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Abstract

The Efficient Market Hypothesis asserts that the stock market prices should reflect all relevant information. News should be incorporated into the price instantaneously. How much time does "instantaneously" imply? While it might imply 0.0 seconds in an ideal world, informed traders need more time in the real world to acquire, process and react to new information, bringing the market to efficiency. In this paper, we focus on measuring how much time the informed traders really need in order to incorporate new information and correct mis-pricing. Thus, we quantify the market efficiency in the modern financial markets. We apply high-frequency price returns of large-cap stocks to two models in order to measure the market speed: the model of information feedback and the model of penalized realized variance. The model of information feedback is an extension of the classical martingale-plus-noise model which incorporates the learning process of informed traders in reacting to new information and correcting mis-pricing. We use the method of moments and the maximum likelihood method and the Kalman filter to estimate the model parameter α which measures the efficiency of informed traders. The model of penalized realized variance studies the autocovariance structure in high-frequency price returns. By determining the maximum lag where positive autocorrelations persist, one can determine the time the market needs to achieve weak-form efficiency. We show that the speed measurements according to these two models are consistent and that high efficiency prevails in large-cap stocks. The speed of each stock varies constantly, and the average speed of each stock has increased over years. The average speed of these large-cap stocks in 2017 varies from 5.5 seconds to 8.5 seconds. One can use these speed measurements to determine the relative efficiency of the stocks. Practitioners may also use these measures to find relatively inefficient stocks and impound their information into the stock prices.

Abstract in German (Zusammenfassung)

Die Markteffizienzhypothese besagt, dass die Aktienpreise alle relevanten Informationen reflektieren sollten. Neue Informationen sollten unverzüglich in die Aktien eingepreist werden. Wie viel Zeit impliziert das Wort "unverzüglich"? Obwohl es 0.0 Sekunden in einer idealisierten Welt impliziert, brauchen die informierten Aktienhändler in der realen Welt mehr Zeit für die Suche nach neuen Informationen und für die Analyse der potentiellen Auswirkungen, um den Markt kollektiv zur Effizienz zu bringen. Diese Arbeit fokussiert sich auf Messungen der Zeitdauer, die die informierten Aktienhändler brauchen, um ihre Informationen in die Aktien einzupreisen und um etwaige Preisverzerrungen zu korrigieren. Wir quantifizieren also die Markteffizienz in dem modernen Finanzmarkt. Zwei Modelle werden zur Messung der Geschwindigkeit des Marktes verwendet: das Modell des Informationsfeedbacks und das Modell der pönalisierten realisierten Varianz. Das Modell des Informationsfeedbacks ist eine Erweiterung des klassischen Martingal-plus-Rauschen Modells und inkludiert den Lernprozess der informierten Händler über neue Informationen und potentielle Preisverzerrungen. Wir verwenden die Momentenmethode und die Maximum-Likelihood-Methode und den Kalman-Filter, um den Parameter α in dem Modell zu schätzen, der die Effizienz der informierten Aktienhändler misst. Das Modell der pönalisierten realisierten Varianz untersucht die Autokovarianzstruktur in den hochfrequenten Preisveränderungen. Der maximale Lag, wo signifikant positive Autokorrelationen zu beobachten sind, deutet auf die Zeitdauer hin, die der Markt braucht um effizient zu sein. Wir zeigen, dass die Messungen der Geschwindigkeit des Marktes nach den zwei Methoden konsistent sind. Hohe Effizienz herrscht in den großkapitalisierten Aktien. Die Geschwindigkeit jeder Aktie variiert mit der Zeit und die durchschnittliche Geschwindigkeit jeder Aktie hat sich über die Jahre erhöht. Die durchschnittliche Geschwindigkeit der großkapitalisierten Aktien in 2017 variiert zwischen 5.5 Sekunden und 8.5 Sekunden. Man kann anhand der Messungen der Geschwindigkeiten die relative Effizienz der Aktien bestimmen. Die Praktiker können dadurch ineffiziente Aktien finden und ihre Informationen in diese Aktien einpreisen.

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Contents

1 Introduction

Ever since the Efficiency Market Hypothesis (EMH) was proposed in the seminal paper [Fama, 1970], many researches have been conducted to determine how efficient the market really is. According to [Fama, 1970], the stock prices should reflect all relevant information. A precondition for this hypothesis is that information costs and transaction costs, the costs of searching for information and the costs of impounding it into the stock prices, should be zero. Otherwise traders would be less incentivized to collect new information and incorporate it into the price. However, in reality, the information and transaction costs are not zero. Therefore, this strong version of the EMH does not hold. In his sequel [Fama, 1991], Fama mentions a weaker and economically more sensible version of the EMH, which asserts that the stock prices reflect the relevant information to the extent that the marginal benefits of acting on information does not exceed the marginal costs of collecting information and trading. [Fama, 1991] also discusses past interesting work related to the EMH and tests the three forms of market efficiency: weak-form efficiency, semi-strong-form efficiency and strong-form efficiency. Weak-form efficiency asserts that it is not possible to gain excess returns by trading upon past information. Semi-strong-form efficiency maintains that one cannot gain excess returns by trading upon public information. Strong-form efficiency asserts that it is not possible to gain excess returns by trading upon insider information. [Fama, 1991] states that the tests in semi-strong-form efficiency, so-called event studies, show the strongest evidence for the market efficiency.

In the meantime, there is a growing literature about irrationalities and psychological quirks observed in the trading behavior of individual traders. In the paper [Black, 1986], individual traders who do not act according to fundamental information are described as noise traders. Black asserts that noise traders trade because they think they are trading on information rather than on noise or because they just enjoy trading. In the field of Behavioral Finance, noise traders are described as irrational traders. Noise traders take long positions in a stock because they believe that the stock price will increase without having conducted a thorough analysis. They may have obtained some information from other investors, or have done some simple analysis. Noise traders are often overconfident in their trading abilities. It has been shown in [Barber and Odean, 2001] that the more often noise traders trade, the poorer the results they achieve. The Disposition Effect (DE) is another characteristic of noise trader's trading behavior. Since noise traders do not value a stock according to its fundamental value, their buying or selling decision is also not based on its intrinsic value. The DE describes a phenomenon in which a noise trader unwinds his long position as soon as the stock rises above the price at which he purchased it, so that he can realize a gain. The DE also describes the behavior of a noise trader when he holds a stock as long as the stock price is below the purchase price, so that a loss is not realized immediately. Therefore, a noise trader might sell an undervalued share and hold an overvalued share, leading to the mis-pricing of shares.

Consistent with these findings, scholars agree that some individuals trade irrationally all the time and all individuals trade irrationally some of the time. Somehow, despite all these observed behavioral quirks and irrationalities, the market becomes efficient, at least efficient enough that active traders have a hard time beating passive investment strategies. But how exactly does it happen and how long does it take?

The EMH does not describe the concrete market mechanisms which produce the hypothesized state of efficiency. Clearly, when new news arrives, the market does not become efficient automatically. The realization of market efficiency depends on individual actions, specifically the actions of informed traders. As soon as informed traders observe any mis-pricing caused by noise traders in the market, they step in and correct it. They also impound their information into the price to achieve market efficiency. It must take some time for informed traders to collect and incorporate new information into the price and recognize and correct the mis-pricing in the market. In this paper, we focus on measuring how long it takes the market to be efficient, so-called market speed. We apply two models to measure the market speed. The first model, the model of information feedback, is developed in [Andersen et al., 2016]. In the recent financial econometrics literature, stock prices are modeled as semi-martingale processes polluted by noise terms. It is also assumed that the semi-martingale price processes are known by active informed traders. However, the information set of informed traders is not perfect. Informed traders need some time to acquire, process and react to new information and recognize and correct the mis-pricing. [Andersen et al., 2016] extend the classical martingale-plus-noise setting by including the process of sequential learning, delayed acquisition of new information and the processing of new information. The inclusion of the learning process serves as a bridge from the financial econometrics literature to the related market microstructure literature. In the model of [Andersen et al., 2016], the speed of processing new information and correcting mis-pricing is described by the parameter α. α measures the proportion of mis-pricing which is corrected by the informed traders in the current period. Therefore, the closer α is to 1, the faster the speed of informed traders is. By using the method of moments and the maximum likelihood method and the Kalman filter, we estimate α and measure the market speed of individual stocks.

The second model we use is the model of penalized realized variance developed in [Sheppard, 2013]. In contrast to the model in [Andersen et al., 2016], the model in [Sheppard, 2013] presents the high-frequency asset prices as martingale-plus-noise processes polluted by endogenous noise terms. Therefore, it does not include the process of learning and delayed responses to new information. Instead, it studies the autocovariance structure in high-frequency price returns and tests their autocorrelations. Significant positive autocorrelations at large lags indicate the predictability of future returns and market inefficiency. They might also imply sluggish incorporation of new information into the price. If active informed traders are slow to impound new information into the market price, the price only converges slowly toward the intrinsic value, and positive autocorrelations in the returns could be observed. In contrast, if the information were impounded into the price immediately, no positive autocorrelation in the price returns could be observed. Therefore, by determining the maximum lag of significant positive autocorrelations, we can measure the time required for the prices to behave like semi-martingales and to incorporate all relevant information. The longer the significant autocorrelations prevail, the lower the market speed is.

We apply two models to measure the market speed in order to compare the measurements with each other and validate their accuracy. The speed measurements based on these two models should be consistent. If informed traders were not efficient in impounding their information into the market price, indicated by low α value in the model of information feedback, and the market price converged only slowly to the intrinsic value, the price returns would display significant positive autocorrelations, which would be captured by the model of penalized realized variance. Thus, the efficiency of informed traders is related to the autocovariance structure in the price returns.

If the EMH holds, we expect the speed measurements obtained by using these two models to be rapid. Informed traders conduct fundamental analysis, which includes discounted cash flow analysis and comparing P/E and EV/EBITDA ratios among stocks, over time in order to detect mispricing and search for new information and trade on it. They aim to earn an above-market return, so that their costs of searching for information will eventually be paid off. Since new information is incorporated into the price immediately according to the EMH, only the fastest informed traders can profit from trading upon the new information. In modern financial markets, high frequency traders represent those exceptionally fast traders. Many high-frequency trading firms locate their trading offices near stock market exchanges in order to get an extremely fast connection to the trading venues, so that their orders can be executed first. In addition, news providers in the financial industry recently started including tags in the feeds, which enable high frequency traders to create algorithms that can automatically identify trading opportunities by screening tags. Therefore, as the technology develops, high-frequency traders become faster and faster. This improvement enables information to be incorporated into the prices immediately, leading to enhanced efficiency in the market and a faster market speed.

Since high-frequency traders can act on public information immediately and adopt algorithms that detect trading opportunities by analyzing patterns of historical price returns and historical information, there are few profitable trading opportunities left for slow traders who also act upon past information or public information. Therefore, weak-form efficiency and semi-strong-form efficiency prevail in the modern financial markets. In addition, investors are not allowed to trade upon material nonpublic information, i.e. insider information, indicating that the strong-form EMH should also prevail in the equity markets. Additionally, over the last decades, there have been significant developments in the technology of financial markets which improved efficiency. The minimum tick size has been reduced to 0.01. [Chordia and Subrahmanyam, 1995] argue that a reduction in the tick size leads to more competition among market makers and causes orders to flow to the least-cost liquidity providers. The emergence of many electronic trading venues leads to a more fragmented market and more competition among exchanges. In order to attract investors, the transaction costs of exchanges have decreased over time, leading to smaller bid-ask spreads. Therefore, market liquidity has also improved over time. Due to the lower transaction costs, informed traders are more willing to impound their information into the prices.

Other research has been conducted to measure market speed from different perspectives. [Chordia et al., 2005] find that the daily returns in the stocks listed on the NYSE are not serially correlated while the order imbalances in the same stocks, defined as total buy orders minus total sell orders, are highly persistent from day-to-day. The authors assert that there must be some astute traders who correctly predict the order imbalance and conduct countervailing trades within the very first day, removing the autocorrelations in the price returns, which would otherwise be induced by the persistent order imbalances. Thus, the authors measure the return predictive ability of order imbalances within the trading day. According to their measurements, the return predictive ability of order imbalances disappears within 5 to 60 minutes. However, in their study, the authors analyze 5-minute returns and order imbalances within 5-minute intervals, which do not represent high-frequency returns and might not measure the market speed accurately due to the low resolution. In addition, since order imbalance can only be observed immediately by NYSE specialists and astute floor traders, [Chordia et al., 2005] actually measure strong-form market efficiency.

[Epps, 1979] studies price adjustments to news for companies in the same industry and finds cross-firm variations in the speed of adjustments to new information which is relevant for all firms in the industry. The informed traders are not able to impound the new information into the stocks simultaneously. If one put these stocks in a portfolio, one would find positive autocorrelations in the portfolio price returns after new information hit the market, since individual stock prices would adjust sequentially. This phenomenon is called non-synchronous trading. [Epps, 1979] concludes that the average lag in response to new information is more than 10 minutes and the predictive power of a price change in one stock does not last much longer than one hour. [Fama, 1991] mentions that the non-synchronous trading effect is more common in portfolios consisting of small stocks. In addition, the event studies discussed in [Fama, 1991] show that stock prices adjust to new announcements related to companies within a day.

[Sheppard, 2013] uses the model of penalized realized variance to study the autocovariance structure of multiple portfolios' price returns. One would expect to observe positive autocorrelations in the portfolio price returns due to the non-synchronous trading effect. By studying 15-second and 60-second price returns, [Sheppard, 2013] finds that the significant autocorrelations in Dow Jones Industrials persist for 0:59 minutes and 2:04 minutes, respectively, in 2017.

In our speed measurements, we take the concrete market mechanisms into account by scrutinizing the efficiency of informed traders in correcting the mis-pricing and impounding new information. In addition, we study the autocovariance structure in the price returns of individual stocks. By implementing the models to individual stocks, we can determine the relative efficiency of these stocks. We use high-frequency price returns in the model estimation to ensure a high resolution in the speed measurements. We first apply 1-second price returns to the model of information feedback and interpret the estimates of market efficiency and the estimated impacts of informed traders and noise traders. Then we apply 5-second price returns of large-cap stocks to both the model of information feedback and the model of penalized realized variance and compare the speed measurements between these models. The model of information feedback is more sophisticated than the model of penalized realized variance, since it includes the sequential learning and delayed responses of informed traders and illustrates actual market mechanisms. However, the parameter α, which indicates the market speed in the model of information feedback, can hardly be interpreted as a speed measure in a time unit, which makes its interpretation somewhat abstract. In the model of penalized realized variance, the speed measurements are obtained in the unit of seconds. Additionally, the model of information feedback does not study the return predictability and test weak-form efficiency directly as the model of penalized realized variance does by studying the autocovariance structure. By showing that the speed measurements based on these two models are consistent, we are able to translate the estimated α into a speed measure in the unit of seconds, which makes it more tangible and concrete. The consistency also confirms the accuracy of the speed measurements. In addition, we measure the development of market speed from 2010 to 2017 based on these two models.

Section 2 discusses the roles of noise traders and dealers in more detail and presents an equilibrium model which illustrates how an informed trader corrects mis-pricing and impounds his information into the price in a single-auction setting. Section 3 describes the models and methodologies used in the paper. Section 4 presents the data and speed measurements. Section 5 concludes.

2 Market Participants and Kyle's Equilibrium Model

In this section, we elaborate on the roles of noise traders and dealers and use an equilibrium model to illustrate the process of incorporation of information into stock prices. [Black, 1986] asserts that if there were no noise trading, there would be very little trading in the market. If all traders were informed, they would not trade with each other, since the trading parties could not both make a profit at the same time. If the market price were undervalued, no party would be willing to sell; if the market price were overvalued, no party would be willing to buy; if the market price were fairly valued, no party would have an incentive to buy or sell, because there would not be any profit from doing so. The No-trade theorem is already stated in [Milgrom and Stokey, 1982]. Therefore, noise trading provides liquidity for the financial market and enables informed traders to incorporate their information into the price by trading. Meanwhile, noise traders are the people who cause the market prices to deviate from the intrinsic value and thus inefficiencies in the market. Thus, as noise trading makes financial markets possible, it makes them also inefficient.

By trading with noise traders, informed traders can exploit their information disadvantage and reap gains. As soon as noise traders cause mis-pricing, informed traders step in and correct the mis-pricing. The interplay between informed traders and noise traders contributes to the price volatility in the stock market.

Noise traders also tend to imitate other investors by copying other people's trades as discussed in [Hirshleifer et al., 1994]. Such behavior, called herding, led to the inflated prices of Internet stocks in the 1990's and inflated housing prices in the U.S. in the 2000's, which resulted in two severe financial crises. One of the most intelligent people in history, Sir Isaac Newton, lost a fortune by engaging in herding. After losing 20,000 Pounds from trading in the stock of South Sea Company, he made the famous quote, "I could calculate the motion of heavenly bodies but not the madness of people". The "madness of people" referred to the herding behavior of people, which led to the extremely inflated price of South Sea's stock and eventually the bursting of the bubble.

In comparison to uninformed traders, dealers do not trade according to their beliefs about future price movements. By offering stocks for the ask price and buying stocks at the bid price, dealers act as liquidity providers to satisfy investors' needs. Dealers have always played crucial roles since the onset of equity markets. Since dealers are the ones who set the bid-ask spread, they practically determine the liquidity of the market. The smaller the bid-ask spread is, the less the investors have to pay to buy a stock and the less costly it is to impound their information into the price. Therefore, liquid markets are able to attract more investors and generate larger trading volumes, and in liquid markets, new information is impounded into the price faster than in illiquid markets. The sluggish incorporation of information in illiquid markets results in a slow convergence to the intrinsic value and thus positive autocorrelations in the price returns.

According to [Kyle, 1985], market liquidity can also be interpreted as tightness, depth and resiliency of the market. If a market is infinitely tight, it is costless to turn over a position in a short period. Depth refers to the ability of the market to absorb a huge trade without having a large impact on the price. Resiliency refers to the speed at which the market price recovers from an uninformative shock. They are all important indicators of the market efficiency. One might also say that "resiliency" is modeled by the parameter α in the model of information feedback, which is estimated later in this paper, since α indicates the speed at which mis-pricing is corrected by informed traders.

In order to illustrate the process of correction of mis-pricing and the interplay between dealers, informed traders, and uninformed traders, we will briefly discuss the model in [Kyle, 1985]. The single auction setting of this model shows how mis-pricing diminishes due to informed trading. In this model, the fundamental value of the stock follows a normal distribution, $p^* \sim N(p,\ \sigma_v^2)$, where p is the current market price, and the trading volume of noise traders also follows a normal distribution, $u \sim N(0,\ \sigma_u^2)$. u is independent from p^*, since noise traders trade randomly. One informed trader knows the true value of p^*, but the dealer and all uninformed traders do not. The informed trader submits an order with a size of x. The dealer observes the total order size of the informed trader and noise traders, $q = x + u$, but he cannot distinguish between the orders submitted by the informed trader and noise traders. He has to consider that he is at a disadvantage when trading with the informed trader. For instance, if the dealer observes a large buy order, he might think that the informed trader considers the stock undervalued and thus charges a higher price. The dealer has to set his price according to the observed order size q to compensate himself for the informational disadvantage. Thus, the dealer's price is denoted as $p(q)$. Therefore, the profit of the informed trader would be

$$\pi = x \cdot (p^* - p(q)).$$

The informed trader tries to maximize the expected profit

$$E(\pi) = E(x \cdot (p^* - p(q)))$$

by submitting an order with the optimal size of $x^*(p^*)$. The dealer sets his price as the expected intrinsic value given the observed order size

$$p^*(q) = E(p^*|x(p^*) + u). \tag{1}$$

One can assume that $p(q)$ is a linear function of q and $x(p^*)$ is a linear function of p^*:

$$p(q) = \theta + \eta q,\ x(p^*) = \rho + \beta p^*. \tag{2}$$

Obviously, if the order size were 0, the dealer would not charge a different price than the current market price, p:

$$p(0) = \theta + \eta \cdot 0 = p \Rightarrow \theta = p.$$

If the intrinsic value were equal to the current market price, the informed trader would not submit any order, since there would be no mis-pricing to correct,

$$x(p) = \rho + \beta p = 0 \Rightarrow \rho = -\beta p.$$

By substituting ρ and θ in (2), one obtains

$$p(q) = p + \eta q,\ x(p^*) = \beta(p^* - p). \tag{3}$$

Given the price set by the dealer, the expected profit of the informed trader would be

$$\begin{aligned} E(\pi) &= E(x(p^* - p(q))) = E(x(p^* - p - \eta q)) = E(x(p^* - p - \eta(x + u))) \\ &= x(p^* - p - \eta x). \end{aligned}$$

In order to obtain the optimal order size which maximizes the expected profit, the first derivative of $E(\pi)$ with respect to x must be equal to 0

$$p^* - p - 2\eta x = 0 \Rightarrow x^*(p^*) = \frac{p^* - p}{2\eta},\ E(\pi) = \frac{(p^* - p)^2}{4\eta}. \tag{4}$$

Given the order submitted by the informed trader according to (3), one can rewrite the optimal price set by the dealer (1) as

$$p^*(q) = E(p^* | \beta(p^* - p) + u), \tag{5}$$

which can be calculated by using the *Projection Theorem*. According to the *Projection Theorem*, if random variables Υ and Π are normally distributed, then the conditional expected value of Υ on Π is

$$E(\Upsilon|\Pi) = E(\Upsilon) + \frac{Cov(\Upsilon, \Pi)}{Var(\Pi)}(\Pi - E(\Pi)).$$

Since p^* is independent from u and the variances of p^* and u are given as σ_v^2 and σ_u^2 respectively, we obtain

$$E(\beta(p^* - p) + u) = 0,\ Cov(p^*, \beta(p^* - p) + u) = \beta\sigma_v^2,\ Var(q) = \sigma_u^2 + \beta^2\sigma_v^2$$

Therefore, we obtain the optimal pricing of the dealer,

$$p^*(q) = p + \frac{\beta\sigma_v^2}{\sigma_u^2 + \beta^2\sigma_v^2} q. \tag{6}$$

In the equilibrium, the informed trader submits an optimal order size conditional on how the market maker sets the price, and the dealer sets the price by assuming the optimal order size submitted by the informed trader, i.e. $x^*(p^*) = x(p^*)$ and $p^*(q) = p(q)$ hold. Therefore, according to (3), (4) and (6), we obtain

$$\beta = \frac{1}{2\eta},\ \eta = \frac{\beta\sigma_v^2}{\sigma_u^2 + \beta^2\sigma_v^2} \Rightarrow \beta = \frac{\sigma_u}{\sigma_v},\ \eta = \frac{\sigma_v}{2\sigma_u} \tag{7}$$

According to (7) and (4), we obtain the expected profit of the informed investor as

$$E(\pi) = \frac{(p^* - p)^2}{2}\frac{\sigma_u}{\sigma_v}.$$

Therefore, if the trading volume of uninformed investors is 0, i.e. $\sigma_u = 0$, the informed investor will not be able to make profits by exploiting his informational advantage, and the dealer will not trade with the informed investor in the absence of noise traders due to his informational disadvantage. This conclusion of the model also shows that the camouflage of noise traders is a prerequisite for the informed trader to reap gains.

According to (2) and (7), the price set by the dealer is

$$p^*(q) = p + \frac{\sigma_v}{2\sigma_u} q.$$

By using $q = x^* + u$, (4) and (7), one obtains the quote

$$p^*(q) = p + \frac{\sigma_v}{2\sigma_u}(\frac{\sigma_u}{\sigma_v}(p^* - p) + u) = p + \frac{\sigma_v}{2\sigma_u} u + \frac{1}{2}(p^* - p). \tag{8}$$

Therefore, one half of the mis-pricing $p^* - p$ is incorporated into the price by the informed trader, leaving the other half of mis-pricing to be corrected in subsequent periods. This formula shows how the market price is updated and converges to the intrinsic value. Since the order could be a buy order or a sell order, i.e. q could be positive or negative, the bid-ask spread is

$$a - b = \frac{\sigma_v}{\sigma_u} Q, \ \textit{where} \ Q = |q|.$$

Therefore, the bid-ask spread is determined by the magnitude of potential mis-pricing and trading volume of noise traders. If the magnitude of potential mis-pricing were large, the bid-ask spread would be bigger and the market would be illiquid, leading to smaller trading volume of informed traders, impacted efficiency and, in turn larger mis-pricing. The informed traders are unwilling to incorporate new information into the price due to the high trading costs. This mechanism can be described as a vicious circle. At the same time, the bid-ask spread decreases in the variance of noise trading volume. Dealer's loss caused by trading with informed traders can be compensated by trading with non-informed traders. In the equilibrium, noise traders suffer a loss which is equal to the gain of the informed trader, and the dealer achieves a break-even result on average. Therefore, the more the dealer trades with noise traders, the more he will be compensated and the smaller bid-ask spread he will ask for. In this sense, noise trading shrinks the bid-ask spread and contributes to market liquidity. However, if noise trading dominates informed trading, the mis-pricing will be too big for informed traders to correct, leading to a persistently-biased market price.

In conclusion, Kyle's model shows the interplay between informed traders, dealers, and uninformed traders. It quantifies the optimal order size submitted by informed traders to impound their information into the price, shows how dealers update their prices according to the observed order size and illustrates the role of noise traders as a camouflage for informed traders. It also shows how the quality of informed trader's information and volume of noise trading impact liquidity and thus market efficiency. Most importantly, this model quantifies how much of the information is incorporated into the price by the informed investor after the single auction. This is relevant because our paper focuses on measuring how quickly information is incorporated into the price and how closely the observed price follows the intrinsic value. However, the single auction equilibrium serves here only as an illustration of the price mechanism in the financial markets. It is not true that there is only one single auction in the modern markets. In a much more complex setting, [Kyle, 1985] shows that information is incorporated into prices at a constant rate in a continuous auction equilibrium. Since there is constantly news regarding a stock, its industry or its economies hitting the market, its intrinsic value might be changing constantly. Therefore, we use high frequency price data to measure the market speed of incorporating new information and correcting mis-pricing.

3 Models and Methodologies

3.1 The Model of Information Feedback

As mentioned, the model in [Andersen et al., 2016] is used in this paper. This model introduces two distinct pricing processes for both the observed market value and the intrinsic value. By introducing the parameter α, the model measures how closely the market price follows the intrinsic value. An α close to 1 indicates that informed traders incorporate new information into the price and correct the mis-pricing at high speeds. Therefore, by estimating α, we can obtain a measure of market speed. We will also discuss the estimates of the other important parameters in the model, such as the variance of informational noise, the variance of non-informational noise, and the variance of the mis-pricing. Informational noise indicates the changes in the intrinsic value. Non-informational noise demonstrates the impact of noise traders on the observed price. These variables quantify the impact of distinct market participants on the market price and the magnitude of mis-pricing, which is an indicator of the market efficiency. The details of the model will be presented in the rest of this subsection.

The model sets the observed price and the fundamental price as

$$
\begin{aligned}
p_{t+1} &= p_t - \alpha \underbrace{(p_t - p_t^*)}_{:=\mu_t} + \epsilon_{t+1},\ \alpha \geq 0, \\
p_{t+1}^* &= p_t^* + \epsilon_{t+1}^*,
\end{aligned}
\tag{9}
$$

respectively, where p is the observed price and p^* is the fundamental price. ϵ^* is the noise in the fundamental price process, and ϵ is the change in the observed price process caused by noise traders. μ, calculated as the difference between the observed price and the fundamental price, is the mis-pricing component. As the formula (9) illustrates, α indicates the proportion of mis-pricing which is eliminated during the current period. If $\alpha = 1$,

$$p_{t+1} = p_t^* + \epsilon_{t+1},$$

this means that the observed price follows the fundamental price closely. If $\alpha > 1$, it means more than 100% of the mis-pricing in the current period is corrected, leading to an overcorrection and a mis-pricing with the opposite sign in the subsequent period. It could be that too many informed traders are trying to correct the mis-pricing at the same time. If $\alpha = 0$,

$$p_{t+1} = p_t + \epsilon_{t+1},$$

which indicates persistent mis-pricing over time and implies that fundamental price changes are not being reflected in the observed market price. It can be shown that the closer α is to 1, the more closely the observed price follows the intrinsic value. Therefore, an α close to 1 means a higher speed at which mis-pricing is corrected by rational arbitrageurs and new information is incorporated by informed traders. The variance of informational noise ϵ^* is denoted $\sigma^2_{\epsilon^*}$ and called informational volatility, and the variance of non-informational noise ϵ is denoted as σ^2_ϵ and called non-informational volatility. Informational noise and non-informational noise have the same expected value of 0 and are both independently and identically distributed, and they are independent from each other. In addition, the change in the fundamental value, ϵ^*_{t+1}, is independent from the level of mis-pricing μ_t in the previous period.

It can be shown that the mis-pricing μ follows an AR(1) process

$$\mu_{t+1} = (1 - \alpha)\mu_t + \epsilon^\mu_{t+1}, \tag{10}$$

where $\epsilon^\mu_{t+1} := \epsilon_{t+1} - \epsilon^*_{t+1}$. The variance of ϵ^μ is denoted as σ^2_μ. Obviously it follows that

$$\sigma^2_\mu = \sigma^2_\epsilon + \sigma^2_{\epsilon^*}.$$

Informational volatility as a proportion of informational- and non-informational volatility is defined as λ, which is calculated as

$$\lambda = \frac{\sigma^2_{\epsilon^*}}{\sigma^2_\mu}.$$

Since μ follows an AR(1) process, its expected value, variance and autocovariance are respectively

$$E[\mu_t] = 0,\ Var[\mu_t] = \frac{\sigma_\mu^2}{\alpha}\frac{1}{2-\alpha},\ E[\mu_{t+\Delta}\mu_t] = (1-\alpha)^\Delta \frac{\sigma_\mu^2}{\alpha}\frac{1}{2-\alpha}. \tag{11}$$

Furthermore, absolute inefficiency, κ, and relative inefficiency, $\bar{\kappa}$, in the market are respectively defined as

$$\kappa := Var(\mu_t) = \frac{\sigma_\mu^2}{\alpha}\frac{1}{2-\alpha}, \quad \bar{\kappa} := \frac{Var(\mu_t)}{\sigma_\mu^2} = \frac{1}{\alpha}\frac{1}{2-\alpha}.$$

As one can see, the variance of the mis-pricing is defined as absolute inefficiency, and the relative inefficiency is defined as absolute inefficiency divided by the summed noise variances. Relative inefficiency shows the degree of variance of mis-pricing for given levels of the variance of informational noise and the variance of non-informational noise. As one can see in the formulas, the closer α is to 1, the smaller the relative inefficiency and the absolute inefficiency are. This is consistent with the mentioned finding that the closer α is to 1, the more closely the observed price follows the intrinsic value, indicating higher market efficiency.

The observed return can be calculated as

$$r_{t+1} = p_{t+1} - p_t = -\alpha\mu_t + \epsilon_{t+1}. \tag{12}$$

Since noise traders trade randomly and independently from mis-pricing, ϵ_{t+1} is independent from μ_t. Therefore, the variance of the observed return can be obtained as

$$Var(r_t) = \sigma_\epsilon^2 + \alpha^2 Var(\mu_t) = \sigma_\mu^2(\frac{2}{2-\alpha} - \lambda). \tag{13}$$

Autocovariance of observed returns can be calculated as

$$\begin{gathered} Cov(r_{t+\Delta}, r_t) = -\phi(\Delta-1)(\alpha\bar{\kappa} - \lambda), \\ where\ \phi(\Delta-1) = \alpha(1-\alpha)^{\Delta-1}(\sigma_{\epsilon^*}^2 + \sigma_\epsilon^2) \geq 0. \end{gathered} \tag{14}$$

The proof is shown in the appendix.

The return over a time interval Δ is defined as

$$r_t^\Delta := p_{t+\Delta} - p_t,$$

and the quadratic variation of prices is defined as

$$\langle p \rangle_T^\Delta := E\left[\sum_{i=0}^{\frac{T}{\Delta}-1} (r_{i\Delta}^\Delta)^2\right].$$

It can be shown that quadratic variation of prices can be calculated as

$$\langle p \rangle_T^{\Delta} = \sigma_{\epsilon^*}^2 T + \sigma_\mu^2 T \phi(\Delta)(\alpha\bar{\kappa} - \lambda),$$
$$where \quad \phi(\Delta) = \frac{2}{\alpha\Delta}(1 - (1-\alpha)^{\Delta}).$$

Its proof is shown in the appendix.

In order to measure the market speed, we need to estimate the value of α in this model. In the following subsections, we introduce the methodologies used to estimate parameters in the model of information feedback.

3.1.1 Methodology: The Kalman Filter and The Maximum Likelihood Method

The first method used to estimate the market speed α and other parameters is the Kalman filter and the maximum likelihood estimation. As shown before in (10), mis-pricing follows an AR(1) process

$$\mu_{t+1} = (1-\alpha)\mu_t + \epsilon_{t+1}^{\mu}.$$

Therefore, one can obtain the following system of linear equations in matrix form by defining a set of variables as X:

$$\begin{aligned} X_{t+1} := \begin{pmatrix} \mu_{t+1} \\ \mu_t \\ \epsilon_{t+1} \end{pmatrix} &= \begin{pmatrix} 1-\alpha & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mu_t \\ \mu_{t-1} \\ \epsilon_t \end{pmatrix} + \begin{pmatrix} \epsilon_{t+1}^{\mu} \\ 0 \\ \epsilon_{t+1} \end{pmatrix} \\ &= \begin{pmatrix} 1-\alpha & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} X_t + \begin{pmatrix} \epsilon_{t+1}^{\mu} \\ 0 \\ \epsilon_{t+1} \end{pmatrix}. \end{aligned}$$

By setting

$$F := \begin{pmatrix} 1-\alpha & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \; v_{t+1} := \begin{pmatrix} \epsilon_{t+1}^{\mu} \\ 0 \\ \epsilon_{t+1} \end{pmatrix},$$

one obtains

$$X_{t+1} = FX_t + v_{t+1}. \tag{15}$$

Since $\epsilon_{t+1}^{\mu} = \epsilon_{t+1} - \epsilon_{t+1}^{*}$, the covariance matrix of the noise term is

$$Q := E(v_t v_t^{'}) = \begin{pmatrix} \sigma_{\epsilon^*}^2 + \sigma_\epsilon^2 & 0 & \sigma_\epsilon^2 \\ 0 & 0 & 0 \\ \sigma_\epsilon^2 & 0 & \sigma_\epsilon^2 \end{pmatrix}. \tag{16}$$

As discussed in (12), the return can be expressed as

$$r_{t+1} = -\alpha\mu_t + \epsilon_{t+1},$$

which can be rewritten as

$$r_{t+1} = \begin{pmatrix} 0 & -\alpha & 1 \end{pmatrix} \begin{pmatrix} \mu_{t+1} \\ \mu_t \\ \epsilon_{t+1} \end{pmatrix} = \begin{pmatrix} 0 & -\alpha & 1 \end{pmatrix} X_{t+1}.$$

By setting

$$H := \begin{pmatrix} 0 \\ -\alpha \\ 1 \end{pmatrix},$$

one obtains

$$r_{t+1} = H^{'} X_{t+1}. \tag{17}$$

According to (15), (16) and (17), we obtain a state space representation of the dynamics of returns as

$$\begin{aligned} r_t &= H^{'} X_t \\ X_{t+1} &= F X_t + v_{t+1}. \end{aligned}$$

The Kalman filter can be used to update a linear projection for the system above, and the state vector here is X_t. The Kalman filter calculates linear least squares forecasts of state vector based on past returns

$$\hat{X}_{t+1|t} = E(X_{t+1}|R_t),\ where\ R_t = (r_1, r_2, \ldots, r_t).$$

The mean squared error (MSE) matrix of the forecast state vector is calculated as

$$P_{t+1|t} := E((X_{t+1} - \hat{X}_{t+1|t})(X_{t+1} - \hat{X}_{t+1|t})^{'}).$$

Starting with $\hat{X}_{1|0}$, we can write it as the unconditional mean of X_1, since there was no observation before:

$$\hat{X}_{1|0} = E(X_1) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}. \tag{18}$$

The associated MSE matrix is calculated as

$$P_{1|0} = E((X_1 - E(X_1))(X_1 - E(X_1))^{'}).$$

According to (11), we know that

$$Var(\mu_t) = \frac{\sigma_\mu^2}{\alpha}\frac{1}{2-\alpha},\ Cov(\mu_t, \mu_{t+1}) = (1-\alpha)\frac{\sigma_\mu^2}{(2-\alpha)\alpha}.$$

Since $\mu_{t+1} = (1-\alpha)\mu_t + \epsilon_{t+1}^{\mu} = (1-\alpha)\mu_t + \epsilon_{t+1} - \epsilon_{t+1}^{*}$ and ϵ_{t+1} and ϵ_{t+1}^{*} are independent from μ_t, the covariance between μ_{t+1} and ϵ_{t+1} is obtained as

$$Cov(\mu_{t+1}, \epsilon_{t+1}) = \sigma_\epsilon^2.$$

Therefore, we obtain the MSE matrix of forecast state vector as

$$P_{1|0} = \begin{pmatrix} \frac{\sigma_\mu^2}{\alpha}\frac{1}{2-\alpha} & (1-\alpha)\frac{\sigma_\mu^2}{(2-\alpha)\alpha} & \sigma_\epsilon^2 \\ (1-\alpha)\frac{\sigma_\mu^2}{(2-\alpha)\alpha} & \frac{\sigma_\mu^2}{\alpha}\frac{1}{2-\alpha} & 0 \\ \sigma_\epsilon^2 & 0 & \sigma_\epsilon^2 \end{pmatrix}. \tag{19}$$

After obtaining the initial values of $X_{t|t-1}$ and $P_{t|t-1}$, we can forecast the return as

$$\hat{r}_{t|t-1} = H^{'}\hat{X}_{t|t-1},$$

and the MSE of forecasted return is obtained as

$$E((r_t - \hat{r}_{t|t-1})(r_t - \hat{r}_{t|t-1})^{'}) = H^{'}P_{t|t-1}H.$$

According to the Kalman filter, the linear projection is updated as

$$\hat{X}_{t+1|t} = F\hat{X}_{t|t-1} + FP_{t|t-1}H(H^{'}P_{t|t-1}H)^{-1}(r_t - H^{'}\hat{X}_{t|t-1}).$$

For $t \geq 1$, the associated MSE matrix of forecast state vector is updated as

$$P_{t+1|t} = F(P_{t|t-1} - P_{t|t-1}H(H^{'}P_{t|t-1}H)^{-1}H^{'}P_{t|t-1})F^{'} + Q.$$

By assuming X_t and v_t are Gaussian, the distribution of r_t conditioned on R_{t-1} is

$$r_t|R_{t-1} \sim N(H^{'}\hat{X}_{t|t-1}, H^{'}P_{t|t-1}H),$$

and its density function can be written as

$$\begin{aligned} f_{r_t|R_{t-1}}(r_t|R_{t-1}) = & \frac{1}{\sqrt{2\pi}}|H^{'}P_{t|t-1}H|^{-1/2} \\ & \cdot exp(-\frac{1}{2}(r_t - H^{'}\hat{X}_{t|t-1})^{'}(H^{'}P_{t|t-1}H)^{-1} \cdot (r_t - H^{'}\hat{X}_{t|t-1})), \\ & for\ t = 1, 2, \ldots, T. \end{aligned}$$

We can build a maximum likelihood function

$$\sum_{t=1}^{T} log\ f_{r_t|R_{t-1}}(r_t|R_{t-1}),$$

and by maximizing this function numerically with respect to the unknown parameters α, σ_ϵ^2 and $\sigma_{\epsilon^*}^2$, one can obtain estimates of these parameters. Thus, the proportion of informational volatility can be estimated as

$$\hat{\lambda} = \hat{\sigma_{\epsilon^*}^2}/(\hat{\sigma_{\epsilon^*}^2} + \hat{\sigma_\epsilon^2}).$$

Furthermore, one can estimate the first-order autocovariance of observed returns according to (14)

$$\hat{\psi} = \hat{\alpha}(\hat{\sigma_{\epsilon^*}^2} + \hat{\sigma_\epsilon^2})(\hat{\lambda} - 1/(2 - \hat{\alpha})).$$

The total inefficiency in the market can be estimated as

$$\hat{\kappa} = (\hat{\sigma_{\epsilon^*}^2} + \hat{\sigma_\epsilon^2})/(\hat{\alpha}(2 - \hat{\alpha})),$$

and the relative inefficiency in the market can be estimated as

$$\hat{\bar{\kappa}} = 1/(\hat{\alpha}(2 - \hat{\alpha})).$$

3.1.2 Methodology: The Method of Moments

The second method which is used to estimate the parameters in the model of information feedback is the method of moments. As in the maximum likelihood method, we estimate the same three parameters α, σ_ϵ^2 and $\sigma_{\epsilon^*}^2$. Therefore, one needs to find three equations in the model in which these three parameters are present. On the other side of equation there should be variables that can be estimated empirically with the sample. These three empirically estimated variables are the variance, the first-order autocovariance and the second-order autocovariance of the price returns. The formulas for variance and autocovariances are given in the model above. As shown in (13), the variance of observed returns is calculated as

$$Var(r_t) = \sigma_\epsilon^2 + \alpha^2 Var(\mu_t) = \sigma_\epsilon^2 + \alpha^2 \frac{\sigma_\mu^2}{\alpha} \frac{1}{2 - \alpha} = \sigma_\epsilon^2 + \frac{\alpha}{2 - \alpha} \sigma_\mu^2.$$

One can calculate the sample variance of the returns, denoted as $\hat{\psi}_0$. As shown in the equation above, the sample variance should be close to the formula expressed in α, σ_ϵ^2 and $\sigma_{\epsilon*}^2$. Therefore, our first moment condition for these three parameters is

$$g_1(\hat{\psi}_0, \alpha, \sigma_\epsilon^2, \sigma_{\epsilon*}^2) = \hat{\psi}_0 - \frac{\alpha}{2-\alpha}(\sigma_\epsilon^2 + \sigma_{\epsilon*}^2) - \sigma_\epsilon^2. \tag{20}$$

According to (14), the first-order and the second-order autocovariances are calculated as

$$Cov(r_{t+1}, r_t) = \alpha\sigma_\mu^2(\lambda - \frac{1}{2-\alpha}),$$
$$Cov(r_{t+2}, r_t) = \alpha(1-\alpha)\sigma_\mu^2(\lambda - \frac{1}{2-\alpha}).$$

Sample autocovariances $\hat{\psi}_1$, $\hat{\psi}_2$ can be calculated by using the observed returns. It follows naturally that sample autocovariances should be close to the formulas above expressed in α, σ_ϵ^2 and $\sigma_{\epsilon*}^2$. Therefore, we obtain the following two moment conditions:

$$g_2(\hat{\psi}_1, \alpha, \sigma_\epsilon^2, \sigma_{\epsilon*}^2) = \hat{\psi}_1 - \alpha(\sigma_\epsilon^2 + \sigma_{\epsilon*}^2)(\lambda - \frac{1}{2-\alpha}),$$
$$g_3(\hat{\psi}_2, \alpha, \sigma_\epsilon^2, \sigma_{\epsilon*}^2) = \hat{\psi}_2 - \alpha(1-\alpha)(\sigma_\epsilon^2 + \sigma_{\epsilon*}^2)(\lambda - \frac{1}{2-\alpha}).$$

Because the derived variables expressed in α, σ_ϵ^2 and $\sigma_{\epsilon*}^2$ should be close to their empirical counterparts, one needs to minimize the absolute values of $g_1(\hat{\psi}_0, \alpha, \sigma_\epsilon^2, \sigma_{\epsilon*}^2)$, $g_2(\hat{\psi}_1, \alpha, \sigma_\epsilon^2, \sigma_{\epsilon*}^2)$ and $g_3(\hat{\psi}_2, \alpha, \sigma_\epsilon^2, \sigma_{\epsilon*}^2)$. Therefore, the estimates of these three parameters α, σ_ϵ^2 and $\sigma_{\epsilon*}^2$ are obtained by solving

$$\hat{\alpha}, \hat{\sigma_\epsilon^2}, \hat{\sigma_{\epsilon*}^2} = arg \min_{\alpha, \sigma_\epsilon^2, \sigma_{\epsilon*}^2} \sum_{i=1}^{3} g_i(\hat{\psi}_i, \alpha, \sigma_\epsilon^2, \sigma_{\epsilon*}^2)^2.$$

By using the obtained estimates of these three parameters, one can obtain estimates of the other parameters including λ, κ, $\bar{\kappa}$ and the first-order autocovariance by using the formulas derived by the model. In the section of empirical results, the estimates obtained by using the method of moments are compared with the ones obtained by using the maximum likelihood method and the Kalman filter. In the next subsection, another measure of market speed is introduced.

3.2 The Model of Penalized Realized Variance

In the paper [Sheppard, 2013], a measure of penalized realized variance is developed to measure how quickly the significant autocorrelations in the price returns disappear. If significant autocorrelations in the price returns persist over time, traders can exploit this characteristic to forecast future price returns in order to reap gains. However, according to the weak-form market efficiency, it should be impossible to achieve above-market returns by trading upon past information. Therefore, we expect significant autocorrelations in price returns to disappear in very short time. In addition, significant positive autocorrelations over time might indicate that new information is only slowly incorporated into the price. The slow convergence to the new intrinsic value is an indication of low market speed and imperfect market efficiency. The reason might be that the stock is not being covered by many analysts or that the trading volume in the market is relatively low. Therefore, how quickly significant autocorrelation disappears is an important indicator of market speed. In the following paragraphs, we discuss the model developed in [Sheppard, 2013].

In this model, the intrinsic value of the stock follows an Itô process

$$dp_t^* = \delta_t dt + \sigma_t dW_t,$$

where δ_t and σ_t are locally bounded processes. The observed price is the sum of the intrinsic value and of the mis-pricing

$$p_t = p_t^* + \mu_t,$$

and the mis-pricing μ_t depends on past changes in the intrinsic value

$$\mu_t = \sum_{j=0}^{h} d_j \Delta p_{i-j}^*, \text{ where } d_0,\ d_1 \ldots d_h \text{ are coefficients.}$$

The integrated variance and integrated quarticity in the intrinsic value are calculated respectively as

$$IV = \int_0^1 \sigma_s^2 ds,\ IQ = \int_0^1 \sigma_s^4 ds.$$

The concept of realized autocovariance is used in this model. The realized autocovariance is used to study the autocovariance structure in the high-frequency price returns. As first described in [Barndorff-Nielsen et al., 2008], the realized autocovariance is calculated as

$$\hat{\gamma}_j(p) = \sum_{i=j+1}^{m} \Delta p_i \Delta p_{i-j}, \tag{21}$$

where m is the number of observed price returns. Under the null hypothesis that $\hat{\gamma}_i = 0$ for all $i \geq j$, IV can be estimated as

$$\widehat{IV} = \hat{\gamma}_0 + 2\sum_{i=1}^{j-1} \hat{\gamma}_i.$$

[Sheppard, 2013] has shown that the quotient between integrated quarticity and squared integrated variance can be estimated as

$$\frac{\widehat{IQ}}{\widehat{IV}^2} = \frac{m^2}{m-j} \frac{\sum_{i=j+1}^{m} (\Delta p_i \Delta p_{i-j})^2}{\hat{\gamma}_0^2}. \tag{22}$$

As one wants to measure how quickly significant autocovariances disappear, one needs to determine the maximum lag which has significant autocovariance. One possibility is to consider a sequence of estimates for integrated variance $\widehat{IV} = \hat{\gamma}_0 + 2\sum_{i=1}^{h} \hat{\gamma}_i$ for $h = 1, 2 \ldots$. Additionally, [Sheppard, 2013] constructs a penalty term. He has shown that if the autocovariance is significantly different from 0, then

$$\hat{\gamma}_j > \sqrt{\frac{2ln\ ln\ m}{m}} \cdot \sqrt{4\int_0^1 \sigma_s^4 ds}.$$

Consequently, the penalized realized variance is constructed as

$$RAC^{pen}(h) = 1 + \sum_{j=1}^{h} \hat{\gamma}_j/\hat{\gamma}_0 - \sqrt{\frac{2ln\ ln\ m}{m}} \cdot \sum_{k=1}^{h} \sqrt{4(1 + \sum_{i=1}^{k} (\hat{\gamma}_i/\hat{\gamma}_0)^2) \int_0^1 \sigma_s^4 ds / (\int_0^1 \sigma_s^2 ds)^2}. \tag{23}$$

By subsitituting (22) into the formula above, we can calculate the penalized realized variance for $h = 1, 2 \ldots$ by using the sample prices. The lag h^* which maximizes the penalized realized variance indicates the maximum duration of significant autocorrelations. By multiplying h^* with the time lapse between two consecutive price points p_i and p_{i+1}, we obtain an estimate of market speed. Therefore, the time lapse between p_i and p_{i+1} serves as the unit of speed measure and cannot be too large. For instance, if one set the time lapse to 15 seconds and the actual speed were 11 seconds, the h^* obtained by applying penalized realized variance would always be 1 and the estimated market speed would be $1 * 15 = 15$ seconds, which would underestimate the market speed by 4 seconds. Therefore, determining the optimal time lapse between p_i and p_{i+1} is crucial when applying penalized realized variance.

4 Empirical Evidence

4.1 Data

In order to obtain accurate market speed measurements and ensure a high resolution, we use high-frequency price returns. It seems sensible to examine frequently traded stocks so that most high-frequency price returns do not equal 0. Because the price data of frequently traded stocks are so voluminous, we use a limited sample of stocks over a limited interval of time. The high-frequency price data of 11 large-cap stocks, including AAPL (Apple), AMZN (Amazon), EBAY (Ebay), GOOG (Google), NFLX (Netflix), TSLA (Tesla), BAC (Bank of America), QCOM (Qualcomm), FB (Facebook), MSFT (Microsoft) and T (AT&T), are used to measure the market speed. We use logarithmic mid-quote prices in the models. These stocks are randomly selected among frequently traded large-cap stocks, and all of them but TSLA are listed on the S&P 500. All these stocks have a market value above 5 billion US dollar. The price data is obtained through LOBSTER[1]. LOBSTER is an efficient reconstructor of limit order books. It records the bid- and ask-prices and their corresponding size whenever a new order is submitted or an existing order is cancelled or executed. It also provides accurate time, size, price, direction and type (e.g. execution or cancellation) of each event (e.g. submission of a new order). The time frame of data is from 2010 to 2017. The price data of the first 50 trading days of each year is used in the estimation of model parameters, so that the development of speed over those years can be observed.

As mentioned in the previous section, the time lapse between p_i and p_{i+1} cannot be too large as a unit of measure, so that the speed will not be underestimated. Meanwhile, in the model of penalized realized variance, the time lapse also cannot be too small, so that most price returns do not equal zero. As shown in (21), price returns of zero lead to inaccurate estimates of realized autocovariances and thus inaccurate speed measurements. After deliberate experimenting, the 5-second time interval works well as a unit of speed measure. It ensures the accuracy of speed measurements since the majority of 5-second price returns of frequently traded large-cap stocks are not zero. Using a 4-second time interval leads to an increased number of price returns equal to 0 and less accurate speed measurements. The 5-second interval is also an adequate unit of measure since most of the stocks have a slower speed than 5 seconds.

In contradiction to the model of penalized realized variance, the model

[1] https://lobsterdata.com/

	2013	2014	2015	2016	2017
AAPL	88.3%	82.9%	78.5%	76.1%	48.7%
AMZN	71.1%	65.5%	74.2%	77.4%	66.7%
GOOG	65.8%	62.6%	66.3%	71.9%	54.1%
NFLX	67.9%	63.7%	66.8%	89.4%	62.2%

Table 1: Proportions of non-zero 5-second returns in AAPL, AMZN, GOOG and NFLX from 2013 to 2017. For each stock, the returns in the first 50 trading days of each year are taken into consideration. Therefore, there are a total of $12 * 60 * 6.5 * 50 = 234,000$ price points for each stock in each year, and the proportion is obtained by dividing the number of non-zero returns by 233,999.

of information feedback can be presented as a state space representation of the dynamics of returns. Price returns with a value of zero do not impact the estimation of the parameters in this model. Therefore, this model allows high-frequency price returns sampled every second as input. In the next subsection, we first use one-second price returns over 10 trading days in the model of information feedback to illustrate the model estimation in a high-frequency context. Then we use 5-second price returns in both models to compare the speed measurements.

In Table 1, the proportions of non-zero 5-second price returns are presented for AAPL, AMZN, GOOG and NFLX from 2013 to 2017. For 50 trading days in a year, there are $50 * 6.5 * 60 * 12 = 234,000$ price points. The numbers of non-zero 5-second price returns are divided by 233,999 to obtain the proportion for each stock and each year.

As we can observe in the table, almost all percentage numbers are above 60%. For AMZN, GOOG and NFLX, the percentage number of non-zero price returns peaks in 2016. The percentage numbers of all four stocks plunge in 2017, coinciding with their respective price increases in 2017. Figure 1 and Figure 2 display the stock prices of AAPL and NFLX in 2016 and 2017, respectively. The stocks prices are mid-quote prices that are sampled every 5-seconds. As one can see, both the prices of AAPL and NFLX display an upward trend in 2017. The estimated standard deviations of logarithmic price returns of AAPL in 2016 and 2017 are $3 * 10^{-4}$ and $1.5 * 10^{-4}$, respectively; the estimated standard deviations of logarithmic price returns of NFLX in 2016 and 2017 are $5.5 * 10^{-4}$ and $2.3 * 10^{-4}$, respectively, indicating higher volatilities in the price returns in 2016. Overall, the large percentage numbers of non-zero 5-second price returns ensure the accuracy of speed measurements in the model of penalized realized variance.

Furthermore, we study the empirical autocorrelations of 5-second price

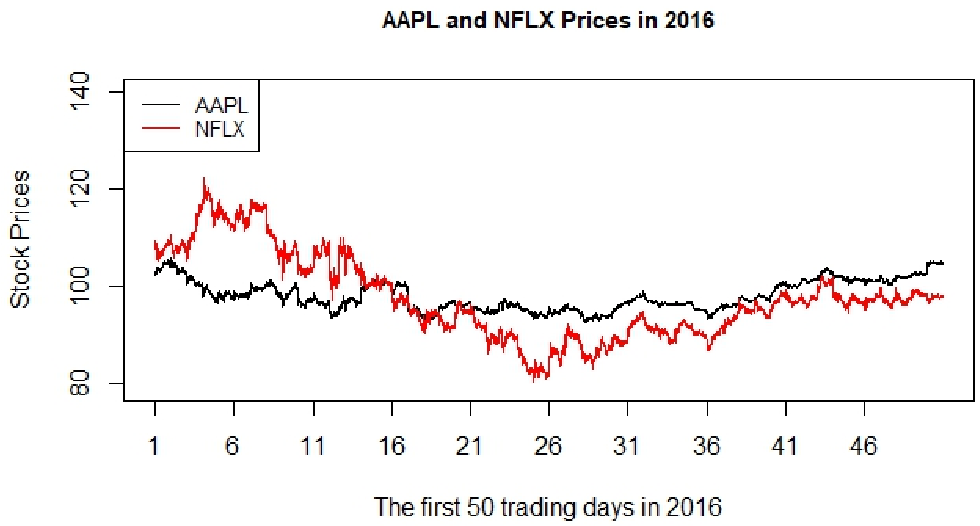

Figure 1: AAPL and NFLX stock prices in the first 50 trading days of 2016

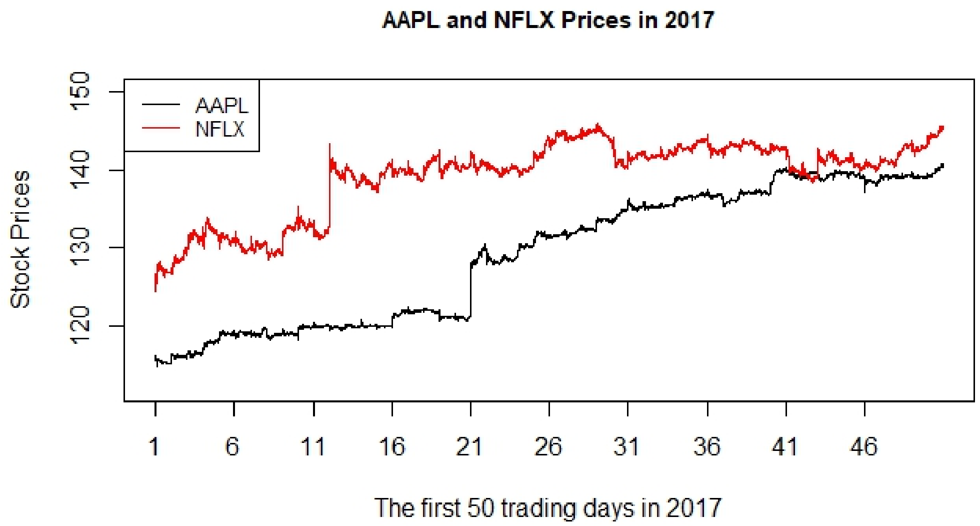

Figure 2: AAPL and NFLX stock prices in the first 50 trading days of 2017

returns in AAPL, AMZN, GOOG and NFLX from 2013 to 2017. We use 30 minutes as a time block to estimate the autocorrelation. Therefore, a time block consists of $12 * 30 = 360$ price points. Over the 50 trading days in each year, there are $13 * 50 = 650$ time blocks and corresponding estimated autocorrelations for each stock. The estimated autocorrelations are assumed to be asymptotically normally distributed $N(0, 1/359)$. Two-sided tests are applied to test the statistical significance of the estimated autocorrelations at the 5% level. In Table 2, the proportions of statistically significant autocorrelations are presented. The proportions are obtained by dividing the number of significant autocorrelations for each stock and each year by 650.

As we can see, the percentage numbers well exceed the number one would expect under the null hypothesis of uncorrelated price returns, and the percentage numbers of significant autocorrelations declined in AMZN, GOOG and NFLX over the years. In addition, one-sided tests are applied to test the statistical significance of positive and negative autocorrelations at the 5% level. In Table 3 and 4, the proportions of statistically significant positive and negative autocorrelations are presented, respectively. The proportions are obtained by dividing the number of positive and negative autocorrelations by 650 for each stock and each year. Although the proportions of positive significant autocorrelations in AAPL, AMZN and GOOG decreased over the years, they still exceed the percentage number one would expect under the null hypothesis of uncorrelated price returns in 2017. This might indicate increased market speed in these stocks over the years, since the positive significant autocorrelation implies return predictability and could signal sluggish incorporation of new information. NFLX shows an increased proportion of positive significant autocorrelations in 2017, which might indicate a slower speed. On the other hand, the proportions of significant negative autocorrelations increased in AAPL, AMZN and GOOG and decreased in NFLX over the years.

We will present the model estimation in the next subsections.

4.2 Empirical Results: The Model of Information Feedback

4.2.1 Estimation of the Market Speed α and the Proportion of Informational Volatility λ

In this subsection, we present the estimates of α and λ in the model of information feedback. Before going directly to the results, we discuss first how informed traders and uninformed traders contribute to continuously varying market speed in different ways. It might be reasonable to assume that

	2013	2014	2015	2016	2017
AAPL	23.7%	17.8%	15.5%	17.2%	21.4%
AMZN	25.2%	20.6%	18.5%	16.6%	15.5%
GOOG	18.5%	16.9%	17.2%	15.4%	15.6%
NFLX	22.8%	19.8%	19.8%	20.2%	19.2%

Table 2: Proportions of significant first-order autocorrelations of 5-second returns in AAPL, AMZN, GOOG and NFLX from 2013 to 2017. The autocorrelations are estimated by using time blocks of 30 minutes. Two-sided tests are used to test their significance at the 5% level. The first 50 trading days of each year are considered in the sample. The proportions are calculated by dividing the numbers of significant autocorrelations by 650.

	2013	2014	2015	2016	2017
AAPL	24.0%	20.6%	8.6%	6.3%	7.0%
AMZN	23.5%	25.7%	13.1%	15.7%	10.8%
GOOG	18.2%	14.0%	14.3%	20.0%	9.0%
NFLX	18.0%	19.2%	7.2%	14.8%	19.2%

Table 3: Proportions of significant positive first-order autocorrelations of 5-second returns in AAPL, AMZN, GOOG and NFLX from 2013 to 2017. The autocorrelations are estimated by using time blocks of 30 minutes. One-sided tests are used to test their significance at the 5% level. The first 50 trading days of each year are considered in the sample. The proportions are calculated by dividing the numbers of significant positive autocorrelations by 650.

	2013	2014	2015	2016	2017
AAPL	7.8%	4.9%	13.5%	17.5%	21.8%
AMZN	7.8%	3.2%	13.2%	6.9%	12.5%
GOOG	9.2%	9.4%	12.2%	5.1%	13.6%
NFLX	15.1%	8.2%	20.2%	13.4%	6.5%

Table 4: Proportions of significant negative first-order autocorrelations of 5-second returns in AAPL, AMZN, GOOG and NFLX from 2013 to 2017. The autocorrelations are estimated by using time blocks of 30 minutes. One-sided tests are used to test their significance at the 5% level. The first 50 trading days of each year are considered in the sample. The proportions are calculated by dividing the numbers of significant negative autocorrelations by 650.

market speed of a single stock varies over time, because it is unlikely that informed investors are always able to correct the mis-pricing at the same speed. The number of informed traders who focus on a certain stock does not stay constant, because they have limited attention, and because they also try to detect mis-pricing in other stocks. When informed investors focus on studying the pricing of certain stocks, the market speed of other stocks might decrease, because there are fewer informed traders who can recognize and correct the mis-pricing when it appears. When informed investors start studying the pricing of a certain stock, the market speed of this stock increases.

The trading behavior of noise traders also contributes to the variation in the market speed. When a substantial amount of noise trading causes the observed price to deviate significantly from the fundamental price, many informed traders and sizable financial resources are needed to correct the mis-pricing. Sometimes there are not so many informed traders or financial resources available, leading to persistent mis-pricing and lower market speed. If the trades of noise traders offset each other or there are not very many noise traders on the market, informed traders can efficiently correct the mis-pricing with their financial resources, leading to a higher market speed. Overall, we expect to see continuously varying market speed α.

As discussed above, we can use 1-second price returns to estimate the parameters in the this model, so we have sufficient price data points to obtain intraday speed measurements. We divide the total time horizon into smaller time blocks and estimate the parameters in each block. In order to ensure a sufficient number of price returns in each time block and the robustness of estimation, we estimate α every 5, 10 and 30 minutes. The mid-prices every second from 2.1.2014 to 15.1.2014 are used in the estimation. If there is no trading activity in the interval of 1 second, the closest previous mid-price is used as an approximation. The mid-price is calculated as the arithmetic mean of the best ask and bid prices. The logarithmic prices are used here. Overall, there are $10 * 6.5 * 3600 = 234000$ price data points for each stock, since each trading day has 6.5 hours. The time horizon lasts $6.5 * 10 * 60 = 3900$ minutes.

The market speed α and the proportion of informational volatility λ are estimated in the non-overlapping blocks of 5 minutes, 10 minutes and 30 minutes. For instance, a 5 minute-block contains $60 * 5 = 300$ price data points, which are used as inputs for the model to estimate the parameters. For every 5-minute block, one obtains a new set of estimates for these parameters. There are $234000/300 = 780$ non-overlapping 5-minute blocks and sets of estimates for these parameters over 10 trading days. For 10-minute blocks, there are $234000/600 = 390$ non-overlapping blocks and sets of esti-

	Estimated α				Estimated λ			
	min	max	mean	sd	min	max	mean	sd
Method of Moments	0.58	1.41	0.88	0.09	0.65	0.95	0.89	0.02
Maximum Likelihood	0.62	1.29	0.92	0.08	0.85	0.94	0.91	0.01

Table 5: Summary Statistics of estimates of AAPL's α and λ over 10 trading days. 5-minute non-overlapping blocks are used here. Therefore, there are 780 distinct periods each with their own respective estimate. Both the results of the method of moments and the maximum likelihood method are illustrated here. Minimum, maximum, mean and standardard deviation of the estimates are shown in the table.

mates; for 30-minute blocks, there are $234000/1800 = 120$ non-overlapping blocks and sets of estimates. Therefore, by using these time blocks, one can measure the varying market speed and proportion of informational volatility.

We apply both the method of moments (MM) and the maximum likelihood method (ML) to the data of 6 stocks including AAPL, AMZN, EBAY, GOOG, NFLX and TSLA. Since the range and the pattern of estimates for α and λ do not vary significantly from stock to stock, for the sake of brevity, only the estimates of AAPL in 5-minute blocks are plotted in Figure 3 and Figure 4. Figure 3 shows the results obtained by using the method of moments. Figure 4 shows the results obtained by using the Kalman filter and the maximum likelihood method. The units on the horizontal axis are minutes, thus, the length of the axis over 10 days is 3900. The key summary statistics of those plotted estimates are shown in Table 5.

As one can see, the estimates obtained by using these two methodologies are similar to each other. This similarity between the results obtained by these two methodologies also applies to other stocks for various time blocks. It is worth mentioning that as the time blocks become bigger, the estimates for all stocks become less volatile, as we will show later. Therefore, larger sample sizes give us a more stable estimation.

As shown in Table 5, the average of the estimated α obtained by using the maximum likelihood method is slightly higher than the average obtained by the MM. Additionally, estimates of α obtained by the MM has a larger range. The minimum value of $\hat{\lambda}$ estimated by the MM is 0.65, much smaller than the minimum value of $\hat{\lambda}$ estimated by the ML with a value of 0.85. Other summary statistics of estimates obtained by these methods do not differentiate much from each other. The difference between the standard deviations of estimates obtained by these two methodologies is subtle.

Most of estimates of α obtained by using the MM are in the range of 0.65 and 1.1, while most of the estimates of λ obtained by using the MM are

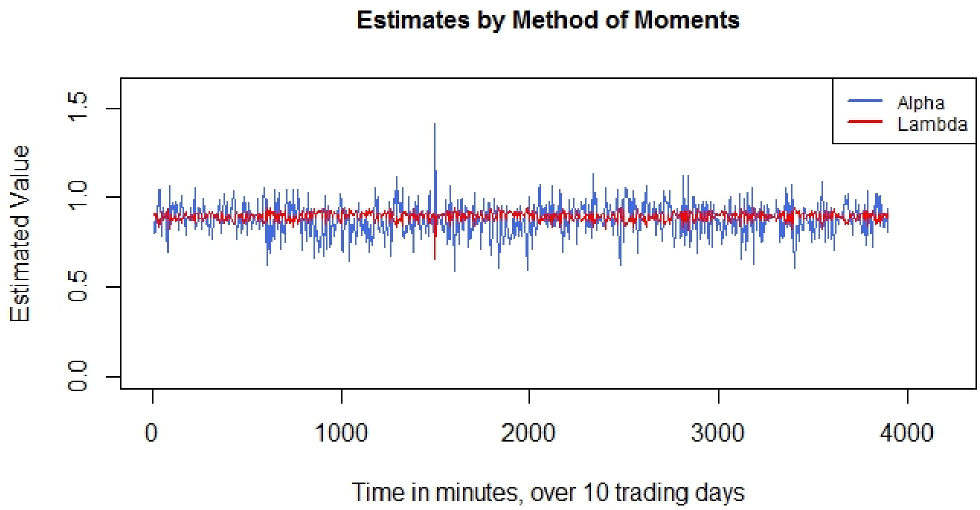

Figure 3: Estimated α and λ of AAPL using the method of moments

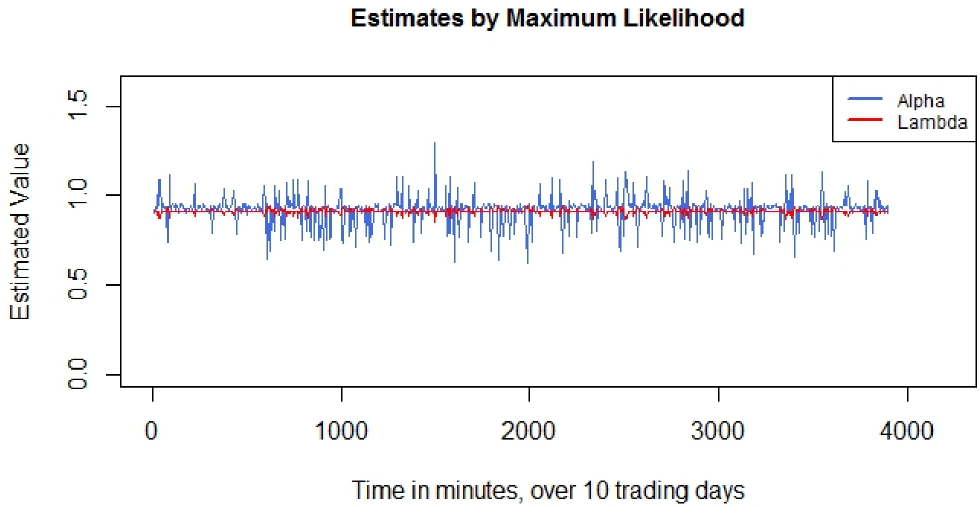

Figure 4: Estimated α and λ of AAPL using the maximum likelihood method and the Kalman filter

between 0.75 and 0.94. The mean of the market speed $\hat{\alpha}$ obtained by using the method of moments is 0.88, showing that mis-pricing is corrected rapidly by informed traders, and the observed price follows the fundamental price closely. The majority of the estimates for α are below 1, which indicates that informed traders tend to underreact to news or mis-pricing rather than overreact. As the market price converges slowly to the intrinsic value, the price returns display positive autocorrelations, which are captured later in the model estimation of penalized realized variance.

The mean proportion of informational volatility is 0.89, which means that the impact of informed trades varies much more than the impact of uninformed trades. The magnitude of changes in the intrinsic value is much larger than the magnitude of non-informational noise. Informed traders play the dominant role in determining the market price by incorporating external information into the price. Unlike noise trading, informed trading impacts the market price depending on positive news, negative news, overvaluation or undervaluation. The impact might change from positive to negative from one period to another, leading to large informational volatilities.

On the other hand, noise traders trade randomly and might offset each other's trades to a large extent, causing little change in aggregated price impact over time. It could also be that biased traders always trade in a similar way. They might use the same biased strategy over time, leading to the similar price impact of noise trading from period to period. A low λ would indicate larger non-informational volatility and varying impact of noise traders. However, in an efficient market, the impact of noise traders should always be low at any time point, so that the mis-pricing is held low. Therefore, a large λ indicates a high market efficiency and a high market speed.

Estimation of α and λ in Different Time Blocks

The results for all six stocks in 5-minute, 10-minute and 30-minute blocks are shown in Table 6. Since the results obtained by these two methods are similar, only the estimates obtained by the method of moments are illustrated in Table 6 for the sake of brevity. It is worth mentioning that in a couple of 5-minute blocks, the mid-price of stocks has stayed constant. For instance, EBAY's mid-price stays the same from 12:00 to 12:05 on 3.1.2014. The constant price in a 5-minute block makes estimation of the parameters impossible. However, this happens very rarely.

As one can see in Table 6, the lowest market speed $\hat{\alpha}$ with a value of 0.27 appears in TSLA within a 5-minute block. TSLA is also the only stock which is not listed on the S&P 500. However, the mean and standard de-

		Estimated α						Estimated λ					
		AAPL	AMZN	EBAY	GOOG	NFLX	TSLA	AAPL	AMZN	EBAY	GOOG	NFLX	TSLA
5 minutes	min	0.58	0.45	0.51	0.58	0.55	0.27	0.65	0.65	0.82	0.79	0.77	0.64
	max	1.41	1.38	1.18	1.2	1.32	1.45	0.95	0.97	0.96	0.95	1	1
	mean	0.88	0.87	0.88	0.89	0.87	0.88	0.89	0.9	0.89	0.89	0.9	0.89
	sd	0.09	0.1	0.08	0.09	0.1	0.1	0.02	0.03	0.02	0.02	0.02	0.03
10 minutes	min	0.65	0.6	0.64	0.68	0.58	0.67	0.82	0.71	0.85	0.83	0.81	0.77
	max	1.14	1.4	1.05	1.11	1.11	1.25	0.94	1	0.94	0.93	0.95	0.94
	mean	0.88	0.87	0.88	0.89	0.87	0.88	0.9	0.9	0.9	0.89	0.9	0.9
	sd	0.07	0.08	0.06	0.07	0.08	0.07	0.02	0.02	0.01	0.02	0.02	0.02
30 minutes	min	0.77	0.69	0.71	0.79	0.74	0.73	0.87	0.83	0.86	0.86	0.85	0.85
	max	0.98	1.11	1	1	1	1.08	0.92	0.93	0.93	0.92	0.93	0.93
	mean	0.88	0.87	0.88	0.89	0.87	0.87	0.9	0.9	0.9	0.89	0.9	0.9
	sd	0.04	0.05	0.04	0.04	0.05	0.05	0.01	0.01	0.01	0.01	0.01	0.01

Table 6: Summary statistics of estimated market speed $\hat{\alpha}$ and proportion of informational volatility $\hat{\lambda}$ for 5-minute, 10-minute and 30-minute blocks. When using 5-minute blocks, there are 780 separate blocks over 10 trading days and thus 780 estimates of α and λ; when using 10-minute blocks, there are 390 separate blocks over 10 trading days and thus 390 estimates of α and λ; when using 30-minute blocks, there are 130 separate blocks over 10 trading days and thus 130 estimates of α and λ. Only the results of the method of moments are presented in the table, and key statistics including minimum, maximum, mean and standard deviation are used to show the characteristics of the distribution of the estimates.

viation of the market speed in TSLA within 5-minute blocks are 0.88 and 0.1 respectively. Therefore, 0.27 seems to be an outlier. The estimates of market speed in AMZN, NFLX and TSLA have the largest standard deviation with a value of 0.1. The mean of market speed for each of the stocks lies between 0.87 and 0.89 when 5-minute blocks are used, indicating a high market speed. Since these stock are all large-cap stocks and 5 of them are listed on the S&P 500, the daily trading volume on these stocks must be high and the markets for these stocks must be very liquid. Positive and negative news is quickly incorporated into the prices by informed investors' trading activities. Also the mis-pricing caused by biased traders is corrected rapidly by informed traders. The high market speed can be explained by the large trading volume and market capitalization of these stocks.

The lowest estimate for the proportion of informational volatility λ with a value of 0.64 is with TSLA in 5-minute blocks. However, the minimum of $\hat{\lambda}$ of some other stocks in 5-minute blocks is 0.65, not much bigger than 0.64. The mean of $\hat{\lambda}$ of these stocks in 5-minute blocks is either 0.89 or 0.9 with a standard deviation of 0.02 or 0.03. Therefore, a value of 0.64 or 0.65 is clearly an outlier. In some 5-minute blocks, λ can be as high as 1, indicating absolute domination of informational volatility.

When comparing the estimated market speed $\hat{\alpha}$ obtained by using 5-minute blocks with $\hat{\alpha}$ obtained by using 10-minute blocks, one can observe that the mean stays the same for every stock. The minimum for each stock is higher when using 10-minute blocks, and the maximum is lower when using 10-minute blocks, except for the maximum of AMZN's estimated α, which increases from 1.38 to 1.4 when switching from 5-minute blocks to 10-minute blocks. In addition, the standard deviation decreases by approximately 23% on average when switching from 5-minute blocks to 10-minute blocks.

When comparing the estimated α obtained by using 10-minute blocks with the estimated α obtained by using 30-minute blocks, one can observe that the mean stays the same for almost all stocks. The minimum of each stock is higher when using 30-minute blocks, and the maximum of each stock is smaller when using 30-minute blocks. The standard deviation decreased by approximately 37% when switching from 10-minute blocks to 30-minute blocks. Clearly, when the time blocks get bigger, the variance and range of estimated market speed $\hat{\alpha}$ decrease and the estimation becomes more stable.

When comparing the estimated λ obtained by using 5-minute blocks with the estimated λ obtained by using 10-minute blocks, one can observe that the mean stays the same for AMZN, GOOG and NFLX. The mean of $\hat{\lambda}$ of AAPL, EBAY and TSLA changes from 0.89 to 0.9 when switching from 5-minute blocks to 10-minute blocks. The minimum of $\hat{\lambda}$ is higher for each stock when using 10-minute blocks, and the maximum is lower for each stock when using 10-minute blocks, except for the maximum of $\hat{\lambda}$ of AMZN, which increases from 0.97 to 1 when switching from 5-minute blocks to 10-minute blocks. The standard deviation of the estimated λ is only slightly smaller when using 10-minute blocks since the standard deviation of $\hat{\lambda}$ obtained by using 5-minute blocks is already quite small with values such as 0.02 and 0.03.

When comparing the estimated λ obtained by using 10-minute blocks with the estimated λ obtained by using 30-minute blocks, one observes that the mean stays the same for each stock. The minimum of $\hat{\lambda}$ is higher when using 30-minute blocks, and the maximum of $\hat{\lambda}$ is lower when using 30-minute blocks. The standard deviation decreases only slightly when switching from 10-minute blocks to 30-minute blocks since the standard deviation is generally small for $\hat{\lambda}$. The standard deviation decreases to 0.01 for all stocks. Clearly, when the blocks get bigger, the variance and the range of the estimated proportion of informational volatility decrease.

It holds for all stocks that the variance of the estimates of λ is much smaller than the variance of the estimates of α, no matter whether 5-minute blocks, 10-minute blocks or 30-minute blocks are used. One can also see that by using bigger time blocks, the estimations of α and λ become less volatile. By evaluating these stocks according to the average market speed $\hat{\alpha}$ in 30-minute blocks, GOOG tends to have the highest market speed over those 10 trading days, and AMZN, NFLX and TSLA have the lowest market speeds. However, the differences in average market speed among those large-cap stocks are not significant. It is also important to notice the varying market speed of individual stocks over time. If one enters the market when the market speed is low, one has more opportunities to incorporate new information into the price and correct the mis-pricing caused by noise traders profitably.

4.2.2 Estimation of the other Parameters

This subsection discusses the estimation of the other parameters in the model of information feedback: estimated informational- and non-informational volatility (σ_ϵ^2, $\sigma_{\epsilon^*}^2$), which quantify the volatilities of price impacts of informed- and uninformed trading, the derived first-order autocovariance according to (14), which serves as an indicator of the accuracy of model estimation, and direct measures of market efficiency κ and $\bar{\kappa}$. We use the same data as in the previous estimation of α and λ. By estimating these parameters, we can quantify the impacts of different market participants and determine how efficient one stock is according to its variance of mis-pricing κ. Since they are not direct measures of market speed, their tables and graphs are illustrated in the appendix.

Estimation of Informational- and Non-Informational Volatility

$\hat{\sigma}_{\epsilon}^2$ and $\hat{\sigma}_{\epsilon^*}^2$ of AAPL, AMZN, EBAY, GOOG, NFLX and TSLA are discussed first. Figure 10 and Figure 11 show the estimates of informational and non-informational volatility of AAPL within 5-minute blocks over 10 trading days from January 1, 2014 to January 15, 2014. Therefore, there are 780 non-overlapping blocks and distinct estimates for both parameters. Both the estimates obtained by using the method of moments and the maximum likelihood method and the Kalman filter are plotted here. For the sake of brevity, only the graphs of AAPL are shown here. The graphs of other stocks are similar in shape to the ones of AAPL.

As shown in the graphs, the estimates of non-informational volatility are obviously smaller than the estimates of informational volatility. As already discussed, the value of $\hat{\lambda}$ is approximately 0.9 on average, showing that on average, informational volatility is nine times as large as non-informational volatility. Since the units on the horizontal axis of the graphs are minutes, the beginnings of trading days display the values 1, $60 * 6.5 + 1 = 391$, $60 * 6.5 * 2 + 1 = 781$, $60 * 6.5 * 3 + 1 = 1171 \ldots$ on the horizontal axis. The graphs show that at the beginning of each trading day, the informational volatility is particularly high. Over the next couple 5-minute periods, the informational volatility declines gradually to the average level. This finding shows that informed traders incorporate their private information into the price at the beginning of the trading day. The non-informational volatility is also relatively high at the beginning of trading day compared to other times of the day, showing that noise traders also incorporate their beliefs into the price when the exchange opens. Summary statistics of AAPL's estimates are shown in Table 7.

Table 7 shows that the minimum, maximum, mean and standard deviation of estimates obtained by the method of moments are similar with the ones obtained by the maximum likelihood method and the Kalman filter. The mean and standard deviation of both $\hat{\sigma}_{\epsilon^*}^2$ and $\hat{\sigma}_{\epsilon}^2$ are slightly higher when the method of moments is used than when the maximum likelihood method is used. Since the informational and non-informational volatility are very small in the logarithmic prices, the estimates of these two volatilities presented in Table 7 and illustrated in the rest of the paper are obtained by multiplying the original estimates by 10^{10} for the sake of clarity. It is not surprising to see that the standard deviation of both informational volatility and non-informational volatility are large relative to their mean, because the volatilities at the beginning of the trading day can be extremely large.

	Estimated $\sigma^2_{\epsilon^*}$				Estimated σ^2_{ϵ}			
	min	max	mean	sd	min	max	mean	sd
method of moments	1.63	646	32.9	44.7	0.25	73.3	3.69	5.02
maximum likelihood	1.96	597	30.8	41.8	0.198	59.4	2.98	4.05

Table 7: Summary Statistics of the estimates of AAPL's $\sigma^2_{\epsilon^*}$ and σ^2_{ϵ} over 10 trading days. 5-minute non-overlapping blocks are used here. Therefore, there are 780 distinct periods and estimates. Both the results of the method of moments and the maximum likelihood method and the Kalman filter are presented here. Summary statistics of the estimates including minimum, maximum, mean and standard deviation are shown here. The numbers presented here are obtained by multiplying the original estimates by 10^{10}.

Estimation of Informational Volatility in Different Time Blocks

The summary statistics of estimates of $\sigma^2_{\epsilon^*}$ and σ^2_{ϵ} for all stocks in 5-minute blocks, 10-minute blocks and 30-minute blocks are presented in Table 10. Since the results obtained by the method of moments are similar with those obtained by the maximum likelihood method and the Kalman filter, only the summary statistics of estimates obtained by the method of moments are shown here for the sake of brevity.

Starting with the results obtained by using 5-minute blocks, we can see that the estimated informational volatility can be as low as 0.29 or as high as 16740. The estimated informational volatility is especially high in NFLX and TSLA, whose mean are 124 and 341 respectively. The results might indicate that informed traders have a lot of information to incorporate into the prices and make larger profits by trading with NFLX and TSLA, because the change in the fundamental price, ϵ^*, can be extremely large from time to time, as indicated by the high informational volatility. The standard deviation of the estimated informational volatility is relatively high for NFLX and TSLA, and the explanation of this phenomenon might lie in the existence of some extremely large values of informational volatility in these stocks.

When switching from 5-minute blocks to 10-minute blocks, one can observe that the minimum of the estimated informational volatility increases, the maximum of $\hat{\sigma}^2_{\epsilon^*}$ decreases, the mean stays approximately the same, and the standard deviation becomes smaller for each stock. The same changes apply to the minimum, the maximum, the mean and the standard deviation of estimated informational volatility when one switches from 10-minute blocks to 30-minute blocks. The only exception is that the minimum of $\hat{\sigma}^2_{\epsilon^*}$ of EBAY in 10-minute blocks is 5.3, bigger than the minimum of $\hat{\sigma}^2_{\epsilon^*}$ of EBAY in 30-minute blocks, which is 3.26. Clearly, by increasing the length of time blocks, one can decrease the range and variance of the estimated $\sigma^2_{\epsilon^*}$.

Estimation of Non-Informational Volatility in Different Time Blocks

Switching to the results of non-informational volatility, one can observe that $\hat{\sigma}^2_\epsilon$ can be as low as 0 or as high as 1892. It is interesting to note that both the minimum and the maximum of $\hat{\sigma}^2_\epsilon$ appear in TSLA. NFLX's minimum of $\hat{\sigma}^2_\epsilon$ is also 0, and its maximum is relatively high with a value of 270. Non-informational volatility with a value of 0 indicates that the impact of noise traders on the observed price stays 0 in a certain period. A relatively high value of $\hat{\sigma}^2_\epsilon$ shows that the impact of noise traders changes over time, as they might hold different beliefs from time to time. It might also indicate that noise traders trade actively on this stock. The mean and standard deviation of $\hat{\sigma}^2_\epsilon$ are relatively high for NFLX and TSLA, which could be explained by the extremely large values of $\hat{\sigma}^2_\epsilon$ of those stocks.

When switching from 5-minute blocks to 10-minute blocks, one can observe that the minimum of the estimated non-informational volatility increases, the maximum $\hat{\sigma}^2_\epsilon$ decreases, the mean stays approximately the same, and the standard deviation becomes smaller. The only exception is the minimum of AMZN's $\hat{\sigma}^2_\epsilon$, which is 0.04 when 5-minute blocks are used and 0 when 10-minute blocks are used. The same changes apply to the minimum, the maximum, the mean and the standard deviation when one switches from 10-minute blocks to 30-minute blocks. The only exception is the minimum of EBAY's $\hat{\sigma}^2_\epsilon$, which is higher when 10-minute blocks are used than when 30-minute blocks are used. Again, by increasing the length of blocks, one can decrease the range and variance of the estimated σ^2_ϵ.

According to the findings above, increasing the time block size can produce less volatile estimates of informational- and non-informational volatility. Based on the average $\hat{\sigma}^2_\epsilon$ and $\hat{\sigma}^2_{\epsilon^*}$ in the 30-minute blocks, TSLA has by far the highest informational and non-informational volatilities, meaning that from time to time, there is large amount of new information incorporated into the stock price of TSLA, and at some time points, noise trading impacts the market price significantly. Significant impacts of noise trading lead to larger mis-pricing and lower market efficiency. GOOG has the lowest average informational- and non-informational volatilities in the 30-minute blocks, showing that there is generally smaller amount of new information to be incorporated into the market price, and that the impact of noise trading on the observed price does not vary significantly over time. The difference between the standard deviations of informational volatility and the standard deviations of non-informational volatility can be explained by the different scale of the estimates of these two parameters.

Derived Autocovariance

According to (14), the first-order autocovariance of returns can be estimated as

$$\hat{\psi} = \hat{\alpha}(\hat{\sigma^2_{\epsilon^*}} + \hat{\sigma^2_{\epsilon}})(\hat{\lambda} - 1/(2 - \hat{\alpha})). \tag{24}$$

Therefore, according to the formula above, one can use $\hat{\alpha}$, $\hat{\lambda}$, $\hat{\sigma^2_{\epsilon^*}}$ and $\hat{\sigma^2_{\epsilon}}$ estimated by the method of moments and the maximum likelihood method and the Kalman filter to estimate the autocovariance. Figure 12, Figure 13 and Figure 14 show the graphs of sample autocovariances, autocovariances estimated by using the method of moments and autocovariances estimated by using the maximum likelihood method and the Kalman filter, respectively. The graphs show estimated first-order autocovariances of AAPL within 5-minute blocks over 10 trading days. One can see that the sample autocovariances and the estimated autocovariances according to (24) are very similar.

The similarity between sample autocovariances and estimated autocovariances according to (24) applies for all stocks and for all blocks with various lengths. For the sake of brevity, the graphs of other stocks with blocks of various lengths are not shown here. The similarity between the sample autocovariances and estimated autocovariances indicates the accuracy of the model, the methodologies used to estimate the parameters and the obtained estimates of the parameters, which in turn strengthen the validity of interpretation derived from those estimates.

Estimation of Total Inefficiency and Relative Inefficiency

As discussed, the total inefficiency in the market, which is the variance of mis-pricing, can be estimated as

$$\hat{\kappa} = (\hat{\sigma^2_{\epsilon^*}} + \hat{\sigma^2_{\epsilon}})/(\hat{\alpha}(2 - \hat{\alpha})), \tag{25}$$

and the relative inefficiency in the market, which is the scaled variance of mis-pricing by informational- and non-informational volatilities, can be estimated as

$$\hat{\bar{\kappa}} = 1/(\hat{\alpha}(2 - \hat{\alpha})).$$

Therefore, we use $\hat{\alpha}$, $\hat{\sigma^2_{\epsilon^*}}$ and $\hat{\sigma^2_{\epsilon}}$ estimated by the method of moments and the maximum likelihood method and the Kalman filter to estimate the total inefficiency and relative inefficiency. Figure 15 and Figure 16 show the estimated total inefficiency and relative inefficiency obtained by using AAPL's prices in 5-minute blocks over 10 trading days. Due to the similarities between the estimates of $\hat{\alpha}$, $\hat{\sigma_{\epsilon^*}^2}$ and $\hat{\sigma_{\epsilon}^2}$ obtained by the method of moments and the estimates obtained by the maximum likelihood method, the derived total inefficiency and relative inefficiency obtained by the method of moments must be similar with the ones obtained by the maximum likelihood method. Therefore, only the estimates obtained by using the method of moments are plotted here. The graphs of the other stocks' $\hat{\kappa}$ in blocks with various lengths share similar patterns as the graphs of AAPL and are thus not shown for the sake of brevity.

Obviously the estimated total inefficiency is higher at the beginning of the trading days. The reason underlying this finding is that both informational volatility and non-informational volatility are higher at the beginning of a trading day, and market speed α is not necessarily higher or lower at the beginning of the trading day than at other times of the day. Therefore, according to (25), the total inefficiency is higher at the beginning of the trading day, indicating larger variances of mis-pricing.

Large informational volatility leads to a large variance of mis-pricing, because high informational volatility implies large intrinsic value changes from time to time, which might not be always incorporated into the price immediately, leading to large mis-pricing at some points in time and thus a large variance of mis-pricing. Informed traders might have limited financial resources to enforce an immediate convergence to the intrinsic value, or they are impeded by the activity of noise traders. It could also be that too many informed traders want to reap the large gains by incorporating their information into the market price, leading to an overreaction by the market. Higher non-informational volatility also leads to a higher variance of mis-pricing, because it implies a large price impact of noise traders at some points in time which might cause significant mis-pricing. Mis-pricing is especially persistent when noise traders continuously trade in the opposite direction of informed traders.

Both absolute inefficiency and relative inefficiency depend on α. In Figure 16, the estimated relative inefficiency does not show any particular pattern, because it depends solely on the estimated market speed $\hat{\alpha}$. A high value of relative inefficiency indicates higher variance of mis-pricing for given levels of informational volatility and non-informational volatility. As discussed in the introduction of the model, α is the proportion of mis-pricing which is eliminated during the current period. If $\alpha > 1$, the mis-pricing is over-corrected. The over-correction can turn negative mis-pricing into positive mis-pricing or positive mis-pricing into negative mis-pricing, leading to higher variance in mis-pricing. If $\alpha < 1$, the mis-pricing from the previous period is not entirely corrected, which might also lead to a higher variance of mis-pricing. Therefore, the closer α is to 1, the smaller the absolute inefficiency and relative inefficiency are.

Estimation of Total Inefficiency in Different Time Blocks

Table 11 shows the summary statistics of the estimates of total inefficiency and relative inefficiency for all stocks and blocks with various lengths, including 5-minute blocks, 10-minute blocks and 30-minute blocks. Due to the similarity between the estimates obtained by using the method of moments and the estimates obtained by using the maximum likelihood method, only the summary statistics of the estimates obtained by using the method of moments are presented in the table. The summary statistics of total inefficiency shown in the table and discussed in the rest of paper are obtained by multiplying the original estimates by 10^{10} for the sake of clarity.

The total inefficiency can be as low as 0.33 or as high as 18940, which is not surprising, since the informational and non-informational volatility of TSLA can have extremely large values, leading to a high total inefficiency. A variance of mis-pricing with a value of 18940 shows that the mis-pricing changes substantially over the period, and mis-pricing can sometimes be large in absolute value. A large mis-pricing in absolute value indicates the inability of informed traders to bring the market price to its intrinsic value. It could be that the trading activity of noise traders dominates the trading activity of informed traders at some points in time. It could also be that informed traders do not have sufficient financial resources available to correct the mis-pricing. For instance, there are not enough informed traders in the market, because the information is hard to obtain, or few analysts conduct analysis about the stock. By knowing that there are limited informed participants, informed traders might also refrain from investing in the stock since the mis-pricing will likely not be corrected in the near future, and the investment will not offer large profits. Since the stocks listed on the S&P 500 are analyzed by investors intensively and TSLA is the only stock here which is not listed on the S&P 500, TSLA might be less studied than the other 5 stocks. The smaller coverage of TSLA might lead to smaller informed trading volume and thus more significant mis-pricing. It could also be the case that TSLA is a popular stock among noise traders, leading to high noise trading volume and large mis-pricing. The relatively higher mean and standard deviation of the estimated total inefficiency of NFLX and TSLA are due to the extremely high values of total inefficiency at some points in time.

When switching from 5-minute blocks to 10-minute blocks, the mean of estimated total inefficiency stays approximately the same, the minimum of estimated total inefficiency increases, the maximum of estimated total inefficiency decreases, and the standard deviation decreases for each stock. The same changes apply to the minimum, maximum, mean and standard deviation of the estimated total inefficiency when one switches from 10-minute blocks to 30-minute blocks. The only exception is EBAY's minimum, which is higher when 10-minute blocks are used than when 30-minutes blocks are used. The smaller range and standard deviation imply that the variability of estimated total inefficiency is smaller when blocks with longer time horizon are used.

Estimation of Relative Inefficiency in Different Time Blocks

Switching to the results of relative inefficiency, one can observe that relative inefficiency can be as low as 1 or as high as 2.12 when 5-minute blocks are used. Relative inefficiency with a value of 2.12 indicates that the variance of mis-pricing can be 2.12 times as large as the sum of informational- and non-informational volatility. It is not surprising that TSLA has the highest relative inefficiency since it was previously shown that the estimated market speed of TSLA can be as low as 0.27, and the relative inefficiency depends solely on α and decreases when α is closer to 1. It follows naturally that a lower market speed or an over-correction leads to lower market efficiency. All stocks reached the minimum level of relative inefficiency at some points in time. The mean of estimated relative inefficiency is low for each stock. The standard deviation of $\hat{\bar{\kappa}}$ is slightly higher for TSLA than for the other stocks, and the reason might lie in those large values of TSLA's $\hat{\bar{\kappa}}$.

When switching from 5-minute blocks to 10-minute blocks, the minimum $\hat{\bar{\kappa}}$ stays the same, the maximum decreases, the mean stays approximately the same, and the standard deviation decreases for each stock. The same changes apply to the minimum, maximum, mean and standard deviation of the estimated relative inefficiency when one switches from 10-minute blocks to 30-minute blocks. It is worth noting that TSLA's maximal $\hat{\bar{\kappa}}$ decreased from 2.12 to 1.12 when we switch from 5-minute blocks to 10-minute blocks, indicating a much lower variability in mis-pricing for a given level of total volatility when larger time blocks are used. Clearly, the range and standard deviation of the estimated $\bar{\kappa}$ decrease when we use blocks with a longer time horizon. Overall, the empirical results show that these large-cap stocks show relatively low levels of market inefficiency.

In the following subsection, we will present the speed measurements obtained by using penalized realized variance and compare them with the estimated α in the model of information feedback.

4.3 Empirical Results: The Model of Penalized Realized Variance

4.3.1 Speed Measurements in 2014

In this subsection, we focus on measuring market speed by using the penalized realized variance. As discussed above, in using the model of penalized realized variance to measure the market speed, we use the 5-second price returns. Time blocks of 5 trading days are used, so that a sufficient amount of data points in each block can be ensured. Therefore, speed measurements based on penalized realized variance can be updated every week. The first 50 trading days of the years from 2010 to 2017 are used in the sample. Thus for each year, there are 10 time blocks and 10 estimates of speed measurements.

According to (23), the penalized realized variances (RAC^{pen}) of various lags are determined for each stock and each time block

$$RAC^{pen}(h) = 1 + \sum_{j=1}^{h} \hat{\gamma}_j/\hat{\gamma}_0 - \sqrt{\frac{2ln\ ln\ m}{m}} \cdot \sum_{k=1}^{h} \sqrt{4(1+\sum_{i=1}^{k}(\hat{\gamma}_i/\hat{\gamma}_0)^2) \int_0^1 \sigma_s^4 ds/(\int_0^1 \sigma_s^2 ds)^2}.$$

The lag which maximizes RAC^{pen} is multiplied by 5 seconds to obtain the concrete speed measurement. Therefore, the bigger the lag is, the slower the market speed is. We also estimate α in the model of information feedback by using the identical time blocks consisting of 5-second price returns over 5 trading days. As we have seen, most of the estimates of α have values below 1, and as discussed before, the closer α is to 1, the more closely the observed price follows the intrinsic value. Therefore, a higher $\hat{\alpha}$ indicates a faster market speed, and the duration of significant autocovariances based on RAC^{pen} should be negatively correlated with the estimated α. α is estimated by both the method of moments (MM) and the maximum likelihood method and the Kalman filter (ML).

Figure 5 and Figure 6 show the speed measurements of AAPL and NFLX in the first 50 trading days of 2014, respectively. Since the estimates of α obtained by the method of moments are highly positively correlated with those obtained by the maximum likelihood method, only the results obtained by the method of moments are presented here.

As one can see, the duration of significant autocovariances based on RAC^{pen} (in short, the duration based on RAC^{pen}) and $\hat{\alpha}$ are negatively correlated in these two stocks as expected. The speed of AAPL plunges to 20 seconds in the time block from the 15th trading day to the 20th trading

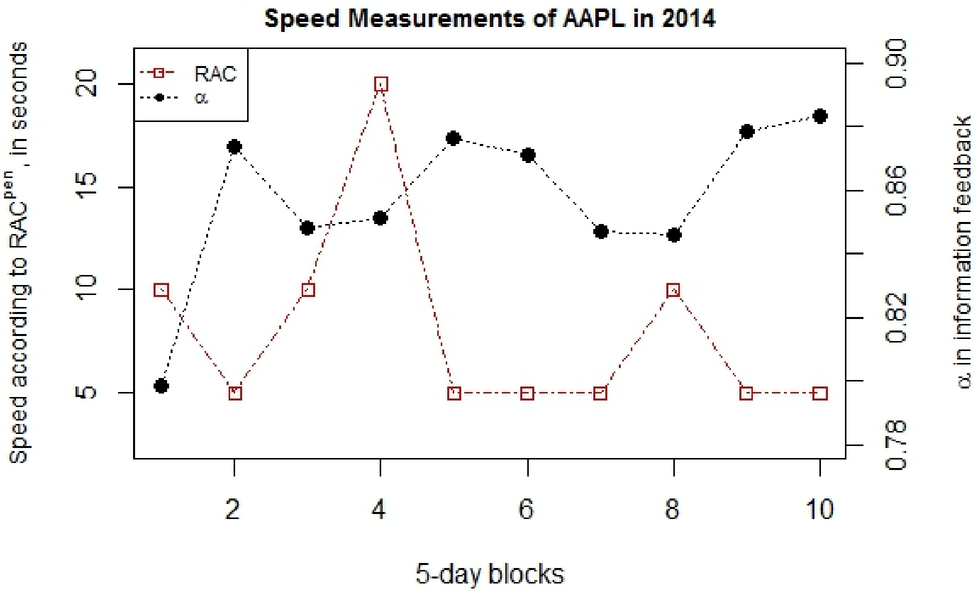

Figure 5: Speed Measurements of AAPL in the first 50 days of 2014

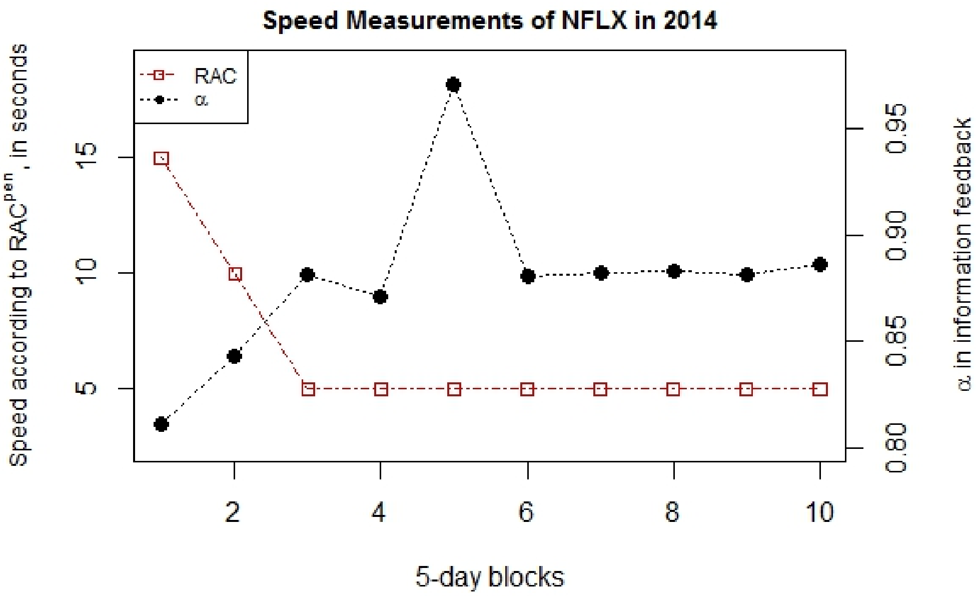

Figure 6: Speed Measurements of NFLX in the first 50 days of 2014

day in 2014 according to RAC^{pen}. $\hat{\alpha}$ in the fourth block is also relatively low compared to the estimates in other blocks. The low market speed might be due to the lack of coverage of informed traders or significant amount of noise trading, leading to a slow convergence toward the intrinsic value and a slow incorporation of new information. In addition, RAC^{pen} indicates a relatively low speed in the first, third and eighth blocks, coinciding with the relatively low $\hat{\alpha}$ values in these blocks. The correlation between the duration based on RAC^{pen} and $\hat{\alpha}$ is -0.45 for AAPL over these 10 time blocks in 2014, showing overall consistency between these two speed measures. The highest market speed can be as fast as 5 seconds according to RAC^{pen}, and $\hat{\alpha}$ can be as high as 0.883 in 2014, indicating immediate elimination of mis-pricing and incorporation of new information into the stock price.

By looking at the speed measurements of NFLX in 2014, the speed is as low as 15 seconds in the first block according to RAC^{pen}. The low speed coincides with the low $\hat{\alpha}$ in the first time block. In the second time block, the speed increases to 10 seconds according to RAC^{pen}, and $\hat{\alpha}$ increases from 0.811 to 0.843. In the subsequent time blocks, RAC^{pen} indicates a fast market speed of 5 seconds, and $\hat{\alpha}$ also rose significantly. The correlation between the duration based on RAC^{pen} and $\hat{\alpha}$ is -0.71 for NFLX in 2014. The negative correlation between the duration based on RAC^{pen} and $\hat{\alpha}$ over those 10 time blocks in 2014 can be observed for the majority of tested stocks, showing the consistency and accuracy of the market speed measurements. One reason why the negative correlation does not hold for all stocks could be that 5-second price returns of large cap stocks still have a lot of zero values, leading to inaccurate speed measurement based on RAC^{pen}. However, if one wants to increase the time lapse between p_t and p_{t+1}, i.e. the time unit in RAC^{pen}, one may underestimate the market speed (as discussed earlier).

By applying these two speed measures to time blocks (5 trading days each), one can determine each week whether the market has been efficient in correcting mis-pricing and incorporating new information. By updating the speed measurements for each stock every week, informed traders can focus on the stocks with a relatively low market speed, so that they have more opportunities to correct mis-pricing and incorporate new information into the prices profitably.

Table 8 shows the average speed measurements of 11 stocks in 2014. For each stock, the average is taken from 10 time blocks which consist of the first 50 trading days in 2014. By calculating these average values, we can compare the speed measurements among these stocks. The correlation between α estimated by the method of moments and α estimated by the maximum likelihood method is very close to 1. Therefore, we mainly focus on the relationship between the speed measurement based on RAC^{pen} and α

	Speed according to RAC^{pen}	α by MM	α by ML
AAPL	8 seconds	0.857	0.919
AMZN	8 seconds	0.854	0.908
EBAY	5.5 seconds	0.896	0.941
NFLX	6.5 seconds	0.879	0.926
TSLA	7.5 seconds	0.857	0.918
BAC	5 seconds	0.888	0.931
QCOM	5.5 seconds	0.892	0.931
FB	5 seconds	0.887	0.93
MSFT	9.5 seconds	0.879	0.928
GOOG	11.5 seconds	0.878	0.926
T	6 seconds	0.928	0.932

Table 8: Average speed measurements of 11 stocks in the first 50 trading days of 2014. Those 50 days are divided into 10 blocks. For each block, i.e. 5 trading days, three distinct speed measurements are estimated. The first column illustrates the speed according to the penalized realized variance. The second and the third columns show $\hat{\alpha}$ in the model of information feedback estimated by the method of moments and the maximum likelihood method and the Kalman filter, respectively. The average is taken from those 10 blocks for each stock and each speed measurement.

estimated by the method of moments. As discussed, the correlation between the duration based on RAC^{pen} and α should theoretically be close to -1.

According to RAC^{pen}, EBAY, BAC, QCOM, FB and T have the highest market speed with 5-6 seconds on average. In addition, the average $\hat{\alpha}$ of these stocks is at least 0.887. This finding shows that in those 5 stocks, mispricing is eliminated in the shortest time and new information is incorporated into the prices relatively fast. RAC^{pen} indicates that GOOG has the lowest market speed, while $\hat{\alpha}$ indicates that AMZN has the lowest market speed. This inconsistency might be due to the large number of price returns equal to 0 as discussed earlier. An important finding is that the correlation between the duration based on RAC^{pen} and α estimated by the method of moments is very close to -1 for most stocks. The speed measurements of the first 8 stocks in Table 8 are illustrated in Figure 7.

One can see that these two measures are almost perfectly negatively correlated. The duration based on RAC^{pen} and $\hat{\alpha}$ for those 8 stocks have a negative correlation of -0.94, confirming the consistency of the speed measurements. If one included MSFT, the correlation would increase to -0.68. If one also included GOOG and T, the correlation would increase to -0.43. Again, the increase in correlation might be due to the large number of price

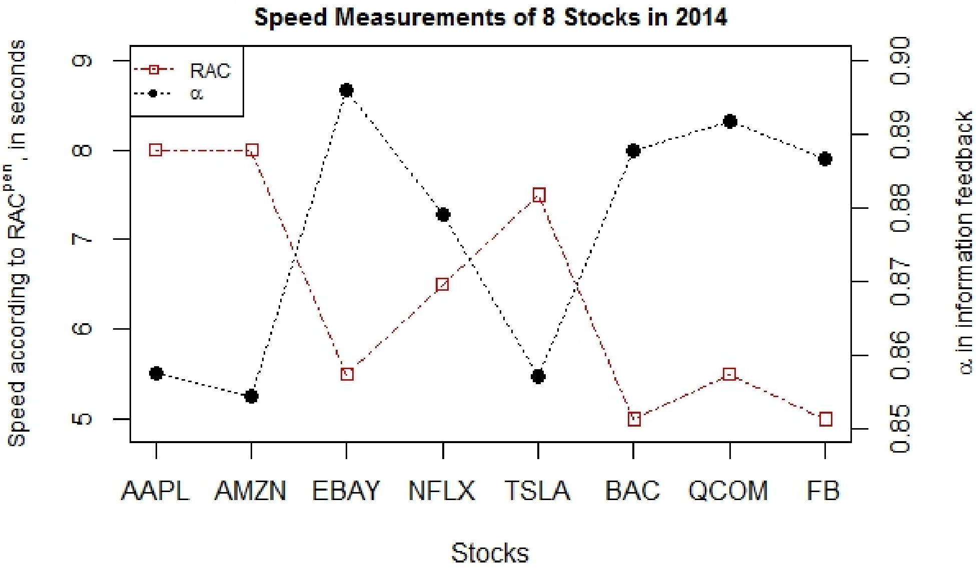

Figure 7: Speed Measurements of 8 stocks in the first 50 days of 2014

returns equal to 0. However, one can see that the overall consistency between these two speed measurements can be retained.

One can use these speed measures to detect stocks with lower average market speed. By trading upon these stocks, one has more opportunities to correct mis-pricing and incorporate new information into the prices profitably. In the subsequent subsection, we present the average market speed measurements of AAPL, AMZN, EBAY, GOOG and NFLX from 2010 to 2017.

4.3.2 Development of Market Speed from 2010 to 2017

Price returns in the first 50 tradings days from 2010 to 2017 are used to estimate speed measurements for AAPL, AMZN, EBAY, GOOG and NFLX. Again, 50 trading days are divided into 10 blocks. In each year, the average speed measurement is taken as the average of 10 estimates from those 10 blocks. As in the previous subsection, the three speed measurements are the duration based on RAC^{pen}, α estimated by the method of moments and α estimated by the maximum likelihood method and the Kalman filter. Table 9 shows three speed measurements for each stock over those eight years. As the correlation between the estimates of α based on the method of moments and the estimates of α based on the maximum likelihood method is very close to 1, we focus on discussing the estimates based on the method of moments for the sake of brevity.

As technology in trading advances every year, one could expect that the market speed has increased over these 8 years. One can indeed observe that most stocks have shown a trend of increased market speed over the past 8 years. According to the speed measurement based on RAC^{pen}, the speed of AMZN increased from 8.5 seconds in 2010 to 6 seconds in 2017, the speed of EBAY increased from 9 seconds in 2010 to 7 seconds in 2017, the speed of GOOG increased from 14.5 seconds in 2010 to 7 seconds in 2017, and the speed of NFLX increased from 14 seconds in 2010 to 8.5 seconds in 2017. Therefore, GOOG displays the largest increase in speed. The speed measurement of AAPL has been volatile in past years, though it also increased from 6 seconds in 2010 to 5.5 seconds in 2017.

Consistent with the finding based on RAC^{pen}, α estimated by the method of moments shows that the market speed of EBAY increased from 0.885 in 2010 to 0.921 in 2017, and the market speed of GOOG increased from 0.861 in 2010 to 0.88 in 2017. The estimated average α for the other three stocks was quite high in 2010 before hitting troughs in the subsequent years. The speed of AAPL was as low as 0.864 in 2011 and 0.857 in 2013 and 2014; the speed of AMZN was as low as 0.868 in 2011 and 0.853 in 2012; the speed of NFLX was as low as 0.834 in 2011. Therefore, from 2011 to 2017, all stocks display a trend of increasing market speed based on the average $\hat{\alpha}$. In order to illustrate the trend of increasing market speed, the speed measurements of EBAY and NFLX are shown in Figure 8 and 9 respectively.

As we can see in the graphs, $\hat{\alpha}$ has been increasing gradually for EBAY, and the speed measurements based on RAC^{pen} also show a trend of increasing speed. By observing the graph of NFLX, we can easily recognize the increased speed based on RAC^{pen}, and $\hat{\alpha}$ shows an upward trend since 2011. According to Table 9, the slowest speed based on RAC^{pen} is 14.5 seconds, shown by GOOG in 2010, and the fastest speed is 5 seconds, shown by NFLX in 2016. Based on $\hat{\alpha}$ estimated by the method of moments, the highest speed is 0.925, shown by EBAY in 2016, and the lowest speed is 0.834, shown by NFLX in 2011. We can see that highest speed is observed in later years and lower speed is observed in earlier years. Over those 8 years, GOOG shows the lowest average speed based on both RAC^{pen} and $\hat{\alpha}$. However, NFLX has the lowest speed in 2017 according to both speed measures. AAPL has the highest average speed over those 8 years according to the measurements based on RAC^{pen} and the second highest speed according to $\hat{\alpha}$. On the other hand, EBAY has the second highest average speed based on RAC^{pen} and the highest speed according to $\hat{\alpha}$. For each stock, these two speed measurements are negatively correlated over those years, indicating consistency and accuracy in these speed measurements.

		2010	2011	2012	2013	2014	2015	2016	2017
AAPL	RAC^{pen}	6 secs	8 secs	6 secs	5.5 secs	8 secs	5.5 secs	8 secs	5.5 secs
	α by MM	0.89	0.864	0.873	0.857	0.857	0.889	0.905	0.887
	α by ML	0.93	0.922	0.924	0.917	0.919	0.932	0.936	0.929
AMZN	RAC^{pen}	8.5 secs	9 secs	12.5 secs	8 secs	8 secs	6 secs	7.5 secs	6 secs
	α by MM	0.893	0.868	0.853	0.861	0.854	0.873	0.87	0.871
	α by ML	0.932	0.922	0.918	0.919	0.908	0.924	0.923	0.923
EBAY	RAC^{pen}	9 secs	7 secs	14 secs	5.5 secs	5.5 secs	7.5 secs	7.5 secs	7 secs
	α by MM	0.885	0.881	0.884	0.886	0.896	0.892	0.925	0.921
	α by ML	0.93	0.927	0.926	0.931	0.941	0.929	0.939	0.937
GOOG	RAC^{pen}	14.5 secs	7 secs	14 secs	11.5 secs	11.5 secs	7 secs	6.5 secs	7 secs
	α by MM	0.861	0.87	0.862	0.855	0.878	0.87	0.862	0.88
	α by ML	0.922	0.925	0.922	0.918	0.926	0.923	0.921	0.928
NFLX	RAC^{pen}	14 secs	10.5 secs	7 secs	7.5 secs	6.5 secs	7 secs	5 secs	8.5 secs
	α by MM	0.875	0.834	0.867	0.867	0.879	0.88	0.873	0.869
	α by ML	0.926	0.912	0.921	0.924	0.926	0.929	0.923	0.922

Table 9: Annual average speed measurements of 5 large-cap stocks. For each year, the first 50 trading days are divided into 10 blocks. 3 types of speed measurements are estimated in every block: speed according to the model of penalized realized variance, α in the model of information feedback estimated by the method of moments (MM) and α estimated by the maximum likelihood method and the Kalman filter (ML). Annual average is taken from these 10 blocks for each year from 2010 to 2017.

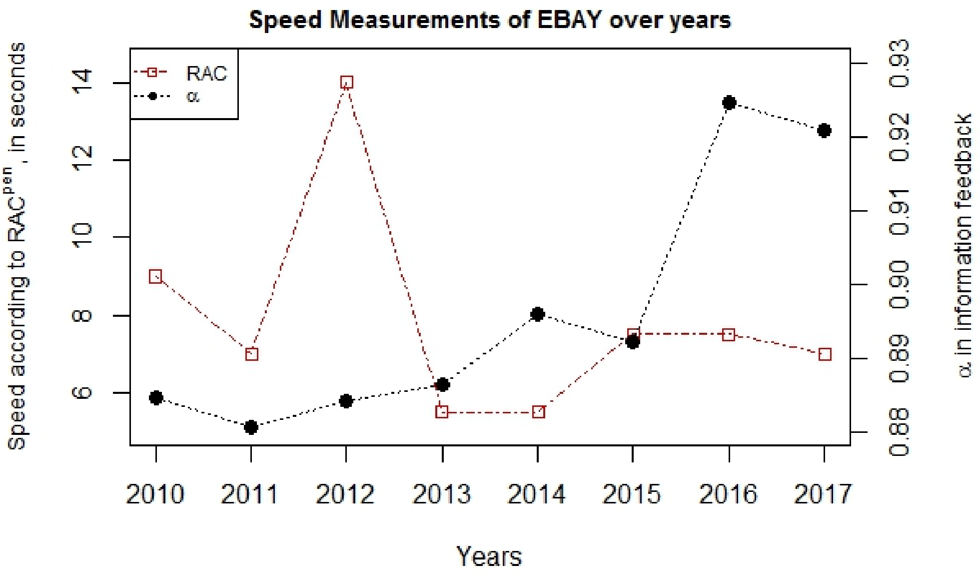

Figure 8: Speed Measurements of EBAY over 2010-2017

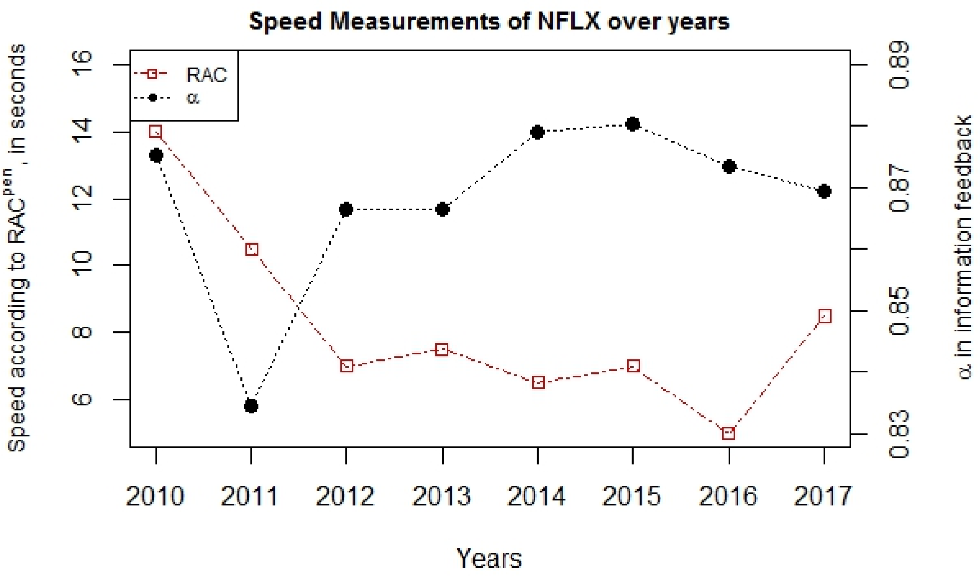

Figure 9: Speed Measurements of NFLX over 2010-2017

5 Conclusion

The Efficient Market Hypothesis asserts that the stock prices should reflect all available information. In an ideal world, efficiency might be achieved instantaneously as soon as new information hits the market. In the real world, informed traders need some time to acquire, process and trade upon new information. During the time interval in which they push the market toward efficiency collectively, the market remains inefficient. A central goal of this paper is to measure the speed of convergence to market efficiency.

We apply two models to measure the market speed: the model of information feedback and the model of penalized realized variance. We compare the measurements and validate their accuracy. The model of information feedback is an extension of the classical martingale-plus-noise model which includes the process of sequential learning by the informed traders. The parameter α in this model indicates the efficiency of informed traders in correcting mis-pricing and acquiring and reacting to new information. This model allows underreaction and overreaction of informed traders, as α can be smaller or bigger than 1. Two methods, the method of moments and the maximum likelihood method, are used to estimate the parameters in this model. The results obtained by using these two methods are fairly similar. As we have seen, the market speed of large-cap stocks is relatively high. Most of the estimates of α have values higher than 0.85 for those tested large-cap stocks. TSLA, the only tested stock which is not listed on the S&P 500, displays low speed measurements such as 0.27. Overall, we can infer from the results that the observed prices follow the intrinsic values closely, and informed traders are efficient in incorporating their information into the price and correcting the mis-pricing caused by noise traders. The majority of the estimates for α are smaller than 1, indicating that the informed traders tend to underreact to mis-pricing or news rather than overreact. As the market price converges slowly to the intrinsic value, the price returns display positive autocorrelations.

Another parameter in the model of information feedback, λ, the proportion of informational volatility in the total volatilities, displays estimates with values over 0.89. This shows that the informed traders are the principal drivers of price changes, indicating high market efficiency. The informed traders receive either positive or negative news about the underlying firm and buy or sell accordingly. They will trade collectively in the same direction due to the validity of their information. On the other hand, noise traders trade randomly. They might trade in the opposite direction among themselves, offsetting the impacts of each other's trades and leading to lower non-informational volatility. We also estimate other important parameters in this model, including $\sigma^2_{\epsilon^*}$ and σ^2_{ϵ}, which quantify the volatilities of price impacts of informed trading and uninformed trading, and κ and $\bar{\kappa}$, which measure the efficiency of individual stocks directly. When we use larger time blocks to estimate the model's parameters, we find that both the range and standard deviation of the estimates decrease, indicating less extreme values.

In the model of penalized realized variance, we study the autocovariance structure in the price returns and focus on measuring the sluggishness of incorporation of new information. We observe that most of the speed measurements for large-cap stocks are under 10 seconds in 2014. The remarkably fast speed of most stocks indicates a high level of market efficiency. By updating market speed measurements of individual stocks weekly, one can get a sense of overall market efficiency in the previous week and detect stocks with low speed. For the majority of the measured stocks, the speed measurement based on penalized realized variance is consistent with $\hat{\alpha}$ in the model of information feedback over the first 50 trading days in 2014. The reason why these two measures are not consistent for a small number of stocks is due to the fact that a large number of 5-second price returns in those stocks are 0, leading to inaccurate speed measurements based on penalized realized variance. One potential solution to mitigate this problem is to decrease the tick size to 0.001 dollars, so that high frequency price changes would not equal 0. In addition, stock prices would reflect stock values more accurately. Since the tick size has decreased over recent decades, a further decrease does not seem implausible. [Chordia et al., 2005] also find that the market speed is higher in 1999 and 2002 than in 1996 after the tick size reduction in 1997 and 2001. Yet a smaller tick size would need to be approved by the market participants and regulatory bodies, and it would require a lot of effort and large expenses in technological development in trading venues and exchanges.

When we look at the cross-sectional comparison of stocks' average speed in 2014, we can see that the average speed measurements based on penalized realized variance are largely consistent with the estimated α in the model of information feedback. For 8 of the 11 stocks, the correlation between the average duration based on RAC^{pen} and the average $\hat{\alpha}$ is -0.94 in 2014. For 11 stocks, the correlation coefficient increases to -0.43. We also show that the speed of all tested stocks has increased since 2010 based on RAC^{pen}. GOOG's speed increased from 14.5 seconds to 7 seconds in 2017 and NFLX's speed increased from 14 seconds to 8.5 seconds. This is consistent with the finding of [Stambaugh, 2014] which asserts that the volume of noise trading has decreased over time. Decreased noise trading leads to smaller mis-pricing and thus faster market speed.

For each stock, the correlation coefficient between the average duration based on RAC^{pen} and the average estimated α from 2010 to 2017 is negative.

After various extensive tests, the model of penalized realized variance and the model of information feedback show consistency in measuring market speed. If informed traders were inefficient in incorporating their information into the market price, indicated by low α value in the model of information feedback, and the market price converged only slowly to the intrinsic value, the price returns would display significant positive autocorrelations, which would be captured by the model of penalized realized variance. Therefore, the efficiency of informed traders is related to the autocovariance structure in the price returns.

Since the model of information feedback includes sequential learning and delayed responses of informed traders, it is more sophisticated than the model of penalized realized variance. Furthermore, it allows high-frequency price returns, such as 1-second returns, in the model estimation. However, α in the model of information feedback is not measured in a time unit, which makes it somewhat abstract. The duration based on RAC^{pen} in the model of penalized realized variance is measured in seconds. Moreover, the model of information feedback does not study return predictability and test weak-form efficiency directly as the model of penalized realized variance does by studying the autocovariance structure. By showing that the speed measurements based on these two models are consistent, we can translate the estimated α in the model of information feedback into a speed measure in the unit of seconds, which makes it more concrete and tangible. The consistency also validates the accuracy of these two speed measurements.

For academic purposes, one may measure the development and current level of market efficiency in individual stocks or the whole market by applying these two models. One can also use the results from the model of information feedback to understand the interplay between informed traders and uninformed traders. In practice, one may use these two speed measures as screening tools to identify stocks with lower speed and trade upon them, so that one can have a greater chance to correct mis-pricing and incorporate information into prices profitably.

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A The Model of Information Feedback: the Derivation of Autocovarainces of Returns

$$Cov(r_{t+\Delta}, r_t) = -\phi(\Delta - 1)(\alpha\bar{\kappa} - \lambda),$$
$$where\ \phi(\Delta - 1) = \alpha(1-\alpha)^{\Delta-1}(\sigma^2_{\epsilon^*} + \sigma^2_{\epsilon}) \geq 0.$$

Proof.

$$Cov(r_{t+\Delta}, r_t) = E((r_{t+\Delta} - E(r_{t+\Delta}))(r_t - E(r_t)))$$

Since $r_{t+1} = -\alpha\mu_t + \epsilon_{t+1}$, $\mu_{t+1} = (1-\alpha)\mu_t + \epsilon^{\mu}_{t+1}$ and μ_t is an AR(1) process, it follows that $E(\mu_t) = 0$ and $E(r_t) = 0$. It also holds that

$$\begin{aligned}
Cov(r_{t+\Delta}, r_t) &= E(r_{t+\Delta} r_t) = E((p_{t+\Delta} - p_{t+\Delta+1})(p_t - p_{t-1})) \\
&= E((\mu_{t+\Delta} - \mu_{t+\Delta-1} + p^*_{t+\Delta} - p^*_{t+\Delta-1})(\mu_t - \mu_{t-1} + p^*_t \\
&\quad -p^*_{t-1})) \\
&= E((-\alpha\mu_{t+\Delta-1} + \epsilon^{\mu}_{t+\Delta} + \epsilon^*_{t+\Delta})(-\alpha\mu_{t-1} + \epsilon^{\mu}_t + \epsilon^*_t)) \\
&= E((-\alpha\mu_{t+\Delta-1} + \epsilon_{t+\Delta})(-\alpha\mu_{t-1} + \epsilon_t)) \\
&= \alpha^2 E(\mu_{t+\Delta-1}\mu_{t-1}) - \alpha E(\mu_{t+\Delta-1}\epsilon_t) - \alpha E(\mu_{t-1}\epsilon_{t+\Delta}) \\
&\quad +E(\epsilon_{t+\Delta}\epsilon_t),
\end{aligned}$$

$$\begin{aligned}
\mu_{t+\Delta} &= (1-\alpha)\mu_{t+\Delta-1} + \epsilon^{\mu}_{t+\Delta} \\
&= (1-\alpha)((1-\alpha)\mu_{t+\Delta-2} + \epsilon^{\mu}_{t+\Delta-1}) + \epsilon^{\mu}_{t+\Delta} \\
&= (1-\alpha)^2\mu_{t+\Delta-2} + (1-\alpha)\epsilon^{\mu}_{t+\Delta-1} + \epsilon^{\mu}_{t+\Delta} \\
&= (1-\alpha)^{\Delta}\mu_t + \sum_{i=0}^{\Delta-1}(1-\alpha)^i\epsilon^{\mu}_{t+\Delta-i} \\
&= (1-\alpha)^{\Delta+1}\mu_{t-1} + \sum_{i=0}^{\Delta}(1-\alpha)^i\epsilon^{\mu}_{t+\Delta-i} \qquad (26) \\
\Rightarrow \quad \mu_{t+\Delta-1} &= (1-\alpha)^{\Delta}\mu_{t-1} + \sum_{i=0}^{\Delta-1}(1-\alpha)^i\epsilon^{\mu}_{t+\Delta-1-i}.
\end{aligned}$$

Since $\epsilon^{\mu}_t = \epsilon_t - \epsilon^*_t$ and ϵ_t and ϵ^*_t are independent from each other and both i.i.d. random noises,

$$E(\mu_{t+\Delta}\epsilon_t) = (1-\alpha)^{\Delta}\sigma^2_{\epsilon}.$$

Therefore, $E(\mu_{t+\Delta-1}\epsilon_t) = (1-\alpha)^{\Delta-1}\sigma_\epsilon^2$.

$$\begin{aligned}
\mu_t &= (1-\alpha)\mu_{t-1} + \epsilon_t^\mu = (1-\alpha)^2\mu_{t-2} + (1-\alpha)\epsilon_{t-1}^\mu + \epsilon_t\mu \\
&= (1-\alpha)^t\mu_0 + \sum_{i=0}^{t-1}(1-\alpha)^i\epsilon_{t-i}^\mu \\
\Rightarrow \quad \mu_{t-\Delta} &= (1-\alpha)^{t-\Delta}\mu_0 + \sum_{i=0}^{t-\Delta-1}(1-\alpha)^i\epsilon_{t-\Delta-i}^\mu .
\end{aligned}$$

The equation above implies that

$$E(\mu_{t-\Delta}\epsilon_t) = 0, \quad E(\mu_{t-\Delta}\epsilon_t^*) = 0. \tag{27}$$

Therefore, $E(\epsilon_{t+\Delta}\mu_{t-1}) = 0$. By properties of an AR(1) process, $E(\mu_{t+\Delta-1}\mu_{t-1}) = (1-\alpha)^\Delta \frac{\sigma_\mu^2}{1-(1-\alpha)^2}$. Therefore,

$$\begin{aligned}
Cov(r_{t+\Delta}, r_t) &= \alpha^2(1-\alpha)^\Delta \frac{\sigma_\mu^2}{(1-(1-\alpha))(1+(1-\alpha))} \\
&\quad -\alpha(1-\alpha)^{\Delta-1}\sigma_\epsilon^2 \\
&= \alpha(1-\alpha)^\Delta \frac{\sigma_\mu^2}{(2-\alpha)} - \alpha(1-\alpha)^{\Delta-1}\sigma_\mu^2 + \alpha(1-\alpha)^{\Delta-1}\sigma_{\epsilon^*}^2 \\
&= \sigma_\mu^2\alpha(1-\alpha)^{\Delta-1}\left(\frac{1-\alpha}{2-\alpha} - 1 + \lambda\right) \\
&= \sigma_\mu^2\alpha(1-\alpha)^{\Delta-1}\left(\lambda - \frac{1}{2-\alpha}\right).
\end{aligned}$$

□

B The Model of Information Feedback: the Derivation of the Quadratic Variation of the Market Price

$$\begin{aligned}
\langle p\rangle_T^\Delta &= \sigma_{\epsilon^*}^2 T + \sigma_\mu^2 T\phi(\Delta)(\alpha\bar{\kappa} - \lambda) \\
where \quad \phi(\Delta) &= \frac{2}{\alpha\Delta}(1-(1-\alpha)^\Delta).
\end{aligned}$$

Proof.

$$\begin{aligned}
\langle p\rangle_T^\Delta &= E\Big(\sum_{i=0}^{\frac{T}{\Delta}-1}(r_{i\Delta}^\Delta)^2\Big) = \sum_{i=0}^{\frac{T}{\Delta}-1} E((p_{i\Delta+\Delta} - p_{i\Delta})^2) \\
&= \sum_{i=0}^{\frac{T}{\Delta}-1} E((\mu_{i\Delta+\Delta} - \mu_{i\Delta} + p_{i\Delta+\Delta}^* - p_{i\Delta}^*)^2).
\end{aligned}$$

By (26) and $p^*_{t+1} = p^*_t + \epsilon^*_{t+1}$, we obtain

$$\langle p\rangle^{\Delta}_{T} = \sum_{i=0}^{\frac{T}{\Delta}-1} E(((1-\alpha)^{\Delta}\mu_{i\Delta} + \sum_{j=0}^{\Delta-1}(1-\alpha)^{j}\epsilon^{\mu}_{i\Delta+\Delta-j} - \mu_{i\Delta} + \sum_{j=1}^{\Delta}\epsilon^{*}_{i\Delta+j})^{2}). \tag{28}$$

As we know by (27), $E(\mu_{i\Delta}\epsilon^*_{i\Delta+j}) = 0$ for $j = 1, \ldots, \Delta$. $\mu_{i\Delta}$ is independent from $\epsilon^{\mu}_{i\Delta+\Delta-j}$, $j = 0, \ldots, \Delta - 1$, due to the nature of $\mu_t = (1-\alpha)\mu_{t-1} + \epsilon^{\mu}_t$ as an AR(1) process. Since $\epsilon^{\mu}_t = \epsilon_t - \epsilon^*_t$ and ϵ_t and ϵ^*_t are independent from each other and are i.i.d. random noises, the only cross term in (28) left to consider is

$$\begin{aligned}
&E(\sum_{j=0}^{\Delta-1}(1-\alpha)^{j}\epsilon^{\mu}_{i\Delta+\Delta-j}\sum_{j=1}^{\Delta}\epsilon^{*}_{i\Delta+j})\\
&= E(\sum_{j=0}^{\Delta-1}(1-\alpha)^{j}(\epsilon_{i\Delta+\Delta-j} - \epsilon^{*}_{i\Delta+\Delta-j})\sum_{j=1}^{\Delta}\epsilon^{*}_{i\Delta+j})\\
&= -(1-\alpha)^{0}\sigma^2_{\epsilon^*} - (1-\alpha)^{1}\sigma^2_{\epsilon^*} - \ldots - (1-\alpha)^{\Delta-1}\sigma^2_{\epsilon^*}\\
&= -\frac{1-(1-\alpha)^{\Delta}}{1-(1-\alpha)}\sigma^2_{\epsilon^*} = -\frac{1-(1-\alpha)^{\Delta}}{\alpha}\sigma^2_{\epsilon^*}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\langle p\rangle^{\Delta}_{T} &= \sum_{i=0}^{\frac{T}{\Delta}-1}\Big(((1-\alpha)^{\Delta}-1)^{2}E(\mu^2_{i\Delta}) + \sum_{j=0}^{\Delta-1}(1-\alpha)^{2j}\sigma^2_{\mu} + \sum_{j=1}^{\Delta}\sigma^2_{\epsilon^*}\\
&\quad -2\frac{1-(1-\alpha)^{\Delta}}{\alpha}\sigma^2_{\epsilon^*}\Big)\\
&= \frac{T}{\Delta}\cdot\Delta\cdot\sigma^2_{\epsilon^*} + \frac{T}{\Delta}\Big(((1-\alpha)^{\Delta}-1)^{2}\frac{\sigma^2_{\mu}}{1-(1-\alpha)^{2}} + \frac{1-(1-\alpha)^{2\Delta}}{1-(1-\alpha)^{2}}\sigma^2_{\mu}\\
&\quad -2\frac{1-(1-\alpha)^{\Delta}}{\alpha}\sigma^2_{\mu}\cdot\lambda\Big)\\
&= T\cdot\sigma^2_{\epsilon^*} + \frac{T}{\Delta}(1-(1-\alpha)^{\Delta})\sigma^2_{\mu}\Big(\frac{1-(1-\alpha)^{\Delta}}{1-(1-\alpha)^{2}} + \frac{1+(1-\alpha)^{\Delta}}{1-(1-\alpha)^{2}} - \frac{2\lambda}{\alpha}\Big)\\
&= T\cdot\sigma^2_{\epsilon^*} + \frac{T}{\Delta}(1-(1-\alpha)^{\Delta})\sigma^2_{\mu}\Big(\frac{2}{\alpha(2-\alpha)} - \frac{2\lambda}{\alpha}\Big)\\
&= T\cdot\sigma^2_{\epsilon^*} + \frac{2T}{\alpha\Delta}(1-(1-\alpha)^{\Delta})\sigma^2_{\mu}\Big(\frac{1}{2-\alpha} - \lambda\Big)\\
&= T\cdot\sigma^2_{\epsilon^*} + \frac{2T}{\alpha\Delta}(1-(1-\alpha)^{\Delta})\sigma^2_{\mu}\Big(\alpha\bar{\kappa} - \lambda\Big).
\end{aligned}$$

$\square$

C Supplementary Figures and Tables

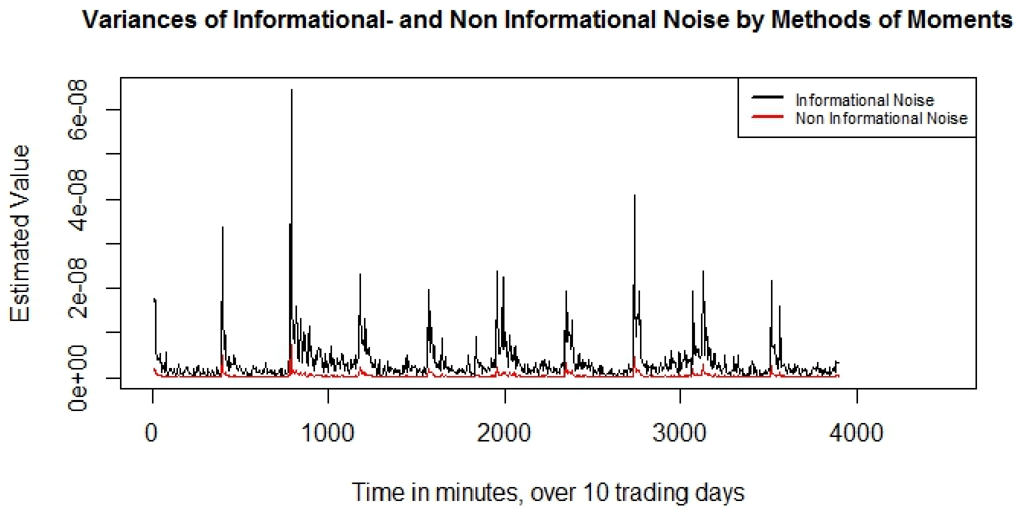

Figure 10: Estimated variances of the informational noise $\sigma^2_{\epsilon^*}$ and the non-informational noise σ^2_{ϵ} of AAPL using the method of moments

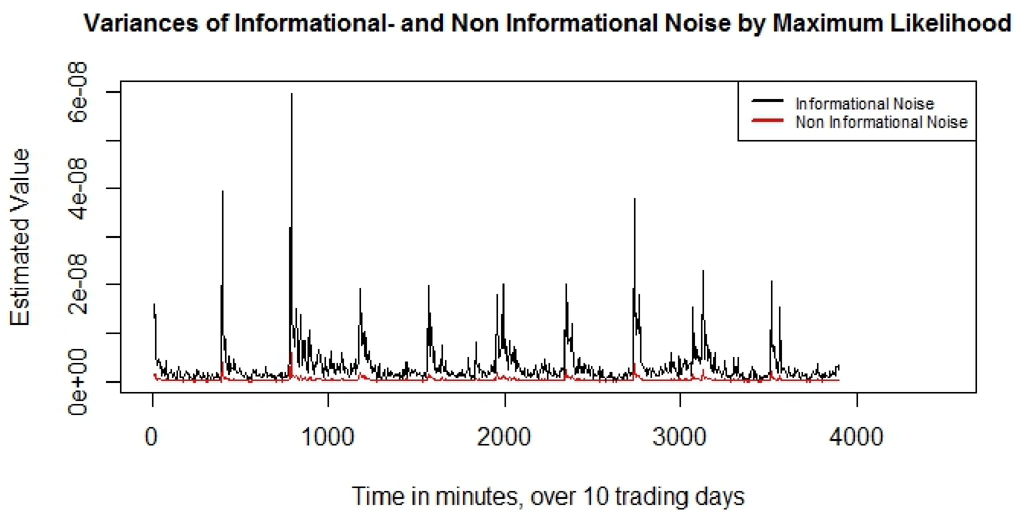

Figure 11: Estimated variances of the informational noise $\sigma^2_{\epsilon^*}$ and the non-informational noise σ^2_{ϵ} of AAPL using the maximum likelihood method and the Kalman filter

		Estimated $\sigma^2_{\epsilon^*}$						Estimated σ^2_{ϵ}					
		AAPL	AMZN	EBAY	GOOG	NFLX	TSLA	AAPL	AMZN	EBAY	GOOG	NFLX	TSLA
5 minutes	min	1.63	0.29	2.28	0.49	0.76	5.28	0.25	0.04	0.27	0.06	0	0
	max	646	621	881	442	2548	16740	73.3	90.5	137	52.2	270	1892
	mean	32.9	55.5	59.8	23.7	124	341	2.69	6.29	6.82	2.8	14.1	37.3
	sd	44.7	72.9	86.6	34.1	193	802	5.02	8.86	10.3	4.18	22.4	86.8
10 minutes	min	2.05	5.81	5.3	1.1	3.73	15.9	0.38	0	0.71	0.13	0.58	2.32
	max	392	556	636	263	1780	12230	41.9	77.4	84.8	34.4	200	1305
	mean	33	55.1	59.8	23.6	123	340	3.69	6.29	6.81	2.8	14.2	37.3
	sd	40.6	66.5	79	30.1	173	748	4.47	8.28	9.23	3.73	20.7	79.5
30 minutes	min	4.2	10.7	3.26	2.76	12.9	26.7	0.59	1.19	0.48	0.31	1.48	3.19
	max	190	357	446	187	922	5335	22	47.7	51.6	22.4	105	563
	mean	33	54.4	59.7	23.7	123	338	3.68	6.28	6.77	2.8	14.1	37.2
	sd	34.2	56.5	68.1	25.5	145	576	3.77	7.05	7.67	3.06	17.1	61.7

Table 10: Summary statistics of estimated informational volatility $\hat{\sigma}^2_{\epsilon^*}$ and non-informational volatility $\hat{\sigma}^2_{\epsilon}$ for 5-minute, 10-minute and 30-minute blocks. When using 5-minute blocks, there are 780 separate blocks over 10 trading days and thus 780 estimates of $\sigma^2_{\epsilon^*}$ and σ^2_{ϵ}; when using 10-minute blocks, there are 390 separate blocks over 10 trading days and thus 390 estimates of $\sigma^2_{\epsilon^*}$ and σ^2_{ϵ}; when using 30-minute blocks, there are 130 separate blocks over 10 trading days and thus 130 estimates of $\sigma^2_{\epsilon^*}$ and σ^2_{ϵ}. Only the results of method of moments are presented in the table, and summary statistics including the minimum, maximum, mean and standard deviation are used to show the characteristics of the distribution of the estimates. The numbers presented in the table are obtained by multiplying the original estimates by 10^{10} for the sake of clarity.

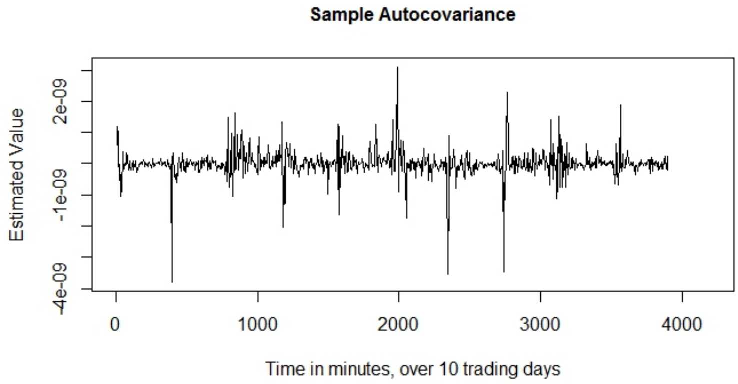

Figure 12: Sample autocovariances of AAPL from 2.1.2014 to 15.1.2014

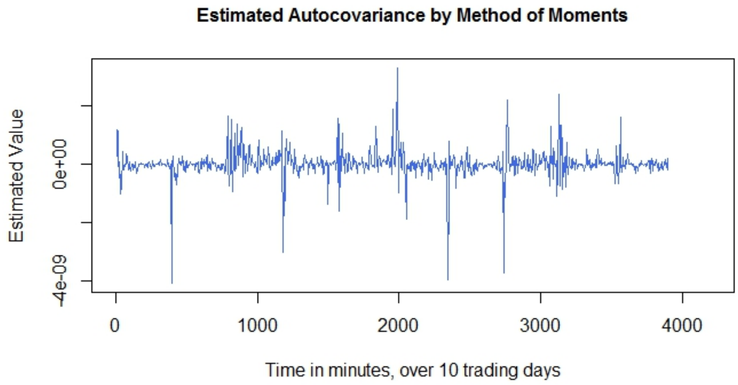

Figure 13: Estimated autocovariances of AAPL using the method of moments

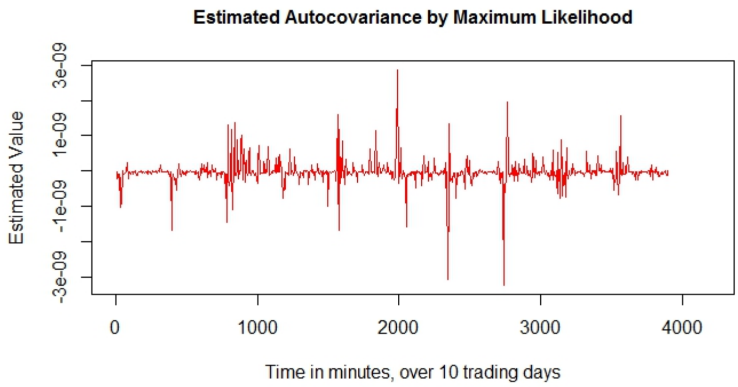

Figure 14: Estimated autocovariances of AAPL using the maximum likelihood method and the Kalman filter

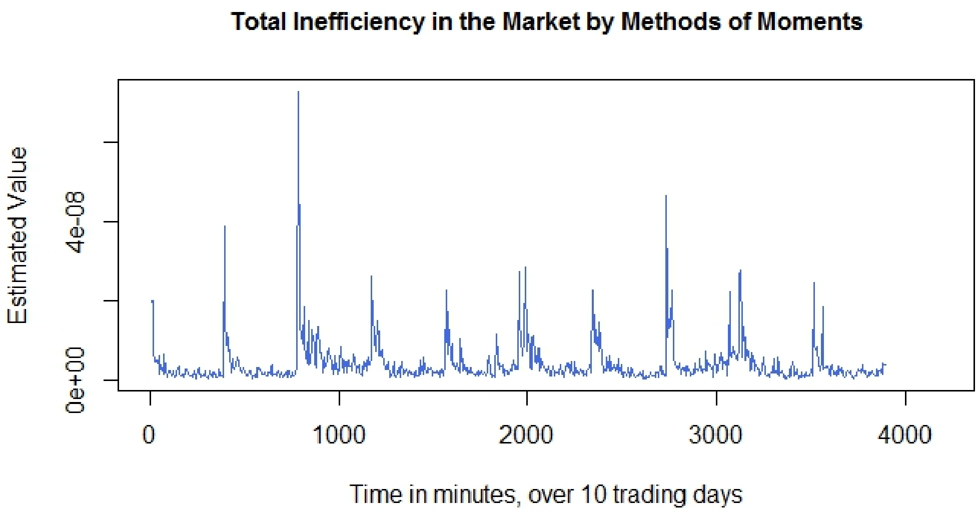

Figure 15: Estimated total inefficiency in AAPL

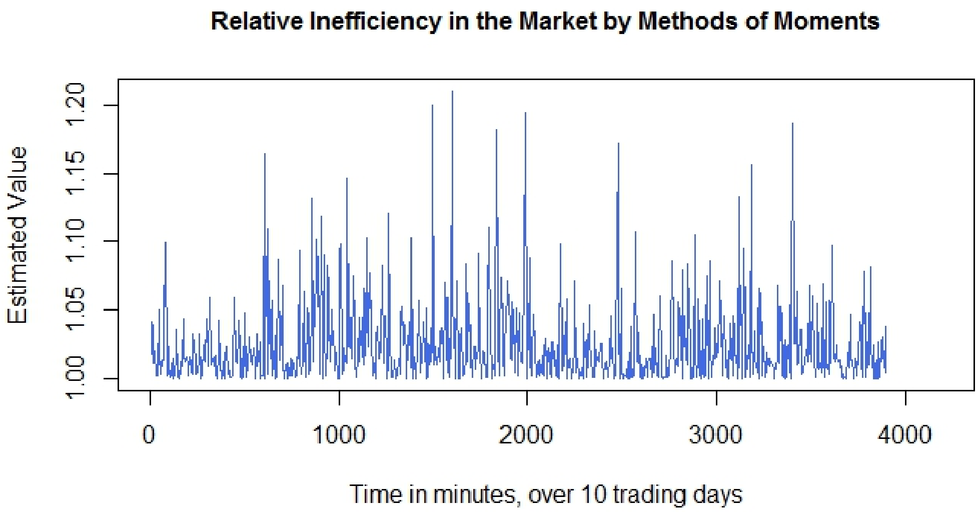

Figure 16: Estimated relative inefficiency in AAPL

		Estimated κ						Estimated $\bar{\kappa}$					
		AAPL	AMZN	EBAY	GOOG	NFLX	TSLA	AAPL	AMZN	EBAY	GOOG	NFLX	TSLA
5 minutes	min	1.88	0.33	2.62	0.55	0.86	6.36	1	1	1	1	1	1
	max	730	713	1018	501	2934	18940	1.21	1.44	1.32	1.22	1.26	2.12
	mean	27.7	63.9	18.3	27	142	392	1.02	1.03	1.02	1.02	1.03	1.03
	sd	51.1	83.8	98.7	39	220	914	0.03	0.04	0.03	0.03	0.04	0.05
10 minutes	min	2.44	6.59	6.03	1.25	4.31	18.3	1	1	1	1	1	1
	max	443	635	721	299	2015	13840	1.14	1.2	1.15	1.11	1.21	1.12
	mean	37.7	62.9	68	26.9	141	388	1.02	1.03	1.02	1.02	1.02	1.02
	sd	46.2	76	89.7	34.3	197	849	0.02	0.03	0.02	0.02	0.03	0.02
30 minutes	min	4.8	12.2	3.74	3.13	14.6	30.2	1	1	1	1	1	1
	max	215	407	504	211	1043	6040	1.06	1.11	1.09	1.05	1.07	1.08
	mean	37.4	61.9	67.7	26.9	140	383	1.02	1.02	1.02	1.01	1.02	1.02
	sd	38.7	64.3	77.2	29	164	653	0.01	0.02	0.01	0.01	0.01	0.01

Table 11: Summary statistics of estimated total inefficiency $\hat{\kappa}$ and relative inefficiency $\hat{\bar{\kappa}}$ for 5-minute, 10-minute and 30-minute blocks. When using 5-minute blocks, there are 780 separate blocks over 10 trading days and thus 780 estimates of κ and $\bar{\kappa}$; when using 10-minute blocks, there are 390 separate blocks over 10 trading days and thus 390 estimates of κ and $\bar{\kappa}$; when using 30-minute blocks, there are 130 separate blocks over 10 trading days and thus 130 estimates of κ and $\bar{\kappa}$. Only the results obtained by using the method of moments are presented in the table, and summary statistics including minimum, maximum, mean and standard deviation are used to show the characteristics of the distribution of estimates. The estimates of total inefficiency presented in the table are obtained by multiplying the original estimates by 10^{10} for the sake of clarity.

List of Figures