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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Chapter 1

Introduction

Broadly speaking, there are three different types of markets considered in my dissertation. The first two types entail asymmetric information for prices due to search frictions, while the third type pays attention to strategic interactions of consumers. Although the first two types have the same feature that there are upstream and downstream firms where downstream firms engage in costly search for acquiring price information of upstream firms, they are different in that the first article considers an auction-based competition downstream, while the second article the price competition, where downstream firms charge prices to consumers with search frictions. My motivating examples of the three different markets are based on public procurement auctions for constructions, the Tsukiji fish market in Japan, and consumers' technology adoptions of video and DVD formats and browsers, respectively.

This dissertation consists of the following essays.

The first essay (Chapter 2) is based on the paper entitled "Intermediary Search for Suppliers in Procurement Auctions". Here I analyze how and to what extent the existence of information frictions for supplier prices influences market outcomes in procurement auctions provided that intermediaries buy their inputs from suppliers and then compete in an auction.

The second essay (Chapter 3) is based on my joint working paper with Daniel García and Maarten Janssen, entitled "The Double Diamond Paradox". Here I ask what happens if retailers engage in search for wholesale prices as well as consumers for retail prices, and then analyze how much the two levels of search frictions influence market outcomes.

The third essay (Chapter 4) is based on the paper entitled "Games with the Total Bandwagon Property". Here I focus on technology adoption of consumers in a market where multiple products are sold by firms whose pricing behaviors are assumed to be fixed, and then examine how consumers adopt technologies when a bandwagon effect prevails in the market and what technology is chosen in the long run.

1.1 Intermediary Search for Suppliers in Procurement Auctions

Most public buildings (including city hall, fire station, and police station), bridges, and highways are constructed through procurement auctions in such a way that a public authority demands goods and services by calling for applications from general contractors who normally implement projects through subcontracts with suppliers for their inputs. In principle, the general contractors serve as intermediaries and find out who is the best (or the cheapest) supplier for suitable materials and labor to their projects. In determining from which suppliers the general contractors obtain the required materials and labor, information frictions regarding price as well as quality can substantially matter for their market participations and transactions. There is the following empirical evidence: (i) prices of raw materials are dispersed even for homogeneous goods, such as ready mixed concrete (Roberts and Supina, 2000), as well as wages (e.g. Mortensen, 2005);[1] (ii) bidders in public procurement auctions have asymmetric costs, which is observed in Austria (Gugler, Weichselbaumer and Zulehner, 2015), Italy (Decarolis, 2014), Japan (Nakabayashi, 2009), and US (Krasnokutskaya, 2011). Case studies for bid rigging have shown that different firms have different comparative advantages across multiple inputs and cost estimations through gathering supplier price information are quite costly so that the only suggested winner among cartel members normally calculates their estimate costs used for proposals; (iii) entry is significantly costly (Li and Zheng, 2009) which is consistent with the known fact that over half of the proposers observe how many potential bidders participate in the auction and afterward drop out without bidding before the auction takes place. The above introduced empirical evidence tells us three important features of such markets: (i) supplier prices are widespread even for homogeneous goods; (ii) intermediaries decide whether to enter after observing through their proposals how many potential bidders participate in the auction; (iii) information acquisition for supplier prices is costly.[2]

[1] For labor in the construction industry, it is well known that construction workers in Japan are vastly different from many old and skilled ones to many young and unskilled ones within every single company, so that different companies suggest different wage payments.

[2] The empirical evidence is model-driven (through date disclosed by national governments) or data-driven (through data used in bid-rigging or cartel cases).

To investigate the above stated market environment, I consider a procurement auction model with entry where entrants are intermediaries who engage in search to purchase their inputs from suppliers. I mainly show three results: (i) the information frictions concerning input prices can break down the market due to supplier induced entry deterrence by a sufficiently high input price even if both the search cost and the entry cost are sufficiently small; (ii) when the object value to search cost ratio is sufficiently large, there is an alternative market outcome with price dispersion. This dispersed price equilibrium is inefficient due to excessive entry, and in addition, as the value or the search cost increases, so does the range of price dispersion; (iii) the procurement cost may rise as the number of bidders increases.

This research contributes to two strands of the literature. First of all, this article is related to the consumer search literature. The most closely related work is the joint working paper (García, Honda and Janssen, 2015), which is the second article of this dissertation, where we incorporate retailer search into vertical oligopoly models together with consumer search and then examine those impacts on market outcomes, whereas this article adds intermediary search into a procurement auction model and then analyze its impact on the auction. The distinctions between two papers are: (i) the second article introduces *two levels* of search frictions into vertical oligopoly models where price competition takes place at *both* the upstream and downstream market, whereas this article introduces *one level* of search frictions into a procurement auction model where upstream firms compete in prices while downstream firms compete in the auction. The commonly observed feature in both models is that price dispersion emerges endogenously at both the upstream and the downstream level without exogenous cost uncertainty, while this common feature is generated by different pricing mechanisms. Secondly, this research is relevant for the procurement auction literature, particularly for the relationship between the number of bidders and the procurement cost. We usually think that the auction tends to be more competitive as there are more potential bidders, resulting in cost-saving for procurements, and therefore the relationship is of interest and has been investigated theoretically and empirically.[3] The recent literature suggests that the relationship may be non-monotonic (Bulow and Klemperer, 2002; Pinkse and Tan, 2005; Li and Zheng, 2009). In understanding the non-monotonic relationship, this paper provides a different explanation by incorporating information frictions for supplier prices to a procurement auction model with entry.

[3]The research in this line has begun in late 1980s by Samuelson (1985); McAfee and McMillan (1987) and has been still studied (e.g. Gentry and Li, 2014).

1.2 The Double Diamond Paradox

The Tsukiji fish market in Japan is the world's largest wholesale fish market. Every day early in the morning many retailers visit this market to purchase high quality fish before opening their stores. There are over 500 stores in the market and wholesalers often do not disclose their information, such as what kind of fish is available and how much it costs, and therefore retailers must literally engage in search for acquiring information about fish price and quality and then negotiate with wholesalers. After the purchase, retailers go back to their stores to sell the purchased fish to consumers. Consumers also engage in search by visiting (super)markets to purchase good fish at a reasonable price. This is just a specific example, but in general we consider that consumers search for retail stores to buy goods and similarly retailers search for manufacturing or wholesale stores to buy goods and sell them to consumers. Of course, transactions between retailers and manufacturers are often based on (long-term) contracts and different from those between consumers and retailers, but in principle both retailers and consumers commonly engage in search.

This chapter is based on the joint working paper (García, Honda and Janssen, 2015). What we do in this paper is to incorporate retailer search and consumer search into a vertical oligopoly model to analyze their impacts on market outcomes. The main result is that social welfare can be much lower than in the double marginalization outcome. This equilibrium outcome entails two novel features: (i) manufacturers randomize two wholesale prices and induce two retail prices, generating bimodal price distribution at both the wholesale and the retail level; (ii) manufacturers extract more retailers' surplus at a higher wholesale price. Interestingly, these features are consistent with an empirically observed phenomenon on sales patterns.

This research contributes to three strands of the literature. Since our paper mainly examines the impact of search frictions on market outcomes, it is closely related to the consumer search literature and the most closely related work is the paper by Janssen and Shelegia (2015b) who incorporate vertical relations into consumer search models where there is a single manufacturer upstream, thereby resulting in no retailer search, whereas our paper adds retailer search into vertical oligopoly models where there are multiple manufacturers, and therefore retailer search matters for market outcomes. Secondly, our paper is related to the literature on price dispersion with cost uncertainty. In this literature, we usually introduce exogenously given cost uncertainty through inflation and exchange rate or productivity shocks, generating price dispersion (e.g. Bénabou and Gertner, 1993; Dana, 1994; Fishman, 1996; Tappata, 2009; Janssen, Pichler and Weidenholzer, 2011), while we derive price dispersion endogenously without exogenous cost uncertainty. Thirdly, our paper is related to the literature on sales patterns. There are a number of papers which attempt to replicate empirically observed sales patterns (e.g. Varian, 1980; Sobel, 1984; Lal and Matutes, 1994; Pesendorfer, 2002; Hong, McAfee and Nayyar, 2002; Hosken and Reiffen, 2004, 2007; Heidhues and Köszegi, 2008, 2011). To the best of my knowledge, our paper is the first to provide a theoretical model to generate bimodal price distributions at both the wholesale and the retail level.

1.3 Games with the Total Bandwagon Property

In the late 1970s and the 1980s, consumers struggled to choose videotape formats of VHS by Matsushita as JVC and Betamax sold by Sony, but VHS prevailed in the end. The striking force behind this market outcome is that consumers tend to adopt the more popular technology, which is called the bandwagon effect (e.g. Rohlfs, 2001). Similar examples are observed for the high definition optical disc formats between Blu-ray Disc by Sony and HD-DVD by Toshiba (Fackler, 2008) and browsers between Internet Explorer by Microsoft and Netscape by Navigator in the late 1990s and between Google Chrome, Mozilla Firefox, Internet Explorer, Safari by Apple, and Opera in recent years (The Economist, Aug 10, 2013).

The Bandwagon effect is a form of groupthinking in social psychology which says that as more people adopt a belief or an action, others are more likely to do the same thing. In mid 19th century, Dan Rice, a famous circus clown, used a wagon with musicians (bandwagon) for his political campaign in the USA, thereby succeeding to get attention, and afterwards many other politicians started using bandwagons for their political campaigns, which in turn became the standard tool for political campaigns at the beginning of 20th century. This is the origin of the term "bandwagon effect". Today the bandwagon effect is observed in markets through fads and trends, such as consumers' product choices and customs.[4] The concept of the bandwagon effect is explicitly introduced by Leibenstein (1950) into economics for consumer demand theory, and has been further investigated theoretically and empirically.[5]

What I do in this article is to consider the total bandwagon property (TBP) introduced by Kandori and Rob (1998) where TBP is used as a bandwagon effect regarding consumer

[4] The bandwagon effect can be interpreted as a network externality and is related to conformity, herd behavior, information cascade, and so on. Note that we can also see the opposite effect known as snob effect that when many people adopt something, a person avoids to have the same thing or be associated with them. Exclusive products, such as designer clothing and rare artworks, are typical examples.

[5] See, among others, Rohlfs (2001) for a comprehensive analysis on the bandwagon effect in high-tech industries.

technology adoption in an evolutionary context as follows. There is a society consisting of a single population, and each consumer in the society observes a product choice profile taken by all other consumers in the last period and chooses one of the products used by some other people as a best response. Formally, TBP is the property imposed on the class of symmetric two-player games under which all best responses against any mixed strategy are in the support of this mixed strategy.

I first show a characterization of games with TBP via the number of Nash equilibria: A symmetric $n \times n$ game has TBP if and only if the game has $2^n - 1$ symmetric Nash equilibria. Furthermore, by considering the generalized TBP to allow for asymmetric games, I extend the characterization to bimatrix games. This characterization result suggests that a game with a bandwagon effect may have so many Nash equilibria that it is hard to select a single equilibrium. With this in mind, the second objective of our paper is to provide a simple equilibrium selection criterion. Such a simple but strong equilibrium selection criterion is the solution concept of 1/2–*dominant equilibrium* proposed by Morris, Rob and Shin (1995), which is a generalization of *risk dominant equilibrium* (Harsanyi and Selten, 1988). It is chosen by various equilibrium selection methods including the "evolutionary learning method" based on the best response dynamics with mutation (Kandori et al., 1993; Young, 1993); the "global game method" (Carlsson and van Damme, 1993); the "incomplete information game method" (Kajii and Morris, 1997); the "perfect foresight dynamics method" (Matsui and Matsuyama, 1995; Hofbauer and Sorger, 1999); the "spatial dominance method" (Hofbauer, 1999). A 1/2–dominant equilibrium needs not to exist in games with TBP. I show the existence of a 1/2–dominant equilibrium for two subclasses of games with TBP: (i) supermodular games; (ii) potential games.

Lastly, I apply our results to a classical experimental game—the minimum-effort game—introduced by Van Huyck, Battalio and Beil (1990), and then show that the minimum-effort game does not satisfy TBP but is a limit case of TBP.

This article contributes to two strands of the literature. First of all, this article is related to the literature on the number of Nash equilibria in games. To the best of my knowledge, this is the first paper to provide a characterization of a class of games via the number of Nash equilibria. Interestingly, this characterization is connected to the conjecture of Quint and Shubik (1997) that any $n \times n$ bimatrix game has at most $2^n - 1$ Nash equilibria, implying that the number of Nash equilibria given by our class of games involving the bandwagon property is exactly the same as the maximum number given by the Quint-Shubik conjecture. Secondly, we provide new insights on equilibrium selection methods.

Chapter 2

Intermediary Search for Suppliers in Procurement Auctions

2.1 Introduction

Today relevant information for various kinds of products and services is easily obtained via online markets but when purchasing, price dispersion is still persistent not only at the retail level but also at the wholesale level.[1] The purpose of this paper is to investigate why price dispersion emerges in the upstream market through information frictions concerning supplier (or input) prices. As an example, consider a public tender bidding or a procurement process, where a public authority demands goods or services from general contractors by calling for their applications and selecting providers via an auction to choose the most cost-efficient provider out of potentially many applicants. General contractors normally implement projects through subcontracts with suppliers. In principle, the general contractors serve as intermediaries and search for suppliers who can provide suitable materials and labor for the project. Because each project requires various materials and skilled labor, a general contractor needs to acquire relevant information about the suppliers. In determining from which suppliers they obtain the required materials and labor, information

[1] For instance, we have empirically observed price dispersion on, among many others, airline tickets (Gerardi and Shapiro, 2009, references therein), books (Clay et al., 2001), electronics (Baye et al., 2004), life insurance (Brown and Goolsbee, 2002) at the retail level, whereas cars (Goldberg and Verboven, 2001), fruits and vegetables (Kano, Kano and Takechi, 2013), gasoline (Lewis, 2009), beer (Marshall, 2013), multiple materials (Roberts and Supina, 2000) at the wholesale level.

frictions regarding price as well as quality can substantially matter for their transactions.[2]

In this paper we analyze how and to what extent the existence of information frictions for supplier prices in an upstream market influences market outcomes provided that intermediaries buy products from suppliers and compete in a downstream market to sell their products to a buyer through an auction. To do this, we consider a market where a buyer organizes an auction for potentially multiple intermediaries to procure a single unit product from a single intermediary. In order to supply the product to the buyer, intermediaries must obtain products from suppliers but price information about the products is not publicly available and therefore intermediaries engage in costly sequential search to acquire price information. We aim to unveil a hidden side behind this market, more specifically, to pin down a pricing mechanism caused by two components: (i) search frictions in the upstream market and (ii) auction-based competition in the downstream market among searchers. If these two components coexist, suppliers selling products to intermediaries should consider not only their purchasing behaviors in search but also their competition in the auction because intermediaries and suppliers have vertical relations and split profits earned in the auction between them. With this in mind, we introduce the simplest possible model to explore the role of the above mentioned two components (i) and (ii) for market outcomes. We assume that suppliers are symmetric and produce homogeneous inputs at the same constant marginal cost while intermediaries are also symmetric in such a way that their search costs are common and they can transform one unit of the input to one unit of the output sold to the buyer for free. In the main analysis, we consider the case of two suppliers and two intermediaries for clarifying the mechanism arising from the combined effect of (i) and (ii) but most of our results are maintained when considering multiple suppliers or multiple intermediaries. Suppliers and intermediaries take their actions sequentially as follows: At the beginning, suppliers simultaneously charge per-unit prices of their products to sell to intermediaries. Next, intermediaries engage in sequential search for price information on materials and labor without knowing which suppliers other intermediaries

[2] Actual data on procurement auctions including bids and estimated prices (or engineer's estimates) clearly exhibits dispersed ratio of bids to estimated prices, which basically attributes to costs, quality, scales, and types of projects as well as potential competitors related to projects, though bids are substantially influenced by cartels if any (see, e.g., Porter and Zona, 1993; Pesendorfer, 2000). For instance, when we consider a public tender bidding for constructions, bid dispersion is notable (e.g., the websites of the Ministry of Land, Infrastructure, and Transportation in Japan for public procurements at the national and regional levels). We have empirically observed that prices of raw materials such as concrete and wood products are dispersed (see, e.g., Roberts and Supina, 2000). Empirical studies to estimate procurement costs support cost dispersion among bidders (see, e.g., Decarolis (2014) for procurements of road construction and maintenance in northern Italy, Gugler et al. (2015) for construction procurements in Austria, Krasnokutskaya (2011) and Krasnokutskaya and Seim (2011) for highway procurement auctions in US, and Nakabayashi (2009) for construction procurement auctions in Japan).

contract with. When all intermediaries end their search activities, intermediaries who have bought products participate in an auction where a single winner is chosen to supply the product to the buyer. It is important to understand that intermediaries are searchers in the upstream market and that they strategically compete in the downstream market via an auction. In order to see how intermediary entry affects supplier pricing behaviors and market outcomes, we allow intermediaries to decide whether or not to enter the market provided that each intermediary's decision is unobservable to others.

The market considered above can break down in such a way that suppliers charge a sufficiently high input price, by which none of intermediaries enter the market, and therefore the auction cannot take place even when the search cost is sufficiently small. This occurs because entering the market is not beneficial for intermediaries as long as suppliers charge a sufficiently high price. Even if a supplier were to lower the price, their beliefs about supplier prices will not change, thus none of intermediaries enter the market. More importantly, alternative market outcomes with price dispersion can arise from an unexplored mechanism based on the combined effect of search frictions and auction-based competition among searchers. This happens typically when the buyer's valuation of the object procured through the auction is large enough relative to the search cost. Intuitively, the higher valuation stimulates supplier competition, lowering input prices, and then it induces intermediary entry. If a fraction of intermediaries enters the market, charging any single price cannot be optimal for suppliers, leading to price dispersion. To explain this, let us look at the two situations where all suppliers commonly charge marginal cost or any single price above marginal cost. Each supplier has an incentive to charge a slightly higher price than marginal cost because charging marginal cost yields zero profit in any case but charging a slightly higher price can yield a positive (expected) profit when all intermediaries buy products from a single supplier due to the search cost, which might be a tiny profit but still better than nothing. In contrast, when all suppliers charge any single price above marginal cost, each supplier has an incentive to slightly cut the price and enable the intermediary who has bought the supplier's product to win in the auction because each supplier is more likely to compete with other suppliers indirectly through intermediaries than being a monopolist who contracts with all intermediaries. Extending the above argument, we can show that suppliers adopt randomized prices as their optimal strategies. This result tells us that search frictions cause a market distortion, which is well known in the search literature, but that its impact depends crucially on the seller-buyer relationship in a market with search frictions. In our environment where sellers are suppliers while buyers are intermediaries instead of consumers, suppliers indirectly sell products to consumers through intermediaries, thereby giving some margin to intermediaries. This seller-buyer relationship mitigates market distortion caused purely by search frictions but it generates price dispersion through a distinct market channel.

An interesting feature of any given equilibrium with intermediary entry is that entry of all potential intermediaries cannot be part of an equilibrium even if the search cost for acquiring input price information is sufficiently small and intermediary entry is costless. This happens because suppliers charge prices to extract intermediary entry surplus through costly *first* search while accounting for intermediary sequential search.

What kind of price dispersion emerges depends heavily on both supplier pricing and intermediary market entry. On the one hand, if suppliers charge competitive prices, it induces intermediaries to enter the market and then ends up with overall low bids in the auction, while on the other hand if suppliers charge high prices, it deters entry and the whole market competition fails to hold, leading to high bids. This gives us two possibilities of price dispersion, one of which produces a competitive outcome while the other an anti-competitive outcome, but the common feature of these two price dispersion is that suppliers cannot charge a price to fully extract an intermediary surplus due to competition in the downstream market, and thereby set all mixed prices below the monopoly price. In fact, we show that if the (object) value to search cost ratio is large enough, there are two dispersed price equilibria, which are basically socially inefficient. The equilibrium with the higher entry rate entails social excess entry, whereas the equilibrium with the lower entry rate causes social insufficient entry. As the search cost converges to zero, input prices converge to the perfectly competitive price if suppliers induce intermediary entry, whereas the prices converge to the monopoly price if suppliers deter entry. Thus, both full participation and no participation would be part of equilibria in the limiting case. To argue if an equilibrium is stable, we utilize the idea of entry stability in the long run where intermediaries more frequently enter the market if they get a positive (expected) profit, otherwise they less frequnetly enter the market, and then show that the equilibrium with the higher entry rate is stable and the equilibrium with the lower entry rate is unstable. Below we discuss the stable equilibrium where suppliers adopt competitive prices and intermediaries excessively enter the market.

Next we examine to what extent the value or the search cost affects price dispersion. Intuitively, the larger value strengthens the price competition among suppliers and its competitive force reduces overall input prices, which enhances intermediary entry rate and prices are more dispersed because of two countervailing effects: (i) stronger competition among suppliers; (ii) stronger competition among intermediaries. As stated above, the first effect leads to the enhancement of intermediary entry indirectly through reduced input prices, which results in the second effect. This supplier induced entry strengthens the monopolist position of suppliers when all intermediaries buy at a single supplier store and enlarges the gap between the monopolist profit and the profit under competition. On the other hand, if the search cost decreases, overall input prices become lower because entry is less costly for intermediaries, increasing entries, and therefore it strengthens supplier comptition, resulting in lower prices. As opposed to the effect of the larger value, the range of price dispersion decreases as the search cost decreases. This occurs because the smaller search cost reinforces the suppliers' competition, which outweighs the countervailing effect of (ii) mentioned above, thereby suppliers are obliged to decrease the payoff gap between the monopolist profit and the profit under competition, in addition to charging lower prices. This implies that a change of the value to search cost ratio itself does not tell how it affects price dispersion, and therefore we need to assess the impact of the value and search cost on price dispersion separately.

Additionally, we investigate how the number of suppliers or intermediaries influences multiple aspects including supplier pricing, intermediary entry, welfare, and address the relationship between market concentration and price dispersion, which heavily relies on which market concentration we measure, downstream or upstream market concentration. Our numerical analysis shows that the relation can be positive or negative.[3] But we can see fundamental number effects on market outcomes. Basically as there are more suppliers, the upstream market becomes more competitive for suppliers, and then more competitive input prices induce more intermediary entries. Thus, the average input price goes down and prices are more dispersed due to the two countervailing effects (i) and (ii) explained above. On the other hand, as there are more potential entrants (as intermediaries), suppliers obtain stronger seller power against intermediaries and therefore suppliers charge overall higher input prices and spread them more because of the effect (ii).

Our contribution is given as follows. We add search frictions to an upstream market instead of a downstream market and explore the role of search frictions for market outcomes under the consideration that downstream price competition takes place via an auction. Then we uncover a hidden pricing mechanism underlying the above mentioned market that causes price dispersion at both the upstream and downstream levels. This is closely related to García, Honda and Janssen (2015) who incorporate two levels of search frictions in both upstream and downstream markets but their mechanism differs from ours.[4] In their model,

[3]Note that the numerical analysis suggests that there are at most two equilibria even if the number of either suppliers or intermediaries increases.

[4] There is a related paper by Gal-Or, Gal-Or and Dukes (2007) who consider an input market where there are a single manufacturer and input suppliers and the manufacturer procures the input from a supplier with incorporating search frictions in such a way that the manufacturer engages in search to acquire input information including quality (as *incomplete* information). They consider the only one manufacturer and do not consider competition among them. In contrast, our paper incorporates only input prices without considering quality but instead introduces multiple intermediaries (corresponding to manufacturers in their model) to analyze the role of their competition for market outcomes.

there is one more level of search frictions in a downstream market for a buyer (corresponding to a unit mass of consumers in their model) to acquire retail price information together with search frictions in an upstream market for intermediaries (retailers) to acquire (wholesale) prices charged by suppliers (manufacturers). They endogenously derive price dispersion due to two levels of search frictions. Contrary to this, price dispersion in our model stems from the combined effect of one level of search frictions and competition among searchers via an auction. In addition, double marginalization is the best market outcome that one can expect in their model but in our model, because there are no search frictions in the downstream market and furthermore price competition downstream is based on an auction, input prices and bids in dispersed price equilibria are below the monopoly price. The common feature of market outcomes in both models is that price dispersion is observed at both the upstream and downstream levels, although it is driven by distinct channels. The main force behind this result is that suppliers generate retail cost uncertainty *endogenously* in the absence of exogenous uncertainty, which contrasts from the literature where we incorporate *exogenous* cost uncertainty, such as inflation and exchange rate or production shocks, to investigate its impact on price dispersion (see, e.g., Bénabou and Gertner, 1993; Dana, 1994; Fishman, 1996; Tappata, 2009; Janssen, Pichler and Weidenholzer, 2011).

Our mechanism underlying price dispersion is similar to Janssen and Rasmusen (2002) who consider Bertrand competition with *exogenous* uncertainty of number of entrants and derive price dispersion. The mechanism behind their result is that each firm believes due to the uncertainty that they can be a monopolist while they might compete with other potential entrants, which expresses an essential part of our mechanism. In our model, however, uncertainty of number of entrants is endogenously determined by strategic pricing of suppliers.

We show that equilibrium price dispersion arises without assuming heterogeneity. This result is also found in the consumer search model of Burdett and Judd (1983), which induces the *fixed sample search* and differs from other consumer search models (Salop and Stiglitz, 1977; Reinganum, 1979; MacMinn, 1980; Rosenthal, 1980; Varian, 1980; Stahl, 1989).[5] They consider a retail market and derive two dispersed price equilibria given that all retailers and consumers are identical. In their model (with fixed sample search), heterogeneous consumer search behaviors in terms of search intensity emerge endogenously, which affects firms' pricing behaviors and causes price dispersion, and then different search intensity distributions among consumers can generate multiple equilibria. By contrast, in our model (with sequential search) consumers use the same search behavior but the com-

[5]For instance, Salop and Stiglitz (1977), Varian (1980), and Stahl (1989) add different types of consumers to induce equilibrium price dispersion, whereas Reinganum (1979) and MacMinn (1980) introduce firms' cost uncertainty into their models to derive price dispersion.

bined effect of search frictions and auction-based competition among searchers generates price dispersion, and then different intermediary entry rates, that are endogenously determined by supplier pricing patterns, can generate multiple equilibria. In fact, our model is essentially different from consumer search models mentioned above in the sense, that search frictions in consumer search models can give firms a chance to fully exploit some fraction of consumers, which makes it possible to charge the monopoly price, but in our model such an attempt of a firm (as supplier) does not work due to competition among searchers downstream, which pushes the upper bound of prices below the monopoly price. Because of this, our model introduces a different type of search frictions, which produces an unexplored mechanism behind price dispersion.

Costly first search is the driving force of our result that entry of all intermediaries cannot be part of equilibrium no matter how small the search cost is. Janssen, Moraga-González and Wildenbeest (2005) are the first paper to introduce costly first search into the consumer search model of Stahl (1989) by which firms exploit consumers (as non-shopper) due to information frictions regarding firm prices and thereby a fraction of consumers decide not to buy the products without searching. On the other hand, in our model, suppliers extract intermediary surplus due to information frictions regarding supplier prices and thereby a fraction of intermediaries does not enter a market while competing with each other if they enter the market. Thus, the market entry decisions of entrants are made in different ways.

The relationship between the winnig bid and number of bidders has long been investigated theoretically and empirically.[6] This is particularly important in the procurement auctions because the lower winning bid reduces the procurement cost.[7] We usually think that the larger number of bidders strengthens the competition among bidders, thereby lowering the procurement cost. But its relationship may be non-monotonic because of countervailing effects against the competition effect. One of the countervailing effects related to our paper is *entry* effect (Li and Zheng, 2009), meaning that a winning bidder may believe that they overestimate the intensity of entry, and it may outweigh the competition effect, resulting in a higher winning bid as the number of (potential) bidders increases.[8] By contrast, in our model equilibrium input prices increase overall as there are more bidders (intermediaries) because suppliers gain larger power against intermediaries, thereby increasing the procurement cost. Moreover, if there are more suppliers, they induce more

[6] See Samuelson (1985), McAfee and McMillan (1987), Levin and Smith (1994), Bulow and Klemperer (2002), Hong and Shum (2002), Pinkse and Tan (2005), Li and Zheng (2009), and Gentry and Li (2014).

[7] In practice, however, "competitive bidding may lead to adverse selection" (Bajari, McMillan and Tadelis, 2009, p.379).

[8] There are other representative countervailing effects known as *winner's curse* effect (Bulow and Klemperer, 2002) and *affiliation* effect (Pinkse and Tan, 2005).

intermediary entries by lowering the average input price while spreading prices more due to the above mentioned two countervailing effects, and therefore more bidders might have higher input prices, leading to higher procurement costs.

The rest of our paper is organized as follows. In Section 3.2 we introduce the model and derive the basic property that all equilibria satisfy. Our main analysis and results are contained in Sections 2.4 and Section 2.5 presents an example to illustrate them. Section 2.6 discusses the relationship between market concentration and price dispersion, namely, how number of firms influences market prices. In Section 2.7, we argue what happens if we introduce two-part tariffs beyond linear contracts. Section 4.6 concludes.

2.2 Model

We consider a market with three types of agents. There are $m(=1,2,\dots)$ suppliers who produce homogenous goods at a constant unit cost that is normalized to zero, there are $n(=1,2,\dots)$ potential intermediaries who buy goods from suppliers, and there is a buyer who buys the good from an intermediary. Initially, intermediaries decides whether to enter the market where a common entry cost is normalized to zero.[9] Each intermediary's decision on market entry is ex ante unobservable to others so that both suppliers and intermediaries know how many intermediaries potentially exist but do not know how many of them actually enter the market. After all intermediaries simultaneously decide whether to enter, suppliers set unit prices of the goods. The prices are fixed afterwords and not negotiable between suppliers and intermediaries, and in addition each supplier price is unobservable to others ex-ante but intermediaries who visit the supplier can observe its price. To observe prices, intermediaries who has entered the market engage in sequential search. The search costs of intermediaries to observe any single price quote are common and given by $s > 0$. This means that if an intermediary visits a supplier to observe its price, this search activity costs s.[10] When an intermediary visits a supplier and decides to buy immediately without continuing to search for an another supplier, the transaction between them is made on a unit-price basis. The transaction is not observable for all other suppliers and intermediaries. When all intermediary entrants end search activities, they compete each other to sell the good to the buyer at a downstream market that is described below. If intermediaries cannot sell the good to the buyer, suppliers who has sold goods

[9] Similarly to marginal cost of productions, as long as an entry cost is common for all intermediaries, our main argument continues to hold and we can straightforwardly add its impact on market outcomes.

[10] Note that the first search is costly as well as other additional search.

to them cannot gain a profit as well.[11] Thus, suppliers' profits highly depend on visiting intermediaries' sales at the downstream market.

Procurement Auction at the Downstream Market

Each intermediary knows his marginal cost, which is a price charged by a supplier whom he has contracted, but he does not know how many competitors he has at the downstream market nor their marginal costs. In this set-up, the cost uncertainty is endogenously induced by suppliers if they randomly choose prices. We consider a procurement (or a reverse) auction, where (i) the buyer procures a single unit good and the buyer's valuation of the good is given by $v > 0$, which is known to all suppliers and intermediaries prior to the auction, and (ii) bidders (as intermediaries who participate in the auction) follow the Dutch auction rule under which the auction begins with the known buyer's valuation v, decreasing the asking price from v to a lower price continuously, and a bidder who keeps participating in the auction without leaving and turns to be the single participant in the auction will be a winner. The winner's payment is determined by this specific rule in such a way that if there is a competition among bidders, the bidder who has the lowest marginal cost will be a winner with paying the second lowest marginal cost of bidders, whereas if the bidder is a single participant in the auction, the bidder will be the winner with v.

The conditions (i) and (ii) are used to model a procurement auction in a tractable way. In practice however, engineers' estimates of (procurement) costs that are part of the buyer's valuation are often not publicly available prior to an auction, which does not fit with (i), and it is natural to consider a first price sealed bid auction (e.g. Pesendorfer, 2000) for public procurement, giving bidders' behaviors a crucial difference from the above considerd specific auction. Several of our main results for existence of an equilibrium (with price dispersion) hold even when incorporating the first price sealed bid auction (with uncertainty of the buyer's valuation) to our model, but the other results, particularly the results for comparative statics, obtains by assuming the specific auction, otherwise our model is not explicitly solvable when suppliers randomize over prices, thereby resulting in cost uncertainty of bidders.[12]

[11] We can consider that the transaction between each supplier and intermediary is based on a kick-buck policy of a contract that an intermediary can give the unsold good back to an supplier without its payment or a contract to guarantee that at any given point in time an intermediary can buy the good at a price charged by the supplier and sell it to the buyer.

[12] Technically speaking, our model with a sealed-bid auction can be considered as the Bertrand competition with *endogenous* cost uncertainty, which generalizes the framework of Spulber (1995) where cost uncertainty is exogeneous. In general, even if cost uncertainty is exogeneously given as an identical and

Sequence of Actions

Suppliers, intermediaries, and a single buyer take their actions sequentially as follows. First, each intermediary decides whether to enter the market. Secondly, each supplier $i(= 1, \dots, m)$ simultaneously charges a per-unit price $w_i \geq 0$ for the good to sell intermediaries.[13] Thirdly, intermediaries who have entered the market engage in sequential search and this search process determines their marginal costs to sell the single unit good to the buyer. Finally, those intermediaries participate in the auction and then the market outcome realizes when the auction ends.

Intermediary Search

Since each intermediary does not know prices charged by suppliers ex-ante, he randomly visits one of the suppliers by incurring a search cost s to observe the price charged by the supplier. If he wants to continue to search, he pays an additional search cost s to randomly visit an another supplier whom he has not visited before. We assume search with free recall, that is, after an intermediary continues to search, he can go back to the supplier whom he has visited before without any cost.[14]

An intermediary's search strategy is characterized by a reservation price strategy, which is basically a binary-choice strategy: there exists a reservation price ρ such that if an intermediary observes a price $w \leq \rho$, he buys immediately at that price, otherwise he continues to search for an additional price.[15] So, the reservation price strategy is defined

independent cost distribution across firms, it is not easy to obtain an explicit bidding strategy. This gives us one reason why we consider the reverse auction above.

[13]We assume that intermediaries make market entry decisions at the first stage but the market outcomes are identical even if the market entry decisions are made after suppliers charge prices because every intermediary's entry decision is unobservable for all others.

[14]The free recall is often assumed explicitly or implicitly in the consumer search literature. Janssen and Parakhonyak (2014) is the first among others to explore what happens if we incorporate costly revisits into a general sequential search model and show that the optimal sequential search rule can substantially differ from the reservation price rule under free recall, but they show that the reservation price equilibrium in the model of Stahl (1989) still remains even under costly revisits mainly because all consumers stop to buy at the first visiting store, although it is not the case in the model of Wolinsky (1986) where some consumers search more than once.

[15]If an intermediary visits all suppliers whose prices are above ρ, he buys at a supplier who sets the lowest price among all suppliers. Notice that by the free recall assumption, going back to a supplier whom he has previously visited is costless.

by

$$\sigma^*(\rho) \quad = \quad \begin{cases} \text{buy at } p, & \text{if } p \leq \rho, \\ \text{continue to search,} & \text{if } p > \rho. \end{cases} \tag{2.1}$$

The optimality of the reservation price strategy for intermediaries follows from the fact that intermediaries have nothing to learn in this model.[16] In an environment without learning, a reservation price strategy is optimal (Kohn and Shavell, 1974).

Equilibrium Concept

The equilibrium concept used in this paper is symmetric *Perfect Bayesian Equilibria* with passive beliefs (see, e.g., McAfee and Schwartz, 1994).[17]

To formally define equilibria, we introduce the following notation. We denote by $F^*(\cdot)$ a price distribution for suppliers with (bounded) support $[\underline{w}, \overline{w}]$ for $0 \leq \underline{w} \leq \overline{w}$ and $\alpha^* \in [0, 1]$ is denoted by the market entry probability. The intermediary's search strategy is determined by a reservation price strategy as discussed above. For an intermediary's strategy in the auction, because of the Dutch auction rule explained before, we can consider a pure strategy $b^*(\cdot)$ as a unique undominated strategy in the auction such that (i) if intermediary $j = 1, \ldots, n$ is a single participant in the auction, he bids v; (ii) if he competes with others in the auction where his marginal cost is w_j, others' costs are w_{-j}, and the lowest cost among others is w^l_{-j}, his (final) bid is $\min\{w^l_{-j}, v\}$ if $w_j \leq w^l_{-j}$ and w_j otherwise. That is, a bidding strategy in the reverse auction can be defined by

$$b^*(w_j, w_{-j}) \quad = \quad \begin{cases} \min\{w^l_{-j}, v\} & \text{if } w_j \leq w^l_{-j}, \\ w_j & \text{otherwise.} \end{cases} \tag{2.2}$$

[16] See Rothschild (1974) for an observation on the (non-)optimality of the reservation price rule when the search environment is (not) stable.

[17] Passive beliefs used in our model are out-of-equilibrium beliefs such that when observing a supplier's deviation from the equilibrium prices, the visiting intermediary believes that other suppliers still stick to charge the equilibrium prices. If we do not restrict out-of-equilibrium beliefs, most of results in the consumer search literature (e.g., the Diamond Paradox (Diamond, 1971) and price dispersion in Stahl (1989)) does not hold in general. This is because we can construct a parsimonious out-of-equilibrium belief instead of passive beliefs. For example, when they observe an unexpectedly higher price than the marginal cost, consumers have a negatively correlated belief that the other firms charges a much lower price to compensate at least a search cost and then continue to search if the unexpected price is slightly higher than the search cost. For related issues, for instance, see García et al. (2015) and Janssen and Shelegia (2015a).

From above, we define the equilibrium concept used in this paper as follows.

Definition 2.1. A symmetric Perfect Bayesian Equilibrium is a supplier price distribution $F^*(\cdot)$ and an intermediary entry, search, and bidding strategy $(\alpha^*, \sigma^*(\rho), b^*(\cdot))$ such that $\sigma^*(\rho)$ is defined by (2.1) and $b^*(\cdot)$ by (2.2), and intermediaries have passive beliefs on supplier prices off the equilibrium path.

2.3 Basic Properties of Equilibria

As benchmarks for our main analysis, we provide basic properties that all equilibria satisfy.

First of all, we consider if no intermediary entry can be an equilibrium where the auction does not take place and the market completely collapses. To do this, suppose that there is no intermediary entry, i.e., $\alpha = 0$ and the buyer's valuation is larger than the search cost, that is, $v - s > 0$ so that intermediary entry can pay off. If suppliers charge any symmetric price $w \in [0, v - s)$, an intermediary has an incentive to unilaterally deviate to enter the market and then buy at w, sell at v, and get a profit $v - w > 0$, which implies that it is a profitable deviation. So, the supplier price below the price $v - s$ and no entry cannot be an equilibrium. In contrast, if suppliers charge a high price $w \geq v - s$, any intermediary has no incentive to enter the market because they cannot gain a positive profit even when selling the good to the buyer in the auction due to costly search, and in addition, each supplier does not have an incentive to deviate because charging any price does not change beliefs of intermediaries for supplier prices off the equilibrium path and there is no entry, thereby giving the deviating supplier zero profit. Thus, there is an equilibrium where the suppliers charge a high price and all intermediaries do not enter the market, no matter what auction we consider and no matter what bidding strategies intermediaries adopt.[18]

Proposition 2.1 (Entry Deterrence and Market Breakdown)**.** *There is an equilibrium such that all suppliers adopt a pure strategy* $w \in [\min\{v - s, 0\}, v]$ *and all intermediaries do not enter the market.*

This implies that there exists a continuum of the market breakdown equilibria where suppliers can deter intermediary entry by charging a sufficiently high price and then the auction does not take place due to no entry.

[18]Here we consider an equilibrium as a *weak* Perfect Bayesian Equilibrium. If we consider a *strong* Perfect Bayesian Equilibrium, however, because of the off-the-equilibrium path where a supplier deviates to a lower price than v and an intermediary has an incentive to enter the market and get a positive payoff when visiting the deviating supplier, $\alpha = 0$ and $w \in [v - s, v]$ cannot be part of an equilibrium.

Next, we examine what happens if competition at both the upstream and downstream markets exists. In doing so, suppose that there are multiple suppliers and multiple intermediaries in the market. Then, we can easily see that both marginal cost pricing and monopoly pricing are not optimal for suppliers. To see this, first let us consider a situation where all suppliers charge prices equal to marginal cost and each supplier's profit is zero. If a supplier deviates to charge a slightly higher price, the deviation is profitable when all intermediaries visit her and stop to buy goods due to costly search, which occurs with some probability. Thus marginal cost pricing is not an equilibrium strategy. Next, let us consider a situation where all suppliers charge the monopoly price v. If a supplier reduces the price a little bit, a visiting intermediary will be a winner in the auction because other intermediaries' bids are higher than his bid and he gets the whole demand, making the supplier's profit larger than under the monopoly price. So, competition among intermediaries gives suppliers an incentive to undercut.

More generally, we can show that any price without randomization is not optimal for suppliers.

Proposition 2.2. *There is no other equilibrium such that suppliers adopt a pure strategy profile except for a continuum of the market breakdown equilibria given in Proposition 2.1.*

For the proof of Proposition 2.2, see the Appendix.[19] This implies that "the 'law of one price' is no law at all" (Varian, 1980, p.651) and instead price dispersion necessarily arises if an equilibrium with market entry exists.

The mechanism behind Proposition 2.2 is reminiscent of Janssen and Rasmusen (2002) who consider Bertrand competition with (exogenous) uncertainty of number of entrants and derive price dispersion. The mechanism behind their result is that each firm believes due to the uncertainty that they can be a monopolist while may compete with other potential entrants, which expresses an essential part of our mechanism. In our model, however, uncertainty of number of (intermediary) entrants is endogenously determined by strategic pricing of suppliers.

2.4 Equilibrium Analysis

Our main purpose in this paper is to pin down a pricing mechanism underlying competition among intermediaries on top of intermediary search. To this aim, we mainly consider the

[19] We consider only symmetric pure strategies for suppliers but we can extend Proposition 2.2 by which asymmetric pure strategies are also shown to be not optimal.

simple case where there are two suppliers and two intermediaries. In the following section below we will discuss the number effect of suppliers or intermediaries on pricing and market outcomes, that is, how upstream or downstream market concentration affects the market prices and welfare.

Assume that suppliers adopt a price distribution F with support $[\underline{w}, \overline{w}]$. Since we consider an equilibrium where intermediaries use a reservation price strategy, the upper bound of the support, $\overline{w}$, should equal the reservation price ρ as an optimal strategy for suppliers.[20] Given that intermediaries enter the market with probability $\alpha \in (0, 1]$ and the other supplier adopts F and randomizes over prices $[\underline{w}, \overline{w}]$, the expected profit of a supplier when charging any given price $w \in [\underline{w}, \overline{w}]$ is given by

$$\Big(\alpha^2\big(\frac{1}{4} + \frac{1}{2}(1 - F(w))\big) + \alpha(1 - \alpha)\Big)w.$$

To understand the above profit, we consider two cases: (i) both of two intermediaries enter the market, which occurs with probability α^2; (ii) only one intermediary enters the market, which occurs with probability $2\alpha(1 - \alpha)$. Note that in a reservation price equilibrium, no intermediary searches more than once. In Case (i), if both of them visit the same supplier, which occurs with probability $(\frac{1}{2})^2$, she can capture the whole demand no matter how intermediaries compete in the auction; if one of them visits her and the other intermediary visits the other supplier, which occurs with probability $\frac{1}{2}$, the intermediary who visits her will win the auction if the other supplier price is above w, which occurs with probability $1 - F(w)$; if no one visits her, which occurs with probability $\frac{1}{4}$, obviously she gets nothing. Similarly, in Case (ii) where only one intermediary enters the market, if the intermediary visits her, which occurs with probability $\frac{1}{2}$, she can capture the whole demand because the intermediary is the only bidder in the auction; otherwise, she gets nothing. Since the expected profits over all prices in the support must be equal as an equilibrium strategy for suppliers, we derive the condition under which the profit of $w \in [\underline{w}, \overline{w}]$ equals that at the upper bound $\overline{w}$.

$$\Big(\alpha^2\big(\frac{1}{4} + \frac{1}{2}(1 - F(w))\big) + \alpha(1 - \alpha)\Big)w \quad = \quad \Big(\frac{\alpha^2}{4} + \alpha(1 - \alpha)\Big)\overline{w}. \tag{2.3}$$

Next, we consider the condition regarding sequential search of intermediaries as follows. Suppose that an intermediary visits a supplier who sets the highest price $\overline{w}$. After this

[20] This is a commonly used argument in the consumer search literature. For the basic arguments used in consumer search models, see, for instance, Varian (1980) and Stahl (1989). For an overview on the consumer search literature, see Baye, Morgan and Scholten (2006).

visit, if he continues to search for an additional price, he gets the expected gain of

$$\Big(\frac{\alpha}{2}+(1-\alpha)\Big)\Big(\overline{w}-\int_{\underline{w}}^{\overline{w}} wdF(w)\Big).$$

To understand the above gain, we consider three cases: (i) the other intermediary enters the market and visits the same supplier who sets $\overline{w}$, which occurs with probability $\frac{\alpha}{2}$; (ii) the other intermediary enters the market but visits the other supplier, which occurs with probability $\frac{\alpha}{2}$; (iii) the other intermediary does not enter the market, which occurs with probability $1-\alpha$. In Case (i), since the other supplier randomizes over $[\underline{w},\overline{w}]$ under distribution F and he expects to observe the average price $\int_{\underline{w}}^{\overline{w}} wdF(w)$ at the other supplier, the gain of continuing to search is given by $\overline{w}-\int_{\underline{w}}^{\overline{w}} wdF(w)$. In Case (ii), he visits the supplier whom the other intermediary has visited and then both intermediaries compete in the auction under the same marginal cost, which means that continuing to search is not beneficial. In Case (iii), since his marginal cost will be replaced by $\int_{\underline{w}}^{\overline{w}} wdF(w)$ and there is no competition between intermediaries in the auction, the expected gain is $\overline{w}-\int_{\underline{w}}^{\overline{w}} wdF(w)$. As a reservation price equilibrium, $\overline{w}$ equals the reservation price ρ and the expected gain via an additional search must be the largest among all supplier prices and moreover it must be below or equal to the search cost, otherwise a visiting intermediary continues to search. Thus, a reservation price strategy imposes the following condition.

$$\Big(\frac{\alpha}{2}+(1-\alpha)\Big)\Big(\rho-\int_{\underline{w}}^{\rho} wdF(w)\Big) \;=\; s. \tag{2.4}$$

Lastly, we consider the market entry decision of intermediaries when suppliers follow the reservation price strategy that satisfies the above two conditions (2.3) and (2.4). To do this, we first need to derive the expected gain of an intermediary when both intermediaries enter the market and one intermediary buys at a supplier while the other intermediary buys at the other supplier provided that both of them search once. In this case, each intermediary gets the following expected gain.

Lemma 2.1 (Expected Revenue in the Auction)**.** *The expected revenue of each intermediary in the reverse auction is given by* $\int_{\underline{w}}^{\overline{w}}(1-F(w))F(w)dw$.

The proof of Lemma 2.1 is given in the Appendix.

Using Lemma 2.1, we do the benefit and cost analysis for intermediary entry as follows.

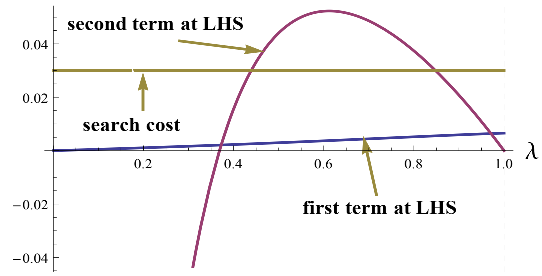

Figure 2.1: The values of three terms of Condition (2.5) when $(v, s) = (1, 0.1)$.

The expected profit of an intermediary who has entered the market is given by

$$\frac{\alpha}{2}\int_{\underline{w}}^{\rho}(1-F(w))F(w)dw + (1-\alpha)\Big(v - \int_{\underline{w}}^{\rho} wdF(w)\Big).$$

To understand the above profit, we consider the same three cases as in Condition (2.4). In Case (i), since the other intermediary visits the same supplier, both intermediaries have the same marginal cost and compete in the auction, resulting in no benefit, while in Case (ii), since the other intermediary visits the other supplier and the marginal cost for each intermediary is considered to be randomly drawn from the distribution F, the expected gain is given by $\int_{\underline{w}}^{\rho}(1-F(w))F(w)dw$.[21] In Case (iii), since there is no competition in the auction, he can obtain the whole demand v under the (expected) cost of $\int_{\underline{w}}^{\rho} wdF(w)$, resulting in an expected gain of $v - \int_{\underline{w}}^{\rho} wdF(w)$. Since visiting a single supplier costs s, which must equal the entry benefit as a mixed strategy $\alpha \in (0, 1]$ for market entry, the following condition on the market entry must hold.

$$\frac{\alpha}{2}\int_{\underline{w}}^{\rho}(1-F(w))F(w)dw + (1-\alpha)\Big(v - \int_{\underline{w}}^{\rho} wdF(w)\Big) \;=\; s. \tag{2.5}$$

For Condition (2.5), the first (resp. second) term at the LHS represents an intermediary's expected profit when the other intermediary enters (resp. does not enter) the market. We can easily incorporate a common transaction fee between a supplier (or an buyer) and an intermediary or a common market entry fee by adding a corresponding term to the right hand side of Condition (2.5). For instance, if we consider parameters $(v, s) = (1, 0.1)$, all terms of (2.5) are illustrated in Figure 2.1. This suggests that given that the good's value

[21]For the derivation of this expected gain, see the Appendix.

v is large enough, if intermediary entry level is not sufficiently high, the monopolist's profit gives a much stronger impact on intermediary entry decisions than the profit under the competition.

From above, as a reservation price equilibrium, we consider a strategy profile such that suppliers adopt the price strategy F, intermediaries follow the market entry strategy α, the reservation price strategy $\sigma^*(\rho)$ in search, and the bidding strategy b^* for which the conditions (2.3), (2.4), and (2.5) hold. By the construction of the strategies, it is optimal for both suppliers and intermediaries to follow their strategies.

Intermediary Entry

For intermediary entry in an equilibrium, we show that it is never optimal for all intermediaries to enter the market because suppliers charge prices to control for intermediary entry through the above two conditions of sequential search and market entry and appropriately extract intermediary surplus.

Proposition 2.3 (Non-Optimality of Full Participation)**.** *There is no equilibrium such that all intermediaries enter the market with probability one.*

For the proof of Proposition 2.3, see the Appendix. This gives us an interesting implication that it is never optimal for intermediaries to enter the market even if search cost is sufficiently small (and there is no market entry fee). The main force driving Proposition 2.3 is costly first search. Janssen, Moraga-González and Wildenbeest (2005) are the first among others to introduce costly first search into the consumer search model of Stahl (1989) by which firms exploit consumers (as non-shopper) due to search frictions and thereby a fraction of consumers decide not to enter a retail market with no search. On the other hand, in our model, suppliers extract intermediary surplus by controlling for sequential search of intermediaries and therefore a fraction of intermediaries do not enter a market where entrants can compete with each other. Thus, the market entry decisions of entrants are made differently in each model.

In a similar way, we can extend Proposition 2.3 to a more general case where there are either many suppliers given two intermediaries or many intermediaries given two suppliers (see Section 2.6).

Existence of Equilibrium with Price Dispersion

We here consider when an equilibrium exists. The main intuition is captured by Figure 2.1 given above where we can easily see that the expected gain of market entry is increasing in values v and if v is large enough relative to the search cost, the entry gain outweighs the search cost. Since Proposition 2.3 tells us that full participation does not pay off and the entry gain must be below the search cost, there is at least an entry level $\alpha \in (0, 1)$ such that the entry gain equals the search cost provided that the ratio of the value to the search cost is large enough. In addition, as shown in the Appendix, intermediary entry under small α is not beneficial because suppliers charge sufficiently high and concentrated prices, while the entry gain is gradually increasing as the entry rate α goes up because the higher entry rate pushes prices down and an intermediary who will be a winner in the auction can receive the larger margin, but after the entry rate reaches a threshold below the rate of full participation, the entry gain stops to increase and keeps decreasing because overall supplier prices decrease but they are more dispersed, which reduces the entry gain in more intense downstream competition. As it turns out, there are two possible equilibrium entry rates if the value is large enough. Thus, we provide a sufficient condition under which there are multiple equilibria as follows.

Proposition 2.4 (Existence of Dispersed Price Equilibrium)**.** *There are at most two dispersed price equilibria. There exist two dispersed price equilibria if the value to search cost ratio is sufficiently large, whereas there is no dispersed price equilibrium if it is sufficiently small.*

This implies that a large market value can produce two possible market outcomes.

By analyzing what happens behind each dispersed equilibrium, we can uncover the mechanism that drives the equilibrium as follows.

Proposition 2.5 (Property of Dispersed Price Equilibrium)**.** *Suppose that the value to search cost ratio is sufficiently large. The equilibrium price distribution under the low entry rate first-order stochastically dominates that under the high entry rate. In addition, the range of price dispersion under the high entry rate is larger than under the low entry rate.*

Proposition 2.5 tells us that one of equilibria is pro-competitive because suppliers charge low prices to compete with each other and accordingly intermediaries get motivated to enter the market, giving rise to a competitive downstream market; the other is anti-competitive because suppliers charge high prices to deter intermediary entry and thereby there are little entry in the downstream market, ending up with an anti-competitive market. See the Appendix for the proof of Proposition 2.4. In the following subsection, we will discuss the stability of two equilibria by accounting for intermediary entry. Notice that Proposition 2.4 and part of Proposition 2.5 carry over in both cases of many suppliers with two intermediaries and many intermediaries with two suppliers (see Section 2.6).

Proposition 2.4 gives us a remarkable feature that price dispersion emerge at both the upstream and downstream levels. The main force behind this is that suppliers generate wholesale price uncertainty *endogenously* in the absence of exogenous uncertainty, which contrasts from the literature where we incorporate *exogenous* cost uncertainty, such as inflation and exchange rate or production shocks, to investigate its impact on price dispersion (see, e.g., Bénabou and Gertner, 1993; Dana, 1994; Fishman, 1996; Tappata, 2009; Janssen et al., 2011). This is also pointed out by García et al. (2015) who incorporate two levels of search frictions in both upstream and downstream markets but their mechanism differs from ours. In their model, there is one more level of search frictions in a downstream market for a buyer (corresponding to a unit mass of consumers in their model) to acquire retail price information together with search frictions in an upstream market for intermediaries (retailers) to acquire (wholesale) prices charged by suppliers (manufacturers). They endogenously derive price dispersion due to two levels of search frictions. Contrary to this, price dispersion in our model emerges due to the combined effect of one level of search frictions and competition among searchers via an auction. In addition, double marginalization is the best market outcome that one can expect in their model, whereas in our model supplier prices and bids in dispersed price equilibria are below the monopoly price because there are no search frictions in the downstream market and furthermore price competition downstream is based on an auction.

Interestingly, the consumer search model of Burdett and Judd (1983) induces a similar pattern to ours that two dispersed price equilibria are generated.[22] Nonetheless, both results obtain by different mechanisms. They consider a retail market and derive two dispersed price equilibria provided that all retailers and consumers are identical ex ante.[23] In their model with *fixed sample search*, heterogeneous consumer search behaviors in terms of search intensity emerge endogenously, which affects firms' pricing behaviors and causes price dispersion, and then different search intensity distributions among consumers can

[22] See also Fershtman and Fishman (1992) and Janssen and Moraga-González (2004) for its extended arguments.

[23] The model of Burdett and Judd (1983) differs from other consumer search models (Salop and Stiglitz, 1977; Reinganum, 1979; MacMinn, 1980; Rosenthal, 1980; Varian, 1980; Stahl, 1989) where we introduce exogenous heterogeneity to consumers or firms. For instance, Salop and Stiglitz (1977), Varian (1980), and Stahl (1989) add different types of consumers to induce equilibrium price dispersion, whereas Reinganum (1979) and MacMinn (1980) introduce firms' cost uncertainty into their models to derive price dispersion.

generate multiple equilibria. By contrast, in our model with *sequential search*, consumers use a symmetric search behavior but the combined effects of search frictions and competition among searchers cause price dispersion, and then different intermediary entry rates that are endogenously determined by supplier pricing patterns can generate multiple equilibria. In fact, our model is essentially different from consumer search models above in the sense that search frictions in consumer search models can give firms a chance to fully exploit some fraction of consumers, which makes it possible to charge the monopoly price, but in our model such an attempt of a firm (as supplier) does not work due to competition among searchers downstream, which pushes the upper bound of prices below the monopoly price. Because of this, our model introduces a different type of search frictions, which generates a new market mechanism of price dispersion.

For convenience, if there are two equilibria, we denote the high and low entry rates by α^H and α^L, respectively, and use them below. Note that $0 < \alpha^L < \alpha^H < 1$ holds due to Propositions 2.3 and 2.4.

Stability of Equilibria

To argue whether or not an equilibrium is stable in terms of intermediary entry, we introduce the following stability concept to reflect *free entry* for the long-run outcome.[24]

Definition 2.2 (Entry Stability)**.** An equilibrium is *entry stable* if intermediaries who choose an entry rate close to the equilibrium rate have an incentive to change their entry rate to the equilibrium rate for equating the entry gain to the entry cost, otherwise it is *entry unstable.*

Applying Definition 2.2 to multiple equilibria guaranteed by Proposition 2.4, we can assess them on a basis of entry stability. Intuitively, Figure 2.1 captures the (in)stability of the equilibrium with the high (low) entry rate. Then we have:

Corollary 2.1 (Stability of Equilibria)**.** *Assume that there are two dispersed price equilibria. The equilibrium with the high entry rate is entry stable, whereas the other with the low entry rate is entry unstable.*

[24]Here we do not explicitly consider a dynamic model to argue whether or not an equilibrium is stable in the long run. See, for instance, Fershtman and Fishman (1992) who use a closely related stability concept to select one of multiple equilibria by explicitly introducing a dynamic version of the consumer search model of Burdett and Judd (1983). But we think that it is natural to incorporate the stability concept into our model.

In the following, we simply call an entry (un)stable equilibrium as (un)stable equilibrium.

Welfare Analysis

So far we have investigated equilibrium prices and market entry. Now we examine social welfare of those equilibria. Suppose that a market entry rate is $\alpha > 0$ for some parameters (v, s). Then, its social welfare is given by

$$2\alpha(1-\alpha)(v-s) + \alpha^2(v-2s)$$

where the first term represents the expected welfare when only one intermediary enters the market while the second term does the expected welfare when both intermediaries enter the market. So, the market entry rate that maximizes social welfare is simply given by

$$\alpha^{\mathrm{SW}} = \frac{v-s}{v}. \tag{2.6}$$

Given this socially optimal entry rate, assume that $\frac{v}{s}$ is large enough and then there are two dispersed price equilibria due to Proposition 2.4. From now on let us focus on the stable equilibrium with the high entry rate. By (2.5), we can arrange the expected payoff of entry to

$$-v(\alpha - \frac{v-s}{v}) + \frac{\alpha}{2}\int_{\underline{w}}^{\rho}(1-F(w))F(w)dw - (1-\alpha)\Big(\rho - \int_{\underline{w}}^{\rho} F(w)dw\Big).$$

Since $\frac{v}{s}$ is large enough, when $\alpha = \frac{v-s}{v}$, which is sufficiently close to 1, the above payoff is positive because the first term is zero and the second term outweighs the third term involving a sufficiently small coefficient $1-\alpha$. Taking account of Propositions 2.3 and 2.4 by which the expected payoff is decreasing in $\alpha \in (\frac{v-s}{v}, 1]$ and negative when $\alpha = 1$, we can show that the stable equilibrium involves $\alpha \in (\frac{v-s}{v}, 1)$, meaning that it is socially inefficient due to excessive entry.

Proposition 2.6 (Inefficient Market Entry)**.** *If the value to search cost ratio is sufficiently large, then the equilibrium is socially inefficient as it involves excessive entry.*

This implies that a large value induces excessive market entry and the competition level in the downstream market exceeds an optimal level. Note that the unstable equilibrium has insufficient entry and is socially inefficient as well as the stable equilibrium, and furthermore it is socially much worse than under excessive entry because supplier pricing is anti-competitive to deter intermediary entry. Therefore we can say that if the downstream market looks highly attractive for suppliers and intermediaries, market entry becomes either insufficient or excessive and the market functions in an inefficient way.[25]

Comparative Statics on Price Dispersion and Entry

We have known from Proposition 2.4 that if the value to search cost ratio is large enough, suppliers charge pro-competitive or anti-competitive prices, which gives two types of market entry and results in two market equilibria. Taken together with Corollary 2.1 showing that only one of equilibria is (entry) stable, we explore how the ratio influences the stable equilibrium in terms of supplier prices, intermediary entry, and welfare, provided that $\frac{v}{s}$ is large enough.

By the argument of Proposition 2.5, intuitively an enhanced value or a search cost reduction induces more competitive supplier prices and thereby intermediaries get more motivated to enter the market, resulting in more competitive market entry.[26] But it is crucial to understand that the dependency of price dispersion on the value or the search cost primarily relies on two countervailing effects: (i) stronger competition among suppliers; (ii) stronger competition among intermediaries. As explained above, the first effect leads to the enhancement of intermediary entry indirectly through reducing input prices, which results in the second effect. This supplier induced entry strengthens the monopolist position of suppliers when all intermediaries buy at a single supplier store and enlarges the gap between the monopolist profit and the profit under competition. Interestingly, when considering the search cost reduction instead of the increased valuation, overall input prices become lower, while they get concentrated on lower prices, which gives the opposite impact of the increased valuation on price dispersion. This is because the smaller search cost reinforces the suppliers' competition, which outweighs the second countervailing effect mentioned above, and therefore suppliers are obliged to decrease the payoff gap between the monopolist profit and the profit under competition, in addition to charging lower prices (see the Appendix). This implies that the increment of the relative ratio does not tell its

[25]Technically speaking, we can show that $\alpha^H - \alpha^{\mathrm{SW}} > 0$ (resp. $\alpha^{\mathrm{SW}} - \alpha^L > 0$) decreases (resp. increases) in v while increases (resp. decreases) in s.

[26]By contrast, in the unstable equilibrium with anti-competitive input prices and insufficient intermediary entry, the larger valuation further pushes overall prices up, which deters intermediary entry more, and then market prices are more concentrated on higher prices because the competition effect becomes weaker and the monopoly effect becomes more influential on supplier pricing.

impact on price dispersion, and therefore we need to access the impact of the valuation and the search cost separately.

Thus, we give the following simple comparative statics result.

Proposition 2.7 (Comparative Statics for the Value to Search Cost Ratio)**.** *Suppose that the value to search cost ratio is sufficiently large. The highest and lowest prices in the stable dispersed price equilibrium decrease with the ratio, and the range of price dispersion increases as v increases while it decreases as s decreases.*

2.5 Example

Here we give an example to overview previous results. Fix exogenous parameters (v, s). Using the conditions (2.3), (2.4), and (2.5), we can derive an equilibrium. Mechanically speaking, (2.3) determines the price distribution F given $(\underline{w}, \overline{w} = \rho, \alpha, s)$, by which (2.4) pins down ρ (together with $\underline{w}$) as a function of (α, s), and then (2.5) determines α as a function of exogenous parameters (v, s). Then, the reservation price is given by

$$\rho = \frac{2s}{(2-\alpha)\left(1 - \frac{4-3\alpha}{2\alpha}\ln\frac{4-\alpha}{4-3\alpha}\right)} \tag{2.7}$$

for which the lowest and highest prices, $\underline{w}$ and $\rho(= \overline{w})$, satisfy

$$\rho = \frac{4-\alpha}{4-3\alpha}\underline{w} \tag{2.8}$$

and the price distribution for any $w \in [\underline{w}, \overline{w}]$ is given by

$$F(w) = \frac{4-\alpha}{2\alpha} - \frac{s}{\alpha(2-\alpha)(\frac{1}{4-3\alpha} - \frac{1}{2\alpha}\ln\frac{4-\alpha}{4-3\alpha})w}, \tag{2.9}$$

while an equilibrium market entry probability corresponds to a solution α such that the condition (2.5) holds. For these derivations, see the Appendix. When $(v, s) = (1, 0.1)$, since the condition (2.5) is described by Figure 2.2, from which two equilibrium entry rates α^H and α^L are pinned down. Both of entry rates are below the rate of full participation, $\alpha = 1$, which is consistent with Propositions 2.3 and 2.4. The two equilibrium entry rates each determine the corresponding price distribution F^H and F^L, respectively (see Figure 2.3) and then we obtains the equilibrium. For the corresponding price density functions of

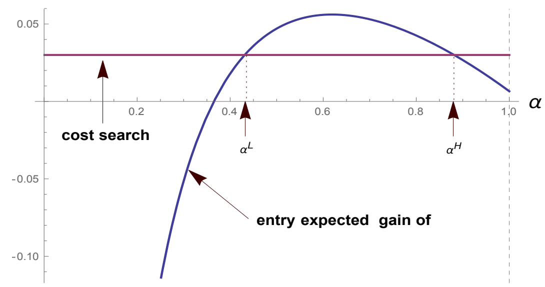

Figure 2.2: Expected gain of entry and search cost for Condition (2.5) and two equilibrium entry rates α^H and α^L when $(v, s) = (1, 0.1)$.

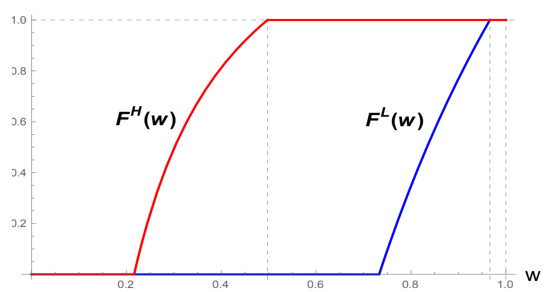

Figure 2.3: Equilibrium price distributions F^H and F^L when $(v, s) = (1, 0.1)$.

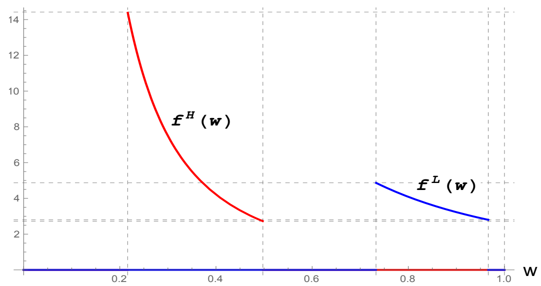

Figure 2.4: Equilibrium price density functions $f^H(w)$ and $f^L(w)$ as $(v, s) = (1, 0.1)$.

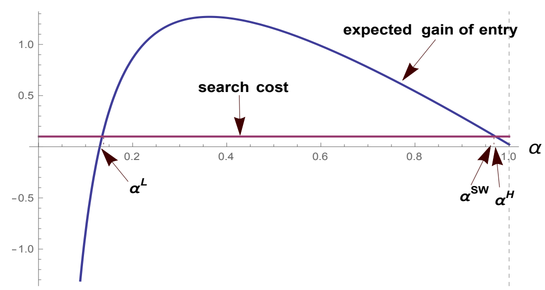

Figure 2.5: Expected gain of entry and search cost for Condition (2.5) and two equilibrium entry rates α^H and α^L when $(v, s) = (3, 0.1)$.

supplier prices denoted by $f^H(w)$ and $f^L(w)$ as α^H and α^L, respectively, see Figure 2.4. One of these two equilibria with the high entry rate α^H is stable and the other with the low entry rate α^L is unstable (see Corollary 2.1). Since $\alpha^H < \alpha^{\mathrm{SW}} = \frac{v-s}{v}$ by Figure 2.2, we can see that market entry even under the stable equilibrium is socially insufficient as it involves insufficient entry. Suppose that the value v increases from 1 to 3 and accordingly the value to search cost ratio increases. Then, the ratio becomes large enough and we can see by Figure 2.5 that the stable equilibrium has excessive entry due to $\alpha^H > \alpha^{\mathrm{SW}}$, which is shown by Proposition 2.6.[27] In addition, we can show that the stable equilibrium prices go down overall and are more dispersed, which follows from Proposition 2.7.

[27]We can show that there is a threshold $(\frac{v}{x})^*$ such that $\alpha^H < \alpha^{\mathrm{SW}}$ if $\frac{v}{s} < (\frac{v}{x})^*$, otherwise $\alpha^H \geq \alpha^{\mathrm{SW}}$.

2.6 Market Concentration and Price Dispersion

So far we have fixed the number of suppliers and intermediaries to be two and analyzed equilibria in the market with the small number of firms. It is natural to ask what happens if market concentration in the upstream or downstream market changes by adding multiple suppliers or multiple intermediaries into the market and look at its impact on market outcomes. Here we briefly summarize the firm number effects as follows. The details are given in the Appendix.

In a situation where there are multiple suppliers given two intermediaries or multiple intermediaries given two suppliers, we can easily find the similarity to the simple case that full participation is never optimal for intermediaries and there are multiple equilibria, all of which are socially inefficient if the value to search cost ratio is sufficiently large.

Proposition 2.8 (Firm Number Effect)**.** *Suppose that there are multiple suppliers and two intermediaries or that two suppliers and multiple intermediaries. If the value to search cost ratio is sufficiently large, there is no equilibrium such that all intermediaries enter the market with probability one, and in addition, there are at least two dispersed price equilibria and all of them are generically inefficient.*

The proof of Proposition 2.8 is given in the Appendix. For multiplicity of equilibria, our numerical analysis shows that there are at most two equilibria as in Proposition 2.4 where there are two suppliers and two intermediaries.

Proposition 2.8 is of interest particularly in procurement auctions. As mentioned in the Introduction, the relationship between the winnig bid and number of bidders is important because the lower winning bid reduces the procurement cost.[28] We usually think that the larger number of bidders strengthens the competition among bidders, thereby lowering the procurement cost. But its relationship may be non-monotonic because of countervailing effects against the competition effect. One of the countervailing effects related to our paper is *entry* effect (Li and Zheng, 2009), meaning that a winning bidder may believe that they overestimate the intensity of entry, and it may outweigh the competition effect, resulting in a higher winning bid as the number of (potential) bidders increases.[29] By contrast, in our model equilibrium input prices increase overall as there are more bidders (intermediaries) because suppliers gain larger power against intermediaries, thereby increasing the

[28]In practice, however, "competitive bidding may lead to adverse selection" (Bajari, McMillan and Tadelis, 2009, p.379).

[29]There are other representative countervailing effects known as *winner's curse* effect (Bulow and Klemperer, 2002) and *affiliation* effect (Pinkse and Tan, 2005).

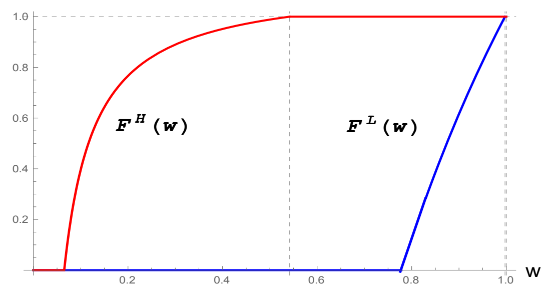

Figure 2.6: Equilibrium price distributions F^H and F^L when $(m, n, v, s) = (10, 2, 1, 0.1)$.

procurement cost. Moreover, if there are more suppliers, they induce more intermediary entries by lowering the average input price while spreading prices more due to the above mentioned two countervailing effects, and therefore more bidders might have higher input prices, leading to higher procurement costs.

In the following, we provide distinct features from the two-supplier and two-intermediary case. Interestingly, as more suppliers are in the market with two intermediaries, social welfare becomes lower than in the simple case because suppliers charge either more pro-competitive prices or more anti-competitive prices so that intermediary entry accordingly becomes either more socially excessive or more insufficient.

As an numerical analysis where there are two equilibria and one of them is (entry) stable whereas the other is unstable, we can find that as there are more suppliers, for the stable equilibrium, the *average* price becomes lower and overall prices are more dispersed. In other words, as there are more suppliers in the stable equilibrium, the average price decreases and the range of price dispersion becomes larger. In contrast, the unstable equilibrium generates the opposite. It is of interest that the upper bound of equilibrium prices is higher as there are more suppliers but the average price becomes lower. This implies that the upstream market competition gets intensified through more competitive prices as there are more suppliers but they spread prices more to extract intermediary surplus by accounting for more intermediary entry even if their market is more competitive. See Figure 2.6. From above, when considering the stable equilibrium, our numerical analysis suggests that there is a positive relationship between *upstream* market concentration (measured by number of suppliers) and price dispersion.

On the other hand, as more intermediaries are in the market with two suppliers, market entry in both equilibria becomes insufficient because the downstream market becomes more competitive and each intermediary find less profitable to compete with more potential entrants. For the stable equilibrium, the *overall* prices go up and are more spread because

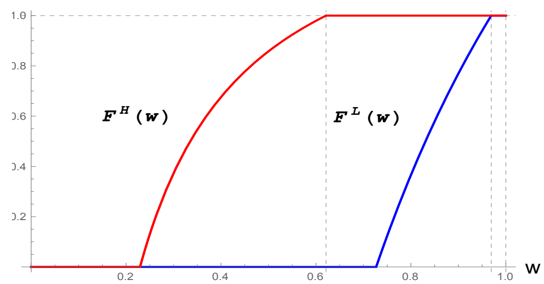

Figure 2.7: Equilibrium Price Distributions F^H and F^L when $(m, n, v, s) = (2, 10, 1, 0.1)$.

as there are more intermediaries, suppliers gain stronger seller power against intermediaries, and thereby charge higher prices overall. For the unstable equilibrium, the prices are almost the same as the previous case because, on top of a lower chance to be a winner in the auction as there are more potential entrants, each intermediary's motivation to enter the market drops further due to supplier entry deterrence, which maintains almost the same total entry size and stops supplier prices rise up. See Figure 2.7. This implies that there is a negative relationship between *downstream* market concentration and price dispersion when suppliers induce intermediary entry via competitive pricing, but there is no significant negative or positive relationship when they deter entry via anti-competitive pricing.

2.7 Two-Part Tariffs

So far we have assumed that suppliers use the linear (pricing) contracts. In practice, we often see simple supply contracts based on combinations of fixed price and cost reimbursement because theoretically optimal contract design (see, e.g., Laffont and Tirole, 1986) is too complicated to implement, whereas a simple contract are good enough to cover a large part of surplus obtained via the optimal contract (see, e.g., Rogerson, 2003; Chu and Sappington, 2007; Carroll, 2015). It is of particularly interest to see what happens if we introduce other contracts that can influence our results shown before. As an example, let us consider a contract on the basis of two-part tariff where all suppliers can simply add a common fee to their per-unit prices previously considered. By allowing for transferring money from intermediaries to suppliers via the two-part tariff, suppliers can yield larger profits with controlling for the market entry of intermediaries by which obviously suppliers extract intermediary surplus more. Interestingly, if there are two equilibria under the per-unit prices with no fixed fees, suppliers can add a fixed fee on top of the unit price and extract intermediary surplus as much as possible with guaranteeing that a fraction of intermediaries enter the market, which results in a unique equilibrium. This result follows from the following argument. First of all, even if we incorporate any kind of fixed fee, there is no pure strategy equililbrium as in Proposition 2.2. Next, suppose that suppliers add a common fixed fee $t \geq 0$ to any given per-unit price. Given that suppliers adopt a mixed price strategy, Conditions (2.3), (2.4), and (2.5) change by the fixed fee to

$$\Big(\alpha^2\big(\frac{1}{4}+\frac{1}{2}(1-F(w))\Big)+\alpha(1-\alpha)\Big)(w+t) \;=\; \Big(\frac{\alpha^2}{4}+\alpha(1-\alpha)\Big)(\overline{w}+t), \quad (2.10)$$

$$\Big(\frac{\alpha}{2}+(1-\alpha)\Big)\Big(\rho-\int_{\underline{w}}^{\rho} wdF(w)-t\Big) \;=\; s, \quad (2.11)$$

and

$$\begin{aligned}&\frac{\alpha}{2}\Big(\int_{\underline{w}}^{\rho}(1-F(w))F(w)dw-\frac{t}{2}\Big)\\ &+(1-\alpha)\Big(v-\int_{\underline{w}}^{\rho} wdF(w)-t\Big)+\frac{\alpha}{4}(-t) \;=\; s \end{aligned}\quad (2.12)$$

where the first term at LHS of (2.12) indicates the expected payoff when two intermediaries visit distinct suppliers and compete in the auction with cost uncertainty, which we derive in a similar manner to Lemma 2.1 (see the Appendix for the derivation), the second term corresponds to the payoff when the other intermediary does not enter the market, and the last term indicates the payoff when two intermediaries visit the same supplier and compete in the auction with the same marginal cost where each intermediary is equally likely to win. As in the example of Section 2.5, we can explicitly derive the equilibrium price distribution with support as a function of entry rates, whereas the equilibrium entry rates are determined by an equation without an explict expression.[30]

Actually, we can show that there is a unique equilibrium with the two-part tariff for suppliers.

Proposition 2.9 (Equilibrium with Two-Part Tariff)**.** *Assume that the value to search cost ratio is sufficiently large. Then, there exists a unique dispersed price equilibrium that involves a fixed fee $t^* > 0$.*

For the proof of Proposition 2.9, see the Appendix. The intuitive argument behind this

[30]See the Appendix for these derivations.

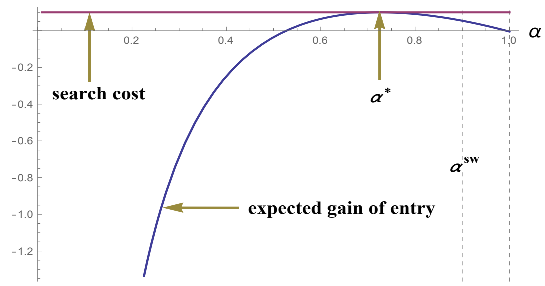

Figure 2.8: Expected gain of entry and search cost under Condition (2.12) and a unique equilibrium entry rate $\alpha^* \approx 0.73$ when $(v, s) = (1, 0.1)$ and $t \approx 0.07$.

result is given as follows. If the value to search cost ratio is sufficiently large, there are two dispersed price equilibria. One of equilibria entails high supplier prices functioning as entry deterrence, which gives suppliers high profits and does not incentivize them to set a tariff because suppliers should reduce high prices when introducing a tariff in order to maintain intermediary entry but it gives rise to induce entry more, resulting in a lower profit. By contrast, the other equilibrium entails low supplier prices inducing entry, which makes supplier profits low and thereby incentivizes them to set a tariff to reduce entry and extract intermediary surplus more as long as intermediary entry is sustained.

Suppose that $(v, s, t) = (1, 0.1, 0)$. By Figure 2.2 in the example, we know that there are two dispersed price equilibria with high and low entry rates α^H and α^L. Note that we can show that the RHS of (2.10) is decreasing in α, that is, the expected payoff of each supplier decreases as the intermediary entry rate increases, thus the payoff of suppliers under α^H is lower than under α^L. Focusing on the stable equilibrium with α^H, let us consider what happens if suppliers can charge a tariff t to maximize their expected payoffs. As the fixed fee increases, the intermediary entry rate goes down.[31] See Figure 2.8 in comparison with Figure 2.2. As it turns out, suppliers who charge low prices inducing intermediary entry have an incentive to add a fixed fee, which deter intermediary entry to some degree, and then increase their profits by extracting intermediary surplus as long as they can sustain the constraint of intermediary entry.[32] Then, they set the unique fixed fee $t^* > 0$ such that the entry gain equals the entry cost for intermediary under the unique entry rate $\alpha^* \in (\alpha^L, \alpha^H)$.

[31]By contrast, the intermediary entry rate goes up under α^L.
[32]On the other hand, under α^L, suppliers do not have an incentive to add a fixed fee.

2.8 Conclusion

Price dispersion has long been explored together with price discrimination mainly on a basis of retail markets. This paper among others addressed a potential impact of intermediaries on market prices, or how information frictions concerning supplier prices that intermediaries confront at the upstream market affect strategic pricing of suppliers, through which market entry of intermediaries changes. More specifically, we investigated intermediary search for inputs in a (cost-based) procurement auction with endogenous entry and then demonstrated that (i) entry deterrence can stem from the monopoly price charged by suppliers, thereby breaking down the market; (ii) market entry of all intermediaries is never realized even when search cost is sufficiently small; (iii) price dispersion emerges when the value to search cost ratio is large enough; (iv) both supplier prices and intermediary bidding depend heavily on market concentration measured by the number of suppliers and intermediaries in the market. The results (iii) and (iv), however, rely on the simplified procurement auction where both suppliers and intermediaries know the buyer's valuation of the object ex ante and more importantly, bidding is descending, by which cost uncertainty among bidders is resolved in the auction. In practice, as the buyer's valuation an engineer's cost estimates is used but often not publicly available before the auction takes place, and in addition it is often the case that the auction rule follows a first price sealed bid auction (with bidder qualifications and multiple rounds).[33] Incorporating the above facts into our model is important for future research.[34] Besides, we discarded buyer (intermediary) power against sellers (suppliers) and assumed that intermediaries should engage in search for input prices, but in fact it is critical to argue to what extent intermediaries have bargaining power against both the buyer and suppliers, by which intermediaries may negotiate with the buyer or suppliers may engage in search for selling their inputs to intermediaries.[35]

[33]As empirical evidence, see, for instance, Decarolis (2014) for public procurements of road construction and maintenance in northern Italy and Kawai and Nakabayashi (2014) for construction projects procured by the national government in Japan.

[34] This extension is closely related to the study by Spulber (1995) who introduces cost uncertainty to the Bertrand competition and shows that firms charge higher prices above the marginal cost due to cost uncertainty. In his model, marginal cost of each firm is independently drawn from an identical distribution, whereas in our model where firms correspond to intermediaries, marginal costs among intermediaries depend on from which suppliers they have bought their goods. A problem in this extension is that we cannot explicitly solve an equilibrium price distribution in general and it would be hard to clarify the linkage between supplier prices and intermediary entry.

[35]For buyer power against sellers, see, for instance, the report by OECD (2007, p.22). For negotiations with the buyer for procurement (of complex projects), see Bajari, Houghton and Tadelis (2014, references therein). At a different market from public procurement, an intermediary (retailer) has often strong bargaining power against suppliers (manufacturers) when an intermediary may have a large platform

The model considered in this paper is limited, to a large extent, by the above mentioned points and many other possible factors, such as product quality in scoring auctions (Che, 1993; Asker and Cantillon, 2008, 2010), but the point here is to address how and to what extent asymmetric information on supplier prices influences (cost-based) procurement through intermediary search and to uncover a pricing mechanism hidden in the market.

2.9 Appendix: Proofs

Proof of Proposition 2.1

Proof. The case of $\alpha = 0$ is shown together with a stronger equilibrium concept. When $\alpha = 0$ and $w \in [0, v - s)$, an intermediary deviates to enter the market and then can buy at w, sell at v, and get a positive profit, which implies that there is a profitable deviation. So, $\alpha = 0$ and $w \in [0, v)$ cannot be an equilibrium. But, if $\alpha = 0$ and $w \in [\min\{v - s, 0\}, v]$, together with any bidding function, this would be an equilibrium (as a weak Perfect Bayesian Equilibrium). □

Remark 2.1. If we strengthen the equilibrium concept and consider a *strong* Perfect Bayesian Equilibrium, no entry cannot be part of an equilibrium, and therefore the market never collapse completely. Assume that $\alpha = 0$ and $w \in [\min\{v - s, 0\}, v]$ are part of an equilibrium. Off the equilibrium path where a supplier deviates to a lower price than $\min\{v - s, 0\}$, an intermediary has a consistent belief off the equilibrium path and thereby is willing to enter the market and get a positive expected payoff when visiting the deviating supplier, thus $\alpha = 0$ and $w \in [\min\{v - s, 0\}, v]$ cannot be sustained as part of an equilibrium.

Costless First Search

Suppose that the first search is *costless* and all intermediaries enter the market, i.e., $\alpha = 1$. It is clear that if there is a single supplier, i.e., $m = 1$, since she is a monopolist in the upstream market, she charges price v and obtains a whole market surplus. On the

where many consumers buy supplier goods or when suppliers have limited channels to sell their goods to consumers, which causes big inventory issues. In this case, as opposed to intermediary search, supplier would engage in search to sign a reasonable contract with a platform to sell goods to consumers indirectly at the platform. Such examples are internet price comparison sites or shopbots for airline tickets (e.g., Stavins, 2001; Gerardi and Shapiro, 2009; Gaggero and Piga, 2011) and supermarkets (Hosken and Reiffen, 2004, 2007, references therein).

other hand, if there is a single intermediary, i.e., $n = 1$, the upstream market is the environment where the logic of Diamond Paradox applies,[36] and therefore all suppliers charge the monopoly price v as before. So, a single supplier or a single intermediary causes the monopoly outcome.

Proposition 2.10 (Monopoly Outcome)**.** *Suppose that the first search is costless and all potential intermediaries enter market. If there is a single supplier or a single intermediary in the market, suppliers charge the monopoly price v.*

This implies that suppliers can extract whole market surplus due to search frictions if there is a monopolist in the upstream or downstream market and market entry including first search is costless.

Proof of Proposition 2.2

Proof. Suppose that all suppliers charge $w = 0$. Then, for a sufficiently small $\varepsilon > 0$, each supplier has an incentive to unilaterally deviate to charge a slightly higher price ε because all intermediaries randomly search and visit the same supplier with some positive probability and the deviating supplier can get a positive expected profit, which is better than under the marginal cost pricing.

Next, suppose that all suppliers charge $w \in (0, p^m]$. For a sufficiently small $\varepsilon > 0$, each supplier has an incentive to unilaterally deviate to a slightly lower price $w - \varepsilon$ because all intermediaries visit multiple suppliers with some positive probability, which is more likely to happen than all intermediaries visit the same supplier, and, since the intermediary who has visited the deviating supplier can bid a slightly lower price than the other intermediaries who has bought at different suppliers and would be the winner in the auction, the deviating supplier can capture the unit demand of the buyer and get a larger expected profit than under the price w. □

[36] The logic of Diamond Paradox used in our set-up is given as follows. Suppose that suppliers charge a symmetric price below the monopoly price. Since an intermediary expects that all suppliers charge the same price, there is no incentive to search more than once. Given this intermediary's behavior, each supplier has an incentive to slightly increase the price because the visiting intermediary incurs the search cost to continue to search. This argument continues to hold until the price reaches the monopoly price. Note that we can extend the above argument to asymmetric price strategy profiles to show that the monopoly price is a unique optimal price.

Proof of Lemma 2.1

Proof. Given that suppliers adopt the price strategy F with support $[\underline{w}, \overline{w}]$, we consider the situation where both intermediaries enter the market and visit distinct suppliers. Each intermediary follows the bidding strategy b^* defined by (2.2). Note that it must be the case that $\rho \leq v$, otherwise the buyer does not purchase in the auction and a supplier gets nothing at that price. Suppose that an intermediary visits a supplier charging a price $w \in [\underline{w}, \overline{w}]$ and the other intermediary visits the other supplier charging a price $w' \in [\underline{w}, \overline{w}]$. Then, the expected gain of each intermediary in the auction is illustrated by Figure 2.9 and given by[37]

$$\begin{aligned}
& \int_{\underline{w}}^{\overline{w}} \int_{w}^{\overline{w}} (w' - w) dF(w') dF(w) \\
= & \int_{\underline{w}}^{\overline{w}} \Big((\overline{w} - wF(w) - \int_{w}^{\overline{w}} F(w')dw' - w(1 - F(w)) \Big) dF(w) \\
= & \; \overline{w} - \int_{\underline{w}}^{\overline{w}} w dF(w) - \int_{\underline{w}}^{\overline{w}} \Big(\int_{\underline{w}}^{w'} dF(w) \Big) F(w')dw' \\
= & \int_{\underline{w}}^{\overline{w}} (1 - F(w))F(w)dw
\end{aligned}$$

where the first equality obtains by integrating by parts and the second equality is given by changing the order of integration. □

Proof of Proposition 2.3

Proof. Substituting (2.4) to (2.5) regarding the common term of search cost, the expected profit of intermediary entry is given by

$$\frac{\alpha}{2} \int_{\underline{w}}^{\rho} (1 - F(w))F(w)dw + (1 - \alpha)\Big(v - \int_{\underline{w}}^{\rho} wdF(w)\Big) - \Big(\frac{\alpha}{2} + (1 - \alpha)\Big)\Big(\rho - \int_{\underline{w}}^{\rho} wdF(w)\Big).$$

When all intermediaries enter the market and $\alpha = 1$, the above is rewritten by

$$\frac{1}{2}\Big(\int_{\underline{w}}^{\rho} (1 - F(w))F(w)dw - \big(\rho - \int_{\underline{w}}^{\rho} wdF(w)\big) \Big) \quad = \quad -\frac{1}{2} \int_{\underline{w}}^{\rho} F^2(w)dw < 0,$$

[37]Since the price distribution F has no mass point over a whole range of prices, the event that both suppliers set the same price occurs with a negligible probability, thus we can ignore such a case.

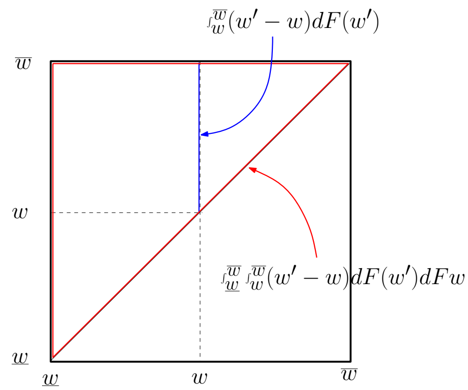

Figure 2.9: The expcted gain of each bidder in the auction with two bidders.

which implies that surely entering the market does not pay off given that no entry gives zero profit. $\square$

Derivation of a solution

From (2.3), given $(\overline{w}, \underline{w}, \alpha)$, we can express F as the function of w by

$$F(w) \quad = \quad 1 - \frac{4-3\alpha}{2\alpha}\frac{\overline{w}-w}{w} = \frac{1}{2\alpha}\Big(4-\alpha-\big(4-3\alpha\big)\frac{\overline{w}}{w}\Big). \tag{2.13}$$

Also note that $F(\overline{w}) = 1$ holds. Since $F(\underline{w}) = 0$ must hold, we derive (2.8) where the coefficient of $\underline{w}$ is increasing in α. Substituting (2.8) into (2.13) above, we derive (2.9).

Next, since $\rho - \int_{\underline{w}}^{\rho} wdF(w) = \int_{\underline{w}}^{\overline{w}} F(w)dw$ and

$$\begin{aligned}
\int_{\underline{w}}^{\overline{w}} F(w)dw &= \frac{4-\alpha}{2\alpha}(\overline{w}-\underline{w}) - \frac{(4-3\alpha)\overline{w}}{2\alpha}\ln\frac{\overline{w}}{\underline{w}} \quad (\because (2.13)) \\
&= \Big(1 - \frac{4-3\alpha}{2\alpha}\ln\frac{4-\alpha}{4-3\alpha}\Big)\overline{w}, \quad (\because (2.8))
\end{aligned} \tag{2.14}$$

together with (2.4), we derive

$$\rho \quad = \quad \frac{2s}{(2-\alpha)\Big(1-\frac{4-3\alpha}{2\alpha}\ln\frac{4-\alpha}{4-3\alpha}\Big)}. \tag{2.15}$$

and in addition, due to (2.8),

$$\underline{w} \;=\; \frac{2s}{(2-\alpha)(4-\alpha)\Big(\frac{1}{4-3\alpha}-\frac{1}{2\alpha}\ln\frac{4-\alpha}{4-3\alpha}\Big)} \tag{2.16}$$

By (2.13) and (2.15), we obtain (2.9).

Finally, we simplify Condition (2.5) as follows. Since (2.5) is rewritten by

$$2\big((1-\alpha)v-s\big) \;=\; -\alpha\int_{\underline{w}}^{\overline{w}} F(w)dw+\alpha\int_{\underline{w}}^{\overline{w}} F^2(w)dw+2(1-\alpha)\int_{\underline{w}}^{\overline{w}} wdF(w)$$

where

$$\begin{aligned}
\int_{\underline{w}}^{\overline{w}} F^2(w)dw &= \frac{1}{4\alpha^2}\int_{\underline{w}}^{\overline{w}}\Big((4-\alpha)^2-2(4-\alpha)(4-3\alpha)\frac{\overline{w}}{w}+(4-3\alpha)^2\overline{w}^2\frac{1}{w^2}\Big)dw\\
&= \frac{1}{4\alpha^2}\Big((4-\alpha)^2(\overline{w}-\underline{w})-2(4-\alpha)(4-3\alpha)\overline{w}\ln\frac{\overline{w}}{\underline{w}}+(4-3\alpha)^2\overline{w}^2\frac{\overline{w}-\underline{w}}{\overline{w}\underline{w}}\Big)\\
&= \frac{1}{4\alpha^2}\Big(2\alpha(4-\alpha)-2(4-\alpha)(4-3\alpha)\ln\frac{4-\alpha}{4-3\alpha}+2\alpha(4-3\alpha)\Big)\overline{w}\\
&= \frac{1}{2\alpha^2}\Big(4\alpha(2-\alpha)-(4-\alpha)(4-3\alpha)\ln\frac{4-\alpha}{4-3\alpha}\Big)\overline{w} \qquad (2.17)
\end{aligned}$$

and

$$\int_{\underline{w}}^{\overline{w}} wdF(w) \;=\; \overline{w}-\int_{\underline{w}}^{\overline{w}} F(w)dw, \tag{2.18}$$

substituting (2.14), (2.17), and (2.18) into (2.5), we get

$$\big((1-\alpha)v-s\big) \;=\; \frac{1}{2\alpha}(4-3\alpha)\Big(\alpha-\ln\frac{4-\alpha}{4-3\alpha}\Big)\overline{w}. \tag{2.19}$$

For any given (v,s), we define the above equation by $g(\alpha)=0$ where

$$g(\alpha) \;=\; 2\alpha\big((1-\alpha)v-s\big)-(4-3\alpha)\Big(\alpha-\ln\frac{4-\alpha}{4-3\alpha}\Big)\overline{w} \tag{2.20}$$

Below we use it to show Proposition 2.4 to guarantee an existence of equilibrium.

Proof of Proposition 2.4

To prove Proposition 2.4, it is enough to show that the equation $g(\alpha) = 0$ has at most two solutions. In so doing, we take the following two steps.

First of all, we show:

Lemma 2.2. *(i) $g(\alpha) < 0$ when α is sufficiently close to 0 or 1 no matter how large parameters are; (ii) if v is large enough, there exists some intermediate $\alpha \in (0,1)$ such that $g(\alpha) > 0$.*

Proof. We first show the observation (i). The first term $2\alpha((1-\alpha)v-s)$ of $g(\alpha)$ is negative for any positive values (v,s) if α is sufficiently close to zero, whereas the second term of $g(\alpha)$ consists of two parts, $(4-3\alpha)(\alpha - \ln\frac{4-\alpha}{4-3\alpha})$ and ρ, both of which are positive as long as $\alpha > 0$ is sufficiently small. Thus, $g(\alpha) < 0$ holds true when α is sufficiently close to 0. On the other hand, since

$$g(1) = -2s - (1-\ln 3)\Big(\frac{4s}{2-\ln 3}\Big) = -2s\Big(1 - 2\frac{\ln 3 - 1}{2-\ln 3}\Big) \approx -2s(0.781) < 0$$

and $g(\alpha)$ is a continuous function of α, if λ is sufficiently close to one, $g(\alpha)$ must be negative for any positive values (v,s).

Next, we show the observation (ii). Since the first term of $g(\alpha)$ is strictly increasing in v for any $\alpha \in (0,1)$ while the second term does not depend on v, if v is large enough, the first term becomes larger than the second term, thus there must exist some intermediate value $\alpha \in (0,1)$ such that $g(\alpha) > 0$. $\square$

Secondly, we decompose function $g(\alpha)$ of (2.20) by

$$g(\alpha) = g_1(\alpha) - g_2(\alpha)$$

where

$$g_1(\alpha) = 2\alpha\big((1-\alpha)v - s\big) \tag{2.21}$$

and

$$g_2(\alpha) = (4-3\alpha)\Big(\alpha - \ln\frac{4-\alpha}{4-3\alpha}\Big)\rho \tag{2.22}$$

and then show that $g_1(\alpha)$ and $g_2(\alpha)$ each satisfy the following properties.

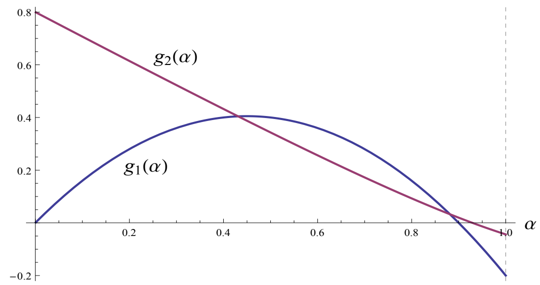

Figure 2.10: Functions $g_1(\alpha)$ and $g_2(\alpha)$ defined by (2.21) and (2.22)

Lemma 2.3. *Function $g_1(\alpha)$ defined by (2.21) is quadratic and concave in $\alpha \in (0,1]$, takes zero at $\alpha = 0, \frac{v-s}{v}$, and strictly increasing in v, whereas function $g_2(\alpha)$ defined by (2.22) is strictly decreasing in $\alpha \in (0,1]$ and does not depend on v.*

The proof of Lemma 2.3 is straightforward and therefore we omit the proof.[38] Figure 2.10 illustrates Lemma 2.3.

From above, we can show Proposition 2.4 as follows.

Proof. By Lemma 2.3, since $g_1(\alpha)$ can be smaller than $g_2(\alpha)$ for the entire interval of $\alpha \in (0,1]$ if $\frac{v}{s}$ is not large enough, $g(\alpha)$ may not have a solution. On the other hand, if $\frac{v}{s}$ is large enough, by Lemmas 2.2 and 2.3, there are exactly two intermediate values α^L and α^H in the open interval $(0,1)$ such that $g_1(\alpha) < g_2(\alpha)$ if $\alpha \in (0,\alpha^L)$ or $\alpha \in (\alpha^H, 1)$, $g_1(\alpha) > g_2(\alpha)$ if $\alpha \in (\alpha^L, \alpha^H)$, $g_1(\alpha^L) = g_2(\alpha^L)$, and $g_1(\alpha^H) = g_2(\alpha^H)$. The above argument implies that there are at most two equilibria and in addition, two equilibria exist if $\frac{v}{s}$ is large enough whereas no equilibrium exists if $\frac{v}{s}$ is small enough. $\square$

Remark 2.2. From above, it is the degenerate case where the unique equilibrium exists.

By substituting the reservation price of (2.7) to Condition (2.3), we can easily show:

Lemma 2.4. *The expected profit of each supplier under (2.3) is decreasing in α.*

This implies that the equilibrium profit of the supplier under α^H is smaller than under α^L.

[38] It is a bit cumbersome to show that function $g_2(\alpha)$ is strictly decreasing in $\alpha \in (0,1]$ but it needs a simple algebra.

Proof of Proposition 2.5

It is easy to prove the former part of Proposition 2.5 as follows. By the entry condition of (2.5), if the value to search cost ratio $\frac{v}{s}$ is large enough, we can easily see that there are two possibilities to equate the entry benefit with its cost: (i) the coefficient $1-\alpha$ is sufficiently small; (ii) the term $v - \int_{\underline{w}}^{\rho} w dF(w)$ is sufficiently small. The first case implies that the high equilibrium entry rate α is sufficiently close to 1, whereby we can approximately derive the highest equilibrium price, whereas in the second case, since $v - \int_{\underline{w}}^{\rho} w dF(w) = v - \rho + \int_{\underline{w}}^{\rho} F(w)dw$ and $F(\cdot) \in [0,1]$, ρ must be sufficiently close to $\underline{w}$ and v with holding $\underline{w} < \rho < v$. This implies that the lowest equilibrium price is close to the monopoly price v and in addition α is sufficiently close to 0. Taken together, the highest equilibrium price under the high entry rate is much lower than under the low entry rate, thus the price distribution under the high entry rate first-order stochastically dominates that under the low entry rate.

To prove the latter part of Proposition 2.5 (as well as the first part), we first show that all equilibrium supplier prices are positive. Next, although equilibria exist only when α takes certain values, we assume that the highest and lowest prices are defined for all values of $\alpha \in (0,1]$ and then derive the property of prices that both of them are decreasing in α while their difference is increasing. This proof for the latter part needs the weaker assumption that there are two equilibria than that used in Proposition 2.5.

Lemma 2.5. *The highest price (2.15) is positive for any $\alpha \in (0,1]$.*

Since $\underline{w}$ is also positive if ρ is positive due to (2.8), Lemma 2.5 implies that the equilibrium prices are all positive for any $\alpha \in (0,1]$, which means that we need not be cautious about the negativity of prices for a whole range of the market entry probability α.

Proof. It is enough to show that the second term of the denominator, $1 - \frac{4-3\alpha}{2\alpha} \ln \frac{4-\alpha}{4-3\alpha}$, is positive for any $\alpha \in (0,1]$. Since the derivative of the second term is given by

$$2\frac{(4-\alpha)\ln\frac{4-\alpha}{4-3\alpha} - 2\alpha}{(4-\alpha)\alpha^2}$$

and one can see that both the denominator and nominator are positive for any $\alpha \in (0,1]$.[39] From this, the above derivative is always positive for any $\alpha \in (0,1]$. Together with the

[39]Regarding the nominator, since 2α and $4-\alpha$ each are linear in α and $\ln\frac{4-\alpha}{4-3\alpha}$ is strictly increasing and convex in α, one can show that the nominator is strictly increasing and convex in $\alpha \in (0,1]$. Taking into account that the nominator takes zero at $\alpha = 0$, the nominator is positive for any $\alpha \in (0,1]$.

observation that the second term is positive for any α sufficiently close to zero, this implies that $1-\frac{4-3\alpha}{2\alpha}\ln\frac{4-\alpha}{4-3\alpha}$ is positive for any $\alpha\in(0,1]$. Since each term in (2.15) is positive for any $\alpha\in(0,1]$, $\rho>0$ holds for any $\alpha\in(0,1]$. □

Remark 2.3. We can further show that $1-\frac{4-3\alpha}{2\alpha}\ln\frac{4-\alpha}{4-3\alpha}$ is strictly increasing and convex in $\alpha\in(0,1]$.

Lemma 2.6. *The highest and lowest prices (2.15) and (2.16) are strictly decreasing in* $\alpha\in(0,1]$ *while the difference of (2.15) and (2.16),* $\overline{w}-\underline{w}$, *is strictly increasing in* $\alpha\in(0,1]$.

Proof. First we can see that $\overline{w}$ is (strictly) decreasing in α by showing that the denominator of (2.15) is increasing in α. Since $\underline{w}=\frac{4-3\alpha}{4-\alpha}\overline{w}$ holds due to (2.8) and the coefficient of $\overline{w}$ is easily shown to be decreasing in α, together with the fact that $\overline{w}$ is decreasing in α, we can see that $\underline{w}$ is also decreasing in α. Secondly, since

$$\begin{aligned}\overline{w}-\underline{w} &= \overline{w}-\frac{4-3\alpha}{4-\alpha}\overline{w}=\frac{2\alpha}{4-\alpha}\overline{w}\\ &= \frac{4\alpha s}{(4-\alpha)(2-\alpha)(1-\frac{4-3\alpha}{2\alpha}\ln\frac{4-\alpha}{4-3\alpha})}\end{aligned}\tag{2.23}$$

from (2.8) and (2.15), we can show that (2.23) is increasing in α. Since the proof is straightforward by taking the derivatives regarding the denominator of (2.15) equals $(2-\alpha)(1-\frac{4-3\alpha}{2\alpha}\ln\frac{4-\alpha}{4-3\alpha})$ and $(2.23)\times\frac{1}{s}=\frac{4\alpha}{(4-\alpha)(2-\alpha)(1-\frac{4-3\alpha}{2\alpha}\ln\frac{4-\alpha}{4-3\alpha})}$ and showing that the former is positive while the latter is negative, we omit the proof. □

Remark 2.4. Since $\overline{w}$ is decreasing in α from above, $F(w)=(2.13)=1-\frac{4-3\alpha}{2\alpha}\frac{\overline{w}-w}{w}$ is increasing in α because both $\frac{4-3\alpha}{2\alpha}$ and $\frac{\overline{w}-w}{w}$ are decreasing in α, thus for $\alpha<\alpha'$, $F(w)$ is larger when α than when α'. This implies that the distribution $F(\alpha^L)$ first-order stochastically dominates $F(\alpha^H)$.

Proof of the latter part of Proposition 2.7

We show that the range of price dispersion decreases in s. Suppose that the value to search cost ratio is sufficiently large. Then, by (2.15) and (2.19), the entry condition of (2.5) is reduced to

$$\frac{v}{s} = \frac{4(3-2\alpha)\alpha+(-16+16\alpha-3\alpha^2)\ln\frac{4-\alpha}{4-3\alpha}}{(1-\alpha)(2-\alpha)(2\alpha-(4-3\alpha)\ln\frac{4-\alpha}{4-3\alpha})}\tag{2.24}$$

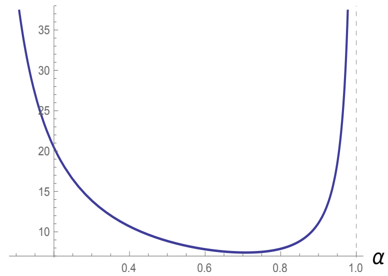

Figure 2.11: The RHS of (2.24).

where the RHS is shown to be increasing in α as long as we consider the stable equilibrium (see Figure (2.24) for its illustration). So, the LHS increases as $\frac{v}{s}$ increases, leading to the higher equilibrium entry rate. If the ratio $\frac{v}{s}$ is sufficiently large, however, this change is sufficiently small. Next, we give the two observations. One of them is that the search cost directly influences the highest and lowest equilibrium prices due to (2.15) and (2.16), thereby changing them proportionally. The second is that the lowest price decreases more than the highest price as the entry rate increases because $\underline{w} = \frac{4-3\alpha}{4-\alpha}$ by (2.8) and its coefficient, $\frac{4-3\alpha}{4-\alpha}$, is decreasing in α. Taken together, the difference between the highest and lowest prices increases as v increases through the increased entry rate, while it decreases as s increases because the decrease coefficient $\frac{4-3\alpha}{4-\alpha}$ makes the decrease of the lowest price (by a smaller search cost) smaller than that of the highest price. Thus, the range of price dispersion decreases as s increases.

Proof of Proposition 2.8

Multiple Suppliers and Two Intermediaries: Equilibrium Conditions and Analysis

We consider a market where there are $m(= 2, 3, \dots)$ suppliers and two intermediaries and analyze equilibria. By a similar manner to the simple case of two suppliers, the three conditions corresponding to (2.3), (2.4), and (2.5) are given by

$$\begin{aligned}\Big(\alpha^2\Big(\frac{1}{m^2}+\frac{2}{m}(1-\frac{1}{m})(1-F(w))\Big)\\ +\frac{2}{m}\alpha(1-\alpha)\Big)w \quad &= \quad \Big(\frac{\alpha^2}{m^2}+\frac{2}{m}\alpha(1-\alpha)\Big)\overline{w}, \qquad (2.25)\end{aligned}$$

$$
\begin{aligned}
\Big(\frac{\alpha}{m} + (1-\alpha)\Big)\Big(\overline{w} - \int_{\underline{w}}^{\rho} w dF(w)\Big) & \\
+\alpha(1-\frac{1}{m})(1-\frac{1}{m-1})\int_{\underline{w}}^{\rho}(1-F(w))F(w)dw &= s, \qquad (2.26)
\end{aligned}
$$

and

$$
\alpha(1-\frac{1}{m})\int_{\underline{w}}^{\rho}(1-F(w))F(w)dw + (1-\alpha)\Big(v - \int_{\underline{w}}^{\rho} w dF(w)\Big) = s. \qquad (2.27)
$$

As in Proposition 2.3, we can show that full participation is not optimal for intermediaries because, by (2.26) and (2.27), the expected profit per firm under full participation (as $\alpha = 1$) is rewritten by

$$
\begin{aligned}
\frac{m-1}{m}\int_{\underline{w}}^{\rho}(1-F(w))F(w)dw & \\
-\frac{1}{m}\Big(\overline{w} - \int_{\underline{w}}^{\rho} w dF(w)\Big) - \frac{m-2}{m}\int_{\underline{w}}^{\rho}(1-F(w))F(w)dw &= -\frac{1}{m}\int_{\underline{w}}^{\overline{w}} F^2(w) < 0,
\end{aligned}
$$

which implies that the profit is negative, thus entering the market is not optimal when $\alpha = 1$. In addition, when α is sufficiently close to zero ($\alpha \approx 0$), the upper and lower bounds of prices are almost the same and then the LHS of (2.27) becomes sufficiently small. Thus, we can easily show that if α is sufficiently close to zero, the expected entry profit when $\alpha \approx 0$ is also negative as same as that when $\alpha = 1$. This tells us that there are multiple equilibria if v is large enough, although we have not shown how many equilibria can exist. Furthermore, given that there are multiple equilibria, we can say that the equilibrium with the closest entry level to full participation is entry stable, whereas the equilibrium with the closest entry level to no entry is entry unstable.

From above, as in Proposition 2.3 for the simple case, we can show that full participation cannot be part of an equilibrium and furthermore if the relative value is large enough, there are multiple equilibria, one of which is entry stable under the highest entry level, while one of which is entry unstable under the lowest entry level. In fact, a numerical analysis shows that there are at most two equilibria.

The expected number of entrants (intermediaries who enter the market) is given by 2α.

When considering the case of $m = 10$ given the other parameters $(n, v, s) = (2, 1, 0.1)$, all terms of (2.27) are illustrated by Figure 2.12, whereas Figure 2.13 describes how the highest and lowest equilibrium prices change as the number of suppliers increases. We can

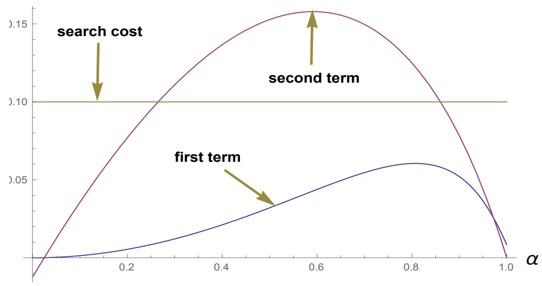

Figure 2.12: The values of three terms of Condition (2.27) when $(m, n, v, s) = (10, 2, 1, 0.1)$.

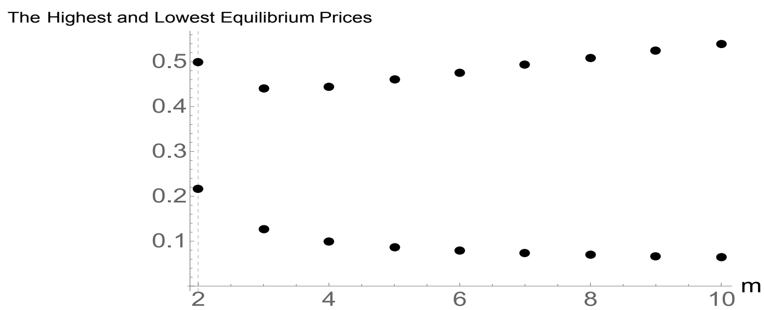

Figure 2.13: The highest and lowest equilibrium prices as m increases from 2 to 10 given $(n, v, s) = (2, 1, 0.1)$. As m increases, the highest equilibrium price becomes lower initially and then higher afterward, whereas the loweset price decreases.

see that the range of prices increases. Similarly, Figure 2.14 shows that the expected number of entrants increases as the number of suppliers increases given that the number of intermediaries is fixed as 2.

Two suppliers and Multiple Intermediaries: Equilibrium Conditions and Analysis

We consider a market where there are two suppliers and $n(= 2, 3, \dots)$ intermediaries and analyze equilibria as in the simple case. The conditions (2.3), (2.4), and (2.5) are modified

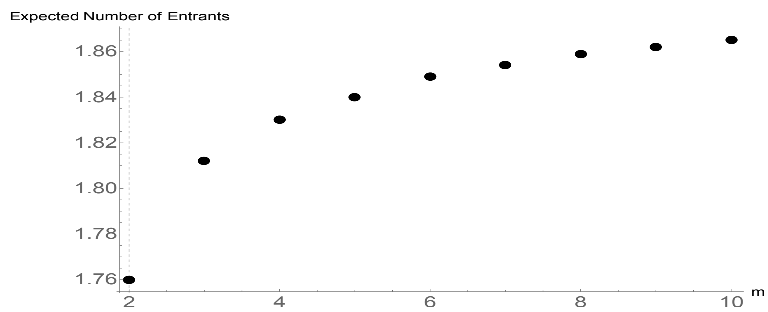

Figure 2.14: The expected number of entrants (intermediaries) as m increases from 2 to 10 given $(n, v, s) = (2, 1, 0.1)$.

to

$$\begin{aligned}\Bigg(\sum_{k=1}^{n}\binom{n}{k}\alpha^k(1-\alpha)^{n-k}\Big((\frac{1}{2})^k \\ +(1-2(\frac{1}{2})^k)(1-F(w))\Big)\Bigg)w &= \Big(\sum_{k=1}^{n}\binom{n}{k}\alpha^k(1-\alpha)^{n-k}(\frac{1}{2})^k\Big)\overline{w}, \qquad (2.28)\end{aligned}$$

$$\sum_{k=0}^{n-1}\Big(\binom{n-1}{k}\alpha^k(1-\alpha)^{n-1-k}(\frac{1}{2})^k\Big)\Big(\overline{w}-\int_{\underline{w}}^{\rho} w dF(w)\Big) = s, \qquad (2.29)$$

and

$$\begin{aligned}\sum_{k=1}^{n-1}\Big(\binom{n-1}{k}\alpha^k(1-\alpha)^{n-1-k}(\frac{1}{2})^k\Big)\int_{\underline{w}}^{\rho}(1-F(w))F(w)dw \\ +(1-\alpha)^{n-1}\Big(v-\int_{\underline{w}}^{\rho} w dF(w)\Big) &= s. \qquad (2.30)\end{aligned}$$

Using the binomial theorem, Conditions (2.28), (2.29), and (2.30) are rewritten by

$$\Big(1-(1-\frac{\alpha}{2})^n+F(w)\big(1+(1-\alpha)^n-(1-\frac{\alpha}{2})^n\big)\Big)w = \Big((1-\frac{\alpha}{2})^n-(1-\alpha)^n\Big)\overline{w},$$

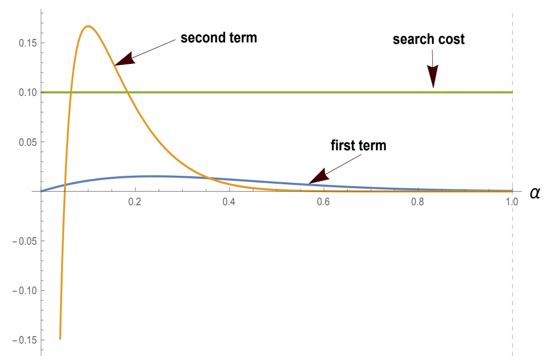

Figure 2.15: The values of three terms of Condition (2.30) when $(m, n, v, s) = (2, 10, 1, 0.1)$.

$$(1 - \frac{\alpha}{2})^{n-1} \int_{\underline{w}}^{\rho} F(w)dw \;\; = \;\; s,$$

and

$$\Big((1 - \frac{\alpha}{2})^{n-1} - (1-\alpha)^{n-1}\Big) \int_{\underline{w}}^{\rho} (1 - F(w))F(w)dw \\ +(1-\alpha)^{n-1}\Big(v - \int_{\underline{w}}^{\rho} wdF(w)\Big) \;\; = \;\; s.$$

By the same way as in the case of multiple suppliers with two intermediaries, we can show that full participation cannot be part of an equilibrium and if the value is large enough there are multiple equilibria including entry stable one with the highest entry level and entry unstable one with the lowest entry level.

The expected number of entrants is given by

$$\sum_{k=0}^{n} \binom{n}{k} \alpha^k (1-\alpha)^{n-k} k \;\; = \;\; n\alpha.$$

In a similar way to the case of multiple suppliers and two intermediaries, let us consider the case of $n = 10$ given the other parameters $(m, v, s) = (2, 1, 0.1)$. Then, all terms of (2.15) are illustrated by Figure 2.15, whereas Figure 2.16 describes how the highest and lowest equilibrium prices change as the number of potential entrants (intermediaries) increases. We can see that the range of prices increases. Similarly, Figure 2.17 shows that the expected number of entrants increases as the number of potential entrants increases

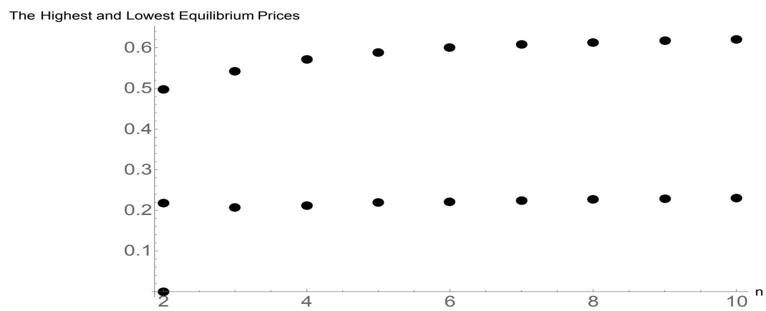

Figure 2.16: The highest and lowest equilibrium prices as n increases from 2 to 10 given $(m, v, s) = (2, 1, 0.1)$.

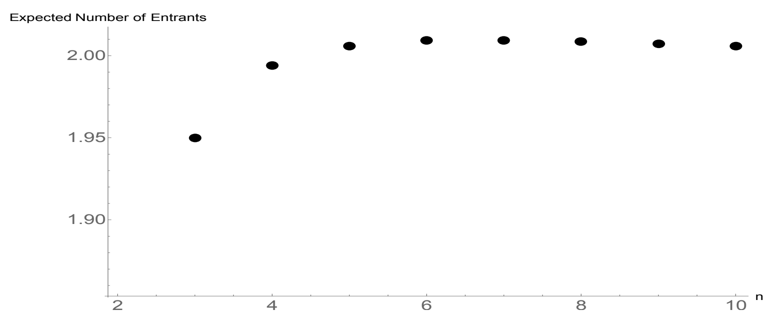

Figure 2.17: The expected number of entrants (intermediaries) as n increases from 2 to 10 given $(m, v, s) = (2, 1, 0.1)$. The number of entrants is maximized when $n = 6$.

given that the number of suppliers is fixed as 2.

Multiple Suppliers and Multiple Intermediaries: Equilibrium Conditions (Not for Publication)

In the case of multiple suppliers and multiple intermediaries, it is hard to derive an explicit solution but we can derive the equilibrium conditions as follows. The essential part of those derivations is to use the inclusion-exclusion principle as counting the number of onto functions

$$\sum_{j=0}^{h} \binom{h}{j} (-1)^j (h-j)^k$$

for any positive integer $k = 1, \ldots,$ which is related to the Sterling number of the second kind. Moreover, we can extend Lemma 2.1 to the case of multiple bidders where the expected revenue of each bidder with $h(=1,2\dots,n-1)$ competitors is given by $\int_{\underline{w}}^{\overline{w}}(1-F(w))^h F(w)dw$ (see, e.g., **?**).

The condition (2.3) is modified to

$$\sum_{k=1}^{n}\binom{n}{k}\alpha^k(1-\alpha)^{n-k}f(k\mid m,n)w \;=\; \sum_{k=1}^{n}\binom{n}{k}\alpha^k(1-\alpha)^{n-k}(\frac{1}{m})^k\overline{w} \tag{2.31}$$

where

$$f(1\mid m,n) \;=\; \frac{1}{m}$$

and for $k=2,\dots,n$, given that we stand from the viewpoint of manufacturer $i=1,2\dots,m$,

$$\begin{aligned} f(k\mid m,n) \;=\; & \overbrace{(\frac{1}{m})^k}^{\text{prob}(k\text{ intermediaries }\to\text{single manufacturer }i)} + \sum_{l=1}^{k-1}\overbrace{\binom{k}{l}(\frac{1}{m})^l(1-\frac{1}{m})^{k-l}}^{\text{prob}(l\to\text{single }i,k-l\to\text{others})} \\ & \times\Big(\sum_{h=1}^{\min\{k-l,m-1\}} \\ & \overbrace{\binom{m-1}{h}\times(\frac{1}{m-1})^{k-l}\Big(\underbrace{\sum_{j=0}^{h}\binom{h}{j}(-1)^j(h-j)^{k-l}}_{k-l\text{ intermediaries }\to h\text{ manufacturers}}\Big)}^{\text{prob}(k-l\text{ intermediaries }\to h\text{ manufacturers out of }m-1)}(1-F(w))^h\Big). \end{aligned}$$

The condition (2.4) is modified to

$$g^S(\lambda) \;=\; s \tag{2.32}$$

where

$$
\begin{aligned}
g^S(\lambda) \;=\; & \sum_{k=0}^{n-1} \overbrace{\binom{n-1}{k}\alpha^k(1-\alpha)^{n-1-k}}^{\text{prob}(k \text{ out of } n-1 \text{ retailers } \to \text{market entry})} \\
& \times \overbrace{(\frac{1}{m})^k}^{\text{prob(all } k\text{retailers entered visits the same manufacturer)}} \qquad (\overline{w} - \int_{\underline{w}}^{\overline{w}} wdF(w)) \\
& + \sum_{k=1}^{n-1} \overbrace{\binom{n-1}{k}\alpha^k(1-\alpha)^{n-1-k}}^{\text{prob}(k \text{ out of } n-1 \text{ retailers } \to \text{market entry})} \\
& \times \sum_{l=0}^{k-1} \overbrace{\binom{k}{l}(\frac{1}{m})^l(1-\frac{1}{m})^{k-l}}^{\text{prob}(l \to \text{single } i, k-l \to \text{others})} \\
& \times \sum_{h=1}^{\min\{k-l,m-2\}} \binom{m-1}{h}(\frac{1}{m-1})^{k-l}\Big(\underbrace{\sum_{j=0}^{h}\binom{h}{j}(-1)^j(h-j)^{k-l}}_{k-l \text{ intermediaries } \to h \text{ manufacturers}} \Big) \\
& \times \underbrace{\frac{m-1-h}{m-1}}_{\text{prob(single retailer sequentially search } \to \text{ manufacturer with no retaier)}} \times \int_{\underline{w}}^{\overline{w}} (1-F(w))^h F(w) dw.
\end{aligned}
$$

The condition (2.5) is modified to

$$
g^M(\lambda) \;=\; s \tag{2.33}
$$

where

$$
\begin{aligned}
g^M(\lambda) \;=\;& (1-\alpha)^{n-1}\Big(v - \int_{\underline{w}}^{\overline{w}} w dF(w)\Big) \\
&+ \sum_{k=1}^{n-1} \overbrace{\binom{n-1}{k} \alpha^k (1-\alpha)^{n-1-k}}^{\text{prob}(k \text{ out of } n-1 \text{ retailers} \to \text{market entry})} \\
&\times \overbrace{(1 - \frac{1}{m})^k}^{\text{prob}(k \to \text{others manufacturers})} \\
&\times \sum_{h=1}^{\min\{k,m-1\}} \binom{m-1}{h} (\frac{1}{m-1})^k \Big(\underbrace{\sum_{j=0}^{h} \binom{h}{j} (-1)^j (h-j)^k}_{k \text{ intermediaries} \to h \text{ manufacturers}} \Big) \\
&\times \int_{\underline{w}}^{\overline{w}} (1-F(w))^h F(w) dw.
\end{aligned}
$$

Derivation of the first term at LHS of (2.12)

As in Lemma 2.1, when two intermediaries visit distinct suppliers and compete in the auction with marginal costs that are independently drawn from the distribution F, the expected payoff of each intermediary is given by

$$
\begin{aligned}
& \int_{\underline{w}}^{\overline{w}} \int_{w}^{\overline{w}} (w' - w - t) dF(w') dF(w) \\
=\;& \int_{\underline{w}}^{\overline{w}} \Big(\overline{w} - wF(w) - \int_{w}^{\overline{w}} F(w')dw' - (w+t)(1-F(w)) \Big) dF(w) \\
=\;& \overline{w} - \int_{\underline{w}}^{\overline{w}} w dF(w) - \int_{\underline{w}}^{\overline{w}} \Big(\int_{\underline{w}}^{w'} dF(w) \Big) F(w') dw' - t \int_{\underline{w}}^{\rho} (1-F(w)) dF(w) \\
=\;& \int_{\underline{w}}^{\rho} (1-F(w))F(w) dw - t \int_{\underline{w}}^{\rho} F(w) dF(w)
\end{aligned}
$$

where

$$\begin{aligned}
\int_{\underline{w}}^{\overline{w}} F(w)dF(w) &= \int_{\underline{w}}^{\overline{w}} \left(\frac{4-\alpha}{2\alpha} - \Big(\frac{4-3\alpha}{2\alpha}\Big)\Big(\frac{\overline{w}+t}{w+t}\Big)\right)\left(\Big(\frac{4-3\alpha}{2\alpha}\Big)(\overline{w}+t)\Big(\frac{1}{w+t}\Big)^2\right)dw \\
&= \Big(\frac{4-\alpha}{2\alpha}\Big)\Big(\frac{4-3\alpha}{2\alpha}\Big)\frac{\overline{w}-\underline{w}}{\underline{w}+t} - \Big(\frac{4-3\alpha}{2\alpha}\Big)^2\frac{(\overline{w}+\underline{w}+2t)(\overline{w}-\underline{w})}{2(\underline{w}+t)^2} \\
&= \Big(\frac{4-\alpha}{2\alpha}\Big) - \frac{(2-\alpha)}{\alpha} = \frac{1}{2}.
\end{aligned}$$

Proof of Proposition 2.9

We first show that the LHS of Condition (2.12) is decreasing in t. Note that Condition (2.12) is the same as Condition (2.5) when $t = 0$.

Lemma 2.7. *The LHS of Condition (2.12) is decreasing in t.*

Proof. The LHS of Condition (2.12) is rewritten by

$$(1-\frac{\alpha}{2})\Big(\int_{\underline{w}}^{\rho} F(w)dw - (\rho+t)\Big) - \frac{\alpha}{2}\Big(\int_{\underline{w}}^{\rho} F^2(w)dw + \rho\Big) + (1-\alpha)v \tag{2.34}$$

where

$$\int_{\underline{w}}^{\rho} F(w)dw = \Big(1 - \Big(\frac{4-3\alpha}{2\alpha}\Big)\ln\Big(\frac{4-\alpha}{4-3\alpha}\Big)\Big)(\rho+t),$$

$$\rho + t = \Big(\frac{2s}{2-\alpha} + t\Big)\frac{1}{1-\Big(\frac{4-3\alpha}{2\alpha}\Big)\ln\Big(\frac{4-\alpha}{4-3\alpha}\Big)},$$

and

$$\int_{\underline{w}}^{\rho} F^2(w)dw = \frac{1}{2\alpha^2}\Big(4\alpha(2-\alpha) - (4-\alpha)(4-3\alpha)\ln\frac{4-\alpha}{4-3\alpha}\Big)(\rho+t).$$

Since

$$\int_{\underline{w}}^{\rho} F(w)dw - (\rho+t) = -\Big(\frac{4-3\alpha}{2\alpha}\Big)\ln\Big(\frac{4-\alpha}{4-3\alpha}\Big)(\rho+t) < 0,$$

together with the simple observation that $\rho + t$ and ρ are increasing in t, (2.34) is shown to be decreasing in t. $\square$

The proof of Proposition 2.9 proceeds as follows.

Proof. By accounting for the two observations that (i) the supplier profit is decreasing in α as shown in Lemma 2.4 (in the appendix) and (ii) the LHS of Condition (2.12) has an inverted-U shape in α as shown in Lemmas 2.2 and 2.3, this implies that there is a unique pair (t^*, α^*) such that the LHS of Condition (2.12) equals the RHS. When considering the equilibrium with α^H for which Condition (2.5) holds as $t = 0$, by allowing for two-part tariffs, suppliers switch from the linear contract to the two-part tariff with the fixed fee t^* in order to increase their profits while extracting intermediary surplus through high entry costs. In contrast, when considering the equilibrium with α^L, suppliers do not change the linear contract because two-part tariffs do not increase their payoffs. $\square$

Derivation of a solution

In a similar way to Section 2.5, we can derive

$$\rho \quad = \quad \Big(\frac{2s}{2-\alpha} + t\Big)\frac{1}{\Big(1 - \frac{4-3\alpha}{2\alpha}\ln\frac{4-\alpha}{4-3\alpha}\Big)} - t \tag{2.35}$$

for which the lowest and highest prices, $\underline{w}$ and $\rho(= \overline{w})$, satisfy

$$\rho \quad = \quad \frac{1}{4-3\alpha}\Big((4-\alpha)\underline{w} + 2\alpha t\Big) \tag{2.36}$$

and the price distribution for any $w \in [\underline{w}, \overline{w}]$ is given by

$$F(w) \quad = \quad \frac{4-\alpha}{2\alpha} - \frac{\frac{2s}{2-\alpha} + t}{2\alpha(\frac{1}{4-3\alpha} - \frac{1}{2\alpha}\ln\frac{4-\alpha}{4-3\alpha})(w+t)}, \tag{2.37}$$

whereas an equilibrium market entry probability corresponds to a solution α such that the condition (2.12) holds.

Chapter 3

The Double Diamond Paradox

3.1 Introduction

For markets to be truly competitive, consumers must be able to compare price offers of different firms (Stigler, 1961). Indeed, it has been established, both theoretically and empirically, that the Law of One Price fails to hold whenever (some) consumers make their purchasing decisions before observing all relevant prices. One source of price dispersion concerns the temporal variation in prices at a given firm, generally referred to as *sales*. Sales are ubiquitous in consumer markets and represent a large share of the observed price variation.[1] Accordingly, a large empirical literature has identified a number of regularities in the temporal distribution of prices. The distribution of retail prices is double-peaked, with two prices being charged most of the time (see Hosken and Reiffen (2004) and Pesendorfer (2002)) and the typical time-series is *V-shaped* with a high price for a number of periods followed by a discrete, short-lived drop which we identify as sales. The bold curve in Figure 3.1 provides an example of such a pattern.[2] In contrast, the equilibrium price distribution generated in the theoretical literature is continuous and monotone.[3]

[1]See Nakamura and Steinsson (2008).

[2]This is the time series of prices reported in the Dominick's Database for one of the most popular Miller Beer Products.

[3]Varian (1980) and Stahl (1989), and a large subsequent literature, analyze static models where consumers differ in their information about other firms' prices and show that random prices can arise in equilibrium. Pesendorfer (2002) presents a model of durable good consumption whereby consumers differ in their willingness to pay and firms optimally vary prices over time in order to price discriminate among consumers (see also Sobel, 1984).

It is not difficult to see that theoretical models that only consider the retail market have difficulties explaining the empirically documented bimodal price distributions. If all consumers only search once, then firms have monopoly power and the Diamond Paradox emerges where firms set the monopoly price; see, for example, Diamond (1971) and Burdett and Judd (1983).[4] On the other hand, if all consumers sample at least two prices, firms will price at marginal cost and again the Law of One Price will hold. As long as some consumers find search costly, this cannot be part of an equilibrium (see, e.g., Stahl, 1989). In markets where some consumers search once and others search twice or more, the equilibrium price distribution is continuous, as any hole in its support would yield incentives for rivals to undercut. Even in markets with product differentiation as in Wolinsky (1986) or Anderson and Renault (1999), bimodal price dispersion will not emerge as a robust phenomenon as firms need to be exactly indifferent between charging two prices.

We argue that bimodal price distributions may be explained as the natural outcome of the vertical interaction between manufacturers and retailers. Indeed, we provide evidence that price variation at the retail level may be induced by manufacturers: sales are associated with lower wholesale prices. While this pattern has so far not been documented in the literature, it is consistent with the price series in the well-known Dominick's database. A typical example is presented by the grey curve in Figure 1.[5] Note that in that Figure retail sales coincide with (or are induced by) contemporaneous reductions in wholesale prices. Note also that retail markups are decreasing in wholesale prices, consistent with incomplete cost pass-through.[6]

To explain bimodal retail and wholesale price distributions, our baseline model has a simple vertical industry structure. Manufacturers sell their product to retailers and retailers resell the product to final consumers. The key feature of the model is that both final consumers *and* retailers engage in sequential search and have to pay a search cost to acquire new information. Final consumers search among retailers, while retailers search among manufacturers. To emphasize that our explanation is not based on search cost heterogeneity, the baseline model has all consumers facing the same consumer search cost

[4]Diamond (1971) showed that, if all consumers have a positive, yet (arbitrary) small, search cost, firms will set monopoly prices in equilibrium. Indeed, if consumers expect any price below the monopoly price to be charged, but instead observe a slightly higher price, they do not have an incentive to continue to search. Thus, no price lower than the monopoly price can be part of the equilibrium price distribution.

[5]See Section 5 for a somewhat more systematic statistical analysis.

[6]Many industries are characterized by incomplete cost pass-through (Weyl and Fabinger, 2013), i.e. higher wholesale prices induce lower margins at the retail level. For instance, using real exchange rates as a source of wholesale price variation, Campa and Goldberg (2005) find that the typical product has a cost pass-through elasticity smaller than one. Our model endogenously generates incomplete cost pass-through, regardless of the shape of demand.

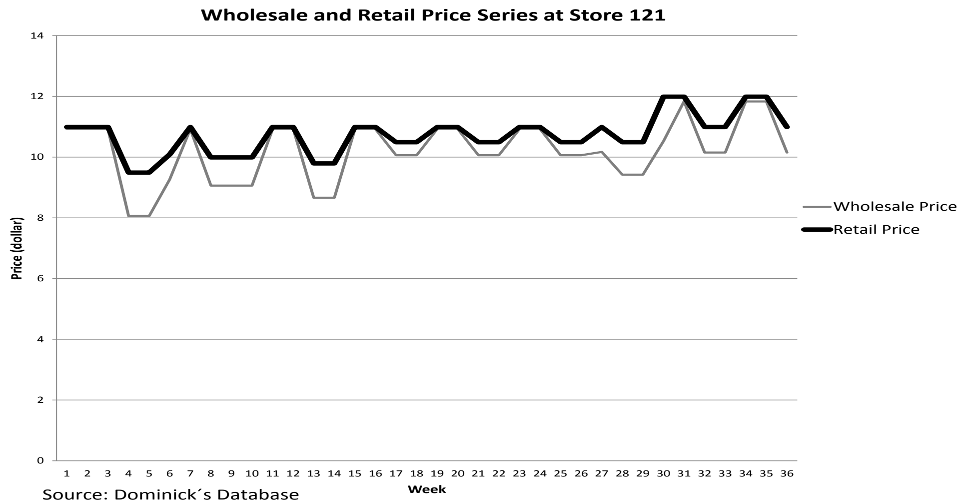

Figure 3.1: Typical Retail and Wholesale Price Series.

and all retailers having identical retailer search cost. We consider homogeneous goods markets so that in the absence of search costs, competition would force prices to be equal to marginal cost.

Retailer and consumer search are important in many real-world markets. While there is a large theoretical and empirical literature documenting the importance of search costs in retail markets, this is the first paper dealing with search in wholesale markets. Even though one may argue that retailers are professional traders with substantial knowledge of market conditions, search frictions in the wholesale market may be substantial if the relevant prices of *alternative* suppliers are hard to identify.[7]

As our baseline model is a simple Diamond model with a vertical industry structure,

[7]Retailers often sign complex contracts with manufacturers, specifying delivery conditions, packages, buy-back policies, branding, and product positioning so that the relevant price may be hard to know ex-ante. In this sense, retailers may be subject to obfuscation practices by manufacturers (see Ellison and Wolitzky, 2012).

It may be argued that retailers often have long-term contracts with their current suppliers and that it is (almost) costless for them to continue their relations with these suppliers. This is perfectly consistent with our paper. In search theoretic terms, this would imply that the first search is for free. What we argue in this paper is that to understand the content of the contracts between retailers and their suppliers we need to understand the cost retailers need to make to be supplied by *alternative* suppliers. Both search and switching costs are relevant in this regard and we will discuss the similarities and differences between the effects of these costs.

one may expect the interaction between manufacturers and retailers to be characterized by the classic double marginalization outcome, where retailers charge the retail monopoly price given the wholesale price that is set by manufacturers, and that given this retail behaviour, manufacturers set their optimal monopoly price as well. Since there is no price dispersion, neither consumers nor retailers have incentives to search. Indeed, we show that double marginalization constitutes an equilibrium for some parameter values, but - more importantly- it is *not* an equilibrium for other parameter values. In particular, under competitive conditions (a relatively large number of retailers or small consumers' search cost) a manufacturer is able to increase its profit by deviating to a price that is equal to the consumers' reservation price, thereby appropriating the retailer's margin. If there are many retailers and the retail market is competitive, the squeezed retailer does not have an incentive to continue to search and will pass-on this higher price to consumers. For low levels of consumer search cost, the (small) reduction in manufacturers' demand is outweighed by the large increase in margin.

Under the competitive conditions where double marginalization is not an equilibrium outcome, manufacturers randomize over two prices: the double marginalization price and a higher price where retailers are squeezed. Because retailers incur a search cost to visit the next manufacturer, they also buy and resell at the highest of these two prices. In a competitive market where retail profits are low, a small retail search cost is sufficient to deter retailers from continuing to search. Retail price strategies are deterministic, with a higher price response at the higher wholesale price. The resulting bimodal retail price distribution is, therefore, induced by manufacturers. This result obtains for small, but strictly positive, consumer search cost so that the retailers' price reaction is limited by consumers' reservation price.

The equilibria that exhibit bimodal price distributions are also interesting from a welfare point of view. As the lowest price over which manufacturers randomize is the double marginalization wholesale price, these equilibria are welfare-inferior to the Double Marginalization Equilibrium. To highlight the idea that search frictions at both levels of the retail chain significantly strengthen the Diamond Paradox, we refer to these equilibria as Double Diamond Equilibria. Interestingly, when the consumer search cost approaches zero, Double Diamond Equilibria exist for all values of the other parameters and they converge to the manufacturer choosing the double marginalization wholesale price with arbitrary small probability and setting the consumer reservation price with probability close to one. Thus, what is typically considered essential for competition, namely the combination of many retailers and small search costs, is disastrous for social welfare in vertically related industries where both retailers and consumers have a small search cost. Numerically, we show that the difference in outcomes between Double Diamond Equilibria and the Double Marginalization Equilibrium can be quantitatively substantial. For example, for linear demand, we show that the total surplus generated in these equilibria can be in the order of 40% lower than the total surplus generated in the Double Marginalization Equilibrium (which is already considered to be low), whereas consumer surplus is more than 60% lower! These results suggest that retailer search has important implications that have so far been unexplored.

Together, the Double Diamond Equilibria and the Double Marginalization Equilibrium span the whole parameter space. In these equilibria, consumers and retailers follow standard reservation price strategies. These equilibria are the main focus of our analysis. For certain demand functions, there exists, however, another type of equilibrium where the manufacturers make more profits than in any of the equilibria mentioned before and where consumers follow a non-reservation price strategy. These equilibria may exist because of the possibility of consumer learning. There exists an important search literature, initiated by Bénabou and Gertner (1993), where consumers learn about the underlying cost structure of firms while observing prices (see also Dana (1994), Fishman (1996), and more recently Tappata (2009) and Janssen et al. (2011)). In these papers, uncertainty about firms' cost is exogenously imposed. In contrast, our paper provides a foundation for retail cost uncertainty through randomized equilibrium pricing of manufacturers in the wholesale market.

It is important to observe that our results can also be re-interpreted in terms of retailers' switching cost. The difference between search and switching costs is that the first relates to an informational friction, while the second does not (see Wilson, 2012). One may argue that in some environments, retailers have a long-term relation with their supplier and do not find it easy to switch to a different supplier. As we go along, we will discuss how these results can be understood in terms of this alternative interpretation.

Notice also that our results do not depend on whether or not the first search is free (provided the first search is not too costly, prohibiting consumers or retailers to enter the market). Thus, long-term relationships between retailers and their suppliers can be accommodated in our analysis.[8] Still, search costs are important to understand the content of long-term contracts in such markets, as they determine retailers' outside options.

As far as we know, there is no literature on search by retailers. There is a small, recent literature dealing with consumer search in markets with an explicit vertical structure. In

[8]If the first visit is costly (see Janssen et al., 2005), then there exists an upper bound on the search cost such that our results hold, as the surplus of consumers and retailers should be positive for the market not to break down. Since most of our important results hold true for small search costs, costly first search is inconsequential.

? and **?**, all retailers acquire their product from one monopolistic manufacturer, while in Lubensky (2013), a single manufacturer deals exclusively with a large number of retailers who face a competitive fringe of independent sellers. There is no retail search in these vertical models.

Our paper also has interesting implications for empirical studies assessing the classical question of the effect of retail concentration on prices (see, e.g., Berger and Hannan (1989) and Bikker and Haaf (2002) for the banking industry, or Cotterill (1986) for the food industry). In our model, retail margins are lower in Double Diamond Equilibria and these equilibria typically exist when there are many retailers in the market. However, this does not imply that competitive retail markets create higher welfare. Manufacturers find it easier to squeeze retailers when retail markets are competitive, resulting in much higher wholesale prices and higher retail prices than in concentrated retail markets.

The rest of the paper is organized as follows. The next section presents the model. Section 3 presents some general characterization results that all equilibria have to satisfy. In particular, we show that the double marginalization equilibrium has the lowest wholesale and retail price in any possible equilibrium. Section 4 shows the conditions that are together necessary and sufficient for a Double Marginalization Equilibrium to exist. Section 5 focuses on Double Diamond Equilibria. It characterizes these equilibria and determines necessary and sufficient conditions for these equilibria to exist. This Section also verifies in some detail the empirical implications of these equilibria and discusses the welfare implications. Section 6 deals with the non-reservation price equilibrium in which manufacturers benefit from active consumer search, while Section 7 treats search cost heterogeneity. Section 8 concludes.

3.2 The Basic Model

To focus on search frictions in a vertical industry structure, consider a standard market for a homogenous product, supplied by $m \geq 2$ manufacturers. Our arguments do not depend on the manufacturers' cost structure, so we normalize cost to be equal to 0. Manufacturer i (*she*) sells the product for a price w_i per unit to $n \geq 2$ retailers, and each retailer (*he*) j resells the product to consumers at a retail price p_j at no additional cost. There is a unit mass of consumers (often referred to as *they*) and each individual consumer has a demand function $D(p)$ so that they derive a surplus of $S(\widetilde{p}) = \int_{\widetilde{p}}^{\infty} D(p)dp$ when buying at price $\widetilde{p}$.[9]

[9]We assume throughout a vertical structure of independent firms signing linear pricing contracts. We believe that this is the natural starting point for any analysis of retailer search. For a discussion of this

The novel feature of our model is that both retailers and consumers engage in costly sequential search in order to discover the relevant prices. Retailers (consumers) have to pay a positive search cost s_R (respectively s_C) for any additional price quotation (either w for retailers or p for consumers) they observe. As we want to understand the different roles played by retailers' and consumers' search costs, we allow $s_R \neq s_C$. For simplicity, we follow most of the search literature and assume that the first price quotation is free, but most of our results continue to hold if the first search is costly. When retailers decide to stop their search and pay a wholesale price w', their marginal cost is w' for each unit sold. When consumers stop searching, they decide to buy at the lowest price p they have observed and buy $D(p)$ units. For most of the paper, we assume that all individuals in a given layer share the same share cost. In Section 7 we show that most of our main results continue to hold with heterogeneous search costs at both levels.

A retailer's search strategy is characterized by a reservation price ρ_R: at any price $w \leq \rho_R$, the retailer buys, otherwise he continues to search. For the first search, each retailer either visits a manufacturer at random or, in the case of long-term relationships, he visits his current supplier. Reservation price strategies are optimal for the retailer since he does not update his beliefs about the distribution of prices posted by the remaining manufacturers, following any history of observed prices (see, e.g., Kohn and Shavell, 1974).[10] Consumers face a slightly different problem, since they may learn about the distribution of manufacturer prices from the observed history of retail prices. For the bulk of this paper, however, we consider equilibria in which consumers' optimal search behaviour is also characterized by a reservation price strategy. In such a case, we denote this reservation price by ρ_C: at any price $p \leq \rho_C$, consumers buy, otherwise they continue to search. For the first search, an equal share of consumers visits each retailer. In Section 6 we provide an equilibrium where consumers do not follow a reservation price strategy. The determination of ρ_R and ρ_C depends on the equilibrium features, and we discuss the details in different sections.

The timing of the interaction is as follows. Manufacturers first set their wholesale prices. Retailers then search among manufacturers according to an optimal search strategy. When all retailers have stopped searching, taking their wholesale price as given, they simultaneously set retail prices without knowing wholesale prices of others. Consumers then engage in sequential search among the given retail prices.

Given some wholesale price w, we define the profit-maximizing price for a monopolist

assumption see Section 8.

[10]See Rothschild (1974) for the observation on the (non)optimality of the reservation price rule when the search environment is (not) stable.

retailer as

$$p^m(w) = \arg\max_{p\geq 0}(p-w)D(p). \tag{3.1}$$

We refer to $p^m(w)$ as the retail monopoly price. Given the retailer's optimal price $p^m(w)$, a single manufacturer would choose the wholesale monopoly price defined by

$$w^m = \arg\max_{w\geq 0} wD(p^m(w)). \tag{3.2}$$

We shall assume throughout that both maximization problems are well-defined. In particular, we assume that manufacturers' and retailers' (monopolistic) profit functions are single-peaked.[11] We refer to w^m as the wholesale monopoly price and $p^m(w^m)$ as the double marginalization (DM) retail price. Together they induce the DM outcome.

Following the vertical contracting literature (see, e.g., McAfee and Schwartz, 1994) we consider symmetric Perfect Bayesian equilibria satisfying a natural extension of passive beliefs to our environment. Namely, we assume that whenever a given player observes an off-the-equilibrium path action, she puts positive probability on *minimal deviation paths* only. For each information set, the minimal deviation path specifies a set of actions for every other player that induces that information set and such that the number of players whose action does not belong to the support of their equilibrium strategy profile is minimal. Notice that passive beliefs satisfy the minimal deviation property.

Definition 3.1. A symmetric perfect Bayesian reservation price equilibrium (RPE) is a wholesale price w^*, a retail search and pricing strategy $(\rho_R^*, p^*(w))$ and a consumer search strategy ρ_C^* such that in every information set, a player's strategy is optimal given the strategies of the other players and beliefs that put zero probability on action profiles that do not belong to a minimal deviation path.[12]

If consumers visit a retailer who has posted an unexpected retail price, they are not sure which firm has actually deviated from the equilibrium strategy: the retailer they have visited or the manufacturer from which the retailer they have visited visits had procured the product. Out-of-equilibrium beliefs are therefore of some importance to determine the

[11]Standard assumptions in $D(p)$ would suffice. For instance, if the elasticity changes smoothly and monotonically with the price, both problems are well-behaved.

[12]A definition like this is often implicitly used in the consumer search literature. Formally, if one does not require reservation prices to be optimal off the equilibrium path, marginal cost pricing may be considered equilibrium behaviour if consumers' reservation prices are equal to marginal cost. By requiring retail and consumer strategies to be optimal for all beliefs that are consistent with a minimal deviation path, the above definition rules out situations like this.

optimal search behaviour of the consumer. If they believes that the retailer has deviated to another retail price, while all manufacturers followed their equilibrium price strategies, then they do not need to update their beliefs about the other retailers when deciding whether to continue their search. If, on the other hand, consumers believe that a manufacturer has deviated to set a different wholesale price, then they have to take into account that other retailers may have visited the same deviating manufacturer, bought at the same wholesale price, and sell at the same out-of-equilibrium retail price.

3.3 General Characterization Results

We first provide a general characterization of some of the properties that all equilibria in this model have to satisfy. This allows us to reduce the space of possible strategy profiles to consider. Our first result shows that in any symmetric equilibrium the firms' prices are not smaller than the DM prices.

Proposition 3.1. *In any symmetric equilibrium, wholesale and retail prices set by manufacturers and retailers are such that* $w^* \geq w^m$ *and* $p^*(w^*) \geq p^m(w^m)$.

This is an important result.[13] It shows that when both consumers and retailers have a positive search cost, we cannot expect competition in the market to result in social welfare levels that are better than those resulting from a market served by a monopolist at both the wholesale and the retail level. Thus, it constitutes a relatively straightforward extension of the Diamond Paradox.

In addition, we show that in any equilibrium, the wholesale monopoly price is always set with some positive probability and if other prices are also set with positive probability, there has to be a non-negligible gap between the wholesale monopoly price and the rest of the support.

Proposition 3.2. *In any symmetric equilibrium, manufacturers set* w^m *with some strictly positive probability. In addition, there exists some* $\varepsilon > 0$ *such that the open set of prices* $(w^m, w^m + \varepsilon)$ *does not belong to the support of the wholesale price distribution.*

The first part of the Proposition is the combination of two basic observations. First, in any equilibrium the retail price is decreasing in the wholesale price. It follows that if

[13]With minor changes in the proof, we can extend Proposition 1 to hold for any (potentially non-symmetric) equilibrium.

a manufacturer chooses the lowest price in the support of her distribution, she guarantees that both retailers and consumers buy without further search. Second, of all prices that can be on the equilibrium path according to Proposition 1, the wholesale monopoly price gives the manufacturer the highest profits if retailers and consumers buy outright. Thus, if the wholesale monopoly price was not set with positive probability on the equilibrium path, it would be optimal to deviate to it.

For the second part, notice that for any wholesale price sufficiently close to the wholesale monopoly price, retailers are able to charge the retailer monopoly price and, given positive search costs, still sell to all incoming consumers. Since, by definition, this yields lower profits to the manufacturer, such prices are never part of the equilibrium price distribution.

3.4 Double Marginalization Equilibrium

In our model, Double Marginalization (DM) is the natural counterpart to the monopoly outcome that arises in Diamond's seminal paper. In the strategy profile that leads to the DM outcome, (i) each manufacturer sets the wholesale monopoly price, (ii) each retailer visits one single manufacturer, buys at that wholesale price, and then sets the retail monopoly price, and (iii) consumers visit one single retailer and buy at that retail price. More formally, we have the following. Each manufacturer chooses w^m. Retailers' behaviour as a function of the wholesale price w is given by

$$\sigma_R^r(w) = \begin{cases} \text{buy and set } p^m(w) & \text{if } w \leq \rho_R \text{ and } p^m(w) \leq \rho_C, \\ \text{buy and set } \rho_C & \text{if } w \leq \rho_R \text{ and } p^m(w) > \rho_C, \\ \text{buy and set } w & \text{if } \rho_C < w < \rho_R, \\ \text{continue to search} & \text{if } w > \rho_R, \end{cases} \tag{3.3}$$

while consumers' optimal search rule $\sigma_C^r(p)$ as a function of the retail price p is given by

$$\sigma_C^r(p) = \begin{cases} \text{buy at } p & \text{if } p \leq \rho_C, \\ \text{continue to search} & \text{if } p > \rho_C. \end{cases} \tag{3.4}$$

If this strategy profile constitutes an equilibrium, there is no active search and no price dispersion. Retailers are effective monopolists in their incoming demand and, therefore, optimize by setting the monopoly retail price $p^m(w)$.

Definition 3.2. A symmetric (pure) strategy profile σ^{DM} is a double marginalization

(DM) strategy profile if $\sigma^{DM} = (w^*, \sigma_R^r(w), \sigma_C^r(p))$ is such that each manufacturer sets $w^* = w^m$ as in (3.2), each retailer follows the strategy $\sigma_R^r(w)$ as in (3.3), and consumers follow the search rule $\sigma_C^r(p)$ as defined by (3.4). If σ^{DM} is an equilibrium, then we call it a Double Marginalization Equilibrium (DME).

Given the construction of these strategies, it is clear that the only profitable deviation may come from a manufacturer who charges a higher wholesale price at the expense of a lower retailer margin. To derive necessary and sufficient conditions for this deviation not to be profitable, we still have to specify the reservation prices ρ_R and ρ_C. As we mentioned in Section 2, retailers' reservation price is relatively easy to characterize. We can compute the wholesale price $\rho_R > w^m$ such that retailers are indifferent between buying the product at that price and continuing to search for a lower (equilibrium) price w^m that they expect from other manufacturers. Given that, after observing ρ_R a retailer would set a retail price equal to $\min\{\rho_C, p^m(\rho_R)\}$. ρ_R is implicitly determined by

$$(\min\{\rho_C, p^m(\rho_R)\} - \rho_R)\frac{D(\rho_C)}{n} = (p^m(w^m) - w^m)\frac{D(p^m(w^m))}{n} - s_R,$$ [14]

where because of the first random search, consumer demand is evenly split among all retailers. The above equation is rewritten as

$$(p^m(w^m)) - w^m)D(p^m(w^m)) - (\min\{\rho_C, p^m(\rho_R\}) - \rho_R)D(\rho_C) = ns_R.$$

The characterization of the consumers' reservation price is more complicated because $\rho_C > p^m(w^m)$. If consumers observe the reservation price (whatever its precise value), then they know that either this retailer has deviated or the manufacturer has deviated that has sold the product to the retailer he has visited. Notice that both interpretations of the deviation are consistent with the minimal deviation property on beliefs as discussed in Section 2.

Assume for now that consumers *blame* the manufacturer for a possible deviation from the equilibrium retail price. In this case, if a consumer continues to search, there is a chance of $\frac{1}{m}$ that the next retailer has bought the product from the same manufacturer. The consumer reservation price ρ_C is then given by

$$\int_{\rho_C}^{\infty} D(p)dp = \frac{1}{m}\int_{\rho_C}^{\infty} D(p)dp + \frac{m-1}{m}\int_{p^m(w^m)}^{\infty} D(p)dp - s_C,$$

[14] If we were to consider retailer switching cost instead of search cost, the same equation would apply as there is no uncertainty for retailers as regards to what price to expect on the next search in the DM strategy profile.

that is,

$$\frac{m-1}{m}\int_{p^m(w^m)}^{\rho_C} D(p)dp = s_C. \tag{3.5}$$

We have two reasons to focus on the case in which consumers blame a manufacturer for a possible deviation. First, our main results suggest that retailer search brings about equilibrium outcomes that may be significantly worse than the DM outcome. We do not want this argument to depend on specific choices of out-of-equilibrium beliefs. Later in this section, we show that the most critical deviation to consider is the manufacturer choosing a wholesale price w equal to the consumer reservation price ρ_C, thereby fully squeezing the retail margins. This deviation is more profitable if the reservation price is low, which happens if consumers blame retailers.[15] In other words, if the double marginalization equilibrium is *not* an equilibrium *if consumers blame the manufacturer*, it is certainly not an equilibrium if consumers blame the retailer. Second, by assuming consumers blame the manufacturer, the next Section provides a clear characterization of all reservation price equilibria. In Section 6, we consider out-of-equilibrium beliefs where consumers blame retailers and show that these beliefs are necessary for other (non-reservation price) equilibria to exist.

We are now in the position to provide a characterization of the conditions under which the Double Marginalization strategy profile is an equilibrium. As argued above, it suffices to focus on potential deviations by manufacturers. Moreover, it suffices to verify whether manufacturers can profitably deviate to a wholesale price that induces retailers to buy and charge the consumers' reservation price (whenever retailers react to a deviation by setting the retail monopoly price, the manufacturer's profit is always lower than in the candidate equilibrium, by definition of the wholesale monopoly price). There are two cases to consider. First, suppose s_C is so large that $\rho_C D(\rho_C) \leq w^m D(p^m(w^m))$.[16] It is clear that in this case no deviation is profitable. Whether or not retailers buy from the manufacturer that deviated, the profit the deviating manufacturer makes is smaller than the equilibrium profit $w^m D(p^m(w^m))$. Formally, we can define $\overline{w}$ as the lowest wholesale price such that if retailers buy and react by selling themselves at $\overline{w}$ and consumers react by buying as well,

[15] If consumers believe the retailer has deviated to an unexpectedly high retail price, then they should expect that all other retailers will set the equilibrium retail price $p^m(w^m)$. Therefore, in this case, the reservation price is given by

$$\int_{p^m(w^m)}^{\rho_C} D(p)dp = s_C. \tag{3.6}$$

Comparing expressions (3.5) and (3.6), it is clear that if consumers blame retailers, their reservation price is lower than if they blame manufacturers.

[16] Note that ρ_C is increasing in s_C and that because of the single peakedness assumption, $\rho_C D(\rho_C)$ is decreasing in s_C for all $\rho_C > p^m$.

the manufacturer is indifferent between setting this price and the wholesale monopoly price, i.e.,

$$\overline{w}D(\overline{w}) = w^m D(p^m(w^m)). \tag{3.7}$$

Whenever consumers observe the retail price $\overline{w}$, their pay-off of buying (resp. continue to search) is given by $\int_{\overline{w}}^{\infty} D(p)dp$ (resp. $\frac{1}{m}\int_{\overline{w}}^{\infty} D(p)dp + (1-\frac{1}{m})\int_{p^m(w^m)}^{\infty} D(p)dp - s_C$) . Thus, we can define a threshold $\overline{s}_C(m)$ by

$$\overline{s}_C(m) = \frac{m-1}{m}\int_{p^m(w^m)}^{\overline{w}} D(p)dp \tag{3.8}$$

at which the consumer is indifferent between continuing to search after observing a price $\overline{w}$ and buying. Thus, for $s_C \geq \overline{s}_C(m)$, the manufacturer's deviation is not profitable, even if retailers and consumers react by buying at this price. In that case the DME exists, while for smaller search costs, we have to inquire into the retailers' decision problem.

Second, suppose then $\rho_C D(\rho_C) > w^m D(p^m(w^m))$, or equivalently that $s_C < \overline{s}_C(m)$. In this case if the retailer still buys at ρ_C (even if fully squeezed), which requires that the retailer search cost is larger than the profit she would make if she continues to search, i.e., $s_R > \frac{1}{n}(p^m(w^m) - w^m)D(p^m(w^m)) \equiv \overline{s}_R(n)$, it is clearly optimal for the manufacturer to deviate and the DME does not exist. But even if $s_R < \frac{1}{n}(p^m(w^m) - w^m)D(p^m(w^m))$, it is still profitable to deviate if, and only if, $\rho_R D(\rho_C) > w^m D(p^m(w^m))$, where ρ_R is defined by

$$(\rho_C - \rho_R)\frac{D(\rho_C)}{n} = (p^m(w^m) - w^m)\frac{D(p^m(w^m))}{n} - s_R. \tag{3.9}$$

From (3.9) it follows that

$$\rho_R D(\rho_C) = \rho_C D(\rho_C) - (p^m(w^m) - w^m)D(p^m(w^m)) + ns_R,$$

and so a manufacturer's gain from deviating is positive if

$$\rho_R D(\rho_C) - w^m D(p^m(w^m)) = \rho_C D(\rho_C) - p^m(w^m)D(p^m(w^m)) + ns_R > 0.$$

As the RHS is strictly increasing in s_R and is positive at $\overline{s}_R(n)$, there is a threshold retailer search cost level $0 < s_R^*(m,n,s_C) < \overline{s}_R(n)$ defined by

$$s_R^*(m,n,s_C) = \frac{1}{n}\Big(p^m(w^m)D(p^m(w^m) - \rho_C D(\rho_C)\Big), \tag{3.10}$$

such that for any $s_R \in (s_R^*(m,n,s_C), \overline{s}_R(n))$ partial squeezing of retailers is optimal and

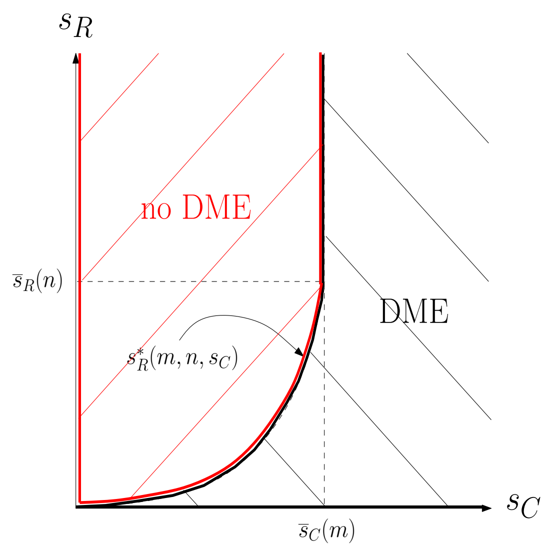

Figure 3.2: The region where the DME does (not) exist for a given pair (m, n).

a double marginalization equilibrium does not exist. If, however, $s < s^*_R(m, n, s_C)$, the deviation is not profitable and the DM strategy profile constitutes an equilibrium.

From the above, we can characterize the existence of the DME as follows.

Proposition 3.3. *For any given m and n the Double Marginalization strategy profile is an equilibrium if and only if s_C and s_R are such that (i) $s > \overline{s}_C(m)$ as defined in (3.8) or (ii) $s < \overline{s}_C(m)$ and $s_R > s^*_R(m, n, s_C)$ as defined in (3.10).*

Note that the threshold value $s^*_R(m, n, s_C)$ is decreasing in n and increasing in s_C (via ρ_C). Importantly, the threshold value approaches 0 when n becomes very large or when s_C is close to 0. In this case, it is profitable for the manufacturer to deviate for almost all retailer search cost levels. In addition, $\overline{s}_C(m)$ is increasing in m. Thus, in markets that are thought of as being competitive because the search frictions are small and there are many retailers and manufacturers, DM is not an equilibrium.

Figure 3.2 illustrates Proposition 3.3. The region to the right of the thick black line indicates the parameter region where the DM strategy profile is an equilibrium. Above the curve $s < s^*_R(m, n, s_C)$ and to the left of $\overline{s}_C(m)$, the strategy profile is not an equilibrium because manufacturers are better off by partially or fully squeezing retailers.

To provide some quantitative illustration of our findings so far, we briefly consider the special case of a linear demand function $D(p) = 1 - p$. As is well-known, the wholesale and retail monopoly prices are equal to $w^m = \frac{1}{2}$ and $p^m(w^m) = \frac{1+w^m}{2} = \frac{3}{4}$, respectively. Accordingly, the profit of manufacturers (resp. retailers) is given by $\frac{1}{8}$ (resp. $\frac{1}{16}$). Social welfare and consumer surplus are given by $\frac{7}{32}$ and $\frac{1}{32}$.

To see for which parameter values this is an equilibrium, we need to compute the consumer and retailer reservation prices, ρ_C and ρ_R, given by (3.5) and (3.9), and calculate $\overline{s}_C(m)$ and $s_R^*(m,n,s_C)$. By (3.5), we derive the consumer reservation price to be $\rho_C(s_C) = 1 - \frac{1}{4}\sqrt{\frac{m-1-32ms_C}{m-1}}$. It follows that $\overline{s}_C(m) = \frac{4\sqrt{2}-5}{32}\frac{m-1}{m}$. Given $\rho_C(s_C)$, using (3.9), we derive the retailer reservation price to be $\rho_R(s_R, s_C) = \rho_C(s_C) - \frac{1-16ns_R}{4}\sqrt{\frac{m-1}{m-1-32ms_C}}$, while $s_R^*(m,n,s_C) = \frac{1}{4n}\left(1 - \frac{8ms_C}{m-1} - \frac{1}{4}\sqrt{1 - \frac{32ms_C}{m-1}}\right)$. It can be easily shown that $s_R^*(m,n,s_C)$ is strictly increasing and convex in s_C and strictly decreasing in both m and n. Proposition 3.3 shows that the DME exists, if and only if, $s_R \leq s_R^*(m,n,s_C)$ or $s_C \geq \overline{s}_C(m)$. In Figure 3.2 we show this region for $m = n = 2$. In this case $\overline{s}_C(2) = \frac{4\sqrt{2}-5}{64}$, which is approximately 33% of the consumer surplus generated in the DME. Thus, the DM strategy profile is *not* an equilibrium for a large set of relevant parameter values.

3.5 Double Diamond Equilibria

We now show that manufacturers may partially or fully squeeze retailers *along the equilibrium path* by setting a wholesale price that is larger than the wholesale monopoly price with positive probability. This will result in retail prices that are higher than the DM price and in welfare that is smaller than the welfare generated in the DME. Interestingly, these equilibria exist whenever the DME does not, i.e., when the consumer search cost is small and the retailer search cost is not too small (depending on how many retailers there are in the market). We call these equilibria Double Diamond Equilibria (DDE) because the combination of search costs for consumers and retailers renders market outcomes that are significantly worse than those in the Diamond model. Moreover, the discontinuity as search costs become negligible is even more severe here than in the original Diamond model. Since manufacturers randomize *in equilibrium* between two prices, consumers' reservation price no longer depends on off-the-equilibrium path beliefs, but is directly pinned down by equilibrium strategies.

The simplest equilibrium generating market outcomes that are worse than the DM outcome has manufacturers randomizing over two wholesale prices: the wholesale monopoly price w^m defined by (3.2) is set with probability γ and a higher wholesale price $w^{dd} > w^m$

is set with probability $1-\gamma$. So, manufacturers adopt a mixed strategy given by

$$\sigma_M = \begin{cases} w^m & \text{w.p. } \gamma \\ w^{dd} & \text{w.p. } 1-\gamma. \end{cases} \tag{3.11}$$

We know that retailers will react to w^m by setting the retail monopoly price p^m. We also know that in any equilibrium where consumers choose reservation price strategies, it must be the case that retailers choose $p(w) = \min\{p^m(w), \rho_C\}$ (if $w \leq \min\{\rho_R, \rho_C\}$). As the manufacturer has to be indifferent between choosing w^m and w^{dd} (and w^m is the unique maximizer of $wD(p^m(w))$), it therefore has to be that in a Double Diamond equilibrium, retailers react to w^{dd} by choosing ρ_C. Thus, we may have two types of Double Diamond Equilibria: one where with positive probability manufacturers fully squeeze retailers and set $w^{dd} = \rho_C$ and one where they partially squeeze retailers and set $w^{dd} = \rho_R < \rho_C$ with positive probability.

Formally, we define a Double Diamond strategy profile as follows.

Definition 3.3. A symmetric (pure) strategy profile σ^{DD} is a Double Diamond (DD) strategy profile if $\sigma^{DD} = (\sigma_M, \sigma_R^r(w), \sigma_C^r(p))$ is such that each manufacturer sets a strategy as in (3.11), each retailer follows the strategy $\sigma_R^r(w)$ as in (3.3), and consumers follow the search rule $\sigma_C^r(p)$ as in (3.4). If σ^{DD} is an equilibrium, then we call it a Double Diamond Equilibrium (DDE).

We first inquire into the conditions for existence of DDE whereby retailers are fully squeezed. In this case, consumers' reservation price ρ_C has to be equal to $\overline{w}$ as defined in (3.7). Given that manufacturers now explicitly randomize, ρ_C is defined as follows. Consumers who visit a retailer charging a retail price ρ_C are indifferent between buying and continuing to search if, and only if,

$$\int_{\rho_C}^{\infty} D(p)dp = \frac{1}{m}\int_{\rho_C}^{\infty} D(p)dp + (1-\frac{1}{m})\Big(\gamma \int_{p^m(w^m)}^{\infty} D(p)dp + (1-\gamma)\int_{\rho_C}^{\infty} D(p)dp\Big) - s_C.$$

To understand the RHS of this expression note that if a consumer continues to search for a lower retail price, there are two cases to consider: (i) with probability $\frac{1}{m}$, another retailer has visited the same manufacturer setting the high wholesale price ρ_C (with the retail price ρ_C as the best response), and (ii) when each other retailer visits one of the other manufacturers, because of manufacturers' randomized pricing, the retailer sets $p^m(w^m)$ with probability γ and ρ_C with probability $1-\gamma$. Thus, after observing the retail price

ρ_C, consumers are indifferent if, and only if,

$$\frac{m-1}{m}\int_{p^m(w^m)}^{\rho_C} D(p)dp = \frac{s_C}{\gamma}. \tag{3.12}$$

As $\rho_C = \overline{w}$, this equation determines γ, the probability manufacturers choose the wholesale monopoly price, i,e.,

$$\gamma = \frac{s_C}{\frac{m-1}{m}\int_{p^m(w^m)}^{\overline{w}} D(p)dp}.$$

Note that this expression is linear in s_C and, as will be important later, that γ approaches 0 when s_C approaches 0. Note that if $s_C < \overline{s}_C(m)$ as defined in the previous section, this equation will always define γ in such a way that it is smaller than 1.

We next consider the retailer strategy. If a retailer visits a manufacturer with a wholesale price of ρ_C, the best he can do is to buy and sell at ρ_C if

$$\gamma(p^m(w^m) - w^m)\frac{D(p^m(w^m))}{n} \leq s_R.^{17}$$

This equation is easily understood: the chance of observing a wholesale price of w^m on the next search equals γ and the expected pay-off that is obtained should be smaller than the search cost.

Thus, a DDE with full squeezing of retailers exists if $s_C < \overline{s}_C(m)$ and

$$\frac{s_C(p^m(w^m) - w^m)D(p^m(w^m))}{\frac{(m-1)n}{m}\int_{p^m(w^m)}^{\overline{w}} D(p)dp} \leq s_R. \tag{3.13}$$

We have the following result.

Proposition 3.4. *For any given m and n, there exists a DDE where retailers are fully squeezed with strictly positive probability if, and only if, $s_C < \overline{s}_C(m)$ and s_R is such that (3.13) holds.*

The lower bound on s_R such that retailers do not want to continue to search goes to 0 if s_C approaches 0 or if the number of retailers n is large. The reason why retailers do not want to continue to search after observing ρ_C if s_C is close to 0 (and s_R is also small, but larger than the threshold value defined by the LHS of (3.13)) is that the probability

[17] If the retailer cost is a switching cost rather than a search cost, and retailers know all manufacturer prices, this condition needs to be modified to $(p^m(w^m) - w^m)\frac{D(p^m(w^m))}{n} \leq s_R$.

with which manufacturers charge the monopoly wholesale price w^m is also close to 0 so that the chance of getting a lower wholesale price and making a profit on the next search is arbitrarily small.

Note that in a full squeezing DDE, γ ranges between 0 and 1, depending on whether s_C is close to 0 or close to $\frac{m-1}{m}\int_{p^m(w^m)}^{\overline{w}} D(p)dp$. For larger s_C values in this range, we have an equilibrium with a "regular" high price and an irregular lower "sales price" that is induced by the manufacturer, very much like what we have observed in Figure 1. At the sales price, retailers' margins are positive (and are in fact equal to the monopoly level given the wholesale price), while they are 0 at the regular price. For low s_C values in this range, we have that the low price is charged most of the time and becomes the regular price.

We next consider the possibility that the manufacturers partially squeeze retailers in a DDE. In this case, it is clear that the following conditions should be satisfied. First, manufacturers should be indifferent between charging w^m and ρ_R :

$$\rho_R D(\rho_C) = w^m D(p^m(w^m)). \tag{3.14}$$

As the RHS is constant, it follows from the single-peakedness of $pD(p)$ and the fact that the manufacturer has to be indifferent between choosing w^m and $\rho_R < \rho_C$ that consumers' reservation price under partial squeezing has to be smaller than under full squeezing.

Second, after observing ρ_C, consumers should be indifferent between buying and continuing to search:

$$\frac{m-1}{m}\int_{p^m(w^m)}^{\rho_C} D(p)dp = \frac{s_C}{\gamma}.$$

This condition is similar to what we saw before in the full squeezing DDE, the only difference being that ρ_C is now determined differently. Finally, at ρ_R retailers should be indifferent between buying and continuing to search for the lower wholesale price w^m :

$$(\rho_C - \rho_R)\frac{D(\rho_C)}{n} = \frac{\gamma}{n}(p^m(w^m) - w^m)D(p^m(w^m)) + \frac{1-\gamma}{n}(\rho_C - \rho_R)D(\rho_C) - s_R,$$

which reduces to

$$(p^m(w^m)D(p^m(w^m)) - \rho_C D(\rho_C)) = \frac{ns_R}{\gamma}. \tag{3.15}$$

To inquire when such a partial squeezing DDE exists, consider for any given m and n all pairs (s_C, s_R) such that (3.10) is satisfied. In the previous section, we have argued that these (s_C, s_R) pairs define the boundary of the region where the DME exists. These

(s_C, s_R) pairs satisfying (3.10) can, however, also be re-interpreted as partial squeezing DDE where $\gamma = 1$. From equation (3.14), (3.15) and (3.12) it is then clear, however, that if $(\rho_R, \rho_C, 1)$ is a solution for the parameters (m, n, s_C, s_R), then for any $\gamma \in (0, 1)$ it should be the case that (ρ_R, ρ_C, γ) is a solution for the parameters $(m, n, s_C^{'}, s_R^{'})$, where $s_C^{'} = \gamma s_C$ and $s_R^{'} = \gamma s_R$. This fact is used in the proof of the next proposition to argue that for small values of s_C and s_R, the partial squeezing DDE exists in the region in between the DME and the full squeezing DDE. Thus, at least one of these types of equilibria always exists. In addition, the next Proposition argues that, for a given set of parameters, the equilibrium is unique in the class of reservation price equilibria where both retailers and consumers follow reservation price strategies.

Proposition 3.5. *For any given set of parameter values m, n, s_C and s_R there exists a unique RPE that is either of the DM or the DD type.*

In the Introduction, we made some observations about the empirical relevance of Double Diamond Equilibria (DDEs) and about their welfare implications. We will now further detail these points.

Empirical Relevance

The price distribution emerging in a DDE provides a micro-foundation for several regularities of real-world consumer markets. First, the distribution of wholesale prices is bimodal, with a high (regular) price and a lower (sale) price. This pattern has been documented for a number of retail markets and is a salient feature of the Dominick's database (see Midrigan, 2011).

Second, DDEs exhibit a positive correlation between wholesale and retail prices, since retail price dispersion is due to manufacturers' incentives to randomize over two different prices. This pattern of contemporaneous correlation has not been documented before. We follow the methodology in Midrigan (2011) and construct a series of regular prices for each of the products sold in one of the supermarkets in Dominick's Database. A price is deemed regular for a given week if it is the modal price in a time-window around that date. Our model predicts that in a cross-section of products, conditional on the regular price, a higher realized cost (which proxies for wholesale price) induces a higher realized retail price. We then run a simple log-log regression on prices on costs, controlling for the regular price:

$$logp_{i,t} = \alpha + \beta logp_{i,t}^{R} + \gamma logc_{i,t} + \epsilon_{i,t}. \tag{3.16}$$

The results are shown in Table 1. As predicted by the model, cost increases are correlated with higher realized prices.[18]

Table 3.1: Estimation Results : Posted Prices, Regular Prices and Wholesale Prices

Variable	**Coefficient**	**(Std. Err.)**
ln regular price	0.901***	(0.000)
ln cost	0.096***	(0.000)
Intercept	0.014***	(0.000)

Third, our model predicts an incomplete pass-through from wholesale to final prices, independently of the shape of the demand curve. This prediction is also confirmed by our data. In particular, we run a simple fixed-effect regression of observed prices on costs:

$$p_{i,t} = \alpha + \beta c_{i,t} + \delta_t + \mu_i + \epsilon_{i,t} \tag{3.17}$$

The estimated coefficient is $0.515 < 1 (p < 0.001)$ (see Table 2). Therefore, an increase in wholesale prices reduces observed markups.

Table 3.2: Estimation Results : Pass-Through

Variable	**Coefficient**	**(Std. Err.)**
cost	0.515	(0.001)
date	0.001	(0.000)
Intercept	1.588	(0.002)
Product Fixed Effects	(Y)	

Welfare

DDEs yield very inefficient outcomes, and this inefficiency is largest in *competitive* conditions. Indeed, DDEs exist when s_C is small and n is relatively large (or s_R is not too

[18]We also run separate regressions for all 29 product categories in the sample and the estimated elasticities range from 0.03 in toothpastes to 0.34 in frozen dinners. Similar results are obtained from a set of category-specific Probit regressions and Linear Probability fixed effects on the probability of sales given costs.

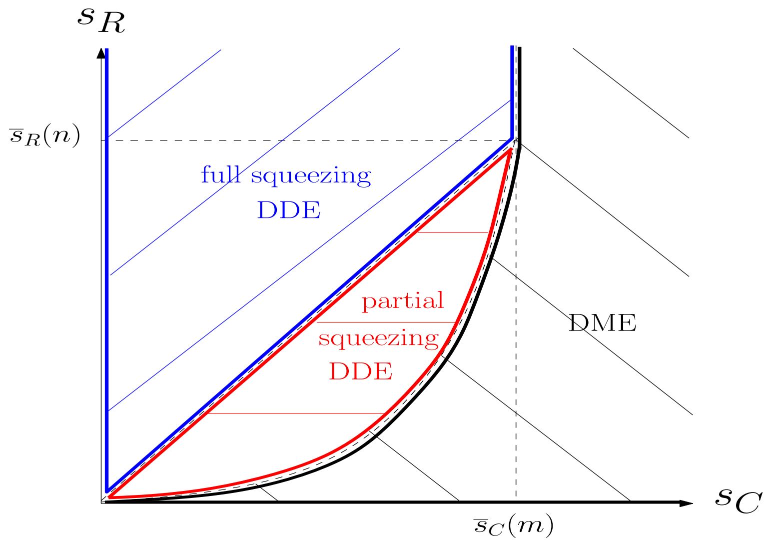

Figure 3.3: Equilibrium for Different Search Costs.

small). Further, as consumer search cost decreases, by (3.12), the equilibrium value of γ also decreases. In particular, when s_C goes to zero, γ converges to 0 and so the randomized market prices converge to $w = p = \rho_C$. Competitive forces, far from improving equilibrium outcomes, lead to equilibria with *lower* social welfare in markets with search frictions in *both layers* of the product chain. In particular, the social welfare loss becomes the largest when the consumer search cost approaches zero.

How much social welfare in the DDEs is lower than that in the DME depends on the shape of the demand function, the number of firms, and the level of search costs. We now illustrate the size of the effects by considering linear demand $D(p) = 1 - p$.

From the above, it follows that a full squeezing DDE exists if $s_C < \overline{s}_C(m) = \frac{4\sqrt{2}-5}{32}\frac{m-1}{m}$ and $s_R > \frac{2m}{(4\sqrt{2}-5)(m-1)n}s_C$. From (3.7) it follows that in this equilibrium $\rho_C = \frac{2+\sqrt{2}}{4}$ and $\gamma = \frac{s_C}{\frac{m}{m-1}\int_{p^m(w^m)}^{\rho_C} D(p)dp} = \frac{32}{4\sqrt{2}-5}\frac{m-1}{m}s_C$. The region where this equilibrium exists is also depicted in Figure 3.3. Total surplus and consumer surplus are decreasing in s_C. When s_C approaches 0, total and consumer surplus are at their lowest level and equal to approximately 0.135 and 0.01, respectively, which is around 38% and 66% lower than the corresponding figures for the double marginalization outcome, which is already known to generate low welfare levels. The remaining area in Figure 3.3 is where the partial squeezing DDE exists.

Figure 3.4 shows how total surplus and consumer surplus vary with changes in s_C and s_R for a given level of the other search cost. In the left panel we show the dependence on

s_C for $s_R = 0.016$ that is such that the equilibrium gradually changes from a DDE with full squeezing to one with partial squeezing and, finally for large enough values of s_C, to the DME. The right panel of Figure 3.4 shows a similar picture for the impact of s_R for a value of s_C equal to 0.005. Notice that the impact of search cost on welfare is large and that the two search costs have opposite effects: welfare is the lowest for large values of s_R and small values of s_C.

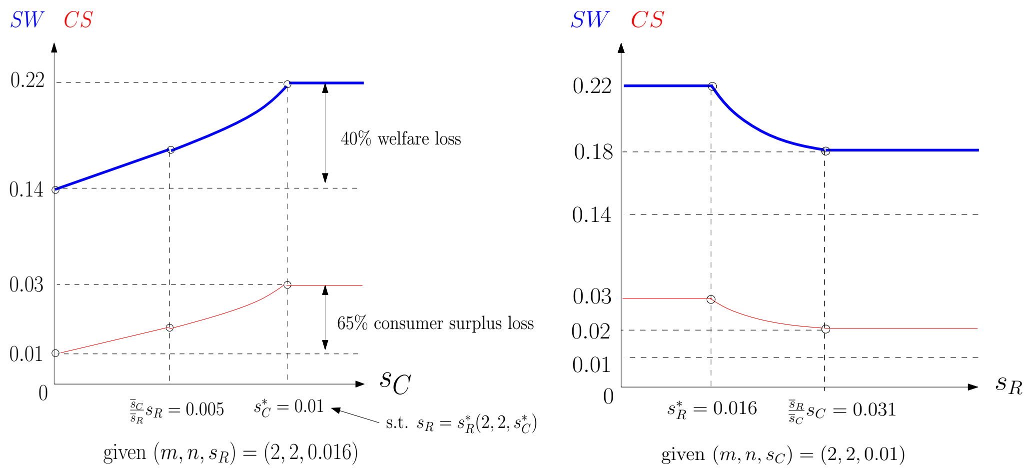

Figure 3.4: Welfare and Consumer Surplus as a Function of Search Costs.

The panels clearly show the discontinuity of total surplus and consumer surplus at a search cost equal to 0 and how the interaction between retail and consumer search severely affects this discontinuity. If both retailer and consumer search costs are equal to 0, we have marginal cost pricing and total surplus and consumer surplus being both equal to 0.5. With retail search costs being equal to 0 and consumer search costs approaching 0, we have the Diamond Paradox with monopoly pricing resulting in total surplus and consumer surplus being equal to 0.375 and 0.125, respectively. With both search costs approaching 0 at a ratio such that the full squeezing DDE prevails, we have a limiting total surplus and consumer surplus of 0.135 and 0.01.

To sum up, in our model, seemingly competitive conditions yield equilibria with large manufacturer's markups and substantial welfare losses. This theoretical prediction is consistent with recent evidence on the Chilean supermarket industry (Noton and Elberg, 2015). Using directly estimated manufacturer's costs, they find substantial markups for *all* coffee suppliers of big-box supermarkets.[19]

[19] The bulk of the literature *infers* manufacturer's costs from equilibrium pricing conditions. See, in

3.6 Non-Reservation Price Equilibria

Until now, we have considered reservation price equilibria (RPEs) where prices are such that retailers and consumers immediately buy at the first price they observe. There is no active search in these equilibria and the manufacturers' profits are always equal to the profit in the DM outcome. We will now show that for certain demand functions there exists another type of equilibrium with active search among consumers. This equilibrium is interesting for two reasons. First, by randomizing between the wholesale monopoly price and a higher price at which consumers continue to search with probability α, manufacturers make more profits than in a DME where they choose a pure strategy (or any other reservation price equilibrium). Manufacturers randomize in such a way that consumers have an incentive to search and by doing so, they increase the number of consumers who buy from them at the wholesale monopoly price. Second, the non-reservation price equilibrium Pareto-dominates the DDE.

We illustrate the possibility of this type of equilibrium by building on the simple structure of the full squeezing DDE where the manufacturer randomizes between w^m and a price $\widehat{w} > w^m$ with probabilities γ and $(1-\gamma)$, respectively, and the retailers buy at wholesale price $w \leq \widehat{w}$ and react by setting the retail monopoly price $p^m(w^m)$ and $\widehat{w}$.[20] Since the different search paths that can arise under an arbitrary number of retailers yield complicated expressions, we concentrate on the simplest case with two retailers, *i.e.*, $n = 2$. The main difference with the analysis so far is that consumers choose a non-reservation price strategy (cf., Janssen et al., 2014) where they buy at prices $p \leq \rho_C$, randomize between continuing to search and buying with probabilities α and $(1-\alpha)$ after observing $\widehat{w} > \rho_C$ and continue to search at all other prices.

To understand the nature of non-reservation price equilibrium (NRPE) in the present context, it is important to return to the fact that the reservation price may depend on out-of-equilibrium beliefs of the consumers regarding who they held responsible for a deviation. In Section 4 we have argued that if consumers blame retailers then the reservation price can be low. To get a NRPE, it has to be the case that the reservation price is off-the-equilibrium path and that consumers blame the retailer for the deviation.[21] Moreover, in

particular, Villas-Boas (2009).

[20]One could also build other non-reservation price equilibria, for example where retailers are partially squeezed, but our construction below is probably the simplest type of non-reservation price equilibrium. The purpose here is not to provide a complete characterization of all equilibria that may exist, but rather to illustrate that there may well be other types of equilibria than the one we focussed on so far.

[21]The first statement follows from the fact that ρ_C cannot be equal to $p^m(w^m)$, while if it were equal to $\widehat{w}$ both manufacturers and retailers would have an incentive to slightly undercut. The sec-

the present context, consumers can also hold beliefs about the wholesale price the deviating retailer has obtained. In particular, it is more likely that the retailer has deviated to a price $p \in (p^m, \widehat{w})$ after having bought at a wholesale price of w^m, rather than after a wholesale price of $\widehat{w}$ as in the latter case the retailer would make a loss by selling at a price below cost. Thus, the consumer believes that the deviating retailer bought at a wholesale price of w^m so that the pay-off of continuing to search equals

$$\frac{1}{m}\int_{p^m(w^m)}^{\infty} D(p)dp + \frac{m-1}{m}\left[(1-\gamma)\int_{\rho_C}^{\infty} D(p)dp + \gamma\int_{p^m(w^m)}^{\infty} D(p)dp\right] - s_C.$$

which (importantly) is larger than the continuation pay-off if they had searched after observing $\widehat{w}$.[22] Thus, the consumer reservation price ρ_C such that the consumer buys at all $p \leq \rho_C$ equals

$$\frac{1+(m-1)\gamma}{m}\int_{p^m(w^m)}^{\rho_C} D(p)dp = s_C. \tag{3.18}$$

To make this into an equilibrium, we need three conditions to hold along the equilibrium path. First, the manufacturer should be indifferent between charging the two different prices. If she charges w^m, she will get the following demand. With probability $\left(\frac{1}{m}\right)^2$ both retailers will visit her and all consumers will buy. With probability $2\left(\frac{1}{m}\right)\left(1-\frac{1}{m}\right)$ exactly one of the retailers will visit her and then a fraction of $\frac{1}{2}(1+\alpha(1-\gamma))$ of all consumers will buy from this retailer. If she charges $\widehat{w}$, the only difference is when exactly one retailer is visiting her, in which case a fraction of $\frac{1}{2}(1-\alpha\gamma)$ of all consumers will buy from this retailer. Thus, the indifference condition for the manufacturer is

$$(m+(m-1)\alpha(1-\gamma))w^m D(p^m(w^m)) = (m-(m-1)\alpha\gamma)\widehat{w}D(\widehat{w}). \tag{3.19}$$

The second condition requires that at the high price $\widehat{w}$ consumers are indifferent between searching and buying. As before, the condition is given by

$$\frac{m-1}{m}\int_{p^m(w^m)}^{\widehat{w}} D(p)dp = \frac{s_C}{\gamma}.$$

The third condition is that retailers should not continue to search after observing $\widehat{w}$. If

ond statement follows from the fact that in this case the reservation price would be implicitly defined by $\frac{2s_C}{\gamma} = \int_{p^m(w^m)}^{\rho_C} D(p)dp$ so that in fact $\rho_C = \widehat{w}$ and the previous argument cuts in.

[22] One could go one step further and say that it is more likely that the retailer has deviated to $\widehat{w} - \varepsilon$ after having observed a manufacturer price of ω^m, then after a manufacturer price of $\widehat{w}$–as in the latter case it makes a loss.

a retailer deviates and continues to search, then there is a probability γ that he will find a lower price at one of the other manufacturers. In that case, he sells to all consumers who visit him first. In addition, due to active consumer search, he can sell to some other consumers who visit the other retailer first, depending on the manufacturer the other retailer visited first. Basically, there are three kinds of manufacturers to be considered: (i) the same manufacturer that he visited first before continuing to search (which happens with probability $\frac{1}{m}$); (ii) the same manufacturer that he visited after continuing to search (which happens with probability $\frac{1}{m}$); (iii) one of the other manufacturers (which happens with probability $1-\frac{2}{m}$). Since some consumers who first visited the other retailer continue to search and visit him in cases (i) and (iii) but not in case (ii), continuing to search gives him the pay-off of

$$\gamma\left(\frac{1}{m}\frac{1+\alpha}{2}+\frac{1}{m}\frac{1}{2}+\frac{m-2}{m}\left[\frac{\gamma}{2}+(1-\gamma)\frac{1+\alpha}{2}\right]\right)\left(p^m(w^m)-w^m\right)D(p^m(w^m)).$$

As they get a profit of 0 if they buy, the retailers' condition not to continue to search reduces to:

$$\frac{\gamma}{2m}\left(m+\alpha\left[1+(m-2)\left(1-\gamma\right)\right]\right)\left(p^m(w^m)-w^m\right)D(p^m(w^m))<s_R. \tag{3.20}$$

These are the conditions that should hold on the equilibrium path when the manufacturer chooses w^m or $\widehat{w}$. To see when it is not optimal for a manufacturer to deviate, we have first to develop the optimal strategy of retailers given such a deviation, and then to consider the optimal deviation profit of a manufacturer. The analysis around these considerations is provided in the Appendix, where we show that for some demand functions, most notably for linear demand, all conditions for a NRPE to exist are satisfied.

Proposition 3.6. *In a vertical industry structure with two retailers, there exists parameter values m, s_C and s_R and a demand function $D(p)$ such that a NRPE exists. Manufacturers make more profit than in any RPE. If a NPRE exists, then generically, there is a continuum of them.*

As only two conditions have to hold with equality and there are three "free parameters" that are determined endogenously, namely α, γ and $\widehat{w}$, it is clear that if there exists an equilibrium for given parameter values such that all inequalities hold strictly, then there must also be another equilibrium in the neighbourhood of these parameter values. That is why generically, there is a continuum of non-reservation price equilibria (NRPEs) if there exists at least one.

We can show the following relationship between a NRPE and a fully squeezing DDE.

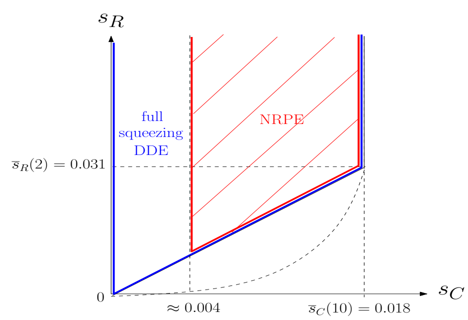

Figure 3.5: An Approximated Parameter Region of the NRPEs given $m = 10$.

Proposition 3.7. *If a NRPE exists, it coexists with and is Pareto-superior to the full squeezing DDE.*

The intuition why the NRPE is Pareto-superior to the fully squeezing DDE is the following. In the NRPE, manufacturers induce consumers to actively search by randomizing wholesale prices. Active search increases the expected profits of manufacturers. In addition, manufacturers set the wholesale monopoly price with a higher probability than in the fully squeezing DDE, i.e., denoting γ^{non} (resp. γ^{DD}) the probability with which the wholesale monopoly price is set in the two respective equilibria, we have $\gamma^{non} > \gamma^{DD}$. Thus, since retailers are fully squeezed at a high wholesale price in both equilibria, the higher probability to get the wholesale monopoly price gives them higher expected profits. Furthermore, since retailers are more likely to set the retail monopoly price, consumers also receive a higher surplus. Note that Proposition 3.7 can be extended to the general case of multiple manufacturers and multiple retailers.

To illustrate these two Propositions, a NRPE exists for the linear demand function $D(p) = 1 - p$ and parameter values $(s_C, m) = (0.009, 10)$ and $s_R \geq 0.016$. For these parameters, there is an equilibrium where $(\gamma, \alpha, \widehat{w}) \approx (0.5, 0.022, 0.85)$ and where $\rho_C = 0.828 (< \overline{w} \approx 0.85)$. For these parameter values a fully squeezing DDE also exists. The parameter region under which the NRPE exists given $m = 10$ is depicted in Figure 3.5.

Table 3.3 summarizes the expected profit for the manufacturers and the retailers and consumer surplus for both NRPE and full squeezing DDE in comparison to the corresponding values in the DM outcome. Each value is expressed as a percentage of the corresponding value in the DM outcome. Note that per Proposition 6, if a NRPE exists, there is a continuum of them, which exists in a neighbourhood of the full squeezing DDE. It is easily verified that all NRPE constitute a Pareto improvement over the full squeezing DDE.

Table 3.3: Welfare Comparison across Equilibria.

	Full Squeezing DDE	NRPE
Manufacturer's Profits	100%	(100%,102.1%]
Retailers' Profits	48.7%	(48.7%,52.9%]
Consumer Surplus	66.3%	(66.3%,70.2%]
Social Welfare	80.5%	(80.5%,83.2%]

3.7 Search Cost Heterogeneity

So far, we have assumed that all consumers have the same search cost and also that all retailers have the same search cost. In this extension, we consider to what extent our results can be generalized to allow for search cost heterogeneity at both levels. It is not our intention to provide a full analysis under search cost heterogeneity, as it is clear that this is beyond the scope of this paper. Rather, we want to demonstrate that the two types of equilibria (DME and DDE) we have discussed in this paper continue to exist if there is some search cost heterogeneity.

For the DME to exist, search costs (s_R,s_C) have to be large enough so that both manufacturers and retailers set their respective DM prices and make profits equal to $\frac{1}{m}w^m D(p^m(w^m))$ and $\frac{1}{n}(p^m(w^m)) - w^m)D(p^m(w^m))$, respectively. Now, if any consumer i faces a sufficiently high search cost ($s_{C_i} \geq \overline{s}_C(m)$), then regardless of the search cost distribution of retailers, DM remains an equilibrium. On the other hand, whenever some consumers have a search cost s_C smaller than $\overline{s}_C(m)$, the DME may continue to exist, but the analysis is somewhat more subtle and it is clear that whether or not the DME exists depends on the search cost distribution of both consumers and retailers.

Consider then the possibility of a DDE existing under search cost heterogeneity. Fix a pair $(s_R^{'}, s_C^{'})$ such that a DDE exists if all retailers and consumers had the same search cost. Consider then a search cost distribution such that all consumers have a search cost larger than $s_C^{'}$ and all retailers have a search cost larger than $s_R^{'}$. Given prices, these search cost distributions translate into distributions of reservation prices ρ_C and ρ_R respectively. Denote these distributions by $F_C(\rho_C)$ and $F_R(\rho_R)$ with lower bounds of the support given by $\rho_C^{'}$ and $\rho_R^{'}$ respectively (corresponding to $s_C^{'}$ and $s_R^{'}$). In a partial squeezing DDE whereby all retailers, resp. all consumers, have the same search cost $s_R^{'}$, resp. $s_C^{'}$, manufacturers make a profit equal to $\rho_R^{'}\frac{D(\rho_C^{'})}{m}$, while retailers' profit equals $(\rho_C^{'} - \rho_R^{'})\frac{D(\rho_C^{'})}{n}$ if they buy at $\rho_R^{'}$ and sell at $\rho_C^{'}$. A full squeezing DDE is a special case where $\rho_C^{'} = \rho_R^{'}$. We shall now inquire whether under search cost heterogeneity it is possible that we have an identical equilibrium outcome with manufacturers and retailers setting the same prices $\rho_R^{'}$ and $\rho_C^{'}$ and all retailers and consumers buying immediately. First, it is clear that consumers will not change their behaviour and will keep on buying immediately as their search costs are not smaller than $s_C^{'}$. Second, retailers will also continue buying, but they may set a different price. For a given wholesale price $\rho_R^{'}$, their profit $\pi_R(p)$ of setting a price p different from $\rho_C^{'}$ (given that their competitors choose $\rho_C^{'}$ or $p^m(w^m)$) is $\pi_R(p) = (1 - F_C(p))(p - \rho_R^{'})\frac{D(p)}{n}$, with $F_C(p) = 0$ for $p < \rho_C^{'}$. To see this, note that if a retailer sets a price $p < \rho_C^{'}$ all consumers will continue to buy, while if he sets a price $p > \rho_C^{'}$ only a fraction $1 - F_C(p)$ will buy, namely those consumers who have a reservation price $\rho_C > p$. It is then clear that it is certainly not optimal to set a price smaller than $\rho_C^{'}$, while it is *not* optimal to increase the price above $\rho_C^{'}$ if $\partial\pi_R(p)/\partial p < 0$, or in other words, if

$$(1 - F_C(p))\left[(p - \rho_R^{'})\frac{\partial D(p)}{\partial p} + D(p)\right] - f_C(p)(p - \rho_R^{'})D(p) < 0 \quad \text{for } p \geq \rho_C^{'}.$$

If $\rho_C^{'} = \rho_R^{'}$, i.e., if we are in a full squeezing DDE, then this condition should also hold at $p = \rho_C^{'}$. However, this condition is never met at $p = \rho_R^{'} = \rho_C^{'}$. Thus, any full-squeezing DDE cannot exist under consumer search cost heterogeneity as retailers will have an incentive to deviate and raise prices above the wholesale price $\rho_R^{'}$. In a partial squeezing DDE, however, the condition can be fulfilled if the hazard rate of the consumer reservation price distribution satisfies

$$\frac{f(p)}{1 - F(p)} > \frac{(p - \rho_R^{'})\frac{\partial D(p)}{\partial p} + D(p)}{(p - \rho_R^{'})D(p)}.$$

It is clear that this condition is more easily satisfied the larger the retailers' margin.

Finally, we need to check whether manufacturers want to deviate. For given strategies of retailers, consumers and other manufacturers, a manufacturer's profit $\pi_M(w)$ of setting a price w different from $\rho_R^{'}$ (given that their competitors choose $\rho_R^{'}$ or w^m) equals $\pi_M(w) = (1 - F_R(w))w\frac{D(\rho_C^{'})}{m}$, with $F_R(w) = 0$ for $w < \rho_R^{'}$. The interpretation is similar to what we had above for the retailer. Thus, it is clearly not optimal to set a price smaller than $\rho_R^{'}$, while it is not optimal to increase the price above $\rho_R^{'}$ if $\partial\pi_M(w)/\partial w < 0$, or in other words, if

$$(1 - F_R(w)) - f_R(w)w < 0,$$

for all $w > \rho_R^{'}$, which is a condition on the hazard rate of the retailers' reservation price distribution.

Thus, we conclude that both the DME and the DDE where retailers are partially squeezed do not depend on all retailers and all consumers having one and the same search cost. These equilibria also exist when the search cost heterogeneity satisfies some distributional requirements that are characterized above in terms of distributional requirements on the respective reservation prices.

3.8 Discussion and Conclusion

This is the first paper to consider costly retailer search. We show that in conjunction with costly consumer search, retailer search has important implications for the functioning of markets. From a welfare perspective, our main result is that markets with small, but positive consumer search cost will produce outcomes that are not better, and often much worse, than the double marginalization outcome. This is especially true when there are many manufacturers and many retailers. Interestingly, the lower bound on retailer search cost for which this is true converges to zero, when the consumer search cost approaches zero or when the number of retailers becomes large. That is, circumstances that we often think of as favouring competitive outcomes may actually generate very low welfare levels. This is true even if there is some heterogeneity in consumer search cost. The discontinuity that is at the heart of the Diamond Paradox is considerably strengthened when we consider the interaction between manufacturers and retailers in search markets.

The welfare results of our paper are surprising and disturbing. As already mentioned, the direct evidence on the side of manufacturer's margins in vertically related industries is very scarce. However, the recent paper by Noton and Elberg (2015) suggests that markups for producers of fairly homogenous goods are substantial. Further, notice that even if we had evidence suggesting that prices are close to marginal cost at the retail level, manufacturer margins may be substantial. Indeed, retail margins in the Double Diamond Equilibria are lower than those in the Double Marginalization Equilibrium, but the welfare generated by the Double Diamond Equilibria is much lower.

From an empirical point of view, our paper generates a new and interesting way to explain retail price distributions. In a given store, the distribution of prices for a given product tends to be concentrated around two price levels: a regular price and a sales price. Existing models of price dispersion typically fail to generate this bimodal nature of the (retail) price distribution. Our model also suggests that it is manufacturers who induce sales by offering discounts to retailers. While there is surprisingly little empirical research on the interaction between retail and wholesale prices, we have provided some suggestive evidence that these decisions are very much related. These results draw attention to the importance of studying price fluctuations at both levels simultaneously and call for a more ambitious empirical study of pricing along the vertical product chain with search frictions.

Our paper also points in the direction of taking search frictions seriously when doing horizontal merger analysis at both the retail and wholesale level. More concentration at the retail level potentially gives retailers more incentive to continue to search for better price offers from manufacturers. Markets with relatively low consumer search costs tend to end up less likely in a Double Diamond Equilibrium if the number of retailers is small. Also, the number of manufacturers may affect consumer reservation prices, as with fewer manufacturers it is more likely that alternative retailers will be supplied by the same manufacturer, making it less beneficial for consumers to search and, therefore, not optimal for manufacturers to deviate. It should be noted, though, that our results are derived in a model where it is the manufacturer that has the power to propose the terms of trade with retailers. It would be interesting to reverse this power and to investigate how our conclusions change when retailers have the power to determine wholesale conditions (as is certainly true in some markets).

We have focused on a simple vertical industry with linear pricing contracts between independent firms. It is not difficult to see that if the first search is costly, allowing for two-part tariffs will result in a market break down as the manufacturers cannot commit to a contract where retailers make a profit. In a sense, this would strengthen our result of the inefficiency of the market, but one could also argue that sequential search models are not well-suited to studying non-linear contracts.[23] Allowing for vertical integration provides firms with incentives to charge simple monopoly prices at the retail level. The reason not to consider vertical integration is that there are many markets where retailers carry products of many different manufacturers, creating barriers for vertical integration. When retailers carry many different products, multi-product search is important, and it would be interesting to combine the analysis of Rhodes (forthcoming) and Zhou (2014) with ours.

3.9 Appendix: Proofs

Proof of Proposition 3.1

Consider an arbitrary symmetric (possibly mixed) equilibrium strategy profile of manufacturers, retailers, and consumers. Let $\underline{w}$ be the lower bound of the wholesale price distribution. Without loss of generality, we assume that given any symmetric strategies

[23] If the first search is costless, two-part tariffs would give rise to double marginalization. However, this would create high risks for the company that is supposed to pay the high fixed fee. These companies may not be willing to bear these risks in a world where there is high uncertainty about demand.

of manufacturers and retailers, there exists a maximum price ρ_C such that for any retail price that is lower than ρ_C, consumers do not continue to search and buy at that price.

First of all, we show that if retailers observe the minimum wholesale price $\underline{w} \leq \rho_C$, it is optimal for retailers to set the optimal retail price $p^m(\underline{w})$ because consumers would not search at that price. This corresponds to Lemma 3.1 below. Next, given Lemma 3.1, we show that if $\underline{w} < w^m$, manufacturers have an incentive to unilaterally deviate from $\underline{w}$ to a slightly higher wholesale price. This corresponds to Lemma 3.2 below. Thus, Lemmas 3.1 and 3.2 imply that $w \geq w^m$ (resp. $p \geq p^m(w^m)$) holds in any equilibrium strategy profile.

Lemma 3.1. *Assume that manufacturers adopt a symmetric strategy whereby they put positive probability on a price $\underline{w} \in (0, w^m)$. If a retailer visits a manufacturer setting $\underline{w}$, it is optimal for the retailer to buy the product and set the retail price at $p^m(\underline{w})$.*

Proof. First of all notice that in such a strategy profile, $\rho_C > p(\underline{w})$, since consumers should be compensated for their search costs. Therefore, for any price in $(p(\underline{w}), \rho_C)$ retailers' profits are just proportional to per-consumer profits. Since per-consumer profits are single-peaked, if $p(\underline{w}) < p^m(\underline{w})$, increasing the price leads to higher profits. On the other hand, any $p(\underline{w}) > p^m(\underline{w})$ is dominated. Thus, $p(\underline{w}) = p^m(\underline{w})$. □

Lemma 3.2. *In any equilibrium, $\underline{w} \geq w^m$.*

Proof. Assume to the contrary that there exists a symmetric equilibrium such that the lower bound of the wholesale prices in the strategy of manufacturers is $\underline{w} \in [0, w^m)$. Notice that whenever retailers visit a manufacturer setting the minimum wholesale price $\underline{w}$, they buy the product from the manufacturer and set the retail price $p^m(\underline{w}) < \rho_C$ to consumers,[24] and then consumers visiting the retailers buy the product.

Suppose that a manufacturer deviates from $\underline{w}$ to a slightly higher wholesale price w that is close enough to $\underline{w}$. Since w is sufficiently close to $\underline{w}$, $p^m(w) < \rho_C$ holds. Since $\underline{w} < w < w^m$ and $p^m(\underline{w}) < p^m(w) < \rho_C$ hold, by single-peakedness of the profit function, it follows that $\underline{w}D(p^m(\underline{w})) < w(p^m(w))$. Thus, the deviating manufacturer can slightly increase the profit by setting a slightly higher wholesale price w

Since we can repeat the above argument until the minimum wholesale price $\underline{w}$ reaches the wholesale monopoly price w^m, it implies that manufacturers never set a wholesale price $\underline{w} \in [0, w^m)$. □

[24] The inequality follows because $p^m(\underline{w})$ is the minimum retail price and consumers have a positive search cost.

Proof of Proposition 3.2

By Proposition 3.1, we do not need to consider a case where a manufacturer sets some wholesale price $w < w^m$ with some probability.

Assume to the contrary that there exists a symmetric equilibrium such that the lower bound of the wholesale prices in the strategy of manufacturers is $\underline{w} > w^m$. When a manufacturer sets $\underline{w}$, a visiting retailer buys the product at $\underline{w}$ and sells the product to consumers at $p^m(\underline{w})$ due to Lemma 3.1, and then a visiting consumer buys the product at $p^m(\underline{w})$ because of $p^m(\underline{w}) < \rho_C$. Given this, each manufacturer has an incentive to deviate to the wholesale monopoly price w^m for which visiting retailers buy and sell at the retail monopoly price $p^m(w^m)$, and then visiting consumers buy at $p^m(w^m)$. This gives the deviating manufacturer the profit given by $w^m D(p^m(w^m))$ that is larger than the profit under $\underline{w}$ given by $\underline{w} D(p^m(\underline{w}))$ due to single-peakedness of the profit function. Thus, the deviation is profitable, a contradiction.

Next, we show that manufacturers do not charge $w' = w^m + \varepsilon$ for a sufficiently small $\varepsilon > 0$. From above, we know that manufacturers set w^m with some positive probability. Given this, assume to the contrary that there exists a symmetric equilibrium such that manufacturers charge a slightly higher wholesale price $w' = w^m + \varepsilon$ with a positive probability for a sufficiently small $\varepsilon > 0$. Since w' is sufficiently close to w^m, a visiting retailer buys the product at w' without continuing to search due to search cost $s_R > 0$ and sells to consumers at $p^m(w')$ that is a little bit above $p^m(w^m)$ because consumers visiting the retailer buy the product at $p^m(w')$ without continuing to search due to search cost $s_C > 0$. From above, the manufacturer setting w' gets the profit given by $w' D(p^m(w'))$ that is smaller than the profit under w^m given by $w^m D(p^m(w^m))$. Thus, each manufacturer has an incentive to deviate to w^m, a contradiction.

Proof of Proposition 3.5

We first restate the Proposition in terms of admissible beliefs. As explained in the text, we shall impose the Minimum Deviation Property. See Appendix B for details.

Lemma 3.3. *There exists a unique RPE Outcome for any belief system satisfying the Minimal Deviation Property.*

Proof. First, in a RPE, there cannot be active search. We show this for the case of consumers, but the same argument works for retailers. For a contradiction, assume that there exists a set of equilibrium retail prices P such that consumers search at P and let $\bar{p}$

be the maximum of those prices. Clearly, by the reservation price property, $\bar{p}$ is also the maximum equilibrium price. Hence, demand at $\bar{p}$ is zero. Hence, either the wholesale price is also $\bar{p}$ or the retailer could profitably deviate to a lower price. But a wholesale price of $\bar{p}$ cannot be optimal for a manufacturer who could charge the double marginalization price. Thus, there is no active search. We are left with two cases to consider. First, if there is price dispersion in equilibrium, both retailers' and consumers' reservation prices are pinned down uniquely in any belief system satisfying the MDP. Since consumers' demand is $D(p)$ for any $p \le \rho_C$, the retailers' optimal price is $\max\{p^m(w), \rho_C\}$ if $w \le \rho_R$, $p = w$ otherwise (provided that $w \le \rho_R$ holds and retailers do not continue to search). This implies that retailers' demand is proportional to $D(p^m(w))$ for any $w < \min\{\rho_R, \rho_C\}$. Clearly, if $w < \min\{\rho_R, \rho_C\}$, then $p(w) = p^m(w)$. Otherwise, $p(w) = \min\{\rho_R, \rho_C\}$. Second, if there is no price dispersion, Proposition 1 implies that the unique equilibrium outcome is the DME outcome. Next, notice that DM is an equilibrium outcome, if and only if, there is no profitable deviation for a manufacturer to charge a higher price than the wholesale monopoly price. Hence, if there exists a pure-strategy equilibrium there does not exist an equilibrium with price dispersion and vice versa. Hence, a RPE is unique. □

Proof of Proposition 3.6

We specify strategies for all the players and then verify that they constitute an equilibrium.

Manufacturers randomize between the wholesale monopoly price w^m and a higher price $\hat{w}$ with probabilities γ and $1 - \gamma$, respectively. Retailers buy and choose $p^m(w)$ if the manufacturer they have visited sets w such that $p^m(w) \le \rho_C$, and choose either ρ_C or $\hat{w}$ (to be determined below) if the manufacturer they visit sets a price w such that $p^m(w) > \rho_C$. Finally, consumers buy at any price $p \le \rho_C$, randomize whether to buy or continue their search if they observe a price $p = \hat{w}$ (so that $1 - \alpha$ is the probability with which they buy) and always continue their search at a price $p > \hat{w}$. If the consumer has visited both manufacturers and both of them have a price $p = \hat{w}$, the consumer simply buys at the last store she visited.

We next have to make sure that deviations are not profitable. Given the retailer strategy, it is clear that if it is optimal for the manufacturer to deviate to a price $w \in (w^m, \hat{w})$ then it must be the case that retailers prefer to set a price ρ_C rather than $\hat{w}$, for otherwise manufacturers could obtain higher profits but setting $\hat{w}$ themselves. Suppose that the other retailer would also choose ρ_C if she meets a manufacturer charging w. Their profit by setting ρ_C is

$$\left(\frac{1}{2}+\frac{m-1}{m}\frac{(1-\gamma)\alpha}{2}\right)(\rho_C-w)D(\rho_C).$$

If one of them chooses $\widehat{w}$ she makes a profit of

$$\left(\frac{1-\alpha}{2}+\frac{m-1}{m}\frac{(1-\gamma)\alpha}{2}\right)(\widehat{w}-w)D(\widehat{w}).$$

These expressions can be understood as follows. If the retailer chooses ρ_C, half of the consumers who first visit him buy immediately, while of the remaining consumers who first visit the other retailer, only a fraction α will come back if this retailer has visited another manufacturer (which happens with probability $\frac{m-1}{m}$) and this manufacturer chooses $\widehat{w}$ (which happens with probability $1-\gamma$). The second expression is obtained by considering that (i) half of the consumers first visit him and buy with probability $1/2$, (ii) if the other retailer has visited another manufacturer (which happens with probability $\frac{m-1}{m}$, he gets half of the consumers who continue to search if the other manufacturer sets the wholesale price $\widehat{w}$.

Thus, the retailer is indifferent between setting ρ_C and $\widehat{w}$ (given that the other retailer chooses ρ_C) if the manufacturer chooses the price $\widetilde{w}$ that is given by

$$\widetilde{w}=\frac{\big(m+(m-1)(1-\gamma)\alpha\big)\rho_C D(\rho_C)-\big(m+\alpha-(m-1)\gamma\alpha\big)\widehat{w}D(\widehat{w})}{\big(m+(m-1)(1-\gamma)\alpha\big)D(\rho_C)-\big(m+\alpha-(m-1)\gamma\alpha\big)D(\widehat{w})}. \tag{3.21}$$

Also, it must be the case that the retailer does not want to just undercut $\widehat{w}$ after a manufacturer has chosen some w. Undercutting $\widehat{w}$ yields a profit of almost

$$\Big(\frac{1+\alpha}{2m}+\frac{m-1}{2m}(1-\gamma)(1+\alpha)\Big)(\widehat{w}-w)D(\widehat{w}).$$

This expression is understood as follows. If the retailer chooses $\hat{w}-\varepsilon$ for a sufficiently small $\varepsilon>0$, all of the consumers who first visit him continue to search because $\hat{w}-\varepsilon$ is an unexpectedly high price above the reservation price ρ_C, but they may come back, depending on the other retailer's price.[25] If the other retailer has visited the same manufacturer and sets $\widehat{w}$, which happens with probability $\frac{1}{m}$, all of the consumers who first visited him come back after continuing to search and buy, because he offers a slightly lower price than the

[25]Notice that when observing $p\in(\rho_C,\hat{w})$ off the equilibrium path, consumers continue to search, though they buy at $p=\hat{w}(>\rho_C)$ with probability $1-\alpha$ on the equilibrium path.

other retailer. In addition, a fraction α of the consumers who first visit the other retailer continues to search and buy from the retailer under consideration. The same argument holds if the other retailer has visited another manufacturer.

Thus, the retail profit of undercutting $\widehat{w}$ is smaller than the retail profit of setting $\widehat{w}$ if

$$m\alpha < (m-1)\gamma. \tag{3.22}$$

From above, for prices $w \leq \widetilde{w}$, the sequential rationality of the retailer requires him to buy and set a price equal to ρ_C, while for wholesale prices $w > \widetilde{w}$ he will react by buying and setting $\widehat{w}$. This fully determines the retail strategy.

Given this strategy, the best profitable deviation for a manufacturer is to set the price $\widetilde{w}$. At lower wholesale prices retailers will react by choosing $\min\{\rho_C, p^m(w)\}$, but it is clearly not optimal to set w such that $p^m(w) < \rho_C$, while for all w such that the retailer sets ρ_C it is best to squeeze the retailer as much as possible. Finally, if the retailer anyway reacts by choosing $\widehat{w}$ it is better to choose $\widehat{w}$ as well. Since retailers sell to the same fraction of consumers at the retail price $p^m(w^m)$, deviating to $\widetilde{w}$ is not profitable if

$$\widetilde{w}D(\rho_C) < w^m D(p^m(w^m)). \tag{3.23}$$

These conditions characterize the equilibrium for arbitrary demand function. We now show by example that such conditions can be met. Suppose that demand is linear $D(p) = 1-p$ and parameter values $(s_C, m) = (0.009, 10)$ and $s_R \geq 0.016$, there exists an NRPE where $(\gamma, \alpha, \widehat{w}) \approx (0.5, 0.021, 0.75)$ with $\rho_C = 0.828$. Moreover, it is easily verified that if all conditions are satisfied for a certain α^*, they are satisfied for any $\alpha < \alpha^*$. This establishes the second part of the Proposition.

Proof of Proposition 3.7

Proof. Assume that a NRPE exists for some parameter constellation. We first show that there also exists a DDE. By (3.19), $\hat{w} < \rho_C^{DD}$ should hold. Given (m, s_C), from (3.12) and (3.18) it follows that $\gamma^{non} > \gamma^{DD}$. In addition, if (3.21) holds we have that

$$\gamma^{DD}\frac{1}{2}(p^m(w^m)-w^m)D(p^m(w^m)) < \gamma^{non}\left(\frac{1}{2}+\frac{\alpha(1+(m-2)(1-\gamma))}{2m}\right)(p^m(w^m)-w^m)D(p^m(w^m)). \tag{3.24}$$

Hence, if an NRPE exists, a DDE also exists.

Next, we show that the NRPE is Pareto-superior to the corresponding DDE. By construction, manufacturers make more profits, both at the lower price (since there is more demand) and at the higher price (by randomization). Thus, the higher price in an NRPE is lower than in the corresponding DDE. Since consumers are indifferent whether to search or not at the higher price in *both equilibria* it must be that $\gamma^{non} > \gamma^{DD}$. Hence, they are better off ex-ante. Finally, in both equilibria retailers are fully squeezed if they visit a manufacturer with the highest price and get the same margin if they observe the lower price $p^m(w^m)$. Thus, since $\gamma^{non} > \gamma^{DD}$ and their expected demand at the lower price is higher, they make higher profits. $\square$

3.10 Appendix 2: Minimal Deviation Property (Not for Publication)

Let I be an information set. Let $\mathcal{I}$ be the set of information sets and given an equilibrium σ^*, let $\mathcal{I}*$ be the set of on-the-equilibrium path information sets. For every $I \notin \mathcal{I}*$, let $A(I)$ be the set of action profiles that lead to I. For each $a \in A(I)$, let $m(a)$ be the number of actions that do not belong to the support of the equilibrium strategy profile σ^*. Let $m^*(I) = \min_{a \in A(I)} m(a)$. Notice that $m^*(I) \geq 1$.

Definition 3.4 (Minimal Deviation Property)**.** A belief system β at information set I satisfies the Minimal Deviation Property (MDP) if and only if $\beta(a; I) > 0$ if and only if $m^*(I) = m(a)$ and $a \in A(I)$.

In words, a belief system satisfies MDP if at every information set, players put positive probability on paths that lead to that information set and that contains the minimal number of deviations from equilibrium strategies.

Remark 3.1. Passive beliefs satisfy MDP but the converse is not true.

Remark 3.2. In Vertical Contracting Games (e.g. McAfee and Schwartz, 1994), MDP is equivalent to Passive Beliefs. In **?** MDP is consistent both with Passive and with Symmetric Beliefs (regarding retailer prices) but not with Wary Beliefs.

Chapter 4

Games with the Total Bandwagon Property

4.1 Introduction

Bandwagon effect is a form of groupthinking in social psychology which says that as more people adopt a belief or an action, others are more likely to do the same thing. We can observe it in markets through fads or trends, such as consumer product choices and customs.[1] This concept is explicitly introduced by Leibenstein (1950) into economics for consumer demand theory, and has been further investigated theoretically and empirically.[2] Here we consider a related but stronger concept, the total bandwagon property (TBP), which is introduced by Kandori and Rob (1998) where TBP is used as a bandwagon effect regarding consumer technology adoptions in an evolutionary context as follows. There is a society consisting of a single population, and each consumer in the society observes a product choice profile taken by all other consumers in the last period and chooses one of products used by some other people as a best response. Formally, TBP is the property imposed on the class of symmetric two-player games under which all best responses against

[1] The bandwagon effect can be interpreted as a network externality and is related to conformity, herd behavior, information cascade, and so on. Note that we can also see the opposite effect known as snob effect that when many people adopt something, a person avoids to have or be associated with the same thing. Exclusive products, such as designer clothing and rare artworks, are typical examples.

[2] For instance, see Granovetter and Soong (1986), Becker (1991), Bikhchandani et al. (1992), Karni and Levin (1994), and Pesendorfer (1995) as related theoretical research; Biddle (1991) and McAllister and Studlar (1991) as empirical research; Plott and Smith (1999) as experimental research on markets. See Rohlfs (2001) for a comprehensive analysis on bandwagon effect in high-tech industries.

any mixed strategy are in the support of this mixed strategy. Our main purpose is to unveil hidden sides of the above mentioned bandwagon effect on games.

We first show a characterization of games with TBP via the number of Nash equilibria: A symmetric $n \times n$ game has TBP if and only if the game has $2^n - 1$ symmetric Nash equilibria. Furthermore, by considering the generalized TBP to allow for asymmetric games, we extend the characterization to bimatrix games. This characterization result suggests that a game with the bandwagon property may have so many Nash equilibria that it is hard to select a single equilibrium. Given no observed history of actions taken in society, when choosing an action, agents cannot have clear selection criteria on which equilibrium is chosen.[3] With this in mind, the second objective of our paper is to provide a simple equilibrium selection criterion. Such a simple but strong equilibrium selection criterion is the solution concept of 1/2–*dominant equilibrium* proposed by Morris, Rob and Shin (1995), which is a generalization of *risk dominant equilibrium* (Harsanyi and Selten, 1988). It is chosen by various equilibrium selection methods including the "evolutionary learning method" based on the best response dynamics with mutation (Kandori et al., 1993; Young, 1993); the "global game method" (Carlsson and van Damme, 1993); the "incomplete information game method" (Kajii and Morris, 1997); the "perfect foresight dynamics method" (Matsui and Matsuyama, 1995; Hofbauer and Sorger, 1999); the "spatial dominance method" (Hofbauer, 1999). A 1/2–dominant equilibrium needs not to exist in games with TBP. I show the existence of a 1/2–dominant equilibrium for two subclasses of games with TBP.

One of them is the class of supermodular games. Supermodularity (strategic complementarity) has been considered to be important in economics (Milgrom and Roberts, 1990; Milgrom and Shannon, 1994; Topkis, 1998; Vives, 1990, 2001). We show that a (generic) symmtric two-player supermodular game with TBP has a unique 1/2–dominant equilibrium, and the equilibrium is either the lowest or the highest strategy profile. This implies that the various equilibrium selection methods consistently predict either the lowest or the highest strategy profile to be chosen in this subclass of games with TBP.

The other is the class of potential games (Hofbauer and Sigmund, 1988; Monderer and Shapley, 1996). We show that if a game with TBP has a potential function with

[3] In the late 1970s and the 1980s, consumers struggled to choose between videotape formats of VHS by Matsushita as JVC and Betamax sold by Sony, but VHS prevailed in the end. The striking force behind this market outcome is that consumers tend to adopt the more popular technology. Similar examples are observed for the high definition optical disc formats between Blu-ray Disc by Sony and HD-DVD by Toshiba (Fackler, 2008) and browsers between Internet Explorer by Microsoft and Netscape by Navigator in the late 1990s and between Google Chrome, Mozilla Firefox, Internet Explorer, Safari by Apple, and Opera in recent years (The Economist, Aug 10, 2013).

a unique potential maximizer, the potential maximizer is a 1/2–dominant equilibrium. More generally, when considering a *local potential maximizer* (Morris, 1999), which is a generalization of *potential maximizer* and chosen by the evolutionary learning method based on the log-linear dynamics (Blume, 1993; Okada and Tercieux, 2012),[4] we can show that if a local potential maximizer (with constant weights) exists in a game, it is a 1/2–dominant equilibrium.

Lastly, we apply our results to a classical experimental game—the minimum-effort game—introduced by Van Huyck, Battalio and Beil (1990) where subjects choose their individual effort levels with incurring constant per-unit effort cost while their benefits are commonly determined by the minimum level of efforts chosen by all subjects, and therefore every subject has no incentive to choose a higher effort level than the other(s). This game is a (symmetric) supermodular coordination game with multiple Pareto-ranked Nash equilibria. We show that the two-player minimum-effort game does not satisfy TBP but is a limit case of TBP.[5] Then we examine the result of their experiment in the two-player case by the selection criterion of the 1/2–dominant equilibrium.

This article contributes to two strands of the literature. First of all, this article is related to the literature on the number of Nash equilibria in games. To the best of my knowledge, this is the first paper to provide a characterization of a class of games via the number of Nash equilibria. Interestingly, this characterization sheds light on the (wrong) conjecture of Quint and Shubik (1997) that any (nondegenerate) $n \times n$ bimatrix game has at most $2^n - 1$ Nash equilibria.[6] Our result implies that the number of Nash equilibria given by our class of games with the bandwagon property is exactly the same as the maximum number given by the Quint-Shubik conjecture. Secondly, we provide new insights on equilibrium selection methods.

The rest of the paper is organized as follows. The next section presents the underlying game considered in this paper. Section 4.3 first gives the characterization result of symmetric games with TBP via the number of Nash equilibria and then extend the charaterization to bimatrix games. Section 4.4 focuses on the equilibrium selection problem for the class

[4]See also Alós-Ferrer and Netzer (2010, 2015).

[5]The two-player minimum-effort game used in their experiment of Van Huyck et al. (1990) is also the knife-edge case for a potential maximizer (Monderer and Shapley, 1996) and a logit equilibrium (Anderson et al., 2001).

[6]Keiding (1997) and McLennan and Park (1999) prove the conjecture in the case of $n \leq 4$ and Quint and Shubik (2002) for games where payoff matrices are identical between two players, while von Stengel (1999) shows that it does not hold in general. In fact, von Stengel (1999) constructs a general lower bound on the maximal number of Nash equilibria based on a technique of polytope theory, and then as its application, he provides a counterexample of an asymmetric 6×6 game with 75 Nash equilibria, which is larger than $2^6 - 1 = 63$. Nonetheless, the case of $n = 5$ is still unknown to the best of our knowledge.

of games with TBP, thereby providing the simple selection criterion consistently chosen by various methods. Section 4.5 applies the above obtained results to the experimental game. Section 4.6 concludes.

4.2 The Underlying Game

We consider a symmetric two-player game $\mathbf{g} = (A, g)$ where $A = \{1, 2, \dots, n\}$ with $|A| = n \geq 2$ is the finite set of pure strategies and $g : A^2 \to \mathbb{R}$ is the symmetric payoff function. We write the set of mixed strategies by the $(n-1)$–dimensional simplex $\Delta = \{x \in \mathbb{R}^n \mid \forall i \in A, x_i \geq 0, \sum_{i \in A} x_i = 1\}$. For any $x \in \Delta$, let $\mathrm{supp}(x) = \{i \in A \mid x_i > 0\}$ be the support of x and $\mathrm{br}(x) = \mathrm{argmax}_{i \in A} \sum_{j \in A} g_{ij} x_j$ be the set of pure strategy best responses against x. When $\mathrm{supp}(x) = S \subseteq A$, we write $x \in \mathring{\Delta}(S)$ instead of $x \in \Delta$. A game is *nondegenerate* if $|\mathrm{br}(x)| \leq |\mathrm{supp}(x)|$ for any $x \in \Delta$, it is a *coordination game* if any symmetric pure strategy profile is a strict Nash equilibrium, and it is a *pure coordination game* if for any $i, j \in A$, $g_{ij} > 0$ if $i = j$, otherwise $g_{ij} = 0$.

Kandori and Rob (1998) introduce the following concept capturing a bandwagon effect regarding consumer technology adoptions in an evolutionary context.

Definition 4.1. A game $\mathbf{g} = (A, g)$ has the total bandwagon property (TBP) if $\mathrm{br}(x) \subseteq \mathrm{supp}(x)$ for any $x \in \Delta$.

TBP is a strong condition when just considering the symmetric game itself, but it is meaningful when considering the following evolutionary model usually adopted in the literature. There is a society made of a single population. Each period each agent of the population is randomly matched with one other agent in the society to play a game $\mathbf{g}$. We assume that each agent is a myopic decision maker and observes an action distribution $x \in \Delta$ taken in the last period, and then believes that the opponent with whom he is randomly matched chooses the same mixed strategy as the observed action distribution x, thereby choosing a best response against the belief x. In this situation, TBP requires that it is the best for the agent to take one of the actions taken in the society instead of taking an action not taken in the society. Note that any game with TBP is a nondegenerate coordination game and in addition that any pure coordination game and slightly perturbed ones satisfy TBP.

TBP is related to the set-valued solution concept, *curb set*, introduced by Basu and Weibull (1991) and further investigated by Ritzberger and Weibull (1995) in an evolutionary context. Since we here focus on symmetric (coordination) games, we simply introduce the definition of curb sets by using subset of strategies instead of subset of strategy profiles in the following way.

Definition 4.2. A subset of strategies $S \subseteq A$ is a *curb set* if for any $x \in \mathring{\Delta}(S)$,

$$\mathrm{br}(x) \ \subseteq \ S \quad \Big(= \bigcup_{x \in \Delta(S)} \mathrm{supp}(x)\Big)$$

From this definition, it is easy to see that TBP is equivalent to the condition that any $S \subseteq A$ is a curb set.[7]

4.3 Characterization

We provide the characterization of games with TBP via the number of NE as follows.

Theorem 4.1. *Let $\mathbf{g} = (A, g)$ be a symmetric $n \times n$ game. The game $\mathbf{g}$ has TBP if and only if $\mathbf{g}$ has $2^n - 1$ symmetric Nash equilibria.*

The proof of Theorem 4.1 is given in the Appendix. Theorem 4.1 gives us the following interesting points. First of all, the class of games with the bandwagon property is characterized by the number of NE. To the best of our knowledge, this is the first paper to provide a characterization of games via the number of Nash equilibria. Secondly, Theorem 4.1 is related to the conjecture of Quint and Shubik (1997) for the number of NE that any nondegenerate $n \times n$ bimatrix game has at most $2^n - 1$ NE including asymmetric ones. In general however, von Stengel (1999) shows that the Quint-Shubik conjecture does not hold. But we can say that the conjecture holds for any symmetric $n \times n$ game with TBP.[8] More precisely, the class of symmetric games with TBP obtains the maximum number of NE given by the Quint-Shubik conjecture, and conversely, the game with the maximum number of NE must have TBP as long as the game is symmetric.

[7]The curb set is not a condition for a class of games but is used for a solution concept, while TBP is a condition for games. Formally, a curb set is defined in an N-person normal-form game and a product set of pure strategies. There are two related set-valued concepts. One of them is *retract* defined by Kalai and Samet (1984), which is a product set of mixed strategies. This concept is used to generalize the concept of NE and give a refinement of NE as in trembling hand perfect equilibrium (Selten, 1975) and proper equilibrium (Myerson, 1978). The other is *pre set* defined by Voorneveld (2004), which is a product set of pure strateiges as in curb set. For a relation among the three concepts, see van Damme (2002) and Voorneveld (2005).

[8]We can show that the game has no asymmetric NE in addition to Theorem 4.1. For proof showing that any symmetric game with TBP has no asymmetric equilibrium, see Remark 4.3 in the Appendix.

4.3.1 Extension to Bimatrix Games

It is natural to extend the notion of TBP to allow for asymmetric games as follows.

We write a bimatrix game as $\mathbf{g} = (\{1,2\}, (A_i)_{i=1,2}, (g^i)_{i=1,2})$ where $A_1 = A_2 = \{1, 2, \ldots, n\}$ is the linearly ordered set of strategies, and $g^i : A^2 \to \mathbb{R}$ the payoff function of player $i = 1, 2$.[9] Since we consider the common set of strategies $A_1 = A_2 = \{1, 2, \ldots, n\}$, we simply denote by $\mathbf{g} = (\{1,2\}, A, (g^i)_{i=1,2})$ a bimatrix game and use notations in almost the same way as in symmetric games. For clarity, we write by $x^i \in \Delta$ a mixed strategy of player i. We denote by $\mathrm{br}_i(x^j) = \mathrm{argmax}_{k \in A} \sum_{h \in A} g^i(k,h) x^j_h$ the set of pure strategy best responses of player i against the opponent j's mixed strategy $x^j \in \Delta$ as $j \neq i$. A game $\mathbf{g}$ is *nondegenerate* if for any $i, j = 1, 2$ and any $x^j \in \Delta$ as $i \neq j$, $|\mathrm{br}_i(x^j)| \leq |\mathrm{supp}(x^j)|$, and a (pure) coordination game is defined in the same way as in symmetric games.

We define the naturally extended TBP to bimatrix games as follows.

Definition 4.3. The game $\mathbf{g} = (\{1,2\}, A, (g^i)_{i=1,2})$ has the generalized TBP (GTBP) if for $i, j = 1, 2$ with $i \neq j$ and any $x^j \in \Delta$, $\mathrm{br}_i(x^j) \subseteq \mathrm{supp}(x^j)$.

Note that a game with GTBP is a coordination game and nondegenerate, and in addition that by allowing for permutations or reordering of strategies, we can consider a slightly larger class of games than those with GTBP, such as a class of anti-coordination games including Hawk-Dove games.

We extend the characterization of Theorem 4.1 to bimatrix games by incorporating restrictions on support of NE as follows.

Theorem 4.2. *Let $\mathbf{g} = (\{1,2\}, A, (g^i)_{i=1,2})$ be an $n \times n$ bimatrix game. The game $\mathbf{g}$ has GTBP if and only if $\mathbf{g}$ has $2^n - 1$ Nash equilibria, each of which gives the same support for both players that is distinct from those of other Nash equilibria.*

See the Appendix for the proof of Theorem 4.2. It turns out that $\mathbf{g}$ has GTBP if and only if both g_1 and g_2 (viewed as symmetric $n \times n$ games) have TBP. This tells us that the conjecture of Quint and Shubik (1997) still holds for the class of bimatrix games with the bandwagon property. The reason why we restrict support of NE is given in the Appendix where we show that the game without the restriction may not have GTBP.

[9] We consider the linearly ordered set of strategies for simple notations. We assume that the size of set of strategies is common for both players by $|A_1| = |A_2| = n$ in order to extend TBP to asymmetric games (see the Appendix for an example to explain why we need the same size of set of strategies).

4.4 The Equilibrium Selection Problem

So far we have shown that any two-player game with the bandwagon property has many NE, so that it seems hard to select a single equilibrium. With this in mind, the objective of this section is to provide a simple equilibrium selection criterion. Such a simple but strong equilibrium selection criterion is the solution concept of 1/2–*dominant equilibrium* proposed by Morris, Rob and Shin (1995), which is a generalization of *risk dominant equilibrium* (Harsanyi and Selten, 1988). It is chosen by various equilibrium selection methods mentioned below. A 1/2–dominant equilibrium needs not to exist in games with TBP. We will show the existence of a 1/2–dominant equilibrium for two subclasses of games with TBP: (i) supermodular games; (ii) potential games.

4.4.1 Half–Dominant Equilibrium

There are various equilibrium selection methods: (1) the evolutionary learning method of best-response dynamics with mutations (Kandori et al., 1993; Young, 1993); (2) the global game method (Carlsson and van Damme, 1993); (3) the incomplete information method (Kajii and Morris, 1997); (4) the perfect foresight dynamics method (Matsui and Matsuyama, 1995; Hofbauer and Sorger, 1999); (5) the spatial dominance method (Hofbauer, 1999), among others. One common property among those methods is that if a two-player game has a 1/2–dominant equilibrium, then it is uniquely selected by all above methods. The 1/2–dominant equilibrium (Morris et al., 1995) is defined as follows.[10]

Definition 4.4. A strategy profile $(i^*, i^*) \in A^2$ is a 1/2–*dominant equilibrium* if for any $x \in \Delta$ with $x_{i^*} \geq 1/2$,

$$\mathrm{br}(x) = \{i^*\}.$$

Note that if a strategy profile is a 1/2–dominant equilibrium, then it is a strict NE. To make the above condition more clear, we can rewrite it by the following equivalent condition: for strategy profile (i^*, i^*) and any strategy profile $(i, j) \in A^2$ with $(i, j) \neq (i^*, i^*)$,

$$\frac{1}{2} g_{i^* i^*} + \frac{1}{2} g_{i^* j} > \frac{1}{2} g_{i i^*} + \frac{1}{2} g_{ij}. \tag{4.1}$$

[10]Note that the following definition is for symmetric (two-player) coordination games but the formal definition is for all games.

Also, note that a game can have at most one 1/2–dominant equilibrium. A game with TBP may have no 1/2–dominant equilibrium.

As another generalization of risk-dominant equilibrium, Kandori and Rob (1998) consider a risk–dominant concept for strategies based on pairwise comparison. Consider a coordination game and two distinct pure strategy NE (i, i) and (j, j). Then, strategy i *pairwise risk dominates* j (i PRD j) if

$$g_{ii} - g_{ji} > g_{jj} - g_{ij}.$$

If i PRD j for any strategy $j \neq i$, then i is *globally pairwise risk dominant* (GPRD). We call a symmetric pure strategy NE in which the strategy is GPRD a *GPRD-equilibrium.* Note that if a game has a GPRD-equilibrium, by definition, the GPRD-equilibrium is unique and also that since a GPRD-equilibrium is a natural extension of risk-dominant equilibrium and a weaker concept than a 1/2–dominant equilibrium, a game may have a GPRD-equilibrium but not a 1/2–dominant equilibrium. But it is easily shown:

Lemma 4.1. *If a GPRD-equilibrium exists in a game with TBP, it is a* 1/2*–dominant equilibrium.*

See the Appendix for the proof of Lemma 4.1.

4.4.2 Supermodularity

In economics, strategic complementality (supermodularity) has been considered to be important (Milgrom and Roberts, 1990), and is defined as follows.

Definition 4.5. A game $\mathbf{g} = (A, g)$ with $A = \{1, 2, \ldots, n\}$ is *supermodular* if for any $i, i', j, j' \in A$ with $i > i'$ and $j \geq j'$,

$$g_{ij} - g_{i'j} \geq g_{ij'} - g_{i'j'}. \tag{4.2}$$

By definition, if a game $\mathbf{g} = (A, g)$ is supermodular, each person's best response is non-decreasing in his opponent's strategies.

Adding supermodularity to games with TBP, we can provide a simple equilibrium selection criterion based on the 1/2–dominance as follows.

Proposition 4.1. *We consider a symmetric $n \times n$ game $\mathbf{g} = (A, g)$ where $A = \{1, \ldots, n\}$ and $g_{11} - g_{n1} \neq g_{nn} - g_{1n}$. If $\mathbf{g}$ has TBP and is supermodular, the game always has a 1/2–dominant equilibrium. The 1/2–dominant equilibrium is either the lowest or the highest strategy profile, $(1,1)$ or (n,n), and given by*

$$\begin{cases} (1,1), & \text{if} \quad g_{11} - g_{1n} > g_{nn} - g_{n1}, \\ (n,n), & \text{if} \quad g_{11} - g_{1n} < g_{nn} - g_{n1}. \end{cases}$$

The proof of Proposition 4.1 is given in the Appendix. It turns out that a GPRD–equilibrium in a supermodular game with TBP is a 1/2–dominant equilibrium. Proposition 4.1 basically tells us that we can guarantee the existence of a 1/2–dominant equilibrium in a subclass of games with TBP and then provide a simple prediction to select a single equilibrium. Note that the condition $g_{11} - g_{n1} \neq g_{nn} - g_{1n}$ is very mild and holds in all generic games.

4.4.3 Potential Games

We consider a potential game (Monderer and Shapley, 1996) or partnership game (Hofbauer and Sigmund, 1988) in order to show a connection of equilibrium selection methods between the potential game method and the other methods introduced in Section 4.4.1, given that the game has TBP. In the literature, it has been shown that the potential game method is consistent with other equilibrium selection methods including the incomplete information method (Ui, 2001; Morris and Ui, 2005; Oyama and Tercieux, 2009), the global game method (Frankel, Morris and Pauzner, 2003), and the perfect foresight dynamics method (Hofbauer and Sorger, 1999, 2002; Oyama, Takahashi and Hofbauer, 2008).[11]

We introduce a potential game (Monderer and Shapley, 1996) for a symmetric two-player game as follows. Given a symmetric game $\mathbf{g} = (A, g)$, a symmetric function $v : A^2 \to \mathbb{R}$ with $v_{ij} = v_{ji}$ for any $i, j \in A$ is a *potential function* of $\mathbf{g}$ if for any $i, i', j \in A$,[12]

$$g_{i'j} - g_{ij} = v_{i'j} - v_{ij}.$$

[11]For the relation between the potential game method, the incomplete information method, and the global game method, see Basteck and Daniëls (2011), Honda (2011), and Oyama and Takahashi (2011).

[12]In fact, Monderer and Shapley (1996) define a potential function in an (possibly asymmetric) N-person game where the symmetry of potential functions does not necessarily hold. For instance, a two-player bimatrix game $\mathbf{g} = (A, (g^i)_{i=1,2})$ as defined in previous section has a potential function $v : A^2 \to \mathbb{R}$ of $\mathbf{g}$ if for any $h = 1, 2$ and any $i, i', j \in A$, $g^h_{ij} - g^h_{i'j} = v_{ij} - v_{i'j}$ If a game is symmetric, by definition, we obtain the symmetry of $v_{ij} = v_{ji}$. Hofbauer and Sigmund (1988) call such a game a (rescaled) partnership game.

Definition 4.6. A pure strategy profile $(i^*, j^*) \in A^2$ is a *potential maximizer* if there exists a potential function $v : A^2 \to \mathbb{R}$ such that $(i^*, j^*) \in \arg\max_{(i,j)\in A^2} v_{ij}$. We call $\mathbf{g}$ a *potential game* if there exists a potential function $v : A^2 \to \mathbb{R}$.

Any potential maximizer is a NE. If a game has TBP, since we know by Theorem 4.1 that every symmetric pure strategy profile is a NE while there is no asymmetric NE, only a symmetric pure strategy profile can be a potential maximizer. It is shown by Monderer and Shapley (1996, Lemma 2.7) that if a game $\mathbf{g} = (A, g)$ has a potential function v, it is unique up to constant, meaning that when taking v and v' as potential functions of $\mathbf{g}$, there exists a constant c such that for any $(i, j) \in A^2$, $v_{ij} - v'_{ij} = c$. This implies that it is enough to focus on a potential function when considering the equilibrium selection criterion of potential games.

We show the existence of a 1/2–dominant equilibrium in a subclass of games with TBP as follows.

Proposition 4.2. *We consider a two-player symmetric game $\mathbf{g} = (A, g)$ with TBP. Suppose that $\mathbf{g}$ has a potential function with a unique potential maximizer. Then, the potential maximizer is a 1/2–dominant equilibrium.*

The proof of Proposition 4.2 is given in the Appendix where we show that the potential maximizer is a GPRD–equilibrium, and then use Lemma 4.1 to show that the potential maximizer is 1/2–dominant equilibrium. Actually we can extend Proposition 4.2 to a generalized potential maximizer introduced below.

Generalized Potential Games and Log-Linear Dynamics

As an equilibrium selection method, we consider the log-linear dynamics introduced by Blume (1993).[13] We briefly explain what is the log-linear dynamics. Let us consider a single population consisting of N players who interact in a normal-form game. The log-linear dynamic is a stochastic process in discrete time and its state space is the set of all pure strategy profiles. An initial strategy profile is chosen according to a distribution. At each subsequent period, only one of players is randomly selected and gets an opportunity to revise his or her strategy. The strategy revisions follow the log-linear stochastic choice rule under which the log likelihood ratio between two strategies is proportional to the difference between payoffs of those actions. The (common) factor of proportionality in the

[13] See also Blume (1997) and Young (1998). Note that this paper takes into account a discrete time version of log-linear dynamics (Blume, 1997) instead of its continuous time version (Blume, 1993).

choice rule is exogenously given and is interpreted as noise of payoff information. The log-linear choice rule generates a time-homogeneous Markov chain on the set of pure strategy profiles, which is irreducible and aperiodic. As the long-run outcome in this dynamic, we consider a unique invariant distribution of the Markov chain when the noise level goes to zero. It is known that if an exact potential function exists in a game, the unique invariant distribution in the log-linear dynamic is explicitly given by a simple closed form.[14] This gives us a powerful tool when using an explicit stationary distribution in an application.[15]

In the following, we introduce a solution concept used in the study of the log-linear dynamics, local potential maximizer (Morris, 1999), which is a generalization of potential maximizer (Monderer and Shapley, 1996). Consider a symmetric game $\mathbf{g} = (A, g)$ with $A = \{1, , \dots, n\}$. Then, we define a simplified local potential maximizer (with constant weights) as in Okada and Tercieux (2012, Definition 1).[16]

Definition 4.7. A pure strategy profile $s^* = (i^*, j^*) \in A^2$ is a *local potential maximizer* (LP-maximizer) of $\mathbf{g}$ if there exists a local potential function $v : A^2 \to \mathbb{R}$ with $v_{s^*} > v_s$ for all $s \in A^2 \backslash \{s^*\}$ such that any $i, j \in A$,

$$\begin{aligned} v_{i+1j} - v_{ij} &\leq g_{i+1j} - g_{ij}, \quad \text{if } i < i^*, \\ v_{i-1j} - v_{ij} &\leq g_{i-1j} - g_{ij}, \quad \text{if } i > i^*, \end{aligned}$$

and similarly

$$\begin{aligned} v_{j+1i} - v_{ji} &\leq g_{j+1i} - g_{ji}, \quad \text{if } j < j^*, \\ v_{j-1i} - v_{ji} &\leq g_{j-1i} - g_{ji}, \quad \text{if } j > j^*. \end{aligned}$$

The notion of LP-maximizer relaxes the equality condition of potential-maximizer by an inequality under a certain requirement on the local relation in terms of order of strategies between v and g. Note that since LP-maximizer is a generalization of the potential maximizer, there is a class of games where an LP-maximizer exists but no potential maximizer does, and also note that a game may have multiple LP-maximizers.[17] A 1/2–dominant

[14]This is because the Markov chain satisfies reversibility, the detailed balance conditions hold for an invariant distribution, and the Gibbs representation of an invariant distribution is applied.

[15]As a suitable application, Young and Burke (2001) consider agricultural contracts of crops in Illinois as a case study to investigate an evolutionary process for the contracts and then provide an explanation why currently adopted contracts according to regions are established and stable.

[16]See Morris (1999) for the detail and Okada and Tercieux (2012, Definition 3) for the simplified version with non-constant weights.

[17]Although Frankel et al. (2003) claim that an LP-maximizer of a supermodular game that satisfies

equilibrium is in general irrelevant for the selection criterion under the log-linear dynamics. It is known that if a local potential maximizer exists in a supermodular game, it is selected in the log-linear dynamics (Okada and Tercieux, 2012).

We provide a relation between LP-maximizer and 1/2–dominant equilibrium as follows.

Proposition 4.3. *Let us consider a supermodular game* $\mathbf{g} = (A, g)$ *with TBP. Suppose that there exists an LP-maximizer* (i, i) *for* $i \in A$*. Then,* (i, i) *is a* 1/2*–dominant equilibrium.*

For the proof of Proposition 4.3, see the Appendix. Together with Proposition 4.1, the above considered LP-maximizer must be either the lowest strategy profile or the highest strategy profile. In a subclass of games with the bandwagon property, the log-linear dynamic selects the 1/2–dominant equilibrium chosen out of many equilibria as well as the best response dynamics with mutation (Kandori et al., 1993; Young, 1993) and others.

4.5 Application to the Minimum-Effort Game

We introduce the minimum-effort (or the weak-link) game defined by Van Huyck, Battalio and Beil (1990) and then examine one of their experimental results showing that no stable outcome obtains in their experiment if subjects repeatedly play the two-player minimum-effort game with the *random matching*. To this aim, we first show that the two-player minimum-effort game does not satisfy TBP but is a limit case of TBP. Then we examine the result of their experiment by the selection criterion of the 1/2–dominant equilibrium.

The Minimum-Effort Game

The two-player minimum-effort game is a symmetric $n \times n$ coordination game $\mathbf{g} = (A, g)$ such that each strategy $i \in A = \{1, 2, \ldots, n\}$ represents an effort level and the payoff of player who takes i given the opponent's effort level j is determined by

$$g_{ij} = a \min\{i, j\} - bi + c \tag{4.3}$$

own-action *quasi-concavity* is unique in terms of noise-independent selection in global games, Oyama and Takahashi (2009, Example 1) provide a counterexample that this claim does not hold. In fact, Oyama et al. (2008) show that an LP-maximizer of a supermodular game that satisfies diminishing marginal return (or own-action *concavity*) is at most one, and so if an LP-maximizer exists in such a game, it is unique and no multiplicity happens.

	1	2	3
1	0.70	0.70	0.70
2	0.60	0.80	0.80
3	0.50	0.70	0.90

Table 4.1: The 3×3 minimum-effort game where $(a, b, c) = (0.20, 0.10, 0.60)$.

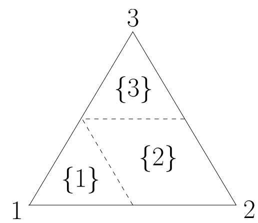

Figure 4.1: The best response regions of the 3×3 minimum-effort game.

where a, b, and c are positive constants such that $a > b > 0$ and $c > 0$ guarantees positive payoffs for all subjects in the experiment for any given effort profile. This payoff function entails an interesting feature in capturing coordination problems that the effort cost of each subject depends only on the individual effort choice while the benefit depends upon the minimum of effort levels chosen by both subjects, and therefore each subject has no incentive to choose a higher effort level than that of the other subject. The minimum-effort game is a symmetric $n \times n$ supermodular coordination game. The lowest strategy profile $(1, 1)$ and the highest strategy profile (n, n) are the most inefficient and the most efficient equilibria in the game, respectively.

Property of the Minimum-Effort Game

We first illustrate that the minimum-effort game does not satisfy TBP but is a limit case of TBP. Consider a symmetric 3×3 minimum-effort game given by Table 4.1. The best response regions of the game are given by Figure 4.1 where we can see that the 3×3 minimum-effort game "almost" satisfies TBP in the sense that $\mathrm{br}(x) \subseteq \mathrm{supp}(x)$ holds for "almost" all $x \in \Delta$ except for one specific point $x^{13} \in \mathring{\Delta}(\{1, 3\})$ violating TBP where $\mathrm{supp}(x^{13}) \subsetneq \mathrm{br}(x^{13})$ holds. Actually we can show that the two-player minimum-effort game does not satisfies TBP but is a limit case of TBP in the following sense.

Lemma 4.2. *For any $l, m, h \in A$ with $l < m < h$, there exists a unique point $x^* \in \mathring{\Delta}(\{l, h\})$*

	1	2	3	4	5	6	7
1	0.70	0.70	0.70	0.70	0.70	0.70	0.70
2	0.60	0.80	0.80	0.80	0.80	0.80	0.80
3	0.50	0.70	0.90	0.90	0.90	0.90	0.90
4	0.40	0.60	0.80	1.00	1.00	1.00	1.00
5	0.30	0.50	070	0.90	1.10	1.10	1.10
6	0.20	0.40	0.60	0.80	1.00	1.20	1.20
7	0.10	0.30	0.50	0.70	0.90	1.10	1.30

Table 4.2: The minimum-effort game used in the experiment.

such that $x_l^* = (a-b)/a$, $x_h^* = 1 - x_l^*$, *and*

$$supp(x^*) \subsetneq br(x^*), \tag{4.4}$$
$$\forall x \in \Delta(\{l,h\})\backslash\{x^*\}, \quad br(x) \subseteq supp(x). \tag{4.5}$$

The proof of Lemma 4.2 is given in the Appendix.

From above, the minimum-effort game contains the knife-edge case for the 1/2–dominant equilibrium in the minimum effort game $\mathbf{g} = (A, g)$ with $|A| = n$ because the 1/2–dominant equilibrium is given by

$$\begin{cases} \{(1,1)\}, & \text{if } a < 2b, \\ \{(n,n)\}, & \text{if } a > 2b. \end{cases} \tag{4.6}$$

Similarly, it is easily shown (Monderer and Shapley, 1996) that the minimum-effort game is a potential game where a potential function $v : A^2 \to \mathbb{R}$ is given by $v_{ij} = a \min\{i,j\} - b(i+j)$ for any $(i,j) \in A^2$ and the potential maximizer by (4.6) as well as the 1/2–dominant equilibrium except for the knife-edge case where all symmetric pure strategy profiles are potential maximizers.

Examination of Experimental Result

Van Huyck et al. (1990) run the experiments for the two-subject case together with the many-subject case by specifying the payoff matrix of the minimum-effort game in such a way that $A = \{1, 2, \ldots, 7\}$ and $(a, b, c) = (0.20, 0.10, 0.60)$, which is described by Table 4.2. At each session in each case, one group of subjects repeatedly play the game under the fix-pair while the other under the random-pair. Their experimental result in the two-subject case shows that the most efficient equilibrium is selected under the fix-pair, while no stable outcome obtains under the random-pair.[18] For the equilibrium selection result in case of the fixed-pair, Van Huyck et al. (1990) apply the repeated game argument to justify the result, which makes sense. For the result in case of the random-pair, however, they do not give a justification. When $a = 2b$ used in the experiment, by (4.6), this is the knife-edge case where there is no 1/2–dominant equilibrium,[19] and therefore we cannot apply the theoretical prediction obtained in the previous sections to the experimental game. The equilibrium selection methods considered in this paper are based on random matching where no convergence is obtained in their experiment.

4.6 Conclusion

This paper considered two-player games with the bandwagon property and then pinned down the underlying characteristic of those games. In doing so, we provided two main results. One of them is a characterization of a class of games via the number of Nash equilibria. To the best of our knowledge, this is the first paper to characterize a class of games by the number of Nash equilibria. It is of interest that the class of games with the maximum number of Nash equilibria given by the Quint-Shubik conjecture is equivalent to that of games with the bandwagon property. Secondly, taking into account that the games with the bandwagon property has too many equilibria to select a single equilibrium, we gave it a simple equilibrium selection criterion that is commonly chosen by various methods. Applying our results to the minimum-effort game, we clarified the property of the game.

4.7 Appendix: Proofs

Proof of Theorem 4.1

We provide proofs for the if part and the only if part of separately.

[18]In the many-subject case, the clear convergence to the most inefficient outcome obtains in their experiment, which is one of well known coordination failure problems. This can be simply explained in a way that, when choosing a higher effort level than the minimum level, a player thinks that all subjects are less likely to coordinate to choose high effort levels as there are more subjects. For details, see Van Huyck et al. (1990).

[19]Similarly, Goeree and Holt (2005) point out that this is the knife-edge case for a logit-equilibrium (Anderson et al., 2001).

	1	2	3	4
1	a	c	b	c
2	c	a	c	b
3	b	c	a	c
4	c	b	c	a

Table 4.3: A symmetric 4×4 coordination game where $a > b$, $a > c$, and $a + b > 2c$.

Remark 4.1. Any game with TBP is nondegenerate, and so the game has the oddness property regarding the number of NE. For this fact, see Shapley (1974, Theorem 2) and Quint and Shubik (1997, Lemma 2.2), among others. Note that any nondegenerate game $\mathbf{g}$ has at most symmetric $2^n - 1$ NE.

Proof of the if part

Proof. Consider a symmetric game $\mathbf{g} = (A, g)$ with $2^n - 1$ symmetric NE. Assume to the contrary that TBP does not hold. By definition, TBP is equivalent to the condition that $\text{br}(x) \backslash \text{supp}(x) = \emptyset$ for any $x \in \Delta$, so that there exists some $x \in \Delta$ such that

$$\text{br}(x) \backslash \text{supp}(x) \neq \emptyset. \tag{4.7}$$

If $x \in \cup_{i \in A} \Delta(\{i\})$, $\text{br}(x) = \text{supp}(x) = j$ holds for some $j \in A$ due to pure strategy symmetric NE, a contradiction to (4.7). Also, if $x \in \text{int}\Delta$, $\text{br}(x) \subseteq A = \text{supp}(x)$ holds, a contradiction to (4.7). Next, we consider the remaining case of $x \in \Delta \backslash (\cup_{i \in A} \Delta(\{i\}) \cup \text{int}\Delta)$ and let $S \subsetneq A$ be the support of x. Assume that $j \in \text{br}(x) \backslash \text{supp}(x) \neq \emptyset$. Since we know that the game has the unique symmetric NE strategy which is completely mixed over $\{j\} \cup S$, we denote it by $x^*|_{\{j\} \cup S}$. Given that the game has all symmetric NE, since (pure strategy) best response sets are convex, the best response region of pure strategy j goes across the entire interior of the face spanned by all pure strategies in $\{j\} \cup \text{supp}(x)$ and prevents the NE $(x^*|_{\{j\} \cup S}, x^*|_{\{j\} \cup S})$ where the best responses of all strategies in $\{j\} \cup \text{supp}(x)$ meet to exist, a contradiction. $\square$

We will see below how the above shown proof works in an example.

Example 4.1. Let us consider a symmetric 4×4 coordination game given by Table 4.3 where there are three parameters a, b, and c such that $a > b$, $a > c$, and $a + b > 2c$.

We can show that the game has $2^4 - 1 = 15$ symmetric NE. For example, when considering $(a, b, c) = (10, 5, 7)$, we can compute all (symmetric) NE of this game. In fact, these NE are given by 4 pure strategy NE, 6 completely mixed strategy NE over S with $|S| = 2$ s.t.

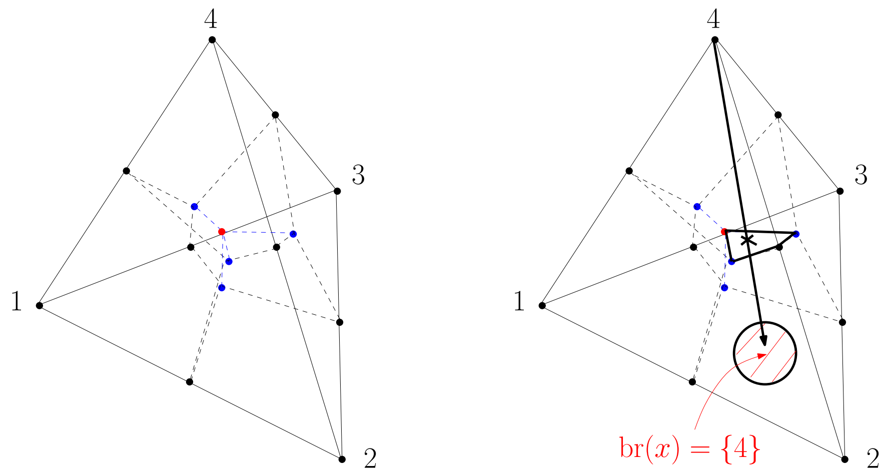

Figure 4.2: The best response regions of the game (Left) and an illustration that the condition $\{4\} \in \mathrm{br}(x)\backslash\mathrm{supp}(x)$ breaks down the best response regions (Right).

$x_i = x_j = 1/2$ for any distinct $i, j \in A$ and $x_k = 0$ for any $k \in A\backslash\{i, j\}$, 4 completely mixed strategy NE over S with $|S| = 3$ s.t. $(3/7, 1/7, 3/7, 0), (0, 3/7, 1/7, 3/7), (3/7, 0, 3/7, 1/7), (1/7, 3/7, 0, 3/7)$ and a unique interior NE, $(1/4, 1/4, 1/4, 1/4)$.

Assume to the contrary that TBP does not hold and therefore there exists some $x \in \Delta(\{1, 2, 3\})$ such that (4.7) holds for x. This corresponds to the remaining case of $x \in \Delta\backslash(\cup_{i\in A}\Delta(\{i\}) \cup \mathrm{int}\Delta)$ for $A = \{1, 2, 3, 4\}$ where $S = \mathrm{supp}(x) = \{1, 2, 3\} \subsetneq A$, and $j = \{4\} \in \mathrm{br}(x)\backslash\mathrm{supp}(x) \neq \emptyset$. The strategy $\{4\} \in \mathrm{br}(x)\backslash\mathrm{supp}(x)$ breaks down the best response regions of the game, which is described by Figure 4.2.

Alternative proof of the if part

Proof. We show the if part as follows. If the number of symmetric NE in symmetric game $\mathbf{g} = (A, g)$ with $|A| = n$ is $2^n - 1$, from Remark 4.1, then the game must have all possible symmetric NE:

$$\{x^* \in \Delta \mid i = 1, \dots, n, S \subseteq A \text{ with } |S| = i, x^* \in \mathring{\Delta}(S)\}. \tag{4.8}$$

From (4.8) with $k = 1$, all symmetric pure strategy profiles are NE, and so $\mathrm{br}(x) \subseteq \mathrm{supp}(x)$ holds for the set of beliefs denoted by $X^1 = \{x \in \Delta \mid |\mathrm{supp}(x)| = 1\}$.

To show that $\mathbf{g}$ has TBP, we have to show that $\mathrm{br}(x) \subseteq \mathrm{supp}(x)$ also holds for the set of beliefs denoted by $X^2 \equiv \Delta\backslash X^1$. For any belief $x \in \mathrm{int}\Delta$, since $\mathrm{br}(x) \subseteq A$ and

$\text{supp}(x) = A$, $\text{br}(x) \subseteq \text{supp}(x)$ holds. So, we consider the remaining set of beliefs $X^2 \backslash \text{int}\Delta$. For any $x \in X^2 \backslash \text{int}\Delta$, we construct x via a sequence of symmetric NE of restricted games, $(x^*|_S)_{S \subseteq \text{supp}(x)}$, as follows. Note that any $x \in X^2 \backslash \text{int}\Delta$ is an n-dimensional vector as $(x_1, \ldots, x_n) \in [0,1]^n$.

Let us consider a sequence of pairs of restricted set of strategies and some positive constants, $\{(S^m, c^m)\}_m$ with $S^m \subseteq A$ and $c^m > 0$ for $m = 0, 1, \ldots, M$ such that the following condition hold:

$$
\begin{aligned}
& S^0 = \text{supp}(x), \quad c^0 = \min_{i \in S^0} \frac{x_i}{x_i^*|_{S^0}} = \frac{x_{i_0}}{x_{i_0}^*|_{S^0}} \quad (i_0 = \arg\min_{i \in S^0} \frac{x_i}{x_i^*|_{S^0}}), \\
& S^1 = S^0 \backslash \arg\min_{i \in S^0} \frac{x_i}{x_i^*|_{S^0}} = S^0 \backslash \{i_0\}, \\
& c^1 = \min_{i \in S^1} \frac{x_i - c^0 x_i^*|_{S^0}}{x_i^*|_{S^1}} = \frac{x_{i_1} - c^0 x_{i_1}^*|_{S^0}}{x_{i_1}^*|_{S^1}} \quad (i_1 = \arg\min_{i \in S^1} \frac{x_i - c^0 x_i^*|_{S^0}}{x_i^*|_{S^1}}), \\
(1 \le m \le M-1) \quad & S^{m+1} = S^m \backslash \arg\min_{i \in S^m} \frac{x_i - \sum_{j<m} c^j x_i^*|_{S^j}}{x_i^*|_{S^m}} = S^m \backslash \{i_m\} = S^0 \backslash \{i_0, i_1, \ldots, i_m\}, \\
& c^{m+1} = \min_{i \in S^{m+1}} \frac{x_i - \sum_{j<m+1} c^j x_i^*|_{S^j}}{x_i^*|_{S^{m+1}}}, \\
& \emptyset \subsetneq S^M \subsetneq \cdots \subsetneq S^1 \subsetneq S^0.
\end{aligned}
$$

We use the above introduced condition by Condition $(*)$ below. For the above defined sequence $\{(S^m, c^m)\}_m$, we can write x by

$$
x = \sum_{m=0}^{M} c^m x^*|_{S^m}.
$$

One can show that

$$
\text{br}(x) = \bigcap_{m=0}^{M} \text{br}(x^*|_{S^m}). \tag{4.9}
$$

Since $\text{br}(x^*|_{S^m}) = \text{supp}(x^*|_{S^m}) = S^m$ and $S^{m+1} \subsetneq S^m$ for $m = 0, 1 \ldots, M-1$, together with (4.9), we have

$$
\text{br}(x) = \bigcap_{m=0}^{M} \text{br}(x^*|_{S^m}) = \bigcap_{m=0}^{M} S^m = S^M \subsetneq S^0 = \text{supp}(x).
$$

	1	2	3	4
1	1,1	0,0	0,0	0,0
2	0,0	2,2	0,0	0,0
3	0,0	0,0	3,3	0,0
4	0,0	0,0	0,0	4,4

Table 4.4: A 4×4 pure coordination game.

Note that if $M = 0$, $\mathrm{br}(x) = \mathrm{supp}(x)$ holds. This implies that $\mathrm{br}(x) \subseteq \mathrm{supp}(x)$ holds for any $x \in X^2 \backslash \mathrm{int}\Delta$.

Thus, since we have shown that $\mathrm{br}(x) \subseteq \mathrm{supp}(x)$ holds for any $x \in \Delta$, the symmetric game $\mathbf{g}$ with $2^n - 1$ symmetric NE has TBP. $\square$

In the following, we will see how the above used iterative construction process works via an example.

Example 4.2 (Symmetric 4×4 pure coordination game)**.** Let us consider the symmetric 4×4 pure coordination game of Table 4.4. Given the fact that any $n \times n$ pure coordination game satisfies TBP and it has $2^4 - 1 = 15$ symmetric NE in the case of $n = 4$, we illustrate how the iterative process to construct a sequence of strategies works above in the proof.

All symmetric NE strategies of the game except for pure strategies are given as follows.

$$(\frac{2}{3}, \frac{1}{3}, 0, 0), (\frac{3}{4}, 0, \frac{1}{4}, 0), (0, \frac{3}{5}, \frac{2}{5}, 0), (0, 0, \frac{4}{7}, \frac{3}{7}), (\frac{4}{5}, 0, 0, \frac{1}{5}), (0, \frac{2}{3}, 0, \frac{1}{3}),$$
$$(\frac{6}{11}, \frac{3}{11}, \frac{2}{11}, 0), (0, \frac{6}{13}, \frac{4}{13}, \frac{3}{13}), (\frac{4}{7}, \frac{2}{7}, 0, \frac{1}{7}), (\frac{12}{19}, 0, \frac{4}{19}, \frac{3}{19}),$$
$$(\frac{12}{25}, \frac{6}{25}, \frac{4}{25}, \frac{3}{25})$$

For any belief $x \in \mathrm{int}\Delta$, it is obvious that $\mathrm{br}(x) \subseteq \mathrm{supp}(x)$ holds because $\mathrm{br}(x) \subseteq A = \{1, 2, 3, 4\} = \mathrm{supp}(x)$.

Next, we consider two beliefs with support $\{1, 2, 3\}$, $x = (\frac{3}{11}, \frac{6}{11}, \frac{2}{11}, 0)$ and $x' = (\frac{1}{3}, \frac{2}{5}, \frac{4}{15}, 0)$, and then we see that both $\mathrm{br}(x) \subseteq \mathrm{supp}(x)$ and $\mathrm{br}(x') \subseteq \mathrm{supp}(x')$ hold by applying the proof method used for the only if part to the example. Let us consider a sequence of $\{(S^m, c^m)\}_m$ with $S^m \subseteq A$ and $c^m > 0$ for each $m = 0, 1, \ldots, M$ such that

Condition $(*)$ holds. Then, for $x = (\frac{3}{11}, \frac{6}{11}, \frac{2}{11}, 0)$, we have

$$
\begin{aligned}
S^0 &= \{1,2,3\}, \quad c^0 = \frac{x_1}{x_1^*|_{S^0}} = \frac{3/11}{6/11} = \frac{1}{2} \quad (i_0 = 1), \\
S^1 &= S^0 \backslash \{i_0\} = \{1,2,3\} \backslash \{1\} = \{2,3\}, \\
c^1 &= \min\left\{ \frac{x_2 - c^0 x_2^*|_{S^0}}{x_2^*|_{S^1}}, \frac{x_3 - c^0 x_3^*|_{S^0}}{x_3^*|_{S^1}} \right\} = \min\left\{ \frac{\frac{6}{11} - \frac{1}{2} \times \frac{3}{11}}{3/5}, \frac{\frac{2}{11} - \frac{1}{2} \times \frac{2}{11}}{2/5} \right\} \\
&= 5 \min\left\{ \frac{3}{22}, \frac{1}{22} \right\} = \frac{5}{22} \quad (i_1 = 3), \\
S^2 &= \{2\}, \quad (M = 2), \\
c^2 &= \min\left\{ \frac{x_2 - c^0 x_2^*|_{S^0} - c^1 x_2^*|_{S^1}}{x_2^*|_{S^2}} \right\} = \left\{ \frac{\frac{6}{11} - \frac{1}{2} \times \frac{3}{11} - \frac{5}{22} \times \frac{3}{5}}{1} \right\} \\
&= \frac{12 - 3 - 3}{22} = \frac{6}{22} = \frac{3}{11} \quad (i_2 = 2),
\end{aligned}
$$

and we can write x by

$$
\begin{aligned}
x &= \sum_{m=0}^{2} c^m x^*|_{S^m} = c^0 x^*|_{S^0} + c^1 x^*|_{S^1} + c^2 x^*|_{S^2} \\
&= \frac{1}{2}(\frac{6}{11}, \frac{3}{11}, \frac{2}{11}, 0) + \frac{5}{22}(0, \frac{3}{5}, \frac{2}{5}, 0) + \frac{3}{11}(0,1,0,0) = (\frac{3}{11}, \frac{6}{11}, \frac{2}{11}, 0),
\end{aligned}
$$

while for $x = (\frac{1}{3}, \frac{2}{5}, \frac{4}{15}, 0)$, similarly we have

$$
\begin{aligned}
\tilde{S}^0 &= \{1,2,3\}, \quad \tilde{c}^0 = \frac{x_1}{x_1^*|_{S^0}} = \frac{1/3}{6/11} = \frac{11}{18} \quad (i_0 = 1), \\
\tilde{S}^1 &= \{2,3\}, \\
\tilde{c}^1 &= \min\left\{ \frac{x_2 - \tilde{c}^0 x_2^*|_{S^0}}{x_2^*|_{S^1}}, \frac{x_3 - \tilde{c}^0 x_3^*|_{S^0}}{x_3^*|_{S^1}} \right\} = \min\left\{ \frac{\frac{2}{5} - \frac{11}{18} \times \frac{3}{11}}{3/5}, \frac{\frac{4}{15} - \frac{11}{18} \times \frac{2}{11}}{2/5} \right\} \\
&= 5 \min\left\{ \frac{7}{90}, \frac{7}{90} \right\} = \frac{7}{18} \quad (i_1 = 2, 3),
\end{aligned}
$$

and as $M = 1$, we can write x' by

$$\begin{aligned} x' &= \sum_{m=0}^{1} \tilde{c}^m x^*|_{S^m} = \tilde{c}^0 x^*|_{S^0} + \tilde{c}^1 x^*|_{S^1} \\ &= \frac{11}{18}(\frac{6}{11}, \frac{3}{11}, \frac{2}{11}, 0) + \frac{7}{18}(0, \frac{3}{5}, \frac{2}{5}, 0) = (\frac{1}{3}, \frac{2}{5}, \frac{4}{15}, 0). \end{aligned}$$

From above, since

$$\begin{aligned} \mathrm{br}(x) &= \cap_{m=0}^{2} \mathrm{br}(x^*|_{S^m}) = \mathrm{br}(x^*|_{S^2}) = \{2\}, \\ \mathrm{br}(x') &= \cap_{m=0}^{2} \mathrm{br}(x^*|_{\tilde{S}^m}) = \mathrm{br}(x^*|_{\tilde{S}^2}) = \{2, 3\}, \end{aligned}$$

and

$$\mathrm{supp}(x) = \mathrm{supp}(x') = \{1, 2, 3\},$$

it follows that both $\mathrm{br}(x) \subseteq \mathrm{supp}(x)$ and $\mathrm{br}(x') \subseteq \mathrm{supp}(x')$ hold.

Proof of the only if part

We provide a proof of the only if part of Theorem 4.1 based on an induction argument and the oddness property of NE below.

Let us consider any symmetric two-player game $\mathbf{g} = (A, g)$ with TBP and $|A| \geq 2$. For any nonempty subset $S \subseteq A$, we define $\mathbf{g}|_S$ by the *restricted game* of $\mathbf{g}$ where the players choose actions only from S. First, we show:

Lemma 4.3. *Let $\mathbf{g} = (A, g)$ be any symmetric two-player game with TBP and $|A| \geq 2$. Then any restricted game $\mathbf{g}|_S$ with $|S| \leq k(= 2, 3, \ldots, n)$ has a unique symmetric Nash equilibrium which is completely mixed over S.*

Remark 4.2. Any restricted game $\mathbf{g}|_S$ has TBP if $\mathbf{g}$ has TBP.

Proof. We show Lemma 4.3 by induction as follows.

(I) It is obvious for $k = 2$.

(II) Suppose that any restricted game $\mathbf{g}|_S$ with $|S| \leq k(= 2, 3, \ldots, n-1)$ has a unique symmetric NE which is completely mixed over S. This implies that any restricted game $\mathbf{g}|_S$ with $|S| = k' \leq k$ has $2^{k'} - 1$ symmetric NE for any $k' = 1, \ldots, k$. Fix the restricted game $\mathbf{g}|_{S'}$ with $|S'| = k + 1$. Given the set of strategies S', we consider all combinations of restricted games with at least one strategy and at most $k + 1$ strategies, which are subsets of S'. The number of the combinations of the restricted games except for $\mathbf{g}|_{S'}$ is $2^{k+1}-2$ and, by the assumption, the corresponding restricted game to each combination has a unique symmetric NE which is completely mixed over the restricted set of strategies. Together with the property of TBP due to Remark 4.2, $\mathbf{g}|_{S'}$ has at least $2^{k+1}-2$ symmetric NE. Since $\mathbf{g}|_{S'}$ has at most symmetric $2^{k+1}-1$ NE due to Remark 4.1, if $\mathbf{g}|_{S'}$ has more symmetric NE than $2^{k+1}-2$, the only possibility is to have a unique symmetric NE which is completely mixed over S. Since the number of NE, $2^{k+1}-2$, is even, by the oddness property of NE, $\mathbf{g}|_{S'}$ must have the unique symmetric NE which is completely mixed over S'. □

Although it is obvious from Lemma 4.3, we show the only if part of Theorem 4.1.

Proof. From Lemma 4.3, since any restricted game $\mathbf{g}|_S$ with $|S| \leq n-1$ has a unique symmetric NE which is completely mixed over S and the number of the restricted games $\mathbf{g}|_S$ with $|S| \leq n-1$ is 2^n-2, the game $\mathbf{g}|_S$ as $|S| = n$, that is, $\mathbf{g} = (A, g)$ has at least 2^n-2 symmetric NE, which do not include an interior NE in Δ, and, again by Lemma 4.3, the game must have a unique interior NE. Thus, $\mathbf{g} = (A, g)$ has $(2^n-2)+1 = 2^n-1$ symmetric NE. □

Remark 4.3. We can show that any symmetric game with TBP has no asymmetric NE of the game as follows. Assume to the contrary that there is an asymmetric NE, $(x^1, x^2) \in \Delta^2$ with $x^1 \neq x^2$. For (x^1, x^2), there are two cases to consider: (i) $\text{supp}(x^1) = \text{supp}(x^2)$ and (ii) $\text{supp}(x^1) \neq \text{supp}(x^2)$. In Case (i), from the proof of the only if part of Theorem 4.1, we know that for any $S \subseteq A$, there is a unique symmetric NE, $x^*|_S$, which is completely mixed over S. Together with this and TBP, $x^1 \neq x^2$ implies that there exists some $j = 1, 2$ such that $\text{br}(x^j) \subsetneq \text{supp}(x^j)$. Although (x^1, x^2) is an equilibrium, $\text{br}(x^j) \subsetneq \text{supp}(x^j)$ implies that some action in x^j of player j should not be chosen, a contradiction. In Case (ii), without loss of generality, there is some $j \in \text{supp}(x^1)$ such that $j \notin \text{supp}(x^2)$. By TBP, for player 1, strategy j is not a best response to player 2's mixed strategy x^2, a contradiction because x^1 must be a best response to x^2.

Extended Characterization: Why do we assume the same size of set of strategies?

Suppose that a game has $|A_1| \neq |A_2|$. Then we show that $|A_1| \neq |A_2|$ can violate GTBP by a counterexample, and therefore we assume that $|A_1| = |A_2|$ when considering games with GTBP. The counterexample given here is a slightly modified version of the game given by Quint and Shubik (2002, Remark 1) where the order of strategies for player 2 is changed.

		Player 2			
		1	2	3	4
Player 1	1	4,4	0,1	2,2	0,3
	2	0,0	4,11/6	0,3/2	2,1

Table 4.5: A 2×4 coordination game.

Example 4.3 (Asymmetric coordination game)**.** Let us consider the 2×4 coordination game given by Table 4.5 where $|A_1| = 2 < 4 = |A_2|$. We can find 5 NE: two symmetric pure strategy profiles and three asymmetric strategy profiles,

$$((1/3, 2/3), (0, 0, 1/2, 1/2)), ((1/2, 1/2), (1/3, 0, 0, 2/3)),$$
$$((1/4, 3/4), (0, 1/3, 2/3, 0)).$$

One can easily observe that GTBP does not hold. For player 2 and some mixed strategy $x^2 \in \Delta(A_2)$, the player 1's pure strategy best responses, $\mathrm{br}_1(x^2)$, are not included in $\mathrm{supp}(x^2)$. For example, for the equilibrium $((1/3, 2/3), (0, 0, 1/2, 1/2))$, $\mathrm{br}_1(x^2) \cap \mathrm{supp}(x^2) = \emptyset$. Note that any two of the above given three mixed strategy NE, x and $\tilde{x}$, in the game satisfy $\mathrm{supp}(x^1) = \mathrm{supp}(\tilde{x}^1)$ but not $\mathrm{supp}(x^2) = \mathrm{supp}(\tilde{x}^2)$.

Proof of Theorem 4.2

We basically arrange the argument used in the proof for Theorem 4.1 to show Theorem 4.2.

Proof of the if part

Proof. It follows from the same argument used in the proof of the if part of Theorem 4.1 for some player $i = 1, 2$ and some $x^j \in \Delta$ for $j \neq i$. □

Proof of the only if part

Proof. Fix any game $\mathbf{g} = (\{1, 2\}, A, (g^i)_{i=1,2})$ where $A = \{1, 2, \ldots, n\}$ and GTBP holds. For any player $i = 1, 2$, we construct the symmetric two-player game by using the player i's set of strategies A and payoff function g^i. We denote it by $\mathbf{g}^i = (A, g^i)$. Since two players have the same set of strategies A and payoff function g^i in game $\mathbf{g}^i$, by GTBP regarding player i, we can show that $\mathbf{g}^i$ satisfies TBP. From Theorem 4.1, if $\mathbf{g}^i$ has TBP, it has $2^n - 1$

		Player 2		
		1	2	3
	1	a_1, b_1	0,0	0,0
Player 1	2	0,0	a_2, b_2	0,0
	3	0,0	0,0	a_3, b_3

Table 4.6: A 3×3 pure-coordination game.

symmetric NE. Note that those symmetric NE are different in terms of supports. As any subset of strategies $S \subseteq A$, we consider the symmetric NE $(x^i, x^i) \in \Delta$ with $\text{supp}(x^i) = S$ in $\mathbf{g}_h$. We write this equilibrium strategy by $x^i|_S$. Since $x^i|_S$ is the equilibrium strategy with $\text{supp}(x^i|_S) = S$ in $\mathbf{g}^i$ for any $i = 1, 2$, strategy profiles $(x^2|_S, x^1|_S)$ is a NE with the same support S for players 1 and 2 in the original game $\mathbf{g}$. For any $i = 1, 2$, let the set of NE in $\mathbf{g}^i$ be NE^i. Since $|\text{NE}^i| = 2^n - 1$, we can easily see that the set of

$$\{(x^2|_S, x^1|_S) \mid S \subseteq A, \text{supp}(x^1|_S) = \text{supp}(x^2|_S) = S\}$$

also has the same number of elements, $2^n - 1$.

From above, if $\mathbf{g}$ has GTBP, $\mathbf{g}$ has $2^n - 1$ NE, each of which gives the same support for two players that is distinct from those of other NE. $\square$

The main point in the above proof is to construct a NE with the same support for two players in the original game by using a NE strategy with the same support in a "fictitious" symmetric game for each player. Note that we need the restrictions on NE in terms of supports, otherwise the equivalence does not hold in general (see below for details).

In the following, we give three examples regarding Theorem 4.2.

Example 4.4 (Asymmetric pure-coordination game)**.** Let us consider the 3×3 pure-coordination game given by Table 4.6 where a_i and b_i for $i = 1, 2, 3$ are positive constants. Since, in the pure coordination game, positive payoffs are given to players if their strategies are the same and zero otherwise, we can easily see that the game satisfies GTBP. We can find $2^3 - 1 (= 7)$ NE including asymmetric ones: three symmetric pure strategy profiles and four mixed strategy profiles,

$$\begin{aligned}
&(1/(b_1 + b_2)(b_2, b_1, 0), 1/(a_1 + a_2)(a_2, a_1, 0)), (1/(b_1 + b_3)(b_3, 0, b_1), 1/(a_1 + a_3)(a_3, 0, a_1)) \\
&(1/(b_2 + b_3)(0, b_3, b_2), 1/(a_2 + a_3)(0, a_3, a_2)), \\
&(1/(b_1 b_2 + b_1 b_3 + b_2 b_3)(b_2 b_3, b_1 b_3, b_1 b_2), 1/(a_1 a_2 + a_1 a_3 + a_2 a_3)(a_2 a_3, a_1 a_3, a_1 a_2)).
\end{aligned}$$

The game with GTBP has $7(=2^3-1)$ NE, and therefore it is consistent with Theorem 4.2.

Remark 4.4. Theorem 4.2 holds for all $n \times n$ pure coordination games and slightly perturbed ones with respect to payoffs as in Example 4.4.[20]

Example 4.5 (Bertrand duopoly market)**.** We consider a Bertrand competition with convex costs, which has been theoretically investigated by Dastidar (1995) and experimentally by Abbink and Brandts (2008) and Argenton and Müller (2012).[21] Here, we assume that there are two firms in a market where each of two firms faces a linear demand and quadratic cost function per unit and then competes in prices, and additionally that the prices are discrete.[22] This Bertrand duopoly market is described by a two-player game $\mathbf{g} = (A, (g_1, g_2))$ where $A = \{p_1, \ldots, p_n\}$ for $0 < p_1 < p_2 < \cdots < p_n$ and $g^i : A^2 \to \mathbb{R}$ is the firm i's payoff function such that given a linear demand function and quadratic cost function of firm i, $D^i : A^2 \to \mathbb{R}$ and $C^i : A^2 \to \mathbb{R}$, for any price profile $p = (p^1, p^2) \in A^2$,

$$g^i(p) \quad = \quad p^i D^i(p) - C^i(p)$$

where for some $a > p_n$, $c^i > 0$, and $j \neq i$,

$$D^i(p) = \begin{cases} a - p^i & \text{if } p^i < p^j, \\ \frac{1}{2}(a - p^i) & \text{if } p^i = p^j, \\ 0 & \text{if } p^i > p^j, \end{cases} \qquad C^i(p) = c^i(D^i(p))^2.$$

Note that the assumption $a > p_n$ is given because each firm's demand under any price profile is non-negative, otherwise a price with $a \leq p_n$ cannot give a positive payoff and so it is never chosen.

Let us consider the special case of $A = \{1, 2, 3\}$ and then find an equivalent condition to GTBP in the game. To show that $\mathrm{br}(x) \subseteq \mathrm{supp}(x)$ holds for any $x \in \Delta$, we first consider the condition under which the game is a coordination game, that is, $\mathrm{br}(x) \subseteq \mathrm{supp}(x)$ holds for any $x \in \Delta(\{k\})$ as $k \in A$. This is equivalent to the conditions of $g^i_{11} > \max\{g^i_{21}, g^i_{31}\} = 0$, $g^i_{22} > \max\{g^i_{12}, 0\}$, and $g^i_{33} > \max\{g^i_{13}, g^i_{23}\}$. Since $a > p_3 = 3$, they are reduced to

$$4 < a, \qquad \frac{2(a+1)}{3a^2 - 10a + 7} < c^i < \frac{2}{a-1}. \tag{4.10}$$

[20]Note that every pure coordination game is nondegenerate.

[21]For instance, price competitions with convex costs are relevant due to adjustment costs of productions in utilities and telecommunications industries (Green and Newbery, 1992; Green, 1996; Wolfram, 1998; Armstrong and Porter, 2007; Hortaçsu and Puller, 2008; Janssen and Karamychev, 2010, among others).

[22]This framework is used by Argenton and Müller (2012) in their experiment.

	1	2	3
1	2,1	2,0	2,0
2	1,0	3,2	3,0
3	0,2	2,0	4,3

Table 4.7: An asymmetric 3×3 coordination game.

In fact, one can easily see that the above derived condition (4.10) also guarantees the condition under which $\mathrm{br}(x) \subseteq \mathrm{supp}(x)$ holds for any $x \in \Delta(\{k, h\})$ as $k, h \in A$ due to the specific payoff structure of this game, and furthermore this implies that the game has a unique interior NE as well. Thus, the game $\mathbf{g} = (A, (g_1, g_2))$ with $A = \{1, 2, 3\}$ has GTBP if and only if the condition (4.10) holds for each $i = 1, 2$. From this, for instance, when $a = 5$, since the right hand side condition of (4.10) is given by $3/8 < c^i < 1/2$, under $(c^1, c^2) = (3/7, 4/9)$, the game satisfies Condition (4.10), that is, the game has GTBP, one can see that there are $2^3 - 1 = 7$ NE in the game, which is consistent with Theorem 4.2.

Games with $2^n - 1$ NE without the restriction on support of NE

Let us consider an asymmetric game $\mathbf{g}$ with $2^n - 1$ NE and then will see that $\mathbf{g}$ does not satisfy GTBP in general. In fact, to guarantee GTBP, $\mathbf{g}$ must be nondegenerate even in the case of $n = 3$ and also must satisfy an additional condition in the case of $n \geq 4$. To show this, we give several counterexamples below.

Case of $n = 3$

First of all we consider the case of three strategies to show that a game must be nondegenerate.

Example 4.6 (Degenerate 3×3 coordination game)**.** We consider the asymmetric 3×3 coordination game given by Table 4.7. From Table 4.7, one can easily see that the game has 3 symmetric pure strategy NE. The best response regions of the game are given by Figure 4.3. By the computation, we can find 4 mixed strategy NE:

$$((2/3, 1/3, 0), (1/2, 1/2, 0)), ((1/2, 0, 1/2), (1/2, 0, 1/2)), ((0, 3/5, 2/5), (0, 1/2, 1/2)),$$
$$((2/7, 3/7, 2/7), (1/2, 0, 1/2)).$$

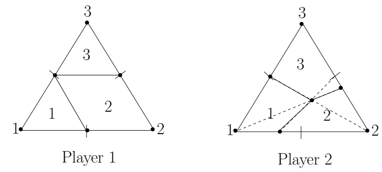

Figure 4.3: The best response regions of the game.

So, the game has $7(= 2^3 - 1)$ NE. But the NE $(x^1, x^2) = ((2/7, 3/7, 2/7), (1/2, 0, 1/2))$ satisfies

$$3 = |\text{supp}(x^1)| \neq |\text{supp}(x^2)| = 2.$$

Since $\text{br}_1(x^2) = \{1, 2, 3\}$ and $\text{supp}(x^2) = \{1, 3\}$, this implies that $\text{supp}(x^2) \subsetneq \text{br}_1(x^2)$, violating GTBP. This is also easily seen by Figure 4.3. Here, since $|\text{br}_1(x^2)| > |\text{supp}(x^2)|$ holds for $x^2 = (1/2, 0, 1/2)$, the game is degenerate.

From this example, if a game with $2^3 - 1$ NE is degenerate, it does not necessarily satisfy GTBP. But we can show that if a game with $2^3 - 1$ NE is nondegenerate, it has GTBP.[23] This implies that the same statement of Theorem 4.1 for (symmetric) TBP also holds for GTBP in the case of $A = \{1, 2, 3\}$.

Case of $n \geq 4$

Next we consider a nondegenerate asymmetric 4×4 game below and then show by an example that the game has $2^4 - 1$ NE but does not satisfy GTBP. This implies that the result in the case of three strategies cannot be carried over to the case of n strategies with $n \geq 4$ in general.

Example 4.7 (Nondegenerate asymmetric 4×4 game)**.** We consider the asymmetric 4×4 game given by Table 4.8. We can find the following 15 NE:[24]

$|\{(x^1, x^2) \in \text{NE} \mid i = 1, 2, |\text{supp}(x^i)| = 1\}| = 2:$

$$((0, 1, 0, 0), (0, 0, 0, 1)), ((0, 0, 1, 0), (1, 0, 0, 0)).$$

[23] The proof is available from the author upon request.

[24] Keiding (1997) and McLennan and Park (1999) show that $2^4 - 1 = 15$ is the maximal number of NE for all 4×4 games.

	1	2	3	4
1	-81,72	36,36	-126,17	126,-3
2	-180,-180	72,-81	-333,-30	297,20
3	20,297	-3,126	42,42	-33,-33
4	-30,-333	17,-126	-33,-33	42,42

Table 4.8: An asymmetric 4×4 game.

$|\{(x^1, x^2) \in \mathrm{NE} \mid i = 1, 2, |\mathrm{supp}(x^i)| = 2\}| = 10:$

$$((11/15, 4/15, 0, 0), (4/15, 11/15, 0, 0), \tag{4.11}$$
$$((0, 0, 1/2, 1/2), (0, 0, 1/2, 1/2)), \tag{4.12}$$
$$\begin{array}{rr} ((51/70, 19/70, 0, 0), (0, 23/27, 4/27, 0), & ((23/27, 0, 0, 4/27), (19/70, 51/70, 0, 0)), \\ ((2/7, 5/7, 0, 0), (0, 0, 19/42, 23/42)), & ((93/112, 0, 0, 19/112), (0, 93/112, 19/112, 0)), \\ ((0, 0, 23/42, 19/42), (2/7, 5/7, 0, 0)), & ((0, 0, 31/59, 28/59), (0, 15/19, 4/19, 0)), \\ ((15/19, 0, 0, 4/19), (0, 0, 28/59, 31/59)), & ((0, 33/53, 20/53, 0), (33/53, 0, 0, 20/53)). \end{array}$$

$|\{(x^1, x^2) \in \mathrm{NE} \mid i = 1, 2, |\mathrm{supp}(x^i)| = 3\}| = 2:$

$$((60/109, 39/109, 10/109, 0), (31/74, 0, 17/111, 95/222)),$$
$$((0, 31/74, 95/222, 17/111), (39/109, 60/109, 0, 10/109)).$$

$|\{(x^1, x^2) \in \mathrm{NE} \mid i = 1, 2, |\mathrm{supp}(x^i)| = 4\}| = 1:$

$$((5/11, 4/11, 5/33, 1/33), (4/11, 5/11, 1/33, 5/33)). \tag{4.13}$$

Since any NE (x^1, x^2) of the game satisfies $|\mathrm{br}_i(x^j)| \leq |\mathrm{supp}(x^j)|$ for any distinct $i, j = 1, 2$, the game is nondegenerate. This nondegenerate game has $15 (= 2^4 - 1)$ NE. But the game does not satisfy GTBP because the game cannot be a coordination game even after any permutation of strategies. Note that $\mathrm{supp}(x^1) = \mathrm{supp}(x^2)$ does not hold for all NE (x^1, x^2) except for (4.11)–(4.13). Also note that for some NE (x^1, x^2) and $(\tilde{x}^1, \tilde{x}^2)$, $\mathrm{supp}(x^i) = \mathrm{supp}(\tilde{x}^i)$ holds for some $i = 1, 2$ while $\mathrm{supp}(x^j) \neq \mathrm{supp}(\tilde{x}^j)$ holds for $j \neq i$.

Example 4.7 implies that a nondegenerate $n \times n$ game with $2^n - 1$ NE may not be a coordination game. So, in order to show that a nongedenerate $n \times n$ game with $2^n - 1$ NE has GTBP, the game must be at least a coordination game. But even under the assumption that a game with $2^n - 1$ NE is a nondegenerate coordination game, the game may not satisfy GTBP because the assumption of coordination game is not enough to satisfy $\mathrm{br}_i(x^j) \subseteq \mathrm{supp}(x^j)$ for any distinct $i, j = 1, 2$ and any $x^j \in \Delta$.

Proof of Lemma 4.1

Proof. Consider a game with TBP. Suppose that there is a GPRD–equilibrium $(i, i) \in A^2$ in the game. By TBP, the GPRD equilibrium satisfies $\mathrm{br}(x) = \{i\}$ for any $x \in \mathring{\Delta}(\{i, j\})$ where $j \neq i$ is any other strategy in A and $x_i = x_j = 1/2$. Since any pure strategy best response set is convex in Δ, this implies that $\mathrm{br}(x) = \{i\}$ holds for any $x \in \Delta$ with $x_i \geq 1/2$, meaning that (i, i) is a 1/2–dominant equilibrium. □

Proof of Proposition 4.1

To show Proposition 4.1, we introduce two notations for convenience as follows. For $x, x' \in \Delta$, we write $x \succ x'$ if x stochastically dominates x', that is, for any $i \in A$, $\sum_{i \leq j \leq n} x_j \geq \sum_{i \leq j \leq n} x'_j$ with strict inequality for at least some i. When we consider opponent's mixed strategies $x \in \mathring{\Delta}(\{i, j\})$ with support size 2 for any two distinct pure strategies $i, j \in A$, we write x^{ij} instead of x for clarity. Given these notations, we show Proposition 4.1 by using the property on supermodular games $\mathbf{g} = (A, g)$ that each player's best response correspondence is non-decreasing in opponent's strategies: for any $x, x' \in \Delta$ with $x \succ x'$, $\min \mathrm{br}(x) \geq \min \mathrm{br}(x')$ and $\max \mathrm{br}(x) \geq \max \mathrm{br}(x')$ where $\min \mathrm{br}(x)$ is the lowest pure strategy best response against opponent's mixed strategy x and $\max\mathrm{br}(x)$ the highest one.[25]

Proof. For the belief $x^{1n} \in \mathring{\Delta}(\{1, n\})$ with $x_1^{1n} = x_n^{1n} = 1/2$ and $x_i^{1n} = 0$ for any $i \in A \backslash \{1, n\}$, TBP implies that

$$\mathrm{br}(x^{1n}) \subseteq \{1, n\}.$$

By the assumption of $g_{11} - g_{n1} \neq g_{nn} - g_{1n}$, $\mathrm{br}(x^{1n}) = \{1\}$ or $\{n\}$. Suppose that $\mathrm{br}(x^{1n}) = \{1\}$. For any $i \in A \backslash \{1\}$, let $x^{1i} \in \mathring{\Delta}(\{1, i\})$ be any belief with $x_1^{1i} = 1/2$ and $x_i^{1i} = 1/2$. Since $x^{1n} \succ x^{1i}$ for any $i \in A \backslash \{n\}$, by supermodularity, it follows that

$$\{1\} = \max \mathrm{br}(x^{1n}) \geq \max \mathrm{br}(x^{1i}), \tag{4.14}$$

[25] For the detail, see Milgrom and Roberts (1990); Milgrom and Shannon (1994); Topkis (1998); Vives (1990, 2001), among others.

which implies that for any $i \in A$ and any belief $x^{1i} \in \mathring{\Delta}(\{1, i\})$,

$$\mathrm{br}(x^{1i}) = \{1\}. \tag{4.15}$$

The condition (4.15) implies that strategy 1 pairwise risk-dominates any distinct strategy i for any $i \in A \backslash \{1\}$, that is, the strategy profile (1,1) is a GPRD-equilibrium. By Lemma 4.1, $(1, 1)$ is a 1/2–dominant equilibrium as well.

To show the uniqueness, suppose that there are two 1/2–dominant equilibria, (i, i) and (j, j). This gives the following two inequalities,

$$\begin{aligned}
\frac{1}{2}g_{ii} + \frac{1}{2}g_{ij} > \frac{1}{2}g_{jj} + \frac{1}{2}g_{ji}, \\
\frac{1}{2}g_{jj} + \frac{1}{2}g_{ji} > \frac{1}{2}g_{ii} + \frac{1}{2}g_{ij},
\end{aligned}$$

a contradiction.

Similarly, if $\mathrm{br}(x^{1n}) = \{n\}$, we can show that (n, n) is the unique 1/2–dominant equilibrium. □

Proof of Proposition 4.2

Proof. Suppose that there is a potential function v such that (i, i) is a unique potential maximizer. For any given $j \neq i$, we have $v_{ii} > v_{jj}$ by definition of potential maximizer and $v_{ij} = v_{ji}$ due to symmetry of potential function, thereby leading to

$$v_{ii} - v_{jj} > 0 = v_{ji} - v_{ij}.$$

Replacing v_{jj} with v_{ij}, the inequality derived above is rewritten by $v_{ii} - v_{ji} > v_{jj} - v_{ij}$. By definition of potential function, this gives $g_{ii} - g_{ji} > g_{jj} - g_{ij}$, thereby leading to

$$\frac{1}{2}(g_{ii} + g_{ij}) > \frac{1}{2}(g_{ji} + g_{jj}).$$

Thus, the unique potential maximizer (i, i) is a GPRD–equilibrium. By Lemma 4.1, (i, i) is also a 1/2–dominant equilibrium. □

Proof of Proposition 4.3

To show Proposition 4.3, we first introduce the solution concept of monotone potential maximizer (Morris and Ui, 2005), which is a generalization of potential maximizer as LP-maximizer.[26] Then, we show that an LP-maximizer with constant weights (introduced in the proof of Proposition 4.3) is an MP-max. Following Morris and Ui (2005), a (strict) MP-max is unique in any generic supermodular game because it is robust to incomplete information in the sense of Kajii and Morris (1997) and the robust equilibrium is unique.[27] While on the other hand, Proposition 4.1 tells us that any symmetric two-player supermodular game with TBP where $g_{1n} - g_{n1} \neq g_{nn} - g_{1n}$ always has a unique 1/2–dominant equilibrium, which is also robust to incomplete information (Kajii and Morris, 1997). Taken together, if an LP-maximizer with constant weights exists in a symmetric two-player supermodular game with TBP, both of the LP-maximizer and the 1/2–dominant equilibrium must be equivalent.

Thus, to prove Proposition 4.3, it is enough to show that an LP-maximizer with constant weights implies an MP-max. In doing so, below we introduce (i) the definition of MP-max and (ii) the equivalent definition of LP-maximizer, which is useful for the proof.

Fix any symmetric game $\mathbf{g} = (A, g)$ with $A = \{1, , \ldots, n\}$. For a function $f : A^2 \to \mathbb{R}$, a mixed strategy $x \in \Delta$, each player $i = 1, 2$, and a non-empty subset of strategies $S \subseteq A$, let

$$\mathrm{br}^i_f(x \mid S) \quad = \quad \arg\max_{s_i \in S} \sum_{j \in A} x_j f_{s_i j}.$$

MP-max is defined for a general class of games, but our interest lies in a simple class of games, and therefore we use its simplified version and refinement, strict MP-max, following Oyama et al. (2008) below.

Definition 4.8. A pure strategy profile $s^* = (s_i^*, s_j^*) \in A^2$ is a *monotone potential maximizer* (MP-max) of $\mathbf{g}$ if there exists a function $v : A^2 \to \mathbb{R}$ with $v(s^*) > v(s)$ for all

[26] We pay attention to a specific LP-maximizer (with constant weights) that is a unique solution if any (Okada and Tercieux, 2012, Proposition 1), but there may exist multiple LP-maximizers in general. See Oyama and Takahashi (2009, Example 1). In fact, they correct the statement of Frankel et al. (2003) in such a way that a (strict) LP-maximizer of a supermodular game with own-action *concavity* instead of own-action *quasiconcavity* is chosen as the noise-independent selection in the global game method (Carlsson and van Damme, 1993).

[27] Similarly, Oyama et al. (2008) show that any "generic" supermodular game has at most one MP-max through the argument of perfect foresight dynamics. They also show that if a supermodular game satisfies own-action concavity, a supermodular game has at most one LP-maximizer.

$s \in A^2 \backslash \{s^*\}$ such that for each $i = 1, 2$ and any $x \in \Delta$,

$$\begin{aligned}
\min \mathrm{br}_v^i(x \mid \{1, \ldots, i^*\}) &\leq \max \mathrm{br}_g^i(x \mid \{1, \ldots, s_i^*\}), \\
\max \mathrm{br}_v^i(x \mid \{s_i^*, \ldots, n\}) &\geq \min \mathrm{br}_g^i(x \mid \{s_i^*, \ldots, n\}).
\end{aligned}$$

Such a function v is called a *monotone potential function* (MP-function) for s^*.

A pure strategy profile $s^* = (s_1^*, s_2^*) \in A^2$ is a *strict MP-max* of $\mathbf{g}$ if there exists a function $v : A^2 \to \mathbb{R}$ with $v(s^*) > v(s)$ for all $s \neq s^*$ such that for each $i = 1, 2$ and any $x \in \Delta$,

$$\begin{aligned}
\min \mathrm{br}_v^i(x \mid \{1, \ldots, s_i^*\}) &\leq \min \mathrm{br}_g^i(x \mid \{1, \ldots, s_i^*\}), \\
\max \mathrm{br}_v^i(x \mid \{s_i^*, \ldots, n\}) &\geq \max \mathrm{br}_g^i(x \mid \{s_i^*, \ldots, n\}).
\end{aligned}$$

Such a function v is called a *strict MP-function* for s^*.

A (strict) MP-max is a (strict) NE, and a potential maximizer is a strict MP-max. Under a generic choice of payoffs, an MP-max is a strict MP-max. A supermodular game can have at most one strict MP-max (Oyama et al., 2008).

Next, we introduce the following equivalent definition for an LP-maximizer (Morris and Ui, 2005, see Definition 11 and Lemma 9).

Definition 4.9. A pure strategy profile $s^* = (s_1^*, s_2^*)$ is an LP-maximizer with a local potential function v if

(i) for any player $i = 1, 2$, any strategy $s_i < s_i^*$, and any $x \in \Delta$ such that $\sum_{j \in A} x_j v_{s_i j} < \sum_{j \in A} x_j v_{s_i + 1 j}$,

$$\sum_{j \in A} x_j g_{s_i j} \leq \sum_{j \in A} x_j g_{s_i + 1 j}. \tag{4.16}$$

(ii) for any player $i = 1, 2$, any strategy $s_i > s_i^*$, and any $x \in \Delta$ such that $\sum_{j \in A} x_j v_{s_i j} < \sum_{j \in A} x_j v_{s_i - 1 j}$,

$$\sum_{j \in A} x_j g_{s_i j} \leq \sum_{j \in A} x_j g_{s_i - 1 j}. \tag{4.17}$$

We show that an LP-maximizer with constant weights is an MP-max.

Proof. We define $\underline{s}_i \equiv \min \mathrm{br}_v^i(x \mid \{1,\ldots,s_i^*\})$ such that $\sum_{j\in A} x_j v_{s_i j} < \sum_{j\in A} x_j v_{\underline{s}_i - 1 j}$ for every $1 \le s_i < \underline{s}_i$ (if any). By (4.16), for any $1 \le s_i < \underline{s}_i$,

$$\sum_{j\in A} x_j g_{s_i j} \le \sum_{j\in A} x_j g_{\underline{s}_i j}. \tag{4.18}$$

Thus, (4.18) implies that $\underline{s}_i \le \min \mathrm{br}_g^i(x \mid \{1,\ldots,s_i^*\})$, that is,

$$\min \mathrm{br}_v^i(x \mid \{1,\ldots,s_i^*\}) \le \min \mathrm{br}_g^i(x \mid \{1,\ldots,s_i^*\}). \tag{4.19}$$

Similarly, by (4.17), we can show that

$$\max \mathrm{br}_v^i(x \mid \{s_i^*,\ldots,n\}) \ge \max \mathrm{br}_g^i(x \mid \{s_i^*,\ldots,n\}). \tag{4.20}$$

Since (4.19) and (4.20) satisfy the definition of MP-max, we have shown that an LP-maximizer with constant weights, s^*, is an MP-max. □

Proof of Lemma 4.2

Proof. Fix any $l, m, h \in A$ with $l < m < h$ and let $S = \{l, h\}$. For any $x \in \mathring{\Delta}(S)$ and $m \in A$ with $l < m \le h$,

$$\begin{aligned}
\sum_{i=1}^n x_i g_{li} - \sum_{i=1}^n x_i g_{mi} = \quad & \big(x_l(a\min\{l,l\} - bl + c) + (1-x_l)(a\min\{l,h\} - bl + c)\big) \\
& - \big(x_l(a\min\{m,l\} - bl + c) + (1-x_l)(a\min\{m,h\} - bm + c)\big) \\
= \quad & x_l b(m-l) + (1-x_l)(a-b)(l-m) \\
= \quad & a(m-l)\big(x_l - \frac{a-b}{a}\big),
\end{aligned}$$

implying that for any $x \in \mathring{\Delta}(S)$ with $S = \{l, h\}$,

$$\arg\max_{m\in\{l,l+1,\ldots,h\}} \sum_{i=1}^n x_i g_{mi} = \begin{cases} \{l\} & \text{if } x_l > x_l^* = (a-b)/a \\ \{l, l+1, \ldots, h\} & \text{if } x_l = x_l^* \\ \{h\} & \text{if } x_l < x_l^*. \end{cases}$$

Thus, $m \notin \mathrm{br}(x)$ holds for any $l, m, h \in A$ with $l < m < h$ and $x \in \Delta(\{l,h\}) \backslash \{x^*\}$. □

Proof of Supermodularity of the Minimum-Effort Game

For any $i, i', j, j' \in A$ with $i > i'$ and $j > j'$,

$$\begin{aligned} & (g(i,j) - g(i',j)) - (g(i,j') - g(i',j')) \\ = \; & a((\min\{i,j\} - \min\{i',j\}) - (\min\{i,j'\} - \min\{i',j'\})) \\ \geq \; & 0 \end{aligned}$$

where the last inequality follows from the property that $\min\{i,j\} - \min\{i',j\})$ is weakly increasing in $j \in A = \{1, \ldots, n\}$. This proves that the minimum-effort game is supermodular. Below we show the above used weakly increasing property by considering all six possible cases.

Case 1: $i > i' \geq j > j'$.

$$\min\{i,j\} - \min\{i',j\} = j - j = j' - j' = \min\{i,j'\} - \min\{i',j'\}.$$

Case 2: $i \geq j > i' \geq j'$.

$$\min\{i,j\} - \min\{i',j\} = j - i' > 0 = \min\{i,j'\} - \min\{i',j'\}.$$

Case 3: $i \geq j > j' > i'$.

$$\min\{i,j\} - \min\{i',j\} = j - i' > j' - i' = \min\{i,j'\} - \min\{i',j'\}.$$

Case 4: $j > i \geq j' > i'$.

$$\min\{i,j\} - \min\{i',j\} = i - i' \geq j' - i' = \min\{i,j'\} - \min\{i',j'\}.$$

Case 5: $j > i > i' \geq j'$.

$$\min\{i,j\} - \min\{i',j\} = i - i' > 0 = j' - j' = \min\{i,j'\} - \min\{i',j'\}.$$

Case 6: $j > j' > i > i'$.

$$\min\{i,j\} - \min\{i',j\} = i - i' = \min\{i,j'\} - \min\{i',j'\}.$$

Remark 4.5. We can straightforwardly extend the proof shown above to the case of multiple players.

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Abstract

Chapter 2: "Intermediary Search for Suppliers in Procurement Auctions."

Entry is important in procurement auctions. Whether firms enter depends crucially on how cheap they can get their inputs from suppliers. In many procurement auctions, entrants are intermediaries who search for the best (or the cheapest) suppliers, but suppliers charge their prices strategically. The market can break down due to supplier induced entry deterrence as long as intermediary search is costly. When both the entry and the search costs are small, however, there is an alternative market outcome with price dispersion. This dispersed price equilibrium is inefficient due to excessive entry. The procurement cost may rise as the number of bidders increases.

Chapter 3: "The Double Diamond Paradox."

We study vertical relations in markets with consumer and retailer search. Retailers search to learn manufacturers' prices. We obtain three important new results. First, we explain why empirical distributions of retail prices are bimodal, with a regular price and a sales price. Second, under competitive conditions (many retailers or small consumer search cost) social welfare is significantly smaller than in the double marginalization outcome. Manufacturers' regular price is significantly above the monopoly price squeezing retailers' markups and providing an alternative explanation for incomplete cost pass-through. Finally, by randomizing to induce active consumer search, manufacturers can increase their profits.

Chapter 4: "Games with the Total Bandwagon Property."

We consider the class of two-player symmetric $n \times n$ games with the total bandwagon property (TBP) introduced by Kandori and Rob (1998). We show that a game has TBP if and only if the game has $2^n - 1$ symmetric Nash equilibria. We extend this result to bimatrix games by introducing the generalized TBP. This sheds light on the (wrong) conjecture of Quint and Shubik (1997) that any $n \times n$ bimatrix game has at most $2^n - 1$ Nash equilibria. As for an equilibrium selection criterion, I show the existence of a 1/2–dominant equilibrium for two subclasses of games with TBP: (i) supermodular games; (ii) potential games. As an application, we consider the minimum-effort game, which does not satisfy TBP, but is a limit case of TBP.

Zusammenfassung

Kapitel 2: ,,Intermediär Suche nach Zulieferern in Ausschreibungsauktionen."

Partizipation ist eine wichtige Komponente von Ausschreibungsauktionen. Ob Unternehmen an Ausschreibungen teilnehmen hängt essentiell davon ab, wie günstig sie Zwischengüter von Zulieferern beziehen können. In vielen Ausschreibungen sind teilnehmende Unternehmen Intermediäre, die den besten (bzw. günstigsten) Zulieferer erst suchen müssen. Gleichzeitig wählen Zulieferer ihre Preise strategisch. Wenn die Suche der Intermediäre kostspielig ist, kann der Markt durch Abschottung von Seiten der Zulieferer zusammenbrechen. Wenn die Suchkosten der Intermediäre hingegen gering sind und Eintritt nicht zu teuer ist, gibt es ein alternatives Marktergebnis mit Preisdispersion. Das entsprechende Gleichgewicht ist aufgrund von exzessivem Firmeneintritt ineffizient. Die Ausschreibungskosten können mit der Anzahl an Bietern ansteigen.

Kapitel 3: ,,Das Doppel Diamond Paradox."

Wir untersuchen vertikale Marktbeziehungen in Märkten mit Konsumenten- und Händlersuche. Händler suchen, um die Preise von Herstellern zu erfahren. Wir erhalten drei neue wichtige Resultate. Erstens erklären wir, warum empirische Preisverteilungen im Einzelhandel bimodal sind, mit einem regulären Preis und Abverkaufspreis. Zweitens ist die Gesamtwohlfahrt unter kompetitiven Verhältnissen (viele Händler oder geringe Konsumentensuchkosten) signifikant geringer als im sogenannten "double marginalization" Marktergebnis. Die regulären Herstellerpreise liegen deutlich über dem Monopolpreis und drücken die Händlermargen, was eine alternative Erklärung für eine unvollständige Kostenweitergabe liefert. Drittens können Hersteller durch Randomisierung ihrer Preise aktive Konsumentensuche induzieren und ihre Profite erhöhen.

Kapitel 4: ,,Spiele mit dem Mitläufer-Effekt"

Wir betrachten die Klasse der symmetrischen 2-Personen $n \times n$ Spiele, die den von Kandori und Rob (1988) eingeführten Mitläufer-Effekt ("total bandwagon property", TBP) aufweisen. Wir zeigen, dass ein symmetrisches $n \times n$ Spiel genau dann TBP erfüllt, wenn es $2^n - 1$ symmetrische Nashgleichgewichte besitzt. Wir bringen dies in Zusammenhang mit der (falschen) Vermutung von Quint und Shubik (1997), dass ein $n \times n$ Bimatrixspiel höchstens $2^n - 1$ Gleichgewichte hat. Zum Problem der Gleichgewichtsauswahl in solchen Spielen wird die Existenz eines 1/2–dominanten Gleichgewichts für zwei Teilklassen von

Spielen mit TBP gezeigt: (i) supermodulare Spiele, (ii) Potentialspiele. Als Anwendung dieser Theorie bringen wir das "minimum-effort" Spiel, das zwar selber nicht TBP erfüllt, aber Grenzfall von Spielen mit dieser Eigenschaft ist.

Curriculum Vitae

Education

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Publications

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