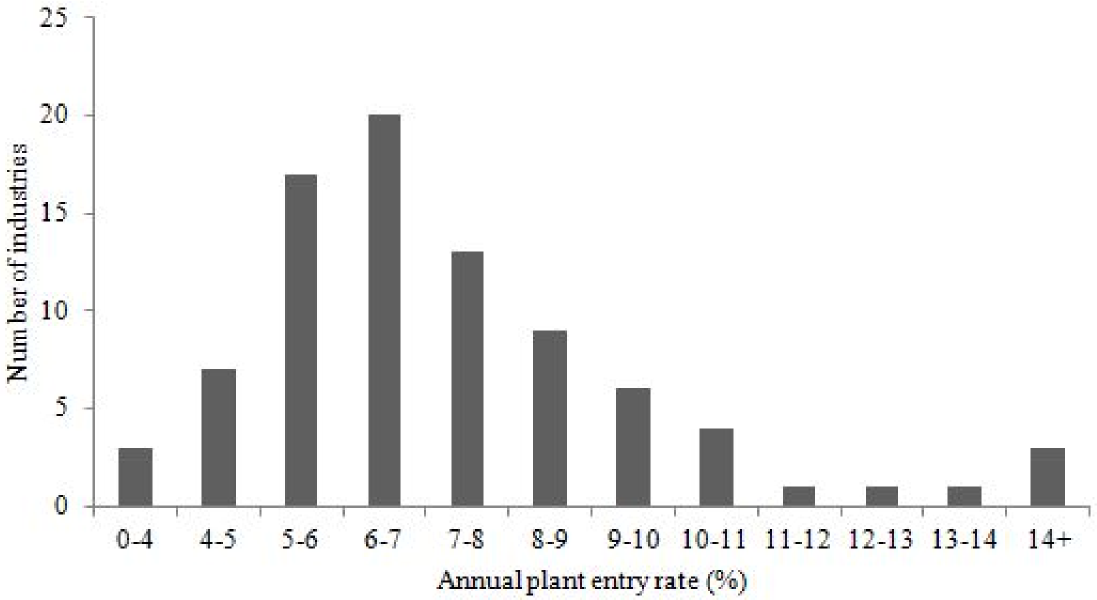
Source: Own calculations using Business Dynamics Statistics of the U.S. Census. For a description of variables see Figure 2.1. NBER recessions are shaded gray. The dates of recessions, retrieved from the website of National Bureau of Economic Research (2012), are January 1980 - July 1980, July 1981 - November 1982, July 1990 - March 1991, March 2001 - November 2001 and December 2007 - June 2009.

Figure 2.7: Sector-specific annual exit rates of firms, 1979-2013



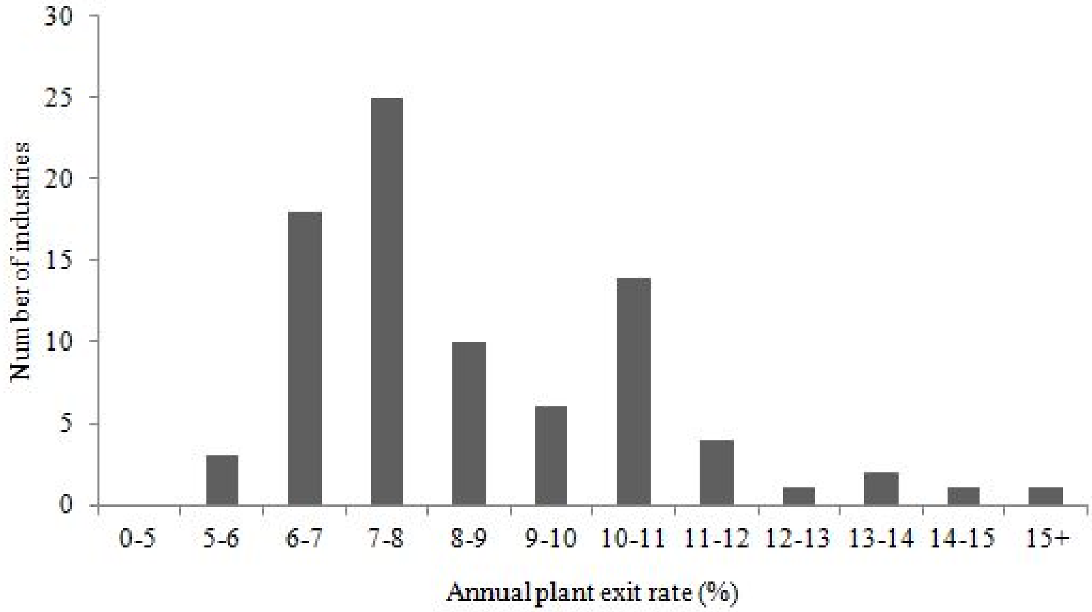
Source: Own calculations using Business Dynamics Statistics of the U.S. Census. For a description of variables see Figure 2.3. NBER recessions are shaded gray. For the dates of recession see Figure 2.6.

Figure 2.8: Distribution of industry-specific plant entry rates in manufacturing



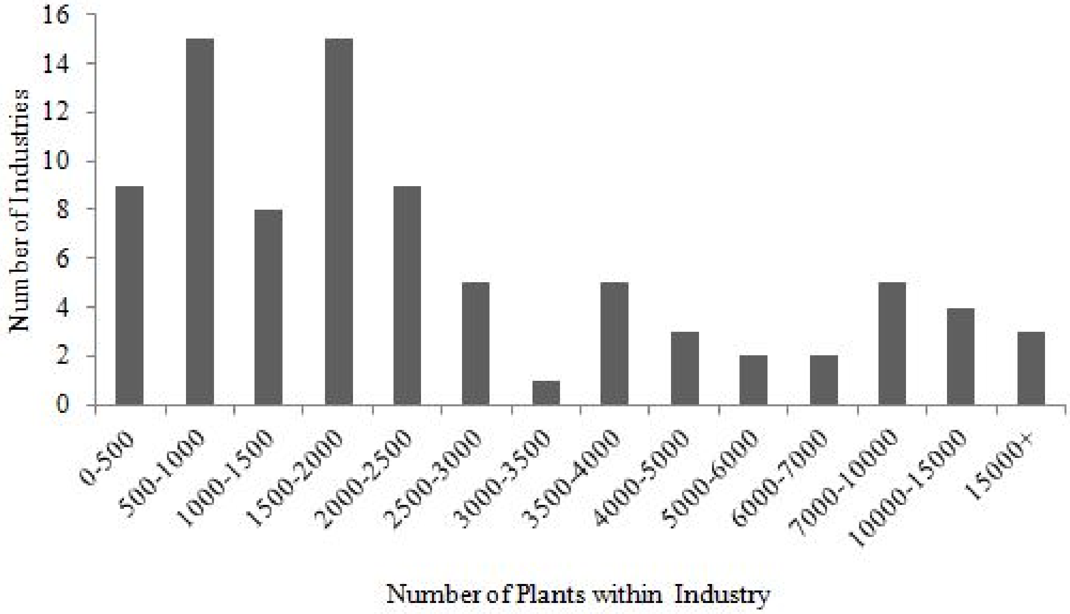
Source: Own calculations using Statistics of U.S. Businesses of the U.S. Census. The plant entry rate is calculated as the fraction of the total number of plants accounted for by the new establishments. The reported figures are averages over the sample period 1998-2012. The total number of industries is 86.

Figure 2.9: Distribution of industry-specific plant exit rates in manufacturing



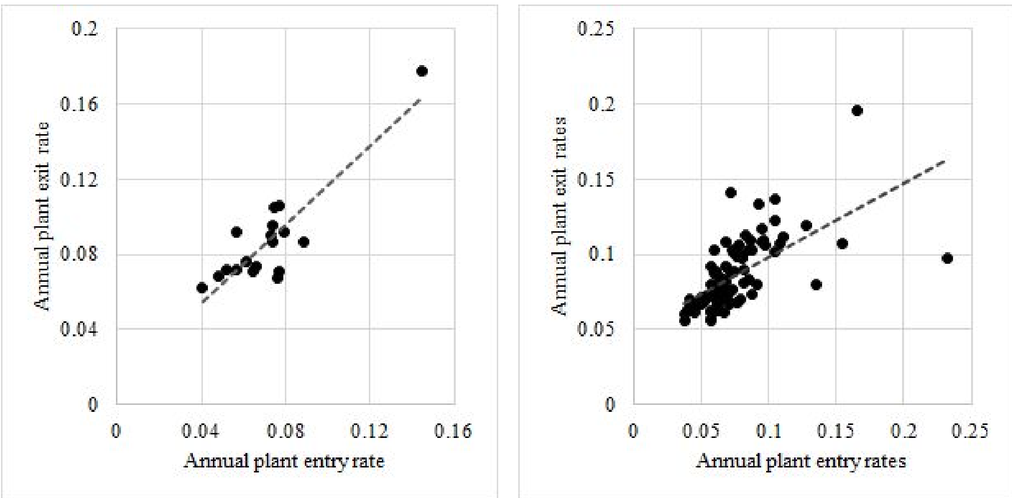
Source: Own calculations using Statistics of U.S. Businesses of the U.S. Census. The plant entry rate is calculated as the fraction of the total number of plants accounted for by the new establishments. The reported figures are averages over the sample period 1998-2012. The total number of industries is 86.

Figure 2.10: Distribution of industry-specific numbers of plants in manufacturing



Source: Own calculations using Statistics of U.S. Businesses of the U.S. Census. The industry-specific number of plants is calculated as an average of annual observations over the time period 1998-2011. The total number of industries is 86.

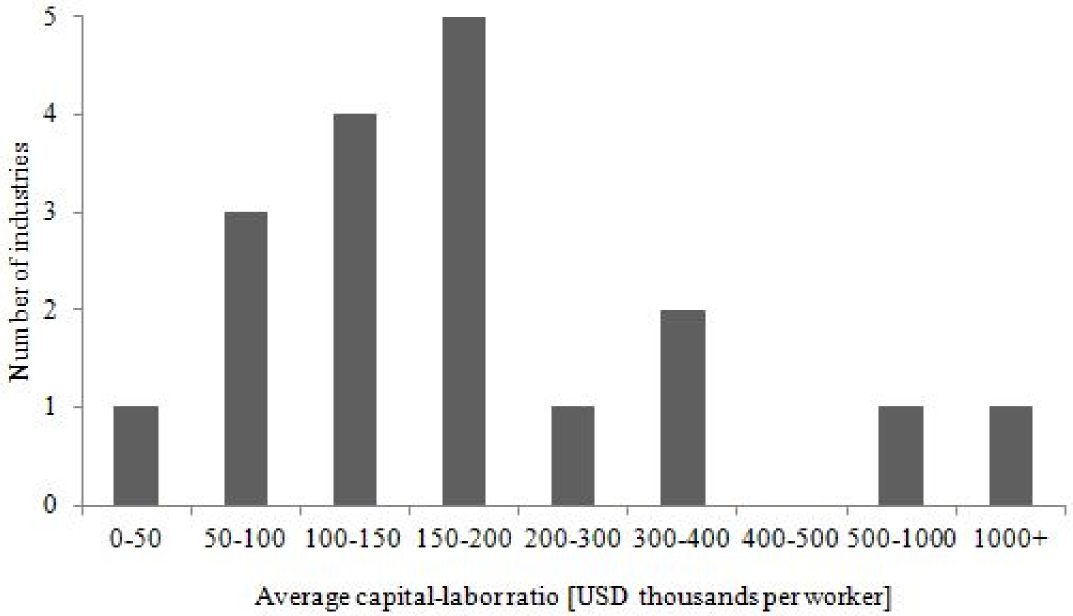
Figure 2.11: Cross-industry correlations between the annual averages of plant entry and exit rates at different levels of disaggregation



(a) 3-digit NAICS industries (b) 4-digit NAICS industries

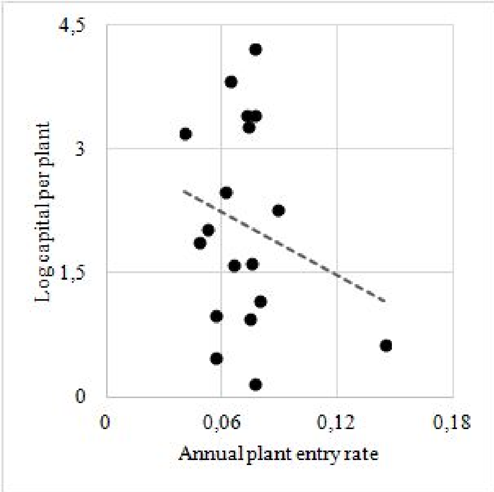
Source: Own calculations using Statistics of U.S. Businesses of the U.S. Census. For a description of variables see Figures 2.8 and 2.9. The total numbers of industries are, respectively, 18 and 86.

Figure 2.12: Distribution of industry-specific levels of capital-labor ratio in manufacturing

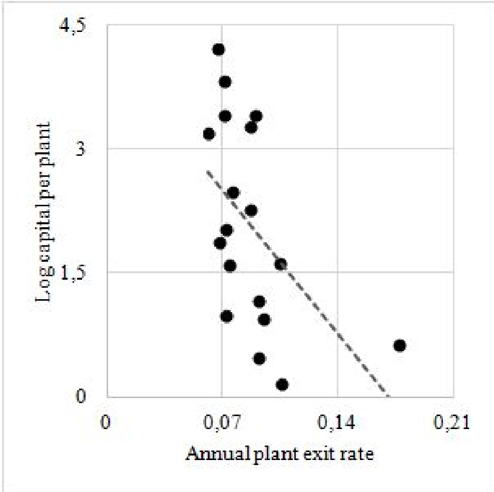


Source: Own calculations based on BEA data. The values used here are calculated as sample averages, over the period 1998-2011, of capital stock values divided by number of full-time employees. The figure represents the number of 3-digit NAICS industries in the manufacturing sector whose values of capital stock per establishment are within the specified range. The total number of industries is 18.

Figure 2.13: Cross-industry correlations between the annual averages of plant entry and exit rates and average capital stocks



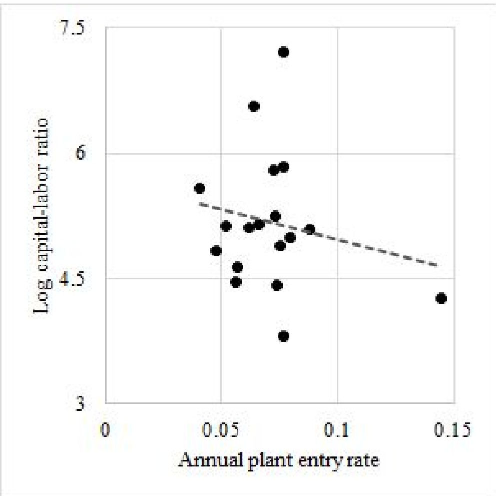
(a) Entry and capital intensity



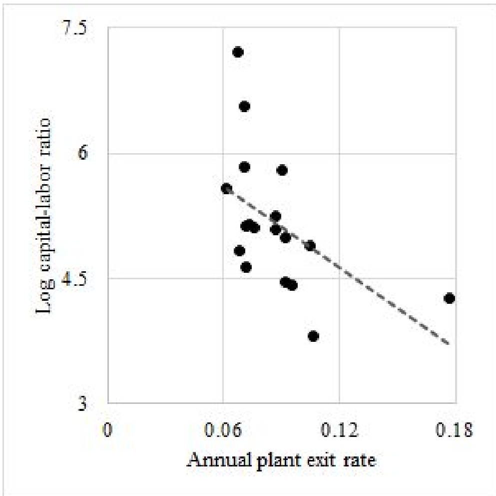
(b) Exit and capital intensity

Source: Own calculations using the datasets Statistics of U.S. Businesses of the U.S. Census and Bureau of Economic Analysis. For a description of variables see Figures 2.8, 2.9 and 2.4. The total number of industries is 18.

Figure 2.14: Cross-industry correlations between the annual averages of plant entry and exit rates and the capital-labor ratios



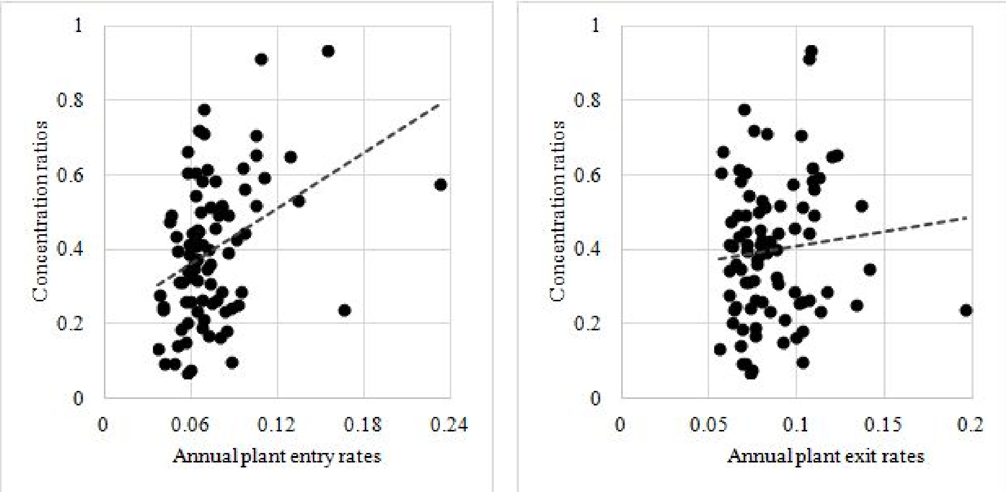
(a) Entry and capital intensity



(b) Exit and capital intensity

Source: Own calculations using the datasets Statistics of U.S. Businesses of the U.S. Census and Bureau of Economic Analysis. For a description of variables see Figures 2.8, 2.9 and 2.12. The total number of industries is 18.

Figure 2.15: Cross-industry correlations between the annual averages of plant entry and exit rates and the concentration ratios



(a) Entry and concentration ratios (b) Exit and concentration ratios

Source: Own calculations using the datasets Statistics of U.S. Businesses and Concentration Ratios of the U.S. Census. For a description of variables see Figures 2.8, 2.9 and 2.5. The total number of industries is 86.

3 Firm entry and exit, investment irreversibility, and business cycle dynamics[17]

3.1 Introduction

The Great Recession in the U.S. economy was accompanied by a low level of entry and a high exits of firms. Using the Business Dynamics Statistics (2013a) data, I have calculated a 23% decrease in the number of firm births and a 14% increase in firm deaths over the period $2007 - 2009$. This decline in the number of firms most likely exacerbated the severity of the recession and contributed to the subsequent slow recovery, as firms adjusting along their extensive margins is crucial for the growth of overall productivity and net job creation[18]. However, these cyclical patterns are by no means specific to the most recent economic downturn. A number of studies, across different countries and for various time periods, have documented similar business cycle movements for various measures of firms' entry and exit[19].

Given the observed co-movements between the mass of businesses and aggregate economic activity, it is natural to investigate the extent to which market entry and exit of firms help explain macroeconomic fluctuations. New businesses contribute to the aggregate dynamics not only by their initial investment, but also by acquiring physical capital in later stages of their life-cycle. However, there is ample evidence that firms find it difficult to detach and liquidate their stock of physical capital, which amounts to a serious source of friction and investment irreversibility. Ramey and Shapiro (2001), examining plant closures in the aerospace industry, document a substantial discount in the price of sold-off capital, relative to its original value adjusted by depreciation, whereas Cooper and Haltiwanger

[17]Earlier versions of the chapter appeared as working papers in Majher (2014), Majher (2015) and Majher (2019a).

[18]See Foster et al. (2001), Foster et al. (2008), and Haltiwanger et al. (2010), Sedláček and Sterk (2014).

[19]Portier (1995) reports pro-cyclicality of net business formation for French data, Sedláček and Sterk (2014) observe pro-cyclical dynamics of entrants' job creation in the U.S. private sector, Devereux et al. (1996) documents counter-cyclicality of a number of business failures in the U.S. economy, and Campbell (1998) presents evidence of counter-cyclicality of plant exit rate in the U.S. manufacturing.

(2006) find for a sample of manufacturing plants that only 10% of plants exhibit negative investment in any given year. The irreversibility of capital implies that firms should view a constant share of their investment as effectively sunk. As a result, this dampens both the entry and post-entry investment activity, and can lead to a postponement of decision to exit by affecting the relative value of staying in business. These interactions of firm investment and disinvestment decisions with partial capital irreversibility are essential for understanding the aggregate effects of firm's extensive margin adjustments.

This paper aims to assess the role of firms' turnover in business cycle dynamics in an environment where investment in physical capital is partially irreversible. I qualitatively and quantitatively evaluate the contribution of firm turnover to the magnitude and persistence of the model's response to aggregate productivity shocks. Towards this end, I extend the framework of heterogeneous-firms, as used by Hopenhayn (1992), to consider aggregate productivity shocks, endogenous entry and exit decisions, and the partial irreversibility of a firm's investment decision. Firms differ in their individual productivity levels, and must decide whether to remain operational or to exit, thereby recovering a fraction of their capital's value. The equations of the model are solved numerically and calibrated to match empirical moments of U.S. private sector data.

This paper contributes to the macro literature by offering a novel framework, distinguished by partial investment irreversibility, for assessing the role of firm entry and exit in business cycle dynamics. I focus on and identify the sunk nature of capital as a characteristic that affects the responses of business' turnover to productivity shocks, as well as subsequent impact on the economic aggregates. The model calibration, which emphasizes a realistic empirical fit of the firm-size distribution, allows me to quantitatively capture how firms vary in optimal investment and exit policies, and the consequences thereof. This is arguably an important part of any quantitative analysis conducted using a heterogeneous-firm model.

Analysis of the behavior of the model reveals that accounting for endogenous entry and exit of firms in a business cycle model with investment irreversibility substantially increases the magnitude and propagation of the response of aggregate variables to exogenous disturbances. A positive aggregate productivity shock increases the number of entrants as well as their individual investment levels. This is due to the fact that higher productivity yields an expected entry value that outweighs the implicit sunk cost associated with investment irreversibility. As a result, exiting businesses are replaced not only by a greater number of new firms, but also by firms that invest more on average. Moreover, as the productivity growth of the surviving firms exceeds the decreases due to the exit of the least productive firms, the output level of any cohort increases in subsequent periods. This built-in selection device enhances the endogenous propagation of economic activity.

The remainder of this chapter is structured as follows. Section 3.2 contains a brief overview of the related literature. Section 3.3 presents the setup of the model economy and an analytical characterization of plants' optimal policies in the steady state case. Section 3.4 describes the calibration procedure and parameter values. Section 3.5 overviews the numerical solution of the model in settings both with and without shocks to the level of aggregate productivity. Section 3.6 summarized and concludes the chapter.

3.2 A brief literature review

This work is connected to several strands of the literature. First and foremost, it is related to papers that analyze the role of firm entry and exit in the business cycle framework. Examples of relevant contributions are Lee and Mukoyama (2012), Sedláček (2014), and in particular Samaniego (2008), Clementi and Palazzo (2013) and Clementi et al. (2014). Focusing on the transition dynamics of an economy after a permanent shock to the aggregate productivity, Samaniego (2008) finds irresponsiveness of plant entry and exit, and subsequently no effect of turnover on aggregate dynamics. However, as Lee and Mukoyama (2012) point out, this result is driven by the assumptions involved in modeling the entry process, in particular giving the entry costs a form, which is tightly linked to the wage rate. This choice of model generates pro-cyclical entry costs, which dampen the response of entrants. Moreover, in this model there is no initial investment decision or scrap value of capital, which are key to my model.

The papers by Clementi and Palazzo (2013) and Clementi et al. (2014) feature a model with a firm-level capital stock subject to a combination of convex and non-convex (fixed) adjustment costs, wherein the former paper features a partial equilibrium analysis, while the latter addresses the dynamics in the general equilibrium. These works show that endogenous firm entry and exit propagate the effects of productivity shocks on aggregate variables. However, these analyses do not incorporate investment irreversibility. Moreover, the entry process is modeled differently, as entrants invest after their initial productivity is known. In my model, entrants are ex-ante identical, and they have to invest before drawing their initial productivity. Finally, my calibration targets the standard deviation of entrant firms' job creation in order to generate a plausible magnitude of the entry process dynamics.

A different approach is taken by those papers that assess the business cycle implications of entry and exit in a framework with imperfect competition. The most recent of these are Jaimovich and Floetotto (2008), Colciago and Etro (2010), Ottaviano (2011) and Bilbiie et al. (2012). Unlike my work, they focus on variations in markup as a possible transmission mechanism. Entry and exit cause a change in the number of firms, which has an effect on price markup because of supply- (tighter competition, implying lower markup) or demand-side considerations (greater variety, increasing the elasticity of substitution). In my model, there is perfect competition, as I focus on the technology-side determinants of entry and exit. Moreover, with the exception of Ottaviano (2011), these papers feature homogeneous firms, so there is no endogenous exit decision, only the exogenously generated business failures.

A second, related, field of the literature concerns papers that assess the implications of non-convex capital adjustment costs for the aggregate dynamics in frameworks with firm heterogeneity. Examples are Veracierto (2002), Khan and Thomas (2008) and Bachmann et al. (2013). The paper by Veracierto is particularly closely related, as the author evaluates the implications of investment irreversibility for business cycle dynamics, eventually concluding it to be quantitatively irrelevant. However, his model has major differences from the model I shall use in the assumptions made concerning plant entry and exit: both start-up and exit processes are purely exogenous; there is no role for initial investment or scrap value of capital; and, moreover, the exit hazard is the same for firms with different productivities.

3.3 The model

My model features an infinite time horizon with a discrete time index t. There are two types of agents present, namely households and firms. Households own plants, supply labor and consume produced goods. Firms, on the other hand, own capital, hire labor from the households, and use these two factors to produce output. The output goods can be used either for consumption or investment, which are perfect substitutes. There are two dimensions of firm heterogeneity: the level of idiosyncratic productivity, which persistently evolves over time, and the stock of capital. The latter can be augmented by investment that is (partially) irreversible. Moreover, the production units consider whether to remain in the market or exit, according to their current state. Finally, businesses can be divided into two groups, namely entrants and incumbents. Entrants have to pay a sunk cost and make their initial investment before becoming operational. They are subsequently relabeled as incumbents after their first period in the market.

At this point I present the assumed sequence of firms' actions. At the beginning of each period, incumbent plants first observe shocks to both the aggregate and the idiosyncratic component of their productivities. Afterwards they hire labor and produce output while paying wages and fixed costs. The next step is for establishments to decide whether to remain in business or exit and receive the scrap value of their capital stock. Moreover, a fraction of establishments is forced to exit exogenously, in order to generate failures among large production units consistent with the empirical evidence. Finally, surviving production units adjust their capital stock by investing subject to the irreversibility constraint and adjustment cost.

The sequence of actions for new firms starts with a draw of the entry cost. Based on its value, the potential entrants decide whether to pay and enter the market, or remain inactive. Subsequently, successful entrants have to purchase their initial stock of physical capital stock, which is done on the capital markets. After this initial investment, the entrant observes the aggregate state and its starting level of productivity is determined. From this point on, the sequence of actions is analogous to that of incumbents. New firms engage in hiring labor and producing output, and decide whether to exit or continue their operation. Finally, they are relabeled as incumbents from the second period of their existence.

The sequence of events for incumbent as well as for newly entering plants is graphically represented in Figure 3.1. The model's choice of timing is intended to refrect several features of real economic activity. Once they observe their current productivity, firms can adjust their capital stock only after producing output, as is standard in business cycle models with physical capital. The exit decision is taken after the production process to reinforce the idea of investment being sunk: a plant with an unfavorable draw of productivity has to engage in production one last time before exiting from the market. In turn, potential entrants have to acquire their initial investment before their starting productivity is revealed. This

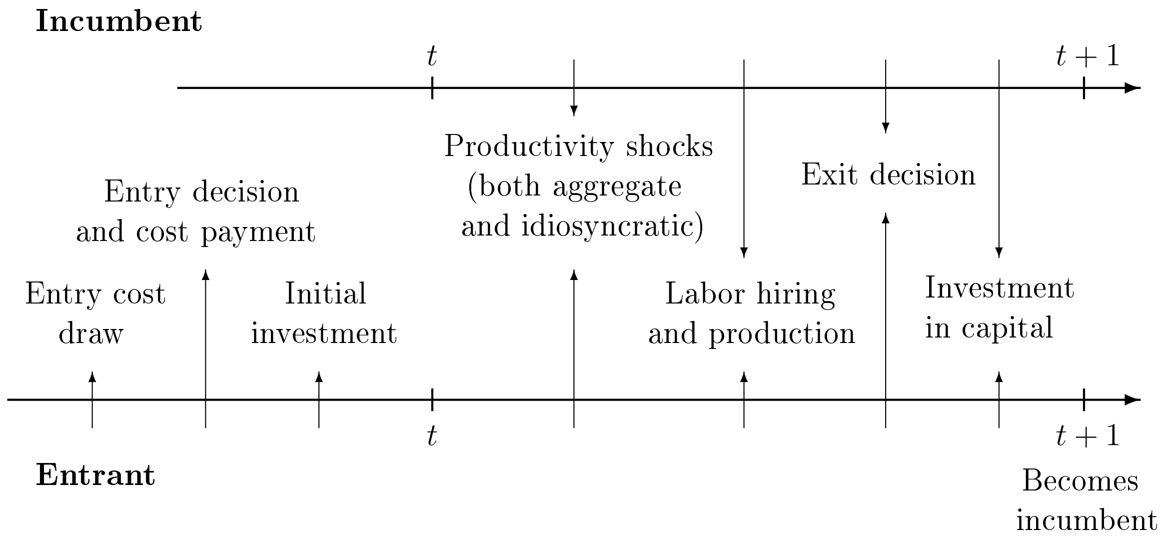


Figure 3.1: Timing of actions

feature, common to most dynamic models involving firm dynamics, captures the uncertainty connected with entry of new plants.

3.3.1 The production side

There is a single final good produced, which can be consumed by households or used by the plant as capital investment. Production by an establishment at time t is given by the production function

$$y_t = f(z_t, k_{t-1}, n_t) \equiv A_t z_t k_{t-1}^{\alpha} n_t^{\eta}, \tag{3.1}$$

where $\alpha, \eta \in (0,1); \alpha + \eta < 1$. I assume decreasing returns to scale in order to have production units choose finite optimal levels of production factors capital k_{t-1} (predetermined) and labor n_t. The terms A_t and z_t denote the aggregate productivity and the plant's idiosyncratic productivity, respectively. The evolution of the logarithm of these productivities over time can be represented by the following AR(1) processes

$$\begin{aligned} \ln A_t &= \rho_A \ln A_{t-1} + \epsilon_t^A, \\ \ln z_t &= \rho_z \ln z_{t-1} + \epsilon_t^z, \end{aligned} \tag{3.2}$$

with $\rho_A, \rho_z \geq 0$, $\epsilon_t^A \sim \mathcal{N}(0, \sigma_A^2)$ and $\epsilon_t^z \sim \mathcal{N}(0, \sigma_z^2)$. Notice that the aggregate and idiosyncratic productivity processes are mutually independent. Let me denote by

$f^A(A_{t+1}|A_t)$ and $f^z(z_{t+1}|z_t)$ the density functions of A_{t+1} and z_{t+1} conditional on A_t and z_t, respectively.

The first production factor is the capital stock, which is owned directly by plants. It can be adjusted by investment and is subject to depreciation over time. The corresponding law of motion is

$$k_t = (1 - \delta^K)k_{t-1} + i_t, \tag{3.3}$$

where $\delta^K \in (0, 1)$ is the depreciation rate and i_t represents gross investment. I assume that establishments face a kind of friction in the form of partial irreversibility, i.e. they are able to recover only a fraction κ of the remaining value of their capital in the case of disinvestment. In addition, my model also features a quadratic cost of capital adjustment in order to smooth a plant's investment behavior over its life-cycle.

The assumption of friction to capital reallocation implies that the capital stock k_t becomes a plant's individual state variable in period t, in addition to the idiosyncratic productivity z_t. The distribution of establishments over the underlying state space $\mathbf{S} = \mathbb{R}_+ \times \mathbb{R}_+$ at the end of a period t is characterized by the measure $\bar{\mu}_t(z_t, k_t)$ defined on the Borel algebra $\mathcal{S}$ on $\mathbf{S}$. However, for convenience I also define the measure $\mu_t(z_t, k_{t-1})$ representing the distribution of establishments after realization of the productivity shocks. This measure, together with the aggregate productivity, describes the aggregate state of the economy $S_t := (A_t, \mu_t)$.

The second factor of production, labor, can be adjusted at the plant level without any costs. In each period, plants hire workers from the households in a frictionless labor market and pay them the wage rate w, which adjusts such that the labor market clears.

At the same time, there is an additional cost associated with the production process that a business has to pay each period: a fixed cost measured in output units and denoted by c^f. It captures the overhead expenditures of production faced by an operating plant. This cost is necessary to obtain endogenous exit decisions of production units, as it forces the least productive establishments to quit rather than to cut their production level to zero.

After determining the plant's technology, I can represent the plant's decision process as a dynamic programming problem. Following Khan and Thomas (2008), I denote the marginal utility of a household's consumption by p_t and use it as a price utilized by firms to value their current cash flow. As a result, a plant's decision problem is expressed in utility terms rather than in physical units.

Using this convention, I now describe the recursive problem of profit maximization of a production unit. First, the value function $V^0(z_t, k_{t-1}; S_t)$ of a plant with capital stock k_{t-1} and a productivity level z_t under the aggregate state S_t is given as follows:

$$V^0(z_t, k_{t-1}; S_t) = p(S_t)\,\tilde{\pi}(z_t, k_{t-1}; S_t) + V^{EX}(z_t, k_{t-1}; S_t). \tag{3.4}$$

Here $\tilde{\pi}$ denotes a plant's contemporaneous profit, net of the investment flows and any associated adjustment costs. I assume that, in each period, plants have to pay a fixed cost c^f, whose value is common for all plants. The plant's contemporaneous profit can be expressed as follows:

$$\tilde{\pi}(z_t, k_{t-1}; S_t) = \max_n \Big\{ A_t z_t k_{t-1}^{\alpha} n^{\eta} - w(S_t) n - c^f \Big\}.$$

Next, V^{EX} depicts a plant's value at the moment when it considers whether to exit or remain operational. This decision is characterized by the following equation

$$V^{EX}(z_t, k_{t-1}; S_t) = \max \Big\{ p(S_t)\, V^X,\ \delta\, p(S_t)\, V^X + (1-\delta) V^{INV}(z_t, k_{t-1}; S_t) \Big\}, \tag{3.5}$$

where V^X represents the value of exit (characterized below) and δ is the probability of an exogenous exit. In equation (3.5), V^{INV} captures a plant's value prior to its investment decision,

$$V^{INV}(z_t, k_{t-1}; S_t) = \max_{k_t} \Big\{ -p(S_t)\, C^K(k_t, k_{t-1}) \ + \ \beta \mathbb{E}\big[V^0(z_{t+1}, k_t; S_{t+1}) | z_t, S_t \big] \Big\}, \tag{3.6}$$

where $C^K(k_t, k_{t-1})$ is a function that represents capital adjustment. Plants use the factor $\beta \in (0, 1)$ from the household utility function to discount future cash flows. Note that the standard stochastic discount factor, $\tilde{\beta} = \beta u_c(c_{t+1}, l_{t+1})/u_c(c_t, l_t)$, is implicitly present in the term $p(S_t)$, which is assumed to equal the current marginal utility of consumption. Furthermore, the expectations operator in equation (3.6) relates to the next period's realizations of both aggregate and idiosyncratic productivities, conditional on their current values. Let me conclude by using $k^*(z_t, k_{t-1}; S_t)$ to denote the plant's optimal choice of the next period's stock of physical capital k_t, subject to its current individual state (z_t, k_{t-1}) and the aggregate state S_t.

3.3.2 A plant's decision margins

Next, I characterize the behavior of the establishment. Specifically, I describe in detail the three decision margins that are present in the model: levels of investment in capital stock, the exit decision, and the entry decision together with the initial capital investment.

Capital adjustment

As represented by equation (3.6), plants choose their future level of capital stock in order to maximize their market value. There $C^K(k_t, k_{t-1})$ is a function of costs associated with the capital investment (or possibly disinvestment) $k_t - (1 - \delta^K)k_{t-1}$. This choice of function incorporates both the adjustment costs C_A and the investment itself (including the irreversibility constraint) C_I, such that $C^K = C_A + C_I$. The underlying functions have the following forms:

$$
\begin{aligned}
C_A(k_t, k_{t-1}) &= \gamma \frac{(k_t - (1 - \delta^K)k_{t-1})^2}{k_{t-1}}, \\
C_I(k_t, k_{t-1}) &= \begin{cases} \kappa(k_t - (1 - \delta^K)k_{t-1}), & \text{if } (k_t - (1 - \delta^K)k_{t-1}) < 0, \\ (k_t - (1 - \delta^K)k_{t-1}), & \text{otherwise.} \end{cases}
\end{aligned}
$$

Note that a parameter value $\kappa = 1$ implies full reversibility, whereas 0 indicates complete irreversibility.

Exit

In each period, after observing the updated current productivity and producing output, each plant considers whether to remain in the market or to exit by comparing the expected value from remaining active with that of the outside option of exiting, V^X. As I treat the scrap value of capital as part of the irreversibility friction, the value of exiting the market depends on the current stock of the plant's capital $(1 - \delta^K)k_{t-1}$. This dependence can be represented by a function $V^X(k_{t-1})$ with the following properties:

$$
V^X(0) = 0, \quad \forall k_1 < k_2 : \ V^X(k_1) \leq V^X(k_2), \quad \text{and} \quad V^X(k) \leq k.
$$

The economic interpretation is straightforward. A production unit without capital cannot recover any value after exiting, and a rise in the capital stock does not decrease scrap value, which cannot be higher than the actual value of the capital itself. In the numerical solution of my model, I assume that an exiting plant receives a fraction κ of its current capital stock net of depreciation, i.e., $V^X(k) = \kappa(1-\delta^K)k$.

The notion of the exit value allows me to rewrite the value function V^{EX} in equation (3.5) as follows:

$$V^{EX}(z_t, k_{t-1}; S_t) = \max\left\{p(S_t)V^X(k_{t-1}), \delta p(S_t)V^X(k_{t-1}) + (1-\delta)V^{INV}(z_t, k_{t-1}; S_t)\right\}. \tag{3.7}$$

Note that in addition to plants exiting endogenously, a fraction δ of the surviving units is forced to exit irrespective of their current state. This modeling feature is introduced to generate exits on the part of large establishments as well, which is consistent with the empirical evidence.

An establishment, characterized by its individual state (z_t, k_{t-1}), exits whenever the expected future profits, including the optimal level of investment, are lower than the scrap value of its capital stock. Using expression (3.6), I characterize the continuation decision of a plant by an indicator function $\chi(z_t, k_{t-1}; S_t)$ as follows:

$$\chi(z_t, k_{t-1}; S_t) = \begin{cases} 1 & \text{if } V^{INV}(z_t, k_{t-1}; S_t) \geq V^X(k_{t-1}), \\ 0 & \text{otherwise.} \end{cases} \tag{3.8}$$

Entry

There is a fixed mass $\mathcal{M}$ of potential entrants. In each period, each of them draws individually a value of entry cost c^E from the time-invariant distribution $G(c^E)$, which is assumed to be uniform over the interval $[0, C^E]$. Based on the draw, it makes a decision on whether to enter or not. Recall that the entry decision takes place one period before the actual output production of entrants. Those who decide to enter in period $t-1$, first have to pay the entry cost and invest in the initial capital stock k^E_{t-1} at the end of period $t-1$. In period t, they observe the aggregate productivity shock and draw their starting productivity z^E_t from a common distribution $g(z)$, independent of their initial investment. I assume that $g(z)$ is log-normal, i.e. $\log z^E_t \sim \mathcal{N}(\bar{z}^E, \sigma^2_E)$. With their productivity revealed, they next hire workers and produce just like the incumbent plants. Finally, they consider whether to remain operational and pursue business activity or to exit from the market.

The investment problem of an entrant can be characterized by the equation

$$V^E(S_{t-1}) = \max_k \left\{ \beta\, \mathbb{E}\left[V^0(z_t, k; S_t | S_{t-1}) \right] - p(S_{t-1})\, k \right\}, \tag{3.9}$$

where the expectations are over the distribution of initial productivity $g(z)$ together with the distribution of aggregate productivity shocks. The associated level of initial investment is selected such that the entrants maximize their expected value of future profits conditional on the current aggregate conditions:

$$k^E_{t-1}(S_{t-1}) = \arg\max_k \left\{ \beta\, \mathbb{E}\left[V^0(z_t, k; S_t | S_{t-1}) \right] - p(S_{t-1})\, k \right\}. \tag{3.10}$$

Since all entrants draw their starting productivity levels from the same distribution and face equal aggregate conditions, they acquire an identical level of capital k^E_{t-1}[20].

Next, I describe the entry decision of a potential entrant by the expression

$$V^{POT}(c^E, S_{t-1}) = \max \left\{ 0, V^E(S_{t-1}) - p(S_{t-1}) c^E \right\}. \tag{3.11}$$

A potential new firm enters if its current draw of entry cost, remunerated by the actual price, is less than the expected value of entry as defined in equation (3.9). This representation allows me to characterize the entry decision of a potential firm by an indicator function $\psi(c^E; S_{t-1})$ as follows:

$$\psi(c^E; S_{t-1}) = \begin{cases} 1 & \text{if } V^E(S_{t-1}) \geq p(S_{t-1}) c^E, \\ 0 & \text{otherwise.} \end{cases} \tag{3.12}$$

3.3.3 Cross-sectional distribution

I shall now present an expression for the distribution of entrants E_t and a transition law for the distribution of all firms $\mu_{t+1} = \Gamma(A_{t+1}, \mu_t)$. Recall that the market-entering plants first draw the entry cost and decide whether to enter, then purchase their start-up stock of capital, and finally are allocated their initial productivity. This leads to the following identity:

$$\forall (Z_t, K_{t-1}) \in \mathcal{S}: \; E_t(Z_t, K_{t-1}; S_{t-1}) = M_{t-1}(S_{t-1}) \int_{Z_t} \mathbb{1}_{\{k^E_{t-1}(S_{t-1}) \in K_{t-1}\}} dg(z), \tag{3.13}$$

[20]Note that the level of capital selected by the entering plants is always finite due to the decreasing returns-to-scale of the technology.

where $\mathbb{1}$ represents the indicator function, and $M_{t-1}(S_{t-1})$ is the number of successful entrants, which depends on the current aggregate state S_{t-1}:

$$M_{t-1}(S_{t-1}) = \mathcal{M} \int_0^{C^E} \psi(c^E; S_{t-1}) \frac{1}{C^E} dc^E. \tag{3.14}$$

The incumbent plants, meanwhile, consider whether to exit before investment decisions and realization of shocks. Recall that $k^*(z_t, k_{t-1})$ denotes the optimal choice of capital for a production unit in the state (z_t, k_{t-1}). The distribution of establishments μ_t evolves according to the following law of motion:

$$\begin{aligned} \forall (Z_{t+1}, K_t) \in \mathcal{S} : \mu_{t+1}(Z_{t+1}, K_t) \;&=\; E_{t+1}(Z_{t+1}, K_t; S_t) + \\ + (1-\delta) \int_{\mathbb{R}^+} \int_{Z_{t+1}} \int_{\mathbf{S}} & \mathbb{1}_{\{z_t > \tilde{z}_t(k_{t-1})\}} \, \mathbb{1}_{\{k^*(z_t, k_{t-1}) \in K_t\}} \, d\mu_t(z_t, k_{t-1}) \, f^Z(z_{t+1}|z_t) \, f^A(A_{t+1}|A_t). \end{aligned} \tag{3.15}$$

3.3.4 The household side

There is a continuum of infinitely long-lived identical households of unit measure that consume output, supply labor, and own the production units, whose profits they claim. At the same time, they have at their disposal a complete set of state contingent assets[21]. The households' objective is to choose consumption c and labor n^h to maximize the expected life-time utility

$$\max_{\{c_t, n_t^h\}_{t=0}^{\infty}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t u(c_t, n_t^h) \tag{3.16}$$

subject to the budget constraint

$$\forall t: \; c_t = w_t n_t^h + \Pi_t,$$

[21]Yet, as there is no heterogeneity across households, these claims are in zero net supply.

where $\beta \in (0,1)$ is the time-invariant discount factor and Π_t stands for plants' profits, which can be specified explicitly in the following way:

$$\begin{aligned}\Pi_t = &\int_{\mathbf{S}} \tilde{\pi}(z_t, k_{t-1}; S_t)\, d\mu(z_t, k_{t-1}) - M(S_t)\, k^E(S_t) - \\ &- (1-\delta) \int_{\mathbf{S}} C^K\Big(k^*(z_t, k_{t-1}; S_t), k\Big) \chi(z_t, k_{t-1}; S_t)\, d\mu(z_t, k_{t-1}) + \\ &+ \int_{\mathbf{S}} \kappa\, (1-\delta^K)\, k_{t-1} \Big(1 - (1-\delta)\chi(z_t, k_{t-1}; S_t)\Big)\, d\mu(z_t, k_{t-1}).\end{aligned}$$

The three lines of the equation capture, respectively, the cash flows from production and entry investment, those associated with investment carried out by the surviving plants, and those from scrapping the capital of the exiting establishments.

The optimization procedure yields the following first-order conditions

$$\begin{aligned} c_t:\ & u_c(c_t, n_t^h) = \lambda_t, \\ n_t:\ & u_n(c_t, n_t^h) = -\lambda_t w_t,, \end{aligned} \tag{3.17}$$

where λ_t is a Lagrange multiplier associated with the budget constraint in time t. These two conditions, together with the assumption $u_c(c_t, n_t^h) = p_t$, yield

$$w_t(A_t, \mu_t) = -\frac{u_n(c_t, n_t^h)}{u_c(c_t, n_t^h)} = -\frac{u_n(c_t, n_t^h)}{p_t(A_t, \mu_t)}.$$

Furthermore, I assume that the labor is indivisible, i.e. workers decide either to work or to remain idle, without adjustments along the intensive margin. Following Hansen (1985) and Rogerson (1988), this assumption yields the utility function of a form $u(c_t, n_t^h) = \log c_t - \theta n_t^h$, which allows me to rewrite the conditions in equations (3.17) as follows:

$$p_t = \frac{1}{c_t}, \qquad w_t = \frac{\theta}{p_t}. \tag{3.18}$$

The assumption of indivisible labor is consistent with modelling choices typically made wihthin studies in this area of research, see e.g. Khan and Thomas (2008) or Bachmann et al. (2013). Furthermore, as a numerical solution of my model shows (see tables 3.4 and 3.5), model-implied statistics characterizing dynamics of the aggregate labor are comparable with the empirical ones. Therefore I argue that this modelling choice is a suitable one with respect to the objective of my analysis.

3.3.5 Recursive equilibrium

In my model, a recursive competitive equilibrium is defined as a set of functions

$$(p, w, V^0, k^*, N^D, \chi, \psi, C, N^H, k^E, M, \Gamma)$$

that satisfy the following conditions:

- Plants maximize their profits: taking $p(A, \mu)$, $w(A, \mu)$ and $\Gamma(A, \mu)$ as given, plants with individual states (z, k) follow optimal policies in investment $k^*(z, k; A, \mu)$, labor demand $N^D(z, k; A, \mu)$ and exit $\chi(z, k; A, \mu)$, such that $V^0(z, k; A, \mu)$ solves (3.4).

- Households maximize their utility: taking $p(A, \mu)$ and $w(A, \mu)$ as given, households choose optimal consumption $C(A, \mu)$ and labor supply $N^H(A, \mu)$ to satisfy (3.18).

- Entry is optimal: taking $p(A, \mu)$, $w(A, \mu)$ and $\Gamma(A, \mu)$ as given, potential entrants follow an entry decision $\psi(A, \mu)$ and choose their initial investment $k^E(A, \mu)$ to satisfy (3.12) and (3.10) respectively, and their mass $M(A, \mu)$ adjusts such that (3.14) holds.

- The goods market clears: the price $p(A, \mu)$ adjusts such that household's consumption $C(A, \mu)$ is equal to the aggregate output minus the fixed cost (first row of (3.19)), minus the cost of entry and the entry investment (second row) , minus the cost of investment of surviving incumbents (third row), plus the disinvestment of exiting production units (fourth row):

$$\begin{aligned} C(A, \mu) = & \int_{\mathbf{S}} \left\{ f\Big(z, k, N^D(z, k; A, \mu); A, \mu\Big) - c^f \right\} d\mu(z, k) - \\ & - \mathcal{M} \int_0^{C^E} \psi(c; S_{t-1}) \, \frac{c}{C^E} \, dc \;-\; M(A, \mu)\, k^E(A, \mu) - \\ & - (1 - \delta) \int_{\mathbf{S}} C^K \Big(k^*(z, k; A, \mu), k\Big) \chi(z, k; A, \mu) \, d\mu(z, k) + \\ & + \int_{\mathbf{S}} \kappa \, (1 - \delta^K) \, k \,\Big(1 - (1 - \delta)\omega(z, k; A, \mu)\Big) \, d\mu(z, k). \end{aligned} \tag{3.19}$$

- The labor market clears: $w(A, \mu)$ adjusts such that the aggregate labor demand equals the labor supply:

$$\int_{\mathbb{S}} N^D(z,k;A,\mu)d\mu(z,k) = N^H(A,\mu), \tag{3.20}$$

- Dynamics are consistent: the transition law $\Gamma(A,\mu)$ for the distribution μ is compatible with the mass of entrants $M(A,\mu)$ and the optimal policies $K(z,k;A,\mu)$, $\omega(z,k;A,\mu)$ and $k^E(A,\mu)$ following (3.15).

3.3.6 Characterization of optimal exit policy

To conclude the outline of my model, I shall now describe a plant's optimal policies with respect to its individual state (z,k) in the steady-state economy. Studying the stationary equilibrium is a simplification with respect to the full model, which incorporates the dynamics arising in the presence of shocks to aggregate productivity. However, I argue that it is instructive to first inspect the characteristics of the stationary environment, as many results can be derived analytically unlike in the framework with aggregate dynamics. Moreover, the firms' optimal exit policies in the steady state can provide useful insights into the economic mechanisms that are at work in the dynamic model. To keep the exposition smooth, all the formal statements and proofs, including the proof of existence and uniqueness of the value function, are presented in Appendix 3.D.

I take equations (3.4) - (3.6) as the starting point and rewrite them as appropriate for the stationary equilibrium. This implies that plant's decision problem is independent of the aggregate state S_t. At the same time, I am able to normalize the final goods price p_t to 1, since it does not change over time and thus does not affect a plant's optimal policies. As a result, the Bellman equation of a plant with a capital stock k and a productivity level z has the following form:

$$V^0(z,k) = \tilde{\pi}(z,k) + \max\Big\{V^X(k), \delta V^X(k) + (1-\delta)\max_{k'}\Big\{-C^K(k',k) + \beta\mathbb{E}\big[V^0(z',k')|z\big]\Big\}\Big\}, \tag{3.21}$$

where $V^X(k)$ represents the exit value $V^X(k) = \kappa(1-\delta^k)k$, $\tilde{\pi}(z,k)$ denotes a plant's contemporaneous profit (net of the capital adjustment),

$$\tilde{\pi}(z,k) = \max_n \Big\{zk^\alpha n^\eta - wn - c^f\Big\}, \tag{3.22}$$

and C^K characterizes any costs related to the adjustment of a plant's stock of physical capital

$$C^K(k',k) = \begin{cases} \gamma\frac{(k'-(1-\delta^K)k)^2}{k} + \kappa(k'-(1-\delta^K)k), & \text{if } (k'-(1-\delta^K)k) < 0, \\ \gamma\frac{(k'-(1-\delta^K)k)^2}{k} + (k'-(1-\delta^K)k), & \text{otherwise.} \end{cases} \tag{3.23}$$

The main analytical result is that, for each level of a plant's capital stock k, it can be shown thatthere exists a unique productivity threshold, $\tilde{z}(k) > 0$, which determines whether the establishment exits or remains operational. Specifically, if a plant with a stock of physical capital k draws a productivity level z lower than $\tilde{z}(k)$, then its optimal policy is to quit production. On the other hand, a high-enough productivity shock increases the value of an establishment above the scrap value of its capital, thus making exiting suboptimal. This pattern is consistent with the empirical evidence showing that less productive plants are more likely to exit (see the survey in Bartelsman and Doms (2000)).

3.4 Calibration

Prior to presenting the model's numerical solution, I have to decide on the parameter values, which are calibrated for correspondence with selected statistics of the annual data for the U.S. private business sector. To make the exposition smooth, I present a detailed description of the data sources and construction of the targeted statistics in Appendix 3.A.

The parameters in my model can be divided into two groups. The first group contains those that can be calibrated on the basis of the literature or direct empirical evidence. The output elasticity with respect to labor η is set to match a labor share of national income of 0.60, and the discount factor β is set in a way that the real annual interest rate equals 4%. Consequently, the depreciation rate δ_K is set to match the annual value 6.02%, which is calculated as the average fraction of depreciated capital based on the Bureau of Economic Analysis (2013) dataset (BEA). At the same time, the fraction of production units exiting for exogenous reasons, δ, is fixed at a value of 0.7% by the exit rate of the largest establishments as calculated from the Business Dynamics Statistics (2013a) dataset (BDS). Finally, the parameter values governing the stochastic process for the aggregate TFP (persistence ρ_A and volatility σ_A) are estimated from the Solow residual using the Bureau of Labor Statistics (2014) dataset (BLS) for the employment level, and the BEA data for the capital stock and output.

The second group of parameters is calibrated such that the selected moments implied by the model economy are consistent with the corresponding statistics extracted from the empirical data. It is important to bear in mind that these parameters interact with the targeted statistics, so I have to calibrate them together. First, I adjust the household labor disutility parameter θ to obtain a labor force participation rate equal to 63.9% (taken from the BLS dataset). Next, the value of the output elasticity with respect to capital α targets a capital-to-output ratio value of 1.76, in accordance with the data provided by BEA. For the parameter $\bar{z}^E$ governing the mean of productivity distribution of entrants, the ratio of the average size of entering plants to the average size of exits (1.07, according to the BDS dataset) is a reasonable target. The standard deviation of the productivity distribution of entrants σ_E is set to match the averagepercentage (19.75%) of units that exit during the first year of their existence (BDS). The fixed cost of production c^f serves as normalization, so I use it to fix the steady state mass of firms at unity. At the same time, the value for the mass of potential entrants $\mathcal{M}$ is set such that the number of entrants equals that of exits in the steady-state equilibrium. Finally, I use the entry cost parameter C^E to discipline how much the number of entrants varies over the business cycle. In particular, it targets a value of 4.36 for the standard deviation of entrants' job creation (relative to that of output).

Finally, I must comment on the calibration of those parameters that characterize the stochastic process of the idiosyncratic TFP (persistence ρ_z and volatility σ_z), and frictions to capital reallocation (the capital quadratic adjustment cost function parameter γ and the degree of irreversibility κ). The literature typically targets selected moments of the investment rate distribution as in Cooper and Haltiwanger (2006) (and see e.g. Khan and Thomas (2008) or Khan and Thomas (2013)).[22] These statistics characterize the behavior of a sample of large manufacturing plants that survived throughout the sample period (17 years). Thus, a comparable sample of production units from the model economy needs to be generated in order to target data moments consistently. In a model with entry and exit, this arguably excludes smaller establishments from the identification process, as they are more prone to exit (and also entry), and puts too much of an emphasis on the large plants, which are typically less subject to turnover risk. However, my focus is on the interaction between the equilibrium distribution of establishments

[22]E.g. the cross-sectional mean and standard deviation of investment rates, or their average serial correlation together with the fraction of plants undertaking positive, negative, or negligible investment.

and the optimal entry and exit policies, which are most pronounced in the lower end of the plant size distribution. The aforementioned approach is, therefore, not suitable for the calibration of my model, given the questions I am seeking to answer.

In order to address the above concerns, I use parameters ρ_z and σ_z to directly adjust the establishment size distribution observed in the stationary equilibrium of my model to match with the empirical evidence. More precisely, I split the model-implied size distribution into nine disjunct bins separated into percentiles corresponding to the size categories presented in the data, and compare their respective employment shares. Details of the data, which are publicly available from the Business Dynamics Statistics (2013b) dataset, are presented in Appendix 3.A.3. As the number of size categories exceeds that of the free parameters, my calibration minimizes a weighted sum of squared differences between the empirical and model employment shares[23]. Next, recall that convex costs of capital adjustment have been introduced to smoothen a firm's growth over its life-cycle. It is therefore appropriate to use a 1.2 ratio of average firm sizes of the second year cohort to the first year cohort in order to fix the parameter value for γ . Finally, in order to discipline the magnitude of plant entry and exit, I use the investment irreversibility parameter κ to align the model-implied turnover rate defined as the sum of entry and exit rates with its empirical counterpart, 0.215.

Table 3.1: Parameter values of the benchmark calibration

η	β	δ_K	δ	ρ_A	σ_A					
0.60	0.96	0.0602	0.007	0.889	0.0145					
θ	α	$\bar{z}^E$	σ_E	c^f	ρ_z	σ_z	γ	κ	C^E	$\mathcal{M}$
1.373	0.205	0	0.115	0.198	0.92	0.175	0.09	0.95	0.14	0.217

The baseline calibration of parameters is presented in Table 3.1. First, note that the production function exhibits returns-to-scale of 0.805, well within the range of empirical estimates. Next, the model-implied degree of reversibility has a value of 0.95, which is reasonable with respect to the numbers discussed by the literature[24]. Finally, my calibration of the transition law of idiosyncratic pro-

[23] The weights used are the normalized inverted standard deviations of the time series observed for the respective size class percentiles.

[24] For loss in value of displaced physical capital relative to its acquisition price and net of depreciation (i.e. equivalent to $1-\kappa$ in my model), Ramey and Shapiro (2001) observe on average $40+\%$ for closing plants in the aerospace industry, Bloom (2009) estimates a value of 33.9%, and Khan and Thomas (2013) obtain 4.6% via calibration. As Khan and Thomas (2013) note, this parameter value is sensitive to the model specification, and therefore a proper measure

ductivity implies a process that is more persistent and volatile compared to the related contributions[25]. This is a consequence of my calibration strategy, and specifically of the emphasis on the distribution of establishment size .

3.5 Numerical results

3.5.1 Stationary equilibrium

I start the numerical analysis by describing the solution of the model's stationary equilibrium. The numerical algorithm that solves the steady-state of the model economy is described in detail in Appendix 3.B.1. Even though this paper addresses the aggregate dynamics, it is instructive to first inspect the characteristics of the stationary environment. In particular, the steady-state distribution of firms and their optimal entry, exit and investment policies can provide useful insights into the economic mechanisms that are at work in the dynamic model.

Table 3.2: Comparison of targeted empirical statistics with the model

Name of statistic	U.S.data	the model
(i) Labor force participation rate	0.639	0.639
(ii) Capital to output ratio	1.76	1.77
(iii) Ratio of entrants-to-exits size	1.06	0.80
(iv) Fraction of exiting entrants	0.198	0.198
(v) Ratio of 2^{nd} year to 1^{st} year firm size	1.214	1.269
(vi) Turnover rate	0.215	0.217
(vii) St. dev. of entry JC	4.357	4.327

Notes: All statistics are calculated as annual averages over the underlying samples. Sources: (i) own calculations using the BLS dataset, time period 1960-2013; (ii) own calculations using the BEA dataset, time period 1960-2013; (iii) - (vii) own calculations using the BDS dataset, time period 1979-2013. Detailed description available in Appendix 3.A.

Table 3.2 contains a comparison of the values of the targeted variables generated by the model with their empirical counterparts, while, Table 3.3 provides an overview of the fit of establishment size distribution. It can be observed that the model-implied distribution tracks it empirical counterpart very closely especially at the bottom end, whereas the fit for the top percentiles of population shares is

of whether it is reasonable should be the model-implied behavior of individual production units rather than the magnitude of the raw number.

[25]E.g. Khan and Thomas (2008) use $\rho_z = 0.859$ and $\sigma_z = 0.022$, Clementi et al. (2014) use $\rho_z = 0.653$ and $\sigma_z = 0.138$, while Clementi and Palazzo (2013) have $\rho_z = 0.55$ and $\sigma_z = 0.22$.

somewhat worse. As I discuss in Appendix 3.A.3, this is due to the assumption of normality for the underlying shock process. Nevertheless, I argue that this does not bias my analysis in a quantitatively significant way, because the economic mechanisms of interest, namely how productions units' between production units' entry and exit decisions are affected by the physical capital, mainly concern the smallest production units.

Table 3.3: Comparison of empirical and model-based (steady-state) establishment size distributions

Size class	Employment shares	Population shares		Size class	Employment shares	Population shares	
		Data	Model			Data	Model
1−4	0.0663	0.4928	0.4840	100−249	0.1523	0.0170	0.0225
5−9	0.0883	0.2236	0.2166	250−499	0.0890	0.0044	0.0069
10−19	0.1123	0.1387	0.1379	500 − 999	0.0687	0.0017	0.0034
20−49	0.1647	0.0903	0.0915	1000+	0.1317	0.0010	0.0025
50−99	0.1264	0.0305	0.0346				

Notes: Data statistics are calculated as the annual data over the underlying sample from the BDS dataset, time period 1979-2013. Detailed description available in Appendix 3.A.3. Model-implied plant population shares are calculated for the corresponding cumulative employment shares, and sizes of the establishments do not exactly match to the respective empirical size classes.

Now I present the distribution of establishments across their individual productivity levels and compare it to its equivalent in the steady-state of a model without firm entry and exit[26]. The comparison is presented in Figure 3.2. It is apparent that the benchmark model features businesses with productivity levels that are higher on average, and less dispersed, than for the framework without plant turnover. This difference is caused by two mechanisms. The first one is a standard selection mechanism, where the least productive establishments choose to exit, thus allowing for a reallocation of resources toward the more productive plants. The second mechanism operates through the entry of new production units, whose productivity is less varied than that of the exiting establishments, thus contributing to the observed discrepancy. All in all, these results indicate that introducing the extensive margin of reallocation into a framework with firm heterogeneity shifts the mass of establishments towards higher productivity levels than would be the case without entry and exit.

[26]Details of the model without firm turnover, together with the corresponding numerical solution, are given in Appendix 3.C.

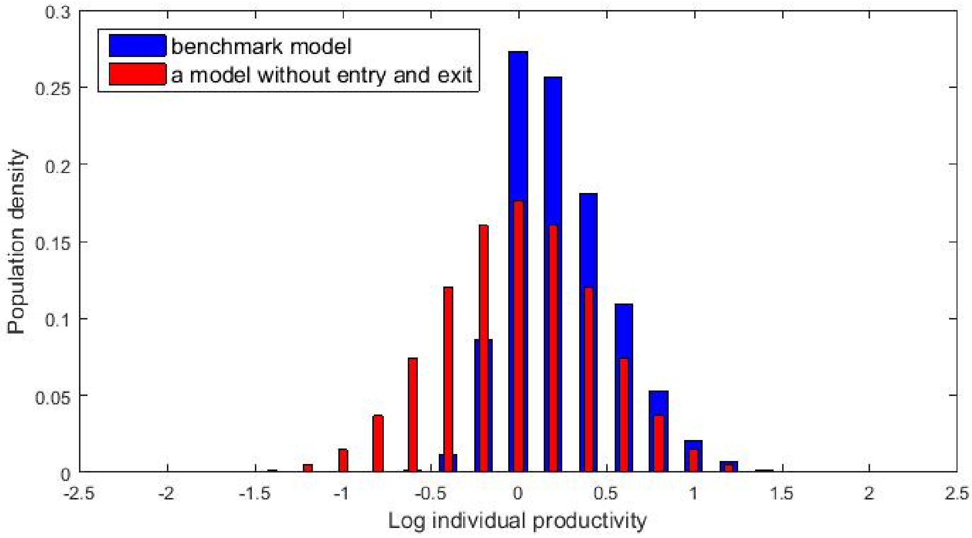


Figure 3.2: Effect of entry and exit on productivity distribution
Notes: The figure captures the steady-state distribution of plants over their individual productivity levels in the benchmark model economy (blue) and in the framework without the entry and exit of establishments (red).

Next, I shall comment on plants' optimal decision rules with respect to investment and exit. This exercise complements the theoretical analysis of the state space characterization of the optimal policies as presented in Section 3.3.6. In Figure 3.3 I display the individual state space split into disjunct regions, which differ from each other by the optimal policy for a given state. More specifically, a firm can choose either to exit or to remain operational. In the latter case, three different adjustments of firm's physical capital stock can take place, namely additional purchase (investment), sale (disinvestment), or inactivity, which is associated with capital depreciation. The patterns observed in the figure are intuitive. In particular, the least productive establishments leave the market for any level of physical capital, whereas those with higher levels of productivity remain in operation and invest. Moreover, the optimal policy changes from disinvestment to inactivity up to further investment as productivity increases, conditional on the stock of capital.

What is a little counterintuitive is the fact that the exit hazard is not monotonic in the level of physical capital conditional on productivity. More precisely, there is a range of productivities where firms with the highest capital stock decide to exit, whereas the ones with somewhat lower size remain active[27]. This feature is

[27]This is also a feature of alternative models with firm entry, exit, and heterogeneity in physical capital, e.g. Clementi et al. (2014).

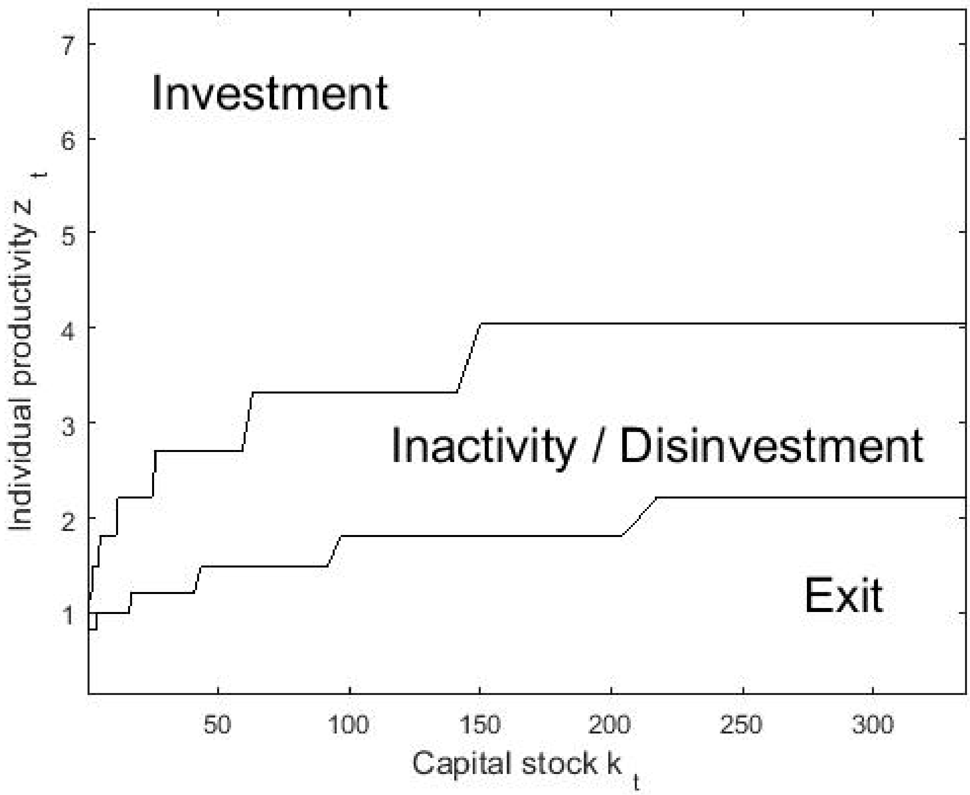


Figure 3.3: Optimal plant policy over the state space
Notes: The figure represents plants' optimal investment and exit policies over the space of their current individual states in the stationary equilibrium.

due to the presence of capital adjustment frictions, as the firms with an excessive stock of capital and insufficient productivity find it optimal to exit rather than to downsize and incur convex costs.

Next, I provide some intuition for the quantitative significance of the economic mechanisms that are my primary interest, in the context of the model. As discussed in the introduction, what lies at the heart of my business cycle analysis of firms' entry and exit is the distribution of business units (both entrants and incumbents) along the dimensions of exit and investment decisions. This is important because occasional synchronization of actions taken at the level of individual firms can lead to large changes in the aggregate outcomes. Figure 3.4 displays the equilibrium mass of firms, and the optimal policies for firms that currently possess the median stock of physical capital. It can be seen that the bulk of the businesses are concentrated at productivity levels just above the exit threshold. This indicates that in the case of a rightward shift of the optimal policy curve,

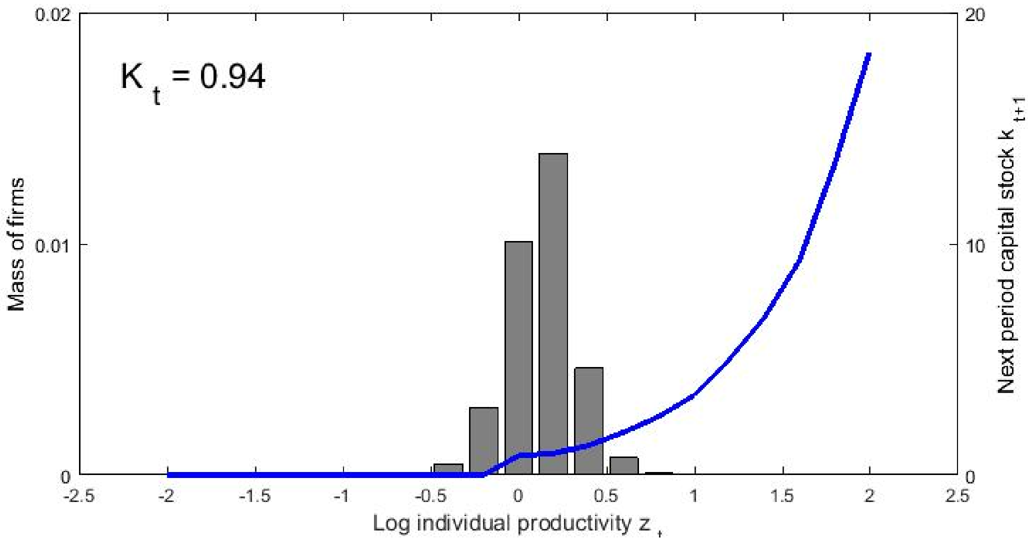
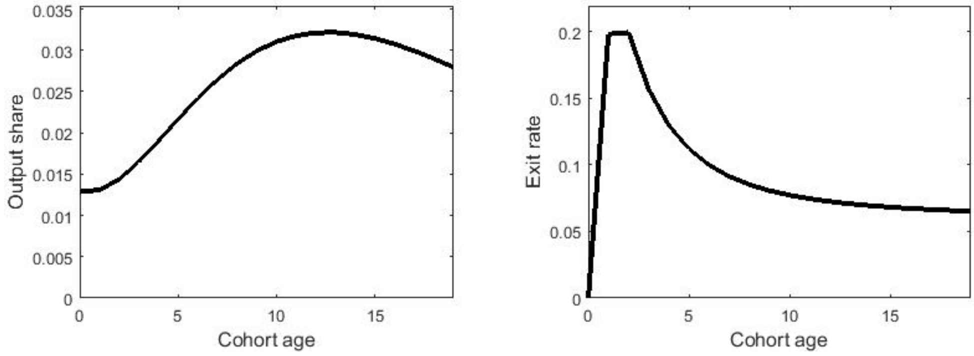


Figure 3.4: Distribution and optimal policies of firms with a median amount of capital stock
Notes: The figure represents the distribution of plants over individual productivity levels (gray bars) and the establishments' optimal exit and investment policies (blue line) in the stationary equilibrium at the median level of current physical capital stock.

as happens under a negative shock to the common productivity component, the number of exiting businesses would increase disproportionately, thus amplifying the drop in economic activity. On the other hand, a leftward shift would decrease the number of exits only moderately. However, a relatively large mass of firms would increase their stock of physical capital, which would boost the aggregate investment.

These observations carry potentially important implications for the dznamics of the business cycle. A relatively high mass of businesses switching into the exit state in the case of negative shock worsens the economic downturn. On the other hand, a comparable number of firms increases their investment in the case of a positive productivity shock. These movements have the potential to amplify the dynamics driven by fluctuations in aggregate productivity.

However, it is important to note that the patterns observed in the stationary equilibrium, or even in the model environment with aggregate fluctuations under partial equilibrium, will not necessarily appear in the dynamics in the full model. As observed in e.g. Khan and Thomas (2008), price movements in the general equilibrium analysis can quantitatively mitigate nonlinear behavior observed in settings where joint supply and demand side considerations are neglected. In any case, the observations above regarding responses to business shockes shows that my model has the potential to rigorously investigate the research question at hand.



(a) Population share

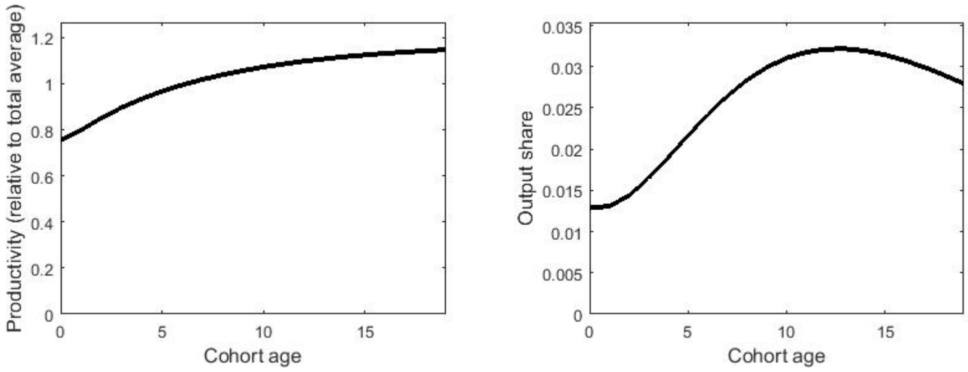
(b) Exit rate

Figure 3.5: Population and exit patterns of firm cohorts

Notes: The figure on the left-hand side represents the age profile of the total plant population share constituted by the establishments from a given age cohort. The figure on the right-hand side depicts the age profile of the exit rate of the plants from a given age cohort. Both patterns are observed in the stationary equilibrium.

Finally, let me present plants' dynamic behavior over their life-cycles. Figure 3.5a depicts how a cohort's share of the total business population varies with cohort age, and figure 3.5b shows cohort exit rates. It can be observed that the number of firms in a given cohort decreases over time, as in each year some of the business units quit production. However, the rate of this outflow, i.e. the exit rate, decreases over time, as the older businesses are on average more productive and thus less likely to find the exit decision optimal. This is captured by the growth of the mean productivity of the cohort, which is displayed relative to the economy-wide average in figure 3.6a. Although initially low, a cohort's mean level of productivity eventually grows above the total average.

These observations imply that a cohort's contribution to the aggregate economic activity is a non-monotonic function of its age. There are two forces at work: a decrease in the mass of firms in the cohort and an increase in their productivity. Figure 3.6b depicts the variation of a cohort's share of total output with its age. It is apparent that the productivity growth channel prevails in the early stage of a cohort's life, as the output share grows until ultimately peaking at 3.2% at the age of 14 years. Afterwards, the mass decrease channel begins to



(a) Relative productivity (b) Output share

Figure 3.6: Production and productivity patterns of firm cohorts

Notes: The figure on the left-hand side represents the age-profile of the mean idiosyncratic productivity of the plants from a given age cohort relative to the economy-wide average. The figure on the right-hand side shows the ratio of the output produced by the establishments in individual age cohorts to the aggregate output, plotted over the cohorts' life cycle. Both patterns are observed in the stationary equilibrium.

dominate and the output share falls, gradually approaching zero.

The observed patterns clearly point to a propagating effect of firm entry on the business cycle dynamics. The above-average number of entrant firms after a positive productivity shock translates into an even higher increase in the economic activity after several years, when the contribution of a given cohort is most pronounced. On the other hand, a drop in the common productivity level leads to *the missing generation effect*, where the loss due to absent entrants becomes especially evident once the cohort's output is at its highest. To summarize, the model exhibits life-cycle dynamics consistent with the empirical evidence, thus proving its suitability for addressing research questions concerning firm dynamics.

3.5.2 The model with aggregate fluctuations

Now I turn to outcomes of the model in an environment where the aggregate total factor productivity is subject to random shocks. In what follows, I first report the standard business cycle statistics for the benchmark model and compare them to the equivalent statistics from the model without entry and exit of firms. Next, I use the impulse-response functions to compare the dynamic behavior of these two frameworks. Finally, I link the observed persistence of responses to the life-cycle patterns of new firms. The numerical algorithm is described in detail in

Appendix 3.B.2.

My numerical solution follows a method developed by Krusell and Smith (1998), as I calculate the approximate equilibrium by assuming that agents track only the first moments of this distribution and use them to forecast both current and future prices. The approximate law of motion is estimated for the aggregate capital stock K_t and the current equilibrium price p_t. The R^2 values of the regressions are 0.9958 and 0.9998 respectively, which is somewhat lower accuracy than for the model without entry and exit, where the R^2 values are 0.9999. This indicates that some additional distribution moment could be added to improve accuracy. A prime candidate is the aggregate level of idiosyncratic productivity, which changes due to entry and exit of firms.

First, I characterize the equilibrium of the benchmark model using the statistics regularly used in the real business cycle literature. In Table 3.4, I provide an overview of the standard deviations of selected aggregate variables expressed relative to the standard deviations of the economy's output level. All the variables have been logged and detrended using the HP-filter prior to calculation of the statistics. The variabilities of the series generated by the two models are roughly similar to each other, with the important exception of aggregate investment. The relative standard deviation of investment in physical capital in the benchmark model is 15% higher than in the framework without entry and exit of firms. This difference indicates a non-negligible amplification effect of firms' adjustment along the extensive margin.

Table 3.4: Standard deviations of variables relative to output

	output	investment	consumption	employment	capital
Data[1]	(0.0144)	3.043	0.777	0.677	0.338
Benchmark Model[2]	(0.0127)	5.657	0.511	0.543	0.354
No Entry and Exit[3]	(0.0125)	4.884	0.508	0.508	0.326

Notes: All time series were logged and detrended by the HP-filter with a smoothing parameter of 6.25. Sources: 1 - own calculations using annual data for the US private sector from the Bureau of Economic Analysis (2013) and Bureau of Labor Statistics (2014) datasets, time period 1960-2013, detailed description available in Appendix 3.A; 2 - outcomes of the benchmark model; 3 - outcomes of the model without entry and exit of establishments.

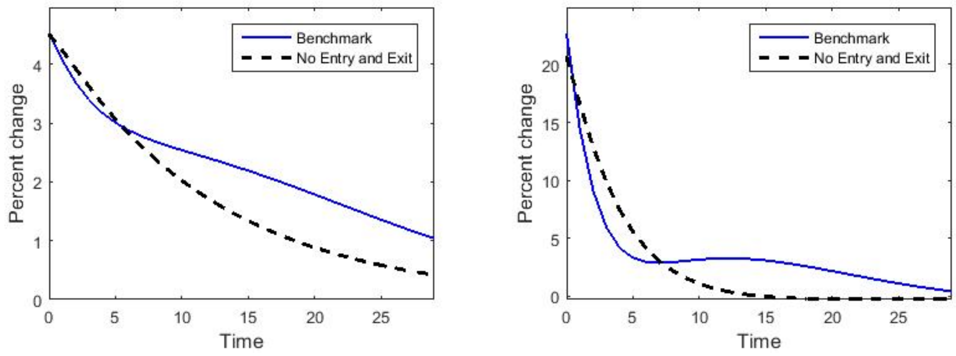
Table 3.5 displays contemporaneous correlations of the aggregateswith output. Here, again, the patterns are very similar across the frameworks, with the benchmark model exhibiting somewhat lower pro-cyclicality of the variables. The calculated statistics are roughly in line with the empirical data, except for the aggregate stock of physical capital, which exhibits a negative correlation with the aggregate output.

Table 3.5: Contemporaneous correlations with output

	investment	consumption	employment	capital
Data	0.878	0.959	0.853	0.352
Benchmark Model	0.962	0.945	0.952	−0.252
No Entry and Exit	0.985	0.984	0.984	−0.320

Notes: See the description of Table 3.4 above.

I shall now present the impulse responses of aggregates to a temporary positive shock to the level of aggregate productivity, and compare them with the equivalent statistics in the framework without entry and exit of firms. The size of the shock is set to one standard deviation. I start with the responses of aggregate output and investment, which are displayed in figures 3.7a and 3.7b. The responses of aggregate consumption and employment are presented in the Appendix, Figures 3.11a and 3.11b.



(a) Output (b) Investment

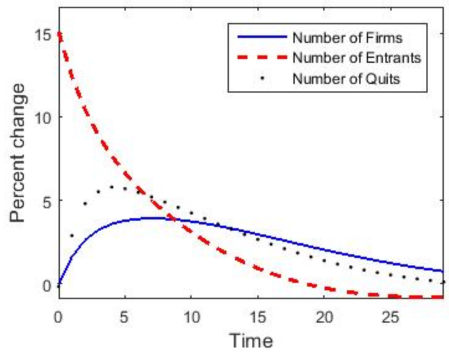
Figure 3.7: Responses of aggregates to a positive TFP shock

Notes: The impulse-responses are calculated as follows: Starting from stationary equilibrium, the aggregate TFP is increased one standard deviation above its steady-state level, to which it is subsequently allowed to return following an AR(1) process. The figure on the left-hand side depicts the responses of aggregate output. The figure on the right-hand side depics the responses of aggregate investment.

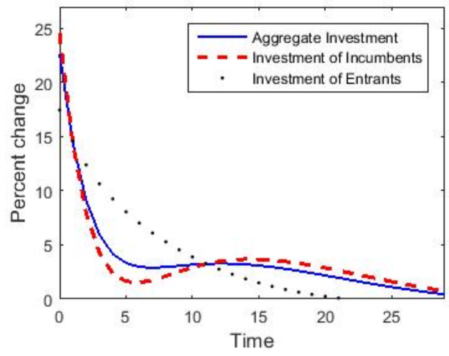
The magnitudes of immediate increases in levels of aggregate output are almost identical for both models. However, the responses diverge over time, as the benchmark model exhibits much higher persistence. For instance, in period 11 after the shock, the deviation of the output from its steady-state value is around 2% for the framework without entry and exit, whereas it is still above 2.5% for the full model. The difference persists long after the shock and peaks at a value of one percentage point in period 20. This superior propagation of the benchmark framework is indicative of an important effect generated by firms adjusting along the extensive margin, which will be confirmed later.

Next, Figure 3.7b shows the impulse response functions of aggregate investment. Again, the model with entry and exit of firms exhibits a hump-shaped pattern of deviations from the steady-state over time. The level of aggregate investment exhibits the strongest response among all the aggregates, increasing by more than 22.5% on impact relative to its steady-state value. This is similar to the framework without entry and exit, where the response is around 20.5%. The response observed in the model without firm turnover is relatively higher in the early periods. However, the deviations in the benchmark framework are larger from period 8 onwards, and the difference again persists long after the shock. Finally, as shown in the Appendix, Figures 3.11a and 3.11b, the impulse response functions of aggregate consumption and employment display qualitatively very similar patterns to the ones seen for aggregate output and investment.

Inspecting the impulse response functions reveals a substantial persistence of the aggregates' reaction to a positive TFP shock in the framework featuring firm turnover. In what follows, I examine the reasons for this increased persistence. I shall begin by reporting the dynamic patterns of firm entry and exit. Figure 3.8a shows the deviations in total number of firms, number of entrants, and number of exiting firms, from their steady-state values. The response of the total mass of firms exhibits a hump-shaped pattern, accumulating gradually after the shock and starting to decrease only once the number of exits exceeds that of entering firms. A qualitatively similar pattern can be observed for the response of the number of firm exits, while the mass of entrants decreases monotonically to its steady-state level. These observations indicate the important role of adjustment of the extensive margin of a firm. The higher magnitudes of entry than exit are concentrated early after the shock and thus provide for a substantial build-up of the total mass of firms. This result can explain the observed higher persistence of the aggegates, due to an increased number of firms taking part in economic activity.



(a) Number of Firms



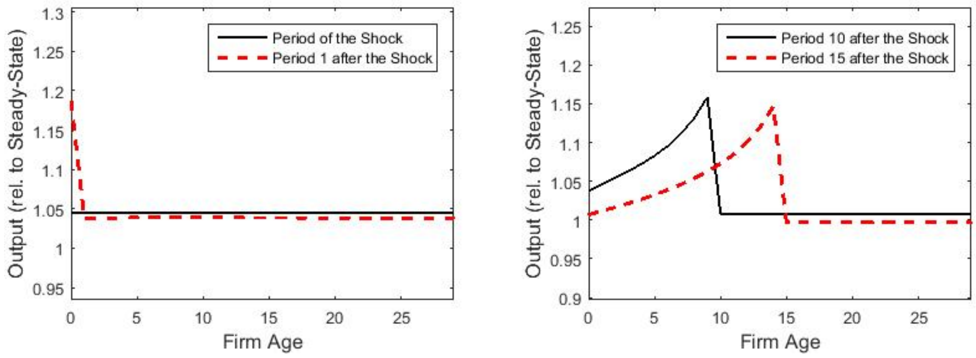
(b) Investment Decomposition

Figure 3.8: Responses to a positive TFP shock

Notes: The impulse-responses are calculated as in Figure 3.7. The figure on the left hand side depicts the responses of the numbers of all firms, entrants, and exiting firms. The figure on the right hand side decomposes the response of aggregate investment into that of investment by incumbents, and by new businesses only.

To assess the instantaneous effect of firm entry, I break down the response of aggregate investment into the fractions accounted for by entrants and older firms. This decomposition is shown in Figure 3.8b. Interestingly, the response of investment by incumbent firms exceeds that of new businesses. The level of investment made by older firms subsequently drops sharply, only to grow again in later periods. On the other hand, capital purchases made by entering firms decrease gradually to the steady-state value. As a result, the excess propagation in the economy is unlikely to be caused by investment realized by new firms on entering the market. This observation necessitates an assessment of firms' economic activity in later periods of their life-cycles.

In Figure 3.9 I plot the total output levels produced by firms of different age groups. These age-specific production levels are displayed for different periods of time elapsed since the shock, and are expressed relative to the corresponding steady-state values (in Appendix Figure 3.12 I present the values in absolute terms). Figure 3.9a shows that the immediate response of aggregate output is evenly distributed across all the age cohorts. Shock-induced additional entrants start to produce only in the first period after the shock (following the timing assumptions), thus bringing about a substantial increase in the output level of that particular age group. Interestingly, this excess production activity persists strongly over the cohort's life-cycle, as represented in Figure 3.9b. In particular, consider the output levels of firms that entered right after the shock, at the age of either ten or fifteen years. Both of these values remain 15% above the production levels of the corresponding cohorts in the stationary scenario. Moreover, firms that entered the market later, after the shock, also exhibit increased levels of total output relative to their steady-state counterparts. On the other hand, the total production of older cohorts is equal to the state of affairs without any shock. This examination of the model data implies that the observed positive deviation of aggregate output ten periods after the shock is entirely attributable to the activity of young cohorts that have entered after the shock.

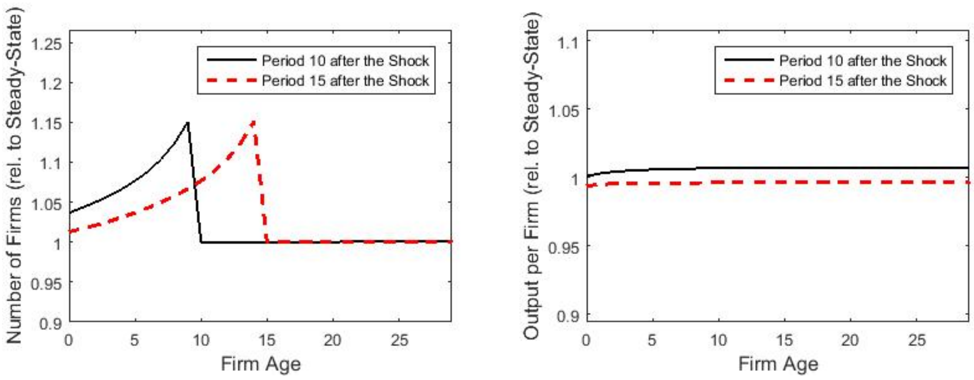


(a) Early periods following the shock (b) Later periods following the shock

Figure 3.9: Relative output of different age cohorts after a positive TFP shock

Notes: The figures represent output produced by firms of different ages in various periods after a positive TFP shock. The values are expressed relative to their steady-state counterparts. The figure on the left-hand side depicts the periods concurrent with and immediately after the shock. The figure on the right-hand side describes later periods.

The above examination linked increased persistence of aggregates in the model to the life-cycle growth of young cohorts, whose entry was affected by a positive TFP shock. In the final part of my analysis, I decompose the levels of total productionof a cohort into two parts: the number of firms of a specific age, shown in Figure 3.10a, and the average level of output per firm of a certain age, displayed in Figure 3.10b, both plotted against age of firm. Comparing these graphs reveals that a major part of the increased output levels observed for young firms in Figure 3.9b is accounted for by a high number of businesses in those age categories. On the other hand, the average output level of firms is not different from its counterpart in the stationary state. These observations allow me to characterize the business cycle propagation mechanism in my model driven by endogenous firm entry as follows. A positive shock to the aggregate productivity level induces an increased number of new firms to enter the market. As productivity reverts to its stationary level, the average level of firm production within these young cohorts becomes similar to that in an environment without the shock. However, the excess number of firms within these cohorts decreases only slowly, and thus provides for persistently high levels of total output.



(a) Number of firms (b) Average output

Figure 3.10: Decomposition of output of different cohorts after a positive TFP shock

Notes: The figure on the left-hand side depicts numbers of firms of different ages. The figure on the right-hand side shows the average levels of output for different age categories. Both of these series are shown in two different periods after the positive TFP shock, and are expressed relative to their steady-state counterparts.

To sum up, introducing firm entry and exit into the model increases slightly the magnitude of the fluctuations in aggregate investment, as indicated by its standard deviation, and provides a substantial boost to the persistence of aggregates' response to productivity shocks. This propagation is driven by a high number of entrants induced by a shock. These additional firms grow according to the prevalent life-cycle patterns in the model economy, and therefore contribute disproportionately to the total levels of production and investment. These outcomes also imply a substantial drag on the aggregate economic activity due to a missing generation of firms that decide not to enter during economic downturns, such as the Great Recession.

3.6 Conclusions

This chapter has analyzed the role of firms' entry and exit in the dynamics of the business cycle. It has emphasized the contribution of investment irreversibility, which affects businesses' decisions to start or shut down production. My analysis reveals that allowing for endogenous adjustments of firms along their extensive margins substantially increases the magnitude and propagation of the response of the aggregate variables to exogenous productivity shocks. An increase in the level of total factor productivity leads to an increase in the aggregate level of output, investment, and employment. The environment with entry and exit of firms shows a larger response than the situation where adjustment along the extensive margin of the firms is absent. These differences are persistent, as they propagate a long time after the shock has hit.

An important feature of my work is the interaction between firms' adjustment along their extensive margin and partial investment irreversibility. These two characteristics jointly determine the dynamic pattern of the firm-size distribution, which drives the main results of my work. However, friction to labor adjustment also determines a firm's decision of whether to enter, grow or exit. Firm dynamics are likely to be affected not only by the costs of adjusting labor *per se*, but also by possible interactions between frictions on capital and labor adjustment. My model currently features fully flexible labor markets only. I consider it a promising avenue for future research to analyze firm turnover in an environment featuring adjustment costs to both capital and labor. This is likely to further improve our understanding of the role of business turnover in the aggregate economic activity.

3.A Data

In the following section, I describe in detail the data sources and the collation of statistics that are used both to discipline the corresponding model-generated moments during the calibration process, and to assess the quantitative patterns of the model economy. I use the annual data for the U.S. private business sector, with the time horizon explicitly described for each data source individually.

3.A.1 Capital stock and production

I start with the data on the aggregate level of output, capital stock, investment and depreciation. This information is provided in the sections GDP & Personal Income (GDPPI) and Fixed Assets (FA) of the U.S. Economic Accounts database of the Bureau of Economic Analysis (2013) over the time period 1947-2013. I construct the relevant aggregate time-series as follows:

(i) Output for the business sector, which I denote by Y_t, can be obtained from the statistic *real gross value added* in GDPPI Table 1.3.6, line 2.

(ii) For the aggregate capital stock, I use data on nonresidential fixed assets and do not consider residential structures or consumption durables. (This is because my focus is on friction to capital reallocation related to firm operations, in which the latter arguably do not play a significant role.) I therefore use *the current-cost net stock of private nonresidential fixed assets* from FA Table 4.1, line 1. As these are expressed in nominal terms, I adjust them using *the chain-type quantity indexes* from FA Table 4.2, line 1, to yield the time-series of aggregate capital stock K_t.

(iii) The data on depreciation was obtained from *the current-cost depreciation of private nonresidential fixed assets* listed in FA Table 4.4, line 1. Again, this time-series needs to be expressed in real value using *the chain-type quantity indexes* from FA Table 4.5, line 1, which yields the time-series of aggregate level of depreciation D_t.

(iv) Finally, the data on nonresidential investment taken from *the investment in private nonresidential fixed assets* listed in FA Table 4.7, line 1, are converted to the real investment I_t using *the chain-type quantity indexes* from FA Table 4.8, line 1.

These time-series are in turn used to construct statistics for the calibration procedure. The capital-to-output ratio is defined as K_t/Y_t, and the value targeted in the calibration is a sample average over the period $1972-2013$. Similarly, I construct the aggregate investment rate and the depreciation rate as sample averages of the respective ratios I_t/K_{t-1} and D_t/K_{t-1}.

Regarding the empirical source of the labor force participation rate: the underlying dataset is the Labor Force Statistics of the Current Population Survey available from the Bureau of Labor Statistics (2014), which presents the *civilian labor force participation rate* at monthly frequency starting from 1948. I calculate the targeted value of this statistic (64.9%) as an average over the period $1972-2013$.

3.A.2 Establishment demography

I shall next describe the data characterizing establishment demography, i.e. entry, growth and exit of production units. The relevant information is available in the dataset of Business Dynamics Statistics (2013a), which provides data on the number of establishments, their entry/exit activity and job flows. This data can be also decomposed into categories according to their age and size characteristics. The sample covers the period 1979-2012 at annual frequency[28].

Let me denote the number of production units of age a in year t by $M_{a,t}$, the employment level of this cohort by $N_{a,t}$, the number of plants exiting in year t by M_t^{EX} and their total employment level by N_t^{EX}. Given this notation, I can calculate, for a given year t, entry and exit rates respectively defined as $EN_t = M_{0,t}/\sum_a M_{a,t}$ and $EX_t = M_t^{EX}/\sum_a M_{a,t}$, and obtain the target average turnover rate as 0.215, which is calculated as the sum of sample averages of entry and exit rates. Subsequently, I can derive the average establishment size as $\bar{N}_t = \sum_a N_{a,t}/\sum_a M_{a,t}$, the average size of an entering unit as $\bar{N}_{0,t} = N_{0,t}/M_{0,t}$, and the average size of an exiting plant as $\bar{N}_t^{EX} = N_t^{EX}/M_t^{EX}$. Consequently, the ratio of the average size of entering establishments relative to the average firm is given by $\bar{N}_{0,t}/\bar{N}_t$, and the ratio of the average size of exiting firms relative to the average plant size can be expressed as $\bar{N}_t^{EX}/\bar{N}_t$. The values targeted in the calibration—0.517 for entry and 0.486 for exit—are calculated as the sample averages of these ratios. The next calibration target is the ratio of the average firm sizes of the second-year cohort to those of the first-year cohort, which has a value of 1.214 and is calculated as $(N_{1,t}/M_{1,t})/(N_{0,t}/M_{0,t})$. The fraction of entering firms that exit during their first year of existence is therefore 19.75%. This statistic is calculated as the sample mean of year-specific exit rates of entrants defined as $1 - M_{1,t+1}/M_{0,t}$. The final BDS-based data moment targeted in the calibration is the standard deviation of entrants' job creation. This statistic is calculated from the logged and detrended time series $N_{0,t}$, where the HP-filter parameter is set to 6.25.

3.A.3 Establishment size distribution

Finally, my calibration procedure aims to discipline the size distribution of establishments generated by the model in a way that is consistent with the data. The data necessary for this can again be found in the dataset of Business Dynamics Statistics (2013a), where numbers and employment levels of production units are shown for distinct size categories. In table 3.6 I present the shares of both statistics across the different size categories, averaged over the entire sample period. For example, the 49.28% share of establishments with an employment level 1 to

[28]The entire available sample spans 1977-2012, however, I omit the first two years following the concern of Moscarini and Postel-Vinay (2012) regarding measurement error in those years.

4 workers indicates that, on average, this proportion of production units had 4 employees or less, on average, over the period $1979-2012$.

size class	$1-4$	$5-9$	$10-19$	$20-49$	$50-99$
establishments	0.4928	0.2236	0.1387	0.0903	0.0305
employment	0.0663	0.0883	0.1123	0.1647	0.1264
size class	$100-249$	$250-499$	$500-999$	$1000+$	
establishments	0.0170	0.0044	0.0017	0.0010	
employment	0.1526	0.0890	0.0687	0.1317	

Table 3.6: Fractions of production units falling into different size classes (averages over 1979-2012)

With respect to the calibration procedure, note that the employment level of a firm in my model is not a discrete variable. I therefore use data to calculate cumulative distributions in both statistics and compare these values against their equivalents from the model. Specifically, I order all the firms in my model economy according to their size, then evaluate the cumulative shares in both their number and labor force, and finally compare with the establishment shares for the employment percentiles corresponding to the available empirical observations, i.e. at 6.63^{th} percentile, 15.46^{th} percentile, etc.

Note that my assumption of normally distributed idiosyncratic productivity shock processes implies that it is not possible to match the long top tail of the empirical size distribution for any reasonable parameter values. However, I believe this does not affect my results significantly, as the main focus is on what proportions of plants find what entry and exit policies optimal, and these are arguably most relevant for the establishments at the bottom end of the size distribution.

3.B Numerical algorithm

In this part I shall outline the numerical routine that solves my model. It follows closely the computational approach used by Chang and Kim (2007) in being a model with heterogeneous households, incomplete asset market and indivisible labor supply, while also incorporating corrective comments on the former by Takahashi (2014). First, I shall describe an algorithm for the stationary equilibrium, which is used for calibration of parameters and as a starting point for the analysis of an economy with aggregate shocks. Next, I shall present the solving procedure for the model economy with fluctuations in aggregate productivity.

3.B.1 Stationary equilibrium

1. As a starting point, I initialize the grid points for an individual firm's capital stock k and idiosyncratic productivity z.

    - The discretization of capital $\{k^1, \ldots, k^{N_k}\}$ is equidistant in log scale and its range is extended sufficiently that, at equilibrium, firms do not reach the upper or lower boundaries. Subsequently, the number of grid points N_k is set so that depreciation spans one grid point[29]. In my benchmark calibration this amounts to $N_k = 160$ grid points between $k^1 = 0.213$ and $k^{160} = 3989.03$.
    - On the other hand, the stochastic process for z_t is approximated on a finite number ($N_z = 21$) of grid points $\{z^j\}_{j=1}^{N_z}$, where the values of the grid points and the transition probabilities $\pi_z(z'|z)$ are calculated using the Rouwenhorst algorithm as presented by Kopecky and Suen (2010).

2. Before calculating the value function and optimal policies, I make a guess at the value for the wage rate w. This variable will be later adjusted in order to pin down the free entry condition. At this stage I can remain agnostic about the value of the price p, because given its multiplicative nature it does not affect optimal policies and solely rescales the value function in the steady-state[30].

3. I compute the steady state value function and firms' optimal policies on each grid point. I first initialize the value function and then update it by iterating in time using firm optimal investment and exit policies and exogenous transition for productivity until convergence is achieved. To increase the speed of the computation, firms are allowed to re-optimize their policy functions only once every hundred time-iterations. I have checked that this simplification does not affect stationary equilibrium outcomes.

4. In the next step I use the steady-state value function and the distribution of entrants' productivity to calculate the optimal level of initial investment by new firms. The value function is interpolated over individual levels of capital using cubic splines. For the steady state, I assume that half of

[29] This means that a depreciation of the capital stock at a grid point k^j yields a value of capital k^{j-1}.

[30] Notice that this is no longer the case once I introduce aggregate uncertainty.

potential entrants are successful, and therefore the threshold level of entry cost is $C^E/2$. If the expected value of entry is not equal to zero (to within a given level of precision, 10^{-8}), I adjust the wage rate w accordingly[31] and the algorithm returns to step 3.

5. Using the optimal policies for investment and exit together with the exogenous transition probabilities for individual productivity, I construct the equilibrium transition matrix $T_{(z,k)}$ that characterizes transitions between the individual states (z, k).

6. I next compute the equilibrium distribution of firms. First, the *probability* distribution μ^* and the entry rate M are obtained by solving the fixed-point problem given by $\mu^* = T'_{(z,k)}\mu^* + M\mu^E$, where μ^E is the probability distribution of entrants over the state space. Subsequently I adjust the mass of firms, which rescales the distribution such that the aggregate labor demand matches the labor supply given by the targeted labor participation rate. Observe that the stationarity of the equilibrium, i.e. entry equals exit, and the targeted unit value for the mass of firms, together yield a value for the mass of potential entrants of $\mathcal{M} = 2 * M$.

7. Finally, the goods market clearing condition (3.19) yields the level of aggregate consumption and thus also the equilibrium price p, which is assumed to equal the average household's marginal utility of consumption. This, together with the equilibrium wage rate w, provides the household labor disutility parameter θ using the labor market clearing condition (3.18).

3.B.2 Equilibrium with aggregate fluctuations

In an economy with aggregate fluctuations, the distribution of firms μ_t becomes a state variable in each individual firm's problem. As this is an infinite-dimensional object, calculation of the equilibrium becomes numerically unfeasible. Therefore, following the method developed by Krusell and Smith (1998), I calculate the approximate equilibrium by assuming that agents track only the first moments of this distribution and use them to forecast both current and future prices. Note that a prediction of the contemporary prices is necessary because they are determined by the aggregate variables, which in turn depend on current actions of the

[31] In the case of a strictly positive (negative) expected value of entry I increase (decrease) the wage rate w.

agents.

The parametric law of motion for the aggregate capital stock K_t is assumed to take the form

$$\log K_{t+1} = b_0 + b_1 \log K_t + b_2 \log A_t, \tag{3.24}$$

and the current equilibrium price p_t is forecast by the rule

$$\log p_t = d_0 + d_1 \log K_t + d_2 \log A_t. \tag{3.25}$$

Note that the first moment of the second individual state variable, the idiosyncratic productivity, is constant by the law of large numbers, and is thus included in the constant terms b_0 and d_0. Moreover, I do not need to forecast the current wage rate w_t, because at equilibrium it is unambiguously determined by the price p_t through the households' first order condition (3.18).

Using this approximation, the algorithm goes as follows:

1. In the first step, I initialize grids for the state variables: each individual firm's capital stock k, idiosyncratic productivity z, aggregate capital stock K and aggregate productivity A. The grids for k and z are identical to those for the calculation of stationary equilibrium.

   - A level of aggregate capital K spans $N_K = 11$ grid points that are equally spaced on the interval $[0.75K^*, 1.25K^*]$, where K^* is the aggregate capital stock in the steady state. During the numerical simulations I make sure that the lower and upper bounds of this interval are never reached.

   - On the other hand, the stochastic process for the aggregate productivity A_t is approximated in a way similar to that of the idiosyncratic productivity process. In particular, there is a finite number ($N_A = 7$) of grids, logarithmically equally spaced, with the transition probabilities $\pi_A(A'|A)$ calculated by the Rouwenhorst algorithm.

2. Before the calculation of the equilibrium value function and optimal policies, I initialize the parameter values for the law of motion (3.24) and the price forecasting function (3.25).

3. Next, I compute the value function and firms' optimal policies for all the grid points. This step is similar to step 3 in the algorithm solving for the equilibrium of the model without aggregate shocks. The difference lies in

the fact that firms' optimization problem during the iterations in time is conditional on the assumed law of motion and forecast function.

4. In the subsequent step I simulate the behavior of the model economy over a large number of time-periods (5500) using the distribution of firms from the stationary equilibrium as a starting point. Note that I cannot use firms' optimal policies as derived in the value-function-iteration step, but need to solve for equilibrium in each step of the simulation. However, I use the value function calculated in step 3 to compute the expected continuation value, which is used in each firm's optimization problem at each step of the simulation[32] Each time period consists of the following sequence of steps:
   (a) At the beginning I observe the distribution of firms, the realization of shock to the aggregate productivity, and the current level of the aggregate capital stock.
   (b) Secondly, I use the transition law for the aggregate capital (3.24) to calculate the next period's aggregate stock of physical capital as perceived by agents in the economy.
   (c) Next, I use the value function obtained in step 3 to calculate the expected continuation value of firms at each individual state. I use linear interpolation over both the aggregate capital and the expected realization of aggregate productivity to allow for values not on the grid points.
   (d) Given the expected continuation value, I iterate over price to obtain clearing in the goods market. First, given an initial value of the price (and of the wage determined immediately through the households' first order condition) I solve the entry problem. Here I obtain the optimal level of entry investment and the threshold level of entry costs, which at the same time yields the current mass of entrants. Second, I solve the equations for incumbent firms and obtain the firms' optimal investment and exit policies. In both problems I use cubic spline interpolation so that firms can choose a level of physical capital not on the grid points. Third, the optimal policies of firms and entrants allow me to calculate the current values of the aggregate variables (output, consumption, investment, and costs). If the aggregate level of consumption does not

[32] See Section II in Takahashi (2014) for more detail.

equal the inverse value of the current price, I change the price and repeat the steps above until convergence is obtained.

(e) In the final step of the simulation, I update the distribution of firms using the current period's optimal investment and exit policies, the transition dynamics of idiosyncratic productivity, and introduction of entrant firms.

5. After the simulation procedure, I estimate the parameters in (3.24) and (3.25) by the ordinary least squarts (OLS) model on the simulated data, discarding the first 500 observations to eliminate the effect of initial conditions. If the estimated values are close to the previous ones, the equilibrium law of motion and forecast function have been found. Otherwise, I update the coefficients with the new estimates and return to step 3.

Lastly, I present the coefficient values in the approximating functions. The approximate law of motion for the aggregate capital stock K_t has form

$$\log K_{t+1} = 0.1151 + 0.8791 \log K_t + 0.3015 \log A_t,$$

and the current equilibrium price p_t is forecast by the rule

$$\log p_t = 0.3278 - 0.4229 \log K_t - 0.7254 \log A_t.$$

The R^2 values of the regressions are 0.9958 and 0.9998 respectively, which is somewhat lower accuracy than for the model without entry and exit, where the R^2 values are 0.9999. This indicates that some additional distribution moment could be added to improve accuracy. A prime candidate is the aggregate level of idiosyncratic productivity, which changes due to entry and exit of firms.

3.C The model without entry and exit

In order to make any statements on the importance of firm entry and exit for the business cycle dynamics, I need to construct a model without firm turnover, which serves as a reference point. In order to construct such an environment, I remove entry and exit decisions of firms from my benchmark model. In addition, the parameters governing exogenous exit δ, entry costs C^E, and fixed costs c^f are set to zero. This yields a fixed mass of firms, whose value is fixed to its full-model equivalent observed at the stationary equilibrium.

The numerical solution is done in a manner very similar to the algorithm used for the benchmark model and described in detail in part 3.B. However, there are certain differences that require clarification. First, in solving for the steady-state equilibrium, the iteration over the wage targets the clearing of the labor market instead of the free-entry condition (see point 4 in section 3.B.1). Second, in the solution of the model with aggregate fluctuations, each step of the simulation requires iterating over the equilibrium price in order to obtain the clearing of the good market. This is somewhat more computationally demanding than the model with firm entry and exit, as there the market clearing in the simulations is ensured by adjustment of the mass of entering firms (see point 4d in section 3.B.2).

I conclude with the specific values of the coefficients in the approximating functions. In both cases, the accuracy is very high, with R^2 well above 0.9999. The approximate law of motion for the aggregate capital stock K_t has the form

$$\log K_{t+1} = 0.1960 + 0.7877 \log K_t + 0.4203 \log A_t,$$

and the current equilibrium price p_t is forecast by the rule

$$\log p_t = 0.0816 - 0.3163 \log K_t - 0.7424 \log A_t.$$

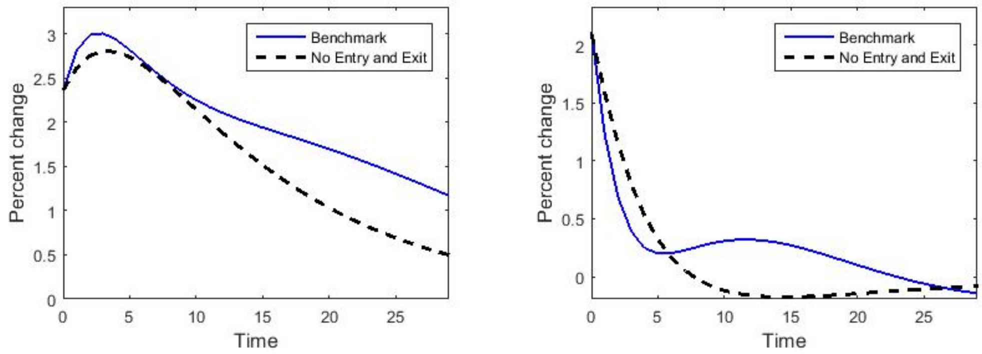
3.C.1 Additional numerical results

The following part contains additional figures characterizing the responses of an economy to a positive TFP shock. First, in Figure 3.11 I present the impulse responses of the levels of aggregate consumption and employment, for both the benchmark framework and the model without firm entry and exit.

Second, Figure 3.12 shows the total output levels produced by firms from different age groups. These age-specific production levels are displayed for various time periods that differ in time elapsed since the shock.

3.D Analytical results and proofs

Now I formally state and prove the results that underlie my characterization of a plant's optimal policies over the state space in the stationary equilibrium as presented in section 3.3.6. First, I provide several auxiliary results regarding the properties of individual components of the Bellman equation. Second, I prove the existence and uniqueness of the value function in the stationary equilibrium.



(a) Consumption (b) Employment

Figure 3.11: Responses of the aggregates to a positive TFP shock

Notes: The impulse-responses are calculated as in Figure 3.7. The figure on the left-hand side depicts the responses of aggregate consumption. The figure on the right-hand side depicts the responses of aggregate employment.

Finally, I characterize the plants' optimal policies with respect to the individual state space.

As a starting point I take the Bellman equation outlined in the expression (3.21). Specifically, the optimization problem of a plant with a capital stock k and a productivity level z has the following form:

$$V^0(z,k) = \tilde{\pi}(z,k) + \max\Big\{V^X(k), \max_{k'}\Big\{ - C^K(k',k) \ + \ \beta\mathbb{E}\big[V^0(z',k')|z\big]\Big\}\Big\}, \tag{3.26}$$

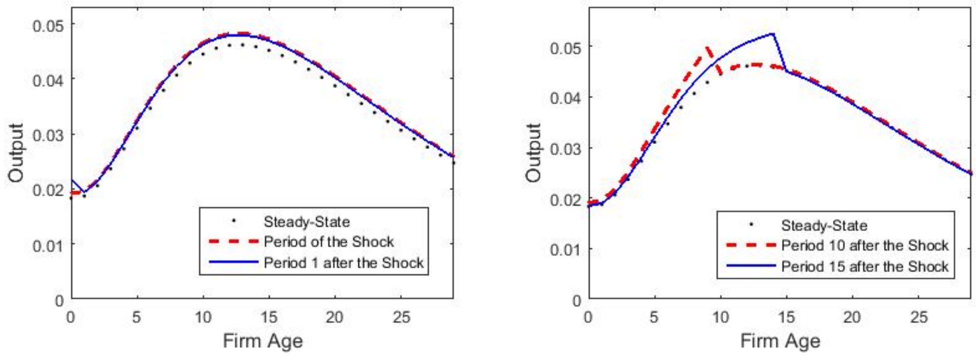
where $V^X(k)$ represents the exit value $V^X(k) = \kappa(1-\delta^k)k$, $\tilde{\pi}(z,k)$ denotes a plant's contemporaneous profit net of the capital adjustment as given in equation (3.22), and C^K characterizes any costs related to adjustment of a plant's stock of physical capital and is explicitly stated in equation (3.23).

Now I derive certain properties of the profit and costs functions with respect to the state variables. The proofs are a trivial algebraic exercise and therefore I skip them to keep the exposition smooth.

Lemma 1. *$\tilde{\pi}(z,k)$ is increasing and concave in k, and increasing and convex in z.*

In later derivations, it is convenient to use the explicit analytical expression for $\tilde{\pi}(z,k)$:

$$\tilde{\pi}(z,k) = \left(\eta^{\frac{\eta}{1-\eta}} - \eta^{\frac{1}{1-\eta}}\right) w^{\frac{-\eta}{1-\eta}} z^{\frac{1}{1-\eta}} k^{\frac{\alpha}{1-\eta}} \ - \ c^f. \tag{3.27}$$



(a) Early periods following the shock (b) Later periods following the shock

Figure 3.12: Output of different age cohorts after a positive TFP shock

Notes: The figures represent output produced by firms of different ages at various periods of time after a positive TFP shock. The figure on the left-hand side depicts periods at the same time as, or just after, the shock. The figure on the right-hand side depicts later periods.

Lemma 2. *(i) Consider any* $k' \in \mathbb{R}^+$. *If* $(1-\delta^K)\gamma \leq \kappa$, *then* $-C^K(k',k)$ *is strictly increasing in* k *for* $k \in \mathbb{R}^+$. *Otherwise,* $-C^K(k',k)$ *is strictly increasing in* k *for* $k \in (0, \frac{k'}{\sqrt{(1-\delta^K)(1-\delta^K-\frac{\kappa}{\gamma})}})$, *and strictly decreasing in* k *for* $k \in (\frac{k'}{\sqrt{(1-\delta^K)(1-\delta^K-\frac{\kappa}{\gamma})}}, \infty)$.
(ii) Consider any $k \in \mathbb{R}^+$. *Then* $-C^K(k',k)$ *is negative for* $k' \in (0,(1-\delta^K-\frac{\kappa}{\gamma})k) \cup ((1-\delta^K)k, \infty)$, *positive for* $k' \in ((1-\delta^K-\frac{\kappa}{\gamma})k, (1-\delta^K)k)$, *strictly increasing in* k' *on* $(0,(1-\delta^K-\frac{\kappa}{2\gamma})k)$, *and strictly decreasing in* k' *on* $((1-\delta^K-\frac{\kappa}{2\gamma})k, \infty)$.

Now I move to the second part of this section, where I prove that a plant's optimization problem, as outlined in the Bellman equation (3.26), has a solution, which is unique. Subsequently, I prove several properties of this value function, which allow me to later to characterize a plant's optimal policies over the state space.

Proposition 1. *Consider the recursive optimization problem of a plant given by* (3.26). *There exists a unique value function* $V^0(z,k)$ *that solves it.*

Proof. First, I reformulate the Bellman equation in a way consistent with the propositions derived in Stokey et al. (1989). Let me introduce a binary variable x, which explicitly represents a plant's exit/survival decision. Specifically, $x = 1$ means that an establishment has decided to remain operational, whereas $x = 0$

indicates exit. Then the equation (3.26) can be equivalently rewritten as follows:

$$\begin{aligned}\tilde{V}^0(z,(k,1)) = &\max_{(k',x)\in\mathbb{R}^+\times\{0,1\}} \Big\{\tilde{\pi}(z,k) \;+\; (1-x+x\delta)V^X(k) \;- \\ &-\; x(1-\delta)C^K(k',k) \;+\; x(1-\delta)\beta\mathbb{E}\big[\tilde{V}^0(z',(k',x))|z\big]\Big\}, \\ \tilde{V}^0(z,(k,0)) = &\;0.\end{aligned} \tag{3.28}$$

Now I can apply Theorem 9.12 from Stokey et al. (1989) to prove the existence and uniqueness of $\tilde{V}^0(z,(k,x))$ (and thus also of $V^0(z,k)$). Assumption 9.1 holds trivially. Assumption 9.2 is ensured by the finite expected value of z and the outside option provided by a plant's exit. Conditions (a) and (b) are also ensured by the finite expected value of z together with the decreasing returns-to-scale production function in capital (see Lemma 1) and the convex capital adjustment costs. Under these conditions, Theorem 9.12 from Stokey et al. (1989) implies the existence of a unique value function $\tilde{V}^0(z,(k,x))$ solving equation (3.28), which in turn provides a unique solution $V^0(z,k)$ to the equation (3.26). □

After establishing the existence and uniqueness of the value function, I prove its several properties, which will be useful for the characterization of a plant's optimal policies. In these proofs I follow the approach taken by Dixit (1997). First, the value function V^0 is shown to be a fixed point of a contraction mapping $\mathcal{M}:\mathcal{V}\to\mathcal{V}$ defined on a complete functional space of continuous functions $\mathcal{V}$ as follows:

$$\mathcal{M}V(z,k) = \max_{k'}\Big\{\tilde{\pi}(z,k)+\max\Big\{V^X(k),\delta V^X(k)-(1-\delta)C^K(k',k)+(1-\delta)\beta\mathbb{E}\big[V(z',k')|z\big]\Big\}\Big\}.$$

Secondly, I demonstrate that $\mathcal{M}$ maps a complete subspace $\mathcal{V}_1\subseteq\mathcal{V}$ onto itself, which implies (via the contraction-mapping theorem) that the fixed point lies in the subspace $\mathcal{V}_1$. However, V^0 is the unique fixed point in $\mathcal{V}$ and therefore it has to be $V^0\in\mathcal{V}_1$.

Proposition 2. *$V^0(z,k)$ is continuous and is increasing in z. Moreover, if $(1-\delta^K)\gamma\leq\kappa$, then $V^0(z,k)$ is increasing in k.*

Proof. Continuity is a trivial consequence of V^0 being the fixed point of $\mathcal{M}$ on $\mathcal{V}$ as outlined above, since $\mathcal{V}$ is the space of continuous functions. Moreover, as $\tilde{\pi}(z,k)$ is strictly increasing in z, $\mathcal{M}$ maps a closed subspace $\mathcal{V}_1$ of functions increasing in z into itself, which yields the desired result. Finally, consider monotonicity with respect to k. Both $\tilde{\pi}(z,k)$ and $V^X(k)$ are strictly increasing in k. At the same time, $(1-\delta^K)\gamma \leq \kappa$ is sufficient for $-C^K(k',k)$ to be increasing in k; therefore I obtain for $k_1 < k_2$

$$\max_{k'}\Big\{-C^K(k',k_2) \ + \ \beta\mathbb{E}\big[V^0(z',k')|z\big]\Big\} \geq -C^K(k_1^*,k_2) \ + \ \beta\mathbb{E}\big[V^0(z',k_1^*)|z\big] >$$
$$> -C^K(k_1^*,k_1) \ + \ \beta\mathbb{E}\big[V^0(z',k_1^*)|z\big],$$

where $k_1^* = \arg\max_{k'}\Big\{-C^K(k',k_1) \ + \ \beta\mathbb{E}\big[V^0(z',k')|z\big]\Big\}$. Repeating the argument about $\mathcal{M}$ mapping a subspace $\mathcal{V}_1$ of functions increasing in k onto itself concludes the proof. □

Strict monotonicity of $V^0(z,k)$ with respect to z and k allows me to prove two corollaries, which will be useful later on.

Corollary 1. *Function* $W(z,k) := \max_{k'}\Big\{-C^K(k',k) \ + \ \beta\mathbb{E}\big[V^0(z',k')|z\big]\Big\}$ *is strictly increasing in* z.

Proof. Consider k, $z_1 < z_2$ and denote $k_1^* := \arg\max_{k'}\Big\{-C^K(k',k) + \beta\mathbb{E}\big[V^0(z',k')|z_1\big]\Big\}$. Observe that the assumed process (AR(1) in logs) of individual productivity is monotone (see Exercise 12.11 in Stokey et al. (1989)) and thus $z_1 < z_2$ imply an inequality $\mathbb{E}\big[V^0(z',k')|z_1\big] < \mathbb{E}\big[V^0(z',k')|z_2\big]$. Using this, simple algebraic steps yield $W(z_2,k) \geq -C^K(k_1^*,k) \ + \ \beta\mathbb{E}\big[V^0(k_1^*,k')|z_2\big] > -C^K(k_1^*,k) \ + \ \beta\mathbb{E}\big[V^0(k_1^*,k')|z_1\big] = W(z_1,k)$. □

Corollary 2. *If* $(1-\delta^K)\gamma \leq \kappa$, *then function* $\mathbb{E}\big[V^0(z',k)|z\big]$ *is strictly increasing in* k.

Proof. Consider z and $k_1 < k_2$. Proposition 2 yields $V^0(z',k_2) - V^0(z',k_1) > 0, \ \forall z'$. However, the mean value of a strictly positive function is necessarily positive. □

Finally, I present and prove the characterization of a plant's optimal policies with respect to its individual state. Consistently with the notation introduced in Section 3.3, $\chi(z,k)$ represents the exit/survival decision of a plant with a productivity level z and current capital k as defined in equation (3.8). Specifically, $\chi(z,k) = 1$ if an establishment decides to remain operational, whereas $\chi(z,k) = 0$ in case of exit. Additionally, $k^*(z,k)$ denotes the optimal choice of the next period capital stock.

I start with the proof of existence of the exit cut-off level of productivity as a function of the current capital stock. More precisely, for each plant there is a well-defined threshold in the productivity, such that a shock that decreases the TFP level below this boundary induces a production unit to quit operation.

Proposition 3. *For any k there exists a unique $\tilde{z}(k) > 0$ such that:*
(i) $\max_{k'} \Big\{ -C^K(k', k) \ + \ \beta \mathbb{E}\big[V^0(z', k') | \tilde{z}(k)\big] \Big\} = \kappa(1 - \delta^k)k$*;*
(ii) $\forall z < \tilde{z}(k)$ $\chi(z, k) = 0$ *and* $\forall z > \tilde{z}(k)$ $\chi(z, k) = 1$*.*

Proof. Consider function $W(z, k)$ as defined in Corollary 1. I proceed by showing that: (a) $\lim\limits_{z \to +\infty} W(z, k) = +\infty$; and (b) $\lim\limits_{z \to 0^-} W(z, k) < \kappa(1 - \delta^k)k$. These properties together with the result from Corollary 1 and a continuity of $W(z, k)$ with respect to z yield the existence and uniqueness of the intersection point $\tilde{z}(k) > 0$ as well as the optimality of the exit (survival) for the productivity levels below (above) it.
(a) First, observe that $W(z, k) \geq \beta \mathbb{E}\big[V^0(z', (1 - \delta^K)k) | z\big]$. Following equation (3.27), $\lim\limits_{z \to +\infty} \tilde{\pi}(z, k) = +\infty$, which implies $\lim\limits_{z \to +\infty} V^0(z, (1 - \delta^K)k) = +\infty$. Subsequently, the persistent Markov structure of the productivity process z yields $\lim\limits_{z \to +\infty} \mathbb{E}\big[V^0(z', (1 - \delta^K)k) | z\big] = +\infty$, which finally implies $\lim\limits_{z \to +\infty} W(z, k) = +\infty$.
(b) Observe that $\lim\limits_{z \to 0} \tilde{\pi}(z, k) = -c^f$ from equation (3.27), and therefore $\lim\limits_{z \to 0} \mathbb{E}\big[\tilde{\pi}(z', k) | z\big] < 0$. Then for every level of the capital stock $\bar{k}$ I can choose the productivity level $\bar{z}$ low enough, so that (starting from the individual state $(\bar{z}, \bar{k})$) conditional on survival a plant generates a negative stream of contemporaneous profits for a sufficiently long time unless the unit always disinvests. However, eventually there is no capital to sell and therefore negative profits prevail. Moreover, disinvestment is associated with an additional convex cost of adjustment relative to exiting. As a result, it is not optimal for a plant to remain operational at the very first decision point, thus $\lim\limits_{z \to 0^-} W(z, k) < \kappa(1 - \delta^k)k$. □

4 The joint adjustment of production factors by heterogeneous plants[33]

4.1 Introduction

The seminal work by Nadiri and Rosen (1969) was the first one to study theoretically the mutual interdependence of plants' demands for capital and labor. Over the last decade, a number of papers (e.g. Sakellaris (2004), Letterie et al. (2004) and Nilsen et al. (2009)) have used plant-level data sets from different countries to document plants' actual factor adjustment before, during, and after periods characterized by infrequent and large, or "lumpy", investment or employment adjustment. A common observation is that, averaged across all plants, abrupt changes in one production factor are *on average* associated with simultaneous adjustments of the other factor in the same direction. For instance, a spike in investment (i.e. a period when the rate of investment exceeds 20%, as defined in Sakellaris (2004)) is on average accompanied by an increase in the level of employment.

Recently, more detailed information has been provided by Letterie et al. (2004), Asphjell et al. (2014) and Hawkins et al. (2015), which suggests that the average positive co-movement hides substantial underlying heterogeneity in factor adjustment practices across individual plants. Most strikingly, for a significant fraction of establishments, the episodes of positive capital investment are associated with a reduction in gross hiring and sometimes even with negative employment growth. The observed proportions of such events (relative to the total episodes of positive investment) range from 24% in Norwegian manufacturing (as documented by Asphjell et al. (2014)) to 36% in Korean manufacturing (Hawkins et al. (2015)).

In this chapter we study the cross-sectional variation in plants' adjustment rates of workforce and capital. Our focus is on episodes when plants simultaneously adjust the two production factors *in the opposite directions* and on the quantitative implication of such behavior for the aggregate economy. We ask how likely plants are to invest in physical capital and decrease their employment level

[33]This chapter is joint work with Monika Gehrig-Merz. An earlier version of the paper appeared as a working paper in Gehrig-Merz and Majher (2019).

at the same time. Subsequently, we quantify the significance of such plants for aggregate employment growth and capital accumulation.

A number of plants investing and laying off workers at the same time is a robust feature of the empirical data. This observation is inconsistent with much of the existing literature on plant dynamics, as models therein assume strict factor complementarity, which would generate factor adjustments that occur simultaneously and go in the same direction. In our model, a plant's simultaneous investment and job destruction results from varying elasticity of factor substitution. We propose a framework where a fraction of plants produces using Leontief production technology, thus keeping their capital-labor ratio intact. This model feature represents any rigidities, which are encountered by plants while changing the capital intensity of their production.

Understanding plants' decisions to simultaneously invest in physical capital and lay-off employees is important, as such behavior is linked to observable dynamics of production factors at the aggregate level. In particular, an increase in a number of plants that at the same time invest and separate from workers yields positive aggregate investment associated with negative growth of aggregate employment, which is a phenomenon frequently observed in recent decades. Our work is well-suited to assessing the contribution of patterns seen at the level of individual plant to these aggregate outcomes, as it uses a quantitative model of an industry with a well defined optimization problem at the micro-level.

In our analysis, we use a quantitative model calibrated to match selected empirical moments documented for the US manufacturing sector. We start from the heterogeneous-plants model of Hawkins et al. (2015), who use a production function with a varying elasticity of factor substitution to explain factor adjustment patterns at the plant level; simultaneous investment and reductions in employment appear in this model. We augment this model by assuming that a fraction of plants produces according to a Leontief production technology, where adjustments preserve the capital-labor ratio[34].

We solve the model numerically by the value function iteration method and calibrate it to plant-level data from the US manufacturing. Our calibration procedure draws on the approach used in Bayer et al. (2015) and targets the empirical average factor substitutability and the average employment growth rate during investment spike periods (documented by Sakellaris (2004)). These data moments are used to identify parameters characterizing the distribution of shocks to the

[34] this feature is analogous to the model by Bayer et al. (2015).

elasticity of factor substitution. We are subsequently able to carry out a model-based evaluation of the *joint* distribution of factor adjustments.

Our simulation results show that, on an annual basis, more than 18% of plants increase their capital stock and separate from workers at the same time. Moreover, two thirds of plants with positive investment and negative employment growth invest at a rate above 10%. These results indicate that the frequency of episodes in which a plant undertakes large investments while reducing workforce is substantial in the US manufacturing sector, thus resembling patterns documented for other countries[35]. The large proportion of plants for which this pattern is observed suggests that these episodes affect the aggregate economy. Indeed, plants separating from workers and investing at a rate of over 10% contribute 38% of the aggregate investment level.

The variations between plants in labor adjustments during periods of heightened investment carry important consequences for plant growth as well as for aggregate outcomes. As plants are able to reorganize their production process in response to shocks, and thus effectively adjust their factors in opposite directions, they take this into account when planning the timing of factor adjustments. Randomly varying factor substitutability serves as an alternative to an *ad hoc* specification to the canonical model (a framework featuring the Cobb-Douglas production function) as in Asphjell et al. (2014), where a negative joint term is included in the adjustment cost function. At the same time, plants have the possibility to increase output production without necessarily hiring new workers. If a substantial number of plants selects this adjustment regime, it is possible to observe an episode of high aggregate investment associated with a low level of aggregate job creation.

Our work contributes to a major strand of the literature featuring dynamic stochastic models with heterogeneous firms that originates in the work of Hopenhayn (1992). Within this field, Bloom (2009) is the first one to allow for the joint *frictional* adjustment of capital and labor when assessing the role of uncertainty shocks for dynamics of aggregate variables. The most recent contributions are the papers by Hawkins et al. (2015) and Bayer et al. (2015). Hawkins et al. (2015) aim to capture in their model themoments of the joint factor adjustment distributions observed in Korean manufacturing. Bayer et al. (2015) assess the determinants of differences in plant-level factor productivities between countries.

[35]See Asphjell et al. (2014) for data on Norwegian manufacturing and Hawkins et al. (2015) for statistics regarding Korean manufacturing.

Furthermore, our work may be linkedto studies that analyze patterns of jointly adjusted production factors either theoretically or empirically. The seminal paper by Dixit and Stiglitz (1977) provides a model framework for assessing the multi-dimensional optimization problem of a firm with frictional changes in the factors' demand levels. Merz and Yashiv (2007) study the dynamic behavior of the aggregate investment and employment in a framework featuring interactions between capital adjustment costs and labor adjustment costs. Empirical data is provided by studies by Letterie et al. (2004), Sakellaris (2004) and Asphjell et al. (2014) using plant-level data from different countries (the Netherlands, the United States, Norway) to provide information on patterns of factor adjustment observed at the micro level.

The remainder of this chapter is structured as follows. Section 4.2 contains an overview of the relevant empirical facts. Section 4.3 presents the setup of the model economy. Section 4.4 describes the calibration procedure and parameter values, and section 4.5 gives and overview of the numerical results of the model solution. Section 4.6 summarized and concludes the chapter.

4.2 Empirical Facts

In recent years, empirical firm- and plant-level data have become increasingly available, thus enabling economists to document key aspects of plant dynamics at the micro-level. This empirical evidence has helped to assess and improve existing theories that incorporate the behavior of heterogeneous plants in an aggregate context. Some data limitations have persisted, as the vast majority of statistics have been documented only for manufacturing (either for the United States or for other countries, such as Norway and Korea). This sector, however, still generates a substantial share of total economic activity despite the decline in its relative importance over recent decades.

The five most notable stylized facts to emerge from these data are summarized below (Table 4.1 provides a comprehensive summary):

(i) Recent evidence suggests that the joint occurrence of a positive investment rate and a negative employment growth rate is a robust feature of the data. Letterie et al. (2004) point out that, for a sample of Dutch manufacturing plants, more than 6% of investment spike episodes are associated with significant job losses - employment growth rates of -10% or below, even though the average employment growth rate for these observations is 5%. Asphjell et al. (2014) show that 24% of plant-year observations exhibit both a positive level of investment and a negative change in employment. Moreover, over 35% of positive investment events are associated with net destruction of jobs. A similar pattern is shown in a study by Hawkins et al. (2015), where 36% of observations characterized by an investment rate above 10% also exhibit a decrease in the employment level[36].

(ii) At any point in time, there is substantial cross-firm and cross-plant heterogeneity in rates of investment, hiring and separations. Cooper and Haltiwanger (2006) find a value of 33.7% for the standard deviation of annual investment rates for a sample of US manufacturing plants observed between 1972 and 1988. [37]. Davis et al. (2010) calculate a value of 48% for the average cross-sectional dispersion of annual employment growth rates for US manufacturing establishments. For non-US countries, Asphjell et al. (2014) document, for a representative sample of non-defaulting Norwegian manufacturing plants, values of 16.9% and 17.7% for the standard deviations of investment rates and employment growth rates, respectively; while Hawkins et al. (2015) report analogous statistics of 22% and 50.5% for Korean manufacturing establishments with at least 10 workers.

(iii) Firms' levels of both physical capital and employment are frequently adjusted by spikes in investment or hiring. Cooper and Haltiwanger (2006) show that 18.6% of plant-year observations exhibit investment rates above the value of 20%. Davis et al. (2010) document averages of 11.7% and 12.1% for the annual rates of job creation and job destruction, respectively. Examining different samples of US plants, Gourio and Kashyap (2007) show that almost 20% of observations with positive plant-year investment rates display rates greater than 20%. In the sample of Korean plants surveyed by Hawkins et al. (2015), 22.5% of observations are associated with an investment rate above 20%, while 15.3% of observations are associated with an employment growth rate above 20%.

(iv) Periods of adjustment inactivity are common. Cooper and Haltiwanger (2006) report that 8% of observations are associated with investment rates below 1% in absolute value. Asphjell et al. (2014) document capital and

[36]This statistic remains high, above 34%, when considering observations with investment rates greater than 20%.

[37]For the record, the mean value of annual investment rates is 12.2%

employment adjustment rates below 2% in absolute value for 30% and 21% of plant-year observations, respectively. Hawkins et al. (2015) detect no adjustment of capital for 48% of observations, while the equivalent value for labor adjustment is 14%.

(v) A close examination of empirical patterns of factor dynamics reveals substantial asymmetry in plant-level adjustments of physical capital. Cooper and Haltiwanger (2006) report 10% of observations as having negative rates of investment, as opposed to 81.5% with positive rates. Asphjell et al. (2014) document that 2% of events are characterized by negative investment, which is disproportionately few relative to the 68% of events characterized by positive investment. In a similar vein, Hawkins et al. (2015) show these percentages in their data to stand at 4.5% for negative investment and 48% for positive investment. Such asymmetry is not observed for labor adjustments. Both Asphjell et al. (2014) and Hawkins et al. (2015) report roughly equal counts of observations with positive and negative changes in employment.

Table 4.1: Summary of empirical data

Study	Country	Investment rate			
		St.dev.	Pos. spike [%]	Inaction [%]	Negative [%]
CH	USA	0.337	18.6	8.1	10.4
GK	USA		20	15	
ASP	Norway	0.171		30	3.0
HMO	Korea	0.505	22.5	47.8	4.4
		Empl. growth rate			$\frac{I}{K} > 0.1$, $\frac{\Delta L}{L} < 0$
		St.dev.	Pos. spike [%]	Inaction [%]	Share [%]
DAV	USA	0.48			
ASP	Norway	0.169		21.3	24.3*
HMO	Korea	0.220	15.3	14.1	36.0

Notes: CH - Cooper and Haltiwanger (2006), DAV - Davis et al. (2010), GK - Gourio and Kashyap (2007), ASP - Asphjell et al. (2014), HMO - Hawkins et al. (2015).
$*$ - Documented share is for $I/K > 0$, $\Delta L < 0$.

The stylized fact (i) is the most noteworth, as it has not been sufficiently taken into account by much of the existing literature on plant dynamics. It is inconsistent with assuming strict factor complementarity, which would generate factor adjustments that occur simultaneously and go in the same direction[38]. Although being a suitable approximation in the aggregate, this simplification does

[38]Notable examples are Dixit (1997) or Bloom (2009).

not capture the substantial amount of heterogeneity in plants' factor adjustment patterns.

In what follows, we will develop a model to address the aggregate implications for investment and employment changes of the heterogeneity in cross-sectional factor adjustment by plants. In order to generate a non-negligible number of plants jointly investing and laying off workers, we deviate from strict factor complementarity of the production function. In our model economy, output is produced by a production technology with varying elasticity of substitution, and some plants produce by the Leontief production technology. To provide an intuition for this assumption, we suppose that plants face rigidities when changing capital intensity of their production process, and, as a result, may not change the capital-labor ratio in each period. The feature of Leontief technology, together with labor-augmenting productivity shocks, yields plants investing and separating from workers at the scale comparable to observations in the empirical data.

Obviously, our approach is not the only way to obtain desired patterns of plants' capital and labor adjustments. An alternative way to model plants that invest and separate from workers would be to introduce lags to investment becoming productive. A plant expecting its productivity level to improve in next period would invest in capital stock while also decreasing its workforce. However, matching both the number of plants that invest and lay-off workers, and the magnitude of factor adjustments by these plants relative to the aggregate flows, would most likely require a huge variation in volatility for the plant-level productivity process.

4.3 A Benchmark Model

In our benchmark framework we augment the model developed in Hawkins et al. (2015) with a Leontief production function while abstracting from endogenous technology choice. The model is set in partial equilibrium to facilitate quantitative evaluation. Heterogeneous plants are the only agents present in the economy. They own physical capital, employ workers, and use these two factors to produce output. A plant can adjust the level of these production factors, subject to adjustment costs. At the same time, each producer faces uncertainty regarding the production technology it will use in a given period. A plant observes its assigned technology type, which is characterized by the elasticity of factor substitution, before adjusting the levels of the production factors. Finally, there are two exogenous sources of cross-plant heterogeneity, namely a factor-neutral demand shifter and labor-augmenting productivity, each of which is subject to idiosyncratic shocks.

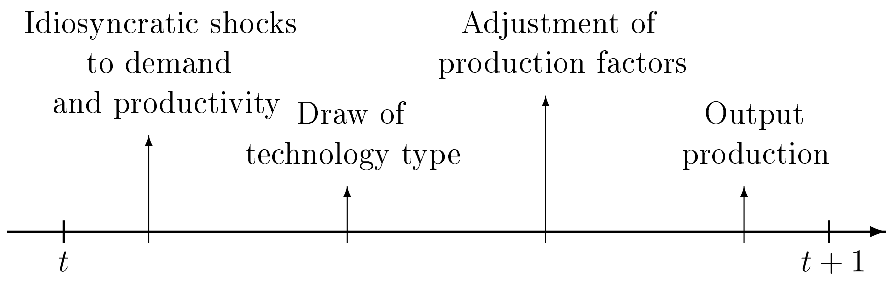


Figure 4.1: Timing of actions

The sequence of plants' actions in the model is represented in Figure 4.1. At the beginning of each period, shocks to both the level of demand for a plant's output and the idiosyncratic productivity component are realized. Next, plants observe the technology type assigned to them, which characterizes their production process. Subsequently, they invest in physical capital and hire or fire workers subject to capital and labor adjustment costs. Finally, plants produce output and pay wages to their employees.

4.3.1 Production Technology

Our model features an infinite time horizon with each period indexed by a discrete time index t. There is a continuum of plants of (in total) unit mass. Each plant produces output following the production technology that is randomly assigned in a given period. There are two types of production function available: (i) a CES production function (h)

$$Y_t^h = \left[\alpha K_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(A_t L_t)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{4.1}$$

and (ii) a Leontief production function (ℓ)

$$Y_t^\ell = \min\left\{K_t, A_t L_t\right\}, \tag{4.2}$$

where K_t and L_t denote, respectively, the stocks of physical capital and labor, $\sigma \geq 0$ is the elasticity of substitution parameter, $\alpha \in (0,1)$ is the share parameter, and A_t represents the level of labor-augmenting productivity. The logarithm of the productivity A_t follows an AR(1) process

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_t^A, \tag{4.3}$$

with $\varepsilon_t^A \sim \mathcal{N}(0, \sigma_A^2)$. The values of the parameters $\rho_A \in (0,1)$ and $\sigma_A > 0$ are common across plants. We represent this transition by the term $f_A(A_t|A_{t-1})$, which denotes the probability density of A_t conditional on A_{t-1}.

Each plant sells its physical output facing monopolistic competition with the price elasticity of demand equal to $1/\epsilon$. Subsequently, the downward-sloping demand curve is given by

$$P_t^i = Z_t^\epsilon \left(Y_t^i\right)^{-\epsilon}, \quad i \in \{h, \ell\}, \tag{4.4}$$

where i Z_t represents a stochastic demand shifter that follows an AR(1) process in logs

$$\ln Z_t = \rho_Z \ln Z_{t-1} + \varepsilon_t^Z, \tag{4.5}$$

with $\varepsilon_t^Z \sim \mathcal{N}(0, \sigma_Z^2)$, and the values of parameters $\rho_Z \in (0,1)$ and σ_Z are equal across competitors. The term $f_Z(Z_t|Z_{t-1})$ denotes the probability density of Z_t conditional on Z_{t-1}.

4.3.2 Factor Adjustment Costs

Physical capital is owned by plants in our model. Its stock at the level of a plant evolves as follows:

$$K_{t+1} = (1 - \delta^K)K_t + I_t, \tag{4.6}$$

where $\delta^K \in (0,1)$ is the rate of capital depreciation and I_t represents the current investment level of a plant.

Each change in the level of production factors by the plant is associated with adjustment costs. Investment is subject to partial irreversibility, which means that used physical capital can be sold only at a discount. More formally, the adjustment cost C_K associated with changing the level of capital from K_{t-1} to K_t is given by

$$C_K(K_t, K_{t-1}) = \begin{cases} (K_t - (1-\delta^K)K_{t-1}), & (K_t - (1-\delta^K)K_{t-1}) \geq 0, \\ \theta(K_t - (1-\delta^K)K_{t-1}), & (K_t - (1-\delta^K)K_{t-1}) < 0, \end{cases} \tag{4.7}$$

where $\theta \in [0,1]$ captures the degree of investment reversibility. Note that in the case of positive investment the adjustment cost equals the investment expenditures.

Regarding employment adjustment, we assume that it is subject to linear costs. This assumption is consistent with the approach prevalent in the literature (see e.g. Dixit and Stiglitz (1977) and Hawkins et al. (2015)) and is consistent with the empirical moments of labor demand distributions discussed in Section 4.2[39]. A plant pays b_1 for any new worker that it hires, or b_2 for any laid-off employee. This can be expressed using the following adjustment cost function C_L:

$$C_L(L_t, L_{t-1}) = \begin{cases} b_1(L_t - L_{t-1}), & (L_t - L_{t-1}) \geq 0, \\ b_2(L_{t-1} - L_t), & (L_t - L_{t-1}) < 0. \end{cases} \tag{4.8}$$

Prior to adjusting the levels of input factors and producing output, a plant draws the type of the technology it will be using in a given period. Following our characterization of output production, there are two types of production functions, CES and Leontief. As the latter is a special case of the former (setting $\sigma = 0$), we can alternatively say that the technology type is characterized by the degree of input substitutability. Each plant can draw either a high or a low value of this parameter, $\sigma^h = \sigma$ or $\sigma^\ell = 0$, respectively. The probability distribution over these types is characterized by the parameter value $\lambda \in [0, 1]$, where

$$\text{Prob}(\sigma = \sigma^h) = \lambda \quad \text{and} \quad \text{Prob}(\sigma = 0) = 1 - \lambda.$$

This feature is analogous to the approach developed by Hawkins et al. (2015). It is supposed to represent the fact that plants are typically not able to freely substitute one production factor for another, and they often face a certain degree of factor complementarity.

Observe that the factor adjustment cost functions (4.7) and (4.8) do not contain a term associated with joint adjustment of factors, and thus do not give rise to interrelated factor demand on the part of plants. In fact, any connection between the factors' input is due to the plants' production function, where variations in the substitutability (or complementarity) of factors affect the behavior of the plant.

[39]We have in mind, in particular, the symmetry of positive and negative labor adjustment occurrences and the high level of distribution variance.

4.3.3 The Plant's Problem

Each plant is characterized by the values of four plant-specific state variables at each point in time t: the level of demand Z_t, labor-augmenting productivity A_t, and the stocks of physical capital K_t and labor L_t. For convenience, we denote the exogenous state variables by $X_t = (Z_t, A_t) \in \mathbb{X} = \mathbb{R}_+ \times \mathbb{R}_+$. In what follows we consider the plant-level variables unless noted otherwise. Moreover, plants discount the future by a constant discount factor β, where $\beta \in (0, 1)$.

We begin our outline of the plant's problem by defining an expression for the plant's revenue $\Pi(X_t, K_t, L_t; w_t)$ using equations (4.1) and (4.4) while also adding a term representing the labor income of workers by the wage rate denoted by w_t:

$$\Pi_t^i(X_t, K_t, L_t) = P_t^i Y_t^i - w_t L_t = Z_t^\epsilon \left(Y_t^i\right)^{1-\epsilon} - w_t L_t, \quad i \in \{C, L\} \tag{4.9}$$

Since the plant is assumed to own its stock of physical capital, it does not need to pay any rental rate for the use of that capital.

Let us denote by V^0 the value of a plant at the beginning of a period, just after the shocks are realized and before it draws its technology type for that period. The value of a plant V^i at the moment it decides on its factor adjustment, depending on the technology type i, is then given as follows

$$\begin{aligned} V^i(X_t, K_{t-1}, L_{t-1}) = \max_{K_t, L_t} \Bigg\{ & - C_K(K_t, K_{t-1}) - C_L(L_t, L_{t-1}) + \\ & + \Pi_t^i(X_t, K_t, L_t) + \beta \mathbb{E}_x \left[V^0\left(X_{t+1}, K_t, L_t\right) \Big| X_t \right] \Bigg\}, \quad i \in \{C, L\}. \end{aligned} \tag{4.10}$$

Recall that investment expenditures are included in the capital adjustment cost function C_K. The optimal choices of physical capital and labor, K^* and L^*, are mutually interrelated and depend on the realized draw of σ.

Finally, we return to V^0, the value of a plant before its draw of technology type, and formulate it as follows

$$\begin{aligned} V^0(X_t, K_{t-1}, L_{t-1}) = & \lambda V^C(X_t, K_{t-1}, L_{t-1}) + \\ & + (1-\lambda) V^L(X_t, K_{t-1}, L_{t-1}). \end{aligned} \tag{4.11}$$

4.3.4 Cross-sectional Distribution

We will now present the transition law for the distribution of all plants μ_t. The formula notation is set to apply at the beginning of a period, i.e. before the draw of idiosyncratic demand and productivity shocks.

Recall that plants' state variables at time t are the demand level Z_t, labor-augmenting productivity A_t, and the stocks of physical capital K_t and labor L_t. For ease of notation, we denote the composite state variable $\xi_t = (Z_t, A_t, K_t, L_t)$, which can take on values in the state space $\mathbf{S} = \mathbb{R}^4_+$. Consequently we also introduce $\mathcal{S}$ the Borel algebra on $\mathbf{S}$.

The following law of motion governs the evolution of the probability distribution of plants :

$$\begin{aligned} &\forall \xi' \in \mathcal{S} : \mu_{t+1}(\xi') = \\ = &\int_{Z'} \int_{A'} \left(\lambda \mathbb{1}_{\{K^*(\xi,\sigma^h) \in K'\}} \mathbb{1}_{\{L^*(\xi,\sigma^h) \in L'\}} + (1-\lambda) \mathbb{1}_{\{K^*(\xi,\sigma^\ell) \in K'\}} \mathbb{1}_{\{L^*(\xi,\sigma^\ell) \in L'\}} \right) \\ &df_Z(Z'|Z)\, df_A(A'|A)\, d\mu_t(\xi), \end{aligned} \tag{4.12}$$

where $\mathbb{1}$ represents the indicator function.

4.3.5 Competitive Partial Equilibrium

The stationary equilibrium of our model economy is defined as a set comprising the decision rules $K^*(\xi, \sigma)$ (the optimal policy for a stock of physical capital) and $L^*(\xi, \sigma)$ (the optimal policy for a level of labor), the value function $V^*(\xi)$ and the probability density function $\mu^*(\xi)$ such that the following conditions hold:

- Firms maximize their profits: $V^*(\xi)$, $K^*(\xi, \sigma)$ and $L^*(\xi, \sigma)$ solve (4.10) and (4.11) for all (ξ, σ),
- The equilibrium distribution of plants is stationary: $\mu^*(z, k)$ satisfies (4.12)

Recall that the equations (4.10) and (4.11) characterize the dynamic optimization problem of a plant. In Appendix 4.A we show formally that a solution to this problem exists and is unique.

4.4 Calibration

We use our benchmark model as a lab to study the determinants of plants' observed investment and hiring or layoff behavior. The complicated structure of the model with non-trivial plant-level heterogeneity does not allow for an analytical solution. We therefore use a numerical solution method, namely dynamic programming, while discretizing the state variables on appropriate grids. We will first discuss our calibration strategy for the model parameters. Subsequently we will characterize the model economy using the regimes of optimal policies for a plant and descriptive statistics of selected variables.

Prior to numerically solving the model, we have to decide on the values for the various parameters it contains. Ultimately, we aim to discipline our model with selected statistics of annual data for US manufacturing. However, fact (i) in section 4.2 - investment spikes and layoffs jointly occurring at the plant level - has been documented only in countries other than the United States. In fact, to our knowledge, there has been no in-depth empirical assessment of patterns of joint capital and labor adjustment at the firm- or plant-level in any sector of the US economy.

We cannot, therefore, follow the calibration procedure by Hawkins et al. (2015), who use selected moments of the joint distribution of factor adjustment rates to identify the parameter values governing changes in production technology. We circumvent this problem by developing a calibration strategy based on that described in the study by Sakellaris (2004), which is feasible given accessible data sources. The identification relates to the patterns of employment growth observed in the period of investment spike. We will discuss this strategy in more detail below.

The parameters in our model can be divided into two groups. The first group contains those that can be calibrated based on the literature or existing empirical evidence. The discount factor β is set in such a way that the real annual interest rate equals 4%. Next, the price elasticity of demand $\frac{1}{\epsilon}$ targets the value of 4, which is consistent with the standard calibrations referred to in Hawkins et al. (2015). Finally, the depreciation rate δ_K is set to an annual value of 10%.

The second group of parameters is calibrated such that selected statistics of the data are matched by the corresponding moments of the model economy. It is important to bear in mind that these parameters have interacting effects on the targeted statistics, and we therefore have to calibrate them jointly. The weight of capital in the production function α is identified by the mean value of capital-to-output ratio. Next, the variances of shock processes driving labor-augmenting and factor-neutral productivity σ_A and σ_z are set such taht the model matches the standard deviations of plant-level investment rates and employment growth rates. At the same time, the persistence parameters of these shock processes, ρ_A and ρ_z, target, respectively, the serial correlation of investment rates and the persistence of profitability shocks as estimated by Cooper and Haltiwanger (2006). The degree of investment irreversibility θ is set to obtain the observed share of plants with negative investment. Finally, the linear labor adjustment cost parameters, b_1 and b_2, are assumed to be equal and target the average annual rate of labor reallocation.

In our identification process we finally arrive at the parameters that are crucial for generating the patterns in micro-level factor adjustment that we are interested in (namely, investing plants creating or destroying jobs). These are the value of factor substitution elasticity σ and the probability of drawing a high substitutability technology λ. When identifying these parameters, Hawkins et al. (2015) make heavy use of the moments documented for the joint distribution of factor adjustment of Korean plants. As we do not have comparable data available for US manufacturing, an alternative approach is needed.

The value of λ is set such that the model is consistent with the average value of 0.5 of the plant-level elasticity of substitution estimated for US manufacturing by Raval (2015) and Oberfield and Raval (2014). Following the approach by Bayer et al. (2015), we can write $\sigma^{AVERAGE} \approx \lambda\sigma + (1-\lambda)0$, as the value of λ governs the shares of plants with different values of elasticity of substitution, namely σ and 0. As a result, we obtain an identification equation $\lambda \approx 0.5/\sigma$. Subsequently, the value of σ targets a difference of 4 percentage points between the mean employment growth rate estimated during an investment spike event, and the mean employment growth rate in years not proximate to the investment spike year. The value of this statistic is obtained from the study by Sakellaris (2004)[40].

Table 4.2: Benchmark calibration

ρ_A	σ_A	ρ_z	σ_z	ϵ	σ	λ
0.62	0.2	0.885	0.05	0.25	3.5	0.143
β	α	b_1	b_2	θ	δ^K	
0.96	0.3	0.001	0.001	0.89	0.1	

The baseline calibration is presented in Table 4.2. The parameters ρ_A and σ_A, which govern, respectively, the persistence and standard deviation of the labor-augmenting shock, have values 0.62 and 0.2. For a factor-neutral shock, the values of ρ_z and σ_z are 0.885 and 0.05. Comparing these two exogenous processes indicates that the factor-augmenting shock is much less persistent and more volatile

[40] See the discussion in Appendix 4.B.

than the factor-neutral one. The additional parameters α and ϵ govern the production function of plants. The capital weight in the CES production function α is given the value 0.3. The parameter value of ϵ is in line with the inverse of the value of 4 typically used for the elasticity of demand.

Finally, we discuss calibration of the parameters that directly relate to plants' adjustments of production factors. Changes in the employment level are subject to the linear costs b_1 and b_2, which equal 0.001 for both hirings and firings. Next, investment in physical capital is subject to partial irreversibility, where the parameter value $\theta = 0.89$ indicates that, upon disinvesting, plants lose 11% of the value of capital stock sold.

Table 4.3: Cross-Sectional statistics

Variable	Data	Model
(i) Serial correlation of investment rates	0.058	0.003
(ii) Standard deviation of investment rates	0.337	0.338
(iii) Persistence of profitability shocks	0.885	0.885
(iv) Share of obs. with neg. investment	0.104	0.093
(v) Annual job reallocation rate	0.238	0.317
(vi) Standard deviation of empl. gr. rate	0.480	0.481
(vii) Capital-to-output ratio	1.79	1.87
(viii) Labor share	0.625	0.494
(ix) Empl. growth rate in investment spike year above normal	0.04	0.037
(x) Average plant-level elasticity of substitution	0.48	0.5

Notes: All statistics are for US manufacturing data observed at annual frequency. Sources: (i)-(iv) Cooper and Haltiwanger (2006), time period: 1972-1988; (v)-(vi) Davis et al. (2010) (1976-2005); (vii) own calculations using Bureau of Economic Analysis (2013), time period: 1972-2005; (viii) own calculations using Federal Reserve Bank of St. Louis (2017), time period: 1972-2005; (ix) Sakellaris (2004), time period: 1974-1991; (x) Raval (2015), years: 1987, 1997.

Table 4.3 contains a comparison of empirical moments selected for calibrating purposes with their counterparts observed for the model economy. Numerical imprecisions and excessive computational cost keep us from achieving a perfect fit. Still, there are some similarities, as well as deviations, that are worth mentioning. Observe, for instance, that the cross-sectional variation of both the investment rates and employment growth rates calculated in an artificial economy is close to the empirical data. This indicates a realistic amount of variation being present in the model. On the other hand, there are non-trivial deviations between the data and the model in the values of the serial correlation of investment rates, as well as in the annual rate of job reallocation.

Table 4.4: Sensitivity Analysis of Calibration Target to Elasticity of Factor Substitution

Value of σ	mean empl. growth rate [spike year rel. to normal year]
1.5	0.258
2	0.175
2.5	0.111
3.5	0.037

Notes: The table contains model-implied values of the mean employment growth rate in years of investment spike relative to years outside of 5-year investment spike windows. The estimation method follows Sakellaris (2004) and is outlined in Appendix 4.B. The share of plants with CES production function λ is calibrated to match the average value of 0.5 of the plant-level elasticity of substitution conditional on given σ.

Let us discuss for a moment the identification strategy concerning the high-substitutability technology. The parameter value of the elasticity of substitution between the production factors σ is 3.5, which yields a value of 0.037 for the mean rate of employment growth in years of investment spike, relative to years that are outside of 5-year windows centered on investment spike years (see Appendix 4.B for more details). Now we may use comparative statics to assess the indentificatory power of these data moments. Table 4.4 shows a sensitivity analysis of the job creation and destruction rates at the upper end of the plant size distribution with respect to the values of factor substitution elasticities.

4.5 Numerical Analysis

In what follows we assess the numerical solution of our model using the parameter values presented above. In particular, our focus is on plants' decisions on adjustment of production factors. In each period, plants decide whether and how much to invest, and how many workers to hire or lay off. These decisions obviously depend on plants' current states, i.e. on their demand and productivity levels as well as on their current holdings of physical capital and workforce. Moreover, optimal investment and hiring or layoff policies are mutually dependent. In this section, we want to investigate this interdependence.

4.5.1 Regimes of Factor Adjustment

We start by qualitatively analyzing how plants' decisions on factor adjustment depend on their current stock of physical capital and workforce. We present an intuitive graphical representation following the approach by Dixit (1997), where an endogenous state space is partitioned based on the directions of factor adjustments, i.e. whether a plant with a given state decides either to increase its levels of factor demand, to keep them unchanged, or to decrease them. Note that this representation is qualitative, as only the signs of the relevant adjustments are considered, while their magnitudes are neglected.

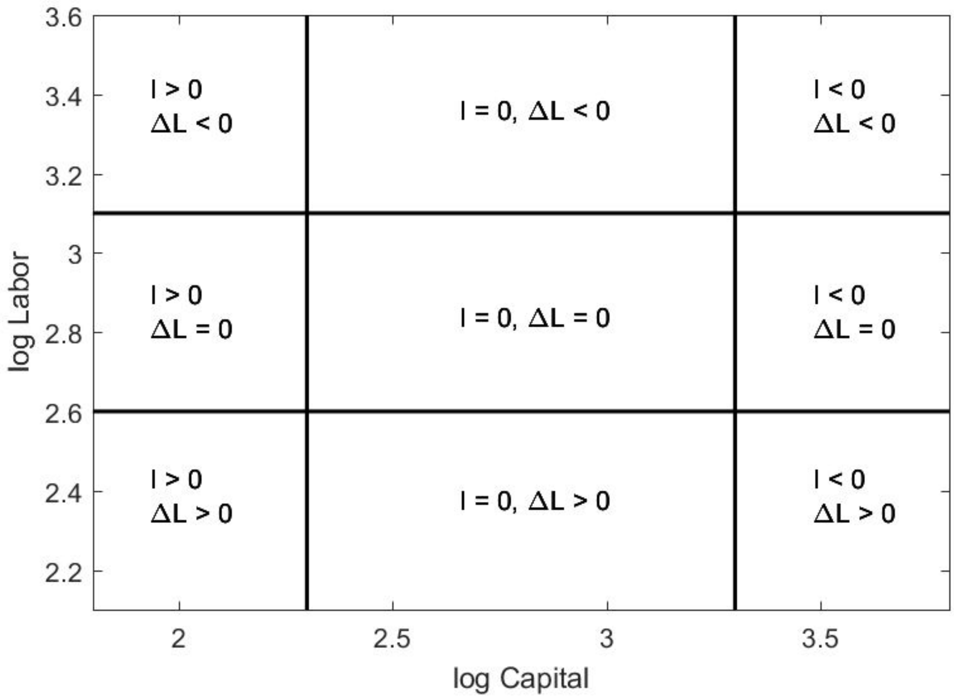


Figure 4.2: State space regions for optimal policy regimes

Notes: The figure represents optimal policy regimes distributed across the endogenous state space (K,L). Firms' policy variables are investment (I) and change of employment (ΔL). The analytical characterization of the boundaries of the optimal policy regime area is presented in Dixit (1997).

In Figure 4.2 we show an example of graphical characterization of plants' optimal investment and employment growth over the endogenous state space. In the central area of the graph, a plant's optimal policy is not to change any of its factor inputs. In regions outside this central area, it is optimal for a plant to adjust one or both of the factors until it reaches a point on the central region's boundary. Note that, in a given time period, it is always optimal for a plant to choose its adjustment so as to land exactly on the border of the inaction region. Falling short of attaining this leaves some optimal adjustment unrealized for a plant, while moving inside the inaction region is associated with additional adjustment costs.

We then use this graphical concept to characterize the optimal factor demand policies calculated for plants in our model economy. Figure 4.3 depicts the space of endogenous state variables - physical capital and employment - partitioned into different regions according to the optimal patterns of factor adjustment. Optimal policies under both production technologies are presented, with the "high sigma" CES technology drawn by the dashed line and the Leontief technology depicted by the solid line. The adjustment policies are evaluated with both exogenous state variables, demand and labor-augmenting productivity, set to their median values. To complement this analysis, in Appendix 4.C we present figures depicting the corresponding state-space regions for different values of demand (Figure 4.8) and labor productivity (Figure 4.9).

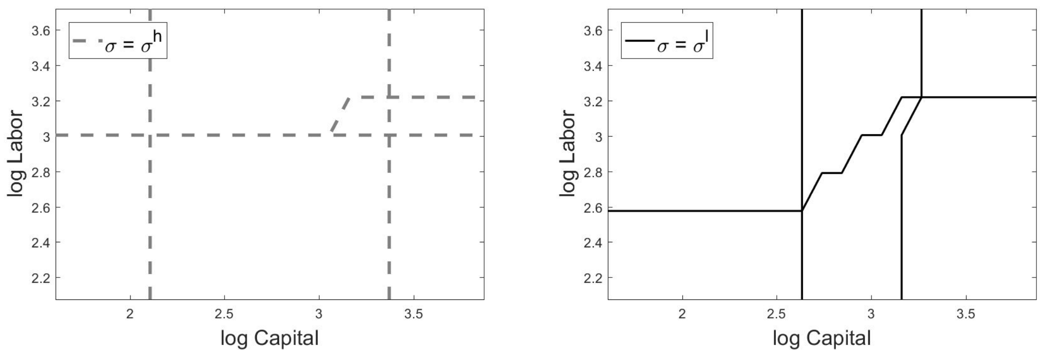


Figure 4.3: Model-based regions for optimal policy regimes

Notes: The figures represent model-based regions for regimes of plants' optimal policies for investment (I) and employment growth (ΔL) over the endogenous state space for both production technologies. The figure on the left hand side represents the regimes for the CES production function with $\sigma > 0$ (σ^h). The figure on the right hand side depicts the regimes for the Leontief production function (σ^l). Levels of the exogenous state variables are set equal to their median values. Characterization of the respective regions is given in Figure 4.2.

Let us use this figure to describe the patterns of optimal factor adjustments in our model economy. First, observe that inaction in investment should be observed more frequently than inaction in labor adjustment, as the respective inaction regions differ considerably in size. This observation is consistent with the empirical data presented in Table 4.1. Note that occurrence of investment inactivity in the model economy results from the feature of irreversibility, as this adjustment friction leads to areas of the state space where a plants' optimal investment policy is inaction.

Second, plants using the CES production technology exhibit inactivity in employment adjustment only in cases of zero investment. Once a plant decides to adjust its capital stock, it will *always* change its level of employment (except for the knife-edge cases). This is understandable given the calibrated parameter values underlying the factor adjustment frictions, where the relative costs of capital adjustments are larger than those of labor. Employment being the more flexible production factor than capital is consistent with the evidence used for calibration, specifically the relative magnitudes of the standard deviations of the respective adjustments (see Table 4.3).

Finally, we assess the adjustment regimes of plants that use the Leontief production technology. The observed patterns appear very different from those for the CES production technology. In particular, investment inactivity occurs less frequently in the case of perfect factor complementarity. Moreover, investment-inactive businesses adjust their employment levels more often. At the same time, we observe for plants investing or disinvesting that some of them keep their workforce intact. These observations seem counterintuitive, since labor is the more flexible factor, and therefore one expects it to adjust whenever the capital stock changes. Yet the above phenomenon arises naturally from the production technology in use.

Perfect complementarity of the production factors implies that it is optimal for plants to keep the two factors' levels at a constant ratio, ceteris paribus. If the capital stock is in the corresponding inactivity range, a plant adjusts its employment level to achieve the optimal factor ratio. Thus, there is no range of labor adjustment inactivity for states with investment inactivity. However, once a plant decides to invest or disinvest, the existing level of employment may be consistent with the newly chosen optimal capital stock, so labor adjustment inactivity may occur.

Let us discuss the implications of these figures for the objective of this work. Given that our interest is in joint occurrence of investment and worker lay-offs, we focus on the north-west region of the graph in Figure 4.3. Comparison of this region for different technologies shows that the area of this region is larger for the Leontief production technology than for the one related to the CES technology. This implies that plants that draw the Leontief technology are more likely to simultaneously invest and separate from workers.

Furthermore, the probability of the Leontief technology draw is given by $1-\lambda$. It follows from our identification strategy (see Section 4.4) that λ is inversely proportional to σ^h. As a result, increase in the elasticity of the substitution parameter leads to a higher number of plants that invest and lay off workforce. Moreover, there is an upper limit to the proportion of plants both investing and separating from workers, which is the proportion observed in the case of the Leontief production technology.

4.5.2 Marginal distributions of the adjustment of the factors

In this section, we shall inspect the distributions of plants across their optimal factor demand policies. First, let us present the marginal distributions of investment rates and employment growth rates, which can be directly compared with the existing empirical evidence that we reviewed in Section 4.2.

The cross-sectional distribution of investment rates is presented in Figure 4.4. As in the data, we observe a substantial positive skewness of the distribution. Positive investments prevail for 41.7% of observations, while negative adjustments of the capital stock happen for only 9.3% of all observations. One difference from the empirical data is too high a share of inactive plants, as the of value 50% observed in the model economy is much larger than the actual statistic for U.S. manufacturing (c.f. Cooper and Haltiwanger (2006)).

At the same time, the magnitude of adjustment is substantial. Conditional on investing, a plant increases its capital stock by (on average) 40%. Moreover, almost 26% of all observations of positive investment exhibit increases of over 50%. Finally, 48% of dis-investing plants decrease their stock of capital by more than 20% in the space of a year.

Figure 4.5, on the other hand, displays the equilibrium distribution of the employment growth rates. The shape of the distribution is roughly symmetric around the no adjustment threshold, which is consistent with the data (c.f. Hawkins et al. (2015)). Inactivity in workforce adjustment is less frequent than in investment adjustment, observed only for 4.1% of observations, indicating that labor is easier to adjust than capital.

The magnitudes of the realized labor adjustments are even larger than those of investment. The standard deviation of the employment growth rates across producers is almost 60%. A plant that is hiring exhibits on average an employment growth rate of 40%, while the average rate for a separating plant is −20%. Additional statistics concerning the marginal distributions of capital and labor adjustment can be found in Table 4.5.

Table 4.5: Statistics on marginal distributions of factor adjustments

Statistic	investment rate	empl. growth rate
	[share of plants]	
Positive adjustment	0.417	0.487
Negative adjustment	0.093	0.472
Inactivity	0.500	0.041
Increase above 20%	0.232	0.220
Increase above 50%	0.108	0.087
Standard deviation	0.337	0.594

Notes: The table presents selected statistics characterizing the marginal distributions of capital and labor adjustment, as implied by our model.

Both marginal distributions provide valuable information on the patterns of factor adjustment at the plant level. The empirical evidence, however, suggests that plants do not choose their optimal levels of capital and labor separately, but rather jointly. Therefore, in the next step we will describe the joint distribution of factor adjustments that our model implies.

4.5.3 Joint distribution of factor adjustments

In this part we present the model-derived moments of the joint distribution of adjustment of the factors. Furthermore, we shall use these statistics to characterize plants' decisions concerning the optimal levels of capital and labor, which are made jointly rather than independently.

In Section 4.2, we reviewed the results of several international works on these matters. However, there is no comparable empirical study on the *joint* dynamics of capital and labor in a cross-section of plants for the U.S. economy. This shortage presents a challenge to our work, as we cannot use the corresponding empirical data to discipline the relevant model parameters. However, we can circumvent this problem by using data on the marginal distribution as an input for the calibration procedure, and subsequently analyze the joint distribution that is an outcome of our model.

Below, we present selected statistics on the joint distribution of factor adjustment as calculated for the model economy. We also compare these results with the studies that report empirical data on non-US economies. Finally, we discuss observed differences in data moments and try to link them to the patterns observed for the marginal distributions of adjustment of factors.

Table 4.6: Plants' Distribution of Factor Adjustment Regimes

	Asphjell et al. (2014)				Model			
	I/K				I/K			
	Negative	Inaction	Positive	Sum	Negative	Inaction	Positive	Sum
$\Delta L/L$								
Positive	0.005	0.098	0.293	0.396	0.035	0.255	0.202	0.492
Inaction	0.003	0.065	0.144	0.213	0.003	0.008	0.014	0.026
Negative	0.011	0.137	0.243	0.391	0.049	0.250	0.183	0.482
Sum	0.020	0.300	0.680		0.088	0.514	0.399	

Notes: The table contains the empirical and model-implied shares of plants for different factor adjustment regimes. The source of the empirical statistics is the study of Norwegian manufacturing plants by Asphjell et al. (2014).

First we look at the fractions of plants that adjust production factors in different regimes. Table 4.6 compares the model-implied distribution with the distribution documented for Norwegian manufacturing plants by Asphjell et al. (2014). Given the differences in the empirical evidence available from different countries, we focus on a qualitative rather than a quantitative comparison between patterns in the data and in the model. As in the data, our model economy exhibits more action in labor adjustment than in investment, which points to capital as a production factor that is costlier to adjust. At the same time, the relative amounts of positive and negative employment adjustments are balanced, while investment rates are skewed towards positive values.

In section 4.2 we noted that the frequent occurrence of joint increases in capital and decreases in workforce is a novel empirical finding, and one largely unaddressed by the literature. According to Table 4.6, our model is capable of replicating the proportion of plants that simultaneously invest and separate from workers. This is comparable to the proportion of plants jointly investing and hiring workers, a pattern that resembles the empirical data on Norwegian manufacturing. Our model framework, with plants using production technologies with different substitution elasticities, is therefore capable of replicating this novel empirical finding.

The occurrence of plants jointly investing and firing is a common phenomenon in our model economy. This is also seen in the distribution of employment rates for plants investing at a rate above 10%, which is presented in Figure 4.6. The shape of the distribution of investing plants is not very different from the distribution observed for all producers. Most notably, a negative employment growth rate is as likely for net investors as for plants overall.[41]

Table 4.7: Statistics on joint distribution of factor adjustments

Statistic	Value	
	Hawkins et al. (2015)	Model
$Prob[\Delta L < 0 \mid I/K > 0.1]$	0.360	0.130
$\mathbb{E}[\Delta \ln L \mid I/K > 0.1, \Delta L < 0]$	-0.186	-0.297
$\mathbb{E}[\Delta \ln L \mid I/K > 0.1, \Delta L > 0]$	0.208	0.214
$Prob[\Delta L < 0 \mid I/K > 0.2]$	0.341	0.086

Notes: The table contains empirical and model-implied shares of plants for selected ranges of the joint factor-adjustment distribution. The source of the empirical statistics is the study of Korean manufacturing plants by Hawkins et al. (2015).

We close this section by presenting some additional statistics on the joint distribution of factor adjustment in Table 4.7. Since there is no empirical evidence on the US manufacturing sector, we compare our model's output to the values reported by Hawkins et al. (2015) for South Korean manufacturing. Two observations stand out. First, the share of plants investing at a rate above 10% and separating from workers is substantial in our model economy - 13% -, although this is smaller than the empirically-observed value. Second, the workforce adjustment is sizable in both directions, exceeding the empirical equivalent. Plants that jointly invest and fire workers decrease their employment level on average by almost 30%, while producers that invest and hire grow on average by more than 20%.

To sum up, the joint factor-adjustment distribution implied by our model economy, which is calibrated to align with the marginal factor-adjustment distributions, is comparable to the empirical facts. We observe some quantitative differences, as the empirical studies focus on countries other than the U.S., but the qualitative patterns are very similar. The model results indicate that the phenomenon of plants investing and decreasing their workforce simultaneously is common in the U.S. manufacturing sector. The positive average co-movements of changes in capital and labor as documented e.g. in Sakellaris (2004) thus hide substantial heterogeneity of plants' adjustment decisions.

[41] A similar pattern is documented empirically by Hawkins et al. (2015) for Korean manufacturing plants.

4.5.4 Aggregate implications

In the final part, we focus on the aggregate implications of the patterns documented for the joint distribution of factor adjustment in our model economy. We want to assess their relevance for the levels of the aggregate variable of plants that separate from workers while increasing their capital stock. We will present and discuss the shares of aggregate investment and output that can be attributed to plants with opposite-direction adjustments of capital stock and workforce.

Let us remark that our model framework does not contain aggregate uncertainty. We cannot, therefore, evaluate the determinants of the aggregate dynamics. However, the shares of aggregate capital accumulation and output production that are attributable to investing plants with negative employment growth may provide important information on the relevance of the economic mechanism captured by our model.

Table 4.8: Statistics on Contributions to aggregate investment

Statistic	Value	
Share of agg. investment due to	Hawkins et al. (2015)	Model
$I/K > 0$ and $\Delta L < 0$	-	0.415
$I/K > 0.1$ and $\Delta L < 0$	0.355	0.383
$I/K > 0.2$ and $\Delta L < 0$	-	0.329

Notes: The table contains empirical and model-implied shares of aggregate investment due to plants with the specified optimal policies. The source of the empirical statistics is the study of Korean manufacturing plants by Hawkins et al. (2015).

First, we evaluate the contribution of plants that adjust their factor demands in opposite directions for aggregate investment. Table 4.8 contains the shares calculated for different thresholds for the plants' investment rates. More than 41% of the aggregate capital accumulation is due to plants that simultaneously decrease their employment levels. When considering lower bounds on the investment rate of 10% or 20%, this percentage decreases to 38% and 33%, respectively. These values imply that plants which lay off workers significantly contribute to aggregate investment.

The only empirical counterpart to these statistics is that provided by Hawkins et al. (2015), who show that more than 35% of the aggregate capital accumulation in Korean manufacturing is accounted for by plants that exhibit, simultaneously, a fall in the employment level and an investment rate above 10%. Note that this is similar in magnitude to our model-implied percentage, albeit slightly lower. These observations illustrate the important role that plants which adjust these factors in the opposite directions play in the economy.

Table 4.9: Distribution of output across plants' factor adjustment regimes

	Model			
	I/K			
	Negative	Inaction	Positive	Sum
$\Delta L/L$				
Positive	0.030	0.304	0.212	0.546
Inaction	0.006	0.003	0.020	0.040
Negative	0.031	0.219	0.175	0.414
Sum	0.067	0.526	0.407	

Notes: The table contains model-implied shares of total output produced by plants in different factor adjustment regimes.

Lastly, we look at the distribution of aggregate output over plants with different factor-adjustment regimes. We use this comparison to present another angle on the contribution of plants simultaneously investing and separateing from workers. Table 4.9 contains the shares of the total output produced by plants in alternative factor adjustment regimes. The highest share is due to investment-inactive plants. Investing establishments produce more than 40% of total output. Out of this portion, plants that lay-off workers provide 43%. The percentages of output for the producers with negative employment growth are slightly below their number as a fraction of total plants (see Table 4.6), yet the contributions are still sizeable..

To sum up, the model results indicate that plants that invest while decreasing the employment level are quantitatively important for outcomes of the aggregate economy in terms of both investment and production. Their contribution is especially important with respect to the aggregate capital accumulation, where plants separating from workers provide more than 40% of the total amount.

4.6 Conclusions

In this chapter, we have studied patterns of investment and labor adjustments in the cross-section of plants. Of particular interest is the frequent occurrence of plants with positive investment and negative employment growth. This pattern is a feature of the empirical data that has not been properly addressed by the majority of the existing literature. We have developed a model framework in which plants use production technologies with different elasticities of factor substitution.

The model is calibrated to match selected data moments for U.S. manufacturing, including statistics on the marginal distributions of factor adjustment.

We calculate in our model economy the statistics that characterize the joint factor-adjustment distribution. As the direct empirical counterpart is missing for the U.S. manufacturing sector, we show that the estimates are broadly consistent with the evidence documented for other countries. The fraction of plants that simultaneously invest and separate from workers is substantial and amounts to almost 20% of all plants in a given year. Moreover, these plants account for 40% of the aggregate investment.

As the frequency of joint opposite-direction adjustments is non-negligible, this phenomenon is likely to play a substantial role in the dynamics of these factors in the aggregate. This idea is further reinforced by the high share of total investment attributed to plants investing while separating from workers. The main question to be addressed is under which conditions growth in the aggregate capital and decline in the aggregate employment can be observed simultaneously. The assessment of these issues in a model with aggregate dynamics is an interesting question for further research.

Finally, we would like to highlight a connection between our work and the study by Odendahl (2018), who develops a method that allows for estimating a multivariate density based on the observed marginal distributions. The methodology is built on a theoretical concept of copula functions and is aimed at the assessment of survey data. The paper is different to ours in terms of approach (statistical) and the area of interest (forecasts based on survey datasets), yet the link between the joint and the marginal distributions is in the both cases the cornerstone of the quantitative exercise.

4.A Existence and uniqueness of a solution

The dynamic optimization problem of a plant is characterized by equations (4.10) and (4.11). In what follows we provide a formal proof that this problem is well defined, i.e. its solution exists and is unique.

First, for any value of exogenous states X define a functional operator $\mathcal{T}^X(V)$ by the RHS of equation (4.10). Next, consider a subspace $\mathcal{J}$ of continuous bounded functions $J : [0, \bar{L}] \times [0, \bar{K}] \to \mathbb{R}$. The upper bounds $\bar{L}$ and $\bar{K}$ are sufficiently large such that $L^* < \bar{L}$ and $K^* < \bar{K}$ for any $L \in [0, \bar{L}]$ and $K \in [0, \bar{K}]$. The existence of these bounds is ensured by the Inada conditions assumed for the production function Y (and thus also for the revenue function Π) and convexity of the factor adjustment costs C_K and C_L (for $b_1 \geq b_2$).

It is easy to see that $\mathcal{T}^X$ maps the subspace $\mathcal{J}$ onto itself, and that it satisfies Blackwell's sufficient conditions for a contraction. Using the contraction mapping theorem, this implies that for any $X \in \mathbb{X}$ there exists a unique fixed point of the mapping $\mathcal{T}$, i.e. there exists a unique function $V : \mathbb{X} \times [0, \bar{L}] \times [0, \bar{K}] \to \mathbb{R}$ satisfying (4.10) and (4.11). At the same time, we obtain the corresponding optimal policy functions K^* and L^*.

4.B Empirical patterns of a plant's joint adjustment of production factors

The paper by Sakellaris (2004) empirically characterizes dynamic decisions of plants in the event of rapid factor adjustment using evidence from data covering manufacturing plants in the US economy. It is one of few studies that analyze *joint* patterns of adjustment of physical capital and labor by plants using data on the micro-level. We use some of the estimated indicators as calibration targets and compare results with the analogous output calculated for our model economy. In what follows we shall therefore review the data, the methodology, and selected results of the study.

Two sources of micro-data are used to construct large samples of U.S. manufacturing plants on an annual basis. The results relevant for our work are estimated for a sample of over ten thousand plants operating in the time period 1974-1991. The values for a number of variables are documented for each plant-year observation. The author identifies occurrences of investment spikes, job creation bursts and job destructions bursts among the observations. Subsequently, he examines changes in different variables (such as output, productivity, physical capital, and material and energy usage) in periods before, during, and after these events.

The paper does not use a structural model for the analysis. Instead, the author uses a simple econometric approach to provide descriptive data. The analysis estimates patterns in the values of plants' indicators over the period within a year of the event. The model is specified for plant i and year s as follows

$$X_{i,s} = \mu_i + \nu_s + \sum_{j=-2}^{+2} \beta_j \times EVENTD_{is}^{t+j} + \beta \times OTHERD_{is} + \varepsilon_{is}, \qquad (4.13)$$

where a dependent variable $X_{i,s}$ is one of the indicators of interest, μ_i stands for a plant fixed effect, ν_s is a year fixed effect, and ε_{is} represents an error term. Next, $EVENTD_{is}^{t+j}$ is a dummy variable indicative of plant i experiencing event D (investment spike, JC burst or JD burst) in year $s-j$. Similarly, $OTHERD_{is}$ represents a dummy variable that equals 1 if plant i experienced event D in year outside of the time-frame $s-2$, $s+2$. Finally, β_j and β are estimated coefficients.

The estimated values of the coefficients β_j stand for the conditional means of the variable X during the specified year within the 5-year time frame centered on the event year. On the other hand, the estimate of the β parameter captures the average level of the variable during normal times, i.e. outside of the period associated with the rapid adjustment.

Note that the absolute magnitudes of the estimates of these coefficients do not have a meaningful interpretation per se, as the fixed effects term is included in the specification. However, the relative magnitudes of the estimates can be presented and interpreted clearly. The author follows this logic when communicating results and presents the estimated values as differences from the β_0 coefficient, which captures the effect in the year of the investment spike. To illustrate this approach, we present the effects seen on the investment rate and stock of physical capital in Figure 4.7a, and for the employment growth rate in Figure 4.7. The results for other indicators of interest and for the burst events of job creation and job destruction can be found in the original work by Sakellaris (2004).

The figures provide illuminating insights into the dynamic behavior of plants shortly before and after implementing a large investment project. First, the values of investment rates are comparable to those observed in normal times in the years before the large investment project, while they remain relatively elevated by approximately 10 percentage points for two years after. Second, plants undergoing an investment spike are initially, on average, smaller in terms of capital stock by around 20% than they are outside of the 5-year time-frame. Once the project is realized, the capital stock grows and is comparable to the non-spike average. Third, the growth rate of employment is heightened relative to normal levels during the entire observed period and peaks in the very year of the investment spike, at a rate $3-4$ percentage points higher. We use the estimated value of this statistic as a target variable in our calibration to identify the the elasticity of the substitution parameter in the CES production function.

Finally, we highlight the patterns estimated for the rates of employment growth, as they provide insights into the joint dynamics of capital and labor adjustment. The estimates reported in Figure 4.7b show that plants treat these factors, on average, as complementary when conducting large investment projects. This conclusion is corroborated by the statistics estimated for job creation and job destruction bursts, as shown in the later parts of Sakellaris (2004). When a plant increases its workforce rapidly (i.e. by 10% or more), it also invests at a rate of 4 percentage points more (on average) than otherwise.

4.C Adjustment Regimes and Exogenous States

This section complements the assessment of plants' factor adjustment regimes in subsection 4.5.1. In what follows we use the representation over the endogenous state space to demonstrate changes brought about by variation in values of the exogenous state variables, namely the level of demand (Figure 4.8) and labor-augmenting productivity (Figure 4.9). In both cases we present the chart for increased values of the exogenous state and compare it with the baseline. Changes in the cases of decreased values are symmetric in the opposite direction and are not presented for the sake of simplicity.

As shown in Figure 4.8, an increase in the level of (factor-neutral) demand shifts the boundaries of the policy regime areas in the north-east direction. This means that there are plants, which are inactive demand-wise at the median level, but choose to invest or hire once their demand level increases. At the same time, we observe plants remaining inactive instead of disinvesting or separating from workers. All in all, the mean values of investment rate and hiring rate are higher for plants with an increased level of demand.

Figure 4.9 displays the adjustment regime areas for different levels of labor-augmenting productivity. The results are comparable to those concerning changes in the demand level, i.e. the average rates of investment and hiring increase as the productivity level grows. One difference compared to the case presented in Figure 4.8 is the observed disappearance of the "inaction" area for the high productivity level. This can be traced to the absence of states, where plants choose inactivity in labor adjustment as their optimal policy.

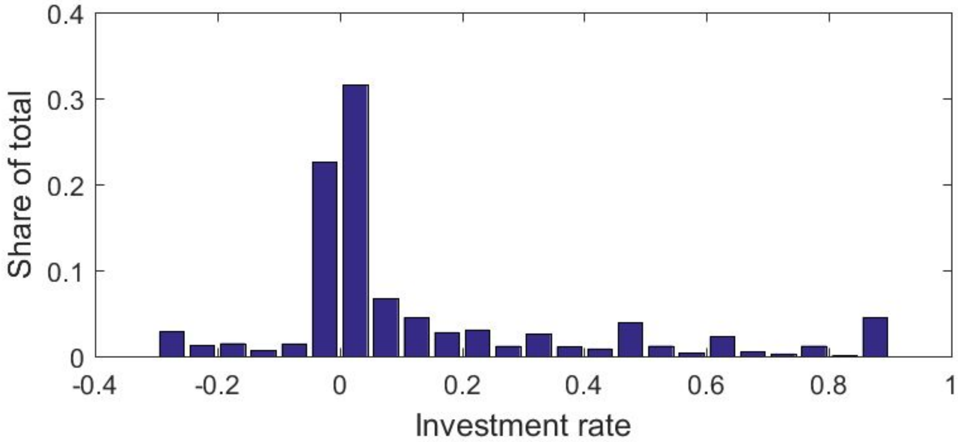


Figure 4.4: Investment rate

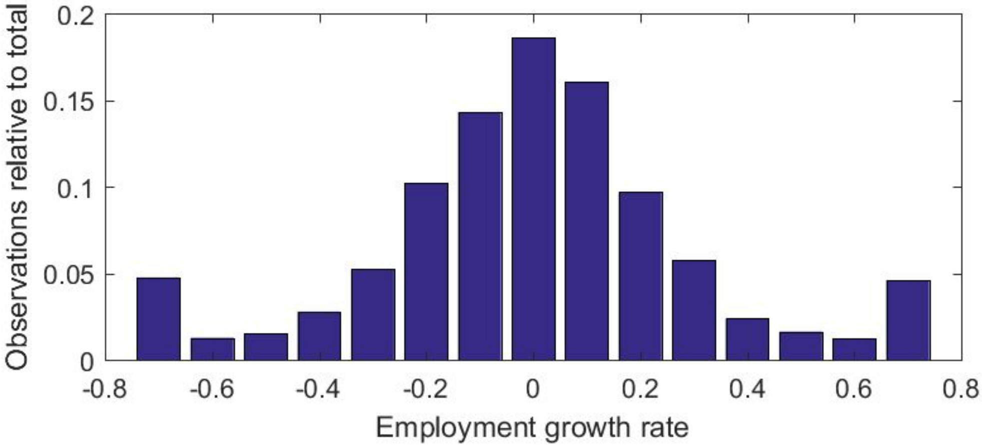


Figure 4.5: Employment growth rate

Notes: The figure in the top panel represents the cross-sectional distribution of plants' investment rates. The figure in the bottom panel depicts the cross-sectional distribution of plants' employment growth rates. The proportions of plants in both distributions are calculated on the basis of the numerical model solution using the benchmark calibration.

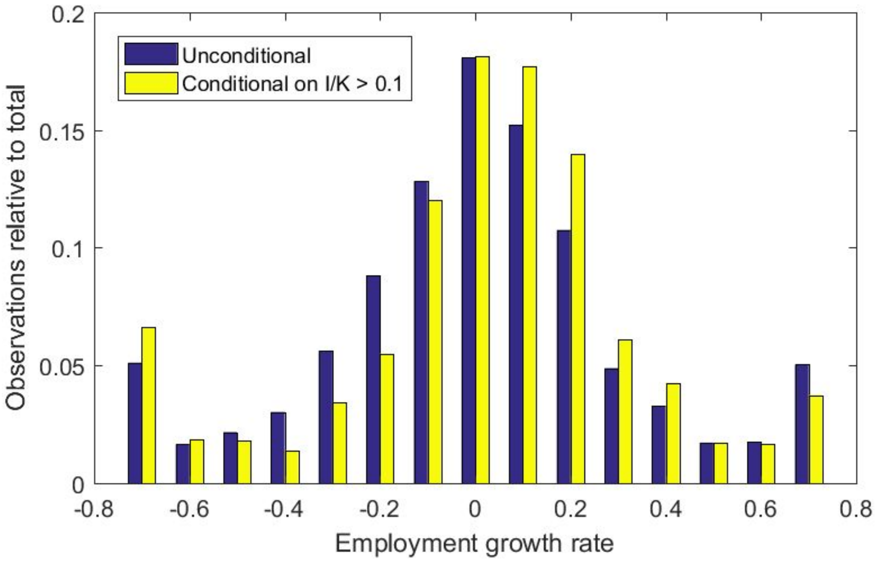
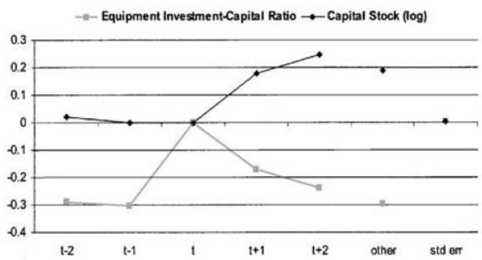
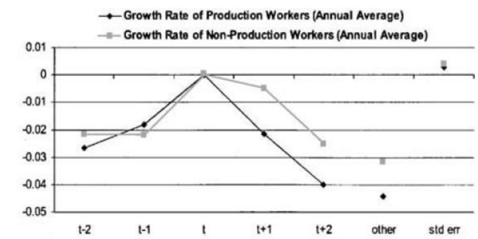


Figure 4.6: Optimal policy regimes over the endogenous state space

Notes: The figure represents distributions of employment growth rates over the cross-section of plants in the model economy. Blue bars show the unconditional distribution. Yellow bars depict the distribution conditional on investment rate being greater than 10%.



(a) Investment rate and capital



(b) Employment growth rate

Figure 4.7: Changes in variables around investment spike

Notes: The figures represent the mean values of investment rate, stock of physical capital (LHS) and employment growth rate (RHS) calculated in time-proximity to an investment spike event. The estimated values are reported as differences from the level in the year of the event. The figures are obtained from the study by Sakellaris (2004) and are estimated using the dataset for a large sample group of U.S. manufacturing plants documented from 1974-1991.

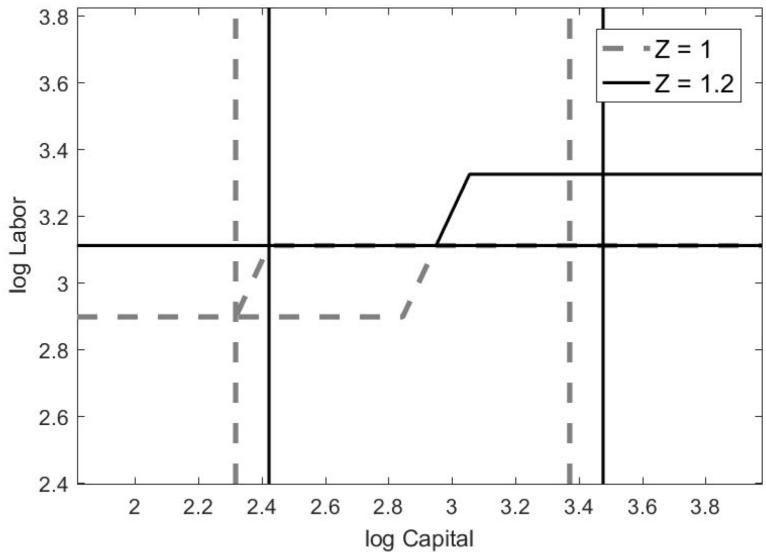


Figure 4.8: Regions of policy regimes for different levels of demand

Notes: The figure represents regimes of plants' optimal policies for investment (I) and employment growth (ΔL) over the endogenous state space for different levels of demand. Technology follows the CES production function. The level of labor productivity is set equal to its median value. Characterization of the respective regions is given in figure 4.2.

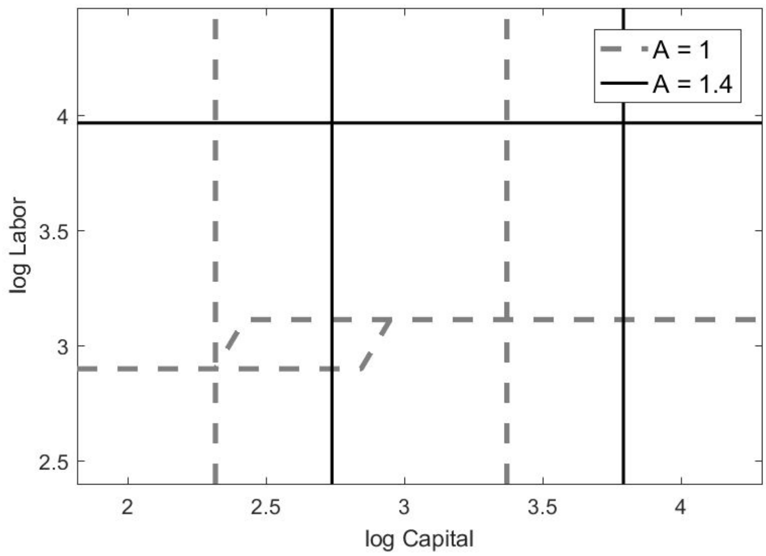


Figure 4.9: Regions of policy regimes for different levels of labor productivity

Notes: The figure represents the regimes of plants' optimal policies for investment (I) and employment growth (ΔL) over the endogenous state space for different levels of labor productivity. Technology follows the CES production function. The level of demand is set equal to its median value. Characterization of the respective regions is given in figure 4.2.

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Abstract

This dissertation addresses the topic of firm heterogeneity and its role in the macroeconomy.

The first paper presents empirical evidence on the inter-sectoral differences in magnitudes of firm entry, growth and exit. Two main patterns of firm dynamics emerge from the data: one sector - manufacturing - displays low firm entry and exit rates and high volatility of age-specific net job creation rates of firms; several broad sectors display the opposite pattern. At a more disaggregated industry level, I document a substantial degree of variation in the rates of plant entry and exit. The correlation coefficients indicate that the magnitudes of sunk entry costs and capital intensity of production are inversely proportional to the relative number of plant exits.

The second paper evaluates the role of firms' entry and exit in the business cycle dynamics of an environment where physical capital is partially sunk. Extending a heterogeneous-firm model a la Hopenhayn (1992) to include aggregate productivity shocks and partially irreversible investment yields substantial endogenous amplification and propagation. A positive aggregate productivity shock increases the number of entrants and their initial investment levels, because the expected entry value outweighs the implicit sunk cost associated with investment irreversibility. The endogenous propagation of an exogenous stimulus arises via a built-in selection mechanism, as the production growth of new businesses over their life-cycle exceeds the decay due to the exits of the least productive firms.

The third paper is a joint work with Monika Merz. We study the economic determinants of the interrelated factor adjustment of heterogeneous plants. We take as our starting point stylized facts documented for the empirical distributions of investment, hiring and separations at the micro level. We augment the heterogeneous plant model of Hawkins et al. (2015) by allowing for random changes in the factor substitutability of the production technology. We use the model to calculate moments of the joint distribution of factor adjustments in US manufacturing, which are not documented by the empirical literature. In our model, 13% of plants fire workers and invest at the same time, while contributing 38% of aggregate investment.

Zusammenfassung

Diese Dissertation besteht aus drei Aufsätzen zur Analyse der heterogener Firmen und ihrer Rolle für die Makroökonomie.

Der erste Aufsatz präsentiert empirische Evidenz zu den Unterschieden im Eintritt, Wachstum und Austritt von Firmen zwischen den Sektoren. Aus den Daten ergeben sich zwei Muster der Unternehmensdynamik: Ein Sektor - das verarbeitende Gewerbe - zeigt niedrige Ein- und Ausstiegsquoten von Unternehmen und hohe Volatilität der altersspezifischen Nettobeschäftigungsquoten von Unternehmen. Andere Sektoren zeigen entgegengesetzte Muster. Auf einer stärker disaggregierten Sektorenebene dokumentiere ich einen erheblichen Unterschied in den Ein- und Ausstiegsquoten von Unternehmen. Die Korrelationskoeffizienten deuten an, dass die Höhe der gesunkenen Einstiegskosten und der Kapitalintensität der Produktion umgekehrt proportional zu einer relativen Anzahl von Firmenausgängen sind.

Der zweite Aufsatz bewertet die Rolle des Ein- und Ausstiegs von Unternehmen für die Dynamik des Konjunkturzyklus in einem Umfeld, in dem Investitionen in physisches Kapital teilweise unumkehrbar sind. Die Erweiterung eines Modells mit heterogenen Firmen a la Hopenhayn (1992) um aggregierte Produktivitätsschocks und teilweise unumkehrbare Investitionen führt zu einer wesentlichen Verstärkung und Fortpflanzung der Dynamik. Ein positiver Produktivitätsschock erhöht die Anzahl der Unternehmen und deren anfängliches Investitionsniveau, da der erwartete Einstiegswert die impliziten Kosten im Zusammenhang mit den unumkehrbaren Investitionen überwiegt. Die endogene Fortpflanzung eines exogenen Stimulus erfolgt über eine eingebaute Auswahlvorrichtung, da das Produktionswachstum neuer Unternehmen über ihren Lebenszyklus den Zerfall aufgrund der Austritte der am wenigsten produktiven Unternehmen übersteigt.

Der dritte Aufsatz wurde zusammen mit Monika Gehrig-Merz verfasst. Wir untersuchen die ökonomischen Determinanten der zusammenhängenden Faktoranpassung bei heterogenen Firmen. Wir bauen unsere Analyse auf empirischen Fakten auf, die für die statistische Verteilung von Investitionen, Einstellungen und Trennungen auf Mikroebene dokumentiert sind. Wir erweitern das Modell heterogener Firmen von Hawkins et al. (2015) um zufällige Änderungen der Faktorsubstituierbarkeit der Produktionstechnologie. In dem Modell berechnen wir Momente der gemeinsamen Verteilung von Faktoranpassungen in dem US-amerikanischen verarbeitende Gewerbe, die in der empirischen Literatur nicht dokumentiert sind. In unserem Modell trennen sich 13% der Firmen von Arbeitnehmern und investieren gleichzeitig. Diese Firmen tragen 38% zu den Gesamtinvestitionen bei.

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Firm entry and exit, investment irreversibility, and business cycle dynamics.
July 2019

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June 2019

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