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„The annuity puzzle“

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Frank Walzer

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1. Introduction

Financing retirement is one of the greatest challenges each government in the world faces. It has achieved a higher priority on the agenda of politicians, especially in the last years, because the baby boom generation entered the retirement phase (Brown, 2004). The topic increased his significance, not only for policymakers, but for many researchers who reinforced their work on it and their approaches have been yield to new reforms and regulations. Some countries as Australia and Mexico, England and China, Chile and Sweden (Feldstein, 1998, Feldstein and Siebert, 2002) already adopted their pension system with reforms (e.g. defined contribution plan) and all performed well afterwards.¹ The US policy also changed their pension system in the past 25 years by exchanging the defined benefit (DB) plans, which were paid out by annuities, for defined contribution (DC) plans, which tends more to invest the savings in bonds or lump-sum pensions (Brown, 2001). However, the US reforms have not achieved the same success as for example in Australia. Moreover, the shift increased the lack of annuities in the portfolios of the retirees, which aggravated the problem of the “annuity puzzle”. It is a significant expression in research literature for pension systems and it is linked to the apparent under-annuitization of households. Despite the theoretical findings in research literature to increase the annuitization level, people invest mostly their savings in bonds or choose a lump-sum pension following empirical findings. The annuity is a monthly paid pension with higher returns, compared to bonds, and expires by the death of the individual. Annuities are exchanged by a stock of money which is paid once and the insurance company shifts resources from people who die young to consumers with a long life. An illustrative example is, if two consumers will get a pension, then the one with the annuity can spend 4000\$ each month as well the individual possessing lump-sum tax or bonds but the owner of the second option will run out of money at the age of 85.²

Further, the comparison of the different pension systems is related to two main problems: how individuals should save and invest, the accumulation phase, and the uncertainty of lifetime of consumers, which is the more crucial one following Brown

¹ Mercer, Australian Centre for Financial Studies 2014;
<http://www.mercer.com/content/dam/merceraustralia/retirement/merceraustralia-global-pension-index/merceraustralia-global-pension-index-infographic.pdf>

Except of China and Mexico, but their bad performance are because of other severe problems.

² http://www.nytimes.com/2011/06/05/business/economy/05view.html?_r=0

(2004). If an individual invests his savings in annuities, the last point can be omitted. While the individual, who invests in lump-sum distributions, faces open questions and concerns referring all to the uncertainty of death. For example, it entails doubts of consuming or saving in the right proportion or the uncertainty of the best investment strategy. This causes many problems because mostly people tend to be myopic and in the retirement phase they cannot afford their desired lifestyle. That is why the older generation could benefit from annuities. Pooling resources leads to a protection against longevity risks and higher stock of money during the retirement phase (Mottola and Utkus, 2007).

However, there would not be an “annuity puzzle” if there are no difficulties and disadvantages regarding annuities. There are several reasons for people to eschew annuities, for example the loss of control, illiquidity, cost and complexity of annuities, bequest motives or the concerns about the insurance companies (Tange-duPrè, 2013). Moreover, there are some advantages in case of lump-sum pension. The life expectancy is uncertain and a lot have doubts that they will live as long as the annuity consultants are forecasting them, because the annuities are only valuable, compared to lump-sum pension, after a precise time of pension. Another reason for lump-sum pension system is that the price for annuity might be too high as a consequence of additional load factors, which in turn are caused by administrative and transaction costs (Bütler and Teppa, 2007). Further intentions to decrease the level of annuities are bequest motives and precautionary savings to cover shocks like unexpected health expenditures (Bütler and Teppa, 2007).

Already almost 30 years ago, the Nobel Prize winner Franco Modigliani included the difficulty of the choice for pensions in his acceptance speech for the award. *“It is a well known fact that annuity contracts, other than in the form of group insurance through pension systems, are extremely rare. Why this should be so is a subject of considerable current interest. It is still ill-understood”* (Brown, 2009). Until now it is a controversial issue with many different opinions and views but it has not been resolved yet. Brown (2001) mentioned this in his work and complains that besides the large literature, there is a lack of empirical evidence on who annuitizes and what the reasons of their decisions are. But within the last years many new contributions were published which created new point of views and approaches to think about.

In the research literature of annuities the most important paper is “Uncertain lifetime, life insurance, and the theory of consumers” (1965) of Menachem Yaari. Many papers refer

on the framework of Yaari (1965) in their calculations as for example the one of Brown (2001). As well the two works “Annuities and individual welfare” (Davidoff, Brown and Diamond, 2005) and “Optimal annuitization with stochastic mortality” (Reichling and Smetters, 2013) which were chosen to be analyzed in this thesis. Besides the opposite conclusion of the papers, this was a central criterion despite the fact that they face many different assumptions.

The intention of the paper of Davidoff et al. (2005) is to receive the results of annuity demand in different market settings. They analyze the behavior of utility-maximizing households in a complete and in an incomplete market setting whether they invest their savings in annuities or bonds. The conclusion of the paper is to invest in annuities, which leads to the existing of the “annuity puzzle”.

The opposite results receive Reichling and Smetters (2013) who state that there is no “annuity puzzle”. They implement the “true annuity puzzle” which advises to invest in bonds. Compared to Davidoff et al. (2005) they include actual health status with stochastic mortality risk which influences the decision of the retirement planning to reduce the level of annuities in the portfolio of an individual.

The purpose of this thesis is to get familiarized with the papers, compare them and conclude by answering the research question:

“Should individuals invest their savings in annuities or bonds?”

The next section consists of a short literature review about the topic of annuities and an introduction to the framework of Yaari (1965). In Section 3 the paper of Davidoff et al. “Annuities and individual welfare” (2005) is described and in the following part the paper of Reichling and Smetters “Optimal annuitization with stochastic mortality” (2013) is summarized. In section 5 the results of both papers are compared and finally a short conclusion is presented.

2. Literature Review

In the analysis of annuities and the so-called “annuity puzzle” the paper of Yaari “Uncertain Lifetime, Life Insurance, and the Theory of the Consumer” (1965) is the central work. It is widely cited and almost everyone who is related to this topic refers to this paper because “he was the first one working with lifetime uncertainty in a lifecycle model” (Huang, Milevsky and Salisbury, 2011). Yaari’s work (1965) received a lot of attention because in those days the presented approach was not very conventional. The conclusion of Yaari was a level of full annuitization, that a rational consumer always invests his total savings in annuities instead of placing them in bonds. In his work he makes some crucial assumptions which are important for the outcomes of his research, for example, the exclusion of bequest motives or the consideration of actuarial fair annuities. In the next section there will be a closer look on the framework.

The assumption to abstract the bequest motives is common and in many following works of the literature about annuity this was kept as a main assumption in their papers (see Brown (2001), Brown et al. (2001), Turra and Mitchell (2004), Koijen et al. (2010) and Ameriks et al. (2008)). The second major assumption of Yaari’s framework (1965), fair annuities, is a central constraint and used in both papers, Davidoff et al. (2005) and Reichling and Smetters (2013), which are compared in this thesis. Finkelstein and Poterba (2002) make a connection to the real world and state that it is used in the UK annuity market. Following by Mitchell et al. (1999) the assumption is used in recent years, but as well Mitchell et al. (1999) as Friedman and Warshawsky (1990) conclude that a fair annuity is expensive due to adverse selection.

There is a group claiming that Yaari’s framework (1965) only holds because of his strong assumptions and that he ignored many types of requirements to split the risk of long-living people. Especially Koijen et al. (2010) worked on the problem of optimal consumption by the constraint of annuity risk management. They conclude that ignoring these risks can lead to an economically costly disadvantage for the individual. Several other papers like Boulier et al. (2001), Deelstra et al. (2003) and Cairns et al. (2006) discover a high relation among annuity risk and the consumption behavior which should be considered in the assumption by working on that topic. Moreover people with less time and a lack of economical knowledge have big troubles deciding what the best pension type is for them. There are several reasons why they may deviate from the optimization behavior. First, people may exhibit bounded rationality (Simon, 1947).

Determining the optimal mix of annuitized and non-annuitized resources requires one to forecast mortality, capital market returns, inflation, future expenditures, income uncertainty, and other factors, as well as appropriately weight these factors according to one's current assessment of future preferences (Brown et al. 2013). Secondly, the statement of Kling, Phaneuf, and Zhao (2012) in their overview of contingent valuation methods concludes that repeating actions cause rationality, which leads to a failure of saving decisions. These assumptions are described in the two papers of Brown, Kapteyn, Luttmer and Mitchell (2013). They are testing the behavior of consumers with Social Security benefits as our choice setting, in an experimental module of the RAND American Life Panel. Further, Brown et al. (2012) question if it is possible for consumers to make a utility-maximizing choice having the opportunity to buy annuities.

On the other side, subsequent literature shows that the origin of under-annuitization is not the result of high expenses for annuities (Mitchell et al., 1999). They found out that annuities are more attractive compared to former days because the decumulation of assets is an advantage for investing in annuities. Another paper which is based on Yaari's framework (1965) was written by Huang, Milevsky and Salisbury (2011). They extended it by stochastic mortality and the utility-maximizing consumer is able to adapt his consumption if he receives different health status. This additional information leads to a welfare enhancement and an increasing demand of annuities.

Furthermore, there is a selection of papers which included longevity-risk sharing consequences and still focus on investing in annuities, for example Davidoff et al. (2005) introduced liquidity constraints or Kotlikoff and Spivak (1981) implemented marriage. Risk-sharing within families of the latter paper leads to three major problems: adverse selection, moral hazard and deception. Despite these defects, there is a high affinity to annuitization because of trust, information and love.

Even a step further than the paper of Davidoff et al. (2005) is the one of Peijnenburg et al. (2010a, 2010b). They yield the result of full annuitization under liquidity constraints and precautionary savings. These two effects together are highly demanding. Only if there is a liquidity shock in early years after the annuitization, their initial results do not hold.

However, there are also additional approaches focusing on the framework of Yaari (1965) which do not lead to full annuitization. They include consequences of longevity-risk sharing like people facing health costs (Sinclair and Smetters, 2004) or there are liquidity constraints after annuitization (Bodie 2003, Turra and Mitchell 2008).

Additionally, charging sales and selecting adversely influence annuities negatively (Finkelstein and Poterba 2004) as well as defined-benefit pensions and social securities (Townley and Boadway 1988, Bernheim 1991).

This extreme controversial topic has many approaches and there are many examples of papers that take the same assumptions into consideration but receive different results. In the paper of Brown et al. (2013) is an overview of these papers. For example, how the optimal annuitization varies in the presence of pre-existing annuitization (Brown 2001, Dushi and Webb 2006), risk-sharing within families (Kotlikoff and Spivak 1981, Brown and Poterba 2000), bequests (Brown 2001, Lockwood 2011), stochastic mortality processes (Reichling and Smetters 2013; Maurer et al. 2013); and broader, portfolio choice issues, including labor income and the types of assets on offer (Inkmann et al. 2011; Koijen et al. 2011; Chai et al. 2011; Horneff et al. 2009, 2010).

In the recent years there are more and more papers, for example Dushi and Webb (2006), Horneff et al. (2009, 2010), Inkmann et al. (2011), Reichling and Smetters (2013), concluding a low level of annuitization. Many of them refuse the annuity puzzle and invent a new puzzle (Brown et al. 2013). Nevertheless Brown et al. (2008) concludes that “as a whole, however, the literature has failed to find a sufficiently general explanation of consumer aversion to annuities.” Even the recently mentioned paper of Reichling and Smetters (2013) shows that in a world with deterministic survival probabilities the result of full annuitization is stronger than assumed.

This literature review shows that there is a highly controversial view among the different papers and it is still a current topic for researches after fifty years of the framework of Yaari (1965) and almost three decades after the Nobel acceptance speech of Franco Modigliani that annuities are “ill-understood” (Brown et al. 2013).

2.1. Framework of the Yaari Model

Although Yaari (1965) considers the uncertainty of lifetime and ignores all the other uncertainties a consumer is normally facing, he starts his study by deriving a framework where lifetime is assumed to be deterministic. The consumer is expected to live T years. c is the consumption plan and a real valued function on the interval $[0, T]$.

The conditions that have to be fulfilled are:

- c is bounded and measurable;
- $c(t) \geq 0$ for all t in $[0, T]$.

The preferences of the consumers are additive separable³ and described by a utility function V ,

$$V(c) = \int_0^T \alpha(t)u(c(t))dt. \quad (1)$$

α is a subjective discount function, which is non-negative. u is an utility associated with the rate of consumption that is a concave real-valued function.

The wealth constraint of the framework is

$$W(T) \geq 0. \quad (2)$$

The net assets of the consumer, $W(t)$, are in the beginning zero. After an accumulation phase of earnings the consumption expenditures imply a reduction of the net assets. Afterwards Yaari (1965) exchanges the certainty of lifetime to a random horizon, meaning that T changes to a random variable. The difference to the previous model is that the utility function is not the same for every T . For solving this problem, the author used the expected utility hypothesis, which says that the consumer always chooses the consumption plan with the optimal expected utility. At this point, the feasibility problem must be solved: the wealth constraint depends also on T and so the consumption plan cannot be used for all values of T . Therefore, Yaari proposes two possible solutions:

- The constrained programming procedure, which replaces the wealth constraint by the probabilistic constraint of

³ "Additive separability is frequently used to represent preferences over consumption at different points in time." (Black, J., Hashimzade, N., and G. Myles, 2012, "Dictionary of Economics", *Oxford University Press*, p.369)

$$\text{prob}[W(T) \geq 0] \geq \lambda. \quad (3)$$

To reach the results that a human being is not dying in debts, the best case is to choose $\lambda = 1$.

- The introduction of a penalty function, which prevents the consumer to violate the wealth constraint. The utility function is added by a term, which includes the wealth constraint and a non-decreasing concave real function. Because of the non-decreasing property only the wealth constraint needs to be considered. A positive amount of wealth increases the utility while a negative would decrease it.

For introducing life insurance or annuities, Yaari (1965) invented an “actuarial note”, which can be bought or sold and is defined by the rate of interest at time t by $R_A(t)$. It must extend the interest rate of regular notes. The result is the assumption,

$$R_A(t) > R_B(t), \quad (4)$$

where $R_B(t)$ is the market interest rate. If someone buys an actuarial note, he actually purchases an annuity. The buyer receives his whole life allowances with a rate that is higher than the market rate of interest. Further, Yaari (1965) assumes that the actuarial rate of interest $R_A(t)$ is fair, which holds for the assumption of

$$R_A(t) = R_B(t) + l(t). \quad (5)$$

$l(t)$ is the hazard rate of dying. The older the consumer gets, the higher this rate will be. The consumer requires higher returns in his late years.

The goal is to maximize the wealth of the consumer under these constraints and to have no debts when he is dying. To get the result of full annuitization the constraint of no bequest motives has to hold. Until his death he needs positive net assets. Thus, these assets are held rather in actuarial notes than in the form of regular notes.

3. Annuities and individual welfare

3.1. Introduction

Davidoff et al. (2005) discuss the properties of annuity demand in different market settings. The authors want to highlight the discrepancy of the observed demand behavior in the real life statistics and the value of annuitization in a standard utility maximization setting. It is related to the work of Yaari (1965) but in a more general setting. Davidoff et al. (2005) extend their work to calculate sufficient conditions for the optimality of full annuitization, meaning that all savings are annuitized. They show that it is important in complete markets to have no bequest motive and that annuities yield a higher rate of return to survivors, which is greater than the return on conventional assets of matching financial risk. Further, some constraints are omitted compared to Yaari's framework (1965), such as consumers to be exponential discounters, for utility to obey expected utility axioms⁴ or for annuities to be actuarially fair. Furthermore, the authors consider a two period model without uncertainty and the Arrow-Debreu case with many future periods.⁵

In the section 3.3 the authors' proof is described that, if markets for either annuities or conventional assets are incomplete, then full annuitization might not be the optimal solution. In that case, partial annuitization is also possible and the authors include the constraint of the bequest motive in this section.

Furthermore, the authors face not only this assumption, but also the standard-of-living to which the consumers have adapted. Hence, the level of annuitization depends on the relation of the standard-of-living to the retirement resources. Both full annuitization and incomplete annuitization are possible.

In the fourth section of the paper, a report about the simulation results is presented. Proving that even with binding liquidity constraints on annuities, consumers are more likely to have a higher part of their savings in annuitization. Finally the paper ends with a conclusion of the performed analysis of annuitization and bonds.

⁴ Von Neumann-Morgenstern expected utility axioms: completeness, transitivity, continuity, independence

⁵ It is a formalized Walrasian economic equilibrium system, and the existence of its competitive equilibrium was proven by Arrow and Debreu in their joint work in 1954. Solving the long-standing problem of proving the existence of equilibrium in a Walrasian system, the Arrow-Debreu Model has been the central piece of the general equilibrium theory of economics since the 1950s. (<http://www.encyclopedia.com/doc/1G2-3045300119.html>)

3.2. Complete Markets

The optimal decision and welfare evaluation of annuities are examined by a dual approach. In order to get these results, the authors minimize the expenditures subject to achieving a given level of utility. In the first section of Davidoff's et al. (2005) paper, not all constraints of the Yaari framework (1965) are considered, but still a setting where full annuitization is optimal, is derived. It starts with complete markets and a two-period model.

3.2.1. The optimality of full annuitization in a two period model with no aggregate uncertainty

On the contrary to the Yaari framework (1965) the periods are divided into two discrete parts. In period 1, the consumer is for sure alive, but in the second period the probability is $1 - q$ for being alive. There are no bequest motives and there is an uncertainty on the survival in period 2. Normally an expected value should be considered, but there is no need that preferences satisfy the axioms for U to be an expected value.

The basic approach of the work of Davidoff et al. (2005) is an expenditure minimization problem over first period consumption with goods of bond and annuity holdings:

$$\begin{aligned} \min_{c_1, A, B} \quad & c_1 + A + B \\ \text{s. t.} \quad & U(c_1, R_A A + R_B B) \geq \bar{U} \\ & B \geq 0. \end{aligned} \tag{6}$$

The expenditures for lifetime consumption are

$$E = c_1 + A + B. \tag{7}$$

A are savings in form of annuities and B are savings in form of bonds. The utility function depends on the consumption in period 1 and 2 and has to be higher than a given utility level. The retirees normally do not have any incomes in their retirement, so the consumption in period 2 is,

$$c_2 = R_A A + R_B B. \quad (8)$$

R_A is the return of annuities, but only if the consumer is alive. As well there are bonds, which return R_B units of consumption. The Arrow-Debreu assumption, consumption is in the consumption possibility space, requires that a dead consumer is not in debt. This is fulfilled with the restriction of

$$B \geq 0. \quad (9)$$

The weak assumption,

$$R_A > R_B, \quad (10)$$

which the authors introduce, leads to positive consumption by buying annuities and selling bonds if the consumer is alive. In the case that he is dead it would leave a debt. This results in an optimality of full annuitization:

- If $B > 0$, the annuitization can be increased while decreasing expenditures and fixing the consumption vector to a constant level.

A proof is that a sale of R_A/R_B of the bond and purchasing one annuity is correct, because of the previous assumption and definition of c_2 . Furthermore the welfare is increasing here.

- is $B = 0$, then it is the optimal solution for the expenditure minimization problem.

A Kuhn-Tucker optimization is used to solve this problem:

$$L = -(c_1 + A + B) + \lambda(U(c_1, R_A A + R_B B) - \bar{U}) + \mu B \quad (11)$$

$$L_{c_1} = -1 + \lambda U_1(c_1, R_A A + R_B B) = 0$$

$$L_A = -1 + \lambda R_A U_2(c_1, R_A A + R_B B) = 0$$

$$L_B = -1 + \lambda R_B U_2(c_1, R_A A + R_B B) + \mu = 0$$

$$\mu B = 0$$

$$\lambda(U(c_1, R_A A + R_B B) - \bar{U}) = 0$$

Set $B > 0 \Rightarrow \mu = 0$:

From the second and third derivation the result is:

$$\lambda R_A U_2(c_1, R_A A + R_B B) = 1 = \lambda R_B U_2(c_1, R_A A + R_B B) \quad (12)$$

$$R_A = R_B$$

Equation (12) contradicts equation (9) ($R_A > R_B$).

Result:

$B = 0$ is optimal because $B > 0$ fails to minimize the problem.

The next step is to calculate the effect on the expenditure minimization problem by loosening or removing constraints which limits annuity possibilities.

To get the result, they include in the previous model a restriction of an upper bound on purchases for the annuities

$$A \leq \bar{A}. \quad (13)$$

Making sure that there is a positive consumption, because the consumers will annuitize as long as possible, they state **Assumption 1**

$$\lim_{c_t \rightarrow 0} \frac{\partial U}{\partial c_t} = \infty \text{ for } t = 1, 2.$$

A small change in $\bar{A}$ would lead to a very small substitution of the annuity and the consumption would be unchanged. If there is an increase in the utility level, then the reduction of costs is

$$1 - (R_A)/(R_B) < 0.^6 \quad (14)$$

If $\bar{A}$ reaches the level that consumers do not hold any bonds, then the price of marginal second-period consumption decreases from $1/R_B$ to $1/R_A$.

The welfare gain of the situation, when it is possible to have no limits in annuitization, consists of two different parts: the savings which finance the same amount of consumption as in the situation with no annuitization, and the savings which adapt the consumption bundle to the new prices. This is the result of the increasing compensating level and the decreasing costs of second-period consumption.

For measuring the welfare gain from no annuities to unlimited annuities, the authors integrate the derivate of the consumption function between the two prices $1/R_A$ and $1/R_B$:

⁶ Resulting from the assumption: $R_A > R_B$

$$E|_{\bar{A}=0} - E|_{\bar{A}=\infty} = - \int_{R_A^{-1}}^{R_B^{-1}} c_2(p_2) dp_2. \quad (15)$$

$c_2(p_2)$ is the compensated demand (Hicks` demand function) yielding from minimizing the expenditures equal to $c_1 + c_2 p_2$ subject to the utility constraint.⁸

To sum up, the consumer who saves more, benefits more of the possibility of full annuitization in a complete market with a two period model because of the decreasing costs in second-period consumption.

3.2.2. The optimality of full annuitization with many periods and many states

The previous described model is the most basic one. But in reality there are many periods and several states of nature. For example, an 80 year old has a lower probability to be alive than a consumer who just entered the pension age. The optimality of full annuitization still is the best opportunity to save considering now many periods and many states. Adding a third period is a simple extension. Period 2 appears by a probability of $1 - q_2$ and period 3 by a probability of $(1 - q_2)(1 - q_3)$. The bonds and annuities are defined as follows,

$$\begin{aligned} E &= c_1 + A_2 + A_3 + B_2 + B_3 \\ c_2 &= R_{B,2}B_2 + R_{A,2}A_2 \\ c_3 &= R_{B,3}B_3 + R_{A,3}A_3. \end{aligned} \quad (16)$$

“Arrow bonds” (B_2 and B_3) and “Arrow annuities” (A_2 and A_3) are the complete set of true Arrow securities of standard theory.

The next step considers the bonds and annuities as vectors for working with future periods and drops the assumption of no aggregate uncertainty. In complete markets there are annuities which pay out every year until death, which leads to the Arrow securities. The gain from allowing more annuitization exceeds state-by-state increases. Concluding this first section of the paper with complete markets as follows:

⁷ $E|_{\bar{A}=0} - E|_{\bar{A}=\infty} = c_1 + B - c_1 - A = B - A$ is negative, because $B < A$

⁸ $\text{Min } c_1 + c_2 p_2 \text{ s.t.: } U(c_1, R_A A + R_B B) - \bar{U}$

A welfare gain is possible with full annuitization if all of the following conditions are fulfilled:

- no possibility of a future trade when the portfolio has been decided by the consumer,
 - If there is the option to purchase only bonds in the second period and at the same time it is not allowed to buy or sell annuities, then it is optimal for the consumer to purchase bonds.
- no bequest motive,
 - This has to hold because annuities are only paid until the death of the individual. If he has any bequeathable motives, then full annuitization would be impossible because he needs other possibilities to bequeath his fortune to his descendants.
- a bundle of annuities exist, which dominate the bundle of bonds. Purchasing annuities with the money of the sold bonds,
 - $R_A > R_B \Rightarrow \text{Buy } A$
- no negative wealth in any state.
 - This holds because of the wealth constraint of the Yaari model (1965) $W(T) \geq 0$

In this part of their work the authors discussed the enhancement of the Yaari results (1965) by complete annuitization to conditions of aggregate uncertainty, actuarially unfair annuity premiums and intertemporally dependent utility that need not satisfy the expected utility axioms. As well as the fact that the rising extent of available annuitization increases welfare for individuals who hold conventional bonds, which is confirming the annuity puzzle.

3.3. Incomplete Markets

To become an incomplete Arrow annuity market, there has to be a purchasing of the Arrow bonds. This is only possible if there are no Arrow annuities, because they are the preferred ones.

As already pointed out on the previous sub chapter, full annuitization result depends on market completeness. However, now the two options for incompleteness are: a setting

with complete Arrow bonds and only some Arrow annuities, or the incompleteness of the securities market.

3.3.1. Incomplete Annuity Markets

In most real world annuity markets, the product consists of a particular “compound” combination of Arrow securities, which are fixed and variable parts that are linked together in the Arrow securities. For analyzing this, the model is kept and the availability of annuities, the options to reinvest and the state of the market are considered.

First, the authors consider a market, where trade occurs all at once. The expenditure minimization problem is now a three-period model with a complete set of bonds, a single available annuity and no option of trade after the first contracting:

$$\begin{aligned} \min_{c_1, A, B} \quad & c_1 + B_2 + B_3 + A \\ \text{s. t. } \quad & U(c_1, R_{B,2}B_2 + R_{A,2}A, R_{B,3}B_3 + R_{A,3}A) \geq \bar{U} \\ & B_2 \geq 0, \quad B_3 \geq 0. \end{aligned} \tag{17}$$

For continuing they state **Assumption 2**, that for any annuitized asset A and any collection of conventional assets B , $R_A A = R_B B \Rightarrow A < \mathbf{l}B$. The last implication is because of the weak assumption (9) $R_A > R_B$.

R_A and B are now vectors, as they consider the model with many periods and many states. R_B is a matrix of returns. The annuity is only a scalar, since only one is available. $\mathbf{l}$ is a row vector of ones, which has the same length as the number of states.

This yields to the result that any consumption vector, which is purchased by annuities, is less expensive when it is financed strictly by annuities than by bonds. An example for this case is, if an annuity costs one unit in the first period and pays $R_{A,2}$ per unit of annuity in the second period and $R_{A,3}$ in the period 3, then this leads to

$$1 < \frac{R_{A,2}}{R_{B,2}} + \frac{R_{A,3}}{R_{B,3}}. \tag{18}$$

Further, costs are decreasing and in the optimum always are some annuity purchases.

⁹ See Appendix A

However, full annuitization is not optimal if it is worth to change the pattern by purchasing a bond. In such case, purchase bonds in the second or third period increases the utility more than the reduction of first-period consumption. It is the case under one of these conditions:

$$U_1(c_1, R_{A,2}A, R_{A,3}A) < R_{B,2}U_2(c_1, R_{A,2}A, R_{A,3}A)^{10} \quad (19)$$

$$U_1(c_1, R_{A,2}A, R_{A,3}A) < R_{B,3}U_3(c_1, R_{A,2}A, R_{A,3}A) \quad (20)$$

This holds because the consumer reduces his consumption in the first period and invests it into bonds or annuities. However, this change in utility is smaller than if he would postpone it into the second or third period and purchase bonds instead of investing the money in consumption in the first period.

In a second step there are several trading opportunities allowed. The option of reinvesting the annuity returns can lead to an increase in purchasing the annuities and even can result in the optimality of full annuitization. Otherwise if there is no trade of annuities, the optimum of the initial wealth would not change. Again the three-period model is the basis of this analysis with an extension of a saving $Z \geq 0$, which occurs in the end of the second period.

$$\min_{c_1, A, B, Z} c_1 + B_2 + B_3 + A \quad (21)$$

$$s. t. U(c_1, R_{B,2}B_2 + R_{A,2}A - Z, R_{B,3}B_3 + R_{A,3}A + (R_{B,3}/R_{B,2})Z) \geq \bar{U}$$

The return on savings between the second and third period equals the bond returns,

$$R_Z = R_{B,3}/R_{B,2}.$$

For not dying in debt and if there is no annuity, the restrictions of nonnegativity of wealth must be taken into account:

$$R_{B,2}B_2 + (R_{B,2}/R_{B,3})R_{B,3}B_3 \geq 0 \quad (22)$$

$$R_{B,3}B_3 + (R_{B,3}/R_{B,2})Z \geq 0. \quad (23)$$

¹⁰ See Appendix B

The first constraint is the restriction from the second period. This is the result of the multiplication of $(R_{B,2}/R_{B,3})$ and some restructuring.

The second one is the constraint of period 3 which is directly showed in the utility function.

If the condition

$$R_{B,2}U_2(c_1, R_{A,2}A, R_{A,3}A) \leq R_{B,3}U_3(c_1, R_{A,2}A, R_{A,3}A) \quad (24)$$

holds, then saving after full annuitization is optimal for the consumer. The bonds in period 3 are dominated by the annuities because of the **Assumption 2**, which implies $R_{B,3} < R_{A,2}R_Z + R_{A,3}$ ¹¹. The positive holdings of B_2 are excluded, because of **Assumption 2** and the previous condition because of this implication:

$$R_{B,2}U_2 < R_{A,2}U_2 + (R_{A,3}/R_{B,3}) R_{B,2}U_2 \leq R_{A,2}U_2 + R_{A,3}U_3^{12} \quad (25)$$

3.3.2. Incomplete securities and annuity markets: The role of liquidity

During a lifetime circumstances may change due to unexpected events for example, an unexpected medical issue or an unexpected inflation in the market. Both examples can reduce the value of an annuity. Mostly in models with incomplete annuitization these unexpected expenditures exist, which are not able to be insured. Davidoff et al. (2005) consider two different cases in order to explore the meaning of total illiquidity of annuities:

- Uninsured medical expenditures
- Inferior returns to annuities

For the first case, the authors use the two-period model. A medical expense, M , is included and differs each period. It occurs with the probability of m . Another extension are the insurance costs of size I , which have a benefit of β .

$$\begin{aligned} & \min_{c_1, A, B, I} c_1 + A + B + I \\ & s. t. \quad (1 - m)U(c_1, R_A A + R_B B) + mU(c_1 - M + \beta I, R_A A + R_B B) \geq \bar{U} \end{aligned} \quad (26)$$

¹¹ See Appendix C

¹² See Appendix D

Full annuitization is only the optimal plan if there is the possibility to insure medical expenses. However, the authors exclude this case and the optimal plan is to purchase bonds. This case only holds in early periods because the older the consumer, the better is to purchase annuities. Medical expenses only happen when you are alive and if this is towards the end, then it is not necessary to possess much money.

To show the effect of the liquidity, they assume that only bonds are sold in the first period with a redemption penalty, but there is no opportunity to sell annuities.

$$\begin{aligned} & \min_{c_1, A, B, I, Z} c_1 + A + B + I & (27) \\ \text{s. t. } & (1 - m)U(c_1, R_A A + R_B B) + mU(c_1 - M + \beta I + \alpha_B Z, R_A A + R_B (B - Z)) \geq \bar{U} \\ & \alpha_B < 1 \end{aligned}$$

Z are bonds sold in early periods and α_B is the part of value achieved by these bonds. The model develops with the assumption, that always annuities dominate bonds, which are not sold, to the form:

$$\begin{aligned} & \min_{c_1, A, B, I} c_1 + A + B + I & (28) \\ \text{s. t. } & (1 - m)U(c_1, R_A A + R_B B) + mU(c_1 - M + \beta I + \alpha_B B, R_A A) \geq \bar{U}. \\ & \alpha_B < 1 \end{aligned}$$

Holding bonds in the portfolio is optimal for the consumer, if

- the differences of returns for bonds and annuities are small;
- the penalty for bonds is small (α_B high, close to 1);
- the medical insurance pricing is unfair.

For example, the savings that a consumer holds are spent almost entirely in the first period if there is a need of medical expense, followed by a high medical insurance and an investment in savings of bonds due to the low penalty restriction. Thus, in the second period, the available resources to invest in annuities are limited. If there is no risk of a first-period medical expense, the consumer can invest, in the second period, a part of his savings in bonds since there is almost no difference to annuity returns.

The next case considers only medical risk in the second period. The expected utility alters to the form:

$$(1 - m)U(c_1, R_A A + R_B B) + mU(c_1, R_A A + R_B B - M + \beta I) \quad (29)$$

There are two options solving the medical insurance problem:

- purchase them at the beginning of period 1;
- purchase an annuity and buy a medical insurance at the beginning of period 2.

The Lagrange has the following form,

$$\begin{aligned} \min_{c_1, A, B, I} L = & -(c_1 + A + B + I) \\ & + \lambda((1 - m)U(c_1, R_A A + R_B B) + mU(c_1, R_A A + R_B B - M + \beta I) - \bar{U}) \end{aligned} \quad (30)$$

In both cases a decrease of pricing of medical insurance is followed by a change in first-period consumption.

$$c_1^* = c_1(\lambda, \beta, m, R_A, R_B) \text{ with } \frac{\partial c_1}{\partial \beta} \neq 0 \quad (31)$$

In the first option a change in investment leads to an alteration in holding annuities even if first-period consumption is constant because the derivation of investments depends on $R_A A$:

$$L_I = -1 + \lambda \beta m U_2(c_1, R_A A + R_B B - M + \beta I) = 0 \quad (32)$$

In the second option the demand for annuities increases and the attitude of buying medical insurance in the second period changes.

$$L_A = -1 + \lambda R_A U_2(c_1, R_A A + R_B B - M + \beta I) = 0^{13} \quad (33)$$

$$\Rightarrow A^* = A(\lambda, \beta, m, R_A, R_B) \text{ with } \frac{\partial A}{\partial \beta} \neq 0 \quad (34)$$

Thus, the decision of consuming and liquidity needs depends on the timing of risk of medical expenses.

The next part describes the other case of illiquidity in the incomplete market setting. Davidoff et al. (2005) examine inferior returns to annuities by adding the assumption of asymmetric information. For example a delay of annuitization would be, in this case,

¹³ $L_A = -1 + \lambda((1 - m)R_A U_2(c_1, R_A A + R_B B - M + \beta I) + mR_A U_2(c_1, R_A A + R_B B - M + \beta I) - M + \beta I) = 0$

optimal. Moshe A. Milevsky and Virginia R. Young (2002) highlight the shortcomings of annuitization with illiquidity. They propose to invest in bonds or postpone the purchase of annuities to later periods. But in the empirical observations, households do not prefer to invest later in annuities. This implies that the reduction of annuity cannot be explained by this argument.

The demand of annuities decreases if the returns of conventional assets are higher. However papers, as for example the one of Mitchell et al. (1999), prove that pricing of annuities is not sufficient for this decrease. It is rather a combination of a high level of previous annuities and a low return of the current annuities.

3.4. Bequests

Until this point bequests were excluded in the assumptions. In this subsection the authors show that individuals still hold annuities, not as many as before, even with bequest motives. In a two period model with uncertain risk of living in period 2, the authors consider this assumption. The bequests can be at the end of the first and second period. The utility consists now of own consumption and the two possible bequest levels. An additional interest is added to the bequest in period 3, for a death in period 2. The framework has the following form:

$$\begin{aligned} \min_{c_1, A, B, c_2} \quad & c_1 + A + B \\ \text{s. t. :} \quad & U(c_1, c_2) + V(R_B B, R_B(R_A A + R_B B - c_2)) \geq \bar{U} \end{aligned} \quad (35)$$

The expectation of a concave value of the bequest, v , calculated in period 3 and with a probability q that the consumer dies before period 2 is the added part of bequests in the previous utility:

$$V(R_B B, R_B(R_A A + R_B B - c_2)) = qv(R_B R_B B) + (1 - q)v(R_B(R_A A + R_B B - c_2)) \quad (36)$$

To prevent annuitization, the value of bonds must exceed the value of annuities by the assumption to exclude the annuities in deriving bonds:

$$R_B R_B \left(q v'(R_B R_B B) + (1 - q) v'(R_B (R_B B - c_2)) \right) > (1 - q) R_B R_A v'(R_B (R_B B - c_2))^{14} \quad (37)$$

This does not hold, if

- $c_2 > 0$
- actuarially fair annuity: $(1 - q) R_A = R_B$, or annuities close to the fair one

Insert the actuarially fair annuity assumption in the inequality:

$$\begin{aligned} R_B R_B \left(q v'(R_B R_B B) + (1 - q) v'(R_B (R_B B - c_2)) \right) &> R_B R_B v'(R_B (R_B B - c_2)) \\ \left(q v'(R_B R_B B) + (-q) v'(R_B (R_B B - c_2)) \right) &> 0 \\ v'(R_B R_B B) &> v'(R_B (R_B B - c_2)) \end{aligned} \quad (38)$$

This is violated because v' is a concave function.

Actuarially fair annuities lead to:

$$v'(R_B (R_A A + R_B B - c_2)) = q v'(R_B R_B B) + (1 - q) v'(R_B (R_A A + R_B B - c_2))^{15} \quad (39)$$

This implies that the second-period consumption is the same as annuitization, $R_A A = c_2$, which is a proof that annuitization is not excluded under bequest motives.

3.5. Simulations

If bonds are completely dominated by annuities, then full annuitization is optimal. This assumption is in practice not realistic. However the case of a partial annuitization has a higher probability to occur. This is the theoretical finding of the paper, which is matching with most of other different publications of this literature. Many papers consider simulations that evaluate utility gains from annuities under various assumptions, as Milvesky and Young (2002) the uncertainty about future asset returns, Brown and

¹⁴ Appendix E (I)

¹⁵ Appendix E (II)

Poterba (2003) risk pooling within couples or Turra and Mitchell (2005) uninsured medical expenditures. Most of the papers consider annuities, which are constraint differently (for example incomplete annuity markets).

It is important to transfer these theoretical results to practical issues. A central policy concern is the optimal consumption level considering annuitization. By analyzing this, the authors used a “stress test” of the results for annuity valuations. They consider several cases, including the extreme ones, to get a wide range of results even if they are not realistic.

3.5.1. A “stress test” of annuity valuation

The theoretical findings point out that welfare gains from annuitization depend on preferences, which are caused by the different options regarding desired consumption and available income. The authors consider in this simulation the various financial positions of households in the retirement, which leads to different structures of the optimal consumption. Further, preferences are not intertemporally additive separable. For a better overview, there is a table included. The first column is the case number and the following three are the relevant parameters. Column (5) is the parts of savings invested in real annuities instead of bonds. Column (6) shows the equivalent variation. It is the increase in wealth, when the optimal fraction of savings is hold in annuities instead of moving it to nonannnnuitized options. The last column reports the welfare gains if there are no longer constraints for annuities. This is the complete market case of the paper described in the current chapter.

A 65-year-old male is evaluated with the assumption that he will die at least by the age of 100. In this utility function the parameter s_t is constant:

$$U = \sum_{t=65}^{100} (1 + \delta)^{-t} \left(\frac{c_t}{s_t} \right)^{1-\gamma} / (1 - \gamma) \quad (40)$$

The real interest rate and discount rate δ is set to 0.03 and the parameter of risk aversion is $\gamma = 2$.

Case 1

In a first observation and described in Table 1 as “Case 1”, it is optimal for a consumer to annuitize all savings. For $r = \delta$, real annuities are the optimal annuity stream. The

benefit of annuitization is small, compared to the other cases. Consumption is not on the optimal path, because of the constraints.

Table 1- Simulated utility gains from access to annuitization (Davidoff et al., 2005)

Case	Habit adjustment speed g	Ratio of initial habit s_{65} to real annuity	Discount rate	% of savings annuitized with real annuities	Welfare gain when optimal % of savings annuitized	Welfare gain when annuity markets complete
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	n/a	n/a	0.03	100	56	56
2	1	1	0.03	100	71	96
3	1	0.5	0.03	100	67	74
4	1	2	0.03	90	51	54
5	1	2	0.1	75	25	30

Diamond and Mirrlees (2000) are showing in their paper, that utility is related rather to past consumption than to the present one. A consumer is behaving differently if he is used to spend less money than someone who lost his house and is forced to save more. This is modeled by the equation of

$$s_t = \frac{s_{t-1} + g c_{t-1}}{1+g}, \quad (41)$$

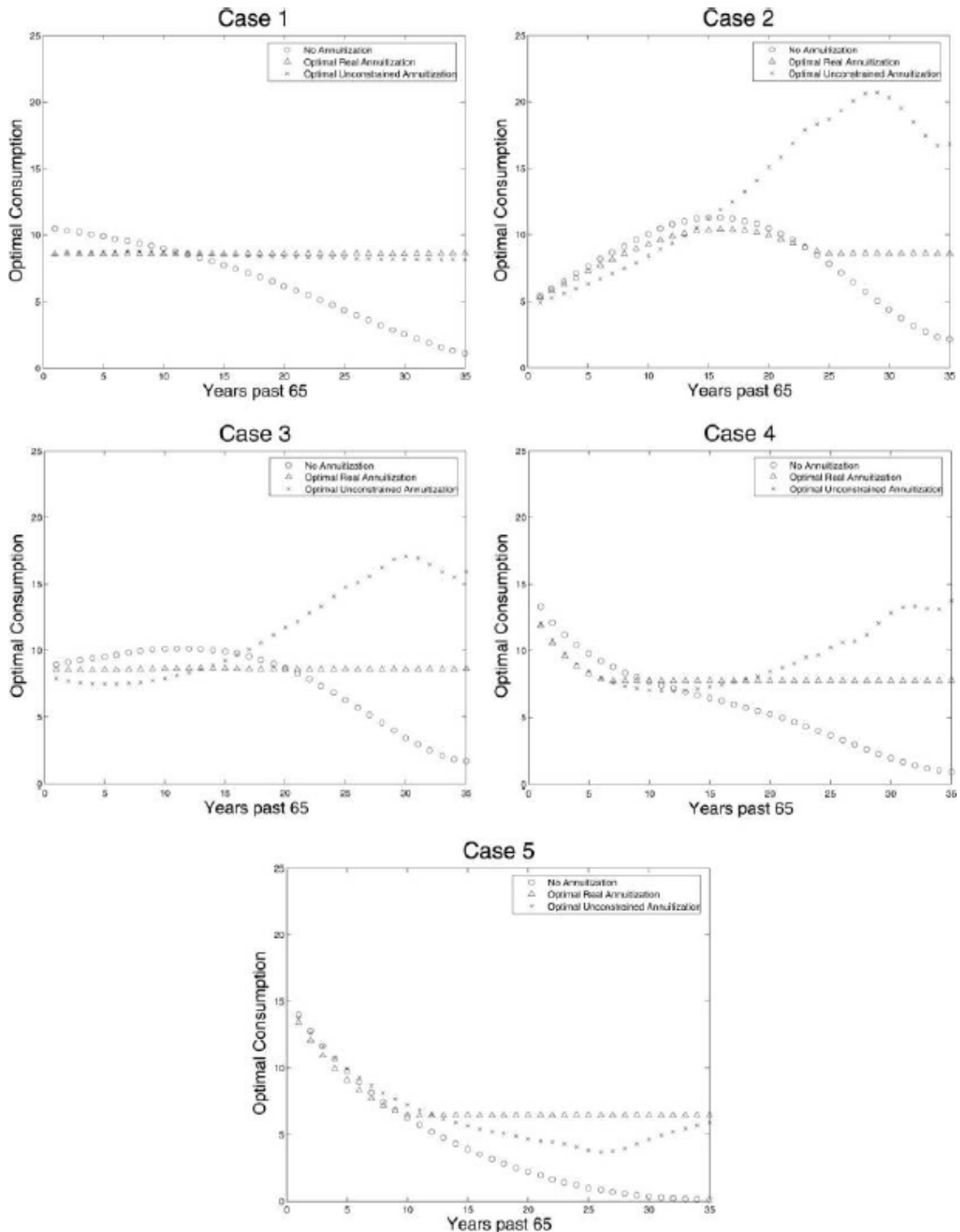
where g is the parameter for the speed of adjustment to the habit level. The authors set an intermediate speed of $g = 1$, because no adjustment speed would consider the additive separable case. And if g is going to infinity the last period consumption is the reference point for present habit.

Case 2

The second case considers a consumer who needs the same resources to live in the retirement phase than he was used to have before. If only real annuities are available, the optimal fraction of savings is 100 percent, which results in the highest possible

return of the consumer. The utility gains are higher than in the previous case, even if the individual can purchase annuities in complete markets. In the early years these assumptions lead to an increasing consumption path in all possible subcases (Figure 1, Case 2).

Figure 1: Optimal consumption trajectories under different annuity availability



In the next cases the authors examine the extreme situations, when the habit level is doubling or reducing to half of the used available amount of money.

Case 3

Case 3 reports the positive case of double the initial habit level. Again, as in the two previous cases, it is optimal to annuitize all wealth. This leads to an equivalent variation of 67 percent. The consumption path is very similar to the Case 2, but in a smoother way.

Case 4

The next two examinations are the cases that the consumer only has half of the wealth to maintain his normal life standard. This reduces the consumption level in the early years to have a more sustainable habit level. For the first time the optimal fraction of savings reduces to 90 percent. The rest is to compensate in the first years the less available amount of money. The equivalent variation and the welfare gains of complete markets also decline compared to the other cases.

Case 5

A more extreme case of the “stress test” is to raise the discount factor to 0.10 and holding the interest rate r fix. This leads to the same results as before but to a greater extent. However, it is still optimal if 75 percent of the savings are annuitized.

These simulations conclude that it is almost impossible to create a case which diminishes the fraction of savings in annuitization below 66 percent.

3.6. Conclusions and Future Directions

Complete annuitization is in complete markets with annuities, which offer positive net premia over conventional assets, the optimal way for an individual to invest his savings. Many assumptions could be relaxed as, for instance, the need for annuities to be actuarially fair or additively separable.

In the incomplete markets, included some constraints, full annuitization does not hold for all cases. However even these variations result in a large fraction of wealth, which is annuitized. The case of a world where only individual mortality is uncertain implies

different values of annuitization. It depends on the individual if it shifts late consumption in earlier times.

The absence of voluntary annuitization in practice is surprising, because the theoretical results prove the opposite. Even in very extreme cases, where the consumption path and income path do not match, the fraction of annuity is still above 50 %. These results suggest that it should be mandatory for individuals to invest in annuities to increase their level of living. The lack of annuities could be from behavioral considerations, which should be modeled to understand the functionality of annuities.

4. Optimal Annuitization with stochastic mortality probability

4.1. Introduction

The goal of Reichling and Smetters (2013) is to achieve results for the Yaari framework (1965) by relaxing the assumption of deterministic mortality probability. They adapt their model by implementing a stochastic mortality probability. This change is linked to the different health status a consumer is facing. Empirical studies show that people save money because of health shocks. The authors find a connection of the stochastic mortality probability to the optimal level of annuitization.

Another difference to the Yaari model (1965) is the occurrence of valuation risk (or principal or resale risk), considered here as long-term bonds.

From the previous literature there are two assumptions for the stochastic mortality probabilities important to regard:

- Negative health shocks that diminish the value of the annuity do not lead to additional costs;
- Agents are patient, they do not discount future utility.

The authors weaken both of these assumptions which imply imperfect annuitization.

Firstly, the value of an annuity decreases after a negative health shock, because to avoid an increase in the marginal utility, the households need to have savings for the correlated-cost shock. This effect is called the correlated-cost channel, which is the reason for other market frictions to reduce annuitization, such as the adverse selection. The insurers adapt their bonus for selection that reduces annuitization in this model. In the simulation this effect shows the large decline of the level of annuitization.

Secondly, Reichling and Smetters (2013) introduce the impatience channel with the following assumptions:

- Households are impatient;
- No correlated costs;
- Valuation risks.

This does not lead to a smooth consumption for the states that are most valued by investors under the presence of annuities.

This model proves the reduction of annuities, hold by younger households with no bequest motives. The reason for this is the high uncertainty about their future health.

The correlated-cost channel risks are higher than the mortality credit. They should reduce the annuities because of possible negative health shocks. This could be reinvested in an annuity with a higher value, which is offered after bad health information.

The model delivers the following results:

- Most households do not annuitize any wealth;
- Positive (negative) annuitization by non-wealthy households is located in households that gain a large (small) mortality credit compared to valuation risks;
- Positive annuitization is more common in wealthy households where the costs, which are affected by a health shock, are small relative to their properties.

Additionally real-world factors as management fees and bequest motives are included in the model. The intention of Reichling and Smetters (2013) is to examine the behavior of the consumers, related to the Milton Friedman's classic billiard ball example.¹⁶

The annuity literature points out the same conclusion as the presence of stochastic mortality probabilities. The industry research and the academic experimental evidence prove that households relate annuities as an increase of their risk rather than a reduction. The paper of Reichling and Smetters (2013) implies that annuities cause a larger expected return from mortality credit with a higher valuation risk because of the presence of stochastic mortality probabilities. A greater level of risk aversion leads to less annuities.

As a first step in the paper of Reichling and Smetters (2013), a three-period model with deterministic survival probabilities is introduced and analyzes the result of Yaari's 100% annuitization result (1965). In the next section the role of stochastic survival probabilities in reducing annuity demand is analyzed and afterwards the authors work on a multiple-period life cycle model. Before the conclusion, the authors introduce simulation evidence that includes various frictions.

4.2. Three-Period Model

In this model an individual can live at most three periods $t, t + 1$ and $t + 2$. The surviving chance is given as $s_t(h)$, which is conditional on state h at time t . State h is included of

¹⁶ Individuals decide correctly if they know how to behave in particular situations because of the results of heuristic or even a bit luck. (Reichling and Smetters, 2013)

a countable set H with a cardinality exceeding 1. This means that there is more than one state and it is modeled of the Markov transitional probability between states as $P(h'|h)$, where $h \in H$ is the current state and $h' \in H$ the one in the future. As stated in many other literatures, the annuity contract considers that a single premium $\pi_{A,t}$ at age t is available and gives one unit in each future period.

For the premium the authors use the following assumption for fair pricing:

$$\pi_{A,t}(h) = \frac{s_t(h)}{(1+r)} + \frac{s_t(h) \sum_{h'} P(h'|h) s_{t+1}(h')}{(1+r)^2} = \frac{s_t(h)}{(1+r)} (1 + \sum_{h'} P(h'|h) \pi_{A,t+1}(h')) \quad (42)$$

The insurers can observe the household's health status because the premium paid at age t is conditioned on the health status h at age t . The term $\sum_{h'} P(h'|h) s_{t+1}(h')$ equals to the expected chance of surviving to period $t+2$, which leads to a change in health status between ages t and $t+1$. At the age of $t+1$ and by using the fact that $A_{t+2}(h) = 0$, because $t+2$ is the last period of living, the premium can be rewritten as,

$$\pi_{A,t+1}(h) = \frac{s_{t+1}(h)}{(1+r)}. \quad (43)$$

The gross annuity rate of return is,

$$1 + R_{A,t}(h) = \frac{1 + \pi_{A,t+1}(h')}{\pi_{A,t}(h)}, \quad (44)$$

where the dividend yield is 1 in this case and the new price $\pi_{A,t+1}(h')$. The old price is denoted as $\pi_{A,t}(h)$.

The net return for a survivor to age $t+1$ is then,

$$R_{A,t}(h'|h) = \frac{1 + \pi_{A,t+1}(h')}{\pi_{A,t}(h)} - 1. \quad (45)$$

4.2.1. Deterministic Survival Probabilities (The Yaari Model 1965)

The mortality probabilities are deterministic and can be stated as a restriction on the stochastic survival probability process,

$$P(h'|h) = \begin{cases} 1, & h' = h \\ 0, & h' \neq h \end{cases} \quad (46)$$

However, survival probabilities are not restricted to be constant across age. That is why the likelihood of survival can decrease with age, which is fully predictable by initial health status h and the current age:

$$s_{t+1}(h) < s_t(h) < 1 \quad (47)$$

Inserting the restriction (46) into equation (42) yields to the function:

$$\pi_{A,t}(h) = \frac{s_t(h)}{1+r} \cdot (1 + \pi_{A,t+1}(h)) \quad (48)$$

The net rate of return to an annuity equals to,

$$R_{A,t}(h) = \frac{1+\pi_{A,t+1}(h)}{\pi_{A,t}(h)} - 1 = \frac{(1+r)}{s_t(h)} - 1. \quad (49)$$

The realized annuity return is independent of the survival probability at age $t + 1$ because it is equal to the single-period annuity. This is sufficient for creating a multi-period annuity.

Annuities always produce a better return than bonds, that is the reason for the expression that annuities statewise dominate bonds if $R_{A,t}(h) > r$. It implies that all people should have annuities for all wealth in the Yaari economy (1965). If $R_{A,t}(h) > r$ for all values of h , then $s_t(h) < 1$.

Any person with preferences of positive marginal utility, prefers a statewise dominant security, which is also the strongest notion of stochastic ordering.

4.2.2. Robustness

The stochastic survival probabilities deliver different mechanisms for many common market frictions to reduce annuitization because the full annuitization result of Yaari (1965) is stronger than mostly expected.

Figure 2: Optimal Annuitization in the Yaari Model

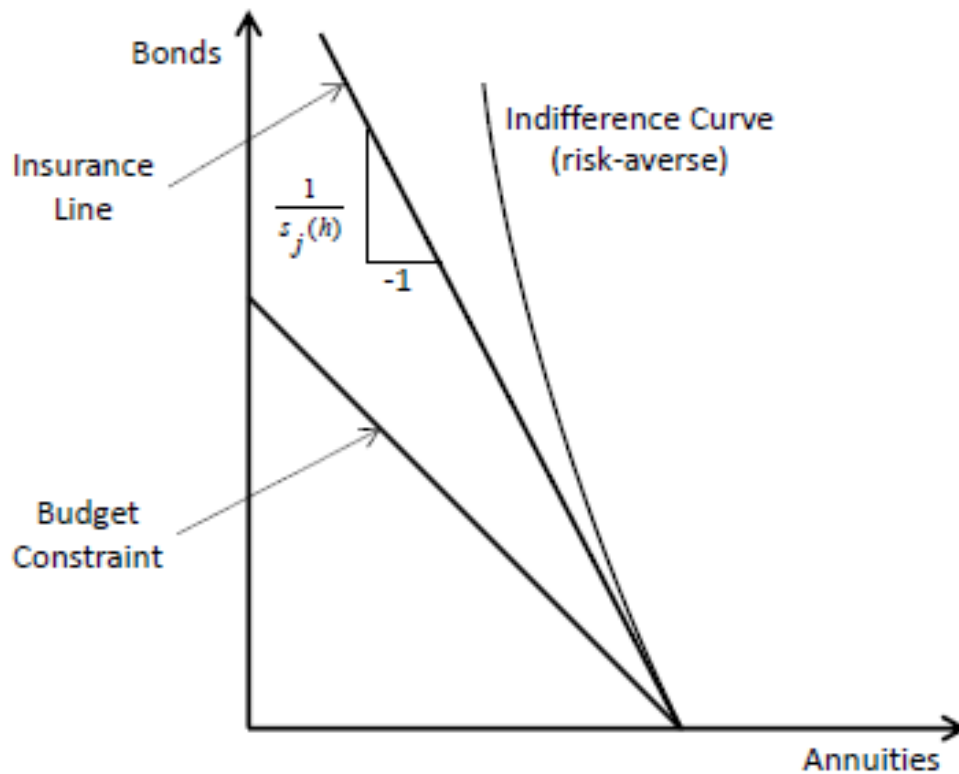


Figure 2 shows the statewise dominance in the Yaari model (1965). The “Budget Constraint” of an investor, who chooses between bonds and annuities with a competitive return that is conditional on her health h at age t , is a straight line with the slope -1 : either invest $\$1$ into bonds or into annuities. The “Insurance Line” marks the tradeoff between bonds and fairly priced annuities that are offered by a competitive annuity provider. The slope is $-\frac{1}{s_t(h)}$, which is steeper than the budget constraint.

$\frac{\$1}{s_t(h)} > \1 invested into bonds at age t is needed to produce the same level of assets at age $t + 1$ as $\$1$ invested into annuities.

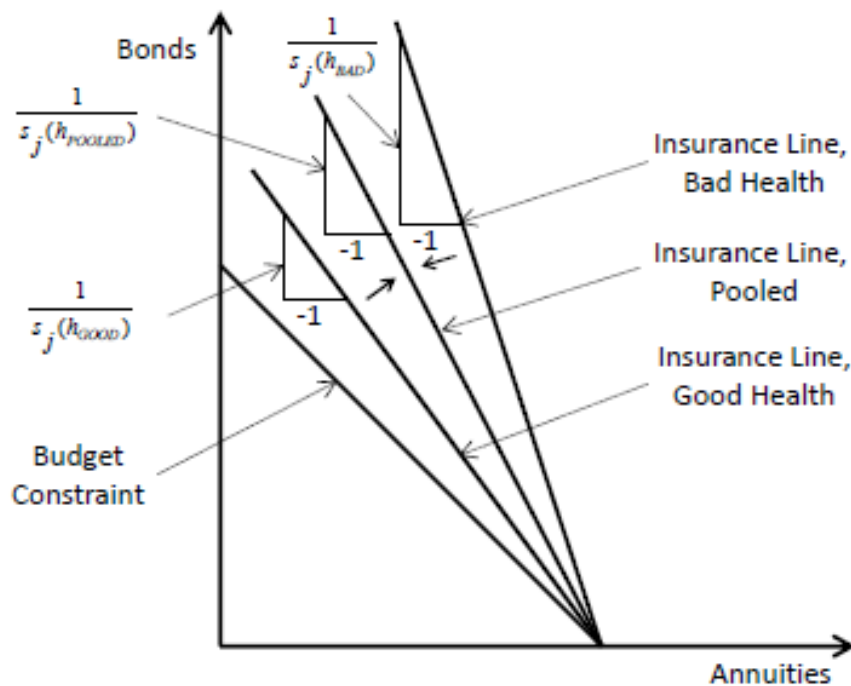
The third line in Figure 2 is the “Indifference Curve (risk averse)” for a risk-averse agent. Risk-averse investors also fully annuitize, because the slope is at least as steep as the insurance line. The investors would require at least $\frac{\$1}{s_t(h)}$ worth of bonds to remain indifferent to a $\$1$ reduction in annuity protection.

Examples:

- **Adverse Selection**

There are two states on health, h_B stands for Bad health and h_G for Good health, and the probability for survival is: $s_t(h_B) < s_t(h_G)$. With the assumption of adverse selection, the insurer cannot distinguish between the different kinds of people. There is now a Pooled Insurance line, which has the effect that households with Good health gain in annuity return and the ones with Bad health loose. But still full annuitization occurs for both types, because it intersects the budget constraint at the full annuitization point. The adverse selection reduces the size of the mortality credit for some households. However, having a smaller mortality credit is preferred to no mortality credit as it occurs in the Yaari model (1965).

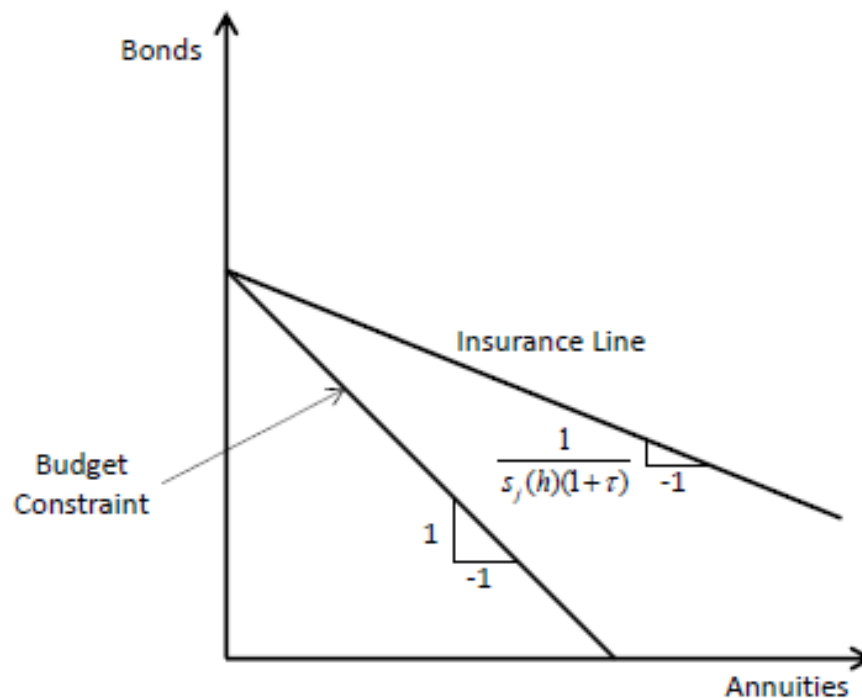
Figure 3: Optimal Annuitization in the Yaari Model with adverse selection



- **Other market imperfections**

Another important market friction is the transaction costs, which can rotate the insurance line. The transaction cost τ reduces the mortality credit and the insurance line rotates downwards. If the mortality credit is even smaller than the transaction costs, then a risk-neutral agent would hold only bonds.

Figure 4: Optimal Annuitization in the Yaari Model with transaction costs



The most other imperfections, as moral hazard, insurance within marriage, uncertain income and uncertain marriage, have no effect on the insurance line.

- **“Liquidity Constraints”**

The assumption of liquidity constraints are known for a reduction of annuitization. If the households annuitize their savings, then they are not able to weaken a health shock. Indeed, the households should invest in short-term bonds to prevent these situations. The “liquidity constraint” appears as a constraint on asset rebalancing. It is not allowed that households can rebalance their existing assets from annuities into bonds if there is an incomplete annuitization. However, only because of the risk of reclassification to survival probabilities, the rebalance of annuity to bond is not competitive.

In the empirical analysis rebalancing is not widespread. But actually there is a direct secondary market for retirement annuities. Easily a person can convert an annuity by purchasing life insurance, which is financed by the original annuity. The present value for the life insurance exceeds the value of the annuity.

In the following sections Reichling and Smetters (2013) show the effects of stochastic mortality probabilities themselves to decrease annuity demand without an additional rebalancing constraint by permitting costless asset rebalancing if stochastic mortality

occurs. The decreasing value of annuities is an important factor for reducing the demand for annuities.

4.3. Stochastic survival probabilities

4.3.1. Stochastic Rankings

Three propositions are considered and proved by the authors:

- Annuities do not always statewise dominate bonds with stochastic probabilities ($p(h'|h) > 0$). The proof follows of inserting equation (42) into the net return (49):

$$\begin{aligned} R_{A,t}(h'|h) &= \frac{1 + \pi_{A,t+1}(h')}{\frac{s_t(h)}{(1+r)}(1 + \sum_{h'} P(h'|h) \pi_{A,t+1}(h'))} - 1 \\ &= \frac{1 + \pi_{A,t+1}(h')}{\frac{s_t(h)}{(1+r)}(1 + E_H(\pi_{A,t+1}(h')))} - 1 \end{aligned} \quad (50)$$

- Consider a set H , $|H| > 1$, with the elements h and h' , where $s_t(h) \approx 1$ and $s_{t+1}(h) \approx 0$.
- Refine H to increase $E_H(A_{t+1}(h')) \approx \infty \Rightarrow R_{A,t}(h'|h) \approx -1$
 $\Rightarrow R_{A,t}(h'|h) < r$

Annuities are not optimal for a wide range of preferences with a positive marginal utility, but it is still optimal for more specific types of preferences as for example when expected utility maximizers are risk-neutral consumers.

- If the mortality rate is positive, then the values of expected returns to annuities are higher than the values of bonds.

The expected annuity returns for a survivor to the next period are:

$$\begin{aligned} E[R_{A,t}(h'|h)] &= \frac{1 + \sum_{h'} P(h'|h) \pi_{A,t+1}(h')}{\pi_{A,t}(h)} - 1 = \frac{(1+r) \pi_{A,t}(h)}{s_t(h) \pi_{A,t}(h)} - 1 = \frac{(1+r)}{s_t(h)} - 1 \\ &> r \end{aligned} \quad (51)$$

if $s_t(h) < 1$.

By considering that:

$$1 + \sum_{h'} P(h'|h) \pi_{A,t+1}(h') = E \left(1 + \pi_{A,t+1}(h') \right) = E \left(\frac{(1+r)\pi_{A,t}(h)}{s_t(h)} \right) = \frac{(1+r)\pi_{A,t}(h)}{s_t(h)} \quad (52)$$

Despite this proposition, there might be cases including risk-averse individuals, where bonds are dominated by annuities.

- Annuities do not generically second-order stochastically dominate (SOSD) bonds if the assumption of stochastic survival probabilities holds.

The proof is showed in the following example b.

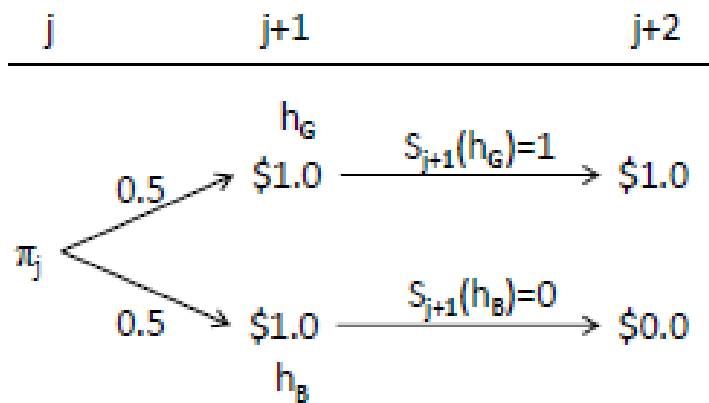
4.3.2. Examples

a. Failure of Statewise Dominance annuities

The three-period setting is still used and, to simplify the examples, the bond net return r is 0. A consumer always survives the first period ($s_t(h) = 1$), which cancels the opportunity to earn any positive mortality credit between the first two periods. In the following period $t + 1$, the health status can be either “Good”, h_G , or “Bad”, h_B , which occurs with the same probability. If the health status is “Good”, then the surviving probability $s_{t+1}(h_G) = 1$. If it is “Bad”, the probability to live in the next period is $s_{t+1}(h_B) = 0$.

Figure 5 shows the payoffs for the annuity:

Figure 5: Annuity Payoff in simple example¹⁷



The premium paid at age t is,

$$\pi_{A,t}(h) = \$1 + 0.5 * \$1 = \$1.5 \quad (53)$$

¹⁷ $\pi_j = \pi_{A,t}; j=t$

At the age of $t + 1$ the consumer receives \$1 of annuities with certainty and in the next period the expected value is also \$1, but it is only paid with a probability of 0.5.

Two cases are possible, if the options to spend \$1.5 in bonds or annuities are considered:

- “Good” health case:

The household lives three periods and the realized net return is bigger than the bond return:

$$R_{A,t}(h) = \frac{1+\pi_{A,t+1}(h)}{\pi_{A,t}(h)} - 1 = \frac{1+1}{1.5} - 1 > 0 = r \quad (54)$$

The value of the annuity is \$2: \$1 paid at the age of $t + 1$ and \$1 paid by certainty at the age of $t + 2$, however the value of the bond is only \$1.5. In this case annuities dominate bonds.

- “Bad” health case:

The realized return is in the status of h_B equal to

$$R_{A,t}(h) = \frac{1+\pi_{A,t+1}(h)}{\pi_{A,t}(h)} - 1 = \frac{1+0}{1.5} - 1 < 0 = r, \quad (55)$$

which is less than the bond return. The annuity pays only \$1 in total, whereas investing in bond yields to \$1.5 at age $t + 1$.

This demonstrates the failure of the statewise dominance of annuities. It depends on the status in which the consumer is located.

b. Failure of Second-Order Dominance

The next examples consider risk-averse investors, who are more likely to prefer annuities because they concentrate on foresighted risk reduction and they try to smooth consumption. He consumes only in the ages $t + 1$ and $t + 2$ with an endowment of \$1.5 in age t . The conditional expected utility is

$$u(c_{t+1}|h_{t+1}) + \beta s_{t+1}(h_{t+1})u(c_{t+2}|h_{t+1}), \quad (56)$$

with a period felicity function of $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, which takes the form of the constant relative risk aversion (CRRA)¹⁸. σ is the risk aversion level and β is the patience level.

The two extreme cases, high patience ($\beta = 1$) and low patience ($\beta = 0$), are examined.

The highly patient case is then separated into two subcases: included or excluded correlated costs.

High Patience ($\beta = 1$):

I. Excluded Correlated Costs:

In this case the values of annuities and bonds are the same as calculated in the previous example for the Good and Bad health case. This results in the following conditional consumption stream:

- Bond: If the status is Good health at age $t + 1$, then $c_{t+1} = c_{t+2} = 0.75$ and in the Bad health status $c_{t+1} = 1.5$.¹⁹
- Annuity: If the status is Good health at age $t + 1$, then $c_{t+1} = c_{t+2} = 1.0$ and in the Bad health status $c_{t+1} = 1.0$.

The annuities transfer 0.5 units of the consumption from the Bad health state²⁰ to the Good health state. This results in a perfectly smooth consumption, which is the aim of the risk-averse consumers.

II. Included Correlated Costs:

Uninsured shocks like uninsured medical expense exceed the problem setting. This only occurs in the Bad health status with an additional expense of \$1. The consumptions are allocated as follows:

- Bond: If the status is Good health at age $t + 1$, then $c_{t+1} = c_{t+2} = 0.75$ and in the Bad health status $c_{t+1} = 0.5$
- Annuity: If the status is Good health at age $t + 1$, then $c_{t+1} = c_{t+2} = 1.0$ and in the Bad health status $c_{t+1} = 0.0$

The consumer prefers the bond investment under the felicity function which satisfies the Inada condition.²¹ The demand of annuities reduces because of different market frictions, which are related to the presence of stochastic probabilities. The correlated

¹⁸ It has been widely used for modeling risk aversion, even if utility function changes from risk-averse to risk-loving. (Peter P. Wakker, 2008)

¹⁹ Calculations: Appendix F

²⁰ Value of annuity – Transfer = consumption in Bad health: $1.5 - 0.5 = 1.0$

²¹ $\frac{\partial u(c \rightarrow 0)}{\partial c} \rightarrow \infty$

costs have an effect on the annuity demand, which is called correlated-cost channel in the paper of Reichling and Smetters (2013).

If the status is Bad health in period $t + 1$, then there is no additional return of annuities in the age of $t + 2$, because the individual will not survive. If there is the probability of surviving in this case, there could be the possibility of buying annuities. But still there might not be enough resources to exclude states with low consumption at the age of $t + 1$.

In the Yaari model (1965) with deterministic survival probabilities the correlated-cost channel is not present because the health status is fixed. Furthermore, a reduction in survival following additional correlated costs yield to incomplete annuitization.

Low Patience ($\beta = 0$):

In this case the consumers are very impatient and the consumption is shifted in early years. Hence, the probability of incomplete annuitization is very high even without additional correlated costs.

The conditional consumptions are allocated as follows:

- Bond: If the status is Good health at age $t + 1$, then $c_{t+1} \rightarrow 1.5$ and $c_{t+2} \rightarrow 0$; in the Bad health status $c_{t+1} = 1.5$
- Annuity: If the status is Good health at age $t + 1$, then $c_{t+1} \rightarrow 2.0$ and $c_{t+2} \rightarrow 0$; in the Bad health status $c_{t+1} = 1.0$.

Regarding the bond investments, the consumption is in both health cases the same ($c_{t+1} = 1.5$), because a fully impatient individual consumes everything in the first period. The annuity investments have different results, because in the Good state the consumption is 2.0. This is the result of borrowing the annuity payment of 1.0 in the first period that will be paid in the age of $t + 2$.²² However, in the Bad health status the consumption is only 1.0, because there is no payment in the second period.

In contrast to the high patience case, the bond investment smooth the consumption and the annuity fail to allocate the consumption fairly.

This impatience channel decreases the level of annuities. A fair life annuity is not the optimal design for impatient households because it is rather based on objective survival probabilities than on a subjectively stochastic one and additionally it is related to the market discount rate. This yields to different allocations than subjectively demanded.

²² The consumption for both periods was $c_{t+1} = c_{t+2} = 1$ in the annuity stream in Good Health in the two examples examined before.

The impatience channel does not exist in the Yaari model (1965), because there is no change in the health status. The level of patience has no effect on how to invest the saving but it has an effect on the level of saving.

4.4. Multi-Period Model

A multi-period model is the basis for the simulation in the next section. In this part the characteristics of the model are described

4.4.1. Individuals

An individual lives maximal t years and the model has an overlapping generation form. The survival probabilities $s(t, h_t)$ last one period and depend on the health status h_t .

- **Health transition probabilities and conditional survival probabilities**

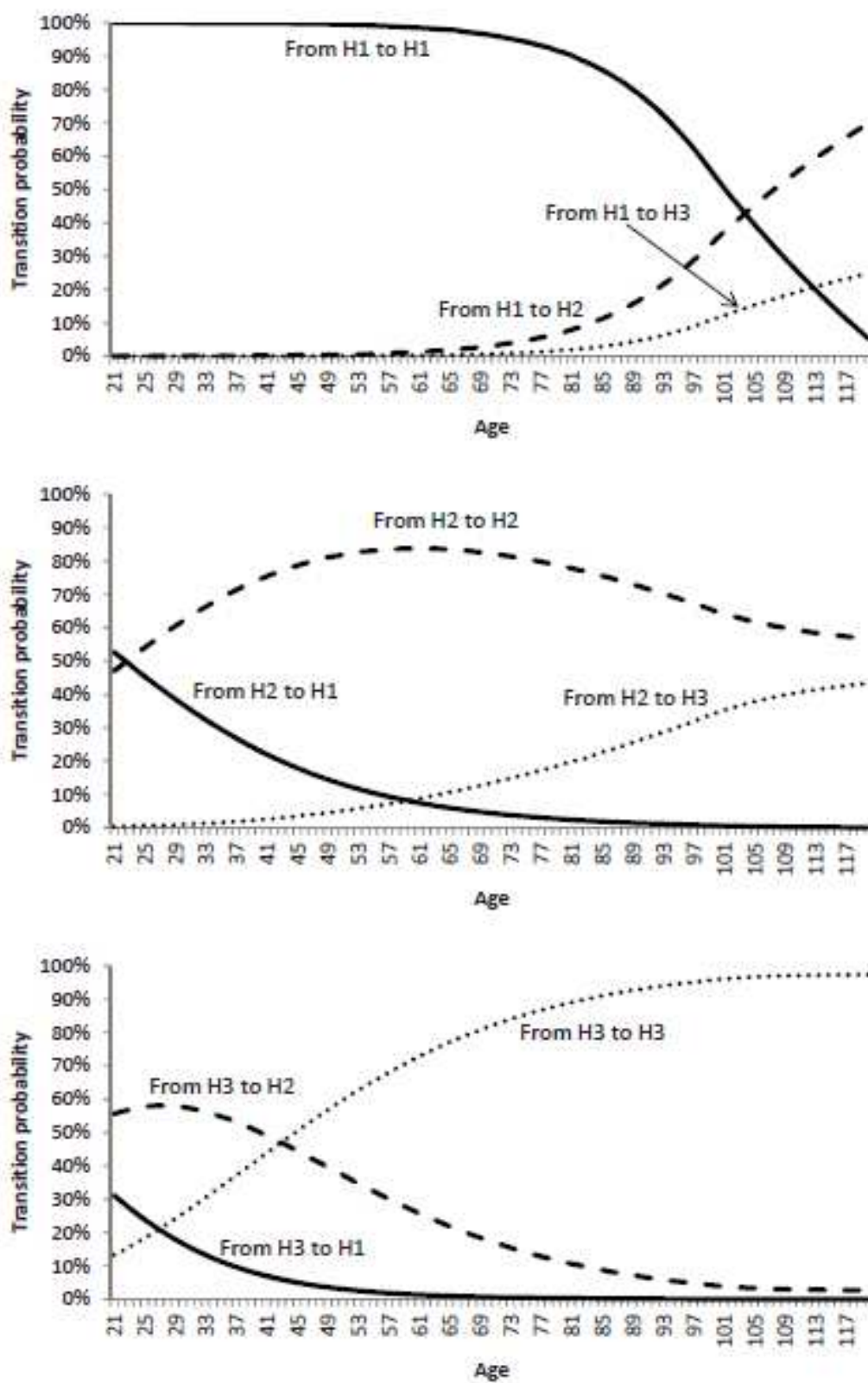
The health state is calculated by a Markov process with an age-dependent transition matrix $P_{mn}(t)$; $m, n = 1, \dots, M$. n is the previous health state and m is the next health state. To simplify the model, the authors used only three states: healthy (h_1), disabled (h_2) and very sick (h_3). The difference between the second and the third state is that a disabled individual is not living in an institution and receiving health effects. Further, the very sick person receives additional payments to cover his long-term expenses for his health. Both of them receive disability benefits until retirement and face a higher mortality rate than the healthy individual. The disabled rate in the model is about 4.2 %.²³

The transition probabilities between the different health states are shown in Figure 6. For example an individual by the age of 27 has a transition probability of 100 % if he was a healthy person. If he was a disabled person, then the transition probabilities changed to 58 % to remain a disabled person, 40 % to improve to the healthy status and 2 % to worsen to the very sick status.²⁴

²³ Based on the actuarial model of Robinson (1996), which was converted by Reichling and Smetters (2013) from eight to three transition states

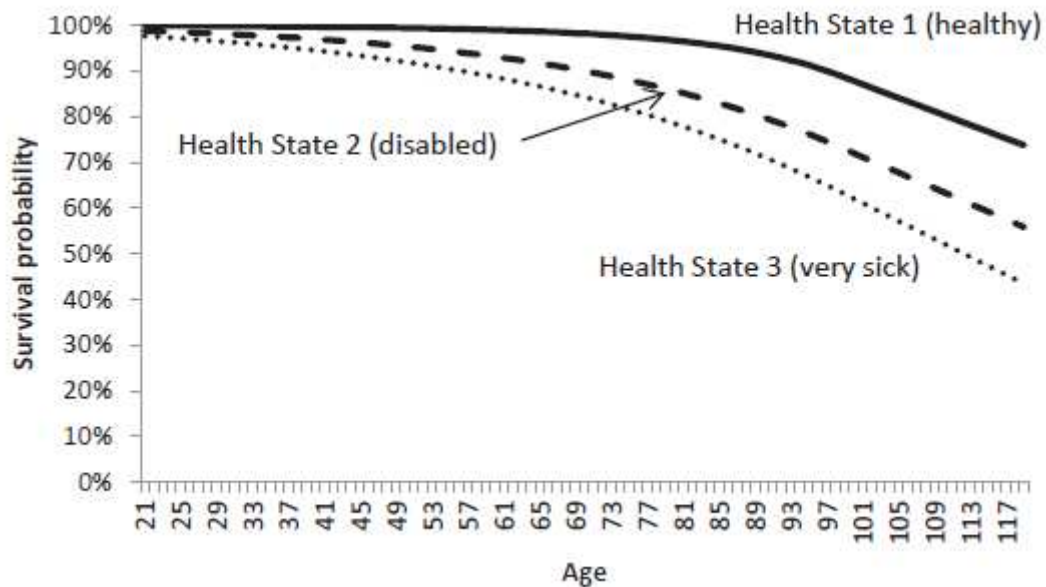
²⁴ Calculations for the Markov process in Appendix G

Figure 6: Health Transition Probabilities



The survival probabilities by age and health state appear in figure 7, which the authors calculated by themselves and compared them to the figures of the Social Security Administration (2005).

Figure 7: Survival Probabilities



• Investment Choices

Households can choose bonds, which have a net return equal to r . This is a secure investment because the aggregate capital stock is deterministic and will not change. Furthermore, they can invest in annuities, which pay \$1 per unit. The single-period annuity return $R_{A,t}$ is demonstrated in figure 8 as a function of age and health-state transition. The return has two parts:

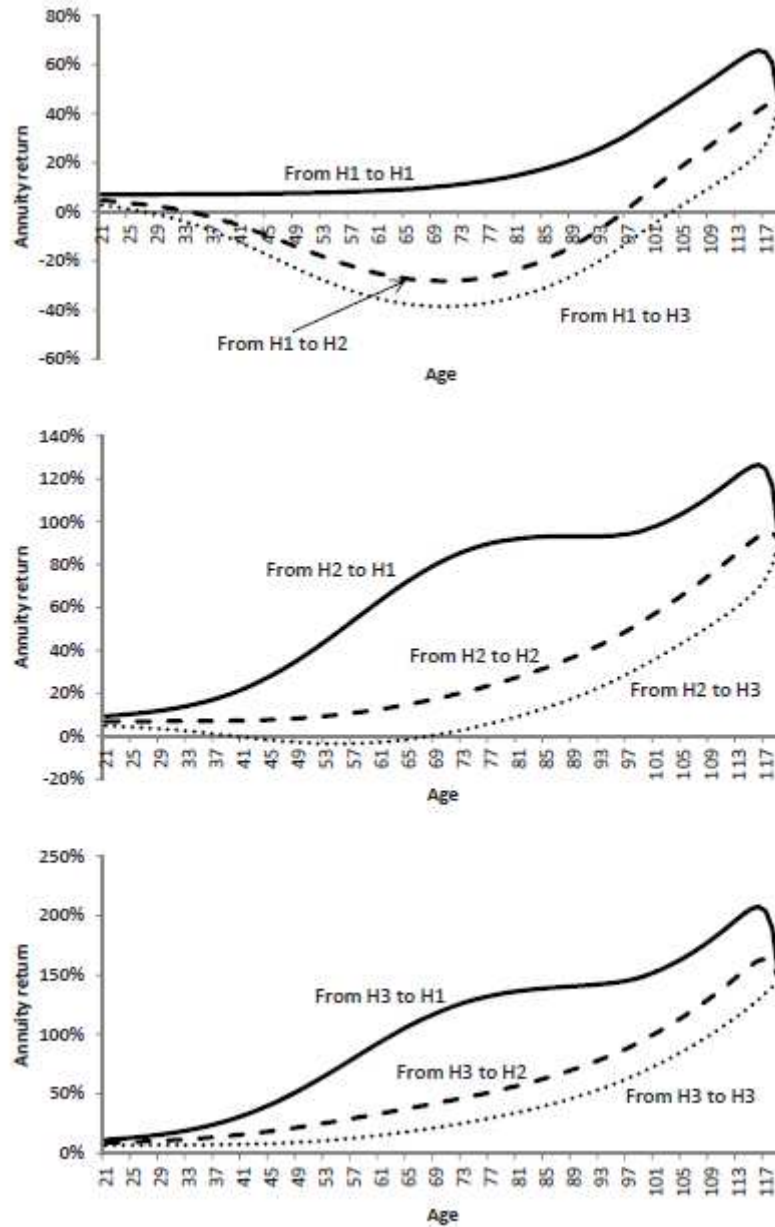
- 1) If the health state is equal for the whole life, the annuity earns the standard mortality credit. This is showed by the increasing lines in figure 8 in every health state.
- 2) If there is a change in the health state, the annuity improves or worsens. The valuation risk (resale risk) increases the probability of selling the annuities if the health status changes. If it is a positive change, then this leads to a rising annuity return. However, if the health status gets worse, then the annuity return will fall, which sometimes can even lead to a negative return, showed in figure 8 in the first graph.

- **Income and Expenses**

The initial amount of money an individual belongs at the beginning of age t is equal to

$$X_t = \varepsilon_t \eta_t I(h = h_1) w(1 - T) + E_t - L_t + TR_t \quad (57)$$

Figure 8: Annuity Returns by age and health state transition



This consists of four different parts:

- 1) Wages and Disability:

- ε_t is an age-related productivity which equals to the average productivity of a worker of age t

- η_t is a random productivity for an individual and has the form of a Markov process with a transition matrix $Q_{kl}(t); k, l = 1, \dots, \Psi$ and Ψ is the highest reachable productivity in the economy.

ε_t and η_t are based on the calculations of Nishiyama and Smetters (2005).

- I is an indicator of the health status. A healthy person has the indicator of $I(h = h_1) = 1$ and is able to work. The other two status have an indicator of $I(h \neq h_1) = 0$
- w is the equilibrium market wage rate per unit of labor and is derived by the production technology.

2) Bequests:

E is the amount of bequest and the bequest of bond holdings are distributed at death. The bequest is positive if the individual receives them and negative if the bequest is given.

3) Uninsured Medical Loss:

L_t is the financial loss, if the individual is in the health state of h_3 . In the working years the individuals are privately insured and the value of $L_t = 0$. In the retirement phase, L_t is the value of nursing home costs that are not paid by Medicare²⁵. The amount is 1.2 times the average wage. The additional costs that are unrelated to the long-term care are excluded in the simulation setting but different parameterizations of the financial loss are considered in the sensitivity analysis.

4) Government Transfers and Taxes:

TR_t is the sum of all government transfers that an individual receives if an uninsured medical loss appears. To finance these transfers, a total tax rate (T) is charged.

• Household Optimization Problem

The utility of the households is based on the size of bequests. To exclude problems these bequests motives are optional and independent of the utility of the recipient:

$$U = \sum_{t=21}^J \beta^t u(c_t) = \sum_{t=21}^J \beta^j \left[\frac{c_t^{1-\sigma}}{1-\sigma} + \xi D_t W_{t+1} \right], \quad (58)$$

²⁵ Public and federal insurance company for older or disabled citizens within the health system of the US (<http://www.medicare.gov/>)

- β is the rate of time preference
- σ is the risk aversion
- c_t is the consumption at age t
- W_t is the amount of wealth that is available for a bequest at age t
- D_t is an indicator that marks the year of death and is 1 in this year, otherwise it is 0
- ξ is a parameter stating the willingness to give a bequest
- $J = 120$, for the maximum age.

The market wage w , interest rate r and the annuity return R_A are given in the individual optimization problem and under the four state variables, wealth W , individual productivity η_t , health h and age t the problem is:

$$V_t(W_t, \eta_t, h_t, t) = \max_{c_t, \alpha_t} \left\{ u(c_t) + \beta s(h_t, t) \int_{h_{t+1}} \int_{\eta_{t+1}} [V_{t+1}(W_{t+1}, \eta_{t+1}, h_{t+1}, t+1)] Q(\eta_t, d\eta_{t+1}) P_t(h_t, dh_{t+1}) \right\} \quad (59)$$

s.t.:

$$W_{t+1} = R(\alpha_t, h_t, h_{t+1})(W_t + X_t - c_t)$$

$$\alpha \leq 1$$

$$0 \leq c_t \leq W_t + X_t$$

- W_t is the remaining asset, after the selling of annuities and bonds.
- $R(\alpha_t, h_t, h_{t+1}) = \alpha_t R_{A,t}(h_t, h_{t+1}) + (1 - \alpha_t)r$ is the portfolio return where $R_{A,t}(h_t, h_{t+1})$ is the annuity return and r the bond return; α_t is the proportion invested in annuities
- X_t is the initial amount of money

The first budget constraint is stated to ensure that bonds are positive ($\alpha \leq 1$). No individual is allowed to die in debts. The second one shows that consumption must be non-negative and is smaller than the earnings an individual receives.

4.4.2. Production

The Output Y is described by a Cobb-Douglas production function

$$Y = \theta K^\lambda N^{1-\lambda} - \delta K, \quad (60)$$

with capital K , labor N and a depreciation rate of $\delta = 0.046$. The capital share of output is $\lambda = 0.32$ and the capital-to-output ratio is $\frac{K}{Y} = 2.8$ with a marginal product of capital of 6.8%.²⁶

4.4.3. General Equilibrium

For a general equilibrium following assumptions have to be fulfilled:

- Household optimization: Households optimize the setting described above, taking as given the set of factor prices and policy parameters.
- Asset Market Clearing: The factor prices are calculated by the production technology with the aggregate levels of saving and labor.
- Policy Balance: The policy parameters are consistent with balanced budget constraints, for example the tax revenue equal spending.
- Bequest Clearing: The given and received bequests are equal.

4.4.4. The implied population, income and wealth distributions

This part shows how different distributions affect the model:

- Population Distribution

It has not a big influence on the model but if a negative annuity holding is assumed, it would depend on the proportion of the population

- Income Distribution

The income Gini coefficient in the model is 0.45, which is close to the real world data 0.47 (U.S. Census Bureau, 2012a).

²⁶ The values are calculated by the support of the paper of Nishiyama and Smetters (2005)

- **Wealth Distribution**

The Gini coefficient for the wealth in this model is 0.61, however the U.S. data is 0.75 (Nishiyama, 2002). This big gap is decreased in the simulation results but not completely.

In most life-cycle models the top 1% of wealthy people is underestimated because the “entrepreneurial spirit” of them is not taken into consideration. Almost all annuities are hold by this part of population because of the high returns of the annuities. If there is an entrepreneurial motive, then it would be also difficult for wealthy people to have both annuities and privately risky equity.

The normal poverty rate for workers (4.2%) is a little bit lower than the data of the U.S. Census Bureau (2012b) (7.2%), because they use a more general definition of “working”. However, the poverty rate among all disabled people of 33.5% in the model is very similar to the empirical figure, which is 28.8%. This is more important for these calculations because disability during working years yield to correlated shocks, which causes a decrease in annuitization. Less work and wages lead to less annuitization. Following the figures, the model is not overestimating the impact of the distributions to the result.

4.5. Simulation Results

The multi-period model is examined with the basic analysis from the upper part and afterwards some sensitivity analyses are done. The important results are compared with a calibrated Yaari model (1965), which excludes health shocks and the mortality rate by age equals to the average mortality rate.

4.5.1. Baseline Model

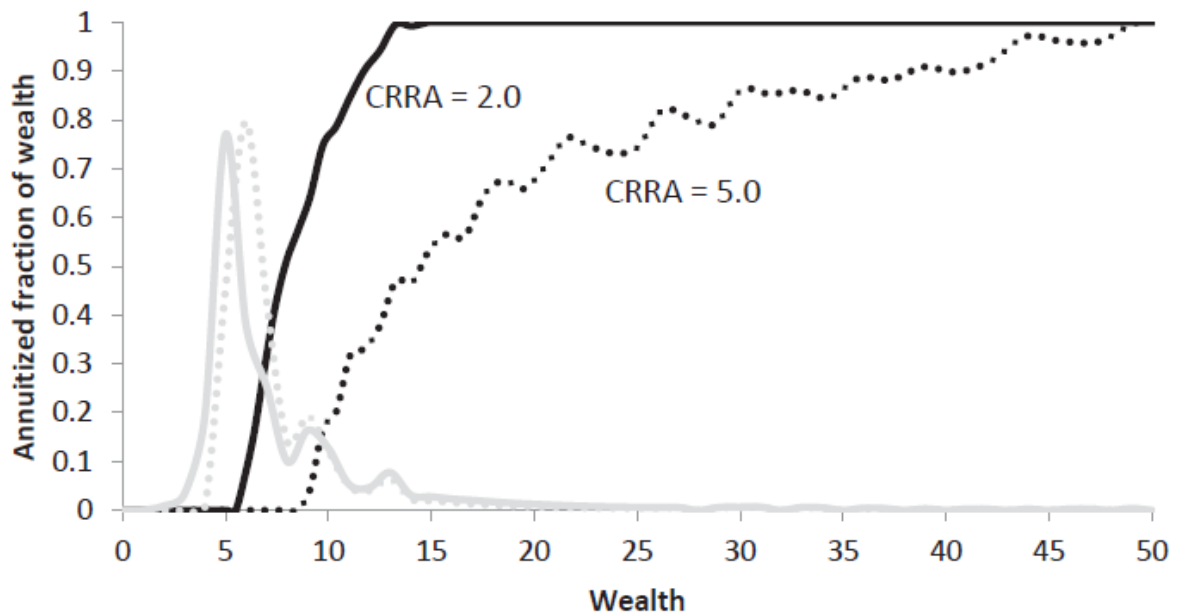
In this baseline model the annuities are non-negative. Two different cases are considered:

- **Annuitization at the age of 65**

In Figure 9 the annuitized fraction of wealth by the age of 65 for a healthy individual, which has the health status $h = 1$, is plotted by the different level of wealth. The wealth is leveled by the national average annual earnings, which is marked here by $\text{wealth}=1$.

The number 5 at the wealth scale represents a wealth amount of 5 times the national average annual earnings.

Figure 9: Annuitized fraction of wealth at age 65 for a healthy person



The darker lines show these facts for households with a constant relative risk aversion rate (CRRA) of 2.0 and 5.0. The higher the rate, the lower is the demand for annuities. For example, with the lower rate no annuities are held until the wealth is around the level of 6. However, by a rate of 5.0 there are no annuities in the portfolios of the households until the wealth level is around 8. This is an effect of the valuation risk, which produces the correlated-cost channel. In the calibrated Yaari model (1965) without health shocks, the households would fully annuitize at all levels of wealth for both rates because there is no correlated-cost channel regarded. Furthermore, if the impatience channel is considered, the result remains on full annuitization. This shows that the impatience channel is not a driver for this result.

The figure 9 shows that wealthy retirees prefer a large fraction of annuitization. They have enough additional assets to pay for any correlated long-term care costs besides the holdings of annuities. Further, they do not have to worry about a decrease of earnings if they change to a bad health status as the workers have to. Unfortunately, most of the households of the retirees have not sufficient resources. The gray lines in Figure 9 show the distribution of 65 years old individuals of their wealth level. Most of the households are located at the wealth level of no annuitization. These results are

driven by the correlated-cost channel, where the values of annuities decrease because of the increase in uninsured health costs.

- Annuitization across the life cycle

Figure 10: Intensive Margin: Annuitized fraction of all wealth by age

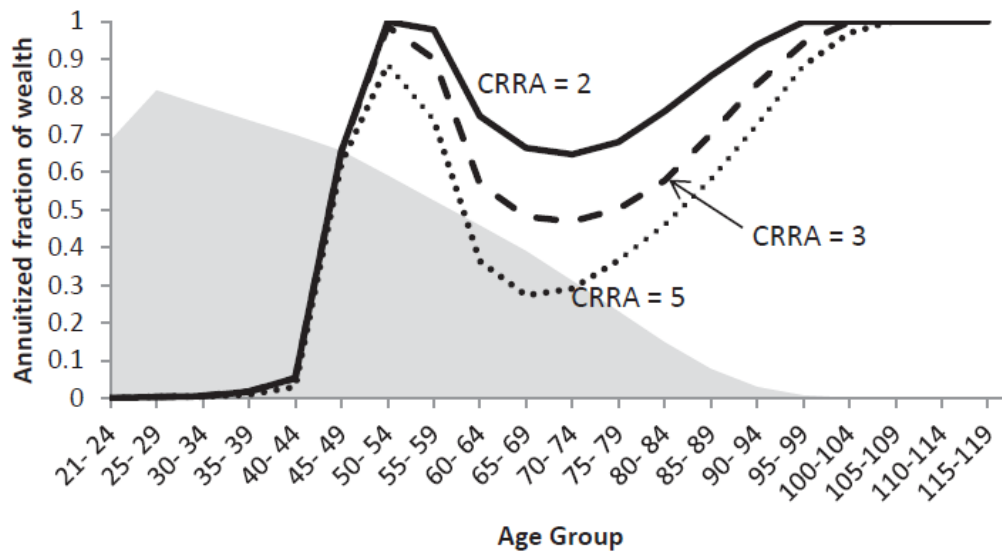


Figure 10 shows the annuitized fraction of all wealth across the life cycle considered all health states. The gray part is the population density of the different age steps.

Figure 11: Extensive Margin: Share of households with any positive amount of annuities by age

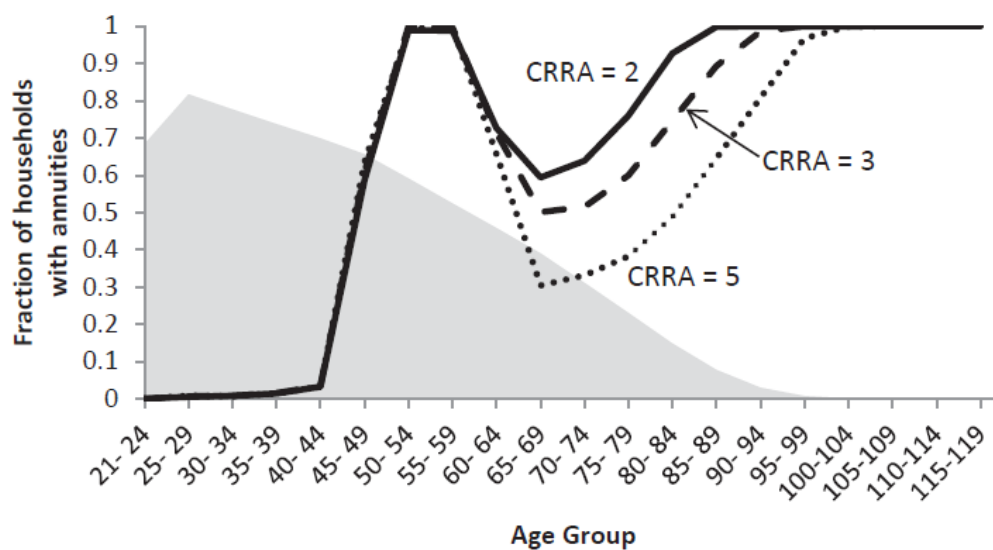


Figure 11 shows the fraction of households, which hold annuities. It also has the same assumptions as Figure 10.

By a constant rate of risk aversion of 3.0 across all ages a wealth fraction of 57% is annuitized. However, this fraction is distributed only on 37% of all households. The rest does not even hold any kind of annuities. The wealthier people have a larger part of the whole amount of available money and can afford to annuitize, as showed above.

Both figures show that annuitization is not monotonic in age. The annuity return has two components, which lead to different results for different groups of ages. The younger generation, who tend to be healthy, has a low risk of mortality. This leads to a small mortality credit earned by annuities, which is smaller than the low probability of falling to a worse health status. This effect reduces the desire of the younger households holding annuities. On the other side, older people prefer to hold all their savings in annuities because in each initial health status they earn such a large mortality credit. A second advantage is the low valuation risk because a large fraction of older households are in the unhealthy status.

4.5.2. Sensitivity Analysis

The authors modify the baseline model with several changes to highlight the importance of long-term care costs, Social Security and the advantage of having a small fraction of annuities in the portfolio. In contrast, the Yaari model (1965) would suggest full annuitization in each case.

- Long-term care costs

Table 4 shows the changes in the annuitized percentage of total wealth and the fraction of households holding annuities if long-term care costs are considered. If these costs are small, then the variation to the baseline model is not so high. However with increasing long-term costs the differentiation increases because in the basic model these non-insured health care costs are excluded.

Especially the retirees are highly affected by these costs because they do not have private insurance as the working households do. If the uninsured long-term care costs are very small, then they invest a large part of their wealth in annuities otherwise they reduce the part dramatically.²⁷ In total, the retirees are only a small part of the population and that is why only 44% of all households invest in annuities with no long-term care costs.

²⁷ 32% annuitized wealth for a CRRA=3 and long-term care costs=1.80 compared to 100% for long-term care costs=0.0

The households in the Yaari model (1965) would annuitize all of their savings in the presence of uninsured expenses because the constraints on asset rebalancing are excluded.

Table 2: Changing Long-Term Care Costs (Reichling and Smetters, 2013)

Long-Term Care Cost	Relative Risk Aversion					
	2	3	5	2	3	5
	Annuitized Percentage of Total Wealth			Percentage of Households with Annuities		
All Households						
1.80	55%	45%	37%	34%	31%	30%
1.50	61%	51%	41%	36%	34%	32%
1.20	67%	57%	45%	39%	37%	33%
0.60	75%	71%	61%	43%	42%	40%
0.00	75%	73%	69%	44%	44%	44%
Retirees Only						
1.80	47%	32%	21%	56%	38%	27%
1.50	56%	40%	25%	64%	48%	31%
1.20	69%	51%	33%	74%	61%	39%
0.60	95%	88%	71%	91%	86%	74%
0.00	100%	100%	100%	91%	91%	89%

- Social Security

Social Security has two opposite effects regarding to invest in annuitization. The negative side to annuitize is that these securities cannot be rebalanced. The more an individual has to pay for Social Security the less amount of money is available for savings against correlated cost shocks. The positive aspect is that within Social Security there are valuable annuities included the average mortality credit.

Mostly these effects equalize each other. For example, if both effects of the Social Security are removed, then the annuitized wealth decrease from 57% in the baseline model to 46% for $CRRA=3.0$. The percentage of households which annuitize at all remains at 36% but the households with full annuitization decline from 24% to 11%. Concluding, Social Security yields to invest in private annuitization. However, in the

Yaari model (1965), with deterministic survival probabilities, Social Security uses the savings of the households but does not change the preferences towards annuities.

- Allowing for short selling

In this subcase the assumption of non-negativity for annuity allocation is removed. A negative annuity is equal to borrow a risk-free rate bond and purchase life insurance to ensure repayment and prevent to die in debt.

Figure 12: Short sales: Annuitized fraction of all wealth by age (Intensive Margin)

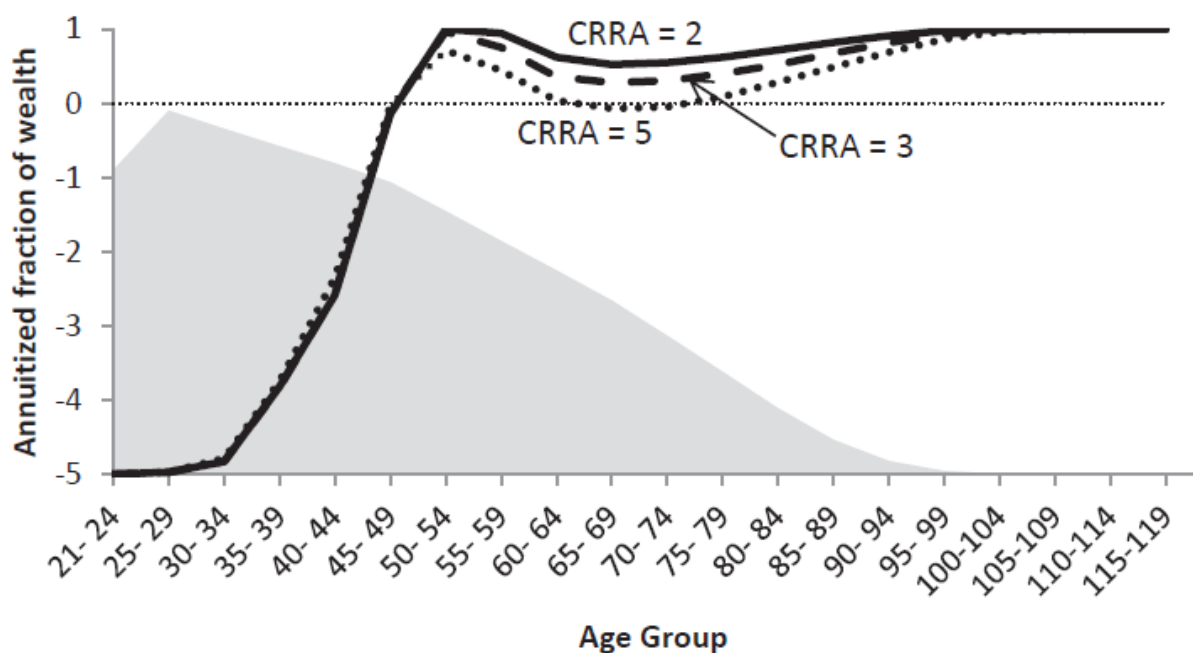


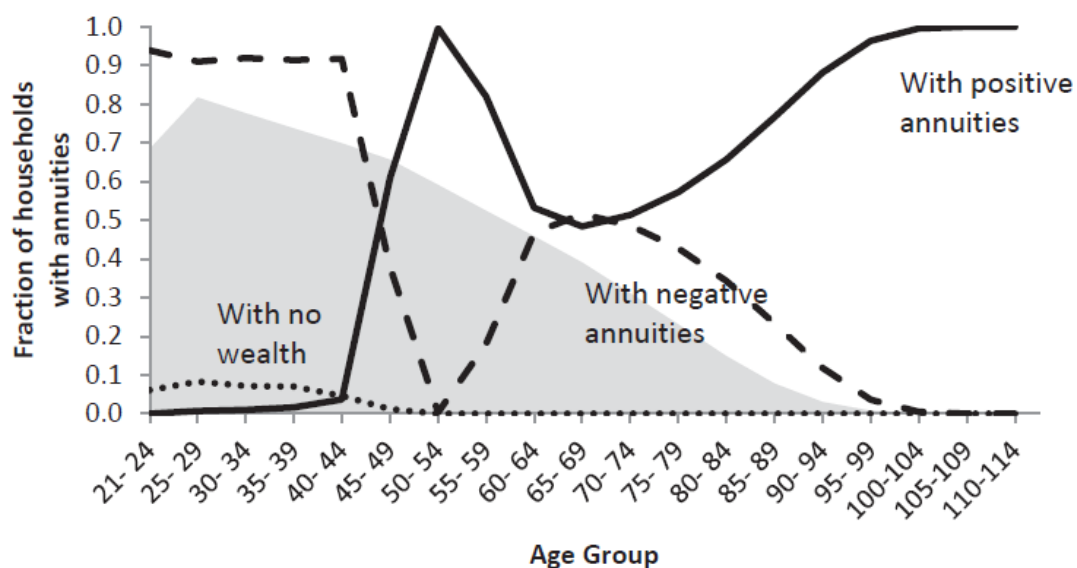
Figure 12 presents the annuitized fraction of wealth by age group. The young generations up to the age of approximately 50 hold a negative (short) position in annuities. Although they have to pay for the mortality credit here, this effect does not influence their decision because the credit is very cheap. The positive effect is to react on a future negative health shock. The short position can be used instead of a long position, which is less expensive than before the health shock. The profits of the short-long trades can be used for example to pay uninsured expenses.

Figure 13 presents the fraction of households with positive, negative or even no-wealth annuities by age group. The pattern is varying within the same age group because of the heterogeneity in the model. They differ in health status, income earnings and the amount of bequests relative to their income.

Following the simulation results, young people should invest in life insurance (short/negative annuities) but only about 17% of households, between 18 and 24, purchase life insurance. It increases to 26% for the age group of 25-34 (LIMRA, 2011) but still it is far too low, according to the figures in the model.

The authors suggest that the “true annuity puzzle” is the lack of young households investing in life insurance (short/negative annuities).

Figure 13: Short sales: Share of households with any (positive or negative) amount of annuities by age (Extensive margin)



4.5.3. Additional Factors reducing annuity demand

In this section Reichling and Smetters (2013) extend the model by additional factors to decrease the annuity level to a very low level and again compare them to the calibrated Yaari model (1965).

- Management Fees

They consider a management fee of 1% for annuities, compared to bonds which have asset costs between 0.1% for normal bonds and 0.9% for special ones.

In the baseline model with stochastic health and mortality probabilities, the level of annuitization decreases to 27% of total wealth. Only 20% of the wealth is annuitized during working ages and 42% during retirement. If the annuities are distributed in different patterns, then 49% of retired household have positive level of annuities but

only 17% of the working individuals have any annuities. Further, 22% of all households invest in annuities, but only 7% of them fully annuitize.

In the Yaari model (1965) 73% of total wealth is annuitized with a risk aversion rate of 3.0 by a full annuitization of retiree's wealth and a 65% fraction of annuitized wealth of non-retirees. A positive level of annuities hold 90% of retirees and the rest has no wealth to annuitize. Only one third of the working households have any annuities. Contrarily to the figures of the baseline model there is a thin line among the extremes in the Yaari calibration (1965). For younger households there is a range from full to zero annuitization, unless the risk aversion rate is too high. For older individuals full annuitization is always the optimal investment because the earned mortality credit exceeds the bond return. A very impressive result is that if a household has a fraction of annuities it fully annuitizes this fraction, which occurs in 43% of the cases.

- Bequest Motives

If there are bequest motives with a 2.5% bequest-GDP ratio and no management fees in the baseline model the fraction of annuitized wealth decreases to 38% and only 35% of households have a positive level of annuities in their portfolio.

In the calibration of the Yaari model (1965) these values do not decrease in the same proportion but they fall also from full annuitization without bequests to a two-third-fraction of annuitized wealth. However, 90% of households hold a positive level of annuities.

- Bequest Motives and Management Fees

If the two assumptions are combined and there are both bequest motives and management fees, then in the baseline model only 22% of total wealth is invested in annuities. The fraction of holding any annuities declines to 19%. By increasing the bequest-GDP ratio these figures decrease even more.

The authors claim that these figures are the most realistic ones they received.

4.6. Conclusions and future directions

The paper concludes that the results of the Yaari framework (1965) with deterministic survival probabilities are very strong and hard to break with a variety of market frictions.

The key driver to receive different results than full annuitization is to consider stochastic survival probabilities.

The simulation results support the assumption of stochastic survival probabilities in order to decrease the fraction of annuitization even under assumptions which have a positive effect to the annuitization level. A major result is also that many young households should even invest more in negative annuities than they have done it yet. Furthermore, there are various different assumptions to consider and discuss. For example differential transaction costs, leading to even higher transaction costs of annuities, which are already higher than bond credits. Another assumption is if the worker risk would increase, which is very low in this model. And as well, if there would be the possibility of asymmetric information that policyholders have more information available than a normal insurer. Following Reichling and Smetters (2013) all these extensions will lead to an even lower level of annuitization.

Another direction for the future work could be how the optimal level of tax and social insurance policies are considering the results of the paper.

5. Comparison of the papers

The main goal of the paper of Davidoff et al. (2005) is to analyze the properties of annuity demand in different market settings. They compare the behavior of households to invest in annuities or bonds in a complete market setting and in an incomplete one. This paper confirms the term of “annuity puzzle”, when full annuitization is the best solution.

The opposite result conclude Reichling and Smetters (2013) who state that the “true annuity puzzle” is a reduction for most households in the annuity demand and a negative net annuity can be possible. Compared to Davidoff et al. (2005), they only consider a complete market setting. In the first part of their paper they consider deterministic survival probabilities and reinforce the full annuitization result of Yaari (1965). However, the major section is when they focus on individuals who know their actual health status with stochastic mortality risk and according to this they make decisions for their future retirement planning. Such a scenario leads to invest the savings rather in bonds than in annuities.

Both referred papers, Davidoff et al. (2005) and Reichling and Smetters (2013), are based on the framework of Yaari (1965). A three-period utility-maximizing model is the central approach of their papers. In contrast to the Yaari model (1965), they do not focus on deterministic survival probabilities. Instead, a random horizon is assumed and they both have stochastic survival probabilities. A difference in the technical part is that Reichling and Smetters (2013) use a premium to value their annuities and receive the net returns for them, whereas Davidoff et al. (2005) do not particularly calculate a premium because the net return for annuities, R_A , is given. In their simulation analysis, both examined households with a constant rate of risk-aversion (CRRA). Davidoff et al. (2005) focus on a rate of 2. Reichling and Smetters (2013) consider three different rates (CRRA=2, 3 and 5).

Davidoff et al. (2005) do not consider different health status, compared to Reichling and Smetters (2013). This is the most important point why the papers conclude in the opposite direction of each other. The results of Davidoff et al. (2005) tend to invest the savings in annuities. During the whole paper they prove that individuals should rather invest in annuities than in bonds. This conclusion is similar to the one of the framework of Yaari (1965) with its 100% annuitization level. Contrarily to them, Reichling and Smetters (2013) achieve throughout their paper results to invest savings in bonds.

According to them, the impatience channel reduces the value of holding annuities. They do not consider in this particular example additional correlated costs, but annuities will fail to smooth consumption. The problem is the objective consideration of survival probabilities and market discount rates to define the annuity premium, which can be very different to the subjective desired one.

This impatience channel is not considered in the paper of Davidoff et al. (2005) because the health status is fixed, as in the Yaari model (1965). They conclude in a different way, taking into account the impatience level of consumers: shifting late to early consumption leads to invest a large fraction of the wealth in a constant real annuity. Another channel Reichling and Smetters (2013) introduce is the cost-correlated one. This occurs when consumers need resources for paying the cost-correlated shock to hold his marginal utility constant. This effect is related to the valuation risk and the different health shocks which lead to a reduction in the annuitization level. Davidoff et al. (2005) do not consider these health shocks therefore this cost-correlated channel is not included in their paper.

The impact of adverse selection is in both papers the same. According to Davidoff et al. (2005), the full annuitization result of the Yaari model (1965) is maintained. This result is also achieved in the model of Reichling and Smetters (2013) in the consideration of the deterministic case. However, the differentiation of the health status causes households with a Bad health status to face a fall of their annuity returns and households with Good health status to raise them.

Unless regarding liquidity constraints in the simulation chapter, Davidoff et al. (2005) consider them in their theoretical analysis. They extend their baseline model by medical expenditures and the ability for repurchasing only bonds and not annuities. The conclusion hereby is that the level of purchasing annuities depends on the timing of buying them. In their final reflection of that section, they conclude that illiquidity is not the main driver for the lack of annuity demand. Reichling and Smetters (2013) also point out in their examination for liquidity constraints that it is important to exclude a rebalancing of their assets from annuities into bonds. Only if this holds, then there is a decrease of annuity demand.

Furthermore, both authors get different results in the same part of their analysis because of the heterogeneity in their model. The different results for households with same age are an effect of the different level of wealth. They have in common that the wealthier the individuals, the higher the level of annuitization. As mentioned before, the

heterogeneity of the time of consuming affects the valuation of annuities in the work of Davidoff et al. (2005). Despite these effects the major part of wealth is invested in annuities. However the opposite effect occurs in the paper of Reichling and Smetters (2013).

In contrast to the paper of Davidoff et al. (2005), the authors of the second paper (Reichling and Smetters, 2013) consider the different behavior of the young and the old generations. They conclude that generally young generations do not tend to invest their savings in annuities. However, older generations are willing to shift all their wealth into annuities because the high mortality credit received by them is higher than the valuation risk. The valuation risk depends on the alteration of the health status. The smaller the deviation the smaller is the valuation risk. This is another point for the heterogeneity in their paper. Even within one group of the same age, the pattern is not homogenous because of different health status, amount of income and the volume of the heritage relative to the running income.

A similar comparison is done in the paper of Davidoff et al. (2005). They examine how different levels of available money do change the investing behavior. In the simulation part they consider the effects if households can invest the double amount of money or only half of it in the retirement phase compared to the working period. In case of doubling the amount and having more money to spend, the annuitization level is 100%. And even in the extreme case of cutting the resources in the retirement to half of the previous ones, the percentage of investing the wealth in annuities still is about 75%. The article of Davidoff et al. (2005) focus in the simulation on the baseline model and only small changes are done within the parameters. There are no additional costs included. Moreover, they relaxed the framework by ignoring bequest motives, learning about health status, liquidity concerns and risks except of longevity. Compared to this, Reichling and Smetters (2013) have another approach. During the entire paper they include learning effects by health status. Additionally, the authors consider bequest motives and/ or management fees. Summing up all these factors lead to a reduction of annuity demand, not only for the younger generation but also for the retirees. The full annuitization level is pushed down to a minimum.

Although the bequest motives are not regarded in the simulation phase, Davidoff et al. (2005) examine this effect in their theoretical part. They can refuse the general assumption that including bequest motives will eliminate every demand for annuities. These results are similar to the one of Yaari (1965). Reichling and Smetters (2013)

calculate an annuitized fraction of two third of wealth for the calibrated Yaari model (1965) in their simulation. This is a very high level of annuities compared to the 38% of annuitization of the savings in the model of Reichling and Smetters (2013) including bequest motives.

A difference of the two papers is the assumption for medical expenditures. The paper of Davidoff et al. (2005) assumes that the unexpected case of illness has no effect on the life expectancy. This is completely different to the paper of Reichling and Smetters (2013). The entire analysis is based on observing the health status and reacting if it changes. This has a strong effect on the life expectancy of the individuals and it leads to different decisions regarding the purchase of annuities. The higher the mortality rate, the higher the credit for an annuity. But in the model the authors also consider a valuation rebalancing after a change in the health state. This could have a positive or a negative effect on the return of an annuity and therefore on the decision between annuities and bonds.

Davidoff et al. (2005) conclude under the assumption of the previous section, medical expenditures, that annuities should be purchased instead of bonds if long-term care insurance is included and connected to the purchase of annuities. In the model of Reichling and Smetters (2013) mostly retirees are affected by this assumption (long-term care costs) and they also invest a large part of their wealth in annuities. If these costs increase, the fraction of annuitized wealth decreases.

The next two enhancements of the baseline model in the paper of Reichling and Smetters (2013) are not taken into consideration in the work of Davidoff et al. (2005). On the one hand there is the Social Security and on the other hand there is the allowance for short selling. The first one has no strong effect on the decision pro or contra annuities. The second one has a major effect. It leads to the so called “true annuity puzzle”, which increases the negative annuitization (=life insurance). They conclude that the aggregate demand for annuitization is negative considering all households.

An extension for future work in the paper of Reichling and Smetters (2013) is the assumption of asymmetric information, whereas Davidoff et al. (2005) regard this point already in their analysis. They conclude that annuities are postponed but the assumption of asymmetric information has not a main impact on the decision. The intention of Davidoff et al. (2005) for future work leads to the direction of behavioral considerations. Consumers should get a better understanding of trading with annuities

and governments should implement mandatory annuities for welfare increasing. However, the ideas of Reichling and Smetters (2013) do not lead in this direction. They focus more on technical extensions as “differential transaction costs” or “more worker risk” which both will lead to decrease the level of annuitization. In addition the authors propose to rethink the structure of the tax and social insurance policies, considering the results of their work.

6. Conclusion

The conclusion of Yaari (1965) and partly Davidoff et al. (2005) to invest all the savings in annuities is not tenable, especially not in a model which includes assumptions as health shocks or behavioral factors. Nevertheless their approach is thoughtful and includes many significant assumptions and conclusions, as for example, the comparison of complete market setting and incomplete market setting or the consideration of bequest motives. However, they do not implement as many important assumptions as it was done by the paper of Reichling and Smetters (2013). They use the same framework as Yaari (1965) but with a significant difference: they exclude the assumption that health shocks are fixed and they implement the existence of different health status with stochastic mortality probabilities. This sustains the actual needs of consumers and the financial planner view. Especially low-level individuals are affected by unexpected shocks because mostly they do not have enough resources to cover them. Consumers prefer liquidity instead of lump sum what is a rational way to protect them against the uncertainty in the future. Therefore, the implementation of health shocks emphasizes the advantages of bonds and highlights the shortcomings of annuities.

Despite these shortcomings, the acceptance and comprehension of annuities is rising in the latest years but still many of the possible annuity buyers have little or no knowledge about the expression “annuity”. The most important advantage about them is the certainty that retirees do not run out of money if they possess an annuity. And this well-feeling is highly valued by old individuals. It is increasing its significance in future years, when the average time of pension last 25 to 30 years.

However, this is an objective average number. There is no certainty that an individual lives as long as his annuity is valuable and he should renounce the secure lump-sum pension of \$100,000 in the beginning of his retirement. This effect belongs to mental accounting and this field also includes loss aversion, what is caused by the fact that an individual first has to give up parts of his savings to purchase annuities and only afterwards he receives money. These approaches of behavioral factors lead to a decline of the demand of annuities.

Both Davidoff et al. (2005) and Reichling and Smetters (2013) excluded behavioral factors in their models, but Davidoff et al. (2005) mentioned this in his remarks for future works as a factor of under-annuitization. Framing effects are also related to the pool of

possible behavioral factors. They decrease the demand for annuities because people are sensitive to framing and starting values which leads to wrong optimization decisions. This argumentation depends on how consumers perceive annuities. On the one hand is the opportunity to regard them as a lifelong insured consumption and on the other hand annuities can be seen as risky investments. Brown et al. (2008) examined these two considerations in an experimental survey and concluded that 70% of individuals would have chosen to invest in annuities if they are described and sold as lifelong insured consumption. But only 21% intend to buy them if annuities are considered as a risk investment.

Furthermore, a related behavioral factor is the illusion of control. Consumers have the common opinion that they are able to control their money more precisely if their wealth stays at the bank accounts. In contrast, by purchasing annuities the individuals “only” receive money monthly which is a loss of control for them.

The US, which is the country with the largest annuity market, changed their pension system from Defined benefits (DB) to Defined contribution (DC). The second option supports the purchase of bonds which is a disadvantage for the annuity products and implies that the access to buy annuities decreased for individuals. Combined with the low knowledge about the product and the limitation of annuities on individual markets (Brown, 2009), these shortcomings induce a displacement of annuities.

In spite of all these weaknesses of annuities, there are reasonable arguments to buy them. The uncertainty of the future is reduced by purchasing an annuity. The individual can live without fears and worries about which investment or consumption strategy he should pursue. Following many researches, most of the retirees who turned to the age of 80 are not capable anymore to manage well their own fortune. That is why annuity-owners live a happier life compared to retirees who rely on liquid savings (Panis, 2004). Moreover, an influential professor of behavioral science and economics at the Booth School of Business at the University of Chicago, Richard Thaler appealed to the honor of the retirees by the quote: “If I buy an annuity, I’m buying my children an insurance policy against me having to move in with them when I run out of money.” (Tange-duPré, 2013) A proud father, who raised all his children with his own money, does not want to rely on his own children.

However, empirical studies showed that only a few people choose annuities if they are available (*“Growing Older in America: the Health and Retirement Study,”* 2007). This might be the reason why annuities are mostly purchased by upper class individuals. In

total, 35% of all households in US receive income in retirement by holding annuities but the individuals with short household assets are the majority which is not capable to purchase them. Only 22% of households with less than \$100,000 household assets invest in annuities. In comparison, those with assets over \$250,000 to \$499,999 are about double (45%) as likely as the low income individuals (LIMRA, 2011). Another proof is the paper of Bütler and Teppa (2007) which found out that about two third of the retirees in Switzerland hold annuities. The caveat of this research is that annuitization is the default option in their analysis and the fact that Switzerland is one of the countries with the highest GDP per capita of the world. In fact, only households who belong to the top half of the income distribution are capable to spend their resources in annuities (Brown, 2009).

Despite these defects the number of annuity holders increased the last years. While in 2005 only \$11.8 billion were financed in annuities in the US market, almost 10 years later the number multiplied to \$235.8 billion (LIMRA, 2014). This rising number is a result of the increasing life expectation and a growing income, which are essential parameters for the decision to purchase annuities.

The most promising explanations of the low demand of annuities are risk-sharing among couples and families and the introduction of liquidity constraints (Brown, 2004). A couple combines their fortune and has higher valued mortality credit than singles. This implies a better position for pricing the annuities and to value the annuities in a lower level. Moreover, the annuity holders intend to lower their level of annuity demand because of the implementation of liquidity constraints, which are mostly highly-priced. Furthermore, a possible bond-driver is the preexisting annuity market because of the Social Service. This covers already a part of the demand. Additionally, bequest motives reduce the demand for annuities in each model because only with savings they are realizable.

A further explanation of under-annuitization and the low level of precautionary savings is the behavior of humans facing unpleasant events. The majority is satisfied with their current state-of-life and do not want to think about future problems, as they will occur during the retirement. It is important to raise the awareness for these problems because the whole Social Security System in each country cannot maintain the standards of today. Particularly the average age of the inhabitants in industrialized countries will increase constantly. Hence, there is the need of precautionary savings to maintain a

convenient standard of life, as for example annuities or other opportunities of private pension.

To conclude, Davidoff et al. (2005) implemented a promising answer to the “annuity puzzle” by the result of partial annuitization in incomplete market settings. There has to be an increase in the demand of annuitization compared to the currently low level of annuitization. However, the conclusion of them to invest at least 66% of the savings in annuities is not a realistic solution. Annuities are still a pension form for rich and healthy people, who can afford them. Nevertheless, if individuals make future plans in the youth of their lives, then annuities are a valuable option.

The paper of Reichling and Smetters (2013) includes many significant assumptions which diminish the demand of annuities. In spite of these implementations, there is still a demand of annuities in their simulation results.

Finally, there should be a hybrid solution with a majority of investments in bonds to cover unexpected health shocks and the desire of the opportunity to realize bequest motives.

Additionally, the financial crisis and the political intention of low interest rates raised up a different investment option. For future work the researcher should focus on the investment of stocks for retirees. This is an interesting option in a portfolio because currently the interest rates tend to 0.

Appendix A

$$\mathbf{R}_A \mathbf{A} = \mathbf{R}_B \mathbf{B} \Rightarrow A < \mathbf{1B}$$

$$\begin{pmatrix} R_{A,2} \\ R_{A,3} \end{pmatrix} A = \begin{pmatrix} R_{B,2} & 0 \\ 0 & R_{B,3} \end{pmatrix} \begin{pmatrix} B_2 \\ B_3 \end{pmatrix} \Rightarrow A < \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} B_2 \\ B_3 \end{pmatrix} \quad (1)$$

$$R_{A,2}A = R_{B,2}B_2 \Rightarrow B_2 = \frac{R_{A,2}}{R_{B,2}}A$$

$$R_{A,3}A = R_{B,3}B_3 \Rightarrow B_3 = \frac{R_{A,3}}{R_{B,3}}A$$

$$\Rightarrow B_2 + B_3 = \frac{R_{A,2}}{R_{B,2}}A + \frac{R_{A,3}}{R_{B,3}}A \quad (2)$$

$$(2) \text{ into } (1) \Rightarrow A < \frac{R_{A,2}}{R_{B,2}}A + \frac{R_{A,3}}{R_{B,3}}A$$

$$1 < \frac{R_{A,2}}{R_{B,2}} + \frac{R_{A,3}}{R_{B,3}}$$

Appendix B

$$\min_{c_1, A, B_2, B_3} L = -(c_1 + A + B_2 + B_3) + \lambda(U(c_1, R_{B,2}B_2 + R_{A,2}A, R_{B,3}B_3 + R_{A,3}A) - \bar{U}) + \mu_2 B_2 + \mu_3 B_3$$

$$L_{c_1} = -1 + \lambda U_1(c_1, R_{B,2}B_2 + R_{A,2}A, R_{B,3}B_3 + R_{A,3}A) = 0 \quad (1)$$

$$L_{B,2} = -1 + \lambda R_{B,2} U_2(c_1, R_{B,2}B_2 + R_{A,2}A, R_{B,3}B_3 + R_{A,3}A) + \mu_2 = 0 \quad (2)$$

$$L_{B,3} = -1 + \lambda R_{B,3} U_3(c_1, R_{B,2}B_2 + R_{A,2}A, R_{B,3}B_3 + R_{A,3}A) + \mu_3 = 0 \quad (3)$$

i. This equilibrium equation has to hold:

$$(1) = (2)$$

$$U_1(c_1, R_{B,2}B_2 + R_{A,2}A, R_{B,3}B_3 + R_{A,3}A) = R_{B,2} U_2(c_1, R_{B,2}B_2 + R_{A,2}A, R_{B,3}B_3 + R_{A,3}A) + \frac{\mu_2}{\lambda} \quad (4)$$

If this equation is violated, then it does not fulfill the constraints.

$$\text{Suppose } U_1(c_1, R_{A,2}A, R_{A,3}A) < R_{B,2} U_2(c_1, R_{A,2}A, R_{A,3}A) \text{ holds.} \quad (5)$$

$$U_1(c_1, R_{A,2}A, R_{A,3}A) - R_{B,2} U_2(c_1, R_{A,2}A, R_{A,3}A) < 0 \quad (6)$$

$$\text{If } B_2 = 0 \text{ is the solution, then } \mu_2 > 0 \Rightarrow \frac{\mu_2}{\lambda} > 0 \quad (7)$$

$$\text{As } \frac{\mu_2}{\lambda} = U_1 - R_{B,2}U_2 \text{ and by (6) } \frac{\mu_2}{\lambda} < 0!$$

This is a contradiction to the assumption (7).

$$\Rightarrow B_2 > 0 \text{ is the solution}$$

ii. This equilibrium equation has to hold:

$$(1) = (3)$$

$$U_1(c_1, R_{B,2}B_2 + R_{A,2}A, R_{B,3}B_3 + R_{A,3}A) = R_{B,3}U_3(c_1, R_{B,2}B_2 + R_{A,2}A, R_{B,3}B_3 + R_{A,3}A) + \frac{\mu_3}{\lambda} \quad (8)$$

If this equation is violated, then it does not fulfill the constraints.

$$\text{Suppose } U_1(c_1, R_{A,2}A, R_{A,3}A) < R_{B,3}U_3(c_1, R_{A,2}A, R_{A,3}A) \text{ holds.} \quad (9)$$

$$U_1(c_1, R_{A,2}A, R_{A,3}A) - R_{B,3}U_3(c_1, R_{A,2}A, R_{A,3}A) < 0 \quad (10)$$

$$\text{If } B_3 = 0 \text{ is the solution, then } \mu_3 > 0 \Rightarrow \frac{\mu_3}{\lambda} > 0 \quad (11)$$

$$\text{As } \frac{\mu_3}{\lambda} = U_1 - R_{B,3}U_3 \text{ and by (10) } \frac{\mu_3}{\lambda} < 0!$$

This is a contradiction to the assumption (11).

$$\Rightarrow B_3 > 0 \text{ is the solution}$$

Appendix C

$$\min_{c_1, A, B, Z} L = -(c_1 + A + B_2 + B_3)$$

$$+ \lambda(U(c_1, R_{B,2}B_2 + R_{A,2}A - Z, R_{B,3}B_3 + R_{A,3}A + (R_{B,3}/R_{B,2})Z) - \bar{U}) \\ + \mu_2 B_2 + \mu_3 B_3$$

$$L_{c_1} = -1 + \lambda U_1(c_1, R_{B,2}B_2 + R_{A,2}A - Z, R_{B,3}B_3 + R_{A,3}A + (R_{B,3}/R_{B,2})Z) = 0 \quad (1)$$

$$L_A = -1 + \lambda[R_{A,2}U_2(c_1, R_{B,2}B_2 + R_{A,2}A - Z, R_{B,3}B_3 + R_{A,3}A \\ + (R_{B,3}/R_{B,2})Z) + R_{A,3}U_3(c_1, R_{B,2}B_2 + R_{A,2}A - Z, R_{B,3}B_3 + R_{A,3}A \\ + (R_{B,3}/R_{B,2})Z)] = 0 \quad (2)$$

$$L_{B,2} = -1 + \lambda R_{B,2}U_2(c_1, R_{B,2}B_2 + R_{A,2}A - Z, R_{B,3}B_3 + R_{A,3}A + (R_{B,3}/R_{B,2})Z) + \mu_2 = 0 \quad (3)$$

$$L_{B,3} = -1 + \lambda R_{B,3}U_3(c_1, R_{B,2}B_2 + R_{A,2}A - Z, R_{B,3}B_3 + R_{A,3}A + (R_{B,3}/R_{B,2})Z) + \mu_3 = 0 \quad (4)$$

$$\begin{aligned}
L_Z = \lambda[-U_2(c_1, R_{B,2}B_2+R_{A,2}A - Z, R_{B,3}B_3+R_{A,3}A + (R_{B,3}/R_{B,2})Z) \\
+ (R_{B,3}/R_{B,2})U_3(c_1, R_{B,2}B_2+R_{A,2}A - Z, R_{B,3}B_3+R_{A,3}A + (R_{B,3}/R_{B,2})Z) = 0
\end{aligned}
\tag{5}$$

Case differentiation:

Case 1:

$$\mu_2 = 0 \text{ and } \mu_3 = 0: \Rightarrow B > 0$$

$$(3) = (4)$$

$$R_{B,2}U_2 = R_{B,3}U_3$$

$$U_2 = \left(\frac{R_{B,3}}{R_{B,2}}\right)U_3$$

$$U_2 = R_Z U_3 \tag{6}$$

Set:

$$(1) = (4)$$

$$U_1 = R_{B,3}U_3$$

$$U_1/U_3 = R_{B,3} \tag{7}$$

Set:

$$(1) = (2)$$

$$U_1 = R_{A,2}U_2 + R_{A,3}U_3$$

Insert (6):

$$U_1 = R_{A,2}R_Z U_3 + R_{A,3}U_3$$

Divide by U_3 :

$$\frac{U_1}{U_3} = R_{A,2}R_Z + R_{A,3}$$

Insert (7):

$$R_{B,3} = R_{A,2}R_Z + R_{A,3}$$

This is not equal the result:

$$R_{B,3} < R_{A,2}R_Z + R_{A,3}$$

Case 2:

$$\mu_2 = 0 \text{ and } \mu_3 < 0: \Rightarrow B = 0$$

Set:

$$(3) < (4)$$

$$\begin{aligned}
R_{B,2}U_2 &< R_{B,3}U_3 \\
U_2 &< \left(\frac{R_{B,3}}{R_{B,2}}\right)U_3 \\
U_2 &< R_ZU_3
\end{aligned} \tag{8}$$

Set:

$$\begin{aligned}
(1) &= (4) \\
U_1 &= R_{B,3}U_3 \\
U_1/U_3 &= R_{B,3}
\end{aligned} \tag{9}$$

Set:

$$\begin{aligned}
(1) &= (2) \\
U_1 &= R_{A,2}U_2 + R_{A,3}U_3
\end{aligned} \tag{10}$$

Insert (8):

$$U_1 < R_{A,2}R_ZU_3 + R_{A,3}U_3 \tag{11}$$

Divide by U_3 :

$$\frac{U_1}{U_3} < R_{A,2}R_Z + R_{A,3} \tag{12}$$

Insert (9):

$$\begin{aligned}
R_{B,3} &< R_{A,2}R_Z + R_{A,3} \\
R_Z &= \frac{R_{B,3}}{R_{B,2}}
\end{aligned}$$

Insert the second term in the inequality:

$$\Rightarrow R_{B,3} < R_{A,2} \frac{R_{B,3}}{R_{B,2}} + R_{A,3}$$

Divide it by $R_{B,3}$:

$$1 < \frac{R_{A,2}}{R_{B,2}} + \frac{R_{A,3}}{R_{B,3}}$$

Appendix D

From Appendix C equation (11):

$$U_1 < R_{A,2}U_2 + R_{A,3}U_3$$

Solutions of the minimization problem in Appendix C:

$$U_1 = R_{B,2}U_2 \quad \text{and} \quad U_3 = \left(\frac{R_{B,2}}{R_{B,3}}\right)U_2$$

$$R_{B,2}U_2 < R_{A,2}U_2 + \left(\frac{R_{B,2}}{R_{B,3}}\right)U_2$$

Condition of dissaving after the full annuitization, $R_{B,2}U_2 \leq R_{B,3}U_3$, implies the second part:

$$R_{B,2}U_2 < R_{A,2}U_2 + R_{A,3}\left(\frac{R_{B,2}}{R_{B,3}}\right)U_2 \leq R_{A,2}U_2 + R_{A,3}U_3$$

Appendix E

(I)

$$V(R_B B, R_B(R_A A + R_B B - c_2)) = qv(R_B R_B B) + (1 - q)v(R_B(R_A A + R_B B - c_2))$$

$$V_A = (1 - q)R_B R_A v'(R_B(R_A A + R_B B - c_2)) = 0 \quad (1)$$

$$V_B = qR_B R_B v'(R_B R_B B) + (1 - q)R_B R_B v'(R_B(R_A A + R_B B - c_2)) = 0 \quad (2)$$

With $A = 0$: $(2) > (1)$

$$R_B R_B \left(qv'(R_B R_B B) + (1 - q)v'(R_B(R_B B - c_2)) \right) > (1 - q)R_B R_A v'(R_B(R_B B - c_2))$$

(II)

$$\begin{aligned} & (1 - q)R_B R_A v'(R_B(R_A A + R_B B - c_2)) \\ &= R_B R_B \left(qv'(R_B R_B B) + (1 - q)v'(R_B(R_A A + R_B B - c_2)) \right) \end{aligned}$$

Insert the assumption: Actuarially fair: $(1 - q)R_A = R_B$

$$\begin{aligned} v'(R_B(R_A A + R_B B - c_2)) &= \left(qv'(R_B R_B B) + (1 - q)v'(R_B(R_A A + R_B B - c_2)) \right) \\ qv'(R_B(R_A A + R_B B - c_2)) &= qv'(R_B R_B B) \end{aligned}$$

Holding this equation, requires assumption: $R_A A \equiv c_2$.

Appendix F

Bond:

Good health:

$$\max_{c_{t+1}, c_{t+2}} u(c_{t+1}|h_{t+1}) + \beta s_{t+1}(h_{t+1})u(c_{t+2}|h_{t+1}) =$$

$$\frac{c_{t+1}^{1-\sigma}}{1-\sigma} + 1 * 1 * \frac{c_{t+2}^{1-\sigma}}{1-\sigma}$$

$$\text{s.t.} \quad c_{t+1} + c_{t+2} \leq 1,5$$

$$\Rightarrow c_{t+1} = 0.75$$

$$\Rightarrow c_{t+2} = 0.75$$

The individual splits the bond investment equally because he lives for sure two periods.

Bad health:

$$\begin{aligned}
 & u(c_{t+1}|h_{t+1}) + \beta s_{t+1}(h_{t+1})u(c_{t+2}|h_{t+1}) = \\
 & \quad \frac{c_{t+1}^{1-\sigma}}{1-\sigma} + 1 * 0 * \frac{c_{t+2}^{1-\sigma}}{1-\sigma} \\
 \text{s.t.} \quad & c_{t+1} + c_{t+2} \leq 1,5 \\
 & \Rightarrow c_{t+1} = 1.5
 \end{aligned}$$

The individual lives only in the first period that is why he consumes everything in the first period.

Annuity:

Good health:

$$\begin{aligned}
 & u(c_{t+1}|h_{t+1}) + \beta s_{t+1}(h_{t+1})u(c_{t+2}|h_{t+1}) = \\
 & \quad \frac{c_{t+1}^{1-\sigma}}{1-\sigma} + 1 * 1 * \frac{c_{t+2}^{1-\sigma}}{1-\sigma} \\
 \text{s.t.} \quad & c_{t+1} + c_{t+2} \leq 2 \\
 & \Rightarrow c_{t+1} = 1 \\
 & \Rightarrow c_{t+2} = 1
 \end{aligned}$$

The individual receives in each period 1 unit for the annuity and consumes this.

Bad health:

$$\begin{aligned}
 & u(c_{t+1}|h_{t+1}) + \beta s_{t+1}(h_{t+1})u(c_{t+2}|h_{t+1}) = \\
 & \quad \frac{c_{t+1}^{1-\sigma}}{1-\sigma} + 1 * 0 * \frac{c_{t+2}^{1-\sigma}}{1-\sigma} \\
 \text{s.t.} \quad & c_{t+1} + c_{t+2} \leq 1 \\
 & \Rightarrow c_{t+1} = 1
 \end{aligned}$$

The individual only lives in the first period and can only get in this period the outcome of the annuity.

Appendix G

$$P_{(m=i|n=h_1)}(j = 27) = \begin{cases} 1, & \text{if } i = h_1 \\ 0, & \text{if } i = h_2 \\ 0, & \text{if } i = h_3 \end{cases}$$

$$P_{(m=i|n=h_2)}(j = 27) = \begin{cases} 0.4, & \text{if } i = h_1 \\ 0.58, & \text{if } i = h_2 \\ 0.02, & \text{if } i = h_3 \end{cases}$$

$$P_{(m=i|n=h_3)}(j = 27) = \begin{cases} 0.2, & \text{if } i = h_1 \\ 0.6, & \text{if } i = h_2 \\ 0.2, & \text{if } i = h_3 \end{cases}$$

3 state Markov process matrix for the age of 27 with m as column vector and n as row vector:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0.4 & 0.58 & 0.02 \\ 0.2 & 0.6 & 0.2 \end{pmatrix}$$

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[pacific/australia/retirement/mercer-melbourne-global-pension-index/mercer-melbourne-global-pension-index-infographic.pdf](http://www.mercer.com/content/dam/mercer/attachments/asia-pacific/australia/retirement/mercer-melbourne-global-pension-index/mercer-melbourne-global-pension-index-infographic.pdf)

Zusammenfassung

In der Magisterarbeit behandle ich das "Annuity Puzzle". Das „Annuity Puzzle“ ist ein weit verbreiteter Begriff in der wissenschaftlichen Literatur für Pensionssysteme. Es behandelt die Fragestellung, warum in der Praxis nur so wenige Individuen auf ein Pensionssystem mit „annuities“ zurückgreifen und im Gegensatz dazu in „bonds“ investieren. Dies ist für viele Forscher nicht verständlich, da in vielen Studien bewiesen wurde, dass eine „full annuitization“ (d.h. sein ganzes Ersparnis in „Annuities“ zu investieren) die bessere Alternative ist.

Es werden in der Magisterarbeit zwei verschiedene Paper behandelt mit unterschiedlichen Ansätzen. Das erste Paper ist von Davidoff et al. (2005) „Annuities and individual welfare“, welches die These vertritt, dass allgemein mehr in „annuities“ investiert werden sollte. Das zweite, „Optimal annuitization with stochastic mortality probabilities“, wurde von Reichling und Smetters (2013) verfasst und erarbeitet viele Argumente für ein Investment in „bonds“. In diesem Abschnitt werden die Arbeiten der Forscher zusammengefasst und die einzelnen Theoreme beschrieben und mit Beweisen belegt.

In dem nächsten Abschnitt werden beide Paper miteinander verglichen. Dabei werden die Gemeinsamkeiten und Unterschiede herausgearbeitet. Es wird auf die Gemeinsamkeit hingewiesen, dass die Paper das gleiche Model (3-Perioden Model mit nutzenmaximierenden Konsumenten) verwenden für die Lösung des Problems. Auf der anderen Seite werden die unterschiedlichen Annahmen herausgearbeitet, welche zu unterschiedlichen Resultate führen.

In dem abschließenden Teil werden die Schlussfolgerungen der Arbeit beschrieben. Zusammenfassend lässt sich sagen, dass ein Individuum einen größeren Anteil an „annuities“ in seinem Portfolio aufnehmen sollte. Jedoch nicht in dem hohen Maße, wie es das Paper von Davidoff et al. (2005) beschreibt. Aufgrund der hohen Unsicherheit im Alter sollte man eine Sicherheit an Vermögen haben (Investment in „bonds“), um einen gesundheitlichen Rückschlag, der mit hohen Kosten verbunden ist, ausgleichen zu können.

Lebenslauf

Name: Frank Walzer
Adresse: Karmeliterhofgasse 2/1/52 1150 Wien
E-Mail: frank_walzer@web.de
Telefon: +49 (0)160 – 97 33 67 61
+43 (0) 699 – 18 32 73 05
Geburtstag/ -ort: 27.01.1988 / Deutschland
Familienstand: Ledig
Staatsangehörigkeit: Deutsch

Studium:

Oktober/08 – September/12 Universität Konstanz Bachelor Economics
März/13 – Juni/15 Universität Wien Master Economics

Schulausbildung:

September/94 – Juli 07 Abitur Hugo-Eckener-Schule Wirtschaftsgymnasium
September/07 – Juni/08 Zivildienst

Berufserfahrung / Praktika:

August/06 & August/07 Ferienjob, ZF Friedrichshafen AG, 88048 Friedrichshafen
Juli/08 & Juli/13 Ferienjob, ZF Friedrichshafen AG, 88048 Friedrichshafen
Januar/12 – Februar/12 Praktikum Controlling, ZF Friedrichshafen AG, 88048 Friedrichshafen
Mai/12 – Juli/12 Praktikum Controlling, ZF do Brasil, Sorocaba, Brasilien
Januar/13 – Februar/13 Praktikum Controlling im Versuchszentrum, ZF Friedrichshafen AG, 88048 Friedrichshafen
September/14 – März/15 Berufspraktikum Accounting & Reporting Raiffeisen International AG
April/15- Externer Mitarbeiter Accounting & Reporting Raiffeisen International AG

Besondere Kenntnisse und Interessen:

Sprachkenntnisse: Englisch fließend, Portugiesisch B1
Französisch Grundkenntnisse, Spanisch Grundkenntnisse
EDV-Kenntnisse: MS-Office
Hobbies: Fussball, Klavier, Skifahren, Beachvolleyball