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Estimation of the NRI using a SVAR and macroeconomic model for EMU data

by

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Abstract

This paper compares two models for estimating the natural rate of interest (NRI), analyses the impact of an uncertainty proxy on the results of one of the models, and compares the main results with those of the literature on which the models are based. The NRI is an indicator for the monetary policy stance of a central bank. It is a latent variable that needs to be estimated, in general dependent on the long term economic growth rate and inflation growth. The NRI plays an essential role in the discussion on the optimal monetary policy, and is therefore an important indicator for policy makers. Complexity and reliability of the used models are therefore important to assess. Two models were used in this paper for the estimation of the NRI: a structural vector autoregressive model (SVAR), based on Brzoza-Brzezina (2003), and a small closed form macroeconomic model, based on Laubach & Williams (2003) and Méssonier & Renne (2004).

The SVAR model is less complex, as it only relies on the real interest rate and inflation growth, and its method for estimation is more straightforward. The results however turn out to be very unreliable, considering their lack of robustness *vis-à-vis* different, but similar, definitions of real interest rates. In general, the SVAR model results show a restrictive monetary policy stance before and an expansionary monetary policy stance after the crisis. Using the intuition of the SVAR model in an estimation which applies a different method also yields unreliable results, hinting towards a problem with the lack of fiscal variables in the model, rather than with the estimation method.

The macro economic model is more complex, but more reliable. This model is also explored more thoroughly in the literature. I found strong hints towards secular stagnation for the periods after the financial crisis in 2008 for the Euro area. Specifically, the output gap and the NRI are negative, weakening the effectiveness of monetary policy. The novelty of my approach is the introduction of an uncertainty shock, proxied by relative excess liquidity reserve data of business banks, which has a major impact on the results. That the NRI is overestimated when not controlling for the uncertainty in the economy after the crisis finds support in my results.

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Chapter 1

Introduction

What is the optimal lending rate a central bank should set? This is the first question a monetary policy institution needs to ask itself, after formulating its general objective, e.g. inflation targeting or interest rate targeting. However, the answer is not trivial. The lending rate is the rate which business banks have to pay when withdrawing liquidity from the central bank. It therefore is an upper bound of interest rates that lead to trade on the interbank market. There are different lending rates for different time horizons. In this paper I use the day-to-day (very short run; which is usually used for policy purposes nowadays), 1-month or 3-month nominal interest rate data. No bank can feasibly set its lending rate on the interbank market higher than the lending rate of a central bank. If the lending rate is increasing, less profitable investments become unattractive, therefore investments decrease and the economy slows down. The opposite holds true when the lending rate is decreasing; at least in theory.

The optimal lending rate depends on many factors. First of all, since I talk about a real interest rate and the central bank directly controls only the nominal part, the real rate obviously depends on expected inflation[1]. Secondly, it depends on the long term or potential state of the economy. Investment should be steered in such a way, that the economy neither overheats nor drifts into a recession. This gives motivation for the concept of a natural rate of interest (NRI). The NRI works as a medium to long run anchor for monetary policy. In short, the optimal lending rate has to capture the willingness of the actors in an economy to invest given the state

[1] To express real interest rates, I use the Fisher equation $r_t = i_t - \pi_t^e$, where r_t and i_t are real and nominal rates respectively and π_t^e are inflation expectations.

the economy is in. I am therefore looking for a latent variable, which has to be estimated.

The idea of such an interest rate is quite old and experienced a revival in the last 20 years. The concept found its beginning in Knut Wicksell's essay “Prices and Interest”, published in 1898. There Wicksell describes a real interest rate, at which inflation growth is zero and investment and savings are equal[2].

> "There is a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor to lower them. This is necessarily the same as the rate of interest which would be determined by supply and demand if no use were made of money and all lending were effected in the form of real capital goods."
>
> Wicksell (1898), p. 102

This proposed rate of interest is the natural rate of interest[3] and was many years later reimagined by the Neo-Wicksellian school of thought, e.g. by Michael Woodford (Woodford (2004)).

It found application in many DSGE models, where it is the interest rate at which inflation is constant and the output gap closed. Since these both features are desirable for an economy, it makes sense that the difference between the lending rate and the NRI, or NRI gap, should be as small as possible, as long as constant inflation is desired. The NRI gives therefore a measure of the stance of monetary policy, i.e. whether it is expansionary or not. It is important to note that for this reason, neither the NRI, nor the lending rate are high or low by themselves, but have to be viewed in context with each other. The NRI can be viewed as a reflection of the real effects of the financial market, as it closes the output gap and is therefore a real phenomenon, but is recovered and driven by using real rate data, which represent a combination of market expectations, risk premia, preferences and the nominal policy rate.

Other than in the application of the concept in Taylor Rule estimations, the NRI in this framework is not time constant. It can be influenced by time-preferences or technology shocks; I will explain this in more detail in chapter 4, where I present a small macroeconomic model.

[2] A different concept of such a neutral level of interest was proposed by the Austrian school of economics; see Oppers (2002). I will not make any further comments on its ideas.

[3] Another place where one can find the expression NRI is in a Taylor Rule. There the NRI depicts the intercept of the interest rate described and is, as the “natural” part of the name suggests, constant over time.

The relevance of a measure of the monetary policy stance of a central bank is easy to see. However, since the NRI is latent and a purely theoretical concept, it is hard to argue that we can find in its estimation the final solution for the implementation of the optimal lending rate. Even more so, considering that reliable forecasts of the NRI are very difficult to optain. On the other hand, complexity of the models is also an important factor for the NRI as a policy variable, since policy making relies on policy makers to easily understand the underlying models of estimation and estimation methods In this paper, I consider two methods for estimating the NRI; on the one hand a comparatively simple SVAR, as proposed by Brzoza-Brzezina (2003), and a small macroeconomic model, as proposed by Laubach & Williams (2003)[4]. I apply these methods to Euro area datasets[5]. As it turns out, my results are very sensitive regarding the used datasets, methods and assumptions. I therefore conclude that the NRI is useful as an indicator for what the monetary policy stance could be, but also that it cannot be used as a robust policy-making tool.

The remainder of this paper is structured as follows: chapter 2 explains the underlying econometric methods, i.e. the SVAR and Kalman Smoother approaches; chapter 3 derives, estimates and interprets the results from the SVAR method; chapter 4 introduces a small macroeconomic model and presents its results; the data used are presented in subsections of chapter 3 and 4; chapter 5 concludes and summarizes. Graphs and tables can be found in the Appendix.

[4] Whereas Brzoza-Brzezina (2003) has not been replicated, the Laubach & Williams (2003) approach was replicated, expanded and used many times since the publication of their model; see Méssonier & Renne (2004), Manrique & Marqués (2004), Pedersen (2015), Lubik & Matthes (2015) to name a few.

[5] The Euro area data are reported in constant composition, meaning the list of countries that are aggregated stays the same (I use the current EA-19 list, which consists of Belgium, Ireland, France, Latvia, Malta, Portugal, Finland, Germany, Greece, Italy, Lithuania, Netherlands, Slovenia, Estonia, Spain, Cyprus, Luxembourg, Austria and Slovakia), and are accumulated weighted national data.

Chapter 2

Econometric methods

2.1 SVAR

This chapter is based on Kilian (2011) and discusses structural vector autoregressive (SVAR) models. Models in this style gained popularity after the downfall of large macroeconomic models, following the Lucas critique (Lucas (1976)). The SVAR models help to pin-down causal relationships, using identification restrictions (IR). IR yield the possibility for the creator of a model to impose economic reasoning on the data, without losing the ability to estimate multiple endogenous variables at the same time. The IR are necessary to decompose the shocks (error terms) into primitive and true ones, where true shocks are uncorrelated and allow for economic interpretation. The same way that the IR introduced to a VAR model are opportunities for economic rational to enter the regressions, they are equally potential sources for mistakes and misinterpretation. Firstly, the assumptions must be well reasoned, thus may be reasonable in some time periods and unrealistic in others, i.e. the model stability hinges on these assumptions. Secondly, impulse response functions of a SVAR are more often than not implied by the restrictions, thus in those cases they are not data driven, but ex ante imposed.

In general, there are different types of IR, namely short run and long run restrictions. Since I use both in the model presented in chapter 3, it is commendable to briefly introduce them both to the extent which is relevant for my purposes.

I start with a SVAR model expressed as

$$B(L)y_t = u_t \tag{2.1}$$

where y_t is a n-dimensional vector and u_t is the vector of structural, by construction uncorrelated, shocks with mean zero. $B(L)$ is a p-dimensional lagpolynomial, i.e.

$$B(L) = (B_0 - B_1 L - ... - B_p L^p) \tag{2.2}$$

or after pre-multiplying both sides by B_0^{-1}

$$B_0^{-1}B(L) = (I - B_0^{-1}B_1 L - ... - B_0^{-1}B_p L^p) \tag{2.3}$$

which I will define as

$$A(L) = (I - A_1 L - ... - A_p L^p). \tag{2.4}$$

Note that these parameters can be estimated via a VAR regression, since

$$B_0^{-1}B(L)y_t = B_0^{-1}u_t \Rightarrow y_t = A(L)^{-1}B_0^{-1}u_t \Rightarrow y_t = A(L)^{-1}\varepsilon_t, \tag{2.5}$$

where ε_t here refers to the primitive shocks, which are not necessarily uncorrelated. We also gain an expression that links the primitive errors to the structural ones.

$$u_t = B_0 \varepsilon_t \tag{2.6}$$

Difficulties arise as one tries to figure out what B_0 actually looks like. B_0 is also called the restriction matrix. The primitive errors can be consistently estimated by using a VAR regression. Before I turn to the problem of recovery, please note that as long as the diagonal entries of B_0 are unrestricted we can normalize the variance-covariance (VCV) matrix of the structural shocks to

$$\mathbf{E}(u_t u_t^{\tau}) = \Sigma_u = I_n. \tag{2.7}$$

This is possible w.l.o.g. since u_t is by construction uncorrelated and every dependent variable is at least in part stochastic, i.e. every equation includes an error term. Combining the last two equations yields

$$\mathbf{E}(\varepsilon_t \varepsilon_t^\tau) = \Sigma_\varepsilon = B_0^{-1} E(u_t u_t^\tau) B_0^{-1\tau} \Leftrightarrow \Sigma_\varepsilon = B_0^{-1} B_0^{-1\tau} \tag{2.8}$$

which can be viewed as a system of nonlinear equations; however, since there are n equations and n^2 entries in the restriction matrix, we cannot uniquely identify all of them. Using the fact that Σ_ε is symmetric by construction, we know that the number of uniquely identifiable entries of the restriction matrix is $n(n+1)\frac{1}{2}$.

2.1.1 Short run restrictions

One elegant, even though sometimes perilous, way of dealing with the missing entries is to restrict the B_0 matrix such that it is lower triangular; effectively, one orthogonalizes the errors with this method. If B_0 (and its inverse for that matter) is lower triangular, we "make" the errors uncorrelated, as one can see when considering equation 2.5. This method is not without problems, if the zero restrictions are not well reasoned. A simple Cholesky decomposition would yield a lower triangular matrix, but the interpretability of the results is not always guaranteed. In the model below, I use exclusively economic reasoning to make B_0 lower triangular, by setting some elements to zero. The source of this restriction is the assumption of information delay, ergo a noninstantanious reaction of one variable to the shock of the other; hence short run restriction.

More details on short run restriction can be reviewed in Kilian (2011), pages 2-18.

2.1.2 Long run restrictions

To show how long run restrictions work, I have to use some Algebra. Note that by definition

$$B_0^{-1} = A(L)B(L)^{-1} \tag{2.9}$$

holds. In the long run lag operator $L = 1$ yields

$$B_0^{-1} = A(1)B(1)^{-1} \Leftrightarrow \Sigma_\varepsilon = A(1)B(1)^{-1}B(1)^{-1\tau}A(1)^\tau \Leftrightarrow A(1)\Sigma_\varepsilon A(1)^\tau = B(1)^{-1}B(1)^{-1\tau}. \tag{2.10}$$

This short derivation is important, since the left hand side of the last equation can be estimated consistently, while the right hand side is unobservable. In order to solve the latter equation I need to put restrictions on the latent part and use numerical methods to recover the remaining entries. Note that $B(1)$ represents the sum of the coefficients, ergo the sum over the impact the shocks have in the long run on the dependent variable. A convenient fact is that economic theory implies binding long run restrictions, such as the neutrality of monetary policy, which I use in my SVAR model.

There are however significant draw-backs to this approach. One requirement is an exact unit root in the observable lag-polynomial (see Kilian (2011), page 23). In the past, many models with persistent $I(0)$ variables turned out to be invalid. Furthermore, in some cases the choice of entering a variable in levels or in differences can make an important impact on the results.

More details on long run restriction can be reviewed in Kilian (2011), pages 19-25.

2.2 Kalman Smoother

The linear discrete Kalman smoother is a tool to estimate unobserved values, given noisy measurements. The goal of this section is to give an idea about how this process works. The main sources for this part of the paper are Särkkä (2013), Nakata & Tonetti (2010) and Welch & Bishop (2001).

The Kalman smoother is a mathematical iterative process that uses a set of equations and data input, starting from some initial value, to minimize the covariance matrix of the error term of the measurement equation. It recovers the true value of a variable by using data around this value with some random error; it therefore separates signal and noise.

We start with a state space model, i.e. a state transition equation, which links states across time, and a measurement equation, which links non-state variables, i.e. the data, to the state variables.

$$x_t = Ax_{t-1} + Bu_t + w_{t-1} \tag{2.11}$$

$$z_t = Cx_t + v_t \tag{2.12}$$

Here, x_t is the vector of state variables, u_t is the input-control vector, w_{t-1} is a white noise process, which is $w_{t-1} \sim \mathcal{N}(0, Q)$, z_t is the vector of measurements, v_t, also called the measurement innovation, is analogous to w_{t-1} with a variance of R and A, B and C are coefficient matrices. Furthermore let $\hat{x}_t^-$ be the *a priori* estimate, i.e. the estimate with information up to, but excluding time period t, and $\hat{x}_t$ be the posterior estimate, i.e. the estimate with information including information from time period t, using the entire period span of the data. Using the two latter definitions, I can formulate the *a priori* error and the posterior error

$$\begin{aligned} e_t^- &= x_t - \hat{x}_t^- \qquad (2.13) \\ e_t &= x_t - \hat{x}_t \qquad (2.14) \end{aligned}$$

with corresponding VCV-matrices $P_t^- = \mathbf{E}\left(e_t^- e_t^{-\tau}\right)$ and $P_t = \mathbf{E}\left(e_t e_t^{\tau}\right)$. The Kalman smoother uses the whole dataset for its predicted values, while the Kalman filter only uses the data points up to and including t.

The Kalman equation links the posterior state to the *a priori* state using the weighted measurement innovation. The weight of the measurement innovation is the Kalman gain, K_t.

$$\hat{x}_t = \hat{x}_t^- + K_t(z_t - Cx_t) \tag{2.15}$$

Rudolf E. Kalman constructed the Kalman gain, K_t, so that it minimizes the posterior VCV-matrix, P_t. Algebraically, the expression for K_t can be derived by plugging equation 2.15 into equation 2.14, computing P_t, deriving the trace and optimizing. At this point, it is obvious how important the Kalman gain is. It can take many forms, but the intuition behind it stays the same; as the variance of the *a priori* error term drops, the less uncertainty there is between the *a priori* and posterior states, ergo K_t decreases, as $\hat{x}_t \approx \hat{x}_t^-$ as a result of P_t^- decreasing. However, as the variance in the measurement equation, see equation 2.12, decreases, the more weight is put on the measurement innovation, ergo K_t increases as R decreases; note that the measurement innovation is normally distributed with mean zero, by assumption, meaning that if the model presented in equation 2.12 comes reasonably close to the true model and R gets small, we add a constant close to zero to the *a priori* estimates. In other words, we trust the measurements more than our predictions. As an example of how the Kalman gain works, Welch & Bishop (2001) give

$$K_t = P_t^- C^\tau (C P_t^- C^\tau + R)^{-1}. \tag{2.16}$$

The computational algorithm can be split into the prediction or time update step and the correction or measurement update step. The prediction step uses equation 2.11 and

$$P_t^- = A P_{t-1} A^\tau + Q. \tag{2.17}$$

As one can see, it links expressions of past states to present states. The correction step gives feedback to the process using new data. This is done in three substeps:

1) computing K_t,

2) updating $\hat{x}_t = \hat{x}_t^- + K_t(z_t - Cx_t)$ using the dataset z_t and equation 2.12,

3) updating $P_t = (I - K_t H)P_t^-$.

The Kalman smoother is therefore a recursive process, which conditions the current state on all past states and the dataset, which implies that initial values have to be given, in order to start the process. Note that in an econometric context often the variables expressed are conditional densities, ergo the optimization maximizes a likelihood function. There is however one problem with this approach. The so-called pile-up problem occurs for low variance variables, e.g. long term growth rates. As long as the actual variance is smaller than T^{-2}, where T denotes the total number of periods, the Kalman smoother estimates the variance as exactly zero. There are some ways to deal with this problem, the most straightforward being to impose a variance ratio according to other empirical papers, similar to how Bayesian econometrics imposes *a priori* distributions. Details on how an actual estimation of the variance is done can be found in section 4.

Chapter 3

The SVAR model

The model presented in this section is based on Brzoza-Brzezina (2003). In order to recover the NRI via a SVAR, I assume that the NRI is the real interest rate at which inflation growth is zero using the short term interest rate data. This approach is used in many NRI estimations using different methods, e.g. Brzoza-Brzezina (2003), Laubach & Williams (2003) or Méssonier & Renne (2004). Furthermore, this approach is consistent with the definition of the NRI given in the introduction. The SVAR method can be applied here, since the NRI is constructed by adding up the true shocks of the NRI on the real rate. Here the identification restrictions of the SVAR (as well as some modeling choices) come into play. This will be clearer before the end of this section.

The main economic insight that I use here is that the NRI is by assumption the real short term interest rate, r^*, at which inflation growth is zero. I start by defining some simple equations.

Equation 3.1 states that there is a (not necessarily different from zero) difference between the real short term interest rate and the NRI.

$$r_t = r_t^* + GAP_t. \tag{3.1}$$

I assume, that the NRI and the GAP are both stationary $AR(1)$ processes[1], hence

[1] Brzoza-Brzezina (2003) uses these assumptions for his model. They can, however, not be verified empirically, since both variables are not observable.

$$r_t^* = \alpha_1(L)r_{t-1}^* + u_{1t} = \beta_1(L)u_{1t} \tag{3.2}$$

and

$$GAP_t = \alpha_2(L)GAP_{t-1} + u_{2t} = \beta_2(L)u_{2t} \tag{3.3}$$

which implies

$$r_t = \beta_1(L)u_{1t} + \beta_2(L)u_{2t} \tag{3.4}$$

Note that $\beta_1(L) = (I - \alpha_1(L))^{-1}$ (and analogously for β_2), which is the resulting polynomial of the transformation from an $AR(1)$ into a $MA(\infty)$ process. Furthermore, in equation 3.2 the NRI is expressed as the weighted sum of the true innovations u_1. Also, the modeling of the NRI and the GAP as $AR(1)$ processes can hardly be proven or disproven empirically, since both variables are latent.

Equation 3.4 is the second part of the SVAR model. For the first part, I use the fact that there is some kind of connection between inflation growth, $\Delta\pi$, and the GAP; since the NRI is defined, in this context, as the interest rate at which inflation growth is zero, which seems plausible. Thus,

$$\Delta\pi_t = \psi(r_t - r_t^*) = \psi GAP_t = \psi\beta_2(L)u_{2t}. \tag{3.5}$$

Using equation 3.1, I recover equation 3.5. For 3.5 to make economic sense, ψ should be negative, the reasoning being that whenever the GAP is greater than zero, the real rate needs to be greater than the NRI, which suggests a restrictive monetary policy (which in turn means less liquidity (for banks), therefore less credit, therefore less demand for the economies output) and therefore slower inflation growth.

Equations 3.5 and 3.4 can now be brought together as my SVAR model

$$\begin{pmatrix} B_{11}(L) & B_{12}(L) \\ B_{21}(L) & B_{22}(L) \end{pmatrix} \begin{pmatrix} \Delta\pi_t \\ r_t \end{pmatrix} = \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix}. \tag{3.6}$$

I proceed as described in section 2.1. The simple VAR model, corresponding to the data on the left hand side of equation 3.6, look as follows

$$\begin{pmatrix} \Delta\pi_t \\ r_t \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} \begin{pmatrix} \Delta\pi_{t-1} \\ r_t - 1 \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \tag{3.7}$$

Equation 3.7 can be transformed into the VMA representation,

$$\begin{pmatrix} \Delta\pi_t \\ r_t \end{pmatrix} = \begin{pmatrix} A_{11}(L) & A_{12}(L) \\ A_{21}(L) & A_{22}(L) \end{pmatrix} \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \tag{3.8}$$

such that $A(L) = (I - CL)^{-1}$. The logic behind these transformations from the SVAR model into its reduced form representation of a VMA corresponds to the derivation shown in equation 2.5. One sees that u_t and ε_t are related to each other by the restriction matrix B_0, which yields $u_t = B_0\varepsilon_t$. Therefore, as soon as all elements of B_0 are known, I can recover u_t, $B_{21}(L) = A_{21}(L)B_{0,11}^{-1} + A_{22}(L)B_{0,21}^{-1}$[2] and finally the NRI, as $r_t^* = B_{21}(L)u_{1t}$. I proceed by assuming that there is no immediate impact of the *GAP* on inflation growth, hence making B_0^{-1} upper triangular, and that there is no long term impact of NRI shocks on inflation growth, hence $B_{11}(1) = 0$. I justify the assumption of the lagged impact of the *GAP* on inflation growth, by arguing that until the change in behaviour of financial institutions based on changes in the interest rate and especially their lending policies, affect demand, some time has to pass; this is especially justifiable when considering monthly data, as I do, essentially arguing that it takes longer than a month, until, on a macroeconomic level, changes in lending behaviour affect markets enough to cause inflation changes. There are many empirical papers supporting this assumption, e.g. Batini & Nelson (2001). The assumption of long term neutrality of NRI shocks is analogous to the assumption of neutrality of money, a widely used and accepted assumption in the economic literature. See Cagan (1972) for, one among many, explanatory texts on this assumption.

Please find the exact derivation of the other entries of the B_0 matrix in the Appendix.

[2] $B_{0,11}^{-1}$ refers here to the first element of B_0^{-1}; analogously for the other elements.

3.1 Data

For this estimation, I use monthly data downloaded from the Eurostat and ECB databases from January 1997 (with one exception) to July 2016. Inflation is expressed as HICP (Harmonized Index of Consumer Prices) data in percent change and seasonally adjusted for the Euro area in differenced form for the first SVAR equation. Nominal interest rates are used for 3 time horizons, namely day-to-day, 1-month and 3-month. In order to deflate the nominal interest rates three methods were used, by defining three cases for inflation expectations:

$$\begin{aligned} \text{naive case:} \quad & \pi^e_{t+1|t} = \pi_t \\ \text{perfect case:} \quad & \pi^e_{t+1} = \pi_{t+1} \\ \text{survey case:} \quad & \pi^e_{t+1} = \pi^{survey}_t. \end{aligned} \tag{3.9}$$

The naive case simply sets the expected inflation for the next month to the inflation in the current month. The perfect case assumes perfect foresight, setting the inflation expectations to the realized value in the next period. The survey case uses a similar alternative as Brzoza-Brzezina (2003). He uses survey data of business and economics professionals, whose expectations were gathered on a monthly basis. The ECB conducts a similar survey and publishes its results on its homepage, however only on a quarterly basis. To account for the two months within each quarter not accounted for in the survey, I assume constant expectations, ergo the expectations from January to April (including) etc. are equal. Furthermore, the survey data are only available from February 1998 on. The three nominal and nine real series are displayed in the Appendix; the nominal interest rate series in figure 6-1, the three deflated day-to-day series in figure 6-2, the three deflated 1-month series in figure 6-3 and the three deflated 3-month series in figure 6-4.

The method to recover inflation expectations I use in section 4.2, i.e. constructing stationary ARMA models using BI and AI criteria and using the predicted values as inflation expectations, cannot be applied in the context of monthly data, used in this section of the paper. This is due to the nonstationarity in the monthly data series, which is not prevalent in the quarterly series.

3.2 Results

Since I use three different kinds of data for interest rates, namely interest rates for a day-to-day, a 1-month and a 3-month time horizon, and inflation expectations, namely the naive, perfect and survey cases, respectively, I run the model nine times, once for each definition of real interest rates[3]. I allow for a structural break in the model at the time of the bankruptcy of Lehman Brothers in September 2008; Chow tests for structural breaks were conducted accordingly[4]. Here, the NRI is recovered by summing up the weighted true shocks of the SVAR, as illustrated by equation 3.6. This means however that for the first data point of the NRI, I need to sum up the shocks some periods before it, therefore, if the series of true shocks starts in period t, the first data point of the NRI is recovered s periods later, where s is the number of true shocks I sum up. Following Brzoza-Brzezina (2003), I choose s to be twelve, ergo using one years worth of true shocks to recover one data point of the NRI. This explains the gap in the results at the beginning of the dataset and after the structural break in 2008. The recovered NRI and Chow test results can be reviewed in the Appendix in figures 6-5 to 6-13 and table 6.1 respectively.

Looking at the figures displaying the NRI results for the naive and survey cases[5], one can see that the variance of the NRI before the crisis is very low, while it is very volatile after the crisis. In the case of perfect foresight, as seen in the three corresponding figures in the Appendix[6], the opposite holds true, displaying a large variance before the crisis and a low one after. This shows that the choice of interest rate and definition of expected inflation makes a large difference. For the most part, the NRI stays inside the unit circle. In general, before the crisis the NRI is below the market rate, except for the time between 2004 and 2006, as real interest rates were particularly low. After the crisis it seems that the market rate is too low, potentially inducing another bubble in some, while trying to fight the deep recession in other Euro area member states[7]. A reason why GDP growth is low, even though the gap between the

[3]That is (1) $r_t = i_t^{day-to-day} - \pi_t^{naive}$, (2) $r_t = i_t^{day-to-day} - \pi_t^{perfect}$, (3) $r_t = i_t^{day-to-day} - \pi_t^{survey}$, (4) $r_t = i_t^{1-month} - \pi_t^{naive}$, (5) $r_t = i_t^{1-month} - \pi_t^{perfect}$, (6) $r_t = i_t^{1-month} - \pi_t^{survey}$, (7) $r_t = i_t^{3-month} - \pi_t^{naive}$, (8) $r_t = i_t^{3-month} - \pi_t^{perfect}$, (9) $r_t = i_t^{3-month} - \pi_t^{survey}$.

[4]The nominal and real interest series, which are the data the error terms tested in the Chow tests are based on, can be reviewed in the Appendix in figures 6-1 to 6-4.

[5]That is figures 6-5, 6-7 6-8, 6-10, 6-11 and 6-13.

[6]That is figures 6-5 6-9 and 6-12.

[7]This also illustrates a general weekness of a NRI estimation for the whole monetary area. While the interest rate might be too low for the area as a whole, it is at the same time comprised of a very heterogenious group of

real rate and the NRI is negative, and therefore monetary policy expansionary, could be the strong fiscal conservatism[8], which is practiced in the Euro area. A reason why GDP has grown rapidly in the periods before the crisis even though monetary policy was restrictive, might be increased public consumption and investment, due to too low borrowing rates for some states. In this case, monetary policy is not bubble inducing, but market failure, since the fiscal stability of some nations was wrongly evaluated, due to their entry into the Euro area, which resulted in overspending and moral hazard. For further analysis of the moral hazard problem and market failure before the crisis, see De Lucia (2011).

The impulse response functions (IRF) of the estimated SVAR models, ergo the reaction functions to a standardized shock, may give a hint on how the challenge of policy making changes before and after the crisis. Please find all IRF depicted in the Appendix in figures 6-5 to 6-13. The general tendency of the IRFs to move towards zero is not surprising, but a result of assumptions. However the sign and the speed at which the IRF tend to zero may give an idea about the dynamics of the model. Unfortunately the IRFs are not very robust vis-à-vis the different definitions of real interest rates. In the survey case the three IRFs, see figures 6-7, 6-10 and 6-13, look very similar. Before the crisis, standardized shocks have a very low and negative transmission rate. After the crisis however, the sign and magnitude change dramatically. In the naive case for 3-month interest rates the picture is quite the same, see figure 6-11, while the IRFs for 1-month interest rate data changes in magnitude, but not in sign, see figure 6-8. The day-to-day interest rates yield two IRFs that are both negative and similar in magnitude, see figure 6-5. It makes sense that the low variance in the early periods of the NRI, when estimated for the naive and survey cases, is in direct relation to the magnitude of the coefficients. It is therefore not surprising that the high variance in the early and low variance in the late periods for the perfect case reflect just that.

In conclusion, the results for the NRI show that unsurprisingly the financial crisis had a major effect on the state of the economy. However, whether it lead to a more or less volatile NRI or a wider or smaller GAP cannot be inferred from the results presented in figures 6-5

countries, where it is plausible that the low interest rates are actually suited or even too high for some struggling economies. A country specific analysis of the NRI is however outside the scope of this paper.

[8] For an overview of the institutional strengh of fiscal conservatism in the Euro area, see European Commission (2016).

to 6-13. The impulse response functions in the same figures also do not present a convincing story, as they change heavily in magnitude and sign with the different definitions of real interest rates. The SVAR model seems to fail to produce NRI estimations that could be used for ex-post policy analysis, at least for Euro area data. I suspect that among the reasons for this, is the fact that this SVAR model tries to recover a variable, that depends on the state of an economy as a whole, using only real interest rates and inflation growth. The major advantage of the SVAR approach, its relative simplicity, apparently does not provide enough scope for a reliable NRI estimation. This however gives motivation for more complicated models and methods, which capture more facets of an economy.

Chapter 4

Closed form macroeconomic model

This section of the paper presents a closed form macroeconomic model, as proposed by Laubach & Williams (2003). After the introduction of the original model, I look at some extensions. The model presented is not micro-founded and is therefore subject to the Lucas Critique (Lucas (1976)). However, unlike the SVAR approach, it allows to put the NRI in the context of actual output, measured by real GDP and its structural component. Here, the NRI is the interest rate at which the output gap[1] is closed, which in turn results in stable inflation. It can be seen as the interest rate consistent with the stable growth path in a Ramsey-Cass-Koopmans model[2]. A framework like this also allows for research into secular stagnation. Secular stagnation is a situation where monetary policy is ineffective and the economy is in a downturn. In other words, as the NRI is negative and inflation is low, the monetary policy rate cannot help in closing the output gap, as long as the gap is negative, resulting in a middle to long term situation with low growth and little chance of recovery. This is especially true in times of fiscal consolidation. The term secular stagnation is coined by Larry Summers (Summers (2014)). He states that when equilibrating forces fail, low demand undercuts potential growth and therefore drives down the NRI below zero, which in turn makes monetary policy ineffective in times of de-

[1] Laubach & Williams (2003), and all other authors mentioned in this chapter, follow the New-Keynesian interpretation of potential output, i.e. potential output is achieved if prices and wages are fully flexible and trade is conducted with prices perfectly matching marginal productivity. This assumption only holds in the long-run. (Bouis et al (2013))

[2] Note that this means that the theoretical basis of the time-series model depends on the corresponding assumptions of a Ramsey model, e.g. a CES-utility function, representative households, clearing markets (consumption equal production), etc. (Romer (2011))

flation. Many authors have found indications that secular stagnation is a real phenomenon that is happening now using similar models to Laubach & Williams (2003), e.g. Bouis et al (2013) or Pedersen (2015). Equation 4.1 intuitively shows how the NRI in the aforementioned papers works[3],

$$r^* = \frac{1}{\sigma}q + n + \theta, \tag{4.1}$$

where σ refers to the intertemporal elasticity of substitution of consumption, q denotes the technological change, n denotes population growth and θ refers to the rate of time preference. Even if one assumes that σ and n are constant over time, there are strong arguments for a time-varying q, e.g. Moores Law[4], and θ, e.g. changes to pension systems, to be time varying. Having established why the NRI can be time-varying in a theoretical context such as this, we can now construct an estimatable time-series model.

Laubach & Williams (2003) use an IS-curve and a Philips-curve as measurement equations. Equation 4.2 depicts the IS-curve of the model. It relates the output gap in percent, i.e. the difference between real log GDP and the log of potential GDP, $\tilde{y}_t$, with its own lags and a serially uncorrelated error term to control for short term and transitory effects, i.e. $A_y(L)$ cannot have a unit root, and the interest rate gap, to model shifts due to the long term relationship between the interest rate gap and the output gap via shifts in the NRI.

Equation 4.3, the Philips-curve, relates core-inflation (Laubach & Williams (2003) use the CPI) to its own lags, other indices of inflation, e.g. import prices and crude oil prices, x_t, a serially uncorrelated error term and the output gap. Even though Laubach & Williams (2003) control for relative prices, which may help to control for external influences on inflation, implicitly they assume a closed economy, especially by not controlling for trade in their model:

$$\begin{aligned} \tilde{y}_t &= A_y(L)\tilde{y}_{t-1} + A_r(L)(r_t - r_t^*) + \varepsilon_{1t}, & (4.2) \\ \pi_t &= B_\pi(L)\pi_{t-1} + B_y\tilde{y}_{t-1} + B_x x_t + \varepsilon_{2t}.^5 & (4.3) \end{aligned}$$

[3] Equation 4.1 is a direct quotation from the introduction of Laubach & Williams (2003).
[4] Moores Law states that computational power doubles every two years.

Moreover, there are three transition equations. As argued in the beginning of this section, r_t^* is determined by the long term growth rate, g_t, and time-preferences, z_t. These two main determinants of the NRI enter the model in equation 4.4, where z_t is modeled as an $AR(1)$ process. Laubach & Williams (2003) allow for equation 4.5 to have a unit root or to be stationary. Furthermore, they model potential output as an $I(2)$ process, effectively setting all coefficients in equations 4.6 and 4.7 to one. The error terms in these equations are serially uncorrelated and uncorrelated with ε_{3t}, not allowing for interdependencies between elements of the NRI, as depicted in equation 4.4. Equations 4.4 to 4.7 depict the transition equations of this economy.

$$r_t^* = C_r g_t + z_t \tag{4.4}$$

$$z_t = D(L) z_{t-1} + \varepsilon_{3t} \tag{4.5}$$

$$y_t^* = E_y(L) y_{t-1}^* + E_g(L) g_{t-1} + \varepsilon_{4t} \tag{4.6}$$

$$g_t = F_g(L) g_{t-1} + \varepsilon_{5t} \tag{4.7}$$

However, unlike Laubach & Williams (2003), Méssonier & Renne (2004) adapt Laubach & Williams (2003) model for Euro area data. Méssonier & Renne (2004) do not assume that potential GDP is an $I(2)$ process, but instead assume GDP to be an $I(1)$ process and restrict equation 4.6 to be an autoregressive process, of differenced potential output on long term growth. Furthermore they do not allow time-preferences to be a random walk process, as Laubach & Williams (2003) do in a case distinction. I follow Laubach & Williams (2003) and assume an $I(2)$ process. They find however that the process is still very persistent. The authors omit any external influence in the measurement equations, i.e. they omit the x_t variable in equation 4.3. They effectively treat the Euro area as a closed economy. For aggregate US or Euro area data the assumption of a closed economy might be acceptable, even if it is still a very strong assumption, but for smaller economies it cannot hold.

A more recent paper, Pedersen (2015), uses a Bayesian econometrics approach to modify the Laubach & Williams (2003) model for an open economy for Danish data[6]. Pedersen (2015)

[6] He states that he tried a Kalman smoother approach, but that he had difficulties identifying the NRI,

uses effective real exchange rate data[7] to model the influence of other economies without modeling them directly. He recognizes that the definition of the NRI as the interest rate that equates savings and investment is not broad enough for a small open economy, since savings and investment need not be equal for the output gap to be closed, due to large current account imbalances. He bases the incorporation of exchange rates on defining a natural exchange rate, i.e. the exchange rate that allows the NRI gap and output gap to be closed[8]. The exchange rate gap enters both measurement equations. In equation 4.2 it would represent the effects of exports and in equation 4.3 it would enter indirectly via the output gap, while the change in exchange rates would capture the affect of foreign activity on domestic inflation[9]. Even though this model expansion is very useful for economies prone to extensive foreign economic influence, it is not fit for a relatively simple estimation method such as the Kalman smoother and introduces additional parameters, making the model less parsimonious. For the Euro area, the assumption of a closed economy is on the one hand reasonable, but on the other hard to defend. It is reasonable, since its net exports, ergo the part which is actually affecting GDP, is comparable with the US, for which models like Laubach & Williams (2003) have performed quite well. However, the share of imports and exports of GDP, are much higher for the Euro area than for the US, resulting in more exposure to price shocks of foreign products[10]. Unfortunately, the used dataset only allows for a control of crude oil prices, but not export or import prices[11].

Furthermore, none of these models take into account the unique situation the financial, and

probably due to his data and/or his model modifications.

[7] Effective exchange rates are computed as foreign price level divided by the nominal domestic exchange rate times the relative price level of the domestic economy.

[8] In order to illustrate how exchange rates influence the whole model-economy, he gives an example: as exchange rates go up the peg with the Euro forces the danish national bank to use their policy rate to decrease nominal interest rates, which results in increased demand, higher inflation, and therefore directly and indirectly to a lower real rate of interest. Since domestic inflation rises, exports as well as inflation growth decrease. This way of thinking holds true for real as well as for nominal variables.

[9] Exchange rates also enter the transition equations, resulting in an expansion of the model from six equations to thirteen.

[10] On July 8th, 2016 the ECB published a report regarding the structure of the EMU and other economies. For 2014 the EMU had 27.7% of its GDP in exports, 23.7% in imports and a trade balance of 3.2%. The US accounted 12.6% and 15.4% of its GDP on exports and imports respectively and had a trade balance of -2.6%. Since the absolute values of the share of net exports on GDP are similar, but the actual numbers of different signs, I assume that the effect of a shock to the current accounts would differ in sign, but not necessarily much in magnitude.

[11] The dataset I use is the ECB AWM (area wide model) dataset, also used by Méssonier & Renne (2004). The assumption of a closed economy is therefore one of the major disputable points in this model.

especially the interbank markets are in since recent years, i.e. the inter-bank market credit dry up[12]. Bouis et al (2013) recognize these circumstances, even if they do not try to model them. They use the standard Laubach & Williams (2003) model (with only minor changes) to estimate the NRI for the Euro area, Swizerland, Sweden, the UK and the US, using output gap data previously estimated by the OECD. Omitting the credit dry up becomes particularly problematic when modelling the NRI. Assuming that the NRI gap is closed, the interest rate the central bank sets should result in the optimal level of investment and output, but only if, firstly, the lower interest rates are passed on from the banks to the consumers and investors, and, secondly, if consumers and investors actually shift their demand in the way theory would suggest it would. In reality however, the recent financial crisis resulted in banks accumulating historic levels of liquidity reserves following the periods after 2008[13]. This is a powerful sign for a significant uncertainty shock. Therefore, taking into account the uncertainty after 2008 the level of the NRI, which actually would close the output gap via investment, would be lower than the original model, not taking into account uncertainty, suggests. I try to model this by introducing an interaction term of a dummy variable, which is zero before 2008 and one afterwards, and relative excess liquidity holdings of business banks as a proxy for uncertainty in the credit market[14]. Using excess reserve liquidity data in this way is a novel approach.

[12] This phenomenon might also be responsible for the low levels of inflation in the last years, as banks are not only intermediaries (as often assumed), but responsible for the creation of money via financing (see Jakab & Kumhof (2015)).

[13] Here Bouis et al (2013) argue that interest rate changes have been passed on to households and that the build up of liquidity reserves is due to other credit supply and demand channels.

[14] One might argue that the interaction of the liquidity data with a dummy variable is not necessary, since the liquidity data is almost zero before 2008 anyways. The liquidity data is however only availiable from 1999 onwards. I choose to use a dummy variable, to eliminate the variance in the liquidity data pre 2008, since I do not lose a lot of information, but gain the ability to use the full length of the other variables datasets in the model. Figure 6-15 in the Appendix shows the liquidity data from the ECB and supports this argument.

4.1 Estimated model

In this section, I present the estimated model[15], describe how I arrived with its form and explain the pile up problem, previously mentioned in section 2.2, and its solution. The state space form can be reviewed in the Appendix in section 6.4, as well as a list of variable names and meanings in section 6.5 in table 6.2.

As mentioned in section 2.2, the pile up problem is the downward bias for the variance estimation of low variance nonstationary variables. There are however ways to deal with this problem[16]. Méssonier & Renne (2004) first impose a ratio for $\lambda_g = \frac{\sigma_5}{\sigma_4}$, where σ_4 and σ_5 are the standard deviations of the error terms in equation 4.6 and equation 4.7 respectively. They then impose a ratio for the coefficient in equation 4.4, $\lambda_\theta = \frac{\theta_r}{\theta_y}$, in reference to equations 4.10 and 4.11. Their intuition behind the second imposition is that there is some relation between the growth rate of potential output and the NRI, which is somewhere between one-to-one and five-to-one. They do not use the Stock & Watson (1998) median-unbiased estimator, which is sometimes used under such circumstances, but use a Lagrange Multiplier test to get a first idea of the values for λ_g and λ_θ and economic reasoning about the significance of some economic variables. Since I use a very similar dataset to theirs I try to use a very similar model as they

[15] I also estimated a smaller model via a Kalman smoother using the intuition from the SVAR model.

$$\Delta\pi_t = \alpha_1 \Delta\pi_{t-1} + \alpha_2(r_t - r_t^*) + \varepsilon_{1t}$$

$$r_t^* = r_t^* + \varepsilon_{2t}$$

Considering the results of this approach using the SVAR method in the previous chapter, it is not surprising that this model did not yield reasonable results. Moreover, the lack of reasonable results does hint towards an inadequacy of the model or economic intuition behind the model, rather than a problem with the SVAR method. The failure of this model again hints towards the need of a model, which encompasses more than just monetary variables, but also real economic components.

[16] In the case of Laubach & Williams (2003) they use the median-unbiased estimator as proposed by Stock & Watson (1998) to estimate variance ratios $\lambda_g = \frac{\sigma_5}{\sigma_4}$, where σ_4 and σ_5 are the stardard deviations of the error terms in equation 4.6 and equation 4.7 respectively, for equation 4.7 and $\lambda_z = \frac{\sigma_3 A_r}{\sigma_1 \sqrt{2}}$, where σ_1 and σ_3 are the standard deviations of the error terms in equation 4.2 and equation 4.5 respectively, and A_r the coefficient from equation 4.2, for the case of nonstationarity in equation 4.5. In a three stage approach they estimate first potential output growth as a constant, while omitting the interest rate terms, secondly imposing the variance ratio λ_g recovered in the first step and introducing the interest rate terms, but assuming z_t to be constant in order to recover λ_g, and thirdly imposing both ratios to arrive at their final results. I tried the model and three stage approach of Laubach & Williams (2003), but did not gain interpretable results. The process either did not converge or the results were too unrealistic to yield any economic insight for variance ratios proposed in the literature. Modeling time preferences as an explicit component of the NRI, apparently is not approriate for the AWM dataset, since its exclusion yields interpretable results.

present, and also their results for the ratios to estimate the following model[17].

In contrast to Laubach & Williams (2003) or Méssonier & Renne (2004), I model the output gap as an $AR(1)$ process, as seen in equation 4.8. Estimations for higher dimensional AR-processes result in a very tight fit for potential output to realized GDP, closing the output gap for most of the observation periods. My approach comes at the cost that potential output is estimated to be almost a straight line. D_2 is a dummy variable, used for notational convenience in separating two model, which only differ in their inclusion of a liquidity shock. The dummy is a vector of zeros with length of the complete sample for the model without a liquidity shock, and a vector of ones for the model including a liquidity shock. liq_t represents the vector of excess liquidity reserve data relative to minimum reserve requirements. The intuition is that the actual output gap should be bigger, when a liquidity shock is introduced, since the NRI gap does not capture the full effect of the uncertainty in the financial market. The raw data for the liq_t variable are downloaded from the ECB homepage and the excess reserves are computed according to Hüttl & Merler (2015). The data can be reviewed in the Appendix in figure 6-15. Thus,

$$\tilde{y}_t = \alpha_1 \tilde{y}_{t-1} + \lambda((1+L)(r_t - r_t^*) + D_2 liq_t) + \varepsilon_{1t}. \tag{4.8}$$

The second measurement equation is a version of the Philips curve, as seen in tificLaubach & Williams (2003), depicted in equation 4.9. I control for autoregressive components of inflation by including three of its lags. Furthermore, I control for foreign price shocks by including imp_t, which is a vector of import price data for the European Union, downloaded from the FRED. I also include a lagged term of the output gap to model the relationship between inflation and the real economic cycle.

$$\pi_t = \beta_1 \pi_{t-1} + \beta_2 \pi_{t-2} + \beta_3 \pi_{t-3} + \beta_4 \tilde{y}_{t-1} + \beta_5 imp_t + \varepsilon_{2t} \tag{4.9}$$

Equation 4.10 shows the construction of the NRI as a constant and weighted long term growth of potential output. Optimally, a liquidity shock would enter equation 4.10 directly. Unfortunately, an introduction of a liquidity shock using data is not possible in this setting, since r_t^* is a

[17] The estimation is done in a two step procedure, similar to Laubach & Williams (2003). First I gain preliminary estimates for potential output by omitting the interest rate terms and then I use these results to estimate the other variables using the ratios λ_g and λ_θ as Méssonier & Renne (2004) do.

by definition an unobservable state variable. A state space form, which is necessary for the used estimation approach, links state variables only to their own lags and other state variables and can therefore not be compromised by observed data. Equations 4.11 to 4.13 show the transition equations of the model economy. Differenced potential output is modeled as a constant and long term growth rate, which is in turn modeled as a random walk. The last equation defines the output gap as the difference between GDP and potential output:

$$
\begin{aligned}
r_t^* &= \mu_r + \theta_r g_t, && (4.10)\\
\Delta y_t^* &= \mu_y + \theta_y g_t + \varepsilon_{3t}, && (4.11)\\
g_t &= g_{t-1} + \varepsilon_{5t}, && (4.12)\\
y_t &= y_t^* + \tilde{y}_r. && (4.13)
\end{aligned}
$$

As mentioned above, I use the ratios computed by Méssonier & Renne (2004) for my estimation of θ_r and σ_5:

$$
\begin{aligned}
\lambda_\theta &= \frac{\theta_r}{\theta_y} = 16.0, && (4.14)\\
\lambda_g &= \frac{\sigma_5}{\sigma_4} = 0.5.
\end{aligned}
$$

4.2 Data

For the model estimation I use the area-wide model (AWM) dataset[18], downloaded from the ECB homepage. The AWM dataset is the result of a weighted sum of data from the 19 European countries using the Euro[19]. I use the log of real GDP as the output variable, HICP seasonally adjusted for inflation and the fitted values of an ARMA model on HICP as inflation expectations. I use the BIC and AIC to construct the ARMA model for this purpose. Both

[18] The AWM, for which the AWM database was created, is an economic long run equilibrium model of the Euro area. It treats the Euro area as if it were a single economy. The purpose of the AWM is to present a medium sized model, with long run supply and short run demand effects that allows for forecasting and simulations.

[19] These countries are Belgium, Ireland, France, Latvia, Malta, Portugal, Finland, Germany, Greece, Italy, Lithuania, Netherlands, Slovenia, Estonia, Spain, Cyprus, Luxembourg, Austria and Slovakia.

criteria favor an $ARMA(3,0)$ model. The details can be reviewed in the Appendix in table 6.4. The dataset is comprised of ECB and/or Eurostat data as well as data from the OECD National Accounts, the OECD main economic indicators, the BIS and the AMECO databases. Typically seasonal variables, including GDP and HICP, are seasonally adjusted. The AWM dataset and model are accompanied by two relevant papers. The database update note, ECB (2015), which gives an overview of the changes and the composition of the dataset, and a thorough working paper presenting the model itself at the time of its creation, Fagan, Henry & Mestre (2001).

In addition I use relative excess liquidity reserve data, also downloaded from the ECB homepage, and import price data from the FRED. The relative excess liquidity reserve data were calculated by summing the deposit facilities and current accounts, minus the sum of the marginal lending facilities and the minimum reserve requirements divided by the minimum reserve requirements. This way of calculating excess reserves is standard and can be reviewed on the ECB homepage or in Hüttl & Merler (2015). The data are depicted in figure 6-15 in the Appendix. The figure shows that the excess liquidity reserves were very close to zero until 2008. At their peak excess liquidity holdings were seven times bigger than the actual minimum reserve requirements. Additionally, the data are a lower bound for excess liquidity holdings, since the calculations do not include any non-standard monetary policy instruments used during and after the crisis, such as quantitative easing.

All data, save for the non-interacted liquidity data, range from the first quarter of 1991 to the third quarter of 2014.

4.3 Results

[20]I first present the results of the model without the liquidity shock, then with liquidity shock and finally a comparison of the results. Please find the state space form of the model in the Appendix in section 6.5, as it is different from the state space form presented in Méssonier & Renne (2004) and important for understanding the estimated dynamics. The re-

[20]At this point, I want to mention that I learned how to code in RATS (the used program) by reading forum posts and codes of already written state space estimating programs. There are very similar models to the one I am estimating and I used some parts of code from these forum entries. The Forum is the RATS Software forum and can be found here: [https://estima.com/forum/]. The codes are available upon request.

sults of the models can be reviewed in the Appendix in figures 6-14 and 6-16[21]. The imposed values for the estimation are based on Méssonier & Renne (2004) and can be found in table 6.3.

The estimation results for the model without liquidity shock are depicted in figure 6-14. The first panel in the upper left corner shows the used GDP data. Next, the middle panel on the left hand side depicts the estimated potential output, or y_t^*. As mentioned above, potential output is estimated as an almost straight line. The output gap, seen in the bottom left panel, shows the EMU in a recession until 1997 and then a boom until the onset of the financial crisis. The period of growth after 1997 might be due to the optimism surrounding the founding of the Euro area in the following years. The decline of the output gap around 2001 seems to me to be effects of the bursting dot-com bubble. In the periods until late 2007, the panel shows another rise in output vis-à-vis potential output. In retrospect, this peaking in 2007 appears to have been an overheating Euro area economy, due to over-investment and consumption. After this, the output gap takes a sharp turn downwards, even reaching negative values in 2012. The top right panel shows the real interest rate and the estimated NRI. The difference between the two can be interpreted as the monetary policy stance. Whenever the real rate is above the NRI, monetary policy stance is restrictive, and expansionary vice versa. Thus, the monetary policy stance in the Euro area was restrictive until 1997, expansionary until the crisis and about neutral at the end of 2014. If the NRI should fall even further after 2014, there is little the central bank can do to close the gap. Considering that the output gap is negative at the same time, this situation would indicate a long term stagnation for the economy of the Euro area. The claim that secular stagnation is prevalent is also supported by the long term growth rate panel on the bottom right hand side, as it shows the growth rate, which is recovered from estimating equation 4.12, being close to zero and at an all time low in the end of 2014. The signs of the coefficients of the model do not raise cause for concern. α_1 can be positive, since it shows the relationship of the output gap to its lag. λ should be negative, since a positive NRI gap, ergo the NRI is smaller than the real rate, indicates a restrictive monetary policy stance and should decrease the output gap. Three coefficients are insignificant, namely α_1, μ_y and β_2. β_3 is defined as $1 - \beta_1 - \beta_2$, based on Laubach & Williams (2003) and Méssonier & Renne (2004). θ_y is imposed based on the results in Méssonier & Renne (2004), set to 0.06.

[21] I will refer to the individual pictures in these figures as panels.

The estimation results for the model including a liquidity shock can be reviewed in figure 6-16. Introducing the liquidity shock parameter and data changes the results. Firstly, comparing the two middle left hand panels in figures 6-14 and 6-16 depicting potential output, one can observe more movement after the introduction of the liquidity reserve variable. A close look reveals that potential output undergoes a slight shift in slope and level at the onset of the crisis and a very minor shift in 2012. This subtle change was not detected by the first model. Secondly, on the bottom left hand side, the output gap seems to have lower peaks than in figure 6-14. The peak in the model without liquidity shock at the beginning of the crisis decreases with the inclusion of the liquidity variables, even below the peak around 2000. Additionally, it is positive for almost all periods before the financial crisis, followed by a drop below zero afterwards. The top right hand side panel shows that the NRI gap is positive for all periods, indicating a restrictive monetary policy stance. This is a somehow surprising result that I cannot explain. Comparing the two middle right hand panels shows that with the inclusion of the liquidity variable the NRI is much smaller and moves within a much smaller interval. This is despite of the much greater estimated long term growth rate, as seen in the bottom right panels of the two figures. This can be explained by the much higher estimated variance for the error term in equation 4.11, showing the differenced potential output equation. During the post crisis periods, the NRI is smaller than in the results of the model without a liquidity shock. This result is in line with the argument in the introduction of section 4. There I argue that excess liquidity holdings are important to include, since not doing so would overestimate the NRI. Furthermore, the NRI cannot be reached using monetary policy instruments alone anymore. This hints heavily to a situation that can be interpreted as secular stagnation. On the other hand, the long term growth rate has its lowest point around the end of the dataset, just below 1%. This is higher than in the model without the liquidity shock, but still very low. The long term growth rate results of the model with a liquidity shock seem more reasonable than those of the model without it, since they show the pre crisis growth periods and sudden drop of the same during the crisis more clearly. Furhtermore, they are more in line with results from other papers, such as Laubach & Williams (2015). As mentioned above, introducing a liquidity shock seems to have increased the variance of the differenced potential output equations error term, which is probably responsible for the smaller variance in all other variables. Regarding the coefficients, the same holds as in the previous paragraph, except for the coefficient of the NRI gap and liquidity shock, λ, in equation 4.8, which increased in statistical significance, from 5% significance to 1% significance, but decreased in magnitude (see tables 6-14 and 6-16 in the Appendix). This further supports the inclusion of the liquidity shock variable. The effect of the NRI gap on the output gap seems to be hampered by relative excess liquidity holdings. However, whether the reduction in the effect on the output gap is directly due to the inclusion of the liquidity variable, the lower estimation of the NRI, the increased movement in potential output or a mixture of the three is hard to pin down.

Regardless of the choice between the two versions of the model, excluding or including a liquidity shock, hints towards secular stagnation are strong. I therefore conclude that monetary policy alone is not enough to kick-start the EMU economy. A coordination of national fiscal and area wide monetary policies might be necessary to quickly recover from the current lack of growth.

Comparing the results of my estimations with those of Méssonier & Renne (2004) is difficult for two main reasons. Firstly, Méssonier & Renne (2004) results stop in the beginning of 2003. Secondly, even if the two models are very similar, the state space forms of our models are different. Méssonier & Renne (2004) include in their transition equations a vector of nominal interest rate and inflation data, whereas I do not. They do this, because they use the IS equation (see equation 4.2) as an transition equation of the output gap, where I use equation 4.11. Using equation 4.11 allows me to avoid including data in my transition equations, which is a requirement for their estimation. This difference in state space modeling might however lessen the comparability of the two models, since the dynamics during the estimation, even if based on the same model, are different. Nonetheless, a broad comparison of the results can yield some insight, since the underlying models and the used datasets[22] are similar. I compare the results for the periods from 1991 (the start of my dataset) to 2003 (the end of their dataset). For the NRI their results show that the NRI moves within an interval from 1% to 6%. This is broadly in line with my results for the NRI without a liquidity shock, as it moves in an only slightly larger interval from a bit below than 1% to a bit above than 6%. Including a

[22] The AWM dataset is reviewed and corrected periodically, including revisions of older data. Therefore the AWM dataset that Méssonier & Renne (2004) use is not identical, but still very similar to the AWM dataset that I use.

liquidity shock however narrows the interval to about a 1.5% range, from slightly below -0.5% to slightly below 1%. The output gap of Méssonier & Renne (2004) moves within a -3% to 1.5% interval. Both of my results for the output gap move within a less negative interval; excluding the liquidity shock, it moves within an interval from -1.5% to 3% and including it within an interval from -0.6 to 1%. The long term growth rate in Méssonier & Renne (2004) moves between 0.3% and 1%. Excluding a liquidity shock, my estimated long term growth rate moves within the same interval. However, Méssonier & Renne (2004) find its lowest point in 2002 and its highest in 1991, whereas in my results it is the other way around. Including a liquidity shock yields much higher results for the compared periods. The long term growth rate moves here in an interval between 1.5% and 3%, with its lowest and highest point in the same years as in my estimation without a liquidity shock. In conclusion, my results without a liquidity shock seem to be comparable in dimensions to Méssonier & Renne (2004). Including a liquidity shock however yields a generally narrower NRI interval, and a higher long term growth rate.

Laubach & Williams (2015) present updated results of the Laubach & Williams (2003) model up until 2014 for US data. Furthermore, Laubach & Williams (2015) use a Kalman filter instead of a Kalman smoother. Their estimates for the NRI show movement between -1% and 4%, for the output gap between -4% and 2%, and for the long term growth rate between 2% and 3.5% from 1991 to 2014. Generally, my results including a liquidity shock are more in line with the estimated situation of the US than the results excluding a liquidity shock are. The US seems to have recovered better after the crisis and results in Laubach & Williams (2015) indicate less clearly a situation one might call secular stagnation at the end of the estimation periods. The US output gap has increased consistently after the crisis, even reaching zero at the end of the sample, indicating a better recovery than my results for the Euro area show, where the output gap is negative in 2014. Also at the end of the estimation period the NRI reaches zero. Therefore monetary policy alone is sufficient for reaching the NRI even in times of low inflation. The long term growth rate Laubach & Williams (2015) recover is also higher than in both models I estimate. In general, the US seems to be in a better position than the Euro area. However, it is not clear how robust the US economy really is towards secular stagnation, considering that after the crisis major fiscal stimuli[23] were launched and very expansionary, even non-standard, such as quantitative easing, monetary policy[24] was conducted in the years after the crisis. How the economy will react to an end of expansionary monetary policy and lower governmental fiscal stimuli cannot be analysed within these models, but are major concerns in both economic areas.

[23] For a description of the stimuli and an analysis of their effects see Shapior & Slemrod (2009)

[24] For a summary and description of the monetary policy measures of the FED see Labonte (2016).

Chapter 5

Conclusion

In this paper I have captured the importance of a fiscal component in the estimation of the NRI, and of the inclusion of an uncertainty component for the post crisis periods. Furthermore, I found indications for secular stagnation in the Euro area.

The results of the SVAR model, based on Brzoza-Brzezina (2003), were not persistent, as seen in the case distinctions between the nine versions of real interest rates. The lack of robustness of the results is not surprising, since the economic basis of the NRI includes a real economic component. Excluding this component leaves the SVAR model lacking in economic foundation. This argument finds some support in the fact that a model using the same economic arguments, but a different method of estimation, namely a Kalman smoother instead of a SVAR, does not yield reasonable results either. Despite that, I found indications that monetary policy, for the most part, is restrictive before the financial crisis and expansionary afterwards.

The estimation of a small macro economic model, following Laubach & Williams (2003) and Méssonier & Renne (2004) provides the opportunity to introduce an excess liquidity of business banks shock as a proxy for the market uncertainty that entered the financial markets after the crisis. The introduction of the relative excess liquidity holdings data has a major impact on the results of the model. Considering the stronger statistical significance of the parameter linking the NRI to the output gap and the generally notable change in results, the results show that the inclusion of some form of indicator of the major uncertainty shock after the crisis, like the relative excess liquidity holdings data I use, is paramount. Specifically, including an uncertainty shock allows for a correction of the NRI *vis-à-vis* investment other than inflation, output and interest rates, i.e. uncertainty, and hence it better shows the weakness of monetary policy instruments in the post crisis periods. The gained results are comparable in dimension to the results of Méssonier & Renne (2004) and Laubach & Williams (2015). The macro economic model estimation hints strongly towards a situation of secular stagnation. Considering this result and the current weakness of monetary policy, I want to point out an old lesson in economic policy making, seen in many models, e.g. the IS-LM/AS-AD model: The effects of fiscal and monetary policies can neutralise each other, rendering a weak net effect for governmental interventions. Both instruments should be used in tandem and for the same goal, be it output maximisation or inflation minimisation, in order to optimally navigate through worrying economic times, such as we are experiencing now in the wake of the financial crisis. I therefore conclude that monetary policy alone is not enough to kick-start the EMU economy. A coordination of national fiscal and area wide monetary policies might be necessary to quickly recover from the current lack of growth.

There are multiple avenues for further research based on the presented models and estimations. Further exploration of the applied models using different data series, such as different inflation definitions and methods for inflation expectations, may be fruitful. Also, an expansion of the models to include trade with foreign economies and uncertainty shocks, using different indicators, seems appropriate. Accepting the result of secular stagnation, other important research questions present themselves. For one, the exploration of options on how to exit a situation like this, in the short term as well as in the long term, is important for economic theory as well as the reality of policy makers. Piketty (2014) argues that long stretches of low overall output growth, as in secularly stagnating economies, and high productivity of capital causes snowballing inequality within and across societies. Such inequalities, so the author, may have major damaging effects on economies and societies as a whole.

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Chapter 6

Appendix

6.1 Derivation of the B_0 matrix

The four restrictions we impose are:

A normalization of the variation of the true innovations, as in section 2.1

$$Var(u_{1t}) = 1 \tag{6.1}$$

$$Var(u_{2t}) = 1 \tag{6.2}$$

No long run effects of NRI shocks to the growth of inflation

$$B_{11}(1) = 0 \tag{6.3}$$

No direct short run impact of the GAP on the growth of inflation

$$B_{0,12} = 0 \tag{6.4}$$

These restrictions make sense economically. At some point, NRI shocks stop affecting inflation growth since the process is stable. Furthermore, it takes some time for the difference between the natural and real interest rate to affect the growth of inflation; with this short run restriction we get a lower-triangular restriction matrix by construction, which ensures orthogonality.

As described in the last paragraph of section 3,

$$\begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} = \begin{pmatrix} B_{0,11} & B_{0,12} \\ B_{0,21} & B_{0,22} \end{pmatrix} \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \Leftrightarrow \begin{pmatrix} B_{0,11} & B_{0,12} \\ B_{0,21} & B_{0,22} \end{pmatrix}^{-1} \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} = \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}$$

$$Var(\varepsilon_{1t}) = Var(B_{0,11}^{-1}u_{1t} + B_{0,12}^{-1}u_{2t}) = Var(B_{0,11}^{-1}u_{1t}) + Var(B_{0,12}^{-1}u_{2t}) + 2Cov(B_{0,11}^{-1}u_{1t}, B_{0,12}^{-1}u_{2t}) = (B_{0,11}^{-1})^2$$

$$\Leftrightarrow B_{0,11}^{-1} = \pm\sqrt{Var(\varepsilon_{1t})} \tag{6.5}$$

since the restrictions displayed in equations (6.1) and (6.4) hold, and the $Cov(u_{1t}, u_{2t}) = 0$ by construction.

For the next element of B_0 we recall the second implication from (2.5) and observe

$$B(L) = A(L)B_0^{-1} = \begin{pmatrix} A_{11}(L) & A_{12}(L) \\ A_{21}(L) & A_{22}(L) \end{pmatrix} \begin{pmatrix} B_{0,11} & B_{0,12} \\ B_{0,21} & B_{0,22} \end{pmatrix}^{-1} \tag{6.6}$$

$$\Leftrightarrow \begin{pmatrix} A_{11}(L)B_{0,11}^{-1} + A_{12}(L)B_{0,21}^{-1} & A_{11}(L)B_{0,12}^{-1} + A_{12}(L)B_{0,22}^{-1} \\ A_{21}(L)B_{0,11}^{-1} + A_{21}(L)B_{0,11}^{-1} & A_{21}(L)B_{0,12}^{-1} + A_{22}(L)B_{0,22}^{-1} \end{pmatrix} = \begin{pmatrix} B_{11}(L) & B_{12}(L) \\ B_{21}(L) & B_{22}(L) \end{pmatrix}$$

Now using (6.3) and (6.5)

$$A_{11}(1)B_{0,11}^{-1} + A_{12}(1)B_{0,21}^{-1} = B_{11}(1) = 0 \Leftrightarrow \tag{6.7}$$

$$\Leftrightarrow B_{0,21}^{-1} = -\frac{A_{11}(1)}{A_{12}(1)}B_{0,11}^{-1} = \mp\frac{A_{11}(1)}{A_{12}(1)}\sqrt{Var(\varepsilon_{1t})} \tag{6.8}$$

$$Var(\varepsilon_{2t}) = Var(B_{0,21}^{-1}u_{1t} + B_{0,22}^{-1}u_{2t}) = (B_{0,21}^{-1})^2 + (B_{0,22}^{-1})^2 + 2Cov(u_{1t}, u_{2t})B_{0,21}^{-1}B_{0,22}^{-1}$$

$$\Leftrightarrow Cov(u_{1t}, u_{2t}) = \frac{(Var(\varepsilon_{2t}) - (B_{0,21}^{-1})^2 - (B_{0,22}^{-1})^2)}{2B_{0,21}^{-1}B_{0,22}^{-1}} \tag{6.9}$$

We use this result here

$$\begin{aligned}
Cov(\varepsilon_{1t},\varepsilon_{2t}) &= Cov(B^{-1}_{0,11}u_{1t}+B^{-1}_{0,12}u_{2t},B^{-1}_{0,21}u_{1t}+B^{-1}_{0,22}u_{2t})^1 = \\
&= B^{-1}_{0,11}B^{-1}_{0,21}+B^{-1}_{0,11}B^{-1}_{0,22}Cov(u_{1t},u_{2t}) = \qquad (6.10)\\
&= \pm\sqrt{Var(\varepsilon_{1t})}B^{-1}_{0,21}+B^{-1}_{0,11}B^{-1}_{0,22}\frac{(Var(\varepsilon_{2t})-(B^{-1}_{0,21})^2-(B^{-1}_{0,22})^2)}{2B^{-1}_{0,21}B^{-1}_{0,22}} \\
\Leftrightarrow\ B^{-1}_{0,22} &= \sqrt{-2\frac{B^{-1}_{0,21}}{B^{-1}_{0,11}}Cov(\varepsilon_{1t},\varepsilon_{2t})+(B^{-1}_{0,21})^2+Var(\varepsilon_{2t})} = \\
&= \sqrt{2\frac{A_{11}(1)}{A_{12}(1)}Cov(\varepsilon_{1t},\varepsilon_{2t})+\left(\frac{A_{11}(1)}{A_{12}(1)}\right)^2 Var(\varepsilon_{1t})+Var(\varepsilon_{2t})}
\end{aligned}$$

Therefore B_0 can be rewritten as

$$B_0^{-1}=\begin{pmatrix} \sqrt{Var(\varepsilon_{1t})} & 0 \\ -\frac{A_{11}(1)}{A_{12}(1)}\sqrt{Var(\varepsilon_{1t})} & \sqrt{2\frac{A_{11}(1)}{A_{12}(1)}Cov(\varepsilon_{1t},\varepsilon_{2t})+\left(\frac{A_{11}(1)}{A_{12}(1)}\right)^2 Var(\varepsilon_{1t})+Var(\varepsilon_{2t})} \end{pmatrix} \qquad (6.11)$$

6.2 Derivation of the $A(L)$ lag-polynomial

$$
\begin{gathered}
A(L) = I - CL - CL^2 - CL^3 - ... = (I - CL)^{-1} \Leftrightarrow \\
\Leftrightarrow (I - CL)A(L) = I \Leftrightarrow \\
\Leftrightarrow (I - CL)(I + A_1 L + A_2 L^2 + A_3 L^3 + ...) = I \Leftrightarrow \\
\Leftrightarrow (I - CL) + (A_1 L - CA_1 L^2) + (A_2 L^2 - A_2 CL^3) + ... = I \Leftrightarrow \\
\Leftrightarrow I + (A_1 - C)L + (A_2 - CA_1)L^2 + (A_3 - A_2 C)L^3 + ... = I \\
I = I \Leftrightarrow A_1 - C = 0 \bigwedge A_2 - CA_1 = 0 \bigwedge A_3 - CA_2 = 0 \bigwedge ...
\end{gathered}
\tag{6.12}
$$

Since C is consistently estimatable, the lag-polynomial of the VMA can be recovered by

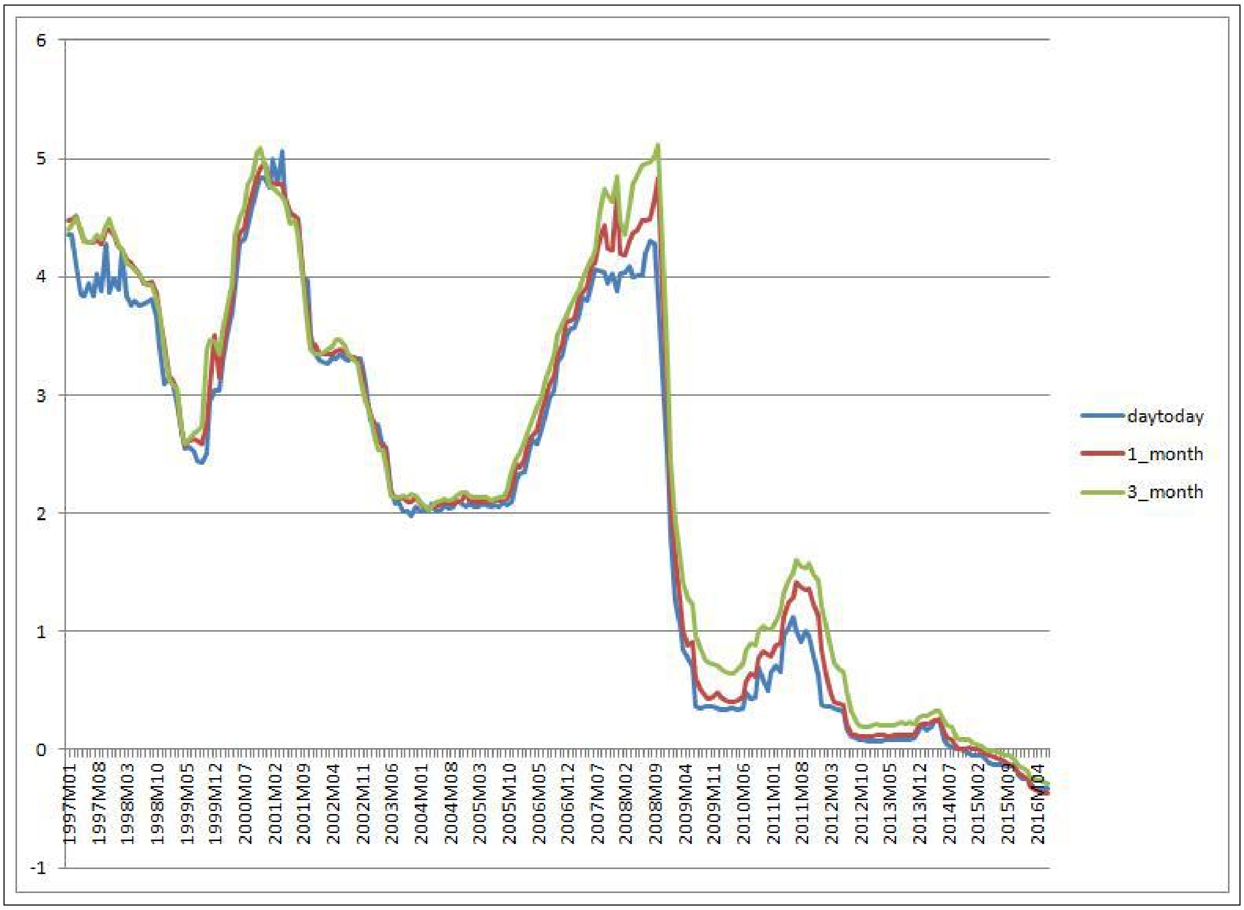
$$
A_i(L) = C^i \tag{6.13}
$$

6.3 SVAR results and graphs

6.3.1 Nominal and real interest rates for three horizons and three inflation expectation

a) Nominal interest rate data

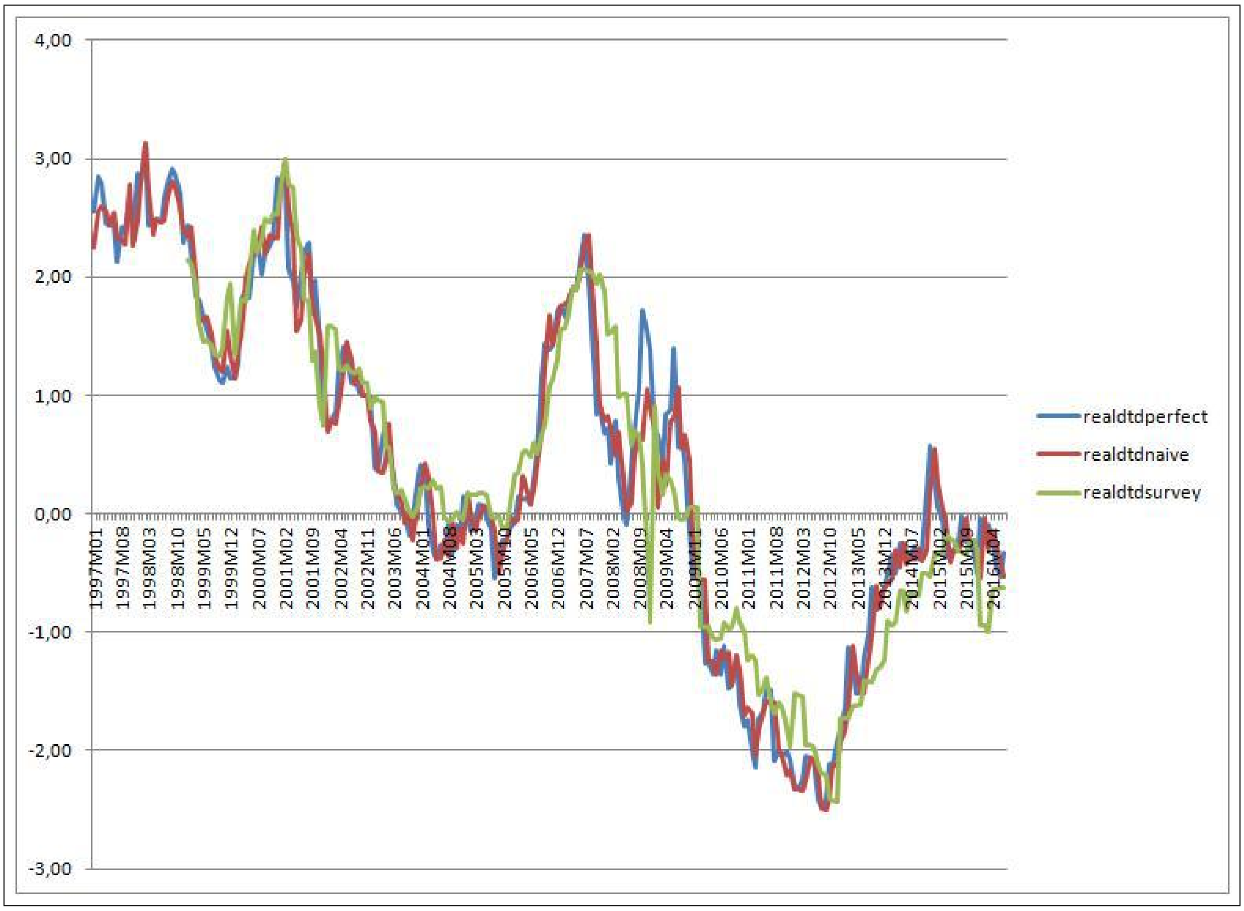
Figure 6-1: **Nominal interest rate data**



Nominal interest rates from 97M1 to 16M4. Blue: Day-to-day nominal interest rate data; Red: 1-month nominal interest rate data; Green: 3-month interest rate data.

b) Real day-to-day interest rate data

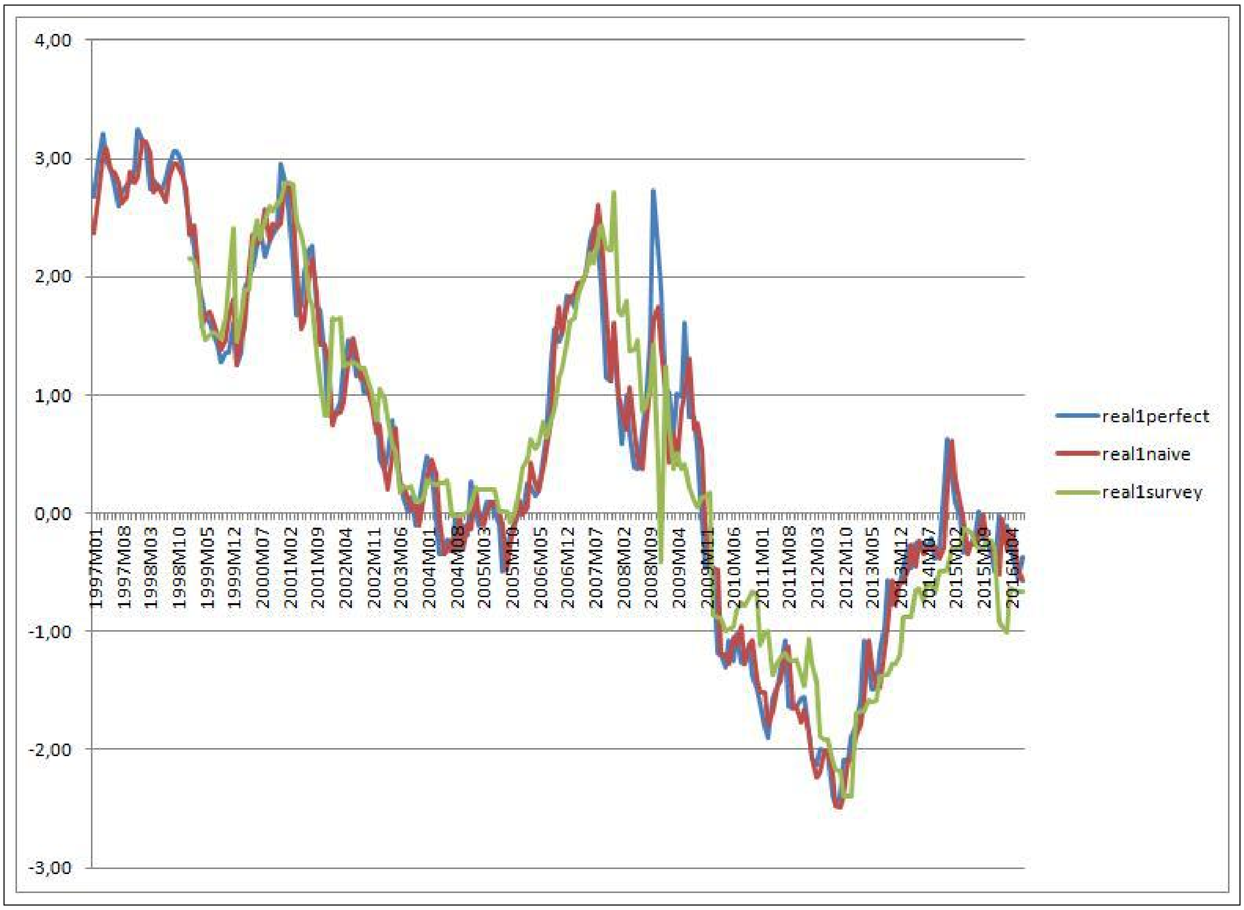
Figure 6-2: **Deflated day-to-day interest rate data**



Real day-to-day interest rate from 97M1 to 16M4. Blue: deflated according to the perfect case; Red: deflated according to the naive case; deflated according to the survey case.

c) Real 1-month interest rate data

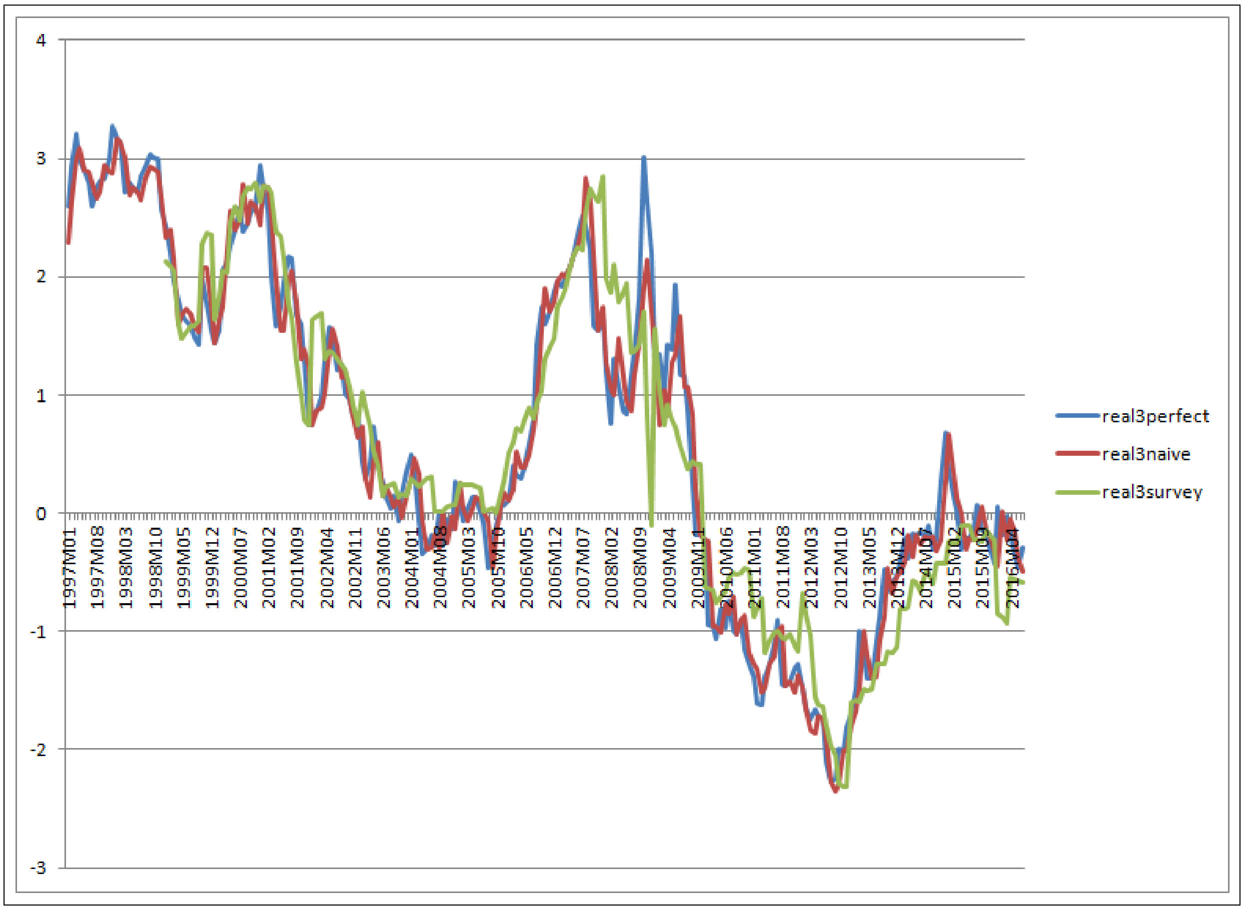
Figure 6-3: **Deflated 1-month interest rate data**



Real 1-month interest rates from 97M1 to 16M4. Blue: deflated according to the perfect case; Red: deflated according to the naive case; deflated according to the survey case.

d) Real 3-month interest rate data

Figure 6-4: **Deflated 3-month interest rate data**



Real 3-month interest rates from 97M1 to 16M4. Blue: deflated according to the perfect case; Red: deflated according to the naive case; deflated according to the survey case.

6.3.2 Chow test results for ε_{1t}

Table 6.1: Chow test results for SVAR model

dtd naive	dtd perfect	dtd survey	1 naive	1 perfect	1 survey	3 naive	3 perfect	3 survey
9.07	20.16	19.52	8.79	19.66	21.44	9.75	19.67	22.07

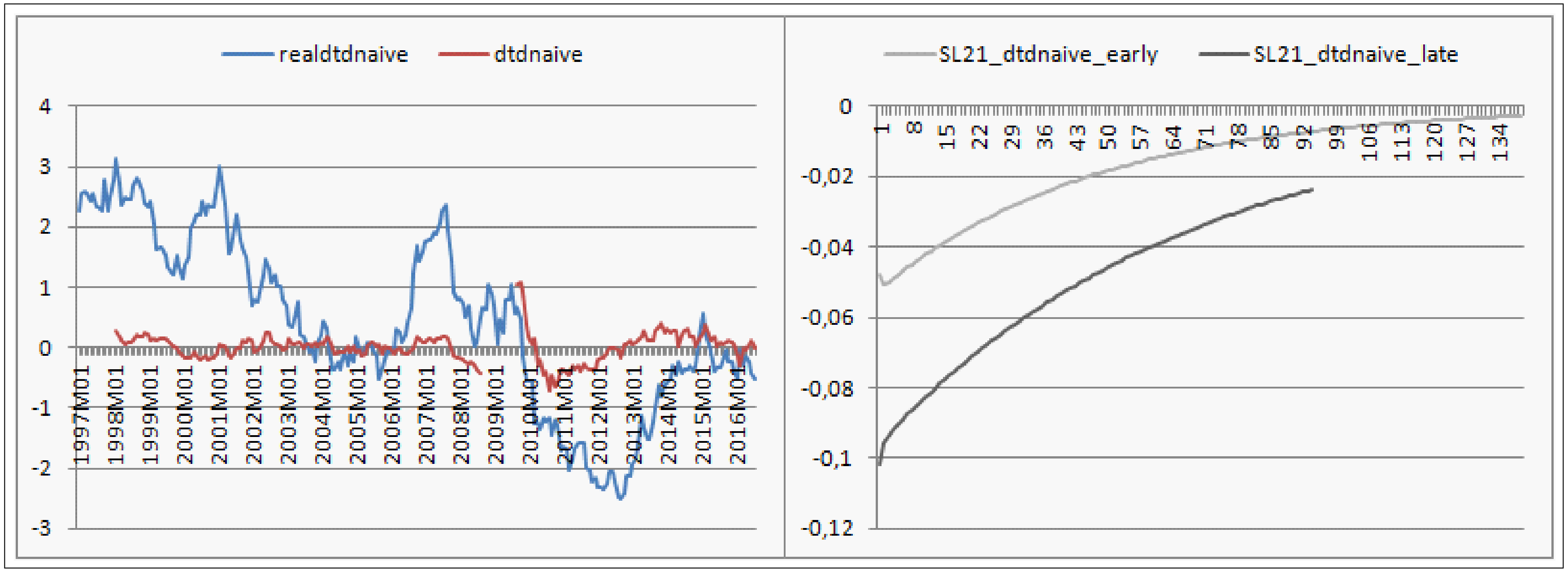
Critical values: $\chi^2_4(10\%) = 7.779$, $\chi^2_4(5\%) = 9.488$, $\chi^2_4(1\%) = 11.143$

Tested break date is September 2008, the month of the Lehman Brothers bankruptcy.

6.3.3 Results for day-to-day interest rate data

a) Day-to-day interest data deflated according to the naive case

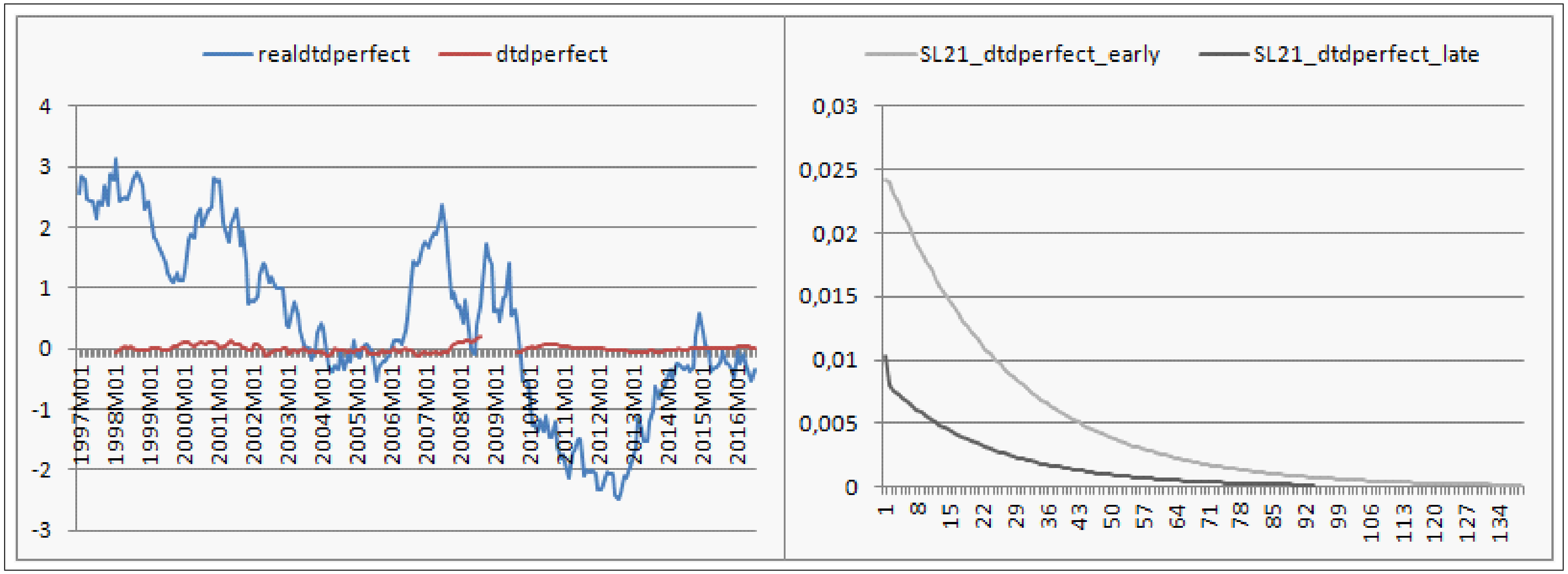
Figure 6-5: **Results: Day-to-day interest rates naive case**



LHS: Real interest (blue) data for day-to-day interest rates deflated according to the naive case and its corresponding NRI (red). RHS: IRF before (light grey) and after (dark grey) the crisis break in Sept. 2008.

b) Day-to-day interest data deflated according to the perfect case

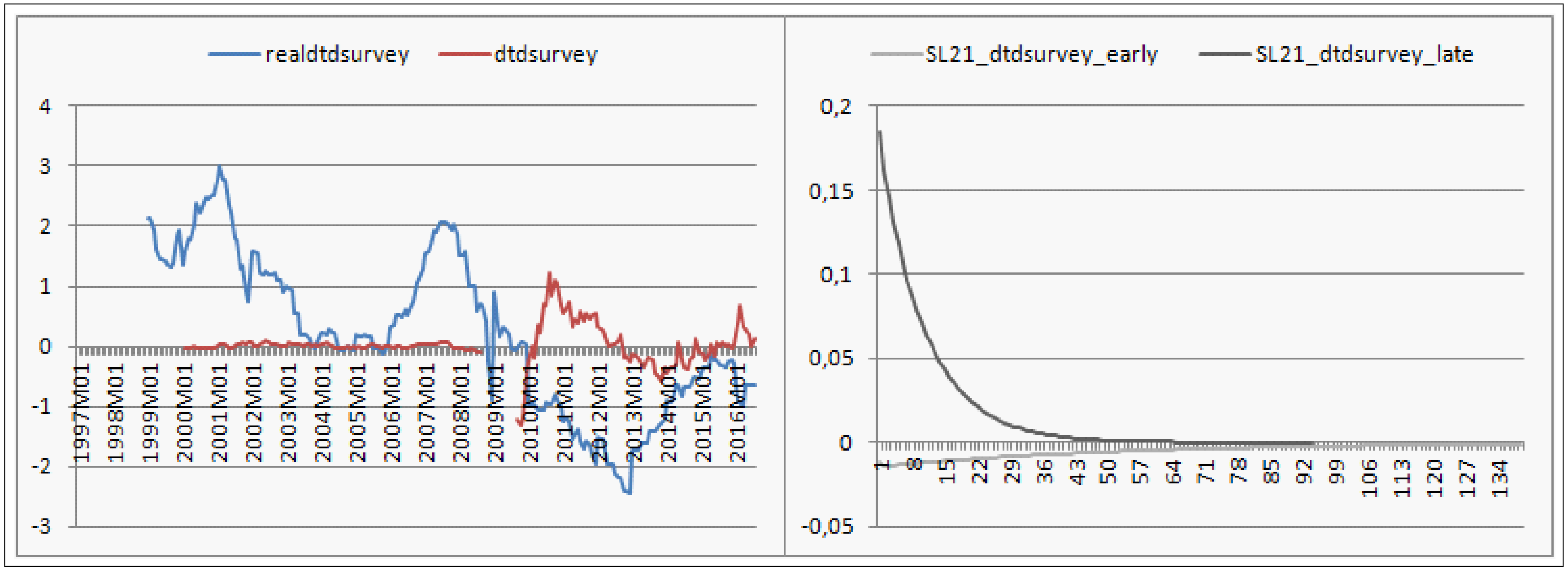
Figure 6-6: **Results: Day-to-day interest rates perfect case**



LHS: Real interest (blue) data for day-to-day interest rates deflated according to the perfect case and its corresponding NRI (red). RHS: IRF before (light grey) and after (dark grey) the crisis break in Sept. 2008.

c) Day-to-day interest data deflated according to the survey case

Figure 6-7: **Results: Day-to-day interest rates survey case**

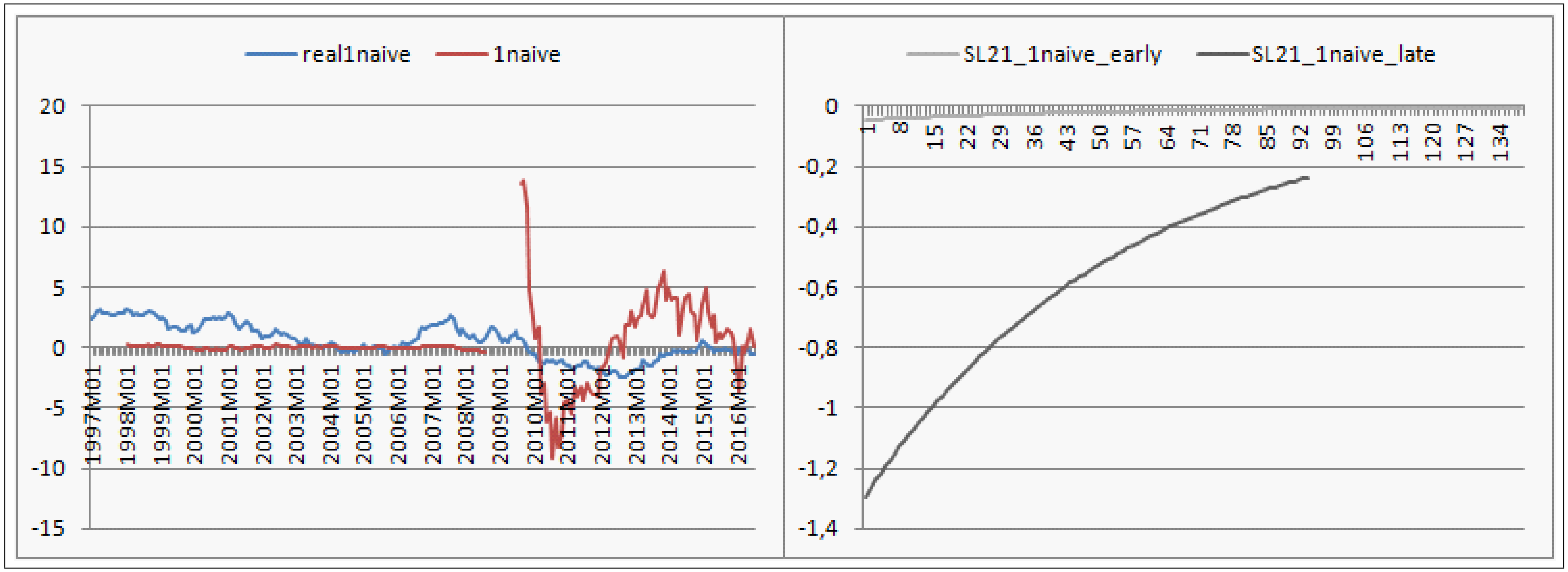


LHS: Real interest (blue) data for day-to-day interest rates deflated according to the survey case and its corresponding NRI (red). RHS: IRF before (light grey) and after (dark grey) the crisis break in Sept. 2008.

6.3.4 Results for 1-month interest rate data

a) 1-month interest data deflated according to the naive case

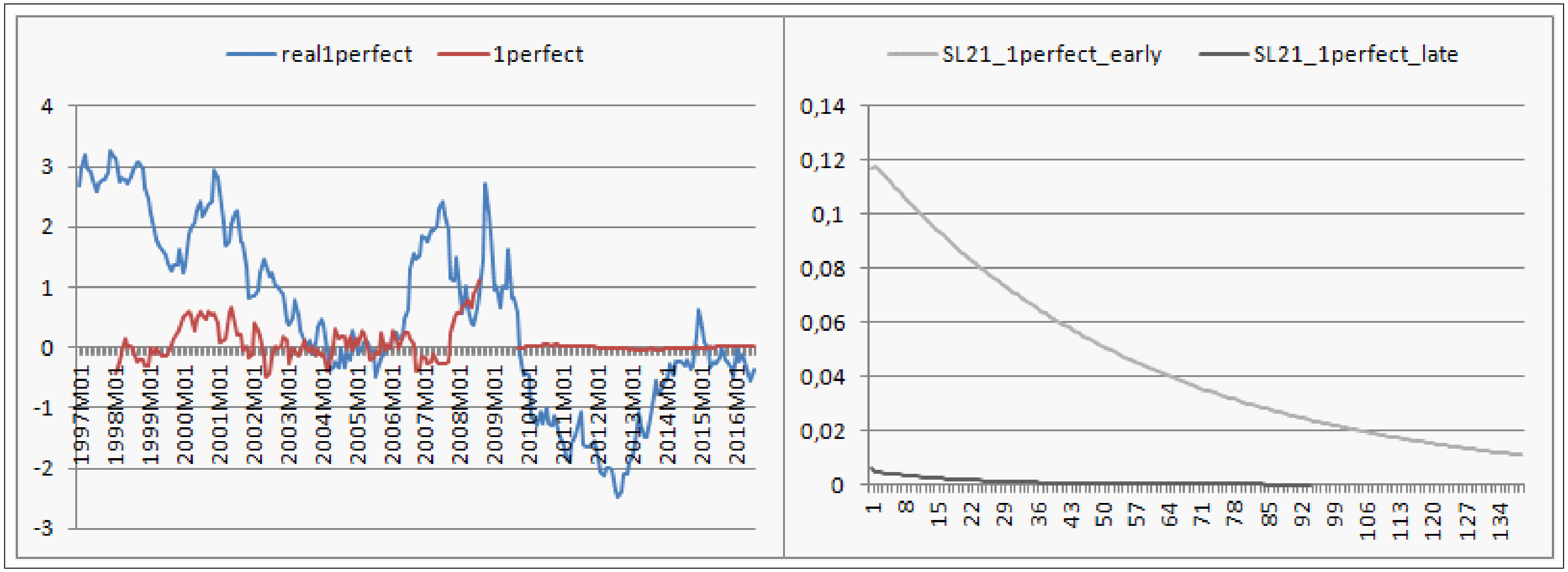
Figure 6-8: **Results: 1-month interest rates naive case**



LHS: Real interest (blue) data for 1-month interest rates deflated according to the naive case and its corresponding NRI (red). RHS: IRF before (light grey) and after (dark grey) the crisis break in Sept. 2008.

b) 1-month interest data deflated according to the perfect case

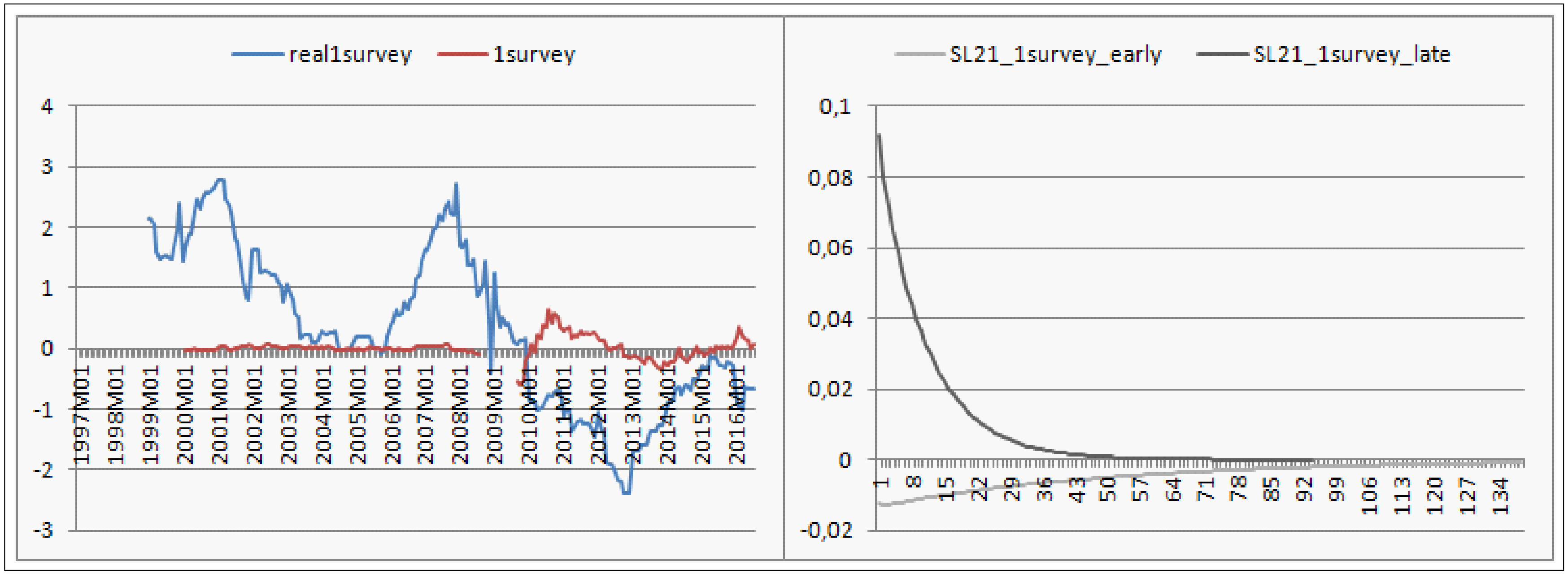
Figure 6-9: **Results: 1-month interest rates perfect case**



LHS: Real interest (blue) data for 1-month interest rates deflated according to the perfect case and its corresponding NRI (red). RHS: IRF before (light grey) and after (dark grey) the crisis break in Sept. 2008.

c) 1-month interest data deflated according to the survey case

Figure 6-10: **Results: 1-month interest rates survey case**

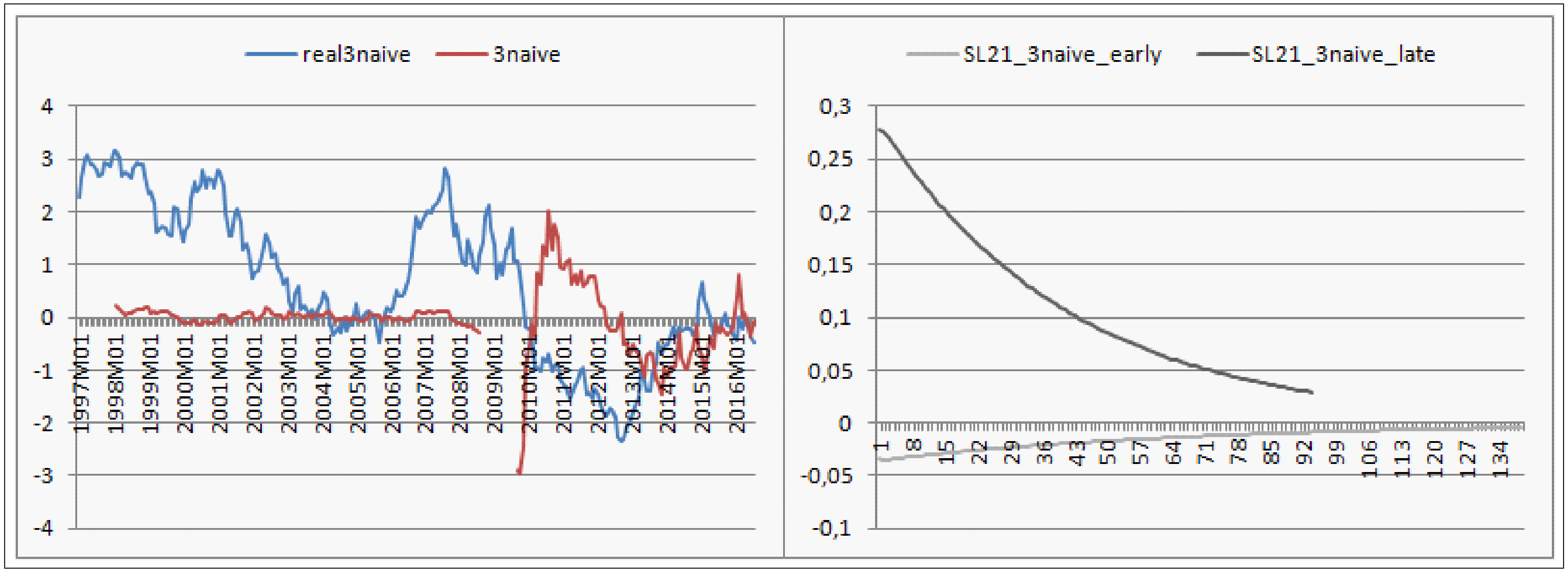


LHS: Real interest (blue) data for 1-month interest rates deflated according to the survey case and its corresponding NRI (red). RHS: IRF before (light grey) and after (dark grey) the crisis break in Sept. 2008.

6.3.5 Results for 3-month interest rate data

a) 3-month interest data deflated according to the naive case

Figure 6-11: **Results: 3-month interest rates naive case**

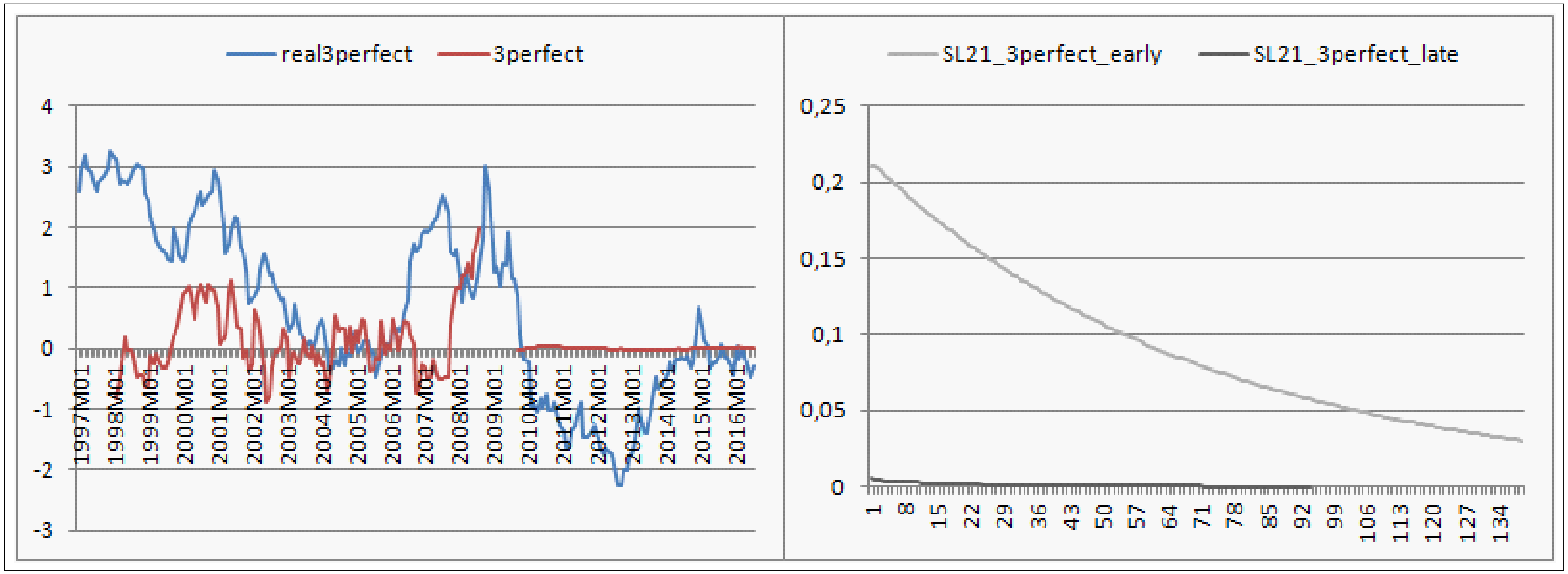


LHS: Real interest (blue) data for 3-month interest rates deflated according to the naive case and its corresponding NRI (red).

RHS: IRF before (light grey) and after (dark grey) the crisis break in Sept. 2008.

b) 3-month interest data deflated according to the perfect case

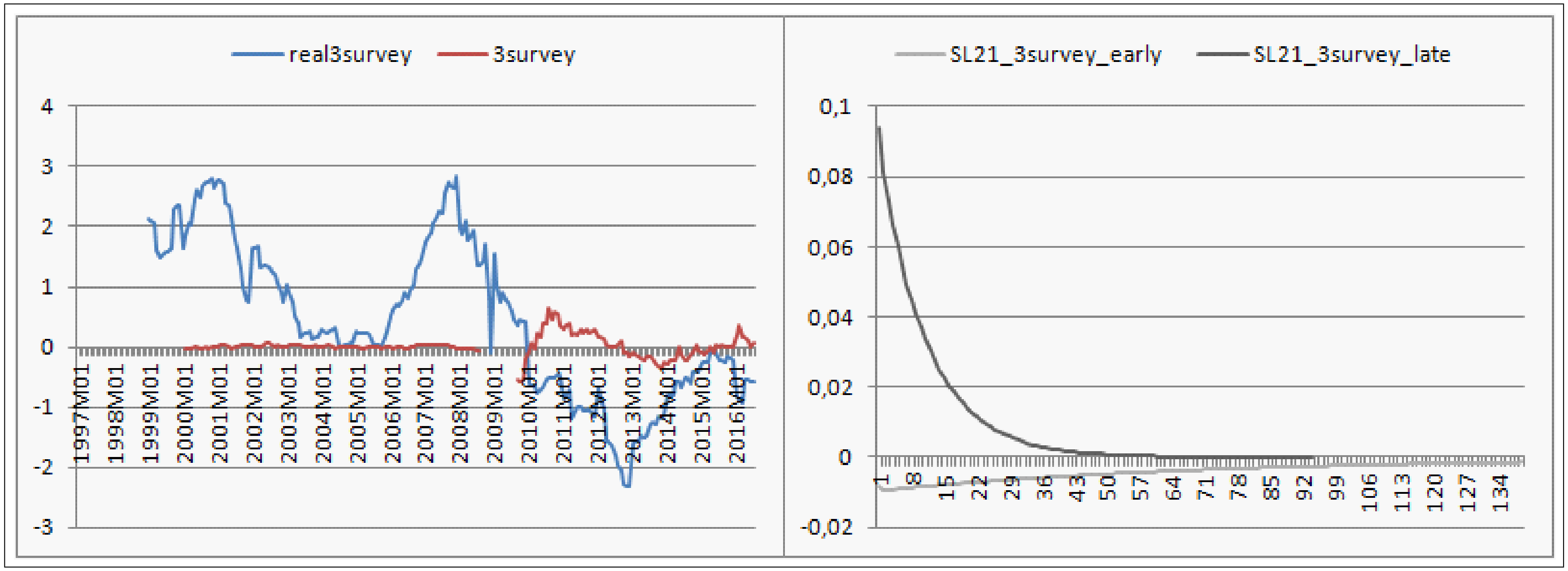
Figure 6-12: **Results: 3-month interest rates perfect case**



LHS: Real interest (blue) data for 3-month interest rates deflated according to the perfect case and its corresponding NRI (red). RHS: IRF before (light grey) and after (dark grey) the crisis break in Sept. 2008.

c) 3-month interest data deflated according to the survey case

Figure 6-13: **Results: 3-month interest rates survey case**



LHS: Real interest (blue) data for 3-month interest rates deflated according to the survey case and its corresponding NRI (red). RHS: IRF before (light grey) and after (dark grey) the crisis break in Sept. 2008.

6.4 Macroeconomic model state space representation

$$\begin{aligned} X_t &= aX_{t-1} + z + Fv_t \\ Y_t &= cX_t + r\hat{Y}_t + w_t \end{aligned}$$

$$\begin{aligned}
X_t &= \begin{bmatrix} y_t^* & y_{t-1}^* & g_t & g_{t-1} & g_{t-2} \end{bmatrix}^\tau \\
z &= \begin{bmatrix} \mu_y & 0 & 0 & 0 & 0 \end{bmatrix}^\tau \\
v_t &= \begin{bmatrix} \varepsilon_{3t} & \varepsilon_{5t} \end{bmatrix}^\tau \\
Y_t &= \begin{bmatrix} \Delta y_t & \pi_t \end{bmatrix}^\tau \\
\hat{Y}_t &= \begin{bmatrix} y_{t-1} & r_{t-1} & r_{t-2} & \pi_{t-1} & \pi_{t-2} & \pi_{t-3} & imp_t & liq_t \end{bmatrix}^\tau \\
w_t &= \begin{bmatrix} \varepsilon_{1t} & \varepsilon_{2t} \end{bmatrix}^\tau
\end{aligned}$$

$$\begin{aligned}
F &= \begin{bmatrix} \theta_y & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}^\tau \\
a &= \begin{bmatrix} 1 & 0 & \theta_y & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \\
c &= \begin{bmatrix} 1 & -\alpha_1 & 0 & -\lambda\theta_r & -\lambda\theta_r \\ 0 & \beta_4 & 0 & 0 & 0 \end{bmatrix} \\
r &= \begin{bmatrix} \alpha_1 & \lambda & \lambda & 0 & 0 & 0 & 0 & D_2\lambda \\ \beta_4 & 0 & 0 & \beta_1 & \beta_2 & \beta_3 & \beta_5 & 0 \end{bmatrix}
\end{aligned}$$

6.5 List of variable names and meanings

Table 6.2: Variable names and meanings

Name	Meaning	Name	Meaning
y_t^*	potential GDP	y_t	real GDP
$\tilde{y}_t$	potential - real GDP	Δy_t	differenced real GDP
g_t	long term growth	r_t	real interest rate
imp_t	import prices	π_t	inflation
liq_t	liquidity reserves		

6.6 Macroeconomic model results and graphs

Table 6.3: Macroeconomic model imposed values

variable	imposed values
$\lambda_\theta = \frac{\theta_r}{\theta_y}$	16.0
$\lambda_g = \frac{\sigma_5}{\sigma_3}$	0.5
θ_y	0.06
β_3	$1 - \beta_1 - \beta_2$
μ_r	3.2

6.6.1 Autoregressive order of π_t for computing $\pi_t^e = \hat{\pi}_t$

Table 6.4: Autoregressive order of inflation for computing expected inflation

	$AR(5)$	$AR(4)$	$AR(3)$	$AR(2)$	$AR(1)$
AIC	-8559.169	-8577.747	-8587.985	-8529.072	-854.431
BIC	-8379.665	-8423.886	-8459.768	-8426.498	-846.738
DW	1.939461	1.947704	1.884083	1.983986	2.016162

Both AIC and BIC select $AR(3)$.

6.6.2 Coefficients without liquidity variable

Table 6.5: Coefficients of the macroeconomic model without a liquidity shock

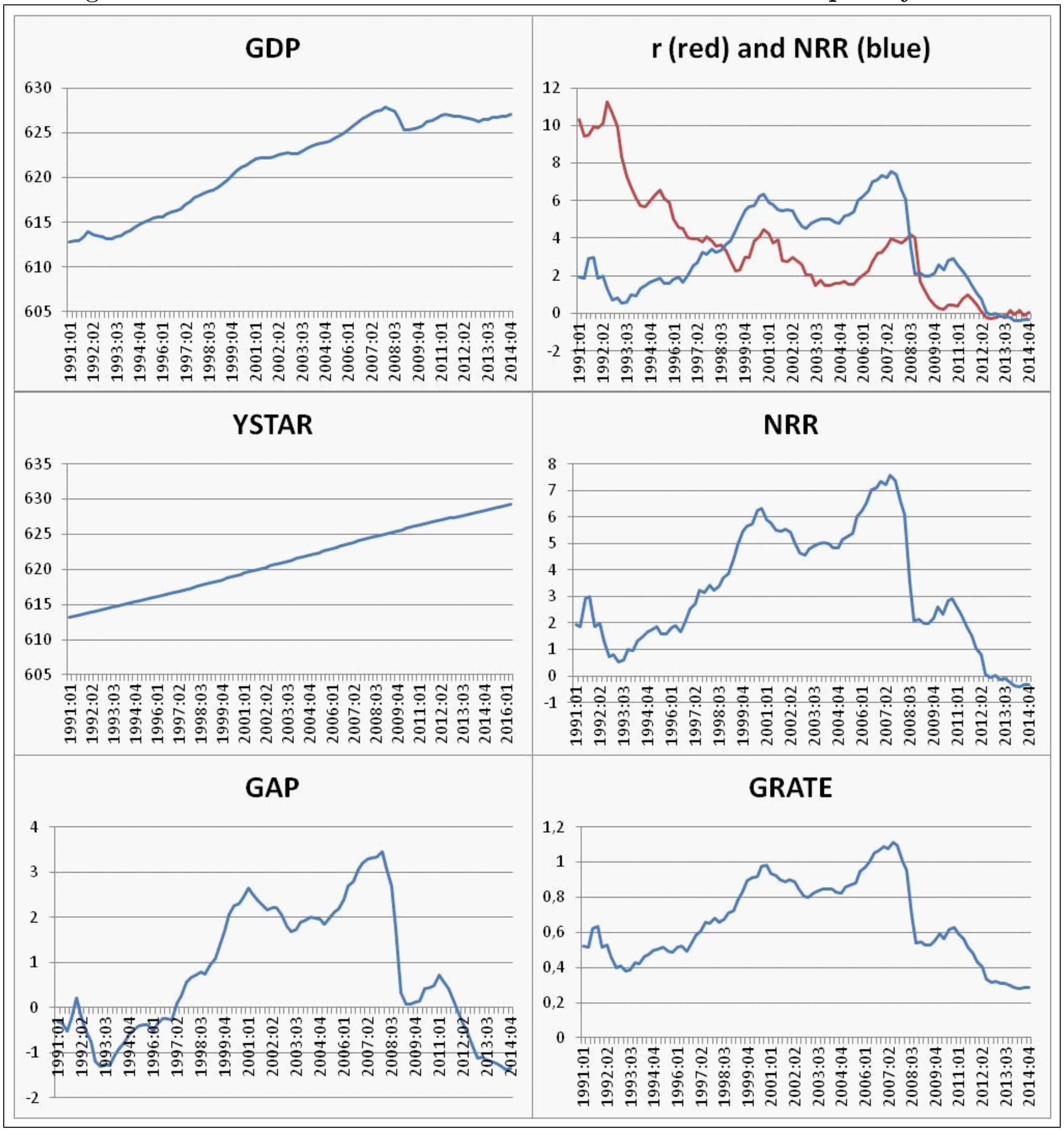
α_1	0.000322901	β_5	-0.051924716^{*}
λ	-3.196897412^{*}	μ_y	0.120989313
β_4	-0.000020155^{**}	σ_1	0.048064540^{**}
β_1	0.636977665^{**}	σ_2	0.002710152^{**}
β_2	-0.040964385	σ_3	0.038469690^{*}

* indicates a 5% significance level;

** indicates a 1% significance level; no * indicates insignificance

6.6.3 Variables without liquidity shock

Figure 6-14: **Results: macroeconomic model without a liquidity shock**

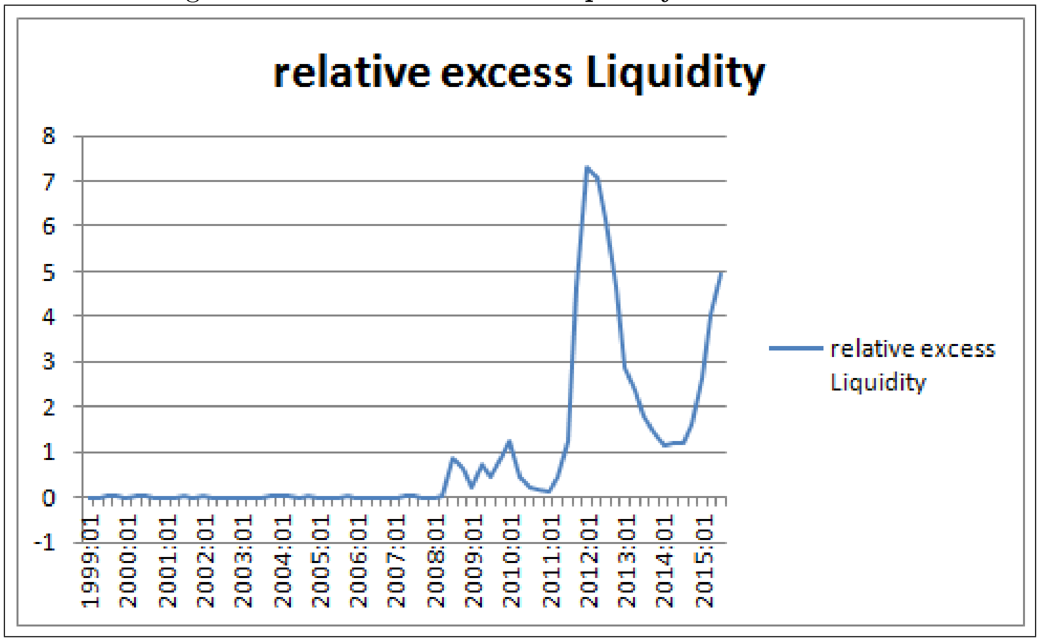


Left: Top: GDP; Middle: Potential output; Bottom: Output Gap.

Right: Top: Real (red) and natural (blue) rates of interest; Middle: Natural real rate; Bottom: long term growth rate.

6.6.4 Liquidity reserve data

Figure 6-15: **Relative excess liquidity reserves data**



Excess liquidity relative to minimum liquidity reserve requirements; Note: These data are not interacted with any dummy variable

6.6.5 Coefficients with liquidity variable

Table 6.6: Coefficients of the macroeconomic model with a liquidity shock

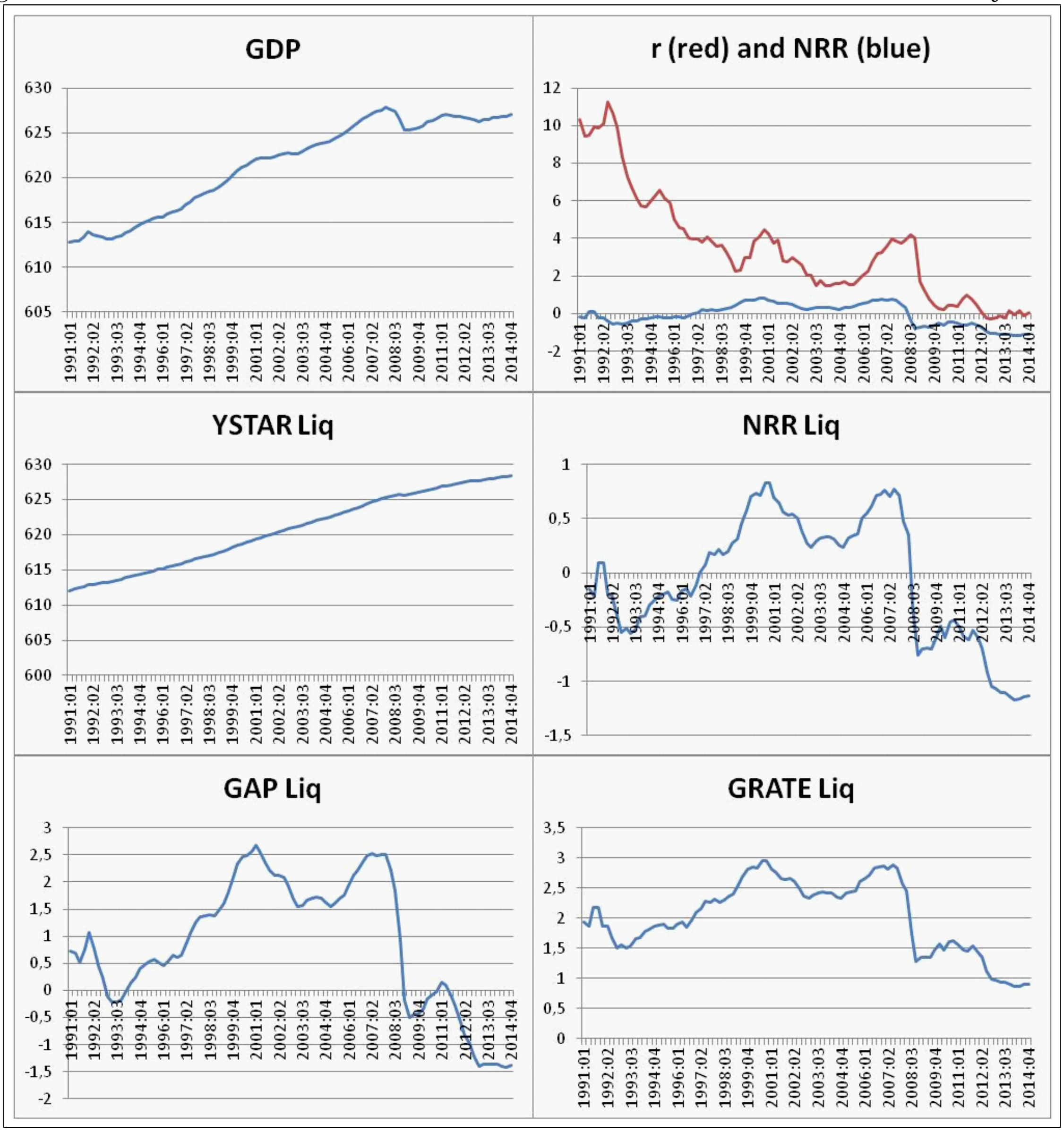
α_1	0.000152049	β_5	-0.051747920^{*}
λ	-1.018168532^{**}	μ_y	0.050557047
β_4	-0.000020190^{**}	σ_1	0.000000430
β_1	0.637358650^{**}	σ_2	0.002710519^{**}
β_2	-0.040963320	σ_3	0.110098935^{**}

* indicates a 5% significance level;

** indicates a 1% significance level; no * indicates insignificance

6.6.6 Variables with liquidity shock

Figure 6-16: **Results: macroeconomic model estimation with the uncertainty shock**



Left: Top: GDP; Middle: Potential output; Bottom: Output Gap.

Right: Top: Real (red) and natural (blue) rates of interest; Middle: Natural real rate; Bottom: long term growth rate.

6.7 Abstract

6.7.1 English Abstract

This paper compares two models for estimating the natural rate of interest (NRI), analyses the impact of an uncertainty proxy on the results of one of the models, and compares the main results with those of the literature on which the models are based. The NRI is an indicator for the monetary policy stance of a central bank. It is a latent variable that needs to be estimated, in general dependent on the long term economic growth rate and inflation growth. The NRI plays an essential role in the discussion on the optimal monetary policy, and is therefore an important indicator for policy makers. Complexity and reliability of the used models are therefore important to assess. Two models were used in this paper for the estimation of the NRI: a structural vector autoregressive model (SVAR), based on Brzoza-Brzezina (2003), and a small closed form macroeconomic model, based on Laubach & Williams (2003) and Méssonier & Renne (2004).

The SVAR model is less complex, as it only relies on the real interest rate and inflation growth, and its method for estimation is more straightforward. The results however turn out to be very unreliable, considering their lack of robustness *vis-à-vis* different, but similar, definitions of real interest rates. In general, the SVAR model results show a restrictive monetary policy stance before and an expansionary monetary policy stance after the crisis. Using the intuition of the SVAR model in an estimation which applies a different method also yields unreliable results, hinting towards a problem with the lack of fiscal variables in the model, rather than with the estimation method.

The macro economic model is more complex, but more reliable. This model is also explored more thoroughly in the literature. I found strong hints towards secular stagnation for the periods after the financial crisis in 2008 for the Euro area. Specifically, the output gap and the NRI are negative, weakening the effectiveness of monetary policy. The novelty of my approach is the introduction of an uncertainty shock, proxied by relative excess liquidity reserve data of business banks, which has a major impact on the results. That the NRI is overestimated when not controlling for the uncertainty in the economy after the crisis finds support in my results.

6.7.2 Deutscher Abstract

Diese Arbeit vergleicht zwei Modelle zur Schätzung der NRI (natürliche Zinsrate), analysiert die Auswirkungen eines Unsicherheitsschocks auf die Resultate und vergleicht diese mit der zugrunde liegenden Literatur. Die NRI ist ein Indikator für die Monetärpolitik einer Zentralbank. Sie ist eine latente Variable, die geschätzt werden muss. Grundsätzlich ist sie vom langfristi-gen Wirtschaftswachstum und Inflationswachstum abhängig. Die NRI spielt eine Schlüsselrolle in der Diskussion der optimalen Monetärpolitik und ist daher auch für Entscheidungsträger relevant. Komplexität und Zuverlässigkeit der verwendeten Modelle sind daher wichtig zu analysieren. Zwei Modelle zur Schätzung werden in dieser Arbeit angewandt, nämlich ein struk-turelles autoregressives Vektorenmodell (SVAR), basierend auf Brzoza-Brzezina (2003) und ein geschlossenes makroökonomisches Modell, basierend auf Laubach & Williams (2003) und Méssonier & Renne (2004).

Das SVAR Modell ist weniger komplex, da es die NRI nur vom Inflationswachstum abhängig macht und die Schätzmethode einfacher ist. Die Resultate sind jedoch unzuverlässig, wenn man ähnliche aber unterschiedliche Realzinsdaten zur Schätzung heranzieht. Grundsätzlich zeigen die Resultate des SVAR Modells eine restriktive Monetärpolitik vor, und eine expansive, nach der Krise. Die Ergebnisse einer Schätzung aufgrund derselben ökonomischen Intuition des SVAR Modells, aber unter Verwendung einer anderen Schätzmethode, sind ebenfalls unzuverlässig. Dies deutet auf eine Schwäche der ökonomischen Intuition, und weniger auf ein Problem mit der Schätzmethode, hin.

Das makroökonomische Modell ist komplexer, aber zuverlässiger. Außerdem ist dieses Modell in der Literatur besser erforscht. Ich habe Hinweise auf säkulare Stagnation für den Zeitraum nach der Finanz- und Wirtschaftskrise für den Euroraum gefunden. Die Outputlücke und die NRI sind negativ, was zu einer Schwächung der monetärpolitischen Instrumente führt. Die Innovation in meiner Herangehensweise ist die Einführung eines Unsicherheitsschocks durch die Verwendung von überschüssigen relativen Liquiditätsbestandsdaten der Geschäftsbanken, als Proxy für Unsicherheit auf den Finanzmärkten. Die Inklusion eines solchen Schocks beeinflusst meine Resultate stark. Diese deuten auf eine Überschätzung der NRI bei Exklusion von relevanten Unsicherheitsschocks, wie während den Krisenjahren, hin.

Felix Lödl

Education

Since Sep 2013	**Universität Wien** Master in Economics; Expected date of graduation: March 2016
Sep 2012-June 2013	**Technische Universität (TU) Wien** Two semesters in the Bachelors' program of mathematics
Aug 2011-Dec 2011	**Miami University, OH, USA** Exchange semester
Sep 2009-Aug 2013	**Wirtschaftsuniversität (WU) Wien** Bachelor in Business, Economics and Social Sciences, major in Economics, completed
Sep 2003-June 2008	**Vienna Business School HAK Untere Augartenstraße**

Job experience

Jan 2014-Sep 2016	**Student representative,** Vienna, Studienvertretung VWL *voting member of the faculty conference, administrational and managerial duties as part of a the student representation*
Aug 2015-now	**Detra – Deutschkurs für Traiskirchen,** Vienna, self-employed, Founder and treasurer *Organization of lessons, lecture rooms and material, finance, fundraising and accounting, instructor*
March 2015-Jun 2015	**Student assistant at University of Vienna**, Vienna, 6 hours per week *Organization and supervision of a series of lectures by different speakers as part of the Bachelors' Curriculum in economics*
Oct 2014-Jan 2015	**Student assistant at University of Vienna,** Vienna, 6 hours per week *Weekly lectures accompanying the master course "mathematics for economists"; in depth look at a selection of topics in linear Algebra and Analysis*
Aug 2014	**BMF,** Vienna, 40 hours per week, internship *composition of summaries of economic texts and reports for department III/1 „policy analysis"*
March 2012-Aug 2013	**Wirtschaftsuniversität (WU) Wien,** Vienna, 6 hours per week IT-Tutor; *first contact for students in case of questions regarding hardware and software, IT-support for university personnel, supervision of lecture rooms, installing and maintaining IT-equipment*

June 2013-Sep 2013 and June 2010-Sep 2010	**ÖAMTC,** Vienna, 21 hours per week, internship *supervision and organisation of roadside assistance across Europe (multilingual)*
July 2011 and July 2007	**Federal Ministry of Finance Austria**, Vienna, 40 hours per week, internship *compiling meeting protocols, preparing and proofreading documents, department reorganisation*
Feb 2011-July 2011	**Caritas,** Vienna, 8 hours per week, cook *Voluntary work in a mobile soup kitchen*
Sep 2008-May 2009	**Civil service with the Vienna police department**, Vienna, 40 hours per week, crossing guard and clerk
Aug 2009	**Volksbank AG,** Vienna, 40 hours per week, internship *assistance to sales personnel, customer care*

IT skills

MS office	Very good knowledge of Word, Excel and PowerPoint, Basic knowledge of Access
Statistical software	Good knowledge of EViews Basic knowledge of R, SPSS, Stata
Programming	Basic Programming knowledge in Scientific WorkPlace, RATS, C and Stata

Languages

German	Native
English	Excellent
French	Basic
Italian	Basic

Awards and scholarships

Okt 2014	Merit scholarship for excellent academic achievements (from Universität Wien) for the study years 2013/14
Aug 2011	Scholarship from the joint study program for overseas exchange semesters
Jan 2011 and Jan 2010	Merit scholarship for excellent academic achievements (from Wirtschaftsuniversität Wien) for the study years 2010/11 and 2009/2010
July 2011	Ranked among the top 5% of students of the WU Wien
Jan 2007	Winner of the national “Schule macht Wirtschaft”-competition of Austrian commercial academies

Vienna, 14 November 2016

Felix Lödl