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Including Tour Costs in a Covering Problem with User Equilibria

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Contents

1 Introduction

After the occurrence of natural disasters such as hurricanes, earthquakes or floods, inhabitants of affected areas are often in urgent need of relief goods. Humanitarian aid organizations that provide victims with commodities like medicine, water or food, need to decide how to effectively distribute these goods, hence in which locations to establish distribution centers (DCs), and how to supply them in order to maximize covered demand. There are several reasons why the routing aspect, referring to the tours vehicles drive to provide beneficiaries with goods, should be considered when evaluating candidate locations for DCs. Roads might not be accessible for trucks after the occurrence of a natural disaster, hence making some locations unfavorable for opening DCs. Compared to other fields, in humanitarian logistics, a relatively short time horizon is being considered and therefore, it is not assumed that the availability of routes changes while the DCs are operating.

This thesis is based on the work of Gutjahr and Dzubur (2016) and aims to extend their covering model with user equilibria (UE) by including the routing aspect in the facility location decisions. Gutjahr and Dzubur (2016) propose a bi-objective, bilevel optimization model in order to determine how many and in which locations to open DCs after the occurrence of a natural catastrophe. The humanitarian aid organization represents the decision maker on the higher decision level who selects the locations in which to open facilities. Beneficiaries represent the decision makers on the lower decision level. They are assumed to walk to the respective DC based on a user equilibrium, resulting from the share of covered demand beneficiaries expect to receive, and the distance to the chosen DC. The authors argue that considering a user equilibrium when determining how people spread over several open DCs reflects real-life situations better than previous models that consider distance only or situations in which the aid-providing organization decides which DCs beneficiaries should visit. The focus of this work lies on the touring aspect which might influence the location decisions, an important aspect that is not considered in the work of Gutjahr and Dzubur (2016) but suggested as an interesting point for future research. By means of complete enumeration, all combinations of open DCs are determined and for every solution, the UE and optimal tour a vehicle takes to supply the DCs with relief goods is calculated. The routing aspect is solved using an exact and heuristic solution approach. By adding transportation costs to the model, total costs increase for the same level of uncovered demand. In other words, aid-providing organizations with a budget limit are now able to supply less people with relief goods than without the consideration of transportation costs.

This work is structured as follows: Section 2 provides an overview of related literature. In section 3, the problem is described, followed by the formulation of the model in section 4. Section 5 presents the proposed solution methods. Computational experiments are analyzed in section 6, following a discussion of the problem including the model's limitations and aspects for further research in section 7. Concluding arguments are given in section 8.

2 Related Literature

Research designated to location-routing problems in emergency logistics is conducted by Wang et al. (2014) who address a multi-objective open location-routing problem in disaster relief after the occurrence of earthquakes. In open location-routing problems, vehicles do not return to the distribution center after visiting the last customer, but wait for further instruction. In emergency logistics, DCs providing relief may change during the operation, that is previous DCs might not be used further and vehicles need to approach other DCs. Split delivery is allowed, that is customers can be visited several times in case demand exceeds the vehicle's capacity. By considering travel time, reliability and total costs, the pareto frontier is computed by means of the Non-dominated sorting genetic algorithm II (NSGA-II) and natural datasets evolution (NSDE). The model is applied to the Great Sichuan Earthquake occurring in 2008 with a number of death close to 70.000.

Abounacer et al. (2014) consider a multi-objective location-transportation problem in disaster response. The location problem, which deals with determining the number and location of distribution centers and the transportation problem, which aims to optimize the shipment of relief goods from the distribution centers to demand nodes, are solved simultaneously. Three objective functions are considered, namely the minimization of total transportation time, the minimization of the number of people who establish and run DCs, and the minimization of uncovered demand. The authors calculate the pareto frontier by means of an epsilon-constraint method. To reduce computation time, the algorithm is slightly altered to an approximation which performs well compared to the exact algorithm.

Tavakkoli-Moghaddam et al. (2010) consider a bi-objective location-routing problem where total costs for setting up facilites, operating facilities, and transportation costs are minimzed, and the total covered demand is maximized. The authors solve the problem by applying a multi-objective scatter search (MOSS) algorithm. In order to evaluate the performance of the algorithm, the problem is also solved using elite tabu search (ETS). Although the authors apply their model not in a disaster relief context, it has common ground with humanitarian logistics since the maximization of covered demand represents one of the two objective functions. Hua-li et al. (2011) solve a bilevel, bi-objective location-routing problem in disaster relief applications. On the upper level, total costs consisting of facility set-up cost and routing costs, are minimized. The lower level addresses the maximization of a service level. The problem is solved by means of a genetic algorithm.

2.1 Location Routing Problems

The classical location-routing problem refers to deciding on optimal DC locations under consideration of the tours vehicles take to visit customers starting at the DC, hence the DC itself is the depot. It is well known that when optimizing facility location and routing problems separately, results can be suboptimal. Location-routing refers to optimizing facility location problems and at the same time taking routing aspects into account. Maranzana (1964) states that "the location of factories, warehouses-and supply points in general, to serve customers distributed over a network of cities-is often influenced by transport costs" . However, research on location-routing problems has only gained significant importance during the past two decades. Prodhon and Prins (2014) name the fact that computers were not able to calculate such complex approaches as main reason. Nagy and Salhi (2007) however point out reasons why researches might deliberately optimize the location and routing problems separately, namely the increased difficulty of the approach and secondly, some authors do not see the point in combining the two approaches due to different time horizons. They argue that routes can change rapidly and unexpectedly and should therefore not be taken into account when optimizing locations. Within the last decade, a great amount of research was conducted on combining these two problems instead of solving the location and routing aspect separately. Nagy and Salhi (2007) and Prodhon and Prins (2014) present an overview of the most significant research on location-routing problems. The classical location-routing problem deals with deciding in which locations to open depots, assigning customers to these depots and determining an optimal tour to supply customers. Many variants of the problem have since been studied. Rath and Gutjahr (2014) establish a two-level location-routing problem with plants and distribution centers to be established, and customers. The main difference compared to true location-routing problems is that the authors consider full truckloads when supplying depots, therefore no real routing aspect is given. Further, customers can be left with uncovered demand and the last customer visited on a tour can be left with partially covered demand. The location of distribution centers is subject to a multi-depot team orienteering problem. Three objectives are considered, namely 1) to minimize the opening costs of distribution centers, 2) to minimize the transportation costs between plants and distribution centers and inventory costs at distribution centers and 3) to maximize total demand covered. To solve the problem, the authors introduce a math heuristic based on the epsilon constraint method. Laporte and Nobert (1981) solve a location-routing problem using an exact algorithm. The authors integrate the optimization of routing in the process of finding the optimal location for a single depot among n possible candidates. They find that in most cases, the optimal point to set up the depot deviates from the center point between customers. Since location-routing problems are relatively difficult to solve, exact methods can only be used for small instances. Nagy and Salhi (2007) present a summary of papers where exact methods are used to solve location-routing problems to optimality. The maximum number of instances is solved by Laporte et al. (1983) with 40 possible facility locations and 40 customers, and by Laporte et al. (1988) with 3 facilities and 80 customers. A large amount of recent research is devoted to heuristic methods. Nagy and Salhi (2007) differentiate between *clustering-based* and *hierarchical* methods. Approaches, where the location problem and the routing problem are solved sequentially are not classified as location-routing problems as the combined problem might not provide the optimal solution. *Clustering-based* methods refer to splitting customers into a number of clusters followed by either first determining the depot location per cluster and the solving the routing problem, or the other way round. *Hierarchical* methods consider the location problem as the main problem and the routing problem as the sub-problem. In each step of solving the location problem, the routing problem is solved on a lower level. Lin et al. (2002) solve a location-routing-loading problem for the distribution of bills and leaflets of a telecommunication provider in Hong Kong. The objective is to minimize total costs consisting of facility set up costs and operating costs. An integrated solution approach is proposed, considering routing and loading decisions when determining locations for depots. First, the minimum number of depots is determined based on customer demand and the capacity of the depots. Complete enumeration of all candidate locations is used, which can be calculated within a reasonable amount of time thanks to the small number of candidate locations (15). For every combination of possible locations, the routing and loading problems are solved. For a starting solution, customer demand nodes are served from the closest depot where capacity is still available. Any unassigned customers are served from those facilities with the smallest volume of work (even though capacity restriction might be exceeded). Then, the savings algorithm by Clarke and Wright is applied to establish vehicle tours, which are then improved by a traveling salesman problem (TSP) algorithm. The authors use threshold accepting and simulated annealing to improve their solutions. Finally, they apply a TSP for another improvement. Vehicles are allowed to take multiple trips. Then, vehicles are assigned to tours by evaluating all allocations. In case the lowest routing cost is smaller that the cost of opening another depot the algorithm stops, otherwise one more facility is opened and again all possible combinations of locations are evaluated. The solution approach has proved to generate cost savings to the company.

Multi-level location routing problems refer to problems where tours are optimized on more than one level, namely among depots and, as in the classical location-routing problem, between depots and customers. Jacobsen and Madsen (1980) solve a two-level loaction-routing problem for the distribution of newspapers. The problem consists of determining the locations of transfer points and establishing a tour from the factory through these points. Further customers are assigned to transfer points or the factory and vehicle routes are established to serve all customers. Ambrosino and Scutellà (2005) solve a location-routing problem with four levels, where the plant represents the first level, level two and three consist of depots, namely distribution centers and transfer points, and customers represent the last level. Apart from facility location and routing decisions, the authors include decisions concerning inventory in the model. Nguyen et al. (2012) consider a two-level location-routing problem with a given depot, capacitated distribution centers to be located and capacitated vehicles that visit customers.

2.2 Bilevel Optimization

Bilevel programs are characterized by a hierarchical relation between two optimization problems, such that the outcome of the decision maker on the upper decision level (leader) is influenced by the behavior of the decision maker on the lower decision maker (follower). Following Colson et al. (2007), the bilevel optimization problem is formulated as follows:

$$\min_{x \in X, y} F(x, y) \tag{1}$$

$$\text{s.t. } G(x, y) \leq 0 \tag{2}$$

$$\min_{y} f(x, y) \tag{3}$$

$$\text{s.t. } g(x, y) \leq 0 \tag{4}$$

where $x \in \mathbb{R}^{n_1}$ and $y \in \mathbb{R}^{n_2}$. Equation 1 refers to the decision of the leader and equation 3 to the decision of the follower. The leader takes the decision variables of both, the upper and the lower optimization problem into account.

When modeling bilevel programs, it can be distinguished between optimistic and pessimistic bilevel optimization, referring to the behavior of the decision maker on the lower decision level. Optimistic bilevel programming deals with situations, in which the follower acts in the best interest of the leader, whereas a pessimistic approach is used when the two agents cannot cooperate or when the decision maker on the upper level is risk-averse and wants to minimize an unfavorable decision of the follower.

Bilevel optimization is first considered by Bracken and McGill (1973) who discuss "mathematical programs with optimization problems in the constraints" and since then, a significant amount of research regarding bilevel programming has been conducted. Fields of application include all kinds of real-life problems such as transportation problems, facility location problems, problems regarding social policies, environmental issues, just to name a few. Colson et al. (2007) provide an overview of bilevel optimization including areas of use. One prominent example is the *toll-setting problem*, which refers to the maximization of the revenue gained by charging tolls for using certain routes in a transportation network. The decision maker on the lower level is represented by the road users who aim to minimize their total transportation costs consisting of travel time, toll costs and other factors. Consequently, the routes for which tolls are charged and the amount of the tolls have to be chosen in way, such that users still use these roads without changing their routes and large revenues are obtained. Bilevel programs are intrinsically hard and therefore relatively hard to solve. Linear bilevel programs can be solved by means of vertex enumeration, in case the optimization problem on the lower level is convex and regular, the Karush-Kuhn-Tucker (KKT) conditions can be used to reformulate the bilevel problem into a single-level problem:

$$\min_{x \in X, y, \lambda} F(x, y) \tag{5}$$

$$\text{s.t. } G(x, y) \leq 0 \tag{6}$$

$$g(x, y) \leq 0 \tag{7}$$

$$\lambda_i \geq 0, \quad i = 1, ..., m_2 \tag{8}$$

$$\lambda_i g_i(x, y) = 0, \quad i = 1, ..., m_2 \tag{9}$$

$$\nabla_y \mathcal{L}(x, y, \lambda) = 0 \tag{10}$$

where

$$\mathcal{L}(x, y, \lambda) = f(x, y) + \sum_{i=1}^{m_2} \lambda_i g_i(x, y) \tag{11}$$

is the Lagrangian function. The problem is the solved by a branch-and-bound algorithm. Further methods for solving linear bilevel programs include complementary pivoting, which is also based on the application of the KKT conditions, and descent methods. Non-linear bilevel optimization problems can be solved by means of penalty functions, where constrained problems are formulated as unconstrained programs with a term penalizing the objective function whenever the constraints are violated. Finally, trust-region algorithms are used to solve non-linear bilevel programs. Therefore, a "trusted" region around the current iterate is considered, where an approximation of the original model is solved.

2.3 Multi-objective Optimization

Definition

Multi-objective optimization problems refer to problems where two or more objectives are considered simultaneously and can be formulated as

$$\min(F_1(x), F_2(x), ..., F_k(x)) \tag{12}$$

$$\text{s.t. } g_i(x) \leq 0, \quad i = 1, ..., m \tag{13}$$

$$h_j(x) = 0, \quad j = 1, ..., n \tag{14}$$

where k refers to the number of objective functions, i to the number of inequality constraints and j to the number of equality constraints.

Pareto Optimality

For optimization problems with only one objective, the solution consist of identifying local optima and, if possible, a global optimum. Solving an optimization problem with two or more objective functions is not that distinct, as improving the objective value of one function might lead to a worsening of the value of the other objective functions. Therefore, there does not exists a single optimal solution but an infinite set of solutions.

Pareto (1971) introduced the concept of pareto optimality, where a point x^* in the feasible space S is defined as pareto optimal if there is no other point

x in S with a better value of at least one objective and equal values of all other objectives. Following Arora (2012), the feasible set S is defined as $S = \{x \mid g_i(x) \leq 0; i = 1 \text{ to } m; \text{and } h_j(x) \leq 0; j = 1 \text{ to } n\}$ and the pareto frontier can formally be expressed as follows: A point x^* in the feasible set S is pareto optimal if and only if there is no other point x in S such that $f(x) \leq f(x^*)$ with at least one $f_i(x) < f_i(x^*)$. By ignoring the latter condition, weakly pareto optimal solutions can be found, hence there is the possibility to improve the value of some objective functions while the values of the other objectives stay unchanged. All solutions part of the pareto fontier are considered equally good and the decision maker has to decide which one to choose. Figure 1 depicts the concept of pareto optimal solutions for a bi-objective minimization problem.

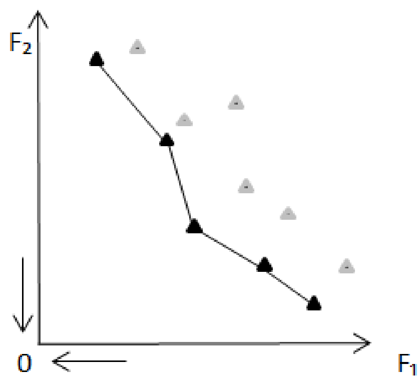

Figure 1: Frontier of pareto-optimal solutions for bi-objective optimization problems.

Solution Methods

Arora (2012) gives a good overview of exact and heuristic solution methods for solving multi-objective optimization problems. A common approach is the *weighted sum* method which can be formulated as

$$U = \sum_{i=1}^{k} w_i f_i(x) \tag{15}$$

where the vector w consists of predefined weights chosen by the decision maker such that $\sum_{i=1}^{k} w_i = 1$ and $x > 0$. There are two ways of using weights, namely i) to pre-define the relative importance of the objective functions and consequently obtaining only one solution or ii) to generate several pareto optimal points through adjustments of the weight vector. The weighted sum method is relatively easy to apply, however there are a few drawbacks as analyzed for example by Das and Dennis (1997). Setting weights before solving the optimization problem might not lead immediately to a favorable solution and consequently the problem has to be solved again with adapted weights. Further, it is not possible to find pareto optimal points in non-convex regions and finally, continuously adjusting weights might lead to a solution that is not uniformly distributed.

The *epsilon constraint* method is another simple approach for solving multi-objective optimization problems. The decision maker decides to minimize the most important objective $F_i(x)$ and defines values $\epsilon_j > 0$ such that the additional constraints $F_j(x) \leq \epsilon_j$ for all $\neq i$ are generated. The major limitation of this approach is that the worst values acceptable for an objective have to be pre-defined by the decision maker which can easily lead to infeasible solutions. Given a feasible solution is found, the solution is weakly pareto optimal. In case the solution is unique, it is pareto optimal.

The *two phases* method is used to solve multi-objective combinatorial optimization problems which generally can be defined as

$$\min\{Cx | Ax = b, x \in \{0, 1\}^n\}. \tag{16}$$

Ehrgott and Gandibleux (2014) solve a two-phases method for the bi-objective combinatorial optimization problem. In phase one, all efficient supported solutions are determined. The results are then used to reduce the search space for finding any other efficient solutions in phase two. This is done by only searching within the triangles generated by connecting the last efficient supported points found in phase one. The method has mainly been studied for bi-objective optimization problems and gets significantly more complex when considering three objectives, as shown by Ehrgott and Gandibleux (2014).

Multiobjective evolutionary algorithms (MOEAs) are of growing importance for solving optimization problems with multiple conflicting objectives. Since they are population-based, they are able to approximate several pareto-optimal points in one single run. Zhou et al. (2011) provide a literature review of evolutionary algorithms studied. One of the standard approaches is the non-dominated sorting genetic algorithm-II (NSGA-II) introduced by Deb et al. (2002). NSGA-II is an elitist algorithm, indicating that characteristics of the fittest candidates (elites) of the parent population can be taken to the next generation. A niche-preserving operator is used to force a consistent spread of points in the criterion space, as evolutionary algorithms are likely to find a limited amount of pareto-optimal solutions clustered closely around each other. Algorithm 1 depicts a pseudocode taken from Jensen (2003).

Algorithm 1 NSGA-II

```
generate P_0 at random
set P_0 = (S_1, S_2, ...) = non-dominated-sort (P_0)
for all S_i ∈ P_0 do
    crowding-distance-assignment (S_i)
end for
set t = 0
while (not done) do
    generate child population Q_t from P_t
    set R_t = P_t ∪ Q_t
    set S = (S_1, S_2, ...) = non-dominated-sort (R_t)
    set P_{t+1}=0
    set i = 1
    while |P_{t+1}| + |S_i| < N do
        crowding-distance-assignment (S_i)
        set P_{t+1} = P_{t+1} ∪ S_i
        set i = i + 1
    end while
    sort S_i on crowding distances
    set P_{t+1} = P_{t+1} ∪ S_i[1 : (N − |P_{t+1}|)]
    set t = t + 1
end while
return S_1
```

3 Problem Description

This work is an extension of the model by Gutjahr and Dzubur (2016) who solve a facility location problem based on user equilibria. In this work, the touring aspect is added to their model, namely the problem of determining the optimal tour a vehicle drives to supply the DCs from the depot. By means of complete enumeration, all possible combinations of open DCs are determined and for every combination, the user equilibrium and the optimal tour are calculated. The problem in this paper deviates from the classical location-routing problem as not the tours from the DCs to be opened to the customers are considered but the tours from a depot to supply the DCs. Beneficiaries are not visited by vehicles but are assumed to walk to the respective DC based on a user equilibrium. The depot from which the distribution centers are supplied is assumed to be located in the main village of a community. The problem therefore deals with determining in which locations to open DCs, taking into account (1) how people spread over opened DCs and (2) the tour a truck drives in order to visit all DCs. The touring aspect is solved by a TSP algorithm which assumes a single uncapacitated vehicle that is available at the depot. The truck covers all DCs in a single tour before returning to the depot.

4 The Model

The basic model is taken from Gutjahr and Dzubur (2016) and extended by the optimization of transports costs. The situation considered is post-disaster relief after the occurrence of a hurricane, flood or earthquake, where humanitarian aid organizations need to decide upon the number and location of capacitated DCs to be set up. People can visit the DCs and receive relief goods. In case demand exceeds capacity, relief goods are shared equally between all people coming to a DC. A specific region is considered and a set of potential locations to establish DCs is given. Further, the population distribution is known, therefore, a set of population centers (PCs), which refer to cities or villages with known numbers of residents, is given. Moreover, the road system with the distances between the villages is assumed to be known. One demand unit refers to one person of a PC. Two objectives are considered, namely the minimization of set-up costs of the DCs and the minimization of the demand that cannot be covered. This work aims to include tour costs in the model which result from supplying the DCs with relief goods from a predetermined depot. Therefore, the first objective is modified and now consists of minimizing total costs, namely the sum of the set-up costs of the DCs and the transportation costs.

4.1 User Equilibrium

Gutjahr and Dzubur (2016) consider a user equilibrium model in order to determine the demand opened DCs will face. People are assumed not to always choose the DC located closest to their hometown but to also take into account up to which amount their demand will be covered at the chosen DC. Beneficiaries might therefore walk to a DC located further away in order to receive full relief instead of visiting a closer DC with high chances of uncovered demand. Two cost types play a role when people decide which DC to visit: traveling costs to reach the selected DC and the costs resulting from the amount of anticipated uncovered demand due to congestion. It is assumed that shortly after the opening of the DCs, a user equilibrium will be formed, that is, the benefit from receiving a greater part of relief commodity compensates for the higher costs of traveling a longer distance. Every person is assumed to minimize the sum of travel costs and costs of uncovered demand. If a user equilibrium is reached, no individual can further reduce their costs by choosing a different route under the condition that all other individuals stick to their decisions. The user equilibrium is basd on the Wardrop equilibrium (see Correa et al. (2010)) occurring in transportation and telecommunication networks, which indicates that no person can reduce their journey time by changing their route without cooperation.

A single relief item is considered which could be, for example, a package filled with medicine, and it is assumed that each beneficiary requires b units of that good. I describes the set of PCs and J the set of DCs. The number of people living in PC i, which equals the demand in this PC, is denoted by d_i $(i \in I)$. The capacity of DC j is denoted by γ_j $(j \in J)$. The factors $a_{ij} \geq 0$ $(i \in I,\ j \in J)$ give the distances from all PCs to all candidate sites for DCs. In case that the demand at DC j exceeds its capacity, it is assumed that the relief goods are split equally between all beneficiaries visiting DC j, that is, every person receives the same share of demand. The number of beneficiaries going to DC j is given by y_j and the expected uncovered demand per person is obtained as follows:

$$\psi(y_j, b, \gamma_j) = (b - \gamma_j / y_j)_+ \tag{17}$$

with $x_+ = x$ if $x \geq 0$ and 0 otherwise. The total costs every beneficiary faces consist of traveling expenses for visiting a DC plus the costs of being only partly provided with relief commodity (or possibly not at all), and is given by the following equation:

$$c_{ij} = \phi(a_{ij}) + p\,\psi(y_j, \underline{,}\gamma_j). \tag{18}$$

The total costs for traveling from PC i to DC j are the sum of the non-decreasing function $\phi(a)$ with $\phi(0) = 0$ and the function for unsatisfied demand multiplied by a weight factor $p > 0$. To represent the possibility of staying at home and not visiting any DC, a dummy DC is introduced. Index 0 is used to refer to the dummy DC, therefore $a_{i0} = 0$ $(i \in I)$ and $\gamma_0 = 0$. Visiting the dummy DC from any PC causes zero travel costs and since the capacity at the dummy DC equals zero, total uncovered demand is b. J_0 refers to the enlarged set of DCs including the dummy DC. The number of inhabitants living in PC $i \in I$ and visiting DC $j \in J$ is given by y_{ij}, the number of people staying at PC i and not visiting any real DC is given by y_{i0}. The equation to obtain the total demand arriving at any DC, including the dummy DC, is given by $\sum_{i \in I} y_{ij} = y_j$ $(j \in J_0)$. The user equilibrium is calculated as follows:

$$\min_y \left\{ \sum_{i \in I, j \in J} \int_0^{y_{ij}} \phi(a_{ij})dz + \sum_{j \in J_0} \int_0^{y_j} p\psi(z, b, \gamma_j)dz \mid y \in Y \right\} \tag{19}$$

subject to

$$\sum_{j \in J_0} y_{ij} = d_i \quad (i \in I) \tag{20}$$

and

$$\sum_{i \in I} y_{ij} = y_j \quad (j \in J_0). \tag{21}$$

The decision maker on the upper level, which can be represented by a humanitarian aid organization, decides upon the number and locations of DCs to be opened. Binary decision variables x_j are introduced, where x_j takes the value 1 if a facility is being established in location j and zero otherwise. Consequently, $X = \{0, 1\}^m$

denotes the feasible region and $x = (x_1, ..., x_m) \in X$ the decision. The opening and operating costs for DC $j \in J$ are given by the factor $\lambda = (\lambda_1, ...\lambda_m)$. The set of opened DCs is denoted by K.

4.2 Solution of the TSP

One uncapacitated vehicle is assumed to deliver relief supply from a predefined depot to all opened DCs. In order to determine the optimal route of the truck, a traveling salesman problem (TSP), as proposed by Miller et al. (1960), is to be solved where $z_{kl} = 1 \;\; (k, l \in K)$ if the optimal tour includes going from DC_k to DC_l and 0 otherwise. The feasible set is denoted by Z. The costs for passing one unit of distance to supply the DCs are given by factor f. The aim of the decision maker is to minimize total costs, consisting of opening costs of the DCs, transportation costs for supplying DCs, and the costs for total uncovered demand. The first objective function, describing the minimization of set-up- and transport costs, can be formulated as

$$F_1(x, z) = \sum_{j \in J} c_j x_j + \sum_{k \in K} \sum_{l \in L} f a_{kl} z_{kl} \tag{22}$$

To ensure every DC is visited exactly once, constraints

$$\sum_{k \in K, k \neq l} z_{kl} = 1 \quad \forall \; l \in K \tag{23}$$

and

$$\sum_{l \in K, l \neq k} z_{kl} = 1 \quad \forall \; k \in K \tag{24}$$

are considered. Further, the following equation eliminates sub-cycles, hence the vehicle is only to return to the depot after having visited all opened DCs:

$$u_k - u_l + p x_{kl} \leq p - 1 \quad \forall \; 1 \leq k \neq l \leq n. \tag{25}$$

Variables u_k and u_l are dummy variables that can take any real numbers for which the inequality is true. The following example is used to explain the subtour elimination constraints in greater detail.

Example

A network of 5 nodes is considered that have to be visited by a single vehicle. Without the subcycle elimination constraints, subtours 1-2-3-1 and 4-5-4 would be a feasible solution, as every node is visited exactly once and also tours of length one are eliminated. Considering the constraints above one would get:

$$\begin{array}{r} u_1 - u_2 + 5 \leq 4 \\ u_2 - u_3 + 5 \leq 4 \\ \hline u_1 - u_3 + 10 \leq 8 \end{array}$$

Because node 1 is included in this subtour, the constraint is not violated as arbitrary reals numbers can be found such that the inequality is true. However, when considering the second subtour (4-5-4), the constraints lead to a contradiction:

$$\begin{array}{r} u_4 - u_5 + 5 \leq 4 \\ u_5 - u_4 + 5 \leq 4 \\ \hline 10 \leq 8 \end{array}$$

By eliminating all subtours that do not include node 1, one gets a single tour for which arbitrary real numbers can be found, such that the constraints hold. In the example below, the tour 1-2-3-4-5-1 is considered.

$$\begin{array}{r} u_1 - u_2 + 5 \leq 4 \\ u_2 - u_3 + 5 \leq 4 \\ u_3 - u_4 + 5 \leq 4 \\ u_4 - u_5 + 5 \leq 4 \\ \hline u_1 - u_5 + 20 \leq 8 \end{array}$$

Consequently, all subtours are eliminated and the result is one single tour visiting all nodes. △

The second objective function, minimizing total uncovered demand, can be written as

$$F_2(x, y) = \sum_{j \in J} (by_j - \gamma_j)_+ + by_0 = \sum_{j \in J} (by_j - \gamma_j)_+. \tag{26}$$

The first term, $(by_j - \gamma_j)$, gives the share of unsatisfied demand resulting from the case where demand is higher than capacity and relief goods are shared equally between all people visiting a DC. The absolute term by_0 is the uncovered demand which arises due to people visiting the dummy DC, that is, staying at home. The bi-objective bilevel program can be written as

$$\min\ (F_1(x, z),\ F_2(x, y)) \tag{27}$$

$$\text{s.t. } x \in X \tag{28}$$

$$z \in Z \tag{29}$$

$$\min g(y) \tag{30}$$

$$\text{s.t. } y \in Y(x) \tag{31}$$

$$g(y) = \sum_{i \in I j \in J} \int_0^{y_{ij}} \phi(a_{ij}) dz + \sum_{j \in J_0} \int_0^{y_j} p\psi(z, b, \gamma_j) dz \tag{32}$$

The equation $g(y)$ represents the decision of the beneficiaries on the lower decision level, who minimize their total costs consisting of the traveling costs of walking to a chosen DC and the costs of being only partially supplied with relief goods.

5 Solution Approach

By means of complete enumeration, all possible combinations of open DCs are determined. Since in at least one location a DC has to be established, there are $2^n - 1$ possibilities for opening DCs, where n stands for the number of PCs or candidate locations. For each of these combinations, the optimal delivery tour and the user equilibrium are determined. The user equilibrium is calculated by means of the Frank-Wolfe algorithm with a fixed step size $\lambda_s = \frac{2}{s+2}$ $(s = 1, 2, ...)$. In this work, the focus lies on determining the optimal route, a vehicle should choose in order to provide the opened DCs with relief items. Therefore, a TSP is being solved by considering two different solution methods.

The first one is an exact algorithm that lists all possible permutations in order to find the shortest route. In this work, listing all ordered combinations of tours is preferred over the use of a mixed-integer programming (MIP) solver due to faster calculation times for small instances. For medium-sized instances, a MIP solver would be preferred over complete enumeration and for large instances, both approaches are unsuitable. It is assumed that distance and transport costs are linear.

The heuristic method proposed to reach results faster is the savings algorithm. The performance of the heuristic method compared to the exact solution approach is measured using the hypervolume indicator. Computations show that if only a small amount of nodes (up to 7 nodes) is being visited by the vehicle, the exact solution algorithm provides results faster than the savings algorithm. For a threshold of 8 or more open DCs, the computation time of the savings algorithm is less. The final procedure is implemented in a way that if 7 or less than 7 DCs are established, the optimal tour is determined using the exact solution algorithm, whereas in the event of more than 7 open DCs, the heursitic procedure is being called.

5.1 Exact Solution Approach

A straight forward way of finding the optimal delivery tour is by evaluating all $(n-1)!$ possible combinations of open DCs.

Algorithm 2 Permutations

```
procedure permutation(n)
    for i = 1 to n do
        s_i = i
    end for
    print(s_1, ..., s_n) //print the first permutation
    for i = 2 to n! do
        m = n − 1
        while (s_m > s_{m+1}) do
            m = m − 1 //find the first decrease working from the right
        end while
        k = n
        while (s_m > s_k) do
            k = k − 1 //find the rightmost element s_k with s_m < s_k
        end while
        swap(s_m, s_k)
        p = m + 1
        q = n
        while (p < q) do
            swap(s_p, s_q) //swap s_{m+1} and s_n, swap s_{m+2} and s_{n−1} and so on
            p = p + 1
            q = q − 1
        end while
        print(s_1, ..., s_n) //print the ith permutation
    end for
end procedure
```

Algorithm 2 is taken from Johnsonbaugh (2009) and lists all permutations of n delivery points in lexicographic order. The input vector contains all DCs opened in the respective iteration. If, for example, in villages 2,3, and 5 DCs are established, the input vector is $v = [2\ 3\ 5]$.

5.2 The Savings Algorithm

The classical vehicle routing problem (VRP) deals with finding optimal routes for a fleet of capacitated vehicles to deliver goods from a central depot to a number of customers. Depending on the number of delivery points, an exact solution method to find the optimal routes can often not be used in practice due to large computation time. Clarke and Wright (1964) propose an approximate algorithm in order to solve the VRP to optimality (or close to optimality) with considerably reduced computation time. They consider a number of vehicles x_i with capacity C_i $(i = 1, ..., n)$ which are to deliver items q_j from a central depot P_0 to a number of customers P_j $(j = 1, ..., M)$. By d_{yz}, the distances between customers and between the depot and customers are denoted and the objective is to minimize the total tour length traveled. The basic concept of the algorithm is to calculate savings gained from adding demand points to routes instead of visiting every demand point separately. In the original problem, capacities are ranked from small to large, $C_{i-1} < C_i$ $(i = 2, ..., n)$, and it is assumed that C_1, which is the vehicle with the smallest capacity, cannot load the full demand. Therefore $C_1 < \sum_{j=1}^{m} q_j$. Further, in the original problem formulation, the number of trucks available is set to infinite. This is done to allow vehicles to carry small loads and to make sure all customers can be visited at the same time. In this work, only one vehicle, that is assumed to be uncapacitated, is used to supply all opened DCs with relief items. Hence, for the TSP the following equation applies: $C_1 \geq \sum_{j=1}^{m} q_j$.

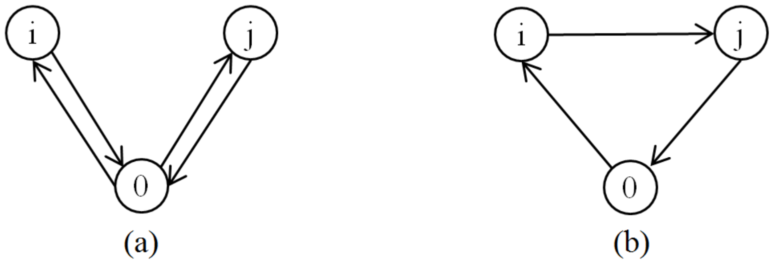

Figure 2: Basic concept of the savings algorithm.

Figure 2 illustrates the basic concept of the savings algorithm. First, as depicted in figure 1(a), the truck starts at the depot, denoted by 0, and is assumed to drive single tours to visit customers i and j, that is returning to the depot after every visit. In the next step as shown in figure 1(b), the two single tours are combined to one route, hence the vehicle goes straight from customer i to customer j before returning to the depot. In order to do so, the capacity of the truck has to be large enough to carry the demand of both customers. Now, the the savings resulting from joining the two single tours can be calculated. The transportation costs between two points k and l are described by c_{ij} and the total transportation costs are denoted by TC_a.

$$TC_a = c_{0k} + c_{k0} + c_{0l} + c_{l0}$$

The total costs for the combined tours are obtained by

$$TC_b = c_{0k} + c_{kl} + c_{l0}$$

The savings, denoted by S_{kl} can easily be calculated by deducting TC_b from TC_a:

$$S_{kl} = TC_a - TC_b = c_{k0} + c_{0l} - c_{kl}$$

At the beginning of the algorithm, the savings between all customer pairs are determined and stored in a descending fashion. Starting from the top of the list, the two demand points with the highest savings are being joined into one route. Going further down the list, two tours are being combined if this can be done without changing any previously established order. Consequently, in order to combine two tours, both entries of the savings pair have to be either the first or last demand point that is being visited by a truck. Further, a customer can only be added to a tour if the total demand does not exceed the capacity of the vehicle. In this work, the latter constraint can be ignored since it is assumed that only one vehicle visits all opened DCs which reduces the VRP to a TSP with unlimited capacity. Algorithm 3 shows a pseudocode describing how the savings algorithm is implemented.

Lysgaard (1997) differentiates between a parallel and a sequential version of the savings algorithm. The algorithm used in this work is the parallel form, where more than one route can be established at the same time before joining them together. The sequential version builds only one route at the time.

Algorithm 3 Savings Algorithm

```
Input: distances between PCs, vector openVec with open DCs
Output: optimal delivery tour
calculate savings and sort in descending order
create an array tours of size (openVec,openVec) where every row contains
a single customer tour
define variable nrTours that counts current tours
while nrTours >1 do
    for every savings pair of the savings matrix do
        find the position of relevant nodes in the matrix tours
        if either of the two nodes is in the middle of a tour then
            continue
        else if both nodes are at the beginning of a tour then
            flip the tour with node 2
            and put it before the tour that includes node 1
        else if node 1 is at the beginning, node 2 is at the end of a tour then
            put tour with node 2 before tour with node 1
        else if node 1 is at the end, node 2 at the beginning of a tour then
            put tour with node 2 after tour with node 1
        else
            flip the tour with node 2
            and put it after the tour that includes node 1
        end if
    end for
    update nrTours
end while
add the depot at the beginning and end of the tour and return optimal tour
```

5.3 The Hypervolume Indicator

In order to compare the performance of the exact solution approach for determining the transport costs with the performance of the savings algorithm, the hypervolume indicator, introduced by Zitzler and Thiele (1998), is used. For multi-objective optimization problems, the hypervolume is an often used indicator to measure the performance of pareto frontiers. The size of the space (or area, in the case of bi-objective optimization) between all points of the pareto frontier and a reference point is determined. Figure 3 illustrates how the hypervolume indicator is used to compare the performance of two pareto frontiers for bi-objective optimization problems where $H_1 = A + C$ and $H_2 = B + C$.

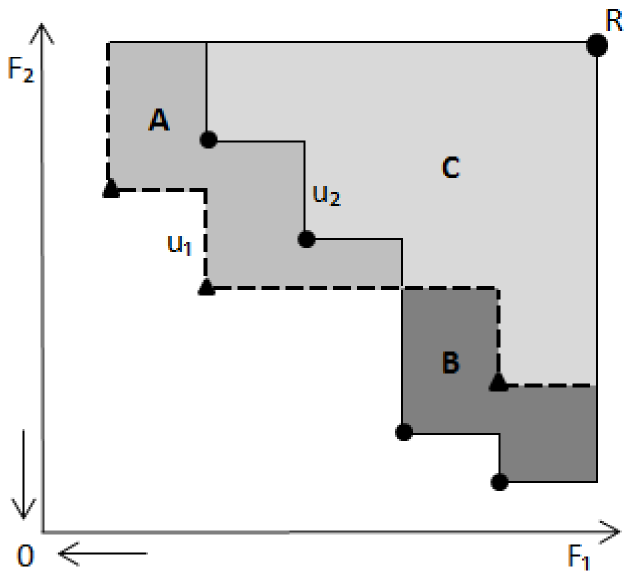

Figure 3: Hypervolume indicator for bi-objetive optimization problems.

The hypervolume of the solution including the exact calculation of the transport costs will be greater (or equal) the hypervolume of the solution using the approximation. The reference point has to be pre-defined and dominated by all pareto-optimal solutions. In order to determine the cost component of the reference point, the sum of the DC set-up costs for all candidate locations and the transport costs resulting from a tour that includes every candidate location, is calculated. The tour costs are determined by means of the savings algorithm. By summing up these costs, an upper bound for the total costs is generated. For the second component of the reference point, total demand of all inhabitants of a community is used, which represents an upper bound for the total uncovered demand. Consequently, a reference point is generated, that is dominated by all points of the two pareto frontiers. Then, the hypervolumes for both approaches are computed and based on the hypervolume of the exact solution approach (100%), the pareto frontier obtained by the savings algorithm is evaluated. The values of total costs and total uncovered demand are therefore normalized by dividing them by the respective values of the reference point. By doing so, it is ensured that both objectives have an equal impact on the calculation of the hypervolume.

6 Computational Experiments

The data analyzed by Gutjahr and Dzubur (2016) in their covering model refer to Thiès, a region in Western Senegal that consists of 32 rural communities, counting approximately 600.000 inhabitants. As this works aims to show how the optimal solution changes when considering the routing decisions to supply the DCs, the same test instances are used. There are between 9 and 31 villages that belong to every community and the population in every village varies between 10 and 19.000 inhabitants. The problem is implemented in MATLAB 8.2.

6.1 Facts about Thiès

The region Thiès is located on the west coast of Senegal, about 70km from the capital Dakar. Thiès is divided into three departments, namely Mbour, Thiès and Tivaouane. The region is further structured in 14 cummunities and 32 rural communities (Au-Senegal.com, Le cœur du Sénégal (2013)). The total area corresponds to 6.670km^2.

According to the National Agency for Statistics and Demography (2015), slightly over 50% of the population are under 19 years old. The mortality rate lies below the national average of 7,7‰ and corresponds to 6,6‰. The human sex ratio is nearly 1:1 (50,1% male and 49.9% female). After Dakar, Thiès is the second highest economically developed region in Senegal thanks to the sectors of agriculture, fishing, tourism, industry, mining, crafts and commerce. The region is relatively flat with a few exceptions such as the Thiès plateau (105 m above sea level) and the Diass massif (90 m above sea level). Due to its location close to the sea, Thiès' climate is subject to ocean currents and also affected by the trade winds occurring between November and March.

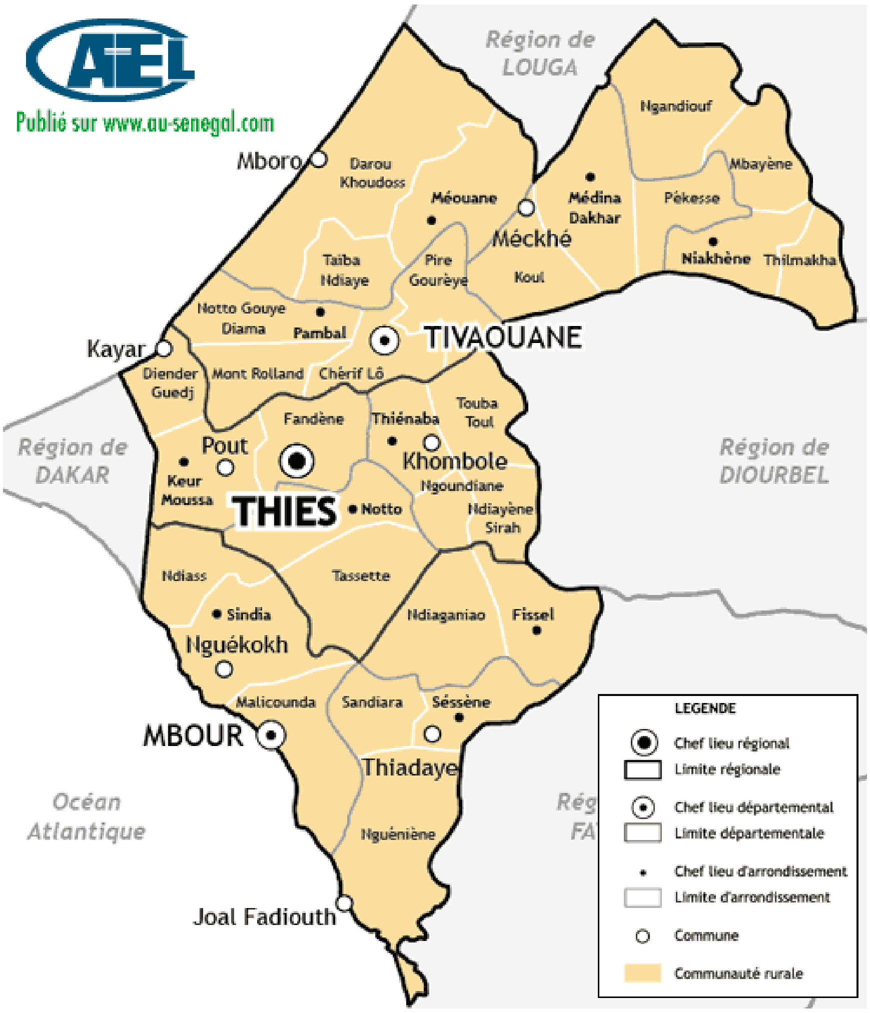

Figure 4: Map of the Region Thiès, taken from Au-Senegal.com, Le cœur du Sénégal (2013).

Senegal is vulnerable to several natural disasters such as floods, increasing sea levels, landslips, drought and fire (United Nations Office for Disaster Risk Reduction (2016)). According to the Global Facility for Disaster Reduction and Recovery (2014), in 2009, heavy rainfalls lead to flooding in the Dakar region with 125.000 people affected in rural communities including Thiès. Another heavy flood occurred in 2009, leading to landslides, uninhabitable houses and partial outage of water supply. In total, 264.000 people were affected with Thiès, again, being among the regions suffering the most.

6.2 Parameter Setting

In this section the parameters chosen for the basic model are introduced. A summary is given in table 11.

Travel costs. Every village is a candidate location for a DC. The distance matrix is symmetrical, indicating that passengers and vehicles traverse the same paths when going from PC i to PC j and going from PC j to PC i. In other words, all paths can be used in either direction. One length unit corresponds to 10m. The traveling costs rise in proportion to distance, whereas one cost unit refers to going two length units, one unit forward and one unit backward respectively. Hence, for visiting a DC e.g. 1km away and therefore passing 200 length units (to get there and to go back), travel costs of 100 units arise. The non-decreasing travel cost function is denoted by $\alpha(a)$.

Costs of uncovered demand. By d_i, the total demand of PC i is denoted. Consequently, every inhabitant of PC i requires exactly one unit of relief good per time period. Therefore, b, indicating the units required by a beneficiary, equals 1. The number of people going from PC i to DC j is denoted by y_{ij}. Since b is set to 1, the value chosen for the parameter p, which denotes the costs for uncovered demand, is the maximum value people would be willing to pay in order to get full supply. As stated above, the costs of the expected uncovered demand for every person are expressed by $p(b - \gamma_j/y_j)_+$. Setting $p = 1000$ and assuming a person receives 50% of their demand, the whole term would equal 500. In case 100% of the demand can be fulfilled, the whole term would equal 0, indicating that costs only arise for partly covered demand. In order to express the relationship between distance and demand, the costs for uncovered demand are expressed in distance e.g. walking a greater distance is the price for receiving more aid. As the costs are expressed in travel cost units, setting p to e.g. 1000 indicates that beneficiaries would walk to a DC located 10km away in order to get the full amount of relief goods, to get half the amount they would be willing to walk to a DC 5km away, and so on. Several factors have an impact on the value of p such as the character of the landscape (e.g. steep/narrow roads that are difficult to traverse), physical fitness, availability of vehicles, urgency of the aid and so on. In accordance with Gutjahr and Dzubur (2016) who tested values between 500 and 5000, $p = 3000$ was chosen in this work. Hence, it is assumed that beneficiaries would walk 30km in order get full supply, 15km to receive half of the desired supply, and so on.

Set-up costs. The capacity and therefore the set-up and operating costs of DCs depend on various factors such as location, availability of buildings or rubb halls and people such as social workers and security guards who establish, run and protect the DC. The fixed costs for establishing a DC are assumed to be 20 cost units per unit of capacity. The variable costs are chosen to be 0,5 cost unit per capacity unit. Considering the planning horizon of 30 days, total variable costs equal 15 cost units. Consequently, total costs for setting-up and running a DC equal 35 cost units per unit of capacity: $\lambda = 35$. Small random deviations are allowed, therefore, total costs are multiplied by 0,5+rand, where *rand* gives a uniformly distributed random number in the interval [0,1]. The capacity is assumed to be 3 times the demand of the PC where the DC is opened, however random deviations are allowed to make the assumption more realistic: $\gamma_j = 3d_j 2rand$.

Regarding the relief phase, general assumptions are made, as the time horizon in which DCs are operation depends on several factors such as severity of the disaster, number of people affected, proximity to unaffected cities and therefore faster access to medical care, just to name a few. In this work, it is expected that the relief phase lasts over a period of 1 month, that is 30 days. Further, it is assumed that DCs operate every day around-the-clock.

Transport costs. Depending on the nature of the natural disaster, the landscape, the condition of roads, the availability of trucks and drivers, etc., decisions about optimal vehicles vary greatly. Generally, after the occurrence of a natural disaster, the probability of damaged or blocked roads is high which is why it is assumed that the smaller the vehicle, the easier to transport goods. On the other hand, trucks need to be sufficiently large to transport a great amount of relief goods which depends, again, on the nature of the relief goods (e.g. drinking water versus medicine). As this work does not focus on a specific kind of disaster, the following generalized assumptions are made: Relief goods are brought to the DCs using trucks with a maximum weight of 12 tons which approximately corresponds to a total length of 9,7 m and total height of 3,8m. The net load that can be carried by such a vehicle is approximately 5 tons. As transport costs per length unit are relatively difficult to estimate, the following general assumptions are made: Transport costs are assumed to be 5% of the set-up and variable costs of a DC and include costs for fuel and depreciation of the vehicle, costs for maintenance, and labor costs. The average demand of all PCs of the whole region, which equals 1.200 units per PC, is taken as a basis for estimating the average set-up and variable costs of a DC. Total costs therefore equal 126.000 cost units and consist of set-up costs to the amount of 72.000 cost units and variable costs amounting to 54.000 units, without consideration of random deviations respectively. Consequently, transport costs equal 5.250 cost units. Further, purchase costs of 150.000 cost units for a vehicle are considered. In order to obtain the transport costs per length unit, it is assumed that a vehicle drives 120km (12.000 length units) a day. Considering the time horizon of 30 days, total costs per length unit therefore equal 0,44 cost units.

Table 1: Parameters used in the basic model.

Variable	Value	Description
α	$0,5$	travel costs for passing one length unit
p	3000	loss caused by one unit of uncovered demand (in travel cost units)
b	1	required amount of relief supply per person
λ_j	$35(0,5 + rand)$	set-up and operating costs of DC_j per unit of capacity
γ_j	$3d_j 2rand$	capacity of DC_j, depending on the demand d_j in PC_j
f	$0,44$	transport costs per length unit

6.3 Test Instances

The model is tested for 14 rural communities, with a number of PCs ranging from 8 to 20. Due to high computational effort, the exact solution method for determining the transport costs is only applied to instances with up to 14 nodes. For communities with a number of villages ranging from 15 to 20, transport costs are solely calculated using the heuristic method. Firstly, it is analyzed how the solution changes by including the routing aspect compared to the original model without consideration of transport costs. Further, the focus lies on comparing the performance of the two approaches used for determining the transport costs, namely the exact solution algorithm and the savings algorithm. Finally, the model is implemented in a way, that a combination of both solution approaches is used in order to keep the computational effort to a minimum.

Inclusion of Transport Costs

Considering the routing decisions results in an upward shift of the pareto frontier, hence given a fixed budget, the number of beneficiaries provided with relief goods is equal or smaller than in the work of Gutjahr and Dzubur (2016). The rightwards shift increases as more distance has to be passed in order to deliver goods to the DCs. This can either result from a larger number of open DCs and/or the establishment of DCs located far from the depot. In case both models provide the same solution, only one DC is opened in the same location as the depot and therefore, no transport costs arise.

Three instances are analyzed to test the effect of transport costs on the solution of the UE model, namely Thienaba (9 PCs), Mbayene (11 PCs), and Sandira (14 PCs). For the first two communities, the optimal routes are determined by means of the exact solution approach, whereas the savings algorithm is applied to calculate the routes for the latter.

Instance Thienaba. Region Thienaba counts 4.592 inhabitants in total, spread over 9 PCs. 70 people live in the smallest village and 2.139 in the largest one. Figure 5 depicts the two pareto frontiers with and without consideration of transport costs for Thienaba. The effects of including the routing aspect in the UE model are further presented in table 2.

Assuming a budget constraint of 50.000 cost units results in an increase of uncovered demand by 119 units. In this case, 3.157 inhabitants do not receive relief supply when the transportation aspect is considered, corresponding to 68,75% of the total population of the region. Without the routing decisions, the share of uncovered demand is 66,16%. It is shown that for this budget, total costs are lower when transportation is considered, than when it is not considered (45.260 compared to 49.871 cost units). Figure 6 depicts an enlarged section of the two pareto frontiers for instance Thienaba. For 50.000 available cost units, the best solution with consideration of transport costs leads to total costs of 45.260, which are significantly lower than the best solution for the model without the transport aspect (49.871 cost units). Consequently, the impact of the routing decisions on uncovered demand depends on the chosen budget constraint. As shown in figure 6 for example, at 46.000 and 55.000 cost units, uncovered demand does not change.

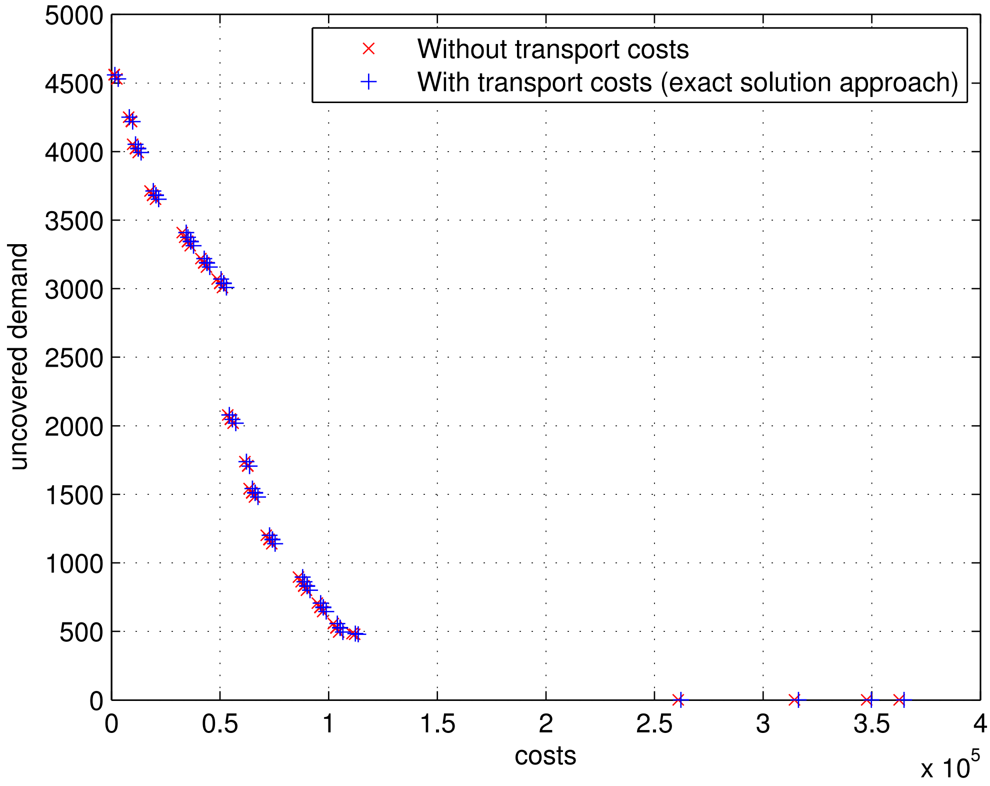

Figure 5: Instance Thienaba. Pareto frontiers of the UE model without consideration of transport costs (red crosses) and with transport costs using an exact solution approach (blue plus signs).

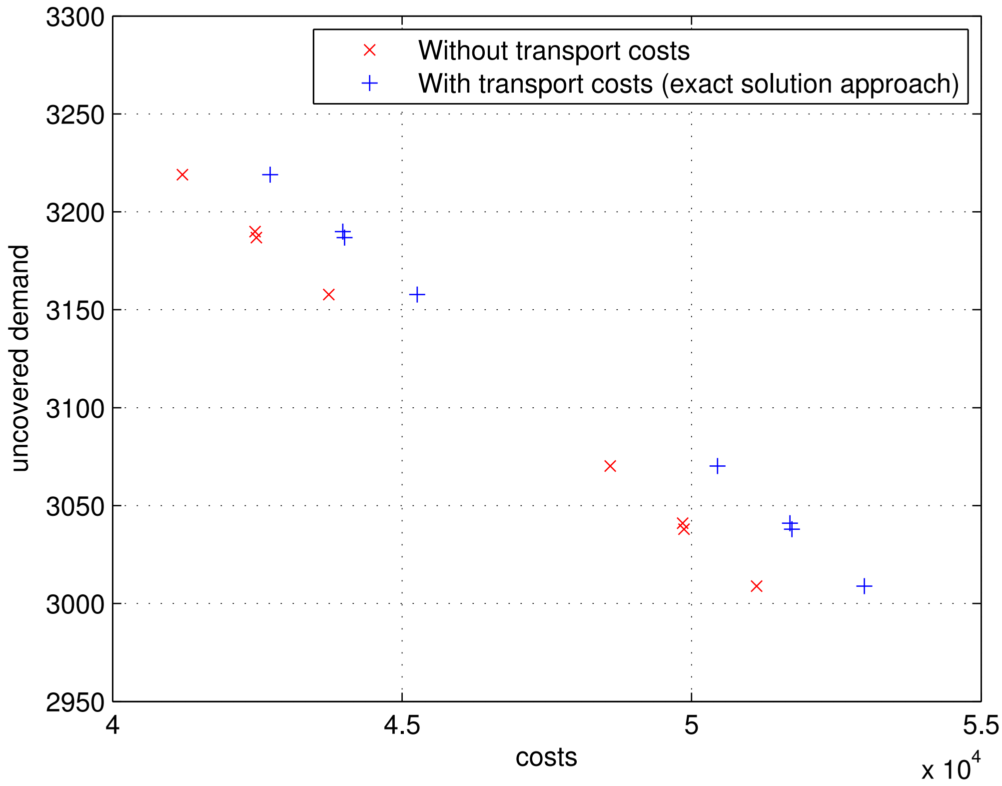

Figure 6: Instance Thienaba. Enlarged section.

Table 2: Instance Thienaba. Effects of transport costs determined by an exact solution approach on uncovered demand for different budget constraints.

	Without transport costs		Including transport costs		
Budget	Costs	Uncov. demand	Costs	Uncov. demand	Increase in uncov. demand
50.000	49.871	3.038	45.260	3.157	119
100.000	97.267	645	98.828	645	0
150.000	112.020	479	113.581	479	0
300.000	260.911	0	262.125	0	0

Instance Mbayene. Figure 7 shows the pareto fronties with and without consideration of transport costs for instance Mbayene. An overview of the analyzed data is given in table 3. Mbayene is a rural community with 11 PCs and a total population of 2.386. The smallest village in the community counts 108 inhabitants and the largest 369.

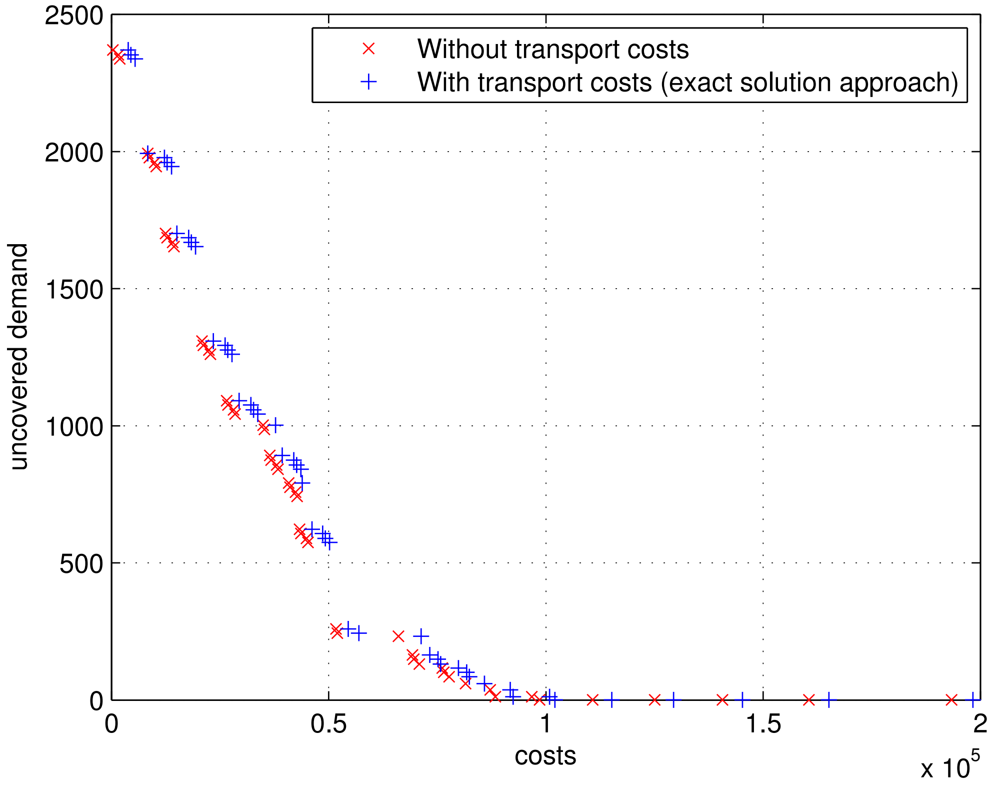

Figure 7: Instance Mbayene. Pareto frontiers of the UE model without consideration of transport costs (red crosses) and with transport costs using an exact solution approach (blue plus signs).

In this example, both models provide the same solution at total costs of 8.365 resulting in 1.993 units of uncovered demand. Hence, in this case only one DC is established in the same location as the depot. Assuming a budget limit of 20.000 costs units leads to total costs of 14.401 units and 1.653 units of uncovered demand if the routing aspect is ignored. Therefore, 69% percent of the population cannot be supplied with relief commodities. By considering transport costs using the exact solution method, total costs increase to 19.381 while uncovered demand does not change. With a budget limit of 40.000 cost units, the optimal solution without consideration of transport costs includes total costs of 38.359 cost units and 841 units of uncovered demand, corresponding to 35% of the total population. When transport costs are included in the model, the optimal solution leads to 39.296 cost units and 892 units of uncovered demand (37% of the population). In case 60.000 cost units can be spent, the optimal solution without the routing decisions includes total costs of 51.996 units and 244 units of uncovered demand(10% of the population), whereas when also calculating routing costs, total costs rise to 56.976 units and uncovered demand stays unchanged. Finally, the instance is analyzed with a budget limit of 80.000 units leading to total costs of 77.743 and 85 units (4% of the population) of uncovered demand without transport costs and total costs of 79.875 and 116 units (5% of the population) of uncovered demand with the routing aspect included.

Table 3: Instance Mbayene: Effects of transport costs determined by an exact solution approach on uncovered demand for different budget constraints.

	Without transport costs		Including transport costs		
Budget	Costs	Uncov. demand	Costs	Uncov. demand	Increase in uncov. demand
20.000	14.401	1653	19.381	1.653	0
40.000	38.359	841	39.296	892	51
60.000	51.996	244	56.986	244	0
80.000	77.743	85	79.875	116	31

Instance Sandira. The third instance used to show the impact of the consideration of transport costs is Sandira, a rural community with 14 PCs and a total population of 8.745 people. The smallest PC counts 77 inhabitants, the largest 1.945. The transport costs are calculated by means of the savings algorithm and the two pareto frontiers are depicted in figure 8. An overview of the analyzed data is presented in table 4.

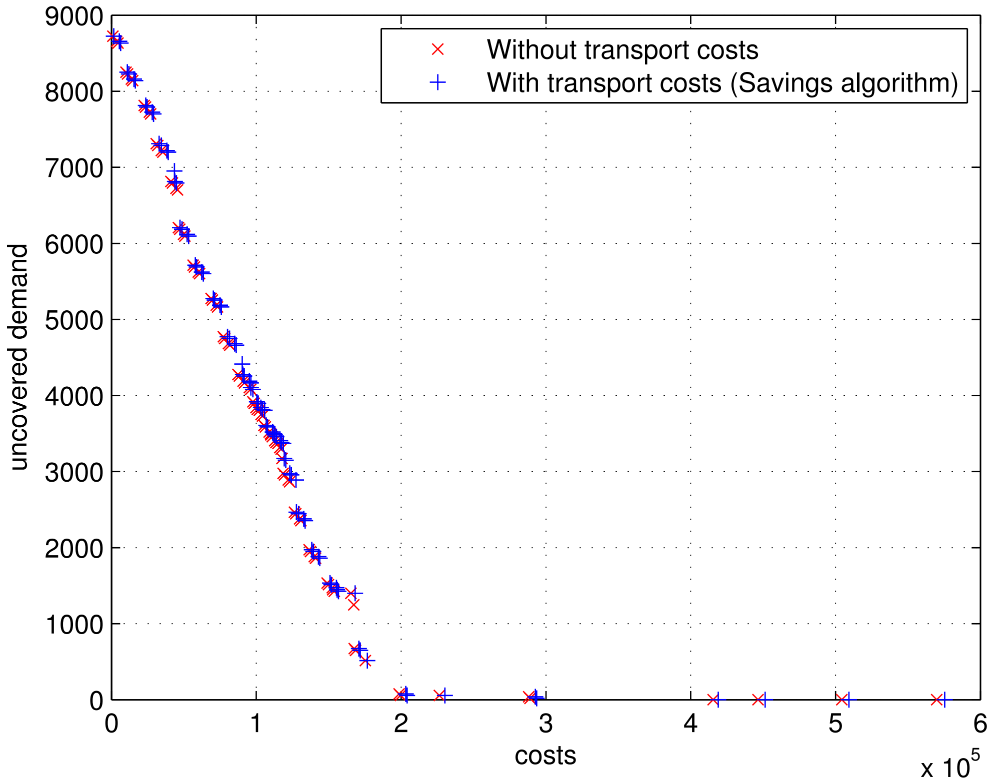

Figure 8: Instance Sandira. Pareto frontiers of the UE model without consideration of transport costs (red crosses) and with transport costs using an exact solution approach (blue plus signs).

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Assuming a budget limit of 50.000 units leads to total costs of 49.764 units and total uncovered demand of 6.118 units without the consideration of transport costs. When the routing aspect is included, the same budget limit leads to total costs of 48.769 and uncovered demand of 6.186 units. Hence, total uncovered demand increases by 86 units. For a budget of 100.000, the optimal solution without routing decisions is 99.793 units of total costs and 3.841 units of total uncovered demand, corresponding to 44% of total demand. By including the routing aspect, total costs are 97.740 and uncovered demand rises by 241 units to 4.082. In this case 47% of all inhabitants do not receive relief supply. With 150.000 units of budget, uncovered demand rises from 1.514 units (17% of total demand) without transport costs to 1.861 (21% of total demand) including transport costs. The increase in uncovered corresponds to 347 units. Finally, a budget limit of 200.000 units is considered, leading to an increase in uncovered demand due to the inclusion of transport costs by 460 units. The share of uncovered demand changes from 0,7% to 6% respectively. Analyzing instance Sandira for different budget constraints suggests that the impact of transportation costs on uncovered demand rises with increasing budget, however, as the previous instances have shown, this phenomenon cannot be generalized.

Table 4: Instance Sandira. Effects of transport costs determined by an exact solution approach on uncovered demand for different budget constraints.

	Without transport costs		Including transport costs		
Budget	Costs	Uncov. demand	Costs	Uncov. demand	Increase in uncov. demand
50.000	49.764	6.118	48.769	6.186	68
100.000	99.793	3.841	97.740	4.082	241
150.000	149.846	1.514	143.851	1.861	347
200.000	199.559	57	176.726	517	460

Comparison of Different Transport Costs

The previously discussed results show that the transport costs do not significantly impact the solution of the UE model, as they are assumed to be relatively low compared to the set-up costs of the DCs. This assumption, however, must not always be true in disaster relief situations as transport costs can rise fast due to factors such as limited availability of vehicles or fuel. In this section, it is analyzed how the solution of the UE model changes, in case transport costs increase by factor 2 and 3.

Table 5 gives an overview of how uncovered demand changes when considering higher transport costs for the same budget limits analyzed in the previous section. For the first constraint of 50.000 cost units, the impact of higher transport costs on uncovered demand is insignificant (3.157 units for $f = 0,44$ and 3.158 units for $f = 0,88$ and $f = 1,32$). In case 100.000 cost units are available for the relief operation, it has been shown that uncovered demand is equal with and without consideration of transport costs for $f = 0,44$. However, when assuming higher transport costs, uncovered demand rises by 29 units for $f = 0,88$ and by 62 units for $f = 1,32$. For 150.000 units of budget, the change is zero.

Table 5: Instance Thienaba. Uncovered demand for different transport cost factors and budget constraints in units.

		With transport costs					
Budget	Without TC	f=0,44	Increase	f=0,88	Increase	f=1,32	Increase
50.000	3.038	3.157	119	3.158	120	3.158	120
100.000	645	645	0	674	29	707	62
150.000	479	479	0	479	0	479	0

For instance Mbayene, uncovered demand increases for every budget constraint analyzed when applying higher transport cost factors. An overview of the data is presented in table 6. Considering, for example, 40.000 cost units available, uncovered demand rises by 202 and 250 units when considering $f = 0,88$ and $f = 0,32$, compared to $f = 0,44$. The share of total uncovered demand therefore changes from 37% of the population to 44% and 46% respectively. When looking at the data for a budget limit of 60.000 cost units, uncovered demand rises by 15 and 346 units when increasing the transport costs by factor 2 and 3. Consequently, 11% and 25% of all inhabitants do not receive relief supply compared to 10% for $f = 0.44$.

Table 6: Instance Mbayene. Uncovered demand for different transport cost factors and budget constraints in units.

		With transport costs					
Budget	Without TC	f=0,44	Increase	f=0,88	Increase	f=1,32	Increase
20.000	1.653	1.653	0	1.702	49	1.961	308
40.000	841	892	51	1.043	202	1.091	250
60.000	244	244	0	259	15	590	346
80.000	85	116	31	164	79	244	159

Table 7: Instance Sandira. Uncovered demand for different transport cost factors and budget constraints in units.

		With transport costs					
Budget	Without TC	f=0,44	Increase	f=0,88	Increase	f=1,32	Increase
50.000	6.118	6.186	68	6.207	89	6.207	89
100.000	3.841	4.082	241	4.082	241	4.104	263
150.000	1.514	1.861	347	1.861	347	1.861	347
200.000	57	517	460	517	460	517	460

Finally, for instance Sandira the increase in transport costs does not have a significant impact on uncovered demand. As presented in table 7, the same or only slightly less people can be supplied with relief commodities when increasing the transport costs by factor 2 and 3. This can be explained by significantly more inhabitants of the community Sandira (8.745) compared to Thienaba (4.592) and

Mbayene (2.386), as the transport costs are still relatively low compared to the set-up and operating costs of the DC which depend on the number of people living in the respective PC.

Comparison of the Exact and Heuristic Solution Approches

The focus on the following computational experiments lies on how the solution method used to determine the transport costs affects the outcome and computation time of the UE model.
Two test instances are analyzed, namely Malicouanda Wolof and Thiadiaye.
Instance Malicouanda Wolof. The first instance to test the model with the exact versus the heuristic approach to determine the optimal route for vehicles refers to the rural community Malicouanda Wolof which consists of 11 PCs that have a population ranging from 151 to 2.532 people. The total number of inhabitants in the rural community is 8.412 people. The depot is established in node 1 and a DC is not necessarily opened in this PC. When applying the hypervolume indicator, both approaches lead to a hypervolume of 0,89125, showing that the heuristic approach for determining the transport costs performs as good as the exact solution approach, but computation time is significantly faster (27,67 seconds compared to 2,72 minutes). The two pareto frontiers are depicted in figure 9. An overview of the analyzed data is presented in table 8.
Malicouanda Wolof counts a significantly higher number of inhabitants compared to Mbayene, which also consists of 11 PCs, and therefore when applying the same budget as before of 40.000 cost units, total uncovered demand reaches 7.132 units and the total cost equal 32.622 units. 85 % of total demand cannot be covered. 4.560 people stay at home, the rest visits the DCs established in nodes 1,2,7 and 9 where 15%, 11%, 36% and 50% of their demand are covered.
The optimal tour for the vehicle supplying the DCs with relief goods obtained by

the exact solution algorithm starts at the depot in PC 1 (where the first DC is provided with relief items), and then continues to visit the open DCs in PCs 2,9 and 7, before returning to the depot.
The savings algorithm provides the same tour but in reversed order. Since the distance matrix is symmetrical, exactly the same distance is being passed. Total distance covered is 5.943 length units and the total cost incurred by transportation of relief goods equals 2.614 cost units.

Table 8: Instance Malicouanda Wolof. Comparison of the exact and heuristic solution approach to determine the optimal route.

	Exact solution algorithm		Savings algorithm	
Budget	Optimal tour	Total distance	Optimal tour	Total distance
40.000	1-2-9-7-1	5.943	1-7-9-2-1	5.943
200.000	1-4-7-9-8-1	8.489	1-9-7-4-8-1	8.489
1.045.000	1-8-9-4-11-3-6-5-10-1	13.120	1-10-9-4-11-3-5-6-8-1	13.121

When introducing a budget limit of 200.000 cost units, total uncovered demand is reduced to 1.099 units which corresponds to 13% of the need of the population. In this case, DCs are opened in PCs 4,7,8 and 9. The optimal route obtained by the exact solution methods is going from the depot to nodes 4, 7, 9, 8, and returning to the depot. The best route found by the savings algorithm is slightly different (1-9-7-4-8-1) but total distance traveled is the same.
In order to guarantee full supply for all beneficiaries (uncovered demand = 0,16 units or 0,0019%), total costs reach more than 1 million units (1.045.000 cost units) and DCs are opened in all PCs except for PC 2 and PC 7. The optimal tour using the exact solution approach is going from node 1 to nodes 8, 9, 4, 11, 3, 6, 5, 10, 1, passing 13.120 distance units in total and hence causing transport

costs of 7.773 cost units. The savings algorithm provides similar results with the best tour going from node 1 to 10, 9, 4, 11, 3, 5, 6, 8, 1 and passing 13.121 distance units. All inhabitants from node 2 visit the DC in node 10 and the people living in PC 7 travel to the DC established in node 9. The high costs of this solution and more than 20.000 relief units being left unused makes this solution not desirable.

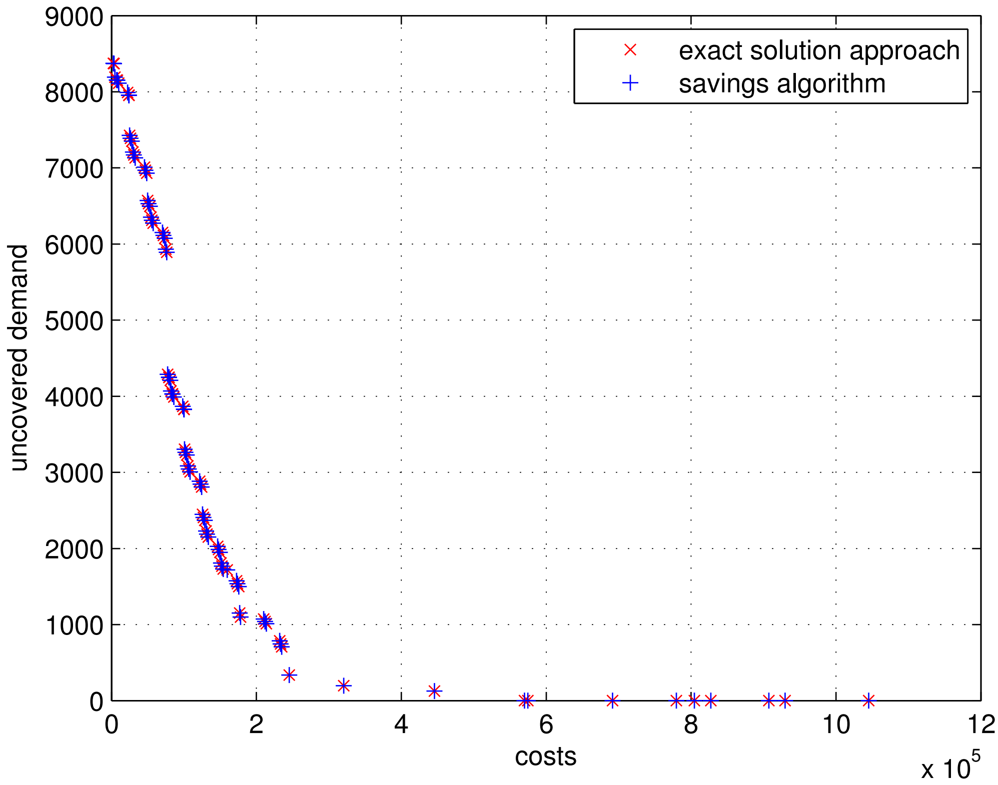

Figure 9: Instance Malicouanda Wolof. Pareto frontiers of the UE model including transport costs using an exact solution approach (red crosses) and using the savings algorithm (blue plus signs).

Instance Thiadiaye. The community Thiadiaye consists of 12 PCs and a total number of inhabitants of 9.947 people. The two pareto frontiers are depicted in figure 10 and table 9 presents an overview of the analyzed data.

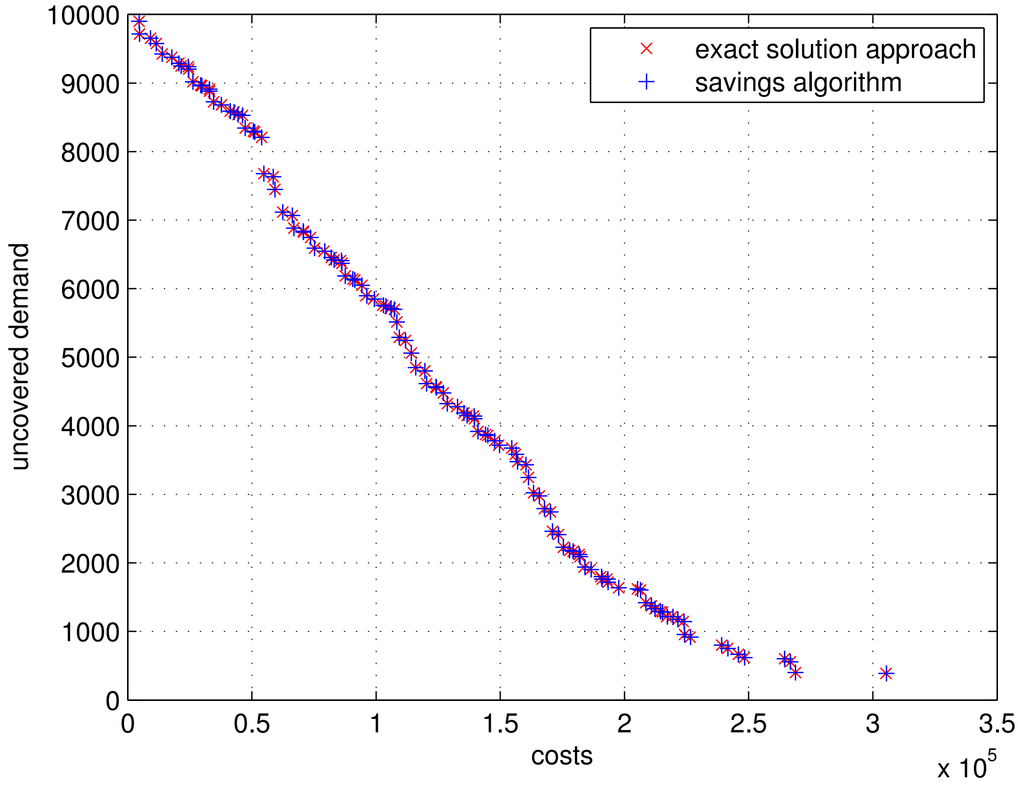

Figure 10: Instance Thiadiaye. Pareto frontiers of the UE model including transport costs using an exact solution approach (red crosses) and using the savings algorithm (blue plus signs).

When assuming a budget limit of 100.000 cost units, the best solution obtained by the exact solution method leads to total costs of 98.981 cost units (transport costs account for 3.128 cost units) whereas total costs are 99.333 cost units when using the savings algorithm (transport costs equal 3.480 cost units). The optimal route is 1-2-12-7-6-5-1, in the best heuristic solution, node 6 is being visited before node

7 and the rest of the tour does not change. This leads to an increase in transport costs of 11,25 %. The ratio between the hypervolumes of both approaches is close to 1 (0,99989), hence the savings algorithm performs as good as the the exact solution method but, again, calculates the results much faster. Figure 10 shows the pareto frontiers of the UE model using the exact versus heuristic approach to calculate transport costs. When considering a budget limit of 250.000 units, both solution approaches lead to open DCs in PC 1, 2, 3, 4, 5, 8 and 12 and the total uncovered demand equals 618 units. The transport costs obtained by the savings algorithm equal 3.831 cost units (optimal route: 1-12-2-8-3-5-4-1) and are only slightly higher than the costs calculated by means of the exact solution algorithm (total costs: 3.747 units, optimal route: 1-4-5-3-2-8-12-1).

Table 9: Instance Thiadiaye. Comparison of the exact and heuristic solution approach to determine the optimal route.

	Exact solution algorithm		Savings algorithm	
Budget	Optimal tour	Total distance	Optimal tour	Total distance
100.000	1-2-12-7-6-5-1	7.109	1-2-12-6-7-5-1	7.909
250.000	1-4-5-3-2-8-12-1	8.516	1-12-2-8-3-5-4-1	10.289

Combination of the Exact and Heuritsic Solution Algorithms

As in disaster relief situations, decisions concerning how to supply beneficiaries with relief goods have to be made fast, it is assumed that a computation time of up to 1 day (24 hours) is acceptable. Therefore, only for rural communities with up 13 nodes the exact solution algorithm can be applied to determine the optimal tour to supply the DCs. For instances that count 14 PCs, computation time is already close to 2,5 days when using the exact solution aspect for the routing decisions. Ideally, the exact and heuristic solution algorithms are combined in a way to minimize overall computation time. For up to 7 nodes, calculating all permutations to determine the optimal tour is faster than using the savings algorithm, whereas for 8 and more open DCs, the heuristic solution approach needs less calculation time. The computation time saved when considering the exact solution approach for up to 7 nodes lies within the range of seconds and therefore, the overall time to calculate the results cannot be improved. However, the solutions obtained by using the combination of both algorithms are more likely to be closer to optimality.

Further Results

Table 10: CPU times and Hypervolumes.

		Exact solution algorithm		Savings algorithm		
Region	# nodes	CPU time	HV	CPU time	HV	HV Ratio
Ndiakhene	8	2,52sec	0,75530	2,18sec	0,75527	0,99997
Thienaba	9	6,22 sec	0,81129	5,10sec	0,81129	1,00000
Ndieyene Shirak	10	23,13sec	0,84566	12,32sec	0,84566	1,00000
Malicouanda Wolof	11	2,40min	0,89125	27,70sec	0,89125	1,00000
Thiadiaye	12	22,06min	0,72185	1,01min	0,72177	0,99989
Tiba Ndiaye	13	4,30h	0,86639	2,32min	0,86638	1,00000
Sandira	14	57,74h	0,85435	4,92min	0,85435	1,00000
Koul	15			12,50min		
Neugeniene	16			26,11min		
Ndiass	17			56,04min		
Nguekhokh	18			2.06h		
Touba Toul	19			4,52h		
Fissel	20			9,37h		

Table 10 provides an overview of the results and computation times of the UE model using the exact and heuristic methods to determine the transport costs. The performance of the savings algorithm compared to the exact solution approach is measured by the hypervolume ratio. The hypervolume of the solution using the heuristic method is equal or only insignificantly larger that the one of the solution with the exact approach, whereas computation times are much faster. Therefore, it is shown that the savings algorithm performs relatively well and is an accurate approach for determining the optimal routes for the problem treated in this work. In humanitarian aid logistics, where decisions commonly need to be made the day after the occurrence of a disaster, the optimal routes using the exact solution approach can only be calculated for communities with up to 13 nodes (4,30h for instance Tiba Ndiaye). For instances with 14 or more nodes, the savings algorithm is preferable due to the much faster calculation times. For instance Sandira with 14 PCs, the problem can be solved about 705 times faster using the savings algorithm, providing a solution almost equally good.

The time complexity for solving the model including the savings algorithm can be expressed by $O((2^n - 1)f(n))$, where $2^n - 1$ refers to the computation time of the complete enumerations and $f(n)$ to the effort needed for the savings algorithm to solve the TSP, and the Frank-Wolfe algorithm to determine the UE.

As listed in table 10, the computation time for solving the model including transport costs attained by the savings algorithm takes insignificantly longer than solving the UE model without the transport aspect.

7 Discussion

In this work, it is assumed that a single uncapacitated vehicle is used to provide all open DCs with relief supply. Depending on the nature of the disaster, this assumption can be realistic when the needed commodities are of small volume and weight such as medicine. In other situations, however, larger relief goods like water, tents and sleeping bags need to be provided. In this case, considering a vehicle routing problem (VRP) with a number of capacitated trucks is more realistic than the TSP. Solving a VRP by means of an exact solution algorithm is generally much more complex than solving the TSP, which is why a heuristic solution method should be preferred. The savings algorithm can easily be applied to solve the problem for several vehicles with capacity limits. Consequently, it has to be determined how the opened DCs are allocated to different routes, and the succession in which the DCs should be visited. The major difference compared to the problem treated in this work is that vehicles have only limited capacities, hence the total demand provided by a vehicle cannot exceed its capacity. The savings algorithm can be applied to solve the problem heuristically while nodes can only be added to a route as long as the total demand is smaller or equal the capacity of the vehicle.

The instances analyzed in this work are relatively small and therefore, determining all possible combinations of open DCs does not require too much computational effort. For larger instances, however, a heuristic approach, like for example NSGA-II, can be solved to approximate the pareto frontier.

8 Conclusion

A relevant aspect when deciding upon the number and locations of distribution centers to be established in disaster relief situations, is the consideration of transport costs to supply these DCs with relief goods. The bi-objective bilevel optimization problem for facility location decisions in disaster areas as proposed by Gutjahr and Dzubur (2016) is extended by the routing aspect and therefore made more realistic. For every possible combination of DCs to be opened, not only the costs resulting from uncovered demand, but also the transport costs to deliver relief supply from a depot to the DCs are calculated. A TSP is solved by means of an exact and heuristic algorithm and the performance of the two approaches are compared. The consideration of routing decisions results in a rightward shift of the pareto frontier, hence assuming the same budget limit, total uncovered demand is higher. The computation of the optimal tour of a vehicle by means of an exact solution approach is relatively time consuming and can only be done for small instances. The savings algorithm calculates the tours significantly faster and the results are close to optimality.

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Appendix

Table 11: Notation used in the basic model.

I	set of n PCs, indexed by i
J	set of m DCs, indexed by j
J_0	set of DCs extended by the dummy DC (index 0)
d_i	demand of PC i ($i \in I$)
b	required amount of relief supply per person
γ_j	capacity of DC j ($j \in J$)
λ_j	set-up and operating costs of DC j ($j \in J$)
a_{ij}	distance between DC i and PC j ($i \in I,\ j \in J$)
p	loss caused by one unit of uncovered demand (in travel cost units)
$\alpha(a)$	costs of passing distance a
$f(a)$	transport costs of the vehicle traveling distance a
x_j	binary variable indicating whether to open a DC in location j or not ($j \in J$)
y_{ij}	flow variable, denoting the people going from PC i to DC j ($i \in I,\ j \in J$)
y_j	demand occurring at DC $j (j \in J)$
F_1	objective function for minimizing DC set-up and transportation costs
F_2	objective function for minimizing total uncovered demand
g	objective function for calculating the user equilibrium

Abstract

This work is an extension of Gutjahr and Dzubur's (2016) bi-objective bilevel facility location model in disaster response situations and focuses on the touring aspect to supply distribution centers (DCs) with relief goods. For every combination of open DCs, the resulting uncovered demand and total costs consisting of set-up and transportation costs are determined. An uncapacitated vehicle starts at the depot and visits all open DCs before returning to the depot. To solve the traveling salesman problem (TSP), an exact and heuristic solution approach are considered. For small instances, the optimal solution is found by calculating all permutations and choosing the route with the smallest total distance. The second approach proposed is the savings algorithm, which calculates results significantly faster and provides solutions close to optimality.

Zusammenfassung

Diese Arbeit ist eine Erweiterung von Gutjahr und Dzuburs (2016) bikriteriellem, bilevel Standortproblem im Bereich des Katastropheneinsatzes mit Schwerpunkt auf der Transportoptimierung, um Distributionszentren (DCs) mit Hilfsgütern zu beliefern. Für jede Kombination an offenen DCs werden der Anteil an nicht abgedecktem Bedarf und die Gesamtkosten, bestehend aus Eröffnungskosten der DCs und Transportkosten, bestimmt. Es wird angenommen, dass ein Fahrzeug mit unbeschränkter Kapazität bei einem Depot startet, alle offenen DCs beliefert und dann wieder zum Depot zurückkehrt. Um das Traveling Salesman Problem (TSP) zu lösen, werden ein exaktes und ein heuristisches Verfahren in Betracht gezogen. Für kleine Instanzen erhält man die optimale Lösung mittels Bestimmung aller Permutationen. Der zweite Lösungsansatz ist der Savings Algorithmus, welcher Ergebnisse wesentlich schneller und nahe der optimalen Lösung liefert.