MASTERARBEIT / MASTER’S THESIS

Titel der Masterarbeit / Title of the Master‘s Thesis

„Re-visiting Models for Balanced Vehicle Routing: An Analysis of Alternative Functions and Metrics“

verfasst von / submitted by

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angestrebter akademischer Grad / in partial fulfilment of the requirements for the degree of

Master of Science (MSc)

Wien, 2017 / Vienna 2017

Studienkennzahl lt. Studienblatt / degree programme code as it appears on the student record sheet:	A 066 915
Studienrichtung lt. Studienblatt / degree programme as it appears on the student record sheet:	Masterstudium Betriebswirtschaft
Betreut von / Supervisor:	o. Univ.-Prof. Dipl.-Ing. Dr. Richard Hartl

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1. Introduction

Due to an increased competition environment in logistics and transportation industry, companies are forced to create an effective way of distribution of the goods to the customers. Besides the pursuing of high profits by lowering the costs, they must consistently improve the quality of services in order to retain and enlarge their customer base so that they would be able to maintain their position in the business. As the enhancement of customer service is usually connected to a cost increase, the companies are facing the challenge of fulfilment of two conflicting objectives.

Considering the high importance of an effective distribution planning, the Vehicle Routing Problem (VRP) belongs to the most studied problems of operations research and is gaining attraction of many authors of research papers. A VRP deals with an optimization of the routes of a vehicle fleet that should deliver the demands to a given set of customers. Besides the optimization of objective function, the VRP must take into consideration several constraints - the capacity of the vehicles cannot be exceeded, each customer must be visited exactly once and each vehicle can make only one tour, whereby it starts and ends its route in a depot. The classical goal of VRP is to deliver all the demands at lowest possible cost.

By adding an equity criteria to the model of VRP, the company can achieve higher quality of services, whether by treating the employees with fairness or by acceleration of delivery process. There are several different motivation for using an equity criteria in the literature. Lee and Ueng (1999) aimed at the improvement of working conditions of the employees. They claimed that the success of a company is dependent on its key resource, the employees, therefore a company must treat its employees fairly. Otherwise, the unsatisfying working conditions can cause a low quality of service. This could be avoided by adding a workload balancing factor into the distribution model. Since the better balanced workloads between employees deliver usually higher cost, it is dependent on a decision maker, to what extent he considers the equity conditions of its employees as important. By adding an equity criteria to the distribution model can be achieved not only the employee satisfaction, but it has also an impact on the level of profits. In the competitive environment, the fast delivery to the customers can influence the amount of sales. If a company delivers the goods or services later than its competitor, it will lose the sale and its profits decrease. Therefore it is necessary to balance the workload of each vehicle to ensure that each customer will be served in the fastest possible time (Norouzi et al., 2012). The time of delivery is also important in the business environments where a delivery of goods or services is required to be promptly due to their nature, such as delivery of news papers or sensitive goods. The consideration of equity criteria is of high importance and should be considered alongside the financial goals (Tsourous et al., 2006).

The researchers who deal with the distribution of public services usually use the equity criteria for improving the resource utilization. For instance, the aim of the school bus routing is to avoid the overcrowding and underutilization. This can be achieved by adding a balancing criteria into the distribution model (Bowerman et al., 1995, Lee and Moore, 1977).

The VRP model introduced in this work considers in addition to the minimization of the total cost for customer delivery also the equity criteria for the employees. The equity consideration that is examined in this work concerns the fairness between the employees that can be achieved by better balanced workload. By reaching the high level of employee satisfaction, the company manages to decrease its costs due to lower fluctuation that is often connected to inadequate working conditions.

The aim of this work is to deliver a satisfactory trade off between economic efficiency and equitable workload allocation. For this purpose we have introduced four bi-objective models for VRP with workload balancing in order to explore several alternatives for defining the workload balance, and investigate the effect of various equity metrics and functions on the resulting VRP solutions.

2. Literature Survey

Most papers about the VRP focus on minimizing the transportation time or the length of the tours in order to increase the cost efficiency, since the organizations aim to maximize their profits. The effort of minimizing the workload between the vehicles is often connected to the increase in cost due to longer transportation time and increased length of the tours, therefore the consideration of the employees welfare is usually not attractive for the corporations. In spite of this, there are papers that focus on decreasing the workload imbalance between the vehicles and thus bring fairness in the assignments of employees.

The workload can be measured in various ways, for instance as the time required for customer delivery, the tour length, number of visited customers or the customers' demand. Similarly, there are many equity functions that can be used for achieving the workload balance improvement. The balancing objective can be presented in a single-objective model, where the optimization of workload balance can be defined as primary objective (PO) or expressed as a constraint in the model, or in bi-objective models, where the reduction of workload imbalance is pursued in addition to cost optimization. In bi-objective models a balancing objective can be modelled as a weighted sum function or is included in a multi-objective model.

Publication	Equity Function			Equity Metric			Optimization Model			
	Min Max	Range	Other	Demand	Stops	Other	MO	WS	CN	PO
Theory-oriented										
Baños et al. (2013)		x		x		x	x			
Bektaş (2012)		x			x				x	
Benjamin and Abdul-Rahman (2016)	x				x	x	x			
Gouveia et al. (2013)		x			x				x	
Dharmapriya et al. (2010)			x	x				x		
Gulczynski et al. (2011)		x			x			x		
Chen and Chen (2008)		x		x					x	
Kara and Bektaş (2005)			x	x					x	
Mourgaya and Vanderbeck (2007)	x			x					x	
Norouzi et al. (2012)		x		x			x			
Ribeiro and Lourenço (2001)			x	x				x		
Tsouros et al. (2006)		x		x			x			
Application-oriented										
Apte and Mason (2006)		x		x	x				x	
Bowerman et al. (1995)			x	x				x		
Groër et al. (2009)			x	x		x			x	
Kritikos and Ioannou (2010)			x	x				x		
Lin and Kwok (2006)		x		x			x			
Liu et al. (2006)		x		x					x	
Mendoza et al. (2009)			x		x		x			

Table 1: Overview of publications dealing with balanced VRPs

We have considered the papers that balanced the workload of the vehicles and used the customers' demand or number of stops as a measure for the workload. Table 1 categorizes the publications of VRP according to the equity functions, equity metrics and optimization models. The papers are divided into two groups, theory and application-oriented.

Theory-oriented publications

Dharmapriya et al. (2010) introduced a bi-objective model for a distribution problem, where two objectives, minimizing the transportation cost and maintaining the load balance among vehicles, were combined into a weighted sum objective function. In his study, the split delivery was allowed, whereby one customer could be served by more vehicles. Chen and Chen (2008) balanced in their paper the workload that was measured by customers' demand and was modelled in a single-objective model as a constraint. They tried to avoid the workload imbalance by assigning the load to each vehicle in the way that the load was kept at an average level.

Kara and Bektaş (2005) presented a model for Minimum Load Constrained VRP, where the balance of loads between the drivers was obtained by bounding the minimum starting or returning load of the vehicles. Lower and upper bounds were used in the paper of Gouveia et al. (2013), where for each vehicle was defined a minimum and maximum number of customers that can be served on each route. Similar approach was used in the paper of Bektaş (2012), where besides the minimization of total cost, the workload in terms of number of visited customers was bounded by a predetermined interval in order to provide balance between the drivers.

Besides the minimization of total distance, Baños et al. (2013) aimed in their work at minimizing the imbalance of workloads, defined by two measures, the travelled distance of used vehicles and the delivered goods by the vehicles. Both workload measures were balanced by minimizing the difference between maximum and minimum workload.

The paper of Gulczynski et al. (2011) involved two variants of VRP, where an existing solution was being improved by reassigning the customers to new patterns in the first one and considering the equity criteria by penalizing the imbalance in the second one. The workload of the vehicles was defined as the number of customers served by each vehicle that is according to the authors the main determinant of workload in practical routing problems.

In the paper of Mourgaya and Vanderbeck (2007) balancing the customers' demand between the vehicles is expressed as a constraint that limits a workload assigned to the vehicles. Norouzi et al. (2012) presented an open vehicle routing problem, where the competition was considered. The goal of this paper was to maximize the sales by the earliest possible arrival to customers and simultaneously increase the load balance by minimizing the difference between the expected load and the average of the expected load allocated to the vehicles.

Benjamin and Abdul-Rahman (2016) analysed the drivers' workload in a Waste Collection VRP, whereby they compared the results of two different analyses - first one balanced the workload in terms of the number of served customers by each vehicle, and the second one measured the workload by total distance travelled.

Ribeiro and Lourenço (2001) took into consideration the human resource management objective that aimed at balancing the working levels. In order to reach higher customer service level, the standard deviation of the workload of each vehicle was minimized. Tsouros et al. (2006) presented in their paper a multi-objective optimization with two objectives - minimizing the maximum deviation between the maximum and the minimum travel times and minimizing the maximum deviation between the maximum and the minimum loading volume of the vehicles.

Application-oriented publications

Bowerman et al. (1995) developed a model for school bus routing problem in Ontario, Canada. He considered three criteria for evaluating the level of public services. Accept efficiency and effectiveness, the public transport should consider an equity criteria that is presented in this paper with the fairness in service distribution for each student. This was achieved by balancing the number of transported students, and the distance travelled by drivers.

A real-world data set of a utility company Routesmart Technologies, Inc. was used in the paper of Groër et al. (2009). The aim was to minimize the route cost of the employees that deliver the bills to the customers, and simultaneously maintain the balance between them. In addition, the researchers had to consider the varying customer base and usage of the meters. Similar problem was introduced by Lin and Kwok (2006) who created a model for a telecommunication service company in Honk Kong, which prints and delivers the bills to its customers. The authors were looking for an optimal trade-off between the minimization of total cost, load imbalance and working time imbalance. The balancing criteria were optimized by minimizing the difference between the maximum and minimum workload.

Apte and Mason (2006) studied delivery operations of San Francisco Public Library in order to make recommendations for improvement of an existing delivery system. Besides the cost optimality of delivery, they aimed at the balanced utilization of truck capacities and balanced routes in terms of visited branches by each vehicle. They claimed that by implementation of the balancing factor into the delivery system, the library will be able to increase the delivery capacity due to better resource utilization.

Kritikos and Ioannou (2010) addressed their paper to the problem of a delivery company in Athens that supplies a large number of supermarkets with their goods. The objective function of their model minimized the route and vehicle cost, as well as the deviation of the loads from the median vehicle load. Their solution method suggests promising results, as the load allocation among all vehicles delivered 23% improvement of the deviation from the median vehicle load.

Mendoza et al. (2009) delivered a decision support system for a water and sewer company in Bogotá, Colombia. Their objectives included optimization of total distance, as well as the workload balance and resource utilization. The balance was measured by the number of auditors visits to customer sites in order to verify if the work was performed properly.

Liu et al. (2006) expressed the equity criteria in their model as a constraint. In their model they used a third party logistics in Taiwan as a data set. They used two balancing criteria, in terms of workload and in terms of delivery time, whereby they determined a tolerance gap between the desired maximum and minimum workload among the drivers.

The aim of this thesis is to extend the research of the multi-objective VRP that considers the workload balancing as a second objective. We present the workload in models with two different equity metrics - the number of served customers and the customers' demand. For each equity metric were used two equity functions, Min Max and Range.

This thesis is structured as follows:

Chapter 3 presents the mathematical model formulation for each model version. Chapter 4 describes the solution methods that were used to solve the proposed models. Chapter 5 is divided into five parts that analyse the results of our computations. In the first part are explored the variances in the number of Pareto-optimal solutions found with different model versions, the second part examines if there is some connection between the number of vehicles used in the models and the number of Pareto-optimal solutions found. The third part explores the development of cost that is caused by the reduction of workload imbalance between the drivers. Some inconsistencies in the workload allocation that were observed in some scenarios are analysed in the fourth part and the last section of the analysis is dedicated to the examination of the amount of identical solutions that were found with different model versions. Conclusions are made in Chapter 6.

3. Modelling the Balanced VRP

The following integer programming formulation was created by modifying the formulation proposed by Baldacci et al. (2007) and Yaman (2006).

Let $G = (V,A)$ be a directed graph with a node set $V = \{0,...,n\}$, where the Node 0 represents the depot and the remaining node set $V' = V \setminus \{0\}$ represents the n customers. Every customer $i \in V'$ has a demand of q_j that should be delivered by a vehicle fleet that is composed of K vehicles, each of them having a capacity Q. The distance between two nodes $i \in V$ and $j \in V \setminus \{i\}$ is defined with the cost c_{ij}.

We have defined two decision variables in our model. The x_{ij}^k is a binary variable that defines, if a vehicle k serves a node j right after node i. If x_{ij}^k is 1, the vehicle k travels directly from customer i to customer j, 0 otherwise. The flow variable u_{ik} specifies the quantity of goods that a vehicle k carries after leaving customer i.

We define with W_{max} and W_{min} the maximal and minimal workloads among all vehicles.

The multi-objective optimization model seeks to satisfy two objectives - the minimization of cost and minimization of the workload imbalance. For the achievement of the second objective, we will examine two different equity functions that should minimize the imbalance:

- Min Max: minimizing the workload of the vehicle with the highest workload
  $f_1 = min\ (W_{max})$
- Range: minimizing the difference between the vehicles with the highest and the lowest workload
  $f_2 = min\ (W_{max} - W_{min})$

The following mathematical formulation is valid for both equity functions and equity metrics:

$\min \sum_{k \in M} \sum_{i,j \in V\ i \neq j} c_{ij} x_{ij}^k$		(1)
s.t.		
$W_{max} \leq \varepsilon_1$		(2a)
$W_{max} - W_{min} \leq \varepsilon_2$		(2b)
$\sum_{k \in M} \sum_{i \in V} x_{ij}^k = 1$	$\forall\, j \in V'$	(3)
$\sum_{i \in V} x_{ip}^k - \sum_{j \in V} x_{pj}^k = 0$	$\forall\, k \in K, \forall\, p \in V'$	(4)
$\sum_{j \in V'} x_{0j}^k = 1$	$\forall\, k \in K$	(5)
$u_{0k} \geq \sum_{i,j \in V} q_i x_{ij}^k$	$\forall\, k \in K$	(6)
$u_{jk} \leq u_{ik} - q_j + Q(1 - x_{ij}^k)$	$\forall\, i, j \in V', \forall\, k \in K$	(7)
$u_{0k} \leq Q$	$\forall\, i \in V, \forall\, k \in K$	(8)
$\sum_{i \in V} \sum_{j \in V'} x_{ij}^k \leq W_{max}$	$\forall\, k \in K$	(9a)
$\sum_{i \in V} \sum_{j \in V'} x_{ij}^k q_j \leq W_{max}$	$\forall\, k \in K$	(9b)
$W_{min} \leq \sum_{i \in V} \sum_{j \in V'} x_{ij}^k \leq W_{max}$	$\forall\, k \in K$	(10a)
$W_{min} \leq \sum_{i \in V} \sum_{j \in V'} x_{ij}^k q_j \leq W_{max}$	$\forall\, k \in K$	(10b)
$u_{ik} \geq 0$		(11)
$x_{ij}^k \in \{0,1\}$		(12)

The objective function (1) minimizes the cost for the whole customer delivery. The ε-constraints are defined by (2a) and (2b). Dependent on the type of equity function, we use in our model either the constraint (2a) for minimizing the maximum workload or the constraint (2b) for minimizing the difference between the maximum and minimum workload. The ε-constraints are further defined by constraints (9a) and (9b) that determine an upper limit for the maximum workload, for models with metric Customers and Demand respectively, and by constraints (10a) and (10b) that determine the bound for difference between the minimum and maximum workload. Because of constraint (3) every customer has to be served exactly once and constraint (4) ensures that if a vehicle visits a customer on its route, it must also depart from it. This constraint guarantees that each vehicle returns after all customers' visits back to the depot. With constraint (5) is guaranteed that each vehicle takes exactly 1 tour. The constraint (6) sums up the total demand of all customers that are included in the route of vehicle k. The constraint (7) computes the remaining demand that a vehicle must still deliver after it visited the customer j. In this way, if the customer j is served right after the customer i the remaining demand of vehicle k that already served the customer j must be lower than or equal to the remaining demand that it has after serving the customer i plus the demand of customer j. The constraint (8) is a capacity restriction constraint that ensures that the total demand that a vehicle should deliver on its route must be lower or equal to its capacity.

4. Solution Method

In the presented work we have implemented the ε-constraint method for producing the Pareto-optimal solutions in a bi-objective optimization problem.

In general, a bi-objective optimization problem can be defined as:

minimize $\{ f_1(x), f_2(x)\}$
subject to $x \in S$

where S is a set of feasible solutions. Since the bi-objective problems involve two objectives that usually stay is conflict, in general no solution can optimize both objectives simultaneously. The outcome of a bi-objective optimization is the set of Pareto-optimal solutions. The definition of Pareto-optimality is as follows:

A solution, $x^* \in X$, is Pareto optimal if there does not exist another solution, $x \in X$, such that $F_i(x) \leq F_i(x^*)$ for all objective functions i, with at least one strict inequality.

In the set of all Pareto-optimal solutions that is commonly referred to as the Pareto frontier, there exists no solution that dominates the other, because any improvement in one objective results in the worsening of the other one.

Two traditional methods for solving bi-objective optimization problems are the weighted-sum method and ε-constraint method.

In the weighted-sum method, all objectives are combined into a single function in which each objective is multiplied by a weight w_i. The weights are assigned to the objective functions according to their relative importance for a decision maker. The weighted-sum model can be defined as follows:

optimize $\sum_{i=1}^{k} w_i f_i(x) \quad \forall\ i \in\ 1,..., k$
subject to $x \in S$,

where $w_i \geq 0$ and $\sum_{i=1}^{k} w_i = 1$.

In the ε -constraint method, introduced in Haimes et al. (1971), one objective function is being optimized, whereas all the other objective functions are defined as a constraints by determining an upper bound to each objective function:

optimize $f_i(\text{x})$

subject to $f_j\ (\text{x}) \leq e_j \quad \forall\, j \in 1,\ldots, k, j \neq i$

$\text{x} \in \text{S},$

where i ∈ {1,...,k} and e_j is an upper bound for the k objectives.

In the weighted-sum method the determining of the weights is of high importance, as it has great influence on the obtained results. The allocation of weights to the objective functions represents a difficult decision for a decision maker, since he has to exactly define on which objective function he should put more emphasis. This becomes even more problematic when the units of measure of different objectives are not comparable, such as time, weight, currency, etc. Another disadvantage of the weighted-sum method is that this method cannot produce unsupported efficient solutions, which limits the number of found solutions only to the supported ones, and can therefore eliminate the possibility of discovery of an optimal solution. An example of unsupported efficient solution is depicted in Figure 1. The solutions in red circles are located within the convex hull of the Pareto-optimal solutions, and therefore no convex combination of weights can identify these solutions as the optimal ones.

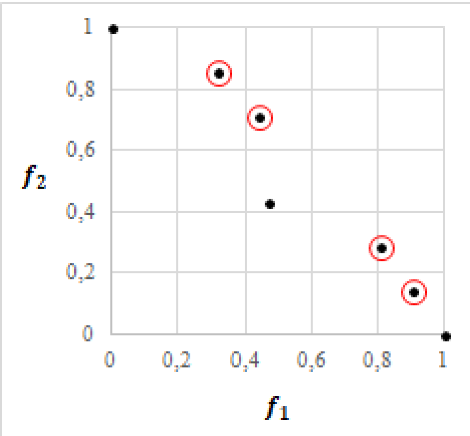

Figure 1: Example of solutions not identifiable with the weighted-sum method

On the other hand, in the ε-constraint method it is only necessary to determine an upper bound to each objective function that is modelled as a constraint, which represents an advantage of ε-constraint method over the weighted-sum. Furthermore, the ε-constraint method is able to find also the unsupported efficient solutions (Mavrotas, 2009). In this work, we have defined the upper bound as a maximum value for a workload and then iteratively decreased the upper bound e_n down to its minimum.

During the analysis of the computational results, we have also observed the lexicographic optimality of our solutions. The lexicographic optimization is a sequential approach that optimizes objectives according to their importance. For the purposes of this work, we have used the lexicographic method by interpretation of the workload of the vehicles. The vehicles were set in lexicographic order according to the level of their workload and subsequently we have identified, if the solutions that were better balanced delivered lexicographically better workload allocation for each vehicle.

5. Analysis

The described VRP was computed in Mosel language by Fico Xpress Optimization Suite. For the calculations we have used the benchmark instance CMT5 that consists of 199 customers with different node coordination and demands. From this instance we have used first three groups of 16 customers and generated three different customer sets, A, B and C, these are depicted in Figure 2. Each set consists of one depot and 16 customers that should be served and have various locations and demands. In the model versions we have used various number of vehicles used for customer delivery, from 2 to 5.

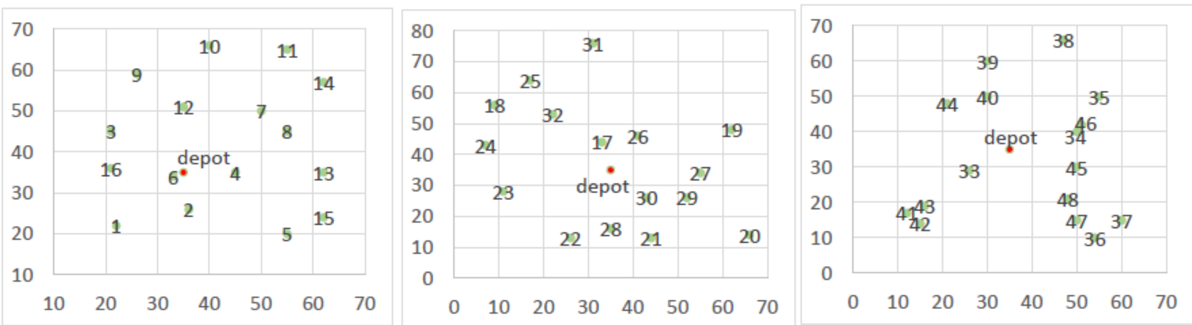

Figure 2: The three customer sets A, B and C

For analysis purposes, we have created four different models that used different equity metrics and functions:

- model with equity metric Customers and equity function Min Max,
- model with equity metric Customers and equity function Range,
- model with equity metric Demand and equity function Min Max,
- model with equity metric Demand and equity function Range.

Equity function Min Max is defined by the vehicle that out of all used vehicles in the fleet delivered the highest workload, whereas the equity function Range is computed as the difference between the vehicle with maximum and the vehicle with minimum workload. In the equity metric Customers the workload for each vehicle is determined by the number of customers that a vehicle serves in its route, and the workload in equity metric Demand is defined as a sum of demands that a vehicle delivers to all customers that are allocated to it.

Although we used in our computations only 3 different sets of customers, the instance set was considerably higher due to variations in the number of used vehicles and equity metrics and functions. Each instance was considered with 4 different fleet sizes, yielding 12 instances that we solved with each of the 4 models. This yielded a total of 48 Pareto fronts and 251 unique solutions. These are depicted in the Table 19, in chapter 5.5.

For purposes of the analysis we have considered always the combined results from all three customers sets to deliver plausible statements about the results.

In this chapter we are going to explore the differences between the models that use different equity metrics and equity functions.

We are going to analyse

- the variances in the number of found solutions when using different model versions,
- the connection between the number of vehicles used in the models and Pareto-optimal solutions found,
- the development of cost that is connected to the imbalance reduction,
- the consistency between balance and lexicographical improvement, and
- to what extent were identified identical solutions by different model versions, in order to determine which model identifies the highest number of unique solutions.

5.1. Number of Pareto-optimal Solutions

Observing the number of Pareto-optimal solutions when using different functions

- minimizing the maximum workload (Min Max)
- minimizing the difference between the maximum and minimum workload (Range)

and using different metrics

- number of customers (Customers)
- demand of the customers (Demand)

we have identified some differences.

5.1.1. Variances Between Different Equity Functions

Figure 3 shows us the total number of solutions found with different types of equity functions divided according to the number of the vehicles used for customer delivery. Except the models, where are only 2 vehicles in the fleet, the function Range identifies more solutions than Min Max, since in the function that balances the difference between maximum and minimum workload there are more possible combinations of splitting the workload between the vehicles. Due to limited possibility of allocation of the customers to the vehicles by the models that use only 2 vehicles for customer delivery, the functions Min Max and Range identify the same number of solutions. The connection between the number of the vehicles and the number of solutions will be discussed in the next chapter.

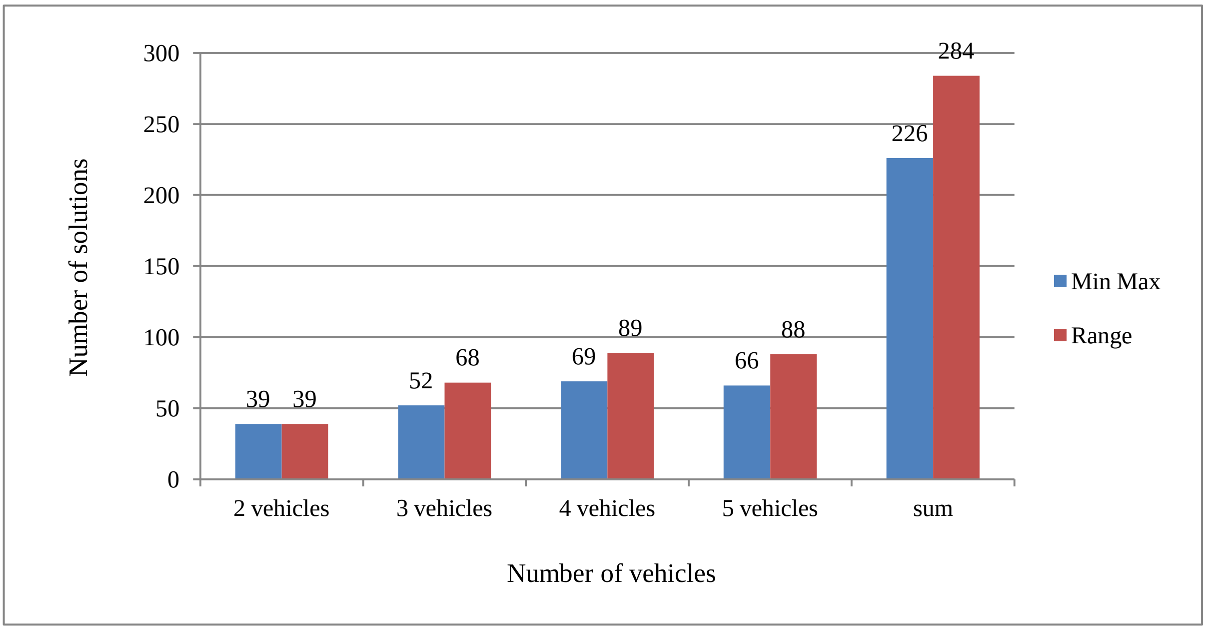

Figure 3: Number of Pareto-optimal solutions using different equity functions

In the models that use more than 2 vehicles for customer delivery, we can find the variance between the number of Pareto-optimal solutions found with different functions, which is caused by the different calculation of the Min Max and Range:

The Min Max value is defined by the vehicle that out of all used vehicles in the fleet serves the highest number of the customers. The optimization of Min Max lies in reduction of the maximum allowed number of served customers by each vehicle. On the other hand, the Range value is calculated as the difference in workload between the vehicle with maximum and the vehicle with minimum number of served customers. Since Range value is optimized by reduction of the difference between the most and the least loaded vehicle, the optimization can be achieved by either decreasing maximum or increasing the minimum number of served customers, or both. Therefore, it is possible that in some situations the Range function identifies several Pareto-optimal solutions, where the maximum number of served customers is not changed.

For illustration, we observe the results of an instance that uses the Customers metric and where 5 vehicles are used to deliver the goods to 16 customers. These are depicted in Table 2.

Since each vehicle must serve at least one customer, it is not possible to allocate the customers between the vehicles and simultaneously decrease the maximum number of served customers under 4. Thus, with the Min Max function there are only 4 Pareto-optimal solutions found. On the other hand, with the Range function, there are two other solutions found when the maximum number of served customers is equal to 4, because the difference in workloads between the vehicles is reduced by increasing the minimum workload.

SolNr	Cost	Equity Function		Allocation of Customers				
		Min Max	Range	W1	W2	W3	W4	W5
1	335,285	7	6	7	6	1	1	1
2	343,724	6	5	6	6	2	1	1
3	358,813	5	4	5	5	4	1	1
4	373,357	4	3	4	4	4	3	1
5	380,076		2	4	4	4	2	2
6	391,981		1	4	3	3	3	3

Table 2: Pareto-optimal solutions of customer set B using 5 vehicles and the Customers metric

In order to support the statement that function Range identifies more solutions than Min Max, we analyse also the models where the metric Demand is used. As it is shown in Table 3 that presents the solutions of a different instance using Demand metric, with the Range function there were found more solutions than with the Min Max. Similar as in the previous example, there are more possible combinations for allocating the customers when we use Range function.

SolNr	Cost	Equity Function		Allocation of Customers				
		Min Max	Range	W1	W2	W3	W4	W5
1	284,599	129	112	129	86	44	27	17
2	286,305	113	86	113	86	44	33	27
3	290,209	96	69	96	86	50	44	27
4	295,708	86	59	86	80	66	44	27
5	298,311		53	86	71	67	46	33
6	302,215		40	86	71	50	50	46
7	303,993	80		80	71	66	65	21
8	306,488	71	38	71	67	67	65	33
9	310,392		21	71	67	65	50	50
10	322,752	67		67	67	65	60	44
11	331,605		17	71	65	57	56	54
12	342,203	66		66	66	66	61	44
13	344,817		10	66	64	60	57	56
14	344,88	65	9	65	65	60	57	56
15	352,535		7	65	61	60	59	58
16	363,521	64		64	64	60	58	57
17	376,294	63	4	63	61	60	60	59
18	384,038	61	1	61	61	61	60	60

Table 3: Pareto-optimal solutions of customer set C using 5 vehicles and the Demand metric

5.1.2. Variances Between Different Equity Metrics

In comparison to the variances between different functions, the variances in the number of Pareto-optimal solutions found when we compare the results according the metrics used in the models are significantly higher. The total number of solutions divided according the different metrics are depicted in the Figure 4.

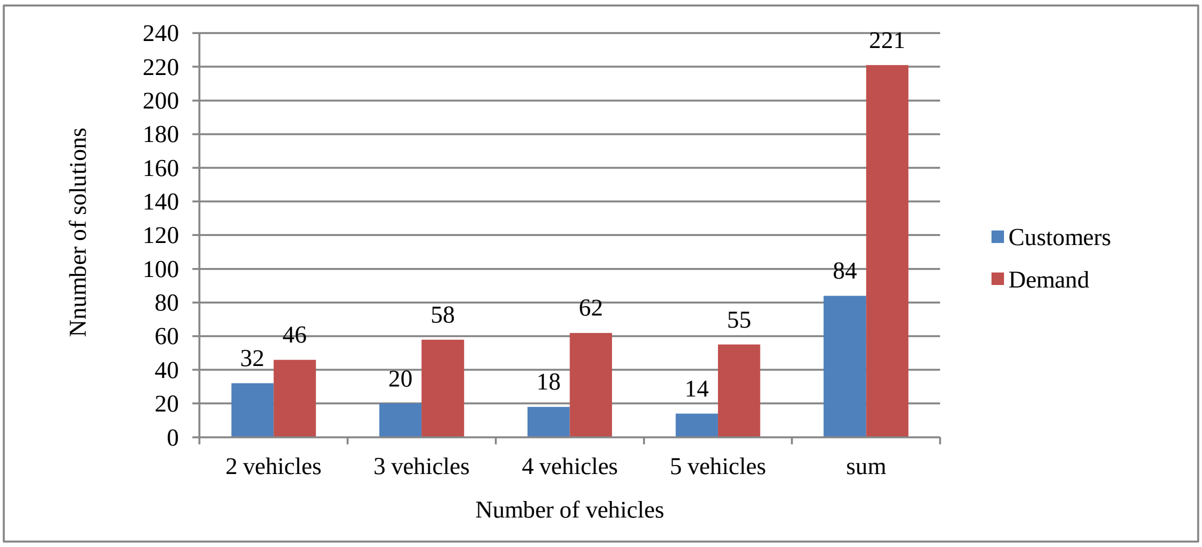

Figure 4: Total number of Pareto-optimal solutions using different metrics

The models that minimize or balance the number of customers identify less solutions independently from the number of vehicles than those that minimize or balance the transported demands between the vehicles. The reason for this tendency can be explained by different definition of workload depending on which metric do we use. While the models with the Customers metric allocate 16 customers to 2 - 5 vehicles, the models with the Demand metric split the whole demand of the customers, in average 315 units, between the vehicles. Therefore the models with the Demand metric allow more possible combinations of splitting the workload between the vehicles, and identify more solutions than the models that use the Customers metric.

For illustration, we compare the results of 4 different models that use function:

- Min Max and Customers metric,
- Min Max and Demand metric,
- Range and Customers metric,
- Range and Demand metric,

for an instance in which 4 vehicles are used to serve the customers.

These solutions are depicted in the Table 4. As we can see, it applies for both equity functions that there are three times as many Pareto-optimal solutions found with the Demand metric. The models where the maximum transported demand is minimized or balanced allow more Pareto-optimal solutions considering the wider scope between the maximum and minimum transported demand that still deliver feasible solutions for the problem. Since the Demand metric identifies more solutions than metric Customers, in most cases the solutions that are found by metric Customers are found also by metric Demand. However, a statement that the solutions of metric Customers are only a subset of the Demand solutions is not correct - a counter-example is solution number 6 in Table 5 that was identified only by metric Customers. The analysis of equality of the solutions generated by different functions and metrics will be delivered in section 5.5.

Equity function Min Max			
SolNr	Cost	Metric	
		Customers	Demand
1	258,903	7	146
2	263,085	6	129
3	264,791		113
4	268,695	5	96
5	274,194		86
6	285,919	4	
9	292,898		84
11	304,684		83
13	311,517		82
14	322,141		79
16	323,837		77
18	381,98		76

Equity function Range			
SolNr	Cost	Metric	
		Customers	Demand
1	258,903	6	119
2	263,085	5	112
3	264,791		80
4	268,695	3	46
5	274,194	2	20
6	285,919	0	
7	287,508		19
8	290,473		17
10	293,392		15
12	305,294		13
13	311,517		11
14	322,141		9
15	323,071		8
16	323,837		3
17	346,041		2
18	381,98		1

Table 4: Variances in the number of found solutions between the same functions and different metrics

5.2. Connection Between Number of Vehicles and Number of Solutions

In this chapter we are going to explore, if there is a connection between the number of vehicles used to serve the customers and the number of solutions found with respective models.

In general it should apply, the more vehicles are used, the more solutions are found, since with more vehicles the models are able to find more possible values for Min Max and Range. Due to the requirement that each vehicle must serve at least one customer, this increasing tendency stops in one point and the number of possibilities starts to decrease.

5.2.1. Metric Customers

In the Figure 5 we can see the number of possible Pareto-optimal solutions for the models with Customers metric that use from 1 to 16 vehicles to serve 16 customers.

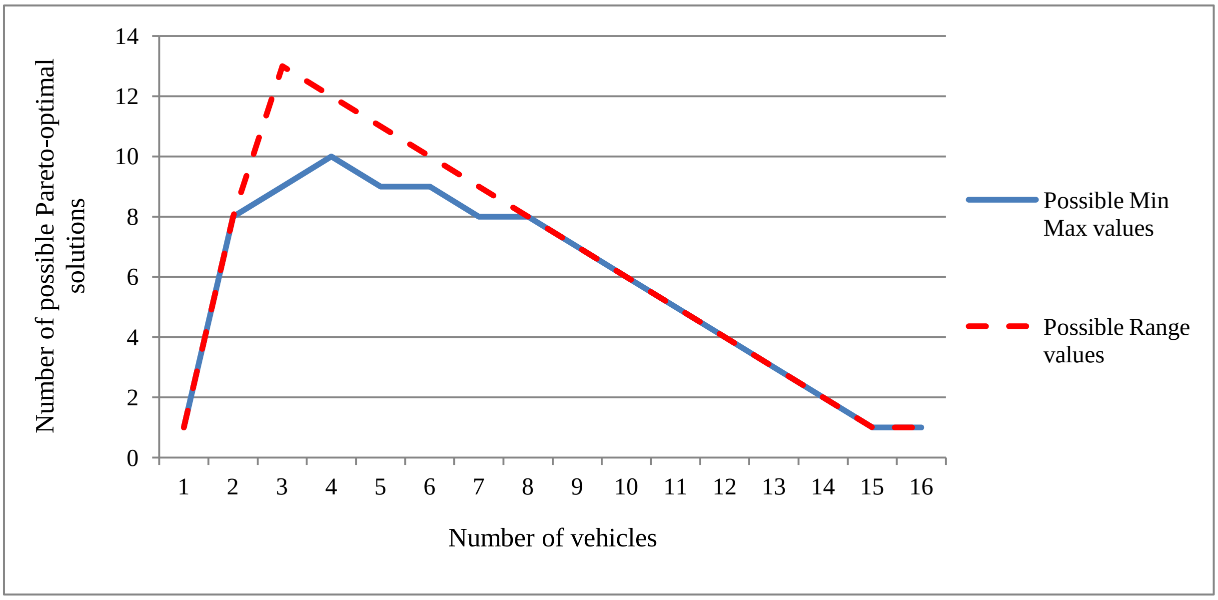

Figure 5: Number of possible Pareto-optimal solutions for models with Customers metric

We obtain the number of possible Min Max values by counting all possibilities between the maximum and minimum value that a Min Max can take. If we use only one vehicle, there is only one possibility of allocating the customers - one vehicle serves all customers. By two vehicles, the number of possible Min Max values rises to 8 - the highest possible Min Max is 15, since the maximum load cannot be 16, otherwise one vehicle would not serve a single customer and the lowest possible is 8, since the maximum load cannot be 7, otherwise two customers would not be served. The models with 3 vehicles are able to find 9 possible Min

Max values, since the maximum possible value for Min Max is 14 and minimum is 6. The models with 4 vehicles are able to find 10 possible values for Min Max. This tendency starts to decrease by the models that use 5 vehicles. The number of possible Min Max value is 9, as the maximum load is 12 (each one of the remaining 4 vehicles serves 1 customer) and the minimum 4 (each one of the remaining 4 vehicles serves 3 customers).

Similar as by models with function Min Max, the models that use 1 and 16 vehicles are able to find only one Range value. By the models that use 2 vehicles for customer delivery, the functions Min Max and Range find due to limited possibility of allocation of the customers to the vehicles the same number of possible values.

Apart from these exceptions, it applies that when balancing customers, the number of possible Range values is equal to the total number of the customers minus the number of used vehicles. This number defines the maximum value that a Range can take. The minimum value (the lowest difference between the maximum and minimum workload) in every model that uses Range function is either 1 or 0, dependently on the number of the vehicles. If the total number of customers is divisible by the number of used vehicles, the minimum Range is 0 and it is not possible to find Range of 1. If the number of customers is not divisible by the number of used vehicles, the minimum Range is 1 and it is not possible to allocate the customers in the way that the Range would be 0. Therefore the number of possible Range values is determined by the maximum value that a Range can take.

For illustration we use the models with 3 and 4 vehicles. The maximum value for Range by the model with 3 vehicles is 13, as the worst balanced allocation is equal to 14-1-1. The best balanced allocation is 6-5-5 that delivers Range of 1, and it is not possible to find also a solution where the Range is 0. Therefore the number of possible Range values by the models that use 3 vehicles for customer delivery is 13. By the model with 4 vehicles is the maximum Range value 12, as the worst balanced allocation is 13-1-1-1. The best balanced allocation is 4-4-4-4, where Range is 0, and it is not possible to find a solution where Range is 1. The number of possible Range values by the models with 4 vehicles is equal to the maximum value that a Range can take, thus it is equal to 12.

The Figure 6 depicts the total number of solutions found in all customer sets with the functions Min Max and Range that use the Customers metric divided according the number of used vehicles. As we can see, up to vehicle 4, there is a positive correlation between the number of used vehicles and number of found solutions. The number of found solutions decreases when we use 5 vehicles for customer delivery.

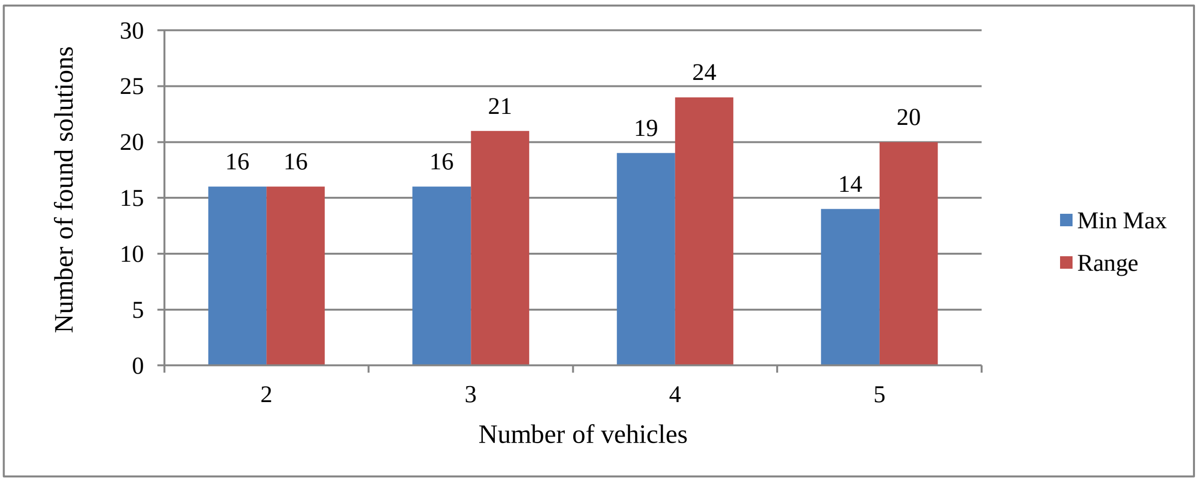

Figure 6: Connection between the number of solutions and number of vehicles by the models with Customers metric

In the Figure 7 we can see the number of solutions found by function Min Max, calculated as an average of all three instances, and the number of all possible values for the Min Max. Since not every Min Max value is also Pareto-optimal, the number of solutions found with Min Max function is lower than the number of all possible Min Max values. On the other hand, we can see that the number of found solutions increases and decreases according to the number of possible values.

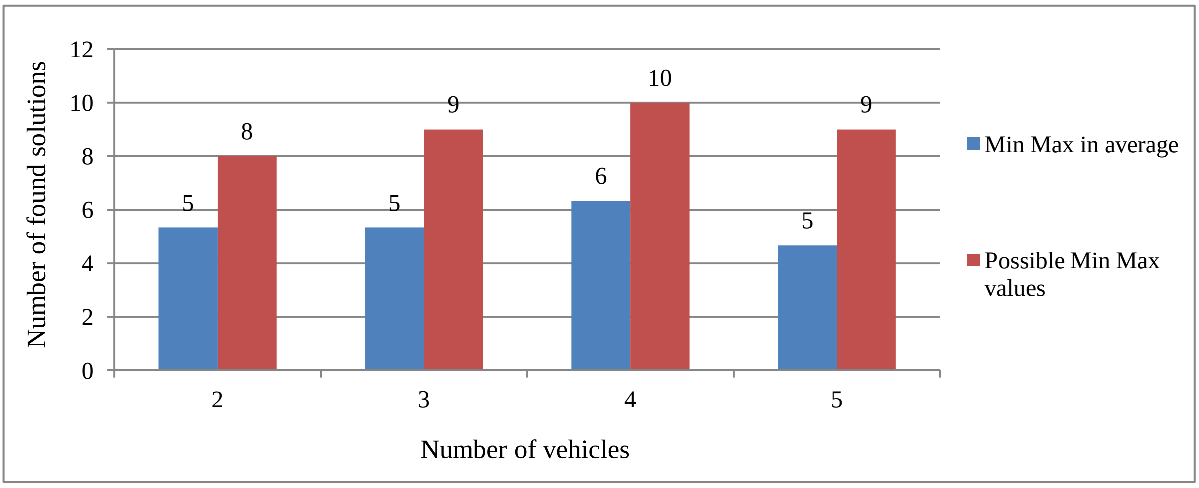

Figure 7: Equity function Min Max and the number of found solutions, number of possible values and their connection to the number of used vehicles

In the Figure 8 we can observe the connection between the average number of found solutions by function Range and the number of possible Range values. The models that use 3 vehicles are able to find the highest number of Range values, but the highest number of solutions found with Range function identifies the model that uses 4 vehicles for customer delivery. In spite of the amount of possible values, there were found more Pareto-optimal solutions by models with 4 vehicles than with 3.

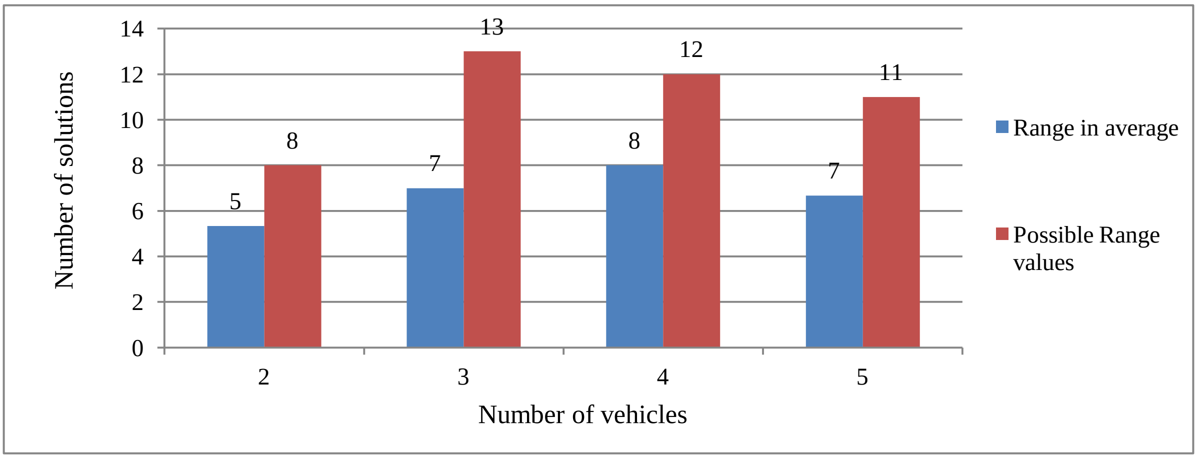

Figure 8: Equity function Range and the number of found solutions, number of possible values and their connection to the number of used vehicles

5.2.2. Metric Demand

The Figure 9 depicts the total number of solutions found in all customer sets with the functions Min Max and Range that use the Demand metric divided according the number of used vehicles. Similar as by the metric Customers, up to vehicle 4, there is a positive correlation between the number of used vehicles and number of found solutions. The number of found solutions decreases when we use 5 vehicles for customer delivery.

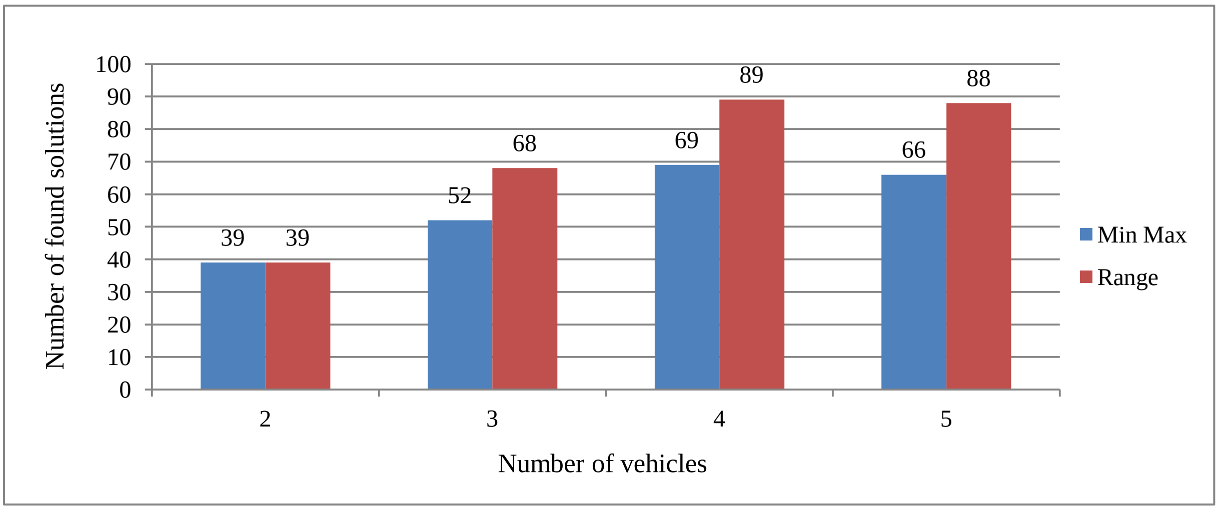

Figure 9: Connection between the number of solutions and number of vehicles by the models with metric Demand

As it was identified in the beginning of this chapter, the increasing tendency of found solutions starts to decrease in one point due to the requirement that each vehicle must serve at least one customer. Although the metric Demand identified significantly higher number of solutions than the metric Customers, there were found similar tendencies between the both metrics, when we observe the connection between the number of used vehicles and solutions found. By both metrics the number of found solutions starts to decrease when the number of used vehicles exceeds the number 4.

5.3. Cost of Equity

In this chapter we are going to analyse the correlation between the cost and the workload balance of the vehicles. It applies to all models that with more balanced workload the cost for the solutions increases, and that different functions, Min Max and Range, deliver similar trends in correlation between the cost and the balance of the vehicles. The goal is to find the solutions, where the high increase in workload balance is accompanied by low cost increase, relative to the cost-optimal solution.

The relative cost increase / imbalance reduction of each solution is calculated as its percentage increase in comparison to the cost optimum:

Relative percentage cost increase of solution n: $\frac{(cost_n - cost_{opt})}{cost_{opt}} * 100$

Relative percentage imbalance reduction of solution n: $\frac{(balance_{opt} - balance_n)}{balance_{opt}} * 100$

In general it applies that the higher number of vehicles is used, the higher costs are incurred. For illustration we observe the tours of the vehicles of the same instance when 2 and 5 vehicles are used for customer delivery. These are depicted in Figure 10. The cost-optimal solution found with model that uses 2 vehicles causes the cost of 199,765 units, whereby each vehicle makes one tour - one vehicle serves only customer number 6 that lies nearest to the depot, and the second vehicle serves all other customers. The cost optimal solution of the model with 5 vehicles causes 24% higher expense for delivery to the customers. When we use more vehicles for customer delivery, each vehicle is allocated to fewer customers and every vehicle must start in and return back to depot, which causes an extra cost for transport.

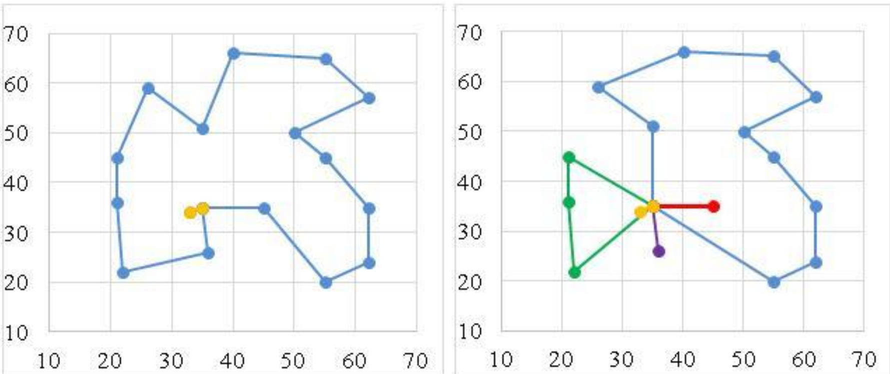

Figure 10: Solution number 1 for model with 2 vehicles (left), with 5 vehicles (right), customer set A

Furthermore, we are going to analyse, if there are variances between the cost development and imbalance reduction, when using different metrics.

5.3.1. Metric Customers

As we can see in the Figure 11 that depicts the percentage increase of cost and percentage reduction of imbalance of the best balanced solutions relative to the cost-optimum by the models that use Customers metric and 2 - 5 vehicles, calculated as an average of all three customer sets, the higher number of vehicles we use, the higher costs are generated by the models. Observing the solutions of the models that use different equity functions, it applies to all models independently of the number of the vehicles that in the best balanced solutions, the Range function generates higher imbalance reduction than the function Min Max. The percentage imbalance reduction of function Range is defined by the divisibility of the number of customers by the number of vehicles. If the number of customers is divisible by number of used vehicles, an identical allocation of customers to each vehicle is allowed, and therefore the Range is equal to zero, and the percentage imbalance reduction of the best balanced solution in comparison to the cost-optimum is 100%. Due to a different calculation of Min Max that is defined by the workload of the vehicle with the highest workload, this function can never be reduced to zero, therefore the percentage imbalance reduction of Min Max cannot achieve 100%. On the other hand, both functions are able to find the same balance optimum, hence the comparison of direct percentages would deliver misleading results. Due to these considerations, we use for imbalance reduction the values normalized between 0 and 1.

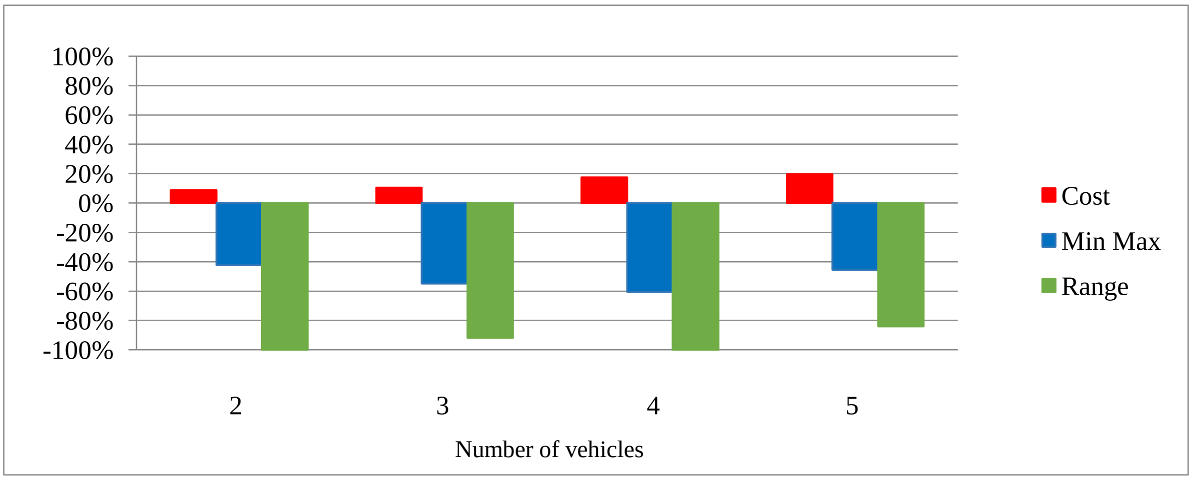

Figure 11: Percentage increase in cost and reduction of imbalance, Customers metric

In the Figure 12 we can see an example of the development of tours, when two objectives, the minimization of cost and imbalance, are being optimized. There are depicted all solutions of an instance that uses 2 vehicles for customer delivery. Comparing the first and the last solution, we can observe, how the balancing factor influences the allocation of customers to the vehicles. The numbers in the graphics represent the number of customers served by specific vehicles.

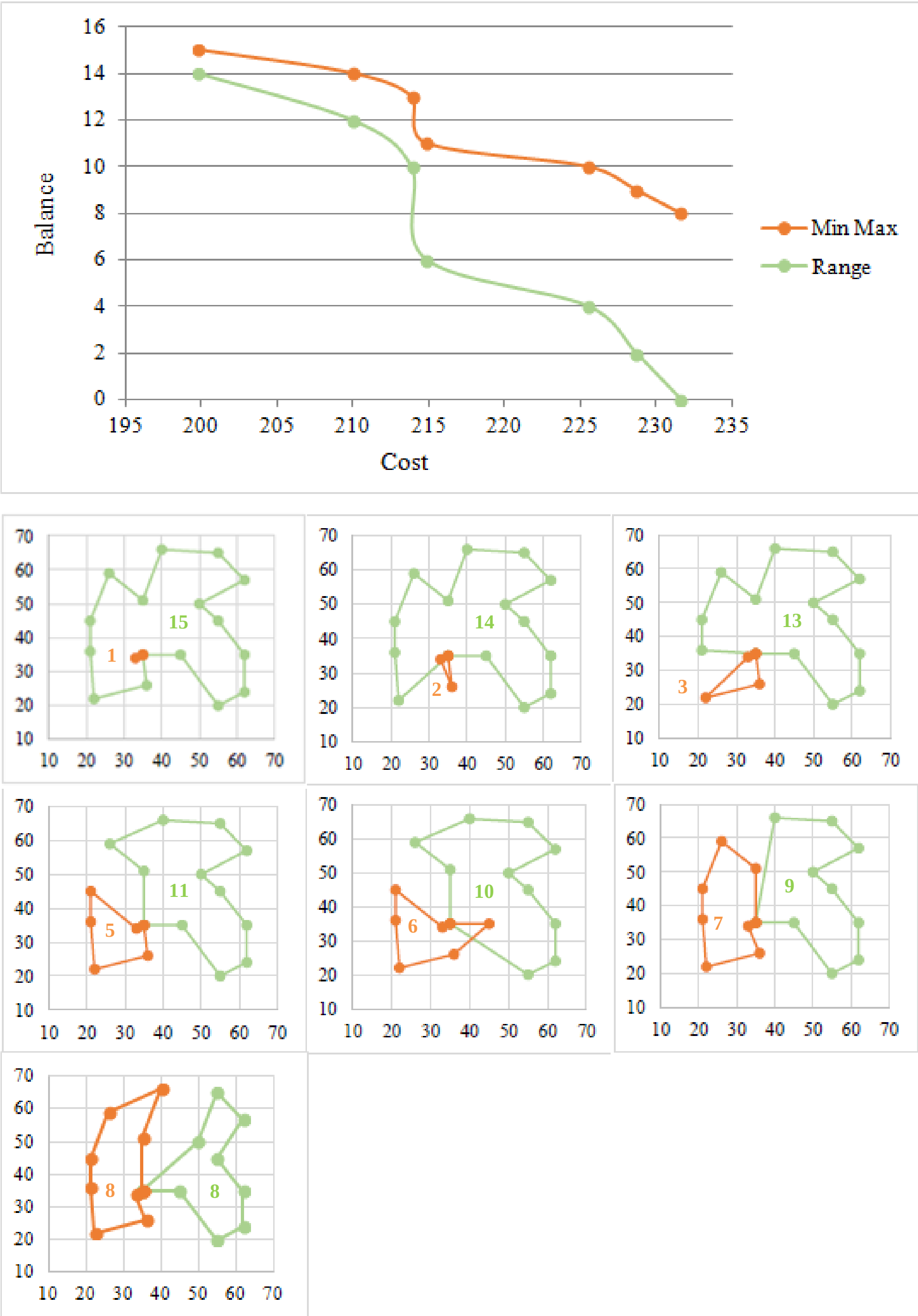

Figure 12: Development of tours of customer set A that uses 2 vehicles

Although the solutions found with both functions are identical in this example, in terms of equity objectives, the different calculations of Min Max and Range cause different percentage imbalance reductions. We therefore normalize the balance objectives to the range [0,1] in order to make imbalance reductions between different objectives comparable.

Furthermore, we are going to observe separately the specific trade-offs that were generated by models that use different customer sets, and different number of vehicles that serve the customers. As the models that use 2 vehicles deliver identical solutions for functions Min Max and Range, the analysis of variances between different functions would not be possible, and therefore we exclude these from the analysis.

Table 5 shows the results of the models of an instance that use the Customers metric and 3 vehicles to customer delivery, and the correlation between the workload imbalance reduction and the increase of cost.

SolNr	Cost	Min Max	Range	W1	W2	W3
1	212,645	14	13	14	1	1
2	217,945	11	10	11	4	1
3	228,078		9	11	3	2
4	228,59	10		10	5	1
5	231,355	9	8	9	6	1
6	234,295	8	7	8	7	1
7	244,405		6	9	4	3
8	245,331	7	3	7	5	4
9	247,609	6	2	6	6	4
10	250,039		1	6	5	5

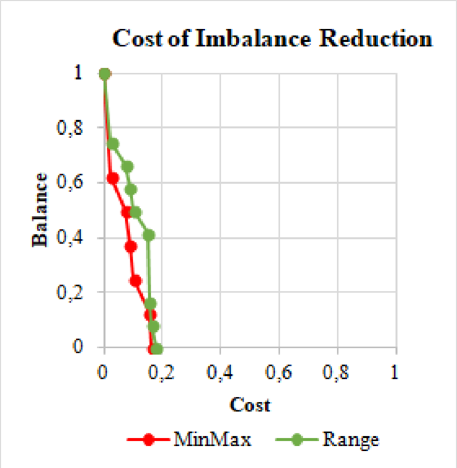

Table 5: Connection between the cost increase and imbalance reduction, customer set A, 3 vehicles

According to the Table 5 it is visible that the trade-off between the first and the second solution delivers an attractive trade-off for both functions. The cost increase of 2,5% reduces the imbalance in function Min Max by 38%, and in function Range by 25%. In the Figure 13 we can see the paths of the vehicles in the solutions 1 and 2. In the solution 1, the vehicle 1 supplies 14 customers and vehicle 2 and 3 only 1 customer. In the solution 2, the vehicle number 2 takes over 3 customers from vehicle 1, what causes only small increase in cost, but high reduction of workload imbalance between the drivers.

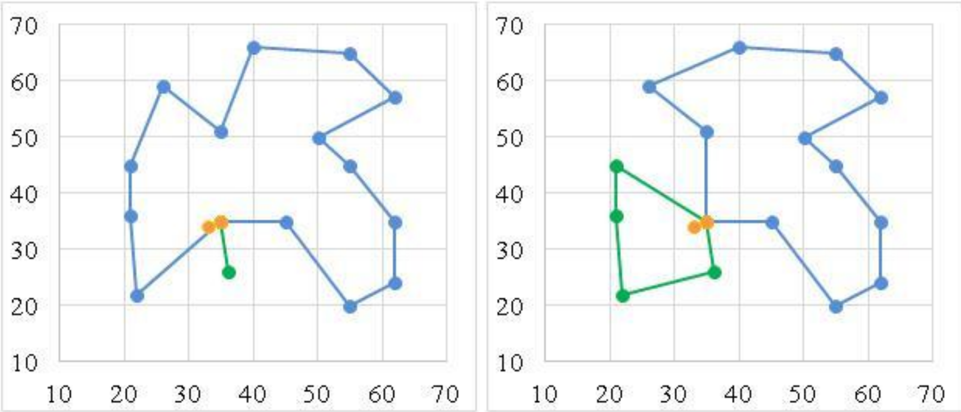

Figure 13: Paths of the vehicles in solution 1 (left) and solution 2 (right), customer set A

The remaining trade-offs of function Min Max do not deliver a significant imbalance reduction. On the other hand, the solution number 7 delivers another interesting trade-off for function Range. The cost increases by less than 0,5% in comparison to the solution number 6, and the imbalance reduces by another 25%. Both solutions are depicted in Figure 14.

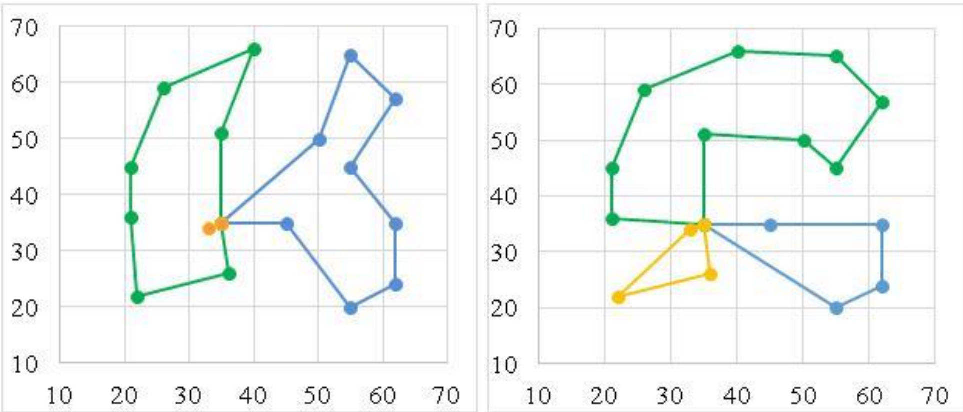

Figure 14: Paths of the vehicles in solution 6 (left) and solution 7 (right), customer set A

The Table 6 presents the solutions of an instance where the trade-offs between cost increase and imbalance reductions are even more significant. The solution number 4 delivers a cost increase of 3,6% relative to the cost-optimum and the imbalance reductions of 75% and 50% for functions Min Max and Range.

SolNr	Cost	Min Max	Range	W1	W2	W3
1	286,326	14	13	14	1	1
2	294,224	13	12	13	2	1
3	294,765	12	11	12	3	1
4	296,672	8	7	8	7	1
5	304,567		6	8	6	2
6	305,111	7	4	7	6	3
7	308,818	6	2	6	6	4
8	313,45		1	6	5	5

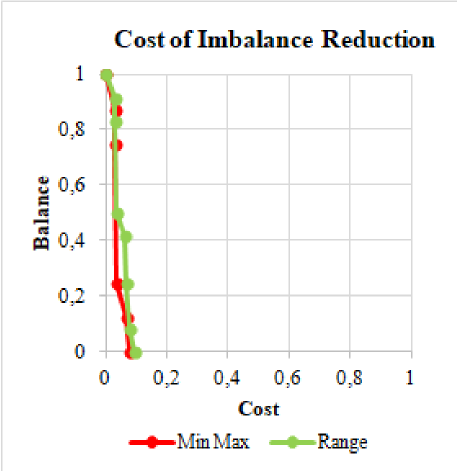

Table 6: Connection between the cost increase and imbalance reduction, customer set B, 3 vehicles

The paths of the vehicles of front B are depicted in the Figure 15. The vehicle number 2 took over 6 customers from the vehicle number 1, which considerably reduces the imbalance and since the customers' location is easily accessible for the vehicle number 2, the cost increase is insignificant.

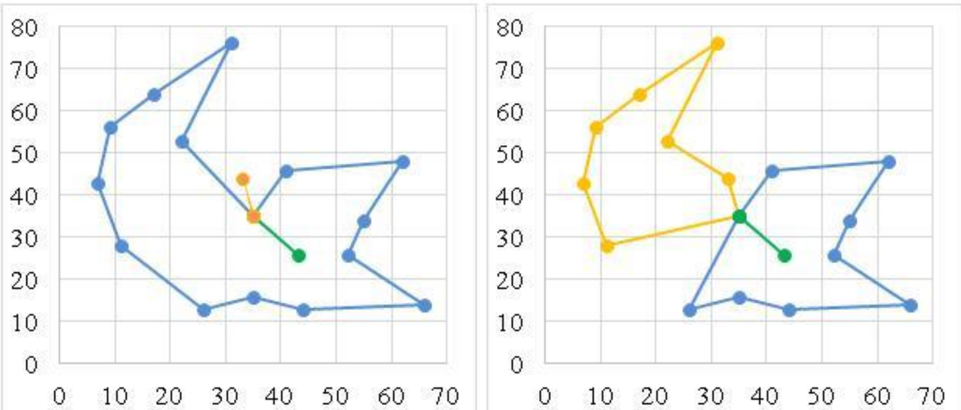

Figure 15: Paths of the vehicles in solution 1 (left) and solution 4 (right), customer set B

The solutions of another instance are depicted in the Table 7. The solution number 2 is very close to being an ideal solution, as the high reduction of imbalance, 83% in the function Min Max and 80% in the function Range, is connected to less than 0,05% increase in cost. This example demonstrates that in some cases the apparently conflicting objective functions (minimization of cost and minimization of imbalance) does not completely stay in conflict.

SolNr	Cost	Min Max	Range	W1	W2	W3
1	237,281	12	11	12	3	1
2	237,389	7	3	7	5	4
3	246,806	6	2	6	6	4
4	247,887		1	6	5	5

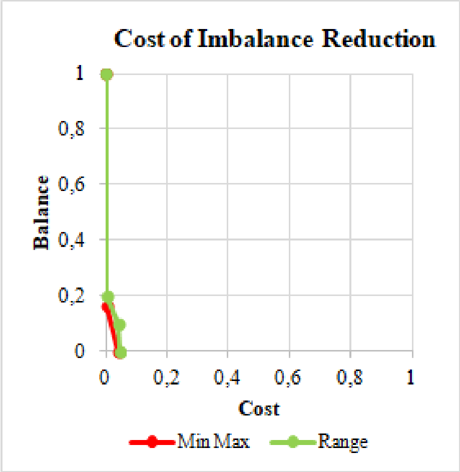

Table 7: Connection between the cost increase and imbalance reduction, customer set C, 3 vehicles

The paths of the solutions 1 and 2 are depicted in the Figure 16. As we can see, the vehicle number 3 gives over 1 customer to vehicle number 2 and took over 5 customers from the vehicle number 1, which causes a high reduction of imbalance but only slight increase of total distance driven by the vehicles.

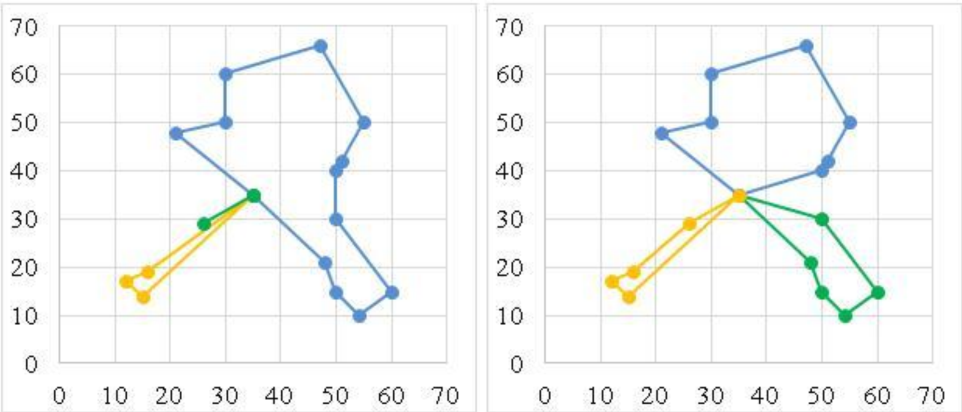

Figure 16: Paths of the vehicles in solution 1 (left) and solution 2 (right), customer set C

Similar as by the models with 3 vehicles, the models with four vehicles in the fleet identify attractive trade-offs that are close to the cost-optimum. The Table 8 indicates that the trade-off between the solution 1 and 2 is the most beneficial for a decision maker. This trade-off is connected to the cost increase of 2%, but delivers imbalance reduction in functions Min Max and Range by 44% and 58% respectively.

SolNr	Cost	Min Max	Range	W1	W2	W3	W4
1	307,728	13	12	13	1	1	1
2	313,883	8	7	8	6	1	1
3	318,074	7	6	7	7	1	1
4	322,322	6	5	6	6	3	1
5	330,22		4	6	6	2	2
6	337,411	5		5	5	5	1
7	345,309		3	5	5	4	2
8	352,643		2	5	4	4	3
9	359,853	4	0	4	4	4	4

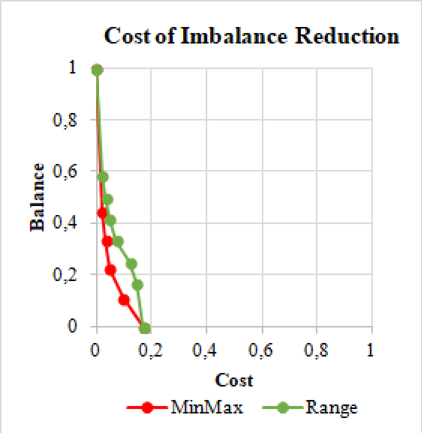

Table 8: Connection between the cost increase and imbalance reduction, customer set B, 4 vehicles

5.3.2. Metric Demand

The Figure 17 depicts the percentage increase of cost and percentage reduction of imbalance of the best balanced solutions relative to the cost-optimum by the models that use Demand metric and 2 - 5 vehicles, calculated as an average of all three customer sets. Similar as by the Customers metric, the more vehicles are used in the model, the higher the corresponding costs, but the relative cost increase by the best balanced solutions is significantly higher than it was by the models that use the metric Customers. The percentage imbalance reduction of function Range reaches almost 100% independently of the number of vehicles, and is higher than the imbalance reduction of function Min Max. In order to deliver comparable results, we use for imbalance reduction the values normalized between 0 and 1.

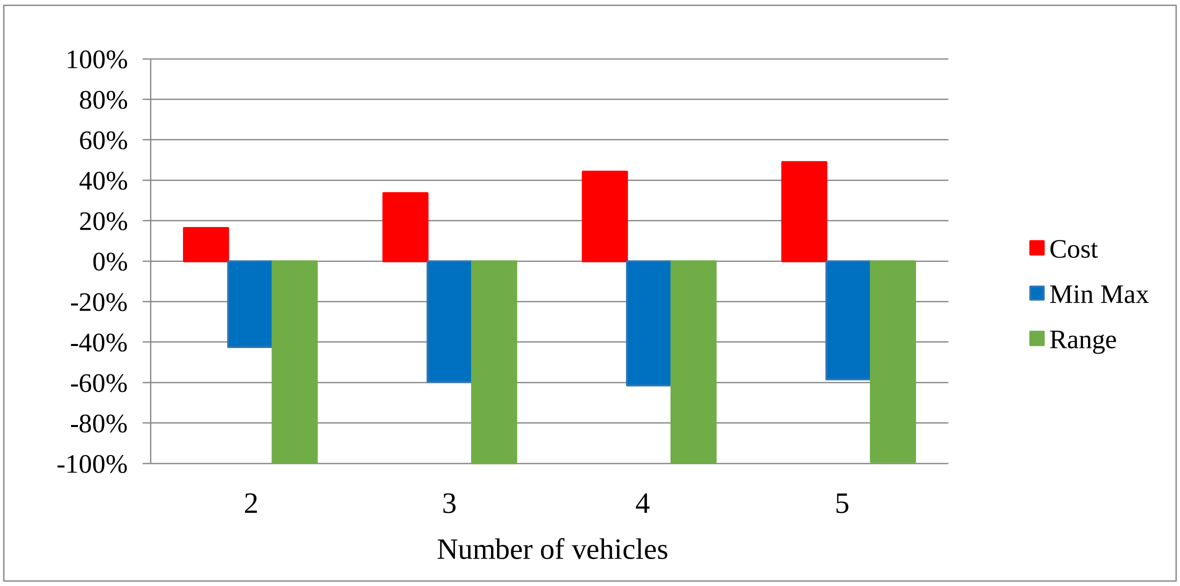

Figure 17: Percentage increase in cost and reduction of imbalance, Demand metric

In the Figure 18 we can observe all solutions of an instance that uses 2 vehicles for customer delivery, and the transformation of the tours of the two vehicles. The numbers in the graphics represent the demand transported by specific vehicles to the customers.

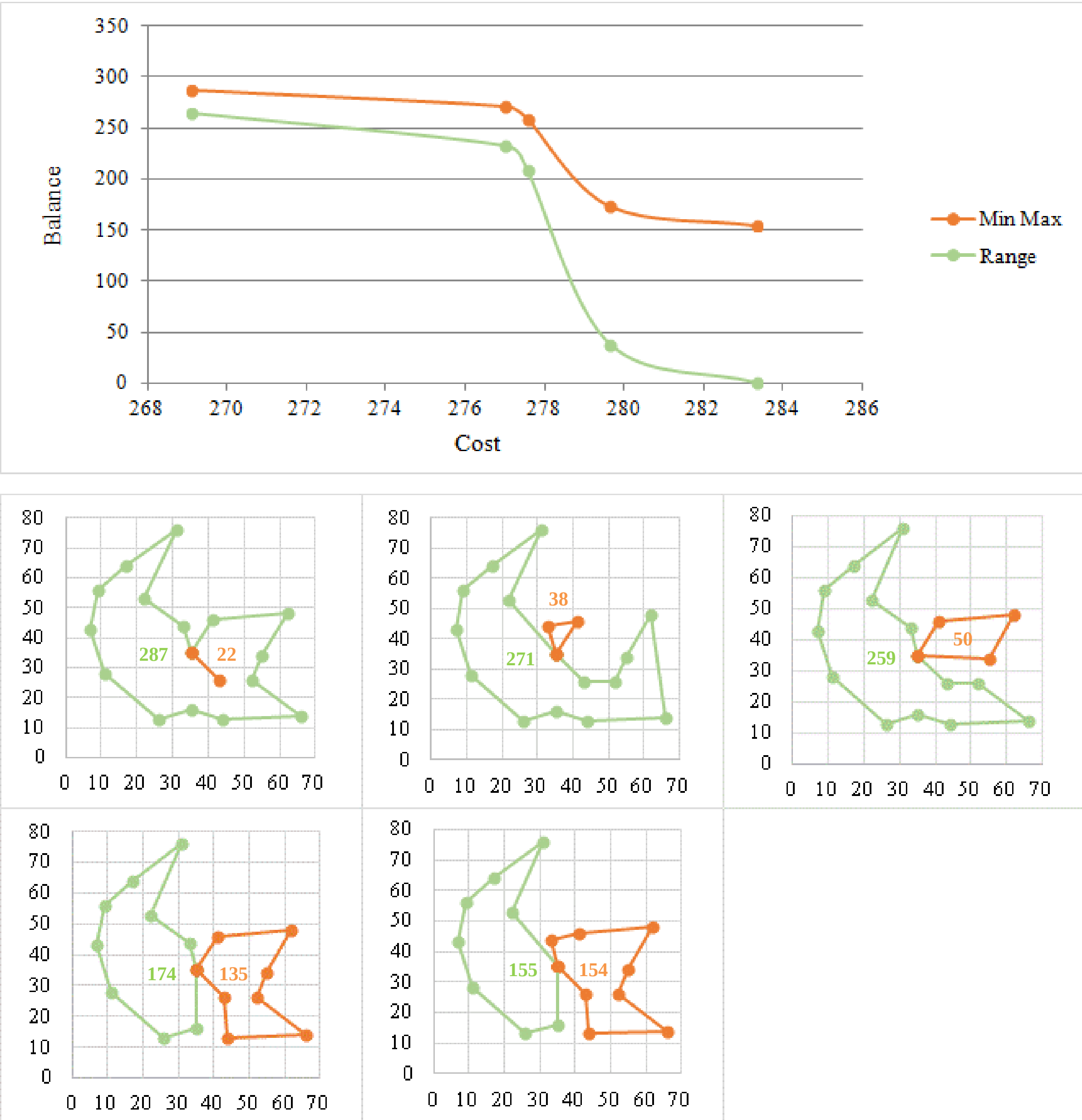

Figure 18: Development of tours of customer set B that uses 2 vehicles

The first two trade-offs deliver only slight reduction of imbalance, since the workloads of the vehicles are not significantly changed. The workloads of the vehicles are essentially changed in third trade-off where the level of imbalance reduction exceeds the increase in cost. Similar as the first two trade-offs, the last one delivers insignificant change in the tours of the vehicles.

Observing the models that use the Demand metric, the trade-offs between cost and imbalance are less attractive than in the models with the Customers metric, since the cost increase of the best balanced solution is significantly higher. On the other hand, we can find some similarities between different metrics. As an example we provide the solutions of an instance using 3 vehicles. These are depicted in the Table 9.

SolNr	Cost	Min Max	Range	W1	W2	W3
1	212,645	289	270	289	26	19
2	215,487	285	266	285	30	19
3	217,945	241	222	241	74	19
4	226,483		215	241	67	26
5	228,078		196	241	48	45
6	228,59	211	192	211	104	19
7	231,355	196	177	196	119	19
8	234,295	170	151	170	145	19
9	239,071	161	142	161	154	19
10	244,405		137	200	71	63
11	244,428		125	170	119	45
12	245,331		103	170	97	64
13	245,701	159		159	156	19
14	247,609	154	87	154	113	67
15	250,039		67	154	93	87
16	251,073	145		145	140	49
17	253,478	140		140	138	56
18	255,073		65	140	119	75
19	256,072	139		139	128	67
20	256,741		45	141	97	96
21	257,668	134		134	118	82
22	258,741	125	28	125	112	97
23	261,019	113	4	113	112	109
24	270,838	112	2	112	112	110
25	292,882		1	112	111	111

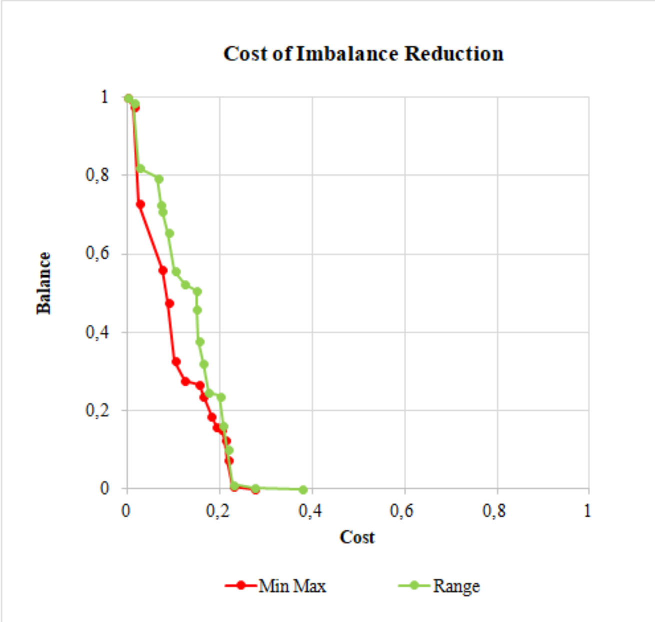

Table 9: Connection between the cost increase and imbalance reduction, customer set A, 3 vehicles

The cost increase of the best balanced solution in comparison to the cost-optimum is 27% by the function Min Max and 38% by the function Range. In comparison to the metric Customers, where the same customer set caused the cost increase by best balanced solutions of 16% and 18% for these functions, the balancing here is two times more expensive. The best trade-off was delivered by the solution number 3. Although the metrics use different definition of workload, this solution is identical to the solution number 2 by the metric Customers that also delivered the most attractive trade-off between the two objective functions. In this example it is visible that the metric Demand delivered lower imbalance reduction than the metric Customers. While the same cost increase of 2,5% causes by metric Demand an imbalance reduction in function Min Max by 27% and in Range by 18%, by metric Customers it is 38% and 25%.

In the Figure 19 we can see the change of the paths and of the workloads between the solutions 1 and 3. The numbers in the graphics represent the demand delivered by each vehicle. The cost increase in this trade-off is insignificant, whereby there is identified a high reduction in workload imbalance between the vehicles.

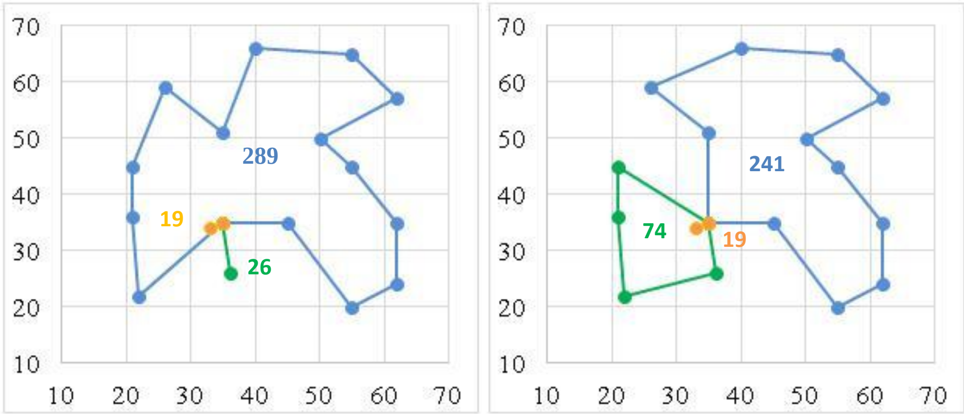

Figure 19: Paths of the vehicles in solution 1 (left) and solution 3 (right), customer set A

In the last trade-offs of the metric Demand it is not worthwhile to balance the workload, as the reduction of imbalance is insignificant and it is exceeded by the cost increase. In the Figure 20 are depicted the solutions number 23 and 24 that are identical for both functions. As we can see, a very slight imbalance reduction causes a cost increase from 261,019 to 270,838 units.

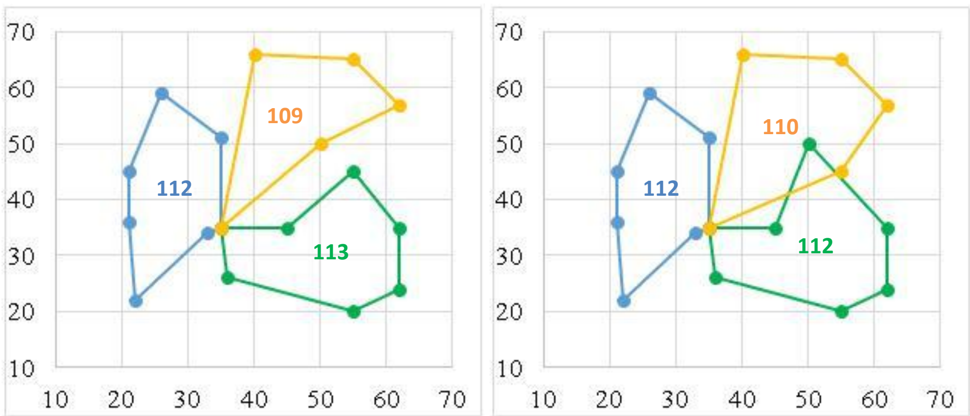

Figure 20: Paths of the vehicles in solution 23 (left) and solution 24 (right), customer set A

In the Table 10 we can see the development of cost increase and imbalance reduction in the models of an instance when we used 4 vehicles to serve the customers. Until solution number 5 it is worthwhile to balance the workload, as the cost increase relative to the cost-optimal solution of only 6% is connected to the balance improvement of 86% by function Min Max and 84% by function Range. In comparison to this solution, the best balanced solution delivers an additional imbalance reduction of 14% by function Min Max and 16% by function Range, whereby the cost increases by 42%.

SolNr	Cost	Min Max	Range	W1	W2	W3	W4
1	258,903	146	119	146	86	44	27
2	263,085	129	112	129	86	71	17
3	264,791	113	80	113	86	71	33
4	268,695	96	46	96	86	71	50
5	274,194	86	20	86	80	71	66
6	287,508		19	86	79	71	67
7	290,473		17	86	77	71	69
8	292,898	84		84	82	71	66
9	293,392		15	86	73	73	71
10	304,684	83		83	82	71	67
11	305,294		13	84	77	71	71
12	311,517	82	11	82	77	73	71
13	322,141	79	9	79	77	77	70
14	323,071		8	79	78	75	71
15	323,837	77	3	77	77	75	74
16	346,041		2	77	76	75	75
17	381,98	76	1	76	76	76	75

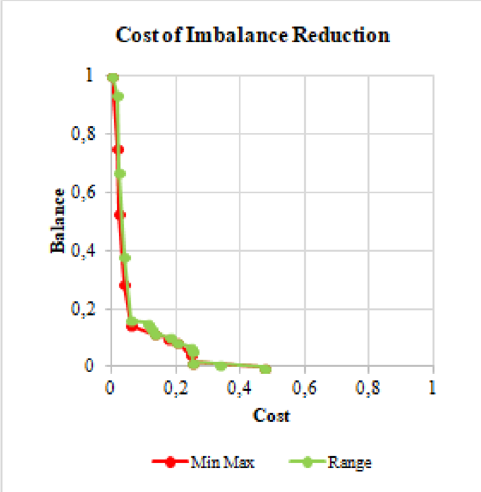

Table 10: Connection between the cost increase and imbalance reduction, customer set C, 4 vehicles

The comparison of the paths of solutions 1, 5 and 17 is depicted in Figure 21.

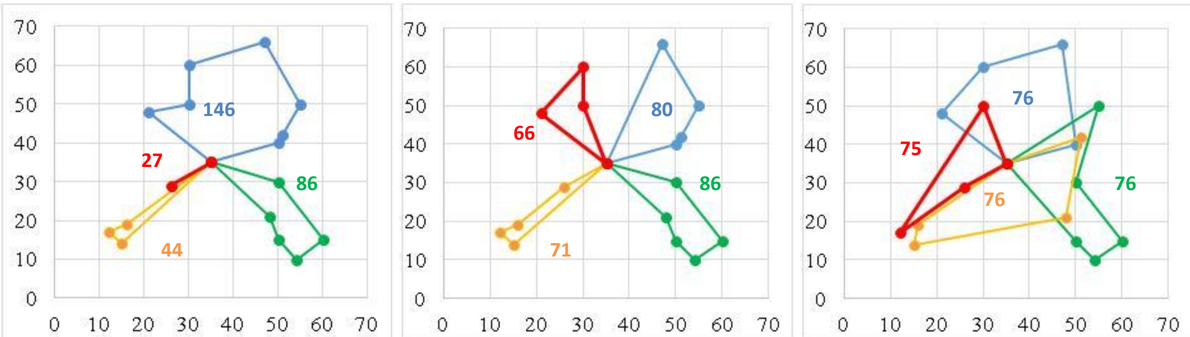

Figure 21: Paths of the vehicles in solution 1 (left), solution 5 (middle) and 17 (right), customer set C

Similar tendency is seen by the model of another instance that uses 5 vehicles for customer delivery. The solutions are presented in Table 11. Until solution number 4 that is identical for both functions, the trade-off between cost increase and imbalance reduction is attractive for a decision maker, as the cost increase of 4% allows significant reduction of imbalance by functions Min Max and Range, 63% and 48% respectively. Every further trade-off causes an average cost increase of 4% by function Min Max and 3% by function Range, and only 5% imbalance reduction in average by both functions.

SolNr	Cost	Min Max	Range	W1	W2	W3	W4	W5
1	284,599	129	112	129	86	44	27	17
2	286,305	113	86	113	86	44	33	27
3	290,209	96	69	96	86	50	44	27
4	295,708	86	59	86	80	66	44	27
5	298,311		53	86	71	67	46	33
6	302,215		40	86	71	50	50	46
7	303,993	80		80	71	66	65	21
8	306,488	71	38	71	67	67	65	33
9	307,911		37	77	71	65	50	40
10	310,392		21	71	67	65	50	50
11	322,752	67		67	67	65	60	44
12	331,605		17	71	65	57	56	54
13	342,203	66		66	66	66	61	44
14	344,817		10	66	64	60	57	56
15	344,88	65	9	65	65	60	57	56
16	352,535		7	65	61	60	59	58
17	363,521	64		64	64	60	58	57
18	376,294	63	4	63	61	60	60	59
19	384,038	61	1	61	61	61	60	60

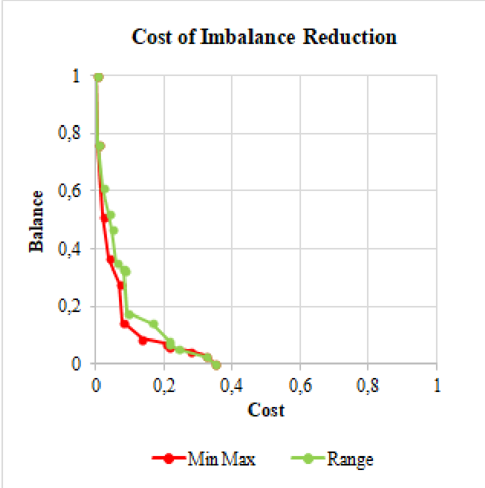

Table 11: Connection between the cost increase and imbalance reduction, customer set C, 5 vehicles

In this section we have discovered that in every front we were able to find a solution, where a small cost increase was connected with high imbalance reduction. Usually, the most attractive trade-offs were found between the initial solutions, close to the cost-optimum, as there was still wide scope of possibilities how to allocate the customers to the vehicles. In practice it could signify that the objectives are not strongly conflicting. This statement is also validated by the fact that the solutions that were close to the balance optimum tended to deliver insignificant imbalance reduction that was connected to the high cost increase.

5.4. Consistency with Lexicographic Preferences

Since the models that minimize the imbalance consider only the most loaded vehicle (models with function Min Max) or the most and the least loaded vehicle (models with function Range), the workload of the vehicles that are in between is not examined by these models. From this reason we are going to analyse the lexicographic optimality of our solutions in order to identify if the solutions that were better balanced delivered also lexicographically better workload allocation for each vehicle.

5.4.1. Metric Customers

The Table 12 represents the solutions of an instance that uses 5 vehicles for customer delivery. As we can see, the solutions 5 and 6 found only with function Range are better balanced and are also lexicographically better than the solution 4 that was found with both functions, since the workload of vehicle 4 is better in solution 5 than in solution 4. The solution 6 is the lexicographic optimum, as well as the best balanced optimum, because it is not possible to improve the workloads of the vehicles or find fairer allocation of customers for this instance without excluding one customer from the delivery.

SolNr	Cost	Equity Metric		Allocation of Customers				
		Min Max	Range	W1	W2	W3	W4	W5
1	335,285	7	6	7	6	1	1	1
2	343,724	6	5	6	6	2	1	1
3	358,813	5	4	5	5	4	1	1
4	373,357	4	3	4	4	4	3	1
5	380,076		2	4	4	4	2	2
6	391,981		1	4	3	3	3	3

Table 12: Pareto-optimal solutions of customer set B using 5 vehicles and the Customers metric

For the instances considered in our study, all best balanced solutions that were computed using function Range and metric Customers, represent also the lexicographic optimum. If we use instances where the number of customers is divisible by the number of vehicles, the best balanced solution delivers the same customers allocation for each vehicle, thus Range is 0. In the instances where the number of customers is not divisible by the number of vehicles, the difference between the minimum and maximum workload between the vehicles is 1, and cannot be improved. In both cases, such allocations represent the lexicographic optima.

5.4.2. Metric Demand

The Table 13 represents the solutions of an instance that uses the metric Demand and 5 vehicles for customer delivery. Here we can see the instances where Range function identifies more solutions without decreasing the maximum workload. The solution number 4 found with function Min Max is lexicographically worse than solutions 5 and 6, but causes lower cost, therefore it was identified as Pareto-optimal solution found with Min Max if the maximum demand is equal to 86. The solutions 5 and 6, found only with Range function, are lexicographically better than solution number 4, as the workload of vehicle with the second highest workload decreases from 80 to 71. If we also observe the workload of the vehicle number 3, the solution number 6 is lexicographically better than the solution number 5, because the workload decreases by 17. On the other hand the solution 6 causes higher cost than the solution 5.

Similar tendency is seen in the solutions 8, 9 and 11. If we observe the maximum workload, the solutions 8, 9 and 11 deliver the same maximum workload, but for the function Min Max, only solution 8 is Pareto-optimal due to cost increase by solutions 9 and 11. From the Range perspective, all solutions are Pareto-optimal, as the difference between the maximum and minimum workload decreases while the cost increases, but the solution 9 is lexicographically better than solution 8, because the workload of the vehicle 3 is improved, and the solution number 11 is lexicographically better than the solution 9, as the workload of vehicle 2 is lower than it is by the solution 9.

SolNr	Cost	Equity Function		Allocation of Customers				
		Min Max	Range	W1	W2	W3	W4	W5
4	295,708	86	59	86	80	66	44	27
5	298,311		53	86	71	67	46	33
6	302,215		40	86	71	50	50	46
8	306,488	71	38	71	67	67	65	33
9	310,392		21	71	67	65	50	50
11	331,605		17	71	65	57	56	54
14	344,88	65	9	65	65	60	57	56
15	352,535		7	65	61	60	59	58

Table 13: Solutions of function Range with the same maximum workload, customer set C

As a result from these instances we can deduce following observations.

From a theoretical point of view,

- the Min Max function identifies always only one solution for a maximum workload that is Pareto-optimal, since the Min Max function only considers the maximum workload and not the workloads of other vehicles, and
- the Range function can identify more solutions with the same maximum workload, since the decrease of the difference between the maximum and minimum workload can be achieved by increasing the minimum workload.

From an empirical point of view, we observe that

- the Range solutions with the same maximum workload are lexicografically better than the Min Max solutions, but are usually connected to higher cost, and
- it applies to Range solutions that have the same maximum workload that the one that is better balanced is usually also lexicografically better.

If we observe all computational results, the last assumptions do not apply for every instance. We have identified several cases, where the solutions with better balanced workload delivered lexicographically worse allocation of the workload. For example we observe the Table 14, where the solution number 13 is lexicographically worse than 12, although it is better balanced.

SolNr	Cost	Equity Function		Allocation of Customers				
		Min Max	Range	W1	W2	W3	W4	W5
12	289,226	98	79	98	94	93	30	19
13	295,344		75	101	99	71	37	26

Table 14: **Solutions of function Range with workload inconsistency, customer set A**

In the previous section we have assumed that a lexicographically better solution can be achieved with an increase of cost, and therefore it can be identified as Pareto-optimal solution - with making one preference criterion better, at least one preference criterion must get worse. But during the calculations we have found also solutions that are connected to higher cost and are lexicographically worse, but still represent a Pareto-efficient solution, what is not consistent with our previous observations and typical expectations with respect to equity.

Since the models that use function Min Max consider only the most loaded vehicle, whose load is in every other solution lower than in the previous one, it is not possible to find a Pareto-optimal solution that is lexicographically worse and causes higher cost. The models that use the function Range consider the difference between the most and the least loaded vehicle, but do not restrict the lower or upper bound of the best and the worst workload, therefore it is not guaranteed that a better balanced solution is also lexicographically better. An example is depicted in Table 15, where the solution number 10 delivers better balance than the solution number 9, but since the workload of the vehicle 1 is higher, it is lexicographically worse.

SolNr	Cost	Equity Function		Allocation of Customers		
		Min Max	Range	W1	W2	W3
9	239,071	161	142	161	154	19
10	244,405		137	200	71	63

Table 15: Solutions of function Range with workload inconsistency, customer set C

In the next section we are going to analyse the solutions that are inconsistent with our expectations of Pareto-optimality according to the metric that we use in the models. As the lexicographical consistency is guaranteed by the models that optimize the function Min Max, the further analysis is based on detection, to what extent the solutions of Range function are consistent with lexicographic preferences.

5.4.3. Inconsistencies in Metric Demand

Out of all solutions found with metric Demand 18% cause higher cost and are lexicographically worse than other Range solutions for the same instance. The Figure 22 depicts the percentages of these solutions on the total sum of solutions found with metric Demand divided according to the number of used vehicles.

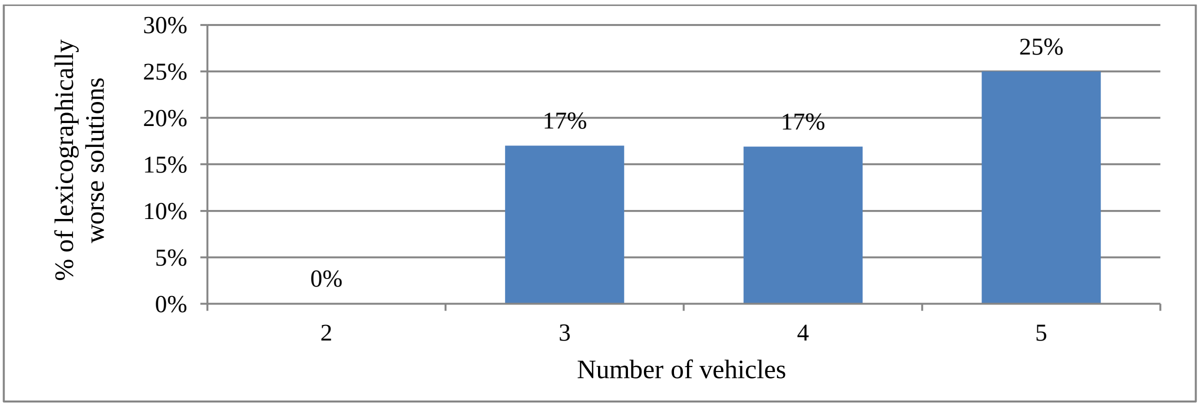

Figure 22: Percentage share of lexicographically worse solutions divided according to the number of used vehicles

In the models that use only 2 vehicles, there was not found a solution of Range that would be lexicographically worse than Min Max, because all solutions found with Range are identical to the solutions of Min Max, and therefore the solutions of Range are lexicographically consistent with those of Min Max. The share on the lexicographically worse solutions rises with the increase of vehicle fleet. With 3 and 4 vehicles there were found 17% lexicographically worse solutions, and the highest share on lexicographically worse solutions identified the models with 5 vehicles, 25%.

For illustration we observe the solutions of an instance that uses 3 vehicles for customer delivery. These are depicted in Table 16.

SolNr	Cost	Equity Function		Allocation of Customers		
		Min Max	Range	W1	W2	W3
9	239,071	161	142	161	154	19
10	244,405		137	200	71	63
11	244,428		125	170	119	45
12	245,331		103	170	97	64

Table 16: Solutions of function Range with workload inconsistency, customer set A

The solutions number 10, 11 and 12 cause higher cost and are lexicographically worse than the solution number 9, as the workload of vehicle 1 is higher in these solutions. From this reason we would assume that these solutions are worse balanced than the solution number 9, and therefore they would not represent Pareto-efficient solutions. Despite this fact, these solutions are Pareto-efficient from the Range perspective, because the difference between the maximum and minimum workload is decreased.

Even more inconsistencies we can find in models that use 4 vehicles to serve the customers. The Table 17 depicts three sets of solutions, where the worse maximum workload is also connected to higher cost. If we observe the solutions 7 and 8, the solution 7 causes lower cost and the same workload of vehicle 1 than the solution 8. But since the workload of vehicle 2 is increased by solution 8, this solution is lexicographically worse. The same applies for solution 9, although here is the worse balance evident by the workload of the first vehicle. Despite the primary appearance that the solutions 8 and 9 are less fair for the worse workloads, the difference between the maximum and minimum workload is decreased, and therefore these solutions represent Pareto-optimal solutions for the equity function Range.

SolNr	Cost	Equity Function		Allocation of Customers			
		Min Max	Range	W1	W2	W3	W4
7	326,372	113	93	113	91	85	20
8	331,095		91	113	104	70	22
9	332,29		79	117	113	38	41
11	337,411	107		107	97	85	20
12	341,604		72	113	85	70	41
13	343,928		71	113	104	50	42
14	344,418	102		102	98	89	20
15	345,309		69	107	97	67	38
16	346,142		50	113	70	63	63

Table 17: Solutions of function Range with workload inconsistency, customer set B

5.4.4. Inconsistencies in Metric Customers

Observing the workload inconsistency in the Customers metric, out of all Range solutions there was found only one that is lexicographically worse than the previous Min Max solution, and is also connected to higher cost. This is depicted in the Table 18, where we can see that even though the difference between the minimum and maximum workload decreases, the workload of the vehicle 1 is increased.

SolNr	Cost	Equity Function		Allocation of Customers		
		Min Max	Range	W1	W2	W3
6	234,295	8	7	8	7	1
7	244,405		6	9	4	3

Table 18: Workload inconsistency by Customers metric

Similar as by the analysis of the variances in the number of found solutions between different metrics in the chapter 5.1., it applies also by the analysis of the consistency with lexicographic preferences that the models with the Customers metric allow less possible combinations of splitting the workload between the vehicles, and therefore identify less solutions that are lexicographically worse and have higher cost than the models that use the metric Demand.

5.5. Agreement Between Different Models

In this chapter we are going to analyse the overlaps in the solutions of the models that use

- different functions and the same metric
- the same function and different metrics.

The percentage overlap is calculated as a share of identical solutions found with models that use different functions/metrics in the total number of unique solutions found with these functions/metrics, per instance.

In the computations we used 2 different equity functions (Min Max and Range) and 2 different equity metrics (Customers and Demand). All solutions are depicted in the Table 19, where the "x" indicates that the solution was found with respective model.

	a	Customers		Demand		b	Customers		Demand		c	Customers		Demand	
	costs	Min Max	Range	Min Max	Range	costs	Min Max	Range	Min Max	Range	costs	Min Max	Range	Min Max	Range
2. vehicles	199,765	x	x	x	x	269,115	x	x	x	x	215,767	x	x	x	x
	208,303			x	x	277,01	x	x	x	x	222,191	x	x	x	x
	209,898	x	x	x	x	277,554	x	x	x	x	222,581	x	x	x	x
	213,808	x	x	x	x	279,624	x	x	x	x	226,655	x	x		
	214,783	x	x	x	x	283,331	x	x	x	x	241,722			x	x
	224,77			x	x						242,076			x	x
	225,428	x	x	x	x						264,612			x	x
	228,608	x	x	x	x										
	231,548	x	x	x	x										
	236,645			x	x										
	238,448			x	x										
	241,932			x	x										
3. vehicles	212,645	x	x	x	x	286,326	x	x	x	x	237,281	x	x	x	x
	215,487			x	x	294,224	x	x	x	x	237,389	x	x	x	x
	217,945	x	x	x	x	294,765	x	x	x	x	246,806	x	x	x	x
	226,483				x	296,672	x	x	x	x	247,887		x		x
	228,078		x		x	302,398			x	x	251,084			x	
	228,59	x		x	x	304,567		x		x	253,497				x
	231,355	x	x	x	x	304,733				x	257,304			x	x
	234,295	x	x	x	x	305,111	x	x	x	x	267,614			x	x
	239,071			x	x	308,818	x	x	x	x	279,469				x
	244,405		x		x	309,161				x	283,773			x	x
	244,428				x	312,868			x	x	319,919			x	x
	245,331	x	x		x	313,45		x		x					
	245,701			x		323,907			x	x					
	247,609	x	x	x	x	326,765				x					
	250,039		x			332,816			x	x					
	251,073			x		335,884			x	x					
	253,478			x	x	368,059			x	x					
	256,072			x	x										
	257,668			x	x										
	258,741			x	x										
	261,019			x	x										
	270,838			x	x										
	292,882				x										

	a	Customers		Demand		b	Customers		Demand		c	Customers		Demand	
	costs	Min Max	Range	Min Max	Range	costs	Min Max	Range	Min Max	Range	costs	Min Max	Range	Min Max	Range
4. vehicles	229,617	x	x	x	x	307,728	x	x	x	x	258,903	x	x	x	x
	230,825	x	x	x	x	313,883	x	x	x	x	263,085	x	x	x	x
	233,667			x	x	318,074	x	x	x	x	264,791			x	x
	234,917	x	x	x	x	320,799				x	268,695	x	x	x	x
	243,455				x	321,781				x	274,194		x	x	x
	244,235	x	x	x	x	322,322	x	x	x	x	285,919	x	x		
	246,327			x	x	326,372			x	x	287,508				x
	247,175	x	x	x	x	330,22		x			290,473				x
	248,327			x	x	331,095				x	292,898			x	
	249,673	x	x			332,29				x	293,392				x
	251,267			x	x	334,27				x	304,684			x	
	251,951	x	x			337,411	x		x		305,294				x
	257,82			x	x	341,604				x	311,517			x	x
	259,805				x	343,928				x	322,141			x	x
	260,094			x		344,418			x		323,071				x
	261,4				x	345,309		x		x	323,837			x	x
	261,774			x		346,142				x	346,041				x
	263,083	x	x	x	x	346,479			x		381,98			x	x
	265,361			x	x	347,862			x	x					
	271,504			x	x	352,643		x							
	274,466		x			354,778				x					
	276,744			x	x	355,275			x	x					
	278,376		x			359,853	x	x	x	x					
	280,654				x	366,607			x	x					
	281,214			x		370,318			x	x					
	282,464				x	373,311			x	x					
	285,07			x		376,598			x	x					
	286,556	x	x		x	387,509				x					
	289,032			x		410,598				x					
	293,789			x	x	416,25			x	x					
	302,038			x	x	433,785				x					
	319,869			x	x										
	331,515			x	x										
5. vehicles	247,797	x	x	x	x	335,285	x	x	x	x	284,599	x	x	x	x
	258,777			x	x	341,011			x	x	286,305			x	x
	259,207	x	x	x	x	343,724	x	x			290,209	x	x	x	x
	261,207	x	x	x	x	344,599			x	x	295,708			x	x
	263,803	x	x			352,497				x	298,311				x
	264,147			x	x	355,638			x	x	302,215		x		x
	266,081	x	x			358,813	x	x	x	x	303,993	x		x	
	272,974			x	x	360,185				x	306,488			x	x
	274,654			x	x	363,006				x	307,911		x		
	277,066	x	x			364,373			x		310,392				x
	277,213			x	x	364,715			x	x	321,631		x		
	279,491			x	x	365,073			x	x	322,752			x	
	285,556			x	x	369,761			x		331,605				x
	286,806				x	370,636			x		342,203			x	
	289,226			x	x	373,357	x	x			344,817				x
	290,662	x	x			373,468				x	344,88			x	x
	295,344				x	376,598				x	352,535				x
	297,029			x		378,486			x		363,521			x	
	297,895			x	x	380,076		x			376,294			x	x
	300,609				x	381,136				x	384,038			x	x
	300,795		x		x	383,363			x						
	304,941				x	384,477			x						
	306,104			x		390,812				x					
	306,525				x	391,981		x							
	308,028				x	393,95				x					
	310,522			x		394,525			x	x					
	312,174				x	403,156				x					
	312,523				x	408,986				x					
	313,758				x	415,551			x	x					
	314,928				x	424,322			x	x					
	315,92				x	454,999				x					
	317,654			x		486,004			x	x					
	318,325				x										
	320,198		x												
	323,971				x										
	327,074			x											
	327,296			x	x										
	329,701			x	x										
	340,833			x	x										
	357,7			x	x										
	363,99			x	x										
	413,471			x	x										

Table 19: Overlap in solutions using different models

By the analysis of overlaps we do not include the first cost-optimal solutions that were found with every model type. These solutions are found by all models that optimize the cost, therefore we exclude these from the analysis.

5.5.1. Comparing the Same Metrics and Different Functions

In order to analyse the overlaps between different functions within the same metrics, we examined the number of solutions that were found

- only with the function Min Max
- only with the function Range
- with both functions,

and their percentage share in the total number of found solutions over all instances.

The outcome is presented in the Table 20. The percentage overlap within a respective metric is calculated as a percentage of the number of solutions that were found with both functions in the total sum of all found solutions. The overlap between the functions in Customers metric reaches 69% and in Demand metric 55%. The solutions that were found only with function Min Max represents 4% of all solutions found in the Customers metric and 13% in the metric Demand. The share of solutions identified only by function Range is higher - these represent 26% of all solutions found in the Customers metric and 32% in the metric Demand.

Metric	Customers				Demand			
Function	Min Max	Range	Both	Sum	Min Max	Range	Both	Sum
# of solutions	3	19	50	72	28	70	121	219
% share	4%	26%	69%		13%	32%	55%	

Table 20: Number of solutions and their percentage share in all solutions found with different functions

For better illustration we vizualize the data from Table 20 in the Figure 23. This depicts the number of solutions found with models that use the metric Customers and Demand. The solutions are divided in three different groups of solutions that were found only with function Min Max, only with function Range, and with both functions.

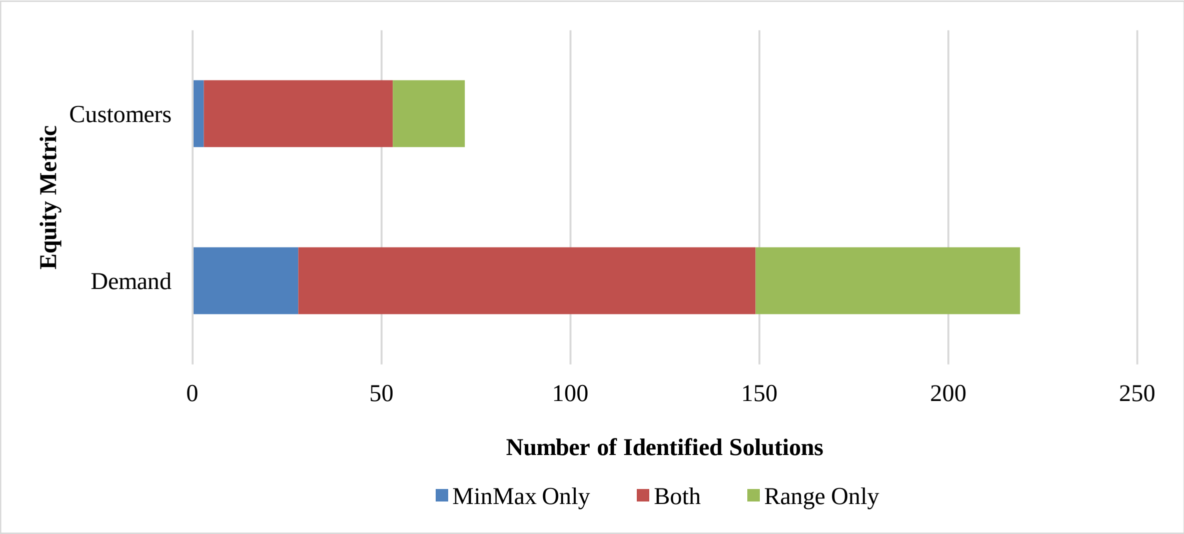

Figure 23: Agreement between different equity functions in absolute terms

It applies for both metrics that the number of solutions found only with the function Min Max is lower than the number of solutions found only with the function Range and that there are more solutions common to both functions than solutions unique to either function.

In order to compare different metrics, we observe the Figure 24 that presents the data of Figure 23 in percentage terms. As we can see, the Customers metric delivers higher percentage overlap between different functions than the metric Demand. This is caused by the fact that the models that use the metric Customers have less room for variety in solutions, as it was detected in the chapter 5.1. Therefore, the solutions found with function Range and Min Max using this metric tend to deliver the same results.

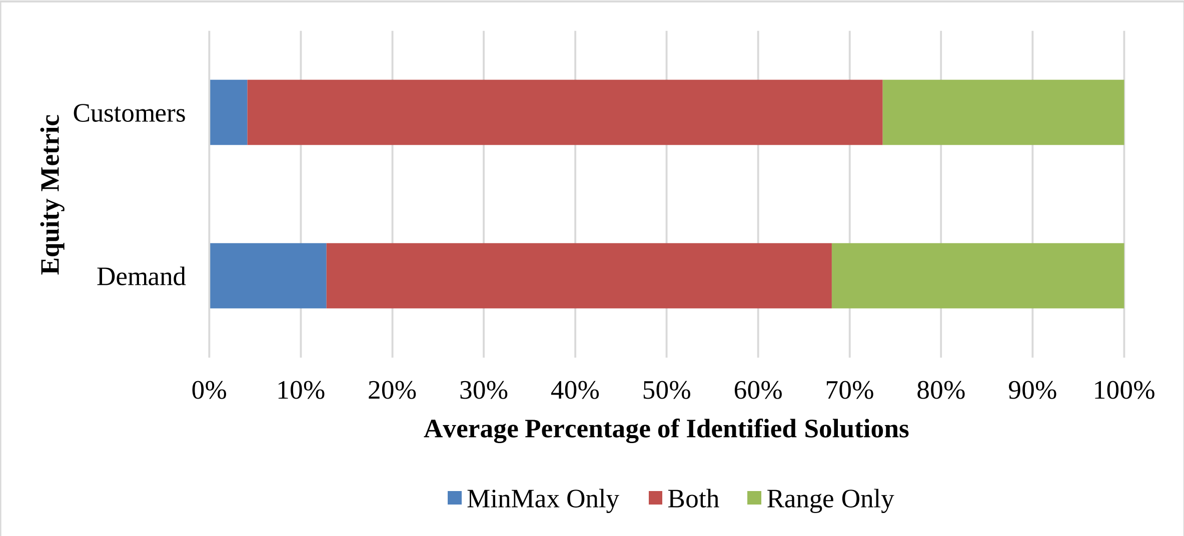

Figure 24: Agreement between different equity functions in percentage terms

In order to analyse the connection between the degree of percentage overlap between different equity functions and the number of vehicles used in the models, we present the Figure 25.

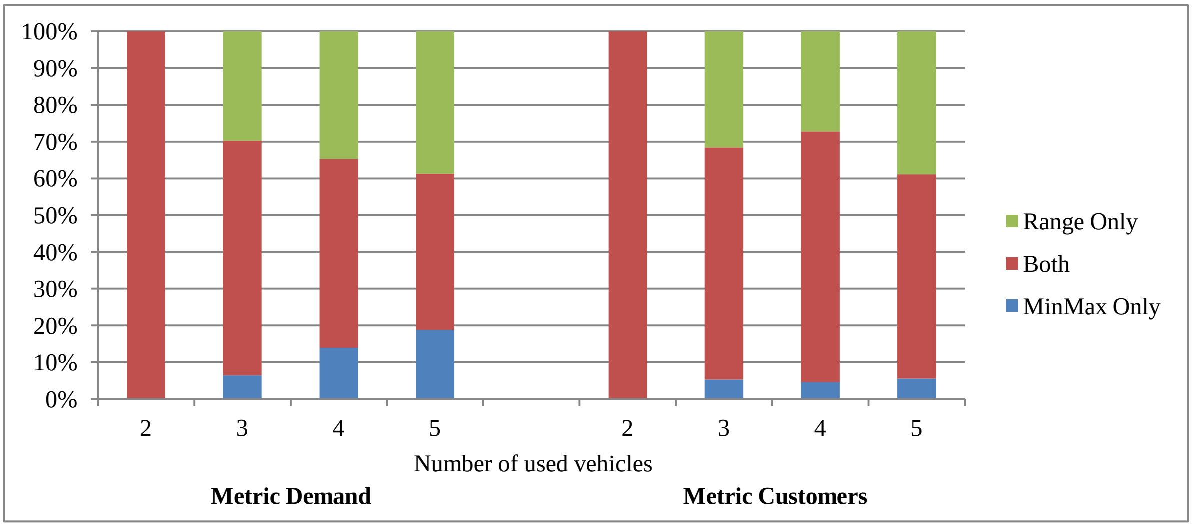

Figure 25: Overlaps of different functions according to the number of vehicles

As it was identified in the chapter 5.1, the models that use only 2 vehicles to serve the customers identify the same number of identical solutions. This applies for both metrics due to limited possibility of allocation of the customers to the vehicles. Therefore, the percentage overlap between both functions is 100%, when we use only 2 vehicles for customer delivery. The proportion of the solutions that were found only with Range function, only with Min Max and with both, is similar by both metrics. It applies for both metrics that the percentage of identical solutions that were found with different function represent the highest share, and that the lowest percentage of solutions that were found only with one of the functions identified Min Max.

5.5.2. Comparing the Same Functions and Different Metrics

Comparable to the diversity in overlaps between different functions, there are also different overlaps in the solutions within the same equity function. In this section we are going to analyse, to what extent we can find identical solutions within function Range and Min Max when different metrics were applied.

Similar as in the previous section, we examined the number of solutions that were found

- only with the metric Customers
- only with the metric Demand

- with both metrics,

and their percentage share in the total number of found solutions.

The outcome is presented in the Table 21. The percentage share of the number of solutions that were found with both metrics in the total sum of all found solutions represents the percentage overlap within a respective function. As we can see, the degree of overlaps is significantly lower than it was by the comparison of different functions and same metrics. The overlap between the metrics in function Min Max reaches 25% and in function Range 23%. The solutions that were found only with Customers metric represents 7% of all solutions found with the function Min Max and 9% with Range. The highest share of found solutions in both functions identifies the metric Demand, 67%.

Function	Min Max				Range			
Metric	Customers	Demand	Both	Sum	Customers	Demand	Both	Sum
# of solutions	12	108	41	161	20	142	49	211
% share	7%	67%	25%		9%	67%	23%	

Table 21: Number of solutions and their percentage share in all solutions found with different metrics

The Figure 26 indicates that the comparison of the models using the same function and different metrics delivers significantly different results than the comparison of the different functions and the same metric. While in the previous section the number of identical solutions found by both functions exceeded the number of solutions found separately, the number of identical solutions found by different metrics is significantly lower. It applies for both functions that the number of solutions found only with the metric Demand represents the majority of all identified solutions. Due to high number of solutions that were found only with the metric Demand, the percentage overlap between the same functions and different metrics is lower.

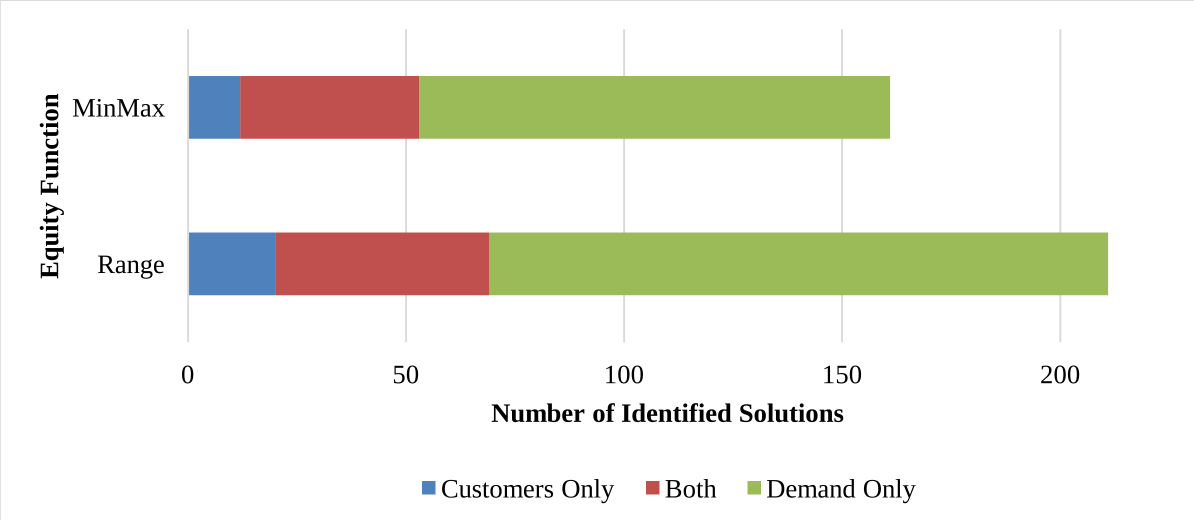

Figure 26: Agreement between different equity metrics in absolute terms

In the chapter 5.1. was identified that a comparison of the results according the metrics we use in the models delivers higher variances between the number of Pareto-optimal solutions due to different definition of workload. As the metric Demand allows significantly more possible combinations of splitting the workload between the vehicles, the differences in the number of found solutions between the metrics are higher and therefore the percentage overlap between the metrics is reduced.

In order to compare different equity functions, we observe the Figure 27 that presents the data of Figure 26 in percentage terms. The percentage share of the solutions that were found with Demand metric in the total sum of solutions is exactly the same by both functions. The percentage share of the solutions that were found only with one of the metrics in the total sum of solutions is almost identical in both functions. The difference is only 2%.

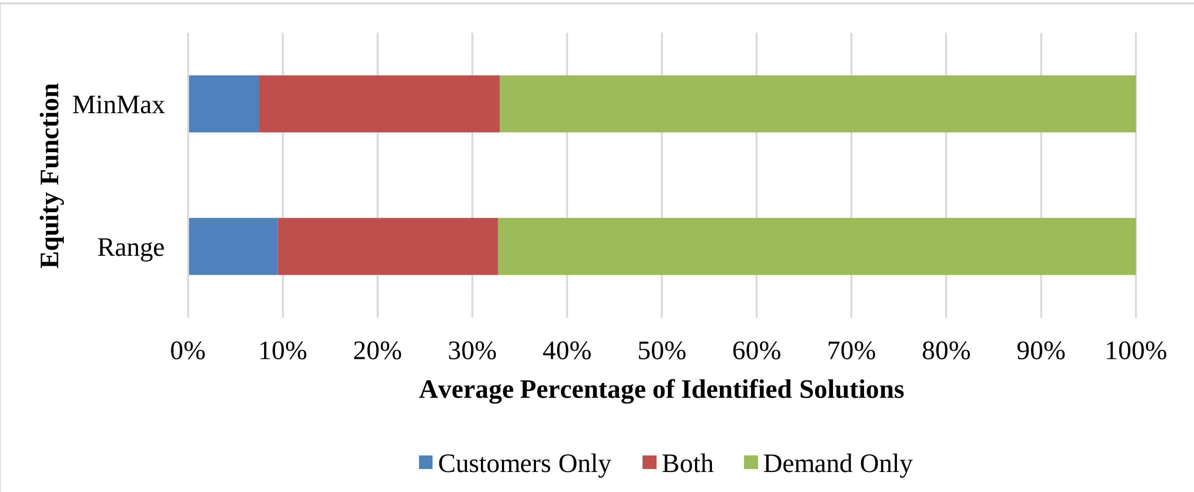

Figure 27: Agreement between different equity metrics in percentage terms

Furthermore, we are going to analyse the connection between the degree of percentage overlap between different equity metrics and the number of vehicles used in the models. The Figure 28 indicates some differences to the comparison of different metrics. The models that use 2 vehicles in the fleet do not find the same number of solutions, but still represent the highest percentage share of solutions that were found with both metrics. Both functions deliver almost identical results - the more vehicles we use the more solutions is found only with one of the metric, which causes a lower percentage overlap between the functions. Since the metric Demand identified more solutions than the Customers metric, the percentage share of the solutions that were found with Demand metric are significantly higher in both functions, independently of the number of vehicles.

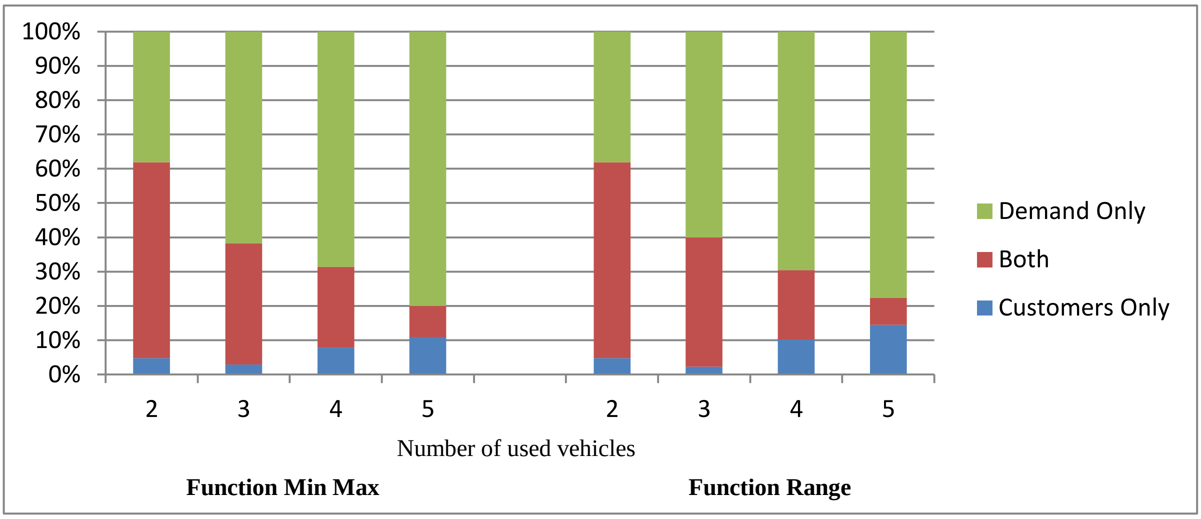

Figure 28: Overlaps of different metrics according to the number of vehicles

Overall, the highest number of identical solutions were found when we compared the solutions of models that use different functions and the same metric. This can be explained by lower difference between the total number of solutions found within a respective metrics. Since the models with the metric Demand identified more than 3 times more solutions than the models with metric Customers, the percentage of overlaps is lower when we compare the models using different metrics.

6. Conclusions

This work revisited the Vehicle Routing Problem (VRP) with workload balancing, whereby different model variants were presented. The computational experiments revealed several differences dependent on what type of model was used.

In the first part of our analysis we have discovered that there are variances in the number of Pareto-optimal solutions found by the models that use different equity functions, as well as by the models that use different equity metrics. The second part studied the connection between the number of vehicles and the number of solutions found with different model types. The correlation between the two conflicting objectives, cost incurred and the workload imbalance of the vehicles, was examined in the third part. In the fourth part we studied the exceptional cases, where better balanced solutions of Range function caused higher cost and were also lexicographically worse than other solutions for the same instance. In the last chapter we have observed the number of identical solutions that were found with different model versions.

The aim of this work was to explore various alternatives for a bi-objective model for VRP with workload balancing. Based on this analysis, we can formulate several managerial implications for decision making:

- Number of unique solutions found: Models that used function Range (metric Demand) identified higher number of unique solutions than the function Min Max (metric Customers), therefore the function Range (metric Demand) is able to find more heterogenous solutions and therefore allows wider selection of trade-offs for a decision maker.
- Fleet size: We found that increasing the number of vehicles increases the number of alternative solutions to some extent, but can also reduce it if too many vehicles are available. This shows the relationship between fleet size decisions and potential for workload balancing.
- Cost of equity: The best balanced solutions of models with equity metric Customers caused lower cost increase relative to the cost-optimum than the models with equity metric Demand, but generally the trade-offs between the cost increase and reduction of workload imbalance were favourable for both metrics. It is possible to reduce imbalance at relatively low extra cost.
- Equity preferences: The models with equity function Range provide more heterogeneous solution sets, but we discovered that some better balanced solutions in

terms of Range can be worse balanced in terms of Min Max. Not all Range solutions were consistent with lexicographic preferences, which may be counter-intuitive to some decision makers.

- Impact of workload model: We analysed the degree of overlap between different models. The highest overlaps were found when comparing the same equity metrics and different equity functions. This suggests that the definition of workload, for example number of customers or total demand, has a bigger impact on the types of solutions found than the choice of equity function, for example Min Max and Range.

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Abstract

Employee satisfaction is a relevant issue for most public and private companies, since their success is dependent on their employees. A very common reason for employee dissatisfaction is the unfair distribution of workload. Since a more balanced allocation of workload to the employees can cause extra operational costs for a company, an attractive trade-off between higher cost and lower workload imbalance is of interest. This is particularly the case in areas such as logistics, where typical optimization and planning methods focus solely on cost minimization.

Using the classical vehicle routing problem (VRP) as a starting point, we consider in addition to the minimization of the total operational cost also various equity criteria for the workload distribution. We define different workload measures and different equity functions, and analyse how these balance objectives affect the resulting trade-offs between minimization of cost and of workload imbalance.

Our analysis reveals that the choice of the workload measure has a significant impact on the resulting VRP solutions. When workload is quantified as the demand served per vehicle, a wider range of compromise solutions is available than when balancing only the number of customers per vehicle, independent of the chosen equity function. However, delivery plans with optimally balanced demands lead to higher costs relative to the cost minimum than plans with optimally balanced customer numbers. However, regardless of the measure it is possible to find compromise solutions with low additional cost and high workload balance – the objectives of cost minimization and workload balance are thus not strongly conflicting. With regard to the chosen balance function, we find that minimizing the range leads to a wider selection of compromise solutions than minimizing the maximal workload, independent of the chosen workload resource. However, workload allocations which improve the range are not always consistent with typical lexicographic minimization, which may appear counter-intuitive to some decision-makers. Finally, we compare all possible compromise solutions found with our alternative models. We observe that a given workload measure often leads to identical delivery plans regardless of the chosen equity function. Therefore the definition of workload has a stronger effect on the available compromise solutions than the choice of the balance function.

Kurzfassung

Mitarbeiterzufriedenheit ist ein relevanter Faktor für die meisten öffentlichen und privaten Unternehmen, da ihr Erfolg von den Arbeitnehmern abhängig ist. Ein häufiger Grund für die Unzufriedenheit von Arbeitnehmern ist eine ungerechte Verteilung von Arbeitslast. Da eine gleichmäßigere Aufteilung der Aufgaben zusätzliche Kosten für den Arbeitgeber verursachen kann, muss ein attraktiver Kompromiss zwischen Kostenerhöhung und Arbeitslastverteilung gefunden werden. Dies ist insbesondere der Fall in Feldern wie die Logistik, wo typische Optimierungs- und Planungsmethoden nur auf Kostenminimierung ausgelegt sind.

Mit dem klassischen Vehicle Routing Problem (VRP) berücksichtigen wir in dieser Arbeit neben der Kostenminimierung für die Lieferung auch verschiedene Balancekriterien für die Arbeitslastverteilung. Wir definieren unterschiedliche Maße für die Arbeitslast und unterschiedliche Balancefunktionen, und untersuchen wie die Balanceziele die resultierenden Kompromisse zwischen Kostenminimierung und Arbeitsverteilung beeinflussen.

Unsere Analyse zeigt, dass die Wahl des Balancekriteriums einen wesentlichen Einfluss auf die resultierenden VRP Lösungen hat. Wenn die Arbeitslast mit den Liefermengen quantifiziert wird, ergibt sich dadurch unabhängig von der Balancefunktion eine breitere Auswahl an Kompromisslösungen als wenn die Kundenanzahl als Arbeitsmaß angenommen wird. Die Lieferpläne mit optimal verteilten Liefermengen verursachen aber höhere Kosten relativ zum Kostenminimum als die Pläne mit gleichverteilter Anzahl der Kunden. Dennoch gibt es mit beiden Arbeitsmaßen Lösungen mit nur geringer Kostensteigerung und gleichzeitig sehr ausgewogener Arbeitslastverteilung – es gibt also keinen großen Zielkonflikt zwischen Kostenminimierung und der gleichmäßigen Verteilung der Arbeitslast. Hinsichtlich der Balancefunktionen, finden wir dass die Minimierung der Spannweite, unabhängig vom gewählten Arbeitsmaß, eine größere Anzahl an Lieferplänen identifiziert als die Minimierung der maximalen Arbeitslast. Dies erlaubt eine breitere Auswahl an Kompromisslösungen. Mit der Minimierung der Spannweite stimmen allerdings nicht alle Arbeitslastverteilungen mit typischen lexikographischen Präferenzen überein, was für manche Entscheidungsträger eventuell kontra-intuitiv erscheinen könnte. Nach einem Vergleich aller möglichen Kompromisslösungen dieser Modelle beobachten wir letztlich, dass derselbe Arbeitsmaß oft zu identischen Plänen unabhängig von der gewählten Balancefunktion führt. Die Definition der Arbeitslast hat also einen stärkeren Einfluss auf die gefundenen Kompromisslösungen als die Wahl der Balancefunktion.