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Chapter 1

Introduction

The thesis consists of three chapters, which deal with a number of issues at the intersection of Macroeconomics, Labor Economics, Development Economics and Monetary Economics. Two main chapters are within the broad literature on labor market informality in developing countries and the third chapter deals with the issue of monetary policy formulation in a particular emerging and developing country setting.

Key question of the first chapter titled “Credit Frictions and the Dynamics of Formal and Informal Employment” is what explains the differential rate of entry into formal and informal employment over the business cycle observed in several developing countries. In particular, the paper documents this differential behavior using Brazilian labor force survey data and attempts to answer the main question using a theoretical model that replicates key patterns of labor reallocation across sectors. Based on observations on the use of external credit by formal and informal firms in Brazil, the study argues that differences in labor market indicators, particularly in entry rates into two types of employment, are driven by differences in the exposure to changes in the cost of credit over the business cycle between two types of firms. The study builds and calibrates to Brazilian data a two-sector dynamic model with search and matching frictions in the labor market and credit frictions in the financial market. The model generates countercyclical risk premium on funds borrowed by firms in the formal sector and successfully replicates a number of patterns of movements of workers between sectors over the business cycle. Policy implications of this theoretical study are in line with findings of several recent empirical studies on developing countries (including Brazil), which suggest that financial conditions matter for sectoral reallocation of workers and policy measures aimed at dampening employment shocks, especially movements from the formal sector, should concentrate on easing financial constraints over the business cycle.

The aim of the second chapter titled “Formal and Informal Employment and Employment Durations in Brazil” is to characterize the determinants of labor market transitions in Brazil by explicitly taking into account possible direct and indirect effects of various covariates on labor market transitions. The study is particularly focused on modeling exits from different employment categories and it models transitions from formal and informal salaried employment and informal self-employment using data on employment durations. For this purpose, the study estimates a competing risks proportional hazard model with unobserved heterogeneity and computes the marginal effect of covariates on the overall probability of exiting to a given destination. Computing marginal effects on overall exit probability allows clear identification of the effect of covariates on labor market transitions in an environment with several labor market states. In an environment with more than two major labor market states (i.e., not only employment and unemployment), traditional methods do not capture direct and indirect effects

of covariates on transitions, which this study does using recently developed estimation methods. Results indicate the strong effect of both individual characteristics and aggregate economic conditions on the probability of transiting from key employment states. In particular, more years of schooling significantly increase the probability of transiting to formal employment both from informal salaried employment and informal self-employment. Firm size is also an important factor with informal salaried workers in larger firms having a higher probability of transiting to formal employment. Business cycle effects also influence transition probabilities with the probability of entering formal employment and informal self-employment being procyclical and the probability of transiting to informal salaried employment exhibiting countercyclical behavior. Findings suggest that increasing the level of formality requires broad-based solutions, which should address not only the quality of the labor force, but also constraints that affect the ability of formal firms to create and retain jobs (including variations in constraints over the business cycle).

The third chapter titled “Monetary Policy Choices Under Currency Substitution” deals with a monetary policy question relevant for a large number of developing and emerging economies. In particular, it studies how monetary policy should be conducted in an environment with more than one currency in circulation (e.g., US dollars along with a domestic currency). In this paper, performance of a number of simple monetary policy rules is analyzed in a dynamic stochastic general equilibrium model with currency substitution. The study establishes that domestic price inflation targeting is superior to consumer price inflation targeting only at low levels of currency substitution. Monetary policy rules with interest-rate smoothing perform notably better than a benchmark rule that responds only to fluctuations in consumer price inflation and output. The analysis also shows strong support for responding to changes in the nominal exchange rate in the Taylor-type rules and this result is robust to variations in the level of currency substitution and changes to other structural features of the model economy (e.g., degree of openness and level of consumption of traded and non-traded goods).

Chapter 2

Credit Frictions and the Dynamics of Formal and Informal Employment

2.1 Introduction

The issue of widespread informal employment and underground economy in developing countries has attracted significant attention of researchers. According to [Schneider et al. \(2010\)](#) the average size of informal economy in developing countries is estimated to be around 35% of official GDP with individual country estimates ranging from about 13% to more than 65%. Share of workers equivalent to these figures is estimated to be employed informally as well.¹ Studies on informality have mainly concentrated on identifying reasons why the informal sector and informal employment exist and what factors in the long-run affect their size and importance. Recent literature has also shifted attention to the cyclical dynamics of informal employment (see [Loayza and Rigolini \(2006\)](#), [Bosch and Maloney \(2008\)](#), [Bosch and Esteban-Pretel \(2012\)](#) and [Fiess et al. \(2010\)](#)). These studies have identified patterns of cyclical fluctuations of worker flows between formal and informal employment and their implications on the dynamics of unemployment over the business cycle. The current study is closely related to this latter stream of literature. It studies cyclical properties of worker flows between formal and informal employment and unemployment. In particular, we analyze worker flows in a large developing country, Brazil, and build a dynamic stochastic general equilibrium model to explain certain empirical observations.

Our data analysis shows that over the business cycle, the job finding rate from unemployment into formal employment is procyclical and more volatile than entry into informal employment. Direct transition from informal to formal employment is also relatively more procyclical and volatile than transition from formal into informal jobs. On the contrary, separations from informal employment exhibit more volatile and countercyclical behavior. Equivalently, the share of labor force employed formally is procyclical, while the share of informal workers is countercyclical. Unemployment rate exhibits strong countercyclical behavior as well. In this study we suggest that this behavior, in particular the behavior of entry into formal employment is driven by varying access to external credit over the business cycle by firms that operate formally. Informal sector, on the other hand, is not directly affected by fluctuations in the financial market and, therefore, we observe this differential behavior of job finding rates between two sectors. To study this mechanism and the ability of a theoretical model to replicate empirical observations we build a dynamic stochastic general equilibrium model, in which both types of employment

¹Informal employment in this study refers to employment in legal economic activities, which does not comply with labor, tax, safety or other relevant regulation due to lack or absence of proper registration of an employee-employer relationship. We discuss issues related to the definition and the measurement of informality in the next section.

arrangements exist and formal firms have access to external credit. The model allows us to study the dynamics of labor market flows between formal and informal employment and unemployment and at the same time explore the effect of financial conditions that also vary over the business cycle and have disproportionately stronger impact on the operation of formal firms.

This paper is closely related to several theoretical and empirical studies in the literature. [Bosch and Maloney \(2008\)](#) analyze worker flows between formal and informal employment based on Brazilian and Mexican employment surveys. They find similar differential behavior of job finding and separation rates between formal and informal employment in both of these countries. In particular, job finding rate from unemployment is more volatile into formal jobs and transition from informality to formal sector employment is procyclical. They establish that an increase in the number of informally employed workers and decline of the number of formally employed workers during recessions are due to lack of access to formal employment. They also show that access to informal employment is less volatile over the business cycle, but informal firms are not capable of accommodating fully the inflow of workers, who are not able to find formal employment. The share of formal employment is found to be procyclical and about half as volatile as output. [Bosch and Esteban-Pretel \(2012\)](#) conduct a similar but more detailed analysis of worker flows using Brazilian panel labor force data, which confirms the findings of the previous study. As detailed below, we also use the recent labor force survey data for Brazil and, in general, confirm the findings of these two studies. In addition, we check if these findings are sensitive to a number of changes in the definition of informal employment.

In another closely related study [Hoek \(2007\)](#) uses a simple reduced form analysis and argues that differences in finding and separation rates over the business cycle in Brazil are mainly driven by institutional differences between formal and informal sectors. In particular, the author argues that associated with formal employment high dismissal costs in Brazil induce formal firms to adjust on the hiring margin in recessions.² On the contrary, absence of similar institutional frictions allows informal labor arrangements to be terminated without significant costs. In another recent study [Bosch and Esteban-Pretel \(2012\)](#) attempt to explain the differential behavior of worker flows based on a different mechanism. They build a continuous time search and matching labor market model, in which workers are homogeneous in their productivity and firms decide to offer a formal or informal employment contract depending on the realization of a match-specific idiosyncratic productivity shock. Aggregate productivity shock has a standard procyclical effect on labor market tightness and job finding probability for both types of contracts. Specifically, higher level of aggregate productivity increases profits overall in the economy and the number of hirings rises. In their model what generates more volatile and procyclical job creation into formal employment is the following: in addition to a positive effect of aggregate productivity, hirings in the formal sector increase due to higher average equilibrium realization of a match-specific productivity shock, which further increases the probability of offering a formal contract by a firm.

In many respects, our analysis is closely related to explanations of the dynamics of the Brazilian labor market presented in [Hoek \(2007\)](#) and [Bosch and Esteban-Pretel \(2012\)](#). Institutional differences between two sectors are important factors that drive the behavior of job finding and separation rates. But we further argue that these institutional differences between formal and informal employment are also important in determining how the effects of aggregate exogenous shocks are amplified on both of these employment arrangements. In particular, we argue that financial frictions directly affect formal sector firms over the business cycle, amplify the effect of exogenous shocks on these firms and, consequently, strongly influence their ability to create new

²[Gonzaga \(2003\)](#) also provides very detailed analysis of dismissal costs in Brazil and their effect on labor turnover.

formal jobs. Thus, this paper contributes to the literature by building a model, in which the credit (or financial frictions) channel is a key amplification mechanism that directly affects the hiring behavior of firms in the formal sector and creates the differential behavior of job finding rates into formal and informal employment.

The argument behind the channel highlighted in this paper is strongly supported by findings of two recent empirical studies on the behavior of labor markets over the business cycle in Latin American countries. [Cravo \(2011\)](#) analyzes the pattern of job creation in Brazil using detailed sectoral data on a large number of formal firms. This study is primarily interested in identifying the type of firms (small or large) that make more adjustments in terms of employment over the business cycle and what factors drive this adjustment behavior of firms. The author finds that in Brazil small formal firms are more sensitive to business cycle fluctuations than large ones.³ In addition, the author finds that changes in credit conditions over the business cycle is the key factor that explains the pattern of adjustment among small firms. In other words, small formal firms are disproportionately harder hit by deteriorating credit conditions during recessions and they adjust by cutting more jobs than large firms. Thus, credit conditions do have an effect on job creation in the formal sector in Brazil and this effect is more pronounced among small formal firms. [Gallego and Tessada \(2012\)](#) uses plant and sector level data on employment for four Latin American countries (including Brazil) and identifies that financial channel is the key transmission mechanism of exogenous shocks to employment and job creation. Financial characteristics of each sector also have important effect on employment outcomes over the business cycle. In particular, authors find that sectors that are more dependent on external financing make stronger adjustments to the level of employment and the rate of job creation.

Some discussion is necessary regarding the difference between the explanation of the differential behavior of job finding rates proposed in this paper and the one suggested by [Bosch and Esteban-Pretel \(2012\)](#). In the latter study, the difference in the rate of job creation is mainly driven by changing over the business cycle probability of offering jobs with formal contracts. Brazilian labor force survey used both in this paper and in [Bosch and Esteban-Pretel \(2012\)](#) does not include related to this issue questions and it is not straightforward to state which of these explanation has more merit. Availability of panel matched employer-employee data could help identify the exact effect and differentiate between cases, when formal firms are genuinely hiring or firing workers over the business cycle or they are simply changing the contractual arrangement of existing and new workers, which is captured as movement of a worker between formal and informal sectors in the labor force survey.⁴ Nevertheless, recent empirical evidence based on aggregate and aggregated data points to the important role of financial frictions in job creation over the business cycle in Latin American countries, which is the key amplification mechanism in our model. The ultimate effect of this mechanism could materialize through different types of decisions. In particular, firms might change the number of workers, change the share of formal and informal workers within a firm or do both. But financial frictions amplify and differentiate the degree of adjustments made across sectors (no matter how exactly they are made) and this is exactly what we aim at capturing in this model.

The paper proceeds as follows. In the next section we review key empirical observations on the dynamics of formal and informal employment and the evolution of job finding and separation rates over the business cycle in Brazil. After laying out key features of the model we discuss several simulation results and the model's ability to explain relevant empirical observations.

³This is in contrast with the findings of, for instance, [Moscarini and Postel-Vinay \(2009\)](#) for the U.S. labor market, where large firms appear to be more sensitive to business cycle fluctuations than small ones.

⁴Additional question in the existing rotating labor force survey asking if an individual is still working in the same company could potentially shed some light on this issue as well.

Section 5 concludes.

2.2 Empirical observations

We use data from the Brazilian Monthly Employment Survey (*Pesquisa Mensal de Emprego*, PME) to examine the business cycle properties of transitions between formal and informal employment and unemployment. PME is one of few labor market surveys in developing countries that allows us to study the dynamics of labor market flows at a relatively high monthly frequency. It is a rotating panel survey of households in major urban areas in Brazil conducted since 1980 by the Brazilian Institute for Geography and Statistics (*Instituto Brasileiro de Geografia e Estatística*, IBGE).⁵ We use the data starting from March 2002 until March 2011. PME is administered since early 1980s but we use only recent data, as the survey has been drastically changed in 2002 and definitions of certain labor market states are not consistently comparable across old and new survey designs. In this respect, our results complement findings of [Bosch and Esteban-Pretel \(2012\)](#), who conduct similar analysis using the PME data from 1982 until the first quarter of 2002. For modeling purposes we equally use our empirical findings and the findings of [Bosch and Esteban-Pretel \(2012\)](#).

Determining the size of the informal economy and share of informal employment in labor force is not a straightforward task. Depending on data availability, researchers define small firms to be in the informal sector or firms within specific industries to be operating informally. Size of the informal economy has also been estimated based on aggregate data and using a number of approaches (see [Schneider et al. \(2010\)](#)). PME data allows us to determine the size of informal employment using several criteria. It is possible to identify if a person holds so-called “work card” (“*carteira de trabalho assinada*”), which is issued to every formally employed worker and such employment relationship complies with labor and other relevant regulation. In addition, the survey provides information on the size of a firm, where a worker is employed, and whether mandatory social security payments are made or not. Based on this information we define informal workers as those employed individuals, who do not hold the work card. The sample is limited to workers of age between 15 and 65, individuals out of the labor force are excluded.

To calculate necessary moments that characterize the cyclical behavior of job finding and separations rates we use the panel structure of PME and calculate gross flows of workers between three labor market states across two months. We use this information to calculate the transition rates between labor market states that we consider in our dataset. Figures 2.2a-2.2c show the evolution of these job finding, separation and transition rates. To estimate the moments of these labor market variables we smooth obtained monthly series using moving average filter, take quarterly averages and detrend the quarterly series using the Hodrick-Prescott filter with the smoothing parameter $\lambda = 1600$. Then, we compute the correlations of labor market variables with smoothed and detrended quarterly GDP series and the volatility of labor market variables relative to the volatility of quarterly GDP series.⁶ Results are reported in Table 2.1.⁷

Job finding rate from unemployment into formal employment is procyclical, while the job

⁵In this survey a household is followed for four consecutive months before it is dropped from the sample for eight months, after which the household is again introduced into the sample for four months. One quarter of households leave the sample every month and they are substituted for by a new set of households of equal amount.

⁶Quarterly GDP series is provided by the IBGE and it is estimated at constant 1995 prices.

⁷We also repeated the analysis using a different definition of informal employment, where following [Bosch and Esteban-Pretel \(2012\)](#) we add to the pool of informal salaried employees self-employed workers, who are not identified as professionals or technicians. Results are presented in Table 2.8 in Appendix 2.A and they are very similar to those presented in Table 2.1.

Table 2.1: Correlation and relative volatility of labor market variables in Brazil.

x	ϕ^F	ϕ^I	λ^F	λ^I	τ^{FI}	τ^{IF}	u	n^F
$\rho(y, x)$	0.49	-0.23	-0.13	-0.41	-0.17	0.18	-0.50	0.28
σ_x/σ_y	4.45	2.94	4.52	5.16	1.43	2.24	2.87	0.58

Notes: Superscripts F and I indicate formal and informal employment, respectively; y refers to real quarterly GDP, x refers to the other variables: ϕ denotes the job finding rate, λ is the job separation rate, τ is the direct transition rate between formal and informal employment, u is the unemployment rate and n^F is the share of formal employment in total employment. Statistics are computed after taking natural logarithm and detrending original series using the HP filter with the smoothing parameter $\lambda = 1600$.

finding rate into informal employment is somewhat countercyclical. Separation rates are both countercyclical but the rate is more countercyclical and volatile from informal employment. Similarly direct transitions from informal to formal employment are procyclical and more volatile. We find that direct transition to informal employment from formal jobs is acyclical. Unemployment rate is countercyclical and about three times more volatile than output series. On the contrary, the share of formal employment in total employment is somewhat procyclical and about half as volatile as output. If we compare these results to estimates of [Bosch and Esteban-Pretel \(2012\)](#) obtained using data for the earlier period (before 2002), cyclical behavior of labor market flows is generally consistent with our estimates. Key difference between our results and estimates of [Bosch and Esteban-Pretel \(2012\)](#) is that the job finding rate into informal employment in a recent period (2002 - 2011) appears to be more volatile and countercyclical (this variable is slightly procyclical for the earlier period). In addition, according to our estimates the relative volatility of the unemployment rate is about two times smaller than the estimate for the period before 2002. Both pro- or countercyclicalities of informal employment (and corresponding finding rate) over the business cycle have been observed in the literature and, thus, difference in the behavior of this rate across two periods is not surprising. Moreover, relative shortness of our sample might be a factor contributing to this particular result.

We argue that this differential behavior of entry into formal and informal employment is driven by changes in the financial conditions over the business cycle, which directly affect formal firms. In the model only formal firms have access to external credit. This assumption is justified by the fact that firms operating informally are usually not able to use the full value of their assets as collateral, which limits their ability to obtain formal credit. Empirical evidence also suggests that higher levels of formality in operations of firms are associated with higher access to formal credit. Empirical evidence to support this assumption can be easily obtained by plotting the ratio of private sector credit to GDP against an estimated size of the informal economy across a number of countries. Figure 2.1 in Appendix 2.A demonstrates this negative relationship for a large number of developing countries. Similar negative relationship between the level of informality and formal credit has also been established using firm level data. For instance, [Dabla-Norris and Koeda \(2008\)](#) use firm level data from a number of transition economies to explore the link between the measure of informality within firms and information on access to and use of external finance. They find that relatively higher level of informality in operations does reduce access to formal bank credit. Interestingly, they establish only one way causality, i.e. informality leads to difficulties in accessing credit and not vice versa. This particular evidence suggests that firm's long-run decision to operate informally is likely to be determined by institutional factors (e.g., labor market regulation) and this decision has a number of negative consequences, one of which is a limited access to formal credit. Based on firm level data for a large number of countries [Gatti and Honorati \(2008\)](#) also find evidence for the link between level of formality within firms

and their ability to access formal credit.⁸

As for evidence on the use of credit and informality in Brazil, we refer to results of the Survey of Urban Informal Economy (*Pesquisa de Economia Informal Urbana*) conducted by the IBGE in 2003. The survey collected information on self-employed workers and small firms with at most five workers. Due to its design this survey provides information that is pertinent only to a limited share of workers surveyed in the PME. Nevertheless, it still provides useful information on the use of external finance in small informal firms and by self-employed workers. According to survey results about 94% of informal firms and self-employed workers used no external finance in their everyday operations within the three months period prior to the survey (IBGE, 2005). In a related empirical study Catão et al. (2009) studies the relationship between formality and access to credit in Brazil. Based on annual household survey data the study analyzes the relationship between access to credit and changes in the share of formal and informal employment in the economy. In particular, the study examines if improved access to formal credit increases the share of formally employed workers by inducing informal firms to operate formally or simply by expanding existing formal firms. Authors establish that the main effect of credit on the level of formalization works by inducing informal firms to operate formally. In addition, the study finds that as access to credit improves, increase in the level of formalization is larger in those sectors that are categorized as more dependent on external funding.

2.3 Model

2.3.1 Economic environment

The model we develop is in the class of real business cycle models augmented with a labor market subject to search and matching frictions. The baseline model in this class is extended to accommodate two production sectors, formal and informal, and the financial sector. Labor markets in both production sectors are assumed to be subject to search and matching frictions. Perfectly competitive financial sector has an access to unlimited source of funds and it lends them to firms in the formal sector. Agents in this economy are households with household members employed in both production sectors, firms that use labor as the only factor of production and infinite number of lenders in the financial sector. In the following subsections, we discuss each of these sectors and agents in more detail, including related assumptions and notation.

2.3.2 Labor market

Labor is homogeneous in terms of its productivity. This way of modeling formal and informal labor markets is similar to recent literature, which does not make special assumptions about the structure of labor markets and abstracts from productivity differences among workers.⁹ In other words, we assume that workers are homogeneous and they are not directed to a specific sector to look for a job. Though it might be considered a strong assumption, its use in case of Brazil can be justified by empirical evidence on transitions between three labor market states over longer time period. Table 2.7 in Appendix 2.A reports transition rates between three employment states that are calculated using the PME data but in this case we compute the transition rates by comparing labor market status of a worker at a given time and after 12 months. Results indicate that the probability of an unemployed individual to be employed formally after one year has been slightly higher than the probability of becoming an informal worker. In addition, the probability of directly transiting from an informal job to formal occupation is significantly

⁸There are also theoretical studies in the literature looking at the relationship between formality, contract enforcement and access to credit. For recent examples, see Straub (2005) and Quintin (2008).

⁹See Charlot et al. (2010) and Ulyssea (2010).

higher than the probability of transiting from formal to informal employment.¹⁰ Apparently, movements across formal and informal employment in the medium-run in Brazil are not very restricted as models with directed search and segmented labor markets suggest.

Labor markets in both production sectors are subject to search and matching frictions. Matching frictions are characterized by the following function of standard Cobb-Douglas form, which maps unemployed household members and vacancies in both sectors to matches:

$$M_t^k = \psi^k u_t^\alpha (v_t^k)^{1-\alpha}, \quad k = I, F, \quad (2.1)$$

where superscripts I and F indicate informal and formal sectors, respectively. The term ψ^k governs the efficiency of the matching technology in both sectors, u_t denotes the unemployment rate and v_t^k denotes vacancies created in period t by firms in sector k . Following closely related literature (Ulyssea, 2010; Charlot et al., 2010) we assume that the matching process is less frictional in the informal sector, i.e. $\psi^F < \psi^I$. This assumption can also be interpreted as capturing the observation that it is, on average, easier to fill vacancies in the informal sector, which usually require fewer skills than in the formal sector. In both labor markets firms find a worker at rate M_t^k/v_t^k , while unemployed workers find employment in either of the sectors at rate M_t^k/u_t .

Let's define the labor market tightness in both sectors as $\theta_t^k \equiv v_t^k/u_t$. Then, using the matching function (2.1) we can define the rate, at which firms fill their vacancies, as $M_t^k/v_t^k \equiv m_t^k = \psi^k (\theta_t^k)^{-\alpha}$ and the job finding rate as $M_t^k/u_t = \theta_t^k m_t^k$. Every period in each sector an exogenous fraction δ^k of employed workers lose their job. As is shown in Table 2.7, separation rates differ across types of employment and, therefore, we assume that $\delta^F < \delta^I$, which implies that job turnover is higher among workers that are employed informally. Employment in each sector evolves according to

$$n_t^k = (1 - \delta^k) n_{t-1}^k + m_{t-1}^k v_{t-1}^k. \quad (2.2)$$

Workers are allowed to be in three states, employed in either formal or informal sector or unemployed. Thus, we define the unemployment rate as $u_t = 1 - \sum_k n_t^k$. Using this definition we can formulate the law of motion for unemployment in this economy as

$$u_t = u_{t-1} + \delta^F n_{t-1}^F + \delta^I n_{t-1}^I - \sum_k m_{t-1}^k v_{t-1}^k. \quad (2.3)$$

To simplify the analysis and to keep the model tractable we assume that all transitions between formal and informal employment occur through unemployment, i.e. direct transitions between formal and informal jobs are not allowed. This is a simplification we make to keep the model tractable and concentrate on the interaction of the financial sector with the real side of the model economy.

2.3.3 Households

There is a representative household in this economy, which consists of a continuum of measure one of family members. We adopt the “large family” framework of Merz (1995) and Andolfatto (1996), which ensures that all household members are able to enjoy the same level of consumption regardless of labor market status of its individual members. Before we formulate the intertemporal problem that a household solves every period, we define the intra-temporal problem of allocating consumption between two consumption goods produced in each of the sectors. Specifically, a household consumes good c_t , which is a composite of goods produced in formal and informal

¹⁰Charlot et al. (2010) and Ulyssea (2010) report similar results.

sectors, c_t^F and c_t^I , and it is defined as $c_t = (\kappa(c_t^F)^\rho + (1 - \kappa)(c_t^I)^\rho)^{1/\rho}$. The parameter $0 < \kappa < 1$ denotes the share of goods produced in the formal sector and ρ is the elasticity of substitution between c_t^F and c_t^I . The household solves the problem of minimizing total costs of purchasing one unit of the composite consumption good. Price of the composite consumption good c_t is fixed to one throughout the model. Relative prices of goods produced by formal and informal firms are denoted by p_t^F and p_t^I , respectively. Solution to this intra-temporal problem yields the following expressions that govern the allocation of consumption across formal and informal goods

$$p_t^F = \kappa \left(\frac{c_t^F}{c_t} \right)^{\rho-1} \quad (2.4)$$

$$p_t^I = (1 - \kappa) \left(\frac{c_t^I}{c_t} \right)^{\rho-1}. \quad (2.5)$$

Employed household members supply labor to firms in both sectors and collect wages, the unemployed receive fixed unemployment benefits and all household members collect returns on state-contingent one period risk-free bonds. Total household revenue, on the other hand, is used for consumption, payment of lump-sum taxes and purchase of one period risk-free bonds. Let's denote the value of a household at time t as $\mathcal{H}_t$. Thus, a household solves the following problem

$$\begin{aligned} \mathcal{H}_t = \max_{c_t, b_t} & \left\{ \frac{c_t^{1-\frac{1}{\zeta}} - 1}{1 - \frac{1}{\zeta}} + \beta \mathbb{E}_t \{ \mathcal{H}_{t+1} \} \right\} \\ \text{subject to} & \\ w_t^F n_t^F + w_t^I n_t^I + u_t \kappa + (1 + r_{t-1})b_{t-1} + \pi_t = c_t + \tau + b_t, \end{aligned} \quad (2.6)$$

where β is a discount factor, w_t^F and w_t^I denote wages, κ is fixed unemployment benefit, b_t denotes real bonds, τ is fixed lump-sum tax payment and π_t is dividend payment due to ownership of firms. Solving this problem yields the following first order conditions

$$\lambda_t^c = c_t^{-\frac{1}{\zeta}} \quad (2.7)$$

$$\lambda_t^c = \beta(1 + r_t) \mathbb{E}_t \lambda_{t+1}^c, \quad (2.8)$$

where λ_t^c is a Lagrange multiplier associated with the household's budget constraint. Let's define the probability of finding a job in one of the sectors as $\phi_t^k = \psi^k(\theta_t^k)^{1-\alpha}$, $k = F, I$. Then, we can reformulate the laws of motion for employment for a household member in each sector as

$$n_{t+1}^F = (1 - \delta^F) n_t^F + \phi_t^F u_t \quad (2.9)$$

$$n_{t+1}^I = (1 - \delta^I) n_t^I + \phi_t^I u_t, \quad (2.10)$$

Using these reformulated laws of motion we determine the following value functions, which define the marginal value of being in each labor market state,

$$\mathcal{H}_{n,t}^F = \lambda_t^c w_t^F + \beta \mathbb{E}_t \left\{ (1 - \delta^F) \mathcal{H}_{n,t+1}^F + \delta^F \mathcal{H}_{u,t+1} \right\} \quad (2.11)$$

$$\mathcal{H}_{n,t}^I = \lambda_t^c w_t^I + \beta \mathbb{E}_t \left\{ (1 - \delta^I) \mathcal{H}_{n,t+1}^I + \delta^I \mathcal{H}_{u,t+1} \right\} \quad (2.12)$$

$$\mathcal{H}_{u,t} = \lambda_t^c \kappa + \beta \mathbb{E}_t \left\{ (1 - \phi_t^F - \phi_t^I) \mathcal{H}_{u,t+1} + \phi_t^F \mathcal{H}_{n,t+1}^F + \phi_t^I \mathcal{H}_{n,t+1}^I \right\}. \quad (2.13)$$

Terms $\mathcal{H}_{n,t}^F$, $\mathcal{H}_{n,t}^I$ and $\mathcal{H}_{u,t}$ denote the value of formal and informal employment and unemployment, respectively. Tax revenues are used by the government to finance the payment of unemployment benefits and the government budget is always balanced.

2.3.4 Financial sector and the firm-lender relationship

In addition to good and labor markets there is a perfectly competitive financial sector, which consists of an infinite number of lenders. These lenders have unlimited access to funds that can be lent only to formal firms. We assume that vacancy creation costs and wages in the formal sector should be paid out before the production in the formal sector occurs. To finance their operations, firms in the formal sector use their net worth, but we also assume that the net worth is not sufficient every period and formal firms should resort to external funds to finance their operational needs. One interpretation of this assumption is that these external funds help a firm gain and maintain access to more productive production technology. Net worth of a formal firm, nw_t , is defined below.

The relationship between formal firms and lenders is modeled as in [Chugh \(2009\)](#) and [Petrosky-Nadeau \(2011\)](#), which model the relationship using the modeling approach proposed in [Carlstrom and Fuerst \(1997\)](#). In other words, the relationship between lenders and borrowers is assumed to be subject to frictions and costly state verification. The advantage of [Chugh \(2009\)](#) and [Petrosky-Nadeau \(2011\)](#) for our model is that in such setup the effect of the financial sector on a firm can be modeled without necessarily modeling the process of capital accumulation and investment. In the model changes in the costs of financing directly affect the hiring decision of firms.

Let's define formal firm's period t operational costs as $\mathcal{C}_{it}^F$ (firms are indexed by i). As we have specified above a formal firm always needs to resort to external finance to be able to operate, i.e. $\mathcal{C}_{it}^F > nw_{it}$. This implies that the size of the loan is defined by $\mathcal{C}_{it}^F - nw_{it}$. To introduce frictions into the borrower-lender relationship we assume that in addition to aggregate productivity shock, formal firms are subject to idiosyncratic productivity shock, ω_{it} , with mean $\mathbb{E}(\omega_{it}) = 1$. We denote by $\Phi(\omega)$ cumulative distribution function and by $\phi(\omega)$ density function of this shock. Idiosyncratic productivity shock is identically and independently distributed across time and formal firms. The actual realization of this shock is revealed after the debt contract between a formal firm and a lender is signed and its value is known only to a formal firm. The idiosyncratic shock introduces uncertainty into the firm-lender relationship and implies that some firms in the formal sector turn out to be more productive and successful than others. This feature of the model can also be easily justified by empirical observation that the share of firms operating formally is relatively higher in technically sophisticated sectors, they operate at a higher capital-labor ratio and they are key drivers of innovation and productivity growth in the economy. These features of the formal sector also imply that there is relatively higher degree of uncertainty in the production process, which stems from introducing and making investments into new technologies. The idiosyncratic shock captures specifically this observation.

Let's define the revenues of a formal firm in terms of a composite consumption good as $\omega_{it}a_t n_{it}^F p_t^F$. Formal firms and lenders sign a 'standard debt contract', which determines the cutoff level of idiosyncratic productivity, $\bar{\omega}_{it}$, and the size of the loan a firm is able to take in period t . This debt contract specifies that if the realization of the idiosyncratic shock is below the threshold level, $\omega_{it} < \bar{\omega}_{it}$ the firm hands over to the lender all the revenues it is able to collect. In case $\omega_{it} \geq \bar{\omega}_{it}$ the firm is able to pay back the loan and it keeps the rest of its revenues in the amount of $(\omega_{it} - \bar{\omega}_{it})a_t n_{it}^F p_t^F$. This is an intra-period loan contract and the loan is paid off before agents move to the next period. To exclude a situation, when a successful firm underreports its revenues and declares bankruptcy (to keep a larger share of revenues), firms that declare bankruptcy are monitored by a lender at a cost. This monitoring cost is a share of revenues that a lender receives after a firm declares bankruptcy and it is denoted by μ_t . Following [Petrosky-Nadeau \(2011\)](#) we assume that the monitoring cost is a function of aggregate productivity and it behaves countercyclically, i.e. $\mu_t = h(a_t)$, $h'(a_t) < 0$ and $h''(a_t) < 0$.

Let's define the probability of being monitored by a lender after observing $\omega_{it} < \bar{\omega}_{it}$ as $G(\bar{\omega}_{it}) = \int_0^{\bar{\omega}_{it}} \omega_{it} d\Phi(\omega)$. Let's also define the lender's share of output before the monitoring costs are paid as $\Gamma(\bar{\omega}_{it}) = \bar{\omega}_{it}(1 - \Phi(\bar{\omega}_{it})) + G(\bar{\omega}_{it})$. The fact that a representative lender operates in a perfectly competitive financial sector and makes zero profit from lending to formal firms implies that it operates as long as the share of revenues it collects equals the amount of the loan it gives out. This condition then allows us to define the lender's participation constraint as

$$(\Gamma(\bar{\omega}_{it}) - \mu_t G(\bar{\omega}_{it})) a_t p_t^F n_{it}^F = C_{it}^F - n w_{it}, \quad (2.14)$$

where $C_{it}^F - n w_{it}$ denotes the size of the loan to a formal firm and $\Gamma(\bar{\omega}_{it}) - \mu_t G(\bar{\omega}_{it})$ is the lender's share of firm's output net of monitoring costs. Since lenders operate in a perfectly competitive financial sector, in equilibrium terms of the debt contract maximize expected profit of a formal firm. Thus, this simplified setup allows us to embed the process of determining terms of the debt contract into formal firm's profit maximization problem.

Restricting access to credit only to firms in the formal sector implies that informal firms are not strongly affected by changes in the cost of credit and financial market conditions. In other words, the implicit assumption is that firms that operate formally and informally are not highly dependent on each other.¹¹ Recent empirical evidence justifies this assumption. Using the 2003 wave of the Brazilian survey of informal firms mentioned above [Paula and Scheinkman \(2010\)](#) check for the existence of the chain effect of formality and informality. The study shows empirically and theoretically that the way how value added tax is currently collected and tax credit is estimated induces formal firms to do business primarily with other formal firms. Consequently, the same mechanism would induce informal firms to do more business with informal firms. We, therefore, argue that due to the evidence that formal and informal firms are not closely integrated though the production chain the effect of varying cost of credit is most likely strongest within the group of formal firms.

2.3.5 Firms

Continuum of firms of size one in formal and informal sectors hire labor from households and produce according to following production technologies

$$y_{it}^F = a_t n_{it}^F \quad (2.15)$$

$$y_{it}^I = \gamma a_t n_{it}^I. \quad (2.16)$$

Firms in both sectors are subject to a common aggregate productivity shock a_t , but we further assume that production technology in the informal sector is strictly inferior to the formal one and we model this assumption by augmenting the production function of informal firms with an exogenous multiplicative parameter $0 < \gamma < 1$.¹² This assumption implies that for a given value of aggregate productivity, workers employed by formal firms engage in a more productive production process, while workers employed by informal firms are a factor in a less productive technology. As it is customary in the literature, the aggregate productivity is assumed to be driven by the following AR(1) process and it is the only source of aggregate disturbance in the model economy: $\log a_t = \varrho \log a_{t-1} + \varepsilon_t$, $0 \leq \varrho < 1$, $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. The value of the aggregate productivity shock is equal across formal and informal sectors, but the overall effect of the aggregate productivity shock across sectors would differ due to existence of credit frictions.

Firms create vacancies every period, announce them to a common pool of the unemployed and after successful matches are formed workers are employed next period. Vacancies in the informal

¹¹Though, as empirical evidence suggests labor flows between two types of employment arrangements are actually not restricted.

¹²There is ample evidence that firms that operate informally are significantly less productive than formal firms.

sector can be opened at lower cost than in the formal sector. On the contrary, formal sector firms incur non-negligible costs every time they decide to open a new formal vacancy. This assumption captures the fact that a formal vacancy should comply with labor, safety and other relevant regulation, which would inevitably incur higher costs to a firm.

Let's denote by $\mathcal{V}_{it}^F$ the value of a formal firm at time t . A firm solves the following problem:

$$\mathcal{V}_{it}^F = \max_{v_{it}^F, \bar{\omega}_{it}} \left\{ (1 - \Gamma(\bar{\omega}_{it})) a_t p_t^F n_{it}^F + \beta \mathbb{E} \left\{ \Lambda_{t,t+1} \mathcal{V}_{it+1}^F \right\} \right\} \quad (2.17)$$

subject to

$$\mathcal{C}_{it}^F - n w_{it} = (\Gamma(\bar{\omega}_{it}) - \mu_t G(\bar{\omega}_{it})) a_t p_t^F n_{it}^F \quad (2.18)$$

$$n_{it+1}^F = (1 - \delta^F) n_{it}^F + m_t^F v_{it}^F, \quad (2.19)$$

where v_{it}^F is the number of vacancies created by a firm, φ^F is a fixed cost of creating one formal vacancy, $\mathcal{C}_{it}^F = w_t^F n_{it}^F + \varphi^F v_{it}^F$ is the total operational cost of a firm and $\Lambda_{t,t+1} = \lambda_{t+1}^c / \lambda_t^c$ is a stochastic discount factor.¹³ Term $1 - \Gamma(\bar{\omega}_{it})$ denotes the firm's share of output after necessary payments to the lender are made. According to this problem, a firm maximizes its value by choosing the number of vacancies to open and by determining the threshold level of the idiosyncratic productivity shock specified in the debt contract. This maximization is performed subject to the lender's participation constraint (2.18) and the law of motion of employment in the formal sector (2.19). Solution to this problem yields the following first order conditions:

$$\lambda_t^\omega = \frac{\Gamma'(\bar{\omega}_{it})}{\Gamma(\bar{\omega}_{it}) - \mu_t G'(\bar{\omega}_{it})} \quad (2.20)$$

$$\frac{\lambda_t^\omega \varphi^F}{m_t^F} = \beta \mathbb{E} \left\{ \Lambda_{t,t+1} \mathcal{V}_{n,it+1}^F \right\}, \quad (2.21)$$

where λ_t^ω is a Lagrange multiplier associated with the lender's participation constraint, which can also be interpreted as the shadow price of external funds. Expression (2.20) governs the evolution of $\bar{\omega}_t$ in equilibrium and (2.21) is formal firm's job creation condition. Taking the derivative of (2.17) with respect to n_{it}^F gives the following envelope condition

$$\mathcal{V}_{n,it}^F = (1 - \Gamma(\bar{\omega}_{it})) a_t p_t^F + \lambda_t^\omega \left((\Gamma(\bar{\omega}_{it}) - \mu_t G(\bar{\omega}_{it})) a_t p_t^F - w_t^F \right) + (1 - \delta^F) \beta \mathbb{E}_t \left\{ \Lambda_{t,t+1} \mathcal{V}_{n,it+1}^F \right\}. \quad (2.22)$$

Then, using the first order condition (2.21) and the envelope condition (2.22) we reformulate the job creation condition of a formal firm to obtain

$$\frac{\lambda_t^\omega \varphi^F}{m_t^F} = \beta \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left\{ \Omega(\bar{\omega}_{it+1}) a_{t+1} p_{t+1}^F - \lambda_{t+1}^\omega w_{t+1}^F + (1 - \delta^F) \frac{\lambda_{t+1}^\omega \varphi^F}{m_{t+1}^F} \right\} \right\}, \quad (2.23)$$

where $\Omega(\bar{\omega}_{it+1}) = 1 - \Gamma(\bar{\omega}_{it+1}) + \lambda_{t+1}^\omega (\Gamma(\bar{\omega}_{it+1}) - \mu_{t+1} G(\bar{\omega}_{it+1}))$.

Informal firms in the economy solve the following simpler problem:

$$\mathcal{V}_{it}^I = \max_{v_{it}^I} \left\{ \gamma a_t p_t^I n_{it}^I - \mathcal{C}_{it}^I + \beta \mathbb{E}_t \left\{ \Lambda_{t,t+1} \mathcal{V}_{it+1}^I \right\} \right\} \quad (2.24)$$

subject to

$$n_{it+1}^I = (1 - \delta^I) n_{it}^I + m_t^I v_{it}^I, \quad (2.25)$$

¹³The latter appears in the firm optimization problem due to ownership of firms by households.

where $\mathcal{C}_{it}^I = w_t^I n_{it}^I + \varphi^I v_{it}^I$ is the total operational cost of an informal firm and φ^I is the fixed cost of creating one informal vacancy. This problem states that an informal firm maximizes the current profit and the discounted future value of the firm by choosing the number of vacancies to create in the current period subject to a law of motion of the labor force employed in the informal sector. Solving this problem yields the job creation condition in the informal sector

$$\frac{\varphi^I}{m_t^I} = \beta \mathbb{E}_t \left\{ \Lambda_{t,t+1} \mathcal{V}_{n,it+1}^I \right\}, \quad (2.26)$$

which using the corresponding envelope condition can be reformulated to

$$\frac{\varphi^I}{m_t^I} = \beta \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left\{ \gamma a_{t+1} p_{t+1}^I - w_{t+1}^I + (1 - \delta^I) \frac{\varphi^I}{m_{t+1}^I} \right\} \right\}. \quad (2.27)$$

Now we are ready to define the evolution of the formal firm's net worth. Net worth is the share of revenues a formal firm keeps after paying off the loan and it is defined as

$$nw_{it+1} = \zeta (1 - \Gamma(\bar{\omega}_{it})) a_t p_t^F n_{it}^F, \quad (2.28)$$

where $0 < \zeta < 1$ is necessary to rule out a situation, when a formal firm is able to self-finance its operational expenses. Using lender's participation constraint (2.14) we can reformulate the expression for the evolution of the net worth as

$$nw_{it+1} = \zeta \left(a_t p_t^F n_{it}^F - (\mathcal{C}_{it}^F - nw_{it}) \left(1 + \frac{\mu_t G(\bar{\omega}_{it}) a_t p_t^F n_{it}^F}{\mathcal{C}_{it}^F - nw_{it}} \right) \right), \quad (2.29)$$

which explicitly shows that the net worth is the difference between formal firm's revenues, costs and debt related payments. It is easy to see that fraction $(\mu_t G(\bar{\omega}_{it}) a_t p_t^F n_{it}^F) / (\mathcal{C}_{it}^F - nw_{it})$ is equivalent to a premium on external funds that a formal firm borrows every period.

2.3.6 Wage bargaining

Wages in both sectors are determined through a standard Nash bargaining mechanism. Let's define the total surplus that formal firms and new matched workers share through the bargaining process as $\mathcal{S}_t^F = \mathcal{W}_t^F + \mathcal{V}_{n,t}^F$. The term $\mathcal{W}_t^F = \frac{\mathcal{H}_{n,t}^F - \mathcal{H}_{u,t}}{\lambda_t^c}$ determines the net value of being formally employed for a household member, where we divide the term $\mathcal{H}_{n,t}^F - \mathcal{H}_{u,t}$ by λ_t^c to represent the net value in terms of a composite consumption good. Using expressions for the marginal value of being in three labor market states (2.11)-(2.13) and the envelope condition for formal employment we determine the total surplus that formal firms and matched workers share as

$$\begin{aligned} \mathcal{S}_t^F &= \Omega(\bar{\omega}_t) a_t p_t^F + (1 - \lambda_t^\omega) w_t^F - \kappa \\ &\quad + (1 - \delta^F) \beta \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left\{ \mathcal{V}_{n,t+1}^F + \mathcal{W}_{t+1}^F \right\} \right\} \\ &\quad - \beta \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left\{ \phi_t^F \mathcal{W}_{t+1}^F + \phi_t^I \mathcal{W}_{t+1}^I \right\} \right\}. \end{aligned} \quad (2.30)$$

Formal firms and newly matched workers solve the following bargaining problem

$$\max_{w_t^F} \left(\mathcal{W}_t^F \right)^\eta \left(\mathcal{V}_{n,t}^F \right)^{1-\eta}, \quad (2.31)$$

where η denotes the bargaining power of workers employed formally. We assume the same value of bargaining power in both sectors. After solving this problem we find that the surplus is shared according to following expressions

$$\mathcal{W}_t^F = x_t \mathcal{S}_t^F \quad (2.32)$$

$$\mathcal{V}_{n,t}^F = (1 - x_t) \mathcal{S}_t^F, \quad (2.33)$$

where $x_t = \frac{\eta}{\eta + \lambda_t^\omega(1-\eta)}$. Simplifying these expressions further and after solving for w_t^F we obtain the following wage equation in the formal sector

$$w_t^F = \eta \left(\frac{\Omega(\bar{w}_t)}{\lambda_t^\omega} a_t p_t^F + \frac{\lambda_t^\omega}{\lambda_{t+1}^\omega} \theta_t^F \varphi^F + \theta_t^I \varphi^I \right) + \eta(1 - \delta^F) \frac{\lambda_t^\omega \varphi^F}{m_t^F} \left(\frac{1}{\lambda_t^\omega} - \frac{1}{\lambda_{t+1}^\omega} \right) + (1 - \eta)\kappa. \quad (2.34)$$

Wage in the formal sector depends not only on terms that are determined in the debt contract but also on the labor market tightness in the informal sector. Equivalently, we define the total surplus, which an informal firm and a matched worker share as $\mathcal{S}_t^I = \mathcal{V}_{n,t}^I + \mathcal{W}_t^I$, where $\mathcal{W}_t^I = \frac{\mathcal{H}_{n,t}^I - \mathcal{H}_{u,t}}{\lambda_t^c}$. Using expressions for the envelope condition in the informal sector and the net value of being employed in the informal sector, we further define the total surplus as

$$\begin{aligned} \mathcal{S}_t^I &= \gamma a_t p_t^I - \kappa \\ &+ (1 - \delta^I) \beta \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left\{ \mathcal{V}_{n,t+1}^I + \mathcal{W}_{t+1}^I \right\} \right\} \\ &- \beta \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left\{ \phi_t^I \mathcal{W}_{t+1}^I + \phi_t^F \mathcal{W}_{t+1}^F \right\} \right\}. \end{aligned} \quad (2.35)$$

Nash bargaining problem identical to (2.31) yields the following sharing conditions

$$\mathcal{W}_t^I = \eta \mathcal{S}_t^I \quad (2.36)$$

$$\mathcal{V}_{n,t}^I = (1 - \eta) \mathcal{S}_t^I. \quad (2.37)$$

Performing the same algebra on equations (2.35)-(2.37) we solve for the following wage equation in the informal sector

$$w_t^I = \eta \left(\gamma a_t p_t^I + \theta_t^I \varphi^I + \frac{\lambda_t^\omega}{\lambda_{t+1}^\omega} \theta_t^F \varphi^F \right) + (1 - \eta)\kappa. \quad (2.38)$$

As in case of the formal sector wage, wage of an informally employed worker is also affected by labor market tightness in the formal sector and changes in financial sector conditions.

2.3.7 Market clearing and equilibrium

In equilibrium goods and factor markets clear and the resource constraints in both sectors are defined by

$$c_t^F + (\varphi^F / p_t^F) v_t^F = y_t^F (1 - \mu_t G(\bar{w}_t)) \quad (2.39)$$

$$c_t^I + (\varphi^I / p_t^I) v_t^I = y_t^I, \quad (2.40)$$

where expression (2.39) states that total output of formal firms net of monitoring cost is used for the consumption of formal goods and creation of formal vacancies. Given the stochastic process for the evolution of the aggregate productivity $\{a_t\}_{t=0}^\infty$, equilibrium in this economy is the sequence of prices and allocations $\{y_t^F, y_t^I, c_t^F, c_t^I, n_t^F, n_t^I, u_t, v_t^F, v_t^I, r_t, p_t^F, p_t^I, w_t^F, w_t^I, \lambda_t^\omega, n w_t, \bar{w}_t\}_{t=0}^\infty$, which for given initial conditions satisfy equations (2.2), (2.3), (2.4), (2.5), (2.7), (2.8), (2.14) (2.15), (2.16), (2.20), (2.23), (2.27), (2.29), (2.34), (2.38), (2.39), (2.40), definitions of the job filling rate in formal and informal sectors, job finding rates, labor market tightness and expressions governing the evolution of firm and lender shares of revenues in the formal sectors.¹⁴

¹⁴Full system of nonlinear equations is provided in Appendix 2.B.

Table 2.2: Parameters and their values

Parameter	Value	Notes
β	0.9723	Time preference parameter
$\Phi(\bar{\omega})$	0.0342	Average quarterly share of non-performing loans to total loans
μ	0.7	Steady state monitoring cost
σ_{ω}	0.55	Standard deviation of the idiosyncratic productivity shock
γ	0.7	Term in the production function of informal firms
φ^F	0.24	Cost of opening a formal vacancy
φ^I	0.08	Cost of opening an informal vacancy
ψ^F	0.32	Efficiency of formal matching
ψ^I	0.48	Efficiency of informal matching
δ^F	0.05	Exogenous rate of separation of formal jobs
δ^I	0.08	Exogenous rate of separation of informal jobs
η	0.5	Bargaining power
α	0.5	Elasticity of the matching function
κ	0.26	Fixed unemployment benefit
$\varkappa$	0.5	Share of formal goods in the composite consumption good
ρ	0.3	Elasticity of substitution between formal and informal goods
ζ	0.85	Parameter of the evolution of net worth
ϱ	0.97	Persistence of the aggregate productivity process
σ_a	0.011	Standard deviation of the aggregate productivity shock

2.4 Analysis of the model

2.4.1 Steady state equilibrium and calibration

In steady state equilibrium the aggregate productivity is constant and equal to $a = 1$. Given an empirically observed value of bankruptcy rate, which is equivalent to $\Phi(\bar{\omega})$ in the model, we calibrate the threshold value of the idiosyncratic productivity shock, $\bar{\omega}$, and its standard deviation, σ_{ω} .¹⁵ Specifically, we choose values of $\bar{\omega}$, σ_{ω} and μ , such that the value of $\Phi(\bar{\omega})$ equals the average quarterly share of non-performing loans in total loans in Brazil. We estimate this value to be about 3.24%.¹⁶ At the same time we target the formal firm's steady state leverage ratio of 0.5, which is within the range of a number of estimates of this ratio for Brazilian firms. This calibration results in values $\sigma_{\omega} = 0.55$ and $\mu = 0.7$. We also target the steady state unemployment rate of approximately 12% and the share of formally and informally employed workers of about 42% and 46%, respectively, as in [Ulyssea \(2010\)](#).

We choose the multiplicative term γ in the production function of informal firms to be equal to 0.7, which corresponds to the estimated 30 percent productivity gap between formal and informal firms in Brazil ([Perry et al., 2007](#)). Cost of creating a formal vacancy is assumed to be significantly higher than cost of creating an informal one. Elasticity of the matching function to unemployment and the bargaining power of workers equal $\alpha = \eta = 0.5$. Share of formal goods in the composite good, $\varkappa$, also equals 0.5. Elasticity of substitution between formal and informal goods, ρ , is chosen to be 0.3 ([Ulyssea, 2010](#)). Time preference parameter equals 0.9723, which corresponds to a real interest rate of about 11% on annualized basis. Parameters of

¹⁵Idiosyncratic productivity shock is assumed to be log-normally distributed.

¹⁶Information on non-performing loans in Brazil is available from the IMF. Specifically, we use the average value of "Non-performing loans to gross total loans" series available in the IMF Financial Soundness Indicators database for the period of 2005:Q1 - 2010:Q4.

the autoregressive productivity process are chosen to be $\varrho = 0.97$ and $\sigma_a = 0.011$. Full list of variables and their corresponding values are given in Table 2.2.

In steady state equilibrium flows into and out of unemployment are equal, i.e. $\delta^F n^F = \phi^F u$ and $\delta^I n^I = \phi^I u$. For given values of $\bar{\omega}$, σ_ω and μ the system of nonlinear equations characterizing the equilibrium can be further simplified to the following system of nine equations in nine unknowns $(\theta^F, \theta^I, c^F, c^I, c, n^F, n^I, p^F, p^I)$, which we numerically solve for the steady state equilibrium:

$$p^F = \frac{\lambda^\omega}{\Omega(\bar{\omega})} \left[\frac{\eta}{1-\eta} (\theta^F \varphi^F + \theta^I \varphi^I) + \kappa + \frac{(1-\beta(1-\delta^F))}{(1-\eta)\beta} \frac{\varphi^F}{m^F(\theta^F)} \right] \quad (2.41)$$

$$p^I = \frac{1}{\gamma} \left[\frac{\eta}{1-\eta} (\theta^F \varphi^F + \theta^I \varphi^I) + \kappa + \frac{(1-\beta(1-\delta^I))}{(1-\eta)\beta} \frac{\varphi^I}{m^I(\theta^I)} \right] \quad (2.42)$$

$$p^F = \kappa (c^F)^{\rho-1} (c)^{1-\rho} \quad (2.43)$$

$$p^I = \kappa (c^I)^{\rho-1} (c)^{1-\rho} \quad (2.44)$$

$$c = p^F c^F + p^I c^I \quad (2.45)$$

$$c^F = (1 - \mu G(\bar{\omega})) n^F - (\varphi^F / p^F) \theta^F (1 - n^F - n^I) \quad (2.46)$$

$$c^I = \gamma n^I - (\varphi^I / p^I) \theta^I (1 - n^F - n^I) \quad (2.47)$$

$$\theta^F = \left(\frac{1}{\psi^F} \frac{\delta^F n^F}{1 - n^F - n^I} \right)^{\frac{1}{1-\alpha}} \quad (2.48)$$

$$\theta^I = \left(\frac{1}{\psi^I} \frac{\delta^I n^I}{1 - n^F - n^I} \right)^{\frac{1}{1-\alpha}}. \quad (2.49)$$

Equations (2.41) and (2.42) are obtained using the job creation condition and the wage equation, respectively for each sector. Equations (2.43) and (2.44) are derived from consumption allocation equations, (2.46) and (2.47) are resource constraints, (2.48) and (2.49) are steady state values of labor market tightness in both sectors.

Table 2.3: Long-run values of labor market variables for different values of steady state monitoring cost μ (in percent of total labor force).

μ	Formal employment	Informal employment	Unemployment
0	44.7	43.5	11.8
0.5	43.0	45.0	12.0
1	41.3	46.3	12.4

Current parametrization results in the value of the steady state premium on external funds of 0.02. Steady state job finding rates for formal and informal employment are 0.17 and 0.30, respectively. The ratio of the unemployment benefit to the formal wage, i.e. replacement ratio, equals 0.66. We conducted a small simulation exercise to understand how steady state values of key labor market variables react to changes in the monitoring cost, μ , which can be interpreted as a variable characterizing the extent of frictions in the financial sector. Results of this exercise are given in Table 2.3. An increase in the value of μ corresponds to higher cost of monitoring formal firms and more severe moral hazard problem in the firm-lender relationship. Changes in the steady state value of the monitoring cost result in relatively sizable movements of workers across formal and informal employment but they do not have large influence on the unemployment rate.

Table 2.4: Correlations with formal sector output, y^F , total output, y , and relative volatilities.

	ϕ^F	ϕ^I	u	n^F	n^I
$\rho(x, y^F)$	0.961	0.054	-0.573	0.464	-0.275
$\rho(x, y)$	0.961	-0.191	-0.358	0.231	-0.031
σ_x / σ_{y^F}	2.420	0.646	0.800	0.379	0.148
σ_x / σ_y	2.614	0.698	0.864	0.410	0.160

Notes: Superscripts F and I indicate formal and informal employment, respectively; ϕ denotes the job finding rate, u is the unemployment rate, n^F is the share of formal employment and n^I is the share of informal employment in total employment. Moments are calculated after detrending simulated series using a Hodrick-Prescott filter with the smoothing parameter $\lambda = 1600$.

In the model lower values of μ make borrowing cheaper and that allows formal firms to expand further and hire more workers. An increase in the monitoring cost indicates the existence of more severe frictions in the financial market that leads to decrease in the share of the labor force employed formally. As a result of rising monitoring cost more workers are employed informally. Nevertheless, not all workers who are not able to find a formal job are employed informally due to existing matching frictions and non-zero hiring cost in the informal labor market.

2.4.2 Model dynamics

We analyze dynamic properties of the model by log-linearizing the resulting system of nonlinear equations around the steady state equilibrium described above. In particular, we are interested in the effect of one standard deviation negative productivity shock on the evolution of key variables. In the analysis of the model dynamics we assume that one standard deviation negative productivity shock leads to about 25% increase in the monitoring cost. Elasticity of the monitoring cost to aggregate productivity shock is calibrated according to this assumption. Impulse responses of key model variables to the productivity shock are shown in Figure 2.3. Negative productivity shock leads to lower productivity in both types of firms but in addition it leads to decrease in the net worth of firms that operate formally, which exacerbates credit frictions. Consequently, higher level of frictions limits the ability of these firms to create formal vacancies and this further limits the ability of workers to enter formal employment. The share of workers employed formally decreases and the share of workers employed informally increases. Due to matching frictions in the informal labor market and non-zero job creation costs, not all workers who are unable to enter formal employment find informal jobs and, thus, unemployment increases. Output of both sectors declines but output of firms that operate informally returns to its steady state value relatively faster than output of formal firms.

To assess the model's performance in replicating empirical observations we compute relevant statistics of the simulated data after applying the Hodrick-Prescott filter with the smoothing parameter $\lambda = 1600$. Correlations of key variables with output and their relative volatilities are shown in Table 2.4. Statistics are calculated using the formal sector and total output, which is the sum of formal and informal sector output (in terms of the composite consumption good).¹⁷

Job finding rate in the formal sector, ϕ^F , is highly procyclical and about three times more volatile than the job finding rate in the informal sector. This result matches well the empirical moments

¹⁷Since official GDP data to a large extent reflects the output of formally operating firms, model statistics are calculated using both types of output.

estimated using the Brazilian data for the period before 2002. The model suggests that credit frictions that directly affect the ability of formal firms to create new vacancies and maintain existing ones make access to formal jobs significantly more volatile than access to informal jobs as it is observed empirically. Job finding rate in the informal sector, ϕ^I , is generally acyclical. Share of workers employed formally is procyclical and more volatile than the share of informal workers. Unemployment rate is countercyclical but less volatile than output, which indicates that the model requires additional amplification mechanism to increase the relative volatility of unemployment. Difficulty in replicating empirically observed moments of the unemployment rate is a common shortcoming of simple real business cycle models with search and matching frictions in the labor market. Though, the model performs well in replicating the differential behavior of job finding rates over the business cycle, overall the volatility of job finding rates in both sectors and of the unemployment rate are lower than observed in the data. Nevertheless, qualitative predictions of the model are in accordance with empirical observations on the dynamics of formal and informal employment over the business cycle.

2.4.3 Robustness checks

We check the sensitivity of our results obtained using the baseline calibration to changes in key model parameters. Specifically, we look at implications of assuming different values for the steady state monitoring cost, μ , fixed unemployment benefits, κ , and standard deviation of the idiosyncratic productivity shock, σ_ω . Results of this sensitivity analysis are reported in Table 2.5.

Panels 2.5a and 2.5b show results of model simulation for two different values of the steady state monitoring cost μ . Lower value of μ represents an economy with relatively lower level of financial frictions, while higher value of μ indicates the existence of a severe moral hazard problem in the firm-lender relationship. Results of simulations in general highlight the importance of financial frictions for generating the differential behavior of job finding rates in both sectors. When we simulate the model under assumption of $\mu = 0.1$, shares of workers and job findings rates in both sectors become highly procyclical, which is in contrast with our empirical observations using Brazilian data. Assuming a high level of the monitoring cost generates the differential behavior of job finding rates and worker shares as observed in the data.

It is known that the value of fixed unemployment benefits strongly affects the cyclical behavior of real business cycle models with search and matching frictions in the labor market. In this regard, we simulate the model with two different values of the fixed unemployment benefits parameter κ . Since unemployment benefits are paid only to formal workers, we choose values of κ such that we observe two different scenarios. In the first scenario, Table 2.5c, we choose the value of κ such that the replacement ratio (ratio of the unemployment benefit to the steady state wage in the formal sector) is approximately equal to 15%. In equilibrium this results in a lower level of steady state unemployment and, consequently, its lower sensitivity to aggregate productivity shocks. As expected, the relative volatility of key variables declines but the cyclical behavior of some variables changes. Job finding rate in the informal sector becomes highly countercyclical and, contrary to intuition, the unemployment rate becomes somewhat procyclical. In the second scenario, results of which are presented in Table 2.5d, we choose the value of κ , such that the replacement ratio equals 80%. Under this calibration the unemployment rate is highly countercyclical as observed in the data. In comparison to our results with baseline calibration, assuming relatively higher value of unemployment benefits induces job finding rates and worker shares in both sectors to be procyclical.

We also simulate the model assuming two different values for the standard deviation of the

Table 2.5: Correlations with formal sector output, y^F , total output, y , and relative volatilities.

(a) $\mu = 0.1$						(b) $\mu = 0.9$					
	ϕ^F	ϕ^I	u	n^F	n^I		ϕ^F	ϕ^I	u	n^F	n^I
$\rho(x, y^F)$	0.993	0.831	-0.682	0.428	0.190	$\rho(x, y^F)$	0.952	0.157	-0.582	0.479	-0.270
$\rho(x, y)$	0.998	0.725	-0.571	0.269	0.345	$\rho(x, y)$	0.956	-0.088	-0.370	0.250	-0.029
σ_x/σ_{y^F}	2.047	0.362	0.761	0.266	0.130	σ_x/σ_{y^F}	2.507	0.666	0.843	0.388	0.136
σ_x/σ_y	2.104	0.372	0.783	0.273	0.134	σ_x/σ_y	2.720	0.723	0.914	0.421	0.147

(c) $\kappa = 0.06$						(d) $\kappa = 0.32$					
	ϕ^F	ϕ^I	u	n^F	n^I		ϕ^F	ϕ^I	u	n^F	n^I
$\rho(x, y^F)$	0.939	-0.835	0.281	0.319	-0.438	$\rho(x, y^F)$	0.964	0.964	-0.707	0.592	0.778
$\rho(x, y)$	0.925	-0.944	0.485	0.058	-0.187	$\rho(x, y)$	0.988	0.894	-0.567	0.424	0.790
σ_x/σ_{y^F}	1.516	1.117	0.372	0.313	0.236	σ_x/σ_{y^F}	3.325	1.269	1.398	0.472	0.137
σ_x/σ_y	1.620	1.194	0.398	0.335	0.252	σ_x/σ_y	3.551	1.355	1.493	0.504	0.146

(e) $\sigma_\omega = 0.15$						(f) $\sigma_\omega = 0.75$					
	ϕ^F	ϕ^I	u	n^F	n^I		ϕ^F	ϕ^I	u	n^F	n^I
$\rho(x, y^F)$	0.659	-0.051	-0.406	0.867	-0.944	$\rho(x, y^F)$	0.926	0.976	-0.637	0.285	0.726
$\rho(x, y)$	0.839	-0.556	-0.001	0.483	-0.668	$\rho(x, y)$	0.921	0.964	-0.598	0.203	0.758
σ_x/σ_{y^F}	4.191	3.344	1.260	0.828	0.536	σ_x/σ_{y^F}	1.665	0.943	0.891	0.181	0.143
σ_x/σ_y	8.660	6.909	2.604	1.711	1.109	σ_x/σ_y	1.617	0.916	0.865	0.175	0.139

Notes: μ is the steady state monitoring costs, κ denotes the fixed unemployment benefit and σ_ω is the standard deviation of the idiosyncratic productivity shock. Superscripts F and I indicate formal and informal employment, respectively; ϕ denotes the job finding rate, u is the unemployment rate, n^F is the share of formal employment and n^I is the share of informal employment in total employment. Moments are calculated after detrending simulated series using a Hodrick-Prescott filter with the smoothing parameter $\lambda = 1600$.

idiosyncratic productivity shock, σ_ω . Given the expected unit value of idiosyncratic productivity shock, changes in σ_ω lead to different steady state values of the threshold value of the idiosyncratic productivity shock, $\bar{\omega}$. In particular, smaller values of σ_ω shift the mode of the log-normal distribution to the right and the latter approaches the normal distribution. This leads to higher value of the threshold level of the productivity shock, $\bar{\omega}$ and the impact of financial frictions strengthens. The opposite is true, when we raise the value of σ_ω . Under this calibration the positive skewness of the log-normal distribution increases and the threshold level of the idiosyncratic productivity shock, $\bar{\omega}$, declines. The latter implies that the impact of financial frictions on formal firms declines as well. This is reflected in the results of simulations reported in Tables 2.5e-2.5f. Under high level of σ_ω and resulting weak effect of financial frictions, the model behaves in a manner very similar to a case, when we assumed the monitoring cost to be $\mu = 0.1$. In this situation, frictions in the financial market are not pronounced and, thus, job finding rates and shares of workers in both sectors are procyclical. Lowering the value of σ_ω and, thus, strengthening the effect of financial frictions on formal firms results in the differential behavior of job finding rates and shares of workers across sectors. In the latter case the volatility of key labor market variables also increases.

Overall, robustness checks suggest that the model simultaneously requires relatively higher level of unemployment benefits to generate the countercyclical unemployment rate and relatively higher level of financial frictions to generate the differential behavior of job finding rates and labor shares across sectors.

Table 2.6: Correlations with formal sector output, y^F , total output, y , and relative volatilities, model with exogenous wage rigidity.

	ϕ^F	ϕ^I	u	n^F	n^I
$\rho(x, y^F)$	0.748	0.306	-0.691	0.816	-0.866
$\rho(x, y)$	0.848	0.120	-0.558	0.691	-0.791
σ_x / σ_{y^F}	3.499	1.938	1.172	0.462	0.154
σ_x / σ_y	4.541	2.515	1.521	0.599	0.200

Notes: Superscripts F and I indicate formal and informal employment, respectively; ϕ denotes the job finding rate, u is the unemployment rate, n^F is the share of formal employment and n^I is the share of informal employment in total employment. Moments are calculated after detrending simulated series using a Hodrick-Prescott filter with the smoothing parameter $\lambda = 1600$.

2.4.4 Role of wage rigidity in the formal sector

Under baseline parametrization the relative volatility of formal wage to the wage in the informal sector, $\sigma_{w^F} / \sigma_{w^I}$, equals 2.22. This seems implausible, since it is very unlikely that the wage is a primary adjustment mechanism over the business cycle in formally operating firms, where contractual restrictions and other labor market regulations would prevent firms from readily adjusting on this margin. To test how the model performs under wage rigidity we exogenously introduce some level of wage rigidity in the formal sector. Specifically, we assume that actual period t wage in the formal sector is determined according to the following equation:

$$w_t^F = zw_{N,t}^F + (1 - z)w_{t-1}^F, \quad (2.50)$$

where $w_{N,t}^F$ equals equation (2.34), the wage in the formal sector determined through the Nash bargaining mechanism. In simulations with this additional wage equation, we choose $z = 0.3$. Other parameter values remain the same.

Impulse responses of key variables to one standard deviation negative productivity shock are shown in Figure 2.4. Qualitative behavior of key endogenous variables in the augmented model is very similar to the behavior of variables in the model with no wage rigidity. As is shown in Table 2.6, correlations of labor market variables with both types of output do not drastically differ from previous results. Major difference is in terms of relative volatilities. Volatility of the job finding rate in the formal sector increases and approaches the value of the relative volatility observed in the data. Relative volatility of the unemployment rate increases as well but still falls short of the empirically observed value. Thus, the model still requires additional amplification mechanism to increase the volatility of the unemployment rate. Introduction of endogenous job destruction might be one of such mechanisms. Endogenous job destruction would additionally make the model suitable for analyzing separation rates of formal and informal jobs, which also exhibit differential behavior over the business cycle as it is observed in the data.

2.5 Conclusions

We build and analyze a dynamic stochastic general equilibrium model, in which a negative aggregate productivity shock leads to a substantial movement of labor force from formal to informal sector over the business cycle. In the model changes of financial conditions over the business cycle directly affect the borrowing ability of formal firms and significantly hinder the creation of new jobs when aggregate productivity declines. We are able to show that over the

business cycle varying conditions that govern the access of formal firms to credit lead to strongly procyclical and volatile labor market tightness and job-finding rate in the formal sector as it is observed empirically. This, on the other hand, leads to countercyclical unemployment and share of informal workers in the economy.

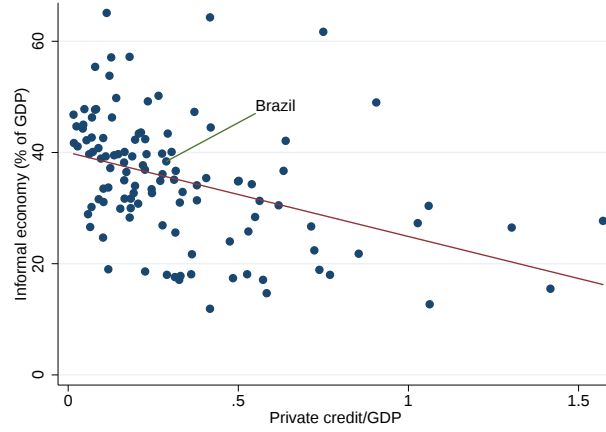
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Appendix 2.A Tables and figures

Figure 2.1: Informality and private credit in 114 developing economies.



Notes: Data on private credit (in percentage of GDP) is available from [Beck et al. \(2009\)](#). Specifically, we use the “Private credit by money banks and other financial institutions / GDP” series. Estimates of the size of informal economy are obtained from [Schneider et al. \(2010\)](#). Observations for year 2005 are plotted.

Table 2.7: Annual transition rates (in %).

	Formally employed	Informally employed	Unemployed
Formally employed	89.5	6.0	4.5
Informally employed	32.2	57.7	10.1
Unemployed	30.9	22.2	46.9

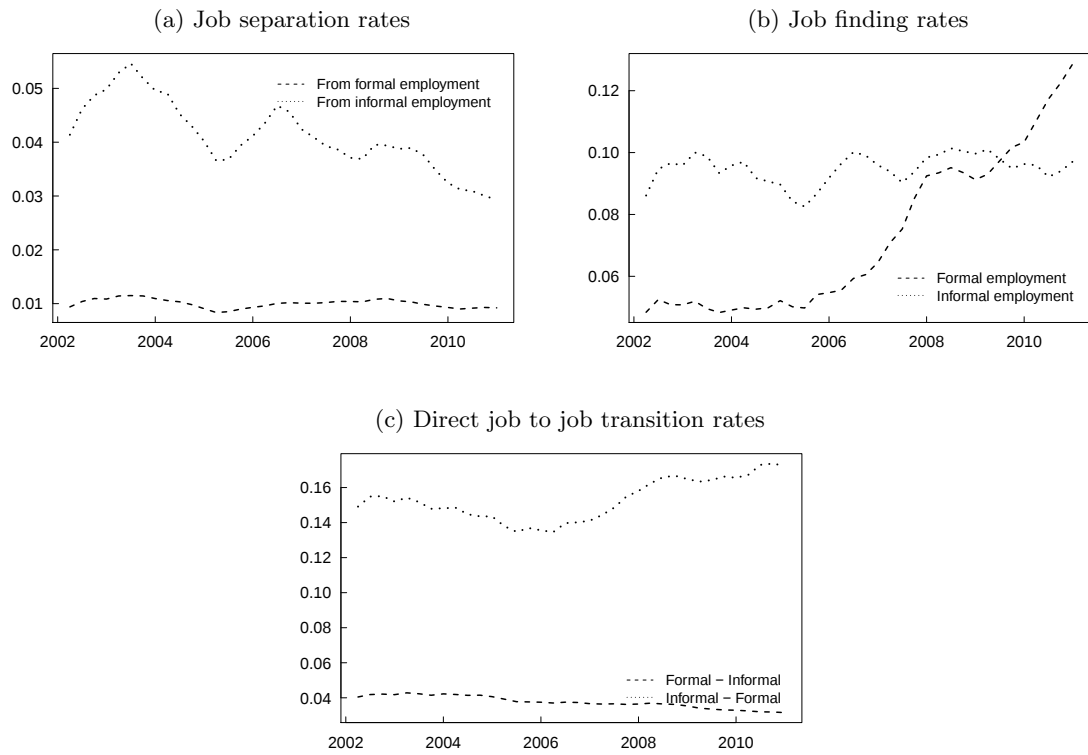
Notes: PME data for 2002 - 2007 is used. Transition rates are computed using information from the fourth and eighth interviews of an individual.

Table 2.8: Correlation and relative volatility of labor market variables in Brazil (extended definition of informal employment).

x	ϕ^F	ϕ^I	λ^F	λ^I	τ^{FI}	τ^{IF}	u	n^F
$\rho(y, x)$	0.50	-0.22	-0.16	-0.37	0.10	0.31	-0.50	0.28
σ_x/σ_y	4.38	3.39	4.33	5.69	2.08	2.54	2.87	0.58

Notes: Superscripts F and I indicate formal and informal employment, respectively; y refers to real quarterly GDP, x refers to the other variables: ϕ denotes the job finding rate, λ is the job separation rate, τ is the direct transition rate between formal and informal employment, u is the unemployment rate and n^F is the share of formal employment in total employment. Statistics are computed after taking natural logarithm and detrending original series using the HP filter with the smoothing parameter $\lambda = 1600$. Informal employment includes both informal salaried employment and informal self-employment.

Figure 2.2: Evolution of labor market variables in Brazil.



Notes: Plotted series are quarterly averages of originally calculated monthly series. All series are smoothed using a moving average filter.

Figure 2.3: Impulse responses to one s.d. negative shock to aggregate productivity.

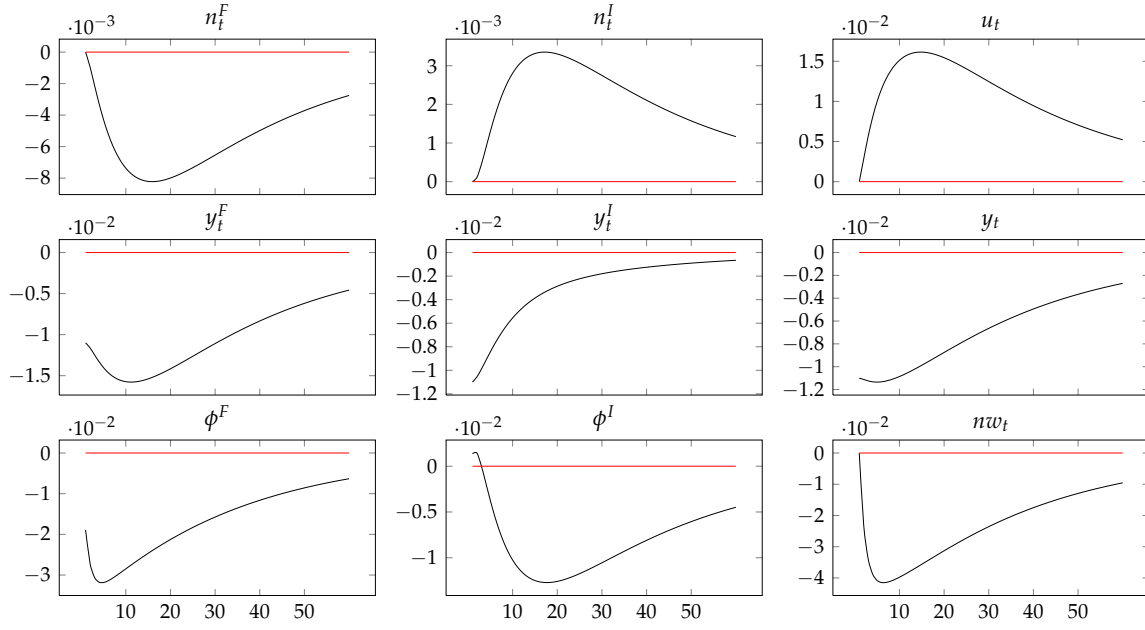
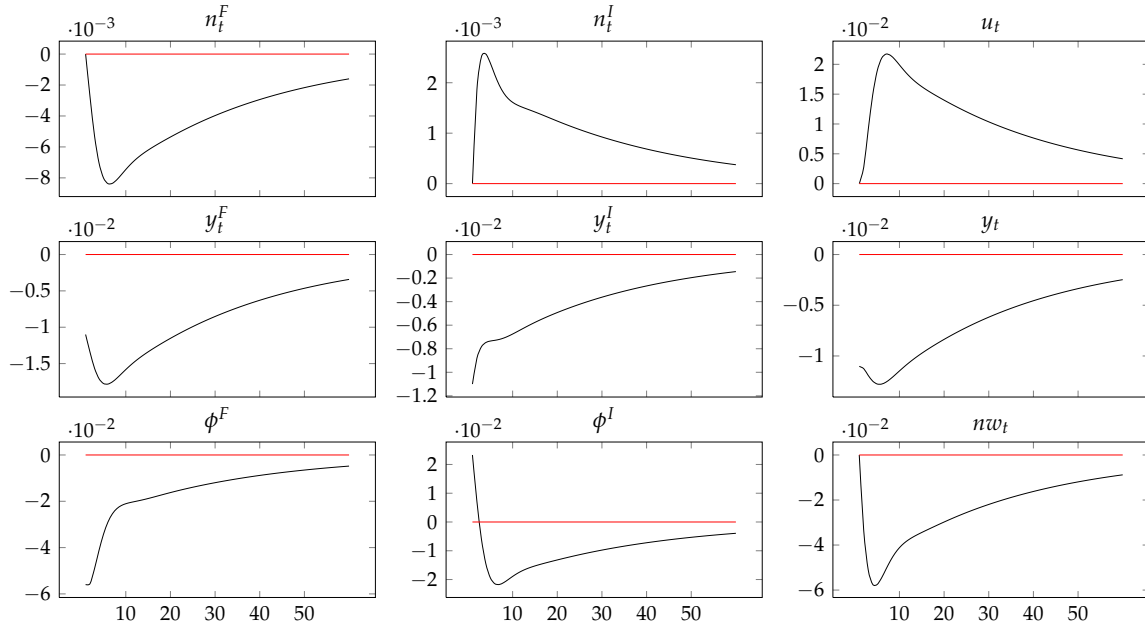


Figure 2.4: Impulse responses to one s.d. negative shock to aggregate productivity, model with exogenous wage rigidity.



Appendix 2.B Nonlinear equations

$$\frac{\lambda_t^\omega \varphi^F}{m_t^F} = \beta \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left\{ \Omega(\bar{\omega}_{t+1}) a_{t+1} p_{t+1}^F - \lambda_{t+1}^\omega w_{t+1}^F + (1 - \delta^F) \frac{\lambda_{t+1}^\omega \varphi^F}{m_{t+1}^F} \right\} \right\} \quad (2.B.1)$$

$$\frac{\varphi^I}{m_t^I} = \beta \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left\{ \gamma a_{t+1} p_{t+1}^I - w_{t+1}^I + (1 - \delta^I) \frac{\varphi^I}{m_{t+1}^I} \right\} \right\} \quad (2.B.2)$$

$$\begin{aligned} w_t^F &= \eta \left(\frac{\Omega(\bar{\omega}_t)}{\lambda_t^\omega} a_t p_t^F + \frac{\lambda_t^\omega}{\lambda_{t+1}^\omega} \theta_t^F \varphi^F + \theta_t^I \varphi^I \right) \dots \\ &\quad + \eta (1 - \delta^F) \frac{\lambda_t^\omega \varphi^F}{m_t^F} \left(\frac{1}{\lambda_t^\omega} - \frac{1}{\lambda_{t+1}^\omega} \right) + (1 - \eta) \kappa \end{aligned} \quad (2.B.3)$$

$$w_t^I = \eta \left(\gamma a_t p_t^I + \theta_t^I \varphi^I + \frac{\lambda_t^\omega}{\lambda_{t+1}^\omega} \theta_t^F \varphi^F \right) + (1 - \eta) \kappa \quad (2.B.4)$$

$$c_t^{-\frac{1}{\xi}} = \lambda_t^c \quad (2.B.5)$$

$$p_t^F = \kappa (c_t^F)^{\rho-1} (c_t)^{1-\rho} \quad (2.B.6)$$

$$p_t^I = (1 - \kappa) (c_t^I)^{\rho-1} (c_t)^{1-\rho} \quad (2.B.7)$$

$$c_t = p_t^F c_t^F + p_t^I c_t^I \quad (2.B.8)$$

$$\lambda_t^c = \beta (1 + r_t) \mathbb{E}_t \lambda_{t+1}^c \quad (2.B.9)$$

$$y_t^F = \frac{1}{(1 - \mu_t G(\bar{\omega}_t))} \left(c_t^F + (\varphi^F / p_t^F) v_t^F \right) \quad (2.B.10)$$

$$y_t^F = a_t n_t^F \quad (2.B.11)$$

$$y_t^I = c_t^I + (\varphi^I / p_t^I) v_t^I \quad (2.B.12)$$

$$y_t^I = \gamma a_t n_t^I \quad (2.B.13)$$

$$n_{t+1}^F = (1 - \delta^F) n_t^F + m_t^F v_t^F \quad (2.B.14)$$

$$n_{t+1}^I = (1 - \delta^I) n_t^I + m_t^I v_t^I \quad (2.B.15)$$

$$u_{t+1} = (1 - \phi_t^F - \phi_t^I) u_t + \delta^F n_t^F + \delta^I n_t^I \quad (2.B.16)$$

$$n w_{t+1} = \zeta \left(a_t p_t^F n_t^F - (C_t^F - n w_t) \left(1 + \frac{\mu_t G(\bar{\omega}_t) a_t p_t^F n_t^F}{C_t^F - n w_t} \right) \right) \quad (2.B.17)$$

$$\theta_t^F = \frac{v_t^F}{u_t} \quad (2.B.18)$$

$$\theta_t^I = \frac{v_t^I}{u_t} \quad (2.B.19)$$

$$m_t^F = \psi^F (\theta_t^F)^{-\alpha} \quad (2.B.20)$$

$$m_t^I = \psi^I (\theta_t^I)^{-\alpha} \quad (2.B.21)$$

$$\phi_t^F = \psi^F (\theta_t^F)^{1-\alpha} \quad (2.B.22)$$

$$\phi_t^I = \psi^I (\theta_t^I)^{1-\alpha} \quad (2.B.23)$$

$$\lambda_t^\omega = \frac{\Gamma'(\bar{\omega}_t)}{\Gamma'(\bar{\omega}_t) - \mu_t G'(\bar{\omega}_t)} \quad (2.B.24)$$

$$w_t^F n_t^F + \varphi^F v_t^F - n w_t = (\Gamma(\bar{\omega}_t) - \mu_t G(\bar{\omega}_t)) a_t p_t^F n_t^F \quad (2.B.25)$$

$$\Gamma(\bar{\omega}_t) = \int_0^{\bar{\omega}_t} \omega d\Phi(\omega) + \int_{\bar{\omega}_t}^{\infty} \bar{\omega}_t d\Phi(\omega) = \bar{\omega}_t(1 - \Phi(\bar{\omega}_t)) + G(\bar{\omega}_t) \quad (2.B.26)$$

$$G(\bar{\omega}_t) = \int_0^{\bar{\omega}_t} \omega d\Phi(\omega) \quad (2.B.27)$$

$$\Gamma'(\bar{\omega}_t) = 1 - \Phi(\bar{\omega}_t) \quad (2.B.28)$$

$$G'(\bar{\omega}_t) = \bar{\omega}_t \Phi'(\bar{\omega}_t) \quad (2.B.29)$$

$$\Omega(\bar{\omega}_t) = (1 - \Gamma(\bar{\omega}_t)) + \lambda_t^\omega (\Gamma(\bar{\omega}_t) - \mu_t G(\bar{\omega}_t)) \quad (2.B.30)$$

$$\mu_t = h(a_t) \quad (2.B.31)$$

Appendix 2.C Log-linear equations

Log-linearized variables are denoted by “hat” and variables without time subscripts are their steady state values. The following system of 21 equations defines the evolution of the following 21 variables $\hat{p}_t^F, \hat{p}_t^I, \hat{c}_t^F, \hat{c}_t^I, \hat{c}_t, \hat{y}_t^F, \hat{y}_t^I, \hat{a}_t, \hat{n}_t^F, \hat{n}_t^I, \hat{v}_t^F, \hat{v}_t^I, \hat{u}_t, \hat{r}_t, \hat{w}_t^F, \hat{w}_t^I, \hat{\theta}_t^F, \hat{\theta}_t^I, \hat{\lambda}_t^\omega, \hat{n}\hat{w}_t, \hat{\bar{w}}_t$.

$$\begin{aligned} \frac{\lambda^\omega \varphi^F(1+r)}{\psi^F(\theta^F)^{-\alpha}} (\hat{\lambda}_t^\omega + \alpha \hat{\theta}_t^F) = & -\frac{r}{1+r} \left[\Omega(\bar{\omega}) p^F - \lambda^\omega w^F + (1 - \delta^F) \frac{\lambda^\omega \varphi^F}{\psi^F(\theta^F)^{-\alpha}} \right] \hat{r}_t \\ & - \Omega'(\bar{\omega}) \bar{\omega} p^F \hat{\bar{w}}_{t+1} + \Omega(\bar{\omega}) p^F (\hat{p}_{t+1}^F + \hat{a}_{t+1}) - \lambda^\omega w^F (\hat{w}_{t+1}^F + \hat{\lambda}_t^\omega) \\ & + \lambda^\omega p^F (\Gamma(\bar{\omega}) - \mu G(\bar{\omega})) \hat{\lambda}_{t+1}^\omega - \lambda^\omega p^F h'(a) G(\bar{\omega}) \hat{a}_{t+1} \\ & + (1 - \delta^F) \frac{\lambda^\omega \varphi^F}{\psi^F(\theta^F)^{-\alpha}} (\hat{\lambda}_{t+1}^\omega + \alpha \hat{\theta}_{t+1}^F) \end{aligned} \quad (2.C.1)$$

$$\begin{aligned} \frac{\varphi^I(1+r)}{\psi^I(\theta^I)^{-\alpha}} \alpha \hat{\theta}_t^I = & -\frac{r}{1+r} \left[\gamma p^I - w^I + (1 - \delta^I) \frac{\varphi^I}{\psi^I(\theta^I)^{-\alpha}} \right] \hat{r}_t + \gamma p^I (\hat{p}_{t+1}^I + \hat{a}_{t+1}) \\ & - w^I \hat{w}_{t+1}^I + (1 - \delta^I) \frac{\varphi^I}{\psi^I(\theta^I)^{-\alpha}} \alpha \hat{\theta}_{t+1}^I \end{aligned} \quad (2.C.2)$$

$$\begin{aligned} \frac{w^F}{\eta} \hat{w}_t^F = & \frac{\Omega(\bar{\omega})}{\lambda^\omega} (\hat{p}_t^F + \hat{a}_t) - \frac{p^F}{\lambda^\omega} (1 - \Gamma(\bar{\omega})) \hat{\lambda}_t^\omega + \frac{\Omega'(\bar{\omega})}{\lambda^\omega} p^F \bar{\omega} \hat{\bar{w}}_t \\ & - h'(a) G(\bar{\omega}) p^F \hat{a}_t + \theta^F \varphi^F (\hat{\theta}_t^F + \hat{\lambda}_t^\omega - \hat{\lambda}_{t+1}^\omega) + \theta^I \varphi^I \hat{\theta}_t^I \\ & + (1 - \delta^F) \frac{\varphi^F}{\psi^F(\theta^F)^{-\alpha}} (\hat{\lambda}_{t+1}^\omega - \hat{\lambda}_t^\omega) \end{aligned} \quad (2.C.3)$$

$$\frac{w^I}{\eta} \hat{w}_t^I = \gamma p^I (\hat{p}_t^I + \hat{a}_t) + \theta^F \varphi^F (\hat{\theta}_t^F + \hat{\lambda}_t^\omega - \hat{\lambda}_{t+1}^\omega) + \theta^I \varphi^I \hat{\theta}_t^I \quad (2.C.4)$$

$$\hat{c}_{t+1} = \hat{c}_t + \xi \frac{r}{1+r} \hat{r}_t \quad (2.C.5)$$

$$\hat{p}_t^F = (\rho - 1) \hat{c}_t^F + (1 - \rho) \hat{c}_t \quad (2.C.6)$$

$$\hat{p}_t^I = (\rho - 1) \hat{c}_t^I + (1 - \rho) \hat{c}_t \quad (2.C.7)$$

$$\hat{c}_t = \frac{p^F c^F}{c} (\hat{p}_t^F + \hat{c}_t^F) + \frac{p^I c^I}{c} (\hat{p}_t^I + \hat{c}_t^I) \quad (2.C.8)$$

$$(1 - \mu G(\bar{\omega})) (\hat{y}_t^F + \hat{p}_t^F) = \frac{c^F}{y^F} (\hat{c}_t^F + \hat{p}_t^F) + \frac{\varphi^F v^F}{p^F y^F} \hat{v}_t^F + h'(a) G(\bar{\omega}) \hat{a}_t + \mu G'(\bar{\omega}) \bar{\omega} \hat{\bar{w}}_t \quad (2.C.9)$$

$$\hat{y}_t^F = \hat{a}_t + \hat{n}_t^F \quad (2.C.10)$$

$$\hat{y}_t^I + \hat{p}_t^I = \frac{c^F}{y^I}(\hat{c}_t^I + \hat{p}_t^I) + \frac{\varphi^I v^I}{p^I y^I} \hat{v}^I \quad (2.C.11)$$

$$\hat{y}_t^I = \hat{a}_t + \hat{n}_t^I \quad (2.C.12)$$

$$\hat{n}_{t+1}^F = (1 - \delta^F) \hat{n}_t^F + \frac{\psi^F u^\alpha (v^F)^{1-\alpha}}{n^F} (\alpha \hat{u}_t + (1 - \alpha) \hat{v}_t^F) \quad (2.C.13)$$

$$\hat{n}_{t+1}^I = (1 - \delta^I) \hat{n}_t^I + \frac{\psi^I u^\alpha (v^I)^{1-\alpha}}{n^I} (\alpha \hat{u}_t + (1 - \alpha) \hat{v}_t^I) \quad (2.C.14)$$

$$u \hat{u}_t = -n^F \hat{n}_t^F - n^I \hat{n}_t^I \quad (2.C.15)$$

$$\hat{\theta}_t^F = \hat{v}_t^F - \hat{u}_t \quad (2.C.16)$$

$$\hat{\theta}_t^I = \hat{v}_t^I - \hat{u}_t \quad (2.C.17)$$

$$\begin{aligned} nw \widehat{nw}_{t+1} = & \zeta \left(p^F n^F \left((1 - \mu G(\bar{\omega})) (\hat{p}_t^F + \hat{a}_t + \hat{n}_t^F) - \mu G'(\bar{\omega}) \bar{\omega} \widehat{\bar{\omega}}_t - h'(a) G'(\bar{\omega}) \hat{a}_t \right) \right. \\ & \left. - w^F n^F (\widehat{w}_t^F + \hat{n}_t^F) - \varphi^F v^F \hat{v}_t^F + nw \widehat{nw}_t \right) \end{aligned} \quad (2.C.18)$$

$$(\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})) \widehat{\lambda}_t^\omega = h'(a) G'(\bar{\omega}) \hat{a}_t - \left(\Gamma''(\bar{\omega}) - \mu G''(\bar{\omega}) - \frac{\Gamma''(\bar{\omega})}{\lambda^\omega} \right) \bar{\omega} \widehat{\bar{\omega}}_t \quad (2.C.19)$$

$$\begin{aligned} \hat{p}^F + \hat{a}_t + \hat{n}_t^F = & \frac{1}{\Gamma(\bar{\omega}) - \mu G(\bar{\omega})} \left(\frac{w^F}{p^F} (\widehat{w}_t^F + \hat{n}_t^F) + \frac{\varphi^F v^F}{p^F n^F} \hat{v}_t^F - \frac{nw}{p^F n^F} \widehat{nw}_t \right) \\ & - \frac{\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})}{\Gamma(\bar{\omega}) - \mu G(\bar{\omega})} \bar{\omega} \widehat{\bar{\omega}}_t + \frac{h'(a) G(\bar{\omega})}{\Gamma(\bar{\omega}) - \mu G(\bar{\omega})} \hat{a}_t \end{aligned} \quad (2.C.20)$$

$$\hat{a}_t = \varrho \hat{a}_{t-1} + \varepsilon_t \quad (2.C.21)$$

Chapter 3

Formal and Informal Employment and Employment Durations in Brazil

3.1 Introduction

In many developing countries up to half of the employed population might be in the informal economy, i.e. be employed in an underground economy or be employed without any formal contractual arrangement. There are several reasons why existence of a large informal labor force can be regarded as a negative phenomenon. For instance, large informality of the labor force leads to a non-negligible share of the population being subject to little or no labor and safety regulations. In addition, informal sector is usually associated with low productive activities and the fact that a large share of the population is engaged in them can be a hindrance to economic growth and growth of labor productivity.

Interest in causes of the existence of informal employment and functioning of informal labor markets in developing countries has resulted in a large body of theoretical and empirical literature.¹ In this study we focus on understanding the dynamics of the labor market in a developing country with a large share of the labor force employed informally, Brazil. Specifically, we study exits from different employment categories by analyzing employment durations and we are primarily interested in understanding what personal characteristics explain estimated transition probabilities.

The analysis is performed by estimating a competing risks duration model to clearly account for the fact that given employment duration may end in different labor market states (competing risks). Duration data analysis is primarily based on modeling the hazard function (or instantaneous rate) of making a certain transition. Interpreting estimated coefficients of the effect of covariates on the hazard of making a certain transition in a competing risks model is not straightforward. In a competing risks environment overall probability of making a certain transition is a function of all hazard functions for making a transition from an origin state and qualitative and quantitative effect of the same covariate on these hazard functions might be different. Thus, in an environment with several possible destinations from a labor market state it is difficult to infer the effect of a covariate on the probability of making a certain transition based only on the estimates of covariate effects on a hazard function. Therefore, we compute marginal effects of covariates on the probability of making a certain transition, which clearly account for the direct effect of a covariate on a specific transition hazard and indirect effect through other cause-specific hazards. In addition, we account for business cycle effects and identify how aggregate economic conditions affect individual labor market transitions. In this paper we

¹See [Perry et al. \(2007\)](#) for an introduction to this issue and review of recent research.

concentrate only on modeling employment durations and, thus, we do not model unemployment durations and transitions from unemployment.²

There are several studies that also analyze labor market transitions in developing countries and in Brazil, in particular. [Bosch and Maloney \(2010\)](#) is one of few studies that carried out a detailed analysis of labor market dynamics in a number of developing countries. They use panel data for three large developing countries (Argentina, Brazil and Mexico), where a large proportion of the labor force is in the informal sector. They estimate continuous-time Markov transition matrices to study the pattern of transitions among several labor market states. Their analysis shows a high degree of turnover in all three countries and it suggests that informal self-employment and informal salaried employment might be two distinct types of employment. In particular, their analysis shows that transitions to informal self-employment are most likely to be voluntary, while entry into informal salaried employment is involuntary. Our study is similar to [Bosch and Maloney \(2010\)](#) in terms of analyzing transitions between different labor market states but it complements it by clearly accounting for the effect of a larger number of covariates. Estimation procedure employed in [Bosch and Maloney \(2010\)](#) does not allow them to fully account for individual heterogeneity present in the data.

Current study is particularly close to [Ulyssea and Szerman \(2007\)](#) and [Gonzaga \(2003\)](#). [Gonzaga \(2003\)](#) uses the same dataset that we do in this study and analyzes the effect of changes in labor legislation on labor turnover in Brazil, specifically, by analyzing the effect on employment durations. [Ulyssea and Szerman \(2007\)](#) also study the determinants of durations of formal and informal salaried employment in Brazil. To estimate the effect of covariates they use the Cox proportional hazard model with competing risks. Interpretation of coefficients of this regression model in an environment with competing risks is difficult and a more straightforward method for the analysis of the effect of covariates should be used, which we do in this paper.

The paper proceeds as follows. In the next section we explain the dataset and the sampling we use to extract information on employment durations. We also present descriptive statistics and brief non-parametric analysis of employment durations. Section 3 explains in detail the econometric model and section 4 presents the estimation results. We extend the model in Section 5 by accounting for unobserved individual heterogeneity and present results of the extended model. Section 6 concludes.

3.2 Data

The data for this study is obtained from the Brazilian Monthly Employment Survey (*Pesquisa Mensal de Emprego, PME*). PME is a monthly rotating panel survey conducted by the Brazilian Institute for Geography and Statistics. We use survey waves starting from March 2002 until August 2011 to build a panel dataset suitable for the duration analysis.³ Each household in this survey is followed for four consecutive months, after which it is dropped from the survey for eight months and reintroduced for four months again. Households are assigned individual identification numbers but individuals within a household have to be matched across survey waves based on a number of their personal characteristics.⁴

We extract information on employment durations using information from the first four interviews of an individual and answers to the retrospective question on employment duration. We do not

²See [Margolis \(2008\)](#) for a recent study of unemployment durations in Brazil.

³Though, the survey is administered since early 1980s, it underwent a major revision in 2002.

⁴To reduce the attrition rate when building a panel of individuals based on personal characteristics we employ the matching algorithm proposed by [Ribas and Soares \(2008\)](#).

use all eight interviews, since the panel with four interviews is subject to less gaps and missing observations. Each individual contributes one duration observation. We record labor market status of an individual in his first interview and follow the individual through the rest of three interviews and record if transition into any other labor market state is made or if a person remained in his initial labor market state. If an individual remained in his initial state during all four interviews we tag his employment duration observation as right-censored, i.e. not having experienced any event of interest within the observation window.

We concentrate on individuals of working age (between 14 and 65 years old) and exclude individuals identified as employers, public employees, employed by military or non-paid workers in their first interview. Thus, we focus on private sector employment only. We also exclude individuals who have two or more jobs. The survey allows the identification of several labor market states, namely salaried employees, self-employed, unemployed and out of labor force. Salaried employees can be further divided into those that are employed formally and informally. Information on whether mandatory social security payments are made is also available. Using this information we identify the following four labor market states, (1) formal salaried workers, whom we identify as salaried employees with a valid work card (*“carteira de trabalho assinada”*), (2) informal salaried workers, who are salaried workers without a valid work card, (3) informal self-employed, whom we identify as self-employed workers not making mandatory social security payments, and (4) unemployed. In case there is no confusion, hereinafter, we refer to informal self-employed workers simply as self-employed. The final sample consists of single employment spells for 392 671 individuals. 68% of spells are right-censored. About half of employment spells are informal employment spells, either informal salaried or self-employed. Table 3.1 provides descriptive statistics for recorded employment spells.

Descriptive statistics reveal notable differences between individuals in formal, informal employment and self-employment. Mean recorded duration of formal employment spells is about twice longer than duration of informal salaried jobs. Mean self-employment spell is the longest and it is mostly due to relatively larger share of long self-employment spells. Workers in formal jobs are relatively more educated and the incidence of male workers is higher. Informal salaried workers appear to be younger than other types of workers. Majority of formal workers are employed in large firms, which is consistent with recent empirical evidence from other countries (for instance, Prata and Quintin (2006) and El Badaoui et al. (2010)). In Table 3.2 we present information on transition dynamics of employment spells.

There is a large turnover of workers between formal and informal employment in our sample. Interestingly, the number of informal salaried and self-employed workers transiting to formal employment actually exceeds the number of formal employment spells ending in the other employment types. Informal employment spells are more likely than others to end in unemployment. Overall, the percentage of censored observations for each type of employment spell also suggests that informal salaried work and self-employment are less stable than formal employment.

Descriptive non-parametric analysis of duration observations also provides interesting insights into the nature of labor market dynamics in Brazil. Figure 3.1 presents the Kaplan-Meier estimator of the survival function for each employment type. To estimate these survival functions we treat all possible exit destinations as failure, i.e. we estimate the survival function by treating all exit types as a similar event.⁵ Shape and level of the survival function for formal employment indicates high survival probability, i.e. relative higher level of stability of formal jobs. Survival function for informal salaried employment declines relatively faster, which on the other hand

⁵Details on these non-parametric estimators are provided in the Appendix 3.A.

Table 3.1: Summary statistics

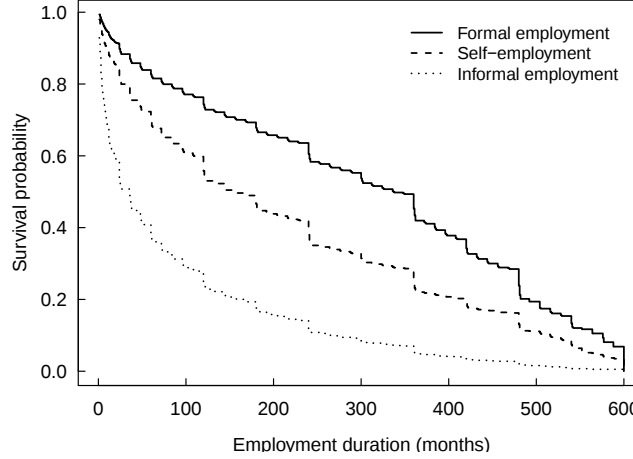
Variable	Sample mean (Std. Dev.)		
	Formal	Informal	Self-empl.
Duration of employment (months)	64.03 (77.16)	30.47 (55.37)	91.10 (102.58)
Age (years)	34.84 (11.13)	30.76 (12.63)	40.78 (12.38)
Male (binary)	0.62	0.49	0.57
Schooling (binary)			
Less than 1 year	0.01	0.02	0.05
1 - 3 years	0.03	0.05	0.09
4 - 7 years	0.19	0.27	0.35
8 - 10 years	0.19	0.25	0.21
11 or more years	0.57	0.40	0.30
Occupation (binary)			
Industry and manufacturing	0.25	0.18	0.16
Construction	0.06	0.13	0.18
Retail trade and related	0.22	0.24	0.32
Financial services and real estate	0.20	0.13	0.07
Education, health and related	0.08	0.07	0.03
Services	0.18	0.24	0.23
Other	0.004	0.01	0.02
Firm size (binary)			
Less than 4 employees	0.12	0.37	
5 - 10 employees	0.06	0.11	
11 or more employees	0.82	0.52	
Number of spells	225 097	79 600	87 974

Table 3.2: Dynamics of employment spells

Origin state	Number of spells	Percentage of spells ending in					Percentage censored
		Formal	Informal	Self-empl.	Unempl.	Out of labor force	
Formal	225,097	-	6.5	3.8	2.4	6.1	81.2
Informal	79,600	21.6	-	10.4	7.4	14.5	46.0
Self-employment	87,974	10.5	11.8	-	4.8	18.6	54.2
Total	392,671	6.7	6.4	4.3	4.0	10.6	68.0

suggests that most of informal salaried jobs are not long-lasting ones.

Figure 3.1: Survival functions for different employment types



For each employment type we also estimate unconditional cumulative incidence functions for every exit destination, which are estimated using the Aalen-Johansen estimator (Aalen et al., 2008). Cumulative incidence function for a given exit destination can be interpreted as the probability of exiting to this destination by duration time t given that no other type of exit occurred. In other words, cumulative incidence function estimates the probability of exiting to a certain labor market state but, in addition, the estimation methodology acknowledges the existence of other destinations. Estimated cumulative incidence functions are presented in Figure 3.2.

Unconditional cumulative incidence functions suggest interesting dynamics of employment spells. In case of formal employment of shorter length the probability of the employment spell ending in informal employment is higher than in other destinations. Interestingly, the incidence of transitions from formal to self-employment highly increases at longer formal employment durations. This might be the indicator of possible voluntary nature of informal self-employment as suggested by Bosch and Maloney (2010). Formal employment is the most likely destination from informal employment at all duration lengths and the incidence of exit to unemployment from informal jobs is higher than from other employment types.

3.3 Econometric Specification

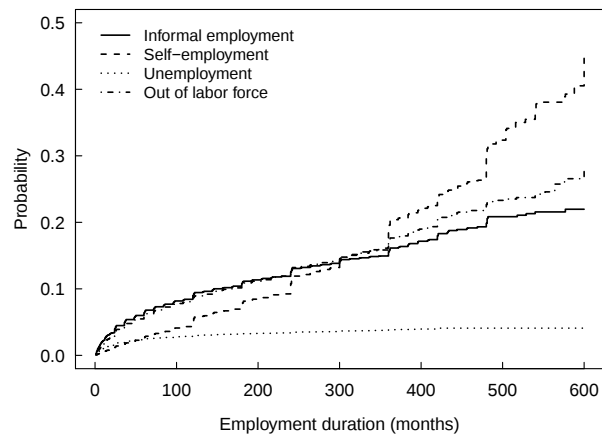
3.3.1 Cause-specific hazard versus cumulative incidence function

In this section we introduce basic components of the duration data analysis and explain why the interpretation of covariate effects in an environment with competing risks is not straightforward and additional inference procedure should be employed.

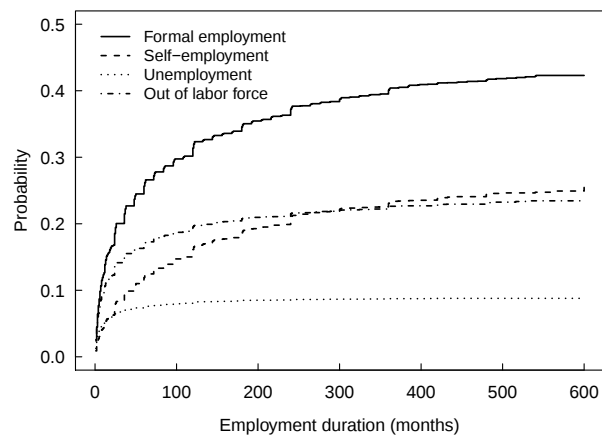
Let's denote by T a random variable for a time in a certain employment state. Each individual provides us information on duration in a type of employment $J \in \{\text{formal salaried employment, informal salaried employment, informal self-employment}\}$ and exit to one of possible destinations $K \in \{\text{formal salaried employment, informal salaried employment, informal self-employment, unemployment, out of labor force}\}$. The set K of possible destinations differs depending on the

Figure 3.2: Cumulative incidence functions

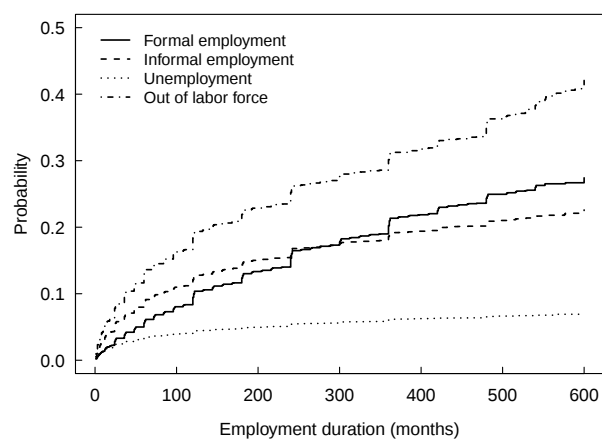
(a) Exits from formal employment



(b) Exits from informal employment



(c) Exits from self-employment



employment type J under consideration and those employment spells that have not ended in any other labor market state are tagged as right-censored.

As it is common in the modeling of time to event data we employ the hazard-based modeling of duration observations. Let's define the cause-specific hazard function for a transition from employment type j to a labor market state k conditional on the set of covariates $\mathbf{x}$ as

$$\lambda_{jk}(t|\mathbf{x}) = \lim_{\Delta t \rightarrow 0} \frac{Pr(t \leq T_j < t + \Delta t, K = k | T_j \geq t, \mathbf{x})}{\Delta t}. \quad (3.3.1)$$

Integrated cause-specific hazard function (or cumulative cause-specific hazard) is determined as

$$\Lambda_{jk}(t|\mathbf{x}) = \int_0^t \lambda_{jk}(s|\mathbf{x}) ds, \quad (3.3.2)$$

which can be further used to define the cumulative hazard function for exits from employment type j as

$$\Lambda_j(t|\mathbf{x}) = \sum_{l \neq j} \Lambda_{jl}(t|\mathbf{x}). \quad (3.3.3)$$

Then, the survival function for employment type j is defined as

$$S_j(t|\mathbf{x}) = Pr(T_j > t|\mathbf{x}) = e^{-\Lambda_j(t|\mathbf{x})}. \quad (3.3.4)$$

Using these definitions we are able to define the overall probability function for leaving an employment state j to a destination k . This probability is a function of both the cause-specific hazard function (3.3.1) and the survival probability (3.3.4) and it is defined as

$$F_{jk}(t|\mathbf{x}) = Pr(T_j \leq t, K = k|\mathbf{x}) = \int_0^t \lambda_{jk}(s|\mathbf{x}) S_j(s|\mathbf{x}) ds. \quad (3.3.5)$$

Expression (3.3.5) is also called the cumulative incidence function or subdistribution function. An important property of this function is that $\lim_{t \rightarrow \infty} F_{jk}(t|\mathbf{x}) = Pr(K = k|\mathbf{x}) < 1$, where the latter can be interpreted as the probability of making transition from state j to state k independent of elapsed duration time. The limit of the cumulative incidence function might also be interpreted as the share of individuals making a transition of interest across the whole population. In many instances, in an environment with competing risks the effect of covariates is inferred only from their effect on the cause-specific hazard function (3.3.1) (primarily because these effects are easy to compute). But it is clear from equation (3.3.5) that the effect of covariates on the probability of making transition from employment state j to some state k depends on the effect of covariates both on the cause-specific hazard function and total survival probability.⁶ What complicates the process of drawing inference about the effect of covariates in (3.3.5) is that the effect of explanatory variables on the survival probability depends on the effect of covariates on all cause-specific hazard functions as it is shown in expressions (3.3.3) and (3.3.4).

Therefore, we estimate the competing risks model for each origin-destination pair and obtain the effect of covariates on cause-specific hazard functions for each possible type of transitions. But to infer the effect of covariates on (3.3.5) we report our results in terms of marginal effects of covariates on the probability of making a specific transition. In particular, we follow the results of Kyyrä (2009), who obtained closed form solutions for marginal effects of covariates on cumulative incidence functions. His results were obtained for a specific and popular in empirical labor economics type of proportional hazard model. In the following sections we explain it in detail and discuss how marginal effects are computed.

⁶This issue is also discussed in detail in Serio (1997) and Cameron and Triverdi (2005, chapters 15 and 19).

3.3.2 A competing risks model

We estimate the following proportional hazard competing risks model for transition from employment state j to labor market state k

$$\lambda_{jk}(t|\mathbf{x}) = \lambda_{0jk}(t)e^{\beta'_{jk}\mathbf{x}}, \quad (3.3.6)$$

where $\lambda_{0jk}(t)$ denotes the baseline hazard, $\mathbf{x}$ is the vector of covariates and β_{jk} is the vector of coefficients. The baseline hazard $\lambda_{0jk}(t)$ is assumed to be of piecewise constant form. Under this assumption time axis (i.e. employment duration) is divided into R intervals $(t_{r-1}, t_r]$, $r = 1, \dots, R$, with $t_0 = 0$ and $t_R = \infty$. Given this structure we can formulate the baseline hazard as

$$\lambda_{0jk}(t) = \sum_{r=1}^R e^{\alpha^r_{jk}} \mathbb{1}_r(t), \quad (3.3.7)$$

where $\mathbb{1}_r(t)$ is an indicator function taking values $\mathbb{1}_r(t) = 1$ if $t \in (t_{r-1}, t_r]$ and $\mathbb{1}_r(t) = 0$, otherwise. A model with piecewise constant baseline hazard assumes that the hazard function is constant within a single time interval. Then, the hazard function (3.3.6) for a specific duration observation $t \in (t_{r-1}, t_r]$ can be formulated as

$$\lambda_{jk}^r(\mathbf{x}) = \lambda_{0jk}^r e^{\beta'_{jk}\mathbf{x}}, \quad (3.3.8)$$

where λ_{0jk}^r is value of the baseline hazard within interval $(t_{r-1}, t_r]$ and it is of exponential form as shown in (3.3.7).

The integrated hazard function is discontinuous at duration time points t_r , $r = 1, \dots, R$ and, thus, given the definition (3.3.2), the cumulative hazard function for transition from state j to state k at duration $t \in (t_{r-1}, t_r]$ can be defined as

$$\begin{aligned} \Lambda_{jk}(t|\mathbf{x}) &= \sum_{u=1}^{r-1} \lambda_{jk}^u(\mathbf{x})(t_u - t_{u-1}) + \lambda_{jk}^r(\mathbf{x})(t - t_{r-1}) \\ &= \sum_{u=1}^{r-1} e^{\alpha^u_{jk} + \beta'_{jk}\mathbf{x}}(t_u - t_{u-1}) + e^{\alpha^r_{jk} + \beta'_{jk}\mathbf{x}}(t - t_{r-1}). \end{aligned} \quad (3.3.9)$$

We also introduce a more convenient notation for terms in the cumulative hazard function. Let's denote by $\Delta_r(t)$ an expression that takes value 0 if observed length of the employment spell is $t < t_{r-1}$, value $t - t_{r-1}$ if duration equals $t \in (t_{r-1}, t_r]$ and value $t_r - t_{r-1}$ if $t > t_r$. Using this notation we rewrite expression (3.3.9) as

$$\Lambda_{jk}(t|\mathbf{x}) = \sum_{r=1}^R e^{\alpha^r_{jk} + \beta'_{jk}\mathbf{x}} \Delta_r(t). \quad (3.3.10)$$

Using definitions (3.3.3) and (3.3.4) the survival probability in state j conditional on covariates $\mathbf{x}$ is determined by

$$S_j(t|\mathbf{x}) = \exp \left\{ - \sum_{l \neq j} \Lambda_{jl}(t|\mathbf{x}) \right\}. \quad (3.3.11)$$

Now we are ready to formulate the likelihood function, which we will use to estimate the parameter vector $\theta_j = (\theta_{j1}, \theta_{j2}, \dots, \theta_{jk})$, where $\theta_{jk} = (\alpha_{jk}^1, \alpha_{jk}^2, \dots, \alpha_{jk}^R, \beta_{jk})$. Let's define the likelihood function for employment duration observations in state j as

$$\mathcal{L}_j(\theta_j) = \prod_{i=1}^n f_j(t_i|\mathbf{x}_i; \theta_j)^{\delta_{ij}} S_j(t_i|\mathbf{x}_i; \theta_j)^{1-\delta_{ij}}, \quad (3.3.12)$$

where n is the total number of individuals observed in state j and δ_{ij} is an indicator variable taking values $\delta_{ij} = 1$ if an individual i exited state j to any other state and $\delta_{ij} = 0$ if an observation is censored. The survival function $S_j(t_i|\mathbf{x}_i; \boldsymbol{\theta}_j)$ in (3.3.12) is defined in (3.3.11). The term $f_j(t_i|\mathbf{x}_i; \boldsymbol{\theta}_j)$ is the probability density function of a duration observation t_i terminating due to any cause at duration time t given that an individual i has not exited to any other state up to t . We can further reformulate this probability density function as

$$f_j(t_i|\mathbf{x}_i; \boldsymbol{\theta}_j) = \prod_{l \neq j} f_{jl}(t_i|\mathbf{x}_i; \boldsymbol{\theta}_j) = \prod_{l \neq j} \lambda_{jl}(t_i|\mathbf{x}_i; \boldsymbol{\theta}_{jl}) S_j(t_i|\mathbf{x}_i; \boldsymbol{\theta}_j) \quad (3.3.13)$$

where $f_{jl}(t_i|\mathbf{x}_i; \boldsymbol{\theta}_j)$ is the probability density function for a transition from state j to state l and $\lambda_{jl}(t_i|\mathbf{x}_i; \boldsymbol{\theta}_j)$ is the corresponding hazard function. Given the assumption of independent competing risk we reformulate the survival function (3.3.11) as

$$S_j(t_i|\mathbf{x}_i; \boldsymbol{\theta}_j) = \prod_{l \neq j} H_{jl}(t_i|\mathbf{x}_i; \boldsymbol{\theta}_{jl}), \quad (3.3.14)$$

where $H_{jl}(t_i|\mathbf{x}_i; \boldsymbol{\theta}_{jl}) = \exp\{-\Lambda_{jl}(t_i|\mathbf{x}_i; \boldsymbol{\theta}_{jl})\}$. Function $H_{jl}(t_i|\mathbf{x}_i; \boldsymbol{\theta}_{jl})$ is a survival function (as a mathematical object) but it actually does not correspond to any observed duration observations (Lawless, 2003).

Due to the sampling scheme we use, i.e. sampling individuals that are already in a certain employment state and observing them for a relatively short period of time, we need to correct for sample selection. This type of sampling is commonly referred to as stock sampling or length-biased sampling. Stock sampling correction is performed by weighting the value of the likelihood function (3.3.12) by the probability of surviving to the duration time observed during the first interview. Thus, to estimate the parameter vector $\boldsymbol{\theta}_j$ we actually use the following likelihood function

$$\mathcal{L}_j(\boldsymbol{\theta}_j) = \prod_{i=1}^n \frac{f_j(t_i|\mathbf{x}_i; \boldsymbol{\theta}_j)^{\delta_{ij}} S_j(t_i|\mathbf{x}_i; \boldsymbol{\theta}_j)^{1-\delta_{ij}}}{S_j(t_i^e|\mathbf{x}_i; \boldsymbol{\theta}_j)}, \quad (3.3.15)$$

where t_i^e is a length of the spell of an individual in state j reported at the first interview (elapsed duration).⁷

Using equations (3.3.13) and (3.3.14) we can rewrite the likelihood function (3.3.15) as

$$\begin{aligned} \mathcal{L}_j(\boldsymbol{\theta}_j) &= \prod_{i=1}^n \prod_{l \neq j} \frac{(\lambda_{jl}(t_i|\mathbf{x}_i; \boldsymbol{\theta}_{jl}) S_j(t_i|\mathbf{x}_i; \boldsymbol{\theta}_j))^{\delta_{ijl}} S_j(t_i|\mathbf{x}_i; \boldsymbol{\theta}_j)^{1-\delta_{ijl}}}{S_j(t_i^e|\mathbf{x}_i; \boldsymbol{\theta}_j)} \\ &= \prod_{i=1}^n \prod_{l \neq j} \frac{(\lambda_{jl}(t_i|\mathbf{x}_i; \boldsymbol{\theta}_{jl}) H_{jl}(t_i|\mathbf{x}_i; \boldsymbol{\theta}_{jl}))^{\delta_{ijl}} H_{jl}(t_i|\mathbf{x}_i; \boldsymbol{\theta}_{jl})^{1-\delta_{ijl}}}{H_{jl}(t_i^e|\mathbf{x}_i; \boldsymbol{\theta}_{jl})} \\ &= \prod_{l \neq j} \prod_{i=1}^n \frac{\lambda_{jl}(t_i|\mathbf{x}_i; \boldsymbol{\theta}_{jl})^{\delta_{ijl}} H_{jl}(t_i|\mathbf{x}_i; \boldsymbol{\theta}_{jl})}{H_{jl}(t_i^e|\mathbf{x}_i; \boldsymbol{\theta}_{jl})} \\ &= \prod_{l \neq j} \mathcal{L}_{jl}(\boldsymbol{\theta}_{jl}), \end{aligned} \quad (3.3.16)$$

where $\mathcal{L}_{jl}(\boldsymbol{\theta}_{jl}) = \prod_{i=1}^n (\lambda_{jl}(t_i|\mathbf{x}_i; \boldsymbol{\theta}_{jl})^{\delta_{ijl}} H_{jl}(t_i|\mathbf{x}_i; \boldsymbol{\theta}_{jl})) / H_{jl}(t_i^e|\mathbf{x}_i; \boldsymbol{\theta}_{jl})$ and δ_{ijl} is an indicator variable taking values $\delta_{ijl} = 1$ if an individual i in state j exits to a destination l and $\delta_{ijl} = 0$, otherwise. This result implies that the parameter vector $\boldsymbol{\theta}_j = (\boldsymbol{\theta}_{j1}, \boldsymbol{\theta}_{j2}, \dots, \boldsymbol{\theta}_{jK})$ with $\boldsymbol{\theta}_{jk} = (\alpha_{jk}^1, \alpha_{jk}^2, \dots, \alpha_{jk}^R, \boldsymbol{\beta}_{jk})$ can be estimated using each likelihood function $\mathcal{L}_{jl}(\boldsymbol{\theta}_{jl}), l \neq j$, separately.

⁷See Wooldridge (2002, chap. 20) for more details on stock sampling and stock sampling correction.

Then, the log-likelihood function for transition observations from state j to state k , which we maximize with respect to the parameter vector $\boldsymbol{\theta}_{jk}$, is given by

$$\begin{aligned} \log \mathcal{L}_{jk}(\boldsymbol{\theta}_{jk}) &= \sum_{i=1}^n \{ \delta_{ijk} \log \lambda_{jk}(t_i | \mathbf{x}_i; \boldsymbol{\theta}_{jk}) + \log H_{jk}(t_i | \mathbf{x}_i; \boldsymbol{\theta}_{jk}) - \log H_{jk}(t_i^e | \mathbf{x}_i; \boldsymbol{\theta}_{jk}) \} \\ &= \sum_{i=1}^n \left\{ \delta_{ijk} \left(\sum_{r=1}^R \alpha_{jk}^r \mathbb{1}_r(t_i) + \boldsymbol{\beta}'_{jk} \mathbf{x}_i \right) - \sum_{r=1}^R e^{\alpha_{jk}^r + \boldsymbol{\beta}'_{jk} \mathbf{x}_i} \Delta_r(t_i) + \sum_{r=1}^R e^{\alpha_{jk}^r + \boldsymbol{\beta}'_{jk} \mathbf{x}_i} \Delta_r(t_i^e) \right\} \\ &= \sum_{r=1}^R n_{jk}^r \alpha_{jk}^r + \sum_{i=1}^n \delta_{ijk} \boldsymbol{\beta}'_{jk} \mathbf{x}_i - \sum_{r=1}^R e^{\alpha_{jk}^r} \left\{ \sum_{i=1}^n e^{\boldsymbol{\beta}'_{jk} \mathbf{x}_i} \Delta_r(t_i) - \sum_{i=1}^n e^{\boldsymbol{\beta}'_{jk} \mathbf{x}_i} \Delta_r(t_i^e) \right\}, \quad (3.3.17) \end{aligned}$$

where $n_{jk}^r = \sum_{i=1}^n \delta_{ijk} \mathbb{1}_r(t_i)$ denotes the number of individuals with duration observations $t_i \in (t_{r-1}, t_r]$ making transition from state j to state k .

3.3.3 Marginal effects

As we have noted above the interpretation of coefficients $\boldsymbol{\beta}_{jk}$ estimated using (3.3.17) is not straightforward in the presence of competing risks. Therefore, we use the interpretation strategy in competing risks models developed recently by Kyyr  (2009), who suggests to compute marginal effects of covariates $\mathbf{x}$ on the overall probability of making transition between states defined in (3.3.5). These marginal effects take into account not only the direct effect of covariates on cause-specific hazard function for transitions from state j to state k estimated using (3.3.17), but also the effect of covariates on other cause-specific hazard functions for transitions from state j to states $l \neq k$. We explain in detail the methodology proposed by Kyyr  (2009) and present the estimation results in the next section.

We need to define the cumulative incidence function for the specific form of the baseline hazard assumed in this study, namely the piecewise constant baseline hazard. Using the notation above we define the cumulative incidence function as

$$\begin{aligned} F_{jk}(t | \mathbf{x}) &= \int_0^t \lambda_{jk}(t | \mathbf{x}) S_j(t | \mathbf{x}) \\ &= \sum_{r=1}^R \pi_{jk}^r(\mathbf{x}) (S_j(t_{r-1} | \mathbf{x}) - S_j(t_r | \mathbf{x})) \mathbb{I}_r(t) + \sum_{r=1}^R \pi_{jk}^r(\mathbf{x}) (S_j(t_{r-1} | \mathbf{x}) - S_j(t | \mathbf{x})) \mathbb{I}_r(t), \end{aligned} \quad (3.3.18)$$

where $\mathbb{I}_r(t)$ is an indicator function taking value $\mathbb{I}_r(t) = 1$ if $t > t_r$ and $\mathbb{I}_r(t) = 0$, otherwise; as defined above $\mathbb{I}_r(t)$ is also an indicator function taking values $\mathbb{I}_r(t) = 1$ if $t \in (t_{r-1}, t_r]$ and $\mathbb{I}_r(t) = 0$, otherwise. Term $\pi_{jk}^r(\mathbf{x}) = \lambda_{jk}^r(\mathbf{x}) / \sum_{l \neq j} \lambda_{jl}^r(\mathbf{x})$ is the probability of transiting to state k conditional on exiting and $S_j(t_{r-1}) - S_j(t_r)$ is the probability of making transition to any of possible destinations in interval $(t_{r-1}, t_r]$.

Marginal effect of a variable x^p , $p = 1, \dots, P$, (in case of a continuous variable), can be computed as

$$\frac{\partial F_{jk}(t | \mathbf{x})}{\partial x^p} = \sum_{r=1}^R \frac{\partial P_{jk}^r(t_r | \mathbf{x})}{\partial x^p} \mathbb{I}_r(t) + \sum_{r=1}^R \frac{\partial P_{jk}^r(t | \mathbf{x})}{\partial x^p} \mathbb{I}_r(t), \quad (3.3.19)$$

where $P_{jk}^r(t_r | \mathbf{x}) = \pi_{jk}^r(\mathbf{x}) (S_j(t_{r-1} | \mathbf{x}) - S_j(t_r | \mathbf{x}))$, $P_{jk}^r(t | \mathbf{x}) = \pi_{jk}^r(\mathbf{x}) (S_j(t_{r-1} | \mathbf{x}) - S_j(t | \mathbf{x}))$ and for

$$t \in (t_{r-1}, t_r]$$

$$\begin{aligned} \frac{\partial P_{jk}^r(t|\mathbf{x})}{\partial x^p} = & \pi_{jk}^r(\mathbf{x}) \left((1 - \pi_{jk}^r(\mathbf{x})) \beta_{jk}^p - \sum_{l \neq j, k} \pi_{jl}^r(\mathbf{x}) \beta_{jl}^p \right) (S_j(t_{r-1}|\mathbf{x}) - S_j(t|\mathbf{x})) \\ & + \pi_{jk}^r(\mathbf{x}) \sum_{l \neq j} \left\{ \beta_{jl}^p (S_j(t|\mathbf{x}) \Lambda_{jl}(t|\mathbf{x}) - S_j(t_{r-1}|\mathbf{x}) \Lambda_{jl}(t_{r-1}|\mathbf{x})) \right\} \end{aligned} \quad (3.3.20)$$

In the partial derivative (3.3.20) the direct effect of a covariate x^p on the cumulative incidence function corresponds to an expression

$$\pi_{jk}^r(\mathbf{x}) \left((1 - \pi_{jk}^r(\mathbf{x})) (S_j(t_{r-1}|\mathbf{x}) - S_j(t|\mathbf{x})) + (S_j(t|\mathbf{x}) \Lambda_{jk}(t|\mathbf{x}) - S_j(t_{r-1}|\mathbf{x}) \Lambda_{jk}(t_{r-1}|\mathbf{x})) \right) \beta_{jk}^p,$$

while the indirect effect of x^p through other cause-specific hazard functions is captured by

$$- \pi_{jk}^r(\mathbf{x}) \sum_{l \neq j, k} \left\{ \left(\pi_{jl}^r(\mathbf{x}) (S_j(t_{r-1}|\mathbf{x}) - S_j(t|\mathbf{x})) - (S_j(t|\mathbf{x}) \Lambda_{jl}(t|\mathbf{x}) - S_j(t_{r-1}|\mathbf{x}) \Lambda_{jl}(t_{r-1}|\mathbf{x})) \right) \beta_{jl}^p \right\}.$$

All terms in expression (3.3.20) are computable using the parameter values $\theta_{jl}, l \neq j$, which we obtain by estimating proportional hazard model (3.3.6) for each origin-destination pair. The marginal effects for binary variables are computed by taking the difference of the cumulative incidence function at different values of the binary value of interest. In other words, the marginal effect of a binary variable x^p on the overall probability of making transition from state j to state k by time t is obtained by computing the difference

$$F_{jk}(t|x^p = 1) - F_{jk}(t|x^p = 0), \quad (3.3.21)$$

while holding the value of other covariates constant.

The asymptotic distribution of the cumulative incidence function $\hat{F}_{jk}(t|\mathbf{x})$ estimated at maximum likelihood estimators $\hat{\theta}_{jk}$ is defined by

$$\sqrt{n} (\hat{F}_{jk}(t|\mathbf{x}) - F_{jk}(t|\mathbf{x})) \xrightarrow{d} N \left(0, \sum_{l \neq j} \hat{\mathbf{v}}_{jl}' \hat{\Sigma}_{jl} \hat{\mathbf{v}}_{jl} \right), \quad (3.3.22)$$

where n is the number of individuals observed in state j , $\hat{\Sigma}_{jl}$ is the covariance matrix for $\hat{\theta}_{jl}$ and

$$\hat{\mathbf{v}}_{jl} = \left(\frac{\partial \hat{F}_{jk}(t|\mathbf{x})}{\partial \hat{\alpha}_{jl}^1}, \dots, \frac{\partial \hat{F}_{jk}(t|\mathbf{x})}{\partial \hat{\alpha}_{jl}^R}, \frac{\partial \hat{F}_{jk}(t|\mathbf{x})}{\partial \hat{\beta}_{jl}^1}, \dots, \frac{\partial \hat{F}_{jk}(t|\mathbf{x})}{\partial \hat{\beta}_{jl}^P} \right).$$

Correspondingly, the asymptotic distribution of the vector of marginal effects $\hat{\psi}_{jk}(t|\mathbf{x})$ estimated using (3.3.19) and (3.3.21) is defined by

$$\sqrt{n} (\hat{\psi}_{jk}(t|\mathbf{x}) - \psi_{jk}(t|\mathbf{x})) \xrightarrow{d} N \left(\mathbf{0}, \sum_{l \neq j} \frac{\partial \hat{\psi}_{jk}(t|\mathbf{x})}{\partial \hat{\theta}_{jl}'} \hat{\Sigma}_{jl} \frac{\partial \hat{\psi}_{jk}(t|\mathbf{x})}{\partial \hat{\theta}_{jl}} \right). \quad (3.3.23)$$

As it is common in the literature, cumulative incidence function $F_{jk}(t|\mathbf{x})$ and marginal effects $\psi_{jk}(t|\mathbf{x})$ are estimated at the sample mean of covariates $\mathbf{x}$.

3.4 Estimation Results

Estimation results are presented in Tables 3.3-3.5. We are primarily interested in understanding the determinants of transitions from informal salaried employment and informal self-employment.

Results for duration observations of informal salaried employment show that male workers are more likely to transit to formal employment or unemployment, while the probability of making transition from informal employment to self-employment or out of labor force for male workers is lower. Age has a significant positive effect on the probability of transiting to formal employment and self-employment but a negative effect of the probability of becoming unemployed. This might be an indicator of recent suggestions in the literature that informal jobs are type of jobs many young workers have at the beginning of their working life but they later switch to formal employment. Firms also might use informal jobs as a screening mechanism of young workers before offering a formal contract. On the other hand, positive value of the marginal effect for transitions to informal self-employment might be indicative of another recent suggestion that informal self-employment is a different than informal salaried type of employment. In particular [Bosch and Maloney \(2010\)](#) suggests that informal self-employment most likely requires some experience and capital and younger workers are less likely to be in this labor market state.

Schooling also appear to be a significant determinant of the probability of exiting informal salaried employment. Higher level of schooling increases the probability of transiting to formal employment, while less educated informal salaried workers appear to make transition to informal self-employment and out of labor force. Firm size also appears to be a significant factor that affects the probability of making informal - formal employment transitions. Being informal salaried worker in a larger firm increases the overall probability of transiting to formal employment. On the other hand, results suggest that working informally in a small firm increases the incidence of making transition to informal self-employment.

In the estimation we use the level of the local unemployment rate (unemployment rate of the metropolitan area) to approximate the effect of business cycle conditions. Given established high countercyclicality of the unemployment rate in Brazil (e.g. see [Bosch and Esteban-Pretel \(2011\)](#)) results in the second column of Table 3.3 suggest that probability of making transition from informal to formal employment is procyclical. In other words, given other factors constant, the probability of transiting from informal to formal employment is higher during periods of high economic growth. This is also true for transitions to out of labor force. Interestingly, transitions from informal salaried employment to informal self-employment are countercyclical. As expected, separations from informal employment are strongly countercyclical.

Estimation results for informal self-employment are overall very similar to estimated effects of covariates on the probability of making transitions from informal salaried employment. Age appears to be a significant factor affecting transitions with workers of higher ages having higher probability of transiting to formal employment. Younger workers, on the other hand, are more likely to make transitions to either informal salaried employment or unemployment. Similarly, more years of schooling increase the probability of transiting to formal employment, while workers with relatively little schooling are more likely to transit to informal salaried employment. As for business cycle effects on transitions from informal self-employment, the behavior is in general similar to the cyclical behavior of transition probabilities from informal salaried employment. Transitions to formal employment appear strongly procyclical, but transitions into informal salaried employment are countercyclical. Transitions into unemployment are also highly countercyclical.

Marginal effects of covariates on the probability of exiting from formal employment are presented in Table 3.5. Results show that male workers are more likely to make transitions to both types of informal employment states and unemployment while the incidence of transiting to out of labor force from formal employment is higher among female workers. Age also appears to be a significant covariate and the probability of transiting to informal employment and to unemployment is higher among younger individuals. Higher age, on the other hand, increases the probability of making transitions to informal self-employment, which again supports the suggestion that informal salaried employment and informal self-employment are different types of informal employment. Transitions to informal self-employment are prevalent among workers of smaller firms. As for cyclical features of the dynamics of formal employment spells, transitions into informal salaried employment appear to be highly countercyclical. On the other hand, the probability of making formal employment - informal self-employment transitions is lower during recessions. This finding again suggests that informal salaried employment might be the type of employment that supports the theory of segmented labor markets in developing countries. In other words, segmented informal sector in the labor market plays the role of a buffer during recessions and workers in this sector queue up for formal sectors jobs. Cyclical features of transitions from formal employment to informal self-employment, on the other hand, suggest that entry into this type of informal employment is voluntary.

3.5 Unobserved heterogeneity

We control for unobserved heterogeneity in this model by introducing the individual fixed effects term that enters the hazard function multiplicatively. As Kyrrä (2009) suggests assuming discrete distribution for the unobserved heterogeneity term still allows using the inference procedure presented in Section 3.3.3.

The hazard function with multiplicative unobserved heterogeneity is defined as

$$\lambda_{jk}(t|\mathbf{x}, \varphi_{jk}) = \lambda_{0jk}(t)e^{\beta'_{jk}\mathbf{x} + \varphi_{jk}}, \quad (3.5.1)$$

where φ_{jk} is the unobserved heterogeneity (or frailty) term with the distribution function $\Phi_{jk}(\varphi_{jk})$. We assume $\Phi_{jk}(\varphi_{jk})$ to be a discrete distribution with two points of support, (ϕ_{1jk}, ϕ_{2jk}) . Moreover, let's assume that $Pr(\varphi_{jk} = \phi_{1jk}) = \pi$ and $Pr(\varphi_{jk} = \phi_{2jk}) = 1 - \pi$. Implicitly, controlling for unobserved heterogeneity in this manner assumes that all employed workers can be divided into two groups based on the realization of an unobserved factor. Values of points of support of the discrete distribution show how different the hazard of making a transition is between these two groups (after controlling for all other observable factors) and probabilities π and $1 - \pi$ approximate sizes of these groups. Then, the cumulative hazard function conditional on the unobserved heterogeneity term is

$$\Lambda_{jk}(t|\mathbf{x}, \varphi_{jk}) = \sum_{r=1}^R e^{\alpha_{jk}^r + \beta'_{jk}\mathbf{x} + \varphi_{jk}} \Delta_r(t). \quad (3.5.2)$$

Using the Laplace transform, the unconditional survival function $H_{jk}(t|\mathbf{x})$ in (3.3.14) with the frailty term integrated out can be defined as

$$H_{jk}(t|\mathbf{x}) = \pi \exp(-\phi_{1jk}\Lambda_{jk}(t|\mathbf{x})) + (1 - \pi) \exp(-\phi_{2jk}\Lambda_{jk}(t|\mathbf{x})). \quad (3.5.3)$$

Equivalently, we determine the unconditional density function as

$$\begin{aligned} f_{jk}(t|\mathbf{x}) &= \pi \phi_{1jk} \lambda_{0jk}(t) e^{\beta'_{jk}\mathbf{x}} \exp(-\phi_{1jk}\Lambda_{jk}(t|\mathbf{x})) \\ &\quad + (1 - \pi) \phi_{2jk} \lambda_{0jk}(t) e^{\beta'_{jk}\mathbf{x}} \exp(-\phi_{2jk}\Lambda_{jk}(t|\mathbf{x})). \end{aligned} \quad (3.5.4)$$

Given these expressions, we can determine the unconditional hazard function as

$$\lambda_{jk}(t|\mathbf{x}) = \frac{\pi\phi_{1jk} \exp(-\phi_{1jk}\Lambda_{jk}(t|\mathbf{x})) + (1-\pi)\phi_{2jk} \exp(-\phi_{2jk}\Lambda_{jk}(t|\mathbf{x}))}{\pi \exp(-\phi_{1jk}\Lambda_{jk}(t|\mathbf{x})) + (1-\pi) \exp(-\phi_{2jk}\Lambda_{jk}(t|\mathbf{x}))} \lambda_{0jk}(t) e^{\beta'_{jk}\mathbf{x}}. \quad (3.5.5)$$

Using more general form the likelihood function given in (3.3.15) we estimate the parameter vector $\tilde{\boldsymbol{\theta}}_{jk} = (\boldsymbol{\theta}_{jk}, \pi, \phi_{1jk}, \phi_{2jk})$ by maximizing the following likelihood function

$$\begin{aligned} \mathcal{L}_{jk}(\tilde{\boldsymbol{\theta}}_{jk}) &= \prod_{i=1}^n \left(\frac{\pi\phi_{1jk} \exp(-\phi_{1jk}\Lambda_{jk}(t_i|\mathbf{x}_i)) + (1-\pi)\phi_{2jk} \exp(-\phi_{2jk}\Lambda_{jk}(t_i|\mathbf{x}_i))}{\pi \exp(-\phi_{1jk}\Lambda_{jk}(t_i|\mathbf{x}_i)) + (1-\pi) \exp(-\phi_{2jk}\Lambda_{jk}(t_i|\mathbf{x}_i))} \lambda_{0jk}(t_i) e^{\beta'_{jk}\mathbf{x}_i} \right)^{\delta_{ijk}} \\ &\times \frac{\pi \exp(-\phi_{1jk}\Lambda_{jk}(t_i^e|\mathbf{x}_i)) + (1-\pi) \exp(-\phi_{2jk}\Lambda_{jk}(t_i^e|\mathbf{x}_i))}{\pi \exp(-\phi_{1jk}\Lambda_{jk}(t_i^e|\mathbf{x}_i)) + (1-\pi) \exp(-\phi_{2jk}\Lambda_{jk}(t_i^e|\mathbf{x}_i))}, \end{aligned} \quad (3.5.6)$$

where we suppressed the dependence of the cumulative hazard function $\Lambda_{jk}(t_i|\mathbf{x}_i)$ on $\tilde{\boldsymbol{\theta}}_{jk}$. We similarly control for stock sampling in (3.5.6), as we did during the estimation of models without unobserved heterogeneity.

Using (3.3.18) the cumulative incidence function conditional on unobserved heterogeneity is defined as

$$\begin{aligned} F_{jk}(t|\mathbf{x}, \boldsymbol{\varphi}_j) &= \int_0^t \lambda_{jk}(t|\mathbf{x}, \boldsymbol{\varphi}_j) S_j(t|\mathbf{x}, \boldsymbol{\varphi}_j) \\ &= \sum_{r=1}^R \pi_{jk}^r(\mathbf{x}|\boldsymbol{\varphi}_j) (S_j(t_{r-1}|\mathbf{x}, \boldsymbol{\varphi}_j) - S_j(t_r|\mathbf{x}, \boldsymbol{\varphi}_j)) \mathbb{I}_r(t) \\ &\quad + \sum_{r=1}^R \pi_{jk}^r(\mathbf{x}, \boldsymbol{\varphi}_j) (S_j(t_{r-1}|\mathbf{x}, \boldsymbol{\varphi}_j) - S_j(t|\mathbf{x}, \boldsymbol{\varphi}_j)) \mathbb{I}_r(t), \end{aligned} \quad (3.5.7)$$

where $\boldsymbol{\varphi}_j$ is the vector of unobserved heterogeneity terms for all hazard models estimated for transitions from a labor market state j . Using (3.3.14) we can define the unconditional total survival function $S_j(t|\mathbf{x})$ as

$$S_j(t|\mathbf{x}) = \prod_{l \neq j} \int H_{jl}(t|\mathbf{x}, \varphi_{jl}) d\Phi_{jl}(\varphi_{jl}). \quad (3.5.8)$$

Integrating out the vector of unobserved heterogeneity terms in (3.5.7) we obtain the following expression for the unconditional cumulative incidence function

$$F_{jk}(t|\mathbf{x}) = \sum_q Pr(\boldsymbol{\varphi}_j = \boldsymbol{\varphi}_j^q) F_{jk}(t|\mathbf{x}, \boldsymbol{\varphi}_j^q), \quad (3.5.9)$$

where the cumulative incidence function is obtained by summing over all q possible realizations of the vector $\boldsymbol{\varphi}_{jk}^q$. This closed form of the cumulative incidence function allows us to use the inference procedure detailed in Section 3.3.3, exactly as we did it for models with no unobserved heterogeneity.

Marginal effects of covariates on the cumulative incidence function estimated from models with unobserved heterogeneity are reported in Tables 3.6-3.8. Though, the magnitude of marginal effects is different, qualitatively results are very similar to results of models with no unobserved heterogeneity. For transitions from informal salaried employment, more years of schooling still appear to have a significant effect of the probability of making a transition to formal employment. Firm size also appears to be a significant factor with employees in larger firms

having higher chances of landing a formal job. The only difference with estimates without unobserved heterogeneity is that sign of the marginal effects of the firm size for transitions to unemployment changed to negative. In other words, working in a larger company as an informal worker increases the chances of transiting to formal employment and decreases the probability of making transition to any other state. Interestingly, the marginal effect of the local unemployment rate on the probability of transiting to formal employment changed its sign from previous estimates but became insignificant. As for transitions to the other states, transitions to informal self-employment appear countercyclical and transitions to out of labor force are procyclical.

Marginal effects for transitions from informal self-employment are also consistent with estimates of marginal effects from models with no unobserved heterogeneity. Women are more likely to make transition to out of labor force and workers of older age are more likely to transit to formal employment or out of labor force. As expected, more schooling significantly increases the chances of landing a formal job and decreases the probability of making transition to any other labor market state. Local unemployment rate also appears to have a significant impact on all transition probabilities. Transitions to formal employment are procyclical, while transitions to informal employment and unemployment are countercyclical. In other words, as suggested above informal salaried employment appears to play a role of a buffer in times of low economic activity even for self-employed workers. As in case of transition from other employment states, probability of transiting to out of labor force decreases during recessions.

Results for formal employment are also consistent with previous estimates but several marginal effects estimated from models with unobserved heterogeneity are not significantly different from zero. More schooling increases the probability of transiting to self-employment and decreases the probability of transition to out of labor force. Effect of schooling on transitions to informal employment becomes insignificant, though with an expected sign. Firm size remains a highly significant factor and workers in larger firms are less likely to transit to informal salaried employment or self-employment. Transitions to informal self-employment are procyclical and to informal salaried employment and unemployment are countercyclical, though the marginal effects for transitions to the latter two labor markets states are insignificant.

3.6 Conclusion

We estimate a competing risks proportional hazard model to characterize the determinants of different types of employment durations in a large developing country, Brazil. Due to difficulty of interpreting coefficients of a proportional hazard model in a competing risks environment we report the estimation results in terms of marginal effects of covariates but not on the hazard of making a certain transition but on the cumulative incidence function of making a transition from one to the other labor market state. Results show the importance of both individual characteristics and aggregate economic conditions on the probability of transiting from key employment states. In particular, more years of schooling significantly increase the probability of transiting to formal employment both from informal salaried employment and informal self-employment. Firm size is also an important factor with informal salaried workers in larger firms having a higher probability of transiting to formal employment. Business cycle effects (approximated by local unemployment rate) also influence transition probabilities with the probability of entering formal employment and informal self-employment being procyclical and the probability of transiting to informal salaried employment exhibiting countercyclical behavior.

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Appendix 3.A Non-parametric estimators

Let's denote by T a continuous random variable for a time to an event (or failure time) and by $t_1 < t_2 < \dots < t_k$ distinct failure time points.⁸ The survival function determines the probability of not failing by time point t_j and is denoted as

$$S(t_j) = Pr(T > t_j). \quad (3.A.1)$$

If there were no censored observations (i.e. observations that have not ended with an event of interest within the observation period) equation $1 - F(t_j)$, where $F(t_j)$ is the cumulative density function of time to an event observations, would be a sufficient estimator of the survival function. Kaplan-Meier estimator is a consistent estimator of the survival function in the presence of censored observations. Let's denote by d_j the number of spells that fail due to an event at t_j and by c_j the number of censored spells. By censored spells we refer only to right-censored spells, i.e. spells that have been observed until time point t_j , but have not failed from the cause of interest and no information is available on the state of a spell after period t_j . Important assumption that has to be made is the independence of the censoring distribution from the distribution of the failure time. Let's denote by n_j the number of spells that are at risk of failing at time t_j . Then, the number of spells at risk at time point t_j can be determined by

$$n_j = (d_j + c_j) + \dots + (d_k + c_k) = \sum_{l|l \geq j} (d_l + c_l). \quad (3.A.2)$$

Let's denote by $\lambda(t_j) = Pr(T = t_j | T > t_{j-1})$ the hazard function of failing at time t_j , which represents the instantaneous probability of failing at t_j given that the individual survived up to this time point. The estimate of the hazard function is the ratio $\hat{\lambda}(t_j) = d_j/n_j$. Given this estimate of the hazard function the Kaplan-Meier estimator of the survival function is defined as

$$\hat{S}(t) = \prod_{j|t_j \leq t} (1 - \lambda(t_j)) = \prod_{j|t_j \leq t} \left(\frac{n_j - d_j}{n_j} \right). \quad (3.A.3)$$

In case of competing risks data, we observe not only duration T , but also an indicator $\delta \in \{1, \dots, p\}$, which specifies the type of failure. Let's denote by $\lambda_m(t_j) = Pr(T = t_j, \delta = m | T > t_{j-1})$ the cause-specific hazard function of failing at t_j due to cause m . Then, the estimate of the cumulative incidence function, which is the probability of observing no event up to time point t_{j-1} and failing due to cause m at t_j is determined as

$$\hat{F}_m(t) = \sum_{j|t_j \leq t} \hat{\lambda}_m(t_j) \hat{S}(t_{j-1}). \quad (3.A.4)$$

Given that the estimate of the cause-specific hazard $\lambda_m(t_j)$ is determined by $\hat{\lambda}_m(t_j) = d_{mj}/n_j$, where d_{mj} denotes the number of spells failed due to cause m at time point t_j , estimator (3.A.4) can be reformulated as

$$\hat{F}_m(t) = \sum_{j|t_j \leq t} \frac{d_{mj}}{n_j} \hat{S}(t_{j-1}). \quad (3.A.5)$$

⁸Discussion in this section is based on [Cameron and Triverdi \(2005\)](#) and [Putter et al. \(2007\)](#).

Appendix 3.B Estimation results

Table 3.3: Marginal effects for the probability of exiting from informal employment

Variable	Exits to			
	Formal empl.	Self-employment	Unemployment	Out of labor force
Male	0.0174*** (0.0052)	-0.0180*** (0.0032)	0.0729*** (0.0049)	-0.0495*** (0.0051)
Age	0.0012*** (0.0002)	0.0023*** (0.0001)	-0.0035*** (0.0002)	0.0000 (0.0002)
White	-0.0237*** (0.0055)	-0.0556*** (0.0032)	0.0603*** (0.0050)	0.0406*** (0.0052)
Currently in school	-0.1650*** (0.0061)	-0.1171*** (0.0038)	0.0392*** (0.0052)	0.2603*** (0.0068)
Schooling: 1-3 years	-0.0335 [†] (0.0202)	-0.0633*** (0.0087)	0.0415** (0.0146)	0.0769*** (0.0179)
Schooling: 4-7 years	-0.0130 (0.0175)	-0.0713*** (0.0075)	0.0452*** (0.0134)	0.0610*** (0.0147)
Schooling: 8-10 years	0.0637*** (0.0179)	-0.0852*** (0.0074)	0.0519*** (0.0137)	-0.0059 (0.0139)
Schooling: 11 or more years	0.1459*** (0.0173)	-0.1021*** (0.0076)	0.0514*** (0.0140)	-0.0715*** (0.0138)
Firm size: 5-10 employees	0.1382*** (0.0089)	-0.1132*** (0.0038)	0.0370*** (0.0066)	-0.0396*** (0.0066)
Firm size: 11 or more employees	0.2001*** (0.0055)	-0.1409*** (0.0033)	0.0347*** (0.0049)	-0.0625*** (0.0051)
Construction	-0.1688*** (0.0084)	0.0130* (0.0066)	0.1392*** (0.0093)	0.0411*** (0.0086)
Retail trade and related	-0.0301*** (0.0075)	-0.0557*** (0.0046)	0.0646*** (0.0069)	0.0434*** (0.0071)
Financial services and real estate	-0.0002 (0.0089)	-0.0640*** (0.0056)	0.0770*** (0.0085)	0.0081 (0.0083)
Education, health care and related	-0.0314** (0.0112)	-0.1082*** (0.0063)	0.0800*** (0.0108)	0.0798*** (0.0110)
Services	-0.0867*** (0.0074)	-0.0380*** (0.0048)	0.1026*** (0.0074)	0.0420*** (0.0072)
Other	-0.1457*** (0.0222)	-0.0260 [†] (0.0143)	0.0696** (0.0212)	0.1239*** (0.0225)
Recife	-0.1213*** (0.0104)	-0.0356*** (0.0071)	0.0245** (0.0085)	0.1577*** (0.0115)
Salvador	0.0092 (0.0139)	-0.0562*** (0.0082)	0.0097 (0.0095)	0.0517*** (0.0130)
Belo Horizonte	-0.0376*** (0.0083)	-0.0521*** (0.0051)	0.0592*** (0.0075)	0.0563*** (0.0077)
Rio de Janeiro	0.1014*** (0.0105)	-0.0054 (0.0072)	0.0077 (0.0083)	-0.0927*** (0.0081)
Sao Paulo	-0.0189* (0.0087)	-0.0604*** (0.0052)	0.0786*** (0.0082)	0.0139 [†] (0.0080)
Unemployment rate (local)	-0.5161*** (0.1126)	0.2059** (0.0663)	1.3191*** (0.1008)	-1.0090*** (0.1046)

Standard errors in parentheses

[†] significant at $p < .10$; * $p < .05$; ** $p < .01$; *** $p < .001$

Table 3.4: Marginal effects for the probability of exiting from informal self-employment

Variable	Exits to			
	Formal empl.	Informal empl.	Unemployment	Out of labor force
Male	0.0058 (0.0041)	0.0883*** (0.0056)	0.0888*** (0.0048)	-0.0798*** (0.0058)
Age	0.0012*** (0.0002)	-0.0059*** (0.0002)	-0.0040*** (0.0002)	0.0087*** (0.0003)
White	-0.0950*** (0.0035)	-0.0068 (0.0054)	0.0505*** (0.0048)	0.1627*** (0.0059)
Currently in school	-0.1913*** (0.0052)	-0.0464*** (0.0080)	0.0415*** (0.0064)	0.3111*** (0.0098)
Schooling: 1-3 years	-0.1093*** (0.0115)	0.0124 (0.0131)	0.0501*** (0.0109)	0.1597*** (0.0138)
Schooling: 4-7 years	-0.0465*** (0.0105)	-0.0061 (0.0114)	0.0593*** (0.0100)	0.1172*** (0.0120)
Schooling: 8-10 years	-0.0045 (0.0131)	-0.0204 [†] (0.0117)	0.0655*** (0.0107)	0.0812*** (0.0123)
Schooling: 11 or more years	0.0899*** (0.0137)	-0.0653*** (0.0114)	0.0740*** (0.0108)	0.0288* (0.0121)
Construction	-0.1473*** (0.0054)	0.0309*** (0.0093)	0.1224*** (0.0092)	0.1105*** (0.0098)
Retail trade and related	-0.1025*** (0.0052)	-0.0011 (0.0079)	0.0528*** (0.0067)	0.1514*** (0.0079)
Financial services and real estate	-0.0911*** (0.0077)	0.0851*** (0.0122)	0.0446*** (0.0089)	0.0772*** (0.0116)
Education, health care and related	-0.1374*** (0.0088)	0.0947*** (0.0151)	0.0681*** (0.0115)	0.0874*** (0.0137)
Services	-0.0879*** (0.0057)	0.0788*** (0.0086)	0.0387*** (0.0067)	0.0842*** (0.0080)
Other	-0.1592*** (0.0113)	0.0671*** (0.0201)	0.0352* (0.0147)	0.1690*** (0.0208)
Recife	-0.2085*** (0.0049)	-0.0591*** (0.0091)	0.0614*** (0.0088)	0.3230*** (0.0112)
Salvador	-0.0789*** (0.0104)	-0.0268* (0.0130)	0.0268** (0.0097)	0.1488*** (0.0145)
Belo Horizonte	-0.1374*** (0.0048)	-0.0017 (0.0082)	0.0579*** (0.0073)	0.1952*** (0.0090)
Rio de Janeiro	-0.0437*** (0.0084)	0.1568*** (0.0119)	-0.0013 (0.0081)	-0.0575*** (0.0108)
Sao Paulo	-0.1356*** (0.0054)	0.0718*** (0.0096)	0.0484*** (0.0076)	0.1095*** (0.0096)
Unemployment rate (local)	-0.4087*** (0.0699)	0.5189*** (0.1101)	1.0050*** (0.0939)	-1.1152*** (0.1166)

Standard errors in parentheses

[†] significant at $p < .10$; * $p < .05$; ** $p < .01$; *** $p < .001$

Table 3.5: Marginal effects for the probability of exiting from formal employment

Variable	Exits to			
	Informal empl.	Self-employment	Unemployment	Out of labor force
Male	0.2320*** (0.0056)	0.0526*** (0.0021)	0.1098*** (0.0044)	-0.0550*** (0.0052)
Age	-0.0014*** (0.0003)	0.0026*** (0.0001)	-0.0042*** (0.0002)	0.0030*** (0.0003)
White	0.2080*** (0.0059)	0.0311*** (0.0022)	0.0951*** (0.0048)	0.0615*** (0.0053)
Currently in school	0.2616*** (0.0083)	-0.0097** (0.0031)	0.0722*** (0.0057)	0.0848*** (0.0076)
Schooling: 1-3 years	0.1913*** (0.0248)	0.0380*** (0.0094)	0.1190*** (0.0260)	0.0512* (0.0201)
Schooling: 4-7 years	0.2092*** (0.0218)	0.0459*** (0.0085)	0.1226*** (0.0227)	0.0123 (0.0168)
Schooling: 8-10 years	0.1931*** (0.0217)	0.0549*** (0.0091)	0.1376*** (0.0236)	0.0056 (0.0167)
Schooling: 11 or more years	0.1868*** (0.0209)	0.0545*** (0.0069)	0.1338*** (0.0200)	-0.0590** (0.0181)
Firm size: 5-10 employees	0.1823*** (0.0122)	-0.0515*** (0.0022)	0.1716*** (0.0138)	0.0858*** (0.0119)
Firm size: 11 or more employees	0.0811*** (0.0079)	-0.2421*** (0.0073)	0.1334*** (0.0052)	0.1298*** (0.0055)
Construction	0.1845*** (0.0112)	0.0703*** (0.0053)	0.0991*** (0.0088)	0.0568*** (0.0095)
Retail trade and related	0.2289*** (0.0080)	0.0231*** (0.0030)	0.0993*** (0.0061)	0.0612*** (0.0069)
Financial services and real estate	0.2397*** (0.0085)	0.0025 (0.0030)	0.1003*** (0.0066)	0.0575*** (0.0074)
Education, health care and related	0.2977*** (0.0118)	0.0018 (0.0039)	0.0900*** (0.0091)	0.0119 (0.0093)
Services	0.2498*** (0.0083)	0.0316*** (0.0033)	0.0979*** (0.0065)	0.0350*** (0.0070)
Other	0.2201*** (0.0359)	0.0306* (0.0121)	0.1284*** (0.0315)	0.0227 (0.0289)
Recife	0.1741*** (0.0114)	0.0463*** (0.0051)	0.0855*** (0.0087)	0.1075*** (0.0105)
Salvador	0.1371*** (0.0138)	0.0614*** (0.0071)	0.0864*** (0.0107)	0.0927*** (0.0132)
Belo Horizonte	0.2003*** (0.0086)	0.0450*** (0.0036)	0.0914*** (0.0067)	0.0655*** (0.0074)
Rio de Janeiro	0.2679*** (0.0105)	0.0556*** (0.0047)	0.0462*** (0.0075)	-0.0214* (0.0086)
Sao Paulo	0.2641*** (0.0092)	0.0261*** (0.0035)	0.0918*** (0.0071)	-0.0105 (0.0076)
Unemployment rate (local)	1.1122*** (0.1193)	-0.1113* (0.0455)	0.4133*** (0.0956)	-1.4142*** (0.1080)

Standard errors in parentheses

† significant at $p < .10$; * $p < .05$; ** $p < .01$; *** $p < .001$

Table 3.6: Marginal effects for the probability of exiting from informal employment (with unobserved heterogeneity)

Variable	Exits to			
	Formal empl.	Self-employment	Unemployment	Out of labor force
Male	0.0034*** (0.0004)	0.0043*** (0.0006)	0.0012 (0.0009)	-0.0088*** (0.0006)
Age	0.0001*** (0.0000)	0.0003*** (0.0000)	-0.0005*** (0.0000)	0.0001*** (0.0000)
White	0.0008 [†] (0.0005)	-0.0000 (0.0005)	-0.0011 (0.0008)	0.0002 (0.0006)
Currently in school	-0.0097*** (0.0006)	-0.0105*** (0.0010)	-0.0007 (0.0016)	0.0209*** (0.0012)
Schooling: 1-3 years	-0.0015 (0.0017)	-0.0017 (0.0011)	-0.0010 (0.0027)	0.0042* (0.0020)
Schooling: 4-7 years	0.0017 (0.0016)	-0.0013 (0.0010)	-0.0037 (0.0024)	0.0033* (0.0016)
Schooling: 8-10 years	0.0096*** (0.0020)	-0.0009 (0.0011)	-0.0085*** (0.0026)	-0.0002 (0.0015)
Schooling: 11 or more years	0.0136*** (0.0019)	-0.0037*** (0.0011)	-0.0029 (0.0025)	-0.0071*** (0.0014)
Firm size: 5-10 employees	0.0147*** (0.0012)	-0.0066*** (0.0008)	-0.0033* (0.0016)	-0.0048*** (0.0007)
Firm size: 11 or more employees	0.0201*** (0.0008)	-0.0053*** (0.0006)	-0.0100*** (0.0011)	-0.0048*** (0.0006)
Construction	-0.0035*** (0.0008)	0.0171*** (0.0016)	-0.0210*** (0.0021)	0.0074*** (0.0012)
Retail trade and related	-0.0000 (0.0006)	0.0003 (0.0007)	-0.0008 (0.0012)	0.0005 (0.0008)
Financial services and real estate	0.0003 (0.0007)	-0.0027** (0.0008)	0.0068*** (0.0013)	-0.0044*** (0.0009)
Education, health care and related	-0.0041*** (0.0007)	-0.0107*** (0.0012)	0.0179*** (0.0017)	-0.0030** (0.0010)
Services	-0.0055*** (0.0006)	0.0005 (0.0007)	0.0070*** (0.0011)	-0.0021** (0.0008)
Other	-0.0090*** (0.0016)	0.0021 (0.0018)	0.0020 (0.0033)	0.0048* (0.0024)
Recife	0.0006 (0.0010)	0.0102*** (0.0015)	-0.0333*** (0.0026)	0.0225*** (0.0021)
Salvador	-0.0153*** (0.0008)	-0.0133*** (0.0013)	0.0444*** (0.0017)	-0.0157*** (0.0009)
Belo Horizonte	0.0050*** (0.0008)	0.0056*** (0.0010)	-0.0176*** (0.0015)	0.0070*** (0.0010)
Rio de Janeiro	0.5212*** (0.0142)	0.2054*** (0.0111)	-0.8670*** (0.0059)	0.1403*** (0.0090)
Sao Paulo	-0.0091*** (0.0006)	-0.0080*** (0.0009)	0.0285*** (0.0013)	-0.0113*** (0.0008)
Unemployment rate (local)	0.0101 (0.0095)	0.0533*** (0.0104)	0.0034 (0.0177)	-0.0668*** (0.0118)

Standard errors in parentheses

[†] significant at $p < .10$; * $p < .05$; ** $p < .01$; *** $p < .001$

Table 3.7: Marginal effects for the probability of exiting from informal self-employment (with unobserved heterogeneity)

Variable	Exits to			
	Formal empl.	Informal empl.	Unemployment	Out of labor force
Male	0.0637*** (0.0076)	0.0760*** (0.0055)	0.0458*** (0.0062)	-0.1856*** (0.0050)
Age	0.0010*** (0.0001)	-0.0041*** (0.0002)	-0.0043*** (0.0003)	0.0074*** (0.0003)
White	0.0203*** (0.0032)	-0.0142*** (0.0028)	-0.0120* (0.0053)	0.0059 (0.0044)
Currently in school	-0.0351*** (0.0055)	-0.0456*** (0.0047)	-0.0219** (0.0078)	0.1026*** (0.0075)
Schooling: 1-3 years	0.0149* (0.0073)	-0.0006 (0.0063)	-0.0111 (0.0123)	-0.0032 (0.0094)
Schooling: 4-7 years	0.0462*** (0.0085)	-0.0116* (0.0058)	-0.0060 (0.0111)	-0.0287*** (0.0085)
Schooling: 8-10 years	0.0705*** (0.0117)	-0.0234*** (0.0065)	0.0048 (0.0118)	-0.0519*** (0.0091)
Schooling: 11 or more years	0.1182*** (0.0165)	-0.0542*** (0.0073)	0.0204 (0.0126)	-0.0843*** (0.0099)
Construction	-0.0149*** (0.0037)	-0.0087 [†] (0.0049)	0.0704*** (0.0095)	-0.0469*** (0.0072)
Retail trade and related	0.0154*** (0.0036)	-0.0093* (0.0037)	-0.0057 (0.0076)	-0.0004 (0.0055)
Financial services and real estate	0.0217*** (0.0051)	0.0522*** (0.0062)	-0.0141 (0.0102)	-0.0599*** (0.0083)
Education, health care and related	-0.0055 (0.0053)	0.0536*** (0.0073)	0.0083 (0.0125)	-0.0564*** (0.0096)
Services	0.0247*** (0.0045)	0.0515*** (0.0046)	-0.0258*** (0.0078)	-0.0503*** (0.0057)
Other	-0.0160* (0.0074)	0.0368*** (0.0099)	-0.0246 (0.0174)	0.0038 (0.0144)
Recife	-0.0435*** (0.0056)	-0.0704*** (0.0053)	0.0085 (0.0102)	0.1053*** (0.0088)
Salvador	0.0307*** (0.0075)	-0.0125 [†] (0.0066)	-0.0155 (0.0127)	-0.0027 (0.0109)
Belo Horizonte	-0.0025 (0.0029)	-0.0182*** (0.0040)	0.0013 (0.0082)	0.0194** (0.0064)
Rio de Janeiro	0.0739*** (0.0111)	0.1983*** (0.0134)	-0.1320*** (0.0188)	-0.1402*** (0.0125)
Sao Paulo	0.0002 (0.0032)	0.0456*** (0.0048)	-0.0097 (0.0087)	-0.0362*** (0.0067)
Unemployment rate (local)	-0.2883*** (0.0554)	0.2277*** (0.0544)	0.9760*** (0.1013)	-0.9154*** (0.0886)

Standard errors in parentheses

[†] significant at $p < .10$; * $p < .05$; ** $p < .01$; *** $p < .001$

Table 3.8: Marginal effects for the probability of exiting from formal employment (with unobserved heterogeneity)

Variable	Exits to			
	Informal empl.	Self-employment	Unemployment	Out of labor force
Male	0.0205*** (0.0054)	0.0067*** (0.0009)	-0.0048 (0.0051)	-0.0224*** (0.0026)
Age	0.0017*** (0.0003)	0.0012*** (0.0001)	-0.0034*** (0.0003)	0.0006*** (0.0001)
White	0.0138** (0.0053)	0.0020* (0.0009)	-0.0132* (0.0054)	-0.0026** (0.0009)
Currently in school	0.0606*** (0.0074)	-0.0128*** (0.0018)	-0.0498*** (0.0078)	0.0021 (0.0014)
Schooling: 1-3 years	0.0107 (0.0227)	0.0044 (0.0034)	-0.0131 (0.0235)	-0.0020 (0.0032)
Schooling: 4-7 years	0.0011 (0.0208)	0.0059 [†] (0.0031)	0.0013 (0.0217)	-0.0083** (0.0026)
Schooling: 8-10 years	-0.0096 (0.0203)	0.0086** (0.0031)	0.0107 (0.0212)	-0.0096*** (0.0027)
Schooling: 11 or more years	-0.0201 (0.0204)	0.0090** (0.0029)	0.0305 (0.0212)	-0.0194*** (0.0036)
Firm size: 5-10 employees	-0.0384** (0.0147)	-0.0429*** (0.0027)	0.0841*** (0.0148)	-0.0029 (0.0019)
Firm size: 11 or more employees	-0.0594*** (0.0130)	-0.0764*** (0.0037)	0.1233*** (0.0143)	0.0126*** (0.0023)
Construction	-0.0182 [†] (0.0109)	0.0162*** (0.0018)	0.0032 (0.0111)	-0.0012 (0.0017)
Retail trade and related	0.0139 [†] (0.0073)	-0.0017 (0.0012)	-0.0092 (0.0075)	-0.0029* (0.0012)
Financial services and real estate	0.0179* (0.0075)	-0.0126*** (0.0014)	-0.0009 (0.0076)	-0.0044*** (0.0013)
Education, health care and related	0.0402*** (0.0101)	-0.0130*** (0.0019)	-0.0165 (0.0103)	-0.0107*** (0.0019)
Services	0.0185* (0.0079)	0.0009 (0.0013)	-0.0121 (0.0084)	-0.0072*** (0.0015)
Other	0.0029 (0.0401)	0.0024 (0.0049)	0.0009 (0.0432)	-0.0062 (0.0047)
Recife	-0.0045 (0.0104)	0.0124*** (0.0020)	-0.0147 (0.0105)	0.0068*** (0.0021)
Salvador	-0.0187 (0.0129)	0.0049* (0.0022)	0.0143 (0.0132)	-0.0005 (0.0021)
Belo Horizonte	0.0005 (0.0078)	0.0079*** (0.0014)	-0.0066 (0.0080)	-0.0018 (0.0012)
Rio de Janeiro	0.3316*** (0.0479)	0.0185*** (0.0043)	-0.3512*** (0.0563)	0.0011 (0.0049)
Sao Paulo	0.0376*** (0.0080)	-0.0037** (0.0013)	-0.0205* (0.0082)	-0.0133*** (0.0019)
Unemployment rate (local)	0.1259 (0.1040)	-0.0464** (0.0180)	0.1159 (0.1078)	-0.1953*** (0.0291)

Standard errors in parentheses

[†] significant at $p < .10$; * $p < .05$; ** $p < .01$; *** $p < .001$

Chapter 4

Monetary Policy Choices Under Currency Substitution

4.1 Introduction

Currency substitution, a practice of using foreign currency for facilitating domestic transactions and as a store of value, is a widespread phenomenon in many developing and emerging economies. Appendix 4.A provides information on the level of currency substitution in a number of countries, which is commonly approximated by the share of foreign currency deposits in a domestic banking system. Estimates show that in some countries the share of foreign-currency denominated deposits might be as high as 90% of total deposits. Though sparse, estimates of the actual use of foreign currency for transactions purposes are of magnitude comparable to the share of foreign-currency denominated deposits.

There are several ways how currency substitution, or dollarization¹, may have an adverse effect on the performance of an economy and effectiveness of policies conducted by its monetary authority. Kokenyne et al. (2010) and Leiderman et al. (2006) provide a detailed discussion of these effects, which basically boil down to weakening of the central bank's ability to control both monetary and exchange rate policies and money in circulation. This, in turn, may have an adverse effect on the ability of a monetary authority to control inflation. In addition, potential seigniorage income that a government can generate may be significantly reduced at high rates of dollarization.

In the current study we address the following related question: what type of monetary policy is preferable in an environment with currency substitution, when a monetary authority explicitly accounts for it while formulating its policy.² To answer this question we build a small open economy dynamic stochastic general equilibrium (DSGE) model with currency substitution and analyze the implications of different monetary policy regimes. We focus on analyzing the performance of monetary policy formulated in the form of inflation targeting rules. In particular, we compare policy rules that respond to consumer price inflation or domestic price inflation assuming that a central bank uses an ad hoc loss function as a criteria to assess the performance of these rules. In addition, we analyze the dynamics of the economy, when a central bank

¹We use terms “currency substitution” and “dollarization” interchangeably. A large number of studies also use the term “financial dollarization”, which refers to an environment, where liabilities of domestic households or firms are issued in foreign currency.

²This gives rise to another related question of empirical relevance of currency substitution. Garcia-Cicco (2008) estimates a large DSGE model with Mexican data and finds that currency substitution plays a minor role in fitting the model to data. On the contrary, Castillo et al. (2006) find a larger role for currency substitution in Peru by estimating a similar DSGE model and establish that a model with currency substitution provides a better fit to the Peruvian data. Garcia-Cicco (2008) explains this by a larger role played by currency substitution in Peru than in Mexico.

targets changes in the nominal exchange rate only, i.e. fixed exchange rate regime. The main contribution of this paper is to study the role of currency substitution in formulating monetary policy rules in a small open economy with two sectors (non-traded and traded goods sectors). The latter structure introduces two types of productivity shocks, a number of relative price terms and a nontrivial relationship of these variables to inflation and output.

Economies with currency substitution have been modeled extensively using a number of approaches. Theoretical analysis of dollarization has been conducted using monetary search-theoretic models as in [Craig and Waller \(2004\)](#), models with spatial separation and limited communication as in [Antinolfi et al. \(2007\)](#) and [Duffy et al. \(2006\)](#) and DSGE models as in [Felices and Tuesta \(2007\)](#) and [Batini et al. \(2008\)](#). While the former two modeling frameworks address more subtle issues pertaining to the existence of dual currency regimes, the latter two studies present models, which are more appropriate to study issues related to the formulation of monetary policy in economies with currency substitution. Current analysis, in general, follows the setup of [Felices and Tuesta \(2007\)](#). In this study, authors build a simple two-country open economy model following among others [Gali and Monacelli \(2005\)](#), in which domestic agents value and hold both domestic and foreign currencies. These two currencies enter the utility function of households through a special monetary aggregate in a non-separable way. This setup allows to have an environment, where foreign currency is used for transactions purposes and as a deposit of value. [Felices and Tuesta \(2007\)](#) build a one sector economy, which limits the range of monetary policy rules that could be analyzed in an environment with currency substitution. Therefore, in this study we closely follow the setup of [Felices and Tuesta \(2007\)](#) and extend it in an important dimension by introducing the sector of non-traded goods.

Possibility to analyze a broader range of policy rules in detail is not the only reason why this additional sector is introduced. There are a number of other reasons why introduction of non-traded goods sector in a small open economy model improves the model's performance in several key dimensions. A number of recent studies in the open economy macroeconomics literature found that accounting for non-traded goods prices is important for understanding movements of the real exchange rate. In addition, it has been shown that price stickiness in non-traded and traded goods sectors significantly differs. For instance, [Burstein et al. \(2006\)](#) show that movements in the relative price of non-traded goods explain a large share of variation in the real exchange rate. Moreover, [Lee \(2009\)](#) shows that assuming different degrees of price stickiness in non-traded and traded goods sectors explains a number of cross-correlation patterns in the data that cannot be shown in models with an equal degree of price stickiness in both sectors. Moreover, the author shows that assuming different degrees of price rigidity is necessary to be able to explain observed dynamics of relative prices of non-traded goods. [Frankel \(2010\)](#) also suggests that using a small open economy model with a non-traded goods sector along with a traded goods sector is more appropriate for studying monetary policy issues in emerging economies. There is empirically determined close connection between nominal and real exchange rates in developing countries and the latter is most likely to be driven by differences in prices of non-traded goods across countries.

The paper is organized as follows. In the following section we describe the model and explain its main assumptions. In section 3, we analyze the dynamics of the model economy by studying impulse responses to a number of shocks originated both in domestic and foreign economies. Section 4 presents the analysis of a number of monetary policy rules under different levels of currency substitution and section 5 concludes.

4.2 Model

The economy consists of two countries, Home and Foreign, with populations of measure $[0, n]$ and $(n, 1]$, respectively. We are interested in the dynamics of a small open economy, equations of which are obtained in the limit of a population measure n . So, in this setup a small open economy model is derived from a two-country model. We assume a home bias in consumption in Home, which endogenizes the real exchange rate in the model and makes the movements of the real exchange rate potentially important for formulating the monetary policy. Such setup and a number of its extensions are studied among others in [De Paoli \(2009\)](#), [Sutherland \(2005\)](#) and [Faia and Monacelli \(2008\)](#).

4.2.1 Households

Home country

A representative household in Home maximizes the following stream of expected utilities, which is a function of the composite consumption index, leisure and real money balances in domestic and foreign currencies. Thus, a household in Home chooses the level of the composite consumption index, amount of real money balances in domestic and foreign currencies and holdings of bonds denominated in home and foreign currencies and maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U \left(C_t, Z_t \left(\frac{M_{H,t}}{P_t}, \frac{M_{F,t} S_t}{P_t} \right), L_t \right). \quad (4.2.1)$$

Households supply total labor hours, $L_t = L_{N,t} + L_{H,t}$, to firms producing Home non-traded, N , and traded goods, H . Term Z_t is a money aggregate, which combines in terms of composite consumption good holdings of real money balances in domestic and foreign currencies, $\frac{M_{H,t}}{P_t}$ and $\frac{M_{F,t} S_t}{P_t}$, respectively.

Asset markets are complete, which implies that households have access to one-period contingent bonds. There are two types of these bonds in the economy, denominated in domestic and foreign currency, respectively. Domestic currency denominated bond, $B_{H,t}(\vartheta_{t+1})$, promises to pay one unit of domestic currency if state of the world $\vartheta_{t+1} \in \Theta$ in the state space Θ occurs and 0, otherwise. Respectively, the foreign currency denominated bond, $B_{F,t}(\vartheta_{t+1})$, promises to pay one unit of foreign currency if $\vartheta_{t+1} \in \Theta$ occurs and 0, otherwise. We denote the period t price of domestic currency denominated one-period contingent bond as $\Xi_t(\vartheta_{t+1})$. Equivalently, $\Xi_t^*(\vartheta_{t+1})$ denotes the price of one-period contingent foreign-currency denominated bond. We assume that households in Home have access to both domestic currency and foreign currency denominated bonds.

Given this structure of assets markets, a representative household in Home maximizes (4.2.1) subject to the sequence of budget constraints

$$P_t C_t + \int \Xi_t(\vartheta_{t+1}) B_{H,t}(\vartheta_{t+1}) d\vartheta_{t+1} + \int \Xi_t^*(\vartheta_{t+1}) S_t B_{F,t}(\vartheta_{t+1}) d\vartheta_{t+1} + M_{H,t} + S_t M_{F,t} \leq \quad (4.2.2) \\ B_{H,t-1}(\vartheta_t) + S_t B_{F,t-1}(\vartheta_t) + M_{H,t-1} + S_t M_{F,t-1} + W_{H,t} L_{H,t} + W_{N,t} L_{N,t} + \Pi_t + T_t,$$

where S_t denotes the nominal exchange rate, Π_t denotes nominal dividends earned due to ownership of firms in Home and T_t is the nominal lump-sum transfer (or tax) that households receive from (or pay to) the government. Let's denote by i_t and i_t^* net returns on domestic and foreign currency denominated bonds, respectively. Then, for any state of the world $\vartheta_{t+1} \in \Theta$, we can define the price of the domestic currency denominated bond as $\int \Xi_t(\vartheta_{t+1}) d\vartheta_{t+1} = \Xi_t = \frac{1}{1+i_t}$. Equivalently, we define the price of the foreign currency denominated bond as $\int \Xi_t^*(\vartheta_{t+1}) d\vartheta_{t+1} =$

$\Xi_t^* = \frac{1}{1+i_t^*}$. Using these expressions for prices of bonds we reformulate the budget constraint (4.2.2) as

$$P_t C_t + \frac{B_{H,t}}{1+i_t} + \frac{S_t B_{F,t}}{1+i_t^*} + M_{H,t} + S_t M_{F,t} \leq B_{H,t-1} + S_t B_{F,t-1} + M_{H,t-1} + S_t M_{F,t-1} + W_{H,t} L_{H,t} + W_{N,t} L_{N,t} + \Pi_t + T_t. \quad (4.2.3)$$

Following Felices and Tuesta (2007) we choose the following functional forms for the utility function and the money aggregate

$$U_t = \frac{\left(\left(v C_t^{\frac{\theta-1}{\theta}} + (1-v) Z_t^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \right)^{1-\rho}}{1-\rho} - \frac{L_t^{(1+\xi)}}{1+\xi} \quad (4.2.4)$$

$$Z_t = \left(\kappa \left(\frac{M_{H,t}}{P_t} \right)^{\frac{\eta-1}{\eta}} + (1-\kappa) \left(\frac{M_{F,t} S_t}{P_t} \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}. \quad (4.2.5)$$

In the utility function (4.2.4), ρ denotes a coefficient of relative risk aversion, ξ is an inverse of the elasticity of labor supply and θ is a degree of complementarity and substitutability between the composite consumption index and the money aggregate. In the money aggregate (4.2.5) η is an elasticity of substitution between domestic and foreign currencies and κ is a preference rate for domestic currency.

Maximizing the discounted sequence of utilities of a representative domestic household (4.2.1) subject to the sequence of budget constraints (4.2.3) yields the following first order conditions

$$\frac{W_{N,t}}{P_t} = \frac{L_t^\xi}{U_{C,t}} \quad (4.2.6)$$

$$\frac{W_{H,t}}{P_t} = \frac{L_t^\xi}{U_{C,t}} \quad (4.2.7)$$

$$U_{C,t} = (1+i_t) \beta \mathbb{E}_t \left\{ U_{C,t+1} \frac{P_t}{P_{t+1}} \right\} \quad (4.2.8)$$

$$U_{C,t} = (1+i_t^*) \beta \mathbb{E}_t \left\{ U_{C,t+1} \frac{P_t}{P_{t+1}} \frac{S_{t+1}}{S_t} \right\}, \quad (4.2.9)$$

$$U_{\frac{M_H}{P},t} = \frac{i_t}{1+i_t} U_{C,t} \quad (4.2.10)$$

$$U_{\frac{M_F}{P},t} = \frac{i_t^*}{1+i_t^*} U_{C,t} \quad (4.2.11)$$

where $U_{C,t}$ is the marginal utility of total consumption, $U_{\frac{M_H}{P},t}$ and $U_{\frac{M_F}{P},t}$ are marginal utilities of real money balances in domestic and foreign currencies, respectively. Given assumed functional forms in (4.2.4) and (4.2.5), we can further define these marginal utility terms as

$$U_{C,t} = v \Omega_t^{\left(\frac{1}{\theta}-\rho\right)} C_t^{-\frac{1}{\theta}} \quad (4.2.12)$$

$$U_{\frac{M_H}{P},t} = \kappa (1-v) \Omega_t^{\left(\frac{1}{\theta}-\rho\right)} Z_t^{\left(\frac{1}{\eta}-\frac{1}{\theta}\right)} \left(\frac{M_{H,t}}{P_t} \right)^{-\frac{1}{\eta}} \quad (4.2.13)$$

$$U_{\frac{M_F}{P},t} = (1-v)(1-\kappa) \Omega_t^{\left(\frac{1}{\theta}-\rho\right)} Z_t^{\left(\frac{1}{\eta}-\frac{1}{\theta}\right)} \left(\frac{S_t M_{F,t}}{P_t} \right)^{-\frac{1}{\eta}} \quad (4.2.14)$$

$$\Omega_t = \left(v C_t^{\frac{\theta-1}{\theta}} + (1-v) Z_t^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \quad (4.2.15)$$

Labor market is fully competitive and labor is mobile across both sectors in Home. This implies that in equilibrium wages in both sectors are equalized, i.e. $\frac{W_{N,t}}{P_t} = \frac{W_{H,t}}{P_t}$. Government rebates seigniorage revenues to the population, so that the government budget is balanced

$$\int_0^n (M_{H,t}^k - M_{H,t-1}^k) dk = \int_0^n T_t^k dk. \quad (4.2.16)$$

Foreign country

In this study we concentrate only on the domestic economy and, thus, for simplicity we assume that households in Foreign do not value holdings of real money balances and purchase only bonds denominated in the foreign currency. A representative household in Foreign chooses the level of composite consumption index and holdings of foreign currency denominated bonds to maximize the following discounted stream of utilities

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U^*(C_t^*, L_t^*), \quad (4.2.17)$$

subject to

$$P_t^* C_t^* + \int \Xi^*(\vartheta_{t+1}) B_{F,t}(\vartheta_{t+1}) d\vartheta_{t+1} = B_{F,t-1} + W_{N,t}^* L_{N,t}^* + W_{H,t}^* L_{H,t}^* + \Pi_t^*.$$

In addition, we assume the following simple functional form for the utility function of households in Foreign

$$U_t^* = \frac{C_t^{*(1-\rho)}}{1-\rho} - \frac{L_t^{*(1+\xi)}}{1+\xi}.$$

Solution to this decision problem for any state of the world $\vartheta_{t+1} \in \Theta$ yields the following first order conditions:

$$\frac{W_{N,t}^*}{P_t^*} = \frac{(L_t^*)^\xi}{U_{C,t}^*} \quad (4.2.18)$$

$$\frac{W_{H,t}^*}{P_t^*} = \frac{(L_t^*)^\xi}{U_{C,t}^*} \quad (4.2.19)$$

$$U_{C,t}^* = (1 + i_t^*) \beta \mathbb{E}_t \left\{ U_{C,t+1}^* \frac{P_t^*}{P_{t+1}^*} \right\}, \quad (4.2.20)$$

where $U_{C,t}^* = (C_t^*)^{-\rho}$ is the marginal utility of total consumption. Wages in both sectors in Foreign are also equalized due to perfect mobility of labor across sectors. For simplicity, we assume that the coefficient of relative risk aversion, ρ , and inverse of the elasticity of labor supply, ξ , are equal across the countries.

4.2.2 Composite consumption and prices indices

Composite consumption index of a representative household is defined as

$$C_t = \left[\alpha^{\frac{1}{\sigma}} C_{N,t}^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} C_{T,t}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad (4.2.21)$$

where $\sigma > 0$ is the elasticity of substitution between non-traded and traded goods in Home, defined as C_N and C_T , respectively. Parameter α is a weight of non-traded goods in the composite index. Equivalently, composite consumption index of households in Foreign is defined as

$$C_t^* = \left[(\alpha^*)^{\frac{1}{\sigma}} (C_{N,t}^*)^{\frac{\sigma-1}{\sigma}} + (1-\alpha^*)^{\frac{1}{\sigma}} (C_{T,t}^*)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad (4.2.22)$$

where α^* captures the preference of households in the foreign economy for non-traded goods produced in Foreign.

We further define the composite index of traded goods in Home and Foreign as

$$C_{T,t} = \left[\omega^{\frac{1}{\phi}} C_{H,t}^{\frac{\phi-1}{\phi}} + (1-\omega)^{\frac{1}{\phi}} C_{F,t}^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \quad (4.2.23)$$

$$C_{T,t}^* = \left[(\omega^*)^{\frac{1}{\phi}} (C_{H,t}^*)^{\frac{\phi-1}{\phi}} + (1-\omega^*)^{\frac{1}{\phi}} (C_{F,t}^*)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}, \quad (4.2.24)$$

where $\phi > 0$ is the elasticity of substitution between Home and Foreign traded goods, ω and ω^* are weights on Home country traded goods consumed in Home and Foreign, respectively. Consumption bundle C_F refers to goods imported from Foreign and C_H^* refers to goods exported from Home to Foreign. Production in both sectors and countries is characterized by monopolistic competition and each consumption bundle of non-traded and traded goods is composed of imperfectly substitutable varieties of goods. These varieties are aggregated into bundles of goods by the following functions

$$\begin{aligned} C_{N,t} &= \left[\left(\frac{1}{n} \right)^{\frac{1}{\varepsilon}} \int_0^n c_{N,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}, & C_{N,t}^* &= \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\varepsilon}} \int_n^1 c_{N,t}^*(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}, \\ C_{H,t} &= \left[\left(\frac{1}{n} \right)^{\frac{1}{\varepsilon}} \int_0^n c_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}, & C_{F,t} &= \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\varepsilon}} \int_n^1 c_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}, \\ C_{H,t}^* &= \left[\left(\frac{1}{n} \right)^{\frac{1}{\varepsilon}} \int_0^n c_{H,t}^*(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}, & C_{F,t}^* &= \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\varepsilon}} \int_n^1 c_{F,t}^*(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}, \end{aligned} \quad (4.2.25)$$

where $\varepsilon > 1$ is the elasticity of substitution between varieties. We assume that $\omega^* = n\tilde{\omega}^*$, where $\tilde{\omega}^*$ is the degree of openness of the domestic economy. This implies that the share of Home traded goods consumed in Foreign increases either in the size of Home or in its degree of openness. Equivalently, the share of goods imported to Home is defined as $(1-\omega) = (1-n)\tilde{\omega}^*$. As stated before, we will derive equilibrium conditions of a small open economy model in the limit of the size parameter n , i.e. by assuming $n \rightarrow 0$.

Solution to the problem of the optimal allocation of total expenditures between non-traded and traded goods gives the following demand functions for each variety j in Home³

$$c_{N,t}(j) = \frac{1}{n} \alpha \left(\frac{p_{N,t}(j)}{P_{N,t}} \right)^{-\varepsilon} \left(\frac{P_{N,t}}{P_t} \right)^{-\sigma} C_t \quad (4.2.26)$$

$$c_{H,t}(j) = \frac{1}{n} \omega(1-\alpha) \left(\frac{p_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} \left(\frac{P_{H,t}}{P_{T,t}} \right)^{-\phi} \left(\frac{P_{T,t}}{P_t} \right)^{-\sigma} C_t \quad (4.2.27)$$

$$c_{F,t}(j) = \frac{1}{1-n} (1-\omega)(1-\alpha) \left(\frac{p_{F,t}(j)}{P_{F,t}} \right)^{-\varepsilon} \left(\frac{P_{F,t}}{P_{T,t}} \right)^{-\phi} \left(\frac{P_{T,t}}{P_t} \right)^{-\sigma} C_t, \quad (4.2.28)$$

³Detailed derivations are provided in Appendix 4.C.

and in Foreign

$$c_{N,t}^*(j) = \frac{1}{1-n} \alpha^* \left(\frac{p_{N,t}^*(j)}{P_{N,t}^*} \right)^{-\varepsilon} \left(\frac{P_{N,t}^*}{P_t^*} \right)^{-\sigma} C_t^* \quad (4.2.29)$$

$$c_{H,t}^*(j) = \frac{1}{n} \omega^* (1 - \alpha^*) \left(\frac{p_{H,t}^*(j)}{P_{H,t}^*} \right)^{-\varepsilon} \left(\frac{P_{H,t}^*}{P_{T,t}^*} \right)^{-\phi} \left(\frac{P_{T,t}^*}{P_t^*} \right)^{-\sigma} C_t^* \quad (4.2.30)$$

$$c_{F,t}^*(j) = \frac{1}{1-n} (1 - \omega^*) (1 - \alpha^*) \left(\frac{p_{F,t}^*(j)}{P_{F,t}^*} \right)^{-\varepsilon} \left(\frac{P_{F,t}^*}{P_{T,t}^*} \right)^{-\phi} \left(\frac{P_{T,t}^*}{P_t^*} \right)^{-\sigma} C_t^*. \quad (4.2.31)$$

Prices indices in Home and Foreign are defined as

$$P_t = \left[\alpha P_{N,t}^{1-\sigma} + (1 - \alpha) P_{T,t}^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad (4.2.32)$$

$$P_{T,t} = \left[\omega P_{H,t}^{1-\phi} + (1 - \omega) P_{F,t}^{1-\phi} \right]^{\frac{1}{1-\phi}} \quad (4.2.33)$$

$$P_t^* = \left[\alpha^* (P_{N,t}^*)^{1-\sigma} + (1 - \alpha^*) (P_{T,t}^*)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad (4.2.34)$$

$$P_{T,t}^* = \left[\omega^* (P_{H,t}^*)^{1-\phi} + (1 - \omega^*) (P_{F,t}^*)^{1-\phi} \right]^{\frac{1}{1-\phi}}, \quad (4.2.35)$$

where the price index for each bundle of varieties of goods is given by

$$\begin{aligned} P_{N,t} &= \left[\left(\frac{1}{n} \right) \int_0^n p_{N,t}(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}, & P_{N,t}^* &= \left[\left(\frac{1}{1-n} \right) \int_n^1 p_{N,t}^*(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}, \\ P_{H,t} &= \left[\left(\frac{1}{n} \right) \int_0^n p_{H,t}(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}, & P_{F,t} &= \left[\left(\frac{1}{1-n} \right) \int_n^1 p_{F,t}(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}, \\ P_{H,t}^* &= \left[\left(\frac{1}{n} \right) \int_0^n p_{H,t}^*(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}, & P_{F,t}^* &= \left[\left(\frac{1}{1-n} \right) \int_n^1 p_{F,t}^*(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}. \end{aligned} \quad (4.2.36)$$

Prices are set in producer currency, which implies that the law of one price holds for traded goods. In other words, $p_{H,t}(j) = S_t p_{H,t}^*(j)$ and $p_{F,t}(j) = S_t p_{F,t}^*(j)$. Due to presence of non-traded goods and home bias in preferences the purchasing power parity does not hold, i.e. $P_t \neq S_t P_t^*$. This implies that the real exchange rate defined as $Q_t = \frac{S_t P_t^*}{P_t}$ is not constant and equal to one. Following [Rychalovska \(2009\)](#) we also define the real exchange rate in terms of relative prices as

$$Q_t = \left(\frac{\omega^* + (1 - \omega^*) P_{FH,t}^{1-\phi}}{\omega + (1 - \omega) P_{FH,t}^{1-\phi}} \right)^{\frac{1}{1-\phi}} \left(\frac{\alpha^* (P_{NT,t}^*)^{1-\sigma} + (1 - \alpha^*)}{\alpha P_{NT,t}^{1-\sigma} + (1 - \alpha)} \right)^{\frac{1}{1-\sigma}}, \quad (4.2.37)$$

where $P_{FH,t} = \frac{P_{F,t}}{P_{H,t}}$ is the terms of trade, i.e. relative price of Foreign to Home traded goods; $P_{NT,t} = \frac{P_{N,t}}{P_{T,t}}$ and $P_{NT,t}^* = \frac{P_{N,t}^*}{P_{T,t}^*}$ are the relative prices of non-traded to traded goods in Home and Foreign. Given that $\alpha \neq \alpha^*$, it is clear from (4.2.37) that dynamics of the real exchange is affected not only by international terms of trade, i.e. relative price of traded goods, but also by domestic terms of trade in both countries, i.e. relative price of non-traded to traded goods.

4.2.3 International risk sharing

Completeness of asset markets implies that in equilibrium returns on domestic currency and foreign currency denominated one-period bonds are equalized. Combining Euler equations (4.2.9) and (4.2.20), which govern the intertemporal allocation of resources in both countries, yields

$$\beta \mathbb{E}_t \left\{ \frac{U_{C,t+1}}{U_{C,t}} \frac{P_t}{P_{t+1}} \frac{S_{t+1}}{S_t} \right\} = \beta \mathbb{E}_t \left\{ \frac{U_{C,t+1}^*}{U_{C,t}^*} \frac{P_t^*}{P_{t+1}^*} \right\}. \quad (4.2.38)$$

This risk sharing condition implies that under the assumption of complete asset markets growth of the marginal utility of consumption across countries (expressed in the same currency units) is equalized. Given the definition of the real exchange rate $Q_t = \frac{S_t P_t^*}{P_t}$, backward recursion of the expression (4.2.38) yields

$$Q_t = k_0 \frac{U_{C,t}^*}{U_{C,t}}, \quad (4.2.39)$$

where $k_0 = Q_0 \frac{U_{C,0}}{U_{C,0}^*}$ is a function of initial conditions and it reflects the difference in the level of the steady state wealth across two countries.

Combining domestic household Euler equations (4.2.8) and (4.2.9) yields the following uncovered interest rate parity condition

$$(1 + i_t) \mathbb{E}_t \left\{ \frac{U_{C,t+1}}{U_{C,t}} \frac{P_t}{P_{t+1}} \right\} = (1 + i^*) \mathbb{E}_t \left\{ \frac{U_{C,t+1}^*}{U_{C,t}^*} \frac{P_t}{P_{t+1}} \frac{S_{t+1}}{S_t} \right\}, \quad (4.2.40)$$

which governs the choice of domestic and foreign currency denominated bonds in the domestic economy.

4.2.4 Relative demand for foreign currency

First order conditions (4.2.10), (4.2.11), (4.2.13) and (4.2.14) yield the following expression for a relative demand for foreign currency

$$\begin{aligned} \tilde{\gamma}_t &= \frac{\frac{S_t M_{F,t}}{P_t}}{\frac{M_{H,t}}{P_t}} \\ &= \left(\frac{i_t^*}{1 + i_t^*} \frac{1 + i_t}{i_t} \frac{\kappa}{1 - \kappa} \right)^{-\eta}. \end{aligned} \quad (4.2.41)$$

It is clear from first order conditions (4.2.10) and (4.2.11) that expressions $\frac{i_t}{1 + i_t}$ and $\frac{i_t^*}{1 + i_t^*}$ in (4.2.41) capture the opportunity cost of holding wealth in domestic and foreign currencies, respectively. Relative demand for foreign currency decreases as the opportunity cost of holding foreign currency increases (interest rate i^* on foreign denominated bonds goes up) and the demand for foreign currency increases as the opportunity cost of holding domestic currency increases.

Modifying expression (4.2.41) also allows us to obtain the following expression for the dollarization ratio, i.e. the ratio of foreign currency in total money supply (where we define total money supply as the sum of wealth held in domestic and foreign currencies)

$$\gamma_t = \left(\left(\frac{i_t^*}{1 + i_t^*} \frac{1 + i_t}{i_t} \frac{\kappa}{1 - \kappa} \right)^\eta + 1 \right)^{-1}, \quad (4.2.42)$$

The effect of changes in interest rates on the ratio of foreign currency in total money supply is analogous to the effect of interest rates on the relative demand for domestic and foreign currency in (4.2.41).

4.2.5 Firms and price-setting behavior

Production of a variety j in this economy occurs through the following linear technology

$$y_{I,t}(j) = A_{I,t}L_{I,t}(j), \quad I = N, H, \quad (4.2.43)$$

where $A_{I,t}$ is a sector-specific productivity parameter. Each firm producing a variety j of non-traded and traded goods is a monopolistic producer of a given variety. Prices in the goods market are assumed to be sticky and the stickiness is modeled using the Calvo-pricing mechanism. This implies that every period domestic firms producing non-traded goods face a constant probability $1 - \psi_N$ of resetting their prices optimally. Equivalently, domestic firms producing traded goods face a constant probability $1 - \psi_H$ of resetting prices optimally. An optimizing firm producing a non-traded variety j chooses a new price of this variety $\tilde{p}_{N,t}(j)$ to maximize the expected discounted stream of profits

$$\max_{\tilde{p}_{N,t}(j)} \mathbb{E}_t \sum_{s=t}^{\infty} \psi_N^{s-t} \Lambda_{t,s} y_{N,s}(j) (\tilde{p}_{N,t}(j) - MC_{N,s}(j))$$

subject to the demand constraint for variety j given by

$$y_{N,s}(j) = \left(\frac{\tilde{p}_{N,t}(j)}{P_{N,s}} \right)^{-\varepsilon} Y_{N,s}.$$

where $MC_{N,t} = \frac{W_t}{A_{N,t}}$ is the nominal marginal cost of labor in non-traded goods sector, $\Lambda_{t,s}$ is a stochastic discount factor and $Y_{N,s}$ is total demand for non-traded goods in Home.⁴ Solution to this problem yields the following first order condition

$$\mathbb{E}_t \sum_{s=t}^{\infty} \psi_N^{s-t} \Lambda_{t,s} y_{N,s}(j) \left(\tilde{p}_{N,t}(j) - \frac{\varepsilon}{\varepsilon - 1} MC_{N,s}(j) \right) = 0,$$

which could also be expressed as

$$\frac{\tilde{p}_{N,t}(j)}{P_{N,t}} = \frac{\mathbb{E}_t \sum_{s=t}^{\infty} (\psi_N \beta)^{s-t} U_{C,s} \mu MC_{N,s}^r \left(\frac{P_{N,s}}{P_{N,t}} \right)^{\varepsilon} Y_{N,s}}{\mathbb{E}_t \sum_{s=t}^{\infty} (\psi_N \beta)^{s-t} U_{C,s} \frac{P_{N,s}}{P_s} \left(\frac{P_{N,s}}{P_{N,t}} \right)^{\varepsilon-1} Y_{N,s}}, \quad (4.2.44)$$

where $\mu = \frac{\varepsilon}{\varepsilon-1}$ is the mark-up over marginal cost and $MC_{N,s}^r = \frac{MC_{N,s}}{P_s}$ is the real marginal cost of labor in the non-traded goods sector.

Under Calvo-pricing mechanism aggregating over the price of each variety of non-traded goods results in the following price index for non-traded goods in Home

$$P_{N,t} = \left[\psi_N P_{N,t-1}^{1-\varepsilon} + (1 - \psi_N) \tilde{p}_{N,t}(j)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \quad (4.2.45)$$

Formulating and solving a similar price-setting problem for firms producing Home traded goods results in the following expression for optimal pricing in the traded goods sector

$$\frac{\tilde{p}_{H,t}(j)}{P_{H,t}} = \frac{\mathbb{E}_t \sum_{s=t}^{\infty} (\psi_H \beta)^{s-t} U_{C,s} \mu MC_{H,s}^r \left(\frac{P_{H,s}}{P_{H,t}} \right)^{\varepsilon} Y_{H,s}}{\mathbb{E}_t \sum_{s=t}^{\infty} (\psi_H \beta)^{s-t} U_{C,s} \frac{P_{H,s}}{P_s} \left(\frac{P_{H,s}}{P_{H,t}} \right)^{\varepsilon-1} Y_{H,s}}, \quad (4.2.46)$$

where $MC_{H,t}^r = \frac{MC_{H,t}}{P_t} = \frac{W_t}{A_{H,t}P_t}$ is the real marginal cost of labor in the traded goods sector. Price index in the home country traded goods sector evolves according to

$$P_{H,t} = \left[\psi_H P_{H,t-1}^{1-\varepsilon} + (1 - \psi_H) \tilde{p}_{H,t}(j)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \quad (4.2.47)$$

⁴Firms are owned by households and, thus, in equilibrium firms discount the expected value of net profits by the intertemporal marginal rate of substitution, i.e. stochastic discount factor, defined as $\Lambda_{t,s} = \beta^{s-t} \frac{U_{C,s}}{U_{C,t}} \frac{P_t}{P_s}$.

4.2.6 Monetary policy

We assume that the monetary authority in the domestic country sets the short-run nominal interest rate according to the following general Taylor-type rule

$$\frac{1+i_t}{1+i} = \left(\frac{1+i_{t-1}}{1+i} \right)^{q_i} \left(\frac{Y_t}{Y_{t-1}} \right)^{q_Y} \left(\frac{\tilde{P}_t}{\tilde{P}_{t-1}} \right)^{q_{\tilde{P}}} \left(\frac{S_t}{S_{t-1}} \right)^{q_S} \left(\frac{Q_t}{Q_{t-1}} \right)^{q_Q}, \quad (4.2.48)$$

where q_i , q_Y , $q_{\tilde{P}}$, q_S and q_Q are positive feedback coefficients, $\tilde{P}_t \in \{P_t, P_{D,t}, P_{N,t}, P_{H,t}\}$ and $\tilde{\pi} \in \{\pi_t, \pi_{D,t}, \pi_{N,t}, \pi_{H,t}\}$ are various types of inflation rates. $\pi_{D,t}$ denotes the domestic inflation rate and it is defined in (4.3.1). Equation (4.2.48) accommodates several monetary policy regimes, including strict consumer price index (CPI) inflation targeting, targeting inflation of non-traded goods prices, traded goods prices or their combination and partial targeting of changes in the nominal exchange rate.

4.2.7 Market clearing conditions

At the variety level, the market clearing in the goods market for non-traded and domestic country traded goods implies that

$$y_{N,t}(j) = nc_{N,t}(j) \quad (4.2.49)$$

$$y_{H,t}(j) = nc_{H,t}(j) + (1-n)c_{H,t}^*(j). \quad (4.2.50)$$

Using results of the optimal expenditure allocation problem given by (4.2.26), (4.2.27) and (4.2.30) we reformulate (4.2.49) and (4.2.50) as⁵

$$y_{N,t}(j) = \alpha \left(\frac{p_{N,t}(j)}{P_{N,t}} \right)^{-\varepsilon} \left(\frac{P_{N,t}}{P_t} \right)^{-\sigma} C_t \quad (4.2.51)$$

$$y_{H,t}(j) = \left(\frac{p_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} \left(\frac{P_{H,t}}{P_{T,t}} \right)^{-\phi} \left(\frac{P_{T,t}}{P_t} \right)^{-\sigma} \left[\omega(1-\alpha)C_t + \frac{1-n}{n} \omega^*(1-\alpha^*)Q_t^\sigma \left(\frac{\omega^*}{\omega + (1-\omega)P_{FH}^{1-\phi}} + \frac{1-\omega^*}{\omega P_{FH}^{\phi-1} + (1-\omega)} \right)^{\frac{\phi-\sigma}{1-\phi}} C_t \right] \quad (4.2.52)$$

Equivalently, goods market clearing in the foreign country requires the following conditions to hold

$$y_{N,t}^*(j) = (1-n)c_{N,t}^*(j) \quad (4.2.53)$$

$$y_{F,t}(j) = nc_{F,t}(j) + (1-n)c_{F,t}^*(j), \quad (4.2.54)$$

which we reformulate using (4.2.29), (4.2.28), (4.2.31) and obtain

$$y_{N,t}^*(j) = \alpha^* \left(\frac{p_{N,t}^*(j)}{P_{N,t}^*} \right)^{-\varepsilon} \left(\frac{P_{N,t}^*}{P_t^*} \right)^{-\sigma} C_t^* \quad (4.2.55)$$

$$y_{F,t}(j) = \left(\frac{p_{F,t}(j)}{P_{F,t}} \right)^{-\varepsilon} \left(\frac{P_{F,t}}{P_{T,t}} \right)^{-\phi} \left(\frac{P_{T,t}}{P_t} \right)^{-\sigma} \left[\frac{n}{1-n} (1-\omega)(1-\alpha)C_t + (1-\omega^*)(1-\alpha^*)Q_t^\sigma \left[\frac{\omega^*}{\omega + (1-\omega)P_{FH}^{1-\phi}} + \frac{1-\omega^*}{\omega P_{FH}^{\phi-1} + (1-\omega)} \right]^{\frac{\phi-\sigma}{1-\phi}} C_t^* \right]. \quad (4.2.56)$$

⁵Detailed derivations are provided in Appendix 4.C.2.

We are primarily interested in obtaining equilibrium equations of a small open economy and, therefore, we determine equilibrium conditions under the assumption of $n \rightarrow 0$. Taking the limit of the size parameter n in equations (4.2.51), (4.2.52), (4.2.55) and (4.2.56) results in the following modified goods market clearing conditions

$$y_{N,t}(j) = \alpha \left(\frac{p_{N,t}(j)}{P_{N,t}} \right)^{-\varepsilon} \left(\frac{P_{N,t}}{P_t} \right)^{-\sigma} C_t \quad (4.2.57)$$

$$y_{H,t}(j) = \left(\frac{p_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} \left(\frac{P_{H,t}}{P_{T,t}} \right)^{-\phi} \left(\frac{P_{T,t}}{P_t} \right)^{-\sigma} \times \left[\omega(1-\alpha)C_t + \tilde{\omega}^*(1-\alpha^*)Q_t^\sigma \left(\omega P_{FH}^{\phi-1} + (1-\omega) \right)^{\frac{\sigma-\phi}{1-\phi}} C_t^* \right] \quad (4.2.58)$$

$$y_{N,t}^*(j) = \alpha^* \left(\frac{p_{N,t}^*(j)}{P_{N,t}^*} \right)^{-\varepsilon} \left(\frac{P_{N,t}^*}{P_t^*} \right)^{-\sigma} C_t^* \quad (4.2.59)$$

$$y_{F,t}(j) = \left(\frac{p_{F,t}(j)}{P_{F,t}} \right)^{-\varepsilon} \left(\frac{P_{F,t}}{P_{T,t}} \right)^{-\phi} \left(\frac{P_{T,t}}{P_t} \right)^{-\sigma} \left[(1-\alpha^*)Q_t^\sigma \left(\omega P_{FH}^{\phi-1} + (1-\omega) \right)^{\frac{\sigma-\phi}{1-\phi}} C_t^* \right] \quad (4.2.60)$$

In these modified market clearing conditions total consumption index of a foreign country C_t^* does have an effect on the production of traded goods in Home and the extent of this effect is determined among other variables by the degree of openness $\tilde{\omega}^*$ of the domestic economy. At the same time consumption in Home has no effect on dynamics of Foreign. This implies that we can treat the equilibrium dynamics of Foreign as exogenous from the perspective of an agent in Home.

Let's define the aggregate output of non-traded and traded goods in Home as

$$Y_{N,t} = \left(\left(\frac{1}{n} \right)^{\frac{1}{\varepsilon}} \int_0^n y_{N,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$Y_{H,t} = \left(\left(\frac{1}{n} \right)^{\frac{1}{\varepsilon}} \int_0^n y_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Equivalently, the aggregate output in non-traded and traded goods sectors in a foreign country are defined by

$$Y_{N,t}^* = \left(\left(\frac{1}{1-n} \right)^{\frac{1}{\varepsilon}} \int_0^n y_{N,t}^*(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$Y_{F,t} = \left(\left(\frac{1}{1-n} \right)^{\frac{1}{\varepsilon}} \int_0^n y_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Then, using our results for market clearing conditions in (4.2.57)-(4.2.60) and definitions of price indices in (4.2.36) we obtain the following expressions for aggregate output of non-traded and

traded goods in both countries

$$Y_{N,t} = \alpha \left(\frac{P_{N,t}}{P_t} \right)^{-\sigma} C_t \quad (4.2.61)$$

$$Y_{H,t} = \omega(1 - \alpha) \left(\frac{P_{H,t}}{P_{T,t}} \right)^{-\phi} \left(\frac{P_{T,t}}{P_t} \right)^{-\sigma} C_t + \tilde{\omega}^*(1 - \alpha^*) \left(\frac{P_{H,t}^*}{P_{T,t}^*} \right)^{-\phi} \left(\frac{P_{T,t}^*}{P_t^*} \right)^{-\sigma} C_t^* \quad (4.2.62)$$

$$Y_{N,t}^* = \alpha^* \left(\frac{P_{N,t}^*}{P_t^*} \right)^{-\sigma} C_t^* \quad (4.2.63)$$

$$Y_{F,t} = (1 - \omega^*)(1 - \alpha^*) \left(\frac{P_{F,t}^*}{P_{T,t}^*} \right)^{-\phi} \left(\frac{P_{T,t}^*}{P_t^*} \right)^{-\sigma} C_t^*. \quad (4.2.64)$$

Households in Home supply labor only to firms in two production sectors and, thus, the labor market clearing condition in Home is given by

$$L_t = L_{N,t} + L_{H,t}. \quad (4.2.65)$$

Equivalently, the labor market clearing condition in the foreign country is given by

$$L_t^* = L_{N,t}^* + L_{H,t}^*. \quad (4.2.66)$$

4.2.8 Equilibrium

We are interested only in the dynamics of a domestic economy and, thus, consider the dynamics of the foreign economy as exogenous. Given the stochastic exogenous processes for the evolution of productivity in both production sectors in Home $\{A_{N,t}, A_{H,t}\}_{t=0}^{\infty}$ and for the evolution of relevant variables in the foreign economy $\{C_t^*, l_t^*, P_t^*, P_{NT,t}^*\}_{t=0}^{\infty}$, the equilibrium in this model is the sequence of allocations and prices $\{C_{N,t}, C_{H,t}, L_{N,t}, L_{H,t}, W_{N,t}, W_{H,t}, P_t, P_{N,t}, P_{H,t}, S_t, Q_t, M_{H,t}, M_{F,t}, B_{H,t}, B_{F,t}, Y_{N,t}, Y_{H,t}\}_{t=0}^{\infty}$, which for given initial conditions satisfy equations (4.2.3), (4.2.6)-(4.2.11), (4.2.16), (4.2.21), (4.2.23), (4.2.26)-(4.2.28), (4.2.32), (4.2.33), (4.2.37), (4.2.39), (4.2.43), (4.2.45), (4.2.47), (4.2.48), (4.2.61), (4.2.62) and (4.2.65).

The dynamics of the model is analyzed by obtaining the first-order approximation of equilibrium conditions around the deterministic steady state, solving the resulting system of equations and examining the response of key endogenous variables to exogenous shocks. The deterministic steady state equilibrium of the model is described in Appendix 4.B.

4.2.9 Log-linear model

In this section we present the linear version of the model, which we obtain by log-linearizing the model equations around the non-stochastic steady state equilibrium. We denote by $x_t = \ln X_t - \ln X$ the deviation of the variable X_t (in its log-transformed form) from its steady state value X . When it is relevant, in the equations below we assume that $n \rightarrow 0$.

Log-linearizing the expression (4.2.44) for the price set by optimizing firms in the non-traded goods sector yields⁶

$$\pi_{N,t} = \Sigma_N (\zeta l_t - a_{N,t} - u_{C,t} - (1 - \alpha)p_{NT,t}) + \beta \mathbb{E}_t \pi_{N,t+1}, \quad (4.2.67)$$

where $\Sigma_N = \frac{(1 - \psi_N \beta)(1 - \psi_N)}{\psi_N}$ and $\pi_{N,t} = \ln(P_{N,t}/P_{N,t-1})$ is the inflation rate. Transforming the similar optimal pricing condition in the traded goods sector results in

$$\pi_{H,t} = \Sigma_H (\zeta l_t - a_{H,t} - u_{C,t} + \alpha p_{NT,t} + (1 - \omega)p_{FH,t}) + \beta \mathbb{E}_t \pi_{H,t+1}, \quad (4.2.68)$$

⁶Detailed derivations are provided in Appendix 4.D.

where $\Sigma_H = \frac{(1-\psi_H\beta)(1-\psi_H)}{\psi_H}$ and $\pi_{H,t} = \ln(P_{H,t}/P_{H,t-1})$ is the inflation rate in the traded goods sector.

Log-linearizing expression (4.2.61) for the aggregate demand in the non-traded goods sector we obtain

$$y_{N,t} = c_t - \sigma(1 - \alpha)p_{NT,t}, \quad (4.2.69)$$

where $p_{NT,t} = p_{N,t} - p_{T,t}$. Equivalently, equation (4.2.62) in the traded goods sector has the following log-linear form

$$y_{H,t} = (\phi + (\phi - \sigma)\omega)(1 - \omega)p_{FH,t} + \alpha\sigma p_{NT,t} + \omega c_t + (1 - \omega)c_t^* + (1 - \omega)\sigma q_t. \quad (4.2.70)$$

Log-linearizing the labor market clearing condition (4.2.65) results in

$$l_t = \alpha(y_{N,t} - a_{N,t}) + (1 - \alpha)(y_{H,t} - a_{H,t}), \quad (4.2.71)$$

where we also used the log-linearized form of the production function (4.2.43).

Given the price structure in this economy, we define the total output in the domestic country as $P_t Y_t = P_{N,t} Y_{N,t} + P_{H,t} Y_{H,t}$. Log-linearizing the latter gives us

$$y_t = \alpha y_{N,t} + (1 - \alpha)y_{H,t} - (1 - \alpha)(1 - \omega)p_{FH,t}, \quad (4.2.72)$$

where we see that the log-deviations of total output from its steady state value depend not only on log-deviations of sectoral output terms but also on movements of the terms of trade.

Log-linearizing the domestic economy Euler equation (4.2.8) and equilibrium risk sharing condition (4.2.39) yields

$$u_{C,t} = i_t - \pi_{t+1} + u_{C,t+1} \quad (4.2.73)$$

$$q_t = -\rho c_t^* - u_{C,t}, \quad (4.2.74)$$

where the marginal utility of consumption in log-linear form is given by

$$u_{C,t} = -\rho c_t + \beta(\theta\rho - 1)(1 - \omega)(i_t(1 - \gamma) + i_t^*\gamma) \quad (4.2.75)$$

The term ω in (4.2.75) is a function of a number of parameters and steady values of endogenous variables and it is defined in (4.D.3). Term γ is the steady state dollarization ratio and, as shown in (4.B.16), it is defined as

$$\gamma = \frac{\left(\frac{\kappa}{1-\kappa}\right)^{-\eta}}{1 + \left(\frac{\kappa}{1-\kappa}\right)^{-\eta}}.$$

Equations (4.2.73) and (4.2.75) specify how exactly the currency substitution affects the dynamics of the economy. These equations indicate that as the steady state dollarization ratio γ in the domestic economy rises, the effect of the Home interest rate on the marginal utility of consumption and overall dynamics of the economy weakens. In other words, this setup allows us to capture the observation that in an environment with currency substitution policy responses of a monetary authority become less effective and the effect of these responses might weaken further as the dollarization ratio in the economy increases. At the same time, higher values of γ imply stronger effect of foreign interest rate shocks on the domestic economy.

Other terms have the following log-linearized forms

$$q_t = \alpha^* p_{NT,t}^* - \alpha p_{NT,t} + \omega p_{FH,t} \quad (4.2.76)$$

$$\Delta s_t = \Delta q_t + \pi_t - \pi_t^* \quad (4.2.77)$$

$$\pi_t = \alpha \pi_{N,t} + (1 - \alpha) \pi_{H,t} + (1 - \alpha)(1 - \omega) \Delta p_{FH,t} \quad (4.2.78)$$

$$\Delta p_{NT,t} = \pi_{N,t} - \pi_{H,t} - (1 - \omega) \Delta p_{FH,t}, \quad (4.2.79)$$

where Δ denotes the first difference of a variable, i.e. $\Delta x_t = x_t - x_{t-1}$. Equations (4.2.76) and (4.2.77) are determined using the definition of the real exchange rate, (4.2.78) is the log-linearized version of the price index of the composite consumption good (4.2.32) and (4.2.79) is obtained from the definition of the relative price of non-traded goods.

The central bank's policy rule has the following general log-linearized form

$$i_t = \varrho_i i_{t-1} + \varrho_y \Delta y_t + \varrho_{\tilde{\pi}} \tilde{\pi}_t + \varrho_s \Delta s_t + \varrho_q \Delta q_t, \quad (4.2.80)$$

where $\tilde{\pi}_t \in \{\pi_t, \pi_{D,t}, \pi_{N,t}, \pi_{H,t}\}$. Evolution of sectoral productivity in the domestic economy is determined by the following AR(1) processes

$$\begin{pmatrix} a_{N,t} \\ a_{H,t} \end{pmatrix} = \begin{pmatrix} \tau_N & 0 \\ 0 & \tau_H \end{pmatrix} \begin{pmatrix} a_{N,t-1} \\ a_{H,t-1} \end{pmatrix} + \begin{pmatrix} \zeta_{N,t} \\ \zeta_{H,t} \end{pmatrix}, \quad (4.2.81)$$

where $\zeta_{N,t} \sim N(0, \varsigma_N)$ and $\zeta_{H,t} \sim N(0, \varsigma_H)$. For simplicity we assume that all variables related to the foreign country also follow AR(1) processes

$$\begin{pmatrix} i_t^* \\ c_t^* \\ \pi_t^* \\ p_{NT,t}^* \end{pmatrix} = \begin{pmatrix} \tau_{i^*} & 0 & 0 & 0 \\ 0 & \tau_{c^*} & 0 & 0 \\ 0 & 0 & \tau_{\pi^*} & 0 \\ 0 & 0 & 0 & \tau_{p_{NT}^*} \end{pmatrix} \begin{pmatrix} i_{t-1}^* \\ c_{t-1}^* \\ \pi_{t-1}^* \\ p_{NT,t-1}^* \end{pmatrix} + \begin{pmatrix} \zeta_{i^*} \\ \zeta_{c^*} \\ \zeta_{\pi^*} \\ \zeta_{p_{NT}^*} \end{pmatrix}, \quad (4.2.82)$$

where $\zeta_{i^*} \sim N(0, \varsigma_{i^*})$, $\zeta_{c^*} \sim N(0, \varsigma_{c^*})$, $\zeta_{\pi^*} \sim N(0, \varsigma_{\pi^*})$ and $\zeta_{p_{NT}^*} \sim N(0, \varsigma_{p_{NT}^*})$.

4.3 Model dynamics

4.3.1 Calibration

We calibrate the model parameters following the literature closely related to this paper (Felices and Tuesta, 2007; Batini et al., 2008). As it is common in the literature we assume that the subjective discount factor β equals 0.99, inverse of the elasticity of labor supply ζ is 2 and the coefficient of relative risk aversion ρ equals 2. Elasticity of substitution across varieties of traded and non-traded goods is calibrated to 6. As for the parameters of the utility function, we choose value of the preference rate for domestic currency, κ , to be 0.47, elasticity of substitution between currencies η to be equal to 4.1, preference rate for consumption in the utility function v to be 0.83 and the degree of complementarity and substitutability between consumption and money aggregate θ to be equal to 2. Following related studies we choose the elasticity of substitution between Home and Foreign traded goods ϕ to be 1.5. Elasticity of substitution between traded and non-traded goods σ is 0.5, which is in line with estimates for a number of Latin American countries (Lorenzo et al., 2005; Neumeyer and Rozada, 2003). Weight on non-traded goods in Home α equals 0.5, while the same parameter in Foreign α^* is chosen to be higher at 0.7 (to capture the common observation of relatively higher share of traded goods in consumption in small countries). Weight on Home-produced traded goods in Home ω equals 0.7. Following recent empirical findings (see Lee, 2009) we assume that prices are more rigid in the non-traded goods sector and assign the value of 0.8 to the probability of not adjusting the price in the non-traded goods sector ψ_N and the value of 0.6 to the parameter ψ_H of the traded goods sector. List of all parameters are given in Table 4.1.

Table 4.1: Benchmark parameter values

Parameter	Value	Description
ρ	2	Coefficient of relative risk aversion
κ	0.47	Preference rate for domestic currency
η	4.1	Elasticity of substitution between currencies
ν	0.83	Preference rate for consumption and money aggregate
θ	2	Degree of complementarity between consumption and money aggregate
β	0.99	Subjective discount factor
ζ	2	Inverse of the elasticity of labor supply
σ	0.5	Elasticity of substitution between traded and non-traded goods
ϕ	1.5	Elasticity of substitution between home and foreign traded goods
α	0.5	Weight on non-traded goods
α^*	0.7	Weight on non-traded goods in a foreign country
ω	0.7	Weight on home traded goods
$\tilde{\omega}^*$	0.3	Degree of openness
ε	6	Elasticity of substitution across varieties
ψ_N	0.8	Probability of not adjusting the price in the non-traded goods sector
ψ_H	0.6	Probability of not adjusting the price in the traded goods sector
ϱ_y	0.5	Weight on output growth rate in the Taylor rule
ϱ_π	1.5	Weight on CPI inflation rate in the Taylor rule
τ_ι	0.9	Persistence parameter in exogenous shock processes, $\iota \in \{N, H, i^*, C^*, \pi^*, p_{NT}^*\}$
ς_ι	0.01	Standard deviation of exogenous shock processes, $\iota \in \{N, H, i^*, C^*, \pi^*, p_{NT}^*\}$

4.3.2 Domestic shocks

The dynamics of the model is analyzed using impulse responses of key endogenous variables to various shocks under three different monetary policy regimes, namely CPI targeting, domestic price inflation targeting (DI targeting) and the fixed exchange regime. We define the domestic inflation in this economy as

$$\pi_{D,t} = \alpha\pi_{N,t} + (1 - \alpha)\pi_{H,t}, \quad (4.3.1)$$

which differs from the definition of CPI in (4.2.78) by not accounting explicitly for movements in the terms of trade. We implement the fixed exchange rate regime in this model by using the following targeting rule

$$i_t = \varrho_s \Delta s_t \quad (4.3.2)$$

and assuming relatively high value of the coefficient ϱ_s . Such setup ensures that any movements of the nominal exchange rate around the steady state value are dampened and it does not react to any types of exogenous shocks.

Figure 4.7 presents impulse responses of key model variables to a unit positive productivity shock in the non-traded goods sector. As one might expect, unit productivity shock in the non-traded goods sector decreases the marginal cost in this sector and leads to a decrease in the non-traded goods inflation and increase in output. Free movement of labor across sectors and wage equalization in equilibrium imply that the marginal cost of labor drops also in the traded goods sector. This leads to a drop in the inflation rate in the traded goods sector and decline in the relative price of non-traded goods. Since the shock hits the productivity of the non-traded goods sector directly, the relative price of non-traded goods declines as well. Both drop in the relative price of non-traded goods and fall in the terms of trade lead to the depreciation of the real exchange rate, which further implies the depreciation of the nominal exchange rate. The economy behaves similarly under all three regimes with minor differences in case of the fixed

exchange rate regime. Not allowing for adjustment in the nominal exchange rate implies that resulting real exchange rate depreciation due to changes in relative prices is achieved mainly through drop in prices as evident from the evolution of the CPI inflation rate and inflation rates in both non-traded and traded goods sectors.

Impulse responses to a unit shock in the traded goods sector are presented in Figure 4.8. Most variables behave in a similar manner as in the previous case. Due to the productivity shock affecting the traded goods sector directly, terms of trade increases and there is a notable increase in the output of traded goods. Prices drop in both sectors and the CPI inflation declines as well. As in case of a productivity shock in the non-traded goods sector, not allowing for adjustment in the nominal exchange rate leads to a large drop in the inflation rate under the fixed exchange rate regime. Subsequently, the relative price of non-traded goods under the fixed exchange rate regime exhibits stronger reaction to the productivity shock. Overall, consumption increases and the positive productivity shock induces a relatively large and persistent increase in the traded goods sector output.

4.3.3 Foreign shocks

The model allows to analyze several types of shocks stemming from a foreign economy. We will first discuss the implications of a positive shock to the relative price of non-traded goods in Foreign; then, examine the dynamics of the model after a negative foreign consumption shock and, finally, analyze the effects of a positive shock to the foreign interest rate.

Positive shock to the relative price of non-traded goods in Foreign may also be interpreted as a positive productivity shock in the traded goods sector of a foreign country. Price of foreign traded goods falls, which is reflected in the fall of the value of the term of trade. This induces a policy of nominal and real exchange rate depreciation to improve the competitiveness of domestic traded goods sector firms. Production of Home traded goods, nevertheless, falls, which reduces the marginal costs and inflation overall in the economy. Relative price of non-traded goods in Home increases, but overall negative effect on the production of non-traded goods is offset by increase in the overall consumption in the economy.

Figure 4.10 presents impulse responses to a negative unit shock to foreign consumption. Through the risk sharing condition decline in foreign consumption induces fall in the domestic consumption as well. This leads to a drop in the output of non-traded goods, which is directly affected by the level of consumption in the economy. Due to the same reason, production of Home traded goods should also fall, but the negative effect of falling level of consumption is offset by an improvement in the terms of trade and attractiveness of traded goods produced in the domestic economy. Both real and nominal exchange rates depreciate. Fixed exchange rate regime induces the largest fluctuations of the relative price of non-traded goods, inflation in both sectors and overall output due to inability of the central bank to depreciate the nominal exchange rate.

We will first discuss implications of a unit shock to foreign interest rate variable, responses to which are presented in Figure 4.11. Positive foreign interest rate shock in this model can also be interpreted as a shock to domestic consumption. Increase in the foreign interest rate increases the opportunity cost of holding foreign currency and the demand for foreign currency decreases. Since consumption and monetary aggregate in this model are substitutes, decrease in the demand for foreign currency leads to higher consumption of the composite good. This has a positive effect on output in both sectors. Higher output increases the demand for labor, which subsequently increases the marginal cost of labor and level of prices. As a result, the real exchange rate appreciates, which also implies the appreciation of the nominal exchange rate. Drop in the terms of trade further increases the relative price of non-traded goods. As

expected, under the fixed exchange rate regime we observe relatively larger fluctuations of prices and output.

We have also examined the role of dollarization in transmitting the foreign interest rate shock to the domestic economy under CPI targeting rule. In particular, we analyzed the impulse responses of model variables under high and low levels of dollarization (Figure 4.12). Results show that the level of currency substitution in the economy determines to what extent the volatility of the foreign interest rates is transmitted to the domestic economy. The economy behaves in a similar manner qualitatively under both high and low levels of currency substitution but the volatility of all endogenous variables increases considerably as the steady state ratio of foreign currency holdings goes up.

4.4 Monetary policy rules analysis

To analyze the performance of various policy rules in the economy with currency substitution, we simulate the model for various values of the steady state dollarization ratio γ and other key variables and identify if the performance of the rules changes with the value of the dollarization ratio.

We assume that the monetary authority is concerned about stabilizing the CPI inflation rate and movements in the aggregate output. In other words, we assume a simple ad hoc variance-based loss function of the following form as a criterion for the assessment of policy rules

$$\mathcal{L}_t = \kappa_\pi \text{var}(\pi_t) + \kappa_y \text{var}(y_t). \quad (4.4.1)$$

In most simulations we compare the performance of different monetary policy rules to the benchmark case, where we assume that the central bank assigns equal weight to fluctuations of the CPI inflation rate and output and resorts to flexible inflation targeting with commonly assumed values for parameters of the Taylor rule $\varrho_\pi = 1.5$ and $\varrho_y = 0.5$. In the simulations below, the value of $\gamma = 0$ refers to an economy with no currency substitution and $\gamma = 1$ refers to an economy with complete transactions dollarization.

Figure 4.1 presents the value of the loss function under benchmark parametrization and different values of feedback coefficient ϱ_π and ϱ_y . As expected the value of ϱ_π greater than approximately 1.5 significantly reduces overall volatility in the economy and values of ϱ_π greater than 1.5 lead to very marginal improvement. As expected, strong response of the central bank to the output volatility increases the overall volatility of the economy, when the reaction to fluctuations in the inflation rate is moderate.

Figure 4.2 presents the results of simulating the model for different values of γ , strength of response to output fluctuations ϱ_y and the degree of interest rate smoothing ϱ_i . As explained above values of the loss function under this simulation are presented relative to values of the loss under benchmark parametrization for given values of γ . In other words, value of the relative loss greater than one indicates higher overall volatility in economy relative to the benchmark. Responding strongly to output growth fluctuations is marginally beneficial at high levels of currency substitution and leads to large increase in the overall volatility at low levels of γ . Results also indicate that some response to output fluctuations at moderate levels of dollarization leads to lower volatility in the economy than in the benchmark case. Simulation also show strong support for interest rate smoothing at all levels of currency substitution.

Figure 4.1: Loss as a function of ϱ_π and ϱ_y .

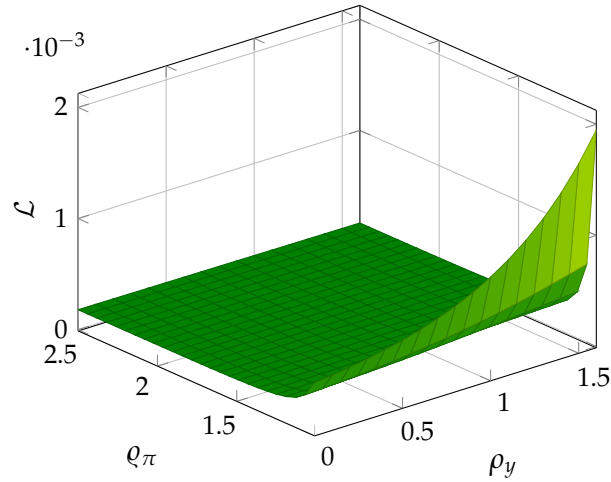
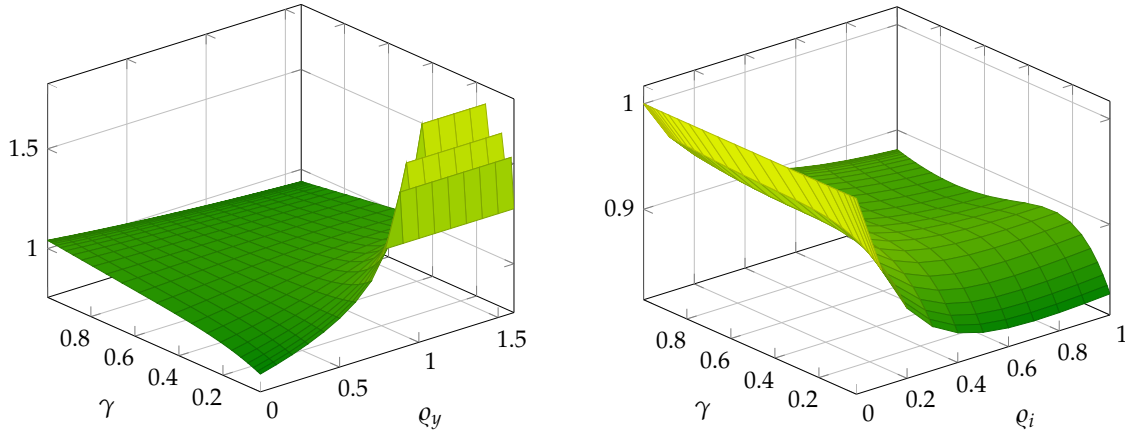


Figure 4.2: Relative loss as a function of γ , ϱ_y and ϱ_i .



Whether a monetary authority in small open economies should respond to fluctuations of nominal and real exchange rates has been a subject of intense research in the literature. Given the importance of this question, we also analyze the relative volatility of the model for different levels of currency substitution and strength of response to either nominal and real exchange rate changes. Results (presented in Figure 4.3) indicate that responding to nominal exchange rate changes reduces the overall volatility in the economy. Simulations show improvements in the relative volatility for almost all levels of dollarization except for extreme case of very high currency substitution and strong response to nominal exchange rate movements. Under the latter scenario, the volatility actually increases relative to the benchmark case. Interestingly, responding to movements in the real exchange rate significantly increases the volatility in the economy relative to the benchmark.

In the following exercise we analyzed the relative volatility in the economy under two different regimes: responding to fluctuations in domestic inflation in the Taylor rule instead of responding to CPI inflation rate and the fixed exchange rate regime (Figure 4.4). These two cases can be interpreted as two extreme approaches to responding to movements in the nominal or real exchange rates. Domestic inflation rate defined in (4.3.1) above does not take into account movements in the real or nominal exchange rates or terms of trade. CPI inflation rate, on

Figure 4.3: Relative loss as a function of γ , ϱ_s and ϱ_q .

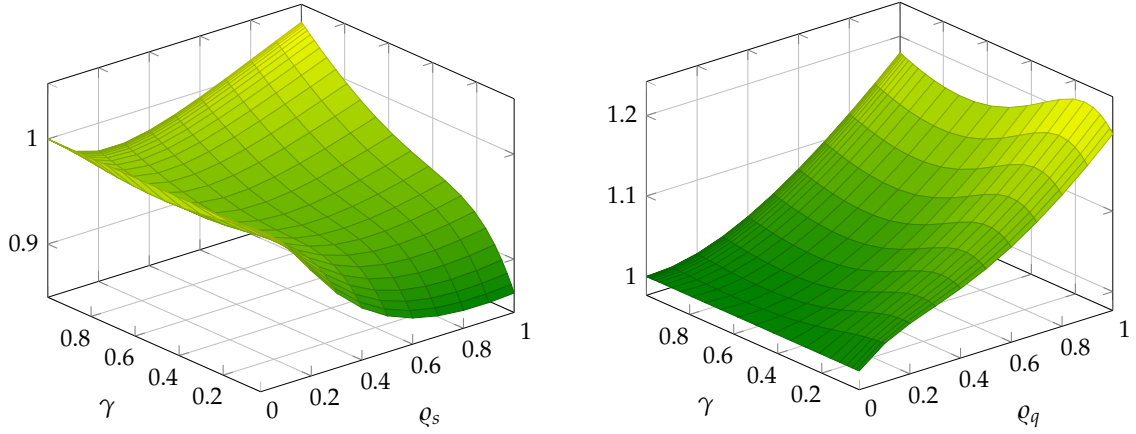
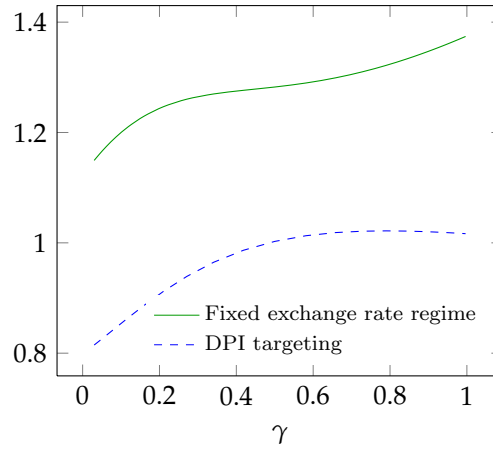


Figure 4.4: Relative loss under domestic price inflation (DPI) targeting and fixed exchange rate regime as a function of γ .



the other hand, does capture the effect of changes in the terms of trade and, thus, effect of movements in the real and nominal exchange rates, as it is shown in (4.2.78). Results of the analysis suggest that under fixed exchange rate regime the value of the loss function increases considerably relative to the benchmark and the relatively volatility is higher at higher levels of currency substitution. Responding to domestic price inflation rate in the rule instead of the CPI inflation rate provides better results under lower levels of dollarization and as the use of foreign currency rises performance of two rules is indistinguishable.

4.4.1 Sensitivity analysis

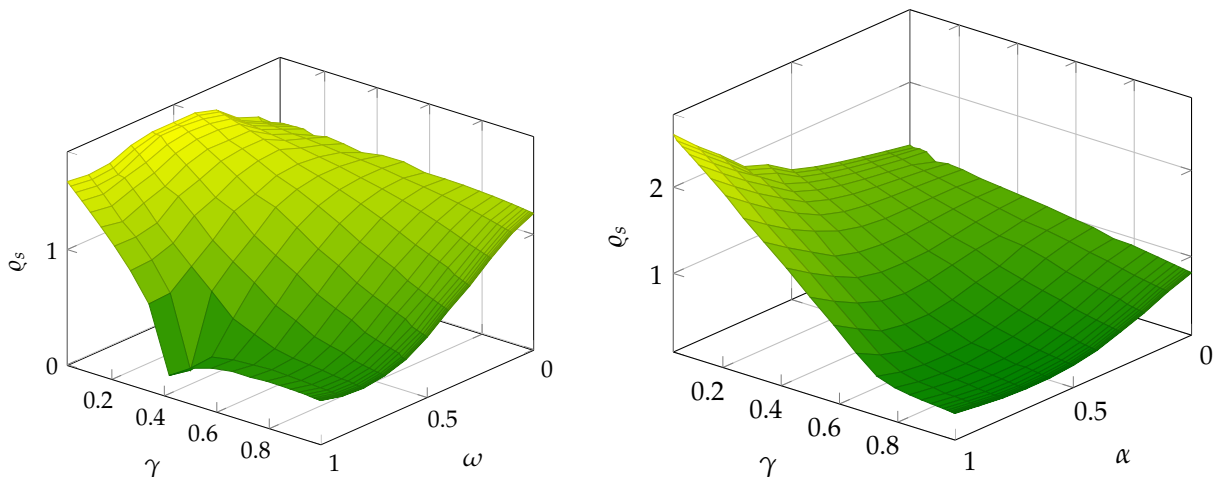
Multisector structure of the model allows us to examine if some results outlined in the previous section are robust to changes in the structure of the economy. In particular, we are interested in understanding how the degree of response to fluctuations in the nominal exchange rate changes as we change certain key structural parameters of the economy. To perform this exercise we minimize the value of the loss function (4.4.1) with respect to the parameter ϱ_s at different values of the dollarization ratio γ , share of non-traded goods in the composite consumption α , and the weight of domestically produced goods in the consumption of traded goods ω . Other parameters in the Taylor rule are chosen as in the benchmark case, i.e. $\varrho_\pi = 1.5$ and $\varrho_y = 0.5$.

Figure 4.5 presents the results of our simulations. Results suggest strong optimal response in the Taylor rule to nominal exchange rate fluctuations under low level of currency substitution.

But this result changes at relatively higher levels of currency substitution and simultaneous lower share of foreign traded goods consumed in the domestic economy. The latter result is most likely observed due to lower level of consumption of foreign traded goods and resulting lower dependence of movements of the inflation rate to changes in the exchange rate. At the same time higher values of ω imply increased importance of productivity shocks in determining the overall volatility in the economy. Nevertheless, feedback coefficient to changes in the nominal exchange rate q_s remains positive under all possible combinations of γ and ω .

As it is evident from expression for movements in the nominal and real exchange rates (4.2.76) and (4.2.77), the share of non-traded goods α in the composite consumption determines how strong the response of both exchange rates is to domestic productivity shocks. Higher share of non-tradable goods leads to more volatile real and nominal exchange rate movements and subsequent more volatile CPI inflation rate. The latter, then, induces the strong optimal response of the central bank to movements in the nominal exchange rate.

Figure 4.5: Optimal q_s as a function of γ , ω and α .



Overall, our results can be summarized as follows. Under CPI targeting regime our simulations provide moderate support for strong response to movements in the output growth rate in the Taylor rule. At the same time, there is strong support for interest rate smoothing at all values of the dollarization ratio. Simulations also show strong support for responding to movements of the nominal exchange rate in the Taylor rule. This result is generally robust to changes in the structure of the economy, i.e. different degrees of currency substitution, changes in the share of foreign traded goods consumed in the domestic economy and changes in the share of non-traded goods. Including domestic inflation in the Taylor rule instead of CPI inflation rate is beneficial only at lower levels of currency substitution. At higher levels of currency substitution the performance of two rules is indistinguishable. In line with results of many related studies, our analysis also makes strong argument against fixing the nominal exchange rate. Under the fixed exchange rate regime, the relative loss increases considerably and this outcome persists at all levels of currency substitution.

4.5 Conclusion

In this study we analyzed the performance of a number of monetary policy rules in a small open economy with currency substitution. Results show strong support for responding to changes in

the nominal exchange rate in the Taylor rule and this result is generally robust to changes in the level of currency substitution and other key structural parameters of the model. Interest rate smoothing also delivers better outcomes in terms of reducing the overall volatility in the economy. Fixing the nominal exchange rate is not beneficial even in an economy with currency substitution as the overall volatility of the economy increases considerably in comparison to other monetary policy regimes.

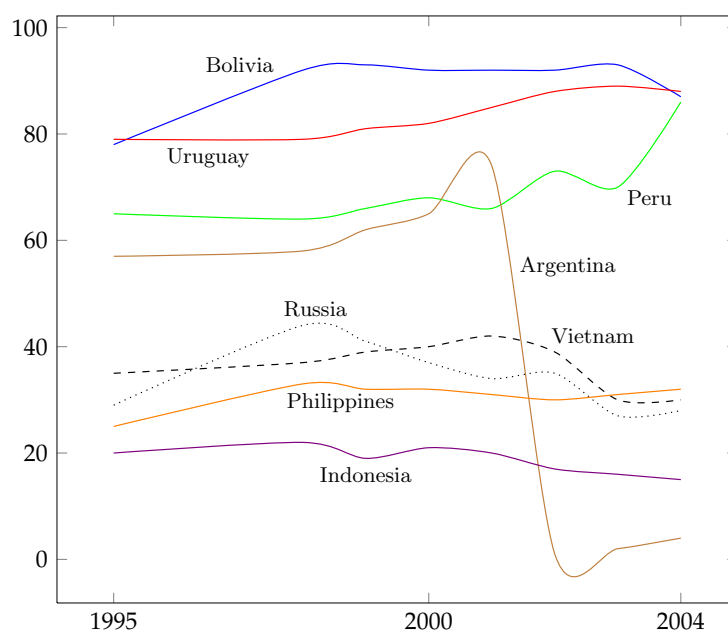
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Appendix 4.A Currency substitution in selected countries

Figure 4.6: Ratio of foreign currency-denominated deposits to total deposits



Source: [Alvarez-Plata and Garcia-Herrero \(2010\)](#)

Appendix 4.B Steady state

Parameters with no time subscript denote their steady state values. We assume that in steady state equilibrium sector-specific productivity shocks are at their unconditional mean value, i.e. $A_N = A_H = 1$. We also assume that in steady state equilibrium terms of trade are equal to unity, i.e. $P_{FH} = 1$, which implies that $P_F = P_H$. Given this assumption, price index for traded goods in the domestic economy (4.2.33) implies that $P_F = P_T$ and $P_H = P_T$. Let's also normalize the steady state level of the overall price index in the domestic economy to one, i.e. $P = 1$ (which also implies that we assume the final composite consumption C to be a numeraire good). Then, price index equation (4.2.32) implies that $P_{NT} = 1$ and $P_N = P_T = P$. Then, from expression (4.2.37) and the definition of the real exchange, we conclude that in steady state equilibrium $Q = 1$ and $S = 1$.

In symmetric steady state equilibrium, all firms set prices such that price of a variety j equals the marginal cost in the economy, which is equal across all firms. Then, equations for price indices (4.2.45) and (4.2.47) also imply that

$$\begin{aligned}\tilde{p}_N(j) &= P_N \quad \forall j \in [0, n] \\ \tilde{p}_H(j) &= P_H \quad \forall j \in [0, n].\end{aligned}$$

Similar assumption on prices in the foreign economy result in the normalization of all price indices in the foreign economy to one.

Given these assumptions on steady state value of price indices, aggregate resource constraints (4.2.61)-(4.2.64) become

$$Y_N = \alpha C \tag{4.B.1}$$

$$Y_H = \omega(1 - \alpha)C + \frac{1 - n}{n}\omega^*(1 - \alpha^*)C^* \tag{4.B.2}$$

$$Y_N^* = \alpha^* C^* \tag{4.B.3}$$

$$Y_F = \frac{n}{1 - n}(1 - \omega)(1 - \alpha)C + (1 - \omega^*)(1 - \alpha^*)C^* \tag{4.B.4}$$

In the limit of the country size measure n , i.e. as $n \rightarrow 0$, resource constraints for traded goods in both countries simplify to

$$Y_H = \omega(1 - \alpha)C + \tilde{\omega}^*(1 - \alpha^*)C^* \tag{4.B.5}$$

$$Y_F = (1 - \omega^*)(1 - \alpha^*)C^*. \tag{4.B.6}$$

From production functions (4.2.43) for both sectors we obtain

$$Y_N = L_N$$

$$Y_H = L_H.$$

Labor market clearing condition (4.2.65) implies that

$$L = L_N + L_H$$

and wages in both sectors are equalized, i.e. $W_N = W_H$.

International risk sharing condition (4.2.39) implies that in steady state equilibrium

$$Q = k_0 \frac{(C^*)^{-\rho}}{U_C} \tag{4.B.7}$$

Assumptions on price indices imply that $Q = 1$, which suggests that $k_0 = U_C(C^*)^\rho$. The world aggregate resource constraint is given by $Y + Y^* = C + C^*$. Assuming zero net exports in steady state equilibrium we obtain the following expression for the relative consumption across two countries

$$\frac{C}{C^*} = \frac{(1 - \alpha^*)\tilde{\omega}^*}{(1 - \alpha)(1 - \omega)}. \quad (4.B.8)$$

Labor market clearing condition in the foreign economy implies that

$$\begin{aligned} L^* &= L_N^* + L_F^* \\ &= Y_N^* + Y_F^* \\ &= \alpha^* C^* + (1 - \alpha^*) C^* \\ &= C^*, \end{aligned} \quad (4.B.9)$$

where we also take into account that $\omega^* \rightarrow 0$ as $n \rightarrow 0$. Foreign country labor market optimality condition implies

$$1 = \frac{\varepsilon}{\varepsilon - 1} \frac{(L^*)^\xi}{(C^*)^{-\rho}}, \quad (4.B.10)$$

which using (4.B.9) gives us the steady state level of consumption in the foreign country

$$C^* = \left(\frac{\varepsilon - 1}{\varepsilon} \right)^{\frac{1}{\xi + \rho}}. \quad (4.B.11)$$

Using the expression for the steady state relative consumption across two countries (4.B.8) we pin down the steady state level of consumption in the domestic economy as follows

$$C = \frac{(1 - \alpha^*)\tilde{\omega}^*}{(1 - \alpha)(1 - \omega)} \left(\frac{\varepsilon - 1}{\varepsilon} \right)^{\frac{1}{\xi + \rho}}. \quad (4.B.12)$$

The value of steady state composite consumption index (4.B.12) is not sufficient for determining the marginal utility of consumption U_C and k_0 , as the former is also a function of monetary aggregate Z , which depends on steady state holdings of real money balances of domestic and foreign currencies.

First order conditions (4.2.10), (4.2.12) and (4.2.13) in steady state give us

$$(1 - v)\kappa\Gamma^{\left(\frac{1}{\eta} - \frac{1}{\theta}\right)} \left(\frac{M_H}{P} \right)^{-\frac{1}{\theta}} = (1 - \beta)vC^{-\frac{1}{\theta}}, \quad (4.B.13)$$

where

$$\Gamma = Z \frac{P}{M_H} = \left(\kappa + (1 - \kappa) \left(\frac{\kappa}{1 - \kappa} \right)^{1 - \eta} \right)^{\frac{\eta}{\eta - 1}}.$$

Rearranging (4.B.13) we obtain the following expression for steady state holdings of the domestic currency

$$\frac{M_H}{P} = \left(\frac{(1 - \beta)vC^{-\frac{1}{\theta}}}{(1 - v)\kappa\Gamma^{\left(\frac{1}{\eta} - \frac{1}{\theta}\right)}} \right)^{-\theta}. \quad (4.B.14)$$

Similarly, using (4.2.11), (4.2.12) and (4.2.14) we obtain an expression for the steady state holdings of Foreign currency, which is a function of only structural parameters

$$\frac{SM_F}{P} = \left(\frac{(1-\beta)vC^{-\frac{1}{\theta}}}{(1-v)(1-\kappa) \left(\Gamma \left(\frac{\kappa}{1-\kappa} \right)^\eta \right)^{\left(\frac{1}{\eta} - \frac{1}{\theta} \right)}} \right)^{-\theta}. \quad (4.B.15)$$

Using these expressions for steady state money holdings we are able to obtain an expression for the steady state dollarization ratio as follows

$$\gamma = \frac{\frac{SM_F}{P}}{\frac{SM_F}{P} + \frac{M_H}{P}} = \frac{\left(\frac{\kappa}{1-\kappa} \right)^{-\eta}}{1 + \left(\frac{\kappa}{1-\kappa} \right)^{-\eta}}. \quad (4.B.16)$$

Let's reformulate the expression for steady state holding of domestic currency (4.B.14) as $\frac{M_H}{P} = \left(\frac{v(1-\beta)}{(1-v)\omega} \right)^{-\theta} \Gamma^{\frac{\theta}{\eta}-1} C$ and expression for holdings of foreign currency (4.B.15) as $\frac{SM_F}{P} = \left(\frac{v(1-\beta)}{(1-v)\omega} \right)^{-\theta} \Gamma^{\frac{\theta}{\eta}-1} C \left(\frac{\kappa}{1-\kappa} \right)^{-\eta}$. Then, we are able to express the steady state money aggregate as

$$Z = \Phi C, \quad (4.B.17)$$

where $\Phi = \left(\frac{v(1-\beta)}{(1-v)\omega} \right)^{-\theta} \Gamma^{\frac{\theta}{\eta}}$.

Then, using (4.B.12), (4.B.11) and the value of the steady state marginal utility of consumption in Home we get

$$k_0 = v \left[\left(v + (1-v)\Phi^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \right]^{1-\rho} \left[\frac{(1-\alpha^*)\tilde{\omega}^*}{(1-\alpha)(1-\omega)} \right]^{\left(\frac{1}{\theta} - \rho \right)}. \quad (4.B.18)$$

Appendix 4.C Detailed derivations

4.C.1 Optimal consumption allocations

The problem of optimal allocation between non-traded and traded goods of a representative household in Home can be formulated as

$$\begin{aligned} \min_{C_{N,t}, C_{T,t}} \quad & P_t C_t = P_{N,t} C_{N,t} + P_{T,t} C_{T,t} \\ \text{s.t.} \quad & C_t = \left[\alpha^{\frac{1}{\sigma}} C_{N,t}^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} C_{T,t}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \end{aligned} \quad (4.C.1)$$

Solving this problem gives

$$C_{N,t} = \alpha \left(\frac{P_{N,t}}{P_t} \right)^{-\sigma} C_t, \quad (4.C.2)$$

$$C_{T,t} = (1-\alpha) \left(\frac{P_{T,t}}{P_t} \right)^{-\sigma} C_t. \quad (4.C.3)$$

After inserting these expressions into the constraint (4.C.1), we obtain the following expression for the overall consumption price index

$$P_t = \left[\alpha P_{N,t}^{1-\sigma} + (1-\alpha) P_{T,t}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}.$$

The problem of optimal consumption allocation between Home and Foreign traded goods can be formulated as

$$\begin{aligned} \min_{C_{H,t}, C_{F,t}} \quad & P_{T,t} C_{T,t} = P_{H,t} C_{H,t} + P_{F,t} C_{F,t} \\ \text{s.t.} \quad & C_{T,t} = \left[\omega^{\frac{1}{\phi}} C_{H,t}^{\frac{\phi-1}{\phi}} + (1-\omega)^{\frac{1}{\phi}} C_{F,t}^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \end{aligned} \quad (4.C.4)$$

Solution to this problem is

$$C_{H,t} = \omega \left(\frac{P_{H,t}}{P_{T,t}} \right)^{-\phi} C_{T,t}, \quad (4.C.5)$$

$$C_{F,t} = (1-\omega) \left(\frac{P_{F,t}}{P_{T,t}} \right)^{-\phi} C_{T,t}. \quad (4.C.6)$$

Using these expressions and constraint (4.C.4) we can obtain the following consumption price index for traded goods in Home

$$P_{T,t} = \left[\omega P_{H,t}^{1-\phi} + (1-\omega) P_{F,t}^{1-\phi} \right]^{\frac{1}{1-\phi}}.$$

Demand function for a differentiated Home produced traded good is determined by solving the following problem

$$\begin{aligned} \min_{c_{H,t}(j)} \quad & P_{H,t} C_{H,t} = \int_0^n p_{H,t}(j) c_{H,t}(j) dj \\ \text{s.t.} \quad & C_{H,t} = \left[\left(\frac{1}{n} \right)^{\frac{1}{\varepsilon}} \int_0^n c_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}. \end{aligned} \quad (4.C.7)$$

Solving this problem yields the following demand function

$$c_{H,t}(j) = \frac{1}{n} \left(\frac{p_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t}. \quad (4.C.8)$$

Inserting (4.C.8) into the constraint (4.C.7) gives the price index for Home traded goods

$$P_{H,t} = \left[\int_0^n \left(\frac{1}{n} \right) p_{H,t}(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}.$$

Solving an equivalent problem for households in Foreign, we obtain the demand function and price index for Foreign traded goods consumed in Home

$$\begin{aligned} c_{F,t}(j) &= \frac{1}{1-n} \left(\frac{p_{F,t}(j)}{P_{F,t}} \right)^{-\varepsilon} C_{F,t} \\ P_{F,t} &= \left[\int_n^1 \left(\frac{1}{1-n} \right) p_{F,t}(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}. \end{aligned} \quad (4.C.9)$$

Optimal consumption of non-traded goods in Home is determined by solving the following problem:

$$\begin{aligned} \min_{c_{N,t}(j)} P_{N,t} C_{N,t} &= \int_0^n p_{N,t}(j) c_{N,t}(j) dj \\ \text{s.t. } C_{N,t} &= \left[\left(\frac{1}{n} \right)^{\frac{1}{\varepsilon}} \int_0^n c_{N,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}. \end{aligned} \quad (4.C.10)$$

Solution to this problem is the following demand function for a variety of non-traded good

$$c_{N,t}(j) = \frac{1}{n} \alpha \left(\frac{p_{N,t}(j)}{P_{N,t}} \right)^{-\varepsilon} \left(\frac{P_{N,t}}{P_t} \right)^{-\sigma} C_t. \quad (4.C.11)$$

Expressions (4.C.3), (4.C.5), (4.C.6), (4.C.8) and (4.C.9) give the following demand functions for Home and Foreign traded goods consumed in Home

$$c_{H,t}(j) = \frac{1}{n} \omega (1 - \alpha) \left(\frac{p_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} \left(\frac{P_{H,t}}{P_{T,t}} \right)^{-\phi} \left(\frac{P_{T,t}}{P_t} \right)^{-\sigma} C_t, \quad (4.C.12)$$

$$c_{F,t}(j) = \frac{1}{1-n} (1 - \omega) (1 - \alpha) \left(\frac{p_{F,t}(j)}{P_{F,t}} \right)^{-\varepsilon} \left(\frac{P_{F,t}}{P_{T,t}} \right)^{-\phi} \left(\frac{P_{T,t}}{P_t} \right)^{-\sigma} C_t. \quad (4.C.13)$$

Equivalent demand function for Foreign are

$$c_{N,t}^*(j) = \frac{1}{1-n} \alpha^* \left(\frac{p_{N,t}^*(j)}{P_{N,t}^*} \right)^{-\varepsilon} \left(\frac{P_{N,t}^*}{P_t^*} \right)^{-\sigma} C_t^*, \quad (4.C.14)$$

$$c_{H,t}^*(j) = \frac{1}{n} \omega^* (1 - \alpha^*) \left(\frac{p_{H,t}^*(j)}{P_{H,t}^*} \right)^{-\varepsilon} \left(\frac{P_{H,t}^*}{P_{T,t}^*} \right)^{-\phi} \left(\frac{P_{T,t}^*}{P_t^*} \right)^{-\sigma} C_t^*, \quad (4.C.15)$$

$$c_{F,t}^*(j) = \frac{1}{1-n} (1 - \omega^*) (1 - \alpha^*) \left(\frac{p_{F,t}^*(j)}{P_{F,t}^*} \right)^{-\varepsilon} \left(\frac{P_{F,t}^*}{P_{T,t}^*} \right)^{-\phi} \left(\frac{P_{T,t}^*}{P_t^*} \right)^{-\sigma} C_t^*. \quad (4.C.16)$$

4.C.2 Aggregate demand

Aggregate demand for a traded good produced in Home is determined by $y_{H,t}(j) = nc_{H,t}(j) + (1-n)c_{H,t}^*(j)$. Thus, using (4.C.12) and (4.C.15) we obtain

$$\begin{aligned}
y_{H,t}(j) &= \omega(1-\alpha) \left(\frac{p_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} \left(\frac{P_{H,t}}{P_{T,t}} \right)^{-\phi} \left(\frac{P_{T,t}}{P_t} \right)^{-\sigma} C_t \\
&\quad + \frac{1-n}{n} \omega^*(1-\alpha^*) \left(\frac{p_{H,t}^*(j)}{P_{H,t}^*} \right)^{-\varepsilon} \left(\frac{P_{H,t}^*}{P_{T,t}^*} \right)^{-\phi} \left(\frac{P_{T,t}^*}{P_t^*} \right)^{-\sigma} C_t^* \\
&= \left(\frac{p_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} \left(\frac{P_{H,t}}{P_{T,t}} \right)^{-\phi} \times \\
&\quad \left[\omega(1-\alpha) \left(\frac{P_{T,t}}{P_t} \right)^{-\sigma} C_t + \frac{1-n}{n} \omega^*(1-\alpha^*) \left(\frac{P_{T,t}}{S_t P_{T,t}^*} \right)^{-\phi} \left(\frac{P_{T,t}^*}{P_t^*} \right)^{-\sigma} C_t^* \right] \\
&= \left(\frac{p_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} \left(\frac{P_{H,t}}{P_{T,t}} \right)^{-\phi} \left(\frac{P_{T,t}}{P_t} \right)^{-\sigma} \times \\
&\quad \left[\omega(1-\alpha) C_t + \frac{1-n}{n} \omega^*(1-\alpha^*) Q_t^\sigma \times \right. \\
&\quad \left. \left(\frac{[\omega^*(S_t P_{H,t}^*)^{1-\phi} + (1-\omega^*)(S_t P_{F,t}^*)^{1-\phi}]^{\frac{1}{1-\phi}}}{[\omega P_{H,t}^{1-\phi} + (1-\omega) P_{F,t}^{1-\phi}]^{\frac{1}{1-\phi}}} \right)^{\phi-\sigma} C_t^* \right] \\
&= \left(\frac{p_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} \left(\frac{P_{H,t}}{P_{T,t}} \right)^{-\phi} \left(\frac{P_{T,t}}{P_t} \right)^{-\sigma} \times \\
&\quad \left[\omega(1-\alpha) C_t + \frac{1-n}{n} \omega^*(1-\alpha^*) Q_t^\sigma \times \right. \\
&\quad \left. \left(\frac{\omega^*}{\omega + (1-\omega) P_{FH}^{1-\phi}} + \frac{1-\omega^*}{\omega P_{FH}^{\phi-1} + (1-\omega)} \right)^{\frac{\phi-\sigma}{1-\phi}} C_t \right]
\end{aligned} \tag{4.C.17}$$

Demand for non-traded goods in Home is determined using

$$y_{N,t}(j) = nc_{N,t}(j) = \alpha \left(\frac{p_{N,t}(j)}{P_{N,t}} \right)^{-\varepsilon} \left(\frac{P_{N,t}}{P_t} \right)^{-\sigma} C_t \tag{4.C.18}$$

Equivalently, using (4.C.13) and (4.C.16) we obtain the following expression for total demand for traded goods produced in Foreign

$$\begin{aligned}
y_{F,t}(j) &= nc_{F,t}(j) + (1-n)c_{F,t}^*(j) \\
&= \left(\frac{p_{F,t}(j)}{P_{F,t}} \right)^{-\varepsilon} \left(\frac{P_{F,t}}{P_{T,t}} \right)^{-\phi} \left(\frac{P_{T,t}}{P_t} \right)^{-\sigma} \times \\
&\quad \left[\frac{n}{1-n} (1-\omega)(1-\alpha) C_t + (1-\omega^*)(1-\alpha^*) Q_t^\sigma \times \right. \\
&\quad \left. \left[\frac{\omega^*}{\omega + (1-\omega) P_{FH}^{1-\phi}} + \frac{1-\omega^*}{\omega P_{FH}^{\phi-1} + (1-\omega)} \right]^{\frac{\phi-\sigma}{1-\phi}} C_t^* \right]
\end{aligned} \tag{4.C.19}$$

Demand for non-traded goods in Foreign is determined by

$$y_{N,t}^*(j) = (1-n)c_{N,t}^*(j) = \alpha^* \left(\frac{p_{N,t}^*(j)}{P_{N,t}^*} \right)^{-\varepsilon} \left(\frac{P_{N,t}^*}{P_t^*} \right)^{-\sigma} C_t^* \quad (4.C.20)$$

To obtain demand equations for a small open economy we assume that $n \rightarrow 0$, which implies that $\omega^* = n\tilde{\omega}^* \rightarrow 0$ and $\omega^* \frac{1-n}{n} = \tilde{\omega}^*(1-n) \rightarrow \tilde{\omega}^*$. Then, (4.C.17) and (4.C.19) can be simplified to

$$y_{H,t}(j) = \left(\frac{p_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} \left(\frac{P_{H,t}}{P_{T,t}} \right)^{-\phi} \left(\frac{P_{T,t}}{P_t} \right)^{-\sigma} \times \left[\omega(1-\alpha)C_t + \tilde{\omega}^*(1-\alpha^*)Q_t^\sigma \left(\frac{1}{\omega P_{FH}^{\phi-1} + (1-\omega)} \right)^{\frac{\phi-\sigma}{1-\phi}} C_t^* \right], \quad (4.C.21)$$

$$y_{F,t}(j) = \left(\frac{p_{F,t}(j)}{P_{F,t}} \right)^{-\varepsilon} \left(\frac{P_{F,t}}{P_{T,t}} \right)^{-\phi} \left(\frac{P_{T,t}}{P_t} \right)^{-\sigma} \left[(1-\alpha^*)Q_t^\sigma \left(\frac{1}{\omega P_{FH}^{\phi-1} + (1-\omega)} \right)^{\frac{\phi-\sigma}{1-\phi}} C_t^* \right] \quad (4.C.22)$$

Demand functions (4.C.21) and (4.C.22) are equivalent to expressions (4.2.58) and (4.2.60) provided in the main text.

4.C.3 Real exchange rate

Using (4.2.32)-(4.2.35) we define the real exchange rate as

$$\begin{aligned} Q_t &= \frac{S_t P_t^*}{P_t} = \frac{S_t [\alpha^* (P_{N,t}^*)^{1-\sigma} + (1-\alpha^*) (P_{T,t}^*)^{1-\sigma}]^{\frac{1}{1-\sigma}}}{[\alpha P_{N,t}^{1-\sigma} + (1-\alpha) P_{T,t}^{1-\sigma}]^{\frac{1}{1-\sigma}}} = \\ &= \frac{S_t P_{T,t}^* [\alpha^* (P_{NT,t}^*)^{1-\sigma} + (1-\alpha^*)]^{1-\sigma}}{P_{T,t} [\alpha P_{NT,t}^{1-\sigma} + (1-\alpha)]^{\frac{1}{1-\sigma}}} = \\ &= \left(\frac{\omega^* + (1-\omega^*) P_{FH,t}^{1-\phi}}{\omega + (1-\omega) P_{FH,t}^{1-\phi}} \right)^{\frac{1}{1-\phi}} \left(\frac{\alpha^* (P_{NT,t}^*)^{1-\sigma} + (1-\alpha^*)}{\alpha P_{NT,t}^{1-\sigma} + (1-\alpha)} \right)^{\frac{1}{1-\sigma}}, \end{aligned}$$

where in the last equality we used the fact that $P_{H,t} = S_t P_{H,t}^*$.

Appendix 4.D Log-linearization

Marginal utility of consumption. Log-linearizing (4.2.12) gives

$$u_{C,t} = \left(\frac{1}{\theta} - \rho \right) \tilde{\Omega}_t - \frac{1}{\theta} c_t, \quad (4.D.1)$$

where $\tilde{\Omega}_t$ refers to a log-linear deviation of Ω_t around its steady state value. To obtain the value of $\tilde{\Omega}_t$ we log-linearize expression $\Omega_t^{\frac{\theta-1}{\theta}} = v C_t^{\frac{\theta-1}{\theta}} + (1-v) Z_t^{\frac{\theta-1}{\theta}}$ using the fact that in steady state equilibrium $Z = \Phi C$. We obtain

$$\tilde{\Omega}_t = \omega c_t + (1-\omega) z_t, \quad (4.D.2)$$

where

$$\omega = \frac{v}{v + (1-v)\Phi^{\frac{\theta-1}{\theta}}}. \quad (4.D.3)$$

Combining (4.D.1) and (4.D.2) we get

$$u_{C,t} = \left(\left(\frac{1}{\theta} - \rho \right) \omega - \frac{1}{\theta} \right) c_t + \left(\frac{1}{\theta} - \rho \right) (1 - \omega) z_t \quad (4.D.4)$$

To obtain z_t we log-linearize expression $Z_t^{\frac{\eta-1}{\eta}} = \kappa \left(\frac{M_{H,t}}{P_t} \right)^{\frac{\eta-1}{\eta}} + (1 - \kappa) \left(\frac{M_{F,t} S_t}{P_t} \right)^{\frac{\eta-1}{\eta}}$, where we use the fact in the steady state equilibrium $\left(\frac{S M_F}{M_H} \right)^{\frac{\eta-1}{\eta}} = \left(\frac{\kappa}{1-\kappa} \right)^{1-\eta}$ and $\frac{Z P}{M_H} = \left(\kappa + (1 - \kappa) \left(\frac{\kappa}{1-\kappa} \right)^{1-\eta} \right)^{\frac{\eta-1}{\eta}}$. Thus, we obtain

$$z_t = (1 - \gamma) m_{H,t} + \gamma m_{F,t}, \quad (4.D.5)$$

where γ denotes the dollarization ratio (defined in (4.B.16)) and $m_{H,t}$ and $m_{F,t}$ refer to log-linearized holdings of real money balances in Home and Foreign currencies. Log-linearizing (4.2.13) and (4.2.14) we get

$$m_{H,t} = \frac{\eta}{\theta} c_t - \eta \beta i_t - \left(\frac{\eta}{\theta} - 1 \right) z_t \quad (4.D.6)$$

$$m_{F,t} = \frac{\eta}{\theta} c_t - \eta \beta i_t^* - \left(\frac{\eta}{\theta} - 1 \right) z_t. \quad (4.D.7)$$

Combining these expression with (4.D.5) and (4.D.4) we obtain

$$u_{C,t} = -\rho c_t + \beta(\theta\rho - 1)(1 - \omega)(i_t(1 - \gamma) + i_t^* \gamma). \quad (4.D.8)$$

Other first order conditions. Log-linearizing domestic Euler equation gives

$$u_{C,t} = i_t - E_t \pi_{t+1} + E_t u_{C,t+1} \quad (4.D.9)$$

Similarly, from a risk sharing condition (4.2.39) we are able to obtain

$$q_t = -\rho c_t^* - u_{C,t}. \quad (4.D.10)$$

Price-setting equations. Before obtaining the log-linearized versions of price setting equations (4.2.44) and (4.2.46) we need to log-linearize the expression for the law of motion of price indices in both non-traded and traded goods sectors. Log-linearizing expressions (4.2.45) around zero-inflation steady-state defined above we obtain

$$r_{N,t} = \frac{\psi_N}{1 - \psi_N} \pi_{N,t}, \quad (4.D.11)$$

where $r_{N,t}$ is the log-linear approximation of $R_{N,t} = \frac{\tilde{p}_{N,t}(j)}{P_{N,t}}$. Using the latter definition we also establish that $r_{N,t} = \tilde{p}_{N,t} - p_{N,t}$. In flexible-price equilibrium price-setting equation (4.2.44) simplifies to $R_{N,t} = \mu MC'_{N,t} \frac{P_t}{P_{N,t}}$, which implies that $r_{N,t} = m c_{N,t} - p_{N,t} + p_t$. We can obtain analogous expressions for the traded goods sector as well. Using these results, log-linearization of (4.2.44) gives

$$\pi_{N,t} = \frac{(1 - \psi_N \beta)(1 - \psi_N)}{\psi_N} (m c_{N,t} - p_{N,t} + p_t) + \beta E_t \pi_{N,t+1}. \quad (4.D.12)$$

Equivalently we obtain a similar expression for the traded goods sector

$$\pi_{H,t} = \frac{(1 - \psi_H \beta)(1 - \psi_H)}{\psi_H} (mc_{H,t} - p_{H,t} + p_t) + \beta E_t \pi_{H,t+1}. \quad (4.D.13)$$

Using the definition of real marginal costs in both sectors, first order conditions (4.2.6) and (4.2.7), (4.D.23) and (4.D.24) we can reformulate (4.D.12) and (4.D.13) as

$$\pi_{N,t} = \Sigma_N (\xi l_t - a_{N,t} - u_{C,t} - (1 - \alpha) p_{NT,t}) + \beta E_t \pi_{N,t+1}, \quad (4.D.14)$$

$$\pi_{H,t} = \Sigma_H (\xi l_t - a_{H,t} - u_{C,t} + \alpha p_{NT,t} + (1 - \omega) p_{FH,t}) + \beta E_t \pi_{H,t+1}, \quad (4.D.15)$$

where $\Sigma_N = \frac{(1 - \psi_N \beta)(1 - \psi_N)}{\psi_N}$ and $\Sigma_H = \frac{(1 - \psi_H \beta)(1 - \psi_H)}{\psi_H}$.

Aggregate demand. Log-linearizing the expression for the aggregate demand in the non-traded goods sector (4.2.61) gives

$$y_{N,t} = c_t - \sigma(1 - \alpha) p_{NT,t}, \quad (4.D.16)$$

where we used the fact that $p_t - p_{N,t} = -(1 - \alpha) p_{NT,t}$ and $p_{NT,t} = p_{N,t} - p_{T,t}$.

Log-linearizing the resource constraint in the traded goods sector in Home (4.2.62) and assuming $n \rightarrow 0$ results in

$$y_{H,t} = \omega c_{H,t} + (1 - \omega) c_{H,t}^*, \quad (4.D.17)$$

where we used the steady state equilibrium condition (4.B.8). Log-linear versions of consumption indices for the demand of the domestically produced traded good in Home (4.2.27) and Foreign (4.2.30) have the following form

$$c_{H,t} = \phi(p_{T,t} - p_{H,t}) + \sigma(p_t - p_{T,t}) + c_t \quad (4.D.18)$$

$$c_{H,t}^* = \phi(p_{T,t}^* - p_{H,t}^*) + \sigma(p_t^* - p_{T,t}^*) + c_t^* \quad (4.D.19)$$

Using expressions for price indices and the fact that $\omega^* \rightarrow 0$ as $n \rightarrow 0$ we can simplify these expressions to

$$c_{H,t} = \phi(1 - \omega) p_{FH,t} + \alpha \sigma p_{NT,t} + c_t \quad (4.D.20)$$

$$c_{H,t}^* = (\phi - \sigma \omega) p_{FH,t} + \sigma q_t + \alpha \sigma p_{NT,t} + c_t^*, \quad (4.D.21)$$

where $p_{FH,t} = p_{F,t} - p_{H,t}$. Then, we can reformulate the resource constraint in the traded goods sector (4.D.17) as

$$y_{H,t} = (\phi + (\phi - \sigma) \omega) (1 - \omega) p_{FH,t} + \alpha \sigma p_{NT,t} + \omega c_t + (1 - \omega) c_t^* + (1 - \omega) \sigma q_t. \quad (4.D.22)$$

Other identities. Log-linearizing the price index of the composite consumption good in Home (4.2.32) results in

$$p_t = \alpha p_{N,t} + (1 - \alpha) p_{T,t}. \quad (4.D.23)$$

Equivalently, from (4.2.33) we obtain

$$p_{T,t} = \omega p_{H,t} + (1 - \omega) p_{F,t}. \quad (4.D.24)$$

Defining overall inflation in the economy as $\pi_t = \ln \left(\frac{P_t}{P_{t-1}} \right)$ we can rewrite (4.D.23) and (4.D.24) as

$$\pi_t = \alpha \pi_{N,t} + (1 - \alpha) \pi_{T,t} \quad (4.D.25)$$

$$\pi_{T,t} = \omega \pi_{H,t} + (1 - \omega) \pi_{F,t}. \quad (4.D.26)$$

Then, from the definition of the terms of trade $P_{FH,t} = \frac{P_{F,t}}{P_{H,t}}$ we obtain

$$\pi_{F,t} = \Delta p_{FH,t} + \pi_{H,t}, \quad (4.D.27)$$

which we use to rewrite (4.D.25) as

$$\pi_t = \pi_{D,t} + (1 - \alpha)(1 - \omega) \Delta p_{FH,t}, \quad (4.D.28)$$

where $\pi_{D,t} = \alpha \pi_{N,t} + (1 - \alpha) \pi_{H,t}$ is the domestic inflation rate. Analogously, we can rewrite (4.D.26) as

$$\pi_{T,t} = \pi_{H,t} + (1 - \omega) \Delta p_{FH,t}. \quad (4.D.29)$$

Log-linearizing the definition of the real exchange rate $Q_t = \frac{S_t P_t^*}{P_t}$ we obtain $q_t = s_t + p_t^* - p_t$. Using the log-linearized price indices above we reformulate this expression as $q_t = \alpha^* p_{NT,t}^* - \alpha p_{NT,t} + s_t + p_{T,t}^* - p_{T,t}$. Then, given the fact that $\omega^* \rightarrow 0$ as $n \rightarrow 0$ we can simplify the latter to

$$q_t = \alpha^* p_{NT,t}^* - \alpha p_{NT,t} + \omega p_{FH,t}. \quad (4.D.30)$$

We also use the definition of the real exchange rate to obtain an expression for the evolution of the nominal exchange rate as follows

$$s_t - s_{t-1} = q_t - q_{t-1} + \pi_t - \pi_t^*. \quad (4.D.31)$$

Using the definition of the relative price of non-traded goods one can easily show that

$$\begin{aligned} \Delta p_{NT,t} &= \pi_{N,t} - \pi_{T,t} = \pi_{N,t} - \omega \pi_{H,t} - (1 - \omega) \pi_{F,t} \\ &= \pi_{N,t} - \pi_{H,t} - (1 - \omega) \Delta p_{FH,t}, \end{aligned} \quad (4.D.32)$$

where in the last expression we used the inflation rate for traded goods produced in the foreign country given in (4.D.27).

Appendix 4.E Impulse response functions

Figure 4.7: Impulse responses to a positive non-traded goods sector productivity shock

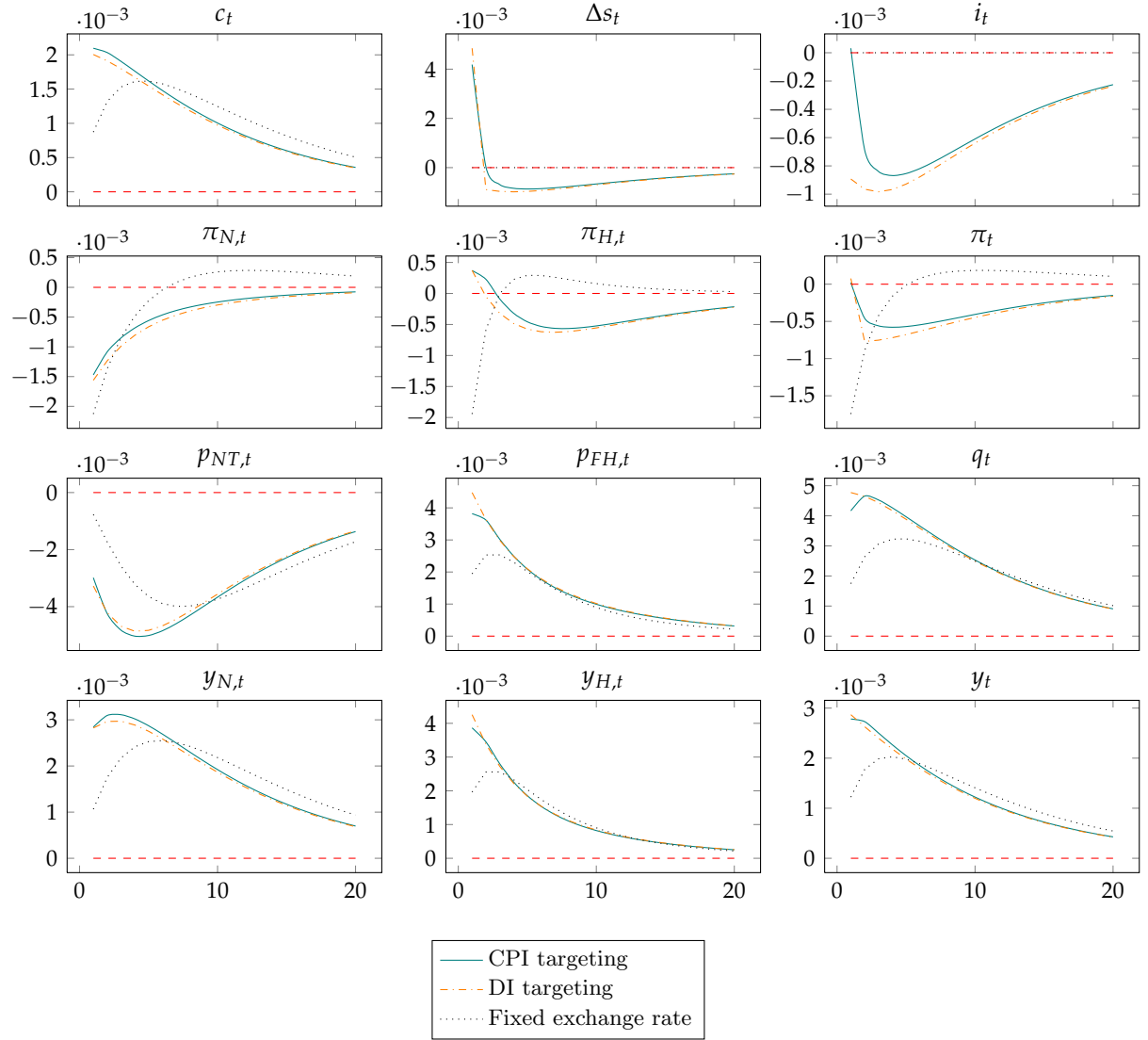


Figure 4.8: Impulse responses to a positive traded goods sector productivity shock

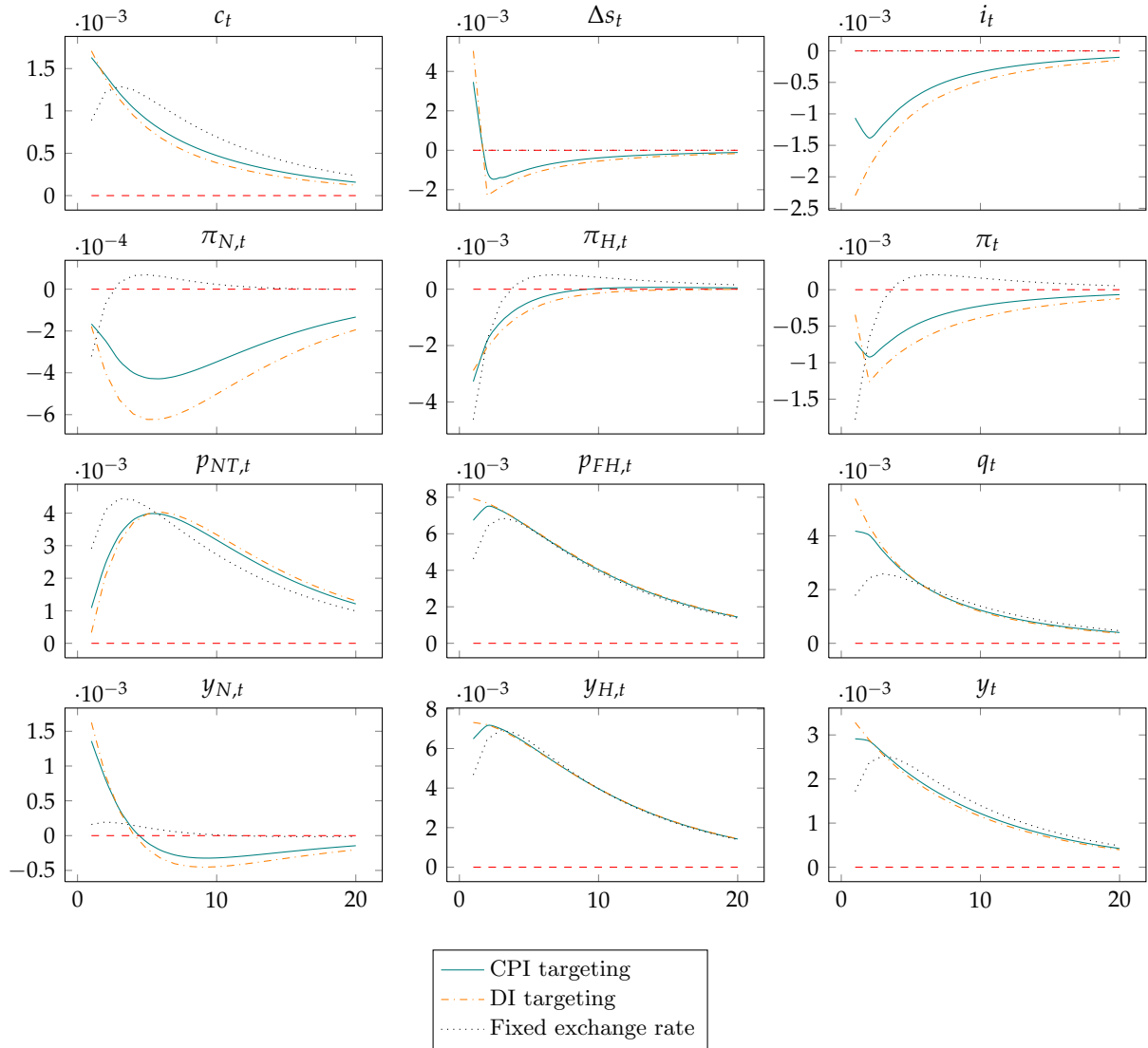


Figure 4.9: Impulse responses to a positive shock to the relative price of non-traded goods in Foreign

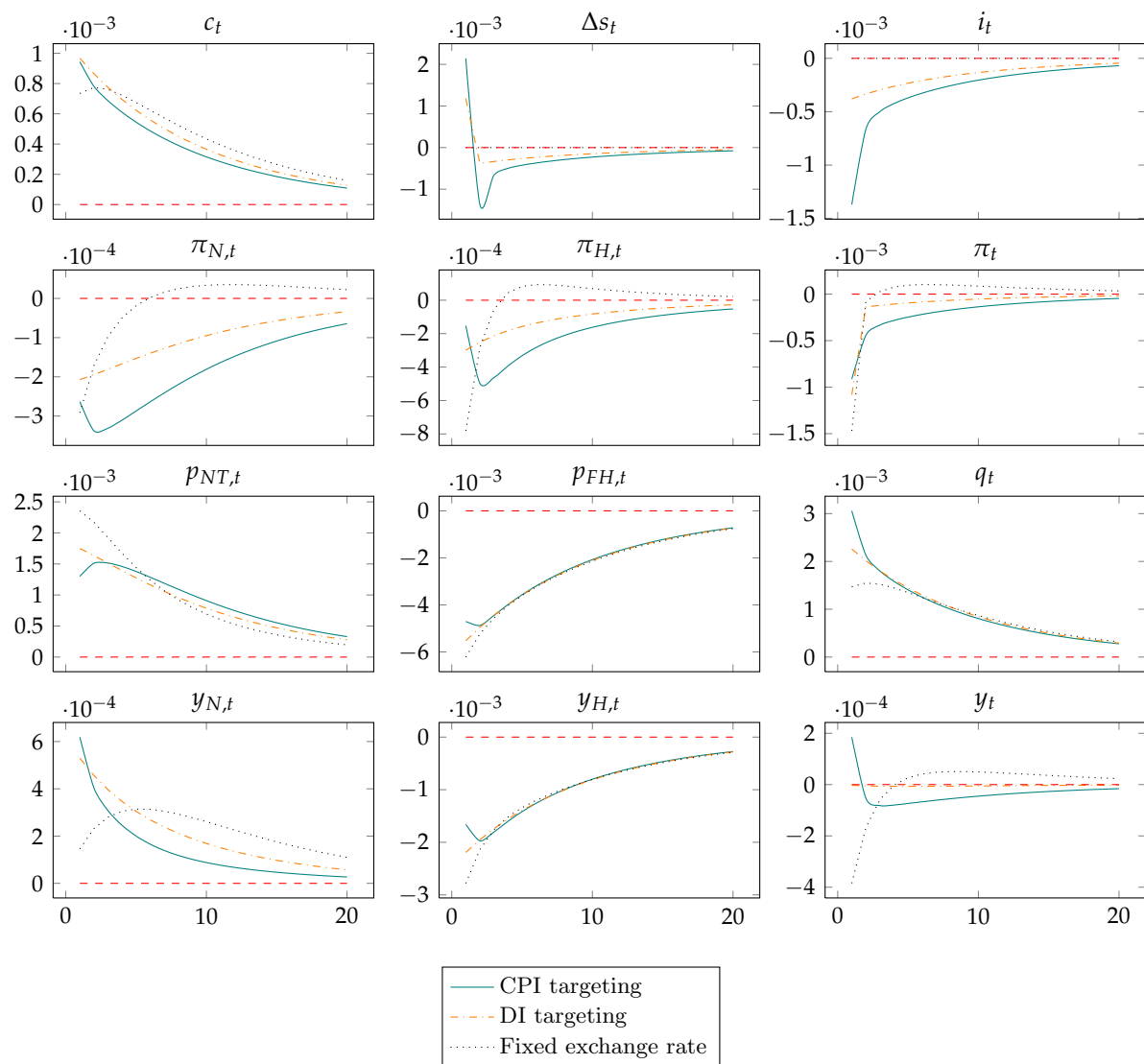


Figure 4.10: Impulse responses to a negative shock to foreign consumption

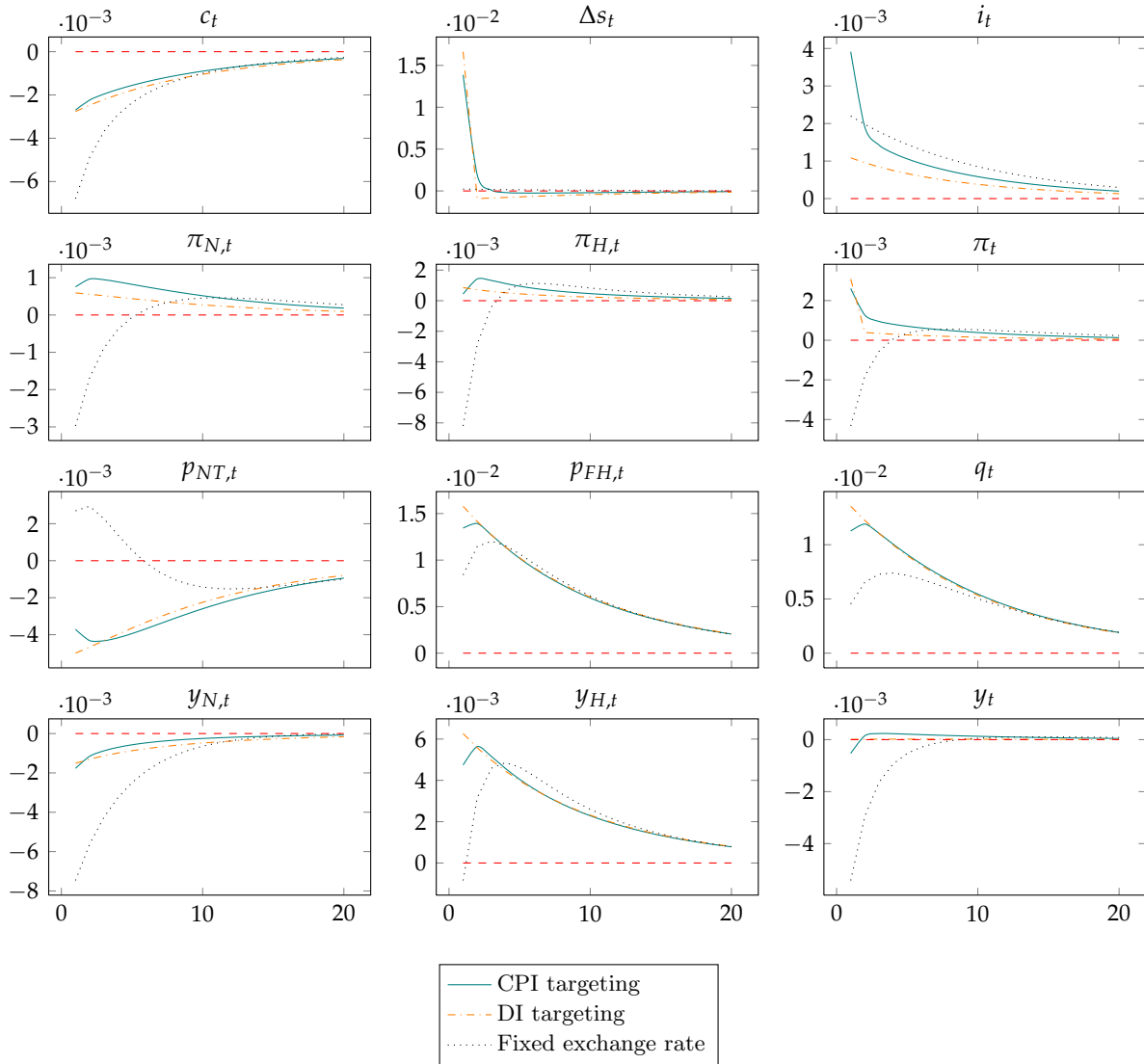


Figure 4.11: Impulse responses to a positive foreign interest rate shock

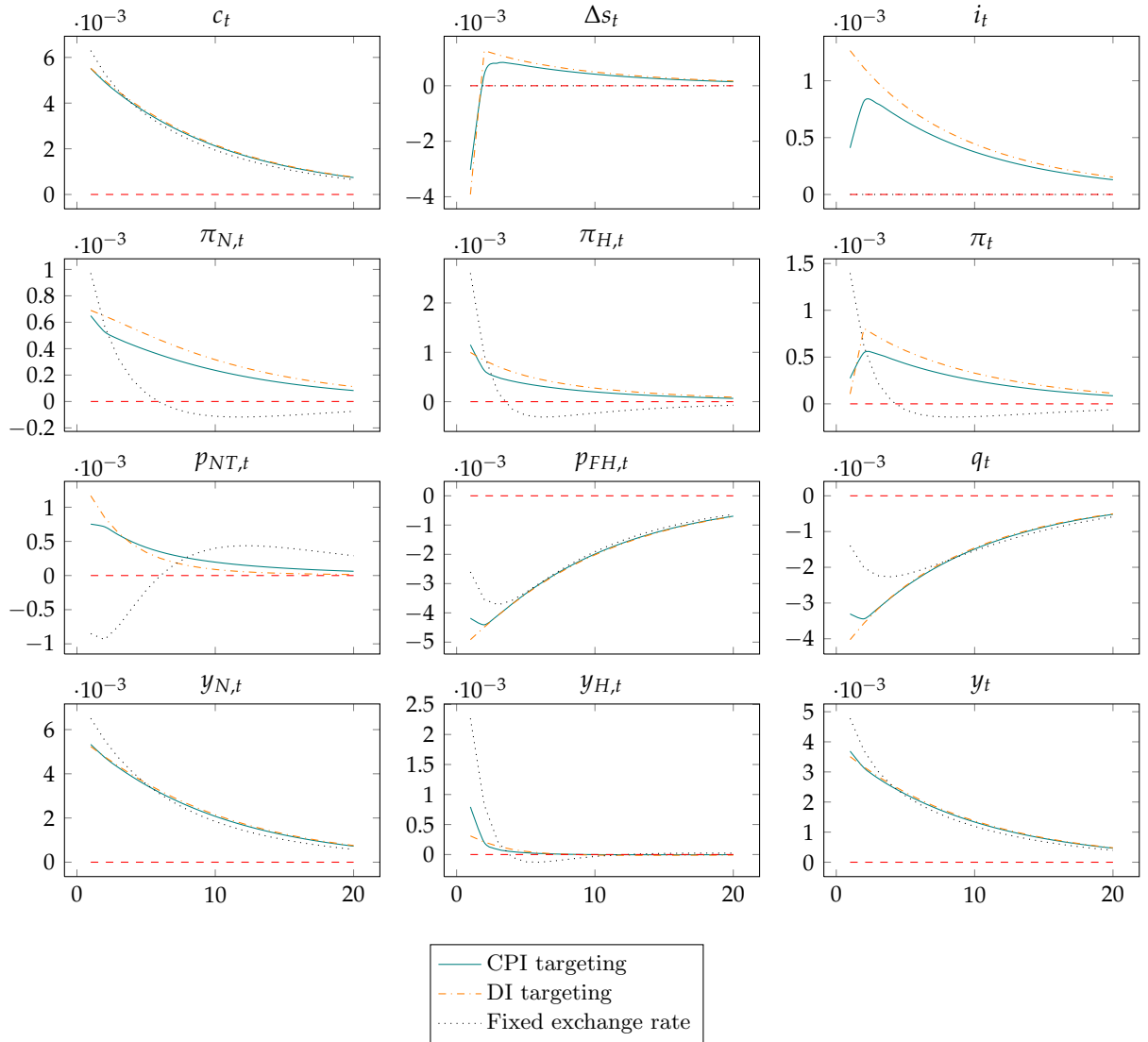
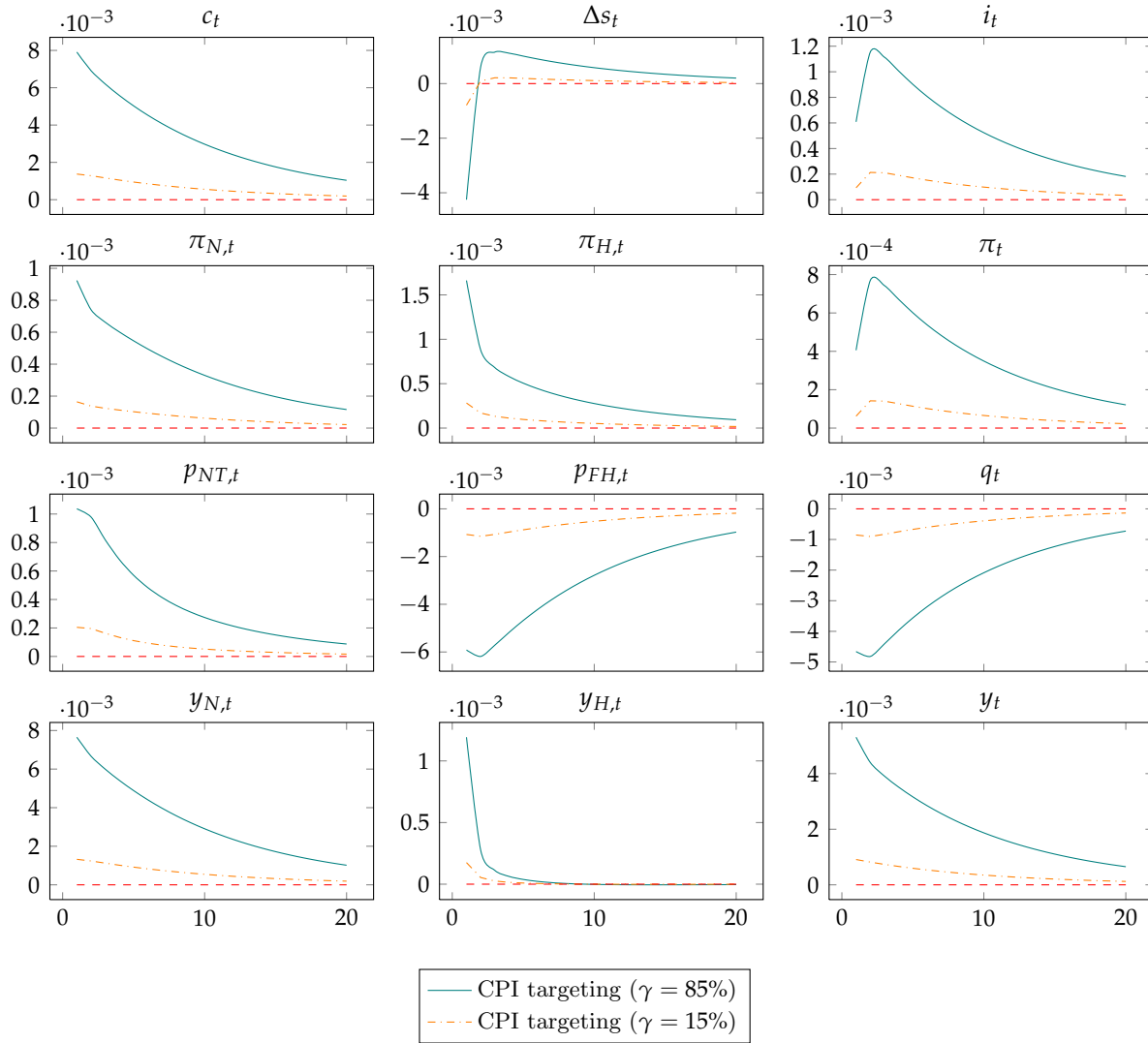


Figure 4.12: Impulse responses to a positive foreign interest rate shock (under high and low levels of steady state dollarization ratio γ)



Abstract

The thesis consists of three chapters, which deal with a number of issues at the intersection of Macroeconomics, Labor Economics, Development Economics and Monetary Economics.

Key question of the first chapter titled "Credit Frictions and the Dynamics of Formal and Informal Employment" is what explains the differential rate of entry into formal and informal employment over the business cycle observed in several developing countries. In particular, the paper documents this differential behavior using Brazilian labor force survey data and attempts to answer the main question using a theoretical model that replicates key patterns of labor reallocation across sectors.

The aim of the second chapter titled "Formal and Informal Employment and Employment Durations in Brazil" is to characterize the determinants of labor market transitions in Brazil by explicitly taking into account possible direct and indirect effects of various covariates on labor market transitions. The study is particularly focused on modeling exits from different employment categories and it models transitions from formal and informal salaried employment and informal self-employment using data on employment durations.

The third chapter titled "Monetary Policy Choices Under Currency Substitution" deals with a monetary policy question relevant for a large number of developing and emerging economies. In particular, it studies how monetary policy should be conducted in an environment with more than one currency in circulation (e.g., US dollars along with a domestic currency). In this paper, performance of a number of simple monetary policy rules is analyzed in a dynamic stochastic general equilibrium model with currency substitution.

Zusammenfassung

Die Dissertation besteht aus drei Kapiteln, die sich mit einer Reihe von Fragen am Schnittpunkt von Makroökonomie, Arbeitsökonomie, Entwicklungsökonomie und Geldtheorie befassen.

Das erste Kapitel mit dem Titel “Credit Frictions and the Dynamics of Formal and Informal Employment” erklärt die über den Konjunkturzyklus schwankenden Übergänge zwischen formeller und informeller Beschäftigung, wie sie in mehreren Entwicklungsländern beobachtet werden. Insbesondere, dokumentiert die Studie das unterschiedliche Verhalten der Beschäftigungsübergänge mit Hilfe von brasilianischen Arbeitsmarktdaten und entwickelt ein theoretisches Modell, das die wichtigsten empirischen Beobachtungen repliziert.

Das zweite Kapitel mit dem Titel “Formal and Informal Employment and Employment Durations in Brazil” charakterisiert die Determinanten der Übergänge auf dem Arbeitsmarkt in Brasilien, indem die möglichen direkten und indirekten Effekte verschiedener erklärender Kovariaten auf die Übergänge explizit berücksichtigt werden. Die Studie konzentriert sich besonders auf verschiedene Beschäftigungskategorien und modelliert die Übergänge zwischen formeller und informeller Erwerbstätigkeit sowie informeller selbstständiger Erwerbstätigkeit, unter mit Daten zur Beschäftigungsdauer.

Das dritte Kapitel mit dem Titel “Monetary Policy Choices Under Currency Substitution” befasst sich mit einer Frage, die für eine große Anzahl von Entwicklungs- und Schwellenländern relevant ist. Insbesondere, untersucht es wie die Geldpolitik in einem Umfeld mit mehr als einer Währung (z.B., US Dollar neben einer heimischen Währung) gestaltet werden soll. Die Studie analysiert das Verhalten einer Reihe von einfachen geldpolitischen Regeln in einem dynamisch-stochastischen allgemeinen Gleichgewichtsmodell, das mit Währungssubstitution erweitert wird.

Resume

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Research Analyst, Institute for Economic Strategies - Central Asia, Almaty, Kazakhstan, various projects and assignments between 2007 and 2010.

SCHOLARSHIPS

Scholarship for PhD studies, University of Vienna, 2007 – 2010

Scholarship for MA studies, Kazakhstan Institute of Management, Economics and Strategic Research, 2005 – 2007

US Department of State Freedom Support Act Undergraduate Exchange Scholarship, 2002 – 2003

State tuition scholarship, Tashkent State University of Economics, 2001 – 2005