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Abstract

This work provides a review on measures for visual attractiveness of routes in the Vehicle Routing Problem and proposes four modelling approaches that aim at improving of the visual beauty of routing plans. The traditional Vehicle Routing Problem with Time Windows is considered and expanded by additional constraints. The first model includes soft constraints similar to what is found in the p-median problem, also known as lollipop routes. In the second model constraints forbidding crossings are added and the third model penalises the use of the arcs that cross each other. The last model tries to enhance the visual appeal of routes by minimising the bending energy. Computational experiments on Solomon's instances are conducted in order to compare the performance of the proposed models with respect to traditional and non traditional quality measures. Moreover, a qualitative survey is carried out in order to involve the subjective opinion of people working in the field of logistics regarding aesthetic aspects of the routing plans.

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Contents

List of Figures	VI
List of Tables	VI
1 Introduction	1
1.1 Problem Description	1
1.2 Motivation	1
2 Literature Review	4
3 Mathematical models	15
3.1 Model 1: Traditional formulation	17
3.2 Model 2: Pretty routes as lollipop constraints	19
3.3 Model 3: Pretty routes as crossing arcs hard constraints	19
3.4 Model 4: Pretty routes as crossing arcs soft constraints	19
3.5 Model 5: Pretty routes as bending energy constraints	20
3.5.1 Two-index bending energy	20
3.5.2 Three-index bending energy	21
4 Results	21
4.1 Test instances	21
4.2 Computational results	22
5 Results of the qualitative survey	40
5.1 Question 1: "Which of the solutions looks nicer (more visually attractive) to you?"	40
5.2 Question 2: "Which of the solutions would you implement in practice and why?"	41
6 Conclusion	43
Bibliography	44
A Deutsche Zusammenfassung	47
B Curriculum Vitae	48

List of Figures

1	Instance 20-RC101: solution of the basic VRPTW model with four routes (left) and solution for the model with crossings constraints with six routes (right).	30
2	Instance 25-R201: solution of the basic VRPTW model with five crossings (left) and solution for the model with crossing soft constraints with three crossings and the same number of tours (right). .	31
3	Instance 25-R201: solution of the crossing arcs hard constraints model with six routes (left) and solution for the crossing arcs soft constraints model with four routes and three crossings (right). . . .	32
4	Instance 20-R101: solution of the crossing arcs hard constraints model with nine routes (left) and solution for the crossing arcs soft constraints model with eight routes and one crossing (right). . . .	33
5	Instance 20-C201: solution of the basic VRPTW model with two routes (left) and solution for the model with lollipop constraints with four routes (right).	34
6	Instance 20-R201: solution of the basic VRPTW model with three routes (left) and solution for the model with bending energy constraints with four routes (right).	35
7	Instance 20-RC201: solution of the model with bending energy $\alpha = 1$ with eight crossings (left) and solution for the model with bending energy $\alpha = 2$ with eleven crossings (right).	36
8	Instance 20-R101: solution of the model with lollipop constraints $\lambda = 1$ with no back and forth routes (left) and solution for the model with lollipop constraints $\lambda = 2$ with four back and forth routes (right).	37
9	Instance 50-R101: solution of the crossing arcs soft constraints model $\mu = 1$ (left) and solution for the crossing arcs soft constraints model $\mu = 2$ (right).	38
10	Instance 20-R105: solution of the basic VRPTW model with one crossing (left) and solution for the model with crossings constraints (right).	39

List of Tables

1	Measures for visual attractiveness	14
2	Results for C1	27
3	Results for C2	28
4	Results for R1	28

5	Results for R2	29
6	Results for RC2	29
7	Important aspects of a routing plan	42

1 Introduction

1.1 Problem Description

The Vehicle Routing Problem (VRP), which was introduced by Dantzig and Ramser [1959], is an important problem that aggregates fields of transportation, distribution, and logistics and affects a wide range of enterprises from many business sectors. Together with its different variants, the VRP has been the concern of study of many researchers from all over the world.

In its simplest form, the VRP refers to the problems of delivery or collection of goods between depots and customers and involves the design of a set of minimum-cost routes, starting and ending at a depot, for a vehicle fleet that services a set of customers with given demands. The basic version of the VRP is the Capacitated Vehicle Routing Problem (CVRP), where the capacity restrictions for vehicles are imposed.

The vehicle routing problem with time windows (VRPTW) is an extension of CVRP which addresses the issue of allowable delivery times, called time windows. Soft time windows can be violated at a penalty, while hard time windows do not permit to arrive at a customer after the latest time to begin the service (Toth and Vigo [2002]). Time windows are very common in the practice since many business organisations work on fixed time schedules and impose delivery deadlines (Solomon [1987]). In the presence of these additional constraints, the problem becomes even more complex than the VRP, which is known to be NP-hard.

1.2 Motivation

Although numerous papers have addressed a welter of VRP extensions, it comes out that only a few of them have explicitly taken into account visual attractiveness of the solution routes. However, the shape of routes becomes of great significance

when it comes to practical applications of the VRP. Kim et al. [2006] argue that it is not practical to have overlapping routes in real-world applications. From a practical perspective, overlapping routes may cause difficulty to manage the operations in practice. Crossing avoidance is a common logic behind human dispatcher while creating routes. MacGregor and Ormerod [1996] observe that people can identify the convex hull of a set of points and that they build tours by connecting points on the convex hull in the order of adjacency. Moreover, Van Rooij et al. [2003] try to justify the same phenomenon with the hypothesis that people achieve this by trying to avoid crossings. Usually, managers and operational planners tend to adopt solutions that are intuitively acceptable, i.e. plans with reasonable shapes of the tours. They prefer routes that are compatible with the historical dispatch system (before the implementation of a vehicle routing software) which they are familiar with. Companies are willing to implement algorithms that represent a progressive transition from their prior business practice. This has been observed and confirmed by researchers who have implemented commercial routing software for industry (ex. Savelsbergh and Sol [1995], Shao et al. [2005], Kim et al. [2006]). According to Poot et al. [2002], visually attractive plans seem to be more logical for the decision makers in a firm and contribute to creation of trust and acceptance among planners and drivers, which leads to faster acceptance of the system. Additionally, Tang and Miller-Hooks [2006] stress that the assignment of drivers to relatively small areas improves the familiarity with the road network and consequently the efficiency of operations, because drivers have regional knowledge and relationships. Furthermore, according to Kant et al. [2008], customers require a limited number of drivers visiting them. Hasle and Bräysy [2014] remark that, in the context of newspaper delivery, it is undesirable to have intersections of service areas because of the risk that neighbouring subscribers can receive their newspaper at different times. Additionally, the authors point out that the visual appeal

of the routes is an important factor in decision whether a specific routing plan will be accepted or rejected. Another reason for the visually appealing routes is the fact, that "pretty" solutions are more robust operationally (Tang and Miller-Hooks [2006]).

The term "visual attractiveness" of the routes can first be found in the paper of Poot et al. [2002]. It belongs to the, so called, non-standard solution quality measures, as opposed to traditional quality measures such as total distance travelled, total number of vehicles used and total transportation costs. A set of tours is said to be visually attractive when it fulfils the following characteristics: each of the tours is compact and tours do not cross each other.

Constantino et al. [2015] assert that it may be not straightforward to measure and describe the attractiveness of the routes in a mathematical way. Therefore, aesthetic aspects of routing plans are often treated as soft constraints. Visually attractive plans may perform worse than other plans with respect to the traditional quality measures, but there have been proofs that some methods deliver good results in terms of both, traditional and non-traditional quality measures (ex. Tang and Miller-Hooks [2006], Lu and Dessouky [2006]).

The goal of this master's thesis is to introduce proposed solution methods and measures that are dealing with the visual attractiveness of the VRP routes and to compare their performance with respect to conventional and non-conventional quality measures.

The remainder of the thesis is structured as follows. Section 2 provides a literature review on measures that have been presented to date to evaluate and increase the visual attractiveness of the routes in VRP. Subsequently, Section 3 is devoted to mathematical formulations of the basic VRPTW model together with the extended models that include the metrics for quantifying and enhancing the visual appeal of the routes. In Section 4 we present computational results of our

study of their performance. Section 5 analyses the results of a qualitative survey investigating the opinions of a number of participants. Section 6 concludes the presented work.

2 Literature Review

Although it is hard to find the research work on visual attractiveness among the literature on VRP, some authors have taken into consideration shapes and compactness of the routes and implemented solution approaches that cope with these aesthetic measures. This section reviews the literature on numerical measures for visual attractiveness and solution methodologies that are employed in order to obtain routing plans with the desired shapes of the routes.

Since the visual attractiveness of the routes is important in the real-life VRP, solution methods which consider the layout of the routes are often developed using real-life data and incorporating different non-standard client-specific constraints, which are rarely mentioned in the literature. Therefore, existing solution methods deal with extensive, somewhat unique problems and are accordingly complex. According to Hollis and Green [2012], tight time windows are the greatest hurdle to visual attractiveness. Since for many problems exact approaches can only solve relatively small sized instances, solution approaches that concern with visually attractive routes are mostly heuristics. The papers are discussed in chronological order.

As mentioned above, Poot et al. [2002] first measure quantitatively the visual attractiveness of routes. They develop a saving-based algorithm and compare obtained solutions with those received using extended sequential insertion algorithm. The authors come to the conclusion that saving-based algorithm outperforms the insertion algorithm in terms of both, traditional and non-traditional

quality measures. In their work, besides standard capacity restrictions, they consider non-standard constraints such as multiple time windows, vehicle type and region constraints. Poot et al. [2002] describe some possible parameters to evaluate the visual attractiveness of the plans (all measures should preferably have low values):

- (a) Average number of customers in a route that are closer to the centre of gravity of another route. Centre of gravity is calculated from the coordinates of the locations of the customers in the trip.
- (b) Average distance to the centre of gravity of the route.
- (c) Average distance between any two customers in the route.
- (d) Total number of crossings between routes.
- (e) Average number of crossings within a route, while crossings that occur on the way from and to the depot are not considered.
- (f) Average number of customers that are in the convex hull of another trip. The convex hull is formed by locations of all customers served in the trip and is defined as the smallest polygon that encloses all the customers in the set.

Poot et al. [2002] remark that, instead of analysing the already obtained results using non-standard quality measures, it would be interesting to incorporate these measures in the optimization function itself.

Kim et al. [2006] remark that overlapping of service areas is related to the crossings of the tours. According to the authors, the route compactness depends on the grouping of the stops into a route. Therefore, the service areas of each vehicle should be preferably concentrated in a geographical region. They develop

a two-phase clustering-based waste collection VRPTW algorithm, which explicitly considers the route compactness. First, the stops are clustered based on their locations and then are moved among clusters in order to enhance the route compactness. The effectiveness of the solution method was confirmed by computational results obtained using real-life data of a waste-management company.

In addition to the traditional VRP constraints, they consider time windows, heterogeneous fleet and lunch breaks of drivers. Furthermore, the vehicles are allowed to renew their capacity at an intermediate facility. They focus on the visual attractiveness of the solution by implementing a shape improvement scheme. The measure to quantify the visual "beauty" is the shape metric, which is defined as follows:

$$S_m = \sum_{\substack{\text{All tours } T_i \\ \text{in a solution}}} \sum_{\substack{\text{All routes } R_j \\ \text{in tour } T_i}} \sum_{\substack{\text{All stops } ST_k \\ \text{in route } R_j}} Dist(ST_k, ST_c), \quad (1)$$

where ST_c is the stop that is closest to the centroid of each route R_j when the distance measure is the street distance. The authors use in addition to shape metric another measure to evaluate the cluster overlap

$$N_h, \quad (2)$$

which counts the number of stops that belong to more than one convex hull of clusters. The measures should take lowest possible values to be more visually attractive.

Tang and Miller-Hooks [2006] define a VRP with solution shape constraints and develop an interactive heuristic which results in solutions with "significantly improved layout while maintaining satisfactory results in terms of conventional VRP measures." [Tang and Miller-Hooks, 2006]

They use real-life data of a package delivery company to test their approach. The results show the effectiveness of the heuristic in terms of conventional and non-conventional VRP measures. As a geographical reference point they propose the *median* of a tour, which is defined as:

$$M_k = \operatorname{argmin}_{\forall i \in R(k)} \left[\sum_{j \in R(k) \setminus \{i\}} d(i, j) \right], \quad (3)$$

where M_k is the median of tour k , $R(k)$ is the set of customers included in tour k , and $d(i, j)$ is the Euclidean distance between customers i and j . In contrast to the centre of gravity, which was used by Poot et al. [2002], the authors use a measure that coincides with an actual customer location.

Tang and Miller-Hooks [2006] propose two measures for the assessment of the visual attractiveness of the tours (smaller values of the measures indicate a better solution shape):

- *Measure One* is the total number of customer locations that are closer to the median of another tour than to the median of the tour in which they are included, computed by:

$$N = \left\{ i \mid d(i, M_{R_i}) > \min_{k \in F \setminus \{R_i\}} d(i, M_k) \quad \forall i \in V \setminus \{0\} \right\}, \quad (4)$$

where R_i denotes the index of the tour to which customer i belongs, F denotes the set of all tours, and $d(i, M_k)$ is the Euclidean distance between customer i and median M_k .

- *Measure Two* is the average Euclidean distance between customers and their routes medians, computed by:

$$\bar{t} = \frac{\sum_{i=1}^n d(i, M_{R_i})}{n}. \quad (5)$$

Lu and Dessouky [2006] present a non-standard measure, Crossing Length Percentage (CLP), defined as the sum of the crossed length of all the crossings divided by the total length of the tour, to quantify the visual attractiveness of the solution. The CLP evaluates the degree of crossings within a trip and is calculated as follows:

$$CLP = \sum_{i=1}^n \min(\beta_i, \lambda - \beta_i) / \lambda, \quad (6)$$

where λ is the total length of the trip and β_i is the route's length of the portion within crossing point cs_i .

The authors state that the number of crossings alone is not a proper quantity measure, since the crossing level depends on how deep the crossings are and whether the multiple crossings entangle each other. The CLP indicates the most entangled portions of a trip. Therefore, it helps in adjusting these sequences and improve the trips. "Maintaining routes with lower CLP at each iteration in an insertion heuristic can avoid the acceptance of unattractive insertions in the earlier stage of creating interim routes." [Lu and Dessouky, 2006]

Matis [2008] proposes other measurements to evaluate the quality of the solution. The route compactness is calculated as follows:

$$COMP_i = \frac{AvgDist_i}{AvgMaxDist_i}, \quad (7)$$

where $AvgDist_i$ is the average distance between two immediate customers in the route i and $AvgMaxDist_i$ is an average distance restricted to the longest 20% of the distances between two immediate customers in the route i . Further, he calculates the average number of customers in a route that are closer to the center of gravity of another route as follows:

$$DGRB_i = 2 \cdot \left(1 - \frac{|\hat{O}_i|}{|O_i|}\right), \quad (8)$$

where $|\hat{O}_i|$ is the number of customers closer to the centre of gravity of another route, and $|O_i|$ is the number of all customers in the route i . Furthermore, the visual attractiveness can be measured as:

$$VA_i = \frac{1}{\frac{NC_i}{5} + \frac{1}{DGRB_i} + \frac{1}{COMP_i} - 1}, \quad (9)$$

where NC_i represents the number of crossings in the route i . In the case of VA_i the result is a number in the range from 0 to 1 and higher value indicates better visual attractiveness; in practice, its value is usually less than 0.5.

Kant et al. [2008] describe an algorithm developed for the Coca-Cola Enterprises that deals with many non-standard constraints, such as vehicle type, time windows, drivers working hours and traffic patterns constraints. The authors do not use explicit measures to quantify the visual "beauty" of the plan. Instead, they add a *clustering function* to their algorithm in order to optimize the visual attractiveness of the solution. They define:

$$Centre\ stop(C) = \frac{n}{2}, \quad (10)$$

where n is the number of stops in the route, and compute the distance to it for each stop in the route. Subsequently, they compute for each route total penalty cost by summing up the distances and multiplying the sum by a *clustering penalty parameter* (CP), which can be tuned manually in order to obtain attractive solutions. This clustering function was incorporated into the cost function.

Hollis and Green [2012] develop a two-phase approach "for producing visually attractive and operationally robust solutions to a real-life VRP with time windows." [Hollis and Green, 2012]

It has been implemented for the daily beverage distribution problem of Schweppes Australia Pty. Ltd. The algorithm uses an augmented objective function, which aims at balancing between minimizing traditional cost measures, whilst delivering a high degree of visual attractiveness of the plan. Additionally to the measures mentioned above, the authors define:

- Centre of gravity of route r : $G(r)$ which is the average of the x and y coordinates of the customers serviced,
- The average distance between two consecutive customers in route r :

$$AD(r) = \sum_{s \in S_r^*} D(s, s^+) / (|S_r| - 1), \quad (11)$$

where S_r^* is the set of customers, excluding the last customer, serviced by route r , and s^+ is the customer serviced immediately after s ,

- The minimum distance between the unrouted customer m and any customer serviced by route r :

$$MD(m, r) = \operatorname{argmin}\{D(m, s) \mid s \in S_r\}, \quad (12)$$

- The measure for the compactness of the route r :

$$Z(r) = \sum_{s \in S_r} D(s, G(r)), \quad (13)$$

and

- The number of customers serviced by route r_1 that are in the convex hull of route r_2 :

$$H(r_1, r_2). \quad (14)$$

Gretton and Kilby [2013] investigate the effects of integration of shape penalties in solutions to the VRP using Large Neighbourhood Search. The first measure is the *compactness penalty* which is defined as the sum of distances of assigned customers to their route medians. The *median* of the route is defined as follows:

$$C_R = \operatorname{argmin}_{i \in R} V(i, R), \quad (15)$$

where i denotes a customer, R is a route and $V(i, R) = \sum_{j \in R} d(i, j)$. In addition, they investigate *bending energy*, which was first proposed by Young et al. [1974]. It is the amount of effort required to form a shape using a linear thin-shelled medium. Liu et al. [2008] propose a new definition using *visual curvature*. Following that, Gretton and Kilby [2013] define:

$$\textit{Bending energy} = \textit{sum of turn angles}. \quad (16)$$

The authors come to the conclusion that minimising the bending energy aims at identifying compact routes in early search.

Constantino et al. [2015] propose a bounded overlapping mixed capacitated arc routing problem (BCARP) and introduce a constraint to reduce the number of intersections between routes. According to them, usable routes should exhibit connectivity and compactness. "The connectivity of a region can be defined as the possibility of travelling between two points in a region without leaving it." [Constantino et al., 2015]

Moreover, compact regions should have preferably shapes which are close to a circle or a square, or the distances between points should be as low as possible. The authors obtain results that are more attractive to implement in practice because of the better shape of the solutions, which is evaluated by the following measures:

$$\text{Connectivity Index (CI)} = \frac{CC}{|routes|}, \quad (17)$$

where CC is the average number of the connected components of the set of tasks in the service zone and $|routes|$ is the number of routes in the solution. For the ideally connected solution the value of CI equals one.

$$\text{Average Task Distance (ATD)} = \frac{1}{|routes|} \sum_{p=1}^P \frac{\sum_{a,b \text{ served by } p} D_{ab}}{|taskpairs|}, \quad (18)$$

where $|taskpairs| = \frac{|tasks| * (|tasks| - |routes|)}{2 * |routes|^2}$ is the approximate average number of tasks assigned to each route and D_{ab} is the minimum time from a to b . The smaller ATD the more compact is the route.

$$\text{Routes Overlapping Index (ROI)} = \frac{NO - |N|}{(\sqrt{|routes|} + \sqrt{|N|} - 1)^2 - |N|}, \quad (19)$$

where N is the number of vertices in the graph, including the depot, $NO = \sum_{i \in N} \sum_{r=1}^{|R|} n_i^r$ is the number of routes a vertex belongs to, and $NO - |N|$ is the node overlapping of the solution and the term in the denominator represents the node overlapping of an "ideal" solution.

Lum et al. [2015] propose another metric for the route compactness that incorporates similar intuition:

$$\text{Hull Overlapp (HO)} = \frac{1}{|R|} \sum_{r_1, r_2 \in R} \frac{\text{intersec}(\text{convex}(r_1), \text{convex}(r_2))}{\text{convex}(r_1)}, \quad (20)$$

where R is the set of routes in the solution, intersec is the area of the intersection,

and *convex* is the convex hull of the points.

According to Lum et al. [2015], the metric HO captures the geographic overlap between the areas of responsibility of the drivers and results in non-overlapping and contiguous routes. The authors describe several weaknesses of the ROI metric, especially for approaches where street networks are used.

The following Table (1) summarises the measures for visual attractiveness used in the literature and lists the authors who have employed them. Additionally, a classification based on multiple criteria is given. First, we distinguish between measurements which use the *median*, the *centre of gravity*, the *customer closest to the centroid* of the trip, or the *centre stop* $C = \frac{n}{2}$ as a routes reference point. Secondly, we capture measures that take into account the *convex hull* of a route or the *turn angles*. Thirdly, we distinguish between measures that consider only one route itself (*single route*) and those which compare many routes at once (*multiple routes*). The next criterion is whether a measure is used to analyse the quality of an already obtained solution and/or to compare different algorithms (*ex-post evaluation*), or whether it has been directly incorporated into the solution approach, for example into the objective function as solution penalty in the form of a *soft constraint* or as a non-standard *hard constraint*, in order to determinate a visually attractive solution. In addition, we check which distance metric, (*Euclidean* or *street distance*) was used and, lastly, we make stand out the measures, which were used in an *exact approach*.

Authors	Measures	<i>point of reference</i>													
		<i>median</i>	<i>centre of gravity</i>	<i>customer closest to the centroid</i>	<i>$C=n/2$</i>	<i>convex hull</i>	<i>turn angles</i>	<i>single route</i>	<i>multiple routes</i>	<i>ex-post evaluation</i>	<i>soft constraints</i>	<i>hard constraints</i>	<i>Euclidean distance</i>	<i>street distance</i>	<i>exact approach</i>
Poot et al. [2002]	(a)-(e)	-	✓	-	-	-	-	✓	-	✓	-	-	✓	-	-
	(f)	-	-	-	-	✓	-	✓	-	✓	-	-	✓	-	-
Kim et al. [2006]	(1)	-	-	✓	-	-	-	-	✓	-	✓	-	✓	-	-
	(2)	-	-	-	-	✓	-	-	✓	-	✓	-	✓	-	-
Tang and Miller-Hooks [2006]	(3), (4)	✓	-	-	-	-	-	✓	-	-	-	✓	✓	-	-
	(5)	✓	-	-	-	-	-	-	✓	-	-	✓	✓	-	-
Lu and Dessouky [2006]	(6)	-	-	-	-	-	-	✓	-	-	-	✓	N/A	-	-
Matis [2008]	(7)-(9)	-	✓	-	-	-	-	✓	-	-	-	✓	✓	-	-
Kant et al. [2008]	(10)	-	-	-	✓	-	-	✓	-	-	✓	-	N/A	-	-
Hollis and Green [2012]	(11)-(13)	-	✓	-	-	-	-	✓	-	-	✓	-	✓	-	-
	(14)	-	-	-	-	✓	-	✓	-	-	✓	-	✓	-	-
Gretton and Kilby [2013]	(15)	✓	-	-	-	-	-	✓	-	-	✓	-	✓	-	-
	(16)	-	-	-	-	-	✓	✓	-	-	✓	-	✓	-	-
Constantino et al. [2015]	(17)-(19)	-	-	-	-	-	-	-	✓	✓	-	-	-	✓	✓
Lum et al. [2015]	(20)	-	-	-	-	✓	-	-	✓	✓	-	-	-	✓	-

Table 1: Measures for visual attractiveness

3 Mathematical models

Several measures to quantify the visual attractiveness of the routes have been considered in the literature. In this section, we present mathematical formulations of both, the basic VRPTW model, as well as the models that include metrics for visual beauty of the solutions. Typical solutions for VRPTW models are usually very unsatisfactory in terms of the visual attractiveness criteria. Therefore, solutions resulting from solving the problem can be very inadequate to implement in practice. As the majority of solution approaches dealing with "pretty" routes are heuristics, we want to consider exact solution methodology in order to evaluate the quality of proposed measures.

Considering the measures given in the Table 1, we can assume that, generally speaking, visually appealing solutions are synonymous with compact and non-overlapping routes. Firstly, "pretty" routes will minimise the distances between the centre of the route and the customers. But, which reference point should be employed as the centre of the route? The authors mentioned above use different reference points. Poot et al. [2002], Matis [2008] and Hollis and Green [2012] propose the centre of gravity, Kim et al. [2006] use a customer that is closest to the centre of gravity, while Tang and Miller-Hooks [2006] and Gretton and Kilby [2013] use median - the customer for which the total Euclidean distance to all other customers is minimal. Moreover, Kant et al. [2008] define the middle stop of a route. According to Tang and Miller-Hooks [2006] routes do not overlap, when all customers are closest to the median of their route. Gretton and Kilby [2013] calculate the median of every route encountered during the search. They give a proof that using accurate medians yields to more compact solutions with reduced number of crossings. Following the latter, we can state that using the right point representing the centre of a route is crucial to achieve compact routes. Therefore, we develop a model that includes, so called, lollipop routes soft constraints. For

that purpose we consider a set of route centres, one per route, and introduce binary variables to the basic VRPTW model (Subsection 3.2).

Furthermore, in order to obtain a visually appealing plan we should reduce the number of inter-route and intra-route crossings. As mentioned above, Lu and Dessouky [2006] propose the measure Crossing Length Percentage as it indicates the most entangled portions within a trip. However, Hollis and Green [2012] remark that, in the presence of time windows, the crossings within a route may be necessary in order to eliminate the waiting time. According to Hollis and Green [2012], inter-route crossings appear as convex hull overlaps between routes. Consequently, it seems plausible to assume that to avoid intersections between tours, the customers served by one vehicle should be concentrated in a geographical region. We develop two models that deal with crossings. Firstly, the model in Subsection 3.3 forbids simultaneous use of intersecting arcs in the solution. Secondly, the Subsection 3.4 presents a model that penalises crossings when they occur in the solution, instead of forbidding them.

Another important measure to enhance the visual attractiveness is the bending energy proposed by Gretton and Kilby [2013]. As pointed out in the previous section, minimising the sum of turn angles mitigates the tendency to select for long and overlapping hulls, and consequently, leads to more visually appealing routes. We introduce the bending energy constraints in the model presented in Subsection 3.5.

Since no methodology is perfect in order to obtain visually appealing routes, we evaluate the models using numerical examples. In the next Section (4) we compare the models with regard to the effectiveness in leading to solutions with the desired structure.

3.1 Model 1: Traditional formulation

The following problem formulation can be found in Toth and Vigo [2002]. We consider the following input data:

- n is the number of customers that have to be visited
- $N = \{1..n\}$ is the set of customers
- A is the set of arcs
- $V = N \cup \{0, n+1\}$ is the set of nodes, including the depot, which is represented by two nodes 0 and $n+1$
- m is the number of available vehicles
- $K = \{1..m\}$ is the set of available vehicles, each with capacity C
- c_{ij} is the travel cost, which corresponds to the Euclidean distance from customer i to j
- d_i is the demand to be delivered to customer i
- s_i is the service time at customer i
- $[a_i, b_i]$ is the time window at customer i
- E is the earliest possible departure from the depot
- L is the latest possible arrival at the depot
- M is a large constant

We use the following decision variables:

- $x_{ijk} \in \{0, 1\}$ takes value 1 if arc (i, j) is used by vehicle k and 0 otherwise

- w_{ik} is the start of service at node i when serviced by vehicle k

The VRPTW can be formulated as follows:

$$\min \sum_{k \in K} \sum_{(i,j) \in A} c_{ij} x_{ijk} \quad (21)$$

subject to:

$$\sum_{k \in K} \sum_{j \in V \setminus \{0\}} x_{ijk} = 1 \quad \forall i \in N \quad (22)$$

$$\sum_{j \in V \setminus \{0\}} x_{0jk} = 1 \quad \forall k \in K \quad (23)$$

$$\sum_{i \in V \setminus \{n+1\}} x_{ijk} - \sum_{i \in V \setminus \{0\}} x_{jik} = 0 \quad \forall k \in K \quad \forall j \in N \quad (24)$$

$$\sum_{i \in V \setminus \{n+1\}} x_{i,n+1,k} = 1 \quad \forall k \in K \quad (25)$$

$$w_{ik} + s_i + c_{ij} - w_{jk} \leq (1 - x_{ijk})M \quad \forall k \in K \quad \forall (i,j) \in A \quad (26)$$

$$a_i \sum_{j \in V \setminus \{0\}} x_{ijk} \leq w_{ik} \leq b_i \sum_{j \in V \setminus \{0\}} x_{ijk} \quad \forall k \in K \quad \forall i \in N \quad (27)$$

$$E \leq w_{ik} \leq L \quad \forall k \in K \quad \forall i \in \{0, n+1\} \quad (28)$$

$$\sum_{i \in N} d_i \sum_{j \in V \setminus \{0\}} x_{ijk} \leq C \quad \forall k \in K \quad (29)$$

$$x_{ijk} \geq 0 \quad \forall k \in K \quad \forall (i,j) \in A \quad (30)$$

$$x_{ijk} \in \{0, 1\} \quad \forall k \in K \quad \forall (i,j) \in A \quad (31)$$

The objective function (21) calculates the total cost, which consist of the total travel distance. Constraints (22) ensure that each customer is assigned to exactly one vehicle route. Next, (23) - (25) are flow constraints. Each vehicle route must start and end at the depot. Additionally, constraints (26) - (28) restrict the schedule feasibility and constraints (29) guarantee the capacity feasibility. Finally, constraints (30) force decision variables x_{ijk} to be equal or greater than zero and

(31) impose binary conditions on the flow variables.

3.2 Model 2: Pretty routes as lollipop constraints

This model is also build upon (21) - (31). We add binary variables γ_{ij} , which take value 1 if node i is centred around node j . We also consider a constant λ to weigh the penalty in the objective function. We modify the objective function as follows:

$$\min \sum_{k \in K} \sum_{(i,j) \in A} c_{ij} x_{ijk} + \lambda \sum_{i \in N} \sum_{j \in V \setminus \{0\}} c_{ij} \gamma_{ij} \quad (32)$$

the following constraints have to be added:

$$\gamma_{jl} \leq \gamma_{il} + 1 - \sum_{k \in K} x_{ijk} \quad \forall i \in N \quad \forall j \in N \quad \forall l \in V \setminus \{0\} \quad (33)$$

$$\sum_{j \in V \setminus \{0\}} \gamma_{ij} \geq 1 \quad \forall i \in N \quad (34)$$

3.3 Model 3: Pretty routes as crossing arcs hard constraints

In the following model the additional input data δ_{ijlm} is considered, which takes the value 1 if arcs (i, j) and (l, m) are intersecting. We add to (21) - (31) the following constraints:

$$\sum_{k \in K} x_{ijk} + \sum_{k \in K} x_{lmk} + \sum_{k \in K} x_{jik} + \sum_{k \in K} x_{mlk} \leq 1 \quad \forall i, j, l, m \mid \delta_{ijlm} = 1 \quad (35)$$

3.4 Model 4: Pretty routes as crossing arcs soft constraints

In this model the use of intersecting arcs is penalised in the objective function. The constant μ weights the penalty. We need additional decision variables α_{ijlm}

which take value 1 if arcs (i, j) and (l, m) are used in the solution, and 0 otherwise. The new objective function is as follows:

$$\min \sum_{k \in K} \sum_{(i,j) \in A} c_{ij} x_{ijk} + \mu \sum_{(i,j,l,m) \in V} \alpha_{ijlm} \quad (36)$$

and we complete the model with the constraints:

$$\alpha_{ijlm} \geq \sum_{k \in K} x_{ijk} + \sum_{k \in K} x_{lmk} + \sum_{k \in K} x_{jik} + \sum_{k \in K} x_{mlk} - 1 \quad \forall i, j, l, m \mid \delta_{ijlm} = 1 \quad (37)$$

3.5 Model 5: Pretty routes as bending energy constraints

In this model, we consider additional input data e_{ijl} , which is the precomputed turn angle between customers i, j and l . We penalise the use of arcs with a high value of turn angle and consider a constant α to weigh the penalty in the objective function.

3.5.1 Two-index bending energy

In this version, we add a new decision variable β_{ij} to the model (21) - (31) and modify the objective function as follows:

$$\min \sum_{k \in K} \sum_{(i,j) \in A} c_{ij} x_{ijk} + \alpha \sum_{(i,j) \in A} \beta_{ij} \quad (38)$$

we also need the following constraints:

$$\beta_{ij} \geq \sum_l \left(e_{ijl} \left(\sum_k x_{ijk} + \sum_k x_{jlk} - 1 \right) \right) \quad \forall i \in V \setminus \{n+1\} \quad \forall j \in V \setminus \{0, n+1\} \quad (39)$$

$$\beta_{ij} \geq 0 \quad \forall (i, j) \in A \quad (40)$$

3.5.2 Three-index bending energy

In the second version, we add a new decision variable β_{ijl} to the model (21) - (31) and modify the objective function as follows:

$$\min \sum_{k \in K} \sum_{(i,j) \in A} c_{ij} x_{ijk} + \alpha \sum_{(i,j) \in A} \sum_{(j,l) \in A} \beta_{ijl} \quad (41)$$

we also need the following constraints:

$$\beta_{ijl} \geq e_{ijl} \left(\sum_k x_{ijk} + \sum_k x_{jlk} - 1 \right) \quad \forall i \in V \setminus \{n+1\} \quad \forall j \in V \setminus \{0, n+1\} \quad \forall l \in V \setminus \{0, n+1\} \quad (42)$$

$$\beta_{ijl} \geq 0 \quad \forall (i,j) \in A \quad \forall (j,l) \in A \quad (43)$$

4 Results

In this section we present results of our computational study on four models: the lollipop constraints model, the bending energy model with two index variables, the crossing arcs hard constraints model, and the crossing arcs soft constraints model. We test the proposed modelling approaches for the purpose of drawing conclusions about their characteristics, solution quality and CPU performance.

4.1 Test instances

We use for the investigation the well known Solomon's instances (Solomon [1987]), which are available on the Solomon's website (Solomon). The instances include following data: the number of vehicles and their capacity, customer coordinates, the demand of customers, service time, and tightness and positioning of time win-

dows. The problem sets are denoted by R1 and R2 (geographical data generated randomly by a random uniform distribution), C1 and C2 (clustered) and RC1 and RC2 (semiclustered). The sets R1, C1 and RC1 have tight time windows, while R2, C2 and RC2 have a long scheduling horizon. These characteristics, coupled with large vehicle capacities, lead to a higher number of customers to be serviced by the same vehicle in the sets R2, C2 and RC2.

To obtain distance data, the Euclidean distance for each pair of nodes is computed. We assume that the travel times correspond to the distances between customers. We believe that this simplification does not affect the validity of our results. Moreover, the vehicle fleet is assumed to be homogeneous. The constant M is always set to 100000. We apply the above presented methods to all sets of test instances. The models were implemented in CPLEX 12.6.3 and executed on 2.67 GHz Intel Xeon CPU. We restrict CPLEX to a single thread. We allocate 64 gigabytes of memory and specify a run time limit of 24 hours for each run. The visualisations of the solution were created with Proute (Tricoire).

4.2 Computational results

Tables (2) - (6) show the results of our computational investigation. Each row in the following tables displays the results for a Solomon's instance. We test 168 Solomon's instances. Since the problems are very CPU intensive, we start the investigation with 20 customers. For the problem sets with 50 customers only a small number of instances could be solved within a given run time restriction of 24 hours and memory usage restriction of 64 gigabytes. We report solution quality and results for the basic VRPTW model, the model with bending energy metric, the model with lollipop constraints and the models with crossings hard and soft constraints. The columns labelled *Dist.* and *CPU(s)* provide absolute values of the travel distance and the run time in seconds, respectively. In the columns labelled

$Dev.(%)$ we give the relative performance of the three methods with respect to the travel distance and the run time, i.e. the per cent deviation from optimal solution. This value is calculated by $100 * (vis.attr. - basic) / basic$, where *basic* is the performance of the basic VRPTW model and *vis.attr.* is the performance of the method with metrics for visual attractiveness. Some of the instances are too time consuming for some of the models. The notation *time limit* means that the problem could not be solved within a given time limit of 24 hours. Furthermore, in some cases, especially for the model with lollipop constraints, the memory usage exceeds 64 gigabytes. We denote the out-of-memory status with *memory*.

We evaluate the solution quality in terms of the travel distance as well as the run time. Additionally, we consider the visual beauty of the solutions as the non-traditional solution quality measure.

Analysing the results with regard to the objective function, it is not surprisingly that the crossing arcs hard constraints method leads to the highest increase in the travel distance. Optimal solutions for the VRPTW include often intersections and forbidding them causes mostly the increase in the travel distance and in the number of vehicle used, because additional routes have to be added to the routing plan, in order to avoid crossings. Figure 1 shows the routing plan produced by the basic VRPTW model and the model with crossings hard constraints at the example of instance 20-RC101.

Instead, the crossing arcs soft constraints method leads to the smallest increase of travel distance. We can observe in Figure 2 that this method results in routing plans where only the best avoidable crossings are ejected. The crossing arcs soft constraints model reduces the number of intersections by two when compared to the basic VRPTW model, keeping the number of tours in the solution at the same level (Figure 2). When avoiding an intersection would cause a high increase in the travel distance, the crossing arcs are still used in the solution. The trade-off

between the number of crossings and the number of routes in the solution can be shown at the example of instance 25-R201 (Figure 3). Another example is the instance 20- R101 (Figure 9), where forbidding intersections leads to nine tours, where two of them are one-customer tours, and penalising the intersections reduces the number of tours by one, while maintaining one crossing in the solution.

Furthermore, results show that the lollipop constraints model produces more compact routes while generating competitive results with respect to the objective function (13% increase in the travel distance on average). However, more compact routes require a higher number of tours. Therefore, when a limited number of vehicles or drivers has to be considered, the utility of the lollipop constraint method can be degraded. Figure 5 shows the comparison on instance 20-C201.

The bending energy method had limited success. For all the clustered problems (C1 and C2), as well as a number of random problems (R1 and R2) the method did not lead to any change in the routing plan. Additionally, for many instances, minimising the bending energy causes an increase of the number of crossings in the solution or to the increase in the number of routes. For some instances, however, the bending energy method leads to more aesthetic routing plans, as it can be shown at the example of instance 20-R201 (Figure 6).

Three of the models, the bending energy method, the lollipop constraints method, and the crossing arcs soft constraints method, are parametrised. The constants α , λ , and μ determine the weight of the penalties in the objective functions. We examined the effect on the solution by setting the parameters as follows: $\alpha_1 = 1$, $\alpha_2 = 2$, $\lambda_1 = 1$, $\lambda_2 = 2$, $\mu_1 = 1$ and $\mu_2 = 2$.

Given the computational results, we can state that increasing α in the model with bending energy causes a change in the solution only for the random (R1 and R2) and semiclustered (RC2) instances. The increase of α leads to an increase in the travel distance by 0,92% on average. For clustered instances (C1 and C2)

the solution remains the same. In terms of the visual attractiveness of the routing plan, higher α increases the number of intersections in the solutions (Example: Instance 20-RC201, Figure 7).

In the case of the model with lollipop constraints, the increase of λ has a significant effect on solution quality (the objective function increases by 23,91% on average). However, we observe that higher λ leads to creation of back and forth routes serving only one customer (Example: Instance R101, Figure 8).

For the crossings arcs soft constraints method setting $\mu = 2$ results in solutions that are the same as either the solutions to hard constraints model or the solutions to the soft constraint model where $\mu = 1$. The only instance where $\mu = 2$ results in another solution is 50-R101. Figure 9 shows the two different solutions. The increased μ reduces the number of crossings by five and at the same time eliminates a back-and-forth tour (0-50-0).

In terms of computation time, the bending energy method seems to be very efficient. On clustered problems, this method was the fastest one. For the C1 and C2 problem sets, the method has an average run time of 21,91 seconds and 517,80 seconds, respectively. Over all problem sets, the method had an average run time of 201,80 seconds. On the other hand, for randomly generated problems and semiclustered problems (R1, R2 and RC2), the crossing arcs hard constraints method leads to the shortest run times. For a number of instances, the method results in a significant decrease in the running time, as compared to the basic model. The running times for the crossing arcs soft constraints model were higher for all of the instances than for the version with hard constraints. The soft constraints method is much slower even if it does not change the routing plan as compared to the basic VRPTW model. The lollipop constraints method leads to the most significant increase in computation time. For many C2 instances, as well as all of the R2 and RC2 instances, the computation time of the lollipop constraints model

exceeds 24 hours. In general, the instances with tighter time windows were more efficient with regard to the computation time.

Table 2: Results for C1

Instance	basic model		bending energy				lollipop			
	Dist.	CPU(s)	Dist.	Dev.(%)	CPU(s)	Dev.(%)	Dist.	Dev.(%)	CPU(s)	Dev.(%)
20-C101	175,37	0,82	175,37	0,00	5,33	550	175,37	0,00	10,90	1229
20-C105	175,37	2,10	175,37	0,00	8,07	284	175,37	0,00	102,27	4770
20-C106	175,37	0,77	175,37	0,00	4,83	527	175,37	0,00	10,97	1325
20-C107	175,37	3,94	175,37	0,00	10,97	178	175,37	0,00	99,98	2438
25-C101	191,81	3,29	191,81	0,00	10,07	206	191,81	0,00	18,48	462
25-C105	191,81	6,05	191,81	0,00	12,86	113	191,81	0,00	116,47	1825
25-C106	191,81	2,03	191,81	0,00	8,38	313	191,81	0,00	24,38	1101
25-C107	191,81	7,20	192,64	0,43	18,77	161	191,81	0,00	141,60	1867
50-C101	363,25	21,66	363,25	0,00	117,90	444	363,25	0,00	1201,99	5449
Average	203,55	5,32	203,64	0,05	21,91	308	203,55	0,00	191,89	2274

Instance	basic model		crossings hard				crossings soft			
	Dist.	CPU(s)	Dist.	Dev.(%)	CPU(s)	Dev.(%)	Dist.	Dev.(%)	CPU(s)	Dev.(%)
20-C101	175,37	0,82	175,37	0,00	10,09	1130	175,37	0,00	10,45	1174
20-C105	175,37	2,10	175,37	0,00	10,59	404	175,37	0,00	34,58	1547
20-C106	175,37	0,77	175,37	0,00	10,17	1221	175,37	0,00	15,34	1892
20-C107	175,37	3,94	175,37	0,00	14,42	266	175,37	0,00	36,42	824
25-C101	191,81	3,29	191,81	0,00	22,69	590	191,81	0,00	57,52	1648
25-C105	191,81	6,05	191,81	0,00	22,43	271	191,81	0,00	63,44	949
25-C106	191,81	2,03	191,81	0,00	23,54	1060	191,81	0,00	54,80	2600
25-C107	191,81	7,20	191,81	0,00	25,03	248	191,81	0,00	58,51	713
50-C101	363,25	21,66	363,25	0,00	288,91	1234	363,25	0,00	1218,93	5528
Average	203,55	5,32	203,55	0,00	47,54	714	203,55	0,00	172,22	1875

Table 3: Results for C2

Instance	basic model		bending energy				lollipop			
	Dist.	CPU(s)	Dist.	Dev.(%)	CPU(s)	Dev.(%)	Dist.	Dev.(%)	CPU(s)	Dev.(%)
20-C201	199,01	0,78	199,01	0,00	4,27	447	265,75	33,54	2375,57	304460
20-C205	199,01	14,95	199,01	0,00	1939,17	12871	memory	-	-	-
25-C201	215,54	1,32	215,54	0,00	6,02	356	286,89	33,10	72166,93	5467092
25-C205	215,54	64,44	215,54	0,00	604,37	838	memory	-	-	-
50-C201	361,80	4,4	361,80	0,00	68,82	1464	time	limit	-	-
Average	238,18	17,18	238,18	0,00	517,80	3042	276,32	33,32	37271,25	2885776

Instance	basic model		crossings hard				crossings soft			
	Dist.	CPU(s)	Dist.	Dev.(%)	CPU(s)	Dev.(%)	Dist.	Dev.(%)	CPU(s)	Dev.(%)
20-C201	199,01	0,78	199,01	0,00	8,43	981	199,01	0,00	11,39	1360
20-C205	199,01	14,95	199,01	0,00	2793,05	18583	199,01	0,00	58981,03	394422
25-C201	215,54	1,32	215,54	0,00	21,98	1565	215,54	0,00	61,53	4561
25-C205	215,54	64,44	215,54	0,00	76,96	19	215,54	0,00	1180,60	1732
50-C201	361,80	4,4	361,80	0,00	396,90	8920	361,80	0,00	1241,28	28111
Average	238,18	17,18	238,18	0,00	659,46	6014	238,18	0,00	12295,18	86037

Table 4: Results for R1

Instance	basic model		bending energy				lollipop			
	Dist.	CPU(s)	Dist.	Dev.(%)	CPU(s)	Dev.(%)	Dist.	Dev.(%)	CPU(s)	Dev.(%)
20-R101	511,25	0,64	511,25	0,00	6,33	889	520,29	1,77	3,97	520
25-R105	432,48	197,22	432,74	0,06	1471,16	646	472,70	9,30	3302,88	1575
25-R101	618,33	1,71	618,33	0,00	14,74	762	634,28	2,58	23,60	1280
25-R105	531,54	3556,65	741,97	39,59	693,73	-80	571,76	7,57	73630,37	1970
50-R101	1046,70	232,23	1048,59	0,18	24755,85	10560	time limit	-	-	-
Average	628,06	797,69	670,58	7,79	5388,36	2555	549,76	5,30	19240,21	1336

Instance	basic model		crossings hard				crossings soft			
	Dist.	CPU(s)	Dist.	Dev.(%)	CPU(s)	Dev.(%)	Dist.	Dev.(%)	CPU(s)	Dev.(%)
20-R101	511,25	0,64	558,06	9,16	6,15	861	515,55	0,84	16,99	2555
25-R105	432,48	197,22	439,34	1,59	14,09	-93	439,34	1,59	144,03	480
25-R101	618,33	1,71	699,08	13,06	10,23	498	627,13	1,42	50,16	2833
25-R105	531,54	3556,65	556,50	4,70	28285,17	-92	538,40	1,29	1238,25	-65
50-R101	1046,70	232,23	1232,98	17,80	163,96	-29	1073,30	2,54	4646,30	1901
Average	628,06	797,69	697,19	9,26	95,92	229	638,74	1,54	1419,15	1541

Table 5: Results for R2

Instance	basic model		bending energy				lollipop			
	Dist.	CPU(s)	Dist.	Dev.(%)	CPU(s)	Dev.(%)	Dist.	Dev.(%)	CPU(s)	Dev.(%)
20-R201	388,12	195,79	390,33	0,57	2070,47	957	time	limit	-	-
25-R201	464,37	60,53	465,18	0,17	850,68	1305	time	limit	-	-
Average	426,25	128,16	427,76	0,37	850,68	1131	-	-	-	-

Instance	basic model		crossings hard				crossings soft			
	Dist.	CPU(s)	Dist.	Dev.(%)	CPU(s)	Dev.(%)	Dist.	Dev.(%)	CPU(s)	Dev.(%)
20-R201	388,12	195,79	403,87	4,06	31,77	-84	395,10	1,80	1929,51	885
25-R201	464,37	60,53	498,20	7,29	200,10	231	467,70	0,72	4736,72	7725
Average	426,25	128,16	451,04	5,67	115,94	73	431,40	1,26	3333,12	4305

Table 6: Results for RC2

Instance	basic model		bending energy				lollipop			
	Dist.	CPU(s)	Dist.	Dev.(%)	CPU(s)	Dev.(%)	Dist.	Dev.(%)	CPU(s)	Dev.(%)
20-RC201	328,33	8,99	329,85	0,46	37,80	320	time	limit	-	-
25-RC201	361,24	36960,91	362,69	0,40	21295,08	-42	time	limit	-	-
Average	344,79	18484,95	346,27	10666,44	0,43	139	-	-	-	-

Instance	basic model		crossings hard				crossings soft			
	Dist.	CPU(s)	Dist.	Dev.(%)	CPU(s)	Dev.(%)	Dist.	Dev.(%)	CPU(s)	Dev.(%)
20-RC201	328,33	8,99	memory	-	-	-	time	limit	-	-
25-RC201	361,24	36960,91	566,22	56,74	90,12	-100	361,24	0,00	78905,99	113
Average	344,79	18484,95	566,22	56,74	90,12	-100	361,24	0,00	78905,99	113

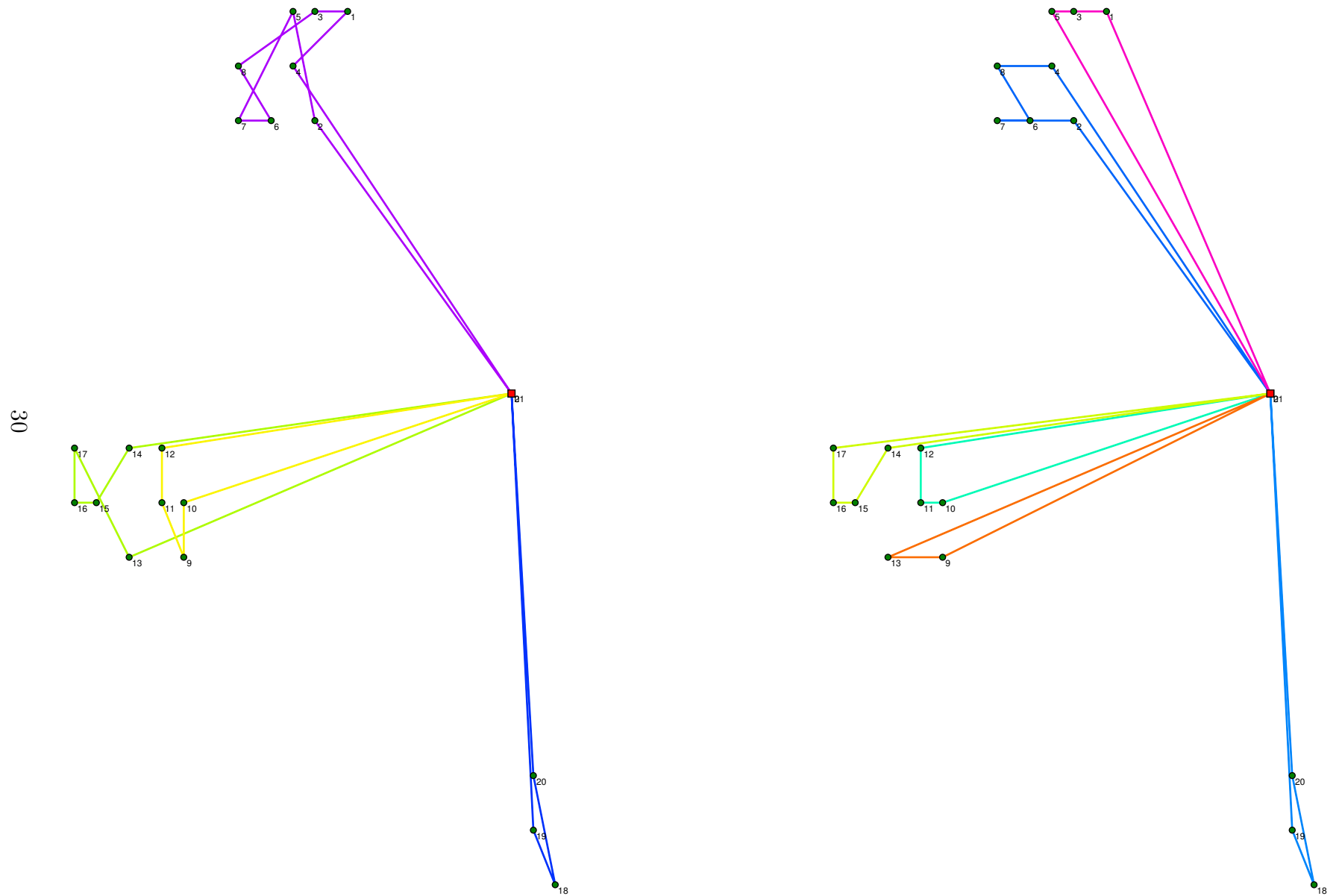


Figure 1: Instance 20-RC101: solution of the basic VRPTW model with four routes (left) and solution for the model with crossings constraints with six routes (right).

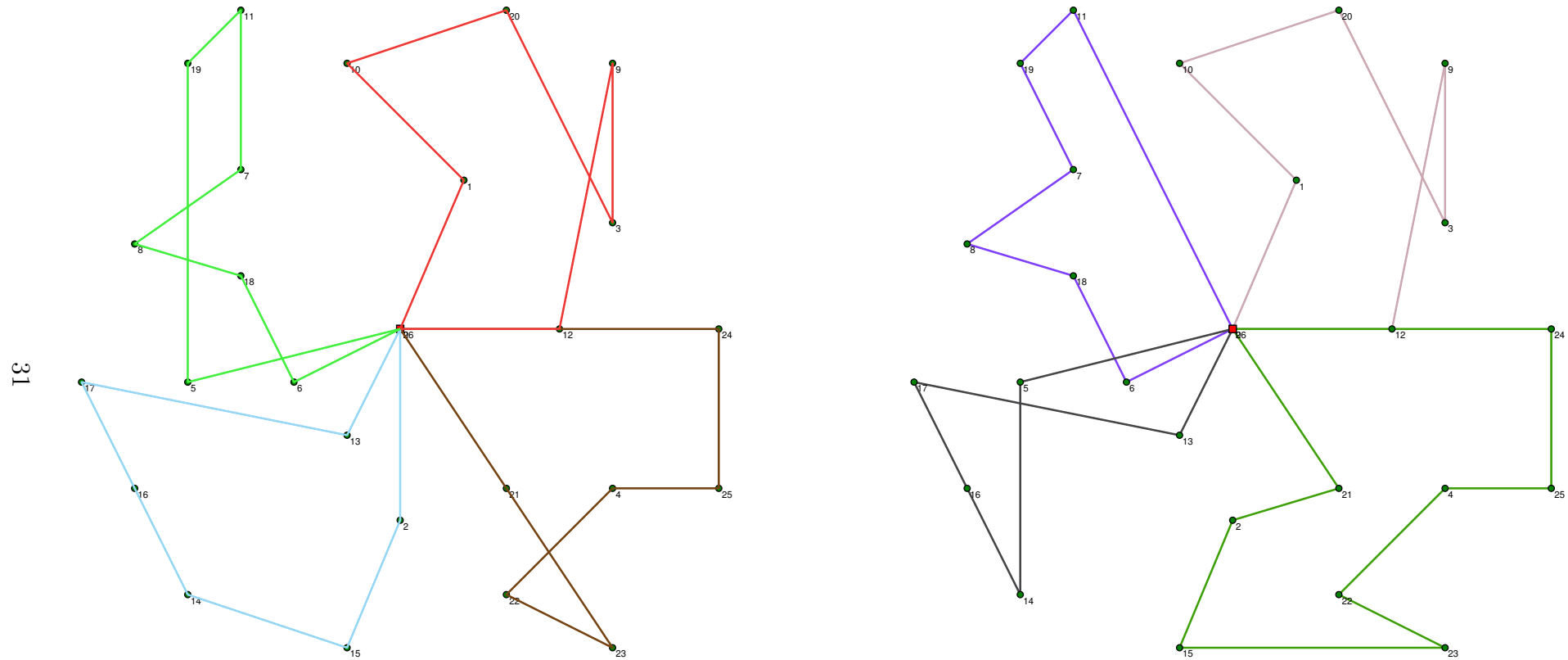


Figure 2: Instance 25-R201: solution of the basic VRPTW model with five crossings (left) and solution for the model with crossing soft constraints with three crossings and the same number of tours (right).

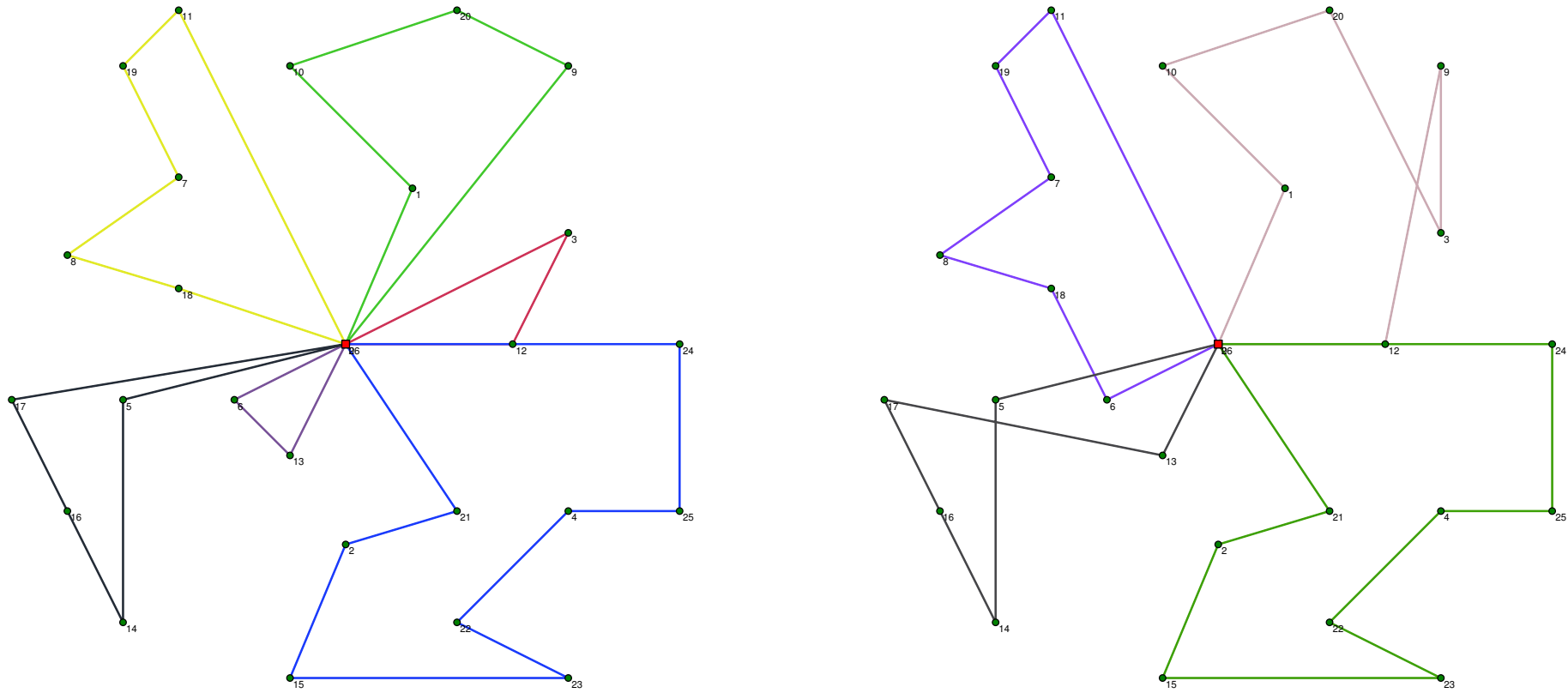


Figure 3: Instance 25-R201: solution of the crossing arcs hard constraints model with six routes (left) and solution for the crossing arcs soft constraints model with four routes and three crossings (right).

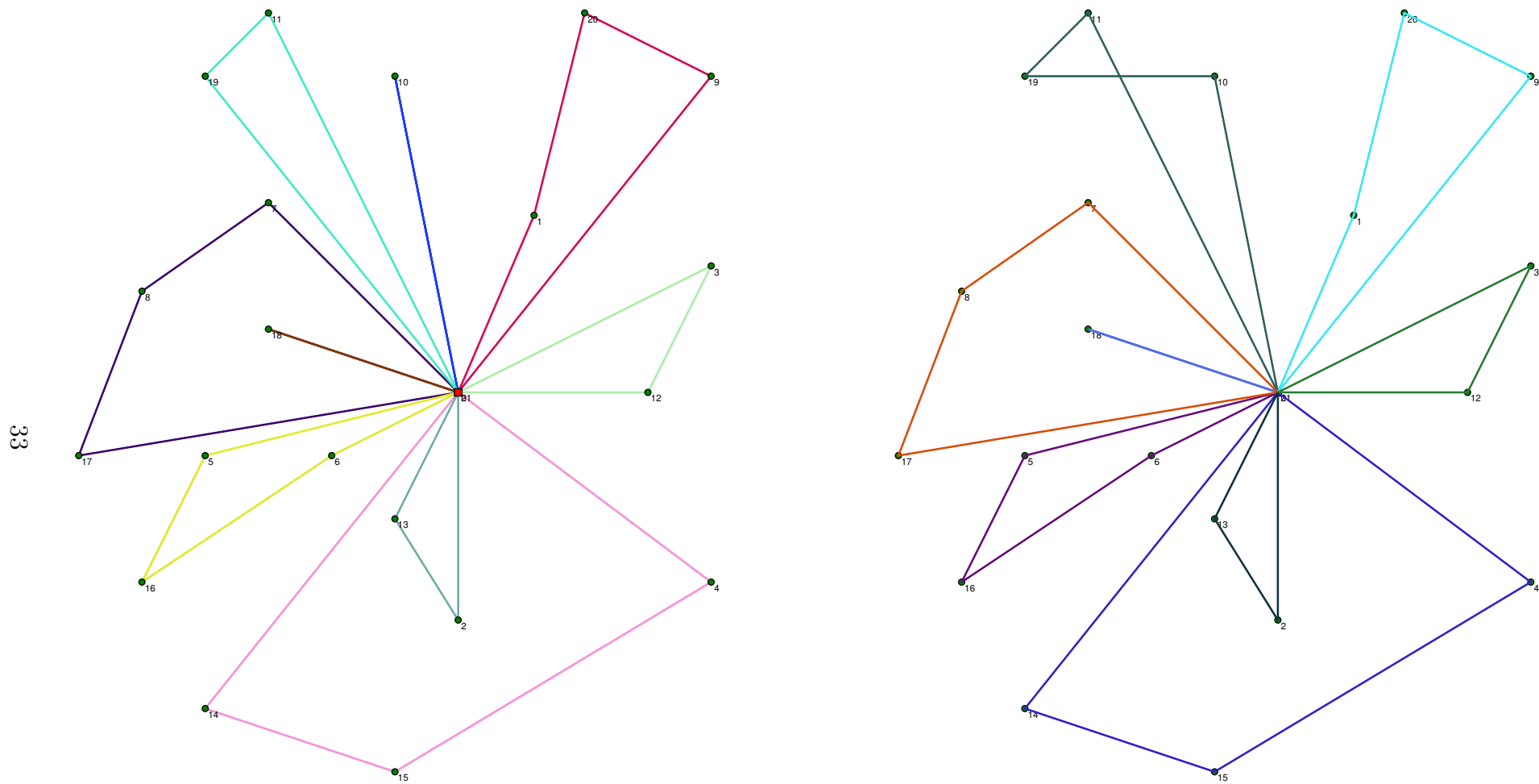


Figure 4: Instance 20-R101: solution of the crossing arcs hard constraints model with nine routes (left) and solution for the crossing arcs soft constraints model with eight routes and one crossing (right).

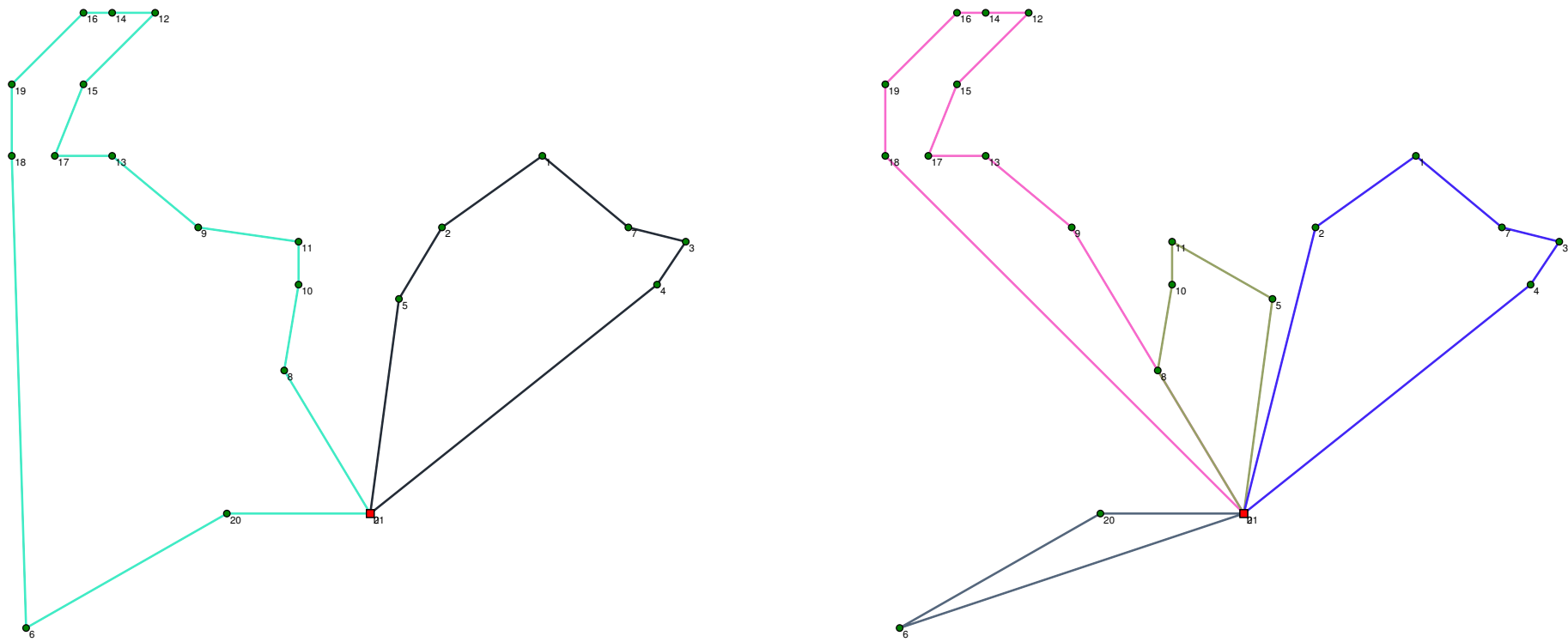


Figure 5: Instance 20-C201: solution of the basic VRPTW model with two routes (left) and solution for the model with lollipop constraints with four routes (right).

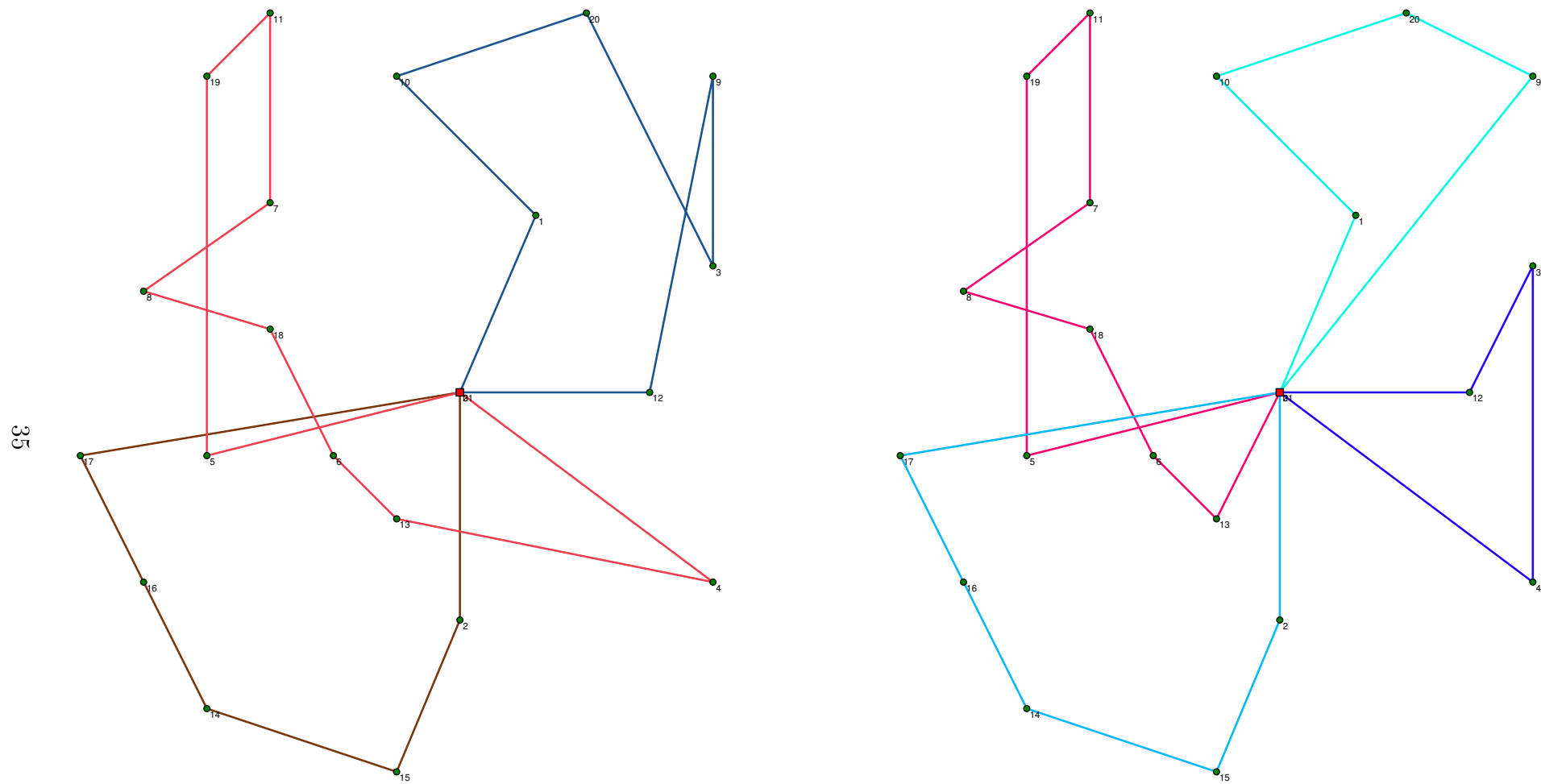


Figure 6: Instance 20-R201: solution of the basic VRPTW model with three routes (left) and solution for the model with bending energy constraints with four routes (right).

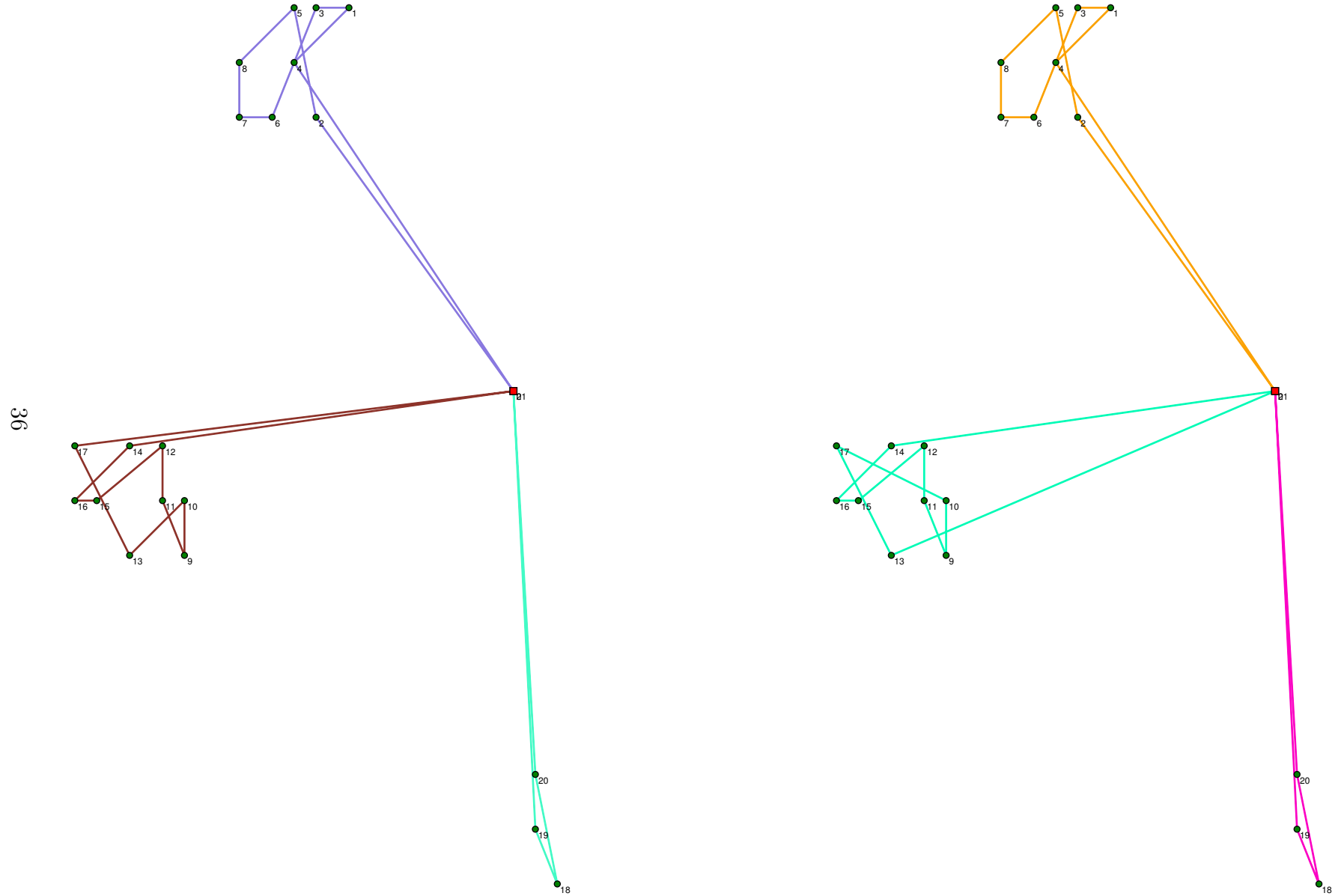


Figure 7: Instance 20-RC201: solution of the model with bending energy $\alpha = 1$ with eight crossings (left) and solution for the model with bending energy $\alpha = 2$ with eleven crossings (right).

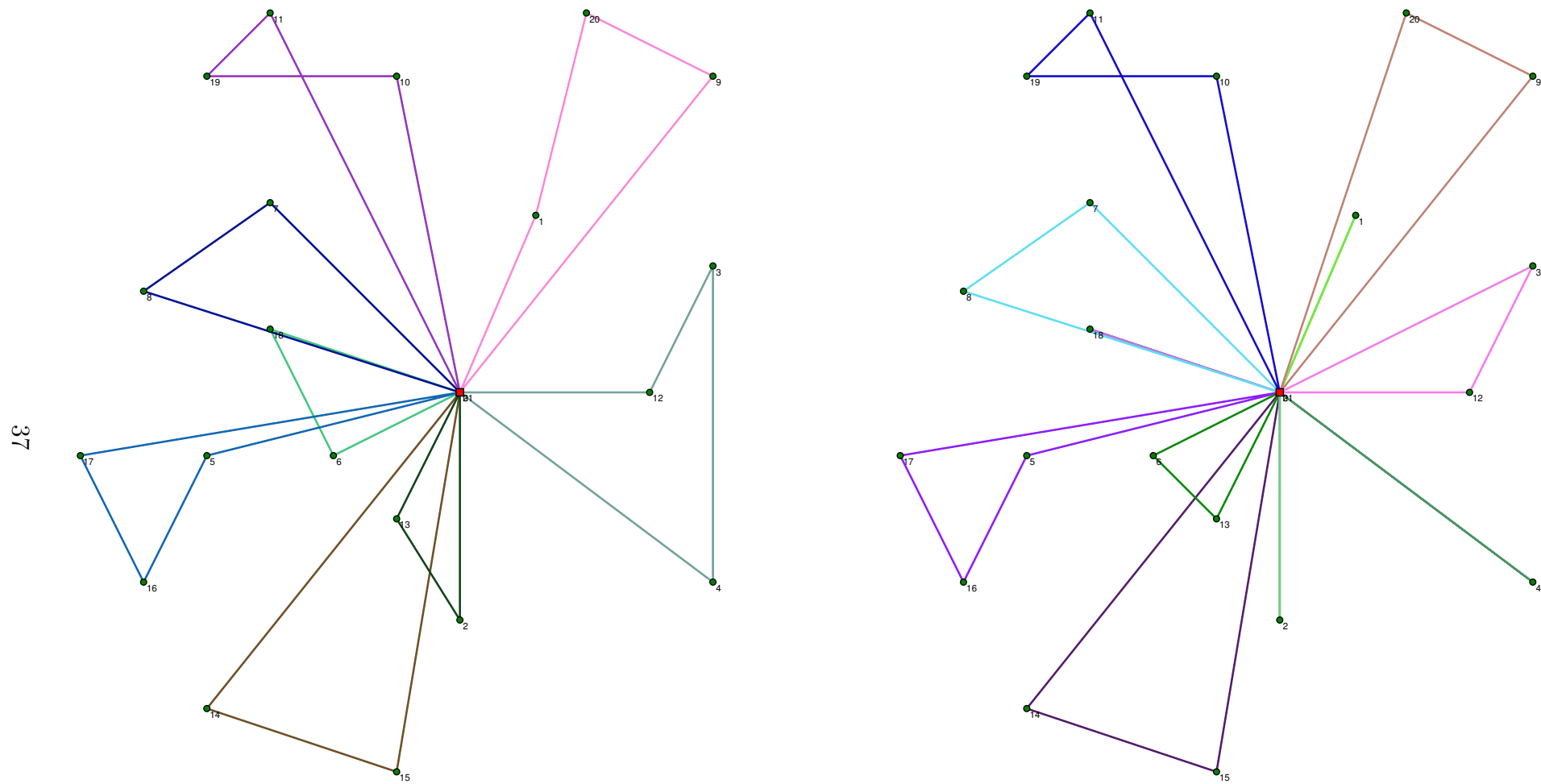


Figure 8: Instance 20-R101: solution of the model with lollipop constraints $\lambda = 1$ with no back and forth routes (left) and solution for the model with lollipop constraints $\lambda = 2$ with four back and forth routes (right).

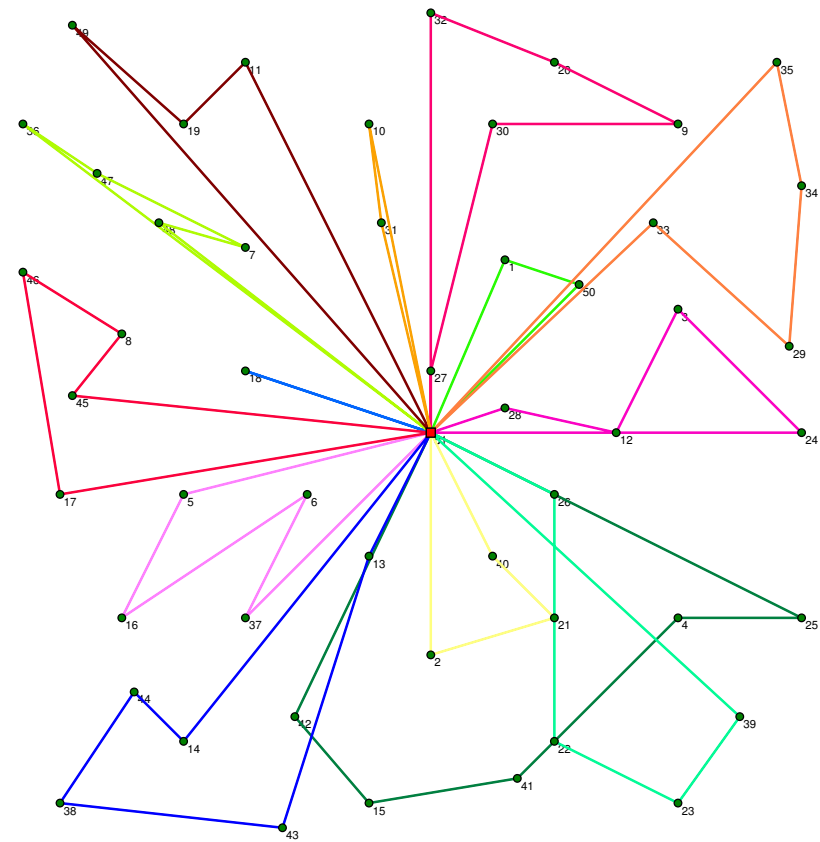
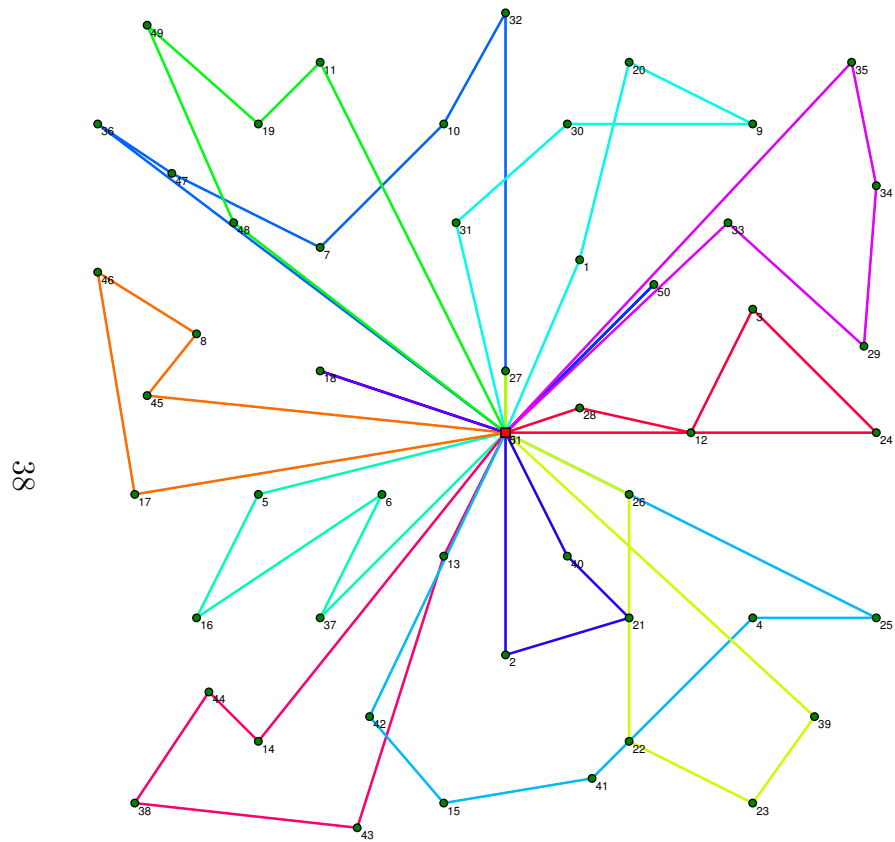


Figure 9: Instance 50-R101: solution of the crossing arcs soft constraints model $\mu = 1$ (left) and solution for the crossing arcs soft constraints model $\mu = 2$ (right).

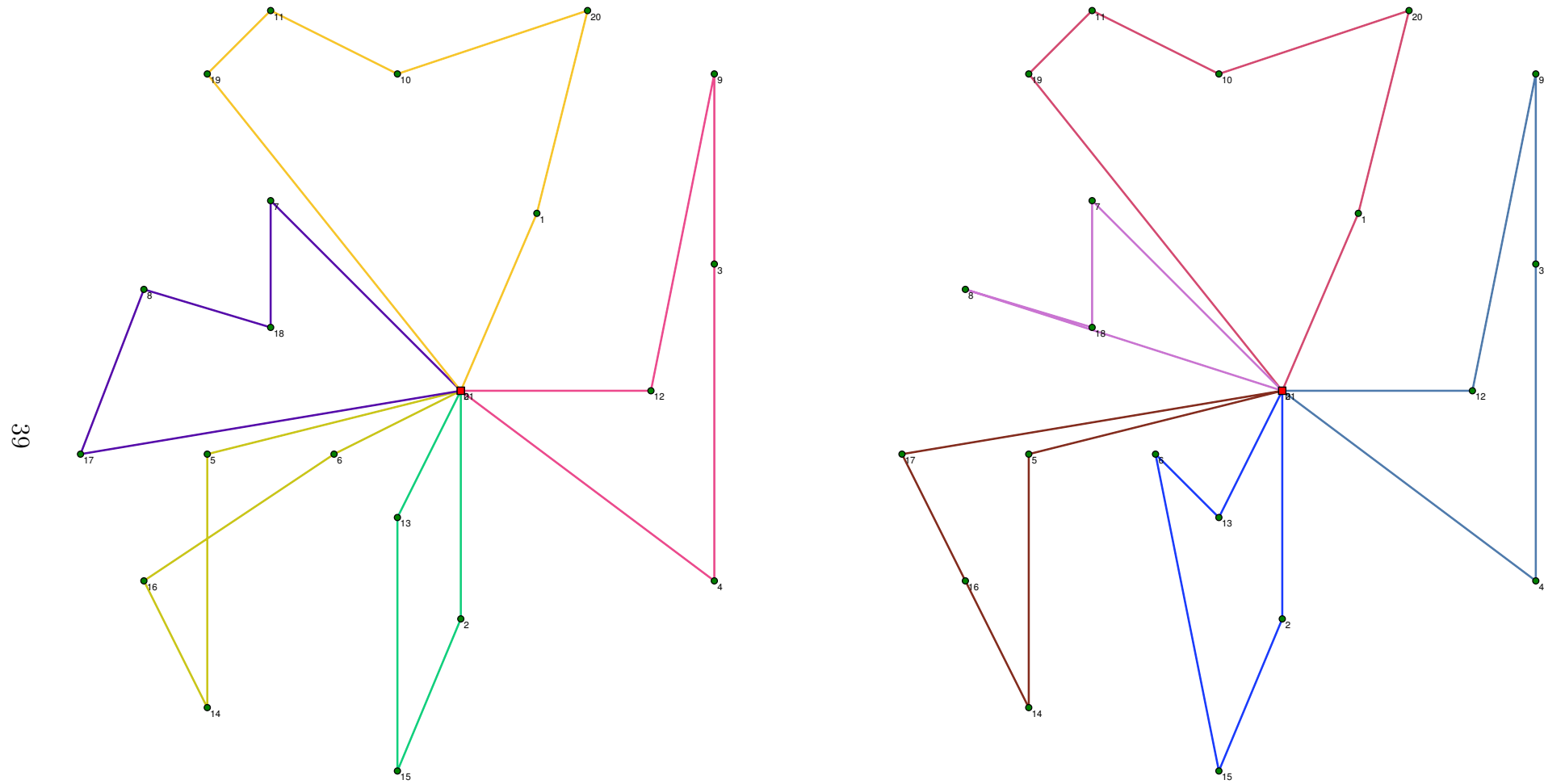


Figure 10: Instance 20-R105: solution of the basic VRPTW model with one crossing (left) and solution for the model with crossings constraints (right).

5 Results of the qualitative survey

We conduct a qualitative survey in order to compare our computational results with subjective opinions of people regarding the aesthetic aspects of solutions. As participants we select the scientific staff of the Faculty of Business, Economics and Statistics of the University of Vienna. All of thirty participants are familiar with the field of logistics, as they are working at the Chair of Productions and Operations Management, as well as the Chair of Productions and Operations Management with International Focus. We show to participants visualised solutions of fourteen problem instances. The solutions include the solution for the basic VRPTW model, the model with bending energy, the model with lollipop constraints and the model with crossings constraints. We ask them for the subjective opinion regarding two aspects:

- Question 1: "Which of the solutions looks nicer (more visually attractive) to you?"
- Question 2: "Which of the solutions would you implement in practice and why?"

5.1 Question 1: "Which of the solutions looks nicer (more visually attractive) to you?"

According to the participants, the method which leads to the most visually attractive routing plans is the crossing avoidance method (46% of those questioned chose solutions without crossings), followed by the basic VRPTW model and lollipop constraints model (33% and 17%, respectively). It is not surprisingly that the bending energy model achieved an acceptance of only 5% of the participants. This is because of the fact, that the bending energy model causes the creation of

(additional) intersections for some problem instances.

It is worth mentioning that, for the clustered instances, the participants find the solution with less routes better, than the solution with more separated clusters (Example: Instance C201, Figure 5). For the random instances, the majority of attendants prefers the solutions without intersections over the solution to the basic VRPTW model. The only exception is the instance 20-R105, where the solution to the basic VRPTW model has been preferred, even if it contains a crossing (Figure 10). For the semiclustered instances, the reverse was true.

5.2 Question 2: "Which of the solutions would you implement in practice and why?"

In this part of our survey we want to examine whether the attendants will prefer the subjective more visually appealing routing plans also in the practice. We want to evaluate which aspects of the solution are of importance for the participants, when it comes to the practical applications.

It seems that the most important factor when choosing a routing plan that should be implemented in practice is the total cost or total distance travelled and the number of vehicles used to supply customers. Therefore, solutions with less routes are, in general, preferred over solutions that look nicer, but use more vehicles. This is one of the most important trade-offs in the crossings avoidance method. Even if the absence of crossings within the routes is an important characteristic of a pretty solution, 45% of participants would implement the solution of the basic VRPTW model which includes intersections, instead. 33% of participants would use the routes that do not cross each other. Another significant aspect of the solutions are, according to the participants, more balanced tours for all drivers. They wish similar distribution of routes lengths for all vehicles and no one-customer-routes. Moreover, for one third of the participants the general visual

appearance of the routing plan seems to play an important part in the practice. They would rather avoid routes that look "messy", "chaotic" or "unclear", and instead choose plans that look "more reasonable", "nicer" and "less complicated". Even if many of the participants name geographic clustering, i.e. clear division of the service area into separated districts, as an important aspect of a solution, only 12% of those questioned prefer solutions delivered by the lollipop constraints model. Also the flexibility of the plan and the vehicle capacity utilisation seem to be of importance. On the one hand, the higher number of tours increases the total cost, but on the other hand, the average duration of deliveries could be reduced. The bending energy model achieved a 10 % acceptance. Table 7 demonstrates important aspects of a solution plan and the percentage of participants that mention the given characteristic in the survey.

Characteristic of the solution	% of participants
number of vehicles (routes)	80
number of crossings (overlaps)	73
total cost of the plan (total distance)	60
balanced routes (similar lengths of the routes)	60
general visual appearance	33
absence of single customer routes	33
geographic clustering of the region	27
flexibility of the routing plans	10
vehicle capacity utilisation	7

Table 7: Important aspects of a routing plan

To summarize, 23% of the participants would implement in practice the solution that looks nicer (more visually attractive) to them. The others would forgo the aesthetic aspects in favour of lower total cost of the plan.

6 Conclusion

In this master's thesis, we investigated the impact of diverse metrics for visual attractiveness of the routes on the quality of the solutions to the VRPTW. We have presented a literature overview on existing measures and developed models which deal with pretty routes. The presented models produced routes with appealing aesthetic qualities with a relative small increase in the objective function as compared to the basic VRPTW model. Our results indicate that the models that reduce the number of crossings proved to be very successful. We believe that our computational study provides a good comparison of methods that could increase the visual beauty of the routes. In the future, it would be interesting to combine the presented modelling approaches with geographical clustering of serviced customers for better visual attractiveness.

References

- M. Constantino, L. Gouveia, M. C. Mourao, and A. C. Nunes. The mixed capacitated arc routing problem with non-overlapping routes. *European Journal of Operational Research*, 244:445–456, 2015.
- G. B. Dantzig and J. H. Ramser. The Truck Dispatching Problem. *Management Science*, 6(1):80–91, October 1959.
- C. Gretton and P. Kilby. A Study of Shape Penalties in Vehicle Routing. *Optimisation Research Group*, 2013.
- G. Hasle and O. Bräysy. *Software Tools and Emerging Technologies for Vehicle Routing and Intermodal Transportation*. In: *Arc Routing: Problems, Methods, and Applications Ed. Paolo Toth and Daniele Vigo*, chapter 16, pages 351–396. SIAM - Society for Industrial and Applied Mathematics, 2014.
- B. Hollis and P. Green. Real-life Vehicle Routing with Time Windows for Visual Attractiveness and Operational Robustness. *Asia-Pacific Journal of Operation Research*, 29(4):29, 2012.
- G. Kant, M. Jacks, and C. Aantjes. Coca-cola Enterprises Optimizes Vehicle Routes for Efficient Product Delivery. *Interfaces*, 38(1):40–50, January-February 2008.
- B.-I. Kim, S. Kim, and S. Sahoo. Waste collection vehicle routing problem with time windows. *Computers & Operations Research*, 33:3624–3642, 2006.
- H. Liu, L. J. Latecki, and W. Liu. A Unified Curvature Definition for Regular, Polygonal, and Digital Planar Curves. *Springer Science + Business Media*, 80: 104–124, 2008.

- Q. Lu and M. M. Dessouky. A New Insertion-based Construction Heuristic for Solving the Pickup and Delivery Problem with Time Windows. *European Journal of Operation Research*, 175(2):672–687, 2006.
- O. Lum, C. Cerrone, B. Golden, and E. Wasil. Compact Routes for the Min-Max K Windy Rural Postman Problem. Presented at the VeRoLog Conference. Vienna, 2015. URL prolog.univie.ac.at/Verolog2015/program-verolog2015.pdf.
- J. N. MacGregor and T. C. Ormerod. Human performance on the traveling salesman problem. *Perception & Psychophysics*, 58:527–539, 1996.
- P. Matis. Decision support system for solving the street routing problem. *Transport*, 23(3):230–235, July 2008.
- A. Poot, G. Kant, and A. Wagelmans. A savings based method for real-life vehicle routing problems. *Journal of the Operational Research Society*, 53:57–68, 2002.
- M. W. P. Savelsbergh and M. Sol. The General Pickup and Delivery Problem. *Transportation Science*, 29(1):17–29, 1995.
- S. Shao, S. Kim, B. Kim, B. Kraas, and A. Popov. Routing Optimization for Waste Mangement. *Interfaces*, 35(1):24–36, 2005.
- M. Solomon. VRPTW Benchmark Problems. URL <http://web.cba.neu.edu/~msolomon/problems.htm>. Last visited 2016-08-31.
- M. Solomon. Algorithms for the vehicle routing and scheduling problems with time window constraints. *Operations Research*, 35:254–265, 1987.
- H. Tang and E. Miller-Hooks. Interactive Heuristic for Practical Vehicle Routing Problem with Solution Shape Constraints. *Transportation Research Record*, 1964:9–18, 2006.

- P. Toth and D. Vigo. *The Vehicle Routing Problem*. Society for Industrial and Applied Mathematics, 2002.
- F. Tricoire. Proute, the python software to visualize routing solutions. URL <https://github.com/fa-bien/proute>. Last visited 2016-08-31.
- I. Van Rooij, U. Stege, and A. Schactman. Convex hull and tour crossings in the Euclidean traveling salesperson problem: Implications for human performance studies. *Memory & Cognition*, 31(2):215–220, 2003.
- I. T. Young, J. E. Walker, and J. E. Bowie. An Analysis Technique for Biological Shape. *Information and Control*, 25:357–370, 1974.

A Deutsche Zusammenfassung

In der vorliegenden Arbeit wird ein Überblick über Maßstäbe für die visuelle Attraktivität von Routen im Vehicle Routing Problem dargestellt. Außerdem werden vier Modellierungsansätze vorgeschlagen, die darauf abzielen die visuelle Schönheit der Lösungen zu verbessern. Dabei wird das traditionelle Vehicle Routing Problem mit Zeitfenstern um zusätzliche Nebenbedingungen erweitert. Das erste Modell beinhaltet „weiche“ Nebenbedingungen, die auch „Lollipop routes“ genannt werden. In dem zweiten Modell werden Nebenbedingungen hinzugefügt, die Überschneidungen zwischen Touren verbieten und das dritte Modell straft die Verwendung von sich überschneidenden Kanten. Der letzte Modellierungsansatz verbessert die visuelle Attraktivität der Lösungen durch die Minimierung von Krümmungsenergie. Die ausgeführten rechnerischen Untersuchungen zeigen die Performance der Modelle bezüglich der traditionellen und nicht traditionellen Qualitätsmaßstäben. Darüber hinaus wird eine qualitative Studie durchgeführt, um die subjektiven Meinungen der Personen bezüglich der ästhetischen Aspekte von Lösungen hinzuzuziehen.