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Zürich, am 14.2.2013

Mag. Michael Markovich

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*To my children
Élise and Laurent*

Abstract

The goal of this dissertation is to analyze whether market participants consider real-option value in price discovery for commodity companies. The model case for this question was Russian companies with their vast exposure to natural resources. The focus was on the Energy and Basic Materials sectors with companies that have an established history and belong to the global peer-group in their sector. We analyze in particular three major companies that fulfill the necessary criteria with regard to data length and sector importance (Gazprom, Lukoil and Norilsk Nickel) and their domestic and international sector peers.

We distinguish between corporate fundamental and time series analysis. From a fundamental standpoint we expected that if real-options are indeed considered for company valuation, the chosen three companies should trade with a premium relative to their peer-group. However, contrary to expectations and despite a better capital structure and higher profitability of the chosen companies, no valuation advantage for selected Russian commodity companies relative to their global peer group could be confirmed within a fundamental multiple analysis framework.

As a consequence we focus on time series analysis for discovering real-option driven price discovery. We derive several hypotheses – separated into unconditional and conditional ones – about expected equity return characteristics of real-option driven companies and conduct complementary stationarity and out-of-sample forecast tests for various data-generating processes. The set of unconditional hypotheses focuses on higher distribution sample moments that indicate real-option pricing, while the conditional hypotheses relate to the return sensitivity of real-option driven commodity companies to changes in commodity prices.

Overall, the results lead us to conclude that there is substantial evidence that real-option pricing is an element of price discovery for Russian commodity companies. Real-option pricing can be observed in the distribution characteristics – in particular in the form of significantly higher skewness values – of individual Russian commodity companies or sectors; it can also be found by analyzing sector-specific sensitivity factors in different

market environments (i.e., during high or low commodity price regimes), since sector sensitivity towards commodity price changes is dynamic (a characteristic that was not confirmed for non-Russian companies) as expected for real-option sensitive companies.

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List of Abbreviations

ABM	Arithmetic Brownian motion
ADF	Augmented Dickey-Fuller
AOUP	Additive Ornstein-Uhlenbeck process
APAC	Asia and Pacific
ATM	At the money
BSM	Black, Scholes and Merton
CAPM	Capital Asset Pricing Model
CCA	Contingent Claim Analysis
CDA	Confirmatory Data Analysis
CDF	Cumulated density function (i.e. distribution function)
CRB	Commodity Research Bureau Index
DCF	Discounted Cash Flow
DGP	Data generating process
DSP	Difference stationary processes
EBIT	Earnings before Interest and Taxes
EBITDA	Earnings before Interest, Taxes, Depreciation and Amortization
EBT	Earnings before Taxes
EEMA	Eastern European Markets
EV	Enterprise value of the company observed in the market
FCF	Free Cash-Flow
GARCH	Generalized Autoregressive Conditional Heteroscedasticity
GBM	Geometric Brownian motion
GJR	Glosten – Jagannathan – Runkle (GARCH model with asymmetric components)
HAC	Heteroskedasticity and autocorrelation
H_0	Null hypothesis
H_1	Alternative hypothesis
ITM	In-the-money
KPSS	Kwiatkowski-Phillips-Schmidt-Sinn
LATAM	Latin America
LnOUP	Log-normal Ornstein-Uhlenbeck process

LTG	Long-term growth rate
MCS	Monte Carlo Simulation
MEA	Middle East and Africa
MICEX	Moscow Interbank Currency Exchange
MPE	Mean percentage error
NA	North America
NCF	Net cash-flow
NFCF	Net free cash-flow (cash-flow to equity holders)
NPV	Net Present Value
OLS	Ordinary Least Squares
OTM	Out-of-the-money
PDF	Probability density function
PDE	Partial differential equation
PEG	Estimated Price to Earnings Ratio to Growth (PEG)
PE	Price / Earnings Ratio
S&P	Standard and Poor's
SDE	Stochastic differential equation
SPGSCI	S&P Global Commodity Index
SPGSCEN	S&P Global Commodity Energy Index
SPGSCIN	S&P Global Commodity Industrial Metal Index
ROA	Real-Option Analysis
RICI	Rogers International Commodity Index
WE	Western Europe
TSP	Trend stationary processes
VAR	Vectorautoregressive
VRA	Variance Ratio Analysis

List of Variables and Parameters

a	Drift parameter
	Location parameter
$\hat{a}$	Estimated location parameter
A	Moment test auxiliary variable
A_t	Investment costs at time t
	Exercise price (costs) of the physical project at time t
Afa_t	Depreciation and amortization at time t
b	Diffusion parameter
	Dispersion parameter
$\hat{b}$	Estimated dispersion parameter
$\sqrt{b_1}$	Unbiased sample statistic for the third distribution moment (skewness)
b_2	Unbiased sample statistic for the fourth distribution moment (kurtosis)
b_2^e	Kurtosis of standardized error (sample statistic)
B	Moment test auxiliary variable
c	Constant
	Regression intercept
	Shape parameter
$\hat{c}$	Estimated shape parameter
$c_{f,t}$	Fix costs in time period t
$c_{i,t}$	Price of a call option at time t on the simulation i
c_t^u	Price of a call option at time t on the underlying u
$\hat{c}_t^u$	Uncertain price of a call option at time t on the underlying u

$c_{v,t}$	Variable unit costs in time period t
$cdf(.)$	Cumulated density function (also called cumulated distribution function)
C	Moment test auxiliary variable
C_t	Cost function at time t
C_t^E	Equity value defined as call option on the company assets at time t
$CE(.)$	Certainty equivalent of a cash-flow or price
$CMDTY_t^i$	Price of commodity i at time t
$Cov(.,.)$	Covariance of two random variables
$d_{i,t}$	Cash-flow during the life-time of the option i at time t
D	Moment test auxiliary variable
D_t	Value of debt capital at time t at market prices
Div_t	Dividend cash-flow at time t
E	Moment test auxiliary variable
E_t	Value of equity capital at time t at market prices
$E_t(.)$	Expectation value of a random variable at time t
$E_t^p(.)$	Expectation value of a random variable at time t under probability distribution p
$E_t^{\bar{p}}(.)$	Expectation value of a random variable at time t under the martingale equivalent measure
$f(x)$	Function of x
F	Moment test auxiliary variable

F_t	Price of a commodity future contract at time t
$F_X(x_i)$	Cumulative distribution function (cdf) showing the probability of X being smaller or equal to x_i
FCF_t	Free cash-flow at time t
g	Growth rate
g_1	Skewness coefficient (standardized skewness statistic)
$\hat{g}_1$	Estimated skewness coefficient (standardized skewness statistic)
g_2	Kurtosis coefficient (standardized skewness statistic)
$\hat{g}_2$	Estimated kurtosis coefficient (standardized skewness statistic)
g_2^e	Excess kurtosis coefficient (standardized excess kurtosis statistic)
$\hat{g}_2^e$	Estimated excess kurtosis coefficient (standardized excess kurtosis statistic)
h	Observation units per year
I_t	Information set
$Index_t$	Commodity price or index at at time t
$Inflation_t$	Inflation at at time t
j	Time constant (stationarity definition)
JB	Jarque-Bara statistic
k	Time constant (variance ratio test)
$\hat{k}_4$	Unbiased sample statistic for the fourth moment about the mean
$k_{E,t}$	Cost of equity capital at time t

$k_{D,t}$	Cost of debt capital at time t
K_0	Net present-value of investment project at $t = 0$
K_t	Net capital-value of investment project at time t
K^i	Strike price i of the financial option, also referred to as exercise price
$K_{u_t}^{c,i}, K_{u_t}^{p,i}$	Strike price i of financial call-(put-) option on the underlying u at time t
$L(\Theta_i x_j)$	Likelihood function for parameter Θ_i based on observations x_j
$\hat{m}_3$	Sample statistic for the third moment about the mean
$\hat{m}_4$	Sample statistic for the fourth moment about the mean
$MSCI_t^i$	Morgan Stanley Capital International index for region i at time t
MPE_t	Mean percentage error at time t
n	Number of shares Number of observations
N	Number of forecasts over fixed time horizon
$N(\cdot)$	Cumulative probability distribution function for a normal distributed random variable
$N'(\cdot)$	Cumulative probability distribution function for a equally distributed random variable
NPV_t^i	Discounted net cash-flows of asset i at time t
p	Lag order
$p_{i,t}$	Put price resulting from simulation i at time t
p_t	Put price at time t

$p_t(z_i)$	Real probability at time t of state i
$\tilde{p}_t(z_i)$	Risk neutral probability at time t of state i
P_t^E	Equity value defined as put function on the company assets at time t
P_t^u	Equity price of the underlying (company) u at time t
P_{it}^u	Equity price of the underlying i for the cross section u at time t
$\hat{P}_t^u$	Uncertain equity price of the underlying (company) u at time t
q_t	Units sold at time t
r	Prevailing risk free interest rate equal to risk neutral drift
$\bar{r}^u$	Annualized mean return of underlying u
r_i^{NPV}	Simulated NPV return i
$r_{i,t}$	Return of asset i at time t
$r_{i,j}^{u,c}, r_{i,j}^{u,p}$	Total return of a call-(put-)option investment on the underlying u with the strike price j
$\bar{r}_{i,j}^{u,c}, \bar{r}_{i,j}^{u,p}$	Mean total return of simulated call or put option investment on the underlying u with the strike price j
r_t^m	Market portfolio return at time t
r_t^{rf}	Risk-free interest rate at time t
r_t^u	Return of the underlying u at time t
R_T	Residual value of project or company
$RMSE_t$	Root mean squared error at time t
$RMSPE_t$	Root mean squared percentage error at time t
s	Tax rate

$\hat{s}(\cdot)$	Estimated sample standard deviation of a stochastic variable
$s_t(\cdot)$	Risk of a stochastic variable measured in standard deviations at time t
S_t^i	Traded market value of equity capital or equity index i at time t
$\hat{S}_t$	Estimated market value of equity capital or equity index at time t
SR_i	Sharpe-ratio of underlying index i
SR_i^{HM}	Sharpe-ratio of underlying index i adjusted for higher moments
SR_i^{AC}	Sharpe-ratio of underlying index i adjusted for autocorrelation
t	Time index
t_n	Continuous time index
T	Time at expiry of investment project Time to maturity of an option
T_N	Number of forecast steps at time t
T_t	Theil's inequality coefficient at time t
T_t^B	Theil's bias coefficient at time t
T_t^V	Theil's variance coefficient at time t
T_t^C	Theil's covariance coefficient at time t
u_t	Random variable at time t Regression error component at time t
U_t	Value of the unlevered (pure equity capital) financed company at time t Turnover at time t
V_0	Value of an underlying risky asset at time $t = 0$

V_t	Value of project at time t
	Value of total company at time t at market prices
VR_k	Variance ratio for the lag k
$Var(.)$	Variance of a random variable
VIX_t	Volatility index for the S&P500 at time t
x_t	Known realization of a random variable in discrete time
$\bar{x}$	Average of sample values x_t
$\hat{x}_t$	Estimated long-term convergence of random variable
$X(t)$	Random variable in (unknown realization in continuous time)
$\bar{X}(t)$	Long-term convergence of random variable (unknown realization in continuous time)
y_t	Observed time series variable
	Known realization of a random variable in discrete time
$\bar{y}_{t+h}$	Average of the time series y_t over a time horizon h
$\hat{y}_{t+h,t}$	Time series forecast at time t over a time horizon h
$\bar{\hat{y}}_{t+h,t}$	Mean of time series forecast at time t over a time horizon h
Y_t	Loan related cash-flow (payment/repayment) at time t
z	z-statistic
z_i	State i , $i = 1 \dots n$
z_t	Known realization of a random variable in discrete time
Z_t	Loan interest expenses at time t
Z'_t	Deductible interest expenses

α_0	Regression intercept
α_i	i -th regression coefficient
β	GARCH diffusion parameter
β_i	i -th regression coefficient
β_2	Population parameter for kurtosis
β_2^e	Kurtosis of standardized error
$\beta_{j,t}$	Beta factor of the underlying j to a market index
β_u^c, β_u^p	Option beta for call-(put) option on the underlying u
γ	GJR-GARCH leverage parameter
γ_2	Population parameter for excess kurtosis
$\Gamma(.)$	Gamma function
$\pi(...)$	Profit/loss function
δ	Instantaneous convenience yield
$\bar{\delta}$	Long-term convergence level for convenience yield
$\hat{\delta}$	Adjusted instantaneous convenience yield
Δ_t	Difference of two random variables at time t
$\Delta_j^{u,c}, \Delta_j^{u,p}$	Call-(put)-option price sensitivity in unit terms (relative to unit change in underlying)
ε_t	Regression error component at time t
ε_{it}	Regression error component for cross section i at time t
ε_t^i	Random variable of process i at time t

$\varepsilon(t)$	Continuous time standard normal variable
η	Mean reversion parameter
$\hat{\eta}$	Estimated mean reversion parameter
η_t	Centered random variable
η_{it}	Fixed effect component for asset i over time t
θ	Risk aversion coefficient
ϑ_i	Sample statistic i
Θ_i	Population parameter i
$\hat{\Theta}_i$	Population estimator for parameter i
κ	Control variable for stochastic decision process
λ_t	Market price of risk at time t
$\hat{\lambda}$	Estimated market price of risk constant
$\hat{\lambda}_t$	Estimated market price of risk at time t
μ	First moment about the origin (location moment, i.e. mean)
$\hat{\mu}$	Estimated first moment about the origin (location moment, i.e. mean)
$\bar{\mu}$	Mean drift of stochastic process
μ_t	Drift of a stochastic process at time t
μ_r^c	r -moment about the mean (central moment of order r)
μ_r^{st}	Standardized r -moment about the mean
$\mu_{r,i}^E$	Expected investment return of asset i
Π_t	Portfolio value at time t

ϖ	Log-likelihood parameter
$\rho(\cdot)$	Correlation coefficient between two stochastic variables
$\hat{\rho}(\cdot)$	Estimated correlation coefficient between two stochastic variables
$\sigma(\cdot)$	Standard deviation of a stochastic underlying
σ_i	Diffusion value of the data generating process i
$\hat{\sigma}_i$	Estimated diffusion value of the data generating process i
σ_t^u	Standard deviation of the underlying u rate of return at time t
$\bar{\sigma}_{i,ann}(\cdot)$	Unconditional annualized GARCH standard deviation for asset i
$\sigma_{i,t}$	Risk of the underlying i measured in standard deviations at time t
$\sigma_j^{u,c}, \sigma_j^{u,p}$	Annualized standard deviation of the return on a call(-put)-option on the underlying u and strike price j
σ_V	Volatility of firm's assets
σ_E	Volatility of firm's equity
$\sigma_{S.E.}$	Standard error of data generating process
$\hat{\sigma}_\varepsilon$	Standard error of regression estimate
$\sigma_{\bar{y}_{t+h,t}}(\cdot)$	Standard deviation of time series between t and h
$\sigma_{\hat{y}_{t+h,t}}(\cdot)$	Standard deviation of forecast time series between t and h
$\sigma^2(\cdot)$	Second moment about the mean (dispersion moment, i.e. variance)
$\sigma_t^2(\cdot)$	GARCH diffusion term at time t
$\bar{\sigma}_t^2(\cdot)$	Unconditional GARCH variance
$\bar{\sigma}_{i,ann}^2(\cdot)$	Unconditional annualized GARCH variance for asset i

ϕ_i	i -th regression coefficient
ϕ_i^d	i -th regression coefficient for regression on $I(1)$ dependent variable
ϕ_i^l	i -th regression coefficient on $I(0)$ dependent variable
φ	Unit-root parameter
χ^2	Test statistic
ξ	Centralized random variable about the mean
Ψ_i	Option price elasticity of option i
ω	Realization of a stochastic process within the probability space $\omega \in \Omega$
Ω	Probability space
Ω_k^c, Ω_k^p	Option price elasticity

1 Introduction

For more than 20 years, investment research has been enriched by an approach called real-option analysis (ROA). The pioneers in using this approach for pricing natural resources were Tourinho (1979) and Brennan and Schwartz (1985). Both of these papers built their assumptions on preceding milestone achievements in contingent claim analysis, mainly by Black and Scholes (1974), Merton (1973), Margrabe (1978), Geske (1979) and Cox, Ross and Rubinstein (1979).¹ Since ROA was first used, it has experienced growing attention in capital budgeting and developed into a powerful tool for managers who want to overcome the difficulties of traditional cash-flow analysis, which are usually conducted on a discounted cash-flow (DCF) basis and net-present-value (NPV) rules.

ROA has been used as an additional tool for considering the value of managerial flexibility and valuing complex investment opportunities. However, it has not become widespread as a valuation device in equity analysis. Although many analysts have used ROA for particular special valuations – e.g. for valuing long-term contracts, investments in land and facilities, or even industrial economic purposes – it never became more than a complementary tool, used far less frequently than NPV and DCF methods.

The reason for this superficial treatment was indisputably the relative value contributed by ROA to the overall company value and its corresponding assessment costs. While most equity value can be explained by traditional DCFs, only a minor portion – especially for financially mature companies with relatively high cash-flow streams – is quantifiable through real-options. Additionally, DCFs are straightforward, while ROA requires an extensive time contribution and is not easily understood by most analysts. Thus, the relationship between value content and effort is usually biased toward DCF

¹ This enumeration is meant to recall the most important scholars on contingent claim analysis and is by no means exhaustive.

analysis, so that ROA becomes desirable only in a highly uncertain environment where the DCF contribution to valuation is not high. Such uncertain environments can arise through a combination of economic, political and/or legal circumstances.

Often in such situations variables and parameters usually necessary to conduct DCF analysis, such as price and demand elasticities, and cost structures or appropriate discount rates, are not easily accessible. Concurrent political and/or legal distortions like autocratic systems or lack of accounting history and transparency can lead to a downward bias in cash-flow estimates. In such an environment, ROA can highlight, i.e. contribute more value to, the overall company value than it otherwise would.²

In transition economies, such as Russia's, foreign investors usually face this kind of uncertain environment. Additionally, key economic sectors such as oil, gas, and mining are prominent settings in which to use real-options to unveil value.

Table 1
P/E Comparisons among Petrol Companies

Corporation	P/E 2009	est. P/E 2010
Sibneft	17.48	4.03
Tatneft	1.46	1.81
Surgutneftegaz	3.88	5.31
Lukoil	3.37	4.54
BP PLC	13.87	14.42
Royal Dutch Petro.	16.42	16.39
Shell PLC	15.45	15.76
ENI SPA	10.24	10.50
OMV	8.31	6.53
Fred Olsen Energy	12.97	7.52

Source: Bloomberg & I/B/E/S as of end Q2 2009. Russian companies are labeled in red.

² We will elaborate on this topic further in Chapter 2, p. 13.

However, looking for example at a key valuation multiple (current and estimated price/earnings ratios or P/Es) of Russian oil companies, relative to comparable Western companies in the same industry groups (see Table 1), reveals that Russian companies trade at a strong discount and have a significant valuation disadvantage despite their real-option value potential.

The main aim of this dissertation is therefore to answer the question of whether market participants consider real-option value in their price discovery for companies in transition economies, with Russia as the major research example. We will formulate how real-option value should affect the distribution of a company's net-present value and hence its equity return distribution pattern. The core of the analysis will focus on the data generating process involving dependent equity return distributions of Russian commodities. After testing the suitability of different data generating processes usually applied for real-option valuation, we will assess whether they differ from processes used for other major international peer-group companies.

The reminder of Chapter 1 explains the main ideas of ROA and its application for equity valuation in general and for some industries in transition economies in particular. We will highlight the differences between standard NPV and ROA, while emphasizing the distinct features of ROA. We will also explain the driving forces behind real-option value, methods of valuation and the intended research plan. This chapter concludes by defining the research process and structure of this dissertation and the different steps necessary to conduct the analysis.

1.1 Overview of traditional valuation approaches

Many conventional methods for equity valuation have been developed and used for equity analysis. They can be divided into two groups—namely, relative and absolute measures of value.

1.1.1 Multiples as relative measures

Asset valuation using multiples can be defined as a “relative measure” valuation tool for which asset prices or other company-specific characteristics (e.g., enterprise value, sales) are normalized (e.g., via sales or earnings) towards a single valuation statistic.

Profit or revenue multiples are very common measures used to assess the value of a company, especially if no long-term positive free cash-flow is measurable. Common multiples used are gross margins (gross profit/net sales), operating margins (operating profit/net sales), price/earnings ratios, EV/EBIT (enterprise value/earnings before interest and taxes) and EBIT-EBITDA margins (EBIT or EBITDA / net sales). All of these relative measures can be subdivided into current and forward-looking multiples.

The above measures have been and still are popular for valuing e-commerce companies. However, these measures are relative in an additional sense: they can be used only as relative ratios within a peer group of similar companies. Current or static multiples are a useful tool to assess the overall over/undervaluation of a company in a relative sense (i.e., relative to other members of the peer group). Table 2 gives an example and an overview over some peer group members of the oil industry.

Table 2
Oil Peer Group: Static valuation multiples

Corporation	$\frac{P}{E}$	$\frac{EBIT}{Sales}$	$\frac{EBITDA}{Sales}$	$\frac{EV}{EBIT}$
Lukoil	3.37	0.28	0.35	12.35
BP PLC	13.87	0.08	0.10	10.44
Royal Dutch Petro.	16.42	0.13	0.17	10.68
Totalfina Elf SA		0.12	0.17	9.77
ENI SPA	10.24	0.22	0.31	6.81
OMV	8.31	0.09	0.13	6.10

Source: Bloomberg & I/B/E/S as of end Q2 2009.

The advantage of this approach is its straightforward simplicity. It assumes that a portfolio of similar companies in general is valued correctly. However, the obvious

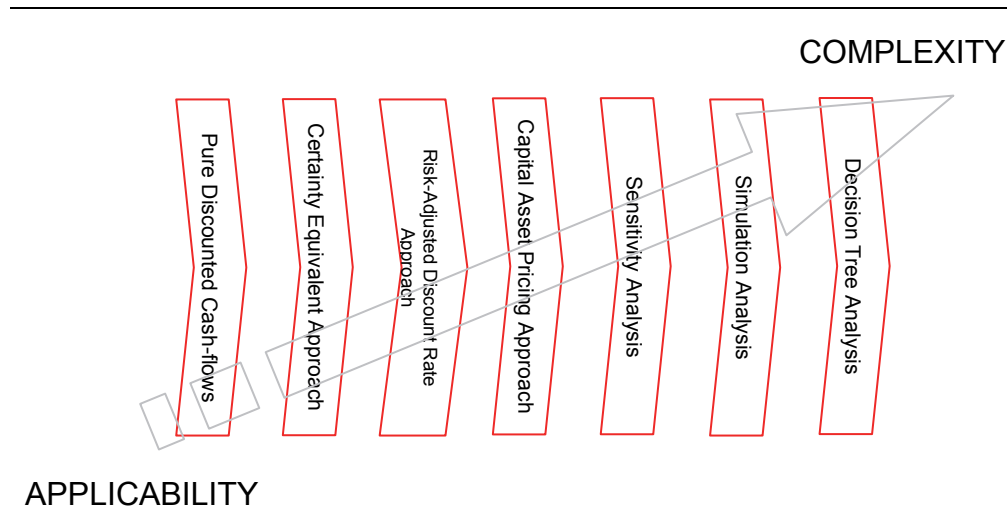
drawback is that it does not assess any precise value of overall equity in future terms. Although equity value can be derived from different key-ratios, this approach is not forward-looking and does not incorporate any macroeconomic or business cycle variables.

Forward priced multiples try to overcome these disadvantages by focusing/anticipating the future value of certain multiples. These measures focus on how a company will look at a given stage in the future. Considering the stage of development of a company (i.e., its growth or maturity) is important in determining which focus management will need to have. For example, during a growth phase, a company will put heavier emphasis on revenue (sales) multiples, while during a maturity stage, profitability multiples will become the predominant measure.

1.1.2 Absolute methods: Calculating companies' net-present-value (NPV)

Above we described some measures that can easily be enumerated and used for valuation purposes. However, given most companies' large cash-flow histories and the mentioned drawbacks of these approaches, other methods are also predominant in equity valuation. All of these methods use free cash-flow (FCF) estimates to determine equity value using standard NPV calculation. The development of standard capital budgeting and NPV techniques is depicted below.

Figure 1
Development of NPV Techniques



The pure discount factor-oriented valuation approach was soon amended by capital risk considerations, which led to the introduction of a more formal risk-return framework known as the capital asset pricing model (CAPM). Sensitivity and simulations extended the valuation tool-box. However, only decision tree analysis opened the door towards option and hence real-option value consideration.

If we compare traditional approaches with ROA, several similarities and distinct differences can be observed. Both approaches discount free cash-flows to the present. However, one crucial difference exists: namely, that NPV calculation using certainty equivalents is a static approach that makes decisions now without considering future flexibility. Thus, the future is considered static, based on the assumption that uncertainty is resolved at a constant rate per period. If managerial flexibility is allowed, however, these methods lead to a downward bias in valuation. Therefore NPV calculation can only be seen as a special, inflexible case of real-option methods or as an approximation of real-option valuation.

Written in a more formal way, NPV makes decisions on the basis of future expectations seen at time $t = \text{now}$. Defining V_T as the project value at expiry and A_t as the cost of the project, one can distinguish the two methods by their decision rule:³

$$\text{NPV rule at } t = 0: \text{Max}[0, E_0(V_T) - A_t] \quad (1)$$

In contrast, an ROA rule states:

$$\text{ROA rule at } t = 0: \text{Max}[0, V_T - A_t] \quad (2)$$

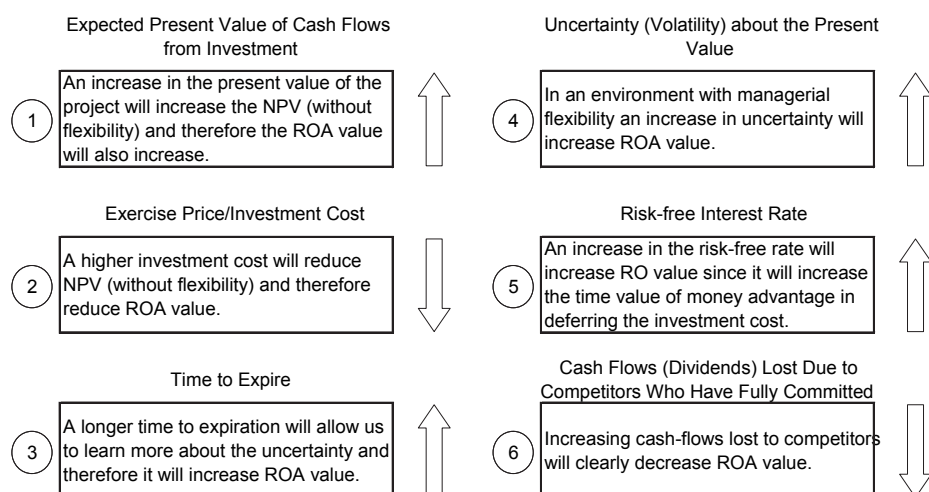
Both methods will lead to the same result only if there is no uncertainty (given as $E_0(V_T) = V_T$).

1.2 Real-option analysis and the case for Russian commodity companies

The search for real-option value in companies is not straightforward, since real- options are not immediately observable. The idea and obstacles behind associating Russian commodity companies with real-option analysis can therefore best be described by looking at some of the major driving forces that generate real-option value. Figure 2 highlights the most important one.

³ Copeland and Antikarov (2001) describe in this context the NPV method as an approach taking the maximum of a set of possible mutually exclusive alternatives, while ROA refers to the expectation of maximums. See Copeland and Antikarov (2001), p. 73.

Figure 2
Environment for Real-Options



Source: Copeland and Antikarov (2001), p. 7. This valuation driver framework will be used again in Chapter 3, p. 38.

The key driving factors – as described in Figure 2 – are similar to conventional options used in daily capital market transactions. Three factors, however, play a predominant role within the above framework. The first one is the change in the net-present-value of the project or company. For Russian commodity companies, this is primarily related to changes in the output price in the form of oil, gas and other natural resources (assuming that costs remain relatively constant), which have been extraordinarily volatile in the past decade.⁴ Volatility is the second driving factor, since it is directly related to management flexibility and hence to optionality in a changing environment. High volatility, an increasing output price of the producing good and change in technology, leads to an environment where the exercise strike price of possible real-options as boundaries for future investments (as distant as they might have appeared years ago) becomes less distant than in the past, and real-options start showing value. This is the environment in which Russian commodity companies should have been for the past decade.

⁴ See Chapter 2, p. 13 for a detailed analysis.

The combination of these key factors is of particular interest for Russian commodity companies, which should have benefited in the past from:

- Higher commodity prices
- Higher uncertainty
- Higher management flexibility (nationalization of the key natural resource sectors) and
- Change in technology (change in strike price)

Note that, outside the company, most of the factors in Figure 2 remain unobservable. The only source of inference is the daily price discovery and a cross-section valuation relative to peer-group companies. The question is therefore whether Russian commodity companies are valued different (i.e., higher) using conventional valuation measures than comparable peer-group companies, and whether the price discovery in daily equity trading gives any indication that real-option value is considered with Russian companies.

This dissertation will therefore concentrate mainly on analyzing these two questions, although emphasis will be placed on the price discovery for Russian commodity related companies relative to the global peer-group.

1.3 Dissertation structure

The dissertation is split into two major parts; the theoretical foundation of the dissertation is covered first. It starts with an analysis for the Russian equity market, whose high dependency on the development of the commodity market is illustrated. Additionally we will conduct a cross valuation of Russian stocks relative to the international peer-group to show how – based on common (cash-flow oriented) valuation multiples – Russian stocks remain undervalued.

The theoretical analysis continues further by laying the foundation for the empirical part by focusing on the change in the NPV distribution if real-options become an element of the valuation. We will describe in detail how and why the NPV distribution, and hence

also the distribution of equity returns, should change if companies possess real-options. A sensitivity analysis of real-option driving factors (e.g., volatility, moneyness, etc.) towards higher return distribution moments concludes this section. The remaining element of this theoretical part covers in detail the importance of the distribution function for real-option valuation.

The empirical part of the dissertation starts with testing different data generation processes for their capability to forecast commodity prices. Since real-options are valued based on certain assumptions about the main underlying data-generating-process (e.g., oil or gas as a major source of the project turnover, and hence the real-option valuation driver), we will conduct a rolling out-of-sample process analysis for major Russia-related commodity prices. The stability and hence forecast quality of a data-generating-process is, however, not the only criterion for a good DGP. The forecast variance directly affects the real-option value. We develop therefore a test framework that inter-relates the option valuation with the variability of the option payoff. The results of this analysis lead to the last chapter in the empirical part, in which the equity return distribution of selected Russian equity companies is tested. This search for real-option indications in the daily price discovery for Russian companies is the last element of the dissertation and is followed by the overall conclusions and outlook for further research. The entire structure is given again in detail on the following two pages.

Theoretical analysis

- Peer-group definition and analysis
- Real-options and the distribution of the net-present-value of a company
 - Option price specific factors and their influence on the payoff distribution of options
- GARCH processes and option pricing
 - Estimated GARCH parameters and their influence on the option distribution parameters
- Stochastic theory definitions crucial for the probability analysis and real-option pricing

Empirical analysis

- Data-generating-processes (DGPs) and their capability of valuing long-term commodity options
 - Calibration of different DGPs
 - Testing different stationarity assumption for commodity prices
 - Out-of-sample performance of different DGPs
 - Testing the influence of inflation and the external value of the USD on commodity prices
- Defining an option price fair-value test for different DGPs
- Unconditional sample tests of higher return distribution moments of Russian and global peer-group companies
- GJR-GARCH(1,1) parameterization of Russian and global peer-group companies
 - Influence of the leverage factor on the return distribution
 - Influence of the news-response factor on the return distribution
 - Associating the unconditional GJR-GARCH(1,1) kurtosis with real-option value
- Commodity price sensitivity (beta) analysis with sector-specific panel data
 - Sensitivity (beta) analysis across peer-group regions
 - Up- vs. down-market beta analysis across peer-group regions

Conclusion

Theoretical analysis and background		Empirical analysis		
The Russian Stock Market: The Value of Russian Commodity Companies - The Russian stock market and commodities - MICEX Structure, composition and dependencies - Valuation relative to the international peer-group	Real-Options, Probability Theory and the Net-Present- Value of the Firm - Definitions, characteristics and valuation - NPV & the distribution of FCFs - The distribution of an option return - Equity price distribution – a theoretical assessment of what to expect in case of real-option value - Stochastic theory and probability analysis for real-option pricing	Stochastic process analysis for commodity prices - Definitions and classifications - Calibration - Data stationarity tests - Out-of-sample performance of different data generating processes (DGPs)	Real-option pricing and process-dependent forecast variance - Forecast mean and variance of different DGPs - Testing the pricing efficiency of different DGPs	Searching for real-option pricing evidence in the Russian equity market - Price discovery in the Russian market - Unconditional distribution (sample moment) analysis - Conditional (GARCH) distribution analysis - Sensitivity (BETA) analysis
Conclusion				

2 The Russian Stock Market: The Value of Russian Commodity Companies Relative to the International Peer Group

2.1 Introduction

The Russian stock market has become increasingly relevant during the first decade of this century. This is primarily explained by the fact that the latest world economic expansion – which was the strongest world economic expansion in global GDP terms in centuries⁵ - was accompanied by a strong rally in commodity prices. The Russian stock market – as we will show in Section 2.2 (see p. 17) – is primarily driven by commodity or commodity-related stocks, and profited accordingly to a very large extent in this world economic expansion. Figure 3 illustrates this relationship by showing the development of the CRB⁶ commodity index along with the Russian main equity index, the MICEX Index.⁷

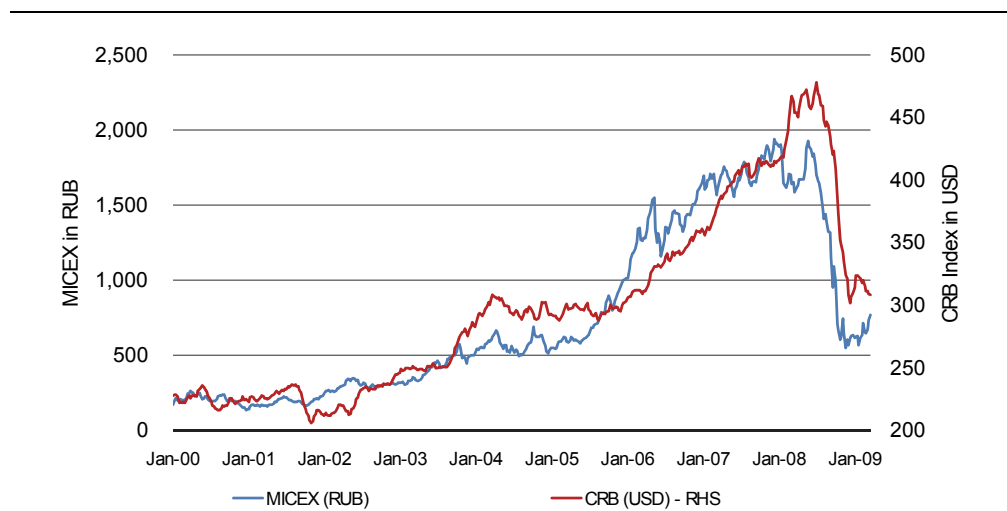
⁵ The author must acknowledge that this expansion was followed by the largest economic contraction since the end of World War II.

⁶ The CRB (Commodity Research Bureau) Index is a global commodity price now calculated by Thomson Reuters/Jefferies.

⁷ The MICEX Index is the real-time cap-weighted Russian composite price index. It comprises the 30 most liquid stocks of the largest and most developed Russian companies from the 10 main economic sectors. The MICEX Index was launched on September 22, 1997 with a base value of 100. It is calculated and disseminated by the MICEX Stock Exchange, the main Russian stock exchange. The MICEX Index currently captures around 80% of the market capitalization of the Russian equity market. Source: Bloomberg. A calculation description can be found under:

http://www.micex.com/articles/file/3273/MICEX_Index_description.pdf

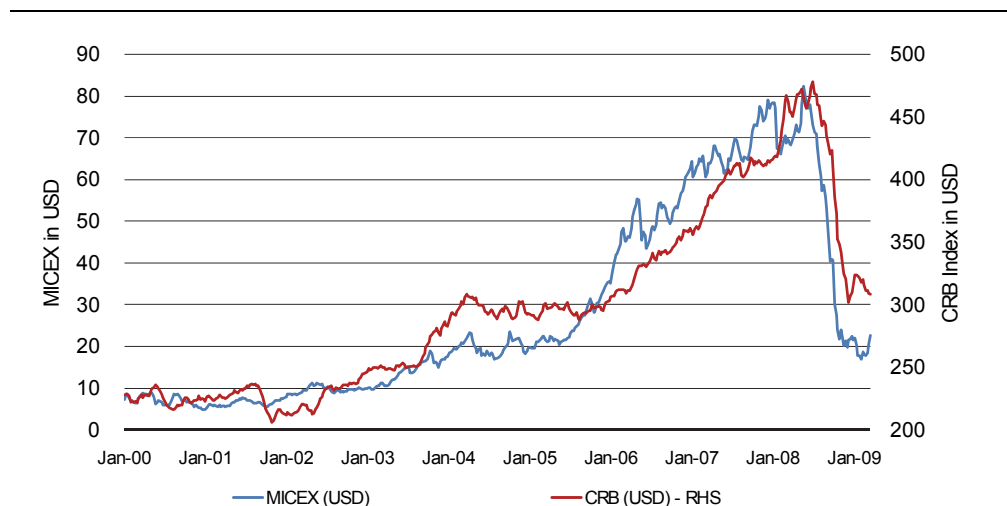
Figure 3
Development of Russian Stock Market Relative to Global Commodities



Russian MICEX stock index relative to CRB commodity index between 2000 and 2009. Source: Bloomberg.

Visual inspection confirms the obvious strong correlation between the development of the Russian MICEX and the CRB commodity index. The correlation is not born out of exchange rate fluctuations, as shown in Figure 4, where the MICEX Index is displayed in USD terms along with the CRB Index.

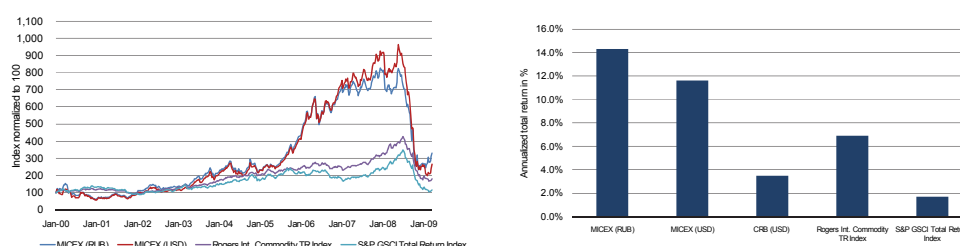
Figure 4
Development of Russian Stock Market in USD Terms Relative to Global Commodities



Russian MICEX stock index in USD relative to CRB commodity index between 2000 and 2009. Source: Bloomberg.

The Russian MICEX Index profited from the global boom in commodity markets between 2001 and mid-2007, and suffered consequently from the collapse in commodity prices between mid-2007 and the beginning of 2009. However, on a total return basis, the MICEX managed to outperform, in USD and RUB terms, main global total-return commodity indices as illustrated in Figure 5. The outperformance in percentage terms of the MICEX Index relative to other commodity total return indices is impressive. After a holding period of slightly more than 9 years, the MICEX Index returned nearly 15% p.a. in RUB terms. In USD terms the index returned twice as much as the Rogers International Commodity Index (RICI)⁸ and five times more than the S&P GSCI Total Return Commodity Index.⁹ The highest outperformance was observed between March 2006 and June 2008, when MICEX performance soured relative to its international peers (especially in USD due to USD weakness in this period relative to RUB).

Figure 5
Total Return Development of Russian Stock Market Relative to Global Commodity Price and Total Return Indices



Panel A: MICEX over time relative to different price and total return indices between 2000 and 2009. Source: Bloomberg.

Panel B: Annualized total return for MICEX and other major price and total return indices. Source: Bloomberg.

⁸ The RICI is a composite total return index. It represents a basket of commodities consumed in the global economy. Source: Bloomberg. Further information can be found under: <http://www.rogersrawmaterials.com/page1.html>.

⁹ Information on the S&P GSCI Total Return index can be found under http://www2.standardandpoors.com/spf/pdf/index/SP_GSCI_Factsheet.pdf and under http://www2.standardandpoors.com/spf/pdf/index/SP_GSCI_FAQ_Web.pdf.

This strong performance raises the question, what are the main driving factors that caused this recent development? Obviously, the Russian market and Russian companies are not known for their leading role in corporate governance or share-holder value orientation, but rather for their government interrelation and for past expropriations from equity holders by the government.

As laid out in the introduction to this dissertation (see Section 1.2, p. 7), there is reason to believe that Russian companies possess large real-options and that the value of these real-options increases with commodity prices, which could explain the strong outperformance of the Russian stock market. Hence, the main question with regard to the Russian stock market becomes the following: Are real-options a valuation driver that is actively priced in the Russian stock market?

Obviously the question cannot be answered directly, since this would require a detailed cash-flow analysis of each MICEX company along with international peers. Such an analysis would need to account for specific real-option possibilities (projects) for each of the analyzed companies in question. Since such a task is beyond any sensible time horizon, there is a need to focus on other, more general and hence broader measures, and above all more observable factors (such as distribution characteristics of daily returns) that indicate the consideration of real-option value in the price discovery of Russian stocks.

This chapter is structured as follows. We will first highlight the main characteristics of the Russian equity market by focusing on the company components of the MICEX Index. We will choose several core companies that fulfill the main requirement for the data analysis conducted in the empirical part of this dissertation. These companies are compared according to industry group with their international peers in order to extract valuation information. This will answer the question whether extraordinary performance can be explained by better fundamental data or not.

2.2 The Russian stock market: An overview

2.2.1 Structure of the Russian stock market

The Russian stock exchange (Moscow Interbank Currency Exchange or MICEX) is broadly represented by four major stock indices, listed in Table 3 along with their major characteristics.

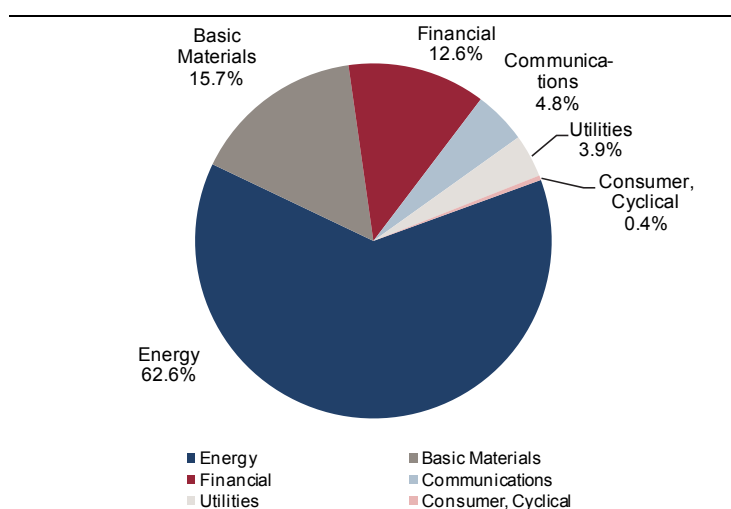
Table 3
Characteristics of Main Russian Equity Indices

	Currency	Inception	Base value	# of members	Market cap	P/E ratio	Dividend yield
MICEX	RUB	Sep. 1997	100	30	31.14 Tr	5.23	2.84
MICEX 10	RUB	Dec. 1997	100	10	175.9 Bln	7.52	7.22
Russian RTS Index	USD	Dec. 1995	100	50	69.3 Bln	5.30	3.07
Russian Traded Index	USD	Oct. 1997	1000	16	109.3 Bln	5.26	2.53

Source: Bloomberg, March 2009.

The MICEX Index represents the stock exchange benchmark and is used as a reference value for the overall development of the market (see Footnote 7, p. 13). The structure of the MICEX and its major constituencies is displayed in Table 4. Splitting the market along the industry sectors unveils the heavy components that define the major development of the MICEX Index. The industry sector weightings are given in Figure 6.

Figure 6
MICEX Sectors



Russian MICEX segmentation along industry sector in % of market capitalization.
Source: Bloomberg, March 2009.

Table 4
MICEX Members

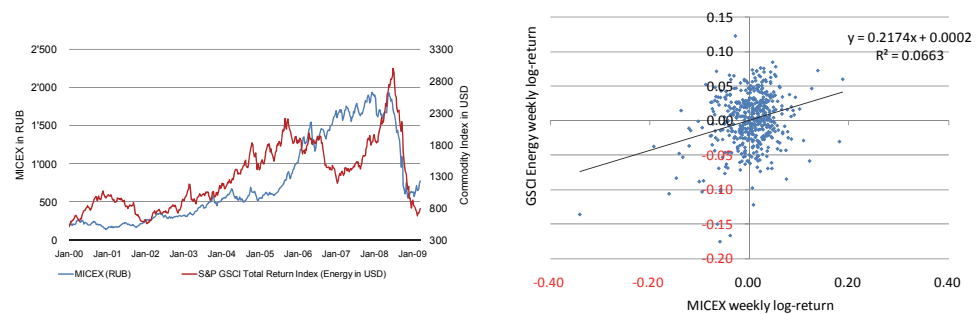
	Ticker	Name	% Weight	Shares in the Index	Last Price	Industry Group	Industry Sector	Issuer Industry
1	LKOH	LUKOIL	16.01	230.8	1,251.99	Oil Comp-Integrated	Energy	Oil&Gas
2	GAZP	Gazprom OAO	14.04	2,053.4	123.39	Oil Comp-	Energy	Oil&Gas
3	ROSN	Rosneft Oil Co	12.04	1,434.0	151.47	Oil Comp-Integrated	Energy	Oil&Gas
4	SBER03	Sberbank	9.91	8,418.9	21.24	Commer Banks Non-	Financial	Banks
5	SNGS	Surgutneftegaz	7.52	5,716.2	23.74	Oil Comp-Integrated	Energy	Oil&Gas
6	NOTK	NovaTek OAO	4.89	1,184.2	74.52	Oil Comp-	Energy	Oil&Gas
7	PLZL	Polyus Gold Co	4.53	68.6	1,190.50	Gold Mining	Basic Materials	Mining
8	GMKN	MMC Norilsk Nickel	3.98	36.2	1,981.80	Metal-Diversified	Basic Materials	Mining
9	TATN3	Tatneft	3.97	871.5	82.10	Oil Comp-Integrated	Energy	Oil&Gas
10	HYDR	RusHydro	3.41	95,555.5	0.64	Electric-Integrated	Utilities	Electric
11	URKA	Uralkali	2.59	722.3	64.70	Agricultural Chemicals	Basic Materials	Chemicals
12	MTSI	Mobile Telesystems	2.41	299.0	145.66	Cellular Telecom	Communications	Telecommunications
13	VTBR	VTB Bank OJSC	2.20	1,479,310.5	0.03	Commer Banks Non-	Financial	Banks
14	NLMK	Novolipetsk Steel	2.13	899.0	42.70	Metal-Iron	Basic Materials	Iron/Steel
15	RTKM	Rostelecom	2.03	123.9	295.34	Telephone-Integrated	Communications	Telecommunications
16	SNGSP	Surgutneftegaz	1.81	4,082.1	7.98	Oil Comp-Integrated	Energy	Oil&Gas
17	SIBN	Gazpromneft OAO	1.52	331.9	82.50	Oil Comp-Integrated	Energy	Oil&Gas
18	PMTL	Polymetal	1.20	100.8	213.91	Silver Mining	Basic Materials	Mining
19	CHMF	Severstal	0.64	90.7	128.21	Steel-Producers	Basic Materials	Iron/Steel
20	MAGN	Magnitogorsk Iron &	0.61	1,340.9	8.24	Metal-Iron	Basic Materials	Iron/Steel
21	TRNFP	AK Transneft OAO	0.57	1.2	8,298.00	Pipelines	Energy	Pipelines
22	SBERP03	Sberbank	0.47	920.0	9.31	Commer Banks Non-	Financial	Banks
23	AFLT	OAO Aeroflot	0.42	244.3	30.80	Airlines	Consumer, Cyclical	Airlines
24	RASP	Raspadskaya	0.26	156.2	30.10	Coal	Energy	Coal
25	URSI	Uralsvyazinform	0.22	12,919.5	0.31	Telecom Services	Communications	Telecommunications
26	OGKE	Fifth Power	0.21	5,305.8	0.73	Electric-Integrated	Utilities	Electric
27	MSNG	Mosenergo OAO	0.19	3,974.9	0.88	Electric-Integrated	Utilities	Electric
28	VTEL	VolgaTelecom	0.11	98.4	19.78	Telecom Services	Communications	Telecommunications
29	OGKC	OGK-3 OJSC	0.07	3,799.0	0.35	Electric-Generation	Utilities	Electric
30	RBCI	RBC Information	0.04	77.0	9.10	Internet Content-	Communications	Internet

Source: Bloomberg as of end of Q2 2009.

The MICEX Index is heavily biased towards commodity-related companies. The energy sector – at 62.6% the highest contributor in terms of market weighting – represents nearly 2/3 of the overall market capitalization. The second largest sector is the Basic Materials sector with 15.7%. Financial equities represent the third largest industry sector; it is important to highlight that most large energy companies also have financial subsidiaries that are used as financing vehicles (thus, the actual financial sector contribution is likely smaller). Thus, the energy and basic materials sector – the focus of our analysis - account for more than $\frac{3}{4}$ of the overall market capitalization of the Russian stock market.

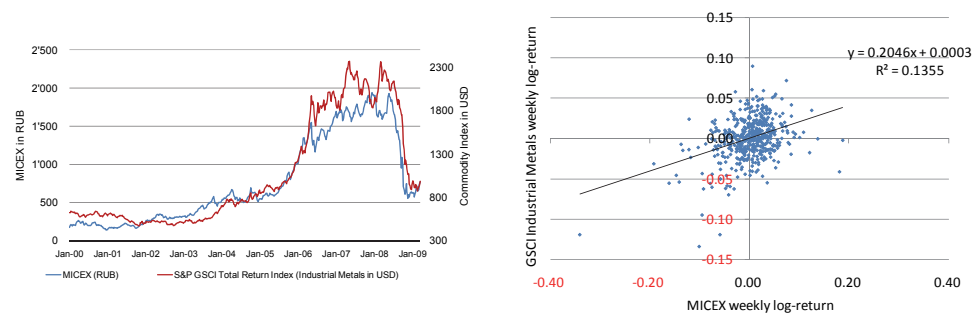
Figure 7

MICEX Index relative to S&P GSCI Energy and Metal Total Return Index



Panel A: MICEX relative to S&P GSCI Energy Total Return Index (2000-2009). Source: Bloomberg.

Panel B: Regression between weekly log-returns of MICEX Index (RUB) and GSCI Energy Total Return Index (2000-2009). Source: Bloomberg.



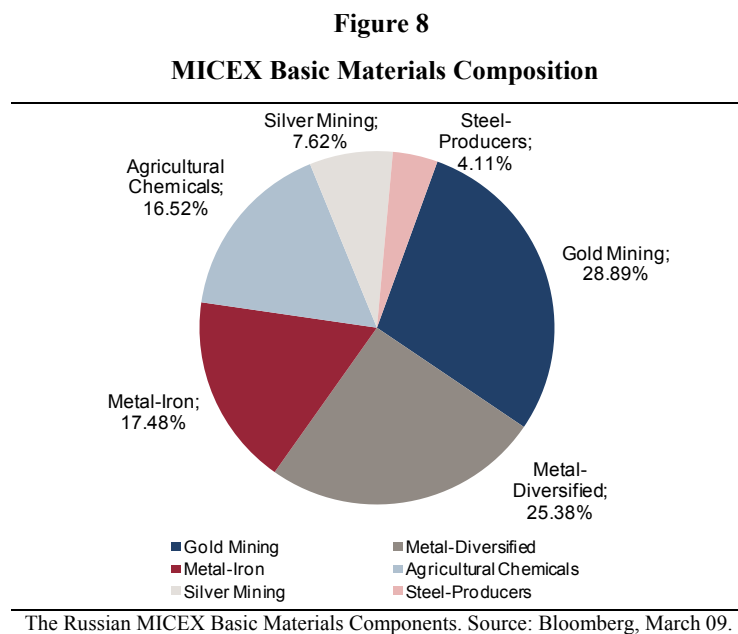
Panel C: MICEX relative to S&P GSCI Industrial Metals total return index (2000-2009). Source: Bloomberg.

Panel D: Regression between weekly log-returns of MICEX Index (RUB) and GSCI Industrial Metals Total Return Index (2000-2009). Source: Bloomberg.

As shown in Table 4, the biggest Russian company in terms of market capitalization is Lukoil, followed by Gazprom and Rosneft. Together with Surgutneftegaz they represent

approximately 50% of the overall market. Figure 7 shows the development of the MICEX relative to the Standard and Poor's GSCI Energy and Industrial Metals total return index. As expected – due to the high contribution of energy and basic materials to the MICEX – both global S&P indices follow the MICEX very closely. Interestingly, the global basic materials contribution (the much smaller of the two commodity sectors) correlates better with the overall MICEX, despite the fact that basic materials account for only 16% in the MICEX (see Panel D).

A closer look at industry subgroups (see Figure 8) shows that most of the Basic Materials companies are Gold, Silver and Steel producers, i.e., industries that are usually in the spotlight of real-option valuation.



The overall conclusion with regard to the Russian stock market is that, due to its large exposure to the commodity industry and the high level of volatility (as illustrated in Figure 3 and Figure 4, see p. 14), the likelihood of real-option generated value is large. In the empirical analysis part of this dissertation, and in particular in Chapter 6, we will focus on the question of whether such considerations are part of the price discovery process for selected MICEX members (see p. 260).

2.2.2 Valuation of Russian stocks relative to peer group

To put the market valuation of Russian stocks into a global perspective, we have related key valuation multiples of selected MICEX members – all major representatives of their own industry group – to their international peer groups. The peer group selection is oriented along the Industry Classification Benchmark (ICB)¹⁰ standard and market capitalization. The data source is Bloomberg, and all data were collected during March 2009. The key valuation multiples are given in Table 5.

Table 5
Used Peer Group Multiples

Name of valuation multiple¹¹
Price to earnings ratio (PE)
Estimated price to earnings ratio to growth (PEG)
Price to sales ratio
Price to cash-flow ratio
Enterprise value (EV) to trailing EBITDA
Sales revenue growth (1Y)
Debt to common equity
Cash dividend coverage
Gross margin
Operating margin
Return on common equity

Source: Bloomberg.

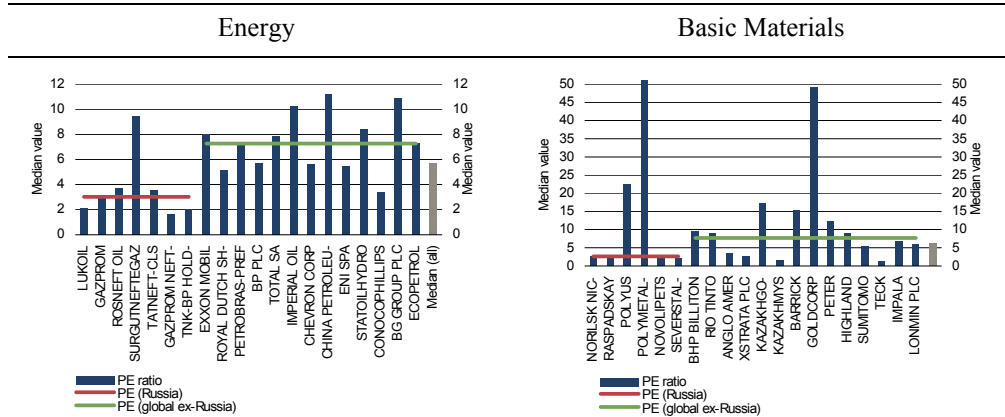
The cross section results for major MICEX companies are shown in Figure 9 and Figure 10. The individual results of selected key companies are shown in Section 2.3, p. 26. The valuation multiples that are price-related (i.e., multiples that relate the current market valuation to company-specific profitability measures, see Figure 9) show that during Q1 of 2009, Russian companies traded cheap relative to their global peers.

¹⁰ See http://www.icbenchmark.com/docs/FTSE_ICB_Fact_Sheet.pdf.

¹¹ The applied Bloomberg definition of the used valuation multiples is summarized in Appendix B – Peer group multiple definition, see p. 338.

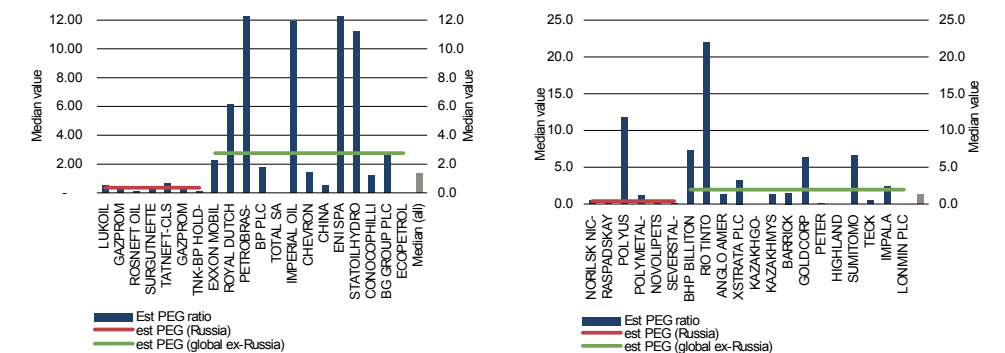
Figure 9

Valuation multiples of selected MICEX companies relative to the international peer group – Part 1



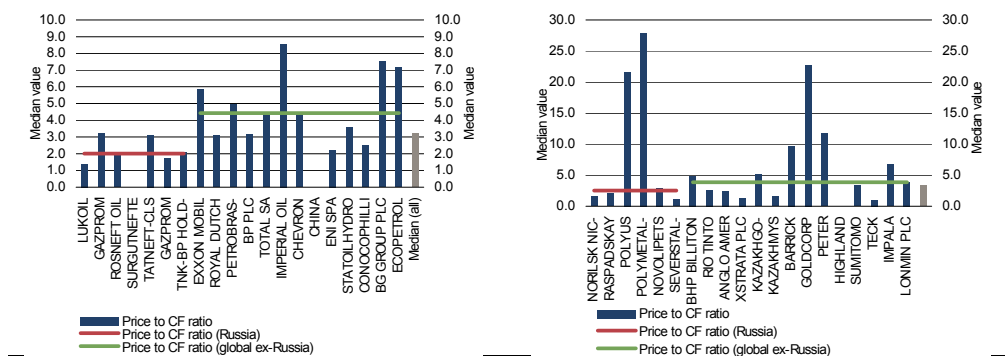
Panel A: Price Earnings ratio (PE ratio) of major MICEX Energy companies relative to the international global peer group. Source: Bloomberg.

Panel B: Price Earnings ratio (PE ratio) of major MICEX Basic Materials companies relative to the international global peer group. Source: Bloomberg.



Panel C: Estimated PE to growth ratio of major MICEX Energy companies relative to the international global peer group. Source: Bloomberg.

Panel D: Estimated PE to growth ratio of major MICEX Basic Materials companies relative to the international global peer group. Source: Bloomberg.



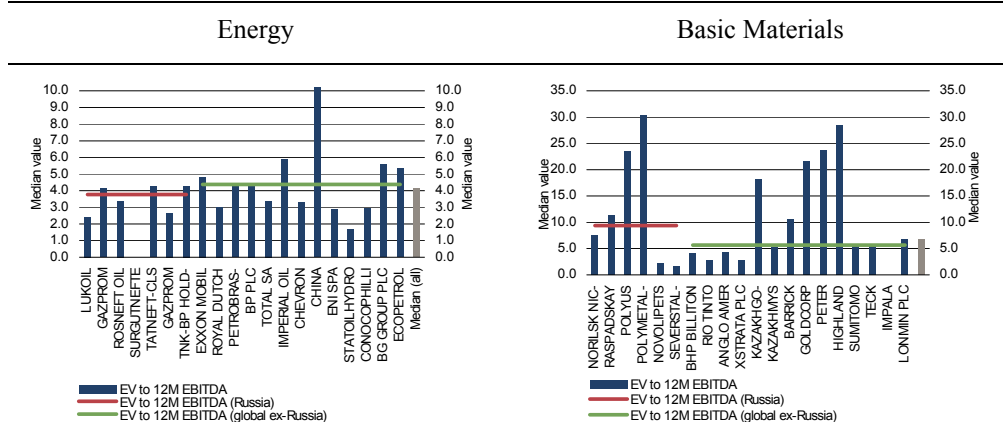
Panel E: Price to cash-flow ratio of major MICEX Energy companies relative to the international global peer group. Source: Bloomberg.

Panel F: Price to cash-flow ratio of major MICEX Basic Materials companies relative to the international global peer group. Source: Bloomberg.

In both of our core sectors (energy and basic materials) P/E for Russian companies was lower than for the global peer group (excluding Russia). The picture is confirmed when one focuses on the price to cash-flow ratio.

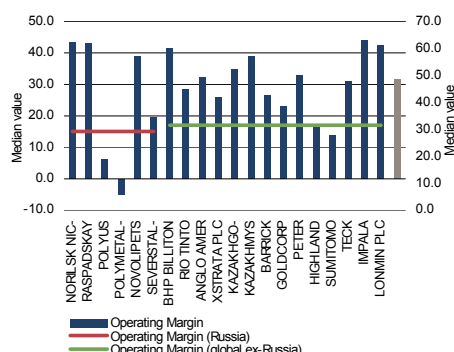
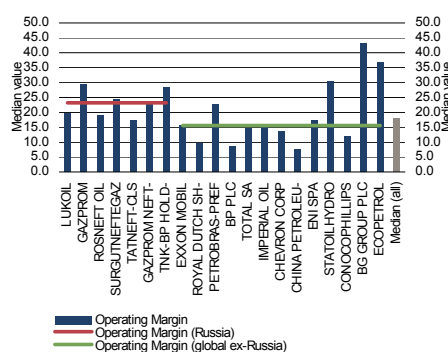
Figure 10

Valuation multiples of selected MICEX companies relative to the international peer group – Part 2



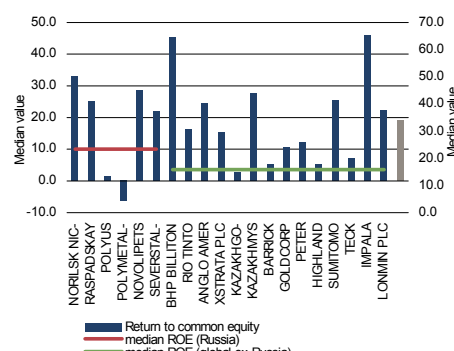
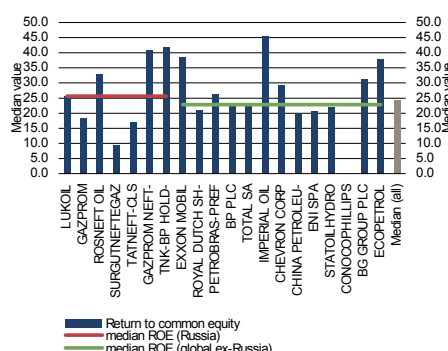
Panel A: EV to 12M rolling EBITDA of major MICEX Energy companies relative to the international global peer group. Source: Bloomberg.

Panel B: EV to 12M rolling EBITDA of major MICEX Basic Materials companies relative to the international global peer group. Source: Bloomberg.



Panel C: Operating Margin of major MICEX Energy companies relative to the international global peer group. Source: Bloomberg.

Panel D: Operating Margin of major MICEX Basic Materials companies relative to the international global peer group. Source: Bloomberg.



Panel E: Return to common equity of major MICEX Energy companies relative to the international global peer group. Source: Bloomberg.

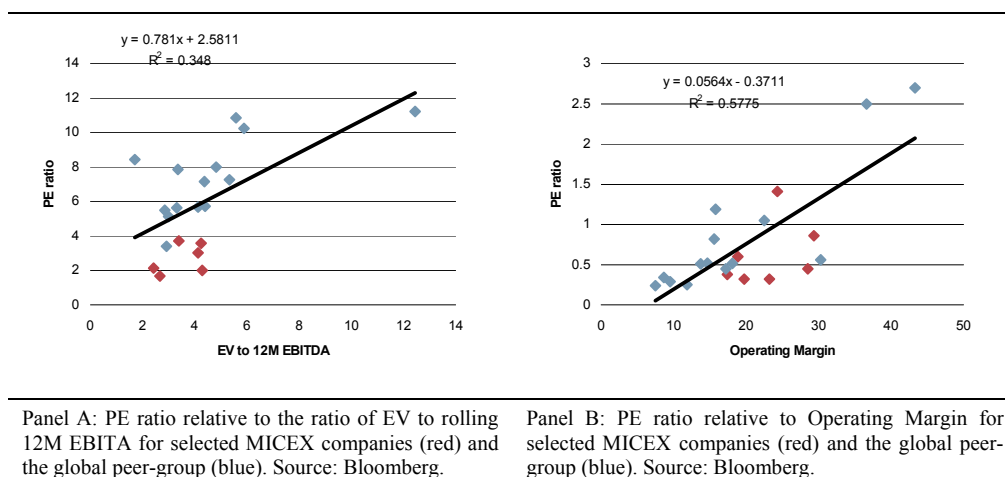
Panel F: Return to common equity of major MICEX Basic Materials companies relative to the international global peer group. Source: Bloomberg.

Forward-looking measures like the estimated price to earnings / growth ratio also show that within the global peer group Russian companies trade “cheap,” i.e., the values are

below those for the global peer-group. In general the undervaluation is more uniform (i.e., broader) in the Energy sector than in the Basic Materials industry group. Since the lower valuation aspect could simply be due to lower profitability, weaker margins and a sub-optimal capital structure, we have calculated additional valuation multiples provided in Table 5. The results are shown in Figure 10.

Panel A shows the enterprise value (as a sum of market capitalization, preferred equity, short-term and long-term interest-bearing debt, less cash and equivalents) relative to EBITDA. According to this non-forward-looking measure, Russian companies are in a similar if not better position than global peers (as reflected by equal or higher multiples). Median operating margins and return on equity are higher or at least similar to the median values of global energy and basic materials companies. Hence, overall the profitability and business dynamics of Russian companies does not explain why their overall valuation is poor relative to the global peer group.

Figure 11
Profitability measures of selected MICEX and global peer-group companies relative to their PE ratio



As further illustration, Figure 11 relates profitability measures to PE ratios, thus bringing the valuation multiples of Figure 9 and Figure 10 together. Again Russian companies (highlighted in red) have PE ratios below the peer-group fair-value regression line, whereas most other global peers (excluding Russia) lie above the fair-value regression.

This illustration confirms the previous assessment that, based on standard peer-group multiple analysis, Russian energy and basic material companies are cheap relative to the international peer group and that this cheapness is not a reflection of lower profitability.

Hence, with regard to general valuation multiples, one can conclude that the Russian Basic Material and the Energy Sector are trading cheap relative to their global peer group, despite the fact that both industry sectors ought to have the highest likelihood of real-option related value. If real-option value is discounted in Russian companies, one might expect Russian companies to trade above the fair-value regression line (higher price despite lower EBITDA and Operating Margin); however, the opposite is the case. This could partially be explained by the strong market correction between 2008 and 2009 due to the global meltdown in equity markets, which has hit Emerging Markets more than developed markets. However, other markets, in particular Asian markets, have been similarly affected by the market correction. Overall the analysis indicates that other methods and measures need to be employed in order to find evidence of real-option related price discovery.

The next section will analyze three major MICEX companies, as representatives of their industry group, that will be used in further analysis. The companies have been chosen based on their importance (specifically, market capitalization within the sector, making them comparable with international peers), their data history (a minimum length of seven years) and their exposure to commodity prices.

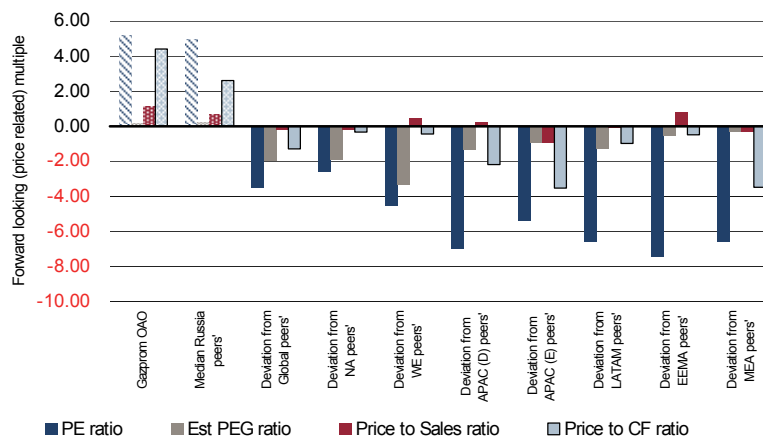
2.3 Comparing key MICEX companies with their international peer groups

2.3.1 Gazprom OAO

Gazprom extracts, transports, stores, and sells natural gas. The Company is the successor to the State-owned gas company and is Russia's largest company. Gazprom owns and operates Russia's Unified Gas Supply System. The Company has a monopoly on supplying gas in the Russian Federation and exports natural gas to Western Europe.¹² It possesses 17% of the world's gas reserves. Gazprom is classified as an Oil Exploration and Production Company within the Energy sector. Its peer group members according to Bloomberg are given in Table 6. Based on these peer group companies we have calculated, as in the previous section, aggregated valuation multiples for each peer group region. Again the valuation multiples defined in Table 5 have been applied. Since these valuation multiples are non-normally distributed within each peer group sub-category, we have used the median of each valuation multiple as a measure of central tendency within each peer group region to aggregate the data and make them comparable with one another.¹³

Figure 12

Forward-looking valuation multiples for Gazprom relative to its global peer group



Forward-looking valuation multiples of Gazprom relative to other global peer groups across regions (measured as the difference between Gazprom and the median value of the peer group). The peer group regions are NA (North America), WE (West Europe), APAC (Asia and Pacific, D for developed and E for emerging), LATAM (Latin America), EEMA (Eastern Europe and Central Asia), MEA (Middle East and Africa). Source: Bloomberg.

¹² Source: Bloomberg.

¹³ For the exact numbers see Appendix I – Peer group members, p. 376.

Table 6
International Peer Group of Gazprom

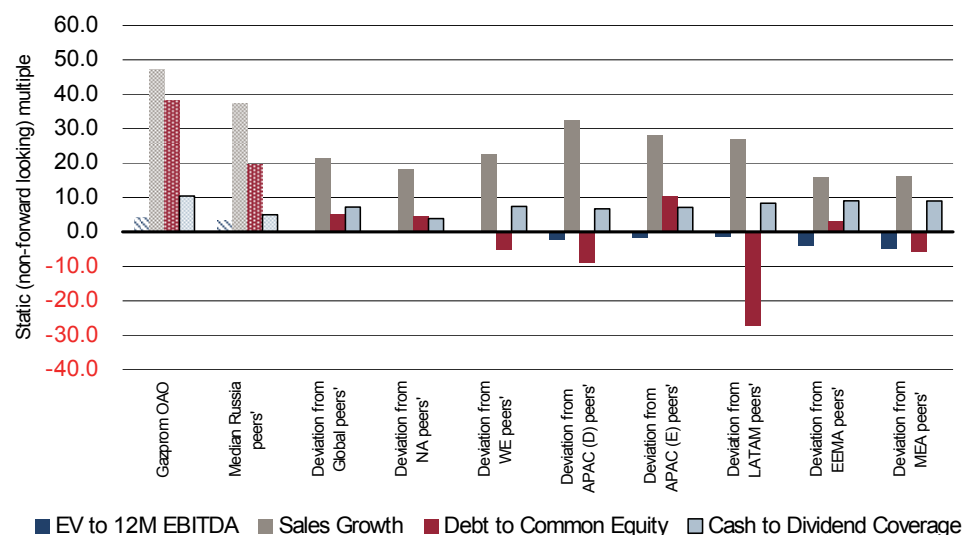
Global peers	North America	Western Europe	Asia-Pacific (D)	Asia-Pacific (E)	Latin America	Eastern Europe	Middle East and Africa
1 OCCIDENTAL PETE	OCCIDENTAL PETE	TULLOW OIL	CNOOC LTD	RELIANCE INDS	OGX-ORD	TUPRAS	RABIGH REFINING
2 CNOOC LTD	ENCANA CORP	CAIRN ENERGY	FORMOSA PETROCHE	OIL & NATURAL GA	RELAPASA	NOVOIL-BRD	SAMIR
3 RELIANCE INDS	APACHE CORP	ADDAX PETROLEUM	WOODSIDE PETRO	PTT PCL	GEOPARK HOLDINGS	UKRNAFTA	PAZ OIL CO
4 ENCANA CORP	CAN NATURAL RES	ROYAL DUTCH SH-A	INPEX	INDIAN OIL CORP	BPC LTD	ROSNEFT OIL	MARIDIVE & OIL S
5 OIL & NATURAL GA	DEVON ENERGY CO	BP PLC	NIPPON OIL CORP	PTT EXPL & PROD	PETROLERA DEL CO	LUKOIL-CLS	ALEXANDRIA MINER
6 ROYAL DUTCH SH-A	XTO ENERGY INC	TOTAL SA	SANTOS LTD	RELIANCE PETROLE	GLOBAL ENVIRONME	SURGUTNEFTEGAZ	AFRIQUIA GAZ
7 CHEVRON CORP	ANADARKO PETROLE	ENI SPA	SK ENERGY CO LTD	CAIRN INDIA	PETROBRAS-PREF	TNK-BP HOLD-BRD	OIL REFINERIES L
8 PETROBRAS-PREF	EOG RESOURCES	STATOILHYDRO ASA	TONENGEN SEKIYU	KMG EP	ECOPETROL	GAZPROM NEFT-CLS	AVNER OIL - LP
9 BP PLC	CHEVRON CORP	BG GROUP PLC	S-OIL CORP	PETRONAS GAS BHD	EMPRESAS COPEC	PGNIG	ISRAMCO NEGEV
10 TOTAL SA	CONOCOPHILLIPS	REPSOL YPF SA	IDEMITSU KOSAN C	OIL SEARCH LTD	YPF SA-D	TATNEFT-CLS	DELEK DRILLIN-LP
11 CHINA PETROLEU-H	IMPERIAL OIL	CEPSA	CNPC HONG KG LTD	OIL & GAS DEVELO	PETROBRAS ENER-B	MOL HUNGARIAN	AABAR INVESTMENT
12 ENI SPA	SUNCOR ENERGY	GALP ENERGIA-B	CALTEX AUST LTD	PAKISTAN PETROLE	TEEKAY OFFSHORE	PKNORLEN	DELEK ENERGY SYS
13 STATOILHYDRO ASA	HESS CORP	OMV AG	ARROW ENERGYLTD	CHINA PETROLEU-H	TEEKAY TANK-CL A	SNP PETROM SA	SASOL LTD
14 CONOCOPHILLIPS	HUSKY ENERGY INC	NESTE OIL OYJ	NEW ZEAL REFININ	GAIL INDIA LTD	SIEM OFFSHORE IN	SLOVNAFT AS	DELEK GROUP LTD
15 BG GROUP PLC	MARATHON OIL	HELLENIC PETRO	NIPPON MINING HO	BHARAT PETROL	SUELOPETROL-CL B	INA INDUSTRIJA	SHELL OMAN MARKE
16 ROSNEFT OIL	SCHLUMBERGER LTD	TRANSOCEAN LTD	SHOWA SHELL	PETRONAS DAGANGA	PROMIGAS	PETROL OFISI	GULF INTERNATIONAL
17 ECOPETROL	HALLIBURTON CO	SAIPEM	COSMO OIL CO	HINDUSTAN PETRO	TEEKAY LNG PARTN	UNIPETROL	MENA HOLDING GRO
18 LUKOIL-CLS	NATL OILWELL VAR	TECHNIP SA	JAPAN PETROLEUM	ESSAR OIL LTD	TRANS GAS SUR-B	DOGAN HOLDING	BURGAN CO FOR WE
19 IMPERIAL OIL	TRANSCANADA CORP	PETROFAC LTD	GS HOLDINGS CORP	CHINA OILFIELD-H		PETROL	NATIONAL GAS & I
20 SCHLUMBERGER LTD	KINDER MORGAN EN	AMEC PLC	WORLEYPARSONS	OFFSHORE OIL-A		EURASIA DRIL-GDR	

Source: Bloomberg as of end of Q1 2009. Asia-Pacific (D): Developed, Asia-Pacific (E): Emerging

Figure 12 shows forward-looking valuation multiples for Gazprom relative to the median value of the different peer group regions. As shown in the previous section, Gazprom is undervalued according to several valuation multiples, especially relative to developed market peer group members. The biggest deviation is found in the PE ratio, followed by the PEG ratio. Based on these valuation multiples, Gazprom seems to trade cheaply (negative deviation from peer group mean across regions) but closer to its Eastern European and Middle East and African peer group members.

Figure 13

Current valuation multiples for Gazprom relative to its global peer groups

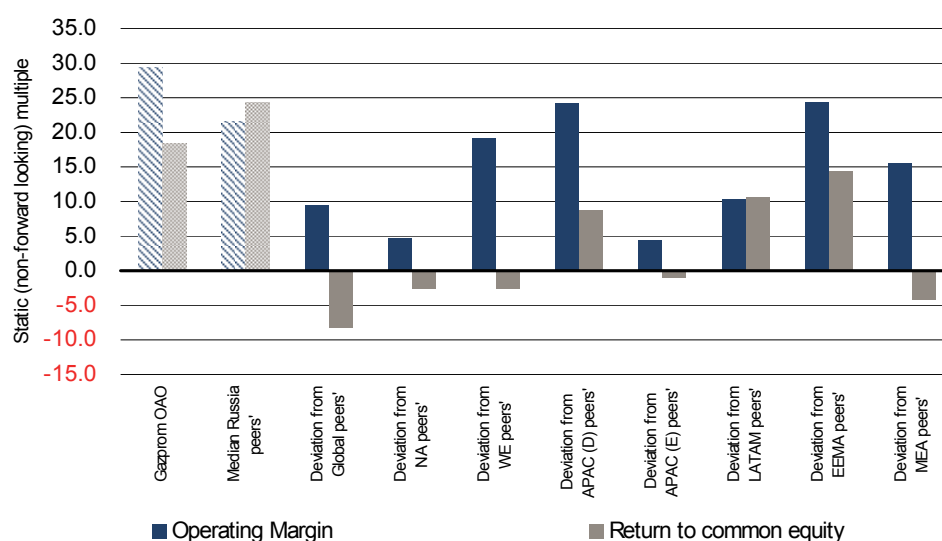


Present (i.e., static) non-forward-looking valuation multiples as indicators for profitability for Gazprom relative to other global peer groups across regions (measured as difference between Gazprom and the median value of the peer group). The peer group regions are NA (North America), WE (West Europe), APAC (Asia and Pacific, D for developed and E for emerging), LATAM (Latin America), EEMA (Eastern Europe and Central Asia), MEA (Middle East and Africa). Source: Bloomberg.

Figure 13 and Figure 14 show different profitability and leverage multiples for Gazprom (again measured in deviation terms from their international peer group families). The sales growth multiples exceed the median values of all measured peer group members, while the EV to EBITDA confirms the pictures of a below fair-value market price for Gazprom. Leverage does not seem to explain the low equity price, since Gazprom has less debt than most other peer group members. The return on equity for Gazprom relative to the North American and Western European companies is the only aggregated

multiple that could justify the lower valuation. However, profitability-wise (measured as operating margin) Gazprom once again exceeds on average all of its global peers. Based on static and forward-looking multiples, Gazprom trades cheap compared to its global peers, despite its low leverage and its relatively high profitability.

Figure 14
Profitability valuation multiples for Gazprom relative to its global peer groups



Present (i.e., static) non-forward-looking profitability multiples for Gazprom relative to other global peer groups across regions (measured as difference between Gazprom and the median value of the peer group). The peer group regions are NA (North America), WE (West Europe), APAC (Asia and Pacific, D for developed and E for emerging), LATAM (Latin America), EEMA (Eastern Europe and Central Asia), MEA (Middle East and Africa). Source: Bloomberg.

2.3.2 Lukoil

Lukoil is classified as Oil Company-Integrated within the Energy sector. Lukoil refines and sells oil and oil products (petrochemicals) and accounts for roughly 18% of Russia's crude oil production. Lukoil explores gas in addition to oil. It processes around 1.1% of the global oil reserves and 2.3% of the global oil production. Lukoil's reserves are mostly located in Russia (Siberia), and it is in terms of market capitalization the second largest Russian company.¹⁴ Peer group members according to the Industry Classification Benchmark (ICB) are given in Table 7.

¹⁴ Source: Bloomberg.

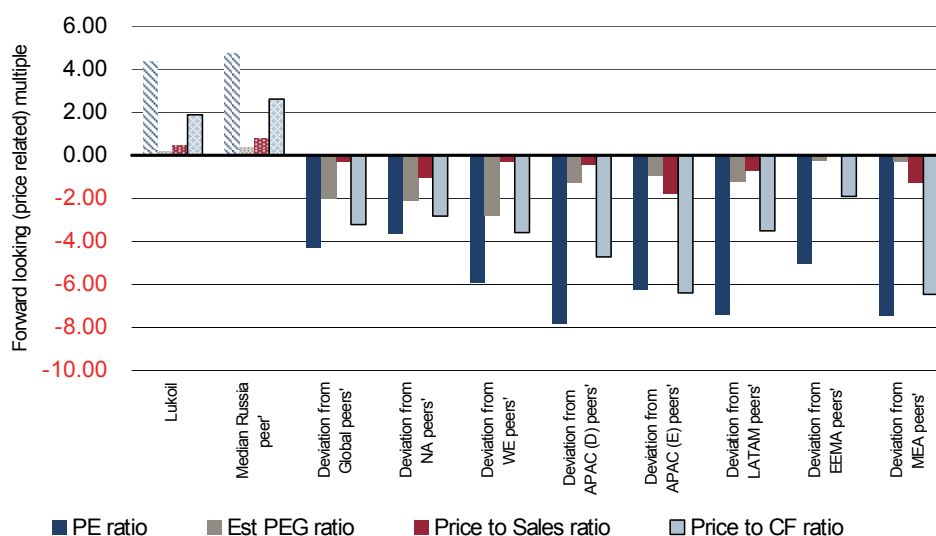
Table 7
International Peer Group of Lukoil

	Global peers	North America	Western Europe	Asia-Pacific (D)	Asia-Pacific (E)	Latin America	Eastern Europe	Middle East and Africa
1	OCCIDENTAL PETE	OCCIDENTAL PETE	TULLOW OIL	CNOOC LTD	RELIANCE INDS	OGX-ORD	TUPRAS	RABIGH REFINING
2	KMG EP	ENCANA CORP	CAIRN ENERGY	FORMOSA PETROCHE	OIL & NATURAL GA	RELAPASA	NOVOIL-BRD	SAMIR
3	WEST SIBERIA-SDR	APACHE CORP	ADDAX PETROLEUM	WOODSIDE PETRO	PTT PCL	GEOPARK HOLDINGS	UKRNAFTA	PAZ OIL CO
4	URALS ENERGY PUB	CAN NATURAL RES	SARAS SPA	INPEX	INDIAN OIL CORP	BPC LTD	UFANEFTEKHIM-CLS	MARIDIVE & OIL S
5	EXXON MOBIL CORP	DEVON ENERGY CO	STATOILHYDRO ASA	NIPPON OIL CORP	PTT EXPL & PROD	PETROLERA DEL CO	ROSNEFT OIL	ALEXANDRIA MINER
6	ROYAL DUTCH SH-A	XTO ENERGY INC	BG GROUP PLC	SANTOS LTD	RELIANCE PETROLE	GLOBAL ENVIRONME	SURGUTNEFTEGAZ	AFRIQUIA GAZ
7	PETROBRAS-PREF	ANADARKO PETROLE	REPSOL YPF SA	SK ENERGY CO LTD	CAIRN INDIA	PETROBRAS-PREF	TNK-BP HOLD-BRD	OIL REFINERIES L
8	BP PLC	EOG RESOURCES	CEPSA	TONENGEN SEKIYU	KMG EP	ECOPETROL	GAZPROM NEFT-CLS	AVNER OIL - LP
9	TOTAL SA	TALISMAN ENERGY	GALP ENERGIA-B	S-OIL CORP	PETRONAS GAS BHD	EMPRESAS COPEC	PGNIG	ISRAMCO NEGEV
10	GAZPROM	IMPERIAL OIL	OMV AG	IDEMITSU KOSAN C	OIL SEARCH LTD	YPF SA-D	TATNEFT-CLS	DELEK DRILLIN-LP
11	ROSNEFT OIL	SUNCOR ENERGY	NESTE OIL OYJ	CNPC HONG KG LTD	OIL & GAS DEVELO	PETROBRAS ENER-B	MOL HUNGARIAN	AABAR INVESTMENT
12	IMPERIAL OIL	HESS CORP	HELLENIC PETRO	CALTEX AUST LTD	PAKISTAN PETROLE	PETROBRAS ENER-B	PKNORLEN	DELEK ENERGY SYS
13	SURGUTNEFTEGAZ	HUSKY ENERGY INC	TRANSOCEAN LTD	ARROW ENERGYLTD	THAI OIL PCL	TEEKAY OFFSHORE	SNP PETROM SA	SASOL LTD
14	TNK-BP HOLD-BRD	MARATHON OIL	SAIPEM	NEW ZEAL REFININ	GAIL INDIA LTD	TEEKAY TANK-CL A	SLOVNAFT AS	DELEK GROUP LTD
15	GAZPROM NEFT-CLS	PETRO-CANADA	TECHNIP SA	NIPPON MINING HO	BHARAT PETROL	SIEM OFFSHORE IN	INA INDUSTRIJA	SHELL OMAN MARKE
16	TATNEFT-CLS	SCHLUMBERGER LTD	PETROFAC LTD	SHOWA SHELL	PETRONAS DAGANGA	SUELOPETROL-CL B	PETROL OFISI	OANDO PLC
17		HALLIBURTON CO	AMEC PLC	COSMO OIL CO	HINDUSTAN PETRO	PROMIGAS	UNIPETROL	GULF INTERNATIONAL
18		NATL OILWELL VAR	FUGRO NV-CVA	JAPAN PETROLEUM	ESSAR OIL LTD	TEEKAY LNG PARTN	DOGAN HOLDING	MENA HOLDING GRO
19		TRANSCANADA CORP	SBM OFFSHORE NV	GS HOLDINGS CORP	CHINA OILFIELD-H	TRANS GAS SUR-B	PETROL	BURGAN CO FOR WE
20		KINDER MORGAN EN	FLUXYS-D	WORLEYPARSONS	OFFSHORE OIL-A		EURASIA DRIL-GDR	NATIONAL GAS & I

Source: Bloomberg as of end of Q1 2009. Developed, Asia-Pacific (E): Emerging

Comparing the major peer group valuation multiples of Lukoil with its global peer groups again confirms the high degree of undervaluation, especially against North American and West European companies (see Figure 15).

Figure 15
Forward-looking valuation multiples for Lukoil relative to its global peer groups

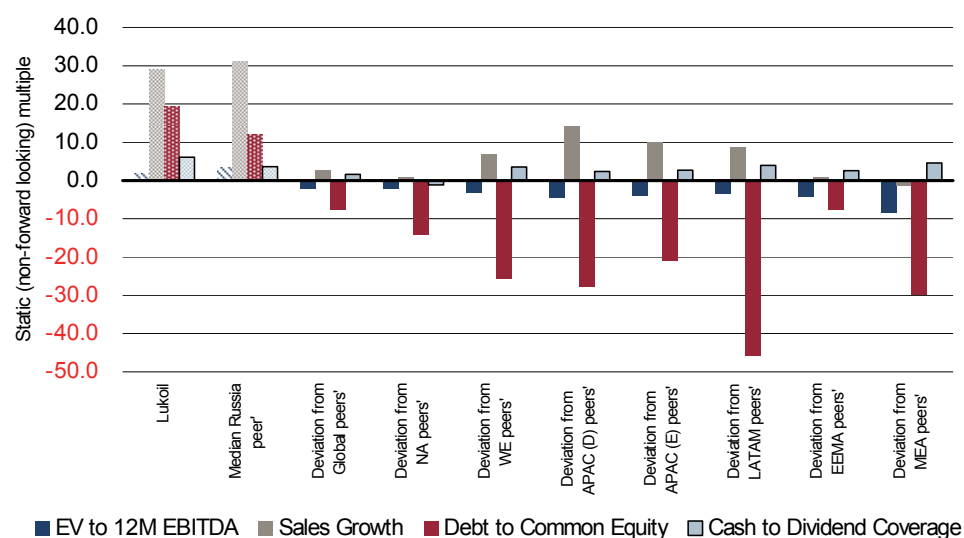


Forward-looking valuation multiples for Lukoil relative to other global peer groups across regions (measured as difference between Lukoil and the median value of the peer group). The peer group regions are NA (North America), WE (West Europe), APAC (Asia and Pacific, D for developed and E for emerging), LATAM (Latin America), EEMA (Eastern Europe and Central Asia), MEA (Middle East and Africa). Source: Bloomberg.

Despite higher sales growth, a less leveraged financial structure and equal or higher profitability with respect to Operating Margin and Return on Equity, the valuation in terms of market price multiples (PE and estimated PEG ratio, Price to Sales and Price to CF ratio) remains far below Lukoil's global peer-group median values as shown in Figure 16. In this regard the results are similar to those of Gazprom which, given its strong similarities to the peer group, should not come as a surprise. Comparing Lukoil with its Russian peer group shows that Lukoil has slightly lower valuation multiples. This is also confirmed by the profitability multiples in Figure 17. The return on equity is higher than the median of most analyzed peer group samples and weaker only than the North American peer group sample.

Figure 16

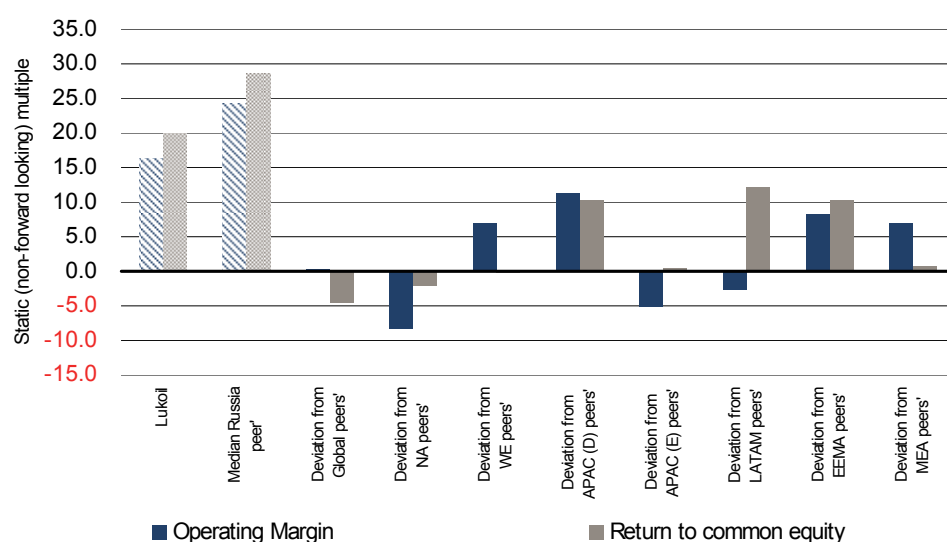
Current valuation multiples for Lukoil relative to its global peer groups



Present (i.e., static) non-forward-looking valuation multiples for Lukoil relative to other global peer groups across regions (measured as difference between Lukoil and the median value of the peer group). The peer group regions are NA (North America), WE (West Europe), APAC (Asia and Pacific, D for developed and E for emerging), LATAM (Latin America), EEMA (Eastern Europe and Central Asia), MEA (Middle East and Africa). Source: Bloomberg.

Figure 17

Profitability valuation multiples for Lukoil relative to its global peer groups



Present (i.e., static) non-forward-looking profitability multiples for Lukoil relative to other global peer groups across regions (measured as difference between Lukoil and the median value of the peer group). The peer group regions are NA (North America), WE (West Europe), APAC (Asia and Pacific, D for developed and E for emerging), LATAM (Latin America), EEMA (Eastern Europe and Central Asia), MEA (Middle East and Africa). Source: Bloomberg.

As with Gazprom, one can conclude that Lukoil is cheaply valued against its global peer group, despite its relatively strong profitability and growth fundamentals.

2.3.3 *Norilsk Nickel*

Norilsk Nickel is the biggest Russian mining company and the world's largest producer of nickel and palladium. Norilsk Nickel explores, extracts, refines and produces platinum, copper cobalt, silver and gold, in addition to various other precious and industrial metals. Norilsk Nickel has major production facilities outside of Russia in South Africa, Australia and North America.¹⁵

Norilsk Nickel is classified under the industry group Metal-Diversified within the Basic Materials sector. Global peer group members according to the Industry Classification Benchmark (ICB) are given in Table 8.

¹⁵ Source: Bloomberg.

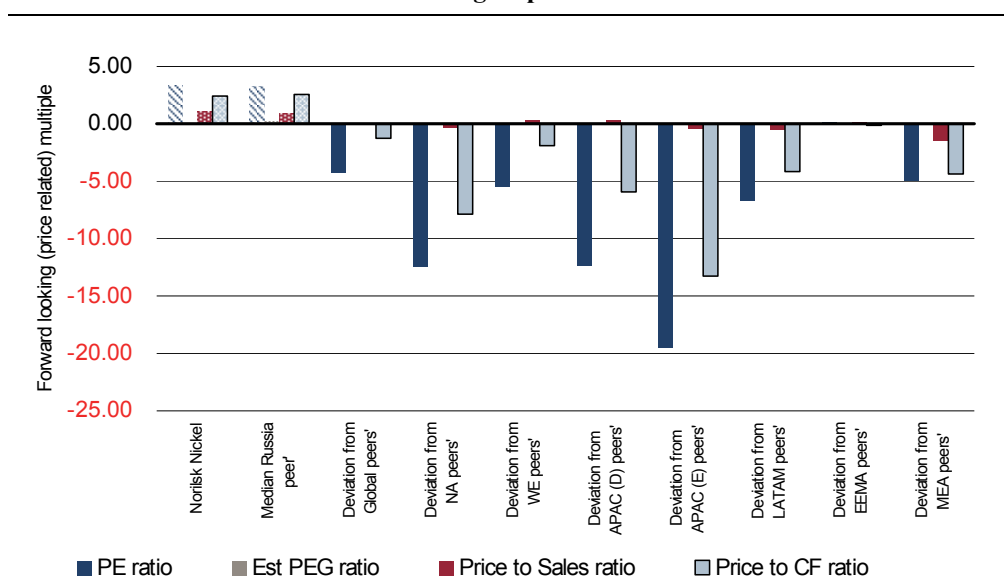
Table 8
International Peer Group of Norilsk Nickel

	Global peers	North America	Western Europe	Asia-Pacific (D)	Asia-Pacific (E)	Latin America	Eastern Europe	Middle East and Africa
1	RASPADSKAYA-CLS	ALCOA INC	NORSK HYDRO ASA	COAL & ALLIED IN	ALUMINUM CORP-H	SOCIEDAD MINERA	POLYUS GOLD-CLS	ARAB POTASH
2	BHP BILLITON LTD	PEABODY ENERGY	LINDE AG	FORMOSA PLASTIC	CHINA COAL ENERG	BUENAVENTURA-INV	POLYMETAL-CLS	SAUDI KAYAN PETR
3	RIO TINTO LTD	CONSOL ENERGY	ANTOFAGASTA PLC	NAN YA PLASTICS	YANZHOU COAL-H	GRUPO MEXICO-B	URALKALI-CLS	YANBU NATIONAL P
4	ANGLO AMER PLC	DU PONT (EI)	EURASIAN NATURAL	FORMOSA CHEM & F	SHANXI XISHAN-A	INDUSTRIAS PENOL	ACRON-CLS	JORDAN PHOSPHATE
5	XSTRATA PLC	AIR PRODS & CHEM	XSTRATA PLC	SUMITOMO CHEM CO	SINOPEC SHANG-H	SOUTHERN CP-INV1	NOVOLIPETSK-CLS	KUMBA IRON ORE L
6	KAZAKHMYS PLC	DOW CHEMICAL	RANDGOLD RES LTD	TORAY INDUSTRIES	MMTC LTD	VOLCAN CIA MIN-B	SEVERSTAL-CLS	SAUDI ARABIAN MI
7	POLYUS GOLD-CLS	NEWMONT MINING	ERAMET	ASAHI KASEI CORP	NMDC LIMITED	MILPO-INVERSION	MAGNITOGORSK-CLS	AFRICAN RAINBOW
8	SUMITOMO MET MIN	KINROSS GOLD	UPM-KYMMENE OYJ	LG CHEM LTD	ZIJIN MINING GRP	MINSUR-INV	MECHEL-CLS	EXXARO RESOURCES
9	TECK RESOURCES-B	AGNICO-EAGLE	STORA ENSO OYJ-R	NEWCREST MINING	LIHIR GOLD LTD	MOLYMET	VSMPO-AVISMA-CLS	ANGLOGOLD ASHANT
10	IMPALA PLATINUM	YAMANA GOLD INC	SYNGENTA AG-REG	SUMITOMO MET MIN	JIANGXI COPPER-H	CMPC		GOLD FIELDS LTD
11	LONMIN PLC	FREEPORT-MCMORAN	AKZO NOBEL	OJI PAPER CO LTD	JINDUICHENG -A	ARACRUZ-PREF B		HARMONY GOLD MNG
12	POLYMETAL-CLS	SOUTHERN COPPER	K+S AG	MITSUBISHI CHEMI	QINGHAI SALT L-A	SUZANO PAPEL-PRA		ANGLO PLATINUM
13	URALKALI-CLS	CAMECO CORP	YARA INTL ASA	NITTO DENKO CORP	QINGHAI SALT-A	FRESNILLO PLC		IMPALA PLATINUM
14	ACRON-CLS	MOSAIC CO/THE	SOLVAY SA-A	ORICA LTD	BAOSHAN IRON & S	SOQUIMICH-B		ISRAEL CHEMICALS
15	NOVOLIPETSK-CLS	ROHM & HAAS CO	DSM (KONIN)	NIPPON STEEL	ANGANG STEEL-H	FOSFERTIL-PREF		SAFCO
16	SEVERSTAL-CLS	ECOLAB INC	GIVAUDAN-REG	JFE HOLDINGS INC	WUHAN IRON & S-A	SID NACIONAL		ISRAEL CORP LTD
17	EVRAZ GROUP-GDR	AGRIUM INC	WACKER CHEMIE AG	SUMITOMO MET IND	STEEL AUTHORITY	GERDAU-PREF		FERTILIZED EGYPT
18	MAGNITOGORSK-CLS	PPG INDS INC	TENARIS SA	CHINA STEEL CORP	PANZHIHUA NEW-A	USIMINAS		MAKHTESHIM-AGAN
19	MECHEL-CLS	SIGMA-ALDRICH	SALZGITTER AG	FORTESCUE METALS	SHANXI TAIGANG-A	MET GERDAU-PREF		ARCELORMITTAL SO
20	VSMPO-AVISMA-CLS	NUCOR CORP	EVRAZ GROUP-GDR	KOBE STEEL LTD	MAANSHAN IRON-H	CAP		EL EZZ ALDEKHELA

Source: Bloomberg as of end Q1 2009.

Figure 18 shows the current relative undervaluation of Norilsk Nickel relative to its global peer group members. Most significant is its undervaluation relative to the Emerging APAC sector. This deviation is generally more pronounced than for Gazprom or Lukoil.

Figure 18
Forward-looking valuation multiples for Norilsk Nickel relative to its global peer groups

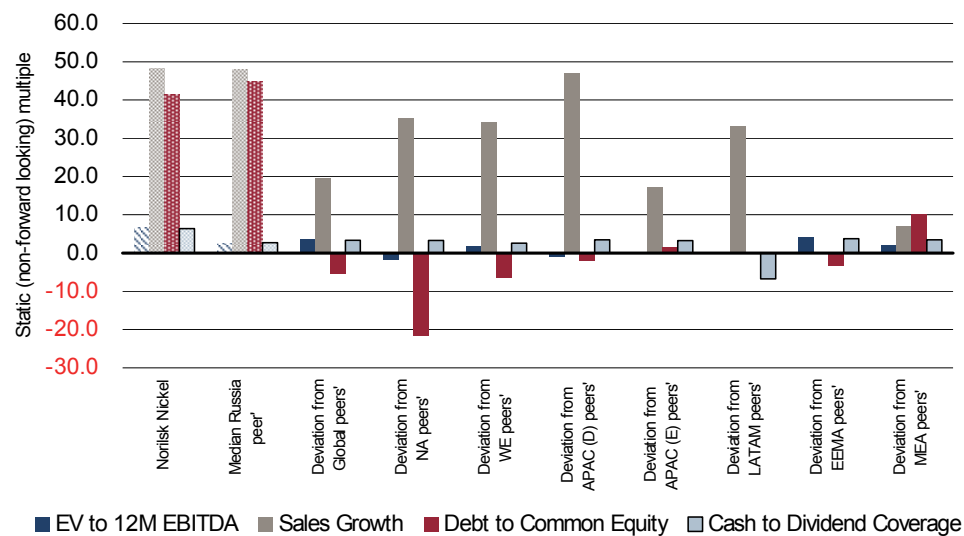


Forward-looking valuation multiples for Norilsk Nickel relative to other global peer groups across regions (measured as difference between Norilsk Nickel and the median value of the peer group). The peer group regions are NA (North America), WE (West Europe), APAC (Asia and Pacific, D for developed and E for emerging), LATAM (Latin America), EEMA (Eastern Europe and Central Asia), MEA (Middle East and Africa). Source: Bloomberg.

Relative to Gazprom and Lukoil, Norilsk Nickel's EV to EBITDA and sales growth are high. As shown in Figure 19 sales growth significantly exceeds the median of its global peers, and its capital structure (measured via debt to common equity or cash to dividend coverage) is favorable compared to global peers.

Figure 19

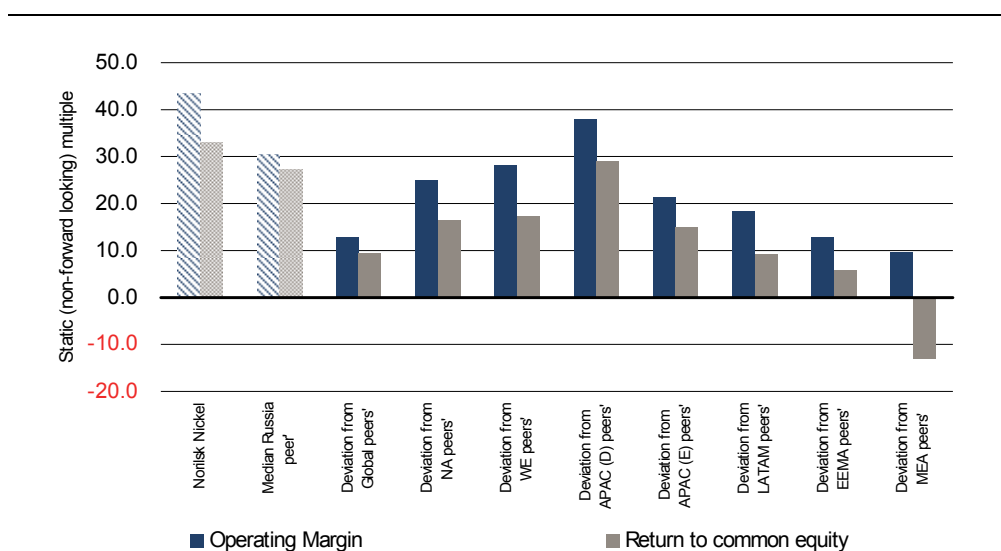
Current valuation multiples for Norilsk Nickel relative to its global peer groups



Present (i.e., static) non-forward-looking valuation multiples for Norilsk Nickel relative to other global peer groups across regions (measured as difference between Norilsk Nickel and the median value of the peer group). The peer group regions are NA (North America), WE (West Europe), APAC (Asia and Pacific, D for developed and E for emerging), LATAM (Latin America), EEMA (Eastern Europe and Central Asia), MEA (Middle East and Africa). Source: Bloomberg.

The profitability of Norilsk Nickel in terms of its Operating Margin and its Return on Equity exceeds most of its local and global peer-group members as illustrated in Figure 20. Despite these advantages in profitability Norilsk Nickel is trading in terms of forward-looking valuation multiples below its global peer group and is hence confirming the picture already observed for Gazprom and Lukoil.

Figure 20
Profitability valuation multiples for Norilsk Nickel relative to its global peer groups



Present (i.e., static) non-forward-looking profitability multiples for Norilsk Nickel relative to other global peer groups across regions (measured as difference between Norilsk Nickel and the median value of the peer group). The peer group regions are NA (North America), WE (West Europe), APAC (Asia and Pacific, D for developed and E for emerging), LATAM (Latin America), EEMA (Eastern Europe and Central Asia), MEA (Middle East and Africa). Source: Bloomberg.

2.4 Critical assessment and conclusion

The peer group valuation approach for the general MICEX Energy and Basic Materials sector, and in particular for the three key Russian companies chosen (Gazprom, Lukoil and Norilsk Nickel), has not given any indication of real-option related price premiums. To the contrary, all three companies trade at a discount relative to their global peer group members. The discount in valuation is more pronounced relative to APAC and LATAM companies and less pronounced relative to North American or Western European peer group companies. The discount in valuation is not explainable by lower profitability multiples or a weaker capital structure. For example, Gazprom, Lukoil and Norilsk Nickel show Operating Margins and Return on Equity above the median of regional peer groups. The same is true for capital structure multiplies, which on average are better (lower leverage and higher cash to dividend coverage) than those of the global peer group.

Based on the peer group valuation multiple results of this section, one cannot conclude that the Russian Energy and Basic Materials sector in general or Gazprom, Lukoil and Norilsk Nickel in particular, show any evidence of valuation premiums that could be associated with real-option value being discounted by market participants.

3 Real-Options, Probability Theory and the Net-Present-Value of the Firm

3.1 Introduction

The following chapter explains the main concept of real-option analysis (ROA) and lays out the basic idea of linking ROA with the concerns of this dissertation. This chapter is hence the necessary theoretical prerequisite for the ideas and assumptions that will be analyzed and tested in the subsequent chapters of this dissertation. The structure of this chapter is as follows: we will start by explaining the basic concept of real-option analysis. Afterwards we will link ROA with the uncertainty aspect of the conventional net-present-value (NPV) calculation by allowing future free cash-flows (FCF) to be uncertain. This is followed by linking the characteristics of option returns to the distribution of NPV and hence the distribution of equity prices, which will help formulate assumptions about how one should expect real-option projects to affect the distribution of NPV and asset returns. The importance of asset distribution is finally revisited at the end of this chapter, which closes with a critical assessment and conclusion.

3.1.1 Real-option example: Definitions and differentiation

A real-option involves the right, but not the obligation, to remain flexible in a given decision-making and action-taking process. Real-options represent a generic term for a bundle of individual decision options (e.g., options to defer, expand, contract, abandon, etc.) that enable management to maneuver through a dynamic environment with respect to capital budget decisions.

Like the better-known financial options (which can be seen as a special type of real-option), real-options are a function of some basic variables, common to nearly all types of real-options:

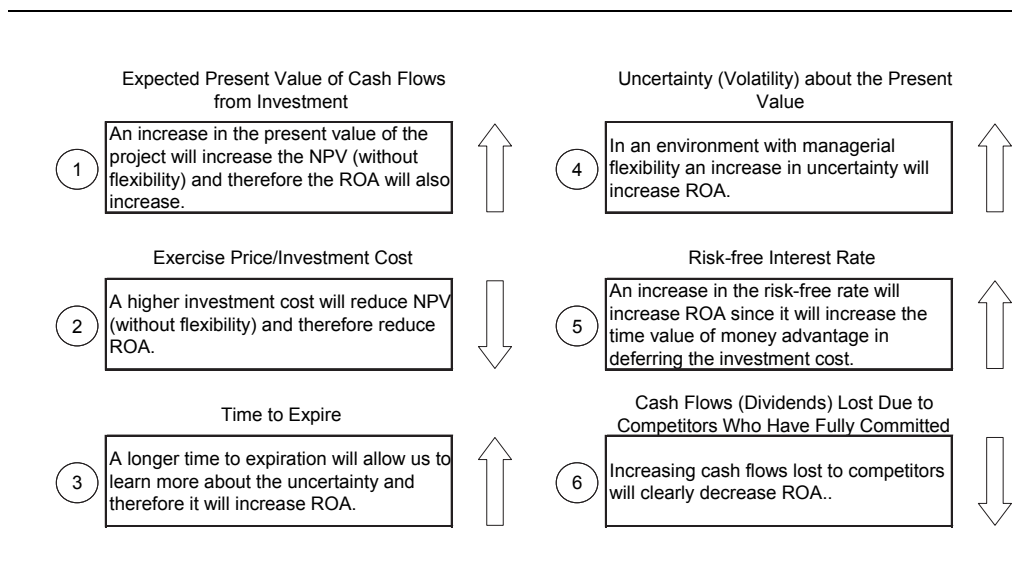
- Value of the underlying risky asset V_0
- Exercise price A_t
- Time to expiration of the option T

- Risk of the underlying asset i at time t (estimated e.g. as standard deviation of its assumed value distribution), $\sigma_{i,t}$
- Risk-free rate r_t^{rf}
- Cash-flows during the lifetime of the option $d_{j,t}$

The variables and their influence on a call-option similar real-option (e.g., option to expand) are summarized in Figure 21.

Figure 21

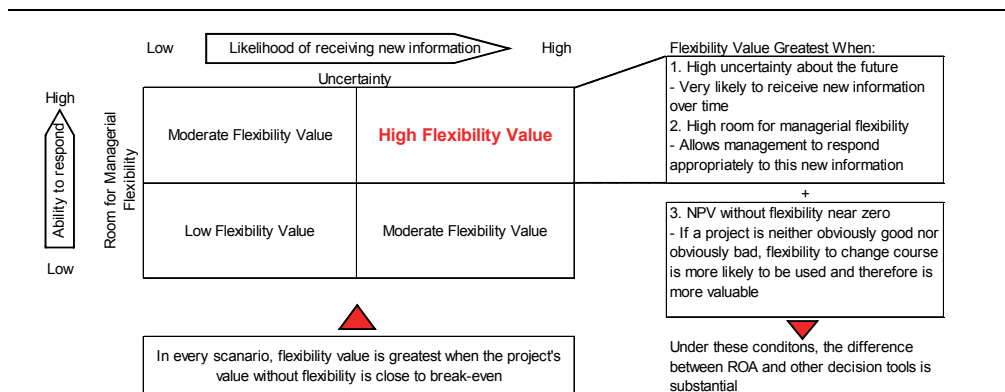
Real-option value drivers



Source: Copeland et al. (2001).

Real-option valuation generally enhances standard NPV or FCF calculation for a given project or undertaking by allowing decision-making flexibility with respect to budget decisions. Most value can be added in cases of high uncertainty, high flexibility of management and cases when the static NPV is very low or close to zero. An optimal environment for the use of real-options is illustrated below:

Figure 22
Environment for Real-options



Source: Copeland et al. (2001).

As highlighted in Figure 22 the value of flexibility and hence the value of a real-option is highly sensitive to the ability to response to a changing environment and uncertainty. An example will illustrate this further.

Excuse 1: Example of a real-option

The following example will illustrate the capabilities of simple real-option analysis: Consider a project which asks for a commitment at time $t = 0$ and generates FCFs starting in year 1 to ∞ from either 50 or 150 units (assuming no fixed costs further). The probability $\tilde{p}$ for both scenarios is set to $\tilde{p} = 0.5$. Assuming complete certainty over the future (i.e., no alteration of prices) and assuming that the risk of future prices is fully diversifiable (i.e., has no relation to other company projects) we can discount all FCFs at the risk-free rate of 10% in our example (see Figure 23).

Figure 23
Comparison of expectation values

$t = 0$		$t = 1$	$t = 2$		$t = \infty$
$A_0 = -1000$	$\tilde{p} = 0.5$	$FCF_1 = 150$	$FCF_2 = 150$		$FCF_2 = 150$
	$1 - \tilde{p} = 0.5$	$FCF_1 = 50$	$FCF_2 = 50$		$FCF_2 = 50$

If the investment outlay for entering the market at $t = 0$ is $A_0 = -1000$ and no further fixed costs are considered, then the static NPV is given by:

$$NPV_{t=0}^{stat} = -1000 + \sum_{t=1}^{\infty} \frac{100}{1.1^t} = 0 \quad (3)$$

Being risk-averse we would not enter into a zero NPV project under these static conditions. Given this result we could ask how much we would be ready to pay for delaying the decision to invest for one year. In one year the uncertainty about the future cash-flows will be resolved (i.e., prices and/or turnover will be resolved); we will invest if and only if cash-flows move to 150. If expected free cash-flows move to 50 we will not invest given that the NPV is negative. This leads to the following new dynamic NPV for time $t = 0$.

$$NPV_{t=0}^{dyn} = \frac{0,5 \left[-1000 + \sum_{t=1}^{\infty} \frac{150}{1.1^t} \right]}{1.1} = 227.27 \quad (4)$$

With this new dynamic NPV we can answer the question how much we would be ready to pay for an option to delay the investment decision for one year instead of making it now or never (i.e., the price of having a flexible investment opportunity). The answer is straightforward: not more than 227.27 units, which in this case is the real-option value of having the flexibility (option to delay or wait) another year before deciding on the project. The overall value of the project is now formally given in Equation (5). This result can easily be related to the factors described in Figure 21.

$$NPV_{t=0}^{dyn} = NPV_{t=0}^{stat} + \text{option to wait} \quad (5)$$

The first to analyze the value of waiting to invest were McDonald and Siegel (1986), who showed that an investment decision based only on the calculation of a static NPV is sub-optimal in the sense that (given uncertainty) a minimum value threshold (above the value of the static NPV) needs to be achieved by the project before an investment decision should be made.

3.1.2 Differences between real options from financial options

Security prices are observable, thus their parameters can be easily estimated or extracted from other tradable options. For real-options, however, different techniques and proxies have to be used to obtain necessary input parameters. The possibility of influencing the value of the option represents another difference. In general the market cannot influence the price of the underlying in a significant way; with real-options, however, the management, as owner of the option, has a greater flexibility to positively/negatively influence the underlying price. Further, the maturity of real-options, which usually have very long durations, differs substantially from standard financial options.

3.1.3 Types of real-options

Out of the sheer uncountable number of theoretical “real world options” some real-options have received special attention among finance scientists. These options, now known as common real-options, are described in Table 9.

Among the real-options summarized in Table 9, only a few can be utilized for standard analysis and hence valuation proposals, as most of the option-types require unique project knowledge that is usually available only inside the company. Options that are likely to occur are options to defer (e.g., defer extraction), options to alter scale (e.g., to increase extraction) and options to abandon (e.g., to stop extraction). These options rely on fixed assets that are observable in company reports and whose utilization can be quantified. Another important element of these options, together with our target industries of oil, gas, mining and energy, is the existence of a trading market for the underlyings in question. For most underlyings even well developed future markets exist, making risk-neutral valuation possible.¹⁶

¹⁶ If, for a given underlying, no future or spot market exists, then equilibrium models like the generalized Merton Model (1973) or the Model from Cox, Ingersoll and Ross (1985) can be used to value general claims.

Table 9
Common real-options

Common Real Options			
Category	Description	Important In	Analyzed By
Option to defer	Management holds a lease on (or an option to buy) valuable land or resources. It can wait (x years) to see if output prices justify constructing a building or plant, or developing a field.	All natural resource extraction industries; real estate development; farming; paper products.	Tourinho (1979); Titman (1985); McDonald & Siegel (1986); Paddock, Siegel & Smith (1988); Ingersoll & Ross (1992).
Time to build option (staged investment)	Staging investment as a series of outlays creates the option to abandon the enterprise in midstream if new information is unfavorable. Each stage can be viewed as an option on the value of subsequent stages, and valued as a compound option.	All R&D intensive industries, especially pharmaceuticals; long-development capital-intensive projects, e.g., large-scale construction or energy-generating plants; start-up ventures.	Majd & Pindyck (1987); Carr (1988); Trigeorgis (1993).
Option to alter operating scale (e.g. expand; to contract: to shut down and restart)	If market conditions are more favorable than expected, the firm can expand the scale of production or accelerate resource utilization. Conversely, if conditions are less favorable than expected, it can reduce the scale of operations. In extreme cases, production may temporarily halt and start up again.	Natural resource industries such as mine operations; facilities planning and construction in cyclical industries; fashion apparel; consumer goods; commercial real estate.	Brennan & Schwartz (1985); McDonald & Siegel (1985); Trigeorgis & Mason (1987); Pindyck (1988).
Option to abandon	If market conditions decline severely, management can abandon current operations permanently and realize the resale value of capital equipment and other assets in secondhand markets.	Capital intensive industries, such as airlines and railroads; financial services; new product introductions in uncertain markets.	Myers & Majd (1990).
Option to switch (e.g. outputs or inputs)	If prices or demand change, management can change the output mix of the facility ("product" flexibility). Alternatively, the same outputs can be produced using different types of inputs ("process" flexibility).	<i>Output shifts:</i> any good sought in small batches or subject to volatile demand, e.g., consumer electronics; toys; specialty paper; machine parts; autos. <i>Input shifts:</i> all feedstock-dependent facilities, e.g., oil, electric power; chemicals; crop switching; sourcing.	Margrabe (1978); Kensinger (1987); Kulatilaka (1988); Kulatilaka & Trigeorgis (1994).
Growth options	An early investment (e.g. R&D, lease on undeveloped land or oil reserves, strategic acquisitions, information network/infrastructure) is a prerequisite or link in a chain of interrelated projects, opening up future growth opportunities (e.g., new generation product or process, oil reserves, access to new market, strengthening of core capabilities). Like interproject compound options.	All infrastructure-based or strategic industries, especially high-tech, R&D, or industries with multiple product generations or applications (e.g. computers, pharmaceuticals); multi-national operations; strategic acquisitions.	Myers (1987); Brealey & Myers (2000); Kester (1984), (1993); Trigeorgis (1990); Pindyck (1991); Chung & Charoenwong (1991).
Multiple interacting options	Real-life projects often involve a "collection" of various options, both upward-potential enhancing calls and downward-protecting put options present in combination. Their combined option value may differ from the sum of separate option values, i.e., they interact. They may also interact with financial flexibility options.	Real-life projects in most industries discussed above.	Brennan & Schwartz (1985); Trigeorgis (1993); Kulatilaka (1993).

Source: Schwartz and Trigeorgis (2001), pp. 104-105.

3.1.4 *Methods for valuing*

3.1.4.1 Black and Scholes

The use of closed-form solutions for option pricing since the discoveries of Black and Scholes (1973) and Merton (1973) – referred to as the BSM approach – remains the most popular approach. However, its usage requires some assumptions, which cannot be expected to hold in a real-option environment. BSM applies to the valuation of European options, which usually do not occur in real-option analysis. There is also only one source of uncertainty in the BSM framework; thus, multi-dimensional stochastic dependencies cannot exist. This BSM contingency on only one underlying is hence the biggest obstacle in using closed-form solutions for real-option applications. The last big obstacle is the assumption that the stochastic process followed by the underlying is known, as with the Brownian motion process for the equity BSM case. In the real-option case, however, the assumption of a Brownian motion process is not necessarily valid.

3.1.4.2 Lattice Methods

Valuing real-options with a lattice approach goes back to the development of binomial option pricing by Cox, Ross and Rubinstein (1979) and Boyle (1986). They introduced risk-neutral valuation by replication portfolios. This approach is extremely useful and intuitively tangible (see simplified binomial lattice on p. 40). The core idea is to construct a hedge portfolio, thus a portfolio that yields independently of the future state of nature at $t=1$, a certain (constant) portfolio (hedge portfolio value $\Pi_{t=1}$) value.

The constructed lattice can then be solved using backward induction, in which we solve for either the optimal hedge ratio (delta) or the risk-neutral probabilities. The risk-neutral probability approach fits the replication portfolio approach and can be seen as a different side of the same coin.¹⁷

¹⁷ For a rigorous treatment of binomial and trinomial trees and risk-neutral valuation see Hull (2000) and Haug (2007).

3.1.4.3 Monte Carlo Simulations

When real-option problems involving valuation and optimum exercising strategies cannot be solved using binomial or trinomial trees, as is often the case in a multi-variable scenario or if path dependencies like taxation have to be considered, then Monte Carlo simulation (MCS) can be used. In practice Monte Carlo simulations are modeled by defining crucial state variables like sale prices, turnover, or simply the cost of commodities like oil or gas, through a certain type of process. Correlation dependencies are additionally defined with respect to the different stochastic driving factors, which then can be linked via Cholesky decomposition into the MCS.¹⁸

MCSs are especially useful if they are constructed under a risk-neutral (equivalent Martingale) measure, where a value V_t at $t = 0$ can be expressed as

$$V_t = E^{\tilde{P}}[V_{t+s} | I_t] \quad (6)$$

and its normalization is given by:

$$V_t = \frac{1}{(1 + r_t^{rf})^s} S_{t+s} \quad (7)$$

Equation (6) states that the best forecast of the unobservable future values of V_{t+s} is V_t (Martingal condition), it is hence a Martingale with respect to the risk-adjusted probability¹⁹ $\tilde{P}$ given the information set I_t . Further, as shown in Equation (7), we can convert all discounted values like, for example, asset prices S_t (which over a very short period usually are assumed to have a trend) or costs, into martingales. Thus, the ratio becomes a Martingale under the risk-adjusted probability measure $\tilde{P}$.²⁰

¹⁸ See Rudolf (2000), pp. 381-387.

¹⁹ These are also often referred to as pseudo-probabilities or martingale probabilities.

²⁰ For an extensive proof see Neftci (2000).

This approach allows for the possibility of operating within a risk-neutral environment.²¹ Thus, risk-adjusted processes can be defined, and simulated path-realizations (payoffs) can be discounted under a risk-neutral measure (the equivalent Martingale measure), resulting in a company (or project) value equal to:

$$V_0 = E^{\tilde{P}} \left(FCF_t e^{-r_t^f T} \right) \quad (8)$$

The free cash-flow (FCF) itself can be defined as function of, among other factors, the turnover U_t and/or cost factor C_t , which themselves are described by stochastic processes that can be correlated to each other.

MCSs have the advantages that, first, they allow path dependencies, and second, correlations between variables can easily be incorporated into the simulation. Such a multi-state variable MCS setup requires, however, assumptions that are not always fulfilled.

One important assumption for using the risk-free interest rate for discounting is related to the tradability of the stochastic state variable (i.e., replicating the stochastic characteristic of the project underlying via a hedge portfolio). This is not an obstacle if the investment project is subject to only a single stage variable (e.g., oil price). However, if the project is dependent on numerous stochastic state variables, of which several cannot be replicated by a traded underlying and remain unobservable, further assumptions need to be made. In such a case, one needs further estimates on the market price of risk of the non-traded state variables in order to run the simulations in a risk-neutral framework.²² A requirement for valuing individual projects in a company value context is the assumption that the company's other projects are well diversified, so that no relation between the real-option project and the overall company exists.

²¹ See Harrison and Kreps (1979) and Harrison and Pliska (1981) for discussions of risk-neutral valuation. The topic is covered also in more detail in Subsection 4.2.3, p. 139.

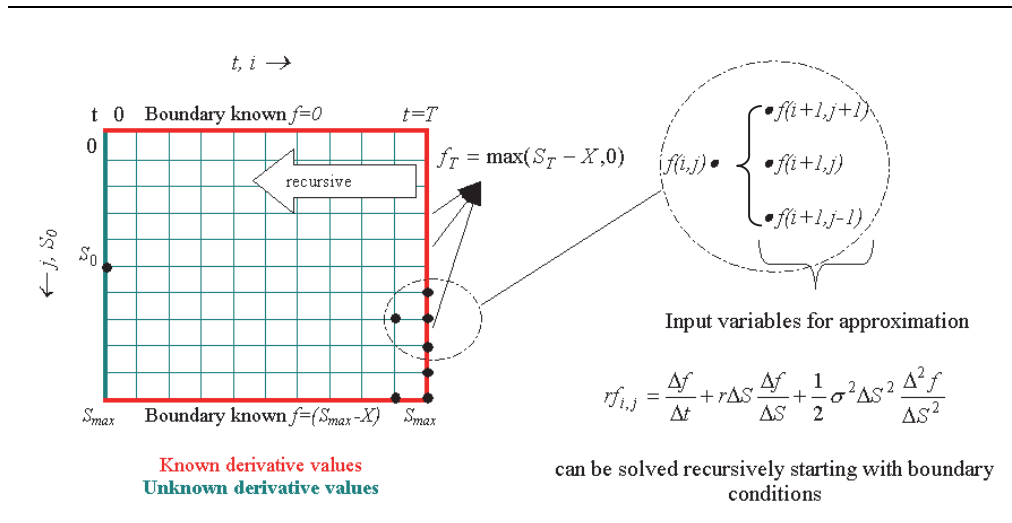
²² For the treatment of non-observable variables in MCS, see Gibson and Schwartz (1990), p. 960.

Another notable drawback to MCS is its inability to solve American options, due to its nature as a forward induction approach.²³ To overcome this obstacle, new approximations and methods have been introduced.²⁴

3.1.4.4 Finite Difference Methods

MCS techniques usually try to simulate the stochastic processes of underlying real-options. These techniques are in general very intuitive and convenient. However, it is also possible to solve real-option problems using finite difference schemes for solving partial differentiation equations. These techniques solve the Black-Scholes-Merton stochastic differentiation equation (SDE) numerically.²⁵ Using a grid of possible underlying price realizations for a certain time horizon and corresponding boundary conditions, a derivative (the real-option) on this underlying (e.g., oil field) can be valued (see Figure 24).²⁶

Figure 24
Finite Difference Method



²³ For a practical application using MCS, see Cortazar and Schwartz (1998), pp. 73-84.

²⁴ Cortazar (2001) gives a good overview of traditional numerical methods, including their advantages and drawbacks.

²⁵ For derivation the Black-Scholes-Merton SDE via Ito's lemma see Hull (2000), p. 246-247.

²⁶ A practical application of the finite difference method with respect to real-option valuation is given in Cortazar and Schwartz (1997), pp. 125-137.

Due to the fact that finite difference methods have a backward induction nature, they have been primarily used for valuing American real-options with stochastic cash-flow payouts.²⁷ Finite difference methods can also be used to solve optimal exercise problems for American put options if dividends are paid.²⁸

3.2 Net-present-value calculation, free cash-flows and the influence of optionality on the project value distribution

3.2.1 Introduction

Each investment project, irrespective of whether the project is real-option related or not, will have several consequences for the balance sheet of a company. In the extreme case, we can define a company balance sheet as solely related to a single investment project, and analyze how this project would change the value of the company if the investment project contained real-options. We will do this by simulating a free cash-flow distribution and hence the main determinant of company market value over a specified horizon. To do this we will assume a simple but realistic real-option related project and conduct a sensitivity analysis that will highlight the differences between considering and not considering the real-option value. The aim of this exercise is to illustrate how real-option value can affect the equity price distribution of companies – assuming that the project NPV is immediately reflected in the market price – and hence help identify real-option pricing in real-world equity markets.

3.2.2 The classical investment project

The investment project consists of only one price, namely oil price, and one quantity component, oil production. Both are assumed to be uncorrelated with each other (which implicitly requires that the company is a price taker and does not have the ability to change prices by altering supply).²⁹ The modeled cash-flow horizon from the oil production is 10 years. Afterwards it is assumed that the net cash-flow (NCF) will grow

²⁷ Approximation methods for American options in a real-option context are analyzed by Geske and Shastri (2001).

²⁸ For an introduction to finite difference methods see Hull (2000) and Tavella & Randall (2000).

²⁹ This assumption is particularly valid for non-OPEC countries like Russia or the Ukraine.

at a rate equal to $g = 2\%$ until infinity, which will affect only the terminal value calculation. The discount rate applied to equity capital $k_{E,t}$ is assumed to follow the CAPM equilibrium rate for a company with a beta factor $\beta_{E,t} = 1.7$ relative to the Bloomberg world equity index.³⁰

Table 10
Variable definitions

Name	Variable	Values for $t = 1 - 10$	Values for $t = 11 - \infty$
Expected market portfolio return	r_t^m	10%	10%
Expected beta relative to market portfolio	$\beta_{E,t}$	1.7	1.7
Risk-free interest rate	r_t^f	3%	3%
Debt capital interest rate	$k_{D,t}$	8%	8%
Cost of equity capital	$k_{E,t}$	14.9%	14.9%
Expected oil price DGP (defined as random walk with drift)	P_t^{oil}	$\mu = 0.195$ $\sigma_{S.E.} = 10$	$\mu = 0.195$ $\sigma_{S.E.} = 10$
Expected oil quantity sold	q_t	25	25
Variable cost in USD	$c_{v,t}$	$p_t \times 75\%$	$p_t \times 75\%$
Fixed costs in USD	$c_{f,t}$	200	200
Investment costs in USD	A_t	1000	0
Project depreciation	Afa_t	Linear over 10Y	0
Loan financing in USD	Y_0	At $t=0$, 400	0
Loan amorization in USD	Y_t	80	0
Interest expenses	Z_t	Linear over 5Y	0
Taxes	s	30%	30%
Terminal growth rate for NCF (after-tax)	g	2%	2%
Residual value for infinitive NCF stream	R_T		R_T

³⁰ The beta factor for this analysis is calculated using the Bloomberg World equity index in USD and the stock price for Lukoil in USD between August 2003 and March 2009. The beta calculation yields identical values for daily and weekly log-returns.

Given the risk-free rate $r_t^{rf} = 3\%$ and the assumed market return $r_t^m = 10\%$, the required return on equity capital $k_{E,t}$ is equal to 14.9% for all periods.³¹ Further investment assumptions are summarized in Table 10.

The project is valued using standard net-present-value calculation (net-method with cash-flows after taxes).³² The corresponding cash-flow and free cash-flow calculations are given in Table 11 along with the net-present-value calculation in Table 12.

Table 11
Cash-flow composition for model company

	2009S	2009E	2010E	2011E	2012E	2013E	2014E	2015E	2016E	2017E	2018E	Terminal value
Riskless yield profile:		3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%
Debt capital rate		8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%
Equity Beta factor		1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70
	running	running	running	running	running	running	running	running	running	running	running	infinite
Product Price	\$56.30	\$60.61	\$65.91	\$65.22	\$67.52	\$69.82	\$72.13	\$74.43	\$76.74	\$79.04	\$79.04	
Sale Quantity	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	
Total Revenues	\$1457.60	\$1515.20	\$1572.80	\$1530.40	\$1688.00	\$1745.60	\$1803.20	\$1860.80	\$1918.40	\$1976.00	\$1976.00	
Total Direct Cost of Goods Sold	\$1093.20	\$1136.40	\$1179.60	\$1222.80	\$1266.00	\$1309.20	\$1352.40	\$1395.60	\$1438.80	\$1482.00	\$1482.00	
Gross Profit	\$364.40	\$378.80	\$393.20	\$407.60	\$422.00	\$436.40	\$450.80	\$465.20	\$479.60	\$494.00	\$494.00	
Operating Expenses	\$200.00	\$200.00	\$200.00	\$200.00	\$200.00	\$200.00	\$200.00	\$200.00	\$200.00	\$200.00	\$200.00	
Sales, General and Admin. Costs	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	
lost revenues due to new project	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	
Cash Flow before Interest and Taxes	\$164.40	\$178.80	\$193.20	\$207.60	\$222.00	\$236.40	\$250.80	\$265.20	\$279.60	\$294.00	\$294.00	
Project Depreciation	\$100.00	\$100.00	\$100.00	\$100.00	\$100.00	\$100.00	\$100.00	\$100.00	\$100.00	\$100.00	\$100.00	
Net Operating Income	\$64.40	\$78.80	\$93.20	\$107.60	\$122.00	\$136.40	\$150.80	\$165.20	\$179.60	\$194.00	\$194.00	
Interest Payments	\$32.00	\$25.60	\$19.20	\$12.80	\$6.40	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	
EBT	\$32.40	\$53.20	\$74.00	\$94.80	\$115.60	\$136.40	\$150.80	\$165.20	\$179.60	\$194.00	\$194.00	
Taxes	\$9.72	\$15.96	\$22.20	\$28.44	\$34.68	\$40.92	\$45.24	\$49.56	\$53.88	\$58.20	\$58.20	
Taxes only attributed to interest payments	\$9.60	\$7.68	\$5.76	\$3.84	\$1.92	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	
Net Income	\$22.68	\$37.24	\$51.80	\$66.36	\$80.92	\$95.48	\$105.56	\$115.64	\$125.72	\$135.80	\$135.80	
Cash Flow before Interest and Taxes	\$164.40	\$178.80	\$193.20	\$207.60	\$222.00	\$236.40	\$250.80	\$265.20	\$279.60	\$294.00	\$294.00	
Operating Cash Flow	\$145.08	\$155.16	\$165.24	\$175.32	\$185.40	\$195.48	\$205.56	\$215.64	\$225.72	\$235.80	\$235.80	
Loan amortization (annuity factor):	5.00	4.00	3.00	2.00	1.00	1.00	4.00	3.00	2.00	1.00		
Loan amortization (standard amortization):	\$80.00	\$80.00	\$80.00	\$80.00	\$80.00	\$80.00	\$80.00	\$80.00	\$80.00	\$80.00	\$80.00	
Loan outstanding:	\$400.00	\$320.00	\$240.00	\$160.00	\$80.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	
NCF (net cash flow)	\$122.68	\$137.24	\$151.80	\$166.36	\$180.92	\$195.48	\$205.56	\$215.64	\$225.72	\$235.80	\$1964.47	
Book value:	\$1000.00	\$900.00	\$800.00	\$700.00	\$600.00	\$500.00	\$400.00	\$300.00	\$200.00	\$100.00	\$0.00	
Investment Outlay	(\$1000.00)	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	
Net Free Cash Flow to Equity holder: NFCF=(NCF-Y)	\$42.68	\$57.24	\$71.80	\$86.36	\$100.92	\$195.48	\$205.56	\$215.64	\$225.72	\$235.80	\$1964.47	

³¹ $k_{E,t} = r_t^{rf} + \left[E \left(r_t^m \right) - r_t^{rf} \right] \beta_{E,t}$ with $\beta_{E,t} = \frac{\text{Cov} \left(r_t^i, r_t^m \right)}{\sigma^2 \left(r_t^m \right)}$.

³² The NPV, i.e. the market value of the partially debt-financed project (K_0), is calculated via:

$$K_0 = -A_0 + Y_0 + \sum_{t=1}^T \frac{NCF - Y_t}{\prod_{\tau=1}^t (1 + k_{E,\tau})} + \frac{R_T}{\prod_{\tau=1}^T (1 + k_{E,\tau})}, \text{ see Fischer (1996), p. 106.}$$

Table 12
Present value calculation

	2009S	2009E	2010E	2011E	2012E	2013E	2014E	2015E	2016E	2017E	2018E	Terminal value
Investment costs	(\$1,000.00)											
Financing costs	\$400.00											
PV of ECFs in period 0	\$1,049.82	37.15	43.36	47.33	49.55	50.39	84.95	77.75	70.99	64.67	58.80	464.89
Total project NPV:	\$449.82											

The project's total net-present-value ($NPV_{t=0}^{1st}$) is 450 (449.82) USD units. Since the project has a positive net-present-value, it should be conducted. The calculation of the free cash-flow stream (Table 13, Panel A) also allows construction of the expected evolution of the income statement along with the future balance sheet (Table 13, Panels B and C). Of special interest is the evolution of the market value of equity E_t (i.e. the sum of equity capital or book value and retained earnings) and the market value of debt capital D_t . Both variables define the total market value of the investment project and hence total value of the overall company V_t at market prices via Equation (9).

$$V_t = E_t + D_t \quad (9)$$

Table 13
Aggregated cash-flow composition, net income calculation and the model company balance sheet

Panel A												
Expected Cash Flows		2009E	2010E	2011E	2012E	2013E	2014E	2015E	2016E	2017E	2018E	2018E
Revenues inflow:		\$1,457.60	\$1,515.20	\$1,572.80	\$1,630.40	\$1,688.00	\$1,745.60	\$1,803.20	\$1,860.80	\$1,918.40	\$1,976.00	
- Variable outflow		(\$1,093.20)	(\$1,136.40)	(\$1,179.80)	(\$1,222.80)	(\$1,266.00)	(\$1,309.20)	(\$1,352.40)	(\$1,395.60)	(\$1,438.80)	(\$1,482.00)	
- Fixed outflow		(\$200.00)	(\$200.00)	(\$200.00)	(\$200.00)	(\$200.00)	(\$200.00)	(\$200.00)	(\$200.00)	(\$200.00)	(\$200.00)	
Cash Flow before Interests and Taxes C		\$164.40	\$178.80	\$193.20	\$207.60	\$222.00	\$236.40	\$250.80	\$265.20	\$279.60	\$294.00	
- Interest Payments		(\$32.00)	(\$25.60)	(\$19.20)	(\$12.80)	(\$6.40)	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	
- Taxation:		(\$9.72)	(\$15.96)	(\$22.20)	(\$28.44)	(\$34.68)	(\$40.92)	(\$45.24)	(\$49.56)	(\$53.88)	(\$58.20)	
Net Cash Flow (NCF)		\$122.68	\$137.24	\$151.80	\$166.36	\$180.92	\$195.48	\$205.56	\$215.64	\$225.72	\$235.80	
Loan amortization (annuity factor):		(\$80.00)	(\$80.00)	(\$80.00)	(\$80.00)	(\$80.00)	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	
Cash Flow to equity holders (NCF)		\$42.68	\$57.24	\$71.80	\$86.36	\$100.92	\$195.48	\$205.56	\$215.64	\$225.72	\$235.80	

Panel B												
Expected Income Statement		2009E	2010E	2011E	2012E	2013E	2014E	2015E	2016E	2017E	2018E	2018E
Revenues:		\$1,457.60	\$1,515.20	\$1,572.80	\$1,630.40	\$1,688.00	\$1,745.60	\$1,803.20	\$1,860.80	\$1,918.40	\$1,976.00	
- Variable costs		(\$1,093.20)	(\$1,136.40)	(\$1,179.80)	(\$1,222.80)	(\$1,266.00)	(\$1,309.20)	(\$1,352.40)	(\$1,395.60)	(\$1,438.80)	(\$1,482.00)	
- Project Depreciation		(\$100.00)	(\$100.00)	(\$100.00)	(\$100.00)	(\$100.00)	(\$100.00)	(\$100.00)	(\$100.00)	(\$100.00)	(\$100.00)	
- Interest Payments		(\$32.00)	(\$25.60)	(\$19.20)	(\$12.80)	(\$6.40)	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	
- other fixed Operating Expenses		(\$200.00)	(\$200.00)	(\$200.00)	(\$200.00)	(\$200.00)	(\$200.00)	(\$200.00)	(\$200.00)	(\$200.00)	(\$200.00)	
NOI (Profit before Taxes)		\$32.40	\$53.20	\$74.00	\$94.80	\$115.60	\$136.40	\$150.80	\$165.20	\$179.60	\$194.00	
- Taxation:		(\$9.72)	(\$15.96)	(\$22.20)	(\$28.44)	(\$34.68)	(\$40.92)	(\$45.24)	(\$49.56)	(\$53.88)	(\$58.20)	
NI (Profit after Taxes)		\$22.68	\$37.24	\$51.80	\$66.36	\$80.92	\$95.48	\$105.56	\$115.64	\$125.72	\$135.80	

Panel C												
Balance sheet Start:		2009S	2009E	2010E	2011E	2012E	2013E	2014E	2015E	2016E	2017E	2018E
Assets												
Machine		\$1,000.00	\$900.00	\$800.00	\$700.00	\$600.00	\$500.00	\$400.00	\$300.00	\$200.00	\$100.00	\$0.00
Cash			\$42.68	\$99.92	\$171.72	\$258.08	\$359.00	\$554.48	\$760.04	\$975.68	\$1,201.40	\$1,437.20
Sum:		\$1,000.00	\$942.68	\$899.92	\$871.72	\$858.08	\$859.00	\$954.48	\$1,060.04	\$1,175.68	\$1,301.40	\$1,437.20
Liabilities												
Debt capital:		\$400.00	\$320.00	\$240.00	\$160.00	\$80.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00
Equity capital:		\$600.00	\$600.00	\$600.00	\$600.00	\$600.00	\$600.00	\$600.00	\$600.00	\$600.00	\$600.00	\$600.00
Retained Earnings:		\$0.00	\$22.68	\$59.92	\$111.72	\$178.08	\$259.00	\$354.48	\$460.04	\$575.68	\$701.40	\$837.20
Sum:		\$1,000.00	\$942.68	\$899.92	\$871.72	\$858.08	\$859.00	\$954.48	\$1,060.04	\$1,175.68	\$1,301.40	\$1,437.20

The conventional net-present-value-based project analysis would usually end here.

3.2.2.1 The distribution analysis of an investment project's NPV

The described investment project in Subsection 3.2.2 has a positive net-present-value and should hence be conducted. However, the static NPV calculation does not reveal anything about the likelihood of future cash-flows. Thus, the likelihood distribution of the net free cash-flows to equity holders (NFCF) expected to materialize is not captured in the calculation. The investment's NPV only gives insight into the distribution mean but not other, higher – distribution moments. The importance of higher moments, however, is essential in financial markets, since it allows for normalized returns based on risk (see also Subsection 3.3.3, p. 103).

Once one allows for uncertainty in the input factors for the net-present-value calculation, it is possible to simulate the project net-present-value and gain greater insight into the project's higher moments. In order to do so, we will first define the assumptions for the elements exposed to uncertainty and afterwards describe the simulation output that will allow further inference about the project's value.

Data-generating process (DGP) for the product price:

The only random variable that will be allowed to fluctuate in our example is the price of oil, P_t^{oil} , which is the product price in Table 10.³³ We have assumed an additive Brownian motion for the price of oil (see Equations (10) and (11)). Although the chosen process theoretically would allow negative oil prices, we have chosen the DGP is in line with the results shown in Chapter 5 (see p. 210).³⁴

$$P_{t+1}^{oil} = P_t^{oil} + \Delta P_t^{oil} \quad (10)$$

³³ It would have been straight forward to allow for a random sales quantity q_t that is negatively correlated with the product price P_t^{oil} and hence closer to a microeconomic demand/supply framework.

³⁴ This assumption also limits additional non-normal inference of the chosen DGP on the project NPV distribution. However, it is clearly straight forward to apply a geometric Brownian motion instead.

$$\Delta P_t^{oil} = a\Delta t + b\sqrt{\Delta t}\varepsilon_{t+1} \quad (11)$$

The Brownian motion process is calibrated as follows:

$$a\Delta t = 0.192 \times 12$$

$$b\sqrt{\Delta t}\varepsilon_{t+1} = 10 \times \varepsilon_{t+1}$$

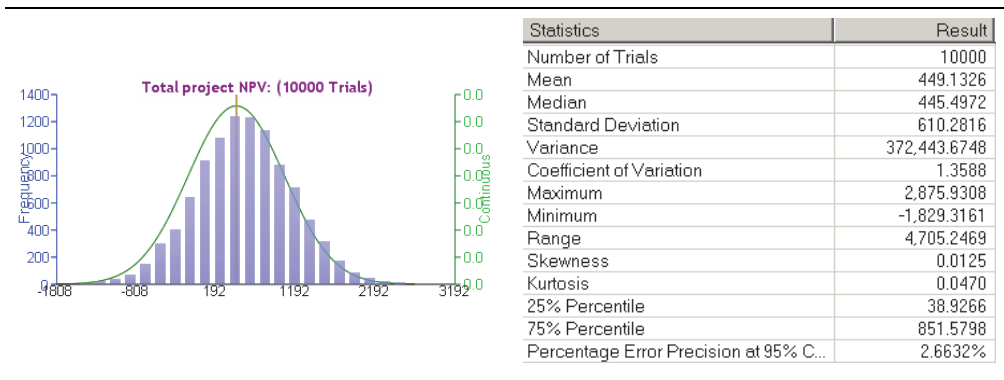
$$\varepsilon_{t+1} = N(0,1)$$

Simulating the net-present-value distribution of the investment project:

Based on this assumption for the DGP, we have simulated the NPV of the described investment project.³⁵ The results are given in Figure 25.

Figure 25

Distribution analysis of simulated NPV



Panel A: Total project NPV distribution (10000 trials). The simulation is conducted with Crystal Ball Add-in for Excel.

Panel B: Distribution statistics for simulated NPV path. The simulation is conducted with Crystal Ball Add-in for Excel. Note: kurtosis statistic indicates excess degree of kurtosis.

Panel A in Figure 25 shows the NPV distribution for 10000 simulations. The NPV is nearly normally distributed, with a mean and median close to the calculated static NPV value. The skewness is close to zero, while the excess kurtosis is slightly positive. The standard deviation of the NPV is around 610 units, which indicates that – assuming a standard normal distribution - the NPV lies between -165 and 1059 units with a 68% probability (i.e., within one standard deviations above and below the mean). This simulation analysis has hence expanded our view to include the higher moments of the

³⁵ All simulations were conducted using the Excel Add-in Crystal Ball.

investment project. Theoretically, one could additionally rank the investment project not only along its mean but also along its standard deviation or even along higher moments. For example, similar to a Sharpe ratio analysis for financial assets, two projects with different NPVs could be normalized for their standard deviations or even for higher moments.³⁶

In order to develop a better understanding of how the NPV is distributed, we have also calculated the standard deviation of the percentage change in the annual free cash-flows, a close proxy for equity market earnings. This measure can be used as project volatility input for valuing real-options.³⁷

The calculation is shown in Table 14. Panels A and B show two random drawings for the net free cash-flows to equity holders (see Table 13, Panel A for the static case).

Table 14
Present value calculation and its link to volatility

Panel A

	2009E	2010E	2011E	2012E	2013E	2014E	2015E	2016E	2017E	2018E
Net Free Cash Flow to Equity holder:	\$42.68	\$83.20	\$148.92	\$77.07	\$99.33	\$205.75	\$155.21	\$159.47	\$138.56	\$193.07
LN(NFCF(t)/NFCF(t-1))		66.75%	58.21%	-65.86%	25.37%	72.82%	-28.19%	2.71%	-14.06%	33.17%
Standard deviation:	47.03%									

Panel B

	2009E	2010E	2011E	2012E	2013E	2014E	2015E	2016E	2017E	2018E
Net Free Cash Flow to Equity holder:	\$42.68	\$57.24	\$71.80	\$86.36	\$100.92	\$195.48	\$205.56	\$215.64	\$225.72	\$235.80
LN(NFCF(t)/NFCF(t-1))		29.35%	22.66%	18.46%	15.58%	66.11%	5.03%	4.79%	4.57%	4.37%
Standard deviation:	19.91%									

Note that this approximation of the return probability finds its limit if negative cash-flows are observed. For this instance we have simulated the one-step-ahead discounted cash-flow volatility (DCF volatility, also called the net-present-value volatility) of the project. The net-present-value volatility is calculated by relating the simulated

³⁶ The concept of investment project discrimination along higher moments will lead to the adjusted Sharpe-ratio analysis covered in Subsection 3.3.3, p. 103.

³⁷ See Mun (2006).

discounted NCF to equity holders in one year (i.e. the NPV that would result in one year's time for each simulation) to today's NPV. It is hence a simulation result distribution.

$$r_i^{NPV} = \ln \left(\frac{NPV_{t=1}}{NPV_{t=0}} \right) \quad (12)$$

This value is superior to the previously used volatility estimate due to its ability to capture changes in the entire project valuation relative to future changes of the uncertainty factor (the product price in our example).³⁸

The simulated change in NPV comes close to changes in the equity value (equity returns), since both elements – future net free cash-flows to equity holders (for the investment project) and future dividends (for equity) – are represented by their discount value today. This analogy will be of importance in the empirical analysis presented in Chapter 6. The standard deviation of the simulated return distribution r_i^{NPV} is therefore another natural candidate as volatility input for the real-option valuation procedure.

Figure 26 shows the results. Panel A shows the simulated return distribution depicted here as discounted cash-flow volatility (% DCF). Panel B shows the summary statistics. A full simulation reveals an average (mean) project NPV return – i.e. the difference between the simulated NPV for time $t=0$ and $t=1$ – of around 14.8%. The median value is slightly lower around 14.4%, which implies positive skew. The excess kurtosis is positive, indicating that the NPV return distribution is leptokurtic. The standard deviation – our proxy for the uncertainty of the investment project return – is around 66.8%, indicating that expected project return lies between -52% and 82% in 68% of the simulations. Note that the mean simulated return is nearly identical to the return on equity defined in Table 10 (see p. 49) at 14.9%, as should be the case.³⁹ The advantage

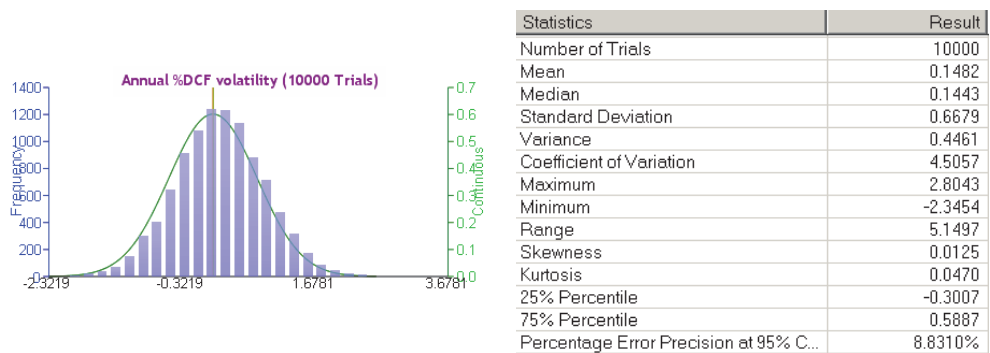
³⁸ For a detailed and critical analysis of different Monte Carlo-based volatility calculations that can be used as real option valuation input, see Mun (2003), pp. 64-65, and Mun (2006), pp. 190-212.

³⁹ Given that $E_0 = E(E_1)/(1+k_E)$.

of this approach, and this is indeed the decisive point, is that we also have a fully simulated distribution of returns with all of the additional higher moments, something that would have not been obtainable from the CAPM framework.⁴⁰

Figure 26

Distribution analysis of simulated % discounted cash-flow (% DCF) volatility



Panel A: Total project % DCF distribution (10000 trials) as proxy for project volatility. The simulation is conducted with Crystal Ball Add-in for Excel.

Panel B: Distribution statistics of simulated % DCF values as proxy for project volatility. The simulation is conducted with Crystal Ball Add-in for Excel. Note: kurtosis statistic indicates excess degree of kurtosis.

Amending the investment assumptions:

To illustrate how real-options can alter the net-present-value distribution, we have amended the investment project described at the beginning of this section to include an additional investment lag. We will assume that after five additional years the company will (not just can) expand its production due to a new facility. The new investment project will lead to a 20% increase in output from q_t 25 units to 30 units of sales. The fixed costs $c_{f,t}$ will stay unchanged. The project will be financed with another loan Y_t of an additional 400 units and an increase in equity capital E_t of 600 units (with no drain out of cash positions) with depreciation over the next five years. Due to diminishing margins, the variable's cost fraction of price $c_{v,t}$ increases from 0.75 to 0.80. The new cash-flow composition for this expansion project is given in Table 15.

⁴⁰ Since the CAPM framework rests on the notion of normally distributed returns, one has no further information on higher moments.

The cash-flows and the present value are calculated here on a standalone basis without incorporating the values of the original project.

Table 15
Cash-flow composition for model company with commitment to expand

	2009S	2009E	2010E	2011E	2012E	2013E	2014E	2015E	2016E	2017E	2018E	Terminal value
Riskless yield profile:		3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%
Debt capital rate		8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%
Equity Beta factor	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	infinf
	running	running	running	running	running	running	running	running	running	running	running	
Product Price	\$58.00	\$58.30	\$60.61	\$62.91	\$65.22	\$67.52	\$69.82	\$72.13	\$74.43	\$76.74	\$79.04	
Sale Quantity		0.0	0.0	0.0	0.0	0.0	30.0	30.0	30.0	30.0	30.0	
Total Revenues		\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$2,094.72	\$2,163.84	\$2,232.96	\$2,302.08	\$2,371.20	
Total Direct Cost of Goods Sold		\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$1,759.51	\$1,839.26	\$1,918.02	\$1,996.77	\$2,075.52	
Gross Profit		\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$314.21	\$324.58	\$314.94	\$305.31	\$295.68	
Operating Expenses		\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	
Sales, General and Admin. Costs		\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	
lost revenues due to new project		\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	
Cash Flow before Interest and Taxes		\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$314.21	\$324.58	\$314.94	\$305.31	\$295.68	
Project Depreciation		\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$200.00	\$200.00	\$200.00	\$200.00	\$200.00	
Net Operating Income		\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$114.21	\$124.58	\$134.94	\$145.31	\$155.68	
Interest Payments		\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$32.00	\$35.00	\$19.20	\$12.80	\$6.40	
EBT		\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$82.21	\$89.58	\$115.74	\$132.51	\$149.28	
Taxes		\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$24.66	\$29.69	\$34.72	\$39.75	\$44.78	
Taxes only attributed to interest payments		\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$9.60	\$7.68	\$5.76	\$3.84	\$1.92	
Net Income		\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$57.55	\$59.28	\$51.02	\$52.76	\$104.50	
Cash Flow before Interest and Taxes		\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$314.21	\$324.58	\$314.94	\$305.31	\$295.68	
Operating Cash Flow		\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$279.95	\$287.29	\$294.46	\$301.72	\$308.98	
Loan amortization (annuity factor):		10.00	9.00	8.00	7.00	6.00	5.00	4.00	3.00	2.00	1.00	
Loan amortization (standard amortization):		\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$80.00	\$80.00	\$80.00	\$80.00	\$80.00	
Loan outstanding:		\$0.00	\$0.00	\$0.00	\$0.00	\$400.00	\$320.00	\$240.00	\$160.00	\$80.00	\$0.00	
NCF (net cash flow)		\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$257.55	\$209.28	\$218.02	\$219.76	\$304.50	\$2,407.64
Book value:		\$0.00	\$0.00	\$0.00	\$0.00	\$1,000.00	\$800.00	\$600.00	\$400.00	\$200.00	\$0.00	\$0.00
Investment Outlay		\$0.00	\$0.00	\$0.00	\$0.00	(\$1,000.00)	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00
Net Free Cash Flow to Equity holder:		\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$177.55	\$189.28	\$201.02	\$212.76	\$224.50	\$2,407.64
NFCF=(NCF-Y)												

The project contributes positively to shareholder equity over the first five years (starting in five years). The positive investment contribution is primarily related to the residual value – which, however, is more distant and hence more subject to uncertainty. The free-cash-flow elements and the net-present-value calculation are illustrated in Table 16.⁴¹

Table 16
Present-value calculation of model company with commitment to expand

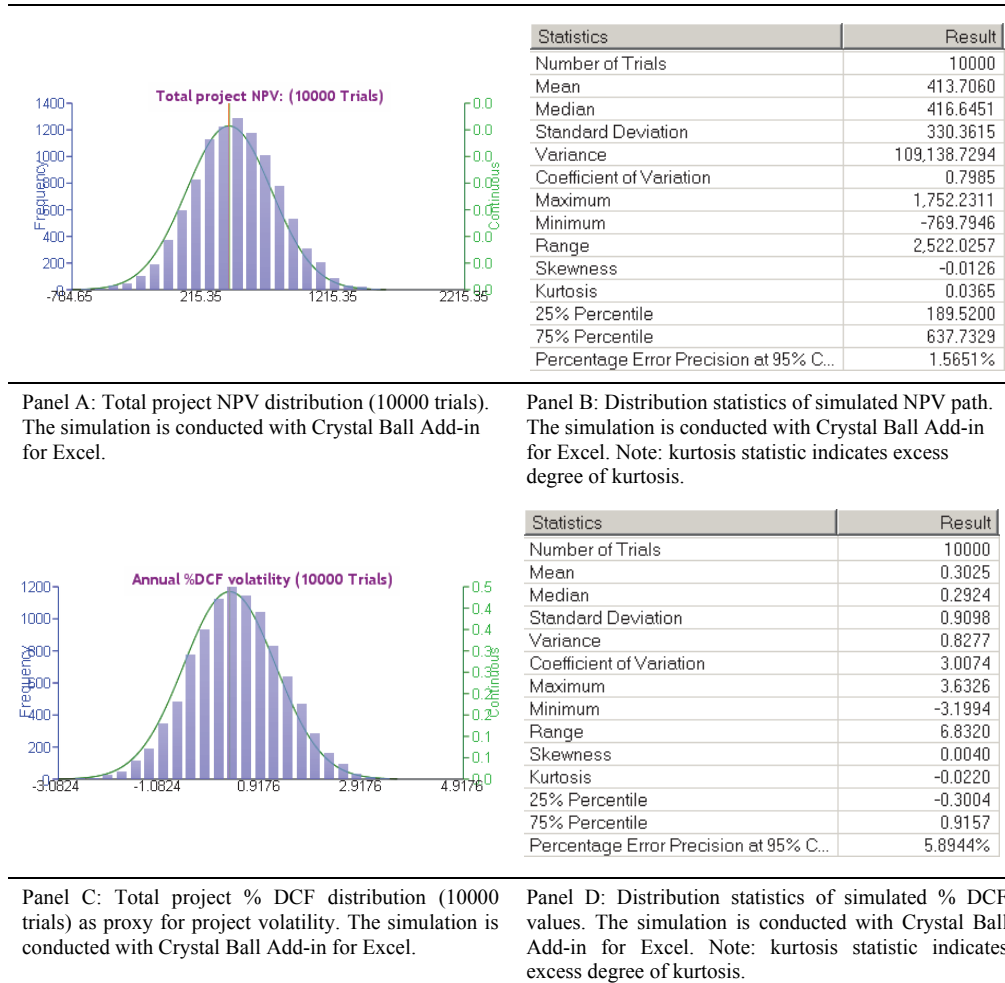
	2009S	2009E	2010E	2011E	2012E	2013E	2014E	2015E	2016E	2017E	2018E	Terminal value
Investment costs	(\$992.61)					(\$992.61)						
Financing costs:	\$345.04	\$0.00	\$0.00	\$0.00	\$0.00	\$345.04						
PV of FCFs in period 0	\$932.19	-	-	-	-	-	\$77.16	\$71.59	\$66.17	\$60.95	\$55.98	\$600.33
Total project NPV:	\$414.62											

⁴¹ Note that the investment and financing costs are already discounted. Since both cash-flows are already known today (with no uncertainty), we have used the risk-free rate as the discount rate.

The overall static net-present-value of the second project ($NPV_{t=0}^{2st}$) is positive with 415 USD. The net-present-value distribution characteristics are given in Figure 27.

Figure 27

Distribution analysis for NPV and % DCF volatility for the model company with commitment to expand



As with the original project, the NPV skewness is close to zero though with a small negative bias. The excess kurtosis is positive though also close to zero. Overall the NPV and the project returns seem to follow a normal distribution.⁴²

⁴² The mean return of the second investment is nearly twice as large as the mean return of the first investment. The reason is that the present value of the liability side is growing negatively with the risk-

Both projects can also be analyzed together instead of using individual net-present-value simulations ($NPV_{t=0}^{total}$). The results are given below. The project now has two investment outlays and two loan financing legs that are amortized continuously over the following years. The free-cash-flow summary and net-present-value calculations are given in Table 17 and Table 18, respectively. A simulation of the overall combined project net-present-value ($NPV_{t=0}^{total}$) is conducted again and shown in Figure 28.

Table 17

Cash-flow composition for model company with initiation project and commitment to expand

	2009S	2009E	2010E	2011E	2012E	2013E	2014E	2015E	2016E	2017E	2018E	Terminal value
Riskless yield profile:	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%
Debt capital rate	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%
Equity Beta factor	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70
	running	running	running	running	running	running	running	running	running	running	running	infinite
Product Price	\$56.00	\$58.30	\$60.61	\$62.91	\$65.22	\$67.52	\$69.82	\$72.13	\$74.43	\$76.74	\$79.04	
Product A Sale Quantity (000s)		25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	
Product B Sale Quantity (000s)		0.0	0.0	0.0	0.0	0.0	30.0	30.0	30.0	30.0	30.0	
Sale Quantity		25.0	25.0	25.0	25.0	25.0	55.0	55.0	55.0	55.0	55.0	
Total Revenues		\$1457.60	\$1515.20	\$1572.80	\$1630.40	\$1688.00	\$1745.60	\$1803.20	\$1860.80	\$1918.40	\$1976.00	
Total Direct Cost of Goods Sold		\$1063.20	\$1136.40	\$1209.60	\$1282.80	\$1356.00	\$1429.20	\$1502.40	\$1575.60	\$1648.80	\$1722.00	
Gross Profit		\$394.40	\$378.80	\$363.20	\$347.60	\$332.00	\$316.40	\$300.80	\$285.20	\$269.60	\$254.00	
Operating Expenses		\$200.00	\$200.00	\$200.00	\$200.00	\$200.00	\$200.00	\$200.00	\$200.00	\$200.00	\$200.00	
Sales, General and Admin. Costs		\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	
Lost revenues due to new project		\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	
Cash Flow before Interest and Taxes		\$194.40	\$178.80	\$163.20	\$147.60	\$132.00	\$116.40	\$100.80	\$85.20	\$69.60	\$54.00	
Project Depreciation		\$100.00	\$100.00	\$100.00	\$100.00	\$100.00	\$100.00	\$100.00	\$100.00	\$100.00	\$100.00	
Net Operating Income		\$64.40	\$78.80	\$93.20	\$107.60	\$122.00	\$136.40	\$150.80	\$165.20	\$179.60	\$194.00	
Interest Payments		\$32.00	\$32.00	\$32.00	\$32.00	\$32.00	\$32.00	\$32.00	\$32.00	\$32.00	\$32.00	
EBT		\$32.40	\$46.80	\$61.20	\$76.00	\$90.00	\$104.40	\$118.80	\$133.20	\$147.60	\$162.00	
Taxes		\$9.72	\$15.96	\$22.20	\$28.44	\$34.68	\$40.92	\$47.16	\$53.40	\$59.64	\$65.88	
Taxes only attributed to interest payments		\$9.60	\$7.68	\$5.76	\$3.84	\$1.92	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	
Net Income		\$22.68	\$32.24	\$41.80	\$51.36	\$60.92	\$70.48	\$80.04	\$89.60	\$99.16	\$108.72	
Cash Flow before Interest and Taxes		\$194.40	\$178.80	\$163.20	\$147.60	\$132.00	\$116.40	\$100.80	\$85.20	\$69.60	\$54.00	
Operating Cash Flow		\$144.00	\$155.16	\$165.24	\$175.32	\$185.40	\$195.48	\$205.56	\$215.64	\$225.72	\$235.80	
Loan amortization (annuity factor):		5.00	4.00	3.00	2.00	1.00	5.00	4.00	3.00	2.00	1.00	
Loan amortization (standard amortization):		\$80.00	\$80.00	\$80.00	\$80.00	\$80.00	\$80.00	\$80.00	\$80.00	\$80.00	\$80.00	
Loan outstanding:		\$400.00	\$320.00	\$240.00	\$160.00	\$80.00	\$400.00	\$320.00	\$240.00	\$160.00	\$80.00	
NCF (net cash flow)		\$122.68	\$137.24	\$151.80	\$166.36	\$180.92	\$453.03	\$474.84	\$496.66	\$518.48	\$540.30	\$4272.11
Book value:		\$1000.00	\$900.00	\$800.00	\$700.00	\$600.00	\$1500.00	\$1200.00	\$900.00	\$600.00	\$300.00	\$0.00
Investment Outlay		(\$1000.00)	\$0.00	\$0.00	\$0.00	(\$1000.00)	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00
Net Free Cash Flow to Equity holder:		\$42.68	\$57.24	\$71.80	\$86.36	\$100.92	\$373.03	\$394.84	\$416.66	\$438.48	\$460.30	\$4272.11
NCF=(NCF-Y)												

Table 18

Present-value calculation of model company with commitment to expand

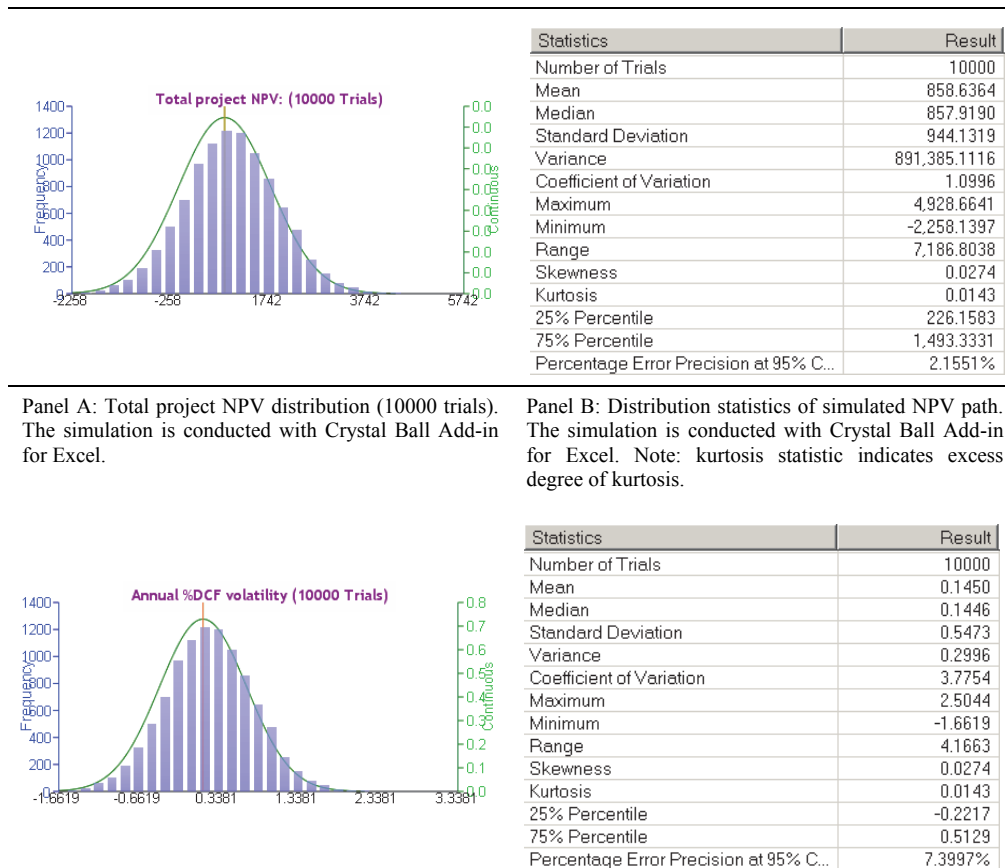
	2009S	2009E	2010E	2011E	2012E	2013E	2014E	2015E	2016E	2017E	2018E	Terminal value
Investment costs		(\$1982.61)				(\$562.61)						
Financing costs:		\$745.04				\$345.04						
PV of FCFs in period 0		\$1982.01	37.15	43.36	47.33	49.55	50.39	162.11	149.34	137.16	125.62	114.77
Total project NPV:		\$864.45										\$1065.23

free rate, while the asset side (the discounted net-free-cash-flows) growth is 14.9% (the discount rate for equity).

Figure 28, Panel A shows that the mean and median values are simply the mean and median sum of the individual simulations ($NPV_{t=0}^{total} = NPV_{t=0}^{1st} + NPV_{t=0}^{2nd}$). The return distribution, and hence the volatility of the overall project, is very much unaffected by adding both projects together (the nearly identical mean return equals the required return on equity). The standard deviation of the simulated return distribution is around 55% and lower than for the stand-alone project (66%; see Panel B in Figure 26). The higher distribution moments, however, are nearly unchanged (with skewness and excess kurtosis around zero).

Figure 28

Distribution analysis for NPV and % DCF volatility for the model company with commitment to expand



Panel A: Total project NPV distribution (10000 trials). The simulation is conducted with Crystal Ball Add-in for Excel.

Panel B: Distribution statistics of simulated NPV path. The simulation is conducted with Crystal Ball Add-in for Excel. Note: kurtosis statistic indicates excess degree of kurtosis.

Panel C: Total project % DCF distribution (10000 trials) as proxy for project volatility. The simulation is conducted with Crystal Ball Add-in for Excel.

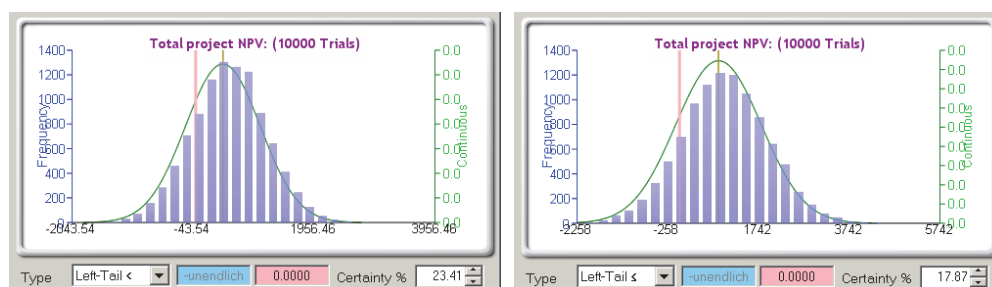
Panel D: Distribution statistics of simulated % DCF values. The simulation is conducted with Crystal Ball Add-in for Excel. Note: kurtosis statistic indicates excess degree of kurtosis.

The overall project – despite the positive (nearly doubled) mean – still has a large probability associated with negative values. As shown in Figure 29, Panel B, the simulated probability for an NPV below 0 is around 17.8%. This, however, is an improvement relative to the expected shortfall risk of the first, stand-alone project, where the below zero NPV probability was 23.4% (see Panel A).

Hence, the overall conclusion that can be drawn from the above example is that the additional project leg does not only increase the NPV (by simply summing the two projects) but leaves the return distribution of the free-cash-flows unchanged. In particular the higher moments remain unaffected if a second project is added into the simulation analysis. This changes, however, if the second investment project becomes optional and not – as in the above example – binding.

Figure 29

Comparing the NPV shortfall risk of the model company with and without a commitment to expand



Panel A: Probability of a negative project NPV of the original investment project based on Monte Carlo simulation (10000 trials). The simulation is conducted with Crystal Ball Add-in for Excel.

Panel B: Probability of a negative project NPV of the amended investment project (original project + expansion project) based on Monte Carlo simulation (10000 trials). The simulation is conducted with Crystal Ball Add-in for Excel.

The real-option to expand

To highlight the distribution consequences of adding optionality to the investment project, we will define the second investment project as an option – to be more precise, an option to expand – and not as a fixed investment decision without opportunities to change. The option to expand is defined as investing in the second project if its NPV in five years is positive and otherwise cancelling the additional investment. The first

project initiated at $t = 0$ is not affected and will continue irrespective of the investment decision for the second project. The interest rate risk that would usually be associated with financing the project in the future is ignored in this example, although it would be straightforward to incorporate randomness to the discount factor. As for most real-option cases we will assume that all other risks associated with the project can be hedged away in the market, which opens the way to discounting the future payoff via the risk-free interest rate.⁴³

The value to expand the investment project at $t = 5$ can be formulated as an option:

$$NPV^{option} = \max\left(NPV_{t=5}^{2nd} / (1 + r_f)^t, 0\right) \quad (13)$$

Further the project value of the second investment – now calculated as an option to invest at stage two – can be decomposed into a static (see Table 16, p. 57) and an option premium component that is related to decision flexibility:

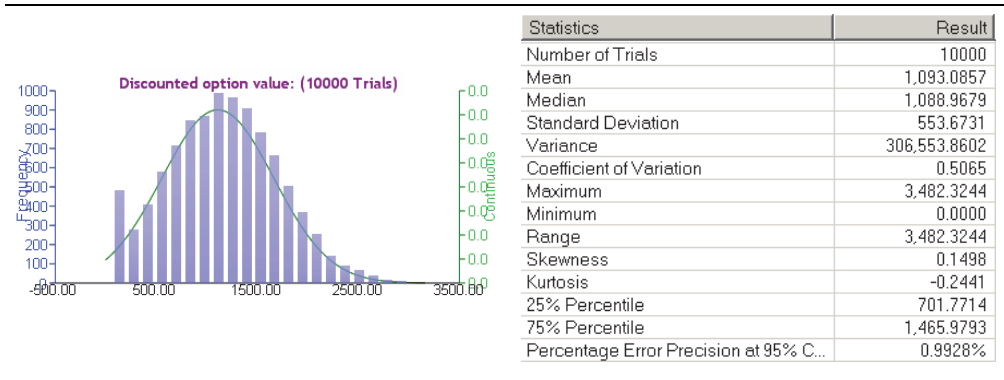
$$NPV^{option} = NPV_{t=0}^{2nd} + NPV^{premium} \quad (14)$$

We have calculated via simulation the net-present-value of the second project assuming managerial flexibility (NPV^{option}) along with its simulated distribution (see Figure 30).

⁴³ This type of clause is often used in real-option theory (see Dixit and Pindyck, 1994); one therefore needs to assume that either the project can be totally replicated by market variables or that the project is part of a well diversified portfolio of projects, which eliminates the entire individual non-systematic (idiosyncratic) risks. Note further that the different interdependencies between the projects and the overall company asset value are usually not disclosed; as pointed out by Trigeorgis (2000), p. 41, investors also should not be willing to pay a premium for diversification between investment projects, given that they can diversify them more effectively in financial markets.

Figure 30

Distribution analysis of second project value after allowing for optionality



Panel A: Total option NPV distribution (10000 trials). The simulation is conducted with Crystal Ball Add-in for Excel.

Panel B: Distribution statistics of simulated option NPV path. The simulation is conducted with Crystal Ball Add-in for Excel. Note: kurtosis statistic indicates excess degree of kurtosis.

The value of the second investment project – now considered as an option to expand – is 1093 units and hence significantly higher than the unconditional (optionless) net-present-value of the second investment project ($NPV_{t=0}^{2nd}$) of 415 units (see Table 16 or Figure 27). Following Equation (14) the difference between the NPV of the investment project, defined as option (NPV^{option}), and the convention present value of the second investment project (NPV^{2nd}) is the value of optionality, i.e., the premium of the option and hence its time value. The project value (NPV^{2nd}) represents the intrinsic value of the option. In this case, since the intrinsic value is positive, the project is in-the-money. Hence, the overall option value of the second investment project defined now as real-option to expand is:

$$NPV^{option} = \text{intrinsic value} + \text{time value} \quad (15)$$

By plugging the intrinsic value as calculated in Table 16, p. 57 into Equation (15), one can quantify the time value of the option with 678 units. It is more than 100% larger than the NPV of the stand-alone second investment project ($NPV_{t=0}^{2st}$) with 415 units.⁴⁴

The enhanced total net-present-value ($NPV_{t=0}^{enhanced}$) now captures the optionality to invest in stage two plus the static net-present-value of the investment decision at stage one ($NPV_{t=0}^{1st}$). Hence, the overall enhanced total net-present-value ($NPV_{t=0}^{enhanced}$) is given by:

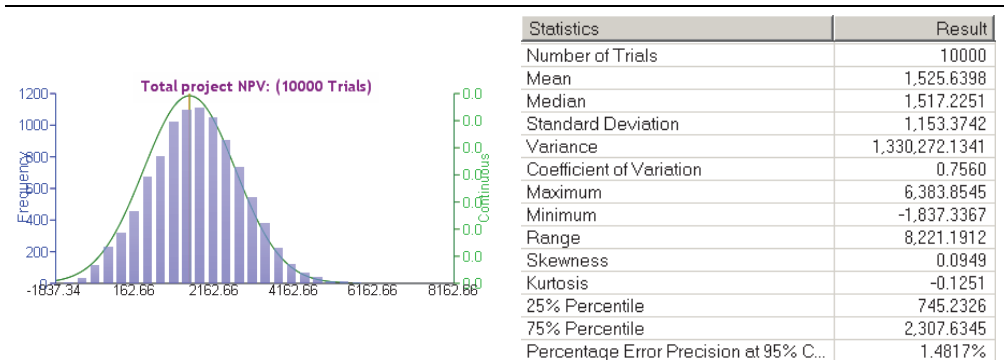
$$NPV_{t=0}^{enhanced} = NPV_{t=0}^{1st} + NPV^{option} \quad (16)$$

The overall value of the total enhanced second investment project ($NPV^{enhanced}$) is now 1543 units, which is nearly twice as large as the simple added simulated project sum ($NPV^{1st} + NPV^{2nd}$) of 858 units (see Figure 28). In addition to the change in the overall project value, there is the change in the NPV distribution. The influence of the real-option on the NPV distribution is shown in Figure 31, where we have simulated the combined (now labeled “enhanced”) NPV for both projects.

⁴⁴ Note: it is also possible to define the overall total project value as project one plus project two with the option to abandon the commitment for project two (put option). In this case such an option or insurance to cancel the second investment project commitment would be equal to 678 units.

Figure 31

Distribution analysis of simulated enhanced project NPV after allowing for optionality



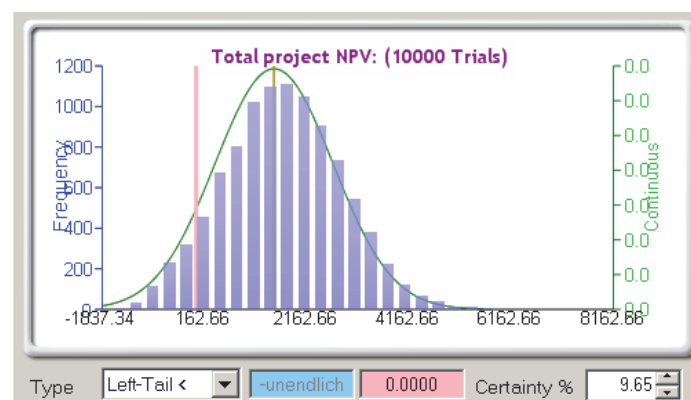
Panel A: Total project NPV distribution (10000 trials). The simulation is conducted with Crystal Ball Add-in for Excel.

Panel B: Distribution statistics of simulated NPV path. The simulation is conducted with Crystal Ball Add-in for Excel. Note: kurtosis statistic indicates excess degree of kurtosis.

The overall NPV now has a much higher skewness than the simple simulated sum of both projects (see Figure 28, Panel B); the kurtosis value is negative and hence much smaller than for the case of no optionality. Together with the positive skewness, this pattern can be attributed to the optionality value of the second investment project. Allowing for optionality has hence helped to increase the uneven moments (mean and skewness) and decrease an even moment (the kurtosis). The standard deviation, however, has increased slightly.

Figure 32

The NPV shortfall risk of the model company with optionality to expand



Probability of a negative NPV of the investment project with option to expand, based on Monte Carlo simulation (10000 trials). The simulation is conducted with Crystal Ball Add-in for Excel.

The shortfall risk of the enhanced NPV has also been reduced, as illustrated in Figure 32. The probability that the project has a negative NPV is further reduced to only 9.65% (compared to the original shortfall risk of the combined simulated NPVs of project one and two of 17.8%, see Figure 29, p. 61).

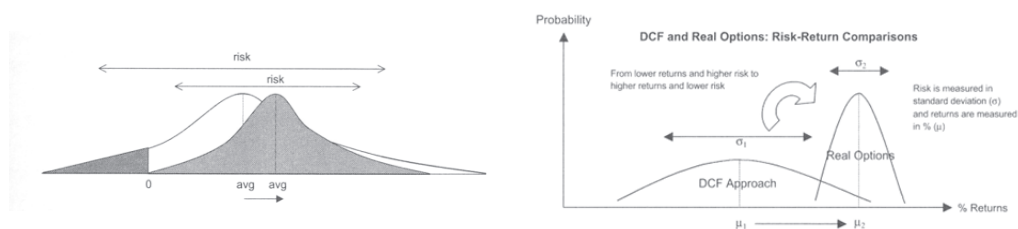
3.2.2.2 Summary and conclusion

One can summarize the results of this investment example in the following way:
Incorporating real-option pricing in an investment project:

- I. enhances the value of the overall project (shift of mean to the right)
- II. reduces the short-fall risk due to the truncation of the left tail of the NPV distribution; it hence increases the skewness and decreases the kurtosis
- III. increases the mean/variance ratio (i.e., the Sharpe ratio) and hence increases the compensation for risk⁴⁵

Figure 33, Panels A and B, illustrate the above observations graphically.

Figure 33
Shaping the probability distribution of the project NPV by incorporating real-options



Panel A: Incorporating real-option analysis decreases the negative tail-risk of the investment and hence increases the positive skewness of the project. See Mun (2006), p. 101.

Panel B: The mean of the overall project value will therefore increase while volatility will usually decline. Both elements result therefore in a more favorable mean/variance (i.e., Sharpe) ratio. See Mun (2006), p. 161.

⁴⁵ See also in this context Section 3.3.3, p. 103.

This simulation experiment does not only illustrate the main advantages of incorporating real-option value into project valuation via value enhancement, it also shows how real-options can alter the return distribution. This is of particular interest for the next sections and chapters, since this finding will provide some guidelines on where and how to search for real-option in the real world. It is obviously unlikely that the market analyzes all the possible real-options that could evolve for a company with potential real-options available. This information is neither available on a micro (i.e., company) level nor time-efficient to obtain. We therefore need to look for other ways and tools to uncover real-option pricing in markets.

One way to do so is to analyze the unconditional distribution moments of different stock market companies. As shown in Figure 33, real-option should lead to more positively skewed and less leptokurtic asset prices. We will conduct an empirical analysis based on this idea in the empirical section of this dissertation in Chapter 6.⁴⁶ However, an unconditional analysis may be too simplistic in the sense that asset-price distributions also depend heavily on the assumed data-generating-process (DGP). With this in mind we will focus on the theory of option pricing and on option pricing sensitive parameter calibration via GARCH modeling in the next section to build a better set of tools to use in the empirical part, in order to search for real-option pricing evidence.

3.2.3 *Option payoff (profit) distribution analysis*

3.2.3.1 Introduction

The expected impact of real-options on the present value distribution can be analyzed by simulation, as shown in the previous section. However, it is difficult to conduct a sensitivity analysis with regard to real-option driving factors in a real investment case. We will therefore use a financial option example and apply Monte Carlo simulation techniques in order to show how changes in key option driving factors like moneyness, volatility and interest rates affect the distribution function of the option payoff. The analysis is important insofar as most of the literature focuses on the impact of the above

⁴⁶ See Chapter 6, p. 260.

factors on the option price but not on the option price distribution. However, our focus is to analyze the price distribution of Russian commodity companies and examine whether their equity price return distribution shows similarities to financial option returns, with respect to higher moments that indicate that the company value is indeed driven by strong optionality. Hence the aim is to give indications on how to expect the equity price-return distribution of stock market companies to change if the overall value of the equity part of the company is subject to changing (i.e. increasing) optionality.

3.2.3.2 Simulating equity price distributions using GARCH diffusion

Assume the following stock-price index⁴⁷ shown in Figure 34. The overall index history between 1963 and 2004 is given in Panel A. We have applied a GARCH(1,1) specification as indicated below to produce an out-of-sample forecast. The forecast Equation is defined by:

$$\ln\left(\frac{S_t}{S_{t-1}}\right) = c + u_t, \quad u_t = \sigma_t \varepsilon_t, \rightarrow u_t | I_t \sim N(0, \sigma_t^2) \quad (17)$$

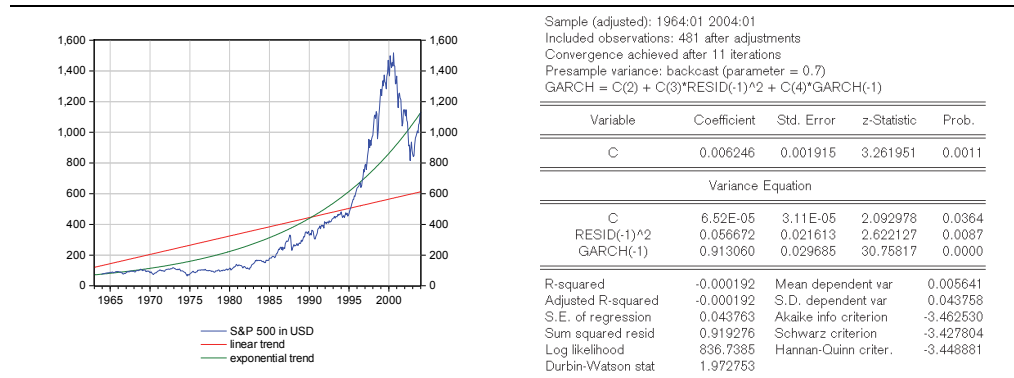
$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (17)\text{-a}$$

The forecast Equation is essentially a random walk with a drift along with an GARCH(1,1) error term (GARCH diffusion). Panel B in Figure 34 shows the estimated GARCH Equation.

⁴⁷ The index is calculated without considering reinvested dividends. The data source is Datastream.

Figure 34

Long-term history of S&P 500 index along with the estimated GARCH(1,1) model on the log-normal return series



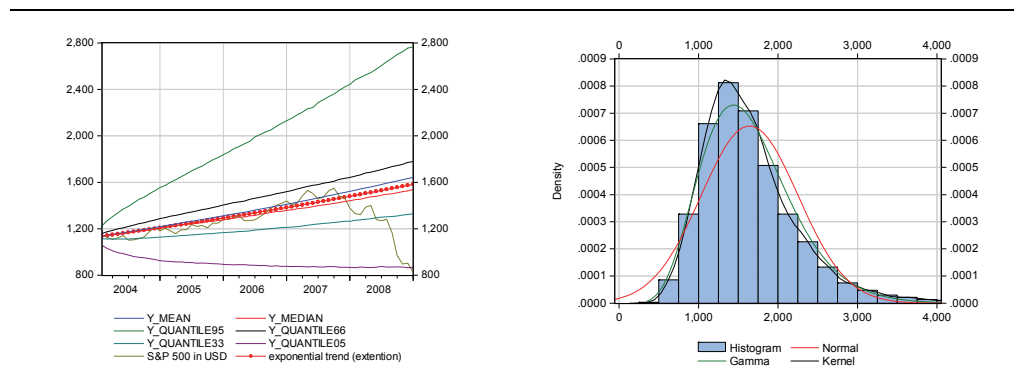
Panel A: S&P 500 equity index between 1963 and 2004 along with an in-sample linear and exponential fit. Data source: Datastream.

Panel B: Estimated GARCH(1,1) Equation for the S&P 500 index between 1964 and 2004.

Panel A in Figure 35 shows the GARCH(1,1) out-of-sample forecast with the associated forecast error around the mean and median forecast. It is very close to a simple extended exponential trend, which is no surprise given the forecast specification (see Equation (17)).

Figure 35

A 5-year (2004 to 2009) out-of-sample forecast for the S&P 500 index and the resulting forecast distribution at the end of the forecast horizon



Panel A: Out-of-sample forecast for S&P 500 index using a GARCH(1,1) data-generating process along with exponential trend forecast.

Panel B: Forecast distribution for the S&P 500 index after a five-year forecast horizon. The histogram is displayed along with a normal and gamma density function.

The forecasts are conducted with monthly data, which in general is not regarded as the best data frequency for equity indices.⁴⁸ The results, however, fit well with other conducted studies (see Engle and Patton, 2001; Engle, 2004; and Duan et al., 2006 for GARCH(1,1) and GJR-GARCH(1,1) model estimates), which can be explained by the long data history used in this sample. Panel B in Figure 35 also shows the estimated price (S&P) distribution over the forecast horizon of five years along with a normal and gamma distribution.⁴⁹

The simulated prices forecasts can be translated into simulated return distribution forecasts (given that the starting value of the simulation $(S_{t=0})$ is known) and also, which will be done in later, into distribution forecasts of future option returns. Figure 36 shows the simulated return distribution forecast for the S&P 500 dataset between 2004-2009 assuming GARCH(1,1) and GJR-GARCH(1,1) diffusion. In analogy to Equation (17) the GJR-GARCH(1,1) diffusion term is given in Equation (18).

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \gamma 1_{\{u_{t-1} < 0\}} u_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (18)^{50}$$

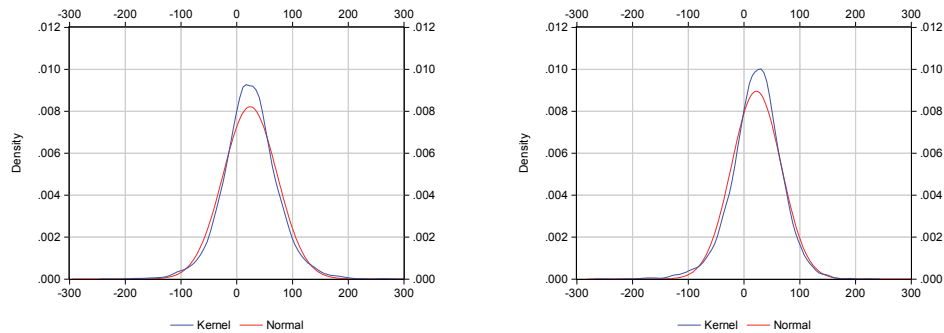
⁴⁸ As pointed out by Alexander (2008a), p. 137, GARCH models on asset returns are almost never estimated with monthly data since the volatility clustering tends to disappear if returns are measured over longer time intervals. However, comparing the results in Figure 35, Panel B with GARCH estimates that use higher frequency data (see Engle and Patton (2001), p. 244) shows that for the S&P 500 case the monthly estimates are nearly the same as weekly or daily estimates, which justifies using them for this forecast example.

⁴⁹ The distributions are fitted with Eviews.

⁵⁰ See Alexander (2008a), p. 150.

Figure 36

**Five-year total return distribution in % for simulated S&P 500 index forecasts
calculated by $\ln(\hat{S}_{t=2009}/S_{t=2004}) \times 100$ using GARCH diffusion variants**



Panel A: Return distribution for GARCH(1,1) simulated forecasts for the S&P 500 index between 2004 and 2009.

Panel B: Return distribution for GJR-GARCH(1,1) simulated forecasts for the S&P 500 index between 2004 and 2009.

Table 19

**Summary statistics for the total return distribution in % for simulated
S&P 500 index forecasts calculated by $\ln(\hat{S}_{t=2009}/S_{t=2004}) \times 100$**

Statistic	Value	Statistic	Value
Mean	24.07066	Mean	22.20647
Median	23.41905	Median	23.85340
Maximum	315.8438	Maximum	227.6383
Minimum	-228.7234	Minimum	-254.5980
Std. Dev.	48.56318	Std. Dev.	44.53292
Skewness	0.092827	Skewness	-0.375884
Kurtosis	4.548005	Kurtosis	4.413276
Jarque-Bera	1012.828	Jarque-Bera	1067.710
Probability	0.000000	Probability	0.000000

Panel A: Return distribution statistics for GARCH(1,1) simulated forecasts for the S&P 500 index between 2004 and 2009.

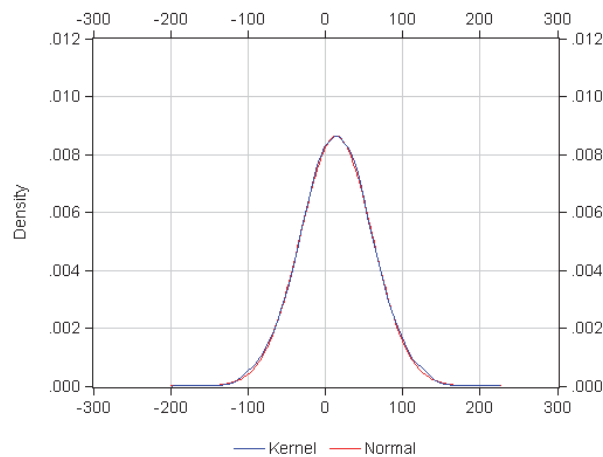
Panel B: Return distribution statistics for GJR-GARCH(1,1) simulated forecasts for the S&P 500 index between 2004 and 2009.

As illustrated, (see realized, i.e., Kernel, distribution relative to normal distribution) the GARCH(1,1) distribution generates a more normal return distribution pattern than does the asymmetric GJR-GARCH(1,1) case. This is explained by the asymmetric response element (γ) in the GJR-GARCH specification, which is often referred to as the

leverage factor.⁵¹ Both GARCH diffusion types can be used to replicate leptokurtic returns. However, GJR-GARCH also captures the negative skewness in asset-price returns, which is closer to the observed reality.

We also repeated the forecast without the GARCH diffusion term, simply using the standard error of the regression as diffusion and solving the resulting OLS regression through simulation. The total return distribution for the non-GARCH diffusion is illustrated in Figure 37 along with the simulation statistics in Table 20. The differences between the GARCH diffusion return distribution and the non-GARCH diffusion become most evident by looking at the Jarque-Bera statistics. The GARCH diffusion variants produce simulated return patterns that are most certainly non-normal, while the results without GARCH diffusion produce simulated return patterns that follow a normal distribution (Jarque-Bera statistic close to 1).

Figure 37
Five-year total return distribution in % for simulated S&P
500 index forecasts calculated by $\ln(\hat{S}_{t=2009}/S_{t=2004}) \times 100$
without GARCH diffusion



Return distribution for simulated forecasts for the S&P 500 index between 2004 and 2009. The diffusion value is defined by the standard error of the OLS regression used.

⁵¹ The asymmetry component in the GJR-GARCH(1,1) specification will be used in Section 6.2.3, p. 285, as a real-option pricing indicator.

Table 20
Summary statistics for the total return distribution
in % for simulated
S&P 500 index forecasts calculated by
 $\ln(\hat{S}_{t=2009}/S_{t=2004}) \times 100$ **without GARCH diffusion**

Statistic	Value
Mean	14.33162
Median	14.33162
Maximum	210.1393
Minimum	-181.4761
Std. Dev.	46.22499
Skewness	-7.90E-16
Kurtosis	3.000897
Jarque-Bera	0.000335
Probability	0.999832

Return distribution statistics for non-GARCH-based simulated forecasts for the S&P 500 index between 2004 and 2009. The return distribution follows very closely a normal distribution pattern.

These results have two important consequences. First, the process of searching for real-option via the return distribution of equity prices is very sensitive to the assumed data-generating-process. We will therefore rely on GARCH, in particular on GJR-GARCH, diffusion-based data-generating processes for the empirical analysis. This distinction will be of special importance when unconditional statistical moments are compared with GARCH-related conditional moments. The second important consequence is more closely related to simulating option return distributions. We will use GARCH diffusion-based Monte Carlos simulation for calculating option prices and hence return distributions in order to highlight how different option drivers like volatility, moneyness, and other factors should affect the return distribution of option investments. This analysis will provide a view of how real-option drivers should influence observed stock market return distribution.

We will adopt the same data set (although this time extending the data from 2008 to 2009, in order to capture the recent downtrend in the equity market) to simulate option prices that have a GARCH(1,1) or GJR-GARCH(1,1) diffusion term. As stressed in the previous paragraph, the focus lies not on forecasting the long-term S&P 500

development, but on showing how different option price parameters such as strike price, volatility, and time to maturity alter the payoff distribution of options, which allows inference on how real-options should affect equity price return distributions.

3.2.3.3 Simulating option payoff distributions using GARCH diffusion

In order to simulate equity prices for option valuation we will first focus on the chosen data generation process (DGP). In this case we will use a standard Ito process with different GARCH diffusion specifications. The simulated option prices and option price distribution characteristics are compared with the closed form Black-Scholes-Merton solution.⁵²

For the Monte Carlo simulation we have assumed a conventional Ito process with a fixed drift rate equal to the prevailing interest rate and anthitetic variance reduction. The applied discrete version of the Ito process (in this particular case, as geometric Brownian motion representation, to which we will refer in the text as conventional Ito process) for a unit time interval is given in Equation (19). The resulting simulation-based forecast results are shown in Figure 38.

$$\ln S_{t+1} - \ln S_t = r - \frac{1}{2}\sigma^2 + \sigma\varepsilon_t, \text{ for } t = 0, 1, 2, \dots \text{ and } \varepsilon_t | I_t \sim N(0, 1) \quad (19)$$

The analogue GARCH(1,1) diffusion version is given in Equation (20)-a.⁵³

$$\ln S_{t+1} - \ln S_t = r - \frac{1}{2}\sigma_{t+1}^2 + \sigma_{t+1}\varepsilon_{t+1}, \text{ for } t = 0, 1, 2, \dots \text{ and } \varepsilon_t | I_t \sim N(0, 1) \quad (20)\text{-a}$$

$$\sigma_{t+1}^2 = \alpha_0 + \alpha_1\sigma_t^2(\varepsilon_t - \lambda)^2 + \beta\sigma_t^2 \quad (20)\text{-b}$$

The diffusion term for the GJR-GARCH specification is given in Equation (21).

⁵² See Black and Scholes (1973) and Merton (1973a).

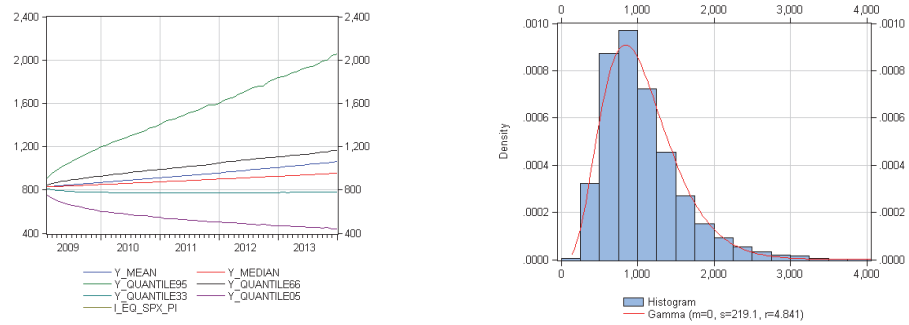
⁵³ See Duan (1995), Duan et al. (2006), and Rachev, Menn and Fabozzi (2005), pp. 345-349 for different discrete versions of GARCH diffusion processes.

$$\sigma_{t+1}^2 = \alpha_0 + \alpha_1 \sigma_t^2 (\varepsilon_t - \lambda)^2 + \gamma \sigma_t^2 \max(0, -\varepsilon_t + \lambda)^2 + \beta \sigma_t^2 \quad (21)^{54}$$

The parameter λ serves in the Duan (1995) version of a discrete GARCH(1,1) diffusion as a risk-premium constant that also enters the mean equation (Equation (20)-a). The same methodology is applied in Duan et al. (2006) for the GJR-GARCH setup. In order to keep the set of variables limited and changes in the variables traceable we will not estimate GARCH-in-mean DGP (λ is therefore zero in the simulation).⁵⁵ Note further that the Duan et al. (2006) GJR-GARCH diffusion version in Equation (21) is identical (except for the time index) to the original Glosten, Jagannathan and Runkle (1993) version in Equation (18).⁵⁶

Figure 38

Five-year (2009 to 2014) out-of-sample forecast for the S&P 500 index and the resulting forecast distribution at the end of the forecast horizon



Panel A: Forecast for S&P 500 index between 2009 and 2013 assuming a conventional Ito process with a constant drift rate $r=5\%$ and $\sigma=20.66\%$ and $S_{t=0}=825.88$.

Panel B: Simulated forecast distribution for S&P 500 index at the end of the forecast horizon (2013) assuming a geometric Brownian motion with a drift rate $r=5\%$ and $\sigma=20.66\%$ and $S_{t=0}=825.88$.

⁵⁴ See Duan et al. (2006), p. 43.

⁵⁵ See Duan (1997) for treatment of risk-premium corrections that lead to GARCH-in-mean versions of discrete stochastic processes. For a practical implementation of a GJR-GARCH(1,1) model for pricing equity options see Christoffersen (2003), pp.133-137.

⁵⁶ See p. 70.

Table 21
GARCH and GJR-GARCH simulation specifications

Panel A					
	α_0	α_1	γ	β	$\bar{\sigma}_{ann.}$
BS numerical (Ito with GARCH(1,1))	5.74E-05 p=(0.0186)	0.11634 p=(0.0000)		0.867522 p=(0.0000)	20.66%
BS numerical (Ito with GJR-GARCH(1,1))	6.48E-05 p=(0.0341)	0.076325 p=(0.0580)	0.071819 p=(0.0364)	0.864820 p=(0.0000)	18.41%
Panel B					
Strike prices in USD	Call prices in USD for different standard and GARCH diffusion specifications				
	BSM analytical	BSM numerical (Ito)	BSM numerical (Ito with GARCH(1,1))	BSM numerical (Ito with GJR-GARCH(1,1))	
$K^{-5} = K^{-4}(1 - \hat{s})$	681.65	322.20	321.82	334.18	325.74
$K^{-4} = K^{-3}(1 - \hat{s})$	708.33	306.56	306.19	318.24	308.98
$K^{-3} = K^{-2}(1 - \hat{s})$	736.05	290.89	290.52	302.26	292.12
$K^{-2} = K^{-1}(1 - \hat{s})$	764.85	275.24	274.96	286.31	275.22
$K^{-1} = K^0(1 - \hat{s})$	794.78	259.67	259.56	270.45	258.35
$ATM = S_0 = K^0$	825.88	244.23	244.31	254.86	241.54
$K^1 = K^0(1 + \hat{s})$	856.98	229.54	229.74	240.08	225.59
$K^2 = K^1(1 + \hat{s})$	889.25	215.08	215.38	225.62	209.90
$K^3 = K^2(1 + \hat{s})$	922.74	200.90	201.31	211.52	194.55
$K^4 = K^3(1 + \hat{s})$	957.49	187.05	187.61	197.83	179.62
$K^5 = K^4(1 + \hat{s})$	993.55	173.57	174.27	184.58	165.16

The BSM analytical values are calculated assuming interest rate at $r=5\%$ and volatility at $\sigma=20.66\%$ (the square root of the GARCH(1,1) diffusion process unconditional variance). No dividend cash-flows have been assumed. The time-to-maturity horizon is set to $T=5$.

The above data-generating process and the two diffusion variants are used in a Monte Carlo simulation to price option on the S&P 500. The simulation setup consists of 10000 simulations and a forecast horizon of $T = 5$ years. The starting value for the simulation is $S_{t=0} = 825.88$. This value also defines the at-the-money strike price and serves as an anchor for defining the moneyness of the option. The diffusion initiation

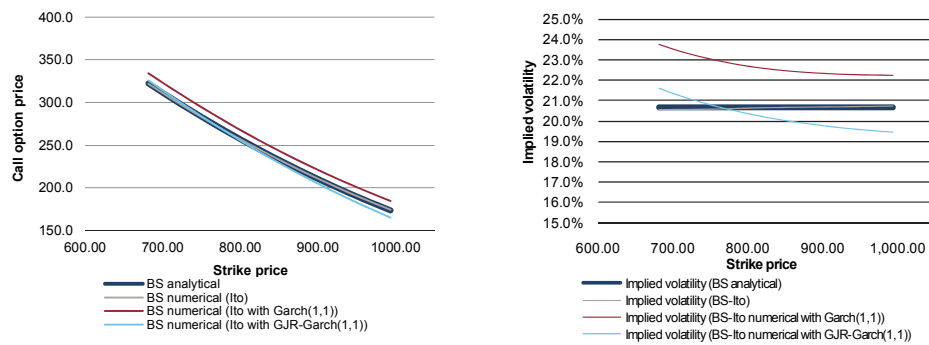
value for both GARCH variants is set equal to the annualized unconditional variance of the appropriate GARCH specification $\bar{\sigma}_{ann}^2$ ⁵⁷.

The resulting call option prices⁵⁸ for the applied diffusion processes and the different out-of-the-money strike prices are given in Table 21. The strike prices K^i are calculated as multiples of a quarter of the past underlying 12-month sample standard deviation of the index return $\hat{s}$.

The call price development (Table 21, Panel B) is illustrated graphically in Figure 39 along with the implied volatility pattern (the volatility skew) for the calculated call-price chain. Among the different specifications, the call price path for the GARCH(1,1) specification is the highest, due to the higher unconditional variance (see the unconditional variance for both diffusion processes in Table 21, Panel A).

Figure 39

Call price and volatility as a function of the strike price and chosen diffusion process



Panel A: Call price across different strike levels and diffusion term assumptions. All prices are in USD terms.

Panel B: Implied volatility across different strike levels and diffusion term assumptions. All prices are in USD terms.

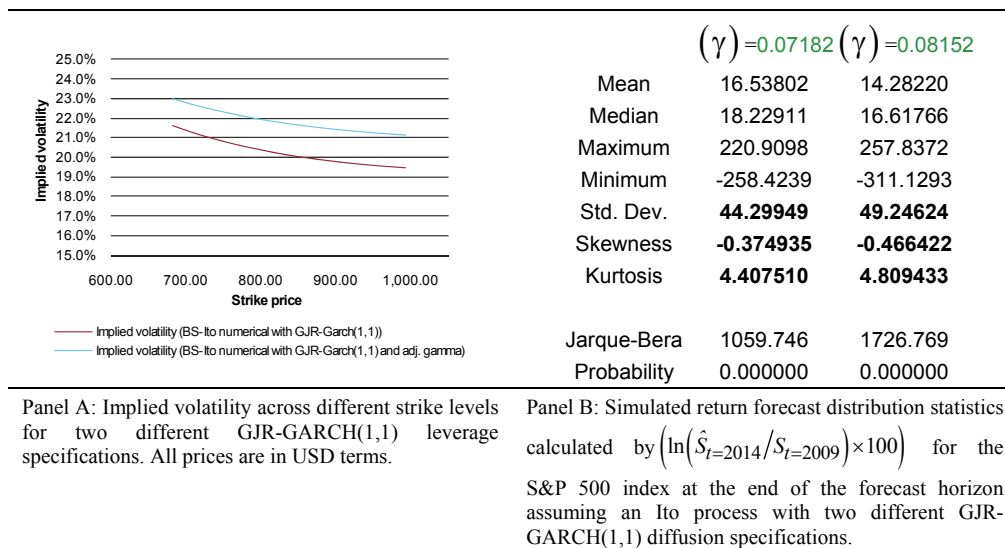
⁵⁷ See Equation (23), p. 82.

⁵⁸ We have calculated only call prices in the simulation. Using the call-put-parity, one could easily extract the appropriate put values. The main purpose of the analysis, however, lies in the implied volatility (which is identical for puts and calls with identical strike price) and the return distribution for call options.

As visible in Figure 39, Panel B, the GARCH-related diffusion processes create volatility skews and hence deviate from the standard Black-Scholes-Merton assumption of constant volatility. The GJR-GARCH(1,1) version, as mention above, has a lower volatility structure than the GARCH(1,1) type estimation, while the conventional Brownian motion simulation (without GARCH) produces implied volatilities across strike prices equal to the input volatility. In order to have the same unconditional (i.e., long-term) variance for all the simulated DGPs we have changed the asymmetry (leverage) factor γ in the GJR-GARCH(1,1) model. The new implied volatility skew is illustrated in Figure 40 along with the distribution moments.

Figure 40

Change in the GJR-GARCH(1,1) simulation-based option price implied volatility due to changes in the leverage factor



As shown in Figure 40 above, the increase in leverage shifts the implied volatility upwards closer to the implied volatility level of the simpler GARCH(1,1) specification. Panel B of Figure 40 shows that the higher leverage factor γ decreases the expected mean return. More interesting, however, is the increase in kurtosis and decrease in skewness of the return distribution which from a return/risk perspective is undesirable given that we observe an increase in even moment (i.e. kurtosis) and decrease in uneven moment (i.e. skewness).

The calibrated GARCH diffusion processes allow calculating and investigating the option price distribution (instead of the stock price distribution or option price value). We will use the same GARCH specifications applied for the option price calculation to analyze how the option prices are distributed and how this distribution changes with different option price determinants. This analysis – as already mentioned in the introduction – will give us a guideline as to how equity prices should trade with respect to higher moments if their value is primarily driven by optionality (out of the money option) and not by cash-flow considerations (in the money options). The analyzed factors that should illustrate this relationship are:

- moneyness,
- leverage factor, and
- volatility

We will test via simulation how these factors affect the distribution of the option payoff. We will use for this analysis the same dataset as for the simulations earlier in this section. The GARCH diffusion chosen for the sensitivity analysis is the previously applied GJR-GARCH(1,1), since it is the type that will most closely replicate the usually observed return distribution characteristics in equity markets (positive excess kurtosis and negative skewness).

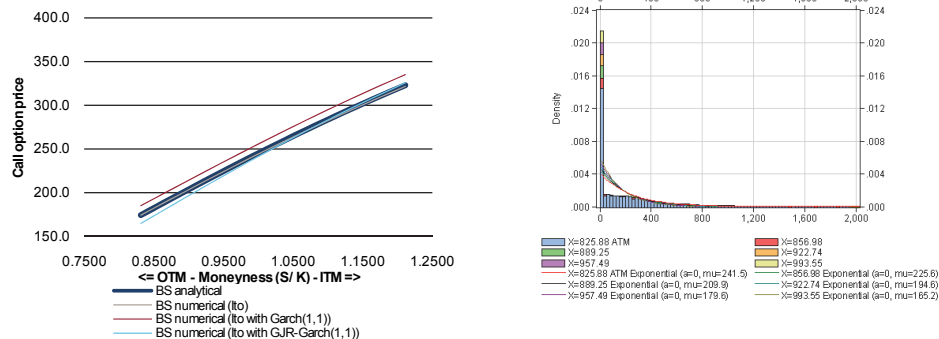
The influence of moneyness on the option price return (payoff):

Moneyness in this context is defined as the ratio between the spot price (S_0) and the option strike price (K^i).⁵⁹ It is hence a measure for differentiating between in-the-money (high moneyness) and out-of-the-money (low moneyness) options. A strong in-the-money option will have a high intrinsic (cash-flows with high certainty) value and hence a different present value probability distribution than an option that is out-of-the-money, where the value is mostly defined by uncertainty and hence optionality. The relation of moneyness to option price is illustrated in Figure 41, Panel A.

⁵⁹ For different definitions of moneyness see Alexander (2008a), pp. 154-155.

Figure 41

Call price and option return distribution as function of moneyness



Panel A: Call price as function of moneyness and different diffusion term assumptions. All prices are in USD terms. The option price increases with moneyness (more in-the-money option).

Panel B: Histogram of the return on option investments in % for different strike prices and hence different assumptions for moneyness, in addition to a fitted exponential distribution function.

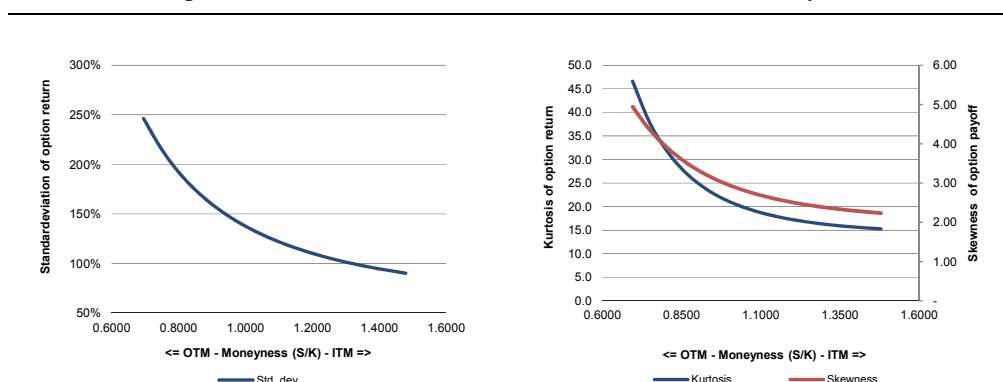
The X-axis in Panel A is labeled with moneyness instead of strike price. As illustrated the call option price appears linear with respect to moneyness. The different diffusion variants are distinguished by the slopes of their curves. The differences, however, are relatively small. Panel B of Figure 41 shows the different simulated payoff distributions for different out-of-the-money call options. With increasing strike price (decreasing moneyness) the payoff becomes more leptokurtic, i.e., the likelihood of zero payment increases. However, the skewness increases too.

A better illustration of how higher-order moments are affected by moneyness is given in Figure 42. Panel A shows the influence of moneyness on the standard deviation of the option payoff. Stronger out-of-the-money options (lower moneyness) show a higher second moment. This pattern is in line with theory, given that high out-of-the-money options represent high beta stock substitutes.⁶⁰ A low moneyness (low likelihood of exercising the option) also leads to a stronger leptokurtic payoff distribution (higher kurtosis and hence higher likelihood of zero excess returns on the option strategy), as shown by looking at Panel B of Figure 42.

⁶⁰ See also Figure 101, p. 229 in Section 5.4 where we will revisit this topic again.

Figure 42

Option return distribution moments as function of moneyness



Panel A: Standard deviation of option return as function of moneyness. Lower moneyness increases the standard deviation of the option investment.

Panel B: Kurtosis and skewness of option returns as function of moneyness. Out-of-the-money options (high optionality value) have higher kurtosis and positive skewness distribution moments.

The kurtosis of the option payoff increases in a non-linear fashion. Also important is the development of the third moment, the skewness. The skewness of a standard (plain vanilla) option is positive. This positive skewness increases with lower moneyness (higher for out-of-the-money call options).

The conclusions that can be drawn for the option price distribution with regard to changes in moneyness are that a decrease in moneyness (higher out-of-the-money options) is usually accompanied by:

- A. higher kurtosis
- B. higher positive skewness
- C. higher standard deviation of the option payoff distribution
- D. higher deviation from the normal distribution

Consequently, should companies contain large, unrealized, out-of-the-money options, we should expect that their stock-price return distribution – if the company value is primarily driven by these options – corresponds as described in points A-D.

The influence of the leverage factor and unconditional volatility:

The leverage factor that is characterized in the GJR-GARCH(1,1) setup (see Equation (18)) as gamma (γ) is another variable that can be estimated for financial time series.

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \gamma 1_{\{u_{t-1} < 0\}} u_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (22)$$

Its influence on the option price distribution, however, is not straightforward, since the leverage factor also changes the long-term unconditional variance and hence the diffusion term in the Monte Carlo simulation. As shown in Equation (23),⁶¹ the leverage factor enters the denominator of the unconditional variance equation. An increase would therefore also raise the long-term unconditional variance.

$$\bar{\sigma}_t^2 = \frac{\alpha_0}{(1 - (\alpha_1 + \beta + 0.5\gamma))} \quad (23)$$

The unconditional variance is important in the general context of GARCH estimation. This become apparent if we acknowledge that a GJR-GARCH(1,1)-based volatility forecast (the forecast for the next period $t + 1$) can be decomposed into:

- a current symmetric innovation component (weighted by α_1),
- an asymmetric innovation component (weighted by $\gamma/2$),
- the previous forecast (weighted by β), and
- the unconditional variance as long-term component (weighted by $1 - \alpha_1 - \beta - \gamma/2$).

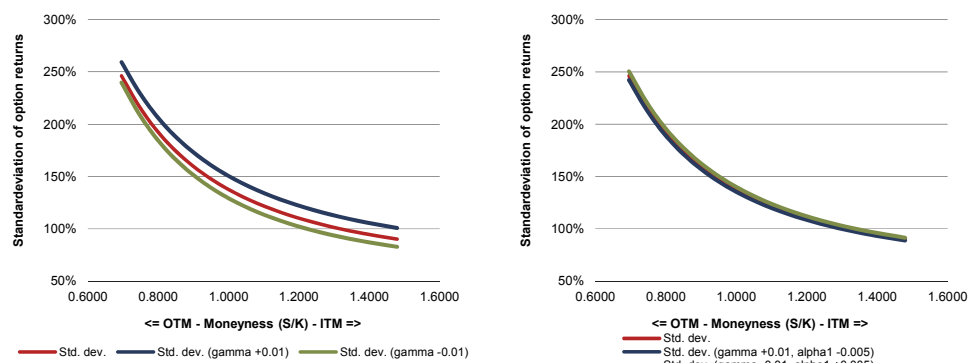
⁶¹ See Alexander (2008b) pp. 147-163 for the calculation of unconditional variances of different non-symmetrical GARCH processes.

For equity the weighting for the previous forecast (β) will usually be the highest value followed by the symmetric and asymmetric components.⁶²

In a first step to test for the influence of the leverage factor, we will therefore change the gamma variable and analyze the changes in the option price distribution moments. Afterwards, in order to isolate the impact of gamma on the option price distribution, we will modify the symmetry weighting α_1 to keep the overall unconditional variance weight $(1 - \alpha_1 - \beta - \gamma/2)$ unchanged.⁶³ The purpose of this exercise is to see if high optionality can be associated with high or low innovation asymmetry.

Figure 43

Change in option return standard deviation relative to moneyness with changed leverage factors



Panel A: The standard deviation of simulated option returns relative to moneyness and a changing GJR-GARCH(1,1) leverage factor.

Panel B: The standard deviation of simulated option returns relative to moneyness and a changing GJR-GARCH(1,1) leverage factor with constant unconditional variance.

⁶² Note that in Table 21, p. 76 the long-term weighting for GARCH(1,1) and GJR-GARCH(1,1) are very similar, while the symmetric component (α_1) for GARCH(1,1) is much higher than in the GJR-GARCH(1,1) calibration. This, however, should not be surprising, since the asymmetry factor (γ) now captures a very large component of the previous volatility.

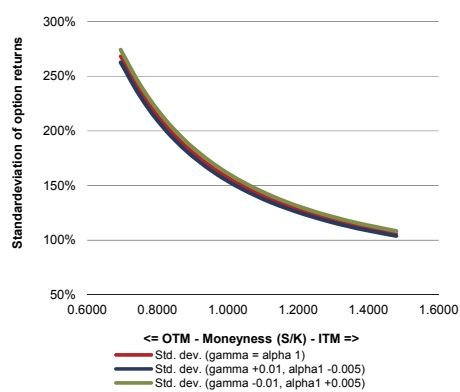
⁶³ A one-unit increase in (γ) is compensated for by a 0.5-unit decrease in (α_1).

Figure 43, Panel A shows the influence of the leverage factor on the option payoff standard deviation as a function of moneyness. Panel B illustrates the same relation with and without adjustments to maintain constant unconditional variance.

A higher leverage factor – keeping everything else unchanged – increases the standard deviation of the option payoff. The development seems unrelated to moneyness (i.e., a parallel shift in the curve). An adjustment to the symmetric innovation term leads to a nearly unchanged, although slightly lower, standard deviation with the parameterization used. Note that given the GJR-GARCH(1,1) calibration in Table 21, the weighting applied to the symmetric innovation is higher than for the asymmetric case, which is one possible reason for the overall stronger influence of the symmetric component. In a further step we have repeated the experiment by setting the symmetric and asymmetric volatility innovation term equal to each other ($\alpha_1 = \gamma = 0.07907$). The results of the sensitivity analysis ($\alpha_1 + 0.005 = \gamma - 0.01$) and ($\alpha_1 - 0.005 = \gamma + 0.01$) for this setup are shown in Figure 44.

Figure 44

Change in option return standard deviation relative to moneyness with changed leverage factors



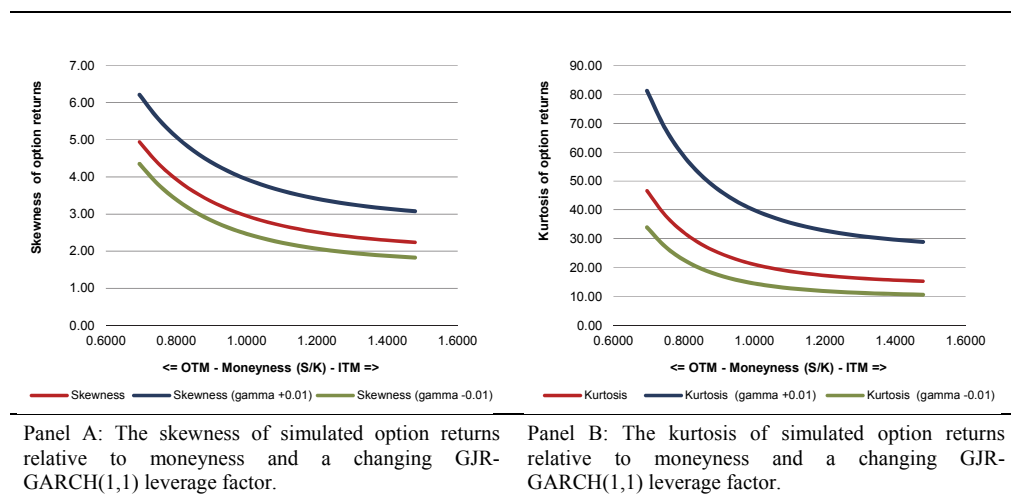
The standard deviation of simulated option returns relative to moneyness and a changing GJR-GARCH(1,1) leverage factor with equal initiation values for the symmetric and asymmetric sensitivity component.

The sensitivity dynamic towards a higher leverage factor is identical for this parameterization. A higher leverage factor with unchanged unconditional variance (i.e., higher symmetry innovation term) lowers slightly the option payoff standard deviation (and vice versa).

The influence of the leverage factor on higher distribution moments is shown in Figure 45 and Figure 46. As illustrated an increased asymmetric factor γ leads ceteris paribus not only to higher unconditional variance but also to higher skewness and higher kurtosis of the option price distribution. Controlling for the unconditional variance (keeping it stable by amending the symmetry response) unveils a similar dynamic as for the standard deviation sensitivity analysis. A higher leverage factor with unchanged unconditional variance leads to lower skewness and lower kurtosis.

Figure 45

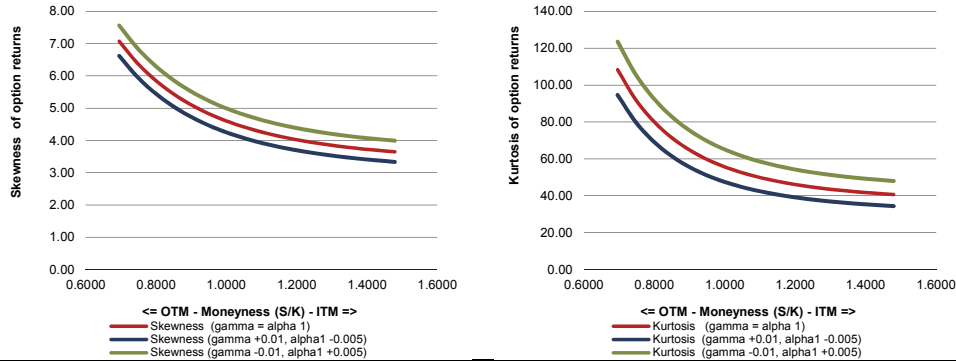
Change in option-payoff skewness and kurtosis relative to the strike price with changed leverage factor



A further possible interpretation of the simulation results is that a higher symmetry response factor (α_1) leads to higher kurtosis and higher positive skewness if the unconditional variance remains unchanged (i.e., by lowering the asymmetry response).

Figure 46

Change in option-payoff skewness and kurtosis relative to the strike price with changed leverage factor and adjusted symmetric response



Overall, the conclusions that can be drawn for the option return distribution with respect to changes in the unconditional variance, changes in the leverage factor, and changes in the symmetry response component, are:

A higher unconditional variance due to higher α_1 or γ (ceteris paribus) leads to:

- A. higher kurtosis
- B. higher skewness
- C. a higher standard deviation

of the option return distribution.

These observations are confirmed even if the increase in the symmetry response α_1 is offset by a decrease in the asymmetry component γ (i.e., the unconditional variance is kept unchanged). However, if the increase in asymmetry component is offset by a decrease in the symmetry response, the sensitivity response is the opposite.

By fitting the GJR-GARCH(1,1) model to Russian equity prices⁶⁴ one would expect that on average, if real-option value is considered in the price discovery, and return distribution due to real-options is affected as in Figure 33 (see p. 66), the GJR-GARCH calibration should show lower asymmetry response values than for companies without real-options. This is because lower asymmetry values lead to a lower standard deviation of the option payoff return. However, the conclusions with respect to skewness and kurtosis are not straightforward. A lower asymmetry value would decrease (in line with the NPV simulation experiment in Subsection 3.2.2, p. 48) the kurtosis of the option return, as illustrated in Figure 45. The skewness, however, should also fall, which is contrary to the NPV simulation experiment. This highlights that the overall influence of the combined symmetric and asymmetric values needs to be taken in consideration.

Overall we can expect that the unconditional volatility – calculated via the estimated GJR-Garch(1,1) parameters – should be higher for Russian equity market companies than for global peers, given the higher expected levels of out-of-the-money optionality in the commodity-driven Russian stock market.

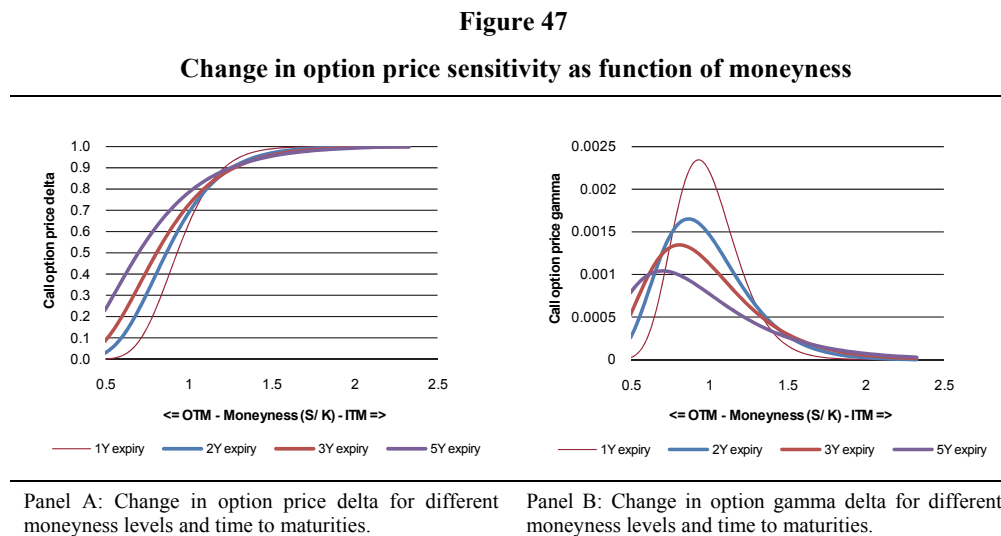
3.2.4 *Option price sensitivity (option beta) as function of moneyness*

Another important element of observed pricing behavior that can be related to optionality is the sensitivity of asset price returns to changes in the underlying driving factor.⁶⁵ The equity price sensitivity of a commodity-driven stock company towards changes in the underlying commodity prices will also be a function of the degree of real-option exposure. Hence, a company that processes strong out-of-the-money real-options should also show a different degree of equity price sensitivity relative to underlying commodity changes. The option price / underlying sensitivity – usually referred to as option delta or first option price derivative as a measure of how one unit change in the underlying changes the option prices – is shown in Figure 46, Panel A.

⁶⁴ The empirical analysis with respect to calibrating GJR-GARCH(1,1) diffusion processes to Russian commodity equity companies is given in Subsection 6.2.3, pp. 285-307.

⁶⁵ The analysis can similarly be related to the concept of turnover beta or cyclicalities for a conventional investment project. See Fischer (1996), pp. 122-129.

Panel B shows the second derivative - usually referred to as option gamma - of the option price relative to changes in the underlying.⁶⁶ It measures the change in delta for each unit change in the underlying.



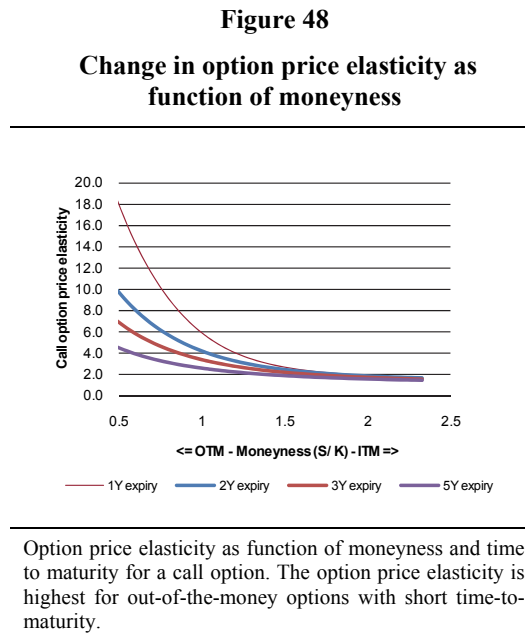
Hence, both elements are important and often used in option theory to relate the change in option price to changes in the underlying asset price. As illustrated in Figure 47, Panel A, an option that is trading in the money will trade nearly identical to the underlying (i.e., sensitivity nearly equal to one). This tendency is amplified the closer the option is to expiry. The sensitivity of delta is illustrated in Panel B of Figure 47. The highest sensitivity of delta relative to moneyness is observed around the strike price. Out-of-the-money options will – similarly to in-the-money options – show lower delta sensitivity towards changes in the underlying assets.

Note that both elements (delta and gamma) will not give us direct insights into the % sensitivity of the option price. What is of particular interest is how the option price changes in percentage terms if the underlying changes one percent. This option price elasticity ψ (Ψ) is defined as:

⁶⁶ For the calculation of the option price delta and gamma within the conventional Black-Scholes-Merton framework see Hull (2000), pp. 311-324.

$$\Psi = \frac{\partial c / \partial S}{\partial S / S} = \frac{\partial C}{\partial S} \frac{S_t}{c_t} = N(d_1) \frac{S_t}{c_t} \quad (24)$$

Figure 48 shows the psi as a function of the moneyness and different option expiry horizons.



The relative price change in percentage terms is highest for highly out-of-the-money options that have a short time to maturity. The price of out-of-the-money options will therefore have higher beta sensitivities than in-the-money options. This sensitivity will even increase if the maturity of the option decreases.

The importance of this observation becomes evident if one replaces the two terms *option price* and *equity* with the term *equity* and *commodity price*-driven asset-value (project NPV). If commodity companies possess real-option value, as reflected in the traded equity price (the same equity price that can be understood as an option on the company's assets), it should be possible to show that these equity prices have a higher beta sensitivity toward changes in the project's NPV relevant underlying commodity factor (the proxy for the change in the project NPV) than comparable companies with no or low or in-the-money optionality.

A further characteristic of out-of-the-money options in this context is the asymmetry in beta sensitivity. Since the gamma function is relatively stable for in-the-money options and relatively unstable (dynamic) for at-the-money and out-of-the-money options, it can be assumed that companies with highly out-of-the-money options have a strongly diverging up- vs. down-market beta, whereas companies with in-the-money real-options should have stable (i.e., nearly identical) up- and down-market betas.

The conclusions that can be drawn from this analysis are that *ceteris paribus* companies with out-of-the-money real-option value that are founded on commodity price developments should have higher equity prices – commodity price beta – than companies without out-of-the-money value or with in-the-money real-option value. The asymmetry in the up- and down-beta factors should be higher for companies with out-of-the-money real-option value. Both hypotheses will be tested in the empirical part of this dissertation in Subsection 6.2.4, p. 307.

3.3 The distribution of equity prices: a theoretical assessment of what to expect in case of real-option value

3.3.1 Introduction

By using a real investment example, we showed in the previous section the influences on the overall project NPV return distribution if real-options are considered. By using conventional financial option price simulation it was further possible to show how the distribution of option returns - analyzing them on a stand-alone basis without a real investment project attached - is influenced by moneyness or GJR-GARCH(1,1) parameters like the leverage factor. However, thus far our reasoning has been based on simulation experiments. To justify the empirical analysis in Chapter 6, further non-experimental theoretical support is needed. The following section will further enhance the theoretical framework and facilitate hypothesis formulation by focusing on observable market price information directly (in the form of traded equity prices) and linking them with contingent claim analysis. This will allow also inference on the overall company value and hence on the asset side of the balance sheet.

3.3.2 Value of equity and the distribution of risk

The price for equity is one of the most often discussed topics in the financial literature. The concepts of dividend cash-flow (constant growth or Gordon growth model)⁶⁷ or free cash-flow analysis (representing the traditional discounted cash-flow valuation) and earnings multiple analysis (representing relative valuation) are the most often applied valuation approaches found in the literature.⁶⁸

However, there is a third valuation methodology that can be utilized for valuation purposes, namely contingent claim analysis (CCA), which was introduced by Black and Scholes (1973) and Merton (1974). The Merton version of valuing equity and debt via contingent claims has found particularly widespread acceptance in the financial

⁶⁷ See Elton et al. (2007), pp. 457-469.

⁶⁸ For a good overview see Damodaran (2002), pp. 11-25.

industry.⁶⁹ To derive the core idea of the Merton approach we start again with a simplified balance sheet (see Table 22).

Table 22
View on the Balance sheet at time $t=0$ at market prices

Total assets at time t	Total liabilities and equity at time t
Value of the company at time t : $V_t = 725$	Present value of total debt at time t : $D_t = 225^{70}$
	Present value of equity at time t : $E_t = 500$

The total value of the company V_t at market prices is given by:

$$V_t = D_t + E_t \quad (25)$$

The total value of a leveraged company can be expressed as the sum of an unleveraged company U_t plus expected future tax savings.⁷¹

⁶⁹ For a good introduction to Merton contingent claim equity and debt valuation, see Cox and Rubinstein (1985), pp. 375-408, Garbade (2001) and Hull, Nelken, and White (2004).

⁷⁰ Note: short-term liabilities (current liabilities) are usually booked in terms of their face value. Long-term liabilities, however, are booked in terms of their refinancing value, i.e., their risk-adjusted discounted face value (see White, Sondhi, and Fried (1998), pp. 475-480). We will treat short- and long-term liabilities, however, as one singular identity (total debt) at market values.

⁷¹ See Fischer (1996a), p. 116.

$$V_t = D_t + E_t = U_t + \sum_{v=t+1}^T \frac{s \cdot E(Z'_v)}{\prod_{\tau=t+1}^v (1 + k_{D,\tau})} \quad (26)$$

The future expected tax savings come from deductible interest rate expenses and are hence a function of the prevailing tax rate. The company value defined in Equation (25) or (26) can be seen as a variant of the traditional valuation approach. This traditional asset valuation approach is static in nature and does not allow for immediate contingent claim analysis other than by Monte Carlo simulation.

The Merton model is applicable to the same balance sheet example. However, our focus will lie first on analyzing the company value at time $t = T$. The time horizon T is assumed to be equal to the maturity (redemption) of all available debt of the company.⁷²

Table 23
Contingency analysis and balance sheet composition

Time	$t = 0$	$t = T$		Representation as call option
Contingency		$V_T \leq D_T$	$V_T > D_T$	
Debt	D_t	V_T	D_T	$D_T = V_T - C_T^E(V_T, D_T)$
Equity	$E_t = nS_t$	—	$E_T = V_T - D_T$	$E_T = C_T^E(V_T, D_T)$

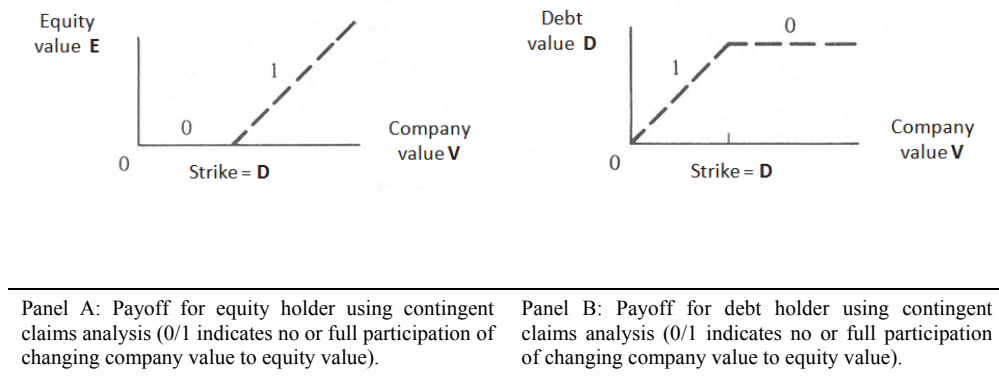
Equity value at market price E_t expressed as share price S_t times number of shares outstanding n . C_T^E is the value of equity expressed as call option on the underlying company asset value S_T with a strike price at D_T .

Table 23 shows the contingencies involved. The equity value of the company E_T is equal to zero, if at time $t = T$ the value of the company falls below the face value of total debt $V_T \leq D_T$. The value of equity will increase linearly if the total company value exceeds the required debt servicing amount $V_T > D_T$. The equity valuation hence becomes contingent on the total asset value, and is therefore equivalent to a call option on the total company asset value with the debt servicing amount as strike price (see

⁷² In practice it is equal to the nominal weighted duration average of the outstanding debt.

Figure 49). The debt holder's position is equivalent to holding the company assets and writing a call option on these assets (i.e., using a covered call strategy).

Figure 49
Payoff profile for equity and debt holder within CCA framework



This analysis is similarly valid for $t < T$. Applying the put-call parity one can write:

$$V_t + P_t^E(V_t, D_T) = D_t e^{-r_t^{rf} T} + C_t^E(V_t, D_T) \quad (27)$$

$$V_t = \left(D_t e^{-r_t^{rf} T} - P_t^E(V_t, D_T) \right) + C_t^E(V_t, D_T) \quad (28)$$

The value of the company according to Equation (28) is equivalent to the value of debt D_t (default-free present value of total debt maturing at time $t = T$) minus the put option value P_t^E on the company assets V_t plus the call value on the company assets C_t^E . This call option on the company asset value can be priced via the traditional Black-Scholes-Merton formula.

$$E_t = nS_t = C_t^E(V_t, D_T) = V_t N(d_1) - D_t e^{-r_t^{rf} T} N(d_2) \quad (29)$$

where:

E_t : market value of equity

nS_t : product of number of shares outstanding and current share price

V_t : market value of the firm's assets

r_t^{rf} : risk-free rate

T : time to maturity of debt

D_T : face value of debt (principal value to be redeemed, i.e., redemption value)

$N(\cdot)$: cumulative normal probability of the unit normal variate d_1

$$d_1: \quad d_1 = \frac{\ln(V_t/D_T) + r_f T}{\sigma_V \sqrt{T}} + \frac{1}{2} \sigma_V \sqrt{T}$$

$$d_2: \quad d_2 = d_1 - \sigma_V \sqrt{T}$$

σ_V : volatility of the firm's assets

Of particular interest is the relation between the market value of the firm's assets V_t and the equity value E_t , since the real-option value of a company should manifest itself on the asset side of the balance sheet. For example, see Figure 50 for the 2008 balance sheet of Gazprom OAO.

Figure 50

Balance sheet composition (assets side) of Gazprom OAO

	31 December	
	2008	2007
Assets		
Current Assets	343,833	279,109
Cash and cash equivalents	3,747	12,025
Restricted cash	23,448	113,911
Short-term financial assets	675,934	697,464
Accounts receivable and prepayments	276,954	245,406
Inventories	115,878	122,558
VAT recoverable	132,281	95,944
Other current assets	1,572,075	1,566,417
Non-current assets		
Property, plant and equipment	4,020,522	3,490,477
Investments in associated undertakings and jointly controlled entities	772,143	670,403
Long-term accounts receivable and prepayments	343,805	402,382
Available-for-sale long-term financial assets	48,186	256,210
Other non-current assets	411,837	406,667
	5,596,493	5,226,139
Total assets	7,168,568	6,792,556

AO Gazprom IFRS consolidated balance sheet as of December 31, 2008 (in millions of Russian rubles).
See OAO Gazprom IFRS consolidated financial statements, December 31, 2008.

If a company possesses substantial real-options, they will likely be a part of non-current assets like property, plants, and equipment. These long-lived operating assets are often tangible in nature (plants, oil fields, machinery, etc.) and booked at their acquisition value. Gains due to asset revaluation or real-option related adjustments are less frequently reported. Accounting principles such as the minimum value principle also lead to a more conservative value assessment than justified by real-option theory. Assets that possess real-option value can even be dropped from the balance sheet, and hence remain unobserved and contribute to underestimation of the overall company value V_t .⁷³ One way to infer how and if real-options are priced in the market is to focus on the equity price behavior via the balance sheet identity.

$$V_t = D_t + E_t \Rightarrow E_t = V_t - D_t \quad (30)$$

Under the assumption that the value of debt D_t is static (i.e., it is equal to a scalar with constant mean and no variance) or only slowly changing, we can apply standard transformation rules between two one-dimensional random variables.⁷⁴ The appropriate transformation function in this case is given by:

$$Y = a + bX \quad (31)$$

⁷³ For example, in their chapter on long-lived asset valuation, White et al. (1998) give the example of the Atlantic Richfield Company (ARCO), which in 1985 removed 8.3 trillion cubic feet of natural gas reserves from its estimate of proven reserves (more than a 50% reduction) due to the strong drop in gas prices and the location of the gas field in northern Alaska; this change made production economically unattractive. See White, Sondhi, and Fried (1998), p. 371. Such a drop in assets was immediately visible in the balance sheet. However, this real-option asset, which would gain in value should gas prices increase again in the future, was likely considered in the future price discovery of the company's equity value.

⁷⁴ See Rinne (2003), pp. 217-218.

In this case the debt value D_t is equal to a and $b = 1$ while $X = E_t$. The transformation for the probability density function (pdf), cumulative density function (CDF), mean, variance, and higher order moments, is given in Table 24.

As can be seen in Table 24 the key distribution properties with respect to higher moments remain unaffected by the linear transformation. Hence, by analyzing the distribution characteristics of E_t we can draw conclusions about the distribution of the unobserved - or, to be more precise, non-traded - part of the balance sheet V_t . Given that we cannot observe V_t and hence possible real-options directly, the theoretical link to observed market prices is vital in our analysis, because it allows us to use this property in the empirical part for inference.

Table 24
Transformation of unidimensional random variables

Transformation	Density	Dist. function	Exp. value	Variance	3 rd moment	4 th moment
$Y = g(X)$	$f_Y(y) =$	$F_Y(y) =$	μ_Y	σ_Y^2	$g_1(Y)$	$g_2(Y)$
$Y = a + bX$ $a, x \in \mathbb{R}, b \neq 0$	$\frac{1}{ b } f_X\left(\frac{y-a}{b}\right)$	$F_X(y-a), \forall b=1$	$= a + b\mu_X$	$= b^2 \sigma_X^2$	$= g_1(Y)$	$= g_2(Y)$

See Rinne (2003), pp. 217-218.

3.3.2.1 Expected distribution characteristics of company assets V_t in case of real-option value

Defining the value of equity as an option on the company assets value enables – as with standard options – the building of first derivatives of the option value $C_t^E(V_t, D_T)$ with respect to other independent variables, and analysis of whether and how this sensitivity is displayed/observed in real markets. The different interdependencies for the option value $C_t^E(V_t, D_T)$ are illustrated best via the Ito differential equation below. A change in the underlying, along with a change in time and higher-order moments, is sufficient enough to fully explain the changes in equity value.

$$dE_t = dC_t^E(V_t, D_T) = \frac{\partial C_t^E}{\partial V_t^E} dV_t + \frac{\partial C_t^E}{\partial t} dt + \frac{1}{2} \frac{\partial^2 C_t^E}{\partial V_t^{E^2}} \sigma_v^2 V_t^2 dt \quad (32)$$

Examining the individual partial derivatives unveils the guidelines along which we can search for real-option pricing in observed (i.e., traded) markets.

Delta:

$$\Delta = \frac{\partial E_t}{\partial V_t} = \frac{\partial C_t^E}{\partial V_t} = N(d_1), \text{ where } 0 \leq N(d_1) \leq 1 \quad (33)$$

Positive changes in the company's asset value (assumed to be driven by changes in the underlying commodity driving factor, such as oil or gas) will have – ceteris paribus – a positive impact on the equity price.

Hypothesis 1:

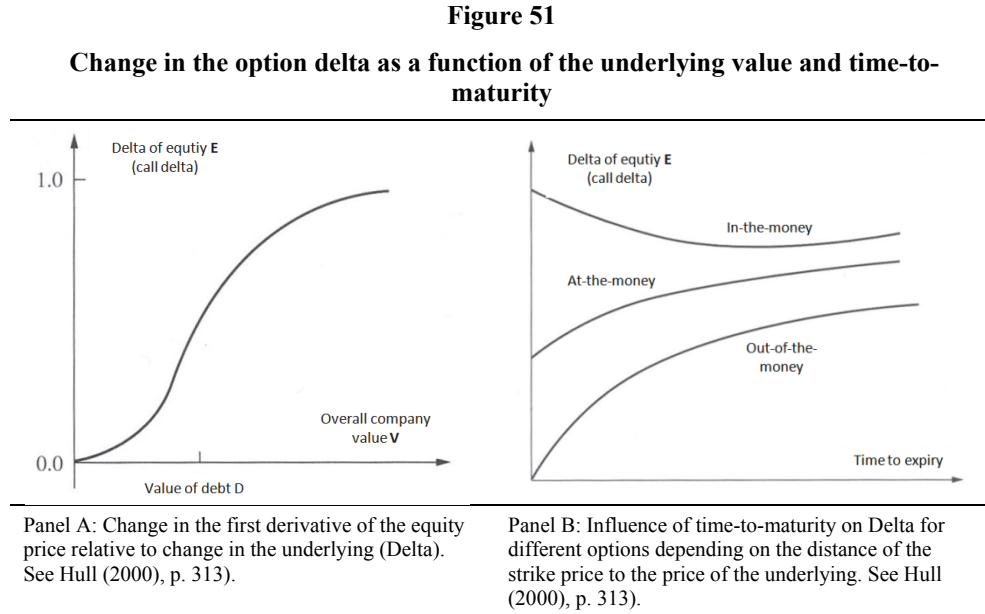
The equity value of a commodity company that contains/possesses real-options will be more sensitive to changes in the business-generating commodity price than for a company with low real-options.

An empirically testable consequence of Hypothesis 1 is that companies with real-options sensitive to commodity price changes – ceteris paribus – should show higher equity price to commodity correlation coefficients (higher beta sensitivity towards commodity price changes, called commodity beta) than low real-option companies, due to higher option elasticity.⁷⁵

Important in this context is the functional dependency of the delta function on the strike price via the cumulative normal probability of the unit normal variable d_1 . The strike price – in our context, the redemption value of the company debt – will change the

⁷⁵ The definition of option price elasticity was introduced in Equation (24), p. 89.

sensitivity of the equity value relative to changes in the company's asset value (as shown in Figure 51).⁷⁶



Companies with strongly in-the-money asset-value to debt structures $V_t \gg D_T$ and hence low leverage will have a stronger equity price delta sensitivity (closer to one) than highly leveraged companies $V_t \ll D_T$. This also applies to the relation of equity sensitivity to the major asset value driver (i.e., oil, gas, etc.), which will govern the overall asset value (especially with respect to reserves) of the company.

A second characteristic of options is closely related to the previously described delta value. It refers to the second derivative (gamma) of the change in price of the option relative to a change in the company's asset value.

Gamma:

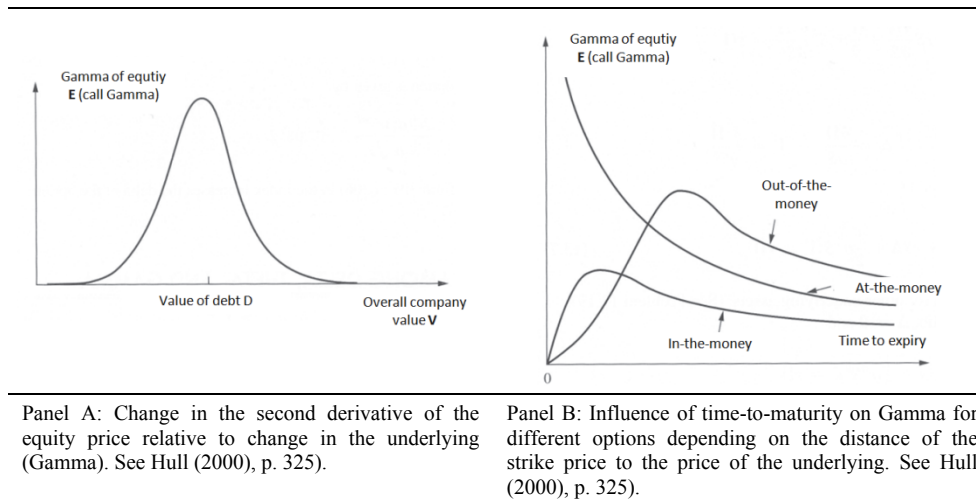
$$\Gamma = \frac{\partial^2 C_t^E}{\partial V_t^2} = \frac{N'(d_1)}{V_t \sigma_V \sqrt{T}}, \text{ with } N'(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \quad (34)$$

⁷⁶ The figures are modified illustrations based on Hull (2000), p. 313.

The second option price derivative will indicate how fast the first derivative (delta) changes with respect to changes in the underlying company value or major contributing asset valuation driver (commodity price in our analysis).

Figure 52

Change in the option gamma as a function of the underlying value and time-to-maturity



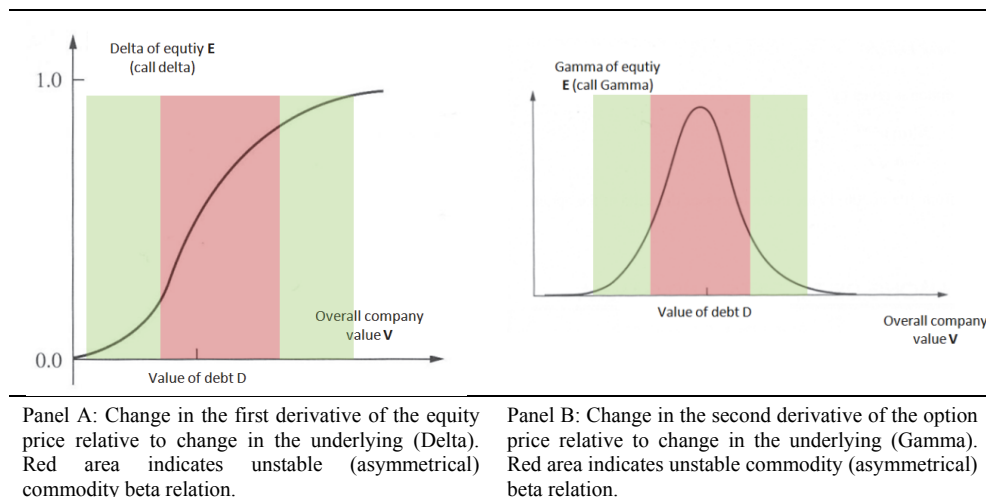
As shown in Figure 52⁷⁷, Panel A, the gamma factor and hence the change in the sensitivity of equity to changes in the underlying market variable (as driver for the overall company value) will be highest around $V_t \approx D_T$. This is also visible in Panel B, where the gamma factor is displayed as a function of time.

The beta factors measured as change in equity to change in underlying company value driving factor (commodity beta) will therefore strongly depend on how far out-of-the-money the real-option projects are. The up- and down-commodity beta factors will therefore be highly asymmetrical, depending on in which part of the gamma curve in Figure 52 the company structure is located. Figure 53 illustrates this relation by indicating expected regions of stable and unstable commodity betas.

⁷⁷The figures are modified illustrations based on Hull (2000), p. 325.

Figure 53

Symmetry of commodity beta as a function of the capital structure



With respect to up- vs. down-commodity beta factor analysis, the following hypothesis can be formulated:

Hypothesis 2:

Companies with identical capital structure but different exposure to real-option valuation should differ in their commodity beta factor sensitivity. Companies with high real-option value exposure should have significantly higher commodity beta asymmetry (stronger difference in up- vs. down-beta) than companies with low real-option exposure.

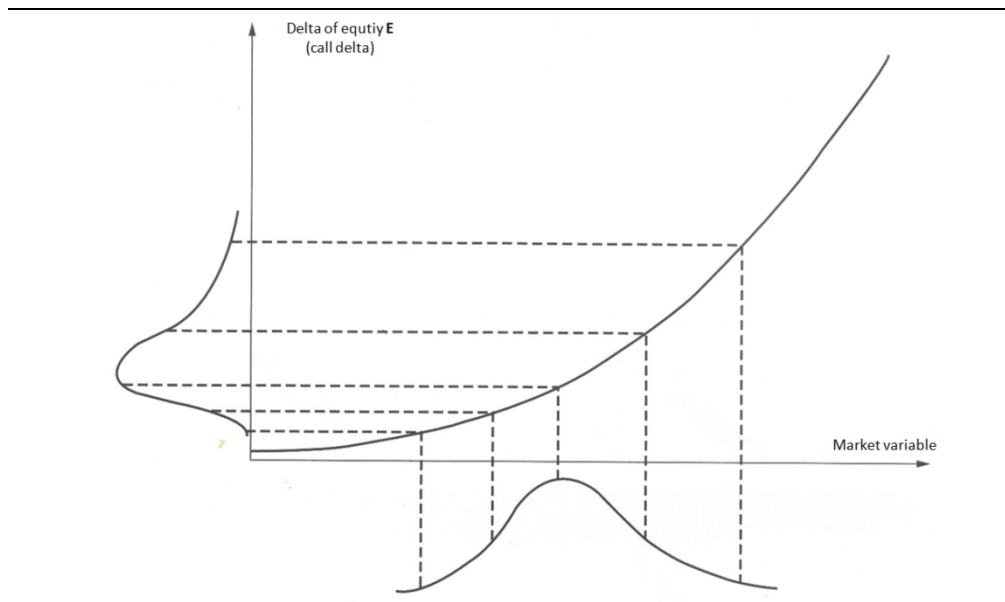
This gamma-related beta asymmetry has, however, consequences on the overall distribution of the total asset value V_t .⁷⁸ Higher optionality leads to higher gamma exposure. Changes in the asset value variable will disproportionately affect the price change in the option and hence the equity value E_t of the company. This leads to

⁷⁸ See Hull (2000), pp. 352-355, for a discussion how gamma exposure influences the probability distribution of asset portfolios.

positive skewness of the firm's assets V_t and consequently also to a more positively skewed return distribution.⁷⁹

Figure 54 shows this relation graphically. Assuming that changes in the company's asset value V_t are primarily driven by changes in the market variable, like oil or gas, that are normally distributed, positive gamma exposure will lead to a positively skewed call value (i.e., equity value).^{80 81} This leads to Hypothesis 3.

Figure 54
Translation of total asset price distribution into probability distribution for equity
(defined as long call position on total assets)



The gamma factor of options leads to a non-linear transformation of the underlying driving factor. The above figure is a modified illustration based on Hull (2000), p. 353.

Hypothesis 3:

The return on the equity value of a commodity company that contains/possesses real-options will be positively skewed. In other words, a company that contains positive

⁷⁹ See also the results of the simulation experiment in Figure 42, p. 81.

⁸⁰ The normality assumption for commodities is supported by the empirical analysis in Chapter 4.

⁸¹ Note, as pointed out by Haug (2007), p. 245, that a lognormal return distribution always implies a non-normal asset value distribution with positive skewness and excess kurtosis.

gamma exposure (e.g., in the form of highly out-of-the money real-options) will show a positively skewed probability distribution of prices changes relative to companies without real-option value.

3.3.2.2 Relating asset volatility to equity volatility

Special attention should be paid to factors that are freely observable in the market or that can at least be deduced from market prices. The most important variable in this regard is equity price volatility and its consequences for the return distribution. As shown by Jones et al. (1984), asset volatility and equity volatility are linked by the following relation:

$$\sigma_E = \sigma_V \frac{\partial E_t}{\partial V_t} \frac{V_t}{E_t} = \sigma_V \Delta \frac{V_t}{E_t} = \sigma_V N(d_1) \frac{V_t}{E_t} \quad (35)$$

Rearranging Equation (35) together with Equation (29), p. 94 yields:

$$\sigma_E = \sigma_V \frac{N(d_1) V_t}{V_t N(d_1) - D_T e^{-r_t^{rf} T} N(d_2)} \quad (36)$$

Equation (36) allows, in conjunction with Equation (29), to obtain the market-implied total company asset value V_t and asset volatility σ_V from $E_t, \sigma_E, D_T, r_t^{rf}$ and T . By analyzing the equity volatility σ_E , either by the observed return distribution of the equity price or via the options market, inferences about the characteristics of the company asset value volatility σ_V are therefore possible, which allows further the formulation of hypothesis tests with regard to the expected influence of real-options on the overall volatility of the company asset value V_t .

3.3.3 From mean-variance towards higher moments: Adjusted Sharpe-ratio analysis and the consequences of a non-normal return distribution

The previous analysis aimed to explain the influence of real-options on the asset and the asset return distribution. Special focus was placed on higher distribution moments (i.e.,

skewness and kurtosis). What follows is the translation of these higher moments into preferences for investors. We will therefore introduce the theory of adjusted Sharpe ratios that is used in the empirical part of this dissertation.⁸²

Adjusted Sharpe-ratios:

The Sharpe-ratio is likely the most frequently calculated investment criterion in financial analysis. It is defined as excess return over the risk-free rate divided by the standard deviation of the return. It is therefore a scale-invariant index of satisfaction as defined by Meucci (2009) and measures the excess return of an investment per unit of risk (also referred to as mean – variance criterion).⁸³

$$SR_i = \frac{E(r_i) - r^{rf}}{\sigma_i} \quad (37)$$

This concept – despite its beautiful simplicity – requires certain assumptions before we can infer the results. The assumptions for deriving the mean-variance criterion according to Alexander (2008a) are listed below:⁸⁴

- Logarithmic (concave) utility function (risk averter, risk aversion increases with wealth)
- Returns are normally distributed

These conditions are further supplemented by:

- Investments are independent (i.e., uncorrelated) from the rest of the portfolio (returns are i.i.d.)⁸⁵

⁸² See Subsubsection 6.2.2.2, p. 273.

⁸³ See Meucci (2009), p. 250.

⁸⁴ See Alexander (2008a), p. 234, and Copeland, Weston, and Shastri (2005), pp. 62-65.

⁸⁵ See Dowd (2000), p. 210, and Bao (2009), p. 153.

All assumptions – especially the return normality assumption and the assumption of identically and independently distributed returns – are unrealistic. Adjustments to the conventional Sharpe-ratio were initially proposed by Dowd (2000) and complemented by proposals from Hodges (1997), P  zier (2008) and Bao (2009). The adjustment process suggested by Dowd (2000) aims to incorporate the correlation of assets into formulation of a modified Sharpe-ratio, while Hodges (1997) and P  zier (2008) focus on incorporating higher distribution moments. Bao (2009) extends the adjustment process further by considering sampling errors due to higher return distribution moments to formulate a general return distribution-adjusted Sharpe-ratio.⁸⁶

Of particular interest is the extension of the mean-variance criterion to higher moments, as shown by Alexander (2008a).⁸⁷ She derives via Taylor-rule expansion the following expression for the certainty equivalent (CE):⁸⁸

$$CE \approx \mu_{r,i}^E - \frac{1}{2} \theta \sigma_i^2 + \frac{g_1}{6} \theta^2 \sigma_i^3 - \frac{g_2 - 3}{24} \theta^3 \sigma_i^4 \quad (38)$$

The expected return on the investment is given by $\mu_{r,i}^E = E(r_{i,t})$; for a risk-averse investor, the risk aversion coefficient is set to $\theta > 1$. The higher moments are represented by skewness (g_1) and kurtosis (g_2). Equation (38) implies that risk-averse investors ($\theta > 1$) will maximize their certainty equivalent by choosing assets with higher uneven moments $\left(\mu_{r,i}^E, \frac{g_1}{6} \theta^2 \sigma_i^3 \right)$ and lower even moments $\left(\frac{1}{2} \theta \sigma_i^2, \frac{g_2 - 3}{24} \theta^3 \sigma_i^4 \right)$.

⁸⁶ For a generalized Sharpe-ratio approach in the context of portfolio optimization see Malevergne and Sornette (2006), pp. 178-181.

⁸⁷ See Alexander (2008a), pp. 235-236.

⁸⁸ The certainty equivalent is used in utility theory for defining the expected wealth-risk premium. A risk-averse investor always maximizes his/her certainty equivalent. See Copeland, Weston, and Shastri (2005), pp. 54-55.

This leads to the following adjusted Sharpe-ratio (using the sample statistics for skewness and kurtosis)⁸⁹:

$$SR_i^{HM} = SR_i + \left(\frac{\hat{g}_1}{6}\right)(SR_i)^2 - \left(\frac{\hat{g}_2 - 3}{24}\right)(SR_i)^3 \quad (39)^{90}$$

The new Sharpe-ratio SR_i^{HM} is the result of the conventional Sharpe-ratio SR_i adjusted for higher moments. A further adjustment in the Sharpe-ratio analysis is necessary if the assets have autocorrelated daily returns. The usual annualized standard deviation calculation assumes that daily returns are independent (addition of independent variances). However, if returns are autocorrelated, this annualizing will underestimate the true variance and hence overestimate the Sharpe-ratio. Also, in order to account for possible return autocorrelation (of order 1), the conventional annualizing factor (usually the square root of the number of measurement units per year) is adjusted according to Equation (40):

$$\sqrt{h + 2 \frac{\hat{\rho}}{(1 - \hat{\rho})^2} [(h - 1)(1 - \hat{\rho}) - \rho(1 - \hat{\rho}^{h-1})]} \quad (40)^{91}$$

With no autocorrelation ($\hat{\rho} = 0$) Equation (40) reduces to $\sqrt{h}$, which is the typical scaling factor for annualizing the standard deviation of independent h-frequency returns.

We will refer to the adjusted Sharpe-ratio analysis in the empirical part of this dissertation, where unconditional moments (mean, standard deviation, skewness and kurtosis) are used to calculate realized Sharpe-ratios for different Russian and global commodity companies. The realized Sharpe-ratios that are calculated via the higher

⁸⁹ See Alexander (2008a), p. 260.

⁹⁰ The derivation of the adjustment factors for skewness and kurtosis are related to the Cornish-Fisher approximation used in the VaR literature. See Dowd (2005), p. 171.

⁹¹ See Alexander (2008a), p. 259.

moment adjustments allow – since the ex-post return is known - extracting the asset standard deviation that would have been necessary to obtain the adjusted Sharpe-ratio directly. They represent an ex-post set of implied standard deviations that can be used for inference (see Subsubsection 6.2.2.2, p. 273).⁹² We will therefor use the adjusted implied standard deviation for inference, since it accounts properly for the high kurtosis of out-of-the-money options, leading to much higher risk of out-of-the-money strategies than suggested by the conducted simulations analysis in Subsection 3.2.3, p. 67.

3.4 Stochastic theory definitions crucial for probability analysis and inferential statistical tests for higher distribution moments

3.4.1 Introduction

Working with random variables and probability functions requires the definition of some basic vocabulary that is necessary for dealing with the “probability space.” Some general properties of random variables, probability distributions, and random sequences are essential for understanding stochastic analysis. The following points will focus on the most important terminology and conceptual frameworks. Also presented here, and of the highest relevance for the empirical analysis presented in Chapter 6, is the formulation of inferential statistical tests for higher moments.

3.4.2 Probability function and moments

A random variable X can be described by the associated probability density function $f(x)$ that allows calculating the probability that the random variable X lies between a and b as given by:

$$P(a < X < b) = \int_a^b f(x) dx \quad (41)$$

The probability density function (PDF) is a function of the variable x . Thus, x represents a certain (fixed) value as input variable, in most cases a threshold (in contrast

⁹² It would have been also possible to obtain adjusted implied volatilities by utilizing analytical option price formulas that account for higher moments. For practical examples see Haug (2007), pp. 244-253.

to the capital letter X , which represents an unknown random variable). The integral, thus the infinite decimal sum of all probabilities attached to different values of x , over all possible x -values is equal to one.

$$\int_{-\infty}^{+\infty} f(x) dx = 1 \quad (42)$$

The probability that the random variable will be lesser or equal to a particular (threshold) value x is given by the cumulative density function (CDF), also called the distribution function. The distribution function is formally given by:⁹³

$$F(x) = P(X \leq x) = \int_{-\infty}^x f(u) du \quad (43)$$

Again, the capital letter X denotes the random variable, while x denotes a certain threshold.⁹⁴ If $F(x)$ is a continuous smooth function, we can inter-relate the cumulative density function $F(x)$ with the probability density function by differentiating $F(x)$ with respect to x to obtain the probability density function (PDF) of the random variable X .

$$\frac{\partial F(x)}{\partial x} = f(x) \quad (44)$$

Each random variable, by being related to the appropriate probability function (either the density function of the random variable $f(x)$ or the distribution function $F(x)$) is characterized by its different moments. The different distribution moments describe:

⁹³ Note: in cases where the integrated interval is denoted by the variable x , the term $f(x)dx$ is replaced by $f(u)du$ to avoid any misinterpretation.

⁹⁴ Often we require the assignment of a threshold x to a given cumulative probability P . Hence, we require the inverse cumulative density function (also called percentile function) which is given by: $F^{-1}(P)$. The inverse cumulative density function is especially important for generating random numbers.

- location (mean, median or mode),
- dispersion (variance, standard deviation or mean absolute deviation),
- asymmetry (skewness) and
- tail/peakness concentration.⁹⁵

The general form for calculating the different probability distribution moments and moments around the mean μ is given by the moment-generating functions below:

$$r^{\text{th}} \text{ moment (about the origin)} = \int_{-\infty}^{\infty} \xi^r f(x) dx, \quad \xi = x \quad (45)$$

$$r^{\text{th}} \text{ central moment (about the mean)} = \int_{-\infty}^{\infty} (x - \mu)^r f(x) dx, \quad \xi = x - \mu \quad (46)$$

The generating function has a finite value if the probability density function $f(x)$ converges towards $|x|^{-r}$. Thus, for increasing values of x the probability density function $f(x)$ should decrease faster than $|x|^{-r}$ which is not always the case (e.g., certain Levy distributions, also known as Pareto-Levy or stable Paretian distributions,⁹⁶ will have infinite higher order moments, see Table 25, p. 112).⁹⁷ The most important first moment for a continuous random variable, the expectation value, is given by:

$$E(X) = \int_{-\infty}^{+\infty} xf(x) dx = \int_{-\infty}^{+\infty} x dF(x) \quad (47)$$

$$\mu := E(X) = \text{Mean (location)}$$

⁹⁵ Referred to as kurtosis, “heaviness of the tails,” or speed of diminishing tail probabilities. See Dowd (2005), p. 46.

⁹⁶ For a good introduction to stable distributions, see Campbell et al. (1997), p. 17 and Rachev et al. (2006), pp. 81-92.

⁹⁷ See Dowd (2005), pp. 47-49.

Other moments – like the central moments (moments about the mean) – are calculated by treating $\xi = X - E(X)$ as analogous to x in Equation (47). The moment about the mean (i.e., the central moment) is defined as:

$$\mu_r^c := E[X - E(X)]^r \quad (48)$$

Central moments for $r = 0$ and $r = 1$ represent special cases with $\mu_0 = 1$ and $\mu_1 = 0$.

The second central moment (variance) is consequently given as:

$$2^{\text{nd}} \text{ moment} = \int_{-\infty}^{\infty} \xi^2 f(x) dx, \quad \xi = x - E(X) \quad (49)\text{-a}$$

$$2^{\text{nd}} \text{ moment} = \int_{-\infty}^{\infty} (x^2 - E(X))^2 f(x) dx \quad (49)\text{-b}$$

$$2^{\text{nd}} \text{ moment} = \int_{-\infty}^{\infty} x^2 f(x) dx - \int_{-\infty}^{\infty} (E(X))^2 f(x) dx \quad (49)\text{-c}$$

$$\begin{aligned} E(X^2) - E((E(X))^2) &= E(X - E(X))^2 = E(X^2 - 2XE(X) + E(X)^2) \\ &= E(X^2) - 2E(X)^2 + E(X)^2 \rightarrow \\ \sigma^2 &:= E(X^2) - E(X)^2 \end{aligned} \quad (49)\text{-d}$$

Other central moments are listed below:⁹⁸

$$E[X - E(X)]^3 : (3^{\text{rd}} \text{ moment about the mean}) \quad (50)$$

$$E[X - E(X)]^4 : (4^{\text{th}} \text{ moment about the mean}) \quad (51)$$

The given Equations highlight the fact that the main characteristics of a random variable – in terms of moments – depend on the probability function associated with the random

⁹⁸ Note that very often in the literature skewness and kurtosis are normalized. Thus, the formed expectation value is divided by σ^3 (for skewness, see Equation (69), p. 128) and σ^4 (for kurtosis, see Equation (80), p. 130), respectively.

variable in question. Some random variables are fully described by the first two moments (e.g., standard normally distributed random variables $N(0,1)$); others, however, require higher moments to describe their probability distributions.

Table 25 summarizes the most important probability distributions and their specific moments when closed-form solutions are available. It distinguishes between unrestricted, asymmetrically restricted and fully restricted distributions. In general all distribution functions are represented by their location (a), dispersion or scale (b) and shape parameter (c). In some cases a fourth parameter will be used (d) to fully define the probability distribution function. Note: the notation (μ, σ) is often used for location and dispersion to define certain probability functions. Here (μ, σ) is used only for describing population estimates or distribution moments.

One further definition that needs to be addressed in this context is the standardization of central moments. A standardized moment is defined as:

$$\mu_r^{st} := E \left[\left(\frac{X - E(X)}{\sigma} \right)^r \right] \quad (52)$$

The special cases are: $\mu_0^{st} = 1$, $\mu_1^{st} = 0$ and $\mu_2^{st} = 1$. Hence, only standardized moments of order higher than two have meaningful informational value to help characterize the probability characteristics of random variables. Note that central moments (c) and standard moments (st) are related to each other via the following Equation. This relation is important for calculating the sample estimates for skewness and kurtosis in the empirical section.

$$\mu_r^{st} = \frac{\mu_r^c}{(\mu_2^c)^{r/2}} = \frac{\mu_r^c}{(\sigma)^r} \quad (53)$$

Table 25
Main random variable distributions and their characteristics

	Probability density function	Parameters	Mean	Variance	Skewness	Kurtosis
	Unrestricted distributions: $-\infty < x < +\infty$		μ	σ^2	g_1	g_2
Normal	$f(x) = \frac{1}{\sqrt{2\pi}b} e^{-0.5\left(\frac{x-a}{b}\right)^2}$	a : location $b > 0$: dispersion	a	b^2	0	3: Rand F-1 (no)
Extreme Value	$f(x) = \frac{1}{b} e^{\left[-c\left(\frac{x-a}{b}\right) - \exp\left(-c\left(\frac{x-a}{b}\right)\right)\right]}$	a : location $b > 0$: dispersion $c \in -1, 1$: shape	$c a + 0.577 b$	$\frac{\pi^2 b^2}{6}$	$\frac{12\sqrt{6} \cdot 1.2020569}{\pi^3} c = 1.139547c$	5.4c Rand F-1 (yes)
Student-t	$f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\Gamma(\nu/2)} \left(1 + \frac{x^2}{\nu}\right)^{-(\nu+1)/2}$	$\nu \in \mathbb{N}$: degrees of freedom	$0 \forall \nu \geq 2$ otherwise infinite	$\frac{\nu}{\nu-2} \forall \nu \geq 3$	$0 \forall \nu \geq 4$	$3 + \frac{\nu}{\nu-4} \forall \nu \geq 5$ $\lim_{\nu \rightarrow \infty} \alpha_4 = 3$ Rand F-1 (as proxy)
Skewed Normal	$f(x) = \frac{2}{b} \phi\left(\frac{x-a}{b}\right) \cdot \Phi\left(c \frac{x-a}{b}\right)$ $\phi(\cdot)$: density of standard N(0,1) variable $\Phi(\cdot)$: cum. distribution function of N(0,1) var.	a : location $b > 0$: dispersion c : shape	$a + b\sqrt{2/\pi}\delta$ with $\delta = a/\sqrt{1+a^2}$	$b^2(1-2\delta^2/\pi)$	$\frac{4-\pi}{2} \frac{(\sqrt{2/\pi}\delta)^3}{(1-2\delta^2/\pi)^{3/2}}$	$(\pi-3) \frac{(\sqrt{2/\pi}\delta)^4}{(1-2\delta^2/\pi)^2}$ Rand F-1 (no)
Logistic	$f(x) = \frac{e^{-(x-a)/b}}{\beta \left(1 + e^{-(x-a)/b}\right)^2}$	a : location $b > 0$: dispersion	a	$\frac{\pi^2 b^2}{3}$	0	4.2 Rand F-1 (yes)
Cauchy	$f(x) = \left[\pi b \left(1 + \left(\frac{x-a}{b}\right)^2\right) \right]^{-1}$	a : location (median) $b > 0$: dispersion	Do not exist	Do not exist	Do not exist	Do not exist Rand F-1 (yes)

Asymmetrically restricted distributions: $x \in [A, \infty)$			μ	σ^2	g_1	g_2
Gamma	$f(x) = \frac{1}{b\Gamma(x)} \left(\frac{x}{b}\right)^{c-1} e^{-x/b} \quad 0 < x < +\infty$	$b > 0$: dispersion $c > 0$: shape	bc	$b^2 c$	$\frac{2}{\sqrt{c}}$	$3 + 6/c$ Rand F-1 (yes)
Exponential	$f(x) = \lambda e^{-\lambda x} \quad 0 < x < +\infty$	$\lambda > 0$: dispersion	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	2	9 Rand F-1 (yes)
Lognormal	$f(x) = \frac{1}{x\sqrt{2\pi}b} e^{-0.5\left(\frac{\ln x - a}{b}\right)^2} \quad 0 < x < +\infty$	a : location $b > 0$: dispersion	$e^{a + \frac{b^2}{2}}$	$e^{2a} e^{b^2} (e^{b^2} - 1)$	$(w+2)\sqrt{(w-1)}, w = e^{a^2}$	$w^4 + 2w^3 + 3w^2 - 3, w = e^{a^2}$ Rand F-1 (yes)
Weibull	$f(x) = \frac{c x^{c-1}}{b^c} e^{-(x/b)^c} \quad 0 \leq x < +\infty$	$b > 0$: dispersion $c > 0$: shape	$b \Gamma\left(1 + \frac{1}{a}\right)$	$b^2 \left[\Gamma\left(1 + \frac{2}{a}\right) - \Gamma^2\left(1 + \frac{1}{a}\right) \right]$	$\frac{\Gamma\left(1 + \frac{3}{a}\right) b^3 - 3\mu\sigma^2 - \mu^3}{\sigma^3}$	$\frac{\Gamma\left(1 + \frac{4}{a}\right) b^4 - 3\sigma^4 - 4\mu\sigma^3\alpha_3 - 6\mu^2\sigma^2 - \mu^4}{\sigma^4}$ Rand F-1 (yes)
Restricted distributions: $x \in [A, B]$			μ	σ^2	α_3	α_4
Beta	$f(x) = \frac{x^{\alpha_1-1} (1-x)^{\alpha_2-1}}{B(\alpha_1, \alpha_2)} \quad 0 < x < 1$	$\alpha_1, \alpha_2 \in \mathbb{R}^+$: shape	$\frac{\alpha_1}{\alpha_1 + \alpha_2}$	$\frac{\alpha_1 \alpha_2}{(\alpha_1 + \alpha_2)^2 (\alpha_1 + \alpha_2 + 1)}$	$2 \frac{\alpha_2 - \alpha_1}{\alpha_1 + \alpha_2 + 2} \sqrt{\frac{\alpha_1 + \alpha_2 + 1}{\alpha_1 \alpha_2}}$	

For a full reference on nearly all important statistical distributions see Evans, Hastings and Peacock (2000).

3.4.3 Estimating distribution parameters and specific distribution tests

In the previous sections we have described the most important probability distribution functions and their characterizing parameters. However, we have thus far not touched on the issue of estimating these parameters. In practice one is confronted with value functions (e.g., present value function of investment projects) or stochastic processes (e.g., for commodities like oil or copper) whose absolute or relative value can be described by stochastic probability density functions (PDFs). Hence, some of the probability density functions in Table 25 can be used to approximate the probability density of future commodity prices or project values. To do so, however, one needs to estimate the PDF parameters like location, dispersion and even asymmetry or tail characteristics if necessary. To estimate a PDF we can apply parametric or non-parametric methods depending on the availability of markets (observed prices) for the underlying in question.⁹⁹ Some of the most popular methods for calibrating probability functions are described in the following sections.

3.4.3.1 Methods of moments

The methods of moments technique extracts population parameters like mean μ (location), variance σ^2 (dispersion), skewness g_1 (shape) and kurtosis g_2 (shape) by analyzing random samples of the entire population. By assuming that the observed random values $X_1, \dots, X_n$ are independent and identically distributed samples of one common probability distribution function $f(x)$, it is possible to statistically infer parameters of that particular PDF by calculating the different moments of the specific sample in question.¹⁰⁰ Given that the probability distribution function depends on one single $f(x|\mu)$ or several parameters $f(x|\mu, \sigma, g_1, g_2)$ one calculates estimators

⁹⁹ For example, the probability density function can be extracted out of traded option prices (non-parametric) or by analyzing historic time series for the underlying in question (parametric estimate).

¹⁰⁰ The random singly observed $X_1, \dots, X_n$ variables could also represent the averages of different sub-samples.

$(\hat{\mu}, \hat{\sigma}, \hat{g}_1, \hat{g}_2)$ for the distribution function in question. The four most common estimators (statistics) for the population distribution parameters - mean, variance, skewness and kurtosis - are given in Table 26:

Table 26
Calculating distribution statistics as unbiased estimators of population parameters (moments)

Moment	Unbiased sample statistic
Mean	$\hat{\mu} = \frac{1}{n} \sum_{i=1}^n x_i$
Variance	$\hat{\sigma}^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \hat{\mu})^2$
Skewness	$\hat{g}_1 = \frac{n}{(n-1)(n-2)} \sum_{i=1}^n \left(\frac{x_i - \hat{\mu}}{\hat{\sigma}} \right)^3$
Kurtosis	$\hat{g}_2^e = \frac{n(n+1)}{(n-1)(n-2)(n-3)} \sum_{i=1}^n \left(\frac{x_i - \hat{\mu}}{\hat{\sigma}} \right)^4 - 3 \frac{(n-1)^2}{(n-2)(n-3)}$

Skewness and kurtosis are in accordance with the definition by Sheskin (2007), pp. 22-29.

However, since sample observations are used (the entire population is unknown) one has to deal with finite sample properties. The prerequisite for doing so is that the calculated parameters are:

- a) unbiased estimators of the true parameters
- b) consistent estimators of the true parameters

where **an estimator is given as a function** with the observed values (defined in functional form - e.g., sample average $\hat{\mu} = f(X_1 \dots X_n)$ - as a formula for building the

average) as input.¹⁰¹ The functional form, hence the ways data are functionally related (e.g., combination in the form of an average), has to follow certain rules. Defining the required parameter in question as Θ we can state that an estimator (i.e., the statistic) $\hat{\Theta}_n = f(X_1 \dots X_n)$ of the parameter Θ is unbiased if on average the calculated (i.e., estimated) value $\vartheta_n = f(x_1 \dots x_n)$ is equal to Θ . In technical terms Equation (54) has to hold:

$$E(\hat{\Theta}_n) = \Theta \quad \forall n \quad (54)$$

Here we see that the unbiasedness of the estimators is related to the sampling distribution (the distribution of the estimate), although some parameter estimators are unaffected by the distribution assumptions (e.g., the mean). If the probability distribution of the estimate (expected value) is equal to the parameter it ought to be estimating, then the estimator is unbiased.¹⁰² For example: defining the parameter Θ as mean μ ($\Theta := \mu$) we know that the estimator of the expected value of random variables $X_1, \dots, X_n$ is known as the sample average of the observed values $x_1, x_2, \dots, x_n$. Thus, the sample averages serves as unbiased estimators for the mean μ . We can state the sample average as estimator (i.e., function) as shown in Equation (56):

$$\Theta := \mu \text{ as definition} \quad (55)$$

$$\hat{\Theta}_n = \text{d} E(\hat{X}_n) \triangleq \frac{1}{n} \sum_{i=1}^n X_i \quad (56)$$

¹⁰¹ Thus, the observed values $x_1, x_2, \dots, x_n$ are plugged into the functional form $f(x_1 \dots x_n)$ to obtain the parameter estimate in question, which is denoted by ϑ_n and is usually a scalar. See, for example, the sample mean estimator in Table 26.

¹⁰² Hence, if we draw infinite samples and calculate individual sample parameters ϑ_n using the estimator $\hat{\Theta}_n$ then their average should converge towards the parameter Θ .

$$E(\hat{X}_n) = \frac{1}{n} \sum_{i=1}^n E(X_i) = \frac{1}{n} \left(\sum_{i=1}^n \mu \right) = \frac{1}{n} n \mu = \mu \forall n \quad (57)$$

Equation (57) shows the unbiased estimator of the population mean, which leads to Equation (58) as the definition of unbiasedness.

$$E(\hat{\Theta}_n) = E(\hat{X}_n) \quad (58)$$

In this context biasness $\mathfrak{B}(\hat{\Theta}_n)$ is defined by:

$$\mathfrak{B}(\hat{\Theta}_n) := E(\hat{\Theta}_n) - \Theta \quad (59)$$

If Equation (59) is different from zero, then the estimator and hence the function $\hat{\Theta}_n = f(X_1 \dots X_n)$ is not appropriate.

Consistency, the second prerequisite, is related to the asymptotic characteristics $(\hat{\Theta}_n \text{ for } n \rightarrow \infty)$ of finite sample analysis. Since the unbiasedness of the estimator $\hat{\Theta}_n = f(X_1 \dots X_n)$ is very sensitive towards the sample size n , it is expected that with an increase in the sample size the difference between the estimator and the real parameter should converge towards zero. Thus, the variance of the estimator should decrease with higher numbers of observations.¹⁰³ Formally this is equal to:

$$\lim_{n \rightarrow \infty} P(|\hat{\Theta}_n - \Theta| > \varepsilon) = 0 \quad (60)$$

¹⁰³ Put another way, the more often an experiment is repeated, the closer to the true value the estimate should become.

As we can see both elements - the unbiasedness and the consistency of the estimator - are related to the sampling distribution (thus the distribution of the estimate). The method of moments requires the sampling distribution estimator to be close to the real parameter and the variance of the estimator to be a finite number that converges towards zero. Based on these assumptions, we can derive, as with the above population mean example, estimators for other distribution moments. The concept of replacing the population parameter with the sample counterpart is identical for distribution parameters.

3.4.3.2 Maximum likelihood estimators

The maximum likelihood theory is based on the definition of likelihood functions. A likelihood function can be interpreted as a joint probability function of individual probability density functions. Let $f(X|\Theta)$ be a probability function of the variable X and the parameter Θ . For a set of observed values $x_1, x_2 \dots x_n$ we can calculate the individual probability density function $f(x_i|\Theta)$, assuming that they are a random sample from the total population with one common distribution parameter Θ , and that they form a joint distribution function as a product of the individual distribution functions $f(x_1, \Theta)$ to $f(x_n|\Theta)$. This joint probability function is called the definition likelihood function $L(\Theta|x)$. For the continuous case we can write:

$$L(\Theta|x) := \prod_{i=1}^n f(x_i|\Theta) = f(x_1|\Theta) \times f(x_2|\Theta) \times \dots \times f(x_n|\Theta) \quad (61)$$

$L \rightarrow$ function of Θ conditional on x vs. $f \rightarrow$ function of x conditional on Θ

It is at this point important to note that $L(\Theta|x)$ is dependent on the parameter Θ and conditional on the pre-set (observed) numerical values of $x_1, x_2 \dots x_n$. Hence by manipulating the value of the function (by changing Θ), the likelihood function – like any other function – can be maximized with respect to Θ . Calculating the first

derivative of the likelihood function results in the maximum likelihood function of Θ . The value that maximizes the likelihood function $L(\Theta|x)$ is called the maximum likelihood estimator $\hat{\Theta}_{\max}$. For a given set of observed values $x_1, x_2 \dots x_n$ any probability function $f(x_n|\Theta)$ can be assumed to form a likelihood function that is maximized with respect to the parameter Θ in question.

The following example will illustrate the methodology. Suppose a sample with three observation $x_1 = 0.5, x_2 = 0.8, x_3 = 0.2$ and assume they come from a population distribution that follows an exponential probability distribution. According to Table 25 (p. 112), the probability distribution function for exponential distributed variables is given by:

$$f(x) = \frac{e^{-x/b}}{b} = \varpi e^{-\lambda x} \text{ with: } \varpi = \frac{1}{b}; 0 < x < +\infty; b > 0 \quad (62)$$

The aim is to calculate an estimate for the distribution parameter $\Theta = \varpi$. Based on the three observations, a likelihood function (a joint distribution function for the sample $n = 3$) can be formed according to Equation (63)-a as:

$$L(\varpi|x_1 = 0.5, x_2 = 0.8, x_3 = 0.2) := \prod_{i=1}^n f(x_i|\Theta) \quad (63)\text{-a}$$

$$L(\varpi|x_1 = 0.5, x_2 = 0.8, x_3 = 0.2) := \varpi e^{-\varpi x_1} \varpi e^{-\varpi x_2} \varpi e^{-\varpi x_3} = \varpi^3 e^{-\varpi(x_1 + x_2 + x_3)} \quad (63)\text{-b}$$

After creating the likelihood function $L(\varpi|x_1, x_2, x_3)$ and differentiating $L(\varpi|x_1, x_2, x_3)$ with respect to the required variable ϖ the result is:

$$\frac{dL}{d\varpi} = e^{-\varpi(x_1 + x_2 + x_3)} (3\varpi^2 - (x_1 + x_2 + x_3)\varpi^3) = 0 \quad (64)$$

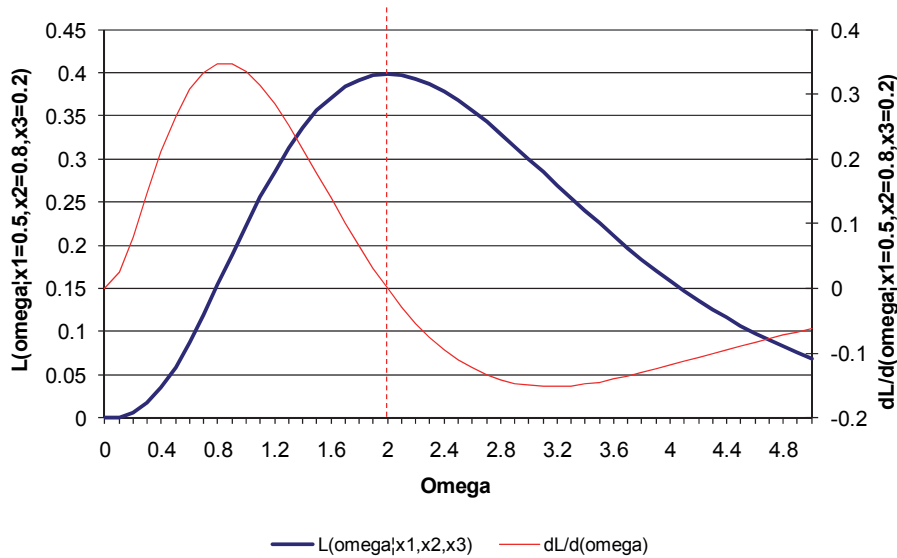
Solving the equation by calculating the appropriate roots results in the maximum likelihood estimate for the parameter ϖ . Equation (65) represents the maximum likelihood estimator.

$$(3\varpi^2 - (x_1 + x_2 + x_3)\varpi^3) = 0 \Rightarrow \hat{\varpi} = \frac{3}{(x_1 + x_2 + x_3)} = \frac{n}{\sum_{i=1}^n x_i} = 2 \quad (65)$$

Figure 55 gives a graphical illustration of the example. Based on the independent sample observations $x_1 = 0.5, x_2 = 0.8, x_3 = 0.2$ and assuming that the random variables are exponentially distributed, the distribution parameter $\Theta = \varpi$ is calculated by maximum likelihood estimate $\hat{\varpi}$.

Figure 55

Searching for the maximum of the likelihood function



The maximum for the likelihood function is found with $\hat{\varpi} = 2$. The development of the first derivative of the likelihood function ($dL / d\varpi$) is shown to cross the zero line at the maximum value of the likelihood function as required in Equation (64).

The maximum likelihood method is very powerful not only for obtaining distribution parameter estimators but also for solving other econometrical problems. The standard OLS estimators for slope and intercept can also be derived by maximum likelihood

calculations.¹⁰⁴ Table 27 gives, for the most commonly used probability distributions, the maximum likelihood estimators for the different distribution parameters.

Several of the maximum likelihood estimates cannot be solved analytically, which makes numerical solutions necessary. For some of these distributions, however, closed form matching moment estimates exist (e.g., for Gamma and Beta distributions).

Table 27
Maximum likelihood estimators for different probability density functions

Probability density function		Maximum likelihood estimator for sample of independent observations $(x_1, x_2 \dots x_n)$	Solution comments
Unrestricted distributions: $-\infty < x < +\infty$			
Normal	$f(x) = \frac{1}{\sqrt{2\pi}b} e^{-0.5\left(\frac{x-a}{b}\right)^2}$	Location: $\hat{a} = \frac{1}{n} \sum_{i=1}^n x_i$ Dispersion: $\hat{b} = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \hat{a})^2}$	Momentum and maximum likelihood estimates available
Extreme Value	$f(x) = \frac{1}{b} e^{\left[-c\left(\frac{x-a}{b}\right) - \exp\left(-c\left(\frac{x-a}{b}\right)\right)\right]}$	$\frac{1}{n} \sum_{i=1}^n e^{-\left(\frac{x_i-a}{b}\right)} = 1$ $\frac{1}{n} \sum_{i=1}^n \frac{x_i - a}{b} \left(1 - e^{-\left(\frac{x_i-a}{b}\right)}\right) = 1$	Numerical solution of simultaneous Equations
Logistic	$f(x) = \frac{e^{-(x-a)/b}}{\beta \left(1 + e^{-(x-a)/b}\right)^2}$	$\frac{2}{n} \sum_{i=1}^n \left(1 + e^{\frac{x_i-a}{b}}\right)^{-1} = 1$ $\frac{1}{n} \sum_{i=1}^n \frac{x_i - a}{b} \frac{1 - e^{\frac{x_i-a}{b}}}{1 + e^{\frac{x_i-a}{b}}} = 1$	Numerical solution of simultaneous Equations
Cauchy	$f(x) = \left[\pi b \left(1 + \left(\frac{x-a}{b}\right)^2\right) \right]^{-1}$	$\sum_{i=1}^n \left\{ 1 + \left[\left(x_i - \hat{a} / \hat{b}\right) \right]^2 \right\}^{-1} = n/2$ $\sum_{i=1}^n x_i \left\{ 1 + \left[\left(x_i - \hat{a} / \hat{b}\right) \right]^2 \right\}^{-1} = n\hat{b}/2$	Numerical solution of simultaneous Equations

¹⁰⁴ See Brooks (2008), pp. 444-448.

Asymmetrically restricted distributions: $x \in [A, \infty)$			μ
			Matching moments solution or numerical solution of simultaneous Equations
Gamma	$f(x) = \frac{1}{b\Gamma(x)} \left(\frac{x}{b}\right)^{x-1} e^{-x/b} \quad 0 < x < +\infty$	Matching moments estimator: $\hat{b} = s^2 / \bar{x}$ $\hat{c} = (\bar{x} / s)^2$	
Exponential	$f(x) = \varpi e^{-\varpi x} \quad 0 < x < +\infty$	Dispersion: $\hat{\varpi} = \frac{n}{\sum_{i=1}^n x_i}$	$\frac{1}{\varpi}$
Lognormal	$f(x) = \frac{1}{x\sqrt{2\pi}b} e^{-0.5\left(\frac{\ln x - a}{b}\right)^2} \quad 0 < x < +\infty$	Location: $\hat{a} = \frac{1}{n} \sum_{i=1}^n \log x_i$ Dispersion: $\hat{b} = \sqrt{\frac{1}{n} \sum_{i=1}^n (\log x_i - \hat{a})^2}$	
Weibull	$f(x) = \frac{c x^{c-1}}{b^c} e^{-(x/b)^c} \quad 0 \leq x < +\infty$	$b > 0$: dispersion $c > 0$: shape $\hat{b} = \left[\left(\frac{1}{n} \sum_{i=1}^n x_i^{\hat{c}} \right) \right]^{1/\hat{c}}$ $\hat{c} = \frac{n}{\left(\frac{1}{\hat{b}} \right) \sum_{i=1}^n x_i^{\hat{c}} \log x_i - \sum_{i=1}^n \log x_i}$	Numerical solution of simultaneous Equations
Restricted distributions: $x \in [A, B]$			μ
			Matching moments solution or numerical solution of simultaneous Equations
Beta	$f(x) = \frac{x^{\alpha_1-1} (1-x)^{\alpha_2-1}}{B(\alpha_1, \alpha_2)} \quad 0 < x < 1$	Matching moments estimator: $\alpha_1 = \bar{x} \left\{ \left[\bar{x} (1-\bar{x}) / s^2 \right] - 1 \right\}$ $\alpha_2 = (1-\bar{x}) \left\{ \left[\bar{x} (1-\bar{x}) / s^2 \right] - 1 \right\}$	

See Evans, Hastings and Peacock (2000), Rachev, Menn and Fabozzi (2005).

3.4.4 Comparing probability distributions

Given a sample set of data, we showed how to calculate specific probability distribution parameters. However, since data can be fitted to any probability distribution, one has to choose among different distribution functions that represent the total population distribution best. Thus, it is necessary to test whether the chosen distribution function is an accurate probability distribution representative for our data sample. Usually different

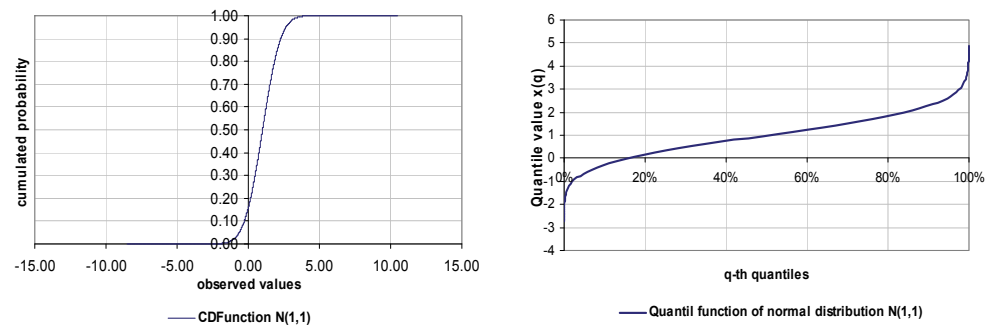
statistical methods have been deployed to solve this problem. Among the most accepted tests are:

- Kolmogorov-Smirnov-Liffiefors-Test
- Cramer-von Mises-Test
- Anderson-Darling-Test

All the above tests belong to the family of goodness-of-fit tests.¹⁰⁵ Without describing different test statistics we will focus instead on fast and usually sufficiently effective ways to analyze whether given distribution samples belong to a specific total population distribution family. We will use quantile-quantile (QQ) plots where the empirical quantiles are plotted against the quantiles of the hypothetical distribution (see Figure 56). Note, that the quantile plots are essentially identical to cumulative distribution plots with switched axes.¹⁰⁶

Figure 56

The relation between the CDF and the quantile to quantile plot



Panel A: A CDF for a normally distributed (N(1,1)) variable.

Panel B: A QQ-plot for a normally distributed (N(1,1)) variable.

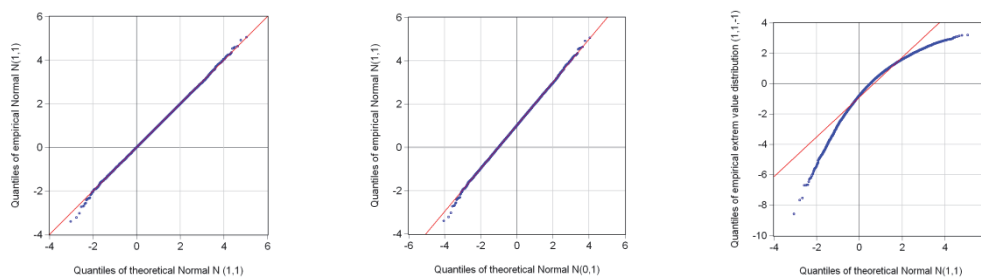
¹⁰⁵ See Rinne (2003), pp. 566-571, and Dowd (2005), pp. 343-349.

¹⁰⁶ A quantile of order p (x_p) is defined as a value (i.e., observation) that is not exceeded by p -% of the observed values. For example, the quantile of order 0.5 ($x_{0.5}$) is defined as the median.

By plotting the empirical quantile function against a theoretical quantile function (theoretical QQ-plot) or another empirical quantile function (empirical QQ-plot) deviations between two distributions become visible. Three different cases are distinguished:

- empirical distribution is identical to the theoretical distribution (same distribution functions and same distribution parameters)
- empirical distribution and theoretical distribution are from the same distribution family but deviate in parameters (different scale or location)
- empirical distribution deviates totally from the theoretical distribution

Figure 57
Interpreting the QQ-plot



Panel A: Identical distribution functions with identical parameters

Panel B: Identical distribution functions with different parameters

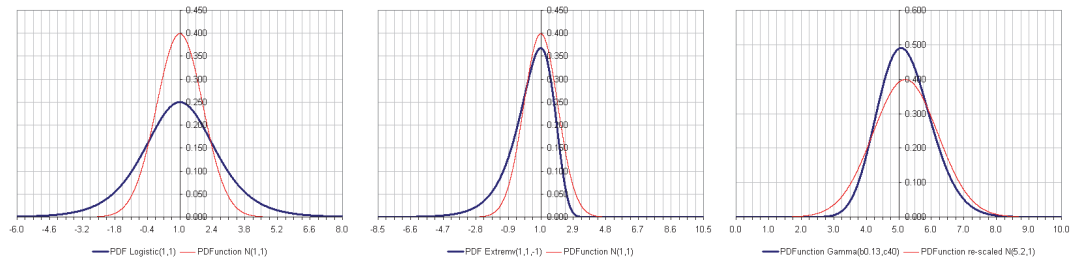
Panel C: Two distribution from different types

Figure 57 shows the QQ-plot for the tree cases. In Panel A the QQ-plot forms a 45°-line which is an indication that the empirical and theoretical distributions are identical. In Panel B, the values are persistently dispersed along the theoretical distribution line. However, the slope is different from 45°, which indicates that the empirical distribution and the theoretical distribution have identical distribution functions with different distribution parameters (different scales and/or locations). In Panel C, the empirical and the theoretical distributions are governed by different distribution functions (in this case an extreme value distribution vs. a normal distribution). Of particular interest is the changing slope for the QQ-plot in Panel C, which is an indication that the tail-

probabilities between the two distribution functions deviate in an asymmetric way. The tail probability analysis is better explained if the location parameter is held constant.

Figure 58

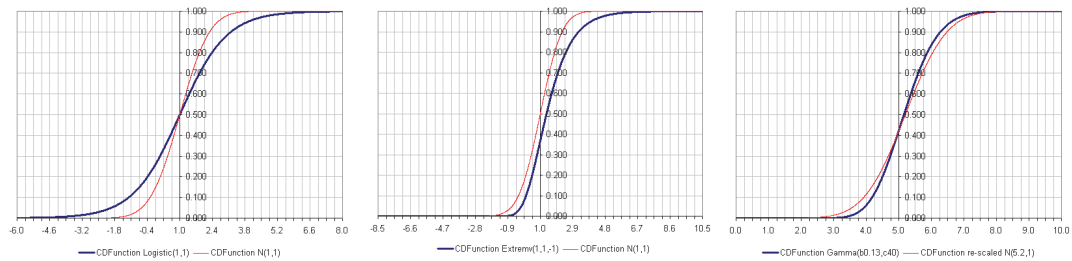
Interpreting the QQ-plot if the location parameter is held constant



Panel A: Logistic vs. Normal PDF

Panel B: Extreme value vs. Normal PDF

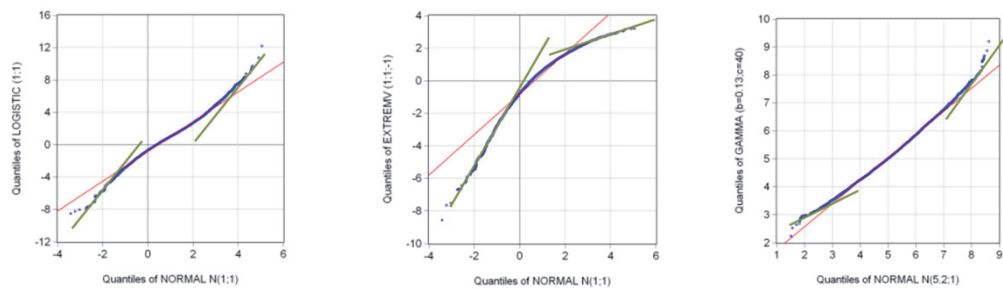
Panel C: Gamma vs. Normal PDF



Panel D: Logistic vs. Normal CDF

Panel E: Extreme value vs. Normal CDF

Panel F: Gamma vs. Normal CDF



Panel G: Logistic vs. Normal QQ-plot

Panel H: Extreme value vs. Normal QQ-plot

Panel I: Gamma vs. Normal QQ-plot

In Figure 58 we have illustrated three differently generated probability distribution functions. Each distribution function is plotted against a normal distribution function

with identical mean. The example also illustrates the corresponding QQ-plot for every single distribution. Obviously, no distribution function is close to a normal distribution function due to strong tail-deviation. The slope of the QQ-plot in Panel G is constant around the mean but steeper at the tail level. This indicates that more tail probability (heavy tails) is associated with the tested distribution relative to the assumed normal distribution. Since the steepness of the QQ-plot at the tail section is similar, the heaviness is symmetrical (see also Panel A). The probability distribution function in Panel B obviously has more tail probability on the left side (negative skew) and less probability on the right side. The QQ-plot therefore has a changing steepness in Panel H. The opposite is true for the PDF shown in Panel C, where the tested distribution has less tail-probability attached at the left side relative to the normal distribution and more probability on the right side as indicated by the changing QQ-plot steepness.¹⁰⁷

3.4.5 *Inferential statistical tests for higher distribution moments*

A large part of the empirical analysis is based on testing distributions and distribution moments. We will therefore explain the main parametric tests that are necessary for inference in this dissertation.

3.4.5.1 Variance test

In order to conduct directional hypothesis tests (one-tailed tests) to answer if and how significant the variances of daily returns between two samples differ, chi-square test can be employed.

The test statistic is given in Equation (66).

$$\chi^2 = \frac{(n-1)\hat{\sigma}^2}{\sigma^2} \tag{66}$$

¹⁰⁷ The QQ-plot analysis is utilized in the empirical section for analyzing different GARCH diffusion specifications and their unconditional kurtosis values (see Subsection 6.2.3, p. 285).

n : number of observations

$\hat{s}^2$: unbiased estimated of the population variance

σ^2 : hypothesized value stated in H_1

The test is formulated as a directional alternative hypothesis test (evaluated as a one-tailed test, see below) relative to a reference value (e.g., the mean or median variance of a reference peer-group):

$$H_0: \sigma^2 = \text{reference value}$$

$$H_1: \sigma^2 > \text{reference value}$$

3.4.5.2 Skewness test¹⁰⁸

Before defining a test for skewness it is necessary to define the skewness statistic, since many different expressions of skewness exist. The sample statistic for the i^{th} moment about the mean is given by:

$$\mu_3^c = \frac{\sum (X - \bar{x})^3}{n} \quad (67)$$

The unbiased estimate of the population parameter estimated by m_3 is usually represented by the computational equation below:¹⁰⁹

$$\hat{m}_3 = \frac{n \sum X^3 - 3 \sum X \sum X^2 + \frac{2(\sum X)^3}{n}}{(n-1)(n-2)} \quad (68)$$

Since m_3 is calculated in cubed units a standardization into a unitless statistic is performed, which yields the skewness coefficient given in Equation (69):

¹⁰⁸ See Sheskin (2007), pp. 207-208.

¹⁰⁹ See Sheskin (2007), p. 20.

$$\hat{g}_1 = \frac{\hat{m}_3}{\hat{s}^3} \quad (69)$$

The skewness coefficient is the skewness measure most often referred to. However, for statistical tests another measure will be employed: namely an adjusted skewness coefficient, given in Equation (70).

$$\sqrt{b_1} = \frac{(n-1)\hat{g}_1}{\sqrt{n(n-1)}} \quad (70)$$

The sample statistic $\sqrt{b_1}$ is often applied by statistical packages as a superior measure, since it further reduces the sample bias of the skewness coefficient.¹¹⁰

The statistical test that is necessary for inference is a z -test statistic (used as an approximation for the sampling distribution of g_1 for $n > 9$). The calculation of the z -statistic requires several calculation steps, shown in Equations (71) to (77).¹¹¹

$$A = \sqrt{b_1} \sqrt{\frac{(n+1)(n+3)}{6(n-2)}} \quad (71)$$

$$B = \frac{3(n^2 + 27n - 70)(n+1)(n+3)}{(n-2)(n+5)(n+7)(n+9)} \quad (72)$$

$$C = \sqrt{2(B-1)} - 1 \quad (73)$$

$$D = \sqrt{C} \quad (74)$$

$$E = \frac{1}{\sqrt{\ln D}} \quad (75)$$

¹¹⁰ For example, Eviews[®] calculates the sample statistic $\sqrt{b_1}$ as a skewness measure.

¹¹¹ This test is conducted according to the guidelines described in Zar (1999), pp. 115-116.

$$F = \frac{A}{\sqrt{\frac{2}{C-1}}} \quad (76)$$

$$z = E \ln \left(F + \sqrt{F^2 + 1} \right) \quad (77)$$

For the hypothesis test, the z -value is compared to the tabulated values for the normal distribution. The skewness test can be formulated as a two-sided (non-directional) statistical test or as a directional (one-tailed) test that is oriented towards testing for either positive or negative skewness. In this dissertation we will test solely for positive skewness (since this is the real-option indicator), i.e., for skewness larger than zero. The one-tailed hypothesis (test for positive skewness) is formally given by:

$$H_0: \sqrt{\beta_1} = 0$$

$$H_1: \sqrt{\beta_1} > 0$$

3.4.5.3 Kurtosis test¹¹²

The applied kurtosis definition is related to the fourth moment about the mean.

$$\mu_4^c = \hat{m}_4 = \frac{\sum (X - \bar{x})^4}{n} \quad (78)$$

The computational Equation usually applied to estimate the kurtosis of a population is given in Equation (79).¹¹³

$$\hat{k}_4 = \frac{(n^3 + n^2) \sum X^4 - 4(n^2 + n) \sum X^3 \sum X - 3(n^2 - n) (\sum X^2)^2 + 12n \sum X^2 (\sum X)^2 - 6(\sum X)^4}{n(n-1)(n-2)(n-3)} \quad (79)$$

¹¹² See Sheskin (2007), pp. 215-216.

¹¹³ Note that k_4 is not an estimate of m_4 ; see Sheskin (2007), p. 25.

Similar to the skewness calculation, the value $\hat{k}_4$ is standardized into a unitless statistic $\hat{g}_2$ (referred to as the kurtosis coefficient) by:

$$\hat{g}_2 = \frac{\hat{k}_4}{\hat{s}^4} \quad (80)$$

The bias in the kurtosis coefficient is further reduced by:

$$\hat{b}_2 = \frac{(n-2)(n-3)\hat{g}_2}{(n+1)(n-1)} + \frac{3(n-1)}{n+1} \quad (81)$$

We will use $\hat{b}_2$ as the measure of kurtosis that is utilized for inference (i.e., hypothesis testing) on the population kurtosis.¹¹⁴ The hypothesis test will be formulated with respect to the population parameter β_2 for which $\hat{b}_2$ is the estimate. Alternatively, it is possible to use $\hat{g}_2$ for defining the hypothesis test (referring then to the population parameter as γ_2 for which $\hat{g}_2$ is the estimate).

The statistical test that is computed is, like the skewness test, cumbersome. The following set of Equations is required.

$$G = \frac{24n(n-2)(n-3)}{(n+1)^2(n+3)(n+5)} \quad (82)$$

$$H = \frac{(n-2)(n-3)g_2}{(n+1)(n-1)\sqrt{G}} \quad (83)$$

$$J = \frac{6(n^2 - 5n + 2)}{(n+7)(n+9)} \sqrt{\frac{6(n+3)(n+5)}{n(n-2)(n-3)}} \quad (84)$$

¹¹⁴ Eviews[®] uses b_2 as the sample statistic to measure kurtosis.

$$K = 6 + \frac{8}{J} \left(\frac{2}{J} + \sqrt{1 + \frac{4}{J^2}} \right) \quad (85)$$

$$L = \frac{1 - \frac{2}{K}}{1 + H \sqrt{\frac{2}{K-4}}} \quad (86)$$

$$z = \frac{1 - \frac{2}{9K} - \sqrt[3]{L}}{\sqrt{\frac{2}{9K}}} \quad (87)$$

Equation (87) shows the calculation of the z -value that provides the approximation for the exact sampling distribution probabilities of $\hat{g}_2$ for $n \geq 20$. The z -value is compared with the tabulated values for the standard normal distribution. The test hypothesis is formulated as a directional (one-tailed) hypothesis test either by γ_2 or by β_2 .

$$H_0: \gamma_2 = 0, \beta_2 = 3$$

$$H_1: \gamma_2 > 0, \beta_2 > 3$$

3.5 Summary and conclusion

Chapter 3 explains the major definitions and concepts surrounding real-option analysis. We have laid out the theoretical framework for analyzing real-option information in markets by the distribution of equity prices. We started with a conventional example of NPV calculation to show how uncertainty and the consideration of real-options affect the distribution of the NPV. This simulation example was extended further to price financial options and to analyze the expected changes in the option price return for different moneyness assumptions. We have also introduced the concept of option return elasticity as a function of moneyness. These simulation-based results have been used to show how optionality affects the option price return distribution and helped to formulate assumptions for searching for real-options in traded equity markets. These results have been further supplemented by a theoretical model that relies on contingent claims analysis (CCA), which helps to link equity distribution to option prices. The results have been used to derive a set of hypotheses that can be tested by empirical analysis.

Furthermore, the distribution moments of price returns have been linked to utility functions and adjusted Sharpe-ratios in order to show how higher distribution moments increase or decrease the value of a project or equity investment for investors.

We have also laid out the different tools for testing distributions and formulating inferential statistical tests that are conducted in the empirical part of this dissertation. The work in this chapter is hence the basis for the empirical analysis in the following chapters.

4 Stochastic process analysis for commodity prices

4.1 Introduction

Since real-option analysis is only applicable in environments where the control variable (e.g., the price of oil or gold or any other underlying that affects the profits of a company directly via the turnover function or indirectly via the cost function) is stochastic, it is necessary to understand different stochastic processes (also referred to as data-generating processes or DGPs) and the implications for their econometric estimations and forecast probability distributions. The importance of the data-generating process is highlighted by the fact that usage of the wrong process can significantly alter the NPV probability and hence value outcome of a real-option. Calculating, for example, the present value of an oil-sensitive project using a DGP that allows negative forecast values for oil can lead to wrong valuation and hence to a wrong decision analysis. Therefore, the choice of the appropriate DGP is likely the most important decision that needs to be made in valuing a real-option project.

We will therefore analyze the most common DGPs and their ability to forecast an aggregated commodity price index, along with forecasting major individual commodity prices like oil, gold, silver, nickel and steel.

The different stochastic processes will be characterized by explaining their major characteristics and their probability distributions (mean, median and variance). Selected DGPs are calibrated afterwards using real commodity prices. These commodity prices are forecasted in order to analyze their forecast distribution. Additional stationary tests are conducted to analyze whether commodity prices are integrated or not, which has consequences for the DGP specifications.

Finally, the selected commodity prices are forecasted on a rolling out-of-sample basis by using different DGPs. The performance of the different DGPs is analyzed by calculating different forecast performance measures, which will help not only find the most applicable process for forecasting commodities, but also choose appropriate DGPs for real-option analysis.

4.2 Defining a stochastic process – terms and definitions¹¹⁵

A stochastic process is a collection of random variables ordered in time (or any other index). In a more formal framework, a one-dimensional stochastic process X can be defined within the probability space Ω as the function $X : \Omega \times T$, where T represents an index (usually time or any other non-random variable). This function is called a stochastic process. The realizations of this stochastic process are referred to as state space (see Equation (88)):

$$X(t, \omega) \mid \omega \in \Omega, t \in T \quad (88)$$

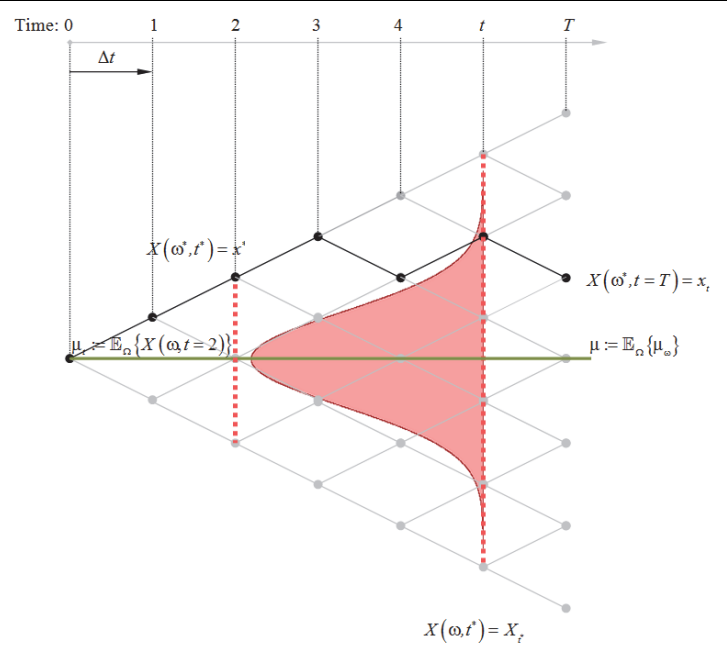
The non-random variable $t \in T$ spans the parameter space with either countable (for discrete time, called point process) or uncountable (for continuous time, called diffusion process) realizations of X .¹¹⁶

A particular realization of $X(t, \omega)$ is depicted by either referring to the trajectory in question (ω^*) or by referring to the index (t^*) or both (ω^*, t^*) or none (ω, t) . The first case with fixed (ω^*) results in a specific time series trajectory $X(\omega^*, t) = x_t$. The second case, which is fixed (t^*) , results in a cross-section of all possible trajectory outcomes at time (t^*) referred to by $X(\omega, t^*) = X_{t^*}$ and hence in a distribution function. Figure 59 gives the graphical interpretation.

¹¹⁵ See Rinne and Specht, (2002), pp. 157-168.

¹¹⁶ We will use $X(t)$ to denote a random variable in continuous time and X_t for a discrete time environment.

Figure 59
Distinguishing between random trajectories and random variable distributions



Defining different realizations of a random stochastic process experiment by either describing an individual trajectory at time t or referring to a cross-section of all possible trajectories at time t by using a probability function. See Rinne and Specht (2002), p. 158.

A stochastic process can therefore be interpreted in two ways. First, it can be seen as a collection of several time series of which one realization is depicted by randomness (one realization of ω). Second, it can be seen as a countable sequence of random variables where each index $t \in T$ has one random variable $(X_1, \dots, X_t, \dots)$.

The joint distribution function of any finite set of random variables that constitutes the random (stochastic) process (i.e., the sequence of random variables) is defined as:

$$F(x_1, x_2, \dots, x_n | t_1, t_2, \dots, t_n) := P[X(t_1) \leq x_1, X(t_2) \leq x_2, \dots, X(t_n) \leq x_n] \quad (89)$$

Thus, it is assumed by definition that with $t \rightarrow \infty$ a stochastic process has a well-defined joint distribution function. Analogous to the distribution function argument, one can state that there exists for every stochastic process at a given point in time t :

- a mean function $\mu_t := E_{\Omega} \{X(\omega, t)\}$ as average of each possible realization (i.e., average of expected trajectories) of $X(t, \omega)$ at time t ¹¹⁷
- a variance function $\sigma_t^2 := E_{\Omega} \{[X(\omega, t) - \mu_t]^2\} = E_{\Omega} \{X(\omega, t)\} - \mu_t^2$ as measure of dispersion of the random variable X_t around the mean at time t
- an autocovariance function of $X(t, \omega^*)$

$$Cov(t_1, t_2) := E_{\Omega} \{[X(\omega, t_1) - \mu(t_1)][X(\omega, t_2) - \mu(t_2)]\}$$

Note that the probability structure of the stochastic process (and hence the stochastic process itself) is under certain conditions¹¹⁸ defined along its joint distribution function. According to the extension theorem of Kolmogorov, a suitable collection of finite-dimensional distributions can define a stochastic process.¹¹⁹ Hence, with increase in time t we usually do not need an infinite dimensional distribution to describe the probability structure of the stochastic process. As pointed out by Rinne and Sprecht (2002), in several cases it is sufficient to know only the first two moments (i.e., stochastic process functions mean and variance) of the distribution function, as in the case of Figure 59, to describe fully the stochastic process.

The above used stochastic process definitions are necessary to define three major properties of stochastic processes, namely: stationary, Markov, and Martingale.

¹¹⁷ We could also calculate a time-independent mean μ_{ω} or the total mean as average of all realizations $\mu := E_{\Omega} \{\mu_{\omega}\}$.

¹¹⁸ These so-called regularity conditions for the stochastic process are defined by symmetry and compatibility. See Maddala and Kim (1998), pp. 8-9.

¹¹⁹ See B. Oksendal (2005), p. 11 and Rinne and Sprecht (2002), p. 159.

4.2.1 Stationary property

After we have defined the most important functions (mean, variance and autocovariance functions) for stochastic processes, we can utilize these functions to define stochastic process stationarity. If the described functions are time-invariant (i.e., outcome does not depend on the point in time when the observation takes place) we can assume stationarity. In this context we define stationarity as:

$$\text{Mean stationarity: } \mu_t = \mu = \text{constant} \quad \forall \quad t \in T$$

$$\text{Variance stationarity: } \sigma_t^2 = \sigma^2 = \text{constant} \quad \forall \quad t \in T$$

$$\text{Covariance stationarity: } \text{Cov}(X(t_1), X(t_2)) = \text{Cov}(X(t), X(t+j)) = \text{constant} \quad \forall \quad t \in T$$

The last stationarity definition – the autocovariance stationarity – states that the covariance is dependent only on the length of the data shift ($t_2 - t_1 = j$) but not on a specific point in time t when the covariance is calculated. The autocovariance is therefore a function of j but not of the point in time t . Depending on how many of the stationarity conditions are satisfied we can further distinguish between strong and weak stationarity. Strong stationarity is formally defined as:¹²⁰

$$F(x_1, x_2, \dots, x_n | t_1, t_2, \dots, t_n) = F(x_1, x_2, \dots, x_n | t_1 + j, t_2 + j, \dots, t_n + j) \text{ with } t_i + j \in T \quad \forall \quad i \quad (90)$$

Strong stationarity indicates that any shift in the observation point does not alter the distribution characteristics (represented here by the cumulative distribution function) of the stochastic process. Hence, all stochastic process functions remain unaffected (i.e., all forms of stationarity listed above are ensured). This is not only limited to the first and second moment, but also extends to higher distribution moments. Thus, the joint distribution function does not change with the observation point but only with the interval length.

¹²⁰ See Rinne (2003), pp. 392-395.

Weak stationarity is – as the name suggests – less rigid. Weak stationarity assumes only that mean and covariance stationarity are assured. Note that autocovariance stationarity leads to variance stationarity, since:

$$\text{Cov}(X(t), X(t)) = \sigma_t = \sigma = \text{constant} \quad \forall \quad t \in T \quad (91)$$

Applying the above definitions, one can conclude that the stochastic process in Figure 59 is only mean-stationary but neither variance- nor covariance-stationary (red-dotted line changes over time).

Related to the concept of stationarity is the concept of ergodicity. Ergodicity of a time series ensures the possibility of estimating the stochastic process distribution moments (called ensemble moments) from moments of one particular time series realization (ω). Ergodicity of a stochastic process is given if time series realization moments converge to the ensemble moments.¹²¹

4.2.2 Markov property

The Markov property of a stochastic process relates to the conditionality of a stochastic process probability distribution on past information. A stochastic process $(X(t_{n+1}))_{t \in T}$ fulfills the Markov property if, for a finite discrete or continuous time sequence $(t_0, t_1, t_2, \dots, t_n, t_{n+1})$, the following condition for its probability distribution is satisfied:¹²²

$$P[X(t_{n+1}) \leq x | X(t_n) = x_n, \dots, X(t_1) = x_1, X(t_0) = x_0] = P[X(t_{n+1}) \leq x | X(t_n) = x_n] \quad (92)$$

Equation (92) states that the probability function of the stochastic process $(X(t_{n+1}))$ at time (t_{n+1}) is conditional only on the last observation at (t_n) and not previous

¹²¹ See Wang (2003), pp. 6-7.

¹²² See Neftci (2000), pp. 108-109.

developments. Hence, the process development after (t_n) is not influenced by the process development before (t_n) . Therefore, a stochastic process with independent increments $(X(t_i) - X(t_{i-1}))$ is also a Markov process.

The knowledge of the past is therefore irrelevant for assessing the development of the future. This assumption becomes crucial for calculating the expected value or variance of the stochastic process; it is vital for pricing of contingent claims, for dynamic programming¹²³ and for numerous other applications in finance.¹²⁴

4.2.3 Martingale property

A sequence of random variables $(X(t_1), \dots, X(t_n), \dots)$ that forms a stochastic process $(X(t_{n+1}))_{t \in T}$ fulfills the Martingale property (i.e., is called Martingale) if for $t_1 < t_2 < \dots < t_n < t_{n+1}$ the following expression holds:

$$E[X(t_{n+1}) | X(t_1) = x_1, X(t_2) = x_2, \dots, X(t_n) = x_n] = x_n \quad (93)$$

Thus, a stochastic process is called Martingale if the expected value of the random variable for (t_{n+1}) is equal to the current (realized) value (x_n) . Thus, the expectation value for (t_{n+1}) is not influenced by past realizations of the stochastic process $(X(t_{n+1}))_{t \in T}$. It follows that a Martingale process shows no trend in its trajectories. In contrast to the Markov property, which is defined within a probability context (as a

¹²³ For dynamic optimization under uncertainty utilizing the Bellman principle, which explicitly relies on the Markov property of stochastic processes, see Dixit and Pindyck (1994), pp. 95-114.

¹²⁴ Note: if a stochastic process is not of Markov type, numerical methods like Monte Carlo simulation cannot be applied, since the partial differential equation (PDE) that is derived from a stochastic differential equation (SDE) – Feynman-Kac theorem (see Shreve (2004, p. 286) – is not properly represented within the Monte Carlo framework. See Neftci (2000), pp. 108-110 and Fries (2007), pp. 46-47.

property of the distribution function for t_{n+1}), the Martingale property is defined as an expectation value (conditional on an information set that contains the past developments of the stochastic process). As with the Markov property, the Martingale property can also be formulated with respect to the stochastic process increments. For a Martingale $E(X(t_{i+1}) - X(t_i)) = 0$ must hold. A process that increases on average is called sub-Martingale. A process that declines on average is called super-Martingale. The expectation value in Equation (93) is then altered respectively.

- Sub-Martingale:

$$E[X(t_{n+1}) | X(t_1) = x_1, X(t_2) = x_2, \dots, X(t_n) = x_n] \geq x_n \quad (94)$$

- Super-Martingale:

$$E[X(t_{n+1}) | X(t_1) = x_1, X(t_2) = x_2, \dots, X(t_n) = x_n] \leq x_n \quad (95)$$

Excuse: Discounting uncertain cash-flows with the risk-free rate – Part I¹²⁵

Martingale theory is helpful for pricing assets with uncertain cash-flows, as in the case of options. The natural question for pricing uncertain cash-flows is how to discount them. As shown in Equation (93), in order for a stochastic process to fulfill the Martingale property it should not display any trend or prediodicity. Translated into asset prices as in the instance of a stock price (S_t) it must hold that:

$$E_t(S_T) = S_t \quad \forall \quad t < T \quad (96)$$

For risk-averse investors, however, we know that Equation (96) will not hold but instead investors will price future equity according to Equation (97).

¹²⁵ See also Neftci (2000), p. 122 in conjuncture with pp. 140-143.

$$E^p \left(e^{-r(T-t)} S_T | S_t \right) > S_t \quad \forall \quad t < T \quad (97)$$

Note that the expectation value is formed under the (real) probability distribution p . In order to make Equation (96) hold, we can either transform the real probability distribution p into an equivalent Martingale measure $\tilde{p}$ (see Equation (98)) by applying for instance the Girsanov Theorem or, if the utility functions of the investors are known, by applying the Radon-Nikodym transformation.¹²⁶

$$E^{\tilde{p}} \left(e^{-r(T-t)} S_T | S_t \right) = S_t \quad (98)$$

Another possibility – one that is intuitively more appealing – is to deduct the expected trend from $e^{-r(T-t)} S$ to make the deviations around the trend unpredictable and hence fulfill the Martingale property (Doob-Meyer decompositions).

The implications of the Martingale property are best illustrated for our purposes when focusing on the example of discounting option prices. Assuming that we can estimate, e.g., by using an arbitrary data-generating process and hence its distribution function p , the expected future price of an underlying S_t , we can state for a given option strike price K that at $t = T$:

$$E^p \left(c_T | I_T \right) = E^p \left(\max(S_T - K, 0) | I_t \right) \quad (99)$$

However, before expiry the question arises whether we can derive the fair market price of the option c_t under the “true,” i.e. non risk-neutral probability distribution p by using the risk-free interest rate for discounting (see Equation (100)).

¹²⁶ See Baz and Chacko (2004), pp. 28-41.

$$c_t = e^{-r(T-t)} E^p \left(\max(S_T - K, 0) | I_t \right) \quad (100)$$

Following Neftci (2000) this is only the case if $e^{-rt} c_t$ is itself a Martingale under information set I_t , i.e. if the option price discounted at the risk-free interest rate is unpredictable. If this is the case we can state:

$$E^p \left(e^{-rT} c_T | I_t \right) = e^{-rt} c_t \rightarrow E^p \left(e^{-r(T-t)} c_T | I_t \right) = c_t \quad \forall \quad t < T \quad (101)$$

This result in Equation (101) is very important and will be referred to in the empirical part of this dissertation, particularly in Section 5.4 (see p. 227).

4.3 Classification of common stochastic data generating processes

This section describes and classifies common stochastic data generating processes on the basis of their major characteristics, as defined in 4.2.1 through 4.2.3. Some of these processes represent building blocks for larger and more complex stochastic processes. We will also show closed form solutions for mean and variance if available.

Since most common stochastic processes used in financial mathematics are non-stationary processes, it is often the case that these processes are defined in terms of changes rather than in levels, by using their stochastic differential form (integrated process I(1) represented in first differences, in contrast to process I(0) represented in level terms).¹²⁷

¹²⁷ Note: often in the literature (μ, σ) is used for location and scale to define certain probability functions.

Here we use $(\hat{\mu}, \hat{\sigma})$ only for describing sample estimates.

Table 28
Most common data generating processes – Part 1

Description	White Noise Process	Random Walk Process	Random Walk with Drift	Wiener Process (Brownian Motion)	Arithmetic Brownian Motion
Stochastic process $(z(\omega, t) \omega \in \Omega, t \in T)$	$z_t = z_0 + \varepsilon_t$ $z_T = z_0 + \varepsilon_T$	$z_{t+1} = z_t + \varepsilon_t$ $z_T = z_0 + \sum_{i=1}^T \varepsilon_i$	$z_{t+1} = z_t + \mu t + \varepsilon_t$ $z_T = z_0 + \mu T + \sum_{i=1}^T \varepsilon_i$	$z_{t+1} = z_t + \sqrt{dt} \varepsilon_t$	$z_{t+1} = z_t + dz$
Stochastic differential equation (SDE)	$\Delta z = \varepsilon_t$	$\Delta z = z_{t+1} - z_t = \varepsilon_t$	$\Delta z = \varepsilon_t + \mu dt$	$dz = \sqrt{dt} \varepsilon_t$	$dz = a dt + b \sqrt{dt} \varepsilon_t$
Markov (persistent probability distribution assumption)	Yes	Yes	Yes	Yes	Yes
Martingale $E(z_{t+1}) = z_t$ (persistent mean assumption)	Yes	Yes	No	Yes	No
Stationary	Yes	I(1): yes I(0): mean (yes), variance (no)	I(1): yes I(0): mean (no), variance (no)	I(1): yes I(0): mean (yes), variance (no)	I(1): yes I(0): mean (no), variance (no)
$E(\Delta z)$	Zero (for Gaussian probability distribution)	0	$\mu \frac{t}{T} \rightarrow \mu \Delta t \rightarrow \mu dt$	0	$a \Delta t$
$E(z_T - z_0)$	0	0	$\mu T - z_0$	0	$a \Delta t N - z_0 = a T - z_0$
Variance: $\text{Var}(\Delta z)$	1	σ^2	$\sigma^2 t$	Δt	$b^2 \Delta t$
Variance: $\text{Var}(z_T - z_0)$	1	$\sigma^2 T$	$\sigma^2 T$	$N \Delta t = T$	$b^2 \Delta t N = b^2 T$
Mean: $E(z_T)$	z_0	z_0	$z_0 + \mu T$	z_0	$z_0 + a T$
Variance: $\text{Var}(z_T)$	1	$\sigma^2 T$	$\sigma^2 T$	$N \Delta t = T$	$b^2 \Delta t N = b^2 T$

Table 29
Most common data generating processes – Part 2

Description	Geometric Brownian Motion	Ito Process as Geometric Brownian Motion in Log Terms	Ornstein-Uhlenbeck Process	Log-normal Ornstein-Uhlenbeck Process	Poisson Process
Stochastic process $(z(\omega, t) \omega \in \Omega, t \in T)$	$z_{t+1} = z_t + dz$	$z_{t+\Delta t} = z_t \exp((\mu - 0.5\sigma^2)\Delta t + \sigma\sqrt{\Delta t}\varepsilon_t)$ $a(z, t) = \mu z_t, b(z, t) = \sigma z_t \rightarrow \text{GBM}$	$z_{t+1} = z_t + dz$	$z_{t+1} = z_t + dz$	$dz = \begin{cases} 1 & \text{if } \varepsilon_t[0,1] > (\lambda t) \text{ with } \approx P(\lambda t) \\ 0 & \text{if } \varepsilon_t[0,1] \leq (\lambda t) \text{ with } \approx P(1 - \lambda t) \end{cases}$
Stochastic differential equation (SDE)	$dz = \mu z dt + \sigma z \sqrt{dt} \varepsilon_t$	$d(\ln z_t) = \ln\left(\frac{z_{t+1}}{z_t}\right) = (\mu - 0.5\sigma^2)\Delta t + \sigma\sqrt{\Delta t}\varepsilon_t$	$dz = \eta(\bar{z} - z_t)dt + \sigma\sqrt{dt}\varepsilon_t$	$dz = \eta z_t(\mu - \ln z_t)dt + \sigma z_t \sqrt{dt}\varepsilon_t$	$P(dz = n = 1) = \exp(-\lambda t) \frac{(-\lambda t)^{n-1}}{(n-1)!}$
Markov (persistent probability distribution assumption)	Yes	Yes	No	No	Yes
Martingale $E(z_{t+1}) = z_t$ (persistent mean assumption)	No $\forall \eta < 0$	No	Yes	Yes	Yes
Stationary	I(1): yes I(0): mean (no), variance (no)	I(1): yes I(0): mean (no), variance (no)	Yes	Yes	I(1): yes I(0): mean (no), variance (no)
$E(\Delta z)$	$\mu z_t \Delta t$	$(\mu - 0.5\sigma^2)\Delta t$	$\eta(\bar{z} - z_t)dt$	$\eta(\alpha - \ln z_t)dt$ with $\alpha = \bar{z} - \frac{1}{2\eta}\sigma^2$	$\lambda \Delta t$
$E(z_T - z_0)$	$z_0(1 + \mu\Delta t)^N - z_0$	$z_0 \exp(\mu T) - z_0$	$(\bar{z} + (z_0 - \bar{z})\exp(-\eta T)) - z_0$	$(\alpha + (\ln z_0 - \alpha)\exp(-\eta T)) - \ln z_0$	$\lambda \Delta t N - z_0$
Variance: $\text{Var}(\Delta z)$	$(\sigma z_t)^2 \Delta t$	$(\sigma z_t)^2 \Delta t$	$\sigma^2 \Delta t$	$\sigma^2 \Delta t$	$\frac{\lambda \Delta t}{T}$
Variance: $\text{Var}(z_T - z_0)$	$z_t^2(1 + 2\mu\Delta t + \sigma^2\Delta t)^N - z_t^2(1 + \mu\Delta t)^{2N}$	$z_0^2 \exp(2\mu T)(\exp(\sigma^2 T) - 1)$	$\frac{\sigma^2}{2\eta}(1 - \exp(-2\eta t))$	$\frac{\sigma}{2\eta}(1 - \exp(-2\eta t))$	λ
Mean: $E(z_T)$	$z_0(1 + \mu\Delta t)^N$	$z_0 \exp(\mu T)$	$\bar{z} + (z_0 - \bar{z})\exp(-\eta T)$	$\alpha + (\ln z_0 - \alpha)\exp(-\eta T)$	$\lambda \Delta t N + z_0$
Variance: $\text{Var}(z_T)$	$z_t^2(1 + 2\mu\Delta t + \sigma^2\Delta t)^N - z_t^2(1 + \mu\Delta t)^{2N}$	$z_0^2 \exp(2\mu T)(\exp(\sigma^2 T) - 1)$	$\frac{\sigma^2}{2\eta}(1 - \exp(-2\eta t))$	$\frac{\sigma}{2\eta}(1 - \exp(-2\eta t))$	λ

4.4 Calibrating the data generating process (DGP) parameters

In the previous sections we described and analyzed several common data generating processes (DGPs) that are applied in standard financial engineering. However, we have not explained how these different DGPs are estimated and hence how one could fit observable data in an econometric framework to these different processes. This section aims to fill that gap. We will describe the econometric estimation procedure for the main real-option relevant DGPs. Furthermore we will fit several commodity prices to different DGPs and test their goodness of fit according to their ability to forecast out of sample. Last but not least, we will test several other DGPs such as trend stationary processes (TSPs), difference-stationary processes (DSPs) and mixture models that combine trend- and difference-stationary processes with each other.

4.4.1 *Estimating conventional DGPs via standard OLS methods*

Most of the described stochastic processes belong to the family of Ito processes. Although there are numerous additional process combinations possible, we will focus only on the most relevant ones for real-option pricing, which can be summarized as follows:

1. Arithmetic (additive) Brownian motion (ABM)
2. Geometric Brownian motion (GBM)
3. Arithmetic (additive) Ornstein-Uhlenbeck process (AOUP)
4. Log-normal Ornstein-Uhlenbeck process (LnOUP)

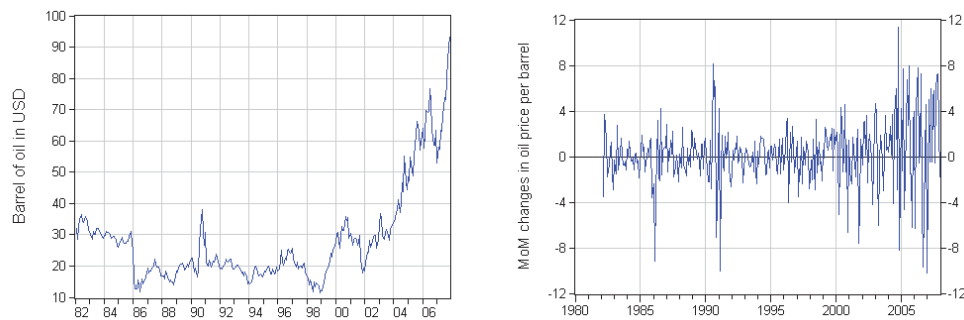
All the above processes can be estimated via OLS or methods of moments.¹²⁸ Going forward, we will use the development of oil price measured in USD between 1982 and

¹²⁸ Note that often general methods of moments (GMM) is applied instead of OLS to account for the possible non-normality of residuals and autocorrelation that usually leads to wrong test statistics in time-series analysis. However, given that the number of moment-matching conditions (i.e. orthogonal conditions) is equal to the number of coefficients estimated (i.e. the model is viewed as fully-identified GMM), the estimated coefficients using GMM would be identical to the OLS estimates. Using OLS we will therefore only need to adjust the standard errors by accounting for heteroskedasticity and serial

2008 to illustrate the different process parameterizations.¹²⁹ The data set frequency used is monthly. The corresponding graph for oil is given in Figure 60.

Figure 60

Development of the oil price (WTI in USD per barrel) over time



Panel A: History of WTI oil in USD between 1982 and 2008. The historical price band between 15 and 30 USD has been broken to the upside since 2003.

Panel B: Monthly differences of WTI oil between 1982 and 2008. Changes in oil prices have become more volatile in recent history.

As illustrated the oil price has experienced a decent run, reaching nearly 100 USD per barrel by the end of 2007. Month-over-month (MoM) changes in the oil price experience strong fluctuation, and it appears that either the oil price is following a time stationary process (rather than a mean reverting process) and/or the changes in the oil price could be of heteroskedastic nature with strong volatility clustering and increasing variance over time. The empirical section of this chapter will analyze the question of stationarity in more detail. For this stage it will be sufficient to acknowledge the development of the level variable (oil) and the first difference of the oil price (the change in the oil price).

4.4.1.1 Estimating arithmetic Brownian motion (ABM) parameters

The additive also called arithmetic Brownian is the simplest of the stochastic processes used. It follows the following stochastic differential equation:

autocorrelation (HAC) via the Newey and West (1987) adjustment. For a detailed discussion, see Hamilton (1994), pp. 416-417.

¹²⁹ The oil prices are based on West Texas Intermediate (WTI) pricing.

$$dX(t) = \mu dt + \sigma dW(t) \quad (102)$$

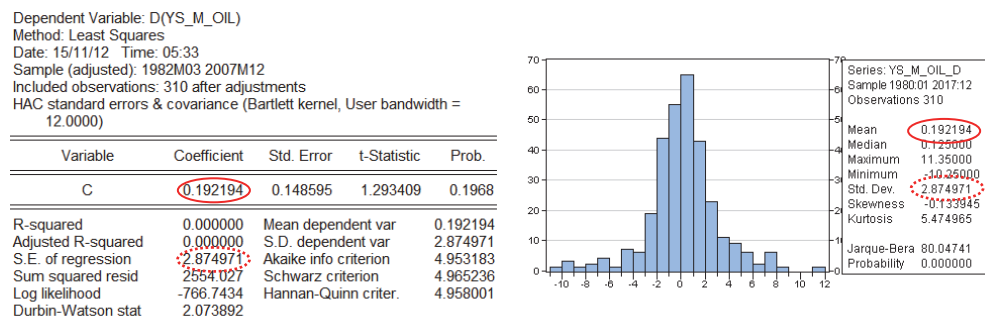
Where $dW(t) = \sqrt{dt}\varepsilon(t)$ and $\varepsilon(t)$ is a standard normal random variable. Its estimation in first differences is straightforward. Applying Euler discretization¹³⁰ one can write the above SDE in its discrete estimation form:

$$\Delta x_t = \mu\Delta t + \sigma\Delta W(t) \rightarrow \Delta x_t = x_t - x_{t-1} = c + \varepsilon_t \quad (103)$$

Since we work with first differences on the left hand side, we assume a stochastic trend in the data. Note that the constant c is actually the trend determinant. As illustrated in Figure 61, the estimated regression intercept c is equal to the mean of the month-over-month changes (see Panel B).¹³¹

Figure 61

Estimating an additive Brownian motion process via OLS

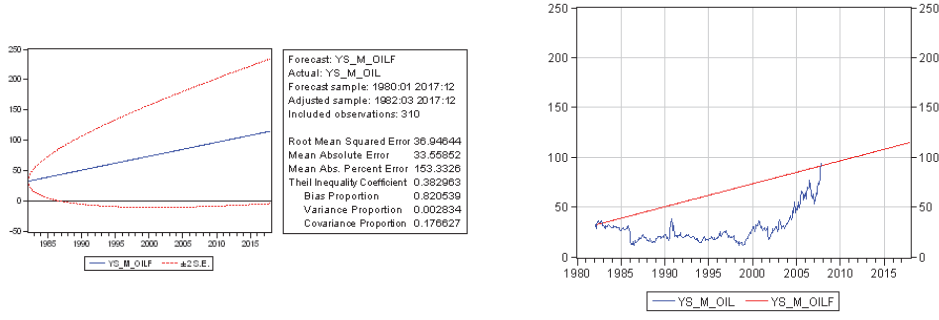


Panel A: OLS estimation results for ABM as data generating process for oil. Newey and West (1987) corrected standard errors with 12-month lags.

Panel B: Distribution characteristics of the underlying first differences.

¹³⁰ An in-depth treatment of discretization methods can be found in Glasserman (2004), pp. 339-376 and Fries (2007), pp. 181-198.

¹³¹ Hence, the process can be calibrated alternatively by calculating directly the first and second moment of the return distribution. For an example of this calibration method see Tsay (2005), pp. 259-261.



Panel C: In-sample and out-of-sample forecast for calibrated ABM along with the standard error forecast boundaries.

Panel D: Forecasting the trend of the ABM over time. The estimated trend component (OLS intercept) matches the last sample observation.

The standard error of estimate is equal to the sample standard deviation (see Panel B). With the intercept c and the standard error of estimate $\hat{\sigma}_\varepsilon$ all the required data to parameterize the arithmetic Brownian motion are available.

$$dX(t) = \mu dt + \sigma dW(t) \quad (104)\text{-a}$$

$$\mu dt \rightarrow c \quad (104)\text{-b}$$

$$\sigma dW(t) \rightarrow \hat{\sigma}_\varepsilon \quad (104)\text{-c}$$

Since we work with monthly data, the time interval dt is equal to one (i.e. one month) and can be omitted. In Figure 61, Panels C and D, we show the out-of-sample estimate and the standard error band for the arithmetic Brownian motion (note that within the arithmetic setup, the underlying theoretically could become negative, which is not realistic to assume for commodities). Due to the constant stochastic trend, the forecast crosses the last observation point (in December 2007) and continues to grow – due to its additive nature – at a constant (linear) rate.

4.4.1.2 Estimating geometric Brownian motion (GBM) parameters

The geometric Brownian motion process is the most commonly used process in financial engineering and real-option literature. The GBM process possesses the advantage of non-negativity, which is required for a vast majority of applications. The SDE is given below:

$$dX(t) = \mu X(t)dt + \sigma X(t)dW(t) \quad (105)$$

Where again $dW(t) = \sqrt{dt}\varepsilon(t)$ and $\varepsilon(t)$ is a standard normal random variable. After the Euler discretization and rearrangement the Equation is expressed in logarithmic terms:

$$\frac{x_t - x_{t-1}}{x_{t-1}} \Rightarrow \ln(x_t/x_{t-1}) = \mu\Delta t + \sigma\Delta W_t \quad (106)$$

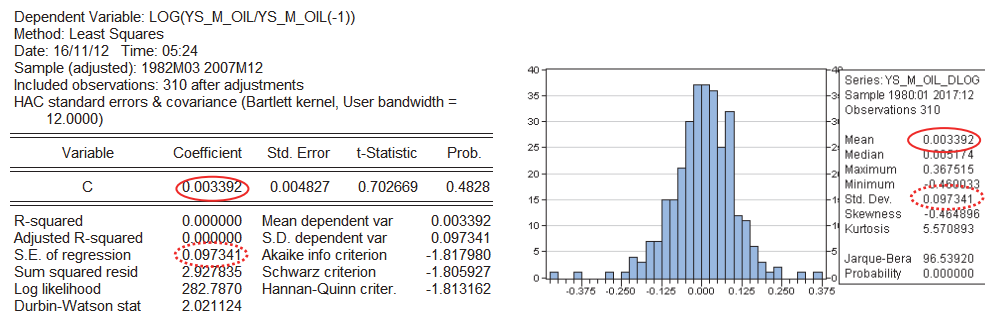
The regression Equation hence becomes:

$$\Delta x_t = c + \varepsilon_t = \ln(x_t) - \ln(x_{t-1}) = c + \varepsilon_t \quad (107)$$

Similar to the arithmetic Brownian motion, the SDE results in a stochastic trend process where the constant rate of change is given by the intercept c . The difference now is that the left hand side of the regression is a logarithmic difference, which translates into an exponential increase when transformed back to non-logarithmic units. In Figure 62 we have calculated the parameter estimates for the oil data series.

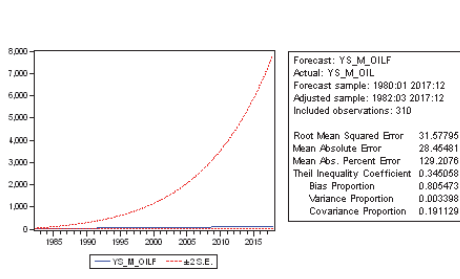
Figure 62

Estimating a geometric Brownian motion process via OLS

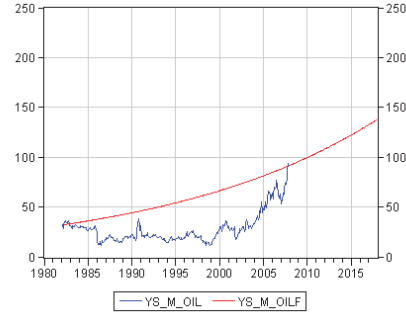


Panel A: OLS estimation results for GBM as data generating process for oil. Newey and West (1987) corrected standard errors with 12-month lags.

Panel B: Distribution characteristics of the underlying first differences.



Panel C: In-sample and out-of-sample forecast for calibrated GBM along with the standard error forecast boundaries.



Panel D: Forecasting the trend of the GBM over time. The estimated trend component (OLS intercept) matches the last sample observation.

The intercept is simply the sample mean, while the standard error of estimate is equal to the sample standard deviation (see Panel B). With the intercept c and the standard error of estimate $\hat{\sigma}_\varepsilon$ we have again all the required data to parameterize the data generation process.

$$\ln X(t) - \ln X(t-1) = \mu dt + \sigma dW(t) \quad (108)\text{-a}$$

$$\mu dt \rightarrow c \quad (108)\text{-b}$$

$$\sigma dW(t) \rightarrow \hat{\sigma}_\varepsilon \quad (108)\text{-c}$$

Similar to the arithmetic Brownian motion, the forecast goes through the last observed value (as expected when working with a stochastic trend data generating process). Hence, the last observation becomes crucial for the entire out-of-sample forecast. This, however, is the major flaw and hence a disadvantage for using both the ABM and the GBM processes. Given the very low significance of the estimated coefficients, the standard error bands evolve towards extreme levels (see Panel C).

4.4.1.3 Estimating arithmetic Ornstein-Uhlenbeck process (AOUP) parameters

Numerous commodities show the tendency of mean reversion. The theory of mean reversion is in this case based on the assumption that commodities are produced at

marginal cost levels (long-term mean price of commodity = long-run marginal cost of production of this commodity¹³²) and that deviations in the price (deviations away from the marginal cost optimality) should be short-lived. In such a case prices should mean revert. The question of mean reversion is linked to the question of stationarity of the time series, which is treated in Section 4.5, p. 163.

The standard arithmetic Ornstein-Uhlenbeck process (AOUP) is given below.

$$dX(t) = \eta(\bar{X} - X(t))dt + \sigma dW(t) \quad (109)$$

Estimates of the mean-revision parameter η and the diffusion parameter σ are obtained by running the following regression:

$$\Delta x_t = x_t - x_{t-1} = c + \phi_{t-1} + \varepsilon_t \quad (110)$$

In contrast to the previously employed regression equations, Equation (110) contains an autoregressive component ϕx_{t-1} . This autoregressive component holds, in addition to the intercept and the standard error of estimate, information necessary for the AOUP process calibration.¹³³ The required AOUP-process variables and their estimators for the autoregressive regression in either I(1)- and I(0)-form are given below:

$$dX(t) = \eta(\bar{X} - X(t))dt + \sigma dW(t) \quad (111)\text{-a}$$

$$\Delta x_t = c + \phi_1^d x_{t-1} + \varepsilon_t \Leftrightarrow x_t = c + \phi_1^l x_{t-1} + \varepsilon_t \quad (111)\text{-b}$$

AR process as I(1) vs AR process in I(0)

¹³² See Dixit and Pindyck (1993), p. 74.

¹³³ The AOUP process could actually be calibrated via three regression equations. It is possible to estimate $x_t = c + \phi_1 x_{t-1} + \varepsilon_t$ or alternatively $x_t = c + \text{AR}(1) + \varepsilon_t$ to obtain an AOUP calibration. The regression output would change accordingly; however, the estimated parameter would uncover the same information as in the Equation (110). See Sodal et al. (2006), p. 12.

$$\text{mean reversion level: } \hat{\eta} = -\ln(1 + \hat{\phi}) \Leftrightarrow \hat{\eta} = -\ln(\hat{\phi}) \quad (111)\text{-c}$$

$$\text{long term mean: } \hat{\bar{x}} = -\frac{\hat{c}}{\hat{\phi}} = \frac{\hat{c}}{(1 - e^{-\hat{\eta}})} \Leftrightarrow \hat{\bar{x}} = \frac{\hat{c}}{1 - \hat{\phi}} \quad (111)\text{-d}$$

$$\text{standard deviation: } \hat{\sigma} = \hat{\sigma}_\varepsilon \sqrt{\frac{2\ln(1 + \hat{\phi})}{(1 + \hat{\phi})^2 - 1}} \quad (111)\text{-e}$$

The closed form solutions for the AOUP parameterization depend on the regression form (either in levels or in delta terms). The entire derivation of the closed form solution is given Appendix D, p. 342.¹³⁴ At this point it is necessary to highlight some important issues with respect to the parameter results. First, since the above AOUP is additive in nature, negative values are possible. This, however, is not plausible of financially traded assets. In order to assure non-negativity of the AOUP it is necessary that the condition $\eta \bar{x} > 0.5\sigma^2$ is fulfilled.¹³⁵ Secondly, if the mean reversion speed η approaches zero, the AOUP becomes a standard Brownian motion with a time dependent (increasing) variance $\sigma^2 t$ (see Table 28, p. 143). If the mean reversion speed becomes negative, the AOUP becomes unstable and does not convert towards a long term mean.¹³⁶ Thirdly, if the mean reversion speed increases $\eta \rightarrow \infty$ the variance from the long term mean $\bar{x}$ becomes constant and converges towards $\sigma^2/2\eta$.

In Figure 63 we have run two OLS regressions on the oil price using the regression Equation (111)-b. The two OLS regressions differ in the sample length. The results in the upper Panels are estimated with a sample until February 2007 while the lower Panel uses a sample until December 2007.

¹³⁴ A maximum likelihood estimator for the additive Ornstein-Uhlenbeck model (i.e., the Vasicek model) is also given in Brigo and Mercurio (2001), p. 53.

¹³⁵ See Baxter and Rennie (1996), p. 156.

¹³⁶ Note: in level terms, where the regression equation is $(x_t) = c + \phi_1^l x_{t-1} + \varepsilon$, the stability condition that guaranties mean reversion is equivalent to $\phi_1^l < 1$. The parameters ϕ_1^l and ϕ_1^d are linked via $\phi_1^l = 1 + \phi_1^d$.

Figure 63

Estimating an additive Ornstein-Uhlenbeck process via OLS

Dependent Variable: D(YS_M_OIL)
Method: Least Squares
Date: 16/11/12 Time: 05:50
Sample (adjusted): 1982M03 2007M02
Included observations: 300 after adjustments
HAC standard errors & covariance (Bartlett kernel, User bandwidth = 12.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.478705	0.354559	1.350140	0.1780
YS_M_OIL(-1)	-0.014640	0.014693	-0.996413	0.3199
R-squared	0.004349	Mean dependent var	0.086633	
Adjusted R-squared	0.001008	S.D. dependent var	2.789790	
S.E. of regression	2.788383	Akaike info criterion	4.895445	
Sum squared resid	2316.974	Schwarz criterion	4.920137	
Log likelihood	-732.3168	Hannan-Quinn criter.	4.905327	
F-statistic	1.301652	Durbin-Watson stat	2.120275	
Prob(F-statistic)	0.254828			

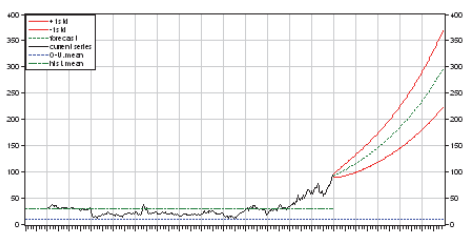


Panel A: Additive Ornstein-Uhlenbeck process estimated in I(1) for oil between March 1982 and February 2007. Newey and West (1987) corrected standard errors with 12-month lags.

Panel B: AOUP is stable and converges towards the long-term mean.

Dependent Variable: D(YS_M_OIL)
Method: Least Squares
Date: 16/11/12 Time: 05:53
Sample (adjusted): 1982M03 2007M12
Included observations: 310 after adjustments
HAC standard errors & covariance (Bartlett kernel, User bandwidth = 12.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.105686	0.432069	-0.244604	0.8069
YS_M_OIL(-1)	0.010553	0.017390	0.606861	0.5444
R-squared	0.002963	Mean dependent var	0.192194	
Adjusted R-squared	-0.000275	S.D. dependent var	2.874971	
S.E. of regression	2.875366	Akaike info criterion	4.956688	
Sum squared resid	2546.461	Schwarz criterion	4.980775	
Log likelihood	-766.2835	Hannan-Quinn criter.	4.966305	
F-statistic	0.915169	Durbin-Watson stat	2.102154	
Prob(F-statistic)	0.339497			



Panel C: Additive Ornstein-Uhlenbeck process estimated in I(1) for oil between March 1982 and December 2007. Newey and West (1987) corrected standard errors with 12-month lags.

Panel D: AOUP is unstable and does not converge towards a long-term mean over time.

The difference (only 10 months of data) in the forecast shows the weaknesses and danger of an AOUP if the underlying is showing a recent trend in the data (like the increase in oil prices during 2007). Due to the positive autoregressive coefficient, the forecasted values do not converge towards a long term mean.

For simulation purposes the following equation for discrete time steps (Δt) can be used:¹³⁷

¹³⁷See Appendix D, p. 342 for deriving the regression expression for an Ornstein-Uhlenbeck process. The simulation equation is simply the regression equation expressed as an expected value equation with random shocks. The random shocks, the last term in the equation, represent the standard error of regression.

$$x_{t+1} = x_t e^{-\eta \Delta t} + \bar{x} (1 - e^{-\eta \Delta t}) + \sigma \sqrt{\frac{1 - e^{-2\eta \Delta t}}{2\eta}} \varepsilon_t \quad (112)$$

4.4.1.4 Estimating log-normal Ornstein-Uhlenbeck process (LnOUP) parameters

The use of arithmetic Ornstein-Uhlenbeck processes has been widely criticized due to its additive nature and the resulting possibility of negative values. The possibility of negative values has already been illustrated and analyzed within the interest rate modeling field, where modeling the logarithm of the interest rate was found as a suitable way to bypass the problem of negativity within an additive Ornstein-Uhlenbeck process (i.e., Vasicek process).¹³⁸ Schwarz (1997) therefore adopted the log-normal form of the Ornstein-Uhlenbeck process to model commodity prices. The stochastic differential Equation of the LnOUP model (not to be confused with the geometric Ornstein-Uhlenbeck model) is of the following form:

$$dX(t) = \eta(\mu - \ln X(t))X(t)dt + \sigma X(t)dW(t) \quad (113)$$

The difference between the geometric OU model and the log-normal OU model is the log term in the drift parameter $\ln X(t)$. Using logarithms will yield, after the Ito transformation, the following equation that can be estimated econometrically.¹³⁹

$$d \ln X(t) = \eta(\alpha - \ln X(t))dt + \sigma dW(t) \quad (114)$$

The reversion speed η is equal to the reversion speed in Equation (111)-a. However, the long term mean μ becomes α :

¹³⁸ Also known as a special form of the Black Karasinski interest rate model; see Brigo and Mercurio (2001), p. 73.

¹³⁹ The proof is given in Appendix F, p. 350.

$$\alpha = \mu - \frac{1}{2\eta} \sigma^2 \quad (115)$$

To estimate the parameters α , η and σ one can apply the regression equation used for the AOUP:

$$\Delta(\ln x) = c + \phi \ln x_{t-1} + \varepsilon_t = \ln x_t - \ln x_{t-1} = c + \phi \ln x_{t-1} + \varepsilon_t \quad (116)$$

Analogous to the AOUP, the required parameters can be extracted according to the following set of equations:

$$\text{mean reversion level: } \hat{\eta} = -\ln(1 + \hat{\phi}) \quad (117)\text{-a}$$

$$\text{long term mean: } \hat{\alpha} = -\frac{\hat{c}}{\hat{\phi}} = \frac{\hat{c}}{(1 - e^{-\hat{\eta}})} \Rightarrow \hat{\mu} = -\frac{\hat{c}}{\hat{\phi}} + \frac{1}{2\hat{\eta}} \sigma^2 \quad (111)\text{-b}$$

$$\text{standard deviation: } \hat{\sigma} = \hat{\sigma}_\varepsilon \sqrt{\frac{2\ln(1 + \hat{\phi})}{(1 + \hat{\phi})^2 - 1}} \quad (117)\text{-c}$$

In Figure 64 we have repeated the out-of-sample regression for the example of oil. The difference between this regression and the AOUP regression (illustrated in Figure 63, p. 153) becomes most obvious for the full sample case. While the AOUP does not converge to a long-term mean (due to the positive autoregression coefficient), the LnOUP manages to do so, although with a much wider forecast error band. The revision speed, however, has decreased to a large extent, and the coefficient estimates have lost their significance compared to the smaller sample that excludes the strong trend in oil prices in 2007.

Figure 64

Estimating a log-normal Ornstein-Uhlenbeck process via OLS

Dependent Variable: D(LOG(YS_M_OIL))

Method: Least Squares

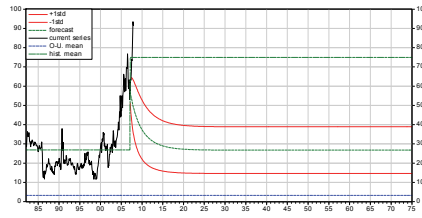
Date: 18/11/12 Time: 15:49

Sample (adjusted): 1982M03 2007M02

Included observations: 300 after adjustments

HAC standard errors & covariance (Bartlett kernel, User bandwidth = 12.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.077705	0.042056	1.847641	0.0656
LOG(YS_M_OIL(-1))	-0.023650	0.013146	-1.799030	0.0730
R-squared	0.009097	Mean dependent var	0.001982	
Adjusted R-squared	0.005772	S.D. dependent var	0.098270	
S.E. of regression	0.097986	Akaike info criterion	-1.801334	
Sum squared resid	2.861190	Schwarz criterion	-1.776643	
Log likelihood	272.2002	Hannan-Quinn criter.	-1.791453	
F-statistic	2.735798	Durbin-Watson stat	1.997548	
Prob(F-statistic)	0.099175			



Panel A: Log-normal Ornstein-Uhlenbeck process estimated in I(1) for oil between March 1982 and February 2007. Newey and West (1987) corrected standard errors with 12-month lags.

Panel B: LnOUP is stable and converges towards the long-term mean.

Dependent Variable: D(LOG(YS_M_OIL))

Method: Least Squares

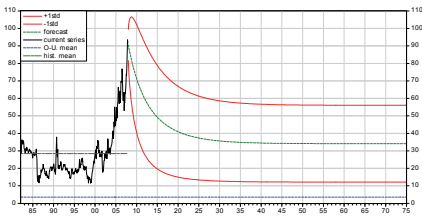
Date: 18/11/12 Time: 15:53

Sample (adjusted): 1982M03 2007M12

Included observations: 310 after adjustments

HAC standard errors & covariance (Bartlett kernel, User bandwidth = 12.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.040831	0.039636	1.030145	0.3038
LOG(YS_M_OIL(-1))	-0.011570	0.012238	-0.945393	0.3452
R-squared	0.002652	Mean dependent var	0.003392	
Adjusted R-squared	-0.000586	S.D. dependent var	0.097341	
S.E. of regression	0.097369	Akaike info criterion	-1.814185	
Sum squared resid	2.920089	Schwarz criterion	-1.790078	
Log likelihood	283.1987	Hannan-Quinn criter.	-1.804548	
F-statistic	0.819142	Durbin-Watson stat	2.003136	
Prob(F-statistic)	0.366139			



Panel C: Log-normal Ornstein-Uhlenbeck process estimated in I(1) for oil between March 1982 and February 2007. Newey and West (1987) corrected standard errors with 12-month lags.

Panel D: LnOUP is stable and converges towards the long-term mean although with a much higher forecast error band compared to Panel B.

4.4.2 Estimating DGPs with trend stationary components

4.4.2.1 Trend stationary processes

The previously tested models have generally been set up as random walk with drift models. The general form was:

$$y_t = c + y_{t-1} + \varepsilon_t \Rightarrow y_t - y_{t-1} = c + \varepsilon_t = \Delta y_t = c + \varepsilon_t \quad (118)$$

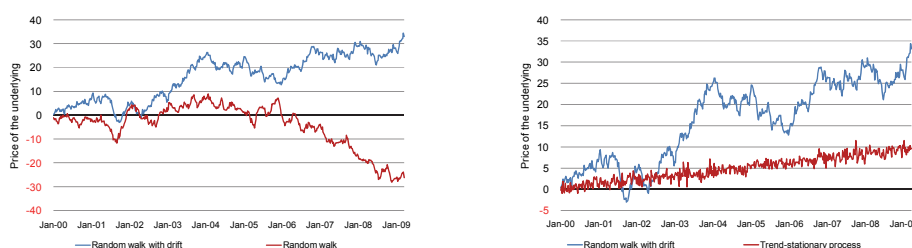
Data generation processes of this form are called stochastic non-stationarity processes, or simple stochastic processes.¹⁴⁰ In Figure 65, Panel A we have illustrated a simple random walk with drift together with a trend stationary process. The trend stationary process was generated using the following DGP:

$$y_t = c + \phi_1 \cdot t + \varepsilon_t \quad (119)$$

The right hand side of the Equation contains a deterministic time trend t on which the dependent variable (the regression variable) is regressed. This has major implications for the forecast (see Figure 65, Panel B) as illustrated by the different forecast dynamic between the stochastic trend and the deterministic trend model. Whereas the stochastic trend model continues its process path along the last observation point (i.e., shocks will have a permanent impact on the trend, with the new trend initiated from the last observation point), shocks in the deterministic trend model will only have a temporary effect, and the forecast will steadily revert to the original trend.

Figure 65

Distinguishing between deterministic and non-deterministic data generating processes



Panel A: Random walk and random walk with drift processes.

Panel B: Random walk with drift compared to time-deterministic (trend-stationary) process.

¹⁴⁰ For a good introduction to stochastic and non-stochastic trend processes, see Diebold (2004), pp. 333-335, and Brooks (2008), pp. 320-326.

Several variants of regression equations that are based on the trend stationarity assumption are applied. Some common are listed below:

$$y_t = c + \phi_1 \cdot t + \varepsilon_t \quad (120)\text{-a}$$

$$\ln y_t = c + \phi_1 \cdot t + \varepsilon_t \quad (120)\text{-b}$$

$$y_t = c + \phi_1 \cdot t + \phi_2 \cdot t^2 + \varepsilon_t \quad (120)\text{-c}$$

$$\ln y_t = c + \phi_1 \cdot t + \phi_2 \cdot t^2 + \varepsilon_t \quad (120)\text{-d}$$

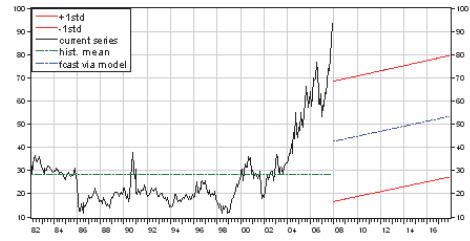
The assumption of a time-deterministic trend in most commodity prices appears plausible if one assumes that commodities are limited resources, which – similar to other assets – are affected by inflation and currency devaluation. However, the stability of the time regression coefficients is problematic, which raises the question of how to estimate models that allow for time trend coefficient instability.¹⁴¹ Depending on the model chosen, forecasts differ substantially. Figure 66 displays forecasts for the four major trend stationary processes mentioned above for $t - 60$ months and for spot forecasts. Among the diverse models, the quadratic trend stationary process seems to offer the best results. Still, together with the in-sample mean average it becomes obvious how the various trend stationarity models, depending on precise specifications, lead to significantly different results. In addition, identical DGP specifications show different forecasts depending on the total sample length. Thus, similar to the AOUP, sample length and recent history play significant roles in the forecast outcome, which reduces the stability and the reliability of the models. Hence, more stable processes should be sought in order to forecast commodity prices for conducting real-option analysis.

¹⁴¹ Commonly used methods to account for time trend instability include multivariate Ornstein-Uhlenbeck processes. Due to the unobservable nature of the changing trend dynamic (time dependence of trend modeled as latent factor), Kalman filtering is often applied; see Pindyck (1999), pp. 1-27.

Figure 66
Trend-deterministic forecast model variants for oil



Panel A: In-sample forecast between 2002 and 2007 for the oil price using $y_t = c + \phi_1 \cdot t + \varepsilon_t$ as DGP.



Panel B: Out-of-sample forecast between 2008 and 2017 for the oil price using $y_t = c + \phi_1 \cdot t + \varepsilon_t$ as DGP.



Panel C: In-sample forecast between 2002 and 2007 for the oil price using $\ln y_t = c + \phi_1 \cdot t + \varepsilon_t$ as DGP.



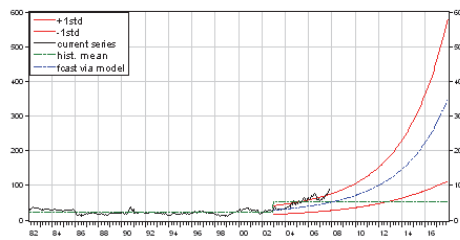
Panel D: Out-of-sample forecast between 2008 and 2017 for the oil price using $\ln y_t = c + \phi_1 \cdot t + \varepsilon_t$ as DGP.



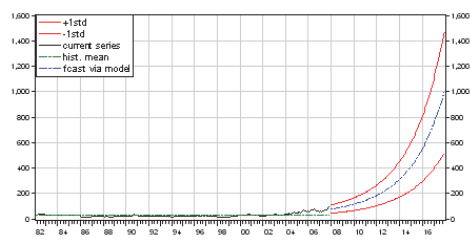
Panel E: In-sample forecast between 2002 and 2007 for the oil price using $y_t = a + b \cdot t + c \cdot t^2 + \varepsilon_t$ as DGP.



Panel F: Out-of-sample forecast between 2008 and 2017 for the oil price using $y_t = a + b \cdot t + c \cdot t^2 + \varepsilon_t$ as DGP.



Panel G: In-sample forecast between 2002 and 2007 for the oil price using $\ln y_t = c + \phi_1 \cdot t + \phi_2 \cdot t^2 + \varepsilon_t$ as DGP.



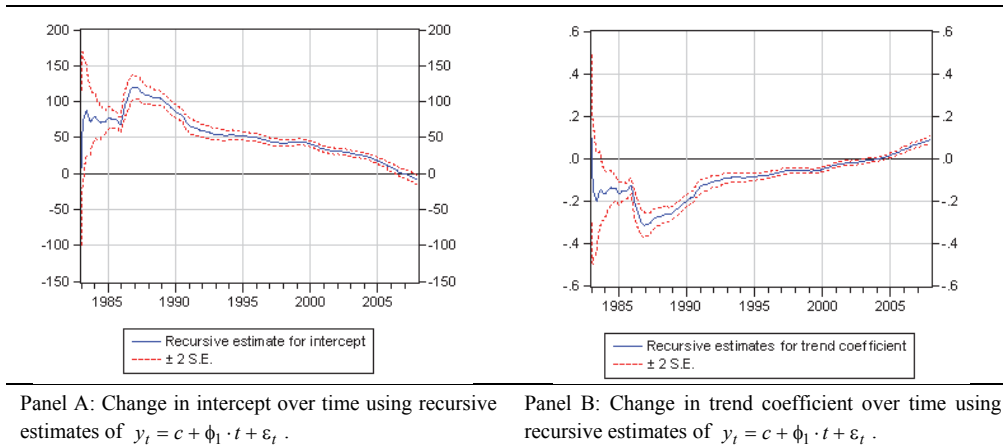
Panel H: Out-of-sample forecast between 2008 and 2017 for the oil price using $\ln y_t = c + \phi_1 \cdot t + \phi_2 \cdot t^2 + \varepsilon_t$ as DGP.

4.4.2.2 Stochastic and deterministic trend (mixture) models

As previously shown, trend stationary (time trend specification) and difference stationary (random walk or mean reverting specification) models often yield ambivalent results. Coefficient stability appears to be a problem for both approaches if the underlying experiences major trend periods. As shown in Figure 63 and Figure 66, both approaches produce totally different forecast dynamics when small adjustments in the sample length are made. This problem can be highlighted by conducting coefficient stability tests. In Figure 67 such a recursive coefficient stability analysis is conducted for the forecast equation $y_t = c + \phi_1 \cdot t + \varepsilon_t$; both illustrations show how the estimated intercept and correlation coefficient change with an increasing sample size without converging to a stable equilibrium.

Figure 67

Coefficient stability test for the forecast equation $y_t = c + \phi_1 \cdot t + \varepsilon_t$ estimated for oil between 1982 and 2008



Therefore alternative DGPs need to be investigated with potentially higher coefficient stability. A first approach comes from the unit-root test literature. One possibility involving nest trending and difference stationarity in one test equation involves running the following regression:¹⁴²

¹⁴² See Brooks (2008), p. 323.

$$\Delta y_t = c + \phi_1 \cdot t + \phi_2 \cdot y_{t-1} + \varepsilon_t \quad (121)$$

This so-called mixture model assumes mean reversion to a linear trend. It hence allows deterministic and stochastic trends, and contains elements of all the previously described data generating processes – the Ornstein-Uhlenbeck process ($\Delta y_t = c + \phi_1 \cdot y_{t-1} + \varepsilon_t$), the Brownian motion ($\Delta y_t = c + \varepsilon_t$) and the previously introduced trend stationary process ($\Delta y_t = c + \phi_1 \cdot t + \varepsilon_t$). Similar to trend stationary process types, several variants of this mixture model are applicable.

$$\Delta(\ln y_t) = c + \phi_1 \cdot t + \phi_2 \cdot \ln y_{t-1} + \varepsilon_t \quad (122)$$

$$\Delta(y_t) = c + \phi_1 \cdot t + \phi_2 \cdot t^2 + \phi_3 \cdot y_{t-1} + \varepsilon_t \quad (123)$$

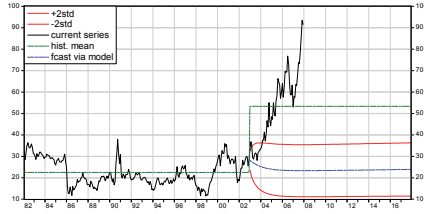
$$\Delta(\ln y_t) = c + \phi_1 \cdot t + \phi_2 \cdot t^2 + \phi_3 \cdot \ln y_{t-1} + \varepsilon_t \quad (124)$$

We have repeated the forecasting experiment analogue to the analysis in Figure 66 for different mixture model specifications. The results are shown in Figure 68.

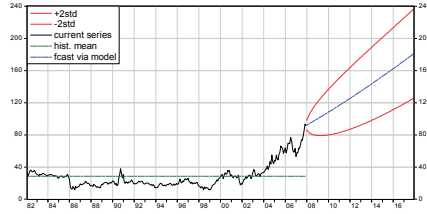
It appears from a solely visual inspection that, similar to the deterministic trend stationary models, the quadratic time mixture models perform better in explaining the development of oil. This accords with similar results by Pindyck (1999), who uses the oil prices history between 1870 and 1999 for his analysis.

Figure 68

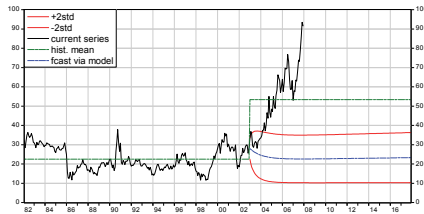
Different mixture model specifications for forecasting oil



Panel A: In-sample forecast between 2002 and 2007 for oil using $\Delta y_t = c + \phi_1 \cdot t + \phi_2 \cdot y_{t-1} + \varepsilon_t$ as DGP.



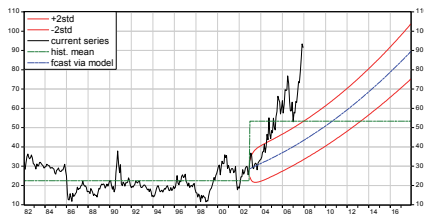
Panel B: Out-of-sample forecast between 2008 and 2017 for oil using $\Delta y_t = c + \phi_1 \cdot t + \phi_2 \cdot y_{t-1} + \varepsilon_t$ as DGP.



Panel C: In-sample forecast between 2002 and 2007 for the oil price using $\Delta(\ln y_t) = c + \phi_1 \cdot t + \phi_2 \cdot \ln y_{t-1} + \varepsilon_t$ as DGP.



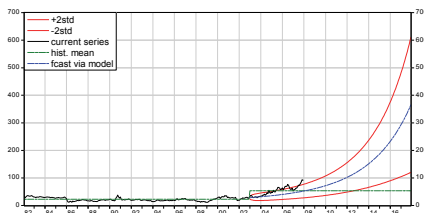
Panel D: Out-of-sample forecast between 2008 and 2017 for the oil price using $\Delta(\ln y_t) = c + \phi_1 \cdot t + \phi_2 \cdot \ln y_{t-1} + \varepsilon_t$ as DGP.



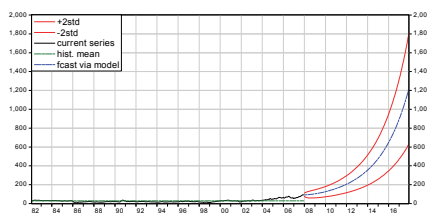
Panel E: In-sample forecast between 2002 and 2007 for the oil price using $\Delta(y_t) = c + \phi_1 \cdot t + \phi_2 \cdot t^2 + \phi_3 \cdot y_{t-1} + \varepsilon_t$ as DGP.



Panel F: Out-of-sample forecast between 2008 and 2017 for the oil price using $\Delta(y_t) = c + \phi_1 \cdot t + \phi_2 \cdot t^2 + \phi_3 \cdot y_{t-1} + \varepsilon_t$ as DGP.



Panel G: In-sample forecast between 2002 and 2007 for the oil price using $\Delta(\ln y_t) = c + \phi_1 \cdot t + \phi_2 \cdot t^2 + \phi_3 \cdot \ln y_{t-1} + \varepsilon_t$ as DGP.

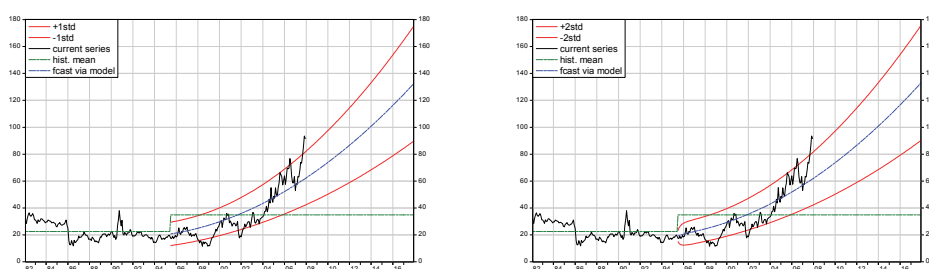


Panel H: Out-of-sample forecast between 2008 and 2017 for the oil price using $\Delta(\ln y_t) = c + \phi_1 \cdot t + \phi_2 \cdot t^2 + \phi_3 \cdot \ln y_{t-1} + \varepsilon_t$ as DGP.

Also noticeable is the fact that, when allowing for a larger out-of-sample period, the quality of the fit increases for the pure deterministic quadratic trend model and the quadratic mixture models (see Figure 69).

Figure 69

Forecasting the oil price out-of-sample using quadratic trend stationary and mixture model specifications



Panel A: Out-of-sample forecast between 1995 and 2017 for the oil price using $y_t = a + b \cdot t + c \cdot t^2 + \varepsilon_t$ as DGP.

Panel B: Out-of-sample forecast between 1995 and 2017 for the oil price using $\Delta(y_t) = c + \phi_1 \cdot t + \phi_2 \cdot t^2 + \phi_3 \cdot y_{t-1} + \varepsilon_t$ as DGP.

Both out-of-sample forecasts are very similar, which should not come as too great a surprise, since both equations defer only marginally. If we extend the trend stationary model with an autoregressive regression coefficient¹⁴³ the results would be identical.

4.5 Testing commodity prices for stationarity

The previous sections have covered the different DGPs that are usually employed in order to forecast and simulate the future development of commodity prices. We have shown how most stochastic differential equations that are usually applied in real-option theory are parameterized, and also explained how alternative models that are more frequently used in standard econometric analysis (rather than in financial engineering), such as trend stationary processes and mixture models, can be employed to forecast the long-term development of commodity prices. In the following section, we will conduct an econometric study on several of the most commonly traded commodities. Our aim is

¹⁴³ I.e., by applying $y_t = c + \phi_1 \cdot t + \phi_2 \cdot t^2 + AR(1) + \varepsilon_t$ as the forecast equation.

to test for stationarity of the different commodity prices since this is of crucial importance for choosing the right DGP for forecasting.

4.5.1 Define the data set

Long-term time series data sets for commodities are usually difficult to obtain. Several commodities have been priced in the past few decades in a relatively sporadic fashion, and liquid markets seem only to have developed during the recent 20 to 30 years. In order to test the stability of the models we have split the analysis into several data sub-samples. The first sample starts with January 1970 and contains only data for gold and silver. The second data sample starts with February 1982 and contains gold, silver, nickel and oil. The third sample starts with January 1987 and contains all of the above commodities plus steel.

In order to illustrate the development of the different tested commodities, the prices are indexed at 100 in January 1987 as shown in Figure 70. A summary statistic for the different sub-samples is given in Table 30.

Figure 70
Price development of major tested commodities over time



Price development of major tested commodities between 1987 and 2008. All prices in USD terms indexed at 100 beginning in 1987.
Data Source: Datastream.

Table 30
Descriptive statistics for major tested commodities, differentiated along the time series inception

Panel A: Analysis in level terms						
		Gold	Silver	Nickel	Oil	Steel
1970-2008	Mean	326	626			
	Median	347	519			
	Std. Dev.	151	403			
	Jarque-Bera	0.4462	0.0000			
1982-2008	Mean	384	631	9418	28	
	Median	374	537	7248	23	
	Std. Dev.	95	247	7582	15	
	Jarque-Bera	0.0000	0.0000	0.0000	0.0000	
1987-2008	Mean	386	592	10482	28	350
	Median	375	514	7848	21	315
	Std. Dev.	99	231	7970	16	110
	Jarque-Bera	0.0000	0.0000	0.0000	0.0000	0.0000
Panel B: Analysis in first differences						
		Gold	Silver	Nickel	Oil	Steel
1970-2008	Mean	1.66	2.67			
	Median	0.25	-0.10			
	Std. Dev.	23.51	153.56			
	Jarque-Bera	0.0000	0.0000			
1982-2008	Mean	1.34	1.76	64.84	0.19	
	Median	-0.53	-1.13	-37.08	0.13	
	Std. Dev.	20.37	61.17	1651.39	2.87	
	Jarque-Bera	0.0000	0.0000	0.0000	0.0000	
1987-2008	Mean	1.51	3.70	114.55	0.30	1.43
	Median	0.03	-0.43	-15.50	0.29	0.00
	Std. Dev.	18.69	50.32	1790.41	3.03	20.17
	Jarque-Bera	0.0000	0.0000	0.0000	0.0000	0.0000
Panel C: Analysis in % changes						
		Gold	Silver	Nickel	Oil	Steel
1970-2008	Mean	0.69%	0.44%			
	Median	0.06%	-0.02%			
	Std. Dev.	5.91%	9.72%			
	Jarque-Bera	0.0000	0.0000			
1982-2008	Mean	0.24%	0.16%	0.48%	0.34%	
	Median	-0.14%	-0.24%	-0.57%	0.52%	
	Std. Dev.	4.77%	7.58%	9.58%	9.73%	
	Jarque-Bera	0.0000	0.0000	0.0000	0.0000	
1987-2008	Mean	0.26%	0.39%	0.89%	0.63%	0.36%
	Median	0.01%	-0.08%	-0.21%	1.35%	0.00%
	Std. Dev.	4.10%	6.74%	10.22%	9.64%	5.44%
	Jarque-Bera	0.0000	0.0000	0.0000	0.0000	0.0000

The descriptive statistics for the five commodities used in the analysis reveal some important results. First, the Jarque-Bera normality test leads us to dismiss the assumption of normal distribution in all three cases. Second, one can observe that nearly all commodities (oil being the only exception) have a lower median than mean – i.e.,

most commodities have the tendency to be skewed to the right. Third, all data show (in first differences) positive means, which will be useful for the unit-root test specification. Finally, since the different sample means appear stable (whether focusing at the $I(0)$ or the $I(1)$ values), one could test stationarity without incorporating a deterministic time trend.¹⁴⁴

4.5.2 *Conducting stationarity tests*

In order to test for stationarity of the different commodity time series, we will apply three different methodologies. The most common method to test for stationarity is the augmented Dickey-Fuller test (ADF test).¹⁴⁵ We will, however, extend the standard unit-root test by incorporating the results from the Kwiatkowski-Phillips-Schmidt-Sinn test (KPSS test) to conduct a conformation stationarity analysis. Additionally we will apply a variance ratio analysis to assess the likelihood of different commodity prices following a random walk.

4.5.2.1 Test specification

The unit root test is specified as an $I(0)$ test (test in levels) without a time trend but with an intercept. It is certainly disputable whether commodity prices underlie a given deterministic time trend.¹⁴⁶ Given that commodity prices are “prices” and hence subject in one form or another to inflation (in this context, asset price inflation), it would appear plausible to assume a constant appreciation factor that is driven by the real value of money (i.e., inflation) and the external value of money (i.e., the trade-weighted USD exchange rate), which should shift commodity prices constantly higher.¹⁴⁷

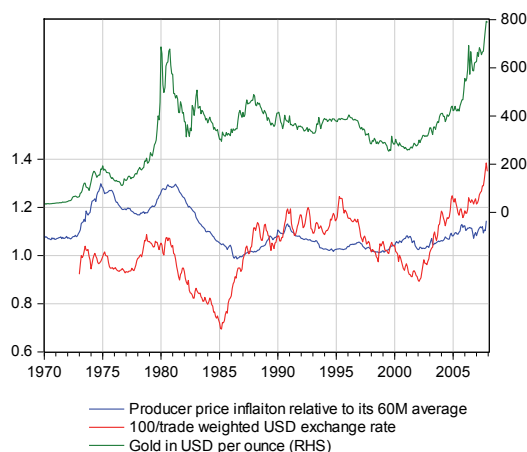
¹⁴⁴ In contrast, Pindyck (1999), p. 8 incorporates a deterministic time trend into his analysis on oil. He uses, however, a much longer time series dataset starting in 1875.

¹⁴⁵ See Maddala and Kim (1998), pp. 74-78.

¹⁴⁶ See Pindyck (1999), p. 8.

¹⁴⁷ There certainly exists a feedback effect between commodity prices and inflation. This cross-dependency will be analyzed further in Subsubsection 4.6.4.2; see p. 202.

Figure 71
**Price development of gold relative to producer price
inflation and trade-weighted USD**



Gold price development between 1970 and 2008 relative to the development of inflation and the external value of its traded currency (i.e., the USD exchange rate). Gold is supported by increasing anticipated inflation and a weaker USD exchange rate. Data source: Datastream.

The influence of inflation becomes additionally clear if one considers that some commodities (especially gold) are often used as inflation hedges and perform extraordinarily well in an environment of negative – inflation-induced – real bond yields and/or a weak USD exchange rate (see Figure 71).¹⁴⁸ To acknowledge the arguments in favor of a time deterministic trend in prices, we will carry out unit-root tests with intercept and with deterministic time trends. The lag-length will be set by minimizing the Schwarz information criterion without exceeding a lag-order higher than four.

¹⁴⁸ The relation to real yields is born out of the fact that gold is a non-interest-bearing asset. Hence, if for example government bond yields adjusted for inflation (i.e., real yields) become negative (inflation expectations higher than nominal bond yields), the attractiveness of gold increases as a competitive asset class.

4.5.2.2 The augmented Dickey-Fuller (ADF) test

The ADF test will be conducted along two OLS regression. The test equation specifications (with stochastic trend and deterministic trend) are given in Equations (125) and (126):

$$\Delta y_t = (\phi - 1)y_{t-1} + \sum_{i=1}^N \phi_i \Delta y_{t-i} + \alpha_0 \quad (125)$$

$$\Delta y_t = (\phi - 1)y_{t-1} + \sum_{i=1}^N \phi_i \Delta y_{t-i} + \alpha_0 + \alpha_1 t \quad (126)$$

The null hypothesis of the test is $\phi=1$, which means the time series has a unit-root (indicating a non-stationary behavior and hence no revision to the mean). We will use the one-sided p-values tabulated by MacKinnon (1996) as criteria in order to reject or accept the null hypothesis of unit-root. The tests are distinguished along the different horizon classes shown in Table 31.

Table 31

ADF unit root test p-values for major commodity prices over different horizons

		Gold	Silver	Nickel	Oil	Steel
1973-2008	intercept	0.7358	0.0059**			
	time trend	0.8222	0.0304*			
1982-2008	intercept	0.9996	0.7059	0.6759	0.9961	
	time trend	1.0000	0.9915	0.6561	0.9902	
1987-2008	intercept	1.0000	0.9988	0.6366	0.9964	0.3504
	time trend	1.0000	0.9988	0.7460	0.9820	0.5382

The p-values are based on MacKinnon (1996)
 * Unit root can be rejected at the 5% level
 ** Unit root can be rejected at the 1% level

As can be seen in the above table, one can reject the unit-root hypothesis only for Silver and conclude that Silver has a stationary time series over the past 35 years. For all other

commodities, we cannot reject the unit-root hypothesis irrespective of the sample chosen.

4.5.2.3 The Kwiatkowski, Phillips, Schmidt and Shin (KPSS) test

Since the ADF test is often criticized for its weak power¹⁴⁹ we extend the unit-root test and calculate an alternative in which the baseline hypothesis assumes stationarity of the commodity time series with the alternative hypothesis of a random walk. This is in contrast to the ADF test, where the null hypothesis assumes non-stationarity of the time series (i.e., a unit root). We use the critical values tabulated by Kwiatkowski et al. (1992) for the Lagrange multiplier (LM) test statistic (see Table 32).¹⁵⁰

Table 32
KPSS unit root test statistic values for major commodity prices over different horizons

		Gold	Silver	Nickel	Oil	Steel
1973-2008	intercept	0.6988**	0.1993			
	time trend	0.2213***	0.1890**			
1982-2008	intercept	0.3318	0.3581*	0.7532***	0.9459***	
	time trend	0.2810***	0.3543***	0.2477***	0.4279***	
1987-2008	intercept	0.4505**	0.6931**	0.7078**	1.3027***	0.4453*
	time trend	0.3681***	0.3175***	0.3337***	0.4410***	0.2910***
The p-values are based on MacKinnon (1996)						
* Stationarity is rejected at the 10% level						
** Stationarity is rejected at the 5% level						
*** Stationarity is rejected at the 1% level						
				intercept	+time trend	
Asymptotic critical values (a): 1% level				0.739	0.216	
5% level				0.463	0.146	
10% level				0.347	0.119	
(a) Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)						

¹⁴⁹ See Maddala and Kim (1998), pp. 98-146.

¹⁵⁰ See Kwiatkowski et al. (1992), p. 166.

Similar to the ADF test in Table 31, there is a strong indication of non-stationarity for most of the tested commodities. The null hypothesis of stationarity is dismissed in nearly all cases (test statistic lies above the critical values). Only for the longer gold and silver time series can one not reject the null hypothesis of stationarity for the intercept test specification.

4.5.2.4 Confirmatory Data Analysis (CDA)

The confirmation analysis was suggested by Kwiatkowski et al. (1992) and Choi (1994). The aim is to compare the results from a unit root test that uses unit root as the null hypothesis and a test that uses stationarity as the null hypothesis. It aims to look for confirmation in both tests. In cases where both tests reject the null hypothesis or fail to reject the null hypothesis, we do not have confirmation. As pointed out by Maddala and Kim (2004)¹⁵¹ it is generally agreed that using both tests together is better than relying on one test only. In the analysis presented we will use the individual results of the ADF and KPSS tests to form an ADF-KPSS confirmatory data analysis.

Comparing the analyses in Table 31 and Table 32 shows that the results from the ADF tests are confirmed by nearly all the KPSS tests (ADF null hypothesis of a unit root – i.e., non-stationarity – cannot be dismissed, while the KPSS null hypothesis of stationarity is dismissed). Hence, the data suggest that – except for silver from 1973-2008 and gold from 1982-2008 – all other commodities do not show stationarity and the hypothesis of unit root (with and without trend specification) is confirmed for nearly all cases.

What is left after this analysis is the question of how stable the assumption of mean reversion is for the case of commodities. A number of studies by Schwartz (1997, 1998), Schwartz and Smith (2000), and Pindyck (1999, 2004a, 2004b) rely on the notion of reversion to the mean in one way or another, and variants of the Ornstein-Uhlenbeck process have been widely used to estimate the future development of commodities in

¹⁵¹ See Maddala and Kim (2004), pp. 86-87 and Cochrane (2005), pp. 410-412.

order to value real-options or other derivatives. It should be questioned whether this simple reliance on reversion to the mean without focusing on the nature of commodity prices is justified. In order to answer this question, we will attempt to normalize the commodity data used in the above analysis and to repeat the tests conducted so far with the normalized data sets.

4.5.2.5 The variance ratio analysis (VRA)

Variance stationarity tests rely on the assumption that a time series that follows a random walk is non-stationary. Hence, if it can be proven that the variance of a time series is time-independent (its variance is not a function of the observation point), the assumption of a random walk can be dismissed (concluding stationarity of the time series). To get a better intuition of the test, recall that the variance of a random walk process X_t is defined as:

$$V(X(t)) = k\sigma_{\Delta}^2 \text{ with } \Delta(t) = X(t) - X(t-1) \quad (127)$$

Hence, the variance of the process is a function of time $t = k$, with σ_{Δ}^2 calculated as the variance of the first differences. If the underlying follows a random walk (benchmark assumption) its variance should increase with time. Stationarity on the other side can be defined via the variance and autocovariance (but not alone; see Subsection 4.2.1, p. 137 for strong and weak definitions of stationarity) of the time series. Of particular interest is the property of autocovariance stationarity (see below):

$$Cov(X(t_1), X(t_2)) = Cov(X(t), X(t+k)) = \text{constant} \quad \forall \quad t \in T \quad (128)$$

$$Cov(X(t), X(t)) = \sigma_{\Delta} = \sigma = \text{constant} \quad \forall \quad t \in T \quad (129)$$

Under autocovariance stationarity one assumes that the covariance between the time series and the k -periods shifted times series is constant, irrespective of the point in time t when the sample is taken. This functional dependency of the lag- k plays a decisive role in the variance ratio test. For the case of a stationary time series, it can be assumed that

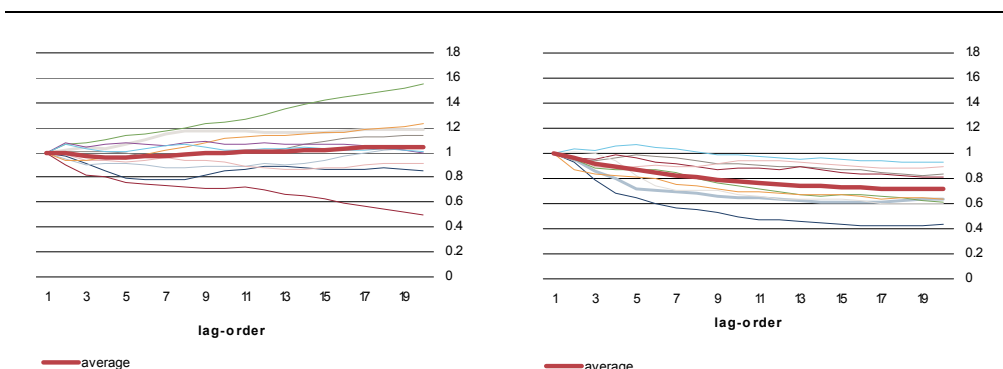
the variance of the difference in the k -lagged data series $\text{Var}(X(t) - X(t-k))$ converges to a constant upper limit with increasing k . Hence, if the time series is mean-reverting (i.e., stationary), the variance will not increase if k increases. Comparing the variance of the calculated k -differential series with the assumed variance of a random walk (denominator in Equation (130)), one can form known discrete realizations of $X(t)$ the so-called variance ratio.

$$VR_k = \frac{\text{Var}(x_t - x_{t-k})}{k\sigma_{x_t - x_{t-1}}^2} \quad (130)$$

The variance ratio is hence a measure of how closely the variance – calculated as the difference between the current series and the k -lagged series values – is equal to the random walk variance over the same period k (given in the denominator). Since the denominator increases with the lag period k , it represents the variance under a random walk assumption. If the numerator converges to a constant, the variance ratio VR_k approaches zero with increasing k , which is an indication that the series is stationary. If the numerator increases linearly with k (which should be the case for a non-stationary time series), the ratio should converge close to one, which would confirm non-stationarity.¹⁵² A stochastic process with trend (random walk with drift) should lead to a variance ratio larger than one.

Figure 72 shows the variance ratio for 10 random simulations of a pure random walk process and a mean-reverting Ornstein-Uhlenbeck process and their average. As illustrated in Panel A (the random walk simulation), with an increase in the lag interval, the variance ratio converges on average to the level of one, while the AOUP process has the tendency to converge towards a constant less than one.

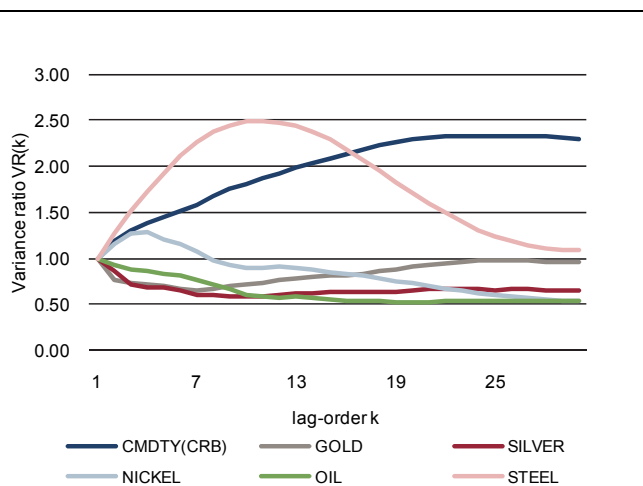
¹⁵² Maddala and Kim (2006), p. 86 point out that the variance ratio can be formulated as an econometric test with the null hypothesis of a random walk (VR=1).

Figure 72**Variance ratio examples for random walk and Ornstein-Uhlenbeck process**

Panel A: Variance ratio as function of lag order k for a simulated random walk example.

Panel B: Variance ratio as function of lag order k for a simulated Ornstein-Uhlenbeck mean-reverting process example.

Figure 73 illustrates the variance ratios for the set of commodities used in the previous unit-root analysis for the sample period 1987.¹⁵³

Figure 73**Variance ratio analysis for major tested commodity prices**

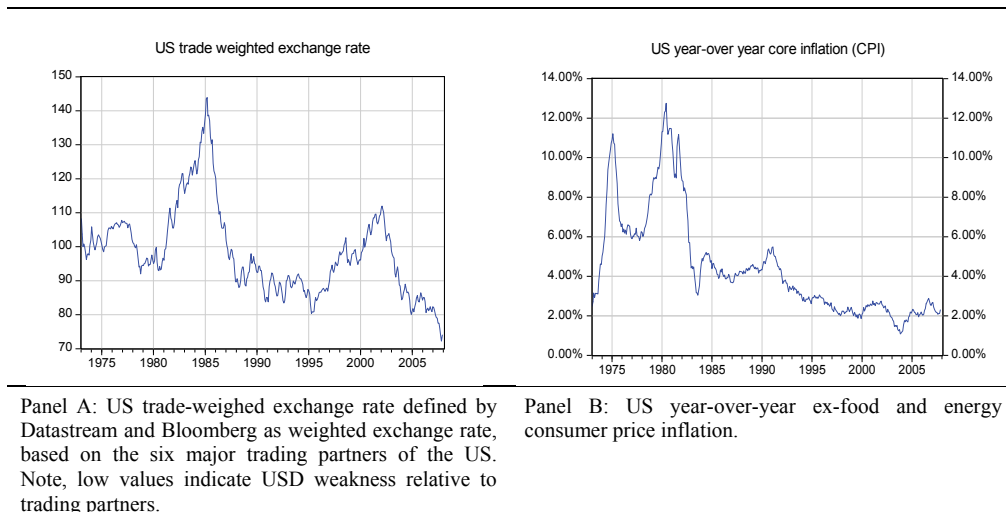
Variance ratio analysis (sample 1987-2008) for different lag-length order (k) calculated on the major tested commodity prices and one commodity total return index.

¹⁵³ The other sample results are given in Appendix G, see p. 352.

The variance ratio analysis that is conducted for the three sample periods used in the previous unit root analysis confirms some of the ADF and KPSS test results. Apparently, oil, silver and nickel show mean-reverting behavior with converging variance ratios towards 0.5 after a lag of 25-30 months. Steel and Gold, however, both converge towards one and hence seem to confirm non-stationarity of the time series. The overall commodity index (CRB Index) shows the weakest link to mean-reversion by increasing above 2, which should not come as a surprise since it is a total return price index rather than a particular mean-reverting commodity. Summarizing the above, one can conclude that the variance ratio analysis more often indicates mean-reverting behavior than other unit root tests indicate. This suggests, as pointed out by Pindyck (1999), that random shocks are more transitory for commodities like oil, silver and nickel.

4.5.3 Normalizing commodity data

As shown in the previous analysis, the assumption of stationarity cannot be justified for most of the tested commodities. Hence, every DGP that relies only on “reversion to the mean” of level $I(0)$ data is faced with opposition. It is therefore rational to question whether it is possible to normalize commodity data in order to work with stationary variables. As illustrated in Figure 71, p. 167, commodity prices like those for Gold seem to be strongly related to the external value of the USD (the trade-weighted exchange rate) and the path of inflation. Figure 74 shows the USD trade-weighted exchange rate and the year-over-year USD core consumer price inflation (core CPI). The negative trend in the trade-weighted exchange rate shows the continuing devaluation of the USD against its major trading partners. The deterioration of the external value of the USD was especially strong during the years between 2005 and 2008. Panel B in Figure 74 shows the US core (ex-food and energy) inflation measured in terms of the consumer price index. After a period of extraordinary high inflation between 1975 and 1985, CPI data have normalized downwards to average 2% year-over-year inflation during the last 25 years.

Figure 74**The external value of the USD and US core inflation over time**

A further normalization method was suggested by Pindyck (1999) that involves taking the data in log form, as is often done with equity indices.

To account for the above normalization variants, we have normalized the remainder of the data in this section by:

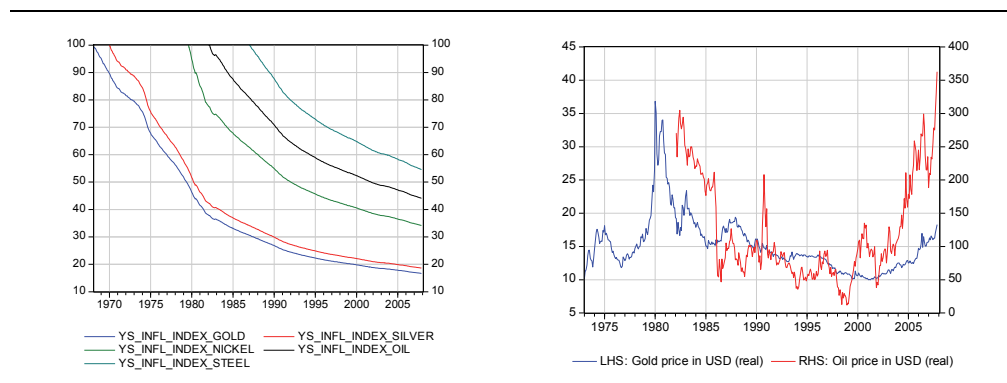
- a) using the USD trade-weighted exchange rate
- b) using the Core USD CPI (consumer price index)
- c) using a natural logarithm transformation

The combination of the above normalization variables leads to the following normalization methods (variants):

- a) Commodity price $\times$ USTW-Index/100
- b) Commodity price $\times$ Inflation-discount-Index/100
- c) Commodity price $\times$ Inflation-discount-Index/100 $\times$ USTW-Index/100
- d) Log(Commodity price)
- e) Log(Commodity price $\times$ Inflation-discount-Index/100) $\times$ USTW-Index/100
- f) Log(Commodity price) $\times$ USTW-Index/100
- g) Log(Commodity price $\times$ Inflation-discount-Index/100 $\times$ USTW-Index/100)

The inflation-discount index requires special attention. It is constructed as a discount function (similar to a present value discount function that “normalizes” future cash-flows to net-present-value). Hence, every commodity index multiplied by its specific discount function (depending on the different start time) yields the commodity index in real terms (thus adjusted for inflation). The inflation-discount indices for the different starting periods are given in Figure 75, Panel A. Panel B of Figure 75 shows the impact of the inflation normalization on gold and oil.

Figure 75
Normalization of gold and oil prices for inflation
(oil and gold prices in real terms)

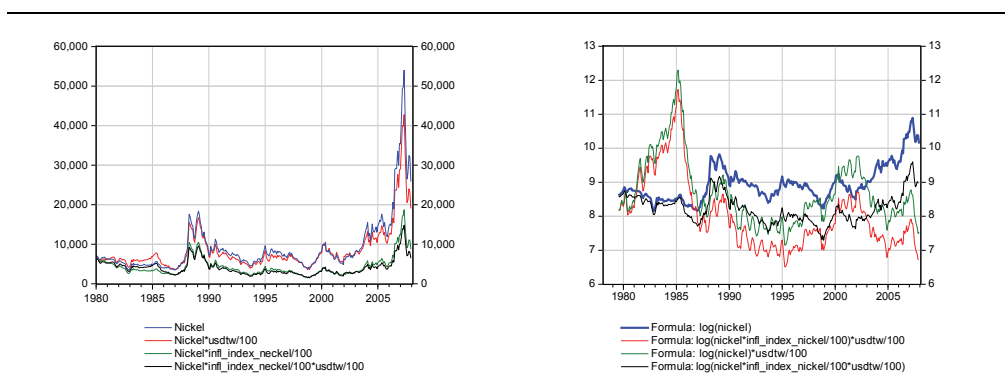


Panel A: Discount factors for commodity prices using the US core consumer price index for different starting years.

Panel B: Inflation-normalized prices for oil and gold over time in USD.

We have the longest time series available for gold; hence, the inflation-discount index starts with 100 first for gold, followed by the index for silver and nickel. Panel B shows the development of gold and oil in real terms (i.e., adjusted for inflation). It is interesting to note that in real terms oil only surpassed its 1983 price level in 2008. Gold, however, remains far below its real-term value highs of 1980.

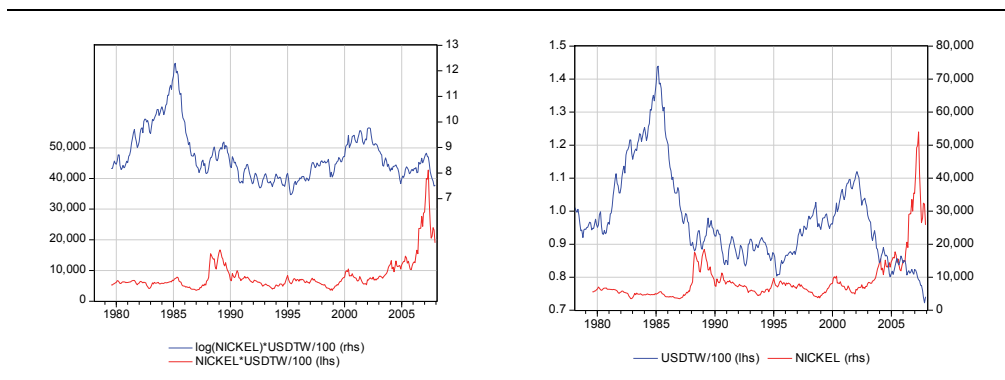
Illustrating the different normalized time series graphically unveils some interesting results. The example for nickel is given in Figure 76 (Panel A shows the non-logarithmic data, while Panel B shows the logarithmic data):

Figure 76**Normalization of nickel by accounting for changes in the external value of the USD and inflation**

Panel A: Price of Nickel normalized for the external value of the USD, for inflation, and for both.

Panel B: Logarithmic price of Nickel normalized for the external value of the USD, for inflation, and for both.

The illustration in Figure 76, Panel A shows that the highest degree of normalization is achieved by looking at the real development of nickel that is additionally adjusted for the external value of the USD. The confirmatory data analysis (see Appendix G, p. 352 for all results) between the ADF and the KPSS test for nickel supports this conclusion. For the log-normalized data, the interpretation is slightly more ambiguous. The bold blue line in Figure 76, Panel B shows nickel in log terms. The log version is much less volatile than the non-logarithmic time series of nickel in Panel A. The normalization of the log-nickel series by the USD trade-weighted exchange rate is therefore much more pronounced than for the non-log case (see Figure 77, Panel A).

Figure 77**Normalizing the non-logarithmic and logarithmic data series of nickel for the external USD value**

Panel A: Non-logarithmic and logarithmic data series of nickel by normalizing for the external value of the USD.

Panel B: External value of the USD (measured as trade-weighted exchange rate) and nickel over time.

The reason for this pattern is found in the overall volatility of the non-log time series of nickel, which limits the degree exchange rate normalization. Panel B of Figure 77 shows the USD trade-weighted exchange rate (divided by 100) and the time series for nickel. The impact of the exchange rate normalization is much stronger today than during 1985. The difference is simply due to the overall level of nickel today vs. 20 years ago. Hence, the exchange rate normalization and the inflation normalization will depend on the overall volatility of the time series.

The normalization problem can be bypassed by taking the logarithm of the already normalized time series. Hence, if we take the logarithm after normalizing for the exchange rate and inflation, the normalization is much more subdued, as shown in Figure 76, Panel B. The question arises as to which method is best in terms of theoretical legitimacy and forecast applicability. If one chooses to model inflation, the trade-weighted exchange rate and commodity prices in a joint-multivariate approach (e.g., vectorautoregressive methods) an individual normalization could be achieved; thus, taking the logarithm of the commodity while leaving the other variables unchanged appears plausible. However, if we choose to work in a univariate framework (single OLS Equation setup), normalization before taking the logarithm is probably the better choice.

Irrespectively of the normalization method chosen, the unit-root test statistics speak for themselves. The results of all ADF and KPSS tests for all the different normalization variants are given in Appendix G, p.352. Regarding normalization performance, the conclusion can be drawn that normalization via the trade-weighted exchange rate for the log-commodity price – among other normalization methods – fulfills its purposes best (see Table 33 and Table 34). For the sample period 1987-2008, stationarity is confirmed for gold, silver, nickel and steel. The results of the confirmatory data analysis are highlighted in green for positive concordance. Only oil continues to show a unit-root if this normalization approach is chosen.

Table 33

ADF unit root test p-values for major commodity prices (in log terms) over different horizons normalized for the external value of the USD

		Gold	Silver	Nickel	Oil	Steel
1973-2008	intercept	0.3467	0.4051			
	time trend	0.5430	0.4423			
1982-2008	intercept	0.6525	0.6222	0.3649	0.3815	
	time trend	0.7370	0.8056	0.5038	0.7631	
1987-2008	intercept	0.0237**	0.0521*	0.0724*	0.3416	0.0438**
	time trend	0.1032	0.1621	0.2296	0.1018	0.1624

The p-values are based on MacKinnon (1996)
 * Unit root can be rejected at the 10% level
 ** Unit root can be rejected at the 5% level
 *** Unit root can be rejected at the 1% level

Table 34

KPSS unit-root test statistics for major commodity prices (in log terms) over different horizons normalized for the external value of the USD

		Gold	Silver	Nickel	Oil	Steel
1973-2008	intercept	0.3469*	0.7039**			
	time trend	0.2427***	0.1994**			
1982-2008	intercept	0.9743***	0.8112***	0.6468**	0.4339*	
	time trend	0.3367***	0.3579***	0.3090***	0.3931***	
1987-2008	intercept	0.1857	0.2854	0.3418	1.2831***	0.2059
	time trend	0.2006**	0.2131***	0.2021**	0.2253***	0.2030**

The p-values are based on MacKinnon (1996)
 * Stationarity is rejected at the 10% level
 ** Stationarity is rejected at the 5% level
 *** Stationarity is rejected at the 1% level

		intercept	+time trend
Asymptotic critical values (a):	1% level	0.739	0.216
	5% level	0.463	0.146
	10% level	0.347	0.119

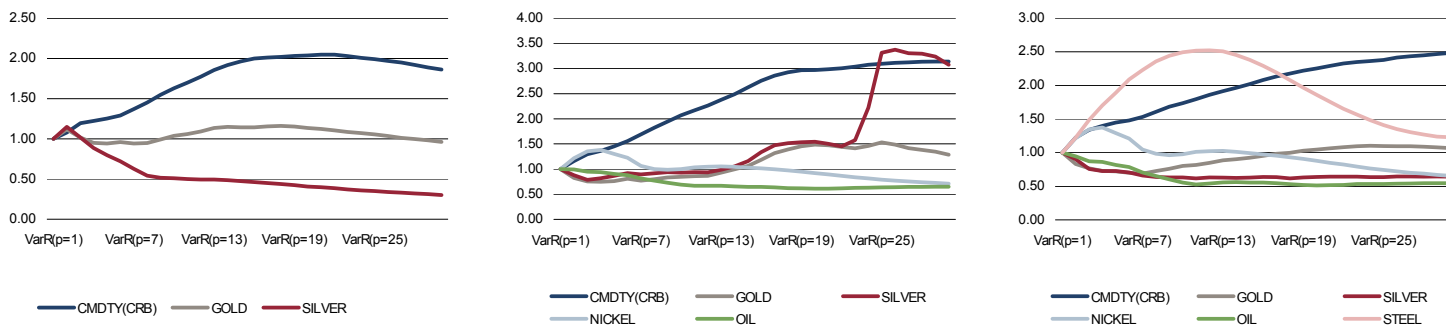
(a) Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)

The other normalization methods yield mixed results. While for most normalization methods the number of non-confirmations (neither unit-root nor stationarity could be confirmed) increases relative to the benchmark (non-normalized dataset) test, none of the other methods is convincing enough.

It is also possible to conduct a variance ratio analysis on the normalized data series. We follow the same guidelines as in Subsubsection 4.5.2.5 (see p. 171) and calculate the variance ratios for the different time series and different sample lengths. The variance ratio analysis of the normalized data series unveils that normalization methods (c) and (g) show the highest tendency towards mean-reversion. In method (c), the product of the commodity index, inflation and exchange rate, four of the six tested series converge towards a stable long-term ratio below one. For method (g), the logarithmic version of method (c), the number is even higher (see Figure 83, Panel C, p. 183). All five tested commodity series converge towards a level below one, with only the commodity price index (CRB) deviating as expected. This confirms the results shown in Table 33 and Table 34 and can be read as additional support for modeling the product of commodity prices, exchange rate, and inflation as a single variable in a univariate time series framework. The variance ratios of the tested series are shown in Figure 78 to Figure 83.

Figure 78

Variance ratio analysis for inflation normalized data set



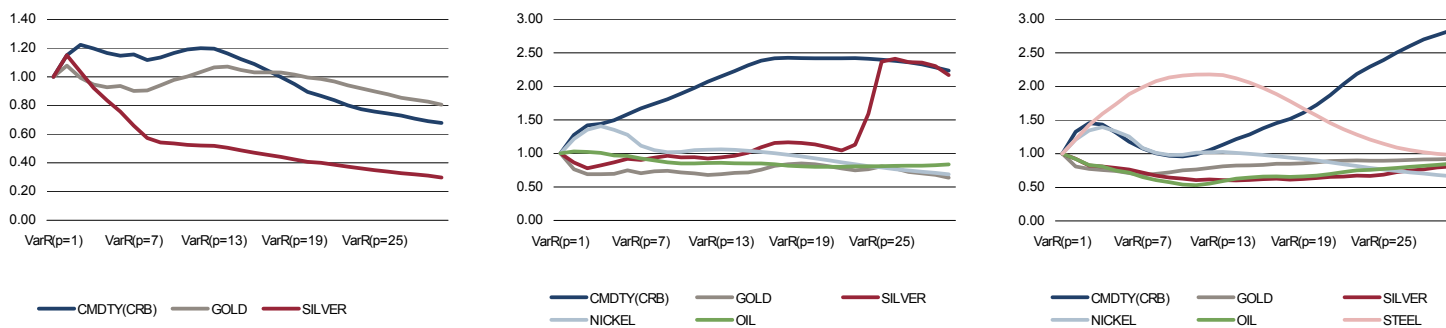
Panel A: Variance ratio analysis (sample 1973-2008)

Panel B: Variance ratio analysis (sample 1982-2008)

Panel C: Variance ratio analysis (sample 1987-2008)

Figure 79

Variance ratio analysis for inflation and external USD value normalized data sets



Panel A: Variance ratio analysis (sample 1973-2008)

Panel B: Variance ratio analysis (sample 1982-2008)

Panel C: Variance ratio analysis (sample 1987-2008)

Figure 80
Variance ratio analysis for logarithmic data sets

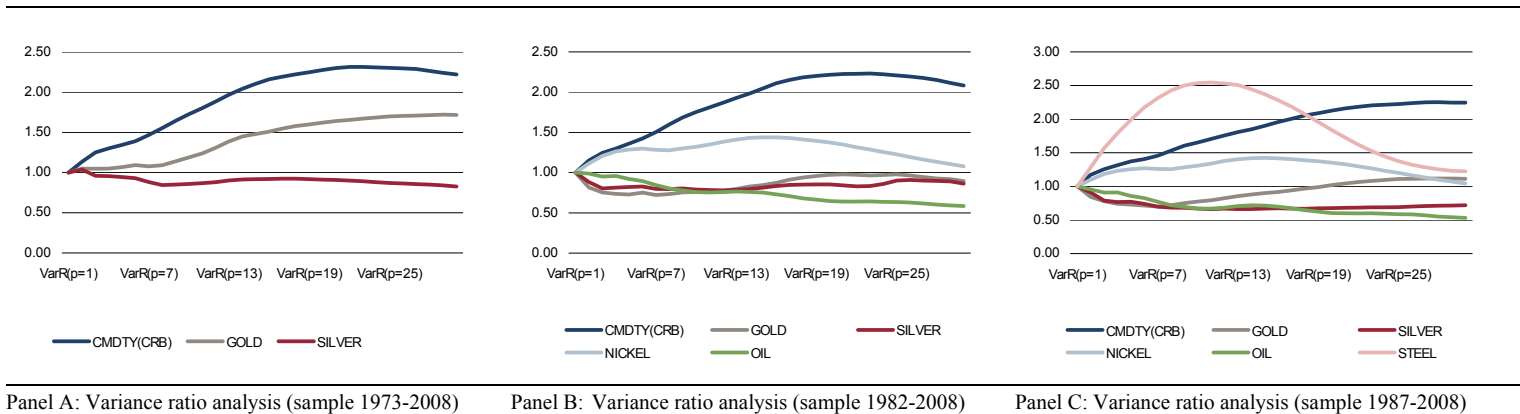


Figure 81
Variance ratio analysis for inflation normalized data set in log terms adjusted for the external USD value

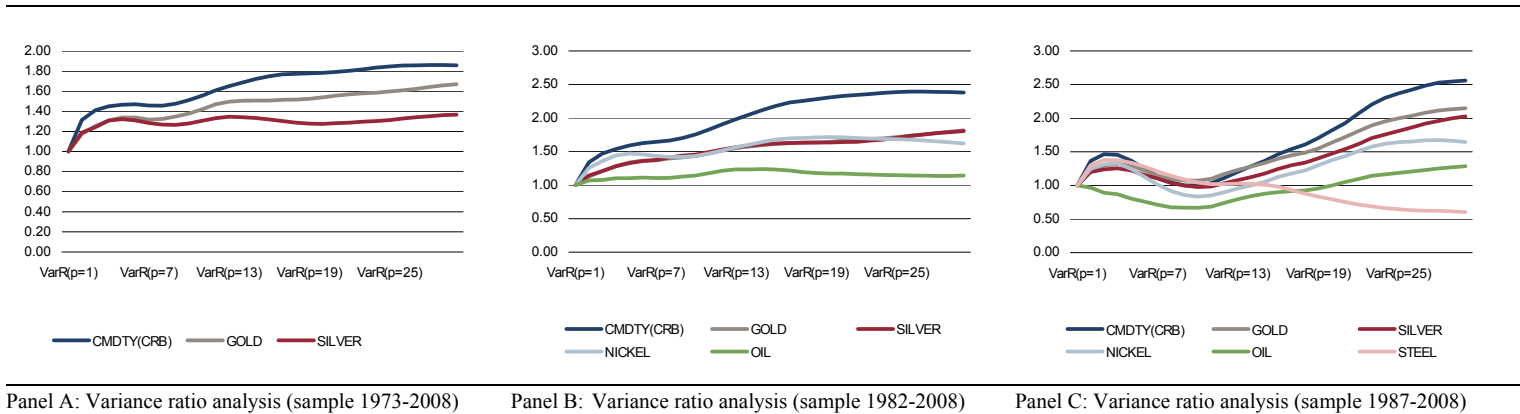
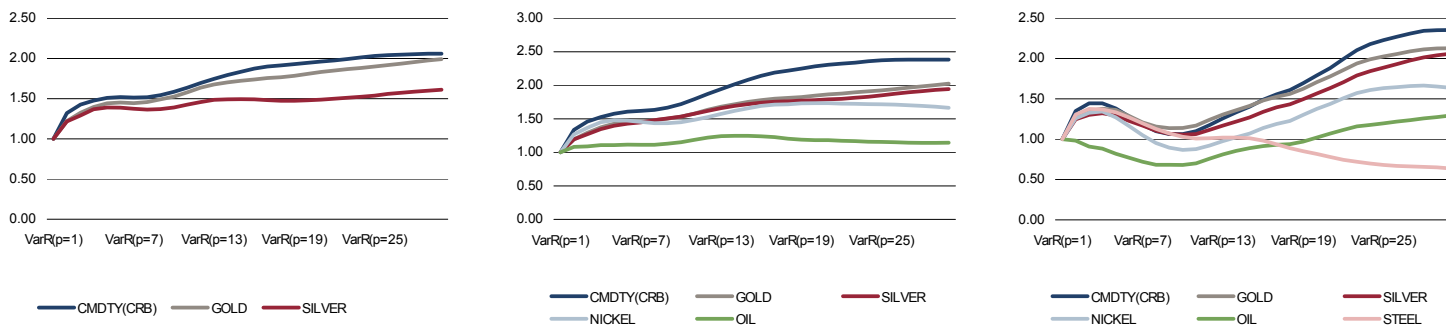
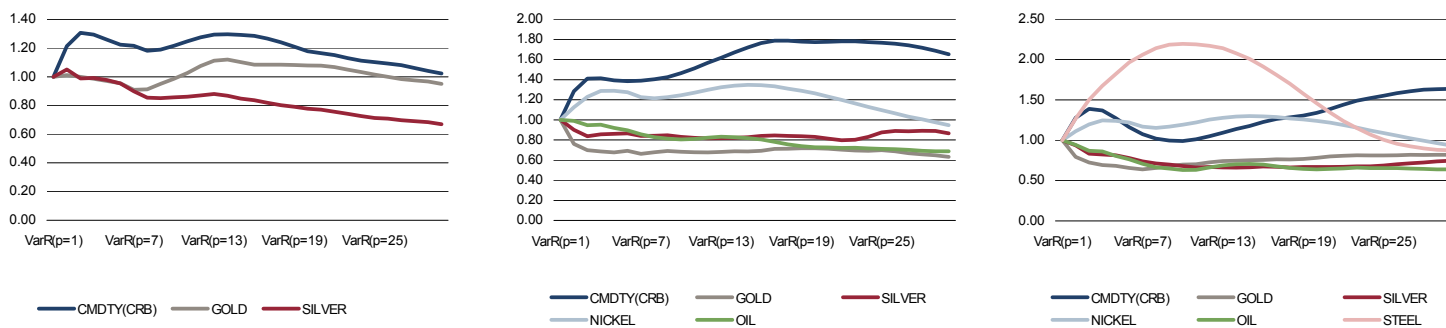


Figure 82**Variance ratio analysis for logarithmic data sets normalized for external USD value**

Panel A: Variance ratio analysis (sample 1973-2008)

Panel B: Variance ratio analysis (sample 1982-2008)

Panel C: Variance ratio analysis (sample 1987-2008)

Figure 83**Variance ratio analysis for logarithmic data sets of inflation and external USD value normalized series**

Panel A: Variance ratio analysis (sample 1973-2008)

Panel B: Variance ratio analysis (sample 1982-2008)

Panel C: Variance ratio analysis (sample 1987-2008)

4.6 Testing the rolling out-of-sample performance of different data generating processes

4.6.1 Introduction

After describing the different characteristics of several data generating processes, we continue to test the suitability of each process for forecasting long-term out-of-sample performance over a rolling horizon of five years. Long-term suitability is crucial in the real-option context, since the quality of the forecast and hence the standard error associated with the forecast is decisive in determining the real-option value of a project. Several authors have focused on the long-run evolution of asset prices (in particular of commodity prices).¹⁵⁴ Their long-term out-of-sample tests, however, focus mostly on the implications for real-option valuation framework (highlighting the differences between certain data generation processes with regard to option valuation based on changes to the implied volatility structure and/or long-term mean convergence), by fitting particular models to currently observable spot or forward prices and using these parameters for option valuation via simulation or closed form solution (if available). However, a rigorous out-of-sample test with respect to the long-term mean of different commodities is missing. In this section of the dissertation we will try to fill this gap.

This section is split into two major parts. First we will analyze several data generating processes in terms of their ability to forecast five years out-of-sample. Secondly, we will repeat the forecasts by using directly normalized commodity prices (see the normalization approach described in Subsection 4.5.3, p. 174) within a univariate OLS framework and indirectly by forecasting non-normalized commodity prices together with normalization factors in a multivariate (vectorautoregressive) framework. The data sample is identical to the unit-root test sample used in Subsection 4.5.2 (see p. 166). However, we have also included a commonly used commodity price index - the CRB Index - as a reference, since a longer time series is available than for most of the other traded commodity indices. It will also allow conducting 10-year out-of-sample tests with gold and silver together.

¹⁵⁴ See Metcalf and Hassett (1995), Pindyck (1999, 2004a, 2004b) and Schwartz (1997, 1998, 2000).

4.6.2 The test procedure

The vertical arrow represents the overall average test statistics measured over the entire forecast path. The horizontal arrow represents the evolution of the individual horizon-specific forecast accuracy measures.

4.6.2.1 Test variable and test statistics definitions

The variable definition for the forecast evaluation is also different for horizontal vs. vertical accuracy measures. The key test variables are:

$\hat{y}_{t+h,t}$: forecast at time t to future point in time h

y_{t+h} : actual (realized) value at time $t + h$

T_N : number of forecasts at time t

N : number of forecasts over fixed horizon h

The out-of-sample statistics are given in Table 36.¹⁵⁵

At this point it is necessary to highlight the fact that there seems to be no standard in econometric literature for calculating the forecast error and hence all other derivative accuracy test statistics. Some practitioners use $y_{t+h} - \hat{y}_{t+h,t}$ for calculating the forecast error¹⁵⁶; however, more often $\hat{y}_{t+h,t} - y_{t+h}$ is used.¹⁵⁷ We will use the latter form for calculating the main accuracy statistics. The set of individual vertical out-of-sample forecast statistics (forecast statistics over the entire forecast horizon) that are generated for each individual forecast path between t and $t + h$ are averaged in order to produce a final overall forecast accuracy measure for the total forecast horizon.¹⁵⁸

¹⁵⁵ A full description and rationale for the given set of test statistics can be found in Diebold (2004), pp. 297-298 and Pindyck and Rubinfeld (1998), p. 210.

¹⁵⁶ See Diebold (2004), p. 297 and Barretro and Howland (2006), p. 398.

¹⁵⁷ See Pindyck and Rubinfeld (1998), p. 210 and Rinne and Specht (2002), p. 129.

¹⁵⁸ As in the case of the Theil's statistics, the results are not always equivalent to averaging the individual horizontal forecast statistics.

Table 36
Forecast accuracy test statistics

Type	Vertical analysis	Horizontal analysis
Mean Error (ME)	$ME_t = \frac{1}{T_N} \sum_{h=1}^{T_N} \hat{y}_{t+h,t} - y_{t+h}$	$ME_h = \frac{1}{N} \sum_{t=1}^N \hat{y}_{t+h,t} - y_{t+h}$
Mean percentage error (MPE)	$MPE_t = \frac{1}{T_N} \sum_{h=1}^{T_N} \frac{\hat{y}_{t+h,t} - y_{t+h}}{y_{t+h}}$	$MPE_h = \frac{1}{N} \sum_{t=1}^N \frac{\hat{y}_{t+h,t} - y_{t+h}}{y_{t+h}}$
Root mean squared error (RMSE)	$RMSE_t = \sqrt{\frac{1}{T_N} \sum_{h=1}^{T_N} (\hat{y}_{t+h,t} - y_{t+h})^2}$	$RMSE_h = \sqrt{\frac{1}{N} \sum_{t=1}^N (\hat{y}_{t+h,t} - y_{t+h})^2}$
Root mean squared percentage error	$RMSPE_t = \sqrt{\frac{1}{T_N} \sum_{h=1}^{T_N} \left(\frac{\hat{y}_{t+h,t} - y_{t+h}}{y_{t+h}} \right)^2}$	$RMSPE_h = \sqrt{\frac{1}{N} \sum_{t=1}^N \left(\frac{\hat{y}_{t+h,t} - y_{t+h}}{y_{t+h}} \right)^2}$
Mean absolute error	$MAE_t = \frac{1}{T_N} \sum_{h=1}^{T_N} \hat{y}_{t+h,t} - y_{t+h} $	$MAE_h = \frac{1}{N} \sum_{t=1}^N \hat{y}_{t+h,t} - y_{t+h} $
Mean absolute percentage error	$MAPE_t = \frac{1}{T_N} \sum_{h=1}^{T_N} \left \frac{\hat{y}_{t+h,t} - y_{t+h}}{y_{t+h}} \right $	$MAPE_h = \frac{1}{N} \sum_{t=1}^N \left \frac{\hat{y}_{t+h,t} - y_{t+h}}{y_{t+h}} \right $
Theil's inequality coefficient	$T_t = \frac{\sqrt{\frac{1}{T_N} \sum_{h=1}^{T_N} (\hat{y}_{t+h,t} - y_{t+h})^2}}{\sqrt{\frac{1}{T_N} \sum_{h=1}^{T_N} (\hat{y}_{t+h,t})^2} + \sqrt{\frac{1}{T_N} \sum_{h=1}^{T_N} (y_{t+h})^2}}$	$T_h = \frac{\sqrt{\frac{1}{N} \sum_{t=1}^N (\hat{y}_{t+h,t} - y_{t+h})^2}}{\sqrt{\frac{1}{N} \sum_{t=1}^N (\hat{y}_{t+h,t})^2} + \sqrt{\frac{1}{N} \sum_{t=1}^N (y_{t+h})^2}}$
Theil-Bias	$T_t^B = \frac{(\bar{\hat{y}}_{t+h,t} - \bar{y}_{t+h})^2}{\frac{1}{T_N} \sum_{h=1}^{T_N} (\hat{y}_{t+h,t} - y_{t+h})^2}$	$T_h^B = \frac{(\bar{\hat{y}}_{t+h,t} - \bar{y}_{t+h})^2}{\frac{1}{N} \sum_{t=1}^N (\hat{y}_{t+h,t} - y_{t+h})^2}$
Theil-variance	$T_t^V = \frac{(\sigma_{\hat{y}_{t+h,t}} - \sigma_{y_{t+h}})^2}{\frac{1}{T_N} \sum_{h=1}^{T_N} (\hat{y}_{t+h,t} - y_{t+h})^2}$	$T_h^V = \frac{(\sigma_{\hat{y}_{t+h,t}} - \sigma_{y_{t+h}})^2}{\frac{1}{N} \sum_{t=1}^N (\hat{y}_{t+h,t} - y_{t+h})^2}$
Theil-covariance	$T_t^C = \frac{2(1 - \rho_{\hat{y}_{t+h,t}, y_{t+h}})(\sigma_{\hat{y}_{t+h,t}} \sigma_{y_{t+h}})}{\frac{1}{T_N} \sum_{h=1}^{T_N} (\hat{y}_{t+h,t} - y_{t+h})^2}$	$T_h^C = \frac{2(1 - \rho_{\hat{y}_{t+h,t}, y_{t+h}})(\sigma_{\hat{y}_{t+h,t}} \sigma_{y_{t+h}})}{\frac{1}{N} \sum_{t=1}^N (\hat{y}_{t+h,t} - y_{t+h})^2}$

4.6.2.2 Requirements for a good model (properties of optimal forecasts)

According to Diebold (2004) key test properties for an optimal forecast are:

1. un-biasness of the forecasts
2. 1-step-ahead errors are white noise
3. h -step-ahead errors are mostly $MA(h-1)$
4. h -step-ahead errors have variances that are non-decreasing in h and converge to the unconditional variance of the process

Additional regression tests would be necessary to assess if these properties are sufficiently fulfilled. For example, to check for the un-biasness of the forecasts, one needs to regress the forecast error series on a constant and test whether the intercept has a mean of zero. If the constant is significantly different from zero, the assumption of un-biasness is violated. Additionally the white noise requirement would be tested with the same regression by observing whether the Durbin Watson statistic is close to 2 (implying no residual autocorrelation).¹⁵⁹

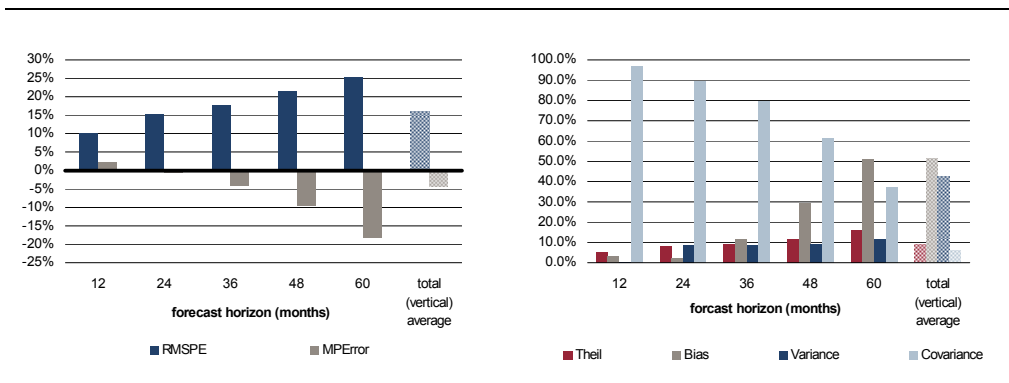
However, testing each individual forecast error series is time-consuming and awkward. It is hence more applicable to use the calculated accuracy test statistics to answer the question of whether the properties of an optimal forecast model are fulfilled or seriously violated. An important insight into the forecast properties can be obtained by looking at the different uniform forecast accuracy measures (i.e., measures that are comparable across models) over time. In Figure 84 we have illustrated the forecast error analysis for the CRB commodity price index using additive Brownian motion as underlying DGP. The RMSPE and MPE are shown in Panel A, while the Theil inequality coefficient analysis (see Table 36) with its corresponding components is shown in Panel B. The MPE illustrated in Panel A shows whether the DGP used for the forecast produces biased forecast errors (test property 1). Ideally this should not be the case (as in this

¹⁵⁹ For a good summary of statistical tests of regression-induced (i.e., regression against forecast errors) forecast evaluation, see Diebold (2004), pp. 305-319.

example between 12 and 36 months). The development of the RMSPE helps to reveal if test property (4) is satisfied or violated (non-decreasing forecast error variance).

Figure 84

Interpreting forecast accuracy measures



Panel A: Increasing RMSPE and low forecast bias (MP error close to zero) as useful criteria in assessing the suitability of the chosen DGP to forecast out-of-sample. In the particular case the DGP seems to be biased in forecasting too low (understating the future price developments of the underlying test series).

Panel B: Forecast accuracy measures based on Theil statistics. A high bias component indicates a trend in forecast error, while a high variance component indicates that the model is not capable of replicating the variability of the time series (forecasts too static relative to underlying development).

The results shown in Panel B give similarly useful information. The Theil inequality coefficient – a normalized RMSE measure – lies between zero and one (with zero indicating a perfect fit). The main advantage of the Theil inequality coefficient, however, is that it can be decomposed into bias, variance and covariance proportions. The bias component T_t^B measures systematic errors in the forecasted data series and should ideally be close to zero. A value above 0.2 is usually taken as indication that the forecasts are biased in one way or another (the model has the persistent tendency to forecast values above/below the realized data path).¹⁶⁰ In such a case the model should be revised. The variance component T_t^V indicates whether the model is capable of replicating the degree of variability of the realized data set. Often realized data are more volatile than the forecasted data path (as in the case of low-order ARIMA or VAR regressions). In such an instance the variance component of the Theil test statistic would be very large. The last part, the covariance component T_t^C , captures the unsystematic

¹⁶⁰ See Pindyck and Rubinfeld (1998), p. 211.

error, i.e., the remaining error after accounting for the average error of the model. Since ideally the bias and variance component should be as close as possible to zero, while all components add up to 1 ($T_t^B + T_t^V + T_t^C = 1$), the covariance component should be relatively high, indicating that most of the errors are unsystematic and not related to a misspecification of the DGP.

Figure 84, Panel B shows the development of the decomposed Theil coefficients over different forecast horizons. Over the horizon of 12 months, the overall Theil inequality coefficient is very low, indicating a solid DGP. The decomposed Theil measures, however, show that with increasing forecast horizon the quality of the DGP deteriorates. The bias component increases steadily, confirming the MPE development in Panel A. This can be read as indication that the desired test properties of un-biasness and white noise of one-step-ahead forecast of an optimal forecast are confirmed for very short forecast horizons but not for the long-term forecasting case. With an increasing forecast horizon, the variance component increases, indicating that the model is incapable of replicating the realized data volatility. The variance component for the vertical analysis is also extraordinarily high. Hence, overall the model test – in this case the additive Brownian motion – should be dismissed as a forecasting model for the commodity price index (CRB Index). Only a full econometric test could conclude if the deviation is statistically significant or not.¹⁶¹

In the following section we will test the different commodities by applying a set of DGPs to test on a rolling basis 60 months out-of-sample between 2003 and 2008. For each regression the full set of test statistics will be presented.

4.6.3 Testing univariate data generating processes

The first part of the analysis focuses on the following univariate data generation processes, given in Table 37, while the second part will focus on normalized data and

¹⁶¹ I.e., test whether 60 months ahead regressed against the realized value with intercept produces a statistically significant intercept.

multivariate methods. The accuracy test statistics of the different out-of-sample forecasts for the six analyzed underlyings are shown in Table 38. To facilitate visual inspection we have highlighted the best DGPs with a green background to help the reader visualize clusters of strong performance (good forecast accuracy).

Table 37
Set of data generating processes for out-of-sample test analysis

a)	Additive Brownian motion (ABM)	$\Delta(y_t) = c + \varepsilon_t$
b)	Geometric Brownian motion (GBM)	$\Delta(\ln y_t) = c + \varepsilon_t$
c)	Additive Ornstein-Uhlenbeck process (AOUP)	$\Delta(y_t) = c + \phi_1 y_{t-1} + \varepsilon_t$
d)	Log-normal Ornstein-Uhlenbeck process (LnOUP)	$\Delta(\ln y_t) = c + \phi_1 \ln y_{t-1} + \varepsilon_t$
e)	Trend (time) stationary process	$y_t = c + \phi_1 \cdot t + \varepsilon_t$
f)	Trend (time) stationary process in log-terms	$\ln y_t = c + \phi_1 \cdot t + \varepsilon_t$
g)	Quadratic trend (time) stationary process	$y_t = c + \phi_1 \cdot t + \phi_2 \cdot t^2 + \varepsilon_t$
h)	Quadratic trend (time) stationary process in log-terms	$\ln y_t = c + \phi_1 \cdot t + \phi_2 \cdot t^2 + \varepsilon_t$
i)	Mixture (stochastic – deterministic) trend process	$\Delta y_t = c + \phi_1 \cdot t + \phi_2 \cdot y_{t-1} + \varepsilon_t$
j)	Mixture (stochastic – deterministic) trend process in log-terms	$\Delta(\ln y_t) = c + \phi_1 \cdot t + \phi_2 \cdot \ln y_{t-1} + \varepsilon_t$

When focusing on the top-performing DGPs (dark green), it becomes clear that, based on RMSPE and Theil's statistic (see Table 39), the mean-reverting additive and log-normal Ornstein-Uhlenbeck process seem to perform good for those asset classes with long price histories (CRB, gold and silver). This would confirm the assumption over the long run that commodity prices are mean-reverting (the additive Ornstein-Uhlenbeck process ranks, in 28% of the tested series, in one of the top percentiles highlighted in green, while the LnOUP process is observed in 17% of the top percentiles highlighted in green). However, especially for shorter time series (beginning in 1982 or 1987), several trend DGPs achieve better forecast accuracy out-of-sample.

Table 38

Forecast accuracy statistics for tested commodity prices using rolling 60-month out-of-sample forecasts between 2003 and 2008

Comdty	MEError	MPEError	RMSE	RMSPE	MAE	MAPE	Theil	Bias	Variance	Covariance
1973-2008										
Random Walk	- 26.26	- 7.87	49.78	16.90	41.02	14.44	9.9%	54.8%	45.2%	
AddBM	- 16.77	- 4.25	45.63	15.88	38.40	13.84	8.9%	51.6%	42.6%	5.8%
GeoBM	- 13.26	- 2.90	44.77	15.79	38.01	13.86	8.7%	50.7%	41.5%	7.8%
AOUP	- 11.83	- 2.53	38.49	13.43	32.23	11.63	7.4%	44.6%	45.4%	10.0%
LNOUN	- 11.43	- 2.38	38.14	13.31	31.89	11.51	7.4%	43.7%	45.7%	10.6%
Trend_t	- 26.04	- 10.92	48.67	19.58	43.39	17.52	9.0%	52.2%	46.4%	1.3%
Trend_t_In	- 24.51	- 12.07	50.32	20.37	44.78	18.19	9.2%	54.2%	44.3%	1.5%
Trend_t_2	- 26.04	- 7.13	57.14	19.53	48.58	17.20	11.4%	50.2%	19.1%	30.6%
Trend_t_2_In	- 21.45	- 5.26	57.40	19.88	49.35	17.68	11.4%	50.9%	20.9%	28.2%
MixModels	- 14.82	- 3.42	44.71	15.59	37.79	13.63	8.7%	49.6%	41.7%	8.7%
MixModels_In	- 11.70	- 2.21	43.80	15.45	37.34	13.59	8.5%	49.1%	40.5%	10.4%
Gold										
1973-2008										
Random Walk	- 74.65	- 16.14	104.02	22.44	82.83	19.18	15.1%	44.3%	55.7%	
AddBM	- 53.97	- 10.53	88.80	19.47	70.74	16.64	12.5%	47.6%	44.4%	8.0%
GeoBM	- 30.43	- 4.20	77.61	17.97	62.51	15.48	10.7%	53.2%	36.0%	10.8%
AOUP	- 40.54	- 6.52	83.52	18.52	65.95	15.70	11.5%	43.3%	44.3%	12.4%
LNOUN	- 40.79	- 6.61	83.58	18.54	66.06	15.73	11.5%	44.0%	43.7%	12.2%
Trend_t	- 64.24	- 25.43	122.95	36.84	107.22	32.78	15.6%	59.5%	39.8%	0.7%
Trend_t_In	- 130.66	- 44.97	164.14	52.01	155.95	49.00	19.3%	68.5%	30.4%	1.1%
Trend_t_2	- 182.94	- 45.63	208.22	49.13	182.94	45.63	36.9%	75.5%	3.0%	21.5%
Trend_t_2_In	- 202.84	- 50.09	235.04	54.98	202.89	50.11	42.9%	71.8%	1.8%	26.3%
MixModels	- 51.35	- 9.22	93.94	20.78	75.04	17.79	13.2%	48.1%	42.5%	9.3%
MixModels_In	- 66.37	- 13.46	101.47	22.01	80.98	18.83	14.6%	45.9%	47.0%	7.2%
Silver										
1973-2008										
Random Walk	- 89.79	- 7.43	197.76	24.31	148.84	20.33	17.0%	37.7%	62.3%	
AddBM	- 62.21	- 2.48	195.35	25.42	149.72	21.52	16.5%	43.7%	45.0%	11.3%
GeoBM	- 44.77	- 0.70	197.82	26.82	153.05	22.78	16.5%	47.4%	39.0%	13.6%
AOUP	- 45.31	- 16.57	198.02	31.43	159.83	27.45	15.7%	51.2%	32.4%	16.4%
LNOUN	- 12.93	- 5.77	179.09	25.21	139.45	21.63	14.7%	45.9%	38.8%	15.3%
Trend_t	- 92.38	- 7.90	190.21	22.31	139.83	19.44	16.2%	31.2%	44.8%	24.0%
Trend_t_In	- 61.10	- 2.38	182.34	22.71	139.41	19.59	15.1%	35.1%	56.9%	8.0%
Trend_t_2	- 318.52	- 50.10	364.38	52.42	318.52	50.10	40.4%	80.1%	7.4%	12.5%
Trend_t_2_In	- 540.56	- 90.73	593.38	93.76	540.56	90.73	80.2%	84.1%	3.1%	12.8%
MixModels	- 131.33	- 15.04	204.98	23.47	146.55	18.45	18.0%	31.4%	30.9%	37.7%
MixModels_In	- 139.87	- 16.70	210.76	24.46	152.71	19.53	18.7%	39.2%	25.2%	35.6%
Nickel										
1982-2008										
Random Walk	- 4,323.55	- 26.93	6,456.09	40.96	4,893.73	36.47	32.5%	45.2%	54.8%	
AddBM	- 4,229.03	- 26.21	6,403.60	41.24	4,867.84	36.73	32.3%	45.2%	53.2%	1.6%
GeoBM	- 4,193.11	- 25.85	6,383.38	41.19	4,852.53	36.66	32.1%	45.4%	52.9%	1.7%
AOUP	- 4,017.02	- 22.82	6,131.21	36.33	4,504.53	31.48	29.3%	38.1%	49.1%	12.8%
LNOUN	- 4,315.68	- 26.24	6,370.95	38.38	4,736.33	33.68	31.3%	43.2%	46.5%	10.3%
Trend_t	- 2,914.54	- 8.66	5,757.13	37.43	4,180.22	31.26	26.3%	31.9%	65.7%	2.4%
Trend_t_In	- 3,136.71	- 11.42	5,813.13	36.81	4,201.40	30.74	26.6%	33.6%	63.1%	3.3%
Trend_t_2	- 5,306.48	- 35.36	7,253.85	45.68	5,543.34	40.48	38.2%	50.3%	31.7%	18.0%
Trend_t_2_In	- 6,315.86	- 46.36	8,205.82	54.10	6,453.21	49.48	46.8%	58.3%	18.7%	25.0%
MixModels	- 3,888.47	- 21.84	6,051.02	36.75	4,482.17	32.06	29.0%	38.1%	55.2%	6.6%
MixModels_In	- 4,028.46	- 23.85	6,168.32	38.43	4,620.88	33.89	30.2%	41.1%	55.9%	3.1%
Oil										
1982-2008										
Random Walk	- 12.12	- 27.45	16.57	38.45	13.59	34.22	27.6%	53.5%	46.5%	
AddBM	- 13.36	- 31.51	17.89	42.41	14.79	37.99	31.4%	55.2%	37.8%	7.0%
GeoBM	- 12.97	- 30.14	17.46	41.02	14.40	36.69	30.1%	54.9%	39.6%	5.4%
AOUP	- 13.64	- 30.16	17.49	37.89	14.34	34.13	28.1%	59.9%	27.3%	12.7%
LNOUN	- 14.32	- 32.26	18.03	39.22	14.88	35.55	29.4%	62.5%	25.6%	11.9%
Trend_t	- 19.18	- 47.95	22.11	51.69	19.35	49.27	40.4%	75.4%	12.6%	6.0%
Trend_t_In	- 18.68	- 46.08	21.60	50.03	18.89	47.68	38.8%	74.6%	20.9%	4.5%
Trend_t_2	- 6.94	- 13.70	10.74	27.04	8.62	22.98	16.3%	46.5%	39.8%	13.7%
Trend_t_2_In	- 6.53	- 11.14	10.95	27.03	8.60	22.29	16.0%	40.5%	44.6%	15.0%
MixModels	- 15.74	- 37.72	19.51	44.45	16.28	40.66	33.8%	65.7%	21.0%	13.3%
MixModels_In	- 16.02	- 38.30	19.64	44.40	16.45	40.76	33.8%	67.2%	21.1%	11.7%
Steel										
1987-2008										
Random Walk	- 88.13	- 15.14	152.81	35.52	121.68	30.65	24.4%	44.1%	55.9%	
AddBM	- 85.78	- 14.30	154.82	36.82	124.52	31.89	24.8%	45.7%	50.4%	3.9%
GeoBM	- 84.71	- 13.96	154.52	36.88	124.43	31.95	24.7%	45.8%	50.3%	3.9%
AOUP	- 49.70	- 3.05	133.01	33.89	105.17	28.20	18.9%	37.3%	46.9%	15.8%
LNOUN	- 58.03	- 5.65	136.85	33.81	108.63	28.68	20.7%	38.7%	48.1%	13.2%
Trend_t	- 83.21	- 11.81	157.14	36.02	124.19	31.19	24.6%	40.4%	42.7%	16.9%
Trend_t_In	- 82.58	- 11.81	155.35	35.55	122.68	30.78	24.3%	40.4%	45.5%	14.1%
Trend_t_2	- 119.58	- 22.64	185.05	41.26	148.80	36.38	31.0%	50.2%	25.8%	24.1%
Trend_t_2_In	- 126.41	- 24.10	194.54	43.44	156.54	38.22	33.0%	50.7%	21.0%	28.3%
MixModels	- 131.40	- 28.05	185.14	40.90	147.85	35.87	31.6%	48.1%	20.3%	31.6%
MixModels_In	- 121.63	- 25.25	175.21	38.53	139.61	33.82	29.3%	46.7%	26.9%	26.3%

The best results are highlighted with a dark green background, whereas DGPs with scores within the top 15% percentile are highlighted in plain green (within the 30% percentile are highlighted with light green). Forecast statistics labeled in green indicate scale-invariant measures (MPE, RMSPE, MAPE, Theil, Bias, Variance and Covariance). All test statistics represent vertical (full sample path) measures; they are hence indicators of the ability to forecast over the entire forecast path of 60 months.

This is confirmed if one focuses on the DGPs that belong to the best 30% percentile. Mean-reverting DGPs in this percentile represent 44.4% of the cases.

Deterministic vs. stochastic trend models:

Trend deterministic models perform especially well in log-quadratic form for oil. Time trend deterministic processes seem to outperform stochastic trend processes for most of the tested underlyings. The additive Brownian motion even drops out of the top quantile for all commodities as shown in Table 39. Linear and quadratic trend deterministic processes seem to perform better than mixture models (models that contain stochastic and deterministic trends).

Table 39
RMSPE and Theil coefficient of accuracy for tested commodity prices using rolling 60-month out-of-sample forecasts between 2003 and 2008

Panel A								
accuracy measure:	RMSPE							
total (vertical) average	Cmddy	Gold	Nickel	Oil	Silver	Steel		
Rwalk	16.90	22.44	40.96	38.45	24.31	35.52		
AddBM	15.88	19.47	41.24	42.41	25.42	36.82		
GeoBM	15.79	17.97	41.19	41.02	26.82	36.88		
AOUP	13.43	18.52	36.30	37.89	31.43	33.59		
LNOUP	13.31	18.54	38.38	39.22	25.21	33.81		
Trend_t	19.58	36.84	37.43	51.69	22.37	36.02		
Trend_t_In	20.37	52.01	36.81	50.03	22.71	35.55		
Trend_t_2	19.53	49.13	45.68	27.04	52.42	41.26		
Trend_t_2_In	19.88	54.98	54.10	27.03	93.76	43.44		
MixModels	15.59	20.78	36.75	44.45	23.47	40.90		
MixModels_In	15.45	22.01	38.43	44.40	24.46	38.53		

Panel B								
accuracy measure:	Theil							
total (vertical) average	Cmddy	Gold	Nickel	Oil	Silver	Steel		
Rwalk	0.10	0.15	0.32	0.28	0.17	0.24		
AddBM	0.09	0.13	0.32	0.31	0.16	0.25		
GeoBM	0.09	0.11	0.32	0.30	0.17	0.25		
AOUP	0.07	0.12	0.29	0.28	0.16	0.20		
LNOUP	0.07	0.12	0.31	0.29	0.15	0.21		
Trend_t	0.09	0.16	0.26	0.40	0.16	0.25		
Trend_t_In	0.09	0.19	0.27	0.39	0.15	0.24		
Trend_t_2	0.11	0.37	0.38	0.16	0.40	0.31		
Trend_t_2_In	0.11	0.43	0.47	0.16	0.80	0.33		
MixModels	0.09	0.13	0.29	0.34	0.18	0.32		
MixModels_In	0.08	0.15	0.30	0.34	0.19	0.29		

The best results are highlighted with a dark green background, whereas DGPs with scores within the top 15% percentile are highlighted in plain green (within the 30% percentile are highlighted with light green). All test statistics represent vertical (full sample path) measures; they are hence indicators of the ability to forecast over the entire forecast path of 60 months.

Theil components:

The Theil inequality coefficient confirms the RMSPE numbers to a large extent (see Table 39, Panel B). It highlights the AOUP and LnOUP as dominant forecasting data generating processes. As expected the minimum Theil values correspond to the smallest mean error (ME). Also important, and similar to comparing the mean errors for the different DGPs, is the Theil bias component. In most cases the mean errors for the tested underlying are negative, indicating that the realized value was much higher than the simulated forecast.¹⁶² This is confirmed by the bias proportion in the Theil statistics (see Table 40). Given that models should be dismissed if they show bias proportions exceeding 20%, the results are surprisingly weak. All common forecast models presented in Table 37, p. 191, fail in this regard and hence do not fulfill one of the key requirements for a good forecasting DGP.

Table 40

Theil bias values as measure of forecast accuracy for tested commodity prices using rolling 60-month out-of-sample forecasts between 2003 and 2008

accuracy measure:	Bias						
total (vertical) average	Cmdty	Gold	Nickel	Oil	Silver	Steel	
Rwalk	0.55	0.44	0.45	0.54	0.38	0.44	
AddBM	0.52	0.48	0.45	0.55	0.44	0.46	
GeoBM	0.51	0.53	0.45	0.55	0.47	0.46	
AOUP	0.45	0.43	0.38	0.60	0.51	0.37	
LNOUN	0.44	0.44	0.43	0.62	0.46	0.39	
Trend_t	0.52	0.59	0.32	0.75	0.31	0.40	
Trend_t_In	0.54	0.69	0.34	0.75	0.35	0.40	
Trend_t_2	0.50	0.76	0.50	0.47	0.80	0.50	
Trend_t_2_In	0.51	0.72	0.58	0.40	0.84	0.51	
MixModels	0.50	0.48	0.38	0.66	0.31	0.48	
MixModels_In	0.49	0.46	0.41	0.67	0.39	0.47	

The best results are highlighted with a dark green background, whereas DGPs with scores within the top 15% percentile are highlighted in plain green (within the 30% percentile are highlighted with light green). All test statistics represent vertical (full sample path) measures; they are hence indicators of the ability to forecast over the entire forecast path of 60 months.

Performance relative to random walk:

Comparing the accuracy measures of the different DGPs with forecast test statistics for a random walk process yields a much more positive picture. The forecast statistics in

¹⁶² Note: the values are negative, since we have used $\hat{y}_{t+h,t} - y_{t+h}$ to specify the forecast error.

Table 38 and Table 39 show that a simple random walk usually underperforms the group of best DGPs. Focusing on the RMSPE as one key measure (see Table 39) shows that, only for the case of steel, the random walk process enters the 30% percentile. Revisiting the results of the Theil bias analysis presented in Table 40 shows that the high bias values are indeed not a particular problem of the different DGPs but are also visible for the random walk process.

Development over time:

Table 41 below shows how the accuracy statistics for the tested DGPs change over time for the different tested commodities. Most tested models show persistent forecasting performance that seems to be independent of the forecast horizon. Other models, however, seem to perform better over the short-term horizon (like mixture models or deterministic time trend models). The forecast statistics for the vertical (full path) analysis is usually confirmed (i.e., if a model performs well over the entire path, it also performs well over individual forecast horizons). However, it is interesting to note that Brownian motion processes generally perform better over short-term horizons than when measured over the entire forecast path.

Table 41
RMSPE as function of forecast horizon for tested commodity prices (rolling out-of-sample forecasts between 2003 and 2008)

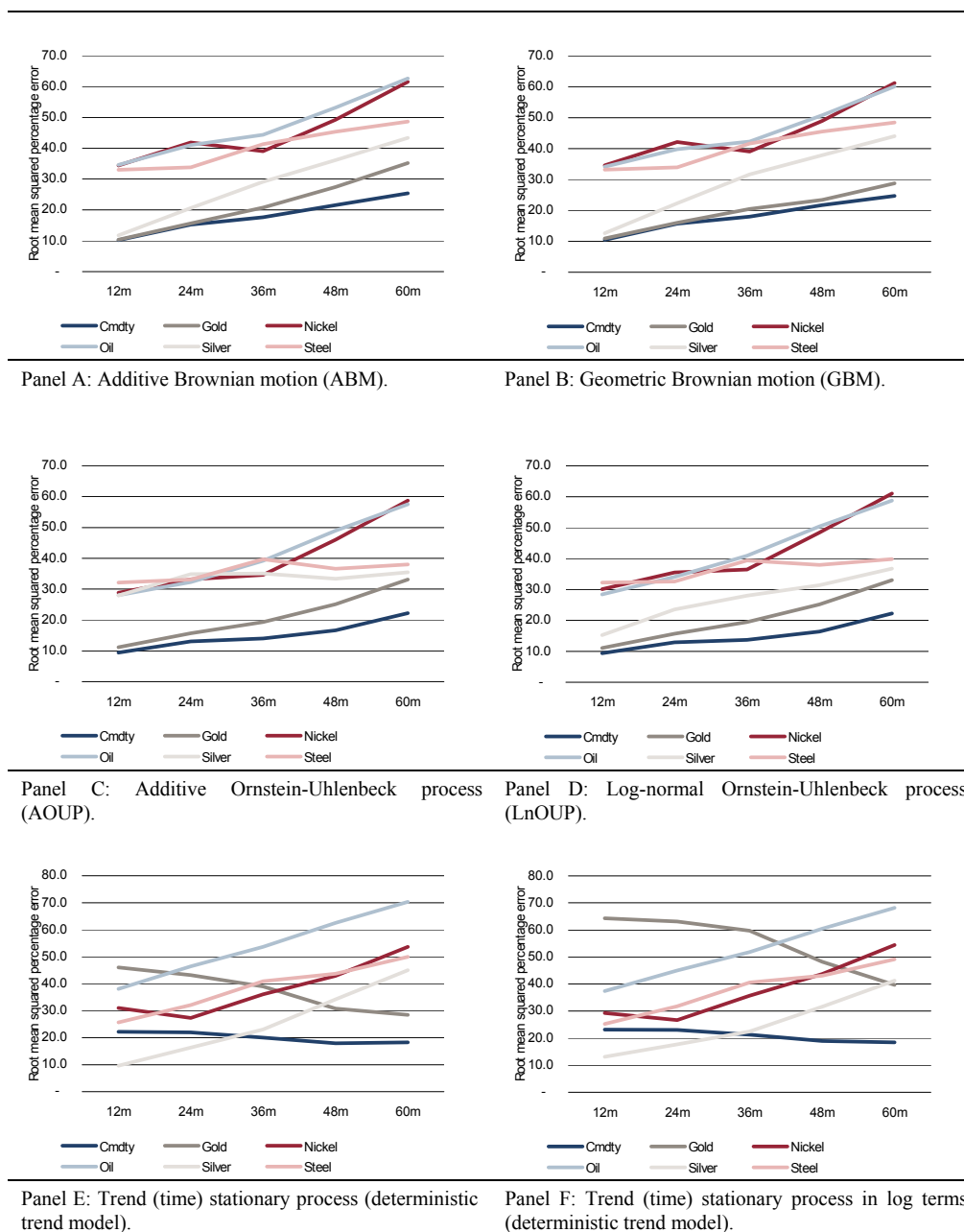
accuracy measure: RMSPE Crndty						
	12m	24m	36m	48m	60m	total (vertical)
1973-2008						
Rwalk	9.65	14.94	17.99	23.04	28.91	16.90
AddBM	10.22	15.29	17.61	21.56	25.41	15.88
GeoBM	10.52	15.69	17.98	21.72	24.65	15.79
AOP	9.41	13.09	14.02	16.67	22.27	13.43
LNOP	9.37	12.90	13.74	16.46	22.28	13.31
Trend_t	22.18	21.95	20.06	17.98	18.28	19.58
Trend_t_in	23.17	23.10	21.29	18.98	18.43	20.37
Trend_t_2	12.31	17.25	20.95	26.03	33.09	19.53
Trend_t_2_in	13.44	17.88	21.15	25.76	32.35	19.88
MixModels	10.31	15.11	17.06	20.27	25.06	15.59
MixModels_in	10.67	15.33	16.94	19.62	24.16	15.45
accuracy measure: RMSPE Nickel						
	12m	24m	36m	48m	60m	total (vertical)
1982-2008						
Rwalk	33.57	40.63	38.67	49.61	62.01	40.96
AddBM	34.39	41.87	39.04	49.28	61.53	41.24
GeoBM	34.53	42.10	39.05	48.95	61.19	41.19
AOP	33.06	33.14	34.66	46.03	58.71	35.35
LNOP	30.06	35.52	36.46	48.46	61.03	38.38
Trend_t	31.04	27.37	36.03	43.98	53.69	37.43
Trend_t_in	29.27	29.27	35.73	43.68	54.44	36.81
Trend_t_2	29.85	36.06	46.03	59.18	70.83	45.68
Trend_t_2_in	32.92	44.19	55.87	70.42	82.20	54.10
MixModels	31.02	35.21	33.11	44.92	57.87	36.75
MixModels_in	32.48	37.92	35.33	46.36	59.13	38.43
accuracy measure: RMSPE Gold						
	12m	24m	36m	48m	60m	total (vertical)
1973-2008						
Rwalk	10.55	17.03	23.79	33.01	42.00	22.44
AddBM	10.55	17.03	20.79	27.46	35.19	19.47
GeoBM	10.95	16.03	20.44	27.46	35.19	19.47
AOP	11.15	15.71	19.11	25.11	33.07	18.52
LNOP	11.08	15.72	19.43	25.15	33.02	18.54
Trend_t	46.00	43.20	38.97	30.75	22.68	36.84
Trend_t_in	64.36	63.18	59.74	48.19	39.68	52.01
Trend_t_2	29.82	41.92	63.08	64.66	73.83	49.13
Trend_t_2_in	31.03	45.31	69.35	73.72	85.71	54.98
MixModels	11.54	17.19	22.38	28.76	36.72	20.78
MixModels_in	11.07	17.27	23.58	31.83	40.59	22.01
accuracy measure: RMSPE Oil						
	12m	24m	36m	48m	60m	total (vertical)
1982-2008						
Rwalk	33.14	37.59	38.66	46.90	55.91	38.45
AddBM	34.63	41.05	44.36	53.19	62.69	42.41
GeoBM	34.11	39.79	42.27	50.80	60.07	41.02
AOP	33.04	32.31	39.21	48.93	57.50	37.89
LNOP	28.43	34.04	40.92	50.42	58.78	39.22
Trend_t	38.07	46.46	53.66	62.54	70.34	51.69
Trend_t_in	37.43	45.02	51.84	60.46	68.15	50.03
Trend_t_2	30.87	30.86	21.72	30.69	30.69	27.94
Trend_t_2_in	30.88	30.83	21.52	30.13	34.90	27.33
MixModels	31.72	38.99	46.25	56.07	64.87	44.45
MixModels_in	31.25	39.09	46.35	55.92	64.40	44.40
accuracy measure: RMSPE Silver						
	12m	24m	36m	48m	60m	total (vertical)
1973-2008						
Rwalk	10.91	18.35	27.19	38.07	45.03	24.31
AddBM	11.71	20.64	29.13	36.15	43.36	25.42
GeoBM	12.63	22.30	31.59	37.87	44.03	26.82
AOP	27.94	34.85	35.04	33.36	35.49	31.43
LNOP	15.23	23.56	28.05	34.25	36.74	25.21
Trend_t	9.62	17.70	23.06	34.25	44.99	22.71
Trend_t_in	13.12	17.70	23.06	31.82	41.26	22.71
Trend_t_2	35.11	44.34	53.97	65.88	75.59	52.42
Trend_t_2_in	65.02	82.43	100.45	116.33	127.76	93.76
MixModels	7.44	16.06	25.51	38.05	49.40	23.47
MixModels_in	8.32	17.67	26.46	38.99	50.23	24.48
accuracy measure: RMSPE Steel						
	12m	24m	36m	48m	60m	total (vertical)
1967-2008						
Rwalk	31.59	32.85	33.66	43.51	46.03	36.52
AddBM	33.00	33.80	41.34	45.36	48.64	36.82
GeoBM	33.16	33.89	41.60	45.49	48.45	36.88
AOP	32.10	33.06	39.70	46.69	50.91	33.86
LNOP	32.20	32.55	38.38	47.96	50.83	33.81
Trend_t	25.96	32.13	40.91	43.68	50.00	36.02
Trend_t_in	27.48	34.91	40.55	43.08	49.17	35.55
Trend_t_2	26.72	33.71	47.64	54.24	60.77	41.26
Trend_t_2_in	27.48	34.91	49.91	57.83	65.18	43.44
MixModels	28.17	34.21	43.73	54.47	63.17	40.90
MixModels_in	27.60	32.79	41.42	50.74	58.61	38.53

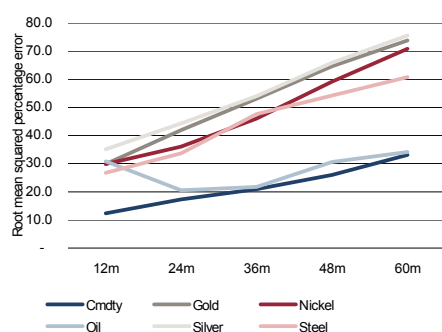
The best results are highlighted with a dark green background, whereas DGPs with scores within the top 15% percentile are highlighted in plain green (within the 30% percentile are highlighted with light green).

The RMSPE analysis over time also enables us to observe if the test property 4 of an optimal forecast (h -step-ahead errors have variances that are non-decreasing in h) is severely violated. A visual inspection of the RMSPE in Figure 85 unveils that for most tested DGPs this is indeed not the case (RMSPEs are increasing with the forecast horizon, confirming test property 4).

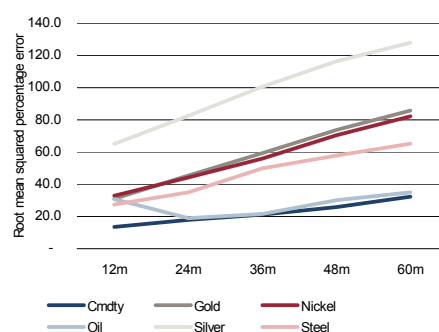
Figure 85

Forecast error (measured as RMSPE) over time for tested commodity prices (rolling out-of-sample forecasts between 2003 and 2008)

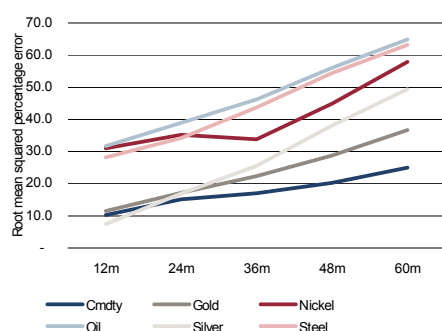




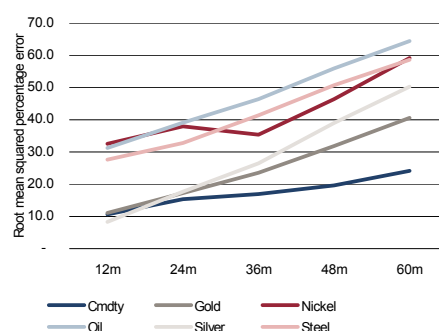
Panel G: Quadratic trend (time) stationary process (deterministic trend model with squared component).



Panel H: Quadratic trend (time) stationary process in log terms (deterministic trend model with squared component).



Panel I: Mixture (stochastic – deterministic) trend process.



Panel J: Mixture (stochastic – deterministic) trend process in log terms.

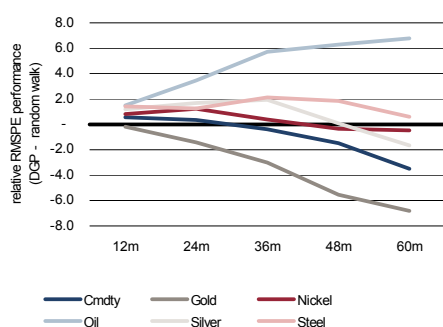
The deterministic trend model, however, seems to be the only exception (see Panel E and F). The RMSPE decreases over time for the commodity index and for gold. Given the importance of the test property, one can conclude that the simple linear time-deterministic process seem problematic for long-term forecasting of commodity prices. The decreasing RMSPE could, however, be a consequence of the steadily increasing sample size in the rolling out-of-sample analysis and the fact that the commodity index, together with gold and nickel, represent the longest time series used in this analysis. A rolling out-of-sample analysis that keeps the sample length constant could change the results. It is also likely that the strong mean-reverting market for commodities observed between 1970 and 2000 - relative to the last eight years - plays a crucial role.

The comparison of the different DGP forecast errors over time relative to the random walk forecast error highlights further important insights into the quality of the tested DGPs. In practice one should apply only DGPs that add value in terms of forecast accuracy relative to the naïve forecast of a random walk. The difference between the

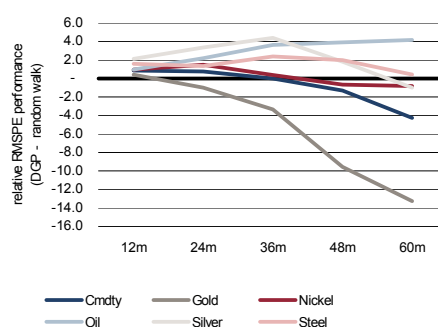
error of the tested DGP forecast and the forecast error of a random walk shows if this is indeed the case. A value smaller than zero indicates that the tested DGP is superior to a simple random walk (adding positive forecast performance). Figure 86 shows the result for all tested DGPs.

Figure 86

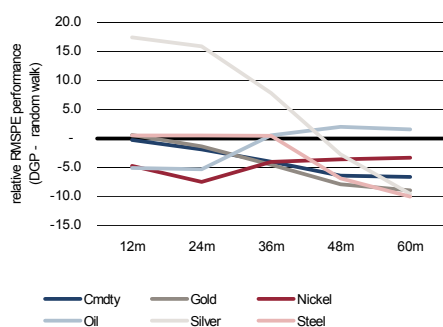
Forecast error (measured as RMSPE) relative to random walk over time for tested commodity prices (rolling out-of-sample forecasts between 2003 and 2008)



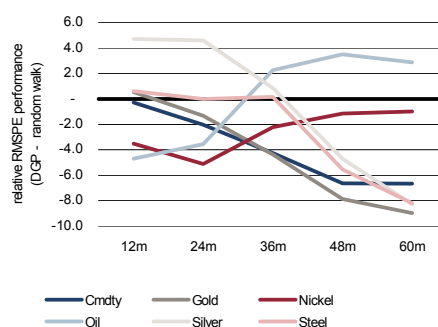
Panel A: Additive Brownian motion (ABM).



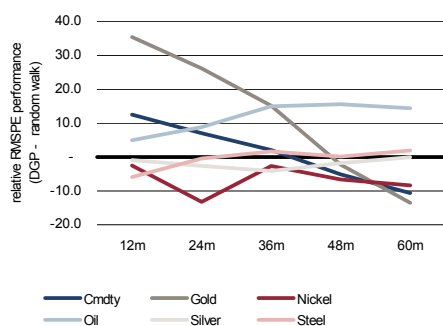
Panel B: Geometric Brownian motion (GBM).



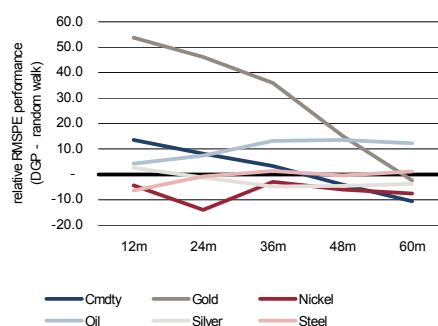
Panel C: Additive Ornstein-Uhlenbeck process (AOUP).



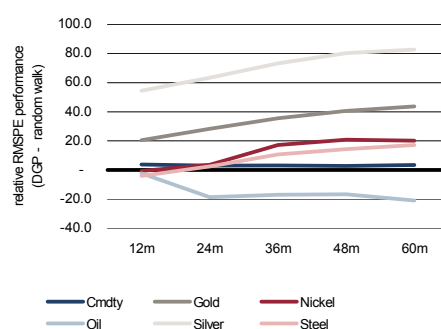
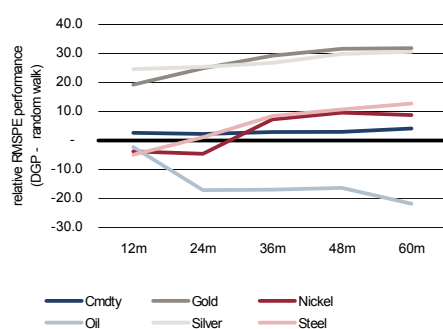
Panel D: Log-normal Ornstein-Uhlenbeck process (LnOUP).



Panel E: Trend (time) stationary process (deterministic trend model).

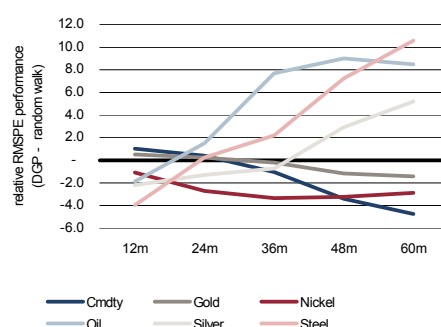
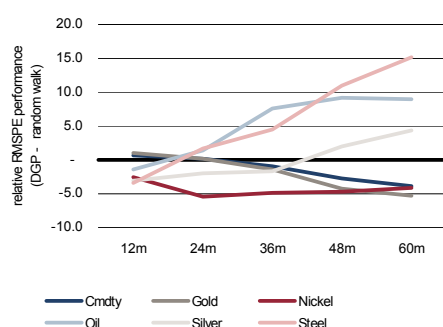


Panel F: Trend (time) stationary process in log terms (deterministic trend model).



Panel G: Quadratic trend (time) stationary process (deterministic trend model with squared component).

Panel H: Quadratic trend (time) stationary process in log terms (deterministic trend model with squared



Panel I: Mixture (stochastic - deterministic) trend process.

Panel J: Mixture (stochastic - deterministic) trend process in log-terms.

Over the short term, instances of positive outperformance (better forecast accuracy than random walk) could be achieved. Models that rely on deterministic trend components (Panel E to Panel J) seem to be the only real exception for industrial commodities (nickel and to some extent oil and steel). Brownian motion process types perform well over the short term for only gold (Panel A and Panel B), while Ornstein-Uhlenbeck performs well over the short term for a variety of commodity prices (see Panel B and Panel C). Only for silver and steel do mean-reversion processes not improve forecast accuracy relative to the random walk assumption. Over the long term (36 to 60 months) the picture reverts partially. The model with deterministic trend components performs well for gold and the aggregated commodity index and in some cases also for nickel. The suitability of non-deterministic DGPs like the AOUP and LnOUP for forecasting commodities over the longer term becomes visible in Panel D, where only the forecast for oil could not be improved relative to the random walk forecast.

4.6.4 Normalized data analysis and multivariate data generating processes

4.6.4.1 Out-of-sample test for normalized commodity prices

In Subsection 4.5.3, p. 174, we have normalized the different commodity data with respect to the US trade-weighted exchange rate and inflation in order to test for stationarity. Since stationarity is essential for improving forecast quality, we have repeated the above test series after adjusting for the trade-weighted USD exchange rate and inflation. The adjustments are shown below:

- Commodity price $\times$ USTW-Index/100
- Commodity price $\times$ USTW-Index/100 $\times$ Inflation-Discount-Index/100

Table 42 and Table 43 show the RMSPE of the normalized DGPs (Panel A) along with the difference from the original forecast error of the non-normalized data series (Panel B).

Table 42

RMSPE for exchange-rate normalized commodity prices using rolling 60-month out-of-sample forecasts between 2003 and 2008

Panel A							
accuracy measure:	RMSPE						
total (vertical) average	Cmdty	Gold	Nickel	Oil	Silver	Steel	
AddBM	7.97	11.27	35.88	39.11	18.35	30.20	
GeoBM	8.12	9.34	35.81	37.18	19.80	30.21	
AOUP	6.51	10.57	31.86	35.25	30.47	25.31	
LNOUP	6.40	10.58	33.59	36.98	20.95	25.61	
Trend_t	8.56	25.43	32.70	65.31	18.10	28.32	
Trend_t_In	8.55	37.90	33.97	56.78	15.61	27.91	
Trend_t_2	21.45	55.60	45.59	25.16	58.16	30.20	
Trend_t_2_In	24.55	74.43	54.56	21.95	118.58	29.63	
MixModels	10.07	16.77	34.78	44.12	24.63	34.59	
MixModels_In	9.30	19.02	35.77	43.51	24.12	32.58	

Panel B							
accuracy measure:	change in RMSPE (normalized (USD TW) - non-normalized accuracy statistic)						
total (vertical) average	Cmdty	Gold	Nickel	Oil	Silver	Steel	
AddBM	-7.91	-8.20	-5.36	-3.30	-7.07	-6.63	
GeoBM	-7.67	-8.63	-5.38	-3.85	-7.02	-6.67	
AOUP	-6.92	-7.95	-4.44	-2.64	-0.97	-8.28	
LNOUP	-6.91	-7.96	-4.79	-2.24	-4.26	-8.20	
Trend_t	-11.02	-11.40	-4.73	13.62	-4.27	-7.70	
Trend_t_In	-11.82	-14.11	-2.84	6.75	-7.10	-7.64	
Trend_t_2	1.92	6.47	-0.09	-1.87	5.74	-11.06	
Trend_t_2_In	4.67	19.45	0.46	-5.08	24.82	-13.80	
MixModels	-5.52	-4.02	-1.97	-0.33	1.16	-6.31	
MixModels_In	-6.15	-2.99	-2.66	-0.90	-0.34	-5.95	

The best results are highlighted with a dark green background, whereas DGPs with scores within the top 15% percentile are highlighted in plain green (within the 30% percentile are highlighted with light green). All test statistics represent vertical (full sample path) measures; they are hence indicators of the ability to forecast over the entire forecast path of 60 months.

The adjustment for the external value of the USD improves the different models significantly (measured in terms of RMSPE). The dominance of the mean-reverting models (AOUP and LNOUP) remains, however, unaffected. Only for Oil is a shift towards a quadratic deterministic DGP observed.

Table 43
RMSPE for inflation and exchange-rate normalized commodity prices using rolling 60-month out-of-sample forecasts between 2003 and 2008

Panel A							
accuracy measure:	RMSPE						
total (vertical) average	Cmdty	Gold	Nickel	Oil	Silver	Steel	
AddBM	15.90	12.03	41.76	49.37	18.25	32.86	
GeoBM	10.76	12.02	38.25	38.85	17.77	31.10	
AOUP	8.48	26.74	26.74	31.48	126.10	27.78	
LNOUP	10.19	14.61	28.91	30.53	28.18	28.55	
Trend_t	22.58	25.43	42.75	100.08	82.71	38.58	
Trend_t_In	18.61	20.07	39.11	60.27	30.66	30.13	
Trend_t_2	16.25	52.99	44.31	25.79	55.64	29.61	
Trend_t_2_In	19.66	138.58	57.86	47.69	215.66	27.88	
MixModels	51.29	35.04	41.02	49.36	91.54	45.18	
MixModels_In	18.70	22.54	38.07	44.82	32.34	34.16	

Panel B							
accuracy measure:	change in RMSPE (normalized (FX+inflation) - non-normalized accuracy statistic)						
total (vertical) average	Cmdty	Gold	Nickel	Oil	Silver	Steel	
AddBM	0.02	-7.44	0.52	6.96	-7.17	-3.97	
GeoBM	-5.02	-5.95	-2.94	-2.17	-9.06	-5.78	
AOUP	-4.94	8.22	-9.56	-6.41	94.67	-5.81	
LNOUP	-3.12	-3.93	-9.46	-8.68	2.97	-5.26	
Trend_t	3.00	-11.40	5.32	48.40	60.34	2.56	
Trend_t_In	-1.75	-31.95	2.30	10.24	7.95	-5.43	
Trend_t_2	-3.28	3.86	-1.37	-1.24	3.22	-11.65	
Trend_t_2_In	-0.22	83.61	3.76	20.66	121.90	-15.56	
MixModels	35.70	14.25	4.27	4.91	68.06	4.28	
MixModels_In	3.25	0.53	-0.36	0.42	7.87	-4.37	

The best results are highlighted with a dark green background, whereas DGPs with scores within the top 15% percentile are highlighted in plain green (within the 30% percentile are highlighted with light green). All test statistics represent vertical (full sample path) measures; they are hence indicators of the ability to forecast over the entire forecast path of 60 months.

If the out-of-sample performance test is conducted on real data that are additionally adjusted for the external value of the USD, the results do not change much. Most interestingly, gold and silver are best explained by an additive or geometric Brownian motion. However, the forecast accuracy for Gold and Silver is nevertheless better for the solo FX adjusted data set than for the inflation and FX adjusted series. Overall one can conclude that an adjustment for the external value of the USD adds significant improvement to the forecast. Inflation, however, affects the forecast performance of the

tested DGPs for only a few cases. The overall forecast accuracy could not be improved relative to the trade-weighted FX adjusted data set.¹⁶³

4.6.4.2 Out-of-sample test for Multivariate data generating processes

The specifications of a vector-autoregressive (VAR) model

After having tested the most important and widely used univariate (single Equation) data generating processes that are found in the literature, we will now focus on multivariate data generating processes. Usually multivariate estimation systems are used in micro and macro finance models in order to jointly describe the relationships among several variables. Often one aims to explain two or more individual Equations (like demand and supply equations) with a variable set that partially affects both equations. Additionally equilibrium conditions are imposed, such as that demand should equal supply. These multivariate models are often called “structural models,” since they are derived from an underlying economic theory. In this section, however, we will apply vectorautoregressive (VAR) models, which are considered atheoretical models that jointly estimate a vector of variables that is governed by an interdependent data generating process. Equation (131) shows a VAR specification as a function of the lag-order p that can be used for forecasting:

$$\begin{bmatrix} \Delta(Index)_{t+1} \\ USDTW_{t+1} \\ \Delta(Inflation)_{t+1} \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} + A_1 \begin{bmatrix} \Delta(Index)_t \\ USDTW_t \\ \Delta(Inflation)_t \end{bmatrix} + \dots + A_p \begin{bmatrix} \Delta(Index)_{t-p+1} \\ USDTW_{t-p+1} \\ \Delta(Inflation)_{t-p+1} \end{bmatrix} + B \begin{bmatrix} X_{1,t+1} \\ X_{2,t+1} \\ X_{3,t+1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{bmatrix} \quad (131)$$

The interdependence between the different factors can be observed separately by running Granger causality tests. When looking at how factors like inflation or the trade-weighted exchange rate improve (enhance) the data generating process of a certain commodity underlying, a Granger causality analysis with different lag lengths can yield

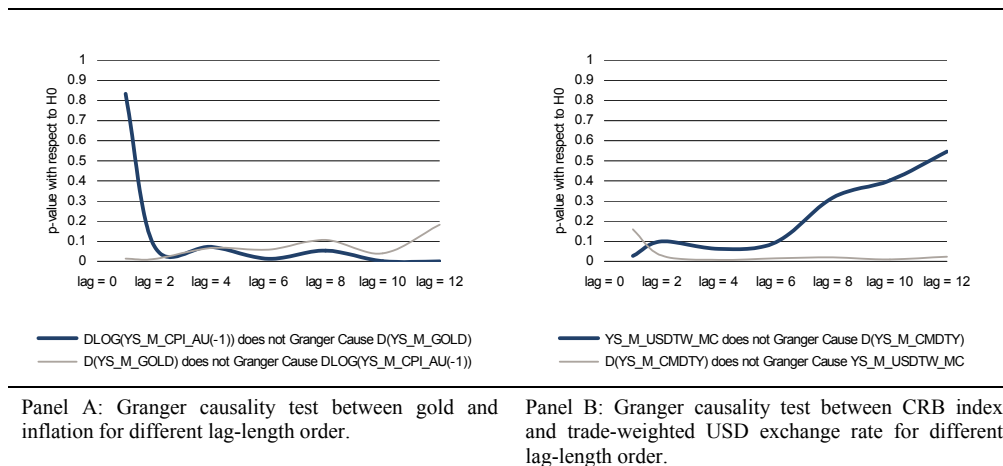
¹⁶³ The results of the Theil statistics and the development of the RMSPE as function of the forecast horizon are given in Appendix G, p. 352.

important insights regarding the importance of these factors in explaining the future development of the underlying in question.¹⁶⁴

The interdependencies among the different variables are shown, for example, for the commodity index in general and gold in particular (see Figure 87).

Figure 87

Granger causality test for gold and the CRB commodity price index with respect to inflation and the trade-weighted exchange rate



Panel A in Figure 87 shows that over the very short run, inflation does not help explain changes in gold. However, by increasing the lag number of the Granger causality test, a significant drop in p-values (signifying that we can dismiss the H_0 hypothesis of no Granger causality between gold and inflation) occurs. The test indicates that incorporating past inflation information into the data generating process for gold could enhance the forecast quality for this underlying. A similar conclusion can be made with respect to the USD-denominated commodity universe and the development of the USD.

¹⁶⁴ As often discussed in the literature, the term “causality” in this context is slightly misleading. Causality in the Granger sense should not be understood as describing a causal relation between the variables (A is causing B and vice versa) but rather as indication that the process A contains supplemental information that helps to explain B and vice versa. A good introduction to the general concept of Granger causality tests can be found in Wooldridge (2003), pp. 627-628. Granger causality tests within a VAR framework are discussed in Lütkepohl (2005), pp. 41-51 and pp. 102-104.

Additional lagged information on the USD trade-weighted index enhances the information content of an autoregressive data generating process for the commodity index (i.e., H_0 hypothesis of no Granger causality can be rejected).

The Granger conditionality test, however, does not give us information on the sign of influence. Hence, we cannot infer from the Granger conditionality test if the factors positively or negatively influence each other. To answer this question we would have to run VAR models, as done in the next section.

Testing the VAR models

The tests are conducted for several VAR specifications. The common denominator is the lag order (p). All the tests will be conducted with a lag order of one to ensure a degree of comparability to the single equation (univariate) models with autoregression components (AOUP, LnOUP, mixture models, etc.). The standard test specification is given in the form below:

$$\begin{bmatrix} \Delta(Index)_t \\ USDTW_t \\ \Delta(Inflation)_t \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} + A_1 \begin{bmatrix} \Delta(Index)_{t-1} \\ USDTW_{t-1} \\ \Delta(Inflation)_{t-1} \end{bmatrix} + \dots + A_p \begin{bmatrix} \Delta(Index)_{t-p} \\ USDTW_{t-p} \\ \Delta(Inflation)_{t-p} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{bmatrix} \quad (132)$$

We will test several variants of this standard VAR specification¹⁶⁵

- a) VarModel I: $\Delta(Index), USDTW, \Delta(Inflation(-1))$
- b) VarModel I_ln: $\log \Delta(Index), USDTW, \log \Delta(Inflation(-1))$
- c) VarModel II: $\log \Delta(Index), USDTW$
- d) VarModel III: $\log \Delta(Index), \log \Delta(Inflation(-1))$

¹⁶⁵ Since the inflation numbers are released with a one-month delay, we adjusted the inflation figures to equal the observation month of the underlying and the trade-weighted exchange rate data.

The out-of-sample test procedure is identical to the test procedure applied for the univariate data generating processes presented in Subsection 4.6.2 (see p. 185). The results for the multivariate models are compared with the results for the univariate cases. In Table 44 the model summary results are presented. The focus is set again on the vertical analysis (model performance over the entire forecast horizon of five years).

Table 44
RMSPE and Theil coefficient of accuracy for tested commodity prices for VAR-
specified forecasting models (rolling 60-month out-of-sample forecasts between 2003
and 2008)

Panel A								
accuracy measure:	RMSPE							
total (vertical) average	Cmnty	Gold	Nickel	Oil	Silver	Steel		
AddBM	15.88	19.47	41.24	42.41	25.42	36.82		
GeoBM	15.79	17.97	41.19	41.02	26.82	36.88		
AOUP	13.43	18.52	36.30	37.89	31.43	33.59		
LNOUP	13.31	18.54	38.38	39.22	25.21	33.81		
Trend_t	19.58	36.84	37.43	51.69	22.37	36.02		
Trend_t_In	20.37	52.01	36.81	50.03	22.71	35.55		
Trend_t_2	19.53	49.13	45.68	27.04	52.42	41.26		
Trend_t_2_In	19.88	54.98	54.10	27.03	93.76	43.44		
MixModels	15.59	20.78	36.75	44.45	23.47	40.90		
MixModels_In	15.45	22.01	38.43	44.40	24.46	38.53		
VarModel_I	17.10	19.23	42.52	41.33	29.30	41.34		
VarModel_I_In	16.87	17.20	42.41	40.54	28.35	41.13		
VarModel_II	16.87	17.30	42.58	40.70	28.58	41.01		
VarModel_III	15.77	17.85	41.25	40.23	26.57	36.85		

Panel B								
accuracy measure:	Theil							
total (vertical) average	Cmnty	Gold	Nickel	Oil	Silver	Steel		
AddBM	0.09	0.13	0.32	0.31	0.16	0.25		
GeoBM	0.09	0.11	0.32	0.30	0.17	0.25		
AOUP	0.07	0.12	0.29	0.28	0.16	0.20		
LNOUP	0.07	0.12	0.31	0.29	0.15	0.21		
Trend_t	0.09	0.16	0.26	0.40	0.16	0.25		
Trend_t_In	0.09	0.19	0.27	0.39	0.15	0.24		
Trend_t_2	0.11	0.37	0.38	0.16	0.40	0.31		
Trend_t_2_In	0.11	0.43	0.47	0.16	0.80	0.33		
MixModels	0.09	0.13	0.29	0.34	0.18	0.32		
MixModels_In	0.08	0.15	0.30	0.34	0.19	0.29		
VarModel_I	0.10	0.12	0.34	0.30	0.19	0.27		
VarModel_I_In	0.09	0.10	0.33	0.30	0.18	0.27		
VarModel_II	0.09	0.10	0.33	0.30	0.18	0.27		
VarModel_III	0.09	0.11	0.32	0.29	0.16	0.25		

The best results are highlighted with a dark green background, whereas DGPs with scores within the top 15% percentile are highlighted in plain green (within the 30% percentile are highlighted with light green). All test statistics represent vertical (full sample path) measures; they are hence indicators of the ability to forecast over the entire forecast path of 60 months.

The out-of-sample test results show that the dominance of the mean-reverting processes is challenged only by the VAR models for gold (Model II: VAR model with trade-weighted exchange rate as second endogenous variable) and partially for oil and silver.

For other underlyings the forecast performance could not be improved. This indicates that inflation and FX considerations play a role for precious metals (gold and silver), whereas industrial metals (steel and nickel) seem rather unaffected.

Out-of-sample test for normalized multivariate data generating processes

As with the univariate data generating processes, we have repeated the multivariate out-of-sample tests with normalized data. We will use in this context only the exchange rate normalized data set (VarModel I and VarModel I_In), since the unit-root test for this normalization variant was most promising (see Table 33, p. 179). The out-of-sample forecast accuracy measures are always related to the normalized data set. For comparison reasons only, scale-insensitive accuracy measures are employed (RMSPE and Theil).

Table 45

RMSPE and Theil coefficient of accuracy for tested normalized commodity prices for VAR specified forecasting models (rolling 60-month out-of-sample forecasts between 2003 and 2008)

Panel A: Forecast accuracy of normalized multivariate DGP with non-normalized univariate DGPs							
accuracy measure:	RMSPE						
total (vertical) average	Cmdty	Gold	Nickel	Oil	Silver	Steel	
AddBM	15.88	19.47	41.24	42.41	25.42	36.82	
GeoBM	15.79	17.97	41.19	41.02	26.82	36.88	
AOUP	13.43	18.52	36.30	37.89	31.43	33.59	
LNOUP	13.31	18.54	38.38	39.22	25.21	33.81	
Trend_t	19.58	36.84	37.43	51.69	22.37	36.02	
Trend_t_In	20.37	52.01	36.81	50.03	22.71	35.55	
Trend_t_2	19.53	49.13	45.68	27.04	52.42	41.26	
Trend_t_2_In	19.88	54.98	54.10	27.03	93.76	43.44	
MixModels	15.59	20.78	36.75	44.45	23.47	40.90	
MixModels_In	15.45	22.01	38.43	44.40	24.46	38.53	
VarModel_I	8.15	11.42	36.02	37.94	19.38	30.15	
VarModel_I_In	8.17	9.64	35.96	36.49	19.33	30.18	

Panel B: Forecast accuracy of normalized multivariate DGP with normalized univariate DGPs							
accuracy measure:	RMSPE						
total (vertical) average	Cmdty	Gold	Nickel	Oil	Silver	Steel	
AddBM	7.97	11.27	35.88	39.11	18.35	30.20	
GeoBM	8.12	9.34	35.81	37.18	19.80	30.21	
AOUP	6.51	10.57	31.86	35.25	30.47	25.31	
LNOUP	6.40	10.58	33.59	36.98	20.95	25.61	
Trend_t	8.56	37.90	32.70	65.31	18.10	28.32	
Trend_t_In	8.55	37.90	33.97	56.78	15.61	27.91	
Trend_t_2	21.45	55.60	45.59	25.16	58.16	30.20	
Trend_t_2_In	24.55	74.43	54.56	21.95	118.58	29.63	
MixModels	10.07	16.77	34.78	44.12	24.63	34.59	
MixModels_In	9.30	19.02	35.77	43.51	24.12	32.58	
VarModel_I	8.15	11.42	36.02	37.94	19.38	30.15	
VarModel_I_In	8.17	9.64	35.96	36.49	19.33	30.18	

The best results are highlighted with a dark green background, whereas DGPs with scores within the top 15% percentile are highlighted in plain green (within the 30% percentile are highlighted with light green). All test statistics represent vertical (full sample path) measures; they are hence indicators of the ability to forecast over the entire forecast path of 60 months.

Table 45, Panel A shows the RMSPE of the normalized VAR data generating processes relative to the univariate non-normalized results. The multivariate DGPs strongly outperform other, non-normalized, univariate DGPs. However, to assess the quality of the VAR-based forecast using normalized commodity prices one needs to compare the results with the forecast accuracy measures calculated for the normalized univariate DGP processes, which is done in Panel B. Panel B reveals that, despite improved forecasting quality, multivariate models add only limited value compared to normalized univariate DGPs. Again the dominance of the Ornstein-Uhlenbeck processes cannot be challenged.

4.6.5 Conclusion

The analysis shows that the assumption of mean reversion for commodity prices (especially over the long run) is justified. We have shown that AOUP and LnOUP outperform stochastic and deterministic trend models over longer forecasting horizons measured by different forecast accuracy measures (especially by looking at the RMSPE and the Theil coefficient of accuracy). Over the short run, trend models – deterministic or stochastic – have a certain justification, especially if the data are normalized in order to account for the external value of the USD, with trend-deterministic processes usually outperforming stochastic trend processes.

A major requirement for a good forecasting process – namely the un-biasness of the forecast – is, however, violated for nearly all tested DGPs, which is an indication that the models chosen are incomplete (omission of important variables).¹⁶⁶ The outperformance of several (best percentile) DGPs relative to the random walk process, however, is an indication that most of the tested DGPs still contribute positively to forecasting commodity prices.

¹⁶⁶ The omission of important variables generally leads to biased and inconsistent coefficient estimates, which leads to biased forecasts. For a detailed discussion on specification error with respect to omitted variables see Pindyck and Rubinfeld (1998), pp. 184-190 and Greene (2008), pp. 133-134.

With respect to normalization, we could show that the forecasting accuracy – measured by RMSPE or Theil's coefficient of accuracy – improves if the data are normalized for the external value of the USD. Normalizing for inflation did not improve the forecast accuracy in a significant way (either in the univariate or in the multivariate DGP case).

4.7 Critical assessment and conclusion

The aim of this section was to find the answer to the question of how and according to what specifications data generating processes for real-option analysis should be applied. The balance sheet simulation example in Subsubsection 3.2.2.1 (see p. 52) has shown that even for the most simple real-option analysis, assumptions for data generating process of the major underlying factor need to be made, and that these assumptions have a crucial impact on the project's real-option value.

This chapter has focused on describing major DGPs that are used to forecast commodity prices. We have shown practical ways of calibrating the different DGPs within an econometric framework. We have also conducted different stationarity tests for commodity prices in order to answer the question of process specification (integrated or non-integrated). The results show that the hypothesis of stationarity can be rejected for nearly all of the tested commodity prices. The notion of reversion to the mean is therefore not confirmed by stationarity tests. Only the applied variance ratio test indicates some justification for some commodities like silver, oil and nickel, which are – as industrial commodities – likely to be more exposed to cyclical factors and hence to correlate with the economic cycle and display some mean reversion. However, if commodity prices are normalized for the external value of the USD the hypothesis of commodity price stationarity cannot be rejected.

Rolling out-of-sample forecasts for the different tested commodities have shown that, despite the stationarity results (rejection of mean reversion), mean reversion DGPs perform better over the long-term horizon than stochastic difference stationary DGPs. Normalization of commodity prices, in order to reflect the change in the external value of the USD, have improved the forecast quality of the different tested DGPs, which is a confirmation of the stationarity tests for normalized commodity prices. Forecasting

normalized commodity prices, however, would require a view on the normalizing underlying (in the particular case of the external value of the USD and/or inflation). Processes like the additive or geometric Brownian motion have disappointed overall with the exception of forecasting over the short-term with normalized data.

5 Real-option pricing and process-dependent forecast variance

5.1 Introduction

After describing and estimating several data generating processes that are applied in standard real-option theory for project valuation, we want to test how the different DGPs affect option pricing if used in a Monte Carlo simulation of price options. Obviously different DGPs produce not only different mean forecasts but also different variance structures. The variance structures have a decisive impact on the real-option value. The degree of convergence towards a long-term variance profile differs between the different DGPs. Hence, the option value and other real-option-related key numbers like the investment threshold (the critical project value at which the rational investor is indifferent between investing and not investing) or disinvestment thresholds (project-abandoning points) differ depending on the variance structure of the data generating process chosen.

The impact of different DGPs on the investment decision under uncertainty has first been covered by Metcalf and Hassett (1995). Their focus was aimed towards entry and exit decisions for investments on a company level by modeling the input prices as AOUP or GBM process. These authors showed in their Monte Carlo simulation analysis that considering different DGPs for input prices results in similar cumulative investments. The main reason is that AOUP results in lower expected mean price levels (mean effect due to process reversion characteristic), which, however, are also accompanied by a lower variance level (variance effect). In contrast, GBM allows for typically higher prices (realized price effect due to stochastic trend) that come with the cost of higher uncertainty (increasing variance due to non-stationary process). The authors conclude that the variance effect and the price effect cancel each other out in their test setting, making the cumulative investments based on both processes nearly identical.

The question we want to answer is if the same can be assumed for pricing options (i.e., the influence on the option value) with different DGPs. In order to do so, we will develop in this chapter a CAPM-based efficiency criterion for option prices that is based on simulated option investment returns and option return distributions. The results are

also linked with two cornerstone papers by Gibson and Schwartz (1990) and Schwartz (1997), who use a state-space econometric approach to calibrate the most common DGPs in order to evaluate real-option projects that are conditional on long-term commodity prices.¹⁶⁷ The Gibson-Schwartz model will be used in this dissertation to highlight the differences between the tested DGP and more sophisticated but also more unstable DGPs (represented here with the Gibson-Schwartz two factor model) that are often found in the literature.

At this point it is important to highlight that our approach should be seen as a mean/variance valuation experiment in a non-risk-neutral world. We do not claim to derive option valuation that is consistent with market prices, nor do we claim to conduct option valuation that is consistent with risk-neutral option pricing theory. However, we do show under which conditions risk-free discounting of contingent claims without imposing equivalent Martingale measures (see Subsection 4.2.3, p. 139) can be conducted, and highlight possible caveats of this approach.

5.2 The Gibson-Schwartz two factor model

The two factor Gibson-Schwartz model, which will serve as a benchmark model for the option valuation, can be seen as a geometric Brownian motion model (factor one) with a stochastic drift component (factor two). The two SDEs used by Gibson-Schwartz are shown in Equations (133) and (134). The correlation between the two error terms is shown in Equation (135).

$$dx = (\mu - \delta)xdt + \sigma_1 x \sqrt{dt} \varepsilon_{1,t} \quad (133)$$

$$d\delta = \eta(\bar{\delta} - \delta)dt + \sigma_2 \sqrt{dt} \varepsilon_{2,t} \quad (134)$$

¹⁶⁷ Successive papers based on Schwarz (1997) have included publications by Cortazar and Schwarz (1998), Schwarz (1998) and Schwarz and Smith (2000). Most of the literature applies the Gibson and Schwarz (1990) two-factor model as the DGP, taking the estimated values in Schwarz (1997) as calibration input.

$$\sqrt{dt}\varepsilon_{1,t}\sqrt{dt}\varepsilon_{2,t} = \rho dt \quad (135)$$

This specified format was introduced by Gibson and Schwartz for valuing commodity options. The stochastic trend for the commodity prices x is determined by an instantaneous drift rate μ and a convenience yield component δ . The convenience yield component itself follows an additive Ornstein-Uhlenbeck process and is hence mean-reverting. The instantaneous convenience yield has the tendency to revert in this setup towards its long-term mean $\bar{\delta}$. The random components of both processes (random increments of the standard Brownian motion) are correlated with ρ . Gibson and Schwartz additionally specify the risk-adjusted process relevant for option valuation, which is given below:¹⁶⁸

$$dx = (r - \delta)xdt + \sigma_1 x \sqrt{dt}\varepsilon_{1,t}^* \quad (136)$$

$$d\delta = \left[\eta(\bar{\delta} - \delta) - \lambda \right] dt + \sigma_2 \sqrt{dt}\varepsilon_{2,t}^* \quad (137)$$

$$\sqrt{dt}\varepsilon_{1,t}^* \sqrt{dt}\varepsilon_{2,t}^* = \rho dt \quad (138)$$

The main difference between the real data generation processes in Equations (133) and (134) is the market price of risk of the convenience yield λ . The adjustment for the market price of risk is a reflection of the inability to hedge the convenience yield risk away. The interpretation of λ becomes clear if one focuses on the Equation (137). An increasing λ decreases the drift for the convenience yield, which leads to an increasing trend in the overall SDE for the commodity x under the equivalent martingale measure.¹⁶⁹ After the adjustment the new SDE for the convenience yield becomes:

¹⁶⁸ In our analysis we will use both variants for valuation by applying different discount rates.

¹⁶⁹ It is plausible to assume that the market price of risk for the instantaneous convenience yields changes over time. Gibson and Schwartz (1990) calculate a negative λ for oil using the sample period 1984-1988. However, in Schwartz (1997) λ is positive for all sub-sample periods tested (1985-1995, 1990-1995, 1993-1996). The estimates are in general associated with high parameter uncertainty.

$$d\delta = \eta(\hat{\delta} - \delta)dt + \sigma_2 \sqrt{dt} \varepsilon_{2,t}^* \quad (139)$$

with:

$$\hat{\delta} = \bar{\delta} - \frac{\lambda}{\eta} \quad (140)$$

The adjusted delta $\hat{\delta}$ represents an adjusted long-term mean for the convenience yields. The advantage of this model is that it allows commodities, which nevertheless remain assets priced on monetary considerations (value of money), to increase in time – for example along with inflation. This is in contrast to earlier assumptions that commodities should be priced as mean-reverting assets.¹⁷⁰

To calibrate the model Schwartz (1997) employs the future market for commodities. Schwartz (1997) shows that the futures price $F(X, T, \delta)$ that follows the joint stochastic processes in Equations (136) and (137) must satisfy the following partial differential Equations:¹⁷¹

$$\frac{1}{2} F_{XX} X^2 \sigma_1^2 + \frac{1}{2} F_{\delta\delta} \sigma_2^2 + F_{X\delta} X \rho \sigma_1 \sigma_2 + F_X X (r - \delta) + F_\delta (\eta(\bar{\delta} - \delta) - \lambda) - F_T = 0 \quad (141)$$

The closed form solution to this PDE for calculating the expected future values is derived by Jamshidian and Fein (1990) and Bjerk Sund (1991). The expected futures value (future price for a contract maturing at time T) is given by:¹⁷²

¹⁷⁰ The model suggested by Pindyck (1999) chooses a similar avenue by assuming that commodity prices follow a linear trend whereby deviations are mean-reverting towards a fixed linear trend.

¹⁷¹ See Gibson and Schwartz (1990), p. 961 for deriving the PDF for a general continuing claim that follows their suggested joint stochastic process.

¹⁷² Deriving a closed form solution for futures prices enables the calculation of initiation values for the state variables X_0 and δ_0 by fitting Equation (142) against the observed futures curve in the market instead of running an iterative numerical method as applied in Gibson and Schwartz (1990). Note that the

$$F(X, T, \delta) = X_0 \exp \left(-\delta_0 \frac{1 - e^{-\eta T}}{\eta} + A(T) \right) \quad (142)$$

with:

$$A(T) = \left(r - \hat{\delta} + \frac{1}{2} \frac{\sigma_2^2}{\eta^2} - \frac{\sigma_1 \sigma_2 \rho}{\eta} \right) T + \frac{1}{4} \sigma_2^2 \frac{1 - e^{-2\eta T}}{\eta^3} + \left(\hat{\delta} \eta + \sigma_1 \sigma_2 \rho - \frac{\sigma_2^2}{\eta} \right) \frac{1 - e^{-\eta T}}{\eta^2} \quad (143)$$

Based on a Kalman filter methodology, Schwartz (1997) extracts the parameters for the joint stochastic process described above for oil, gold and copper. The estimated parameters for oil are given in Table 46.

Table 46
Gibson – Schwartz parameter values

Parameters	Values	Standard errors
μ	0.082	0.120
η	1.187	0.026
α	0.090	0.086
σ_1	0.212	0.011
σ_2	0.187	0.012
ρ	0.845	0.024
λ	0.093	0.101
r	0.05	
Z_0	17.39	
δ_0	-0.057	

Source: Schwarz (1997), p. 939. The data were estimated during the sample period 1/15/1993-5/16/1996. The state variables were initialized on January 2, 1998.

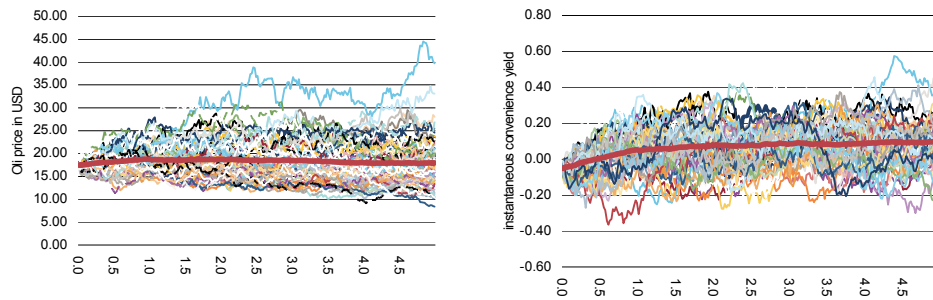
Based on the parameter estimates in Table 46, one can apply Monte Carlo simulation for the real and risk-neutral world. The simulation paths for the real data generating

two required variables (oil price X_0 and instantaneous convenience yield δ_0) are actually not observable in the market; hence the calculated values are proxies. See Gibson and Schwartz (1990), p. 962.

process and the risk-neutral version of the Gibson and Schwartz (1990) model are given in Figure 88 and Figure 89.

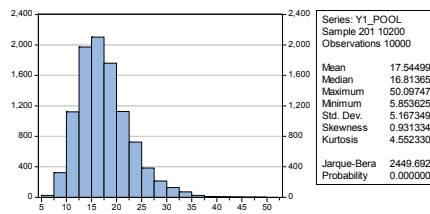
Figure 88

Simulated path for oil and convenience yield between 1998 and 2002 using Gibson-Schwartz two factor (real) model specification

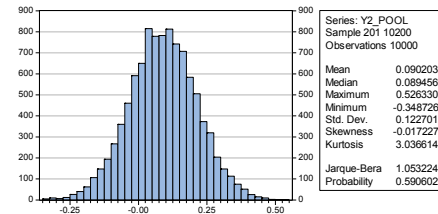


Panel A: Simulated (real) 250 paths for oil over a forecast horizon of five years.

Panel B: Simulated (real) 250 paths for the oil price convenience yield over a forecast horizon of five years.



Panel C: Distribution of the simulated oil price at five year forecast horizon using the Gibson-Schwartz real process specification (10000 simulations).

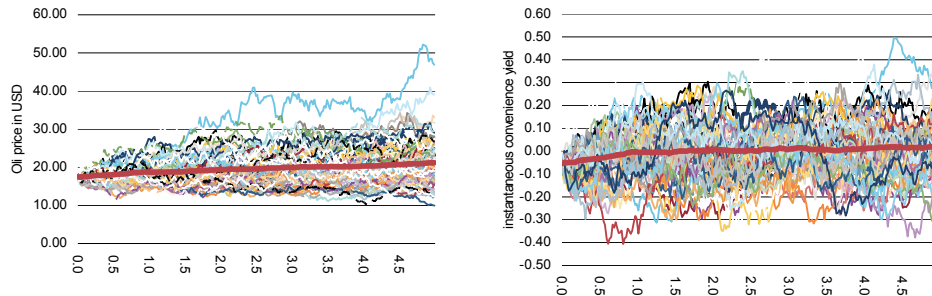


Panel D: Distribution of the simulated oil price convenience yield at five year forecast horizon using the Gibson-Schwartz real process specification (10000 simulations).

Panel C in Figure 88 shows the major advantage of the model, namely that the price of oil is skewed to the right, underscoring the capability of the model to replicate the real distribution characteristics of the oil price. The instantaneous convenience yield has a normal distribution characteristic (see Jarque-Bera statistic in Panel D), which is in line with economic theory that the convenience yield should move along the business cycle.

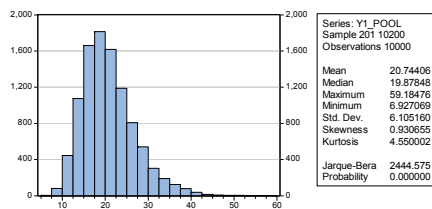
Figure 89

Simulated path for oil and convenience yield between 1998 and 2002 using Gibson-Schwartz two factor (risk-neutral) model specification

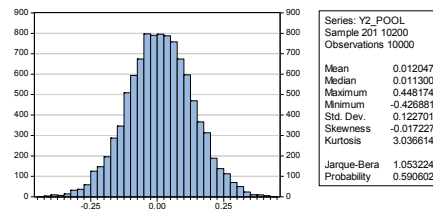


Panel A: Simulated (real) 250 paths for oil over a forecast horizon of five years.

Panel B: Simulated (real) 250 paths for the oil price convenience yield over a forecast horizon of five years.



Panel C: Distribution of the simulated oil price at five year forecast horizon using the Gibson-Schwartz risk-neutral process specification (10000 simulations).

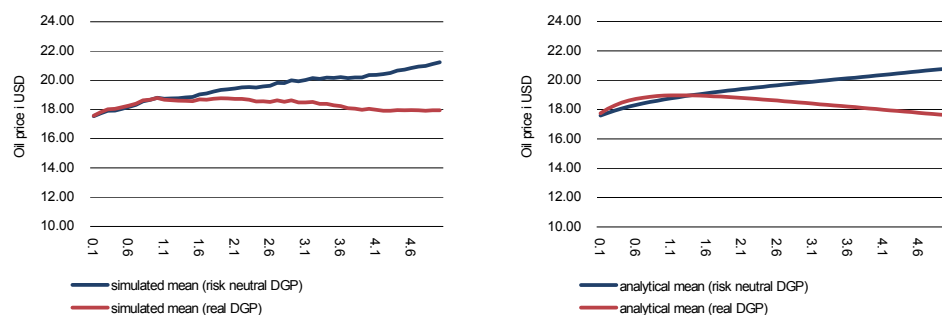


Panel D: Distribution of the simulated oil price convenience yield at five year forecast horizon using the Gibson-Schwartz risk-neutral process specification (10000 simulations).

The risk-neutral Monte Carlo simulations are illustrated in Figure 89. The major difference between the real and the risk-neutral data generation process – based on the estimated parameters in Schwartz (1997) – becomes obvious if we compare the simulated mean for oil over the horizon of five years (1998-2002). As shown in Figure 90, the simulated average price path (average over 250 simulations) for the risk-neutral process lies significantly above the simulated real data generating process for oil. The reason is the positive price of risk component within the convenience yields equation. The positive price of risk component leads to a lower adjusted long-term convenience yield convergence, which increases the drift component in the oil price SDE.

Figure 90

Illustrating the difference in the mean forecast path for oil between 1998-2002 for the risk-neutral and real model calibration (simulated and analytical solution)



Panel A: Simulated (250 simulations) mean path over time for the oil price using the risk-neutral and the real model calibration.

Panel B: Analytical mean path over time for the oil price using the risk-neutral and the real model calibration.

Since within the risk-neutral framework the estimated value for λ is set equal to 0.093, with the initiation state variable $\delta = -0.057$, the adjusted long-term mean for the convenience yield $\hat{\delta}$ is indeed lower than in the real DGP. What follows is the replication of the forecast distribution for the different tested alternative and more simple DGPs applied in the previous chapter. This will allow for a direct comparison of the forecast distribution characteristics between the more sophisticated Gibson-Schwartz model and the tested and simpler alternative models.

5.3 Comparing forecast distributions of different DGP models

5.3.1 Assuming stable data generating process parameters

The real-option value of an investment project does not depend only on the strike price, the risk-free interest rates, and the time to maturity, but also on the data generating process that is chosen to describe the path evolution of the underlying asset. The choice of the specific DGP has consequences for the calculated option price via the mean and variance characteristics of the simulated price paths.

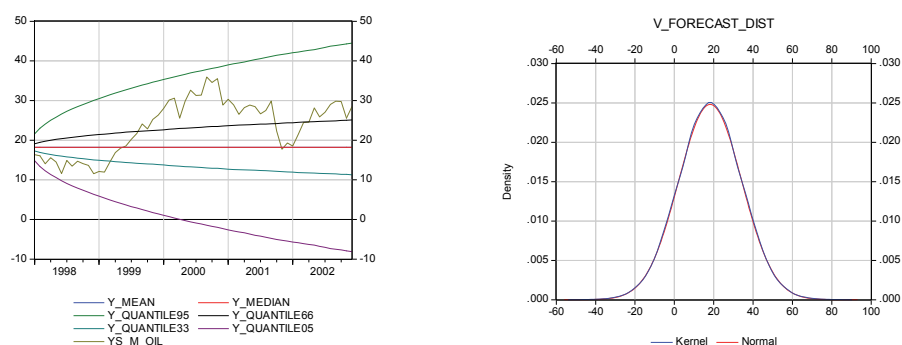
To illustrate the case, we will simulate the previously used DGPs for the commodity oil, from January 1998 to December 2002, to match the sample used in Gibson and Schwartz (1997).¹⁷³ The tested DGPs are:

- Random walk
- Gibson and Schwartz (1997) two-factor data generating process
- Additive Brownian motion (ABM)
- Geometric Brownian motion (GBM)
- Additive Ornstein-Uhlenbeck process (AOUP)
- Log-normal Ornstein-Uhlenbeck process (LnOUP)
- Mixture model: mean revision to trend model (MixModel)

The generated simulation path distribution is compared to the realized oil price path over time. Additionally, we will compare the different results with the assumption of normally distributed simulation paths. Furthermore, we will highlight the impact of parameter uncertainty for the simulation path distribution of the tested DGPs.

Figure 91 below illustrates a simulated random walk path as the simplest DGP. The mean and median values are identical, and the forecast is constant over time (equal to the last observed in-sample value). The fitted probability distribution (Kernel density estimation) is symmetrical and, as expected, equal to the normal distribution function.

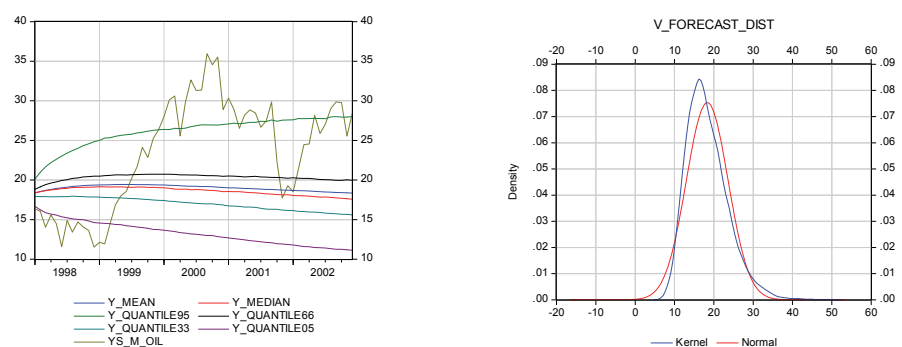
¹⁷³ The simulation is based on an antithetic variance reduction technique and is conducted 10000 times. The simulations are programmed using Eviews[®]. The Kernel density estimations are calculated using Eviews[®].

Figure 91**Probability distribution of random walk forecast for oil between 1998 and 2002**

Panel A: Forecast mean and median together with the forecast probability quantiles (5%, 33%, 66% and 95%) for a random walk DGP.

Panel B: Forecast distribution in 5 years (Kernel density) compared with a fitted normal distribution.

As shown in Panel A in Figure 91, the realized oil price does not breach the 5% to 95% quantile range and remains close to the 33% and 66% quantile corridor. So the forecast distribution is wide; however, given the ex-post view on the realized path this seems appropriate.

Figure 92**Forecast probability distribution for oil between 1998 and 2002 using Gibson-Schwartz DGP (real process calibration)**

Panel A: Forecast mean and median together with the forecast probability quantiles for the Gibson-Schwartz DGP.

Panel B: Forecast distribution in 5 years (Kernel density) compared with a fitted normal distribution.

Figure 92 shows the graphical illustration of the simulated path mean and median with the corresponding quantiles for the Gibson-Schwartz model introduced in Section 5.2. It is equivalent to the simulation paths in Figure 88 (Panel A). However, Figure 92 makes the picture more accessible and also contains the realized oil price path.

The simulated path distribution is positively skewed (skewed to the right) indicating that the probability that the realized path is above the estimated mean forecast is below 50%. More important, however, is the fact that several times the realized oil price breaches the 95% quantile, indicating that the model was not capable of replicating the volatility of the forecasted underlying, which is a warning that option prices valued using this process would likely have been undervalued (i.e., too cheap).

Figure 93

Forecast probability distribution for oil between 1998 and 2002 using additive Brownian motion as DGP

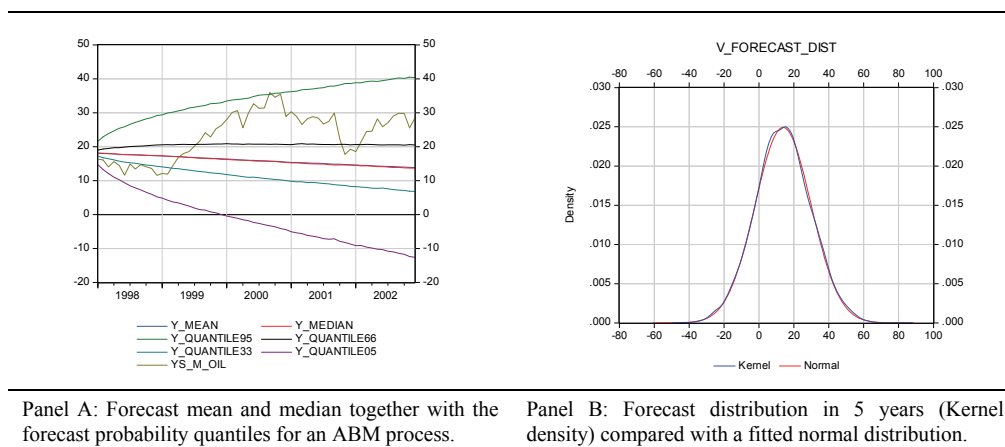
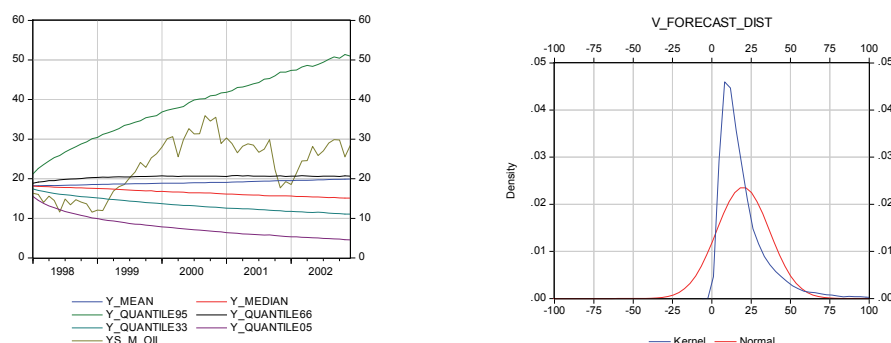


Figure 93 shows the simulated forecast paths for the additive Brownian motion DGP. The advantages and drawbacks are visible in the distribution charts (Panel B). Assuming an ABM allows for the possibility of negative asset prices. One advantage is the symmetric distribution of the simulation paths, which simplifies computation.

Figure 94

Forecast probability distribution for oil between 1998 and 2002 using geometric Brownian motion as DGP



Panel A: Forecast mean and median together with the forecast probability quantiles for a GBM process.

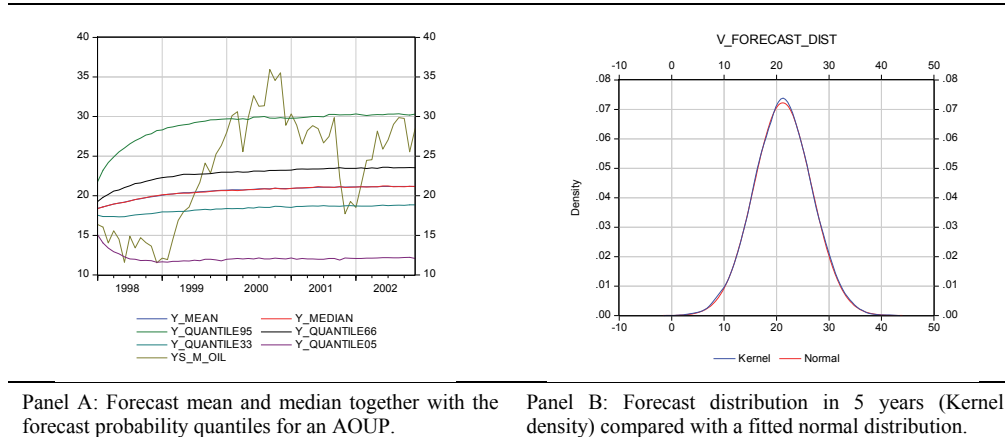
Panel B: Forecast distribution in 5 years (Kernel density) compared with a fitted normal distribution.

As illustrated in Figure 94, the geometric Brownian motion process eliminates the possibility of negative asset prices. The realized oil price path never approaches the 5% to 95% percentile barrier. The simulated forecast path distribution is also skewed to the right (positive skewness). With its skewness it comes closest to the Gibson-Schwartz forecast distribution.

Figure 95 shows the additive Ornstein-Uhlenbeck forecast results. The forecasted paths are – despite the theoretical possibility of negative values – all positive. The trajectory distribution is symmetric and close to a normal distribution. Similar to the Gibson-Schwartz model, the realized oil price path crosses the 95% quantile of the forecast distribution, which is an indication that the DGP does not properly capture the possible volatility of the underlying.

Figure 95

Forecast probability distribution for oil between 1998 and 2002 using an additive Ornstein-Uhlenbeck process as DGP



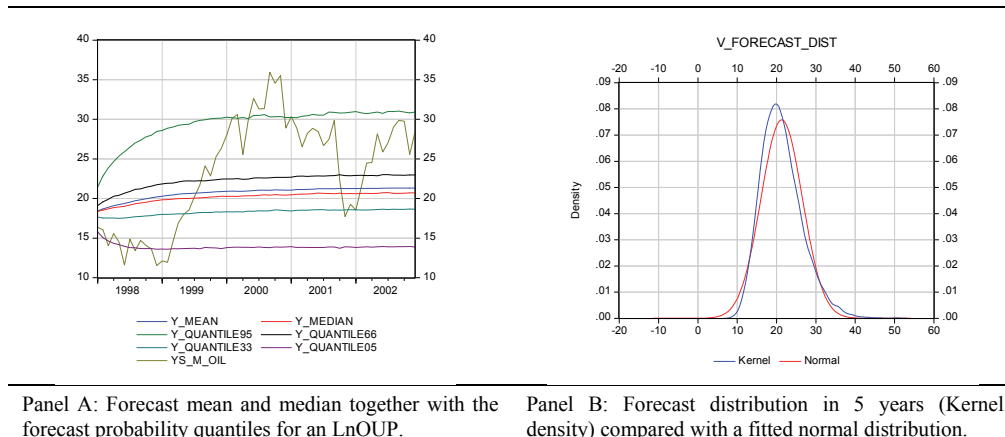
Panel A: Forecast mean and median together with the forecast probability quantiles for an AOUP.

Panel B: Forecast distribution in 5 years (Kernel density) compared with a fitted normal distribution.

The log-normal Ornstein-Uhlenbeck process assumption also produces only positive forecast trajectories for oil. The forecast dispersion, however, is higher than for the additive O-U case. The trajectory distribution of the simulated forecasts is skewed to the right (positive skewness). Similar to the AOUP model, the LnOUP is not able to properly capture possible asset volatility (path crosses the 95% quantile).

Figure 96

Forecast probability distribution for oil between 1998 and 2002 using a log-normal Ornstein-Uhlenbeck process as DGP



Panel A: Forecast mean and median together with the forecast probability quantiles for an LnOUP.

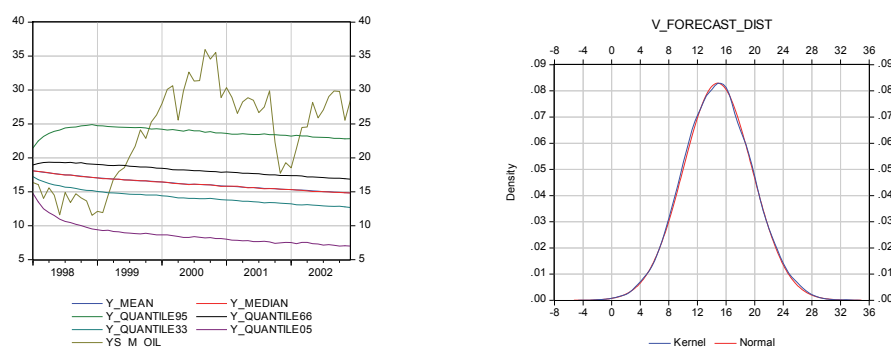
Panel B: Forecast distribution in 5 years (Kernel density) compared with a fitted normal distribution.

The last tested DGP is the mixture model (mean revision towards trend). Since the trend is pointing downwards, this model would also possibly forecast negative asset values. Since the revision to the mean around a trend is set up in an additive framework (similar

to the standard AOUP), the forecast distribution of the simulation values (see Figure 97, Panel B) is normal. The ability to capture the future underlying probability is the weakest for the mixture model. The oil price is several times far beyond the 95% quantile of the forecast distribution. In this regard the likelihood of under-pricing options (for using mixture models as DGPs to simulate future option price payoffs) is likely the highest, followed closely by the Gibson-Schwartz model. The assessment for mixture models does not change if oil is taken in log terms (see Figure 98).

Figure 97

Forecast probability distribution for oil between 1998 and 2002 using mixture model as DGP

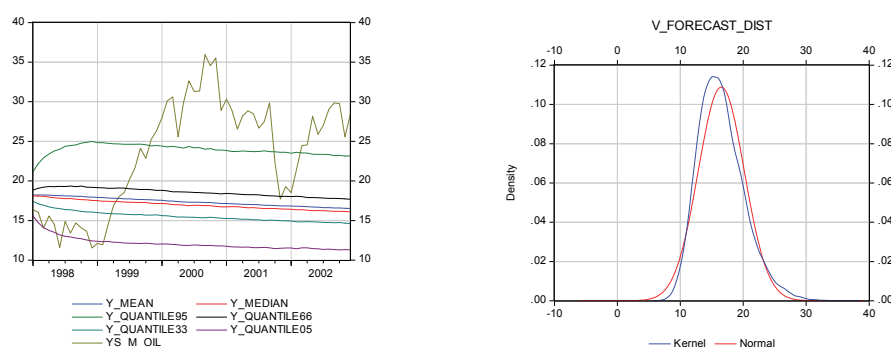


Panel A: Forecast mean and median together with the forecast probability quantiles for a mixture model.

Panel B: Forecast distribution in 5 years (Kernel density) compared with a fitted normal distribution.

Figure 98

Forecast probability distribution for oil between 1998 and 2002 using mixture model in logarithmic terms as DGP



Panel A: Forecast mean and median together with the forecast probability quantiles for a log-specified mixture model.

Panel B: Forecast distribution in 5 years (Kernel density) compared with a fitted normal distribution.

Beside the too-narrow forecast probability interval, the skewness of the 5-year forecast distribution is the only interesting feature of the log-normal mixture model specification.

The results indicate that several of the tested DGPs – which have shown dominance relative to other DGPs with respect to forecast accuracy (AOUP or LnOUP) – fail to account properly for the asset volatility in the test sample between 1998 and 2002. That is, one can assume that valuing options based on price simulations with mean reversion DGPs would lead to undervaluation due to incorrectly captured asset volatility. The results shown confirm in this regard the results by Metcalf and Hassett (1995) that mean reversion processes impose – relative to ABM – a too-low volatility regime. More simple DGPs like ABM or GBM outperform in this regard even the most sophisticated DGP applied (Gibson-Schwartz process).

5.3.2 Assuming instability of the data generating process parameters

The impact of parameter uncertainty plays a very important role in regression-based forecast and hence project valuation. Accounting for parameter uncertainty could partially correct the often too-optimistic forecast error confidence intervals discussed in point 5.3.1. Thus far, we have assumed that the forecast parameters applied in the Monte Carlo simulations are stable. Based on parameter uncertainty, which is measured by the coefficient standard error given for each estimated DGP, one can conduct Monte Carlo simulations by altering the DGP parameter for each simulated path. The resulting forecast probability distribution and mean-median paths differ substantially from the stable parameter Monte Carlo forecasts. Note that the inclusion of parameter uncertainty does not affect the median forecast. This is due to the assumption that the regression parameters are:

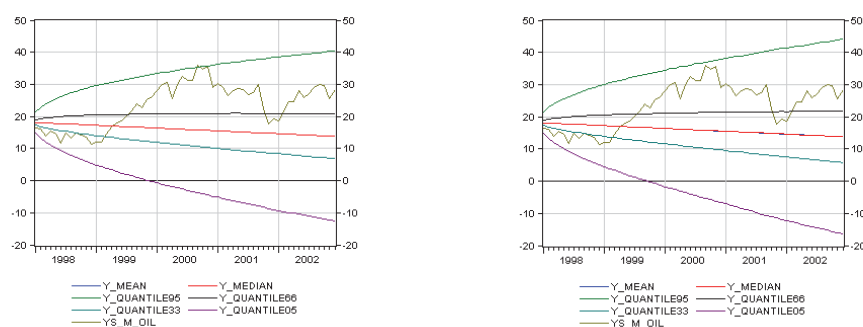
- a) normally distributed¹⁷⁴ and
- b) uncorrelated with each other.

¹⁷⁴ For small sample sizes, it is more appropriate to assume that the parameters are t-distributed. However, due to the large sample, we will work under the assumption of normally distributed regression coefficients.

Hence, parameter uncertainty directly affects the probability distribution via higher moments. Figure 99 shows the different forecast intervals for some of the DGPs applied in point 5.3.1, compared to using the same DGPs but allowing for parameter uncertainty.

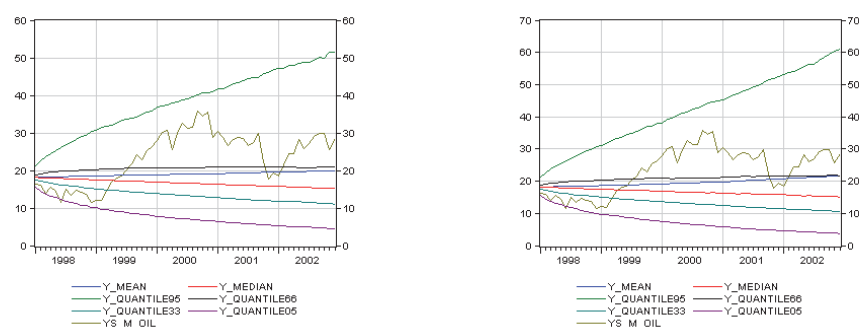
Figure 99

Comparing forecast confidence intervals between 1998-2002 for oil price forecasts by using different DGPs and allowing for parameter uncertainty



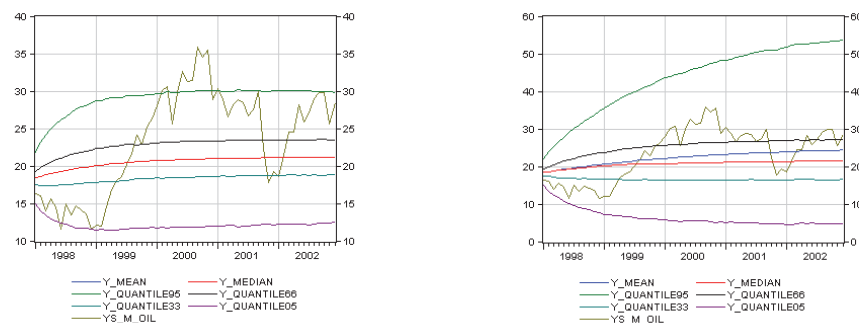
Panel A: Forecast probability quantiles for an ABM process without parameter uncertainty.

Panel B: Forecast probability quantiles for an ABM process with parameter uncertainty.



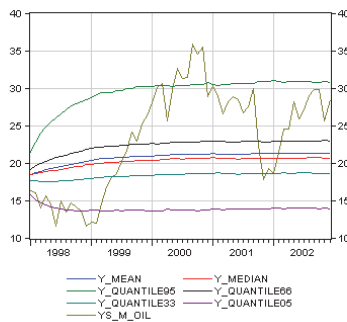
Panel C: Forecast forecast probability quantiles for a GBM process without parameter uncertainty.

Panel D: Forecast probability quantiles for a GBM process with parameter uncertainty.

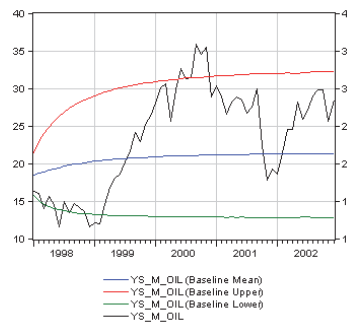


Panel E: Forecast probability quantiles for an AOUP without parameter uncertainty.

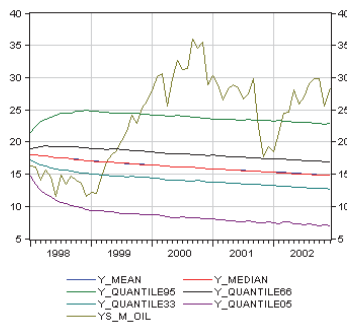
Panel F: Forecast probability quantiles for an AOUP with parameter uncertainty.



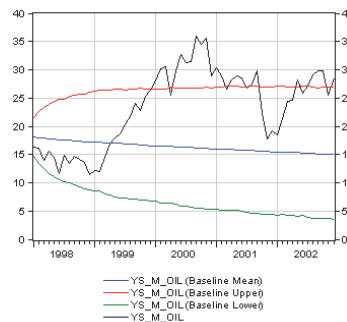
Panel G: Forecast probability quantiles for an LnOUP without parameter uncertainty.



Panel H: Forecast probability quantiles for an LnOUP with parameter uncertainty.



Panel I: Forecast probability quantiles for a mixture model without parameter uncertainty.



Panel J: Forecast probability quantiles for a mixture model with parameter uncertainty.

The analysis conducted shows the strength of parameter uncertainty on the resulting probability distribution of the underlying process and hence on any valuation that is based on such a distribution. The difference between the tested (no parameter uncertainty vs. parameter uncertainty) DGPs seems largest for the AOUP case, where allowing for parameter uncertainty increases the forecast confidence intervals to a sufficiently large extent. One can expect that the impact on option valuation is large, especially if far out of the money options are evaluated. Also important to notice is the fact that the mean forecast (next to the unchanged median forecast) is not altered significantly, which shows that allowing for parameter uncertainty leads primarily to a variance effect but not to a mean effect.¹⁷⁵

¹⁷⁵ The differentiation between variance and mean effect is discussed in this chapter's introduction; see Section 5.1, p. 210.

To illustrate and test how the different DGPs and parameter uncertainty affect the option value, we will conduct an option valuation analysis based on a set of at-the-money and out of the money options that are held to maturity. The aim is to quantify possible misvaluation of options if the DGP is misspecified. The results will be compared with the assumption of a pure random walk option valuation, which will serve as a benchmark.

5.4 Testing different DGPs according to their option pricing efficiency

5.4.1 Introduction

What is the best data generating process to value real-options or options on commodities in general? The answer to this question is difficult, because it first requires a definition of what is “best.” In order to answer this question, we will need to review major theorems of portfolio theory and option pricing.

According to risk-neutral option pricing theory, the price of an option is not related to the real drift but is primarily a function of uncertainty. Hence, the real drift does not affect the price of an option if the real drift can be hedged away.¹⁷⁶ One way to define which process is best suited for valuing options is to look at how far an investor is compensated for the risk he/she takes by investing in options. The link from option risk to risk compensation leads to portfolio theory and allows analyzing this question in a portfolio, and hence a benchmarking risk/return context. Relating option returns to global asset portfolio returns allows conducting a performance analysis of the different DGPs and determining their efficiency, with efficiency being the criterion for “best.”

Efficiency – in a Markowitz/CAPM portfolio context – is derived from the assumption that investors are rational.¹⁷⁷ One immediate implication of the rationality assumption – assuming that two assets or portfolios of assets have the same risk structure – is that the investor will choose the asset or portfolio of assets that offers the highest expected

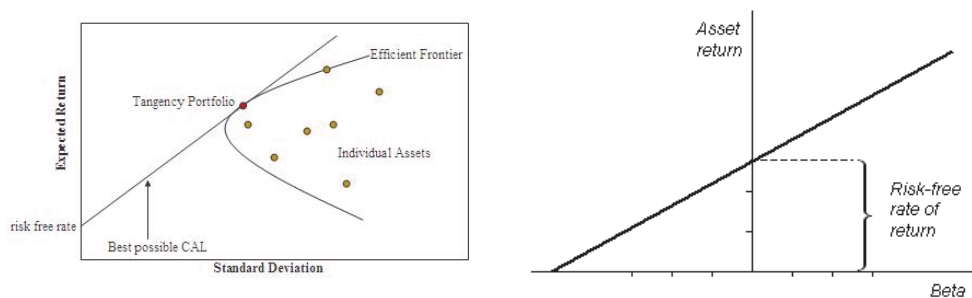
¹⁷⁶ This leads straight to replication (hedge) portfolios that ought to generate the equivalent of the risk-free return.

¹⁷⁷ See Fischer (1996a), pp. 44-45.

return. Hence, the asset or portfolio choice will be efficient (i.e., the portfolio is efficient) if no other asset or portfolio of assets exists that offers a higher return for the given amount of risk. The concept of portfolio efficiency is illustrated in the form of the efficient frontier in Figure 100, Panel A.¹⁷⁸ Each portfolio along the efficient frontier is efficient since, for a given amount of risk, no higher return can be expected.

Figure 100

Tangency portfolio and the efficiency frontier together with the relation of asset returns to risk



Panel A: Efficient frontier and the tangency portfolio as function of risk/return of individual assets (capital market line).

Panel B: Asset return as function of risk measured in form of the market beta (security market line).

Panel B in Figure 100 shows how, in a CAPM portfolio context, the asset return (in our case the return on a real-option investment) can be related to the market risk and hence to the risk/return structure of a global benchmark portfolio. It is possible to apply, for this purpose, the concept of relative asset performance (relative to a global benchmark) to analyze how different DGPs lead to different option prices that can be ranked according to their risk-related relative performance. Hence, the normalization of option returns towards risk will in this case be conducted by normalization relative to the simulated option return volatility that itself depends on the DGP chosen.¹⁷⁹ This will

¹⁷⁸ For a complete introduction to general portfolio theory with respect to the efficiency frontier, see Elton, Gruber et al. (2007), pp. 38-65 and Copeland, Weston and Shastri (2005), pp. 101-144.

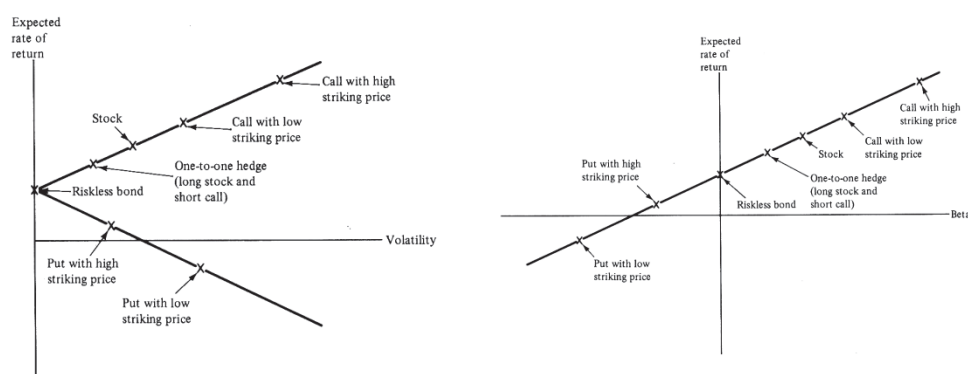
¹⁷⁹ A possible other methodology for normalizing option returns is to use the forecast accuracy statistic RMSPE calculated in Section 4.6 of each DGP. This would allow a combined mean (i.e., how accurate the DGP is in forecasting the underlying trend) and variance effect (i.e., how well the DGP captures the

allow us to calculate an efficiency measure in the form of risk adjusted option returns for different at-the-money and out-of-the-money option strategies for different DGPs.

Efficiency can be understood in this context by looking at Figure 101. Option returns above or below the security market line in Panel B of Figure 101 would indicate that the DGP is not efficient. Our aim is to replicate this relation but to replace the element of expected rate of return with realized rate of return and the element of volatility with the DGP dependend simulated option return volatility. Both values are obtained by rolling out-of-sample simulations for each tested DGP. For each simulation an option price and hence an option return (ex-post) is calculated, which is then normalized with the simulated option return volatility of the DGP or the underlying global benchmark beta.¹⁸⁰

Figure 101

Risk vs. expected return relation for option strategies as function of moneyness illustrated as capital market line and security market line



Panel A: The expected return on option strategies is – via the capital market line – similarly related to risk like any other asset class. Source: Cox and Rubinstein (1985), p. 192.

Panel B: The expected return on option strategies can be illustrated in the form of a security market line for the underlying. Source: Cox and Rubinstein (1985), p. 192.

volatility dynamics of the underlying) analysis. However, we have not applied this methodology here and leave this for future research.

¹⁸⁰ The data specification and data sample are identical to those used for the rolling out-of-sample analysis shown in Subsection 4.6.3, p. 190.

This procedure affords us the opportunity to evaluate whether the different DGPs result in option prices that are properly related relative to the selected underlying (commodity in our case), and also (by applying CAPM) if the option is properly priced relative to the overall market. Elements above the capital or security market line would indicate that the option return was too high and hence the option price was too low. Elements below the security market line would indicate the opposite, namely that the option was too expensive and hence the return on the option strategy – relative to the forecast error of the DGP – was too low. The same DGP can result in a too expensive at-the-money option price valuation, while out-of-the-money options can be too cheap if the forecast distribution of the DGP is too narrow.

Note that the analysis is conducted in a non-risk-neutral framework. If we instead assumed risk-neutral pricing, which would imply using the risk-neutral drift component for each DGP instead of the real drift, we would have to consider that – irrespectively of the chosen DGP – the option prices would be identical, since it is common practice to calibrate different option pricing models (which assume different DGPs) to currently traded market prices (otherwise the non-arbitrage condition would be violated). Hence, simulating risk-neutral prices and therefore option payoffs would have no value. Each simulation would need to yield the same option price observed in the market and hence, ex post, the same option return. The only difference would come from different hedging strategies associated with different DGP-induced hedge ratios.

5.4.2 Realized option return and simulated option return risk

The rolling out-of-sample analysis for the different DGPs allows calculation of the realized returns after a pre-defined holding horizon. Since we are interested in the performance of each DGP relative to the analyzed underlying commodity and relative to a global benchmark, we need to describe and hence formalize the performance calculation. The method of calculating the option alpha and option beta is especially important in this context. The variable definitions are given in Table 47:

Table 47
Variable definitions for calculating option returns and option price sensitivities
for a 5-year rolling out-of-sample experiment

Name	Definition	Calculation
Option price (call/put) $c_{i,t}, p_{i,t}$	Price of the option (call or put) for an underlying simulation i in USD terms	Monte Carlo simulation
Underlying price u_t	Price of the underlying in USD terms	-
Option return (call/put): $r_{i,j}^{u,c,p}$	Total return of option investment on underlying u , simulation i and strike price j calculated over the entire holding horizon	$r_{i,j}^{u,c,p} = \frac{\text{option payoff}}{\text{option price}}$
Mean option return (call/put): $\bar{r}^{u,c,p}$	Mean total return of option investment on underlying u and strike price j calculated over the entire holding horizon	$\bar{r}_j^{u,c,p} = \frac{\sum_{i=1}^n r_i^{u,c,p}}{n}$
Option risk (call/put): $\sigma_j^{u,c,p}$	Annualized standard deviation of the option investment rate of return for the underlying u and the strike price j	$\sigma_j^{u,c,p} = \sqrt{\frac{\sum_{i=1}^n (r_i^{u,c,p} - \bar{r}_j^{u,c,p})^2}{n-1}}$
Underlying mean return: $\bar{r}^u$	Annualized mean return over time t for underlying u over holding horizon n	$\bar{r}^u = \frac{\sum_{i=1}^n r_i^u}{n}$
Underlying risk: σ^u	Annualized standard deviation of the underlying rate of return	$\sigma^u = \sqrt{\frac{\sum_{i=1}^n (r_i^u - \bar{r}^u)^2}{n-1}}$
Option delta: $\Delta_j^{u,c}, \Delta_j^{u,p}$	Option price sensitivity in unit terms (relative to unit change in underlying)	$\Delta_j^{u,c} = \frac{\partial c_j^u}{\partial u},$ $\Delta_j^{u,p} = \frac{\partial p_j^u}{\partial u}$
Option elasticity: Ω_k^c, Ω_k^p	Option price elasticity measured in percentage terms (relative to percentage change in underlying)	$\Omega_k^c = \Delta_u^c \frac{c^u}{u},$ $\Omega_k^p = \Delta_u^p \frac{p^u}{u}$
Option beta: β_u^c, β_u^p	Measure of covariance of the rate of return on option with the rate of return on the market, divided by variance of the rate of return on the market	$\beta_j^{u,c} = \Omega_k^{u,c} \beta_u$ $\beta_j^{u,p} = \Omega_k^{u,p} \beta_u$

See Cox and Rubinstein (1985), pp. 185-196.

In addition to the given variable definitions, we need the following set of equations in order to relate the introduced variables to each other:

Table 48

Option return risk, option price sensitivity and excess rate of return definitions

Call option risk for underlying u and strike j : $\sigma_j^{u,c}$	$\sigma_j^{u,c} = \Omega_j^{u,c} \sigma^u$	(1)
Put option risk for underlying u and strike j : $\sigma_j^{u,p}$	$\sigma_j^{u,p} = -\Omega_j^{u,p} \sigma^u$	(2)
Expected excess rate of return on call option on underlying u and strike price j :	$r_j^{u,c} - r_t^{rf} = \Omega_j^{u,c} (r_t^u - r_t^{rf})$	(3)
Expected excess rate of return on put option on underlying u and strike price j :	$r_j^{u,p} - r_t^{rf} = \Omega_j^{u,p} (r_t^u - r_t^{rf})$	(4)
Expected excess rate of return on underlying u :	$r_j^u - r_t^{rf} = \beta^u (r_t^m - r_t^{rf})$	(5)
Expected excess rate of return on call option on underlying u and strike price j :	$r_j^{u,c} - r_t^{rf} = \Omega_j^{u,c} \beta^u (r_t^m - r_t^{rf})$	(6)

See Cox and Rubinstein (1985), pp. 185-196.

With the above set of Equations and variable definitions, it is now possible to conduct a performance analysis. The only missing ingredient required for the calculation is the individual option elasticity, which is itself a function of the chosen option pricing formula and corresponding strike price (see calculation of elasticity as a function of the option delta in Table 47). However, rearranging Equation (1) in Table 48 allows us to solve for the option price elasticity. In order to do so one needs to calculate the option return volatility $\sigma_j^{u,c}$. Since we will simulate the underlying price path to calculate the option payoff and hence the option price (by discounting the option payoff with the current five year discount rate for risk-free assets)¹⁸¹, it is possible to use this procedure also for calculating the return distribution of the option investment and use it as proxy for the option return volatility $\sigma_j^{u,c}$.¹⁸² We will use this simulated volatility as a proxy for the expected option price volatility in all subsequent calculations.

¹⁸¹ The theoretical legitimation for discounting a risky payoff that is not based on risk-neutral simulation with the risk-free interest rate is discussed in the Excuse on p. 234.

¹⁸² Of course, this will not represent the realized option price return volatility, but instead the simulated option price return volatility for each given data generating process.

Now the calculation of the overall option alpha is straightforward. Since we measure the efficiency of the option price relative to its associated underlying, and the underlying relative to the market portfolio (applying CAPM), we can rearrange Equations (5) and (6) in Table 46 to calculate the appropriate option alphas. The overall (total) option alpha will be the sum of the individual option-to-stock alpha (option alpha) and the individual stock to global market alpha (stock alpha).¹⁸³ The relation becomes clearer if Equation (5) in Table 48 is rewritten to allow for an alpha term.¹⁸⁴

$$\tilde{r}_{t,t+60}^u - \tilde{r}_{t,t+60}^{rf} = \tilde{\alpha}_{t,t+60}^s + \tilde{\beta}_{t,t+60}^u \left(\tilde{r}_{t,t+60}^m - \tilde{r}_{t,t+60}^{rf} \right) \quad (144)$$

and for a call option:

$$\tilde{r}_{t,t+60,j}^{u,c} - \tilde{r}_{t,t+60}^{rf} = \tilde{\alpha}_{t,t+60}^c + \Omega_j^{u,c} \left(\tilde{r}_{t,t+60}^u - \tilde{r}_{t,t+60}^{rf} \right) \quad (145)$$

The above Equations can be solved for $\tilde{\alpha}_{t,t+60}^s$ and $\tilde{\alpha}_{t,t+60}^c$ since all other elements are known ex post. By replacing $\left(\tilde{r}_{t,t+60}^u - \tilde{r}_{t,t+60}^{rf} \right)$ in Equation (145) by the right hand side of Equation (144) one can calculate the total option alpha that will become the sum of the individual option alpha and the stock alpha, since:

$$\tilde{r}_{t,t+60,j}^{u,c} - \tilde{r}_{t,t+60}^{rf} = \tilde{\alpha}_{t,t+60}^c + \Omega_j^{u,c} \left(\tilde{\alpha}_{t,t+60}^s + \tilde{\beta}_{t,t+60}^u \left(\tilde{r}_{t,t+60}^m - \tilde{r}_{t,t+60}^{rf} \right) \right) \quad (146)$$

and

$$\tilde{r}_{t,t+60,j}^{u,c} - \tilde{r}_{t,t+60}^{rf} = \tilde{\alpha}_{t,t+60}^c + \Omega_j^{u,c} \tilde{\alpha}_{t,t+60}^s + \Omega_j^{u,c} \tilde{\beta}_{t,t+60}^u \left(\tilde{r}_{t,t+60}^m - \tilde{r}_{t,t+60}^{rf} \right) \quad (147)$$

¹⁸³ See Cox and Rubinstein (1985), pp. 190-191.

¹⁸⁴ Note: the superscript (~) will be used to highlight realized values, while the superscript (^) will be used for simulated values.

gives the overall (total) option alpha $\tilde{\alpha}_{t,t+60}^{total,c}$ for a call option on the underlying u after time $t + 60$ and a strike price of j as:

$$\tilde{\alpha}_{t,t+60}^{total,c} = \tilde{\alpha}_{t,t+60}^c + \Omega_j^{u,c} \tilde{\alpha}_{t,t+60}^s \quad (148)$$

Hence, the total alpha for the call is defined as:¹⁸⁵

$$\tilde{\alpha}_{t,t+60}^c + \Omega_j^{u,c} \tilde{\alpha}_{t,t+60}^s = \tilde{r}_{t,t+60,j}^{u,c} - \tilde{r}_{t,t+60}^{rf} - \Omega_j^{u,c} \tilde{\beta}_{t,t+60}^u \left(\tilde{r}_{t,t+60}^m - \tilde{r}_{t,t+60}^{rf} \right) = \tilde{\alpha}_{t,t+60}^{total,c} \quad (149)$$

Excuse: Discounting uncertain cash-flows with the risk-free rate – Part II

We have revisited basic Martingale theory and the related assumptions necessary for risk-free discounting in an earlier Excuse (see p. 140). In order to translate a distribution function generated by a specific data-generating process into a risk-neutral distribution function we will need either assumptions on the investors utility function or the Grisanov theorem (if specifications for the assumed DGP exist). Both approaches are cumbersome and difficult (to say the least) to implement.

Practitioners have therefore looked for other, simpler methods that would allow using risk-free discounting. The formulation of the CAPM allowed for discounting uncertain cash-flows or prices in general by utilizing the concept of certainty equivalents. The certainty equivalent (CE) can be seen as adjusted expected future price or cash-flow that accounts for uncertainty via the market price of risk and volatility.¹⁸⁶ The adjusted certainty equivalent allows discounting at the risk-free interest rate.¹⁸⁷ Equation (150) shows the calculation of the certainty equivalent derived within the CAPM.

¹⁸⁵ The calculation for the put alpha is analogous.

¹⁸⁶ The name “certainty equivalent” is thus used in order to indicate the treatment of an uncertain price or cash-flows with certainty, which hence allows for risk-free discounting.

¹⁸⁷ Note that the calculation of a certainty equivalent is similar to adjusting a process with a drift to a process without drift. The transformation of a process with a drift (trend), as in the case of a sub-

$$CE[E(P_t^u)] = E(P_t^u) - \frac{E(r_t^m) - r_t^{rf}}{\sigma^2(r_t^m)} \text{Cov}(P_t^u, r_t^m) \quad (150)$$

The certainty equivalent can now be discounted by the risk-free interest rates as shown in Equation (151).¹⁸⁸

$$P_{t-1}^u = \frac{E(P_t^u) - \frac{E(r_t^m) - r_t^{rf}}{\sigma^2(r_t^m)} \text{Cov}(P_t^u, r_t^m)}{1 + r_t^{rf}} \quad (151)$$

Equation (151) is further reduced by defining the excess market return over the risk-free interest rates relative to the market variance as market price of risk λ_t , leading to Equation (152):

$$P_{t-1}^u = \frac{E(P_t^u) - \lambda_t \text{Cov}(P_t^u, r_t^m)}{1 + r_t^{rf}} \quad (152)$$

The question arises whether Equation (152) can be used for derivative pricing. Indeed, as shown by Rubinstein (1976), the CAMP certainty equivalent concept can be used under certain assumptions as an alternative route to deriving the Black-Scholes-Merton option equation.¹⁸⁹ The price of an uncertain cash-flow (e.g., a dividend flow of the company u) can be calculated according to Rubinstein (1976) as:

Martingale to a Martingale (process without a drift, see Subsection 4.2.3, p. 139), is called Doob-Meyer decomposition and would similarly lead to risk-free discounting. For further information on the Doob-Meyer decomposition, see Neftci (2000), pp. 123-146.

¹⁸⁸ Trigeorgis (2000), pp. 32-52, Fischer (1996a), p. 79, and Copeland, Weston, and Shastri (2005), pp. 156-157.

¹⁸⁹ An example for pricing options with the CAPM is shown by Rees (2005), pp. 6-10.

$$P_o^u = \sum_{t=1}^{\infty} \frac{E(Div_t^u) - \hat{\lambda}_t \rho(Div_t^u, -(1+r_t^m)^t) \sigma(Div_t^u)}{(1+r_t^{rf})^t} = \sum_{t=1}^{\infty} \frac{E(Div_t^u) - \hat{\lambda}_t Cov(Div_t^u, -(1+r_t^m)^t)}{(1+r_t^{rf})^t} \quad (153)$$

where

$$\hat{\lambda}_t = \sqrt{(1+\hat{\lambda})^t - 1} \text{ and } \hat{\lambda} \equiv \frac{\sigma((1+r_t^m)^{-1})}{E((1+r_t^m)^{-1})} \quad (154)$$

The current price of the underlying u over time, given by P_t^u and the current price of a call c_t^u on the underlying, are defined in Equation (155) and Equation (156):¹⁹⁰

$$P_{t-1}^u = \frac{E(\hat{P}_t^u) - \hat{\lambda}_t Cov(\hat{P}_t^u, -(1+r_t^m)^{-t})}{(1+r_t^{rf})^t} \quad (155)$$

The price of a call with time to expiration t :

$$c_{t-1}^u = \frac{E(\hat{c}_t^u) - \hat{\lambda}_t Cov(\hat{c}_t^u, -(1+r_t^m)^{-t})}{(1+r_t^{rf})^t} \quad (156)$$

The uncertain call payoff $\hat{c}_t^u$ is itself a function of the uncertain price of the underlying at expiry $\hat{P}_t^u$ and the strike price $K_u^{c,i}$ as shown in Equation (157):

$$\hat{c}_t^u = \max(0, \hat{P}_t^u - K_u^{c,i}) \quad (157)$$

Most important is the covariance between the underlying and the market return. As can be seen in Equation (152) and more clearly in Equation (154) and Equation (155), if the

¹⁹⁰ See Cox, Ross and Rubinstein (1979), pp. 212-215 in conjunction with Rubinstein (1976), pp. 414-416.

covariance between the underlying and the market return is close to zero, the adjustment for uncertainty drops to zero. Because oil prices are considered uncorrelated to the market portfolio, this often leads to risk-free discounting of commodity real-option projects in general and oil-related real-option projects in particular.¹⁹¹

To show that the covariance matrix between the market return – here represented by the reciprocal of the S&P 500 index return plus one (see Equation (155)) – and the individual commodity prices used in this dissertation are indeed very low to zero, we have calculated the variance-covariance matrix between the S&P 500 index and the commodity prices previously used in the stationarity analysis in Subsection 4.5.1, p. 164. We have used the sample between 1987 and 2008 in order to include all variables and have a balanced dataset. The results are shown in Table 49.

Table 49
The variance-covariance matrix for tested commodities and the S&P 500 index as market portfolio representative

Panel A							
-1/(1+DLOG(S&P500))	-1/(1+DLOG(S&P500))	DLOG(GOLD)	DLOG(SILVER)	DLOG(NICKEL)	DLOG(OIL)	DLOG(STEEL)	DLOG(CRB Index)
	0.00177	-0.00021	0.00028	0.00054	-0.00046	-0.00025	0.00001
DLOG(GOLD)	-0.00021	0.00167	0.00161	0.00122	0.00084	0.00013	0.00016
DLOG(SILVER)	0.00028	0.00161	0.00451	0.00207	0.00065	0.00048	0.00023
DLOG(NICKEL)	0.00054	0.00122	0.00207	0.01056	0.00104	0.00097	0.00048
DLOG(OIL)	-0.00046	0.00084	0.00065	0.00104	0.00922	0.00064	0.00020
DLOG(STEEL)	-0.00025	0.00013	0.00048	0.00097	0.00064	0.00294	0.00017
DLOG(CRB Index)	0.00001	0.00016	0.00023	0.00048	0.00020	0.00017	0.00043
Panel B							
-1/(1+DLOG(S&P500))	-1/(1+DLOG(S&P500))	DLOG(GOLD)	DLOG(SILVER)	DLOG(NICKEL)	DLOG(OIL)	DLOG(STEEL)	DLOG(CRB Index)
	0.02120	-0.00250	0.00335	0.00652	-0.00545	-0.00304	0.00012

Data history between 1987 and 2008 using monthly data. The values shown in Panel A are unannualized. Panel B shows the annualized variance-covariance vector between the S&P 500 and the tested commodity prices.

As can be seen in Panel A, the variance-covariance matrix contains values close to zero. Also worth noting is the different correlation signs between the S&P 500 and the tested commodity prices, which indicate that indeed no general tendency across commodity prices relative to the S&P 500 is observable. Panel B shows the annualized variance-covariance vector for the S&P 500 relative to commodity prices. Even the annualized

¹⁹¹ See Rees (2005), p. 7.

values remain close to zero, which indicates that indeed the adjustment factors necessary in order to allow for risk-free discounting in Equation (163) and Equation (164) are small and can be neglected in particular over the long-run. Hence, we acknowledge for our purposes this small error and discount at the risk-free rate.

However, the assumptions required to derive the same valuation equation as in Black and Scholes (1973) are logarithmic representative utility functions, and rely on the assumption that $(1+r_t^m)$ and $\hat{P}_t^u$ are jointly lognormal distributed.¹⁹² No other restrictions are placed on the rates of return on other securities. However, the log-normality assumption of underlying price $\hat{P}_t^u$ poses a serious restriction with regard to the question of whether the process can be deterministic or stochastic in nature. Having the restriction that the underlying growth rate of $\hat{P}_t^u$ needs to follow a GBM we effectively exclude other data-generating processes, particularly those that allow for deterministic trend components and hence prohibit risk-free discounting for these processes. We therefore need to highlight again that our analysis using risk-free discounting will not lead to fair option prices if the underlying processes are not assumed to follow the assumption stated above. This remains the main caveat of our mean/variance valuation experiment.

5.4.3 *Describing the test structure*

We will focus in the first part of the analysis – for explanatory purposes – only on the commodity price of oil between 1998 and 2003 to test different DGP out-of-sample. This sample period is also applied in Schwartz (1997); hence, the generated parameter estimates can be applied within the Gibson and Schwartz (1990) two-factor model framework, enabling a direct performance comparison with other, less sophisticated models. The calibration period for the DGP is February 1982 through December 1997. The second part of the analysis will include other commodity underlying (using the

¹⁹² The assumption of logarithmic utility functions is omitted in Cox, Ross, and Rubinstein (1979), who simply set the measure of constant proportional risk aversion (CPRA), introduced in Rubinstein (1976), equal to 1, which has identical implications. The same was done in Rubinstein (2006).

identical sample sizes as in Section 4.6) and a rolling out-of-sample option test procedure to calculate an average performance over the last five years (i.e., 60 months).

For both, the first and the second parts of the test are structured in the following way:

1. calculate the option price for the commodity in question over a time to maturity horizon of five years¹⁹³
 - a. calculate the five-year zero coupon yield for US Treasuries
 - b. calculate the annualized sample standard deviation of the log-returns for the underlying in question σ_t^u
 - c. calculate the at-the-money (ATM) strike prices for calls and puts $(K_{u_t}^{c,0} = K_{u_t}^{p,0})$ based on the last in-sample observation value u_t
 - d. calculate the out-of-the-money (OTM) strike prices for puts and calls based on the following procedure:

Table 50

Defining at-the-money and out-of-the-money strike prices

Call	Put
ATM strike: $K_{u_t}^{c,0}$	ATM strike: $K_{u_t}^{p,0}$
OTM strike 1 to 5: OTM strike 1 to 5:	
$K_{u_t}^{c,1} = K_{u_t}^{c,0} + K_{u_t}^{c,0} \sigma_t^u$	$X_{u_t}^{p,1} = X_{u_t}^{p,0} + X_{u_t}^{p,0} \sigma_t^u$
$K_{u_t}^{c,2} = K_{u_t}^{c,1} + K_{u_t}^{c,1} \sigma_t^u$	$X_{u_t}^{p,2} = X_{u_t}^{p,1} + X_{u_t}^{p,1} \sigma_t^u$
$K_{u_t}^{c,3} = K_{u_t}^{c,2} + K_{u_t}^{c,2} \sigma_t^u$	$X_{u_t}^{p,3} = X_{u_t}^{p,2} + X_{u_t}^{p,2} \sigma_t^u$
$K_{u_t}^{c,4} = K_{u_t}^{c,3} + K_{u_t}^{c,3} \sigma_t^u$	$X_{u_t}^{p,4} = X_{u_t}^{p,3} + X_{u_t}^{p,3} \sigma_t^u$
$K_{u_t}^{c,5} = K_{u_t}^{c,4} + K_{u_t}^{c,4} \sigma_t^u$	$X_{u_t}^{p,5} = X_{u_t}^{p,4} + X_{u_t}^{p,4} \sigma_t^u$

¹⁹³ Note that five-year total returns and volatilities are used for the calculation (and not p.a. return or standard deviation data). The reason for not using annualized data is that negative option returns, if they are, for example, equal to -100% (total loss of option investment), cannot be shown in annualized terms over the entire five-year rolling horizon.

2. calculate the simulated return distribution of the different ATM and OTM option investments for puts and calls ($\sigma_j^{u,c}$: as the annualized standard deviation of the rate of return for puts and calls with strike price j)
3. calculate the five-year average annualized return volatility of the tested underlying $\sigma_{t,t+60}^u$
4. calculate the simulated option elasticity for the different strike prices by utilizing the relation $\sigma_j^{u,c} = \Omega_j^{u,c} \sigma_{t,t+60}^u$.
5. calculate the total return of the underlying in question for the corresponding investment horizon
6. calculate the total return of the used market portfolio (CRB commodity price index as representative market index)
7. calculate the realized profits/losses from the different ATM and OTM option investments for puts and calls
8. calculate the expected return of the option strategy given the realized return of the underlying ($r_{t,t+60}^u$) and the option beta (i.e., the option investment risk) for the analyzed investment horizon, e.g. for a call option:

$$r_{t,t+60,j}^{u,c} = r_{t,t+60}^{rf} + \Omega_j^{u,c} (r_{t,t+60}^u - r_{t,t+60}^{rf}) .$$
9. calculate the expected return of the option strategy given the realized return of a global market portfolio ($r_{t,t+60}^m$) and the option beta for the analyzed investment horizon, e.g., for a call option:

$$r_{t,t+60,j}^{u,c} = r_{t,t+60}^{rf} + \Omega_j^{u,c} \beta_{t,t+60}^u (r_{t,t+60}^m - r_{t,t+60}^{rf}) .$$
10. calculate the option alpha
11. repeat the preceding steps on a rolling out-of-sample basis
12. repeat the preceding steps with different data generation processes

The above test structure is used to calculate rolling option alphas for different DGPs and different underlyings over time.

5.4.3.1 Testing different DGPs for oil

Following the test procedure for the non-rolling out-of-sample analysis shown before, we have defined the option strike prices via historical volatility and the last observed in-sample price of the tested underlying (see Table 51 for oil).

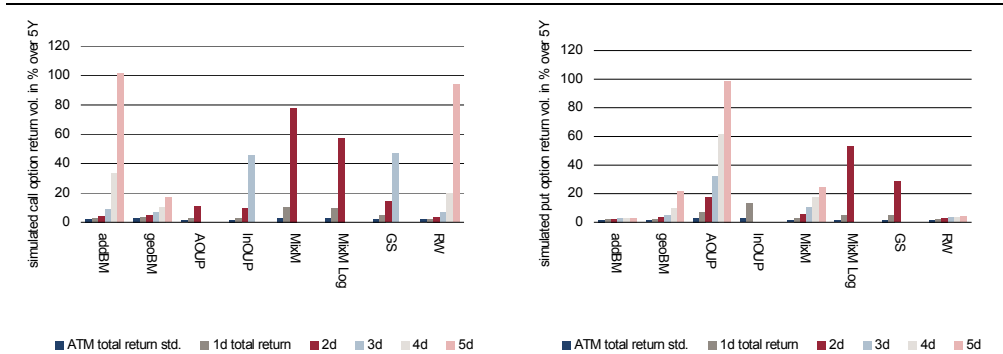
Table 51
Set of out-of-the-money strike prices for oil series

Call strike		Put strike	
ATM: $K_{u_t}^{c,0}$	18.19	ATM: $K_{u_t}^{p,0}$	18.19
$K_{u_t}^{c,1}$	24.17	$K_{u_t}^{p,1}$	12.21
$K_{u_t}^{c,2}$	32.10	$K_{u_t}^{p,2}$	8.20
$K_{u_t}^{c,3}$	42.65	$K_{u_t}^{p,3}$	5.51
$K_{u_t}^{c,4}$	56.66	$K_{u_t}^{p,4}$	3.70
$K_{u_t}^{c,5}$	75.27	$K_{u_t}^{p,5}$	2.48

The next step is the simulation of the option prices and the estimated distribution of the option price payoffs. This step is necessary to derive a numerical proxy for the option elasticity. This calculation is largely sensitive to the applied DGP. Incorporating parameter uncertainty will additionally increase this sensitivity. Figure 102 below shows the different estimated strike dependent call and put return uncertainty proxy values for different analyzed data generating processes.

Figure 102

Numerically approximated option return volatility for call and put options on oil over 5-year investment horizon assuming different DGPs



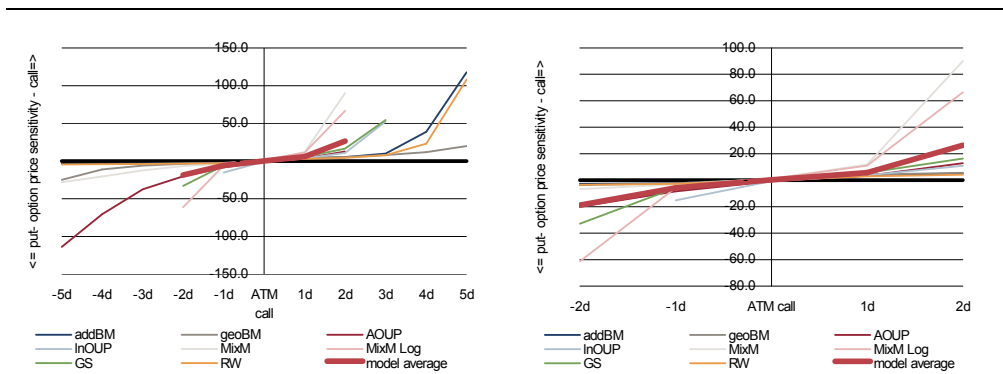
Panel A: Call option return volatility over five years based on simulated option payoffs and different DGPs.

Panel B: Put option return volatility over five years based on simulated option payoffs and different DGPs.

Highly OTM option strike prices produce high option return volatilities for oil. This is primarily due to the very low option values associated with some DGPs. DGPs that are trend mean reverting are in this case especially affected due to their narrower distribution probability of forecast values (see Section 5.3, p. 217). Figure 103 shows the calculated option price sensitivity (i.e., elasticity) for different strike prices.

Figure 103

Option price sensitivity (elasticity) for options on oil assuming different DGPs as function of the distance to the ATM strike price



Panel A: Option price sensitivity (elasticity) measured as % change of the option price per % change in the underlying commodity (broad strike price sample).

Panel B: Option price sensitivity (elasticity) measured as % change of the option price per % change in the underlying commodity (narrow strike price sample).

Note that the illustrations in Figure 103 are the equivalent to Figure 101, Panel B. As illustrated the risk compensation per beta unit increases significantly for higher OTM options. With this information one can calculate the expected returns for each option

strike price based on the illustrated option elasticities (the equivalent to the equity price beta), which will then be related to the observed (i.e., realized) option return. To calculate the option returns, one needs first the option prices. Table 52 shows – based on the simulated forecast distribution – the calculated option prices for different strike price levels for call and put options:

Table 52
Forecast simulation-based prices for five-year put and call options on oil

Panel A						
Price in USD	ATM call	1d	2d	3d	4d	5d
ABM	3.47	1.98	0.82	0.19	0.01	0.00
GBM	4.95	3.45	2.20	1.25	0.64	0.28
AOUP	3.13	0.80	0.04	#NV	#NV	#NV
LnOUP	3.01	0.83	0.09	0.00	#NV	#NV
MixModel	0.54	0.04	0.00	#NV	#NV	#NV
MixModel Log	0.64	0.06	0.00	#NV	#NV	#NV
Gibson-Schwartz	1.66	0.40	0.05	0.00	#NV	#NV
Random Walk	5.82	3.78	1.85	0.60	0.09	#NV
Panel B						
Price in USD	ATM put	1d	2d	3d	4d	5d
ABM	6.85	4.35	3.06	2.36	1.96	1.73
GBM	3.61	1.28	0.37	0.09	0.02	0.00
AOUP	0.80	0.10	0.01	0.00	0.00	0.00
LnOUP	0.57	0.01	#NV	#NV	#NV	#NV
MixModel	3.16	0.69	0.14	0.04	0.01	0.01
MixModel Log	1.95	0.09	0.00	#NV	#NV	#NV
Gibson-Schwartz	1.55	0.11	0.00	#NV	#NV	#NV
Random Walk	4.69	2.83	1.91	1.44	1.19	1.03

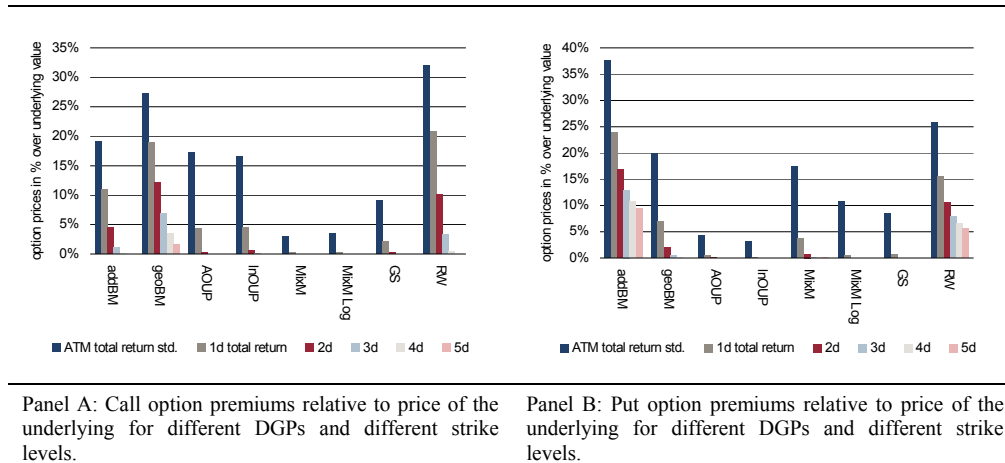
Prices are given in USD units per option contract on one barrel of oil. Panel A shows the prices for call options. Panel B shows the prices for put options. The strike price values (d1 to d5) are shown in Table 51.

Options with far OTM strike prices are valued close to zero (marked with #NV). As a consequence, if the underlying experiences high deviations from the starting value (as is the case for oil over this sample period), the return on this option investment will be incredibly high. A better illustration of the option price sensitivity to the chosen strike price and DGP is given in Figure 104, where the option price is calculated as % premium over the underlying price. Again, the low price of OTM options for mean-

reverting DGPs becomes visible. The lowest price is achieved via the use of mixture models that are explained by the strong process persistence resulting from the deterministic component (lower values are forecasted for much longer, with the deterministic trend as shown in Figure 66, p. 159 and Figure 68, p. 162) and the more narrow forecast distribution. Both effects lead to option prices that are only a small fraction of the ABM or GBM derived option prices. However, the most sophisticated DGP (the Gibson-Schwartz process) leads also to option prices below other – less sophisticated – models.

Figure 104

Calculated put and call option prices for the underlying oil in % terms of the underlying

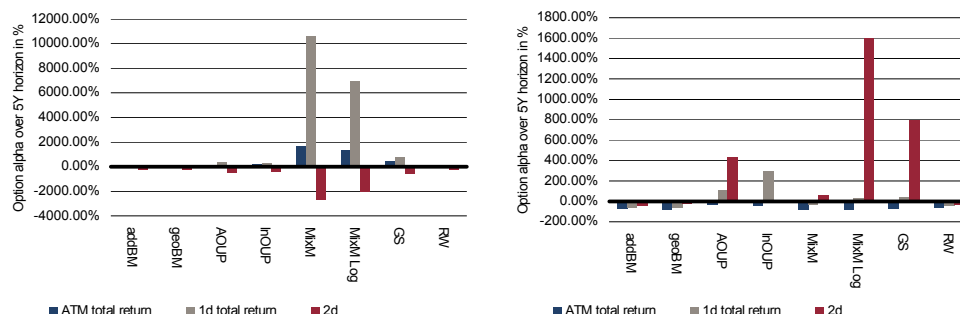


The calculated option prices allow the calculation of option returns and a corresponding normalization with risk using the estimated option return volatilities (applying the described CAPM-based option alpha and beta calculation). The results for the initial oil sample are given in Figure 105.

In this case the return would have been for an ATM option of 197% over five years. The beta for the option vs. underlying return was estimated at 2.65 ATM and increased further OTM. Together with a risk-free accumulated return of 28.4% over the same horizon we can calculate the option alpha (the excess option return given the underlying oil) as equal to roughly 93%.

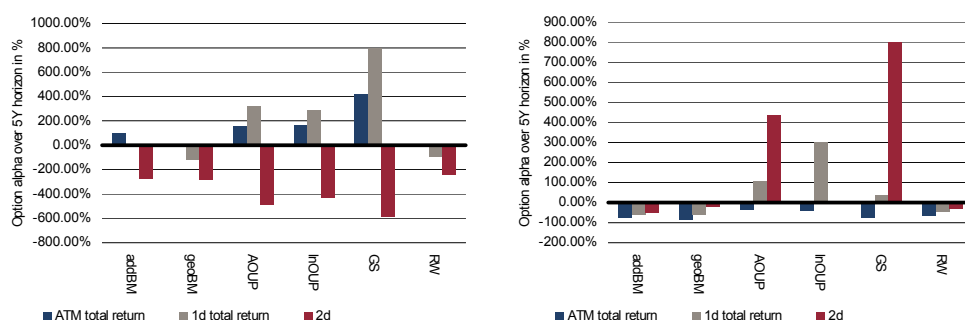
Figure 105

Option alpha over five years in % return units for different DGPs for put and call options



Panel A: Option alpha in % over five years for call option investments using different DGP and different strike levels.

Panel B: Option alpha in % over five years for put option investments using different DGP and different strike levels.



Panel C: Option alpha in % over five years for call option investments using different DGPs, excluding deterministic trend models (average over different strike levels).

Panel D: Option alpha in % over five years for put option investments using different DGPs, excluding deterministic trend models (average over different strike levels).

In this context, a DGP is called efficient if the generated option strategies produce on average zero alphas (no excess return relative to option and underlying risk profile). DGPs with the tendency to produce strong positive or negative option alpha strategies are hence called inefficient.

The option alpha is highest for those DGPs that produce the smallest option values. Comparing the option prices in Table 52 to the alpha generation in Figure 105, Panel A for call options, reveals that an option valuation that utilizes a mixture model as its data generation process would have resulted in call option prices that were too small. However, inference about DGPs with deterministic trends is limited and should be drawn with required caution. Given the ex-post development of the underlying (in this

case for oil), for our five year out-of-sample test for oil this would have resulted in the highest option alpha among all tested DGPs. Hence, one can conclude that for the given sample period (1998-2002), options on oil – using mixture models as DGPs for the option price simulation – would have been priced too low (the price should have been much higher in order to match the risk-adjusted expected return of the option investment). This becomes clear if one recalls Equation (158)¹⁹⁴ in combination with the calculation of the option investment return shown in Equation (159):

$$\tilde{r}_{t,t+60,j}^{u,c} - \tilde{r}_{t,t+60}^{rf} = \tilde{\alpha}_{t,t+60}^c + \Omega_j^{u,c} \left(\tilde{r}_{t,t+60}^u - \tilde{r}_{t,t+60}^{rf} \right) \quad (158)$$

$$\tilde{r}_{t,t+60,j}^{u,c} = \frac{\max(u_{t+60} - K_{u_t}^{c,j}, 0)}{c_t^j} - 1 \quad (159)$$

The return of the option strategy $\tilde{r}_{t,t+60,j}^{u,c}$ is known at $t + 60$. A positive (negative) call option alpha $\tilde{\alpha}_{t,t+60}^c$ indicates that relative to the underlying u_t the call price c_t^j is too low (high), and thus the option is under priced, i.e., the return is too high and vice versa. In this case a real-option project that relied on a mixture model DGP for project valuation (ceteris paribus) would have understated the real-option value.

For the value assessment of puts (Figure 105, Panels B and D) the conclusion is similar. Since put options have a negative elasticity relative to the underlying (i.e., a negative option delta), a positive alpha indicates that the put price p_t^j was too cheap relative to the underlying u_t .¹⁹⁵ Note that for this particular sample trial, where – given the oil price increase – all puts expired as worthless, the option return is -100%. This, however, does not lead necessarily to negative option strategy alphas, given that for puts we estimate negative beta coefficients (see Figure 106). Hence, if given the development of

¹⁹⁴ See also Equation (3) and Equation (4) in Table 48, p. 232.

¹⁹⁵ The expected negative return was not achieved adequately given the risk profile, because the option price was too low. A higher option price would hence have increased the negative return.

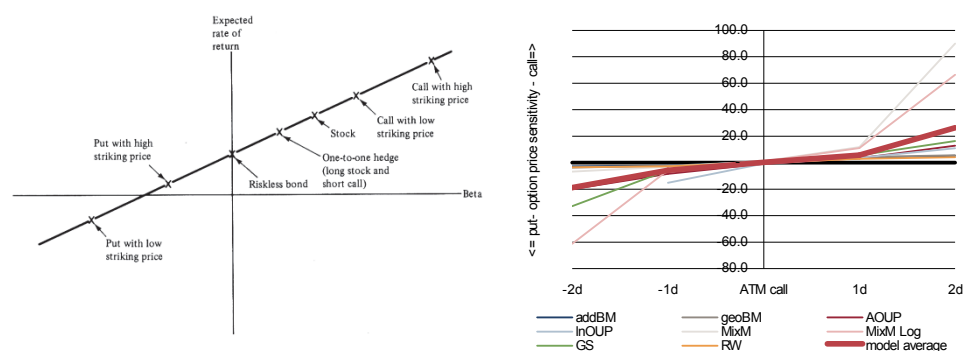
the underlying and the estimated beta the expected negative return exceeds 100%, the option alpha for puts is positive.

Interpreting the results from Figure 105 above, one can conclude that in a real-option project over this sample period, the value of the put structure (*ceteris paribus*) would have been underestimated for out-of-the money options if an AOUP, a mixture process or a GS process had been used as the DGP.

It is in general valid to say that DGPs used for option valuation that result in option returns with positive alpha (irrespective of put or call) will lie above the security market line and indicate that they are underpriced relative to the underlying. For illustrative purposes see again the interrelation of the option beta relative to the expected rate of return and the dependence of the option price sensitivity (option beta/underlying beta) relative to the moneyness of put and call options in Figure 106.

Figure 106

Expected return to risk relation for option strategies together with the option price sensitivity (elasticity) for the tested DGPs and different moneyness levels



Panel A: Security market line for option strategies. Higher out-of-the-money options should yield higher beta and hence higher expected returns. See Cox and Rubinstein (1985), p. 192.

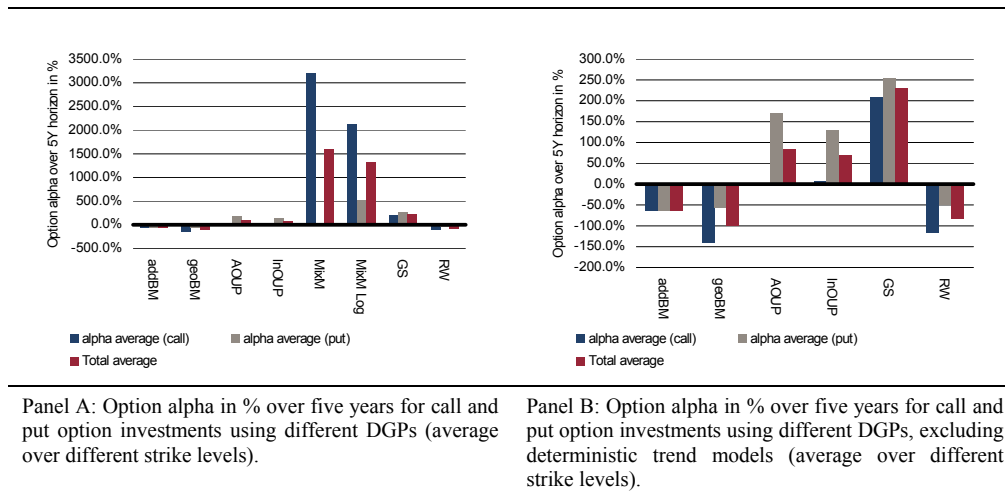
Panel B: The simulation shows that out-of-the-money options result in higher option price sensitivities, which is a confirmation of the theoretical relation shown in Panel A.

If the return on the option investment is too high, which is the case for positive alpha options, then the price for these options was also too low, and a higher price would have been justified given the risk-profile of the option. For a real-option project, one can similarly conclude that if the project is valued using a DGP that produces options with positive alpha, then the optionality value is likely to be underestimated.

In order to make an assessment on which data generation process was most efficient in this sample period for pricing options and hence real-option projects, we will average the option alpha values for ATM, and for one delta and two delta OTM options. The results are given in Figure 107.

Figure 107

Average total option alpha over five years in % return units for different DGPs for put and call options



When excluding models with deterministic trend components, which will be excluded due to the severe violation of critical assumptions necessary for risk-free discounting (see p. 234), the following conclusion can be drawn from this first non-rolling simulation sample (Figure 107, Panel B): on average the additive and geometric Brownian motion led to an overpricing of the option value (for both puts and calls). The results are similar for the random walk process. Ornstein-Uhlenbeck processes (in their additive or log-normal versions) led to close to fair value evaluations of calls. However, the put valuation was overpriced. On average the additive Brownian motion performed best for oil in this sample test.

Since this analysis was conducted over only one observation period (forecast between 1998 and 2002), we will continue in the next section with a rolling out-of-sample evaluation. The rolling analysis will enable the calculation of rolling alphas for each underlying and each DGP. Analyzing these results will give much better intuition

regarding which data generation process performs best for option valuation purposes and which DGP should be used to value commodity-related real-options.

5.4.3.2 Testing different data generating processes for commodities between 1998 and 2007 over a rolling five-year forecast horizon

We have repeated the test from Subsubsection 5.4.3.1 on a rolling horizon for several DGPs and six different commodity underlyings used already in the previous analysis (see Subsection 4.5.1, p. 164).¹⁹⁶ The strike price definition is identical to the one shown previously, and the test structure is equivalent to the one shown in Subsection 5.4.3, p. 238. However, some further calculations are necessary in order to aggregate rolling results. The aggregation is conducted by calculating the following:¹⁹⁷

- a) Median
- b) Mean
- c) Trimmed mean (excluding the largest and smallest 10% of observations)

The variables that are averaged over the entire rolling forecast horizon are:

- the option elasticity
- the expected option return (given the realized return of the underlying)
- realized option alpha relative to the performance of the underlying
- the expected option return (given the realized return of the benchmark and the corresponding rolling beta between the underlying and the benchmark)
- realized option alpha relative to the performance of the underlying and the benchmark index

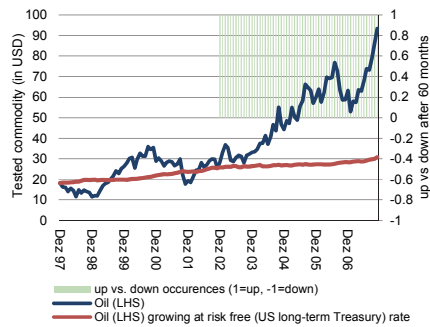
¹⁹⁶ All DGP are re-estimated after each month to include the latest set of data. The Gibson-Schwartz model is therefore excluded, since our own estimates were unstable over time.

¹⁹⁷ It is important to highlight that averaging the results over a rolling five-year forecast horizon introduces the problem of autocorrelation of the results. This problem is acknowledged although ignored for the sake of simplicity. It would certainly have been possible to conduct the entire analysis on a monthly forecast horizon basis (instead over five years). This would have eliminated the the strong autocorrelation problem. However, we will leave this to future research.

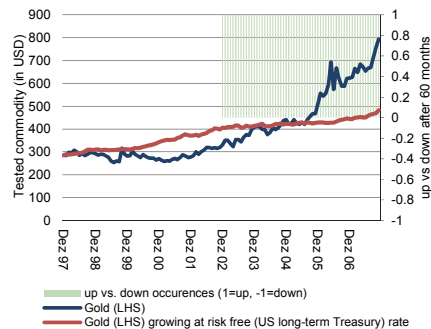
We will distinguish among five different OTM call and put prices together with the ATM valuation.

Figure 108

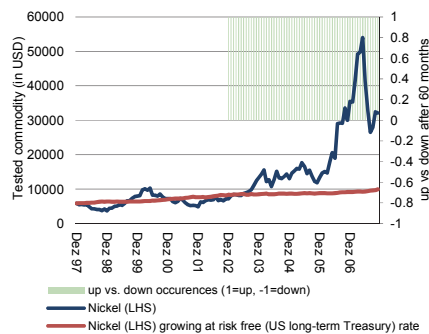
Development of tested commodity prices between 1998 and 2007 showing occurrences when the price of the underlying exceeds (falls) above (below) its 60-month history



Panel A: Oil price development. Prices between the end of 2002 and end of 2007 always exceed the price level between 1998 and 2002.



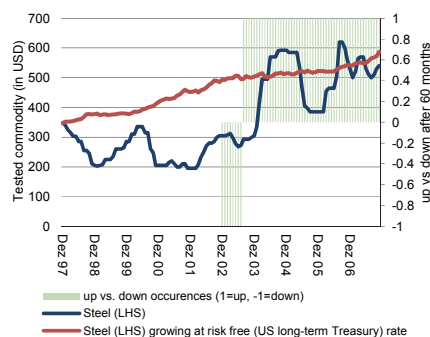
Panel B: Gold price development. Prices between the end of 2002 and end of 2007 always exceed the price level between 1998 and 2002.



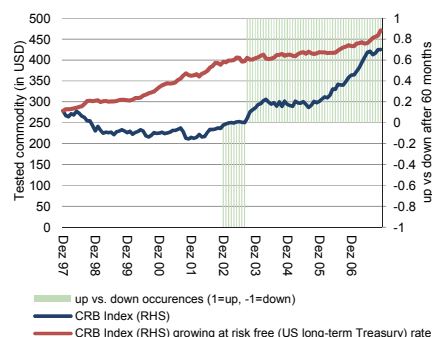
Panel C: Nickel price development. Prices between the end of 2002 and end of 2007 always exceed the price level between 1998 and 2002.



Panel D: Silver price development. Prices between the end of 2002 and end of 2007 nearly always exceed the price level between 1998 and 2002.



Panel E: Steel price development. Prices between the end of 2002 and end of 2007 nearly always exceed the price level between 1998 and 2002.



Panel F: Commodity index price development. Prices between the end of 2002 and end of 2007 nearly always exceed the price level between 1998 and 2002.

Figure 108 shows the historical development of the tested underlyings for the out-of-sample period used along with a hypothetical development if we choose to inflate each commodity price with the realized return data of an interest-bearing asset, in our case using the price index of intermediate U.S. Treasury bonds.

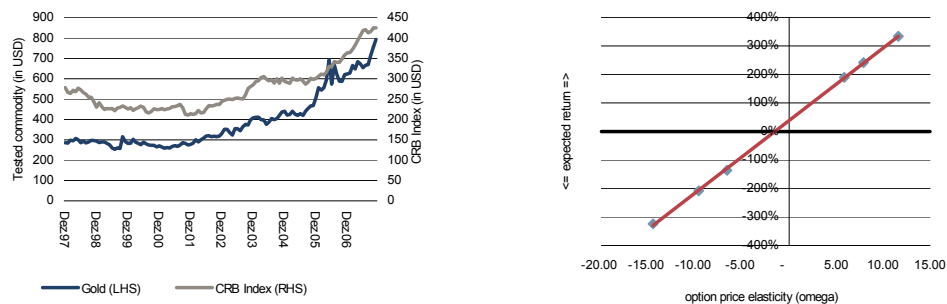
The bar charts (up/down occurrences) highlight how often ATM calls and puts would have been called based on the price history (60 months in the past). Since the last ten years have been dominated by a strong commodity bull market, the bias for exercised options certainly lies on the call side (more call exercises due to the strong commodity price rally). The results of the analysis are summarized for the example of gold in Figure 109.¹⁹⁸

Panel A shows the development of the underlying (in this example for gold) over time together with the commodity price index (CRB index, which represents the benchmark index in the analysis). Panel B shows the expected total return of the option given the ex-post realization of the underlying (gold). The most important information is given in Panels C and D, where the generated option alpha median (the ex-post calculated return relative to the expected option investment return) is illustrated for call and put investments using different data generation processes. They represent the rolling out-of-sample versions of Figure 105, given that we now calculate the median of all individual realizations over the rolling sample. Panels E and F illustrate the average across the strike prices and hence represent the rolling out-of-sample version of Figure 107. Panels G and H show the same information as in Panels E and F, except that instead of taking the median of all realized option alphas, we use a trimmed mean across all out-of-sample observations.

¹⁹⁸ For illustrative purposes we have dropped the results for strike prices higher than two delta standard deviations away from the ATM level. The results for the full set of strike prices are shown together with the other tested underlyings in Appendix H, see p. 364.

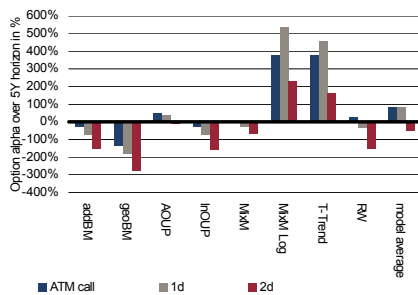
Figure 109

Testing different DGPs for valuing options on gold over a forecast horizon of five years (60 months out-of-sample between 2003-2007); the option valuation is conducted via MC simulation (10000 trails) using different DGPs

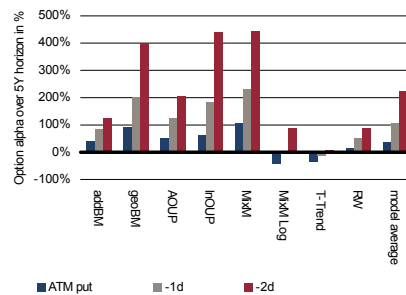


Panel A: Development of the underlying along with the benchmark index for commodities over time.

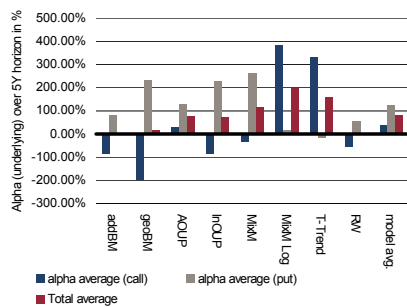
Panel B: Expected median return as function of the option price elasticity and ex-post development of the underlying.



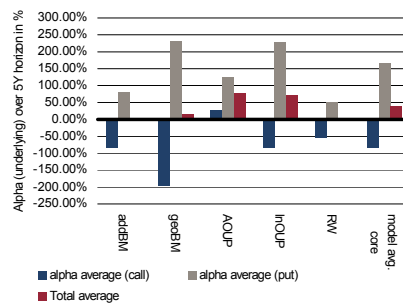
Panel C: Median call option alpha returns over 5Y investment horizon for different DGPs and strike levels.



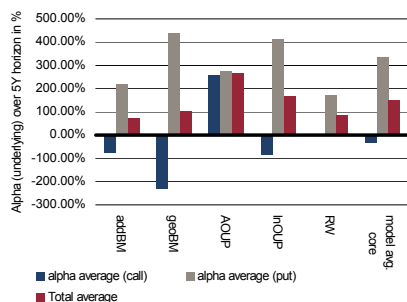
Panel D: Median put option alpha returns over 5Y investment horizon for different DGPs and strike levels.



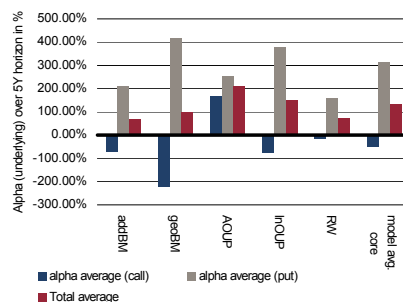
Panel E: Median alpha for call and put option (average over different strikes levels).



Panel F: Median alpha for call and put options (average over different strikes levels) excluding determ. models.



Panel G: Trimmed mean alpha for call and put options (average across strikes levels) over rolling period.



Panel H: Trimmed mean alpha for call and put options excluding deterministic models over rolling period.

When analyzing the results for all DGPs in Figure 109 together with the results in Appendix H, several conclusions for this particular sample set (1998-2007, excluding models with deterministic trend) can be drawn:

- 1) on average call options have been too expensive
- 2) put options have been too cheap
- 3) the extent of over- or undervaluation increases for OTM strike prices - increasing positive or negative option alpha, i.e., if options are too cheap (expensive) they are even cheaper (more expensive) for stronger OTM strike prices
- 4) the misvaluation is strongest for data generating processes that contain a deterministic time trend (inference, however, is limited)
- 5) mean reversion DGPs also seem to produce strong option price undervaluation, due to their narrow forecast probability distributions; in general AOUPs produce more efficient option price estimates than LnOUPs
- 6) some commodities, however, are exceptions in that sense, and do not follow this general conclusion (example: gold and silver where the calculated call and also partially put prices were too expensive)

Cross analysis:

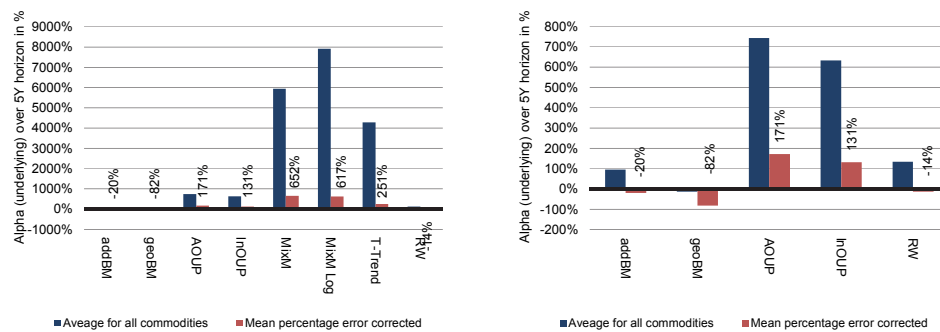
To facilitate an aggregated analysis that allows a cross-sectional view, we have grouped the different forecast methods and averaged their option alphas across different underlying. The results are given in Figure 110 for calls and Figure 111 for puts.

This aggregated view confirms some of the previously drawn conclusions; however, it highlights that over different strike prices the geometric Brownian motion seems to be the most efficient DGP relative to the other tested DGPs. It also shows that the additive Brownian motion results in more efficient option valuation than the random walk-based option prices. Mean-reverting processes and trend stationary variants seem highly inefficient. Processes with deterministic trend components also produce unrealistically low call option prices on a rolling basis, due to the strong process persistence (high negative out-of-sample bias, see mean-percentage error in Table 38, p. 192) and the

narrower forecast distribution. This is particularly the case for oil and steel, for which estimated call prices have been only a small fraction of the ABM or GBM derived option prices, leading – without adjustment – to implausibly high alpha values.

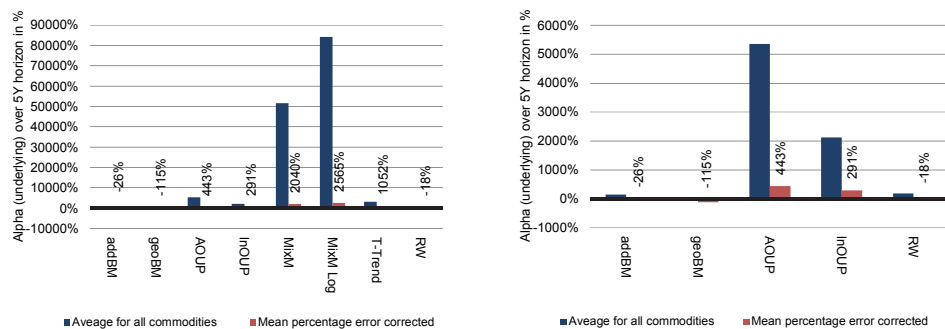
Figure 110

Average of call option alpha returns across all tested commodities for different moneyness levels over 5Y investment horizon and different DGPs



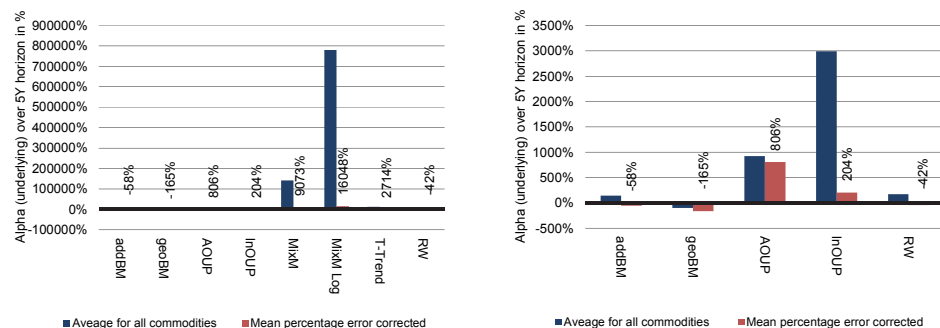
Panel A: Average call option alpha for ATM strike level across all tested commodities.

Panel B: Average call option alpha for ATM strike level across all tested commodities (excluding MixModels and deterministic trend models).



Panel C: Average call option alpha for OTM (1 delta) strike level across all tested commodities.

Panel D: Average call option alpha for OTM (1 delta) strike level across all tested commodities (excluding MixModels and deterministic trend models).



Panel E: Average call option alpha for OTM (2 delta) strike level across all tested commodities.

Panel F: Average call option alpha for OTM (2 delta) strike level across all tested commodities (excluding MixModels and deterministic trend models).

To mitigate this problem of generating senselessly low option prices, and hence excess returns that can exceed 1000% or more, we amend our analysis here in the following way. Although our experiment aims to compare the out-of-sample “mean” quality of each tested DGP with its variance structure, it also useful to think of isolating solely the variance component of each DGP.¹⁹⁹

Following Doob-Meyers’ subtracting the expected trend (i.e. the predictable process) will result in obtaining the variance component; this, however, is more easily said than done. Especially when the trend is time invariant the predictable process component is difficult to estimate, which is the reason why we usually encounter transformations of the real probability distribution via Grisanov to obtain the distribution under the equivalent Martingale measure in the literature rather than the Doob-Meyers decomposition.

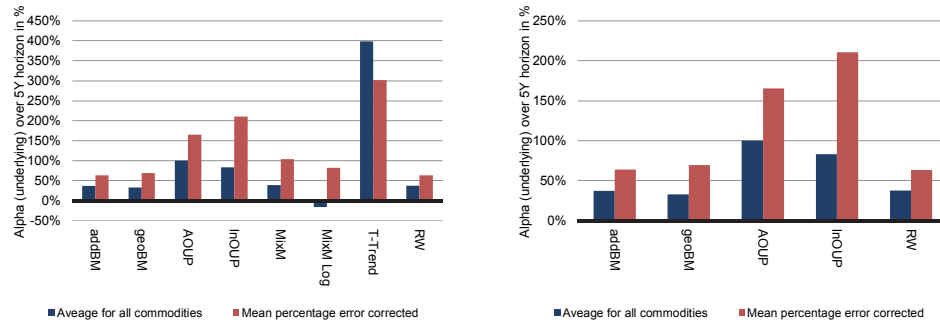
However, we can still follow the spirit of Doob-Meyers by adjusting each simulated path (under the real probability measure) in the Monte Carlo simulation that is used to calculate the option payoff, by the out-of-sample mean percentage error (MPE, see Table 36 p. 187 as proxy for average mispricing) of each tested DGP that was obtained in Subsection 5.4.3 (see Table 38, p. 192).²⁰⁰ This would allow compensation for the bias in the option price that resulted primarily from a too-distant point-forecast and allow for a comparison of the variance structures on an adjusted level. The results for the adjusted option valuation exercise are displayed in Figure 110 and Figure 111 in red.

¹⁹⁹ Indeed, in line with the portfolio replication approach (see Black and Scholes (1973) for option pricing, and following the argument laid out by Doob-Meyers (see p. 140, and Footnote 187, p. 234), only the unpredictable component (i.e. the variance component) should be of relevance for the option price. However, as shown in the example in Figure 99 (see p. 225) different process assumptions lead to different variance distributions, which consequently lead to different option prices.

²⁰⁰ Note that the sample length specification is identical for both experiments.

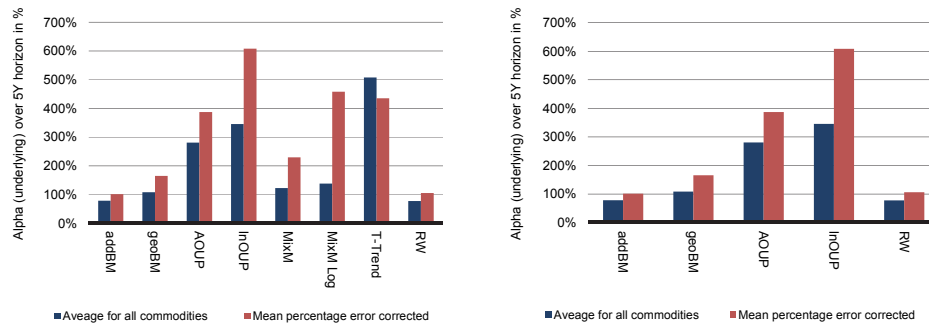
Figure 111

Average of put option alpha returns across tested commodities for different moneyneess levels over 5Y investment horizon and different DGPs



Panel A: Average put option alpha for ATM strike level across all tested commodities.

Panel B: Average put option alpha for ATM strike level across all tested commodities (excluding MixModels and deterministic trend models).



Panel C: Average put option alpha for OTM (1 delta) strike level across all tested commodities.

Panel D: Average put option alpha for OTM (1 delta) strike level across all tested commodities (excluding MixModels and deterministic trend models).



Panel E: Average put option alpha for OTM (2 delta) strike level across all tested commodities.

Panel F: Average put option alpha for OTM (2 delta) strike level across all tested commodities (excluding MixModels and deterministic trend models).

The overall conclusions hardly change with respect to the DGPs with deterministic trend components (all remain inefficient, however, now at a more reasonable level). However, with regard to the other processes, slight changes are observable. First,

AOUP and LnOUP are now more efficiently priced than before the adjustment; however, their variance structure continues to produce option prices that are too low (high positive alpha) compared to other processes. Second, the Random Walk and additive Brownian motion, now close to identical in their pricing, become more efficient for call option (at-the-money and out-of-the-money) than the geometric Brownian motion.

The aggregated analysis for put option prices leads to similar conclusions as in the cross-sectional analysis for call options. However, the dispersion of DGP performance efficiency is much narrower. The simple Brownian motion type DGPs seem to be more efficient compared to the other tested approaches (lowest option alpha). This remains true even after the mean percentage error corrections.

Additive Brownian motion processes seem to have a minimal lead in terms of price efficiency. Random walk and additive Brownian motion are again very close (as expected). Their outperformance increases relative to geometric Brownian motion with increasing OTM strike prices. One significant difference compared to the aggregated call option results in Figure 110 is that trend-deterministic DGPs perform significantly better.

5.4.4 Conclusion

Based on the above analysis one can conclude that additive Brownian motion and geometric Brownian motion DGPs perform well for valuing long-term options. This result holds even for out-of-the-money options for both call and put options. Alternative models like the AOUP or LnOUP disappoint with respect to the employed option price efficiency analysis and usually understate the value of the option. Hence, despite their theoretical attractiveness as DGPs for valuing long-term commodity contracts, their forecast probability distribution (convergence of forecast variance) leads to significant undervaluing of long-term option value. This observation continues to be the case even after correcting for the possible point-forecast bias of each process. Mixture models and pure deterministic trend models perform, however, even worse. Results for the last two model specifications, however, should be read with special caution.

5.5 Critical assessment and conclusion

The analysis in Chapter 5 has shown that the ability of a DGP to forecast accurately the future path of an underlying is not a sufficient property for valuing real-options or options in general. DGPs that have performed well for forecasting out-of-sample (see Subsection 4.6.5, p. 207) do not necessarily qualify as good DGPs for long-term option valuation. The reason for this is the DGP-dependent forecast distribution. We have illustrated for different DGPs the evolution of the forecast distribution over time (see Subsection 5.3, p. 217). We have shown, using the example of oil development, that – for the given price history (1998-2002) – most of the DGPs would have failed to account for the realized price volatility observed for this commodity. Even more sophisticated DGPs like the Gibson-Schwartz two-factor model fail to account properly (as measured by the forecast confidence intervals) for the realized volatility of oil in that period. However, allowing for parameter uncertainty would have mitigated the problem for some DGPs as shown in Subsection 5.3.2, p. 224.

In order to combine the results from the forecast accuracy tests (mean effect) with the results of the forecast probability distribution (variance effect), we have calculated expected option investment returns for long-term options for a variety of strike prices. These expected returns are compared with the realized returns that are a function of the original option price (that was simulated over five years on a monthly rolling basis) and the price of the underlying at expiry.

The results of the analysis have shown that AOUP, LnOUP, mixture models and deterministic trend models produce option price excess returns that are not explained by the risk structure of the underlying (long-term options are undervalued and hence inefficiently priced, i.e., corresponding returns of buying these mispriced options result in investment returns that are too high and hence not in line with the risk structure of the underlying). The mispricing of the aforementioned DGPs increases with the distance from the ATM strike level. We therefore conclude that using ABM or GBM processes helps to ensure that long-term real-option projects are not undervalued. Using ABM and GBM process with normalized commodity prices (as discussed in Subsection 4.6.4, p. 200) could unify the results of Chapter 4 and Chapter 5.

To confirm the test results presented in this chapter, the tests for DGP efficiency for option pricing could be extended for valuing short-term contracts (one month or three months). This would allow using realized option return volatility instead of simulated option return volatility.

Using the risk-free interest rate for discounting future option payoffs, which are generated by non-risk-neutral DGP calibrations, is certainly a critical point in the presented analysis. This simplification seems tolerable for data-generating processes that follow a stochastic trend, given the low covariance between the tested commodities and the broadest representative for a world market portfolio (the S&P 500) as shown in the Excuse on page 234. However, for processes that incorporate deterministic trend components, the results should be read with caution. To mitigate the bias that results from calculating option prices under the real probability measure, we have also conducted an option valuation experiment which adjusts for the point-forecast bias and allows better comparison of the variance components of each tested process. Despite the much more plausible results, the overall conclusion favoring GBM/ABM relative to OUP/LnOUP and models with deterministic trend components, however, remains unaffected.

6 Searching for real-option pricing evidence in the Russian equity market using time-series analysis

6.1 Introduction

Searching for real-option pricing evidence involves the complexities that result from looking at aggregated information found only in the daily traded market value of a company. It is therefore plausible to search for confirmation of real-option pricing by analyzing the pricing history of commodity driven companies with the aim of confirming theory-derived hypotheses. We will therefore apply different statistical tests that aim to address whether our working assumption that real-option value is indeed an element of valuation within Russian commodity companies can be confirmed or not.

The chapter is structured as follows. In the first part we will focus primarily on unconditional distribution moment analysis. We will test whether the return distribution pattern of Russian companies is significantly different from that of other similar, non-Russian, peer-group companies. In the second part we will impose conditionality by assuming – based on the results from Chapter 4 and Chapter 5 – a geometric Brownian motion with a GJR-GARCH diffusion process as the data generating process for commodity driven companies, in order to infer from the estimated diffusion parameters real-option pricing evidence. In the last part of this chapter we will analyze the commodity price sensitivity of Russian MICEX companies on a panel basis and compare the results with those of different global peer-group panels.

6.2 The Analysis

6.2.1 Dataset description and selection

The dataset used in this analysis was collected between March 2003 and March 2009. The data source, unless otherwise specified, was Bloomberg. We will additionally distinguish between the periods of March 2003-2008 and March 2003-2009. The reason for this is the strong bull market in commodity-related equities between 2003-2008 vs. the commodity market crash between 2008 and 2009. Hence, the first sample size is five years and exhibits a strong bull market. The second sample size is six years and contains not only the bull market period but also the following crash in the commodity markets. Consequently we will use only companies that have a price history longer than

six years. Each company is analyzed in local currency terms. This has the advantage that currency effects are minimized in the analysis. For the commodity price sensitivity analysis using panel data we will additionally distinguish between the sample period 2007-2008 (commodity price high) and 2008-2009 (commodity price low).

To limit the impact of market capitalization we will compare only Russian companies with the appropriate peer group. This decreases the number of possible candidates from the MICEX Index from 30 to three. The MICEX companies for the return distribution analysis are given in Table 53.

Table 53

MICEX companies with commodity business exposure and time series history > 6 years

Company name	Industry group	Sector	Issuer Industry
Lukoil	Oil Company-Integrated	Energy	Oil & Gas
Gazprom OAO	Oil Comp.-Exploration and Production	Energy	Oil & Gas
MMC Norilsk Nickel	Metal-Diversified	Basic Materials	Metal-Diversified

Source: Bloomberg

For the Panel data analysis we will use the extended set of Russian companies (see Table 54). The companies are ordered along their market capitalization starting with the highest capitalized company first.²⁰¹

The international peer group is selected according to ICB classification methodology (see Footnote 10, p. 21). Only the first 20 peer-group companies (ranked by the market capitalization by Bloomberg for each peer-group sample) of each MICEX reference company will be used in the analysis. If the length of a time series is below 5 years, it will be dropped out of the analysis.

²⁰¹ See also Table 4, p. 18 for a complete list of MICEX members.

Table 54
MICEX companies with commodity business exposure

Company name	Industry group	Sector	Issuer Industry
Lukoil	Oil Company-Integrated	Energy	Oil & Gas
Gazprom OAO	Oil Comp.-Exploration and Production	Energy	Oil & Gas
Rosneft Oil	Oil Company-Integrated	Energy	Oil & Gas
Surgutneftegaz	Oil Company-Integrated	Energy	Oil & Gas
Polyus Gold Co	Gold Mining	Basic Materials	Gold Mining
MMC Norilsk Nickel	Metal-Diversified	Basic Materials	Metal-Diversified
Tatneft	Oil Company-Integrated	Energy	Oil & Gas
Gazpromneft (Sibneft) OAO	Oil Company-Integrated	Energy	Oil & Gas
Novolipetsk Steel	Metal-Iron	Basic Materials	Metal-Iron
Polymetal	Silver Mining	Basic Materials	Mining
Severstal	Steel-Producers	Basic Materials	Iron/Steel

Source: Bloomberg

We have separated the peer-group companies along their geographical positions. There are therefore 7 world regions and one overall global peer-group:

- Global peer-group (Global)
- North America (NA)
- Western Europe (WE)
- Eastern Europe (EEMA)
- Asia-Pacific - Developed (APAC-D)
- Asia-Pacific - Emerging (APAC-E)
- Latin America (LATAM)
- Middle East and Africa (MEA)

The sensitivity analysis (beta factors) will be conducted relative to individual commodity prices and additionally relative to the most important commodity indices for energy and basic materials. The possible indices are listed below.

Table 55
Commodity prices and commodity price indices used in the analysis

Underlying	Currency	Full name	Bloomberg code
Oil (WTA)	USD	Generic 1st 'CL' Future	CL1 Comdty
Natural Gas	USD	Generic 1st 'NG' Future	NG1 Comdty
Gold	USD	GOLD SPOT \$/OZ	GoldS Comdty
Silver	USD	SILVER SPOT \$/OZ	Silv Comdty
Platinum	USD	PLATINUM SPOT \$/OZ	PLAT Comdty
Palladium	USD	PALLADIUM SPOT \$/OZ	PALL Comdty
Nickel	USD	LME NICKEL 3MO (\$)	LMNIDS03 Comdty
CRB commodity index	USD	Commodity Research Bureau/Reuters	CRB CMDT INDEX
S&P Industrial Metal index	USD	S&P GSCI Industrial Metals	SPGCINTR Index
S&P Energy index	USD	S&P GSCI Energy	SPGCENTR Index

Source: Bloomberg

6.2.2 Unconditional distribution analysis

The unconditional distribution analysis will describe the probability characteristics of daily log-returns for different Russian commodity stocks. The guidelines for this analysis were presented in Chapter 3 by describing the expected equity and option-price distribution characteristic for companies with real-option value. We will focus, in the unconditional analysis, on testing the following statistics:

- a) Jarque-Bara normality
- b) Variance
- c) Skewness
- d) Kurtosis
- e) Higher moment-adjusted variance

The analysis will – where possible – rely on formulating H_0 hypothesis tests and also rank the results according to their significance levels (p-values). We will introduce each

analysis by using the MICEX Russian stock index sample before we show the results for the global peer group of the individual company in question.²⁰²

To facilitate the hypothesis tests (in order to find an appropriate value to formulate the H_0 hypothesis) we have also calculated the aggregated mean and median²⁰³ of the first to fourth distribution moments on a stacked sample basis. The results for the six- and five-year Gazprom peer group data sample are given in Table 56 below.

Table 56
Median and mean of different central moments for the Gazprom peer group
(stacked data sample)

Sample 2003-2009	Central moments (stacked Gazprom peer data)			
	1st	2nd	3rd	4th
Gazprom OAO	0.2587	46.4%	0.1730	15.4708
Median	0.1499	40.1%	-0.1540	10.9113
Mean	0.1523	41.7%	-0.0206	18.4076

Sample 2003-2008	Central moments (stacked Gazprom peer data)			
	1st	2nd	3rd	4th
Gazprom OAO	0.4884	35.6%	-0.0444	9.5200
Median	0.2413	29.9%	-0.0096	5.6924
Mean	0.2221	27.1%	0.0699	8.2886

The data are measured in log-returns. The data frequency is in days. The different regional peer group members are stacked in one data sample.

The monthly log-returns of the stacked data sample are on average negatively skewed with a positive kurtosis above three, which together indicate a strong deviation from a normal distribution. Table 57 shows the descriptive statistics for higher moments along the different peer-group regions.

²⁰² The mean and median of the different unconditional moments along with other statistics are given for each individual peer group in Appendix J, pp. 382-427.

²⁰³ The sample was adjusted (cleaned) for possible double or triple appearances of individual companies in different peer groups.

Table 57

Central higher moments of individual Gazprom peer group sample

Mean (2003-2009)				Mean (2003-2008)			
Region (ex-Russia)	Std. p.a.	Skewness	Kurtosis	Region (ex-Russia)	Std. p.a.	Skewness	Kurtosis
Global	35.4%	-0.1131	11.1761	Global	27.6%	-0.1948	5.3518
North America	39.3%	-0.4749	13.0578	North America	28.5%	-0.1545	4.4988
West Europe	35.2%	-0.3580	19.8062	West Europe	27.6%	0.2419	10.8950
Asia Developed	40.8%	-0.0423	9.0550	Asia Developed	35.4%	0.1172	7.4464
Asia Emerging	43.5%	0.3898	19.9112	Asia Emerging	40.7%	0.5268	21.8895
Latin America	51.6%	0.2441	18.4358	Latin America	48.5%	0.4748	15.3530
Easten Europe	44.7%	0.1897	20.7808	Easten Europe	40.3%	0.3197	20.1646
Middle East and Africa	41.0%	0.7501	27.3472	Middle East and Africa	34.1%	0.0615	10.3143
Median (2003-2009)				Median (2003-2008)			
Region (ex-Russia)	Std. p.a.	Skewness	Kurtosis	Region (ex-Russia)	Std. p.a.	Skewness	Kurtosis
Global	35.0%	-0.0671	9.9787	Global	28.0%	-0.2344	5.1433
North America	40.7%	-0.5046	12.6890	North America	29.8%	-0.1956	3.9746
West Europe	35.0%	-0.1300	10.7913	West Europe	28.4%	-0.0415	6.0979
Asia Developed	38.3%	0.0463	8.2886	Asia Developed	31.9%	-0.0415	6.0486
Asia Emerging	42.6%	-0.0661	8.5291	Asia Emerging	40.0%	0.0911	8.1693
Latin America	40.0%	-0.1122	13.9329	Latin America	34.6%	0.1830	9.1319
Easten Europe	42.6%	-0.0702	11.3069	Easten Europe	35.5%	0.0326	10.0977
Middle East and Africa	39.6%	0.0465	17.1790	Middle East and Africa	33.0%	0.0378	9.1661

Calculation based on daily log-returns.

One can observe that when looking at the distribution moments - if splitt along the corresponding peer group - developed regions (Global, North America and West Europe) appear to show more negative skewness and lower kurtosis values than other developing regions (Eastern Europe, Latin America, Asia - Emerging and Middle East/Africa). This already confirms the so far purely theoretically formulated expectations in Chapter 3 that developed companies trade rather like “in-the-money” options while developing companies with high real-option value trade similar to “out-of-the-money” options.

The stacked summary results for the Lukoil and Norilsk Nickel peer group are given in Table 58, Panels A and B.

Table 58
Median and mean of different central moments for the Lukoil and Norilsk Nickel
peer group (stacked data sample)

Panel A: Lukoil				
Sample 2003-2009	Central moments (stacked Lukoil peer data)			
	1st	2nd	3rd	4th
LUKOIL	0.1679	45.9%	0.1257	17.8115
Median	0.1357	40.1%	-0.1569	10.4135
Mean	0.1402	40.9%	-0.1299	14.6267
Sample 2003-2008	Central moments (stacked Lukoil peer data)			
	1st	2nd	3rd	4th
LUKOIL	0.2784	33.5%	-0.1508	5.6924
Median	0.2555	31.6%	-0.0831	6.5966
Mean	0.2431	34.2%	-0.0009	8.7619
Panel B: Norilsk Nickel				
Sample 2003-2009	Central moments (stacked Norilsk Nickel peer data)			
	1st	2nd	3rd	4th
NORILSK NICKEL	0.1431	53.3%	-1.1576	21.0232
Median	0.1485	44.4%	-0.1863	8.9933
Mean	0.1522	43.7%	-0.2512	10.5717
Sample 2003-2008	Central moments (stacked Norilsk Nickel peer data)			
	1st	2nd	3rd	4th
NORILSK NICKEL	0.4177	42.6%	-0.3022	7.8051
Median	0.3113	36.4%	-0.0190	5.8687
Mean	0.3144	35.6%	-0.0245	7.1078

The data are measured in log-returns. The data frequency is in days. The different regional peer group members are stacked in one data sample.

Table 59
Central higher moments of individual Lukoil peer group sample

Mean (2003-2009)				Mean (2003-2008)			
Region (ex-Russia)	Std. p.a.	Skewness	Kurtosis	Region (ex-Russia)	Std. p.a.	Skewness	Kurtosis
Global	36.9%	0.0015	14.3787	Global	29.5%	-0.2213	8.6788
North America	39.3%	-0.4833	12.6699	North America	28.5%	-0.1669	4.6232
West Europe	35.9%	-0.6459	20.8446	West Europe	28.9%	0.2044	13.4124
Asia Developed	39.9%	-0.0027	8.4911	Asia Developed	35.0%	0.1286	7.5385
Asia Emerging	39.8%	-0.0923	8.7322	Asia Emerging	36.8%	-0.0187	8.9884
Latin America	55.0%	-0.2373	12.3571	Latin America	50.7%	0.0735	9.4697
Easten Europe	38.8%	-0.1711	11.3933	Easten Europe	34.0%	-0.0114	9.4198
Middle East and Africa	41.5%	1.0123	28.8380	Middle East and Africa	34.8%	0.0662	9.4302
Median (2003-2009)				Median (2003-2008)			
Region (ex-Russia)	Std. p.a.	Skewness	Kurtosis	Region (ex-Russia)	Std. p.a.	Skewness	Kurtosis
Global	31.4%	-0.0298	13.0245	Global	23.3%	-0.2161	4.9877
North America	41.0%	-0.4176	12.6778	North America	29.8%	-0.1914	4.0941
West Europe	37.1%	-0.4204	10.3964	West Europe	28.8%	-0.0055	6.3955
Asia Developed	37.7%	0.0588	8.1581	Asia Developed	31.6%	-0.0595	5.9943
Asia Emerging	41.9%	-0.1049	8.2748	Asia Emerging	37.3%	0.0678	8.1541
Latin America	40.0%	-0.2518	10.0844	Latin America	34.6%	0.1134	8.9302
Easten Europe	42.2%	-0.0702	9.4505	Easten Europe	34.9%	0.0326	8.0182
Middle East and Africa	39.6%	0.1499	16.9068	Middle East and Africa	33.0%	0.0303	9.1661

Calculation based on daily log-returns.

The peer group summary results for Lukoil and Norilsk Nickel are very similar to those for the Gazprom sample. All show a negative skewness and a positive kurtosis, with the five-year pre-crisis sample (2003-2008) showing lower skewness and lower kurtosis than the six-year (2003-2009) sample. The median values for the 2003 to 2008 sample show the same pattern as Gazprom (negative skewness and lower kurtosis for the developed peer group, in contrast to the emerging peer group sample). However, one small difference that emerges between the two industries (Oil and Gas vs. Mining and Metal) is observed if the aggregated sample statistics are split along the peer group regions. Table 60 shows the results for the different peer group sample for Norilsk Nickel.

Table 60

Central higher moments of individual Norilsk Nickel peer group sample

Mean (2003-2009)				Mean (2003-2008)			
Region (ex-Russia)	Std. p.a.	Skewness	Kurtosis	Region (ex-Russia)	Std. p.a.	Skewness	Kurtosis
Global	47.9%	-0.5413	16.2494	Global	35.5%	0.1075	6.3158
North America	45.3%	-0.3846	11.8668	North America	34.2%	-0.0913	6.3091
West Europe	37.5%	-0.1139	9.4584	West Europe	30.1%	0.0431	7.4305
Asia Developed	36.4%	-0.0907	8.4604	Asia Developed	31.1%	-0.0669	6.8235
Asia Emerging	53.4%	-0.1105	8.7141	Asia Emerging	47.7%	-0.1845	9.7466
Latin America	42.6%	-0.2228	10.2298	Latin America	34.9%	-0.0236	5.9872
Easten Europe				Easten Europe			
Middle East and Africa	44.0%	-0.3061	9.9930	Middle East and Africa	36.6%	0.1240	7.0457
Median (2003-2009)				Median (2003-2008)			
Region (ex-Russia)	Std. p.a.	Skewness	Kurtosis	Region (ex-Russia)	Std. p.a.	Skewness	Kurtosis
Global	47.8%	-0.3139	11.7576	Global	36.3%	0.0891	5.2253
North America	46.1%	-0.4046	11.7981	North America	35.6%	-0.0659	4.6366
West Europe	35.4%	-0.1578	8.9815	West Europe	27.0%	-0.0247	6.4505
Asia Developed	37.1%	-0.1291	7.9170	Asia Developed	30.5%	-0.0592	5.5738
Asia Emerging	53.5%	-0.0495	7.6924	Asia Emerging	48.0%	0.0187	7.0812
Latin America	44.7%	-0.1442	8.5971	Latin America	36.4%	-0.0088	5.8896
Easten Europe				Easten Europe			
Middle East and Africa	44.2%	-0.1506	8.6186	Middle East and Africa	34.9%	0.0216	5.8477

Calculation based on daily log-returns.

Now most of the tested peer groups show a similar skewness and kurtosis (negative skewness and positive kurtosis). Developing markets show – contrary to the oil and gas industry – the tendency for higher kurtosis for the median 2003-2009 sample.

This small deviation from the apparent general rule is also visible if one aggregates the average values for standard deviation, skewness and kurtosis. Table 61 shows the distribution moments grouped along developed and developing regions (ex-Russia). As expected the peer-group statistics for developed regions have a smaller skewness and a larger kurtosis. The only exception to this rule is the mining peer group between 2003

and 2009, where the kurtosis for the developed regions is higher than for the developing regions.

Table 61

Median of different central moments for Gazprom, Lukoil and Norilsk Nickel peer group companies across all regions (stacked data sample)

Panel A							
Central moments (stacked Gazprom peer data - ex Russia)							
Median (2003-2009)	Std. p.a.	Skewness	Kurtosis	Median (2003-2008)	Std. p.a.	Skewness	Kurtosis
developed (ex-Russia)	38.02%	-0.1961	10.5896	developed (ex-Russia)	30.02%	-0.0929	5.3737
developing (ex-Russia)	41.23%	-0.0505	12.7369	developing (ex-Russia)	35.75%	0.0861	9.1412
Panel B							
Central moments (stacked Lukoil peer data - ex Russia)							
Median (2003-2009)	Std. p.a.	Skewness	Kurtosis	Median (2003-2008)	Std. p.a.	Skewness	Kurtosis
developed (ex-Russia)	38.60%	-0.2597	10.4108	developed (ex-Russia)	30.05%	-0.0855	5.4946
developing (ex-Russia)	40.95%	-0.0693	11.1791	developing (ex-Russia)	34.93%	0.0610	8.5672
Panel C							
Central moments (stacked Norilsk Nickel peer data ex-Russia)							
Median (2003-2009)	Std. p.a.	Skewness	Kurtosis	Median (2003-2008)	Std. p.a.	Skewness	Kurtosis
developed (ex-Russia)	39.51%	-0.2305	9.5655	developed (ex-Russia)	31.04%	-0.0499	5.5536
developing (ex-Russia)	47.47%	-0.1148	8.3027	developing (ex-Russia)	39.77%	0.0105	6.2728

Calculation based on daily log-returns.

All of the above aggregations indicate that developed countries trade (measured along the distribution pattern of daily log-returns) like in-the-money options while companies in developing countries with higher real-option value trade like out-of-the-money options. This can be seen as additional confirmation of our working hypothesis, formulated in Subsection 3.2.3, that – beyond our Russia-related focus – companies in developing markets should have higher real-optionalities and hence trade more similarly to out-of-the-money options (higher volatility, higher skewness and kurtosis) than companies in developed regions (in-the-money companies).

6.2.2.1 Testing for normality (Jarque-Bera-Test)

As shown in Subsection 3.2.3, p. 67, the return distribution of an option payoff is non-normal. The deviation from log-normal returns increases with the out-of-the-money distance of the option. Companies with high out-of-the-money real-option value will therefore have highly non-normal log-returns. We use the standard Jarque-Bera (JB) test statistic to calculate the appropriate p-value to produce an absolute value rank for the different analyzed daily log-returns. The formula for the JB statistic is given in Equation

(160). The significance and critical values for the JB statistic are calculated based on a chi-square distribution with two degrees of freedom.²⁰⁴

$$JB = N \left(\frac{skewness^2}{6} + \frac{kurtosis^2 - 3}{24} \right) \quad (160)$$

Table 62 shows the test results along with other test statistics for tested MICEX companies.

Table 62
Testing for normality – Jarque-Bera test statistic for selected MICEX companies using the test sample between 2003 and 2009

Name	Mean	Std. p.a.	Skewness	Kurtosis	Autocorr.	Jarque-Bera	JB P-Value
LUKOIL	16.79%	45.9%	0.1257	17.8115	-0.0062	14327.86	0.0000
Gazprom OAO	25.87%	46.4%	0.1730	15.4708	0.0528	10161.95	0.0000
Surgutneftegaz	14.84%	54.4%	1.4269	28.7914	0.0505	43963.47	0.0000
MMC Norilsk Nickel	14.31%	53.3%	-1.1576	21.0232	0.1407	21559.02	0.0000
Tatneft	18.97%	55.7%	0.6670	21.1472	0.1167	21618.18	0.0000
Surgutneftegaz Pref.	1.98%	49.6%	1.2243	42.9567	0.0908	104632.3	0.0000
Gazpromneft OAO	2.61%	48.2%	-0.8215	12.6283	0.0926	6229.124	0.0000
AK Transneft OAO	-4.70%	50.4%	-0.2196	21.6671	0.1594	22764.18	0.0000

Calculation based on daily log-returns.

The Jarque-Bera statistic is extremely high, which leads us to dismiss the normality distribution hypothesis with near-certainty (see JB P-Value column). The analysis also presents the skewness and kurtosis values for the different underlying. Although the kurtosis and skewness hypothesis tests are conducted in the next section, a first view indicates that the non-normality is primarily born out of high kurtosis values. The skewness values are positive for most of the MICEX companies tested. These results, however, change if the shorter sample (five-year sample between March 2003 and 2008) is used as shown in Table 63. The skewness values are all negative, while the kurtosis values appear closer to normal than in the 2003-2009 sample.

²⁰⁴ The variable N represent the number of observation; skewness and kurtosis are calculated along the lines of Sheskin (2007), p. 219.

Table 63

Testing for normality – Jarque-Bera test statistic for selected MICEX companies using the test sample between 2003 and 2008

Name	Mean	Std. p.a.	Skewness	Kurtosis	Autocorr.	Jarque-Bera	JB P-Value
LUKOIL	27.84%	33.5%	-0.1508	5.6924	0.0246	399.4267	0.0000
Gazprom OAO	48.84%	35.6%	-0.0444	9.5200	0.0290	2313.701	0.0000
Surgutneftegaz	16.40%	41.9%	-0.2394	8.5088	0.0299	1663.874	0.0000
MMC Norilsk Nickel	41.77%	42.6%	-0.3022	7.8051	-0.0122	1276.323	0.0000
Tatneft	34.39%	41.2%	-0.1013	6.0634	0.0661	512.8893	0.0000
Surgutneftegaz Pref.	10.35%	38.7%	-0.3853	8.3409	0.0466	1584.586	0.0000
Gazpromneft OAO	13.75%	38.8%	-0.7036	8.6619	0.0332	1852.227	0.0000
AK Transneft OAO	23.71%	36.1%	0.1189	5.6759	0.0949	392.7281	0.0000

Calculation based on daily log-returns.

These results have to be interpreted against the backdrop of record high global commodity markets.²⁰⁵ The year 2008 was the peak for most commodity indices. Given the arguments laid out in Subsection 3.2.3, a negative skewness can be seen as indication that real-options are in-the-money which, given the extraordinarily high commodity prices, appears plausible. These results, however, need to be related to other global commodity companies in order to carry out any inferences about the assumption of real-option pricing in the Russian stock market. The results in Table 64 show the Jarque-Bera and distribution moment statistics for the global peer group of Gazprom (sample 2003-2009).

²⁰⁵ See Figure 4 and Figure 5, pp. 14-15 for the development of major commodity indices over time.

Table 64
Testing for normality – Jarque-Bera test statistic for Gazprom global peer group companies (test sample between 2003 and 2009)

Name	Mean	Std. p.a.	Skewness	Kurtosis	Autocorr.	Jarque-Bera	JB P-Value
ENI SPA	0.73%	27.4%	0.8120	21.0044	-0.0984	21336.93	0.0000
CHEVRON CORP	12.02%	30.0%	0.1623	18.9132	-0.1362	16540.58	0.0000
LUKOIL	16.79%	45.9%	0.1257	17.8115	-0.0062	14327.86	0.0000
Gazprom OAO	25.87%	46.4%	0.1730	15.4708	0.0528	10161.95	0.0000
OCCIDENTAL PETROLEUM CORP	21.94%	40.7%	-0.3958	13.6870	-0.1050	7498.021	0.0000
ROYAL DUTCH SHELL PLC-A SHS	-2.09%	26.2%	0.2237	13.2331	-0.0447	6850.157	0.0000
TOTAL SA	3.20%	27.2%	0.3923	12.8159	-0.0920	6331.137	0.0000
SCHLUMBERGER LTD	13.53%	40.1%	-0.8635	12.6666	-0.0746	6295.811	0.0000
IMPERIAL OIL LTD	16.52%	34.4%	-0.0671	12.7857	-0.0750	6253.521	0.0000
CONOCOPHILLIPS	6.77%	34.2%	-0.5046	10.9876	-0.0704	4232.227	0.0000
ENCANA CORP	12.70%	37.0%	-0.4320	9.9787	-0.0539	3228.619	0.0000
BP PLC	1.77%	26.2%	0.2999	9.4042	-0.0739	2701.34	0.0000
RELIAANCE INDUSTRIES LTD	30.36%	39.1%	-0.6731	9.0221	0.0398	2486.174	0.0000
CNOOC LTD	22.55%	45.2%	0.0713	8.2886	0.0766	1827.486	0.0000
OIL & NATURAL GAS CORP LTD	18.10%	39.2%	-0.5063	7.7385	0.0776	1532.953	0.0000
PETROBRAS - PETROLEO BRAS-PR	26.58%	39.0%	-0.1698	7.7032	0.0293	1451.759	0.0000
BG GROUP PLC	23.14%	33.7%	-0.0247	7.6171	-0.0087	1392.017	0.0000
CHINA PETROLEUM & CHEMICAL-H	17.83%	47.1%	0.1068	7.2384	0.0174	1175.869	0.0000
STATOILHYDRO ASA	13.57%	35.0%	-0.3546	6.9093	-0.0483	1030.694	0.0000
mean	14.84%	36.5%	-0.0855	11.7514	-0.0312	6139.742	0.0000
median	16.52%	37.0%	-0.0247	10.9876	-0.0483	4232.227	0.0000

Calculation based on daily log-returns. Ranked according to the Jarque-Bera statistic.

The highest deviation from normality in the global peer group is observed - except for Chevron - for emerging market companies (i.e., Russia, Latin America), indicating again that companies with possible real-options trade differently in markets than traditional commodity (in this case oil) companies. The picture looks similar if we shorten the sample from six to five years. The results are given in Table 65 below.

Table 65
Testing for normality – Jarque-Bera test statistic for Gazprom global peer group companies (test sample between 2003 and 2008)

Name	Mean	Std. p.a.	Skewness	Kurtosis	Autocorr.	Jarque-Bera	JB P-Value
Gazprom OAO	48.84%	35.6%	-0.0444	9.5200	0.0290	2313.701	0.0000
RELIAANCE INDUSTRIES LTD	46.02%	33.2%	-0.5108	8.1844	0.0008	1519.409	0.0000
OIL & NATURAL GAS CORP LTD	27.88%	37.2%	-0.4974	7.8137	0.0743	1314.757	0.0000
CNOOC LTD	34.93%	37.7%	0.1401	6.6279	0.0169	720.504	0.0000
CHINA PETROLEUM & CHEMICAL-H	29.56%	41.8%	0.1431	6.5228	0.0257	679.766	0.0000
ROYAL DUTCH SHELL PLC-A SHS	2.90%	18.6%	-0.4511	6.1208	-0.0406	574.274	0.0000
LUKOIL	27.84%	33.5%	-0.1508	5.6924	0.0246	399.427	0.0000
PETROBRAS - PETROLEO BRAS-PR	37.50%	31.6%	0.0616	5.6505	0.0216	383.115	0.0000
IMPERIAL OIL LTD	24.49%	26.1%	-0.2953	5.2381	-0.0067	291.563	0.0000
BG GROUP PLC	28.92%	24.9%	0.3130	5.2064	-0.0472	286.231	0.0000
STATOILHYDRO ASA	21.43%	28.8%	-0.2344	5.1433	-0.0244	261.924	0.0000
ENCANA CORP	23.54%	28.0%	-0.3158	5.0323	0.0042	246.468	0.0000
SCHLUMBERGER LTD	29.75%	29.8%	-0.2167	4.8974	-0.0001	206.125	0.0000
ENI SPA	10.62%	18.6%	-0.2966	4.7802	-0.0650	191.603	0.0000
TOTAL SA	8.63%	20.0%	-0.1824	4.7373	-0.0826	171.489	0.0000
BP PLC	4.47%	19.9%	-0.0976	4.0832	-0.0539	65.926	0.0000
CONOCOPHILLIPS	22.41%	24.9%	-0.3919	3.7044	0.0243	60.432	0.0000
OCCIDENTAL PETROLEUM CORP	31.59%	28.0%	-0.1956	3.8648	-0.0388	49.024	0.0000
CHEVRON CORP	19.56%	20.7%	-0.2839	3.3732	-0.0595	25.128	0.0000
mean	25.31%	28.4%	-0.1846	5.5891	-0.0104	513.730	0.0000
median	27.84%	28.0%	-0.2167	5.2064	-0.0001	286.231	0.0000

Calculation based on daily log-returns. Ranked according to the Jarque-Bera statistic.

Russian commodity companies remain highly non-normal compared to the entire global peer group sample. In this case Gazprom – the anchor for the peer-group definition – has the highest JB P-value. The results for the other two analyzed companies (Lukoil and Norilsk Nickel) are shown in Table 66 and Table 67.

Table 66

Testing for normality – Jarque-Bera test statistic for Lukoil global peer group companies

Panel A							
Name	Mean	Std. p.a.	Skewness	Kurtosis	Autocorr.	Jarque-Bera	JB P-Value
WEST SIBERIAN RESOURCES-SDR	2.99%	73.4%	-0.2784	27.5322	-0.0208	39314.6	0.0000
Tatneft	18.97%	55.7%	0.6670	21.1472	0.1167	21618.2	0.0000
EXXON MOBIL CORP	11.32%	28.3%	0.0076	17.8681	-0.1805	14433.3	0.0000
SURGUTNEFTEGAZ	14.84%	60.8%	0.2227	15.7852	-0.1030	10685.5	0.0000
GAZPROM OAO	25.87%	46.4%	0.1730	15.4708	0.0528	10162.0	0.0000
LUKOIL	16.79%	50.7%	0.1625	13.9228	-0.0877	7796.6	0.0000
OCCIDENTAL PETROLEUM CORP	21.94%	40.7%	-0.3958	13.6870	-0.1050	7498.0	0.0000
ROYAL DUTCH SHELL PLC	-2.09%	26.2%	0.2237	13.2331	-0.0447	6850.2	0.0000
TOTAL SA	3.20%	27.2%	0.3923	12.8159	-0.0920	6331.1	0.0000
IMPERIAL OIL LTD	16.52%	34.4%	-0.0671	12.7857	-0.0750	6253.5	0.0000
Gazpromneft OAO	2.61%	48.2%	-0.8215	12.6283	0.0926	6229.1	0.0000
BP PLC	1.77%	26.2%	0.2999	9.4042	-0.0739	2701.3	0.0000
PETROBRAS - PETROLEO BRAS-PR	26.58%	39.0%	-0.1698	7.7032	0.0293	1451.8	0.0000
Panel B							
Name	Mean	Std. p.a.	Skewness	Kurtosis	Autocorr.	Jarque-Bera	JB P-Value
WEST SIBERIAN RESOURCES-SDR	19.10%	71.1%	-0.2367	35.5119	-0.0348	57531.8	0.0000
SURGUTNEFTEGAZ	16.40%	47.8%	-0.1597	11.2946	-0.0687	3749.4	0.0000
GAZPROM OAO	48.84%	35.6%	-0.0444	9.5200	0.0290	2313.7	0.0000
Gazpromneft OAO	13.75%	38.8%	-0.7036	8.6619	0.0332	1852.2	0.0000
LUKOIL	27.25%	37.2%	-0.1770	6.3338	-0.0957	611.6	0.0000
ROYAL DUTCH SHELL PLC	2.90%	18.6%	-0.4511	6.1208	-0.0406	574.3	0.0000
Tatneft	34.39%	41.2%	-0.1013	6.0634	0.0661	512.9	0.0000
PETROBRAS - PETROLEO BRAS-PR	37.50%	31.6%	0.0616	5.6505	0.0216	383.1	0.0000
IMPERIAL OIL LTD	24.49%	26.1%	-0.2953	5.2381	-0.0067	291.6	0.0000
TOTAL SA	8.63%	20.0%	-0.1824	4.7373	-0.0826	171.5	0.0000
EXXON MOBIL CORP	18.50%	20.4%	-0.3731	4.2236	-0.0721	111.8	0.0000
BP PLC	4.47%	19.9%	-0.0976	4.0832	-0.0539	65.9	0.0000
OCCIDENTAL PETROLEUM CORP	31.59%	28.0%	-0.1956	3.8648	-0.0388	49.0	0.0000

Calculation based on daily log-returns. Panel A shows the results for the sample 2003-2009, while Panel B shows the results for the sample 2003-2008.

Table 67
Testing for normality – Jarque-Bera test statistic for Norilsk Nickel global peer group companies

Panel A							
Name	Mean	Std. p.a.	Skewness	Kurtosis	Autocorr.	Jarque-Bera	JB P-Value
SUMITOMO METAL MINING CO LTD	15.22%	48.1%	-0.0944	6.7302	0.0095	910.8	0.0000
ANGLO AMERICAN PLC	2.17%	46.9%	-0.2978	12.4826	0.0136	5894.2	0.0000
BHP BILLITON LTD	20.50%	36.3%	-0.2862	7.2644	-0.0398	1208.7	0.0000
MMC NORILSK NICKEL	14.31%	53.3%	-1.1576	21.0232	0.1407	21559.0	0.0000
IMPALA PLATINUM HOLDINGS LTD	15.67%	47.5%	-0.3698	6.0397	0.0726	639.0	0.0000
LONMIN PLC	8.23%	49.9%	0.9437	26.6608	0.0336	36785.0	0.0000
NOVOLIPETSK STEEL	25.92%	67.6%	-0.7334	17.9785	-0.0337	14789.1	0.0000
RIO TINTO LTD	6.99%	43.1%	-2.8303	46.4773	-0.0261	125511.2	0.0000
TECK RESOURCES LTD-CLS B	-2.18%	57.1%	-1.0654	13.3076	0.1150	7233.5	0.0000
XSTRATA PLC	12.93%	54.1%	-0.3300	11.0326	0.1031	4241.3	0.0000

Panel B							
Name	Mean	Std. p.a.	Skewness	Kurtosis	Autocorr.	Jarque-Bera	JB P-Value
SUMITOMO METAL MINING CO LTD	33.30%	41.9%	-0.2874	6.5329	-0.0178	697.2	0.0000
ANGLO AMERICAN PLC	22.51%	33.8%	0.0845	5.9410	-0.0380	472.2	0.0000
BHP BILLITON LTD	27.36%	28.4%	-0.1513	4.5096	-0.0350	129.0	0.0000
MMC NORILSK NICKEL	41.77%	42.6%	-0.3022	7.8051	-0.0122	1276.3	0.0000
IMPALA PLATINUM HOLDINGS LTD	35.81%	40.5%	0.2194	4.4598	0.0369	126.4	0.0000
LONMIN PLC	28.65%	36.1%	0.8013	12.6344	-0.0110	5190.8	0.0000
NOVOLIPETSK STEEL	52.03%	45.1%	0.1987	7.9069	-0.0469	1318.8	0.0000
RIO TINTO LTD	27.36%	29.0%	0.2594	8.7568	0.0050	1818.0	0.0000
TECK RESOURCES LTD-CLS B	39.60%	37.6%	-0.1593	3.2279	0.0498	8.4	0.0154
XSTRATA PLC	50.04%	36.5%	0.0937	4.4641	-0.0046	118.6	0.0000

Calculation based on daily log-returns. Panel A shows the results for the sample 2003-2009, while Panel B shows the results for the sample 2003-2008.

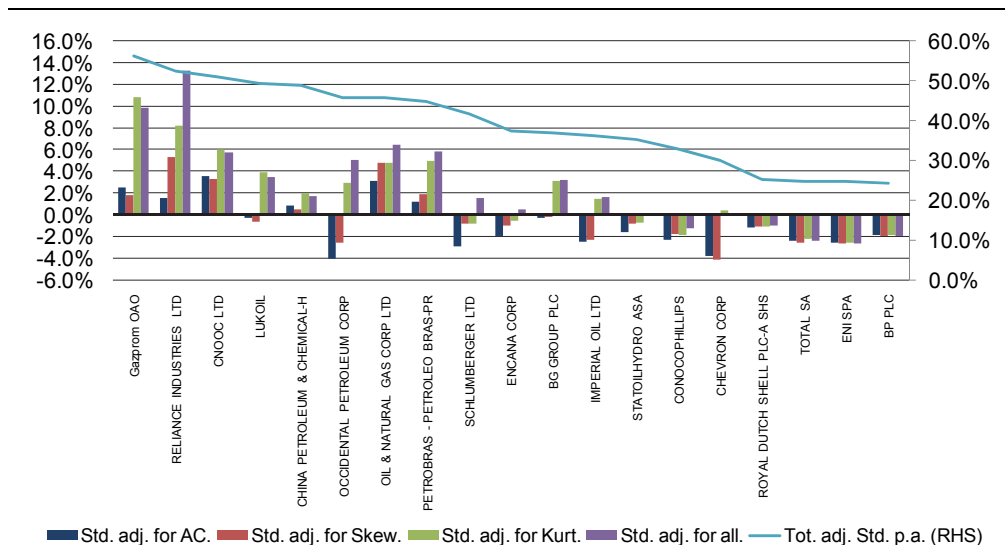
6.2.2.2 Adjusting the historical sample standard deviation (as proxy for market risk) for non-normality of log-returns (reverse Sharpe ratio analysis)

In order to relate the measured standard deviation to other higher moments (i.e., skewness and kurtosis) one can utilize a reverse Sharpe ratio analysis as described in Subsection 3.3.3, p. 103. The analysis is inspired by the assumption that companies with high real-option value (out-of-the-money projects) should trade with higher standard deviations than companies with low real-option value (in-the-money projects). Since, however, the standard deviation as risk measures underestimate the true risk of an underlying if the return distribution is not normal (and the deviation from normality was confirmed with the previously applied Jarque-Bera tests), an adjustment for higher moment needs to be made in order to render the standard deviation comparable across different companies. Figure 112 shows the results ranked according to the overall adjusted standard deviation (Tot. adj. Std. p.a.) that was calculated based on the reversed Sharpe ratio analysis. Individual adjustments for autocorrelation, skewness and kurtosis are also illustrated. The difference between the total adjusted standard deviation

and the unconditional sample standard deviation is labeled Std. adj. for all. Gazprom has the highest adjusted standard deviation, with the adjustments coming primarily from the high kurtosis.

Figure 112

Risk of different Energy sector companies measured by the standard deviation adjusted for higher moments and autocorrelation of the log-returns (2003-2009)

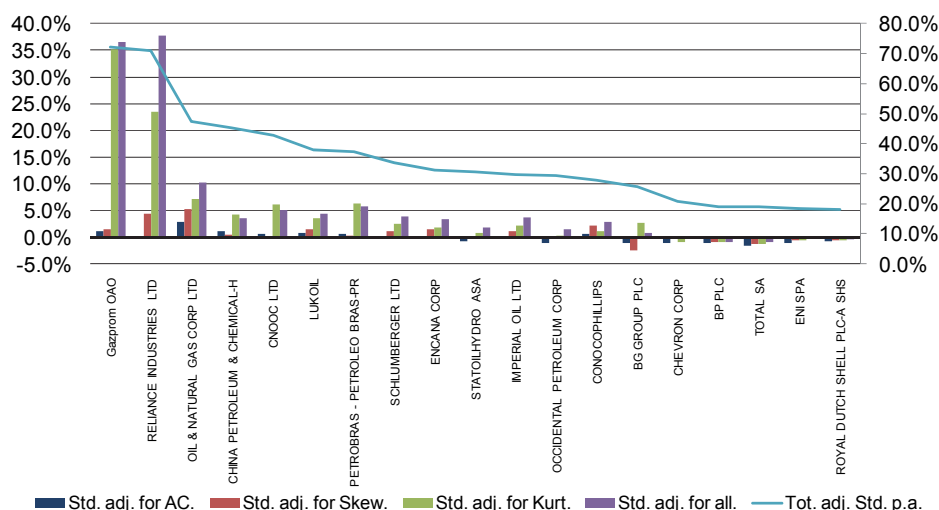


Companies represent the global peer group for Gazprom. The LHS shows the adjustments for the historical standard deviation in % terms. The RHS shows the aggregated adjusted standard deviation for the companies.

Hence, if we argue that assets are normally distributed (mean – standard deviation as distribution determinants), the higher moment adjustment would confirm our expectation that Russian companies have a higher risk (measured by the implied standard deviation) than other members of the peer group. The results are striking insofar as developed oil companies (Royal-Dutch-Shell, Total, ENI and BP) all have negative volatility adjustments, just as expected. Recalling the analysis in Figure 106 (see p. 247) of the relation between strike-price and option return risk, we see confirmation of the assumption of Russian companies trading with much higher risks due to option type log-returns. The results are also confirmed over the five-year sample (see Figure 113).

Figure 113

Risk of different Energy sector companies measured by the standard deviation adjusted for higher moments and autocorrelation of the log-returns (2003-2008)

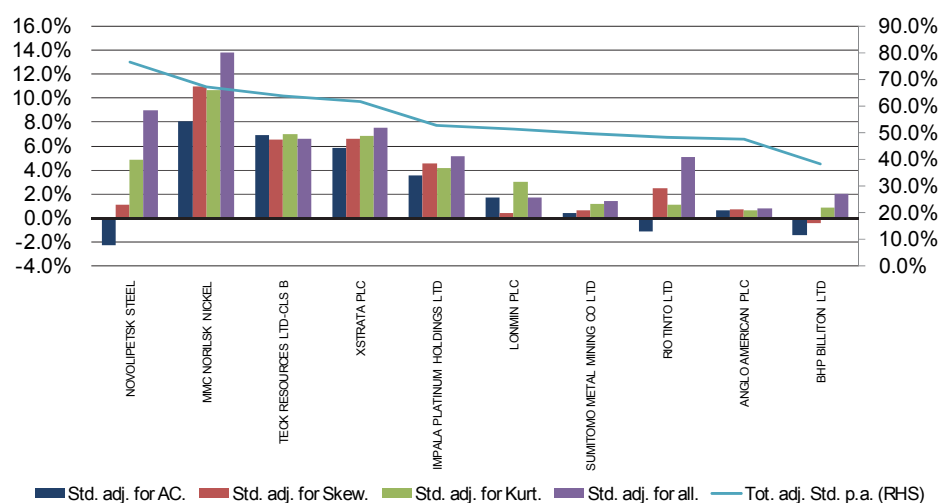


Companies represent the global peer group for Gazprom. The LHS shows the adjustments for the historical standard deviation in % terms. The RHS shows the aggregated adjusted standard deviation for the companies.

These results are confirmed if we repeat the analysis with the Norilsk Nickel peer group. As shown in Figure 114 and Figure 115, the highest adjusted volatility due to the adjustment for higher log-return moments is calculated for the two Russian stock companies.

Figure 114

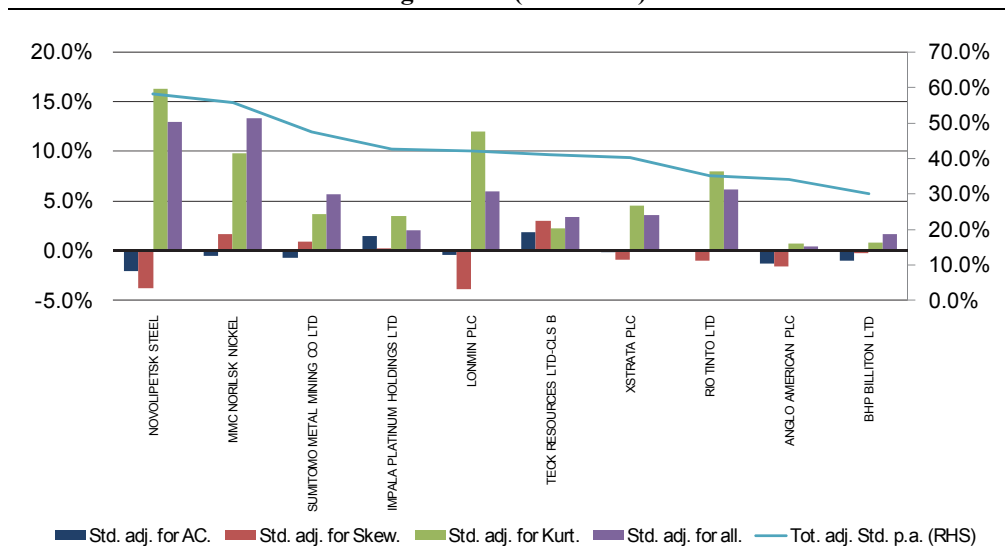
Risk of different Industrial Metals sector companies measured by the standard deviation adjusted for higher moments and autocorrelation of the log-returns (2003-2009)



Companies represent the global peer group for Norilsk Nickel. The LHS shows the adjustments for the historical standard deviation in % terms. The RHS shows the aggregated adjusted standard deviation for the companies.

Figure 115

Risk of different Industrial Metals sector companies measured by the standard deviation adjusted for higher moments and autocorrelation of the log-returns (2003-2008)



Companies represent the global peer group for Norilsk Nickel. The LHS shows the adjustments for the historical standard deviation in % terms. The RHS shows the aggregated adjusted standard deviation for the companies.

One needs to acknowledge that the above analysis is primarily of a descriptive nature since – beside the JB-P-value – no test statistics have been calculated nor was any statistical inference conducted. The following section aims to add a more formal quantitative reasoning to the analysis.

6.2.2.3 Variance test

We will conduct directional hypothesis tests (one-tailed tests) in order to answer whether and how much the variance of daily returns of the international peer group deviates from that of Russian stock market companies. The results of the unconditional analysis for all appropriate MICEX members (see Table 68) show that on average the annualized volatility for this MICEX core group is higher than the mean or median of all the calculated unconditional standard deviations observed in the stacked Gazprom peer group (see Table 56, p. 264).

Table 68

Variance ratio test for Gazprom and selected MICEX companies relative to Gazprom peer group median (across all peer group regions) as H_0

Sample 2003-2009	LUKOIL	Gazprom OAO	Surgutnefteg az	MMC Norilsk Nickel	Tatneft	Surgutneftegaz Pref.	Gazpromneft OAO	Novolipetsk Steel
Std. p.a.	45.9%	46.4%	54.4%	53.3%	55.7%	49.6%	48.2%	67.6%
Skewness.	0.13	0.17	1.43	-1.16	0.67	1.22	-0.82	-0.73
Kurtosis	17.81	15.47	28.79	21.02	21.15	42.96	12.63	17.98
Jarque-Bera	14328	10162	43963	21559	21618	104632	6229	14789
JB P-Value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Observations tot.	1567	1567	1567	1567	1567	1567	1567	1567
Observations p.a.	261	261	261	261	261	261	261	261
H0 for daily st.dev.	2.48%	2.48%	2.48%	2.48%	2.48%	2.48%	2.48%	2.48%
Daily sample st.dev.	2.84%	2.87%	3.37%	3.30%	3.45%	3.07%	2.98%	4.18%
Chi-squared statistic	2048	2100	2878	2767	3018	2396	2262	4448
Variance Ratio P-Value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Sample 2003-2008	LUKOIL	Gazprom OAO	Surgutnefteg az	MMC Norilsk Nickel	Tatneft	Surgutneftegaz Pref.	Gazpromneft OAO	Novolipetsk Steel
Std. p.a.	33.5%	35.6%	41.9%	42.6%	41.2%	38.7%	38.8%	45.1%
Skewness.	-0.15	-0.04	-0.24	-0.30	-0.10	-0.39	-0.70	0.20
Kurtosis	5.69	9.52	8.51	7.81	6.06	8.34	8.66	7.91
Jarque-Bera	399	2314	1664	1276	513	1585	1852	1319
JB P-Value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Observations tot.	1306	1306	1306	1306	1306	1306	1306	1306
Observations p.a.	261	261	261	261	261	261	261	261
H0 for daily st.dev.	1.85%	1.85%	1.85%	1.85%	1.85%	1.85%	1.85%	1.85%
Daily sample st.dev.	2.07%	2.21%	2.59%	2.64%	2.55%	2.40%	2.40%	2.79%
Chi-squared statistic	1635	1854	2558	2645	2480	2189	2200	2965
Variance Ratio P-Value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Calculation based on daily log-returns. The MICEX companies are selected based on their peer-group specification.

This is interesting insofar as – in contrast to the test conducted here – it would also allow for testing variance or distribution equality between samples (Russian sample vs. the global sample). The test conducted here, however, is based on a simple chi-square test for a population variance. The test statistic is given in Equation (161):

$$\chi^2 = \frac{(n-1)\hat{s}^2}{\sigma^2} \quad (161)$$

n : number of observations

$\hat{s}^2$: unbiased estimated of the population variance

σ^2 : hypothesized value stated in H_1

The H_0 hypothesis will be related to the median of the stacked sample variance depending on the sample horizon (five or six years). Since we are interested in testing whether the variance of daily Gazprom log-returns (σ^2) is higher than for the peer group

median, the test is formulated as a directional alternative hypothesis test (a one-tailed test):

$$H_0: \sigma^2 = \text{peer group median variance}$$

$$H_1: \sigma^2 > \text{peer group median variance}$$

The results of the test are shown in Table 68. All the relevant MICEX members along with the peer group defining company (Gazprom) have a significantly higher daily volatility than the peer group median (variance ratio p-values close to zero).²⁰⁶

It is therefore possible to conclude that, measured over two samples periods (2003-2009 and 2003-2008), Gazprom and all other tested core MICEX companies have a higher daily variance and hence daily standard deviation than the Gazprom defining peer group (which leads us to reject the H_0 hypothesis at a 1% level of significance). The same conclusion holds for the variance of Lukoil (using the sample 2003-2009), since the test statistic for Lukoil – by applying the Gazprom peer group median as H_0 – is still highly significant.²⁰⁷ However, the five-year sample median (2003-2008) for the Lukoil stacked peer-group data is slightly higher compared to Gazprom. We have therefore repeated the test using the higher median value as the H_0 reference. The results are shown in Table 69 below.

²⁰⁶ In line with the suggestion from the literature (see Sheskin (2007), p. 19), we have used the median as the measure of central tendency. Although not shown here, the test statistic for the one-tailed test remains highly significant if H_0 is set equal to the peer-group sample mean (instead of using the median).

²⁰⁷ The mean and median of the Lukoil peer group (2003-2009) are slightly lower than for the Gazprom six-year sample (compare Table 56 and Table 58).

Table 69

Variance ratio test for Lukoil and selected MICEX companies relative to Lukoil peer group median (across all peer group regions between 2003-2008) as H_0

Sample 2003-2008	LUKOIL	Gazprom OAO	Surgutnefteg az	MMC Norilsk Nickel	Tatneft	Surgutneftegaz Pref.	Gazpromneft OAO	Novolipetsk Steel
Std. p.a.	33.5%	35.6%	41.9%	42.6%	41.2%	38.7%	38.8%	45.1%
Skewness.	-0.15	-0.04	-0.24	-0.30	-0.10	-0.39	-0.70	0.20
Kurtosis	5.69	9.52	8.51	7.81	6.06	8.34	8.66	7.91
Jarque-Bera	399	2314	1664	1276	513	1585	1852	1319
JB P-Value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Observations tot.	1306	1306	1306	1306	1306	1306	1306	1306
Observations p.a.	261	261	261	261	261	261	261	261
H1 for daily st.dev.	1.96%	1.96%	1.96%	1.96%	1.96%	1.96%	1.96%	1.96%
Daily sample st.dev.	2.07%	2.21%	2.59%	2.64%	2.55%	2.40%	2.40%	2.79%
Chi-squared statistic	1464	1660	2290	2368	2221	1960	1970	2654
Variance Ratio P-Value	0.0014	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Calculation based on daily log-returns. The MICEX companies are selected based on their peer-group specification.

Now H_0 is rejected for Lukoil at a 5% level of significance but not at a 1% level. The tests for the Norilsk Nickel defining peer-group median variances are shown in Table 70. Again the variance ratio tests indicate that the variance of Norilsk Nickel is significantly higher than the stacked peer-group median variance (leading us to reject H_0 at a 1% level of significance). Also noticeable is the observation that, within the MICEX sample, oil and gas companies (Lukoil, Gazprom, etc.) have lower variances than Norilsk Nickel or Novolipetsk Steel.

Table 70

Variance ratio test for Gazprom and selected MICEX companies relative to Norilsk Nickel peer group median (across all peer group regions) as H_0

Sample 2003-2009	LUKOIL	Gazprom OAO	Surgutnefteg az	MMC Norilsk Nickel	Tatneft	Surgutneftegaz Pref.	Gazpromneft OAO	Novolipetsk Steel
Std. p.a.	45.9%	46.4%	54.4%	53.3%	55.7%	49.6%	48.2%	67.6%
Skewness.	0.13	0.17	1.43	-1.16	0.67	1.22	-0.82	-0.73
Kurtosis	17.81	15.47	28.79	21.02	21.15	42.96	12.63	17.98
Jarque-Bera	14328	10162	43963	21559	21618	104632	6229	14789
JB P-Value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Observations tot.	1567	1567	1567	1567	1567	1567	1567	1567
Observations p.a.	261	261	261	261	261	261	261	261
H0 for daily st.dev.	2.75%	2.75%	2.75%	2.75%	2.75%	2.75%	2.75%	2.75%
Daily sample st.dev.	2.84%	2.87%	3.37%	3.30%	3.45%	3.07%	2.98%	4.18%
Chi-squared statistic	1671	1713	2348	2257	2462	1954	1845	3629
Variance Ratio P-Value	0.0326	0.0053	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Sample 2003-2008	LUKOIL	Gazprom OAO	Surgutnefteg az	MMC Norilsk Nickel	Tatneft	Surgutneftegaz Pref.	Gazpromneft OAO	Novolipetsk Steel
Std. p.a.	33.5%	35.6%	41.9%	42.6%	41.2%	38.7%	38.8%	45.1%
Skewness.	-0.15	-0.04	-0.24	-0.30	-0.10	-0.39	-0.70	0.20
Kurtosis	5.69	9.52	8.51	7.81	6.06	8.34	8.66	7.91
Jarque-Bera	399	2314	1664	1276	513	1585	1852	1319
JB P-Value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Observations tot.	1306	1306	1306	1306	1306	1306	1306	1306
Observations p.a.	261	261	261	261	261	261	261	261
H0 for daily st.dev.	2.25%	2.25%	2.25%	2.25%	2.25%	2.25%	2.25%	2.25%
Daily sample st.dev.	2.07%	2.21%	2.59%	2.64%	2.55%	2.40%	2.40%	2.79%
Chi-squared statistic	1103	1251	1726	1785	1674	1477	1485	2000
Variance Ratio P-Value	1.0000	0.8555	0.0000	0.0000	0.0000	0.0006	0.0004	0.0000

Calculation based on daily log-returns. The MICEX companies are selected based on their peer-group specification.

The higher test variance for the Norilsk sample leads to not rejecting H_0 for Lukoil and Gazprom (the chi-squared test statistic is below the required threshold) in the five-year sample.

6.2.2.4 Skewness test

Positive skewness in log-returns indicates that positive deviations from the mean occur with higher probability than in the standard normal distribution case. This, as a stand-alone argument, would indicate that investors look for assets that have a positively skewed return structure.²⁰⁸ The theoretical skewness assumption for equity log-normal returns, however, is typically that skewness equals zero. Nevertheless, the non-normality of log-returns is usually undisputed. This is especially true for companies

²⁰⁸ For a theoretical justification of this argument, see Subsection 3.3.3, p. 103 and in particular Equation (38), p. 105.

with high optionality in their cash-flows. As shown in the summary statistics in Table 62 to Table 67, all tested companies display positive or negative skewness depending on the time horizon used for the calculation.

As highlighted in Subsection 3.2.3, pp. 67-87, the return distribution of a single call is positively skewed. In order to find further confirmation of real-option pricing in Russian equity companies, we have conducted statistical tests to either confirm or dismiss the hypothesis that select Russian companies have positively skewed daily returns. We have again distinguished between two sample periods in order to capture the pre- and post-correction phase in Russian equities.

As shown in Subsubsection 3.4.5.2, p.127, the statistical test for skewness is formulated as a z-value test and compared with the tabulated values for a normal distribution. In this example we will test solely for positive skewness (since this is the real-option indicator), i.e., for skewness larger than zero. Skewness is depicted as $\sqrt{\beta_1}$.²⁰⁹ The test is formally described by:

$$\begin{aligned} H_0: \sqrt{\beta_1} &= 0 \\ H_1: \sqrt{\beta_1} &> 0 \end{aligned}$$

Since all tested peer group samples have an aggregated median skewness smaller than zero (see Table 56, p. 264, and Table 58, p. 266), the hypothesis testing for a higher skewness than zero is sufficient to allow us to come to the conclusion of positive skewness, if individually tested daily log-returns of Russian companies are significantly positively skewed and more positively (i.e., significantly more positively) skewed than their aggregated peer-group median.

²⁰⁹ Note that for estimation and inference on $\sqrt{\beta_1}$ we employ its sample statistic $\sqrt{b_1}$ as shown in Equation (70), p. 128.

Table 71
Testing for Gazprom, Lukoil and Norilsk Nickel against the hypothesis of positive skewness
of its daily log-returns

Panel A							
Sample (2003-2009)	LUKOIL	Gazprom OAO	Surgutneftegaz	MMC Norilsk Nickel	Tatneft	Surgutneftegaz Pref.	Gazpromneft OAO
m3	2.88E-06	4.11E-06	5.44E-05	-4.16E-05	2.73E-05	3.55E-05	-2.18E-05
g1	0.1258	0.1731	1.4282	-1.1587	0.6677	1.2255	-0.8223
sqrt(b1)	0.1257	0.1730	1.4269	-1.1576	0.6670	1.2243	-0.8215
Observations tot.	1'567	1'567	1'567	1'567	1'567	1'567	1'567
Observations p.a.	261	261	261	261	261	261	261
z-statistic	2.0324	2.7884	17.6052	-15.2542	9.8988	15.8703	-11.7593
P-Value	0.0211	0.0026	0.0000	1.0000	0.0000	0.0000	1.0000
Panel B							
Sample (2003-2008)	LUKOIL	Gazprom OAO	Surgutneftegaz	MMC Norilsk Nickel	Tatneft	Surgutneftegaz Pref.	Gazpromneft OAO
m3	-1.34E-06	-4.77E-07	-4.17E-06	-5.54E-06	-1.69E-06	-5.31E-06	-9.78E-06
g1	-0.1510	-0.0445	-0.2397	-0.3026	-0.1014	-0.3858	-0.7044
sqrt(b1)	-0.1508	-0.0444	-0.2394	-0.3022	-0.1013	-0.3853	-0.7036
Observations tot.	1'306	1'306	1'306	1'306	1'306	1'306	1'306
Observations p.a.	218	218	218	218	218	218	218
z-statistic	-2.2253	-0.6584	-3.5038	-4.3892	-1.4996	-5.5266	-9.4643
P-Value	0.9870	0.7449	0.9998	1.0000	0.9331	1.0000	1.0000

Calculation based on daily log-returns. The MICEX companies are selected based on their peer-group specification.

The statistical test shows for the 2003-2009 sample that most Russian oil and gas companies have a significantly higher skewness than zero and hence a significantly higher skewness than the median of their peer-group members (H_0 can be rejected at a 97.5% confidence level for Lukoil, Gazprom, Surgutneftegaz and Tatneft). The H_0 for Norilsk Nickel, Gazpromneft and Novolipetks Steel, however, cannot be rejected. Similarly H_0 cannot be rejected for either of the companies if the test is applied for the pre-crisis sample (2003-2008), where global commodity prices have soared and likely real-optionality was valued to be in-the-money.²¹⁰

It can be therefore concluded that – as measured between 2003-2009 – major Russian oil and gas producers have a statistically significant positive skewness, and that Lukoil and Gazprom in particular have a statistically significantly higher skewness than the median skewness of their international peer group members.

²¹⁰ The detailed test statistic calculations are given in Appendix K, pp. 427-429.

6.2.2.5 Kurtosis test

A return distribution that is characterized by a positive kurtosis above 3 (excess kurtosis) is referred to as a leptokurtic distribution and is regarded as further indication for non-normality of asset returns. As shown in Subsection 3.2.3, the return distribution of options is characterized by a high kurtosis. The kurtosis of the return distribution increases with the distance of the strike price, i.e., out-of-the-money options show a return distribution that is more leptokurtic than options that are at-the-money. This relation is the rationale for testing the kurtosis of the return distribution of Russian equity companies relative to other peer members.

The hypothesis test will reference the kurtosis population parameter β_2 for which b_2 is the estimate.²¹¹ The test hypothesis is formulated as a directional (one-tailed) hypothesis test either by γ_2 or by β_2 :

$$\begin{aligned} H_0: \gamma_2 &= 0, \beta_2 = 3 \\ H_1: \gamma_2 &> 0, \beta_2 > 3 \end{aligned}$$

The results for the tested Russian companies are given in Table 72. The statistical tests show for the 2003-2009 and for the 2003-2008 sample that all Russian oil and gas companies, and Norilsk Nickel as the only representative of the mining industry, have a significantly higher kurtosis than 3 (H_0 can be rejected at a 99% confidence level) indicating that all the tested Russian companies have significantly leptokurtic log-returns.

²¹¹ The expression for the β_2 sample statistic is given in Subsubsection 3.4.5.3, p. 129.

Table 72
Testing for Gazprom, Lukoil and Norilsk Nickel against the hypothesis of leptokurtic daily log-returns

Panel A							
Sample (2003-2009)	LUKOIL	Gazprom OAO	Surgutneftegaz	MMC Norilsk Nickel	Tatneft	Surgutneftegaz Pref.	Gazpromneft OAO
k4	9.65E-06	8.54E-06	3.32E-05	2.14E-05	2.57E-05	3.56E-05	7.65E-06
g2	14.8627	12.5145	25.8777	18.0847	18.2091	40.0884	9.6630
b2	17.8115	15.4708	28.7914	21.0232	21.1472	42.9567	12.6283
Observations tot.	1'567	1'567	1'567	1'567	1'567	1'567	1'567
Observations p.a.	261	261	261	261	261	261	261
z-statistic	18.6776	17.9037	20.9719	19.5251	19.5540	22.5608	16.6889
P-Value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Panel B							
Sample (2003-2008)	LUKOIL	Gazprom OAO	Surgutneftegaz	MMC Norilsk Nickel	Tatneft	Surgutneftegaz Pref.	Gazpromneft OAO
k4	4.98E-07	1.55E-06	2.49E-06	2.33E-06	1.31E-06	1.77E-06	1.90E-06
g2	2.7074	6.5496	5.5346	4.8282	3.0797	5.3661	5.6883
b2	5.6924	9.5200	8.5088	7.8051	6.0634	8.3409	8.6619
Observations tot.	1'306	1'306	1'306	1'306	1'306	1'306	1'306
Observations p.a.	218	218	218	218	218	218	218
z-statistic	9.3486	13.5348	12.7447	12.0969	9.9545	12.5985	12.8740
P-Value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Calculation based on daily log-returns. The MICEX companies are selected based on their peer-group specification.

In order to determine whether the kurtosis is higher than the median value of the corresponding peer-group sample, we have repeated the test with the following alternative hypothesis definition.

$$H_0: \beta_2 = 10.9$$

$$H_1: \beta_2 > 10.9$$

The threshold of $H_0: \beta_2 = 10.9$ for the kurtosis test is chosen from Table 56, since it is the most restrictive peer-group median value for the three tested core peer-groups in the 2003-2009 sample. The test is repeated with $H_0: \beta_2 = 6.6$ (the highest peer-group median for kurtosis among the tested 2003-2008 samples).

Table 73 shows that the H_0 hypothesis for an individual kurtosis value of 10.9 for the full 2003-2009 sample and $H_0: \beta_2 = 6.6$ for the 2003-2008 pre-crisis sample can be rejected for most of the tested Russian companies. For the six-year sample, one can confirm that all tested Russian companies have a statistically significant higher kurtosis than the corresponding peer-group median kurtosis value.

Table 73

Testing for Gazprom, Lukoil and Norilsk Nickel against the hypothesis of a kurtosis value $\beta_2 > 10.9$ for daily log-returns (sample 2003-2009) and $\beta_2 > 6.6$ (sample 2003-2008)

Panel A							
H0: 10.9	LUKOIL	Gazprom OAO	Surgutneftegaz	MMC Norilsk Nickel	Tatneft	Surgutneftegaz Pref.	Gazpromneft OAO
z-statistic	7.2476	5.2895	13.0322	9.4018	9.4752	16.8362	2.2841
P-Value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0112
Panel B							
H0: 6.6	LUKOIL	Gazprom OAO	Surgutneftegaz	MMC Norilsk Nickel	Tatneft	Surgutneftegaz Pref.	Gazpromneft OAO
z-statistic:	-1.4905	4.1514	2.8591	1.8678	-0.8810	2.6296	3.0646
P-Value	0.9320	0.0000	0.0021	0.0309	0.8108	0.0043	0.0011

Calculation based on daily log-returns. The MICEX companies are selected based on their peer-group specification.

Worth noticing, however, is that it is not possible to formulate a similar conclusion for the pre-crisis sample. Lukoil and Tatneft, although significantly leptokurtic, show a kurtosis value that is not statistically different from the sample corresponding peer group median.

6.2.3 Conditional (GARCH) distribution analysis

The analysis of the log-returns has thus far been unconditional, i.e., no specific assumptions about the DGP have been imposed other than that the log-returns are independent, which allowed us to calculate the sample statistics in Subsection 6.2.2. However, if this assumption is violated and indeed the data are driven by other, alternative DGPs, then the inference based on the sample statistics could be misleading.

We have therefore imposed some assumptions on the DGP for stock prices, by assuming that all tested equity companies are governed by a geometric Brownian motion that has a GJR-GARCH(1,1) controlled diffusion term (see Subsubsection 3.2.3.2, p. 68). The particular focus lies on the GJR-GARCH(1,1) diffusion characteristics.

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \gamma 1_{\{e_{t-1} < 0\}} u_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (162)^{212}$$

²¹² See Alexander (2008b), p. 150.

The parameterizations of the GJR-GARCH(1,1) specification allows for the calculation of the unconditional long-term variance of the stock price.²¹³ As pointed out by Verhoeven and McAleer (2003), the GJR-GARCH specification should allow inference about the distribution of the unconditional log-returns. A positive leverage factor (γ) indicates negatively skewed log-returns and vice versa.²¹⁴ This was also confirmed by the simulation example in Subsection 3.2.3 (see in particular Figure 40, Panel B, p. 78). In order to obtain further evidence on the pricing of real-option value in Russian equity companies that have exposure to the commodity sector, we will calibrate the above data generation process on the basis of the daily observable log-normal returns of the previously used Russian and global equity market sample.

We will additionally calculate the unconditional kurtosis for the GJR-GARCH(1,1). Based on the simulation examples in Section 3.2, one can expect that companies with high real-option value should display price returns that are more leptokurtic (show a higher kurtosis) than companies without real-options (or with in-the-money real-options). The Equation for calculating the unconditional GJR-GARCH(1,1) kurtosis values is given below.²¹⁵

$$b_2 = b_2^e \frac{1 - \left(\beta^2 + 2\alpha_1\beta + \beta\gamma + \alpha_1\gamma + \alpha_1^2 + \frac{1}{4}\gamma^2 \right)}{1 - \left(\beta^2 + 2\alpha_1\beta + \beta\gamma + b_2^e\alpha_1\gamma + b_2^e\alpha_1^2 + \frac{1}{2}b_2^e\gamma^2 \right)} \quad (163)$$

GJR – Garch(1,1) parameter: α_1, γ, β

Standardized error kurtosis: $b_2^e = \varepsilon_i / \sqrt{\sigma_i}$

The condition necessary for the existence of the unconditional fourth moment of the error component (ε_i) for the applied data generation process is given below.

²¹³ See Equation (23), p. 82.

²¹⁴ See Verhoeven and McAleer (2004), p. 352.

²¹⁵ See Verhoeven and McAleer (2004), p. 353.

$$\left(\beta^2 + 2\alpha_1\beta + \beta\gamma + b_2^e\alpha_1\gamma + b_2^e\alpha_1^2 + \frac{1}{2}b_2^e\gamma^2 \right) < 1 \quad (164)$$

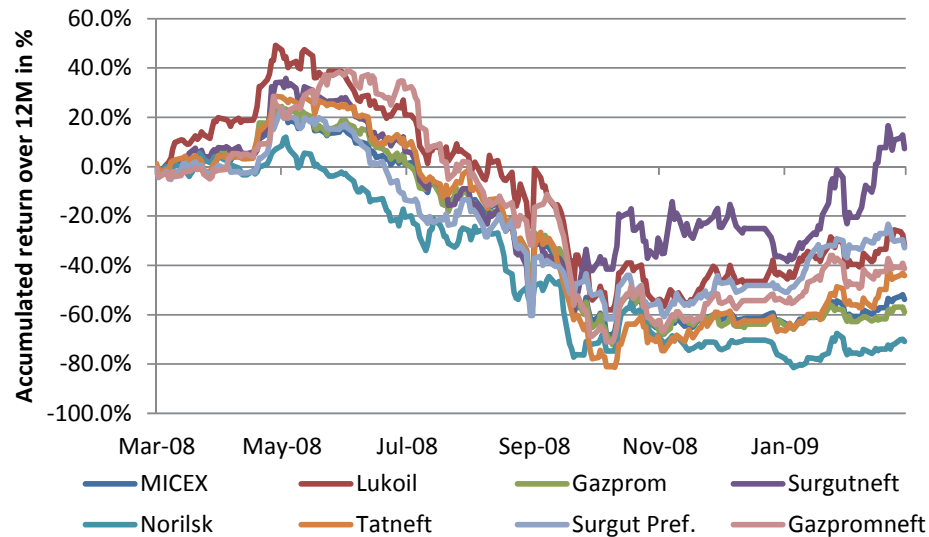
The unconditional second moment for a GJR-GARCH(1,1) process (see Equation (23), p. 82) is calculated as well. The necessary condition for a data set will be considered in the analysis only if both conditions – the existence of an unconditional second and fourth moment – are satisfied.

MICEX results:

Using the six-year sample period (i.e., including the full market crash in 2009), the stability condition for the unconditional kurtosis expressed in Equation (164) and/or the stability condition for the unconditional variance expressed in Equation (23)-a (see p. 82) are not fulfilled for most of the tested MICEX companies. The necessary condition of the existence of an unconditional kurtosis is especially invalidated for the sample period in which the MICEX Index lost more than 54% in only 12 months (between March 18th 2008 and March 18th 2009, which is the end of the sample period). The correction appears even more severe when looking at the individual names. Gazprom lost nearly 60%, while Lukoil lost around 32% (though with a maximum drawdown of 58% during the same period) between March 2008 and March 2009. Other peer group members performed similarly (see Figure 116). The mining sector was even harder hit, with Norilsk Nickel at –71%.

Figure 116

Absolute performance over 12 months of selected MICEX companies from March 2008 to March 2009

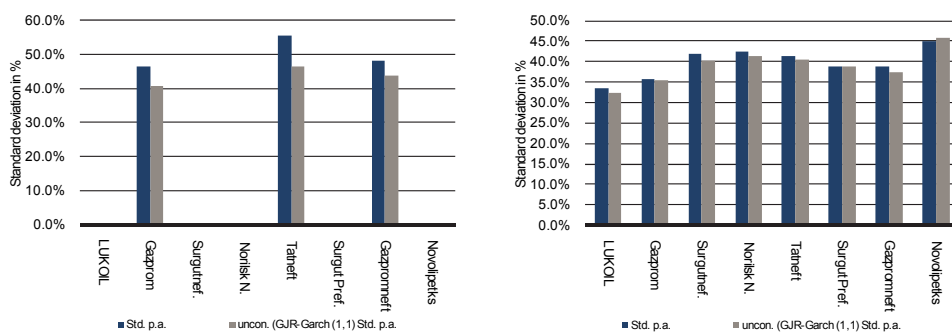


Data source: Moscow Stock Exchange

Such strong moves in the log-returns have produce GJR-GARCH(1,1) calibrations that are not stable. Figure 117, Panel A shows the MICEX members that have yielded stable results despite the exceptional market volatility. Panel B shows the estimated unconditional GJR-GARCH(1,1) standard deviation p.a. for the same companies, this time measured only between 2003-2008.

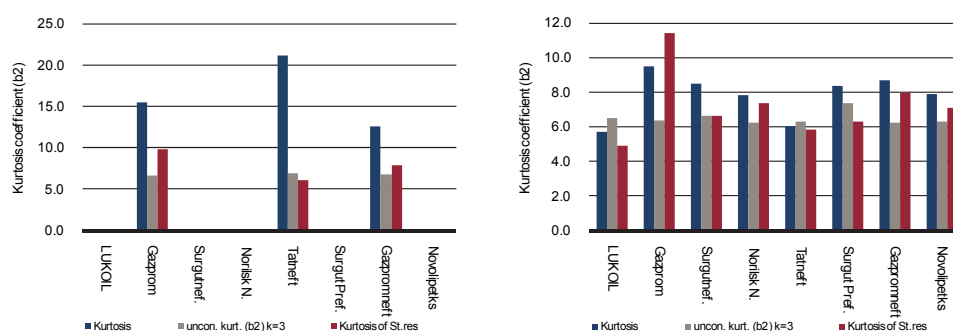
Figure 117

Unconditional GJR-GARCH(1,1) volatility in % p.a. and kurtosis for tested MICEX companies (calibration over five and six years)



Panel A: Standard deviation in % p.a. for companies with valid stability conditions for unconditional variance and kurtosis (2003-2009).

Panel B: Standard deviation in % p.a. for companies with valid stability conditions for unconditional variance and kurtosis (2003-2008).



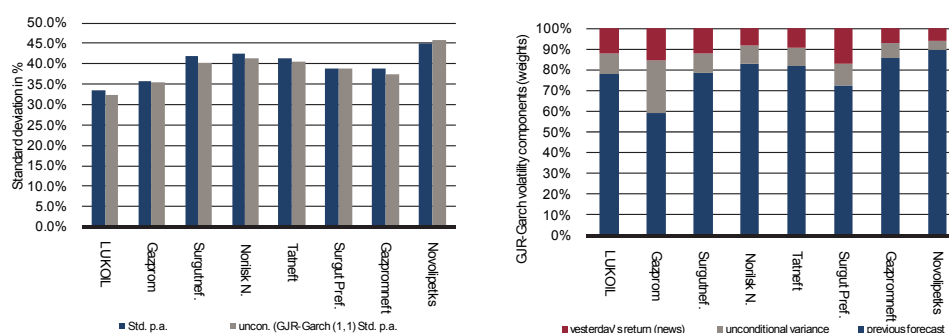
Panel C: Kurtosis for companies with valid stability conditions for unconditional variance and kurtosis (2003-2009).

Panel D: Kurtosis for companies with valid stability conditions for unconditional variance and kurtosis (2003-2008).

Overall the six-year sample produces unstable and hence unreliable GJR-GARCH(1,1) parameterizations. In those cases where the GJR-GARCH parametrization over six years was stable the remaining model kurtosis (i.e. the standardized residual kurtosis) stays high, highlighting the inability of the GJR-GARCH(1,1) model to explain the market return distribution.²¹⁶ We will hence limit the analysis to results generated with the five-year sample between March 2003 and 2008, where the model results are more uniform and stable.

Figure 118

Measures of standard deviation for selected MICEX companies – a GJR-GARCH(1,1) decomposition (2003-2008)



Panel A: The sample standard deviation together with the unconditional GJR-GARCH(1,1) estimates.

Panel B: Variance decomposition using the GJR-GARCH(1,1) parametrization as input variables.

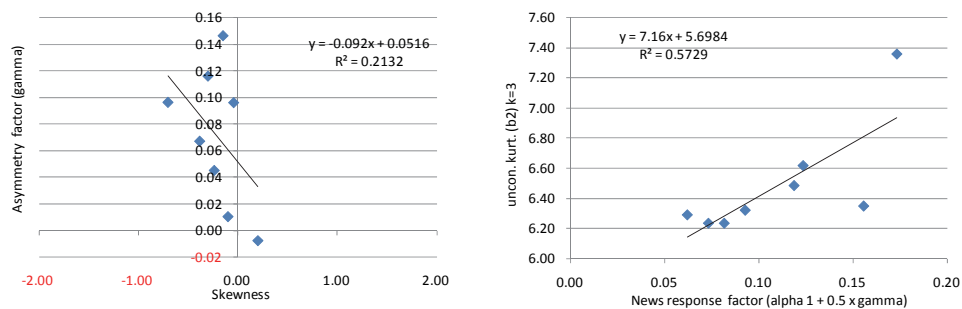
²¹⁶ See a QQ-plot analysis for the three stable GJR-GARCH(1,1) parametrizations in Appendix L, p. 429.

Figure 118 shows the observed sample standard deviation relative to the unconditional GJR-GARCH(1,1) estimate for volatility. Both are very close to each other, indicating that the model can capture the long-term volatility of the individual equity log-returns, which legitimizes the inference of the GJR-GARCH(1,1) model. The decomposition of the GJR-GARCH(1,1) volatility components unveils the fact that Gazprom has the lowest volatility persistency (i.e., the lowest dependency on the previous forecast) and the highest dependency on the unconditional variance within the MICEX sample. The responsiveness of volatility to yesterday's news (i.e., yesterday's returns) is also high for Gazprom relative to its MICEX counterparts.

As was highlighted before, the leverage factor within the GJR-GARCH(1,1) setup should reflect the extent of skewness (negative γ -factor for low positive skewness and vice versa) in log-returns. It could therefore be considered a proxy indicator of real-option pricing (companies with high real-option value should have positively skewed log-returns and hence a smaller GJR-GARCH(1,1) gamma parameterization than equity log-returns of companies without real-option value).

Figure 119

Relating GJR-GARCH(1,1) parameter estimates to distribution characteristics



Panel A: Skewness for log-returns of selected MICEX companies relative to GJR-GARCH asymmetry factors.

Panel B: GJR-GARCH news response factor and its relation to the unconditional GJR-GARCH kurtosis.

Figure 119, Panel A shows that this relation (positive skewness – low asymmetry factor) is indeed confirmed for the MICEX sample. Since high kurtosis is also described as real-option pricing indicator, we have also related the calculated unconditional GJR-GARCH(1,1) kurtosis to the news sensitivity factor. Panel B in Figure 119 shows how

the unconditional GJR-GARCH(1,1) kurtosis is functionally related to the news responsive factor—i.e., the higher the news responsiveness, the higher is the unconditional GJR-GARCH(1,1) kurtosis, and hence the higher is the indication that strong outliers will be observed, which we see as indication of real-option value pricing.

In order to compare the above described factors (unconditional GJR-GARCH(1,1) variance, kurtosis, leverage factor and news responsiveness) with the international peer-group, we have repeated the previous analysis shown in Subsection 6.2.2 (see p. 263) for all the different peer-group settings that have already been applied there. The analysis will again treat each tested company (Gazprom, Lukoil and Norilsk Nickel) within the context of its own peer group, differentiated only between the different regions.

Gazprom results

The summary results are shown in Table 74. In the used test sample (2003-2008) the sample estimate for the standard deviation lies above the international peer group. The sample estimate for the skewness and kurtosis are also above those of the international peer group but also above the values for the MICEX peer group, indicating further that the market is pricing real-option value for Gazprom.²¹⁷ The estimated sample standard deviation for the MICEX peer-group, however, is lower than for Gazprom.

²¹⁷ Note: the results are similar to those shown in Table 61, (see p. 268). However, the sample shown here is smaller due to the exclusion of companies that have shown unstable GJR-GARCH(1,1) parameterizations for the five- and six-year sample periods. Hence, the results in Subsection 6.2.2 and those presented here can deviate from each other.

Table 74

Summary of Gazprom peer group statistics together with the unconditional GJR-GARCH(1,1) standard deviation and kurtosis

Sample 2003-2009										
Central moments (stacked Gazprom peer data - ex Russia)										
	2nd	3rd	4th	Alpha 1	Gamma	Alpha 1 +0.5 Gamma	Beta 1	uncon. (GJR-Garch (1,1)) Std. p.a.	uncon. GJR-Garch (1,1) kurt. (b2)	uncon. GJR-Garch(1,1) kurt. (b2) k=St.res
Median	36.9%	-0.1300	9.7528	0.0469	0.0469	0.0724	0.9032	34.2%	7.0490	10.3776
Mean	36.2%	-0.0937	10.8778	0.0540	0.0496	0.0788	0.8799	33.3%	7.2330	33.9884
Sample 2003-2008										
Central moments (stacked Gazprom peer data - ex Russia)										
	2nd	3rd	4th	Alpha 1	Gamma	Alpha 1 +0.5 Gamma	Beta 1	uncon. (GJR-Garch (1,1)) Std. p.a.	uncon. kurt. (b2)	uncon. kurt. (b2) k=St.res
Median	29.9%	-0.1399	5.6505	0.0516	0.0277	0.0678	0.8802	31.4%	6.3113	6.9293
Mean	31.0%	-0.0090	7.4260	0.0569	0.0350	0.0743	0.8505	31.6%	6.4785	13.9522
Sample 2003-2009										
Central moments (stacked Gazprom peer data - Russia)										
	2nd	3rd	4th	Alpha 1	Gamma	Alpha 1 +0.5 Gamma	Beta 1	uncon. (GJR-Garch (1,1)) Std. p.a.	uncon. GJR-Garch (1,1) kurt. (b2)	uncon. GJR-Garch(1,1) kurt. (b2) k=St.res
Gazprom	46.4%	0.1730	15.4708	0.0478	0.0580	0.0768	0.8873	40.7%	6.5881	70.5893
Median Russia Peer-Group	#NV	#NV	#NV	0.0511	0.0767	0.0895	0.8640	#NV	#NV	70.5893
Mean Russia Peer-Group	#NV	#NV	#NV	0.0566	0.0763	0.0947	0.8655	#NV	#NV	67.6875
Sample 2003-2008										
Central moments (stacked Gazprom peer data - Russia)										
	2nd	3rd	4th	Alpha 1	Gamma	Alpha 1 +0.5 Gamma	Beta 1	uncon. (GJR-Garch (1,1)) Std. p.a.	uncon. kurt. (b2)	uncon. kurt. (b2) k=St.res
Gazprom	35.6%	-0.0444	9.5200	0.1072	0.0963	0.1554	0.5898	35.5%	6.3481	30.5601
Median Russia Peer-Group	40.0%	-0.1951	8.1239	0.0766	0.0818	0.1175	0.8020	39.6%	6.3343	12.7511
Mean Russia Peer-Group	38.3%	-0.2795	7.2795	0.0693	0.1101	0.1244	0.7778	37.6%	6.6919	23.6203

The Gazprom peer group is constructed as a stacked sample using companies from all international peer group regions. The particular regions are defined in Subsection 6.2.1, pp. 260-263. The Russian peer group contains only MICEX companies.

The GJR-GARCH(1,1)-based unconditional higher moments for the test sample confirm some of the results shown in Subsection 6.2.2. The GJR-GARCH(1,1) estimate for the unconditional standard deviation continues to lie above the global peer group but below the MICEX peer group median. The unconditional GJR-GARCH(1,1) kurtosis for Gazprom is slightly above the global peer group mean and median but below the MICEX mean.

A more detailed look at the different peer group results unveils the expected pattern of high skewness for developing market companies with the highest mean and median value for Asia - Emerging, Latin America and Eastern Europe. The sample kurtosis and the standardized residual kurtosis is nearly twice as high for the developing peer group and supports the argument of real-option pricing (see Table 75) in those markets.

Table 75

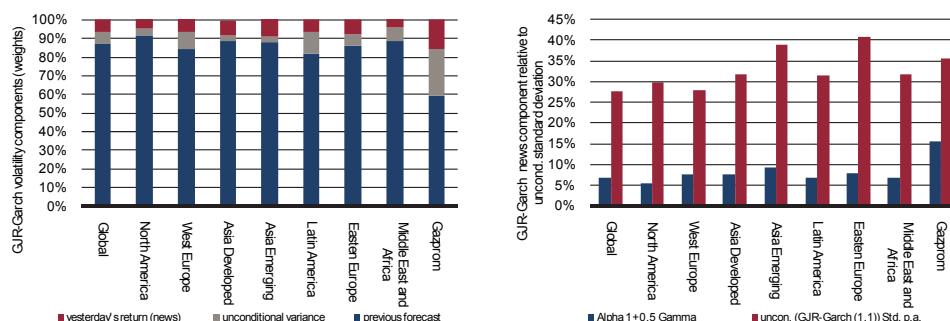
Summary of Gazprom peer group statistics across regions together with the unconditional GJR-GARCH(1,1) standard deviation and kurtosis

Mean (2003-2008)										
Region (ex-Russia)	Alpha 1	Gamma	Alpha 1+0.5 Gamma	Beta 1	uncon. (GJR-Garch (1,1)) Std. p.a.	Std. p.a.	Skewness	Kurtosis	uncon. kurt. (b2) GJR-Garch(1,1)	Kurtosis of St.res GJR-Garch(1,1)
Global	0.0377	0.0589	0.0671	0.8813	26.9%	0.2642	-0.1830	5.2025	6.5557	4.2894
North America	0.0436	0.0283	0.0577	0.8877	28.3%	0.2715	-0.1693	4.3162	6.5376	3.9368
West Europe	0.0466	0.0692	0.0812	0.8456	26.5%	0.2639	-0.1238	5.9007	6.3919	5.0458
Asia Developed	0.0572	0.0328	0.0736	0.8751	32.3%	0.3184	0.0425	6.8065	6.5652	6.5526
Asia Emerging	0.0768	0.0246	0.0891	0.8435	38.3%	0.3789	0.0289	7.3824	6.5805	7.2914
Latin America	0.0802	0.0151	0.0877	0.7470	34.3%	0.3405	0.1072	11.2365	6.3810	10.6492
Easten Europe	0.0662	0.0297	0.0810	0.8100	41.9%	0.4066	0.4850	14.7413	6.4011	14.5202
Middle East and Africa	0.0787	-0.0121	0.0727	0.8601	32.2%	0.3165	0.0028	8.3862	6.3509	7.9121
Gazprom	0.1072	0.0963	0.1554	0.5898	0.3555	0.3564	-0.0444	9.5200	6.3481	11.4282
Median (2003-2008)										
Region (ex-Russia)	Alpha 1	Gamma	Alpha 1+0.5 Gamma	Beta 1	uncon. (GJR-Garch (1,1)) Std. p.a.	Std. p.a.	Skewness	Kurtosis	uncon. kurt. (b2) GJR-Garch(1,1)	Kurtosis of St.res GJR-Garch(1,1)
Global	0.0289	0.0704	0.0668	0.8738	27.5%	0.2705	-0.2256	5.0878	6.2950	4.0774
North America	0.0364	0.0174	0.0543	0.9183	29.8%	0.2832	-0.2030	3.9285	6.2792	3.7162
West Europe	0.0351	0.0635	0.0772	0.8439	27.9%	0.2787	-0.1400	5.1748	6.2776	4.9917
Asia Developed	0.0616	0.0379	0.0747	0.8869	31.6%	0.3137	-0.1399	5.6389	6.4332	4.7569
Asia Emerging	0.0770	0.0199	0.0929	0.8791	38.8%	0.3939	0.1190	7.3090	6.4435	6.3182
Latin America	0.0608	0.0129	0.0672	0.8185	31.6%	0.3210	0.1134	8.9507	6.2167	7.9091
Easten Europe	0.0663	0.0193	0.0775	0.8631	40.9%	0.4078	0.0095	7.2719	6.4305	6.1658
Middle East and Africa	0.0494	-0.0199	0.0673	0.8846	31.9%	0.3120	-0.0984	8.2912	6.2797	7.5419
Gazprom	0.1072	0.0963	0.1554	0.5898	35.5%	0.3564	-0.0444	9.5200	6.3481	11.4282
Mean (2003-2008)										
developed	0.0491	0.0434	0.0709	0.8695	29.0%	28.5%	-0.0835	5.6745	6.4982	5.1784
developing	0.0755	0.0143	0.0826	0.8152	36.7%	36.1%	0.1560	10.4366	6.4284	10.0932
Grouped GJR-Garch(1,1) median parameters (stacked Gazprom peer data - ex Russia)										
Median (2003-2008)	Alpha 1	Gamma	Alpha 1+0.5 Gamma	Beta 1	uncon. (GJR-Garch (1,1)) Std. p.a.	Std. p.a.	Skewness	Kurtosis	uncon. kurt. (b2) GJR-Garch(1,1)	Kurtosis of St.res GJR-Garch(1,1)
developed (ex-Russia)	0.0443	0.0396	0.0687	0.8830	29.8%	29.2%	-0.1610	4.9141	6.3300	4.4883
developing (ex-Russia)	0.0634	0.0080	0.0762	0.8613	35.8%	35.9%	0.0359	7.9557	6.3426	6.9838

The Gazprom peer group is constructed as a stacked sample using companies from all international peer group regions. The particular regions are defined in Subsection 6.2.1, pp. 260-263. The Russian peer group contains only MICEX companies.

Figure 120

A GJR-GARCH(1,1) decomposition of the unconditional volatility for Gazprom and its global peer group (2003-2008)



Panel A: Variance decomposition using the GJR-GARCH(1,1) parametrization as input variables.

Panel B: Unconditional GJR-GARCH(1,1) volatility estimate together with its news response component.

The persistence of a volatility shock is lowest for Gazprom relative to the different peer group sub-samples (see Figure 120, Panel A). The same is true for the news sensitivity factor (see Figure 120, Panel B). The sensitivity for news is highest for Gazprom, which explains the relatively high unconditional GJR-GARCH(1,1) volatility.

With respect to kurtosis and skewness, it can be observed that the results in Table 75 do not totally follow the expectations for real-option pricing formulated in Subsubsection 3.2.3.3, p. 74. The observed kurtosis is relatively high (as measured on a sample estimate basis and on a GJR-GARCH(1,1) unconditional estimate basis) and confirms our expectation for companies with higher out-of-the-money real-option value. Other measures (GJR-GARCH(1,1) unconditional kurtosis and the estimated kurtosis of the standardized GJR-GARCH(1,1) residuals) partially confirm this result. It is worth highlighting that the unconditional GJR-GARCH(1,1) kurtosis is very uniform for all the tested peer-groups, i.e., the unconditional GJR-GARCH(1,1) kurtosis for Gazprom is nearly identical to the median of the tested peer-group values. Hence, after inducing a data generating process that ought to better fit the non-normal nature of stock market returns, the differential in the kurtosis values disappears, at least if measured along the calculated unconditional GJR-GARCH(1,1) kurtosis.

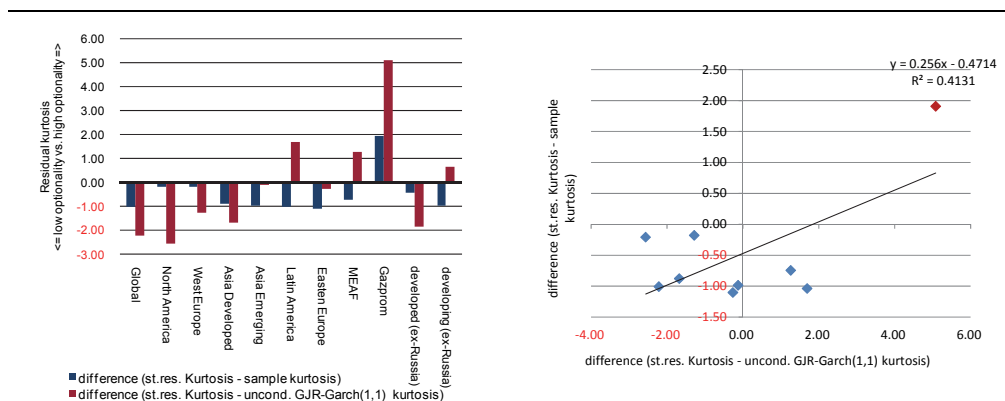
However, this is not the case if one looks at the kurtosis of the standardized GJR-GARCH(1,1) residuals. If the GJR-GARCH(1,1) data generation process with standard

normal distributed error terms is the true underlying data process, the standardized residuals should have a value of three (all the non-normality in the form of excess kurtosis is captured by the GJR-GARCH term, the remaining standardized residuals are $N(0,1)$ distributed). Any deviation from this value would be an indication that the process chosen is not capturing the full dynamics of the underlying equity price. In the case of real-option value in companies – which should increase the non-normality of the diffusion term – such a deviation can be expected to be large and positive (i.e., the GJR-GARCH(1,1) model cannot capture all the excess kurtosis-implied fat tails). The extraordinarily high standardized residual kurtosis for Gazprom seems to confirm this view.

The results in Figure 121, Panel A confirm that after the calibration of the GJR-GARCH(1,1) DGP the standardized residual for Gazprom continues to be positive. The difference between the standardized GJR-GARCH(1,1) diffusion kurtosis and the sample kurtosis (both are positively correlated in this case, see Panel B) confirms this general result. Hence, despite using an asymmetric DGP that should capture fat tails better than conventional random walk or GARCH(1,1) processes, a considerable amount of excess kurtosis exists, indicating that much higher fat-tail information (our indicator for real-option value) is priced for Gazprom.

Figure 121

Explained vs. unexplained kurtosis for Gazprom (2003-2008 sample)



Panel A: The difference between the standardized residual kurtosis and the unconditional or sample kurtosis as real-option valuation indicator.

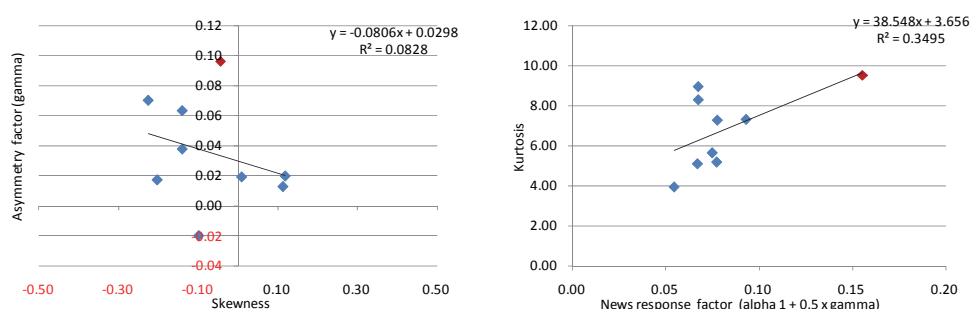
Panel B: Kurtosis measured as sample statistic seems to capture already to a large extent the proper size of leptokurtic log-returns behavior.

A further indication of real-option pricing in markets is observed beyond Gazprom for the peer group of the Middle East and Africa, Eastern Europe and Latin America together with Asia - Emerging (regions where one could expect higher real-option value). The values for the different peer groups (also grouped together under the label “developing” vs. “developed” in Table 75) are higher than for the Western European or North American sample. Irrespective of the chosen measure (sample kurtosis of log-normal returns or standardized residual GJR-GARCH(1,1) kurtosis) the results indicate that global companies and companies of developed countries in particular have lesser fat tails, which is likely a consequence of the lower real-option value of these companies.

The skewness characteristic of Gazprom and the global peer group is tested implicitly via the asymmetry factor γ , since a higher γ value should correspond to a lower skewness and vice versa. The stacked peer group regression in Figure 122, Panel A confirms this relation, although with a low degree of significance. Although the skewness for Gazprom was tested to be significantly above the peer group mean and median (see Subsubsection 6.2.2.3, Table 68, p. 277), the gamma factor that results from the GJR-GARCH(1,1) parameterization is – contrary to the formulated expectation – higher than each peer group median.

Figure 122

Relating higher sample moments with GJR-GARCH(1,1) parameters for the Gazprom peer group sample (2003-2008 sample)



Panel A: Asymmetry factor vs. skewness based on the values shown in Table 75.

Panel B: Kurtosis as function of the news response factor.

The news response factor for Gazprom, however, corresponds to the expectation formulated for the real-option value (high news sensitivity as indicator for high OTM

real-option exposure, see Subsection 3.2.3, Figure 45 and Figure 46, p. 85). Panel B in Figure 122 shows that the news response factor is highly correlated with the estimated sample kurtosis.

Lukoil result:

The results (see Table 76) for Lukoil and the respective peer groups do not look as uniform as in the Gazprom case. Lukoil has, similar to Gazprom, a higher estimated sample standard deviation and a higher unconditional GJR-GARCH(1,1) standard deviation than the global peer group, but a lower value than the MICEX peer group members. The biggest difference between the two test samples is visible for higher moments. Lukoil has a higher negative skewness than Gazprom or the Lukoil peer group members. Its kurtosis is smaller than for Gazprom and similar to the aggregated peer group data. Hence, at first glance the expected features for real-option pricing evidence seem to be missing for Lukoil.

The GJR-GARCH(1,1) parameterization for the Lukoil example shows – in line with the sample estimate for skewness – a relatively high gamma value (nearly five times the value of the peer group median), indicating a high price sensitivity towards negative news. The unconditional GJR-GARCH(1,1) kurtosis is similarly uniform across the tested peer group members and nearly identical to the value calculated for Lukoil.

Looking at the individual peer group regions in Table 77 confirms the analysis of the unconditional sample estimates. The Lukoil data show the highest news sensitivity parameterization across all aggregated peer group median values (as driven primarily by the high gamma value). Its unconditional GJR-GARCH(1,1) standard deviation is below the value for developing countries. The same is true for the skewness and kurtosis. Most striking is the kurtosis value of the GJR-GARCH(1,1) standardized residual, which again confirms that Lukoil should be referred to as a developed country company rather than an emerging country company.

Table 76

Summary of Lukoil peer group statistics together with the unconditional GJR-GARCH(1,1) standard deviation and kurtosis

Sample 2003-2009										
Central moments (stacked Lukoil peer data - ex Russia)										
	2nd	3rd	4th	Alpha 1	Gamma	Alpha 1 +0.5 Gamma	Beta 1	uncon. (GJR-Garch (1,1)) Std. p.a.	uncon. GJR-Garch (1,1) kurt. (b2)	uncon. GJR-Garch(1,1) kurt. (b2) k=St.res
Median	38.4%	-0.2367	9.1733	0.0521	0.0410	0.0724	0.9025	34.7%	6.8164	10.7694
Mean	37.5%	-0.2135	10.0776	0.0650	0.0357	0.0829	0.8563	35.2%	7.1177	16.0449
Sample 2003-2008										
Central moments (stacked Lukoil peer data - ex Russia)										
	2nd	3rd	4th	Alpha 1	Gamma	Alpha 1 +0.5 Gamma	Beta 1	uncon. (GJR-Garch (1,1)) Std. p.a.	uncon. GJR-Garch (1,1) kurt. (b2)	uncon. GJR-Garch(1,1) kurt. (b2) k=St.res
Median	30.2%	-0.0947	5.6589	0.0560	0.0245	0.0709	0.8822	31.1%	6.3170	7.1736
Mean	30.9%	-0.0593	6.5337	0.0614	0.0302	0.0765	0.8500	31.5%	6.4945	12.8706
Sample 2003-2009										
Central moments (stacked Lukoil peer data - Russia)										
	2nd	3rd	4th	Alpha 1	Gamma	Alpha 1 +0.5 Gamma	Beta 1	uncon. (GJR-Garch (1,1)) Std. p.a.	uncon. GJR-Garch (1,1) kurt. (b2)	uncon. GJR-Garch(1,1) kurt. (b2) k=St.res
LUKOIL	45.9%	0.1257	17.8115	0.0545	0.1006	0.1048	0.8625	39.6%	7.5332	#NV
Median Russia Peer-Group	47.3%	0.1493	16.6411	0.0511	0.0767	0.0895	0.8640	42.2%	6.8303	70.5893
Mean Russia Peer-Group	49.0%	0.0360	16.7645	0.0566	0.0763	0.0947	0.8655	42.6%	6.9455	67.6875
Sample 2003-2008										
Central moments (stacked Lukoil peer data - Russia)										
	2nd	3rd	4th	Alpha 1	Gamma	Alpha 1 +0.5 Gamma	Beta 1	uncon. (GJR-Garch (1,1)) Std. p.a.	uncon. GJR-Garch (1,1) kurt. (b2)	uncon. GJR-Garch(1,1) kurt. (b2) k=St.res
LUKOIL	33.5%	-0.1508	5.6924	0.0452	0.1467	0.1185	0.7781	32.4%	6.4842	8.3421
Median Russia Peer-Group	40.0%	-0.1951	8.1239	0.0766	0.0818	0.1175	0.8020	39.6%	6.3343	12.7511
Mean Russia Peer-Group	38.3%	-0.2795	7.2795	0.0693	0.1101	0.1244	0.7778	37.6%	6.6919	23.6203

The Lukoil peer group is constructed as a stacked sample using companies from all international peer group regions. The particular regions are defined in Subsection 6.2.1, pp. 260-263. The Russian peer group contains only MICEX companies.

Table 77

Summary of Lukoil peer group statistics across regions together with the unconditional GJR-GARCH(1,1) standard deviation and kurtosis

Mean (2003-2008)										
Region (ex-Russia)	Alpha 1	Gamma	Alpha 1+0.5 Gamma	Beta 1	uncon. (GJR-Garch (1,1)) Std. p.a.	Std. p.a.	Skewness	Kurtosis	uncon. GJR-Garch (1,1) kurt. (b2)	Kurtosis of St.res GJR-Garch(1,1)
Global	0.0347	0.0539	0.0617	0.8893	24.3%	0.2350	-0.2191	4.8455	6.5771	4.1771
North America	0.0473	0.0253	0.0600	0.8837	28.6%	0.2747	-0.1895	4.5488	6.5479	4.2517
West Europe	0.0544	0.0604	0.0846	0.8383	29.1%	0.2880	-0.0784	6.6033	6.4368	5.7677
Asia Developed	0.0646	0.0328	0.0810	0.8604	32.3%	0.3184	0.0425	6.8065	6.5652	6.5526
Asia Emerging	0.0745	0.0283	0.0887	0.8267	37.9%	0.3740	0.0146	7.4899	6.4846	7.6181
Latin America	0.0842	0.0198	0.0941	0.7436	35.6%	0.3499	-0.0114	9.5209	6.4179	8.0563
Easten Europe	0.0854	0.0232	0.0970	0.8449	37.1%	0.3703	0.1444	6.3480	6.5506	5.4983
Middle East and Africa	0.0787	-0.0121	0.0727	0.8464	32.2%	0.3165	0.0028	8.3862	6.3509	7.9121
LUKOIL	0.0452	0.1467	0.1185	0.7781	0.3244	0.3347	-0.1508	5.6924	6.4842	4.8931
Median (2003-2008)										
Region (ex-Russia)	Alpha 1	Gamma	Alpha 1+0.5 Gamma	Beta 1	uncon. (GJR-Garch (1,1)) Std. p.a.	Std. p.a.	Skewness	Kurtosis	uncon. GJR-Garch (1,1) kurt. (b2)	Kurtosis of St.res GJR-Garch(1,1)
Global	0.0297	0.0529	0.0664	0.8674	20.1%	0.2039	-0.1956	4.7373	6.2094	3.9068
North America	0.0406	0.0160	0.0572	0.9183	29.8%	0.2832	-0.2030	3.9746	6.2808	3.7173
West Europe	0.0548	0.0468	0.0888	0.8345	28.9%	0.2860	-0.0117	6.1643	6.2974	5.2527
Asia Developed	0.0616	0.0379	0.0747	0.8869	31.6%	0.3137	-0.1399	5.6389	6.4332	4.7569
Asia Emerging	0.0718	0.0207	0.0871	0.8417	38.3%	0.3835	0.1167	7.3590	6.4421	7.1277
Latin America	0.0635	0.0172	0.0677	0.7831	32.3%	0.3263	0.0616	8.7490	6.2463	6.0170
Easten Europe	0.0828	0.0193	0.0891	0.8690	38.2%	0.3771	0.0728	5.7458	6.5826	4.8115
Middle East and Africa	0.0494	-0.0199	0.0673	0.8846	31.9%	0.3120	-0.0984	8.2912	6.2797	7.5419
LUKOIL	0.0452	0.1467	0.1185	0.7781	32.4%	0.3347	-0.1508	5.6924	6.4842	4.8931
Mean (2003-2008)										
developed (ex-Russia)	0.0554	0.0395	0.0752	0.8608	30.0%	29.4%	-0.0751	5.9862	6.5166	5.5240
developing (ex-Russia)	0.0807	0.0148	0.0881	0.8154	35.7%	35.3%	0.0376	7.9363	6.4510	7.2712
Grouped GJR-Garch(1,1) median parameters (stacked Lukoil peer data - ex Russia)										
Median (2003-2008)	Alpha 1	Gamma	Alpha 1+0.5 Gamma	Beta 1	uncon. (GJR-Garch (1,1)) Std. p.a	Std. p.a.	Skewness	Kurtosis	uncon. GJR-Garch (1,1) kurt. (b2)	Kurtosis of St.res GJR-Garch(1,1)
developed (ex-Russia)	0.0523	0.0335	0.0736	0.8799	30.1%	29.4%	-0.1182	5.2592	6.3371	4.5756
developing (ex-Russia)	0.0669	0.0093	0.0778	0.8446	35.2%	35.0%	0.0382	7.5362	6.3877	6.3745

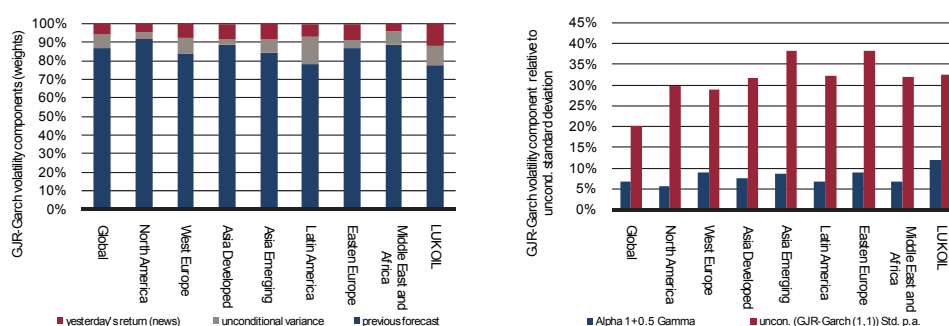
The Lukoil peer group is constructed as a stacked sample using companies from all international peer group regions. The particular regions are defined in Subsection 6.2.1, pp. 260-263. The Russian peer group contains only MICEX companies.

Peer groups that would also qualify as real-option valuation regions (Asia - Emerging, Latin America and Middle East and Africa) show the expected pattern of high kurtosis (measured as sample estimate or as GJR-GARCH(1,1) standardized residual analysis), above average skewness and lower GJR-GARCH(1,1) gamma value.

The GJR-GARCH(1,1) volatility components are illustrated in Figure 123, Panel A. Lukoil shows a relatively low volatility persistence and high news dependency. Panel B of Figure 123 shows the news component in absolute terms along with the unconditional GJR-GARCH(1,1) standard deviation. Both values are relatively high compared to the aggregated peer group data.

Figure 123

A GJR-GARCH(1,1) decomposition of the unconditional volatility for Gazprom and its global peer group (2003-2008)



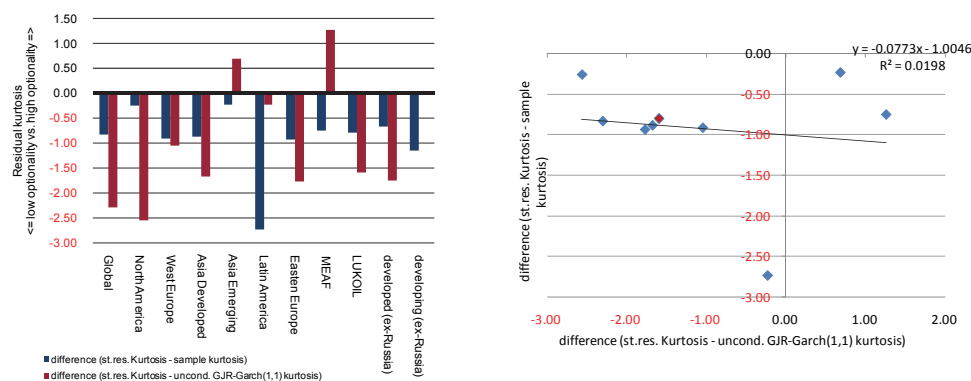
Panel A: Variance decomposition using the GJR-GARCH(1,1) parameterization as input variables.

Panel B: Unconditional GJR-GARCH(1,1) volatility estimate together with its news response component.

The relation between the observed sample kurtosis and the GJR-GARCH(1,1) kurtosis is relatively weak (see Figure 124, Panel B compared to Figure 121 for the Gazprom case). Panel A of Figure 124 confirms again that, based on this measure, low real-option value pricing is observed in Lukoil equity prices.

Figure 124

Explained vs. unexplained kurtosis for Lukoil (2003-2008 sample)

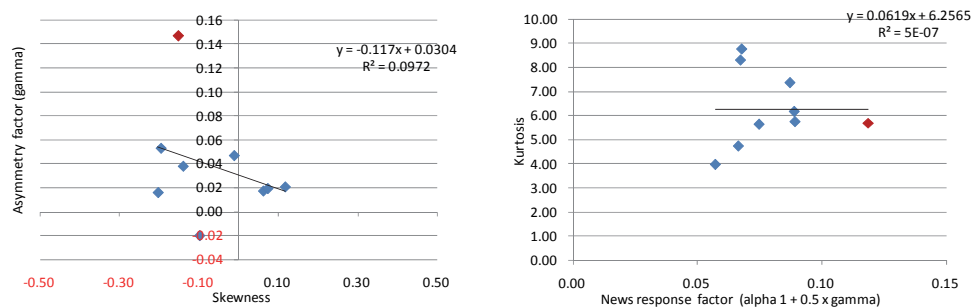


Panel A: The difference between the standardized residual kurtosis and the unconditional or sample kurtosis as real-option valuation indicator.

Panel B: Kurtosis measured as sample statistic seems to capture to a large extent the proper size of leptokurtic log-return behavior.

Figure 125

Relating higher sample moments with GJR-GARCH(1,1) parameters for the Lukoil peer group sample (2003-2008 sample)



Panel A: Asymmetry factor vs. skewness based on the values shown in Table 77.

Panel B: Kurtosis as function of the news response factor.

The inference about the relation between skewness and news asymmetry is limited, as is the case for the relation between the news response factor and the unconditional sample kurtosis.

Overall none of the expected characteristics (except for the unconditional GJR-GARCH(1,1) standard deviation) for real-option pricing formulated in Subsection 3.2.3 have been observed for Lukoil in the tested sample period. Based on this set of data, Lukoil belongs to the peer group of developed market energy companies or has large negative contingencies that weigh negatively on the price discovery.

Norilsk Nickel result:

Norilsk Nickel represents the only non-oil and gas company in the tested Russian company sample. Table 78 shows the summary results for Norilsk Nickel relative to the global peer group. The estimated higher sample moments are – with the exception of skewness – higher than the median aggregate values for the global peer group. The unconditional GJR-GARCH(1,1) standard deviation is higher as well. The unconditional GJR-GARCH(1,1) kurtosis value, however, is lower.

The GJR-GARCH(1,1) parameter estimates highlight – similar to Lukoil and Gazprom – a more pronounced asymmetry value and a higher volatility persistence. Compared to the peer group sub-samples (see Table 79), Norilsk Nickel matches the expected characteristics only partially. The unconditional GJR-GARCH(1,1) standard deviation and standardized residual kurtosis are above grouped values for developed and developing countries. The asymmetry factor (gamma) for the GJR-GARCH(1,1) setup, however, is again higher than the median value for developed and developing regions. This is confirmed by looking at the sample skewness estimate, where Norilsk is more negatively skewed than most of the tested companies.

Table 78

Summary of Norilsk Nickel peer group statistics together with the unconditional GJR-GARCH(1,1) standard deviation and kurtosis

Sample 2003-2009										
Central moments (stacked Norilsk Nickel peer data - ex Russia)										
	2nd	3rd	4th	Alpha 1	Gamma	Alpha 1 +0.5 Gamma	Beta 1	uncon. (GJR-Garch (1,1)) Std. p.a.	uncon. GJR-Garch (1,1) kurt. (b2)	uncon. GJR-Garch(1,1) kurt. (b2) k=St.res
Median	44.8%	-0.2267	8.7954	0.0349	0.0411	0.0588	0.9211	42.2%	6.7557	11.0016
Mean	43.7%	-0.2296	9.4173	0.0383	0.0458	0.0612	0.9096	41.9%	6.9626	27.1586
Sample 2003-2008										
Central moments (stacked Norilsk Nickel peer data - ex Russia)										
	2nd	3rd	4th	Alpha 1	Gamma	Alpha 1 +0.5 Gamma	Beta 1	uncon. (GJR-Garch (1,1)) Std. p.a.	uncon. GJR-Garch (1,1) kurt. (b2)	uncon. GJR-Garch(1,1) kurt. (b2) k=St.res
Median	36.7%	-0.0513	5.4429	0.0433	0.0251	0.0655	0.9020	36.7%	6.3355	7.6792
Mean	35.7%	-0.0821	6.5536	0.0478	0.0321	0.0639	0.8842	36.6%	6.5374	15.4794
Sample 2003-2009										
Central moments (stacked Norilsk Nickel peer data - Russia)										
	2nd	3rd	4th	Alpha 1	Gamma	Alpha 1 +0.5 Gamma	Beta 1	uncon. (GJR-Garch (1,1)) Std. p.a.	uncon. GJR-Garch (1,1) kurt. (b2)	uncon. GJR-Garch(1,1) kurt. (b2) k=St.res
Norilsk N.	--	--	--	0.0484	0.1276	0.1122	0.8323	--	--	--
Median Russia Peer-Group	--	--	--	0.0466	0.0720	0.0826	0.8857	--	--	--
Mean Russia Peer-Group	--	--	--	0.0310	0.0480	0.0550	0.5905	--	--	--
Sample 2003-2008										
Central moments (stacked Norilsk Nickel peer data - Russia)										
	2nd	3rd	4th	Alpha 1	Gamma	Alpha 1 +0.5 Gamma	Beta 1	uncon. (GJR-Garch (1,1)) Std. p.a.	uncon. GJR-Garch (1,1) kurt. (b2)	uncon. GJR-Garch(1,1) kurt. (b2) k=St.res
Norilsk N.	42.6%	-0.3022	7.8051	0.0234	0.1163	0.0816	0.8282	41.5%	6.2339	12.1026
Median Russia Peer-Group	43.8%	-0.0517	7.8560	0.0446	0.0544	0.0718	0.8620	43.7%	6.2617	11.0359
Mean Russia Peer-Group	29.2%	-0.0345	5.2373	0.0297	0.0363	0.0478	0.5746	29.1%	4.1745	7.3573

The Norilsk Nickel peer group is constructed as a stacked sample using companies from all international peer group regions. The particular regions are defined in Subsection 6.2.1, pp. 260-263. The Russian peer group contains only MICEX companies.

Table 79

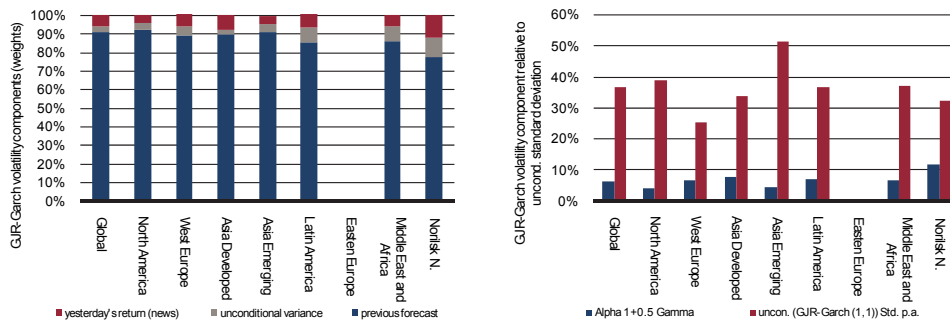
Summary of Norilsk Nickel peer group statistics across regions together with the unconditional GJR-GARCH(1,1) standard deviation and kurtosis

Mean (2003-2008)										
Region (ex-Russia)	Alpha 1	Gamma	Alpha 1+0.5 Gamma	Beta 1	uncon. (GJR-Garch (1,1)) Std. p.a.	Std. p.a.	Skewness	Kurtosis	uncon. GJR-Garch (1,1) kurt. (b2)	Kurtosis of St.res GJR-Garch(1,1)
Global	0.0457	0.0360	0.0637	0.9150	37.7%	0.3644	-0.0334	4.8559	7.1582	3.8456
North America	0.0269	0.0407	0.0472	0.9126	36.2%	0.3636	-0.1451	6.1494	6.3320	4.7632
West Europe	0.0433	0.0476	0.0671	0.8843	30.0%	0.3023	-0.0611	6.7024	6.4029	6.5922
Asia Developed	0.0643	0.0251	0.0769	0.8863	33.2%	0.3213	-0.1876	6.2130	6.9136	6.0655
Asia Emerging	0.0461	0.0121	0.0522	0.8712	52.6%	0.4741	-0.1420	8.7351	6.6042	8.6307
Latin America	0.0526	0.0508	0.0780	0.8390	34.6%	0.3403	-0.0766	5.7892	6.3781	5.5914
Easten Europe	--	--	--	--	--	--	--	--	--	--
Middle East and Africa	0.0588	0.0113	0.0645	0.8742	37.9%	0.3775	0.0859	7.1820	6.3152	7.1053
Norilsk N.	0.0234	0.1163	0.0816	0.8282	0.4148	0.4257	-0.3022	7.8051	6.2339	7.3749
Median (2003-2008)										
Region (ex-Russia)	Alpha 1	Gamma	Alpha 1+0.5 Gamma	Beta 1	uncon. (GJR-Garch (1,1)) Std. p.a.	Std. p.a.	Skewness	Kurtosis	uncon. GJR-Garch (1,1) kurt. (b2)	Kurtosis of St.res GJR-Garch(1,1)
Global	0.0404	0.0400	0.0625	0.9096	36.7%	0.3705	-0.0334	4.4868	6.8549	3.7759
North America	0.0234	0.0344	0.0418	0.9216	38.8%	0.3794	-0.0805	4.6660	6.2094	4.5698
West Europe	0.0464	0.0272	0.0674	0.8912	25.5%	0.2661	-0.0513	5.9379	6.3498	5.5630
Asia Developed	0.0639	0.0227	0.0774	0.9008	33.8%	0.3331	-0.0618	5.4098	6.8654	4.3479
Asia Emerging	0.0470	0.0003	0.0453	0.9094	51.4%	0.4755	0.0232	6.4124	6.5330	6.3682
Latin America	0.0351	0.0589	0.0717	0.8533	36.6%	0.3644	-0.0185	5.8896	6.2735	4.8641
Easten Europe										
Middle East and Africa	0.0540	0.0136	0.0655	0.8634	37.1%	0.3685	-0.0046	5.8477	6.2202	5.3674
Norilsk N.	0.0234	0.1163	0.0816	0.8282	41.5%	0.4257	-0.3022	7.8051	6.2339	7.3749
Mean (2003-2008)										
developed (ex-Russia)	0.0448	0.0378	0.0637	0.8944	33.1%	32.9%	-0.1313	6.3549	6.5495	5.8070
developing (ex-Russia)	0.0525	0.0247	0.0649	0.8615	41.7%	39.7%	-0.0442	7.2354	6.4325	7.1091
Grouped GJR-Garch(1,1) median parameters (stacked Norilsk peer data - ex Russia)										
Median (2003-2009)	Alpha 1	Gamma	Alpha 1+0.5 Gamma	Beta 1	uncon. (GJR-Garch (1,1)) Std. p.a.	Std. p.a.	Skewness	Kurtosis	uncon. GJR-Garch (1,1) kurt. (b2)	Kurtosis of St.res GJR-Garch(1,1)
developed (ex-Russia)	0.0446	0.0281	0.0622	0.9045	32.7%	32.6%	-0.0645	5.3379	6.4749	4.8269
developing (ex-Russia)	0.0454	0.0242	0.0608	0.8754	41.7%	40.3%	0.0000	6.0499	6.3422	5.5332

The Norilsk Nickel peer group is constructed as a stacked sample using companies from all international peer group regions. The particular regions are defined in Subsection 6.2.1, pp. 260-263. The Russian peer group contains only MICEX companies.

Figure 126

A GJR-GARCH(1,1) decomposition of the unconditional volatility for Norilsk Nickel and its global peer group (2003-2008)



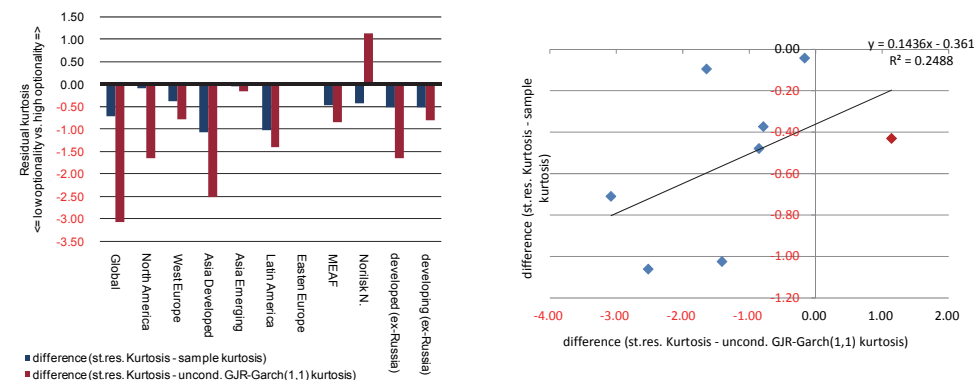
Panel A: Variance decomposition using the GJR-GARCH(1,1) parameterization as input variables.

Panel B: Unconditional GJR-GARCH(1,1) volatility estimate together with its news response component.

Nearly identical to Gazprom and Lukoil are the GJR-GARCH(1,1) volatility component weighting relations. The previous forecast has – identical to Gazprom and Lukoil – the lowest contribution, highlighting that yesterday's return (the news component) has a relatively high impact on today's volatility (see Figure 126, Panels A and B).

Figure 127

Explained vs. unexplained kurtosis for Norilsk Nickel (2003-2008 sample)



Panel A: The difference between the standardized residual kurtosis and the unconditional or sample kurtosis as real-option valuation indicator.

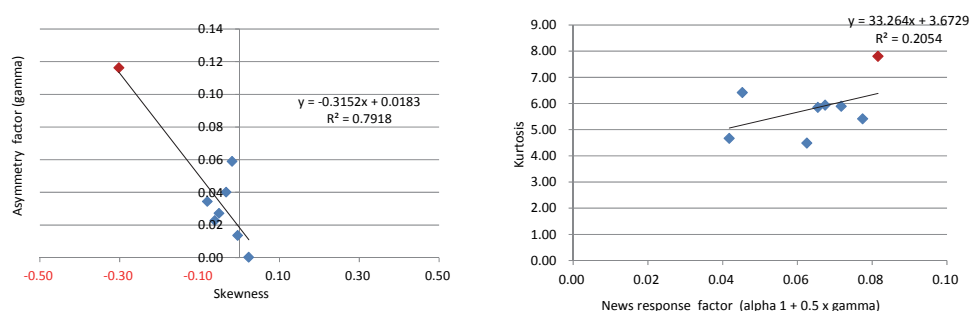
Panel B: Kurtosis measured as sample statistic seems to capture to a large extent the proper size of leptokurtic log-return behavior.

The strongest indication of real-option pricing is found in Figure 127, Panel A. The remaining excess kurtosis for Norilsk Nickel GJR-GARCH(1,1) standardized residuals is very high, indicating that the GJR-GARCH(1,1) specification as DGP is not good

enough to allow for proper inference on its parameters. The difference between the unconditional GJR-GARCH(1,1) kurtosis and the standardized residual GJR-GARCH(1,1) kurtosis is positive only for the Norilsk data set.

Figure 128

Relating higher sample moments with GJR-GARCH(1,1) parameters for the Norilsk Nickel peer group sample (2003-2008 sample)



Panel A: Asymmetry factor vs. skewness based on the values shown in Table 79.

Panel B: Kurtosis as function of the news response factor.

Figure 128, Panel A confirms the assumption that skewness and the GJR-GARCH(1,1) asymmetry factor (gamma) are negatively correlated. Similar to the Gazprom example we find that the relationship between news-response and kurtosis for Norilsk Nickel corresponds to the formulated expectation for real-option pricing (high news sensitivity as an indicator for high OTM real-option exposure; see Subsection 3.2.3, p. 67).

Overall, the Norilsk data partially (as with Gazprom) confirm the expectation of real-option pricing taking place in equity pricing behavior. The very high remaining kurtosis value of the standardized GJR-GARCH(1,1) residual is especially worth highlighting as the strongest real-option indicator. The skewness, and hence asymmetry, in its daily returns towards positive surprises, however, cannot be inferred from the GJR-GARCH parameters.

6.2.4 Sensitivity (Beta) analysis with sector-specific panel data

The price sensitivity of commodity-related companies with respect to changes in the specific major commodity underlying or commodity index can be used as a real-option

pricing indicator. We therefore formulated in Section 3.3 two hypotheses that can be tested via standard regression methods.

The tests are conducted within three different sample periods. The first sample will include all available data between March 2003 and March 2009 (which is the same sample as used for the unconditional analysis). To analyze the impact of possible moneyiness implications on real-option valuation, we will test two additional samples, one containing the market data before the commodity price crash between 2007 and 2008 and one sample after the crash between 2008 and 2009. The two additional samples will allow testing for changes in the commodity price sensitivity for high and low commodity price regimes.

The analysis is conducted via panel data regression.²¹⁸ Hence the sensitivity of Russian commodity companies is tested in panel format against changes in the underlying commodity price (instead of testing each individual company separately). This allows aggregating the MICEX companies as one pooled representative and comparing them with the different pooled global peer-group samples on a panel basis rather than on an individual company basis.²¹⁹ This will allow for a more uniform analysis between the peer-group regions. Another advantage of using panel regression instead of individual regressions is that it allows for the use of unbalanced data sets, which permits the inclusion of (mostly Russian) companies with shorter time series histories that had been dropped from the previous analyses. A third advantage is the opportunity to apply Wald specification tests on a larger, i.e., pooled set of data, which increases efficiency and helps to reduce the Type II error (failing to reject H_0 although it is false) of the analysis. All estimated coefficient standard errors and t-statistics are corrected for

²¹⁸ For an introduction to panel data analysis see Baltagi (2008), pp. 1-55.

²¹⁹ We will actually use two pooled data sets for the MICEX companies. One pool consists of companies from the energy industry, while the second pool consists of companies from the Basic Materials industry.

heteroscedasticity and contemporaneous correlation using White's (1980) robust covariances.²²⁰

6.2.4.1 Standard Beta factors analysis

The first hypothesis formulated in Subsubsection 3.3.2.1 is shown again in the paragraph below:

Hypothesis 1:

The equity value of a commodity company that contains/possesses real-options will be more sensitive to changes in the business-generating commodity price than for a company with low real-options.

This hypothesis can be translated into a testable regression equation. We propose the following test equation (see Equation (165)) for our analysis:

$$d\log(P_t^u) = c + \beta_1 d\log(CMTDY_t^i) + \beta_2 d\log(MSCI_t^{world}) + \beta_3 d(VIX_t) + \varepsilon_t \quad (165)$$

The endogenous variable in the above test regression is the log-change in the price of the company (P_t^u). The exogenous variables are the log-change of the commodity price or commodity index ($CMTDY_t^i$), the log-change of the MSCI World total return index ($MSCI_t^{world}$) and the absolute change of the S&P 500 volatility index (VIX_t). The last two variables are necessary to ensure that the equation is sufficiently defined (i.e., to ensure that the systematic risk component for the return is captured) in order to avoid specification errors due to omitted variables.

²²⁰ I.e. the applied estimator is robust to cross-sectional correlation and heteroskedasticity. For a summary on the calculation of different robust covariances for panel regressions and a discussion of the procedure used see Wooldridge (2002), pp. 147-153.

If Hypothesis 1 is correct, we can expect the regression coefficient β_1 (the % change in the price of the company P_t^u given a 1% change in the underlying commodity price or index) for average Russian companies to be significantly higher than for comparable non-Russian companies. The regression Equation (165) is translated into panel format by allowing for an additional cross-sectional dimension in the exogenous variable. Since we want to estimate a model without an intercept we have eliminated the constant c from Equation (165).²²¹ Note that the endogenous variables do not also have a cross-sectional dimension.

$$d\log(P_t^u) = \beta_1 d\log(CMTDY_t^i) + \beta_2 d\log(MSCI_t^{world}) + \beta_3 d(VIX_t) + \varepsilon_{it} \quad (166)$$

The regression equation is estimated using the following pooled data sets:

Table 80

Pooling MICEX companies according to their industry group

Data panel P_t^u	Panel members
P_t^{Energy}	Lukoil, Gazprom, Rosneft, Surgutneftegaz, Taneft, Gazpromneft
P_t^{BasicM}	Polyus Gold, Norilsk Nickel, Novolipetsk Steel, Polymetal, Severstal
$P_t^{GL,NA,WE,AD,AE,EE,LA,MEA}$	Companies grouped according to their peer-group region (ex- Russia)

The global peer-group panels definition is found in 6.2.1, p. 260. The peer-group members are selected for Gazprom, Lukoil and Norilsk Nickel.

²²¹ It would have been possible to use the fixed effect component in conjunction with the constant term. However, since the constant term is often regarded as a proxy for the risk-free return within a standard CAPM-based beta equity regression framework, we have dropped it, because interpretation for using panel data with different currencies and hence different risk-free rates is unclear and could theoretically affect the model results by allowing currency components to change the variable sensitivity. Tests with intercept and fixed effects for the MICEX panel show, however, that if the panel data are from one currency family, the regression betas remain nearly unchanged (i.e., the MICEX results remain the same independently of whether the test setting is defined with or without an intercept).

Analysis of Energy sector:

The regression results for MICEX energy panel estimations are shown below for the different test sample lengths, along with the results for the Gazprom International peer-groups.²²²

Table 81
Estimating a panel regression equation on MICEX energy companies and the international Gazprom peer-group panel dataset

Panel A										
2003-2009	MICEX	GL	NA	WE	AD	AE	LA	EE	MEA	Avg. †
Oil sensitivity	0.1032***	0.1525***	0.2529***	0.1151***	0.0352***	0.0413***	0.0583***	0.0417***	0.018**	0.0894
MSCI world sensitivity	1.4273***	0.9751***	0.8423***	1.1401***	0.9091***	0.7625***	0.6979***	0.8246***	0.5168***	0.8335
chg. in Vix Index	0.0034***	0.0002**	-0.0023***	0.0019***	0.0029***	0.0015***	0.0004	0.0016***	0.0015***	0.0010
R-squared	0.1738	0.2873	0.3805	0.2524	0.0723	0.0591	0.0377	0.0582	0.0248	
Panel B										
2007-2008	MICEX	GL	NA	WE	AD	AE	LA	EE	MEA	Avg. †
Oil sensitivity	0.077***	0.2025***	0.3119***	0.1323***	-0.005	0.0493*	0.1226***	0.0443**	0.0365*	0.1118
MSCI world sensitivity	1.2617***	1.0354***	0.6705***	1.2704***	1.0426***	1.0281***	0.849***	0.9259***	0.5938***	0.9270
chg. in Vix Index	0.003***	0.0009***	-0.0023***	0.0029***	0.0036***	0.0034***	0.0017***	0.0026***	0.0016***	0.0018
R-squared	0.2417	0.2229	0.3799	0.2959	0.0781	0.0512	0.0336	0.0924	0.0492	
Panel C										
2008-2009	MICEX	GL	NA	WE	AD	AE	LA	EE	MEA	Avg. †
Oil sensitivity	0.1321***	0.144***	0.201***	0.148***	0.0357***	0.0295**	0.0486**	0.0381**	0.0219	0.0833
MSCI world sensitivity	1.58***	1.0093***	0.9839***	1.1841***	0.9538***	0.6695***	0.714***	0.8417***	0.5556***	0.8640
chg. in Vix Index	0.0042***	0.0003	-0.0019***	0.0021***	0.003***	0.0007***	0.0005	0.0015***	0.0017***	0.0010
R-squared	0.2403	0.4227	0.4870	0.3764	0.1321	0.1234	0.0974	0.1002	0.0390	

Peer-group definition: GL (global), NA (north America), WE (west Europe), AD (Asia developed), AE (Asia emerging), LA (latin America), EE (eastern Europe), MEA (middle east and Africa); † average is build across the peer-group regions excluding MICEX and global peer-group companies. All estimated coefficient standard errors and t-statistics are corrected for cross-section heteroscedasticity and contemporaneous correlation.

Coefficient significant at 10% level (*)

Coefficient significant at 5% level (**)

Coefficient significant at 1% level (***)

Nearly all variables are highly significant, even at the 99% confidence level. The result show further that the MICEX energy panel group is more sensitive to market risk (MSCI world) than, for example, the NA or WE panel. On average, sensitivity towards changes in oil price is roughly in line with the ex-Russia sample average. The sensitivity

²²² The standard errors and t-statistics, together with the p-values, are given in Appendix M, p. 431.

towards changes in the market volatility (VIX coefficient) is highest for the MICEX family and far beyond the ex-Russia sample average (three times as high) for that coefficient (which could also be seen as a real-option indicator). More interestingly, however, is how the sensitivity of the MICEX energy pool differs between the 2007-2008 and 2008-2009 sample. As expected (see Hypothesis 2 in Subsubsection 3.3.2.1, p. 97) sensitivity towards changes in oil price is smaller if oil price is trading close to record levels (real-options are in-the-money), as was the case during late 2008. After oil prices collapsed between 2008 and 2009, the sensitivity changed again (real-options were again out-of-the-money). The results are very similar for the Lukoil global peer-group panel (see Table 93).

Table 82

Estimating a panel regression equation on MICEX energy companies and the international Lukoil peer-group panel dataset

Panel A										
2003-2009	MICEX	GL	NA	WE	AD	AE	LA	EE	MEA	Avg.†
Oil sensitivity	0.1032***	0.154***	0.2819***	0.1039***	0.0226***	0.0269***	0.0547***	0.0262***	0.0044	0.0843
MSCI world sensitivity	1.4273***	0.9698***	0.7556***	1.0444***	0.8339***	0.6384***	0.7091***	0.7798***	0.4151***	0.7682
chg. in Vix Index	0.0034***	-0.0004**	-0.0029***	0.0016***	0.003***	0.0014***	0.0002	0.0014***	0.0014***	0.0007
R-squared	0.1738	0.2674	0.4226	0.2118	0.0592	0.0474	0.0351	0.0814	0.0137	
Panel B										
2007-2008	MICEX	GL	NA	WE	AD	AE	LA	EE	MEA	Avg.†
Oil sensitivity	0.077***	0.2272***	0.3528***	0.1153***	-0.0138	0.0395	0.1308**	0.0306	0.0383*	0.1151
MSCI world sensitivity	1.2617***	0.9277***	0.5693***	1.2608***	0.9746***	0.9138***	0.9552***	0.958***	0.5701***	0.8912
chg. in Vix Index	0.003***	-0.0003	-0.003***	0.003***	0.0035***	0.0029***	0.0016**	0.0019***	0.0016***	0.0014
R-squared	0.2417	0.3192	0.3969	0.2462	0.0648	0.0511	0.0343	0.1338	0.0416	
Panel C										
2008-2009	MICEX	GL	NA	WE	AD	AE	LA	EE	MEA	Avg.†
Oil sensitivity	0.1321***	0.1453***	0.2137***	0.1277***	0.0136	-0.0016	0.0341	0.0288*	-0.0014	0.0700
MSCI world sensitivity	1.58***	1.0308***	0.9181***	1.0791***	0.8718***	0.5529***	0.7363***	0.8074***	0.4351***	0.8040
chg. in Vix Index	0.0042***	-0.0002	-0.0025***	0.0017***	0.0031***	0.0007***	0.0004	0.0016***	0.0016***	0.0008
R-squared	0.2403	0.4675	0.5269	0.3292	0.1095	0.0898	0.0893	0.1450	0.0197	

The above table shows the regression coefficients if not labeled differently. Peer-group definition: GL (global), NA (North America), WE (Western Europe), AD (Asia Developed), AE (Asia Emerging), LA (Latin America), EE (Eastern Europe), MA (Middle east and Africa); † average is built across the peer-group regions excluding MICEX and global peer-group companies. All estimated coefficient standard errors and t-statistics are corrected for cross-section heteroscedasticity and contemporaneous correlation.

Coefficient significant at 10% level (*)

Coefficient significant at 5% level (**)

Coefficient significant at 1% level (***)

The results in Table 81 and Table 82, together with the regression coefficient standard errors (see Appendix M, p. 431), allow testing the following set of hypotheses, which can be used for inference into real-option pricing:

Table 83
Formulated hypothesis tests with regard to oil price sensitivity of MICEX energy companies

Test No.	H₀	H₁	t-statistic (Gazprom/Lukoil)
I	$\beta_{1,\{2003-2009\}}^{MICEX} = \beta_{1,\{2003-2009\}}^{average}$	$\beta_{1,\{2003-2009\}}^{MICEX} > \beta_{1,\{2003-2009\}}^{average}$	1.93**/2.44***
II	$\beta_{1,\{2007-2008\}}^{MICEX} = \beta_{1,\{2007-2008\}}^{average}$	$\beta_{1,\{2003-2009\}}^{MICEX} < \beta_{1,\{2003-2009\}}^{average}$	-0.90/-0.91
III	$\beta_{1,\{2008-2009\}}^{MICEX} = \beta_{1,\{2008-2009\}}^{average}$	$\beta_{1,\{2003-2009\}}^{MICEX} < \beta_{1,\{2003-2009\}}^{average}$	2.05**/2.60***
IV	$\beta_{1,\{2008-2009\}}^{MICEX} = \beta_{1,\{2007-2008\}}^{MICEX}$	$\beta_{1,\{2008-2009\}}^{MICEX} > \beta_{1,\{2007-2008\}}^{MICEX}$	1.96**/1.96**

The t-statistic values are calculated using the standard errors in Appendix M, p. 431. All test statistics are conducted as directional, one-tailed hypothesis tests.

Reject H₀ at 90% confidence level (*)

Reject H₀ at 95% confidence level (**)

Reject H₀ at 99% confidence level (***)

Using the results from Table 83 allows us to confirm the assumptions formulated in Hypothesis 1 (see Subsubsection 3.3.2.1), since the oil price sensitivity of the MICEX energy pool is significantly higher than the oil price sensitivity across the peer-group regions (see test statistic under No. I in Table 83). The hypothesis that the MICEX energy pool has lower oil price sensitivity during the oil-price high (in-the-money option) than the average peer-group pool cannot be confirmed (No. II). However, the hypothesis that the MICEX energy pool has a higher sensitivity towards changes in the oil price compared to the global peer-group after oil has collapsed (see test No. III) can be confirmed. Overall the oil price sensitivity of the MICEX energy pool is significantly larger after oil prices collapsed (2008-2009) than during the oil price high during 2007-2008 (see test No. IV).

Analysis of Basic Materials sector:

The results for the Basic Materials sector are shown in Table 84. Since the Basic Materials sector is much more heterogeneous, containing everything from gold to steel, we have used the S&P SPGCIN total return index for industrial metals as a measure of beta sensitivity.

Table 84

Estimating a panel regression equation on MICEX basic materials companies and the international Norilsk Nickel peer-group panel dataset

Panel A										
2003-2009	MICEX	GL	NA	WE	AD	AE	LA	EE	MEA	Avg.†
SPGCIN sensitivity	0.1853***	0.2036***	0.1949***	0.2028***	0.0422***	0.0569***	0.1512***	--	0.1412***	0.1241
MSCI world sensitivity	1.4493***	1.2395***	0.8768***	1.1164***	0.8861***	1.0002***	0.8999***	--	0.6379***	0.8321
chg. in Vix Index	0.0039***	0.0029***	-0.0031***	0.0016***	0.0029***	0.0031***	-0.0011***	--	0.0018***	0.0010
R-squared	0.1569	0.1631	0.2908	0.2500	0.0934	0.0583	0.2219	--	0.0381	
Panel B										
2007-2008	MICEX	GL	NA	WE	AD	AE	LA	EE	MEA	Avg. †
SPGCIN sensitivity	0.1286***	0.2748***	0.1229***	0.2454***	0.0796***	0.0739*	0.1634***	--	0.136***	0.1370
MSCI world sensitivity	1.1664***	1.3915***	0.9185***	1.4814***	0.9021***	1.0381***	1.1668***	--	0.5494***	0.9310
chg. in Vix Index	0.0035***	0.0042***	-0.0029***	0.0032***	0.0028***	0.004***	0.0003	--	0.0018***	0.0017
R-squared	0.1849	0.1938	0.3230	0.3290	0.0904	0.0415	0.2547	--	0.0366	
Panel C										
2008-2009	MICEX	GL	NA	WE	AD	AE	LA	EE	MEA	Avg. †
SPGCIN sensitivity	0.1811**	0.2752***	0.1712***	0.3311***	0.0336	0.0239	0.1712***	--	0.2048***	0.1514
MSCI world sensitivity	1.5559***	1.2387***	0.936***	1.0631***	0.914***	1.0561***	0.9006***	--	0.6604***	0.8461
chg. in Vix Index	0.0044***	0.003***	-0.0028***	0.0015***	0.0031***	0.0032***	-0.0011***	--	0.0019***	0.0011
R-squared	0.1622	0.1825	0.3837	0.3572	0.1330	0.1032	0.3422	--	0.0848	

The above table shows the regression coefficients if not labeled differently. SPGCIN (S&P GSCI Industrial Metals total return index). Peer-group definition: GL (global), NA (North America), WE (Western Europe), AD (Asia Developed), AE (Asia Emerging), LA (Latin America), EE (Eastern Europe), MA (Middle East and Africa); † average is built across the peer-group regions excluding MICEX and global peer-group companies. All estimated coefficient standard errors and t-statistics are corrected for cross-section heteroscedasticity and contemporaneous correlation.

Coefficient significant at 10% level (*)

Coefficient significant at 5% level (**)

Coefficient significant at 1% level (***)

The deployed panel regressions have significant coefficients for nearly all tested peer-group regions. The same is true for the MICEX basic materials sector. However, testing the same hypothesis set as for the MICEX energy sector does not lead to similar conclusions.

Although the test to confirm the assumptions with regard to real-option pricing formulated in Hypothesis 1 is confirmed (Test No. I, rejects H_0 at the 95% confidence level), the other test H_0 cannot be rejected (see Table 85).

Table 85
Formulated hypothesis tests with regard to SPCGIN price sensitivity of MICEX basic materials companies

Test No.	H_0	H_1	t-statistic
I	$\beta_{1,\{2003-2009\}}^{MICEX} = \beta_{1,\{2003-2009\}}^{average}$	$\beta_{1,\{2003-2009\}}^{MICEX} > \beta_{1,\{2003-2009\}}^{average}$	2.318**
II	$\beta_{1,\{2007-2008\}}^{MICEX} = \beta_{1,\{2007-2008\}}^{average}$	$\beta_{1,\{2003-2009\}}^{MICEX} < \beta_{1,\{2003-2009\}}^{average}$	0.280
III	$\beta_{1,\{2008-2009\}}^{MICEX} = \beta_{1,\{2008-2009\}}^{average}$	$\beta_{1,\{2003-2009\}}^{MICEX} < \beta_{1,\{2003-2009\}}^{average}$	0.625
IV	$\beta_{1,\{2008-2009\}}^{MICEX} = \beta_{1,\{2007-2008\}}^{MICEX}$	$\beta_{1,\{2008-2009\}}^{MICEX} > \beta_{1,\{2007-2008\}}^{MICEX}$	0.694

The t-statistic values are calculated using the standard errors in Appendix M, p. 431. All test statistics are conducted as directional one-tailed hypothesis tests.

Reject H_0 at 90% confidence level (*)

Reject H_0 at 95% confidence level (**)

Reject H_0 at 99% confidence level (***)

6.2.4.2 Up- vs. down-market beta analysis

The previous analysis has assumed that the beta sensitivity towards changes in the commodity price is identical for up and down movements of the commodity price or index. However, it is possible to extend the test regression in Equation (166) in order to account for differences in up and down movements. This is especially important for testing the following Hypothesis derived in Subsubsection 3.3.2.1:

Hypothesis 2:

Companies with identical capital structure but different exposure to real-option valuation should differ in their commodity beta factor sensitivity. Companies with high real-option value exposure should have significantly higher commodity beta asymmetry (stronger difference in up- vs. down-beta) than companies with low real-option exposure.

To test the above hypothesis we have extended Equation (166) into Equation (167):

$$\begin{aligned} d\log(P_{it}^u) = & \beta_1 d\log(CMTDY_t^i) 1_{\{d\log(CMTDY_t^i) > 0\}} + \beta_2 d\log(CMTDY_t^i) 1_{\{d\log(CMTDY_t^i) \leq 0\}} + \\ & + \beta_3 d\log(MSCI_t^{world}) + \beta_4 d(VIX_t) + \varepsilon_{it} \end{aligned} \quad (167)$$

The sensitivity of equity prices changes towards changes in the commodity price or index has now become dependent on whether the commodity prices increase or decrease. The up beta factor is given by β_1 while the down beta factor is given by β_2 . According to Hypothesis 2, one would expect that up and down beta sensitivities (β_1, β_2) are different if real-option pricing takes place in the market.

Analysis of the MICEX Energy sector:

The results shown in Table 86 and Table 87 seem to confirm at first glance Hypothesis 2. The asymmetry in the oil price sensitivity factors is indeed exactly as required if one assumes a real-option pricing dynamic for the equity price. The difference between the up and down beta coefficients is highest for the MICEX panel for the 2003-2009 sample (see Table 86 and Table 87, Panel A with $\beta_1 > \beta_2$). During the oil price high, the sensitivity difference reverted as expected, becoming negative and indicating that the market was priced with real-option projects being in-the-money and oil price trading consistently above 100 USD/barrel (see Panel B, $\beta_2 > \beta_1$). After the oil price collapsed to a level below 40 USD/barrel, the sensitivity towards oil price changes reverted again, with a nearly 15% difference in the oil price sensitivity on the stock price of the MICEX energy group (see Panel C, $\beta_1 \square \beta_2$).

Note that the results in Table 86 and Table 87 allow testing a similar set of hypotheses as shown in Table 85. Since the results are likely comparable, we will not repeat the tests and will instead analyze whether indeed the up and down oil price sensitivity coefficients are significantly different and, more importantly, different as predicted by Hypothesis 2 (up sensitivity larger than down sensitivity).

Table 86
Estimating a panel regression equation on MICEX energy companies and the international Gazprom peer-group panel dataset
with up and down oil price sensitivity

Panel A										
2003-2009	MICEX	GL	NA	WE	AD	AE	LA	EE	MEA	Avg. †
Oil up sensitivity	0.1231***	0.1449***	0.2397***	0.1158***	0.0429***	0.0376***	0.05***	0.0458***	0.0226**	0.0792
Oil down sensitivity	0.0826***	0.1602***	0.2664***	0.1144***	0.0273***	0.0452***	0.0669***	0.0375***	0.0133	0.0815
MSCI world sensitivity	1.4314***	0.9738***	0.8401***	1.1402***	0.9104***	0.7618***	0.6964***	0.8253***	0.5176***	2.6825
chg. in Vix Index	0.0034***	0.0002**	-0.0023***	0.0019***	0.0029***	0.0015***	0.0004	0.0016***	0.0015***	0.0664
R-squared	0.1741	0.2874	0.3807	0.2524	0.0724	0.0591	0.0378	0.0583	0.0248	
Wald test b1=b2 F-statistic	3.1015	3.0465	11.2265	0.0261	1.5377	0.3335	0.5714	0.3373	0.4237	
Wald p-value	0.0783	0.0809	0.0008	0.8716	0.2150	0.5636	0.4497	0.5614	0.5151	
Panel B										
2007-2008	MICEX	GL	NA	WE	AD	AE	LA	EE	MEA	Avg. †
Oil up sensitivity	0.0552*	0.2086***	0.3227***	0.1269***	-0.0434	0.0899***	0.1353***	0.0279	0.005	0.0949
Oil down sensitivity	0.1182***	0.1932***	0.2954***	0.1405***	0.0537	-0.0127	0.1033	0.0693**	0.0845***	0.1049
MSCI world sensitivity	1.2188***	1.0361***	0.6717***	1.2698***	1.0383***	1.0326***	0.8504***	0.924***	0.5902***	-1.3232
chg. in Vix Index	0.0029***	0.0009***	-0.0023***	0.0028***	0.0035***	0.0034***	0.0018***	0.0025***	0.0016***	0.1541
R-squared	0.2365	0.2229	0.3801	0.2960	0.0793	0.0520	0.0336	0.0927	0.0504	
Wald test b1=b2 F-statistic	1.4183	0.2249	1.5772	0.2607	4.8191	3.7926	0.1546	1.2323	4.0663	
Wald p-value	0.2339	0.6354	0.2092	0.6097	0.0282	0.0515	0.6942	0.2670	0.0438	
Panel C										
2008-2009	MICEX	GL	NA	WE	AD	AE	LA	EE	MEA	Avg. †
Oil up sensitivity	0.2012***	0.1339***	0.1773***	0.1533***	0.0527***	0.0047	0.0244	0.0315	0.0298	0.0677
Oil down sensitivity	0.0539	0.1552***	0.2274***	0.1421***	0.0167	0.0571***	0.0757***	0.0454**	0.013	0.0825
MSCI world sensitivity	1.6284***	1.0023***	0.9672***	1.1878***	0.9658***	0.6521***	0.697***	0.8371***	0.5612***	3.5720
chg. in Vix Index	0.0043***	0.0002	-0.002***	0.0021***	0.0031***	0.0007***	0.0005	0.0015***	0.0017***	-0.7063
R-squared	0.2412	0.4229	0.4876	0.3764	0.1326	0.1244	0.0979	0.1003	0.0391	
Wald test b1=b2 F-statistic	7.7659	1.1951	6.7104	0.2997	1.9418	4.9372	1.7960	0.2082	0.2612	
Wald p-value	0.0054	0.2744	0.0096	0.5841	0.1636	0.0263	0.1803	0.6482	0.6093	

Peer-group definition: GL (global), NA (North America), WE (Western Europe), AD (Asia Developed), AE (Asia Emerging), LA (Latin America), EE (Eastern Europe), MA (Middle East and Africa); † average is built across the peer-group regions excluding MICEX and global peer-group companies. Coefficient significant at 10% level (*), at 5% level (**) and at 1% (***) level. Standard errors and t-statistics are corrected for cross-section heteroscedasticity and contemporaneous correlation.

Table 87

Estimating a panel regression equation on MICEX energy companies and the international Lukoil peer-group panel dataset
with up and down oil price sensitivity

Panel A										
2003-2009	MICEX	GL	NA	WE	AD	AE	LA	EE	MEA	Avg. †
Oil up sensitivity	0.1231***	0.152***	0.2646***	0.1087***	0.0343***	0.0282***	0.0386*	0.0287***	0.0116	0.0735
Oil down sensitivity	0.0826***	0.1561***	0.2997***	0.0991***	0.0106	0.0256***	0.0711***	0.0237**	-0.003	0.0753
MSCI world sensitivity	1.4314***	0.9694***	0.7526***	1.0452***	0.836***	0.6386***	0.7063***	0.7802***	0.4163***	3.0291
chg. in Vix Index	0.0034***	-0.0004**	-0.0029***	0.0016***	0.003***	0.0014***	0.0001	0.0014***	0.0014***	0.4441
R-squared	0.1741	0.2674	0.4229	0.2119	0.0594	0.0474	0.0352	0.0814	0.0138	
Wald test b1=b2 F-statistic	3.1015	0.0825	18.8202	0.9321	3.2664	0.0362	1.2902	0.1089	0.8550	
Wald p-value	0.0783	0.7740	-	0.3343	0.0707	0.8491	0.2560	0.7414	0.3552	
Panel B										
2007-2008	MICEX	GL	NA	WE	AD	AE	LA	EE	MEA	Avg. †
Oil up sensitivity	0.0552*	0.2164***	0.3627***	0.1132***	-0.0545*	0.0612*	0.1295*	0.0007	-0.0023	0.0872
Oil down sensitivity	0.1182***	0.2437***	0.3376***	0.1184***	0.0485	0.0063	0.1328	0.0763**	0.1005***	0.1172
MSCI world sensitivity	1.2188***	0.9265***	0.5705***	1.2606***	0.9701***	0.9162***	0.9551***	0.9546***	0.5656***	-1.0757
chg. in Vix Index	0.0029***	-0.0003	-0.003***	0.003***	0.0035***	0.003***	0.0016**	0.0019***	0.0015***	-0.1776
R-squared	0.2365	0.3194	0.3971	0.2462	0.0660	0.0514	0.0343	0.1349	0.0435	
Wald test b1=b2 F-statistic	1.4183	0.5018	1.1658	0.0284	4.8475	1.0474	0.0009	2.9779	5.1621	
Wald p-value	0.2339	0.4788	0.2803	0.8662	0.0277	0.3062	0.9762	0.0845	0.0232	
Panel C										
2008-2009	MICEX	GL	NA	WE	AD	AE	LA	EE	MEA	Avg. †
Oil up sensitivity	0.2012***	0.1483***	0.1837***	0.1326***	0.0336*	-0.0163	0.0045	0.0181	0.0094	0.0522
Oil down sensitivity	0.0539	0.142***	0.2471***	0.1223***	-0.0086	0.0147	0.0671*	0.0407*	-0.0135	0.0671
MSCI world sensitivity	1.6284***	1.0329***	0.8971***	1.0826***	0.8858***	0.5427***	0.7156***	0.8***	0.4427***	3.9845
chg. in Vix Index	0.0043***	-0.0002	-0.0026***	0.0017***	0.0031***	0.0006**	0.0004	0.0016***	0.0016***	-0.3167
R-squared	0.2412	0.4675	0.5279	0.3293	0.1102	0.0902	0.0900	0.1452	0.0198	
Wald test b1=b2 F-statistic	7.7659	0.0508	10.3736	0.2159	2.7443	1.5458	1.6613	0.5450	0.4030	
Wald p-value	0.0054	0.8217	0.0013	0.6422	0.0977	0.2138	0.1976	0.4604	0.5256	

Peer-group definition: GL (global), NA (North America), WE (Western Europe), AD (Asia Developed), AE (Asia Emerging), LA (Latin America), EE (Eastern Europe), MA (Middle East and Africa); † average is built across the peer-group regions excluding MICEX and global peer-group companies. Coefficient significant at 10% level (*), at 5% level (**) and at 1% (***) level. Standard errors and t-statistics are corrected for cross-section heteroscedasticity and contemporaneous correlation.

Wald coefficient restriction test:

To answer this question we have conducted preliminary Wald coefficient restriction tests. For the overall sample length (2003-2009) the equality of the beta coefficients for the MICEX energy panel is rejected at the 90% confidence level (see Wald test statistic and p-value). For the 2007-2008, sample coefficient equality – despite the nearly 6% difference between the up and down beta sensitivity – cannot be rejected. However, for the 2008-2009 sample, coefficient equality can be rejected with 99% confidence for the MICEX energy panel. Note that, despite coefficient equality being rejected for other peer-group regions, it is only the MICEX sample in which coefficient equality is rejected with $\beta_1 \neq \beta_2$.

Testing the MICEX energy panel for the coefficient restriction $\beta_1 - \beta_2 = 0$.²²³

Table 88 shows the formulated hypotheses for the different sample periods. In order to test the different H_0 it is necessary to obtain the standard error for $\beta_1 - \beta_2$.

Table 88
Formulated hypothesis tests with regard to oil price sensitivity of MICEX energy companies using coefficient restrictions

Test No.	H_0	H_1	t-statistic
I	$\beta_{1,\{2003-2009\}}^{MICEX} - \beta_{2,\{2003-2009\}}^{MICEX} = 0$	$\beta_{1,\{2003-2009\}}^{MICEX} - \beta_{2,\{2003-2009\}}^{MICEX} > 0$	1.761*
II	$\beta_{1,\{2007-2008\}}^{MICEX} - \beta_{2,\{2007-2008\}}^{MICEX} = 0$	$\beta_{1,\{2007-2008\}}^{MICEX} - \beta_{2,\{2007-2008\}}^{MICEX} < 0$	-1.303
III	$\beta_{1,\{2008-2009\}}^{MICEX} - \beta_{2,\{2008-2009\}}^{MICEX} = 0$	$\beta_{1,\{2008-2009\}}^{MICEX} - \beta_{2,\{2008-2009\}}^{MICEX} > 0$	2.734***

²²³ For formulation and theory on testing hypotheses about a single linear combination of the parameters see Wooldridge (2003), pp. 139-142.

The t-statistic values are calculated using the standard errors in Appendix M, p. 431. All test statistics are conducted as directional one-tailed hypothesis tests. The t-statistics are obtained from Table 89.

Reject H_0 at 90% confidence level (*)
 Reject H_0 at 95% confidence level (**)
 Reject H_0 at 99% confidence level (***)

Since the original test regression shown in Equation (167) does not show the required standard error, one needs to rearrange the Equation (167) as shown in Equation (168).

$$\begin{aligned} \text{dlog}(P_{it}^u) = & \phi_1 \text{dlog}(CMTDY_t^i) 1_{\{\text{dlog}(CMTDY_t^i) > 0\}} + \\ & + \beta_2 \left[\text{dlog}(CMTDY_t^i) 1_{\{\text{dlog}(CMTDY_t^i) > 0\}} + \text{dlog}(CMTDY_t^i) 1_{\{\text{dlog}(CMTDY_t^i) \leq 0\}} \right] + \\ & + \beta_3 \text{dlog}(MSCI_t^{world}) + \beta_4 \text{d}(VIX_t) + \varepsilon_{it} \end{aligned} \quad (168)$$

The auxiliary variable $\phi_1 = \beta_1 - \beta_2$ is used to extract the required standard error of $\beta_1 - \beta_2$. The results for the three sample periods are shown below in Table 89. The appropriate t-statistics are inserted in Table 88.

Table 89
Extracting the standard errors for restriction tests used for the MICEX
energy peer-group panel

Panel A				
2003-2009	Coefficient	Std. Error	t-statistic	Prob.
auxiliary variable	0.0404	0.0230	1.7611	0.0783
(Oil up+Oil down)	0.0826	0.0166	4.9786	0.0000
MSCI world	1.4314	0.0426	33.5757	0.0000
chg. in Vix Index	0.0034	0.0003	13.2617	0.0000
R-squared	0.1741			
Panel B				
2007-2008	Coefficient	Std. Error	t-statistic	Prob.
auxiliary variable	-0.0629	0.0483	-1.3025	0.1929
(Oil up+Oil down)	0.1182	0.0380	3.1062	0.0019
MSCI world	1.2188	0.0649	18.7855	0.0000
chg. in Vix Index	0.0029	0.0004	7.8473	0.0000
R-squared	0.2365			
Panel C				

2008-2009	Coefficient	Std. Error	t-statistic	Prob.
auxiliary variable	0.1473	0.0539	2.7337	0.0063
(Oil up+Oil down)	0.0539	0.0403	1.3374	0.1813
MSCI world	1.6284	0.0984	16.5409	0.0000
chg. in Vix Index	0.0043	0.0005	7.8506	0.0000
R-squared	0.2412			

Based on the results in Table 89 it is possible to reject the H_0 for the test formulated in I and III of Table 88, confirming with 90% and 99% confidence that the up beta sensitivity, in the sample period 2003-2009 and 2008-2009 for equity prices of the MICEX energy panel with respect to changes in the oil price, are significantly higher than the down beta sensitivity. The H_0 for test number II cannot be rejected.

Overall, the above results allow us, in conjunction with the results from Table 85, to accept the formulated Hypothesis 2 and conclude that there is strong econometrical (i.e. test-based) indication that MICEX energy companies trade with real-option value attached.

Analysis of the MICEX Basic Materials sector:

The results for the MICEX basic materials sector are similar to the results from the MICEX energy panel. As shown in Table 90, Panel A, the commodity price up sensitivity of the MICEX basic materials sector is the largest among the tested peer-groups (it is significantly larger than the peer-group average, with a t-statistic of 2.41).²²⁴ The difference between the up and down sensitivity is also very large. The difference increases to nearly 17% for the 2008-2009 sample (see Panel C).

Wald coefficient restriction test:

The Wald coefficient restriction tests show that the hypothesis of equality of the beta coefficients ($H_0: \beta_1 = \beta_2$) cannot be rejected despite the large difference between the up and down commodity price sensitivities (see Wald p-values in Table 90).

²²⁴ The t-statistic is calculated using the standard error information given in Appendix M, p. 431.

Table 90
Estimating a panel regression equation on MICEX basic materials companies and the international Norilsk Nickel peer-group panel dataset
with up and down commodity index sensitivity

Panel A										
2003-2009	MICEX	GL	NA	WE	AD	AE	LA	EE	MEA	Avg. †
SPGCIN up sensitivity	0.2169***	0.1733***	0.205***	0.2152***	0.0197	0.0305	0.1775***	0***	0.1449***	0.1133
SPGCIN down sensitivity	0.1564***	0.2321***	0.1855***	0.191***	0.0634***	0.0819***	0.1263***	0***	0.1377***	0.1123
MSCI world sensitivity	1.4533***	1.2373***	0.8775***	1.1172***	0.8845***	0.9984***	0.9018***	0***	0.6382***	2.4140
chg. in Vix Index	0.0039***	0.0029***	-0.0031***	0.0016***	0.0029***	0.0031***	-0.0011***	0***	0.0018***	1.0679
R-squared	0.1572	0.1634	0.2908	0.2501	0.0937	0.0584	0.2222	-	0.0381	
Wald test b1=b2 F-statistic	1.1543	4.3142	1.2824	2.1469	7.3567	3.7556	6.9558	-	0.0793	
Wald p-value	0.2827	0.0378	0.2575	0.1429	0.0067	0.0526	0.0084	-	0.7782	
Panel B										
2007-2008	MICEX	GL	NA	WE	AD	AE	LA	EE	MEA	Avg. †
SPGCIN up sensitivity	0.1619***	0.3308***	0.1382***	0.2543***	0.0439	0.0188	0.2261***	0***	0.2387***	0.1314
SPGCIN down sensitivity	0.0908*	0.2062***	0.1041***	0.2352***	0.1208***	0.1374**	0.0911***	0***	0.0177	0.1009
MSCI world sensitivity	1.1696***	1.3981***	0.9204***	1.4822***	0.8989***	1.0331***	1.1724***	0***	0.5586***	0.6016
chg. in Vix Index	0.0035***	0.0042***	-0.0029***	0.0032***	0.0028***	0.0041***	0.0002	0***	0.0017***	-0.1890
R-squared	0.1856	0.1951	0.3231	0.3291	0.0911	0.0421	0.2567	0	0.0417	
Wald test b1=b2 F-statistic	1.0718	3.8727	0.9622	0.2347	3.2180	2.3351	9.1388	-	19.6216	
Wald p-value	0.3007	0.0492	0.3267	0.6281	0.0729	0.1266	0.0025	-	-	
Panel C										
2008-2009	MICEX	GL	NA	WE	AD	AE	LA	EE	MEA	Avg. †
SPGCIN up sensitivity	0.2815**	0.1871***	0.243***	0.3712***	-0.0319	-0.0337	0.2071***	0***	0.1219**	0.1254
SPGCIN down sensitivity	0.1132	0.334***	0.1232***	0.304***	0.0779***	0.0628	0.1469***	0***	0.261***	0.1394
MSCI world sensitivity	1.5655***	1.2297***	0.9433***	1.0669***	0.9077***	1.0505***	0.904***	0***	0.6524***	1.4223
chg. in Vix Index	0.0044***	0.003***	-0.0028***	0.0015***	0.003***	0.0032***	-0.0011***	0***	0.0019***	-0.2824
R-squared	0.1632	0.1836	0.3845	0.3576	0.1344	0.1038	0.3424	-	0.0862	
Wald test b1=b2 F-statistic	1.5950	3.1867	6.2283	2.2259	6.5489	2.2078	1.4135	-	5.7776	
Wald p-value	0.2068	0.0744	0.0126	0.1358	0.0105	0.1374	0.2346	-	0.0163	

SPGCIN (S&P GSCI Industrial Metals total return index). Peer-group definition: GL (global), NA (North America), WE (Western Europe), AD (Asia Developed), AE (Asia Emerging), LA (Latin America), EE (Eastern Europe), MA (Middle East and Africa); † average is built across the peer-group regions excluding MICEX and global peer-group companies. Coefficient significant at 10% level (*), at 5% level (**) and at 1% (***) level. Standard errors and t-statistics are corrected for cross-section heteroscedasticity and contemporaneous correlation.

Table 91
Extracting the standard errors for restriction tests used for the MICEX
basic materials peer-group panel

Panel A				
2003-2009	Coefficient	Std. Error	t-statistic	Prob.
auxiliary variable sensitivity	0.0605	0.0563	1.0744	0.2827
(SPGCIN up+ SPGCIN down) sensitivity	0.1564	0.0413	3.7849	0.0002
MSCI world sensitivity	1.4533	0.0701	20.7387	-
chg. in Vix Index	0.0039	0.0004	10.2652	-
R-squared	0.1572			
Panel B				
2007-2008	Coefficient	Std. Error	t-statistic	Prob.
auxiliary variable sensitivity	0.0711	0.0686	1.0353	0.3007
(SPGCIN up+ SPGCIN down) sensitivity	0.0908	0.0534	1.6995	0.0895
MSCI world sensitivity	1.1696	0.0880	13.2984	-
chg. in Vix Index	0.0035	0.0005	7.5338	-
R-squared	0.1856			
Panel C				
2008-2009	Coefficient	Std. Error	t-statistic	Prob.
auxiliary variable sensitivity	0.1684	0.1333	1.2629	0.2068
(SPGCIN up+ SPGCIN down) sensitivity	0.1132	0.0929	1.2182	0.2234
MSCI world sensitivity	1.5655	0.1393	11.2369	-
chg. in Vix Index	0.0044	0.0007	6.0145	-
R-squared	0.1632			

Testing the MICEX basic materials panel for the coefficient restriction $\beta_1 - \beta_2 = 0$: We have also tested, in accordance with the MICEX energy sample, coefficient restrictions in order to confirm or reject Hypothesis 2 for MICEX basic materials companies. The necessary standard errors are given in Table 91.

The results show that, despite the large difference between the up and down price sensitivity of the MICEX basic material panel with respect to changes in the commodity index, the coefficient restriction $\beta_1 - \beta_2 = 0$ cannot be rejected (as expected given the previous Wald restriction test results). Therefore Hypothesis 2 is not confirmed by analyzing MICEX Basic Materials companies.

6.3 Critical assessment and conclusion

Chapter 6 represents the empirical part of this dissertation that is related to statistical inference. The objective was to test the different derived hypotheses in the theoretical part of the dissertation, if possible, by formulating statistical hypothesis tests. We have distinguished between an unconditional analysis, primarily related to testing distribution moments, and conditional analysis, which was particularly centered on commodity price sensitivity tests. For both cases (distribution moment test and commodity price sensitivity tests) it was possible to calculate test statistics in order to test the derived Hypotheses 1 to 3 in Section 3.3, pp. 91-107.

The results have shown that most employed tests have helped to confirm Hypotheses 1 to 3, especially for the MICEX energy sector but also to a sufficient extent for the basic materials sector.

Obtaining evidence for the GJR-GARCH(1,1)-based inference was only partially successful, since the asymmetry (leverage) factor within the GARCH diffusion did not display the required characteristics necessary to infer that real-option pricing is taking place.

However, the GJR-GARCH calibration was certainly undermined by the strong market volatility that made the calibration for most tested companies for the 2003-2009 sample impossible (violation of GJR-GARCH(1,1) stability conditions).

The panel data regression results for MICEX basic materials companies was also confronted by the strong heterogeneity of the data sample, given that gold, platinum, nickel, and steel producers were included in the data panel. Regressing the basic materials panel against price changes in the SPGSCI Precious Metals Index could have yielded other results. However, the index could not be used due to the short time series history.

7 Summary, conclusion and remaining comments

We began this dissertation with the goal of analyzing whether market participants consider real-option value in their price discovery for companies in transition economies. The model case for this question was Russia and Russian companies with their vast exposure to natural resources. The focus was on the Energy and Basic Materials sectors with companies that have a long established history and belong to the global peer-group in their sector. Three companies from the MICEX family (Gazprom OAO, Lukoil and Norilsk Nickel) met these criteria. These companies constitute the anchor for further data analysis and data selection (e.g., for the peer-group selection or the commodity prices study).

7.1 Fundamental valuation

The first element of the analysis was a relative value multiple-based valuation between the chosen companies for our analysis and their major peer-groups. The expectation was that if real-options are indeed considered for company valuation, the chosen three companies should trade with a premium relative to their peer-group. After describing and analyzing the three major companies that fulfill the necessary criteria in Chapter 2, we have come to the conclusion that, based on standard multiple analysis, no real-option values seem to be discounted for the three companies analyzed. They have all traded cheap relative to their different regional peer-groups, and this despite a better capital structure and higher profitability. The summary results of this multiple analysis are shown in Table 92.

Table 92
Fundamental indicators for real-option valuation

Analysis		General	Gazprom	Lukoil	Norilsk Nickel
Panel A					
Fundamental analysis	Price to earnings ratio	Figure 9	Figure 12	Figure 15	Figure 18
	<i>Indication for real-option valuation</i>	NIF	NIF	NIF	NIF
	Forward looking price to earnings ratio		Figure 12	Figure 15	Figure 18
	<i>Indication for real-option valuation</i>		NIF	NIF	NIF
	Estimated price to earnings ratio / growth	Figure 9			
	<i>Indication for real-option valuation</i>	NIF			
	Price to sales ratio		Figure 12	Figure 15	Figure 18
	<i>Indication for real-option valuation</i>		NIF	NIF	NIF
	Price to cash-flow ratio	Figure 9	Figure 12	Figure 15	Figure 18
	<i>Indication for real-option valuation</i>	NIF	NIF	NIF	NIF
	Panel B				
	EV to trailing EBITDA	Figure 10+11	Figure 13	Figure 16	Figure 19
	Operating Margin	Figure 10+11	Figure 14	Figure 17	Figure 20
	Return on common equity		Figure 14	Figure 17	Figure 20

NIF (no indication for real-option pricing found), IF (indication for real-option pricing found).

The above table shows the different analyzed valuation multiples (together with the chart number where the analysis is summarized) and the conclusion with respect to real-option pricing evidence.

7.2 Price return analysis for companies with real-options

After no valuation advantage for selected Russian commodity companies could be confirmed by fundamental analysis, we have turned to analyzing the return distribution characteristics of Russian equity-traded companies. we have used three theoretical frameworks to derive different hypotheses on how real-option companies trade. They represent the theoretical foundation of this dissertation.

The first framework was to define an investment project with and without real-option considerations (see Subsection 3.2.2, pp. 48-67), and to simulate the free-cash-flow distribution and hence NPV by stochastic simulation. Using this example we have shown that real-options:

- A. enhance the value of the overall project (shift of mean to the right)
- B. reduce the shortfall risk due to the truncation of the left tail of the NPV distribution; it hence increases the skewness and decreases the kurtosis
- C. increase the mean/variance ratio (i.e., the Sharpe ratio) and hence increases the compensation for risk.

The second framework aims to bridge the gap between standard NPV analyses, optionality and return distribution by estimating a GJR-GARCH model using equity log-returns and applying it for option pricing (hence looking solely at option price distribution outside of the whole investment project context). This methodology allows observing the influence of option valuation factors on the option value and hence optioning return distribution. The sensitivity analysis is more transparent than using a standard investment project with several cross-dependencies to extract the impact of changing input variables on the investment return distribution.

The following major conclusions can be drawn with respect to the influence of moneyness, asymmetry component (i.e., leverage factor), volatility and option return characteristics. A decreasing moneyness (option being out-of-the-money due to, e.g., low commodity prices) should (see Figure 42, p. 81):

- A. Increase the standard deviation of the return distribution
- B. Increase the skewness of the return distribution
- C. Increase the kurtosis of the return distribution
- D. Lead to an overall stronger non-normal return distribution

An increasing leverage GJR-GARCH component leads to (see Figure 46, p. 86):

- E. Decreasing skewness of the return distribution
- F. Decreasing kurtosis of the return distribution

The analysis has unveiled that companies with out-of-the-money real-option value should have a smaller estimated asymmetric GJR-GARCH component (which also corresponds to higher skewness) than companies without in-the-money real-option value.

Within the same analytical setup we have analyzed further the impact of optionality and moneyness on the beta sensitivity between underlying commodity factor and the option and equity price return (see Section 3.2.4, p. 87). The results show that optionality leads to:

- G. Higher option and hence equity return sensitivity towards changes in the underlying commodity price

Further, we have shown the impact of moneyness on the up and down beta sensitivity of option and hence equity returns to changes in the underlying commodity prices in case of optionality (see Figure 48, p. 89), which leads us to conclude that:

- H. Companies with optionality (i.e., out-of-the-money optionality) should have a higher asymmetry between the up and down market beta than companies without optionality (i.e., in-the-money optionality).

The third framework is likely the most comprehensive approach for combining equity valuation with option theory (see Section 3.3, p. 91). In contrast to the previous frameworks the results are not primarily based on simulation experiments but instead directly on contingent claims analysis. The analysis allows extracting three statistically testable hypotheses that are in line with the observations and conclusions derived from the simulation experiments shown before:

Hypothesis 1:

The equity value of a commodity company that contains/possesses real-options will be more sensitive to changes in the business-generating commodity price than for a company with low real-options.

Hypothesis 2:

Companies with identical capital structure but different exposure to real-option valuation should differ in their commodity beta factor sensitivity. Companies with high real-option value exposure should have significantly higher commodity beta asymmetry (stronger difference in up- vs. down-beta) than companies with low real-option exposure.

Hypothesis 3:

The return on the equity value of a commodity company that contains/possesses real-options will be positively skewed. In other words, a company that contains positive gamma exposure (e.g., in the form of highly out-of-the money real-options) will show a positively skewed probability distribution of price changes relative to companies without real-option value.

We have summarized the above listed assumptions and hypotheses in Table 93 to allow for an aggregated view on the results of the empirical analysis. The results are split between the two major applied sample periods (five-year and six-year data sample). Results that are backed by statistical tests (i.e., hypotheses that allow statistical inference) are marked with a (*). The results show that in most cases indications for real-option pricing can be found in time series data. In particular the unconditional analysis and the corresponding moment tests indicate a significantly different distribution family for the tested MICEX companies (Gazprom, Lukoil and Norilsk Nickel) relative to their international peer-group, which is a strong indication that real-option pricing considerations indeed play a role in daily price discovery.

Table 93
Time series analysis indicators for real-option valuation

Analysis		General	Gazprom	Lukoil	Norilsk Nickel	
Panel A						
Unconditional Analysis	Distribution Analysis	Jarque-Bara normality test (<i>non-normal</i>)	Table 64	Table 66	Table 67	
		Indication for real-option valuation	IF / IF	IF / IF	IF / IF	
		Variance test (<i>variance above peer-group</i>)	Table 68	Table 69	Table 70	
		Indication for real-option valuation	IF* / IF*	IF* / IF*	IF* / IF*	
		Skewness test (<i>above peer-group</i>)	Table 71	Table 71	Table 71	
		Indication for real-option valuation	NIF* / IF*	NIF* / IF*	NIF* / NIF*	
		Kurtosis test (<i>above peer-group</i>)	Table 72	Table 73	Table 73	
		Indication for real-option valuation	IF* / IF*	IF* / IF*	NIF* / IF*	
	Return/Risk	Improved Sharpe ratios	Figure 112-113	Figure 112-113	Figure 114-115	
		Indication for real-option valuation	NIF / NIF	NIF / NIF	NIF / NIF	
		Higher moments adj. Sharpe ratios (<i>higher adj. risk</i>)	Figure 112-113	Figure 112-113	Figure 114-115	
		Indication for real-option valuation	IF / IF	IF / IF	IF / IF	
Panel B						
Conditional Analysis	GJR-GARCH(1,1) Analysis	Unconditional standard deviation (<i>higher std.</i>)	Table 74	Table 76	Table 78	
		Indication for real-option valuation	IF / NA	IF / NA	IF / NA	
		Kurtosis of stand. Residual (<i>higher residual kurtosis</i>)	Table 75, 77, 79	Table 74	Table 76	Table 78
		Indication for real-option valuation	IF / NA	IF / NA	NIF / NA	IF / NA
		Asymmetry factor analysis (<i>lower asymmetry coefficient</i>)	Table 75, 77, 79	Table 74	Table 76	Table 78
		Indication for real-option valuation	IF / NA	NIF / NA	NIF / NA	NIF / NA
	Sensitivity Analysis	Commodity price sensitivity (<i>commodity beta factor high</i>)	Table 83	Table 83	Table 85	
		Indication for real-option valuation	IF*	IF*	IF*	
		Asymmetry in commodity price sensitivity (<i>high asymmetry</i>)	Table 86, 88	Table 86, 88	Table 89	
		Indication for real-option valuation	IF*	IF*	NIF*	

The analysis distinguishes between five-year and six-year sample history (2003-2008/2003-2009 sample). NIF (no indication for real-option pricing found), IF (indication for real-option pricing found), NA (no analysis conducted); * conducted tests allow statistical inference.

However, it was not possible to confirm the expected improvement in the Sharpe ratios, which is likely due to the market correction between 2008 and 2009 whereby MICEX companies suffered extraordinarily (see Figure 116, p. 288).

The results of the conditional analysis are slightly more ambiguous. The GJR-GARCH(1,1) analysis, which was conducted to allow for higher moment inference similar to the analysis in the unconditional section, unveiled only partially real-option pricing characteristics. The expected lower GJR-GARCH(1,1) asymmetry factor – although lower for most emerging market peer-groups relative to development markets – did not show the expected characteristics. This could be explained by the fact that, due to stability reasons, only the five year sample was analyzed. Since commodity prices peaked in this period, this likely led to most options being in-the-money, which explains the relatively high asymmetry factor. The very high unconditional standard deviation and kurtosis for the tested MICEX companies, however, suggest that real-option characteristics can be found even if a conditional data generating process is imposed.

The commodity price sensitivity analysis has yielded the most convincing evidence of real-option pricing for the Russian Energy and Basic Materials sectors. For both sectors it was possible to test Hypothesis 1 and Hypothesis 2 via statistical inference. The results have shown that Hypothesis 1 can be accepted for both tested sectors. Hypothesis 2 was accepted for the Russian Energy sector only.

Overall, the results summarized in Table 93 show that indeed real-option pricing is an element of price discovery for Russian commodity companies. Real-option pricing can be observed in the distribution characteristics of individual Russian commodity companies; it can also be found by analyzing sector-specific sensitivity factors in different market environment (i.e., during high or low commodity price regimes), since sector sensitivity towards commodity price changes is dynamic (a characteristic that was not confirmed for non-Russian companies) as expected for real-option sensitive companies.

7.3 Complementary real-option analysis

7.3.1 Testing for unit-root

Modeling commodity prices requires first formulating a view of time series stationarity. Most forecast models assume that commodity prices are stationary, which would allow forecasting in level terms. Most of the applied stationarity tests in this dissertation, however, do not confirm this notion for most of the tested commodities (see Table 31, p. 168 and Table 32, p. 169).

We have also tested different commodity prices that have been normalized for inflation and the external value of the USD. Exchange rate normalization in particular has been useful in achieving stationarity for several of the tested commodity time series, with the exception of oil, which continued to show a unit-root (Table 33, p. 179 and Table 34, p. 179).

7.3.2 Testing different DGPs for forecasting out-of-sample

We have conducted several out-of-sample forecast tests for different commodity prices by using a variety of DGPs. We have also used inflation and exchange rate normalized commodity prices in order to account for possible cross-effects that could influence the forecast quality. The results show that mean-reverting DGPs indeed perform better than other tested DGPs over the long-term, which makes them the first choice for forecasting commodity prices. Over the short-term, additive and geometric Brownian motion forecast specifications perform well, too (see Table 38, p. 192 and Table 41, p. 195).

Overall, all applied DGPs display an unacceptable forecast bias if rolling out-of-sample forecasts are conducted (see Table 41, p. 195). Despite this inconvenient bias, nearly all tested DGPs outperform a random walk over a five-year forecast horizon (see Figure 86, p. 198).

7.3.3 Testing different DGPs for real-option valuation

Forecasting commodity prices and valuating real-options on commodity projects differ insofar as the probability distribution (i.e., forecast variance) that is attached to each

DGP plays a different role depending on whether the analysis is related to forecasting or pricing options.

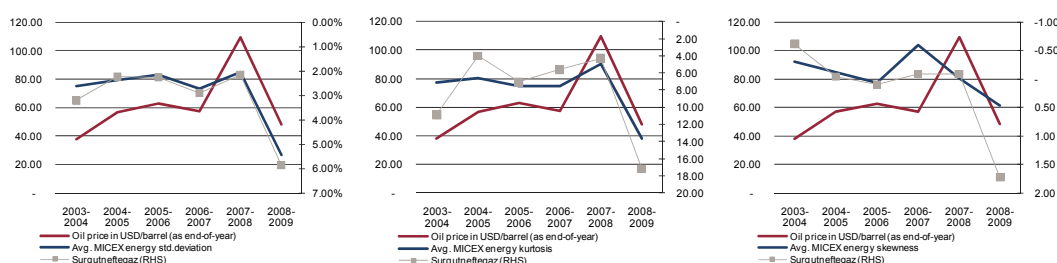
We have shown in this analysis that using mean-reversion DGPs for valuing real-options leads to misevaluation, due to the much too narrow DGP probability distribution (see Figure 110, p. 254 and Figure 111, p. 256). Simple additive or geometric Brownian motion processes perform much better and are preferable as DGPs for real-option valuation. However, we need to acknowledge that inference on this experiment is limited given the very restrictive assumption needed to conduct the analysis.

7.4 Future related research

The modeling of equity return distribution moments as a function of exogenous variables such as oil price (or any other commodity price) should be at the forefront of future research. It would help improve the pricing of financial options, especially on companies with real-option projects. As shown in Figure 129, the log-return characteristics of the average MICEX Energy sector change over time with changes in oil price (as should be the case, if indeed real-option valuation is considered). The increasing oil price (with real-options getting closer to in-the-money) is accompanied by a lower standard deviation, lesser kurtosis and lesser skewness.

Figure 129

Distribution moments of the MICEX Energy sector as a function of the oil price over time



Panel A: Standard deviation of daily-log-returns over time for the MICEX Energy sector and Surgutneftegaz relative to the development of oil.

Panel B: Skewness of daily-log-returns over time for the MICEX Energy sector and Surgutneftegaz relative to the development of oil.

Panel C: Kurtosis deviation of daily-log-returns over time for the MICEX Energy sector and Surgutneftegaz relative to the development of oil.

Instead of using a discrete rolling window to calculate the different return characteristics, one could envisage a more continuous approach that incorporates the return information of entire sectors and allows forecasting the development of distribution moments of equity returns as a function of exogenous variable such as oil. A natural starting point for modeling data-generating processes for commodity companies along with their value driving underlying is to apply bivariate GARCH models like those proposed by Baba et al. (1990), also known as BEKK models. Such models would allow extraction and comparison of conditional correlation patterns for the peer-group sample and commodity prices between individual companies.

Modeling conditional moments beyond variance has already been proposed in the literature (see Harvey and Siddique, 1999, 2000 for models of conditional skewness, and Premaratne and Bera, 2000, who propose a model for stock return data that captures time-varying behavior of mean, variance, skewness and kurtosis). However, most likely we would similarly be confronted with model stability issues for Russian companies like the one encountered in Subsection 6.2.3.

If one could overcome stability issues – maybe by extending the data sample or by applying better-fitting algorithms – it would also be straightforward to extend the significance tests for kurtosis and skewness toward the extracted standardized GJR-GARCH(1,1) residuals. This would be of particular interest, given that the asymmetry factor could help eliminate possible differences in company leverage and hence clean – i.e. purge – the time series from factors that could have biased the analysis.

A further avenue for research on real-option-driven price discovery has been initiated recently by Trigeorgis and Lambertides (2013), who link the growth vs. value risk-return puzzle, first documented by Fama and French (1992), with real-option theory by introducing a growth-option variable. Forming value vs. growth portfolios of companies using the set of peer-group data applied in this dissertation theoretically allows us to conduct a similar analysis.

Further research efforts could also be conducted with respect to GARCH option price modeling. Currently market-priced options on Russian energy companies could be used for GARCH calibration and implied return-distribution analysis. In particular, the response sensitivity of GARCH models with respect to changes in commodity prices (i.e., modeling the diffusion characteristics of GARCH models as a function of commodity prices) is a worthy subject for further analysis.

Appendix A – Comparing different valuation techniques in finance

Table 94

A summary of different valuation techniques in finance

Name	Proponents	Calculating equity value (EQV)	Advantage	Flaws
Certainty Equivalent Approach	Robichek and Myers (1966)	$EQV = \sum_{t=1}^T \frac{\overline{FCF_t}}{(1+r)^t}$ $\overline{FCF_t} = E(FCF_t) - p_t$ p_t represents a correction of the expected free cash-flow to adjust for risk. The correction/deduction allows further discounting with a risk-free interest rate.	Decomposition into effects of time (time value under certainty) and risk aversion.	Assumes constant risk resolution over time (i.e., riskiness and premium increase over time constantly; difficult to determine if risk per period changes).
Risk Adjusted Discount Rate		$EQV = \sum_{t=1}^T \frac{E(FCF_t)}{(1+k)^t}$ $k = r + p'$ Discount rate premium: p'	Discount rate accounts for both time value and risk.	Same as with CEA. If k stays constant we implicitly assume that risk is resolved continuously at the same rate.
Capital Asset Pricing Model	Fama (1977)	Total risk = market risk + firm-specific risk = Systematic risk + non-systematic risk; only systematic risk is rewarded; $E(r_j) = r_f + \beta[E(r_m) - r_f]$ $p' \equiv E(r_j) - r_f = [E(r_m) - r_f]\beta_j$ $p' = [E(r_m) - r_f] \frac{\text{cov}(r_j, r_m)}{\text{Var}(r_m)}$ $p' = \lambda \text{cov}(r_j, r_m)$ $p_t = \lambda \text{cov}(FCF_t, r_{mt})$ λ = market price of risk = excess return per additional unit of risk in the market $EQV_{t-1} = \frac{E_{t-1}(FCF_t) - \lambda \text{cov}(FCF_t, r_{mt})}{1 + r_t}$ $EQV = \sum \frac{E(c_t)}{(1+k)^t}$ According to Fama (1977) the present value of a future-free cash-flow is the current expected value discounted at the	Gives insight of required return given non-diversifiable risk and correlation to market.	Use of one discount rate assumes that relevant total risk increases at a constant rate through time or that the same incremental uncertainty is resolved in each period; Thus, risk-adjusted discount rate evolves in a deterministic fashion through time (non-stochastic); thus, λ and β develop deterministically through time; only uncertainty is size of FCF. Betas are not stable through the lifetime of the project; difficult to estimate even in

risk-adjusted discount rate given by the *CAPM*.

absence of real-options.

Sensitivity Analysis

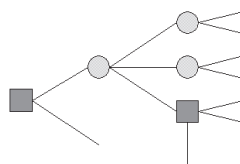
The analysis starts from a base scenario, which was originally used for valuing the company. After key variables have been initialized the equity value sensitivity concerning these key variables is evaluated. Sensitivity analysis is not a direct measure of equity value; however, it helps to assess the riskiness of a variable for the equity valuation without being concerned about the variance of the variable it self.

Easy to use; helpful to identify the risk of one variable, and misestimating certain key variables, which doesn't allow for correlated analysis; if variables serially dependent can make errors larger with time.

Decision Tree Analysis

DTA is a structured approach, which aims to split overall valuation problems and scenarios into sequential or parallel sub-problems (branches), which add up with other problems to overall trees. The result is a map of decision nodes at discrete points of time where uncertainty will be resolved. This leads to different possible outcomes of future FCFs.

They enable the consideration of option value and are therefore an improvement over standard NPV-based FCF analysis.** Assumes same risk-adjusted discount rate for any pattern. This implies, similar to *CEA*, *RADA* and *CAPM* that risk per period is constant and uncertainty is resolved continuously at a constant rate over time through the tree.



Too many branches lead to “decision bush analysis.”

These trees enable the introduction of conditional probability functions instead of static probability assumptions. Usually such trees, which picture sequences of decisions, can be solved using a backward induction procedure where the expanded NPV (i.e., the static NPV + abandonment value) is maximized.

A continuous time version of *DTA* would be an improvement.

Sources: Fischer (1996a), Damodaran (2002) and Copeland, Weston and Shastri (2005).

* For an all-equity financed company.

** Trigeorgis and Mason (1987) discuss real-option valuation as an economically corrected version of decision-tree analysis.

Appendix B – Peer group multiple definition²²⁵

Estimated price to earnings ratio to growth (PEG): A company's estimated P/E ratio for the specified period divided by the company's estimated long term growth (LTG) rate.

The price/cash-flow ratio: The ratio of a stock's price divided by the cash-flow per share. Average shares outstanding is used when calculating cash-flow per share. Cash-flow per share is calculated on a trailing 12-month basis where available. Trailing values are calculated by adding the most recent four quarters.

Enterprise Value: The sum of market capitalization, preferred equity, and short- and long-term interest-bearing debt, less cash and equivalents. Used as a proxy for the takeover value of a firm. Often used in conjunction with EBITDA (earning before interest, taxes, depreciation, and amortization) as an alternative to the P/E ratio.

Operating Margin: A determination of the expense margin, operating margin is operating income as a percentage of net sales. Operating income is equal to net sales minus cost of goods sold minus selling, general, and administrative expenses.

Gross margin (in percentage): Calculated using the following formula:

$$\frac{\text{Net Sales} - \text{Cost of Goods Sold}}{\text{Net Sales}} \times 100 \quad (169)$$

Gross margin is left blank when a company has service-oriented businesses and breakdown of operating expenses between cost of goods sold, selling and general and administrative expenses is not reported.

²²⁵ The source of all multiple definitions is Bloomberg.

Return on equity (in percentage): Calculated as trailing 12-month net income (losses) minus trailing 12-month cash-preferred dividends, divided by average of total common equity, times 100.

Total Common Equity = Share capital and additional amount paid in capital + retained earnings

Additional Paid In Capital = Share premium

Average of Total Common Equity: The average of the beginning balance and ending balance. If either the beginning or ending common equity is negative, ROE is not calculated.

Cash Dividend Coverage: Calculated using the following formula:

$$\frac{\text{Income before XO} - \text{Minority Interest} - \text{Cash Preferred Dividends}}{\text{Cash Common Dividends}} \times 100 \quad (170)$$

Cash Dividend Coverage is not computed if the numerator is negative.

Appendix C – Deriving a PDE for the Bellman Equation in Continuous Time

We start with the Bellman Equation for optimality in a continuous time- and risk-neutral framework (see Equation (171)):

$$rV(x, t) = \max_{\kappa} \left[\pi(x, \kappa, t) + \frac{1}{dt} E(dV | \kappa) \right] \quad (171)$$

and the SDE for an Ito process²²⁶

$$dx = a(x, \kappa, t)dt + b(x, \kappa, t)dz \quad (172)$$

The function $V(x, t)$ – value for the firm/asset – is described as the stochastic state variable x and time t , whereas the immediate profit/loss function $\pi(x, \kappa, t)$ depends on x, κ and t . Note further that dV is a function of x and t and its expectation value $E(dV)$ is still conditional on κ . The control variable κ can be either zero or one or any other discrete function. Further, since $dV(x, t)$ is not differentiable in time we use Ito's lemma and derive the stochastic differential Equation for the process dV as a function of the underlying stochastic state variables x and t . According to Ito's lemma the PDE for dV is described by:

$$dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial t} dt + \frac{1}{2} \frac{\partial^2 V}{\partial x^2} (dx)^2 \quad (173)$$

²²⁶ Note that this SDE deviates from a standard Ito process. The drift and diffusion parameters are functions of not only the state variable x and time t but also the control variable u . However, the control variable is not a stochastic variable in this context. Ito's Lemma can be "easily" extended for more than one stochastic state variable.

Taking the SDE in Equation (172) and substituting dx in Equation (173) results in a new, extended SDE as given below:

$$dV = \frac{\partial V}{\partial x} a(x, \kappa, t) dt + \frac{\partial V}{\partial t} dt + \frac{\partial V}{\partial x} b(x, \kappa, t) dz + \frac{1}{2} \frac{\partial^2 V}{\partial x^2} b^2(x, \kappa, t) dt \quad (174)$$

Note that dx^2 results in $b^2 dt$ as dt goes to zero given that the random term $\varepsilon \left(dz = \varepsilon_t \sqrt{dt} \right)$ is $N(0,1)$ distributed, which leads to the variance of $E(\varepsilon^2) = 1$.²²⁷ Rearranging Equation (174) results in a new process for dV with extended drift and diffusion parameters.

$$dV = \left[\frac{\partial V}{\partial x} a(x, \kappa, t) + \frac{\partial V}{\partial t} + \frac{1}{2} \frac{\partial^2 V}{\partial x^2} b^2(x, \kappa, t) \right] dt + \frac{\partial V}{\partial x} b(x, \kappa, t) dz \quad (175)$$

Taking the expectation value $E(dV)$ of the above process and considering $E(dz) = 0$ we obtain the following result:

$$E(dV) = \left[\frac{\partial V}{\partial x} a(x, \kappa, t) + \frac{\partial V}{\partial t} + \frac{1}{2} \frac{\partial^2 V}{\partial x^2} b^2(x, \kappa, t) \right] dt \quad (176)$$

The expected value is simply the drift of the given process. Now Equation (171) can be substituted through Equation (176), which yields Equation (177) from the text.

$$rV(x, t) = \max_{\kappa} \left[\pi(x, \kappa, t) + \frac{\partial V}{\partial x} a(x, \kappa, t) + \frac{\partial V}{\partial t} + \frac{1}{2} \frac{\partial^2 V}{\partial x^2} b^2(x, \kappa, t) \right] \quad (177)$$

²²⁷ See Hull (2000), p. 236 for a further proof.

Appendix D – Deriving the OLS-based parameterization for an arithmetic Ornstein-Uhlenbeck process

To find the parameter values of the popular arithmetic Ornstein-Uhlenbeck process, we again start with the stochastic differential Equation:

$$dz = \eta(\bar{z} - z_t)dt + \sigma\sqrt{dt}\varepsilon_t \quad (178)$$

As given in Table 29, page 144, closed-form solutions exist for the expectation value $E(z_T)$ and a variance $\text{Var}(z_T)$ of an arithmetic O-U process (see Table 95 below). They are the starting point for the derivation.

Table 95
Ornstein-Uhlenbeck Process

Description (moments)	Ornstein-Uhlenbeck Process
Mean: $E(z_T)$	$\bar{z} + (z_0 - \bar{z})\exp(-\eta T)$
Variance: $\text{Var}(z_T)$	$\frac{\sigma}{2\eta}(1 - \exp(-2\eta t))$

Starting with the expectation value we rewrite the mean Equation:

$$E(z_{0+T}) = \bar{z} + (z_0 - \bar{z})e^{(-\eta T)} \quad (179)$$

$$E(z_{0+T}) = \bar{z} + z_0 e^{(-\eta T)} - \bar{z} e^{(-\eta T)} \quad (180)$$

We set time T equal to 1 and $z_0 = z_t$.

$$E(z_{t+1}) = \bar{z} + z_t e^{(-\eta)} - \bar{z} e^{(-\eta)} \quad (181)$$

After deducting z_t from both sides we rearrange and write:

$$E(z_{t+1}) - z_t = \bar{z} + z_t e^{(-\eta)} - \bar{z} e^{(-\eta)} - z_t \quad (182)$$

$$E(z_{t+1}) - z_t = (1 - e^{(-\eta)})\bar{z} + (e^{(-\eta)} - 1)z_t \quad (183)$$

Since one needs historical data to conduct the parameterization the time index is adjusted to obtain:

$$z_t - z_{t-1} = (1 - e^{(-\eta)})\bar{z} + (e^{(-\eta)} - 1)z_{t-1} \quad (184)$$

Equation (184) already possesses the elements of an autoregressive data-generating process. We have a constant term and a lagged dependent variable. Writing the above Equation in conjunction with an OLS model unveils the relation:

$$\begin{aligned} z_t - z_{t-1} &= (1 - e^{(-\eta)})\bar{z} + (e^{(-\eta)} - 1)z_{t-1} \\ &\Downarrow \\ z_t - z_{t-1} &= c + \phi z_{t-1} + \varepsilon_t \end{aligned} \quad (185)$$

The OLS autoregressive factor ϕ contains information about the mean reversion speed η , while the intercept c contains information about the long-term mean convergence level $\bar{z}$.

$$\begin{aligned} \phi &= (e^{(-\eta)} - 1) \Rightarrow \hat{\eta} = -\ln(1 + \phi) \\ c &= (1 - e^{(-\eta)})\bar{z} \Rightarrow \hat{\bar{z}} = \frac{c}{(1 - e^{(-\eta)})} = \frac{c}{-\phi} \end{aligned} \quad (186)$$

To obtain a parameter estimate σ of the diffusion process we utilize the closed-form solution for the variance of the estimated underlying $\text{Var}(z_T)$.

$$\text{Var}(z_T) = \frac{\sigma}{2\eta} (1 - \exp(-2\eta t)) \quad (187)$$

Since we have for the OLS regression a standard error σ_ε , we can set Equation (187) equal to the standard error of the regression and solve for σ .

$$\hat{\sigma} = \hat{\sigma}_\varepsilon \sqrt{\frac{2 \ln(1 + \phi)}{(1 + \phi)^2 - 1}} \quad (188)$$

Appendix E – Deriving the stochastic differential Equation for a geometric Ornstein-Uhlenbeck process via Ito

The standard Ornstein-Uhlenbeck process is defined as:

$$dz = z_{t+1} - z_t = \eta(\bar{z} - z_t)dt + \sigma\sqrt{dt}\varepsilon_t \quad (189)$$

In this context, the SDE is of an additive nature and would allow process negativity, which is theoretically impossible for most assets under consideration. To avoid negativity one can use the geometric form of the Ornstein-Uhlenbeck process (similar to the geometric form of a standard Brownian motion). The geometric version of the O-U process is given below:

$$dz = z\eta(\bar{z} - z_t)dt + z\sigma\sqrt{dt}\varepsilon_t \quad (190)$$

In this form a geometric O-U process complies with the definition of an Ito process:

$$\begin{aligned} dz &= a(z, t)dt + b(z, t)\sqrt{dt}\varepsilon_t \\ a(z, t) &= z\eta(\bar{z} - z_t) \\ b(z, t) &= z\sigma \end{aligned} \quad (191)$$

Following Ito's Lemma²²⁸ we know that every derivative function $G(z, t)$ of a process that belongs to the family of Ito processes (like the geometric O-U process) can be written as the following stochastic differential Equation:

$$dG = \left(\frac{\partial G}{\partial z}a + \frac{\partial G}{\partial t} + \frac{1}{2} \frac{\partial^2 G}{\partial z^2}b^2 \right) dt + \frac{\partial G}{\partial z}b\sqrt{dt}\varepsilon_t \quad (192)$$

²²⁸ See Hull (2000), p. 229.

In case of the geometric O-U process we define the derivative function as:

$$G = \ln z \Rightarrow dG = \ln z_{t+1} - \ln z_t = \ln \left(\frac{z_{t+1}}{z_t} \right) \quad (193)$$

Applying Ito's Lemma:

$$\begin{aligned} \frac{\partial G}{\partial z} a &= \frac{1}{z} z \eta (\bar{z} - z_t) \\ \frac{\partial^2 G}{\partial x^2} b^2 &= -\frac{1}{z^2} z^2 \sigma^2 \\ \frac{\partial G}{\partial t} &= 0 \end{aligned} \quad (194)$$

The partial derivative elements are reassembled in the Ito stochastic differential Equation, which results in the following Equation:

$$dG = \left(\eta (\bar{z} - z_t) - \frac{1}{2} \sigma^2 \right) dt + \sigma \sqrt{dt} \varepsilon_t = d \ln(z_t) = \ln \left(\frac{z_{t+1}}{z_t} \right) \quad (195)$$

Note that the diffusion process is not altered by the geometric nature of the process. The drift term remains path-dependent, which is the general nature of mean-reverting processes. A further simplification of the stochastic differential Equation in (195) gives us additional insight as to how to understand the process dynamic. After multiplying the variance correction term with the mean reversion speed η we can rearrange Equation (195):

$$d \ln z_t = \left(\eta \bar{z} - \eta z_t - \frac{1}{2} \eta \sigma^2 \right) dt + \sigma \sqrt{dt} \varepsilon_t \quad (196)$$

with:

$$\alpha = \bar{z} - \frac{1}{2} \sigma^2 \quad (197)$$

to finally get:

$$d \ln z_t = \eta (\alpha - z_t) dt + \sigma \sqrt{dt} \varepsilon_t \quad (198)$$

Similar to the geometric Brownian motion, the expectation value for the underlying z governed by a geometric O-U process can be written as:

$$E(z_{t+\Delta t}) = z_t e^{\left(\left(\eta(\bar{z} - z_t) - \frac{1}{2}\sigma^2 \right) \Delta t + \sigma \sqrt{\Delta t} \varepsilon_t \right)} \quad (199)$$

The parameterization of the above process, however, is not as straightforward as in the additive O-U case. The problem arises due to the mixture of parameters in log and non-log form. Recalling the results from Appendix D – Deriving the OLS-based parameterization for an arithmetic Ornstein-Uhlenbeck process in order to estimate the O-U parameters, we need the following format:

$$dz = \eta(\bar{z} - z_t) dt + \sigma \sqrt{dt} \varepsilon_t \quad (200)$$

The expectation value and variance of the underlying variable are shown in Table 96.

Table 96
Ornstein-Uhlenbeck Process

Description (moments)	Ornstein-Uhlenbeck Process
Mean: $E(z_T)$	$\bar{z} + (z_0 - \bar{z}) \exp(-\eta T)$
Variance: $\text{Var}(z_T)$	$\frac{\sigma}{2\eta} (1 - \exp(-2\eta t))$

The transformation of the mean equation to an OLS regression equation was established by:

$$\begin{aligned}
 z_t - z_{t-1} &= (1 - e^{(-\eta)}) \bar{z} + (e^{(-\eta)} - 1) z_{t-1} \\
 &\Downarrow \\
 z_t - z_{t-1} &= c + \phi z_{t-1} + \varepsilon_t
 \end{aligned} \quad (201)$$

The left- and right-hand side of the Equations are specified in identical terms (either both logarithmic or both non-logarithmic). Hence, in order to use the OLS parameterization framework, one needs to keep the O-U setup additive. The transformation of the geometric process into an additive version can be conducted via taking the logarithm ($G = \ln z_t$) and running the OLS regression on the log values:

$$\begin{aligned} d \ln z_t &= \eta^* (\alpha^* - \ln z_t) dt + \sigma \sqrt{dt} \varepsilon_t \\ &\Downarrow \\ dG &= \eta^* (\alpha^* - G_t) dt + \sigma \sqrt{dt} \varepsilon_t \end{aligned} \quad (202)$$

Although this simple transformation appears attractive, it allows us to conduct only limited inference on the parameters of the original geometric O-U stochastic differential equation $dz = z\eta(\bar{z} - z_t)dt + \sigma\sqrt{dt}\varepsilon_t$ given in Equation (190). The parameters η, α in Equation (198) are the precise parameters of the geometric O-U specification that we are looking for. We know that after the conducted Ito transformation the original geometric O-U SDE becomes $d \ln z_t = \eta(\alpha - z_t)dt + \sigma\sqrt{dt}\varepsilon_t$. Setting Equation (198) equal to (202) ($d \ln z_t = dG$) yields:

$$\eta(\alpha - z_t)dt + \sigma\sqrt{dt}\varepsilon_t = \eta^*(\alpha^* - \ln z_t)dt + \sigma\sqrt{dt}\varepsilon_t \quad (203)$$

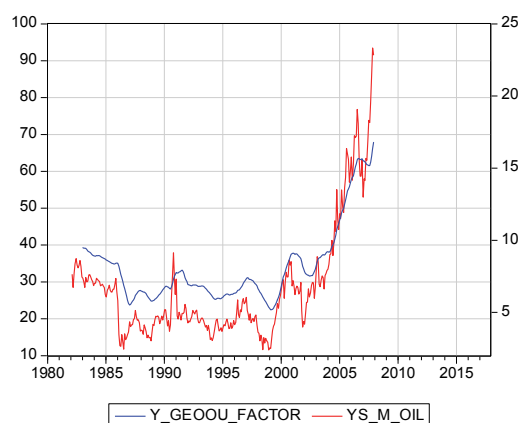
The solution to this Equation is found by setting:²²⁹

$$\begin{aligned} \eta &= \eta^* \frac{\ln z_t}{z_t} \\ \alpha &= \alpha^* \frac{z_t}{\ln z_t} = \bar{z} - \frac{1}{2\eta} \sigma^2 \end{aligned} \quad (204)$$

²²⁹ See www.puc-rio.br/marco.ind/revers.html

Hence, although it is straightforward to estimate the parameters α^*, η^* via the standard OLS regression described in Appendix D, the geometric O-U process defined in Equation (198) has path-dependent mean reversion speed (η) and mean level (α), making the parameters not constant. Practitioners argue that quotient $z_t / \ln z_t$ is fairly stable and can be approximated by recent historical data.²³⁰ Such an argument was certainly valid between 1985 and 2005; however, recent history shows us that this stability seems to have ended, with oil reaching 100 USD per barrel, making such an approximation sub-optimal.²³¹

Figure 130
Oil price relative to geometric Ornstein-Uhlenbeck $z_t / \ln z_t$
coefficient



In Figure 130 we have illustrated the adjustment factor $z_t / \ln z_t$ calculated with 12-month moving average prices. As illustrated, the adjustment factor was fairly stable during the last 20 years, however, since 2002 this stability has come to an end.

²³⁰ See www.puc-rio.br/marco.ind/revers.html.

²³¹ The first time WTI Crude futures reached 100 USD per barrel was in January 2008.

Appendix F – Deriving the stochastic differential Equation for a log-normal Ornstein-Uhlenbeck process via Ito

As described in Appendix E, the derivation of the geometric Ornstein-Uhlenbeck process poses some problems regarding the possibility of deriving path-independent parameters for the geometric O-U stochastic differential Equation. The alternative adopted by Schwarz (1997) and others is to use instead the geometric O-U process as an alternative version with similar characteristics. To recall, the geometric O-U process is given by:

$$dz = z\eta(\bar{z} - z_t)dt + z\sigma\sqrt{dt}\varepsilon_t \quad (205)$$

The alternative to this standard process introduced primarily by Dixit and Pindyck (1994)²³² is the following stochastic differential Equation:

$$dz = z\eta(\mu - \ln z_t)dt + z\sigma\sqrt{dt}\varepsilon_t \quad (206)$$

The use of the logarithm of the current observation $\ln z_t$ instead of z_t has the advantage that throughout the Ito transformation no path dependency (time dependency) is introduced. By setting $G = \ln z_t$ the Ito transformation yields the following result:

$$dG = \left(\frac{\partial G}{\partial z}a + \frac{\partial G}{\partial t} + \frac{1}{2} \frac{\partial^2 G}{\partial z^2}b^2 \right) dt + \frac{\partial G}{\partial z}b\sqrt{dt}\varepsilon_t \quad (207)$$

$$dG = \left(\eta\bar{z} - \eta \ln z_t - \frac{1}{2} \frac{\eta}{\sigma^2} \sigma^2 \right) dt + \sigma\sqrt{dt}\varepsilon_t \quad (208)$$

²³² See Dixit and Pindyck (1994), p. 85 for alternative stochastic processes.

As in Appendix E, by setting

$$\alpha = \mu - \frac{1}{2\eta} \sigma^2 \quad (209)$$

we end up with the following final stochastic differential Equation:

$$dG = d \ln z_t = \eta(\alpha - \ln z_t) dt + \sigma \sqrt{dt} \varepsilon_t \quad (210)$$

$$dG = \eta(\alpha - G_t) dt + \sigma \sqrt{dt} \varepsilon_t \quad (211)$$

Since the underlying variable is now represented in logarithmic terms on both sides of the equation, it is possible to run the conventional OLS regression applied for the additive O-U process in Appendix D.

$$\ln z_t - \ln z_{t-1} = c + \phi \ln z_{t-1} + \varepsilon_t \quad (212)$$

The calculation of the parameters η and G is analogous to that in Equation (185). As for the additive O-U case we calculate:

$$\begin{aligned} \phi &= (e^{(-\eta)} - 1) \Rightarrow \hat{\eta} = -\ln(1 + \phi) \\ c &= (1 - e^{(-\eta)})\alpha \Rightarrow \hat{\alpha} = \frac{c}{(1 - e^{(-\eta)})} = \frac{c}{-\phi} = \mu - \frac{1}{2\eta} \sigma^2 \end{aligned} \quad (213)$$

The mean revision speed is identical to the additive O-U case. The long-term mean factor, however, has to be adjusted due to the Ito transformation of the log process. Similar to the log-normal Brownian motion case, we can calculate the expectation value for $t + \Delta t$ as:

$$E(z_{t+\Delta t}) = z_t e^{\left(\left(\eta(\alpha - \ln z_t) - \frac{1}{2} \sigma^2 \right) \Delta t + \sigma \sqrt{\Delta t} \varepsilon_t \right)} \quad (214)$$

Appendix G – Stationarity and out-of-sample test results for major commodity prices between 1973-2008

Stationarity tests:

Bold numbers indicate non-confirmation of ADF and KPSS tests. Green highlighted values indicate stationarity confirmation of ADF and KPSS test. Other numbers indicate non-stationarity confirmed by ADF and KPSS tests.

Table 97

ADF unit root tests for major commodity prices over different horizons
(Normalized data via USD trade-weighted exchange rate)

Formula: $ys\_m\_comdy * ys\_infl\_index\_comdy / 100$

		Gold	Silver	Nickel	Oil	Steel
1973-2008	intercept	0.2658	0.0041***			
	time trend	0.5243	0.0181**			
1982-2008	intercept	0.8896	0.3507	0.4436	0.9001	
	time trend	0.9960	0.7756	0.4633	0.9271	
1987-2008	intercept	0.9931	0.7898	0.4493	0.9586	0.0958*
	time trend	0.9983	0.7449	0.5834	0.7508	0.2193

The p-values are based on MacKinnon (1996)

* Unit root can be rejected at the 10% level

** Unit root can be rejected at the 5% level

*** Unit root can be rejected at the 1% level

		Gold	Silver	Nickel	Oil	Steel
1973-2008	intercept	0.4178*	0.3525*			
	time trend	0.2853***	0.2078**			
1982-2008	intercept	0.5420**	0.5141**	0.6952**	0.6627**	
	time trend	0.4059***	0.3984***	0.2587***	0.4532***	
1987-2008	intercept	0.4425*	0.6968**	0.7122**	1.4210***	0.4498*
	time trend	0.3812***	0.2989***	0.3287***	0.4328***	0.2914***

The p-values are based on MacKinnon (1996)

* Stationarity is rejected at the 10% level

** Stationarity is rejected at the 5% level

*** Stationarity is rejected at the 1% level

	intercept	+time trend
Asymptotic critical values (a): 1% level	0.739	0.216
5% level	0.463	0.146
10% level	0.347	0.119

(a) Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)

Table 98
ADF unit root tests for major commodity prices over different horizons
(Normalized data via inflation)

Formula: $ys\_m\_cmdty * ys\_infl\_index\_cmdty / 100$

		Gold	Silver	Nickel	Oil	Steel
1973-2008	intercept	0.2376	0.0069**			
	time trend	0.3172	0.0039*			
1982-2008	intercept	0.4410	0.2058	0.6986	0.7172	
	time trend	0.9903	0.7306	0.8365	0.9495	
1987-2008	intercept	0.8394	0.7399	0.6975	0.9638	0.7399
	time trend	0.9999	0.9558	0.8839	0.9477	0.9558

The p-values are based on MacKinnon (1996)

* Unit root can be rejected at the 10% level

** Unit root can be rejected at the 5% level

*** Unit root can be rejected at the 1% level

		Gold	Silver	Nickel	Oil	Steel
1973-2008	intercept	0.9221***	1.1820***			
	time trend	0.1741**	0.1716**			
1982-2008	intercept	1.2913***	0.9346***	0.3122	0.4482**	
	time trend	0.3303***	0.3817***	0.2273***	0.4409***	
1987-2008	intercept	0.8240***	0.3464	0.3881*	0.9717***	0.5106**
	time trend	0.3964***	0.3328***	0.3363***	0.4421***	0.3050***

The p-values are based on MacKinnon (1996)

* Stationarity is rejected at the 10% level

** Stationarity is rejected at the 5% level

*** Stationarity is rejected at the 1% level

	intercept	+time trend
Asymptotic critical values (a): 1% level	0.739	0.216
5% level	0.463	0.146
10% level	0.347	0.119

(a) Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)

Table 99

ADF unit root tests for major commodity prices over different horizons
(Normalized data via USD trade-weighted exchange rate and inflation)

Formula: $ys\_m\_comdy*ys\_infl\_index\_comdy/100*ys\_m\_usdtw\_mc/100$

		Gold	Silver	Nickel	Oil	Steel
1973-2008	intercept	0.3655	0.0116**			
	time trend	0.2832	0.0028***			
1982-2008	intercept	0.3823	0.2753	0.0950*	0.3075	
	time trend	0.9626	0.6955	0.6094	0.8658	
1987-2008	intercept	0.0975*	0.2554	0.1603	0.7240	0.1810
	time trend	0.9677	0.6989	0.3729	0.5257	0.3653

The p-values are based on MacKinnon (1996)

* Unit root can be rejected at the 10% level

** Unit root can be rejected at the 5% level

*** Unit root can be rejected at the 1% level

		Gold	Silver	Nickel	Oil	Steel
1973-2008	intercept	1.2061***	1.3099***			
	time trend	0.2266***	0.1912**			
1982-2008	intercept	1.4572***	1.0779***	0.2434	0.6240**	
	time trend	0.4645***	0.4035***	0.2472***	0.4450***	
1987-2008	intercept	1.0255***	0.3739**	0.3589**	1.0190***	0.6502**
	time trend	0.4079***	0.3185***	0.3298***	0.4303***	0.2993***

The p-values are based on MacKinnon (1996)

* Stationarity is rejected at the 10% level

** Stationarity is rejected at the 5% level

*** Stationarity is rejected at the 1% level

	intercept	+time trend
Asymptotic critical values (a): 1% level	0.739	0.216
5% level	0.463	0.146
10% level	0.347	0.119

(a) Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)

Table 100
ADF unit root tests for major commodity prices over different horizons
(Normalized data via logarithm)

Formula: $\log(y_{s\_m\_comdy})$

		Gold	Silver	Nickel	Oil	Steel
1973-2008	intercept	0.0267**	0.0710*			
	time trend	0.1155	0.2327			
1982-2008	intercept	0.9766	0.6287	0.8094	0.7859	
	time trend	0.9966	0.8841	0.7540	0.6731	
1987-2008	intercept	0.9963	0.8961	0.6075	0.8627	0.2138
	time trend	0.9996	0.9071	0.7953	0.5866	0.4296

The p-values are based on MacKinnon (1996)

* Unit root can be rejected at the 10% level

** Unit root can be rejected at the 5% level

*** Unit root can be rejected at the 1% level

		Gold	Silver	Nickel	Oil	Steel
1973-2008	intercept	0.9272***	0.1970**			
	time trend	0.3076***	0.2175***			
1982-2008	intercept	0.2723	0.3689*	0.8598***	0.8555***	
	time trend	0.2692***	0.3696***	0.2109**	0.4166***	
1987-2008	intercept	0.3939*	0.6570**	0.6483**	1.3382***	0.3471*
	time trend	0.3698***	0.3099***	0.3293***	0.4076***	0.2775***

The p-values are based on MacKinnon (1996)

* Stationarity is rejected at the 10% level

** Stationarity is rejected at the 5% level

*** Stationarity is rejected at the 1% level

	intercept	+time trend
Asymptotic critical values (a): 1% level	0.739	0.216
5% level	0.463	0.146
10% level	0.347	0.119

(a) Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)

Table 101

ADF unit root tests for major commodity prices over different horizons
(Normalized data via logarithm, USD trade-weighted exchange rate and inflation – Variant I)

Formula: $\log(ys\_m\_comdty*ys\_infl\_index\_comdty/100)*ys\_m\_usdtw\_mc/100$

		Gold	Silver	Nickel	Oil	Steel
1973-2008	intercept	0.7451	0.6698			
	time trend	0.5071	0.4419			
1982-2008	intercept	0.7733	0.6605	0.4693	0.3399	
	time trend	0.8858	0.8594	0.5385	0.8019	
1987-2008	intercept	0.0133**	0.0221**	0.0601*	0.0959*	0.0817*
	time trend	0.0346**	0.1024	0.1967	0.0978*	0.1080

The p-values are based on MacKinnon (1996)

* Unit root can be rejected at the 10% level

** Unit root can be rejected at the 5% level

*** Unit root can be rejected at the 1% level

		Gold	Silver	Nickel	Oil	Steel
1973-2008	intercept	1.3344***	1.5631***			
	time trend	0.2297***	0.2142**			
1982-2008	intercept	1.2807***	1.0715***	0.9260***	0.7709***	
	time trend	0.3900***	0.3912***	0.3355***	0.4195***	
1987-2008	intercept	0.8120***	0.2679	0.2017	0.7389**	0.5314**
	time trend	0.2208***	0.2175***	0.2112**	0.3001***	0.2096**

The p-values are based on MacKinnon (1996)

* Stationarity is rejected at the 10% level

** Stationarity is rejected at the 5% level

*** Stationarity is rejected at the 1% level

	intercept	+time trend
Asymptotic critical values (a): 1% level	0.739	0.216
5% level	0.463	0.146
10% level	0.347	0.119

(a) Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)

Table 102

ADF unit root tests for major commodity prices over different horizons
(Normalized data via logarithm, USD trade-weighted exchange rate and inflation – Variant II)

Formula: $\log(ys\_m\_cmdty) * ys\_m\_usdtw\_mc / 100$

		Gold	Silver	Nickel	Oil	Steel
1973-2008	intercept	0.3467	0.4051			
	time trend	0.5430	0.4423			
1982-2008	intercept	0.6525	0.6222	0.3649	0.3815	
	time trend	0.7370	0.8056	0.5038	0.7631	
1987-2008	intercept	0.0237**	0.0521*	0.0724*	0.3416	0.0438**
	time trend	0.1032	0.1621	0.2296	0.1018	0.1624

The p-values are based on MacKinnon (1996)

* Unit root can be rejected at the 10% level

** Unit root can be rejected at the 5% level

*** Unit root can be rejected at the 1% level

		Gold	Silver	Nickel	Oil	Steel
1973-2008	intercept	0.3469*	0.7039**			
	time trend	0.2427***	0.1994**			
1982-2008	intercept	0.9743***	0.8112***	0.6468**	0.4339*	
	time trend	0.3367***	0.3579***	0.3090***	0.3931***	
1987-2008	intercept	0.1857	0.2854	0.3418	1.2831***	0.2059
	time trend	0.2006**	0.2131***	0.2021**	0.2253***	0.2030**

The p-values are based on MacKinnon (1996)

* Stationarity is rejected at the 10% level

** Stationarity is rejected at the 5% level

*** Stationarity is rejected at the 1% level

	intercept	+time trend
Asymptotic critical values (a): 1% level	0.739	0.216
5% level	0.463	0.146
10% level	0.347	0.119

(a) Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)

Table 103

ADF unit root tests for major commodity prices over different horizons
(Normalized data via logarithm, USD trade-weighted exchange rate and inflation – Variant III)

Formula: $\log(ys\_m\_comdty \cdot ys\_infl\_index\_comdty / 100 \cdot ys\_m\_usdtw\_mc / 100)$

		Gold	Silver	Nickel	Oil	Steel
1973-2008	intercept	0.4103	0.4952			
	time trend	0.1779	0.4290			
1982-2008	intercept	0.4484	0.3576	0.3066	0.2745	
	time trend	0.9963	0.9206	0.6229	0.7511	
1987-2008	intercept	0.2425	0.2959	0.5578	0.4679	0.1159
	time trend	0.9876	0.7393	0.8245	0.3610	0.2707

The p-values are based on MacKinnon (1996)

* Unit root can be rejected at the 10% level

** Unit root can be rejected at the 5% level

*** Unit root can be rejected at the 1% level

		Gold	Silver	Nickel	Oil	Steel
1973-2008	intercept	1.4329***	1.7505***			
	time trend	0.2679***	0.2815**			
1982-2008	intercept	1.5031***	1.1371***	0.2680	0.5593**	
	time trend	0.4435***	0.4415***	0.2729***	0.4576***	
1987-2008	intercept	1.0296***	0.3535*	0.3547*	0.8938***	0.6410**
	time trend	0.4076***	0.3082***	0.3396***	0.4127***	0.3177***

The p-values are based on MacKinnon (1996)

* Stationarity is rejected at the 10% level

** Stationarity is rejected at the 5% level

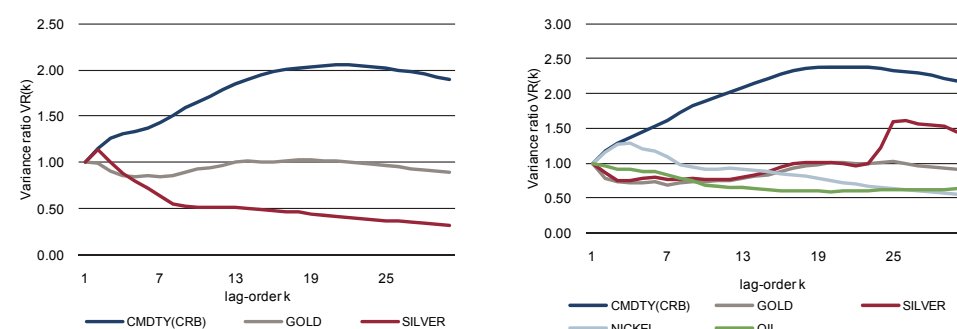
*** Stationarity is rejected at the 1% level

	intercept	+time trend
Asymptotic critical values (a): 1% level	0.739	0.216
5% level	0.463	0.146
10% level	0.347	0.119

(a) Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)

Figure 131

Variance ratio for major commodities and CRB commodity index



Panel A: Variance ratio analysis (sample 1973-2008)

Panel B: Variance ratio analysis (sample 1982-2008)

Out-of-sample tests for normalized commodity prices:

Table 104

Theil coefficient of accuracy with bias component for exchange-rate normalized commodity prices using rolling 60-month out-of-sample forecasts between 2003 and 2008

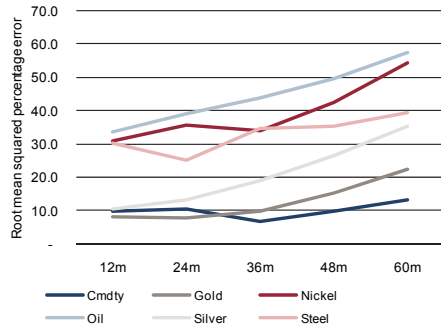
Panel A								
accuracy measure:	Theil							
total (vertical) average	Cmdty	Gold	Nickel	Oil	Silver	Steel		
AddBM	0.04	0.07	0.27	0.28	0.11	0.19		
GeoBM	0.04	0.05	0.27	0.26	0.11	0.19		
AOUP	0.03	0.06	0.25	0.25	0.14	0.15		
LNOUN	0.03	0.06	0.26	0.26	0.11	0.16		
Trend_t	0.04	0.15	0.24	0.53	0.13	0.20		
Trend_t_In	0.04	0.15	0.26	0.44	0.11	0.19		
Trend_t_2	0.12	0.40	0.36	0.12	0.44	0.20		
Trend_t_2_In	0.14	0.59	0.45	0.09	0.91	0.19		
MixModels	0.05	0.10	0.27	0.33	0.17	0.24		
MixModels_In	0.05	0.12	0.28	0.32	0.17	0.23		

Panel B								
accuracy measure:	Bias							
total (vertical) average	Cmdty	Gold	Nickel	Oil	Silver	Steel		
AddBM	0.49	0.46	0.48	0.55	0.39	0.47		
GeoBM	0.47	0.43	0.48	0.55	0.44	0.47		
AOUP	0.30	0.41	0.42	0.61	0.59	0.34		
LNOUN	0.29	0.42	0.46	0.65	0.50	0.36		
Trend_t	0.49	0.74	0.40	0.86	0.41	0.37		
Trend_t_In	0.50	0.74	0.45	0.84	0.26	0.37		
Trend_t_2	0.83	0.87	0.64	0.46	0.87	0.47		
Trend_t_2_In	0.83	0.85	0.69	0.18	0.89	0.46		
MixModels	0.57	0.50	0.47	0.64	0.60	0.49		
MixModels_In	0.55	0.55	0.49	0.67	0.56	0.48		

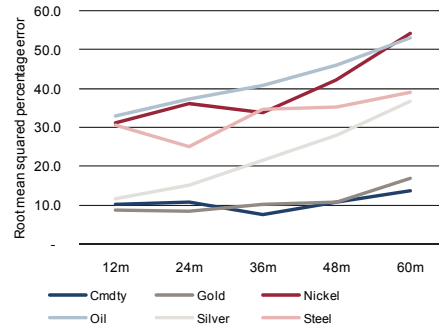
The best results are highlighted with a dark green background, whereas DGPs with scores within the top 15 percentile are highlighted in plain green (within the 30th percentile are highlighted with light green). All test statistics represent vertical (full sample path) measures; they are hence indicators of the ability to forecast over the entire forecast path of 60 months.

Figure 132

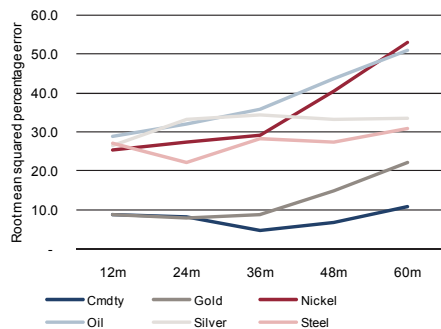
Forecast error (measured as RMSPE) over time for tested exchange-rate normalized commodity prices (rolling out-of-sample forecasts between 2003 and 2008)



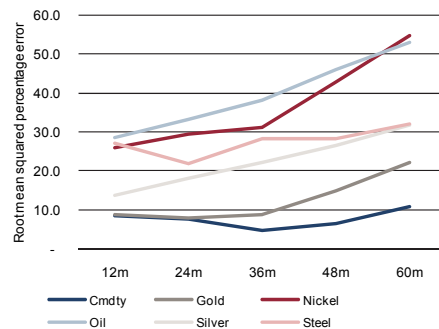
Panel A: Additive Brownian motion (ABM).



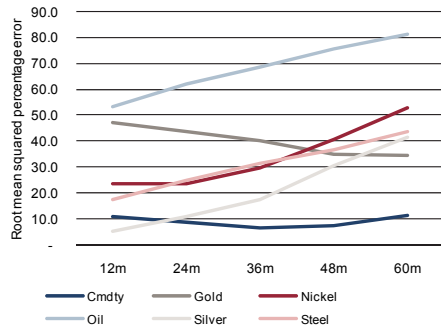
Panel B: Geometric Brownian motion (GBM).



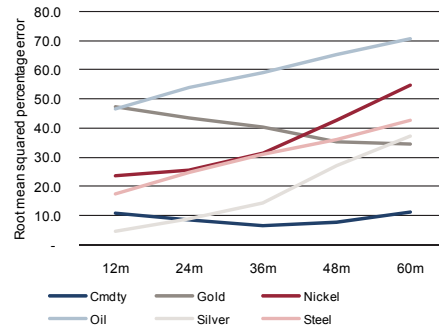
Panel C: Additive Ornstein-Uhlenbeck process (AOUP).



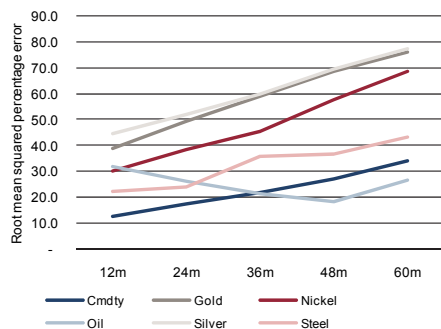
Panel D: Log-normal Ornstein-Uhlenbeck process (LnOUP).



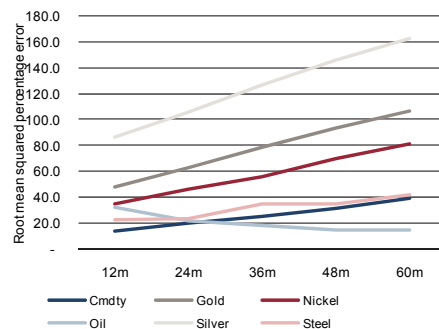
Panel E: Trend (time) stationary process (deterministic trend model).



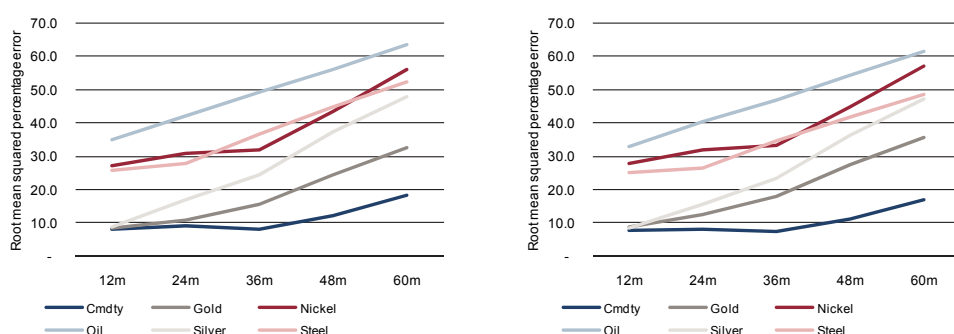
Panel F: Trend (time) stationary process in log-terms (deterministic trend model).



Panel G: Quadratic trend (time) stationary process (deterministic trend model with squared component).



Panel H: Quadratic trend (time) stationary process in log terms (deterministic trend model with squared component).



Exchange rate and inflation normalization:

Table 105

Theil coefficient of accuracy with bias component for exchange-rate and inflation-normalized commodity prices using rolling 60-month out-of-sample forecasts between 2003 and 2008

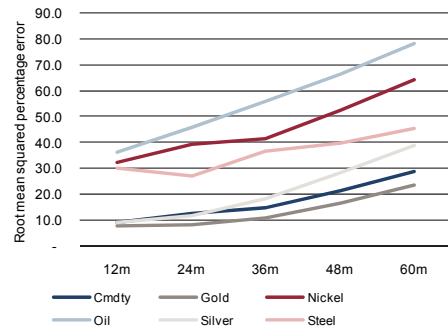
Panel A							
accuracy measure:	Theil						
total (vertical) average	Cmdty	Gold	Nickel	Oil	Silver	Steel	
AddBM	0.09	0.07	0.32	0.37	0.12	0.21	
GeoBM	0.06	0.07	0.29	0.27	0.11	0.20	
AOUP	0.04	0.18	0.18	0.21	0.37	0.16	
LNOUP	0.05	0.07	0.21	0.20	0.13	0.17	
Trend_t	0.12	0.11	0.33	0.89	0.67	0.28	
Trend_t_In	0.10	0.10	0.30	0.47	0.21	0.21	
Trend_t_2	0.09	0.37	0.35	0.12	0.41	0.18	
Trend_t_2_In	0.11	0.94	0.47	0.18	0.98	0.16	
MixModels	0.33	0.23	0.32	0.37	0.70	0.33	
MixModels_In	0.10	0.14	0.29	0.33	0.22	0.24	

Panel B							
accuracy measure:	Bias						
total (vertical) average	Cmdty	Gold	Nickel	Oil	Silver	Steel	
AddBM	0.62	0.47	0.52	0.61	0.35	0.49	
GeoBM	0.59	0.47	0.50	0.57	0.34	0.48	
AOUP	0.53	0.23	0.23	0.53	0.91	0.43	
LNOUP	0.59	0.47	0.33	0.52	0.55	0.44	
Trend_t	0.48	0.63	0.58	0.91	0.89	0.59	
Trend_t_In	0.83	0.50	0.56	0.86	0.79	0.43	
Trend_t_2	0.81	0.88	0.64	0.47	0.88	0.45	
Trend_t_2_In	0.87	0.89	0.71	0.69	0.92	0.41	
MixModels	0.80	0.68	0.55	0.63	0.83	0.57	
MixModels_In	0.69	0.61	0.52	0.68	0.68	0.49	

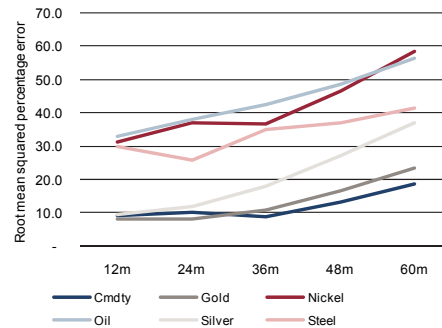
The best results are highlighted with a dark green background, whereas DGPs with scores within the top 15 percentile are highlighted in plain green (within the 30th percentile are highlighted with light green). All test statistics represent vertical (full sample path) measures; they are hence indicators of the ability to forecast over the entire forecast path of 60 months.

Figure 133

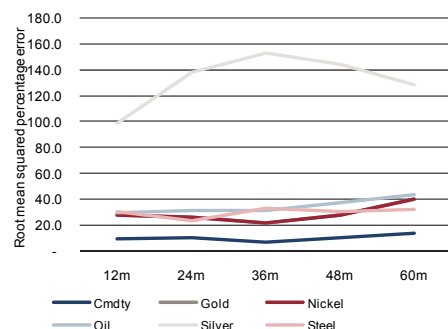
Forecast error (measured as RMSPE) over time for tested exchange-rate and inflation-normalized commodity prices (rolling out-of-sample between 2003 and 2008)



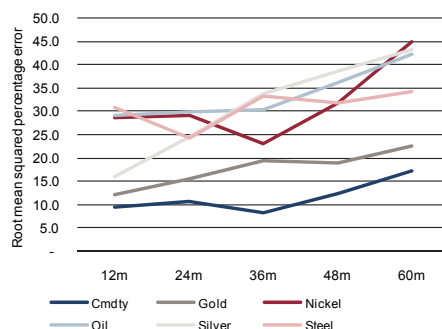
Panel A: Additive Brownian motion (ABM).



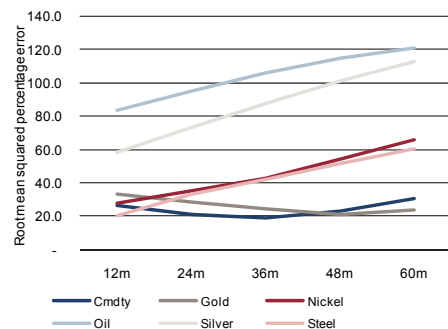
Panel B: Geometric Brownian motion (GBM).



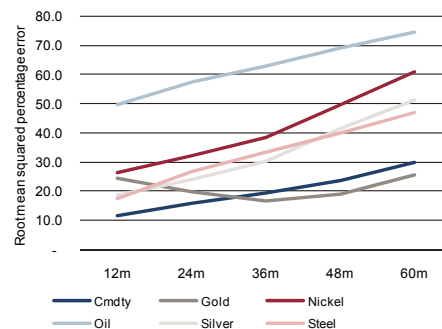
Panel C: Additive Ornstein-Uhlenbeck process (AOU).



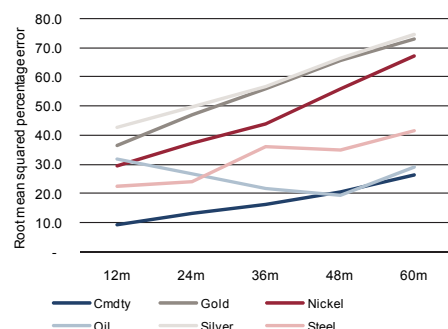
Panel D: Log-normal Ornstein-Uhlenbeck process (LnOUP).



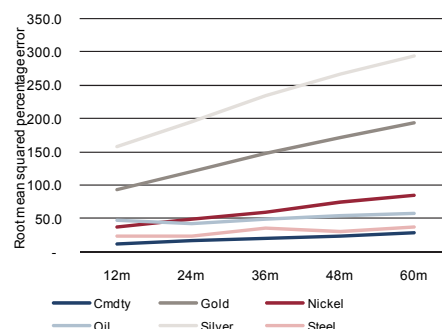
Panel E: Trend (time) stationary process (deterministic trend model).



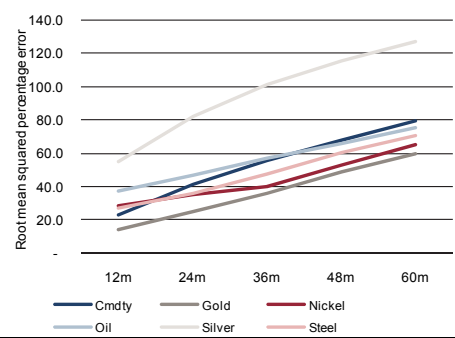
Panel F: Trend (time) stationary process in log-terms (deterministic trend model).



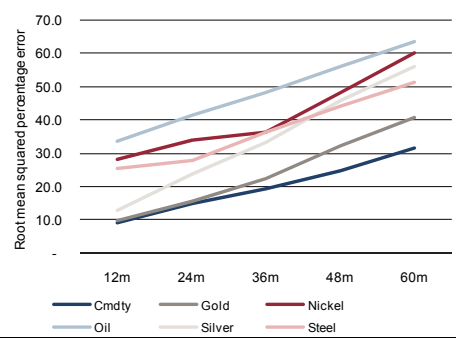
Panel G: Quadratic trend (time) stationary process (deterministic trend model with squared component).



Panel H: Quadratic trend (time) stationary process in log terms (deterministic trend model with squared component).



Panel I: Mixture (stochastic – deterministic) trend process.

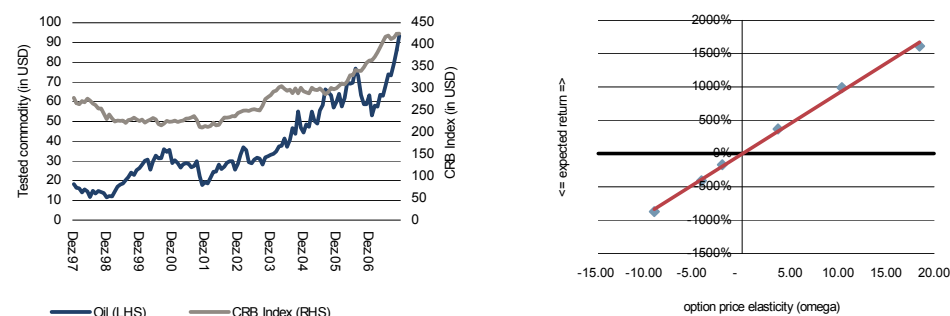


Panel J: Mixture (stochastic – deterministic) trend process in log-terms.

Appendix H – Out-of-sample tests for option data generation processes (1973-2008)

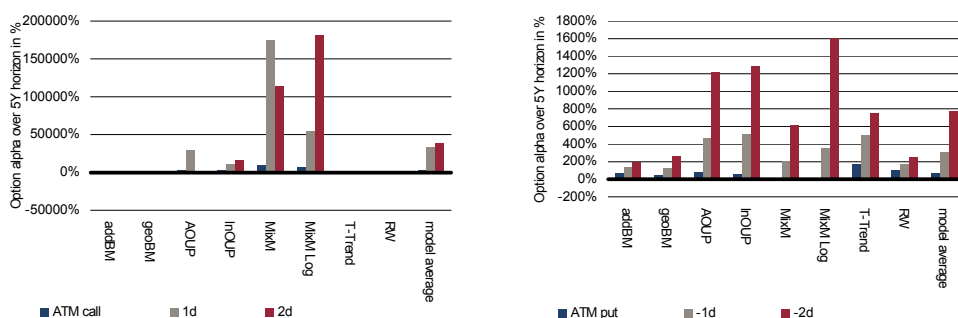
Figure 134

Testing different DGPs for valuing options on oil over a forecast horizon of five years (60-month out-of-sample between 2003-2007); the option valuation is conducted via Monte Carlo simulation (10,000 trials) using the different DGPs



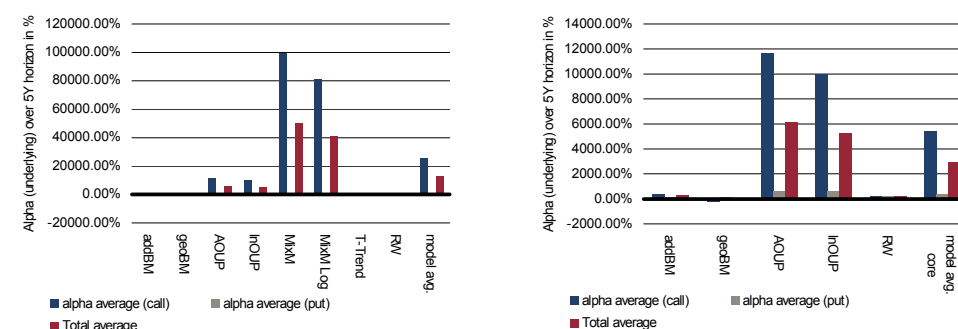
Panel A: Development of the underlying along with the benchmark index for commodities over time.

Panel B: Expected median return as function of the option price elasticity and ex-post development of the underlying.



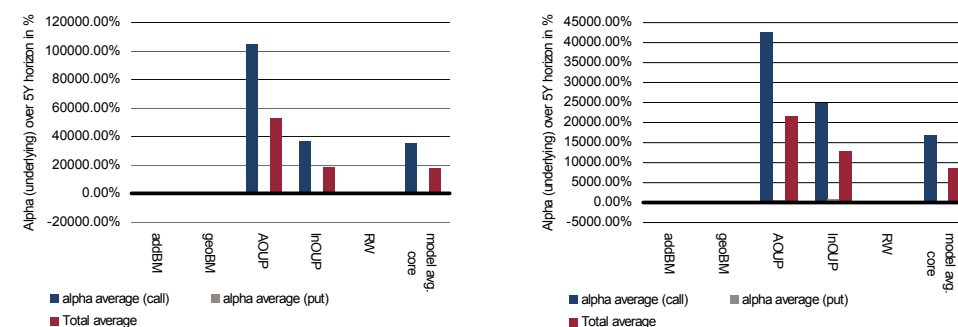
Panel C: Generated call option alpha returns over 5Y investment horizon for different DGPs and strike levels.

Panel D: Generated put option alpha returns over 5Y investment horizon for different DGPs and strike levels.



Panel E: Average median call and put option alpha returns over 5Y investment horizon for different DGPs.

Panel F: Average median call and put option alpha returns over 5Y investment horizon for main DGPs.

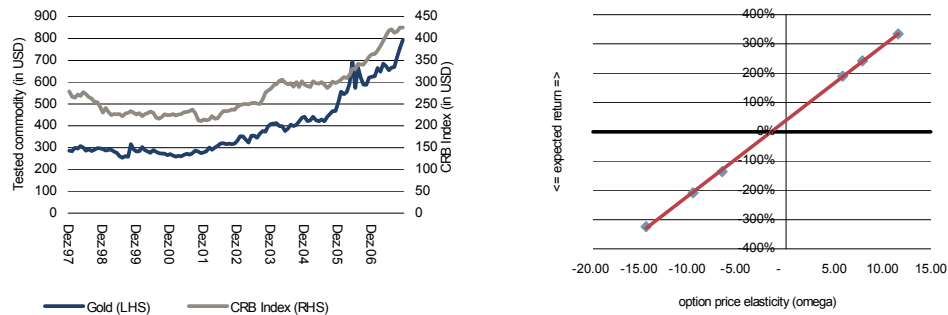


Panel G: Average mean call and put option alpha returns over 5Y investment horizon for main DGPs.

Panel H: Average trimmed mean call and put option alpha returns over 5Y investment horizon for main DGPs.

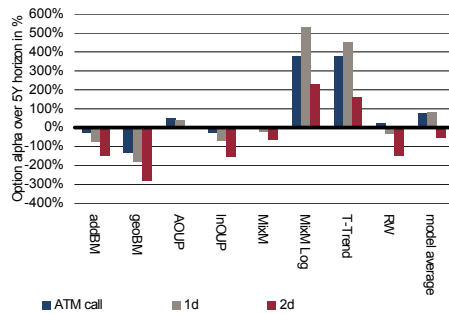
Figure 135

Testing different DGPs for valuing options on gold over a forecast horizon of five years (60-month out-of-sample between 2003-2007); the option valuation is conducted via Monte Carlo simulation (10,000 trials) using the different DGPs

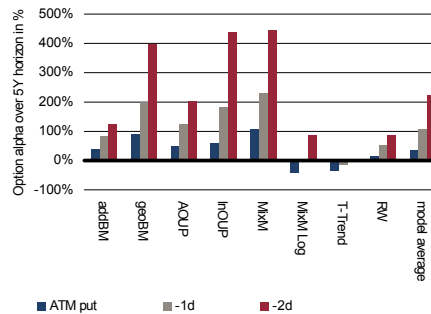


Panel A: Development of the underlying along with the benchmark index for commodities over time.

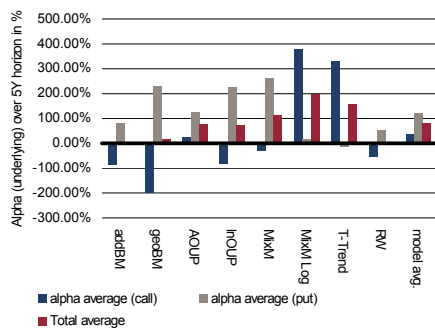
Panel B: Expected median return as function of the option price elasticity and ex-post development of the underlying.



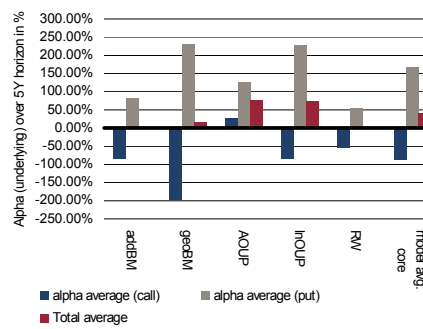
Panel C: Generated call option alpha returns over 5Y investment horizon for different DGPs and strike levels.



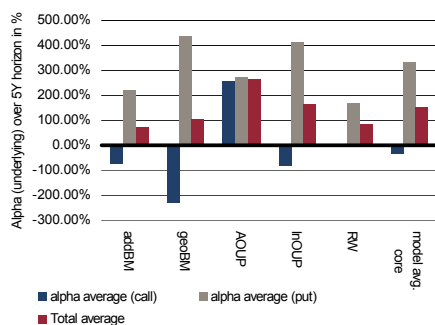
Panel D: Generated put option alpha returns over 5Y investment horizon for different DGPs and strike levels.



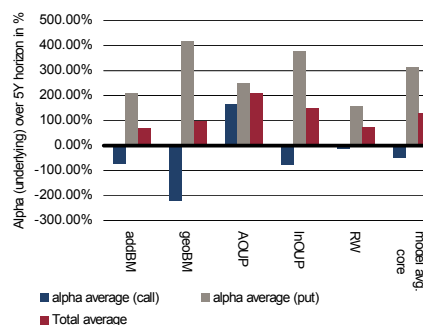
Panel E: Average median call and put option alpha returns over 5Y investment horizon for different DGPs.



Panel F: Average median call and put option alpha returns over 5Y investment horizon for main DGPs.



Panel G: Average mean call and put option alpha returns over 5Y investment horizon for main DGPs.



Panel H: Average trimmed mean call and put option alpha returns over 5Y investment horizon for main DGPs.

Figure 136

Testing different DGPs for valuing options on silver over a forecast horizon of five years (60-month out-of-sample between 2003-2007); the option valuation is conducted via Monte Carlo simulation (10,000 trials) using the different DGPs

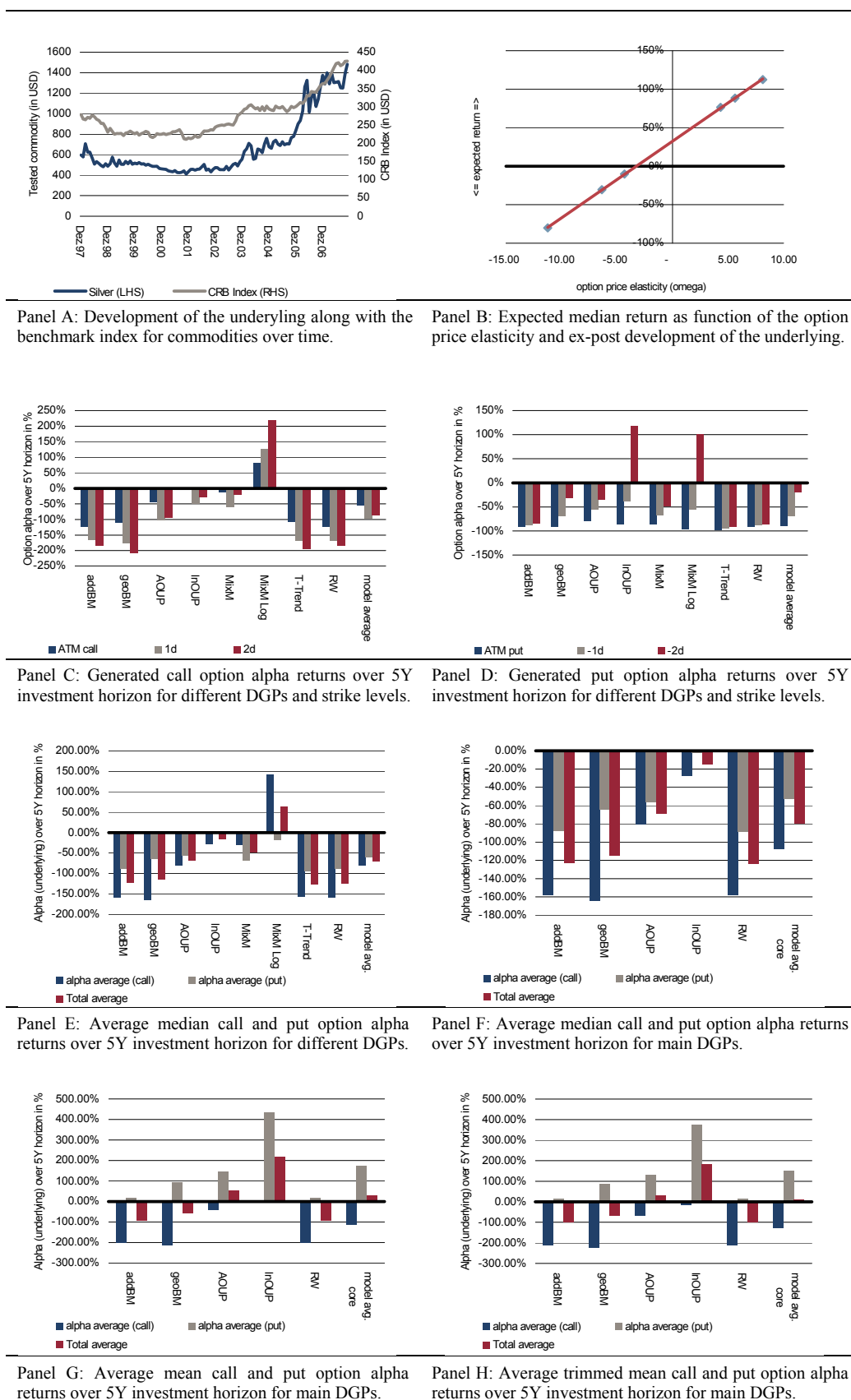
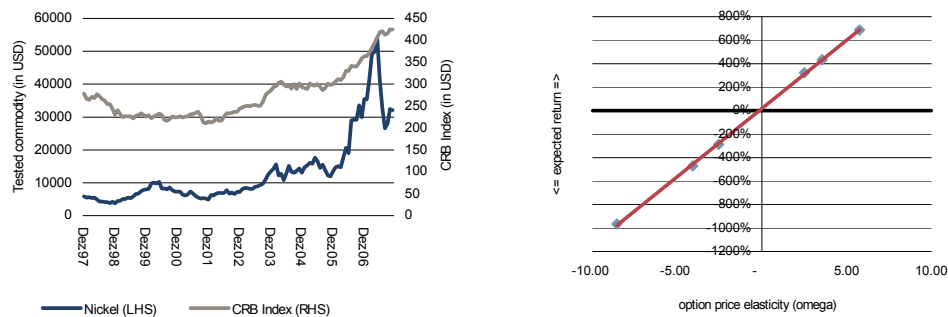


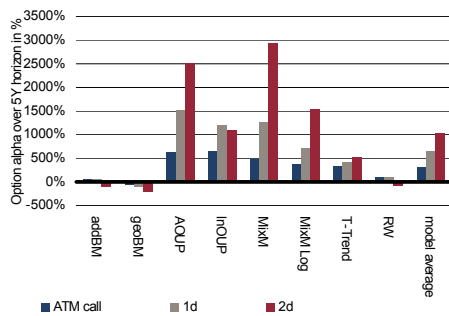
Figure 137

Testing different DGPs for valuing options on nickel over a forecast horizon of five years (60-month out-of-sample between 2003-2007); the option valuation is conducted via Monte Carlo simulation (10,000 trials) using the different DGPs

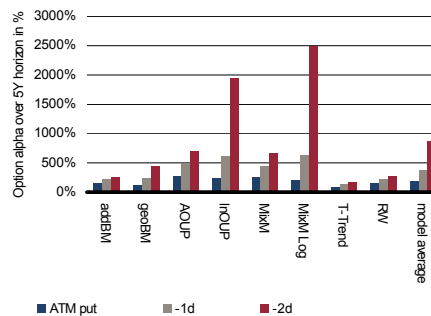


Panel A: Development of the underlying along with the benchmark index for commodities over time.

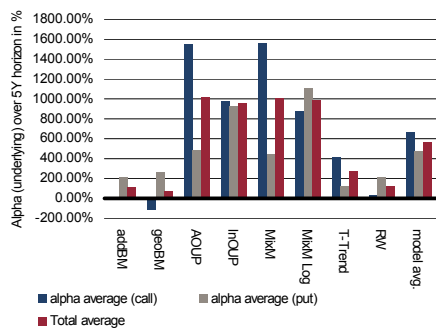
Panel B: Expected median return as function of the option price elasticity and ex-post development of the underlying.



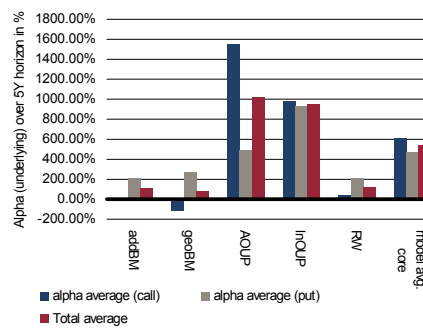
Panel C: Generated call option alpha returns over 5Y investment horizon for different DGPs and strike levels.



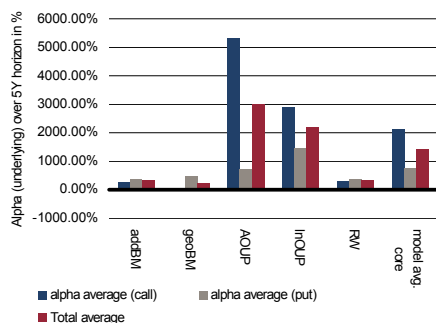
Panel D: Generated put option alpha returns over 5Y investment horizon for different DGPs and strike levels.



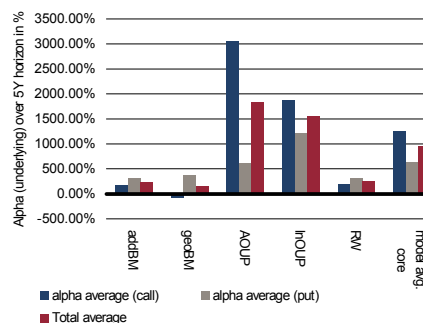
Panel E: Average median call and put option alpha returns over 5Y investment horizon for different DGPs.



Panel F: Average median call and put option alpha returns over 5Y investment horizon for main DGPs.



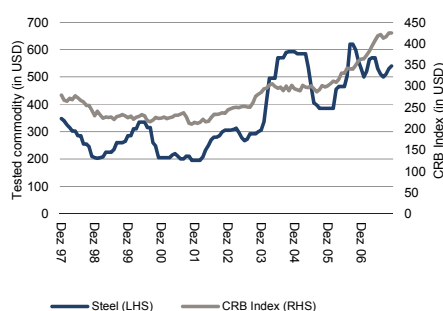
Panel G: Average mean call and put option alpha returns over 5Y investment horizon for main DGPs.



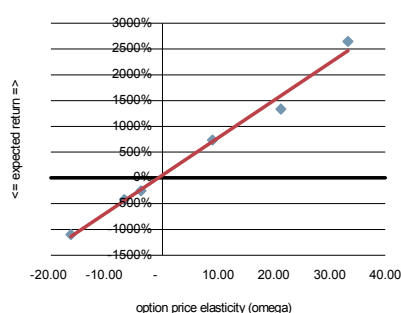
Panel H: Average trimmed mean call and put option alpha returns over 5Y investment horizon for main DGPs.

Figure 138

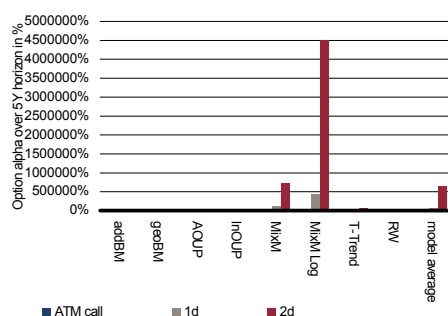
Testing different DGPs for valuing options on steel over a forecast horizon of five years (60-month out-of-sample between 2003-2007); the option valuation is conducted via Monte Carlo simulation (10,000 trials) using the different DGPs



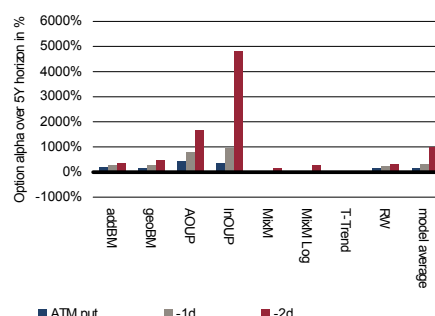
Panel A: Development of the underlying along with the benchmark index for commodities over time.



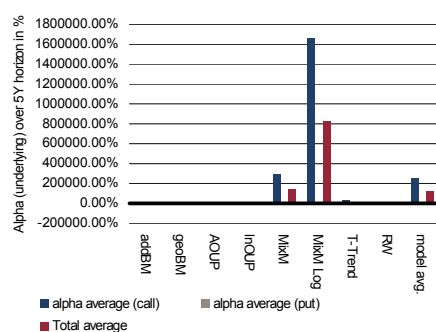
Panel B: Expected median return as function of the option price elasticity and ex-post development of the underlying.



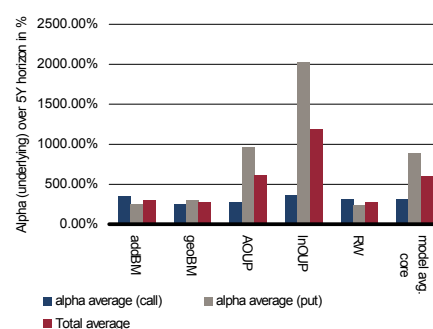
Panel C: Generated call option alpha returns over 5Y investment horizon for different DGPs and strike levels.



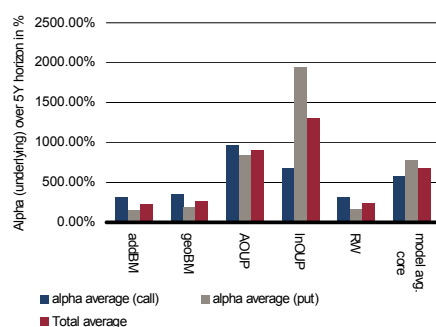
Panel D: Generated put option alpha returns over 5Y investment horizon for different DGPs and strike levels.



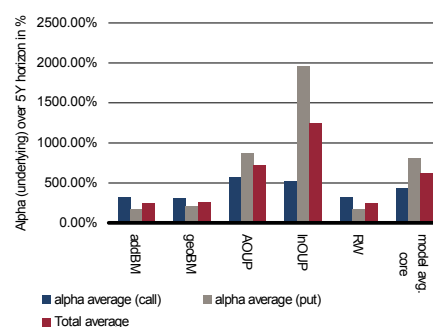
Panel E: Average median call and put option alpha returns over 5Y investment horizon for different DGPs.



Panel F: Average median call and put option alpha returns over 5Y investment horizon for main DGPs.



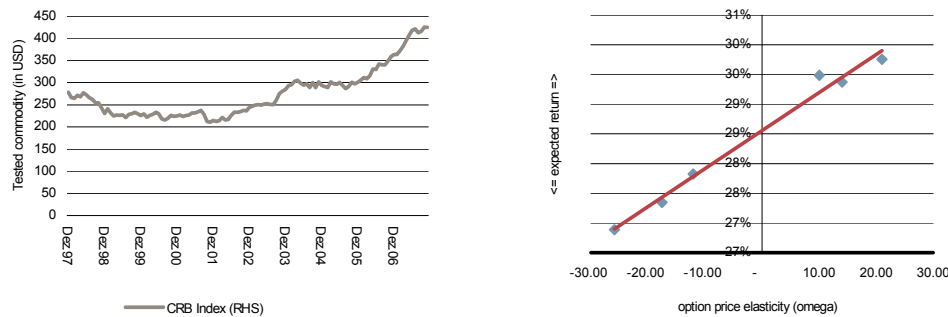
Panel G: Average mean call and put option alpha returns over 5Y investment horizon for main DGPs.



Panel H: Average trimmed mean call and put option alpha returns over 5Y investment horizon for main DGPs.

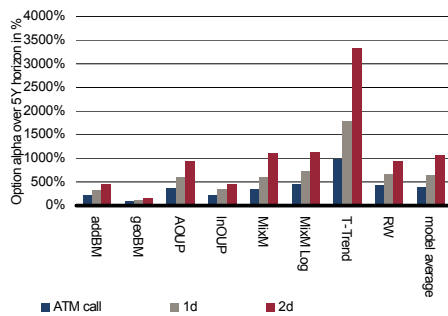
Figure 139

Testing different DGPs for valuing options on CRB index over a forecast horizon of five years (60-month out-of-sample between 2003-2007); the option valuation is conducted via Monte Carlo simulation (10,000 trials) using the different DGPs

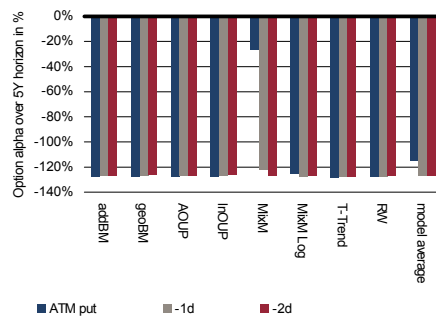


Panel A: Development of the underlying along with the benchmark index for commodities over time.

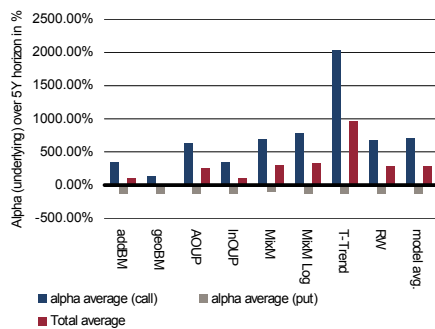
Panel B: Expected median return as function of the option price elasticity and ex-post development of the underlying.



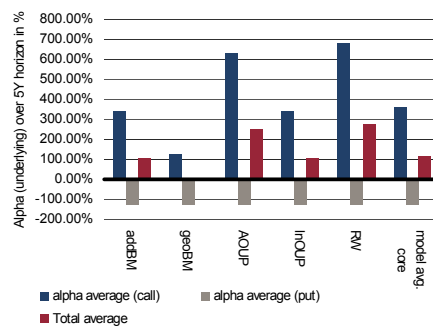
Panel C: Generated call option alpha returns over 5Y investment horizon for different DGPs and strike levels.



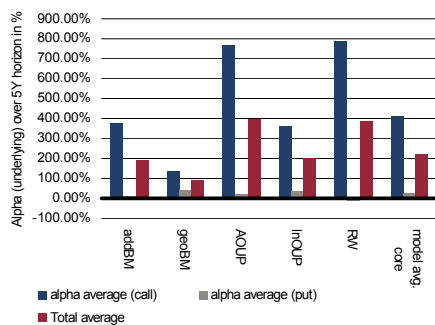
Panel D: Generated put option alpha returns over 5Y investment horizon for different DGPs and strike levels.



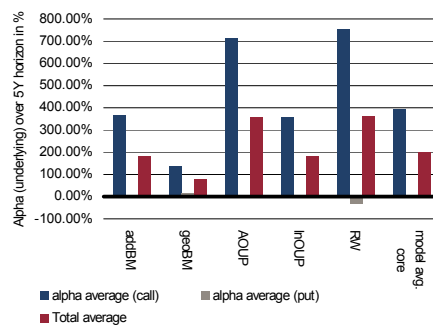
Panel E: Average median call and put option alpha returns over 5Y investment horizon for different DGPs.



Panel F: Average median call and put option alpha returns over 5Y investment horizon for main DGPs.



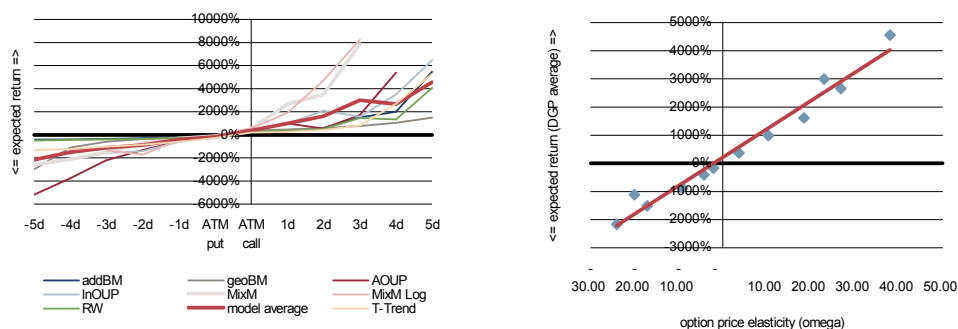
Panel G: Average mean call and put option alpha returns over 5Y investment horizon for main DGPs.



Panel H: Average trimmed mean call and put option alpha returns over 5Y investment horizon for main DGPs.

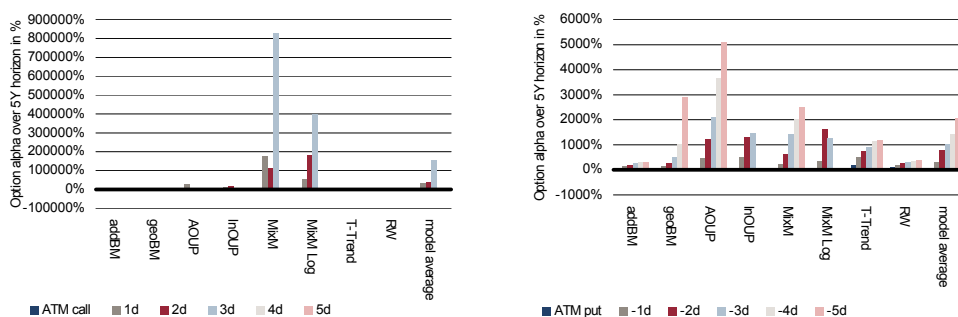
Figure 140

Testing different DGPs for valuing options on oil over a forecast horizon of five years (60 months out-of-sample between 2003-2007); the option valuation is conducted via Monte Carlo simulation (10,000 trails) using different DGPs



Panel A: Expected median return over 5Y horizon for different out-of-the-money puts and calls (full strike set).

Panel B: Expected median return as function of the option price elasticity and ex-post development of the underlying.



Panel C: Option alpha in % over five years for call option investments using different DGP and different strike levels.

Panel D: Option alpha in % over five years for put option investments using different DGP and different strike levels.

MEDIAN AVERAGE OPTION ALPHA VS UNDERLYING												
	ATM call	1d	2d	3d	4d	5d	ATM put	-1d	-2d	-3d	-4d	-5d
addBM	255%	444%	282%	-885%	-2019%	-5573%	89%	140%	192%	258%	294%	313%
geoBM	-81%	-158%	-374%	-786%	-1147%	-1595%	47%	129%	266%	493%	1011%	2889%
AOUP	3352%	29854%	1710%	-1535%	-5482%	#N/A	83%	465%	1215%	2102%	3672%	5097%
InOUP	2779%	11050%	15980%	-1110%	-3600%	-6573%	56%	518%	1286%	1467%	#N/A	#N/A
MxM	9526%	175374%	114103%	829488%	#N/A	#N/A	9%	211%	620%	1413%	2021%	2483%
MxM Log	7235%	54108%	182115%	400895%	#N/A	#N/A	0%	349%	1608%	1239%	#N/A	#N/A
T-Trend	35%	57%	1%	-522%	-2619%	-5360%	176%	501%	755%	915%	1123%	1200%
RW	190%	309%	34%	-711%	-1427%	-4164%	107%	175%	250%	295%	342%	395%
model average	2911%	33880%	39231%	153104%	-2716%	-4653%	68%	311%	774%	1023%	1411%	2063%

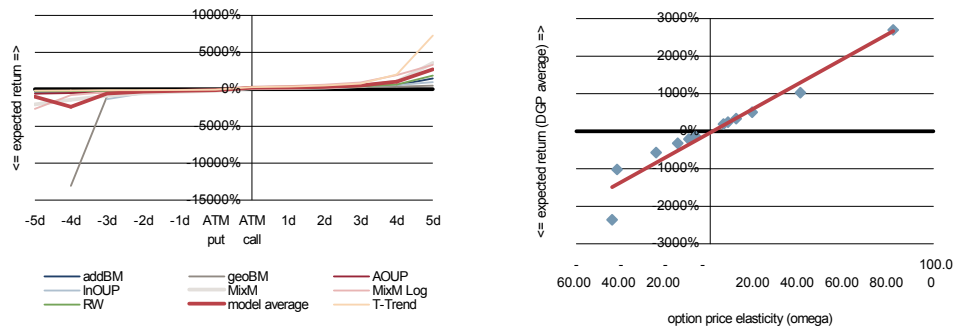
Trimmed mean (20%) AVERAGE OPTION ALPHA VS UNDERLYING												
	ATM call	1d	2d	3d	4d	5d	ATM put	-1d	-2d	-3d	-4d	-5d
addBM	251%	432%	673%	2%	-2478%	-5447%	81%	155%	228%	289%	339%	377%
geoBM	-92%	-168%	-363%	-739%	-1113%	-1576%	55%	144%	291%	534%	1091%	3411%
AOUP	5012%	94252%	28867%	206%	-5328%	#N/A	133%	531%	1271%	2331%	3672%	4897%
InOUP	3262%	15674%	55798%	8003%	-3416%	-6439%	129%	914%	1482%	2059%	#N/A	#N/A
MxM	13453%	321710%	155063%	846513%	#N/A	#N/A	19%	238%	772%	1653%	2500%	3133%
MxM Log	9801%	78268%	224470%	481016%	#N/A	#N/A	17%	468%	1619%	1333%	#N/A	#N/A
T-Trend	35%	56%	39%	-411%	-3479%	-5443%	229%	467%	764%	1062%	1272%	1447%
RW	178%	316%	453%	-238%	-2167%	-3837%	105%	189%	269%	336%	392%	435%
model average	3987%	63817%	58125%	166794%	-2997%	-4549%	96%	388%	837%	1200%	1544%	2283%

MEAN AVERAGE OPTION ALPHA VS UNDERLYING												
	ATM call	1d	2d	3d	4d	5d	ATM put	-1d	-2d	-3d	-4d	-5d
addBM	255%	459%	873%	3003%	-2767%	-5447%	89%	168%	245%	311%	364%	406%
geoBM	-93%	-165%	-354%	-721%	-1143%	-1644%	62%	155%	308%	563%	1151%	3809%
AOUP	7444%	192066%	115474%	203747%	-5328%	#N/A	185%	619%	1387%	2477%	3825%	5063%
InOUP	3892%	19144%	86912%	15880%	-3437%	-6439%	213%	1370%	1733%	2124%	#N/A	#N/A
MxM	17179%	1236394%	245365%	846513%	#N/A	#N/A	37%	284%	904%	1816%	2801%	3331%
MxM Log	12247%	101318%	688960%	481016%	#N/A	#N/A	40%	668%	1767%	1459%	#N/A	#N/A
T-Trend	38%	61%	73%	-108%	-789%	-5472%	244%	495%	834%	1142%	1376%	1573%
RW	178%	338%	643%	2163%	-2430%	-3953%	112%	200%	286%	357%	416%	461%
model average	5143%	193702%	142243%	193937%	-2649%	-4591%	123%	495%	933%	1281%	1656%	2440%

Panel E: Average mean, median and trimmed mean of call and put option alpha return over 5Y investment horizon for main DGPs.

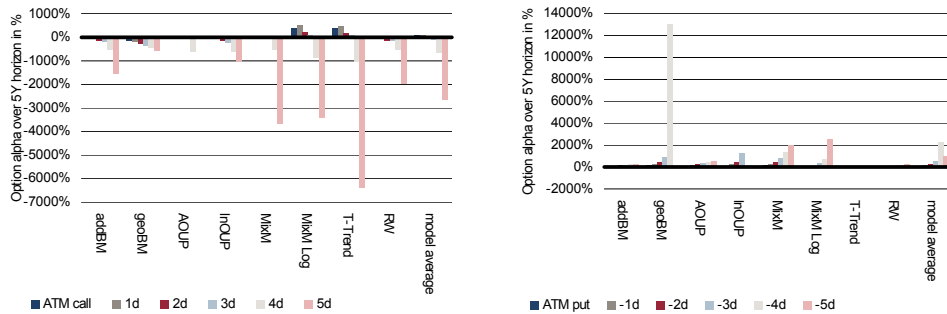
Figure 141

Testing different DGPs for valuing options on gold over a forecast horizon of five years (60 months out-of-sample between 2003-2007); the option valuation is conducted via Monte Carlo simulation (10,000 trails) using different DGPs



Panel A: Expected median return over 5Y horizon for different out-of-the-money puts and calls (full strike set).

Panel B: Expected median return as function of the option price elasticity and ex-post development of the underlying.



Panel C: Option alpha in % over five years for call option investments using different DGP and different strike levels.

Panel D: Option alpha in % over five years for put option investments using different DGP and different strike levels.

MEDIAN AVERAGE OPTION ALPHA VS UNDERLYING												
	ATM call	1d	2d	3d	4d	5d	ATM put	-1d	-2d	-3d	-4d	-5d
addBM	-29%	-73%	-152%	-184%	-545%	-1561%	39%	82%	123%	164%	201%	234%
geoBM	-133%	-181%	-279%	-378%	-459%	-581%	92%	201%	399%	834%	12969%	#N/A
AOUP	50%	3%	-8%	17%	-622%	#N/A	50%	124%	204%	304%	412%	510%
InOUP	-26%	-70%	-157%	-238%	-608%	-1058%	61%	184%	438%	1227%	#N/A	#N/A
MixM	-6%	-25%	-64%	-17%	-504%	-3653%	107%	231%	445%	797%	1332%	1963%
MixM Log	378%	535%	229%	98%	-878%	-3402%	-42%	3%	87%	264%	700%	2550%
T-Trend	376%	454%	164%	82%	-1043%	-6372%	-35%	-15%	6%	28%	49%	68%
RW	25%	-34%	-151%	-162%	-520%	-1960%	16%	53%	88%	122%	153%	182%
model average	79%	80%	-52%	-98%	-647%	-2655%	36%	108%	224%	468%	2259%	918%

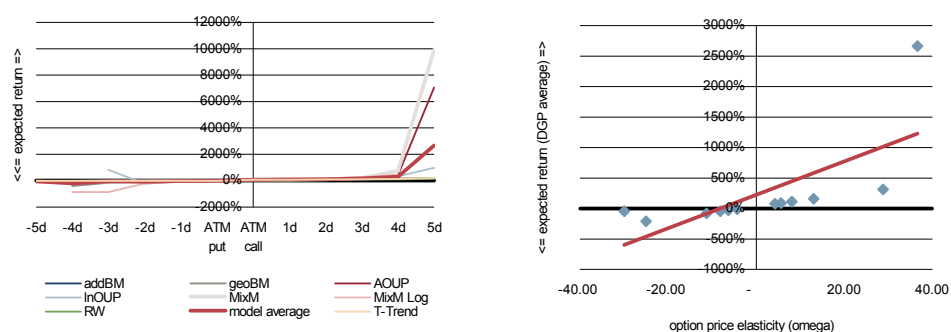
Trimmed mean (20%) AVERAGE OPTION ALPHA VS UNDERLYING												
	ATM call	1d	2d	3d	4d	5d	ATM put	-1d	-2d	-3d	-4d	-5d
addBM	-30%	-62%	-124%	-169%	-581%	-2292%	134%	207%	283%	360%	428%	495%
geoBM	-161%	-211%	-293%	-399%	-584%	-804%	211%	375%	670%	1327%	13471%	#N/A
AOUP	86%	131%	284%	1045%	1586%	#N/A	133%	248%	373%	538%	726%	915%
InOUP	-28%	-60%	-140%	-248%	-733%	-1661%	149%	323%	665%	1715%	#N/A	#N/A
MixM	24%	51%	175%	818%	1170%	-3653%	199%	389%	695%	1186%	1849%	2683%
MixM Log	410%	602%	725%	876%	-672%	-4977%	30%	112%	273%	593%	1388%	4184%
T-Trend	412%	577%	795%	1446%	488%	-6507%	32%	71%	112%	154%	196%	237%
RW	22%	-5%	-60%	-61%	-580%	-3314%	98%	158%	221%	283%	343%	394%
model average	92%	128%	170%	414%	12%	-3315%	123%	235%	412%	769%	2629%	1485%

MEAN AVERAGE OPTION ALPHA VS UNDERLYING												
	ATM call	1d	2d	3d	4d	5d	ATM put	-1d	-2d	-3d	-4d	-5d
addBM	-45%	-69%	-114%	-155%	-550%	-2401%	144%	217%	295%	372%	442%	510%
geoBM	-170%	-221%	-299%	-408%	-601%	-839%	222%	392%	695%	1367%	13471%	#N/A
AOUP	98%	188%	480%	2056%	238048%	#N/A	150%	269%	398%	566%	757%	948%
InOUP	-41%	-70%	-137%	-261%	-766%	-1705%	166%	351%	721%	1857%	#N/A	#N/A
MixM	33%	87%	308%	1484%	28826%	-3653%	210%	393%	677%	1134%	1770%	2630%
MixM Log	425%	647%	883%	1222%	88%	-5143%	35%	119%	283%	612%	1448%	4222%
T-Trend	424%	607%	934%	1929%	4292%	-6670%	37%	77%	119%	162%	206%	249%
RW	9%	0%	-20%	11%	-392%	-3476%	108%	169%	232%	295%	356%	408%
model average	92%	146%	254%	735%	33618%	-3413%	134%	248%	427%	796%	2636%	1495%

Panel E: Average mean, median and trimmed mean of call and put option alpha return over 5Y investment horizon for main DGPs.

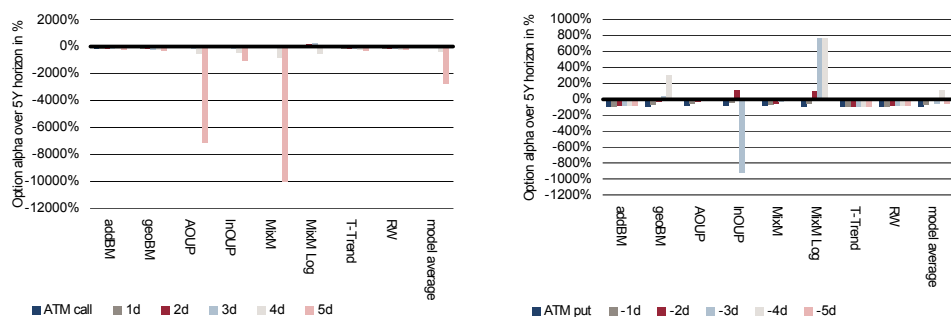
Figure 142

Testing different DGPs for valuing options on silver over a forecast horizon of five years (60 months out-of-sample between 2003-2007); the option valuation is conducted via Monte Carlo simulation (10,000 trails) using different DGPs



Panel A: Expected median return over 5Y horizon for different out-of-the-money puts and calls (full strike set).

Panel B: Expected median return as function of the option price elasticity and ex-post development of the underlying.



Panel C: Option alpha in % over five years for call option investments using different DGP and different strike levels.

Panel D: Option alpha in % over five years for put option investments using different DGP and different strike levels.

MEDIAN AVERAGE OPTION ALPHA VS. UNDERLYING											
	ATM call	1d	2d	3d	4d	5d	ATM put	-1d	-2d	-3d	-5d
addBM	-124%	-167%	-183%	-193%	-210%	-246%	-91%	-87%	-84%	-82%	-80%
geoBM	-111%	-176%	-207%	-228%	-260%	-308%	-92%	-69%	-32%	45%	306%
AQUP	-45%	-103%	-95%	-164%	-53%	-7160%	-79%	-56%	-35%	-18%	-7%
InOUP	-5%	-50%	-28%	-147%	-443%	-1070%	-86%	-39%	118%	-922%	#NV
MxM	-11%	-60%	-20%	-35%	-855%	-10019%	-86%	-67%	-49%	-33%	-20%
MxM Log	80%	126%	219%	233%	-543%	#NV	-96%	-55%	100%	769%	769%
T-Trend	-106%	-169%	-194%	-208%	-236%	-295%	-98%	-94%	-92%	-90%	-89%
RW	-122%	-167%	-185%	-194%	-212%	-250%	-92%	-88%	-86%	-84%	-82%
model average	-55%	-96%	-87%	-117%	-412%	-2764%	-90%	-70%	-20%	-52%	114%

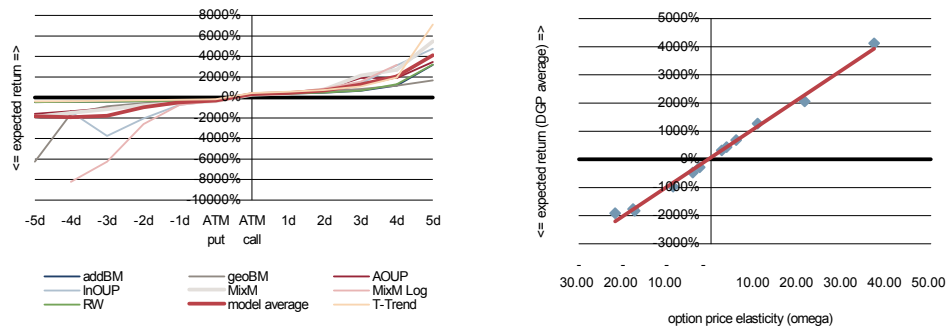
Trimmed mean (20%) AVERAGE OPTION ALPHA VS. UNDERLYING											
	ATM call	1d	2d	3d	4d	5d	ATM put	-1d	-2d	-3d	-5d
addBM	-176%	-209%	-241%	-292%	-354%	-434%	6%	15%	23%	29%	32%
geoBM	-163%	-218%	-285%	-401%	-546%	-708%	12%	68%	179%	400%	1030%
AQUP	-45%	-78%	-75%	-158%	-927%	-6951%	62%	128%	192%	240%	274%
InOUP	-6%	-27%	-13%	-196%	-975%	-2664%	53%	211%	863%	-1362%	#NV
MxM	-12%	-25%	41%	78%	-1549%	-10048%	29%	75%	125%	168%	202%
MxM Log	128%	207%	399%	318%	-1412%	#NV	4%	115%	594%	2652%	2717%
T-Trend	-162%	-207%	-251%	-325%	-426%	-560%	-16%	-7%	0%	4%	7%
RW	-175%	-209%	-242%	-295%	-359%	-442%	3%	12%	20%	25%	29%
model average	-76%	-96%	-84%	-159%	-818%	-3115%	19%	77%	249%	270%	613%

MEAN AVERAGE OPTION ALPHA VS. UNDERLYING											
	ATM call	1d	2d	3d	4d	5d	ATM put	-1d	-2d	-3d	-5d
addBM	-176%	-204%	-233%	-282%	-348%	-408%	13%	18%	26%	31%	34%
geoBM	-158%	-208%	-273%	-391%	-563%	-733%	27%	71%	182%	402%	928%
AQUP	-37%	-53%	-38%	-43%	-772%	-6845%	87%	147%	210%	257%	291%
InOUP	4%	-3%	8%	-160%	-913%	-2630%	78%	245%	986%	-1089%	#NV
MxM	-6%	-7%	66%	221%	-1454%	-10057%	47%	85%	133%	174%	206%
MxM Log	137%	230%	449%	400%	-1427%	#NV	19%	127%	638%	2776%	2799%
T-Trend	-160%	-199%	-240%	-310%	-418%	-527%	-10%	-5%	1%	5%	8%
RW	-174%	-204%	-234%	-284%	-351%	-412%	10%	16%	23%	28%	31%
model average	-71%	-81%	-62%	-106%	-781%	-3087%	34%	88%	275%	323%	614%

Panel E: Average mean, median and trimmed mean of call and put option alpha return over 5Y investment horizon for main DGPs.

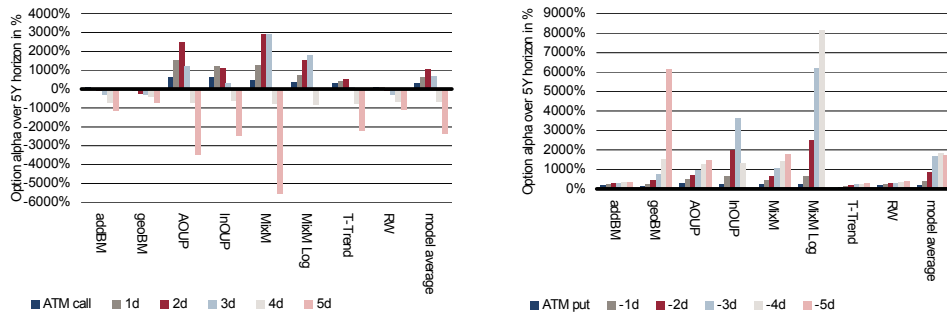
Figure 143

Testing different DGPs for valuing options on nickel over a forecast horizon of five years (60 months out-of-sample between 2003-2007); the option valuation is conducted via Monte Carlo simulation (10,000 trails) using different DGPs



Panel A: Expected median return over 5Y horizon for different out-of-the-money puts and calls (full strike set).

Panel B: Expected median return as function of the option price elasticity and ex-post development of the underlying.



Panel C: Option alpha in % over five years for call option investments using different DGP and different strike levels.

Panel D: Option alpha in % over five years for put option investments using different DGP and different strike levels.

MEDIAN AVERAGE OPTION ALPHA VS UNDERLYING												
	ATM call	1d	2d	3d	4d	5d	ATM put	-1d	-2d	-3d	-4d	-5d
addBM	62%	56%	-91%	-294%	-701%	-1120%	161%	212%	261%	299%	328%	351%
geoBM	-42%	-96%	-210%	-303%	-411%	-711%	125%	243%	436%	759%	1496%	6146%
AOUP	625%	1520%	2507%	1193%	-729%	-3460%	272%	488%	702%	938%	1241%	1485%
InOUP	640%	1194%	1097%	298%	-606%	-2440%	233%	615%	1949%	3630%	1309%	#N/
MxM	484%	1265%	2932%	2886%	-784%	-5528%	249%	440%	655%	1026%	1410%	1772%
MxM Log	366%	718%	1536%	1791%	-819%	#N/	207%	622%	2489%	6161%	8123%	#N/
T-Trend	321%	411%	516%	106%	-762%	-2195%	84%	129%	168%	215%	253%	285%
RW	85%	92%	-76%	-292%	-674%	-1056%	160%	218%	262%	302%	334%	358%
model average	318%	645%	1026%	673%	-686%	-2359%	186%	371%	865%	1666%	1812%	1733%

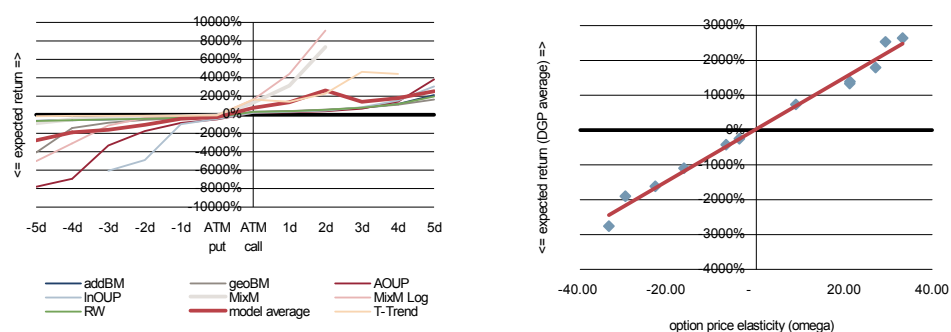
Trimmed mean (20%) AVERAGE OPTION ALPHA VS UNDERLYING												
	ATM call	1d	2d	3d	4d	5d	ATM put	-1d	-2d	-3d	-4d	-5d
addBM	96%	150%	238%	655%	3059%	114381%	234%	313%	384%	444%	493%	531%
geoBM	-38%	-53%	-102%	-169%	-287%	-472%	195%	348%	596%	1017%	1991%	7446%
AOUP	711%	1989%	6470%	88604%	53786%	-5275%	350%	598%	888%	1209%	1648%	1996%
InOUP	704%	1523%	3380%	11135%	92581%	225606%	295%	766%	2586%	6625%	1310%	#N/
MxM	596%	1672%	6414%	105512%	174377%	-3871%	314%	573%	968%	1572%	2250%	2925%
MxM Log	543%	1238%	3285%	11359%	88682%	#N/	281%	812%	3790%	7635%	8966%	#N/
T-Trend	396%	616%	1112%	2775%	12973%	403170%	163%	230%	295%	355%	406%	447%
RW	113%	174%	262%	703%	3246%	135030%	231%	308%	378%	437%	485%	521%
model average	390%	914%	2632%	27572%	53552%	124081%	258%	494%	1236%	2412%	2194%	2311%

MEAN AVERAGE OPTION ALPHA VS UNDERLYING												
	ATM call	1d	2d	3d	4d	5d	ATM put	-1d	-2d	-3d	-4d	-5d
addBM	127%	230%	467%	1448%	8319%	309711%	294%	388%	472%	545%	604%	650%
geoBM	-29%	-30%	-48%	-61%	-54%	12%	250%	429%	720%	1215%	2360%	8306%
AOUP	869%	2642%	12424%	1490821%	110183%	-5275%	419%	713%	1059%	1447%	1983%	2391%
InOUP	878%	2107%	5713%	24854%	277804%	757577%	358%	904%	3116%	7882%	1310%	#N/
MxM	746%	2266%	11664%	294753%	379778%	-3871%	383%	693%	1167%	1898%	2728%	3563%
MxM Log	655%	1591%	4739%	19420%	232101%	#N/	352%	990%	4639%	8928%	8966%	#N/
T-Trend	463%	761%	1506%	4344%	26538%	1755069%	216%	296%	374%	446%	508%	560%
RW	146%	262%	519%	1606%	9607%	3357274%	290%	382%	464%	534%	591%	636%
model average	482%	1228%	4623%	229648%	130535%	881499%	320%	599%	1501%	2862%	2381%	2684%

Panel E: Average mean, median and trimmed mean of call and put option alpha return over 5Y investment horizon for main DGPs.

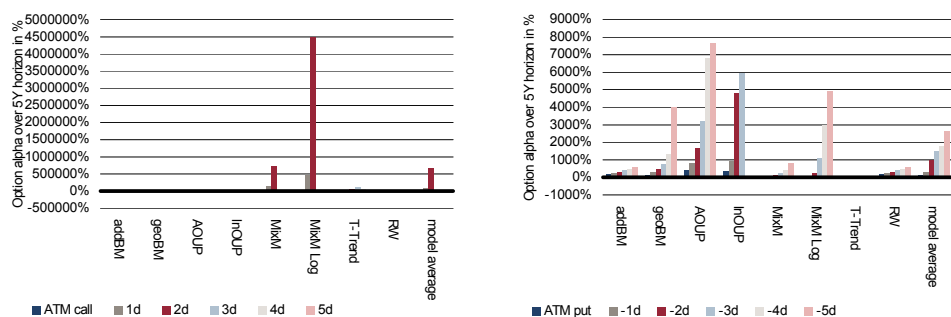
Figure 144

Testing different DGPs for valuing options on steel over a forecast horizon of five years (60 months out-of-sample between 2003-2007); the option valuation is conducted via Monte Carlo simulation (10,000 trails) using different DGPs



Panel A: Expected median return over 5Y horizon for different out-of-the-money puts and calls (full strike set).

Panel B: Expected median return as function of the option price elasticity and ex-post development of the underlying.



Panel C: Option alpha in % over five years for call option investments using different DGP and different strike levels.

Panel D: Option alpha in % over five years for put option investments using different DGP and different strike levels.

MEDIAN AVERAGE OPTION ALPHA VS. UNDERLYING												
	ATM call	1d	2d	3d	4d	5d	ATM put	-1d	-2d	-3d	-4d	-5d
addBM	191%	322%	532%	822%	670%	-76%	174%	252%	331%	406%	476%	563%
geoBM	191%	257%	312%	219%	285%	-455%	154%	273%	455%	756%	1338%	3996%
AOU	115%	215%	477%	1444%	3935%	7876%	404%	793%	1674%	3212%	6835%	7683%
InOUP	187%	302%	587%	1397%	3083%	1043%	364%	927%	4795%	5954%	#N/A	#N/A
MxM	25270%	132952%	733729%	#N/A	#N/A	#N/A	-17%	46%	136%	262%	419%	812%
MxM Log	38978%	448777%	4497643%	#N/A	#N/A	#N/A	-38%	44%	252%	1073%	2985%	4938%
T-Trend	33005%	21627%	53357%	99941%	40190%	#N/A	-28%	-8%	7%	22%	36%	49%
RW	191%	282%	458%	770%	804%	64%	163%	237%	324%	411%	494%	575%
model average	12286%	75592%	660887%	17432%	8161%	1690%	147%	320%	997%	1512%	1798%	2659%

Trimmed mean (20%) AVERAGE OPTION ALPHA VS. UNDERLYING												
	ATM call	1d	2d	3d	4d	5d	ATM put	-1d	-2d	-3d	-4d	-5d
addBM	181%	284%	483%	884%	1547%	1593%	115%	170%	234%	301%	369%	439%
geoBM	190%	287%	425%	598%	679%	216%	98%	184%	328%	557%	1028%	3624%
AOU	165%	377%	1147%	6058%	15916%	22821%	418%	754%	1437%	2749%	6006%	8486%
InOUP	184%	392%	981%	2700%	8156%	13720%	419%	1091%	4381%	4739%	#N/A	#N/A
MxM	34590%	337175%	733153%	#N/A	#N/A	#N/A	-34%	16%	93%	194%	346%	580%
MxM Log	64179%	790337%	5858918%	#N/A	#N/A	#N/A	-47%	42%	299%	1539%	2744%	4938%
T-Trend	56433%	82272%	181420%	601005%	90783%	#N/A	-40%	-25%	-10%	5%	20%	35%
RW	176%	285%	491%	922%	1583%	1343%	114%	170%	233%	299%	365%	430%
model average	19512%	151426%	847127%	102028%	19778%	7939%	130%	300%	874%	1298%	1554%	2647%

MEAN AVERAGE OPTION ALPHA VS. UNDERLYING												
	ATM call	1d	2d	3d	4d	5d	ATM put	-1d	-2d	-3d	-4d	-5d
addBM	178%	279%	477%	888%	1672%	2385%	112%	155%	210%	267%	322%	377%
geoBM	212%	329%	520%	810%	1177%	1559%	96%	168%	302%	511%	918%	3430%
AOU	204%	537%	2155%	21901%	80311%	31948%	440%	742%	1349%	2472%	5646%	8557%
InOUP	214%	493%	1333%	3929%	10061%	20087%	430%	1063%	4338%	4739%	#N/A	#N/A
MxM	43655%	501412%	843383%	#N/A	#N/A	#N/A	-29%	19%	91%	181%	301%	462%
MxM Log	81378%	167509%	777096%	#N/A	#N/A	#N/A	-39%	55%	342%	1773%	2754%	4938%
T-Trend	195885%	131136%	231572%	1105969%	182801%	#N/A	-43%	-30%	-15%	2%	18%	35%
RW	181%	286%	492%	929%	1688%	1818%	118%	165%	223%	282%	340%	397%
model average	40238%	288873%	1106362%	189071%	46285%	11560%	136%	292%	855%	1278%	1471%	2599%

Panel E: Average mean, median and trimmed mean of call and put option alpha return over 5Y investment horizon for main DGPs.

Appendix I – Peer group members

Table 106
International Peer Group of Gazprom OAO

	Global peers	North America	Western Europe	Asia-Pacific (D)	Asia-Pacific (E)	Latin America	Eastern Europe	Middle East and Africa
1	OCCIDENTAL PETE	OCCIDENTAL PETE	TULLOW OIL	CNOOC LTD	RELIANCE INDS	OGX-ORD	TUPRAS	RABIGH REFINING
2	CNOOC LTD	ENCANA CORP	CAIRN ENERGY	FORMOSA PETROCHE	OIL & NATURAL GA	RELAPASA	NOVOIL-BRD	SAMIR
3	RELIANCE INDS	APACHE CORP	ADDAX PETROLEUM	WOODSIDE PETRO	PTT PCL	GEPARK HOLDINGS	UKRNAFTA	PAZ OIL CO
4	ENCANA CORP	CAN NATURAL RES	ROYAL DUTCH SH-A	INPEX	INDIAN OIL CORP	BPC LTD	ROSNEFT OIL	MARIDIVE & OIL S
5	OIL & NATURAL GA	DEVON ENERGY CO	BP PLC	NIPPON OIL CORP	PTT EXPL & PROD	PETROLERA DEL CO	LUKOIL-CLS	ALEXANDRIA MINER
6	ROYAL DUTCH SH-A	XTO ENERGY INC	TOTAL SA	SANTOS LTD	RELIANCE PETROLE	GLOBAL ENVIRONME	SURGUTNEFTEGAZ	AFRIQUIA GAZ
7	CHEVRON CORP	ANADARKO PETROLE	ENI SPA	SK ENERGY CO LTD	CAIRN INDIA	PETROBRAS-PREF	TNK-BP HOLD-BRD	OIL REFINERIES L
8	PETROBRAS-PREF	EOG RESOURCES	STATOILHYDRO ASA	TONENGEN SEKIYU	KMG EP	ECOPETROL	GAZPROM NEFT-CLS	AVNER OIL - LP
9	BP PLC	CHEVRON CORP	BG GROUP PLC	S-OIL CORP	PETRONAS GAS BHD	EMPRESAS COPEC	PGNIG	ISRAMCO NEGEV
10	TOTAL SA	CONOCOPHILLIPS	REPSOL YPF SA	IDEMITSU KOSAN C	OIL SEARCH LTD	YPF SA-D	TATNEFT-CLS	DELEK DRILLIN-LP
11	CHINA PETROLEU-H	IMPERIAL OIL	CEPSA	CNPC HONG KG LTD	OIL & GAS DEVELO	PETROBRAS ENER-B	MOL HUNGARIAN	AABAR INVESTMENT
12	ENI SPA	SUNCOR ENERGY	GALP ENERGIA-B	CALTEX AUST LTD	PAKISTAN PETROLE	TEEKAY OFFSHORE	PKNORLEN	DELEK ENERGY SYS
13	STATOILHYDRO ASA	HESS CORP	OMV AG	ARROW ENERGYLTD	CHINA PETROLEU-H	TEEKAY TANK-CL A	SNP PETROM SA	SASOL LTD
14	CONOCOPHILLIPS	HUSKY ENERGY INC	NESTE OIL OYJ	NEW ZEAL REFININ	GAIL INDIA LTD	SIEM OFFSHORE IN	SLOVNAFT AS	DELEK GROUP LTD
15	BG GROUP PLC	MARATHON OIL	HELLENIC PETRO	NIPPON MINING HO	BHARAT PETROL	SUELOPETROL-CL B	INA INDUSTRIJA	SHELL OMAN MARKE
16	ROSNEFT OIL	SCHLUMBERGER LTD	TRANSOCEAN LTD	SHOWA SHELL	PETRONAS DAGANGA	PROMIGAS	PETROL OFISI	GULF INTERNATION
17	ECOPETROL	HALLIBURTON CO	SAIPEM	COSMO OIL CO	HINDUSTAN PETRO	TEEKAY LNG PARTN	UNIPETROL	MENA HOLDING GRO
18	LUKOIL-CLS	NATL OILWELL VAR	TECHNIP SA	JAPAN PETROLEUM	ESSAR OIL LTD	TRANS GAS SUR-B	DOGAN HOLDING	BURGAN CO FOR WE
19	IMPERIAL OIL	TRANSCANADA CORP	PETROFAC LTD	GS HOLDINGS CORP	CHINA OILFIELD-H		PETROL	NATIONAL GAS & I
20	SCHLUMBERGER LTD	KINDER MORGAN EN	AMEC PLC	WORLEYPARSONS	OFFSHORE OIL-A		EURASIA DRIL-GDR	

Source: Bloomberg as of end of Q1 2009.

Table 107
Valuation multiples for Peer Group of Gazprom OAO

	PE ratio	Est PEG ratio	Price to Sales ratio	Price to CF ratio	EV to 12M EBITDA	Sales Growth	Debt to Common Equity	Cash to Dividend Coverage	Operating Margin	Return to common equity
Gazprom OAO	5.19	0.14	1.10	4.41	4.13	47.21	38.23	10.44	29.36	18.43
Median Russia peers	4.96	0.23	0.67	2.61	3.42	37.34	19.59	4.95	21.61	24.31
Deviation from Global peers	-3.53	-2.00	-0.22	-1.29	-0.12	21.23	4.96	7.23	9.49	-8.26
Deviation from NA peers	-2.62	-1.93	-0.22	-0.33	0.07	18.29	4.42	3.91	4.76	-2.65
Deviation from WE peers	-4.55	-3.36	0.45	-0.44	-0.18	22.58	-5.22	7.41	19.21	-2.66
Deviation from APAC (D) peers	-7.00	-1.34	0.21	-2.19	-2.12	32.25	-8.99	6.75	24.26	8.68
Deviation from APAC (E) peers	-5.42	-0.98	-0.92	-3.53	-1.65	28.10	10.29	7.11	4.46	-1.05
Deviation from LATAM peers	-6.62	-1.29	-0.10	-0.98	-1.31	26.81	-27.08	8.34	10.35	10.57
Deviation from EEMA peers	-7.46	-0.54	0.79	-0.48	-3.83	15.74	3.06	9.02	24.38	14.40
Deviation from MEA peers	-6.59	-0.33	-0.33	-3.49	-4.74	16.23	-5.79	8.99	15.61	-4.16

Source: Bloomberg as of end of Q1 2009.

Table 108
International Peer Group of Lukoil

	Global peers	North America	Western Europe	Asia-Pacific (D)	Asia-Pacific (E)	Latin America	Eastern Europe	Middle East and Africa
1	OCCIDENTAL PETE	OCCIDENTAL PETE	TULLOW OIL	CNOOC LTD	RELIANCE INDS	OGX-ORD	TUPRAS	RABIGH REFINING
2	KMG EP	ENCANA CORP	CAIRN ENERGY	FORMOSA PETROCHE	OIL & NATURAL GA	RELAPASA	NOVOIL-BRD	SAMIR
3	WEST SIBERIA-SDR	APACHE CORP	ADDAX PETROLEUM	WOODSIDE PETRO	PTT PCL	GEOPARK HOLDINGS	UKRNAFTA	PAZ OIL CO
4	URALS ENERGY PUB	CAN NATURAL RES	SARAS SPA	INPEX	INDIAN OIL CORP	BPC LTD	UFANEFTEKHIM-CLS	MARIDIVE & OIL S
5	EXXON MOBIL CORP	DEVON ENERGY CO	STATOILHYDRO ASA	NIPPON OIL CORP	PTT EXPL & PROD	PETROLERA DEL CO	ROSNEFT OIL	ALEXANDRIA MINER
6	ROYAL DUTCH SH-A	XTO ENERGY INC	BG GROUP PLC	SANTOS LTD	RELIANCE PETROLE	GLOBAL ENVIRONME	SURGUTNEFTEGAZ	AFRIQUIA GAZ
7	PETROBRAS-PREF	ANADARKO PETROLE	REPSOL YPF SA	SK ENERGY CO LTD	CAIRN INDIA	PETROBRAS-PREF	TNK-BP HOLD-BRD	OIL REFINERIES L
8	BP PLC	EOG RESOURCES	CEPSA	TONENGEN SEKIYU	KMG EP	ECOPETROL	GAZPROM NEFT-CLS	AVNER OIL - LP
9	TOTAL SA	TALISMAN ENERGY	GALP ENERGIA-B	S-OIL CORP	PETRONAS GAS BHD	EMPRESAS COPEC	PGNIG	ISRAMCO NEGEV
10	GAZPROM	IMPERIAL OIL	OMV AG	IDEMITSU KOSAN C	OIL SEARCH LTD	YPF SA-D	TATNEFT-CLS	DELEK DRILLIN-LP
11	ROSNEFT OIL	SUNCOR ENERGY	NESTE OIL OYJ	CNPC HONG KG LTD	OIL & GAS DEVELO	PETROBRAS ENER-B	MOL HUNGARIAN	AABAR INVESTMENT
12	IMPERIAL OIL	HESS CORP	HELLENIC PETRO	CALTEX AUST LTD	PAKISTAN PETROLE	PETROBRAS ENER-B	PKNORLEN	DELEK ENERGY SYS
13	SURGUTNEFTEGAZ	HUSKY ENERGY INC	TRANSOCEAN LTD	ARROW ENERGY LTD	THAI OIL PCL	TEEKAY OFFSHORE	SNP PETROM SA	SASOL LTD
14	TNK-BP HOLD-BRD	MARATHON OIL	SAIPEM	NEW ZEAL REFININ	GAIL INDIA LTD	TEEKAY TANK-CL A	SLOVNAFT AS	DELEK GROUP LTD
15	GAZPROM NEFT-CLS	PETRO-CANADA	TECHNIP SA	NIPPON MINING HO	BHARAT PETROL	SIEM OFFSHORE IN	INA INDUSTRIJA	SHELL OMAN MARKE
16	TATNEFT-CLS	SCHLUMBERGER LTD	PETROFAC LTD	SHOWA SHELL	PETRONAS DAGANGA	SUELOPETROL-CL B	PETROL OFISI	OANDO PLC
17		HALLIBURTON CO	AMEC PLC	COSMO OIL CO	HINDUSTAN PETRO	PROMIGAS	UNIPETROL	GULF INTERNATIONAL
18		NATL OILWELL VAR	FUGRO NV-CVA	JAPAN PETROLEUM	ESSAR OIL LTD	TEEKAY LNG PARTN	DOGAN HOLDING	MENA HOLDING GRO
19		TRANSCANADA CORP	SBM OFFSHORE NV	GS HOLDINGS CORP	CHINA OILFIELD-H	TRANS GAS SUR-B	PETROL	BURGAN CO FOR WE
20		KINDER MORGAN EN	FLUXYS-D	WORLEYPARSONS	OFFSHORE OIL-A		EURASIA DRIL-GDR	NATIONAL GAS & I

Source: Bloomberg as of end of Q1 2009.

Table 109
Valuation multiples for Peer Group of Lukoil

	PE ratio	Est PEG ratio	Price to Sales ratio	Price to CF ratio	EV to 12M EBITDA	Sales Growth	Debt to Common Equity	Cash to Dividend Coverage	Operating Margin	Return to common equity
Lukoil	4.37	0.19	0.46	1.88	1.85	29.14	19.49	6.06	16.37	19.98
Median Russia peers	4.73	0.36	0.76	2.61	3.42	31.03	11.94	3.66	24.32	28.63
Deviation from Global peers	-4.30	-2.03	-0.32	-3.23	-2.26	2.84	-7.55	1.59	0.32	-4.54
Deviation from NA peers	-3.65	-2.14	-1.06	-2.83	-2.22	0.99	-14.20	-1.14	-8.26	-2.17
Deviation from WE peers	-5.95	-2.82	-0.29	-3.60	-3.24	6.94	-25.73	3.55	7.00	-0.28
Deviation from APAC (D) peers	-7.82	-1.29	-0.44	-4.72	-4.40	14.18	-27.73	2.37	11.27	10.23
Deviation from APAC (E) peers	-6.24	-0.97	-1.78	-6.40	-3.93	10.03	-20.95	2.73	-5.10	0.50
Deviation from LATAM peers	-7.44	-1.25	-0.74	-3.52	-3.59	8.74	-45.82	3.96	-2.64	12.12
Deviation from EEMA peers	-5.02	-0.27	-0.03	-1.91	-4.36	0.90	-7.66	2.58	8.29	10.28
Deviation from MEA peers	-7.45	-0.29	-1.26	-6.46	-8.41	-1.51	-29.82	4.63	6.94	0.69

Source: Bloomberg as of end of Q1 2009.

Table 110
International Peer Group of Norilsk Nickel

	Global peers	North America	Western Europe	Asia-Pacific (D)	Asia-Pacific (E)	Latin America	Eastern Europe	Middle East and Africa
1	RASPADSKAYA-CLS	ALCOA INC	NORSK HYDRO ASA	COAL & ALLIED IN	ALUMINUM CORP-H	SOCIEDAD MINERA	POLYUS GOLD-CLS	ARAB POTASH
2	BHP BILLITON LTD	PEABODY ENERGY	LINDE AG	FORMOSA PLASTIC	CHINA COAL ENERG	BUENAVENTURA-INV	POLYMETAL-CLS	SAUDI KAYAN PETR
3	RIO TINTO LTD	CONSOL ENERGY	ANTOFAGASTA PLC	NAN YA PLASTICS	YANZHOU COAL-H	GRUPO MEXICO-B	URALKALI-CLS	YANBU NATIONAL P
4	ANGLO AMER PLC	DU PONT (EI)	EURASIAN NATURAL	FORMOSA CHEM & F	SHANXI XISHAN-A	INDUSTRIAS PENOL	ACRON-CLS	JORDAN PHOSPHATE
5	XSTRATA PLC	AIR PRODS & CHEM	XSTRATA PLC	SUMITOMO CHEM CO	SINOPEC SHANG-H	SOUTHERN CP-INV1	NOVOLIPETSK-CLS	KUMBA IRON ORE L
6	KAZAKHMYS PLC	DOW CHEMICAL	RANDGOLD RES LTD	TORAY INDUSTRIES	MMTC LTD	VOLCAN CIA MIN-B	SEVERSTAL-CLS	SAUDI ARABIAN MI
7	POLYUS GOLD-CLS	NEWMONT MINING	ERAMET	ASAHI KASEI CORP	NMDC LIMITED	MILPO-INVERSION	MAGNITOGORSK-CLS	AFRICAN RAINBOW
8	SUMITOMO MET MIN	KINROSS GOLD	UPM-KYMMENE OYJ	LG CHEM LTD	ZIJIN MINING GRP	MINSUR-INV	MECHEL-CLS	EXXARO RESOURCES
9	TECK RESOURCES-B	AGNICO-EAGLE	STORA ENSO OYJ-R	NEWCREST MINING	LIHIR GOLD LTD	MOLYMET	VSMPO-AVISMA-CLS	ANGLOGOLD ASHANT
10	IMPALA PLATINUM	YAMANA GOLD INC	SYNGENTA AG-REG	SUMITOMO MET MIN	JIANGXI COPPER-H	CMPC		GOLD FIELDS LTD
11	LONMIN PLC	FREEPORT-MCMORAN	AKZO NOBEL	OJI PAPER CO LTD	JINDUICHENG -A	ARACRUZ-PREF B		HARMONY GOLD MNG
12	POLYMETAL-CLS	SOUTHERN COPPER	K+S AG	MITSUBISHI CHEMI	QINGHAI SALT L-A	SUZANO PAPEL-PRA		ANGLO PLATINUM
13	URALKALI-CLS	CAMECO CORP	YARA INTL ASA	NITTO DENKO CORP	QINGHAI SALT-A	FRESNILLO PLC		IMPALA PLATINUM
14	ACRON-CLS	MOSAIC CO/THE	SOLVAY SA-A	ORICA LTD	BAOSHAN IRON & S	SOQUIMICH-B		ISRAEL CHEMICALS
15	NOVOLIPETSK-CLS	ROHM & HAAS CO	DSM (KONIN)	NIPPON STEEL	ANGANG STEEL-H	FOSFERTIL-PREF		SAFCO
16	SEVERSTAL-CLS	ECOLAB INC	GIVAUDAN-REG	JFE HOLDINGS INC	WUHAN IRON & S-A	SID NACIONAL		ISRAEL CORP LTD
17	EVRAZ GROUP-GDR	AGRIUM INC	WACKER CHEMIE AG	SUMITOMO MET IND	STEEL AUTHORITY	GERDAU-PREF		FERTILIZED EGYPT
18	MAGNITOGORSK-CLS	PPG INDS INC	TENARIS SA	CHINA STEEL CORP	PANZHIHUA NEW-A	USIMINAS		MAKHTESHIM-AGAN
19	MECHEL-CLS	SIGMA-ALDRICH	SALZGITTER AG	FORTESCUE METALS	SHANXI TAIGANG-A	MET GERDAU-PREF		ARCELORMITTAL SO
20	VSMPO-AVISMA-CLS	NUCOR CORP	EVRAZ GROUP-GDR	KOBE STEEL LTD	MAANSHAN IRON-H	CAP		EL EZZ ALDEKHELA

Source: Bloomberg as of end of Q1 2009.

Table 111
Valuation multiples for Peer Group of Norilsk Nickel

	PE ratio	Est PEG ratio	Price to Sales ratio	Price to CF ratio	EV to 12M EBITDA	Sales Growth	Debt to Common Equity	Cash to Dividend Coverage	Operating Margin	Return to common equity
Norilsk Nickel	3.35	N.A.	1.04	2.42	6.80	48.22	41.41	6.41	43.41	32.96
Median Russia peer	3.23	0.19	0.91	2.55	2.57	48.00	44.83	2.67	30.55	27.23
Deviation from Global peers	-4.29	N.A.	0.05	-1.25	3.51	19.51	-5.48	3.35	12.80	9.48
Deviation from NA peers	-12.41	N.A.	-0.37	-7.87	-1.78	35.20	-21.60	3.32	25.08	16.45
Deviation from WE peers	-5.44	N.A.	0.30	-1.91	1.68	34.08	-6.36	2.62	28.29	17.33
Deviation from APAC (D) peers	-12.38	N.A.	0.29	-5.94	-0.84	47.05	-1.90	3.45	38.04	28.94
Deviation from APAC (E) peers	-19.51	N.A.	-0.45	-13.26	-0.00	17.28	1.42	3.26	21.37	15.01
Deviation from LATAM peers	-6.68	N.A.	-0.50	-4.17	0.00	33.19	0.16	-6.74	18.34	9.28
Deviation from EEMA peers	0.12	N.A.	0.14	-0.13	4.23	0.22	-3.42	3.74	12.87	5.74
Deviation from MEA peers	-4.93	N.A.	-1.45	-4.37	2.04	7.12	10.14	3.46	9.64	-13.11

Source: Bloomberg as of end of Q1 2009.

Appendix J – Peer group descriptive statistics

Table 112

Summary of Gazprom global peer group sample statistics (2003-2009) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
CHINA PETROLEUM & CHEMICAL-H	17.83%	47.1%	0.1068	7.2384	0.0174	1175.869	0.0000	0.82%	0.51%	2.02%	1.69%	48.8%
CNOOC LTD	22.55%	45.2%	0.0713	8.2886	0.0766	1827.486	0.0000	3.59%	3.32%	6.00%	5.71%	50.9%
BG GROUP PLC	23.14%	33.7%	-0.0247	7.6171	-0.0087	1392.017	0.0000	-0.29%	-0.20%	3.10%	3.22%	37.0%
BP PLC	1.77%	26.2%	0.2999	9.4042	-0.0739	2701.34	0.0000	-1.86%	-1.95%	-1.83%	-1.92%	24.3%
CONOCOPHILLIPS	6.77%	34.2%	-0.5046	10.9876	-0.0704	4232.227	0.0000	-2.32%	-1.74%	-1.83%	-1.24%	33.0%
CHEVRON CORP	12.02%	30.0%	0.1623	18.9132	-0.1362	16540.58	0.0000	-3.83%	-4.15%	0.44%	0.00%	30.0%
ENCANA CORP	12.70%	37.0%	-0.4320	9.9787	-0.0539	3228.619	0.0000	-1.94%	-1.00%	-0.55%	0.47%	37.5%
ENI SPA	0.73%	27.4%	0.8120	21.0044	-0.0984	21336.93	0.0000	-2.57%	-2.67%	-2.55%	-2.65%	24.8%
TOTAL SA	3.20%	27.2%	0.3923	12.8159	-0.0920	6331.137	0.0000	-2.39%	-2.60%	-2.22%	-2.43%	24.8%
Gazprom OAO	25.87%	46.4%	0.1730	15.4708	0.0528	10161.95	0.0000	2.51%	1.78%	10.82%	9.82%	56.3%
IMPERIAL OIL LTD	16.52%	34.4%	-0.0671	12.7857	-0.0750	6253.521	0.0000	-2.48%	-2.30%	1.43%	1.66%	36.1%
LUKOIL	16.79%	45.9%	0.1257	17.8115	-0.0062	14327.86	0.0000	-0.28%	-0.63%	3.89%	3.47%	49.3%
LUKOIL-CLS	15.54%	49.6%	-0.3841	17.6002	0.0417	13956.39	0.0000	2.11%	3.12%	5.11%	6.25%	55.9%
GAZPROM OAO-SPON ADR	17.44%	54.6%	-0.5967	16.9500	0.0857	12798.85	0.0000	4.88%	6.66%	8.01%	9.99%	64.5%
OIL & NATURAL GAS CORP LTD	18.10%	39.2%	-0.5063	7.7385	0.0776	1532.953	0.0000	3.16%	4.74%	4.74%	6.45%	45.6%
OCCIDENTAL PETROLEUM CORP	21.94%	40.7%	-0.3958	13.6870	-0.1050	7498.021	0.0000	-4.06%	-2.55%	2.90%	5.05%	45.8%
PETROBRAS - PETROLEO BRAS-PR	26.58%	39.0%	-0.1698	7.7032	0.0293	1451.759	0.0000	1.16%	1.92%	4.92%	5.84%	44.9%
ROYAL DUTCH SHELL PLC-A SHS	-2.09%	26.2%	0.2237	13.2331	-0.0447	6850.157	0.0000	-1.14%	-1.06%	-1.07%	-0.99%	25.2%
RELIANCE INDUSTRIES LTD	30.36%	39.1%	-0.6731	9.0221	0.0398	2486.174	0.0000	1.58%	5.30%	8.19%	13.29%	52.4%
SCHLUMBERGER LTD	13.53%	40.1%	-0.8635	12.6666	-0.0746	6295.811	0.0000	-2.87%	-0.82%	-0.78%	1.53%	41.6%
STATOILHYDRO ASA	13.57%	35.0%	-0.3546	6.9093	-0.0483	1030.694	0.0000	-1.65%	-0.82%	-0.72%	0.15%	35.2%

Table 113

Summary of Gazprom global peer group sample statistics (2003-2008) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
CHINA PETROLEUM & CHEMICAL-H	29.56%	41.8%	0.1431	6.5228	0.0257	679.7658	0.0000	1.09%	0.39%	4.30%	3.50%	45.3%
CNOOC LTD	34.93%	37.7%	0.1401	6.6279	0.0169	720.504	0.0000	0.64%	-0.16%	6.15%	5.11%	42.8%
BG GROUP PLC	28.92%	24.9%	0.3130	5.2064	-0.0472	286.2305	0.0000	-1.14%	-2.56%	2.60%	0.71%	25.6%
BP PLC	4.47%	19.9%	-0.0976	4.0832	-0.0539	65.92615	0.0000	-1.04%	-0.97%	-0.99%	-0.92%	19.0%
CONOCOPHILLIPS	22.41%	24.9%	-0.3919	3.7044	0.0243	60.4318	0.0000	0.61%	2.16%	1.20%	2.83%	27.7%
CHEVRON CORP	19.56%	20.7%	-0.2839	3.3732	-0.0595	25.1279	0.0000	-1.19%	-0.22%	-0.88%	0.12%	20.8%
ENCANA CORP	23.54%	28.0%	-0.3158	5.0323	0.0042	246.4684	0.0000	0.12%	1.41%	1.89%	3.36%	31.4%
ENI SPA	10.62%	18.6%	-0.2966	4.7802	-0.0650	191.6033	0.0000	-1.17%	-0.62%	-0.67%	-0.10%	18.5%
TOTAL SA	8.63%	20.0%	-0.1824	4.7373	-0.0826	171.4891	0.0000	-1.58%	-1.32%	-1.28%	-1.01%	19.0%
Gazprom OAO	48.84%	35.6%	-0.0444	9.5200	0.0290	2313.701	0.0000	1.05%	1.41%	35.12%	36.49%	72.1%
IMPERIAL OIL LTD	24.49%	26.1%	-0.2953	5.2381	-0.0067	291.5628	0.0000	-0.17%	1.09%	2.18%	3.69%	29.8%
LUKOIL	27.84%	33.5%	-0.1508	5.6924	0.0246	399.4267	0.0000	0.83%	1.54%	3.57%	4.40%	37.9%
LUKOIL-CLS	33.47%	34.9%	-0.3804	6.8274	0.0283	828.6533	0.0000	1.00%	3.25%	6.78%	9.85%	44.7%
GAZPROM OAO-SPON ADR	46.38%	41.3%	-0.0415	6.1574	0.0317	542.8696	0.0000	1.33%	1.65%	9.19%	9.64%	50.9%
OIL & NATURAL GAS CORP LTD	27.88%	37.2%	-0.4974	7.8137	0.0743	1314.757	0.0000	2.86%	5.31%	7.17%	10.20%	47.4%
OCCIDENTAL PETROLEUM CORP	31.59%	28.0%	-0.1956	3.8648	-0.0388	49.02409	0.0000	-1.06%	0.01%	0.34%	1.53%	29.5%
PETROBRAS - PETROLEO BRAS-PR	37.50%	31.6%	0.0616	5.6505	0.0216	383.1145	0.0000	0.69%	0.31%	6.35%	5.82%	37.4%
ROYAL DUTCH SHELL PLC-A SHS	2.90%	18.6%	-0.4511	6.1208	-0.0406	574.2741	0.0000	-0.74%	-0.52%	-0.67%	-0.45%	18.1%
RELiance INDUSTRIES LTD	46.02%	33.2%	-0.5108	8.1844	0.0008	1519.409	0.0000	0.03%	4.47%	23.48%	37.74%	71.0%
SCHLUMBERGER LTD	29.75%	29.8%	-0.2167	4.8974	-0.0001	206.1251	0.0000	-0.00%	1.11%	2.54%	3.86%	33.7%
STATOILHYDRO ASA	21.43%	28.8%	-0.2344	5.1433	-0.0244	261.9238	0.0000	-0.69%	0.17%	0.85%	1.81%	30.6%

Table 114

Summary of Gazprom North American peer group sample statistics (2003-2009) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
APACHE CORP	12.94%	40.1%	-0.4033	12.2855	-0.0787	5671.934	0.0000	-3.03%	-2.14%	-1.20%	-0.21%	39.9%
ANADARKO PETROLEUM CORP	10.10%	41.3%	-0.6924	15.6443	-0.0325	10563.87	0.0000	-1.32%	-0.12%	0.08%	1.36%	42.7%
CANADIAN NATURAL RESOURCES	22.97%	45.5%	-0.8599	12.7184	-0.0172	6359.734	0.0000	-0.77%	2.78%	4.58%	9.08%	54.6%
CONOCOPHILLIPS	6.77%	34.2%	-0.5046	10.9876	-0.0704	4232.227	0.0000	-2.32%	-1.74%	-1.83%	-1.24%	33.0%
CHEVRON CORP	12.02%	30.0%	0.1623	18.9132	-0.1362	16540.58	0.0000	-3.83%	-4.15%	0.44%	0.00%	30.0%
DEVON ENERGY CORPORATION	11.68%	40.1%	-0.2209	10.9113	-0.0625	4099.208	0.0000	-2.43%	-1.99%	-1.19%	-0.73%	39.4%
ENCANA CORP	12.70%	37.0%	-0.4320	9.9787	-0.0539	3228.619	0.0000	-1.94%	-1.00%	-0.55%	0.47%	37.5%
EOG RESOURCES INC	19.05%	42.4%	-0.2436	10.4135	-0.0671	3603.966	0.0000	-2.75%	-1.96%	0.30%	1.21%	43.6%
Gazprom OAO	25.87%	46.4%	0.1730	15.4708	0.0528	10161.95	0.0000	2.51%	1.78%	10.82%	9.82%	56.3%
HALLIBURTON CO	9.24%	42.4%	-0.6072	12.7182	-0.0274	6262.608	0.0000	-1.14%	-0.19%	-0.29%	0.71%	43.1%
HESS CORP	23.81%	46.5%	-0.8877	13.0314	-0.0702	6776.017	0.0000	-3.15%	0.69%	3.11%	8.20%	54.7%
HUSKY ENERGY INC	20.86%	32.6%	-0.0011	6.8340	-0.0359	959.7457	0.0000	-1.15%	-1.14%	1.23%	1.23%	33.9%
IMPERIAL OIL LTD	16.52%	34.4%	-0.0671	12.7857	-0.0750	6253.521	0.0000	-2.48%	-2.30%	1.43%	1.66%	36.1%
KINDER MORGAN ENERGY PRTNRS	3.98%	20.9%	-0.1569	21.9779	-0.0030	23521.95	0.0000	-0.06%	0.04%	0.56%	0.67%	21.6%
LUKOIL	16.79%	45.9%	0.1257	17.8115	-0.0062	14327.86	0.0000	-0.28%	-0.63%	3.89%	3.47%	49.3%
MARATHON OIL CORP	12.96%	41.7%	-0.5740	15.2463	-0.0754	9877.99	0.0000	-3.02%	-1.74%	-0.67%	0.77%	42.5%
NATIONAL OILWELL VARCO INC	16.94%	54.9%	-0.7276	12.6890	-0.0256	6267.613	0.0000	-1.38%	0.75%	0.87%	3.20%	58.1%
GAZPROM OAO-SPON ADR	17.44%	54.6%	-0.5967	16.9500	0.0857	12798.85	0.0000	4.88%	6.66%	8.01%	9.99%	64.5%
OCCIDENTAL PETROLEUM CORP	21.94%	40.7%	-0.3958	13.6870	-0.1050	7498.021	0.0000	-4.06%	-2.55%	2.90%	5.05%	45.8%
SCHLUMBERGER LTD	13.53%	40.1%	-0.8635	12.6666	-0.0746	6295.811	0.0000	-2.87%	-0.82%	-0.78%	1.53%	41.6%
SUNCOR ENERGY INC	15.13%	43.8%	-0.7381	10.3536	-0.0229	3672.954	0.0000	-0.99%	0.96%	0.72%	2.82%	46.6%
TRANSCANADA CORP	6.28%	19.9%	-0.3195	11.5434	-0.0995	4792.229	0.0000	-1.88%	-1.54%	-1.07%	-0.70%	19.2%
XTO ENERGY INC	22.31%	42.7%	-0.8433	11.8777	-0.0459	5331.555	0.0000	-1.91%	1.49%	3.16%	7.50%	50.2%

Table 115

Summary of Gazprom North American peer group sample statistics (2003-2008) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
APACHE CORP	27.51%	28.2%	-0.2111	3.3335	0.0070	15.75032	0.0004	0.20%	1.20%	0.57%	1.60%	29.8%
ANADARKO PETROLEUM CORP	21.79%	28.3%	-0.2030	3.7597	-0.0076	40.37168	0.0000	-0.21%	0.54%	0.33%	1.12%	29.4%
CANADIAN NATURAL RESOURCES	34.96%	32.6%	-0.1497	3.7824	0.0715	38.18894	0.0000	2.41%	3.31%	3.59%	4.55%	37.2%
CONOCOPHILLIPS	22.41%	24.9%	-0.3919	3.7044	0.0243	60.4318	0.0000	0.61%	2.16%	1.20%	2.83%	27.7%
CHEVRON CORP	19.56%	20.7%	-0.2839	3.3732	-0.0595	25.1279	0.0000	-1.19%	-0.22%	-0.88%	0.12%	20.8%
DEVON ENERGY CORPORATION	29.60%	29.8%	0.0084	4.3587	-0.0246	100.4673	0.0000	-0.72%	-0.76%	1.09%	1.05%	30.8%
ENCANA CORP	23.54%	28.0%	-0.3158	5.0323	0.0042	246.4684	0.0000	0.12%	1.41%	1.89%	3.36%	31.4%
EOG RESOURCES INC	36.62%	32.7%	0.1921	7.0239	-0.0224	889.1522	0.0000	-0.72%	-1.85%	8.28%	6.44%	39.2%
Gazprom OAO	48.84%	35.6%	-0.0444	9.5200	0.0290	2313.701	0.0000	1.05%	1.41%	35.12%	36.49%	72.1%
HALLIBURTON CO	26.48%	30.4%	-0.0831	3.9285	0.0334	48.41704	0.0000	1.03%	1.40%	1.92%	2.31%	32.7%
HESS CORP	38.36%	31.1%	-0.0648	4.2833	-0.0255	90.53087	0.0000	-0.78%	-0.36%	2.06%	2.56%	33.6%
HUSKY ENERGY INC	33.81%	26.0%	0.0552	3.8808	0.0576	42.88257	0.0000	1.54%	1.23%	3.15%	2.81%	28.8%
IMPERIAL OIL LTD	24.49%	26.1%	-0.2953	5.2381	-0.0067	291.5628	0.0000	-0.17%	1.09%	2.18%	3.69%	29.8%
KINDER MORGAN ENERGY PRTRNS	8.59%	15.4%	-0.3139	8.1255	0.0415	1451.031	0.0000	0.65%	1.11%	1.70%	2.22%	17.6%
LUKOIL	27.84%	33.5%	-0.1508	5.6924	0.0246	399.4267	0.0000	0.83%	1.54%	3.57%	4.40%	37.9%
MARATHON OIL CORP	28.88%	29.8%	-0.4112	4.3358	0.0127	133.906	0.0000	0.38%	2.50%	2.00%	4.36%	34.1%
NATIONAL OILWELL VARCO INC	33.58%	38.6%	-0.1249	4.2136	-0.0196	83.54079	0.0000	-0.75%	-0.03%	0.82%	1.60%	40.2%
GAZPROM OAO-SPON ADR	46.38%	41.3%	-0.0415	6.1574	0.0317	542.8696	0.0000	1.33%	1.65%	9.19%	9.64%	50.9%
OCCIDENTAL PETROLEUM CORP	31.59%	28.0%	-0.1956	3.8648	-0.0388	49.02409	0.0000	-1.06%	0.01%	0.34%	1.53%	29.5%
SCHLUMBERGER LTD	29.75%	29.8%	-0.2167	4.8974	-0.0001	206.1251	0.0000	-0.00%	1.11%	2.54%	3.86%	33.7%
SUNCOR ENERGY INC	28.15%	29.9%	-0.1872	3.6815	0.0124	32.90004	0.0000	0.37%	1.28%	1.13%	2.09%	32.0%
TRANSCANADA CORP	11.74%	15.6%	0.2267	3.9746	-0.0834	62.86809	0.0000	-1.25%	-1.68%	-0.85%	-1.30%	14.3%
XTO ENERGY INC	39.48%	31.8%	-0.2377	3.5252	0.0023	27.31228	0.0000	0.07%	1.72%	1.18%	2.95%	34.8%

Table 116

Summary of Gazprom Western European peer group sample statistics (2003-2009) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
Gazprom OAO	25.87%	46.4%	0.1730	15.4708	0.0528	10161.95	0.0000	2.51%	1.78%	10.82%	9.82%	56.3%
LUKOIL	16.79%	45.9%	0.1257	17.8115	-0.0062	14327.86	0.0000	-0.28%	-0.63%	3.89%	3.47%	49.3%
AMEC PLC	17.07%	35.9%	-0.1300	10.3964	-0.0260	3576.321	0.0000	-0.92%	-0.55%	1.85%	2.29%	38.2%
BG GROUP PLC	23.14%	33.8%	-0.0247	7.6171	-0.0087	1392.017	0.0000	-0.29%	-0.20%	3.10%	3.22%	37.0%
BP PLC	1.77%	26.2%	0.2999	9.4042	-0.0739	2701.34	0.0000	-1.86%	-1.95%	-1.83%	-1.92%	24.3%
CIA ESPANOLA DE PETROLEOS SA	8.26%	25.0%	-6.0662	130.7678	0.2164	1075472	0.0000	6.13%	17.55%	24.80%	62.26%	87.3%
CAIRN ENERGY PLC	30.82%	45.5%	1.8401	35.4000	0.0601	69424.73	0.0000	2.81%	-5.10%	61.60%	29.21%	74.7%
HELLENIC PETROLEUM SA	5.24%	31.1%	0.1137	6.1076	0.0408	633.914	0.0000	1.29%	1.19%	1.40%	1.30%	32.4%
ENI SPA	0.73%	27.4%	0.8120	21.0044	-0.0984	21336.93	0.0000	-2.57%	-2.67%	-2.55%	-2.65%	24.8%
TOTAL SA	3.20%	27.2%	0.3923	12.8159	-0.0920	6331.137	0.0000	-2.39%	-2.60%	-2.22%	-2.43%	24.8%
GAZPROM OAO-SPON ADR	17.44%	54.6%	-0.5967	16.9500	0.0857	12798.85	0.0000	4.88%	6.66%	8.01%	9.99%	64.5%
OMV AG	12.51%	39.7%	-0.5770	8.5389	0.0508	2090.046	0.0000	2.06%	3.30%	2.94%	4.24%	43.9%
ROYAL DUTCH SHELL PLC-A SHS	-2.09%	26.2%	0.2237	13.2331	-0.0447	6850.157	0.0000	-1.14%	-1.06%	-1.07%	-0.99%	25.2%
REPSOL YPF SA	-1.12%	27.5%	-0.9325	15.5006	0.0071	10429.85	0.0000	0.19%	0.02%	0.22%	0.04%	27.5%
TRANSOCEAN LTD	16.07%	43.3%	-0.7305	11.1110	-0.0187	4434.798	0.0000	-0.80%	1.25%	1.36%	3.63%	46.9%
SAIPEM	12.96%	37.4%	-0.4204	10.7913	-0.0102	4009.625	0.0000	-0.38%	0.55%	1.16%	2.17%	39.6%
STATOILHYDRO ASA	13.57%	35.0%	-0.3546	6.9093	-0.0483	1030.694	0.0000	-1.65%	-0.82%	-0.72%	0.15%	35.2%
TECHNIP SA	10.25%	40.5%	-0.4327	9.7807	-0.0091	3050.885	0.0000	-0.36%	0.39%	0.39%	1.17%	41.6%
TULLOW OIL PLC	38.10%	42.4%	0.4977	10.3766	-0.0155	3617.46	0.0000	-0.65%	-3.59%	13.68%	8.51%	51.0%

Table 117

Summary of Gazprom Western European peer group sample statistics (2003-2008) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
Gazprom OAO	48.84%	35.6%	-0.0444	9.5200	0.0290	2313.701	0.0000	1.05%	1.41%	35.12%	36.49%	72.1%
LUKOIL	27.84%	33.5%	-0.1508	5.6924	0.0246	399.4267	0.0000	0.83%	1.54%	3.57%	4.40%	37.9%
AMEC PLC	26.55%	28.4%	0.1434	6.0979	-0.0505	526.7123	0.0000	-1.40%	-2.02%	2.45%	1.64%	30.1%
BG GROUP PLC	28.92%	24.9%	0.3130	5.2064	-0.0472	286.2305	0.0000	-1.14%	-2.56%	2.60%	0.71%	25.6%
BP PLC	4.47%	19.9%	-0.0976	4.0832	-0.0539	65.92615	0.0000	-1.04%	-0.97%	-0.99%	-0.92%	19.0%
CIA ESPANOLA DE PETROLEOS SA	25.55%	19.4%	2.2807	30.1155	-0.0183	41142.22	0.0000	-0.35%	-6.78%	-37.88%	-55.94%	-36.5%
CAIRN ENERGY PLC	44.48%	39.1%	3.6061	66.3321	0.0727	221093.2	0.0000	2.94%	-13.40%	-60.60%	-70.98%	-31.9%
HELLENIC PETROLEUM SA	13.05%	27.3%	0.2323	4.8532	0.0057	198.6403	0.0000	0.16%	-0.34%	0.64%	0.13%	27.4%
ENI SPA	10.62%	18.6%	-0.2966	4.7802	-0.0650	191.6033	0.0000	-1.17%	-0.62%	-0.67%	-0.10%	18.5%
TOTAL SA	8.63%	20.0%	-0.1824	4.7373	-0.0826	171.4891	0.0000	-1.58%	-1.32%	-1.28%	-1.01%	19.0%
GAZPROM OAO-SPON ADR	46.38%	41.3%	-0.0415	6.1574	0.0317	542.8696	0.0000	1.33%	1.65%	9.19%	9.64%	50.9%
OMV AG	28.07%	32.4%	-0.2563	4.0582	0.0593	75.23761	0.0000	1.98%	3.22%	3.02%	4.34%	36.8%
ROYAL DUTCH SHELL PLC-A SHS	2.90%	18.6%	-0.4511	6.1208	-0.0406	574.2741	0.0000	-0.74%	-0.52%	-0.67%	-0.45%	18.1%
REPSOL YPF SA	10.52%	21.0%	-0.5152	9.8968	0.0223	2646.165	0.0000	0.47%	1.41%	2.06%	3.16%	24.2%
TRANSOCEAN LTD	36.33%	34.0%	-0.0055	3.4487	-0.0203	10.95996	0.0042	-0.68%	-0.65%	0.08%	0.11%	34.1%
SAIPEM	29.84%	29.5%	0.1489	6.3955	-0.0847	632.2098	0.0000	-2.39%	-3.11%	3.22%	2.17%	31.7%
STATOILHYDRO ASA	21.43%	28.8%	-0.2344	5.1433	-0.0244	261.9238	0.0000	-0.69%	0.17%	0.85%	1.81%	30.6%
TECHNIP SA	25.15%	32.1%	-0.8600	11.5570	-0.0812	4145.485	0.0000	-2.50%	1.61%	7.78%	15.61%	47.7%
TULLOW OIL PLC	42.16%	34.0%	0.3279	6.2306	0.0280	591.3516	0.0000	0.96%	-1.20%	9.47%	6.18%	40.2%

Table 118

Summary of Gazprom Asia Developed peer group sample statistics (2003-2009) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
Gazprom OAO	25.87%	46.4%	0.1730	15.4708	0.0528	10161.95	0.0000	2.51%	1.78%	10.82%	9.82%	56.3%
LUKOIL	16.79%	45.9%	0.1257	17.8115	-0.0062	14327.86	0.0000	-0.28%	-0.63%	3.89%	3.47%	49.3%
S-OIL CORPORATION	20.25%	38.8%	0.3042	6.2050	-0.0265	694.8547	0.0000	-1.01%	-2.01%	0.50%	-0.58%	38.2%
CNPC HONG KONG LIMITED	27.23%	46.0%	0.0714	7.3495	0.0281	1236.545	0.0000	1.31%	0.99%	4.33%	3.96%	50.0%
NIPPON OIL CORP	-0.83%	38.3%	-0.2648	9.5269	-0.0563	2799.791	0.0000	-2.09%	-2.13%	-2.09%	-2.13%	36.2%
SHOWA SHELL SEKIYU KK	2.75%	30.8%	-0.2964	9.2134	-0.0589	2543.584	0.0000	-1.76%	-1.62%	-1.69%	-1.55%	29.2%
COSMO OIL COMPANY LTD	9.84%	36.9%	0.0761	6.1868	-0.0090	664.6032	0.0000	-0.33%	-0.45%	0.03%	-0.10%	36.8%
TONENGENERAL SEKIYU KK	3.76%	23.5%	0.2557	8.5969	-0.0008	2062.38	0.0000	-0.02%	-0.18%	0.12%	-0.04%	23.5%
NIPPON MINING HOLDINGS INC	8.85%	46.9%	-0.2877	6.2075	0.0434	693.3417	0.0000	2.07%	2.50%	2.29%	2.72%	49.6%
CNOOC LTD	22.55%	45.2%	0.0713	8.2886	0.0766	1827.486	0.0000	3.59%	3.32%	6.00%	5.71%	50.9%
ARROW ENERGY LTD	55.46%	72.8%	0.4842	13.7637	-0.0761	7625.69	0.0000	-5.33%	-9.53%	23.98%	15.57%	88.4%
CALTEX AUSTRALIA LIMITED	25.91%	37.1%	-0.3092	5.5945	0.0395	464.4738	0.0000	1.49%	2.87%	3.47%	5.00%	42.1%
NEW ZEALAND REFINING CO LTD	24.98%	27.1%	0.0463	12.4679	0.0992	5853.33	0.0000	2.83%	2.64%	14.17%	13.80%	40.9%
GAZPROM OAO-SPON ADR	17.44%	54.6%	-0.5967	16.9500	0.0857	12798.85	0.0000	4.88%	6.66%	8.01%	9.99%	64.5%
SANTOS LTD	17.32%	36.6%	-0.2623	8.0276	-0.0095	1668.353	0.0000	-0.34%	0.43%	1.48%	2.33%	38.9%
WORLEYPARSONS LTD	39.15%	44.1%	0.2274	10.2064	0.1097	3404.219	0.0000	5.11%	3.67%	16.67%	14.49%	58.5%
WOODSIDE PETROLEUM LTD	20.86%	34.1%	-0.1540	7.2409	0.0421	1180.488	0.0000	1.46%	2.00%	3.77%	4.38%	38.4%

Table 119

Summary of Gazprom Asia Developed peer group sample statistics (2003-2008) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
Gazprom OAO	48.84%	35.6%	-0.0444	9.5200	0.0290	2313.701	0.0000	1.05%	1.41%	35.12%	36.49%	72.1%
LUKOIL	27.84%	33.5%	-0.1508	5.6924	0.0246	399.4267	0.0000	0.83%	1.54%	3.57%	4.40%	37.9%
S-OIL CORPORATION	26.26%	39.0%	0.4010	5.9399	-0.0460	505.3396	0.0000	-1.75%	-3.43%	0.66%	-1.23%	37.8%
CNPC HONG KONG LIMITED	37.32%	42.9%	0.3961	7.0061	0.0639	907.4847	0.0000	2.82%	0.48%	8.55%	5.61%	48.5%
NIPPON OIL CORP	4.76%	31.4%	-0.0918	5.2039	-0.0691	266.1488	0.0000	-2.09%	-2.02%	-2.02%	-1.95%	29.4%
SHOWA SHELL SEKIYU KK	6.01%	24.5%	-0.1399	6.0486	-0.0079	510.0132	0.0000	-0.19%	-0.05%	0.00%	0.14%	24.7%
COSMO OIL COMPANY LTD	11.25%	30.8%	-0.1562	4.7131	0.0104	165.0005	0.0000	0.32%	0.62%	0.62%	0.92%	31.7%
TONENGENERAL SEKIYU KK	3.22%	20.0%	-0.0272	6.8932	0.0165	824.9655	0.0000	0.33%	0.35%	0.42%	0.43%	20.4%
NIPPON MINING HOLDINGS INC	17.44%	41.7%	-0.4503	5.5151	0.0192	388.3466	0.0000	0.80%	2.15%	1.57%	2.97%	44.7%
CNOOC LTD	34.93%	37.7%	0.1401	6.6279	0.0169	720.504	0.0000	0.64%	-0.16%	6.15%	5.11%	42.8%
ARROW ENERGY LTD	58.37%	72.2%	0.5298	16.2491	-0.0954	9613.286	0.0000	-6.56%	-11.34%	44.37%	30.10%	102.3%
CALTEX AUSTRALIA LIMITED	35.33%	31.9%	-0.2885	5.6389	0.0537	397.0516	0.0000	1.75%	3.54%	6.40%	8.74%	40.6%
NEW ZEALAND REFINING CO LTD	29.29%	25.7%	0.5782	10.5884	0.0933	3206.277	0.0000	2.51%	-0.05%	17.09%	11.46%	37.2%
GAZPROM OAO-SPON ADR	46.38%	41.3%	-0.0415	6.1574	0.0317	542.8696	0.0000	1.33%	1.65%	9.19%	9.64%	50.9%
SANTOS LTD	17.27%	29.1%	-0.2319	4.8751	-0.0212	203.0305	0.0000	-0.61%	0.07%	0.23%	0.96%	30.1%
WORLEYPARSONS LTD	58.77%	36.3%	1.3879	15.0209	0.1067	8282.547	0.0000	4.09%	-6.08%	-726.80%	109.22%	145.6%
WOODSIDE PETROLEUM LTD	32.84%	27.1%	-0.2474	5.2189	0.0262	281.2374	0.0000	0.72%	2.14%	4.83%	6.71%	33.8%

Table 120

Summary of Gazprom Asia Emerging peer group sample statistics (2003-2009) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
Gazprom OAO	25.87%	46.4%	0.1730	15.4708	0.0528	10161.95	0.0000	2.51%	1.78%	10.82%	9.82%	56.3%
LUKOIL	16.79%	45.9%	0.1257	17.8115	-0.0062	14327.86	0.0000	-0.28%	-0.63%	3.89%	3.47%	49.3%
CHINA OILFIELD SERVICES-H	19.47%	60.5%	0.1520	8.1018	0.0204	1705.443	0.0000	1.24%	0.75%	2.58%	2.07%	62.5%
CHINA PETROLEUM & CHEMICAL-H	17.83%	47.1%	0.1068	7.2384	0.0174	1175.869	0.0000	0.82%	0.51%	2.02%	1.69%	48.8%
OFFSHORE OIL ENGINEERING-A	35.70%	41.9%	0.0182	6.1680	0.0314	655.3554	0.0000	1.33%	1.22%	5.61%	5.48%	47.4%
BHARAT PETROLEUM CORP LTD	8.08%	43.0%	0.1512	5.7675	0.0360	506.0527	0.0000	1.57%	1.37%	1.74%	1.54%	44.5%
ESSAR OIL LTD	47.54%	82.0%	7.9955	188.1076	0.1216	2253903	0.0000	10.62%	-27.01%	-171.74%	-348.02%	-266.0%
GAIL INDIA LTD	25.40%	45.5%	-0.1160	16.8558	-0.0373	12538.42	0.0000	-1.66%	-1.16%	8.88%	9.64%	55.1%
HINDUSTAN PETROLEUM CORP	-2.88%	42.8%	-0.1145	6.5153	0.0732	810.264	0.0000	3.25%	3.19%	3.27%	3.22%	46.0%
INDIAN OIL CORPORATION LTD	15.44%	42.5%	-0.3235	8.9315	0.1720	2324.482	0.0000	8.02%	8.87%	9.22%	10.11%	52.6%
GAZPROM OAO-SPON ADR	17.44%	54.6%	-0.5967	16.9500	0.0857	12798.85	0.0000	4.88%	6.66%	8.01%	9.99%	64.5%
OIL & NATURAL GAS CORP LTD	18.10%	39.2%	-0.5063	7.7385	0.0776	1532.953	0.0000	3.16%	4.74%	4.74%	6.45%	45.6%
OIL SEARCH LTD	34.82%	44.0%	-0.2641	8.7833	-0.0062	2202.035	0.0000	-0.27%	1.32%	7.61%	9.84%	53.9%
PETRONAS DAGANGAN BHD	16.22%	20.8%	0.4467	8.2748	-0.0760	1868.791	0.0000	-1.52%	-2.66%	2.03%	0.45%	21.3%
PETRONAS GAS BHD	5.91%	19.8%	0.1615	7.5702	-0.1989	1370.527	0.0000	-3.60%	-3.76%	-3.18%	-3.34%	16.4%
PTT PCL	20.30%	38.0%	-0.1049	9.5094	-0.0111	2769.473	0.0000	-0.42%	-0.06%	2.81%	3.23%	41.2%
PTT EXPLOR & PROD PUBLIC CO	21.88%	39.9%	-0.0274	10.2808	-0.0596	3461.348	0.0000	-2.30%	-2.20%	2.01%	2.13%	42.0%
RELIANCE INDUSTRIES LIMITED	30.36%	39.1%	-0.6731	9.0221	0.0398	2486.174	0.0000	1.58%	5.30%	8.19%	13.29%	52.4%
RELIANCE PETROLEUM LTD	-0.77%	50.4%	-0.6653	9.7141	0.0963	3058.902	0.0000	5.09%	5.00%	5.09%	5.01%	55.4%

Table 121

Summary of Gazprom Asia Emerging peer group sample statistics (2003-2008) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
Gazprom OAO	48.84%	35.6%	-0.0444	9.5200	0.0290	2313.701	0.0000	1.05%	1.41%	35.12%	36.49%	72.1%
LUKOIL	27.84%	33.5%	-0.1508	5.6924	0.0246	399.4267	0.0000	0.83%	1.54%	3.57%	4.40%	37.9%
CHINA OILFIELD SERVICES-H	37.60%	52.2%	0.3481	5.6672	-0.0583	413.5033	0.0000	-2.95%	-5.04%	0.46%	-1.92%	50.3%
CHINA PETROLEUM & CHEMICAL-H	29.56%	41.8%	0.1431	6.5228	0.0257	679.7658	0.0000	1.09%	0.39%	4.30%	3.50%	45.3%
OFFSHORE OIL ENGINEERING-A	49.76%	39.4%	0.1144	6.8662	0.0015	816.2439	0.0000	0.06%	-0.87%	13.65%	11.99%	51.4%
BHARAT PETROLEUM CORP LTD	13.10%	40.5%	0.3426	6.5966	0.0289	729.4641	0.0000	1.18%	0.45%	1.81%	1.05%	41.6%
ESSAR OIL LTD	78.35%	83.0%	9.2860	213.2458	0.1336	2424165	0.0000	11.89%	-41.35%	-102.12%	-108.73%	-25.7%
GAIL INDIA LTD	34.61%	44.7%	0.0292	19.4485	-0.0102	14722.78	0.0000	-0.45%	-0.62%	31.53%	31.03%	75.7%
HINDUSTAN PETROLEUM CORP	-2.37%	41.2%	-0.1573	7.3090	0.0787	1015.77	0.0000	3.36%	3.30%	3.39%	3.32%	44.5%
INDIAN OIL CORPORATION LTD	20.54%	42.5%	-0.3990	9.9451	0.1943	2659.428	0.0000	9.20%	10.60%	11.67%	13.22%	55.7%
GAZPROM OAO-SPON ADR	46.38%	41.3%	-0.0415	6.1574	0.0317	542.8696	0.0000	1.33%	1.65%	9.19%	9.64%	50.9%
OIL & NATURAL GAS CORP LTD	27.88%	37.2%	-0.4974	7.8137	0.0743	1314.757	0.0000	2.86%	5.31%	7.17%	10.20%	47.4%
OIL SEARCH LTD	39.70%	37.3%	-0.2617	8.1541	-0.0408	1460.483	0.0000	-1.49%	0.33%	11.34%	14.76%	52.1%
PETRONAS DAGANGAN BHD	20.65%	20.3%	0.4397	8.9248	-0.0320	1952.275	0.0000	-0.64%	-2.04%	6.71%	4.12%	24.5%
PETRONAS GAS BHD	6.57%	20.5%	0.1227	7.4090	-0.1857	1061.114	0.0000	-3.49%	-3.63%	-3.01%	-3.15%	17.3%
PTT PCL	40.01%	34.5%	0.0678	10.7979	-0.0395	3309.908	0.0000	-1.33%	-1.78%	28.47%	26.88%	61.4%
PTT EXPLOR & PROD PUBLIC CO	36.25%	34.9%	0.1190	9.7324	-0.0564	2469.534	0.0000	-1.91%	-2.61%	14.96%	13.37%	48.3%
RELIANCE INDUSTRIES LIMITED	46.02%	33.2%	-0.5108	8.1844	0.0008	1519.409	0.0000	0.03%	4.47%	23.48%	37.74%	71.0%
RELIANCE PETROLEUM LTD	31.49%	48.4%	-0.7571	13.6145	0.1241	6255.716	0.0000	6.40%	10.68%	15.78%	21.74%	70.1%

Table 122

Summary of Gazprom Latin America peer group sample statistics (2003-2009) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
Gazprom OAO	25.87%	46.4%	0.1730	15.4708	0.0528	10161.95	0.0000	2.51%	1.78%	10.82%	9.82%	56.3%
LUKOIL	16.79%	45.9%	0.1257	17.8115	-0.0062	14327.86	0.0000	-0.28%	-0.63%	3.89%	3.47%	49.3%
EMPRESAS COPEC SA	10.30%	24.1%	0.2733	6.9718	0.1380	1049.477	0.0000	3.58%	3.11%	4.23%	3.74%	27.8%
GLOBAL ENVIRONMENTAL ENERGY	-39.87%	163.1%	0.5409	7.9443	-0.0980	1672.556	0.0000	-15.22%	-11.53%	-12.97%	-9.17%	153.9%
GAZPROM OAO-SPON ADR	17.44%	54.6%	-0.5967	16.9500	0.0857	12798.85	0.0000	4.88%	6.66%	8.01%	9.99%	64.6%
PETROBRAS ENERGIA PARTICIP-B	-1.68%	40.1%	-0.3338	12.2244	0.0127	5584.719	0.0000	0.51%	0.42%	0.54%	0.44%	40.6%
PETROBRAS ENERGIA SA-B	-2.93%	39.9%	-0.4492	19.1262	0.0123	17032.02	0.0000	0.49%	0.27%	0.63%	0.41%	40.3%
PETROBRAS - PETROLEO BRAS-PR	26.58%	39.0%	-0.1698	7.7032	0.0293	1451.759	0.0000	1.16%	1.92%	4.92%	5.84%	44.9%
PROMIGAS S.A.	31.12%	27.2%	0.8952	22.8522	-0.0069	25941.32	0.0000	-0.19%	-4.15%	-309.34%	328.94%	356.2%
PETROLERA DEL CONOSUR SA	2.07%	59.3%	-0.4147	15.6413	0.0848	10478.73	0.0000	5.24%	5.38%	5.28%	5.42%	64.7%
SUELOPETROL S.A.C.A.-CL B	54.27%	48.6%	3.4445	62.6490	0.1661	235406.2	0.0000	8.83%	-11.37%	-95.56%	-133.14%	-84.6%
TRANSPORTADORA DE GAS SUR-B	2.76%	41.2%	-0.0547	7.7144	0.0215	1451.938	0.0000	0.89%	0.92%	0.93%	0.95%	42.1%
YPF S.A.-D	5.34%	33.2%	-1.2908	21.5311	0.0747	22856.38	0.0000	2.57%	3.76%	3.20%	4.43%	37.7%

Table 123

Summary of Gazprom Latin America peer group sample statistics (2003-2008) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
Gazprom OAO	48.84%	35.6%	-0.0444	9.5200	0.0290	2313.701	0.0000	1.05%	1.41%	35.12%	36.49%	72.1%
LUKOIL	27.84%	33.5%	-0.1508	5.6924	0.0246	399.4267	0.0000	0.83%	1.54%	3.57%	4.40%	37.9%
EMPRESAS COPEC SA	21.07%	21.2%	0.2008	4.5157	0.1159	133.7865	0.0000	2.61%	1.92%	3.85%	3.09%	24.3%
GLOBAL ENVIRONMENTAL ENERGY	-21.69%	160.4%	0.6679	9.1113	-0.0791	2129.474	0.0000	-12.18%	-9.73%	-11.37%	-8.89%	151.5%
GAZPROM OAO-SPON ADR	46.38%	41.3%	-0.0415	6.1574	0.0317	542.8696	0.0000	1.33%	1.65%	9.19%	9.64%	50.9%
PETROBRAS ENERGIA PARTICIP-B	9.93%	32.6%	-0.9146	14.7715	0.0009	7722.445	0.0000	0.03%	1.62%	1.58%	3.33%	36.0%
PETROBRAS ENERGIA SA-B	7.67%	37.2%	-0.4049	16.4879	0.0002	9935.334	0.0000	0.01%	0.53%	0.92%	1.47%	38.7%
PETROBRAS - PETROLEO BRAS-PR	37.50%	31.6%	0.0616	5.6505	0.0216	383.1145	0.0000	0.69%	0.31%	6.35%	5.82%	37.4%
PROMIGAS S.A.	37.03%	27.5%	0.9374	23.2457	-0.0037	22496.07	0.0000	-0.10%	-4.88%	-78.03%	-110.31%	-82.8%
PETROLERA DEL CONOSUR SA	17.93%	54.8%	0.8020	9.1524	0.0374	2199.8	0.0000	2.08%	-0.22%	3.57%	1.15%	56.0%
SUELOPETROL S.A.C.A.-CL B	61.77%	52.6%	3.2224	54.5261	0.1728	146732.9	0.0000	9.99%	-11.70%	-109.86%	-163.87%	-111.3%
TRANSPORTADORA DE GAS SUR-B	17.51%	36.6%	0.0104	8.7490	-0.0159	1798.541	0.0000	-0.57%	-0.60%	1.59%	1.56%	38.1%
YPF S.A.-D	18.84%	30.9%	0.1653	7.3195	-0.0063	1021.255	0.0000	-0.19%	-0.70%	2.04%	1.45%	32.4%

Table 124

Summary of Gazprom Europe Emerging peer group sample statistics (2003-2009) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
Gazprom OAO	25.87%	46.4%	0.1730	15.4708	0.0528	10161.95	0.0000	2.51%	1.78%	10.82%	9.82%	56.3%
LUKOIL	16.79%	45.9%	0.1257	17.8115	-0.0062	14327.86	0.0000	-0.28%	-0.63%	3.89%	3.47%	49.3%
DOGAN SIRKETLER GRUBU HLDGS	10.28%	47.9%	-0.0079	6.4307	0.0159	768.4659	0.0000	0.77%	0.78%	1.08%	1.09%	49.0%
LUKOIL-CLS	15.54%	49.6%	-0.3841	17.6002	0.0417	13956.39	0.0000	2.11%	3.12%	5.11%	6.25%	55.9%
MOL HUNGARIAN OIL AND GAS NY	9.71%	36.8%	-0.0702	11.3069	0.0541	4506.674	0.0000	2.04%	2.15%	2.90%	3.02%	39.8%
NOVOIL-BRD	42.37%	67.6%	5.2807	75.5876	0.0292	351302.1	0.0000	1.99%	-22.29%	-645.68%	100.01%	167.6%
GAZPROM OAO-SPON ADR	17.44%	54.6%	-0.5967	16.9500	0.0857	12798.85	0.0000	4.88%	6.66%	8.01%	9.99%	64.6%
PETROL	7.33%	27.5%	0.2392	12.5483	0.1107	5967.602	0.0000	3.22%	2.93%	3.93%	3.63%	31.2%
POLSKI KONCERN NAFTOWY SA	3.86%	34.9%	-0.2367	5.0369	0.0162	285.5313	0.0000	0.57%	0.72%	0.60%	0.76%	35.7%
PETROL OFISI AS	7.71%	44.3%	0.5573	8.8327	0.0648	2302.384	0.0000	2.96%	2.25%	3.26%	2.55%	46.8%
GAZPROM NEFT-CLS	1.12%	50.5%	-4.0147	55.1521	-0.0061	181792.7	0.0000	-0.31%	0.46%	-0.25%	0.51%	51.0%
SLOVNAFT AS	8.16%	30.6%	-1.5000	22.4306	-0.0480	25238.46	0.0000	-1.43%	0.76%	0.54%	3.05%	33.6%
SURGUTNEFTEGAZ	14.84%	60.8%	0.2227	15.7852	-0.1030	10685.54	0.0000	-5.95%	-6.49%	-3.72%	-4.31%	56.5%
SNP PETROM SA	6.21%	42.6%	-0.2569	9.4505	0.0434	2733.938	0.0000	1.88%	2.15%	2.12%	2.39%	45.0%
TATNEFT-CLS	17.39%	59.4%	-3.1466	55.3932	0.0136	181814.6	0.0000	0.81%	11.55%	14.21%	30.95%	90.3%
TUPRAS-TURKIYE PETROL RAFINE	11.76%	42.8%	-0.0631	7.2289	0.0474	1168.674	0.0000	2.07%	2.19%	2.62%	2.74%	45.5%
UKRNAFTA	20.00%	74.2%	-1.6543	50.4620	-0.1764	147793.4	0.0000	-12.06%	-6.01%	3.96%	13.78%	87.9%
UNIPETROL AS	14.99%	42.2%	-0.2012	19.2739	0.1136	17302.38	0.0000	5.08%	5.59%	8.54%	9.13%	51.4%

Table 125

Summary of Gazprom Europe Emerging peer group sample statistics (2003-2008) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
Gazprom OAO	48.84%	35.6%	-0.0444	9.5200	0.0290	2313.701	0.0000	1.05%	1.41%	35.12%	36.49%	72.1%
LUKOIL	27.84%	33.5%	-0.1508	5.6924	0.0246	399.4267	0.0000	0.83%	1.54%	3.57%	4.40%	37.9%
DOGAN SIRKETLER GRUBU HLDGS	24.91%	44.6%	0.1512	5.2079	0.0001	270.2601	0.0000	0.00%	-0.61%	1.32%	0.67%	45.3%
LUKOIL-CLS	33.47%	34.9%	-0.3804	6.8274	0.0283	828.6533	0.0000	1.00%	3.25%	6.78%	9.85%	44.7%
MOL HUNGARIAN OIL AND GAS NY	29.27%	29.3%	0.1130	4.5516	0.0924	133.7774	0.0000	2.83%	2.29%	4.66%	4.05%	33.3%
NOVOIL-BRD	64.69%	62.4%	5.5946	80.3878	0.0343	332707.1	0.0000	2.17%	-29.02%	-91.38%	-112.18%	-49.7%
GAZPROM OAO-SPON ADR	46.38%	41.3%	-0.0415	6.1574	0.0317	542.8696	0.0000	1.33%	1.65%	9.19%	9.64%	50.9%
PETROL	29.42%	20.3%	0.4877	10.1444	0.0611	2829.305	0.0000	1.27%	-0.88%	28.16%	18.49%	38.8%
POLSKI KONCERN NAFTOWY SA	16.23%	31.3%	-0.0668	3.9292	0.0096	47.95826	0.0000	0.30%	0.48%	0.63%	0.81%	32.1%
PETROL OFISI AS	17.80%	41.6%	0.6498	10.0977	0.0395	2833.303	0.0000	1.67%	-0.17%	3.95%	1.91%	43.5%
GAZPROM NEFT-CLS	20.97%	42.6%	-0.7783	14.3931	-0.0031	7195.28	0.0000	-0.13%	2.78%	5.43%	9.19%	51.7%
SLOVNAFT AS	23.76%	28.8%	-0.8885	18.5002	-0.0193	13245.85	0.0000	-0.55%	3.47%	23.15%	38.56%	67.4%
SURGUTNEFTEGAZ	16.40%	47.8%	-0.1597	11.2946	-0.0687	3749.39	0.0000	-3.17%	-2.73%	-0.99%	-0.50%	47.3%
SNP PETROM SA	29.50%	35.5%	-0.0136	8.0182	0.0171	1370.403	0.0000	0.61%	0.68%	6.47%	6.56%	42.0%
TATNEFT-CLS	39.49%	44.5%	-0.4522	7.7163	0.0478	1254.927	0.0000	2.17%	5.35%	9.80%	14.15%	58.7%
TUPRAS-TURKIYE PETROL RAFINE	26.50%	39.9%	0.0326	6.2836	0.0077	586.9403	0.0000	0.31%	0.16%	2.85%	2.68%	42.6%
UKRNAFTA	52.22%	75.0%	-1.9748	56.6451	-0.2202	157448.6	0.0000	-14.99%	9.10%	-161.65%	-136.30%	-61.3%
UNIPETROL AS	36.07%	34.9%	-0.5682	18.0452	0.1220	12387.94	0.0000	4.53%	8.27%	48.15%	66.70%	101.6%

Table 126

Summary of Gazprom Middle East and Africa peer group sample statistics (2003-2009) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
Gazprom OAO	25.87%	46.4%	0.1730	15.4708	0.0528	10161.95	0.0000	2.51%	1.78%	10.82%	9.82%	56.3%
LUKOIL	16.79%	45.9%	0.1257	17.8115	-0.0062	14327.86	0.0000	-0.28%	-0.63%	3.89%	3.47%	49.3%
AVNER OIL & GAS LTD - LP	18.30%	32.1%	1.1311	16.4057	-0.0031	12067.81	0.0000	-0.10%	-3.21%	7.04%	2.48%	34.6%
DELEK DRILLIN - LP	15.65%	36.2%	1.2940	18.4515	0.0021	16025.66	0.0000	0.07%	-3.01%	5.00%	1.07%	37.3%
DELEK ENERGY SYSTEMS LTD	16.29%	45.0%	2.6528	40.3024	0.0163	92689.41	0.0000	0.74%	-5.49%	11.96%	2.62%	47.7%
DELEK GROUP LTD	0.56%	46.6%	-1.4611	44.9352	0.1231	115376.7	0.0000	6.11%	6.25%	6.12%	6.26%	52.8%
AFRIQUIA GAZ	31.66%	34.9%	-0.4026	9.1733	0.0259	2530.558	0.0000	0.91%	3.17%	9.91%	13.50%	48.4%
ISRAMCO NEGEV 2 LP	41.69%	55.0%	6.7797	123.6025	-0.0494	961668.6	0.0000	-2.64%	-27.44%	-78.86%	-95.53%	-40.6%
NATIONAL GAS & INDUSTRIALIZA	-1.35%	56.0%	0.0703	17.4079	-0.0118	13555.09	0.0000	-0.65%	-0.64%	-0.63%	-0.62%	55.4%
OANDO PLC	13.71%	40.8%	0.2295	6.7149	0.1886	914.8348	0.0000	8.55%	8.03%	9.15%	8.62%	49.5%
GAZPROM OAO-SPON ADR	17.44%	54.6%	-0.5967	16.9500	0.0857	12798.85	0.0000	4.88%	6.66%	8.01%	9.99%	64.5%
SAMIR	14.34%	29.9%	-0.1931	5.7596	0.1375	506.9729	0.0000	4.42%	4.88%	5.12%	5.61%	35.5%
SASOL LTD	17.83%	38.4%	0.0227	5.6268	0.0453	450.6607	0.0000	1.78%	1.71%	2.66%	2.59%	41.0%
SHELL OMAN MARKETING	16.97%	22.1%	-0.5256	22.8365	0.0806	25763.44	0.0000	1.85%	3.44%	18.84%	23.71%	45.8%

Table 127

Summary of Gazprom Middle East and Africa peer group sample statistics (2003-2008) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
Gazprom OAO	48.84%	35.6%	-0.0444	9.5200	0.0290	2313.701	0.0000	1.05%	1.41%	35.12%	36.49%	72.1%
LUKOIL	27.84%	33.5%	-0.1508	5.6924	0.0246	399.4267	0.0000	0.83%	1.54%	3.57%	4.40%	37.9%
AVNER OIL & GAS LTD - LP	4.56%	27.7%	0.3625	9.0735	0.0188	2035.858	0.0000	0.52%	0.25%	0.71%	0.44%	28.1%
DELEK DRILLIN - LP	5.74%	29.9%	0.3567	7.5090	0.0721	1134.023	0.0000	2.23%	1.89%	2.42%	2.08%	32.0%
DELEK ENERGY SYSTEMS LTD	12.85%	35.1%	0.7101	10.5690	0.0038	3227.219	0.0000	0.13%	-1.33%	1.67%	0.09%	35.2%
DELEK GROUP LTD	11.47%	29.5%	-0.0727	6.3095	0.0458	597.1673	0.0000	1.38%	1.52%	1.98%	2.12%	31.6%
AFRIQUIA GAZ	38.75%	35.9%	-0.4273	9.3657	0.0485	2244.846	0.0000	1.78%	4.76%	16.49%	22.43%	58.3%
ISRAMCO NEGEV 2 LP	14.67%	33.5%	-0.4885	13.7850	-0.0858	6381.458	0.0000	-2.75%	-1.51%	0.76%	2.31%	35.8%
NATIONAL GAS & INDUSTRIALIZA	5.70%	57.5%	0.1333	17.9539	-0.0227	12172.46	0.0000	-1.29%	-1.41%	-0.93%	-1.05%	56.5%
OANDO PLC	36.39%	37.5%	0.5070	9.2587	0.1322	2187.53	0.0000	5.31%	2.44%	15.26%	10.97%	48.5%
GAZPROM OAO-SPON ADR	46.38%	41.3%	-0.0415	6.1574	0.0317	542.8696	0.0000	1.33%	1.65%	9.19%	9.64%	50.9%
SAMIR	19.86%	29.2%	-0.2943	6.0699	0.1583	531.7041	0.0000	5.02%	6.03%	6.57%	7.66%	36.8%
SASOL LTD	30.98%	32.5%	-0.1241	4.4082	0.0674	111.2672	0.0000	2.26%	2.91%	3.96%	4.68%	37.2%
SHELL OMAN MARKETING	23.40%	19.9%	0.1172	23.3117	0.0398	22453.31	0.0000	0.81%	0.36%	-289.51%	-397.62%	-377.7%

Table 128

Summary of Lukoil global peer group sample statistics (2003-2009) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
BP PLC	1.77%	26.2%	0.2999	9.4042	-0.0739	2701.34	0.0000	-1.86%	-1.95%	-1.83%	-1.92%	24.3%
TOTAL SA	3.20%	27.2%	0.3923	12.8159	-0.0920	6331.137	0.0000	-2.39%	-2.60%	-2.22%	-2.43%	24.8%
GAZPROM OAO	25.87%	46.4%	0.1730	15.4708	0.0528	10161.95	0.0000	2.51%	1.78%	10.82%	9.82%	56.3%
IMPERIAL OIL LTD	16.52%	34.4%	-0.0671	12.7857	-0.0750	6253.521	0.0000	-2.48%	-2.30%	1.43%	1.66%	36.1%
LUKOIL	16.79%	50.7%	0.1625	13.9228	-0.0877	7796.641	0.0000	-4.25%	-4.70%	-1.31%	-1.82%	48.9%
OCCIDENTAL PETROLEUM CORP	21.94%	40.7%	-0.3958	13.6870	-0.1050	7498.021	0.0000	-4.06%	-2.55%	2.90%	5.05%	45.8%
PETROBRAS - PETROLEO BRAS-PR	26.58%	39.0%	-0.1698	7.7032	0.0293	1451.759	0.0000	1.16%	1.92%	4.92%	5.84%	44.9%
ROYAL DUTCH SHELL PLC	-2.09%	26.2%	0.2237	13.2331	-0.0447	6850.157	0.0000	-1.14%	-1.06%	-1.07%	-0.99%	25.2%
Gazpromneft OAO	2.61%	48.2%	-0.8215	12.6283	0.0926	6229.124	0.0000	4.67%	5.03%	4.72%	5.08%	53.3%
SURGUTNEFTEGAZ	14.84%	60.8%	0.2227	15.7852	-0.1030	10685.54	0.0000	-5.95%	-6.49%	-3.72%	-4.31%	56.5%
Tatneft	18.97%	55.7%	0.6670	21.1472	0.1167	21618.18	0.0000	6.90%	4.86%	11.57%	9.22%	64.9%
WEST SIBERIAN RESOURCES-SDR	2.99%	73.4%	-0.2784	27.5322	-0.0208	39314.59	0.0000	-1.50%	-1.36%	-1.38%	-1.24%	72.2%
EXXON MOBIL CORP	11.32%	28.3%	0.0076	17.8681	-0.1805	14433.34	0.0000	-4.71%	-4.73%	-0.79%	-0.81%	27.5%

Table 129

Summary of Gazprom global peer group sample statistics (2003-2008) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
BP PLC	4.47%	19.9%	-0.0976	4.0832	-0.0539	65.92615	0.0000	-1.04%	-0.97%	-0.99%	-0.92%	19.0%
TOTAL SA	8.63%	20.0%	-0.1824	4.7373	-0.0826	171.4891	0.0000	-1.58%	-1.32%	-1.28%	-1.01%	19.0%
GAZPROM OAO	48.84%	35.6%	-0.0444	9.5200	0.0290	2313.701	0.0000	1.05%	1.41%	35.12%	36.49%	72.1%
IMPERIAL OIL LTD	24.49%	26.1%	-0.2953	5.2381	-0.0067	291.5628	0.0000	-0.17%	1.09%	2.18%	3.69%	29.8%
LUKOIL	27.25%	37.2%	-0.1770	6.3338	-0.0957	611.6015	0.0000	-3.39%	-2.57%	-0.04%	0.96%	38.2%
OCCIDENTAL PETROLEUM CORP	31.59%	28.0%	-0.1956	3.8648	-0.0388	49.02409	0.0000	-1.06%	0.01%	0.34%	1.53%	29.5%
PETROBRAS - PETROLEO BRAS-PR	37.50%	31.6%	0.0616	5.6505	0.0216	383.1145	0.0000	0.69%	0.31%	6.35%	5.82%	37.4%
ROYAL DUTCH SHELL PLC	2.90%	18.6%	-0.4511	6.1208	-0.0406	574.2741	0.0000	-0.74%	-0.52%	-0.67%	-0.45%	18.1%
Gazpromneft OAO	13.75%	38.8%	-0.7036	8.6619	0.0332	1852.227	0.0000	1.31%	2.99%	2.45%	4.23%	43.1%
SURGUTNEFTEGAZ	16.40%	47.8%	-0.1597	11.2946	-0.0687	3749.39	0.0000	-3.17%	-2.73%	-0.99%	-0.50%	47.3%
Tatneft	34.39%	41.2%	-0.1013	6.0634	0.0661	512.8893	0.0000	2.81%	3.40%	6.53%	7.22%	48.4%
WEST SIBERIAN RESOURCES-SDR	19.10%	71.1%	-0.2367	35.5119	-0.0348	57531.79	0.0000	-2.42%	-1.66%	5.62%	6.57%	77.7%
EXXON MOBIL CORP	18.50%	20.4%	-0.3731	4.2236	-0.0721	111.7665	0.0000	-1.42%	-0.19%	-0.45%	0.91%	21.3%

Table 130

Summary of Lukoil North American peer group sample statistics (2003-2009) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
APACHE CORP	12.94%	40.1%	-0.4033	12.2855	-0.0787	5671.934	0.0000	-3.03%	-2.14%	-1.20%	-0.21%	39.9%
ANADARKO PETROLEUM CORP	10.10%	41.3%	-0.6924	15.6443	-0.0325	10563.87	0.0000	-1.32%	-0.12%	0.08%	1.36%	42.7%
CANADIAN NATURAL RESOURCES	22.97%	45.5%	-0.8599	12.7184	-0.0172	6359.734	0.0000	-0.77%	2.78%	4.58%	9.08%	54.5%
DEVON ENERGY CORPORATION	11.68%	40.1%	-0.2209	10.9113	-0.0625	4099.208	0.0000	-2.43%	-1.99%	-1.19%	-0.73%	39.4%
ENCANA CORP	12.70%	37.0%	-0.4320	9.9787	-0.0539	3228.619	0.0000	-1.94%	-1.00%	-0.55%	0.47%	37.5%
EOG RESOURCES INC	19.05%	42.4%	-0.2436	10.4135	-0.0671	3603.966	0.0000	-2.75%	-1.96%	0.30%	1.21%	43.6%
HALLIBURTON CO	9.24%	42.4%	-0.6072	12.7182	-0.0274	6262.608	0.0000	-1.14%	-0.19%	-0.29%	0.71%	43.1%
HESS CORP	23.81%	46.5%	-0.8877	13.0314	-0.0702	6776.017	0.0000	-3.15%	0.69%	3.11%	8.20%	54.7%
HUSKY ENERGY INC	20.86%	32.6%	-0.0011	6.8340	-0.0359	959.7457	0.0000	-1.15%	-1.14%	1.23%	1.23%	33.9%
IMPERIAL OIL LTD	16.52%	34.4%	-0.0671	12.7857	-0.0750	6253.521	0.0000	-2.48%	-2.30%	1.43%	1.66%	36.1%
KINDER MORGAN ENERGY PRTNRS	3.98%	20.9%	-0.1569	21.9779	-0.0030	23521.95	0.0000	-0.06%	0.04%	0.56%	0.67%	21.6%
LUKOIL	16.79%	50.7%	0.1625	13.9228	-0.0877	7796.641	0.0000	-4.25%	-4.70%	-1.31%	-1.82%	48.9%
MARATHON OIL CORP	12.96%	41.7%	-0.5740	15.2463	-0.0754	9877.99	0.0000	-3.02%	-1.74%	-0.67%	0.77%	42.5%
NATIONAL OILWELL VARCO INC	16.94%	54.9%	-0.7276	12.6890	-0.0256	6267.613	0.0000	-1.38%	0.75%	0.87%	3.20%	58.1%
OCCIDENTAL PETROLEUM CORP	21.94%	40.7%	-0.3958	13.6870	-0.1050	7498.021	0.0000	-4.06%	-2.55%	2.90%	5.05%	45.8%
PETRO-CANADA	2.99%	36.7%	-0.3673	14.0646	-0.0091	8028.602	0.0000	-0.33%	-0.15%	-0.22%	-0.03%	36.7%
SCHLUMBERGER LTD	13.53%	40.1%	-0.8635	12.6666	-0.0746	6295.811	0.0000	-2.87%	-0.82%	-0.78%	1.53%	41.6%
SUNCOR ENERGY INC	15.13%	43.8%	-0.7381	10.3536	-0.0229	3672.954	0.0000	-0.99%	0.96%	0.72%	2.82%	46.6%
TALISMAN ENERGY INC	11.88%	42.4%	-0.2652	11.9704	-0.0548	5272.274	0.0000	-2.25%	-1.72%	-0.90%	-0.33%	42.1%
TRANSCANADA CORP	6.28%	19.9%	-0.3195	11.5434	-0.0995	4792.229	0.0000	-1.88%	-1.54%	-1.07%	-0.70%	19.2%
XTO ENERGY INC	22.31%	42.7%	-0.8433	11.8777	-0.0459	5331.555	0.0000	-1.91%	1.49%	3.16%	7.50%	50.2%

Table 131

Summary of Lukoil North American peer group sample statistics (2003-2008) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
APACHE CORP	27.51%	28.2%	-0.2111	3.3335	0.0070	15.75032	0.0004	0.20%	1.20%	0.57%	1.60%	29.8%
ANADARKO PETROLEUM CORP	21.79%	28.3%	-0.2030	3.7597	-0.0076	40.37168	0.0000	-0.21%	0.54%	0.33%	1.12%	29.4%
CANADIAN NATURAL RESOURCES	34.96%	32.6%	-0.1497	3.7824	0.0715	38.18894	0.0000	2.41%	3.31%	3.59%	4.55%	37.2%
DEVON ENERGY CORPORATION	29.60%	29.8%	0.0084	4.3587	-0.0246	100.4673	0.0000	-0.72%	-0.76%	1.09%	1.05%	30.8%
ENCANA CORP	23.54%	28.0%	-0.3158	5.0323	0.0042	246.4684	0.0000	0.12%	1.41%	1.89%	3.36%	31.4%
EOG RESOURCES INC	36.62%	32.7%	0.1921	7.0239	-0.0224	889.1522	0.0000	-0.72%	-1.85%	8.28%	6.44%	39.2%
HALLIBURTON CO	26.48%	30.4%	-0.0831	3.9285	0.0334	48.41704	0.0000	1.03%	1.40%	1.92%	2.31%	32.7%
HESS CORP	38.36%	31.1%	-0.0648	4.2833	-0.0255	90.53087	0.0000	-0.78%	-0.36%	2.06%	2.56%	33.6%
HUSKY ENERGY INC	33.81%	26.0%	0.0552	3.8808	0.0576	42.88257	0.0000	1.54%	1.23%	3.15%	2.81%	28.8%
IMPERIAL OIL LTD	24.49%	26.1%	-0.2953	5.2381	-0.0067	291.5628	0.0000	-0.17%	1.09%	2.18%	3.69%	29.8%
KINDER MORGAN ENERGY PRTRNS	8.59%	15.4%	-0.3139	8.1255	0.0415	1451.031	0.0000	0.65%	1.11%	1.70%	2.22%	17.6%
LUKOIL	27.25%	37.2%	-0.1770	6.3338	-0.0957	611.6015	0.0000	-3.39%	-2.57%	-0.04%	0.96%	38.2%
MARATHON OIL CORP	28.88%	29.8%	-0.4112	4.3358	0.0127	133.906	0.0000	0.38%	2.50%	2.00%	4.36%	34.1%
NATIONAL OILWELL VARCO INC	33.58%	38.6%	-0.1249	4.2136	-0.0196	83.54079	0.0000	-0.75%	-0.03%	0.82%	1.60%	40.2%
OCCIDENTAL PETROLEUM CORP	31.59%	28.0%	-0.1956	3.8648	-0.0388	49.02409	0.0000	-1.06%	0.01%	0.34%	1.53%	29.5%
PETRO-CANADA	12.02%	26.1%	-0.6272	7.3278	0.0579	1104.824	0.0000	1.55%	2.87%	2.53%	3.94%	30.1%
SCHLUMBERGER LTD	29.75%	29.8%	-0.2167	4.8974	-0.0001	206.1251	0.0000	-0.00%	1.11%	2.54%	3.86%	33.7%
SUNCOR ENERGY INC	28.15%	29.9%	-0.1872	3.6815	0.0124	32.90004	0.0000	0.37%	1.28%	1.13%	2.09%	32.0%
TALISMAN ENERGY INC	19.27%	31.1%	-0.1831	3.8959	-0.0055	50.97513	0.0000	-0.17%	0.43%	0.28%	0.90%	32.0%
TRANSCANADA CORP	11.74%	15.6%	0.2267	3.9746	-0.0834	62.86809	0.0000	-1.25%	-1.68%	-0.85%	-1.30%	14.3%
XTO ENERGY INC	39.48%	31.8%	-0.2377	3.5252	0.0023	27.31228	0.0000	0.07%	1.72%	1.18%	2.95%	34.8%

Table 132

Summary of Lukoil Western European peer group sample statistics (2003-2009) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
AMEC PLC	17.07%	35.9%	-0.1300	10.3964	-0.0260	3576.321	0.0000	-0.92%	-0.55%	1.85%	2.29%	38.2%
BG GROUP PLC	23.14%	33.7%	-0.0247	7.6171	-0.0087	1392.017	0.0000	-0.29%	-0.20%	3.10%	3.22%	37.0%
CIA ESPANOLA DE PETROLEOS SA	8.26%	25.0%	-6.0662	130.7678	0.2164	1075472	0.0000	6.13%	17.55%	24.80%	62.26%	87.3%
CAIRN ENERGY PLC	30.82%	45.5%	1.8401	35.4000	0.0601	69424.73	0.0000	2.81%	-5.10%	61.60%	29.21%	74.7%
HELLENIC PETROLEUM SA	5.24%	31.1%	0.1137	6.1076	0.0408	633.914	0.0000	1.29%	1.19%	1.40%	1.30%	32.4%
FLUXYS-D	17.42%	25.9%	0.0542	7.2113	-0.1030	1158.721	0.0000	-2.53%	-2.69%	-0.00%	-0.19%	25.7%
FUGRO NV-CVA	18.85%	38.7%	-1.1010	18.7047	0.0182	16419.94	0.0000	0.71%	4.50%	7.65%	12.99%	51.7%
LUKOIL	16.79%	50.7%	0.1625	13.9228	-0.0877	7796.641	0.0000	-4.25%	-4.70%	-1.31%	-1.82%	48.9%
OMV AG	12.51%	39.7%	-0.5770	8.5389	0.0508	2090.046	0.0000	2.06%	3.30%	2.94%	4.24%	43.9%
REPSOL YPF SA	-1.12%	27.5%	-0.9325	15.5006	0.0071	10429.85	0.0000	0.19%	0.02%	0.22%	0.04%	27.5%
TRANSOCEAN LTD	16.07%	43.3%	-0.7305	11.1110	-0.0187	4434.798	0.0000	-0.80%	1.25%	1.36%	3.63%	46.9%
SBM OFFSHORE NV	2.25%	37.1%	-1.4246	23.4555	0.0488	27849.72	0.0000	1.85%	2.39%	1.96%	2.51%	39.6%
SAIPEM	12.96%	37.4%	-0.4204	10.7913	-0.0102	4009.625	0.0000	-0.38%	0.55%	1.16%	2.17%	39.6%
STATOILHYDRO ASA	13.57%	35.0%	-0.3546	6.9093	-0.0483	1030.694	0.0000	-1.65%	-0.82%	-0.72%	0.15%	35.2%
TECHNIP SA	10.25%	40.5%	-0.4327	9.7807	-0.0091	3050.885	0.0000	-0.36%	0.39%	0.39%	1.17%	41.6%
TULLOW OIL PLC	38.10%	42.4%	0.4977	10.3766	-0.0155	3617.46	0.0000	-0.65%	-3.59%	13.68%	8.51%	51.0%

Table 133

Summary of Lukoil Western European peer group sample statistics (2003-2008) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
AMEC PLC	26.55%	28.4%	0.1434	6.0979	-0.0505	526.7123	0.0000	-1.40%	-2.02%	2.45%	1.64%	30.1%
BG GROUP PLC	28.92%	24.9%	0.3130	5.2064	-0.0472	286.2305	0.0000	-1.14%	-2.56%	2.60%	0.71%	25.6%
CIA ESPANOLA DE PETROLEOS SA	25.55%	19.4%	2.2807	30.1155	-0.0183	41142.22	0.0000	-0.35%	-6.78%	-37.88%	-55.94%	-36.5%
CAIRN ENERGY PLC	44.48%	39.1%	3.6061	66.3321	0.0727	221093.2	0.0000	2.94%	-13.40%	-60.60%	-70.98%	-31.9%
HELLENIC PETROLEUM SA	13.05%	27.3%	0.2323	4.8532	0.0057	198.6403	0.0000	0.16%	-0.34%	0.64%	0.13%	27.4%
FLUXYS-D	22.26%	25.5%	-0.0179	7.3053	-0.1152	1008.721	0.0000	-2.78%	-2.71%	1.94%	2.04%	27.6%
FUGRO NV-CVA	35.49%	29.7%	-1.8798	25.4979	-0.0198	28312.44	0.0000	-0.58%	17.42%	-103.42%	-67.14%	-37.5%
LUKOIL	27.25%	37.2%	-0.1770	6.3338	-0.0957	611.6015	0.0000	-3.39%	-2.57%	-0.04%	0.96%	38.2%
OMV AG	28.07%	32.4%	-0.2563	4.0582	0.0593	75.23761	0.0000	1.98%	3.22%	3.02%	4.34%	36.8%
REPSOL YPF SA	10.52%	21.0%	-0.5152	9.8968	0.0223	2646.165	0.0000	0.47%	1.41%	2.06%	3.16%	24.2%
TRANSOCEAN LTD	36.33%	34.0%	-0.0055	3.4487	-0.0203	10.95996	0.0042	-0.68%	-0.65%	0.08%	0.11%	34.1%
SBM OFFSHORE NV	15.50%	27.6%	-0.2170	9.0471	-0.0375	2000.15	0.0000	-1.01%	-0.44%	1.47%	2.16%	29.8%
SAIPEM	29.84%	29.5%	0.1489	6.3955	-0.0847	632.2098	0.0000	-2.39%	-3.11%	3.22%	2.17%	31.7%
STATOILHYDRO ASA	21.43%	28.8%	-0.2344	5.1433	-0.0244	261.9238	0.0000	-0.69%	0.17%	0.85%	1.81%	30.6%
TECHNIP SA	25.15%	32.1%	-0.8600	11.5570	-0.0812	4145.485	0.0000	-2.50%	1.61%	7.78%	15.61%	47.7%
TULLOW OIL PLC	42.16%	34.0%	0.3279	6.2306	0.0280	591.3516	0.0000	0.96%	-1.20%	9.47%	6.18%	40.2%

Table 134

Summary of Lukoil Asia Developed peer group sample statistics (2003-2009) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
S-OIL CORPORATION	20.25%	38.8%	0.3042	6.2050	-0.0265	694.8547	0.0000	-1.01%	-2.01%	0.50%	-0.58%	38.2%
CNPC HONG KONG LIMITED	27.23%	46.0%	0.0714	7.3495	0.0281	1236.545	0.0000	1.31%	0.99%	4.33%	3.96%	50.0%
NIPPON OIL CORP	-0.83%	38.3%	-0.2648	9.5269	-0.0563	2799.791	0.0000	-2.09%	-2.13%	-2.09%	-2.13%	36.2%
SHOWA SHELL SEKIYU KK	2.75%	30.8%	-0.2964	9.2134	-0.0589	2543.584	0.0000	-1.76%	-1.62%	-1.69%	-1.55%	29.2%
COSMO OIL COMPANY LTD	9.84%	36.9%	0.0761	6.1868	-0.0090	664.6032	0.0000	-0.33%	-0.45%	0.03%	-0.10%	36.8%
TONENGENERAL SEKIYU KK	3.76%	23.5%	0.2557	8.5969	-0.0008	2062.38	0.0000	-0.02%	-0.18%	0.12%	-0.04%	23.5%
NIPPON MINING HOLDINGS INC	8.85%	46.9%	-0.2877	6.2075	0.0434	693.3417	0.0000	2.07%	2.50%	2.29%	2.72%	49.6%
CNOOC LTD	22.55%	45.2%	0.0713	8.2886	0.0766	1827.486	0.0000	3.59%	3.32%	6.00%	5.71%	50.9%
ARROW ENERGY LTD	55.46%	72.8%	0.4842	13.7637	-0.0761	7625.69	0.0000	-5.33%	-9.53%	23.98%	15.57%	88.4%
CALTEX AUSTRALIA LIMITED	25.91%	37.1%	-0.3092	5.5945	0.0395	464.4738	0.0000	1.49%	2.87%	3.47%	5.00%	42.1%
LUKOIL	16.79%	50.7%	0.1625	13.9228	-0.0877	7796.641	0.0000	-4.25%	-4.70%	-1.31%	-1.82%	48.9%
NEW ZEALAND REFINING CO LTD	24.98%	27.1%	0.0463	12.4679	0.0992	5853.33	0.0000	2.83%	2.64%	14.17%	13.80%	40.9%
SANTOS LTD	17.32%	36.6%	-0.2623	8.0276	-0.0095	1668.353	0.0000	-0.34%	0.43%	1.48%	2.33%	38.9%
WORLEYPARSONS LTD	39.15%	44.1%	0.2274	10.2064	0.1097	3404.219	0.0000	5.11%	3.67%	16.67%	14.49%	58.5%
WOODSIDE PETROLEUM LTD	20.86%	34.1%	-0.1540	7.2409	0.0421	1180.488	0.0000	1.46%	2.00%	3.77%	4.38%	38.4%

Table 135

Summary of Lukoil Asia Developed peer group sample statistics (2003-2008) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
S-OIL CORPORATION	26.26%	39.0%	0.4010	5.9399	-0.0460	505.3396	0.0000	-1.75%	-3.43%	0.66%	-1.23%	37.8%
CNPC HONG KONG LIMITED	37.32%	42.9%	0.3961	7.0061	0.0639	907.4847	0.0000	2.82%	0.48%	8.55%	5.61%	48.5%
NIPPON OIL CORP	4.76%	31.4%	-0.0918	5.2039	-0.0691	266.1488	0.0000	-2.09%	-2.02%	-2.02%	-1.95%	29.4%
SHOWA SHELL SEKIYU KK	6.01%	24.5%	-0.1399	6.0486	-0.0079	510.0132	0.0000	-0.19%	-0.05%	-0.00%	0.14%	24.7%
COSMO OIL COMPANY LTD	11.25%	30.8%	-0.1562	4.7131	0.0104	165.0005	0.0000	0.32%	0.62%	0.62%	0.92%	31.7%
TONENGENERAL SEKIYU KK	3.22%	20.0%	-0.0272	6.8932	0.0165	824.9655	0.0000	0.33%	0.35%	0.42%	0.43%	20.4%
NIPPON MINING HOLDINGS INC	17.44%	41.7%	-0.4503	5.5151	0.0192	388.3466	0.0000	0.80%	2.15%	1.57%	2.97%	44.7%
CNOOC LTD	34.93%	37.7%	0.1401	6.6279	0.0169	720.504	0.0000	0.64%	-0.16%	6.15%	5.11%	42.8%
ARROW ENERGY LTD	58.37%	72.2%	0.5298	16.2491	-0.0954	9613.286	0.0000	-6.56%	-11.34%	44.37%	30.10%	102.3%
CALTEX AUSTRALIA LIMITED	35.33%	31.9%	-0.2885	5.6389	0.0537	397.0516	0.0000	1.75%	3.54%	6.40%	8.74%	40.6%
LUKOIL	27.25%	37.2%	-0.1770	6.3338	-0.0957	611.6015	0.0000	-3.39%	-2.57%	-0.04%	0.96%	38.2%
NEW ZEALAND REFINING CO LTD	29.29%	25.7%	0.5782	10.5884	0.0933	3206.277	0.0000	2.51%	-0.05%	17.09%	11.46%	37.2%
SANTOS LTD	17.27%	29.1%	-0.2319	4.8751	-0.0212	203.0305	0.0000	-0.61%	0.07%	0.23%	0.96%	30.1%
WORLEYPARSONS LTD	58.77%	36.3%	1.3879	15.0209	0.1067	8282.547	0.0000	4.09%	-6.08%	-726.80%	109.22%	145.6%
WOODSIDE PETROLEUM LTD	32.84%	27.1%	-0.2474	5.2189	0.0262	281.2374	0.0000	0.72%	2.14%	4.83%	6.71%	33.8%

Table 136

Summary of Lukoil Asia Emerging peer group sample statistics (2003-2009) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
CHINA OILFIELD SERVICES-H	19.47%	60.5%	0.1520	8.1018	0.0204	1705.443	0.0000	1.24%	0.75%	2.58%	2.07%	62.5%
OFFSHORE OIL ENGINEERING-A	35.70%	41.9%	0.0182	6.1680	0.0314	655.3554	0.0000	1.33%	1.22%	5.61%	5.48%	47.4%
BHARAT PETROLEUM CORP LTD	8.08%	43.0%	0.1512	5.7675	0.0360	506.0527	0.0000	1.57%	1.37%	1.74%	1.54%	44.5%
GAIL INDIA LTD	25.40%	45.5%	-0.1160	16.8558	-0.0373	12538.42	0.0000	-1.66%	-1.16%	8.88%	9.64%	55.1%
HINDUSTAN PETROLEUM CORP	-2.88%	42.8%	-0.1145	6.5153	0.0732	810.264	0.0000	3.25%	3.19%	3.27%	3.22%	46.0%
INDIAN OIL CORPORATION LTD	15.44%	42.5%	-0.3235	8.9315	0.1720	2324.482	0.0000	8.02%	8.87%	9.22%	10.11%	52.6%
LUKOIL	16.79%	50.7%	0.1625	13.9228	-0.0877	7796.641	0.0000	-4.25%	-4.70%	-1.31%	-1.82%	48.9%
OIL & NATURAL GAS CORP LTD	18.10%	39.2%	-0.5063	7.7385	0.0776	1532.953	0.0000	3.16%	4.74%	4.74%	6.45%	45.6%
OIL SEARCH LTD	34.82%	44.0%	-0.2641	8.7833	-0.0062	2202.035	0.0000	-0.27%	1.32%	7.61%	9.84%	53.9%
PETRONAS DAGANGAN BHD	16.22%	20.8%	0.4467	8.2748	-0.0760	1868.791	0.0000	-1.52%	-2.66%	2.03%	0.45%	21.3%
PETRONAS GAS BHD	5.91%	19.8%	0.1615	7.5702	-0.1989	1370.527	0.0000	-3.60%	-3.76%	-3.18%	-3.34%	16.4%
PTT PCL	20.30%	38.0%	-0.1049	9.5094	-0.0111	2769.473	0.0000	-0.42%	-0.06%	2.81%	3.23%	41.2%
PTT EXPLOR & PROD PUBLIC CO	21.88%	39.9%	-0.0274	10.2808	-0.0596	3461.348	0.0000	-2.30%	-2.20%	2.01%	2.13%	42.0%
RELIANCE INDUSTRIES LTD	30.36%	39.1%	-0.6731	9.0221	0.0398	2486.174	0.0000	1.58%	5.30%	8.19%	13.29%	52.4%

Table 137

Summary of Lukoil Asia Emerging peer group sample statistics (2003-2008) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
CHINA OILFIELD SERVICES-H	37.60%	52.2%	0.3481	5.6672	-0.0583	413.5033	0.0000	-2.95%	-5.04%	0.46%	-1.92%	50.3%
OFFSHORE OIL ENGINEERING-A	49.76%	39.4%	0.1144	6.8662	0.0015	816.2439	0.0000	0.06%	-0.87%	13.65%	11.99%	51.4%
BHARAT PETROLEUM CORP LTD	13.10%	40.5%	0.3426	6.5966	0.0289	729.4641	0.0000	1.18%	0.45%	1.81%	1.05%	41.6%
GAIL INDIA LTD	34.61%	44.7%	0.0292	19.4485	-0.0102	14722.78	0.0000	-0.45%	-0.62%	31.53%	31.03%	75.7%
HINDUSTAN PETROLEUM CORP	-2.37%	41.2%	-0.1573	7.3090	0.0787	1015.77	0.0000	3.36%	3.30%	3.39%	3.32%	44.5%
INDIAN OIL CORPORATION LTD	20.54%	42.5%	-0.3990	9.9451	0.1943	2659.428	0.0000	9.20%	10.60%	11.67%	13.22%	55.7%
LUKOIL	27.25%	37.2%	-0.1770	6.3338	-0.0957	611.6015	0.0000	-3.39%	-2.57%	-0.04%	0.96%	38.2%
OIL & NATURAL GAS CORP LTD	27.88%	37.2%	-0.4974	7.8137	0.0743	1314.757	0.0000	2.86%	5.31%	7.17%	10.20%	47.4%
OIL SEARCH LTD	39.70%	37.3%	-0.2617	8.1541	-0.0408	1460.483	0.0000	-1.49%	0.33%	11.34%	14.76%	52.1%
PETRONAS DAGANGAN BHD	20.65%	20.3%	0.4397	8.9248	-0.0320	1952.275	0.0000	-0.64%	-2.04%	6.71%	4.12%	24.5%
PETRONAS GAS BHD	6.57%	20.5%	0.1227	7.4090	-0.1857	1061.114	0.0000	-3.49%	-3.63%	-3.01%	-3.15%	17.3%
PTT PCL	40.01%	34.5%	0.0678	10.7979	-0.0395	3309.908	0.0000	-1.33%	-1.78%	28.47%	26.88%	61.4%
PTT EXPLOR & PROD PUBLIC CO	36.25%	34.9%	0.1190	9.7324	-0.0564	2469.534	0.0000	-1.91%	-2.61%	14.96%	13.37%	48.3%
RELIANCE INDUSTRIES LTD	46.02%	33.2%	-0.5108	8.1844	0.0008	1519.409	0.0000	0.03%	4.47%	23.48%	37.74%	71.0%

Table 138

Summary of Lukoil Latin America peer group sample statistics (2003-2009) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
EMPRESAS COPEC SA	10.30%	24.1%	0.2733	6.9718	0.1380	1049.477	0.0000	3.58%	3.11%	4.23%	3.74%	27.8%
GLOBAL ENVIRONMENTAL ENERGY	-39.87%	163.1%	0.5409	7.9443	-0.0980	1672.556	0.0000	-15.22%	-11.53%	-12.97%	-9.17%	153.9%
LUKOIL	16.79%	50.7%	0.1625	13.9228	-0.0877	7796.641	0.0000	-4.25%	-4.70%	-1.31%	-1.82%	48.9%
PETROBRAS ENERGIA PARTICIP-B	-1.68%	40.1%	-0.3338	12.2244	0.0127	5584.719	0.0000	0.51%	0.42%	0.54%	0.44%	40.6%
PETROBRAS ENERGIA SA-B	-2.93%	39.9%	-0.4492	19.1262	0.0123	17032.02	0.0000	0.49%	0.27%	0.63%	0.41%	40.3%
PETROBRAS - PETROLEO BRAS-PR	26.58%	39.0%	-0.1698	7.7032	0.0293	1451.759	0.0000	1.16%	1.92%	4.92%	5.84%	44.9%
PETROLERA DEL CONOSUR SA	2.07%	59.3%	-0.4147	15.6413	0.0848	10478.73	0.0000	5.24%	5.38%	5.28%	5.42%	64.7%
TRANSPORTADORA DE GAS SUR-B	2.76%	41.2%	-0.0547	7.7144	0.0215	1451.938	0.0000	0.89%	0.92%	0.93%	0.95%	42.1%
YPF S.A.-D	5.34%	33.2%	-1.2908	21.5311	0.0747	22856.38	0.0000	2.57%	3.76%	3.20%	4.43%	37.7%

Table 139

Summary of Lukoil Latin America peer group sample statistics (2003-2008) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
EMPRESAS COPEC SA	21.07%	21.2%	0.2008	4.5157	0.1159	133.7865	0.0000	2.61%	1.92%	3.85%	3.09%	24.3%
GLOBAL ENVIRONMENTAL ENERGY	-21.69%	160.4%	0.6679	9.1113	-0.0791	2129.474	0.0000	-12.18%	-9.73%	-11.37%	-8.89%	151.5%
LUKOIL	27.25%	37.2%	-0.1770	6.3338	-0.0957	611.6015	0.0000	-3.39%	-2.57%	-0.04%	0.96%	38.2%
PETROBRAS ENERGIA PARTICIP-B	9.93%	32.6%	-0.9146	14.7715	0.0009	7722.445	0.0000	0.03%	1.62%	1.58%	3.33%	36.0%
PETROBRAS ENERGIA SA-B	7.67%	37.2%	-0.4049	16.4879	0.0002	9935.334	0.0000	0.01%	0.53%	0.92%	1.47%	38.7%
PETROBRAS - PETROLEO BRAS-PR	37.50%	31.6%	0.0616	5.6505	0.0216	383.1145	0.0000	0.69%	0.31%	6.35%	5.82%	37.4%
PETROLERA DEL CONOSUR SA	17.93%	54.8%	0.8020	9.1524	0.0374	2199.8	0.0000	2.08%	-0.22%	3.57%	1.15%	56.0%
TRANSPORTADORA DE GAS SUR-B	17.51%	36.6%	0.0104	8.7490	-0.0159	1798.541	0.0000	-0.57%	-0.60%	1.59%	1.56%	38.1%
YPF S.A.-D	18.84%	30.9%	0.1653	7.3195	-0.0063	1021.255	0.0000	-0.19%	-0.70%	2.04%	1.45%	32.4%

Table 140

Summary of Lukoil Europe Emerging peer group sample statistics (2003-2009) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
DOGAN SIRKETLER GRUBU HLDGS	10.28%	47.9%	-0.0079	6.4307	0.0159	768.4659	0.0000	0.77%	0.78%	1.08%	1.09%	49.0%
LUKOIL	16.79%	50.7%	0.1625	13.9228	-0.0877	7796.641	0.0000	-4.25%	-4.70%	-1.31%	-1.82%	48.9%
MOL HUNGARIAN OIL AND GAS NY	9.71%	36.8%	-0.0702	11.3069	0.0541	4506.674	0.0000	2.04%	2.16%	2.90%	3.02%	39.8%
PETROL	7.33%	27.5%	0.2392	12.5483	0.1107	5967.602	0.0000	3.22%	2.93%	3.93%	3.63%	31.2%
POLSKI KONCERN NAFTOWY SA	3.86%	34.9%	-0.2367	5.0369	0.0162	285.5313	0.0000	0.57%	0.72%	0.60%	0.76%	35.7%
PETROL OFISI AS	7.71%	44.3%	0.5573	8.8327	0.0648	2302.384	0.0000	2.96%	2.25%	3.26%	2.55%	46.8%
GAZPROM NEFT OAO	2.61%	48.2%	-0.8215	12.6283	0.0926	6229.124	0.0000	4.67%	5.03%	4.72%	5.08%	53.3%
SLOVNAFT AS	8.16%	30.6%	-1.5000	22.4306	-0.0480	25238.46	0.0000	-1.43%	0.76%	0.54%	3.05%	33.6%
SURGUTNEFTEGAZ	14.84%	60.8%	0.2227	15.7852	-0.1030	10685.54	0.0000	-5.95%	-6.49%	-3.72%	-4.31%	56.5%
SNP PETROM SA	6.21%	42.6%	-0.2569	9.4505	0.0434	2733.938	0.0000	1.88%	2.15%	2.12%	2.39%	45.0%
TATNEFT	18.97%	55.7%	0.6670	21.1472	0.1167	21618.18	0.0000	6.90%	4.86%	11.57%	9.22%	64.9%
TUPRAS-TURKIYE PETROL RAFINE	11.76%	42.8%	-0.0631	7.2289	0.0474	1168.674	0.0000	2.07%	2.19%	2.62%	2.74%	45.5%
UNIPETROL AS	14.99%	42.2%	-0.2012	19.2739	0.1136	17302.38	0.0000	5.08%	5.59%	8.54%	9.13%	51.4%

Table 141

Summary of Lukoil Europe Emerging peer group sample statistics (2003-2008) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
DOGAN SIRKETLER GRUBU HLDGS	24.91%	44.6%	0.1512	5.2079	0.0001	270.2601	0.0000	0.00%	-0.61%	1.32%	0.67%	45.3%
LUKOIL	27.25%	37.2%	-0.1770	6.3338	-0.0957	611.6015	0.0000	-3.39%	-2.57%	-0.04%	0.96%	38.2%
MOL HUNGARIAN OIL AND GAS NY	29.27%	29.3%	0.1130	4.5516	0.0924	133.7774	0.0000	2.83%	2.29%	4.66%	4.05%	33.3%
PETROL	29.42%	20.3%	0.4877	10.1444	0.0611	2829.305	0.0000	1.27%	-0.88%	28.16%	18.49%	38.8%
POLSKI KONCERN NAFTOWY SA	16.23%	31.3%	-0.0668	3.9292	0.0096	47.95826	0.0000	0.30%	0.48%	0.63%	0.81%	32.1%
PETROL OFISI AS	17.80%	41.6%	0.6498	10.0977	0.0395	2833.303	0.0000	1.67%	-0.17%	3.95%	1.91%	43.5%
GAZPROM NEFT OAO	13.75%	38.8%	-0.7036	8.6619	0.0332	1852.227	0.0000	1.31%	2.99%	2.45%	4.23%	43.1%
SLOVNAFT AS	23.76%	28.8%	-0.8885	18.5002	-0.0193	13245.85	0.0000	-0.55%	3.47%	23.15%	38.56%	67.4%
SURGUTNEFTEGAZ	16.40%	47.8%	-0.1597	11.2946	-0.0687	3749.39	0.0000	-3.17%	-2.73%	-0.99%	-0.50%	47.3%
SNP PETROM SA	29.50%	35.5%	-0.0136	8.0182	0.0171	1370.403	0.0000	0.61%	0.68%	6.47%	6.56%	42.0%
TATNEFT	34.39%	41.2%	-0.1013	6.0634	0.0661	512.8893	0.0000	2.81%	3.40%	6.53%	7.22%	48.4%
TUPRAS-TURKIYE PETROL RAFINE	26.50%	39.9%	0.0326	6.2836	0.0077	586.9403	0.0000	0.31%	0.16%	2.85%	2.68%	42.6%
UNIPETROL AS	36.07%	34.9%	-0.5682	18.0452	0.1220	12387.94	0.0000	4.53%	8.27%	48.15%	66.70%	101.6%

Table 142

Summary of Lukoil Middle East and Africa peer group sample statistics (2003-2009) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
AVNER OIL & GAS LTD - LP	18.30%	32.1%	1.1311	16.4057	-0.0031	12067.81	0.0000	-0.10%	-3.21%	7.04%	2.48%	34.6%
DELEK DRILLIN - LP	15.65%	36.2%	1.2940	18.4515	0.0021	16025.66	0.0000	0.07%	-3.01%	5.00%	1.07%	37.3%
DELEK ENERGY SYSTEMS LTD	16.29%	45.0%	2.6528	40.3024	0.0163	92689.41	0.0000	0.74%	-5.49%	11.96%	2.62%	47.6%
DELEK GROUP LTD	0.56%	46.6%	-1.4611	44.9352	0.1231	115376.7	0.0000	6.11%	6.25%	6.12%	6.26%	52.8%
AFRIQUIA GAZ	31.66%	34.9%	-0.4026	9.1733	0.0259	2530.558	0.0000	0.91%	3.17%	9.91%	13.50%	48.4%
ISRAMCO NEGEV 2 LP	41.69%	55.0%	6.7797	123.6025	-0.0494	961668.6	0.0000	-2.64%	-27.44%	-78.86%	-95.53%	-40.6%
LUKOIL	16.79%	50.7%	0.1625	13.9228	-0.0877	7796.641	0.0000	-4.25%	-4.70%	-1.31%	-1.82%	48.9%
NATIONAL GAS & INDUSTRIALIZA	-1.35%	56.0%	0.0703	17.4079	-0.0118	13555.09	0.0000	-0.65%	-0.64%	-0.63%	-0.62%	55.4%
OANDO PLC	13.71%	40.8%	0.2295	6.7149	0.1886	914.8348	0.0000	8.55%	8.03%	9.15%	8.62%	49.5%
SAMIR	14.34%	29.9%	-0.1931	5.7596	0.1375	506.9729	0.0000	4.42%	4.88%	5.12%	5.61%	35.5%
SASOL LTD	17.83%	38.4%	0.0227	5.6268	0.0453	450.6607	0.0000	1.78%	1.71%	2.66%	2.59%	41.0%

Table 143

Summary of Lukoil Middle East and Africa peer group sample statistics (2003-2008) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
AVNER OIL & GAS LTD - LP	4.56%	27.7%	0.3625	9.0735	0.0188	2035.858	0.0000	0.52%	0.25%	0.71%	0.44%	28.1%
DELEK DRILLIN - LP	5.74%	29.9%	0.3567	7.5090	0.0721	1134.023	0.0000	2.23%	1.89%	2.42%	2.08%	32.0%
DELEK ENERGY SYSTEMS LTD	12.85%	35.1%	0.7101	10.5690	0.0038	3227.219	0.0000	0.13%	-1.33%	1.67%	0.09%	35.2%
DELEK GROUP LTD	11.47%	29.5%	-0.0727	6.3095	0.0458	597.1673	0.0000	1.38%	1.52%	1.98%	2.12%	31.6%
AFRIQUIA GAZ	38.75%	35.9%	-0.4273	9.3657	0.0485	2244.846	0.0000	1.78%	4.76%	16.49%	22.43%	58.3%
ISRAMCO NEGEV 2 LP	14.67%	33.5%	-0.4885	13.7850	-0.0858	6381.458	0.0000	-2.75%	-1.51%	0.76%	2.31%	35.8%
LUKOIL	27.25%	37.2%	-0.1770	6.3338	-0.0957	611.6015	0.0000	-3.39%	-2.57%	-0.04%	0.96%	38.2%
NATIONAL GAS & INDUSTRIALIZA	5.70%	57.5%	0.1333	17.9539	-0.0227	12172.46	0.0000	-1.29%	-1.41%	-0.93%	-1.05%	56.5%
OANDO PLC	36.39%	37.5%	0.5070	9.2587	0.1322	2187.53	0.0000	5.31%	2.44%	15.26%	10.97%	48.5%
SAMIR	19.86%	29.2%	-0.2943	6.0699	0.1583	531.7041	0.0000	5.02%	6.03%	6.57%	7.66%	36.8%
SASOL LTD	30.98%	32.5%	-0.1241	4.4082	0.0674	111.2672	0.0000	2.26%	2.91%	3.96%	4.68%	37.2%

Table 144

Summary of Norilsk Nickel global peer group sample statistics (2003-2009) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
SUMITOMO METAL MINING CO LTD	15.22%	48.1%	-0.0944	6.7302	0.0095	910.8044	0.0000	0.46%	0.70%	1.21%	1.46%	49.6%
ANGLO AMERICAN PLC	2.17%	46.9%	-0.2978	12.4826	0.0136	5894.167	0.0000	0.64%	0.75%	0.68%	0.79%	47.7%
BHP BILLITON LTD	20.50%	36.3%	-0.2862	7.2644	-0.0398	1208.706	0.0000	-1.41%	-0.40%	0.87%	2.02%	38.3%
MMC NORILSK NICKEL	14.31%	53.3%	-1.1576	21.0232	0.1407	21559.02	0.0000	8.08%	10.97%	10.69%	13.84%	67.1%
IMPALA PLATINUM HOLDINGS LTD	15.67%	47.5%	-0.3698	6.0397	0.0726	638.9881	0.0000	3.56%	4.55%	4.18%	5.19%	52.7%
LONMIN PLC	8.23%	49.9%	0.9437	26.6608	0.0336	36784.95	0.0000	1.70%	0.44%	3.03%	1.70%	51.6%
NOVOLIPETSK STEEL	25.92%	67.6%	-0.7334	17.9785	-0.0337	14789.05	0.0000	-2.23%	1.10%	4.88%	9.00%	76.6%
RIO TINTO LTD	6.99%	43.1%	-2.8303	46.4773	-0.0261	125511.2	0.0000	-1.11%	2.47%	1.11%	5.09%	48.2%
TECK RESOURCES LTD-CLS B	-2.18%	57.1%	-1.0654	13.3076	0.1150	7233.493	0.0000	6.96%	6.58%	6.99%	6.61%	63.7%
XSTRATA PLC	12.93%	54.1%	-0.3300	11.0326	0.1031	4241.26	0.0000	5.87%	6.59%	6.82%	7.56%	61.6%

Table 145

Summary of Norilsk Nickel global peer group sample statistics (2003-2008) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
SUMITOMO METAL MINING CO LTD	33.30%	41.9%	-0.2874	6.5329	-0.0178	697.1873	0.0000	-0.74%	0.92%	3.66%	5.70%	47.6%
ANGLO AMERICAN PLC	22.51%	33.8%	0.0845	5.9410	-0.0380	472.2345	0.0000	-1.26%	-1.57%	0.77%	0.42%	34.2%
BHP BILLITON LTD	27.36%	28.4%	-0.1513	4.5096	-0.0350	128.9931	0.0000	-0.97%	-0.26%	0.86%	1.67%	30.0%
MMC NORILSK NICKEL	41.77%	42.6%	-0.3022	7.8051	-0.0122	1276.323	0.0000	-0.51%	1.70%	9.84%	13.32%	55.9%
IMPALA PLATINUM HOLDINGS LTD	35.81%	40.5%	0.2194	4.4598	0.0369	126.4413	0.0000	1.52%	0.25%	3.46%	2.07%	42.6%
LONMIN PLC	28.65%	36.1%	0.8013	12.6344	-0.0110	5190.811	0.0000	-0.39%	-3.85%	12.04%	5.97%	42.1%
NOVOLIPETSK STEEL	52.03%	45.1%	0.1987	7.9069	-0.0469	1318.815	0.0000	-2.06%	-3.72%	16.31%	12.99%	58.1%
RIO TINTO LTD	27.36%	29.0%	0.2594	8.7568	0.0050	1818.038	0.0000	0.15%	-0.99%	7.98%	6.17%	35.1%
TECK RESOURCES LTD-CLS B	39.60%	37.6%	-0.1593	3.2279	0.0498	8.353002	0.0154	1.91%	2.99%	2.29%	3.39%	41.0%
XSTRATA PLC	50.04%	36.5%	0.0937	4.4641	-0.0046	118.5566	0.0000	-0.17%	-0.93%	4.58%	3.61%	40.1%

Table 146

Summary of Norilsk Nickel North American peer group sample statistics (2003-2009) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
ALCOA INC	-22.22%	45.9%	-0.3232	14.1003	0.0230	8072.324	0.0000	1.06%	-0.10%	6.49%	5.04%	50.9%
AGNICO-EAGLE MINES	21.70%	53.2%	-0.7937	12.9140	-0.0366	6581.796	0.0000	-1.91%	1.13%	2.19%	5.75%	59.0%
AGRIUM INC	17.82%	46.2%	-0.8594	11.9517	0.0037	5424.873	0.0000	0.17%	2.87%	2.87%	5.91%	52.1%
AIR PRODUCTS & CHEMICALS INC	5.04%	30.1%	-0.4861	11.3362	-0.0464	4598.965	0.0000	-1.36%	-0.95%	-1.05%	-0.63%	29.5%
PEABODY ENERGY CORP	23.64%	57.5%	-0.5382	9.7166	0.0228	3021.106	0.0000	1.32%	3.52%	4.11%	6.52%	64.1%
CAMECO CORP	19.81%	45.0%	-0.2013	6.2125	-0.0080	684.4183	0.0000	-0.36%	0.32%	0.85%	1.57%	46.6%
CONSOL ENERGY INC	21.32%	59.6%	-0.7867	9.8643	0.0291	3238.059	0.0000	1.75%	4.68%	3.95%	7.10%	66.7%
DU PONT (E.I.) DE NEMOURS	-9.77%	28.3%	-0.5051	12.0849	-0.0644	5455.47	0.0000	-1.76%	-2.56%	-0.32%	-1.21%	27.1%
DOW CHEMICAL	-20.63%	33.4%	-1.5904	19.9949	-0.1154	19518.55	0.0000	-3.64%	-8.26%	11.73%	1.89%	35.3%
ECOLAB INC	4.71%	25.6%	-0.1585	11.6445	-0.1543	4885.655	0.0000	-3.67%	-3.55%	-3.30%	-3.17%	22.4%
FREEPORT-MCMORAN COPPER	14.83%	55.9%	-0.5484	8.8356	-0.0159	2302.025	0.0000	-0.88%	0.51%	0.11%	1.55%	57.5%
MMC NORILSK NICKEL	14.31%	53.3%	-1.1576	21.0232	0.1407	21559.02	0.0000	8.08%	10.97%	10.69%	13.84%	67.1%
KINROSS GOLD CORP	14.87%	55.1%	0.2452	8.7553	-0.0923	2178.374	0.0000	-4.85%	-5.45%	-3.78%	-4.40%	50.7%
NEWMONT MINING CORP	7.81%	42.0%	0.4136	12.6032	-0.0757	6065.963	0.0000	-3.05%	-3.59%	-2.42%	-2.96%	39.0%
NUCOR CORP	22.80%	50.4%	-0.2325	10.3432	-0.1023	3534.846	0.0000	-4.90%	-4.00%	-1.11%	-0.05%	50.3%
SOUTHERN COPPER CORP	31.24%	55.5%	0.0979	9.5094	0.0251	2769.055	0.0000	1.41%	0.90%	6.47%	5.87%	61.4%
PPG INDUSTRIES INC	-4.70%	28.3%	-0.0663	12.1861	-0.1108	5510.742	0.0000	-2.97%	-3.02%	-2.63%	-2.68%	25.6%
SIGMA-ALDRICH	7.71%	27.2%	-0.5373	16.7475	-0.0964	12415.1	0.0000	-2.50%	-1.79%	-1.04%	-0.24%	27.0%
YAMANA GOLD INC	17.42%	76.6%	-0.0516	14.8020	-0.1166	9094.917	0.0000	-8.43%	-8.28%	-6.17%	-6.01%	70.5%

Table 147

Summary of Norilsk Nickel North American peer group sample statistics (2003-2008) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
ALCOA INC	12.35%	29.5%	-0.0805	4.6660	0.0424	152.4539	0.0000	1.27%	1.44%	1.62%	1.79%	31.3%
AGNICO-EAGLE MINES	27.98%	41.3%	-0.8050	11.7264	-0.0093	4284.878	0.0000	-0.38%	3.75%	8.01%	14.15%	55.4%
AGRIUM INC	29.53%	33.7%	-0.1093	5.2140	0.0082	269.3412	0.0000	0.28%	0.82%	2.82%	3.46%	37.1%
AIR PRODUCTS & CHEMICALS INC	16.28%	21.2%	0.0015	4.2540	-0.0351	85.56904	0.0000	-0.73%	-0.73%	-0.03%	-0.04%	21.2%
PEABODY ENERGY CORP	41.67%	40.9%	-0.0512	4.2304	0.0600	82.94915	0.0000	2.52%	2.88%	4.67%	5.07%	45.9%
CAMECO CORP	34.69%	37.5%	-0.2729	3.9989	0.0556	70.51223	0.0000	2.14%	3.78%	3.44%	5.20%	42.7%
CONSOL ENERGY INC	43.30%	41.2%	-0.2325	4.7525	0.0445	178.8962	0.0000	1.87%	3.61%	5.30%	7.34%	48.5%
DU PONT (E.I.) DE NEMOURS	4.08%	19.5%	-0.0225	5.8299	-0.0061	435.8991	0.0000	-0.12%	-0.10%	-0.02%	0.00%	19.5%
DOW CHEMICAL	5.86%	22.9%	-0.1851	6.2038	-0.0421	566.0047	0.0000	-0.94%	-0.76%	-0.73%	-0.54%	22.3%
ECOLAB INC	11.66%	19.0%	0.1888	4.6944	-0.1349	163.9928	0.0000	-2.40%	-2.76%	-1.80%	-2.18%	16.8%
FREEMONT-MCMORAN COPPER	36.94%	41.5%	-0.4766	4.1624	-0.0096	122.9717	0.0000	-0.40%	2.76%	1.28%	4.71%	46.2%
MMC NORILSK NICKEL	41.77%	42.6%	-0.3022	7.8051	-0.0122	1276.323	0.0000	-0.51%	1.70%	9.84%	13.32%	55.9%
KINROSS GOLD CORP	19.82%	43.0%	0.0860	4.3015	-0.0614	93.78691	0.0000	-2.56%	-2.84%	-2.02%	-2.31%	40.7%
NEWMONT MINING CORP	14.12%	31.4%	-0.0052	4.4956	-0.0381	121.7276	0.0000	-1.17%	-1.16%	-0.75%	-0.74%	30.7%
NUCOR CORP	41.95%	37.9%	-0.0505	4.2787	-0.0643	89.53709	0.0000	-2.36%	-2.00%	0.49%	0.91%	38.8%
SOUTHERN COPPER CORP	52.66%	43.8%	-0.1833	4.5282	0.0229	134.3918	0.0000	1.01%	2.68%	5.33%	7.34%	51.2%
PPG INDUSTRIES INC	5.11%	20.3%	-0.0939	4.6072	-0.0673	142.4861	0.0000	-1.32%	-1.24%	-1.23%	-1.14%	19.2%
SIGMA-ALDRICH	18.65%	19.1%	0.6319	12.2214	-0.0783	4714.165	0.0000	-1.44%	-3.20%	11.83%	6.78%	25.9%
YAMANA GOLD INC	30.24%	72.6%	0.0166	19.3992	-0.1256	14634.55	0.0000	-8.57%	-8.66%	2.95%	2.83%	75.4%

Table 148

Summary of Norilsk Nickel Western European peer group sample statistics (2003-2009) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
AKZO NOBEL	6.24%	32.5%	0.6040	13.4179	-0.0206	7181.542	0.0000	-0.66%	-1.28%	-0.12%	-0.76%	31.8%
ANTOFAGASTA PLC	24.59%	48.0%	0.1410	9.6304	-0.0617	2875.565	0.0000	-2.87%	-3.44%	1.16%	0.48%	48.5%
KONINKLIJKE DSM NV	2.07%	29.1%	-0.8655	13.9435	0.0091	8014.995	0.0000	0.26%	0.57%	0.33%	0.63%	29.7%
ERAMET	36.70%	49.9%	0.2972	8.4160	-0.0013	1938.259	0.0000	-0.06%	-1.82%	6.88%	4.62%	54.5%
GIVAUDAN-REG	4.48%	22.0%	-0.3739	9.8723	-0.0771	3120.106	0.0000	-1.63%	-1.34%	-1.34%	-1.05%	20.9%
MMC NORILSK NICKEL	14.31%	53.3%	-1.1576	21.0232	0.1407	21559.02	0.0000	8.08%	10.97%	10.69%	13.84%	67.1%
LINDE AG	13.56%	30.3%	0.2893	8.3629	-0.1008	1899.693	0.0000	-2.91%	-3.55%	-1.32%	-2.04%	28.3%
NORSK HYDRO ASA	6.86%	42.4%	-0.1578	11.6270	-0.0839	4865.811	0.0000	-3.41%	-3.23%	-2.97%	-2.78%	39.6%
RANDGOLD RESOURCES LTD	34.30%	50.1%	0.1367	5.9184	-0.0035	560.9694	0.0000	-0.17%	-0.94%	2.87%	2.00%	52.1%
K+S AG	34.18%	43.5%	-0.4190	8.9815	-0.0636	2381.836	0.0000	-2.67%	-0.14%	5.96%	9.73%	53.3%
SOLVAY SA	-3.56%	24.9%	-0.3679	7.5843	-0.0173	1407.53	0.0000	-0.43%	-0.64%	-0.33%	-0.54%	24.4%
STORA ENSO OYJ-R SHS	-19.76%	35.4%	0.1539	7.9013	-0.0131	1574.699	0.0000	-0.46%	0.06%	1.98%	2.57%	38.0%
SYNGENTA AG-REG	23.35%	31.2%	-0.0666	7.8791	-0.0259	1555.483	0.0000	-0.79%	-0.53%	3.34%	3.68%	34.9%
SALZGITTER AG	30.87%	48.1%	-0.4381	11.6530	-0.0602	4938.831	0.0000	-2.80%	-0.43%	6.31%	9.77%	57.9%
TENARIS SA	21.61%	42.4%	-0.3982	9.8035	-0.0271	3063.565	0.0000	-1.13%	0.36%	2.34%	4.09%	46.5%
UPM-KYMMENE OYJ	-15.93%	32.5%	-0.2431	6.8851	-0.0176	1000.934	0.0000	-0.57%	-1.20%	0.77%	0.09%	32.6%

Table 149

Summary of Norilsk Nickel Western European peer group sample statistics (2003-2008) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
AKZO NOBEL	19.30%	27.0%	0.7211	13.0349	-0.0701	5592.887	0.0000	-1.83%	-3.95%	6.36%	2.73%	29.8%
ANTOFAGASTA PLC	35.79%	36.6%	-0.0513	5.1801	-0.0296	259.2163	0.0000	-1.06%	-0.76%	2.55%	2.92%	39.5%
KONINKLIJKE DSM NV	8.99%	22.6%	-0.4439	8.9523	-0.0385	1970.874	0.0000	-0.85%	-0.16%	0.11%	0.86%	23.4%
ERAMET	69.30%	40.6%	0.2782	5.7707	-0.0335	434.5851	0.0000	-1.34%	-4.31%	20.70%	13.76%	54.4%
GIVAUDAN-REG	13.70%	18.2%	0.3715	7.4606	-0.1098	1112.776	0.0000	-1.89%	-2.70%	0.58%	-0.48%	17.7%
MMC NORILSK NICKEL	41.77%	42.6%	-0.3022	7.8051	-0.0122	1276.323	0.0000	-0.51%	1.70%	9.84%	13.32%	55.9%
LINDE AG	26.73%	25.3%	0.2237	5.9379	-0.1283	480.5713	0.0000	-3.05%	-4.01%	1.72%	0.32%	25.6%
NORSK HYDRO ASA	29.73%	31.0%	0.7191	11.2916	-0.0592	3853.755	0.0000	-1.78%	-4.95%	14.48%	7.23%	38.2%
RANDGOLD RESOURCES LTD	38.53%	44.3%	-0.0247	4.4867	0.0049	120.4138	0.0000	0.22%	0.38%	2.38%	2.56%	46.9%
K+S AG	48.88%	34.0%	-0.3402	9.3056	-0.0914	2188.805	0.0000	-2.96%	0.08%	55.44%	86.49%	120.5%
SOLVAY SA	7.23%	20.8%	-0.1903	7.5353	-0.0582	1127.19	0.0000	-1.17%	-0.94%	-0.66%	-0.41%	20.4%
STORA ENSO OYJ-R SHS	-3.69%	26.6%	-0.0729	5.0100	-0.0501	221.0011	0.0000	-1.30%	-1.34%	-1.25%	-1.30%	25.3%
SYNGENTA AG-REG	32.02%	24.9%	0.1373	5.3436	-0.0596	302.9896	0.0000	-1.43%	-2.15%	3.79%	2.74%	27.6%
SALZGITTER AG	55.39%	39.4%	-0.5762	10.5689	-0.1055	3189.735	0.0000	-3.94%	2.32%	115.21%	408.78%	448.2%
TENARIS SA	41.17%	33.6%	0.2378	5.1284	0.0389	258.8295	0.0000	1.33%	-0.23%	6.23%	4.22%	37.8%
UPM-KYMMENE OYJ	-4.35%	26.2%	-0.3429	6.4505	-0.0239	673.4666	0.0000	-0.62%	-0.86%	-0.51%	-0.76%	25.4%

Table 150

Summary of Norilsk Nickel Asia Developed peer group sample statistics (2003-2009) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
LG CHEM LTD	14.49%	43.3%	-0.0373	5.6193	0.0692	448.3213	0.0000	3.09%	3.18%	3.59%	3.69%	47.0%
FORMOSA PLASTICS CORP	7.67%	27.4%	-0.3327	7.9527	-0.0012	1630.428	0.0000	-0.03%	0.40%	0.42%	0.86%	28.3%
NAN YA PLASTICS CORP	6.29%	27.3%	-0.3372	7.3767	0.0777	1280.395	0.0000	2.20%	2.56%	2.45%	2.81%	30.1%
FORMOSA CHEMICALS & FIBRE	6.58%	27.5%	-0.3835	8.1386	0.0706	1762.427	0.0000	2.01%	2.44%	2.33%	2.76%	30.3%
CHINA STEEL CORP	5.28%	28.8%	-0.4808	8.4368	0.0054	1990.332	0.0000	0.16%	0.59%	0.38%	0.81%	29.6%
TORAY INDUSTRIES INC	6.97%	33.8%	0.3635	7.8813	-0.0161	1590.185	0.0000	-0.54%	-0.95%	-0.24%	-0.66%	33.2%
ASAHI KASEI CORP	2.41%	35.5%	0.2033	7.4353	-0.0645	1295.185	0.0000	-2.21%	-2.29%	-2.18%	-2.26%	33.2%
OJI PAPER CO LTD	-3.91%	32.2%	0.3120	12.9827	-0.0197	6531.983	0.0000	-0.63%	-0.42%	-0.42%	-0.22%	31.9%
SUMITOMO CHEMICAL CO LTD	-1.79%	39.8%	-0.5130	9.9497	-0.0352	3222.219	0.0000	-1.37%	-1.52%	-1.35%	-1.50%	38.3%
NIPPON STEEL CORP	9.58%	40.1%	0.3358	10.3824	-0.0418	3587.871	0.0000	-1.64%	-2.17%	-0.89%	-1.44%	38.6%
SUMITOMO METAL INDUSTRIES	21.17%	44.1%	0.0892	8.1339	0.0334	1722.978	0.0000	1.49%	1.18%	3.70%	3.35%	47.5%
KOBE STEEL LTD	9.22%	42.2%	0.0425	6.3162	0.0071	718.5149	0.0000	0.30%	0.23%	0.58%	0.51%	42.7%
JFE HOLDINGS INC	7.43%	44.9%	-0.2210	7.0024	-0.0064	1058.672	0.0000	-0.28%	-0.01%	-0.08%	0.20%	45.1%
NITTO DENKO CORP	-6.57%	38.7%	0.0532	6.2036	0.0097	670.8315	0.0000	0.37%	0.43%	0.52%	0.58%	39.2%
MMC NORILSK NICKEL	14.31%	53.3%	-1.1576	21.0232	0.1407	21559.02	0.0000	8.08%	10.97%	10.69%	13.84%	67.1%
NEWCREST MINING LIMITED	29.14%	44.7%	-0.2677	7.0912	0.0351	1111.562	0.0000	1.59%	2.93%	4.94%	6.48%	51.2%
ORICA LTD	8.96%	32.8%	-0.2776	14.4643	0.0116	8601.358	0.0000	0.38%	0.80%	1.58%	2.03%	34.8%

Table 151

Summary of Norilsk Nickel Asia Developed peer group sample statistics (2003-2008) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
LG CHEM LTD	14.72%	40.1%	-0.0618	6.0089	0.0829	493.4957	0.0000	3.46%	3.61%	4.09%	4.25%	44.4%
FORMOSA PLASTICS CORP	19.74%	24.7%	-0.1149	6.6432	0.0211	725.1585	0.0000	0.52%	0.91%	3.11%	3.57%	28.3%
NAN YA PLASTICS CORP	19.59%	24.7%	-0.4137	7.5587	0.0575	1168.131	0.0000	1.45%	2.88%	4.58%	6.38%	31.0%
FORMOSA CHEMICALS & FIBRE	20.10%	25.3%	-0.3645	8.3183	0.0305	1568.085	0.0000	0.78%	2.06%	4.73%	6.44%	31.8%
CHINA STEEL CORP	19.74%	26.0%	-0.7138	11.1132	0.0001	3692.814	0.0000	0.00%	2.58%	6.30%	10.39%	36.4%
TORAY INDUSTRIES INC	15.53%	29.2%	0.2947	5.1499	0.0004	270.4059	0.0000	0.01%	-0.73%	0.77%	-0.01%	29.2%
ASAHI KASEI CORP	10.79%	29.3%	0.0280	4.3800	-0.0300	103.7991	0.0000	-0.86%	-0.91%	-0.62%	-0.67%	28.6%
OJI PAPER CO LTD	-1.63%	25.0%	0.3763	5.3325	-0.0486	326.8948	0.0000	-1.18%	-1.08%	-1.17%	-1.07%	23.9%
SUMITOMO CHEMICAL CO LTD	10.35%	31.7%	-0.7761	9.1479	-0.0000	2187.851	0.0000	-0.00%	1.40%	0.89%	2.37%	34.1%
NIPPON STEEL CORP	22.77%	33.3%	-0.0566	4.8717	-0.0378	191.3375	0.0000	-1.23%	-1.01%	0.08%	0.32%	33.6%
SUMITOMO METAL INDUSTRIES	37.10%	37.7%	0.0276	5.4098	0.0199	316.1722	0.0000	0.75%	0.58%	4.72%	4.52%	42.2%
KOBE STEEL LTD	26.66%	37.9%	0.0093	4.7912	0.0059	174.6178	0.0000	0.22%	0.18%	1.67%	1.62%	39.5%
JFE HOLDINGS INC	21.12%	36.7%	-0.1377	4.2992	0.0109	95.97743	0.0000	0.40%	0.89%	1.06%	1.57%	38.2%
NITTO DENKO CORP	6.16%	33.5%	0.0431	3.8004	0.0295	35.26317	0.0000	1.00%	0.95%	1.04%	0.99%	34.4%
MMC NORILSK NICKEL	41.77%	42.6%	-0.3022	7.8051	-0.0122	1276.323	0.0000	-0.51%	1.70%	9.84%	13.32%	55.9%
NEWCREST MINING LIMITED	38.53%	36.7%	-0.3986	5.7378	-0.0129	442.4568	0.0000	-0.47%	2.28%	4.90%	8.57%	45.3%
ORICA LTD	24.18%	25.9%	1.1875	16.6132	0.0239	10391.46	0.0000	0.63%	-3.43%	24.27%	11.51%	37.4%

Table 152

Summary of Norilsk Nickel Asia Emerging peer group sample statistics (2003-2009) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
QINGHAI SALT LAKE POTASH -A	38.79%	42.0%	-0.0495	6.6858	0.0907	887.6147	0.0000	3.98%	4.31%	9.62%	10.03%	52.0%
SHANXI TAIGANG STAINLESS-A	17.09%	47.5%	-1.0590	16.9937	0.0963	13078.51	0.0000	4.80%	8.00%	8.27%	11.92%	59.5%
SHANXI XISHAN COAL & ELEC-A	31.93%	53.5%	-0.6955	10.8288	0.0744	4128.035	0.0000	4.12%	8.08%	10.54%	15.46%	68.9%
YANZHOU COAL MINING CO-H	16.86%	56.9%	-0.0497	8.3991	0.0900	1903.942	0.0000	5.35%	5.49%	6.39%	6.54%	63.4%
ALUMINUM CORP OF CHINA LTD-H	18.47%	59.8%	0.3472	7.8267	0.0881	1552.575	0.0000	5.50%	4.45%	6.57%	5.48%	65.3%
MAANSHAN IRON & STEEL-H	23.49%	56.9%	0.5683	9.3610	0.1146	2726.16	0.0000	6.92%	4.77%	9.29%	6.98%	63.9%
SINOPEC SHANGHAI PETROCHEM-H	8.33%	52.1%	-0.1378	6.0506	0.0353	612.5869	0.0000	1.87%	2.06%	2.03%	2.22%	54.4%
ANGANG STEEL CO LTD-H	31.72%	61.9%	0.1982	7.5515	0.1105	1362.834	0.0000	7.23%	6.20%	10.11%	8.99%	70.9%
JIANGXI COPPER COMPANY LTD-H	32.74%	63.1%	0.0812	7.6924	0.0833	1439.347	0.0000	5.47%	5.03%	8.67%	8.19%	71.3%
WUHAN IRON & STEEL CO LTD-A	18.10%	50.6%	-0.6930	11.2069	0.0591	4523.016	0.0000	3.07%	5.24%	5.24%	7.60%	58.2%
BAOSHAN IRON & STEEL CO-A	2.04%	40.6%	-0.1385	6.3216	0.0550	725.3743	0.0000	2.29%	2.34%	2.31%	2.35%	43.0%
MMC NORILSK NICKEL	14.31%	53.3%	-1.1576	21.0232	0.1407	21559.02	0.0000	8.08%	10.97%	10.69%	13.84%	67.1%
LIHIR GOLD LTD	14.68%	48.9%	0.0619	6.8796	0.0057	983.7103	0.0000	0.28%	0.13%	1.00%	0.84%	49.8%
STEEL AUTHORITY OF INDIA	37.19%	60.1%	0.1298	7.4855	0.0191	1318.046	0.0000	1.15%	0.36%	5.69%	4.77%	64.9%

Table 153

Summary of Norilsk Nickel Asia Emerging peer group sample statistics (2003-2008) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
QINGHAI SALT LAKE POTASH -A	53.57%	40.2%	0.2236	5.8320	0.0086	447.3255	0.0000	0.35%	-1.56%	10.85%	7.87%	48.1%
SHANXI TAIGANG STAINLESS-A	33.92%	43.1%	-1.5375	26.8870	0.0676	31564.16	0.0000	3.01%	13.72%	56.71%	125.43%	168.6%
SHANXI XISHAN COAL & ELEC-A	41.12%	48.8%	-1.0638	15.9246	0.0523	9336.334	0.0000	2.61%	11.11%	29.64%	51.31%	100.1%
YANZHOU COAL MINING CO-H	32.89%	48.0%	0.0187	6.0834	0.0688	517.4367	0.0000	3.41%	3.31%	6.27%	6.15%	54.1%
ALUMINUM CORP OF CHINA LTD-H	42.82%	53.6%	0.5924	8.5546	0.0924	1755.34	0.0000	5.18%	1.24%	13.41%	8.33%	61.9%
MAANSHAN IRON & STEEL-H	36.29%	47.1%	0.1324	5.4429	0.0748	328.5583	0.0000	3.65%	2.86%	6.44%	5.56%	52.7%
SINOPEC SHANGHAI PETROCHEM-H	17.01%	48.0%	-0.0860	6.8392	0.0415	803.6902	0.0000	2.03%	2.27%	2.97%	3.23%	51.2%
ANGANG STEEL CO LTD-H	53.62%	52.5%	0.1413	5.2321	0.0805	275.4682	0.0000	4.40%	3.16%	9.52%	8.05%	60.6%
JIANGXI COPPER COMPANY LTD-H	53.19%	56.1%	0.0512	7.9158	0.0802	1315.566	0.0000	4.68%	4.23%	15.98%	15.35%	71.5%
WUHAN IRON & STEEL CO LTD-A	38.35%	46.5%	-0.9172	16.0701	0.0235	9478.92	0.0000	1.10%	7.79%	27.08%	44.36%	90.9%
BAOSHAN IRON & STEEL CO-A	18.60%	36.4%	-0.0598	7.0812	0.0317	907.1573	0.0000	1.17%	1.36%	2.80%	3.01%	39.4%
MMC NORILSK NICKEL	41.77%	42.6%	-0.3022	7.8051	-0.0122	1276.323	0.0000	-0.51%	1.70%	9.84%	13.32%	55.9%
LIHIR GOLD LTD	23.98%	42.5%	-0.1584	5.9855	-0.0325	490.4918	0.0000	-1.35%	-0.71%	0.46%	1.16%	43.7%
STEEL AUTHORITY OF INDIA	60.58%	57.7%	0.2651	8.8569	-0.0071	1881.996	0.0000	-0.41%	-2.97%	21.09%	16.33%	74.0%

Table 154

Summary of Norilsk Nickel Latin America peer group sample statistics (2003-2009) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
ARACRUZ CELULOSE SA-PREF B	-24.96%	44.1%	-1.1089	18.7630	0.1654	16544.28	0.0000	7.98%	3.74%	17.23%	11.44%	55.5%
EMPRESAS CMPC SA	8.92%	24.0%	-0.2851	6.9320	0.1761	1030.665	0.0000	4.66%	5.09%	5.12%	5.57%	29.6%
CIA SIDERURGICA NACIONAL SA	31.99%	50.3%	-0.3075	8.5971	-0.0038	2070.12	0.0000	-0.19%	1.51%	5.08%	7.16%	57.4%
FERTILIZANTES FOSFATADOS-PRF	26.81%	36.7%	-0.1714	18.7373	0.0062	16177.89	0.0000	0.23%	1.01%	19.73%	21.58%	58.3%
GERDAU SA-PREF	24.05%	45.5%	-0.1200	6.2745	0.0726	703.8252	0.0000	3.42%	3.91%	5.09%	5.61%	51.1%
GRUPO MEXICO SAB DE CV-SER B	34.90%	45.4%	-0.2656	9.0051	0.1012	2372.901	0.0000	4.83%	6.43%	11.73%	13.80%	59.2%
MMC NORILSK NICKEL	14.31%	53.3%	-1.1576	21.0232	0.1407	21559.02	0.0000	8.08%	10.97%	10.69%	13.84%	67.1%
METALURGICA GERDAU SA-PREF	27.71%	44.7%	-0.0845	6.6900	0.0764	890.8912	0.0000	3.54%	3.94%	6.12%	6.56%	51.3%
MINSUR SA-INVERSIONES	12.97%	35.8%	-0.5456	14.7134	0.1857	9035.949	0.0000	7.37%	8.58%	9.36%	10.69%	46.5%
INDUSTRIAS PENOLES SAB DE CV	34.85%	51.5%	-0.0690	8.0502	0.1483	1666.477	0.0000	8.27%	8.67%	12.87%	13.34%	64.9%
SOCIEDAD MINERA CERRO VERDE	61.44%	51.2%	0.0383	9.4059	0.1493	2679.687	0.0000	8.28%	7.89%	31.97%	31.21%	82.4%
SOC QUIMICA Y MINERA CHILE-B	36.70%	38.0%	0.1292	11.7590	0.1321	5013.532	0.0000	5.38%	4.60%	20.73%	19.31%	57.3%
SUZANO PAPEL E CELULOSE SA	8.24%	36.9%	0.0385	7.2001	0.0111	1152.194	0.0000	0.41%	0.36%	0.73%	0.68%	37.5%
USINAS SIDERURGICAS DE MINAS	33.99%	49.5%	-0.1442	6.8606	0.0430	978.5318	0.0000	2.17%	3.00%	6.03%	6.99%	56.5%

Table 155

Summary of Norilsk Nickel Latin America peer group sample statistics (2003-2008) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
ARACRUZ CELULOSE SA-PREF B	9.95%	31.2%	0.1709	4.1760	-0.0035	81.611	0.0000	-0.11%	-0.39%	0.05%	-0.24%	31.0%
EMPRESAS CMPC SA	19.20%	21.7%	-0.0185	5.0780	0.1849	235.0559	0.0000	4.44%	4.50%	5.72%	5.79%	27.5%
CIA SIDERURGICA NACIONAL SA	52.21%	42.3%	-0.6414	7.8218	0.0090	1354.699	0.0000	0.38%	6.80%	18.73%	32.76%	75.1%
FERTILIZANTES FOSFATADOS-PRF	39.58%	24.7%	0.2380	6.0145	0.0415	506.8201	0.0000	1.04%	-0.44%	11.92%	8.99%	33.7%
GERDAU SA-PREF	47.24%	37.7%	-0.1082	3.8719	0.0613	43.92067	0.0000	2.37%	3.24%	4.51%	5.47%	43.1%
GRUPO MEXICO SAB DE CV-SER B	57.98%	36.9%	-0.3178	4.8031	0.1083	198.8952	0.0000	4.22%	7.53%	11.45%	16.10%	53.0%
MMC NORILSK NICKEL	41.77%	42.6%	-0.3022	7.8051	-0.0122	1276.323	0.0000	-0.51%	1.70%	9.84%	13.32%	55.9%
METALURGICA GERDAU SA-PREF	52.23%	36.4%	-0.0088	4.1833	0.0524	76.20784	0.0000	1.95%	2.03%	5.81%	5.90%	42.3%
MINSUR SA-INVERSIONES	33.14%	29.6%	0.1575	8.6876	0.1471	1765.711	0.0000	4.71%	3.86%	14.45%	13.06%	42.7%
INDUSTRIAS PENOLES SAB DE CV	55.03%	43.8%	-0.3128	7.1266	0.0832	947.9446	0.0000	3.79%	6.85%	18.00%	23.25%	67.0%
SOCIEDAD MINERA CERRO VERDE	87.08%	48.4%	0.3151	8.9088	0.1390	1921.486	0.0000	7.23%	3.01%	91.88%	67.79%	116.2%
SOC QUIMICA Y MINERA CHILE-B	34.00%	25.9%	0.3742	5.8896	0.0729	484.8576	0.0000	1.96%	-0.02%	8.05%	5.16%	31.1%
SUZANO PAPEL E CELULOSE SA	28.18%	31.3%	0.0187	4.2621	0.0323	86.75948	0.0000	1.02%	0.93%	2.37%	2.27%	33.5%
USINAS SIDERURGICAS DE MINAS	64.76%	43.4%	-0.1737	7.0098	0.0033	881.4994	0.0000	0.14%	2.10%	25.64%	30.70%	74.1%

Table 156

Summary of Norilsk Nickel Europe Emerging peer group sample statistics (2003-2009) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
MMC NORILSK NICKEL	14.31%	53.3%	-1.1576	21.0232	0.1407	21559.02	0.0000	8.08%	10.97%	10.69%	13.84%	67.1%
NOVOLIPETSK STEEL-CLS	25.92%	67.6%	-0.7334	17.9785	-0.0337	14789.05	0.0000	-2.23%	1.10%	4.88%	9.00%	76.6%

Table 157

Summary of Norilsk Nickel Europe Emerging peer group sample statistics (2003-2008) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
MMC NORILSK NICKEL	41.77%	42.6%	-0.3022	7.8051	-0.0122	1276.323	0.0000	-0.51%	1.70%	9.84%	13.32%	55.9%
NOVOLIPETSK STEEL-CLS	52.03%	45.1%	0.1987	7.9069	-0.0469	1318.815	0.0000	-2.06%	-3.72%	16.31%	12.99%	58.1%

Table 158

Summary of Norilsk Nickel Middle East and Africa peer group sample statistics (2003-2009) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
ARCELORMITTAL SOUTH AFRICA	24.26%	44.7%	-0.9315	15.4353	0.0567	10323.18	0.0000	2.60%	6.69%	10.07%	15.64%	60.3%
ANGLO PLATINUM LTD	8.28%	47.6%	-0.4103	5.7012	0.0949	520.3818	0.0000	4.73%	5.30%	4.88%	5.46%	53.0%
ANGLOGOLD ASHANTI LTD	6.06%	40.6%	0.3670	6.6714	0.0084	915.2708	0.0000	0.34%	-0.03%	0.48%	0.11%	40.7%
ARAB POTASH COMPANY	37.52%	40.9%	0.2820	6.5120	0.1480	826.0655	0.0000	6.55%	4.85%	11.33%	9.28%	50.2%
AFRICAN RAINBOW MINERALS LTD	19.32%	43.9%	-0.6237	8.6186	0.0755	2162.753	0.0000	3.43%	5.53%	5.35%	7.63%	51.5%
EXXARO RESOURCES LTD	37.72%	43.0%	0.2460	11.1461	0.1719	4348.512	0.0000	8.12%	6.62%	19.71%	17.47%	60.5%
GOLD FIELDS LTD	4.02%	47.9%	0.1420	6.8917	0.0491	994.1073	0.0000	2.41%	2.31%	2.46%	2.36%	50.3%
MMC NORILSK NICKEL	14.31%	53.3%	-1.1576	21.0232	0.1407	21559.02	0.0000	8.08%	10.97%	10.69%	13.84%	67.1%
HARMONY GOLD MINING CO LTD	0.50%	49.2%	-0.0064	6.3753	0.0986	743.8384	0.0000	5.09%	5.09%	5.10%	5.10%	54.3%
ISRAEL CHEMICALS LTD	31.08%	35.0%	-0.8472	11.8238	0.0840	5271.054	0.0000	3.06%	8.02%	15.43%	24.52%	59.5%
ISRAEL CORP LIMITED/THE	19.38%	44.2%	-1.3831	16.3880	0.0457	12202.35	0.0000	2.06%	7.00%	7.08%	13.23%	57.4%
JORDAN PHOSPHATE MINES	40.64%	46.4%	0.1278	5.1965	0.1339	319.2738	0.0000	6.66%	5.81%	9.67%	8.72%	55.1%
MAKHTESHIM-AGAN INDUSTRIES	9.51%	35.7%	-0.1506	13.8188	0.0845	7648.03	0.0000	3.14%	3.38%	4.22%	4.47%	40.1%
SAUDI ARABIAN FERTILIZER CO	28.40%	53.5%	-0.7906	15.3302	-0.0053	10089.77	0.0000	-0.28%	3.75%	8.84%	14.44%	67.9%

Table 159

Summary of Norilsk Nickel Middle East and Africa peer group sample statistics (2003-2008) for daily log-returns together with return autocorrelation and kurtosis-skewness adjusted Sharpe ratio equivalent standard deviation

Name	Mean	Std. p.a.	Skewness	Kurtosis (RHS)	Autocorr.	Jarque-Bera	JB P-Value	Std. adj. for AC.	Std. adj. for Skew.	Std. adj. for Kurt.	Std. adj. for all.	Tot. adj. Std. p.a.
ARCELORMITTAL SOUTH AFRICA	53.16%	34.9%	-0.0194	4.7051	0.1093	158.285	0.0000	4.03%	4.21%	9.98%	10.21%	45.1%
ANGLO PLATINUM LTD	30.12%	39.9%	-0.0549	3.9614	0.0700	50.95068	0.0000	2.89%	3.16%	3.75%	4.04%	43.9%
ANGLOGOLD ASHANTI LTD	1.97%	34.3%	0.0216	4.1323	0.0047	69.87103	0.0000	0.16%	0.15%	0.16%	0.16%	34.5%
ARAB POTASH COMPANY	52.66%	33.4%	0.7943	7.6741	0.0672	1326.154	0.0000	2.32%	-3.52%	28.46%	12.84%	46.3%
AFRICAN RAINBOW MINERALS LTD	36.71%	34.3%	0.0638	5.3499	0.0626	301.3895	0.0000	2.21%	1.82%	6.22%	5.75%	40.0%
EXXARO RESOURCES LTD	58.49%	36.9%	0.8566	11.1779	0.1821	3798.961	0.0000	7.42%	0.40%	72.37%	37.69%	74.5%
GOLD FIELDS LTD	7.64%	40.4%	-0.0046	4.1666	0.0410	74.06253	0.0000	1.69%	1.69%	1.75%	1.76%	42.1%
MMC NORILSK NICKEL	41.77%	42.6%	-0.3022	7.8051	-0.0122	1276.323	0.0000	-0.51%	1.70%	9.84%	13.32%	55.9%
HARMONY GOLD MINING CO LTD	1.30%	44.9%	-0.2861	5.8477	0.1071	459.0991	0.0000	5.07%	5.13%	5.07%	5.14%	50.0%
ISRAEL CHEMICALS LTD	46.73%	26.9%	0.1931	6.0328	-0.0019	508.6218	0.0000	-0.05%	-1.48%	16.54%	12.93%	39.9%
ISRAEL CORP LIMITED/THE	46.20%	32.4%	0.2931	5.9474	0.0229	491.4468	0.0000	0.75%	-1.37%	11.15%	7.57%	39.9%
JORDAN PHOSPHATE MINES	60.43%	39.7%	0.4547	5.1498	0.0715	296.4911	0.0000	2.93%	-1.20%	12.30%	6.27%	45.9%
MAKHTESHIM-AGAN INDUSTRIES	24.89%	27.3%	-0.5687	13.9066	0.0169	6543.429	0.0000	0.46%	3.04%	16.43%	23.18%	50.5%
SAUDI ARABIAN FERTILIZER CO	45.10%	50.0%	-0.1318	13.5425	-0.0547	6051.879	0.0000	-2.65%	-1.64%	28.78%	31.62%	81.6%

Appendix K – Skewness and kurtosis test statistic details

Table 160

Testing daily log-returns of major MICEX companies against the hypothesis of positive skewness

Panel A							
Sample (2003-2009)	LUKOIL	Gazprom OAO	Surgutneftegaz	MMC Norilsk Nickel	Tatneft	Surgutneftegaz Pref.	Gazpromneft OAO
m3	2.88E-06	4.11E-06	5.44E-05	-4.16E-05	2.73E-05	3.55E-05	-2.18E-05
g1	0.1258	0.1731	1.4282	-1.1587	0.6677	1.2255	-0.8223
sqrt(b1)	0.1257	0.1730	1.4269	-1.1576	0.6670	1.2243	-0.8215
Observations tot.	1,567	1,567	1,567	1,567	1,567	1,567	1,567
Observations p.a.	261	261	261	261	261	261	261
z-statistic	2.0324	2.7884	17.6052	-15.2542	9.8988	15.8703	-11.7593
P-Value	0.0211	0.0026	0.0000	1.0000	0.0000	0.0000	1.0000
A	2.0346	2.8009	23.1032	-18.7429	10.8003	19.8242	-13.3020
B	3.0226	3.0226	3.0226	3.0226	3.0226	3.0226	3.0226
C	1.0113	1.0113	1.0113	1.0113	1.0113	1.0113	1.0113
D	1.0056	1.0056	1.0056	1.0056	1.0056	1.0056	1.0056
E	13.3526	13.3526	13.3526	13.3526	13.3526	13.3526	13.3526
F	0.1528	0.2104	1.7351	-1.4076	0.8111	1.4888	-0.9990
z-statistic	2.0324	2.7884	17.6052	-15.2542	9.8988	15.8703	-11.7593
10% conf. z-stat.	1.2816	1.2816	1.2816	1.2816	1.2816	1.2816	1.2816
5% conf. z-stat.	1.6449	1.6449	1.6449	1.6449	1.6449	1.6449	1.6449
1% conf. z-stat.	2.3263	2.3263	2.3263	2.3263	2.3263	2.3263	2.3263

Panel B							
Sample (2003-2008)	LUKOIL	Gazprom OAO	Surgutneftegaz	MMC Norilsk Nickel	Tatneft	Surgutneftegaz Pref.	Gazpromneft OAO
m3	-1.34E-06	-4.77E-07	-4.17E-06	-5.54E-06	-1.69E-06	-5.31E-06	-9.78E-06
g1	-0.1510	-0.0445	-0.2397	-0.3026	-0.1014	-0.3858	-0.7044
sqrt(b1)	-0.1508	-0.0444	-0.2394	-0.3022	-0.1013	-0.3853	-0.7036
Observations tot.	1,306	1,306	1,306	1,306	1,306	1,306	1,306
Observations p.a.	218	218	218	218	218	218	218
z-statistic	-2.2253	-0.6584	-3.5038	-4.3892	-1.4996	-5.5266	-9.4643
P-Value	0.9870	0.7449	0.9998	1.0000	0.9331	1.0000	1.0000
A	-2.2301	-0.6566	-3.5401	-4.4692	-1.4983	-5.6979	-10.4046
B	3.0271	3.0271	3.0271	3.0271	3.0271	3.0271	3.0271
C	1.0135	1.0135	1.0135	1.0135	1.0135	1.0135	1.0135
D	1.0067	1.0067	1.0067	1.0067	1.0067	1.0067	1.0067
E	12.2186	12.2186	12.2186	12.2186	12.2186	12.2186	12.2186
F	-0.1831	-0.0539	-0.2907	-0.3670	-0.1230	-0.4679	-0.8544
z-statistic	-2.2253	-0.6584	-3.5038	-4.3892	-1.4996	-5.5266	-9.4643
10% conf. z-stat.	1.2816	1.2816	1.2816	1.2816	1.2816	1.2816	1.2816
5% conf. z-stat.	1.6449	1.6449	1.6449	1.6449	1.6449	1.6449	1.6449
1% conf. z-stat.	2.3263	2.3263	2.3263	2.3263	2.3263	2.3263	2.3263

Calculation based on daily log-returns. The MICEX companies are selected based on their peer-group specification.

Table 161

Testing daily log-returns of major MICEX companies against the hypothesis of leptokurtic daily log-returns

Panel A								
Sample (2003-2009)	LUKOIL	Gazprom OAO	Surgutneftegaz	MMC Norilsk Nickel	Tatneft	Surgutneftegaz Pref.	Gazpromneft OAO	
k4	9.65E-06	8.54E-06	3.32E-05	2.14E-05	2.57E-05	3.56E-05	7.65E-06	
g2	14.8627	12.5145	25.8777	18.0847	18.2091	40.0884	9.6630	
b2	17.8115	15.4708	28.7914	21.0232	21.1472	42.9567	12.6283	
Observations tot.	1,567	1,567	1,567	1,567	1,567	1,567	1,567	
Observations p.a.	261	261	261	261	261	261	261	
z-statistic	18.6776	17.9037	20.9719	19.5251	19.5540	22.5608	16.6889	
P-Value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
G	0.0152	0.0152	0.0152	0.0152	0.0152	0.0152	0.0152	
H	120.2868	101.2820	209.4333	146.3628	147.3698	324.4427	78.2043	
J	0.3679	0.3679	0.3679	0.3679	0.3679	0.3679	0.3679	
K	244.4584	244.4584	244.4584	244.4584	244.4584	244.4584	244.4584	
L	0.0829	0.0969	0.0493	0.0691	0.0687	0.0324	0.1220	
z-statistic	18.6776	17.9037	20.9719	19.5251	19.5540	22.5608	16.6889	
10% conf. z-stat.	1.2816	1.2816	1.2816	1.2816	1.2816	1.2816	1.2816	
5% conf. z-stat.	1.6449	1.6449	1.6449	1.6449	1.6449	1.6449	1.6449	
1% conf. z-stat.	2.3263	2.3263	2.3263	2.3263	2.3263	2.3263	2.3263	

Panel B								
Sample (2003-2008)	LUKOIL	Gazprom OAO	Surgutneftegaz	MMC Norilsk Nickel	Tatneft	Surgutneftegaz Pref.	Gazpromneft OAO	Novolipetsk Steel
k4	4.98E-07	1.55E-06	2.49E-06	2.33E-06	1.31E-06	1.77E-06	1.90E-06	2.81E-06
g2	2.7074	6.5496	5.5346	4.8282	3.0797	5.3661	5.6883	4.6380
b2	5.6924	9.5200	8.5088	7.8051	6.0634	8.3409	8.6619	7.6157
Observations tot.	1,306	1,306	1,306	1,306	1,306	1,306	1,306	1,306
Observations p.a.	218	218	218	218	218	218	218	218
H0 for daily volatility								
z-statistic	9.3486	13.5348	12.7447	12.0969	9.9545	12.5985	12.8740	11.9055
P-Value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
G	0.0182	0.0182	0.0182	0.0182	0.0182	0.0182	0.0182	0.0182
H	20.0098	48.4074	40.9054	35.6844	22.7618	39.6597	42.0413	34.2786
J	0.4022	0.4022	0.4022	0.4022	0.4022	0.4022	0.4022	0.4022
K	205.7946	205.7946	205.7946	205.7946	205.7946	205.7946	205.7946	205.7946
L	0.3310	0.1702	0.1952	0.2175	0.3032	0.2001	0.1910	0.2244
z-statistic	9.3486	13.5348	12.7447	12.0969	9.9545	12.5985	12.8740	11.9055
10% conf. z-stat.	1.2816	1.2816	1.2816	1.2816	1.2816	1.2816	1.2816	1.2816
5% conf. z-stat.	1.6449	1.6449	1.6449	1.6449	1.6449	1.6449	1.6449	1.6449
1% conf. z-stat.	2.3263	2.3263	2.3263	2.3263	2.3263	2.3263	2.3263	2.3263

Calculation based on daily log-returns. The MICEX companies are selected based on their peer-group specification.

Appendix L – GJR-GARCH(1,1) estimates

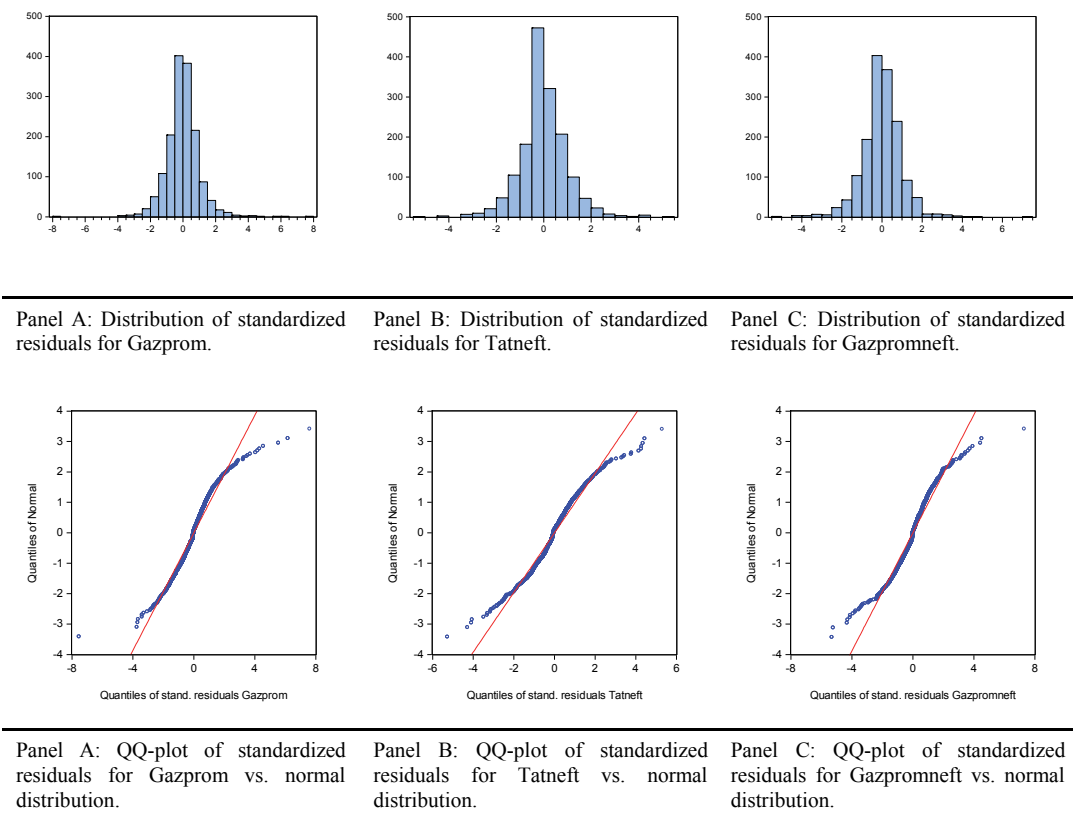
Table 162
Summary of GJR-GARCH(1,1) estimation results for major MICEX companies (using daily log-returns)

Panel A: 2003-2009								
	LUKOIL	Gazprom OAO	Surgutneftegaz	MMC Norilsk Nickel	Tatneft	Surgutneftegaz Pref.	Gazpromneft OAO	Novolipetsk Steel
Mean	16.8%	25.9%	14.8%	14.3%	19.0%	2.0%	2.6%	25.9%
Median	14.2%	26.1%	0.0%	0.0%	0.0%	0.0%	11.2%	0.0%
Std. p.a.	45.9%	46.4%	54.4%	53.3%	55.7%	49.6%	48.2%	67.6%
Skewness	0.1257	0.1730	1.4269	-1.1576	0.6670	1.2243	-0.8215	-0.7334
Kurtosis	17.8115	15.4708	28.7914	21.0232	21.1472	42.9567	12.6283	17.9785
Autocorr.	-0.0056	0.0532	0.0509	0.1409	0.1165	0.0911	0.0929	-0.0337
Jarque-Bera statistics								
Jarque-Bera	14327.86	10161.95	43963.47	21559.02	21618.18	104632.30	6229.12	14789.05
JB P-Value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Observations tot.	1567	1567	1567	1567	1567	1567	1567	1567
Observations p.a.	261	261	261	261	261	261	261	261
GARCH (1,1) analysis								
Alpha 0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Alpha 1	0.0545	0.0478	0.1014	0.0484	0.0791	0.1476	0.0450	0.0448
Gamma	0.1006	0.0580	0.0598	0.1276	0.0513	0.0832	0.0953	0.0164
Beta 1	0.8625	0.8873	0.8255	0.8323	0.8466	0.7587	0.8654	0.9391
uncon. var.	0.0006	0.0006	0.0009	0.0009	0.0008	0.0009	0.0007	0.0017
uncon. Std. p.a.	39.63%	40.71%	48.52%	47.75%	46.37%	47.33%	43.68%	67.41%
GJR-Garch impl. Kurtosis								
uncon. kurt. (g2) k=3	4.5516	3.6034	5.0609	3.9112	3.9007	10.1500	3.7922	4.6397
uncon. kurt. (b2)	7.5332	6.5881	8.0409	6.8949	6.8844	13.1138	6.7763	7.6211
Kurtosis of St.res	5.5604	9.8670	7.1833	7.4320	6.0373	6.5859	7.8500	8.2616
uncon. kurt. (b2) k=St.res	1,294.4960	70.5893	-21.4503	-253.4420	16.0298	-6.1002	116.4434	-25.9723
uncon. kurt. (b2) k=St.res	1,294.4960	70.5893	-21.4503	-253.4420	16.0298	-6.1002	116.4434	-25.9723
Kurt. stability condition <1	1.000	0.990	1.028	1.003	0.964	1.110	0.994	1.005
Panel B: 2003-2008								
	LUKOIL	Gazprom OAO	Surgutneftegaz	MMC Norilsk Nickel	Tatneft	Surgutneftegaz Pref.	Gazpromneft OAO	Novolipetsk Steel
Mean	27.8%	48.8%	16.4%	41.8%	34.4%	10.4%	13.8%	52.0%
Median	32.1%	36.5%	0.0%	0.0%	0.0%	0.0%	16.9%	0.0%
Std. p.a.	33.5%	35.6%	41.9%	42.6%	41.2%	38.7%	38.8%	45.1%
Skewness	-0.1508	-0.0444	-0.2394	-0.3022	-0.1013	-0.3853	-0.7036	0.1987
Kurtosis	5.6924	9.5200	8.5088	7.8051	6.0634	8.3409	8.6619	7.9069
Autocorr.	0.0258	0.0300	0.0308	-0.0115	0.0658	0.0472	0.0339	-0.0469
Jarque-Bera statistics								
Jarque-Bera	399.43	2313.70	1663.87	1276.32	512.89	1584.59	1852.23	1318.82
JB P-Value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Observations tot.	1306	1306	1306	1306	1306	1306	1306	1306
Observations p.a.	261	261	261	261	261	261	261	261
GARCH (1,1) analysis								
Alpha 0	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000
Alpha 1	0.0452	0.1072	0.1006	0.0234	0.0874	0.1393	0.0249	0.0657
Gamma	0.1467	0.0963	0.0453	0.1163	0.0106	0.0673	0.0966	-0.0075
Beta 1	0.7781	0.5898	0.7855	0.8282	0.8186	0.7270	0.8550	0.8957
uncon. var.	0.0004	0.0005	0.0006	0.0007	0.0006	0.0006	0.0005	0.0008
uncon. Std. p.a.	32.44%	35.55%	40.18%	41.48%	40.41%	38.95%	37.52%	45.95%
GJR-Garch impl. Kurtosis								
uncon. kurt. (g2) k=3	3.5022	3.3656	3.6353	3.2509	3.3377	4.3791	3.2512	3.3068
uncon. kurt. (b2)	6.4842	6.3481	6.6168	6.2339	6.3204	7.3578	6.2341	6.2895
Kurtosis of St.res	4.8931	11.4282	6.6395	7.3749	5.8510	6.3160	7.9845	7.1249
uncon. kurt. (b2) k=St.res	8.3421	30.5601	13.6148	12.1026	7.7638	50.4162	13.3996	9.9693
uncon. kurt. (b2) k=St.res	8.3421	30.5601	13.6148	12.1026	7.7638	50.4162	13.3996	9.9693
Kurt. stability condition <1	0.885	0.834	0.915	0.895	0.872	0.976	0.917	0.941

Calculation based on daily log-returns. The MICEX companies are selected based on their peer-group specification.

Figure 145

GJR-GARCH (1,1) standardized residuals analysis for selected MICEX energy companies together with QQ-plot distribution analysis (2003-2009)



Appendix M – Beta sensitivity test statistic details

Table 163

Estimating a panel regression equation on MICEX energy companies and the international Gazprom peer-group panel dataset

std.error										
	MICEX	GL	NA	WE	AD	AE	LA	EE	MA	
2003-2009 Oil sensitivity	0.0118	0.0045	0.0041	0.0046	0.0065	0.0067	0.0115	0.0073	0.0074	
MSCI world sensitivity	0.0426	0.0164	0.0149	0.0169	0.0235	0.0243	0.0418	0.0267	0.0268	
chg. in Vix Index	0.0003	0.0001	0.0001	0.0001	0.0001	0.0001	0.0003	0.0002	0.0002	
2007-2008 Oil sensitivity	0.0242	0.0164	0.0110	0.0135	0.0223	0.0266	0.0411	0.0189	0.0199	
MSCI world sensitivity	0.0643	0.0436	0.0292	0.0359	0.0594	0.0707	0.1094	0.0501	0.0529	
chg. in Vix Index	0.0004	0.0002	0.0002	0.0002	0.0003	0.0004	0.0006	0.0003	0.0003	
2008-2009 Oil sensitivity	0.0280	0.0104	0.0103	0.0109	0.0138	0.0126	0.0204	0.0163	0.0176	
MSCI world sensitivity	0.0944	0.0350	0.0348	0.0367	0.0465	0.0424	0.0688	0.0549	0.0594	
chg. in Vix Index	0.0005	0.0002	0.0002	0.0002	0.0003	0.0002	0.0004	0.0003	0.0003	
t-values										
	MICEX	GL	NA	WE	AD	AE	LA	EE	MA	
2003-2009 Oil sensitivity	8.7246	33.8713	61.7527	24.8218	5.4486	6.1627	5.0775	5.6687	2.4392	
MSCI world sensitivity	33.5254	59.5070	56.5250	67.5617	38.6832	31.3549	16.6965	30.8296	19.2689	
chg. in Vix Index	13.2387	2.5045	-25.6676	18.4781	20.8214	10.0333	1.5179	9.9753	9.0198	
2007-2008 Oil sensitivity	3.1852	12.3620	28.4495	9.7905	-0.2240	1.8538	2.9803	2.3488	1.8324	
MSCI world sensitivity	19.6324	23.7627	22.9958	35.3590	17.5525	14.5350	7.7580	18.4690	11.2178	
chg. in Vix Index	8.3960	3.7250	-14.1914	14.1399	10.7177	8.4818	2.8402	9.1276	5.4344	
2008-2009 Oil sensitivity	4.7109	13.8975	19.5058	13.6151	2.5917	2.3485	2.3891	2.3412	1.2436	
MSCI world sensitivity	16.7397	28.8233	28.2562	32.2393	20.5096	15.7858	10.3780	15.3247	9.3584	
chg. in Vix Index	7.8148	1.3089	-9.9227	10.0766	11.5462	2.9966	1.3636	4.9439	5.1643	
p-values										
	MICEX	GL	NA	WE	AD	AE	LA	EE	MA	
2003-2009 Oil sensitivity	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0147	
MSCI world sensitivity	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
chg. in Vix Index	0.0000	0.0123	0.0000	0.0000	0.0000	0.0000	0.1291	0.0000	0.0000	
2007-2008 Oil sensitivity	0.0015	0.0000	0.0000	0.0000	0.8228	0.0638	0.0029	0.0189	0.0670	
MSCI world sensitivity	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
chg. in Vix Index	0.0000	0.0002	0.0000	0.0000	0.0000	0.0000	0.0045	0.0000	0.0000	
2008-2009 Oil sensitivity	0.0000	0.0000	0.0000	0.0000	0.0096	0.0189	0.0170	0.0193	0.2138	
MSCI world sensitivity	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
chg. in Vix Index	0.0000	0.1906	0.0000	0.0000	0.0000	0.0027	0.1728	0.0000	0.0000	

Calculation based on daily log-returns. The MICEX companies are selected based on their peer-group specification. Peer-group definition: GL (global), NA (North America), WE (Western Europe), AD (Asia Developed), AE (Asia Emerging), LA (Latin America), EE (Eastern Europe), MEA (Middle East and Africa). All estimated coefficient standard errors and t-statistics are corrected for cross-section heteroscedasticity and contemporaneous correlation.

Table 164

Estimating a panel regression equation on MICEX energy companies and the international Lukoil peer-group panel dataset

		std.error								
		MICEX	GL	NA	WE	AD	AE	LA	EE	MA
2003-2009	Oil sensitivity	0.0118	0.0074	0.0042	0.0051	0.0067	0.0068	0.0147	0.0077	0.0081
	MSCI world sensitivity	0.0426	0.0268	0.0152	0.0186	0.0244	0.0247	0.0535	0.0280	0.0296
	chg. in Vix Index	0.0003	0.0002	0.0001	0.0001	0.0001	0.0001	0.0003	0.0002	0.0002
2007-2008	SPGCEN sensitivity	0.0242	0.0195	0.0118	0.0158	0.0236	0.0271	0.0550	0.0221	0.0228
	MSCI world sensitivity	0.0643	0.0518	0.0313	0.0420	0.0628	0.0720	0.1463	0.0588	0.0608
	chg. in Vix Index	0.0004	0.0003	0.0002	0.0002	0.0004	0.0004	0.0008	0.0003	0.0003
2008-2009	SPGCEN sensitivity	0.0280	0.0148	0.0105	0.0118	0.0136	0.0133	0.0258	0.0162	0.0192
	MSCI world sensitivity	0.0944	0.0499	0.0354	0.0399	0.0458	0.0448	0.0872	0.0549	0.0648
	chg. in Vix Index	0.0005	0.0003	0.0002	0.0002	0.0003	0.0003	0.0005	0.0003	0.0004
		t-values								
		MICEX	GL	NA	WE	AD	AE	LA	EE	MA
2003-2009	Oil sensitivity	8.7246	20.8978	67.7020	20.3450	3.3570	3.9669	3.7218	3.4105	0.5378
	MSCI world sensitivity	33.5254	36.1588	49.8538	56.1706	34.1786	25.8465	13.2654	27.8483	14.0070
	chg. in Vix Index	13.2387	-2.4823	-32.2097	14.4572	20.5840	9.1756	0.4688	8.6236	7.6864
2007-2008	SPGCEN sensitivity	3.1852	11.6666	30.0130	7.2930	-0.5838	1.4579	2.3775	1.3840	1.6781
	MSCI world sensitivity	19.6324	17.9119	18.2134	30.0001	15.5157	12.6970	6.5272	16.2931	9.3824
	chg. in Vix Index	8.3960	-0.8623	-16.9503	12.8115	10.0340	7.2847	1.9748	5.8687	4.7094
2008-2009	SPGCEN sensitivity	4.7109	9.8341	20.3760	10.8062	1.0055	-0.1213	1.3222	1.7736	-0.0747
	MSCI world sensitivity	16.7397	20.6425	25.9105	27.0241	19.0352	12.3342	8.4447	14.7166	6.7146
	chg. in Vix Index	7.8148	-0.5634	-12.7038	7.6780	11.9838	2.6324	0.8460	5.2181	4.3620
		p-values								
		MICEX	GL	NA	WE	AD	AE	LA	EE	MA
2003-2009	Oil sensitivity	0.0000	0.0000	0.0000	0.0000	0.0008	0.0001	0.0002	0.0007	0.5907
	MSCI world sensitivity	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	chg. in Vix Index	0.0000	0.0131	0.0000	0.0000	0.0000	0.0000	0.6393	0.0000	0.0000
2007-2008	SPGCEN sensitivity	0.0015	0.0000	0.0000	0.0000	0.5594	0.1450	0.0175	0.1665	0.0935
	MSCI world sensitivity	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	chg. in Vix Index	0.0000	0.3886	0.0000	0.0000	0.0000	0.0000	0.0484	0.0000	0.0000
2008-2009	SPGCEN sensitivity	0.0000	0.0000	0.0000	0.0000	0.3147	0.9035	0.1862	0.0763	0.9404
	MSCI world sensitivity	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	chg. in Vix Index	0.0000	0.5732	0.0000	0.0000	0.0000	0.0085	0.3977	0.0000	0.0000

Calculation based on daily log-returns. The MICEX companies are selected based on their peer-group specification. Peer-group definition: GL (global), NA (North America), WE (Western Europe), AD (Asia Developed), AE (Asia Emerging), LA (Latin America), EE (Eastern Europe), MEA (Middle East and Africa). All estimated coefficient standard errors and t-statistics are corrected for cross-section heteroscedasticity and contemporaneous correlation.

Table 165
Estimating a panel regression equation on MICEX basic materials companies and the international Norilsk Nickel peer-group panel dataset

std.error									
	MICEX	GL	NA	WE	AD	AE	LA	EE	MA
2003-2009 SPGCIN sensitivity	0.0313	0.0155	0.0094	0.0091	0.0088	0.0145	0.0106	-	0.0140
MSCI world sensitivity	0.0700	0.0360	0.0219	0.0211	0.0208	0.0342	0.0250	-	0.0330
chg. in Vix Index	0.0004	0.0002	0.0001	0.0001	0.0001	0.0002	0.0001	-	0.0002
2007-2008 SPGCIN sensitivity	0.0390	0.0361	0.0199	0.0225	0.0243	0.0441	0.0254	-	0.0284
MSCI world sensitivity	0.0879	0.0829	0.0456	0.0510	0.0552	0.1000	0.0576	-	0.0644
chg. in Vix Index	0.0005	0.0004	0.0002	0.0003	0.0003	0.0005	0.0003	-	0.0003
2008-2009 SPGCIN sensitivity	0.0757	0.0467	0.0272	0.0256	0.0244	0.0369	0.0288	-	0.0329
MSCI world sensitivity	0.1391	0.0863	0.0504	0.0470	0.0448	0.0678	0.0528	-	0.0604
chg. in Vix Index	0.0007	0.0005	0.0003	0.0003	0.0002	0.0004	0.0003	-	0.0003
t-values									
	MICEX	GL	NA	WE	AD	AE	LA	EE	MA
2003-2009 SPGCIN sensitivity	5.9187	13.1253	20.7351	22.4062	4.7790	3.9150	14.2477	-	10.0745
MSCI world sensitivity	20.7105	34.4317	39.9617	52.8597	42.5980	29.2178	35.9699	-	19.3336
chg. in Vix Index	10.2531	14.2045	-24.8603	12.7960	24.0804	15.6758	-7.8784	-	9.6308
2007-2008 SPGCIN sensitivity	3.2945	7.6129	6.1890	10.9194	3.2735	1.6758	6.4360	-	4.7918
MSCI world sensitivity	13.2698	16.7767	20.1358	29.0606	16.3482	10.3827	20.2646	-	8.5321
chg. in Vix Index	7.5756	9.7010	-12.2558	12.0539	9.6688	7.7090	0.8558	-	5.2563
2008-2009 SPGCIN sensitivity	2.3916	5.8978	6.2833	12.9318	1.3769	0.6471	5.9516	-	6.2267
MSCI world sensitivity	11.1819	14.3504	18.5760	22.5973	20.4080	15.5810	17.0428	-	10.9266
chg. in Vix Index	5.9753	6.5127	-10.4495	5.8621	12.8356	8.8084	-3.8544	-	5.9993
p-values									
	MICEX	GL	NA	WE	AD	AE	LA	EE	MA
2003-2009 SPGCIN sensitivity	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	-	0.0000
MSCI world sensitivity	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000
chg. in Vix Index	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000
2007-2008 SPGCIN sensitivity	0.0010	0.0000	0.0000	0.0000	0.0011	0.0939	0.0000	-	0.0000
MSCI world sensitivity	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000
chg. in Vix Index	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3922	-	0.0000
2008-2009 SPGCIN sensitivity	0.0169	0.0000	0.0000	0.0000	0.1686	0.5176	0.0000	-	0.0000
MSCI world sensitivity	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000
chg. in Vix Index	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	-	0.0000

Calculation based on daily log-returns. The MICEX companies are selected based on their peer-group specification. Peer-group definition: GL (global), NA (North America), WE (Western Europe), AD (Asia Developed), AE (Asia Emerging), LA (Latin America), EE (Eastern Europe), MEA (Middle East and Africa). All estimated coefficient standard errors and t-statistics are corrected for cross-section heteroscedasticity and contemporaneous correlation.

Appendix N – Abstract (German)

Ziel dieser Dissertation ist, eine Antwort auf die Fragestellung zu finden, ob Marktteilnehmer in ihrer Preisfindung für Rohstoffunternehmen Realoptionen berücksichtigen oder nicht. Als Modellfall dienen russische Rohstoffproduzenten; im Besonderen liegt der Schwerpunkt auf Unternehmen des Energie- und Metallurgiesektors, die über eine ausreichende Historie verfügen und innerhalb ihres Sektor zu den wichtigsten Vertretern der Branche gehören (Teil der globalen Peergroup). In der Analyse ermitteln wir drei russische Rohstoffproduzenten, die die Anforderungen bezüglich Historie und Wichtigkeit erfüllen. Diese Unternehmen (Gazprom, Lukoil und Norilsk Nickel) bilden gemeinsam mit ihrer globalen und regionalen Peergroup die Datengrundlage für die Analyse.

Wir unterscheiden zwischen Fundamentaldaten- und Zeitreihen-Analyse. Von einem fundamentalen Bewertungsstandpunkt heraus erwarteten wir, dass, sollten Realoptionen tatsächlich eine Rolle in der Unternehmensbewertung spielen, die drei analysierten russischen Unternehmen mit einer Bewertungsprämie gegenüber ihrer Peergroup handeln. Tatsächlich konnte jedoch – entgegen unseren Erwartungen – bei den analysierten Unternehmen trotz besserer Kapitalisierung und höherer Profitabilität kein Bewertungsvorteil mittels Multiplikatormethode im Vergleich zu ihren Peergroups festgestellt werden.

Infolgedessen konzentrieren wir uns im Verlauf der Dissertation auf die Zeitreihenanalyse, um nach Indizien zu forschen, die den Einfluss von Realoptionen auf die Preisfindung von Rohstoffunternehmen bestätigen. Wir leiten hierzu mehrere Hypothesen bezüglich der erwarteten Charakteristika von Aktienpreisrenditen für Unternehmen ab, die durch Realoptionen beeinflusst werden. Zusätzlich dazu nehmen wir diverse Stationaritätstests vor und überprüfen unterschiedliche Daten generierende Prozesse auf ihre „out-of-sample“-Qualität. Für die Inferenz unterscheiden wir zwischen einem nichtkonditionalen und einem konditionalen Ansatz. Innerhalb des nichtkonditionalen Ansatzes testen wir höhere Verteilungsmomente der Aktienpreisrenditen, die Preisfindung unter Berücksichtigung von Realoptionen indizieren. Innerhalb des konditionalen

Ansatzes testen wir die Renditesensitivität von Rohstoffunternehmungen bezüglich der Änderung von Rohstoffpreisen in unterschiedlichen Marktphasen.

Unsere Ergebnisse lassen darauf schließen, dass es in Summe substantielle Hinweise der Berücksichtigung von Realoptionen in der Preisbildung von russischen Rohstoffunternehmen gibt. Realoptionskonforme Preisbildung wird vor allem durch die vergleichsweise höhere positive Verteilungsschiefe der observierten Renditen russischer Rohstoffunternehmen evident; ein weiteres starkes Indiz ist die dynamische Preissensitivität von russischen Rohstoffunternehmen zum generellen Rohstoffmarkt während veränderter Preisphasen (hohe/tiefe Sensitivität während tiefer/hoher Rohstoffpreise), das im Einklang mit unseren Erwartungen für Unternehmen steht, die Realoptionswerte besitzen.

Appendix O – CV and academic development

Personal Data:

Michael Markovich, Mag.

Born on 08.05.1974, in Vienna

Marital status: married, two children

Academic career

Since 2005: Admission to Doctoral Studies (Social and Economic Sciences,
Business Administration, A 084 151, UniStG, before AHStG),
First submission of dissertation draft in 2010

Main research courses (Rigorosenfächer):

- Empirical Methods
- Corporate Finance and Empirical Asset Pricing
- Economic Theory

Other research projects and working papers:

- Sovereign Credit Analysis:
2011, Debt Sustainability Measures and Their Explanatory
Power for Estimating government Bond Yields, Working
Paper
2011, Government Debt and Corporate Financing Costs:
Evidence from the Recent Sovereign Debt Crisis, Term Paper
- Central Bank Reaction Function Analysis:
2012, Central Bank Reaction Functions with Equity Valuation
Variables “Everyone Can Agree On”, Working Paper
- Bond Term Premium Analysis:
2012, Predicting Yields for Predicting Returns, together with
Alberto Plazzi (USI), Working Paper

SSRN page:

http://papers.ssrn.com/sol3/cf_dev/AbsByAuth.cfm?per_id=1807436

09/00: Master of Social and Economic Sciences (with merit) and European
Master of Business Science (with distinction)

10/99 - 06/00: Diploma of higher education – European Master of Business Science
(EMBS) Great Britain in accountancy and finance at the Heriot-Watt
University of Edinburgh

Study priority:

- Financial engineering and operational finance
- Capital market research
- Economics

Master thesis: Share Repurchase and Analysis under the Perspective of Capital
Market Research (with distinction)

10/93 - 06/99: Business Administration at the Centre of Business Studies,
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Current career

Since 04/05: With Credit Suisse Private Banking in Zurich; since 2007 Head of
Global Interest Rate Research.

9/03-03/05: Bank Austria Creditanstalt Vienna: interest rate derivatives trader -
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11/01-07/03: RZB in Vienna as trainee for Investment Banking and Treasury;
trader / prop - trader for interest rate derivatives (swaps – swaptions)
– financial engineering.

Literature index

- Alexander, C. (2008a). *Quantitative Methods in Finance*. New York: Wiley & Sons, Ltd.
- Alexander, C. (2008b). *Practical Financial Econometrics*. New York: Wiley & Sons, Ltd.
- Alexander, C. (2008c). *Pricing, Hedging and Trading Financial Instrument*. New York: Wiley & Sons, Ltd.
- Baltagi, B. H. (2008). *Econometric Analysis of Panel Data* (4 ed.). Chichester: John Wiley & Sons Ltd.
- Bao, Y. (2009). Estimation Risk-Adjusted Sharpe Ratio and Fund Performance Ranking under a General Return Distribution. *Journal of Financial Econometrics*, 7(2), 152-173.
- Barreto, H., & Howland, F. (2006). *Introductory Econometrics*. New York: Cambridge University Press.
- Baxter, M., & Rennie, A. (1996). *Financial Calculus: An Introduction to Derivative Pricing*. Cambridge : Cambridge University Press.
- Baz, J., & Chacko, G. (2004). *Financial Derivatives - Pricing, Applications, and Mathematics*. Cambridge: Cambridge.
- Bjerkhund, P. (1991). Contingent Claims Evaluation when the Convenience Yield is Stochastic: Analytical Results. *Norwegian School of Economics - Working Paper*.
- Black, F., & Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, 81, pp. 637-654.
- Boyle, P. (1986). Option Valuation Using a Three Jump Process. *International Options Journal*, S. 7-12.
- Brennan, M. J., & Schwartz, E. S. (1985). Evaluating Natural Resource Investments. *Journal of Business* 58, 2, pp. 135-157.
- Brigo, D., & Mercurio, F. (2001). *Interest Rate Models - Theory and Practice*. Heidelberg: Springer.
- Brooks, C. (2008). *Introductory Econometrics for Finance* (2 ed.). Cambridge: Cambridge University Press.
- Campbell, J., Lo, A., & MacKinlay, C. (1996). *The Econometrics of Financial Markets*. Princeton: Princeton University Press.

-
- Choi, I. (1994). Residual-Based Tests for the Null of Stationary with Application to US Macroeconomic Time Series. *Econometric Theory*, 10, 720-746.
- Christoffersen, P. (2003). *Elements of Financial Risk Management*. San Diego: Academic Press.
- Cochrane, J. (2005). *Asset Pricing* (Revised Edition ed.). Pinceton and Oxford: Princeton University Press.
- Copeland, T., & Antikarov, V. (2001). *Real Options a practitioners's guide*. London: Texere Publishing Limited.
- Copeland, T., Weston, F., & Shastri, K. (2005). *Financial Theory and Corporate Policy*. Boston: Pearson Addison Wesley.
- Cortazar, G., & Schwartz, E. (1997). Implementing a Real Option Model for Valuing an Undeveloped Oil Field. *International Transactions in Operational Research*, 4(2), pp. 125-137.
- Cortazar, G., & Schwartz, E. (1998). Monte Carlo Evaluation Model of an Undeveloped Oil Field. *Journal of Energy Finance & Development*, 3(1), pp. 73-84.
- Cortazar, O. (2001). Simulation and Numerical Methods in Real Option Valuation. In L. Trigeorgis, & E. Schwartz, *Real Options and Investment Under Uncertainty* (pp. 601-620). Cambridge-Massachusetts: The MIT Press.
- Cox, J., & Rubinstein, M. (1985). *Option Markets*. New Jersey: Prentice-Hall, Inc.
- Cox, J., Ingersoll, J., & Ross, S. (1985). An Intertemporal General Equilibrium Model of Asset Prices. *Econometrica* 53 (March), pp. 385-407.
- Cox, J., Ross, S., & Rubinstein, M. (1979). Option pricing: A simplified approach. *Journal of Financial Econometrics* 7, pp. 239-263.
- Damodaran, A. (2002). *Investmetn Valuation - Tools and Techniques for Determining the Vlaue of Any Asset* (2 ed.). New York: Wiley & Sons, Ltd.
- Diebold, F. (2004). *Elements of Forecasting* (3 ed.). Louiseville: Thomson.
- Diebold, F. (2004). *Elements of Forecasting*. Louiseville: Thomson.
- Dixit, A., & Pindyck, R. (1994). *Investment under Uncertainty*. Princeton: Princeton University Press.
- Dowd, K. (2000). Adjusting for risk: An improved Shapre ratio. *International Review of Economics and Finance*(9), 209-222.
- Dowd, K. (2005). *Measuring market risk*. Chichester: John Wiley & Sons Ltd.

- Duan, J.-C. (1995). The Garch Option Pricing Model. *Mathematical Finance*, 5(1), pp. 13-32.
- Duan, J.-C., Gauthier, G., Simonato, J.-G., & Sasseville, C. (2006). Approximating the GJR-GARCH and EGARCH option pricing models analytically. *Journal of Computational Finance*, 9(3), 41-69.
- Elton, E., Gruber, M., Brown, S., & Goetzmann, W. (2007). *Modern Portfolio Theory and Investment Analysis* (7 ed.). New York: Wiley & Sons, Ltd.
- Engle, R. (2004). Risk and Volatility: Econometric Models and Financial Practice. *The American Economic Review*, 94(3), 405-420.
- Engle, R., & Patton, A. (2001). What good is a volatility model? *Quantitative Finance*, 1, 237-245.
- Evans, M., Hastings, N., & Peacock, B. (2000). *Statistical Distributions* (3 ed.). New York: John Wiley & Sons, Inc. .
- Fama, E., & French, K. (1992). The Cross-section of Expected Stock Returns. *Journal of Finance*, 47, pp. 427-466.
- Fischer, E. (1996a). *Finanzwirtschaft für Fortgeschrittene*. München-Wien: Oldenbourg.
- Fries, C. (2007). *Mathematical Finance - Theory, Modeling, Implementation*. New Jersey: Wiley & Sons, Inc. .
- Garbade, K. (2001). *Pricing Corporate Securities as Contingent Claims*. Cambridge: The MIT Press.
- Geske, R. (1979). The Valuation of Compound Options. *Journal of Financial Economics*, pp. 63-81.
- Geske, R., & Shastri, R. (2001). Valuation by Approximation: A Comparison of Alternative Option Valuation Techniques. In E. Schwartz, & L. Trigeorgis, *Real Options and Investment Under Uncertainty* (pp. 103-134). Cambridge-Massachusetts: The MIT Press.
- Gibson, R., & Schwartz, E. (1990). Stochastic Convenience Yield and the Pricing of Oil Contingent Claims. *The Journal of Finance*, (3), pp. 959-976.
- Glasserman, P. (2004). *Monte Carlo Methods in Financial Engineering*. New York: Springer.
- Greene, W. (2008). *Econometric Analysis* (6 ed.). New Jersey: Pearson.

-
- Hamilton, J. (1994). *Time Series Analysis*. Princeton, New Jersey: Princeton University Press.
- Harrison, J., & Kreps, D. (1979). Martingales and Arbitrage in Multiperiod Securities Markets. *Journal of Economic Theory* 2, pp. 381-408.
- Harvey, C., & Siddique, A. (1999). Autoregressive Conditional Skewness. *Journal of Financial and Quantitative Analysis* 34(4), pp. 465-477.
- Harvey, C., & Siddique, A. (2000). Conditional Skewness in Asset Pricing Tests. *Journal of Finance* 55, pp. 1263-1295.
- Haug, E. (2007). *The Complete Guide to Option Pricing Formulas*. New York: McGraw-Hill.
- Hodges, S. (1997). A generalisation of the Sharpe ratio and its applications to valuation bounds and risk measures. (U. o. Warwick, Ed.) *Financial Options Research Centre*.
- Hull, J. (2000). *Options, Futures & Other Derivatives*. London: Prentice-Hall International (UK).
- Hull, J., Nelken, I., & White, A. (2004/05). Merton's Model, Credit Risk and volatility Skews. *Journal of Credit Risk*, 1(Winter), 3-28.
- Jamshidian, F., & Fein, M. (1990). Closed-Form Solutions for Oil Futures and European Options in the Gibson-Schwartz Model. *Merrill Lynch Capital Markets Working Paper*.
- Jones, P., Mason, S., & Rosenfeld, E. (1984). Contingent Claims Analysis of Corporate Capital Structures: an Empirical Investigation. *The Journal of Finance*, 39(July), 611-625.
- Kwiatkowski, D., Phillips, P., Schmidt, P., & Shin, Y. (1992). Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root? *Journal of Econometrics*, 54(1-3), 159-178.
- Lütkepohl, H. (2005). *New Introduction to Multiple Time Series Analysis*. Heidelberg: Springer.
- MacKinnon, J. (1996). Numerical distribution functions for unit root and cointegration tests. *Journal of Applied Econometrics*, 11, 601-618.
- Maddala, G., & Kim, I.-M. (2006). *Unit Roots, Cointegration, and Structural Change* (6 ed.). Cambridge: Cambridge University Press.

- Malevergne, Y., & Sornette, D. (2006). Multi-moment Method for Portfolio Managment: Generalised Capital Asset Pricing Model in Homogeneous and Heterogeneous Markets. In E. Jurczenko, & B. Maillet, *Multi-moment asset allocation and pricing models* (pp. 165-193). West Sussex: John Wiley & Sons Ltd.
- McDonald, R., & Siegel, D. (1986, Nov.). The Value ofWaiting to Invest. *The Quarterly Journal ofEconomics*, pp. 707-727.
- Merton, R. (1973a). Theroy of Rational Option Pricing. *Bell Journal of Economics and Management Science*, 4, 373-416.
- Merton, R. (1973b). An Intertemporal Capital Asset Pricing Model. *Econometrica*, 41(5), pp. 141-183.
- Metcalf, G., & Hassett, K. (1995). *Investment Under Alternative Return Assumptions: Comparing Random Walks and Mean Reversionq*. Massachusetts: National Bureau of Economic Research.
- Meucci, A. (2009). *Risk and Asset Allocation*. Heidelberg: Springer Finance.
- Mun, J. (2003). *Real Options - Analysis Course*. New Jersey: Wiley.
- Mun, J. (2006). *Real Options Analysis*. New Jersey: Wiley.
- Neftci, S. (2000). *An Introduction to the Mathematics of Financial Derivatives*. San Diego: Academic Press.
- Oksendal, B. (2005). *Stochastic Differential Equations - An Introduction with Applications* (6 ed.). Heidelberg: Springer.
- Pézier, J. (2008). Maximum certain equivalent returns and equivalent performance criteria. *ICMA Centre Discussion Paper in Finance*.
- Pindyck, R. (1999). The long-Run Evolution of Energy Prices. *The Energy Journal*, 20(2), 1-27.
- Pindyck, R. (2004a). Volatility in Natural Gas and Oil Markets. *The Journal of Energy and Development*, 30(1), 1-19.
- Pindyck, R. (2004b). Volatility and Commodity Price Dynamics. *The Journal of Futures Markets*, 24(11), 1029-1047.
- Pindyck, R., & Rubinfeld, D. (1998). *Econometric Models and Economic Forecasts* (4 ed.). Singapore: Irwin/McGraw-Hill.
- Premarante, G., & Bera, A. (2000). Modeling Asymmetry and Excess Kurtosis in Stock Return Data. *Working Paper*.

-
- Rachev, S., Menn, C., & Fabozzi, F. (2005). *Fat-Tailed and Skewed Asset Return Distributions*. New Jersey: John Wiley & Sons, Inc.
- Rees, M. (2005). Shall the Twain Ever Meet? *Wilmott Magazin*(5).
- Rinne, H. (2003). *Taschenbuch der Statistik* (3 ed.). Frankfurt am Main: Harri Deutsch GmbH.
- Rinne, H., & Specht, K. (2002). *Zeitreihen - Statistische Modellierung, Schätzung und Prognose*. München : Franz Vahalen.
- Rubinstein, M. (1976). The Valuation of Uncertain Income Streams and the Pricing of Options. *The Bell Journal of Economics*, 7(2), 407-425.
- Rubinstein, M. (2006). Foreword. In E. Jurczenko, & B. Maillet, *Multi-moment asset allocation and pricing models* (pp. xvii-xix). New York: John Wiley & Sons Ltd.
- Rudolf, M. (2000). Monte Carlo Simulation im Risikomanagement. *Wirtschaftswissenschaftliches Studium*, pp. 381-387.
- Schwartz, E. (1997). The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging. *The Journal of Finance*, 52(3), 923-973.
- Schwartz, E. (1998). Valuing Long-Term Commodity Assets. *Financial Management*, 27(1), 57-66.
- Schwartz, E., & Moon, M. (2000). Rational Pricing of Internet Companies. pp. 62-75.
- Schwartz, E., & Smith, J. (2000). Short-Term Variations and Long-Term Dynamics in Commodity Prices. *Management Science*, 46(7), 893-911.
- Sheskin, D. (2007). *Handbook of Parametric and Nonparametric Statistical Procedures*. Boca Raton: Chapman & Hall/CRC.
- Shreve, S. (2004). *Stochastic Calculus for Finance II - Continuous-Time Models*. Heidelberg: Springer.
- Sodal, S., Koekebakker, S., & Adland, R. (2006). Value based trading of real assets in shipping under stochastic freight rates. *Unpublished Manuskript*, 1-27.
- Tavella, D., & Randall, C. (2000). *Pricing Financial Instruments - The Finite Difference Method*. New York: Wiley & Sons, Ltd.
- Tourinho, O. A. (1979). The Option Value of Reserves of Natural Resources. *Research Program in Finance Working Paper, University of California at Berkeley* 94, pp. 50-62.

- Trigeorgis, L. (2000). *Real Options: Managerial Flexibility and Strategy in Resource Allocation*. Cambridge-Massachusetts: The MIT Press.
- Trigeorgis, L. (2001). Real Options: an Overview. In E. Schwartz, & L. Trigeorgis, *Real Options and Investment Under Uncertainty* (pp. 103-134). Cambridge-Massachusetts: The MIT Press.
- Trigeorgis, L., & Lambertides, N. (2013). The Role of Growth Options in Explaining Stock Returns. *Journal of Financial and Quantitative Analysis*, p. forthcoming.
- Trigeorgis, L., & Mason, S. (1987). Valuing Managerial Flexibility. *Midland Corporate Finance Journal*, S. pp. 14-21.
- Tsay, R. (2005). *Analysis of Financial Time Series*. Hoboken: John Wiley & Sons, Inc.
- Verhoeven, P., & McAleer, M. (2004). Fat tails and asymmetry in financial volatility models. *Mathematics and Computers in Simulation*, 64, 351-361.
- Wang, P. (2003). *Financial Econometrics*. London: Routledge.
- White, G., Sondhi, A., & Fried, D. (1998). *The Analysis and Use of Financial Statements* (2 ed.). New York: John Wiley & Sons, Inc.
- White, H. (1980). A Heteroskedasticity-Consistent Covariance Matrix and a Direct Test for Heteroskedasticity. *Econometrica* 48, pp. 817-838.
- Wooldridge, J. (2002). *Econometric Analysis of Cross Section and Panel Data*. London, England: The MIT Press.
- Wooldridge, J. (2003). *Introductory Econometrics* (2 ed.). Mason: Thomson.