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Chapter 1

Introduction

Motivation

In the beginning of the 90ies, the second generation of wireless systems (2G), with its main application of voice transmission, was introduced in many countries all over the world. About a decade later, the number of mobile cellular subscriptions in the European Union exceeded the number of its citizen, proving the great success of mobile communications. As pointed out by The Economist (2015), 80% of adults will have a smartphone in their pocket by 2020. Smartphones usually require data transmission in addition to voice transmission. Such data transmission was already possible in 2G (GPRS) but the data rates were so low that only 3G provided high enough data rates for practical applications. However, the Internet with its increasing number of network applications requires even higher data rates so that each generation of wireless systems has to increase it. While the main objective of 2G was to build a robust and relatively simple system, 3G and 4G focused on increasing the data rate by using a large range of possible methods. One promising method which has not been included in the standard so far (because of technical challenges) is cognitive radio, ITU Report ITU-R M2225 (2011). The basic idea of cognitive radio is to dynamically access a large bandwidth and in particular to utilize unused spectrum. Currently, dynamic spectrum allocation only happens at user level within a single network operator. Depending on the instantaneous throughput demand of subscribers, the available time-frequency resources are scheduled to specific users in a dynamic way. In contrast to that, the bandwidth of different network operators is fixed and assigned by a regulatory authority based on an auction. For example, the auction conducted in Austria in 2013 generated more than two billion € for the government (`https://www.rtr.at`). Thus, the network operators pay a large sum in order to rent a certain bandwidth but the actual utilization of this spectrum is relatively low, usually between 15% to 85%, Cabric et al. (2004), because user only sporadically access the data connection. Cognitive radio allows a better utilization of the scarce spectrum and many authors, such as Wunder et al. (2014), expect that it will be employed in the next generation of wireless systems (5G), at least to some extent. However, because the standardization process for 5G has not yet started and only a few research projects have been conducted so far no one knows for sure how 5G will eventually looks like. Note that in wireless communication, standardization is of utmost importance due to the underlying technical complexity, as history proves: Europe concentrated in setting formal standards in 2G while the USA relied on market competition. The European style of standardization turned out to be better so that European firms dominated the 2G market, Kano (2000). Another way to increase the spectrum utilization, already happening today, can be realized by a Mobile Virtual Network Operator (MVNO). Here, the MVNO uses the network infrastructure and the spectrum of an already existing mobile network operator and adds value such as brand appeal or distribution channels. The mobile network operator, on the other hand, has the advantage of higher spectrum utilization, an increased market share or that he can focus on the technical aspects of his network so that he does not need to compete in the fierce brand competition. However, the possible gain depends on the market structure as investigated in Shin and Bartolacci (2007).

Literature Overview

Although cognitive radio has been investigated in academia for more than a decade now and the underlying components are well-known technologies, the market is still very small and there are hardly any market-ready products available. Medeisis and Minervini (2013) identify possible reasons for the stalling innovation of cognitive radio, such as a slow standardization process, where not only technological advancements but also business and politic related aspects have to be considered, or the fact that traditional operators might want to avoid disturbing the status quo. In order to revitalize the innovation process for cogntive radio technologies they suggest that the government should assign a dedicated frequency band with less strict regulatory requirements. Basaure et al. (2014) use the Coase theorem to argue that a regulatory scheme which clearly defines spectrum rights and allows transactions between participants, results in an optimal spectrum assignment. The authors claim that the Coase theorem can be gradually achieved by introducing dynamic spectrum management. They also perform agent-based simulations to investigate different regulatory regimes where they show the great potential of a dynamic spectrum management system. They also stress that the market structure has a high impact on the decision whether firms want to resell their spectrum. In particular, if a firm has a dominant position in the market, it might be unwilling to resell the spectrum. However, they did not provide any deeper analyses on this subject. The authors also empathize the importance of standardization because it greatly reduces the transaction costs. Caicedo and Weiss (2010) investigate spectrum trading markets by means of agent-based simulations. As already mentioned above, firms which have a dominant position in the market might be unwilling to sell their spectrum. The authors claim that more than 5 to 6 network operators are required for a market to be competitive, so that spectrum reselling takes place. However, this is a quite specific result and only true for the strict conditions the authors assumed. Nguyen et al. (2011) study the effects of adding an unlicensed spectrum to an already existing allocation of licensed spectrums, owned by incumbent network operators. The social welfare then depends on the size of the additional unlicensed spectrum and can even decrease due to strong congestion. Such effect is related to the tragedy of the commons. Peha and Panichpapiboon (2004) use simulations to assess dynamic spectrum allocation which they call real-time secondary markets for spectrum. They include the signal to interference ratio in their consideration and claim that everyone is better off by employing dynamic spectrum access. However, interdependencies, as for example the fact that a secondary network operator influences the profit of the primary operator, are not included in their considerations. Grønsund et al. (2013) simulates a scenario in which a mobile network operator offloads his traffic by deploying cognitive radio femtocells. In particular, they provide a comprehensive cost overview and propose a detailed business case scenario in which they show that the profit of a network operator can be increased by deploying cognitive radio femtocells. Niyato and Hossain (2008) identify that the price secondary users have to pay in order to get access to the spectrum is a key issue in cognitive radio. They propose three different pricing strategies for which they analyze the outcome and possible stability criteria. Books which provide a comprehensive overview of cognitive radio while also taking economical aspects into account are for example Hossain et al. (2009) or Medeisis and Holland (2014).

Scientific Goal

The current literature lacks a clear analytical model to describe dynamic spectrum allocation and usually relies on simulations. I therefore develop a model that describes mobile wireless networks and, in particular, includes the following important aspects:

- Cellular structure of mobile networks,
- Dynamic spectrum allocation (statistical multiplexing) to model a round robin scheduler which delivers a throughput measure.

I then apply this model to answer the following questions:

1. When should a mobile network operator allow a MVNO?
2. Do mobile network operators want to share their spectrum with each other?

Chapter 2

Model

2.1 Cellular Network

The great success of mobile wireless networks is owed to the cellular structure: the user throughput can easily be increased by increasing the number of base stations, Rappaport et al. (1996). Besides 2G, a possible candidate for mobile wireless networks was Iridium which uses satellites in order to provide coverage. 2G on the other hand deploys the base stations on the ground which is much more cost efficient, especially if the number of base station has to be increased due to an increasing demand. 2G therefore became the dominant technology in the 90ies while Iridium only has a few selected customers like explorers on an expedition where the deployment of ground base stations is not economically feasible.

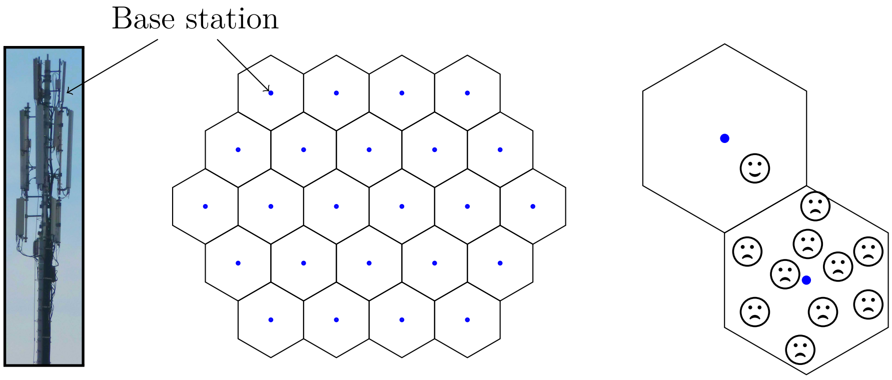

Fig. 2.1 Geographical deployment of base stations in a cellular network. All users within a cell are served by the same base station. They also have to share the available spectrum so that the individual user throughput decreases if the number of users increases.

Figure 2.1 shows the structure of a cellular network. I assume that the population is uniformly distributed so that a hexagonal deployment of base station is the most efficient lattice (which is a typically assumption in the literature). Note however, that in real world scenarios, a non-uniform population and objects, such as buildings, lead to a base station deployment that is different from such simplified model. All users within a cell are served by the same base station. Due to a uniformly distributed population, the number of subscribers per cell n is given by:

$$n = \frac{N}{B}, \tag{2.1}$$

whereas B is the number of base stations and N the total number of subscribers.

The base stations transmit an electromagnetic signal at a certain power level while the mobile station receives a signal which is corrupted by channel induced multipath propagation and noise. One of the most remarkable results in information theory is the channel capacity, that is, the rate at which error free transmission is possible, Tse and Viswanath (2005). For an additive white Gaussian noise channel, the capacity C is given by:

$$C = B_W \log_2(1 + \mathrm{SNR}), \tag{2.2}$$

and depends on the bandwidth B_W linearly and the signal-to-noise ratio (SNR) logarithmically. If the mobile station is close to the base station, the SNR will be high. Conversely, if the mobile station is far away from the base station it will will be low. Because the main focus of this thesis is the effect of dynamic spectrum allocation, I keep the model simple by ignoring that the capacity depends on the distance between the base station and the mobile station. Instead, I assume an average capacity which is independent of the geographical location. Another reason for ignoring such SNR dependency is the fact that the maximal throughput (capacity) is limited by the standard due to technical challenges, so that there no longer exists a SNR dependency on the distance if the SNR is sufficiently high. Interference also plays an important role in reality. The transmitted signal of one base station is also received by mobile stations located in neighboring cells. In 2G, such interference is mainly mitigated by using clustered cells, that is, several cells form a cluster and the available bandwidth is split between them so that each cell within a cluster has a unique frequency range. In 3G and 4G, on the other hand, the interference is mainly mitigated by cooperation between base stations. Similar to the SNR, I assume that the average capacity expression also captures the effects of interference. As indicated in Equation (2.2), the

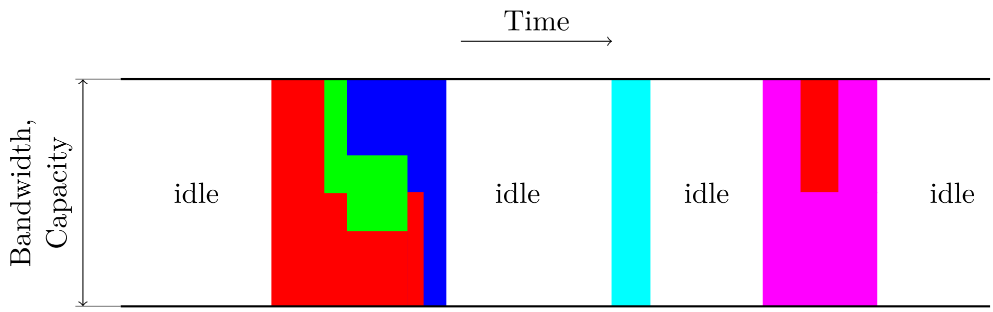

Fig. 2.2 Dynamic spectrum allocation (statistical multiplexing) means that the capacity is shared (equally) among active users. The instantaneous user throughput then depends on how many users are active and is thus a statistical variable. Note that it can also happen that there are no active users, that is, idle times.

capacity depends only on the bandwidth once the SNR and the inference is averaged out. Bandwidth and capacity have thus the same meaning in our model (besides a scaling factor).

2.2 Dynamic Spectrum Allocation

The idea of dynamic spectrum allocation (statistical multiplexing) is based on the fact that not all users are active at the same time. There are even times when all users are inactive. As mentioned in Section 2.1, each cell consist of n subscribers which all share the same transmission bandwidth. However, only a small subset of these subscribers are active users (they transmit data). If static spectrum allocation would be used, every user would only get a throughput of C/n, even if only one user is active. In dynamic spectrum allocation on the other hand, such user would get the whole bandwidth. Only when all users are active at the same time they have to share the available bandwidth according to C/n. Thus, statistical multiplexing can never be worse than static frequency multiplexing (ignoring practical issues such as signaling overheads). Figure 2.2 illustrates the basic concept of dynamic spectrum allocation. Different colors correspond to different active users (here we have six active users) and the area of the color determines the transmitted data size. Depending on how many users are active, the available capacity is shared (equally) among them so that the user throughput becomes a statistical variable. I model the throughput as continuous flows which are limited by the data size and the available bandwidth.

In reality, such flows are not continuous but discrete. Let us for example consider Long Term Evolution (LTE) downlink which employs Orthogonal Frequency Division

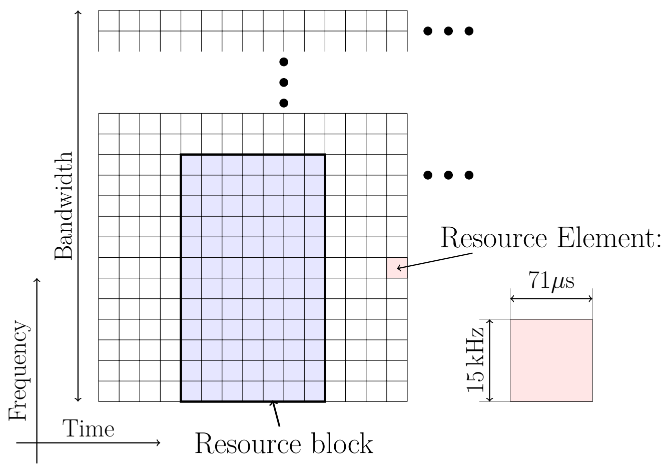

Fig. 2.3 In LTE, the smallest possible unit that can be assign to a user is a resource block, that is, 180 kHz × 0.5 ms.

Multiplexing (OFDM) as modulation technique, 3GPP (2010). OFDM can be thought of transmitting symbols over a rectangular time-frequency grid whereas each of such resource elements has a size of approximately 15 kHz × 71μs, see Figure 2.3. For a 10 MHz LTE signal, only 9 MHz can be used, so that 9 MHz/15kHz=600 subcarriers are available. In theory, each of this subcarriers could be assigned to a specific user at each time interval of 71 μs. However, in order to keep the signaling overhead small, the base station assigns only resource blocks consisting of 12 subcarriers × 7 OFDM symbols, corresponding to 180 kHz × 0.5 ms. There exist different strategies to assign the resource blocks to specific users, Muller et al. (2013), whereas round robin scheduling is the most practical one due to its simplicity. Here, all active users get the same share of the time-frequency resources without taking the channel quality into account. Our dynamic spectrum allocation model that shares the bandwidth equally reflects exactly such round robin scheduling.

Each resource element carriers a maximum of 6 bits (defined by the standard), so that the maximal data rate (capacity) for a single-input-single-output 10 MHz LTE system is given by 6 bits × 600 subcarriers/71μs = 50Mbit/s. In reality, however, the data rate is lower because coding, pilot symbols and signaling overheads have to be included.

2.2.1 Poisson Process

The starting time for the data transmission is modeled by a Poisson process. The reason behind this assumption is illustrated in Figure 2.4 and Equation (2.3). In Figure

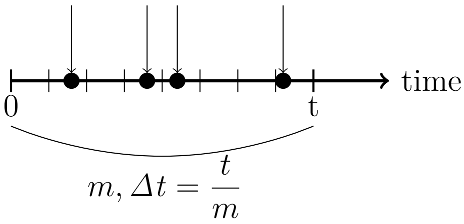

Fig. 2.4 The time interval $[0, t]$ is split into m smaller intervals Δt.

2.4, a large time interval t is split into m small time intervals Δt. The probability that a data transmissions start in Δt is given by $\lambda\,\Delta t$, whereas λ represents the arrival rate. Because $\lambda\,\Delta t$ is very small, only one data transmission can start within the interval Δt. By assuming that the arrival process is independent between different intervals, the probability that k transmission starts within the time interval t is given by a binomial distribution. Finally, by letting m go to infinity, we obtain:

$$P(k \text{ in } [0,t]) = \lim_{m\to\infty} \left\{ \binom{m}{k} (\lambda\,\Delta t)^k (1-\lambda\,\Delta t)^{m-k} \right\} \tag{2.3}$$

$$= \frac{(\lambda t)^k}{k!} \mathrm{e}^{-\lambda t}, \tag{2.4}$$

which is a Poisson process. Equation (2.4) thus describes the probability that k transmissions start within the time interval $[0, t]$.

Lemma 1. *Suppose the statistically independent random variables X_1 and X_2 are Poisson distributed with $\lambda_1 t$ and $\lambda_2 t$. Then, the distribution of the random variable $Y = X_1 + X_2$ is again Poisson distributed with $\lambda = \lambda_1 + \lambda_2$.*

Proof. Lemma 1 can straightforwardly be obtained by the fact that the probability mass function (pmf) of the sum of two statistically independent random variables is given by the convolution of the individual pmfs:

$$P_Y(k) = \sum_{i=-\infty}^{\infty} P_{X_1}(i) P_{X_2}(k-i) \tag{2.5}$$

$$= \sum_{i=0}^{k} \mathrm{e}^{-\lambda_1 t} \frac{(\lambda_1 t)^i}{i!} \mathrm{e}^{-\lambda_2 t} \frac{(\lambda_2 t)^{(k-i)}}{(k-i)!} \tag{2.6}$$

$$= \mathrm{e}^{-(\lambda_1+\lambda_2)t} \sum_{i=0}^{k} \binom{k}{i} \frac{(\lambda_1 t)^i (\lambda_2 t)^{(k-i)}}{k!} \tag{2.7}$$

$$= \mathrm{e}^{-(\lambda_1+\lambda_2)t} \frac{[(\lambda_1+\lambda_2)t]^k}{k!}. \tag{2.8}$$

□

Lemma 1 provides an important property of a Poisson processes and states that, if there are n user and the starting time of each user is modeled by a Poisson process with arrival rate λ, then the overall starting time follows also a Poisson process with arrival rate $n\lambda$.

For a numerical example, I assume a capacity of $C = 45\,\text{Mbit/s}$ and that the average user needs 1 GB data per month. Furthermore, the size $\tilde{\sigma}$ of the data packets is exponential distributed with mean $\sigma = 1\,\text{MB}$. On average, the number of connection buildups in one month is given by $1\,\text{GB}/1\,\text{MB} = 1000$, so that the arrival rate λ becomes:

$$\lambda = 1000\,\text{month}^{-1} = 413 \times 10^{-6}\,\text{s}^{-1}. \tag{2.9}$$

The required service time μ^{-1} (to complete the data transmission at full capacity) depends on the capacity and the size of the data packet according to $\mu^{-1} = \frac{\tilde{\sigma}}{C}$. Because of this linear relationship, the service time is also exponential distributed with service rate:

$$\mu = \frac{45\,\text{Mbit/s}}{1\,\text{MB}} = 5.625\,\text{s}^{-1}. \tag{2.10}$$

2.2.2 Average User Throughput

As already mentioned earlier, the user throughput is a statistical variable which depends on the number of active users. As performance measure I define the average user throughput as average packet size over average time needed to transmit packets. Waiting time and capacity are interchangeable. For example, one can transmit from $0 \ldots T/2$ at rate zero and from $T/2 \ldots T$ at a rate C, or one can transmit at rate $C/2$ over the whole time interval. In both cases, the same amount of data is transmitted in the intervall $0 \ldots T$. The mean response time of our process sharing system can therefore be modeled by a queuing system (Erlang C) with one service, that is, M/M/1 queue. The analyses of such queue system dates back to the beginning of the 20th century and describes the waiting time in telephone networks. The basic idea is the same as in the derivation of the Poisson process: the probability that in the next time interval Δt, the number of current calls is increased by one, is given by $\lambda\,\Delta t$.

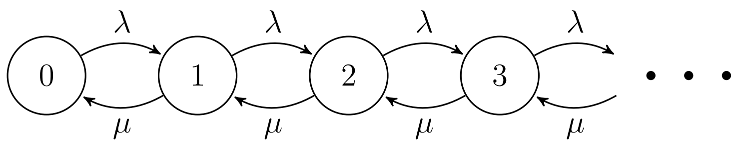

Fig. 2.5 Markov chain that describes the birth-death process of a M/M/1 queue. The states represent the number of active users.

Similar, the probability that one call is cleared in the same time interval is given by $\mu\,\Delta t$. Interpreting this as a birth–death process and writing such model as a Markov chain for the limit case of $\Delta t \longrightarrow 0$, results in a Markov chain as shown in Figure 2.5. The calculation of the steady state equilibrium of this Markov chain then gives us the probability that k users are active:

$$P(k \text{ users active}) = \rho^k(1-\rho), \tag{2.11}$$

where ρ is the traffic load. If there are n users who all have the same arrival rate λ, then the overall arrival rate is given by $n\lambda$ (see Lemma 1) and therefore the load ρ by:

$$\rho = \frac{\lambda}{\mu} = \frac{n\lambda\sigma}{C}. \tag{2.12}$$

Note that a steady state equilibrium exists only if $0 \leq \rho < 1$. The M/M/1 queue system delivers a certain average waiting time. Because our throughput measure is defined as expected packet size over expected response time, the average user throughput γ for our dynamic spectrum allocation system becomes, Fred et al. (2001):

$$\gamma = C(1-\rho) = C - n\lambda\sigma. \tag{2.13}$$

By comparing Equation (2.13) with Equation (2.11) we see that the throughput is given by the capacity times the probability that no user is active ($k = 0$). An important property of γ is the following: Suppose m companies of equal size merge (they have the same number of subscribers and the same bandwidth). According to Equation (2.13), the throughput then increases by a factor of m. As comparison, for static frequency multiplexing the throughput would stay constant. Thus, from this point of view, a single firm provides a higher throughput than many firms because a single firm can better exploit the statistical multiplexing gain.

Suppose there exist two groups of users, primary users and secondary users. I assume that a spectrum holder can give access rights to these user groups in two

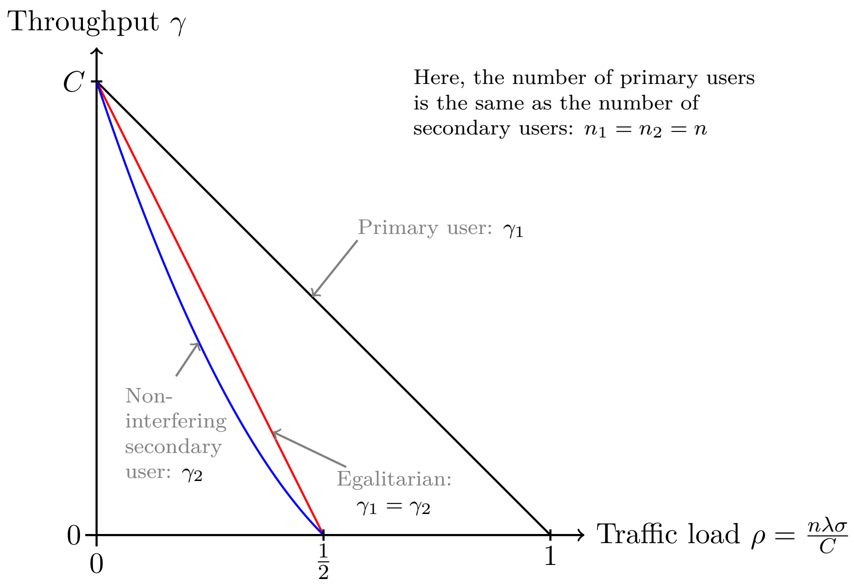

Fig. 2.6 The average user throughput for primary and secondary users for the case of discriminatory access and egalitarian access.

different ways:

- Discriminatory (non-interfering): the primary users have priority over the secondary users. This means, as soon as a primary user wants to have access, the instantaneous throughput of the secondary user becomes zero. Only when no primary users are active, secondary users can transmit/receive data.
- Egalitarian: The primary and secondary user all have the same priority.

Suppose primary users generate a traffic load ρ_1 while secondary users generate a traffic load ρ_2. Let us first consider the egalitarian case. Because both users have the same priority, we simply have to increase the traffic load by $\rho_1 + \rho_2$, so that the average user throughput becomes:

$$\gamma_1 = \gamma_2 = C(1 - [\rho_1 + \rho_2]) = C - (n_1 + n_2)\lambda\sigma. \tag{2.14}$$

On the other hand, in case of discriminatory access, the primary user throughput γ_1 and the secondary user throughput γ_2 are different and given by:

$$\gamma_1 = C(1 - \rho_1) = C - n_1\lambda\sigma \tag{2.15}$$

$$\gamma_2 = C(1 - [\rho_1 + \rho_2])(1 - \rho_1) = (C - [n_1 + n_2]\lambda\sigma)\left(1 - \frac{n_1\lambda\sigma}{C}\right). \tag{2.16}$$

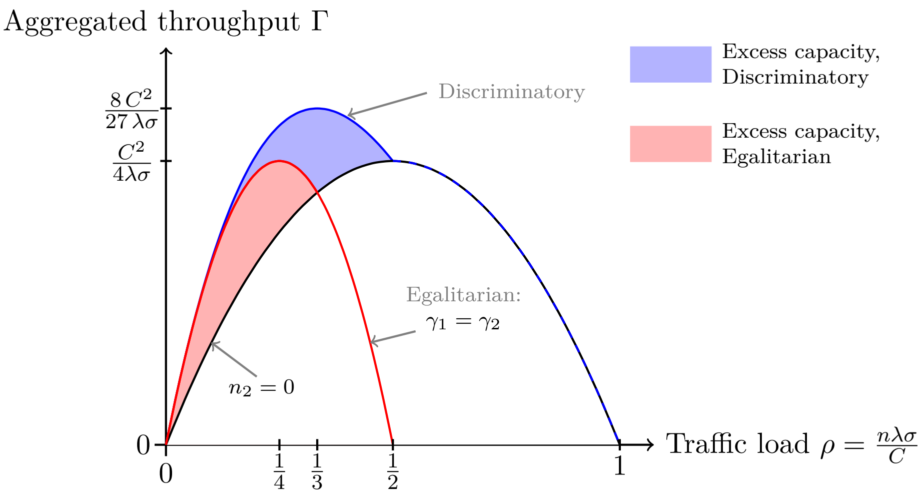

Fig. 2.7 The aggregated throughput, a possible measure for social optimality. Discriminatory access allows a better utilization of the excess capacity than egalitarian access.

The proof of Equation (2.15) is straightforward: because secondary user do not affect primary users, we have the same situation as if secondary users would not exist. The proof of Equation (2.16) can be obtained by using the Equations in Altman et al. (2006) for the special case of $g_1 = 1$, $g_2 = 0$ and by considering the throughput definition given in Fred et al. (2001), which relates the average waiting time to the throughput. The interpretation of Equation (2.16) is as follows:

$$\gamma_2 = \Pr(\text{no active users } 2 \wedge \text{no active users } 1)\, \underbrace{C \Pr(\text{no active users } 1)}_{\text{primary user througput}} . \tag{2.17}$$

Figure 2.6 plots the user throughput for the case of egalitarian access, that is, Equation (2.14), as well as for the case of discriminatory access, that is, Equations (2.15) and (2.16). To keep the illustration simple I assume that both user groups have the same arrival rate $n\lambda$.

Besides the individual user throughput shown in Figure 2.6, a good measure for social optimality is the aggregated throughput which I define as:

$$\Gamma = n_1\gamma_1 + n_2\gamma_2. \tag{2.18}$$

Figure 2.7 shows the aggregated throughput, given by Equation (2.18), for the case of $n = n_1 = n_2$ (if not stated otherwise). In general, for an increasing traffic load, the aggregated throughput increases, reaches a maximum and then decreases again due to congestion of the system. Let us assume that in the beginning there are no secondary users, that is, $n_2 = 0$ (the black curve). If an operator then decides to allow secondary users in his network, he can do this either by discriminatory access (blue curve) or by egalitarian access (red curve), as explained in this section. For a small traffic load, discriminatory and egalitarian access show a similar performance. However, the aggregated throughput for the egalitarian case soon decreases due to strong congestion and even becomes worse than the case where $n_2 = 0$. This is an interesting observation and shows that the network operator should deny access to secondary users as soon as the traffic load is higher than 1/3 (for the egalitarian case), if he aims at maximizing the aggregated throughput (of course this comes at the expense of secondary users). If the network operator employs discriminatory access, the performance can never be worse than for $n_2 = 0$ because secondary users only access the network if no primary users are active. The shaded blue area describes the excess capacity which is used by secondary users. For a traffic load greater than 1/2, this excess capacity becomes zero because primary users occupy the bandwidth more than half of the time, so that secondary users would completely congest the remaining resources. However, if the number of secondary users can be decreased, $n_2 < n_1$, it is possible to preserve an excess capacity even for such higher traffic loads.

2.3 Demand Function

In order to keep the model simple, I assume a linear demand function which not only includes the price (as usually in literature), but also captures the quality of the mobile network, that is, the average user throughput. The number of subscribers N_{M} for a monopoly is given by:

$$N_{\mathrm{M}} = \alpha_{\mathrm{M}} \gamma_{\mathrm{M}} - \beta_{\mathrm{M}} p_{\mathrm{M}}, \tag{2.19}$$

whereas γ_{M} describes the throughput, that is, the quality of our wireless system, and p_{M} the price a subscriber has to pay in order to get access to the mobile network (which provides an average data transmission rate of γ_{M}). The parameters α_{M} and β_{M} capture how much the demand changes if the quality or the price changes. For example, suppose $\alpha = 200\,000\,\mathrm{Subscriber/MB/s}$, then, a throughput increase of 1MB/s generates 200 000 new subscribers. Such assumption makes sense because we expect that a higher data rate also increases the number of subscribers. Consider for example the case of video streaming. A high data rate allows a better quality of video streams so that people who did not paid for data access before, because the price was too high for the given quality of the video stream, now want to pay for data access. Another possible reason could be that high data rates tempt users to replace their cable access in favor of mobile access. There are a large range of possible reasons why a higher data rate leads to a higher demand. However, such investigation is beyond the scope of this thesis so that I assume all these possible reasons are captured by the product $\alpha_\mathrm{M}\gamma_\mathrm{M}$. Similar as α_M, suppose $\beta = 180\,000\,\mathrm{subscriber}/€$. If the price is decreased by one €, the number of subscribers increases by 180 000. The reason for such price behavior can be found in standard textbooks such as Mas-Colell et al. (1995).

Let us now consider the case of a duopoly. We extend the monopoly demand function, Equation (2.19), as suggested in Banker et al. (1998) by:

$$N_1 = \alpha_{11}\gamma_1 - \alpha_{12}\gamma_2 - \beta_{11}p_1 + \beta_{12}p_2 \tag{2.20}$$

$$N_2 = \alpha_{22}\gamma_2 - \alpha_{21}\gamma_1 - \beta_{22}p_2 + \beta_{21}p_1. \tag{2.21}$$

Compared to the monopoly, the quality and the price of one firm now affect the demand of the other firm. For example, suppose $\alpha_{22} = 200\,000\,\mathrm{Subscriber/MB/s}$ and $\alpha_{12} = 100\,000\,\mathrm{Subscriber/MB/s}$. If the second firm increases the average throughput γ_2 by $1\,\mathrm{MB/s}$, then the number of subscribers increases by 200 000 for firm 2 while for firm 1 it decreases by 100 000. Thus, 100 000 customers of firm 1 switch to firm 2 because the quality gets relatively better and 100 000 additional customers are acquired (which were neither subscribers of firm 1 or firm 2 before). A similar effect applies on the price. Such demand functions capture exactly the behavior we expect, although the assumption of a linear dependency might not be true in reality. However, because every function can be approximated by a linear function such simple model makes sense in order to investigate basic effects. According to our demand functions, it can happen that $\gamma_2 < \gamma_1$ and at the same time $p_2 > p_1$, thus, firm 2 provides a lower data rate at a higher prices than firm 1. Nevertheless, there might exist a demand for firm 2. Can this happen in reality? The answer is yes. Consider, for example different deployment strategies of base stations for the two firms. The average throughput γ assumes that the signal-to-noise ratio is averaged over all geographical locations, see Section 2.1. It can happen that the deployment of base stations of firm 2 is more advantageous for a small subset of subscribers, so that they prefer firm 2 over firm 1 even though the price is higher. Another possibility might be that changing the network operator might require some time effort which the remaining subscribers of firm 2 do not want to invest. All these effects could be modeled in more detail but is beyond the scope of this thesis. Note that I follow the conclusion in Banker et al. (1998) and assume $\alpha_{12} < \alpha_{11}$, $\alpha_{21} < \alpha_{22}$, $\beta_{12} < \beta_{11}$ and $\beta_{21} < \beta_{22}$.

To compare the duopoly with the monopoly, one can set $\gamma_2 = 0$ and $p_2 = 0$ which delivers the same Equation as in case of a monopoly. However, depending on how large $\alpha_{12}\gamma_2$ and $\beta_{12}p_2$ were at the beginning, the new demand N_1 could be higher or lower then before setting γ_2 and p_2 to zero. The reason for this behavior is the large range of possible values of α_{xx} and β_{xx}. To circumvent this problem I define the property **comparable** as the case where $N_\mathrm{M} = N_1 + N_2$ for $p_1 = p_2 = p_\mathrm{M}$ and $\gamma_1 = \gamma_2 = \gamma_\mathrm{M}$. This restricts the domain of α_M and β_M to:

$$\alpha_\mathrm{M} = \alpha_{11} - \alpha_{21} + \alpha_{22} - \alpha_{12} \tag{2.22}$$

$$\beta_\mathrm{M} = \beta_{11} - \beta_{21} + \beta_{22} - \beta_{12} \tag{2.23}$$

Such comparable demand function have another advantage, if we want to analyze our model over a large range of possible competition parameters β_{12} and β_{21}. To understand the problem, let us assume that β_{12} and β_{21} are close to β_{11} and β_{22}. Both firms can then jointly increase their prices which generates a much higher profit than in case of $\beta_{12} = \beta_{21} = 0$. Thus, the Nash equilibrium prices might be unreasonably high due to this multiplier effect. By keeping α_M and β_M constant and by using comparable demand functions, Equations (2.22) and (2.23), we can straightforwardly avoid such problems.

Chapter 3

Reference Markets

In this chapter, I use the basic elements developed in Chapter 2 to model the behavior of mobile network operators. For simplicity, I assume that there are no marginal costs, that is, additional subscribers do not increase the costs of a network operator. The strategic choices a network operator faces are:

- Price p: each subscriber has to pay the price p in order to have access to the mobile network. This price in combination with the demand determines the profit of the operator.
- Number of base station B: The network operator has to deploy base stations in order to provide a certain data rate, see Section 2.1. Each base station increases the user throughput but also generates costs of size F for the operator.

3.1 Monopoly

As described in Chapter 2, the number of subscribers per cell is given by $n = N/B$ with N being the overall number of subscribers and B the number of base stations. Combining the monopoly demand, Equation (2.19), and the average user throughput, Equation (2.13), leads to:

$$N = \alpha \left(C - \frac{N}{B} \lambda \sigma \right) - \beta p. \tag{3.1}$$

Note that the number of subscribers appears on the left side as well as on the right side of the equation because the quality of the system is affected by the number of subscribers, that is, a larger number of subscribers decreases the throughput because user have to share the same resources (statistically). Rewriting Equation (3.1) with respect to N delivers:

$$N = \frac{\alpha}{1+\frac{\alpha\lambda\sigma}{B}}C - \frac{\beta}{1+\frac{\alpha\lambda\sigma}{B}}p. \tag{3.2}$$

A monopolist maximizes the profit by choosing a price of:

$$p_{\mathrm{M}} = \arg\max_{p} \{Np - FB\} \tag{3.3}$$

$$p_{\mathrm{M}} = \frac{\alpha C}{2\beta}, \tag{3.4}$$

which is independent of the number of base stations. Note that the monopolist price p_{M} depends linearly on the capacity C. Thus, if the capacity is doubled, for example because the operator obtains more bandwidth, the price is also doubled. The reason for such behavior is the underlying linear demand function. In reality, we might expect that the price does not scale linearly with the maximal data rate (bandwidth) due to diminishing marginal utility. However, in order to keep the model simple and to investigate basic effects of statistical multiplexing, such assumption makes sense. Note also that the price depends on the ratio of α and β. The monopolist can further increase the profit by choosing the optimal number of base stations according to:

$$B_{\mathrm{M}} = \arg\max_{B} \left\{ \left[\frac{\alpha}{1+\frac{\alpha\lambda\sigma}{B}}C - \frac{\beta}{1+\frac{\alpha\lambda\sigma}{B}}p_{\mathrm{M}} \right] p_{\mathrm{M}} - FB \right\} \tag{3.5}$$

$$B_{\mathrm{M}} = p_{\mathrm{M}}\sqrt{\frac{\alpha\beta\lambda\sigma}{F}} - \alpha\lambda\sigma. \tag{3.6}$$

The product $\lambda\sigma$ determines how much data one user needs on average over a certain time, for example, 1 GB per month. Equation (3.6) shows an interesting property: for an increasing $\lambda\sigma$, the number of base stations B_{M} increases, reaches a maximum at $\lambda\sigma = \frac{Cp_{\mathrm{M}}}{8F}$, and decreases again. Note also that Equation 3.6 can become negative which is clearly not feasibly and only a mathematical result of the underlying linear functions. I assume throughout the whole thesis that the variables are chosen in such a way that everything is always positive. The maximal profit for the monopolist is given by:

$$\pi_{\mathrm{M}} = \frac{C^2\alpha^2}{4\beta} + F\alpha\lambda\sigma - \sqrt{\frac{C^2F\alpha^3\lambda\sigma}{\beta}}, \tag{3.7}$$

and the average throughput a user experiences by:

$$\gamma_{\mathrm{M}} = C - \sqrt{F\lambda\sigma\frac{\beta}{\alpha}}. \tag{3.8}$$

Let us consider the following simple example:

Capacity:	$C = 45\,\mathrm{MB/s}$
Traffic:	$\lambda = 0.5 \cdot 10^{-3}\,\mathrm{s}^{-1}$
	$\sigma = 1\,\mathrm{MB}$
Demand:	$\alpha = 200\,000\,\mathrm{subscriber/MB/s}$
	$\beta = 180\,000\,\mathrm{subscriber/€}$
Bast station costs	$F = 100\,000\,€/\mathrm{Basestation}$

Using Equations (3.4) and (3.6), the monopolist choses the price p_{M} and the number of base station B_{M} according to:

$$\begin{aligned} p_{\mathrm{M}} &= 25\,€ \\ B_{\mathrm{M}} &= 235\,\mathrm{Basestations}, \end{aligned}$$

so that the user throughput γ_{M}, the monopoly profit π_{M} and the number of subscribers N_{M} become:

$$\begin{aligned} \gamma_{\mathrm{M}} &= 38.2\,\mathrm{MB/s} \\ \pi_{\mathrm{M}} &= 55.4\,\mathrm{Million}\,€ \\ N_{\mathrm{M}} &= 3.16\,\mathrm{Million\ subscriber} \end{aligned}$$

3.2 Duopoly

Similar as in the monopoly, we can write the demand for the duopoly by inserting Equation (2.13) in Equations (2.20) and (2.21):

$$N_1 = \alpha_{11}\left(C_1 - \frac{N_1}{B_1}\lambda\sigma\right) - \alpha_{12}\left(C_2 - \frac{N_2}{B_2}\lambda\sigma\right) - \beta_{11}p_1 + \beta_{12}p_2 \tag{3.9}$$

$$N_2 = \alpha_{22}\left(C_2 - \frac{N_2}{B_2}\lambda\sigma\right) - \alpha_{21}\left(C_1 - \frac{N_1}{B_1}\lambda\sigma\right) - \beta_{22}p_2 + \beta_{21}p_1. \tag{3.10}$$

Combining Equations (3.9) and (3.10) and solving them with respect to N_1 and N_2, leads to:

$$N_1 = \tilde{\alpha}_{11}C_1 - \tilde{\alpha}_{12}C_2 - \tilde{\beta}_{11}p_1 + \tilde{\beta}_{12}p_2 \tag{3.11}$$

$$N_2 = \tilde{\alpha}_{22}C_2 - \tilde{\alpha}_{21}C_1 - \tilde{\beta}_{22}p_2 + \tilde{\beta}_{21}p_1, \tag{3.12}$$

with

$$\tilde{\alpha}_{11} = \frac{\alpha_{11} + \frac{\lambda\sigma}{B_2}\left(\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}\right)}{\left(1 + \alpha_{11}\frac{\lambda\sigma}{B_1}\right)\left(1 + \alpha_{22}\frac{\lambda\sigma}{B_2}\right) - \alpha_{12}\alpha_{21}\frac{(\lambda\sigma)^2}{B_1B_2}} \tag{3.13}$$

$$\tilde{\alpha}_{12} = \frac{\alpha_{12}}{\left(1 + \alpha_{11}\frac{\lambda\sigma}{B_1}\right)\left(1 + \alpha_{22}\frac{\lambda\sigma}{B_2}\right) - \alpha_{12}\alpha_{21}\frac{(\lambda\sigma)^2}{B_1B_2}} \tag{3.14}$$

$$\tilde{\alpha}_{22} = \frac{\alpha_{22} + \frac{\lambda\sigma}{B_1}\left(\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}\right)}{\left(1 + \alpha_{11}\frac{\lambda\sigma}{B_1}\right)\left(1 + \alpha_{22}\frac{\lambda\sigma}{B_2}\right) - \alpha_{12}\alpha_{21}\frac{(\lambda\sigma)^2}{B_1B_2}} \tag{3.15}$$

$$\tilde{\alpha}_{21} = \frac{\alpha_{21}}{\left(1 + \alpha_{11}\frac{\lambda\sigma}{B_1}\right)\left(1 + \alpha_{22}\frac{\lambda\sigma}{B_2}\right) - \alpha_{12}\alpha_{21}\frac{(\lambda\sigma)^2}{B_1B_2}}, \tag{3.16}$$

and

$$\tilde{\beta}_{11} = \frac{\beta_{11} + \frac{\lambda\sigma}{B_2}\left(\beta_{11}\alpha_{22} - \alpha_{12}\beta_{21}\right)}{\left(1 + \alpha_{11}\frac{\lambda\sigma}{B_1}\right)\left(1 + \alpha_{22}\frac{\lambda\sigma}{B_2}\right) - \alpha_{12}\alpha_{21}\frac{(\lambda\sigma)^2}{B_1B_2}} \tag{3.17}$$

$$\tilde{\beta}_{12} = \frac{\beta_{12} + \frac{\lambda\sigma}{B_2}\left(\beta_{12}\alpha_{22} - \alpha_{12}\beta_{22}\right)}{\left(1 + \alpha_{11}\frac{\lambda\sigma}{B_1}\right)\left(1 + \alpha_{22}\frac{\lambda\sigma}{B_2}\right) - \alpha_{12}\alpha_{21}\frac{(\lambda\sigma)^2}{B_1B_2}} \tag{3.18}$$

$$\tilde{\beta}_{22} = \frac{\beta_{22} + \frac{\lambda\sigma}{B_1}\left(\beta_{22}\alpha_{11} - \alpha_{21}\beta_{12}\right)}{\left(1 + \alpha_{11}\frac{\lambda\sigma}{B_1}\right)\left(1 + \alpha_{22}\frac{\lambda\sigma}{B_2}\right) - \alpha_{12}\alpha_{21}\frac{(\lambda\sigma)^2}{B_1B_2}} \tag{3.19}$$

$$\tilde{\beta}_{21} = \frac{\beta_{21} + \frac{\lambda\sigma}{B_1}\left(\beta_{21}\alpha_{11} - \alpha_{21}\beta_{11}\right)}{\left(1 + \alpha_{11}\frac{\lambda\sigma}{B_1}\right)\left(1 + \alpha_{22}\frac{\lambda\sigma}{B_2}\right) - \alpha_{12}\alpha_{21}\frac{(\lambda\sigma)^2}{B_1B_2}}. \tag{3.20}$$

I assume a two-stage game: In a fist step, the firms choose the number of base stations simultaneously. In a second step, they choose the price simultaneously. The reason why the number of base stations are chosen in a first step is the fact that the deployment of base stations reflects a long term decision and takes a long time. Due to perfect information and a finite game, we can find a subgame perfect Nash equilibrium by backward induction. The best response functions of the second stage can be written as:

$$p_1(p_2) = \arg\max_{p1}\{p_1 N_1\} \tag{3.21}$$

$$p_1(p_2) = \frac{\tilde{\alpha}_{11}C_1 - \tilde{\alpha}_{12}C_2 + \tilde{\beta}_{12}p_2}{2\tilde{\beta}_{11}}, \tag{3.22}$$

and

$$p_2(p_1) = \arg\max_{p2}\{p_2 N_2\} \tag{3.23}$$

$$p_2(p_1) = \frac{\tilde{\alpha}_{22}C_2 - \tilde{\alpha}_{21}C_1 + \tilde{\beta}_{21}p_1}{2\tilde{\beta}_{22}}, \tag{3.24}$$

so that the subgame perfect Nash equilibrium becomes:

$$p_1^* = \frac{(2\tilde{\beta}_{22}\tilde{\alpha}_{11} - \tilde{\beta}_{12}\tilde{\alpha}_{21})C_1 - (2\tilde{\beta}_{22}\tilde{\alpha}_{12} - \tilde{\beta}_{12}\tilde{\alpha}_{22})C_2}{4\tilde{\beta}_{11}\tilde{\beta}_{22} - \tilde{\beta}_{12}\tilde{\beta}_{21}} \tag{3.25}$$

$$p_2^* = \frac{(2\tilde{\beta}_{11}\tilde{\alpha}_{22} - \tilde{\beta}_{21}\tilde{\alpha}_{12})C_2 - (2\tilde{\beta}_{11}\tilde{\alpha}_{21} - \tilde{\beta}_{21}\tilde{\alpha}_{11})C_1}{4\tilde{\beta}_{11}\tilde{\beta}_{22} - \tilde{\beta}_{12}\tilde{\beta}_{21}}. \tag{3.26}$$

Equations (3.25) and (3.26) show that the equilibrium prices ($p_1^* = p_2^*$) are the same for both firms if the underlying capacities, demand, and costs are the same for both firms. We can use the subgame perfect Nash equilibrium prices p_1^* and p_2^*, Equations (3.25) and (3.26), to find the best response functions according to:

$$B_1(B_2) = \arg\max_{B1}\{p_1^* N_1 - B_1 F_1\} \tag{3.27}$$

$$B_2(B_1) = \arg\max_{B2}\{p_2^* N_2 - B_1 F_1\}, \tag{3.28}$$

which deliver the Nash equilibrium number of base stations. Unfortunately, Equations (3.27) and (3.28) cannot be formulated in closed form, so that numerical methods have to be used. Figure 3.1 shows a numerical example of the best response functions and the corresponding Nash equilibrium. In particular, we see that for $\alpha_{12} = \alpha_{21} = 0$ (the throughout of firm 2 does not influence the demand of firm 1 and vice versa) the optimal number of base stations becomes independent of the competition. Indeed, for the special case of $\alpha_{12} = \alpha_{21} = 0$ we even find closed form solutions for the Nash

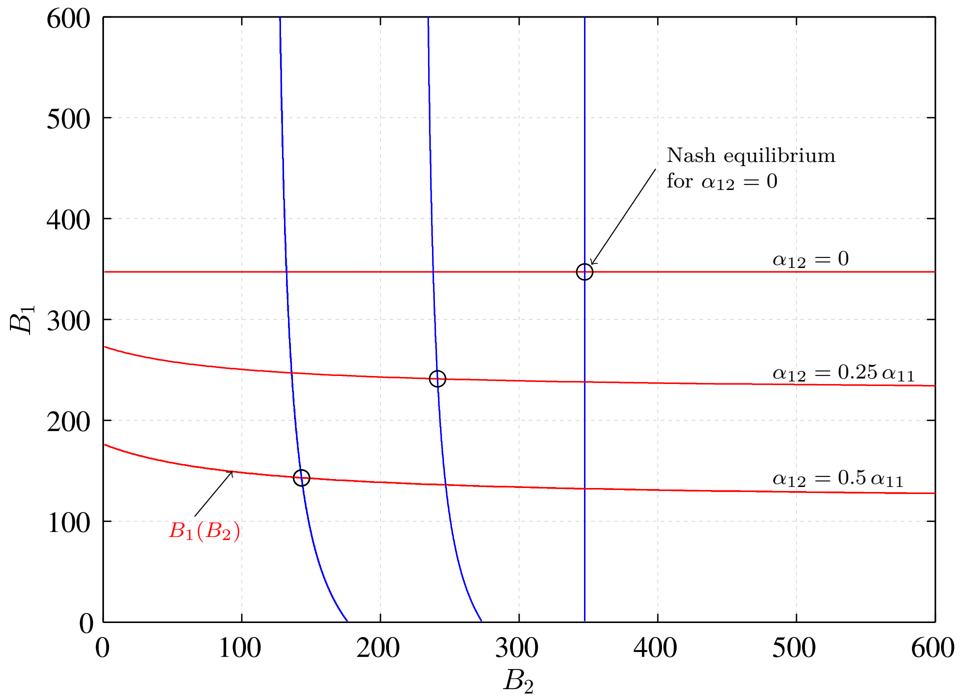

Fig. 3.1 Best response functions of the first stage in the duopoly. The Nash equilibrium determines how many base stations are deployed.

equilibrium by:

$$p_1^* = \frac{2\beta_{22}\alpha_{11}C_1 + \beta_{12}\alpha_{22}C_2}{4\beta_{11}\beta_{22} - \beta_{12}\beta_{21}} \tag{3.29}$$

$$p_2^* = \frac{2\beta_{11}\alpha_{22}C_2 + \beta_{21}\alpha_{11}C_1}{4\beta_{11}\beta_{22} - \beta_{12}\beta_{21}}, \tag{3.30}$$

and

$$B_1^* = p_1^* \sqrt{\frac{\alpha_{11}\beta_{11}\lambda\sigma}{F_1}} - \alpha_{11}\lambda\sigma \tag{3.31}$$

$$B_2^* = p_2^* \sqrt{\frac{\alpha_{22}\beta_{22}\lambda\sigma}{F_2}} - \alpha_{22}\lambda\sigma. \tag{3.32}$$

By comparing the Nash equilibrium number of base stations for $\alpha_{12} = \alpha_{21} = 0$, Equations (3.31) and (3.32), to the monopoly case, Equation (3.6), we see that they have the same structure. Furthermore, by considering the case of comparable demand (Section 2.3) and assuming the same capacities, costs and the same demand parameters (α_{xx} and β_{xx}) for both firms, the Nash equilibriums of Equations (3.29) to (3.32) transform to:

$$p_1^* = p_2^* = \frac{(1-\epsilon)}{\left(1-\frac{\epsilon}{2}\right)}\, p_\mathrm{M} \tag{3.33}$$

$$B_1^* = B_2^* = \frac{1}{2}\left[\frac{\sqrt{1-\epsilon}}{\left(1-\frac{\epsilon}{2}\right)}\underbrace{p_\mathrm{M}\sqrt{\frac{\alpha_\mathrm{M}\beta_\mathrm{M}\lambda\sigma}{F}}-\alpha_\mathrm{M}\lambda\sigma}_{B_\mathrm{M}}\right], \tag{3.34}$$

whereas the variable ϵ describes the ratio between how much the demand increases if the price of the competitor increases to how much the demand decreases if the own price increases, that is, $\epsilon = \frac{\beta_{12}}{\beta_{11}}$. Because ϵ is limited between zero and one, $0 \leq \epsilon < 1$, the Nash equilibrium prices are always smaller than the monopoly price, $p_1^* = p_2^* \leq p_\mathrm{M}$, and the numbers of base stations in the duopoly is smaller then the number of base stations in the monopoly, divided by two, that is, $B_1^* = B_2^* \leq \frac{B_\mathrm{M}}{2}$. The reason why I assume the same capacities for a monopoly and a duopoly in this considerations, $C_1 = C_2 = C_\mathrm{M}$, is the fact that the bandwidth and therefore the capacity is limited by the standard. For example, LTE allows a maximal bandwidth of 20 MHz. The newer releases of the LTE-Advanced standards now include also the so called carrier aggregation which allows to increase the available bandwidth. However, implementing a new technology usually takes a long time so that the market penetration of carrier aggregation is still relatively low. By setting the variable ϵ to zero, the duopoly becomes similar to the monopoly case. In particular, the strategic choices of the monopoly and duopoly are $p_1^* = p_2^* = p_\mathrm{M}$ and $B_1^* + B_2^* = B_\mathrm{M}$, so that the throughput is also the same $\gamma_1^* = \gamma_2^* = \gamma_\mathrm{M}$. However, in the duopoly we need overall twice the bandwidth as in the monopoly case. Thus, a monopoly is more bandwidth efficient than a duopoly, as already pointed out in Section 2.2.2.

Let us again consider a simple example, similar to the monopoly case. The parameters are chosen so that the duopoly demand functions are comparable to the monopoly case, see Section 2.3 for the definition of comparability.

Capacity:	$C_1 = C_2 = 45\,\mathrm{MB/s}$
Traffic:	$\lambda = 0.5 \cdot 10^{-3}\,\mathrm{s}^{-1}$
	$\sigma = 1\,\mathrm{MB}$
Demand:	$\alpha_{11} = \alpha_{22} = 200\,000\,\mathrm{subscriber/MB/s}$
	$\alpha_{12} = \alpha_{21} = 100\,000\,\mathrm{subscriber/MB/s}$

$$\beta_{11} = \beta_{22} = 180\,000\,\text{subscriber}/€$$
$$\beta_{12} = \beta_{21} = 90\,000\,\text{subscriber}/€$$

Bast station costs: $F = 100\,000$ €/Basestation

Using numerical simulation on Equations (3.27) and (3.28), for the first stage, and using Equations (3.25) and (3.26), for the second stage, leads to the following Nash equilibrium:

$$p_1^* = p_2^* = 20\,€$$
$$B_1^* = B_2^* = 143\,\text{Basestations.}$$

This Nash equilibrium results in the following user throughput, profits and number of subscribers:

$$\gamma_1^* = \gamma_2^* = 38\,\text{MB/s}$$
$$\pi_1^* = \pi_2^* = 25.8\,\text{Million}\,€$$
$$N_1^* = N_2^* = 1.99\,\text{Million subscriber}$$

Compared to the monopoly, Section 3.1, the price is lower in the duopoly while the performance of the system (throughput) is almost the same. Thus, the consumers are better of. The lower price leads to a higher number of total subscribers $N_1^* + N_2^*$. In contrast to the case of $\alpha_{12} = \alpha_{21} = 0$, the total number of base stations is now higher in the duopoly, $B_1^* + B_2^* > B_{\mathrm{M}}$, so that the negative effects of a higher number of subscribers on the throughput is compensated, leading to almost the same throughput as in the monopoly. However, we always have to keep in mind that the benefits for the consumers comes at the expense of a worse spectrum efficiency because the duopoly needs twice the bandwidth of the monopoly here.

3.3 Social Optimality

Maximizing Social Welfare, $\pi = -FB$

One possible method to investigate social optimality is the well-known social welfare, measured by the aggregated Marshallian surplus, Mas-Colell et al. (1995). With respect to social welfare, a monopoly is inefficient because the price is higher then the marginal costs, leading to a deadweight loss. Due to marginal costs of zero in our model, the social welfare is maximize by setting the price to zero:

$$p_{\mathrm{maxW}} = 0. \tag{3.35}$$

However, a price of zero leads to no revenues while the deployment of base stations still generate costs, resulting in a (negative) profit of $\pi = -FB$. Thus, the government has to pay for the deployment of base stations by using taxes obtained from other sources. The number of base stations that maximize the social welfare (consumer surplus plus producer surplus, which is negative and given by FB) for $p_{\mathrm{maxW}} = 0$, is found by:

$$B_{\mathrm{maxW}} = \arg\max_{B} \left\{ \frac{1}{2} \left(\frac{\alpha C}{\beta} \frac{\alpha C}{1 + \frac{\alpha\lambda\sigma}{B}} \right) - FB \right\} \tag{3.36}$$

$$B_{\mathrm{maxW}} = \frac{C}{\sqrt{2}} \sqrt{\frac{\alpha^3 \lambda \sigma}{F\beta}} - \alpha\lambda\sigma \tag{3.37}$$

$$B_{\mathrm{maxW}} = \sqrt{2} B^{\mathrm{M}} + \alpha\lambda\sigma(\sqrt{2} - 1). \tag{3.38}$$

From a social welfare point of view, Equations (3.35) and (3.38) show that a monopolist chooses a price that is too high and at the same time does not deploy enough base station.

Maximizing Social Welfare 2, $\pi = 0$

A price of zero, as considered in the previous subsection, leads to a negative profit which has to be compensated by the government using taxpayer money. In order to avoid this drawback, I allow a price that is higher than the marginal costs and chosen such that the profit is zero, $\pi = Np - FB = 0$. Of course, two possible prices satisfy this condition whereas maximizing the social welfare requires to chose the lower price.

Such price can be calculated by:

$$p_{\mathrm{maxW}_2} = p_{\pi=0} = \frac{\alpha C - \sqrt{\alpha^2 C^2 - 4F\beta(B + \alpha\lambda\sigma)}}{2\beta}, \tag{3.39}$$

and depends on the number of base station in an intermediate step. Because the producer surplus is zero, maximizing the social welfare becomes equivalent to maximizing the consumer surplus by choosing the optimal number of base stations according to:

$$B_{\mathrm{maxW}_2} = \arg\max_B \left\{ \frac{1}{2} \left(\frac{\alpha C}{\beta} - p_{\pi=0} \right) N(p_{\pi=0}) \right\} \tag{3.40}$$

$$B_{\mathrm{maxW}_2} = \arg\max_B \left\{ \frac{\left(\alpha C + \sqrt{\alpha^2 C^2 - 4F\beta(B + \alpha\lambda\sigma)} \right)^2}{8\beta \left(1 + \frac{\alpha\lambda\sigma}{B} \right)} \right\}. \tag{3.41}$$

Equation (3.41) can easily be solved in closed-form. However, the resulting equation is quite lengthy so that I omit them at this point. By inserting the optimal number of base stations, Equation (3.41), into Equation (3.39), we obtain the price that maximizes the social welfare under the constrain that the profit is zero.

Maximizing Aggregated Throughput

Another method to determine social optimality is to maximize the aggregated throughput, see Section 2.2.2. Again, I assume that the price is chosen so that the profit is zero, resulting in the same price as before, that is, Equation (3.39). The aggregate throughput, $\Gamma = N(p_{\pi=0})\gamma(p_{\pi=0})$, can then be written as:

$$\Gamma_{\pi=0} = \frac{B \left(B \left(C\sqrt{\alpha^2 \mathrm{C}^2 - 4\beta F(\alpha\lambda\sigma + B)} + \alpha C^2 + 2\beta F\lambda\sigma \right) + 2\alpha\beta F\lambda^2\sigma^2 \right)}{2(\alpha\lambda\sigma + B)^2}. \tag{3.42}$$

By maximizing Equation (3.42) with respect to B, we obtain the number of base stations that maximizes the aggregated throughput (under the constrain of zero profit):

$$B_{\max\Gamma} = \arg\max_B \left\{ \Gamma_{\pi=0}(B) \right\}. \tag{3.43}$$

Expressing Equation (3.43) in closed-form is possible but again omitted due to its lengthy nature. Inserting Equation (3.43) in (3.39) delivers the throughput maximizing price under the constrain that the profit is zero.

There exist an interesting relationship between maximizing the social welfare and maximizing the aggregated throughput, both under the constrain of zero profits: they have both the same number of subscribers, $N_{\max\Gamma} = N_{\max W_2}$. Per definition, we then have the following relationship $\Gamma_{\max\Gamma} > \Gamma_{\max W_2}$. This implies that maximizing the aggregated user throughput also leads to a higher individual throughput compared to maximizing the social welfare, $\gamma_{\max\Gamma} > \gamma_{\max W_2}$, which in turn is only possible if the number of base stations is higher, $B_{\max\Gamma} > B_{\max W_2}$, and consequently, $p_{\max\Gamma} > p_{\max W_2}$.

In later chapters I will mainly use the aggregates throughput as social optimality criteria. There are two reasons for that: Firstly, the demand functions depend on the quality so that the underlying theory in standard textbooks regarding aggregated Marshallian surplus might not be straightforwardly applicable. Secondly, accurate approximations of the real demand function by a linear demand function only work for prices close to some operation point (due to a Taylor approximation). The aggregated Marshallian surplus, however, uses a large price range which could lead to large errors.

Let us again consider a simple example by using the same parameters as in Section 3.1:

Capacity:	$C = 45\,\text{MB/s}$
Traffic:	$\lambda = 0.5 \cdot 10^{-3}\,\text{s}^{-1}$
	$\sigma = 1\,\text{MB}$
Demand:	$\alpha = 200\,000\,\text{subscriber/MB/s}$
	$\beta = 180\,000\,\text{subscriber/€}$
Bast station costs	$F = 100\,000\,\text{€/Basestation}$

The three social optimality conditions discussed in this section are labeled by “maxW”, “maxW2”, and “maxΓ”. The price and number of base stations (BS) have to be chosen according to:

$$p_{\max W} = 0\,\text{€} \qquad p_{\max W_2} = 5.6\,\text{€} \qquad p_{\max\Gamma} = 9.9\,\text{€}$$
$$B_{\max W} = 374\,\text{BS} \qquad B_{\max W_2} = 347\,\text{BS} \qquad B_{\max\Gamma} = 616\,\text{BS}$$

and lead to the following user throughput γ, number of subscribers N and aggregated throughput Γ:

$$\gamma_{\max W} = 35.5\,\text{MB/s} \qquad \gamma_{\max W_2} = 36.1\,\text{MB/s} \qquad \gamma_{\max\Gamma} = 40\,\text{MB/s}$$
$$N_{\max W} = 7.1\,\text{Million} \qquad N_{\max W_2} = 6.2\,\text{Million} \qquad N_{\max\Gamma} = 6.2\,\text{Million}$$
$$\Gamma_{\max W} = 252.2\,\text{TB/s} \qquad \Gamma_{\max W_2} = 223.8\,\text{TB/s} \qquad \Gamma_{\max\Gamma} = 248\,\text{TB/s}$$

The aggregated throughputs of all three social optimality criteria, as defined in this section, are approximately twice as high as for the monopoly (121 TB/s).

Due to a higher number of subscribers, the aggregated throughput for maximizing the social welfare (if negative profits are allowed) is higher than for maximizing the aggregated throughput. However, we have to keep in mind that the negative profit has to be compensated by, for example, the government. Maximizing the aggregated throughput under the constrain of zero profit requires a higher number of base stations compared to maximizing the social welfare (roughly 1.7 times higher) and to the case of a monopoly (2.5 times higher).

Chapter 4

MVNO

In Europe, Mobile Virtual Network Operator (MVNO) gained popularity while in Asia they have a dismal track record because the possible gain of MVNOs depend strongly on the market structure, as pointed out in Shin and Bartolacci (2007). In my model, different demand functions can be modeled by setting the parameters α_{xx} and β_{xx} accordingly, see Section 2.3, which lead to different market structures. In order to model the decision whether a MVNO should be allowed, I assume that the incumbent starts as a monopoly who can then decide if he should resell the right to access his network infrastructure and spectrum to a MVNO, or not. In the short term, I assume that the number of base stations is fixed and given by B^{M}, see Equation (3.6). I then assume a sequential game with 4 stages (see also Figure 4.1):

1. The incumbent decides whether he should resell his network infrastructure to a MVNO or not.

2. The incumbent sets the price p_1 his subscribers have to pay in order to get access to the network and thus a certain throughput.

3. The incumbent sets the price p_R the MVNO has to pay in order to use the network infrastructure (and spectrum) of the incumbent.

4. The MVNO sets the price p_2 his subscribers have to pay.

As discussed in Section 2.2.2, I assume that the incumbent can offer two possible quality (throughput) levels: Firstly, the case of egalitarian access where the subscribers of the incumbent and the subscribers of the MVNO have the same priority and thus the same throughput. Secondly, the case of discriminatory access where the subscribers

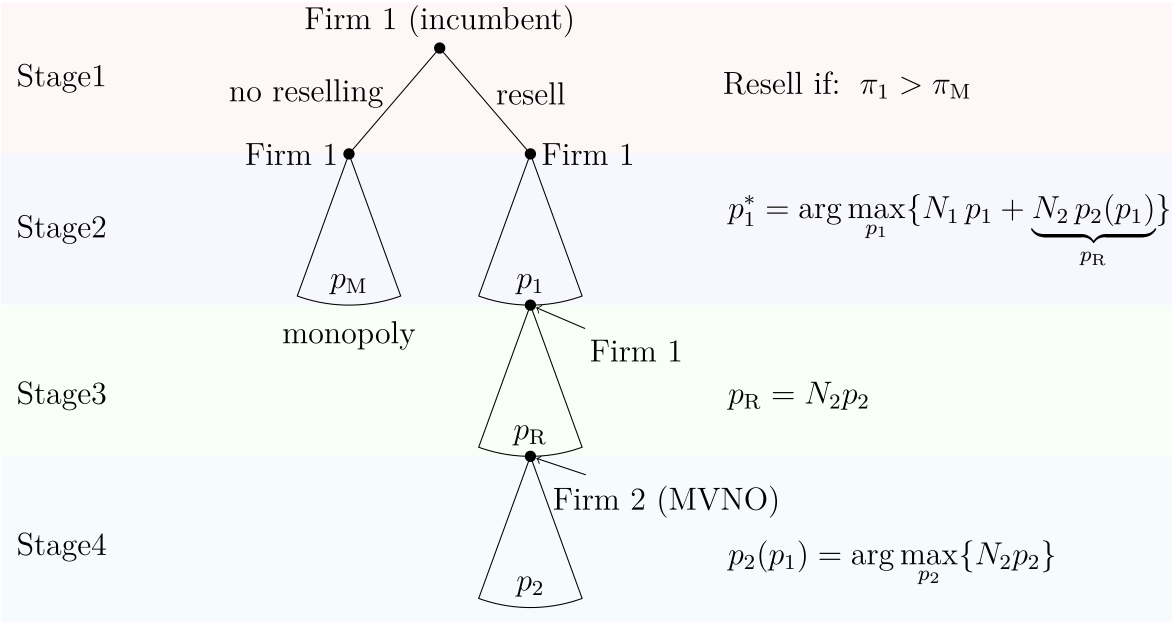

Fig. 4.1 Extensive-form of the MVNO game in the short term. Firm 1 decides if it should allow a MVNO and sets the price p_{M} respectively p_1. If Firm 1 allows a MVNO, the games continues. The incumbent sets the price p_{R} the MVNO has to pay. Finally, the MVNO sets the price p_2.

of the incumbent have priority over the subscribers of the MVNO. I also include long term considerations for which I assume a similar setup as for the short term except that in a first stage the number of base stations can be chosen by the incumbent.

4.1 Egalitarian

In the egalitarian case, subscribers from both, the incumbent and the MVNO, have the same throughput, that is, $\gamma_1 = \gamma_2$, (see Section 2.2.2). The egalitarian throughput, Equation (2.14), together with the demand for a duopoly, Equations (2.20) and (2.21), deliver the demand for the incumbent N_1 and for the MVNO N_2:

$$N_1 = \alpha_{11}\left(C_1 - \frac{N_1 + N_2}{B_1}\lambda\sigma\right) - \alpha_{12}\left(C_1 - \frac{N_1 + N_2}{B_1}\lambda\sigma\right) - \beta_{11}p_1 + \beta_{12}p_2 \tag{4.1}$$

$$N_2 = \alpha_{22}\left(C_1 - \frac{N_1 + N_2}{B_1}\lambda\sigma\right) - \alpha_{21}\left(C_1 - \frac{N_1 + N_2}{B_1}\lambda\sigma\right) - \beta_{22}p_2 + \beta_{21}p_1. \tag{4.2}$$

Compared to the duopoly, Section 3.2, the throughput of firm 1 now also depends on the demand of firm 2 and vice versa. To keep the equations compact, I assume, without loss of generality, a symmetric demand, that is, $\alpha_{11} = \alpha_{22}$, $\alpha_{12} = \alpha_{21}$,$\beta_{11} = \beta_{22}$ and $\beta_{12} = \beta_{21}$. Combining Equations (4.1) and (4.2) delivers the demand by:

$$N_1 = \frac{C_1(\alpha_{11} - \alpha_{12}) - \beta_{11}p_1 + \beta_{12}p_2 - (p_1 - p_2)(\alpha_{11} - \alpha_{12})(\beta_{11} + \beta_{12})\frac{\lambda\sigma}{B_1}}{1 + 2(\alpha_{11} - \alpha_{12})\frac{\lambda\sigma}{B_1}} \tag{4.3}$$

$$N_2 = \frac{C_1(\alpha_{11} - \alpha_{12}) - \beta_{11}p_2 + \beta_{12}p_1 + (p_1 - p_2)(\alpha_{11} - \alpha_{12})(\beta_{11} + \beta_{12})\frac{\lambda\sigma}{B_1}}{1 + 2(\alpha_{11} - \alpha_{12})\frac{\lambda\sigma}{B_1}}. \tag{4.4}$$

Similar as in the case of a duopoly, we find the subgame perfect Nash equilibrium by backward induction, for which we need the best response function of the MVNO, given by:

$$p_2(p_1) = \arg\max_{p_2}\{N_2p_2\} \tag{4.5}$$

$$p_2(p_1) = \frac{C_1(\alpha_{11} - \alpha_{12}) + \beta_{12}p_1 + p_1(\alpha_{11} - \alpha_{12})(\beta_{11} + \beta_{12})\frac{\lambda\sigma}{B_1}}{2\beta_{11} + 2(\alpha_{11} - \alpha_{12})(\beta_{11} + \beta_{12})\frac{\lambda\sigma}{B_1}}. \tag{4.6}$$

The incumbent maximizes the profit if the reselling price is set to $p_{\mathrm{R}} = N_2p_2$, so that all the potential profit of the MVNO is captured by the incumbent. The overall profit of the incumbent can then be maximized by setting the price p_1 according to:

$$p_1^* = \arg\max_{p_1}\{N_1p_1 + N_2p_2(p_1)\} \tag{4.7}$$

$$p_1^* = \frac{2B_1C_1\frac{(\beta_{11}+\beta_{12})}{(\alpha_{11}-\alpha_{12})}(2\lambda\sigma(\alpha_{11} - \alpha_{12}) + B_1)}{\lambda^2\sigma^2(\beta_{11} + \beta_{12})^2 + B_1^2\frac{\left(4\beta_{11}^2-3\beta_{12}^2\right)}{(\alpha_{11}-\alpha_{12})^2} + 2B_1\lambda\sigma\frac{(\beta_{11}+\beta_{12})(4\beta_{11}-3\beta_{12})}{(\alpha_{11}-\alpha_{12})}}. \tag{4.8}$$

For the short term, this price already reflects the equilibrium price for the case that the incumbent grants access rights (reselling) which happens if the profit of allowing a MVNO is higher than in case of a monopoly. Thus, the incumbent will resell his spectrum and infrastructure if:

$$\pi_1 > \pi_{\mathrm{M}} \tag{4.9}$$

$$p_1^*N_1 + p_2(p_1^*)N_2 > p_{\mathrm{M}}N_{\mathrm{M}}, \tag{4.10}$$

whereas the monopoly demand parameters are chosen so that $\alpha_{\mathrm{M}} = \alpha_{11}$ and $\beta_{\mathrm{M}} = \beta_{11}$. Later in this section I will also consider comparable demand parameters as defined in Section 2.3.

To quantify social optimality, I consider the aggregated throughput Γ which was

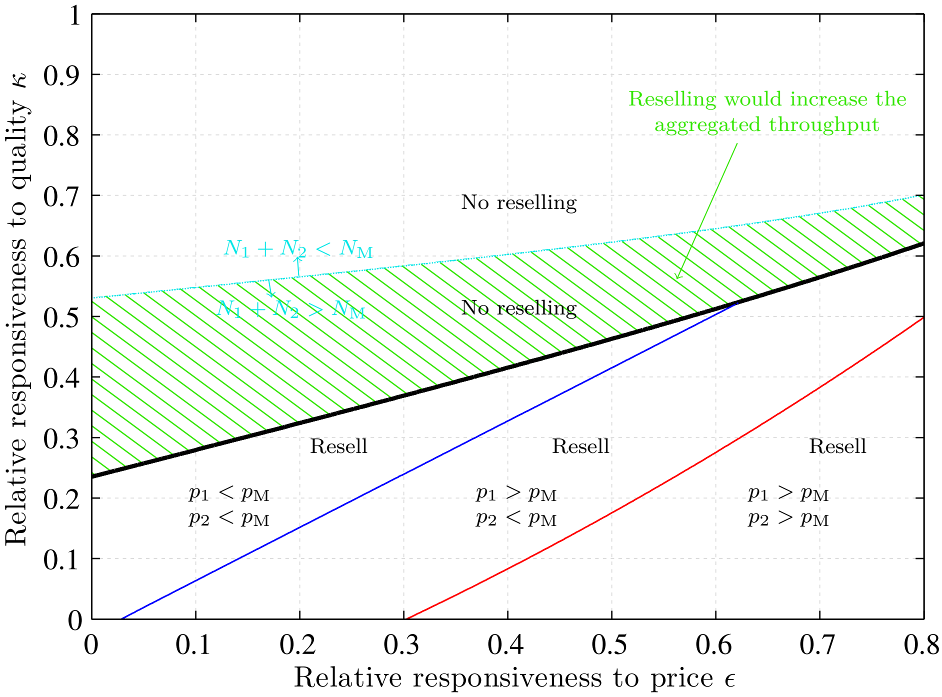

Fig. 4.2 Reselling regions for the case of **egalitarian** sharing in the **short term**, using the following parameters: $C = 45\,\mathrm{MB/s}$, $\lambda\sigma = 0.5\,\mathrm{kB/s}$, $\alpha_{11} = 200\,000\,\mathrm{subscriber/MB/s}$, $\beta_{11} = 180\,000\,\mathrm{subscriber/€}$ and $F = 100\,000\,\mathrm{€/Basestation}$.

described in Section 3.3. Reselling increases the aggregated throughput if:

$$\Gamma_R = N_1\gamma_1 + N_2\gamma_2 > N_M\gamma_M = \Gamma_M. \tag{4.11}$$

For a compact description, let us further define the relative responsiveness to quality κ and the relative responsiveness to price ϵ by:

$$\kappa = \frac{\alpha_{12}}{\alpha_{11}} \tag{4.12}$$

$$\epsilon = \frac{\beta_{12}}{\beta_{11}}, \tag{4.13}$$

whereas $0 \leq \kappa < 1$ and $0 \leq \epsilon < 1$, see Banker et al. (1998).

Figure 4.2 shows an example of how the reselling decision, Equation (4.10), depends on κ and ϵ. For all responsiveness values above the bold black curve, a monopoly generates a higher profit than reselling the infrastructure to a MVNO so that the incumbent does not resell his infrastructure. The monopoly demand does not depend

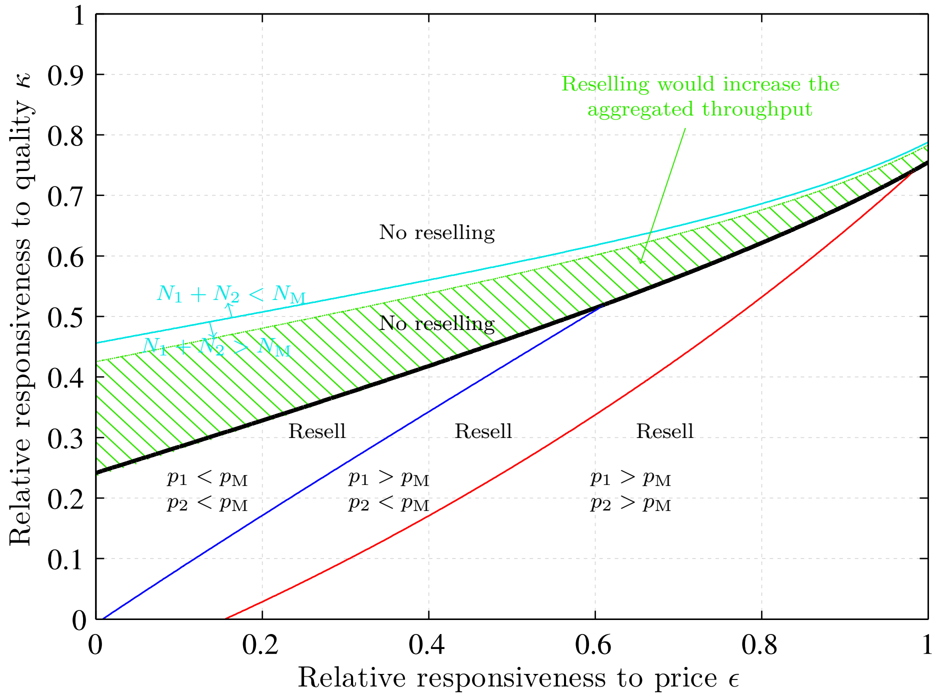

Fig. 4.3 Reselling regions for the case of **egalitarian** sharing in the **long term**, using the following parameters: $C = 45\,\text{MB/s}$, $\lambda\sigma = 0.5\,\text{kB/s}$, $\alpha_{11} = 200\,000\,\text{subscriber/MB/s}$, $\beta_{11} = 180\,000\,\text{subscriber/€}$ and $F = 100\,000\,\text{€/Basestation}$.

on the relative responsiveness parameters, so that the price and therefore all other variables are constant in this area. The area under the bold black curve correspond to the responsiveness values for which the incumbent allows a MVNO. The prices p_1 and p_2 then depend on the responsiveness parameters and can be higher or lower than the monopoly price. The shaded green area in the figure shows at which responsiveness values a MVNO would increase the aggregated throughput (social optimality) compared to a monopoly. However, the incumbent refuses to share his infrastructure, so that the government should intervene if the aim is to maximize the aggregated throughput. Note also that the domain of ϵ is limited here because values higher than 0.8 lead to a negative demand of N_1. Such extreme cases can simply be investigated by setting the price N_1 to zero and re-evaluating the profit maximizing decisions of the incumbent and the MVNO. However, because such extreme cases do not provide any additional insights into the system I exclude them from my considerations.

In the long term, I additionally assume that the incumbent is able to change the number of base stations at the beginning. Similar as in the duopoly, Section 3.2, there is no closed form solution for the equilibrium, so that numerical methods have to be used. Figure 4.3 shows how the reselling regions depend on κ and ϵ in the long term. Compared to Figure 4.2, ϵ is no longer limited by 0.8 because the increased degree of freedom (in terms of deploying additional base stations) prevents that the demand becomes negative. Also, the green area, for which reselling would increase the aggregated throughput compared to the monopoly, is smaller in the long term and its boundary no longer coincide with the isoquant curves of equal aggregated demand, that is, $N_1+N_2 = N_{\mathrm{M}}$. However, overall, the behavior in the short term is very similar as in the long term.

Comparable demand

In Section 2.3, I defined the property of comparable demand as the case when the aggregated duopoly demand, $N_1 + N_2$, is the same as the monopoly demand, N_{M}, if the prices and the quality levels (throughput) are the same, leading to the following comparable responsiveness values:

$$\alpha_{11} = \frac{\alpha_{\mathrm{M}}}{2(1-\kappa)} \tag{4.14}$$

$$\beta_{11} = \frac{\beta_{\mathrm{M}}}{2(1-\epsilon)}. \tag{4.15}$$

The case of comparable demand shows some remarkable properties: Firstly, by inserting the comparable responsiveness values, Equations (4.14) and (4.15), into the subgame perfect Nash equilibrium prices, Equations (4.6) and (4.8), we observe that, after some rearranging, the equilibrium prices are independent of κ. Therefore, the profit is also independent to changes in the relative responsiveness to quality (throughput). Secondly, extensive numerical evaluation shows that the profit of the incumbent is always higher in case of a monopoly than by allowing a MVNO. Thus, **for comparable demand, the incumbent never wants to resell his spectrum and infrastructure**. The reason for such behavior lies in the fact that the incumbent loses market power because parts of the profit can only be obtained through the MVNO. An intervention of the government, forcing the monopoly to resell his spectrum and infrastructure, would be justified because the equilibrium prices are below the monopoly price, $p_2^* < p_1^* < p_{\mathrm{M}}$, and the aggregated throughput is higher than in case of a monopoly.

4.2 Discriminatory

Here I assume that the primary users have priority over the secondary users, see Section 2.2.2. The demand can then be written as:

$$N_1 = \alpha_{11}\left(C_1 - \frac{N_1}{B_1}\lambda\sigma\right) - \alpha_{12}\left(C_1 - \frac{N_1+N_2}{B_1}\lambda\sigma\right)\left(1 - \frac{N_1}{B_1C_1}\lambda\sigma\right) - \beta_{11}p_1 + \beta_{12}p_2 \tag{4.16}$$

$$N_2 = \alpha_{22}\left(C_1 - \frac{N_1+N_2}{B_1}\lambda\sigma\right)\left(1 - \frac{N_1}{B_1C_1}\lambda\sigma\right) - \alpha_{21}\left(C_1 - \frac{N_1}{B_1}\lambda\sigma\right) - \beta_{22}p_2 + \beta_{21}p_1. \tag{4.17}$$

One can calculate N_1 and N_2 out of the equations above. However, because the resulting equations are quite lengthy I omit them at this point. Following along the same line as in the egalitarian case, the subgame perfect Nash equilibrium conditions can be written as:

$$p_2(p_1) = \arg\max_{p_2}\{N_2p_2\} \tag{4.18}$$

$$p_1^* = \arg\max_{p_1}\{N_1p_1 + N_2p_2(p_1)\}. \tag{4.19}$$

Unfortunately, there does not exist a closed-form solution so that we have to rely on numerical methods. Figure 4.4 shows an example for the same parameters as in Section 4.1. Compared to the egalitarian case, there is a higher price differentiation, $p_1^* - p_2^*$, leading also to a larger area of the prices $p_1^* > p_{\mathrm{M}} > p_2^*$. The intuition behind this behavior is the fact that discriminatory access leads to different average throughputs, $\gamma_1 > \gamma_2$, which results in higher price differentiation compared to the egalitarian case. Clearly, the profit for discriminatory access is higher than for egalitarian access due to a higher overall throughput, so that the incumbent would prefer discriminatory access over egalitarian access.

Although I focus here on MVNO, the model in this section can also be used to describe cognitive radio in a more general sense, where a spectrum holder faces the decision of whether he should allow access to his spectrum or not. In particular, by replacing $\frac{N_2}{B_1}\lambda\sigma$ by $\frac{N_2}{B_2}\lambda\sigma$ we are able to model the number of base stations B_2, similar as in the duopoly. For the important special case of $\kappa = \epsilon = 0$, that is, the two firms operate in completely separate markets, the spectrum holder will always increase the price p_1 compared to the monopoly price p_{M}. To get an intuition behind this effect, let us recall that the profit is given by $N_1p_1 + N_2p_2$. The first term is maximized by

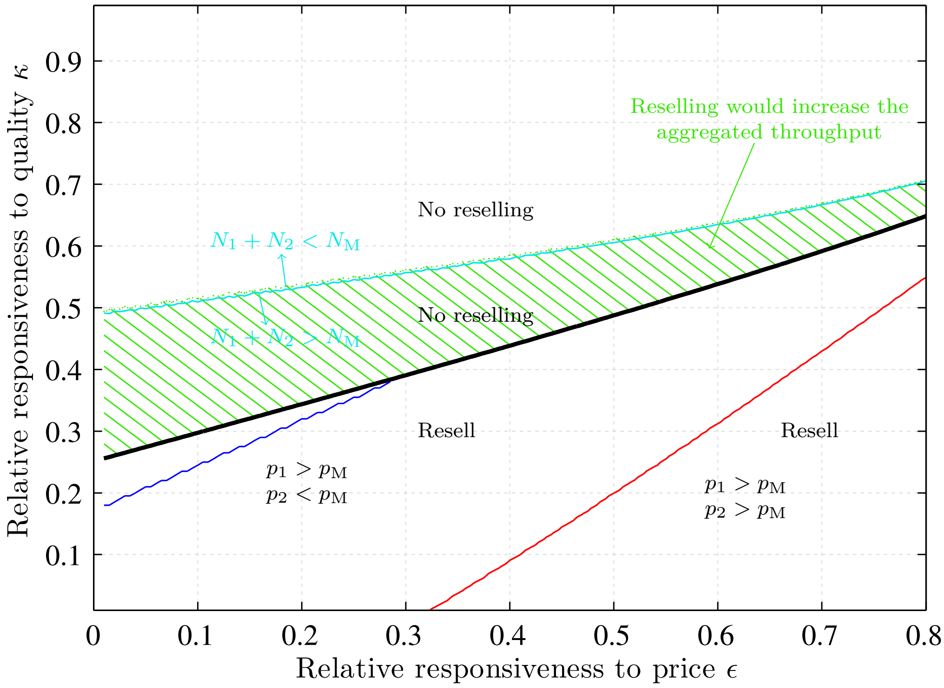

Fig. 4.4 Reselling regions for the case of **discriminatory** access in the **short term**, using the following parameters: $C = 45\,\mathrm{MB/s}$, $\lambda\sigma = 0.5\,\mathrm{kB/s}$, $\alpha_{11} = 200\,000\,\mathrm{subscriber/MB/s}$, $\beta_{11} = 180\,000\,\mathrm{subscriber/€}$ and $F = 100\,000\,\mathrm{€/Basestation}$.

p_M. If we now decrease the price $p_1 < p_M$, the first term $N_1 p_1$ clearly decreases, but also the second term $N_2 p_2$ decreases because a lower price p_1 leads to a higher demand N_1 which in turn decreases the quality for the secondary users and consequently the demand N_2. Thus, the price p_1 has to be larger than p_M.

Comparable demand

Let us again consider the case of comparable demand, see Equations (4.14) and (4.15). Figure 4.5 shows an example for the same parameters as before. In contrast to the case of egalitarian access, the incumbent now has an incentive to resell his spectrum and infrastructure for low relative price responsiveness values ϵ, that is, a scenario in which the price competition is relatively low. The reason for such behavior lies again in the fact that, by allowing secondary users to access the network, the overall throughput increases which compensates the loss of market power and leads to a higher profit. Similar to the egalitarian case, the aggregated throughput for allowing an MVNO is

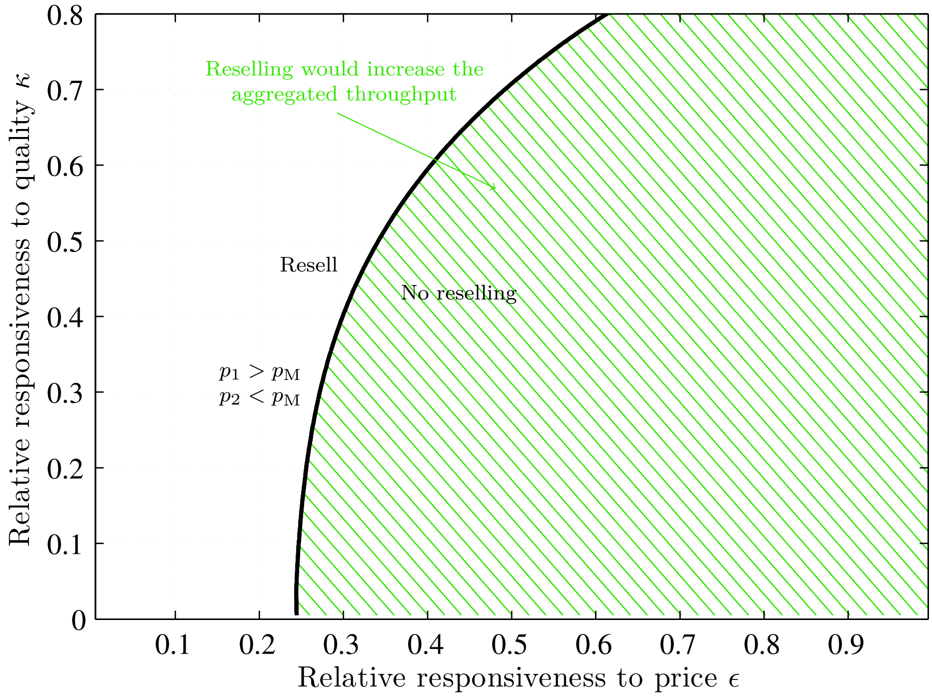

Fig. 4.5 Reselling regions for the case of **discriminatory** access in the **short term** and for **comparable** demand, using the following parameters: $C = 45\,\mathrm{MB/s}$, $\lambda\sigma = 0.5\,\mathrm{kB/s}$, $\alpha_{11} = 200\,000\,\mathrm{subscriber/MB/s}$, $\beta_{11} = 180\,000\,\mathrm{subscriber/€}$ and $F = 100\,000\,€/\mathrm{Basestation}$. In contrast to the egalitarian case, there exists regions where the incumbent wants to resell the spectrum.

always higher than for the monopoly, so that the government might want to intervene, forcing the incumbent to resell his spectrum and infrastructure.

Chapter 5

Spectrum Sharing

As already shown in Section 2.2, doubling the capacity while the number of subscribers also doubles (two firms merge), improves the overall performance, that is, the throughput is also doubled, because a higher capacity allows a better exploitation of the statical multiplexing gain. This simple fact leads to the question whether two firms want to share their spectrum, that is, can they increase their profit by spectrum sharing? Such cognitive radio techniques, which allow a flexible access to the available bandwidth, have been intensely investigated in recent years from a technical point of view and might be included in the next wireless communication standard (5G).

In order to answer the question whether two firms want to share their spectrum, I consider a similar situation as in Section 3.2, that is, both firms make strategic decision simultaneously, which is in contrast to Chapter 4 where we have a leader, the incumbent, and a follower, the MVNO, because there is a clear hierarchy, allowing us to model the behavior as Stackelberg competition. Here, on the other hand, both firms are equal. If no spectrum sharing is applied, Section 3.2 directly describes the behavior of the firms in the duopoly. The case of spectrum sharing can also be modeled in the same way as in Section 3.2, except that the throughput equations have to be replaced in order to describe spectrum sharing. Similar as previously done in this thesis, I assume that the spectrum sharing can be done in two possible way, either by egalitarian access or by discriminatory access, see Section 2.2.2. In the short term, I assume that the numbers of base stations are given by the Nash equilibrium of no spectrum sharing, see Section 3.2. For a fair comparison, the fix costs, that is, the costs for deploying the base stations, have to be paid independently of whether the spectrum is shared or not. In the long term, the number of base stations can also be changed in case of spectrum sharing. Note that, in contrast to Chapter 4, each firm

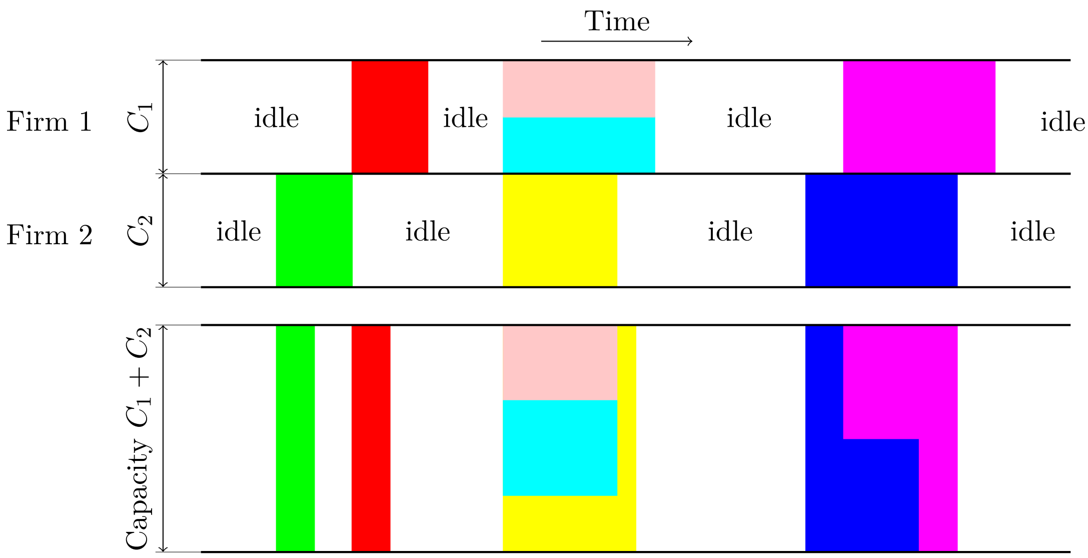

Fig. 5.1 Egalitarian spectrum sharing: the spectrum of both firms is combined and equally shared between active users. In most cases, the instantaneous user throughput increases. However, it might happen that the instantaneous throughput is lower as indicated by the yellow data packet. Also, if the spectrum endowment is highly unsymmetrical, say $C_1 >> C_2$, sharing might reduce the throughput of firm 1.

has its own infrastructure and only the spectrum is shared.

5.1 Egalitarian

Figure 5.1 depicts the case of egalitarian spectrum sharing. Here, the capacities of both firms are combined to a larger capacity $C_1 + C_2$. If a data transmission starts, the combined capacity is then equally shared between the users. Note that it can happen that the transmission of a packet takes longer than in case of no sharing, as indicated by the yellow packet in Figure 5.1, so that the throughput is lower. By applying the duopoly demand functions, Equations (2.20) and (2.21), and using the fact that subscribers of both firms experience the same throughput, $\gamma_1^{\mathrm{ES}} = \gamma_2^{\mathrm{ES}}$, given by Equation (2.14), we can write the demand as:

$$N_1 = (\alpha_{11} - \alpha_{12}) \left(C_1 + C_2 - \frac{N_1}{B_1}\lambda\sigma - \frac{N_2}{B_2}\lambda\sigma \right) - \beta_{11}p_1 + \beta_{12}p_2 \tag{5.1}$$

$$N_2 = (\alpha_{22} - \alpha_{21}) \left(C_1 + C_2 - \frac{N_1}{B_1}\lambda\sigma - \frac{N_2}{B_2}\lambda\sigma \right) - \beta_{22}p_2 + \beta_{21}p_1. \tag{5.2}$$

Similar as in Section 3.2, we have to rearrange Equations (5.1) and (5.2), so that the demand reads:

$$N_1 = \tilde{\alpha}_{11} C_1 - \tilde{\alpha}_{12} C_2 - \tilde{\beta}_{11} p_1 + \tilde{\beta}_{12} p_2 \tag{5.3}$$

$$N_2 = \tilde{\alpha}_{22} C_2 - \tilde{\alpha}_{21} C_1 - \tilde{\beta}_{22} p_2 + \tilde{\beta}_{21} p_1, \tag{5.4}$$

with

$$\tilde{\alpha}_{11} = \frac{\alpha_{11} - \alpha_{12}}{1 + (\alpha_{11} - \alpha_{12})\frac{\lambda\sigma}{B_1} + (\alpha_{22} - \alpha_{21})\frac{\lambda\sigma}{B_2}} \tag{5.5}$$

$$\tilde{\alpha}_{12} = \frac{\alpha_{12} - \alpha_{11}}{1 + (\alpha_{11} - \alpha_{12})\frac{\lambda\sigma}{B_1} + (\alpha_{22} - \alpha_{21})\frac{\lambda\sigma}{B_2}} \tag{5.6}$$

$$\tilde{\alpha}_{22} = \frac{\alpha_{22} - \alpha_{21}}{1 + (\alpha_{11} - \alpha_{12})\frac{\lambda\sigma}{B_1} + (\alpha_{22} - \alpha_{21})\frac{\lambda\sigma}{B_2}} \tag{5.7}$$

$$\tilde{\alpha}_{21} = \frac{\alpha_{21} - \alpha_{22}}{1 + (\alpha_{11} - \alpha_{12})\frac{\lambda\sigma}{B_1} + (\alpha_{22} - \alpha_{21})\frac{\lambda\sigma}{B_2}}, \tag{5.8}$$

and

$$\tilde{\beta}_{11} = \frac{\beta_{11} + \frac{\lambda\sigma}{B_2}(\alpha_{11}\beta_{21} + \alpha_{22}\beta_{11} - \alpha_{12}\beta_{21} - \alpha_{21}\beta_{11})}{1 + (\alpha_{11} - \alpha_{12})\frac{\lambda\sigma}{B_1} + (\alpha_{22} - \alpha_{21})\frac{\lambda\sigma}{B_2}} \tag{5.9}$$

$$\tilde{\beta}_{12} = \frac{\beta_{12} + \frac{\lambda\sigma}{B_2}(\alpha_{11}\beta_{22} + \alpha_{22}\beta_{12} - \alpha_{12}\beta_{22} - \alpha_{21}\beta_{12})}{1 + (\alpha_{11} - \alpha_{12})\frac{\lambda\sigma}{B_1} + (\alpha_{22} - \alpha_{21})\frac{\lambda\sigma}{B_2}} \tag{5.10}$$

$$\tilde{\beta}_{22} = \frac{\beta_{22} + \frac{\lambda\sigma}{B_1}(\alpha_{22}\beta_{12} + \alpha_{11}\beta_{22} - \alpha_{21}\beta_{12} - \alpha_{12}\beta_{22})}{1 + (\alpha_{11} - \alpha_{12})\frac{\lambda\sigma}{B_1} + (\alpha_{22} - \alpha_{21})\frac{\lambda\sigma}{B_2}} \tag{5.11}$$

$$\tilde{\beta}_{21} = \frac{\beta_{21} + \frac{\lambda\sigma}{B_1}(\alpha_{22}\beta_{11} + \alpha_{11}\beta_{21} - \alpha_{21}\beta_{11} - \alpha_{12}\beta_{21})}{1 + (\alpha_{11} - \alpha_{12})\frac{\lambda\sigma}{B_1} + (\alpha_{22} - \alpha_{21})\frac{\lambda\sigma}{B_2}}. \tag{5.12}$$

In particular, Equations (5.3) and (5.4) have the same structure as Equations (3.11) and (3.12) of Section 3.2. The only differences are the equivalent demand parameters $\tilde{\alpha}_{xx}$ and $\tilde{\beta}_{xx}$. Because the Nash equilibriums of Section 3.2 are given explicitly by $\tilde{\alpha}_{xx}$ and $\tilde{\beta}_{xx}$, we can directly reuse these results, so that the best response functions and the Nash equilibriums of our egalitarian spectrum sharing system are given by Equations (3.21) to (3.28).

Let us assume for both firms the same capacities, demand functions, and costs. By comparing the duopoly demand in Section 3.2 to the case of egalitarian spectrum sharing, we immediately see that for the same prices, the demand and therefore also the profit will increase. Of course, the prices will also increase due to a higher offered quality which also leads to a small decrease of the demand. However, the overall profit will increase compared to the case of no spectrum sharing. Thus, if both firms have the same endowment in terms of capacities, they will always choose to share their spectrum. However, if the capacities are different, say $C_2 < C_1$, things change. Figure (5.2) shows how the profit is affected by the capacity of firm 2 whereas I consider otherwise the same parameters as in Section 3.2, that is, $C_1 = 45\,\mathrm{MB/s}$, $\lambda\sigma = 0.5\,\mathrm{kB/s}$, $\alpha_{11} = 200\,000\,\mathrm{subscriber/MB/s}$, $\alpha_{12} = 100\,000\,\mathrm{subscriber/MB/s}$, $\beta_{11} = 180\,000\,\mathrm{subscriber/€}$, $\beta_{12} = 90\,000\,\mathrm{subscriber/€}$ and $F = 100\,000\,\mathrm{€/Basestation}$. The dotted lines reflect the profit if no spectrum sharing is applied and the solid lines the profit in case of egalitarian spectrum sharing. Of particular interest is the area right of the blue point (black vertical line). Here, spectrum sharing increases the profit for both firms compared with no sharing, that is, $\pi_1^{\mathrm{ES}} > \pi_1$ and $\pi_2^{\mathrm{ES}} > \pi_2$, so that both firms have an incentive to share their spectrum. The area right of the green point on the other hand reflects a scenario where the aggregated profit of spectrum sharing is larger than that of no sharing, $\pi_1^{\mathrm{ES}} + \pi_2^{\mathrm{ES}} > \pi_1 + \pi_2$. Between the green and the blue point, firm 1 does not have an initial incentive to share its spectrum because the profit is lower. However, if firm 2 is able to compensates firm 1 for agreeing to share the spectrum, both firms can end up at a higher profit whereas the final distribution then depends on the bargaining output. Note that in case of spectrum sharing, the profit of firm 1 is lower then that of firm 2 although firm 1 has more market power and therefore a higher revenue, but also deploys more base stations which increases the overall costs. In this sense, firm 2 uses the spectrum of firm 1 in a parasitic way without contributing a sufficient number of base stations to reduce the traffic load. The higher market power of firm 1 is related to the number of base stations because the quality depends on N_x/B_x and $B_1 > B_2$, so that subscribers of firm 1 have less influence on the throughput than subscribers of firm 2. In the long term, both firms will deploy the same number of base stations because they offer the same quality, so that the price and the profit will also be the same for both firms. The profit for spectrum sharing is higher than for the case of no sharing, so that in the long term, both firms always want to share their spectrum.

Figure 5.3 depicts the equilibrium prices for both firms. If no spectrum sharing is used, we have $p_1 > p_2$ because firm 1 offers a higher quality. On the other hand, we have opposite prices if egalitarian spectrum sharing is employed, $p_2^{\mathrm{ES}} > p_1^{\mathrm{ES}}$, because the higher market power of firm 1 allows to increase the profit by increasing the

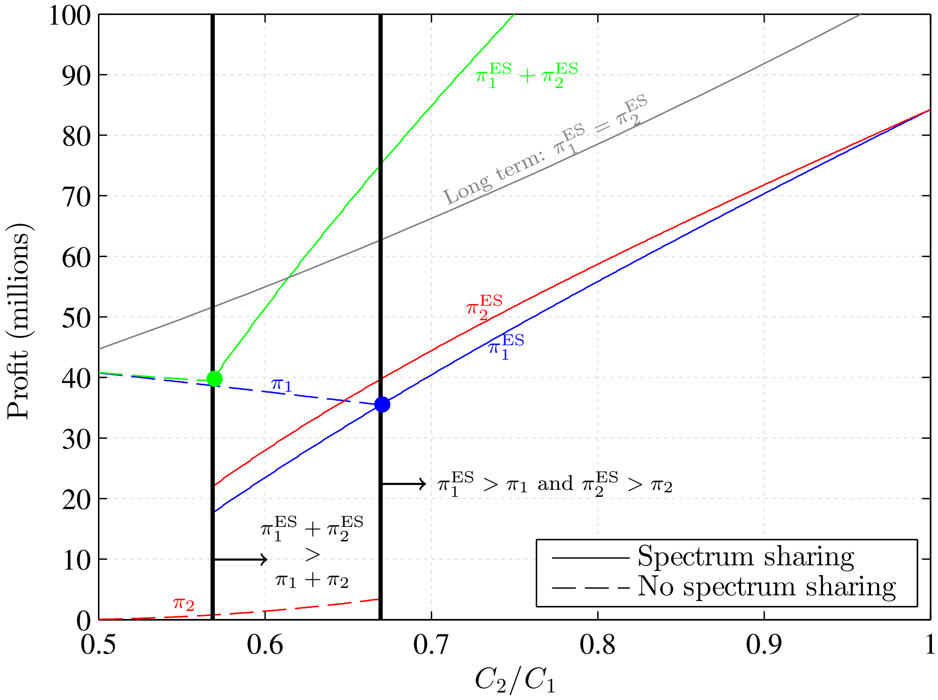

Fig. 5.2 Profits for egalitarian spectrum sharing as well as the case of no spectrum sharing. For capacity values C_2 larger than indicated by the blue point, both firms have an incentive to share their spectrum.

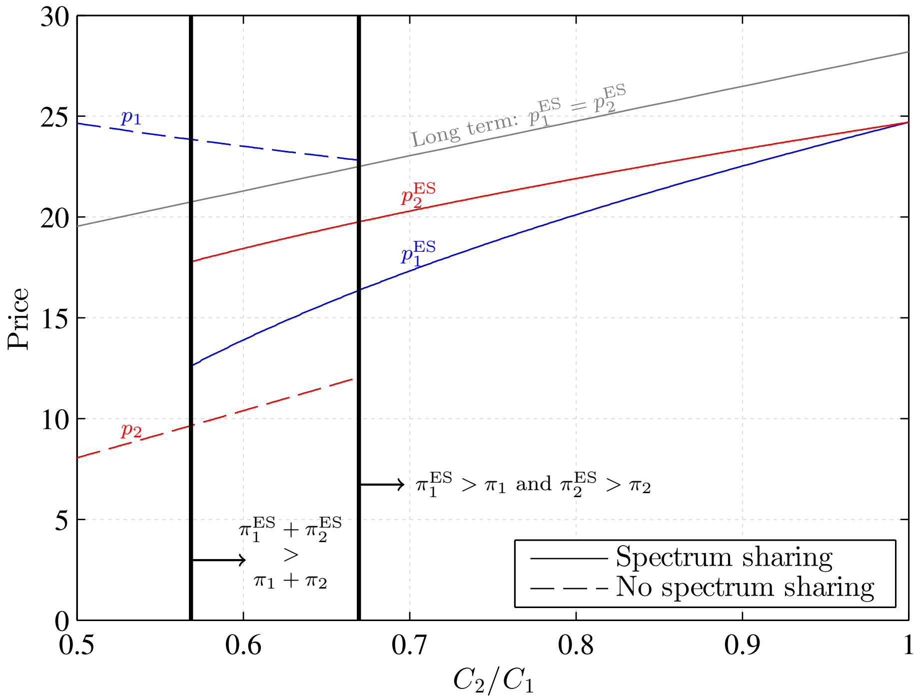

Fig. 5.3 Nash equilibrium prices for egalitarian spectrum sharing as well as the case of no spectrum sharing.

number of subscribers through a lower price (the quality is the same for both firms). To get an intuition of this fact, let us recall that the quality depends on N_x/B_x and that $B_1 > B_2$. Thus, by increasing the demand N_1 through a decreasing price p_1, the quality is relatively less affected then by increasing the demand N_2. In the long term, the prices are higher than in the short term because more base stations are deployed which increase the quality and consequently allow the firms to set a higher prices.

5.2 Discriminatory

Here, I assume that firm 1 can access the capacity of firm 2 in an opportunistic way, that is, only when no users of firm 2 are active. Thus, subscribers of firm 1 are secondary users of the capacity of firm 2, as described in Section 2.2.2. Conversely, the same concept applies for firm 2, that is, subscribers of firm 2 are secondary users of the capacity of firm 1. Unfortunately, the throughput equations of Section 2.2.2 cannot be straightforwardly used to describe such discriminatory spectrum sharing. To see why this is the case, let us consider the throughput in Figure 2.6. A straightforward approach would be to simply add both throughput measures. For a traffic load larger than $\frac{1}{2}$, we immediately see the problem of such a simple throughput definition: it coincides with the primary users throughput due to congestion. However, because "secondary users" have a own primary channel, they could distribute parts of their load to the primary channel, so that congestion is avoided, leading to a higher throughput. We therefore need a more evolved method to describe discriminatory spectrum sharing whereas such load distribution lies in the heart of it. Figure 5.4 shows the basic concept for firm 1: A data packet of size σ is split using the packet size distribution factor $0 \leq \delta_1 \leq 1$, so that the primary channel has to process a packet of size $\delta_1\sigma$ while the secondary channel has to deal with the remaining packet of size $(1 - \delta_1)\sigma$. Because the throughput is defined as the average data size over the average time needed to transmit the packet, we can use Equation (2.15) to determine the time $T_1^{\text{Pri.}}$ a packet of size $\delta_1\sigma$ requires to be transmitted over the primary channel. In the same way, Equation (2.16) can be used to determine the required time $T_1^{\text{Sec.}}$ for the secondary channel which has to process a packet of size $(1 - \delta_1)\sigma$. Doing so allows us to write

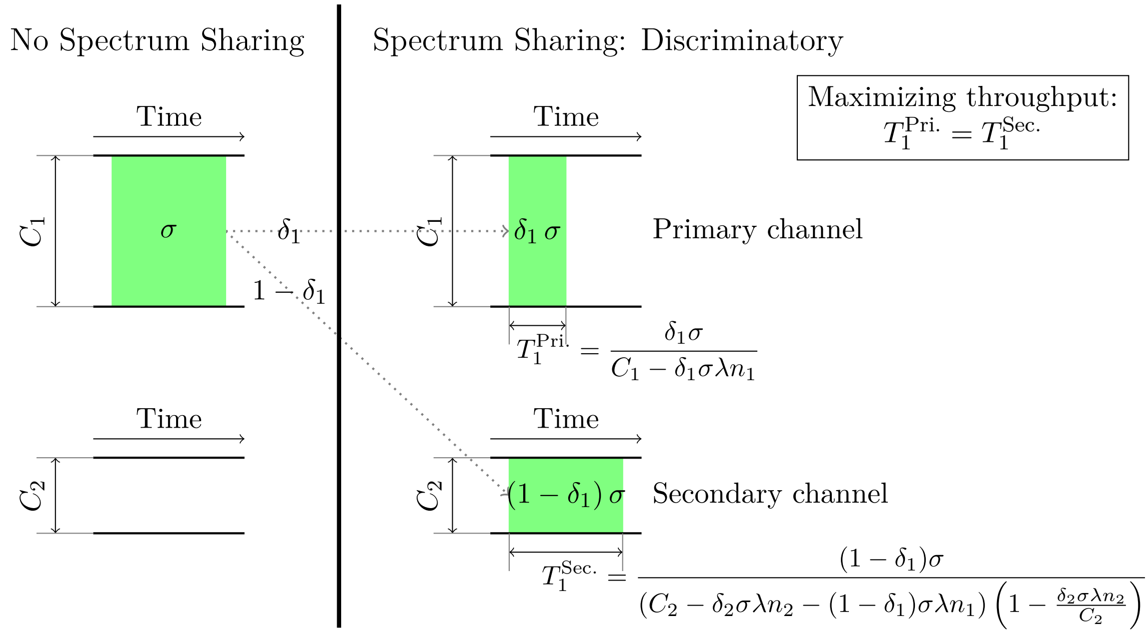

Fig. 5.4 A packet of size σ is split between primary and secondary channel. Because the required time in the secondary channel depends also on the choices of the other firm, we have to analyze the outcome using game theory. The Nash equilibrium then corresponds to the optimal load distribution between primary an secondary channel. Only one packet is depicted in the figure but of course there are in general more packets which all access the channel either by egalitarian spectrum sharing or by discriminatory spectrum sharing.

the required times as:

$$T_1^{\text{Pri.}} = \frac{\delta_1\sigma}{C_1 - \delta_1\sigma\lambda n_1} \tag{5.13}$$

$$T_1^{\text{Sec.}} = \frac{(1-\delta_1)\sigma}{(C_2 - \delta_2\sigma\lambda n_2 - (1-\delta_1)\sigma\lambda n_1)\left(1 - \frac{\delta_2\sigma\lambda n_2}{C_2}\right)}. \tag{5.14}$$

Different average times $T_1^{\text{Pri.}} \neq T_1^{\text{Sec.}}$ implies that the load distribution is not optimal because changing it could decrease the overall required time which is given by $T_1^{\text{Pri.}}$ if $T_1^{\text{Pri.}} > T_1^{\text{Sec.}}$ or $T_1^{\text{Sec.}}$ if $T_1^{\text{Sec.}} > T_1^{\text{Pri.}}$. Thus, the throughput is maximized by choosing the packet size distribution δ_1 so that $T_1^{\text{Pri.}} = T_1^{\text{Sec.}}$. A further challenge of such discriminatory spectrum sharing is the fact that the required time $T_1^{\text{Sec.}}$ also depends on the load distribution δ_2 of firm 2. Such interdependency can be best analyzed in the framework of game theory. By setting $T_1^{\text{Pri.}} = T_1^{\text{Sec.}}$ and rewriting it with respect to δ_1, we obtain the best response functions by:

$$\delta_1(\delta_2) = \frac{1}{2} + \frac{\delta_2 n_2}{2n_1} + \frac{C_1 C_2}{2\delta_2\sigma^2\lambda^2 n_1 n_2} + \frac{C_2^2}{2\delta_2\sigma^2\lambda^2 n_1 n_2} - \frac{C_2}{\sigma\lambda n_1} - \\ - \frac{\sqrt{\left(C_2(C_1+C_2) - 2C_2\delta_2\sigma\lambda n_2 + \delta_2\sigma^2\lambda^2 n_2(n_1+\delta_2 n_2)\right)^2 - 4C_1C_2\delta_2\sigma^2\lambda^2 n_1 n_2}}{2\delta_2\sigma^2\lambda^2 n_1 n_2} \tag{5.15}$$

$$\delta_2(\delta_1) = \frac{1}{2} + \frac{\delta_1 n_1}{2n_2} + \frac{C_2 C_1}{2\delta_1\sigma^2\lambda^2 n_2 n_1} + \frac{C_1^2}{2\delta_1\sigma^2\lambda^2 n_2 n_1} - \frac{C_1}{\sigma\lambda n_2} - \\ - \frac{\sqrt{\left(C_1(C_2+C_1) - 2C_1\delta_1\sigma\lambda n_1 + \delta_1\sigma^2\lambda^2 n_1(n_2+\delta_1 n_1)\right)^2 - 4C_2C_1\delta_1\sigma^2\lambda^2 n_2 n_1}}{2\delta_1\sigma^2\lambda^2 n_2 n_1}. \tag{5.16}$$

Unfortunately, there does not exist a general closed-form solutions for the Nash equilibrium of Equations (5.15) and (5.16). For the special case of $C = C_1 = C_2$ and $n = n_1 = n_2$, however, we find a Nash equilibrium at $\delta_1^* = \delta_2^* = \frac{C}{(2C-\sigma\lambda n)}$. For more general cases we have to rely on numerical methods. Figure 5.5 shows an example of the best response functions, Equations (5.15) and (5.16), for the case of $C = 1$ and $n = 0.5$. The point where the two functions intersect is the Nash equilibrium. Figure 5.5 also shows other Nash equilibriums for different capacities and different numbers of subscribers whereas for the sake of clarity the best response functions are not shown. Such Nash equilibrium tells the firm how it should split the data packets between primary and secondary channel so that the throughput is maximized. As illustrated in Figure 5.4, the average user throughputs can then be found by:

$$\gamma_1^{\mathrm{DS}} = \frac{\sigma}{T_1^{\mathrm{Pri.}}} = \frac{1}{\delta_1^*}\left(C_1 - \delta_1^*\sigma\lambda n_1\right) \tag{5.17}$$

$$\gamma_2^{\mathrm{DS}} = \frac{\sigma}{T_2^{\mathrm{Pri.}}} = \frac{1}{\delta_2^*}\left(C_2 - \delta_2^*\sigma\lambda n_2\right). \tag{5.18}$$

Again, only for the special case of $C = C_1 = C_2$ and $n = n_1 = n_2$ we can express Equations (5.17) and (5.18) in closed-form, which turns out to be the same as in the case of egalitarian spectrum sharing, $\gamma_1^{\mathrm{DS}} = \gamma_2^{\mathrm{DS}} = \gamma_1^{\mathrm{ES}} = \gamma_2^{\mathrm{ES}} = 2(C - n\lambda\sigma)$, see Section 5.1. For the special case of $C_1 = 1.5$, $C_2 = 0.5$ and $\sigma\lambda = 1$, Figure 5.6 shows how the average user throughput depends on the number of users n. The corresponding packet size distribution (Nash equilibrium) is given by the blue curve in Figure 5.5. By employing discriminatory spectrum sharing, firm 2 gains relatively more throughput than firm 1 while the absolute throughput of firm 1 is still higher,

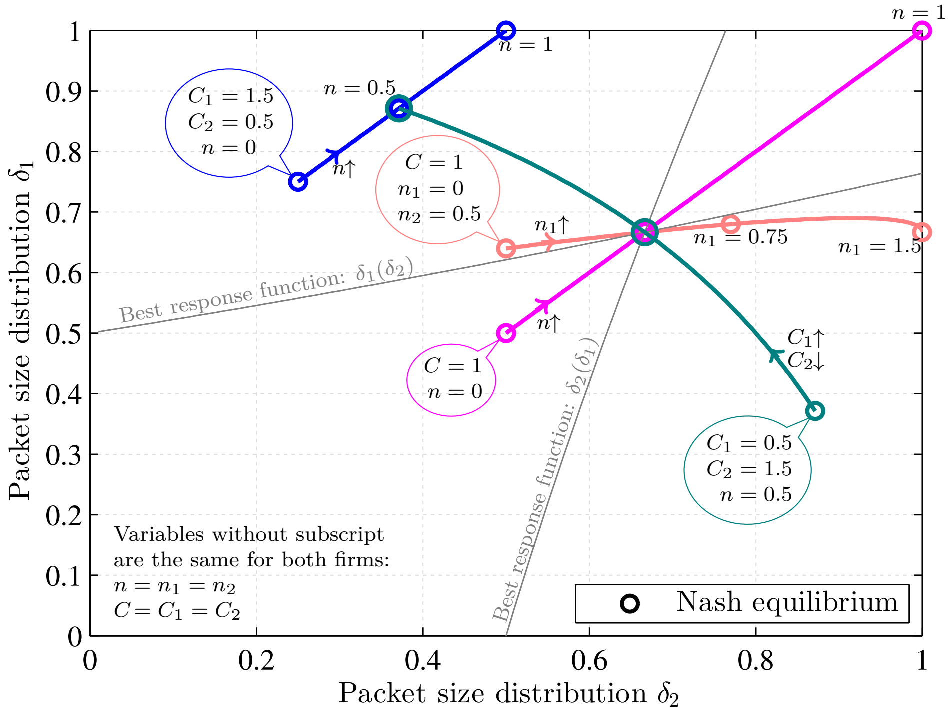

Fig. 5.5 Best response functions and Nash equilibriums for $\sigma\lambda = 1$ and a few selected values of C and n. The Nash equilibirum gives the throughput maximizing packet size distribution between primary and secondary channel.

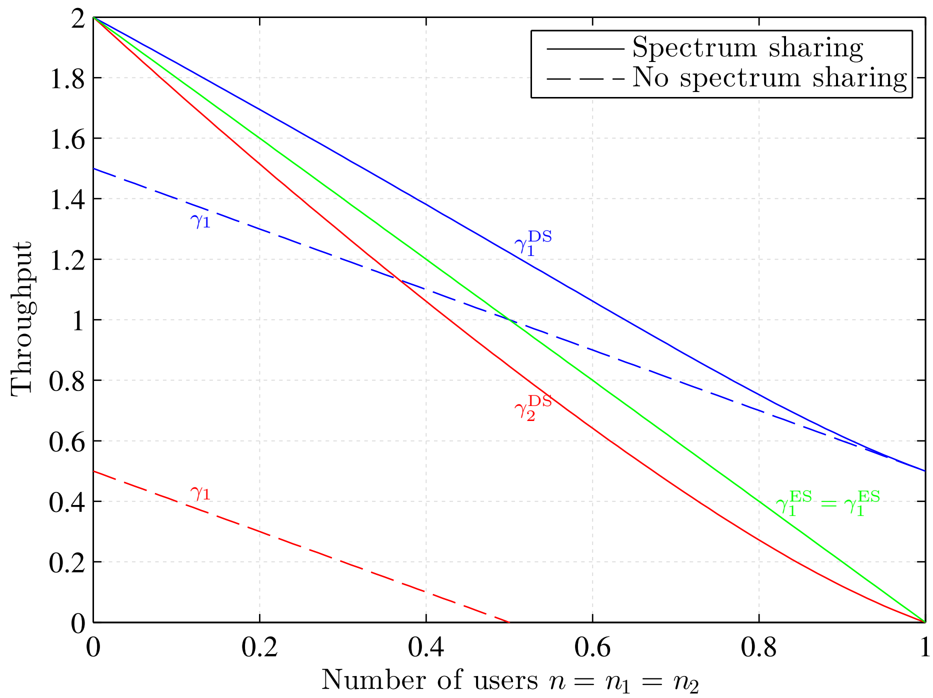

Fig. 5.6 Average user throughput in case of no spectrum sharing, egalitarian spectrum sharing (ES) and discriminatory spectrum sharing (DS), for the special case of $C_1 = 1.5$, $C_2 = 0.5$ and $\lambda\sigma = 1$. The packet size distribution which maximizes the throughput is shown in Figure 5.5 (blue curve).

as illustrated in Figure 5.6. For a small number of users n, discriminatory spectrum sharing leads to approximately the same throughput as egalitarian sharing, $\gamma_1^{\mathrm{ES}} = \gamma_2^{\mathrm{ES}}$. Only if the traffic load increases, the differences become more sever and at some point, the throughput for discriminatory access becomes the same as no spectrum sharing for firm 1, while for firm 2 it becomes zero due to strong congestion.

Once we have the throughput for the case of discriminatory spectrum sharing, Equations (5.17) and (5.18), we can insert them into the demand equations of Section 2.3, leading to:

$$N_1 = \alpha_{11}\gamma_1^{\mathrm{DS}} - \alpha_{12}\gamma_2^{\mathrm{DS}} - \beta_{11}p_1 + \beta_{12}p_2 \tag{5.19}$$

$$N_2 = \alpha_{22}\gamma_2^{\mathrm{DS}} - \alpha_{21}\gamma_1^{\mathrm{DS}} - \beta_{22}p_2 + \beta_{21}p_1. \tag{5.20}$$

Unfortunately, Equations (5.19) and (5.20) cannot be solved in closed form, so that we have to rely on numerical methods. Following along the same line as in Section 3.2 and 5.1, with the difference that all equations have to be evaluated numerically, we can find the Nash equilibrium prices for discriminatory spectrum sharing.

Figure 5.7 and 5.8 show the same plots as in Section 5.1. In particular, I use the same parameters, that is, $C_1 = 45\,\mathrm{MB/s}$, $\lambda\sigma = 0.5\,\mathrm{kB/s}$, $\alpha_{11} = 200\,000\,\mathrm{subscr./MB/s}$, $\alpha_{12} = 100\,000\,\mathrm{subscriber/MB/s}$, $\beta_{11} = 180\,000\,\mathrm{subscriber/€}$, $\beta_{12} = 90\,000\,\mathrm{subscriber/€}$ and $F = 100\,000\,\mathrm{€/Basestation}$. Compared to egalitarian spectrum sharing (Section 5.1), discriminatory spectrum sharing results in approximately the same profit for firm 2 while for firm 1 it is much higher. Both firms will always want to share their spectrum because the profits in case of discriminatory access are higher than in case of no spectrum sharing. The reason why firm 1 performs much better now can be explained by the number of base stations and the underlying access scheme. In case of egalitarian access, the number of base stations for firm 2 is too low, leading to a traffic load which is too high, hurting in particular subscribers of firm 1. In discriminatory access this negative effect is avoided because subscribers of firm 2 can only access the spectrum when no subscriber of firm 1 is active. Of particular interest is the case when $C_1 = C_2$. Here, the throughput for discriminatory access is the same as for egalitarian access if the number of subscribers is the same, $N_1 = N_2$. However, as shown in Figures 5.7 and 5.8, the profits as well as the prices are higher in case of discriminatory access than for egalitarian access. The reason for this behavior lies in the underlying best response functions. Even if the absolute values would result in the same profit, the fact that the best response functions are different for both cases, leads

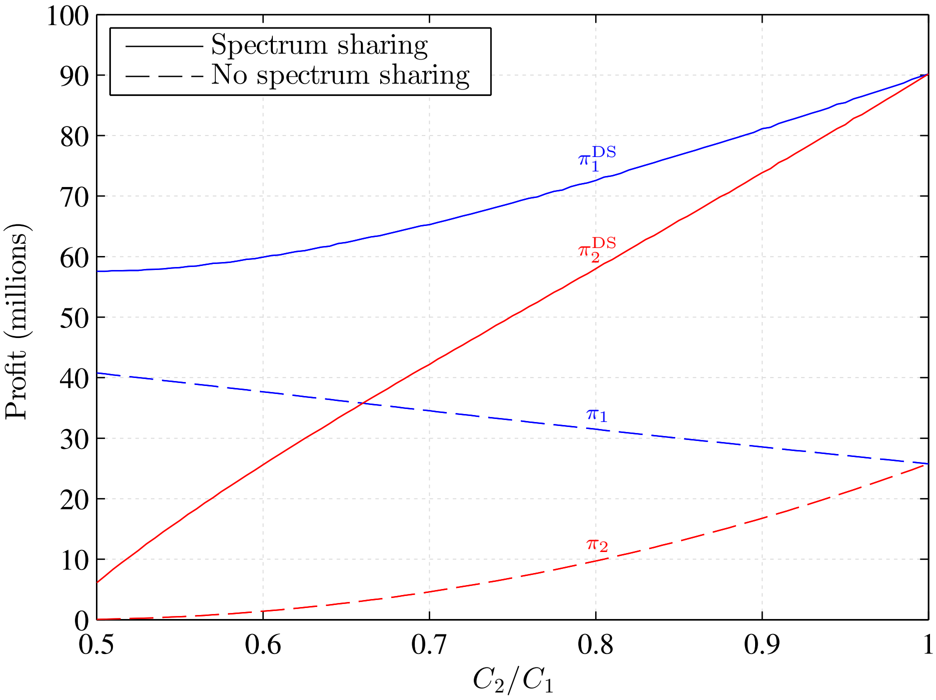

Fig. 5.7 Profits for discriminatory spectrum sharing as well as the case of no spectrum sharing. Here, both firms have an incentive to share their spectrum.

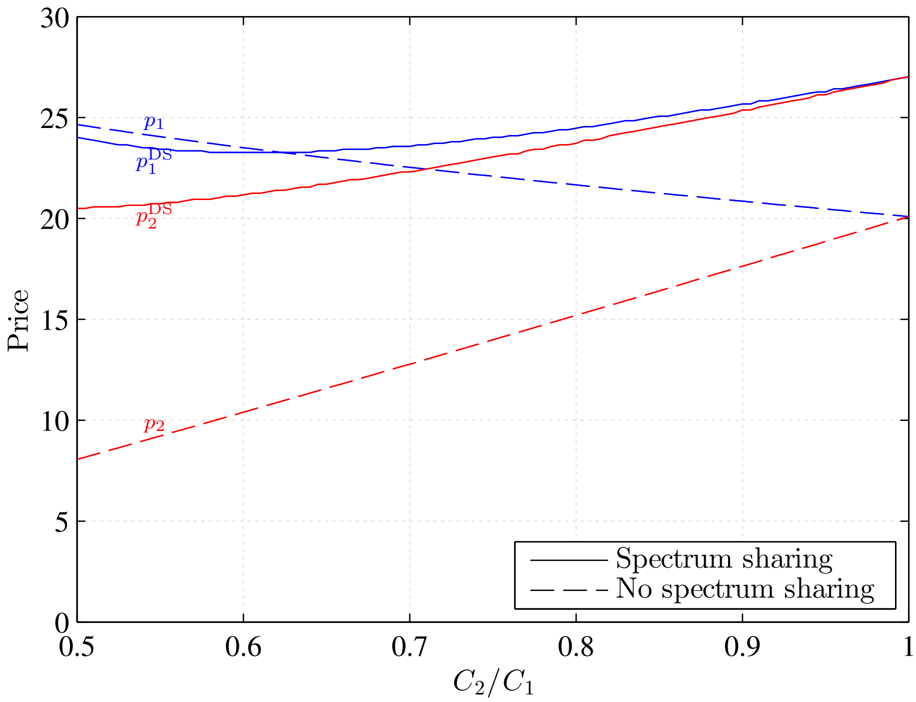

Fig. 5.8 Nash equilibrium prices for discriminatory spectrum sharing as well as the case of no spectrum sharing.

to a Nash equilibrium which delivers a higher profit for both firms if discriminatory access is used.

Chapter 6

Conclusion

I showed that dynamic spectrum allocation can be modeled as a continuous flow of data, which enables us to find closed-form solutions for the user throughput. Such findings are important because they provide analytical insights and allow performance evaluations of our system. By doubling the bandwidth while the number of subscribers also doubles (e.g., two firms merge), the throughput is also doubled. Thus, a larger spectrum increases the throughput even when the number of users also increases, because a higher capacity allows a better exploitation of the statical multiplexing gain. In order to increase spectrum utilization, a promising method is discriminatory access where secondary user can access the capacity in an opportunistic way only when no primary user is active. The aggregated user throughput, a measure for social optimality, is then always higher than in case of no spectrum sharing. Note that the aggregated throughput has a maximum for a certain traffic load. If the traffic load is too high, the network operator should exclude certain users in order to avoid congestion.

For a monopoly as well as a duopoly, firms will always choose a price that is too high with respect to the social optimal solution. Similar, they will not deploy enough base stations. Of the three social optimality criteria I consider, the aggregate throughput is the most important one because the linear approximation of the demand might only be accurate close to some operating point, decreasing the validity of the aggregated Marshallian surplus. Maximizing the aggregated throughput requires a higher price and a higher number of base stations than maximizing the aggregated Marshallian surplus (zero profit). Note that the number of subscriber is the same in both cases.

Whether a monopolistic network operator wants to share his spectrum and infrastructure depends on the demand parameters, whereas the equilibrium prices can be higher or lower relative to the monopoly price. Because discriminatory spectrum access offers two quality levels, we observe a higher price differentiation compared to the egalitarian case. The large range of possible outcomes was the main motivation to define the property of comparable demand, defined as those demand parameters for which the aggregated duopoly demand is the same as the monopoly demand if prices and quality levels are the same. For a compareable demand and in case of egalitarian access, the monopolist never wants to resells his spectrum and infrastructure. On the other hand, in case of discriminatory access, the monopolist allows a MVNO if the price competition is relatively low. Sharing the spectrum usually increases the aggregated throughput. In particular, for a comparable demand function, the aggregated throughput will always be increased. Thus, if the government aims at maximizing the social optimality, it should force the monopolist to allow a MVNO.

If two firms have the same endowment in terms of spectrum, they can increase their profit by spectrum sharing. In the short term, I assume that the number of base stations is fixed and given by the Nash equilibrium of no spectrum sharing. In particular, if the capacity for firm 2 is much lower than that of firm 1, the number of base stations will also be lower. In case of egalitarian spectrum sharing, this implies that firm 1 is able to potentially support more subscribers than firm 2 which results in a lower price for firm 1 compared to firm 2. Although firm 1 has more market power compared to firm 2, the profit can be lower because the higher number of base stations generates also higher costs. It can happen that the capacity of firm 2 is so low that firm 1 refuses to share the spectrum because the higher traffic load in case of sharing would decrease the profit of firm 1. In the long term, such traffic load can be reduced by deploying more base stations so that both firms are better off and always want to share their spectrum. In case of discriminatory sharing, some users have priority so that the congestion effect we see in the egalitarian case is mitigated. Also, different levels of quality allow to increase the price for firm 1, so that it is possible that this price is higher than the price of firm 2.

While I considered two specific application scenarios in this thesis, a MVNO and spectrum sharing in a duopoly, the basic elements proposed here can also be used in further works to answer a large range of possible questions. The linear demand functions can easily be reformulated in matrix notation, so that an extension to more than two firms becomes straightforward. One can then ask the question how the number of firms influence the decision whether firms want to share their spectrum. Nonlinear demand functions might also provide additional insights into the system, in particular, if they capture the effect of diminishing marginal utility. I expect that firms will be more reluctant to share their spectrum because the potential throughput increase no longer has such a large effect on the demand. Practical implementation issues such as signaling overhead, interference, and the performance dependency on the geographical location could also be included in future works.

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Abstract

The spectrum of a mobile network operator is currently fixed and assigned by a regulatory authority based on an auction. By allowing a more dynamic allocation of the spectrum, as proposed in cognitive radio, it is possible to improve the performance in terms of user throughput. In this thesis, I derive a novel closed-form solution for the user throughput in the context of dynamic spectrum allocation, by reinterpreting and reformulating well-known results from queueing theory. I assume two specific access schemes: Firstly, egalitarian access, where all active users share the spectrum equally. Secondly, discriminatory access, where primary users have priority over secondary users who can only access the channel if no primary user is active. I also extend my throughput equations so that they describe spectrum sharing among different network operators. In particular, discriminatory spectrum sharing is analyzed using game theory for which I derive a Nash equilibrium which determines the distribution of data packets between primary channel and secondary channel.

By assuming a linear demand function that depends not only on the price but also on the quality (throughput), I am able to describe the behavior of firms in case of a monopoly and a duopoly. My model predicts the number of base stations and the price subscribers have to pay. These outcomes are then compared to the socially optimal solutions whereas I propose three different social optimality criteria.

The basic elements discussed so far are combined in order to answer the question whether a monopolistic network operator has an inventive to share his spectrum and infrastructure with a mobile virtual network operator. I answer this question by assuming a sequential game which resembles the classical Stackelberg leadership model. The outcome then depends strongly on the underlying demand functions.

Finally, I consider the case of a duopoly and answer the question whether the firms want to share their spectrum. In most cases, they have an incentive to share their spectrum because the additional throughput generates a large additional demand which increases the profit for both firms. However, for the case of egalitarian sharing and an unsymmetrical spectrum endowment, the additional traffic load could be so high that one firm refuses to share the spectrum.

Kurzfassung

Das Spektrum eines Mobilfunkbetreibers wird derzeit statisch zugewiesen wobei hierfür die Regulierungsbehörde eine Auktion durchführt. Durch eine dynamische Allokation des Spektrums, wie beispielsweise in Cogentive Radio vorgeschlagen, kann der Datendurchsatz für Benutzer erhöht werden. In dieser Magisterarbeit wird eine neuartige Gleichung hergeleitet, die den Datendurchsatz für so eine dynamische Zuweisung des Spektrums beschreibt. Hierzu werden bekannte Resultate der Warteschlangentheorie umgeformt und uminterpretiert. Ich nehme an, dass der Zugriff auf das Spektrum auf zwei mögliche Arten erfolgen kann: Erstens, egalitärer Zugriff, bei dem alle aktiven Benutzer den gleichen Anteil am Spektrum erhalten. Zweitens, diskriminierender Zugriff, bei dem primäre Benutzer Priorität gegenüber sekundären Benutzern haben, die auf das Spektrum nur zugreifen können, wenn kein primärer Benutzer aktiv ist. Die Gleichungen werden erweitert, sodass sie auch das dynamische Mitbenützen des Spektrums von unterschiedlichen Netzbetreibern beschreiben. Insbesondere wird diskriminierender Zugriff anhand der Spieltheorie analysiert wobei das resultierende Nash Gleichgewicht angibt, wie Datenpakete zwischen primären und sekundären Kanal aufgeteilt werden müssen.

Die Annahme einer linearen Nachfragefunktion, die nicht nur vom Preis sondern auch von der Qualität (dem Datendurchsatz) abhängt, ermöglicht es mir das Verhalten von Firmen im Falle eines Monopols und eines Duopols zu beschreiben. Mein Modell prognostiziert die Anzahl der Basisstationen sowie den Preis den Mobilfunkkunden bezahlen müssen. Dieses Ergebnis wird dann mit dem sozialen Optimum verglichen, wobei ich drei unterschiedliche Optimalitätskriterien vorschlage.

Die bisher vorgestellten Elemente werden dann verwendet um eine Antwort auf die Frage zu finden, ob ein monopolistischer Netzbetreiber sein Spektrum und seine Infrastruktur einem mobilen virtuellen Netzbetreiber zur Verfügung stellen wird. Ich nehme hierzu ein sequentielles Spiel an, welches Ähnlichkeiten zum klassischen Stackelbergmodell aufweist. Das Ergebnis hängt dann stark von der zugrundeliegenden Nachfragefunktion ab.

Abschließend nehme ich wieder ein Duopol an und beantworte die Frage, ob Netzbetreiber ihr Spektrum gegenseitig mitbenutzen wollen. In den meisten Fällen haben Netzbetreiber einen Anreiz ihr Spektrum gegenseitig mitzubenutzen, da der zusätzliche Datendurchsatz eine zusätzliche Nachfrage generiert wodurch beide Netzbetreiber ihren Gewinn erhöhen können. Wenn jedoch ein egalitärer Zugriff verwendet wird und die Bandbreite der Netzbetreiber unsymmetrisch ist, könnte die zusätzliche Belastung des Spektrums so hoch sein, dass ein Netzbetreiber das Mitbenützen seines Spektrums verweigert.

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Doubly-Selective MMSE Channel Estimation and ICI Mitigation for OFDM Systems, R. Nissel and M. Rupp, IEEE International Conference on Communications (ICC), London, UK, 8-12. June 2015

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