



universität
wien

DIPLOMARBEIT

Titel der Diplomarbeit

“Implementing a rating migration
Credit Value at Risk model
including business cycle effects”

Verfasser

Günther Hahn

Angestrebter akademischer Grad

Magister der Sozial- und Wirtschaftswissenschaften
Magister rerum socialium oeconomicarumque
(Mag.rer.soc.oec.)

Wien, im Oktober 2008

Studienkennzahl lt. Studienblatt:
Studienrichtung lt. Studienblatt:
Betreuer:

A 175
Wirtschaftsinformatik
Univ.-Prof. Mag. Dr. Wolfgang Aussenegg

Mein Dank gilt Dr. Stephan Kossmeier, der mich dem Thema Kreditrisiko nahe gebracht hat und meinem hervorragenden Betreuer Univ. Prof. Mag. Dr. Wolfgang Aussenegg, der es mir selbst unter starkem Termindruck ermöglichte die Arbeit abzuschließen.

Meinen ArbeitskollegInnen und Vorgesetzten bei der Österreichischen Volksbanken A.G., allen voran Ing. Gerhard Bayerl danke ich für die außerordentliche Förderung und das mir entgegengebrachte Verständnis insbesondere während der Abschlussphase.

Meiner Familie, meinen Eltern, Josefa und Gerhard, meiner Schwester Karin, Ihrem Freund Martin und meinen Schwiegereltern danke ich für die Ermutigungen und nicht nur mentale Unterstützung während meiner Studienzeit.

Diese Arbeit widme ich meiner Frau Doris - ich danke Ihr von ganzem Herzen für Ihre Liebe und Geduld.

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1. Introduction

The credit value at risk and the business cycle - How can the effects of the business cycle be integrated in a credit migration approach model for the credit value at risk, such as the CreditMetrics framework? To answer this question, one first has to focus on targets of risk models.

1.1. Targets of Risk Models

Banks fulfil the economical function of financial intermediaries in economies. As highly regulated institutions, banks are not only subject to detailed risk disclosure requirements by banking supervision (Basel II), but also by shareholders and other stakeholders. It is common for banks to disclose overall risk exposure broken down into market, credit and operational risk in periodical financial statements.

The goal of financial risk management is not avoiding risk, but comprehending and visualising the coherence between different risk factors and risk exposure. It is the business of financial institutions to take market and credit risk intentionally in order to earn higher than risk free returns. As profit oriented organisations, efficient comprehensive risk management is vital to banks. First this involves identifying the relevant risk categories, such as credit, market, liquidity and operational risk. Risk management performs the tasks of risk assessment, monitoring and mitigation. Another risk management role is to ensure that the total risk taken is no greater than the bank's ability to absorb worst case losses within a specified confidence interval.

These goals can be reached by establishing profound risk models, that allow capital allocation and controlling of capital adequacy based on risk adjusted performance measures. This in turn renders possible strategic management and benchmarking the performances of exposures across different asset and risk classes, profit centres or business opportunities in general.

Capital, basically defined as the difference between the market value of assets and the market value of liabilities, plays the role of a buffer against insolvency. It secures a firm's economic survival, even if serious, unexpected losses have to be sustained. From an internal bank perspective, capital represents an ideal metric for aggregating risks

across different risk types. Capital helps to provide confidence to stakeholders, like investors or rating agencies on the financial capacity of a bank. A firm's credit rating can be seen as a measure of its capital adequacy and is generally linked to a specific probability that a firm will enter default over some period of time (cf. Aziz and Rosen, 2004).

From an external, regulator viewpoint, the primary objective of regulatory capital adequacy requirements is to constitute a level playing field across all financial institutions and to push international capital adequacy standards. Another aim is to safeguard the security and viability of the banking system. As national governments act as guarantors, they have a strong interest in ensuring that banks remain capable of meeting their obligations and in minimising systematic effects on the economy.

From a historical perspective, the imposition of the Basel I Accord by the Basel Committee on Banking Supervision in 1988 (BCBS 1988) proved to be successful in its objective of increasing worldwide capital levels and thereby reducing the overall level of risk in the global banking system. Thanks to regulatory capital requirements banks have to bear their share of the burden that would otherwise be borne by national governments. However, lack of differentiation in the framework lead to what is termed in the literature as regulatory capital arbitrage, as in depth described in Jones (2000). This comprises instruments or processes, which allow reducing regulatory capital, without an equivalent reduction of the actual risk being taken.

In 1999 the BCBS issued a proposal intended to replace the existing capital adequacy framework. Following some consultative documents, in June 2004 the final version of the Basel II Accord (BCBS 2004a) formulated new regulatory, more risk-sensitive rules that first assess default risk of bank's loan portfolios and then deduce the corresponding minimum regulatory capital requirements for credit risk. Basel II consists of three pillars: minimum capital requirements, supervisory review, and market discipline principles.

Concerning credit risk modelling, Basel II puts a strong emphasis on development of risk management and encourages ongoing improvements in banks' internal risk measurement abilities. For instance paragraph 748 of the Basel II Accord (BCBS 2004a) states that "Supervisors should assess the degree to which internal targets and processes

incorporate the full range of material risks faced by the bank. Supervisors should also review the adequacy of risk measures used in assessing internal capital adequacy and the extent to which these risk measures are also used operationally in setting limits, evaluating business line performance, and evaluating and controlling risks more generally. Supervisors should consider the results of sensitivity analyses and stress tests conducted by the institution and how these results relate to capital plans.” This implies that e.g. banks choosing the internal ratings-based approach are required to demonstrate that they use the outputs of their models not just for minimum capital requirements, but also to actively manage their business.

1.2. Problem Statement

How does the business cycle influence the Credit value at risk when calculated via the CreditMetrics framework? Why is it worthwhile to ask such a question and to extend the framework and integrate a business cycle aspect?

To use the words of Crouhy et al. (2000), in “order to measure credit risk of derivative securities, the next generation of credit models should allow at least for stochastic interest rates, and possibly default and migration probabilities which depend on the state of the economy, e.g. the level of interest rates and the stock market” (Crouhy et al. 2000, p.62). For the purpose of illustration they continue by stating that according “to Standard & Poor’s, only 17 out of more than 6700 rated corporate bond issuers it has rated, defaulted on US \$ 4.3 billion worth of debt in 1997, compared with 65 on more than US \$ 20 billion in 1991.” When having a close look at the record of defaults from 1985 to 1997, it can be seen that in 1990 and 1991, when the world economies were in recession, the frequency of defaults was quite large. In the years around 1996, characterised by a sustained growth economy, the default rate has declined dramatically.

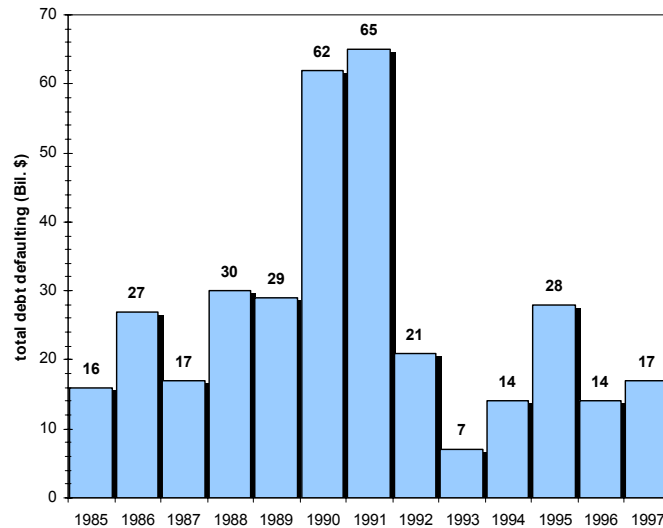


Figure 1 Global corporate defaults in bn. \$ (Crouhy, Galai, and Mark 2000, p.63)

Figure 1 illustrates the worldwide record of defaults in billions of Dollars from 1985 to 1997, and thereby allows identifying the possible influence of the business cycle on the default probability.

Vazza et al. (S&P, 2008) offer not only a more exhaustive history of global corporate defaults, but as well statements on the latest implications of the sub-prime mortgage defaults, remarking that “despite the liquidity disruption in the credit markets, the incidence of corporate defaults in 2007 remained low, maintaining the trend prevalent in the past few years. However, tightening credit conditions are likely to increase in 2008. Although our latest default study only covers defaults through year-end 2007, early reports are indicating that the default rate already began to climb in January 2008.”

Year	Total Defaults	Investment-grade defaults	Speculative-grade defaults	Default rate (%)	Investment-grade default rate (%)	Speculative-grade default rate (%)	Total debt defaulting (bn. \$)
1981	2	0	2	0,14	0	0,62	0,06
1982	18	2	15	1,19	0,18	4,41	0,9
1983	12	1	10	0,76	0,09	2,93	0,37
1984	14	2	12	0,91	0,17	3,25	0,36
1985	19	0	18	1,1	0	4,31	0,31
1986	34	2	30	1,72	0,15	5,66	0,46
1987	19	0	19	0,95	0	2,79	1,6
1988	33	0	30	1,43	0	3,96	3,3
1989	42	2	34	1,69	0,14	4,53	7,28
1990	69	2	56	2,74	0,14	8,09	21,15
1991	93	2	65	3,26	0,14	11,04	23,65
1992	39	0	32	1,49	0	6,08	5,4
1993	26	0	14	0,6	0	2,5	2,38
1994	20	1	15	0,62	0,05	2,1	2,3
1995	35	1	29	1,04	0,05	3,51	8,97
1996	20	0	16	0,51	0	1,79	2,65
1997	23	2	20	0,62	0,08	1,98	4,93
1998	58	4	49	1,28	0,14	3,68	11,27
1999	108	5	91	2,1	0,17	5,45	39,38
2000	136	7	108	2,41	0,23	6,05	43,28
2001	229	8	173	3,73	0,26	9,64	118,79
2002	225	13	158	3,49	0,41	9,19	190,92
2003	121	3	89	1,87	0,1	4,88	62,89
2004	56	1	39	0,77	0,03	2,01	20,66
2005	39	1	30	0,57	0,03	1,41	42
2006	30	0	26	0,46	0	1,14	7,13
2007	22	0	21	0,36	0	0,86	8,15

Table 1 Global Corporate Default Summary, Vazza et al. (S&P, 2008)

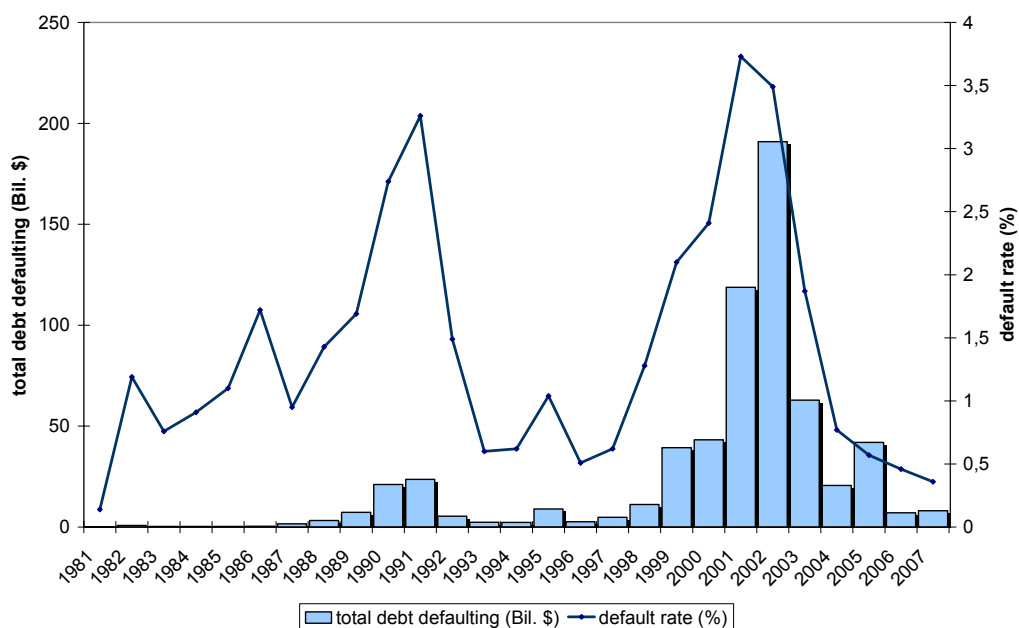


Figure 2 Global Corporate Default Summary, data source: Vazza et al. (S&P, 2008)

As illustrated in Figure 2 relevant variables, such as the average default rate tend to increase during recessionary phases of the business cycle. To quote Hamilton and Raj (2002), “The normal behaviour of economies is occasionally disrupted by dramatic events that seem to produce quite different dynamics for the variables that economists study. Chief among these is the business cycle, in which capitalist economies depart from their normal growth behaviour and a variety of indicators go into decline.” The authors explicate that interest in the asymmetrical characteristic of the business cycle has a long history in economics, dating back to Keynes. They summarise that economics addresses the asymmetry feature of business cycles by distinguishing different characteristics of expansions and contractions. As will be detailed in section 7, Markov switching models can be used to describe the evolution of the economy through business cycles.

1.3. Results Overview

In the implementation of the credit risk model within this diploma thesis, a regime switching behaviour of the business cycle is integrated. It comprises two different states, expansion and contraction. For each scenario, given an initial state, the business cycle will either switch from e.g. initial expansion into recession or remain in the initial state.

The business cycle regime is modelled to either influence the credit rating transition probabilities, the credit spreads used for bond revaluation, both or none of the parameters. It turned out that switching the transition probabilities had by far a much larger effect on the credit value at risk compared to only switching credit spreads. For instance starting in the high business environment and assuming a certain migration into the low economic state lead to a 67% increase of the 95% credit value at risk due to transition matrix switching, whereas the effect due to credit spread switching resulted in an increase of only about 7%. Similarly the combined mode resulted in a 74% increase.

In a second approach, for all four modes a sensitivity analysis of the credit value at risk on the business cycle is performed for sample portfolios. For both initial business cycle stages, *ceteris paribus* the probability of switching to or remaining in the favourable business environment is increased. As intuitively expected, it is shown that the credit value at risk increases with worsening economical conditions represented by conditional credit migration matrices and differing credit spreads. For a sample portfolio starting in

the expansionary business cycle environment setting the probability of remaining in the favourable state to 0, resulted in an increase of the 99% credit value at risk of about 36%, compared to remaining in the “high” business cycle state. Analogically the 95% credit value at risk even rises by about 68%. Further detailed results are presented in section 8.

2. Review of Credit Risk Management

2.1. Credit Risk

Managing credit risk is a particularly important task in banks and other financial institutions. As globally, institutions are taking on increasing amounts of credit risk, credit exposures have multiplied and credit risk has become a key risk management challenge. Regulators specify minimum levels of capital that banks are required to keep in order to reflect the credit risks they are bearing. This capital has to be held in addition to the capital that banks are compelled to keep for market risk. In the beginning, regulators stipulated certain minimum levels for balance sheet ratios such as equity to debt. As time passed by, this approach became inappropriate in the late 1980s, because off balance trades, such as swaps and options, which do not appear in balance sheets, began to account for a significant proportion of the total credit risk. So even when it comes to derivative pricing in the over the counter market, the risk of counter party default must not be neglected. As the over the counter market has grown enormously since the mid 1980s, the quantification and management of credit risk has become an increasingly important activity for financial engineers. Regulators now apply meliorated formulas for determining credit risk capital that reflects both, on- and off-balance sheet contracts.

One has to take into account many different aspects of credit risk. At first, one problem is to determine how the prices of contracts should be adjusted in order to reflect the risk that the counter party might default. A bond issued by a company with a good credit rating generally sells for a higher price, than a similar bond issued by a company with a poor credit rating, because the latter company is associated with a higher chance of default. For the very same reason, a swap entered into with a company that possesses a good credit rating should be worth more than a similar contract entered into with a company that has a poor credit rating. Methods for quantifying credit risk allow to price

credit risk per transaction per counterparty and to carry out stress and scenario analysis in order to perform worst case credit loss estimates.

Regarding credit risk emanating from the OTC market, a countertrend can be observed that involves more and more OTC products being cleared centrally. Exchanges have successfully organised their trading rules in order to ensure that the contracts are always fulfilled. The implementation of margin accounts and margin calls due to daily mark to the market technique has proven to be a successful strategy. One effect of the sub-prime mortgage defaults that started in 2007 was turmoil in the global financial markets, entailing rising anxiety for counterparty default risk and involving rising funding liquidity risk. Approvals of counterparty credit lines tend to become more difficult. This in turn afforded an opportunity for exchanges and investment banks to establish centrally cleared trading facilities for derivatives. For instance in July 2008 the CME group, head of the Chicago Mercantile Exchange, Chicago Board of Trade and New York Mercantile Exchange, announced to expand its OTC Clearing Capabilities to the Interest Rate Swap Market. They offer clearing services by which they guarantee the credit risk to both sides of a transaction by acting as a central counterparty and taking out counterparty risk. This also allows cross market netting of trades, thereby momentarily reducing outstanding positions. Daily mark to market substantially reduces potential credit exposure by reducing the risk horizon to one day, compared to mark to market being performed only at the end of a transaction. A central counterparty holding an appropriate high rating lowers required capital for counterparty risk. Furthermore, as regulators push the industry to reduce operational risk created through OTC products, introducing a central clearing counterparty significantly reduces the manual effort that is necessary due to bilateral negotiations. This will diminish the need to ratify a transaction with documentation as prescribed by the International Swaps and Derivatives Association (ISDA). Post trade processing of trades, including validating, confirming and matching transactions, handling trade discrepancies between counterparties, the hazard due to backlogs or valuation disputes, missing margin calls in case of collateral management, ratifying and maintaining ISDA agreements with various counterparties is a huge manual effort for banks and accordingly involves operational risk.

One of the committees of the BIS, the Committee on Payment and Settlement Systems, analysed arrangements and risk management practices in the OTC derivatives market.

This report discusses inter alia “the potential for significant market disruptions from the closeout of OTC derivatives transactions following the default of a large market participant” (CPSS, 2007). In March 2008 this aspect was almost tested by the wake the collapse of Bear Stearns had on the financial markets. Fender and Hördahl (2008) notice that the “near collapse and subsequent takeover of Bear Stearns on 14-18 March highlighted the risks that banks face in such situations. On the one hand, the Federal Reserve-facilitated takeover of Bear Stearns by JPMorgan was generally perceived by investors as signalling that large banks would not be allowed to fail, and this helped restore order in other markets. On the other hand, the speed with which Bear Stearns’ access to market liquidity had collapsed underscored the vulnerability of other banks in this regard”. This analysis is exacerbated by the typical characteristics of credit default events: Although they are scarce, they can result in heavy losses.

2.2. Credit Value at Risk

For pricing purposes it is useful to perform analysis that allows calculating the present value of expected credit losses on a contract or a given portfolio of contracts. The risk of default of a single debtor is the calculation basis for loan pricing. Earned credit premia should compensate expected losses. For risk management purposes financial institutions are rather interested in calculating a complete probability distribution for the credit losses of the whole loan portfolio, since the economic viability of the bank largely depends on the size of credit related losses. The Value at Risk provides an estimate of a specific quantile loss arising from movements in the market variables. A credit value at risk measure can be similarly defined to the Value at Risk measure. This is a measure designed to provide a solution for the following question: “What is the credit loss that one is 99% confident that will not be exceeded in the next year?” Of course this question can be generalised to the following form: “What credit loss is such that we are x% certain it will not be exceeded in time T?”

Once portfolio analysis is in place, marginal risk analysis may be performed in order to price individual loans based on their marginal contribution to the total value of the risk measure for the entire portfolio. As describe above, bank regulators intend to base credit risk capital on a credit value at risk measure. It can be defined, whether credit losses are only considered when counterparty defaults or if the effects of deteriorating counter-

party credit ratings are also included in the risk measure in order to reflect the revalued prices of the outstanding instruments.

2.3. Portfolio Market Risk vs. Portfolio Credit Risk

As equity portfolio pricing is widely explored in the asset pricing literature, one might endeavour to translate established theories on the pricing of credit portfolios. However one has to pay attention to the fundamental differences between credit risks and market risks. As Gupton et al. (1997) outline, in comparison to market Value at Risk, credit value at risk poses two major difficulties:

2.3.1. Portfolio Return Distributions

The first problem is concerned with the fact that equity returns are often assumed to be relatively symmetric, whereas on the other hand, credit returns are by nature highly skewed and fat tailed. This stems from the fact that earnings of credit portfolios tend to be upward limited, since obligors will only repay the debt owed plus interest above the risk free interest rate. The fat downside tail of credit returns reflects possible defaults, resulting in rather huge losses compared to the limited possible earnings of loans. Limited positive effects can be expected from any improvement in credit quality, while there is substantial downside risk due to consecutive downgrades and default. One can summarise that credit returns exhibit small profits with a huge probability, but significant losses with a rather small probability. This characteristic is often referred to as fat tails or leptokurtosis of the distribution.

To characterise market risks, mean and standard deviation of portfolio returns are adequate to estimate distribution percentiles of interest, since normal Gaussian distributions are judged appropriate to represent market returns. The difference between typical market and credit returns is depicted in Figure 3.

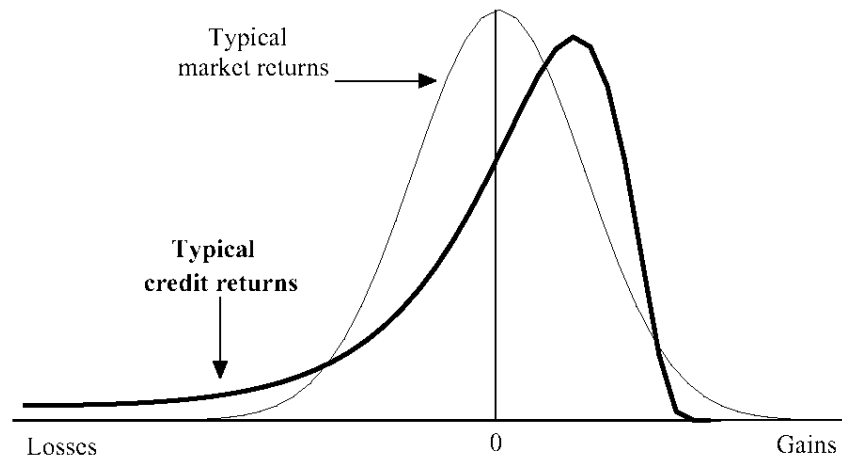


Figure 3 Distribution of credit and market returns (Gupton et al., 1997)

Thus for credit risk, mean and standard error are no more sufficient to quantify percentile values that are necessary to attain a Value at Risk. The calculation of the credit value at risk requires a simulation of the whole distribution of portfolio value returns. A large, granular portfolio should result in a smooth, skewed return distribution as illustrated in Figure 3.

2.3.2. Portfolio Diversification Effects

The second problem Gupton et al. (1997) list, concerns the difficulty of modelling correlations. As CreditMetrics assesses risk on the portfolio level it also needs to address correlation of credit quality moves across obligors, in order to incorporate diversification effects and from another perspective, to detect possible concentration risks. For the purpose of measuring the portfolio diversification effect, one needs to estimate the correlations in credit quality changes for all pairs of obligors. Correlations for market risk factors, such as equities can be directly estimated from historical quoted market prices. However this is not the case for credit correlations, due to missing liquid market data.

CreditMetrics makes use of equity returns as a proxy for asset returns, which itself results from simplifying assumptions on the capital structure of the obligor and on the generating process for equity returns. Further the approach does not estimate pair wise correlations between obligors, but groups them into indices per industry and country. Section 5.2 will present in detail, how credit correlation is introduced in the framework. This is clearly another key feature of CreditMetrics, but as far as the implementation

within this diploma thesis is concerned, correlations are only regarded as known input parameters into the model.

2.4. Concentration Risk

Huge exposures in one obligor or a group of correlated obligors, e.g. within the same industry, strongly linked sectors or geographical region drives concentration risk. Interestingly, for European banks investments in U.S. mortgage products was partly motivated in order to achieve a wider diversification and thereby reduce concentration risk. In the wake of the current financial crises, a return to the respective core business of banks is observable. Studies on financial crises, such as BCBS (2004b) highlight that concentration risk often played a major role in bank failures. From a regulatory perspective, the internal rating based approach in the Basel II Accord builds on the key assumption of the Asymptotic Single-Risk Factor (ASRF) model. It assumes only one single source of systematic risk and that portfolios are granular to the point, so that a large single exposures can only result in a small part of the whole portfolio risk profile, i.e. idiosyncratic risk is assumed to be diversified away at the portfolio level. The BIS working paper No. 15 (BCBS, 2006b) points out that concentration risk violates these assumptions, as “Name concentration implies less than perfect granularity of the portfolio, while sectoral concentration implies that risk may be driven by more than one systematic component (factor).” The paper continues by appraising that “Historical experience shows that concentration of credit risk in asset portfolios has been one of the major causes of bank distress. This is true both for individual institutions as well as banking systems at large”. One can deduce that a portfolio credit risk model has to integrate correlations in order to reflect diversification effects and concentration risks.

3. Credit Risk Pricing Models

In the academic literature two primary credit risk pricing approaches can be distinguished: First the structural or firm value models and second the reduced form models. The basic distinction between the two methodologies is the type of input variables they use. The structural approach employs company idiosyncratic data and interprets debt as a contingent claim on the firm’s value. A structural approach models the way the firm value changes over time. The relationship between firm value and debt value then implies default risk of a specific obligor. On the other hand the reduced form models of

default presume default to be unpredictable and model it via a stochastic hazard intensity function. This circumvents problems stemming from the difficulty of observing the firm's asset and debt value processes. Instead, the reduced form models deduce default risk from market prices, credit spreads, and sometimes rating transitions. Both credit risk pricing models are employed to estimate default probabilities, which is an important task required for all credit risk models. Zhou (2001) combines the benefits of both approaches by modelling the evolution of firm value as a jump-diffusion process.

From an information theoretic perspective Jarrow and Protter (2004) opine that reduced-form models are more appropriate, since they "assume that the modeller has the same information set as the market - incomplete knowledge of the firm's condition". Whereas structural models assume that the modeller has complete information of all the firm's assets and liabilities. The authors summarise, that both models are viewed as competing and that there was a heated debate in the professional and academic literature trying to assess which type of models performs better for which purpose. This debate mainly revolved around default prediction and hedging performance (see Jarrow et al. 2003).

It is worthwhile to mention that several other credit pricing methods were published: For instance, Fisher (1959) proposed a risk factor premium method, linking credit spread to various company specific risk factors. Kao (1996) utilises obligor's financial fundamentals and two correlated binomial trees modelling the interest rate process and commercial mortgage default process in order to price commercial mortgage backed securities.

3.1. Structural Credit Pricing Model

The structural credit risk pricing approach, also known as the firm value approach was built on the classical Black and Scholes (1973) option pricing theory. The structural approach was first introduced by Merton (1974) and later extended by Black and Cox (1976), Geske (1977), and Vasicek (1984). As described in chapter 5.2 the CreditMetrics approach builds upon this framework, and hence it is in brief presented below. Merton links the default process of a company to the value of the company's assets.

The first assumption is made concerning the simple capital structure of the firm, as it is financed exclusively by equity S_t and a single zero coupon debt instrument maturing at

time T with a face value of F , and current market value B_t . This implies the following balance sheet of the firm:

Assets		Liabilities/Equity	
Risky Assets: V_t		Equity: S_t	
		Debt: $B_t(F)$	
Total:	V_t	V_t	

Figure 4 Balance sheet of a firm in Merton's model

As Altman et al. (2001) point out the “payment to the debtholders at the maturity of the debt is therefore the smaller of two quantities: the face value of the debt or the market value of the firm's assets. Assuming that the company's debt is entirely represented by a zero-coupon bond, if the value of the firm at maturity is greater than the face value of the bond, then the bondholder gets back the face value of the bond. However, if the value of the firm is less than the face value of the bond, the equityholders get nothing and the bondholder gets back the market value of the firm. The payoff at maturity to the bondholder is therefore equivalent to the face value of the bond minus a put option on the value of the firm, with a strike price equal to the face value of the bond and a maturity equal to the maturity of the bond”. Since this approach regards defaultable debt as a contingent claim on the value of the firm, this family of models are often referred to as contingent claim approach. Crouhy et al. (2004) relate the price of eliminating credit risk to the put option, by clarifying that “by purchasing the put on the assets of the firm for the term of the debt, with a strike price equal to the face value of the loan, the bank (the lender) can completely eliminate all the credit risk and convert the risky corporate loan into a riskless loan. Thus, the cost of eliminating the credit risk associated with providing a loan to the firm is the value of this put option.”

The most important part of a structural pricing model is the firm value, V_t , represented through a diffusion process resembling the firm's profitability and cash flows. The firm's asset value is presumed to follow a standard geometric Brownian motion, which is defined by

$$V_t = V_0 \cdot e^{(\mu - \frac{\sigma^2}{2})t + \sigma \sqrt{t} Z_t}, \quad (1)$$

where V_t equals the firm's asset value at time t , μ and σ^2 are the mean and the variance of the expected instantaneous rate of return of the firm's assets. μ can also be interpreted as the constant drift of the process that expresses the alteration of V_t as a percentage. Z_t is a normally distributed random variable. This equals the following, more common statements describing the dynamics of V_t :

$$dV_t = \mu V_t dt + \sigma V_t dW_t \quad (2)$$

$$\frac{dV_t}{V_t} = \mu dt + \sigma dW_t \quad (3)$$

This Geometric Brownian Motion incorporates the following characteristics: If V_t starts from a positive value V_0 , then V_t will only evaluate in positive values. Hence it is often used to model price as well as interest rate processes. The probability distribution of V_s , given V_t is a lognormal distribution with expected value and variance as follows:

$$E(V_s) = V_t \cdot e^{\mu(s-t)} \quad (4)$$

$$Var(V_s) = V_t^2 \cdot e^{2\mu(s-t)} \cdot (e^{\sigma^2(s-t)} - 1) \quad (5)$$

This indicates that the probability distribution of $\ln(V_s)$, given $\ln(V_t)$ is a normal distribution with expected value and variance as defined by the following equations:

$$E = \ln(V_t) + \mu(s-t) - \frac{\sigma^2}{2}(s-t) \quad (6)$$

$$Var = \sigma^2(s-t) \quad (7)$$

As the variance of a prognosis of V_t increases infinitely with increasing t , the process is non stationary. Moreover it is noteworthy that the percentage changes in prices are independent and identically normally distributed random variates. This simple firm value process presumes the company's business opportunities and activities to remain fixed as time passes.

Under the assumptions necessary to apply the Black Scholes model to equity or debt instruments (cf. Galai et al., 2004), one can calculate the current value of the put P_0 as:

$$P_0 = -N(-d_1)V_0 + Fe^{-rt}N(-d_2), \quad (8)$$

where $N(\bullet)$ is the cumulative standard normal distribution and

$$d_1 = \frac{\ln(V_0 / F) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}} = \frac{\ln(V_0 / Fe^{-rt}) + \sigma^2 T / 2}{\sigma\sqrt{T}} \quad (9)$$

$$d_2 = d_1 - \sigma\sqrt{T}. \quad (10)$$

Thus, in the structural model the price of credit risk is an increasing function of asset volatility σ and the time to debt maturity T , but on the contrary a decreasing function of the risk free interest rate r . In the Merton model, the default spread π_T , that is the risk premium associated with holding the bond, can be computed as:

$$\pi_T = -\frac{1}{T} \ln \left(N(d_2) + \frac{V_0}{Fe^{-rt}} N(d_1) \right) \quad (11)$$

As can be intuitively reasoned in such a lender-borrower structure, the structural model assumes that default of the firm only occurs at maturity of the debt obligation, if the value of the firm's assets, V_t are worth less than the promised payment to the bondholders, F . Due to the fact that the market's valuation of assets vary over time, just as well varies the issuer's ability to meet its obligations, that are backed by the assets. A firm's default risk is thus linked to the variability in the firm's asset value and default is endogenously designed. In other words the trigger point of default is a predictable stopping time without a sudden surprise. In a structural pricing model, the following company idiosyncratic factors influence the debt value: the initial value of the firm, the expected growth rate of the firm value, the volatility of the firm value, coupons or dividends, the term of the loan, and the hierarchy of debt structure and the leverage. The distribution of the asset's value at time T , the maturity of the zero coupon debt is displayed in Figure 5.

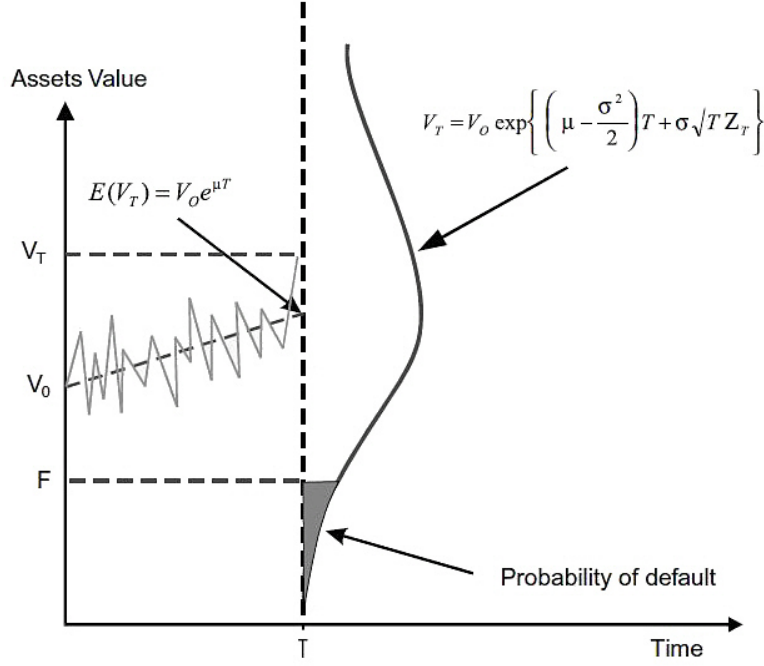


Figure 5 Distribution of the firm's assets value at maturity of debt obligation (Crouhy et al. 2000)

One of the by-products of the contingent claim approach to credit risk pricing is a thorough measure of credit risk: The expected default frequency also going by the name of probability of default is represented by the shaded area below F in Figure 5. For the purpose of compiling this estimate, one first has to estimate the distribution of the firm's market value versus the book value of its liabilities. The expected default frequency is the distance to default represented by the number of standard deviations to the default point in the distribution.

Giesecke (2004) summarised that default probabilities $p(T)$ in structural credit risk pricing models are given by:

$$p(T) = P[V_T < F] = P[\sigma W_T < \log L - mT] = N\left(\frac{\log L - mT}{\sigma\sqrt{T}}\right) \quad (12)$$

where $L = \frac{F}{V_0}$ represents the initial leverage ratio, m is short for $m = \mu - \frac{\sigma^2}{2}$ and

$N(\bullet)$ is the standard normal distribution function.

Lots of refinements of the first generation structural models have been developed. Some extensions address strict core assumptions that are built into the model. Such restrictions

include the flat and static yield curve, the lack of existence of taxes, and default triggered only at maturity, if $V < F$. For instance Black and Cox (1976) incorporate subordinated debt in order to allow more complex capital structures, whereas Geske (1977) integrate interest paying debt. In order to exogenously define default time, some variations of the model, for instance Longstaff and Schwartz (1995), assume default to be triggered when the firm value breaches a constant boundary. Hence default is allowed to happen even prior to maturity, and V can be less than F without inflicting default. In further amendments to the structural approach, Longstaff and Schwartz (1995) expound that just like the firm value process also interest rates can be modelled stochastically.

Criticism on the structural approach often focuses on the fact that in practice the firm value is hard to estimate, even if the stocks are traded, only the equity value process can be observed. This problem becomes more striking, if the firm's securities are illiquid or the firm is privately owned, since the asset volatility needs to be estimated. Nystrom and Skoglund (2003) mention another practical challenge which is the fact that most firms possess a rather complex liability structure that is difficult to model explicitly and price simultaneously. Kao (2000) notes that employment of the structural approach to areas such as municipal and sovereign bonds remains questionable and that the model requires plenty of input data, such as debt structures and contractual terms. Accordingly the model is sometimes considered as computationally laborious. Kafetzaki-Boulamatsis and Tasche (2001) detect another deficiency specific to the Merton model, which concerns the credit spreads: "If spreads followed the dynamics given by the model, they would tend to zero as the maturity of the bond approaches. In practice, experience shows that this is not the case. Spreads are usually bounded away from zero over all the time to maturity of the bond." They disclose another caveat which is the implication that according to the model the value of the firm's debt should decrease whenever the risk-free interest rate increases, which stands in opposition to experience. To summarise, Nystrom and Skoglund (2003) accord that the "sound foundation on fundamentals makes the structural models a nice framework for theoretical reasoning but the complexity of the real world makes their full implementation almost impossible."

3.2. Reduced Form Credit Pricing Model

The reduced form approach to credit risk pricing makes no assumption on the reasons of defaults. Instead, as Giesecke (2004) explains “the dynamics of default are exogenously given through a default rate, or intensity. In this approach, prices of credit sensitive securities can be calculated as if they were default free using an interest rate that is the riskfree rate adjusted by the intensity.” As the reduced form approach does not model default risk linked to the obligor’s financial characteristics, it does not require information on the company’s financial fundamentals, but takes as input what can be directly observed on the market: bond prices and spreads. Models of this family have been primarily presented in Jarrow and Turnbull (1995), Jarrow, Lando and Turnbull (1997), Duffie and Singleton (1997, 1998, 1999), and Jarrow and Turnbull (2000).

Reduced form models configure default as a stochastic process and default event as totally unpredictable or inaccessible random stopping time, i.e. they treat default as a sudden surprise. In other words, the arrival process of default is the time of the first jump of a stochastic process with random intensity. The intensity model is then calibrated to the market, providing prices for both defaultable and risk free instruments. Applying exogenously defined default and recovery rates, prices of defaultable and risk free assets are directly linked.

Kao (2000) classifies reduced form models into three main branches: default based, rating transition based, and spread models.

3.2.1. Default based Approach

This type of model was initially proposed by Jarrow and Turnbull (1995) and connects the price of a defaultable bond $B(t, T)$ at time t with maturity at time T , with the price of a default free bond $P(t, T)$. Since the authors base this approach on their former solution for a foreign currency pricing problem, the linkage between the risk free and risky asset is modelled as an exchange rate or conversion factor, which depends on default and recovery, i.e. they decompose the payoff from a risky instrument into a certain and a risky payoff, as summarised in the following equation:

$$\begin{aligned}
B(t, T) &= P(t, T) \cdot \{[1 - q(t, T)] + q(t, T) \cdot \phi\} \\
&= P(t, T) - P(t, T) \cdot [(1 - \phi)q(t, T)]
\end{aligned}
\tag{13}$$

Here, $q(t, T)$ is the default probability at time t with a final maturity of T and ϕ stands for the recovery rate. Kao (2000) summarises that the default process can either be modelled via a square root diffusion process or a jump diffusion process and the jump intensity can be characterised as constant or time varying, i.e. dependent on state variables, like economic conditions or company parameters. The recovery rate varies between a fraction of the debt's face value and the market value of a default free security at termination.

3.2.2. Rating Transition Approach

This approach is exposed in Jarrow, Lando, and Turnbull (1997) and incorporates a refinement of Jarrow and Turnbull (1995). The authors resolve the problem that credit spreads vary without default itself necessarily happening by specifying the bankruptcy process as a finite state Markov process in the firm's credit ratings. They treat default as the consequence of credit migration rather than as a sudden occurrence, build a term structure of credit risk spreads and describe their evolution through time. This in turn also allows pricing of specific credit derivatives, whose payoffs depend directly on the rating or the occurrence of other credit events. The core equation of the model remains the same like in the default based approach, except that it includes the default probabilities for multiple rating categories, that are denoted by the subscript i :

$$B(t, T) = P(t, T) - P(t, T)[(1 - \phi) \cdot q_i(t, T)] \tag{14}$$

The article presents a solution for discrete time case for credit ratings migrations as well as for the continuous time case. In the continuous time case, default time distribution is modelled by a continuous time, time homogeneous Markov chain with a finite state space comprising states that describe the different credit rating classes.

3.2.3. Spread Approach

Duffie and Singleton (1997) advocated a multi factor econometric model of the interest rate swap yields, including the possibility of counterparty default. They develop a default and liquidity adjusted instantaneous short rate that is they find a risk adjusted short rate process that allows the development of a term structure model for swap markets,

decomposing spreads into two components, default and recovery. The risk-less short rate and the mean loss rate do not enter the pricing model directly, but implicitly through the default adjusted short rate. They conclude that credit and liquidity factors are important sources impacting the variation of swap spreads.

In a more recent article, Jeanblanc and Le Cam (2008) categorise well known variations of the reduced form models into two main branches, depending on whether the information of the default free assets - a sub filtration of the filtration containing the whole information of the financial market - was introduced or not. They name the two classes “intensity approach” and “hazard process approach”.

Reduced form models are often considered mathematically more traceable by practitioners, but on the other hand, as Arora et al. (2005) argue, such models suffer from other weaknesses including the lack of clear economic rationale for defining the nature of the default process and that since “this type of model reflects a framework not directly rooted in an explanation of why a firm defaults diagnosing how to improve performance of these models can be challenging”. Kao (2000) subsumes arguments brought forward by critics of reduced form models: The most critical drawback is the need for existence of market prices for traded debt instruments. Hence the model is difficult to apply to private debt and commercial or industrial loans. Most of the models exclusively use aggregate market or sector data about default rates, the default term curve, the rating transition matrix, and the recovery rate implying that company specific risk can not be evaluated directly and financial fundamentals are ignored.

4. Credit Value at Risk Models

In addition to the proposed academic frameworks, currently four major industry-sponsored, sophisticated Credit value at risk methodologies are prevalent. Crouhy et al. (2000) provide a thorough review of these models:

As first they mention the CreditMetrics methodology as developed by JP Morgan. This credit migration approach heavily builds on credit rating migration matrices.

KMV (now Moody’s KMV) established another industry sponsored model that extends the structural approach, as introduced by Merton (1974). The Vasicek-Kealhofer model

(Vasicek, 1984) distinguishes short and long term liabilities and also models the default process as endogenous by linking it to the financial structure of the firm. In this approach, default is modelled to incur in case the obligor's asset values drops below a critical limit, which is in the original Vasicek-Kealhofer model the sum of short term liabilities and only half the amount of long term liabilities. Moody's KMV combines the model with an empirical distribution of distance-to-default in order to estimate the Expected Default Frequency credit measure.

The third type of models is known as the actuarial approach as proposed by Credit Suisse Financial Products (CSFB, 1997) and marketed as PortfolioRisk+ (formerly CreditRisk+). Credit default events for individual bonds or loans are modelled as an event that is modelled via a Poisson process. This model does not aim to explain causes of default and only focuses on the default case, omitting the risk arising from changes in the obligor's credit rating.

The fourth reviewed model is called CreditPortfolioView and was developed by McKinsey. According to Crouhy et al.(2000), it uses a discrete time, multi period setting, with default probabilities being modelled conditional on macro economic variables, such as unemployment, government expenses, the level of interest rates, and the growth rate of the economy (see Wilson, 1997a, 1997b and 1998).

As the model presented in this thesis will mainly make use of the CreditMetrics methodology, as proposed by JP Morgan, section 5 will provide further details of this framework. Section 7 will then cover the adaptations to the CreditMetrics framework that were implemented in order to introduce the effect of the business cycle into the model.

Credit Risk Models Building Blocks

According to Kao (2000) the recent interest in credit risk pricing has resulted in many model variations, but all the credit risk pricing models have three basic building blocks in common: the interest rate process, the default or rating transition process, and the asset recovery process. Each of these processes can be presented in different manners. It may involve continuous or discrete time, be deterministic or stochastic, or incorporate diffusion or jump diffusion. Furthermore, the implementation of the elements of a proc-

ess can vary: The implementation method of choice can be based on a lattice, for instance a binomial one, or it can make use of finite differencing or simulations.

4.1. Interest Rate Process

The way interest rates develop over time is described by the term structure of interest rates. Numerous stochastic models, with different variations and empirical studies deal with interest rate processes describing the evolution of interest rates. The most popular models include those of Vasicek (1977) and Cox, Ingersoll and Ross (1985). Although CreditMetrics and the framework implemented in this diploma thesis apply deterministic interest rates, for the sake of illustration, the square root diffusion framework developed by Cox-Ingersoll-Ross is described, in order to present a stochastic interest rate process.

The Cox-Ingersoll-Ross one factor interest rate process describes the future evolution of the instantaneous short rate via the following stochastic differential equation:

$$dr_t = k(\mu - r_t)dt + \sigma\sqrt{r_t} \cdot dW_t, \quad (15)$$

where W_t is a Wiener process representing the random market risk factor that depends on the volatility σ and the current interest rate level r_t . The drift factor $k(\mu - r_t)$ assures mean reversion of r_t towards the long run average interest rate μ . The mean reverting factor k governs the speed at which the interest rate reverts back to its mean. The standard deviation factor $\sigma\sqrt{r_t}$ constitutes the extension to the Vasicek model and guarantees that the modelled interest rate cannot reach negative values. Thereby at low interest rate levels, the effect of random shocks is diminished, empowering the drift factor to push the rate again upwards towards its long term average value.

Das (2002) appends a jump component to the interest rate process so that potential market shocks or information surprises, such as unexpected central banks interventions can be taken into account. The basis process is described by

$$dr_t = k(\mu - r_t)dt + \nu dz + Jd\pi(h), \quad (16)$$

where J is the jump size that can either be constant or derived from a probability distribution. The dynamics of interest rate evolves with two independent random terms, the diffusion dz and a Poisson process $\pi(h)$ governing the arrival of jumps, where h embodies the number of jumps per year. The jump diffusion process has proven to be an efficient statistical description for empirical characteristics of short interest rate dynamics.

4.2. Default Process

Just like the remaining credit model elements, the default process as well can be deterministic, or follow diffusion or jump diffusion process. As mentioned in section 3, some models attempt to describe the economic mechanism behind the default process, whereas others apply intensity based approaches, resulting in unpredictable default. In structural models, the definition of the default trigger point can vary. No matter how the default process is modelled, it should provide output that is consistent with empirical results.

While some models regard default to be the only credit event driving an instrument's price of credit, other approaches also consider the effects of rating transitions or distress. Then the trigger point specifies how and when a firm incurs default or experiences any other credit event. The trigger point may be described by the ratio between the value of debt and the firm value. The evolution of the firm value in such settings has already been discussed in a more detailed manner in chapter 3.1. Alternatively, the trigger point may be explained by an endogenously or exogenously specified boundary or by the hitting time of a jump process with an intensity measure. The default trigger point needs to be linked to the timing of default with respect to maturity. Default may only happen at or even prior to debt maturity.

For the purpose of constructing or testing the quality of a default process, one has first to define what a credit risk event is.

4.2.1. Definition of Default

Standard & Poors defines default as follows: (cf. Vazza et al., S&P, 2008): “A default is recorded on the first occurrence of a payment default on any financial obligation, rated or unrated, other than a financial obligation subject to a bona fide commercial dispute; an exception occurs when an interest payment missed on the due date is made within the grace period. Preferred stock is not considered a financial obligation; thus, a missed preferred stock dividend is not normally equated with default. Distressed exchanges, on the other hand, are considered defaults whenever the debt holders are coerced into accepting substitute instruments with lower coupons, longer maturities, or any other diminished financial terms.”

4.2.2. Default Probabilities

Intuitively, the simplest way to represent default probabilities is the one period default probability. Schönbucher (2004b) describes a one step default tree: starting at time T_0 , two different outcomes can occur at Time T_1 : Either default is reached with default probability p_1 or survival with survival probability $(1-p_1)$. To expand this tree to several points in time one has to specify marginal default probabilities for each period. These local probabilities are conditional probabilities, i.e. conditional on survival until the previous point in time, otherwise they are meaningless. This allows expressing cumulative default probabilities between any two points in time and thereby building a term structure of default probabilities. It quickly becomes more convenient to consider cumulative survival probabilities, as for survival, only one path has to be traced across the tree. Reducing the step size of the periods, one can specify a default hazard rate function $\lambda(t)$ for all $t \geq 0$, so that the survival probability can be calculated as

$$P(0, T) = \exp \left\{ - \int_0^T \lambda(t) dt \right\}. \quad (17)$$

Conversely, given a term structure of survival probabilities, that is a set of $P(0, T)$ for different time horizons, one can infer the corresponding default intensity function:

$$\lambda(T) = - \frac{\partial}{\partial T} \ln P(0, T) \quad (18)$$

$\lambda(T)$ is the default probability per unit of time, evaluated at time T . It is often referred to as default rate or default hazard rate at time T , as it is the instantaneous probability of default in a continuous time setting.

4.3. Recovery Process

Just alike the other elements, the recovery rate process can be modelled as constant, deterministic, dependent on external variables or stochastic. Within the CreditMetrics framework, the recovery rate is modelled via a beta distributed stochastic variable, as will be shown in chapter 5.1.2. The recovery rate of an obligor at default time usually takes values from zero to one. The opposite, the Loss Given Default is given by one minus the recovery rate. Usually the recovery rate is greater than zero, since some values can typically be recovered during bankruptcy proceedings.

From the three elements of a credit risk pricing model, recovery in the event of default involves a high level of complexity, since recovery rates are closely linked with legal bankruptcy procedures that involve various uncertainties. Bankruptcy procedures usually ask creditors to register their legal claim amounts owed by the insolvent party with the bankruptcy court. Schönbucher (2004a) points out that the creditor's claim amount need not equal the market value, for instance for bonds and loans it covers the nominal amount plus interest payments that are currently due, but not the value of future coupon payments. On the other hand for OTC derivatives according to the ISDA standard definitions, the current replacement value of the contract is honoured, assuming a counterparty holding the same rating like the defaulted counterparty had before the default. Creditor's claims are then grouped into priority classes and it is assured to treat all claim holders in the very same class equal. The outcome of a bankruptcy process may either be liquidation or reorganisation with the goal of keeping the defaulter a going concern.

Schönbucher (2004a) distinguishes two different recovery definitions: In case of market value recovery, "the recovery rate is the market value per unit of legal claim amount of defaulted debt, some short time (e.g. 1 or 3 months) after default". Using post default bid prices is as well common practice in the credit default swaps market, "since prices observed shortly after default are generally accepted as the market's estimate of discounted expected ultimate recovery rates" (cf. Emery et al, 2008). In other words, these prices reflect the market's expectations of ultimate recovery rates. For instance Moody's

KMV publishes recovery rates based on observed market prices 30 days after default. This way also satisfies investors, who prefer to liquidate their positions in defaulted bonds shortly after default. The other approach is the settlement value recovery, which defines that “the recovery rate is the value of the default settlement per unit of legal claim, discounted back to the date of default and after subtracting legal and administrative costs” (cf. Schönbucher 2004a). This way is sometimes referred to as ultimate or discounted recovery approach, as the recovered values are discounted back to the date of default. This view should include cash settlement, equity or preferred stocks as well as restructured debt of the distressed obligor that is assigned to the creditor after the default process has been settled. It is more appropriate for small and medium sized enterprises. Ultimate recovery rates are considerably higher than those based on post default prices. Hence it can be worthwhile for a bank to hold the claims until the insolvency proceedings are settled. The choice, of which recovery rates to use, depends on the investor’s policy concerning defaulted exposures.

	Post Default Trading Prices Recovery Rates			Ultimate Debt Recovery Rates		
Lien Position	2007	2006	1982-2007	2007	2006	1982-2007
Bank Loans	-	-	-	90.82	94.42	82.04
Senior Secured	67.74	76.02	70.47	-	-	-
Senior Unsecured	-	-	54.02	-	-	-
Bonds						
Senior Secured	80.54	74.63	51.89	62.37	91.98	65.77
Senior Unsecured	51.02	55.02	36.69	100.00	53.75	38.95
Senior Subordinated	54.47	41.41	32.42	69.75	52.79	29.11
Subordinated	-	56.11	31.19	0.00	40.12	26.51
Junior Subordinated	-	-	23.95	-	100.00	16.25

Table 2 Historical Average Recovery Rates (in %) source: Moody’s KMV (Emery et al. 2008)

As expected, recovery rates deteriorate with declining seniority of the debt. As illustrated by the one year averages, recovery rates possess a wide level of uncertainty. Emery et al. (2008) also comment on the effects of outliers on the averages, for instance in 2007 they reported “one defaulter that had an extremely low recovery rate of 0.3% on its senior unsecured bonds”, which influenced the 2007 average. On the other side, the

unfiltered observation of post default prices can also lead to extremely high recovery rates, even above 100.

Empirical research noticed and tested the connection of recovery rates with further different cause variables. Manning (2004) notes that, “recovery outcomes vary significantly across industries. Those industries in which assets tend to be more liquid and more tangible enjoy higher liquidation values”. This reflects the fact that assets need to be liquidated post default and thereby depend largely on the market liquidity for such assets. Renault and Scaillet (2004) report the mean and standard deviation of recovery rates by industry, as listed in Table 3. They highlight that, “Sectors with a lot of real assets such as the utility sector exhibit the highest mean recovery rate while Telecom bond investors tend to suffer the highest losses in default”.

Industry	Number of Observations	Mean Recovery (%)	Standard Deviation (%)
Utility	55	68.37	20.82
Insurance	33	39.79	26.70
Telecom	14	24.73	7.53
Transport	39	37.52	27.12
Financial	24	29.70	24.63
Chemicals	22	36.51	26.33
High Tech	14	50.46	22.15
Automotive	79	42.98	21.36
Building	43	39.69	29.46
Consumer	155	36.80	21.21
Leisure	86	42.91	27.06
Energy	59	45.56	25.61

Table 3 Historical Average Recovery Rates (in %) per industry, source: Renault and Scaillet (2004)

Friedman and Sandow (2003) focus on the ultimate recovery rate and estimate a conditional probability distribution of the discounted recovery rate using a maximum utility approach. They test as explanatory variables the quality of collateral, debt below class, debt above class and overall default rates and find that the “better the collateral, the debt superiority or the economic environment, the more likely is a high recovery.” They corroborate the findings of previous studies, such as Bos et al. (2002) by acknowledging

that “the effect of collateral, debt below class and debt above class on recoveries is very strong” and observing that recoveries are highly uncertain.

Moreover Moody’s KMV publishes average historical senior unsecured bond recovery rates that vary depending on the issuer’s rating prior to default and how much time the issuer spent in that rating prior to default (see Emery et al., 2008, p. 21).

5. CreditMetrics Methodology

First, one has to state that CreditMetrics is a trademark of JP Morgan. The CreditMetrics technical document (Gupton et al., 1997) provides a detailed exposition of the methodology and is illustrated with numerous examples.

To quote the CreditMetrics technical document (1997), “CreditMetrics is a tool for assessing portfolio risk due to changes in debt value caused by changes in obligor credit quality.” A distinction between market and credit risk must be drawn. “CreditMetrics estimates portfolio risk due to credit events. To state the matter differently, it measures the uncertainty in the forward value of the portfolio at the risk horizon caused by the possibility of obligor credit quality changes – both up and down grades, and default.” The CreditMetrics framework does not value market risk, since forward values and exposures are simply derived from deterministic forward curves. The only uncertainty in CreditMetrics refers to credit migration, that is the process of moving up or down in the credit spectrum. In other words, credit risk is analysed independently of market risk. The CreditMetrics paradigm is based on the estimation of the forward distribution of the changes in value in a portfolio of loan and bond type products at a given horizon, usually one year.

A whole view on the CreditMetrics risk measurement framework is provided by Figure 6. It splits the framework’s basic ideas into four building blocks. The building blocks are named “value at risk due to credit” for a single financial instrument, then “portfolio value at risk due to credit” which accounts for portfolio diversification effects, and the two supporting functions “correlations”, which derives the asset return correlations that are used to generate the joint transition probabilities and “exposures” which produces the future exposures of derivative securities. These building blocks will be briefly described in the following chapters.

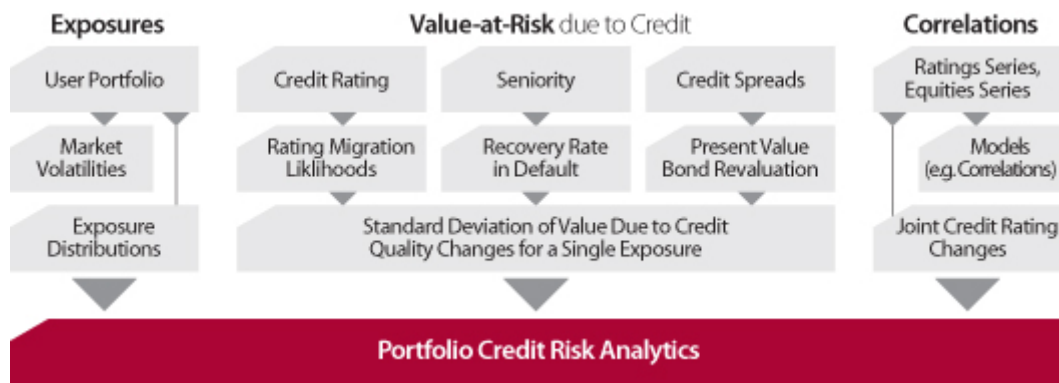


Figure 6 Schema of the CreditMetrics framework (Source: RiskMetrics, 2008)

5.1. Single Instrument Credit Value at Risk

This chapter illustrates the building block applied by CreditMetrics for calculating the credit value at risk for a single exposure. Basically, as described in more detail in the Credit Metrics technical document (Gupton et al., 1997) this encompasses the following four components:

5.1.1. Credit Rating System

The first step includes the specification of a rating system with rating categories, together with the probabilities of migrating from one credit quality to another over the credit risk horizon. Schönbucher (2004b) distinguishes three different credit risk assessment methods: agency ratings, internal ratings and market implied default probabilities.

The goal of every credit rating system is the accurate assessment of obligor credit risk by appraising the probability that an obligor defaults. It is the core business of rating agencies to analyse the credit quality of issuers of debt instruments. The use of credit rating systems is heavily encouraged by the Basel II Accord by claiming that “all IRB banks must produce their own estimates of PD and must adhere to the overall requirements for rating system design”. (BCBS, 2004a) Further rating system design requirements are provided, e.g. “A qualifying IRB rating system must have two separate and distinct dimensions: (i) the risk of borrower default, and (ii) transaction-specific factors”.

The rating transition matrix is the key component of the model proposed by JP Morgan and is described in more detail in chapter 6. It is worthwhile to mention that an arbitrary

rating system can be utilized, be it Moody's or Standard & Poor's or any proprietary rating system internal to a bank. Regardless of how the rating categories are constructed and how many categories there are, since the goal is to quantify credit risk, it is necessary to specify transition probabilities for each category. Summarized in brief, one can state that the transition matrix defines the probabilities of the states of the world that may exist for each credit in one period from now.

The obligor's credit rating implies both, its probability of default and its likelihood for possible credit quality migrations. An assumption made by CreditMetrics is that all issuers are credit homogeneous within the same rating class, resulting from the same transition and default probabilities within the same rating category. The reliance on rating transitions probabilities is often challenged to be a major weak point of the model. Credit quality changes are assumed to be identical with credit rating changes, implying that the default rate only changes, if the rating is adjusted. Generally, default rates evolve continuously, whereas ratings are revised in a discrete manner. This time lag exists naturally, because rating agencies simply need time to upgrade or downgrade companies, based on periodical revisions of credit risk.

A rating system needs not necessarily output a categorisation of obligors into a set of ordinal ranking classes. Other rating approaches, such as these proposed by KMV or Kamakura characterise each issuer by its own default probability. For instance an obligor's expected default frequency as published by Moody's KMV, is a function of the current asset value, asset return distribution (especially asset volatility) and capital structure (leverage) of the rated firm. As the EDF can vary on a continuous scale, it offers a cardinal ranking of obligors. As these ratings usually represent point in time snapshots, they do not try to smooth business cycle effects.

In practice, market implied default probabilities are derived out of observed benchmark security prices. Concerning the information that can be found in quoted prices, Huang et al. (2003) conclude that, "for investment grade bonds of all maturities, credit risk accounts for only a small fraction - typically around 20%, and, for Baa-rated bonds, in the 30% range - of the observed corporate-Treasury yield spreads, and it accounts for a smaller fraction of the observed spreads for bonds of shorter maturities. For junk bonds,

however, credit risk accounts for a much larger fraction of the observed corporate-Treasury yield spreads.”

Implied probabilities should be used for pricing credit sensible securities, because they include the risk premia that is expected to be paid for the associated credit risk of the calibration securities. In contrast for risk management purposes, capital allocation, and value at risk calculations historical probabilities are appropriate, because risk aversion can be added later on (see Schönbucher, 2004b).

5.1.2. Default and Recovery Rate

This part of the framework comprises the valuation of the instrument in the state of default. If the credit quality migration moves into default, that is the issuer defaults at the end of the risk horizon, the methodology assumes that not the total investment is lost. A total loss of a credit exposure will only be incurred in the least, most improbable cases. The likely remaining net value of recoveries is modelled to depend on the seniority class of the debt. Hence it is the seniority of the bond that influences its value in the case of default. The value of the instrument is set at a percentage of the face value, namely the recovery rate, which is modelled as a stochastic variable.

As described in section 4.3 practical challenges exist, when one tries to grasp recovery rates. Often, no objective values can be observed and even if market prices are available, they may stem from highly illiquid distressed debt markets. Further uncertainties result in rather high volatilities of empirically observed recovery rates.

Numerous literature reporting historical recovery rates in the state of default are available. The results of three representative studies, which also include estimates of standard deviations, namely these of Altman and Kishore (1996), Carty and Lieberman (1996) and Renault and Scaillet (2004) are depicted in Table 4. These studies by and large estimated similar mean and standard errors for recovery rates.

Seniority	Altman and Kishore (1996) Carty and Lieberman (1996) Renault and Scaillet(2004)								
	Number of observations			Mean Recovery (%)			Standard Deviation (%)		
Senior Secured	85	115	82	57.89	53.80	56.31	22.99	26.86	23.61
Senior Unsecured	221	278	225	47.65	51.13	46.74	26.71	25.45	25.57
Senior Subordinated	177	196	-	34.38	38.52	-	25.08	23.81	-
Subordinated	214	226	174	31.34	32.74	35.35	22.42	20.18	24.64
Junior Subordinated	-	9	142	-	17.09	35.03	-	10.90	22.09

Table 4 Recovery statistics per seniority (% of face value)

Table 4 exhibits differing recovery rates for junior subordinate classes, whereas the senior secured and senior unsecured debt show nearly identical values. These studies restricted themselves to recoveries of default cases in the United States. Since recovery rates depend on bankruptcy law and procedural details, these historically estimated recovery rates should not be directly used for European cases. Although the study of recovery rates by Renault and Scaillet is based on the largest sample of defaults, Carty and Lieberman (1996) provide estimates for all seniority classes, so their figures are used in the implementation of the framework.

All these studies point to one common characteristic of historical recovery rates, which is the generally wide uncertainty, due to their rather huge standard deviations. The CreditMetrics framework incorporates the wide uncertainty and the general shape of the recovery rate distribution by applying the beta distribution, which is fully parameterised by its mean and standard deviation. The beta probability density function

$$f(x|\alpha, \beta) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1} (1-x)^{\beta-1} I_{(0,1)}(x) \quad (19)$$

requires two parameters $\alpha > 0$ and $\beta > 0$, where $I_{(0,1)}(x)$ stands for the indicator function and $B(\alpha, \beta)$ stands for the Beta function that is defined as follows:

$$B(\alpha, \beta) = \int_0^1 x^{\alpha-1} (1-x)^{\beta-1} dx \quad (20)$$

It is worthwhile to mention that the beta function can be expressed by the gamma function:

$$B(\alpha, \beta) = \frac{\Gamma(\alpha) \cdot \Gamma(\beta)}{\Gamma(\alpha + \beta)} \quad (21)$$

The mean and standard deviation of the beta distribution are defined as follows:

$$\mu = \frac{\alpha}{\alpha + \beta} \quad (22)$$

$$\sigma = \frac{\alpha \cdot \beta}{(\alpha + \beta + 1)(\alpha + \beta)^2} \quad (23)$$

As Table 4 illustrates mean and standard deviation of recovery rates, one has to fit the beta distribution parameters α and β to these values of μ and σ by using

$$\alpha = \frac{\mu^2 - \mu^3 - \mu\sigma^2}{\sigma^2}, \quad (24)$$

and

$$\beta = \frac{\alpha}{\mu} - \alpha. \quad (25)$$

Figure 7 displays the full representation of the beta distribution with the parameters estimated by Carty and Lieberman (1996) that were stated in Table 4, whereas Figure 8 depicts the distributions obtained by the use of parameters estimated by Altman and Kishore (1996) and finally Figure 9 is based on data by Renault and Scaillet (2004).

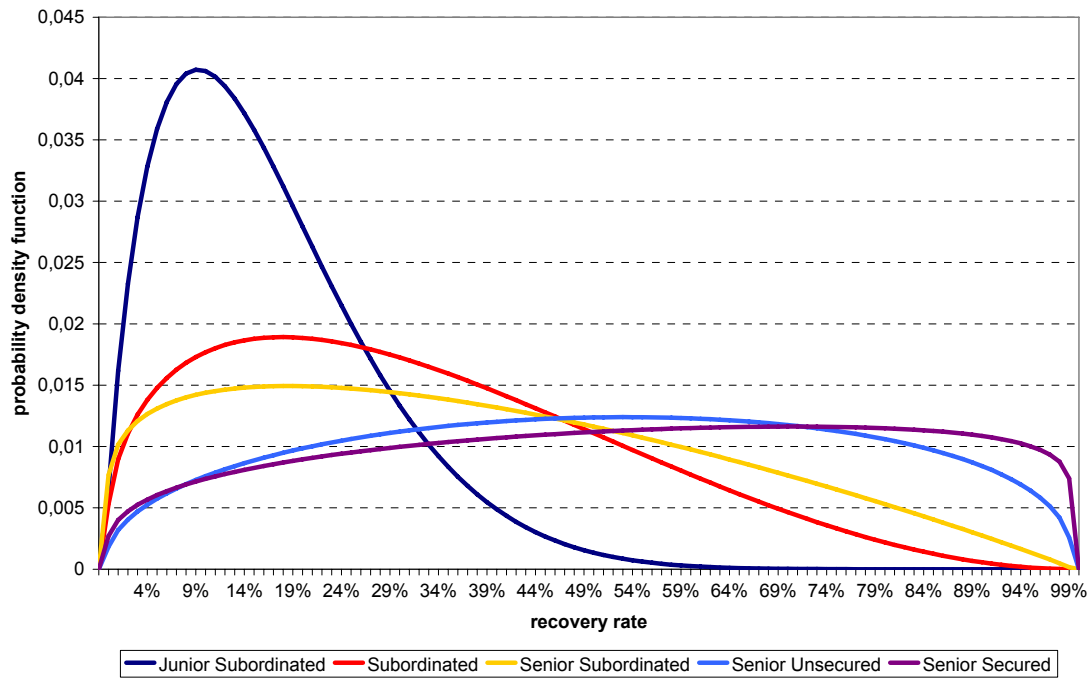


Figure 7 Beta distributions per seniority class (parameters estimated by Carty and Lieberman, 1996)

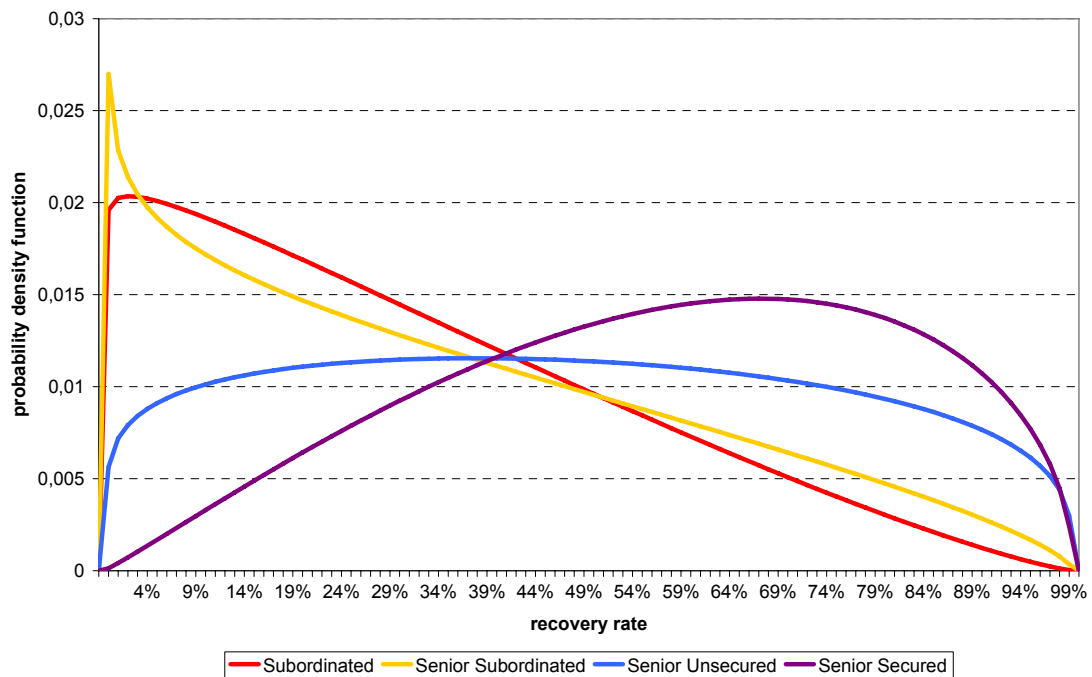


Figure 8 Beta distributions per seniority class (parameters estimated by Altman and Kishore 1996)

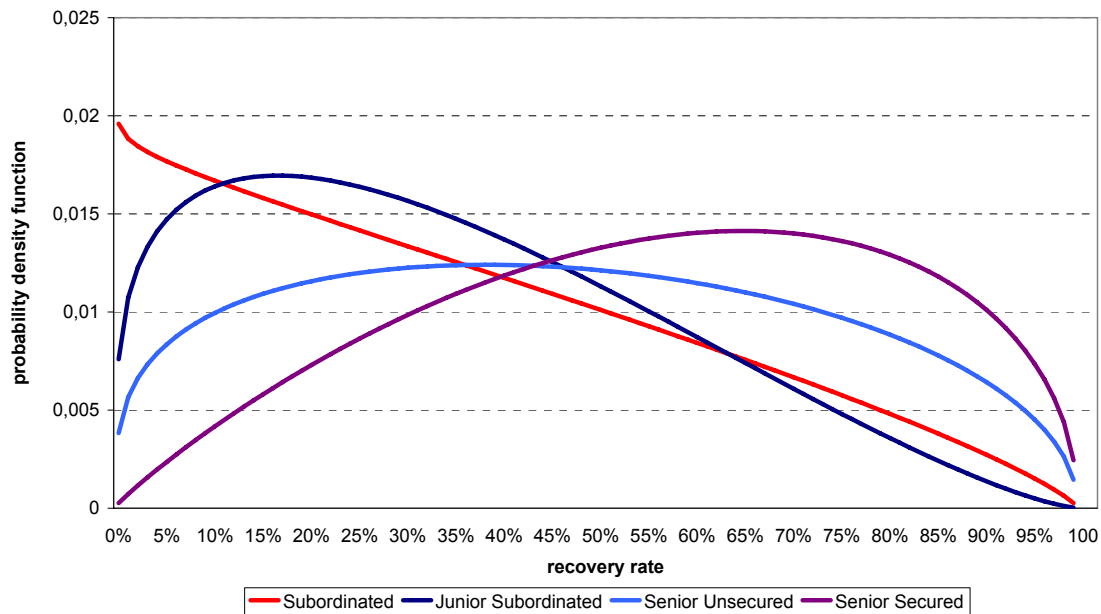


Figure 9 Beta distributions per seniority class (parameters estimated by Renault and Scaillet, 2004)

As loss rates are generally bounded between 0 and 100 percent of the amount exposed, the beta distribution comes in handy as it only returns values between 0 and 1 and is hence frequently used in connection with random percentages. Furthermore, as depicted above the beta distributions prove to be flexible as to their kurtosis and skewness.

5.1.3. Present Value Revaluation

This step includes the valuation of the instrument for each migration state, except default. A straightforward present value bond revaluation is calculated for each rating category and each bond taking into account the remaining future cash flows that result from holding the bond at the end of the risk horizon. As described in this building block, the CreditMetrics methodology does explicitly not account for market risk, as forward values are calculated from deterministic forward interest rate curves. In other words, the only uncertainty stems from credit rating migration and defaults.

For the purposes of the bond's revaluation, it is necessary to provide a credit spread corresponding to each rating category so that the forward zero curve for each credit rating category can be obtained, because different credit spreads for each rating category determine the new value of the bond upon up and downgrade. One may observe different main credit risk determinants for investment grade instruments and sub investment

grade exposures. For sub investment grade positions the largest part of credit risk stems from default events, whereas credit spread changes due to rating changes also impact credit risk of investment grade exposures.

All bonds of obligors holding the same rating are marked to market with the same credit rating specific spread curve. Depending on the rating system, if n possible credit qualities exist, n spread curves are required to price the bond in all possible states. The spot value of a bond at $t=0$ is calculated using the spot zero curve, whereas the forward price of the bond at the end of the risk horizon is calculated using the forward zero curve. The forward zero curve is calculated as of the risk horizon and is applied to the remaining future cash flows from year one to the maturity of the bond.

For a bond with five years to maturity at time $t=0$, thus with four remaining future cash flows at the end of the risk horizon, this procedure results in the following simple equation, defining the forward value of a bond at the end of the risk horizon using forward zero rates per rating category:

$$V_{RC} = c + \frac{c}{1 + r_{RC}^{12}} + \frac{c}{(1 + r_{RC}^{13})^2} + \frac{c}{(1 + r_{RC}^{14})^3} + \frac{f + c}{(1 + r_{RC}^{15})^4} \quad (26)$$

V_{RC} is the value of the bond at the end of 1 year, for the bond migrating to rating class RC, where RC must not be the default rating class (for computation of the bond value in case of default refer to equation (55)); c stands for the annual coupon the bond pays; f stands for the face value of the bond that is repaid at maturity; r represents the forward rate of the corresponding rating category for the specified period, e.g. r_{AAA}^{13} represents the 1x3 forward zero rate for the rating category “AAA”.

It is worthwhile to mention that CreditMetrics calculates dirty prices, i.e. prices including the first coupon payment. Once this revaluation is performed for each possible rating category, one obtains differing values of each bond at the end of the risk horizon.

5.1.4. Credit Risk Measurement

The probabilities from step 5.1.1 and the instrument’s values from step 5.1.2 in case of default and 5.1.3 in remaining cases are combined in the calculation of the forward distribution of the changes in value due to credit rating migration. Assessment of credit

risk due to credit quality changes may be based on the standard deviation of the new instrument values at the end of the risk horizon. Since probabilities of all possible outcomes are known for the single exposure, it is possible to calculate standard deviation or value at risk measures at various quantile levels.

5.2. Portfolio Credit Value at Risk

So far, the presented CreditMetrics methodology illustrated the treatment of a single, stand alone exposure. The following building block is concerned with portfolio diversification effects. For the sake of simplicity, this chapter demonstrates how to obtain the distribution of values for a two bond portfolio. Just as it is the case for a single bond, one needs to specify the portfolio's possible values at the end of the risk horizon and the possibilities of achieving each of these values, in order to characterise the distribution of portfolio values.

One can calculate the distribution of two bonds for each on a stand alone basis as described in section 5.1. This provides n possible values for each bond and n possible probabilities for each bond, with n equalling the number of rating classes. Next, one can combine the possible values for the individual bonds to obtain the year end values for the portfolio as a whole. Since either of the bonds can have any of n values at the risk horizon as a result of rating migration, and one needs to consider all possible combinations of states, the portfolio of two bonds can take on n^2 different values.

First, the portfolio's values in each of the n^2 states at the risk horizon can be obtained by simply adding the individual bonds' values:

$$V_{RC1,RC2}^{12} = V_{RC1}^1 + V_{RC2}^2 \quad (27)$$

$V_{RC1,RC2}^{12}$ is the portfolio's value, in case the first bond moves to rating category RC1 and the second bond migrates to rating category RC2. V_{RC1}^1 stands for the stand alone value of the first bond, when moving to rating category RC1 and V_{RC2}^2 corresponds to the stand alone value of the second bond, when migrating to rating class RC2.

Second, the probability of reaching each of these n^2 values has to be assessed. These n^2 joint probabilities must fit each set of n likelihoods that were obtained for the bonds on

a stand alone basis, as the joint probabilities should sum up to the stand alone probabilities. If the rating migrations of both bonds are statistically independent of each other, this task can be simply performed. Under the assumption of independence, the joint probability is the product of the individual likelihoods:

$$P_{RC1,RC2}^{12} = P_{RC1}^1 \cdot P_{RC2}^2 \quad (28)$$

, where $P_{RC1,RC2}^{12}$ stands for the probability that the first bond of the portfolio migrates to rating class RC1 and the second bond of the portfolio migrates to rating class RC2. Hence this is the probability corresponding to the portfolio value of $V_{RC1,RC2}^{12} \cdot P_{RC1}^1$ represents the stand alone probability of the first bond to migrate to rating class RC1 on a stand alone basis, whereas P_{RC2}^2 is the probability of the second bond to migrate to rating class RC2 on a stand alone basis.

Regretfully, the assumption of independence is unsophisticated and will generally underestimate credit risk, as rating migrations actually correlate. Indeed, actual correlations between the changes in credit quality are different from zero, as correlations are noticeable for firms in the same industry sector, in the same region or firms that are closely linked in a supply chain. Obligors in unrelated business sectors may exhibit lower default correlations. Furthermore rating changes may be driven by common macroeconomic factors, as the number of defaults seems to vary over the years. Of course one can try to estimate joint rating change likelihoods or joint default probabilities directly from credit rating time series, but the scarce data on joint defaults makes this task a difficult one. Contra wise assuming a positive correlation of value one, may overstate risk, but represent an expedient stress testing scenario.

5.2.1. Asset Value Model

Solving the issue of credit migration correlations involves the employment of a correlation model of the firm's assets, from which one can derive joint default and migration probabilities. CreditMetrics has chosen the firm's asset value process to drive credit rating changes and default, as the value of an obligor's assets indicates its ability to pay its debt. This suggested framework is also going by the name of the option pricing approach to the valuation of corporate securities, which was initially developed by Merton (1974) and is presented in more detail in section 3.1. A further assumption of the Credit-

Metrics methodology is that equity prices act as a proxy for the asset value of the firm, because the latter is not directly observable. First the approach estimates the correlations between equity returns of different obligors, and then the model deduces the correlations between changes in credit quality directly from the joint distribution of percentage equity returns.

The CreditMetrics approach extends Merton's model to include changes in credit quality additional to default by assuming that there is a series of levels for the asset value returns that will determine the company's credit rating at the end of the risk horizon. This assumption includes the existence of asset levels, such that one can construct a mapping from asset value to credit rating. These asset levels are referred to as asset value thresholds. Figure 10 depicts the relationship of thresholds with rating transitions for a currently BB rated obligor in a rating system that comprises the rating classes default, CCC, B, BB, BBB, A, AA, and AAA. Due to scaling issues, the thresholds for classes A, AA, and AAA, namely $Z(A)$, and $Z(AA)$ are not visible in Figure 10. It is noteworthy to mention that since all values above $Z(AA)$ result in a transition to the best rating class AAA, there is no need for an asset return threshold $Z(AAA)$.

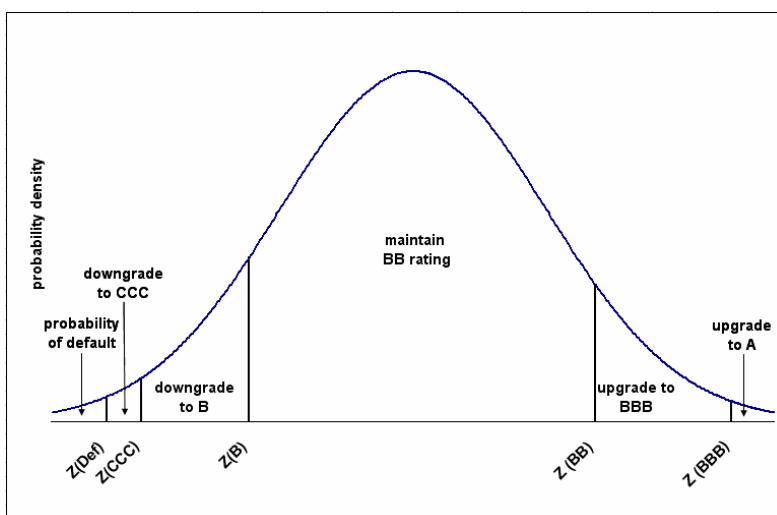


Figure 10 Distribution of asset returns with rating change thresholds

Assuming the asset return thresholds for a company are known, one only needs to model the company's change in asset value in order to describe its credit rating evolution. To perform this task, the model claims that percentage changes in asset value that are the asset returns, symbolised by R are standard normally distributed. It is worthwhile to lay emphasis on the fact that the normalised asset returns are standard normally

distributed with mean 0 and unit variance, whereas the asset values themselves are log normally distributed, due to the use of the Geometric Brownian Motion.

At first view, the transition matrix merely defines transition and default probabilities for each credit rating class, but due to the preceding description of asset thresholds, it can be argued that there is an asset return threshold $Z(Def)$, so that if the asset return $R < Z(Def)$, the obligor will default. If one pursues this idea it can be stated that if $Z(Def) < R < Z(CCC)$, then the obligor will migrate to CCC, and so on. Since Credit-Metrics assumes that R is standard normally distributed, the probabilities for default or credit rating transition respectively can be calculated as follows:

$$P\{Def\} = P\{R < Z(Def)\} = \Phi(Z(Def)), \quad (29)$$

$$P\{CCC\} = P\{Z(Def) < R < Z(CCC)\} = \Phi(Z(CCC)) - \Phi(Z(Def)), \quad (30)$$

where $\Phi(\bullet)$ denotes the cumulative standard normal distribution and $P\{Def\}$ is the default probability. Obviously, (24) allows solving for $Z(Def)$ and all other asset return thresholds in the following manner:

$$Z(Def) = \Phi^{-1}(P\{Def\}) \quad (31)$$

$$Z(CCC) = \Phi^{-1}(P\{Def\} + P\{CCC\}) \quad (32)$$

$$Z(RC_i) = \Phi^{-1}\left(\sum_{i=1}^{i-1} P\{RC_i\}\right) \quad (33)$$

Taking into account given transition probabilities (31) to (33) describe the calculation of asset return thresholds. Supposing that $i+1$ rating classes exist, the last equation above describes the calculation of the last asset return threshold. It states that in order to calculate the threshold of the last rating class the sum of the transition probabilities to lower rating classes needs to be determined. Then the threshold equals the inverse of the cumulative normal distribution at the sum's value. In this manner, a set of thresholds can be calculated for each rating class.

(29) equals the following statement that takes the critical asset value V_{Def} into account

$$P\{Def\} = P\{V_t \leq V_{Def}\} \quad (34)$$

and allows expressing the probability of default via asset values.

This can be transformed into a normalised threshold $Z(\text{Def})$ so that the area in the left tail below $Z(\text{Def})$ is $P\{\text{Def}\}$. Indeed, according to the Geometric Brownian Motion expressed in equation (1), default occurs when Z_t satisfies the following equation, which describes the probability of default according to Merton's model:

$$\begin{aligned}
P\{\text{Def}\} &= P\left\{\frac{\ln(V_{\text{Def}}/V_0) - (\mu - \sigma^2/2)t}{\sigma\sqrt{t}} \geq Z_t\right\} = \\
&= P\left\{Z_t \leq -\frac{\ln(V_0/V_{\text{Def}}) + (\mu - \sigma^2/2)t}{\sigma\sqrt{t}}\right\} = \\
&= P\{Z_t \leq -d_2\} = \Phi(-d_2),
\end{aligned} \tag{35}$$

In (35) the normalised return r is standard normally distributed:

$$r = \frac{\ln(V_t/V_0) - (\mu - \sigma^2/2)t}{\sigma\sqrt{t}} \tag{36}$$

Hence $Z(\text{Def})$ is the threshold point in the standard normal distribution corresponding to a cumulative probability of $P\{\text{Def}\}$. The critical asset value V_{Def} , which triggers default is such that $Z(\text{Def}) = -d_2$. d_2 is known as the “distance to default” and defined as follows:

$$d_2 = \frac{\ln\left(\frac{V_0}{V_{\text{Def}}}\right) + \left(\mu - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \tag{37}$$

One can state that CreditMetrics generalises Merton's framework by slicing the distribution of percentage asset returns into bands in such a way that if one draws randomly from this distribution, one can exactly reproduce the migration frequencies occurring in the applied transition matrix. The credit rating thresholds for a given rating class correspond to its transition probabilities from the transition matrix. In Figure 10 the left tail of the distribution on the left hand side of the $Z(\text{Def})$ threshold corresponds to the probability of default for the given rating class. Further, the area between $Z(\text{Def})$ and $Z(\text{CCC})$ matches the transition probability of a BB rated obligor to migrate to credit rating CCC and so on. In this notation, the area at the right hand side of the highest

threshold, namely $Z(AA)$ corresponds to the probability of migrating from the initial rating BB to the best credit rating AAA.

However, only the threshold levels are required to obtain migration probabilities, and the thresholds are calculated without the need to observe the asset value, its mean or variance. Only if the critical asset value V_{Def} needs to be calculated, one needs to estimate the expected asset return and asset return volatility. Gupton et al. (1997) illustrate this issue by providing the following example: consider two obligors that have the same rating, but the first obligor's asset volatility is ten times greater than the other's. The authors explicate that "credit risk is the same to either obligor. One obligor does have a more volatile asset process, but this just means that its asset return thresholds are greater than those of the other firm. In the end, the only parameters which affect the risk of the portfolio are the transition probabilities for each obligor and the correlations between asset returns." This allows the CreditMetrics methodology to consider standardised asset returns and the only parameter that remains necessary to estimate is the correlation between asset returns.

Yet, to this point, in case of one obligor CreditMetrics only needs the transition probabilities to describe the evolution of credit rating changes. The benefit of the asset value process is exclusively when considering multiple obligors.

To describe the evolution of two credit ratings jointly, CreditMetrics assumes that the two asset returns are bivariate normally distributed. The bivariate normal probability density function $N(\mu_x, \mu_y, \sigma_x^2, \sigma_y^2, \rho)$, $\mu_x \in R, \mu_y \in R, \sigma_x^2 > 0, \sigma_y^2 > 0, \rho \in (-1,1)$ reads as follows:

$$f(x, y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_x}{\sigma_x}\right)^2 - 2\rho\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right) + \left(\frac{y-\mu_y}{\sigma_y}\right)^2\right]\right\} \forall (x, y) \in R^2 \quad (38)$$

The only variable that needs to be specified is the correlation between the two asset returns ρ . Once ρ is known, the thresholds can be used to compute the probability for both obligors of being in any combination of ratings. Thereby the way credit ratings move together can be seen by calculating the joint distribution:

$$f(r_1, r_2, \rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)}[r_1^2 - 2\rho \cdot r_1 \cdot r_2 + r_2^2]\right\}, \quad (39)$$

where r_1 and r_2 are the standardised asset returns of firm 1 and firm 2 respectively. If the first firm's credit rating is supposed to be BB, and the second firm's to be A, one can for instance calculate the probability that both remain in the same rating class. This is the likelihood that the asset return for the first obligor falls between $Z_1(B)$ and $Z_1(BB)$, while at the same time the asset return for the second obligor falls between $Z_2(BBB)$ and $Z_2(A)$:

$$P_{BB,A}^{12} = P\{Z_1(B) < r_1 < Z_1(BB), Z_2(BBB) < r_2 < Z_2(A)\} = \int_{Z_1(B)}^{Z_1(BB)} \int_{Z_2(BBB)}^{Z_2(A)} f(r_1, r_2, \rho) dr_1 dr_2, \quad (40)$$

where $Z_1(\bullet)$ and $Z_2(\bullet)$ refer to the asset return thresholds for the credit rating of firm 1 and of firm 2, respectively. In a setting of a credit rating system with n rating classes (including default), this proceeding provides n^2 possible joint transition probabilities for two obligors. These n^2 probabilities are sufficient to calculate the credit value at risk for a portfolio containing solely bonds issued by these two obligors. The matrix, containing the joint migration probabilities for firm 1 and 2 is illustrated in Figure 11.

Rating firm 1	Rating of firm 2					
	AAA	AA	...	CCC	Default	Total
AAA	$P_{AAA,AAA}^{12}$	$P_{AAA,AA}^{12}$	...	$P_{AAA,CCC}^{12}$	$P_{AAA,Def}^{12}$	P_{AAA}^1
AA	$P_{AA,AAA}^{12}$	$P_{AA,AA}^{12}$	...	$P_{AA,CCC}^{12}$	$P_{AA,Def}^{12}$	P_{AA}^1
...	...	...	...	...	...	...
CCC	$P_{CCC,AAA}^{12}$	$P_{CCC,AA}^{12}$	...	$P_{CCC,CCC}^{12}$	$P_{CCC,Def}^{12}$	P_{CCC}^1
Default	$P_{Def,AAA}^{12}$	$P_{Def,AA}^{12}$	...	$P_{Def,CCC}^{12}$	$P_{Def,Def}^{12}$	P_{Def}^1
Total	P_{AAA}^2	P_{AA}^2	...	P_{CCC}^2	P_{Def}^2	100

Figure 11 Joint rating change probabilities for two obligors

As Figure 11 points out, the sums for each obligor are exactly that obligor's transition probabilities from the transition matrix. This is true, since the most right column and the last row describe the marginal totals, i.e. the marginal distribution for the joint rating change distribution. The joint transition probabilities depend only on the ratings of the

two obligors and on the correlation between them, but not on the particular obligors themselves.

5.2.2. Default Correlation

Asset return correlation and the joint default probability of two obligors are technically linked in the CreditMetrics framework. However default correlations are not used explicitly in the calculations. Gupton et al. (1997) explicate the worst case event for a portfolio containing two obligors that is that both obligors default. In case the asset returns are independent, as described in (28) the joint default probability is simply the product of the individual default probabilities. The CreditMetrics technical document continues describing the contrary case that is if the asset returns are perfectly correlated with $\rho = 1$, then every time the first obligor defaults, so also does the second obligor. To be more specific consider two obligors whose probabilities of default are P_{Def}^1 and P_{Def}^2 , respectively. For a given asset return correlation ρ , the joint probability of default for obligor one and two is denoted $P_{Def,Def}^{12}$. Then joint default correlation between two obligors ρ_{Def}^{12} is given by

$$\rho_{Def}^{12} = \frac{P_{Def,Def}^{12} - P_{Def}^1 \cdot P_{Def}^2}{\sqrt{P_{Def}^1 \cdot (1 - P_{Def}^1) \cdot P_{Def}^2 \cdot (1 - P_{Def}^2)}}. \quad (41)$$

Merton's model defines the joint probability of default expressed via asset values:

$$P_{Def,Def}^{12} = P\{V_1 \leq V_{Def1}, V_2 \leq V_{Def2}\}, \quad (42)$$

where V_1 and V_2 represent the asset values of obligor 1 and obligor 2, respectively at time t , and V_{Def1} and V_{Def2} refer to the corresponding critical values that trigger default. Furthermore the joint probability of default can be explained via both distances to default:

$$P_{Def,Def}^{12} = P\{r_1 \leq -d_2^1, r_2 \leq -d_2^2\} = N(-d_2^1, -d_2^2, \rho) \quad (43)$$

Where r_1 and r_2 stand for the normalised asset returns of obligors 1 and 2, respectively, as defined in (36). Further d_2^1 and d_2^2 denote the corresponding distance to default as defined in (37), $N(x, y, \rho)$ denotes the cumulative standard bivariate normal distribu-

tion where ρ is the correlation coefficient between x and y , which is in this case the asset return correlation.

The following figure illustrates the effect of asset return correlation on the joint default probability for given distances to default. The first obligor's distance to default equals $d_1^1 = -3.24$ and the second obligor's distance to default is $d_2^2 = -2.3$. This setting results in a probability of default of $\Phi(-3.24) = 0.06\%$ for the first obligor and of $\Phi(-2.3) = 1.07\%$ for the second obligor, where $\Phi(\bullet)$ denotes the cumulative distribution for the standard normal distribution.

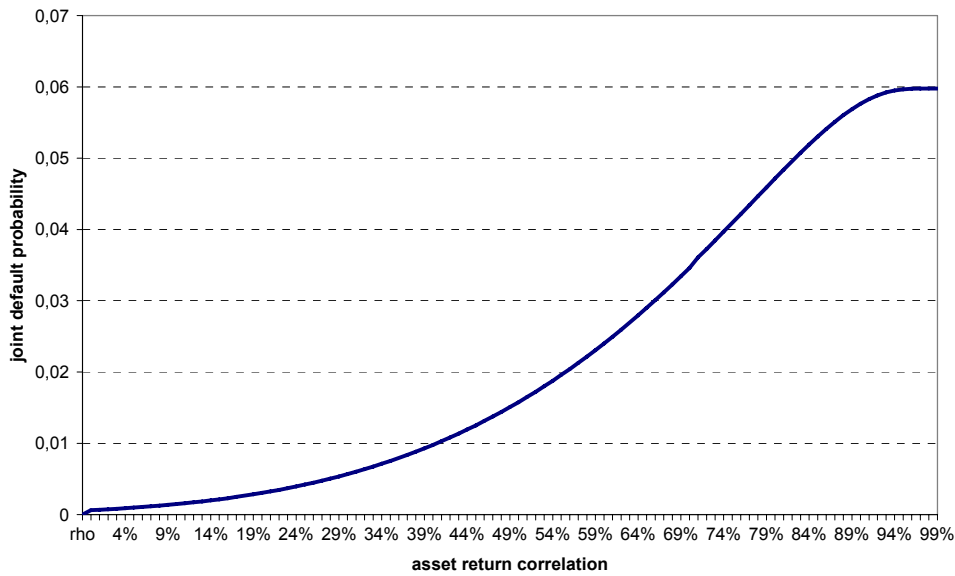


Figure 12 Probability of joint defaults as a function of asset return correlation

This is the graphical representation of (43) for the asset return correlation varying from 0 to 1. According to Crouhy et al. (2000), assuming asset return correlation to equal 20% implies a joint probability of default of 0.0054%. Further, this entails a default correlation of 1.89%. The ratio of asset return correlations to default correlations is approximately ten to one for asset return correlations ranging from 20% to 60%. According to Gupton et al. (1997), the translation from asset to default correlation lowers the correlation significantly, stating that asset return correlations in the range from 40% to 60% will typically translate into default correlations of 2% to 4%.

5.2.3. Asset Return Correlations

The joint probability of default is quite sensitive to pair wise asset return correlations, which leads to the necessity to estimate correctly these data in order to be able to assess precisely diversification effects within a portfolio. Hence the statistical procedure used to estimate asset return correlations, as described in detail by Crouhy et al (2000) will be summarised briefly in this section. The impact of correlations on the credit value at risk is stronger for low credit quality than for high grade portfolios. In fact, when the credit quality of the portfolio deteriorates the expected number of defaults increases, and this number is magnified by an increase in default correlations.

CreditMetrics obtains asset return correlations from a structural model that links correlation to fundamental factors. In order to avoid sampling errors inherent in simple historical correlations and to achieve a better accuracy in forecasting correlations, a structure is imposed on the return correlations. Furthermore a practical need to reduce dramatically the number of correlations to be calculated is observable. Assuming that a bank is dealing with n different counter parties leads to $n \cdot (n-1)/2$ different correlations that need to be estimated. As the number of required correlations grows with $O(n^2)$ this number quickly becomes unmanageable. Multi factor models of asset returns help to reduce the number of correlations to be calculated to those between the limited number of common factors affecting asset returns.

The multi factor model assumes that the firm's asset returns are driven by a set of common also known as systematic risk factors and idiosyncratic factors. The model's idiosyncratic factors are either sector-, country- or firm-specific. Since they are neither correlated with each other, nor with the common factors, they do not contribute to asset return correlations. Furthermore asset return correlations between two firms are solely explained by the factors that are common to all firms. By means of portfolio diversification merely the risks associated with the idiosyncratic risk factors can be diversified away, while risk contribution of common factors is non diversifiable.

According to the multi factor model the asset return generating process for all firms is as follows:

$$r_k = x_k + \beta_{1k} I_1 + \beta_{2k} I_2 + \varepsilon_k \text{ for } k = 1, \dots, n \quad (44)$$

Here n corresponds to the number of obligors, r_k stands for the asset return for firm k , x_k is the component of asset return independent of common factors, I_1, I_2 are the common factors, β_{1k}, β_{2k} are the expected changes in r_k , given a change in common factors I_1 and I_2 , respectively, and ε_k is the idiosyncratic risk factor with zero mean. Furthermore the idiosyncratic risk factor is assumed to be uncorrelated with all the common factors as well as with the idiosyncratic risk factors of the other obligors.

From this asset return generating process one can derive the variance and covariance of asset returns for the firms:

$$\text{var}(r_k) \equiv \sigma_k^2 = \beta_{1k}^2 \text{var}(I_1) + \beta_{2k}^2 \text{var}(I_2) + \text{var}(\varepsilon_k^2) + 2\beta_{1k}\beta_{2k} \text{cov}(I_1, I_2) \quad (45)$$

$$\text{cov}(r_i, r_j) \equiv \sigma_{ij} = \beta_{1i}\beta_{1j} \text{var}(I_1) + \beta_{2i}\beta_{2j} \text{var}(I_2) + (\beta_{1i}\beta_{2j} + \beta_{2i}\beta_{1j}) \text{cov}(I_1, I_2) + \text{cov}(\varepsilon_i, \varepsilon_j) \quad (46)$$

Hence, it follows that the asset return correlation between firms i and j is given by ρ_{ij} :

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} \quad (47)$$

The multifactor model described above allows the logical deduction of the asset return correlation between any number of firms. In order to derive these measures, one only needs to estimate all β_{ik} that are $2 \cdot n$ parameters, and the covariance matrix for the common factors that comprises 3 parameters. In general for m common factors the number of parameters to be estimated is:

$$m \cdot n + \frac{m(m-1)}{2} \quad (48)$$

Hence, in a multifactor model, the number of parameters that need to be estimated grows with $O(n)$, in contrast to the previous example that grows with $O(n^2)$. For illustration purposes assume a model that encompasses 1000 obligors. In the first case this would lead to 499500 different asset return correlations that need to be estimated, whereas the second two factor model only requires 2003 asset return correlations.

5.3. Exposures

The task of this CreditMetrics building block is to deliver the future exposures at risk at the end of the risk horizon. This element allows extending credit risk analysis to other types of instruments, such as credit derivatives or market-driven instruments, including swaps and forwards. For interest rate derivatives, one has to assess of exposures that depend on future interest rates and thus one has to model dynamics of future interest rates, that is one has to link credit risk and market risk factors. For instance, credit risk exposure stemming from an interest rate swap can be either positive or negative, depending on future interest rates. This implicitly involves a kind of optionality due to the fact that the bank faces a loss on the transaction in case of counter party default only if the deal has a positive market value for the bank, i.e. the swap counterparty owes money to the bank on a net present value basis.

For bonds and similar products, such as loans, receivables, letters of credit, and commitments to lend this step comprises the use of a forward pricing model as described in section 5.1.

However, since this CreditMetrics building block refers to the pricing of derivative products and the framework that was implemented for this diploma thesis does not deal with derivatives, but only with a portfolio consisting of bonds, this building block will not be described in depth in this thesis. For a further discussion of calculating forward risk free values of derivatives in the CreditMetrics framework, refer to section 4.5 of the CreditMetrics technical document (Gupton et al., 1997).

Concerning exposure at default estimation from a regulatory perspective, the Basel II Accord (BCBS, 2006a) specifies in paragraph 474 that “Banks estimates of EAD should reflect the possibility of additional drawings by the borrower up to and after the time a default event is triggered”. This specifically refers to instruments such as commitments to lend, whose takedown rates tend to increase prior an obligor’s possible default. Studies like Asarnow and Marker (1995) explore the fact that usage levels of facilities such as commitments to lend tend to increase prior default, interestingly to a larger degree for previously higher rated, than lower rated obligors. However, for a bond portfolio like in this thesis, the exposure at default is the market value of the bonds respectively the replacement value of the investments.

6. Credit Rating Migration Matrices

As shown in chapter 5, the CreditMetrics methodology does not only account for the case of default. Attention is not only focused on the probability of default, but as well on the likelihood of moving from one credit level or rating to another. One convenient way of expressing this information is through a transition matrix. Credit ratings published by agencies such as Moody's and Standard and Poor's play an increasingly significant role in financial markets, since the primary resource for these probabilities has been the rating agencies. The importance of agency ratings may procedure results in two differen even become greater, if they are applied as a principal element in calculating banks' regulatory capital as is suggested in proposals recently issued by the BCBS.

Agency ratings of credit quality play an important role in the CreditMetrics framework. Since mark-to-market values are not observable for non-marketed loans or illiquid bonds, measuring risk is difficult. A common approach is to link value changes to transitions in ratings. One may then measure risk by having a look at the joint distribution of rating transitions for the loans and bonds which make up the portfolio of interest. The matrix of different rating transition probabilities plays a crucial role in such calculations and shall hence be presented.

Each element in this matrix is the probability that an issuer of a given initial rating moves to some other rating or default over a given period of time. For the transition matrix contains no information about correlations in the ratings transitions of different issuers, knowledge about the ratings transition matrix applicable to a group of issuers is only the first step in credit risk modelling.

Unconditional transition matrices can be calculated by employing historical datasets. Unconditional refers to the fact, that during the calculation of these historical probabilities, the state of the surrounding business environment was omitted.

The basic assumption behind such an approach is that, for a given sample, the probability of a transition from rating i to rating j is a constant parameter $p_{i,j}$. This amounts exactly to saying that, for a given initial rating, transitions to different possible future ratings or default follow a constant parameter temporally independent multinomial proc-

ess. Estimations may then be executed by taking the fraction of occasions in the sample on which an obligor starts the year in state i and ends it in state j .

1 year	Global Average Transition Rates, 1981-2007 (%)								
To From	AAA	AA	A	BBB	BB	B	CCC/C	Default	NR
AAA	88.53	7.70	0.46	0.09	0.09	0.00	0.00	0.00	3.15
AA	0.60	87.50	7.33	0.54	0.06	0.10	0.02	0.01	3.84
A	0.04	2.07	87.21	5.36	0.39	0.16	0.03	0.06	4.67
BBB	0.01	0.17	3.96	84.13	4.03	0.72	0.16	0.23	6.61
BB	0.02	0.05	0.21	5.32	75.62	7.15	0.78	1.00	9.84
B	0.00	0.05	0.16	0.28	5.92	73.00	3.96	4.57	12.05
CCC/C	0.00	0.00	0.24	0.36	1.02	11.74	47.38	25.59	13.67

Table 5 Global average one year transition rates in %, including the “not rated” category (data from 1981 to 2007) source: Vazza et al. (S&P, 2008)

As an example the historical frequency of annual transitions based on observations by Standard & Poor’s from 1981 to 2007 is listed. It exhibits a basic unconditional rating 1 year transition matrix. Each entry represents a sample’s absolute realised frequency of transitions from the initial rating, which is given on the left hand side of the matrix, to a given terminal rating, which is given along the top of the matrix divided by the total number of issuers that began in the initial rating category in question. Similar transition matrices are also regularly published by other rating agencies, e.g. Moody’s (Emery et al., 2008).

6.1. Withdrawn Ratings

Different approaches exist on how to handle the observed rating transitions to the “N.R.” category, which comprises withdrawn credit ratings. It depends on how one intends to interpret the information content such a transition provides, as usually causes for withdrawn ratings are unknown. Further, subsequent probabilities of defaults or re-ratings of “not rated” issuers are generally not published; therefore it is not possible to assess what happens to “not rated” issuers.

As a conservative approach, in order to remain on the safe side, one may suggest that a transition to the “not rated” category implies a decline in the obligor’s credit quality. One would then adjust the downgrade and default probability for each cohort by its

probability of migrating to “not rated”, i.e. one would only increase in proportion to their values the probabilities for rating degradations and default in order to distribute the probability mass of the “not rated” category. Of course in such a conservative approach, one may overestimate credit risk, which may turn out costly in case of an internal ratings based approach. On the other hand, in practice, there must not be unrated obligors in a banks credit portfolio, as for instance under the Basel II Accord, banks are forced to rate their counterparties prior conclusion of a business transaction.

Alternatively one could distribute the “not rated” probability mass to all other transition probabilities weighted with their current transition probabilities. This represents a rather liberal interpretation of rating withdrawals.

Another common method that has according to Bangia et al. (2002) become industry standard simply interprets transitions to “not rated” as non-information, i.e. it distributes the “not rated” probability among all the remaining states, including default.

6.2. Monotonicity

As rating grades are changed sporadically, a typical characteristic of transition matrices is the high probability mass on the diagonal. As a consequence obligors are most likely to preserve their current rating, which illustrates the strong rating stability that rating agencies usually aim for. As extreme jumps over several rating categories can only be observed scarcely, as expected, the second largest likelihoods can usually be found close to the diagonal. One can assume that the further one moves away from the diagonal, the smaller is the probability of such a rating transition. This characteristic is commonly referred to as monotonicity, see for instance Bangia et al. (2002).

However, the transition matrix depicted in Table 5 exhibits some exceptions to this rule. For instance the default probabilities for A, BBB, BB, and B rated issuers are larger than the likelihood of moving to the CCC rating class. When inspecting medium quality rating categories and smaller time horizons, Bangia et al. (2002) note that such violations are weaker or even non existent. The authors put forward the idea that the violations are a result of rating activities within an interval and that in longer transition horizons such activities can not be observed. They further argue that one has to keep in mind the effect noisy data can have on the calculated frequencies. For instance the zero

entries in line one only mean that no AAA rated obligor was observed to move to B, CCC, C or default within one year according to the underlying historical observations.

When focusing on speculative grade rated obligors, it can be argued that “certain CCC rated firms are “do or die” type firms. Their very risky nature makes them highly default prone, but if successful they have significant chance of skipping a few categories on their way to higher ratings”. After all, rating agencies consider low rated obligors to be exceptionally volatile, and the risky nature of these obligors manifests itself in high probability occurrence of different states (see Lando, 1998).

As intuitively expected, the transition matrix exhibits higher default risk and higher migration volatility for lower quality ratings, hence ratings reflect indeed an ordering with respect to the default probability. Vazza (S&P, 2008) outlines this behaviour as follows: “Transition studies have repeatedly confirmed that higher ratings tend to be more stable and that speculative-grade debt generally experiences more rating volatility.” Specifically the matrix states that default likelihood increases exponentially with decreasing grade.

6.3. Transition Horizon

Conditional upon a given grade at time T , the transition matrix M is an explanation of the probabilities of being in any of the various grades at $T+1$. As a consequence it fully describes the probability distribution of grades at $T+1$, given the grade at T . In theory transition matrices can be estimated for any transition horizon that is considered necessary. For instance short term transition matrices will best represent the rating process, given a historical rating dataset that is based on a quarterly monitoring pattern. Bangia et al. (2002) clarify, that “the shorter the measurement interval, the fewer rating changes will be omitted. However shorter duration also results in less extreme movements, as large movements are often achieved via some intermediary steps”.

However, much of the credit agency and academic data is published on annual basis. This is a convention rather than a requirement. It is important to mention, that there is nothing about the methodology presented in this diploma thesis that requires a one year horizon.

As illustrated below, with increasing transition horizon the violation of monotonicity for the default rates becomes more conspicuous, since default is an absorbing state.

5 years	Global Average Transition Rates, 1981-2007 (%)								
To From	AAA	AA	A	BBB	BB	B	CCC/C	Default	NR
AAA	53.93	23.71	4.93	1.02	0.13	0.13	0.03	0.29	15.81
AA	1.73	51.60	23.56	4.18	0.63	0.37	0.04	0.31	17.57
A	0.11	5.83	53.09	15.41	2.52	0.99	0.18	0.70	21.16
BBB	0.05	0.81	10.70	46.72	8.56	3.02	0.48	2.75	29.90
BB	0.02	0.14	1.56	12.16	27.76	11.09	1.58	10.64	35.05
B	0.03	0.06	0.54	2.08	10.53	20.51	2.84	24.53	38.88
CCC/C	0.00	0.00	0.25	1.10	3.39	10.17	3.73	50.17	31.19

Table 6 Global average five year transition rates in %, including the “not rated” category (data from 1981 to 2007) source: Vazza et al. (S&P, 2008)

As an alternative to estimate transition matrices for different time horizons from empirically observed ratings migrations and defaults, literature offers another procedure to derive transition matrices with varying time horizons from a given matrix. In order to transform any given transition matrix with respect to a specific horizon into a transition matrix describing another horizon, two assumptions have to be fulfilled (see Schönbucher, 2004b):

6.4. Time Invariance

The probability of a transition from rating class i at time t to rating class j at time $T > t$ does not depend on the calendar dates t and T , but only on time via the length $T-t$ of the time interval. Generally, the assumption that future will be like the past is not uncommon in statistical analysis. Although common, the time invariance assumption is rather restrictive as it forbids time dependent phenomena like business cycle effects (see section 7.2).

6.5. Markov Property

The probability of a transition from rating class i to rating class j from time t to $T > t$, only depends on the rating class that the obligor belongs to at time t (class i) and the rating class that it is assumed to end up in at time T (class j) and no other external vari-

ables. The Markov property assumption also contradicts some empirical evidence, such as the rating momentum, as outlined in section 6.6.2. Furthermore, Nickell et al. (2000) analysed the effect of different variables on the transition matrix: for instance the industry and country of the obligor and the length of time that has elapsed since the issuance of the bond. The authors diagnose industry heterogeneity as for instance issuers belonging to the financial industry possess more volatile ratings than industrial issuers. When concentrating on the obligors' domiciles they state that Japanese rating transition probabilities were consistent with less volatile ratings than those of US and UK.

However, under the time invariance and Markov property assumptions, the n year transition matrix can be obtained given the one year transition matrix by multiplying n times with itself.

$$M_{(n)} = M_{(1)}^n \quad (49)$$

For technical reasons, to obtain a square matrix a default row has to be added and the "Not Rated" category column removed from Table 6. Different approaches concerning the handling of N.R. data are presented in Chapter 6.1. Assuming time invariance enables to apply the same transition matrix for all years, whereas the Markov property allows multiplying probabilities by declining correlation.

6.6. Risk Horizon

Crouhy et al. (2004) state that the choice of the risk horizon is somewhat arbitrary, but it is usually one year as it corresponds to the planning cycle and the average time it would require to recapitalise the bank, if it were to suffer a major unexpected loss.

Bangia et al. (2002) mention that the application purpose determines the matrix's transition horizon and that for the calculation of credit risk a one year transition horizon has become standard. Employing as a convention a one year risk horizon is not unlike the common convention of annualised interest rates.

A major goal of credit risk figures is to support management in choosing adequate risk mitigation actions, such as hedging positions via credit derivatives, introducing netting agreements, tightening credit limits and enhancing collateral management processes. Any specified risk horizon, *ceteris paribus*, will result in risk measures pointing to the

same direction and will support similar qualitative actions. Hence the choice of the risk horizon is not likely to make an appreciable difference. Although credit risk mitigation actions can be executed in daily or even shorter terms, assessing credit risk on a farther horizon is appropriate.

A credit portfolio model must be defined and parameterised consistently with the definition of economic capital used by the bank. This includes among other parameters the time horizon or holding period. For credit value at risk measurement, it is common practice to assume a one year horizon. Rosen (2004, p.317) lists several reasons in favour of a one year horizon:

- It accords with firm's accounting cycles.
- It is a reasonable period over which the firm will typically be able to renew any capital depleted through losses
- It coincides with a reasonable period over which actions can be taken to mitigate losses for various assets
- Credit reviews are usually performed annually, as the Basel II Accord requires banks to "review the loss characteristics and delinquency status of each identified risk pool on at least an annual basis" (paragraph 427, BCBS, 2004a)
- The borrower might be updating its financial information only on an annual basis.

When facing other applications, such as the pricing of credit derivatives, shorter horizons may be required. As many different security types bear credit risk, one of the arguments in favour of multiple credit risk horizons is that they allow the practitioner to calculate risk at horizons tailored to each credit security type. One can calculate credit risk related figures for each security type at its specifically tailored risk horizon. For instance a specific risk horizon can be applied for loans and a distinctive one for swaps, due to the different liquidities of these financial products. Nevertheless the risk estimates for these different sub portfolios cannot be aggregated if there is an incongruity in time horizons, as comparisons or aggregations of risk measures for different business opportunities must be made applying the same risk horizon.

6.6.1. Credit Rating Definitions

Because the model strongly relies on migration matrices and therefore on the issuer credit rating classifications, it is worthwhile to mention the credit rating definitions, as for instance established by Standard & Poor's. In their issuer credit rating definition Standard & Poor's (S&P, 2008) proclaims that a "Standard & Poor's issuer credit rating is a current opinion of an obligor's overall financial capacity (its creditworthiness) to pay its financial obligations. This opinion focuses on the obligor's capacity and willingness to meet its financial commitments as they come due."

The Standard & Poor's rating universe comprises ordinal ratings from AAA to CC. Obligors rated BBB or better are subsumed under the term "investment grade", whereas worse ratings are regarded as "speculative grade".

AAA	An obligor rated 'AAA' has extremely strong capacity to meet its financial commitments. 'AAA' is the highest issuer credit rating assigned by Standard & Poor's.
AA	An obligor rated 'AA' has very strong capacity to meet its financial commitments. It differs from the highest-rated obligors only to a small degree.
A	An obligor rated 'A' has strong capacity to meet its financial commitments but is somewhat more susceptible to the adverse effects of changes in circumstances and economic conditions than obligors in higher-rated categories.
BBB	An obligor rated 'BBB' has adequate capacity to meet its financial commitments. However, adverse economic conditions or changing circumstances are more likely to lead to a weakened capacity of the obligor to meet its financial commitments.
BB	An obligor rated 'BB' is less vulnerable in the near term than other lower-rated obligors. However, it faces major ongoing uncertainties and exposure to adverse business, financial, or economic conditions which could lead to the obligor's inadequate capacity to meet its financial commitments.
B	An obligor rated 'B' is more vulnerable than the obligors rated 'BB', but the obligor currently has the capacity to meet its financial commitments. Adverse business, financial, or economic conditions will likely impair the obligor's capacity or willingness to meet its financial commitments.
CCC	An obligor rated 'CCC' is currently vulnerable, and is dependent upon favourable business, financial, and economic conditions to meet its financial commitments.
CC	An obligor rated 'CC' is currently highly vulnerable.

Table 7 Standard & Poor's long term issuer credit rating definitions (S&P 2008)

Obligors rated 'BB', 'B', 'CCC', and 'CC' are regarded as having significant speculative characteristics. 'BB' indicates the least degree of speculation and 'CC' the highest. While such obligors will likely have some quality and protective characteristics, these may be outweighed by large uncertainties or major exposures to adverse conditions.

Public rating agencies update ratings regularly in order to reflect changes in the obligor's ability to meet its financial obligations. Clearly, when it comes to rating changes, rating agencies keep an eye on rating stability and rating accuracy balance. Ratings are intended to represent long term views, so the effect of business cycle variations may average out as rating agencies attempt to rate rather through the cycle than with the cycle.

6.6.2. Rating Momentum

The Markov property forbids the use of any information except the current rating, since all one needs to know to determine transition probabilities is the current rating class, implying that the history of the obligor's rating is irrelevant. This contradicts some empirical studies, such as by Altman and Kao (1992b), who were among the first to document a significant serial path dependence of credit ratings. Bangia et al. (2002) observe that “analysis reveals path dependence, which is a clear violation of first-order Markov behaviour. In a first-order Markov chain process next period's distribution is only dependent on the present state and not on any developments in the past. In other words, transitions have only a one-period memory.” They remark that “This momentum hypothesis is supported by the data as most downgrade probabilities for the down-momentum matrix are larger than the corresponding values in the unconditional matrix. The exact opposite is true for the up-momentum matrix, which exhibits smaller downgrade probabilities than the unconditional matrix”. Finally they diagnose that “most striking finding, however, is the extreme difference in average default rates. The down momentum average default rate is nearly five times as large as the unconditional one, whereas the up momentum average default rate is less than one fifth of the unconditional expectation. Thus the default probability is most sensitive to a prior downgrading history.”

Likewise, Hamilton and Cantor (2004) observe that rating class transition probabilities differ for issuers that received current rating through an upgrade compared to a down-

grade by summarising that “a rating change tends to be followed by another rating action in the same direction” and exemplifying that “results show that past upgrades are more likely to result in future upgrades than downgrades”. Figlewski et al. (2006) notice based on their analysis that “there is a "ratings drift" or "momentum" effect, by which a firm that has been downgraded (upgraded) in the recent past has a higher intensity of default or of being downgraded (upgraded) again than a firm in the same rating category that has not experienced a recent downgrade (upgrade).” More recently, Frydman and Schuermann (2008) find that “two firms with identical current credit ratings can have substantially different transition probability vectors”. The authors propose and estimate a model consisting of a mixture of two Markov chains that turns out to statistically dominate a single Markov chain model.

Integrating the research findings back on a portfolio perspective, Bangia et al. (2002) acknowledge that the effect on a portfolio of risky debt would be much smaller as different momentums of several issuers might cancel one another.

6.6.3. Aging Effect

Studies by Altman and Kao (1992a and 1992b), that are based on time ratings for individual bond issues emphasize the impact of the length of time since issue on risk. They assert that default risk is increasing during the first three or four years of an issue’s life and that this effect disappears thereafter. This commonly known “ageing effect” is also examined in depth by Helwege and Kleiman (1996) and Jonsson and Fridson (1996). Similarly, Shcherbakova (2008) asserts that “When it comes to Caa credit rating category, the probability of default increases during the first three years of the issuer’s rating period, it declines thereafter indicating that currently insolvent firms that have been in business longer, have greater associated likelihoods of meeting their financial obligations. Essentially there appears to be some sort of a reversion to the mean, where probability of default increases with time for investment-grade obligors and declines with time for speculative-grade obligors.”

Equally, Figlewski et al. (2006) diagnose that “there is also evidence of an "aging" effect, such that the intensity of occurrence for a credit event depends on how long the firm has been rated. In particular, a recently rated firm has lower default intensity than a seasoned firm in the same ratings class. Similarly, a recently rated speculative grade

firm in a B or Ba category has a lower intensity of being upgraded to the investment class.”

Fledelius et al. (2004) find that “transition intensities strongly depend on the direction of the previous move, but that this dependence vanishes after 2-3 years”. This relationship is illustrated for rating class A1 in Figure 13.

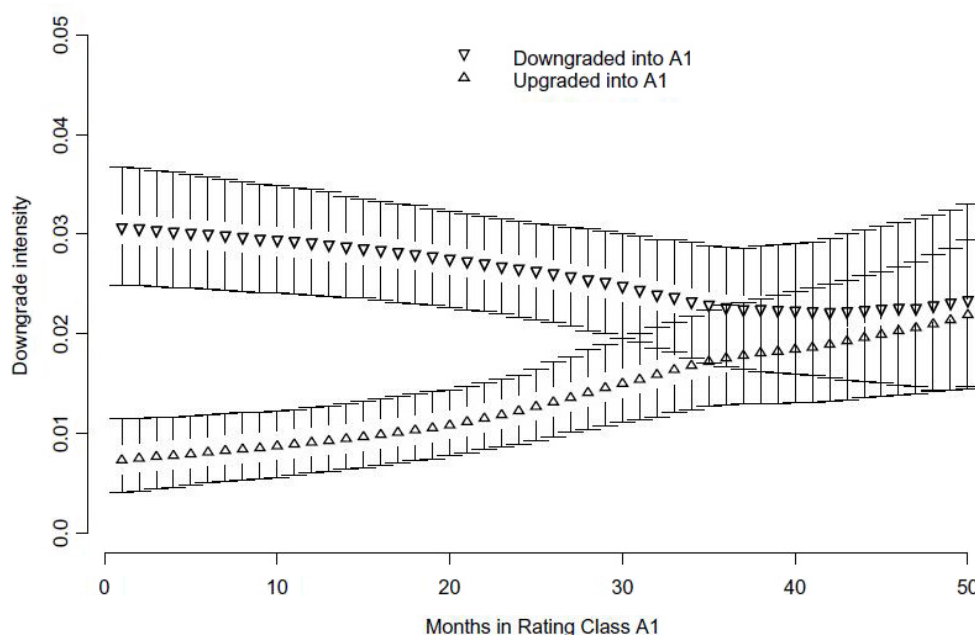


Figure 13 Downgrade Rating intensity as function of time with respect to direction firms came to rating class A1 (source Fledelius et al., 2004)

Since Nickell et al. (2000) used notional, obligor specific senior, unsecured ratings histories that do not include associated maturities or issue dates, they cannot focus on the impact that the time which has elapsed since the bond’s initial issue or the period remaining until maturity has on transition probabilities.

7. Business Cycle Effects

Various elements of a credit risk management framework can be enhanced in order to incorporate the effects of the business cycle on the respective parameters. This may include effects on the probabilities of default or the whole rating migration matrix, if applicable. Further the loss given default or its reverse, the recovery rate may vary with different business cycle regimes. Finally, the exposure at default may also vary, but will be assumed as a given input parameter within this diploma thesis.

A prerequisite is to model the state of the business cycle, and then the variables can be valued depending on the simulated or expected state of the business cycle.

7.1. Business Cycle Regime Switching

As introduced in section 1.2, it is common practice for macroeconomists to analyse the evolution of economic activity by means of the business cycle. Economic cycles describe the way real economic activity fluctuates around its long term growth trend. Cycles allow dissecting the evolution of aggregate economic activity - starting from peak - into recession, leading to trough and recovery or expansion. Macroeconomic factors, such as the GDP growth rate, investment growth rate, labour productivity and unemployment rate vary with the condition of the economy. Hamilton and Raj (2002) provide an overview of the literature on Markov switching models and describe their mathematical structure. To illustrate the basics of such an approach, assume that the growth rate of real GDP in quarter t is represented by y_t . A simple way to describe its evolution is to apply a first order auto regression process, such as

$$y_t = c_1 + \phi_1 y_{t-1} + \varepsilon_t, \quad (50)$$

where ϕ_1 is a time series component and ε_t is normally distributed with mean 0 and variance σ^2 . Assuming that equation (50) describes the dynamic behaviour of GDP growth rates during normal times, a forecast for the next period, given the information at time t is given by

$$\hat{y}_{t+1|t} = c_1 + \phi_1 y_t \quad (51)$$

To attain a more adequate forecasting rule during recessionary phases one might apply different parameters c_2 and ϕ_2 .

$$y_t = c_2 + \phi_2 y_{t-1} + \varepsilon_t \quad (52)$$

Introducing a regime indicator s_t leads to

$$y_t = c_{s_t} + \phi_{s_t} y_{t-1} + \varepsilon_t, \quad (53)$$

where $s_t = 2$ if the economy is in a recession at time t and $s_t = 1$ otherwise. In order to obtain a description of the dynamics of y_t , a probabilistic description of how the economy changes from one state of the economy to another is required. According to Hamilton and Raj (2002), the simplest model is a Markov chain, characterised by:

$$\Pr(s_t = j \mid s_{t-1} = i, s_{t-2} = k, \dots, y_{t-1}, y_{t-2}, \dots) = \Pr(s_t = j \mid s_{t-1} = i) = p_{ij} \quad (54)$$

In a regular Markov model both, the history of regime indicators s_t and the values of y_t are visible to the observer, whereas the term “hidden Markov model” refers to the fact that the econometrician can only observe y_t directly, but can not directly observe the realisations of the discrete valued Markov chain s_t . The unknown values of s_t can only be deduced from the values of the observable parameters. According to Hamilton and Raj (2002) autoregressive regime switching processes are designed to capture the asymmetries observed in the business cycle and therefore appropriate to model economic regime changes.

In their study, Bangia et al. (2002) separate the economy into two different states, expansion and contraction. The authors make use of the categorisations published by the National Bureau of Economic Research (NBER), which identify peaks, troughs, expansions and recessions for the US economy since 1854. The NBER (Hall et al. 2003) views a recession as a “significant decline in economic activity spread across the economy, lasting more than a few months, normally visible in real GDP, real income, employment, industrial production, and wholesale-retail sales.” From this history, the study estimates a two state regime switching process, resulting in a simple regime transition matrix that shows the probabilities of moving either from the current business state to the other state or remaining in the same. They distinguish two time windows, one ranging from 1959 to 1998 and another only focusing on more recent data from 1981 to 1998.

		1959 - 1998		1981-1998	
From \ To		Expansion	Recession	Expansion	Recession
Expansion		84.8%	15.2%	85.0%	15.0%
Recession		57.5%	42.4%	69.2%	30.8%

Table 8 Quarterly Regime Switching matrices (source: Bangia et al. 2002)

In the implementation of the credit risk model within this diploma thesis, the behaviour of the business environment is simulated. Starting from a selectable initial business cycle state, business cycle transition probabilities are used in order to simulate the business environment. For each scenario, the business cycle will either switch into the other regime or remain in the initial state. The simulated state of the business cycle at the end of the risk horizon implies which conditional transition probabilities will be applied for modelling credit migration changes and which credit spreads will be applied for the forward valuation of the bonds. This procedure results in two different asset return thresholds for each rating class, one for each business cycle state. In other words, the unconditional view of credit risk within the CreditMetrics framework will be extended via a conditional view.

7.2. Conditional Transition Probabilities

Various studies put forward the idea that default and credit migration probabilities are linked to the state of the economy. Intuitively, when the economy worsens, downgrades and defaults increase and when economic conditions improve the contrary can be observed. One can conclude that credit cycles follow the business cycles.

In reference only to the default probability Gordy (2002) defines the “unconditional default probability, also known as its PD or expected default frequency, is the probability of default before some horizon given all information currently observable.” He continues by distinguishing that the “conditional default probability is the PD we would assign the obligor if we also knew what the realized value of the systematic risk factors at the horizon would be.”

Nickell et al. (2000) highlight, that “the distribution of ratings changes plays a crucial role in many credit risk models and as these distributions vary across time ”...,” ignoring such dependencies may lead to inaccurate assessments of credit risk.” Various studies have documented the empirical findings that ratings transition probabilities vary according to the stage of the business cycle. Nickell et al. (2000) quantify the connection of ratings transition probabilities on different factors, among others: the industry, the domicile of the obligor and on the stage of the business cycle. They employ an ordered probit model in order to examine the incremental impact of these factors. The authors conclude that business cycle effects are important particularly for low rated bor-

rowers. Their results corroborate the intuitive idea that downgrades are much more likely during recessions than in boom phases of the business cycle.

Bangia et al. (2002) were among the first to propose the deployment of conditional credit rating transition matrices as a link between asset quality and macroeconomic conditions. They pursued the idea of considering the influence of macroeconomic activity primarily for credit portfolio stress testing purposes. Based on data published by the NBER, they distinguish two different states of the business cycle, namely expansion and recession and then estimated conditional transition matrices, depending on the state of the economy. To allow comparisons, the unconditional quarterly transition data is also pictured in the following table:

1/4 year	US Unconditional / Expansion / Contraction Transition Matrix (%)							
To From	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	97.92	1.95	0.10	0.02	0.01	-	-	-
	98.21	1.66	0.11	0.02	0.02	-	-	-
	97.99	1.76	0.25	-	-	-	-	-
AA	0.16	97.95	1.75	0.10	0.01	0.02	0.00	-
	0.15	98.08	1.61	0.12	0.01	0.03	0.01	-
	0.18	96.89	2.79	0.05	0.09	-	-	-
A	0.02	0.57	97.91	1.34	0.10	0.06	0.00	0.00
	0.02	0.53	98.06	1.21	0.11	0.06	0.00	0.00
	0.02	0.88	96.44	2.59	0.07	-	-	-
BBB	0.01	0.07	1.37	96.90	1.38	0.23	0.02	0.03
	0.01	0.07	1.47	96.94	1.25	0.22	0.02	0.02
	0.04	0.04	1.11	96.31	2.33	0.07	-	0.11
BB	0.01	0.03	0.17	1.87	95.35	2.26	0.18	0.13
	0.01	0.03	0.19	1.93	95.31	2.25	0.16	0.12
	-	0.06	0.06	1.39	94.98	2.72	0.42	0.36
B	-	0.02	0.07	0.11	1.66	95.72	1.46	0.96
	-	0.02	0.07	0.10	1.70	95.91	1.31	0.88
	-	0.06	0.06	0.11	0.72	95.02	2.27	1.77
CCC	0.04	-	0.16	0.20	0.41	3.28	87.18	8.72
	0.05	-	0.19	0.23	0.47	3.57	87.32	8.17
	-	-	-	-	-	1.20	85.60	13.20

Table 9 US unconditional, expansion and contraction quarterly transition probabilities in %
(source: Bangia et al., 2002)

Transformed to a yearly horizon using equation (49) and keeping in mind the equations' underlying assumptions, the differences between unconditional, expansion and contraction migration matrices become even more obvious.

1 year	US Unconditional / Expansion / Contraction Transition Matrix (%)							
To From	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	91.95	7.33	0.57	0.10	0.04	0.00	0.00	0.00
	93.04	6.28	0.57	0.10	0.08	0.01	0.00	0.00
	92.22	6.53	1.20	0.05	0.01	0.00	0.00	0.00
AA	0.60	92.12	6.59	0.51	0.06	0.08	0.00	0.00
	0.57	92.60	6.09	0.56	0.06	0.12	0.03	0.01
	0.67	88.28	10.09	0.59	0.34	0.01	0.00	0.00
A	0.08	2.15	92.06	4.97	0.47	0.25	0.01	0.01
	0.08	2.01	92.62	4.50	0.49	0.25	0.01	0.01
	0.09	3.19	86.80	9.29	0.58	0.03	0.00	0.02
BBB	0.04	0.31	5.09	88.42	4.94	1.00	0.09	0.15
	0.04	0.31	5.47	88.55	4.48	0.96	0.09	0.11
	0.15	0.21	3.99	86.38	8.17	0.59	0.06	0.48
BB	0.04	0.13	0.77	6.68	83.01	7.95	0.72	0.71
	0.04	0.13	0.86	6.89	82.87	7.93	0.64	0.64
	0.00	0.23	0.31	4.89	81.67	9.37	1.55	1.94
B	0.00	0.08	0.29	0.59	5.84	84.40	4.50	4.30
	0.00	0.08	0.30	0.56	5.99	85.07	4.06	3.90
	0.00	0.22	0.23	0.45	2.49	81.76	6.73	8.16
CCC	0.13	0.01	0.54	0.70	1.55	10.12	58.00	28.92
	0.16	0.02	0.65	0.81	1.77	11.07	58.37	27.16
	0.00	0.00	0.00	0.01	0.04	3.55	53.82	42.58

Table 10 US unconditional, expansion and recession yearly transition probabilities in % (data from Bangia et al., 2002 converted to yearly horizon)

As illustrated in the tables above, downgrade and default probabilities tend to increase during recessionary business cycle phases, especially for sub investment grade rated obligors. However, since the study was based on actual historical rating migration observations, it also comes with some unexpected outliers, e.g. the upgrade probability for B rated firms to AA is higher during downturns, than during expansions.

7.3. Loss Given Default

Concerning recovery rates or loss given defaults, the Basel II guidelines for capital-requirements instructs banks in paragraph 468 to “estimate an LGD for each facility that aims to reflect economic downturn conditions where necessary to capture the relevant risks. This LGD cannot be less than the long-run default-weighted average loss rate given default calculated based on the average economic loss of all observed defaults within the data source for that type of facility. In addition, a bank must take into account the potential for the LGD of the facility to be higher than the default-weighted average during a period when credit losses are substantially higher than average” (BCBS, 2006a). So the accord defines a rather stringent Loss Given Default, prescribing it must be estimated based on economic downturn values, instead of historical averages over the business cycle or simply most recent values.

7.3.1. Default Probabilities and Recovery Rates Correlation

Recent improvements in modelling recovery rates have been achieved by concentrating on their stochastic properties as well on their correlation with interest rates or default probabilities.

Altman et al. (2005) depict recovery rates as a function of supply and demand for distressed debt. As the supply of defaulted bonds is largely driven by the default rate and absolute amount of defaults, supply will rise when default rates rise, which according to supply and demand rules imply a decrease in the price of distressed debt and consequently also a decline of recovery rates. The demand side for distressed bonds and bank loans is proxied by the size of the high yield bond market. They rely on the data from Altman and Jha (2003), who expound that the demand side, represented by niche investors, distressed asset or alternative investment managers, also known as vultures grew only slowly since the 1990s, whereas the supply side grew enormously. Concerning the supply side, Altman and Jha (2003) estimated that “the size of the U.S. distressed and defaulted public and private debt markets swelled from about \$300 billion (face value) at the end of 1999 to about \$940 billion by year end 2002”. Their univariate and multivariate models manage to explain the variance of recovery rates for the most part.

7.4. Exposure at Default

Exposure at default will be assumed as a preassigned constant within the implementation part of this diploma thesis, since it is directly linked to input factors that describe the composition of the bond portfolio. Allen and Saunders (2003) provide an overview of various structural as well as reduced form model studies that deal with cyclical effects of the EAD, which manifest themselves in an increase in “the likelihood of commitment takedown and the extent of commitment usage during economic downturns when credit is tight and credit-constrained firms are experiencing liquidity crises”.

From a regulator’s view, the Basel II Accord (BCBS, 2006a) prescribes for exposure at default estimates under the advanced approach, that they “must be an estimate of the long-run default-weighted average EAD for similar facilities and borrowers over a sufficiently long period of time, but with a margin of conservatism appropriate to the likely range of errors in the estimate”. Indeed the Basel II Accord also incorporates a more refined approach, where appropriate by stating that “for exposures for which EAD estimates are volatile over the economic cycle, the bank must use EAD estimates that are appropriate for an economic downturn, if these are more conservative than the longrun average.”

8. Calculation Results

For the purpose of investigating the sensitivity of credit value at risk to the business cycle, three different sample portfolios have been evaluated in this diploma thesis.

8.1. Model Settings

In order to account for business cycle effects, a two state regime switching behaviour of the business cycle was introduced. Starting from an initial state, the business cycle either switched into the high or low state with a given probability. This probability manifests itself in a proportion of the generated scenarios either being evaluated in the high or low business cycle environment. This regime switching model comprises credit rating migration matrices for expansion and recession business cycle phases, as reported by Bangia et al. (2002). Since the risk horizon was set to one year, the transition matrices were transformed to annual values making use of equation (49), with results illustrated in Table 10.

Further the business cycle regime switching mechanism was modelled to have an impact on the credit spreads used for bond revaluations. It turned out to be relatively difficult to retrieve credit spreads for all seven main Standard & Poor's rating classes for expansion and recession phases, since whole datasets or histories of datasets are only scarcely available. Mostly only historical investment grade spreads are quoted. Kindly Riskmetrics provided me with a term structure of credit spreads as of January 2002 and October 2004 for various industries. The data from January 2002 was chosen to represent the low business cycle state, whereas the other was used for the rather favourable business environment. These datasets were averaged out across all industries to represent average credit spreads as depicted in the following figures.

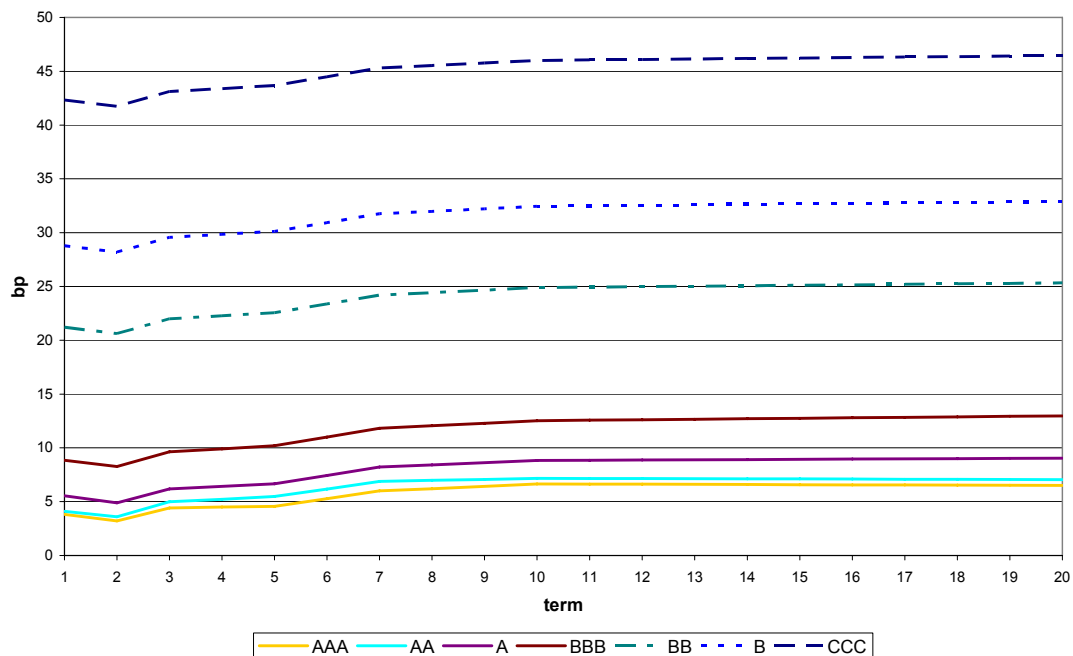


Figure 14 Term structure of credit spreads for expansionary business cycle state

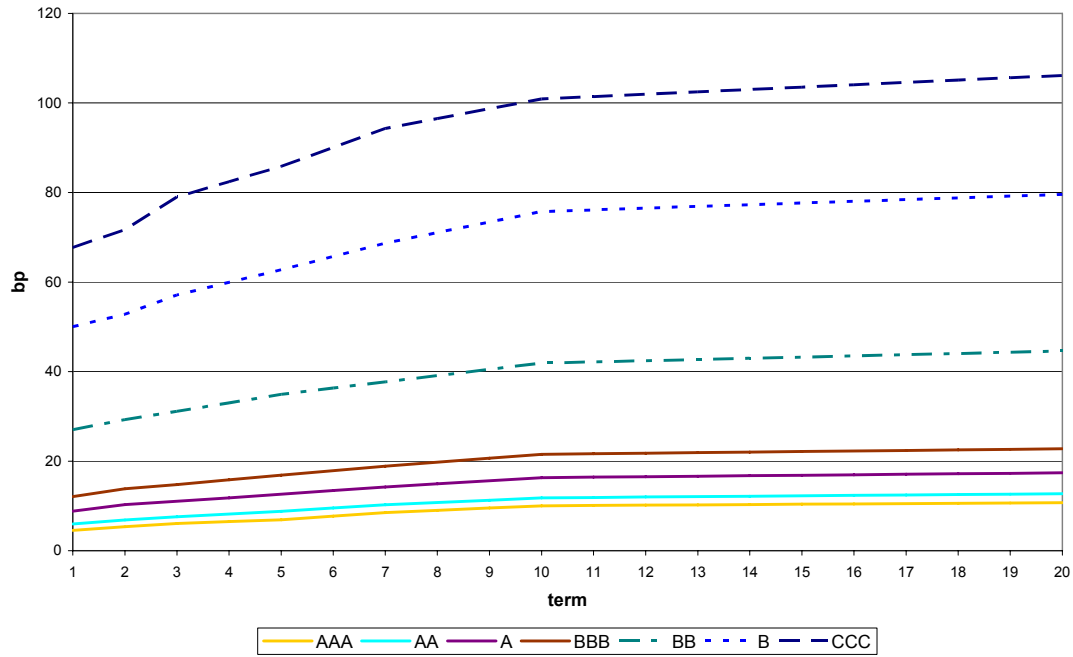


Figure 15 Term structure of credit spreads for recessionary business cycle state

This leads to generally higher credit spreads for the unfavourable business state, which is a plausible assumption for the test cases. For three sample rating classes, namely AAA, BBB and CCC the term structure of credit spreads for both different business cycle regimes are compared in Figure 16.

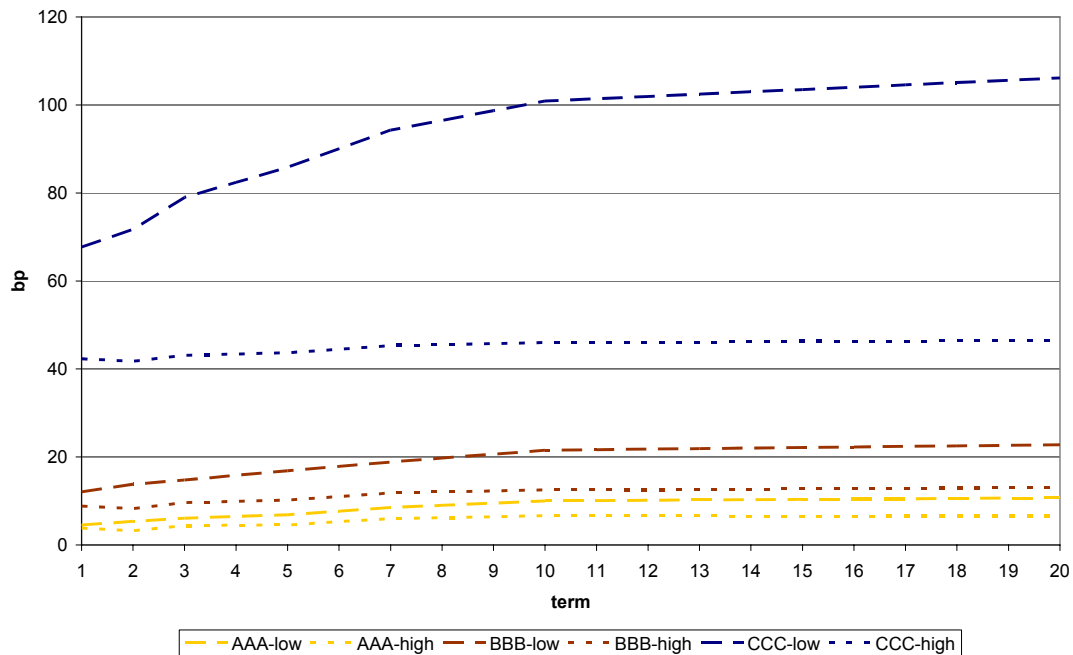


Figure 16 Term structure of credit spreads in high and low regime

The absolute differences in basis points between the high and low regime, which turned out to be strictly greater zero for all terms are illustrated in Figure 17.

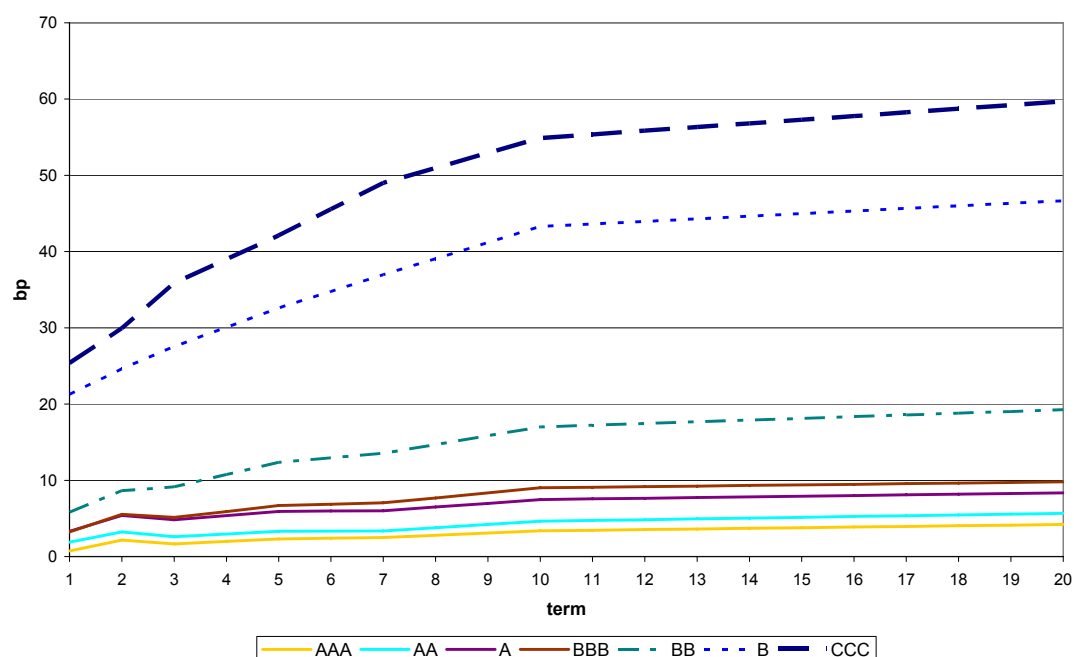


Figure 17 Absolute differences of credit spreads between high and low regime

However, in practical applications, several specific credit spreads of the corresponding instruments that make up the position are required.

Concerning recovery rates, the data from Carty and Lieberman (1996) as depicted in Figure 7 were applied. For simplicity reasons, all bonds were assumed to hold the seniority “senior unsecured”.

8.2. Example Portfolios

The first portfolio consisted of 20 different bonds from 20 different issuers with credit ratings ranging from AAA to CCC. Bond’s maturities ranged from two up to eight years, the exposure varied from 600,000 to 10 million monetary units, with a total exposure of 55.6 million. Annual coupons ranged from five up to ten percent.

Asset return correlations of the 20 issuers varied from 65% down to 10%. In accordance with the sample portfolio in Gupton et al. (1997), asset return correlations were divided into several blocks, basically representing issuers from rather uncorrelated to more correlated industrial sectors.

1	0.45	0.45	0.45	0.15	0.15	0.15	0.15	0.15	0.15	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
0.45	1	0.45	0.45	0.15	0.15	0.15	0.15	0.15	0.15	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
0.45	0.45	1	0.45	0.15	0.15	0.15	0.15	0.15	0.15	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
0.45	0.45	0.45	1	0.15	0.15	0.15	0.15	0.15	0.15	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
0.15	0.15	0.15	0.15	1	0.35	0.35	0.35	0.35	0.35	0.2	0.2	0.2	0.2	0.2	0.15	0.15	0.15	0.1	0.1
0.15	0.15	0.15	0.15	0.35	1	0.35	0.35	0.35	0.35	0.2	0.2	0.2	0.2	0.2	0.15	0.15	0.15	0.1	0.1
0.15	0.15	0.15	0.15	0.35	0.35	1	0.35	0.35	0.35	0.2	0.2	0.2	0.2	0.2	0.15	0.15	0.15	0.1	0.1
0.15	0.15	0.15	0.15	0.35	0.35	0.35	1	0.35	0.35	0.2	0.2	0.2	0.2	0.2	0.15	0.15	0.15	0.1	0.1
0.15	0.15	0.15	0.15	0.35	0.35	0.35	0.35	1	0.35	0.2	0.2	0.2	0.2	0.2	0.15	0.15	0.15	0.1	0.1
0.15	0.15	0.15	0.15	0.35	0.35	0.35	0.35	0.35	1	0.2	0.2	0.2	0.2	0.2	0.15	0.15	0.15	0.1	0.1
0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.2	1	0.45	0.45	0.45	0.45	0.2	0.2	0.2	0.1	0.1
0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.45	1	0.45	0.45	0.45	0.2	0.2	0.2	0.1	0.1
0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.45	0.45	1	0.45	0.45	0.2	0.2	0.2	0.1	0.1
0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.45	0.45	0.45	1	0.45	0.2	0.2	0.2	0.1	0.1
0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.45	0.45	0.45	0.45	1	0.2	0.2	0.2	0.1	0.1
0.1	0.1	0.1	0.1	0.15	0.15	0.15	0.15	0.15	0.15	0.2	0.2	0.2	0.2	0.2	1	0.55	0.55	0.25	0.25
0.1	0.1	0.1	0.1	0.15	0.15	0.15	0.15	0.15	0.15	0.2	0.2	0.2	0.2	0.2	0.55	1	0.55	0.25	0.25
0.1	0.1	0.1	0.1	0.15	0.15	0.15	0.15	0.15	0.15	0.2	0.2	0.2	0.2	0.2	0.55	0.55	1	0.25	0.25
0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.25	0.25	0.25	1	0.65
0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.25	0.25	0.25	0.65	1

Table 11 Sample portfolio correlation matrix

The remaining two portfolios consisted of equal investments of 100 monetary units in 100 different bonds, each issued by a different obligor. Thereby they represent a well diversified portfolio with low concentration risk which fulfils the Basel II assumption of the ASRF model as outlined in section 2.4. For the second portfolio, credit ratings were assumed to be distributed among all rating classes with weightings representing an overall average market portfolio. Standard & Poor's, (Vazza et al. 2006) reported the rating classifications of new issuers for each year since 1981. In total this lead to the following credit rating distribution of the second sample portfolio.

Rating	Sample Portfolio 1	Sample Portfolio 2 "Average Market Portfolio"	Total Number of Issuers (S&P, 2006)	Sample Portfolio 3 "Sub Investment Grade Portfolio"
AAA	1	3	329	3
AA	1	9	864	5
A	5	16	1,680	10
BBB	4	20	2,013	5
BB	2	20	2,085	35
B	6	30	3,022	36
CCC	1	2	220	6
Total	20	100	10,213	100

Table 12 Distribution of ratings in sample portfolios

The third sample portfolio was chosen to represent a portfolio that is more invested in sub investment grade bonds. It was chosen in order to be able to illustrate, whether business cycle effects tend to have larger impacts on sub investment grade investments.

8.3. Results

Four different modes were evaluated. In the first mode, business cycle effects were ignored, with all parameters staying unchanged, i.e. remaining in the initial environment. In the second mode the simulated state of the business cycle at the end of the risk horizon determined only which conditional transition probabilities to use for modelling credit migration changes. Hence this only affected the rating thresholds for each obligor. In the third mode only varying credit spreads, depending on the business environment were applied for the forward valuation of the portfolio. And finally in the fourth mode, both the transition probabilities as well as the credit spreads were switched depending on the simulated business cycle state. In other words, the unconditional perspective of credit risk within the CreditMetrics framework was extended via a conditional view.

It was found that switching the credit rating transition matrix rigorously drives credit risk compared to switching credit spreads alone, if measured with the credit value at risk. For example, only switching the credit rating transition probabilities due to the business cycle state, starting in the high business environment state and setting the probability of remaining in the high state to 35% resulted in a 25% increase of the credit value at risk on the 99% level compared to remaining in the high regime. Observing only the credit spread effect merely leads to a 2.9% increase. On the 95% quantile level, the changes are similarly: 47% compared to 5.6%.

In a further step, a sensitivity analysis of the credit value at risk on the business cycle was conducted for all four modes for all sample portfolios. For both initial business cycle stages, *ceteris paribus* the probability of switching to or remaining in the favourable business environment was increased.

8.3.1. Average Market Portfolio

Figure 18 depicts the 99% credit value at risk for the average market sample portfolio 2 with a total investment of 10,000 monetary units. It shows the sensitivity of the credit value at risk for all four modes mentioned above.

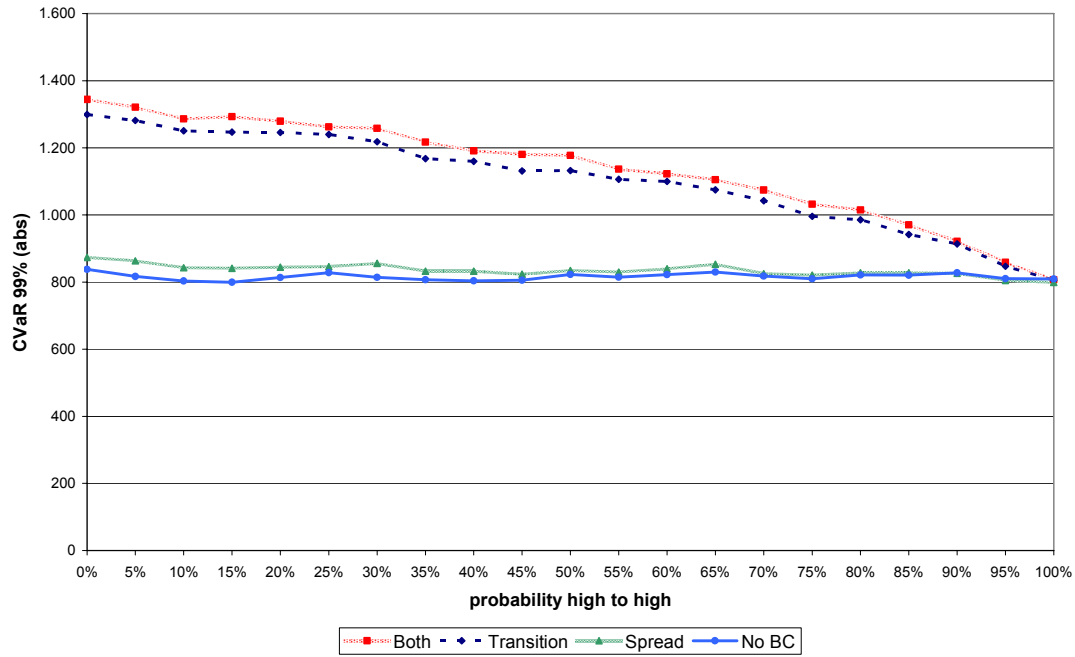


Figure 18 Average market portfolio CVaR absolute (99%) - initial state high

This unambiguously illustrates the effect of the probability of remaining in the favourable business state on the credit value at risk. With an increasing probability of remaining in the high state, the credit value at risk decreases. It points out that the effect of switching credit rating transition matrices mainly drives credit risk and introduces the negative slope. Activating both effects results in a higher credit value at risk, than only considering conditional transition matrices. Figure 19 illustrates similar results for the 95% credit value at risk.

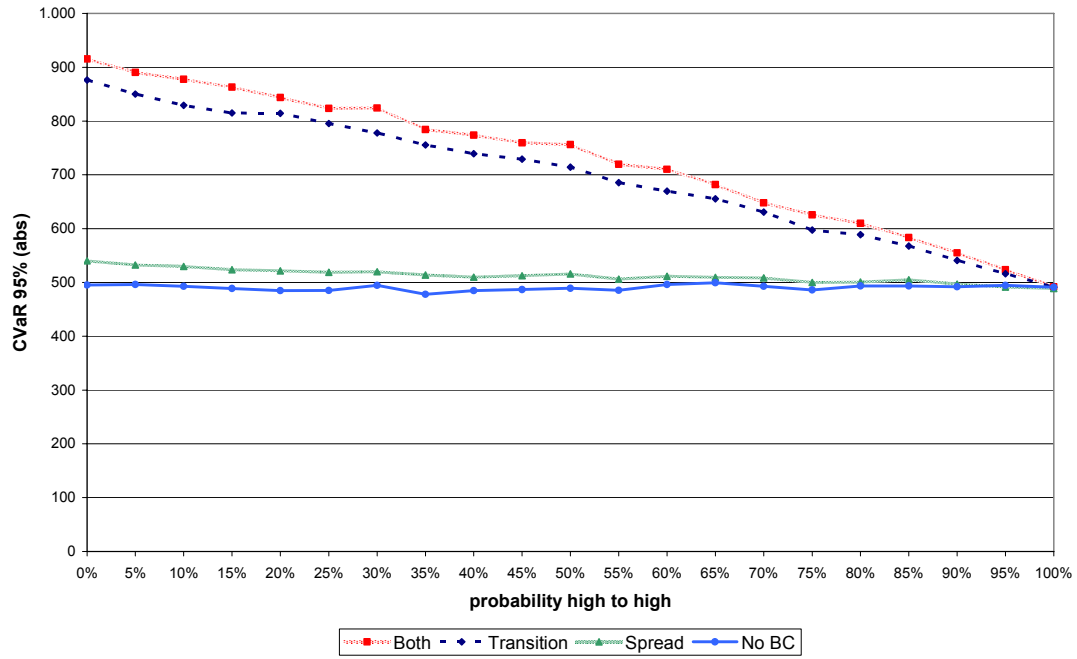


Figure 19 Average market portfolio CVaR absolute (95%) – initial state high

Figure 20 zooms on percental differences between a constant business regime and solely conditional transition matrices for both levels of the credit value at risk.

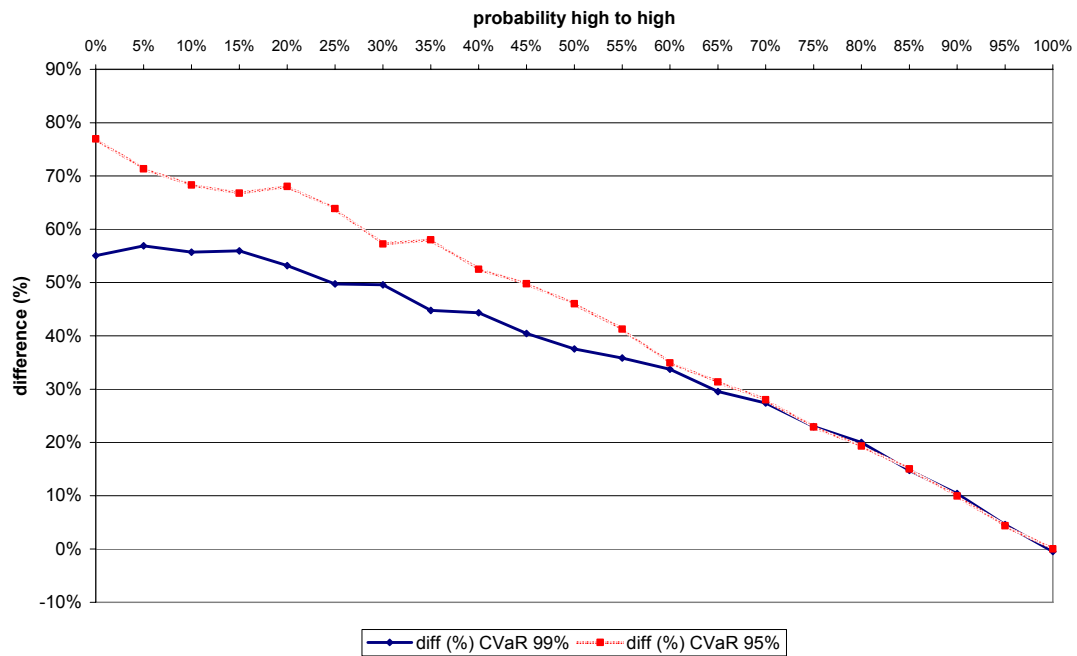


Figure 20 CVaR percentage difference: no business cycle vs. transition effect

Compared to a constant environment the credit value at risk (95% quantile) increases more than 70% in case of likely moves into the recessionary environment. On the 99% quantile level, credit risk increases more than 50%.

Figure 20 focuses on the credit spread effect, which shows smaller effects on the credit value at risk, but as well a clearly negative slope of the curve for both quantile levels.

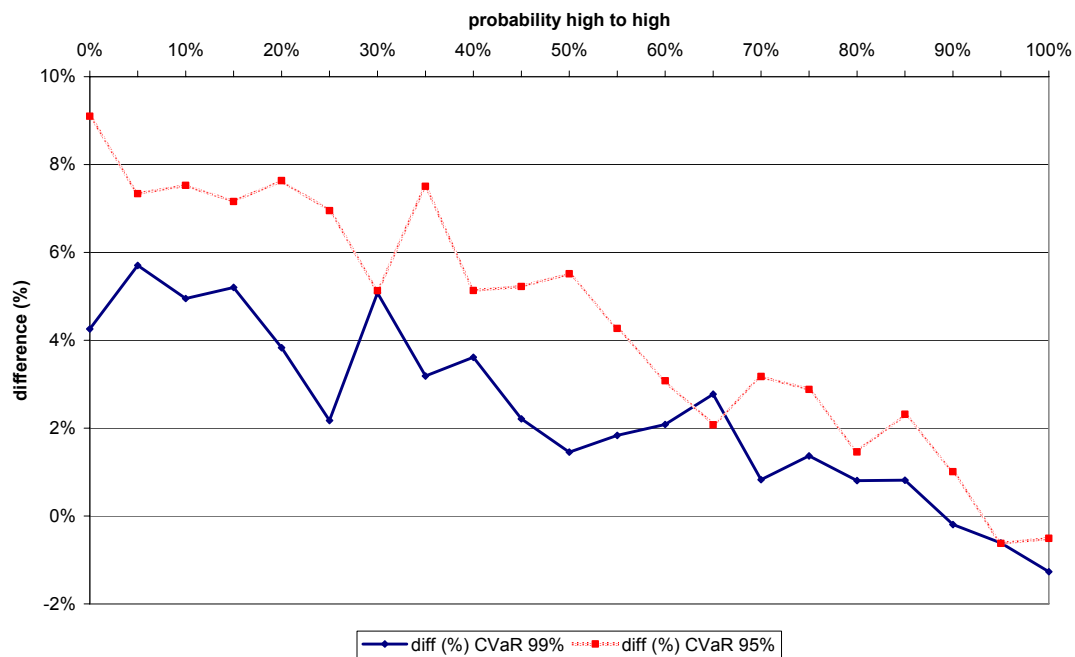


Figure 21 CVaR percentage difference: no business cycle vs. spread effect

Finally, Figure 22 summarises both effects and exhibits the percentage difference to a constant environment.

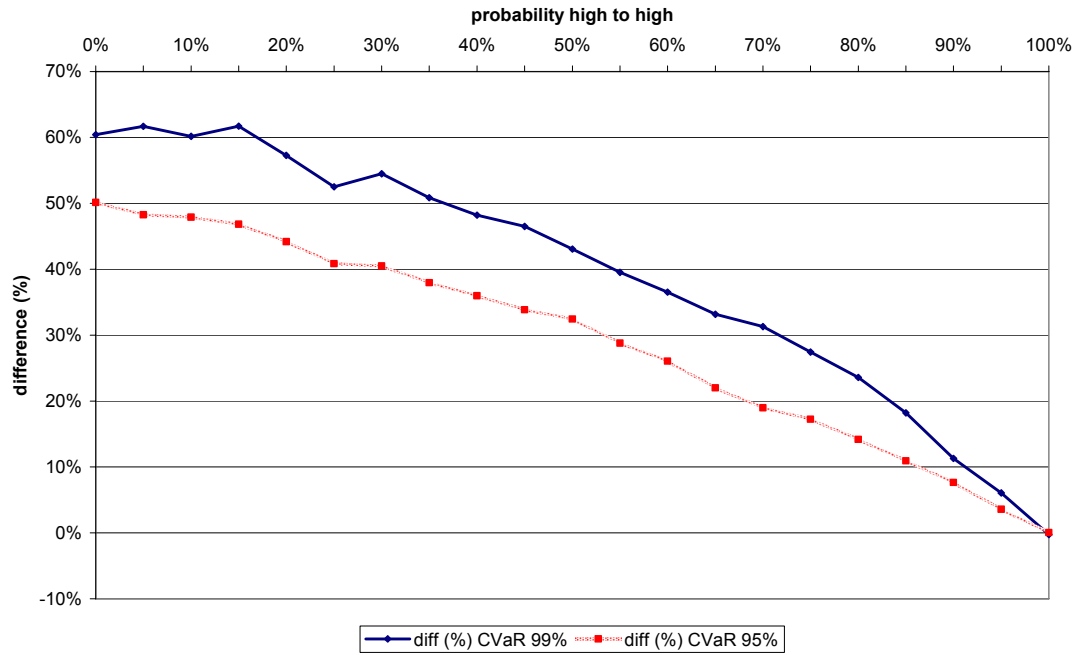


Figure 22 CVaR percentage difference: no business cycle vs. both effects

All results and figures were based on simulations starting in the favourable business environment. Conversely, starting in the recessionary phase and changing *ceteris paribus* the probability of switching from the low state to the high state returns similar results.

8.3.2. Sub Investment Grade Portfolio

As expected, for the sub investment grade portfolio the credit value at risk turned out to be generally higher than for the average market portfolio. Starting in the expansionary business cycle state the sub investment grade portfolio displayed a 99% credit value at risk of 1766 monetary units in case the probability to remaining in the high state is set to 0. Switching this probability to 1, results in a credit value at risk of 1,102. This leads to an absolute difference of 664 units. For the average market sample portfolio this difference only amounts to 537 monetary units. Besides that the sub investment grade sample portfolio did not exhibit a higher sensitivity towards the regime switching probabilities.

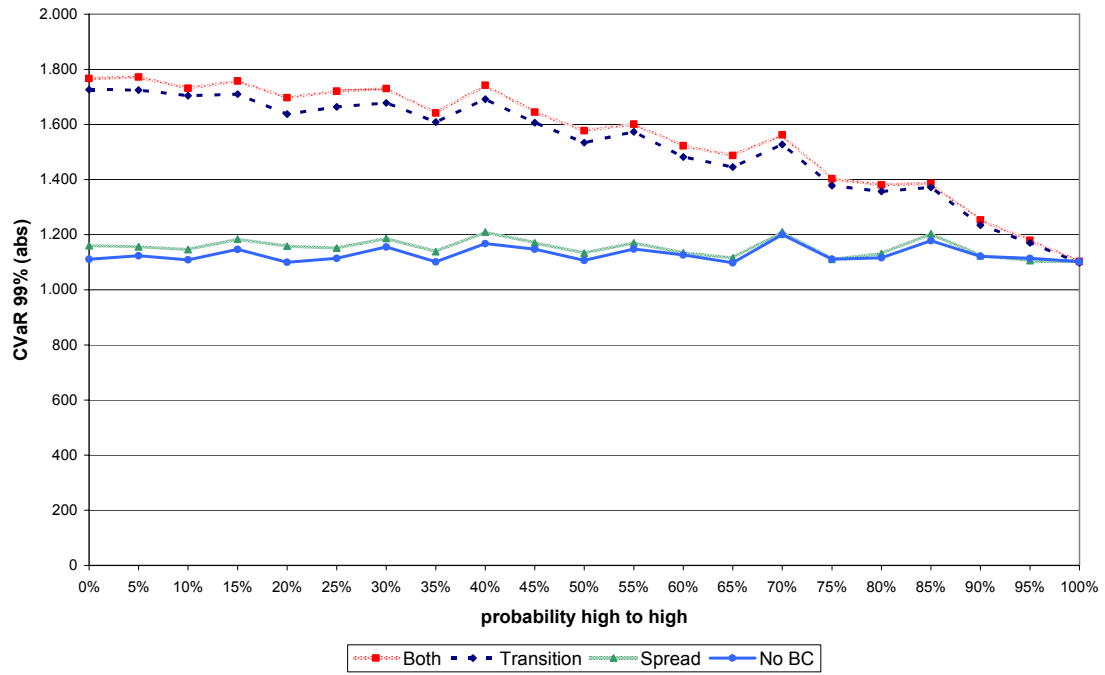


Figure 23 Sub investment grade portfolio CVaR absolute (99%) – initial state high

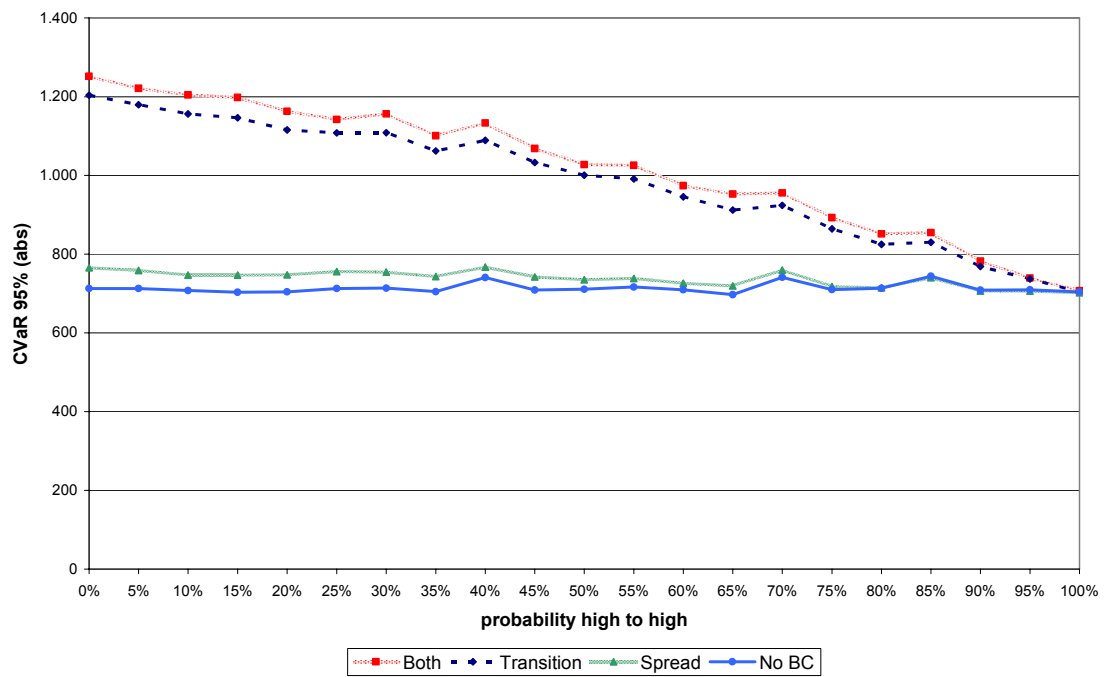


Figure 24 Sub investment grade portfolio CVaR absolute (95%) – initial state high

9. Implementation

First of all, one has to state that JP Morgan itself offers a software implementation of CreditMetrics. This desktop software is marketed under the name “Credit Manager” and has been developed to provide a flexible analysis and reporting tool. It enables users to generate a value at risk report to quantify the amount of credit risk across their portfolio of interest. Credit Manager allows expressing such reports in aggregate or broken down by country, industry, maturity, rating, or product type.

Since the CreditMetrics methodology involves the use of a Monte Carlo simulation, the framework had to be transformed into executable code. Due to the fact that Microsoft Excel is widely spread especially among financial engineers, an implementation on the basis of Visual Basic for Applications(VBA) was chosen. Furthermore Microsoft Excel allows the user to enter input variables in a convenient way and use the models output for arbitrary purposes.

9.1. Monte Carlo Simulation

The analytic approach that is outlined in chapter 5 for a portfolio consisting of bonds issued by only two obligors is not manageable for larger portfolios. Instead, a Monte Carlo simulation needs to be implemented in order to generate the full distribution of the portfolio values at the credit horizon. The implementation of a Monte Carlo simulation may be seen as a compromise: On the one hand, the whole distribution of portfolio value changes can be pictured; on the other hand, the simulation method introduces noise into results. However, as the number of scenarios is increased, errors cancel each other out. Further improvements to reduce random noise and achieve variance reduction in Monte Carlo simulations, such as importance sampling and antithetic sampling exist. Analytic formulas have the advantage that they require fewer computations and do not introduce random noise into the output risk estimates.

However, for larger portfolios Gupton et al. (1997) argue that the analytical approach quickly becomes unmanageable, since the number of pair wise joint transition probabilities grows exponentially with the number of obligors in the portfolio. Another drawback of the analytical approach is that it can not output the whole distribution of profit and loss due to credit and hence not all statistics, such as percentile levels can be calculated.

Gupton et al. (1997) structure the Monte Carlo simulation into the following steps: generation of scenarios, portfolio revaluation, and computation of descriptive statistics for the forward distribution of portfolio values.

9.1.1. Scenario Generation

This section outlines the generation of scenarios resulting in future obligor's credit ratings. The steps makes use of the asset value model examined in chapter 5.2.

First, given the transition probabilities for each rating class from the transition matrix, the asset return thresholds for each rating category are calculated as described by equation (33). In order to describe how the asset values of the obligors move jointly, it is assumed that the asset returns for each obligor are standard normally distributed. Given the asset return correlation for each pair of obligors, the asset return scenarios are generated according to their joint multivariate normal distribution. A standard technique to produce correlated normally distributed variates is the Cholesky decomposition, also going by the name of Cholesky factorisation. Each scenario contains one standardised asset return for each obligor, according to the joint normal distribution.

Then, for each scenario and each obligor the standardised asset return is mapped to the corresponding new rating class at the end of the risk horizon, according to the threshold levels that have been derived before. Consequently, a standardised asset return falling between two thresholds determines the obligor's credit rating at the end of the risk horizon and a standardised asset return that falls below $Z(\text{Def})$, the default threshold, causes a default scenario for the specific obligor. This mapping procedure yields a table that contains the future credit ratings for each obligor and for each scenario.

9.1.2. Portfolio Valuation

This step includes the following activities in case of no default scenarios: Given the spread curves which apply for each credit rating and the future credit ratings of each obligor from chapter 9.1.1, each bond is revaluated for each scenario according to equation (26). In other words, the new rating implies the new value. For a given bond, the value will be the same in scenarios with the same non default credit rating.

On the other hand, in case of a simulated default, the bond revaluation requires recovery rates as discussed in chapter 5.1. Since recovery rates are not deterministic within the

CreditMetrics framework, but modelled to be beta distributed, this step involves drawing a stochastic recovery rate for the defaulted bond, depending on its seniority.

For each default scenario, a random recovery rate is generated according to a beta distribution with the specified input parameters mean and variance. Each default scenario receives an independently drawn recovery rate, which determines the value of the defaulted bond for each default scenario.

$$V_{Def} = RR \cdot f \quad (55)$$

RR stands for the randomly drawn recovery rate and f stands for the bond's face value. This defines the value of a bond in a default scenario. Since the obligor defaulted at the end of the risk horizon and the CreditMetrics methodology calculates dirty prices at the end of the risk horizon, there is no impact of interest rates on the value of the bond in case of default.

Summing up the bonds' values for each scenario according to (27), results in a table comprising the simulated portfolio values at the end of the risk horizon. In the end, by application of the procedure presented above a portfolio value for each scenario is obtained.

9.1.3. Statistics Computation

Repeating the procedure mentioned above for a large number of scenarios, allows plotting the distribution of the portfolio values. Hence the percentiles of the portfolio's future values distribution can be easily derived.

9.2. Object Oriented Data Model

As the recent versions of VBA allow the programmer to implement object-oriented code, the CreditMetrics methodology was converted into a so called Class Diagram. In the field of software engineering, such Class Diagrams are commonly applied in early stages of the software development process, when analysing the requirements of a software package.

Class Diagrams identify classes that are depicted in rectangles. These rectangles include compartments for a class's class name, attributes and functions. Classes are an impor-

tant element of object-oriented systems as they help to identify the compulsory vocabulary in complex systems. They represent a description of numerous objects that have the same attributes, operations and relationships with other objects. For instance, Figure 25 depicts the identified entity bond.

9.2.1. Bond

As a bond is usually equipped with a certain coupon, coupon period, face value, and maturity the following attributes were put into the second compartment.

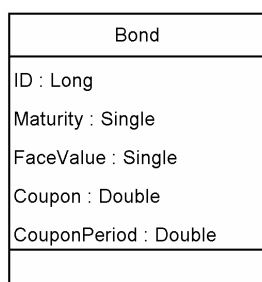


Figure 25 UML style class representing the entity bond

This figure is consistent with the graphical Uniform Modelling Language, as the attributes are usually inserted into the rectangle's second compartment, whereas the class's name is put into the first compartment. The attribute ID was inserted in order to assign an identification number to each bond within the portfolio. As each attribute has a certain data type, these data types are stated after a colon. The different types of data types include Integer, Single, Double, and String. As UML is independent of the programming language, the ranges of valid data for each attribute can vary from programming language to programming language. For Microsoft Excel VBA the valid data ranges are depicted in Table 13.

Data Type	Valid Range	Memory Requirement per variable (in byte)
Byte	0-255	1
Boolean	True or False	2
Integer	-32768-+32767	2
Single	decimal number	4
Double	decimal number	8
String	Character String	10 plus 2 per character

Table 13 VBA data types and ranges

When transforming the important entities that occur in the CreditMetrics methodology into classes, one can identify the following classes: obligor, bond, seniority, rating and rates.

9.2.2. Obligor

Identifiable attributes for obligors are as follows: its name, which is stored in the variable issuer of type String, its ID, and its thresholds. Within the framework, for each obligor a set of thresholds will be calculated, whereas for each rating class one threshold will be calculated.

9.2.3. Rating

The following attributes constitute the class “Rating”: The rating’s name, again its ID, its transition probabilities, and its credit spreads. The transition probabilities are these that are represented in the transition matrix. The corporate credit spreads are quoted for predefined time horizons.

9.2.4. Seniority

Within the model the following attributes are stored per seniority: The seniority’s name, e.g. “AAA”, its mean and its variance of its recovery rates’ distribution as the framework defines the recovery rate via a stochastic process.

The Uniform Modelling Language (UML) allows software engineers to develop Class Diagrams that put different classes in relation with other classes. Such associations are indicated by lines that put two rectangles in a relationship. UML admits different types of relations, whereby the association relation is the simplest form of relating two classes. Generally speaking it represents a “has a” relationship, e.g. an obligor has exactly one rating. Thus, in the Class diagram depicted in Figure 26 a line is drawn between the obligor and the rating class. Depending on the direction of reading, this line can be translated into two different statements: On the one hand it connotes that each obligor has one rating, and on the other hand each rating class has at least one or more obligors. The fact that each rating class has one or more obligors is denoted by the symbol “1..*”, which is put close to the obligor class in Figure 26. This element is known as the cardinality or multiplicity of relationships. Such symbols can be omitted, in case it

states a “to one” relationship, hence no “1” symbol is depicted close to the rating class and the relationship line.

These statements can be constructed for each class that was identified above: Each obligor issues at least one or more bonds that are within the portfolio of interest, whereas each bond is linked to precisely one obligor. Moreover, each bond is of one certain seniority, while on the contrary every instance of the class seniority can be linked to one or more bonds.

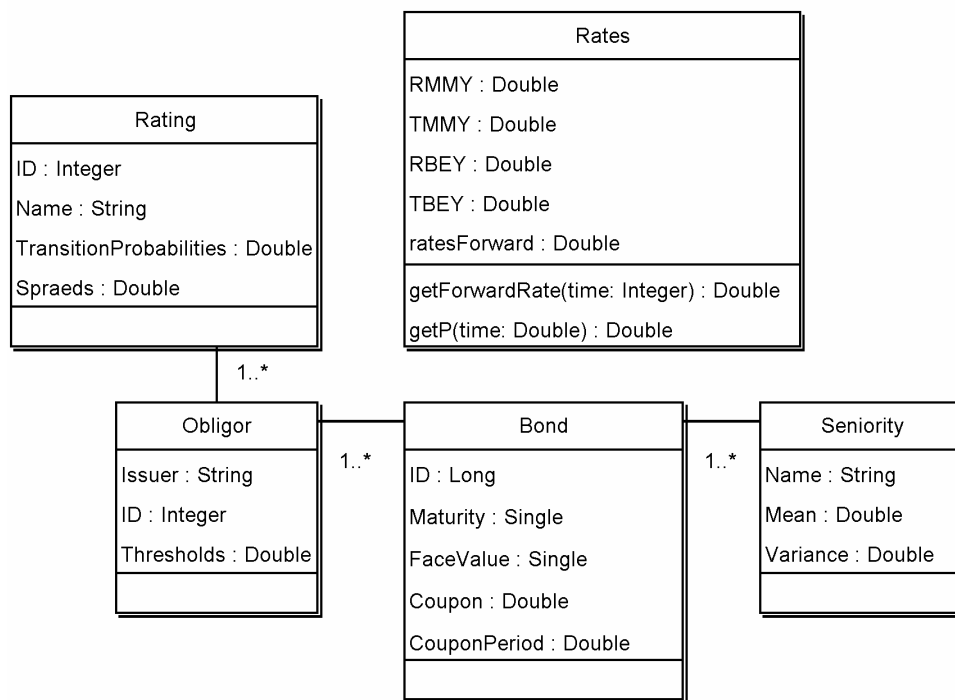


Figure 26 Core class diagram of the implementation

For the interest rates represent a fundamental building block for the credit risk pricing framework and the CreditMetrics paradigm applies deterministic interest rates, the **Rates** class owns attributes that fully define the interest rate curve. These attributes include Bond Equivalent Rates, Money Market Rates and the implied Forward Rates. Furthermore the class **Rates** offers two functions. The first operation, “`getForwardRate`” takes a given time as an argument and delivers the desired forward rate. The second function, “`getP`” requires as well a given time and delivers the corresponding discount factor, named “`P`”.

The classes presented in Figure 26 essentially compose the whole database for the implementation. All required data are stored within these objects at run time of the programme.

9.2.5. Simulation

What misses yet is a class for performing the task at hand, which is basically calculating the credit value at risk of the portfolio. Therefore the class “Simulation” was established, which is responsible for reading the model input dataset and performing the required actions. Hence the Simulation class comprises the three following essential functions: “ReadData” reads the input data and stores it in the data model described above, “Simulate” performs the calculations and finally “FormatResults” creates the output.

The Simulation class’s attributes are as follows: the alpha that is used within the Cholesky decomposition for creating correlated random numbers, the correlations for each pair of obligors, the drawn random numbers, the correlated random numbers, the simulated new ratings, the generated present values for each product in the portfolio, and the present value of the portfolio.

The classes comprising the model are summarised in a class “Model” that will be employed by the class “Simulation” in order to retrieve the required data about bonds, transition probabilities and others. Furthermore, the class “Model” will comprise the attribute defining the number of scenarios that will be simulated. Hence the Class Diagram changes to its final version as depicted in Figure 27.

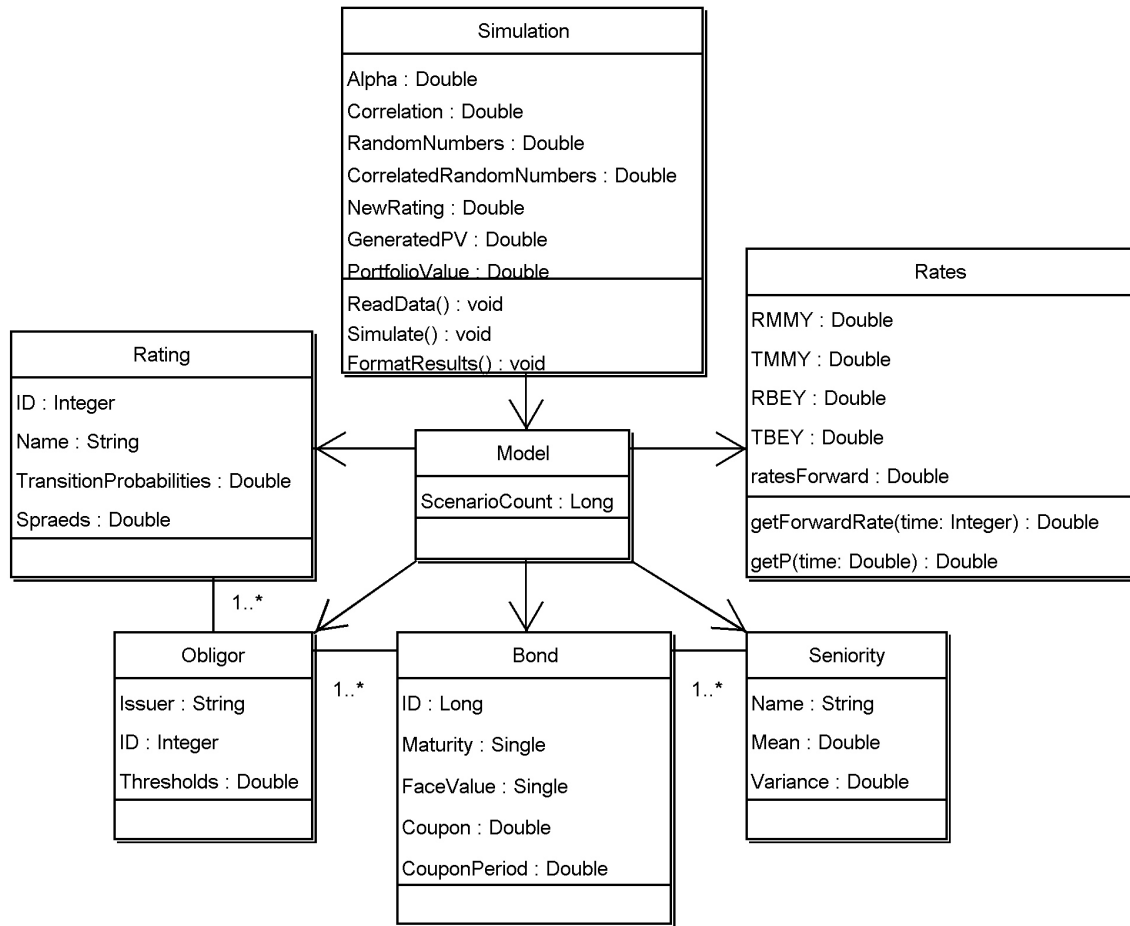


Figure 27 Complete class diagram of the implementation

Now, the “Rates” class is in relationship with the model and constitutes a basic building block as promoted by Kao (2000) and therefore could easily be substituted with a different interest rate process.

9.3. Time Horizon Variation

In order to apply the methodology presented in this diploma thesis for computing credit risk on different time horizons, some minor modifications would need to be made in both, the framework as well as some input parameters. Due to market conventions and reasons listed in chapter 6.6, a default horizon of one year is set. However, if a varied time horizon is required, the following changes must be put in place:

9.3.1. Parameter Modifications

The probabilities of credit quality migration stated within the transition matrix, must be restated to the new risk horizon. One way of performing this operation is to multiply the

short horizon transition matrices to obtain the transition matrix for a longer horizon. For instance a two year transition matrix can be obtained by multiplying the one year transition matrix with itself. For a more detailed illustration, refer to chapter 7.2, where a quarterly transition matrix is transformed into a one year transition matrix.

Unfortunately, this methodology ignores the issue of autocorrelation in the credit quality changes over multiple time horizons. A zero autocorrelation would indicate that successive credit quality moves are statistically independent between adjoining periods. However, autocorrelation exists, when time series observations have a non zero correlation over time. The issue of time period interdependencies can also arise for the credit quality migrations. For instance, Altman and Kao (1992b) find that there is a positive autocorrelation in Standard & Poor's downgrades, so a downgrade implies a higher probability of a downgrade in the consecutive period. This equals the assumption, that agency credit ratings typically exhibit move persistence behaviour and are positively auto correlated during downgrades. For completeness, one has to mention that upgrades do not better predict future upgrades, but they predict a quiet period that includes no transition.

9.3.2. Framework Modifications

The credit instrument revaluation formulas must be changed in order to perform the revaluation computation for the newly defined time horizon.

10. Abstracts

10.1. English

The quantification of credit risk has become more and more important for financial institutions in the last years. This diploma thesis first describes what distinguishes market risk from credit risk and reviews the most prevalent credit risk pricing models, which are the structural and reduced form approaches. It provides an overview of the latest developments in credit risk modelling and points to the appropriate regulatory requirements defined by the Basel Committee on Banking Supervision in the Basel II accord.

Since this diploma thesis includes the implementation of a credit rating migration based credit value at risk model, it describes the essential constituents of a portfolio credit risk model: the interest rate, default and recovery processes. The developed software following the CreditMetrics framework is capable of assessing credit risk of a credit or bond portfolio by making use of Monte Carlo simulations. This approach highly relies on credit rating systems, as provided by external rating agencies, such as Standard & Poors or Moody's KMV or bank internal rating as required by the Basel II Accord. Since credit rating migration matrices are the major key element of this credit risk model, their key characteristics are discussed. As most of credit risk stems from defaults, studies on recovery rates are reviewed and distributions of stochastic recoveries are presented. Since credit risk is addressed on a portfolio level, the diploma thesis demonstrates how diversification effects, default correlations and asset return correlations are linked and accounted for.

This thesis introduces business cycle effects into the model by means of regime switching. This comprises a two state Markov switching model, containing expansion and recessionary business cycle phases. The business cycle regime is modelled to have an effect on transition probabilities via conditional transition matrices and on the credit spreads. The literature also provides evidence that loss given default and exposure vary with the business cycle. Since various input parameters worsen during recessions this allows a more detailed examination of the well known fat tails of credit portfolio losses.

The thesis then describes results of a sensitivity analysis that shows the influence of the regime switching model on the credit value at risk. The results are broken down into its

constituents, namely an effect due to switching the conditional credit migration matrix and another effect due to switching credit spreads.

Finally the diploma thesis explains how the implementation of the model was performed by utilising Monte Carlo simulation and in an interdisciplinary notion illustrates key elements via object oriented UML diagrams.

10.2. German

Die Quantifizierung von Kreditrisiken gewann in den letzten Jahren an Bedeutung. Diese Diplomarbeit beschreibt die Unterschiede zwischen Marktrisiken und Kreditrisiken und gibt einen Überblick über gängige Kreditrisikobewertungsmodelle, die „reduced-form models“ und „structural models“. Weiters wird auf aktuelle Entwicklungen im Bereich der Kreditrisikomodellierung eingegangen und bei passenden Punkten auf die im Basel II Rahmenvertrag definierten regulatorischen Vorschriften des Basler Ausschusses für Bankenaufsicht verwiesen.

Diese Diplomarbeit umfasst die Implementierung eines Kredit - Rating basierten „Credit Value at Risk“ Modells und beschreibt daher die Hauptkomponenten eines Kreditrisikoportfoliomodells: die Zinsstruktur, den Ausfallprozess und den Wiedergewinnungsquotenprozess. Die entwickelte Software basiert auf dem CreditMetrics Ansatz, welcher das Kreditrisiko eines Anleihen oder Kreditportfolios mittels Monte-Carlo Simulationen messen kann. Ein Kernelement dieses Modells sind Ratingsysteme und Rating Migrationsmatrizen. In der implementierten Software können sowohl externe Ratingsysteme von Ratingagenturen, wie beispielsweise Standard & Poors oder Moody's KMV, als auch bankinterne Ratings verwendet werden. Weiters werden Eigenschaften von Kredit Rating Migrationsmatrizen und ein Überblick über Studien zu Wiedergewinnungsquoten erläutert. Weiters wird gezeigt, wie der CreditMetrics Ansatz den Einfluss von Ausfallkorrelationen auf das Kreditrisiko modelliert, sodass Diversifikationseffekte berücksichtigt und Konzentrationsrisiken erkannt werden können.

Diese Diplomarbeit zeigt, wie Effekte des Wirtschaftszyklus mittels Regimewechseln in das Modell eingebracht werden können. Dies wird mittels eines dualen Regime Modells, welches Expansions- und Rezessionsphasen des Wirtschaftszyklus umfasst, umgesetzt. Im implementierten Modell wird angenommen, dass der Status des Wirtschaftszyklus Einflüsse auf Rating Migrationsmatrizen und Kreditrisikoprämien hat. Es gibt Studien, die besagen, dass auch der Verlust bei Konkurs (LGD) und die Entwicklung der Exposures vom Wirtschaftszyklus beeinflusst werden könnten. Da sich einige Eingangswerte des Modells während Rezessionen verschlechtern, erlaubt die Hereinnahme von Rezessions-Szenarien in die Monte-Carlo Simulation eine gründlichere Analyse der so genannten „Fat-Tails“ von Kreditportfolio-Verlusten.

Es wurde eine Sensitivitätsanalyse durchgeführt, die die Stärke des Einflusses des Wirtschaftszyklus Regimewechsels auf den „Credit Value at Risk“ zeigt. Die Ergebnisse werden in zwei Bestandteile zerlegt: Den Effekt aufgrund des Wechsels der Migrationsmatrizen und den Kreditprämieneffekt.

Schließlich wird gezeigt, wie die Implementierung mittels Monte-Carlo Simulationen durchgeführt wurde und in einer interdisziplinären Herangehensweise wird das Modell mittels objektorientierten UML Diagrammen illustriert.

11. Appendices

11.1. List of Figures

Ich habe mich bemüht, sämtliche Inhaber der Bildrechte ausfindig zu machen und ihre Zustimmung zur Verwendung der Bilder in dieser Arbeit eingeholt. Sollte dennoch eine Urheberrechtsverletzung bekannt werden, ersuche ich um Meldung bei mir.

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CV

Ausbildung

- 1986 – 1990 Volksschule Sonnenuhrgasse
www.schulen.wien.at/schulen/906041
- 1990 – 1999 Haydn - Gymnasium
www.bg-haydn.asn-wien.ac.at
Abschluss AHS Matura mit gutem Erfolg
Neusprachliches Realgymnasium
Englisch 8 Jahre
Französisch 5 Jahre
Latein 5 Jahre
- Oktober 1999 Beginn des Studiums der Wirtschaftsinformatik an der TU Wien und Universität Wien
winf.at
- Dezember 2001 Erstes Diplom mit Auszeichnung bestanden

Berufliche Tätigkeiten

- Sommersemester 2002
Studienassistent der Übung „Dataengineering für Wirtschaftsinformatiker“
Institut für Datenbanken und Artificial Intelligence der TU Wien
www.dbai.tuwien.ac.at/education/deue
- Dezember 2002 – August 2004
IT Support
Österreichische Volksbanken AG
www.oevag.volksbank.at
- August 2004 –
Programmierung Marktrisiko
Österreichische Volksbanken AG
www.volksbank.com

Praktika

- 1997/1998 www.psk.at
- 1999/2000 www.post.at
- 1999-2002 www.kwp.at
- 2001/2002 Projekt im Rahmen des Studiums bei www.kmb.co.at
- 2002/2003 Softwareentwicklung www.pse.siemens.at