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„Classically scale invariant extensions of the Standard Model“

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Carla Schuler, BSc

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Abstract

In this thesis, an extension of the Standard Model (SM) with classical scale invariance is studied, where spontaneous symmetry breaking occurs due to quantum corrections and the generation of the masses of all particles is described by the Coleman-Weinberg mechanism. To achieve a possible theoretically viable scale-invariant version of the SM, the SM Higgs sector is enlarged by a complex scalar field, being a singlet with respect to the SM gauge group and the SM gauge group is extended by an additional abelian $\mathrm{U}(1)_{\mathrm{z}}$ factor, resulting in the prediction of a new heavy color-singlet and electrically-neutral Z' boson. The introduction of right-handed neutrino fields gives the possibility to generate neutrino masses via the see-saw mechanism, with heavy neutrino Majorana masses in the TeV region. Assuming generation-independent $\mathrm{U}(1)_{\mathrm{z}}$ charges for the observed quarks and leptons, it is found that the addition of three right-handed neutrino fields leads to a set of solutions for the $\mathrm{U}(1)_{\mathrm{z}}$ charges with one free parameter being consistent with anomaly cancellation conditions and viable fermion mass terms. In the scalar sector an analysis of the scalar effective potential up to one-loop level is performed, using the perturbative framework introduced by Gildener and Weinberg.

Zusammenfassung

In dieser Arbeit wird eine Erweiterung des Standardmodells (SM) mit klassischer Skaleninvarianz untersucht, bei welcher spontane Symmetriebrechung aufgrund von Quantenkorrekturen auftritt und die Erzeugung der Massen aller Teilchen durch den Coleman-Weinberg-Mechanismus beschrieben wird. Um eine mögliche, in der Theorie realisierbare, skaleninvariante Version des SM zu bilden, wird der Higgs-Sektor des SM um ein komplexes Skalarfeld, welches ein Singulett bezüglich der SM Eichgruppe ist und die SM Eichgruppe um einen Abel'schen $\mathrm{U}(1)_{\mathrm{z}}$ Faktor erweitert. Daraus resultiert die Vorhersage eines neuen schweren, farbneutralen und elektrisch neutralen Z' Bosons. Die Einführung rechtshändiger Neutrinofelder ermöglicht die Erzeugung von Neutrino-Majorana-Massen durch den Seesaw-Mechanismus im TeV Bereich. Bei Annahme von generationsunabhängigen $\mathrm{U}(1)_{\mathrm{z}}$ Ladungen der beobachteten Quarks und Leptonen, stellt sich heraus, dass eine Ergänzung durch drei zusätzliche Neutrinofelder zu einer einparametrigen Lösungsschar für die $\mathrm{U}(1)_{\mathrm{z}}$ Ladungen führt. Diese ist mit den Bedingungen für die Anomaliefreiheit und der Existenz von Fermionmassentermen konsistent. Im skalaren Sektor wird eine Analyse des skalaren effektiven Potentials auf Einschleifenniveau durchgeführt, bei welcher nach einer störungstheoretischen Methode nach Gildener und Weinberg vorgegangen wird.

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Contents

1 Introduction

The Standard Model (SM) [1–3], the theory of strong and electroweak interactions, is a renormalizable field theory which describes the elementary particles and their interactions and realizes the famous Higgs mechanism [4–6], accounting for the generation of the masses of the known particles. Despite the model can record a series of extraordinary successes in explaining the experimental data collected in the past decades [7], since its formulation, many phenomena are unaccounted. The experimental observation of neutrino oscillations [8–10], describing neutrinos as massive particles, is one indication for an extension of the SM to a theory Beyond the Standard Model (BSM), where possible new physical degrees of freedom may arise at a scale Λ higher than the electroweak scale.

Theories with classical scale invariance at tree-level form a promising possibility of extensions of the SM. In these theories the generation of the particles' masses is realized through the Coleman-Weinberg (CW) mechanism [11], and spontaneous symmetry breaking (SSB) occurs as a consequence of quantum corrections, rather than a negative mass term of a scalar field, the SM Higgs field, in the Lagrangian. Since a formulation of the SM with classical scale invariance cannot be achieved due to the experimentally measured mass values of the Higgs boson [12] and the top quark [13], the SM particle content needs to be extended. In modern theoretical particle physics, many extensions of the SM, with an modified Higgs-Sector with possible additional real or complex scalar fields and/or an extension of the SM gauge group by additional (non-)abelian factors, are studied [14–48].

In this thesis an extension of the SM with classical scale invariance is discussed, where the SM gauge group is extended by an additional abelian $\mathrm{U}(1)_\mathrm{z}$ factor and the SM Higgs-Sector is enlarged by a complex scalar singlet field. Due to the new $\mathrm{U}(1)_\mathrm{z}$ gauge group a new color-singlet and electrically neutral gauge boson, commonly referred to as Z' boson, appears in the particle spectrum.

When analyzing the most general gauge symmetry leading to an additional Z' boson, it is observed that the electric charges of the observed fermions imply, that the abelian part of the gauge group of the model may be taken to $\mathrm{U}(1)_\mathrm{z} \times \mathrm{U}(1)_\mathrm{Y}$, where the $\mathrm{U}(1)_\mathrm{Y}$ denotes the SM hypercharge gauge group. Following the assumptions of ref. [49], the kinetic mixing between $\mathrm{U}(1)_\mathrm{Y}$ and the new $\mathrm{U}(1)_\mathrm{z}$ may be taken to vanish at any particular scale, without loss of generality, which is simplifying the analysis of the theory.

The introduction of three additional right-handed neutrino fields provides the possibility of the generation of neutrino masses via the see-saw mechanism [50–53]. With an $\mathrm{U}(1)_\mathrm{z}$ symmetry breaking scale chosen to be in the TeV region, neutrino Majorana masses of $\mathcal{O}(\mathrm{TeV})$ arise. Assuming generation independent $\mathrm{U}(1)_\mathrm{z}$ charges for the observed quarks and leptons, it is found that the addition of three right-handed neutrino fields leads to a set of solutions for the $\mathrm{U}(1)_\mathrm{z}$ charges with one free para- meter being consistent with anomaly cancellation conditions and viable fermion mass terms [49].

In the scalar sector a perturbative analysis of the scalar effective potential of the model up to one-loop order is performed. The scalar effective potential is calculated up to one-loop level and its properties are studied using the method suggested by Gildener and Weinberg (GW) [54] to investigate the perturbative validity of the model, when taking into account constraints given by experimentally measured values of quantities of the known SM particles. With the experimental values of the masses of the Higgs boson, the top quark and the W- and Z-bosons, a constraining relation for the mass of the new Z' boson and the heavy neutrino Majorana masses can be formulated.

The thesis is organized as follows. Section 2 provides an overview of the general structure of the SM, its problems and a short introduction to theories with classical scale invariance. This is followed by the introduction of the concepts of generating functionals and the scalar effective potential as the mathematical tools to describe the CW mechanism in Section 3. Section 4 gives a discussion of the framework of GW for the perturbative analysis of the properties of the effective potential for massless renormalizable theories with an arbitrary number of weakly coupled scalar fields. The properties of the model under consideration are presented in Section 5. The scalar and gauge sector are studied and the anomaly cancellation conditions for the $\mathrm{U}(1)_{\mathrm{z}}$ charges of the particle content of the model are discussed. This is succeeded by a short section about Yukawa interactions, fermion masses and the principles of the see-saw mechanism. Finally, the calculation of the couplings of gauge boson-scalar interactions is presented and the section closes with a short summary.

2 The Standard Model

The Standard Model (SM) [1–3] is a chiral local gauge theory of the electroweak and strong interactions with the underlying non-abelian gauge symmetry group

$$\mathrm{G_{SM}} = \mathrm{SU(3)_C} \times \mathrm{SU(2)_L} \times \mathrm{U(1)_Y}, \tag{1}$$

where $\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ describes the electromagnetic and weak interactions and $\mathrm{SU(3)_C}$ denotes the symmetry group of the strong interactions [55] .

2.1 Symmetries, Lie groups and Representations of SU(N)

This short mathematical subsection is based on refs. [56] and [57].

In general a symmetry is a certain transformation ϕ of a physical system that maps allowed configurations of the field space $\mathcal{M}$ of the system in other allowed, „equivalent“ configurations,

$$\phi : \mathcal{M} \to \mathcal{M}. \tag{2}$$

In the context of elementary particle physics it is focused on symmetries given by the Lie group SU(N) that acts on the field space of the Lagrangian systems of the SM and leads to conservation laws via Noether's Theorem.

The special unitary group SU(N) is given by

$$\mathrm{SU(N)} = \{A \in \mathrm{Mat(N,\mathbb{C})},\ AA^\dagger = \mathbb{1}, \det A = 1\}. \tag{3}$$

A Lie group G is a **continuous** group, containing infinitely many different transformations that are related in a differentiable way, such that the maps

$$\mu : \quad G \times G \to G \tag{4}$$
$$(g_1, g_2) \mapsto g_1 \cdot g_2, \qquad g_1, g_2 \in G \tag{5}$$

and

$$\nu : \quad G \to G \tag{6}$$
$$g \mapsto g^{-1}, \qquad g \in G \tag{7}$$

are smooth.

SU(N) is a **connected** Lie group. Any two points in this space equipped with "coordinates" can be connected by a curve lying wholly in it. The group elements can be described by a finite set of real continuous parameters,

$$g(\vec{\theta}) = g(\theta_1, \theta_2, ...\theta_n), \qquad \theta_i \in \mathbb{R} \quad \forall\, i \in 1, ..., n \tag{8}$$

This description connects each element to the identity by a path within the group, with the identity element given by

$$e = g(\vec{0}). \tag{9}$$

The multiplication law can be written in the form

$$g(\vec{\theta}) \cdot g(\vec{\theta'}) = g(\vec{f}(\vec{\theta}, \vec{\theta'})), \tag{10}$$

where $\vec{f}$ is a mapping of the parameter space into itself and a smooth function with respect to its variables. The following relations for $\vec{f}$ hold:

$$\vec{f}(0, \vec{\theta}) = \vec{f}(\vec{\theta}, 0) = \vec{\theta}, \tag{11}$$

$$\vec{f}(\vec{\theta}, \vec{f}(\vec{\varphi}, \vec{\chi})) = \vec{f}(\vec{f}(\vec{\theta}, \vec{\varphi}), \vec{\chi}), \tag{12}$$

$\vec{f}(\vec{\theta}, \vec{\theta'}) = 0$ if $g^{-1}(\vec{\theta})$ is parametrized as $g(\vec{\theta'})$.

A **linear representation** of a group is a specific realization of the group and its multiplication law on a vector space. In the language of group theory a representation of the group G is a group homomorphism

$$\pi : G \to GL(V), \tag{13}$$

which associates each group element to a specific element in $GL(V)$, the group of linear transformations, acting on the elements in the vector space V. If the vector space is real, complex (i.e. $V = \mathbb{R}^n$ or $V = \mathbb{C}^n$, essentially) it is referred to a real, complex representation, respectively. Since in physics the vector space under consideration is the Hilbert space of physical states, representations by operators and matrices are of interest.

As a consequence of Wigner's Theorem [58], any symmetry (like a rotation, translation, Lorentz transformation,...) that is continuously connected to the identity, must be represented by a linear unitary operator $U(g(\vec{\theta}))$.

For a Lie group, these operators can be represented in at least a finite neighborhood of the identity by a power series,

$$U(g(\vec{\theta})) = 1 + i\theta^A T^A + \frac{1}{2}\theta^B\theta^C T^{BC} + ..., \qquad (A, B, C = 1, ..., N) \tag{14}$$

where T^A, the generators of the Lie group, and T^{BC} are Hermitian operators and independent of $\vec{\theta}$.

According to previous equations, the expansion of f^A to second order must take the form

$$f^A(\vec{\theta}, \vec{\theta'}) = \theta^A + \theta'^A + \frac{1}{2}\theta^B\theta'^C c^{BCA} + ... \tag{15}$$

with c^{BCA} being real coefficients.

From the relation $U(g(\vec{\theta}))U(g(\vec{\theta}')) = U[g(\vec{f}(\vec{\theta},\vec{\theta}'))]$ and comparison of coefficients, one obtains

$$T^{BC} = -T^B T^C - ic^{BCA}T^A. \tag{16}$$

This leads to the commutation relation

$$[T^B, T^C] = if^{BCA}T^A, \tag{17}$$

with the anti-symmetric so called structure constants $f^{BCA} \equiv -c^{BCA} + c^{CBA}$. The set of commutation relations defines the Lie algebra of the Lie group, which encodes the structure of the corresponding Lie group.

A **Lie algebra** $\tilde{g}$ over $\mathbb{R}$ (resp. $\mathbb{C}$, etc., ... any field space) is a vector space over $\mathbb{R}/\mathbb{C}$ with an operation (Lie bracket),

$$[.,.] : \tilde{g} \times \tilde{g} \to \tilde{g}, \tag{18}$$

which is bilinear and satisfies

$$[X, X] = 0, \tag{19}$$

implying

$$[X, Y] = -[Y, X], \qquad \text{"}\textit{antisymmetry}\text{"} \tag{20}$$

and the Jacobi identity

$$[X, [Y, Z]] + [Z, [X, Y]] + [Y, [Z, X]] = 0. \tag{21}$$

In the case of the Lie algebra corresponding to the Lie group SU(N), the Lie bracket is the commutator:

$$[X, Y] = XY - YX. \tag{22}$$

structure constants. Let $\tilde{g}$ be a finite dimensional Lie algebra and let the generators T^A build a basis for $\tilde{g}$ (as a vector space). Then for each A and B, $[T^A, T^B]$ can be written uniquely in the form

$$[T^A, T^B] = i \sum_C f^{ABC} T^C. \tag{23}$$

The f^{ABC} are the structure constants in (17) and determine uniquely the bracket operation on $\tilde{g}$.

Knowing the structure of the group, given by $\vec{f}(\vec{\theta},\vec{\theta}')$, one can calculate the second order term of $U(g(\vec{\theta}))$ from the generators T^A appearing in the first order term. The commutation relation in (17) is the only condition that this process can be continued. Hence the complete power series for $U(g(\vec{\theta}))$ may be calculated from an infinite sequence of relations like for T^{BC} in (16). So $U(g(\vec{\theta}))$ is uniquely determined in at least a finite neighborhood of the identity, such that Eq. (10) holds, providing $\vec{\theta}, \vec{\theta'}$ and $\vec{f}(\vec{\theta}, \vec{\theta'})$ lie in this neighborhood.

The number of generators of a Lie group does not depend on the representation of the Lie group, but the form of the generators in general does. The structure constants are independent of the representation.

In the following, the focus lies on the **fundamental** and **adjoint representations** of SU(N). The fundamental representation of SU(N) is its defining matrix representation. In the Hilbert space of physical states, fields or states transforming according to the fundamental representation of SU(2) and SU(3) form doublets and triplets, respectively. The proton and neutron transform as a SU(2) isospin doublet $\begin{pmatrix} p \\ n \end{pmatrix}$ and the left-handed fields in the SM are $\mathrm{SU(2)_L}$ doublets (e.g. $\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$). Each quark flavour $\begin{pmatrix} u_g & u_b & u_r \end{pmatrix}$ lives in the fundamental representation of $\mathrm{SU(3)_C}$.

The Lie groups SU(2) and SU(3) are the non-abelian factors of the SM gauge group, whereas U(1) is abelian. The gauge fields transform according to the adjoint representation.

Any unitary matrix M can be written in terms of a Hermitian matrix H:

$$M = \exp iH. \tag{24}$$

From the relations

$$\det(\exp A) = \exp(\mathrm{tr}A) \tag{25}$$

and

$$\det M = 1, \tag{26}$$

it follows

$$\mathrm{tr}H = 0. \tag{27}$$

Since there are $N^2 - 1$ traceless $N \times N$ matrices, an element M of SU(N) can be written as

$$M = \exp(i \sum_{n=1}^{N^2-1} \theta^n T^n), \tag{28}$$

with $\theta^n \in \mathbb{R}$, $(T^n)^\dagger = T^n$ and $\mathrm{tr}(T^n) = 0$.

Therefore the generators of the fundamental SU(2) representation are three 2×2 traceless Hermitian matrices, in general chosen to be a half times the Pauli matrices, $\frac{1}{2}\sigma^i$. The Pauli matrices are given by

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{29}$$

SU(3) has a number of eight generators T^A, chosen to be $T^A = \frac{1}{2}\lambda^A$, where the λ^As are the so called Gell-Mann matrices, given by

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{30}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{31}$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \tag{32}$$

With the choices for the SU(2) and SU(3) generators given above, the normalization for the generators reads:

$$\mathrm{Tr}[T^A T^B] = \frac{1}{2}\delta^{AB}. \tag{33}$$

If a set of fields transforms according to the fundamental representation, the set of complex conjugate fields transforms according to the **anti-fundamental representation**, whose generators read

$$T^A_{antifund,ij} = -(T^A_{fund})^*_{ij} = -T^A_{fund,ji}. \tag{34}$$

If quark fields $q_{k,\alpha}$, (with $\alpha =$ red, blue, green, and the flavour index $k = 1, 2, 3$), transform according to the fundamental representation of $\mathrm{SU}(3)_C$ and anti-quarks transform according to the anti-fundamental representation, the basic color singlet states are given by $\sum_{\alpha,k} |q_{k,\alpha}\, \overline{q}_{k,\alpha}\rangle$, describing a meson state and $\sum_{\alpha,\beta,\gamma,k,l,m} \epsilon^{\alpha,\beta,\gamma} |q_{k,\alpha}\, q_{l,\beta}\, q_{m,\gamma}\rangle$, describing a baryon state.

The **adjoint representation** of SU(N) acts on the vector space spanned by the generators T^A. Since SU(N) has $N^2 - 1$ generators, the adjoint representation is $N^2 - 1$ dimensional and its generators are $(N^2 - 1) \times (N^2 - 1)$ matrices.

The T^A's being the generators of the SU(N) representation $U(g)$, the adjoint representation satisfies:

$$U(g) T^B U^{-1}(g) = \sum_{D=1}^{N^2-1} T^D\, U^{adj}_{DB}(g). \tag{35}$$

By expanding $U(g)$ in the neighborhood of the identity,

$$U(g) = \mathbb{1} + i\theta^A T^A + ..., \tag{36}$$

one gets for the adjoint representation,

$$U^{adj}_{DB}(g) = \mathbb{1} + i\theta^A (T^A_{adj})_{DB} + ..., \tag{37}$$

with $(T^A_{adj})_{DB}$ being the generators of the adjoint representation. For the commutation relation for the generators of the fundamental representation, one has

$$\underbrace{[T^A, T^B]}_{=if^{ABD}\,T^D} = \sum_{D=1}^{N^2-1} T^D (T^A_{adj})_{DB}. \tag{38}$$

Thus, it follows that

$$(T^A_{adj})_{DB} = if^{ABD}. \tag{39}$$

In analogy to the fundamental representation the commutation relation for the adjoint representation reads:

$$[T^B_{adj}, T^C_{adj}] = if^{BCA} T^A_{adj}. \tag{40}$$

For the normalization one obtains

$$\mathrm{Tr}[T^A_{adj}\, T^B_{adj}] = \sum_{C,D} f^{CDA} f^{CDB} \equiv T_A \delta^{AB}, \tag{41}$$

where $T_A = N$.

2.2 General framework of the SM

Following refs. [59] and [60], sections 2.2, 2.3 and 2.4 give a short overview of the framework of the SM.

One component to construct a gauge-invariant SM Lagrangian is the covariant derivative given by

$$D_\mu = \partial_\mu + igT^i W^i_\mu + ig_Y Y B_{Y\mu} + ig_S \mathcal{T}^A G^A_\mu, \tag{42}$$

where $T^i, \mathcal{T}^A$ and Y denote the generators of the $\mathrm{SU}(2)_L$, $\mathrm{SU}(3)_C$ and $\mathrm{U}(1)_Y$ gauge groups, respectively. G^A_μ are the eight gluon fields, mediators of the strong interactions and W^i_μ, $B_{Y\mu}$ give the four vector boson fields mediating the electroweak interactions.

The SM fermion and scalar field content, with three generations of left- and right-handed quark fields, Q_L, u_R, d_R, and lepton fields, L, l_R, (flavour indices are understood for the fermion fields), and the complex Higgs doublet H, is given in Tab.1.

The $\mathrm{U(1)_Y}$ hypercharge Y is assigned such, that the relation

$$Q = T_{3L} + \frac{Y}{2} \tag{43}$$

holds. Q denotes the electric charge and T_{3L} is the third weak isospin component, the eigenvalue of T_3.

Name	$\mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y}$
$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$(\mathbf{3},\mathbf{2},\frac{1}{3})$
u_R	$(\overline{\mathbf{3}},\mathbf{1},\frac{4}{3})$
d_R	$(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})$
$L = \begin{pmatrix} \nu_L \\ l_L \end{pmatrix}$	$(\mathbf{1},\mathbf{2},-1)$
l_R	$(\mathbf{1},\mathbf{1},-2)$
$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$	$(\mathbf{1},\mathbf{2},1)$

Table 1: Fermion and scalar particle content of the SM.

The most general gauge invariant Lagrangian including the kinetic terms for the fermions, the Higgs field and the gauge fields plus the gauge interactions of fermionic particles and the Higgs boson reads

$$\begin{aligned} \mathcal{L}_{\mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y}} &= \sum_{\psi^j} i\overline{\psi}^j \not{D} \psi^j + (D^\mu H)^\dagger (D_\mu H) \\ &\quad -\frac{1}{4} G^A_{\mu\nu} G^{A\mu\nu} - \frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu} - \frac{1}{4} B_{Y\mu\nu} B^{\mu\nu}_Y , \end{aligned} \tag{44}$$

where $\psi^j = Q_L, u_R, d_R, L, \nu_R, l_R$ and $\not{D} \equiv \gamma_\mu D^\mu$. The pure Yang Mills terms are given by

$$G^{A\mu\nu} = \partial^\mu G^{A\nu} - \partial^\nu G^{A\mu} + g_S f^{ABC} G^{B\mu} G^{C\nu}, \tag{45}$$

$$W^{i\mu\nu} = \partial^\mu W^{i\nu} - \partial^\nu W^{i\mu} + g\epsilon^{ijk} W^{j\mu} W^{k\nu}, \tag{46}$$

$$B^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu, \tag{47}$$

with f^{ABC} and ϵ^{ijk} being the structure constants of $\mathrm{SU(3)_C}$ and $\mathrm{SU(2)_L}$, respectively as introduced in Sect. 2.1, where ϵ^{ijk} is the Levi-Cita symbol in three dimensions.

2.3 Higgs Sector of the SM - Electroweak Symmetry Breaking

The scalar potential $V(H)$ of the SM Lagrangian is of the form

$$V(H) = m^2 (H^\dagger H) + \frac{\lambda}{2} (H^\dagger H)^2, \tag{48}$$

where H is the SM complex Higgs doublet in the fundamental spin-$\frac{1}{2}$ representation of SU(2),

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}, \qquad \phi_i \in \mathbb{R}, \;\; i = 1, ..., 4. \tag{49}$$

To assure that $V(H)$ is bounded from below, the parameter λ is chosen to be $\lambda > 0$. The choice of $m^2 < 0$ leads to a minimum of the potential away from the origin, as illustrated in Fig.1, and the scalar field acquires a vacuum expectation value (VEV),

$$\langle 0|H|0\rangle = \langle H\rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{50}$$

where, without loss of generality, v is taken to be $v > 0$. This phenomenon is known as spontaneous symmetry breaking (SSB).

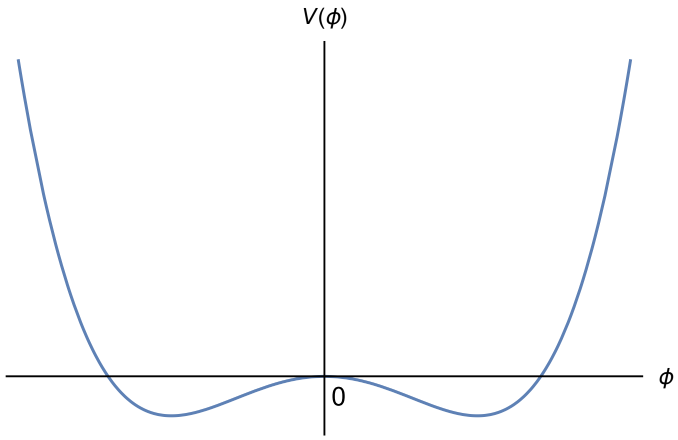

Figure 1: $V(\phi) = \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4$ for a real scalar field $\phi(x)$, with $m^2 < 0$.

In the SM the gauge group of the electroweak interactions $\mathrm{SU(2)_L \times U(1)_Y}$ is spontaneously broken down to $\mathrm{U(1)_{EM}}$,

$$\mathrm{SU(2)_L \times U(1)_Y \to U(1)_{EM}}. \tag{51}$$

Goldstone's-Theorem for gauge theories [61, 62] says that for each generator of a gauge group G whose symmetry transformation is broken by the vacuum state of the Higgs field, there is a corresponding massless Goldstone boson. The gauge boson associated to this broken generator acquires a mass through the dynamics of the Goldstone boson.

In the case of the SM there are three broken generators of the gauge group of weak interactions $\mathrm{SU(2)_L \times U(1)_Y}$, which results in three massive gauge bosons, the $W^{\pm}$- and the Z-boson, whereas the generator of the $\mathrm{U(1)_{EM}}$ is unbroken and the photon remains massless and the massless state does not appear in the spectrum of the theory.

Expanding $V(H)$ around its vacuum expectation value,

$$H \to \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}, \tag{52}$$

with the standard choice of the VEV

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \tag{53}$$

and the Higgs doublet in unitary gauge, which is a specific choice of gauge,

$$H \to e^{-i\alpha^i(x)T^i} e^{-i\beta(x)Y} H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h(x) \end{pmatrix}, \tag{54}$$

such that $h(x)$ is a real scalar field and the massless Goldstone bosons become part of the massive gauge bosons and vanish from the particle spectrum, yields, (omitting the constant term)

$$V = \frac{1}{2} 2\lambda v^2 h^2 + \lambda v h^3 + \frac{1}{4}\lambda h^4, \tag{55}$$

with the Higgs mass

$$m_h^2 = 2\lambda v^2. \tag{56}$$

The kinetic term of the Higgs field gives the gauge boson mass spectrum,

$$\begin{aligned} (D^\mu H)^\dagger (D_\mu H) &= \frac{1}{2}\partial^\mu h \partial_\mu h + \frac{(v+h)^2}{8} \left[g^2 (W^{1\mu} - iW^{2\mu})(W^1_\mu + iW^2_\mu) \right. \\ &\quad \left. + (g_Y B^\mu - g W^{3\mu})^2 \right]. \end{aligned} \tag{57}$$

Introducing the gauge boson mass eigenstates,

$$W^\pm_\mu = \frac{W^1_\mu \mp i W^2_\mu}{\sqrt{2}}, \tag{58}$$

and

$$\begin{pmatrix} A_{\gamma\mu} \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B_{Y\mu} \\ W^3_\mu \end{pmatrix}, \tag{59}$$

with

$$\sin\theta_W = \frac{g_Y}{\sqrt{g_Y^2 + g^2}}, \; \cos\theta_W = \frac{g}{\sqrt{g_Y^2 + g^2}}, \tag{60}$$

where θ_W denotes the Weinberg angle, one obtains the masses for the W- and the Z-bosons from

$$(D^\mu H)^\dagger (D_\mu H) = \frac{1}{2}\partial^\mu h \partial_\mu h + \frac{(v+h)^2}{4} g^2 W^{+\mu} W^-_\mu + \frac{(v+h)^2}{8} (g_Y^2 + g^2) Z^\mu Z_\mu, \tag{61}$$

with,

$$m_W^2 = \frac{1}{4} g^2 v^2, \tag{62}$$

$$m_Z^2 = \frac{1}{4}(g_Y^2 + g^2)v^2. \tag{63}$$

The definition of the Fermi coupling constant

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}, \tag{64}$$

together with Eq. (62) yields

$$v = \sqrt{\frac{1}{\sqrt{2}G_F}}; \tag{65}$$

hence from the experimentally measured value of G_F [7], one obtains a vacuum expectation value of the SM Higgs field of $v = 246\,\mathrm{GeV}$.

The interaction Lagrangian between gauge bosons and fermions, written in the mass eigenstate basis for vector fields includes the terms

$$\mathcal{L}_{cc} + \mathcal{L}_{nc} + \mathcal{L}_{qg}, \tag{66}$$

where the charged current terms are

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} W_\mu^+ \left(\overline{\nu}_L \gamma^\mu e_L + \overline{u}_L \gamma^\mu d_L\right) + \text{h.c.}, \tag{67}$$

and the neutral current Lagrangian reads

$$\mathcal{L}_{nc} = \sum_{\psi^i} \left[e A_\mu Q^i \overline{\psi^i} \gamma^\mu \psi^i + \frac{g}{\cos\theta_W} Z_\mu \left(T_{3L}^i - \sin^2\theta_W Q^i) \overline{\psi^i} \gamma^\mu \psi^i \right) \right], \tag{68}$$

where $\psi^i = \{u_L, d_L, u_R, d_R, \nu_L, e_L, e_R\}$ and

$$e = g_Y \cos\theta_W = g \sin\theta_W \tag{69}$$

is the unit electric charge. Finally, one has the gluon quark interaction terms with

$$\mathcal{L}_{qg} = g_S \left(\overline{u} \gamma^\mu \mathcal{T}^A u + \overline{d} \gamma^\mu \mathcal{T}^A d \right) G_\mu^A, \tag{70}$$

where u and d denote the full Dirac spinors, including left- and right-handed components.

2.4 Yukawa interactions and Fermion masses

Since fermion mass terms do not respect chiral gauge symmetry, fermion masses are generated introducing Yukawa interaction terms listed up below:

$$\mathcal{L}_{\mathrm{Y}} = -\overline{L}_i \lambda_{ij}^l H l_{jR} - \overline{Q}_{iL} \lambda_{ij}^u \tilde{H} u_{jR} - \overline{Q}_{iL} \lambda_{ij}^d H d_{jR} + \text{h.c.}\,. \tag{71}$$

L_i and Q_{iL} denote the left-handed lepton and quark doublets with

$$L_i = \begin{pmatrix} \nu_L \\ l_L \end{pmatrix}_i , \tag{72}$$

$$Q_{iL} = \begin{pmatrix} u_L \\ d_L \end{pmatrix}_i , \tag{73}$$

and l_{iR}, u_{iR} and d_{iR} denote the right-handed lepton, up-quark and down-quark fields respectively, $(i = 1, 2, 3)$. $\lambda^{l,u,d}$ are in general complex-valued 3×3 matrices, containing the Yukawa couplings and $\tilde{H} = i\sigma_2 H^*$, with hypercharge $Y = -1$. For reasons of simplicity the flavour indices are suppressed in the following.

2.4.1 Charged leptons

The Yukawa term for charged leptons in unitary gauge after SSB breaking is given by

$$\mathcal{L}_{\mathrm{Y},cl} = -\frac{1}{\sqrt{2}} \bar{l}_L \lambda^l \, l_R \, (v + h) + \text{h.c.} \, . \tag{74}$$

From Eq. (74) one obtains the mass matrix of the leptons

$$M^l = \frac{1}{\sqrt{2}} v \lambda^l \, . \tag{75}$$

The mass matrix M^l can be diagonalized by a bi-unitary transformation

$$U_L^{l\dagger} M^l U_R^l = \hat{M}^l, \tag{76}$$

with $\hat{M}^l$ diagonal and positive, yielding the physical lepton fields $\hat{l} = \hat{l}_L + \hat{l}_R$ via the relation

$$l_{L,R} = U_{L,R}^l \, \hat{l}_{L,R} \, . \tag{77}$$

Therefore the Yukawa term for the charged leptons written in mass eigenstates reads

$$\mathcal{L}_{\mathrm{Y},cl} = -\bar{\hat{l}} \hat{M}^l P_R \, \hat{l} \left(1 + \frac{h}{v} \right) + \text{h.c.} \, , \tag{78}$$

with the projection operator P_R on the right-handed component of the lepton field $\hat{l}$. Thus the Yukawa coupling of leptons to the Higgs field h is proportional to the lepton mass m_k^l, with $\hat{M}_{kj}^l = m_k^l \delta_{kj}$,

$$g_{h\bar{l}l,k} = \frac{m_k^l}{v} . \tag{79}$$

2.4.2 Quark fields

The case of quark fields can be formulated in analogy to the case of charged leptons, with the difference, that the relation between weak eigenfields and states with a definite mass is different for up-type and down-type quarks $u^i_{L,R}$ and $d^i_{L,R}$. Following the example of the charged leptons, the Yukawa term for the quark fields in unitary gauge reads:

$$\Delta\mathcal{L}_{\mathrm{Y},q} = -\frac{1}{\sqrt{2}}\left(\overline{u}_L\lambda^u\, u_R + \overline{d}_L\lambda^d\, d_R\right)(v+h) + \mathrm{h.c.}\ . \tag{80}$$

Like in the case of charged leptons the diagonalization of the mass matrices for the up-type quarks, $M^u = \frac{1}{\sqrt{2}}\lambda^u v$, and for the down-type quarks, $M^d = \frac{1}{\sqrt{2}}\lambda^d v$,

$$U_u^\dagger M^u W_u = \hat{M}^u\ , \ \text{ and }\ U_d^\dagger M^d W_d = \hat{M}^d\ , \tag{81}$$

with $\hat{M}^u$ and $\hat{M}^d$ diagonal and positive, yields the mass eigenstates with a definite mass via the relations

$$u_L = U_u\hat{u}_L \quad , \quad u_R = W_u\hat{u}_R\ , \tag{82}$$

$$d_L = U_d\,\hat{d}_L \quad , \quad d_R = W_d\,\hat{d}_R\ . \tag{83}$$

In terms of quark field mass eigenstates, the Yukawa terms are given by

$$\Delta\mathcal{L}_{\mathrm{Y},q} = -\left(\overline{\hat{u}}_L\,\hat{M}^u\,\hat{u}_R + \overline{\hat{d}}_L\,\hat{M}^d\,\hat{d}_R\right)\left(1+\frac{h}{v}\right) + \mathrm{h.c.}\,, \tag{84}$$

and the coupling constants of the quarks to the Higgs field h are proportional to the mass of the quarks m^q_k, with

$$g_{h\bar{q}q,k} = \frac{m^q_k}{v}. \tag{85}$$

Fermionic kinetic terms and neutral current interaction terms remain unchanged under the transformation of the fermions to mass eigenstates , since all dependence on the transformation matrices cancel exactly, whereas the charged current interaction Lagrangian is affected,

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}}W^+_\mu\left(\overline{\nu}_L\gamma^\mu U^l_L\hat{l}_L + \overline{\hat{u}}_L\gamma^\mu\,\underbrace{U_u^\dagger U_d}_{:=V_{\mathrm{CKM}}}\,\hat{d}_L\right) + \mathrm{h.c.}, \tag{86}$$

where V_{CKM} denotes the unitary Cabbibo-Kobayashi-Maskawa (CKM) matrix [63].

The CKM matrix is a 3×3 complex matrix with 9 free parameters; 3 real angles and 6 complex phases. Since there are 6 left-handed quark fields, 5 phases can be removed by phase redefinitions. With one complex phase left, V_{CKM} violates CP-symmetry and there are interactions of quarks with $W^\pm$ bosons that generate transitions between different flavor generations (i.e. the $K^0 - \overline{K}^0$ mixing of the neutral Kaon system), where the entries of V_{CKM} describe the transition probabilities between the individual quark generations.

Referring to the leptonic term, one is free to redefine the left-handed neutrino field by

$$\nu_L = U_L^l \hat{\nu}_L. \tag{87}$$

Thus no interactions between gauge bosons and leptons involving mixing of different generations appear.

Treating neutrinos as massless particles in the SM excludes CP violating processes with gauge bosons and leptons, since the kinetic terms of the leptons remain unchanged under the transformation of the leptons to mass eigenstates. But measurements of neutrino oscillations [8–10] indicate the existence of massive neutrinos. The introduction of right-handed neutrino fields ν_R that are singlets under the SM gauge group,

$$\nu_R \sim (\mathbf{1},\mathbf{1},0), \tag{88}$$

denoted as sterile neutrinos, is a common method to generate neutrino masses. In the following discussion of the model under consideration the generation of neutrino masses via the see-saw mechanism is illustrated in Sect. 5.

The introduction of right-handed neutrino fields for the generation of neutrino masses implicates interactions between leptons and gauge bosons generating transitions between different generations, described by the Pontecorvo-Maki-Nakagawa-Sakata matrix [64, 65], the lepton mixing matrix U_{PMNS}, denoting a unitary matrix with the same number of free parameters as V_{CKM} (three angles and one complex phase, the CP-violating phase), defined in analogy to the case of the quarks,

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} W_\mu^+ \left(\overline{\hat{\nu}}_L \gamma^\mu \underbrace{U_\nu^\dagger U_L^l}_{:=U_{\mathrm{PMNS}}} \hat{l}_L + \overline{\hat{u}}_L \gamma^\mu V_{\mathrm{CKM}}\, \hat{d}_L \right) + \text{h.c.}\,, \tag{89}$$

with $\nu_L = U_\nu \hat{\nu}_L$.

2.5 Problems of the SM - The Hierarchy Problem

Although the SM achieves a successful description of three of the four fundamental forces of the universe, the elementary particles and their interactions, and explains the experimental data collected in the past decades at the LEP collider, the TEVATRON and in a number of low-energy experiments with remarkable precision [7, 24], it has to face many challenges and unsolved questions. The Model does not provide a description of gravity, since the theory of perturbative quantum gravity breaks down before reaching the Planck scale [66]. Neutrinos are accounted as massless in the classical formulation of the SM, which contradicts the experimental observation of neutrino oscillations [8–10]. The absence of an explanation of Dark Matter and Dark Energy and the question why there is more matter than antimatter leads to an inconcistency of the SM to the Standard Model of cosmology. In particular, the Higgs boson gives rise to a problem called the Hierarchy or Naturalness Problem of the Higgs mass, which will be explained shortly in the following.

The depicted problems need the SM to be extended. The SM in its present structure can be described as an effective theory [66], embedded in a more fundamental high-energy theory, denoted as a theory Beyond the Standard Model (BSM), where non-SM particles and their interactions emerge at an energy scale Λ. This BSM theory might provide an explanation of the microscopic origin of "mysteries" of the SM such as the masses of neutrinos, the origin of flavour or of electroweak symmetry breaking (EWSB) in the form of predictions of the operators and their coefficients in the SM Lagrangian in terms of more fundamental parameters of the theory [66].

The Higgs mass operator $c\Lambda^2 H^\dagger H$, with $c = \frac{m_h^2}{2\Lambda^2}$, is the only scale-dependent parameter in the SM Lagrangian and sets the scale of EWSB. In a high-energy embedding of the SM in a more fundamental theory with an energy scale $\Lambda \gg$TeV, (i.e. $\Lambda = M_{\text{GUT}} \sim 10^{15}$ GeV) [66], the parameter c appears to be, $c \ll 1$, with the observed mass of the Higgs boson of $m_h \simeq 125$ GeV [12]. This describes an enormous hierarchy between the electroweak scale and the new-physics scale Λ, with $m_h \ll \Lambda$. The essence of the Hierarchy Problem is the question why the electroweak scale can be light within an embedding of the SM in a theory with other possible heavy (scalar) degrees of freedom emerging at a high-energy scale Λ [29].

2.6 Extensions of the SM with classical scale invariance

There are various ideas to adress the Naturalness Problem discussed in the previous part of this section. One proposal, investigated by Krzysztof Nicolai and Hermann Meissner in 2006 in [21], is a theory without any scale, by imposing classical scale invariance on the tree-level SM Lagrangian. To recover EWSB, the classical conformal symmetry is broken due to quantum corrections to the scalar effective potential via the Coleman-Weinberg (CW) mechanism [11]. The proposals of Nicolai and Meissner rely only on the ingredients known already to be there in the SM [21], (meaning the models are not based on any "beyond the SM" scenarios like GUT or Supersymmetry), and a classically unbroken conformal symmetry is the basic reason of the existence of small mass scales, emerging due to a conformal anomaly [21].

To ensure that the exact conformal symmetry of the tree-level Lagrangian is not violated explicitly by counterterms, introduced when adopting a UV cut-off regularization scheme for the computation of quantum corrections [67] , dimensional regularization [68, 69] is used, which respects this invariance as it respects gauge invariance.

All divergent integrals are regulated by the replacement

$$\int \frac{d^4k}{(2\pi)^4} \to \mu^{2\epsilon} \int \frac{d^{4-2\epsilon}k}{(2\pi)^{4-2\epsilon}}, \tag{90}$$

where the renormalization scale μ is some mass scale, which explicitly breaks scale invariance. Since μ appears in an „evanescent“ exponent [21] in (90), renormalized quantities depend only logarithmically on the renormalization scale [21]. Hence the divergent terms are of the same form as the ones in the tree level Lagrangian and can thus be absorbed by counterterms. Therefore the effective action contains no mass terms or terms with a polynomial dependence on μ at any order in perturbation theory [21].

As a purely technical example serves the massless ϕ^4-theory, also discussed in [11] with the classical action,

$$S = \int d^4x \left(\partial_\mu \phi \partial_\mu \phi + \frac{\lambda}{4!}\phi^4 \right),$$

being invariant under the scale transformation

$$x \to \rho x,$$
$$\phi \to \frac{1}{\rho}\phi,$$

with the scale parameter ρ. The effective potential V_{eff} at one-loop order in the $\overline{\text{MS}}$ scheme reads

$$V_{eff} = \frac{\lambda}{4!}\phi^4 + \frac{\lambda^2\phi^4}{64\pi^2}\left(\ln \frac{\frac{\lambda}{2}\phi^2}{\mu^2} - \frac{3}{2} \right).$$

One-loop corrections break scale invariance

$$\int d^4x\, \phi^4 \ln \phi^2 \to \int d^4x\, \phi^4 \left(\ln \phi^2 - \ln \rho^2 \right)$$

and supposedly cause the appearence of a "new" minimum of the effective potential away from the origin, as illustrated in Fig. 2, (for more details see ref. [11]).

In a scale-invariant formulation of the SM where EWSB is achieved via the CW-mechanism, the mass of the Higgs boson m_h, generated at one-loop level is expressed in terms of the masses of the other particles [11],

$$m_h^2 = \frac{1}{8\pi^2 \langle \varphi^2 \rangle} \left(6m_W^4 + 3m_Z^4 - 4\sum_i m_{fi}^4 \right), \tag{91}$$

where $m_{W,Z,f}$ denote the masses of the W- and Z- bosons and the Dirac fermions, respectively and $\langle \varphi \rangle$ is the SM vacuum expectation value of $\langle \varphi \rangle = 246$ GeV. The fermion contribution in Eq. (91) is negative since fermion loops contribute with a minus sign

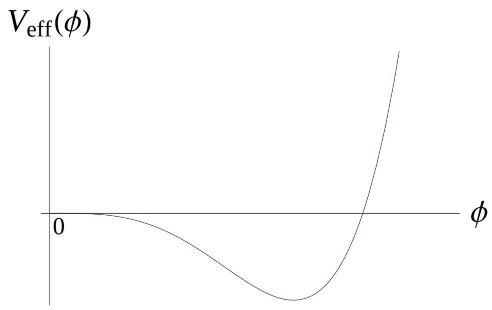

Figure 2: The one-loop effective potential of massless ϕ^4-theory.

and the sum runs over all internal indices. In their work in [11], CW accounted the contributions of fermions as negligible since the top quark had not been discovered at this period of time and gained a prediction of $m_h \sim 10$ GeV [21].

Since the originally predicted value of m_h in [11] is not consistent with the experimentally observed Higgs mass of ~ 125 GeV [12], and the incorporation of the CW mechanism in the SM fails because of the large top-quark mass, of $m_t \simeq 173$ GeV [13], that leads to an effective scalar potential that is not bounded from below [54], the SM particle needs to be enlargend by additional scalar and/or vector degrees of freedom giving a positive contribution to Eq. (91).

Besides the proposed model by Nicolai and Meissner in [21], there are many other extensions of the SM with classical scale invariance, explaining EWSB via the CW-mechanism, see for example [14–20, 22–48]. The SM extension by one real scalar field as proposed in [21] is probably ruled out [25, 29], since no set of parameters can be found for which a perturbative description of spontaneous symmetry breaking is achieved.

3 The Coleman-Weinberg-Mechanism - Spontaneous Symmetry Breaking induced by Radiative Corrections

The Coleman-Weinberg-Mechanism describes the occurrence of spontaneous breakdown of continuous symmetries of a renormalizable field theory as the consequence of quantum corrections rather than a negative mass term of a scalar field in the Lagrangian. In their original paper (ref.[11]) Coleman and Weinberg studied the theory of massless scalar electrodynamics and performed a perturbative analysis of the effective potential up to one-loop order illustrating that the scalar field aquires a vacuum expectation value away from the origin and both the scalar boson and the photon become massive as a consequence of radiative corrections.

3.1 Generating Functionals and the Effective Potential

The study of spontaneous symmetry breakdown is performed by searching for the minima of the effective potential of the theory [11], the negative sum of all the nonderivative terms in the expansion of a functional called the effective action in powers of momentum. This is commonly carried out in the semiclassical approximation [11], which includes the tree-level terms and the one-loop corrections. The effective potential is a function, defined such that its minima gives, without any approximation, the vacuum states of the theory [11]. There exists an expansion for the effective potential in powers of the quantity $\hbar$, which refers to a diagramatic expansion, such that the first term in this expansion gives the semiclassical approximation. Hence this concept enables one to survey all possible vacuum states simultaneously and to compute higher-order terms before a decision is taken which vacuum the theory finally picks [11].

In the following the general formalism is illustrated by considering the theory of a single scalar field φ whose dynamics are described by the Lagrangian $\mathcal{L}(\varphi, \partial_\mu \varphi)$.

The quantities, which fully describe a quantum field theory are the time-ordered correlation functions of the basic fields of the model
$\langle 0|T\phi(x_1)\ldots\phi(x_n)|0\rangle$ [67]. These quantities are generated by the functional [67]

$$Z[f] = \sum_{n=0}^{\infty} \frac{1}{n!} \int d^4x_1 \ldots d^4x_n f(x_1) \ldots f(x_n) \langle 0|T\phi(x_1)\ldots\phi(x_n)|0\rangle \,. \tag{92}$$

The functional $Z[f]$ depends on an arbitrary test function $f(x)$, which is referred to as an external field added to the Lagrangian,

$$\mathcal{L}(\varphi, \partial_\mu \varphi) \to \mathcal{L} + f(x)\varphi(x). \tag{93}$$

It is assumed that $f(x)$ tends to zero for $|x| \to \infty$, so that the above integrals are convergent [67].

The generating functional $Z[f]$ can be written as the exponential of the functional $W[f]$ and is defined as the vacuum to vacuum transition amplitude in the presence of an external field f, which is interpreted as the probability amplitude for the external field not to create any particles [67]:

$$Z[f] = \exp\left(iW[f]\right) = \langle 0|0\rangle_f = \langle 0|T\exp\left(i\int d^4x f(x)\phi(x)\right)|0\rangle. \tag{94}$$

The functional $W[f]$ expanded in terms of the external field yields the connected parts of the correlation functions [67],

$$W[f] = \sum_{n=1}^{\infty}\frac{i^{(n-1)}}{n!}\int d^4x_1\ldots d^4x_n f(x_1)\ldots f(x_n)\langle 0|T\phi(x_1)\ldots\phi(x_n)|0\rangle_c. \tag{95}$$

The correlation functions may be represented as a path integral [67],

$$\langle 0|T\phi(x_1)\ldots\phi(x_n)|0\rangle = \frac{1}{\mathcal{N}}\int[d\varphi]\mathrm{e}^{iS[\varphi]}\varphi(x_1)\ldots\varphi(x_n), \tag{96}$$

with $\phi(x)$ being operaters and $\varphi(x)$ being c-number fields, where $S[\varphi]$ is the classical action of the model and $\mathcal{N} = \int[d\varphi]\mathrm{e}^{iS[\varphi]}$.

Thus the path integral representation of $Z[f]$ reads [67]:

$$Z[f] = \exp\left(iW[f]\right) = \frac{1}{\mathcal{N}}\int[d\varphi]\mathrm{e}^{iS[\varphi]}\,\mathrm{e}^{i\int d^4x f(x)\varphi(x)}. \tag{97}$$

Therefore the functional W[f] describes the response of the system to the perturbation generated by the external field [67]. It represents an average value of the corresponding classical action [67].

The effective action $\Gamma[\varphi_c]$ itself is definded by a functional Legendre transformation [11]

$$\Gamma[\varphi_c] = W[f] - \int d^4x f(x)\varphi_c(x), \tag{98}$$

where the field φ_c is defined by [11]

$$\varphi_c(x) = \frac{\delta W}{\delta f(x)}. \tag{99}$$

From the definition of $\Gamma[\varphi_c]$ one can see that [11]

$$\frac{\delta\Gamma}{\delta\varphi_c(x)} = -f(x). \tag{100}$$

Similar to Eq. (92), the effective action can be expanded in terms of the field φ_c [11]:

$$\Gamma = \sum_n \frac{1}{n!} \int d^4x_1 \dots d^4x_n \Gamma^{(n)}(x_1 \dots x_n) \varphi_c(x_1) \dots \varphi_c(x_n). \tag{101}$$

The coefficients $\Gamma^{(n)}(x_1 \dots x_n)$ represent the sum of all one-particle-irreducible (1PI) Feynman diagrams with n external lines [11]. Therefore the effective action $\Gamma(\varphi_c)$ is the generating functional of all 1PI Green functions.

An expansion of $\Gamma[\varphi_c]$ in powers of momenta instead of powers of φ_c gives [11]

$$\Gamma = \int d^4x [-V(\varphi_c) + \frac{1}{2}(\partial_\mu \varphi_c)^2 Z(\varphi_c) + \dots]. \tag{102}$$

$V(\varphi_c)$ is an ordinary function and is called the effective potential [11].

The comparison of the expansions (101) and (102) shows that the nth derivative of V is the sum of all 1PI graphs with n vanishing external momenta [11].

3.2 The Loop Expansion of the Effective Potential

Since the effective potential V is unknown as its computation requires an infinite summation of Feynman diagrams, it is important to know a sensible approximation method for V. One such method is the loop expansion [11]. This refers to a summation of all the tree graphs at the lowest order, followed by the summation of all the graphs with one loop at the next order, etc. This expansion is equivalent to an expansion of V in powers of the parameter $\hbar$, where the quantity $\hbar$ multiplies the total Lagrangian [11]:

$$\mathcal{L}(\varphi, \partial_\mu \varphi, \hbar) \equiv \frac{1}{\hbar} \mathcal{L}(\varphi, \partial_\mu \varphi). \tag{103}$$

Thus the Lagrangian is independent of $\hbar$ and the loop expansion is unaffected by shifts of fields [11].

3.3 The one-loop contributions to V

Following the description of Coleman and Weinberg [11] the effective potential up to one-loop order in a non-abelian, general massless renormalizable gauge-field theory is computed, where the Lagrangians of interest involve a set of real spinless boson fields φ^i, which are collected in the vector $\vec{\varphi}$, a set of Dirac bispinor fields ψ^j and a set of real vector fields A^k_μ.
The one-loop approximation of the effective potential is given by suming up the terms

$$V(\vec{\varphi}) = V_0 + V_s + V_f + V_g, \tag{104}$$

where V_0 is the tree-level potential in the Lagrangian, and V_s, V_f and V_g denote the contributions from scalar, fermion and gauge field loops. The expressions for these one-loop contributions, computed in Landau gauge and renormalized in the $\overline{\text{MS}}$-scheme [11, 70] are listed below.

- V_s:

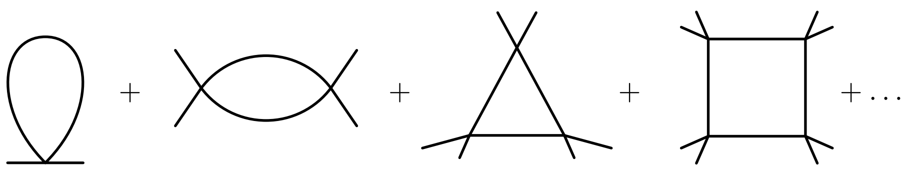

Figure 3: The one-loop contribution of scalar fields to the effective potential.

  The contribution of spinless scalar fields to the one-loop approximation of V comes from the infinite series of polygon graphs shown in Fig. 3, where scalar fields run around the loop [11, 70]:

$$V_s = \frac{1}{64\pi^2}\mathrm{Tr}\left[m_S^4(\vec{\varphi})\left(\ln\frac{m_S^2(\vec{\varphi})}{\mu^2} - \frac{3}{2}\right)\right]. \tag{105}$$

  In this expression, m_S^2 is a real symmetric matrix and a quadratic function of $\vec{\varphi}$ and is given by

$$m_S^2(\vec{\varphi})_{ij} = \frac{\partial^2 V_0}{\partial\varphi_i \partial\varphi_j}. \tag{106}$$

  Evaluated at the vacuum expectation value $\langle\vec{\varphi}\rangle$, $m_S^2(\langle\vec{\varphi}\rangle)$ gives the squared mass matrix of the scalar fields. The trace refers to the sum over all internal indices.

- V_g:
  For the computation of the contribution of the gauge fields the summation of the polygon graphs in Fig. 4 is considered.

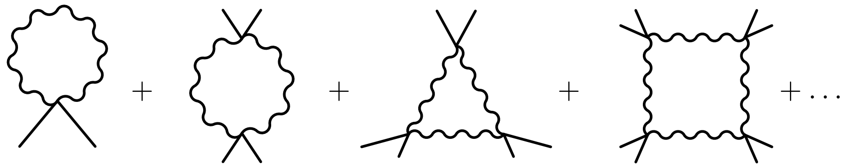

Figure 4: The contribution of the gauge fields to the one-loop approximation.

The expression for V_g reads [11, 70]

$$V_g = \frac{3}{64\pi^2}\mathrm{Tr}\Big[m_V^4(\vec{\varphi})\Big(\ln\frac{m_V^2(\vec{\varphi})}{\mu^2} - \frac{5}{6}\Big)\Big], \tag{107}$$

where the matrix $m_V^2(\vec{\varphi})$ is like in the case of the scalar fields a real symmetric matrix and a quadratic function of $\vec{\varphi}$. Evaluated at $\langle\vec{\varphi}\rangle$, $m_V^2(\langle\vec{\varphi}\rangle)$ gives the squared vector boson mass matrix. For minimally coupled vector bosons $m_V^2(\vec{\varphi})$ can be written as

$$m_V^2(\vec{\varphi})_{ij} = g_i g_j (T_i\vec{\varphi}, T_j\vec{\varphi}), \tag{108}$$

where T_i is the representation of the ith infinitesimal transformation of the gauge group, and g_i is the coupling constant of the associated gauge field. The factor of 3 in the expression (107) corresponds to the polarization degrees of freedom of the vector bosons.

- V_f:
The contribution of the fermion fields to the one-loop approximation of V involves diagrams with an even number of external lines, as shown in Fig. 5, since the trace of an odd number of γ matrices vanishes.

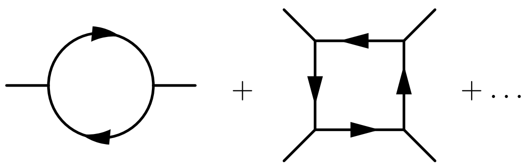

Figure 5: The fermion one-loop approximation for the effective potential V.

For the computation one defines a Yukawa coupling term in the most general form

$$\mathcal{L} = \cdots - \frac{1}{2}\overline{(\chi_R)^c}\, m_F(\vec{\varphi})\chi_R + \text{h.c.}\ldots, \tag{109}$$

where $m_F(\vec{\varphi}) = (m_F)^T(\vec{\varphi})$ and $m_F(\langle\varphi\rangle)$ is the fermion mass matrix, being a linear function of $\vec{\varphi}$. χ_R is a right-handed fermion field column vector,

$$\chi_R \equiv \begin{pmatrix} (\psi_L)^c \\ \psi_R \end{pmatrix}, \tag{110}$$

where the charge-conjugated fields are defined by

$$(\psi_L)^c \equiv C\,\overline{\psi_L}^T, \qquad (\psi_R)^c \equiv C\,\overline{\psi_R}^T, \tag{111}$$

and C denotes the charge-conjugation matrix.

After summing up all the diagrams one obtains [11, 70]

$$V_f = -\frac{1}{32\pi^2}\mathrm{Tr}\left[(m_F(\vec{\varphi})m_F^*(\vec{\varphi}))^2\left(\ln\frac{m_F(\vec{\varphi})m_F^*(\vec{\varphi})}{\mu^2} - \frac{3}{2}\right)\right]. \tag{112}$$

The trace in (112) runs over all internal indices and one can also recognize the minus sign for fermion loops.

4 The Gildener-Weinberg Analysis of the Effective Potential

The Gildener - Weinberg (GW) framework [54] deals with massless (scale invariant) models having an arbitrary number of weakly coupled scalar fields. It provides a method to analyze the properties of the effective potential V and to find a local minimum of V in a region where the use of perturbation theory is valid [54].

Following the discussion of [24, 54], the framework of GW is reviewed hereinafter.

Let us consider a general renormalizable gauge theory with an arbitrary set of n real weakly-coupled real scalar fields ϕ_i being the components of an n-dimensional field multiplet $\mathbf{\Phi}$. As mentioned in section 3.1 spontaneous symmetry breaking is studied by analyzing the effective potential $V(\mathbf{\Phi})$ as a function of the c-number real scalar field components ϕ_i of the field multiplet $\mathbf{\Phi}$. The potential is defined such that

$$V(0) = 0. \tag{113}$$

The theory is assumed to be classically scale invariant so that the scalar effective potential at tree level is of the form

$$V_0(\mathbf{\Phi}) = \frac{1}{24} f_{ijkl}\phi_i\phi_j\phi_k\phi_l, \tag{114}$$

where the quartic coupling constants f_{ijkl} are defined as

$$\left.\frac{\partial^4 V(\mathbf{\Phi})}{\partial\phi_i\partial\phi_j\partial\phi_k\partial\phi_l}\right|_{\mathbf{\Phi}\sim\Lambda} \equiv f_{ijkl}, \tag{115}$$

with Λ being the renormalization scale with the dimension of a mass.

To search for the local minima of the potential by means of ordinary perturbation theory, the tree-level potential V_0 is considered on the unit sphere $N_iN_i = 1$. There the minimum of $V_0(\mathbf{N})$ is found at a specific unit vector $\mathbf{N} = \mathbf{n}$:

$$\left.\frac{\partial V_0(\mathbf{N})}{\partial N_i}\right|_{\mathbf{N}=\mathbf{n}} = \frac{1}{6} f_{ijkl}(\Lambda) n_j n_k n_l = 0. \tag{116}$$

Then a specific renormalization scale Λ_W is chosen such that the minimum of the real continuous function $V_0(\mathbf{N})$ on the unit sphere $N_iN_i = 1$ takes the value zero:

$$\min_{N_iN_i=1} (f_{ijkl}(\Lambda_W) n_i n_j n_k n_l) = 0. \tag{117}$$

Due to its structure in Eq. (114), $V_0(\mathbf{\Phi})$ will attain a minimum value of zero everywhere on the ray $\mathbf{\Phi}^{\mathbf{flat}} = \varphi\mathbf{n}$, which represents the flat direction .

It is to be noted that Eq. (117) imposes only a single constraint on the parameters in f_{ijkl}, independent of how many free parameters f_{ijkl} contains and it is expected, that even for generic initial values of the f_{ijkl}, it should be possible to find a value of Λ at which Eq. (117) is satisfied [54].

As it is demanded that on $\mathbf{\Phi}^{\mathbf{flat}}$ not only lies a stationary point but a minimum of $V(\mathbf{\Phi})$, it is therefore required that the eigenvalues of the Hessian matrix P, defined as

$$P_{ij} \equiv \left.\frac{\partial^2 V_0(\mathbf{N})}{\partial N_i \partial N_j}\right|_{\mathbf{N}=\mathbf{n}} = \frac{1}{2} f_{ijkl} n_k n_l, \tag{118}$$

are either positive or equal to zero:

$$P_{ij} u_i u_j = \frac{1}{2} f_{ijkl} u_i u_j n_k n_l \geq 0 \qquad \forall\, u_i. \tag{119}$$

Adding higher order quantum corrections $\delta V(\mathbf{\Phi})$ to the potential gives the potential a small curvature in the radial direction, which picks out a specific value $\langle\varphi\rangle$ at the minimum on the ray $\mathbf{\Phi}^{\mathbf{flat}} = \varphi \mathbf{n}$, and also a small shift $\delta\mathbf{\Phi}$ in a direction away from $\mathbf{\Phi}^{\mathbf{flat}}$ in field space is produced at this minimum.

At one-loop order the stationary condition for the effective scalar potential $V = V_0 + \delta V$ reads

$$\nabla\big(V_0(\mathbf{\Phi}) + \delta V(\mathbf{\Phi})\big)\big|_{\mathbf{\Phi}=\mathbf{n}\langle\varphi\rangle+\delta\mathbf{\Phi}} = \mathbf{0}. \tag{120}$$

By treating the shift $\delta\mathbf{\Phi}$ as a parameter of one-loop order, one gets

$$P \cdot \delta\mathbf{\Phi}\langle\varphi\rangle^2 + \nabla\delta V(\mathbf{\Phi})|_{\mathbf{\Phi}=\mathbf{n}\varphi} = \mathbf{0}, \tag{121}$$

where P is the Hessian matrix defined in Eq. (118).

The perturbative minimization condition (121) uniquely determines $\delta\mathbf{\Phi}$, except for terms in directions along eigenvectors of P with eigenvalue zero. These eigenvectors include the flat direction $\mathbf{n}$ itself, since Eq. (116) gives

$$P \cdot \mathbf{n} = 0, \tag{122}$$

as well as the directions of the Goldstone bosons, that may result from the spontaneous symmetry breaking of any continuous symmetries.

By contracting Eq. (121) with $\mathbf{n}$ the first term in this relation vanishes and one gets the minimization condition along the flat direction:

$$\mathbf{n} \cdot \nabla\delta V(\mathbf{\Phi})|_{\mathbf{\Phi}=\mathbf{n}\langle\varphi\rangle} = \left.\frac{d\delta V(\mathbf{n}\varphi)}{d\varphi}\right|_{\varphi=\langle\varphi\rangle} = 0. \tag{123}$$

This equation will be used to find the VEV of φ, $\langle\varphi\rangle$, to one-loop order in perturbation theory.

Along the flat direction $\mathbf{\Phi}^{\mathbf{flat}} = \varphi\mathbf{n}$, the one-loop potential $\delta V(\varphi\mathbf{n})$ may be put in the form [24, 54]

$$\delta V(\varphi\mathbf{n}) = A\varphi^4 + B\varphi^4 \ln\frac{\varphi^2}{\Lambda_W^2}, \tag{124}$$

where A and B are dimensionless constants that read in the $\overline{\mathrm{MS}}$ scheme [16, 54]:

$$\begin{aligned} A &= \frac{1}{64\pi^2\langle\varphi\rangle^4}\Bigg\{\mathrm{Tr}\Bigg[m_S^4\Bigg(-\frac{3}{2}+\ln\frac{m_S^2}{\langle\varphi\rangle^2}\Bigg)\Bigg] + 3\mathrm{Tr}\Bigg[m_V^4\Bigg(-\frac{5}{6}+\ln\frac{m_V^2}{\langle\varphi\rangle^2}\Bigg)\Bigg] \\ &\quad -4\mathrm{Tr}\Bigg[m_F^4\Bigg(-\frac{3}{2}+\ln\frac{m_F^2}{\langle\varphi\rangle^2}\Bigg)\Bigg] - 2\mathrm{Tr}\Bigg[m_N^4\Bigg(-\frac{3}{2}+\ln\frac{m_N^2}{\langle\varphi\rangle^2})\Bigg)\Bigg]\Bigg\}, && (125) \\ B &= \frac{1}{64\pi^2\langle\varphi\rangle^4}\left(3\mathrm{Tr}m_S^4 + \mathrm{Tr}m_V^4 - 4\mathrm{Tr}m_F^4 - 2\mathrm{Tr}m_N^4\right). && (126) \end{aligned}$$

$m_{S,V,F,N}$ denote the tree-level scalar, vector, dirac fermion and (neutrino) majorana fermion mass matrices, respectively, evaluated at $\langle\varphi\rangle\mathbf{n}$. The trace is taken over the mass matrices as well as over all internal degrees of freedom (colour, flavour...).

According to Eq. (123) one can see that the potential (124) has a nontrivial stationary point at a value of $\langle\varphi\rangle$ given by

$$\ln\frac{\langle\varphi\rangle^2}{\Lambda_W^2} = -\frac{1}{2} - \frac{A}{B}. \tag{127}$$

From Eq. (124) it follows that the stationary point specified by (127) is not a minimum unless $B > 0$ and in the further analysis it will be assumed that this is the case.

With the use of the relation (127) one can see immediately that the value of the one loop potential at its stationary point in Eq. (124) is less than that at the origin $V(0) = 0$:

$$V(\mathbf{n}\langle\varphi\rangle) = \delta V(\mathbf{n}\langle\varphi\rangle) = -\frac{1}{2}B\langle\varphi\rangle^4 < 0. \tag{128}$$

That means that if the stationary point found in Eq. (123) is indeed a local minimum, it is one with broken symmetry.

In the following discussion it is shown that this stationary point is a local minimum of the one-loop order effective potential.

At the tree-level, the squared masses of the scalar bosons are given by the eigenvalues of the matrix,

$$(m_S^2)_{ij} = \left.\frac{\partial^2 V_0(\mathbf{\Phi})}{\partial\phi_i\partial\phi_j}\right|_{\mathbf{\Phi}=\langle\varphi\rangle\mathbf{n}} = \langle\varphi\rangle^2 (P)_{ij}. \tag{129}$$

From the assumptions taken above, it is clear that the Hessian matrix P only has positive eigenvalues, except for a set of zero eigenvalues corresponding to the Goldstone bosons associated with the spontaneous symmetry breaking of compact symmetries of the theory and one zero eigenvalue with eigenvector $\mathbf{n}$. Thus the model contains at tree-level a set of massive scalars, a set of massless Goldstone bosons plus one massless scalar, called "scalon" [54], which is associated with the spontaneous symmetry breakdown of scale invariance.

The scalon does not remain massless when turning on higher order terms in perturbation theory. The one-loop correction $\delta V(\mathbf{\Phi})$ to the effective scalar potential shifts the squared mass matrix to

$$\left(m_S^2 + \delta m_S^2\right)_{ij} = \left.\frac{\partial^2\big(V_0(\mathbf{\Phi}) + \delta V(\mathbf{\Phi})\big)}{\partial\phi_i\partial\phi_j}\right|_{\mathbf{\Phi}=\langle\varphi\rangle\mathbf{n}+\delta\mathbf{\Phi}}. \tag{130}$$

To first order in small quantities, this reads

$$(\delta m_S^2)_{ij} = \left.\frac{\partial^2 \delta V(\mathbf{\Phi})}{\partial\phi_i\partial\phi_j}\right|_{\mathbf{\Phi}=\langle\varphi\rangle\mathbf{n}} + \langle\varphi\rangle f_{ijkl} n_k \delta\phi_l. \tag{131}$$

For the stationary point $\mathbf{n}\langle\varphi\rangle + \delta\mathbf{\Phi}$ to be a local minimum of the potential, one has to reveal whether the eigenvalues of the matrix $m_S^2 + \delta m_S^2$ are positive definite. The eigenvalues of m_S^2 corresponding to the massive scalars are positive definite, so the corresponding eigenvalues of $m_S^2 + \delta m_S^2$ are also positive-definite as long as the perturbation δm^2 is small. The masses of the massive scalars are then given by the expectation value of m_S^2 with respect to an eigenvector $\tilde{\mathbf{n}}$ that is perpendicular to the flat direction $\mathbf{n}$. The Goldstone bosons remain massless provided $\delta V(\mathbf{\Phi})$ respects the same global symmetries as $V_0(\mathbf{\Phi})$.

Therefore only the scalon is left. The mass of the scalon m_h^2 is calculated by taking the expectation value of δm_S^2 with respect to the eigenvector $\mathbf{n}$. Using Eq. 116 and contracting $(\delta m_S^2)_{ij}$ with n_i and n_j one gets:

$$m_h^2 = n_i n_j (\delta m_S^2)_{ij} = n_i n_j \left.\frac{\partial^2 \delta V(\mathbf{\Phi})}{\partial\phi_i\partial\phi_j}\right|_{\mathbf{\Phi}=\langle\varphi\rangle\mathbf{n}} = \left.\frac{d^2 \delta V(\varphi\mathbf{n})}{d\varphi^2}\right|_{\varphi=\langle\varphi\rangle} = 8B\langle\varphi\rangle^2, \tag{132}$$

where (124) and (127) is used to at the last relation in (132). The scalon is referred to as the pseudo-Goldstone boson of the anomalously broken scale invariance. It is massless at tree-level when the theory is scale invariant and aquires a non-zero mass when scale invariance is spontaneously broken by quantum corrections.

Since it is assumed that $B > 0$ holds the proof that $\mathbf{n}\langle\varphi\rangle + \delta\mathbf{\Phi}$ is indeed a local minimum of the effective scalar potential is completed.

5 U(1) Extensions of the SM

The apparatus introduced by Gildener and Weinberg to analyze the effective potential of scale invariant models with an arbitrary number of weakly coupled scalar fields provides an efficient method to study the SM as a scale invariant theory. This cannot be achieved with the experimentally measured values of the top quark pole-mass of $m_t \simeq 173\,\mathrm{GeV}$ [13] and the Higgs boson mass of $m_h \simeq 125\,\mathrm{GeV}$ [12], as the symmetry breaking condition, $B > 0$, is not fulfilled.

To rescue the idea of a scale invariant version of the SM the particle content has to be enlarged by additional fields whose contribution to the parameter B is positive. This is realized by an extended Higgs sector with additional scalar fields or an extension of the SM gauge group, $\mathrm{G_{SM}}$, by (non-)abelian factors.

Among the other possibilities given in [14–48], models with an U(1) extended gauge group provide an extension of the SM with classical scale invariance. An example of such models is analyzed in this section.

In the model under consideration the SM gauge group is extended by an additional abelian $\mathrm{U(1)_z}$ factor and is spontaneously broken down to $\mathrm{SU(3)_C \times U(1)_{EM}}$,

$$\underbrace{\mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y}}_{=\mathrm{G_{SM}}} \times \mathrm{U(1)_z} \to \mathrm{SU(3)_C \times U(1)_{EM}}, \tag{133}$$

where $\mathrm{G_{SM}}$ denotes the SM gauge group. Due to the additional broken generator of the abelian $\mathrm{U(1)_z}$ gauge group a new color-singlet and electrically-neutral, heavy massive gauge boson, which appears as Z' in the literature [38, 49, 71–76] is expected.

To generate neutrino masses and to explain the smallness of the experimentally measured neutrino mass values [7] with respect to the masses of other fundamental fermions, the see-saw mechanism is implemented [77]. Therefore n_R additional right-handed neutrino fields ν_R, with an $\mathrm{U(1)_z}$ charge z_ν, but being singlets with respect to $\mathrm{G_{SM}}$, are added.

The Higgs Sector is enlarged by a complex scalar S, which is a singlet under $\mathrm{G_{SM}}$.

5.1 General framework

The Lagrangian of the model is given by

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{\mathrm{SM}} - \frac{1}{4} B_{z\mu\nu} B_z^{\mu\nu} + (D_s^\mu S)^\dagger (D_{s,\mu} S) \\ &\quad - \left(\overline{L} \Delta \tilde{H} \nu_R + \frac{1}{2} \overline{\nu_R^c} \Gamma_\nu S \nu_R + \text{h.c.} \right) \\ &\quad + \overline{\nu}_R \, i(\partial\!\!\!/ + i g_z B\!\!\!/_z) \nu_R - V_0(H, S),\end{aligned} \tag{134}$$

with the covariant derivatives

$$D_\mu = \partial_\mu + igT^i W^i_\mu + ig_Y Y B_{Y\mu} + ig_z z B_{z\mu} + ig_S \mathcal{T}^A G^A_\mu, \tag{135}$$

$$D_{s,\mu} = \partial_\mu + ig_z B_{z\mu}, \tag{136}$$

and the gauge field B_z of the additional $\mathrm{U(1)_z}$ gauge group, with gauge coupling g_z and charge z.

$\mathcal{L}_{\mathrm{SM}}$ contains the SM terms without the Higgs potential which is included in the tree-level scalar potential $V_0(H, S)$.

The vacuum expectation value of the complex scalar field S, charged under $\mathrm{U(1)_z}$, is responsible for the spontaneous symmetry breakdown of the additional abelian gauge group $\mathrm{U(1)_z}$.

For the implementation of the see-saw mechanism, expression (134) includes a Yukawa term and a Majorana type Yukawa term with right-handed neutrinos ν_R, where L denotes the SM left-handed lepton doublet and Δ and Γ_ν are $n_L \times n_R$ and $n_R \times n_R$ matrices respectively.

The see-saw mechanism produces heavy neutrino Majorana masses m_R, which are assumed to be in the TeV region. Therefore, as m_R is proportional to the vacuum expectation value of S, v_S, the spontaneous symmetry breaking is required to occur at the TeV scale.

5.2 The Scalar Sector - Symmetry Breaking

The scale invariant tree-level effective scalar potential $V_0(H, S)$ takes the form

$$V_0(H, S) = \frac{\lambda_H}{2}(H^\dagger H)^2 + \frac{\lambda_S}{2}(S^* S)^2 - \lambda_{HS}(H^\dagger H)(S^* S), \tag{137}$$

with λ_H, λ_S and λ_{HS} being dimensionless coupling constants.

The spontaneous symmetry breakdown of scale invariance is described by the CW-mechanism which is discussed in section 3. Both the Higgs doublet H and the complex scalar singlet S acquire vacuum expectation values (VEVs),

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_H + h' \end{pmatrix}, \qquad S = \frac{1}{\sqrt{2}}(v_S + s'), \tag{138}$$

with $v_H = 246\,\mathrm{GeV}$. The scalar fields H and S can be written in unitary gauge, as the scalar potential $V_0(H, S)$ is invariant under U(1) symmetry transformations .

To guarantee the validity of the perturbative description of the effective potential, the coupling constant of an ordinary $\frac{\lambda}{4!}\phi^4$ term in ϕ^4-theory, need to fulfill the condition

$\lambda \ll (4\pi)^2$. Thus for the coupling constants $\lambda_{H,S,HS}$ with the expressions of the scalar fields H and S in (138) it holds:

$$\lambda_H \ll 48, \qquad \lambda_S \ll 48, \qquad \lambda_{HS} \ll 24. \tag{139}$$

The effective potential $V_0(H, S)$ of the scalar fields is analyzed using the method suggested by Gildener and Weinberg (see section 4). Hence V_0 is considered along a particular flat direction $\mathbf{n}$, characterized by the condition

$$\lambda_H(\Lambda_W)\lambda_S(\Lambda_W) = \lambda_{HS}^2(\Lambda_W). \tag{140}$$

Eq.(140) shows a phenomenon called dimensional transmutation [11], where a dimensionless parameter gets traded for a dimensional one. In this case the dimensionless coupling constant λ_{HS} is expressed in terms of λ_S and λ_H and gets traded for the renormalization scale Λ_W with the dimension of a mass.

After acquiring a VEV the scalar fields mix, which is expressed in terms of a mixing angle α. The VEVs v_H and v_S are given by

$$v_H = \langle\varphi\rangle \sin\alpha, \ \text{ and } \ v_S = \langle\varphi\rangle \cos\alpha, \tag{141}$$

$$\text{with } \ \sin^2\alpha = \frac{\lambda_{HS}}{\lambda_H + \lambda_{HS}}, \tag{142}$$

where $\langle\varphi\rangle$ denotes the vacuum expectation value along the ray $\mathbf{\Phi}^{\mathbf{flat}} = \varphi\mathbf{n}$, with $\mathbf{n} = (\cos\alpha, \sin\alpha, 0, 0, 0, 0)$ and the mixing angle α being fixed by a relation in terms of the coupling constants, with $\lambda_{H,S,HS} \equiv \lambda_{H,S,HS}(\Lambda_W)$ from now on.

The matrix m_S^2, which gives the scalar mass matrix at zeroth-order when evaluated at $\langle\varphi\rangle\mathbf{n}$, is obtained by

$$(m_S^2)_{ij} = \frac{\partial^2 V_0}{\partial\phi_i\partial\phi_j}, \tag{143}$$

with

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 + i\phi_2\\ \phi_3 + i\phi_4\end{pmatrix}, \quad \text{and } S = \frac{1}{\sqrt{2}}\left(\phi_5 + i\phi_6\right), \tag{144}$$

and reads

$$m_S^2 = \frac{1}{2}\left(\begin{array}{c|c} \begin{array}{l}(\lambda_H\phi^T\phi - \lambda_{HS}\phi_S^T\phi_S)\mathbb{1}_4\\ +2\lambda_H\phi\phi^T\end{array} & -2\lambda_{HS}\phi\phi_S^T \\ \hline -2\lambda_{HS}\phi_S\phi^T & \begin{array}{l}(\lambda_S\phi_S^T\phi_S - \lambda_{HS}\phi^T\phi)\mathbb{1}_2\\ +2\lambda_S\phi_S\phi_S^T\end{array}\end{array}\right), \tag{145}$$

where

$$\phi := \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix}, \quad \text{and } \phi_S := \begin{pmatrix} \phi_5 \\ \phi_6 \end{pmatrix}. \tag{146}$$

By diagonalizing the 6×6 matrix m_S^2 in Eq. (145) and evaluating it at $\langle\varphi\rangle\mathbf{n}$, one obtains the scalar mass spectrum, consisting at tree-level of one massive scalar, 4 massless would-be Goldstone bosons and the "scalon", which is massless at tree-level.

Thus the scalar mass matrix , neglecting the contributions of the would-be Goldstone bosons, is given by

$$\tilde{m}_S^2(\langle\varphi\rangle\mathbf{n}) = \begin{pmatrix} F_+|_{\langle\varphi\rangle\mathbf{n}} & 0 \\ 0 & F_-|_{\langle\varphi\rangle\mathbf{n}} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2\lambda_{HS}\langle\varphi\rangle^2 & 0 \\ 0 & 0 \end{pmatrix}, \tag{147}$$

with

$$\begin{aligned} F_\pm \quad = \quad & \frac{1}{4}\Big[(3\lambda_H - \lambda_{HS})\phi^T\phi + (3\lambda_S - \lambda_{HS})\phi_S^T\phi_S \pm \qquad (148) \\ & \pm\sqrt{(3\lambda_H + \lambda_{HS})\phi^T\phi - (3\lambda_S + \lambda_{HS})\phi_S^T\phi_S + 16\lambda_{HS}^2\phi^T\phi\phi_S^T\phi_S}\Big]. \end{aligned}$$

The scalar mass eigenstates h and s, corresponding to the mass eigenvalues $F_+|_{\langle\varphi\rangle\mathbf{n}}$ and $F_-|_{\langle\varphi\rangle\mathbf{n}}$ respectively, expressed in terms of the mixing angle α, read:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_H + h' \end{pmatrix} \quad = \quad \frac{1}{\sqrt{2}}(h\cos\alpha + s\sin\alpha + v_H)\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{149}$$

$$S = \frac{1}{\sqrt{2}}(v_S + s') \quad = \quad \frac{1}{\sqrt{2}}(s\cos\alpha - h\sin\alpha + v_S). \tag{150}$$

Thus one can write

$$(s', h', 0, 0, 0, 0) = (s + \langle\varphi\rangle)\mathbf{n} + h\,\mathbf{n}_\perp, \tag{151}$$

where the vector that collects the real scalar fields h' and s' of the model is split up into a contribution along the flat direction $\mathbf{n}$, the mass eigenstate s, which is referred to as the scalon, with $m_s^2 = 0$ at tree-level and a contribution that is perpendicular to $\mathbf{n}$, the mass eigenstate h, with tree-level mass
$m_h^2 = \lambda_{HS}\langle\varphi\rangle^2$.

Since the model contains two scalar mass eigenstates h and s, there arises the question which one of these states, h or s, is to be identified with the physical observed Higgs with the experimentally measured mass of $m_h \simeq 125\,\text{GeV}$, from a combined measurement of ATLAS and CMS [12].

From the assumption of the heavy neutrino Majorana masses $m_R \propto v_S$ to be in the TeV region follows

$$\frac{v_H^2}{v_S^2} = \tan^2\alpha \ll 1, \tag{152}$$

$$\cos\alpha \sim 1, \qquad \sin\alpha \ll 1. \tag{153}$$

Relation (152) and the fact that the deviation of the experimentally measured values of the coupling constants of the SM particles to the Higgs boson is rather small [78, 79], lead to the conclusion, that, when looking at the expression of the Higgs boson doublet H in Eq. (149), the mass eigenstate h correspondes to the observed Higgs boson with the measured mass of $m_h \simeq 125\,\mathrm{GeV}$.

5.3 The one-loop effective potential

According to Eq. (124) in section 4, the one-loop effective potential $\delta V(\varphi\mathbf{n})$ can be put in the following form along the flat direction $\mathbf{n}$ [24, 54],

$$\delta V(\varphi\mathbf{n}) = A\varphi^4 + B\varphi^4 \ln\frac{\varphi^2}{\Lambda_W^2}, \tag{154}$$

with the dimensionless constants A and B in $\overline{\mathrm{MS}}$ [24, 54]:

$$\begin{aligned}
A &= \frac{1}{64\pi^2\langle\varphi\rangle^4}\Bigg\{m_h^4\bigg(-\frac{3}{2}+\ln\frac{m_h^2}{\langle\varphi\rangle^2}\bigg) + 6m_W^4\bigg(-\frac{5}{6}+\ln\frac{m_W^2}{\langle\varphi\rangle^2}\bigg) \\
&\quad +3m_Z^4\bigg(-\frac{5}{6}+\ln\frac{m_Z^2}{\langle\varphi\rangle^2}\bigg) + 3m_{Z'}^4\bigg(-\frac{5}{6}+\ln\frac{m_{Z'}^2}{\langle\varphi\rangle^2}\bigg) \\
&\quad -12m_t^4\bigg(-\frac{3}{2}+\ln\frac{m_t^2}{\langle\varphi\rangle^2}\bigg) - 2\sum_{r=1}^{3}\bigg[m_R^4\bigg(-\frac{3}{2}+\ln\frac{m_R^2}{\langle\varphi\rangle^2}\bigg)\bigg]\Bigg\},
\end{aligned} \tag{155}$$

$$B = \frac{1}{64\pi^2\langle\varphi\rangle^4}\left(m_h^4 + 6m_W^4 + 3m_Z^4 + 3m_{Z'}^4 - 12m_t^4 - 2\sum_{r=1}^{3} m_R^4\right). \tag{156}$$

The fermion contribution to the one-loop effective potential $\delta V(\varphi\mathbf{n})$ consists of the contribution of the top quark mass m_t and the contribution of the heavy neutrino Majorana masses m_R, since the other fermion contributions are negligible.

The mass of the scalon m_s^2 can be computed in terms of other particle masses according to Eq. (132):

$$m_s^2 = 8B\langle\varphi\rangle^2. \tag{157}$$

The symmetry breaking condition requires that $B > 0$, which imposes a constraint on the Majorana masses of the neutrinos m_R,

$$3m_{Z'}^4 - 2\sum_{r=1}^{3} m_R^4 > (317)^4\,\mathrm{GeV}^4, \tag{158}$$

with the observed top quark pole-mass of $m_t \simeq 173\,\mathrm{GeV}$, from combined measurements by the Tevatron and the LHC experiments [13], the Higgs boson mass of $m_h \simeq 125\,\mathrm{GeV}$ [12] and the experimentally measured values of the W- and Z- boson masses of $m_W \simeq 80\,\mathrm{GeV}$ and $m_Z \simeq 91\,\mathrm{GeV}$ [80].

5.4 The Gauge Sector

In general an extension of the SM gauge group with one additional abelian factor is given by $\mathrm{SU(3)_C} \times \mathrm{SU(2)_L} \times \mathrm{U(1)_1} \times \mathrm{U(1)_2}$, whose generators are associated with the gluon octet, the $W^{\pm}$ bosons, and three $\mathrm{SU(3)_C} \times \mathrm{U(1)_{EM}}$ singlets: the photon, the Z boson and a Z' boson.

In the abelian gauge group $\mathrm{U(1)_1} \times \mathrm{U(1)_2}$, gauge invariance allows a kinetic mixing term, with the mixing parameter ε,

$$\mathcal{L}_{kin} = -\frac{1}{4}B_{1\mu\nu}B_1^{\mu\nu} - \frac{1}{4}B_{2\mu\nu}B_2^{\mu\nu} - \frac{1}{2}\sin\varepsilon\, B_{1\mu\nu}B_2^{\mu\nu}, \tag{159}$$

with $B_{1\mu}$ and $B_{2\mu}$ being the gauge fields of the gauge groups $\mathrm{U(1)}_1$ and $\mathrm{U(1)}_2$ respectively. Without loss of generality the kinetic mixing between the $\mathrm{U(1)}_1$ and $\mathrm{U(1)}_2$ gauge fields may be diagonalised at any particular scale [49]. Therefore for reasons of simplification in the analysis of the effective theory, a basis is chosen for the $\mathrm{U(1)_1} \times \mathrm{U(1)_2}$ gauge fields, in which the kinetic terms are diagonal and canonically normalized.

One can always perform a certain SO(2) transformation on the $\mathrm{U(1)_1} \times \mathrm{U(1)_2}$ gauge fields, such that the scalar field S does not couple to one of the resulting gauge fields of the corresponding abelian gauge group, i.e. $\mathrm{U(1)}_1$ and the charge $q_{1,S} = 0$ [49]. By rescaling the gauge coupling constants the charges of the Higgs doublet under $\mathrm{U(1)}_1$, $q_{1,H}$, and of the scalar field S under $\mathrm{U(1)}_2$, $q_{2,S}$, are defined to be

$$q_{1,H} = 1, \quad q_{2,S} = 1. \tag{160}$$

In general the usual SM hypercharge gauge group $\mathrm{U(1)_Y}$ is not identified with either $\mathrm{U(1)_1}$ or $\mathrm{U(1)_2}$. But the electric charges of the observed fermions imply that with the definition of $q_{1,H}$ and $q_{2,S}$ in expression (160), $\mathrm{U(1)_1}$ corresponds to $\mathrm{U(1)_Y}$. As in expression (133), $\mathrm{U(1)}_2$ is labelled by $\mathrm{U(1)}_z$ and the kinetic part of the Lagrangian $\mathcal{L}_{kin}$ of the $\mathrm{U(1)_Y} \times \mathrm{U(1)_z}$ gauge fields, $B_{Y\mu}$ and $B_{z\mu}$, is given by

$$\mathcal{L}_{kin} = -\frac{1}{4}B_{Y\mu\nu}B_Y^{\mu\nu} - \frac{1}{4}B_{z\mu\nu}B_z^{\mu\nu}. \tag{161}$$

After spontaneous symmetry breaking, the mass terms for the electrically-neutral $\mathrm{SU}(2)_\mathrm{L}\times \mathrm{U}(1)_\mathrm{Y}\times \mathrm{U}(1)_\mathrm{z}$ gauge bosons, W^3_μ, $B_{Y\mu}$ and $B_{Z\mu}$, can be extracted from the kinetic terms for the scalar fields of the model H and S [49] :

$$\frac{v_H^2}{8}(gW^{3\mu}+g_Y B_Y^\mu + z_H g_z B_z^\mu)(gW^3_\mu + g_Y B_{Y\mu} + z_H g_z B_{z\mu}) + \frac{v_S^2}{8} g_z^2 B_z^\mu B_{z\mu}. \tag{162}$$

This can be written as

$$\Delta\mathcal{L}_{mass} = \frac{1}{2}\left(B_{Y\mu}, W^3_\mu, B_{z\mu}\right) M^2 \begin{pmatrix} B_{Y\mu} \\ W^3_\mu \\ B_{z\mu} \end{pmatrix}, \tag{163}$$

with

$$M^2 = \frac{1}{4}\begin{pmatrix} g_Y^2 v_H^2 & -g_Y g v_H^2 & g_Y g_z z_H v_H^2 \\ -g_Y g v_H^2 & g^2 v_H^2 & -g g_z z_H v_H^2 \\ g_Y g_z z_H v_H^2 & -g g_z z_H v_H^2 & g_z^2\left(v_S^2 + v_H^2 z_H^2\right) \end{pmatrix}. \tag{164}$$

To isolate the mass eigenstate corresponding to the massless physical photon field, an orthogonal transformation U is performed on the gauge fields [49, 60],

$$\begin{pmatrix} B_{Y\mu} \\ W^3_\mu \\ B_{z\mu} \end{pmatrix} = U \begin{pmatrix} A_{\gamma\mu} \\ \tilde{Z}_\mu \\ \tilde{Z}'_\mu \end{pmatrix}, \tag{165}$$

$$U = \begin{pmatrix} \cos\theta_W & -\sin\theta_W & 0 \\ \sin\theta_W & \cos\theta_W & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{166}$$

with

$$\sin\theta_W = \frac{g_Y}{\sqrt{g_Y^2+g^2}},\ \cos\theta_W = \frac{g}{\sqrt{g_Y^2+g^2}}, \tag{167}$$

where θ_W denotes the Weinberg angle, which brings the mass matrix M^2 into a block-diagonal form [60],

$$U^T M^2 U = \begin{pmatrix} 0 & 0 & 0 \\ 0 & m^2_{\tilde{Z}} & \Delta^2 \\ 0 & \Delta^2 & m^2_{\tilde{Z}'} \end{pmatrix}, \tag{168}$$

with

$$
\begin{aligned}
\Delta^2 &= -\frac{1}{4} g_z z_H \sqrt{g^2 + g_Y^2}\, v_H^2, && (169) \\
m_{\tilde{Z}}^2 &= \frac{1}{4}\left(g^2 + g_Y^2\right) v_H^2, && (170) \\
m_{\tilde{Z}'}^2 &= \frac{1}{4} g_z^2 \left(v_S^2 + v_H^2 z_H^2\right). && (171)
\end{aligned}
$$

It is to be noted that the $\mathrm{U}(1)_z$ charge of the SM Higgs doublet, z_H, in the parameter Δ^2 in (169) is responsible for the mass mixing of the SM Z and the Z' boson at tree-level.

Performing another orthogonal transformation on the gauge fields yields the mass eigenstates of the photon, the observed SM Z boson and the Z' boson, which is the heavy neutral gauge boson not yet discovered [49, 60],

$$
\begin{pmatrix} B_{Y\mu} \\ W_\mu^3 \\ B_{z\mu} \end{pmatrix} = D \begin{pmatrix} A_{\gamma\mu} \\ Z_\mu \\ Z'_\mu \end{pmatrix}, \tag{172}
$$

$$
D = \begin{pmatrix} \cos\theta_W & -\sin\theta_W & 0 \\ \sin\theta_W & \cos\theta_W & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}, \tag{173}
$$

$$
M_{Z,Z'}^2 = D^T M^2 D = \mathrm{diag}\left(0, m_Z^2, m_{Z'}^2\right), \tag{174}
$$

where the angle θ satisfies the relation [49, 60]

$$
\tan 2\theta = \frac{2\Delta^2}{m_{\tilde{Z}}^2 - m_{\tilde{Z}'}^2}, \tag{175}
$$

and the mass eigenvalues of the Z boson, m_Z, and the Z' boson, $m_{Z'}$, are given by [49]

$$
m_{Z,Z'}^2 = \frac{g^2 v_H^2}{4\cos^2\theta_W}\left(\frac{1}{2}\left[\left(\cot^2\alpha + z_H^2\right) t_z^2 \cos^2\theta_W + 1\right] \mp \frac{z_H t_z \cos\theta_W}{\sin 2\theta}\right), \tag{176}
$$

where $\cot^2\alpha = \frac{v_S^2}{v_H^2}$ and $t_z \equiv \frac{g_z}{g}$.
As one can see in Eq. (176), the Z' boson is assumed to be heavier than the observed Z boson.

5.4.1 Fermion interactions

The interaction Lagrangian $\mathcal{L}_{int}$ of fermions with gauge bosons of the neutral electroweak $\mathrm{SU(2)_L \times U(1)_Y \times U(1)_z}$ sector reads

$$\begin{aligned}
\mathcal{L}_{int} &= g_Y J_Y^\mu B_{Y,\mu} + g J_3^\mu W_\mu^3 + g_z \tilde{J}_z^\mu B_{z,\mu} \\
&= \left(J_Y^\mu, J_3^\mu, \tilde{J}_z^\mu\right) \begin{pmatrix} g_Y & 0 & 0 \\ 0 & g & 0 \\ 0 & 0 & g_z \end{pmatrix} \begin{pmatrix} B_{Y,\mu} \\ W_\mu^3 \\ B_{z,\mu} \end{pmatrix},
\end{aligned} \tag{177}$$

where the fermionic currents $J_A^\mu, (A = Y, 3, z)$, are defined by

$$J_A^\mu = q_A^f \overline{\psi} \gamma^\mu \psi, \tag{178}$$

with the corresponding charges q_A^f.

$\mathcal{L}_{int}$ written in terms of mass eigenstates of the gauge bosons $A_{\gamma\mu}, Z_\mu$ and Z'_μ takes the form

$$\mathcal{L}_{int} = \left(J_Y^\mu, J_3^\mu, \tilde{J}_z^\mu\right) \begin{pmatrix} g_Y & 0 & 0 \\ 0 & g & 0 \\ 0 & 0 & g_z \end{pmatrix} D \begin{pmatrix} A_{\gamma\mu} \\ Z_\mu \\ Z'_\mu \end{pmatrix}, \tag{179}$$

where the matrix D is defined in Eq. (173).

Thus the currents $J_A^\mu, (A = Y, 3, z)$, transform with D^T,

$$\begin{pmatrix} J_\gamma^\mu \\ J_Z^\mu \\ J_{Z'}^\mu \end{pmatrix} = D^T \begin{pmatrix} g_Y & 0 & 0 \\ 0 & g & 0 \\ 0 & 0 & g_z \end{pmatrix} \begin{pmatrix} J_Y^\mu \\ J_3^\mu \\ \tilde{J}_z^\mu \end{pmatrix}, \tag{180}$$

with

$$J_\gamma^\mu = \underbrace{\frac{g g_Y}{\sqrt{g^2 + g_Y^2}}}_{:=e} \underbrace{\left(J_Y^\mu + J_3^\mu\right)}_{:=J_Q^\mu}, \tag{181}$$

$$J_Z^\mu = \frac{g}{\cos\theta_W} \left(J_3^\mu - \sin^2\theta_W J_Q^\mu\right) \cos\theta + g_z \tilde{J}_z^\mu \sin\theta, \tag{182}$$

$$J_{Z'}^\mu = -\frac{g}{\cos\theta_W} \left(J_3^\mu - \sin^2\theta_W J_Q^\mu\right) \sin\theta + g_z \tilde{J}_z^\mu \cos\theta, \tag{183}$$

where e denotes the SM electromagnetic coupling constant and J_Q denotes the SM electromagnetic current.

Therefore $\mathcal{L}_{int}$ can be rewritten in the following way:

$$\mathcal{L}_{int} = J^{\mu}_{\gamma} A_{\gamma\mu} + J^{\mu}_{Z} Z_{\mu} + J^{\mu}_{Z'} Z'_{\mu}. \tag{184}$$

Since the physical photon field A_γ needs to couple to the electromagnetic current J_Q as defined in the SM with the coupling constant e, and since $J_\gamma = eJ_Q$, as given in expression (181), it follows that $\mathrm{U}(1)_Y$ in the abelian part of the gauge group of the model, $\mathrm{U}(1)_Y \times \mathrm{U}(1)_z$, corresponds indeed to the SM hypercharge gauge group.

The covariant derivative of the electroweak $\mathrm{SU(2)_L \times U(1)_Y \times U(1)_z}$ sector in terms of the mass eigenstates of the vector bosons reads

$$\begin{aligned}
D_\mu = \partial_\mu \quad &+ \quad i\frac{g}{\sqrt{2}}\left(W^+_\mu \overbrace{(\tau_1 + i\tau_2)}^{=:\tau^+} + W^-_\mu \overbrace{(\tau_1 - i\tau_2)}^{=:\tau^-} \right) \\
&+ \quad i\left(\frac{g}{\cos\theta_W}\left(\tau_3 - \sin^2\theta_W Q\right)\cos\theta + g_z z\sin\theta \right) Z_\mu \\
&+ \quad i\left(-\frac{g}{\cos\theta_W}\left(\tau_3 - \sin^2\theta_W Q\right)\sin\theta + g_z z\cos\theta \right) Z'_\mu \\
&+ \quad ieQA_{\gamma\mu}\,,
\end{aligned} \tag{185}$$

with $Q = \tau_3 + \frac{Y}{2}$ being the SM electric charge and τ_i, $(i = 1, 2, 3)$, being the generators of the $\mathrm{SU}(2)_L$ gauge group.

5.5 $\mathrm{U(1)_z}$ charges and examples of U(1) SM extensions

In the model under consideration the SM fermion fields with three generations of quarks, Q_{iL}, u_{iR}, d_{iR}, and leptons L_i, l_{iR}, $i = 1, 2, 3$, and a number of n_R additional right-handed neutrino fields ν_R, as introduced in section 5.1, which are singlets under $\mathrm{G_{SM}}$ and electrically neutral, are charged under $\mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_z}$. The $\mathrm{U(1)_z}$ charges of these are defined by constraints of anomaly cancellation conditions and fermion mass constraints, which are imposed in the following section, based on [49, 60, 81].

5.5.1 Anomaly cancellation

A symmetry is said to be anomalous if the associated current J^μ is conserved classically but not at the quantum level and $\partial_\mu J^\mu \neq 0$. Symmetries of local gauge theories are required to be anomaly free, otherwise the Ward identity will be violated and unphysical degrees of freedom could be produced.

The gauge anomaly is related to triangle diagrams such as shown in Fig. 6

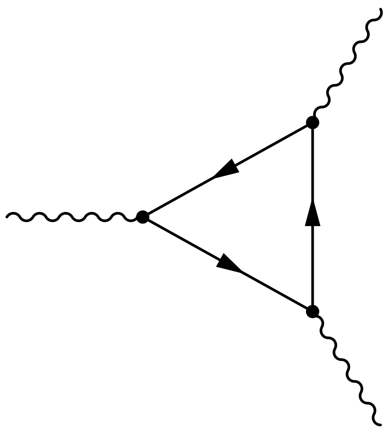

Figure 6: Triangle diagram with a fermion running around the loop and gauge bosons at the external legs.

The contribution of such diagrams is proportional to an expression of a trace [60]

$$D^{\alpha\beta\gamma} = \mathrm{Tr}[\{T^\alpha, T^\beta\}T^\gamma], \tag{186}$$

where $T^{\alpha,\beta,\gamma}$ are the generators of the corresponding gauge group.

The condition for the cancellation of anomalies reads

$$D^{\alpha\beta\gamma} = 0. \tag{187}$$

Hence certain constraints on the z charges of the fermion fields, which are assumed to be generation independent, are imposed:

$$\begin{aligned}
&[\mathrm{SU}(2)]^2[\mathrm{U}(1)_\mathrm{Z}] && \mathrm{Tr}[\{T^i, T^j\}z] &= 0 && (188)\\
&[\mathrm{SU}(3)]^2[\mathrm{U}(1)_\mathrm{Z}] && \mathrm{Tr}[\{\mathcal{T}^a, \mathcal{T}^b\}z] &= 0 && (189)\\
&[\mathrm{U}(1)_\mathrm{Y}]^2[\mathrm{U}(1)_\mathrm{Z}] && \mathrm{Tr}[Y^2 z] &= 0 && (190)\\
&[\mathrm{U}(1)_\mathrm{Y}][\mathrm{U}(1)_\mathrm{Z}]^2 && \mathrm{Tr}[Y z^2] &= 0 && (191)\\
&[\mathrm{U}(1)_\mathrm{Z}]^3 && \mathrm{Tr}[z^3] &= 0 && (192)
\end{aligned}$$

with $T^{i,j}$, $\mathcal{T}^{a,b}$ and Y being the generators of the SM $\mathrm{SU}(2)_\mathrm{L}$, $\mathrm{SU}(3)_\mathrm{C}$ and $\mathrm{U}(1)_\mathrm{Y}$ gauge group respectively.

With the SM hypercharges of the fermion fields the constraints on the fermionic $\mathrm{U}(1)_\mathrm{z}$ charges listed above are given by [49, 60]:

$$\begin{aligned}
&[\mathrm{SU}(2)]^2[\mathrm{U}(1)_\mathrm{Z}] && z_l + 3z_q &= 0 && (193)\\
&[\mathrm{SU}(3)]^2[\mathrm{U}(1)_\mathrm{Z}] && 2z_q - z_d - z_u &= 0 && (194)\\
&[\mathrm{U}(1)_\mathrm{Y}]^2[\mathrm{U}(1)_\mathrm{Z}] && \frac{2}{3}z_q + 2z_l - \frac{16}{3}z_u - \frac{4}{3}z_d - 4z_e &= 0 && (195)\\
&[\mathrm{U}(1)_\mathrm{Y}][\mathrm{U}(1)_\mathrm{Z}]^2 && 2z_q^2 + 4z_u^2 - 2z_d^2 - 2z_l^2 - 2z_e^2 &= 0 && (196)\\
&[\mathrm{U}(1)_\mathrm{Z}]^3 && 3\left(6z_q^3 + 2z_l^3 - 3z_u^3 - 3z_d^3 - z_e^3\right) - n_R\, z_\nu^3 &= 0 && (197)
\end{aligned}$$

$z_{q,l,u,d,e,\nu}$ denotes the $\mathrm{U}(1)_\mathrm{z}$ charge of the left-handed quark and lepton fields and the right-handed up- and down quark, lepton and neutrino fields respectively.

Making use of the relations obtained from Eq. (193), (194) and (195),

$$\begin{aligned}
z_l &= -3z_q, && (198)\\
z_d &= 2z_q - z_u, && (199)\\
z_e &= -\left(2z_q + z_u\right), && (200)
\end{aligned}$$

Eq. (196) is automatically satisfied.

The $[\mathrm{U}(1)_\mathrm{Z}]^3$ anomaly condition in (197) leads to

$$n_R\, z_\nu^3 = 3\left(-4z_q + z_u\right)^3. \tag{201}$$

To allow the SM Yukawa interactions for the quarks and the leptons, the $\mathrm{U}(1)_\mathrm{z}$ charge of the SM Higgs doublet z_H is constrained to

$$z_H = -z_q + z_u. \tag{202}$$

The existence of the Yukawa interaction term with ν_R

$$\overline{L}\Delta\tilde{H}\nu_R, \tag{203}$$

as introduced in section 5.1, requires z_ν to be

$$z_\nu = -4z_q + z_u. \tag{204}$$

With this constraint the mixed gravitational-$\mathrm{U(1)_z}$ anomaly cancellation condition,

$$\mathrm{grav}^2\mathrm{U}(1)_\mathrm{Z} \qquad 3\,(6z_q + 2z_l - 3z_u - 3z_d - z_e) - n_R\, z_\nu = 0, \tag{205}$$

which is related to diagrams with two gravitons and a gauge boson at the external legs and requires the trace of the z charges to vanish, $\mathrm{Tr}(z) = 0$, is automatically satisfied.

With (204), Eq. (201) reads:

$$n_R\, z_\nu^3 = 3\, z_\nu^3. \tag{206}$$

Hence the number of additional right-handed neutrino fields, n_R, is fixed to $n_R = 3$.

By choosing the normalization $z_S = 1$, the Yukawa-type Majorana mass term $\frac{1}{2}\overline{\nu_R^c}\Gamma_\nu S \nu_R$ defines z_ν to be $z_\nu = -\frac{1}{2}$.

The anomaly cancellation conditions in (193), (194), (195), (196) and (197) and the fermion mass constraints in (202) and (204) lead to a set of solutions for the $\mathrm{U(1)_z}$ charges of the fermion fields with one free parameter, which is chosen to be z_H, shown in Table 2 [49, 60].

Name	T_{3L}	Y	z
$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$\frac{1}{3}$	$\frac{1}{3}z_H + \frac{1}{6}$
u_R	0	$\frac{4}{3}$	$\frac{1}{3}\,(4z_H + \frac{1}{2})$
d_R	0	-$\frac{2}{3}$	-$\frac{1}{3}\,(2z_H - \frac{1}{2})$
$L = \begin{pmatrix} \nu_L \\ l_L \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	-1	$-z_H - \frac{1}{2}$
ν_R	0	0	-$\frac{1}{2}$
l_R	0	-2	$-2z_H - \frac{1}{2}$
$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	1	z_H
S	0	0	1

Table 2: Fermion and scalar gauge charges of the $\mathrm{U(1)_z}$ extension of the SM.

5.6 Yukawa interactions and Fermion masses

The Yukawa terms listed up in the following Yukawa interaction Lagrangian $\mathcal{L}_{\mathrm{Y}}$ are responsible for the generation of fermion masses after spontaneous symmetry breaking:

$$\begin{aligned}\mathcal{L}_{\mathrm{Y}} &= -\overline{L}_i \lambda^l_{ij} H l_{jR} - \overline{L}_i \Delta_{ij} \tilde{H} \nu_{jR} \\ &\quad -\overline{Q}_{iL} \lambda^u_{ij} \tilde{H} u_{jR} - \overline{Q}_{iL} \lambda^d_{ij} H d_{jR} \\ &\quad -\frac{1}{2} \overline{\nu^c_{iR}} \Gamma_{\nu,ij} S \nu_{jR} + \text{h.c.}\,.\end{aligned} \tag{207}$$

L_i and Q_{iL} denote the left-handed lepton and quark doublets with

$$L_i = \begin{pmatrix} \nu_L \\ l_L \end{pmatrix}_i , \tag{208}$$

$$Q_{iL} = \begin{pmatrix} u_L \\ d_L \end{pmatrix}_i , \tag{209}$$

and l_{iR}, u_{iR}, d_{iR} and ν_{iR} denote the right-handed lepton, up-quark, down-quark and neutrino fields respectively, $(i = 1,2,3)$. $\lambda^{l,u,d}, \Delta$ and Γ_ν are in general complex-valued 3×3 matrices, containing the Yukawa couplings and $\tilde{H} = i\sigma_2 H^*$. For reasons of simplicity the flavour indices are omitted in the following discussion.

5.6.1 Charged leptons

The Yukawa term for charged leptons in terms of scalar mass eigenstates in unitary gauge in Eq. (149) is given by

$$\mathcal{L}_{\mathrm{Y},cl} = -\frac{1}{\sqrt{2}}\, \bar{l}_L \lambda^l\, l_R\, (v_H + h\cos\alpha + s\sin\alpha) + \text{h.c.}\ . \tag{210}$$

From Eq. (210) one obtains the mass matrix of the leptons

$$M^l = \frac{1}{\sqrt{2}} v_H \lambda^l\,. \tag{211}$$

Diagonalizing the mass matrix M^l by

$$U^{l\dagger}_L M^l U^l_R = \hat{M}^l, \tag{212}$$

with $\hat{M}^l$ diagonal and positive, yields the physical lepton fields $\hat{l} = \hat{l}_L + \hat{l}_R$ via the relation

$$l_{L,R} = U^l_{L,R}\, \hat{l}_{L,R}\,. \tag{213}$$

Therefore the Yukawa term for the charged leptons written in mass eigenstates reads

$$\Delta\mathcal{L}_{\mathrm{Y},cl} = -\bar{\hat{l}} \hat{M}^l P_R\, \hat{l} \left(1 + h\frac{\cos\alpha}{v_H} + s\frac{\sin\alpha}{v_H}\right) + \text{h.c.}\,, \tag{214}$$

with the projection operator P_R on the right-handed component of the lepton field $\hat{l}$. Hence the Yukawa coupling of the SM leptons to the physical Higgs field h is proportional to the lepton mass m_k^l, with $\hat{M}_{kj}^l = m_k^l \delta_{kj}$,

$$g_{h\bar{l}l,k} = \frac{m_k^l}{v_H} \cos\alpha. \tag{215}$$

5.6.2 Quark fields

The case of quark fields can be formulated in analogy to the case of charged leptons with the difference that the relation between weak eigenfields and states with a definite mass is different for up-type and down-type quarks $u_{L,R}^i$ and $d_{L,R}^i$. Following the example of the charged leptons the Yukawa term for the quark fields in terms of scalar mass eigenstates in unitary gauge reads:

$$\mathcal{L}_{\mathrm{Y},q} = -\frac{1}{\sqrt{2}} \left(\overline{u}_L \lambda^u \, u_R + \overline{d}_L \lambda^d \, d_R \right) (v_H + h\cos\alpha + s\sin\alpha) + \text{h.c.} \; . \tag{216}$$

Like in the case of charged leptons the diagonalization of the mass matrices for the up-type quarks, $M^u = \frac{1}{\sqrt{2}}\lambda^u v_H$, and for the down-type quarks, $M^d = \frac{1}{\sqrt{2}}\lambda^d v_H$,

$$U_u^\dagger M^u W_u = \hat{M}^u \, , \;\; \text{and} \;\; U_d^\dagger M^d W_d = \hat{M}^d \, , \tag{217}$$

with $\hat{M}^u$ and $\hat{M}^d$ diagonal and positive, yields the mass eigenstates with a definite mass via the relations

$$u_L = U_u \hat{u}_L \quad , \quad u_R = W_u \hat{u}_R \, , \tag{218}$$

$$d_L = U_d \, \hat{d}_L \quad , \quad d_R = W_d \, \hat{d}_R \, . \tag{219}$$

In terms of scalar and quark field mass eigenstates the Yukawa terms are given by

$$\mathcal{L}_{\mathrm{Y},q} = -\left(\overline{\hat{u}}_L \, \hat{M}^u \, \hat{u}_R + \overline{\hat{d}}_L \, \hat{M}^d \, \hat{d}_R \right) \left(1 + h\frac{\cos\alpha}{v_H} + s\frac{\sin\alpha}{v_H} \right) + \text{h.c.} \, , \tag{220}$$

and the coupling constants of the SM quarks to the physical Higgs field h are proportional to the mass of the quarks m_k^q, with

$$g_{h\overline{q}q,k} = \frac{m_k^q}{v_H} \cos\alpha. \tag{221}$$

5.6.3 Neutrino masses and the See-Saw mechanism

The addition of a number of 3 right-handed neutrino fields ν_R and a scalar field S, which is a singlet under the SM gauge group $\mathrm{G_{SM}}$, to the particle content of the SM allows the construction of a Yukawa term and a Majorana-type Yukawa term with right-handed neutrinos and the generation of neutrino masses via the see-saw mechanism [50–53],

$$\mathcal{L}_{Y,\nu} = -\left(\overline{L}\Delta\tilde{H}\nu_R + \frac{1}{2}\overline{\nu_R^c}\,\Gamma_\nu S\,\nu_R\right) + \text{h.c.}\,. \tag{222}$$

Following the procedure of the examples of the charged leptons and the quarks, the Yukawa terms for the neutrinos in terms of scalar mass eigenstates are given by

$$\begin{aligned}\mathcal{L}_{Y,\nu} \quad = \quad & -\frac{1}{\sqrt{2}}\,\overline{\nu}_L\Delta\,\nu_R\,(v_H + h\cos\alpha + s\sin\alpha) \\ & -\frac{1}{2\sqrt{2}}\,\overline{\nu_R^c}\,\Gamma_\nu\,\nu_R\,(v_S - h\sin\alpha + s\cos\alpha) + \text{h.c.}\;.\end{aligned} \tag{223}$$

To review the principles of the see-saw mechanism it will be focused on the mass terms for neutrinos contained in this part of the Yukawa Lagrangian, following the discussion of [77, 82],

$$\mathcal{L}_{m_\nu} = \mathcal{L}^D + \mathcal{L}^M = -\overline{\nu}_L m_D \nu_R - \frac{1}{2}\overline{\nu_R^c}\,M_R\nu_R + \text{h.c.}, \tag{224}$$

where $\mathcal{L}^D$ denotes the Dirac part and $\mathcal{L}^M$ the Majorana part of the mass terms, with $m_D = \frac{1}{\sqrt{2}}\Delta v_H$ and $M_R = \frac{1}{\sqrt{2}}\Gamma_\nu v_S$ being complex valued 3×3 mass matrices.

The charge-conjugated fields are defined by

$$(\nu_L)^c \equiv C\,\overline{\nu_L}^T, \qquad (\nu_R)^c \equiv C\,\overline{\nu_R}^T, \tag{225}$$

where C denotes the charge-conjugation matrix.

Hence one has

$$\overline{(\nu_L)^c} = -\nu_L^T\,C^{-1}, \qquad \overline{(\nu_R)^c} = -\nu_R^T\,C^{-1}. \tag{226}$$

It is to notice that $(\nu_L)^c$ and $(\nu_R)^c$ are right-handed and left-handed components, respectively [77].

From the anticommutation property of fermion fields and the antisymmetry of the charge-conjugation operator C it follows the relation

$$\nu_{iL}^T\,C^{-1}\nu_{jL} = \nu_{jL}^T\,C^{-1}\nu_{iL}, \qquad i,j = 1,2,3 \tag{227}$$

and an analogous relation holds for the right-handed fields. Using this relation one can show, that the matrix M_R is symmetric:

$$\sum_{i,j}\overline{(\nu_{iR})^c}(M_R)_{ij}\,\nu_{jR} = -\sum_{i,j}\nu_{jR}^T\,C^{-1}(M_R)_{ij}\,\nu_{iR} = \sum_{i,j}\overline{(\nu_{iR})^c}(M_R)_{ji}\,\nu_{jR}. \tag{228}$$

Taking account of the anticommutation property of fermion fields one obtains for the Dirac mass term:

$$\sum_{ij}\overline{\nu_{iL}}\,(m_D)_{ij}\,\nu_{jR} = -\sum_{ij}\nu_{jR}^T\,(m_D)_{ij}\,\overline{\nu_{iL}}^T = \sum_{i,j}\overline{(\nu_{jR})^c}(m_D)_{ji}^T\,(\nu_{iL})^c\,. \tag{229}$$

With the definition of an $3+3$ component neutrino field [82]

$$\omega_R \equiv \begin{pmatrix} (\nu_L)^c \\ \nu_R \end{pmatrix}, \tag{230}$$

the Dirac-Majorana mass term $\mathcal{L}_{m_\nu}$ can then be written in the form

$$\mathcal{L}_{m_\nu} = \mathcal{L}^D + \mathcal{L}^M = \frac{1}{2}\omega_R^T C^{-1} M_\nu \, \omega_R + \text{h.c.}, \tag{231}$$

where the matrix $M_\nu = M_\nu^T$ is given by

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}. \tag{232}$$

Diagonalization of the mass matrix M_ν by a unitary transformation [82]

$$U^\dagger M_\nu U^* = \hat{M}_\nu, \tag{233}$$

with $\hat{M}_\nu$ diagonal and non-negative, yields the (tree-level) mass eigenfields $\hat{\omega}_R$

$$\omega_R = U^* \hat{\omega}_R, \tag{234}$$

and [82]

$$\mathcal{L}^D + \mathcal{L}^M = \frac{1}{2}\hat{\omega}_R^T C^{-1} \hat{M}_\nu \, \hat{\omega}_R + \text{h.c.} = -\frac{1}{2}\overline{\hat{\omega}_R^c} \hat{M}_\nu \, \hat{\omega}_R + \text{h.c.} = -\frac{1}{2}\overline{N} \hat{M}_\nu N, \tag{235}$$

with

$$N \equiv \begin{pmatrix} \nu_1 \\ \vdots \\ \nu_i \\ \vdots \end{pmatrix} = \hat{\omega}_R + (\hat{\omega}_R)^c, \qquad i = 1, \cdots, 6 \tag{236}$$

with ν_i satisfying the Majorana condition

$$(\nu_i)^c = \nu_i. \tag{237}$$

Thus, in general if considering a Dirac-Majorana mass term, the diagonalization of the mass term in (231) yields fields of Majorana-particles as states with definite masses. A Majorana particle is its own antiparticle and has spin $\frac{1}{2}$. Since the mass term in Eq. (231) is not invariant under any global phase transformation [77], it is a natural consequence that the mass eigenfields of a general Dirac and Majorana mass term are fields of Majorana particles, since there are no conserved quantum numbers that allow to distinguish a particle from its antiparticle [77].

By an appropriate decomposition of the matrix U [82]

$$U = \begin{pmatrix} U_L \\ U_R^* \end{pmatrix}, \tag{238}$$

the weak eigenfields $\nu_{L,R}$ can be expressed in terms of mass eigenfields

$$\begin{aligned}\nu_L &= U_L\,\hat{\omega}_R^c = U_L P_L N \qquad (239)\\ \nu_R &= U_R\,\hat{\omega}_R = U_R P_R N, \qquad (240)\end{aligned}$$

with $P_{R,L} = \frac{1}{2}\left(\mathbb{1} \pm \gamma_5\right)$ being the projectors on the right- and left-handed field components respectively.

5.6.4 The one-generation case

Considering the case with one generation of right- and left-handed neutrino fields $\nu_{R,L}$, respectively, the Dirac and Majorana mass term is given by

$$\begin{aligned}\mathcal{L}^D + \mathcal{L}^M &= -\overline{\nu}_L m_D \nu_R - \frac{1}{2}\overline{\nu_R^c} M_R \nu_R + \text{h.c.} \qquad (241)\\ &= \frac{1}{2}\omega_R^T C^{-1} M_\nu\, \omega_R + \text{h.c.}, \qquad (242)\end{aligned}$$

with the $1+1$ component neutrino field ω_R and the complex 2×2 mass matrix M_ν

$$\omega_R \equiv \begin{pmatrix} (\nu_L)^c \\ \nu_R \end{pmatrix}, \qquad M_\nu \equiv \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix}, \qquad (243)$$

where m_D and M_R are in general complex parameters, with $m_D \ll M_R$.

The mass eigenvalues of M_ν read [82]

$$\begin{aligned}m_1 &= \sqrt{\frac{1}{4}M_R^2 + m_D^2} - \frac{1}{2}M_R \simeq \frac{m_D^2}{M_R}, \qquad (244)\\ m_2 &= \sqrt{\frac{1}{4}M_R^2 + m_D^2} + \frac{1}{2}M_R \simeq M_R. \qquad (245)\end{aligned}$$

Diagonalization of M_ν is done using the unitary matrix U

$$U = \begin{pmatrix} i\cos\beta & \sin\beta \\ -i\sin\beta & \cos\beta \end{pmatrix}, \qquad (246)$$

with [82]

$$\sin\beta = \sqrt{\frac{m_1}{m_1+m_2}} \simeq \frac{m_D}{M_R}, \qquad \cos\beta > 0, \qquad (247)$$

as described in the previous section:

$$U^\dagger M_\nu U^* = \hat{M}_\nu = \text{diag}(m_1, m_2). \qquad (248)$$

With the scale of M_R being much larger than that of m_D, $m_D \ll M_R$, the see-saw mechanism leads to the generation of one light Majorana neutrino, with $m_{light} = m_1 \simeq \frac{m_D^2}{M_R}$ and one heavy Majorana neutrino, with $m_{heavy} = m_2 \simeq M_R$.

The see-saw mechnanism is in general based on the assumption that the Majorana mass term for the right-handed neutrinos violates the conservation of the total lepton number L on a scale Λ that is much larger than the scale of electroweak symmetry breaking [77]. Depending on the specific model under consideration the scale Λ can take values in the TeV range, or can be as high as the grand unification scale of $\sim 10^{15}$ GeV or even the Planck scale of $\sim 10^{19}$ GeV [77].

5.6.5 The case with $n_L = 3$ and $n_R = 3$

In the case of 3 left- and right-handed neutrino fields, the see-saw mechanism leads to a mass spectrum of 3 light neutrino Majorana particles with masses m_k and 3 very heavy neutrino Majorana mass eigenfields with masses of the order of the high symmetry breaking scale, $M_k \sim \Lambda$, with $m_k \ll M_k$, $(k = 1, 2, 3)$.

The Dirac-Majorana mass matrix M_ν

$$M_\nu \equiv \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}, \tag{249}$$

where the matrix M_R is such, that its eigenvalues are much larger than the elements of m_D, $m_D \ll M_R$, is block-diagonalized by the unitary transformation [77, 83]

$$B^\dagger M_\nu B^* = \begin{pmatrix} M_{light} & 0 \\ 0 & M_{heavy} \end{pmatrix}, \tag{250}$$

with [77, 83]

$$B = \begin{pmatrix} \sqrt{\mathbb{1} - JJ^\dagger} & J \\ -J^\dagger & \sqrt{\mathbb{1} - J^\dagger J} \end{pmatrix}, \tag{251}$$

where $J \simeq m_D (M_R)^{-1}$ in the lowest order of the expansion in $\frac{m_D}{M_R}$.

The matrices for the light and heavy neutrino masses are given by [84, 85]

$$M_{light} \simeq -m_D \frac{1}{M_R} m_D^T, \qquad M_{heavy} \simeq M_R, \tag{252}$$

where the accuracy for M_{heavy} is $\mathcal{O}\left(\frac{1}{M_R} m_D^T m_D\right)$ and for M_{light} $\mathcal{O}\left(\frac{m_D^3}{M_R^2}\right)$ [84].

For the light neutrino masses the mass eigenvalues depend on the specific form of m_D and M_R [77]. Independent of which case is considered the see-saw mechanism implies the hierarchical relation

$$m_1 \ll m_2 \ll m_3, \tag{253}$$

for the three light neutrino masses [77]. Via the see-saw mechanism light neutrino masses emerge naturally. Therefore this mechanism represents a model with great attractiveness, explaining the smallness of observed neutrino masses with respect to the masses of the other fundamental fermions [77], $\frac{m_k}{m_e} \sim 10^{-6}$, where m_e is the mass of the electron [86].

Using the diagonalization procedure described above and the relation

$$\begin{pmatrix} 0 & m_D \\ m_D^T & -\tan^2\alpha M_R \end{pmatrix} = -\tan^2\alpha \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} + \left(1+\tan^2\alpha\right) \begin{pmatrix} 0 & m_D \\ m_D^T & 0 \end{pmatrix}, \tag{254}$$

the Yukawa terms for the neutrinos in terms of mass eigenstates read

$$\begin{aligned} \Delta\mathcal{L}_{Y,\nu} = -\frac{1}{2} \quad \Bigg[\quad & \overline{N}\hat{M}_\nu N \left(1 + \frac{s}{v_H}\sin\alpha - \frac{h}{v_H}\tan^2\alpha\cos\alpha\right) \\ & + \overline{N}\left(U_L^\dagger m_D U_R P_R + U_R^T m_D^T U_L^\dagger P_R + U_R^\dagger m_D^\dagger U_L P_L \right. \\ & \left. + U_L^T m_D^* U_R^* P_L\right) N \left(1+\tan^2\alpha\right) \frac{h}{v_H}\cos\alpha \quad \Bigg], \end{aligned} \tag{255}$$

where $\hat{M}_\nu$ is the 6 dimensional diagonal and non-negative mass matrix with 3 light and 3 heavy neutrino Majorana mass eigenvalues m_k and M_k ,
$(k = 1, 2, 3)$.

5.7 Gauge boson - scalar interactions

The interactions of gauge bosons with scalar fields arise from

$$\Delta\mathcal{L} = (D^\mu H)^\dagger (D_\mu H) + (D^\mu S)^\dagger (D_\mu S), \tag{256}$$

with the scalar fields in unitary gauge

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_H + h' \end{pmatrix}, \qquad S = \frac{1}{\sqrt{2}} \left(v_S + s' \right), \tag{257}$$

and the covariant derivative D_μ in mass eigenstates in Eq. (185).

It will be focused on terms of $D_\mu H$ without terms containing $W^\pm$ since they stay the same as in the SM.

Relevant terms from $\Delta\mathcal{L}$ are given by

$$\begin{aligned} & \frac{1}{8}(v_H + h')^2 \left(\left[\frac{g}{\cos\theta_W} \cos\theta - 2 g_z z_H \sin\theta \right] Z_\mu \right. \\ & \left. - \left[\frac{g}{\cos\theta_W} \sin\theta + 2 g_z z_H \cos\theta \right] Z'_\mu \right)^2 \\ + \; & \frac{1}{8}(v_S + s')^2 g_z^2 \left(\sin\theta Z_\mu + \cos\theta Z'_\mu \right)^2 . \end{aligned} \tag{258}$$

Using (174) to express the mass eigenvalues of the Z boson, m_Z, and the Z' boson, $m_{Z'}$ in terms of the angle θ,

$$m_Z^2 = m_{\tilde{Z}}^2 \cos^2\theta + 2\Delta^2 \sin\theta\cos\theta + m_{\tilde{Z}'}^2 \sin^2\theta \tag{259}$$

$$m_{Z'}^2 = m_{\tilde{Z}}^2 \sin^2\theta - 2\Delta^2 \sin\theta\cos\theta + m_{\tilde{Z}'}^2 \cos^2\theta, \tag{260}$$

where $m_{\tilde{Z}}^2$, $m_{\tilde{Z}'}^2$ and Δ^2 are given in Eqs. (169-171), Eq. (258) reads

$$\begin{aligned} \Delta\mathcal{L} = \; & \frac{1}{2} \left(m_{Z'}^2 Z'_\mu Z'^\mu + m_Z^2 Z_\mu Z^\mu \right) \left(1 + \frac{h'}{v_H} \right)^2 \\ & - \frac{1}{8} g_z^2 \cot^2\alpha \, v_H^2 \left(Z_\mu \sin\theta + Z'_\mu \cos\theta \right)^2 \left(\frac{2h'}{v_H} + \frac{h'^2}{v_H^2} \right) \\ & + \frac{1}{8} g_z^2 \cot^2\alpha \, v_H^2 \left(Z_\mu \sin\theta + Z'_\mu \cos\theta \right)^2 \left(\frac{s'^2}{v_H^2} \tan^2\alpha + \frac{2s'}{v_H} \tan\alpha \right), \end{aligned} \tag{261}$$

with $\tan\alpha = \frac{v_H}{v_S}$. Written in terms of scalar mass eigenstates, h and s,

$$h' = h\cos\alpha + s\sin\alpha, \tag{262}$$

$$s' = s\cos\alpha - h\sin\alpha, \tag{263}$$

one yields for $\Delta\mathcal{L}$:

$$\begin{aligned}
\Delta\mathcal{L} \;=\; & \frac{1}{2}\left(m_{Z'}^2 Z'_\mu Z'^\mu + m_Z^2 Z_\mu Z^\mu\right)\left(1+\frac{h\cos\alpha + s\sin\alpha}{v_H}\right)^2 \\
& -\frac{1}{8}g_z^2\cot^2\alpha\, v_H^2\left(\sin\theta Z_\mu + \cos\theta Z'_\mu\right)^2 \times \\
& \quad \times\left(\frac{2(h\cos\alpha + s\sin\alpha)}{v_H} + \frac{(h\cos\alpha + s\sin\alpha)^2}{v_H^2}\right) \\
& +\frac{1}{8}g_z^2\left(\sin\theta Z_\mu + \cos\theta Z'_\mu\right)^2\left((s\cos\alpha - h\sin\alpha)^2\right. \\
& \quad \left. +2\cot\alpha\, v_H(s\cos\alpha - h\sin\alpha)\right).
\end{aligned} \tag{264}$$

From expression (264) one obtains the gauge boson - scalar interaction couplings given in the following.

Gauge boson - Higgs couplings:

$$hZZ: \quad \left(\frac{m_Z^2}{v_H} - \frac{1}{4}g_z^2 v_H \frac{\sin^2\theta}{\sin^2\alpha}\right)\cos\alpha \tag{265}$$

$$hZ'Z': \quad \left(\frac{m_{Z'}^2}{v_H} - \frac{1}{4}g_z^2 v_H \frac{\cos^2\theta}{\sin^2\alpha}\right)\cos\alpha \tag{266}$$

$$hZ'Z: \quad -\frac{1}{2}g_z^2 \frac{v_H}{\sin^2\alpha}\sin\theta\cos\theta \tag{267}$$

$$h^2ZZ: \quad \frac{1}{2}\frac{m_Z^2}{v_H^2}\cos^2\alpha + \frac{1}{8}g_z^2(1-\cot^2\alpha)\sin^2\theta \tag{268}$$

$$h^2Z'Z': \quad \frac{1}{2}\frac{m_{Z'}^2}{v_H^2}\cos^2\alpha + \frac{1}{8}g_z^2(1-\cot^2\alpha)\cos^2\theta \tag{269}$$

$$h^2Z'Z: \quad \frac{1}{4}g_z^2(1-\cot^2\alpha)\sin\theta\cos\theta \tag{270}$$

Gauge boson - scalon couplings:

$$sZZ: \quad \frac{m_Z^2}{v_H}\sin\alpha \tag{271}$$

$$sZ'Z': \quad \frac{m_{Z'}^2}{v_H}\sin\alpha \tag{272}$$

$$sZ'Z: \quad - \tag{273}$$

$$s^2ZZ: \quad \frac{1}{2}\frac{m_Z^2}{v_H^2}\sin^2\alpha \tag{274}$$

$$s^2Z'Z': \quad \frac{1}{2}\frac{m_{Z'}^2}{v_H^2}\sin^2\alpha \tag{275}$$

$$s^2Z'Z: \quad - \tag{276}$$

Mixed Gauge boson - scalar couplings:

$$hsZZ: \quad \frac{m_Z^2}{v_H^2}\cos\alpha\sin\alpha - \frac{1}{4}g_z^2\cot\alpha\sin^2\theta \tag{277}$$

$$hsZ'Z': \quad \frac{m_{Z'}^2}{v_H^2}\cos\alpha\sin\alpha - \frac{1}{4}g_z^2\cot\alpha\cos^2\theta \tag{278}$$

$$hsZ'Z: \quad -\frac{1}{2}g_z^2\cot\alpha\sin\theta\cos\theta \tag{279}$$

Summary and Conclusion

The model considered in this thesis provides a possible extension of the SM with classical scale invariance, where the origin of mass is described by quantum corrections and a light electroweak scale emerges naturally, being consistent with the experimentally observed Higgs mass of $m_h \simeq 125\,\mathrm{GeV}$. With a symmetry breaking scale Λ, chosen to be in the TeV region, new physical degrees of freedom are expected to arise at the TeV scale. The extension of the SM gauge group by an additional $\mathrm{U}(1)_\mathrm{z}$ factor results in the prediction of a new heavy Z' boson with mass $m_{Z'} \sim \mathcal{O}(\mathrm{TeV})$ and the introduction of three right-handed neutrino fields being singlets under the SM gauge group G_SM, leads to heavy Majorana masses in the TeV region. The generation of neutrino masses via the see-saw mechanism leads to a natural emergence of light neutrino masses with respect to the masses of the other fundamental fermions. Apart from the introduction of the right-handed neutrinos to the particle content of the model, no extra exotic fermions are included.

The scalar sector of the model is enlarged by an additional complex scalar field, being a singlet with respect to the SM gauge group, with an $\mathrm{U}(1)_\mathrm{z}$ charge, chosen to be $z_s = 1$, which is responsible for the spontaneous symmetry breaking of the additional gauge group $\mathrm{U}(1)_\mathrm{z}$. The effective scalar potential is calculated up to one-loop order, using dimensional regularization for the computation of quantum corrections and the contributions from scalar, vector and fermion fields are given in the $\overline{\mathrm{MS}}$ scheme. Throughout the analysis of the effective potential, the constraints for the parameters of the model to guarantee the validity of the perturbative description of the effective potential are worked out and a constraining relation for the mass of the new Z' boson and the heavy neutrino Majorana masses can be formulated, when taking into account the experimental values of the masses of the Higgs boson, the top quark and the W- and Z-bosons. To discuss the stability of the effective potential, a study of the behavior of the dimensionless coupling constants λ_H, λ_S and λ_{HS} by an analysis by renormalization group equations can be performed in further examinations.

It is found that the electric charges of the observed fermions imply, that the gauge group of models with an additional abelian factor, leading to a new color-singlet and electrically neutral gauge boson, is $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y} \times \mathrm{U}(1)_\mathrm{z}$, with $\mathrm{U}(1)_\mathrm{Y}$ being the SM hypercharge gauge group. $\mathrm{U}(1)_\mathrm{z}$ is a new spontaneously broken symmetry and the basis of the $\mathrm{U}(1)_\mathrm{Y} \times \mathrm{U}(1)_\mathrm{z}$ gauge fields is chosen to be kinetically diagonal, without loss of generality [49]. The addition of three right-handed neutrino fields leads to a set of solutions for the $\mathrm{U}(1)_\mathrm{z}$ charges for the fermions with one free parameter, chosen to be z_H, being consistent with anomaly cancellation conditions and the existence of fermion mass terms [49]. Furthermore it turns out, that the mixed gravitational- $\mathrm{U}(1)_\mathrm{z}$ anomaly cancellation condition is automatically satisfied, if one requires the existence of a Yukawa interaction term for the right-handed neutrinos. The choice of a $\mathrm{U}(1)_\mathrm{z}$ charge for the SM Higgs field of $z_H \neq 0$, leads to the mass mixing of the SM Z and the Z' boson at tree-level and the mass and the couplings of the Z boson mass eigenstate are affected [49]. Therefore bounds for the parameters primarily describing the properties of the

Z' boson, its gauge coupling g_z, the VEV of the complex scalar singlet field v_s and the $\mathrm{U(1)_z}$ charge of the Higgs field z_H [49], coming from electroweak precision measurements at the Z-pole have to be considered in the discussion of these models.

The class of SM extensions with an additional abelian gauge group $\mathrm{U(1)_z}$ provides possible prospects for further future research on the SM as a classically scale invariant theory, in exploring the allowed Z' parameter space bounded by constraints coming from experimental data and theoretical conditions and in studying models with different z charge assignments to the particles. By now, no new physics at the TeV scale has been discovered, which also could form a potential direction for further future investigations at collider facilities such as the LHC.

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