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„The Cost of Capital in Valuation: An Empirical
Investigation of the Arbitrage Pricing Theory“

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Abbreviations and Symbols

APT	arbitrage pricing theory
APV	adjusted present value
ATX	Austrian Traded Index
BC	bankruptcy cost
CAPEX	capital expenditures
CAPM	capital asset pricing model
CCF	capital cash flow
CF_t	cash flow at time t
$Cov()$	covariance
D	market value of debt
DA	depreciation and amortization
DCF	discounted cash flow
Div_t	dividend at time t
DPS	dividend per share
E	market value of equity
$E()$	expected value
EBIT	earnings before interest and taxes
EVA	economic value added
FCF	free cash flow
FCFE	free cash flow to equity
FCFF	free cash flow to the firm
f_i	price of systematic factor i on capital markets
g	expected growth rate in perpetuity
GDP	gross domestic product
HML	high minus low
I	interest payments
ITS	interest tax shield
MRP	market risk premium
NPR	new debt issuances
P	stock price
PBV	price / book value
PD	Probability of default
PE	price / earnings

PEG	price / earnings / growth
PR	debt repayments
PS	price / sales
PV	present value
r	required rate of return
r_D	cost of debt
r_E	cost of equity
r_F	risk-free rate
r_M	market return
r_{PE}	cost of preferred equity
r_U	unlevered cost of equity
Rec	recovery rate
ROE	return on equity
SMB	small minus big
T	tax rate
t	time t
U	unanticipated return
Var()	variance
VIF	variance inflation factor
WACC	weighted average cost of capital
WC	change in working capital
YTM	yield to maturity
β_i	asset risk factor for systematic factor i
ε	unsystematic portion of stock returns

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1 Introduction

The valuation of assets and the cost of capital as one of the key elements in various valuation methods play an important role in the course of every investment decision, may it be the investment into a project, a publicly listed conglomerate, or the acquisition of a privately held company.

Thus, the first part of this paper will focus on the theoretic concepts underlying company valuation and the cost of capital and its estimation. As such, chapter 1 will present various valuation techniques, such as discounted cash flow valuation, which shall be emphasized, relative valuation, and liquidation and accounting valuation. The idea of the cost of capital and different levels of a required rate of return as a function of the risk involved with the provision of capital shall be covered in chapter 2. Chapter 3 will provide an outline of several models that can be used to estimate the cost of equity as a function of one or several risk factors when providing capital and taking an ownership position in a company. In this context, both single-factor models like the capital asset pricing model as well as multifactor models such as the arbitrage pricing theory and the three-factor model will be presented.

The application of these models in practice may not always happen in absolute accordance with the theoretical framework, and different methods may have certain advantages and disadvantages when used in practice. While the capital asset pricing model is widely used in practice, the more general approach of the arbitrage pricing theory may allow for more flexibility when estimating a company's cost of capital. As such, the latter part of this paper will present a test on the arbitrage pricing theory and its behavior when used in practice.

The purpose of this empirical investigation is to analyze the functioning of the arbitrage pricing theory under the constraints of a small capital market, and to examine whether the risk and thus the cost of capital for such a market's assets can be reflected through a number of factors, and which macroeconomic variables show a fit with such factors, assuming that a structure is identifiable and that a factor extraction can be done. The approach to this investigation shall be presented under Methodology in chapter 7, while chapter 8 will follow with an outline of Results and Analysis.

2 Valuation

2.1 Assets and Values

Often described as a broad and dynamic field, directly affecting persons and organizations alike, Finance can be defined as the art and science of managing money. Yet it is not a separate, cut off discipline, but closely interlinked with other business areas, and almost every decision in business life will eventually have a financial aspect to it (Brigham & Ehrhardt, 2008; Gitman, 2006). Titman and Martin (2008) suggest that, among other things, a key contributor to the ultimate success or failure of a firm is the evaluation and selection of profitable investments. In this environment of overall importance, valuation is not only at the center, but can be considered the heart of finance (Copeland, Weston & Shastri, 2004; Damodaran, 2005).

Ehrhardt (1994) suggests that adding value to the firm is the ultimate goal, and that the outcome of a valuation process will indicate which decision has to be made. As a result, he identifies the core question of how to define value. Mayo (2001) gives a simple answer to this question, stating that the value of an asset is constituted by the present value (PV) of its future benefits. Ergo, valuation is the process of determining what an asset is currently worth. Gitman (2006) uses a similar definition of valuation, but points out that these benefits are only expectations in the process of linking risk and return.

It is apparent that a value can be attributed to every kind of asset, no matter if financial or real. The differences between assets can be significant, and the details and difficulties of each valuation will depend on the underlying asset. Despite these differences, the core fundamentals remain unchanged, and the same basic principles determine the values of all assets (Damodaran, 2002; Damodaran 2006). Before taking a detailed look into valuation methods, one should first consider in which situations a valuation could take place.

2.2 Use of Valuation

Titman and Martin (2008) describe two possible states of growing and expanding businesses, both requiring the necessity of valuing certain assets. Companies can either assemble the assets themselves, or they acquire productive assets by buying an al-

ready existing firm. In the first case, the problem associated is that of a project valuation, whereas the latter will lead to an enterprise valuation. Damodaran (2005) equally considers the question of how best to increase firm value through investment, financing, and dividend decisions a valuation objective in corporate finance. Yet he extends the tasks of valuation to problems such as finding firms trading at less than their true value in portfolio management, or analyzing the deviation of prices from value when studying the efficiency of markets.

2.2.1 Valuation in Portfolio Management

In portfolio management, the role of valuation is likely to increase with the activity of an investor. When trading on information, focus will be on the relationship between information through company announcements and the resulting changes in value. Other investors will use valuation when identifying the potential for additional value in poorly managed companies, and then pursue management to change their conduct of business in an effort to attain those higher values. Another may resort to fundamental analysis and the assumption, that a firm's value is related to its financial characteristics, and that this relationship is stable over time and can be measured (Damodaran, 2006). With reference to the time value of money and value as the current worth of future benefits, Block and Hirt (1994) as well as Gitman (2006) explain that the basic concept of valuation can actually be customized for calculating the value of several specific securities, namely bonds, common stock, and preferred stock.

2.2.2 Valuation in Acquisition Analysis

In the case of acquisition analysis, the valuation process is essential to the bidder in deciding on a fair value before making an offer, as well as to the target company in assessing a realistic value for itself before accepting or declining the bid. Particular features to consider in takeover valuation are the potential of synergies on a combined value, and the effect on value from restructuring and changes in management (Damodaran, 2002). For a firm with shares traded on a stock exchange the market price is indicative of the company's value, even if security prices are subject to fluctuations. Smaller firms that are not owned by the general public will not have a market price for their stock. In fact, their true value may be unknown to the owners unless a liquidation or sale occurs, and the only indication of the firm's worth might come from equity as shown on accounting statements (Mayo, 2001). Feldman (2005) slightly offsets this

argument and proposes the fair market value standard to estimate the value of a private firm. As he explains, this standard embodies three features: First, the assumption of a hypothetical transaction, mimicking the process that happens between the two parties engaging in a real market transaction. Second, the hypothetical parties involved are understood to be willing buyers and sellers, not forced to transact but with the means and ability to do so, as well as the right to withdraw, as opposed to the case of a liquidation. Last, both the imaginary buyers and sellers have to be reasonably informed, meaning that they are aware of the entity's true financial condition, hold expectations of future performance consistent with those of the market, and are able to accurately process disclosed information. The outcome of such a hypothetical transaction is an exchange price reflecting real market conditions, therefore setting a fair value for the firm (Koller, Goedhart & Wessels, 2005).

2.2.3 Valuation in Corporate Finance

Firms are constantly confronted with decisions determining their capital expenditures and their financing, and the respective investment strategy adopted will influence future growth and profitability (Levy & Sarnat, 1978). Titman and Martin (2008) mention that the evaluation of new investment opportunities can range from small-scale capital budgeting exercises to acquisitions of an entire firm. Damodaran (2006) also points to the fact that during the life cycle of a firm, valuation will have a role in every single phase. This is the case for small firms approaching private equity groups or venture capitalists to finance their expansion, as well as for larger companies determining an offer price before going public. Finally, if maximization of value is the ultimate objective in corporate finance, an outline of the interrelation between corporate strategy, financial decisions, and firm value has to be made, a view backed by other authors (Block & Hirt, 1994; Brigham & Ehrhardt, 2008; Gitman, 2006; Koller et al., 2005). This aspect of value creation shall be covered in a later segment.

2.3 Valuation Techniques

One can choose from a wide range of models that often come with quite different assumptions concerning the fundamentals determining the value of an asset. Yet, some of these models can be classified into groups, as they share some common characteristics. In particular, four general approaches can be identified, namely discounted cash flow (DCF) valuation, liquidation and accounting valuation, relative valuation, and con-

tingent claim valuation (Damodaran, 2005). Of these, discounted cash flow valuation is probably closest to the concept of value as the present value of an asset's future benefits. While there shall also be a general presentation of some of the other approaches, namely liquidation and accounting valuation as well as relative valuation, the focus will first be put on discounted cash flow valuation.

2.3.1 Discounted Cash Flow Valuation

Gitman (2006) portrays cash flows, timing, and a measure of risk determining the required return as the three key inputs to the discounted cash flow valuation process, which is effectively an implementation of the time value of money technique. Cash flows are the expected returns during the ownership period and can range from periodic (e.g. annual) to sporadic cash flows or even to just one single cash flow. Only together with the exact timing of the cash flows can the return expected from the asset be fully defined. A cash flow's associated risk level can have significant effect on its value, and higher risk can be incorporated into a valuation process by using a higher discount rate. The ideas of the discount rate and the measure of risk will be mentioned several times in the following and will be covered in detail in later sections. This basic approach can be formulated in such a way, that

$$Value = \sum_{t=1}^{t=n} \frac{CF_t}{(1+r)^t} \quad (1)$$

where CF_t is the cash flow at time t , n is the number of periods or the life time of the asset, and r is the required rate of return (Brealey, Myers & Allen, 2006; Titman & Martin, 2008).

Koller et al. (2005) identify several frameworks for DCF-based valuation, hinting that each model may have certain benefits in practice. Indicating that both the enterprise discounted cash flow model and the discounted economic profit model discount future streams at the weighted average cost of capital (WACC), they recommend these models when a company's debt ratio remains relatively stable. For companies facing significant changes in their capital structure, they propose the adjusted present value (APV). While also mentioning capital cash flow and equity cash flow models, Koller et al. (2005) look at these models as easier victims to mistakes in implementation because performance and capital structure are commingled in the cash flow. Damodaran (2005)

slightly differs in his system to group DCF models and categorizes four alternate approaches used in practice. First, to discount expected cash flows at a risk-adjusted rate; Second, to undertake a risk-adjustment on cash flows and use the risk-free rate to discount the resulting certainty equivalents. He equally considers the APV as a third approach, making value a function of the financing decision, and states that valuation on the basis of excess returns is a fourth DCF method, linking value directly to the quality of investment decisions. While the aforementioned authors may identify seemingly dissimilar subgroups of DCF models, some of the differences exist merely in the names of these classes (Feldman, 2005; Koller et al., 2005; Titman & Martin, 2008).

Equity Discounted Cash Flow Valuation

Probably the most basic discounted cash flow model is the dividend discount model, although there are already several ways of implementing just this one single subtype of discounted cash flow model. Damodaran (2005) describes it as the oldest model and one being less and less used due to its conservative estimates of value. This is because in the model's original design, dividends are the only factor determining the value of a stock apart from the discount rate, as even the price of a stock at the end of the holding period will equally be determined by its future dividends. Gitman (2006) also explains that possible capital gains from selling stocks at higher prices than the purchase prices come effectively from selling the rights to their future dividends. Following this assumption, the value of a stock can be formulated as

$$Value = \sum_{t=1}^{t=\infty} \frac{E(Div_t)}{(1 + r_E)^t} \quad (2)$$

where $E(Div_t)$ is the expected dividend in period t , and r_E is the cost of equity (Damodaran, 2002). This works for the value of equity as well as the value of a single share, considering that the input (Div_t) could either be total dividends or just the dividend per share (DPS). The basic model is even flexible enough to allow for changes in the discount rate.

The accuracy of projections of future dividends in absolute numbers will probably decrease with the length of the estimation period, and can in no way be guaranteed in perpetuity, but at the same time publicly traded firms can last forever at least in theory.

To allow for valuations in this theoretical state, a simple model for firms in stable growth is as follows (Damodaran, 2002; Titman & Martin, 2008):

$$Value_0 = \frac{E(Div_1)}{r_E - g} \quad (3)$$

where $E(Div_1)$ is the expected dividend in the following period, r_E is again the cost of equity, and g is the expected growth rate in perpetuity. As Damodaran (2005) points out, a number of variations of the basic models have developed in practice over time. Two-stage or multi-stage models enable one to incorporate different phases of a company's life, such as periods of higher growth in the beginning, and decreasing and stable growth at later stages. When using a constant growth model, one should first consider that in perpetuity the growth rate can never beat the growth rate of the economy, and second that the assumption of such a growth rate implicitly holds for all performance measures in order to keep payout ratios stable.

In variants of the dividend discount model, the cash flows to be discounted can either be extended by factors such as stock buybacks, or deduced from earnings, or calculated from residual cash flows. This is to reflect that, contrary to the dividend discount model's implicit assumptions, firms may not pay out as dividends what they could afford to pay out. Yet, accumulated cash and alternative ways of returning cash to stockholders also influence the value of equity, therefore extended equity valuation models try to capture what could potentially be paid out as dividends (Brealey et al., 2006; Damodaran, 2005; Gitman, 2006).

The incorporation of stock buybacks can simply be achieved by first adding them to dividends, and then calculating a modified dividend payout ratio by dividing the sum by net income. To avoid distortions from unbalanced stock buybacks, this payout ratio can be estimated by looking at average numbers over a period of a few years. Also, in case buybacks are made with the intention of increasing financial leverage, new debt in the form of long term issues can be subtracted from this calculation (Damodaran, 2002):

$$\begin{aligned} & \text{Modified Dividend Payout Ratio} \\ &= \frac{\text{Dividends} + \text{Stock Buybacks} - \text{Long Term Debt Issues}}{\text{Net Income}} \end{aligned} \quad (4)$$

Damodaran (2005) proposes the approach of discounting a company's earnings if one considers cash flows as too difficult to estimate and wishes to counter the fact that the company may pay lower dividends than it could afford to. At the same time it should be taken into account that any assumed growth would have to be created by reinvesting at least a portion of those earnings, thus discounting growing earnings will lead to over-valuation of the stock.

Another alternative to value what could potentially be paid as dividends is the Free Cash Flow to Equity (FCFE) model. The assumption behind this model is that all cash available after reinvestment needs and debt payments will be paid out to the company's owners. To calculate the FCFE, one has to subtract all capital expenditures (CAPEX) – investments fixed assets and non-operating assets – and the change in non-cash working capital (WC) as well as the difference between debt issuances (NPR) and repayments (PR) – increases in non-equity financing minus decreases – from net income – calculated as EBIT after interest payments I and after application of the tax rate T – and added-back non-cash expenses such as depreciation (DA) (Damodaran, 2006; Titman & Martin, 2008):

$$\begin{aligned} FCFE = & \text{Net Income} + \text{Depreciation} - \text{Capital Expenditures} \\ & - \text{Change in Non-Cash Working Capital} \\ & + (\text{New Debt Issuances} - \text{Debt Repayments}) \end{aligned}$$

or

$$FCFE = (EBIT - I)(1 - T) + DA - CAPEX - WC + NPR - PR \quad (5)$$

FCFE is then used to value the equity portion of a company as follows:

$$\text{Equity Value} = \sum_{t=1}^{t=\infty} \frac{FCFE_t}{(1 + r_E)^t} \quad (6)$$

As with the dividend discount model, this approach can be altered to account for different growth phases or perpetual growth, with the same principles concerning perpetual growth rates holding for the FCFE approach as for the constant growth dividend model

(Damodaran, 2005). Koller et al. (2005) point out that this equity model becomes difficult to implement when a company's debt-to-value ratio is changing over time, as changes in leverage would have to be reflected by adjusting the cost of equity.

As mentioned earlier, the difference between extended equity valuation models and the classic dividend discount model lies in the choice of cash flow that will be discounted. For all equity valuations models holds the fact that the cash flows have to be discounted at the cost of equity. Differing from this are models valuing entire businesses, where cash flows are discounted at the firm's cost of capital. These models shall be presented in the following (Koller et al., 2005; Titman & Martin, 2008).

Firm Discounted Cash Flow Valuation

Instead of directly valuing the owners' claims against a company's cash flows, enterprise discounted cash flow models value the operating cash flows of a firm, embedding tax benefits of debt and expected additional risk associated with this debt into the outcome. An advantage of this approach is that individual projects, single business units and entire companies can be valued according to a consistent methodology (Koller et al., 2005).

Varying definitions of the expected after-tax cash flow are in use, but a common approach to value a company's operations is to discount the free cash flow to the firm (FCFF), defined as after-tax operating income – calculated as EBIT after taxes – plus added-back depreciation minus capital expenditures and change in non-cash working capital (Gitman, 2006). This calculation is consistent with FCFE as in equation (5) apart from the non-equity financing aspects. Consequently, the discount rate has to be the firm's cost of capital, representing a combined required return to the debt and equity holders, but this will be covered in detail in the subsequent sections.

Adding non-operating assets – such as excess cash, marketable securities, non-consolidated subsidiaries, or other equity investments – to the value of operating assets gives the enterprise value of a company. As a last step to arrive at the value of equity, one has to net out the market value of all non-equity financial claims, including fixed- and floating rate debt, capitalized leases, unfunded pension plans and health care obligations, employee options, and preferred stock (Damodaran, 2005; Titman & Martin, 2008):

$$FCFF = \text{After-Tax Operating Income} + \text{Depreciation} - \text{Capital Expenditures} \\ - \text{Change in Non-Cash Working Capital}$$

or

$$FFCF = EBIT(1 - T) + DA - CAPEX - WC \quad (7)$$

FCFF is then used to value the equity portion of a company as follows:

$$\text{Firm Value} = \sum_{t=1}^{t=\infty} \frac{FCFF_t}{(1 + \text{Cost of Capital})^t} \quad (8)$$

Changes in the financing mix result in an adjusted debt ratio and are therefore reflected in the valuation through the discount rate rather than the cash flows – which is an advantage when leverage is expected to change significantly over time, since the estimation of debt repayments and issuances into the future becomes more and more complex (Koller et al., 2005).

As with the aforementioned equity models, the FCFF model can be subject to alterations to account for assumptions about the growth rate and different phases in the company's life. It should once more be noted that in perpetuity the growth rate cannot exceed that of the economy, and that some of the firm characteristics and especially the inputs for reinvestment, namely capital expenditures and depreciation, have to be in line with the stable growth rate. Further, the use of a constant cost of capital for the firm implies the assumption of an unvarying debt ratio (Koller et al., 2005; Titman & Martin, 2008).

Adjusted Present Value

In contrast to capturing the effects of debt financing in the discount rate as it is done in the conventional entity valuation approach, the value of benefits and costs of debt is calculated separately from operations in adjusted present value (APV) models. The value of the firm therefore consists of the enterprise value as if the company was all-equity financed, plus the value of any cash flows associated with debt borrowing. The

latter can further be split into positive effects, such as interest tax shields, and negative effects, such as issue costs and expected bankruptcy costs (Koller et al., 2005).

Damodaran (2005) presents three steps to arrive at the complete APV approach. First, discounting the free cash flows to firm at the unlevered cost of equity gives the value of the unlevered firm. Apart from the discount rate, this computation of the first part is in analogy to the entity approach as presented in equation (8) and can equally be made subject to various alterations concerning growth.

$$\text{Unlevered Firm Value} = \frac{FCFF_0(1 + g)}{r_U - g} \quad (9)$$

The discount rate r_U is the unlevered cost of equity, another variant to be explained in detail in later sections.

Second, tax benefits from a given level of debt are calculated as a function of the firm's tax rate and then discounted to reflect the risk of this cash flow. If both the tax rate and the debt amount in absolute terms are constants, and the pre-tax cost of debt is used as the discount rate, this leads to a simplification, such that the tax benefit value equals the amount of debt times the tax rate.

$$\text{Value of Tax Benefits} = \sum_{t=1}^{t=\infty} \frac{\text{Tax Rate}_t \times \text{Interest Rate}_t \times \text{Debt}_t}{(1 + r)^t} \quad (10)$$

Third, to estimate the impact of the respective debt level on default risk and expected bankruptcy costs the present value of bankruptcy costs (BC) has to be multiplied with the probability of default.

$$\begin{aligned} \text{Present Value of Expected Bankruptcy Cost} \\ = \text{Probability of Bankruptcy} \times \text{Present Value of Bankruptcy Costs} \end{aligned} \quad (11)$$

Combining all three steps, the resulting general approach to APV valuation looks as follows:

$$\text{Levered Firm Value} = \frac{FCFF_0(1+g)}{r_U - g} + (T \times D) - PV(E(BC)) \quad (12)$$

Koller et al. (2005) as well as Titman and Martin (2008) propose the APV approach when debt does not grow in line with firm value but significant changes to the capital structure are made. Damodaran (2005) argues that computing the impact of debt is easier in absolute than in proportional terms, and that firms define their debt target not as a ratio of market value, but in absolute value terms. According to the author, the major difficulty of the APV lies in the calculation of expected bankruptcy costs. This is because neither the probability of default, nor direct costs, such as court-related fees, nor indirect costs, such as the loss of customers and suppliers or other reactions from stakeholders, can be estimated directly. It should also be noted that in the case of too much debt, a company may not be able to fully use the tax shields due to the lack of sufficient profits. Under significant probability for distress only the expected tax shields should be calculated by deducting cumulative default probability from promised tax shields (Koller et al., 2005).

Capital Cash Flow

Koller et al. (2005) present a variation for cases when a company actively manages its capital structure to a target debt-to-value level. The resulting interest tax shield (ITS) and the free cash flow (FCF) – together forming the capital cash flow (CCF) – will then be discounted by the unlevered cost of equity as follows:

$$\begin{aligned} \text{Value} &= \sum_{t=1}^{t=\infty} \frac{FCF_t}{(1+r_U)^t} + \sum_{t=1}^{t=\infty} \frac{ITS_t}{(1+r_U)^t} \\ \text{Value} &= \sum_{t=1}^{t=\infty} \frac{FCF_t + ITS_t}{(1+r_U)^t} \end{aligned} \quad (13)$$

This much can be said that while the FCFF model treats tax shields through a combined discount rate representing equity and debt capital, tax shields in the capital cash flow model are quite apparently valued in the cash flow (Koller et al., 2005; Ruback, 2000).

Excess Return Models

In excess return models, the value of a business is expressed as the sum of two components, namely the capital invested in the firm today, plus the present value of excess return cash flows from current and future projects. Consequently, cash flows in this approach are split into two corresponding parts. The normal return cash flow has to be earned to satisfy the required rate of return (either the cost of capital or the cost of equity), while excess returns, which can be positive or negative, are defined as all earnings either above or below this cash flow. This approach is in line with the net present value rule and the idea that in order to add value to a business, an investment's returns must exceed its cost of capital (Damodaran, 2005; Titman & Martin, 2008).

Koller et al. (2005) present a common variant of excess return models which is called economic value added (EVA) or economic profit. It is defined as such that EVA equals the return on invested capital minus the cost of capital, multiplied with capital invested, or rewritten as operating income after taxes, minus cost of capital times capital invested. The value of the firm can then be defined as follows:

$$Value_0 = Invested\ Capital_0 + \sum_{t=1}^{t=\infty} \frac{Economic\ Profit_t}{(1 + Cost\ of\ Capital)^t} \quad (14)$$

In a simplified version, assuming a state of constant growth, this can be written as:

$$Value_0 = Invested\ Capital_0 + \frac{Economic\ Profit_1}{Cost\ of\ Capital - g} \quad (15)$$

Damodaran (2005) splits the right-hand side of these equations into three components and describes firm value as capital invested today, plus economic value added on these assets already in place, plus economic value added on any future projects.

Certainty Equivalent Models

In the models of discounted cash flow valuation presented so far, the risk from future cash flows influencing today's value was taken into account through adjustment of the discount rate. Alternatively, instead of having discount rates corresponding with a certain level of risk, one can also make adjustments to the expected cash flows directly, in a sense taking away their risky portion. As the resulting certainty equivalents already

incorporate the respective risk level associated with future expected cash flows, they can be discounted at the basic discount rate, usually the risk-free rate (Brealey et al., 2005; Titman & Martin, 2008). Damodaran (2005) presents three ways to arrive at the certainty equivalent of an expected cash flow, namely utility models, risk and return models, and cash flow haircuts.

The first type works on the basis of the differences in utility functions and willingness to accept risk for different individuals. The considerable difficulties with this method lie in the precise specification of a utility function, and the necessity to account for all possible scenarios in order to correctly calculate the certainty equivalents for a certain asset's cash flows.

The second approach, risk and return models, works the same way that discount rates are adjusted for risk just that this adjustment is made on the cash flow. As such, the certainty equivalent is calculated by discounting the cash flow with a risk premium. Using a compounded risk premium will produce exactly the same results as when adjusting the discount rate. Only in the case that risk premiums are calculated as the absolute difference between risk-adjusted and risk-free rate there will be divergence in the final values.

Last, cash flow haircuts work in a way that literally the risky portion of a cash flow is taken away to arrive at the certainty equivalent. When subjectively adjusting the uncertain returns from an asset, too speculative cash flows can either be replaced with conservative estimates or be completely ignored, such that only the predictable returns are taken into consideration. Yet, there is no definition by how much the cash flow should be reduced to qualify as a certainty equivalent, and different individuals may very well not have the same view of how to correctly make such a subjective assessment.

2.3.2 Accounting and Liquidation Valuation

One of the general assumptions usually underlying discounted cash flow valuation is that the business to be valued will continue to exist. This is why, apart from the assets currently owned by a company, future investments and growth opportunities are also taken into consideration for value. As such a going concern valuation might not always be appropriate, certain methods concentrate more on the already existing assets and assess each asset's value separately.

Book Value Based Valuation

Damodaran (2005) refers to the original intention of accounting as a means of providing a measure of a company's true earnings potential and a reliable estimate of its assets' and equity's value through the profit and loss statement as well as the balance sheet, but the treatment of historical costs has developed differently with respect to various asset classes. In detail each of the following is certainly subject to a respective country's accounting rules, but while book value is mostly still related to the original cost for fixed assets, current assets sometimes receive treatment more related to market value, and neither of the two approaches might work for new categories such as brand names. Effectively, book value will not be the optimal measure for all firms, but the more mature, the higher the share of fixed assets, and the lower the growth opportunities, the more reasonable an approximation of the true value of a firm it will be. Certain methods have been developed to include earnings into valuation models based on book value. The residual income model leans on the basic dividend discount model by putting expected dividends in relation to book value, as the book value of equity at the start of a period must equal the book value of equity at the start of the previous period plus net income minus all dividends paid out (Damodaran, 2005):

$$\text{Book Value of Equity}_t = \text{Book Value of Equity}_{t-1} + \text{Net Income}_t - \text{Dividends}_t \quad (16)$$

$$\begin{aligned} \text{Value of Equity}_0 &= \text{Book Value of Equity}_0 \\ &+ \sum_{t=1}^{t=\infty} \frac{(\text{Net Income}_t + r_{E_t} \times \text{Book Value of Equity}_{t-1})}{(1 + r_E)^t} \end{aligned} \quad (17)$$

The more recent development has been towards fair value accounting, possibly in an effort to return to the idea of balance sheets bearing more resemblance to a firm's true value. On the one hand, this connection may indeed exist and therefore provide more useful information to investors, on the other hand, the potential for misuse and manipulation could increase with the use of fair value accounting, and techniques such as marking to market will only reflect what already has happened in the market before (Damodaran, 2005; Gitman, 2006; Koller et al., 2005).

Liquidation Valuation

Under the circumstance of liquidation, the assets of a company have to be sold under a very short time horizon, which may result in a discount depending on the asset's characteristics, the number of potential buyers, and the general state of the economy. A relationship between book value and liquidation value may be expressed in terms of assuming that the latter will be a specific percentage of book value. To estimate the liquidation value as a fraction of a discounted cash flow value may be more difficult, simply because of the underlying growth assumptions in going concern valuation. Naturally, liquidation valuation and the expectation of discounts due to urgent disposal of assets are more appropriate for companies already finding themselves in financial distress (Damodaran, 2005; Gitman, 2006; Koller et al., 2005).

Damodaran (2002) and Koller et al. (2005) point out that while the focus on the future is by far not as high in liquidation and accounting valuation as it is in discounted cash flow models, both methods have in common that the basic approach is to estimate the value of a company by directly examining this company and the assets it owns. In order to identify the true earnings potential, discounted cash flow models emphasize a company's growth opportunities, while liquidation and accounting valuation put more weight on the ability to generate returns from the currently existing assets. Departing from this fundamental approach of direct examination, one may consider estimating the value of a business by looking at the market and finding out about the price of similar companies. This approach shall be presented in the following.

2.3.3 Relative Valuation

In relative valuation, as the name suggests, an asset is put into relation to comparable assets and one tries to estimate its value by looking at the price of these other assets. This approach differs considerably from the previously presented methods. Both in discounted cash flow models and accounting models the attempt is made to correctly identify the value of an asset from its potential to produce earnings or cash flows, no matter whether the results may be correct or not. In relative valuation, this search for intrinsic value is not at all taken into account, as one relies completely on the ability of the market to correctly price an asset. As a result, relative valuation will only provide an indication for the true value of an asset as long as the market is not consistently over- or underpricing an entire group of similar assets (Damodaran, 2002; Koller et al., 2005; Titman & Martin, 2008).

When using relative valuation, a few key considerations have to be kept in mind. First, it is essential to find a suitable group of comparable assets that has already been priced by the market. As there are probably not two businesses that perfectly look like another, there is a potential for difficulties and open questions. Second, as divergence in such things as size or the number of shares outstanding is more than likely, market values have to be adapted to reflect the same measure, just like units in natural sciences do. In the last step, adjustments should be made to reflect prevailing differences. This can almost be done using common sense, for instance high growth will normally be preferable to low growth (Damodaran, 2002; Koller et al., 2005; Titman & Martin, 2008).

Standardized Multiples

Some standardized measures have developed over time that can be applied universally, while others may only be appropriate for a certain industry. Multiples represent the common unit to allow for comparison, often relating market value to a company's earnings, book value, or revenues. The various models are most easily explained at the hand of the dividend discount model assuming a business in stable growth, where the value of equity equals the expected dividend for the next period discounted by the cost of equity minus the perpetual growth rate, as presented in equation (3). Consistent with the most basic equity models in discounted cash flow valuation, value can be seen as a function of earnings. As such, the corresponding multiple will stand for the ratio of price paid to the earnings generated by an asset, although the outcome can vary due to whether future or current earnings are used in the calculation. After dividing both sides of the stable growth model by current earnings, the next period's dividend can be displayed as the payout ratio multiplied by the growth factor. Thus, the price / earnings (PE) ratio is defined as follows (Damodaran, 2002; Koller et al., 2005; Titman & Martin, 2008):

$$PE = \frac{P_0}{EPS_0} = \frac{Payout\ Ratio \times (1 + g)}{r_E - g} \quad (18)$$

Setting price in relation to book value gives an indication of how over- or undervalued a stock is with regards to the assets the company owns. When dividing through book value of equity, the reformulated right-hand side numerator turns into return on equity (ROE) times the previously stated payout ratio after growth. The resulting definition of

price / book value (PBV) ratio is as follows (Damodaran, 2002; Koller et al., 2005; Titman & Martin, 2008):

$$PBV = \frac{P_0}{BV_0} = \frac{ROE \times Payout Ratio \times (1 + g)}{r_E - g} \quad (19)$$

The ratio of a firm's market value to its revenues presents a multiple less affected by accounting rules but rather reflecting profit margins, thus making it more applicable when comparing firms across markets with diverging accounting systems. The price / sales (PS) ratio is defined as a function of the operating margin multiplied with the payout ratio and the growth factor (Damodaran, 2002; Koller et al., 2005; Titman & Martin, 2008):

$$PS = \frac{P_0}{Sales_0} = \frac{Profit Margin \times Payout Ratio \times (1 + g)}{r_E - g} \quad (20)$$

Comparable Firms and Further Adjustments

The use of multiples as an indication for the value of a business only makes sense as long as these multiples are derived from companies with similar characteristics, which already have been priced by the market. This includes components such as cash flows, growth potential, and the level of risk. Additional criteria, like a company's size with regards to total assets, may further be considered. The implicit assumption that companies from the same industry have comparable profiles in terms of these characteristics can be observed in practice, yet there is no definition stating that multiples must be derived from firms within the same sector. When the industry is taken as an appropriate selection criterion, the number of comparable companies increases, but this will also result in a group of more diverse firms. The alternative is to accept a smaller group of comparable companies, but with more precisely matching characteristics (Feldman, 2005; Koller et al., 2005; Titman & Martin, 2008).

In most cases, the firm to be valued will still differ from those chosen as comparable companies on some points, and some last refinements have to be made to control for these differences. This can be done through subjective adjustments, modified multiples, or with the help of statistical techniques. Subjective adjustments cover issues

such as the choice of how best to estimate the average multiple for an industry, or the interpretation and explanation of a deviating firm multiple. Modification of a multiple often refers to the growth-adjustment of the PE ratio through division by an expected growth rate in earnings. The implicit assumptions underlying the resulting price / earnings / growth (PEG) ratio are first that, apart from growth, all other measures influencing firm value are comparable, and second that the firms share the same risk level. The last methods of refinement are statistical techniques such as sector or market regressions. With their help, more complex relationships between fundamentals and multiples can be solved in such a way that the multiple is defined as a dependent variable influenced by various independent factors (Feldman, 2005; Koller et al., 2005; Titman & Martin, 2008).

Koller et al. (2005) as well as Titman and Martin (2008) point out that while discounted cash flow models are independent from the market's perception, the underlying assumption of relative valuation is that the market is correctly assessing the value of a business, at least on average. Because of this, results from the two approaches will usually differ to some extent, but divergence can go as far as contrary outcomes in the question of whether a stock is over- or undervalued.

3 The Cost of Capital

When looking back at the various discounted cash flow valuation models presented earlier, different discount rates have to be used depending on the approach taken. Equity valuation models use the cost of equity as required rate of return, while the firm's cost of capital is the right denominator when taking an enterprise valuation approach. In excess return models, either the cost of equity or the firm's cost of capital qualify as the correct discount rate, depending on how exactly the valuation model is designed. In the adjusted present value approach itself different discount rates are used for the separate components constituting a company's total value, as the value of the unlevered firm is calculated by discounting at the unlevered cost of equity, while the right discount rate for the value of tax benefits is assumed to be the cost of debt. The unlevered cost of equity is also the accurate discount rate in capital cash flow valuation, yet it is the risk-free rate that has to be used when choosing certainty equivalent models.

There are mainly two reasons for this necessity of different discount rates: First, the attempt to forecast a company's development and estimate the value of business opportunities comes hand in hand with the element of risk, as the future can hardly be accurately predicted. As such, risk has already been identified as one of the crucial elements in every valuation earlier (Gitman, 2006). Second, it has been said that no company is just like another. The specific needs of operations and external factors such as regulation will also leave an imprint on the financing of the firm, and capital structures may differ significantly. Together, risk and capital structure account for the adjustment of discount rates depending on the company observed and the valuation approach taken. These two elements shall therefore briefly be covered, before the differences in discount rates will be presented in detail.

3.1 Basics

Before thinking about the cost of capital, one should try to define the two components of this widely used expression. Cost is defined as an amount given or required as payment (Oxford English Dictionary, 2012). For the definition of capital, Armitage (2005) states three meanings in common use, where differences result from different levels of perspective: From an individual person's viewpoint, capital might have the same meaning as savings or wealth, more exactly their various assets minus personal borrowing; For a company, capital is the sum of debt and equity tied up in the firm; Looking at the

whole economy, capital describes the tangible and intangible real assets in the production process.

The second, namely the company's position, is the one being followed throughout this paper. It should be mentioned that besides a company also a single project, an operation or a division each have capital, since such business units can be seen as discrete entities and therefore could also exist as independent companies, with the ability to issue debt and equity (Armitage, 2005).

3.1.1 Opportunity Cost and Hurdle Rate

Levy and Sarnat (1978) now define the cost of capital as the minimum required rate of return on new investment. A possibly even more accurate way is to describe it as the minimum expected rate of return needed to attract the required capital for funding a project (Armitage, 2005). Finally, Young and O'Byrne (2002) explain it as the rate of return a capital provider would expect to receive if the capital were invested elsewhere.

One can see that the cost of capital is actually an opportunity cost, namely the rate of return from the next-best alternative. Only if a project's expected return is higher than an equally risky alternative's, capital will be committed to the original project and wealth will be increased. Because markets are assumed to be efficient, equally risky assets will present exactly the same expected rate of return to investors. This single market rate for each risk-level suggests seeing the cost of capital as a hurdle rate, as investors could always earn this minimum expected rate of return from other assets with the same risk. Wealth will therefore only be increased if the expected rate of return for a project is higher than the market rate (Armitage, 2005).

3.1.2 Certainty and Uncertainty

Focusing on expected rates of return already indicates an important element of the cost of capital, namely that these rates are based on the future (Young & O'Byrne, 2002). This explains why different risk-levels have to be taken into account at all: The future is uncertain.

Under certainty, the outcome of projects or operations are known and there is no risk involved, therefore companies anticipate not a range of possible returns, but one exact result for a prospective profit. Because all investment opportunities bear the same, so to say no risk, they will share the same required rate of return, which is, to be more

precise, the risk-free rate. Uncertainty describes a situation under which future profits are not exactly known. Only a variety of alternative outcomes – subject to different states of nature – and their probabilities are known (Armitage, 2005; Levy & Sarnat, 1978). When explaining the meaning of risk in finance and valuation, Damodaran (2006) refers to the Chinese symbol for risk, which is a combination of the symbols for danger and opportunity. This perfectly points to the tradeoff investors are facing, as the chance for higher returns comes hand in hand with an increased risk.

As a result, the involvement of risk asks for the adjustment of discount rates, since different projects will have different levels of risk. This is because rational investors are risk-averse, meaning they better like less risk than more. For providing capital and engaging in riskier projects they require payment, namely in the form of higher expected returns (Young & O'Byrne, 2002). The critical element of how to adequately calculate such compensation will be covered in subsequent sections.

3.1.3 Capital Structure

As was said before, the sum of debt and equity tied up in a company constitute this firm's capital. Yet, this is only a simplification, as there is a wide range of financing tools that managers can choose from besides classic debt and equity capital. Koller et al. (2005) name various forms of debt financing such as straight debt, convertible bonds, and commodity-linked bonds as well as other types of structured debt. In addition to common and preferred shares, which can be seen as traditional equity instruments, they list employee stock options, convertible preferred stock, or tracking stocks.

It should be noted that for the estimation of discount rates the definition of a firm's invested capital includes only those parts of debt and equity that are interest-bearing. Accounts payable, unfunded pension liabilities and leases, or all other non-interest-bearing liabilities are explicitly excluded. While these sources of financing do not influence a company's cost of capital and are therefore left out from calculation, they still affect overall firm value through their influence on future cash flows (Titman & Martin, 2008).

There are two major differences between debt and equity that have considerable effect on a company's cost of capital: First, the diverging nature of claims to cash flows that arise from either debt or equity holdings. While, at least in the case of basic forms of debt financing, debtholders have to be paid a fixed amount of interest per period, equity

holders are only entitled to what remains after these interest expenses as well as tax obligations. Second, the order of these payments directly affects their treatment with regards to taxes. Debtholders' claims are serviced before taxes are calculated and, as such, interest payments are tax-deductible. On the other hand, as shareholders receive their dividends well after taxes have been paid, payments to equity investors lack this characteristic of tax-deductibility. The former of these two differences has a direct effect on the rates of return required by equity and debt holders, with the required rate of return to debt holders normally being lower due to priority of their claims. The latter, namely divergence in tax treatment, albeit having no direct influence on the required rates of return significantly affects a company's overall cost of capital (Titman & Martin, 2008).

3.2 Value Creation

It would be a narrow approach to view the cost of capital as nothing more than a discount rate when valuing investments or companies. Seeing capital as a resource that has to be paid for will raise awareness for the necessity of capital budgeting and the requirement to invest into projects where present value exceeds initial costs in order to increase wealth. A more efficient use of resources, a resulting lower amount of capital in the books, and an improved allocation of this capital will equally contribute to value, for capital charges are going to decrease (Armitage, 2005; Young & O'Byrne, 2002).

Readjusting a company's capital structure can decrease the company's WACC and enhance value. As interest payments are tax deductible, an increase in the leverage ratio will let the WACC decrease. This cannot be done infinitely though. As fixed payments from interest are increasing with additional debt, a higher fraction of operating profit will have to be paid to debtholders. This increases the default risk, as a year with unexpectedly low earnings would put the company in a situation where they cannot satisfy all interest claims. The interest rate debtholders require will increase with a higher leverage ratio in order to keep their expected required rate at the same level. In the case of default, there will be additional direct expenses to lawyers, courts, accountants and investment banks in case of reorganization, as well as indirect costs from the loss of confidence and resulting departure of customers. Further, a possible reduction of control would certainly not be in line with the goal of shareholders' wealth maximization (Brealey et al., 2006; Young & O'Byrne, 2002).

As Titman and Martin (2008) point out, there can be considerable differences between what should be done based on academic theories, and what really is done in practice. While there are established models of how discount rates used to evaluate investment opportunities should be determined, managers may face internal corporate hurdle rates well above these discount rates. This may serve as additional motivation or as insurance against too optimistic estimations. In the case of budget constraints such hurdle rates may have the desired effect, but a company might lose the chance to create additional value in a situation where there would be capital on hand.

3.3 Discount Rates

3.3.1 Weighted Average Cost of Capital

As mentioned earlier, companies are financed with various forms of capital, each carrying a certain amount of risk. The resulting required returns confront the company with different costs and weighting each cost with the financing form's fraction of total capital gives a weighted average cost of capital (Young & O'Byrne, 2002). The following, simplified formula for the WACC can be found in slightly differing forms in various textbooks (Brealey, Myers & Allen, 2006; Koller et al., 2005):

$$WACC = r_E \times \frac{E}{D + E} + r_D \times \frac{D}{D + E} \times (1 - T) \quad (21)$$

where r_E and r_D stand for the cost of equity and debt, and T is the tax rate. Since E and D are the market values of equity and debt, $D + E$ gives the firm's total market value. It is a simplified formula because there can be various categories of equity or debt as well as other types of financing (Levy & Sarnat, 1978). Koller et al. (2005) clearly state that costs from all sources must be included, with additional terms representing any other financing form's required rate of return and weighting.

In the simple case, WACC is calculated from the cost of equity and the after-tax cost of debt, weighted by the percentage of equity, respectively debt of total value, at market values. Weightings based on historical book values do not reflect the actual cost of raising capital today, since the amount of cash current investors could raise by selling their holdings is only measured by market values. An alternative way is to use the

company's target capital structure, as current weights may not be in line with those necessary to succeed in the future. The advantage of target capital structures is that short-term changes, e.g. a stock price movement, that have yet to be rebalanced are not affecting the calculation and therefore cannot falsify the company's cost of capital (Armitage, 2005; Koller et al., 2005; Young & O'Byrne, 2002).

3.3.2 Cost of Debt

Already Ehrhardt (1994) had raised the question if the average rate on existing debt should be used or the rate one would have to pay when issuing new debt, and if the cost of debt should be adjusted for expected bankruptcy costs. He draws attention to the fact that for an estimation of a proposed project's cost of capital, historical borrowing rates are inappropriate if one can no longer borrow at these conditions. Still, Young and O'Byrne (2002) deny the use of the more accurate expected rate and recommend the pre-tax rate paid to the company's lenders as the cost of debt, additionally pointing out that if debt financing comes from various sources, the cost of debt is a weighted average itself.

For estimation of the current borrowing rate, Ehrhardt (1994) suggests calculating the yield to maturity (YTM) of the company's outstanding debt in case it is publicly traded and looking at yields on similar bonds if it is not. The yield to maturity can be calculated using the following bond valuation equation (Bodie, Kane & Marcus, 2005):

$$Bond\ Price = \sum_{t=1}^{t=n} \frac{Coupon}{(1 + YTM)^t} + \frac{Par\ Value}{(1 + YTM)^n} \quad (22)$$

Koller et al. (2005) equally propose the use of the yield to maturity on the company's long-term bonds, at the same time mentioning that this is only an approximation to the expected cost of debt itself, as the yield actually represents a promised rate of return. Armitage (2005) further explains this problem, stating that in case of no default, the actual rate of return to the lender will have exceeded the expected rate of return. The promised rate equals the expected rate only if the default risk is zero. Under the possibility of default, the promised rate incorporates compensation for the expected loss from default, therefore being higher than the rate the lender actually expects to receive. At the same time the author acknowledges the difficulty of estimating the expected cost

of debt in practice, whereas Koller et al. (2005) simply describe the inconsistency as immaterial for highly rated, so to say investment-grade debt. For a one-period bond issue, Titman and Martin (2008) offer the following approach to calculate the cost of debt for debt with default risk, stating that expected cash flows have to mirror the probability of default (PD) and the respective recovery rate (Rec) on outstanding debt should it really come to bankruptcy:

$$\text{Bond Price} = \frac{(\text{Coupon} + \text{Par Value}) \times (1 - \text{PD}) + (\text{Coupon} + \text{Par Value}) \times \text{PD} \times \text{Rec}}{(1 + r_D)} \quad (23)$$

As has been said before, companies can resort to hybrid forms of financing such as convertible bonds. Due to the debtholders' right of converting such bonds into equity under certain circumstances, this type of debt normally carries lower interest rates. As a result, using only the bond valuation will underestimate the cost of debt, ignoring the value of the option to exchange. The true value of such a bond must capture both components, namely the value of the straight bond plus the value of the conversion feature (Titman & Martin, 2008):

$$\text{Convertible Bond Value} = \text{Straight Bond Value} + \text{Call Option Value} \quad (24)$$

Thus, the cost of debt for convertible bonds can be understood as a weighted average of the cost of issuing a straight bond and the cost of the exchange option. Bodie et al. (2005) acknowledge the idea of treating a convertible bond's value as the sum of the aforementioned components, yet they identify several reasons that make practical implementation difficult. This can be because of increasing conversion prices and resulting changes of the option's exercise price, or because dividends from the stock may complicate the valuation of the option price. Further, convertible bonds can be designed in a way that the firm as issuer holds a call option and therefore has the right to repurchase the bond, making it virtually impossible to determine the actual maturity and thus the value of the bond.

Finally, adjusting the cost of debt to an after-tax rate in the WACC formula captures the value of the interest tax shields (Brealey et al., 2006). In other words, specific cost of debt lies below the expected cost, since interest payments are tax deductible (Levy &

Sarnat, 1978). The marginal tax rate should be calculated consistently, but adjustments might be necessary as the future marginal tax rate can be different, depending on the timing of future tax payments (Koller et al., 2005).

3.3.3 Cost of Equity

Like the cost of debt, also the cost of equity is an opportunity cost, since the expected return on the company's shares can be achieved by investing in other assets on the same risk-level. Neither the risk, nor the expected rate of return can be detected under normal conditions though, as both do not depend on known outcomes, but on forecasts of future returns (Armitage, 2005). The difference to the cost of debt now is that the rate equity investors require cannot be directly observed, as analogue contracts defining the terms of repayment or distribution to equity-holders do not exist. Besides the impossibility of inquiring millions of shareholders, finding the cost of equity by directly asking investors to state their desired rate is not a viable option, as they might not even be able to articulate a precise required return (Young & O'Byrne, 2002).

One should remember that these difficulties do not exist for the cost of preferred equity, as holders of straight preferred stock receive a fixed dividend each period. As such, the cost of preferred equity r_{PE} can easily be calculated as a function of the current stock price (Titman & Martin, 2008):

$$r_{PE} = \frac{Div_{PE}}{P_{PE}} \quad (25)$$

Taking a similar approach to common equity, one can now try to estimate the cost of equity by using a dividend growth model. Instead of estimating future dividends into infinity, a constant growth rate for dividend payments is assumed. In such a case, the cost of equity will simply be the expected dividend yield plus the growth rate (Ehrhardt, 1994):

$$r_E = \frac{E(Div)}{P} + g \quad (26)$$

Armitage (2005) acknowledges this basic idea of creating an analogy between dividends and the interest payments on debt. At the same time, he points to the cost of

equity's characteristic as an ex ante conception and the fact that its estimation in advance is based on expectations. It will therefore not be conditional on actual observed cash flows, which are likely to deviate from the expected ones, and consequently result in differences between expected returns and actual distribution to shareholders. Titman and Martin (2008) also point to the possibility of such a divergence and note that even for the cost of preferred equity the promised dividend does not necessarily have to equal the return an investor expects to receive. This is because payments may be suspended in the case of financial constraint, or a company might even go bankrupt, resulting in a lower priority of preferred stockholders' claims to the firms' assets than those of debtholders (Bodie et al., 2005). As such, the promised dividend presents an upper limit on the cost of preferred equity.

One can see that the use of dividends as an indicator can turn out to be problematic, due to the backward look when estimating the cost of equity and the upward bias when estimating the cost of preferred equity. As the cost of equity is indispensable for calculation of the WACC, one will have to resort to another option, namely deducting the required rate of return from observation of capital markets. As we are assuming effective capital markets, certain models will give an estimate for the pricing of risky assets (Young & O'Byrne, 2002). Because the cost of equity is such a crucial element, the following section will cover methods of how to make a calculation from capital market observation.

4 Calculating the Cost of Equity

4.1 Capital Asset Pricing Model

4.1.1 Theoretical Framework

The most widely used model of how risky assets are priced by the capital market is the capital asset pricing model (CAPM). As with every model, a number of assumptions have to be made in order to successfully build the CAPM. In an effort to summarize them, it can be said that all investors are risk-averse, have homogenous expectations, act under a one-period horizon and are confronted with the same situation of a perfect, frictionless capital market. This restrictive approach is necessary to change the focus from an individual's investment to a situation where everybody invests in a comparable way. Only examination of such collective behavior in the market allows finding the equilibrium relationship between risk and return (Sharpe, Alexander & Bailey, 1999).

As a result of these stringent assumptions, investors in the model will invest either into a risk-free rate, or a market portfolio which comprises all existing assets. Since investment decisions are conditional on individual utilities and distinct preferences concerning risk and return, investors put different weights on the risk-free rate and the market portfolio. The combination of risky securities will be the same for all investors though, leading to a linear relationship between risk and return, depending only on the weighting of the investor's portfolio (Harrington, 1987; Sharpe et al., 1999).

4.1.2 Calculation

The above mentioned linearity then leads to the following formula with which an asset's expected return can be measured (Harrington, 1987; Koller et al., 2005; Sharpe et al., 1999):

$$E(r) = r_F + \beta \times (E(r_M) - r_F) \quad (27)$$

where $E(r)$ denotes the expected rate of return, r_F the risk-free rate, $E(r_M)$ the expected market return and β a factor specific to the asset's risk. Ehrhardt (1994) indicates that a stock's expected return equals the company's cost of equity. Consequently, after measuring the risk of the company's stock, the CAPM formula can be used to convert

that risk into the cost of equity. Harrington (1987) and Young and O'Byrne (2002) explain the logic as follows: The risk-free rate is the minimum return an investor would expect to receive from any asset, but additional compensation is required for risky assets. This compensation ($E(r_M) - r_F$) is called the market risk premium (MRP), a price paid to all investors in the stock market. MRP will be adjusted for beta, the asset's risk factor.

After this simple explanation, several problems must be pointed out, which exactly relate to the components of the CAPM formula. The capital asset pricing model is based on expectations and not on past events, but since such expectations cannot be observed, one will have to resort to estimates. Additionally, no guidance for actual implementation is provided in the model. This leaves questions marks behind the appropriate risk-free rate, the market risk premium and the calculation of beta (Koller et al., 2005).

How these elements can be determined in practice to allow for the use of CAPM will be addressed in the following.

4.1.3 Risk-Free Rate

The assumption of a risk-free rate in the CAPM raises a couple of questions that should be mentioned before talking about ways to estimate such a rate. Harrington (1987) raises the question if a risk-free asset exists at all, and if all investors can borrow and lend at such a rate.

Sharpe et al. (1999) see government securities as riskless, because government can always choose to print money when necessary, virtually creating certainty on promised repayments. Still, they acknowledge a level of uncertainty concerning the purchasing power of such repayments, since nominal returns might differ from real returns due to inflation. The argument made about borrowing and lending at such rates is even harder to offset, because the assumption of free access to such risk-free assets creates a falsified image of the world. Yet, relaxing this theory would possibly affect the linearity of the CAPM or, even worse, lead back to investor-specific situations, therefore the borrowing-lending assumption has to be accepted to keep the model's integrity (Harrington, 1987).

Ehrhardt (1994) mentions the yield on a short-term Treasury bill as the most widely used proxy for the risk-free rate or, to be more exact, the Treasury bill with maturity closest to one month. However, he presents arguments for the 13-week-bill most re-

cently auctioned, stating that the profound trading for this security might give more confidence in the reported yield. Similarly, Harrington (1987) describes this 90-day Treasury bill as virtually the only proxy employed as the riskless asset. At the same time, attention is drawn to the fact that the Treasury bill is not pure market rate, because influence can be taken through interest rate control or money supply.

According to Koller et al. (2005), nowadays the 10-year government bond is the most common security taken as proxy for the risk-free rate (for U.S.-based corporate valuations). They emphasize the ideal situation of using different government bonds with maturities similar to the timing of expected cash flows, but admit that such a matching of maturities is seldom done in practice. Also, the use of local government bond yields is recommended, but importance has to be given to the fact that only default-free bonds work when estimating the risk-free asset. They equally call attention to the wrong estimation resulting from the use of short-term Treasury bills when valuing companies or long-term projects, hence reject this approach and oppose the above mentioned authors. Their explanation for this error is that with CAPM, expected returns are typically calculated for the next month.

4.1.4 Beta

As stated earlier, investors are assumed to be risk-averse in the world of the CAPM. Additionally, they opt to be diversified, so to say, to not invest into just one stock, but a portfolio of such. Under the concept of the CAPM, a security's risk can be split into two parts. It can be observed that while share prices of all stocks listed on an exchange often increase or decrease together, sometimes a single stock develops into a direction differing from the market. This is because variations in a company's stock price on one hand depend on circumstances affecting the market as a whole, such as announcements like an economy's current growth in gross domestic product (GDP), but on the other hand they are conditional on occurrences uniquely affecting the company or its industry.

While investors can do nothing against market movements, diversifying their portfolios will cancel out company-related stock price changes, if sufficiently many stocks are included in the portfolio. Hence, the component of risk conditional on overall fluctuations has a systematic relationship with the market portfolio, whereas the company-specific part of the risk does not at all offer such a systematic connection (Brealey et al., 2006; Levy & Sarnat, 1978; Young & O'Byrne, 2002).

$$\begin{aligned}
\text{Total Risk} &= \text{market risk} + \text{company-specific risk} \\
&= \text{non-diversifiable risk} + \text{diversifiable risk} \\
&= \text{systematic risk} + \text{unsystematic risk}
\end{aligned}$$

Because the company-specific, unsystematic risk can easily be diversified away, investors cannot expect to be paid for such a risk. The market will only offer payment for bearing the systematic risk resulting from investment into the market portfolio. Beta now measures to what extent the market and stock move together, or in other words, how sensitive the volatility of a company's stock price is to market movements. This sensitivity is caught in a proportional reward for taking on the market risk (Sharpe et al., 1999; Young & O'Byrne, 2002).

For the calculation of a stock's beta, the covariance of the stock's return and the market's return is divided by the variance of the market's return.

$$\beta = \frac{\text{Cov}(r, r_M)}{\text{Var}(r_M)} \quad (28)$$

It is easily visible that the beta for the market portfolio equals 1. Risky stocks with a beta greater than 1 are known as aggressive stocks, and will amplify the overall market development. Less volatile stocks are known as defensive stocks and have a beta between 0 and 1. A negative beta indicates a stock usually swinging into the opposite direction from market movement (Brealey et al., 2006; Sharpe et al., 1999).

Besides the possibility of retrieving betas from a published source, estimations of beta can be made using a regression analysis, thus regressing the returns of the company's stock against the market returns. Koller et al. (2005) recommend the use of at least 60 data points or otherwise the result could be biased. Several other important questions in practice remain as follows (Ehrhardt, 1994; Harrington, 1987; Young & O'Byrne, 2002; Koller et al., 2005):

First, future expectations are still unobservable, hence the measurement period of historical returns for estimation of beta becomes even more crucial. Longer samples increase statistical significance, but a too long timeframe could include information unlikely to still affect the relationship between market and stock returns in the future.

Second, related to the length of historical observation is the difficulty of choosing an appropriate return interval. Most often, daily and monthly returns are chosen as frequency, but one might also prefer to use weekly or annual returns. However, shorter intervals tend to be noisier, especially if the stock is rarely traded.

Last, an appropriate market index has to be found as proxy for the market portfolio. Preferably, the index includes a large number of securities, so that a higher grade of diversification is reached, and weighs the comprised stocks by value, as this is what the theory underlying the CAPM demands.

A final, brief thought on betas should be given to the case that the cost of capital has to be calculated for divisions or companies that are not traded on a stock exchange. Young and O'Byrne (2002) suggest using the betas of comparable companies in the same or similar industry. First, assuming pure equity financing, these levered betas β_L have to be unlevered. Simply restructuring the formula, the average of the unlevered betas β_U should then be relevered to account for the company's current or target capital structure.

$$\beta_U = \frac{\beta_L}{1 + (1 - T) \times \frac{D}{E}} \quad (29)$$

4.1.5 Market Risk Premium

The only remaining component of the CAPM formula is the market risk premium ($E(r_M) - r_F$), equal to the amount by which the market return is expected to exceed the risk-free rate (Ehrhardt, 1994). Cornell (1999) indicates that this does not explain what the risk premium for an individual stock is, but what the stock's risk premium relative to the market portfolio is. This relation is achieved by multiplication with the stock's beta. The reason for calculating the beta and MRP instead of directly estimating an individual stock's equity risk premium is attributed to a combination of factors. On short sample periods, variances, and as a result the beta, can be estimated more accurately than means, hence average risk premiums in general. As returns on a single security are more volatile than those on the market, the market risk premium is comparably easier to calculate than an individual security's premium (Cornell, 1999).

According to Koller et al. (2005), the MRP can be estimated with various methods, namely using the historical excess returns, using regression analysis, or using a discounted cash flow valuation. Due to the general difficulty of finding the MRP and continuing lack of precision as well as variations of these approaches, their implementation will not be clarified in detail and only the overall functioning will briefly be explained.

Assuming that the level of risk aversion has not changed, one can employ historical excess returns as a proxy for future premiums. Without any existing trends in the risk premium, it is advisable to use the longest period possible and calculate the average of all past premiums by comparing historical market returns to risk-free securities to achieve an annual number. Long-term government bonds should be used when matching for the cost of capital of long-term investments as discussed earlier (Ehrhardt, 1994; Koller et al., 2005).

Regression analysis can be used since arguments are made that the MRP is predictable by means of observable variables like dividend-to-price, earnings-to-price or book-to-market ratio. Hence, excess market returns are regressed against such financial ratios to estimate the market risk premium (Koller et al., 2005).

The DCF approach makes use of a dividend growth model. As stock prices should keep up with growing dividends in the long run, the expected market return is measured by adding the average long-term growth rates in dividends to the average dividend yield (Brealey et al., 2006; Cornell, 1999).

As a result of the number of methods to estimate the MRP, combined with different assumptions undertaken and varying interpretations concerning the appropriate inputs, there is not one unanimously accepted market risk premium, but an ongoing discussion about what could be an accurate value. Actual development concerning the current situation on capital markets all over the world was not taken into account for this paper, but one might still wonder if the debate as a result has not further increased, instead of ceased.

The variations are apparent when looking at the results already Koller et al. (2005) obtain when taking different approaches: Using historical data over 100 years, they observe a downward trend of annualized excess return from 6.2 percent to 5.5 percent with increasing holding period. With the regression method, they calculate a negative expected market risk premium for several years. Finally, applying the constant dividend growth model, MRP turns out to be just under 5 percent. In the end, a range from 4.5 to 5.5 percent is declared as appropriate (Koller et al., 2005).

Equally, Young and O'Byrne (2002) state a widely used MRP of 5 percent, plus or minus one percentage point. Brealey et al. (2006) take no official position, but believe that for the U.S., a risk premium in the range from 5 to 8 percent would be fitting, and Damodaran (2002) uses an MRP of either 4 or 5.5 percent for most examples in his textbook.

In a comparison of risk premiums produced by competing approaches, Cornell (1999) lists the results from various authors' calculations using different types of analysis. The estimations for the premium over bills range from roughly 4.6 to 9.2 percent, whereas the calculations for the premium over bonds spread from around 2 to 7.4 percent. In a survey by Bruner, Eades, Harris and Higgins (1998), 37 percent of inquired firms use a fixed rate between 5 and 6 percent as their MRP.

Young and O'Byrne (2002) point out that high MRPs are a result from the bullish markets since the 1980s, as equity investors required large premiums over the returns from bonds. The problem arising is that future earnings and EVAs reflected through the stock price can actually only satisfy such high premiums when growth rates reach heights observed in booming markets. In other words, lower required growth rates can only result from a decrease of the market risk premium. At the time, Glassman and Hassett (1999) already declare a rate of 3 percent for the MRP, much lower than estimates from other sources.

They further expect a movement to the level of zero, effectively resulting in disappearance of premiums over bonds. One argument for this is the statement that stocks are a safer long-term investment with regards to purchasing power than bonds. Other reasons could be investors' better education and information concerning financial markets. Because of being smarter and calmer, they require less expected excess return to compensate for their fear. Also, shareholder pressure along with global competition and computer technology forced companies to reorganize and increase efficiency. Other factors are improvement in government's monetary and fiscal management, as well as more liberal tax and regulatory framework (Glassman & Hassett, 1999).

Armitage (2005) points to the wide range of premiums currently used and the impossibility of giving a definite answer which method of measuring MRP is correct. He emphasizes that the choice from such an array can have considerable impact on the cost of equity, hence the cost of capital, and might even make a bigger difference than any other aspect, like tax adjustments or estimation of beta.

The previous presentation of several diverse views is not to state what is right and wrong, but to highlight what effect the choice of method, measurement period, interval, etc. can have on the outcome, and thus the cost of capital, and how this outcome can differ depending on the assumptions made. This holds not only for the calculation of market risk premiums, but also for the other factors in the CAPM.

4.1.6 Problems and Limitations

Some of the difficulties with regard to the assumptions of the CAPM as well as the three components in the model have already been mentioned, but they will again be listed here briefly. Economic models are simplifications of the reality, but those simplifications are needed to interpret what is happening around us (Brealey, 2006). The CAPM is such an economic model, and therefore has a set of assumptions to keep it working. Some of the problems stem from the fact that, although based on Nobel Prize-winning theory, it offers no instructions for practical implementation. An example of this is the concept as a one-period model, but at the same time no information is given on how long this period is supposed to be, leaving all decisions concerning timeframe to the user. Similarly, all the choices for estimation of appropriate risk-free asset, market risk premium and beta are taken by those using the model (Koller et al., 2005).

The assumptions underlying the CAPM are very restrictive and in certain cases obviously not true, as in the real world such things as taxes, transaction costs or inflation do indeed exist (Harrington, 1987). Likewise, Armitage (2005) describes the assumption that investors know the contingent future returns of an asset and their probabilities and can therefore calculate that asset's variance and covariance as hopelessly unrealistic. Still, he acknowledges the combination of stringent theoretical basis with practicality, stating this as the reason for the model's success in finance.

Besides such trouble caused from lack of guidelines, the model is subject to two more serious drawbacks.

First, the CAPM is based on expected returns. As expectations are unobservable though, one has to resort to apparent actual returns. Such actual returns should impound expectations, but they are also exposed to noise, so to say unexpected events that hide whether, on average, investors have received their expected return. Hence, it is impossible to compare the success of different models (Brealey et al., 2005).

Second, Roll (1977) points out that the true market portfolio includes all risky assets, such as stocks, bonds, commodities, real estate, but also human capital. This makes

the CAPM virtually untestable, as every time a proxy for the market portfolio is tested, it will actually be a joint test of the following two hypotheses: the truth of the CAPM, and if the chosen proxy is efficient, meaning that no subset from the proxy gives higher risk-adjusted returns than the proxy itself. In the end, it cannot be found out whether the CAPM is actually correct, because instead of judging the model as wrong, one might simply have chosen an inappropriate proxy for the market portfolio (Armitage, 2005; Fama & French, 2004; Roll, 1977; Young & O'Byrne, 2002).

4.2 Multifactor Models

Multifactor models, as opposed to the standard version of the CAPM, explain expected returns dependent on correlation with two or more risk factors (Armitage, 2005). The idea behind this approach is that the relationship between risk and return may be more complex and require a security's required rate of return to be defined as a function of more than just its correlation with the market, as expressed through the beta coefficient (Brigham & Ehrhardt, 2008). Ehrhoff (1994) gives an example where two companies are both affected by two types of costs, but each with a different sensitivity to these factors. Therefore the addition of a unique component to stock returns, besides the common factors, could be suggested. Two such models will be introduced in the following.

4.2.1 Arbitrage Pricing Theory

The arbitrage pricing theory (APT) of Ross (1976) is an alternative method of calculating a risky asset's rate of return. While the CAPM builds from the question if a portfolio is efficient, the original approach in the APT is different, because it starts with the assumption that a stock's return is conditional on several macroeconomic factors as well as noise (Brealey et al., 2006).

Then again, there is one common element to the logic of the CAPM, namely that investors are only paid for bearing non-diversifiable risk. In APT, returns are assumed to depend on predictable and surprise elements:

$$r = E(r) + U \tag{30}$$

The total return r is the sum of the predictable component $E(r)$, the expected return, and the unanticipated component U , also called surprise component. The unanticipated component is nothing else than a company-specific element. Because one part of the total return is predictable, it should already be impounded in the company's stock price. As then only the surprise element U will cause the share price to move, this is the only element attached with risk (Young & O'Byrne, 2002).

Here the analogy to the CAPM can be drawn: Just like the unsystematic, company-specific risk in the CAPM can be diversified away, also the company-specific effects of the unanticipated component in APT are diversifiable by investing not just into one stock, but into a portfolio (Brealey et al., 2006).

Only the systematic, non-diversifiable macroeconomic factors remain from the unanticipated component:

$$r = E(r) + \beta_1 \times f_1 + \dots + \beta_i \times f_i + \varepsilon \quad (31)$$

where β_i is the asset risk factor for systematic factor i , f_i is the price of this systematic factor i on capital markets, and ε is the unsystematic portion of stock returns. In other words, the realized return on any stock equals its expected return, plus increases or decreases resulting from unexpected changes in fundamental economic factors times the sensitivity of the stock to these changes, plus a random term reflecting changes that are unique to the firm (Brigham & Ehrhardt, 2008). Young & O'Byrne (2002) equally stress the importance that only the surprise changes in macroeconomic indicators can present possible systematic risk factors, as the expected portion of macroeconomic effects will already be impounded in $E(r)$.

Similar to the lack of guidance in the CAPM, also in the original APT it is neither stated or known what these multiple systematic risk factors represent, nor how many there should be (Harrington, 1987).

Sharpe et al. (1999) summarize several factors that later have been identified or suggested by various authors, including the following: growth rate in industrial production, rate of inflation (both expected and unexpected), spread between long-term and short-term interest, spread between low-grade and high-grade bonds, growth rate in aggregate sales in the economy, rate of return on the S&P 500, growth rate in gross domestic product, rate of interest, rate of change in oil prices, rate of growth in defense spending.

To sufficiently describe the systematic risks influencing stock returns, as many risk factors as necessary can be chosen. One should keep in mind that for deriving a cost of equity, all the betas have to be measured and the factors have to be priced in relation to the risk of unanticipated changes borne by the investor.

Young and O'Byrne (2002) describe this process as even more calculated than for the CAPM. Unanticipated changes in factors have to be derived by comparison of expected and actual values (e.g. for inflation). Betas would be calculated using time series regressions, but it is necessary to compute them for a large number of stocks in the same market in order to be able to work out the price of the factors by cross-sectional regression afterwards. All of this has to be done over a long time horizon so to avoid statistically biased results.

Koller et al. (2005) call the APT extremely powerful in theory but elusive in practice. They attribute this to disagreement of how many and which factors to use, and how to measure them. Young and O'Byrne (2002) show that the measurement is an extremely complex process. They acknowledge the high grade of explanation of stock price movements and increased understanding of risk exposures the arbitrage pricing theory offers, but point to the fact that application of APT in practice is far more difficult than the CAPM.

4.2.2 Three-Factor Model

According to Fama and French (1992), stock returns are inversely related to the size of a company, measured by market capitalization, and are positively related to a firm's book-to-market ratio. This is because small firms are more sensitive to changes in business conditions, and firms with high ratios of book to market value are more likely to be in financial distress. Applying a multifactor model based on this information, the cost of equity for a company can be calculated as follows:

$$E(r) = r_F + \beta_M \times (E(r_M) - r_F) + \beta_{SMB} \times E(SMB) + \beta_{HML} \times E(HML) \quad (32)$$

where $E(r)$ is the expected return on the firm's stock, r_F is the return on one-month treasury bills, and $E(r_M)$ is the expected market return. Small minus big (SMB) represents a risk factor associated with small size, so $E(SMB)$ is the expected premium on

small companies. High minus low (HML) stands for a risk due to a high book-to-market ratio, hence $E(HML)$ is the expected premium on companies with a high such ratio. β_M , β_{SMB} and β_{HML} are the company's betas, measuring the sensitivity to the respective risk premiums, which are calculated by averaging historic values (Armitage, 2005; Koller et al., 2005).

4.3 Estimating the Cost of Capital in Practice

To finalize, a brief overview shall be given which methods are preferred for use in practice. Bruner et al. (2001) find out that CAPM is the most widely used model for estimating the cost of equity. 81 percent of firms participating in the survey stated the use of this method, another 4 percent make use of a modified CAPM. A small minority mentioned to use multi-factor asset-pricing models, such as the APT.

Graham and Harvey (2001) get similar results, as 73.5 percent of respondents declare to always or almost always use the CAPM for their cost of capital calculation, just below 40 percent state to use arithmetic average historical returns, and slightly more than 30 percent make use of a multi-beta CAPM like the APT.

5 Problem Specification

After review of the theoretical framework to valuation as well as to the cost of capital and several different methods to calculate the cost of equity, a number of questions can be imagined regarding these methods. While a lot of questions will already have been answered, research does not cease and new problems may arise that have yet to be evaluated, or it may be thought of a new way to analyze older questions in a different environment.

The application of economic models in practice is an area where new problems may constantly arise, due to significant and often sudden changes in the external environment compared to the mostly static assumptions around the theoretic conception of a model. As such, one might want to pursue answers to questions such as whether the cost of capital, on average, changes depending on the method of estimation, whether the variation or range in results when calculating the cost of equity is affected by the choice of method, or whether the number of risk factors used for the estimation ultimately has an influence on the resulting cost of equity. Also, thinking of stock market turbulences such as the ones caused by the different mortgage, financial, and debt crises over the recent years, one might want to analyze the influence of such phases of significant economic downturn, and, likewise, of recovery and strong growth at other times, on the cost of equity and notably to the aspect of how the different models react to such deviations from the average or known structures, given that models have different exposure to factors such as volatility or correlation, depending on if and how these factors are reflected in a model.

The remainder of this paper will investigate an application of the arbitrage pricing theory in practice. In this context, the principal question that shall be examined is if and how the arbitrage pricing theory functions and can be used as a method to estimate the cost of capital in a limited environment? As such, an empirical investigation will be undertaken on the Austrian stock market, examining if a multifactor model such as the arbitrage pricing theory might be suitable for such a market, and which variables should be considered as factors when building a model based on the APT in order to estimate the cost of capital for one or several companies.

6 Purpose

The initial concept of the arbitrage pricing theory as well as early research on this topic has been based on large sample groups of returns from the US stock markets. Compared to the capital asset pricing model, the underlying theory to which is the existence of a single market portfolio including all assets that can be valued, the arbitrage pricing theory does not define a market portfolio, nor does it indicate any constraints concerning the applicability of the model on markets of different sizes.

Due to this nature of the APT and the approach that more factors and notably such reflecting macroeconomic indicators can be included in the model, one can assume that the approach of the arbitrage pricing theory can easily be adapted to markets of different nature and size. As such, an analysis of the model in the environment of the Austrian stock market will be presented in the following chapters of this paper.

The first question that has to be answered is how many factors to include in a model in order to reasonably reflect any significant influence on a single stock or the stock market in general. The second question is which economic variables can be identified as fitting factors, and as such could be imagined as suitable factors in a model used to estimate the cost of equity of a company on the Austrian stock market. Also, it shall be investigated whether changes to the length of the observation period and/or the size of the sample group have an influence on the results, in particular on the number of factors chosen for the model as well as the economic variables that are ultimately fitting as factors.

The analysis of the aforementioned questions shall help to understand how the arbitrage pricing theory functions within the limits of a small stock market, and how such a small market influences the results, notably the number of factors and the economic variables chosen as factors. While no direct comparison of models and methods will be undertaken, sometimes analysis of a single approach can be sufficient to interpret how this method would fare next to alternative approaches. As such, the question of how the arbitrage pricing theory functions in a limited environment can be interpreted slightly differently as how this method compares to alternatives that are more regularly used in practice, notably the CAPM, when estimating the cost of equity.

The following section, Methodology, presents the framework to the empirical investigation, while Results and Analysis are covered in the chapter thereafter.

7 Method

7.1 Data

Stock market data has been collected from the 39 stocks listed in the prime market of the Vienna Stock Exchange (Wiener Börse) on December 31, 2010 and the analysis is built in part around the two main stock indices, specifically the Austrian Traded Index (ATX) and the ATX Prime. All the stocks in the prime market form the components of the index ATX Prime, which has been calculated since May 7, 1996. A subset of 20 stocks from the prime market is used to calculate the Austrian Traded Index, which is available from January 7, 1986 on. To ensure comparability, no analysis has been conducted outside of the period from May 7, 1996 until December 31, 2010. It should be mentioned that only one company listed in the prime market at the end of 2010 had stocks traded continuously since inception of the ATX.

Of the 39 stocks considered for the investigation, 18 have been traded throughout the whole period of analysis, while trading for the last of the 39 has begun on May 21, 2008. Stock returns have been calculated on daily, weekly, and monthly closing values.

Data for the economic variables that will be used during the tests against the factors was collected from the statistic database of the Austrian National Bank (Oesterreichische Nationalbank, OeNB). The data has been collected on a total number of 41 economic variables, which can be classified into nine types of variables: stock market returns, secondary market government bond yields, the EURIBOR, long-term government bond yields, economic sentiment indicators, labor market indicators (unemployment rates), an index on industrial production, inflation indicators, and commodity prices. As secondary market and long-term government bond yields as well as the EURIBOR are all different types of interest rates, these three were grouped under a class called interest rates, which leaves us with seven variable classes.

Table 7.1 Economic Variables – Data

Variable Class	Variable Name	Description	Type	Calculation	Region
Stock Market Returns	ATX_Return	Stock Market Index	Index	Relative Change	Austria
	ATXPrime_Return	Stock Market Index	Index	Relative Change	Austria
Interest Rates	Secondary Market Gov't Bond Yields	Austrian Bond Yields	Yield	Absolute Yield	Austria
	ATGovBond_YChan	Austrian Bond Yields	Yield	Absolute Change	Austria
	EURBOR	Euro Area Money Market Interest Rates	Rate	Absolute Rate	Euro Area
	Euribor12M_Change	Euro Area Money Market Interest Rates	Rate	Absolute Change	Euro Area
	ATLTbond_Yield	Long-Term Government Bond Yields	Rate	Absolute Rate	Austria
	ATLTbond_YChan	Long-Term Government Bond Yields	Rate	Absolute Change	Austria
	EULTbond_Yield	Long-Term Government Bond Yields	Rate	Absolute Rate	Euro Area
	EULTbond_YChan	Long-Term Government Bond Yields	Rate	Absolute Change	Euro Area
Economic Sentiment Indicators	EUEconSent_Indicator	Economic Sentiment Indicator	Indicator	Value of Indicator	Euro Area
	EUEconSent_ChangeRel	Economic Sentiment Indicator	Indicator	Relative Change	Euro Area
	EUEconSent_ChangeAbs	Economic Sentiment Indicator	Indicator	Absolute Change	Euro Area
Labor Market Indicators	UnempNatDef_Rate	Unemployment rate in %, National definition	Rate	Absolute Rate	Austria
	UnempNatDef_Change	Unemployment rate in %, National definition	Rate	Absolute Change	Austria
	UnempEurDef_Rate	Unemployment rate in %, Eurostat definition	Rate	Absolute Rate	Austria
	UnempEurDef_Change	Unemployment rate in %, Eurostat definition	Rate	Absolute Change	Austria
Industrial Production	ATProd_Index	Production Index	Index	Index Value	Austria
	ATProd_ChangeA	Production Index	Index	Relative Change (Year on Year)	Austria
	ATProd_ChangeM	Production Index	Index	Relative Change (Month on Month)	Austria
	EUProd_Index	Production Index, EU definition	Index	Index Value	Austria
	EUProd_ChangeA	Production Index, EU definition	Index	Relative Change (Year on Year)	Austria
	EUProd_ChangeM	Production Index, EU definition	Index	Relative Change (Month on Month)	Austria
Inflation Indicators	Inflation_FPI_Index	Producer price index	Index	Index Value	Austria
	Inflation_FPI_Change	Producer price index	Index	Absolute Change	Austria
	Inflation_WPI_Index	Wholesale price index	Index	Index Value	Austria
	Inflation_WPI_Change	Wholesale price index	Index	Absolute Change	Austria
	Inflation_CPI_Index	Consumer price index	Index	Index Value	Austria
	Inflation_CPI_Change	Consumer price index	Index	Absolute Change	Austria
	Inflation_HICP_Index	Harmonised index of consumer prices	Index	Index Value	Euro Area
	Inflation_HICP_Change	Harmonised index of consumer prices	Index	Absolute Change	Euro Area
	Inflation_Wage_Index	Negotiated standard wage rate index	Index	Index Value	Austria
	Inflation_Wage_Change	Negotiated standard wage rate index	Index	Absolute Change	Austria
Commodity Prices	Commod_OilArabPrice	Crude oil price, Arabian Light	Price	Absolute Price	Global
	Commod_OilArabChange	Crude oil price, Arabian Light	Price	Relative Change	Global
	Commod_OilBrentPrice	Crude oil price, North Sea Brent	Price	Absolute Price	Global
	Commod_OilBrentChange	Crude oil price, North Sea Brent	Price	Relative Change	Global
	Commod_IndexHWWITotal	HWWI commodity price index, including energy	Index	Index Value	Global
	Commod_IndexHWWITotalChange	HWWI commodity price index, including energy	Index	Absolute Change	Global
	Commod_IndexHWWIExEnerg	HWWI commodity price index, excluding energy	Index	Index Value	Global
	Commod_IndexHWWIExEnergChange	HWWI commodity price index, excluding energy	Index	Absolute Change	Global

7.2 Factor Extraction

Due to the nature of the data set, the factor extraction has been conducted on two groups of stock returns, differing in the number of stocks included and in the analyzed period, with the statistics software SPSS. First, on a group of 18 stocks traded between May 7, 1996 and December 31, 2010. Second, on the complete group of 39 stocks traded in the period from May 21, 2008 until December 31, 2010. For each group, the extraction process has been done separately six times, twice each on daily, weekly, and monthly returns, with the difference being the handling of missing values. These were either excluded pairwise or listwise, meaning an exclusion of only pairs in the correlation matrix as opposed to exclusion of a complete data point (Norusis, 2004a).

The factor extraction has been done using two different methods, specifically Principal Axis Factoring and Principal Components Analysis. With Principal Axis Factoring, factors are extracted from the original correlation matrix and the initial estimates of the communalities are squared multiple correlation coefficients which are placed in the diagonal. The old communality estimates in the diagonal are then replaced by new communalities using these factor loadings. This iteration is continued until the convergence criterion for extraction is satisfied by the communality changes. Principal Components Analysis extracts factors by building linear combinations without correlation out of the observed variables. Maximum variance is attributed to the first component, while gradually smaller portions of variance are explained by the following components which are showing no correlation with each other. This method obtains the initial factor solution and can be used in cases of a singular correlation matrix (Gorsuch, 1983; Kline, 1994; Norusis, 2004b).

To simplify the interpretation of the extracted factors they have been rotated using the Varimax Method. This orthogonal rotation brings the number of variables with high loadings on each factor to a minimum. The use of a rotation keeps the cumulative percentage of variation that extracted components explain at the same level, but allows for a more even spread of this variation over the components. Considerable changes in the individual variance totals support the argument that interpretation is easier for the rotated component matrix than the unrotated matrix (Kline, 1994; Norusis, 2004a).

Two tests have been run parallel to each extraction, namely the Kaiser-Meyer-Olkin Measure of Sampling Adequacy and Bartlett's Test of Sphericity. The former tests the

extent of partial correlations among variables, while the latter test is done to check if the correlation matrix is identical to an identity matrix. More precisely, the Kaiser-Meyer-Olkin Measure of Sampling Adequacy shows the fraction of variance in variables that might be attributable to one or several underlying factors. A factor analysis probably will not yield very valuable results if this statistic shows values below 0.50, whereas values close to 1.0 are a sign that it may be useful to conduct a factor analysis on the data. The test of whether the correlation matrix resembles an identity matrix as used in Bartlett's Test of Sphericity gives indication if the variables are unrelated. This would make them not suitable for further structure detection, and only significance levels below 0.05 suggest usefulness of a factor analysis (Gorsuch, 1983; Kline, 1994; Norusis, 2004b).

Last, to help decide how many of the extracted components should be kept, a scree plot has been used to show the amount of variance related to each factor. As such, the optimal number of factors can be identified by plotting each component's eigenvalue from the initial solution. Normally, such a plot has a characteristic separation between a steep and a shallow slope, where only the large factors of the steep part contribute significant amounts to the solution (Cattell, 1966; Norusis, 2004a).

The goal of the factor analysis is to find a limited number of components representing the original variables, while at the same explaining a large part of their variation, justifying the substitution. The initial solution of a factor analysis always shows as many components as there are variables, with the sum of eigenvalues equaling the number of components in the correlation analysis. Only components with eigenvalues greater than 1 will be extracted, as only these components explain a fraction of total variance larger than what their own variance accounts for. The variance that the extracted components account for in the initial solution does not differ depending on whether Principal Axis Factoring or Principal Components Analysis has been chosen as extraction method. Principal Axis Factoring also shows the cumulative variability explained by the factors in the extracted solution. Generally, the level of variance explained in the extracted solution lies below that of the initial solution, as the factor model simply cannot explain a certain number of aspects that are unique to the total variance or the original variables in general. In other words, the extraction comes with a loss of information whose extent depends on how well the factors represent the overall set of variables (Gorsuch, 1983; Norusis, 2004b).

7.3 Testing the Economic Variables

It is necessary to test if the extracted factors correspond to some economic variables, which then could be used to explain returns on stocks and ultimately to build a model to estimate a stock's expected rate of return. We thus search for economic variables that sufficiently explain the given returns. As such, each of the extracted factors will be tested against a number of economic variables by means of a linear regression, and the results will be analyzed to see if the elements correspond to each other. Most of the economic variables are not available on either daily or weekly basis. As such, testing the economic variables on the extracted factors in a regression will only be conducted on stock returns on a monthly basis. Also, the linear regression will be done several times, that is on each the four factors extracted from the analysis of monthly returns from the small group of stock returns over the longer period, as well as on the nine factors from monthly returns for the large group of stock returns over the shorter period (Draper & Smith, 1981; Weisberg, 1985).

The process of testing the economic variables against each extracted factor in a linear regression has been repeated five times, though changes have been made to the procedure with regards to which and how many of the economic variables were entered into the regression. The regressions on the extracted factors were conducted in decreasing order regarding the amount of variation each factor explained in the previously done factor analysis. As an example, factor 1 always explains more variation than factor 3 and as such could be interpreted as having a higher importance in the overall conception of the model. Therefore, it can be regarded as more important to find the best corresponding economic variable for the first factors than for the last ones. The number of variables entered into the regressions is determined as follows:

In the first regression set, all 41 variables were entered into each regression against the four, respectively nine factors.

In the second regression set, the first predictor from each previous regression was not entered into the following regressions. This is to reflect the idea that the same economic variable cannot be a factor more than once in the model.

In the third regression set, not only the first predictors were left out from regressions on the following factors, rather was the complete variable class not entered into the analysis. For instance, if the first predictor in the linear regression on factor 2 would be a secondary market government bond yield, all interest rate-related economic variables would be omitted from the following regressions on factor 3 and further factors. This

has been done to avoid models with more than one factor representing a certain type of economic variable, such as interest rates or inflation.

In the fourth regression set, all predictors from the previous regressions were not entered into the following regressions. This is a narrower variant of regression set 2 and as such an extension of the idea that the same economic variable cannot be a factor more than once in the model.

The criteria for inclusion of economic variables were most restrictive in the fifth set of regressions, as the variable classes from all the predictors in previous regression outputs were left out from the following factor regressions.

These criteria caused, to a different extent, a reduction in the number of variables entered. While in the first set of linear regressions the number of variables entered stays at 41 and in the second set is reduced by one for each previously regressed factor, the number of variables entered diminishes more strongly in the remaining three sets of regressions. For the linear regressions on the four factors extracted from the small group of stocks over the longer period, the number of variables entered for factor 4 in the fifth (and last) set of regressions has decreased to 10, due to most of the types of economic variables appearing as factors in the previous linear regressions (Draper & Smith, 1981; Weisberg, 1985).

For the linear regressions on the nine factors extracted from the large sample of stock returns over the shorter period, this feat is even more pronounced. In the fifth set of regressions, the number of variables entered for the regression on factor 3 has already diminished to 6, with each of them coming from the single remaining type of economic indicator. For factor 4 and following, no more economic variables are available for entry, as all of the different types of variables were part of the preceding regression sets (Draper & Smith, 1981; Weisberg, 1985).

There are different methods in SPSS for entering variables into a linear regression and the choice of method will affect the design of the linear regression model as well as the number of factors this model is composed of (Norusis, 2004a; Norusis, 2004b).

One option that does not yield useful results is to enter all variables into the regression at once. While this may be applicable in certain situations, it does not help in the current case, as the intention is to use the linear regression in order to find one or several economic indicators which show the highest correlation with a certain extracted factor, and as such could be used in a factor model explaining stock market returns. The Enter

method in SPSS does not reduce the number of economic indicators and is thus not further pursued when testing the economic variables (Norusis, 2004a; Norusis, 2004b). A better choice to enter variables into the analysis is using the Stepwise method. Of all independent variables not yet included in the regression equation the one with the smallest probability of F is entered next, as long as this probability is small enough. In case the probability of an independent variable already in the equation becomes too large, this variable will be removed. The number of variables entered into the regression equation thus depends on their probabilities of F, as the procedure stops when no variables are eligible for either removal or inclusion (Norusis, 2004a; Norusis, 2004b). Forward Selection is an alternate version of the Stepwise method, and variables are entered into the regression equation only if they meet the selection criterion. The difference between the two methods lies in the order of inclusion, which is based on the absolute correlation (i.e., positive or negative) between an independent variable and the dependent variable, and those independent variables with higher absolute correlation are included first. The method terminates when no variable satisfies the criterion for entry (Norusis, 2004a; Norusis, 2004b). It should be noted that stepwise and forward selection do yield exactly the same results in the current case, and as such there will be a combined presentation of their results (Norusis, 2004a; Norusis, 2004b).

7.4 Methodological difficulties

While this is ultimately one of the aspects that shall be analyzed with regard to the functioning of the arbitrage pricing theory, it should be pointed out that it is in the nature of a smaller sized stock market that the number of stocks that can be observed is smaller, thus resulting in smaller sample groups and potentially weaker statistical results. In the given case, while data is available for a considerably long period for some of the stocks which are components of the ATX stock market index, the problematic of a small sample group is amplified in one of the sample groups, because for a large number of stocks observed the available data is only available for a much shorter period, mostly because these stocks started trading at some point during the given observation period.

Also, one should consider the influence of the observed stocks on the stock market indices, due to the index being composed of a small number of stocks. As a result, the structure and movement of an index will appear to be more similar to that of a single

component than in a very large index. As an obvious result, the correlation between the given stock market indices and the extracted factors from the sample group may be larger than for a broad index.

Ultimately, this could be much more of a problem for the CAPM, where the market portfolio is supposed to include every asset that can be assessed a value, than for the APT, where neither the number of factors nor their type are clearly defined. Nevertheless, common logic suggests that correlation between a factor extracted from a small sample group and a stock market index composed of almost the same sample group should be high.

Further, there is a certain degree of incoherence in the available data with regard to the frequency and intervals of data points for stock market data and macroeconomic variables. As such, stock market information is available on a level of up to daily frequency (or even higher when considering single ticks and trades that are exercised), while monthly information is the highest frequency for most of the economic variables considered in the current analysis, with some other variables only available on a quarterly or even yearly basis. While daily information would be too volatile and too dependent on very short-term influences, an analysis on the basis of weekly information might have been interesting.

Thus, the absence of some variables – notably growth rates of gross domestic product – in the following analysis because of this non-availability of monthly data may have a significant influence on the final results with regard to which variables seem to fit to the extracted factors. An analysis on the basis of yearly intervals may be interesting and can certainly be considered as one where influences from too frequent data points are nonexistent, but for the given analysis and the period observed from 1996 until 2010 the period observed would probably be too short to be considered as capable to deliver statistically sound results.

Last, it should be pointed out that there has been no analysis of the small sample group over a short observation period. This analysis has not been undertaken given that, ex ante, the results from this sample group should be or can be assumed to be weaker than those of either the small sample group over the long period or the large sample group over the short period. The ideal case to analyze the effects of changes to the observation period and the size of the sample group would have been to base the evaluation on the large sample group over a long observation period, and to examine the mentioned effects by comparison to the two sets that were ultimately analyzed. As has been pointed out, this analysis could not be undertaken given that not all the stocks were trading throughout the entire observation period.

8 Results and Analysis

8.1 Factor Extraction

In total, the factor analysis has been conducted twelve times, with variations being made on the data set as well as the extraction criteria. Such variations were the length of the observation period in combination with the number of stocks whose returns were analyzed. The small group included 18 stocks, whose returns were observed in the longer timeframe between May 7, 1996 and December 31, 2010. The large group consisted of the 39 stocks included in the ATX Prime on December 31, 2010 and their stock returns were analyzed from May 21, 2008 on. As has been mentioned earlier, factor analysis has been done on daily, weekly, and monthly returns, and twice each with either pairwise or listwise exclusion of missing values.

The factor analysis could be done without occurrences for any of the six extractions from the small group of stocks. For the large group though, which was also analyzed over a much shorter period, extraction could not be done on weekly and monthly stock returns with pairwise exclusion of missing values, as the respective correlation matrixes were not positive definite. Further, only Principal Components Analysis could be conducted on monthly returns with listwise exclusion of missing values.

The Kaiser-Maier-Olkin Measure of Sampling Adequacy shows relatively high values for almost all of the nine extractions on which the test could be performed, ranging from 0.874 to 0.928, with the only exception coming at 0.514 for the extraction from 39 stocks with pairwise exclusion of missing values. What can be observed is that the Kaiser-Maier-Olkin Measure of Sampling Adequacy shows higher values when choosing listwise exclusion rather than pairwise exclusion of missing values. The high values suggest that there might indeed be a large enough fraction of variance in variables attributable to underlying components that the extraction of factors could yield useful results.

Bartlett's Test of Sphericity indicates that the variables are unrelated and confirms that they are suitable for structure detection in a factor analysis, as significance is at a level of 0.00 for all nine extractions on which the test could be performed.

Table 8.1 Factor Extraction – Results

18 stocks traded between May 7, 1996 and December 31, 2010						
Daily Returns			Weekly Returns		Monthly Returns	
Listwise	Pairwise		Listwise	Pairwise	Listwise	Pairwise
Principal Axis Factoring						
Kaiser-Meyer-Olkin Measure of Sampling Adequacy	0.905	0.891	0.901	0.893	0.874	0.874
Bartlett's Test of Sphericity - Approx. Chi-Square	6411,126	5388,740	2550,808	2220,379	1139,723	1037,332
Bartlett's Test of Sphericity - df	153	153	153	153	153	153
Bartlett's Test of Sphericity - Significance	0.000	0.000	0.000	0.000	0.000	0.000
Number of Initial Eigenvalues >1	4	4	3	3	4	4
Total Variance Explained - Initial Eigenvalues	47.530	43,737	44,633	42,214	59,632	57,281
Total Variance Explained - Sums of Squared Loadings	32.593	28,897	34,549	31,804	48,660	46,106
Number of factors identifiable in scree plot	2	2	2	2	2	2
Principal Components						
Initial Eigenvalues >1	4	4	3	3	4	4
Total Variance Explained - Initial Eigenvalues	47.530	43,737	44,633	42,214	59,632	57,281
Number of factors identifiable in scree plot	2	2	2	2	2	2

39 stocks traded between May 21, 2008 and December 31, 2010						
Daily Returns			Weekly Returns		Monthly Returns	
Listwise	Pairwise		Listwise	Pairwise	Listwise	Pairwise
Principal Axis Factoring						
Kaiser-Meyer-Olkin Measure of Sampling Adequacy	0.928	0.514	0.899			
Bartlett's Test of Sphericity - Approx. Chi-Square	6827,185	6126,323	3248,144			
Bartlett's Test of Sphericity - df	741	741	741			
Bartlett's Test of Sphericity - Significance	0.000	0.000	0.000	Correlation matrix not positive definite. Extrac-	Correlation matrix not positive definite. Extrac-	Correlation matrix not positive definite. Extrac-
Number of Initial Eigenvalues >1	9	9	9	tion cannot be done.	tion cannot be done.	tion cannot be done.
Total Variance Explained - Initial Eigenvalues	57.868	49,310	67,957			
Total Variance Explained - Sums of Squared Loadings	45.375	n.a.	58,182			
Number of factors identifiable in scree plot	2	2	2			
Principal Components						
Initial Eigenvalues >1	9	9	9	Correlation matrix not positive definite. Extrac-	9	Correlation matrix not positive definite. Extrac-
Total Variance Explained - Initial Eigenvalues	57.868	49,310	67,957	tion cannot be done.	83,463	positive definite. Extrac-
Number of factors identifiable in scree plot	2	2	2		2	tion cannot be done.

The goal of the factor analysis is to find a limited number of components representing the original variables, while at the same explaining a large part of their variation, justifying the substitution. The initial solution of a factor analysis always shows as many components as there are variables, with the sum of eigenvalues equaling the number of components in correlation analysis. Only components with eigenvalues greater than 1 will be extracted, as only these components explain a fraction in the total variance larger than the part their own variance causes (Norusis, 2004a; Norusis, 2004b).

There is a considerable difference in the number of factors with an eigenvalue greater than 1 in the initial solution between the small group and the large group. In the initial solution, only 4 factors have eigenvalues greater than 1 in the factor analysis on daily and monthly stock returns for the group of 18 stocks, and only three factors have an eigenvalue greater than 1 for the analysis of weekly returns of the same group.

For the large group of 39 stocks, the number of factors with eigenvalues greater than 1 in the initial solution is 9 for analyses on daily, weekly, and monthly returns. For both group of stocks there are no differences in the number of factors with eigenvalues greater than 1 between listwise and pairwise deletion of missing values at the respective intervals (daily, weekly, or monthly) of returns. Also, both Principal Components Analysis and Principal Axis Factoring yield the same number of factors with eigenvalues greater than 1.

The extraction of a reduced number of components will come with a loss of information concerning the total variance explained, with the extent of this loss depending on how well the extracted components actually represent the complete set of variables (Norusis, 2004a; Norusis, 2004b). It can be seen that there are considerable differences in the variance explained depending on the size of the group of stock returns analyzed as well as the interval of these stock returns. While the fraction of variance explained ranges between 42.2% and 47.5% in the analysis of daily and weekly returns from the small group of 18 stocks, this share is more than 10% higher in the analysis of monthly stock returns for the same group of stocks, as can be seen under Total Variance Explained - Initial Eigenvalues. Also, the part of variance explained is slightly higher when the component extraction is done with listwise deletion of missing values as opposed to pairwise deletion for any of the three intervals of stock returns.

The component extraction for the large group of 39 stocks shows considerably higher fractions of variance explained, especially when choosing listwise exclusion of missing values. In this setting, the extracted components explain a variance of 57.9% for daily returns, 68.0% in the sample of weekly returns, and 83.4% when analyzing monthly

stock returns. As pointed out above, the extraction could not be done for weekly and monthly returns with pairwise deletion of missing values. Still, the pattern of a lower fraction of variance explained when choosing pairwise deletion appears to hold, as for the analysis of daily stock returns the fraction of variance explained lies more than 8% lower with pairwise exclusion of missing values as opposed to listwise exclusion.

The variance that the extracted components account for in the initial solution does not differ depending on whether Principal Axis Factoring or Principal Components Analysis has been chosen as extraction method. Principal Axis Factoring also shows the cumulative variability explained by the factors in the extracted solution. Generally, the level of variance explained in the extracted solution lies below that of the initial solution, as the factor model simply cannot explain a certain number of aspects that are unique to the total variance or the original variables in general. In other words, the extraction comes with a loss of information whose extent depends on how well the factors represent the overall set of variables (Gorsuch, 1983; Kline, 1994).

The cumulative variability, as indicated under Total Variance Explained - Sum of Squared Loadings, lies considerably lower than the variance explained in the initial solution for all the extractions conducted, with the difference being 10% or more in most of the cases. This loss of variation explained lies at very similar values under pairwise and listwise deletion of missing values when looking at the small group of stock returns observed. With the additional loss of information, the cumulative variability explained by the extracted factors is still lower and lies between 28.9% and 34.5% for the four, respectively three factors from the analysis of daily and weekly returns of the small group of 18 stocks.

Corresponding to the already higher level of variance explained in the initial solution for the analysis of monthly stock returns from the same group of variables, the cumulative variability explained by the four extracted factors lies at 48.7% and 46.1% under listwise, respectively pairwise exclusion of missing values. The factor analysis on the large group of 39 stocks shows that the cumulative variability explained by the nine extracted factors is about 45.4% in the analysis of daily returns, and 58.2% in the analysis of weekly returns. These are the only values available as not all the analyses could be conducted on the large group of stock returns with regards to the different intervals of stock returns.

Building a scree plot from the initial extraction helps describe the relative contribution of a factor and the variance it explains to the total amount of variance explained by the

factor model. Only the components on the steep slope on the left-hand side of a scree plot contribute considerable portions in explaining this variance. The number of components with initial eigenvalues greater than 1 was 4 in the extraction from daily and monthly stock returns and 3 for the analysis of weekly returns from the small group of 18 stocks, while 9 components had initial eigenvalues greater than 1 in the extractions that could be performed on the large group of 39 stocks. The scree plots from the extractions conducted show no difference between pairwise or listwise deletion of missing values, and no difference between Principal Components Analysis and Principal Axis Factoring (Cattell, 1966; Norusis, 2004a; Norusis, 2004b).

The number of factors according to the scree plots is lower than the number of initial components with eigenvalues greater than 1, as can be seen in appendices I, II and III. According to these plots, no more than 2 components should be extracted in most of the cases. The ideal number of factors to be extracted cannot be perfectly identified in some cases, as it appears that a second drop occurs after a few additional components, even though these drops have by far not the same magnitude of the initial first drops.

8.2 Testing the Economic Variables

The results of the linear regressions show that the numbers of independent economic variables entered into the regression equation in order to maximally explain the extracted factors under the given circumstances differ from case to case, depending on the respective factor, on the sample size together with the length of the observation period of the underlying dataset, and on the number of variables available for potential entry into the equation.

Between one and six predictors per linear regression equation explain one extracted factor for the small group of stock returns over the longer period of observation. Between one and four economic variables are entered into the equation for the regressions on the extracted factors from the large group of stock returns over the shorter observation period.

Table 8.2 Economic Variables – Fit with Factors

18 stocks traded between May 7, 1998 and December 31, 2010												
Factor	Variables Entered	Predictors	Regression Set 1	Variables Entered	Predictors	Regression Set 2	Variables Entered	Predictors	Regression Set 3	Variables Entered	Predictors	Regression Set 4
1	41	1 ATXPrime_Return 2 ATX_Return 3 Inflation_FPI_Change		41	1 ATXPrime_Return 2 ATX_Return 3 Inflation_FPI_Change		41	1 ATXPrime_Return 2 ATX_Return 3 Inflation_FPI_Change		41	1 ATXPrime_Return 2 ATX_Return 3 Inflation_FPI_Change	
2	41	1 ATXPrime_Return 2 ATLBTbond_Ychange 3 ATGovBond_Ychange 4 ATX_Return 5 UnempEurDef_Change		40	1 ATX_Return 2 ATLBTbond_Ychange 3 ATGovBond_Ychange 4 ATProd_Chana 5 UnempEurDef_Change		39	1 EUConSent_ChangeRel 2 ATLBTbond_Ychange 3 Euribor12M_Change 4 ATProd_Chana 5 UnempEurDef_Change		38	1 EUConSent_ChangeRel 2 ATLBTbond_Ychange 3 Euribor12M_Change	
3	41	1 ATX_Return 2 Euribor12M_Change 3 Commod_OilArabChange		39	1 EUConSent_ChangeRel 2 Euribor12M_Change 3 Commod_OilArabChange		36	1 Euribor12M_Change 2 Euribor12M_Rate 3 UnempEurDef_Change		35	1 EUConSent_ChangeAbs	
4	41	1 ATXPrime_Return 2 ATXPrime_Return 3 UnempNatDef_Change 4 Commod_IndexHWMTotalChange 5 EUConSent_ChangeRel 6 UnempEurDef_Rate		38	1 UnempEurDef_Rate 2 Commod_OilArabChange 3 UnempNatDef_Change 4 ATProd_Chana 5 Inflation_WFI_Change		28	1 UnempEurDef_Rate 2 Commod_OilArabChange 3 UnempNatDef_Change 4 ATProd_Chana 5 Inflation_WFI_Change		34	1 UnempEurDef_Rate 2 UnempNatDef_Change 3 ATProd_Chana	
5												
6												
7												
8												
9												

39 stocks traded between May 21, 2008 and December 31, 2010												
Factor	Variables Entered	Predictors	Regression Set 1	Variables Entered	Predictors	Regression Set 2	Variables Entered	Predictors	Regression Set 3	Variables Entered	Predictors	Regression Set 4
1	41	1 ATX_Return 2 Inflation_FPI_Change 3 Euribor12M_Change 4 EUConSent_ChangeAbs		41	1 ATX_Return 2 Inflation_FPI_Change 3 Euribor12M_Change 4 EUConSent_ChangeAbs		41	1 ATX_Return 2 Inflation_FPI_Change 3 Euribor12M_Change 4 EUConSent_ChangeAbs		41	1 ATX_Return 2 Inflation_FPI_Change 3 Euribor12M_Change 4 EUConSent_ChangeAbs	
2	41	1 Commod_OilBrentChange 2 Commod_OilArabChange 3 UnempNatDef_Change		40	1 Commod_OilBrentChange 2 Commod_OilArabChange 3 UnempNatDef_Change		39	1 Commod_OilBrentChange 2 Commod_OilArabChange 3 UnempNatDef_Change		37	1 Commod_OilBrentChange 2 Commod_OilArabChange 3 UnempNatDef_Change	
3	41	1 Inflation_Wage_Change		39	1 Inflation_Wage_Change		31	1 Inflation_Wage_Change		34	1 Inflation_Wage_Change	
4	41	1 Inflation_WFI_Change		38	1 Inflation_WFI_Change		21	1 EUConSent_ChangeRel		33	1 Inflation_WFI_Change	
5	41	1 ATX_Return		37	1 ATXPrime_Return		18	-		32	1 ATXPrime_Return	
6	41	1 ATX_Return 2 ATXPrime_Return		36	-		18	-		31	-	
7	41	1 Inflation_HQF_Index 2 Inflation_FPI_Change		36	1 Inflation_HQF_Index 2 Inflation_FPI_Change		18	-		31	1 Inflation_HQF_Index	
8	41	-		35	-		18	-		30	-	
9	41	-		35	-		18	-		30	-	

While there appear to be some differences in the number of independent variables entered for each factor from the smaller observed group, the number of variables per factor seems to be more coherent for the large sample group. For the larger sample group, it can be noted that the number of predictors included in the regression equation in order to maximally explain the respective observed factor is higher for the first factors observed and decreases for later observed factors. While four predictors are entered into the regression equation for the factor 1 and three for the factor 2 (throughout the five regression sets), no more than two variables are included in the regression equation for factors 3 to 7. For factors 8 and 9 no predictors were entered into the regression equation, meaning that none of the economic variables satisfied the selection criterion.

One can also observe that the fit between an economic variable or a certain type of economic variables and one of the four factors seems to be more pronounced for the larger group. This means that, for each factor, the economic variables entered as predictors remain the same for the most part, regardless of the regression set and thus the variables available for entry. As such, factors 3, 4, and 7 appear to be best explained by inflation indicators, whereas the indices reflecting stock market returns are the only variables corresponding to factors 5 and 6. Similarly, the same two economic variables reflecting changes in oil price are entered as predictors for factor 2 throughout the five regression sets, with a third predictor for changes in unemployment equally remaining unchanged. Only factor 1 seems to correspond to several variable types, as the four predictors entered into the regression equation are from four different variable classes, specifically stock market index, inflation, interest rate, and economic sentiment. Whether the stronger coherence in economic variables is a result or the reason for the low number of predictors observed for each factor cannot be exactly said, but it clearly appears that the two facts are linked, at least when compared to the results from the small sample group. One explanation could be that, due to the shorter observation period, the factors may have less of a defined structure and correlation with economic indicators can thus be recognized only for a lower number of such variables.

For the smaller sample group, there appears to be less coherence in the variables entered as predictors in the regression equation. While some similarities in the variables entered throughout the five regression sets are visible, it appears to a much lesser extent that one factor can be defined as showing correlation to certain variables. As such, four of the five predictors entered for factor 2 in the first and second regression set stay the same, and the predictors for regression sets 3 to 5 are equally the same, yet only

one variable is entered as a predictor throughout all five regression sets. Similarly, factor 4 happens to have the same five variables as predictors in regression sets 2 and 3, three of which constitute the predictors in regression set 5 and are equally entered as predictors in the first regression set for factor 4. It can be noted that, throughout factors 1 to 4, the first predictor entered in the first regression set is a variable representing stock market movement. Given that the first set of regressions is the only one where all variables are available for entry and that the type of variable corresponding most to the first extracted factors from both sample groups is one or another index reflecting the development of the country's capital markets suggests that the overall stock market movement is, under the approach taken during this investigation, the most important factor when trying to explain the returns of a single stock from the analyzed sample group of stocks on the Austrian stock market.

Regression set 1 gives the strongest indication as to the variables' correlation with the extracted factors, given that it is the only regression set where all variables are available for entry for each of the factors. The regression set most likely reflecting the approach when building a prediction model is the third regression set, where all the variables from the type of variable entered as the first predictor are omitted from entry for the remaining factors. This is that only the first variable from all the predictors entered for a factor would be considered for the prediction model. While this results in a lower correlation with the respective factor, it seems unlikely that a factor model would be built where each factor itself is made up of several constituents (Gorsuch, 1983; Kline, 1994). For both sample groups, results from the regression equation do not change for factor 1 throughout the five approaches, as the conditions for entry at the start of the process remained the same for the different methods.

From a statistical perspective (the statistical results are provided in detail in appendices IV and V), the ANOVA table gives a first indication of the acceptability of the model. The values of the F statistic, as indicated in the ANOVA tables, are less than 0.05 for all the factors under all five regression sets and for each of the different cases with regard to how many predictors are included in the regression equation for each factor. These low significance values mean that the variation explained by the variables entered as predictors into the regression equation is not due to chance. The ANOVA table equally provides information on the Sum of Squares for Regression and Residuals, and thus the components of the R^2 statistic. While information on how much of the factors' variation is explained by the variables entered as predictors is thus visible in the ANO-

VA table, the strength of the relationship is easier to assess by directly looking at the R^2 values (Norusis, 2004a; Weisberg, 1985).

For all the extracted factors, the results from the different regression sets do not indicate a very strong relationship with the economic variables. The low numbers for the coefficient of determination, R^2 , show that the economic variables entered as predictors lack the power of explaining most of the factors' variation (Draper & Smith, 1981; Norusis, 2004b).

For the small sample group, the highest R^2 for any of the four first predictors from regression set 1 is 0.367 from factor 4, indicating that slightly more than one third of this factor's variation can be explained by entering one single economic variable into the regression equation, in this case the variable `ATX_Return`.

When taking into consideration all the predictors entered into the regression equation, the highest R^2 under the first regression set is 0.641 for factor 1. The lowest R^2 for this regression set when including all predictors is 0.306 from the five predictors for factor 2. With the exception of factor 1, the results for which remain unchanged throughout the five regression sets due to the total number of variables available for entry for this factor, these numbers mostly decrease for later regression sets, as the number of economic variables available for entry is gradually reduced for factors 2 and higher, and as this reduction of available variables is more and more pronounced for later regression sets, due to the more strict assumptions with regard to the reduced number of variables available for entry.

For the large sample group, the highest R^2 value from the first predictors under regression set 1 is 0.473 for factor 1. Considering all the predictors entered for each factor, the highest R^2 is 0.742, equally for factor 1. Subsequently, these numbers are also the highest R^2 under all the other regression sets, given that the results for factor 1 do not change with regard to the regression set and that the results for the other factors in later regression sets rather indicate a decrease in correlation between economic variables and the factors.

The Adjusted R-squared is a measure compensating for complexity of the model and can thus be regarded as a more fair comparison in terms of model performance. Its values are always lower than those of the corresponding R^2 . In the given case, this holds true and the Adjusted R-squared are slightly lower than the previously presented R^2 . With regard to the overall signal of the results, the Adjusted R-squared confirm what could already be seen in the R^2 , specifically the low correlation between the ex-

tracted factors and the economic variables that were entered as predictors (Draper & Smith, 1981; Norusis, 2004a).

That the addition of further predictors does not add much to explaining a factor's variation is also evidenced by the low numbers for R^2 Change. A variable presenting a good predictor can be identified by a large R^2 Change associated with the inclusion of this variable into the regression equation. In the given case, the addition of further predictors does not help to increase the total R^2 for the factor to significantly higher levels. Even if this was the case, it has been pointed out that a factor made up by several types of economic variables would result in a much more complex prediction model, and can thus not be regarded as desirable. As a result, one can proceed in such a way that only the first predictor for each factor is regarded as pertinent with regard to the variable or, in a broader sense, the type of economic variable type associated with this factor. This is ultimately the approach of regression sets 2 and 3, where some or all variables apart from the one included as first predictor are still available for inclusion for the remaining factors (Norusis, 2004b; Weisberg, 1985).

That the additional predictors do not always add much strength to the relationship with the extracted factor can also be seen in the coefficients table. While further predictors may appear to have high coefficients, the relative importance of significant predictors can be determined by looking at the standardized coefficients. When observing the results from the given sample groups, it can be seen that the additional predictors mostly have lower standardized coefficients than the first predictor entered into the regression equation. This feat is especially pronounced in cases where two economic variables from the same type are already included in the equation, and an additional predictor from a different variable class is then entered into the equation. It can also be seen that in those cases where two variables from the same variable type are entered into the regression, the second variable is entered as a predictor with a negative coefficient, while the coefficient of the first variable – the one that was already included in the equation – increases significantly. This can be interpreted as such, that, while one variable from a certain variable type manages to explain a particular amount of variation, a pair of variables from the same type shows a stronger relationship with the extracted factor when the variables are entered with a positive and a negative sign. While this combination of one positive and one negative sign before two variables from the same type is apparently offsetting any large addition to the total correlation with the extracted factor, it appears that the results can be solidified (Draper & Smith, 1981; Norusis, 2004b; Weisberg, 1985).

For several factors extracted from the large sample group, the question of multicollinearity does not have to be considered, as only one economic variable was entered into the regression equation for each of these factors. For those factors where more than one economic variable has been entered into the regression equation as a predictor, it can be observed that the partial correlation is higher than the respective zero-order correlation for all of these predictors. While the part correlation then drops off below the zero-order correlation for a few predictors entered into the regression equation for factor 1, the values of this statistic remain higher for all the multiple predictors entered into the regression equations for factors 2, 6 and 7.

For the small sample group, the comparison of zero-order and partial as well as part correlation vary slightly more than those of the large sample group. Both partial correlation and part correlation are higher in some cases, and lower in other cases. There are several cases where partial correlation is high first, with a drop from zero-order correlation to part correlation, but also some cases where part correlation stays higher than the zero-order correlation.

For the small sample group, it can be seen that the values for partial and part correlations are for the most part not too different from the zero-order correlation, indicating that the additional predictors do not create a problem with multicollinearity. While under most regression sets and for all four factors both the partial and part correlation for the first predictor decrease when additional predictors are entered, these two correlation statistics are, in absolute terms, mostly higher than the respective zero-order correlation for the additional predictors. From this perspective alone, the addition of further economic variables as predictors in the regression equation seems favorable, as evidenced by the fact that these additional predictors actually explain a share of the factors' variation that has not yet been explained by other variables (Draper & Smith, 1981; Norusis, 2004a; Weisberg, 1985).

As can be expected, the tolerance and variance inflation factors (VIF) statistics largely decrease respectively increase when two or more economic variables from the same type were entered into the regression equation. While tolerance and VIF stay low for predictors from other variable classes, suggesting that they are not affected by multicollinearity, the weak numbers in the two statistics for predictors from the same type of economic variables indicate that the addition of another predictor from an already considered variable class does not add much strength to the regression equation with regard to how much more variation in the factor this new predictor could explain (Norusis,

2004a; Norusis, 2004b). This confirms the initial assumption that each factor should not have more than one constituent from a certain variable type, and thus proves that the approach chosen under regression sets 3 and 5, namely the omission of already included variable types for further predictors, was correct.

8.3 Summarizing the Results

Given that the assumptions underlying regression set 3 are the ones most closely reflecting the approach needed to build or conceive a model predicting stock market returns which can in turn be used to calculate the cost of capital for such a stock, the first variables corresponding to the extracted factors from the small, respectively large sample group are as follows:

Table 8.3 Economic Variables – First Predictors

Small sample group, long period	Large sample group, short period
ATXPrime_Return	ATX_Return
EUEconSent_ChangeRel	Commod_OilBrentChange
Euribor12M_Change	Inflation_Wage_Change
UnempEurDef_Rate	EUEconSent_ChangeRel

A model with four factors is a result similar to what has been found in previous papers, where the recommended number of factors for the arbitrage pricing theory lies between three and five.

Both in the small and the large sample group we can find variables representing stock market movement (ATXPrime_Return, ATX_Return) as well as an index reflecting changes in the economic sentiment (EUEconSent_ChangeRel). For the remaining factors from the small sample group, the variables fitting best are Euribor12M_Change and UnempEurDef_Rate, reflecting the changes in interest rates and the unemployment rate, respectively. For the large sample group, a fitting economic variable could only be found for two more (out of seven remaining) factors. Commod_OilBrentChange and Inflation_Wage_Change reflect changes in oil prices and the change in inflation rates.

Both the results from the small and from the large sample group suggest that a factor extraction and the ensuing testing of economic variables for a fit with the extracted factors can be undertaken on sample groups from a small stock market. While the results

between the two sample groups appear to differ with regard to the number of factors extracted, the large sample group's factors show high coherence with regard to fitting economic variables throughout the five regression sets, as these variables hardly change. Also, it can be seen for the large sample group that some variables fit to several factors, and in general that the factors extracted from both sample groups ultimately respond to very similar economic variables.

Overall, the results do not show very high levels of correlation, meaning that only a limited level of total variance can be explained throughout the different steps of the analysis. The results certainly appear to have some explanatory power, but one might question their ability to explain all aspects of structure and variation in the sample groups in a convincing fashion. It is interesting to note that for both the small and the large sample group the economic variables linked to overall development and returns of the stock market (as expressed through the stock market indices) appear to be the variables closest to the extracted factors.

As has been pointed out before, the variable type reflecting stock market movement shows the highest correlation with the extracted factors from the small sample group when the number of variables available for entry is not reduced. For the large sample group, this variable type likewise is the one best fitting for three of the extracted factors, with variables reflecting inflation best fitting for three other factors. This relatively strong influence of variables reflecting the movement and development of the stock market in general point to a similarity with the capital asset pricing model, the single-factor model where the only factor reflects the difference between the rate of return that can be achieved on the stock market and the return that can be achieved from investment into a riskless asset.

A multi-factor model where the most important factor is equally a variable representing the returns from a stock market index and where further factors reflecting other variables do not add much to the total variance explained by the model is a result that does win in a convincing fashion over an established model that, while simpler, causes less potential disagreements over the outcome – at least when considering the application of the model in practice.

The process of building a model to predict stock market returns which can in turn be used to estimate the cost of capital by use of the arbitrage pricing theory is, while a broad one considering the underlying assumptions, an approach consuming time and resources. At the same time, the results are, while methodically and fundamentally

correct, at the most, if not even less, as convincing as a similar estimation when using the capital asset pricing model.

When thinking about the application of such an approach in practice, the different types of macroeconomic variables that could be considered as potential factors as well as the number of factors in a model may largely influence the outcome of a cost of capital calculation and as such, given the weight of this element, the final result of any type of discounted cash flow valuation. While any type of valuation is dependent on the underlying approach and methodology and can thus be discussed, a method which in itself can be used in various different ways does not necessarily solidify the final result.

9 Conclusion

Knowing the true value of an asset such as the stock of a publicly listed group or the share in a private company is essential for every investor in order to take the right investment decisions. Depending on the circumstances and the type of asset to be valued, an investor may choose from a wide range of valuation techniques, and different approaches under relative valuation and discounted cash flow valuation were presented in the first chapter of this paper. An emphasis was given to the various discounted cash flow models and the necessary components to undertake such a calculation. Apart from estimates for the future cash flows from an asset or a company, the key element to every discounted cash flow valuation is the use of an appropriate discount rate, and as such the concept of the cost of capital was analyzed in detail. The different levels of risk involved with various types of financing necessitate the adjustment of discount rates in order to reflect the respective compensation required by the providers of equity and debt capital and ultimately lead to the calculation of a weighted average cost of capital.

To estimate the required rate of return for an investment into a company's equity or stock, investors use different models incorporating one or several factors that represent the measure of risk in capital markets, beta, and the price for this risk, the premium. While some approaches, such as the capital asset pricing model, resort to a single factor determining the price for an investment into an asset, the concept of the arbitrage pricing theory or the three-factor model is to estimate the cost of capital as a function of several influencing factors, such as different macroeconomic variables. The importance and wide use of the capital asset pricing model in practice are undeniable, even though not all the elements of the theoretic concept might be respected when applying the model in a business context, notably due to the difficulty of establishing a market portfolio in line with the original idea and theory.

In this context, multifactor models which are similar to the capital asset pricing model's original idea but have been adapted in order to incorporate several factors may present a useful alternative when estimating the cost of capital in practice. Notably the arbitrage pricing theory's underlying conception allows for more flexibility by letting the user take into account several macroeconomic factors that may be responsible for the price of a stock or the value of an asset in general. Nevertheless, this absence of constraints

combined with the lack of guidance may also make the practical application of this model more difficult. As such, an empirical investigation was undertaken to examine the functioning of the arbitrage pricing theory on a small capital market, by testing the model on a series of data on the Austrian stock market.

The results suggest that the model generally also works under the constraints of a small capital market and for a limited group of stocks that can be taken as a sample and for which the model may be used when estimating the cost of capital. As such, the data from a small stock market is sufficient to allow for identification of a structure and the extraction of factors. In the given investigation, the extraction of four factors is in line with previous research on the topic, where data from larger sample groups of stocks was used. Some of the macroeconomic variables that correlate with the extracted factors and as such might be considered when estimating the cost of capital are indicators such as the stock market movement, an index reflecting economic sentiment, and variables reflecting interest rates, but also such that represent oil prices or inflation.

While the necessary structure allowing for factor extraction and the fit of these factors with different indicators can be identified in principle, the results do not show a lot of strength from a statistical perspective. Also, the stock market movement as reflected by an index is by far the variable corresponding best to the extracted factors, which may be a sign that a single-factor model with only this factor, such as the capital asset pricing model, may be sufficient. More so, notably from a practical perspective, the significant amount of work necessary to identify the number of factors and the corresponding variables may not seem justifiable given the relative weakness of the statistical results. As such, while the concept of the arbitrage pricing theory still seems tempting in theory, its application in practice indeed proves difficult compared to the relatively more simple capital asset pricing model.

References

- Armitage, S. (2005). *The Cost of Capital: Intermediate Theory*. New York: Cambridge University Press.
- Block, S. B., & Hirt, G. A. (1994). *Foundations of financial management* (7th ed.). Burr Ridge, Illinois: Irwin.
- Bodie, Z., Kane, A., & Marcus, A. J. (2005). *Investments* (6th ed.). Boston, Massachusetts: McGraw-Hill.
- Brealey, R., Myers, S., & Allen, F. (2006). *Principles of Corporate Finance* (8th ed.). New York: McGraw-Hill/Irwin.
- Brigham, E. F., & Ehrhardt, M. C. (2008). *Financial management: theory and practice* (12th ed.). Mason, Ohio: South-Western.
- Bruner, R. F., Eades, K. M., Harris, R. S., & Higgins, R. C. (1998). Best Practices in estimating the cost of capital: Survey and Synthesis. *Financial Practices and Education*, 8(1), 13-28.
- Cattell, R. B. (1966). The scree test for the number of factors. *Journal of Multivariate Behavioral Research*, 1(2), 245-276.
- Copeland, T. E., Weston, J. F., & Shastri, K. (2004). *Financial theory and corporate policy* (4th ed.). Boston, Massachusetts: Pearson Addison Wesley.
- Cornell, B. (1999). *The Equity Risk Premium: The Long-Run Future of the Stock Market*. New York: John Wiley & Sons, Inc.
- Damodaran, A. (2002). *Investment Valuation: Tools and Techniques for Determining the Value of Any Asset* (2nd ed.). New York: John Wiley & Sons, Inc.
- Damodaran, A. (2005). Valuation Approaches and Metrics: A Survey of the Theory and Evidence. *Foundations and Trends® in Finance*, 1(8), 693-784.
- Damodaran, A. (2006). *Damodaran on Valuation: Security Analysis for Investment and Corporate Finance* (2nd ed.). Hoboken, New Jersey: John Wiley & Sons.

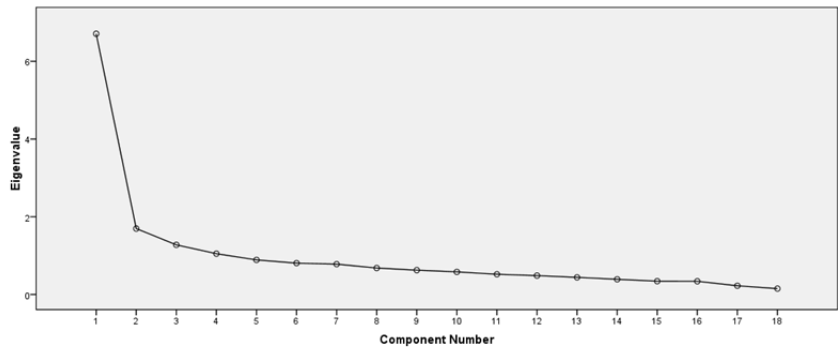
- Draper, N. R., & Smith, H. (1981). *Applied regression analysis* (2nd ed.). New York: John Wiley and Sons.
- Ehrhardt, M. (1994). *The Search for Value: Measuring the Company's Cost of Capital*. Boston, Massachusetts: Harvard Business School Press.
- Fama, E. F., & French, K. R. (1992). The Cross-section of Expected Stock Returns. *The Journal of Finance*, 47(2), 427-466.
- Fama, E. F., & French, K. R. (2004). The Capital Asset Pricing Model: Theory and Evidence. *The Journal of Economic Perspectives*, 18(3), 25-46.
- Feldman, S. J. (2005). *Principles of private firm valuation*. Hoboken, New Jersey: Wiley.
- Gitman, L. J. (2006). *Principles of managerial finance* (11th ed.). Boston, Massachusetts: Pearson Addison Wesley.
- Glassman, J. K., & Hassett, K. A. (1999, March 30). Are stocks overvalued? Not a chance. *Wall Street Journal*, p. 28.
- Gorsuch, R. L. (1983). *Factor Analysis* (2nd ed.). Hillsdale, New Jersey: Lawrence Erlbaum Associates.
- Graham, J. R., & Harvey, C. R. (2001). The theory and practice of corporate finance. *Journal of Financial Economics*, 60(2-3), 187-243.
- Harrington, D. R. (1987). *Modern Portfolio Theory, The Capital Asset Pricing Model, and Arbitrage Pricing Theory: A User's Guide* (2nd ed.). Englewood Cliffs, New Jersey: Prentice Hall, Inc.
- Kline, P. (1994). *An Easy Guide to Factor Analysis*. London: Routledge.
- Koller, T., Goedhart, M., & Wessels, D. (2005). *Valuation: Measuring and Managing the Value of Companies*. Hoboken, New Jersey: John Wiley & Sons, Inc.
- Levy, H., & Sarnat, M. (1978). *Capital Investment and Financial Decisions*. London: Prentice-Hall International, Inc.
- Mayo, H. B. (2001). *Financial institutions, investments and management: an introduction* (7th ed.). Fort Worth, Texas: Harcourt College Publishers.

- Norusis, M. (2004a). *SPSS 13.0 Guide to Data Analysis*. Upper Saddle-River, New Jersey: Prentice Hall, Inc.
- Norusis, M. (2004b). *SPSS 13.0 Statistical Procedures Companion*. Upper Saddle-River, New Jersey: Prentice Hall, Inc.
- Oxford English Dictionary (2012). *Cost*. Oxford: Oxford University Press. Retrieved June 1, 2012, from <http://oxforddictionaries.com/definition/cost?view=uk>
- Roll, R. (1977). A Critique of the Capital Asset Pricing Theory's Tests, Part 1: On Past and Potential Testability of the Theory. *Journal of Financial Economics*, 4(2), 129-176.
- Ross, S. A. (1976). The arbitrage theory of capital asset pricing. *Journal of Economic Theory*, 13(3), 341-360.
- Ruback, R. S. (2000). Capital Cash Flows: A Simple Approach to Valuing Risky Cash Flows. *Financial Management*, 31(2), 85-103.
- Sharpe, W. F., Alexander, G. J., & Bailey, J. V. (1999). *Investments* (6th ed.). Englewood Cliffs, New Jersey: Prentice Hall, Inc.
- Titman, S., & Martin, J. D. (2008). *Valuation*. Boston, Massachusetts: Pearson Addison-Wesley.
- Weisberg, S. (1985). *Applied Linear Regression* (2nd ed.). New York: John Wiley and Sons.
- Young, S. D., & O'Byrne, S. F. (2002). *EVA® and value-based management: A practical guide to implementation* (6th ed.). New York: McGraw-Hill.

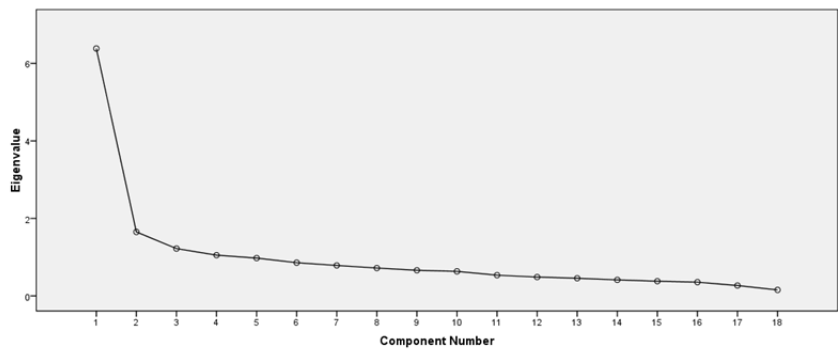
Appendices

Appendix I: Scree Plots – Monthly Returns

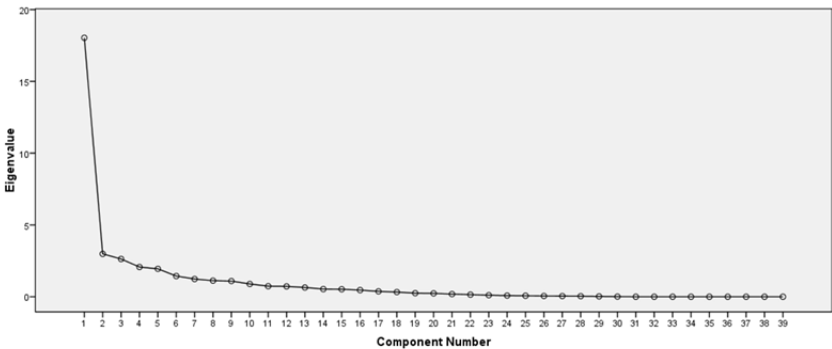
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Monthly Returns – Small sample group (18 stocks) – Pairwise extraction



Monthly Returns – Large sample group (39 stocks) – Listwise extraction

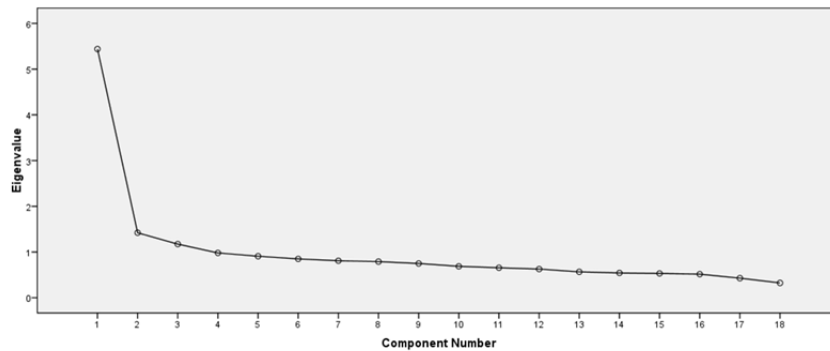


Monthly Returns – Large sample group (39 stocks) – Pairwise extraction

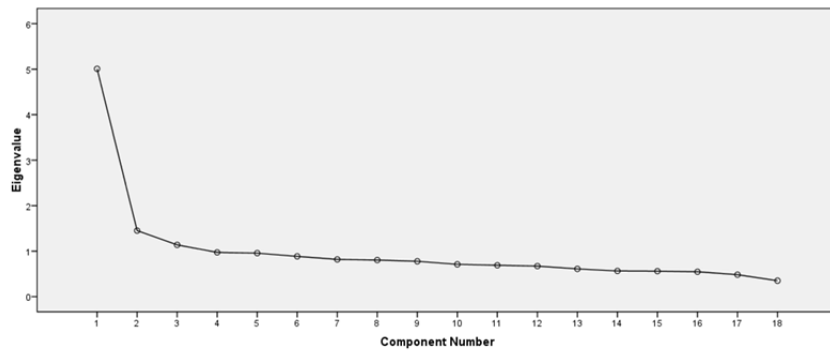
Correlation matrix not positive definite. Extraction cannot be done.

Appendix II: Scree Plots – Weekly Returns

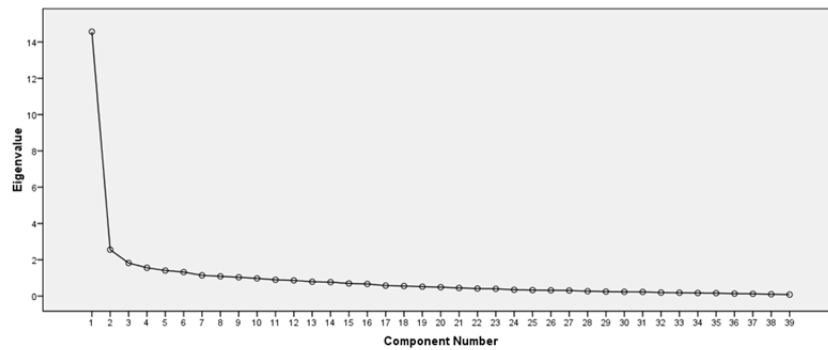
Weekly Returns – Small sample group (18 stocks) – Listwise extraction



Weekly Returns – Small sample group (18 stocks) – Pairwise extraction



Weekly Returns – Large sample group (39 stocks) – Listwise extraction

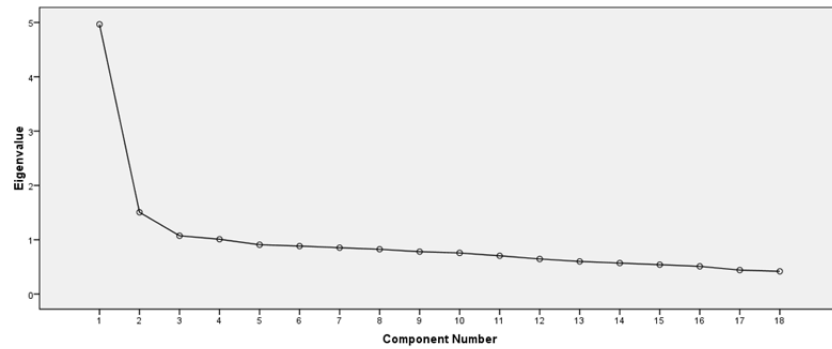


Weekly Returns – Large sample group (39 stocks) – Pairwise extraction

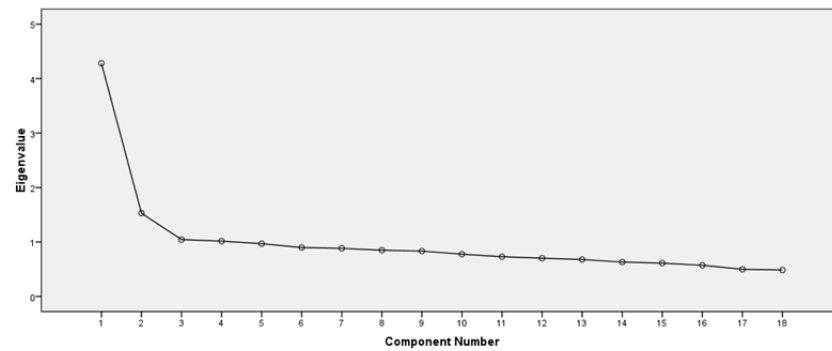
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Appendix III: Scree Plots – Daily Returns

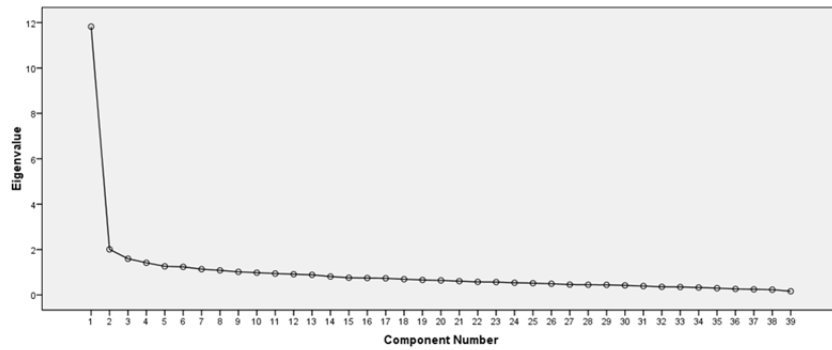
Daily Returns – Small sample group (18 stocks) – Listwise extraction



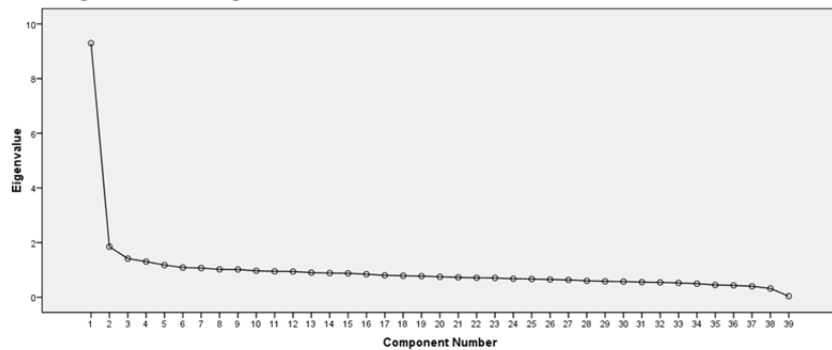
Daily Returns – Small sample group (18 stocks) – Pairwise extraction



Daily Returns – Large sample group (39 stocks) – Listwise extraction



Daily Returns – Large sample group (39 stocks) – Pairwise extraction



Appendix IV: Statistical Results – Small Sample Group

18 stocks traded between May 7, 1986 and December 31, 2010 - Regression Set 1																													
Factor	Predictor	Variable	ANOVA						Model Summary						Coefficients														
			Sum of Squares			df	Regression			Residual	Total	Mean Square	F	Significance	R	R ²	Adjusted R ²	Standard Error	RF Change	F Change	df1	df2	Spn. F Change	Durbin-Watson	Stand. Coeff. Beta	Correlations Zero-order	Partial	Part	Tolerance
1	1	ATXPrime_Return	44.528	98.730	143.257	1	111	112	44.528	0.889	50.062	0.000	0.558	0.311	0.305	0.94310943	0.311	50.062	1	111	0.000	0.000	2.071	5.057	0.558	0.722	0.625	0.015	65.458
	2	ATX_Return	86.139	54.118	143.257	2	110	112	44.570	0.492	90.592	0.000	0.789	0.622	0.615	0.70141546	0.311	90.676	1	110	0.000	2.071	-4.538	0.484	-0.684	-0.561	0.015	65.457	
	3	Inflation_HPI_Change	91.869	51.388	143.257	3	109	112	30.623	0.471	64.955	0.000	0.801	0.641	0.631	0.68662196	0.019	5.791	1	109	0.018		-0.138	-0.141	-0.225	-0.138	0.989	1.001	
2	1	ATXPrime_Return	12.563	104.706	117.269	1	111	112	12.563	0.943	13.318	0.000	0.327	0.107	0.099	0.97123604	0.107	13.318	1	111	0.000		2.028	0.327	0.282	0.245	0.015	68.782	
	2	ATLTbond_Ychange	19.536	97.733	117.269	2	110	112	9.768	0.888	10.994	0.000	0.408	0.167	0.151	0.94259321	0.059	7.848	1	110	0.006		0.795	0.270	0.334	0.295	0.138	7.267	
	3	ATGovBond_Ychange	28.473	88.796	117.269	3	109	112	9.491	0.815	11.651	0.000	0.493	0.243	0.222	0.90257561	0.076	10.970	1	109	0.001	2.089	-0.830	0.182	-0.284	-0.228	0.131	7.655	
	4	ATX_Return	32.749	84.520	117.269	4	108	112	8.187	0.783	10.462	0.000	0.528	0.279	0.253	0.88464338	0.036	5.464	1	108	0.021		-1.837	0.295	-0.228	-0.196	0.014	70.109	
	5	UnempEurDef_Change	35.886	81.383	117.269	5	107	112	7.177	0.761	9.436	0.000	0.553	0.306	0.274	0.87211771	0.027	4.125	1	107	0.045		-0.168	-0.146	-0.193	-0.164	0.949	1.054	
3	1	ATX_Return	27.040	88.768	115.808	1	111	112	27.040	0.800	33.812	0.000	0.485	0.233	0.227	0.89426844	0.233	33.812	1	111	0.000		0.496	0.483	0.487	0.462	0.089	1.150	
	2	Euribor3M_Change	33.089	82.720	115.808	2	110	112	16.544	0.752	22.001	0.000	0.535	0.286	0.273	0.85671737	0.052	8.044	1	110	0.005	1.879	0.273	0.315	0.301	0.261	0.912	1.096	
	3	Control_OilkrabChange	36.289	79.519	115.808	3	109	112	12.096	0.730	16.581	0.000	0.560	0.313	0.294	0.85412792	0.028	4.387	1	109	0.039		-2.160	0.067	-0.197	-0.166	0.830	1.205	
4	1	ATX_Return	43.026	74.358	117.384	1	111	112	43.026	0.670	64.229	0.000	0.605	0.367	0.361	0.81846948	0.367	64.229	1	111	0.000		2.890	0.605	0.457	0.322	0.014	68.802	
	2	ATXPrime_Return	53.220	64.164	117.384	2	110	112	26.610	0.583	45.619	0.000	0.673	0.443	0.443	0.763774661	0.087	17.476	1	110	0.000		-2.135	0.654	-0.372	-0.250	0.014	72.647	
	3	UnempEurDef_Change	57.074	60.311	117.384	3	109	112	19.025	0.553	34.383	0.000	0.697	0.486	0.472	0.74389944	0.033	6.964	1	109	0.000	2.071	0.214	0.281	0.311	0.205	0.920	1.087	
	4	Control_IndexHWWTotalChange	61.257	56.127	117.384	4	108	112	15.314	0.520	39.468	0.000	0.722	0.322	0.504	0.72089944	0.036	8.050	1	108	0.005		0.310	0.240	0.384	0.260	0.707	1.414	
	5	EUEconSent_ChangeRel	66.492	50.893	117.384	5	107	112	13.398	0.476	27.959	0.000	0.753	0.566	0.546	0.68966010	0.045	11.006	1	107	0.001		-0.339	0.069	-0.378	-0.256	0.968	1.762	
	6	UnempEurDef_Rate	71.395	45.989	117.384	6	106	112	11.899	0.434	27.427	0.000	0.780	0.608	0.586	0.65867772	0.042	11.303	1	106	0.001		0.227	0.332	0.310	0.204	0.908	1.237	
18 stocks traded between May 7, 1986 and December 31, 2010 - Regression Set 2																													
Factor	Predictor	Variable	ANOVA						Model Summary						Coefficients														
			Sum of Squares			df	Regression			Residual	Total	Mean Square	F	Significance	R	R ²	Adjusted R ²	Standard Error	RF Change	F Change	df1	df2	Spn. F Change	Durbin-Watson	Stand. Coeff. Beta	Correlations Zero-order	Partial	Part	Tolerance
1	1	ATXPrime_Return	44.528	98.730	143.257	1	111	112	44.528	0.889	50.062	0.000	0.558	0.311	0.305	0.94310943	0.311	50.062	1	111	0.000	0.000	2.071	5.057	0.558	0.722	0.625	0.015	65.458
	2	ATX_Return	86.139	54.118	143.257	2	110	112	44.570	0.492	90.592	0.000	0.789	0.622	0.615	0.70141546	0.311	90.676	1	110	0.000	2.071	-4.538	0.484	-0.684	-0.561	0.015	65.457	
	3	Inflation_HPI_Change	91.869	51.388	143.257	3	109	112	30.623	0.471	64.955	0.000	0.801	0.641	0.631	0.68662196	0.019	5.791	1	109	0.018		-0.138	-0.141	-0.225	-0.138	0.989	1.001	
2	1	ATX_Return	10.182	107.087	117.269	1	111	112	10.182	0.965	10.554	0.002	0.295	0.087	0.079	0.95221564	0.087	10.554	1	111	0.002		0.357	0.265	0.361	0.328	0.841	1.189	
	2	ATLTbond_Ychange	17.234	100.035	117.269	2	110	112	8.617	0.836	9.476	0.000	0.383	0.147	0.131	0.95362895	0.060	7.765	1	110	0.006		0.884	0.270	0.366	0.333	0.142	7.046	
	3	ATGovBond_Ychange	26.149	91.120	117.269	3	109	112	8.716	0.836	10.427	0.000	0.472	0.223	0.202	0.91431175	0.076	10.664	1	109	0.001	2.104	-0.719	0.182	-0.288	-0.265	0.135	7.392	
	4	ATProd_Change	29.560	87.710	117.269	4	108	112	7.390	0.812	9.099	0.000	0.502	0.252	0.224	0.90118030	0.029	4.200	1	108	0.043		-0.198	-0.219	-0.222	-0.192	0.940	1.064	
	5	UnempEurDef_Change	33.214	84.055	117.269	5	107	112	6.643	0.786	8.456	0.000	0.532	0.323	0.290	0.86632109	0.031	4.652	1	107	0.033		-0.183	-0.146	-0.222	-0.177	0.934	1.070	
3	1	EUEconSent_ChangeRel	16.643	99.165	115.808	1	111	112	16.643	0.893	18.630	0.000	0.379	0.179	0.136	0.94518708	0.144	18.630	1	111	0.000	1.991	0.379	0.379	0.379	0.379	1.000	1.000	
	1	UnempEurDef_Rate	12.932	104.452	117.384	1	111	112	12.932	0.941	13.743	0.000	0.332	0.110	0.102	0.97005672	0.110	13.743	1	111	0.000		0.259	0.332	0.294	0.253	0.955	1.048	
	2	Control_OilkrabChange	19.745	97.639	117.384	2	110	112	9.972	0.888	11.122	0.000	0.410	0.168	0.153	0.94241317	0.058	7.675	1	110	0.007		0.506	0.295	0.392	0.351	0.481	2.080	
4	3	UnempEurDef_Change	29.511	87.874	117.384	3	109	112	9.837	0.806	12.020	0.000	0.501	0.251	0.231	0.89787538	0.083	12.114	1	109	0.001	2.158	0.269	0.214	0.333	0.291	0.946	1.057	
	4	ATProd_Change	33.960	83.424	117.384	4	108	112	8.490	0.772	10.991	0.000	0.538	0.289	0.263	0.87888737	0.038	5.761	1	108	0.026		0.269	0.176	0.288	0.248	0.949	1.178	
	5	Inflation_WPI_Change	37.751	79.633	117.384	5	107	112	7.550	0.744	10.145	0.000	0.567	0.322	0.290	0.86268970	0.032	5.094	1	107	0.036		-0.260	0.096	-0.213	-0.180	0.477	2.096	

18 stocks traded between May 7, 1998 and December 31, 2010 - Regression Set 3																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																									
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			Sum of Squares		df	Regression		Residual	Total	R ²	Adjusted R ²	Standard Error	Change Statistics		Stand. Coeff. Beta	Correlations	Collinearity Statistics																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																								
Regression	Residual	Total	Regression	Residual		F	Significance						F Change	df1			df2	Sign. F Change	Wilson	Zero-order	Partial	Part	Tolerance	VIF																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																	
1	1	ATXPrime_Return	44.528	98.730	143.257	1	111	112	44.528	0.689	50.062	0.000	0.588	0.311	0.305	0.94310943	0.311	90.676	1 110	0.000	2.071	5.057	0.588	0.722	0.625	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458	0.015	66.458

Appendix V: Statistical Results – Large Sample Group

39 stocks traded between May 21, 2008 and December 31, 2010 - Regression Set 1																					
Factor	Predictor	ANOVA					Model Summary					Coefficients									
		Sum of Squares		df	Mean Square		R	R ²	Adjusted R ²	Standard Error	Change Statistics			Durbin-Watson							
		Regression	Residual		Total	Regression					Residual	F	Significance		R Change	F Change	dF1	dF2	Sign.	F Change	Walson
1	1	14.181	15.819	30.000	1	29	30	14.181	0.545	25.997	0.000	0.688	0.473	0.455	0.73859661	0.473	25.997	1	29	0.000	0.000
	2	17.283	12.717	30.000	2	28	30	8.642	0.545	19.028	0.000	0.759	0.576	0.546	0.67391687	0.103	6.831	1	28	0.014	1.752
	3	20.840	9.160	30.000	3	27	30	6.947	0.339	20.477	0.000	0.833	0.695	0.661	0.55244743	0.119	10.465	1	27	0.003	0.003
	4	22.256	7.744	30.000	4	26	30	5.594	0.298	18.682	0.000	0.861	0.742	0.702	0.54573554	0.047	4.755	1	26	0.038	0.038
2	1	9.129	20.871	30.000	1	29	30	9.129	0.720	12.684	0.001	0.552	0.304	0.280	0.84833215	0.304	12.684	1	29	0.001	0.001
	2	14.344	15.656	30.000	2	28	30	7.172	0.559	12.827	0.000	0.691	0.478	0.441	0.74761819	0.174	9.327	1	28	0.005	2.346
	3	16.870	13.130	30.000	3	27	30	5.623	0.486	11.563	0.000	0.750	0.562	0.514	0.69735118	0.094	5.194	1	27	0.031	0.031
3	1	8.665	21.335	30.000	1	29	30	8.665	0.736	11.778	0.002	0.537	0.289	0.264	0.85772625	0.289	11.778	1	29	0.002	2.166
4	1	7.673	22.327	30.000	1	29	30	7.673	0.770	9.966	0.004	0.506	0.256	0.230	0.87743984	0.256	9.966	1	29	0.004	1.987
5	1	4.599	25.401	30.000	1	29	30	4.599	0.876	5.251	0.029	0.392	0.153	0.124	0.93586201	0.153	5.251	1	29	0.029	2.782
6	1	3.943	26.057	30.000	1	29	30	3.943	0.899	4.388	0.045	0.363	0.131	0.101	0.94790957	0.131	4.388	1	29	0.045	2.569
	2	8.866	21.134	30.000	2	28	30	4.433	0.755	5.873	0.007	0.544	0.296	0.245	0.86878953	0.164	6.523	1	28	0.016	0.016
7	1	4.263	25.737	30.000	1	29	30	4.263	0.887	4.803	0.037	0.377	0.142	0.113	0.94206556	0.142	4.803	1	29	0.037	2.507
	2	8.084	21.916	30.000	2	28	30	4.042	0.783	5.164	0.012	0.519	0.269	0.217	0.88470277	0.127	4.883	1	28	0.035	0.035
8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

39 stocks traded between May 21, 2008 and December 31, 2010 - Regression Set 2																					
Factor	Predictor	ANOVA					Model Summary					Coefficients									
		Sum of Squares		df	Mean Square		R	R ²	Adjusted R ²	Standard Error	Change Statistics			Durbin-Watson							
		Regression	Residual		Total	Regression					Residual	F	Significance		R Change	F Change	dF1	dF2	Sign.	F Change	Walson
1	1	14.181	15.819	30.000	1	29	30	14.181	0.545	25.997	0.000	0.688	0.473	0.455	0.73859661	0.473	25.997	1	29	0.000	0.000
	2	17.283	12.717	30.000	2	28	30	8.642	0.545	19.028	0.000	0.759	0.576	0.546	0.67391687	0.103	6.831	1	28	0.014	1.752
	3	20.840	9.160	30.000	3	27	30	6.947	0.339	20.477	0.000	0.833	0.695	0.661	0.55244743	0.119	10.465	1	27	0.003	0.003
	4	22.256	7.744	30.000	4	26	30	5.594	0.298	18.682	0.000	0.861	0.742	0.702	0.54573554	0.047	4.755	1	26	0.038	0.038
2	1	9.129	20.871	30.000	1	29	30	9.129	0.720	12.684	0.001	0.552	0.304	0.280	0.84833215	0.304	12.684	1	29	0.001	0.001
	2	14.344	15.656	30.000	2	28	30	7.172	0.559	12.827	0.000	0.691	0.478	0.441	0.74761819	0.174	9.327	1	28	0.005	2.346
	3	16.870	13.130	30.000	3	27	30	5.623	0.486	11.563	0.000	0.750	0.562	0.514	0.69735118	0.094	5.194	1	27	0.031	0.031
3	1	8.665	21.335	30.000	1	29	30	8.665	0.736	11.778	0.002	0.537	0.289	0.264	0.85772625	0.289	11.778	1	29	0.002	2.166
4	1	7.673	22.327	30.000	1	29	30	7.673	0.770	9.966	0.004	0.506	0.256	0.230	0.87743984	0.256	9.966	1	29	0.004	1.987
5	1	4.235	25.765	30.000	1	29	30	4.235	0.888	4.766	0.037	0.376	0.141	0.112	0.94259045	0.141	4.766	1	29	0.037	2.813
6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	1	4.263	25.737	30.000	1	29	30	4.263	0.887	4.803	0.037	0.377	0.142	0.113	0.94206556	0.142	4.803	1	29	0.037	2.507
	2	8.084	21.916	30.000	2	28	30	4.042	0.783	5.164	0.012	0.519	0.269	0.217	0.88470277	0.127	4.883	1	28	0.035	0.035
8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

39 stocks traded between May 21, 2008 and December 31, 2010 - Regression Set 1

Factor	Predictor	Variable	ANOVA						Model Summary						Coefficients						Collinearity Statistics																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																							
			Sum of Squares		df		Mean Square		F	Significance	R	R ²	Adjusted R ²	Standard Error	Change Statistics				Durbin-Watson	Stand. Coeff. Beta	Zero-order	Partial	Part	Tolerance	VIF																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																			
Regression	Residual	Total	Regression	Residual	Total	Regression	Residual	df							df	df	df	df								df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df	df

39 stocks traded between May 21, 2008 and December 31, 2010 - Regression Set 4

Factor	Predictor	Variable	ANOVA						Model Summary						Coefficients						Collinearity Statistics										
			Sum of Squares		df		Mean Square		F	Significance	R	R ²	Adjusted R ²	Change Statistics				Durbin-Watson	Stand. Coeff. Beta	Zero-order	Partial	Part	Tolerance	VIF							
Regression	Residual	Total	Regression	Residual	Total	Regression	Residual	R ² Change						F Change	df 1	df 2	Sign. F Change														
1	1	ATX_Return	14.181	15.819	30.000	1	29	30	14.181	0.545	25.997	0.000	0.688	0.473	0.455	0.73856661	0.473	25.997	1	29	0.000					0.884	0.688	0.767	0.607	0.471	2.122
	2	Inflation_PPI_Change	17.283	12.717	30.000	2	28	30	8.642	0.454	19.028	0.000	0.759	0.576	0.546	0.67391687	0.103	6.831	1	28	0.014					-0.591	-0.324	-0.678	-0.468	0.628	1.592
	3	Eurlor12M_Change	20.840	9.160	30.000	3	27	30	6.947	0.339	20.477	0.000	0.833	0.695	0.661	0.58244743	0.119	10.485	1	27	0.003					0.881	0.125	0.624	0.406	0.489	2.047
	4	ELExconSent_ChangeAbs	22.256	7.744	30.000	4	26	30	5.564	0.298	18.682	0.000	0.861	0.742	0.702	0.54573554	0.047	4.755	1	26	0.038					-0.353	0.368	-0.383	-0.217	0.378	2.643
2	1	Commod_OilRentChange	9.129	20.871	30.000	1	29	30	9.129	0.720	12.684	0.001	0.552	0.304	0.280	0.84833215	0.304	12.684	1	29	0.001					3.388	0.552	0.651	0.567	0.028	35.648
	2	Commod_OilCrabChange	14.344	15.656	30.000	2	28	30	7.172	0.559	12.827	0.000	0.691	0.478	0.441	0.7476189	0.174	9.327	1	28	0.005					-2.782	0.470	-0.581	-0.472	0.029	34.702
	3	UnempNatDef_Change	16.870	13.130	30.000	3	27	30	5.623	0.488	11.563	0.000	0.759	0.562	0.514	0.69735118	0.084	5.194	1	27	0.031					0.314	0.007	0.402	0.290	0.857	1.168
3	1	Inflation_Wage_Change	8.665	21.335	30.000	1	29	30	8.665	0.736	11.778	0.002	0.537	0.289	0.264	0.85773825	0.289	11.778	1	29	0.002					0.537	0.537	0.537	0.537	1.000	1.000
4	1	Inflation_WPI_Change	7.673	22.327	30.000	1	29	30	7.673	0.770	9.966	0.004	0.506	0.256	0.230	0.87743884	0.256	9.966	1	29	0.004					0.506	0.506	0.506	0.506	1.000	1.000
5	1	ATXPrime_Return	4.235	25.765	30.000	1	29	30	4.235	0.888	4.766	0.037	0.376	0.141	0.112	0.94258045	0.141	4.766	1	29	0.037					0.376	0.376	0.376	0.376	1.000	1.000
6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	1	Inflation_HQCP_Index	4.263	25.737	30.000	1	29	30	4.263	0.887	4.803	0.037	0.377	0.142	0.113	0.94206556	0.142	4.803	1	29	0.037					0.377	0.377	0.377	0.377	1.000	1.000
8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

39 stocks traded between May 21, 2008 and December 31, 2010 - Regression Set 5

Factor	Predictor	Variable	ANOVA					Model Summary					Coefficients				Collinearity Statistics														
			Sum of Squares		df		Significance	Mean Square		R	Adjusted R ²	Standard Error	Change Statistics			Stand. Coeff. Beta	Correlations		Tolerance	VIF											
Regression	Residual	Total	Regression	Residual	Total	Regression		Residual	R Change				F Change	df 1	df 2		Sign. F Change	Zero-order			Partial	Part									
1	1	ATX_Return	14,181	15,819	30,000	1	29	30	14,181	0.545	25,997	1	29	0.000	0.688	0.473	0.455	0.73955661	0.473	25,997	1	29	0.000	0.894	0.688	0.767	0.607	0.471	2.122		
	2	Initiation_PRR_Change	17,283	12,717	30,000	2	28	30	8,642	0.454	19,028	0.000	0.759	0.576	0.546	0.67391687	0.103	6,831	1	28	0.014	-0.591	-0.324	-0.676	-0.468	0.628	1.592				
	3	Eurobor12M_Change	20,840	9,160	30,000	3	27	30	6,947	0.339	20,477	0.000	0.833	0.695	0.661	0.59244743	0.119	10,485	1	27	0.003	0.581	0.125	0.624	0.406	0.489	2.047				
	4	EUConSent_ChangeAbs	22,256	7,744	30,000	4	26	30	5,564	0.298	18,882	0.000	0.861	0.742	0.702	0.54573554	0.047	4,755	1	26	0.038	-0.353	0.368	-0.393	-0.217	0.378	2.643				
2	1	Commod_OBrentChange	9,129	20,871	30,000	1	29	30	9,129	0.720	12,684	0.001	0.552	0.304	0.280	0.84835215	0.304	12,684	1	29	0.001	0.691	0.478	0.441	0.74778189	0.174	9,327	1	28	0.005	2.346
	2	Commod_OVArabChange	14,344	15,656	30,000	2	28	30	7,172	0.559	12,827	0.000	0.691	0.478	0.441	0.74778189	0.174	9,327	1	28	0.005	-2.782	0.470	-0.981	-0.472	0.029	34.702	1.168			
	3	UnempNatBerl_Change	16,870	13,130	30,000	3	27	30	5,623	0.486	11,563	0.000	0.750	0.562	0.514	0.69735118	0.084	5,194	1	27	0.031	0.314	0.007	0.402	0.290	0.857	1.168				
3	1	EUProd_Index	4,598	25,402	30,000	1	29	30	4,598	0.876	5,249	0.029	0.391	0.153	0.124	0.93592082	0.153	5,249	1	29	0.029	-0.391	-0.391	-0.391	1.000	1.000					
4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		

Abstract

This paper presents different methods to evaluate companies and investment opportunities and focuses on the cost of capital and notably its estimation, as this is one of the most important elements in every valuation process. As such, an analysis of the arbitrage pricing theory and its application in practice is undertaken to find out whether this method can be used to estimate the cost of capital in the environment of a small capital market.

Depending on the circumstances, investors can resort to a range of valuation techniques which can be more or less suitable for a certain situation. Along with these different approaches comes the necessity to find the appropriate discount rate, or the cost of capital when evaluating a company. There are several ways to estimate a company's cost of capital, with the common point being the relationship between the risk of an investment and the return an investor expects or requires. While some estimation models assume a single risk factor, other methods, including the arbitrage pricing theory, allow incorporating several factors.

In an empirical investigation, the functioning of the arbitrage pricing theory on a small capital market is examined by testing the model on a series of data on the Austrian stock market. The results suggest that a structure and certain factors as well as a fit with common macroeconomic variables can ultimately also be identified in data from small capital markets, thus encouraging the use of the model under such constraints. Nevertheless, the results do not show very high levels of strength from a statistical perspective, and the factors' correlation to the general movement of the stock market raises the question whether a similar result could not be achieved with a simpler model.

Zusammenfassung

Die vorliegende Arbeit stellt unterschiedliche Methoden zur Bewertung von Unternehmen und Investitionsmöglichkeiten vor und legt den Schwerpunkt hierbei auf die Kapitalkosten und insbesondere deren Schätzung, als diese eines der entscheidenden Elemente in jedem Bewertungsprozess sind. In diesem Zusammenhang erfolgt eine Analyse der Arbitrage Pricing Theory und eine Untersuchung, ob diese Methode zur Schätzung der Kapitalkosten unter den Rahmenbedingungen eines kleinen Kapitalmarktes angewendet werden kann.

Abhängig von der Ausgangslage können Investoren auf eine Reihe von Bewertungsmethoden zurückgreifen, die je nach Situation mehr oder weniger angemessen sind. Gemeinsam mit diesen unterschiedlichen Ansätzen besteht die Notwendigkeit den entsprechenden Zinssatz beziehungsweise die entsprechenden Kapitalkosten, im Falle der Bewertung eines Unternehmens, zu ermitteln. Es gibt mehrere Möglichkeiten um die Kapitalkosten eines Unternehmens zu schätzen, wobei allen Ansätzen der Zusammenhang zwischen dem Risiko einer Investition und der erwarteten Rendite oder Mindestrendite eines Investors gemein ist. Während manche Modelle von einem einzigen Risikofaktor ausgehen, erlauben andere Ansätze, darunter die Arbitrage Pricing Theory, mehrere Faktoren zu berücksichtigen.

In einer empirischen Untersuchung wird die Funktionsweise der Arbitrage Pricing Theory anhand eines Datensatzes zum österreichischen Aktienmarkt betrachtet. Die Ergebnisse legen nahe, dass sich auch aus Daten eines kleinen Kapitalmarktes eine Struktur und zugrundeliegende Faktoren erkennen sowie ein Bezug zu gängigen makroökonomischen Variablen herstellen lassen, was letztendlich zur Anwendung des Modells auch unter eingeschränkten Rahmenbedingungen animiert. Gleichwohl ist die statistische Bedeutsamkeit der Ergebnisse nicht auf sehr hohem Niveau, und die Korrelation der Faktoren mit der allgemeinen Bewegung des Aktienmarktes lässt die Frage aufkommen, ob ein ähnliches Ergebnis nicht auch mit einem einfacheren Schätzmodell erzielt werden könnte.

Curriculum Vitae

Name: Matthias KRIMMEL
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10/2005 - 06/2012 Diploma Studies in International Business Administration
University of Vienna, Austria
Specializations: Corporate Finance, Management Accounting

01/2008 - 06/2008 Semester abroad at Jönköping International Business School
University of Jönköping, Sweden

09/1996 - 06/2004 Bundesgymnasium und Bundesrealgymnasium Mödling Keimgasse
Grammar school specializing in modern languages, A-Levels

Work Experience:

Since 11/2011 Paris Corporate Finance – Paris, France
Junior Analyst (Internship), M&A and Corporate Finance Advisory

02/2011 - 07/2011 Credit Suisse – Vienna, Austria
Junior Analyst (Internship), Corporate Advisory / M&A

07/2010 - 10/2010 Raiffeisen-Holding Niederösterreich-Wien – Vienna, Austria
Internship, Investment Management / Investment Controlling

10/2009 - 06/2010 University of Vienna – Faculty of Business, Economics and Statistics
Teaching Assistant, Department of Management Accounting

08/2006 and 08/2007 Deloitte – Vienne, Austria
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Computer Skills: MS Office (Word, Excel, Powerpoint), Bloomberg, Mergermarket,
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