universität wien

DISSERTATION / DOCTORAL THESIS

Titel der Dissertation /Title of the Doctoral Thesis

„Combinatorial Auction-based Allocation of Collaborative Transportation“

verfasst von / submitted by

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angestrebter akademischer Grad / in partial fulfilment of the requirements for the degree of

Doctor of Philosophy (PhD)

Wien, 2019 / Vienna, 2019

Studienkennzahl lt. Studienblatt / degree programme code as it appears on the student record sheet:	A 794370146
Dissertationsgebiet lt. Studienblatt / field of study as it appears on the student record sheet:	Economics/Business Administration: Management
Betreut von / Supervisor:	o. Univ.-Prof. Mag. Dr. Rudolf Vetschera

Abstract

The rapid development of the transportation industry has motivated companies and researchers to develop methods in order to reduce costs and increase productivity. This has led collaborative relationships to become one main trend in transportation. This work focuses specifically on the study of request exchange mechanisms that offer carriers the opportunity to improve their sets of requests and obtain efficiency gains.

First, a new type of problem scenario is introduced, combining various attributes: a pickup and delivery problem with multiple regions, multiple depots and multiple transportation modes. This includes an extensive literature review and mathematical models from simple and well-known scenarios to the multi-region multi-depot pickup and delivery problem. Second, regression-based estimation models are presented. These models provide fast predictions for the travel distance for various problems. This is very helpful for evaluating the costs for servicing requests. Finally, frameworks for auction-based and posted price request-exchange mechanisms are presented. All mechanisms are tested within the same scenario and their results are compared aiming to obtain insights regarding the efficiency gains they offer and the amount of information required for their application.

Zusammenfassung

Die schnelle Entwicklung der Transportindustrie hat Unternehmen und Forscher motiviert, Methoden zu entwickeln, um Kosten zu senken und die Produktivität zu steigern. Dies hat Kooperationsbeziehungen zu einem Haupttrend in der Transportsforschung gemacht. Diese Arbeit konzentriert sich speziell auf die Untersuchung von Anfrage-Austauschmechanismen, die Transportunternehmen die Möglichkeit bieten, ihre Anfragesätze zu verbessern und Effizienzgewinne zu erzielen.

Zunächst wird eine neue Art von Problemszenario vorgestellt, in dem verschiedene Attribute kombiniert werden: ein Abhol und Zustellproblem mit mehreren Regionen, mehreren Depots und mehreren Verkehrsträgern. Dazu gehören eine umfassende Literaturrecherche und mathematische Modelle von einfachen und bekannten Szenarien bis hin zum Multi-Region-Multi-Depot-Abhol-und-Zustellproblem. Zweitens werden auf Regression basierende Schätzmodelle vorgestellt. Diese Modelle bieten schnelle Vorhersagen für die Fahrstrecke bei verschiedenen Problemen. Dies ist sehr hilfreich für die Wertung der Transportanfrageskosten. Abschließend werden Frameworks für Auktionsbasierte und Festpreis Anfrage-Austauschmechanismen vorgestellt. Alle Mechanismen werden im selben Szenario getestet und ihre Ergebnisse verglichen. Ziel ist es, Einblicke in die Effizienzgewinne zu erhalten, die sie dem Netzwerk bieten, sowie in die Informationsmenge, die für ihre Anwendung erforderlich ist.

A la familia.

Preface

This thesis is a collection of most of the work I have carried out in the last few years. As part of the FEAT (Fair and Efficient Allocation of Transportation) project, my motivation was to look for different ways of collaboration between carriers.

As an initial study, my project colleagues and I put together an extensive literature review, presented a typology and a general model for multi-region routing problems. We also proposed future (by now, some past and present) research topics, which include collaborative approaches. This first paper is presented as the second chapter of this dissertation.

Evaluating requests, in order to obtain their costs, is a fundamental part of collaboration mechanisms. Depending on the level of collaboration, this process might incur very large computational times. The third chapter of this dissertation presents regression-based models that include distances, time windows, as well as quantities and capacities. These models provide estimations for the total travel distance for various problems, with a relatively low error rate, very quickly.

The fourth chapter of the dissertation presents auction-based and posted price mechanisms for the exchange of requests. The mechanisms were tested within the same scenario. Individual and centralized auction-based mechanisms yielded similar results on average, outperforming the posted price mechanisms. Nevertheless, the latter have some properties that could be considered advantageous under some assumptions, mainly related to trust issues and strategic behavior opportunities.

This dissertation can be read altogether or, if you consider chapters 2-4, one chapter at a time. I hope you enjoy it.

Acknowledgements

I want to start by thanking my supervisor Prof. Rudolf Vetschera for giving me the chance to work with him. His suggestions, comments and guiding have been fundamental for getting things done. Thanks for always being open for (sometimes silly) questions and very useful discussions.

I would also like to thank the FEAT project team and the OPIM research group for listening to me in every meeting/presentation and providing some great insights and feedback to improve my research outputs. A big shout-out to the secretaries, for their enormous and helpful work, they definitely made my life easier.

My huge appreciation to all the colleagues I have met during this journey for the fun moments, drinks, meals and chats we have shared.

Last, but most important, thanks to Juan, Graciela, Laura and Alicia, for always having my back, and for all the love and support they have given me day after day.

Publications

Parts of this dissertation have been published or submitted for publication.

Chapter 2 is published as:

- Dragomir, A. G., **Nicola, D.**, Soriano, A., & Gansterer, M. (2018). Multidepot pickup and delivery problems in multiple regions: a typology and integrated model. *International Transactions in Operational Research*, 25(2), 569-597.

Chapter 3 is published as:

- **Nicola, D.**, Vetschera, R., & Dragomir, A. (2019). Total distance approximations for routing solutions. *Computers & Operations Research*, 102, 67-74.

Chapter 4 has been submitted as:

- **Nicola, D.** (2019). Mechanisms for Request Exchange in Collaborative Transportation. *Manuscript submitted for publication.*

Additionally, some material was published as:

- **Nicola, D.** (2019). Collaborative Transportation Mechanisms: A Request Exchange Approach. *19th International Conference on Group Decision and Negotiation, Local Proceedings*, June 11-15, 2019, Loughborough, UK, paper 73.

- **Nicola, D.** (2017). Combinatorial Auction Mechanism for Allocation of Transportation in Collaborative Networks. *Doctoral Consortium, Group Decision and Negotiation*, 14 August 2017, Stuttgart-Hohenheim, Germany, 51-57.

Contents

List of Tables

List of Figures

Chapter 1

Introduction

The transportation industry is a highly competitive environment in which companies need to maintain or lower prices; leading to the reduction of costs in order to obtain satisfactory profit levels. Companies that are not productive enough usually struggle to stay in business, in fact, many transportation companies have been closed due to low revenue/cost ratios [87].

From an operational perspective, one way to contribute to cost reduction is to develop solution methods that aim to optimize the use of resources. Collaboration mechanisms also present several options that help increase efficiency. For example, large companies could hire smaller shippers to perform part of their requests; similar size firms could benefit from sharing vehicles, depots and even human resources.

In less-than-truckload transportation, companies might also look for other approaches to improve their sets of customers. This would allow them to focus on the ones that offer a higher marginal profit, i.e. the ones that can be serviced at lower costs by the company. One particular issue is the cost caused by returning trips, which are usually empty and do not generate any revenue. These empty trips affects companies and the environment, by increasing emissions of harmful elements as well as other externalities, such as noise pollution and traffic. Thus, the avoidance of empty trips helps achieve economic and environmental goals.

In this work, the main focus is the study of collaborative mechanisms to reallocate requests between carriers that operate in the same geographical regions, looking to improve the overall efficiency.

1.1 Request-exchange Mechanisms

A request refers to the transportation of one package from a pick-up node to a destination node. Depending on the routing problem, a request might also have other attributes like time windows, demands, etc. The cost of servicing a request might be different for different carriers, i.e. requests have no standard value. Moreover, a request that is costly for one carrier might be serviced by another carrier at a relatively low cost. This cost difference is the main motivation for the use of request-exchange mechanisms.

The cost of servicing a request not only depends on the carrier's resources costs (e.g. vehicles, depots, personnel) but also on other requests being serviced by the carrier due to interdependencies or synergies that exist between them. These synergies might be different according to the specific problem. For example, consider a simple routing problem with two requests that are geographically close from each other. The costs for servicing both requests together by the same vehicle in the same tour are a lot lower than servicing the same requests in different tours, therefore, the synergy between these two requests would be very high. In different scenarios, the synergy between the same two requests could be lower. For example, if time windows and demands are added to the picture. Very different time windows might result in long waiting times and large demands might require different tours when servicing the requests. Furthermore, there is a strong motivation for grouping or bundling requests that are being exchanged, in order to exploit the synergies between them.

Carriers need to evaluate their own requests and decide which requests to exchange, how to construct bundles, which requests to obtain, and perhaps more important, prices to pay and charge for this exchange. All these decisions will depend on the mechanism being used for exchange. Mechanisms can not only be compared in terms of efficiency. As the decisions being made are collective, informational aspects are also important. Furthermore, the trade-off between efficiency gains and information requirements is explored for the proposed mechanisms. Depending on the party setting the prices, two main types of mechanisms are considered here: auction-based and posted price mechanisms.

1.1.1 Auction-based Mechanisms

An auction is a market institution with explicit rules that determine the allocation of resources based on bids from the market participants [56]. For the specific case of request-exchange mechanisms, carriers bid on requests they want to service. Therefore, it is better to select the lowest bid, i.e. the carrier with the lowest cost for servicing the requests.

As mentioned in Section 1.1, synergies between requests should be exploited by bundling them. For this reason, combinatorial auctions are used. Combinatorial auctions are a special type of auctions in which bidders can place bids on combinations of items being bought/sold. For this work, a centralized and a individual approach are included. In the first, a central institution plays the role of the auctioneer and re-allocates requests to all the participants. The bundles of requests might contain requests belonging to more than one carrier. In the latter, each carrier is the auctioneer of its own requests and allocates its requests to the other participants. In both cases, carriers have to specify their bid to acquire the transport requests. Carriers who pass on requests might provide reservation prices.

1.1.2 Posted Price Mechanisms

Posted price mechanisms differ from auction-based mechanisms in one fundamental aspect: the prices for requests are set by the carriers passing them on. The new assignment of bundles to carriers aims to decrease operational costs, this is a well known problem in operations research, called the assignment problem. The solution for this problem reduces to solving an integer linear program. One early example of the problem can be found in [76].

For this type of mechanisms, carriers need not to reveal their costs but only specify if they are willing to service a bundle of requests at the price determined by the carrier who owns the bundle. From an informational point of view, except for the carrier owning a bundle, carriers do not need to reveal their specific costs, therefore, less is revealed by the carriers when using these mechanisms compared to auction-based mechanisms.

1.2 Dissertation Structure

The main focus of this dissertation is the study of request-exchange mechanisms. Since the effects of such exchange mechanisms might depend on the type of problems they are applied to, the first part of the dissertation deals with an overview of these problems, considering realistic scenarios. Second, cost estimation methods for some of these problems are studied. Specifically, approximation models are proposed and tested. Finally, frameworks for various mechanisms are proposed, implemented, tested and compared. The entire work has been divided in three successive chapters, which can be read all together, but the way they are presented allows them to be read as stand-alone work. The contents of each chapter are described as follows.

Chapter 2 starts by providing definitions that are necessary in transportation problems. It presents the transition from single-region multi-depot problems to multi-region pickup and delivery problems, going through all the basic mathematical models and existing literature in each field, including information about already used instances. All models focus on the travel distance as a main component of the (variable) costs. This chapter aims to provide a clear understanding of the market for which collaboration mechanisms are proposed. An examination of possible extensions to the presented problems is also included.

Chapter 3 focuses on obtaining estimations for the travel distance for various problems. Carriers involved in a competitive bidding/pricing process need rapid information about costs. Solving routing problems might require high computational efforts and time, therefore, estimation models that achieve good and fast approximations of travel distances are required. Models are proposed for various problems ranging from the a simple traveling salesman problem to a two-region multi-depot pickup and delivery problem. Models are adjusted to estimate optimal solutions and also lower quality solutions, depending on the problem.

Chapter 4 presents request-exchange mechanisms. Every mechanism is described along with considerations that are taken for their application. Requests are evaluated for bidding or for pricing, depending on the mechanism. This evaluation is performed by estimation models that are based on models proposed in Chapter 3. Variables included in these models are considered as request selection and bundling generation criteria. Variants of auction-based and posted price mechanisms are tested in a two-region scenario, in which requests have a pickup node and a delivery node in different regions. The results are compared in terms of efficiency gains and number of requests being exchanged in one-round and multi-round settings. The aim of this chapter is to obtain insights regarding the efficiency improvement and the amount of information required by every mechanism.

Chapter 5 contains concluding summaries of the included papers.

Chapter 2

Multi-depot pickup and delivery problems in multiple regions: A typology and integrated model

Abstract

The rapid development experienced by the transportation industry in the last decades has led to many configurations of networks and therefore to an explosion of variants in transportation problems, motivating researchers to look at broader logistic problems, beyond the basic vehicle routing problems. This work introduces a new type of problem scenario combining various attributes: a pickup and delivery problem with multiple regions, multiple depots and multiple transportation modes. We provide definitions, a literature review and a step-by-step construction of the mathematical models from a simple and well-known scenario to the multi-region multi-depot pickup and delivery problem (MR-MDPDP). For each step the relevant literature is examined. Furthermore we suggest possible extensions for prospective research.

2.1 Motivation

When thinking about realistic transportation networks four main features come to mind: multiple depots, multiple regions, paired pickup and deliveries, and multi-modality. The first three are the core pillars of the problems we are examining. The literature so far has mostly looked at transportation problems within one single region, although in reality transportation between different regions is common practice since the beginning of modern globalization. Also, the so often made assumption that the routing problem can be simplified to a vehicle routing problem might be justified from an academic point of view but is rather oversimplified for practical applications. Moreover, almost all companies operating on a somewhat larger scale will have a network structure that contains at least two depots, especially if transportation takes place between two geographically separated regions. The multi-modality aspect usually occurs by default by operating in different regions and by the need for a switch of transportation modes for the last-mile delivery.

The great advances in infrastructures, computation power and solution algorithms have increased the possibilities for all carriers, specially when facing long distance services. Therefore, we believe that research in the transportation field should start to look at more complex problem structures, like the ones described in this work. This paper is aimed to serve as an introduction of this new kind of problems that have been hardly studied in their entirety in the literature. Furthermore, we want to ascertain which works consider at least part of the main building blocks. We are therefore specifically looking at multi-depot pickup and delivery problems in multiple regions to provide a foundation for further research in this field.

This research will not only benefit big transportation companies, but also by small carriers who are willing to form a collaborative network system to be able to compete with those big companies in the long distance transportation market. The paper presents the transition from single-region multi-depot problems to multi-region pickup and delivery problems, going through all the basic mathematical models and existing literature in each field. This aims to provide a better understanding of the underlying multi-region transportation problem. Our goal is to gradually introduce the building blocks of our problem to allow the reader a deeper understanding of it. Our basis are multi-depot pickup and delivery problems.

The paper is structured as follows: in Section 2.2 we will give a short problem introduction including all necessary definitions. Section 2.3 gives an overview of the relevant literature. Section 2.4 and 2.5 introduces in detail different variations of multi-depot pickup and delivery problems in one or several regions. We will present mathematical models for each problem variation and give a literature review over work already done in these areas including information about already used instances. In Section 2.6 we conclude our work and in Section 2.7 we examine possible extensions to the basic problem presented before.

2.2 Problem introduction

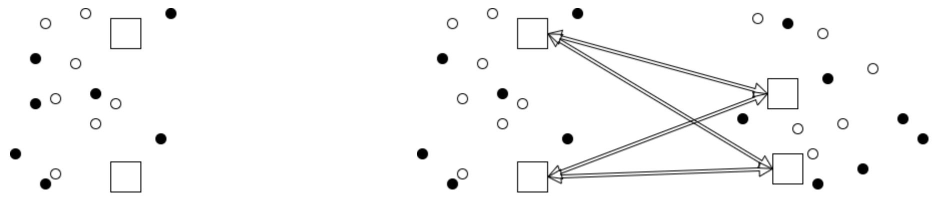
Before going into detail we need to provide some definitions:

We assume a two-dimensional plane. All nodes within this plane are represented by a complete graph $G(V, E)$, where V represents a set of nodes and A represents a set of arcs connecting them. A single arc connects a node i to a node j, with a transportation cost of d_{ij}. This cost is usually represented by the distance between both end points of the arc.

Our definitions are based on the nomenclature provided by Parragh et al. [66]. We define a **region** as a group of nodes. The nodes in different regions are geographically separated. A **pickup node** is a customer, located at some specific location, where goods have to be collected by a vehicle. A **delivery node** is a customer where goods have to be dropped off. In unpaired problems, the goods are homogeneous and some quantity is collected and dropped off. There is no need to deliver specific goods. We are either looking at unpaired or at paired pickup and delivery nodes. For **unpaired nodes** the goods collected or dropped off are either transported between customers or have the depot as their origin or destination. **Paired nodes** are called a **request**. A request can have its pickup and delivery node within the same region or in different regions. A request therefore consists of exactly one pickup and one delivery node with a specific quantity. The pickup node has to be served before the delivery node. Since the goods are considered heterogeneous, the delivery node expects exactly these goods and not any goods just like it. Two modes of transportation are considered. **Short-haul vehicles** are used within a region. They are of small capacity and are mainly used for the last-mile delivery. **Long-haul vehicles** are used for transporting goods between regions. They allow the consolidation of goods and have a much bigger capacity than the short-haul vehicles. Long-haul vehicles can be slow, fast, cheap or expensive in any combination. A **depot** is the place where a vehicle starts and ends their tours. A tour starts and ends at the same depot. Also, vehicles are assumed to stay at the depot when they are not being used.

Figure 2.1 shows two cases of regions. All figures in this work use the following representation: the squares are depots, the circles are nodes. An empty (white) circle is a pickup node, a full (black) circle is a delivery node. Routes fulfilled by a short-haul vehicle are depicted as dashed lines, long-haul routes are double lines.

In general, the multi-depot multi-region version of the vehicle routing problems (VRP) is closely related to network design problems (including location routing problems), two- or multi-echelon VRPs, multi-modal transportation and VRP with intermediate facilities. All of these topics are closely related to real-world problems and have been studied in the literature. However, no clear problem definition or consistent typology exists. Our goal is to provide structure and formulations for this generalization of existing problems. For a literature review on VRPs with multiple depots we refer the interested reader to the publication by Montoya-Torres et al. [58]. The works mentioned there build the basis for the problem we are examining but they do not contain the core characteristics



(a) One region with 2 Depots (b) Two regions with 4 depots and long-haul

Figure 2.1: One Region and Two Regions Pickup and Delivery Representations
Figure 3.1a shows one region with two depots where all nodes can be served by either depot. Figure 3.1b shows two regions with two depots each. The depots belonging to different regions are connected by long-haul lanes. The delivery node of a corresponding pickup can be in the same or in another region. Black nodes are pickup points, while white nodes are the paired delivery points.

of examining solely pickup and delivery problems or dealing with multiple regions. For a literature review on multimodal freight transportation planning we refer the reader to the work by Steadie Seifi [80]. Multi-modal problems often incorporate multiple depots, pickup and delivery problems, or multiple regions but almost never all three. The literature review by Montoya-Torres et al. [58] gives a very good overview but does not integrate all building blocks.

2.3 Literature

Our goal was to find literature that fits exactly to our problem: multiple depot pickup and delivery problem with inter-depot routes in multiple regions. However, to the best of our knowledge, the exact problem we are studying here, has not been yet addressed in the existing literature. Therefore we set the boundary at literature dealing with multi-depot problems with an underlying pickup and delivery routing problem. We loosened this boundary if a paper dealt with other important characteristics (inter-depot routes or multiple regions) even if it was "only" a vehicle routing problem. Therefore we also included in our literature multi-depot VRPs if they seemed fitting and had an overall network structure that contained some features of the problem classes. Because of this, literature about the two-echelon routing problem and work dealing with multiple modes became relevant for this work. We want to give structure to a new problem class because there has already been some work done but it seems uncoordinated and looked at from very different point of views. We collected the relevant work to show the potential of this problem. Most of the work mentioned here used real-life instances provided by a company which had a need for a solution. Therefore the importance becomes evident.

Table 2.1 gives an overview on the included literature. The individual papers are presented in detail in their corresponding sections. The papers are sorted first by the section they appear in this work and second by year of publication.

The first block of columns shows the underlying routing problem. This refers to the way the routing is dealt with within a region. We divide the papers according to the following classification: VRPs (**VRP**), paired pickup and delivery problems (**paired P&D**), pickup and delivery problems with backhauls (**backhauls**), and pickup and delivery problems with mixed backhauls (**mixed backhauls**). In the standard VRP each customer node has a certain quantity. It does not matter if this quantity has to be delivered or picked up since it is the same type for all nodes. Please note, that we did include literature that was similar enough to our problem, even if it only dealt with the standard VRP. The paired pickup and delivery problem (see the definitions by Parragh et al. [66]) contains for each transportation request a pickup and delivery node that belong together. The delivery can be done directly (within the same tour as the pickup) or be forced to go over a depot. The VRP with backhauls or mixed backhauls belongs to a different class of problems. Here, all goods that are delivered to a customer are loaded at the depot and all goods that are picked up at a customer are unloaded at the depot. There is no link between the pickup and delivery nodes (see the definitions by Parragh et al. [65]). In the mixed backhaul problem the pickup and delivery at the customers can be fulfilled in any sequence. However the standard backhaul problem (also called the VRP with clustered backhauls) specifies that all deliveries have to occur before the first pickup.

The second block of columns shows constraints and other problem specifications. Only constraints that were mentioned in the relevant literature are included in the table, apart from the vehicle capacity constraint, which was excluded because it appeared in every paper. These are however not all possible constraints since many problem variations have not been studied yet. For further suggestions we refer the reader to Section 2.7. The third block of columns shows whether the solution method applied was an exact method or a heuristic. Because of the difficult nature of the problems it is not surprising that heuristics were used more often. Also, it shows the number of locations (nodes) for the biggest instances solved. If only the number of requests where given, the assumption is made that each request consists of two locations and this number was added to the table.

2.4 Single region problems

Single region problems are the basic building block for our new problem class. According to the previous definition of region, single-region problems encompass the vast majority of families of VRPs studied so far. The main elements of this setting are a set of customers or nodes that are to be visited and a set of depots from and to which routes start and arrive. Many configurations and restrictions of this setting can be found in the literature, leading to a broad spectrum of problems. The most representative problems of this setting are the classical travelling salesman problem and VRPs. Both problems have one unique depot. In Laporte [55], the VRP is described in detail and solution algorithms for the VRP are presented. For the purpose of this paper we are interested in problems where the number of depots is greater than one. Routing problems with one depot can be extended to several depots, leading to the family of multi-depot problems. One of the classic problems among them is the multi-depot VRP, which is an extension of the aforementioned VRP. A formal description of this type of problem can be found in the works by Polacek et al. [70] and

	Year	VRP	paired P&D	backhauls	mixed backhauls	capacitated depots	limited number of vehicles	time windows	heterogeneous vehicles	multimodality	compatibility constraints	exact	heuristic	instance size	section
Currie and Salhi [23]	2003	✓				✓	✓	✓	✓			✓	✓	200	2.4
Polacek et al. [70]	2004	✓					✓	✓					✓	288	2.4
Dondo and Cerdá [29]	2007	✓						✓	✓				✓	100	2.4
Min et al. [57]	1992			✓		✓							✓	161	2.4.1
Irnich [50]	2000			✓			✓	✓	✓				✓	484	2.4.1
Nagy and Salhi [61]	2005				✓		✓						✓	249	2.4.1
Bettinelli et al. [9]	2014		✓				✓	✓	✓			✓		144	2.4.2
Detti et al. [28]	2016		✓			✓		✓	✓		✓		✓	246	2.4.2
Gendron and Semet [41]	2009	✓				✓			✓			✓		701	2.4.3
Perboli and Tadei [68]	2010	✓				✓	✓			✓		✓		50	2.4.3
Crainic et al. [20]	2011	✓				✓	✓			✓			✓	50	2.4.3
Perboli et al. [69]	2011	✓				✓	✓			✓			✓	50	2.4.3
Hemmelmayr et al. [47]	2012	✓				✓	✓			✓			✓	50	2.4.3
Nguyen et al. [62]	2012	✓				✓				✓			✓	200	2.4.3
Ghilas et al. [42]	2016		✓				✓	✓				✓		50	2.4.3
Ghilas et al. [43]	2016		✓			✓	✓	✓	✓	✓			✓	100	2.4.3
Breunig et al. [11]	2016	✓				✓	✓			✓			✓	200	2.4.3
Bard et al. [3]	1998	✓					✓					✓	✓	20	2.4.3.1
Angelelli and Speranza [1]	2002	✓				✓	✓						✓	150	2.4.3.1
Crevier et al. [22]	2007	✓					✓						✓	288	2.4.3.1
Goel and Gruhn [44]	2008	✓					✓	✓	✓		✓		✓	2500	2.4.3.1
Muter et al. [60]	2014	✓					✓					✓		50	2.4.3.1
Salhi and Sari [75]	1997	✓							✓				✓	360	2.7.3
Ceselli et al. [15]	2009	✓				✓	✓	✓	✓		✓	✓		100	2.7.4

Table 2.1: General classifications of the literature sorted by section and year

Renaud et al. [71]. The main elements of these settings remain the same as for the single-depot problems, only the graph $G(V, E)$ is extended to include all depots. The many existent typologies of problems in this family arise from the diversification of factors like the nature of customers, their relation, time constraints and many others. In this work we focus on pickup and delivery problems. We present some of the more relevant variants of this problem within the single-region setting, along with their mathematical model, before extending them to the multi-region setting.

Multi-depot vehicle routing problems

Dondo and Cerdá [29], consider a heterogeneous fleet of vehicles in a multi-depot scenario with time windows. The vehicles differ in capacity and speed. The underlying routing problem is a capacitated vehicle routing problem. Although they mention both pickups and deliveries, only one option is chosen at a time. The proposed method is a region-based optimization including three phases. In the first phase, a set of cost effective regions is identified, while in a second phase, regions are assigned to vehicles and tours are sequenced using a region-based MILP formulation. The third phase orders nodes within regions and schedules arrival times for the vehicles at customer locations by solving a MILP model. They test the computational performance of their algorithm on instances from Solomon's work found in [77]. In addition, new test instances have been generated. They have up to 100 nodes and can solve many problems up to optimality or near-optimality. Even though the work deals only with a simple vehicle routing problem it is mentioned because of the region based solution approach which makes it interesting for our topic.

Another application presented by Currie and Salhi [23] is a full-load pickup and delivery problem with time windows. They have heterogeneous vehicles and products. In their problem they consider the delivery requirements of a large construction company. Goods have to be transported from multiple depots to a large number of customers. Since the problem consists only of deliveries it can be classified as a capacitated vehicle routing problem. The problem ignores vehicle load splits as they very rarely occur. Trips are made between locations, where the product is collected, and the depots. Each location is visited exactly once, as several requests for the same location are considered as a single requests for a location. Loading and unloading times are assumed to be constant, irrespective of the product to be transported. A constant cleaning time for vehicles, as well as waiting times might be added to the routes. 'Dayworks', which are situations in which the vehicles remain at a location for the rest of the day after their arrival, are also included. External vehicles might be hired by the company if the fleet is not sufficient to perform all trips.

To solve the problem described in Currie and Salhi's work [23], the authors present a first formulation as a 0-1 LP followed by a hybrid algorithm that chooses between a greedy heuristic and one based on regret costs. The greedy heuristic uses an index of stringency to select the next request for insertion, where requests with the lowest index values are assigned to vehicles first. Stringency includes different attributes such as quantity and length of time windows. The regret

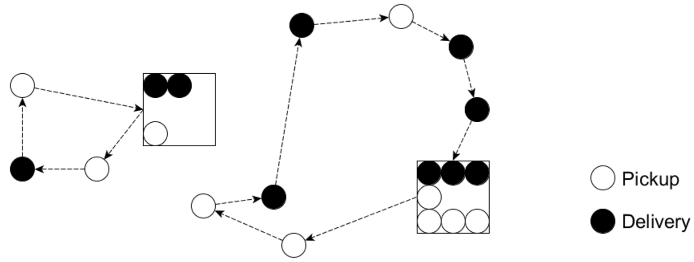


Figure 2.2: A single-region multi-depot problem with mixed backhauls
For every pickup in a tour, a delivery is done at the depot, and for every delivery in a tour, the goods are picked up at the depot as well. Note, that the pickup and delivery sequence within the tour can be mixed.

costs method calculates a penalty cost for each request, which is obtained from the cost of forfeiting the opportunity of inserting the request in its best slot. They randomly generate their instances and solve instance sizes up to 200 location sites and 500 request. These instances can be found on: *http://www.mat.bham.ac.uk/S.Salhi/others.*

2.4.1 Multi-depot vehicle routing problem with mixed backhauls

The multi-depot vehicle routing problem with mixed backhauls consists of a set of unpaired pickup and delivery customers. Unpaired nodes are independent of each other, there is no relationship between the pickup and the delivery locations. By definiton this means that all pickup goods are transported to a depot, and all delivery goods have to be loaded in the vehicle at a depot.

The underlying routing problem has been defined in different ways in the literature. In [65], Parragh et al. refer to the problem as VRP with Mixed Linehauls and Backhauls, while Salhi et al. name it Multiple Depot VRPs with Backhauling in [74]. Traditionally, the VRP with backhauls has been associated with pickup and delivery problems where pickup and delivery customers are not paired and a node has positive or negative quantities. In the simplified version (with clustered backhauls) the delivery points must always be visited before the pickup of goods to avoid infeasibility concerning the vehicle capacity restriction. However, in this section we present the more general case where no precedence relations have to be satisfied, that is, pickup and delivery points can be visited in any order. Therefore, we want to differentiate from the pure backhauling problems by referring to the problem as multi-depot VRP with mixed backhauls (MDVRPMB). Figure 2.2 gives a visual representation of the VRP with mixed backhauls can be found .

As described above, the main characteristics of this problem are that the nodes in graph $G(V, E)$ could be either pickup or delivery customers, and that they can be visited in any order. The objective is to visit all customers at minimum cost while satisfying the capacity restrictions of the vehicles. A set of vehicle is defined in advance, but it is assumed that this number is sufficient to fulfill all transportation requests.

The mathematical model is based on the work by Montoya et al. [58].

Sets and Parameters

N = set of customers,

L = set of depots,

V = set of all nodes, $N \cup L$,

K_l = set of vehicles in depot l,

K = $\cup_{l \in L} K_l$ = set of all vehicles,

$$q_i = \text{quantity load for the node} \begin{cases} +q_i, & \text{if it is a pickup customer,} \\ -q_i, & \text{if it is a delivery customer.} \end{cases}$$

Q^{SH} = capacity of each vehicle,

c_{ij} = travel cost between nodes i and j.

Decision Variables

$$x_{ijk} = \begin{cases} 1, & \text{if vehicle } k \text{ travels directly from node } i \text{ to node } j, \\ 0, & \text{otherwise.} \end{cases}$$

S_{ik} = loading amount of vehicle k after visiting customer i.

For simplicity in the notation, copies of depot l representing start and end points of the routes are denoted as l^0 and l^1.

$$\text{minimize} \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} c_{ij} x_{ijk} \qquad \text{Subject to} \tag{2.1}$$

$$\sum_{i \in N} \sum_{k \in K} x_{ijk} = 1 \qquad \forall j \in V \tag{2.2}$$

$$\sum_{i \in V} x_{ihk} - \sum_{j \in V} x_{hjk} = 0 \qquad \forall k \in K, h \in N \tag{2.3}$$

$$\sum_{i \in V} x_{ilk} = \sum_{j \in V} x_{ljk} \leq 1 \qquad \forall l \in L, k \in K_l \tag{2.4}$$

$$S_{jk} = \sum_{i \in V} x_{ijk} \cdot (S_{ik} + q_i) \qquad \forall j \in V, k \in K \tag{2.5}$$

$$0 \leq S_{ik} \leq Q^{SH} \qquad \forall i \in V,\ k \in K \tag{2.6}$$

$$\sum_{i \in S} \sum_{j \in S} x_{ijk} \leq |S| - 1 \qquad \forall k \in K, S \subseteq N, |S| \geq 2 \tag{2.7}$$

$$x_{ijk} \in \{0, 1\} \qquad \forall i, j \in V, k \in K \tag{2.8}$$

$$S_{ik} \in \mathbb{N} \qquad \forall i \in V, k \in K$$

Equation (2.1) represents the objective function, which is to minimize costs. Constraint (2.2) ensures that each node is visited once, constraint (2.3) that the vehicle entering and leaving a node should be the same (route continuity). Each short-haul tour originates and terminates at the corresponding depot (2.4). Loading restrictions for backhauls (2.5). Constraint (2.5) makes the model non-linear. For simplification purposes, linearization of the constraint is not included in the model but on the appendix. Since both pickup and delivery nodes are visited in any order, it is not sufficient to sum up over all quantities within a route. After every visited node, the quantity of goods in the vehicle has to be within our bounds (2.6). Constraint (2.7) is used for sub tour elimination.

Related problems

Different types of this model (with certain variants and assumptions) have been worked on. For example, the MDVRP with backhauling, was early proposed by Min et al. in [57]. They do not consider mixed backhauls because they assume rear-loaded trucks for their problem where reshuffling of loads is tedious and not easily possible. Therefore all deliveries are fulfilled before the first pickup is allowed. Both their vehicles and depots have a limit on capacity. For the depots this means that they are able to only serve a limited amount of loadings and unloadings per day. Their main decision problems are to determine the fleet size (the number of available vehicles is not limited), to allocate the vehicles to the depots, to allocate the customers to vehicles and the routing of the vehicles.

Irnich [50] looks at a kind of multi-depot pickup and delivery problem. The author here considers multiple depots and heterogeneous vehicles. The goods are delivered to and from a hub, instead of the depots of the particular vehicles. This is because of the particularity that all pickup or delivery locations serve as the depots for the vehicles. We classified it as a problem with backhauls under the assumption, that the hub is actually the depot and the nodes are just simple customers. However it is clear that it does not quite fit since the hub does not correspond to the definition of a depot. Their main decision is the assignment of a request to a specific vehicle. The routing is far less important.

Nagy and Salhi [61], look at a multi-depot VRP with mixed backhauls and simultaneous pickup and deliveries where a customer can both receive and send goods at the same time. Although their work focuses mainly on single-depot problems, they manage to extend this approach to a multi-depot scheme. Their vehicles are capacitated and they have a maximum route length constraint.

Methods and instances

Min et al. in [57] decompose their problem and use a three phase heuristic method. Its first phase aggregates customers (delivery nodes) and vendors (pickup nodes) to capacitated regions; in the second phase, customers and vendors are assigned to a depot and a route; the third phase designs the individual vehicle routes. Each phase takes the output of the previous phase as an input. The data and instances used were provided by the transportation division of a large U.S. company and altered to ensure confidentiality. Their data contained 161 potential nodes to visit. The input data can be found in the appendix of their published paper.

Irnich [50] formulated their problem as a network model. The goal is to find a set of trips which fulfills all requests while minimizing costs. They use a set partitioning/covering model which implies a two phase algorithm. The solution approach first enumerates relevant route-vehicle combinations; then a set T of relevant trips for each route-vehicle combination are enumerated, checked for dominance, and, if dominated, eliminated from T. Finally they solve a set covering problem with the corresponding remaining set T. The algorithm was implemented in a decision support system used by the Deutsche Post AG. They have up to 22 locations with 242 requests in total and up to 6 different vehicle types. They do not provide a comparison to exact solutions, only to weak lower bounds. However, compared to previously manually planned solutions, they where able to reduce cost by 15 per cent on average. As far as we know, the exact data has not been made public.

The multi-depot problems from Nagy and Salhi [61] are solved by dividing the customer set into borderline and non-borderline customers. For the non-borderline customers, the assignment is straightforward, whereas the borderline customers need an explicit assignment to their nearest depot. A customer is borderline if it is situated roughly midway between two depots. For each depot they find a weakly feasible solution and finally insert the borderline customers into the vehicle routes one at a time. Their instances have up to 249 customers and up to 5 depots. They also reimplemented other methods from the literature to compare against and show that their methods outperforms the others in relation to solution quality, but not necessarily in relation to computational times.

2.4.2 Multi-depot paired pickup and delivery problem

This problem represents the basic extension to a multi-depot scheme of the classic pickup and delivery problem (PDP). In PDP, customers are not independent any more, but they are paired to form requests. Each request consists thus of two customers or locations, one where the goods have to be picked up and a second one where goods are to be delivered. This problem differs from the one presented in Section 2.4.1 because the order in which customers are visited is now constrained. No delivery customer can be visited before its matching pickup customer. For further information about PDP problems we refer the interested reader to the work by Parragh et al. [65] and by Berbeglia et al. [7]. Despite the extensive research conducted on single depot PDP and its variants, very little literature exists for the multi-depot case (MDPDP). A visual representation of the VRP with paired P&D can be found in Figure 2.3.

<u>New Sets and Parameters</u>

R	$=$	set of requests, $=\{1,..\text{n}\}$,
N	$=$	set of pickup and delivery points, $=\{1,..n,\ n{+}1,...,2n\}$.

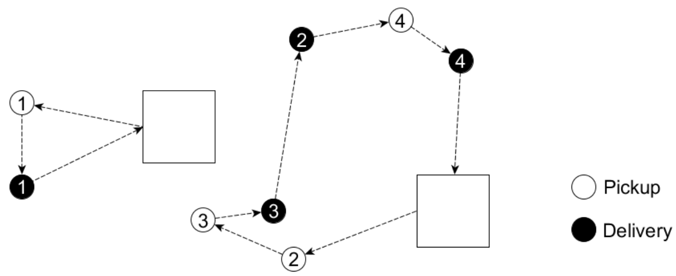


Figure 2.3: A single-region multi-depot VRP with paired P&D
Here the delivery has to be after the pickup within the same route and vehicle. The nodes belonging together share the same number.

New decision variables

t_{ik} = fulfillment time at node i on tour (vehicle) k.

The precedence relation between pickup and delivery customers is modeled by explicitly considering the visiting time as a decision variable. The pickup and delivery nodes have to be always within the same route, it is not possible to store a package at the depot.

$$\sum_{j \in N} x_{ijk} - \sum_{j \in N} x_{i+n,j,k} = 0 \qquad \forall i \in R, k \in K \tag{2.9}$$

$$t_{ik} \leq t_{i+n,k} \qquad \forall i \in \{1,..,n\},\ \forall k \in K \tag{2.10}$$

$$t_{jk} = \sum_{i \in V} x_{ijk} \cdot (t_{ik} + c_{ij}) \qquad \forall j \in V, k \in K \tag{2.11}$$

$$t_{ik} \in \mathbb{Z} \qquad \forall i \in V, k \in K \tag{2.12}$$

Constraint (2.9) ensures that both pickup and delivery locations are served by the same vehicle. The precedence constraints for P&D are included by (2.10). (2.11) makes sure that the time continuity is given in a tour.

Related problems

Bettinelli et al. in [9] present a multi-depot heterogeneous pickup and delivery problem with soft time windows. They find a set of routes of minimum length for a fleet of vehicles with limited capacity to serve a set of customers. Routes start and end at a depot from a given set of depots. Customers have a given demand that has to be served by a single vehicle from a heterogeneous fleet. The violations of the time windows are penalized. The pickup and delivery of a package belonging to one request has to be done by the same vehicle.

Detti et al. [28] present a multi-depot dial-a-ride problem with heterogenous vehicles. They extend the problem by introducing compatibility constraints. The authors study a problem in healthcare, where patients have to be transported in vehicles depending on their needs. A fleet of heterogeneous vehicles are located in geographically distributed depots. Patients may ask to be transported by a specific vehicle, which is called patient's preferences. The problem consists on assigning transportation services to vehicles and finding a route for them. Among the constraints included in this problem we can find vehicle's capacity, patient-vehicle compatibility, pick up and delivery time windows, patients' preferences, precedence constraints, quality and timing of the service provided.

Methods and instances

As for the problem described by Bettinelli et al. in [9], the authors propose a branch-and-cut-and-price algorithm. First, a set of columns is generated for every customer, representing optimal paths when customers are served one at a time. A dummy column with a very high cost is included in order to ensure feasibility at each node of the search tree. They solve the linear relaxation of the restricted master problem (RMP), and then search for columns which are not in the RMP, but have negative reduced cost. If this column does not exist, the solution is optimal for the linear relaxation of the master problem and provides a lower bound to the problem. Their next step is a dynamic programming heuristic to provide an upper bound on the amount of dual prizes that could be collected when completing the path. To strengthen the lower bound they add violated 2-path inequalities. The branching policies used are branching on the number of vehicles and branching on arcs. This approach is tested on instances derived from Solomon's work presented in [77], solving instances up to 144 customers. They manage to prove optimality for instances up to 75 customers.

In Detti et al. [28], the proposed solution methods include variable neighbourhood search and tabu search algorithms. A mixed integer linear programming formulation is also presented. These algorithms are compared based on their computational results for large real-world and random instances. For the tabu search, in a first phase, an initial solution is generated through a fast insertion heuristic. Then, a tabu search scheme iteratively tries to improve this solution. At each iteration, a neighbourhood is generated by perturbing the current solution. The whole process is repeated until a stopping criterion is satisfied. As for the variable neighbourhood search algorithm, two steps are proposed. First, an initial solution is generated through a fast insertion heuristic. Then, a local search step is performed on this initial solution. The second step of the algorithm is an iterative procedure. At each iteration of the algorithm, a new randomly generated solution is created in the current neighbourhood. A local search step is applied to this solution. If it is feasible and the best so far, it replaces the initial solution. If only one violation on any single type of constraint occurs, the solution is changed by an adjusting procedure to make it feasible. These methods are tested on random instances based on real-life data. The instances have up to 246 transportation

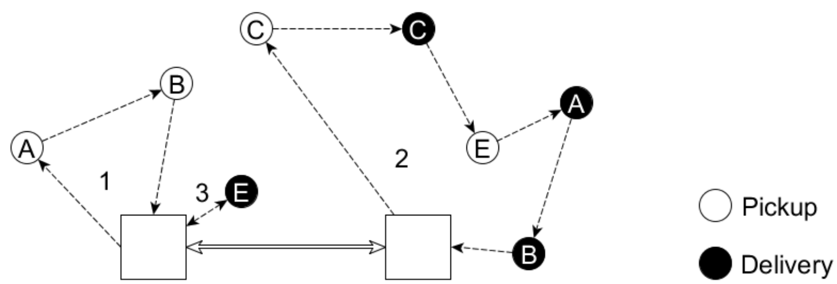


Figure 2.4: Inter-depot routes by adding a separate (faster, cheaper) vehicle between the depots
Each request (pickup and delivery pair) has a letter assigned. The short-haul vehicles visit the pickup nodes first and then deliver to the corresponding delivery nodes. The optimal routes are shown and numbered according to their sequence of execution. All transportation requests except one (C) are transported over the inter-depot route. Request C is picked up and directly delivered by the same vehicle.

services (requests) and 313 vehicles with 4 different types, distributed over 17 depots and belonging to 29 non-profit organizations.

2.4.3 Multi-depot pickup and delivery problem with inter-depot routes

Here we present one variant of the multi-depot problem where depots are connected by inter-depot lanes. Figure 2.4 shows a simple version of an inter-depot route connecting two depots in a single regions with paired pickup and delivery nodes. This setting makes sense when the vehicle type used for inter-depot transportation is different from the one used for servicing the customer nodes, exploiting the advantages of a faster or cheaper connection. Therefore, this problem lies also in the family of multi-modal transportation. The inter-depot lane can be a permanent connection or opened when needed. The connection allows the consolidation of goods and a more efficient and maybe even faster transportation per unit of distance. It can be operated on a fixed schedule and would therefore mimic a third party providing the service, or on a flexible schedule which would mimic long-haul vehicles at the carriers' own disposition. The costs can be fixed (full-truckload transportation) or variable depending on the goods transported. To represent different modes of transportation, multiple parallel lanes can be added to a model, for example a cheap and slow lane for transportation by ship and an expensive and fast lane for transportation by plane. To the best of our knowledge, this problem has not been previously addressed in this setting in literature. However it is included in this work as a basis for the development of multi region transportation problems presented in Section 2.5.

This mathematical model inherits the main characteristics of the previous models without inter-depot routes, and adds inter-depot related sets, variables and constraints. Also, decision variables regarding the beginning and ending times of the routes have to be defined to model the time consistency between the short- and long-haul vehicles.

New sets and parameters

H_l = set of inter-depot vehicles in depot $l \in L$,

H = set of inter-depot vehicles, $\cup_{l\in L} H_l$,

Q^{LH} = capacity of each inter-depot vehicle,

c_{lp} = cost of traveling between depots l and $p \in L$.

New decision variables

$$y_{ilph} = \begin{cases} 1, \text{if request } i \text{ is transported from depot } l \text{ to depot } p \text{ with vehicle } h, \\ 0, \text{otherwise.} \end{cases}$$

$$z_{lph} = \begin{cases} 1, \text{if inter-depot vehicle } h \text{ is used for a transport between depots } l \text{ and } p, \\ 0, \text{otherwise.} \end{cases}$$

m_h = starting time of vehicle h, $h \in H$,

b_k = beginning time of tour $k \in K$,

e_k = ending time of tour $k \in K$.

The adjusted objective function includes the costs of transporting between depots, which may be different from the costs of the vehicles used for the routes:
Constraints to add:

$$\text{minimize} \sum_{i\in V}\sum_{j\in V}\sum_{k\in K} c_{ij}x_{ijk} + \sum_{l\in L}\sum_{p\in L}\sum_{h\in H_l} c_{lp}z_{lph} \tag{2.13}$$

Constraints to add:

$$\sum_{j\in N\cup\{l\}}\sum_{k\in K_l} x_{ijk}e_k \le \sum_{h\in H_l} y_{ilph}m_h + M(1 - \sum_{j\in N\cup\{p\}}\sum_{k'\in K_p} x_{(i+n)jk'}) \quad \forall i \in R, l, p \in L \tag{2.14}$$

$$\sum_{j\in N\cup\{l\}}\sum_{k'\in K_p} x_{(i+n)jk'}b_{k'} \ge \sum_{l\in L\setminus\{p\}}\sum_{h\in H_l} y_{ilph}(m_h + d_{lp}) \quad \forall i \in R, p \in L \tag{2.15}$$

$$\sum_{i\in R}\sum_{h\in H_l} y_{ilph}q_i \le Q^{LH} \quad \forall l, p \in L \tag{2.16}$$

$$\sum_{i\in R} y_{ilph} \le M \cdot z_{lph} \quad \forall l, p \in L, h \in H_l \tag{2.17}$$

$$y_{ilph}, z_{lph} \in \{0, 1\} \quad \forall i \in V,\ l, p \in L \tag{2.18}$$

$$m_h \in \mathbb{Z} \quad \forall h \in H$$

Starting time of the inter-depot vehicle transporting goods of request i should be greater than the arriving time of the short-haul route serving the pickup customer corresponding to the request (2.14). The big M component is necessary to allow for requests being transported directly from pickup to delivery, without going through the inter-depot route. Starting time of the short-haul route delivering goods of request i should be greater than the arriving time of the long-haul route transporting the goods of the request (2.15). Constraint (2.16) for the quantity on the inter-depot vehicle. Constraint (2.17) ensures that requests are only assigned to long-haul transports that are performed.

Constraints (2.14) and (2.15) make the model non-linear and therefore intractable for a commercial solver. However they can be linearized, as shown in the appendix. In spite of this the model remains NP-hard and can only be solved to optimality for small instances.

Related problems

As far as we know, this type of multi-depot problem has not been yet addressed in the literature in this form. Its main feature is the existence of two different ways of transportation in two stages, which requires the combination of a scheduling and a routing problem. Extensive literature exists for similar problems, where two dependent decisions are to be made. For example, two-echelon vehicle routing problems (2E-VRP) and location routing problems (LRP) are frequently studied in the literature. Also another concept of MDPDP with inter-depot routes has been addressed in the literature, where inter-depot routes are understood as a vehicle starting and ending the route in a different depot, thus staying away from multi-modal transportation. A detailed explanation is given in Section 2.4.3.1.

Gendron and Semet [41] consider a multi-echelon LRP for a fast delivery service, with three levels of intershipment facilities (hubs, depots, satellites). From satellites, the products are sorted and delivered to the customers. Transportation between levels is done in different transportation modes.

In Crainic et al. [20], Perboli and Tadei [68], Perboli et al. [69], and Breunig et al. [11] a two-echelon vehicle routing problem (2E-VRP) is presented. They define one central hub, a set of intermediate depots called satellites and a set of customers with demands. The freight stored at the central hub must transit through the satellites and then be delivered to the customers. Two different types of vehicles are considered according to the level they serve. Neither route size nor number of visited customers are limited.

In Hemmelmayr et al. [47], the 2E-VRP is also considered. Satellites locations are assumed to be known and a limit on the number of vehicles for both levels is considered. They are also able to solve an LRP with their approach, by transforming the 2E-VRP to an LRP. Capacity limit and opening costs are considered for the depots.

Ghilas et al. [42, 43] use a very similar type of inter-depot routes in their work. Their inter-depot route is a scheduled active public transportation line travelling in the city of Amsterdam transporting both passengers and packages. All vehicles from a depot can serve all customer nodes without restrictions, but they have the possibility to shorten their routes by consigning the packages to a public scheduled line. At the end of the line, the packages have to be picked up by another vehicle belonging to another depot and delivered to their destination. They prove that introducing scheduled lines leads to a decrease in transportation cost compared to the solutions obtained without them.

Methods and instances

Gendron and Semet [41] present both an arc-based and a path-based formulation for tackling the problem. Furthermore, they develop a binary relaxation as an alternative, for which both formulations are equivalent. The computational results are obtained from 32 instances generated based on real data, for 93 depots, 320 satellites and 700 customers. The linear programming relaxations are solved using CPLEX with a 2 hours CPU time limit.

For the 2E-VRP solution, Perboli and Tadei [68] present new inequalities in an exact model. Inequalities are then inserted in a Branch & Cut framework with a 10000 seconds computing time limit for each instance.

Crainic et al. [20] present heuristics to solve the 2E-VRP. An initial solution is obtained by a first-clustering second-routing strategy. Then a local search is performed over the clustering of customers around satellites. A final perturbation move is performed until a feasible solution is reached.

Perboli et al. [69] present two interesting math-based heuristics to solve the 2E-VRP. The first one forces assignment variables to be fixed to a value and, if the model is infeasible, they unfix some variables and restart the process. The second heuristic method, solves a semi-continuous 2E-VRP using a MIP solver with a 60 seconds time limit and obtains a list of best integer solutions found. For every solution in the list, they consider the customer-satellite assignments and solve the VRP instances with a time limit of 5 seconds.

Crainic et al. [20], Perboli and Tadei [68] and Perboli et al. [69] use instances from some previous work of theirs, which covers up to 50 customers and 5 satellites.

An adaptive large neighborhood search is proposed in Hammelmayr et al. [47]. Its main feature is that it allows for infeasibilities in the initial solution construction phase. Destroy and repair algorithms are used for improving the solution and making it feasible. The authors use the same instances as Crainic et al. [20] and Perboli et al. [69].

Breunig et al. [11] make use of a large neighborhood search heuristic. The method uses several destroy operators and only one repair operator. They also work on collecting the instances for the problem, obtaining different sets from several previous works and modifying them to a common

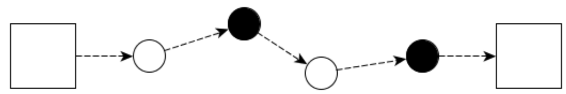


Figure 2.5: Inter-depot routes by allowing vehicles have different start and end depots

structure. Instances with up to 200 customers are solved. Results show a very good performance in comparison to the results from Hemmelmayr et al. [47].

Ghilas et al. [42, 43] use both exact and heuristic methods for their problem. They decided for a branch-and-price approach as an exact method and for an adaptive large neighborhood search heuristic method. They managed to obtain exact results for small and medium size instances with up to 50 requests. In addition they studied the effects of the number of scheduled lines, the scheduled lines frequency, and of the time windows on the operating costs. Their instances contain up to 100 requests and 3 scheduled lines.

2.4.3.1 Interpretations of inter-depot routes

So far, existing literature has referred to the inter-depot routes as routes in which vehicles start and ends in different depots or when another depot than the starting depot is used as a replenishment or unloading point along the route. This interpretation leaves us with a quite different problem than the previously proposed one. Multi-modal transportation stays out of scope and it is no longer a two-level decision problem.

The first option mentioned consist of vehicles starting in a depot and ending at a different location (depot or customer) within the same region. A practical application would be to allow the vehicles to return to their start depot on another day. Figure 2.5 shows a simple route with two pickup and two delivery nodes which starts and ends at different depots. From this example it is clear that by forcing a vehicle to return to the start depot, a considerable increase in distance has to be condoned.

The paper by Goel et al. [44] deals with a rich VRP that incorporates several constraints found in real-life applications. These locations can require a pickup, a delivery, or simply a service, where no quantity of goods change hands. They do not assume that vehicles are stationed at a depot, but simply become available at a specific location, to a specific time with some predefined load. Also, vehicles do not return to a depot, but an end location for the tour is given. Therefore we classified the problem as one with inter-depot routes, since every location can be a depot. During the computation, some requests get canceled while others became known only then. In addition to the inter-depot routes, they consider several constraints like time window restrictions or drivers' working hours, among others.

They make use of a reduced large neighborhood search strategy, with several and specific neighborhoods that allow them to go from one solution to another. They solved self generated instances with up to 500 vehicles and 2500 orders.

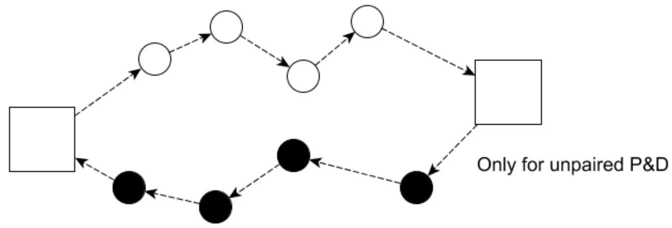


Figure 2.6: Inter-depot routes by allowing vehicles to visit depots within their route
In the first part of the route it picks up all the white nodes and delivers all or part of the goods to the nearest depot (different from the starting depot). From there, it picks up the goods that have to be delivered to the black nodes. At the end of the day it returns to the depot it started from.

Another approach that can be considered as a problem with inter-depot routes consists in allowing a vehicle to visit another depot during its tour while still returning to its own depot at the end of the day. This approach makes sense for unpaired pickup and delivery problems or capacitated VRP with backhauls. Especially vehicles with a tight capacity might profit from the possibility to unload or replenish the goods to be transported during their route. This scenario also applies, for example, to electric vehicles that need to use recharging stations during their route, as stated in the work by Hiermann et al. [48]. Figure 2.6 shows an example of a route in which a vehicle visits a depot during its route while transporting backhauls.

This problem has been introduced by Crevier et al. [22] where depots act as replenishment facilities, in a real-life grocery distribution network. Vehicles can have a single-depot route, which starts and ends at the same depot, or inter-depot routes which connects two different depots. Nevertheless, returning to its original depot is mandatory. They tackle the problem with a three-phase heuristic: route generation, determination of least cost feasible rotations and post-optimization phase. They adapted instances from the literature and generated benchmark instances with 48 to 288 customers and 3 to 6 depots.

Muter et al. [60] tackle the same problem and allow vehicles to stop at intermediate depots for replenishment.They solve their problem by a branch-and-price algorithm. Moreover, two different pricing sub problems are implemented and compared. The traditional one-level column generation scheme is inferior to their developed two-level decomposition scheme. They work on instances from the literature containing between 3 and 7 depots and up to 50 customers, solving some of them to optimality. They show that allowing inter-depot routes leads to an improvement of cost, although making the problem harder to solve.

Bard et al. [3], and Angelelli and Speranza [1] allow the replenishment at intermediate facilities, but differing from the previous works in that depots cannot act as intermediate facilities, hence being different kind of locations.

Angelelli and Speranza [1] propose a tabu search algorithm to iteratively move from one solution to another within the same neighborhood. A new solution is chosen minimizing a penalty that is

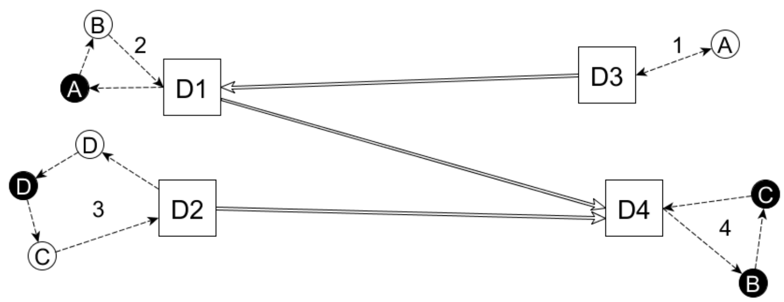


Figure 2.7: MR-MDPDP with two regions and two depots per region
Requests A, B and C are transported over long-haul lanes. Request D is an intra-region request served only by depot 2. The SH routes are numbered in their order of execution. The order is the following: Route 1 picks up request A, which then travels from D3 to D1 over the LH. Route 2 delivers A and picks up B. B travels over the LH from D1 to D4. Route 3 picks up D and C and immediately delivers D within the same route. Request C travels over the LH to D4. After both B and C have arrived in D4 they are both delivered by route 4.

related to the objective function. Instances from previous literature are solved, as well as random generated ones with up to 150 customers, 4 satellites and 5 vehicles.

The solution method proposed in Bard et al. [3] starts by relaxing the vehicle routing problem with satellite facilities linear program. Subsequently, a branch and cut approach is implemented. They solve self-generated instances with up to 20 customers and 2 satellites.

2.5 Multi-region pickup and delivery problems

In this section we introduce a new type of problems, the multi-region pickup and delivery problems. The main characteristic of these problems is that regions are independent of each other, meaning that customers in different regions cannot be visited by a vehicle from a depot situated in another region. Especially, we are interested in describing the multi-depot version of these problems (MR-MDPDP), where more than one depot is located in each region. This problem constitutes a special case of the MDPDP with inter-depot routes explained in Section 2.4.3. From the application point of view, we think it seems to be more realistic than the MDPDP with inter-depot routes since there exist many cases where transportation between two separated regions is unpractical for the vehicles performing the short distance transportation, and no other option than multi-modal transportation is left.

In MR-MDPDP, all transportation requests have a pickup and a delivery node. These nodes can be within the same region (intra-region requests) or they can be in different regions (inter-region requests). Customer locations (either pickup or delivery) are visited by short-haul vehicles. The goods from inter-region requests have to be transported from the region where the pickup customer is located, to the delivery customer region. This inter-region transportation is done by long-haul vehicles. Long-haul or inter-region vehicles connect the regions and transport the consolidated goods. They have to be scheduled so that all requests are fulfilled and cost minimized. In the case where there is only one depot in each region the inter-region vehicles represent a standing connection between the regions which has to be used due to a lack of alternatives. However, the departure times can still be either fixed (scheduled) or flexible (up to optimization). If multiple depots are present in each region, the additional decision occurs, which of the possible long-haul connections to use. Since each depot can be connected to every other depot of any other region, several possibilities arise. Figure 2.7 shows a simple representation of a solution of a MR-MDPDP with two regions and two depots in each region.

To the best of our knowledge, this problem has not been considered in the literature yet. However, all related problems and literature detailed for the MDPDP with inter-depot routes in Section 2.4.3 apply to the MR-MDPDP, and even in a greater degree, since in this case multi-modal transportation is required and we face a pure two-level decision problem like the problems from 2E-VRP and LRP families.

The characteristics of this problem induce the need of introducing an index for the region in most of the sets and decision variables in the mathematical model. We also have to make a distinction between requests going over inter-region transportation and requests for which pick up and delivery are in the same region. Since the introduction of this problem is one of the main goals of this paper and due to the several changes that need to be made on the mathematical formulation, we present the complete model for the multi-region problems.

<u>Sets</u>

$$\begin{aligned}
U &= \text{set of regions},\\
R^I &= \text{set of inter-region requests, with customers located in different regions},\\
R^u &= \text{set of intra-region requests, with both customers in region } u,\\
N^u &= \text{customer points in region } u,\\
L^u &= \text{set of depots in region } u,\\
V^u &= N^u \cup L^u,\\
K_l^u &= \text{set of short haul vehicles in depot } l \text{ of region } u,\\
K^u &= \cup_{j \in L^u} K_j^u,\\
H_l^u &= \text{set of long haul vehicles in depot } l \text{ of region } u,\\
H^u &= \cup_{j \in L^u} H_j^u.
\end{aligned}$$

Parameters

$$c_{ij}^u = \text{distance between points } i \text{ and } j \text{ in region } u,$$

$$c_{lp}^{uw} = \text{distance between depots } l \in L^u \text{ and } p \in L^w,$$

$$d_i^u = \begin{cases} 1, & \text{if request } i \text{ starts in region } u, \\ 0, & \text{otherwise.} \end{cases}$$

$$q_i^u = \text{quantity to serve at customer } i \text{ in region } u \begin{cases} > 0, \text{if it is a pick up point,} \\ < 0, \text{ if it is a delivery point,} \\ 0, \text{if customer } i \text{ is not in region } u. \end{cases}$$

$$Q^{SH} = \text{capacity limitation for short-haul vehicles,}$$

$$Q^{LH} = \text{capacity limitation for long-haul vehicles.}$$

Decision Variables

$$x_{ijk}^u = \begin{cases} 1, \text{arc } i - j \text{ in region } u \text{ is covered by vehicle } k \in K^u, \\ 0, \text{otherwise.} \end{cases}$$

$$y_{ilph}^{uw} = \begin{cases} 1, \text{request } i \in R^I \text{ transported from depot } l \in L^u \text{ to depot } p \in L^w \text{ in long-haul vehicle } h \in H_l^u, \\ 0, \text{otherwise.} \end{cases}$$

$$z_{lph}^{uw} = \begin{cases} 1, \text{vehicle } h \in H_l^u \text{ is used for transporting between depots } l \in L^u \text{ and } p \in L^w, \\ 0, \text{otherwise.} \end{cases}$$

$$S_{ik}^u = \text{quantity loaded in tour } k \in K^u \text{ after visiting customer } i,$$

$$t_{ik}^u = \text{fulfillment time at customer } i \text{ from region } u \text{ with vehicle } k \in K^u,$$

$$b_k^u = \text{beginning time of route performed by vehicle } k \in K^u,$$

$$e_k^u = \text{ending time of route performed by vehicle } k \in K^u,$$

$$m_h^u = \text{starting time of inter-region trip with vehicle } h \in H^u.$$

For simplicity in the notation, copies of depot $l \in L^u$ representing starting and ending depot respectively in each region will be denoted as l_0^u and l_1^u. Indexes in N^u are constituted first with the customers belonging to inter-region requests and then with pickup and delivery customers belonging to intra-region requests in u (R^u).

$$\text{minimize} \quad \sum_{u \in U} \sum_{i,j \in V^u} \sum_{k \in K^u} c_{ij}^u \cdot x_{ijk}^u + \sum_{u \in U} \sum_{w \in \bar{u}} \sum_{l \in L^u} \sum_{p \in L^{\overline{u}}} \sum_{h \in H_l^u} c_{lp}^{uw} \cdot z_{lph}^{uw} \tag{2.19}$$

Subject to

$$\sum_{j\in V^u}\sum_{k\in K^u} x^u_{ijk} = 1 \qquad \forall u\in U, i\in N^u \tag{2.20}$$

$$\sum_{i\in V^u} x^u_{ihk} - \sum_{i\in V^u} x^u_{hik} = 0 \qquad \forall u\in U, h\in N^u, k\in K^u \tag{2.21}$$

$$\sum_{i\in V^u} x^u_{l^u_0,ik} = \sum_{i\in V^u} x^u_{i,l^u_1,k} \leq 1 \qquad \forall u\in U, l\in L^u, k\in K^u_l \tag{2.22}$$

$$t^u_{jk} = \sum_{i\in V} x^u_{ijk}\cdot\left(t^u_{ik} + c^u_{ij}\right) \qquad \forall u\in U, j\in V, k\in K \tag{2.23}$$

$$b^u_k = t^u_{l^u_0,k} \qquad \forall u\in U, k\in K^u, l\in L^u$$

$$e^u_k = t^u_{l^u_1,k} \qquad \forall u\in U, k\in K^u, l\in L^u \tag{2.24}$$

$$t^u_{ik} < t^u_{i+|R^u|,k} \qquad \forall u\in U, i\in R^u, k\in K^u \tag{2.25}$$

$$S^u_{jk} = \sum_{i\in V^u} x^{ut}_{ijk}\cdot(S^u_{ik} + q^u_i) \qquad \forall u\in U, j\in N^u, k\in K^u \tag{2.26}$$

$$0 \leq S^u_{ik} \leq Q^{SH} \qquad \forall u\in U, i\in V^u, k\in K^u \tag{2.27}$$

$$\sum_{i\in R^I} y^{uw}_{ilph} \leq M\cdot z^{uw}_{lph} \qquad \begin{array}{l}\forall u\in U, w\in U\setminus u, l\in L^u\\ \forall p\in L^w, h\in H^u_l\end{array} \tag{2.28}$$

$$d^u_i\cdot\sum_{j\in V^u}\sum_{k\in K^u_l} x^u_{ijk}e^u_k \leq \sum_{h\in H^u_l}\sum_{p\in L^w} y^{uw}_{ilph}m^u_h \qquad \forall u\in U, w\in U\setminus u, i\in R^I, l\in L^u \tag{2.29}$$

$$\sum_{j\in V^w}\sum_{k\in K^w_p} x^w_{ijk}b^w_k \geq d^u_i\cdot\sum_{l\in L^u}\sum_{h\in H^u_l} y^{uw}_{ilph}(m^u_h + c^{uw}_{lp}) \qquad \forall u\in U, w\in U\setminus u, p\in L^w, i\in R^I \tag{2.30}$$

$$\sum_{i\in N^u} y^{uw}_{ilph}q^u_i \leq Q^{LH} \qquad \begin{array}{l}\forall u\in U, w\in U\setminus u, l\in L^u\\ \forall p\in L^w, h\in H^u\end{array} \tag{2.31}$$

$$x^u_{ijk}\in\{0,1\} \qquad \forall i,j\in V, k\in K \tag{2.32}$$

$$y^{uw}_{ilph}, z^{uw}_{lph}\in\{0,1\} \qquad \forall i\in V,\ l,p\in L, h\in H, u,w\in U$$

$$t^u_{ik}, S^u_{ik}, b^u_k, e^u_k\in\mathbb{Z} \qquad \forall i\in V, k\in K$$

$$m_h\in\mathbb{Z} \qquad \forall h\in H$$

Constraint (2.20) ensure that all customers in region u are visited once and only once by a vehicle from region u. A vehicle arriving at a customer must leave it in the same tour (2.21). A vehicle belonging to depot l in region u has to arrive at depot l, if it leaves it (2.22). Constraint (2.23) refers to time continuity constraints: the arriving time to customer j with vehicle k equals the time the previous customer of vehicle k was serviced plus the travel time between i and j. Constraints modeling the beginning and end time of vehicle k: Start time equals the time when vehicle k leaves

the depot. End time is when vehicle k arrives at the depot (2.24). Time precedence constraint for intra-region pickup and delivery requests (2.25). Constraints (2.26) and (2.27) model the load of a short-haul vehicle when visiting customer j and ensure the load stays below the short-haul capacity limitation. Constraint (2.28) binds decision variable y and z. Constraints (2.29) and (2.30) bind decision variables x and y. If inter-region request i starts in region u, the end time of the vehicle k visiting the pickup customer must be smaller than the departing time of the long-haul vehicle h transporting request i. Equivalently, the beginning time of vehicle k' visiting the delivery customer must be greater than the arriving time of the long-haul vehicle h to depot p. The load on a long-haul vehicle cannot exceed the long-haul vehicle capacity limitation (2.31).
The linearization of constraints (2.26), (2.29) and (2.30) is shown in the appendix.

Methods

As previously stated, the MR-MDPDP is a two-level decision problem in which interrelated scheduling and routing decisions have to be made. Any change in the scheduling of requests traveling in long-haul vehicles affects the routes in both regions and vice versa. Therefore we face a problem with two simpler underlying problems. The routing problem corresponds to a MDVRPMB (see Section 2.4.1). It is based on the classical capacitated VRP, which is itself a NP-hard problem (see Cordeau et al. [19]). Therefore the MR-MDPDP also belongs to the category of NP-hard problems. This means that an exact approach to the solution is only possible for instances of very small size, and therefore an heuristic approach is needed for solving problems of a realistic size.

In order to assess how far we can get with the use of commercial solvers, the previous model has been implemented and applied to instances of different sizes. A time limit of two hours was set for every run. All computational results and data description can be found as online supplements to this publication. In general, within the specified computational time we are able to solve instances with up to 4 inter-region requests and 4 intra-region requests.

Consequently, heuristic algorithms are the only feasible option for dealing with bigger instances. Many types of algorithms have been used for problems with similar two-level decision structure, like the families of 2E-VRP and LRP. Looking at the most relevant 2E-VRP literature (presented in Section (2.4.3)), however, it is possible to observe that most of heuristics used consist of two-stage heuristics due to the fact that they provide convenient solution representation. The interrelated variables make it difficult to store a solution in a format required to use them as chromosomes in a population for genetic based algorithms or as labels in a column generation scheme. Hence, large neighborhood heuristics arise as common practice to solve these problems, since the work is done mainly on one single solution. Also, most of the papers make use of two phase algorithms for obtaining initial solutions or to explore neighborhood spaces. That is, they separate the scheduling and the routing problems and consider one of them as an independent problem. This first problem is solved, and afterwards the second one is approached according to the solution obtained from the first phase. However we found an interesting case in the work from Ghilas et al. [42], where a branch-and-price algorithm is successfully applied, getting solutions that outperform in quality those from the ALNS approach, and in time those from CPLEX. However, as earlier mentioned, they need to design a complex labeling system, far from the standard labeling systems applied in general VRP problems.

Everything stated above suggests that probably the best way of approaching a solution method for the MR-MDPDP would be by making use of large neighborhood heuristics, as well as two-phase heuristics with a more particular design focused on each type of problem.

2.6 Conclusion

Multi-region problem scenarios are becoming more common everyday. Different real-world applications include intra-region as well as inter-region transportation, in which, regions are connected by vehicles of larger capacity, different speed and cost.

In-town logistics are constantly evolving in order to reduce costs and increase productivity. While big trucks and trains are an advantage when traveling from one city to another, smaller trucks are used for pickup and delivery operations inside a city. Other multi-modal models have to be used in certain cities, as the only transportation means that are allowed in some neighborhoods are small electric vehicles.

This work introduces this new type of logistic problems which main focus lies on pickup and deliveries in multiple geographically separated regions. The characteristics of these problems lead to the necessity of dealing with other issues like multi-modal transportation and multi-depot networks. An exhaustive examination of already existing literature in related topics is also presented, justifying the multi-region scenarios as a logical evolution step towards more realistic problems in transportation logistics and setting a basis for further research in this direction. This work also contains a step-by-step construction of mathematical models, which highlights the relation between classical transportation problems and the new introduced logistic networks.

2.7 Outlook and possible extensions

As logistics operations are centralized to save resources, n-regions multi-modal problems are an attractive scenario for further research. Companies can use big vehicles like airplanes, trains and trucks to connect main depots to a central location, smaller trucks can be used to reach smaller regional depots and from these, smaller depots, stores, dealers and/or customers can be visited with small vehicles routes. It is very rare to find models that are capable of solving this kind of problems.

Multi-depot, multi-region problems are of particular importance in collaborative settings. Although all cited studies assume to have one single decision maker who aims for a centralized solutions, this assumption does not necessarily hold for collaborative vehicle routing problems. Probably many companies are involved in such collaborative networks, and therefore decentralized decision making has to be investigated.

Collaborative Networks have been studied and developed over the last years. In works like Berger and Bierwirth [8], Gansterer and Hartl [37], Wang and Kopfer [85]. These studies were, however, focusing on single depot per carrier settings. In a multiple depot approach, collaborative networks could include collaborative depots that could be used by all participants. When thinking about a multiple region structure, collaboration might be even more beneficial, by allowing carriers to share depots and short-haul vehicles on every region, and by splitting costs in combined long-haul trips.

Real world applications of collaborative networks can be found in Montoya et al. [59] and Buijs et al. [13], where a quantification of the obtained improvement suggests that collaboration is a plausible and recommendable practice. Once collaboration is performed, a final aspect to take into account is profit sharing. Guajardo and Rönnqvist in [45], present a recent overview on cost and profit sharing allocation methods.

Besides solution methods, collaborative networks could be improved with managerial implications suggesting best practices in this field. The decision on how much information does a company need to share in order to get the maximum advantage is one of the main questions.

Furthermore we introduce some possible extensions for the problems. All extensions are presented with respect to the model in Section 2.5. These show the extensive gap between already existing literature and a rough approximation to reality, making these problems an attractive field for future research since many real world extensions are not considered so far.

2.7.1 Time windows

Time windows have already been used a lot in the literature and can be added in many different ways. Usually the nodes or customers to visit have soft or hard time windows for the visit. A soft time window that is not met, results in penalties that impair the objective value. A hard time window that is not met, results in an infeasible solution. Time windows can also be considered on the depot opening hours. A clear example of a routing problem with time windows can be found in Dondo and Cerda [29].

Some changes should be done to the presented models when including time windows. Let $[a_i, f_i]$ be the time window for customer i.

- Soft time windows. The objective function must be modified to account for penalty costs. Let ϕ represent the relative penalty cost. The new objective cost would be:

$$\begin{aligned}\text{minimize} \quad & \sum_{u\in U}\sum_{i,j\in V^u}\sum_{k\in K^u} c^u_{ij}\cdot x^u_{ijk} + \sum_{u\in U}\sum_{w\in\bar{u}}\sum_{l\in L^u}\sum_{p\in L^{u*}}\sum_{h\in H^u_l} c^{uw}_{lp}\cdot z^{uw}_{lph} \\ & + \sum_{u\in U}\sum_{i\in V^u}\sum_{k\in K^u} \phi\cdot max(0, t^u_{ik} - f_i)\end{aligned} \tag{2.33}$$

- Hard time windows. New constraints have to be added.

$$a_i \leq \sum_{k \in K^u} t^u_{ik} \leq f_i \qquad \forall u \in U, i \in V^u \tag{2.34}$$

2.7.2 Multiple periods

Another common extension in routing problems is to consider multiple periods. Generally this problems introduce the possibility for requests to be performed on different days. That induces another degree of freedom to the combinatorial problem, widening the solution space. Servicing a customer on different days can lead to significant improvements in the objective function. Storage of goods in depots is a realistic option for these problems. The work of Vidal et al. [82] contains a clear explanation of multiple period models. A hybrid genetic metaheuristic is developed to solve a variety of problems within the multi-depot and periodic VRP families.

Multiple periods induce a change on the variables of the model. A temporal index should be added to all variables that could be dependent on it. If we name o the new index representing the days in the planning horizon, the variables of our model would be the following:

$x^u_{ijko}, y^{uw}_{ilpho}, z^{uw}_{lpho}, S^u_{iko}, t^u_{iko}, b^u_{ko}, e^u_{ko}, m^u_{ho}$.

2.7.3 Heterogeneous vehicles

A problem with an heterogeneous fleet is a problem where vehicles with different problem-relevant characteristics exist. It refers to vehicles within the same mode of transportation and performing the same kind of service. Vehicles can be different if they have different capacities, fixed or variable cost, range (especially for electric vehicles), driving time restrictions or restrictions concerning the arcs they can travel, different speed or different requirements for the drivers (not all drivers have a license for all vehicle types). In a realistic scenario, a company will generally have an heterogeneous fleet.

Regarding heterogeneous fleets, the following works have to be mentioned: The work by Dondo and Cerdá [29] is a multi-depot VRP with heterogeneous fleet and time windows. Their vehicles differ in capacity, cost and speed. Irnich [50] works on a single region multi-depot VRP with an heterogeneous fleet. The vehicles in the fleet differ by cost and speed (regarding driving and loading), and capacity. Salhi and Sari [75] consider a single region multi-depot capacitated VRP with the objective to construct vehicle routes and determine the composition of the vehicle fleet. The vehicles in the fleet differ in capacity and cost. The aim of the authors is to construct a set of routes and determine the vehicle fleet composition minimizing costs.

The changes on our model derived from the use of heterogeneous fleet would be reflected on the parameters and sets. Also we would need an extra index for the variables. Assume we have n vehicle types for the short-haul routes and r vehicle types for long-haul routes. The new elements would be:

$K^u_{ln}, K^u_n, H^u_{lr}, H^u_r$, sets of short-haul and long-haul vehicles of each type in depot l,

$c^u_{ijn}, c^{uw}_{lpr}, Q^{SH}_n, Q^{LH}_r$, costs of transportation and capacities for each vehicle type,

$x^u_{ijkn}, y^{uw}_{ilphr}, z^{uw}_{lphr}, S^u_{ikn}, t^u_{ikn}, b^u_{kn}, e^u_{kn}, m^u_{hr}$, variables containing information about the vehicle type.

2.7.4 Multi-attribute vehicle routing problems

Multi-attribute or rich VRPs are labels for VRPs with additional constraints that aspire to express a more realistic problem structure and decision choices (multiple depots, fleets and commodities that take place in multiple periods and have to consider driver work rules, traffic congestion and much more).

A rich VRP can be found in the work by Ceselli et al. [15] who use a heterogeneous fleet of vehicles, multiple depots, multiple capacities, time windows associated with depots and customers, incompatibility constraints between goods, depots, vehicles, and customers; maximum route length and duration; upper limits on the number of consecutive driving hours and compulsory drivers' rest periods; the possibility of skipping some customers and using express courier services instead of the given fleet to fulfill some orders; the option of splitting up the orders; and the possibility of open routes that do not terminate at depots. Vidal et al. [83] present a survey on heuristics for multi-attribute VRPs. However, in spite of their extensive pool of constraints, both papers do not consider multi-modality, the possibility of consolidation, or more complex network structures.

These considerations would lead to many different changes in our model from Section 2.5, depending on which attributes are added to the problem.

Chapter 3

Total Distance Approximations for Routing Solutions

Abstract

In order to make strategic, tactical and operational decisions, carriers and logistic companies need to evaluate scenarios with high levels of accuracy by solving a large number of routing problems. This might require relatively high computational efforts and time. In this paper, we present regression-based estimation models that provide fast predictions for the travel distance in the Traveling Salesman Problem (TSP), the Capacitated Vehicle Routing Problem with Time Windows (CVRP-TW), and the Multi-Region Multi-Depot Pickup and Delivery Problem (MR-MDPDP). The use of general characteristics such as distances, time windows, capacities and demands, allows us to extend the models and adjust them to different problems and also to different solution methods. The resulting regression models in most cases achieve good approximations of total travel distances except in cases where strong random noise is present, and outperform previous models.

3.1 Introduction

Many exact and heuristic algorithms have been developed to solve transportation problems in short computational time. However, there are situations in which it is not necessary or possible to apply such algorithms. For example, carriers who need to respond to an inquiry by a potential customer or who are involved in a competitive bidding process need rapid information about costs, but not about the actual routing. This information must often be obtained very quickly for a large number of transportation requests, for example in a combinatorial auction, when bids must be made for many bundles of requests. Combinatorial auctions were frequently proposed as part of collaboration mechanisms to improve the efficiency of the transportation system [8, 37].

Several approaches for approximating travel distances in transportation problems have been developed, which we will review in section 3.2. Previous methods were mostly based on analytically derived approximation formulas [e.g., 5], which were sometimes augmented by empirically estimated parameters [e.g., 17]. This approach limits the domain of problem classes for which approximations can be developed to comparatively simple problems, in which tour lengths can be approximated analytically. Only few authors [e.g., 14, 49] so far considered a more empirical approach, but also limited their work to problems like the traveling salesman problem or vehicle routing problems without complex, real world constraints.

Our work takes an empirical perspective, initially considering a large set of (possibly redundant) potential variables, and then with the help of statistical methods identifying variables that are particularly useful in approximating total costs. Some of these variables, like distances between customers, are present in all routing problems. Other variables, like time windows and capacity constraints, are added to approximate the solution of problems containing those constraints. Previous studies have mostly reported the quality of approximations in terms of goodness of fit to the problem instances for which the model was estimated. We complement this in-sample evaluation with an out-of-sample evaluation, in which we analyze how well our models are able to predict total costs for new problem instances.

Our empirical approach requires actual solutions of some problems to estimate the parameter values. The approach therefore does not necessarily approximate the optimal solution to a problem, but the solution that can be found by the algorithm applied. For many of the applications envisioned, this is exactly the information that is required. For example, in a competitive bidding process, the price charged for performing a set of requests needs to cover the actual costs that will be incurred. If the algorithm used for generating the actual schedule delivers a solution that causes higher than optimal costs, these are the costs that need to be covered. Of course, having a better routing algorithm would make the carrier more competitive, but pricing should be based on the actual planning process.

Although this empirical approach is adaptable to different solution methods, this does not imply that the same empirical model can be applied to approximate solutions found by any algorithm. It is quite possible that the solution quality of different algorithms also depends on different characteristics of the problem, thus different empirical models will be needed to approximate solutions of the algorithms. However, the general method remains the same, even if other variables are included in the models.

In line with much of the existing literature [35, 24], we consider (variable) costs to be roughly equivalent to travel distance. We develop approximations for travel distance in different types of transportation problems of increasing complexity, ranging from a standard Traveling Salesman Problem (TSP) to the recently introduced [30] Multi-Region Multi-Depot Pickup and Delivery Problem (MR-MDPDP). This gradual approach allows us to study whether the presence of additional constraints like vehicle capacity or time windows influences the quality of approximations in general. We also study how such factors can be represented in the approximation to make it more accurate even for a complex setting. In our view, the ability to incorporate additional problem characteristics into the approximation in a rather straightforward manner is a considerable advantage of an empirical approach. Thus, we consider it important to determine the potential benefit of adding such additional parameters to the model.

The remainder of the paper is structured as follows: In Section 3.2, we present a literature review on approximation models for different types of transportation problems. Section 3.3 introduces the regression based approximation for the different types of logistic problems we study in this paper. Computational experiments and results are presented in Section 3.4, and Section 3.5 concludes the paper by summarizing its results and presenting an outlook onto future research.

3.2 Literature Review

Bearwood et al. [5] are among the first authors who developed a distance approximation. They demonstrated that for a set of n nodes in a compact and convex area A, the length of the TSP tour asymptotically converges to $c\sqrt{nA}$ when $n \to \infty$, c being a constant. Christofides and Eilon [17] approximated the average TSP route length by $100 \times c\sqrt{n}$. Further distance approximations for the TSP were developed by Chien [16], Kwon et al. [53], and Hindle and Worthington [49]. They all recognized the need for additional parameters representing the shape of the area or distances between customers. Chien [16] modified the area factor A by including the area of the smallest rectangle that covers all customers and also includes distance related measures like the average distance to the depot. Kwon et al. [53] used regression models and neural networks to improve the TSP approximations. They considered a rectangular length/width ratio as well as a shape factor. Hindle and Worthington [49] used two different models depending on how customers are distributed over the area. Their first model considered a uniform distribution of locations. The second model used "demand surfaces" to represent different densities of customers across regions. In a recent paper on the approximation of TSP travel distances, Çavdar and Sokol [14] proposed an approximation based on coordinates of customers. Their approximation was based on the standard deviations of coordinates as well as the standard deviations of distances between customers and the average-point of the area. Stein [81] considered the bus problem, a TSP with additional constraints in which pickup and delivery nodes are paired. They extended the results of Bearwood et al. [5] and estimated the length of the optimal tour either for one or for several buses.

The first published approximations for the Capacitated Vehicle Routing Problem (CVRP) were developed in the 1960s by Webb [86], who studied the correlation between route length and customer-depot distance. Eilon et al. [31] proposed approximations to the length of the CVRP based on the shape and area of delivery, distances between customers and the depot, and capacity of vehicles. Daganzo [24] approximated CVRP tour length as

$$CVRP(n) \approx 2rn/Q + 0.57\sqrt{nA} \tag{3.1}$$

where n is the number of customers customers, r is the average distance between the customers and the depot and Q is the maximum number of customers that can be served by a vehicle. Robusté et al. [73] tested Daganzo's approximation and proposed adjustments based on the shape of the area, in particular for elliptic zones. Erera [32] extended Daganzo's approximation for stochastic versions of the CVRP. Langevin and Soumis [54] developed an approximate method for planning pickup and delivery zones assigned to vehicles. Their approach included an estimation of the increase in the number of vehicles used if zones are allowed to overlap.

Time window constraints were first considered in the late 1980s by Daganzo [25] by dividing the time horizon in periods and clustering the customers in rectangles. The problem was simplified by assigning customer time windows to a time period. A more recent contribution for VRP's with time windows is provided by Figliozzi [34], who approximated the distance for serving n customers using a known number m of routes as

$$VRP(n) \approx b\frac{n-m}{n}\sqrt{An} + m2r. \tag{3.2}$$

The parameter b was estimated by linear regression. Figliozzi [35] studied approximations to the average length of VRP's when the number of customers, time window constraints and demand levels vary. A detailed literature review of models that use continuous approximation in distribution management can be found in Franceschetti et al. [36].

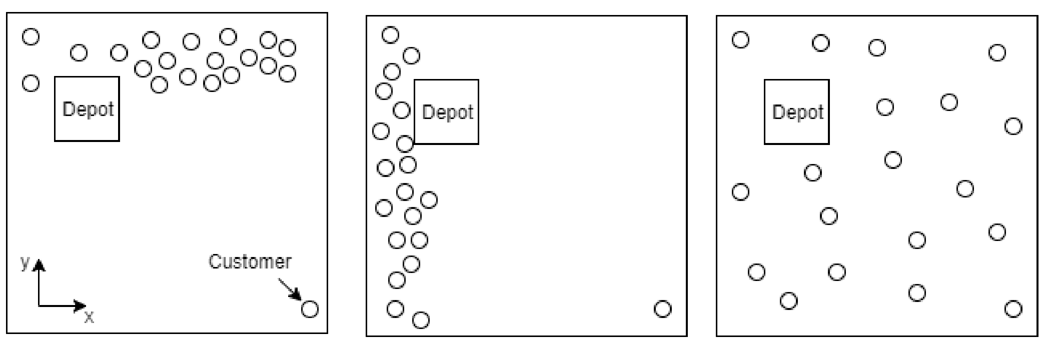
3.3 Regression Based Approximations

This section introduces the problems that are studied in this paper and describes the variables that are included in the estimation models presented in Section 3.4. We consider three classes of problems in increasing order of complexity: The Traveling Salesman Problem (TSP), the Capacitated Vehicle Routing Problems with Time Windows (CVRP-TW), and the Multi Region Multi-Depot Pickup and Delivery Problem (MR-MDPDP).

These problems are also different with respect to the solution methods that can be applied. For small TSP, an optimal solution can be found via exact methods, for the MR-MDPDP, only one heuristic is available. This allows us to study how well the regression approach can adapt to solutions and methods of different quality. We also study this question in more detail for the CVRP-TW, where we apply the regression approach both to known optimal solutions and to solutions obtained with a simple heuristic.

3.3.1 Traveling Salesman Problem

The first models for the TSP approximated the total tour length using the factor $\sqrt{nA}$. Since the total travel distance depends on the distance between nodes, we include different distance measures in the estimation model. To represent different distributions of nodes in the area, we also include dispersion measures such as the variance of distances and maximum distances to the average-node. Based on the approach proposed in [14], we also include the product of the variances of coordinates. Figure 3.1 illustrates the purpose of including this interaction term. The problems presented in figures 3.1a and 3.1b both have solutions with a small total travel distance, but variances in the two coordinates differ considerably. As Figure 3.1c shows, large travel distances will occur if variances in both coordinates are sufficiently high, this effect is covered by including the product.



(a) VarX>>VarY (b) VarX<<VarY (c) VarX $\approx$ VarY

Figure 3.1: TSP nodes distribution scenarios in a similar area
Figure 3.1a shows a distribution of nodes along the x axis. Figure 3.1b shows a distribution along the y axis. Figure 3.1c shows a distribution in which nodes are uniformly distributed along both axis, leading to similar values for their variances.

We therefore consider the following variables:

- Number of requests n (indicated by Req in the tables).
- Distances between nodes: These include the minimum (MinP) and maximum (MaxP) distance across all pairs of nodes and the variance of distances (VarP). Furthermore, we consider the sum of distances to the nearest (SumMinP) and to the farthest (SumMaxP) neighbor of each node.

- Distances to the average-node: Distances to the node located at the average x and y coordinates are aggregated across nodes by considering the minimum (MinM), the maximum (MaxM), the sum (SumM) and the variance (VarM).

- Product of variances of x and y coordinates of nodes (VarX×VarY).

3.3.2 Capacitated Vehicle Routing Problem with Time Windows

In the CVRP-TW, each customer has a known demand, which has to be served by a given fleet of homogeneous vehicles of limited capacity, so that each customer is visited once by one vehicle within a given time window. The presence of time windows can have a strong effect on the total travel distance. In one extreme case, customers could be visited in exactly the same sequence as in the optimal tour without time windows, if time windows are large enough or happen to correspond exactly to that sequence. In the other extreme, time windows may require to visit a customer that is far from the previous customer, and then come back to a customer that is located close to the first one. We use two types of measures to represent how much the presence of time windows constrains the problem: The first directly refers to the length of time windows of individual customers, where we consider the sum and the variance. Furthermore, we use the average overlap and variance of overlaps of time windows between pairs of customers to measure how strongly time windows restrict routing possibilities.

The presence of time windows also influences which links between customers can actually be used. If it is not possible to visit customer i after customer j, then the travel distance from i to j cannot influence the solution. We therefore include travel distance d_{ij} between customers i and j in the calculation of distance-related measures only if the condition

$$STW_i + serv_i + d_{ij} \leq ETW_j \tag{3.3}$$

is fulfilled. In (3.3), $serv_i$ is the service time, and STW_i and ETW_i are the starting and ending times of the time window of customer i. A vehicle is allowed to arrive early to a node and wait until the customer's time window starts.

Since the capacity of each vehicle is limited, additional vehicles will be required once the capacity of a vehicle is exceeded. This increases the total tour length, and we therefore have to include these effects in the model. This is achieved by adding two more variables. The total-demand/vehicle-capacity ratio provides information on the minimum number of tours necessary to perform all requests. The vehicle-capacity/average-demand ratio represents the average number of requests that can be included in one tour.

Previous studies have included the number of requests, the average distance to the depot and the ratio between the number of requests and the vehicle capacity. In addition, we include the following variables:

- Sum (SumD) and variance (VarD) of distances between customers and the depot.

- Sum (SumTW) and variance (VarTW) of the lengths of time windows.
- Minimum (MinPosDist), maximum (MaxPosDist), sum (SumPosDist) and variance (VarPosDist) of distances between customers. Only distances that fulfill condition (3.3) are included in these calculations.
- Sum (SumOverlap), average (AvgOverlap) and variance (VarOverlap) of overlaps of time windows betwen customers.
- Total-demand/vehicles-capacity ratio (Q/Cap) and vehicles-capacity/average-demand ratio (Cap/AvQ).

3.3.3 Multi-Region Multi-Depot Pickup and Delivery Problem

The MR-MDPDP has not been studied as much in previous literature as the TSP and the CVRP-TW. It extends the other problems by providing a hierarchical structure of the transportation system. Customers (pickup and delivery nodes) are located in different regions. A request is fulfilled in three phases: pickup, performed by a short-haul vehicle, a long-haul trip, performed by a larger vehicle, and delivery, again performed by a short-haul vehicle. Requests have different demands and customers have time windows in which they can be visited. The short-haul routing for each region can therefore be classified as a Vehicle Routing Problem with Mixed Backhauls (VRPMB) according to the definitions by Parragh et al. [65]. The objective is to minimize total costs of short-haul and long-haul trips. Although the distance between regions is fixed and there is no routing involved, costs of long-haul trips can vary because different schedules require a different number of trips. Figure 3.2 shows a graphical representation of the problem. In the present paper, we consider only two regions. This problem already provides a sufficiently rich environment to test how well regression based models can approximate optimal solutions in such a complex transportation problem.

New and more complex problems like the MR-MDPDP include multi-modal transportation using different vehicles. Therefore, we have to relate demand to the capacities of all types of vehicles involved. Furthermore, timing of long-haul trips might influence the total distance. For example, if the scheduled departure times are fixed in a way that orders cannot be consolidated, more long-haul trips will be necessary than if the long-haul schedule can be flexibly adjusted.

Even small instances of this problem are quite difficult to solve exactly in reasonable time. Therefore, heuristic solution methods have to be used and the regression approximates the costs in the solutions obtained by these methods, which might be higher than the optimal costs. Different algorithms might find different solutions, but the regression would work in a similar way. However, as we already indicated, we cannot study differences between algorithms for this particular problem, since no alternative algorithms are available.

The dependent variable of the model is the total distance, including long-haul and short-haul trips. The model is very complex, so many properties of the problem can influence the objective

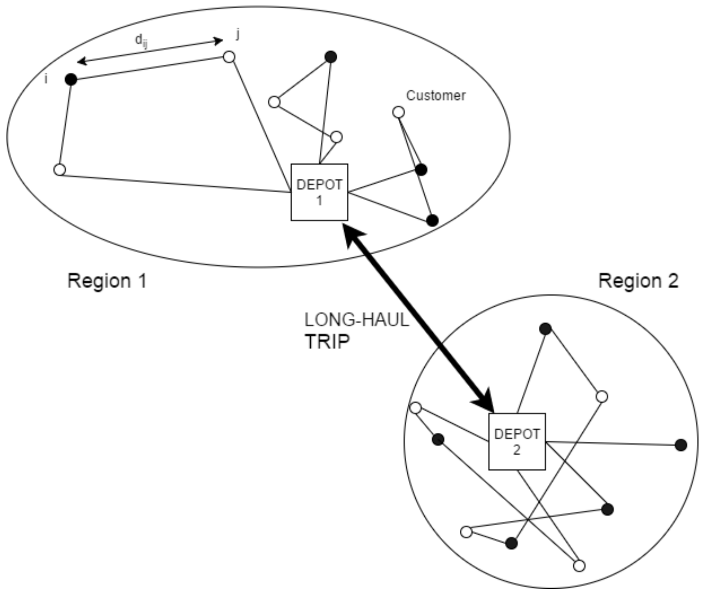


Figure 3.2: 2R-MDPDP Diagram

Requests have paired pickup (white) and delivery (black) nodes in opposite regions. To be serviced, they are part of a vehicle tour in every region starting and ending at the region depot. Both depots are connected by long-haul trips.

value. In particular, we have to distinguish between variables related to long- and short-haul transportation, and variables that refer to single regions or both regions simultaneously. In addition to the variables considered in the previous models, we include the following variables in the initial regression model:

- Long-haul schedule type: a binary variable taking the value of 0 if the schedule is fixed and 1 if it is flexible (BinFlexFix).
- Distances to the depots: in addition to the sum and variance, we also consider the minimum (MinD) and maximum (MaxD).
- Sum of distances between customers (SumP).
- The sum of demands is zero, as it is a pickup and delivery problem. Therefore, we include the quantity-capacity information using the total-pickup-quantities/capacity ratio (SumQ/Cap). When both regions are considered together, the quantity-capacity information is introduced as the variance-of-quantities/capacity ratio (VarQ/Cap).

3.4 Computational Experiments and Results

Our computational experiments are based on instances that are either randomly generated or obtained from literature. Each set of instances was split into two parts, a training set and an evaluation set. Parameters of the regression models were first estimated for the training set, and then the resulting regression equations were applied to the evaluation set. Thus, we obtain data on the within-sample and out-of-sample fit of the model. In line with previous research [16, 53, 34, 35], the regression model does not contain a constant parameter, since a problem without customers obviously has an objective value of zero. After generating a model including all possible variables, we apply forward and backward stepwise regression to obtain models with a smaller set of parameters.

We apply two measures for the quality of approximation. The Mean Percentage Error (MPE) is defined as:

$$MPE = \frac{1}{k}\sum_{i=1}^{k}\frac{D_i - E_i}{D_i} \times 100\% \tag{3.4}$$

and the Mean Absolute Percentage Error (MAPE) as:

$$MAPE = \frac{1}{k}\sum_{i=1}^{k}\frac{|D_i - E_i|}{D_i} \times 100\% \tag{3.5}$$

where D_i is the total travel distance obtained by the solution method, E_i the approximation obtained by the regression models for observation i, and k is the number of observations.

3.4.1 Traveling Salesman Problem

For the TSP, the groups of instances, solution methods to obtain the travel distance and the estimation models with their results are presented in this section.

3.4.1.1 Instances and Solution Method

Instances for this problem contain between 25 and 1000 nodes located in a square of 200×200 units. Travel distance between nodes is assumed to be equal to the Euclidean distance. All instances for this problem were randomly generated. The first group of TSP problems consisted of 260 instances (130 in the training set and 130 in the evaluation set) with 25 to 50 customers. Problems were solved to optimality using the commercial solver IBM CPLEX version 12.6.3.

To test approximation to the TSP problem for larger instances, we generated three groups of 400 instances (200 for training, 200 for evaluation) with 50 to 1000 customers. The three groups differed in the locations of nodes. In one group (random instances, R), nodes were uniformly distributed across the entire area. The second group (C) contained clustered instances, where two clusters 1/9 the size of the entire area were created and located at opposite corners within the area and nodes were uniformly distributed within these clusters. The last group (random-clustered, RC) was a mixed group, in which the set of nodes was randomly fractioned in three smaller sets, two were uniformly distributed in each cluster and one was uniformly distributed across the entire area. As these instances are too large to solve optimally, we used the Lin-Kernighan (LK) implementation by Helsgaun [46].

3.4.1.2 Regression Parameters and Results

The models estimated for the small instances are presented in Table 3.1. Model 1 was created using a backward stepwise regression and Model 2 is based on a forward stepwise regression. In both models, a threshold of $p = 0.05$ for entering/removal was used. The best models for each of the three groups of larger instances are presented in Tables 3.2, 3.3 and 3.4. For comparison, we also add the models of Hindle and Worthington [49] (HW) and the model of Çavdar and Sokol [14] (CS).

Parameter	Model 1	Model 2	HW	CS
Req	***7.077		6.98	
MinP				
MaxP	***1.309	***1.369		
SumMinP	***0.478	***0.388		
SumMaxP		***0.072		
VarP		***-7.4 E-5		
VarX × VarY	**3.23 E-6			
MinM				
MaxM		***-1.581		
SumM		***0.101		
VarM	***0.0899	***0.127		
Constant			59.18	
$\ln(Req)$			186.48	
$\sqrt{Req(cstdev_x cstdev_y)}$				*** 2.879
$\sqrt{Req(stdev_x stdev_y)\frac{A}{C_x C_y}}$				4.4 E-5
R2 (adjusted)	0.9988	0.9989	0.7153	0.9958
MPE (in sample)	0.07%	0.07%	-0.33%	0.23%
MAPE (in sample)	2.76%	2.57%	4.79%	5.23%
MPE (out of sample)	0.64%	0.88%	0.78%	0.34%
MAPE (out of sample)	2.60%	2.73%	4.42%	5.55%

$*p < 0.1; **p < 0.05; ***p < 0.01$

Table 3.1: TSP Estimation Models for Small Instances

In sample, for the small instances, our regression models provide a considerably better approximation than the previous models. Out-of-sample, the MPE is comparable to the value of the model of Hindle and Worthington [49], and higher than in the model of Çavdar and Sokol [14]. However, the MAPE of our models is considerably better. Approximation errors in the model of Çavdar and Sokol [14] are more evenly distributed around zero, but on average, the absolute error is larger. For larger instances, all models perform similarly for the R instances, providing small estimation errors. However, our models outperform the other two models for the RC and C instances.

Parameter	Model 1	HW	CS
Req	0.198	*** 2.33	
MinP	***-33.818		
MaxP	***3.064		
SumMinP	***1.063		
SumMaxP	-0.003		
VarP			
VarX × VarY	** -1.3 E-5		
MinM	**-2.635		
MaxM	-0.652		
SumM	*0.0188		
VarM	-0.088		
Constant		***-1073.8	
ln(Req)		***539.392	
$\sqrt{Req(cstdev_x cstdev_y)}$			***2.70883
$\sqrt{Req(stdev_x stdev_y)\frac{A}{C_x C_y}}$			***-1.65 E-6
R2 (adjusted)	0.9998	0.9972	0.9996
MPE (in sample)	-0.11%	0.03%	0.54%
MAPE (in sample)	1.59%	1.85%	2.02%
MPE (out of sample)	0.12%	-0.01%	0.40%
MAPE (out of sample)	1.77%	1.79%	2.15%

*:$p < 0.1$; **:$p < 0.05$; ***:$p < 0.01$

Table 3.2: TSP Estimation Models for Large R Instances

Parameter	Model 1	HW	CS
Req		***1.573	
MinP	***-17.268		
MaxP	***2.107		
SumMinP	***1.1147		
SumMaxP			
VarP			
VarX × VarY			
MinM	***-6.597		
MaxM			
SumM	***0.012		
VarM	***-0.090		
Constant		***-1507.87	
ln(Req)		***623.38	
$\sqrt{Req(cstdev_x cstdev_y)}$			***2.33239
$\sqrt{Req(stdev_x stdev_y)\frac{A}{C_x C_y}}$			2.43 E-6
R2 (adjusted)	0.9996	0.8872	0.9779
MPE (in sample)	-0.23%	-1.29%	0.41%
MAPE (in sample)	1.81%	9.66%	13.25%
MPE (out of sample)	-0.37%	-0.38%	1.32%
MAPE (out of sample)	2.12%	10.32%	13.43%

*:$p < 0.1$; **:$p < 0.05$; ***:$p < 0.01$

Table 3.3: TSP Estimation Models for Large RC Instances

Parameter	Model 1	HW	CS
Req	***0.157	***1.228	
MinP	***-36.069		
MaxP	***1.194		
SumMinP	***0.987		
SumMaxP			
VarP	***-4.41 E-7		
VarX × VarY			
MinM			
MaxM			
SumM	***0.025		
VarM	***0.157		
Constant		*-757.049	
$\ln(Req)$		***350.698	
$\sqrt{Req(cstdev_x cstdev_y)}$			***1.409
$\sqrt{Req(stdev_x stdev_y)\frac{A}{C_x C_y}}$			***9.9 E-6
R2 (adjusted)	0.9997	0.8798	0.9665
MPE (in sample)	-0.15%	-0.15%	1.20%
MAPE (in sample)	1.72%	11.16%	17.02%
MPE (out of sample)	0.56%	0.80%	-0.17%
MAPE (out of sample)	1.91%	11.62%	25.46%

*:$p < 0.1$; **:$p < 0.05$; ***:$p < 0.01$

Table 3.4: TSP Estimation Models for Large C Instances

3.4.2 Capacitated Vehicle Routing Problem with Time Windows

Section 3.4.2.1 describes the instances and solutions for the CVRP-TW. Estimation models and their results for optimal and Nearest Neighbor Solutions are presented in Section 3.4.2.2.

3.4.2.1 Instances and Solution Method

For this problem, the instances proposed in Solomon [77] for 25, 50 and 100 customers are used. These instances have a hierarchical objective, minimizing first the number of vehicles and then the total distance. The dependent variable corresponds to the travel distance reported for the optimal solutions. Instances and solutions can be found at *http://web.cba.neu.edu/~msolomon/home.htm*. Additional solutions were obtained from Baldacci et al. [2].

The Solomon data set contains six groups of instances, clustered instances (C), random instances (R) and random-clustered instances (RC), which are all further separated into instances with long and short planning horizons (subgroups C1 and C2 etc.). In works like [35], separate models were generated for each group of instances. This approach implicitly assumes that when applying the model to a new instance, one is able to correctly identify the level of clustering as well as whether the time horizon is long or short. In this paper, we present models for instance groups C, R and RC separately, as they differ considerably in the distribution of customers. Groups 1 and 2 are merged by including variables that relate to capacity.

3.4.2.2 Regression Parameters and Results

Table 3.5 presents the best performing approximation models for the optimal solutions of the three groups of problems. The model for instances of type C approximates the solution with very small errors. For instances of type R, our results are comparable to results obtained in previous papers like [35]. The approximation error for problems of type RC is considerably larger. These problems seem to be particularly difficult, since instances that are very similar in all the variables considered sometimes have very different objective values. However, this effect can, to a certain extent, be corrected when calibrating the models by considering the in-sample approximation error of each instance. For the present estimation, we removed one instance (RC205) that had a particularly large in-sample error. RC205 represents a situation where, because of the arrangement of time windows across nodes, one would expect a large total travel distance. Time windows in this problem are rather short and have little overlap. Still the total travel distance is rather short because neighboring nodes also have time windows that allow to visit them within a short time span.

These results show that there might always be additional factors not included in the model that increase the variance of travel distance and thus make prediction more difficult. Such influences might come from the problem, but also from the solution algorithm used. In particular, simple algorithms tend to find good solutions in some instances, but also very bad solutions in other instances. This creates a large variance in solutions that is hard to predict. Table 3.6 illustrates this effect with models that approximate solutions obtained by a nearest neighbor insertion algorithm. As it was expected, a less powerful solution algorithm like this one leads to a high variability in objective values, which makes approximation difficult. Our models perform acceptably only for the R type instances. For the C and RC type, the approximation errors are considerably larger.

Parameter	R	RC	C
Req		**-18.713	***25.422
SumD	***0.901	***0.653	
VarD	**-0.168	***0.06	***-0.071
VarX $\times$VarY			***1.29 E-7
SumMinP			***4.86 E-4
SumTW	**4.393 E-3	***-7.79 E-3	***-9.06 E-5
MinPosDist	**-25.907		
MaxPosDist	***0.143		
SumPosDist	***-3.63 E-3	***-3.99 E-3	
AvgOverlap	*** -0.035		
VarOverlap		***4.12 E-7	
Q/Cap	***63.251	***57.212	***-12.963
Cap/AvQ		***4.612	
R2 (adjusted)	0.9980	0.9989	0.9999
MPE (in sample)	-0.26%	-0.20%	0.09%
MAPE (in sample)	3.22%	2.51%	0.37%
Out-of-Sample			
MPE (out of sample)	0.01%	3.38%	0.1%
MAPE (out of sample)	4.45%	6.47%	0.5%

*:$p < 0.1$; **:$p < 0.05$; ***:$p < 0.01$

Table 3.5: CVRPTW Estimation Models for Optimal Solutions

Parameter	R	RC	C
Req	***27.882	87.652	-23.252
SumD		-0.914	***2.439
VarD		-0.022	-0.035
VarX $\times$VarY			
SumTW		-0.032	*-0.014
VarTW		2.22 E-5	**3.21 E-6
MinPosDist	***-19.706		
MaxPosDist			
SumPosDist	***-3.33 E-3	-9.2 E-4	***-7.2 E-3
VarPosDist		-4.4 E-5	* -4.7 E-6
SumMinPosDist	***0.925		
SumOverlap		8.41 E-5	**9.62 E-5
AvgOverlap	***-0.218	-0.0281	0.156
VarOverlap		3.31 E-7	-3.47 E-9
Q/Cap	***30.763	-160.62	***-130.449
Cap/AvQ		-9.906	***7.624
R2 (adjusted)	0.9990	0.9930	0.9979
MPE (in sample)	9.94 E-3%	-0.81%	0.04%
MAPE (in sample)	3.12%	6.90%	3.31%
MPE (out of sample)	-0.96%	-4.11%	0.97%
MAPE (out of sample)	4.78%	12.06%	6.45%

*:$p < 0.1$; **:$p < 0.05$; ***:$p < 0.01$

Table 3.6: CVRPTW Estimation Models for Nearest Neighbor Solutions

3.4.3 Multi-Region Multi-Depot Pickup and Delivery Problem

The instances for this problem are described in Section 3.4.3.1. Since no standard method for this problem class is available, instances were solved using a specifically developed heuristic approach described in Section 3.4.3.2. The models, which estimate the solutions obtained by this particular solution method, and their results are presented in Section 3.4.3.3.

3.4.3.1 Instances

For this problem, instances with 10, 25, 50 and 100 customers were randomly created. We consider two regions with a size of 100×100 distance units. Each customer has a transportation request from a randomly located pickup node in one region to a randomly created delivery node in the other region. Demand values are randomly assigned to each customer. Two types of time windows are used: broad time windows, with a length of 75% of the planning period for all customers, and tight time windows, with a length of 6.25% of the planning period for all customers. For every instance, we also considered two types of long-haul schedules: a fixed schedule, in which long-haul vehicles leave at fixed points in time, and a flexible schedule, in which long-haul vehicles can leave at any time. A total of 444 instances is used for this problem, split evenly into training and evaluation sets.

3.4.3.2 Solution Method

Due to problem complexity, a decomposition approach is used. In the first step, an exact solution for the long-haul assignment is determined using the commercial solver IBM CPLEX version 12.6.3. The long-haul assignment fulfills all requests and minimizes long-haul costs. This long-haul information is used as input for the second step, in which the short-haul routing is performed. The short-haul routing can be performed for each region independently and can therefore be classified as Vehicle Routing Problem with Mixed Backhauls (VRPMB) [65]. The solution method starts by assuming single-customer routes for each node. It then uses the pilot method [84] with an underlying savings algorithm [18]. Since we are dealing with a problem with time windows, the standard savings calculation

$$s_{ij} = d_{i0} + d_{0j} - d_{ij} \tag{3.6}$$

is changed to

$$s_{ij} = w_1 \cdot (d_{i0} + d_{0j} - d_{ij}) - w_2 \cdot waitingTime \tag{3.7}$$

where w_1 and w_2 are weights to account for distances and waiting time, referred as $waitingTime$. Waiting time is considered because the savings algorithm merges routes without re-evaluating sequence of nodes within the combined route. If a vehicle arrives at a node before the time window opens, waiting time occurs. It is time that is not spent productively and too much waiting time could lead to infeasible solutions. A long waiting time makes joining the two routes less attractive.

We use the pilot method to determine the order in which the savings are selected. This method "looks ahead" to determine which of the available immediate decisions is the best in the long run. In contrast to a greedy approach, which chooses the (locally) best savings, or a pure random approach, we try out the best m savings. This approach creates m solution candidates, which correspond to the m best savings in the first iteration and the best savings for the next n iterations. The best partial solutions after $n-1$ iterations are chosen for evaluation of the current step. This is iterated until the entire solution is built. Figure 3.3 shows the basic principle of the "looking ahead" approach.

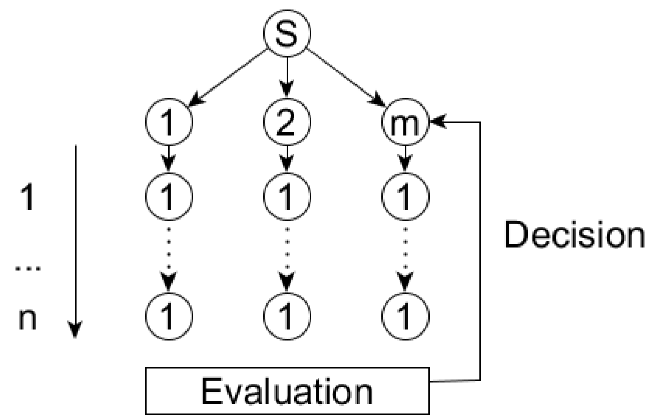


Figure 3.3: The basic principle of the pilot method showing $1..m$ solution copies of solution S with $1..n$ looking ahead steps.

Since a vehicle can leave the depot multiple times, the tours obtained by the pilot method and savings algorithm are combined to vehicle routes via a heuristic proposed by Battarra et al. [4]. This is also necessary if at this point the number of vehicles needed is greater than the number of available vehicles. All tours are initially sorted by non-decreasing starting time, breaking ties according to minimum route duration. Tours are appended in this order to vehicles that are already in use. If a tour cannot be added anywhere, a new vehicle is used. Vehicles are selected to minimize the idle time spent at the depot.

This solution method's average computation time for the instances described in Section 3.4.3.1 ranged from 21.13 seconds for problems with 10 customers to 162.17 seconds for problems with 100 customers.

3.4.3.3 Regression Parameters and Results

The results for models considering variables for each region separately are presented in Table 3.7. Model 1 is the full model that includes all variables. Models 2 and 3 are obtained by forward and backward stepwise procedures. In both cases, a threshold value of $p = 0.05$ was used. The models achieve a very good approximation with out-of-sample MAPE values below 2%.

Table 3.8 illustrates the advantage of considering each region individually. In that table, we combine distances from both regions into one variable, which results in a much higher MAPE value. Considering each region individually allows the model to evaluate the complexity of the VRP, and thus the likely contribution to the total objective value, in each region separately. In contrast, similar average values across both regions could be generated by very different scenarios in the two regions, leading to high variance of solutions and thus a considerably worse approximation overall.

Parameter	Model 1	Model 2	Model 3
Req	**19.617		***17.017
BinFlexFix	***-214.185	***-210.349	***-213.9
SumTW	**-8.8 E-4		**-9.536 E-4
MinD1	-0.4448		
MaxD1	***2.964	***3.081	***3.495
SumD1	***1.936	***1.901	***1.887
VarD1	-3.598 E-3		
MinD2	*2.231		**2.656
MaxD2	-0.299		
SumD2	***1.917	***1.937	***1.907
VarD2	**1.008 E-2		***9.72 E-3
MinP1	-0.628		
MaxP1	*3.2	***2.318	
SumP1	-3.34 E-3		
VarP1	3.02 E-4		
MinP2	-0.394		
MaxP2	*2.808		***4.289
SumP2	1.339 E-3		
VarP2	-6.7 E-4		**-5.316 E-4
VarX × VarY1	2.36E-07		
VarX × VarY2	2.E-06	***3.69 E-7	***2.077
MinM1	1.071		
MaxM1	-1.491		
SumM1	*-0.409		
VarM1	-0.017		
MinM2	0.337		
MaxM2	-0.228	***4.744	
SumM2	*-0.539		***-0.741
VarM2	*-2.992 E-2		**-4.204 E-2
SumTW × Req	*1.13 E-5		***1.245 E-5
BinFlexFix × Req	***5.189	***5.014	***5.137
SumQ/Cap	-2.077		
R2(adjusted)	0.9998	0.9998	0.9998
MPE (in sample)	4.74 E-5%	0.12%	0.04%
MAPE (in sample)	1.44%	1.49%	1.46%
MPE (out of sample)	-0.32%	-0.12%	-0.21%
MAPE (out of sample)	1.57%	1.50%	1.55%

*: $p < 0.1$; **: $p < 0.05$; ***: $p < 0.01$

Table 3.7: 2R-MDPDP Estimation Models with variables for every region separately

Parameter	Model 1
Req	***118.303
BinFlexFix	-134.489
SumTW	***5.006 E-3
VarQ/Cap	**-289.528
FlexReq	**4.533
MinD	-0.639
MaxD	*-7.691
SumD	0.318
VarD	0.011
MinP	0.553
MaxP	**8.317
SumP	0.007
VarP	***-4.206 E-3
VarXVarY	5.71E-07
MinM	13.253
MaxM	-2.393
SumM	0.102
VarM	**0.079
R2(adjusted)	0.9916
MPE (in sample)	-0.67%
MAPE (in sample)	7.91%
MPE (out of sample)	-0.61%
MAPE (out of sample)	9.30%

*:$p < 0.1$; **:$p < 0.05$; ***:$p < 0.01$

Table 3.8: 2R-MDPDP Estimation Model for two region-combined variables

3.5 Conclusions

A fast and accurate approximation of travel distances that can be obtained without solving a complex transportation problem is important for many applications. In this paper, we have presented an empirical approach to estimate total travel distances for different routing problems using different properties as predictors in regression models. Our models covered a wide range of transportation problems, ranging from conceptually simple traveling salesman problems to complex multi-region transportation problems involving a wide spectrum of real-life constraints. Although the quality of approximation varies between different problem classes, in many cases we obtain very good approximations, which are more accurate than previous models. In particular, we obtained good approximation results for a new class of problems, the 2R-MDPDP problem, for which no previous results were available. This shows that an empirical approach makes it possible to cover problems that include additional characteristics that could not be taken into account in previous models.

Furthermore, we were also able to show that models estimated on one set of instances can reliably predict travel distances for other, previously unknown problems. This is a particularly important result in view of potential applications of our models. Another important feature of the empirical approach for this type of applications is that our models can not only be fitted to optimal solutions, but also reliably predict the travel distances that can be obtained with heuristic methods like the one we have utilized for the 2R-MDPDP problem.

Apart from the fact that our models provide a very good approximation in many cases, the regression models also provide additional insight about which variables have a strong influence on total travel distances for the different types of transportation problems. By applying a stepwise regression approach, we were able to identify significant drivers of travel distances (and costs) out of a large set of possible influence factors. An interesting result here is that the number of requests and the area in which customers are located are not present in all the models, and that the total travel distance can also be estimated based mainly on information about distances between customers. Information about which problem characteristics are strong drivers of travel distances is not only important to obtain better approximations, but can also support carriers in their decision making. Knowing which factors influence total travel distances can help carriers to select those requests that can be fulfilled efficiently and thus to focus their acquisition efforts. Our models therefore can support carriers in many ways in their planning and decision making for efficient operations.

Our results indicate not only the usefulness, but also the limitations of such an empirical approach. For some types of problems, approximation errors were considerably larger than for other problems. An empirical approach relies on the stability of the relationship between explanatory variables and the value to be predicted. Any effect that is not explicitly captured in the model can be considered as noise that makes the empirical prediction more difficult. Apart from systematic factors omitted from the model specification, such noise can also result from some inherent randomness of the problem, which makes travel distances to a certain extent unpredictable. One possible source of such randomness might be the particular structure of the problem, as became evident in the case of the random-clustered CVRP problems. Here the partly systematic, partly random location of customers makes it difficult to approximate total costs. Another source of randomness may be the algorithm used to solve the problem, as became evident in the CVRP solved with the nearest neighbor heuristic. There, the highly variable performance of the algorithm makes prediction difficult. However, as the example of the 2R-MDPDP has shown, it is sometimes possible to overcome this high level of randomness by considering relevant variables at a more detailed level, here by considering information about both regions separately. This shows that a systematic analysis of possible variables to be included in such models is a worthwhile exercise, that can lead to a highly reliable prediction of travel distances and costs.

Chapter 4

Mechanisms for Request Exchange in Collaborative Transportation

Abstract

This paper presents frameworks for auction-based and posted price mechanisms for the exchange of requests between carriers that are members of a collaborative network operating in the same geographical areas. All mechanisms are tested within the same scenario and by comparing their results; the aim is to obtain insights regarding the efficiency gains they offer to the network and the amount of information required for their application. The mechanisms' main objective is to give carriers the chance to improve their original sets of requests in order to increase their overall efficiency. Due to the nature of routing problems, requests usually have different costs for different carriers, depending on the location of the depots as well as on the other requests that have to be serviced. The exchange mechanism should find a new allocation such that carriers obtain requests that have higher value (i.e. lower cost) for them. Results obtained here show that individual auction-based mechanisms provide similar results, on average, to centralized auction-based mechanisms, both outperforming posted price mechanisms. Nevertheless, this improvement difference comes at the cost of providing a larger amount of information contained in the carriers' bids.

4.1 Introduction

The competitive environment in the transportation industry motivates companies and researchers to continuously study and test mechanisms in order to reduce operational costs and increase productivity. Over the past decades, efforts have mainly been focused on the development of exact and heuristic methods which provide high quality and even optimal solutions in terms of traveled distance for a broad list of problems. However, the application of other initiatives like collaborative relationships, which have recently been considered as one main trend in transportation [79], has been shown to reduce operational inefficiencies significantly.

While large companies might benefit from economies of scope when using a central planning approach for sub-units of their own, small and mid-sized firms are constantly searching for innovations that allow them to reach satisfactory profit levels. Moreover, these smaller carriers need to be productive enough to stay in business, in fact, from 1980 to 1999, around fifty thousand carriers closed due to low revenue/cost ratios [87]. One innovative approach which increases productivity is horizontal collaboration. In Europe, specifically in the less-than-truckload market, collaborative networks of small and medium companies occupied six out of the top ten carrier organizations in the early 2000's [51], suggesting that formation of collaborative networks is a well known practice in transportation.

Horizontal collaboration in transportation is conducted in many ways, including sharing vehicles, locations and other resources, (re)allocating tasks, as well as exchanging requests. In this paper, mechanisms for request exchange between competing carriers operating in the same geographical areas are considered. The mechanisms aim to give these carriers the chance to improve their original sets of requests, in order to increase their efficiency. At the same time they require limited information, in contrast to central planning approaches in which a central decision maker needs access to all the network's members' information.

Due to the nature of routing problems, requests generally have different costs for different carriers, depending on the location of the depots as well as on the other requests that have to be serviced. The exchange mechanism aims to allocate requests to carriers, such that requests are serviced at a lower cost. An exchange takes place when the parties involved agree on a price for re-allocating a particular group of requests. In order to do so, either sellers can offer prices to be accepted by buyers, or buyers can set prices to be accepted by sellers. Considering the first case, an auction-based process would allow sellers to build their offers based on their own costs and express them as bids, and buyers would choose the best option to accept. In the latter case, buyers would post a price based on their own costs, and sellers could accept to provide their service at that specific price or not, without revealing their exact costs. In both exchange mechanisms, carriers act as buyers and sellers at the same time, looking for other carriers to service some of their requests while also selling some of their operational capacity to service other carriers' requests.

In this article, variants of these mechanisms are described along with some special considerations that have to be taken into account in the design of the mechanisms' processes. The proposed mechanisms are tested within the same scenario, and by comparing their results, the aim is to obtain insights regarding the improvement they offer to the network and the amount of information required for their application. Moreover, the trade-off between information requirement and efficiency gain is explored.

Thanks to the development of information and communication technologies and tools, the implementation of any of the proposed mechanisms is plausible. Results may help carriers interested in collaboration as well as third-party companies that might be interested in acting as a central institution (when needed). The proposed mechanisms and their processes could be used in other collaborative assignment markets, where costs for servicing a customer are different for firms that are willing to be part of a collaborative exchange network.

The rest of the paper is structured as follows: a literature review is presented in Section 4.2. Section 4.3 provides a detailed description of the proposed mechanisms and presents considerations that are taken for each one. Computational experiments and comparison of results are presented in Section 4.4. Section 4.5 concludes the paper with a discussion about its findings.

4.2 Literature Review

To date, several studies have addressed different ways of collaboration in transportation. For example, [88] studied the allocation of tasks considering cost minimization and fairness. Another type of collaboration is presented in [33], where in the Shipper Collaboration Problem, shippers identify sets of lanes that are submitted to carriers as a bundle in order to obtain better rates. A recent study of a centralized collaboration approach in a two-region setting with multiple depots can be found in [78]. Gansterer and Hartl [39] presented an extensive review of collaborative vehicle routing research. They provided a well organized overview of this subject and identified centralized planning approaches, as well as auction-based and non-auction-based approaches, as major streams of research in collaborative transportation.

With regard to the re-assignment of requests, one can think of a collaborative network of carriers acting as buyers, sellers or both. Shapley and Shubik [76] presented an assignment game model for a two-sided market in which indivisible products are exchanged for money, however, players in this model acted either as sellers or as buyers, but not both at the same time; based on this study, different models have been investigated over the last years, [64] for example. In the specific case of transportation and logistics, particularly in less-than-truckload transportation, Wang and Kopfer [85] presented a decentralized request exchange mechanism in which only vehicle routes are taken into account for exchange. Research has also been conducted on the exchange of single requests and bundles of requests, starting with Krajewska and Kopfer [52], who presented a collaboration model based on combinatorial auctions and game theory.

A complete framework for an auction-based request exchange mechanism was provided in [8], which was composed of different processes in the following order:

1. Forming a request candidate set.
2. Composition of bundles from the candidate set.
3. Determination of marginal profits.
4. Assignment of bundles to carriers.
5. Profit sharing.

These authors stated that, since bundle reassignments outperformed single request reassignments, the only way to reduce decentralization costs is to increase the amount of data that is centrally known.

Within the same framework, Gansterer and Hartl [37] compared different request selection strategies. It can be observed in their results that strategies which considered geographical properties dominated pure profit-based strategies, reflecting the difficulty in obtaining the real marginal value of a request. Similar geographical properties are included in models (e.g. [63]) as direct components of an estimation of the travel costs.

Building bundles of requests exploits the synergies that exist between requests, as a matter of fact, collaborative transportation is usually mentioned as one of the most common applications of combinatorial auctions (e.g. [21]). Considering all possible combinations of requests in bundle composition, like in [8, 37], would result in either limiting the number of requests to be exchanged, or facing an unmanageable number of bundles that could be potentially reassigned. This complexity was addressed by Gansterer and Hartl in [38], where they defined the Bundling Generation Problem and proposed a method that efficiently generates a limited number of attractive bundles with a small loss in solution quality. Another way of reducing this complexity is the use of multi-round auctions, as presented for example in [26], where the sets of items that firms are willing to exchange are decomposed and offered in multiple rounds. Both, limiting the number of (attractive) bundles and running a mechanism for multiple rounds combined, present an interesting and promising approach.

In previous works, such as [8, 37] marginal profits were determined by assigning each request a marginal value, resulting from the difference between routing solutions with and without the request. Nicola et al. [63] presented regression-based approximations for routing problems that provide fast estimations, which could become a useful tool for evaluating the cost of servicing a particular request or bundle of requests. Buer [12] dealt with strategies for truthful bidding in combinatorial transportation auctions and he also stated that strategic bidding in difficult scenarios like these collaborative transportation problems results in a non-trivial activity.

A strategic component might be added to the process of determining marginal profits in game theoretic approaches. Pekeč and Rothkopf [67] suggested that single-round combinatorial auctions are resistant to collusive conspiracies by bidders, therefore these settings encourage participation. Gansterer et al. [40] proposed incentive compatible exchange mechanisms and showed that desirable properties (i.e. efficiency, incentive compatibility, individual rationality, and budget balance) cannot be fulfilled at the same time.

The problem of assigning bundles to carriers in such a way that the operational costs are minimized is a well known problem in operations research, namely the assignment problem, which reduces to solving an integer linear program similarly as presented in [76]. For auction-based mechanisms, this allocation of requests to carriers is performed based on bids that are generated by participating carriers and is known as the Winner Determination Problem [27, 67]. A final step to be addressed in collaboration mechanisms is profit sharing, for which Guajardo and Rönnqvist [45] presented an extensive review of cost allocation methods.

Behavioral components are not considered in this paper but should be considered for applications and future research. Experimental studies of exchange mechanisms complement theoretical approaches and provide insights on potential expected behavior from participants. For example, Rief and Van Dinther [72] studied the behavior of participants in both an asymmetric and a symmetric information scenarios, via an experimental study modeled as an ultimatum game in a supply chain environment. Bellantuono et al. [6] examined e-procurement of logistics services experimentally, comparing a multi-attribute auction and a negotiation exchange mechanism, providing practical implications on the use of both. Bichler et al. [10] analyzed bidding behavior in two common sealed-bid split-award auctions experimentally; some of their results suggest that risk aversion has a significant impact in underbidding.

4.3 Request Exchange Mechanisms

The exchange mechanisms described in this section correspond to double-side mechanisms, where carriers that collaborate with each other are willing to procure services from other carriers to fulfill some of their requests and also sell part of their operational capacity to fulfill other carriers' requests. No extra income from outside the network is generated by the exchange mechanisms. The possible benefit carriers should obtain as a group by altering their sets of requests is reflected in lower operational costs, increasing their net profit. Carriers face different decisions depending on the mechanism they use for collaboration. Before the application of a mechanism, carriers have an initial allocation of requests, referred to as their original set of requests. The starting point for any mechanism is the evaluation of these original sets and the selection of some requests they want to exchange.

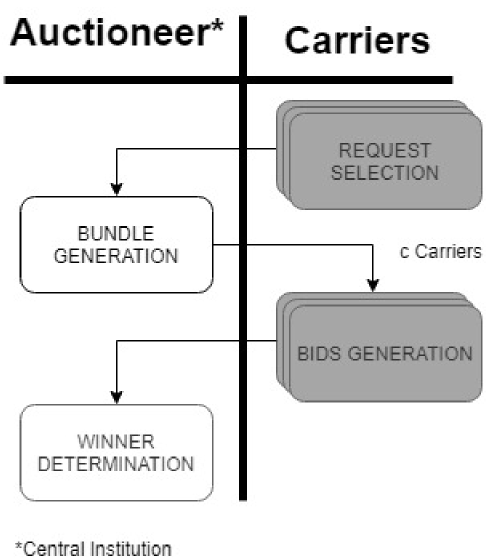
Mechanisms differ from each other according to which party (buyer or seller) is setting the prices, the allocation decision maker (centralized or decentralized) and/or other minor rules. Auction-based and posted price mechanisms described in this section follow a similar framework to the one mentioned in Section 4.2, presented in [8, 37].

4.3.1 Auction-based Mechanisms

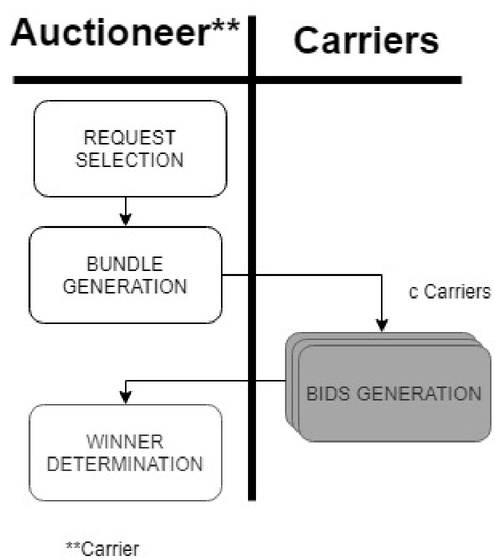
Starting with an initial request allocation, request exchange is performed following these steps:

1. Request Selection.
2. Bundle Generation.
3. Bid Generation.
4. Winner Determination.
5. Exchange and Payment.[1]

Carriers select which requests they are willing to exchange. The bundle generation is performed by the auctioneer. Bundles might contain requests from one or several carriers, depending on the auction format that is being used. The bid generation is always performed by the carriers involved in the mechanism. The winner determination is solved by the auctioneer. The processes differ between centralized and individual auctions, as detailed in Sections 4.3.1.1 and 4.3.1.2 respectively. Figure 4.1 shows the steps and their decision makers for centralized and individual auction mechanisms.



(a) Centralized Auctions Mechanism



(b) Individual Auctions Mechanism

Figure 4.1: Decisions in Central and Individual Auctions Mechanisms

[1]This step is a direct consequence of the winner determination step.

4.3.1.1 Centralized Auctions

The centralized auctions mechanism requires a central institution acting as the auctioneer. The entire mechanism follows the steps mentioned in Section 4.3.1 as the output from one process becomes an input for the next one. A round of centralized auctions is defined as the application of all the steps mentioned in section 4.3.1, starting with the selection of requests and ending with the reassignment of these requests based on bids made by carriers.

4.3.1.1.1 Request Selection

In [8], request j was evaluated based on its marginal value, obtained as the difference between the tour length when including request j and the tour length when excluding request j. As mentioned in section 4.2, results presented in [37] showed that strategies taking into account geographical properties performed better than pure profit-based strategies. In [63], we proposed regression-based estimation models of the form:

$$E = \sum_i \kappa_i \times K_i \tag{4.1}$$

where E is the estimation of the expected travel distance to be obtained by the problem solution and κ_i is the value of the regression parameter for variable K_i. Variables K_i can provide various measures to be considered either individually or combined as part of a selection criteria list. In line with [37], these variables correspond to geographical properties such as distance between nodes. Apart from the selection criteria, a selection threshold is necessary. The selection process works as follows:

1. The carrier chooses a selection criterion.
2. The carrier sets a threshold for that particular selection criterion or the number of requests to be selected.
3. The carrier selects requests that exceed that specific threshold and submits them to be exchanged by the mechanism being used.

Carriers must either perform the request selection themselves or provide information to an external party to perform this process for them. The latter option might not be desirable as carriers might want to keep some information for themselves or they might consider certain customers to be "special" and would want to hang on to their requests even when it would be efficient to exchange them. The number of selected requests depends on the value of the threshold considered for selection. Carriers might select a large number of requests to be exchanged if their initial sets are costly, but also might select a few or no requests if the costs for their initial sets are low. This decision would of course depend on each carrier's own experience and relevant information.

4.3.1.1.2 Bundle Generation

Requests have different values when they are grouped into bundles. Results obtained in previous research (e.g. [8, 37]) show that bundle exchange yields better results than single request exchange. For centralized auctions, bundling is performed by the central institution acting as the auctioneer. As all possible combinations might result in an unmanageable number ($2^n - 1$ for n requests), only attractive bundles should be generated. The bundles should have a size within some determined limits, as some carriers might not want to be assigned neither too large nor too small bundles due to capacity constraints. To generate attractive bundles, generation criteria should take into account properties from models like Equation (4.1). Bundles would start with one request, referred to as a seed request, then other requests (previously sorted according to the selected generation criteria in relation to the seed request) would be added one at a time. The requests submitted by each carrier are also combined to a bundle. Thus, if no better re-allocation is possible, all requests can be sent back to their original owners to guarantee a feasible solution.

4.3.1.1.3 Bid Generation

When the bundle generation process has been completed, carriers are asked to bid on all the bundles offered by the auctioneer. Carriers then have to evaluate every bundle individually and decide how much to bid for it based on two main factors:

1. The carrier's available capacity.

2. The carrier's cost for servicing the requests in the bundle.

As for available capacity, no carrier would be willing to end up with any requests that cannot be handled. This is avoided by reporting extremely high costs for bundles that are too large. In case a bundle's size does not exceed the carrier's available capacity, the expected marginal servicing cost for that particular bundle could be obtained from the difference between the carrier's routing solutions including and excluding the bundle. These solutions can be obtained by any method the carrier considers appropriate. However, it should be noted that the number of offered bundles, even when not all possible combinations ($2^n - 1$ for n requests) are considered, might be large, needing large computational efforts and time. For example, if the solution method the carrier uses takes 1 second per bundle, and there are 4000 offered bundles, over one hour would be needed to evaluate their individual marginal cost, therefore, approximation methods like Equation 4.1 are highly recommended for this process. It is assumed that carriers do not incur in strategic bidding behavior as it is not a trivial action to take [12], and also might be unprofitable to do so [8, 37].

4.3.1.1.4 Winner Determination

The central institution has to reassign bundles to carriers based on their bids. A fundamental link between the winner determination, bundle generation and bidding generation is the number of bundles a carrier will obtain, as the carriers' capacities should not be exceeded. Another important consideration is compensation payments, as they can influence carriers' bidding behavior. Carriers might be willing to service costly bundles if they are compensated for it. As carriers are asked to bid on their submitted set of requests (reserve prices), the auctioneer has information about their individual cost before collaboration, as well as the group's expected profit after collaboration, this information could be taken into account for profit sharing. The auctioneer has to allocate bundles to carriers minimizing the total cost. This problem, known as the Winner Determination Problem (WDP) (e.g.,[27, 67]), is formulated as follows:

C	set of carriers
R	set of requests
B	set of bundles
p_{bc}	bid for bundle b offered by carrier c
q_{rb}	binary parameter indicating if request r is in bundle b
x_{bc}	binary variable indicating if bundle b is assigned to carrier c

$$min \sum_{c \in C} \sum_{b \in B} p_{bc} x_{bc} \quad s.t. \tag{4.2}$$

$$\sum_{b} x_{bc} \leq 1 \; \forall c \in C \tag{4.3}$$

$$\sum_{c} x_{bc} \leq 1 \; \forall b \in B \tag{4.4}$$

$$\sum_{c} \sum_{b} x_{bc} q_{rb} = 1 \; \forall r \in R \tag{4.5}$$

$$x_{bc} \in \{0, 1\} \tag{4.6}$$

The objective function (4.2) minimizes the costs of reassignment of bundles to carriers. As in the approach presented in [37], a carrier can receive at most one bundle (4.3). This means carriers do not end up with requests that might exceed their capacity. A bundle can only be assigned to at most one carrier (4.4). Every request has to be re-allocated exactly once (4.5). Constraints (4.6) define the decision variable x_{bc} as binary. This problem does not represent high complexity and can be solved to optimality using exact methods.

4.3.1.2 Individual Auctions

Depending on the timing of events, individual auctions could be simultaneous or sequential. In the first variant, all participants conduct auctions at the same time, while in the latter variant auctions are conducted one after the other, taking turns to do so. In one-round auctions, or in a limited number of rounds setting, simultaneous auctions would require a centralized control, either performed by the carriers as a group or by a central institution, to ensure that a carrier c gets a set R_c of new requests assigned only when other carriers are willing to service some of c's requests, so c has enough available capacity to service all requests in R_c. Individual simultaneous auctions are not included in this paper, as they can be considered a special case of centralized auctions, in which bundles are built from requests submitted by only one carrier.

Individual sequential auctions mechanisms, on the other hand, have one carrier at a time reassigning its own requests. One round of individual auctions is considered to be completed when all carriers in the network have had the turn to be the auctioneer once. Each turn within the round contains all the steps mentioned in Section 4.3.1.

4.3.1.2.1 Request Selection

This process is performed by the carrier acting as the auctioneer, following the same steps as described in Section 4.3.1.1.1. In the proposed mechanism, carriers can only exchange their own requests within a particular round. This means they are not allowed to exchange requests they have been assigned in previous turns within the same round. This makes the request selection process easier, being performed once per round, and also provides information to carriers about their available capacity in that particular round so they can adjust their bids to avoid obtaining bundles they cannot handle.

4.3.1.2.2 Bundle Generation

This process is also performed by the carrier acting as the auctioneer. The bundles, as in centralized auctions, should have a size within some determined limits, however, in individual sequential auctions, these limits might be more flexible, as carriers take turns being the auctioneer and they can get small bundles and fill their capacity partially in every turn. In this case bundles contain requests that belong exclusively to the auctioneer, hence no profit sharing is necessary.

4.3.1.2.3 Bid Generation

Following the bundle generation, all carriers bid on all offered bundles. The auctioneer sets reserve prices for the offered bundles, which can be considered as the auctioneer's own bids. In this process, carriers consider:

1. The carrier's (expected) available capacity.

2. The carrier's (expected) cost for servicing the requests in the bundle.

It should be noted that if a carrier is, or has already been the auctioneer in a particular round, they know which of their requests are being reassigned and therefore know their available capacity. On the other hand, not having been the auctioneer yet, a carrier c would bid on bundles that do not exceed its expected capacity, assuming its selected requests could be assigned to another carrier when it is c's turn to be the auctioneer. If a bundle does not exceed the carrier's capacity, the cost for servicing it can be obtained from the marginal cost for every bundle, as described in Section 4.3.1.1.3.

4.3.1.2.4 Winner Determination

The auctioneer assigns the bundles to the other carriers and itself, by solving the WDP from Section 4.3.1.1.4. Furthermore, for one round of individual sequential auctions in a network composed of $|C|$ carriers, $|C|$ WDP have to be solved, one for every carrier's turn. Only after a round is completed, requests are assigned to the carriers, avoiding reselling in the same round and giving carriers the chance to balance their capacity by adjusting their bids and reserve prices. For example, when having the last turn to be the auctioneer, a carrier might set its reserve prices very high, so that the exchange of requests above its capacity is enforced.

4.3.2 Centralized Posted Price Exchange Mechanisms

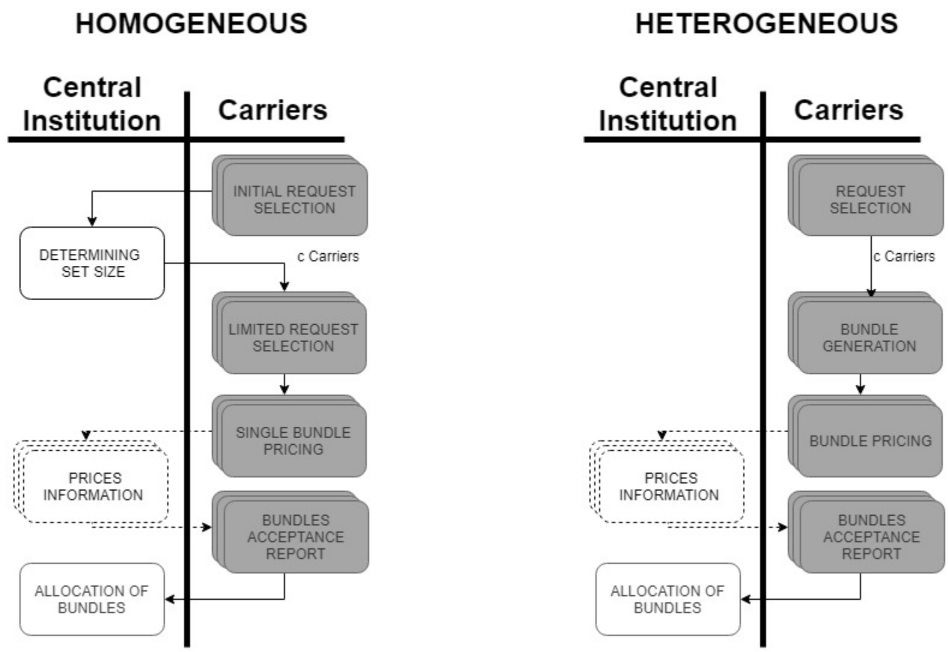
A posted price exchange mechanism fundamentally differs from an auction-based mechanism in one aspect: carriers set the prices for their selected requests themselves. If carriers want to maintain the number of requests they service unchanged, they can either exchange the same number of requests or provide information about the maximum number of requests they would want to be allocated. The first option is referred to as homogeneous posted price whereas the latter option is referred to as heterogeneous posted price.

4.3.2.1 Homogeneous Centralized Posted Price Exchange

In this mechanism carriers exchange either all of their (limited) selected requests or none. The mechanism is composed of these following steps:

1. Initial Request Selection.
2. Determining Set Size.
3. Limited Request Selection.
4. Single Bundle Pricing.
5. Bundle Acceptance Report.
6. Allocation of Bundles.

A central institution is needed to determine the size of bundles being exchanged and deciding the best re-allocation of bundles. Figure 4.2a shows the logical order of these steps, divided according to the party in charge of the decision making process.



(a) Homogeneous Posted Price Exchange (b) Heterogeneous Posted Price Exchange

Figure 4.2: Decisions in Posted Price Exchange Mechanisms

4.3.2.1.1 Initial Request Selection

Carriers perform an initial request selection following the process described in Section 4.3.1.1.1.

4.3.2.1.2 Determining Set Size

This process, performed by a central institution, takes as input the sizes of the sets of requests from the carriers' initial request selection and determines the set size that all carriers will submit.

4.3.2.1.3 Limited Request Selection

Carriers repeat their request selection processes, this time selecting the number of requests that has been set in the previous step (Section 4.3.2.1.2). As initial capacity levels are unknown for the central institution and other carriers, the mechanism tries in this way to keep a carrier's capacity utilization unchanged. These limits could be changed with information from carriers willing to service larger sets of requests.

4.3.2.1.4 Single Bundle Pricing

Each carrier sets a price for one single bundle, containing all its selected requests. Similar to the bidding generation in auction-based mechanisms, the marginal cost for this bundle can be obtained. In this case, nevertheless, as only one bundle is being evaluated, the marginal cost can be obtained by using the actual routing solution method the carrier usually uses.

4.3.2.1.5 Bundle Acceptance Report

The central institution gathers all bundles from the carriers and the posted prices for them. These bundles are offered to all other carriers for further evaluation. A carrier would be willing to accept a bundle only if it will be better off than with its initial allocation, this would occur if the carrier's servicing cost for a bundle b is lower than b's posted price and lower than its own bundle's servicing cost. This means the carrier's cost after exchange will be lower than its initial cost.

4.3.2.1.6 Allocation of Bundles

The main function for the central institution is allocating bundles back to the carriers. This allocation is performed based on the information provided by the carriers regarding their own bundles' posted prices and their acceptance willingness report. This Bundle Allocation Problem (BAP) is defined as follows:

C	set of carriers
R	set of requests
B	set of bundles
s_{bc}	binary parameter indicating that carrier c submitted bundle b
o_{bc}	parameter indicating the size of the bundle b, submitted by carrier c
a_{bc}	binary parameter indicating that carrier c is willing to accept bundle b
q_{rb}	binary parameter indicating if request r is in bundle b
x_{bc}	binary variable indicating if bundle b is assigned to carrier c

$$min \sum_{c \in C} \sum_{b \in B} s_{bc} x_{bc} \quad s.t. \tag{4.7}$$

$$\sum_{b} x_{bc} \leq 1 \; \forall c \in C \tag{4.8}$$

$$\sum_{c} x_{bc} \leq 1 \; \forall b \in B \tag{4.9}$$

$$x_{bc} \leq a_{bc} \; \forall b \in B, \; \forall c \in C \tag{4.10}$$

$$\sum_{c} \sum_{b} x_{bc} q_{rb} = 1 \; \forall r \in R \tag{4.11}$$

$$x_{bc} \in \{0, 1\} \tag{4.12}$$

The objective function (4.7) minimizes the reassignment of bundles back to their original carriers, therefore, favoring exchange. A carrier can get at most one bundle (4.8). In this way, carriers do not end up with unwanted requests that might exceed their capacity. A bundle can only be assigned to at most one carrier (4.9). A bundle can only be allocated to a carrier that is willing to accept it (4.10). Every request has to be allocated exactly once (4.11). Constraints (4.12) define the decision variable x_{bc} as binary. Alternatively, the objective function (4.7) can be replaced by:

$$min \sum_{c \in C} \sum_{b \in B: s_{bc}=1} o_{bc} x_{bc} \quad s.t. \tag{4.13}$$

4.3.2.2 Heterogeneous Centralized Posted Price Exchange

This mechanism does not limit the number of requests carriers select to be exchanged. Therefore, its process differs from the homogeneous variant, being composed of the following steps:

1. Request Selection.
2. Bundle Generation.
3. Bundle Pricing.
4. Bundle Acceptance Willingness Report.
5. Allocation of Bundles.

The order in which these steps are performed, divided according to the party in charge of the decision making process, is shown in Figure 4.2b.

4.3.2.2.1 Request Selection

Carriers perform an initial request selection following the process described in Section 4.3.1.1.1.

4.3.2.2.2 Bundle Generation

This process is performed by every carrier. Only attractive bundles should be generated. The process is similar to bundle generation in individual auctions presented in Section 4.3.1.2.2, setting first some limits for sizes of bundles and then generating bundles considering the same criteria. In this case, bundles contain requests that belong exclusively to one carrier.

4.3.2.2.3 Bundle Pricing

Each carrier sets a price for every one of its bundles. Similar to the bidding generation in auctions-based mechanisms, the marginal cost for these bundles can be obtained using estimation methods, as a large number of bundles is expected.

4.3.2.2.4 Bundle Acceptance Report

This step is performed by every carrier following the same process as in Section 4.3.2.1.5.

4.3.2.2.5 Allocation of Bundles

As in Section 4.3.2.1.6, the allocation of bundles to carriers can be performed by solving the BAP. Although in this case more than one bundle can be assigned to a carrier, replacing constraints (4.8) with:

$$\sum_{b}\sum_{r} x_{bc}q_{rb} \leq Q_c \ \forall c \in C \tag{4.14}$$

where Q_c is a quantity limit reported by carrier c as the highest number of requests that carrier c is willing to be assigned.

4.4 Computational Experiments

The auction-based mechanisms from Section 4.3.1 and the posted price mechanisms from Section 4.3.2 are analyzed in a Two-Region scenario with paired nodes. Regions S and T are connected by a single long-haul trip that connects a carrier's depots in both regions. The objective of each carrier is to minimize the total distance their vehicles travel. As the number of long haul trips remains the same, improvement is obtained from shortening short-haul or in-region tours. It is assumed that carriers taking part of a collaborative network are willing to exchange requests. A request is composed of a pick-up node, a delivery node, and a quantity transported between them. Both nodes are serviced by the same carrier. One tour is performed by every carrier in each region and it is assumed that their total capacity is being used, so they have no availability for more requests than those they start with. Moreover, a Traveling Salesman Problem (TSP) is solved for each region by using the Lin-Kernighan (LK) implementation by Helsgaun [46] for the original assignments and for the assignments obtained by every mechanism. The difference between both solutions is reported as the obtained improvement for both regions short-haul trips.

The mechanisms are tested in 30 cooperation networks with four carriers each. Every carrier services 25 customers, each with a pick-up node in one region and a delivery node in another region. All requests have the same demand. The regions are generated based on the instances from [63], as squares of 200 × 200 distance units. The travel distance between nodes corresponds to the Euclidean distance. Nodes coordinates are randomly generated from a uniform distribution.

4.4.1 Auction-based Mechanisms Parameters

4.4.1.1 Request Selection Criteria

In results presented in [37], criteria that includes geographical properties such as distances to depots, outperformed pure profit-based criteria in terms of efficiency gains. In line with this, models presented in [63] include distances between nodes as variables for distance estimation. Distance criteria for request selection with a selection threshold are included. In this paper, carriers choose one criterion from the ones shown in Table 4.1 for any mechanism and any round.

Criteria	Threshold
Distance to depot in region S (Dist S)	$\eta\times$ Maximum value **
Distance to depot in region T (Dist T)	$\eta\times$ Maximum value
Distance to depots in both regions (Dist)	$\eta\times$ Maximum value
Multi-Criteria* in region S (Multi S)	$\eta\times$ Maximum value
Multi-Criteria* in region T (Multi T)	$\eta\times$ Maximum value
Multi-Criteria* in both regions (Multi)	$\eta\times$ Maximum value
Random selection (Rand)	Requests selected randomly***

* Corresponds to minimum and maximum distances to other nodes weighted with parameters from estimation models like Equation 1 [[63]]($SumMinP$ & $SumMaxP$) and the distance to the average node weighted with the parameter $SumM$.

**Maximum value for every criterion accordingly. $\eta \in [0, 1]$, for results shown here, $\eta = 0.7$

***Same number of requests as selected using *Dist*

Table 4.1: Request Selection Criteria & Thresholds

For both auction formats, bundle generation is performed with the same criteria. The limits for the bundles size correspond to the minimum and maximum number of submitted requests by carriers for the central auctions mechanism. For the individual auctions mechanism, the bundle size limits are 1 and the number of selected requests. The generation criteria are:

- Minimum distance to seed request's node in region S.
- Minimum distance to seed request's node in region T.
- Minimum total distance to seed request's nodes in both regions.
- Minimum distance to last added request's node in region S.
- Minimum distance to last added request's node in region T.
- Minimum total distance to last added request's nodes in both regions.
- Minimum distance to the bundle's average node in region S.
- Minimum distance to the bundle's average node in region T.
- Minimum total distance to the bundle's average nodes in both regions.

Algorithm 1 shows the process that is followed to generate a set of bundles from a set of n requests that has been selected. This set of n requests is referred as a pool of requests, while the resulting set of bundles is referred as a list of bundles. The requests submitted by each carrier are also combined to a bundle and added to this list of bundles.

Algorithm 1: Bundle Generation

```
Input: PoolOfRequests, minimum, maximum
Output: ListOfBundles
initialization;
for Criterion : Criteria do
    for size ← minimum to maximum by 1 do
        for SeedRequest : PoolOfRequests do
            Bundle ← empty;
            Add SeedRequest to Bundle;
            drop SeedRequest from PoolOfRequests;
            while BundleSize < size do
                sort PoolOfRequests(Criterion);
                Add first request in PoolOfRequests to Bundle;
                drop first request from PoolOfRequests;
            Add Bundle to ListOfBundles;
```

For the bid generation, an estimation model is used. This model is based on *Model 2 for TSP small instances* presented in [63]. For these experiments, the same model is used for bids generation for all carriers. In the central auctions mechanism, carriers also bid on bundles that have higher costs that their own bundles, assuming they could be compensated for it. In the individual auctions mechanism, carriers balance their capacities every round by adjusting their bids, i.e. setting high reserve prices in bundles they want to get rid of; this is done in order to make a comparison between mechanisms after any number of rounds. This is also rational for carriers when not knowing if a next round will take place or not. However, the individual auctions mechanism could have different rules, allowing carriers to exceed their capacity in one round, as long as they have the chance to exchange or transfer requests in future rounds.

The WDP is solved to optimality using the commercial solver IBM CPLEX version 12.6.3. For the individual sequential auctions mechanism, carriers are assigned turns randomly according to a uniform distribution for every round, i.e. order can change between rounds.

4.4.2 Centralized Posted Price Exchange Parameters

Request selection is performed in the same way as in Section 4.4.1.1. For the homogeneous variant, the limit for request selection determined by the central institution is the smallest number of requests submitted by a carrier. The evaluation of bundles for pricing and acceptance is performed by the carriers based on marginal cost obtained from solutions for each region by using the Lin-Kernighan (LK) implementation by Helsgaun [46].

As for the heterogeneous variant, the limits for the bundle size limits are 1 and the number of selected requests by every carrier. The bundle generation criteria are the same as in Section 4.4.1

and the process is the same as in Algorithm 1. The bundle pricing and acceptance is performed using an estimation model based on *Model 2 for TSP small instances* presented in [63]. The allocation of bundles for both variants is performed according to an optimal BAP solution obtained by using the commercial solver IBM CPLEX version 12.6.3.

4.4.3 Results

The computational experiments use the same instances for all mechanisms. All instances are run on an Intel Core i7-4790 3.60 GHz. Results for one single round are presented in table 4.2, for Centralized Auctions (CenAuc), Individual Auctions (IndAuc), Homogeneous Centralized Posted Price Exchange (HomPosPri) and Heterogeneous Centralized Posted Price Exchange (HetPosPri) mechanisms. These results are obtained when all carriers use the same request selection criterion. Results shown in the table include the total average improvement, the maximum and minimum improvements obtained by a carrier for the indicated criterion and mechanism, the average improvement for every region, the average number of sent and exchanged requests per carrier, and the computational time for all 30 instances.

Criterion	Mechanism	Total Improv. (%)			Avg. Improv./Reg. (%)		Avg. Req./Car.		Comp. Time
		Avg.	Max.	Min.	Region S	Region T	Sent	Exchanged	(s)
Dist S	CenAuc	8.52	21.30	-2.85	15.27	1.77	8.78	7.32	77.71
	IndAuc	7.32	20.74	-7.28	11.42	3.22	8.78	7.11	34.69
	HomPosPri	4.06	14.61	0.00	6.29	1.84	6.27	4.73	27.95
	HetPosPri	2.10	13.77	-19.38	4.18	-0.24	8.78	8.78	25.79
Dist T	CenAuc	9.65	26.08	-1.53	1.67	17.63	8.28	7.14	68.79
	IndAuc	8.80	21.49	-9.09	2.61	14.98	8.28	7.34	30.31
	HomPosPri	4.43	12.93	0.00	1.03	7.82	5.23	4.78	28.21
	HetPosPri	3.48	19.90	-13.54	-0.23	6.91	8.28	8.28	21.55
Dist	CenAuc	7.66	20.60	-3.87	6.65	8.67	9.62	7.60	124.91
	IndAuc	7.76	25.80	-4.98	6.77	8.75	9.62	7.96	45.62
	HomPosPri	3.65	15.24	0.00	3.55	3.74	5.93	4.74	28.59
	HetPosPri	2.95	17.78	-11.98	1.92	3.72	9.62	9.62	33.49
Multi S	CenAuc	7.88	19.79	-5.71	13.78	1.98	6.28	4.52	63.54
	IndAuc	6.21	23.84	-8.24	9.76	2.67	6.28	4.60	31.66
	HomPosPri	3.01	13.82	0.00	4.71	1.30	2.61	2.13	30.26
	HetPosPri	1.95	18.82	-11.50	3.98	-0.22	6.28	6.13	21.82
Multi T	CenAuc	7.48	18.59	-4.35	1.72	13.24	5.96	4.23	64.51
	IndAuc	5.55	16.88	-12.07	1.66	9.43	5.96	4.35	25.84
	HomPosPri	3.13	12.74	0.00	0.83	5.42	2.33	2.06	31.09
	HetPosPri	2.31	18.55	-17.92	-0.50	4.96	5.96	5.63	21.09
Multi	CenAuc	9.61	23.81	-2.39	9.17	10.04	8.81	6.38	121.77
	IndAuc	7.40	20.42	-7.16	6.77	8.02	8.81	6.46	52.90
	HomPosPri	2.92	13.55	0.00	2.91	2.92	4.77	2.99	30.53
	HetPosPri	2.41	15.91	-14.28	1.33	3.15	8.81	8.69	42.47
Rand	CenAuc	3.67	17.27	-8.90	2.79	4.55	13	10.06	49.81
	IndAuc	7.75	24.58	-5.03	7.28	8.22	13	9.5	27.52
	HomPosPri	2.06	11.01	0.00	2.16	1.96	10	5.08	28.44
	HetPosPri	0.47	15.58	-14.64	-0.17	0.79	13	13	55.62

Table 4.2: One round results: carriers use the same request selection criterion

If more rounds are run, carriers would have the chance to exchange new requests every round. It should be noted that by using the same selection criterion round after round, as soon as no exchange is made in a y_{th} round, the same requests will be selected for the $(y+1)_{th}$ round, getting the same zero request exchange, so no further improvement will be obtained. All mechanisms were applied in a multi-round setting (50 rounds), in which carriers randomly selected one selection criterion from Section 4.4.1.1 every round.

These results are shown in table 4.3, which includes the average, maximum and minimum improvements, as well as the average number of exchanged requests compared to the initial allocation (before the mechanism is applied). Round by round comparisons are shown in table 4.4 which includes the average, maximum and minimum improvements, and the average number of exchanged requests. It can be observed that the application of the mechanisms can make a carrier worse-off when comparing to the previous round allocation, but when comparing to the initial allocation, after some rounds the minimum improvement, i.e. the smallest improvement obtained by a carrier, is positive for all the mechanisms except for the heterogeneous posted price.

Round	Centralized Auctions				Individual Auctions				Homogeneous Centralized Posted price				Heterogeneous Centralized Posted price			
	Improvement %			Avg. Exc.	Improvement %			Avg. Exc.	Improvement %			Avg. Exc.	Improvement %			Avg. Exc.
	Avg.	Max.	Min.	Requests	Avg.	Max.	Min.	Requests	Avg.	Max.	Min.	Requests	Avg.	Max.	Min.	Requests
1	9.29	23.27	-3.13	6.38	7.83	21.12	-9.11	6.62	2.39	12.78	0.00	3.23	2.41	15.98	-14.27	8.69
2	12.10	29.41	-3.20	9.52	11.86	24.64	-2.34	9.96	4.56	16.01	0.00	5.49	3.82	20.62	-10.44	11.80
3	13.98	28.87	-9.68	11.97	14.75	32.41	-3.82	12.49	5.79	16.01	0.00	6.36	4.54	21.89	-12.46	13.78
4	15.55	33.06	-9.61	13.52	15.87	33.17	-5.02	14.18	6.52	16.01	0.00	7.08	4.18	20.70	-14.09	15.23
5	16.62	36.04	-9.61	14.54	17.87	34.19	-0.75	14.96	7.05	16.01	0.00	7.49	4.89	25.65	-13.84	16.18
6	17.68	36.12	-9.98	15.14	18.44	32.78	-1.80	16.00	7.65	18.89	0.00	8.01	5.18	26.06	-12.67	16.90
7	18.51	37.11	-9.61	15.53	19.64	31.29	-1.08	16.66	8.30	18.89	0.00	8.33	5.40	25.09	-17.59	17.33
8	19.19	37.18	-1.94	15.86	20.26	35.21	3.93	17.11	9.23	23.14	0.00	8.76	5.26	28.50	-12.79	18.01
9	19.92	37.11	0.21	16.23	20.89	35.19	3.36	17.09	9.78	23.14	0.00	9.14	5.17	24.49	-15.08	18.61
10	20.28	37.09	2.28	16.40	21.26	35.51	0.78	17.54	10.17	23.14	0.37	9.27	5.60	26.78	-14.23	18.58
11	20.67	37.18	0.95	16.62	21.31	37.77	-0.32	17.91	10.44	23.14	0.37	9.41	6.17	26.97	-18.74	18.63
12	21.16	37.18	-3.34	16.68	21.38	37.77	0.52	18.01	10.85	23.14	1.01	9.54	6.66	24.17	-8.43	18.65
13	21.47	37.11	-3.26	16.78	21.85	37.68	-1.44	18.17	11.08	24.28	1.01	9.71	6.59	29.03	-10.59	18.78
14	21.75	37.18	-3.34	16.86	21.92	37.88	-1.81	18.23	11.26	24.28	1.01	9.86	5.77	20.51	-13.53	18.83
15	21.87	35.48	-3.30	17.08	21.61	35.77	-1.12	18.21	11.53	24.69	1.01	9.92	6.40	27.62	-13.69	18.89
16	22.17	37.18	-3.30	17.13	22.31	36.84	2.80	18.23	12.01	24.69	1.01	10.03	6.42	25.10	-8.25	18.96
17	22.51	37.13	-3.29	17.14	22.35	41.77	-0.36	18.36	12.16	27.53	1.01	10.03	5.83	24.35	-12.77	18.76
18	22.72	37.24	1.82	17.15	22.44	39.96	0.82	18.33	12.30	27.53	1.01	10.09	6.32	26.39	-13.41	18.98
19	22.70	37.24	1.72	17.27	23.19	39.54	2.91	18.38	12.55	27.53	1.01	10.14	5.18	24.96	-14.15	19.23
20	22.90	36.52	1.82	17.24	23.07	41.36	1.32	18.38	12.68	27.53	1.01	10.28	6.64	26.85	-11.71	19.11
21	23.23	38.07	1.82	17.34	22.92	41.48	2.35	18.47	12.99	27.53	1.01	10.39	6.86	30.09	-15.31	19.11
22	23.27	38.42	4.81	17.43	23.21	41.64	0.66	18.56	13.13	27.53	1.01	10.55	5.61	27.23	-13.09	19.15
23	23.44	38.24	4.71	17.45	22.64	41.64	1.20	18.62	13.35	27.53	1.88	10.66	6.11	28.32	-10.86	19.25
24	23.37	38.24	3.70	17.55	22.94	45.53	5.06	18.59	13.51	27.53	1.88	10.73	6.07	26.21	-11.08	19.28
25	23.55	38.20	3.63	17.67	23.01	46.39	-11.51	18.66	13.67	27.53	1.88	10.74	5.88	28.18	-13.80	18.94
26	23.68	38.20	3.59	17.68	22.75	46.06	-4.29	18.61	13.93	27.53	1.96	10.88	5.95	26.01	-9.73	19.05
27	23.76	38.20	3.70	17.83	23.08	46.39	3.39	18.96	14.03	27.53	2.08	10.98	5.29	29.70	-13.50	19.32
28	23.88	38.20	3.90	17.86	23.46	43.97	-0.65	18.81	14.14	27.53	2.08	11.06	5.63	27.47	-15.12	19.27
29	23.88	38.20	3.90	17.86	24.01	43.83	1.75	18.80	14.39	27.53	2.85	11.13	5.63	26.12	-13.58	19.41
30	24.00	38.24	6.06	17.88	24.25	43.86	-1.26	18.83	14.60	27.53	2.85	11.24	5.40	27.80	-13.88	19.19
31	24.01	38.25	6.06	17.90	24.41	43.86	5.64	18.79	14.78	28.14	2.85	11.43	6.23	26.06	-13.81	19.06
32	24.19	38.25	6.06	17.94	24.20	43.86	1.17	18.74	14.88	28.14	2.85	11.45	6.41	29.29	-11.53	19.28
33	24.12	38.25	6.06	17.98	23.44	44.10	-0.72	18.79	15.06	28.14	2.85	11.64	6.21	22.76	-12.45	19.10
34	24.33	38.29	6.06	18.02	23.02	44.10	-5.36	18.70	15.17	28.14	2.85	11.63	6.31	24.99	-14.71	19.03
35	24.45	38.20	6.06	18.03	23.06	43.97	-8.04	18.73	15.18	28.14	2.85	11.67	6.37	27.70	-16.61	19.08
36	24.47	38.24	6.06	18.05	23.26	43.86	2.45	18.75	15.25	28.14	2.85	11.65	7.12	30.16	-15.25	19.21
37	24.54	38.42	5.95	18.06	23.67	43.97	6.51	18.82	15.29	28.14	2.85	11.67	6.72	26.15	-15.31	19.21
38	24.51	38.20	6.06	18.12	24.17	43.97	7.20	18.78	15.29	28.14	2.85	11.67	6.39	26.94	-13.25	19.44
39	24.54	38.42	5.51	18.14	23.99	43.97	7.18	18.76	15.36	28.14	2.85	11.68	5.99	26.18	-11.12	19.48
40	24.54	38.42	5.51	18.14	23.90	43.05	6.04	18.68	15.42	28.14	2.85	11.69	7.42	24.68	-14.09	19.32
41	24.72	38.20	5.56	18.19	23.45	43.35	5.84	18.65	15.42	28.14	2.85	11.69	6.59	28.34	-16.08	19.56
42	24.72	38.20	5.56	18.19	23.27	39.98	1.90	18.80	15.46	28.14	2.85	11.70	6.27	28.16	-12.07	19.43
43	24.82	38.20	6.06	18.20	23.54	41.79	5.19	18.80	15.50	28.14	2.85	11.72	5.40	22.02	-14.02	19.51
44	25.00	40.09	5.51	18.13	23.12	36.89	-3.84	18.93	15.52	28.14	2.85	11.79	5.97	26.46	-14.67	19.42
45	24.97	39.65	6.06	18.14	23.63	36.92	-1.29	18.92	15.52	28.14	2.85	11.79	6.30	28.55	-12.48	19.40
46	25.02	40.28	13.51	18.18	23.56	37.71	5.31	19.04	15.59	28.14	2.85	11.83	6.70	28.11	-11.30	19.40
47	25.02	40.28	13.51	18.18	22.65	37.74	8.05	19.08	15.68	28.14	2.85	11.86	5.28	21.98	-14.16	19.55
48	25.02	40.28	13.51	18.18	22.59	38.36	6.58	19.16	15.75	28.14	2.85	11.84	5.30	21.78	-10.19	19.57
49	25.02	40.28	13.51	18.18	22.29	43.54	3.00	19.10	15.82	28.14	2.85	11.86	5.82	23.17	-17.32	19.51
50	25.10	40.28	13.52	18.17	22.57	41.21	6.89	19.14	15.82	28.14	2.85	11.86	6.04	25.10	-19.88	19.49

Table 4.3: 50 rounds accumulated results

Carriers use any request selection criterion. The average, minimum and maximum improvement is presented for every mechanism, as well as the accumulated average number of exchanged requests, compared to the initial allocation.

Round	Centralized Auctions				Individual Auctions				Homogeneous Centralized Posted price				Heterogeneous Centralized Posted price			
	Improvement %			Avg. Exc. Requests	Improvement %			Avg. Exc. Requests	Improvement %			Avg. Exc. Requests	Improvement %			Avg. Exc. Requests
	Avg.	Max.	Min.		Avg.	Max.	Min.		Avg.	Max.	Min.		Avg.	Max.	Min.	
1	9.29	23.27	-3.13	6.38	7.83	21.12	-9.11	6.62	2.39	12.78	0.00	3.23	2.41	15.98	-14.27	8.69
2	2.81	17.86	-8.32	4.51	4.03	15.84	-15.27	5.48	2.17	13.91	0.00	3.36	1.41	12.77	-0.73	7.99
3	1.88	16.82	-16.43	4.30	2.89	16.90	-11.88	4.96	1.23	10.85	0.00	1.68	0.72	10.85	-1.22	8.17
4	1.57	20.36	-13.56	3.67	1.12	14.38	-23.73	4.46	0.73	11.97	0.00	1.11	-0.37	11.97	-1.09	8.42
5	1.07	17.17	-20.09	2.48	2.01	14.32	-16.26	3.88	0.52	9.11	0.00	0.69	0.71	8.95	-0.70	8.10
6	1.06	15.69	-13.99	2.22	0.56	14.08	-20.23	3.99	0.60	12.34	0.00	0.83	0.30	10.24	-0.70	7.59
7	0.83	12.74	-11.01	1.34	1.20	17.91	-22.19	3.07	0.65	8.02	0.00	0.80	0.22	6.66	-1.23	6.99
8	0.69	20.60	-15.93	1.47	0.63	18.14	-15.40	2.92	0.93	10.77	0.00	0.98	-0.15	10.95	-1.14	7.90
9	0.73	17.35	-20.64	1.33	0.63	11.46	-22.96	2.31	0.56	10.32	0.00	1.00	-0.09	6.24	-1.19	8.21
10	0.36	12.31	-8.12	0.93	0.37	15.81	-22.02	2.53	0.38	10.44	0.00	0.46	0.44	12.50	-1.56	7.48
11	0.40	14.71	-18.99	1.30	0.04	17.05	-22.26	2.73	0.28	5.54	0.00	0.39	0.57	9.15	-1.14	7.24
12	0.49	18.95	-17.30	0.38	0.07	12.57	-33.04	2.38	0.40	6.97	0.00	0.43	0.49	8.07	-1.20	6.90
13	0.31	7.46	-11.56	0.44	0.48	17.16	-26.04	2.55	0.23	6.70	0.00	0.43	-0.07	6.57	-1.06	7.55
14	0.29	10.17	-15.33	0.56	0.07	17.21	-24.10	2.28	0.18	5.13	0.00	0.34	-0.82	8.73	-1.54	7.36
15	0.12	9.63	-8.57	0.45	-0.32	18.40	-20.68	2.38	0.28	7.21	0.00	0.27	0.62	6.23	-1.32	7.29
16	0.29	8.02	-10.29	0.42	0.70	18.34	-18.90	2.08	0.47	12.04	0.00	0.38	0.02	3.57	-1.21	6.87
17	0.34	16.21	-9.75	0.44	0.04	15.05	-28.96	2.26	0.15	8.64	0.00	0.07	-0.59	5.57	-1.46	7.11
18	0.21	8.22	-1.53	0.25	0.09	13.63	-24.51	1.83	0.14	11.33	0.00	0.16	0.50	4.84	-1.68	7.11
19	-0.02	6.67	-9.98	0.52	0.75	18.69	-25.01	1.94	0.25	7.75	0.00	0.30	-1.14	4.99	-1.22	7.58
20	0.20	10.97	-1.94	0.20	-0.12	16.17	-22.64	1.58	0.13	4.74	0.00	0.20	1.45	9.94	-1.44	6.25
21	0.34	8.93	-13.60	0.51	-0.15	15.38	-23.13	1.94	0.31	6.77	0.00	0.28	0.22	5.76	-1.20	7.37
22	0.04	9.48	-11.77	0.68	0.29	19.46	-20.90	1.98	0.14	6.91	0.00	0.35	-1.24	11.28	-1.08	7.17
23	0.18	16.81	-7.88	0.29	-0.58	12.24	-19.72	1.94	0.23	4.83	0.00	0.23	0.49	4.89	-0.65	7.76
24	-0.07	11.46	-22.52	0.46	0.30	14.34	-20.20	1.77	0.16	6.59	0.00	0.24	-0.04	7.13	-1.26	7.93
25	0.18	9.99	-9.09	0.35	0.07	13.45	-36.48	1.84	0.16	8.26	0.00	0.22	-0.18	4.53	-0.80	7.52
26	0.13	6.57	-8.53	0.27	-0.26	13.43	-22.77	1.93	0.26	8.92	0.00	0.37	0.07	6.32	-0.86	7.54
27	0.08	11.20	-7.30	0.37	0.34	18.23	-19.20	2.45	0.09	2.75	0.00	0.34	-0.66	12.80	-1.70	7.36
28	0.12	8.73	-5.23	0.13	0.38	17.09	-34.11	2.10	0.11	4.69	0.00	0.15	0.34	3.87	-1.16	6.72
29	0.00	0.00	0.00	0.00	0.55	15.56	-13.01	1.81	0.25	5.63	0.00	0.49	0.00	5.56	-1.16	6.94
30	0.16	7.34	-4.47	0.34	0.23	13.89	-37.70	1.65	0.21	6.73	0.00	0.18	-0.23	3.87	-1.02	6.86
31	0.01	4.87	-2.43	0.08	0.16	11.34	-12.97	1.58	0.18	4.50	0.00	0.50	0.83	4.12	-1.72	6.98
32	0.18	7.76	-4.58	0.25	-0.21	13.71	-16.75	1.68	0.10	5.30	0.00	0.12	0.18	2.27	-0.91	6.85
33	-0.07	10.53	-14.14	0.32	-0.76	12.94	-22.36	2.12	0.18	7.43	0.00	0.33	-0.19	2.77	-1.64	6.91
34	0.20	12.77	-10.50	0.29	-0.42	13.02	-44.42	2.18	0.12	4.85	0.00	0.05	0.10	5.18	-1.05	6.57
35	0.13	9.66	-5.95	0.15	0.03	12.09	-18.92	1.59	0.02	2.05	0.00	0.07	0.05	1.59	-1.16	6.97
36	0.01	4.16	-2.33	0.19	0.20	15.09	-17.82	2.03	0.09	3.04	0.00	0.10	0.75	5.43	-0.88	7.57
37	0.07	9.20	-3.34	0.08	0.42	12.56	-17.83	1.54	0.04	4.01	0.00	0.07	-0.40	3.84	-1.23	7.06
38	-0.02	9.81	-20.04	0.24	0.50	16.36	-21.52	1.91	0.00	0.00	0.00	0.00	-0.33	3.34	-1.91	6.11
39	0.03	12.68	-8.22	0.15	-0.19	9.40	-16.13	1.83	0.07	3.12	0.00	0.03	-0.40	7.11	-0.79	6.68
40	0.00	0.00	0.00	0.00	-0.09	11.86	-10.11	1.86	0.06	4.61	0.00	0.05	1.43	6.26	-1.39	6.74
41	0.17	13.47	-6.93	0.23	-0.45	15.04	-26.28	1.88	0.00	0.00	0.00	0.00	-0.83	5.14	-1.71	6.93
42	0.00	0.00	0.00	0.00	-0.17	12.83	-41.30	2.23	0.04	5.46	0.00	0.13	-0.31	1.79	-1.16	7.04
43	-0.02	0.72	-1.00	0.02	0.26	17.17	-22.47	1.75	0.04	3.25	0.00	0.03	-0.87	5.57	-1.13	7.06
44	0.18	10.65	-9.41	0.28	-0.42	14.49	-46.44	2.12	0.02	4.17	0.00	0.15	0.58	4.38	-1.58	7.43
45	-0.03	6.20	-7.15	0.17	0.51	14.12	-17.54	1.38	0.00	0.00	0.00	0.00	0.33	1.35	-1.16	7.68
46	0.05	8.82	-7.33	0.18	-0.07	16.32	-23.88	1.87	0.04	5.30	0.00	0.11	0.39	5.44	-0.78	6.76
47	0.00	0.00	0.00	0.00	-0.91	18.70	-23.49	2.85	0.08	2.36	0.00	0.05	-1.42	4.31	-1.57	6.88
48	0.00	0.00	0.00	0.00	-0.05	14.76	-25.00	2.54	0.08	8.66	0.00	0.12	0.02	5.15	-0.67	6.97
49	0.00	0.00	0.00	0.00	-0.30	16.62	-30.15	2.48	0.07	4.53	0.00	0.03	0.53	4.59	-1.39	7.77
50	0.08	0.60	-0.17	0.01	0.28	22.30	-44.55	2.48	0.00	0.00	0.00	0.00	0.21	3.03	-1.45	6.89

Table 4.4: 50 rounds (marginal) results

Carriers use any request selection criterion. The average, minimum and maximum improvement is presented for every mechanism, as well as the average number of exchanged requests, compared to the allocation obtained in the previous round.

Figure 4.3 shows the cumulative average improvement (more intense lines) and the minimum improvement (less intense lines) for every mechanism and for every round. Figure 4.4 shows the cumulative average number of exchanged requests. For both auction-based and for the homogeneous posted price mechanisms, the improvement and the number of exchanged requests have a similar shape. This is not the case for the heterogeneous posted price mechanism, which seems to have a large exchange of requests that does not have the same effect on the improvement that is obtained by the mechanism.

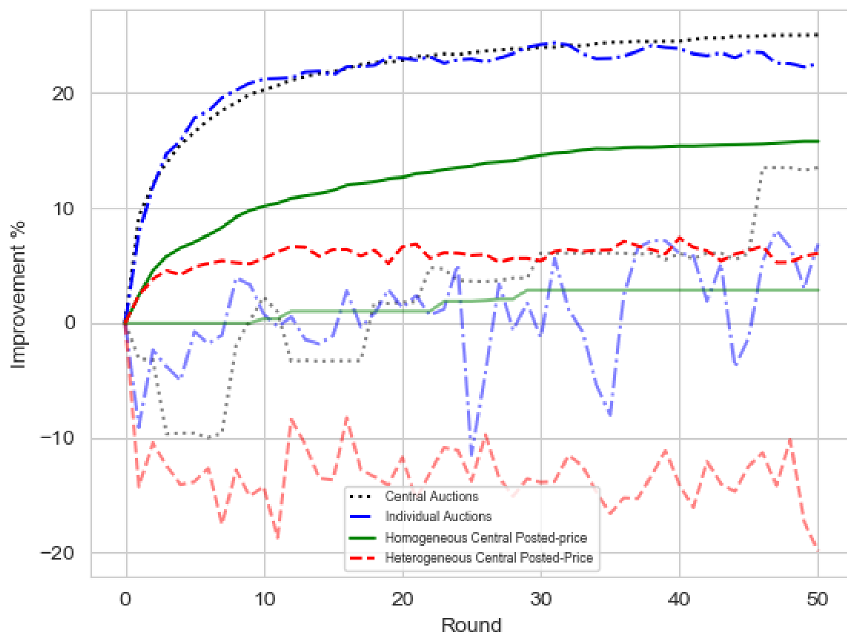


Figure 4.3: Accumulated average improvement obtained after every round (compared to the initial allocation)

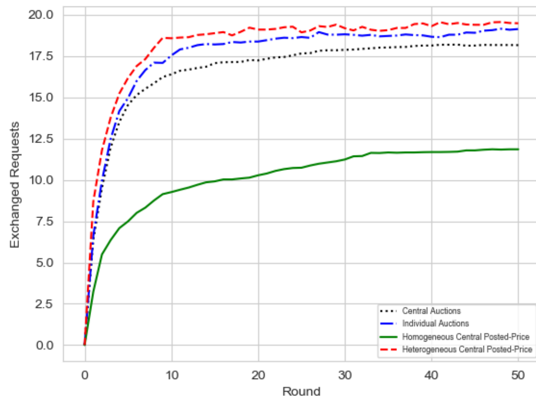
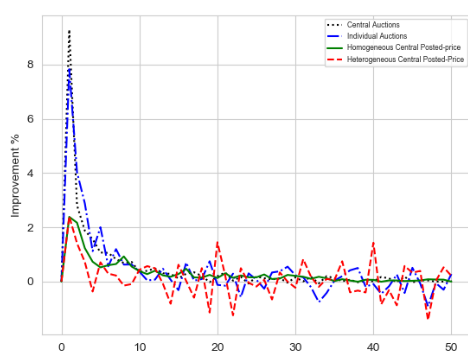
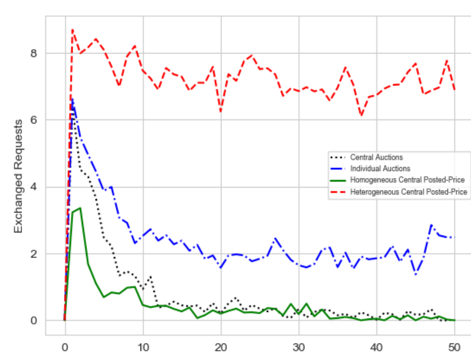


Figure 4.4: Accumulated average exchanged requests in every round (compared to the initial allocation)

Figure 4.5a shows the average improvement for every mechanism and Figure 4.5b shows the average number of exchanged requests, compared to the previous round allocation. This improvement and number of exchanged requests is larger for the initial rounds, followed by a rapid decrease and further stabilization at lower values for all mechanisms but the heterogeneous posted price one. When applying this latter mechanism, the average number of requests being exchanged is as high as in previous rounds (also higher than when using other mechanisms), but the improvement provided by this relatively large exchange is either small or negative compared to the previous round.



(a) Average improvement obtained after every round (compared to the previous round allocation)



(b) Average exchanged requests in every round (compared to the previous round allocation)

Figure 4.5: 50 Rounds Average Improvement and Exchanged Requests (compared to the previous round allocation)

4.5 Conclusions

In this paper, various request-exchange mechanisms are described, tested and compared within the same scenario. All of the mechanisms decrease the travel distances on average, reducing costs and increasing the network's efficiency. However, there is no guarantee that the application of these mechanisms results in all members of the network being better off, as reflected by the lowest improvement obtained by the carriers.

By comparing the average improvement obtained by each mechanism, it can be observed that the auction-based mechanisms outperform the posted price ones. In both of the auction-based variants, carriers must reveal their actual valuations in terms of bids, whereas the posted price mechanisms only require carriers to express whether a price is high enough for them to accept servicing a bundle. Revealing less information (i.e. using posted price mechanisms) represents an improvement as low as 25 % of the improvement obtained by the best auction-based mechanism in one round, while in multi-round approaches it goes up to around 60 % of the improvement provided by an auction-based mechanism for the scenario being considered.

The centralized auctions mechanism delivers a higher average improvement in a one-round setting for most of the considered selection strategies. The mechanism yields similar results for most rounds in a multi-round setting when carriers choose different request selection strategies.

The lowest improvement obtained by a carrier is negative for some rounds, while the maximum improvement reaches around 40%, this difference can be decreased by profit sharing methods or compensation payments which can be decided centrally. In one-round settings, this mechanism presents a scenario in which behaving strategically is not a trivial task, nevertheless, when the mechanism is applied in a multi-round setting, carries might infer some information about other carriers' costs, motivating strategic behavior. The impact of these possible strategic gains is unclear and presents further research opportunities.

The individual auctions mechanism provides very interesting results in a multi-round approach. Its results are, on average, similar to the central auctions mechanism's results. However, the order in which carriers act as auctioneers seems to affect individual results as the minimum improvement is very unstable. In this paper, opportunistic behavior has not been considered, but it is clear that carriers would have a strong motivation to raise their bids when it is likely that the carrier acting as the auctioneer is willing to get rid of requests due to capacity constraints. Finding solutions for this kind of situations presents an interesting opportunity for further research. If strategic behavior could be avoided, individual auction mechanisms could be preferable to central auctions mechanisms not only because costs for a central institution and profit-sharing methods do not have to be considered, but also because they are faster and easier to apply.

Posted price mechanisms present lower improvement in the number of rounds considered in this work. It should be mentioned that, in the homogeneous variant, carriers know the exact cost for the allocations they may end up with, having the chance to guarantee they will not be worse off after collaboration. In some situations, when keeping information from competitors is critical, this mechanism is a recommendable alternative. Strategical pricing might not be a good idea as carriers would risk finding no carrier interested in servicing those bundles at those particular prices, therefore missing the chance for exchanging and improving their set of requests.

The heterogeneous variant yields the worse results. On an individual level, carriers might end up facing losses caused by unforeseen unwanted exchanges. Carriers seem to report the willingness to accept bundles after evaluating them one by one, but when many bundles are allocated together, servicing costs might end up being higher than the carriers' costs for their original set of requests.

Each mechanism requires different levels of information and different considerations for its performance. Apart from an economic benefit, different factors play a role when selecting cooperation mechanisms, trust for example. Moreover, a trade-off between improvement and amount of information being revealed exists; this is fundamental when deciding which mechanism is the best option depending on the actual situation carriers are in.

Chapter 5

Conclusions

Collaborative transportation has become a main trend in research due to its effectiveness in reducing operational inefficiencies. This work focuses on obtaining overall efficiency gains for a group of carriers collaborating with each other by exchanging requests. Moreover, carriers can lower their operational costs significantly by improving their sets of requests.

This dissertation starts by studying the structure of various routing problems in a step-by-step approach. In this part, travel distance is considered a main part of the costs for a carrier, therefore, the shorter this distance, the better it is for a carrier in terms of costs. Next, a fast way of predicting the travel distance is presented. By using regression-based estimation models, a carrier is able to evaluate a large number of requests (and their combinations) in relatively low computational time. These models are used for evaluation of requests in variants of auction-based and posted price mechanisms. The models also provide information about variables that are key drivers of the travel distance. Finally, the mentioned mechanisms are tested to obtain results and insights regarding their application. Following, conclusions for every chapter are summarized.

Chapter 2 provides a study of multi-region problems, which are a common scenario nowadays. Several real-world applications include intra-region and inter-region transportation, performed by vehicles with different capacity, speed and cost. This chapter introduces this new type of logistic problems, which main focus lies on pickup and deliveries in multiple geographically separated regions. An exhaustive examination of already existing literature in related topics is also presented, justifying the multi-region scenarios as a logical evolution step towards more realistic problems in transportation logistics and setting a basis for further research in this direction. A step-by-step construction of mathematical models highlights the relation between classical and the new introduced transportation problems. Collaborative networks are identified here as an interesting extension to multi-region problems. These networks could be improved with managerial implications suggesting best practices in this field.

Chapter 3 presents an empirical approach to estimate travel distances for various routing problems using different properties as predictors in regression models. The models cover a wide range of transportation problems, from conceptually simple traveling salesman problems to complex multi-region transportation problems involving a wide spectrum of real-life constraints. The quality of approximation varies between different problem classes, however, the obtained estimations are more accurate than models presented in previous works. Particularly, good approximation results are obtained for the 2R-MDPDP, for which no previous results were available. This shows that an empirical approach makes it possible to cover problems that include additional characteristics that could not be taken into account in previous models.

The models are generated using one set of instances and reliably predict travel distances for other, previously unknown instances. This result is important in view of potential applications of the models. Another important feature of this empirical approach is that the models can be fitted to optimal solutions, as well as to solutions obtained with heuristic methods, as is the case for the 2R-MDPDP.

Apart from the fact that the models provide very good approximations, the regression models also provide additional insight about which variables have a strong influence on total travel distances for the different types of transportation problems. By applying a stepwise regression approach, significant drivers of travel distances (and costs) are identified. This information is not only important to obtain better approximations, but can also support carriers in their decision making for efficient operations. For example, this information can be considered in collaborative approaches for selecting requests to be exchanged and selecting criteria for bundle generation.

Results also indicate the limitations of such an empirical approach. For some types of problems, approximation errors were considerably larger than for other problems. As an empirical approach relies on the stability of the relationship between explanatory variables and the value to be predicted, any effect that is not explicitly captured in the model can be considered as noise that makes the empirical prediction more difficult. Sources of such randomness might be the structure of the problem and/or the algorithm used to solve the problem. However, as the example of the 2R-MDPDP shows, it is sometimes possible to overcome this high level of randomness by considering relevant variables at a more detailed level, here by considering information about both regions separately.

Chapter 4 presents various request-exchange mechanisms. While all of the mechanisms decrease the travel distances on average, their application does not guarantee all members of the network to be better off. Auction-based mechanisms outperform posted price ones, but they require carriers to reveal their actual valuations in terms of bids, whereas the posted price mechanisms only require carriers to express whether a price is high enough for them to accept servicing a bundle. Moreover, revealing less information (i.e. using posted price mechanisms) results in lower improvement.

The centralized auctions mechanism presents higher average improvement in a one-round setting for most of the considered selection strategies. These results are similar for most rounds in a multi-round setting when carriers choose different request selection strategies. In one-round settings, strategic behavior is said to be avoided by this type of mechanisms, nevertheless, when the mechanism is applied for multiple rounds, carries might be motivated to incur in strategic actions by inferring some information about other carriers' costs.

The individual auctions mechanism provides very interesting results. Its results are, on average, similar to the central auctions mechanism's results. However, the order in which carriers act as auctioneers seems to affect individual results as the minimum improvement is very unstable. If strategic behavior could be avoided, individual auction mechanisms could be preferable to central auctions mechanisms not only because costs for a central institution and profit-sharing methods do not have to be considered, but also because they are faster and easier to apply.

Posted price mechanisms present lower improvement in the number of rounds considered in this work. When using the homogeneous variant, carriers know the exact cost for the allocations they may end up with, having the chance to guarantee they will not be worse off after collaboration. In some situations, when keeping information from competitors is critical, this mechanism is a recommendable alternative. The heterogeneous variant yields the worse results, mainly because carriers end up facing losses caused by unforeseen unwanted exchanges.

Apart from the required levels of information, other factors, like trust, could play a role when selecting the cooperation mechanism to use. It is somehow clear that a trade-off between improvement and amount of information being revealed exists.

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Appendix A

Linearization of non-linear constraints in Chapter I

Some constraints in the previously presented models are not linear. Here we present the linearization of these constraints, in order to provide full information about the way the models can be implemented.

All non-linear constraints can be grouped in two main equation classes.

1. The first one applies to time continuity and capacity constraints. They appear in constraints 11, 23, 5 and 26. An additional variable A_{ijk} is defined, and big M notation is used.
   Original constraint:
   $$S_{jk} = \sum_{i \in V} x_{ijk} \cdot (S_{ik} + q_i), \quad \left(t_{jk} = \sum_{i \in V} x_{ijk} \cdot (t_{ik} + c_{ij}) \right) \qquad \forall j \in V, k \in K,$$
   New constraints:
   $$x_{ijk} + A_{ijk} = 1 \qquad \forall i, j \in V, k \in K,$$
   $$S_{ik} + q_i - S_{jk} \leq M A_{ijk} \qquad \forall i, j \in V, k \in K,$$
   $$S_{ik} + q_i - S_{jk} \geq -M A_{ijk} \qquad \forall i, j \in V, k \in K,$$
2. The second linearization applies to the flow consistency constraints for the models with inter depot routes 14, 15, 29 and 30. Two new decision variables are defined: α_{ik}, β_{ilh}.
   Original basic constraint:
   $$\sum_{j \in N \cup \{l\}} \sum_{k \in K_l} x_{ijk} e_k \leq \sum_{h \in H_l} y_{ilph} m_h \qquad \forall i \in R^I, l, p \in L,$$

New constraints:

$$\sum_{j \in N} x_{ijk} e_k = \alpha_{ik} \qquad \forall i \in R^I, k \in K, (Non-linear)$$

$$\sum_{l' \in L \setminus l'} y_{ill'h} m_h = \beta_{ilh} \qquad \forall i \in R^I, l \in L, h \in H^l, (Non-linear)$$

$$\sum_{k \in K^l} \alpha_{ik} \leq \sum_{h \in H^l} \beta_{ilh} \qquad \forall i \in R^I, l \in L,$$

The first two of the new equations have to be linearized too. We show the linearization of the first one, and the same method applies to the second one. Big M notation is again used.

$$\alpha_{ik} \leq M \sum_{j \in N} x_{ijk} \qquad \forall i \in R^I, k \in K,$$

$$\alpha_{ik} \leq e_k \qquad \forall i \in R^I, k \in K,$$

$$\alpha_{ik} \geq e_k - M(1 - \sum_{j \in N} x_{ijk}) \qquad \forall i \in R^I, k \in K,$$