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„Capacitated Lot Sizing and Scheduling – An alternative approach for the parallel machine case.“

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A INTRODUCTION

1 Background

Nowadays the global economy opens new markets for almost every industry. New competitors enter the market and force a strong challenge and competition. Many companies are not longer able to win this challenge by reducing the prices of their products. Instead they try to increase the quality of the products and services. They try to fulfil customers' needs. Therefore the trend changes towards customized variants of products instead of standardized goods. Thus, the number of different products increases. At the same time delivery times and production costs have to be reduced to a minimum. [2]

Due to this new complexity of the manufacturing processes, planning problems are growing in size and complexity. Because of this new market situation and global competition manufacturing companies are forced to improve their processes continuously and to elaborate new planning tools and planning systems for their production. Production systems are concerned with the effective management of the total flow of goods, from the acquisition of raw material to the delivery of finished goods. [7] The tactical and operational planning process is a key factor for success or failure for manufacturing companies. [3]

Production planning systems are refined into micro structures where every segment performs a set of operations. Raw materials and component parts are floating through this system in order to be processed and assembled until a final product comes out. Production planning and scheduling is one of the most challenging subjects for the management. The goal is to find a feasible production plan which meets the request of customers or other facilities and provides release dates and capacities for all products including component parts. [1]

2 Problem Statement

A production planning system determines which product to produce on which machine at what time and helps the companies to deploy their capacities and to secure a high service level for their customers. [2]

Production plans can be evaluated by an objective function, which measures setup and holding costs and tries to find the optimum.

The subject of most planning activities is to optimize the trade off between low set up costs (favouring large production costs) which occur when a machine has to be adapted from product A to product B and low holding costs (favouring a lot for lot production and sharing resources) which occur when items are hold in inventory. The manufacturing process is triggered by orders from customers or other facilities. Large inventories, lead to high opportunity costs of capital. So costs of storing goods and holding items in inventory should be avoided. Scarce resources, however, lead to high set up costs. If there is low inventory, orders have to be produced just in time, therefore set up actions must take place to prepare proper operations and production is delayed which causes again opportunity costs.

Finally the problem of short term production planning turns out to be a lot sizing and scheduling problem where sequence decisions have to be made due to sharing common resources. To solve this problem it is important to consider that operations can only be executed if parts which are subject of this particular operation are available. So a production plan must respect the precedence relations of operations. Each operation produces an item, and every item is the output of an operation. Multi level structures and multi item problems must be taken into account. Another key element is scarce capacity. Producing an item requires a certain amount of one or more resources with limited capacity per time unit, for example manpower or machine time. [1]

3 Structure

This paper is divided into five main parts. Chapters A and B give a short introduction into the subject matter. Further chapter B lists up some common definitions which relate to this subject in particular and gives a brief insight into production planning concepts and manufacturing processes.

Chapter C introduces general and special case lot sizing models and scheduling procedures from existing literature. The basic ideas, different ways and approaches for lot sizing, scheduling and production planning are described. Whereas chapter D presents several solution methods which are used to solve lot sizing and scheduling models for different production structures. This part of the paper also includes the advantages and disadvantages and the quality of the solution approaches.

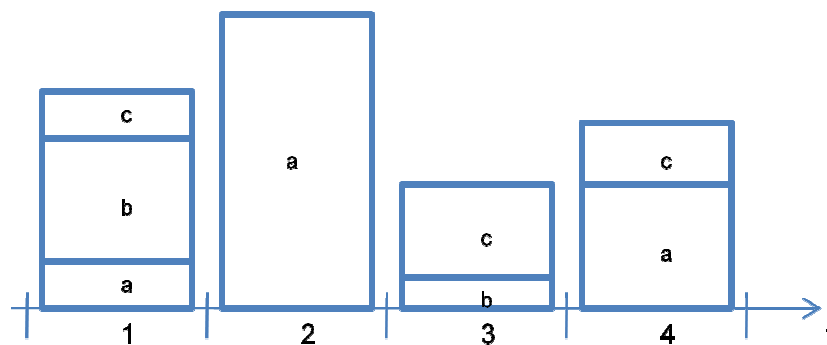
Finally chapter E presents the case study. It contains a detailed description of the procedure as well as the data used in order to get some feasible solutions. Last but not least the results of the case study are interpreted and conclusions are made.

B LOT SIZING

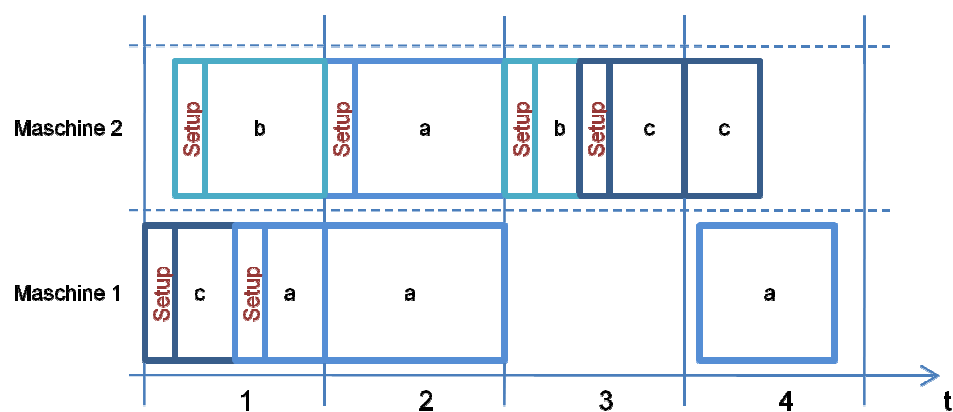
1 Aims and Objectives of Lot Sizing

Part of the production planning process is to determine lot sizes. One goal of manufacturing companies is to coordinate lot sizes and sequences of products in the manufacturing process in a way that total costs are minimized.

“Lot Sizing: What quantities in which period?”



“Scheduling: Which machine and sequence?”



(Daniel Quadt, Lot-Sizing and Scheduling for Flexible Flow Lines, 2004)

Figure 1: Lot sizing and scheduling [2]

Figure (1) shows the relationship between lot sizing and scheduling. Three products *a*, *b* and *c* have to be produced on two machines in four periods. Lot sizing is concerned with product quantities and scheduling considers machine and product sequences.

A lot size is the number of product units produced at once. Demand volumes of several periods can be combined in one period, because setting up a machine to produce a certain item needs time and money. Producing a product in a period other than its demand period leads either to inventory holding or back order costs. So the objective of a lot sizing process is to find optimal production quantities that balance the trade off between set up, inventory holding and back order costs.

- *Inventory*: A product volume is produced before its demand period. It is stored in inventory until its delivery date in the demand period. Costs occur for every period and unit of inventory.
- *Back Orders*: A product is produced after its demand period. The product is tardy and therefore back order costs incur for every unit and period of the delay. If capacity is scarce some products have to be back ordered. The question is which demand should be backordered and which not? [2]

This classical optimization problem tries to minimize all relevant costs. Lot sizing is part of short term production planning and includes a time frame of one to several weeks. With consideration of given goals, lot size planning is therefore, examining, whether it is meaningful, to combine production orders into a lot and in which sizes lots should be planned for manufacturing. [4, 9]

2 Production Management

The basis of production management decision and production planning is an aggregate plan covering the whole structure of a production company. This plan includes different time frames and decisions concerning all levels of an enterprise.

2.1 Supply Chain Management (SCM)

The SCM concept is founded on a hierarchical planning structure. The main focus is on supporting the material flow across the supply chain (SC) and related business functions as procurement, production, transport, distribution and sales. A SC is the organizational network of a company, where different processes are linked together on downstream and upstream levels. It represents a new strategy on how to link organizational units to best serve customer needs and to improve competitiveness. SCM should integrate organizational units along the SC and coordinate material, information and financial flow. All activities along the SC should be designed according to the needs of the customers with the aim of improving competitiveness.

These hierarchical planning processes can be presented by a “Supply Chain Planning Matrix”. It shows the interdependence of the material flow across the SC and the related business functions. It describes planning tasks which can be considered as different levels of aggregation and planning ranging from aggregate long term planning to detailed short term planning. [16]

- *Strategic planning (Strategic Network Planning)* is concerned with establishing managerial policies. The goal is to satisfy external demands and customers orders. Strategic planning is concerned with designing and planning production and distribution facilities and the capacities of sites and warehouses. These managerial decisions force a long time planning horizon. [7, 16]

- *Tactical planning* or “*Master Planning*” deals with the effective allocation of resources (e.g. production, storage and distribution capacities, work force and financial resources). It is done after the structure of the SC is fixed. It looks for efficient ways to fulfill demand forecasts over a medium time horizon. Master planning means to find a balance between demand forecasts and given capacities as well as assigning demands to facilities. Costs and revenues associated with these operations can be taken into account. Typical decisions to be made within this context are about the utilization of work force (e.g. regular and overtime), the accumulation of inventories, selection of transportation alternatives and definition of distribution channels. [7, 16]
- *Operational planning* includes detailed *production planning*. Typical decisions at this level are for example the assignment of customers’ orders to machines, lot size decisions, sequencing these orders at the workshop, inventory control, dispatching and processing of orders. These considerations are of a short time horizon. [7] Production planning and detailed scheduling is done separately in each site. [16]

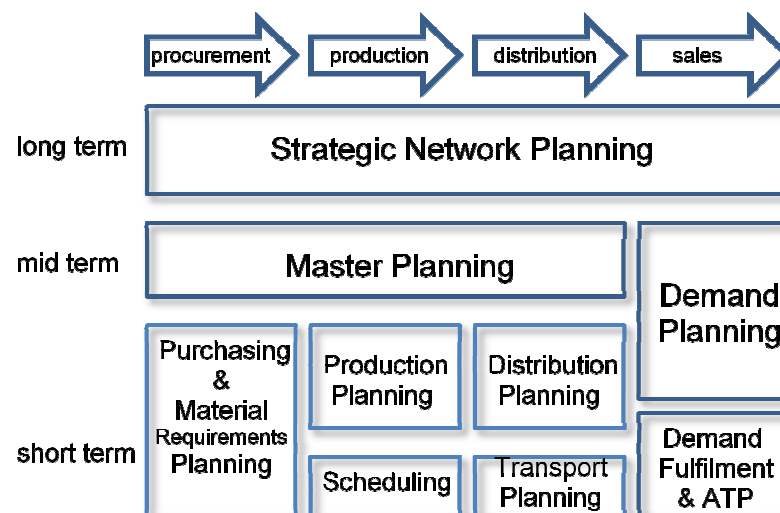


Figure 2: The Supply Chain Planning Matrix [16]

SCM is driven by forecasted demand and planned customer orders. Expected demands are inputs to several modules. The structure of a SC assumes a planning horizon of several years. The module purchasing and material requirement planning is dependent on master planning and short term production planning. Required quantities of items are derived from a bill of material explosion which also gives information about which operations have to be performed and at which point in time. The flow of goods between sites and customers is part of distribution planning. A distribution network should be established. This is also part of short term planning. Order execution and order delivery as well as due date setting and shortage planning are parts of the module demand fulfillment. Remaining quantities are available-to-promise quantities (ATP), which are used to promise due dates for new incoming orders. [16]

2.2 Production Planning

Production planning systems are usually split into long, medium and short term planning. In order to get viable plans these three phases have to be connected in some way. In general, long term production planning uses long and medium term demand forecasts and coordinates them with supply, production and staff deployment. Capacities are usually aggregated on a factory level or a product type level. A typical time frame is one to several years with a period length of one to three months. Medium term planning is more detailed. Inputs are short term forecasts and given customers' orders. It usually covers the end products. A typical time frame is one to several months and is mostly divided into weekly periods. [2]

Process planning means to prepare production flows chronologically and quantitatively. On the basis of a certain production output, which is specified by the production program and by results from material requirement planning, production orders are formed and taken into account for the manufacturing process. Production orders are work instructions for the production of a given

quantity of pre-, intermediate, and final products. [4] Production planning refers to a hierarchical structure including the following steps:

- *Aggregate Planning*

The aggregate planning is the overall production planning and coordination. It includes information about capacities, resources, demand forecasts and workforce level. All production decisions are based on the data of the aggregate planning.

- *Master Production Schedule (MPS)*

The MPS is a delivery plan for the manufacturing organization. It lists the exact amounts and delivery timings for each end product. [11]

- *Material Requirement Planning (MRP)*

The MRP is based on the MPS, on an inventory status record and on the bill of material (BOM). It determines material requirements and timings for each phase of production and includes a detailed capacity planning for component parts. It determines the timing of order releases and lot sizes can be determined. [11]

- *Operation Scheduling*

“Scheduling is the process of organizing, choosing and timing resource usage to carry out all the activities necessary to produce the desired outputs at the desired times , while satisfying a large number of time and relationship constraints among the activities and the resources.”

(Morton and Pentico, 1993) [11]

2.3 Operation and Job Scheduling

Scheduling copes with the detailed planning of when and on which machine to produce a product unit or a job. It is the ordering of operations to machines. [2] Operation scheduling observes routing relationships and other restrictions. [7] Scheduling specifies the time each job starts and completes on each machine, as well as any additional resources needed. What is more, scheduling should determine the best sequence of the jobs. [11] Scheduling procedures cover a single period of the lot sizing problem between one shift and a few weeks. Objectives of scheduling procedures are time oriented like minimizing completion time of the last job (makespan), due date fulfilment, minimizing the tardiness of jobs or minimizing the mean flow time through a system. [2]

The release time or arrival time of a job is the time at which the job is released to the shop floor. It is the earliest time at which the first operation of a job can start to be processed on a machine. The due date is the time by which the last operation of the job should be completed. The completion time of a job is the time at which processing of the last operation of the job is completed. [7] A job's flow time is the time between the beginning of a job's first operation until the finishing of the last operation. [2] An operation is an elementary task to be performed. The amount of processing required by an operation is called processing time. In most cases set up times are included in the processing times.

A job is a set of operations that are correlated with precedence restrictions. These restrictions define the routings which is the ordering of operations within jobs. A machine is a part of the equipment and is capable to perform an operation. Most job shop problems deal with a number of machines on which a number of jobs have to be processed. Sequencing means that for each machine in the shop one has to establish the order in which the jobs are waiting to be processed on machines. The job arrival process can be classified as static or dynamic. In the static case all jobs to be processed in

the shop arrive simultaneously at some time. In the dynamic shop jobs arrive periodically. [7]

Scheduling problems can be denoted with a “three field notation” – $\alpha \mid \beta \mid \gamma$. α specifies the machine configuration e.g. single machine, identical machines in parallel or flexible flow shop with a given number of stages in series. β specifies any processing restrictions and constraints like jobs with due dates or release dates or precedence restrictions. γ describes the objective to be minimized. [15]

3 Problem Classification

Material and lot size planning is based on a given product program and in case of multi level demand planning on a given product structure. On all levels of demand and product planning capacities have to be taken into account. With the help of this information and with the MPS demands for different products can be calculated. [6]

3.1 Demand Definition

- *Primal or External Demand*
This kind of demand is market driven. It is the aggregate supply for end products and customer orders. [5, 6]
- *Secondary or Internal Demand*
It is the demand of raw material and component parts. It is determined through the product structure and the primal demand. [6]
- *Deterministic Demand*
Demand is assumed to be known for sure during the whole planning period. [12]
- *Stochastic Demand*
It is possible for demand to be constant in expectation but still be random. Random also means uncertain or stochastic. [19]

- *Stationary Demand*

Demand is assumed to be constant and can be calculated with the EOQ (Economic Order Quantity) formula. Models with stationary demand are easier to solve because parameters are constant over the whole planning horizon. [12]

- *Dynamic Demand*

Demand varies over the planning horizon (e.g. Wagner Whitin). If demand changes over some periods, capacities have to adapted, too. [12]

3.2 Resource Constraints

- *Capacitated Model*

This kind of lot sizing models deal with scarce resources like time, quantities or money. Interdependencies between items have to be taken into account because machines on which items are produced have limited capacities. [5] (multi item problem)

- *Uncapacitated Model*

Solving these models means that any restrictions and interdependencies can be ignored. (single item problem)

3.3 Product Structure

The product structure concerns the connection between products, i.e. between final products, pre-products, component parts. The product structure is mostly shown in the BOM. This list contains the quantities of all components needed to manufacture a product. The product structure can be described graphically with the help of a spanning tree or a “Gozinto” graph or tabulated through parts list or with a linear programming system. [6]

- *Linear Structure*
Each item has at most one direct successor and at most one direct predecessor.
- *Convergent Structure*
Each item has at most one direct successor and more than one direct predecessor.
- *Divergent Structure*
Each item has at most one direct predecessor but more than one direct successor.
- *General Structure*
The characteristics of some or all product structures are combined. [6]

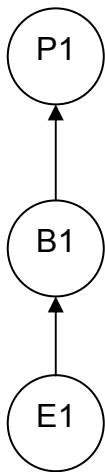


Figure 3: Linear Structure [6]

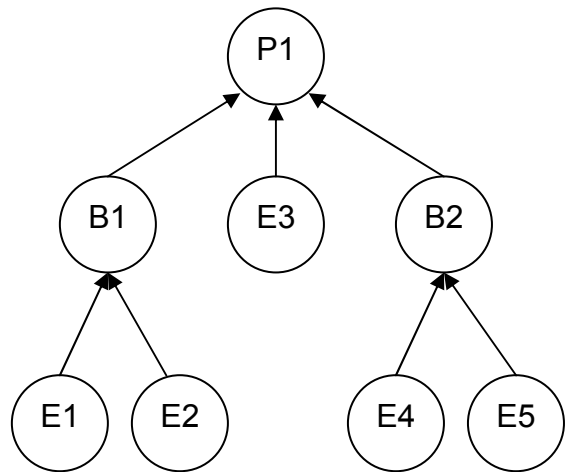


Figure 4: Convergent Structure [6]

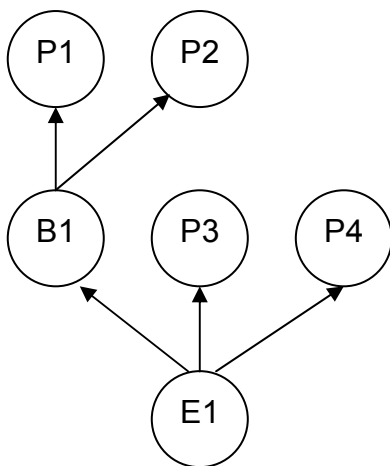


Figure 5: Divergent Structure [6]

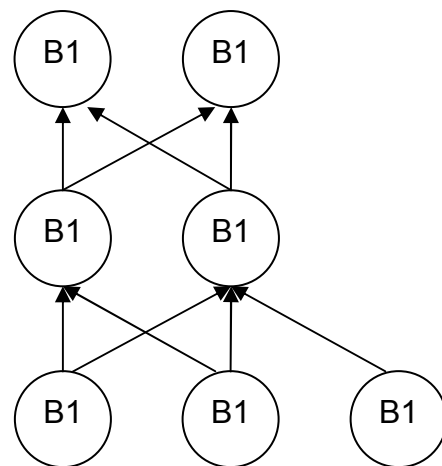


Figure 6: General Structure [6]

3.4 Single Item and Multi Item Problem

Lot sizing and scheduling problems can also differ from the number of different products which are produced. The question is if only a single product is produced or if multiple products are produced. The production of more than one product has high impact on the set up of machines, for example if different products are produced one behind the other on one machine. Single item lot sizing problems plan the lot size of exactly one item. In multi item problems lot sizing has to be done for several products simultaneously. Most multi item problems are combined with a multi level production structure. [8, 14]

3.5 Single Level and Multi Level Production

The number of production levels is the number of workstations which an item has to run through within the production process or the number of activities which are necessary to manufacture an item. A multistage production system is a production process in which component parts have to be obtained by manufacturing or by purchasing. Afterwards they are assembled to subassemblies and at last into the finished goods. All end products within a product structure are on the first disposition level. Products which have direct impact on end products are on the second disposition level of a product structure and so on. In multi level models all sub- and end products are combined through the product structure. Items within a single level model do not have any interdependencies. Only one work process is necessary to produce this item. [7, 14]

3.6 Shop Configuration

- The simplest configuration is the *single machine shop*. Each job consists only of one single operation which should be processed on a single machine. Now a sequence in which the jobs have to be processed should be determined. Also an assembly line can be viewed as one big machine. More complex job shop models allow more than one machine. [7]
- The *flow shop* contains a number of machines and the jobs are strictly ordered in sequences of operations. It comprises a series of machines that perform operations on a job as it is progressed down the line. [15] All movements between machines within the shop must be in a uniform direction. In the general flow shop some jobs may have less than m operations. So neither all jobs have to begin processing on the same machine nor have to be completed on the same machine nor should two sequentially numbered operations of a job require two neighbouring machines. [7] The flow shop configuration is common in many manufacturing and assembly facilities. [15]
- The *general job shop* has m machines. The sequence of all jobs is not constant over the entire processing time. Each job has its own way to be processed on the machines. If the jobs are passed on to the next machine sequencing has to be done again. Jobs are in general multistage and can have any number of operations. Most models, however, assume that each job has exactly m operations and requires each machine only once. Job shops can be classified into *open and closed job shops*. In an open job shop all products are made to order. No inventory is accumulated. In a closed shop demand forecasts for finished products are given. Production runs in batches and inventory is carried. [7, 9]

- *Continuous production systems* involve the manufacturing of a few families of technologically related products in large quantities through fixed routings. Demand is large and constant. Production takes place in form of a production line.
- *Intermitted production systems* involve batch production of many products which share several processing centres. Machines are timeshared between many different items and switch from one item to another according to some demand and manufacturing processes.
- The *parallel machine shop* is for example an extension of the single machine case. The job still consists of only one operation but there are several machines that work in parallel and a certain job can be processed on any of these machines. There are just different processing times if the machines are not identical. [7]

C LOT SIZING MODELS AND SCHEDULING PROCEDURES

1 The Interdependence of Lot Sizing and Scheduling

A lot sizing procedure calculates how many units of a product should be produced in one period. This calculation needs information about set up times which are dependent on machine assignment and on a product sequence. This data is determined by a scheduling procedure. The relationship between lot sizing and scheduling is obvious if set up times are sequence dependent but also if set up times are sequence independent, in case that the last product of a period is continued in the next period without a new set up. So lot sizing needs detailed information about set up state from the scheduling procedure. Vice versa the scheduling procedure can only find an assignment and a sequence if production volumes from the lot sizing phase are given. [2] This is also true for production lines or parallel machines; capacity that is finally available for production on a specific line is known only when the size and the sequence of the lots have been determined. Lot sizing, line assignment and lot scheduling have to be done simultaneously. [10] To coordinate these interdependencies it is unavoidable to find an integrative solution procedure that makes it possible to solve the lot sizing and the scheduling problem at the same time. The characteristics of lot sizing and scheduling are different from all different kinds of production facilities. Therefore specific solution procedures have to be developed. [2]

2 Sequence Dependent Set Up

Lot sizing and scheduling problems with sequence dependent set up time consider production processes where machines have to be set up individually for each operation. Sequence dependent set up (SDS) is often combined with the availability of more than one machine for a certain production step. Models including sequence dependent set up times also involve the problem of assigning jobs to different, individual machines. [3] The objectives of SDS scheduling are mostly time oriented. The problem of prescribing a sequence can be driven by the objective of minimizing makespan or by due date fulfillment like minimizing lateness or weighted tardiness. [15] Final items can be assigned to a few set up families or batches. Changeovers within a family can be disregarded, whereas changeovers between two items of different families can incur significant set up costs and set up times that are sequence dependent. [10] Product family set up is sequence independent. [2]

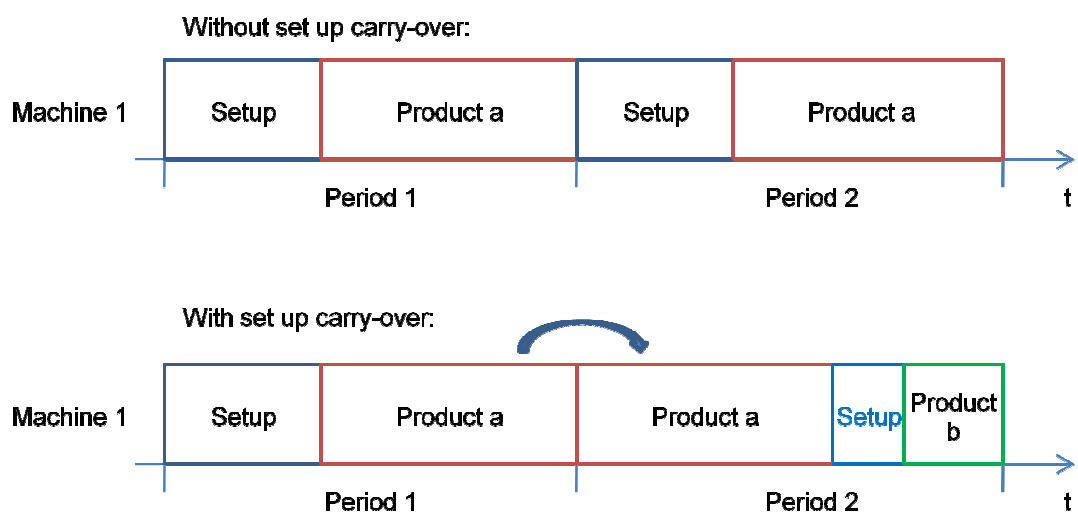


Figure 7: Set up carry-over [2]

The set up for product *a* can be saved, because the set up state from period one can be carried over. Thus, additional capacity which is not required for another set up can be used to start with another product *b* in the same period. So solution becomes significantly different when set up carry over is used. [2] Most practical scheduling problems involve set up times or costs.

SDS addresses a variety of machine configurations like single machine, parallel machine, flow shop or job shop systems. The MIP Models which are presented in chapter 5 give attention to lot sizing and scheduling but they do not incorporate SDS. [15]

3 Small Bucket and Big Bucket Models

In small bucket problems at most one set up per period is possible. The set up state from one period can be carried over to the next period. [3] The carry over set up state from one period to another means that the last product of a period and the first product of the next period are the same. No set up has to take place that saves capacity. [2] This fact requires the production of only one item in one period. The time periods of such models have to be very small, meaning that for a short planning horizon a big number of time periods have to be considered. [3] A critique against small bucket models is that for real world problem sizes the number of periods is too large. This argument may be true for mathematical programming approaches but for common heuristics it is not true because instances with hundreds of periods can nowadays be solved easily on personal computers. [1]

In most standard big bucket lot sizing problems a set up has to be performed in each period in which production takes place. No set up carry over is possible which may lead to no feasible solution because too much capacity is needed. The inclusion of set up carry over leads to a certain sequence of products in a period and incorporates characteristics of small bucket models into a big bucket model. [2] Big bucket models allow more than one set up per period and for different items. The carryover of set up states from one period to the next is hardly possible. [3] In big bucket models it is difficult to combine lot sizing and scheduling. New researches on big bucket models try to incorporate lot sizing and scheduling.

4 Historical Review

4.1 The Economic Order Quantity Model

The classical economic order quantity (EOQ) model was one of the first approaches which dealt with lot sizing. It assumes a single level production process with no capacity constraints and it is based on a single item problem. Demand is assumed to be stationary (continuous demand with constant rate). [1] It does not take any interaction between individual items and production scheduling into account.

4.2 The Economic Lot Scheduling Problem

The economic lot scheduling problem (ELSP) is a single level, multi item problem, because it considers capacity constraints, meaning that resources are shared in common by several items. This model still assumes stationary demand. [1] The ELSP can incorporate lot sizing and scheduling simultaneously. [14]

4.3 The Wagner Whitin Problem

The uncapacitated lot sizing problem (ULSP) is the simplest dynamic lot sizing problem and was first introduced by Wagner and Whitin (WW). The WW model is the first formulation of a lot sizing problem for dynamic demand. [3] It assumes that demand is given by period and varies over time. Capacity is ignored which means that this single level problem is a single item problem. [1] Wagner and Whitin also developed an exact algorithm based on dynamic programming to solve this problem. Consequently, a large number of heuristics were developed with the idea of minimizing average set up costs and inventory cost over several periods. [3]

New models, however, combine capacitated and dynamic approaches and scheduling is integrated with lot sizing decisions. [1] These basic ideas, mentioned above, were taken as a starting point to develop different extensions and features. [3]

5 Single Level Lot Sizing and Scheduling

5.1 The Capacitated Lot Sizing Problem (CLSP)

The CLSP considers one or several items (multi item problem) which can be produced on the same machine in one time period. [17] Taking capacity constraints (e.g. machine capacity) into account it can be seen as an extension to the WW problem. The CLSP is a large bucket problem because several items can be produced per period. The planning horizon is mostly less than six months and a period represents a time slot of one week. The CLSP integrates no scheduling decisions. [1] This model can also be extended with set up times. [17]

Table 1

Decision variables for the CLSP:

I_{jt}	Inventory for item j at the end of period t
q_{jt}	Production quantity for item j in period t
x_{jt}	Binary variable which indicates whether a set up for item j occurs in period t ($x_{jt} = 1$) or not ($x_{jt} = 0$)

Table 2

Parameters for the CLSP:

C_t	Available capacity for the machine in period t
d_{jt}	External demand for item j in period t
h_j	Non-negative holding costs for item j
I_{j0}	Initial inventory for item j
J	Number of items
p_j	Capacity needs for producing one unit of item j

s_j Non-negative set up costs for item j
 T Number of periods

Objective function

$$\text{Min} \sum_{j=1}^J \sum_{t=1}^T (s_j x_{jt} + h_j I_{jt}) \quad (1)$$

The objective is to minimize the sum of set up and holding costs.

The objective function must be subjected to the following constraints:

$$\begin{aligned} I_{jt} &= I_{j(t-1)} + q_{jt} - d_{jt} \\ j &= 1, \dots, J \\ t &= 1, \dots, T \end{aligned} \quad (2)$$

This equation represents the inventory balance. The inventory in period t must be equal to the last period's inventory plus the quantity produced in period t minus the demand of item j in period t .

$$p_j q_{jt} \leq C_t x_{jt}, \quad j = 1, \dots, J, \quad t = 1, \dots, T \quad (3)$$

The production of an item can only take place if the machine is set up for that particular item.

$$\sum_{j=1}^J p_j q_{jt} \leq C_t, \quad t = 1, \dots, T \quad (4)$$

This restriction is the capacity constraint.

$$x_{jt} \in \{0, 1\}, \quad j = 1, \dots, J, \quad t = 1, \dots, T \quad (5)$$

The set up variables are defined to be binary.

$$I_{jt}, q_{jt} \geq 0, \quad j = 1, \dots, J, \quad t = 1, \dots, T \quad (6)$$

Inventory and production quantity are assumed to be non-negative. [1]

5.2 The Discrete Lot Sizing and Scheduling Problem (DLSP)

The macro periods of the CLSP are divided into micro periods. Therefore the DLSP is a small bucket problem. The DLSP includes the “all or nothing” assumption which says that only one item may be produced per period and if so production uses the full capacity. As a small bucket problem only one item can be produced by period. One period in the DLSP corresponds to hours or shifts. Because of these short periods set up takes place only if the production of a new lot begins. [1] Due to the characteristics of this model the sequencing of jobs is done indirectly and scheduling can be incorporated. [14] The DLSP uses the same decision variables and parameters like the CLSP. The CLSP, however, raises set up costs in every period in which production takes place. So a new decision variable and a new parameter show the set up state in a certain period. [1] The DLSP model can be extended to the parallel machine case or for multiple alternative machines. Moreover, set up times, sequence dependent or independent, can be incorporated. [13, 17]

Table 3

New decision variable for the DLSP:

y_{jt} Binary variable which indicates whether the machine is set up for item j in period t or not

New parameter for the DLSP:

y_{j0} Binary value which indicates whether the machine is set up for item j at the beginning of period 1 or not ($y_{j0} = 1$, $y_{j0} = 0$)

Objective function

$$\text{Min} \sum_{j=1}^J \sum_{t=1}^T (s_j x_{jt} + h_j I_{jt}) \quad (7)$$

The objective is to minimize the sum of set up and holding costs. The objective function and most of the constraints equal to the CLSP formulation.

Subject to

$$\begin{aligned} I_{jt} &= I_{j(t-1)} + q_{jt} - d_{jt} \\ j &= 1, \dots, J \\ t &= 1, \dots, T \end{aligned} \quad (8)$$

$$p_j q_{jt} = C_t y_{jt}, \quad j = 1, \dots, J, \quad t = 1, \dots, T \quad (9)$$

This equation represents the “all or nothing” assumption. In contrast to the CLSP the left and the right side must be equal.

$$\sum_{j=1}^J y_{jt} \leq 1, \quad t = 1, \dots, T \quad (10)$$

At most one item can be produced per period, in combination with constraint (9) capacity limits are taken into account.

$$x_{jt} \geq y_{jt} - y_{j(t-1)}, \quad j = 1, \dots, J, \quad t = 1, \dots, T \quad (11)$$

These inequalities figure out the beginning of a new lot.

$$y_{jt} \in \{0,1\}, \quad j = 1, \dots, J, \quad t = 1, \dots, T \quad (12)$$

This condition defines the set up state to be binary.

$$I_{jt}, q_{jt}, x_{jt} \geq 0, \quad j = 1, \dots, J, \quad t = 1, \dots, T \quad (13)$$

In contrast to the CLSP a non-negativity constraint for x_{jt} is sufficient. [1]

5.3 The Continuous Setup Lot Sizing Problem (CSLP)

This model is more realistic than the DLSP and gives up the “all or nothing” rule. Nevertheless only one item may be produced by period – small bucket problem. The decision variables and the parameters are equal those of the DLSP. [1] The CSLP can be extended to sequence dependent and sequence independent set up costs. It can also be formulated for the parallel machine case. [21]

Objective function

$$\text{Min} \sum_{j=1}^J \sum_{t=1}^T (s_j x_{jt} + h_j I_{jt}) \quad (14)$$

Subject to

$$\begin{aligned} I_{jt} &= I_{j(t-1)} + q_{jt} - d_{jt} \\ j &= 1, \dots, J \\ t &= 1, \dots, T \end{aligned} \quad (15)$$

$$p_j q_{jt} \leq C_t y_{jt}, \quad j = 1, \dots, J, \quad t = 1, \dots, T \quad (16)$$

Production quantities can now be of any continuous size, capacity constraints must not be violated.

$$\sum_{j=1}^J y_{jt} \leq 1, \quad t = 1, \dots, T \quad (17)$$

$$x_{jt} \geq y_{jt} - y_{j(t-1)}, \quad j = 1, \dots, J, \quad t = 1, \dots, T \quad (18)$$

$$y_{jt} \in \{0,1\}, \quad j = 1, \dots, J, \quad t = 1, \dots, T \quad (19)$$

$$I_{jt}, q_{jt}, x_{jt} \geq 0, \quad j = 1, \dots, J, \quad t = 1, \dots, T \quad (20)$$

There is an important aspect why the CSLP differs from the DLSP. In the DLSP set up costs occur whenever a new lot begins. A lot for item j is completed in period t . Another lot for the same item is scheduled in period $t' > t$. In the period between, however, the machine can be idle, and set up for item j has to take place again. In the CSLP set up costs occur only once

$$Y_{j(t+1)} = \dots = y_{j(t'-1)} = 1,$$

this fact does not contradict to

$$q_{j(t+1)} = \dots = q_{j(t'-1)} = 0$$

as it does in the DLSP. [1]

5.4 The Proportional Lot Sizing and Scheduling Problem (PLSP)

Considering the CSLP remaining capacity which is not used in full in a period is left unused. The PLSP as a small bucket problem tries to avoid this by using remaining capacity for scheduling a second item in the same period. If two items are produced in one period it has to be clear in which order they are produced. So the set up state decision variable is interpreted in a new manner. y_{jt} is the set up state of the machine at the end of a period. The set up state can only change one time in a period. The set up from the previous period can be carried over to the next period. The machine can be set up either at the beginning or at the end of a period and at most two items per period may be produced. This model uses the variables and the parameters of the DLSP.

Objective function

$$\text{Min} \sum_{j=1}^J \sum_{t=1}^T (s_j x_{jt} + h_j I_{jt}) \quad (21)$$

Subject to

$$\begin{aligned} I_{jt} &= I_{j(t-1)} + q_{jt} - d_{jt} \\ j &= 1, \dots, J \\ t &= 1, \dots, T \end{aligned} \quad (22)$$

$$p_j q_{jt} \leq C_t (y_{j(t-1)} + y_{jt}), \quad j = 1, \dots, J, \quad t = 1, \dots, T \quad (23)$$

These inequalities make sure that production of an item in a certain period can only take place if the machine is set up at the beginning or at the end of that period.

$$\sum_{j=1}^J p_j q_{jt} \leq 1, \quad t = 1, \dots, T \quad (24)$$

This constraint is introduced to limit the total capacity requirement per period, whenever more than one item is produced in one period.

$$\sum_{j=1}^J y_{jt} \leq 1, \quad t = 1, \dots, T \quad (25)$$

$$x_{jt} \geq y_{jt} - y_{j(t-1)}, \quad j = 1, \dots, J, \quad t = 1, \dots, T \quad (26)$$

$$y_{jt} \in \{0,1\}, \quad j = 1, \dots, J, \quad t = 1, \dots, T \quad (27)$$

$$I_{jt}, q_{jt}, x_{jt} \geq 0, \quad j = 1, \dots, J, \quad t = 1, \dots, T \quad (28)$$

Like in the CSLP formulation, idle periods between two lots of the same item do not cause additional set up costs. [1]

5.5 The General Lot Sizing and Scheduling Problem (GLSP)

New researches returned to take large bucket models into account. In contrast to the CLSP lot sizing and scheduling is done simultaneously. The GLSP uses the same parameters as the DLSP. The main idea of the GLSP is based on lot sizing with stationary demand. Each lot is assigned to a position number in order to define a sequence. The GLSP introduces a new user defined parameter which restricts the number of lots per period $(1, \dots, N_1, N_1 + 1, \dots, N_T)$.

Table 4

New parameter for the GLSP:

N_t Maximum number of lots in period t

Table 5

Decision variables for the GLSP:

I_{jt}	Inventory for item j at the end of period t
q_{jn}	Production quantity for item j at position n
x_{jn}	Binary variable which indicates whether a set up for item j occurs at position n or not
y_{jn}	Binary variable which indicates whether the machine is ready to produce item j at position n or not
$F_t = 1 + \sum_{\tau=1}^{t-1} N_\tau$	This notation denotes the first position in period t
$L_t = F_t + N_t - 1$	This notation denotes the last position in period t
$N = \sum_{t=1}^T N_t$	Total number of positions and maximum number of lots that can be built

Objective function

$$\text{Min } \sum_{j=1}^J \sum_{n=1}^N s_j x_{jn} + \sum_{j=1}^J \sum_{t=1}^T h_j I_{jt} \quad (29)$$

Subject to

$$I_{jt} = I_{j(t-1)} + \sum_{n=F_t}^{L_t} q_{jn} - d_{jt}, \quad j = 1, \dots, J, \quad t = 1, \dots, T \quad (30)$$

The inventory balance.

$$p_j q_{jn} \leq C_t y_{jn}, \quad j = 1, \dots, J, \quad t = 1, \dots, T, \quad n = F_t, \dots, L_t \quad (31)$$

These inequalities guarantee that if a lot for item j is scheduled at position n the machine is in the correct set up state.

$$\sum_{j=1}^J \sum_{n=F_t}^{L_t} p_j q_{jn} \leq C_t, \quad t = 1, \dots, T \quad (32)$$

Capacity constraint.

$$\sum_{j=1}^J y_{jn} \leq 1, \quad n = 1, \dots, N \quad (33)$$

This equation forces a unique set up state.

$$x_{jn} \geq y_{jn} - y_{j(n-1)}, \quad j = 1, \dots, J, \quad n = 1, \dots, N \quad (34)$$

These inequalities determine the position at which a set up takes place.

$$y_{jn} \in \{0,1\}, \quad j = 1, \dots, J, \quad n = 1, \dots, N \quad (35)$$

Binary condition for the set up state variable.

$$I_{jt} \geq 0, \quad j = 1, \dots, J, \quad t = 1, \dots, T \quad (36)$$

$$q_{jn}, x_{jn} \geq 0, \quad j = 1, \dots, J, \quad n = 1, \dots, N \quad (37)$$

These two last equations are the non-negativity constraints.

If $N_t = 1$ in all periods then the GSLP equals to the CLSP. [1]

6 Continuous Time Lot Sizing and Scheduling (BSP)

A continuous time horizon, as in the EOQ and ELSP models, may be used for dynamic demand conditions as well. Each demand is characterized by its size and deadline. Demands are seen as jobs and demand size determines the processing time of a job. Capacity, like the speed of a machine is constant, so the processing time of a job does not depend on the schedule. Jobs are not allowed to be split. As a consequence each demand must be processed in one piece. Jobs concerning the same item can be grouped together to form one lot and set up costs can be saved. This problem can rather be seen as a batching and scheduling problem (BSP) than as a lot sizing problem.

If there are N demands to be fulfilled one can say that $1, \dots, N$ are the job numbers. 0 and $N + 1$ are the numbers of dummy jobs, which are scheduled as the first and the last job. A solution of the BSP is characterized by the sequence in which jobs are scheduled and by each job's completion time. [1]

Table 6

Decision variables for the BSP:

r_n	Completion time of Job n
x_{nk}	Binary variable which indicates that job n is scheduled right before job k

Table 7

Parameters for the BSP:

B	A big number
f_n	Deadline for job n
h_j	Holding costs for item j
$j(n)$	The item for which job n represents demand
N	Number of jobs
p_n	Processing time of job n
s_{ji}	Sequence dependent set up costs for items.

Objective function

$$\text{Min } \sum_{n=0}^N \sum_{\substack{K=1 \\ K \neq n}}^N s_{j(n)j(k)} x_{nk} + \sum_{j=n}^N h_{j(n)} p_n (f_n - r_n) \quad (38)$$

The objective is to minimize the total sum of set up and holding costs.

The x_{nk} variable makes it easy to incorporate sequence dependencies into the model. The holding costs are calculated by multiplying the holding costs of the corresponding item with the processing time of the job and with the earliness of the job. Demand is fulfilled if the whole job which represents that particular demand is processed.

Subject to

$$\sum_{\substack{k=1 \\ k \neq n}}^{N+1} x_{nk} = 1, \quad n = 0, \dots, N \quad (39)$$

This formulation makes sure that each job has exactly one successor. Only job $N + 1$ has none.

$$\sum_{\substack{k=1 \\ k \neq n}}^N x_{kn} = 1, \quad n = 1, \dots, N + 1 \quad (40)$$

Each job has exactly one predecessor, only job 0 has none.

$$r_n + p_k \leq r_k + B(1 - x_{nk}), \quad n = 0, \dots, N \quad k = 1, \dots, N + 1 \quad (41)$$

Jobs cannot overlap. Equation (39) together with (40) and (41) define a total order among all jobs.

$$r_n \leq f_n, \quad n = 1, \dots, N \quad (42)$$

No backlogging exists.

$$x_{nk} \in \{0,1\}, \quad n = 0, \dots, N, \quad k = 1, \dots, N + 1 \quad (43)$$

Binary condition.

$$r_n \geq 0, \quad n = 1, \dots, N + 1 \quad (44)$$

Non-negativity condition for the decision variable.

$$r_0 = 0 \quad (45)$$

The completion time for the dummy is zero.

In this BSP Model formulation, idle periods among jobs for the same item do not cause additional set ups (similar to CSLP, PLSP, GLSP). [1]

7 Multi Level Lot Sizing and Scheduling (MLLS)

Multi level lot sizing models take several levels of the production process into account. These models concentrate not only on the production of a final product as a single step but split it into several production steps according to the BOM. [3] The production planning is not only done for the end product but also for the component parts and for subassemblies that are needed to make the end product. Production at one level leads to demand on a lower level. [17] Primal demand, the demand of the end product which is market driven and secondary demand have to be taken into account. These models are more complex and harder to solve because of the interdependencies of all different items. [3] Researches where multi level lot sizing and scheduling is done simultaneously are based on quite general assumptions like no capacity constraints, only one bottleneck machine, only two product levels or a general product structure and multiple machines. [1] The problem, however, can also be extended with capacity constraints, e.g. only at the final level, or with set up times. [17]

7.1 Manufacturing Resource Planning (MRP II)

The MRP II approach is implemented in most commercial production planning and control systems. It tries to get a feasible production plan in a stepwise manner including three phases. Unfortunately this concept provides long lead times, high work in process and backlogging in practice. New researches try to find more accurate solution procedures for multi level structures. [1]

- *Phase 1*

Starting with the end items lot sizes are generated level by level for all items in a multi level structure. Capacity restrictions are ignored.

- *Phase 2*

The basic results calculated in phase 1 mostly exceed available capacities in some periods. So lots are shifted to find a plan which meets

capacity restrictions. This procedure takes no precedence relations among items into account.

- *Phase 3*

Sequence decisions are made and orders are released to the shop floor. Single level models (e.g. CLSP, DLSP, GLSP,...) can be solved level by level for a multi level structure by applying the MRP II concept.

Using a solution procedure for the CLSP during phase 1 capacities would not be violated. In this case phase 2 is not necessary. In a multi level structure it is easy to use the CLSP on a level by level basis but it does not lead to a feasible solution and phase 3, the scheduling phase, is not incorporated either. Applying the DLSP level by level with the presence of multi level precedence constraints among all items does not guarantee a feasible solution although the DLSP does combine all three phases. Incorporating minimum lead times is not a problem because of having short time periods. Concerning the CLSP minimum lead times must be ignored or overestimated. This causes high total lead times. Also the GLSP can integrate phase 1 to 3. Although it is designed for a single level structure, solution procedures for the GLSP can be implemented level by level. A feasible solution does not come up, either. As a large bucket problem integrating minimum lead time is difficult. In contrast to these single level models, the multi level PLSP integrates all three phases and above all it includes a multi level structure. The traditional MRP II concept can be replaced by the multi level PLSP. [1]

7.2 The Multi Level PLSP

This model uses the same decision variables as the single level PLSP. Some parameters are redefined and some are new for the multi level structure. Regarding some researches on several variants of the multi level PLSP it can be proven that the multi level DSLP and the multi level CSLP are special cases of the multi level PLSP.

Table 8

Parameters for the PLSP:

a_{ji}	“Gozinto value”: Its value is zero if item i is not an immediate successor of item j . Otherwise it is the quantity of item j that is directly needed to produce one item i
C_{mt}	Available capacity of machine m in period t
d_{jt}	External demand for item j in period t
h_j	Non-negative holding costs for having one unit of item j one period in inventory
I_{j0}	Initial inventory for item j
$\mathfrak{S}_m$	Set of all items that share the machine m ; i.e. $\mathfrak{S}_m = \{j \in \{1, \dots, J\} m_j = m\}$
J	Number of items
M	Number of machines
m_j	Machine on which item j is produced
p_j	Capacity needs for producing one unit of item j
s_j	Non-negative set up costs for item j
S_j	Set of immediate successors of item j ; i.e. $S_j = \{i \in \{1, \dots, J\} a_{ji} > 0\}$
T	Number of periods
v_j	Positive and integral lead time of item j
y_{j0}	Unique initial set up state

Objective function

$$\text{Min } \sum_{j=1}^J \sum_{t=1}^T (s_j x_{jt} + h_j I_{jt}) \quad (46)$$

Again set up costs and inventory holding costs should be minimized.

Subject to

$$I_{jt} = I_{j(t-1)} + q_{jt} - d_{jt} - \sum_{i \in S_j} a_{ji} q_{it}, \quad j = 1, \dots, J, \quad t = 1, \dots, T \quad (47)$$

This constraint guarantees inventory balance. At the end of a period t inventory equals what was on inventory at the end of period $t - 1$ plus what is produced minus external and internal demand.

$$I_{jt} \geq \sum_{i \in S_j} \sum_{T=t+1}^{\min\{t+v_{j,T}\}} a_{ji} q_{iT} , \quad j = 1, \dots, J, \quad t = 0, \dots, T-1 \quad (48)$$

Production has to keep to positive lead times to be able to fulfil internal demand.

$$p_j q_{jt} \leq C_{mjt} (y_{j(t-1)} + y_{jt}) , \quad j = 1, \dots, J, \quad t = 1, \dots, T \quad (49)$$

$$\sum_{j \in \mathfrak{S}_m} p_j q_{jt} \leq C_{mt} , \quad m = 1, \dots, M, \quad t = 1, \dots, T \quad (50)$$

$$\sum_{j \in \mathfrak{S}_m} y_{jt} \leq 1, \quad m = 1, \dots, M, \quad t = 1, \dots, T \quad (51)$$

$$x_{jt} \geq y_{jt} - y_{j(t-1)} , \quad j = 1, \dots, J, \quad t = 1, \dots, T \quad (52)$$

$$y_{jt} \in \{0,1\} , \quad j = 1, \dots, J, \quad t = 1, \dots, T \quad (53)$$

$$I_{jt}, q_{jt}, x_{jt} \geq 0 , \quad j = 1, \dots, J, \quad t = 1, \dots, T \quad (54)$$

[1]

8 Lot Sizing and Scheduling on Parallel Machines

Lot sizing and scheduling on parallel machines is a challenging subject for new researches. Very important facts are the consideration of positive set up times and sequence dependencies as well as backlogging. [1]

When producing standard products a large number of final items have to be produced on several parallel production lines, for example automatic flow lines consisting of many independent work stations. [10] Single machine models tend to put together all demands of several periods to produce the whole volume in a single period. Assuming that there is only one machine and enough capacity, this method saves set up time and cost. Producing more units on parallel machines means that more machines have to be set up which leads to higher set up costs. A better solution would be to produce in that period when demand is needed, saving inventory and back order

costs. Fewer parallel machines would be needed and the set up state of each machine could be carried over between periods. [2]

8.1 Characteristics and Assumptions

Simultaneous lot sizing and scheduling of several products on non-identical parallel production lines (heterogeneous machines) can be restricted by limited capacity of the production line. Capacity may be reduced further by sequence dependent set up times. Each production line can be considered as a single planning unit (machine). All production lines offer the same services and those can be used alternatively. Nevertheless, production lines do not have to be technically identical. The capacity that is finally available for production on a specific line is known only when the size and the sequence of the lots have been determined. This planning problem relates to a single stage scheduling on parallel machines.

Over the last years significant progress has been made in solving single line problems with sequence dependent set up times. Simultaneous lot sizing and scheduling of parallel production lines is not often discussed in literature. The broadest formulation among single line models is the GLSP including sequence dependent set up times. Now it is obvious to extend the GLSP for parallel production lines. [10]

8.2 GLSP for Parallel Production Lines (GLSPPL)

A given number of items should be scheduled on different parallel production lines over a finite planning horizon consisting of several macro periods. A macro period is divided into a fixed number of non overlapping micro periods with variable length. The production lines are scheduled independently and scheduling is done separately for each line. The length of a micro period is a decision variable, expressed by the quantity produced in the micro period on a line. A sequence of consecutive micro periods, in which the same item is produced on the same line, defines the lot. A lot may continue over several

micro and macro periods, independent of the time structure of the macro periods. Due to the fixed number of micro periods a lot may contain idle micro periods where no quantity is produced. The set up state is conserved if after an idle period the same item is produced on the same line again. [10]

Table 9

Parameters for the GLSPPL:

$j = 1, \dots, J$	Number of products to be scheduled
$l = 1, \dots, L$	Number of parallel production lines
$t = 1, \dots, T$	Number of macro periods
S_{lt}	The set of micro periods s belonging to macro period t and production line l
$s = 1, \dots, S^l$	Order of micro periods
K_{lt}	Capacity (time) of production line l available in macro period t
a_{lj}	Capacity consumption (time) needed to produce one unit of product j on line l
m_{lj}	Minimum lot size of product j (unit) if produced on line l
h_j	Holding costs of product j (per unit and per macro period)
c_{lj}	Production costs of product j (per unit) on line l
s_{lij}	Set up costs of a changeover from product i to product j on line l
st_{lij}	Set up time of a changeover from product i to product j on line l (time)
d_{jt}	Demand for product j in macro period t (units)
I_{j0}	Initial inventory of product j at the beginning of the planning horizon (units)
y_{lj0}	Equals 1, if line l is set up for product j at the beginning of the planning horizon; otherwise 0

Table 10

Decision variables for the GLSPPL:

$I_{jt} \geq 0$	Inventory of product j at the end of macro period t
$x_{ljs} \geq 0$	Quantity of item j produced in micro periods s on line l

$y_{ljs} \in \{0,1\}$ Set up state: $y_{ljs} = 1$, if line l is set up for product j in micro period s ; 0 otherwise

$z_{lijs} \geq 0$ Takes on 1, if a changeover from product i to product j takes place on line l at the beginning of micro period s ; 0 otherwise

Objective function

$$\text{Min } \sum_j h_j I_{jt} + \sum_{l,i,j,s} s_{lij} z_{lijs} + \sum_{l,j,s} c_{lj} x_{ljs} \quad (55)$$

The objective function minimizes inventory holding costs, sequence dependent set up costs and line specific production costs. If production costs c_{lj} are identical with all lines ($c_{lj} = c_j$) then total production costs $\sum_{j,t} c_j d_{jt}$ are irrelevant for optimization and can be ignored.

Subject to

$$I_{jt} = I_{j,t-1} + \sum_{l,s \in S_{lt}} x_{ljs} - d_{jt} \quad \forall t, j \quad (56)$$

The inventory balancing constraint together with $I_{jt} \geq 0$ ensures that demand is met without backlogging. The parallel production lines are linked together only with this equation.

$$\sum_{j,s \in S_{lt}} a_{lj} x_{ljs} \leq K_{lt} - \sum_{i,j,s \in S_{lt}} s_{tij} z_{tjis} \quad \forall l, t \quad (57)$$

Capacity is reduced by set up times.

$$x_{ljs} \leq \frac{K_{lt}}{a_{lj}} y_{ljs} \quad \forall l, j, s \quad (58)$$

Because of constraint (58) together with constraint (60), production can only take place if the line is set up for the right product which should be produced. Moreover it is secured that only one set up state is defined per line and micro period.

$$x_{ljs} \geq m_{lj} (y_{ljs} - y_{lj,s-1}) \quad \forall l, j, s \quad (59)$$

Minimum lot sizes are introduced in order to avoid set up changes without product changes.

$$\sum_j y_{ljs} = 1 \quad \forall l, s \quad (60)$$

$$z_{ljs} \geq y_{li,s-1} + y_{ljs} - 1 \quad \forall l, i, j, s \quad (61)$$

This constraint connects set up state indicators and changeover indicators.

$$s_{lik} + s_{lkj} \geq s_{lij} \quad \forall i, j, k = 1, \dots, J \quad (62)$$

The triangle inequality, e.g. a certain product sequence i, j requires cleaning at the changeover to avoid contamination. A changeover has to be started and finished within the same macro period. [10]

8.3 Lot Sizing and Scheduling for Flexible Flow Lines

Flexible flow lines can be seen as a special parallel machine case. A flexible flow line (FFL) is a flow shop with parallel machines or a multiprocessor flow shop. A FFL is a flow line with several parallel machines on some or all production stages. Multiple products follow the same linear path through the system. A production unit cannot be produced by two machines at the same time. This means that at each stage throughout the system one of the parallel machines has to be chosen for producing the item. All machines of a stage are identical and can produce all products with the same processing time. But processing times can vary for different products and production stages.

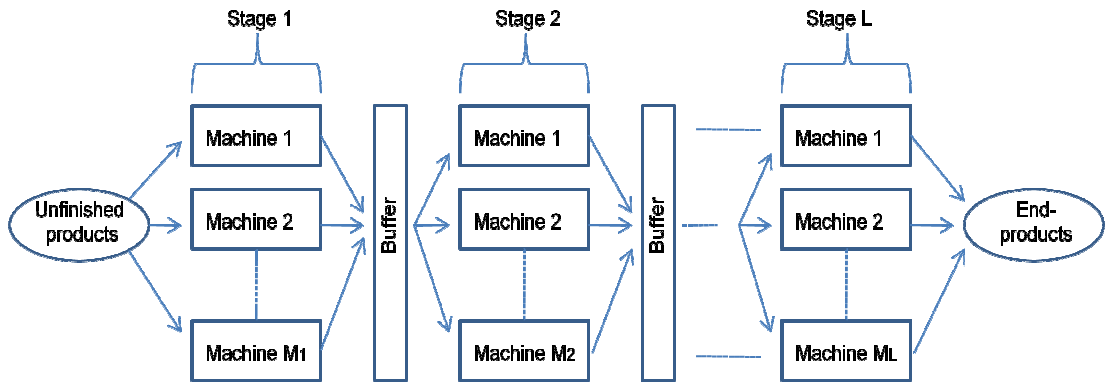


Figure 8: Schematic view of a flexible flow line [2]

Figure (8) shows a schematic view of a FFL with L production stages and M_l machines on stage l . Between stages buffers are located, so that intermediate products can be stored. FFL are very often found in the process industry like automotive, chemical, cosmetic, electronic, food, paper, printing, textile and wood processing industry.

The problem focuses on production quantities, machine assignments and product sequences. Products are aggregated to product families, to similar products and to product variants. Products of a product family have the same processing time on any machines. Set up within a product family is also called intra family set up. Intra family set up costs are lower than set up costs between families, because products within a family are very similar. The demand volume of a product can be divided into several product units. It has to be determined on which machine each product unit should be produced and how many machines should be set up for each product. A Job consists of several operations, one at each production stage. The buffer between stages is limited. Transportation times between stages are negligible. As a simplification of the lot sizing and planning problem, machines can be assumed to never break down. [2]

D SOLUTION APPROACHES FOR LOT SIZING AND SCHEDULING

Most lot sizing problems are hard to solve and various solution techniques have been used to solve them. The lot sizing and scheduling problems described in chapter C can be solved exactly or heuristically. The choice of an appropriate solution technique depends on the available software, on a cost benefit point of view and on the complexity of the problem. Further, one has to take care of the robustness and on the quality of solutions which are obtained by the different methods. [13] First, different solution techniques for lot sizing models are introduced. Afterwards the computational complexity of lot sizing models and algorithms which may give some good solutions to several single level and multi level problems are discussed. In chapter 4 solution and scheduling procedures for the parallel machine case and for a flexible flow line are presented. Chapter 5 describes some solution and sequencing procedures for different machine configurations with SDS.

1 Solution Techniques

1.1 Exact Solution Methods

This kind of solution technique is able to solve a problem to the optimum. [14]

1.1.1 Linear Programming

The advantage of formulating a problem as a linear program (LP) is that optimal solutions can be found efficiently even for larger problems. These problems can be solved on computers by using the Simplex method. The Simplex method is an algorithm that moves sequentially from one extreme point to the other until it reaches the optimum [19]

1.1.2 Dynamic Programming

Dynamic programming (DP) can be used to solve optimization problems with a computer program. The main problem is split into subproblems and these small problems are also tried to be solved to the optimum. DP algorithms are developed for single item problems and cannot be directly extended to multi item problems. [17]

1.1.3 “Branch & Bound”

Branch and Bound (B&B) is a technique that computes the lower and upper bounds of the objective value by using a heuristic. It splits the problem into subproblems and solves them by relaxing some constraints by using for example Lagrange Relaxation. [14]

1.2 Heuristics

Heuristics are reasonable methods to solve a lot sizing or scheduling model but they do not necessarily give the optimal solution. [19] Heuristics are often used to solve the simple ELSP or they are used as lot sizing rules in MRP systems. Lots of heuristics are based on an average cost criterion. With the help of this criterion one decides if future demand is produced in the actual period or if a new lot is set up in another period. Heuristics usually provide a good starting solution for optimal algorithm or they are necessary if optimal algorithms fail to solve very large or complex problems. For standard problems traditional heuristics are even better than meta heuristics. [17] Heuristics are mostly built on three steps. [14]

1.2.1 Opening Procedure

The opening procedure tries to find a first good solution of the problem. There are many heuristics which can be used to find a starting solution, some of them are listed and explained below.

- *ABC Heuristic*

The ABC heuristic uses several cost criteria to determine whether or not to include a next period. It is based on several rules for determining the order and the importance of the items. [17]

- *Dixon Silver Heuristic*

The Dixon and Silver heuristic selects that item for which a one period increase in the supply, results in the largest decrease in average cost per unit time or by unit of capacity absorbed. [17]

- *Silver Meal Procedure*

This procedure is based on the assumption that average costs per time unit can be minimized if an optimal lot size is defined. [5]

- *Least Unit Cost*

The production amount of a period is raised by future demand as long as the average costs per unit are reduced. This procedure is done at each iteration step until a result is found which cannot be minimized anymore. [5]

- *Part Period*

The demand of all periods is put together to one lot until set up costs are near the same like inventory costs. This procedure is done stepwise at each iteration level until the optimization criteria is reached. [5]

- *Groff*

This procedure is based on a decision rule. The lot size of a period is increased with the demand of future periods until the increase of average inventory costs exceeds the decrease of average set up costs per period. [5]

1.2.2 Optimization Method

If a first valid solution is found, one can try to improve the first solution by implementing an optimization method like exchanging or elimination of lots or by implementing a neighbourhood or local search strategy.

1.2.3 Meta Heuristics

Meta heuristics can be seen as local search methods or optimization procedures. They have become more and more popular to solve complex combinatorial problems. These heuristics are very flexible and can handle large problems. They are mostly used for extensions of the standard lot sizing problem. They have already been applied to a wide variety of lot sizing models. Common meta heuristics are for example Simulated Annealing (SA), Tabu Search (TS), these two methods are based on neighbourhood definition, Genetic Algorithm (GA) uses genetic parameters to explore the solution space and Treshold Accepting (TA). [17]

1.3 Other Solution Methods

1.3.1 Hybrid Algorithms

These combinations of different methods try to integrate elements from several solution approaches into a more powerful algorithm. [17]

1.3.2 Decomposition and Lagrange Relaxation

These are two techniques to find improved lower and upper bounds for the objective value. The basic idea is to divide the problem into smaller subproblems which are easier to solve and ensure to obtain a good approximation of the overall problem. The subproblems that result from these two approaches are equal. Most decomposition schemes relax the capacity constraint. A disadvantage of both techniques is that they have to be extended with heuristics or with a B&B algorithm to obtain a feasible solution. For multi level problems relaxation of the capacity constraint results in an uncapacitated single level, multi item problem. Lagrange Relaxation is only

for a limited number of iterations and there is no guarantee that the optimal bound can be found. [17]

1.3.3. Shortest Path and Warehouse Location Problem

Lot sizing problems can be reformulated as a network or shortest path problem where the arcs define costs (the shortest path within a network has to be found with the help of several algorithms) or as a warehouse location problem which can be solved with the add and drop algorithm. This formulation can be used for the single item uncapacitated lot sizing problem. Both the simple plant location and the network formulation can be extended to the multi level case. [12, 17]

2 Solution Approaches for the Single Level Structure

The solution of single item problems is very important because they appear as core structures in more complex problems. [17]

2.1 The ELSP

This lot sizing model is known to be NP hard. A restricted version of the problem can be solved optimally by a DP algorithm and heuristics give an acceptable solution for the original problem. Finding an optimal solution is, however, time consuming. [21] The ELSP can be formulated as a network problem. [17]

2.2 The CLSP

Solving the CLSP optimally is NP hard, meaning that there is no algorithm solving the problem exactly with reasonable computational and time effort. If the model considers set up times the feasibility problem turns out to be NP complete. [1] In case of 0 set up costs, or constant set up costs and equal capacity in each period the problem can be solved efficiently by a DP

algorithm. [13] Many heuristics including an opening and optimization procedure have been developed but there are only a few approaches which solve the CLSP optimally for small problems. [1, 4] A basic solution approach for the CLSP Model, including set up times, can be reached by implementing the B&B algorithm. Using the Lagrange Relaxation upper and lower bounds can be calculated for the objective value. Neglecting capacity constraints the model can be solved optimally and with reasonable time effort. [4] Due to the fact that the CLSP does not take scheduling decisions into account, the usual approach solves the CLSP first and afterwards scheduling is done for each period separately step by step. [1]

2.3 The CSLP

Although the CSLP is NP hard relatively large problem instances have been solved to the optimum within reasonable amount of time. An exact algorithm to solve this problem is based on relaxation of capacity constraints in combination with DP. An optimal production schedule for multiple items is obtained by a heuristic combining capacity constraint relaxation and subgradient optimization. A meta strategy can also be applied. An extension of the CSLP with sequence dependent set up costs cannot be solved. [13]

2.4 The DLSP

Regarding the DLSP an optimal solution is obtained in polynomial time, therefore the DLSP is NP hard. Considering set up times or parallel machines the problem turns out to be NP complete. [13] The all or nothing assumption makes efficient implementations of mathematical programming approaches possible. [1] The DLSP can also be solved by Lagrange Relaxation and decomposition. The single and multi item DLSP can be reformulated as shortest path problems. Researches on meta strategies to solve the standard DLSP are scarce. [17]

2.5 The GLSP

The GLSP Model focuses on a single production line. First a basic set up pattern is determined by a local search procedure. Secondly neighbourhood operation which include changes in the set up pattern e.g. deletion of a lot of the current solution or an exchange of two lots, can improve the first solution. A candidate for a new solution is accepted if the costs of the current solution can be minimized. There are two local search procedures, TA and SA, which can be implemented. [10]

3 Solution Approaches for the Multi Level Structure

Solution procedures dealing with multi level structures are scarce. Most researches are based on quite general assumptions, e.g. no capacity restrictions. There have been attempts to improve heuristics where methods for the single level problem are applied level by level to get a feasible plan. Some sparse literature tries to integrate hierarchically some lot sizing and some scheduling procedures. [1]

3.1 Uncapacitated MLLP

Some heuristic approaches solve the MLLP approximately, however, ignoring capacity constraints. Meta heuristics can be implemented to solve a MLLP without capacities for example the multi level, multiple machine PLSP can be solved with a GA. Another approach to solve a general case of the MLLP uses the search technique SA, computational requirements, however, grow rapidly when the problem size increases. [17]

3.2 Capacitated MLLP

Finding a feasible solution for a multi level problem including capacity restrictions is NP hard. That is why it is not possible to solve this problem to the optimum. Most heuristics for solving the MLLP with a general product structure are based on decomposition in combination with cost adaption procedures. MLLPs with a general product structure are heuristically harder to handle than MLLPs with demand for end items only. [5, 13]

3.2.1 Product Based Decomposition

Product based decomposition makes use of the Lagrange Relaxation and decomposes the capacitated multi level problem into several uncapacitated single item lot sizing problems. Afterwards the subproblems are reunited to an adequate end solution for the basic problem. Decomposition starts by planning the end item using a single product heuristic, like the Silver Meal procedure or the WW procedure. The single item problems can also be solved efficiently by using a shortest path algorithm. The second step plans the demand of the next item according to the disposition structure. This demand is based on demand decisions concerning the end item. Lots are determined depending on the disposition levels. [5, 11, 12, 13]

3.2.2 Time Oriented Decomposition

Using a time oriented decomposition, the subproblems are solved step by step over the whole planning horizon. Connections between all items on several disposition levels can be taken into account. Another solution procedure takes all products simultaneously into account and extends the planning horizon stepwise. This heuristic leads to a near optimal solution. [11]

3.2.3 Other Solution Approaches

Different heuristics try to solve the capacitated MLLP. Most of them are based on the Silver Meal criterion. This criterion can also be extended to set up times. Another heuristic starts from a lot for lot schedule and smoothes the capacity profile using marginal cost savings as a priority index. A starting point can also be a WW schedule, minimizing costs can be achieved by shifting production. DP based on heuristic gives good solutions, either. Also meta heuristics can be implemented to solve the capacitated MLLP with or without set up times. [17] SA can be used to optimize the capacitated MLLP. Therefore the SA algorithm used for the uncapacitated case is extended. The use of TS can be an alternative to the use of SA. Solutions from SA are slightly better than from TS, both methods, however, achieve reasonably good quality solutions. Concerning the MLLP with capacity constraints and only a single bottleneck machine a B&B algorithm using Lagrange to generate lower bounds was established. Computation time, however, grows rapidly with the size of the problem. [13]

4 Solution Approaches for the Parallel Machine Structure

Two small time bucket models for non-identical production lines have already been solved successfully by Lagrange Relaxation. Only one product can be produced per micro period. In the first model the all or nothing assumption is valid and set up times are formulated as a production loss during set up periods. The second model relaxes the all or nothing assumption and set up times are disregarded. The BSP also can be extended to identical production lines but results are not satisfying. [10] Extending the CLSP for a parallel machine structure can be solved by using an appropriate meta strategy. [17] The DLSP seems to be NP complete for the parallel machine case, a feasible schedule, however, for the single item problem can be obtained in polynomial time, by scheduling demand period by period at the fastest available machine not yet used. This problem can also be reformulated as a transportation problem. [13]

A parallel production line problem with sequence dependent set up costs but no set up times can be solved by a sequence splitting model. The entire schedule is split into a predefined number of subsequences, decomposing the overall planning problem into subproblems. Column generation and B&B are the basic elements of the solution heuristic. These heuristics for parallel production lines, however, cover a rather inflexible time structure (small time buckets) or ignore sequence dependent set up times. So there is still a need for new, powerful and more general solution procedures. [10]

4.1 The GLSPPL

A heuristical solution approach combining the local search metastrategies, TA and SA with dual reoptimization, has already been successful in the single machine case. These GLSP solution procedures have to be thought over for several production lines and the local search metastrategies have to be modified. Two solution procedures for the GLSPPL, both combining again TA and SA and dual reoptimization have been developed.

In the GLSPPL model demand can be satisfied by more than one production line, more than one line can produce the same item. The set up patterns of the different lines, however, can be fixed separately. The set up sequence for a production line is drawn at random from a uniform distribution. A candidate is refused anyway if its set up pattern does not ensure for any production line or macro period, that net capacity (set up times are already subtracted) is high enough to satisfy the minimum lot sizes for a given set up pattern. The remaining problem for several identical production lines can be formulated as a Network Flow Problem. A network consists of a set of nodes and a set of arcs. Flows on each arc have to be determined so that total costs are minimized. An arc includes per unit costs. Capacity nodes and dummy nodes stand for the limited capacity of a production line. Demand nodes represent the demand of products in a macro period. Due to minimum lot sizes, ending inventory nodes are introduced to that model. A generalized Network Flow Algorithm is hard to solve but it is more efficient than a solution procedure for

linear programs. A relaxation algorithm can be implemented in order to solve the Network Flow Problem and to refuse unacceptable candidates at an early stage. This algorithm introduces a maximum number of iterations per candidate in order to minimize computational time. A candidate is cancelled regardless of its objective value if the number of maximum iterations is exceeded. [10]

4.2 Flexible Flow Lines

Unfortunately there are no solution approaches which combine lot sizing and scheduling decisions for FFL. The lot sizing and the scheduling problem for FFL has only been considered separately. Few studies concentrate on standalone lot sizing problems, but there is more literature which emphasizes on standalone FFL scheduling problems. Most studies with emphasize on scheduling do not pay attention to set up times between jobs. Assuming that all jobs belong to different products, set up times are sequence independent and can be added to process times.

There are two studies considering a standalone lot sizing problem for FFL. The multi level job shop problem is extended by including alternative routings on parallel machines. This problem can be solved by using a Lagrange Relaxation procedure. A FFL lot sizing problem can also be solved heuristically by assuming that if a product is produced in one period, the complete production volume has to be assigned to a single machine. So lot splitting is not allowed. Both approaches consider set up times and determine the production volume per product and period. Each unit is assigned to a machine, but they do not take interdependencies between lot sizing and scheduling into account. [2]

Scheduling procedures for FFL can be differed according to their solution approach. Scheduling FFL is NP hard for optimality criteria like minimizing makespan even when set up times are negligible. So computation time is long and costly for medium and large size problems. Nevertheless a number

of optimal solution procedures mostly based on B&B algorithms for small instances have been developed. Some other researches on scheduling FF lines tend to solve this problem heuristically using methods like local search and meta heuristics or decomposition. [2] Sequencing jobs within a family can be done by a heuristic and sequencing families is done by B&B. [15] Algorithms that treat the loading and sequencing part sequentially even find an optimal solution.

Although standalone lot sizing procedures determine production volumes per product and period and assign a machine to each product unit, they do not sequence the products loaded on a machine. Whereas scheduling methods solve machine assignment and sequencing but they do not care about different demands per period and about calculating lot sizes. Now it is important to develop an approach which combines lot sizing and scheduling for FFL and integrates these interdependencies. [2]

An integrative solution approach has been developed. The objective is to find a schedule that minimizes all relevant costs and the mean flow time by splitting the problem hierarchically into three phases. First the bottleneck machine is planned, then the schedule is rolled out and at last products are assigned to slots. [2]

5 Solution Approaches for Sequence Dependent Set Up

Optimization methods including B&B, DP and MIP solvers or hybrids including heuristics are often used to solve single machine configurations with SDS. Sometimes they are also used to solve parallel machine and job shop configurations. Most formulations of SDS problems are MIP models. Runtimes are very excessive for these NP hard problems, even for instances of modest size. Heuristics are very common to solve SDS scheduling problems because of their difficulty but they do not guarantee a good performance for problems involving SDS. Meta heuristics have been widely used for all machine configurations mainly for objective functions including

makespan or due dates (maximum lateness or tardiness). In this case the performance of TS improves with the number of machines in a parallel machine configuration.

SDS is very closed to a Travelling Salesman Problem. Heuristics based on TSP algorithm have been developed for single machine, parallel machine and flow shop configurations. Most researches address to the objective of minimizing makespan. Research on due date related objectives is scarce. Other heuristics like decomposition, simulation and list scheduling approaches have also been implemented to these problems. B&B can be used to determine the next job to be processed or it can be applied each time a schedule decision has to be made. [15]

5.1 Single Machine Configuration

Prescribing a sequence for a single machine with SDS and minimizing makespan as the objective is NP hard. This problem is equivalent to the TSP, a node represents a job and each directed arc gives the travel time or distance between the nodes it connects. A number of different methods can be used to solve it near to the optimum. E.g. B&B, DP, MIP, hybrids and meta strategies. The problem of minimizing weighted completion time in combination with SDS can be solved by combining B&B with DP. By using B&B algorithm for minimizing maximum lateness, run times increase rapidly for problems with more than 15 operations. Minimizing total tardiness can be solved by heuristics and B&B. B&B may be preferable to solve smaller problems. Hybrid approaches give good solutions to some of the sequencing problems with SDS. [15]

5.1.1 CLSP with SDS

Regarding the CLSP with SDS, setups may be carried over from one period to the next and are preserved over idle periods. [15] This extension of the CLSP can be solved by using an appropriate meta strategy. [17] A MIP can be formulated considering only efficient sequences (the total set up cost of a sequence is less than of another sequence). Instances ranging from 3 products and 15 periods to 10 products and 3 periods can be solved optimally using B&B. [15]

5.1.2 DLSP with SDS

The DLSP with SDS can be formulated as a TSP. A procedure is devised to solve it. It determines lower bounds by using Lagrange Relaxation in combination with a heuristic. Problems up to ten products and 150 periods can be solved. An exact solution method applies a DP algorithm to the problem formulated as a TSP. Instances of moderate size can be optimized. [15]

5.1.3 GLSP with SDS

The GLSP Model with SDS times can be formulated as a MIP. It has been successfully solved by combining local search meta strategies with dual reoptimization of subproblems. The GLSP with SDS is NP complete. Finding a feasible initial solution to start the neighbourhood search is very difficult. Solving this model to optimality is a big problem because one runs out of computation times. Solving it heuristically is because of the solution quality not desirable. [10, 15]

5.2 Parallel Machine Configuration

Little research has been done on optimizing parallel machine configuration with SDS. Minimizing total earliness and tardiness for uniform machines operating at a different speed can be formulated as a MIP, which is able to solve small instances.

A decomposition method can be used to solve larger instances by separating the formulation into an Integer Master Problem which prescribes job assignments to machines and the sequence at each machine, and a LP subproblem which prescribes the exact completion time of each job.

A hybrid describes a heuristic for minimizing makespan with SDS, where each job can be split into segments that can be processed in parallel on different machines. First the problem is decomposed into independent single machine problems with SDS where makespan should be minimized. These single problems are modelled as TSPs and solved by B&B algorithm. Afterwards an improvement method is employed step by step taking set up times and job splitting into account.

Several other heuristics have been employed on a parallel machine configuration with the aim of minimizing makespan or total weighted completion time where machines are in parallel but unrelated and each job has a unique processing time. Other heuristics are able to solve the problem with the objective of minimizing weighted tardiness. One heuristic for example performs well in general but results get worse if the number of jobs increases and results improve if the number of machines increases. List Scheduling Algorithms are heuristics used to prescribe schedules. A schedule is constructed by assigning each job in listed order to the first machine that becomes idle or to the machine that can finish it at the earliest time. This algorithm can be used to minimize makespan and maximum lateness. [15]

5.3 Flow Shop Configuration

There are various possibilities to sequence the jobs to machines in a flow shop e.g. permutation of jobs or prescribing different sequences at each machine. There are two points at which decisions must be made. The first decision is entry point sequencing, determining the permutation of jobs at the first work center. Secondly dispatching determines which available job should be assigned to a machine as it becomes available. These decisions are addressed independently using a first in, first out dispatching. The objective is to minimize the average time that jobs spend in the system.

Flow shop configurations with SDS are mostly NP hard and permutation schedule does not automatically minimize makespan. A good solution can also be reached by implementing preference relations and then implying an improvement scheme to reduce total weighted completion time and total weighted tardiness. Random search and local search strategies can also be implemented to find good sequences. [15]

5.4 Job Shop Configuration

The job shop scheduling problem is even without SDS NP hard. Limited research has been done on this problem including SDS. B&B can only solve small instances because run time increases rapidly with problem size. A hybrid including a MIP and Lagrange Relaxation tried to solve the problem to obtain a near optimal solution. Dispatching rules and simulation models have been used to schedule job shops with SDS.

SDS scheduling is definitely a challenging topic for future research. Although lots of research has been done on this topic. A number of real world instances should receive more attention. [15]

E CASE STUDY OF SEMI-CONDUCTOR INDUSTRY

1 Introduction

This part of the paper deals with a simplified “Lot Sizing and Scheduling Problem”. Based on the problem described in chapter 2 an alternative approach was developed. The assumptions and characteristics of the MIP model will be presented in chapter 3. Chapter 3 also includes a comparison of standard lot sizing problems and the new model approach and tries to stress their common characteristics and differences. Afterwards the implementation of this optimization problem is shown in chapter 4. The objective function and the constraints are translated into a so called “MOSEL Language” and are tried to be optimized with a program named “XPRESS MP”. The main part of this chapter, however, is to prepare some realistic data to show how the MIP works and if it is possible to get some feasible solutions by using the optimization program. Therefore data from the semi-conductor industry was modified and the essential information needed for the model was filtered out. The interpretation of the solution values and the positive and negative aspects of the optimization problem are given in chapter 5. Finally sections 6, 7 and 8 contain a summary of the results, a conclusion and acknowledgement. The main objective of the approach is to show that the scheduling of jobs can be influenced by different cost structures and that time and money can be saved by implementing a good strategy for scheduling jobs to machines.

2 Problem Statement – Bottleneck planning for a complex workshop

In a workshop products are manufactured step by step on different workstations (machines). The sequence of the machines is not fixed. Each product has its own manufacturing plan, which includes different operations on the machines. Each operation can be executed on specific machines. A product can be processed on the same machine several times. Further, the product can consist of several subassemblies, which are manufactured separately on the basis of their own production plan. Moreover some operations need certain component parts (resources) which can vary dependent on the product type or they can be the same for each product.

The main goal is to recognize scarce resources or bottlenecks (number of component parts or machines) in time to ensure that due dates are met. There are different possibilities to prevent a bottleneck situation.

- *Supplement of resources*: The acquisition of additional machines or workforce can be a measurement against scarce resources.
- *Outsourcing*: Certain jobs or operations can be done by other workshops or facilities.
- *Smoothing of capacities*: By interchanging one job for another a bottleneck can be avoided easily. Therefore a precise analysis of the production schedule is essential.

This general problem statement can lead to different bottleneck situations. Most bottlenecks are, however, very specific and can be solved easily by recognizing missing component parts or by exchanging jobs. The complexity of this problem situation can be further minimized by giving up some restrictions or constraints. [18]

3 MIP Approach

3.1 Simplified Assumptions

The problem situation described in chapter 2 can be formulated as an alternative MIP approach by relaxing some assumptions. Formulating this problem with the help of a standard lot sizing model is difficult due to the computational time effort. No satisfying and feasible solutions can be reached. The main goal of the simplified approach is due date fulfillment of jobs. For each job a due date and a latest date concerning completion time are assumed. Moreover each job has a certain release date on which production can start. Each job has a fixed processing time independent of the machine on which it is manufactured. No inventory but backorder costs are taken into account. The time horizon is divided into periods which are measured as days or shifts. Another simplification of the model is the absence of component parts. If component parts are taken into account, the set up of machines has to be defined, either. The definition of set ups would lead to more binary variables. Consequently computational time effort to optimize the problem would increase again. Job splitting is not allowed. If the production of a job starts on a certain machine it has to be finished on the same machine. So a job is produced by only one machine.

3.2 Common Aspects with Standard Lot Sizing Problems

The basic problem described in chapter 2 can be seen as a capacitated multi level, multi item lot sizing problem. Multiple products can be produced on several machines. The production planning is not only done for the end product but also for the component parts. Machine capacity and component parts are scarce resources. Moreover set up times or set up costs have to be considered because some items can be produced on several machines more than one time and they do not have to be produced in the same sequence.

Whereas the simplified MIP approach in chapter 3 is a capacitated single level, multi item lot sizing problem. The given jobs are not dependent on any subassemblies. A set of machines is given on which the products can be produced. So a parallel machine case with limited machine capacity can be considered. The set up state of a machine can be carried over from one period to the next period. Set up times or costs are ignored. This alternative approach can neither be classified as a small bucket nor as a big bucket model. A period represents a time slot of mostly one day or shifts. Regarding the data which is going to be used to solve this approach one can observe that the processing time of jobs is often longer than one period. These are characteristics which speak for a small bucket structure. It is, however, possible that more than one job is produced by one machine in one time period like in a big bucket model. Set up is not taken into account what makes it again difficult to make a specific assignment.

Based on the facts mentioned above the alternative approach has lots of characteristics in common with the standard lot sizing problem CLSP. The CLSP is a large bucket problem where several items can be produced per period in one time period. Multi items and capacities are taken into account. Whereas the DSLP, a small bucket problem uses the “all or nothing” assumption which is not true for the alternative approach. The production of several items in one period is possible and it does not matter if it is produced for 100% or just a part of it. So the CSLP would be a better choice because it is more realistic than the DLSP. It gives up the “all or nothing” assumption but still only one item can be produced per period. The new model can hardly be compared to the PLSP because this problem considers set up states. The PLSP allows the production of two items in one period to use the full capacity of one period. The new approach does not have any restrictions according to the number of jobs which are allowed to be produced in one period or to which percentage they are produced. The only restriction is the machine capacity. In the continuous time lot sizing and scheduling problem the processing time of a job does not depend on the machine like in the new approach. Jobs are not allowed to be split which means in contrast to the alternative approach demand has to be produced in one piece. In the new

model the processing of an item can be interrupted. Job splitting is also not allowed but it is defined in another way. A job must be produced by only one machine.

3.3 Model Description

Table (11)

Parameters:

i	Job
k	Machine
t	Period
R_i	Release date of job i
F_i	Due date of job i
L_i	Latest date of job i
p_i	Process time of job i
S_i	Set of machines on which job i can be produced
c_{it}	Cost to produce job i in period t

Table (12)

Decision variables:

x_{ikt}	Time (in period) to produce job i on machine k in period t
y_{ik}	Binary variable

Objective function

$$\text{Min } \sum_i \sum_k \sum_t c_{it} x_{ikt} \quad (63)$$

Subject to

$$\sum_k y_{ik} = 1 \quad \forall i \quad (64)$$

$$\sum_k \sum_t x_{ikt} = p_i \quad \forall i \quad (65)$$

$$x_{ikt} \leq y_{ik} \quad \forall i, k \quad (66)$$

$$\sum_i x_{ikt} \leq 1 \quad \forall k, t \quad (67)$$

$$x_{ikt} = 0 \quad \forall i, k, t < R_i \quad (68)$$

$$y_{ik} = 0 \quad \forall i, k \notin S_i \quad (69)$$

$$0 \leq x_{ikt} \leq 1 \quad (70)$$

$$y_{ik} \in \{0,1\} \quad (71)$$

The objective function (63) minimizes the overall costs of production. The costs c_{it} of producing a job are assumed to be a fictive value. These values are summed up for each job and for each period in which the job is produced on a machine. Constraint (64) ensures that each job is assigned to only one machine. The total processing time of a job must equal the sum of processing times of the job in each period it is manufactured (65). Constraint (66) says that the production of a job can only take place if the machine is set up for that job. Constraint (67) is the capacity constraint. Each machine has a capacity of 1. The sum of processing times of all jobs which are produced on one machine during one period has to be smaller or equal to 1. Constraint (68) makes sure that the processing of a job cannot start until the job is released to the workshop. The binary condition (69) is 0 if a machine does not belong to the set of machines which are capable to produce a job. It is 1 if the machine which is chosen to produce a job belongs to this set of machines. Constraint (70) makes sure that the processing time of a job in a period has to be greater than 0 and smaller or equal to 1 if it is produced on a machine during a period. The processing time of a job is equal to 0 if it is not produced in a certain period. Constraint (71) defines the binary variable. The binary variable is 0 if a job is not produced on a machine, whereas it is 1 if a job is assigned to a machine.

The model's main objective is to calculate a minimal objective value that comprises an optimal assignment of jobs to machines to meet due dates and to save costs.

For each job i , c_{it} has to be at its minimum in period $F_i - \frac{p_i}{2}$. The value of c_{it} is dependent on the time distance to this minimum period. The higher the distance of the production period to the minimum period the higher the value of c_{it} . The value of c_{it} should reach its maximum for periods beyond F_i and L_i . These values can also be seen as penalty costs. If production and/ or completion of a job are far from its given due date, the value of c_{it} has to be

much higher than as if jobs are produced and completed in time. These penalty costs should control production processes to get a feasible schedule which encourages the production and completion of jobs around a certain time range. Taking these considerations into account, three variants of the alternative approach have been developed. Each variant is based on another strategy to come up with a value for c_{it} .

3.4 Variants of the Objective Value

Three variants to prescribe and calculate the objective value have been determined. Besides the formulation of c_{it} , the MIP does not change at all.

3.4.1 Variant 1 (V1)

$$c_{it} = \begin{cases} \varepsilon_1 & t \leq F_i \\ \varepsilon_2 & F_i > t \leq L_i \\ C & t > L_i \end{cases} \quad (72a)$$

ε_1 and ε_2 should be fixed at a low value, the value of ε_2 should be a little higher than the value of ε_1 . Factor C should be of a very high value in contrast to ε_1 and ε_2 . The values were distributed like this to achieve a completion time not later than a given due date or still within a given latest date. Completing a job after its latest date leads to very high penalty costs.

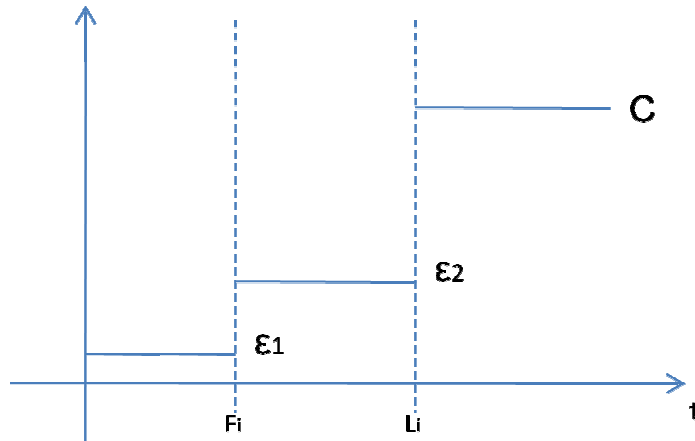


Figure 9: Development of the objective value by using V1

Figure (9) shows the development of the objective value c_{it} dependent on the production period by using V1. The objective value is minimized if each job is produced not later than F_i . Using this variant the value of c_{it} does not change for a period before F_i . So the interdependency of the production period and the cost value is not always obvious. If a job is produced not later than F_i , c_{it}

is fixed with a value of ε_1 . Producing a job between F_i and L_i fixes c_{it} by ε_2 . If a job is produced later than ε_2 the objective value is C .

3.4.2 Variant 2 (V2)

$$c_{it} = \begin{cases} \varepsilon_1 \times (F_i - p_i - t) & t \leq (F_i - p_i) \\ 0 & (F_i - p_i) \leq t \leq F_i \\ \varepsilon_2 \times (t - F_i) & F_i < t \leq L_i \\ C & t > L_i \end{cases} \quad (72b)$$

ε_1 and ε_2 should be fixed at a low value again, the value of ε_2 should be little higher than the value of ε_1 . Factor C should be of a very high value in contrast to ε_1 and ε_2 .

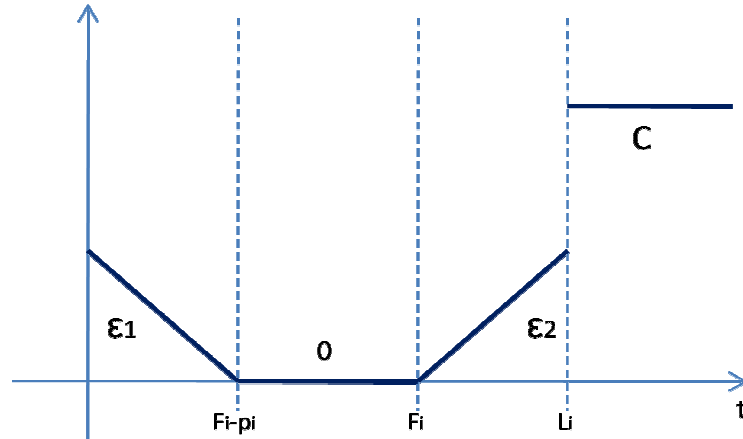


Figure 10: Development of the objective value by using V2

Figure (10) shows the interdependency of the production period and the objective value by using V2. This linear cost curve includes the possibility of reaching an objective value of 0. No production costs arise if the processing of a job starts in period $F_i - p_i$ and completes not later than its due date. Before period $F_i - p_i$ the value of c_{it} decreases linear by the factor ε_1 . Past period F_i the value of c_{it} increases again linear by ε_2 .

3.4.3 Variant 3 (V3)

$$c_{it} = \begin{cases} \varepsilon_1 \left(t - F_i + \frac{p_i}{2}\right)^2 & t \leq F_i \\ \varepsilon_2 \left(t - F_i + \frac{p_i}{2}\right)^2 & F_i < t \leq L_i \\ C & t > L_i \end{cases} \quad (72c)$$

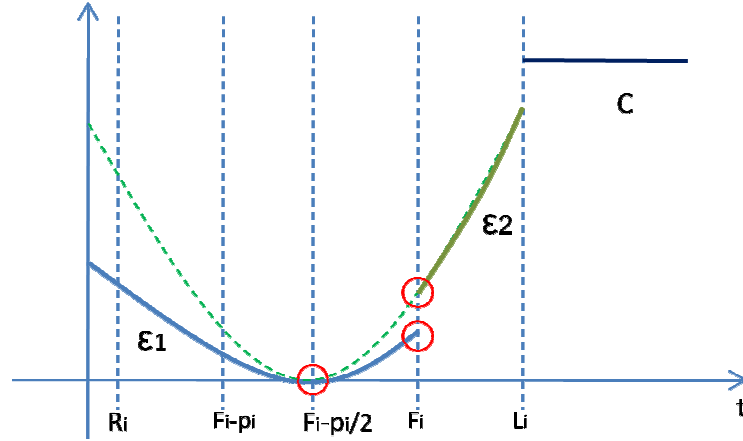


Figure 11: Development of the objective value by using V3

Figure (11) shows the interdependency of the production period and the objective value by using V3. Regarding this curve a minimum of the objective value is reached if jobs are produced around period $F_i - \frac{p_i}{2}$ (minimum). Reaching a value of 0 is hardly possible. For that, processing of a job has to start at period $F_i - \frac{p_i}{2}$ and complete in the same period. Consequently a minimal value of c_{it} can be reached by producing and completing a job between period F_i and $F_i - p_i$. If a job is produced in time, costs increase quadratic by factor ε_1 with the distance to the minimum. If a job is produced past the desired due date but still before the given latest date, costs increase again quadratic by a higher factor ε_2 with the distance to the minimum period. If the production period of a job is higher than its given latest date, c_{it} is fixed by C .

4 Implementation with „XPRESS MP“

Based on the problem statement described in chapter 2 and on the simplified assumptions in chapter 3 a MIP approach has been developed. This alternative approach was implemented with an optimization program called “XPRESS MP”. Therefore the parameters, decision variables, the objective function and its constraints had to be adapted to the program and had to be rewritten in “MOSEL Language”.

The aim of the implementation is to optimize the real world data. By formulating the model for “XPRESS MP” some additional commands had to be considered. Due to these commands solutions and the corresponding data can be presented in a more informative way after the optimization procedure. Additional information about the schedule, the processing of jobs and the time structure can be gained.

4.1 Data Reference

It is not sure if the MIP approach and the assumptions described in chapter 3 are able to solve real world problems. So the implementation of the model should be tested with realistic data. Job recordings of the semi-conductor industry are listed in an Access data base. The chair of “Production and Operations Management” has this data base at its disposal. The data base includes lots of information about different jobs to be scheduled, produced and delivered over several years. This documentation includes, for example, detailed information about the structure of subassemblies and about the workshop in which they are produced. It also includes information about machine configurations. Due to the size of information just a limited number of jobs were taken into account. The data which is essential to start the optimization was filtered out.

- job number ... i
- estimated arrival ... release date R_i
- target date ... due date F_i

- latest date ... L_i
- process time ... p_i
- tester ID ... machine number k

Additional reduction of the data was reached by taking a time horizon of only three weeks into account. The number of jobs released during three independent weeks was considered. Without a reduction of the data volume “XPRESS MP” would not be capable to optimize the problem within a reasonable computational time effort. No feasible solution can be calculated.

4.2 Preparation of Input Data

After having filtered out the essential data of the Access data base the information has to be prepared with the help of Excel. Otherwise “XPRESS MP” would not be able to load the data for the optimization process. An Excel sheet with the input data needed was configured. One table includes all numbers of jobs which should be processed, together with its release dates, target dates, latest dates and processing times. A second table includes a matrix with the set of machines.

i	Ri	Fi	Li	pi
1	3	4	4	0.894
2	3	3	3	0.339
3	3	3	3	0.323
4	3	3	3	0.260
5	3	5	5	2.206
6	3	5	5	2.240
7	3	4	4	1.596
8	3	4	4	0.996
9	3	5	5	2.023
10	17	19	19	2.385
11	5	7	7	2.426
12	3	5	5	2.467
13	3	9	9	6.187
14	3	4	4	1.240
15	3	3	3	0.224

Figure 12: Extract of the Excel sheet showing the input data for week 1

Figure (12) shows an extract (the first 15 jobs of week 1) of the Excel sheet with the data of an input file. The number of jobs is listed in the first column. The release date, target date, latest date and processing time is listed in the other columns. The dates and the processing time are given in periods.

job i	machine k																																																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48		
1		1	1	1					1		1	1	1	1	1					1	1	1					1		1					1						1										
2		1	1	1					1		1	1	1	1	1					1	1	1						1		1				1						1										
3		1									1					1																																		
4			1	1							1	1			1																										1									
5		1									1					1		1	1																												1			
6		1									1					1		1	1																												1			
7	1		1	1							1													1	1																	1								
8	1		1	1							1													1	1																	1								
9	1		1	1							1													1	1																			1						
10																																		1			1				1	1		1	1	1	1	1		1
11																																		1			1			1	1		1	1	1	1	1		1	
12																																		1			1			1	1		1	1	1	1	1		1	
13		1									1					1																																		
14				1							1																																							
15				1							1																																							

Figure 13: Extract of the Excel sheet showing the input data for week 1

Figure (13) shows an extract (the first 15 jobs of week 1) of the matrix that prescribes information about the set of machines on which a job can be produced. The first column includes the number of jobs and the first row includes all numbers of machines which are available to produce the jobs. For example job number 1 can be produced on machine 2, 3, 4, 9, 11 - 15, 20 – 22, 28, 30, 34 and 41. Each week has its own independent input file including the data and the matrix described in figure (12) and (13). These input files are read and optimized separately by “XPRESS MP”.

Each week was transformed into periods. One time period corresponds to a shift. On the first day of the week the first shift starts at 6.00 a.m. 1 day includes 3 shifts and 1 shift corresponds to 8 hours. So one week can be divided into 21 periods. The release dates, target dates, latest dates and processing times of jobs had to be changed from the format given in the Access data base into shifts by using an Excel calculation.

As a consequence jobs with a target date greater than 21 periods were filtered out and not taken into account. The target date of all jobs should be within a time period of 21 shifts. If the latest date period of a job was greater than 21 periods the target period of a job was assumed to be also the latest period in which the job has to be completed. A further constraint of the MIP implementation was that all jobs which are released within one week have to be completed in the same week. It was not always possible to schedule the jobs to a given number of machines so that they can be completed within 21 periods. Consequently the time period of one week was extended to 24 periods to ensure that all jobs of one week can be produced within this given time range. The implementation of the MIP is based on a time horizon of 24 shifts per week to produce and complete a given number of jobs.

4.3 Preparation of Output Data

Jobs which are released within one week can be manufactured by a set of different machines. Each job however, is assigned to only one machine. The processing of a job can be distributed on several periods. The processing of a job can also be interrupted. Idle periods can occur among periods with positive production. Idle periods arise if the processing of a job on a machine is interrupted to produce another job on the same machine. More than one job can be produced on the same machine within one period. The capacity of a machine however, must not be exceeded.

The input data of each week is optimized and scheduled separately with the different variants for c_{it} by “XPRESS MP”. After having optimized the MIP approach successfully two different files with the output data are formulated. These output files include the following information and values of the model:

		machine k																																															
job i		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
1															1																																		
2															1																																		
3																	1																																
4																1																																	
5																			1																														
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12																																																1	
13																	1																																
14				1																																													
15												1																																					

Figure 14: Extract - Output file „setup V1” for week 1

Which job is produced on which machine after having optimized the model by using V1 is written down in the output file „setup V1“. The matrix of the output file defines which machine is set up for producing which job. This matrix is part of the production schedule. Column A of the matrix lists up all jobs which have to be produced in week 1. Row 1 lists up the number of all machines. The grey cell with the number 1 signalizes that a machine is set up for a certain job. Figure (14) shows an extract of the output file „setup V1“, including the first 15 jobs of week 1. For example job number 1 and 2 are produced on machine number 14. Machine 1 is set up to produce job 8 and 9. The solution of the objective function, the minimized objective value, is written down separately in the output file.

job i	period t																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1				100%																				
2			18.15%																	81.85%				
3													100%											
4																				100%				
5															45.34%		9.32%					45.34%		
6			44.64%		44.64%															10.73%				
7																			37.33%					62.67%
8				98.07%																				1.93%
9			49.43%	1.15%	49.43%																			
10																	16.14%	41.93%	41.93%					
11					41.23%	17.55%	41.23%																	
12			18.94%	40.53%	40.53%																			
13			6.01%	16.16%	16.16%	13.18%	16.16%	16.16%	16.16%															
14												11.25%					8.12%			80.63%				
15																						100%		

Figure 15: Extract - Output file „time V1” for week 1

Output file „time V1“ contains information about the time structure of production. With the help of a matrix one can see which jobs are produced in which periods and how their production is distributed to the 24 periods. The matrix shows how many percents of a job are produced in each period of positive production. Figure (15) shows an extract of the output file „time V1“. The shortcut of the matrix represents the time schedule of the first 15 jobs in week 1. For example job number 1 is produced for 100% in period 4, 18.15% of job 2 is produced in period 3 and 81.85% in period 20 and so on.

These output files are formulated for each week and for each variant. Roughly speaking there are 6 output files for each week, 3 different “setup” files and three different “time” files for V1, V2, V3. These output files are the basis for additional calculation and analysis. In an additional matrix one can see how many periods with positive production a job needs to be completed and how many idle periods occur from the beginning of a production till its completion. This matrix gives insight if a job is produced without interruption or with idle periods. A good job scheduling prefers less idle periods to get a more homogenous production of jobs. This analysis makes clear how many periods a job really needs to be produced. The number of these periods does not have to be the same as the processing time (in periods) of a job. As already mentioned it is possible that only little parts of a job are produced over several periods. By filtering out the first period of production and the last period of production one can find out if a job is completed in time or too late. Consequently the number of jobs which are in time or too late can be summed up. This is all together important information about the production schedule. The production schedules are however, very dependent on the used variant for calculating the objective value. All three output files of one week are therefore of a different character.

4.4 Test Example

This section presents simplified input data to show how the optimization of the MIP approach with “XPRESS MP” works. This short overview shows how the input data has to be prepared for “XPRESS MP” and which information the output data includes and how it is presented.

	k			
i	1	2	3	4
A	1	1		1
B		1		1
C	1	1	1	1

Table 13: Test input data for the optimization program „XPRESS MP“

i	R_i	F_i	L_i	p_i
A	1	2	4	2
B	1	3	4	3
C	1	2	3	2

Table 14: Test input data for the optimization program „XPRESS MP“

This simplified data includes 3 jobs (i) A, B, C and 3 machines (k) 1, 2 and 3. The jobs have to be produced and completed within a time horizon of 4 periods (t). Job A can be produced on a set of machines including machine 1, 2 and 4. Job B can be produced on machine 2 or 4 and job C’s set of machines includes all four machines. This information is illustrated in table (13). Table (14) gives information about the release date, target date, latest date and about the processing time of all jobs. Job A for example is released for production in period 1 (R_i) and should be finished in period 2 (F_i). Job A has a processing time of 2 periods (p_i) and it must not be finished later than period 4 (L_i).

Solution: 15.375

Setup:

i	k			
	1	2	3	4
A	1			
B				1
C			1	

Table 15: Test output data „setup“

Time:

i	t			
	1	2	3	4
A	0.50	0.50		
B	0.33	0.33	0.33	
C	0.50	0.50		

Table 16: Test output data „time“

This simplified data was optimized by using V3. Therefore ε_1 was fixed with a value of 0.5, ε_2 with a value of 1 and C with 100. Table (15) shows a minimal objective value of 15,375. The matrix includes the set up of machines. Job A is produced on machine 2, machine 4 is set up for job B and machine 3 is set up for job C. Table (16) shows another matrix presenting the time structure. 50% of job A are produced in period 1 and the other 50% are produced in period 3. One third of job B is produced in period 1, one third in period 2 and the last third in period 3. 50% of job C are produced in period 1 and 50% of job C are produced in period 2. Job A is completed in period 2 on time. The production of job B can also meet its due date. Job C is also on time and completed in period 2. All three jobs are produced without interruption, no idle periods, between the beginning of the production process till its end.

5 Results

The following chapter presents the results of optimizing the data of 3 different weeks by using 3 variants to calculate the objective value. By implementing V1, the values of ε_1 , ε_2 and C were fixed with 0, 3 und 100. In V2 c_{it} is calculated by using the values 0.1, 0.1 und 100. In V3 the values of ε_1 , ε_2 and C were fixed with 0.5, 1 and 100. As already mentioned, the time horizon to produce and complete the jobs released within one week was fixed by 24 periods.

5.1 Week 1

The input data of week 1 cannot exactly be calculated. The data volume is too big to get a feasible solution. The computational time effort is too high to optimize the MIP with the given number of jobs and machines. The optimization process was interrupted after a computational time of 1 hour. The difference of the gained solution to the optimal value is 1%. This means that the solution can be improved by 1% if the computational time is extended. 247 jobs have to be produced in week 1. 48 machines are available to produce these jobs. The first jobs, 192, are released in period 3. The last job is released in period 18. 1 job has the highest processing time of 15,46 periods.

5.1.1 Output V1

Optimizing the MIP by using V1 an objective value of 12.548,1 can be gained. 114 jobs are completed after the given latest date (L_i). 122 jobs finish production before or exactly at their target date, F_i . 11 jobs can be completed after their target date but still before or exactly at given latest date, which means that these jobs are still in time. Regarding the job scheduling one can observe that 93 jobs are produced with idle periods between the first and the last period of production. 2 jobs stand out because their production could be

finished within 2 periods but 20 idle periods occur between the first and the last (second) period with positive production.

		period t																								Fi	Li	
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24			
job i																												
1				4																						4	4	
2			3																		20						3	3
3													13														3	3
4																					20						3	3
5															15		17						22				5	5
6			3		5																20						5	5
7																				19					24	4	4	
8				4																					24	4	4	
9			3	4	5																						5	5
10																		17	18	19							19	19
11					5	6	7																				7	7
12			3	4	5																						5	5
13			3	4	5	6	7	8	9																		9	9
14												12					17				20						4	4
15																							22				3	3

Figure 16: Extract –Output file “time V1” for week 1

Figure (16) shows an extract of the first 15 jobs of the output file “time V1” in week 1. This matrix is an extension of the matrix “time V1”. One can filter out the first and the last period of production. The last two columns include the desired due date and the latest date of the jobs. If the last production period is written in green the job is completed in time. If the last production period is written in red the job is completed too late. The actual completion period can be compared to the periods in the last two columns to see if the jobs are finished in time or too late. Further the periods with positive production and idle periods can be filtered out. Job number 2, for example, is processed in period 3 and 20, but 20 idle periods occur between the first and the last. The processing of job 13 starts in period 3 and is completed in period 9 with 7 periods of positive production and no idle periods. Job 15 is produced to 100% in period 22.

5.1.2 Output V2

Using V2 to optimize the MIP approach an objective value of 12.470,1 can be realized. 113 jobs are completed too late after L_i . 123 jobs are completed before or exactly at their target date F_i . 11 jobs are in time. Their production is finished past F_i but before or exactly at their latest date L_i . 78 jobs are produced with idle periods during their processing time. One job has 19 idle periods although only 2 periods are necessary to produce this job.

5.1.3 Output V3

Using V3 to optimize the MIP an objective value of 18.957,2 can be realized. 73 jobs are finished too late. 152 jobs are completed not later than their due date and 22 jobs are still in time. 71 jobs are scheduled with idle periods during their processing time. 1 job has to be underlined because its production starts in period 3 and is finished in period 24 but 18 idle periods are in between.

5.2 Week 2

67 jobs have to be processed in week 2. 45 machines are available to produce these jobs. The first 20 jobs are released in period 1. The last job is released in period 17. 5 jobs have the highest processing time of 13,84 periods.

5.2.1 Output V1

Optimizing the model by using V1 an optimal value of 184,236 can be reached. 1 job is finished too late and 65 jobs are finished before or exactly at F_i . 1 job is finished between its target date but still before or exactly at its latest date. 17 jobs are produced with idle periods. 2 jobs have the highest number of idle periods during their processing time. 10 idle periods occur within 4 periods with positive production.

5.2.2 Output V2

Using V2 an objective value of 163,569 can be realized. 1 job is too late, 65 jobs are on time. 1 job is completed after F_i but not later than L_i . 1 job is scheduled with idle periods between 9 periods with positive production.

5.2.3 Output V3

Using V3 an objective value of 9.427,41 can be realized. 8 jobs are completed too late and 53 jobs are on time. 6 jobs are completed after F_i but not later than L_i . 6 jobs are scheduled with idle periods. 3 jobs have 3 idle periods between 17 periods with positive production.

5.3 Week 3

73 jobs have to be processed in week 3. 52 machines are available to produce these jobs. The first 9 jobs are released in period 1. The last job is released in period 18. 1 jobs have the highest processing time of 10,49 periods.

5.3.1 Output V1

Using V1 an objective value of 5,08019 can be realized. No job is completed too late. 69 jobs are completed before or exactly at their target date F_i , these jobs are all on time. 4 jobs are finished after F_i but before or exactly at L_i . 8 jobs are scheduled with idle periods. By using this variant it is positive that only 2 jobs have a maximum of 3 idle periods. Their processing however, needs 8 periods with positive production.

5.3.2 Output V2

Using V2 an objective value of 0,189 can be realized. No job is completed too late and 69 jobs are on time. 4 jobs are completed after F_i but not later than L_i . No job is scheduled with idle periods.

5.3.3 Output V3

Using V3 an objective value of 2.031,4 can be realized. 1 job is completed too late and 67 jobs are on time. 5 jobs are completed after F_i but not later than L_i . No job is produced with idle periods during its processing time.

6 Summary of Results

		WEEK 1			WEEK 2			WEEK 3		
number of jobs to be processed	i	247			67			73		
number of machines on which the jobs can be produced	k	48			45			52		
		V1	V2	V3	V1	V2	V3	V1	V2	V3
	ε1, ε2, C	0, 3, 100	0.1, 0.1, 0.1	0.5, 1, 100	0, 3, 100	0.1, 0.1, 0.1	0.5, 1, 100	0, 3, 100	0.1, 0.1, 0.1	0.5, 1, 100
value of the objective function		12.548,1	12.470,1	18.957,2	184,236	163,569	9.427,41	5.0802	0,189147	2.031,4
number of jobs which are finished too late	t > Li	114	113	73	1	1	8	0	0	1
number of jobs which are on time	t ≤ Fi	122	123	152	65	65	53	69	69	67
number of jobs which are finished with delay	Fi < t ≤ Li	11	11	22	1	1	6	4	4	5
number of jobs which are produced with idle periods	Xikt = 0	93	78	71	17	1	6	8	0	0

Table 17: Results of optimizing the MIP approach with „XPRESS MP“

Regarding the results presented in chapter 5 and 6 it is important to underline that the value of the objective function is not the main focus of the case study. As already mentioned this value can be compared to penalty costs. Its aim is to force a good time structure of production and a good job schedule to ensure due date fulfillment. If due dates cannot be met because of a bad schedule a high value of the objective function would be the penalty. Whereas due date fulfillment minimizes the objective value. Table (17) shows that the scheduling of jobs is very dependent on the used variant to calculate the objective value. The sequencing and timing of jobs changes because of the shape of the cost curve. Consequently the starting and completion time of jobs changes either.

The scheduling, sequencing and timing of jobs are also dependent on the number of jobs and machines which are available for production. Table (17) illustrates that the use of V1 causes the highest number of idle periods in every week. By implementing V1 to optimize the MIP a homogenous and continuous job scheduling is hardly possible. If the model also includes machine set up cost or set up time, lots of idle periods would lead to high production costs. An idle period occurs if the production of one job is interrupted to produce another job on the same machine. Set up takes place whenever another job is produced on the same machine. V2 causes negative results in week 1 but that is influenced by the high job volume in this week. V2 realizes good results in week 2 and 3 because fewer jobs have to be processed. Week 3 is planned very successfully by using V2 and V3. No idle periods occur during the production of all jobs and only 1 job is completed too late. As a consequence a homogenous and continuous production flow is possible.

7 Conclusions

The aim throughout the paper was to show that lots of different approaches and models exist in the literature to solve lot sizing and scheduling problems for different production systems. Nevertheless it is still difficult to implement these models to real world situations. Finding feasible solutions or solutions of good quality turns out to be very hard. Real world data can be optimized by giving up different constraints or by simplifying standard lot sizing and scheduling models. As it is shown in the case study an alternative MIP approach can lead to some good results.

Studying the results of the case study one can say that the goal was reached to a great extent. The case study proved that the scheduling and sequencing of jobs can be influenced and controlled by a well formulated objective function. Moreover the implementation of this approach ensures due date fulfillment to a high percentage and production costs can be saved. A negative aspect of the model and its implementation is that optimizing high data volume is difficult because of the computational time effort. A number of changes and adaptations were necessary to finally make it work. This problem may be solved by using a computer with higher computational speed and stronger capacity.

Furthermore this paper demonstrates that production planning can be improved by implementing some lot sizing and scheduling procedures. Production flows can be optimized and time and money can be saved. Lot sizing and scheduling is still challenging because many extensions like multi level structures or flexible flow lines are very difficult to solve.

I personally think that production planning and lot sizing and scheduling in particular are a very interesting area. Taking the different aspects into account I am convinced that this topic still provides a promising field for further research.

8 Acknowledgement

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F APPENDIX

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2 Tables

2.1 Input data W2

Table (18) includes the input data of week 2. It lists up job 248 to job 314 with the release dates, target and lates dates and the processing times. Table (19) shows the set of machines on which each job of week 2 can be produced.

i	Ri	Fi	Li	pi
248	1	1	1	0.838
249	1	1	1	0.848
250	1	1	1	0.348
251	14	14	14	0.031
252	14	14	14	0.043
253	13	13	13	0.018
254	2	4	4	2.385
255	7	16	16	9.043
256	14	15	15	0.430
257	4	4	4	0.414
258	2	2	2	0.833
259	1	2	2	1.036
260	7	8	8	0.854
261	7	17	17	9.071
262	16	18	18	1.603
263	1	17	19	3.070
264	7	17	19	1.486
265	1	1	1	0.911
266	10	21	23	0.597
267	7	15	16	6.490
268	1	5	29	4.103
269	12	14	14	2.310
270	14	16	16	2.171
271	3	7	7	4.400

272	1	1	1	0.334
273	1	2	2	0.808
274	2	2	2	0.589
275	9	9	9	0.276
276	9	10	10	0.668
277	10	10	10	0.487
278	13	13	13	0.298
279	13	14	14	0.720
280	14	14	14	0.524
281	7	11	11	4.369
282	4	4	4	0.376
283	2	11	11	9.137
284	15	20	20	4.711
285	1	2	2	0.462
286	9	10	10	1.325
287	10	13	13	3.208
288	13	15	15	2.335
289	2	5	5	2.543
290	5	6	6	1.265
291	4	5	5	1.107
292	5	8	8	2.678
293	8	10	10	1.950
294	4	5	5	1.161
295	7	7	7	0.205

296	2	4	4	1.515
297	4	5	5	0.753
298	17	18	18	0.782
299	7	20	20	12.600
300	7	10	10	3.058
301	7	17	17	9.925
302	7	19	22	0.468
303	1	15	15	13.839
304	1	15	15	13.839
305	1	2	2	1.107
306	1	2	2	1.107
307	1	15	15	13.839
308	1	15	15	13.839
309	1	15	15	13.839
310	1	15	15	13.286
311	16	16	20	3.545
312	16	16	21	7.616
313	1	15	16	2.613
314	1	15	16	4.718

Table 18: Input data for week 2

k

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45			
248		1	1	1			1		1	1	1	1	1					1	1	1	1					1		1				1						1										
249		1	1	1			1		1	1	1	1	1					1	1	1	1					1		1				1						1										
250		1		1			1				1	1	1					1	1	1						1		1			1							1										
251		1	1	1			1		1		1	1	1					1	1	1						1		1			1	1		1		1	1	1	1	1	1	1	1	1		1		
252		1	1	1			1		1		1	1	1					1	1	1						1		1			1	1		1		1	1	1	1	1	1	1	1	1	1		1	
253		1	1	1			1		1		1	1	1					1	1	1						1		1			1	1		1		1	1	1	1	1	1	1	1	1	1		1	
254	1	1	1	1		1	1		1		1		1	1	1				1	1			1			1		1	1	1		1	1						1									
255			1	1					1	1			1																																	1		
256		1	1	1			1		1		1	1	1					1	1	1						1		1			1	1		1		1	1	1	1	1	1	1	1	1	1		1	
257		1	1	1			1		1		1	1	1					1	1	1						1		1			1	1		1		1	1	1	1	1	1	1	1	1	1		1	
258		1	1	1			1		1		1	1	1					1	1	1						1		1			1	1		1		1	1	1	1	1	1	1	1	1	1		1	
259		1	1	1			1		1	1	1	1	1	1				1	1	1	1					1		1					1						1									
260		1					1		1		1		1					1		1						1		1				1								1								
261			1	1					1	1			1																																	1		
262	1	1		1	1	1	1	1	1	1	1	1	1	1	1			1	1	1	1	1	1	1	1	1	1	1	1	1																1		
263	1							1	1				1										1		1																					1		
264							1							1				1									1		1																	1		
265			1	1					1	1			1																																	1		
266		1	1	1			1		1	1	1	1	1	1				1	1	1						1		1				1													1			
267																	1	1																													1	
268		1	1	1			1		1	1	1	1	1	1				1	1	1						1		1			1	1		1	1	1	1	1	1	1	1	1	1	1	1		1	
269		1	1	1			1		1	1	1	1	1	1				1	1	1						1		1				1														1		
270		1	1	1			1		1	1	1	1	1	1				1	1	1						1		1					1															
271																															1			1			1	1		1	1	1	1	1			1	

[illegible]

Table 19: Input data for week 2 – Set of machines

2.2 Output data W2

Table (20) is the output file “setup V2” for week 2. This matrix illustrates which job is produced on which machine after the optimization process with “XPRESS MP” by using variant 2 to calculate the objective value. Table (21) is the output file “time V2”. It shows how the production of jobs is distributed over 24 periods and how many percentage of a job is processed in each period. Table (22) includes the production periods of jobs. The actual completion period can be compared to the given due and latest date. Jobs with idle periods among the production process can be filtered out. Table (23) lists up statistical information about production, scheduling and timing of jobs in week 2 by using V2.

[illegible]

t																								
i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
248	100%																							
249	100%																							
250	100%																							
251														100%										
252														100%										
253													100%											
254		16.14%	41.93%	41.93%																				
255							11.06%	3.77%	11.06%	11.06%	11.06%	11.06%	7.77%	11.06%	11.06%	11.06%								
256															100%									
257				100%																				
258		100%																						
259	96.50%	3.50%																						
260								100%																
261								11.02%	11.02%	11.02%	11.02%	11.02%	11.02%	11.02%	0.78%	11.02%	11.02%							
262																	62.37%	37.63%						
263														32.58%	32.58%	32.58%	2.27%							
264																67.28%	32.72%							
265	100%																							
266																					100%			
267									15.41%	15.41%	15.41%	15.41%	15.41%	15.41%	7.55%									
268	24.37%	24.37%	24.37%	24.37%	2.50%																			
269												43.29%	20.82%	35.89%										
270														7.87%	46.06%	46.06%								
271			22.73%	9.10%	22.73%	22.73%	22.73%																	

272	100%																							
273		100%																						
274		100%																						
275									100%															
276										100%														
277										100%														
278													100%											
279														100%										
280														100%										
281							22.89%	22.89%	8.45%	22.89%	22.89%													
282				100%																				
283		10.95%	1.49%	10.95%	10.95%	10.95%	10.95%	10.95%	10.95%	10.95%	10.95%													
284																3.79%	21.23%	21.23%	21.23%	11.29%	21.23%			
285		100%																						
286										24.55%	75.45%													
287											31.18%	31.18%	31.18%	6.47%										
288														42.82%	42.82%	14.35%								
289			39.33%	39.33%	21.35%																			
290					79.07%	20.93%																		
291				9.65%	90.35%																			
292						37.34%	37.34%	25.33%																
293										51.28%	48.72%													
294				13.86%	86.14%																			
295							100%																	
296			41.03%	58.97%																				
297					100%																			

298																		100%						
299								7.94%	7.94%	7.94%	7.94%	7.94%	7.94%	7.94%	7.94%	7.94%	7.94%	7.94%	7.94%	4.76%				
300							32.70%	32.70%	32.70%	1.90%														
301								10.08%	10.08%	10.08%	10.08%	10.08%	9.32%	10.08%	10.08%	10.08%	10.08%							
302																			100%					
303		7.23%	7.23%	7.23%	7.23%	7.23%	7.23%	7.23%	7.23%	6.06%	7.23%	7.23%	7.23%	7.23%	7.23%									
304		7.23%	7.23%	6.06%	7.23%	7.23%	7.23%	7.23%	7.23%	7.23%	7.23%	7.23%	7.23%	7.23%	7.23%									
305	9.68%	90.32%																						
306	9.68%	90.32%																						
307		7.23%	7.23%	7.23%	7.23%	7.23%	7.23%	7.23%	7.23%	7.23%	7.23%	7.23%	6.06%	7.23%	7.23%									
308		7.23%	7.23%	7.23%	7.23%	7.23%	7.23%	7.23%	7.23%	7.23%	6.06%	7.23%	7.23%	7.23%	7.23%									
309		6.19%	7.23%	7.23%	7.23%	7.23%	7.23%	7.23%	7.23%	7.23%	7.23%	7.23%	7.10%	7.23%	7.23%									
310		7.53%	7.53%	7.53%	7.53%	7.53%	3.73%	7.53%	7.53%	7.53%	7.53%	7.53%	7.53%	7.29%	6.18%									
311																28.21%	28.21%	28.21%	15.38%					
312																13.13%	13.13%	13.13%	13.13%	13.13%	13.13%		8.09%	13.13%
313													38.27%	23.46%	38.27%									
314											21.20%	21.20%	15.21%	21.20%	21.20%									

Table 21: Output file “time V2” for week 2

t																											
i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	Fi	Li	
248	1																								1	1	
249	1																								1	1	
250	1																								1	1	
251														14											14	14	
252														14											14	14	
253													13												13	13	
254		2	3	4																					4	4	
255							7	8	9	10	11	12	13	14	15	16									16	16	
256															15										15	15	
257				4																					4	4	
258		2																							2	2	
259	1	2																							2	2	
260								8																	8	8	
261								8	9	10	11	12	13	14	15	16	17								17	17	
262																	17	18							18	18	
263														14	15	16	17								17	19	
264																16	17								17	19	
265	1																								1	1	
266																					21				21	23	
267									9	10	11	12	13	14	15										15	16	
268	1	2	3	4	5																				5	29	
269												12	13	14											14	14	
270														14	15	16									16	16	
271			3	4	5	6	7																		7	7	
272	1																								1	1	
273		2																							2	2	
274		2																							2	2	
275									9																9	9	
276										10															10	10	
277										10															10	10	
278													13												13	13	
279														14											14	14	
280														14											14	14	
281							7	8	9	10	11														11	11	
282				4																					4	4	
283		2	3	4	5	6	7	8	9	10	11														11	11	
284															15	16	17	18	19	20					20	20	
285		2																							2	2	
286									9	10															10	10	
287										10	11	12	13												13	13	
288													13	14	15										15	15	
289			3	4	5																				5	5	
290					5	6																			6	6	

	periods with pos. prod.	idle per.	number idle per.	first period of prod.	last period of prod.	latest date	i > Li	target date	i <= Fi	Fi < i <= Li
248	1	N	0	1	1	1	0	1	1	0
249	1	N	0	1	1	1	0	1	1	0
250	1	N	0	1	1	1	0	1	1	0
251	1	N	0	14	14	14	0	14	1	0
252	1	N	0	14	14	14	0	14	1	0
253	1	N	0	13	13	13	0	13	1	0
254	3	N	0	2	4	4	0	4	1	0
255	10	N	0	7	16	16	0	16	1	0
256	1	N	0	15	15	15	0	15	1	0
257	1	N	0	4	4	4	0	4	1	0
258	1	N	0	2	2	2	0	2	1	0
259	2	N	0	1	2	2	0	2	1	0
260	1	N	0	8	8	8	0	8	1	0
261	10	N	0	8	17	17	0	17	1	0
262	2	N	0	17	18	18	0	18	1	0
263	4	N	0	14	17	19	0	17	1	0
264	2	N	0	16	17	19	0	17	1	0
265	1	N	0	1	1	1	0	1	1	0
266	1	N	0	21	21	23	0	21	1	0
267	7	N	0	9	15	16	0	15	1	0
268	5	N	0	1	5	29	0	5	1	0
269	3	N	0	12	14	14	0	14	1	0
270	3	N	0	14	16	16	0	16	1	0
271	5	N	0	3	7	7	0	7	1	0
272	1	N	0	1	1	1	0	1	1	0
273	1	N	0	2	2	2	0	2	1	0
274	1	N	0	2	2	2	0	2	1	0
275	1	N	0	9	9	9	0	9	1	0
276	1	N	0	10	10	10	0	10	1	0
277	1	N	0	10	10	10	0	10	1	0
278	1	N	0	13	13	13	0	13	1	0
279	1	N	0	14	14	14	0	14	1	0
280	1	N	0	14	14	14	0	14	1	0
281	5	N	0	7	11	11	0	11	1	0
282	1	N	0	4	4	4	0	4	1	0
283	10	N	0	2	11	11	0	11	1	0
284	6	N	0	15	20	20	0	20	1	0
285	1	N	0	2	2	2	0	2	1	0
286	2	N	0	9	10	10	0	10	1	0
287	4	N	0	10	13	13	0	13	1	0
288	3	N	0	13	15	15	0	15	1	0
289	3	N	0	3	5	5	0	5	1	0
290	2	N	0	5	6	6	0	6	1	0
291	2	N	0	4	5	5	0	5	1	0

292	3	N	0	6	8	8	0	8	1	0
293	2	N	0	9	10	10	0	10	1	0
294	2	N	0	4	5	5	0	5	1	0
295	1	N	0	7	7	7	0	7	1	0
296	2	N	0	3	4	4	0	4	1	0
297	1	N	0	5	5	5	0	5	1	0
298	1	N	0	18	18	18	0	18	1	0
299	13	N	0	8	20	20	0	20	1	0
300	4	N	0	7	10	10	0	10	1	0
301	10	N	0	8	17	17	0	17	1	0
302	1	N	0	19	19	22	0	19	1	0
303	14	N	0	2	15	15	0	15	1	0
304	14	N	0	2	15	15	0	15	1	0
305	2	N	0	1	2	2	0	2	1	0
306	2	N	0	1	2	2	0	2	1	0
307	14	N	0	2	15	15	0	15	1	0
308	14	N	0	2	15	15	0	15	1	0
309	14	N	0	2	15	15	0	15	1	0
310	14	N	0	2	15	15	0	15	1	0
311	4	N	0	16	19	20	0	16	0	1
312	8	J	1	16	24	21	1	16	0	0
313	3	N	0	13	15	16	0	15	1	0
314	5	N	0	11	15	16	0	15	1	0

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4 Abstract

Das Thema der Diplomarbeit ist „Capacitated Lot Sizing and Scheduling – An alternative approach for the parallel machine case“. Das Ziel ist die Optimierung von Produktionsaufträgen aus der Halbleiterindustrie hinsichtlich ihrer Bearbeitung auf parallelen Maschinen mit beschränkten Kapazitäten. Ein alternatives MIP Model zur Losgrößen- und Reihenfolgeplanung von Aufträgen wird in das Optimierungsprogramm „Xpress MP“ implementiert. Ausgewählte Daten werden in das Programm eingelesen und optimiert. Die gewonnenen Ergebnisse zeigen, dass durch die Anwendung dieses Modells Fertigstellungstermine von Aufträgen eingehalten und Produktionskosten minimiert werden können.

Die Arbeit basiert auf der Beschreibung der Produktionsprogrammplanung. Hauptaufgabe der Produktionsplanung ist, herauszufinden welches Produkt auf welcher Maschine zu welchem Zeitpunkt produziert werden soll. Losgrößen und Maschinenbelegung müssen, unter Berücksichtigung verschiedener Produktstrukturen, knapper Ressourcen und eines hohen Service Levels, bestimmt werden.

Die Diplomarbeit gliedert sich in fünf Hauptkapitel. Kapitel A beinhaltet eine Einführung zum Thema, Hintergrund und Problematik werden kurz beschrieben. In Kapitel B werden die Ziele der Losgrößen- und Reihenfolgeplanung definiert. Auf den Ablauf und die Vorgehensweise bei der Produktionsplanung und auf die Aufgaben des Produktionsmanagements wird näher eingegangen. In Kapitel C werden diverse Losgrößen- und Reihenfolgemodelle präsentiert. Es werden Modelle für verschieden Produkt- und Produktionsstrukturen vorgestellt. Der Zusammenhang von Losgrößen-, Reihenfolgeplanung und Maschinenbelegung wird beschrieben. Kapitel D stellt verschiedene Methoden und deren Qualität zur Lösung der Modelle vor. Kapitel E beschäftigt sich mit der Fallstudie („Case Study of Semi-Conductor Industry“). Das Kapitel beginnt mit einer Einleitung und der Problemstellung. Im Anschluss werden der alternative MIP Ansatz und dessen Eigenschaften

definiert. Die Datenquellen und die Vorgehensweise bei der Aufbereitung der Daten werden beschrieben. Daraufhin wird auf die Implementierung und Optimierung des neuen Modells mit dem Optimierungsprogramm „Xpress MP“ genau eingegangen. Das Kapitel endet mit der Präsentation und Interpretation der gewonnenen Ergebnisse.

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