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1. Introduction

Open-economy macroeconomics' development has traditionally been related to the advancement in its closed-economy counterpart. Consequently, the evolution of models considering monetary policy in both fields resembles strong similarities. Under autarky, the IS-LM model was dominant during most of the postwar period. In an open environment, its extension, the Mundell-Fleming model, was the leading theoretical framework. The Mundell-Fleming model was developed by Fleming (1962) and Mundell (1963, 1964). Similarly to the IS-LM model, it is an ad hoc model. The equilibrium relations are not derived from utility maximizing agents, but are constructed as an explanation of observed behavior of some aggregates. Lucas (1976) criticizes exactly such models in his famous critique. As he points out, such models cannot be used for forecasting or policy evaluation as they do not explicitly describe the decision process of the agents. Hence, they fail to capture the change in the estimated parameters, which is the result of the effect the change in policy has on agents' behavior (see Lucas (1976), p. 41). Indeed, several such shortcomings of the Mundell-Fleming model are described in the literature. For example, the capital account is specified as a flow function of the interest rate levels, which implies that capital flows uniformly forever, even if there is constant domestic-foreign interest rate differential (see Obstfeld (2000), p. 8). This is not reasonable if investors are rational and profit maximizing. However, this model's combination of Keynesian pricing with international market segmentation made it, in practice, a good approximation and was widely used by policy makers (see Obstfeld (2000), p. 2).

Nevertheless, international macroeconomists aimed at designing a model on sound microeconomic basis as Robert Lucas suggested in his critique. Similar to their closed-economy colleagues, they focused on intertemporal dynamic optimization by the economic subjects. In both fields of macroeconomics, however, economists "assumed away the awkward reality of sticky prices" (see Obstfeld and Rogoff (1995), p. 624). In closed economy setting the Real Business Cycle models were developed. In international environment, models like Sachs (1981), a two-period model, and Obstfeld (1982), infinite planning period model, were designed. These open-economy models can be used to make predictions and to evaluate policies. However, they fail to explain some empirical

observations, e.g. the effect of policies on the exchange rates. This is primarily, because models with markets without frictions fail to explain most international finance issues (see Obstfeld and Rogoff (1995), p. 625).

Logically, models that combine the two approaches were developed, the so-called New Keynesian models. These models combine the rational expectations and intertemporal optimization with nominal rigidities and imperfect markets. They perform well empirically. Furthermore, their conclusions are consistent with the fundamental core of beliefs central banks have concerning monetary economics: monetary policies have significant effect on real economic activity, which persist for several years; there is no significant long term trade-off between inflation and real activity; eliminating inflation leads to substantial gains; credibility of the monetary policies matter (see Goodfriend and King (1997), pp. 233-234). The models can be used for evaluating policies and making credible predictions in accordance with Lucas's critique. Consequently, these models have become the common modeling framework for both closed- and open-economy monetary analysis in what Goodfriend and King call "The New Neoclassical Synthesis" (see Goodfriend and King (1997), p. 233).

In international macroeconomics, the new trend in research was initiated by Obstfeld and Rogoff (1995), as it is now commonly recognized (see Lane (2001), p. 236). This is the first contribution that combines rigorously all the described above features. Subsequently, there has been a substantial amount of such models that, however, vary in many other aspects of their setting. As these models are commonly used to evaluate and predict the effects of monetary policy rules, it is important to fully understand how a difference in the settings of the model affects the findings of the model. Furthermore, what setting is the most appropriate for which real-world situation? For example, it is logical to think that a model, which presents the country as one of a continuum of economies and therefore its policy decisions do not affect the world equilibrium, is more applicable to a country such as Croatia than the US. This thesis's goal is to examine precisely these issues. For the purpose, two of the most prominent New Keynesian open-economy models are considered. In the second section of this paper the original Obstfeld and Rogoff model is examined. The third section studies Gali and Monacelli's (2005) model. In the last section, the explained models are compared and jointly analyzed.

2. Obstfeld and Rogoff Model

An extensive part of the following is based on Obstfeld and Rogoff (1995), thus there will not be specific references to this source.

2.1 Households and Firms

The world is inhabited by continuum of infinitely-lived individuals $z \in [0,1]$, which are both producers and consumers. Every individual is a monopolistic producer. There are basically no firms, or equivalently one can think of an individual as a firm when discussing production. There are two countries, individuals $z \in [0,n]$ are residents of Home and individuals $z \in (n,1]$ are located in Foreign. Hence, n determines the relative size of the two economies. All individuals have identical preferences and, within a country, the same constraints. Therefore, a representative national individual j can be analyzed. From here on in order to avoid confusion, the representative individual is denoted with j , while z is mostly used to denote the goods produced by the individuals. This individual maximizes the present discounted value of his/her life-long utility given by:

$$U^j = \sum_{t=0}^{\infty} \beta^t \left[\log C_t^j + \frac{\chi}{1-\varepsilon} \left(\frac{M_t^j}{P_t} \right)^{1-\varepsilon} - \frac{\kappa}{2} y_t(j)^2 \right] \quad (1)$$

where:

β is a discount factor.

$C_t^j = [\int_0^1 c(z)^{(\theta-1)/\theta} dz]^{\theta/(\theta-1)}$ is a consumption index with $\theta > 1$, where $c(z)$ is home individual's consumption of good z .

M_t^j is the money holdings of the individual in home currency.

$P_t = [\int_0^1 p(z)^{1-\theta} dz]^{1/(1-\theta)}$ is a consumption based price index for an agent in Home, where

$p(z)$ is the domestic-currency price of good z .

$y_t(j)$ is individual j 's effort.

Assuming this utility function contains several smaller assumptions. First, it assumes that consumers' preferences for goods exhibit Constant Elasticity of Substitution (CES),

as $[\int_0^1 c(z)^{(\theta-1)/\theta} dz]^{\theta/(\theta-1)}$ is a CES function. Elasticity of substitution between goods is equal to θ (the derivation of this is done in Mathematical Appendix A.1, p. 59). The price index, P_t , which is defined as the minimum expenditure for which a consumer can obtain 1 unit of the consumption index, is derived from this function (see Mathematical Appendix A.2, p. 59). This assumption may at first seem not excessively damaging to the model's credibility, but one must note that, together with a further assumption made of no trade costs, this excludes home biases or border effects, facts strongly supported by empirical literature (see Wolf (2000), pp. 561-562 and McCallum (1995), p. 622). Second, the utility of an agent depends on his/her consumption of goods, $\log C_t^j$, and effort of producing goods, $-\frac{\kappa}{2} y_t(j)^2$, positively on the former and negatively on the latter. This in general is an unproblematic assumption, and the functions' forms are widely used in the literature. The third component of the utility function, $\frac{\chi}{1-\varepsilon} \left(\frac{M_t^j}{P_t} \right)^{1-\varepsilon}$,

represent dependence on real money holdings in domestic currency. Hence, the model is a Money-in-the-Utility-Function (MIU) model. This is one of the typical ways to incorporate money in an economy resulting in a positive demand for money (see Walsh (2010), p. 34). One of the other ways, treating money as an asset in order to transfer resources intertemporally, is still a possibility for this model as specified so far. However, a riskless asset yielding positive interest is later introduced, making this function of money obsolete. Practically, the MIU models perform robustly, helping to explain many monetary phenomena. Theoretically, there is a logical puzzle, especially for utility functions of the type $U^j = \sum_{t=0}^{\infty} \beta^t u[C_t^j, \frac{M_t^j}{P_t}, y_t(j)]$. Money on its own is useless, i.e. if it

does not result in less work or more consumption now or in the future. If there is no infinite horizon one can interpret MIU function just as more money is better than less, with some presumption of later use. However, when the utility function is of the type

$U^j = \sum_{t=0}^{\infty} \beta^t u[C_t^j, \frac{M_t^j}{P_t}, y_t(j)]$, one can hold constant the path of consumption and effort

for all t (until infinity). Still, an increase in money holdings paradoxically increases

individual's utility. Nevertheless, MIU function is a useful shortcut to ensure positive demand for money (see Walsh (2010), pp. 33-36).

Utility function (1), results in an individual j 's demand for product z in period t given by: $c_t(z) = [\frac{p(z)}{P_t}]^{-\theta} C_t$ (see Mathematical Appendix A.3, p. 60).

The representative individual maximizes the present discounted value of his life-long utility subject to his/her budget constraint, which is given by:

$$P_t F_t^j + M_t^j = P_t(1+r_{t-1})F_{t-1}^j + M_{t-1}^j + p_t(j)y_t(j) - P_t C_t^j - P_t T_t^j \quad (2)$$

Where: F_t^j is the only financial asset, a real riskless bond denominated in the composite consumption good, traded at integrated complete world capital market.

r_t is the real interest rate of this bond between periods t and $t+1$

T_t^j is real lump-sum taxes paid to the home government.

The budget constraint is straightforward, it only must be noted that F_t^j, M_t^j denote the bonds and money holdings of the individual just before period $t+1$.

One can completely analogously derive all the above results for a foreign agent, since preferences are identical in both countries and there are no trade costs. In the following, variables noted with asterisk are concerning the foreign country. For example, M_t^* is money holdings in a foreign currency. Since there are no trade costs, the law of one price holds. Let E denote the nominal exchange rate, the home-currency price of foreign currency. Then the law of one price implies $p(z) = Ep^*(z)$. This assumption is often used as it is supported by theoretical logic, however a number of empirical studies have found that it is debatable (see Haskel and Wolf (2001), p. 557 and Isard (1977), p. 942). $p(z) = Ep^*(z)$ implies $P = EP^*$ (see Mathematical Appendix A.4, p. 61).

The government completely finances its per capita purchases, G , by taxes and seigniorage, i.e. $G_t = T_t + \frac{M_t - M_{t-1}}{P_t}$ and $G_t^* = T_t^* + \frac{M_t^* - M_{t-1}^*}{P_t^*}$. For simplicity, it is assumed that G is a composite of government consumption of individual private goods $g(z)$ with the same weights as private consumption, i.e. $G_t^j = [\int_0^1 g(z)^{(\theta-1)/\theta} dz]^{\theta/(\theta-1)}$. Then

the demand faced by each individual producer is obtained by adding up private and government demand:

$$y_t(z) = \left[\frac{p_t(z)}{P_t} \right]^{-\theta} (C_t^w + G_t^w)$$

where $C_t^w = nC_t + (1-n)C_t^*$ and $G_t^w = nG_t + (1-n)G_t^*$ are, respectively, world private consumption demand and world government consumption demand.

As this paper focuses on monetary theory and does not discuss fiscal policy, specifying a government in fiscal terms is not really necessary. The other two models presented do not discuss government purchases. This model can be easily modified not to include a government demand, e.g. $G_t = T_t + \frac{M_t - M_{t-1}}{P_t} = 0$, or equivalently,

$-PT_t = M_t - M_{t-1}$, all government revenues from seigniorage are distributed in terms of transfers to the population. This might be an useful simplification, however the goal of this paper is to compare the two models without modifications in order to keep authenticity of the analysis.

2.2. Equilibrium

The representative individual maximizes the present discounted value of his life-long utility given by (1) subject to (2). The first-order conditions for this maximization problem are (see Mathematical Appendix A.5, p. 61):

$$C_{t+1}^j = \beta(1+r_t)C_t^j$$

$$\frac{M_t^j}{P_t} = \left(\chi C_t^j \frac{1+i_t}{i_t} \right)^{1/\varepsilon}$$

$$y_t(j)^{(\theta+1)/\theta} = \frac{\theta-1}{\theta\kappa} \frac{1}{C_t^j} (C_t^w + G_t^w)^{1/\theta}$$

where $1+i_t = \frac{P_{t+1}}{P_t}(1+r_t)$ defines the nominal interest rate in home at time t . The nominal

interest rate for foreign is defined in the same manner and, since $P_t = EP_t^*$ and the real

interest rate is globally the same, the connection between the two nominal interest rates

$$\text{can be expressed as: } 1+i_t = \frac{E_{t+1}}{E_t}(1+i_t^*)$$

The three first-order conditions for foreign are analogical:

$$C_{t+1}^{j*} = \beta(1+r_t)C_t^{j*}$$

$$\frac{M_t^{j*}}{P_t^*} = \left(\chi C_t^{j*} \frac{1+i_t^*}{i_t^*} \right)^{1/\varepsilon}$$

$$y_t^*(j)^{(\theta+1)/\theta} = \frac{\theta-1}{\theta\kappa} \frac{1}{C_t^{j*}} (C_t^w + G_t^w)^{1/\theta}$$

The first conditions are the standard Euler equations that give the optimal consumption path, in this case, fully described by the consumption index, the real interest rate and the discount factor. The second conditions are the money market equilibrium conditions. They equate the utility gained from consuming a unit of the consumption index in t with the utility obtained by keeping the equivalent amount as money holdings in t and then consuming it in $t+1$. The last, third, conditions equalize the utility gained from additional revenue from producing an additional unit of good j to the disutility from the effort required for the production.

The other condition, which insures that the above first-order conditions are indeed maximizing, is the Transversality Condition given by $\lim_{t \rightarrow \infty} \frac{1}{(1+r_t)^t} \left(F_t^j + \frac{M_t^j}{P_t} \right) = 0$ (see Mathematical Appendix A.5). This is the so-called “no Ponzi scheme” condition. It assures that no country or individual will continue to borrow forever (the limit cannot be negative) and it assures that no country or individual will continue to lend forever (the limit cannot be greater than zero). Both cases must be discarded as no rational agent will finance someone else’s consumption forever, by doing so, in fact making a gift. Therefore, it is fully reasonable to impose this condition onto the agents, i.e. assume it must hold, otherwise the agents are not rational. The first order conditions, the budget constraint and the transversality condition fully characterize the equilibrium.

Due to monopoly pricing and endogenous output, this model does not yield a simple closed-form solution for the general paths. The system could be analyzed through

numerical simulations, however, the intuition of the model is preserved, and thus it is far more convenient, if a linear approximation is studied. In order to linearize the system, a well-defined flexible-price steady state around which to approximate is necessary. The most convenient such steady state is the symmetric one with all exogenous variables held constant (see Obstfeld and Rogoff (1996), p. 667).

In the steady state, all the endogenous variables are constant, e.g. $C_{t+1}^j = C_t^j$. Then it is clear that from the consumption Euler equation it follows: $\bar{r} = \frac{1-\beta}{\beta}$, which is the world real steady-state interest rate. It is also constant. Symmetric steady state means that all producers are symmetric, i.e. they set the same price and output quantity in equilibrium. Let $\bar{p}(h)$ be the home currency price of a home good and equivalently $\bar{p}^*(f)$ be the foreign currency price of a foreign good. Let $\bar{y}$ and $\bar{y}^*$ be the corresponding output levels. Then modifying the budget constraint gives (see Mathematical Appendix A.6, p. 64):

$$\bar{C} = \bar{r}\bar{F} + \frac{\bar{p}(h)\bar{y}}{\bar{P}} - \bar{G}$$

and for foreign:

$$\bar{C}^* = -\bar{r}\left(\frac{n}{1-n}\right)\bar{F} + \frac{\bar{p}^*(f)\bar{y}^*}{\bar{P}^*} - \bar{G}^*$$

Note that for the second equation it is taken use of the fact that world net foreign assets must be zero, as domestic nominal money supply must equal domestic nominal money demand, i.e. $nF + (1-n)F^* = 0$. Furthermore, one must stress that $\frac{\bar{p}(h)}{\bar{P}}$ and $\frac{\bar{p}^*(f)}{\bar{P}^*}$ are not generally equal to 1. They are only equal to one if the two countries have equal wealth. Otherwise, it is not the case, as the terms of trade vary with relative wealth.

It is important here to observe that the world financial market clearing condition ($nF + (1-n)F^* = 0$) leads to global goods market clearing condition given by:

$$C_t^w = n \frac{p_t(h)y_t}{P_t} + (1-n) \frac{p_t^*(f)y_t^*}{P_t^*} - G_t^w \quad (\text{see Mathematical Appendix A.7, p. 65})$$

One can easily show, by just plugging in C_t^w , that the values obtained for $\bar{C}$ and $\bar{C}^*$ satisfy this condition.

The special case of no foreign assets and no government spending has a closed form solution. Variables denoted with subscript 0 concern this case. Then: $\bar{F}_0 = \bar{F}_0^* = 0$ and $\bar{G}_0 = \bar{G}_0^* = 0$. The solution of the model is then given by:

$$\bar{y}_0 = \left(\frac{\theta - 1}{\theta \kappa} \right)^{1/2}$$

$$\bar{C}_0 = \bar{C}_0^* = \bar{y}_0 = \bar{y}_0^* = \bar{C}_0^w$$

$$\frac{\bar{M}_0}{\bar{P}_0} = \frac{\bar{M}_0^*}{\bar{P}_0^*} = \left(\frac{1 - \beta}{\chi} \right)^{-1/\varepsilon} \bar{y}_0^{1/\varepsilon}$$

(see Mathematical Appendix A.8, p. 66)

After obtaining the solution to this steady state, the whole model can be log-linearized around it. The linearization is implemented by expressing the model in terms of deviations from the obtained steady path. In the following variables denoted by hats stand for percentage changes from this path, e.g. $\hat{X}_t \equiv \frac{dX_t}{\bar{X}_0}$, where X is some variable of the

model. In order to obtain the final set of equations that fully describe the model in these terms, first all the equations should be rewritten (see Mathematical Appendix A.9, p. 67):

Purchasing power parity equation:

$$P = EP^* \Rightarrow \hat{E} = \hat{P} - \hat{P}^*$$

Price indexes:

$$P = \left[\int_0^n p(z)^{1-\theta} dz + \int_n^1 [Ep^*(z)]^{1-\theta} dz \right]^{1/(1-\theta)} \Rightarrow \hat{P}_t = n\hat{p}_t(h) + (1-n)(\hat{E}_t + \hat{p}_t^*(f))$$

$$P^* = \left[\int_0^n \left[\frac{p(z)}{E} \right]^{1-\theta} dz + \int_n^1 p^*(z)^{1-\theta} dz \right]^{1/(1-\theta)} \Rightarrow \hat{P}_t^* = n(\hat{p}_t(h) - \hat{E}_t) + (1-n)\hat{p}_t^*(f)$$

Global goods market equilibrium:

$$C_t^w = n \frac{p_t(h)y_t}{P_t} + (1-n) \frac{p_t^*(f)y_t^*}{P_t^*} - G_t^w \Rightarrow$$

$$\Rightarrow \hat{C}_t^w = n\hat{C}_t + (1-n)\hat{C}_t^* = n(\hat{y}_t + \hat{p}_t(h) - \hat{P}_t) + (1-n)(\hat{y}_t^* + \hat{p}_t^*(f) - \hat{P}_t^*) - \frac{dG_t^w}{\bar{C}_0^w} \quad (3)$$

Demand faced by producers:

$$y_t = \left[\frac{p_t(h)}{P_t} \right]^{-\theta} (C_t^w + G_t^w) \Rightarrow \hat{y}_t = \theta(\hat{P}_t - \hat{p}_t(h)) + \hat{C}_t^w + \frac{dG_t^w}{\bar{C}_0^w} \quad (4)$$

$$y_t^* = \left[\frac{p_t^*(f)}{P_t^*} \right]^{-\theta} (C_t^w + G_t^w) \Rightarrow \hat{y}_t^* = \theta(\hat{P}_t^* - \hat{p}_t^*(f)) + \hat{C}_t^w + \frac{dG_t^w}{\bar{C}_0^w} \quad (5)$$

First order conditions determining output:

$$y_t^{(\theta+1)/\theta} = \frac{\theta-1}{\theta\kappa} \frac{1}{C_t} (C_t^w + G_t^w)^{1/\theta} \Rightarrow (\theta+1)\hat{y}_t = -\theta\hat{C}_t + \hat{C}_t^w + \frac{dG_t^w}{\bar{C}_0^w} \quad (6)$$

$$y_t^{*(\theta+1)/\theta} = \frac{\theta-1}{\theta\kappa} \frac{1}{C_t^*} (C_t^w + G_t^w)^{1/\theta} \Rightarrow (\theta+1)\hat{y}_t^* = -\theta\hat{C}_t^* + \hat{C}_t^w + \frac{dG_t^w}{\bar{C}_0^w} \quad (7)$$

Modified budget constraints:

$$\bar{C} = \bar{r}\bar{F} + \frac{\bar{p}(h)\bar{y}}{\bar{P}} - \bar{G} \Rightarrow \hat{\bar{C}} = \bar{r} \frac{d\bar{F}}{\bar{C}_0^w} + \hat{\bar{p}}(h) + \hat{\bar{y}} - \hat{\bar{P}} - \frac{d\bar{G}}{\bar{C}_0^w} \quad (8)$$

$$\bar{C}^* = -\bar{r} \left(\frac{n}{1-n} \right) \bar{F} + \frac{\bar{p}^*(f)\bar{y}^*}{\bar{P}^*} - \bar{G}^* \Rightarrow \hat{\bar{C}}^* = -\bar{r} \left(\frac{n}{1-n} \right) \frac{d\bar{F}}{\bar{C}_0^w} + \hat{\bar{p}}^*(f) + \hat{\bar{y}}^* - \hat{\bar{P}}^* - \frac{d\bar{G}^*}{\bar{C}_0^w} \quad (9)$$

Consumption Euler equations:

$$C_{t+1} = \beta(1+r_t)C_t \Rightarrow \hat{C}_{t+1} = (1-\beta)\hat{r}_t + \hat{C}_t \quad (10)$$

$$C_{t+1}^* = \beta(1+r_t)C_t^* \Rightarrow \hat{C}_{t+1}^* = (1-\beta)\hat{r}_t + \hat{C}_t^* \quad (11)$$

Money demand equations:

$$\frac{M_t}{P_t} = \left(\chi C_t \frac{1+i_t}{i_t} \right)^{1/\varepsilon} \Rightarrow \hat{M}_t - \hat{P}_t = \frac{1}{\varepsilon} \hat{C}_t - \frac{\beta}{\varepsilon} \left(\hat{r}_t + \frac{\hat{P}_{t+1} - \hat{P}_t}{1-\beta} \right) \quad (12)$$

$$\frac{M_t^*}{P_t^*} = \left(\chi C_t^* \frac{1+i_t^*}{i_t^*} \right)^{1/\varepsilon} \Rightarrow \hat{M}_t^* - \hat{P}_t^* = \frac{1}{\varepsilon} \hat{C}_t^* - \frac{\beta}{\varepsilon} \left(\hat{r}_t + \frac{\hat{P}_{t+1}^* - \hat{P}_t^*}{1-\beta} \right) \quad (13)$$

It is important to note that equations (3)-(7) hold at any point, so clearly they as well hold when the economy is in a steady state, hence they remain valid when time-subscripted changes are interchange by steady state changes. Then there are seven equations for the seven variables, $\hat{y}$, $\hat{y}^*$, $\hat{\bar{C}}$, $\hat{\bar{C}}^*$, $\hat{\bar{p}}(h) - \hat{\bar{P}}$, $\hat{\bar{p}}^*(f) - \hat{\bar{P}}^*$, and $\hat{\bar{C}}^w$ (notice that $\hat{\bar{p}}(h) - \hat{\bar{P}}$

and $\hat{p}^*(f) - \hat{P}^*$ always appear in this way (or as $\hat{P} - \hat{p}(h) = -(\hat{p}(h) - \hat{P})$) in the above equations, hence they could be considered as variables). Solving this system of equations gives the new real steady state path. The solution for consumption in home and foreign is (see Mathematical Appendix A.10, p. 72):

$$\hat{C} = \left(\frac{\theta+1}{2\theta} \right) \frac{\bar{r}d\bar{F}}{\bar{C}_0^w} - \left(\frac{\theta+1-n}{2\theta} \right) \frac{d\bar{G}}{\bar{C}_0^w} + \left(\frac{1-n}{2\theta} \right) \frac{d\bar{G}^*}{\bar{C}_0^w}$$

$$\hat{C}^* = - \left(\frac{n}{1-n} \right) \left(\frac{\theta+1}{2\theta} \right) \frac{\bar{r}d\bar{F}}{\bar{C}_0^w} + \left(\frac{n}{2\theta} \right) \frac{d\bar{G}}{\bar{C}_0^w} - \left(\frac{\theta+n}{2\theta} \right) \frac{d\bar{G}^*}{\bar{C}_0^w}$$

An exogenous increase in home per capita bond holding increases home private consumption in the new steady state, but by less than the income per period from these bonds $\bar{r}d\bar{F}$. This is the case, since output is endogenous and individuals change their consumption-leisure decision. Higher wealth leads to higher consumption, which on its turn leads to a fall in marginal utility from consumption. Then the marginal disutility from working will be higher than the marginal utility from consumption and it will be optimal for the individual to work less. Thus, some of the additional income is used for additional consumption and some is used to compensate the income, already used for consumption, lost due to less work. An increase in home government spending decreases home private consumption. This is due to the fact that, although, some of the government purchases are of home goods, some are of foreign. However, all of the purchases are financed by taxes at home. The positive effect on output is offset by the higher taxes. An increase in foreign government spending increases home private consumption, as some of the purchases are home goods. Home output increases to meet the higher demand and thus consumption. The same analysis is valid for foreign, just reversed.

The solution of the model for output is:

$$\hat{y} = - \frac{\theta}{1+\theta} \hat{C} + \left(\frac{1}{2(1+\theta)} \right) \frac{d\bar{G}^w}{\bar{C}_0^w}$$

$$\hat{y}^* = - \frac{\theta}{\theta+1} \hat{C}^* + \left(\frac{1}{2(1+\theta)} \right) \frac{d\bar{G}^w}{\bar{C}_0^w}$$

The output rises with government spending. The result for home government spending holds, since it finances its expenditures with lump-sum taxes. Higher lump-sum taxes

change the consumption-leisure decision, individuals consume less and produce more. One can show (by substituting the result for $\hat{\bar{C}}$) that output falls, if home per capita foreign assets increase. This is again due to the fact that if individuals get wealthier, they would work less.

The solution of the model for the prices is:

$$\hat{p}(h) - \hat{p}^*(f) - \hat{E} = -\frac{1}{\theta}(\hat{y} - \hat{y}^*) = \frac{1}{1+\theta}(\hat{\bar{C}} - \hat{\bar{C}}^*)$$

If output in home increases relatively to the one in foreign, the prices in home relative to foreign fall, i.e. the terms of trade fall. The opposite is true, if home private consumption increases relatively to foreign private consumption.

As prices are flexible, money supply does not affect real variables and, hence, money demand equations do not play a role in determining the real variables. Since inflation and the real interest rate do not change in the steady state, equations (12) and (13) give the solution to the steady state price levels:

$$\hat{P}_t = \hat{M}_t - \frac{1}{\varepsilon} \hat{C}_t$$

$$\hat{P}_t^* = \hat{M}_t^* - \frac{1}{\varepsilon} \hat{C}_t^*$$

Flexible-price models are in general unable to replicate the magnitude and the persistency of the effects of monetary shocks on real variables, which are commonly observed in the real world. This model is not an exception. In order to properly capture these effects, nominal rigidities are introduced (see Walsh (2010), p. 408). In this model, this is done by presetting nominal producer prices, $p_t(h)$ and $p_t^*(f)$. The prices are set one period in advance and are hold fixed during this period. After the period, if there is no new shocks, prices fully adjust to the flexible-price levels.

Price stickiness is not an artificial theoretical convenience, but considerably supported by theoretical logic and empirical evidence. Theoretically, there are many reasons why prices should be rigid, such as menu costs, fixed durable contracts, cost-based pricing, lagged information upon decision on pricing, pricing points, etc. An extensive survey of all the theories is done in Blinder et al. (1998). Empirically, a number of studies have supported these theories, including: Blinder et al. (1998) - an extensive

survey of firms and some econometric data analysis; Golosov and Lucas (2007) examine disaggregated CPI data from greater New York area, Blinder (1991) surveys price setters; Kashyap (1995) studies catalog prices in the US.

Price stickiness might be an established fact, however, how rigid prices are and in what manner they adjust is still an open question. Blinder et al. report that the median number of price changes for a product in a year is 1.4. Furthermore, almost half of the prices change no more frequent than once a year and, when there are regular price reviews, they are almost always annual (see Blinder et al. (1998), p. 298). Golosov and Lucas show that in their sample 21.9% of all prices in the CPI change every month (see Golosov and Lucas (2007), p. 183). Blinder finds that 55% of prices change no more often than annually and only 15% change more frequently than quarterly (see Blinder (1991), p. 93). Kashyap shows that in the sample studied the average time between price changes is 15 months. (see Kashyap (1995), p. 252). These studies examine the dynamics of prices (the observed price changes), yet it is hard to find out if and how quickly prices converge to the flexible steady state based only on the observation of price changes. The first difficulty is that prices are “sticky” relatively to some unmeasured, perhaps unmeasurable, Walrasian norm (see Blinder et al. (1998), p. 296). It is problematic to determine convergence when the state to which the system should be converging is unknown. Second, it is unclear what triggered these changes. If there is identical shock to all producers, they should adjust simultaneously, which is obviously not the case. Third, it must be known with what lag producers act after a shock. Blinder examines many different kinds of shocks and finds that, while there is quite of variation, the mean lags cluster around 3-4 months range (see Blinder (1991), p. 94).

The model assumes that prices remain fixed for one period and then adjust 100%. This means that producers react after 1 period to the shock, which is consistent with the empirical evidence if the period is one quarter. After this period prices adjust fully. This in practice means a change of all product prices after 1 period, which would on average not be problematic if the period is at least one year, as it is seen from the data above. The change of prices in the model is simultaneous, since in the model there is one single identical to all shock. While in reality there are many shocks affecting different producers differently, and, hence, prices never change simultaneously. To sum up, the quite

restricting assumption about price stickiness in this model harms the richness of the dynamics, as prices simultaneously adjust immediately after one period, and it makes it hard for the model to fit the data, as there is inconsistency between its and the empirical price dynamics.

With sticky prices, output becomes demand driven for shocks that are not huge. This is the case, since in this model every good is produced by a single producer, a monopolist. The monopolist maximizes profits by equating marginal revenue to marginal cost. The profit maximizing conditions here are equations (6) and (7). Equating marginal revenue to marginal cost by a monopolist always results in a price above marginal cost, as the monopolist faces downward sloping demand curve (except when demand is perfectly elastic, which is here not the case). Then, with prices fixed, the producer would benefit from every additional sell as it would increase profits by price minus marginal cost. The producer would increase his/her output if there is demand. This holds until marginal cost equals the price. Thus, the shock must not be huge, as then the output might not be fully demand determined if demand rises by so much that satisfying it means output must increase to a level at which marginal cost exceeds the price. A fall in demand will obviously lead to a fall in output. As a result equations (6) and (7) need not hold. Output is fully determined by the demand schedules (4) and (5).

The fact that $p_t(h)$ and $p_t^*(f)$ are fixed, does not mean that the aggregate price level does not change. It changes with the nominal exchange rate. If there is a nominal depreciation ($\hat{E}_t > 0$), the home price index increases, since the domestic currency price of foreign goods becomes higher. From the two previously derived equations:

$$\hat{P}_t = n\hat{p}_t(h) + (1-n)(\hat{E}_t + \hat{p}_t^*(f)) \quad \Rightarrow \quad \hat{P} = (1-n)\hat{E} \quad (14)$$

$$\hat{P}_t^* = n(\hat{p}_t(h) - \hat{E}_t) + (1-n)\hat{p}_t^*(f) \quad \Rightarrow \quad \hat{P}^* = -n\hat{E} \quad (15)$$

Note that the variables on the right-hand side have no time indexes, but are also not variables in a steady state. From here on, such variables, with hats and no time index or overbars, denote short-run changes.

Combining equations (4) and (5) with equations (14) and (15) gives the short-run aggregate demand:

$$\hat{y} = \theta(1-n)\hat{E} + \hat{C}^w + \frac{dG^w}{\bar{C}_0^w} \quad (16)$$

$$\hat{y}^* = -\theta n\hat{E} + \hat{C}^w + \frac{dG^w}{\bar{C}_0^w} \quad (17)$$

Equations (10)-(13) always hold. However, the new steady state is reached immediately after one period, meaning that variables with the subscript $t+1$ can be replaced by the steady-state changes. The variables with subscript t are now interpreted as short-run changes:

$$\hat{\bar{C}} = (1-\beta)\hat{r} + \hat{C} \quad (18)$$

$$\hat{\bar{C}}^* = (1-\beta)\hat{r} + \hat{C}^* \quad (19)$$

$$\hat{M} - \hat{P} = \frac{1}{\varepsilon}\hat{C} - \frac{\beta}{\varepsilon}\left(\hat{r} + \frac{\hat{\bar{P}} - \hat{P}}{1-\beta}\right) \quad (20)$$

$$\hat{M}^* - \hat{P}^* = \frac{1}{\varepsilon}\hat{C}^* - \frac{\beta}{\varepsilon}\left(\hat{r} + \frac{\hat{\bar{P}}^* - \hat{P}^*}{1-\beta}\right) \quad (21)$$

The last two equations to complete the model come from the budget constraints. While the modified intertemporal budget constraints ensure that in the long-run current accounts are balanced, in the short-run the current account deficit or surplus is given by:

$$F_t - F_{t-1} = r_{t-1}F_{t-1} + \frac{p_t(h)y_t}{P_t} - C_t - G_t$$

$$F_t^* - F_{t-1}^* = r_{t-1}^*F_{t-1}^* + \frac{p_t^*(h)y_t^*}{P_t^*} - C_t^* - G_t^*$$

Log-linearizing gives (see Mathematical Appendix A.11, p. 75):

$$\frac{d\bar{F}}{\bar{C}_0^w} = \hat{y} - \hat{C} - (1-n)\hat{E} - \frac{dG}{\bar{C}_0^w} \quad (22)$$

$$\frac{d\bar{F}^*}{\bar{C}_0^w} = \hat{y}^* - \hat{C}^* + n\hat{E} - \frac{dG^*}{\bar{C}_0^w} = -\left(\frac{n}{1-n}\right)\frac{d\bar{F}}{\bar{C}_0^w} \quad (23)$$

The system that fully describes the short-run is given by the 10 equations (14)-(23) in the 10 variables, $\hat{C}, \hat{C}^*, \hat{y}, \hat{y}^*, \hat{P}, \hat{P}^*, \hat{C}^w, \hat{E}, \hat{r}$ and $d\bar{F}$. For the long-run, it is enough to

notice that the economy returns to the steady state, which was already solved, and that after the short-run the wealth situation has changed due to the current account, $d\bar{F}$.

The model is now ready to be solved for a permanent monetary shock. Notice that in all the equations the effects of monetary shocks and of change in government spending are additive, therefore, nothing is lost if it assumed that the government does not change its spending, i.e. $dG = d\bar{G} = dG^* = d\bar{G}^* = 0$. This is one of the strong features of this model - it allows monetary and fiscal policy to be analyzed separately or in a policy mix. As in the case for the long-run steady state, it makes sense first to solve for the differences between home and foreign variables. This is again done in the mathematical appendix, however, this time it is beneficial to report some of the results in the main text, as it enhance the intuition of the model and some of the main predictions of the model can be more easily understood. The equations that follow are derived in Mathematical Appendix A.12, p. 76.

The money supply shock is permanent, i.e. $\hat{M} - \hat{M}^* = \hat{\bar{M}} - \hat{\bar{M}}^*$, there is an increase (or decrease) in relative home money supply that occurs in the short-run and stays the same when the economy reaches its new steady state after one period. From the Euler equations (18) and (19), it is easily seen that $\hat{C} - \hat{C}^* = \hat{\bar{C}} - \hat{\bar{C}}^*$. This means that all shocks have permanent effect on the difference between countries' consumptions. The difference is the same in the short-run and in the long-run. However, this does not imply that the consumption is the same in the two periods, since the short-run real interest rate might be different from the steady state real interest rate. From the money demand equations, one can show that: $(\hat{M} - \hat{M}^*) - \hat{E} = \frac{1}{\varepsilon}(\hat{C} - \hat{C}^*) - \frac{\beta}{\varepsilon(1-\beta)}(\hat{\bar{E}} - \hat{E})$. Notice, that relative short-run money demand does not depend on output differences, but rather on consumption differences. In an open economy setting this is really important, since individuals can smooth consumption through borrowing or lending in the international financial market. The long-run nominal exchange rate after the shock is given by:

$\hat{\bar{E}} = (\hat{\bar{M}} - \hat{\bar{M}}^*) - \frac{1}{\varepsilon}(\hat{\bar{C}} - \hat{\bar{C}}^*)$, combining this with the short-run money demand gives:

$\hat{E} = (\hat{M} - \hat{M}^*) - \frac{1}{\varepsilon}(\hat{C} - \hat{C}^*)$. As $\hat{M} - \hat{M}^* = \hat{\bar{M}} - \hat{\bar{M}}^*$ and $\hat{C} - \hat{C}^* = \hat{\bar{C}} - \hat{\bar{C}}^*$, this means that

the exchange rate jumps immediately to its new steady state value, $\hat{\bar{E}} = \hat{E}$. This is to be expected, since the differences in money supply and consumption remain constant. In fact from the short-run aggregate demand equations, (16) and (17), the current account equations, (22) and (23) and the solutions for the difference in consumptions in the long-

run it follows that: $\hat{E} = \left(\frac{2\theta + \bar{r}(\theta + 1)}{\bar{r}(\theta^2 - 1)} \right) (\hat{C} - \hat{C}^*)$ The equation shows by how much the

home currency must depreciate in order for home relative output to increase enough to justify a permanent rise in relative home consumption. Home relative output increases when home currency depreciates, as can be seen from equations (16) and (17):

$\hat{y} - \hat{y}^* = \theta \hat{E}$. Notice that $\hat{E} = (\hat{M} - \hat{M}^*) - \frac{1}{\varepsilon}(\hat{C} - \hat{C}^*)$ expresses the nominal exchange

rate as a function of the difference in consumption, which is linear and downward

sloping, while $\hat{E} = \left(\frac{2\theta + \bar{r}(\theta + 1)}{\bar{r}(\theta^2 - 1)} \right) (\hat{C} - \hat{C}^*)$ is again a linear function in the difference in

consumption, but it is upward sloping ($\theta > 1$). One can express the change in the exchange rate as a function of the change in money supply:

$$\hat{E} = \left(\frac{\varepsilon [2\theta + \bar{r}(\theta + 1)]}{\varepsilon [2\theta + \bar{r}(\theta + 1)] + \bar{r}(\theta^2 - 1)} \right) (\hat{M} - \hat{M}^*) = (**)$$

Observe that, since $\theta > 1$, the following inequality holds:

$$\hat{E} = \left(\frac{\varepsilon [2\theta + \bar{r}(\theta + 1)]}{\varepsilon [2\theta + \bar{r}(\theta + 1)] + \bar{r}(\theta^2 - 1)} \right) (\hat{M} - \hat{M}^*) < \hat{M} - \hat{M}^*$$

The exchange rate rises with home relative money supply, but by less than the money shock. This is the case, since relative consumption in Home increases as well. The depreciation switches world demand toward domestic products, which leads to short-run rise in relative home income. Individuals in Home do not consume the whole additional income, but, as typical for open-economy models, they intertemporally smooth the increase in consumption by running a current account surplus, as shown later.

The difference in consumption is a function of relative money supply as well:

$$(\hat{C} - \hat{C}^*) = \left(\frac{\varepsilon \bar{r}(\theta^2 - 1)}{\varepsilon[2\theta + \bar{r}(\theta + 1)] + \bar{r}(\theta^2 - 1)} \right) (\hat{M} - \hat{M}^*) = (*)$$

All the information obtained so far can be summarized in a graph, which makes understanding the model more intuitive and clear. This is done in Figure 1, where the

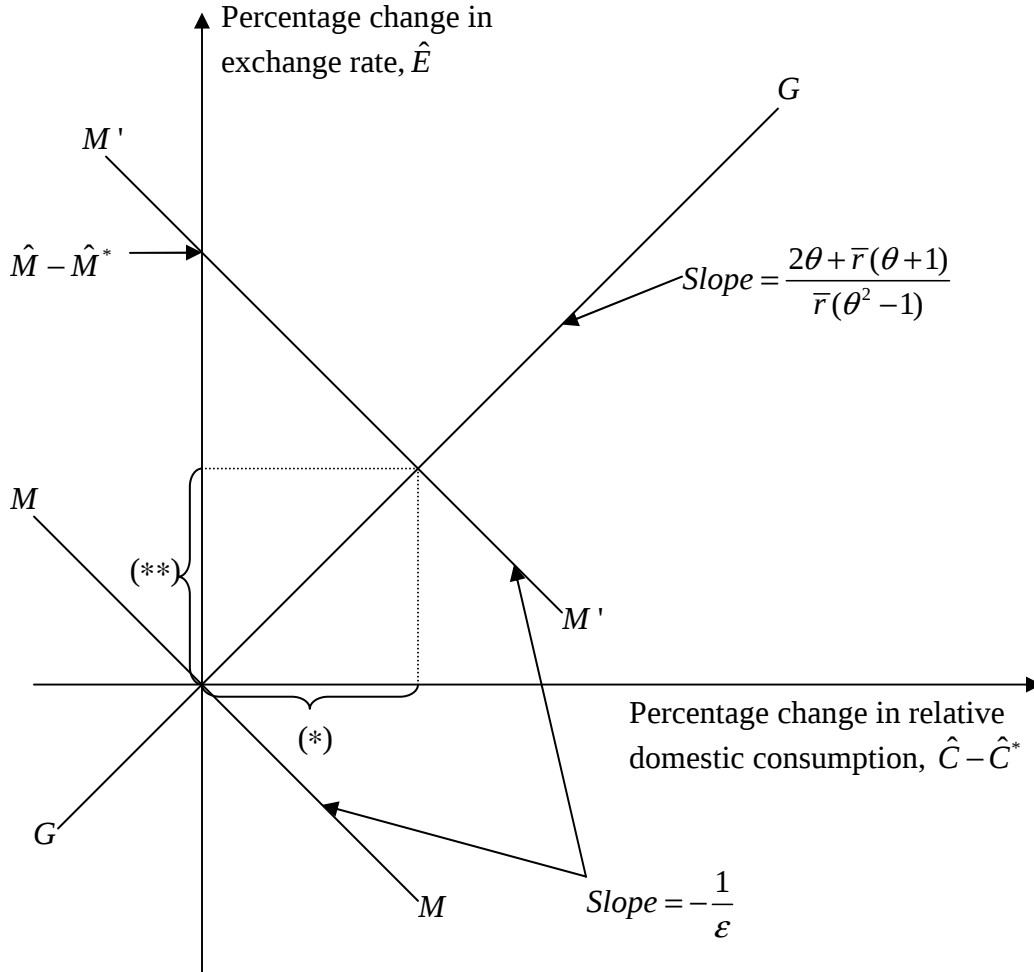


Figure 1: Permanent Shock in Domestic Relative Money Supply

MM schedule is given by $\hat{E} = (\hat{M} - \hat{M}^*) - \frac{1}{\varepsilon}(\hat{C} - \hat{C}^*)$. The vertical intercept of the MM line equals the relative percentage increase in home money supply. Thus, the MM schedule passes through the origin before the monetary shock. The GG schedule is given by $\hat{E} = \left(\frac{2\theta + \bar{r}(\theta + 1)}{\bar{r}(\theta^2 - 1)} \right) (\hat{C} - \hat{C}^*)$. It is called GG , since if the model is solved for

fiscal shocks, this is the curve that shifts. Obviously, it too passes through the origin. The monetary shock shifts the initial MM line to $M'M'$. The intersection of the $M'M'$ schedule and the GG schedule is the short-run equilibrium. Notice that if $\theta \rightarrow \infty$, i.e. domestic and foreign goods are perfect substitutes and perfectly competitive economy is approached, GG becomes flat. There will not be any exchange rate change.

The model can be solved for the current account:

$$\frac{d\bar{F}}{\bar{C}_0^w} = \left(\frac{2\theta(1-n)\varepsilon(\theta-1)}{\varepsilon[2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2-1)} \right) (\hat{M} - \hat{M}^*)$$

A permanent increase in relative home money supply leads to positive current account, as explained earlier. Individuals at home smooth their consumption path and do not consume all their additional income. The current account is inversely proportional to the size of the country. The bigger home is (the greater n is), the less positive affect on the current account an increase in home money supply has (the smaller $\frac{d\bar{F}}{\bar{C}_0^w}$ is).

The current account is all that is needed in order to solve for the new long run steady state. The results are:

For consumption:

$$\text{Home: } \hat{C} = \left(\frac{\bar{r}(1-n)\varepsilon(\theta^2-1)}{\varepsilon[2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2-1)} \right) (\hat{M} - \hat{M}^*)$$

$$\text{Foreign: } \hat{C}^* = - \left(\frac{n\bar{r}\varepsilon(\theta^2-1)}{\varepsilon[2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2-1)} \right) (\hat{M} - \hat{M}^*)$$

For output:

$$\text{Home: } \hat{y} = - \left(\frac{\theta\bar{r}(1-n)\varepsilon(\theta-1)}{\varepsilon[2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2-1)} \right) (\hat{M} - \hat{M}^*)$$

$$\text{Foreign: } \hat{y}^* = \left(\frac{\theta n\bar{r}\varepsilon(\theta-1)}{\varepsilon[2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2-1)} \right) (\hat{M} - \hat{M}^*)$$

For terms of trade:

$$\hat{p}(h) - \hat{p}^*(f) - \hat{E} = \left(\frac{\varepsilon\bar{r}(\theta-1)}{\varepsilon[2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2-1)} \right) (\hat{M} - \hat{M}^*)$$

If there is a positive monetary shock in Home, consumption is permanently higher for home residents, while output in the long-run is permanently lower, since home individuals are wealthier. As they have obtained wealth (the current account), they change their consumption-leisure decision, as discussed before. Home individuals' consumption increases, but not by the whole additional income, as individuals choose to work less. Thus, the long-run steady state home output falls. Exactly the opposite is true for foreign.

There is an improvement in the terms of trade in the long-run for home. This is again the case, because the country has become richer and, therefore, works less. However, short-run and long-run terms of trade effects work in opposite directions. In the long-run they might improve by $\left(\frac{\varepsilon \bar{r}(\theta - 1)}{\varepsilon[2\theta + \bar{r}(\theta + 1)] + \bar{r}(\theta^2 - 1)} \right) (\hat{M} - \hat{M}^*)$, but in the short-run they deteriorate by $\left(\frac{\varepsilon[2\theta + \bar{r}(\theta + 1)]}{\varepsilon[2\theta + \bar{r}(\theta + 1)] + \bar{r}(\theta^2 - 1)} \right) (\hat{M} - \hat{M}^*)$, the value of $\hat{E}$, as $\hat{p}(h)$ and $\hat{p}^*(f)$ are fixed. It is easily seen that the short term effect is stronger. This is to be expected as the whole effect in the long-run comes only from the interest payments, $\frac{\bar{r}d\bar{F}}{\bar{C}_0^w}$.

The effects from the monetary shock last longer than the sticky price period, in fact, they last infinitely long. The second part of this statement is due to some specifications of this model, infinitely-lived individuals and intertemporally separable utility. The first part, however, is a more general result, typical for such models. This is due to the fact that, if short-run nominal rigidities exist, monetary shocks will result in international capital flows. The resulting transfers have effects on real variables after the period of stickiness is over.

The last thing left is to solve the model for short run variables:

The short-run interest rate is:

$$\hat{r} = - \left(\varepsilon + \frac{\beta}{(1 - \beta)} \right) \hat{M}^w, \text{ where } \hat{M}^w = n\hat{M} + (1 - n)\hat{M}^*$$

The real interest rate falls if world money supply increases. Hence, it increases global consumption demand.

The short-run output is:

$$\begin{aligned}\hat{y} &= \left[n(\varepsilon(1-\beta) + \beta) + (1-n)\theta \left(\frac{\varepsilon[2\theta + \bar{r}(\theta+1)]}{\varepsilon[2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2 - 1)} \right) \right] \hat{M} + \\ &+ \left[(1-n)(\varepsilon(1-\beta) + \beta) - (1-n)\theta \left(\frac{\varepsilon[2\theta + \bar{r}(\theta+1)]}{\varepsilon[2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2 - 1)} \right) \right] \hat{M}^* \\ \hat{y}^* &= \left[n(\varepsilon(1-\beta) + \beta) - n\theta \left(\frac{\varepsilon[2\theta + \bar{r}(\theta+1)]}{\varepsilon[2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2 - 1)} \right) \right] \hat{M} + \\ &+ \left[(1-n)(\varepsilon(1-\beta) + \beta) + n\theta \left(\frac{\varepsilon[2\theta + \bar{r}(\theta+1)]}{\varepsilon[2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2 - 1)} \right) \right] \hat{M}^*\end{aligned}$$

The short-run consumption is:

$$\begin{aligned}\hat{C} &= (\varepsilon(1-\beta) + \beta) \hat{M}^w + (1-n) \left(\frac{\varepsilon \bar{r}(\theta^2 - 1)}{\varepsilon[2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2 - 1)} \right) (\hat{M} - \hat{M}^*) \\ \hat{C}^* &= (\varepsilon(1-\beta) + \beta) \hat{M}^w - n \left(\frac{\varepsilon \bar{r}(\theta^2 - 1)}{\varepsilon[2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2 - 1)} \right) (\hat{M} - \hat{M}^*)\end{aligned}$$

A positive home monetary shock increases the short-run output of home, as it is expected. Home consumption increases as well. The more interesting question is what happens to foreign variables. Alternatively, what happens to home short-run variables when there is a positive foreign monetary shock? Unfortunately, the answer is ambiguous and depends on the parameters, or, more precisely, on ε . The special case where $\varepsilon = 1$, the output equation simplifies to:

$$\hat{y} = \frac{2(n + \theta(1-n)) + (\theta+1)\bar{r}}{2 + (\theta+1)\bar{r}} \hat{M} + \frac{2(1-n)(1-\theta)}{2 + (\theta+1)\bar{r}} \hat{M}^*$$

If there is an increase in money supply in foreign, home short-run output falls, since $\theta > 1$. It is easily seen from the solution for $\hat{y}$, that as ε gets large enough, the effect of a foreign positive monetary shock on home output turns positive. From the first order conditions of the individual's decision problem, one can see that ε is inversely related to the interest elasticity of money demand. Money demand is interest inelastic if ε is large.

3. Gali and Monacelli Model

An extensive part of the following is based on Gali and Monacelli (2005), thus there will not be specific references to this source.

3.1 Households

The world consists of a continuum of economies represented by a unit interval. Let Home be the economy that is modeled, as before, variables without superscripts refer to this economy. However, in this model there is not one foreign country, but infinitely many. In the following, variables denoted with asterisk are regarding a single foreign country, one among the infinitely many that make up the world economy, $* \in [0,1]$. Variables with w-superscripts again stand for world aggregates.

As Home has zero weight in the world aggregates, its policy changes do not affect the rest of the world. This is rather different from the previous model with only two countries. However, one could think that the two models are asymptotically equivalent in this respect, if $n \rightarrow 0$. From the solutions obtained in part 2 of this paper, one can see that, if $n \rightarrow 0$, the changes of foreign long- and short-run output and consumption due to a monetary shock in Home tend to zero. That is, home monetary policy changes do not affect Foreign, which is the rest of the world in the case of two-country world.

All the world economies have the same individuals' preferences, technology and market structure. The first two points of this assumption are equivalent to the one of identical utility functions made in Obstfeld and Rogoff's model, since there the utility function represents not only the preferences concerning consumption, but also the technology with which goods are produced from labor, as individuals are producers and consumers. The third point of the assumption, identical market structures, is the same for both models.

As assumed, all individuals have identical preferences and, within a country, the same constraints. Therefore, a representative national individual can be analyzed. This individual maximizes the expected present discounted value of his/her life-long utility given by:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right] \quad (24)$$

where:

β is a discount factor.

$C_t \equiv \left[(1-\alpha)^{1/\eta} (C_t^H)^{(\eta-1)/\eta} + \alpha^{1/\eta} (C_t^F)^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}$ is a composite consumption index,

where:

$C_t^H \equiv \left(\int_0^1 c_t(z)^{(\mu-1)/\mu} dz \right)^{\mu/(\mu-1)}$ is an index of domestic goods consumption, where $z \in [0,1]$

is the goods' variety.

$C_t^F \equiv \left(\int_0^1 (C_t^*)^{(\gamma-1)/\gamma} d* \right)^{\gamma/(\gamma-1)}$ is an index of imported goods, where

$C_t^* \equiv \left(\int_0^1 c_t^*(z)^{(\mu-1)/\mu} dz \right)^{\mu/(\mu-1)}$ is the quantity of goods imported from * and consumed by the home representative individual.

N_t denotes hours of labor.

As seen before, $\mu > 1$ is the constant elasticity of substitution (see Mathematical Appendix A.1, p. 59). However, here it is the elasticity of substitution between goods produced within a country, while in the Obstfeld and Rogoff's model θ is the elasticity of substitution between all goods. That is why a different parameter is used. γ is the elasticity of substitution between goods produced in different foreign countries. $\eta > 0$ is the elasticity of substitution between home and foreign goods. Finally, $\alpha \in [0,1]$ inversely measures the home bias, as seen from the definition of C_t . Therefore, it is a measure of openness. If $\alpha = 0$, home consumers would not buy any foreign goods. If $\alpha = 1$, they would buy only foreign goods. The presence of a home bias in this model is a welcome expansion compared to Obstfeld and Rogoff's model, since, as discussed before, home bias and border effects are strongly supported by the empirical literature.

Note that in $C_t^F \equiv \left(\int_0^1 (C_t^*)^{(\gamma-1)/\gamma} d* \right)^{\gamma/(\gamma-1)}$, Home is also included. However, this is irrelevant, since the economy has a zero measure.

Let $P_t^H \equiv \left(\int_0^1 p_t(z)^{1-\mu} dz \right)^{1/(1-\mu)}$ be the price index as defined and derived before (see Mathematical Appendix A.2) for domestically produced goods. Correspondingly,

$P_t^* \equiv \left(\int_0^1 p_t^*(z)^{1-\mu} dz \right)^{1/(1-\mu)}$ is the price index for goods imported from * in domestic currency. Then the domestic demand function for each good is given by:

$$c_t(z) = \left(\frac{p_t(z)}{P_t^H} \right)^{-\mu} C_t^H \quad \text{for domestic goods}$$

$$c_t^*(z) = \left(\frac{p_t^*(z)}{P_t^*} \right)^{-\mu} C_t^* \quad \text{for goods from country *}.$$

The derivation is equivalent to the one in Mathematical Appendix A.3, p. 59.

The individual optimizes his/her allocation of expenditures on foreign goods by country

of origin, which yields: $C_t^* = \left(\frac{P_t^*}{P_t^F} \right)^{-\gamma} C_t^F$, where $P_t^F \equiv \left(\int_0^1 (P_t^*)^{1-\gamma} d* \right)^{1/(1-\gamma)}$ is the price index for imported goods in domestic currency. This is derived in exactly the same way as the demand function, but from the function $C_t^F \equiv \left(\int_0^1 (C_t^*)^{(\gamma-1)/\gamma} d* \right)^{\gamma/(\gamma-1)}$.

In order to obtain the last important optimal allocation of expenditures between goods, precisely between domestic and foreign goods, one must first note that, given the demand functions, total expenditures on home goods are given by $\int_0^1 p_t(z)c_t(z)dz = P_t^H C_t^H$, on goods from country * by $\int_0^1 p_t^*(z)c_t^*(z)dz = P_t^* C_t^*$, on aggregated foreign goods by $\int_0^1 P_t^* C_t^* d* = P_t^F C_t^F$. Then the optimal allocation of expenditures between home and foreign goods is given by:

$$C_t^H = (1-\alpha) \left(\frac{P_t^H}{P_t} \right)^{-\eta} C_t$$

$$C_t^F = \alpha \left(\frac{P_t^F}{P_t} \right)^{-\eta} C_t$$

where $P_t \equiv \left[(1-\alpha)(P_t^H)^{1-\eta} + \alpha(P_t^F)^{1-\eta} \right]^{1/(1-\eta)}$ is the Consumer Price Index (CPI) (see Mathematical Appendix B.1, p. 82). One can show that total expenditures on

consumption are given by: $P_t^H C_t^H + P_t^F C_t^F = P_t C_t$, which shows that P_t is indeed the CPI. The derivation is again done in Appendix B.1.

The budget constraints subject to which the individual maximizes his/her utility are given by¹:

$$\int_0^1 p_t(z) c_t(z) dz + \int_0^1 \int_0^1 p_t^*(z) c_t^*(z) dz d^* + E_t [Q_{t,t+1} D_{t+1}] = D_t + W_t N_t + T_t$$

where W_t is the nominal wage, T_t denotes lump-sum transfers, D_{t+1} is the nominal payoff in period $t+1$ of the portfolio held at the end of period t (it includes shares in firms). All these variables are expressed in units of domestic currency. $Q_{t,t+1}$ is a stochastic discount factor for one period ahead nominal payoffs. D_{t+1} and $Q_{t,t+1}$ are explained in more detail later on. Assume, in addition, that the economic agents have access to a complete set of contingent claims, traded internationally, i.e. complete international asset markets with Arrow-Debreu securities. Note that here T_t denotes lump-sum transfers in nominal terms, not in real, as it is in Obstfeld and Rogoff's model. There the equivalent is $P_t T_t$. This is a trivial fact and here it is kept in nominal terms for simplicity, as most variables in this model are nominal, while in the first model most were real.

More importantly, observe that there is no money in the utility function as well as in the budget constraint. This is a cashless limit economy. Money is not introduced at all. One can think of money as a unit of account, but not as an asset. Monetary policy takes the form of interest rate rules. This noticeably differs from the Obstfeld and Rogoff's model, where monetary policy was executed through changes in the money supply. These are, historically, the two main approaches used in conducting monetary policy. In recent years money growth targeting has almost entirely been abandoned in favor of interest rate rules. Growth of money supply targeting was extensively used in the 1970s and was widely credited for the 1978-1981 disinflation period in the United States. However, after

¹ The budget constraint is given with an equal sign (=) instead of smaller or equal ($\leq$), since, if the decision problem is solved with smaller or equal sign, from the first-order condition with respect to consumption comes that the Lagrangian multiplier must be greater than 0, as consumption cannot be negative (the problem is solved with a bit modified budget constraint, as seen shortly). This, together with the complementary slackness condition, assures that the budget constraint holds with equality sign. Therefore, for brevity and simplicity, one can skip all these steps and start directly with the problem as given above.

this period, money growth targets became long-run considerations for the Federal Reserve in the US and, currently, most central banks have adopted this approach (see Taylor (1999a), p. 335). This is not a coincidence. Several studies have shown that, if some conditions are met, which is usually the case for advanced economies, interest rate rules are superior to money growth rules (see Poole (1970), pp. 205-206 and Carlstrom and Fuerst (1995), p. 266). Superior has different meaning in two papers. In Poole (1970), equivalent monetary policies in the two approaches are compared by measuring the volatility of output (the lower the volatility is, the better). Equivalent monetary policies means that if the interest rate is set on some level (when using interest rate rules), there is a corresponding expected level of money growth, and if money growth is set on this level (when using money supply rules), the corresponding expected level of the interest rate is exactly the same as before. This means that if the model was deterministic the two policies would have been indistinguishable (see Poole (1970), pp. 203-204). Carlstrom and Fuerst use a fully specified model with utility maximizing agents and then compare different monetary approaches on utility basis. Nevertheless, when inflation is really high or negative, interest rate rules perform worse than money supply rules (see Taylor (1999b), p. 661). Then again, this was hardly the case in developed countries during the last decades and, as a consequence, it is normal to expect that central banks in these countries would prefer interest rate rules.

Deciding to use interest rate rules is not equivalent to eliminating money altogether. To understand why it is possible not to introduce money at all, one must realize that when the money supply or the interest rate is the policy instrument the other one is endogenous to the model. If the central bank uses an interest rate rule, then it must vary the money supply in order to achieve its desired interest rate setting (see Taylor (1999b), p. 661). Similarly, a money growth rule has implications for the interest rate. In fact, in Obstfeld and Rogoff's model it was shown that the short-run real interest rate is given by

$$\hat{r} = -\left(\varepsilon + \frac{\beta}{(1-\beta)}\right)\hat{M}^w \text{ and the long-run by } \bar{r} = \frac{1-\beta}{\beta}.$$

The long-run (steady state) interest rate is determined only by the model's parameters (the discount factor), since prices are flexible and money has no effect on it. The short-run real interest rate, however, is a (invertible) function of the money supply. Because the model is deterministic and there is

no uncertain financial intermediation, the two policy approaches are interchangeable. The central bank can easily transfer its money supply rules to real (or nominal, as one can show) interest rate rules. In reality, “a fixed money growth rule will generally imply a reaction of interest rates to the inflation rate and to real output similar in form though not necessary similar in size to interest rate policy rules” (see Taylor (1999b), p. 661). However, if the central bank follows an interest rate rule, it varies money supply in such a way that the interaction of money supply with money demand leads to the desired interest rate. What feeds back into the model is the interest rate and it is the interest rate that affects the model’s variables. If it is assumed that the central bank can in some way set the interest rate to the desired levels, it is not necessary to specify in exactly what way this happens. Changes in money growth do not feed back into the model and, hence, money growth need not be computed and could actually be ignored (see Taylor (1999b), p. 661).

Another important difference between the two models is that Obstfeld and Rogoff’s model is, as mentioned above, a deterministic model, while Gali and Monacelli’s model is stochastic. In the former, agents have a perfect-foresight. They know exactly what their investment in assets will yield and what their output will be in the future (as well as all other values of variables). The central bank might surprise them with an unanticipated monetary shock, as it was modeled in part 2 of this paper, but immediately after the shock the agents know what the values of the variables will be in the future with certainty. There is no intrinsic uncertainty. Only a state of nature is possible. In the latter model, this is not the case. The agents can only expect what will happen in the future. As seen later on, in this model, the productivity of the workers follows a random process (AR(1)), and, as mentioned previously, there is a stochastic discount factor. What the future value of one of these variables will be depends on the state of nature, which follows some distribution. The agent has some rational expectations about what might be the value and he/she knows that the deviations from it depend on the states of nature. However, the expected value need not be realized, and once there is a specific realization of some of the random variables (or processes), the decision problem of the individual changes. He/she must condition his/her expectations on the fact that some state of nature has already occurred. The view of the world in Gali and Monacelli’s model is obviously more

realistic, but what really makes a difference is the fact that when there is uncertainty and individuals form expectations about the future, the model results will depend on the expectations for the future of the economic agents. Forward-looking relationships between variables can be formed, such as new Keynesian Phillips curve, where current inflation depends on expected future inflation. This is widely used connection, supported by some empirical literature (see Chadha et al. (1992), p. 326 and Stock and Watson (1999), p. 327). However, there are some studies reaching the conclusion that the forward-looking elements are not significant (see Fuhrer (1997), p. 349).

The first two terms of the left-hand side of the budget constraint denote total expenditures on consumption goods. Hence, using the revealed fact that total expenditures on consumption goods are given by $P_t C_t$, the budget constraint becomes:

$$P_t C_t + E_t [Q_{t,t+1} D_{t+1}] = D_t + W_t N_t + T_t \quad (25)$$

For some of the following, Gali (2008) is used as a source.

Notice that D_{t+1} is the nominal payoff of a portfolio in period $t+1$, not the price of a bond in period t . $E_t [Q_{t,t+1} D_{t+1}]$ is the price of this nominal payoff in period t . Let there be H possible states of nature that could be realized in $t+1$ (it is not really necessary to specify H , it could be infinity as well) conditional on the state of nature in period t . It was assumed that there is a complete asset market. There is an Arrow-Debreu security² for every state of nature. Then in order to obtain a certain payoff of D_{t+1} in period $t+1$, the agent must buy D_{t+1} Arrow-Debreu securities for each possible state of nature. Let the price in period t of an Arrow-Debreu security be $V_{t,t+1}(h)$, where h is the state of nature for which the security pays a unit of currency in period $t+1$. Then the price of a portfolio in period t risklessly yielding D_{t+1} units of currency in period $t+1$ is given by

$$\sum_{h=1}^H V_{t,t+1}(h) D_{t+1} \text{ or, equivalently, } E_t \left(\frac{V_{t,t+1}(h)}{\xi_{t,t+1}(h)} D_{t+1} \right), \text{ where } \xi_{t,t+1}(h) \text{ is the probability that}$$

² An Arrow-Debreu Security is an one-period security that yields one unit of currency in the period after the purchase of the security, if a specific state of nature occurs in this period, and nothing, if any other state of nature occurs.

state of nature h occurs conditional on the state on nature in period t . Hence, the one-period stochastic discount factor $Q_{t,t+1}$ is defined as: $Q_{t,t+1}(h) \equiv \frac{V_{t,t+1}(h)}{\xi_{t,t+1}(h)}$.

The domestic individual maximizes (24) subject to (25), which yields the two first-order conditions (see Mathematical Appendix B.2, p. 84):

$$C_t^\sigma N_t^\varphi = \frac{W_t}{P_t}$$

$$\beta \frac{1}{E_t[Q_{t,t+1}]} E_t \left[\frac{P_t}{P_{t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \right] = 1$$

In order for these conditions to be indeed maximizing, it is assumed that the individuals are constrained by a no-Ponzi-scheme condition.

Notice that $E_t[Q_{t,t+1}] = E_t \left(\frac{V_{t,t+1}(h)}{\xi_{t,t+1}(h)} \right) = \sum_{h=1}^H V_{t,t+1}(h)$, this means that this is the price in period t of a portfolio consisting of one Arrow-Debreu security for each state of nature, i.e. a portfolio yielding riskless one unit of currency in period $t+1$. Then the nominal riskless interest rate must be given by $E_t[Q_{t,t+1}](1+i_t) = 1$. Hence, $1+i_t \equiv \frac{1}{E_t[Q_{t,t+1}]}$.

This might look different from the setting in Obstfeld and Rogoff's model, but the nominal interest rate is exactly equivalent. In the first model in this paper, the individual is buying a portfolio (of bonds) for a nominal price of $P_t F_t$ in period t , which yields $P_{t+1}(1+r_t)F_t$ in period $t+1$. Therefore, the nominal interest rate is defined as $(1+i_t)P_t F_t = P_{t+1}(1+r_t)F_t$, or as stated before $1+i_t = \frac{P_{t+1}}{P_t}(1+r_t)$. In the currently discussed model, the individual buys a nominal portfolio (of Arrow-Debreu securities) for $E_t[Q_{t,t+1}]$ and gets 1 in the following period. Hence, $1+i_t \equiv \frac{1}{E_t[Q_{t,t+1}]}$.

Generally, the two asset markets, one with riskless bonds and one with a full set of Arrow-Debreu securities, are not equivalent. Assuming that there is a complete market for a full set of Arrow-Debreu securities is a stronger assumption than assuming there is a riskless bond. As discussed, one can always create a riskless bond out of Arrow-Debreu

securities by just buying securities for each state of nature. However, when there is an uncertainty, the Arrow-Debreu securities can provide insurance against unfavorable states of nature, which the riskless bond cannot. With Arrow-Debreu securities, the individual can not only smooth consumption intertemporally, but also across different states of nature. In Obstfeld and Rogoff's model, however, the two assumptions are equivalent, since there is no uncertainty, i.e. there is only one state of nature. Arrow-Debreu securities are totally unrealistic, there are no such assets in the real world. On the contrast, one can think of some real world financial assets as riskless (if not perfectly riskless) bonds, such as US Treasuries or German Bonds. Both models assume perfect financial capital markets. This assumption does matter, even though there is no capital accumulation in these models. For a list of studies that show the major effects of asset markets imperfections on the current account, future output, etc. for models with capital accumulation see Antras and Caballero (2009), p. 704. Even when there is no investment in fixed capital, and only consumption, financial capital imperfections limit how much individuals can smooth consumption, thus possibly affecting the current account, consumption, output, etc. (see Obstfeld and Rogoff (1996), p. 359).

Using $1+i_t \equiv \frac{1}{E_t[Q_{t,t+1}]}$, one can rewrite the second first-order condition as:

$$\beta(1+i_t)E_t\left[\frac{P_t}{P_{t+1}}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\right]=1$$

Then the two first-order condition can be log-linearized (in the case of the second, a log-linear approximation), (see Mathematical Appendix B.3, p. 86):

$$C_t^\sigma N_t^\varphi = \frac{W_t}{P_t} \quad \Rightarrow \quad \sigma c_t + \varphi n_t = w_t - p_t \quad (26)$$

$$\beta(1+i_t)E_t\left[\frac{P_t}{P_{t+1}}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\right]=1 \quad \Rightarrow \quad c_t = E_t[c_{t+1}] - \frac{1}{\sigma}[i_t - E_t(\pi_{t+1}) - \rho] \quad (27)$$

where small case letters denote the natural logarithms of the respective variables, i_t is still the nominal interest rate (not the log), $\pi_t \equiv p_t - p_{t-1}$ is the CPI inflation, and $\rho \equiv -\ln \beta$. Note that $c_t \neq c_t(z)$ and $p_t \neq p_t(z)$.

Next, define the bilateral terms of trade between Home and country * as $S_t^* = \frac{P_t^*}{P_t^H}$,

i.e. the price of country * goods in terms of domestic goods. Then the effective terms of

trade are given by: $S_t = \frac{P_t^F}{P_t^H} = \left[\int_0^1 (S_t^*)^{1-\gamma} d^* \right]^{1/(1-\gamma)}$, which can be approximated (up to first

order) by log-linearizing around a perfect-foresight symmetric steady state. This symmetric state is derived later on, as, in order to solve for a symmetric steady state, the model should be fully specified (more relationships here, the firms' side, and equilibrium conditions). It is a lot more convenient if the relationships are now log-linearized with a leap of faith, than postponing it after the symmetric steady state derivation. Furthermore, after discussing the Obstfeld and Rogoff's model, it should be straightforward to see that there should be such a steady state. In fact, the symmetric steady state is basically equivalent. Again, all countries are assumed to be symmetric (for both firms and households), there are no foreign assets (before $\bar{F}_0 = \bar{F}_0^* = 0$), and no government spending, as here there is no government at all (before $\bar{G}_0 = \bar{G}_0^* = 0$). The last feature of the steady-state, perfect-foresight, is again shared by both models. In the first model, it is the general setting. In the currently discussed model, it is corresponding to assuming a fixed and known (and symmetric) technology of production for firms in all countries. Therefore, as in the first model, in the following, variables denoted with subscript 0 concern this case. In this symmetric steady state the Purchasing Power Parity (PPP) holds, i.e. $P_0^H = P_0^F$, and $S_0 = \frac{P_0^F}{P_0^H} = 1 = S_0^*$ for all $* \in [0,1]$. Then (see Mathematical

Appendix B.4, p. 87):

$$S_t = \frac{P_t^F}{P_t^H} = \left[\int_0^1 (S_t^*)^{1-\gamma} d^* \right]^{1/(1-\gamma)} \Rightarrow s_t = p_t^F - p_t^H = \int_0^1 s_t^* d^*$$

The price index can be linearized around the steady state as well:

$$P_t \equiv \left[(1-\alpha)(P_t^H)^{1-\eta} + \alpha(P_t^F)^{1-\eta} \right]^{1/(1-\eta)} \Rightarrow p_t = (1-\alpha)p_t^H + \alpha p_t^F$$

where small case variables are the natural logarithms of the corresponding normal variables. s_t is called the log effective terms of trade.

Combining the two equations above gives:

$$p_t = (1-\alpha)p_t^H + \alpha(s_t + p_t^H) = p_t^H + \alpha s_t$$

Define domestic goods inflation as $\pi_t^H = p_t^H - p_{t-1}^H$. Given the above equation, the following must hold: $\pi_t = \pi_t^H + \alpha \Delta s_t$. The difference between CPI inflation and domestic goods inflation is proportional to the percentage (remember that the variables are in logarithmic form) change in the terms of trade. The coefficient of proportionality is given by the coefficient of openness α .

As in Obstfeld and Rogoff's model, it is assumed that the law of one price holds: $p_t^*(z) = E_t^* p_{t,*}^*(z)$ for all $z, * \in [0,1]$, where $p_t^*(z)$ is, as before, the price of country $*$'s good z in domestic currency, E_t^* is the bilateral nominal exchange rate (the price of country $*$'s currency in terms of home currency), $p_{t,*}^*(z)$ is the price of country $*$'s good z in terms of country $*$'s currency (the producer's currency). As previously,

$$p_t^*(z) = E_t^* p_{t,*}^*(z) \Rightarrow P_t^* = E_t^* P_{t,*}^*,$$

where $P_{t,*}^* \equiv \left(\int_0^1 (p_{t,*}^*(z))^{1-\mu} dz \right)^{1/(1-\mu)}$ (the result is obtained by just factoring out E_t^*).

Plugging this into the definition of P_t^F and log-linearizing around the steady state gives (see Mathematical Appendix B.4, p. 87):

$$p_t^F = \int_0^1 e_t^* d* + \int_0^1 (p_{t,*}^*) d* = e_t + p_t^w$$

where, $p_t^F \equiv \ln P_t^F$; $e_t^* \equiv \ln E_t^*$; $p_{t,*}^* \equiv \int_0^1 (\ln p_{t,*}^*(z)) dz$, which is the log domestic price index for country $*$; $e_t \equiv \int_0^1 e_t^* d*$, which is the log nominal effective exchange rate;

$p_t^w \equiv \int_0^1 (p_{t,*}^*) d*$, which is the log world price index. For the world as a whole, there is no

difference between CPI and domestic goods price level, nor for the corresponding inflation rates, because of $p_t = p_t^H + \alpha s_t$ (when aggregated among all countries, the terms of trade cancel out).

Combining the above result with $s_t = p_t^F - p_t^H$, gives:

$$s_t = e_t + p_t^w - p_t^H$$

Let $B_t^* \equiv \frac{E_t^* P_{t,*}}{P_t}$ be the bilateral real exchange rate with country *. Note that here the

notation is a bit different than the one used for the Obstfeld and Rogoff's model. Roughly said, variables with superscript * are concerning country * from the perspective of Home or are concerning some bilateral relationship between Home and *, while variables with second subscript * are concerning country * from country *'s perspective. The notation is used like this, since any possible approach could not be fully consistent with the one used in Obstfeld and Rogoff's model and variables from Home's perspective are used a lot more often and, hence, for convenience they are the one with a superscript. Therefore, $P_{t,*}$ is the CPI index of country * in country *'s currency ($P_{t,*}^*$ is the price only for goods produced in country *, not the whole CPI). Let the log real effective exchange rate be given by: $b_t \equiv \int_0^1 b_t^* d^*$, where, as usual, $b_t^* \equiv \ln B_t^*$. Then:

$$b_t = \int_0^1 b_t^* d^* = \int_0^1 (e_t^* + p_{t,*} - p_t) d^*, \text{ recall now that for the world as a whole, there is no difference between CPI and domestic goods price level, i.e. } p_t^w = \int_0^1 (p_{t,*}) d^*.$$

$$b_t = e_t + p_t^w - p_t, \text{ using the fact that } s_t = e_t + p_t^w - p_t^H \text{ gives:}$$

$$b_t = s_t + p_t^H - p_t, \text{ and using the approximation } p_t = p_t^H + \alpha s_t \text{ results in:}$$

$$b_t = (1 - \alpha)s_t. \quad \text{The log real effective exchange rate depends only on the log effective terms of trade and on the coefficient of openness.}$$

Since international integrated complete asset markets are assumed, it must be that the Arrow-Debreu securities have the same price everywhere. Then some foreign country faces the same first-order optimization condition as Home:

$$\frac{V_{t,t+1}}{E_{t,*} P_{t,*}} C_{t,*}^{-\sigma} = \xi_{t,t+1} \beta C_{t+1,*}^{-\sigma} \frac{1}{E_{t+1,*} P_{t+1,*}}, \text{ the only difference is that the price of the Arrow-}$$

Debreu security is defined in Home currency and, correspondingly, it pays one unit of Home currency in the following period. Thus, both must be converted in foreign currency. The dependence on states of nature is dropped here, since the equations must hold for any state of nature and it is irrelevant for which one the derivation is done. This

expression can be rewritten as: $Q_{t,t+1} = \beta \frac{P_{t,*}}{P_{t+1,*}} \frac{C_{t+1,*}^{-\sigma}}{C_{t,*}^{-\sigma}} \frac{E_{t,*}}{E_{t+1,*}}$. Combining this, the condition

for home and the definition of bilateral real exchange rate gives (see Mathematical Appendix B.5, p. 90):

$C_t = \vartheta^* C_{t,*} (B_t^*)^{1/\sigma}$, where ϑ^* is a constant depending on the initial conditions. Without loss of generality one can assume identical initial condition – no net foreign assets and identical ex-ante environment, i.e. the steady state conditions. Then $\vartheta^* = 1$ for all $*$. Furthermore, as shown later, in the steady state $C_t = C_{t,*} = C_t^w$. Taking the natural logarithm of both sides of $C_t = \vartheta^* C_{t,*} (B_t^*)^{1/\sigma}$ and the integrating over i gives:

$$c_t = c_t^w + \frac{1}{\sigma} b_t = c_t^w + \frac{1-\alpha}{\sigma} s_t, \text{ using } b_t = (1-\alpha)s_t \text{ (which is again an approximation),}$$

where $c_t^w = \int_0^1 (c_{t,*}) d*$ is the log index of world consumption.

Therefore, because of international integrated complete asset markets, home consumption can be fully described by world consumption, the terms of trade and the index of openness.

3.2 Firms

There is a unit interval of firms in Home, each producing a differentiated good with a linear technology given by the production function:

$y_t(z) = A_t N_t(z)$, where z is the firm (which produces good z), and $a_t \equiv \ln A_t$ follows an AR(1) process $a_t = \rho_a a_{t-1} + \varepsilon_t$, where $E_t[\varepsilon_t] = 0$. Then the log real marginal costs are given by:

$mc_t = -v + w_t - p_t^H - a_t$, where $v \equiv -\ln(1-\tau)$, τ is an employment subsidy, which is not necessary for the purposes of this paper, but as in the case of Obstfeld and Rogoff's model the analysis is kept authentic. To obtain the real marginal cost, just take the derivative with respect to $N_t(z)$ of the total cost function, which is the labor divided by its productivity, times the real home wage, and times one minus the employment subsidy, and then take the natural logarithm. The production technology is, obviously, different

from the one in the first model, since here the producer can only expect, but not be certain, what he/she will be able to produce in future periods given some resources (labor), while in Obstfeld and Rogoff's model the individual (who is also a producer) knows exactly how much output will come out of his/her future effort.

Define $Y_t \equiv \left(\int_0^1 y_t(z)^{(\mu-1)/\mu} dz \right)^{\mu/(\mu-1)}$, an index of aggregate domestic output, which is

analogous to the index of domestic consumption goods. Notice that $N_t \equiv \int_0^1 N_t(z) dz$, then the index of aggregate domestic output can be log-linearized around the steady state, giving (see Mathematical Appendix B.6, p. 91): $y_t = a_t + n_t$

In order to finish with the supply side specification, one must determine how prices are set. This model assumes Calvo pricing as initiated by Calvo (1983), which means that in each period a constant fraction of randomly selected firms from the unit interval of firms can set new prices. The remaining fraction must stick with their old prices. Let a fraction of $1-\varpi$ be able to set new prices, where $\varpi \in [0,1]$, correspondingly, a fraction of ϖ must keep prices unchanged. The probability for each firm to be able to set a new price is unaffected by what happened in the past, i.e. it does not matter how long ago the firm was not able to re-optimize, the probability of being able to do so in the current period is still $1-\varpi$. In order to justify price stickiness, one can quote the same factors as in part 2 of this paper. However, one must again ask, if this is a proper way to model price behavior. It was shown that the way price dynamics are modeled in Obstfeld and Rogoff's model is not consistent with the empirical evidence. Recall that in the first model all prices are fixed for one period and then all adjust. In terms of the setting here this means that ϖ will not be a constant, but rather $\varpi_t = 1$ and $\varpi_{t+1} = 0$. If the period is a year, the average period a price stays fixed is a year, which is roughly consistent with the data, even if the dynamics are not. What about the current model? There are many ways to check if it is consistent with price dynamics given ϖ and the length of the period. If the period is a quarter one must look at what fraction of the prices change every quarter. As already stated, Blinder finds that 15% of prices change more frequently than quarterly (see Blinder (1991), p. 93). That is $1-\varpi = 0.15$ and $\varpi = 0.85$. All the studies mentioned in part 2 find that the average period prices stay fixed is somewhat longer than one year.

What is the average period prices stay fixed in the Calvo model? Notice that in the first period $1-\varpi$ of the prices change, the length of their period being fixed is 1 period. In the second period $1-\varpi$ of the remaining fraction ϖ change, the duration of their period being fixed is 2 periods. By continuing, one can see that the average period a price stays fixed is given by $(1-\varpi)(1+2\varpi+3\varpi^2+4\varpi^3\dots)$, or $(1-\varpi)\sum_{k=0}^{\infty}k\varpi^{k-1}$. To calculate this

sum, just notice that $\sum_{k=0}^{\infty}\varpi^k = \frac{1}{1-\varpi}$, as $\varpi < 1$. Differentiate both sides with respect to ϖ :

$\sum_{k=0}^{\infty}k\varpi^{k-1} = \frac{1}{(1-\varpi)^2}$. Therefore, the average time a price stays fixed in the Calvo model is

given by $\frac{1}{1-\varpi}$ periods. If the period is a quarter and $\varpi = 0.75$, prices remain fixed on

average for a year. More serious econometric studies concerning the value of ϖ exist. Gali and Gertler estimate the value of ϖ (there θ , which is usually used, but in this paper θ was already used for something else) and obtain a value of 0.8 (see Gali and Gertler (1999), p. 210). Eichenbaum and Fisher perform an extensive analysis. In their main case scenario, they obtain values between 0.83 and 0.89, depending on the data (GDP deflator or price deflator for personal consumption expenditures) and on the time period (first part and second of observations, and full sample) (see Eichenbaum and Fisher (2007), p. 2040). Then they perform a series of robustness checks by varying the econometric model (estimating tens of models), and get a wider interval of values for ϖ , from 0.56 to 0.89, with most values cluster around 0.75-0.86 (see Eichenbaum and Fisher (2007), pp. 2044-2045). To conclude, the Calvo pricing might be artificial, meaning that there is no economic reason backing the assumption that firms price their products in this way, but it fits extremely well the data. One can see that almost all the studies approximately agree with each other for a quarterly Calvo pricing with $\varpi \in [0.75, 0.85]$.

Under Calvo pricing a firm will optimally set prices by a rule that can be the log-linearized so that (see Mathematical Appendix B.7, p. 92):

$$\tilde{p}_t^H = \bar{h} + (1-\varpi\beta)\sum_{k=0}^{\infty}(\varpi\beta)^k(mc_{t+k}^n),$$

where $\tilde{p}_t^H$ is the log newly set domestic price, $mc_{t+k}^n = \ln MC_{t+k}^n = \ln \frac{(1-\tau)W_t}{A_t}$ the log

nominal marginal cost, and $\hbar \equiv \ln \left(\frac{\mu}{\mu-1} \right)$.

For future references, it is useful to derive one more equation from the optimal price setting. This is as well done in Mathematical Appendix B.7:

$$\pi_t^H = \beta E_t (\pi_{t+1}^H) + \lambda (mc_t + \hbar) \quad (28)$$

where $\lambda \equiv \frac{(1-\varpi)}{\varpi} (1-\varpi\beta)$ (note that λ is not a Lagrangian multiplier)

3.3 Equilibrium

In equilibrium supply and demand for Home goods must be equal. For a specific good z this means:

$$y_t(z) = c_t^H(z) + \int_0^1 c_{t,*}^H(z) d* ,$$

where the first term on the right-hand side gives the home consumption of home goods and the second term gives the consumption of home goods from all foreign countries. Using the demand functions and allocation relationships obtained before, one can rewrite

this equation. Note that $c_t(z) = \left(\frac{p_t(z)}{P_t^H} \right)^{-\mu} C_t^H$ and $C_t^H = (1-\alpha) \left(\frac{P_t^H}{P_t} \right)^{-\eta} C_t$, combining

these two gives: $c_t(z) = (1-\alpha) \left(\frac{p_t(z)}{P_t^H} \right)^{-\mu} \left(\frac{P_t^H}{P_t} \right)^{-\eta} C_t$. The easiest way to find the second

term is to see what the demand of Home for a specific foreign good is, and then just rewrite it from the perspective of a foreign country, which is possible because of the

assumption of symmetric preferences across countries. Notice that $c_t^*(z) = \left(\frac{p_t^*(z)}{P_t^*} \right)^{-\mu} C_t^*$,

$C_t^* = \left(\frac{P_t^*}{P_t^F} \right)^{-\gamma} C_t^F$ and $C_t^F = \alpha \left(\frac{P_t^F}{P_t} \right)^{-\eta} C_t$. Combining these gives:

$c_t^*(z) = \alpha \left(\frac{p_t^*(z)}{P_t^*} \right)^{-\mu} \left(\frac{P_t^*}{P_t^F} \right)^{-\gamma} \left(\frac{P_t^F}{P_t} \right)^{-\eta} C_t$. Rewriting this for foreign country leads to:

$$c_{t,*}^H(z) = \left(\frac{p_t(z)}{P_t^H} \right)^{-\mu} \left(\frac{P_t^H}{E_t^* P_{t,*}^F} \right)^{-\gamma} \left(\frac{P_{t,*}^F}{P_{t,*}} \right)^{-\eta} C_{t,*}$$

Then the equilibrium relation for good z is given by:

$$y_t(z) = \left(\frac{p_t(z)}{P_t^H} \right)^{-\mu} \left[(1-\alpha) \left(\frac{P_t^H}{P_t} \right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_t^H}{E_t^* P_{t,*}^F} \right)^{-\gamma} \left(\frac{P_{t,*}^F}{P_{t,*}} \right)^{-\eta} C_{t,*} d^* \right]$$

In order to obtain the equilibrium aggregate domestic output, one must substitute the above equation into the definition of aggregate domestic output

$$Y_t = \left(\int_0^1 y_t(z)^{(\mu-1)/\mu} dz \right)^{\mu/(\mu-1)}, \text{ which eventually leads to (see Mathematical Appendix B.8, p. 99):}$$

$$Y_t = \left(\frac{P_t^H}{P_t} \right)^{-\eta} C_t \left[(1-\alpha) + \alpha \int_0^1 (S_{t,*} S_t^*)^{\gamma-\eta} (B_t^*)^{\eta-1/\sigma} d^* \right]$$

After specifying the model in its entirety, it is now possible to derive the steady state around which some of the equations above are log-linearized. This is done in Mathematical Appendix B.9, p. 100.

Now that the symmetric steady state is obtained, one can linearize without leaps of faith the equation giving the equilibrium in the domestic goods market. The result is (see Mathematical Appendix B.10, p. 104):

$$y_t = c_t + \alpha \gamma s_t + \alpha \left(\eta - \frac{1}{\sigma} \right) b_t = c_t + \frac{\alpha \omega}{\sigma} s_t$$

where $\omega \equiv \sigma \gamma + (\sigma \eta - 1)(1 - \alpha)$.

For each country $*$, a similar condition hold, i.e. $y_{t,*} = c_{t,*} + \frac{\alpha \omega}{\sigma} s_{t,*}$ for each $*$. In order to obtain the world goods market clearing condition, one must aggregate over all the countries: $y_t^w \equiv \int_0^1 y_{t,*} d^* = \int_0^1 c_{t,*} d^* + \frac{\alpha \omega}{\sigma} \int_0^1 s_{t,*} d^* = \int_0^1 c_{t,*} d^* \equiv c_t^w$. Note that here it is again

used the fact that $\int_0^1 s_{t,*} d^* = 0$. Using this, and a previously obtained result,

$c_t = c_t^w + \frac{1-\alpha}{\sigma} s_t$, to plug into the linearized home goods clearing condition, gives:

$$y_t = c_t + \frac{\alpha\omega}{\sigma} s_t = c_t^w + \frac{1-\alpha}{\sigma} s_t + \frac{\alpha\omega}{\sigma} s_t = y_t^w + \frac{1}{\sigma_\alpha} s_t$$

$$\text{where } \sigma_\alpha \equiv \frac{\sigma}{1-\alpha+\alpha\omega}$$

The results just obtained allow the linearized Euler equation (27) to be transformed in the following way (see Mathematical Appendix B.11, p. 105):

$$y_t = E_t[y_{t+1}] - \frac{1}{\sigma_\alpha} \left[i_t - E_t(\pi_{t+1}^H) - \rho \right] + \alpha\Theta E_t[\Delta y_{t+1}^w]$$

$$\text{where } \Theta = (\sigma\gamma - 1) + (\sigma\eta - 1)(1 - \alpha) = \omega - 1$$

It is interesting to observe that the sensitivity of output to interest rate changes is influenced by the degree of openness, as $\sigma_\alpha \equiv \frac{\sigma}{1-\alpha+\alpha\omega} = \frac{\sigma}{1+\alpha(\omega-1)} = \frac{\sigma}{1+\alpha\Theta}$. If

$\Theta > 0$, an increase in openness raises the sensitivity, since σ_α gets smaller. In this case, if the interest rate is increased, there is an additional, to the direct negative effect on demand and output, negative effect of real appreciation (which makes consumers shift their demand towards foreign goods). The effect's magnitude is reduced by an increase in the CPI inflation relatively to domestic inflation (which makes consumer allocate more expenditures on home goods.), but not enough.

Let the current account be defined as $ca_t = \frac{1}{Y_0} (Y_t - \frac{P_t}{P_t^H} C_t)$ in terms of domestic output, as a fraction of the steady state output (similarly to the approach in Obstfeld and Rogoff's model, where the current account was given by $\frac{d\bar{F}}{\bar{C}_0^w}$). $P_t^H Y_t = P_t C_t$ implies

balanced trade. Linearizing this expression is quite straightforward, additionally, recalling that $p_t = p_t^H + \alpha s_t$, then $ca_t = y_t - c_t - \alpha s_t$. Modifying the expression, using

$$y_t = c_t + \frac{\alpha\omega}{\sigma} s_t, \text{ gives: } ca_t = \alpha \left(\frac{\omega}{\sigma} - 1 \right) s_t. \text{ The current account depends only on the terms}$$

of trade and the parameters' values. Notice that when $\frac{\omega}{\sigma} = 1$, which is possible, there is always balanced trade. This is quite strange, as then the current account is independent of the policy makers. There was no such situation in Obstfeld and Rogoff's model, where

the current account is always affected by a change in policy. If it is predetermined that there is always a balanced trade, the model is seriously impaired. There will be no international net capital flows with resulting transfers, thus the possibility that a monetary shock has a longer affect (or in this model, stronger effect, as with Calvo pricing, some prices stay theoretically forever fixed) than for the period that prices are fixed has vanished. Furthermore, if this is the case, the model departs from reality in a noticeable way.

The dynamics of domestic inflation are described by the previously derived equation (28): $\pi_t^H = \beta E_t(\pi_{t+1}^H) + \lambda(mc_t + \bar{h})$, where $\lambda \equiv \frac{(1-\varpi)}{\varpi}(1-\varpi\beta)$. The real marginal cost is

given by: $mc_t = -v + w_t - p_t^H - a_t$, as previously shown. Add and subtract p_t :

$$mc_t = -v + (w_t - p_t) + (p_t - p_t^H) - a_t, \text{ remember that } p_t = p_t^H + \alpha s_t \text{ and equation (26)}$$

$\sigma c_t + \varphi n_t = w_t - p_t$, which assures labor market clearing.

$$mc_t = -v + \sigma c_t + \varphi n_t + \alpha s_t - a_t, \text{ now use } y_t = a_t + n_t \text{ and } c_t = c_t^w + \frac{1-\alpha}{\sigma} s_t$$

$$mc_t = -v + \sigma c_t^w + \varphi y_t + \left(\alpha + \sigma \frac{1-\alpha}{\sigma} \right) s_t - (1+\varphi)a_t, \text{ world goods market clearing ensure}$$

that $c_t^w = y_t^w$.

$$mc_t = -v + \sigma y_t^w + \varphi y_t + s_t - (1+\varphi)a_t$$

The marginal cost increases with output and decreases with productivity. These two effects are obvious and equivalent to their counterparts in the closed economy setting. In open economy setting, however, the marginal cost also increases with world output and the terms of trade (when treated separately, as if they are not connected). Both variables influence positively the real wage through the wealth effect on labor supply resulting from their effect on domestic consumption.

Finally, use $y_t = y_t^w + \frac{1}{\sigma_\alpha} s_t$ to eliminate s_t from the equation:

$$mc_t = -v + (\sigma - \sigma_\alpha) y_t^w + (\varphi + \sigma_\alpha) y_t - (1+\varphi)a_t \quad (29)$$

Here, one can see even better how the different variables affect the marginal cost. Domestic output affects the marginal cost through its impact on employment (captured by

φ) and on terms of trade (captured by σ_α). World output, as discussed, affects marginal cost through its affect on consumption and, thus, on the real wage (captured by σ) and, now, on the terms of trade, again captured by σ_α . Recall that σ_α is a function of the coefficient of openness. It is not clear if world output raises the marginal cost or not when terms of trade are treated as endogenous, as it depends on the values of σ_α and σ .

The output gap is defined as $x_t \equiv y_t - y_t^n$, where y_t^n is the log natural output, which means the output when prices are flexible. To obtain y_t^n use the obtained fact that under flexible prices $mc_t = -\hbar$ and plug into (29), which gives (see Mathematical Appendix B.12, p. 106):

$$y_t^n = \Omega + \alpha \Psi y_t^w + \Gamma a_t$$

$$\text{where } \Omega \equiv \frac{\nu - \hbar}{\varphi + \sigma_\alpha} \quad ; \quad \Gamma \equiv \frac{1 + \varphi}{\varphi + \sigma_\alpha} \quad ; \quad \Psi \equiv -\frac{\Theta \sigma_\alpha}{\varphi + \sigma_\alpha}$$

Furthermore, combining equations (28) and (29) (it is derived in the same appendix B.12.) yields the so-called New Keynesian Phillips Curve (NKPC):

$$\pi_t^H = \beta E_t(\pi_{t+1}^H) + \kappa_\alpha x_t \quad (30)$$

$$\text{where } \kappa_\alpha \equiv \lambda(\varphi + \sigma_\alpha)$$

The degree of openness affects the sensitivity of domestic inflation to the output gap again through σ_α . The output gap affects the inflation rate by its impact on employment (captured by φ) and on the terms of trade (captured by σ_α).

Finally, modify $y_t = E_t[y_{t+1}] - \frac{1}{\sigma_\alpha} [i_t - E_t(\pi_{t+1}^H) - \rho] + \alpha \Theta E_t[\Delta y_{t+1}^w]$ to obtain a dynamic IS-type of relation in terms of the output gap. This is again done in appendix B.12.

$$x_t = E_t[x_{t+1}] - \frac{1}{\sigma_\alpha} [i_t - E_t(\pi_{t+1}^H) - r^n] \quad (31)$$

where $r^n \equiv \rho - \sigma_\alpha \Gamma (1 - \rho_a) a_t + \alpha (\Theta + \Psi) E_t[\Delta y_{t+1}^w]$ is the natural real rate of interest. One can see that this is the natural rate of interest from the fact that if there is no output gap, i.e. $x_t = 0$ for all t , output is always equal to its natural level, it must be that

$i_t - E_t(\pi_{t+1}^H) = r^n$ for all t (note that according to the Fisher identity $i_t - E_t(\pi_{t+1}^H)$ is the real interest rate).

Equations (30) and (31) construct the non-policy block of this model. They involve two endogenous variables, the output gap- x_t and domestic inflation - π_t^H , and one exogenous variable – the nominal interest rate i_t . World output is also exogenous, but it is treated more as a parameter than as a variable from here on, since it cannot be affected by Home's policies. In order to complete the model, it must be specified how the central bank determines the nominal interest rate. As already discussed, the market for money is ignored in this model, and it is assumed that the central bank can set any nominal interest rate it wants, without specifying exactly how it achieves that. Immediately, the question why does not the central bank choose a rule $i_t = r^n$ arises. Surely, then $x_t = \pi_t^H = 0$ for all t is an equilibrium. There are two problems with this rule, the first one concerns the real world, the second the particular model. First, the natural rate of interest is, in reality, something that is not usually observable, thus it is hard for a central bank to follow such a rule. The more serious problem is, however, that in this model, even if the central bank knows exactly what r^n is, this is not a rule that can be followed mechanically. This is the case, as $x_t = \pi_t^H = 0$ is only one of the possible equilibria, and it is not unique. Multiple equilibria must be avoided, otherwise the model is nonsensical. In order to understand when there are multiple equilibria and when there is a unique equilibrium, let the central bank follow a Taylor rule, as proposed by Taylor (1993) (see Taylor (1993), p. 202). $i_t = \rho + h_\pi \pi_t^H + h_x x_t + \delta_t$ This is the most famous and used interest rate rule. The non-negative parameters h_π and h_x show how strongly the central bank reacts on, correspondingly, inflation and output gap. Let $\delta_t \equiv \rho_\delta \delta_{t-1} + \varepsilon_t^\delta$, $\rho_\delta \in [0,1)$, be an AR(1) process, which represents an exogenous disturbance, i.e. the monetary shock. Then the whole model is described by the three equations (30), (31) and the Taylor rule:

$$\pi_t^H = \beta E_t(\pi_{t+1}^H) + \kappa_\alpha x_t$$

$$x_t = E_t[x_{t+1}] - \frac{1}{\sigma_\alpha} \left[i_t - E_t(\pi_{t+1}^H) - r^n \right]$$

$$i_t = \rho + h_\pi \pi_t^H + h_x x_t + \delta_t$$

Using the Taylor rule to eliminate the interest rate from equation (31) transforms the system into:

$$\pi_t^H = \beta E_t(\pi_{t+1}^H) + \kappa_\alpha x_t$$

$$x_t = E_t[x_{t+1}] - \frac{1}{\sigma_\alpha} \left[\rho + h_\pi \pi_t^H + h_x x_t + \delta_t - E_t(\pi_{t+1}^H) - r^n \right]$$

A system of two expectational difference equations. Expressing the system explicitly and stating it in matrix form gives (see Mathematical Appendix B.13, p. 107):

$$\begin{pmatrix} x_t \\ \pi_t^H \end{pmatrix} = A_\alpha \begin{pmatrix} E_t(x_{t+1}) \\ E_t(\pi_{t+1}^H) \end{pmatrix} + b_\alpha (r^n - \rho - \delta_t)$$

where:

$$A_\alpha \equiv \Xi_\alpha \begin{pmatrix} \sigma_\alpha & 1 - \beta h_\pi \\ \sigma_\alpha \kappa_\alpha & \kappa_\alpha + \beta(\sigma_\alpha + h_x) \end{pmatrix} ; \quad b_\alpha \equiv \Xi_\alpha \begin{pmatrix} 1 \\ \kappa_\alpha \end{pmatrix} ; \quad \Xi_\alpha \equiv \frac{1}{\sigma_\alpha + h_x + \kappa_\alpha h_\pi}$$

Now, one can discuss the problem with multiple equilibria. The problem is that both x_t and π_t^H are non-predetermined (jump) variables. Therefore, the solution is unique if and only if both eigenvalues of A_α are in the unit circle (see Blanchard and Kahn (1980), p. 1308). For exactly this problem, the necessary and sufficient condition for this to hold is given by: $\kappa_\alpha(h_\pi - 1) + (1 - \beta)h_x > 0$ (see Bullard and Mitra (2002), p. 1115). After establishing this fact, examining the proposed interest rate rule $i_t = r^n$ shows that it is just a special case of the general formula of the Taylor rule. However, in this special case it holds that $h_\pi = 0$ and $h_x = 0$, thus $\kappa_\alpha(h_\pi - 1) + (1 - \beta)h_x = -\kappa_\alpha < 0$.

In Obstfeld and Rogoff's model a permanent money shock is modeled, that is here equivalent to a permanent increase of the interest rate. However, there is a key difference. In the first model in this paper, the central bank does not react on any developments in the economy. Its monetary policy is totally exogenous. Here, this would correspond to a policy with $h_\pi = 0$ and $h_x = 0$, e.g. $i_t = \rho + \delta_s$, where δ_s is the increase, but, as it was just shown, such a policy cannot be used in Galí and Monacelli's model. In the currently discussed model, an active central bank must be modeled, which responds to the

economy developments. If there is a shock through δ_t , the interest rate does not change by δ_t , but also accounts for the central bank's endogenous response to changes in the output gap and the inflation rate. Qualitatively, there should generally not be a difference, but, quantitatively, the results could be quite different. One can again quote John Taylor, as earlier in this paper: In reality, “a fixed money growth rule will generally imply a reaction of interest rates to the inflation rate and to real output similar in form though not necessary similar in size to interest rate policy rules” (see Taylor (1999b), p. 661). One must keep this in mind when analyzing the results.

In order to find how variables react to a monetary shock, it is useful to eliminate all other shocks, otherwise the effects of the different shocks would indistinguishably mix. Assume that the change in world output is zero. Furthermore, assume that monetary policy does not affect the natural level of output and this level stays the same (equivalent to assuming that the technology of production stays the same). This assumptions imply $r^n = \rho$. This is, because

$r^n \equiv \rho - \sigma_\alpha \Gamma(1 - \rho_a) a_t + \alpha(\Theta + \Psi) E_t [\Delta y_{t+1}^w]$, as $\Delta y_{t+1}^w = 0$, this turns to:
 $r^n = \rho - \sigma_\alpha \Gamma(1 - \rho_a) a_t$. It was assumed that $\Delta y_{t+1}^n = 0$, but $y_t^n = \Omega + \alpha \Psi y_t^w + \Gamma a_t$ and $y_{t+1}^n = \Omega + \alpha \Psi y_{t+1}^w + \Gamma a_{t+1}$, combining these two gives $\Delta y_{t+1}^n = \Gamma(\rho_a - 1) a_t$. Hence, it must be that $r^n = \rho$. Note that the fact that $\Gamma(\rho_a - 1) a_t = 0$ implies that technology stays constant.

The system of equations that must be solved for x_t and π_{t+1}^H is given by:

$$\begin{aligned} \pi_t^H &= \beta E_t (\pi_{t+1}^H) + \kappa_\alpha x_t \\ x_t &= E_t (x_{t+1}) - \frac{1}{\sigma_\alpha} \left[\rho + h_\pi \pi_t^H + h_x x_t + \delta_t - E_t (\pi_{t+1}^H) - r^n \right] \end{aligned}$$

The solution is found again in appendix B.13. It takes the following form:

$$\begin{aligned} x_t &= -(1 - \beta \rho_\delta) H_\delta \delta_t \\ \pi_t^H &= -\kappa_\alpha H_\delta \delta_t \end{aligned}$$

$$\text{where } H_\delta \equiv \frac{1}{(1 - \beta \rho_\delta) [\sigma_\alpha (1 - \rho_\delta) + h_x] + (h_\pi - \rho_\delta) \kappa_\alpha} > 0$$

A contractionary monetary shock ($\varepsilon_t^\delta > 0$) affects both the output gap and the inflation rate negatively. Notice that H_δ is decreasing in both h_π and h_x , and hence the effect of the shock is decreasing in them. Therefore, if the central bank reacts strongly to both deviation of output and inflation, it would stabilize them a lot faster when there is monetary shock. There is no tradeoff between inflation stabilization and output stabilization when stabilizing a monetary shock. The optimal thing for a central bank that faces only such shocks is to have as strong a response as possible.

It is now fairly easy to solve for all other variables in the model. The most important step is to realize that, since the natural level of output is unaffected by the monetary shock and stays constant, the response of output matches that of the output gap. Furthermore, the results below are responses to the monetary shock, the deviation caused by the shock from their value before that. Some variables may experience permanent changes, some might revert back to their original level. This is not equivalent to the “hat” notation used in part 2 of this paper, since the variables are still the logarithms. Note that a single one period shock is modeled, i.e. $\varepsilon_t^\delta \neq 0$ in period t and $\varepsilon_t^\delta = 0$ in later periods.

Therefore, since $y_t = y_t^w + \frac{1}{\sigma_\alpha} s_t$, the effect of the monetary shock on the terms of trade is

given by $s_t = \sigma_\alpha x_t = -\sigma_\alpha (1 - \beta \rho_\delta) H_\delta \delta_t$. Here are the effects on the following variables

$$ca_t = \alpha \left(\frac{\omega}{\sigma} - 1 \right) s_t = -\alpha \left(\frac{\omega}{\sigma} - 1 \right) \sigma_\alpha (1 - \beta \rho_\delta) H_\delta \delta_t$$

$$\pi_t = \pi_t^H + \alpha \Delta s_t = -\kappa_\alpha H_\delta \delta_t - \alpha \sigma_\alpha (1 - \beta \rho_\delta) H_\delta \Delta \delta_t$$

$$c_t = y_t - \frac{\alpha \omega}{\sigma} s_t = -(1 - \beta \rho_\delta) H_\delta \delta_t + \frac{\alpha \omega}{\sigma} \sigma_\alpha (1 - \beta \rho_\delta) H_\delta \delta_t = -\left(1 - \frac{\alpha \omega}{\sigma} \sigma_\alpha \right) (1 - \beta \rho_\delta) H_\delta \delta_t$$

$$b_t = (1 - \alpha) s_t = -\sigma_\alpha (1 - \alpha) (1 - \beta \rho_\delta) H_\delta \delta_t$$

$$\text{From } s_t = e_t + p_t^w - p_t^H \Rightarrow \Delta e_t = \Delta s_t - \pi_t^H = -\sigma_\alpha (1 - \beta \rho_\delta) H_\delta \Delta \delta_t - \kappa_\alpha H_\delta \delta_t$$

$$i_t = \rho + h_\pi \pi_t^H + h_x x_t + \delta_t = (1 - h_\pi \kappa_\alpha H_\delta - h_x (1 - \beta \rho_\delta) H_\delta) \delta_t$$

$$r_t = i_t - E_t(\pi_{t+1}^H) = (1 - h_\pi \kappa_\alpha H_\delta - h_x (1 - \beta \rho_\delta) H_\delta) \delta_t + \kappa_\alpha H_\delta \delta_{t+1} =$$

$$= (1 - h_\pi \kappa_\alpha H_\delta - h_x (1 - \beta \rho_\delta) H_\delta) + \kappa_\alpha H_\delta \rho_\delta \delta_t = (1 - (h_\pi - \rho_\delta) \kappa_\alpha H_\delta - h_x (1 - \beta \rho_\delta) H_\delta) \delta_t$$

As seen from the equations above, an expansionary monetary shock ($\varepsilon_t^\delta < 0$) increases consumption by less than output, because of a deterioration of the terms of trade. The intuition behind this is straightforward. Given an expansionary shock, s_t becomes larger. s_t is the logarithm of $S_t = \frac{P_t^F}{P_t^H}$. That is, the relative price of foreign goods increases. This is the case, since an expansionary shock increases home output, while the outputs of foreign countries stay the same. Therefore, the relative price of home goods decreases with the effect depending on η , the elasticity of substitution between home and foreign goods. A lower relative price of home goods means that the real amount of goods home consumers can buy from the revenue of their exports falls. Hence, the consumption does not increase as much as output. Another important thing to note is that CPI inflation increases faster than home goods inflation, again because of deterioration of the terms of trade. The effects are, however, more easily seen when the model is simulated and plotted, as displayed in the next part of this paper.

4. Assessment

In this section, the two models' results are compared, to each other and to the real world data. In order to perform the comparison quantitatively and not only qualitatively, the two models must be simulated. This also makes the qualitative analysis easier. Before simulated, the models must be calibrated. In obtaining the parameters' values, various microeconomic studies are used and, generally, the main case scenario for the estimated value is chosen. Studies or cases in a study that give extreme values for a parameter are avoided. The parameters are assigned the following values taken from the quoted studies:

$\varepsilon = 3.846$, since from the money demand equations (12) and (13), one can calculate that:

$$\varepsilon = \frac{1}{(\text{interest elasticity of money demand})} (1+i) \approx \frac{1}{(\text{interest elasticity of money demand})}$$

for small i (even if i is 0.1 (10% interest, which is quite high), ε becomes 4.23. The later obtained results are quite robust in terms of this parameter and, thus, this approximation is not of a great importance). Hafer and Hein estimate interest elasticities of money

demand for different periods in the US. Here, their estimation of 0.26 is used (see Hafer and Hein (1984), p. 251). This gives $\varepsilon = 3.846$.

$\beta = 0.955$, which is the annual discount factor. Coller and Williams find that individuals discount payoffs, which are one year in the future, by a rate between 4.08% and 5.13% (see Coller and Williams (1999), p. 117). Here, a 4.5% is chosen. Note that an annual discount factor of 0.955 means a quarterly discount factor of 0.989.

$\theta = \mu = \gamma = 1.88$, the elasticity of substitution between all goods in Obstfeld and Rogoff's model, θ , is assumed to be equal to the elasticity of substitution between goods produced within a country, μ , and the elasticity of substitution between goods produced in different foreign countries, γ , in Gali and Monacelli's model. This is the case, since the value for θ is obtained from a study examining goods in general (all goods, foreign and home, making no difference between the two, as in Obstfeld and Rogoff's model), and there is no logical reason why consumers should have different, from the general case, preferences when they consider goods from the same country or that they should favor some foreign country's goods instead of the goods of some other foreign country. The value is taken from a paper that studies price elasticities of demand (recall that for these particular functions (CES functions of this type) the elasticity of substitution between goods is also the price elasticity of demand). A not-extreme value of 1.88 is chosen (see Tellis (1988), p. 334).

$\eta = 0.81$, consumers do, however, differentiate between foreign and home goods in the real world. There are many studies examining this phenomenon and they always find lower elasticity of substitution between home and foreign goods compared to the elasticity of substitution for all good. Here the value is taken from a study by Blonigen and Wilson (see Blonigen and Wilson (1999), p. 8).

$n = 0.23$, which is roughly the size of US GDP relatively to World GDP. The biggest economy in the world is chosen for the size of Home in Obstfeld and Rogoff's model, since if the model is interpreted as "Foreign equals the rest of the world", changes in policy of a small country have negligible effects on Foreign and much of the insights of the model are lost. Furthermore, the effects of Foreign policy on Home are unrealistically strong (mainly because there is no home bias in this model).

$h_{\pi}=1.5$ and $h_x=0.5$, the strength of the reaction of the central bank to inflation and output gap in Gali and Monacelli's model is set the same as Taylor estimated it in his initial contribution (see Taylor (1993), p. 202).

$\alpha=0.55$, which is the coefficient of openness in Gali and Monacelli's model. A value of 0.55 corresponds to a rather open economy.

$\varpi=0.8$, which agrees with the already discussed studies concerning quarterly Calvo pricing. This particular value is found by Gali and Gertler (see Gali and Gertler (1999), p. 210).

$\rho_{\delta}=0.933$, which measures the persistency of a monetary shock. The parameter's value is calculated by using the fact the half-life of a monetary shock is found to be around 10 quarters (see Christiano et al. (2005), p. 41). The effect of a monetary shock is precisely halved after 10 quarters if $\rho_{\delta}=0.933$.

The last two parameters that must be determined are the ones from the utility function in Gali and Monacelli's model: $U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right]$. It is assumed that $\varphi=1$, a quadratic loss from labor effort, just as in Obstfeld and Rogoff's model, and that $\sigma=0.9$. For the latter parameter, a value different from 1 is chosen, since, although a value of one corresponds to the logarithmic case, similarly to Obstfeld and Rogoff's model, it is a particularly special case, and there might be some insights that occur only for this situation. As the model is solved for the general case, it is practical not to restrict it only to the logarithmic one.

One must stress that this is not a proper calibration, as the values are taken from across the literature. To be fully consistent one must investigate what the values of all the parameters for one single country are. For example, it could be quite possible that no country that is that open ($\alpha=0.55$), pursues such a monetary policy ($h_{\pi}=1.5$ and $h_x=0.5$), even though both phenomena are separately observed. This argument can be extended to many of the parameters' values, even the ones that depend mostly on human nature. Unfortunately, an estimation of all these parameters for a single country at a single point of time (as parameters change over time) is out of the scope of this paper.

Therefore, with this disclaimer note, it is assumed that such countries, as the ones described by the parameters above, exist.

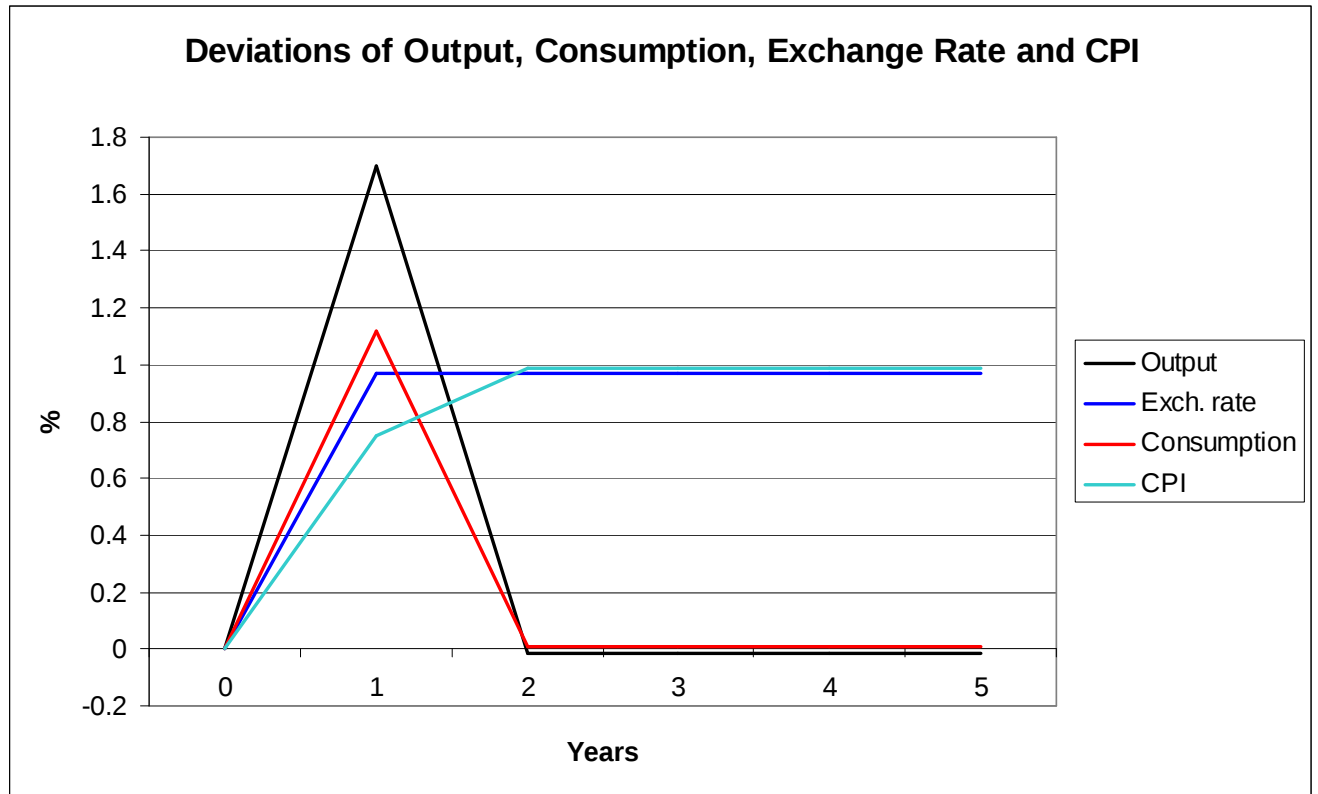


Figure 2: Effects of a permanent one percent increase in home money supply on Output, Exchange rate, Consumption and Consumer Price Level.

Figure 2 shows the effects of a one percent increase in home money supply in Obstfeld and Rogoff's model. Figure 3 displays the simulation of a unit expansionary monetary shock ($\varepsilon_1^\delta = -1$ and $\varepsilon_t^\delta = 0$ for $t > 1$) in the setting of Gali and Monacelli's model. Before comparing the models' results, it must be again stated that oranges and apples are compared. In the first model, the central bank does not react to developments in the economy, while, in the second model, the central bank actively interacts with the economic environment. Therefore, the effects from the monetary shocks are not equivalent in magnitude. Only the relative size of two variables in one of the models can be compared with the relative size of two variables in the other model, e.g. consumption rises by some fraction of the increase in output in Obstfeld and Rogoff's model and by some other fraction in Gali and Monacelli's model. The sheer fact that output increases by 1.7% from the steady state in the first period after the shock in Obstfeld and Rogoff's

model and by approximately 0.7% in Gali and Monacelli's model does not yield further information concerning the comparison between the models.

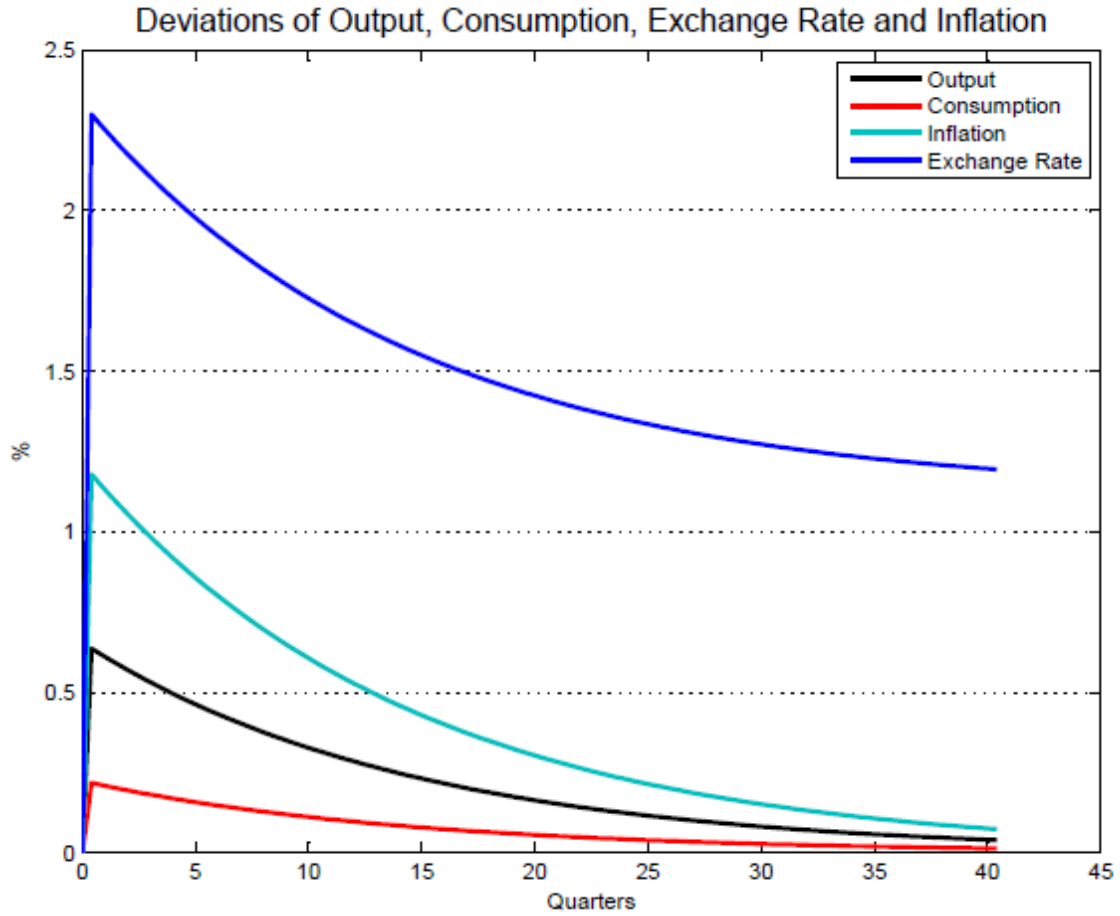


Figure 3: Effects of a unit expansionary monetary shock ($\varepsilon_1^\delta = -1$) on Output, Exchange rate, Consumption and Consumer Price Inflation rate.

Furthermore, Obstfeld and Rogoff's model is simulated by setting one period equal to a year, otherwise (one period = one quarter), as already discussed, the price dynamics does not make sense. Moreover, the results would be quite strange, since the short term effects of the shock would disappear after only one quarter. Gali and Monacelli's model is simulated on a quarterly basis, as typical for such models.

The most notable difference is in terms of the models' dynamics. As, in the first model, prices are fixed only for one period, the graph displays kinks and the economy converges from the short-run state to its long-run steady state for just one period. In the second model, the economy converges slowly to the long-run steady state. However, in both cases there are only two phases after the shock: the first phase, which displays the

increases after the shock and is equal to one period for both models, and a second phase of convergence to the long-run steady state. In Obstfeld and Rogoff's model the second phase is again equal to one period, while in Gali and Monacelli's model it is much longer.

If the length of the convergence is put aside, the models exhibit strong resemblance. In both models, an expansionary monetary shock results in an increase in output and in a smaller increase in consumption. Both of which converge in the long-run to values close to zero. In Obstfeld and Rogoff's model consumption stays permanently higher and output permanently lower, because of interest payments, as discussed before, but the effects are relatively small compare to the short-run fluctuations. In Gali and Monacelli's model, consumption and output deviations from the steady state asymptotically converge to zero, however, that also means that they stay positive for any finite horizon, even if the values become negligibly small.

Furthermore, in both models the price level rises permanently and the rise occurs in both phases. The graphs are here slightly confusing, as in figure two the consumer price level is shown, and in figure three the inflation rate. This is done, since the two models are structured in such a way that it is straightforward to solve for the displayed variables and the literature usually depicts them in this manner. It is clear from figure 2 that there is inflation in periods 1 and 2. After these two periods the price level reaches a value close to one, but remains below one, because of the permanent increase in consumption (recall the long-run money demand equations). After period 2, there is no inflation in Obstfeld and Rogoff's model. In figure 3, inflation starts in period one and diminishes in future periods, asymptotically converging to zero. This is equivalent to the price level increasing in period one and then in increasing further at a diminishing rate to some value (which could be found if the model is solved for the price level).

The only significant difference, besides the length of convergence, is in the effect of the policy shock on the nominal exchange rate. In Obstfeld and Rogoff's model, the exchange rate jumps immediately to its long-run steady state value. In Gali and Monacelli's model, this is not the case. The exchange rate either overshoots or undershoots its long-run steady state value. One can show that which of the two happens depends on the parameters' values (see Gali (2008), ch. 7, p. 22). Then the exchange rate converges to its long-run steady state value.

How do these results compare to the real world? Obviously, Obstfeld and Rogoff's model is not consistent with the dynamics of a real economy, but dynamics aside, do the real world economies exhibit the same patterns. Figure 4 and figure 5 show a vector autoregression (VAR) studies done respectively by Christiano, Eichenbaum and Evans (see Christiano et al. (2005), pp. 6-7) and by Bouakez, Cardia and Ruge-Murcia (see Bouakez et al. (2005), p. 1083).

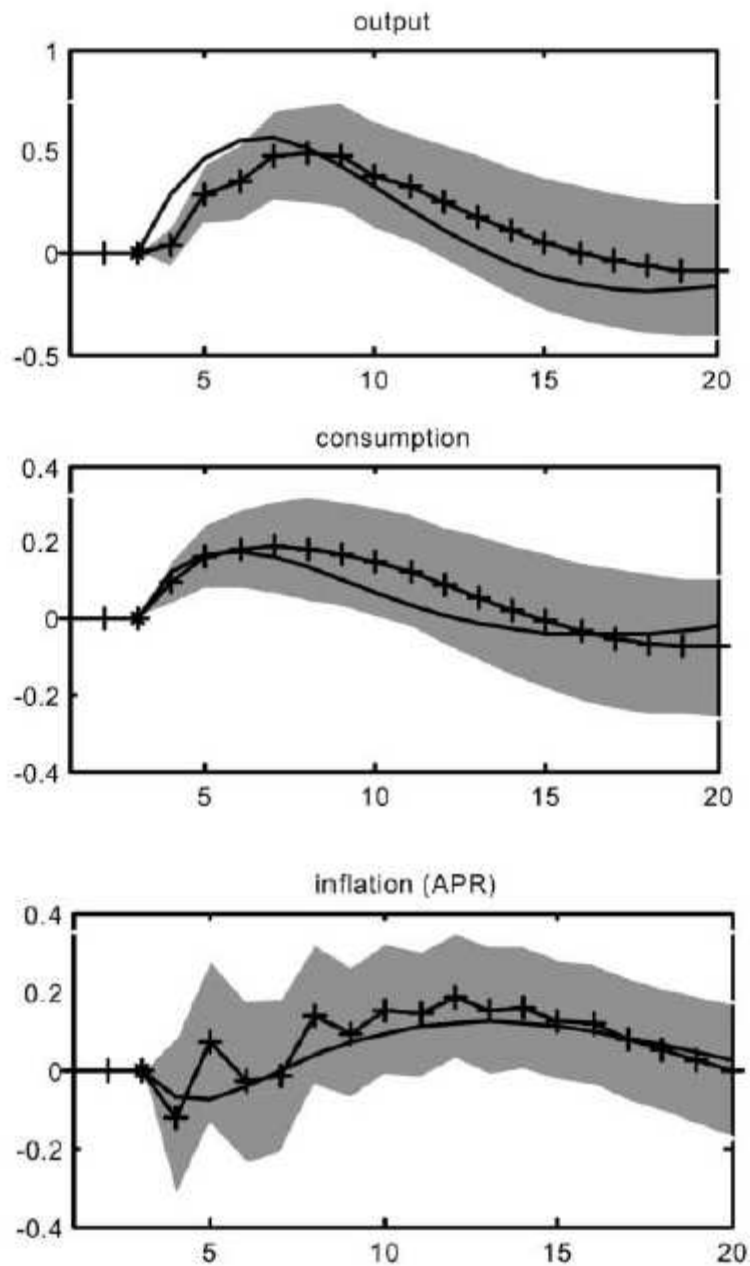


Figure 4: The effects of an expansionary monetary shock on output, consumption and inflation, estimated by a VAR (the line with plus signs). The shaded areas are the 95% confidence intervals. Periods are quarters. The effects are measured in percentages. Note: the line without plus signs is a simulation of a model, which is not important for this paper

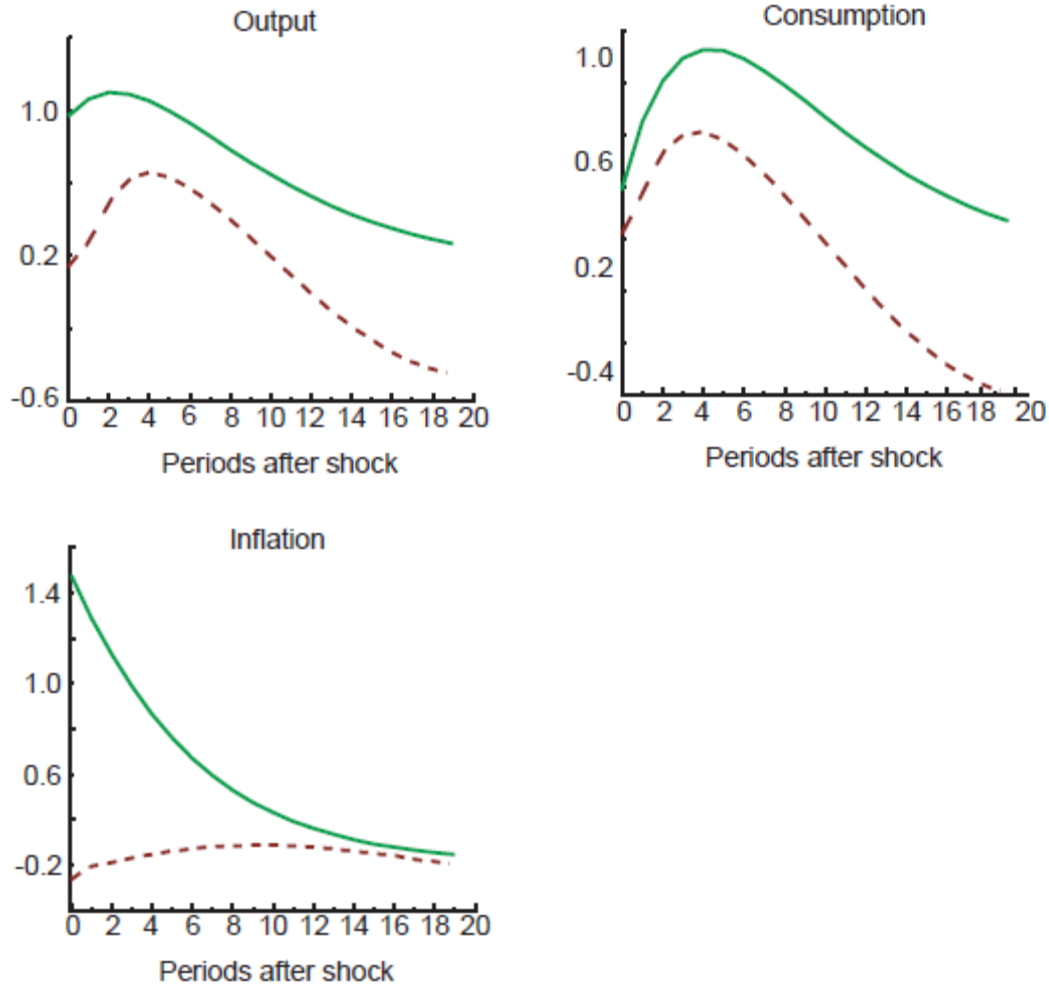


Figure 5: The effects of a one percent money supply shock on output, consumption and inflation, estimated by a VAR (the dashed line). Periods are quarters. The effects are measured in percentages. Note: the green continuous line is a simulation of a model, which is not important for this paper.

The models' results are roughly consistent with these studies when output and consumption are considered. Although, in both models the maximum deviation from the steady state occurs immediately after the shock, while the estimates show that the maximum is reached after a few quarters. In figure 4, output and consumption behave

exactly as predicted by the models, with output increasing more than consumption and then both converge to zero. Output slightly overshoots the zero and goes to negative, an effect precisely consistent with Obstfeld and Rogoff's insights. The study displayed in figure 2 finds similar patterns in output and consumption, however, here consumption rises by more than output and then overshoots the zero and turns negative. Both effects are not predicted by the models. Inflation behaves quite strangely in both studies. At first, it is negative and then turns positive before starting to converge to zero. The negative turn in the beginning is not explain by the models in this paper. However, it must be noted that both studies are using US data. In the models described here, almost the entire (in Obstfeld and Rogoff's model - the entire) positive deviation of inflation in the beginning is explained by exchange rate fluctuations. As the US is huge and quite close economy, these effects are smaller than for smaller and more open economies.

One must also state that studies examining the effects of monetary policy do not always agree with each other. For example, in a famous paper, Bernanke and Mihov find persistency of output deviations after a monetary shock, which is not consistent with most of macroeconomic theory (including the two models discussed in this paper). Figure 6 displays one of their results, if some other money aggregate is used, the results are similar (see Bernanke and Mihov (1998), p. 155).

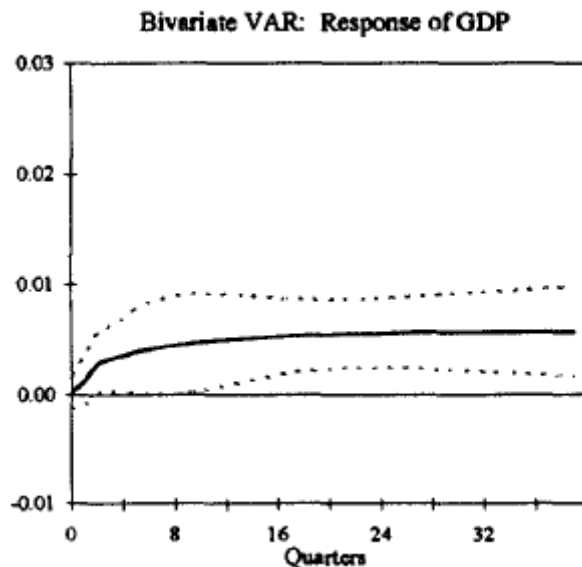


Figure 6: Effect of a M2 increase on GDP, estimated by a bivariate VAR. Dashed lines denote the 95% confidence intervals.

When considering nominal exchange rates, the problem of estimating the response to a monetary shock is quite severe. This is the case, since nominal exchange rates are extremely volatile. They might change with full percentage points in a day, which is not the case for the other estimated aggregates. Furthermore, in general, exchange rates are not driven by fundamentals in the short-run. Even a rumor in the financial markets can dramatically change the exchange rate. Meese and Rogoff find that no estimating procedure performs better than a random walk (or in continuous setting – Brownian motion) when it comes to estimating nominal exchange rates (see Meese and Rogoff (1983), pp. 20-21). Nevertheless, there are studies that try to measure the effect of a monetary shock on the exchange rate. Figure 7 displays a study by Eichenbaum and Evans, note that here a contractionary shock is modeled (see Eichenbaum and Evans (1995), p. 995). Hence, in order to compare to the models in this paper, just multiply the results by -1, which is slightly sloppy but should be, in general, approximately true.

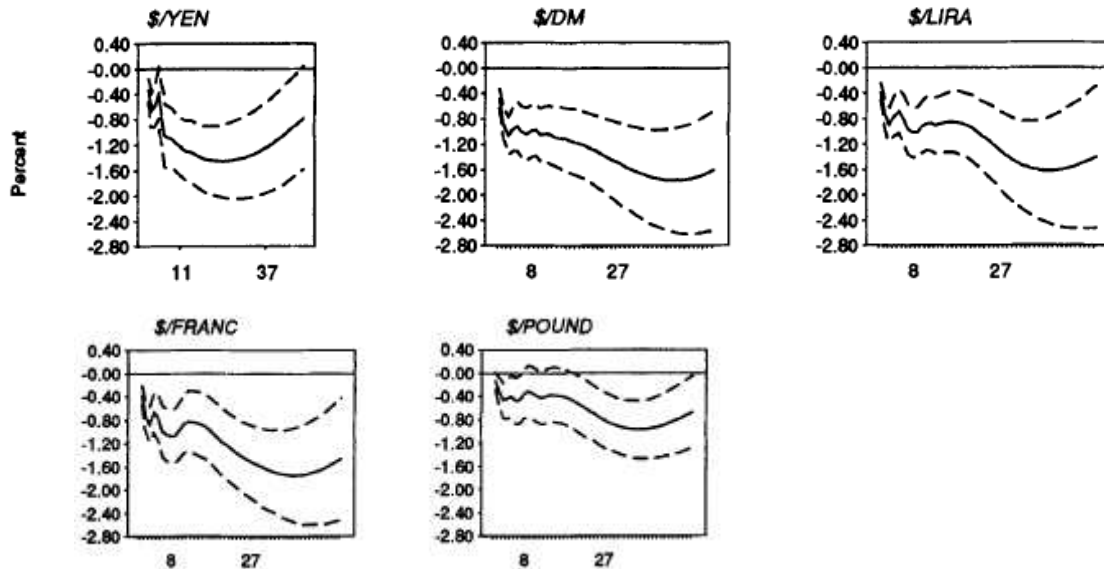


Figure 7: Effects of a contractionary monetary shock on the bilateral nominal exchange rates between the USD and, respectively, the Yen, the DM, the Lira, the Franc and the Pound, as estimated by a VAR. The periods are months. Dashed lines denote the 95% confidence intervals.

Eichenbaum and Evans look only on bilateral exchange rates. Figure 8 displays a study by Kim where the trade-weighted exchange rate is considered. This time the monetary shock is expansionary (see Kim (2003), p. 364).

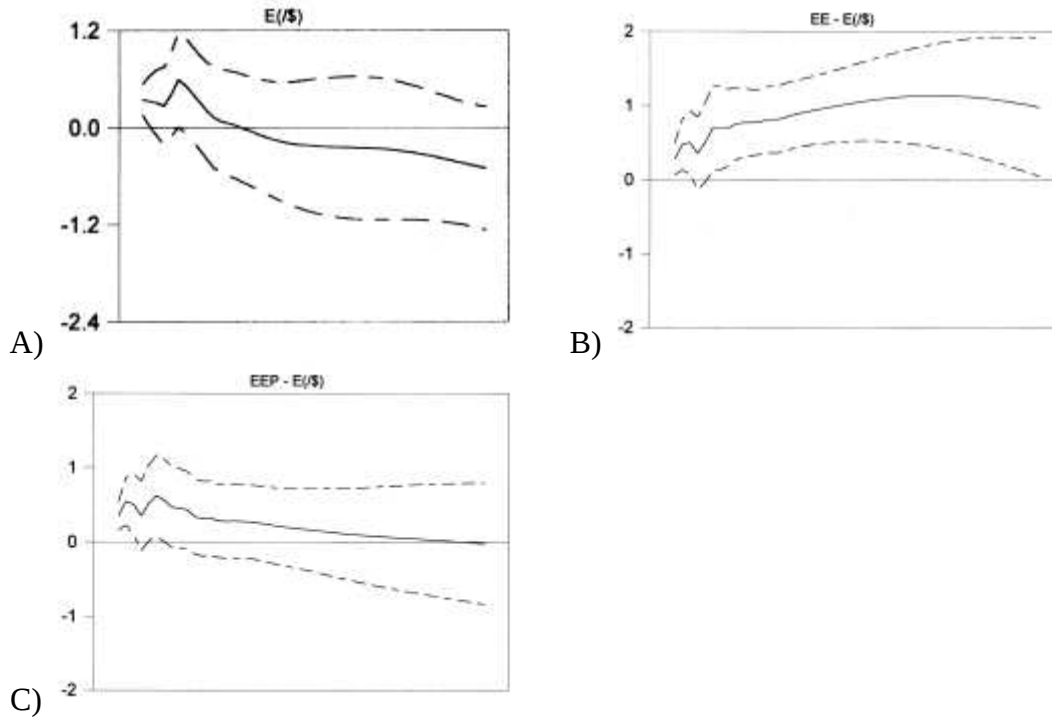


Figure 8: Effects of an expansionary monetary shock on the trade-weighted nominal exchange rate, as estimated by three different VARs (A, B, C). Dashed lines denote the 95% confidence intervals.

The displayed results are slightly different from each other, but they all share some features. No study shows that exchange rates jump immediately to their new long-run values. On the contrary, there is strong evidence that this is not the case. Therefore, this insight from Obstfeld and Rogoff's model is not consistent with the data. Furthermore, all studies show a peak of the deviation after some periods and then convergence to some lower value. That is consistent with the simulation of Gali and Monacelli's model when the exchange rate overshoots, although there the peak occurs immediately after the shock.

One last aspect to consider is the magnitude of the effect of the exchange rate on the inflation rate. In Obstfeld and Rogoff's model, all of the inflation in the period directly after the shock is caused by the change in the interest rate. Afterwards, in the convergent phase, all of it is caused by the change in home prices. In Gali and Monacelli's model, it is harder to judge exactly what fraction of inflation is due to exchange rate fluctuations from the graph. However, a precise answer can be obtained as the model can be simulated also for home goods inflation. Nevertheless, you can see from the graph that the peak in the inflation rate is just after the shock, when only a small fraction of home

prices change, but the exchange rate is at its peak as well. Therefore, much of the inflation rate deviation is due to the change in the exchange rate. In Obstfeld and Rogoff's model around 75% of inflation is caused by the change in the exchange rate. In Gali and Monacelli's model the fraction is less, but still quite high. There is extensive literature on exchange rate pass-through effects in imported goods inflation and its effect on the inflation rate in general. Studies like Choudhri and Hakura (2006), Tulk (2004), etc. find significant pass-through effects, especially for smaller, more open, and less developed economies. However, these effects are a lot smaller than 75% even for the most open developing economies. Usually, there is significantly less than 100% pass-through in imported goods prices (see Tulk (2004), p. 7) and, consequently, much less in consumer price inflation (see Choudhri and Hakura (2006), p. 628). The effect is so significant in Obstfeld and Rogoff's model, because of the earlier discussed flaw of the model, which is that there is no home bias.

To sum up, the models fit rather well to the real world data when output and consumption are considered, if the flawed dynamics of Obstfeld and Rogoff's model is put aside. This is not the case when inflation is examined. Gali and Monacelli's model roughly match the exchange rate data when the coefficients are such that the exchange rate overshoots its long-run value immediately after the shock. Obstfeld and Rogoff's model fails to explain the dynamics of interest rate changes.

4. Conclusion

The two models discussed in this thesis represent two of the three prominent ways to model the effects of monetary policy in an open economy setting while combining rational expectations and intertemporal optimization with nominal rigidities and imperfect markets. Obstfeld and Rogoff's initial contribution was followed by many two-country models with one period in advance preset prices. Notably, by Corsetti and Pesenti (2001), where the flaw in Obstfeld and Rogoff's model of no home bias is fixed, making the results more credible. However, models with one period in advance preset prices cannot describe the more complex dynamics of a real world economy. Models with Calvo pricing are better suited for this. Gali and Monacelli's paper is one of the key contributions when a small open economy with Calvo pricing is analyzed. Another paper

by Clarida , Gali and Gertler (2001) introduces also cost-push shocks in this setting, which could also be done in the Gali and Monacelli's model, a not extremely complicated extension. Kollmann (2002) analyzes even more general case with several sources of shocks.

The problem with models like Gali and Monacelli's is that not only the home economy has a zero measure, but all the economies in the world do so. This means that policies in a country do not affect any other countries. In many real world situations this is not really the case (e.g. policy in the US should have no effect on Canada). In this situation a two-country model can be used to examine the effects. However, if one wants to keep Calvo pricing in the model, models like Obstfeld and Rogoff's cannot be used. Therefore, the third prominent way to model monetary policy in open economy setting is developed. Contributions, such as Clarida, Gali and Gertler (2002) and Benigno and Benigno (2006), combine a two-country model with Calvo pricing. This field in the literature is not analyzed in this paper, but it is equally important to the first two.

The fact that these three branches of literature are already initiated does not mean that there is no scope for further research. As seen in this thesis, there are many things that could be improved in the existing models. Furthermore, new models can be developed by relaxing some of the strong assumptions in the existing models. For example, a model with capital market imperfections can lead to rather different results or a model with central banks with different strengths of commitment in different countries. Many of these ideas are already treated in the literature, but rarely in fully specified models with all the essential features discussed in this thesis.

Mathematical Appendix A

A.1. CES Function

Let an individual has fixed income equal to I and let he/she gains utility from higher values of $C = [\int_0^1 c(z)^{(\theta-1)/\theta} dz]^{\theta/(\theta-1)}$, then the agent maximizes $C = [\int_0^1 c(z)^{(\theta-1)/\theta} dz]^{\theta/(\theta-1)}$, subject to $\int_0^1 p(z)c(z)dz = I$, which is his/her budget constraint. Take the Lagrangian:

$$L = [\int_0^1 c(z)^{(\theta-1)/\theta} dz]^{\theta/(\theta-1)} - \lambda [\int_0^1 p(z)c(z)dz - I]$$

$$\text{FOC: } \frac{\partial L}{\partial c(z_i)} = \frac{\theta}{\theta-1} [\int_0^1 c(z)^{(\theta-1)/\theta} dz]^{1/(\theta-1)} \frac{\theta-1}{\theta} c(z_i)^{-1/\theta} - \lambda p(z_i) = 0$$

$[\int_0^1 c(z)^{(\theta-1)/\theta} dz]^{1/(\theta-1)} c(z_i)^{-1/\theta} = \lambda p(z_i)$, taking this also for z_j and dividing gives:

$$\frac{[\int_0^1 c(z)^{(\theta-1)/\theta} dz]^{1/(\theta-1)} c(z_i)^{-1/\theta}}{[\int_0^1 c(z)^{(\theta-1)/\theta} dz]^{1/(\theta-1)} c(z_j)^{-1/\theta}} = \frac{\lambda p(z_i)}{\lambda p(z_j)}$$

$$\frac{c(z_i)^{-1/\theta}}{c(z_j)^{-1/\theta}} = \frac{p(z_i)}{p(z_j)} \Rightarrow \left[\frac{c(z_i)}{c(z_j)} \right]^{-1/\theta} = \frac{p(z_i)}{p(z_j)} \Rightarrow \left[\frac{c(z_j)}{c(z_i)} \right]^{1/\theta} = \frac{p(z_i)}{p(z_j)}$$

$$\frac{1}{\theta} \ln \left[\frac{c(z_j)}{c(z_i)} \right] = \ln \left[\frac{p(z_i)}{p(z_j)} \right] \Rightarrow \ln \left[\frac{c(z_j)}{c(z_i)} \right] = \theta \ln \left[\frac{p(z_i)}{p(z_j)} \right]$$

Since elasticity of substitution is defined as $E_s = \frac{d \ln[c(z_j)/c(z_i)]}{d \ln[p(z_i)/p(z_j)]}$, it is clear from the

equation above that $E_s = \theta$.

A.2 Price index

The price index is defined as the minimum expenditure for which a consumer can obtain 1 unit of the consumption index, hence the problem is:

$$\min \int_0^1 p(z)c(z)dz \quad \text{subject to } [\int_0^1 c(z)^{(\theta-1)/\theta} dz]^{\theta/(\theta-1)} = 1$$

the solution to this is quite similar to the derivation of the Elasticity of substitution.

$$L = \int_0^1 p(z)c(z)dz - \lambda \{ [\int_0^1 c(z)^{(\theta-1)/\theta} dz]^{\theta/(\theta-1)} - 1 \}$$

$$\frac{\partial L}{\partial c(z_i)} = p(z_i) - \lambda \frac{\theta}{\theta-1} \left[\int_0^1 c(z)^{(\theta-1)/\theta} dz \right]^{1/(\theta-1)} \frac{\theta-1}{\theta} c(z_i)^{-1/\theta} = 0, \text{ rearranging and taking the}$$

same for z_j and dividing gives:

$$\frac{p(z_i)}{p(z_j)} = \frac{c(z_i)^{-1/\theta}}{c(z_j)^{-1/\theta}} \Rightarrow \frac{c(z_i)}{c(z_j)} = \frac{p(z_i)^{-\theta}}{p(z_j)^{-\theta}} \Rightarrow c(z_i) = c(z_j) \frac{p(z_i)^{-\theta}}{p(z_j)^{-\theta}}, \text{ we plug in this in}$$

$$\text{the constraint } \left[\int_{i=0}^1 c(z_i)^{(\theta-1)/\theta} dz \right]^{\theta/(\theta-1)} = 1$$

$$\left[\int_{i=0}^1 c(z_j)^{(\theta-1)/\theta} \left[\frac{p(z_i)^{-\theta}}{p(z_j)^{-\theta}} \right]^{(\theta-1)/\theta} dz \right]^{\theta/(\theta-1)} = 1$$

$$c(z_j) \left[\int_{i=0}^1 \frac{1}{p(z_j)^{1-\theta}} p(z_i)^{1-\theta} dz \right]^{\theta/(\theta-1)} = 1$$

$$c(z_j) = \frac{p(z_j)^{-\theta}}{\left[\int_{i=0}^1 p(z_i)^{1-\theta} dz \right]^{\theta/(\theta-1)}}, \text{ multiply both sides by } p(z_j) \text{ and integrate:}$$

$$\int_{j=0}^1 p(z_j) c(z_j) dz_j = \frac{\int_{j=0}^1 p(z_j)^{1-\theta} dz}{\left[\int_{i=0}^1 p(z_i)^{1-\theta} dz \right]^{\theta/(\theta-1)}} = \left[\int_0^1 p(z)^{1-\theta} dz \right]^{1/(1-\theta)}$$

$\int_0^1 p(z) c(z) dz$ is the minimum expenditure to buy one unit, that is what was denoted by

$$P_t. \text{ Hence, } P_t = \left[\int_{i=0}^1 p(z_i)^{1-\theta} dz \right]^{1/(1-\theta)}$$

A.3. Demand function

The beginning of this derivation is exactly the same as the one in A.1 and A.2. The individual has some fixed amount for expenditure on consumption I , and maximizes

$$C = \left[\int_0^1 c(z)^{(\theta-1)/\theta} dz \right]^{\theta/(\theta-1)}, \text{ because higher values of } C_t \text{ result in higher utility, given}$$

utility function (1). The problem is again: $\max C = \left[\int_0^1 c(z)^{(\theta-1)/\theta} dz \right]^{\theta/(\theta-1)}$, subject to

$$\int_0^1 p(z) c(z) dz = I. \text{ From here we get:}$$

$$c(z_i) = c(z_j) \frac{p(z_i)^{-\theta}}{p(z_j)^{-\theta}}, \text{ plugging in this in the constraint as in A.2. gives:}$$

$$c(z_j) \frac{1}{p(z_j)^{-\theta}} \left[\int_{i=0}^1 p(z_i)^{1-\theta} dz \right]^{\theta/(\theta-1)} = I, \text{ divide both sides by } P_t$$

$$c(z_j) \frac{1}{p(z_j)^{-\theta}} \frac{\left[\int_i^1 p(z_i)^{1-\theta} dz \right]^{\theta/(\theta-1)}}{P_t} = \frac{I}{P_t}, \text{ using the definition of } P_t$$

$$c(z_j) \frac{1}{p(z_j)^{-\theta}} \frac{\left[\int_i^1 p(z_i)^{1-\theta} dz \right]^{\theta/(\theta-1)}}{\left[\int_i^1 p(z_i)^{1-\theta} dz \right]^{1/(1-\theta)}} = \frac{I}{P_t}, \text{ clearly } \frac{I}{P_t} = C_t$$

$$c(z_j) \frac{P_t^{-\theta}}{p(z_j)^{-\theta}} = C_t$$

$$c(z_j) = \left[\frac{p(z_j)}{P} \right]^{-\theta} C_t \text{ gives the demand function for } z_j$$

A.4. Price indexes' connection:

As $p(z) = Ep^*(z)$, the price index for home can be rewritten as:

$$P = \left[\int_0^1 p(z)^{1-\theta} dz \right]^{1/(1-\theta)} = \left[\int_0^n p(z)^{1-\theta} dz + \int_n^1 [Ep^*(z)]^{1-\theta} dz \right]^{1/(1-\theta)}$$

analogously for foreign:

$$P^* = \left[\int_0^1 p^*(z)^{1-\theta} dz \right]^{1/(1-\theta)} = \left[\int_0^n \left[\frac{p(z)}{E} \right]^{1-\theta} dz + \int_n^1 p^*(z)^{1-\theta} dz \right]^{1/(1-\theta)}$$

Then:

$$EP^* = E \left[\int_0^n \left[\frac{p(z)}{E} \right]^{1-\theta} dz + \int_n^1 p^*(z)^{1-\theta} dz \right]^{1/(1-\theta)} = \left[\int_0^n p(z)^{1-\theta} dz + \int_n^1 [Ep^*(z)]^{1-\theta} dz \right]^{1/(1-\theta)} = P$$

A.5. The individual's decision problem:

$$\max \sum_{t=0}^{\infty} \beta^t \left[\log C_t^j + \frac{\chi}{1-\varepsilon} \left(\frac{M_t^j}{P_t} \right)^{1-\varepsilon} - \frac{\kappa}{2} y_t(j)^2 \right]$$

$$\text{subject to: } P_t F_t^j + M_t^j = P_t(1+r_{t-1})F_{t-1}^j + M_{t-1}^j + p_t(j)y_t(j) - P_t C_t^j - P_t T_t^j$$

The individual decides how much to consume - C_t^j , how much money to hold - M_t^j , how much real bonds to hold - F_t^j , and how much to work - $y_t(j)$. World demand, both private and government, is taken as given.

The budget constraint can be modified, since, as shown $y_t(j) = \left[\frac{p_t(j)}{P_t} \right]^{-\theta} (C_t^w + G_t^w)$:

$y_t(j)p_t(j)^\theta = P_t^\theta (C_t^w + G_t^w)$, multiplying both sides by $y_t(j)^{\theta-1}$ gives:

$$y_t(j)^{\theta-1} y_t(j) p_t(j)^\theta = y_t(j)^{\theta-1} P_t^\theta (C_t^w + G_t^w)$$

$$(y_t(j) p_t(j))^\theta = y_t(j)^{\theta-1} P_t^\theta (C_t^w + G_t^w)$$

$y_t(j) p_t(j) = y_t(j)^{(\theta-1)/\theta} P_t (C_t^w + G_t^w)^{1/\theta}$, substituting this into the budget constraint:

$$P_t F_t^j + M_t^j = P_t(1+r_{t-1})F_{t-1}^j + M_{t-1}^j + P_t y_t(j)^{(\theta-1)/\theta} (C_t^w + G_t^w)^{1/\theta} - P_t C_t^j - P_t T_t^j$$

Expressing explicitly for C_t^j :

$$P_t C_t^j = P_t(1+r_{t-1})F_{t-1}^j + M_{t-1}^j + P_t y_t(j)^{(\theta-1)/\theta} (C_t^w + G_t^w)^{1/\theta} - P_t T_t^j - P_t F_t^j - M_t^j$$

$$C_t^j = (1+r_{t-1})F_{t-1}^j + \frac{M_{t-1}^j}{P_t} + y_t(j)^{(\theta-1)/\theta} (C_t^w + G_t^w)^{1/\theta} - T_t^j - F_t^j - \frac{M_t^j}{P_t}$$

Plugging in this into the utility function in order to obtain an unconstrained maximization problem:

$$\max \sum_{t=0}^{\infty} \beta^t \left[\log \left((1+r_{t-1})F_{t-1}^j + \frac{M_{t-1}^j}{P_t} + y_t(j)^{(\theta-1)/\theta} (C_t^w + G_t^w)^{1/\theta} - T_t^j - F_t^j - \frac{M_t^j}{P_t} \right) + \frac{\chi}{1-\varepsilon} \left(\frac{M_t^j}{P_t} \right)^{1-\varepsilon} - \frac{\kappa}{2} y_t(j)^2 \right]$$

Then the three first order conditions (Euler equations) are:

With respect to F_t^j :

$$\begin{aligned} & \beta^t (-1) \frac{1}{(1+r_{t-1})F_{t-1}^j + \frac{M_{t-1}^j}{P_t} + y_t(j)^{(\theta-1)/\theta} (C_t^w + G_t^w)^{1/\theta} - T_t^j - F_t^j - \frac{M_t^j}{P_t}} + \\ & + \beta^{t+1} (1+r_t) \frac{1}{(1+r_t)F_t^j + \frac{M_t^j}{P_{t+1}} + y_{t+1}(j)^{(\theta-1)/\theta} (C_{t+1}^w + G_{t+1}^w)^{1/\theta} - T_{t+1}^j - F_{t+1}^j - \frac{M_{t+1}^j}{P_{t+1}}} = 0 \end{aligned}$$

$$\beta^{t+1} (1+r_t) \frac{1}{C_{t+1}^j} = \beta^t \frac{1}{C_t^j}$$

$$C_{t+1}^j = \beta(1+r_t)C_t^j$$

With respect to M_t^j :

$$\begin{aligned}
& \beta^t \frac{-1}{P_t} \frac{1}{(1+r_{t-1})F_{t-1}^j + \frac{M_{t-1}^j}{P_t} + y_t(j)^{(\theta-1)/\theta} (C_t^w + G_t^w)^{1/\theta} - T_t^j - F_t^j - \frac{M_t^j}{P_t}} + \beta^t \frac{1}{P_t} \frac{\chi}{1-\varepsilon} (1-\varepsilon) \left(\frac{M_t^j}{P_t} \right)^{-\varepsilon} + \\
& + \beta^{t+1} \frac{1}{P_{t+1}} \frac{1}{(1+r_t)F_t^j + \frac{M_t^j}{P_{t+1}} + y_{t+1}(j)^{(\theta-1)/\theta} (C_{t+1}^w + G_{t+1}^w)^{1/\theta} - T_{t+1}^j - F_{t+1}^j - \frac{M_{t+1}^j}{P_{t+1}}} = 0 \\
& - \frac{1}{P_t} \frac{1}{C_t^j} + \frac{1}{P_t} \chi \left(\frac{M_t^j}{P_t} \right)^{-\varepsilon} + \beta \frac{1}{P_t} \frac{1+i_t}{1+r_t} \frac{1}{C_{t+1}^j} = 0, \text{ since } 1+i_t = \frac{P_{t+1}}{P_t} (1+r_t) \Rightarrow P_{t+1} = P_t \frac{1+i_t}{1+r_t} \\
& \chi \left(\frac{M_t^j}{P_t} \right)^{-\varepsilon} = \frac{1}{C_t^j} - \beta \frac{1+r_t}{1+i_t} \frac{1}{C_{t+1}^j}, \text{ substituting } C_{t+1}^j = \beta(1+r_t)C_t^j \text{ gives} \\
& \chi \left(\frac{M_t^j}{P_t} \right)^{-\varepsilon} = \frac{1}{C_t^j} - \beta \frac{1+r_t}{1+i_t} \frac{1}{\beta(1+r_t)C_t^j} \\
& \chi \left(\frac{M_t^j}{P_t} \right)^{-\varepsilon} = \frac{1}{C_t^j} \left(1 - \frac{1}{1+i_t} \right) \\
& \left(\frac{M_t^j}{P_t} \right)^{-\varepsilon} = \frac{1}{\chi C_t^j} \frac{i_t}{1+i_t} \Rightarrow \left(\frac{M_t^j}{P_t} \right)^{\varepsilon} = \chi C_t^j \frac{1+i_t}{i_t} \\
& \frac{M_t^j}{P_t} = \left(\chi C_t^j \frac{1+i_t}{i_t} \right)^{1/\varepsilon}
\end{aligned}$$

With respect to $y_t(j)$:

$$\begin{aligned}
& \frac{\theta-1}{\theta} \beta^t y_t(j)^{[(\theta-1)/\theta]-1} (C_t^w + G_t^w)^{1/\theta} \frac{1}{(1+r_{t-1})F_{t-1}^j + \frac{M_{t-1}^j}{P_t} + y_t(j)^{(\theta-1)/\theta} (C_t^w + G_t^w)^{1/\theta} - T_t^j - F_t^j - \frac{M_t^j}{P_t}} - \\
& - \beta^t \kappa y_t(j) = 0 \\
& \frac{\theta-1}{\theta} y_t(j)^{-1/\theta} (C_t^w + G_t^w)^{1/\theta} \frac{1}{C_t^j} - \kappa y_t(j) = 0 \\
& y_t(j) y_t(j)^{1/\theta} = \frac{1}{\kappa} \frac{1}{C_t^j} \frac{\theta-1}{\theta} (C_t^w + G_t^w)^{1/\theta}
\end{aligned}$$

$$y_t(j)^{(\theta+1)/\theta} = \frac{\theta-1}{\theta\kappa} \frac{1}{C_t^j} (C_t^w + G_t^w)^{1/\theta}$$

The Transversality Conditions are given by:

The Transversality Condition concerning the real bonds is:

$$\lim_{t \rightarrow \infty} \beta^t \frac{-1}{(1+r_{t-1})F_{t-1}^j + \frac{M_{t-1}^j}{P_t} + y_t(j)^{(\theta-1)/\theta} (C_t^w + G_t^w)^{1/\theta} - T_t^j - F_t^j - \frac{M_t^j}{P_t}} F_t^j = 0$$

$$\lim_{t \rightarrow \infty} \beta^t \frac{1}{C_t^j} F_t^j = 0, \text{ however } C_{t+1}^j = \beta(1+r_t)C_t^j = \beta^2(1+r_t)^2 C_{t-1}^j, \text{ continuing this until } t=0$$

gives: $C_t^j = \beta^t(1+r_t)^t C_0^j$, then the Transversality Condition is:

$$\lim_{t \rightarrow \infty} \beta^t \frac{1}{\beta^t(1+r_t)^t C_0^j} F_t^j = 0, \text{ canceling out gives:}$$

$$\lim_{t \rightarrow \infty} \frac{1}{(1+r_t)^t} F_t^j = 0$$

Transversality Condition for money holdings is given by:

$$\lim_{t \rightarrow \infty} \beta^t \frac{1}{C_t^j} \frac{M_t^j}{P_t} = 0 \Rightarrow \lim_{T \rightarrow \infty} \beta^T \frac{C_t^j}{\beta^T(1+r_t)^T} \frac{M_t^j}{P_t} = 0$$

$$\lim_{t \rightarrow \infty} \frac{1}{(1+r_t)^t} \frac{M_t^j}{P_t} = 0$$

Combining these two gives the condition imposed onto the agents:

$$\lim_{t \rightarrow \infty} \frac{1}{(1+r_t)^t} \left(F_t^j + \frac{M_t^j}{P_t} \right) = 0$$

A.6. Modifying the budget constraint:

Given that no Ponzi schemes can exist (insured by the Transversality Condition) the budget constraint in a steady state for home must hold in the following form:

$$\bar{P}\bar{F} + \bar{M} = \bar{P}(1+\bar{r})\bar{F} + \bar{M} + \bar{p}(h)\bar{y} - \bar{P}\bar{C} - \bar{P}\bar{T}, \text{ expressing for } \bar{C} \text{ gives}$$

$$\bar{C} = \bar{r}\bar{F} + \frac{\bar{p}(h)\bar{y}}{\bar{P}} - \bar{T}, \text{ however } G_t = T_t + \frac{M_t - M_{t-1}}{P_t}, \text{ hence in this case it is}$$

$$\bar{G} = \bar{T} + \frac{\bar{M} - \bar{M}}{\bar{P}} = \bar{T}, \text{ therefore } \bar{T} \text{ can be interchanged with } \bar{G}.$$

$$\bar{C} = r\bar{F} + \frac{\bar{p}(h)\bar{y}}{\bar{P}} - \bar{G}$$

Analogously for foreign:

$$\bar{C}^* = r\bar{F}^* + \frac{\bar{p}^*(f)\bar{y}^*}{\bar{P}^*} - \bar{G}^* . \text{ World net foreign assets must be zero, as domestic nominal}$$

money supply must equal domestic nominal money demand, i.e. $nF + (1-n)F^* = 0$. Then

$$F^* = -\frac{n}{1-n}F , \text{ plugging in this into the equation gives:}$$

$$\bar{C}^* = -r\left(\frac{n}{1-n}\right)\bar{F} + \frac{\bar{p}^*(f)\bar{y}^*}{\bar{P}^*} - \bar{G}^*$$

A.7. Global goods market clearing condition:

The budget constraint for a representative agent of home is given by:

$$P_t F_t + M_t = P_t(1+r_{t-1})F_{t-1} + M_{t-1} + p_t(h)y_t - P_t C_t - P_t T_t$$

for a representative agent of foreign:

$$P_t^* F_t^* + M_t^* = P_t^*(1+r_{t-1})F_{t-1}^* + M_{t-1}^* + p_t^*(f)y_t^* - P_t^* C_t^* - P_t^* T_t^* , \quad \text{divide both by the}$$

corresponding price indexes:

$$F_t + \frac{M_t}{P_t} = (1+r_{t-1})F_{t-1} + \frac{M_{t-1}}{P_t} + \frac{p_t(h)y_t}{P_t} - C_t - T_t$$

$$F_t^* + \frac{M_t^*}{P_t^*} = (1+r_{t-1})F_{t-1}^* + \frac{M_{t-1}^*}{P_t^*} + \frac{p_t^*(f)y_t^*}{P_t^*} - C_t^* - T_t^* , \text{ rearrange and use the fact that}$$

$$G_t = T_t + \frac{M_t - M_{t-1}}{P_t} = 0 \text{ and } G_t^* = T_t^* + \frac{M_t^* - M_{t-1}^*}{P_t^*} :$$

$$C_t = (1+r_{t-1})F_{t-1} - F_t + \frac{p_t(h)y_t}{P_t} - G_t$$

$$C_t^* = (1+r_{t-1})F_{t-1}^* - F_t^* + \frac{p_t^*(f)y_t^*}{P_t^*} - G_t^* , \text{ take a population weighted average and add:}$$

$$\text{Left-hand side: } nC_t + (1-n)C_t^* = C_t^w$$

Right-hand side:

$$n[(1+r_{t-1})F_{t-1} - F_t] + (1-n)[(1+r_{t-1})F_{t-1}^* - F_t^*] + n \frac{p_t(h)y_t}{P_t} + (1-n) \frac{p_t^*(f)y_t^*}{P_t^*} - nG_t - (1-n)G_t^*$$

Using the financial market clearing condition to cancel out the real bonds and the fact that $G_t^w = nG_t + (1-n)G_t^*$ gives:

$$C_t^w = n \frac{p_t(h)y_t}{P_t} + (1-n) \frac{p_t^*(f)y_t^*}{P_t^*} - G_t^w$$

A.8. Closed Form Solution:

As $\bar{F}_0 = \bar{F}_0^* = 0$ and $\bar{G}_0 = \bar{G}_0^* = 0$

$$\bar{C}_0 = r\bar{F}_0 + \frac{\bar{p}_0(h)\bar{y}_0}{\bar{P}_0} - \bar{G}_0 = \frac{\bar{p}_0(h)\bar{y}_0}{\bar{P}_0} \quad \text{and} \quad \bar{C}_0^* = -r\left(\frac{n}{1-n}\right)\bar{F}_0 + \frac{\bar{p}_0^*(f)\bar{y}_0^*}{\bar{P}_0^*} - \bar{G}_0^* = \frac{\bar{p}_0^*(f)\bar{y}_0^*}{\bar{P}_0^*}$$

Furthermore, $\frac{\bar{p}_0(h)}{\bar{P}_0} = \frac{\bar{p}_0^*(f)}{\bar{P}_0^*} = 1$, as the wealth of the countries is the same ($\bar{F}_0 = \bar{F}_0^* = 0$).

(Formally, that is: $\frac{\bar{p}_0(h)}{\bar{P}_0} = \frac{\bar{p}_0(h)}{\left[\int_0^n p(h)^{1-\theta} dh + \int_n^1 [Ep^*(f)]^{1-\theta} df \right]^{1/(1-\theta)}}$. Since $\bar{p}_0(h)$ and

$\bar{p}_0^*(f)$ are the same for all h and f respectively, this can be rewritten as:

$$\frac{\bar{p}_0(h)}{\bar{P}_0} = \frac{\bar{p}_0(h)}{\left[n\bar{p}_0(h)^{1-\theta} + (1-n)[E\bar{p}_0^*(f)]^{1-\theta} \right]^{1/(1-\theta)}}. \text{ In this symmetric equilibrium with equal}$$

country wealth, any two goods produced anywhere in the world have the same price

when measured in the same currency, that is $\bar{p}_0(h) = E\bar{p}_0^*(f)$. Then $\frac{\bar{p}_0(h)}{\bar{P}_0} = \frac{\bar{p}_0(h)}{\bar{p}_0(h)} = 1$

One must again stress that this is not the case when wealth is not the same in both countries, since, as seen later on, terms of trade vary with wealth, because individuals change their consumption-leisure decision with wealth.)

Then $\bar{C}_0 = \bar{y}_0$ and $\bar{C}_0^* = \bar{y}_0^*$. However, as the individuals in this case in the two countries face identical optimization, it must be that $\bar{y}_0 = \bar{y}_0^*$ and, thus, $\bar{C}_0 = \bar{C}_0^*$. From $\bar{C}_0 = \bar{C}_0^*$ it follows that $\bar{C}_0 = \bar{C}_0^* = \bar{C}_0^w$. Then the condition for the output is:

$$\bar{y}_0^{(\theta+1)/\theta} = \frac{\theta-1}{\theta\kappa} \frac{1}{C_0} (C_0^w + G_0^w)^{1/\theta} = \frac{\theta-1}{\theta\kappa} \frac{1}{\bar{y}_0} (\bar{y}_0)^{1/\theta}$$

$$\frac{\bar{y}_0^{(\theta+1)/\theta}}{\bar{y}_0^{(1-\theta)/\theta}} = \frac{\theta-1}{\theta\kappa} \Rightarrow \bar{y}_0^{(\theta+1-1+\theta)/\theta} = \frac{\theta-1}{\theta\kappa}$$

$$\bar{y}_0 = \left(\frac{\theta-1}{\theta\kappa} \right)^{1/2}$$

The equation that determines real money holdings is:

$$\frac{M_t^j}{P_t} = \left(\chi C_t^j \frac{1+i_t}{i_t} \right)^{1/\varepsilon}, \text{ where } 1+i_t = \frac{P_{t+1}}{P_t} (1+r_t), \text{ as at the steady state } P_{t+1} = P_t, \text{ it follows}$$

$i_t = \bar{r}$. The real interest rate is given by $\bar{r} = \frac{1-\beta}{\beta}$, then

$$\frac{1+i_t}{i_t} = \frac{1+\bar{r}}{\bar{r}} = \frac{1+\frac{1-\beta}{\beta}}{\frac{1-\beta}{\beta}} = \frac{\frac{1}{\beta}}{\frac{1-\beta}{\beta}} = \frac{1}{1-\beta}. \text{ Hence,}$$

$$\frac{\bar{M}_0}{\bar{P}_0} = \left(\bar{y}_0 \frac{\chi}{1-\beta} \right)^{1/\varepsilon} = \left(\frac{1-\beta}{\chi} \right)^{-1/\varepsilon} (\bar{y}_0)^{1/\varepsilon}. \text{ The derivation is analogous for } \frac{\bar{M}_0^*}{\bar{P}_0^*} \text{ and, since}$$

$$\bar{y}_0 = \bar{y}_0^*, \frac{\bar{M}_0}{\bar{P}_0} = \frac{\bar{M}_0^*}{\bar{P}_0^*}.$$

A.9. Log-linearization of the model:

Purchasing power parity:

$P = EP^*$, take natural logarithm of both sides

$\ln(P) = \ln(EP^*) \Rightarrow \ln(P) = \ln E + \ln P^*$, totally differentiate (one must take into account that the initial value is at the steady state denoted by 0 subscripts):

$$\frac{dP}{\bar{P}_0} = \frac{dE}{\bar{E}_0} + \frac{dP^*}{\bar{P}_0^*} \Rightarrow \hat{P} = \hat{E} + \hat{P}^* \Rightarrow \hat{E} = \hat{P} - \hat{P}^*$$

Price Indexes:

As all producers are symmetric $P = \left[\int_0^n p(z)^{1-\theta} dz + \int_n^1 [Ep^*(z)]^{1-\theta} dz \right]^{1/(1-\theta)}$ is equivalent to:

$$P_t = \left\{ np_t(h)^{1-\theta} + (1-n) [E_t p_t^*(f)]^{1-\theta} \right\}^{1/(1-\theta)}, \text{ take natural logarithm of both sides}$$

$\ln P_t = \frac{1}{1-\theta} \ln \left\{ n p_t(h)^{1-\theta} + (1-n) \left[E_t p_t^*(f) \right]^{1-\theta} \right\}$, totally differentiate:

$$\frac{dP}{\bar{P}_0} = \frac{1}{1-\theta} \frac{(1-\theta) n \bar{p}_0(h)^{-\theta}}{n \bar{p}_0(h)^{1-\theta} + (1-n) \left[E_0 \bar{p}_0^*(f) \right]^{1-\theta}} dp_t(h) + \frac{1}{1-\theta} \frac{(1-\theta)(1-n) \bar{p}_0^*(f)^{1-\theta} \bar{E}_0^{-\theta}}{n \bar{p}_0(h)^{1-\theta} + (1-n) \left[E_0 \bar{p}_0^*(f) \right]^{1-\theta}} dE_t +$$

$$+ \frac{1}{1-\theta} \frac{(1-\theta)(1-n) \bar{p}_0^*(f)^{-\theta} \bar{E}_0^{1-\theta}}{n \bar{p}_0(h)^{1-\theta} + (1-n) \left[E_0 \bar{p}_0^*(f) \right]^{1-\theta}} dp_t^*(f), \text{ as } \bar{p}_0(h) = \bar{E}_0 \bar{p}_0^*(f) \text{ (this is true, since in}$$

this steady state the two countries have the same wealth, hence $\frac{\bar{p}_0(h)}{\bar{P}_0} = \frac{\bar{p}_0^*(f)}{\bar{P}_0^*} = 1$, and as

$\bar{P} = E\bar{P}^*$, it must be that $\bar{p}_0(h) = \bar{E}_0 \bar{p}_0^*(f)$ also.) the equation becomes:

$$\hat{P}_t = \frac{n \bar{p}_0(h)^{-\theta}}{n \bar{p}_0(h)^{1-\theta} + (1-n) \left[\bar{p}_0(h) \right]^{1-\theta}} dp_t(h) + \frac{(1-n) \bar{p}_0^*(f)^{1-\theta} \bar{E}_0^{-\theta}}{n \bar{p}_0(h)^{1-\theta} + (1-n) \left[\bar{p}_0(h) \right]^{1-\theta}} dE_t +$$

$$+ \frac{(1-n) \bar{p}_0^*(f)^{-\theta} \bar{E}_0^{1-\theta}}{n \bar{p}_0(h)^{1-\theta} + (1-n) \left[\bar{p}_0(h) \right]^{1-\theta}} dp_t^*(f)$$

$$\hat{P}_t = \frac{n \bar{p}_0(h)^{-\theta}}{\bar{p}_0(h)^{1-\theta}} dp_0(h) + \frac{(1-n) \bar{p}_0^*(f)^{1-\theta} \bar{E}_0^{-\theta}}{\bar{p}_0(h)^{1-\theta}} dE_t + \frac{(1-n) \bar{p}_0^*(f)^{-\theta} \bar{E}_0^{1-\theta}}{\bar{p}_0(h)^{1-\theta}} dp_t^*(f)$$

$$\hat{P}_t = n \frac{dp_t(h)}{\bar{p}_0(h)} + (1-n) \left[\frac{\bar{p}_0^*(f)^{1-\theta} \bar{E}_0^{-\theta}}{\left(\bar{E}_0 \bar{p}_0^*(f) \right)^{1-\theta}} dE_t + \frac{\bar{p}_0^*(f)^{-\theta} \bar{E}_0^{1-\theta}}{\left(\bar{E}_0 \bar{p}_0^*(f) \right)^{1-\theta}} dp_t^*(f) \right]$$

$$\hat{P}_t = n \hat{p}_t(h) + (1-n)(\hat{E}_t + \hat{p}_t^*(f))$$

Completely analogical is the derivation of the Price index for foreign:

$$\hat{P}_t^* = n(\hat{p}_t(h) - \hat{E}_t) + (1-n) \hat{p}_t^*(f)$$

Global goods market equilibrium:

$$C_t^w = n \frac{p_t(h) y_t}{P_t} + (1-n) \frac{p_t^*(f) y_t^*}{P_t^*} - G_t^w$$

The left-hand side is given by: $C_t^w = n C_t + (1-n) C_t^*$, take natural logarithm of both sides

$\ln C_t^w = \ln (n C_t + (1-n) C_t^*)$, totally differentiate:

$$\frac{dC_t^w}{\bar{C}_0^w} = \frac{n}{n \bar{C}_0 + (1-n) \bar{C}_0^*} dC_t + \frac{1-n}{n \bar{C}_0 + (1-n) \bar{C}_0^*} dC_t^*, \text{ one must recall that } \bar{C}_0 = \bar{C}_0^* = \bar{C}_0^w$$

$$\frac{dC_t^w}{\bar{C}_0^w} = n \frac{dC_t}{\bar{C}_0} + (1-n) \frac{dC_t^*}{\bar{C}_0^*} \Rightarrow \hat{C}_t^w = n\hat{C}_t + (1-n)\hat{C}_t^*$$

The right-hand side:

$$C_t^w = n \frac{p_t(h)y_t}{P_t} + (1-n) \frac{p_t^*(f)y_t^*}{P_t^*} - G_t^w, \text{ take natural logarithm of both sides}$$

$$\ln C_t^w = \ln \left(n \frac{p_t(h)y_t}{P_t} + (1-n) \frac{p_t^*(f)y_t^*}{P_t^*} - G_t^w \right), \text{ totally differentiate:}$$

$$\hat{C}_t^w = \frac{n \frac{\bar{p}_0(h)}{\bar{P}_0}}{n \frac{\bar{p}_0(h)\bar{y}_0}{\bar{P}_0} + (1-n) \frac{\bar{p}_0^*(f)\bar{y}_0^*}{\bar{P}_0^*} - \bar{G}_0^w} dy_t + (...) dp_t(h) + (...) dP_t + (...) dy_t^* + (...) dp_t^*(f) + (...) dP_t^* + (...) dG_t^w$$

where (...) is the first partial derivative with respect to the variable of the following differential (it is explicitly solved only for y_t , the other cases are as straightforward).

$$\text{Here, it is important to remember that } \bar{C}_0 = \bar{C}_0^* = \bar{y}_0 = \bar{y}_0^* = \bar{C}_0^w \text{ and } \frac{\bar{p}_0(h)}{\bar{P}_0} = \frac{\bar{p}_0^*(f)}{\bar{P}_0^*} = 1.$$

Then the above equation can be simplified to:

$$\hat{C}_t^w = n \frac{dy_t}{\bar{y}_0} + n \frac{dp_t(h)}{\bar{p}_0(h)} - n \frac{dP_t}{\bar{P}_0} + (1-n) \frac{dy_t^*}{\bar{y}_0^*} + (1-n) \frac{dp_t^*(f)}{\bar{p}_0^*(f)} - (1-n) \frac{dP_t^*}{\bar{P}_0^*} - \frac{dG_t^w}{\bar{C}_0^w}$$

$$\hat{C}_t^w = n(\hat{y}_t + \hat{p}_t(h) - \hat{P}_t) + (1-n)(\hat{y}_t^* + \hat{p}_t^*(f) - \hat{P}_t^*) - \frac{dG_t^w}{\bar{C}_0^w}$$

Demand faced by producers:

$$y_t(z) = \left[\frac{p_t(z)}{P_t} \right]^{-\theta} (C_t^w + G_t^w), \text{ take into account that all producers are symmetric and take}$$

natural logarithm of both sides:

$$\ln y_t = -\theta \ln \left[\frac{p_t(h)}{P_t} \right] + \ln(C_t^w + G_t^w),$$

$$\ln y_t = \theta (\ln P_t - \ln p_t(h)) + \ln(C_t^w + G_t^w), \text{ totally differentiating gives:}$$

$$\hat{y}_t = \theta (\hat{P}_t - \hat{p}_t(h)) + \hat{C}_t^w + \frac{dG_t^w}{\bar{C}_0^w}$$

Analogously for foreign: $\hat{y}_t^* = \theta(\hat{P}_t^* - \hat{p}_t^*(f)) + \hat{C}_t^w + \frac{dG_t^w}{\bar{C}_0^w}$

First order conditions determining output:

$y_t^{(\theta+1)/\theta} = \frac{\theta-1}{\theta\kappa} \frac{1}{C_t} (C_t^w + G_t^w)^{1/\theta}$, take natural logarithm of both sides

$\frac{\theta+1}{\theta} \ln y_t = \ln \frac{\theta-1}{\theta\kappa} + \ln \frac{1}{C_t} + \frac{1}{\theta} \ln(C_t^w + G_t^w)$, multiply both sides by θ and totally

differentiate:

$$(\theta+1)\hat{y}_t = -\theta\hat{C}_t + \hat{C}_t^w + \frac{dG_t^w}{\bar{C}_0^w}$$

Same for foreign: $(\theta+1)\hat{y}_t^* = -\theta\hat{C}_t^* + \hat{C}_t^w + \frac{dG_t^w}{\bar{C}_0^w}$

Modified budget constraints:

$\bar{C} = r\bar{F} + \frac{\bar{p}(h)\bar{y}}{\bar{P}} - \bar{G}$, take natural logarithm of both sides:

$\ln \bar{C} = \ln \left(r\bar{F} + \frac{\bar{p}(h)\bar{y}}{\bar{P}} - \bar{G} \right)$, totally differentiate:

$$\hat{\bar{C}} = \frac{r d\bar{F}}{r\bar{F}_0 + \frac{\bar{p}_0(h)\bar{y}_0}{\bar{P}_0} - \bar{G}_0} + (\dots) d\bar{p}(h) + (\dots) d\bar{y} - (\dots) d\bar{P} - (\dots) d\bar{G}, \text{ one must here again}$$

recall the fact that: $\bar{C}_0 = \bar{C}_0^* = \bar{y}_0 = \bar{y}_0^* = \bar{C}_0^w$; $\frac{\bar{p}_0(h)}{\bar{P}_0} = \frac{\bar{p}_0^*(f)}{\bar{P}_0^*} = 1$ and $\bar{F}_0 = \bar{F}_0^* = 0$

$$\hat{\bar{C}} = \bar{r} \frac{d\bar{F}}{\bar{C}_0^w} + \hat{\bar{p}}(h) + \hat{\bar{y}} - \hat{\bar{P}} - \frac{d\bar{G}}{\bar{C}_0^w}$$

For foreign, repeating the same procedure leads from $\bar{C}^* = -\bar{r} \left(\frac{n}{1-n} \right) \bar{F} + \frac{\bar{p}^*(f)\bar{y}^*}{\bar{P}^*} - \bar{G}^*$

$$\text{to } \hat{\bar{C}}^* = -\bar{r} \left(\frac{n}{1-n} \right) \frac{d\bar{F}}{\bar{C}_0^w} + \hat{\bar{p}}^*(f) + \hat{\bar{y}}^* - \hat{\bar{P}}^* - \frac{d\bar{G}^*}{\bar{C}_0^w}$$

Consumption Euler equations:

$C_{t+1} = \beta(1+r_t)C_t$, take natural logarithm of both sides and totally differentiate:

$$\hat{C}_{t+1} = \frac{dr_t}{1+\bar{r}_0} + \hat{C}_t = \frac{dr_t}{1+\frac{1-\beta}{\beta}} + \hat{C}_t = \beta dr_t + \hat{C}_t = (1-\beta)\hat{r}_t + \hat{C}_t \quad (\text{note that } \bar{r} = \frac{1-\beta}{\beta} \text{ or}$$

$$\beta = \frac{1-\beta}{\bar{r}})$$

$$\text{Same for foreign: } \hat{C}_{t+1} = (1-\beta)\hat{r}_t + \hat{C}_t$$

Money demand equations:

$$\frac{M_t}{P_t} = \left(\chi C_t \frac{1+i_t}{i_t} \right)^{1/\varepsilon} \Rightarrow \frac{M_t}{P_t} = \left(\chi C_t \left(\frac{1}{i_t} + 1 \right) \right)^{1/\varepsilon} \quad \text{recall that } P_{t+1} = P_t \frac{1+i_t}{1+r_t}, \text{ from where}$$

$$\text{one can express } i = \frac{(1+r_t)P_{t+1}}{P_t} - 1 = \frac{(1+r_t)P_{t+1} - P_t}{P_t}. \text{ Then:}$$

$$\frac{M_t}{P_t} = \left(\chi C_t \left(\frac{P_t}{(1+r_t)P_{t+1} - P_t} + 1 \right) \right)^{1/\varepsilon} = \left(\chi C_t \left(\frac{(1+r_t)P_{t+1}}{(1+r_t)P_{t+1} - P_t} \right) \right)^{1/\varepsilon}, \text{ take natural logarithm of}$$

both sides:

$$\ln \frac{M_t}{P_t} = \frac{1}{\varepsilon} \left(\ln \chi + \ln C_t + \ln \left(\frac{(1+r_t)P_{t+1}}{(1+r_t)P_{t+1} - P_t} \right) \right)$$

$$\ln \frac{M_t}{P_t} = \frac{1}{\varepsilon} \left(\ln \chi + \ln C_t + \ln((1+r_t)P_{t+1}) - \ln((1+r_t)P_{t+1} - P_t) \right)$$

$$\ln \frac{M_t}{P_t} = \frac{1}{\varepsilon} \left(\ln \chi + \ln C_t + \ln(1+r_t) + \ln P_{t+1} - \ln((1+r_t)P_{t+1} - P_t) \right), \text{ totally differentiate:}$$

$$\hat{M}_t - \hat{P}_t = \frac{1}{\varepsilon} \hat{C}_t + \frac{1}{\varepsilon} \left(\frac{1}{1+\bar{r}_0} dr_t + \hat{P}_{t+1} - \frac{\bar{P}_0}{(1+\bar{r}_0)\bar{P}_0 - \bar{P}_0} dr_t - \frac{1+\bar{r}_0}{(1+\bar{r}_0)\bar{P}_0 - \bar{P}_0} dP_{t+1} + \frac{1}{(1+\bar{r}_0)\bar{P}_0 - \bar{P}_0} dP_t \right)$$

$$\hat{M}_t - \hat{P}_t = \frac{1}{\varepsilon} \hat{C}_t + \frac{1}{\varepsilon} \left(\frac{1}{1+\bar{r}_0} - \frac{1}{\bar{r}_0} \right) dr_t + \frac{1}{\varepsilon} \left(1 - \frac{1+\bar{r}_0}{\bar{r}_0} \right) \hat{P}_{t+1} + \frac{1}{\varepsilon} \frac{1}{\bar{r}_0} \hat{P}_t$$

$$\hat{M}_t - \hat{P}_t = \frac{1}{\varepsilon} \hat{C}_t + \frac{1}{\varepsilon} \left(\frac{\bar{r}_0 - 1 - \bar{r}_0}{\bar{r}_0(1+\bar{r}_0)} \right) dr_t - \frac{1}{\varepsilon} \frac{1}{\bar{r}_0} \hat{P}_{t+1} + \frac{1}{\varepsilon} \frac{1}{\bar{r}_0} \hat{P}_t, \quad \text{recall that } \bar{r}_0 = \frac{1-\beta}{\beta}, \text{ hence:}$$

$$\frac{1}{\bar{r}_0} = \frac{1}{\frac{1-\beta}{\beta}} = \frac{\beta}{1-\beta}. \text{ Then:}$$

$$\hat{M}_t - \hat{P}_t = \frac{1}{\varepsilon} \hat{C}_t - \frac{1}{\varepsilon} \left(\frac{1}{\bar{r}_0(1+\bar{r}_0)} \right) dr_t - \frac{1}{\varepsilon} \frac{\beta}{1-\beta} (\hat{P}_{t+1} - \hat{P}_t)$$

$$\bar{r}_0(1+\bar{r}_0) = \frac{1-\beta}{\beta} \left(1 + \frac{1-\beta}{\beta} \right) = \frac{1-\beta}{\beta} + \frac{(1-\beta)^2}{\beta^2} = \frac{\beta(1-\beta) + (1-\beta)^2}{\beta^2} = \frac{\beta - \beta^2 + 1 - 2\beta + \beta^2}{\beta^2}$$

$$\bar{r}_0(1+\bar{r}_0) = \frac{1-\beta}{\beta^2}. \text{ Substitute in the equation:}$$

$$\hat{M}_t - \hat{P}_t = \frac{1}{\varepsilon} \hat{C}_t - \frac{1}{\varepsilon} \left(\frac{\beta^2}{1-\beta} \right) dr_t - \frac{1}{\varepsilon} \frac{\beta}{1-\beta} (\hat{P}_{t+1} - \hat{P}_t), \text{ recall that } \frac{\beta}{1-\beta} = \frac{1}{\bar{r}_0}:$$

$$\hat{M}_t - \hat{P}_t = \frac{1}{\varepsilon} \hat{C}_t - \frac{\beta}{\varepsilon} \left(\hat{r}_t + \frac{\hat{P}_{t+1} - \hat{P}_t}{1-\beta} \right)$$

$$\text{Analogically for foreign: } \hat{M}_t^* - \hat{P}_t^* = \frac{1}{\varepsilon} \hat{C}_t^* - \frac{\beta}{\varepsilon} \left(\hat{r}_t^* + \frac{\hat{P}_{t+1}^* - \hat{P}_t^*}{1-\beta} \right)$$

A.10. Solving the linearized model:

First solve for the difference between home and foreign variables. Subtracting (5) from (4) gives:

$$\hat{y} - \hat{y}^* = \theta \left(\hat{P} - \hat{p}(h) \right) + \hat{C}^w + \frac{d\bar{G}^w}{\bar{C}_0^w} - \theta \left(\hat{P}^* - \hat{p}^*(f) \right) - \hat{C}^{*w} - \frac{d\bar{G}^w}{\bar{C}_0^w}$$

$$\hat{y} - \hat{y}^* = \theta \left(\hat{P} - \hat{p}(h) - \hat{P}^* + \hat{p}^*(f) \right), \text{ recall that } \hat{E} = \hat{P} - \hat{P}^*$$

$$\hat{y} - \hat{y}^* = \theta \left(\hat{E} - \hat{p}(h) + \hat{p}^*(f) \right)$$

Then subtracting (7) from (6):

$$(\theta+1)(\hat{y} - \hat{y}^*) = -\theta \hat{C} + \hat{C}^w + \frac{d\bar{G}^w}{\bar{C}_0^w} + \theta \hat{C}^* - \hat{C}^{*w} - \frac{d\bar{G}^w}{\bar{C}_0^w}$$

$$(\theta+1)(\hat{y} - \hat{y}^*) = -\theta \hat{C} + \theta \hat{C}^*$$

$$\hat{y} - \hat{y}^* = -\frac{\theta}{\theta+1} (\hat{C} - \hat{C}^*)$$

Subtracting (9) from (8)

$$\hat{\bar{C}} - \hat{\bar{C}}^* = \bar{r} \frac{d\bar{F}}{\bar{C}_0^w} + \hat{p}(h) + \hat{y} - \hat{P} - \frac{d\bar{G}}{\bar{C}_0^w} + \bar{r} \left(\frac{n}{1-n} \right) \frac{d\bar{F}}{\bar{C}_0^w} - \hat{p}^*(f) - \hat{y}^* + \hat{P}^* + \frac{d\bar{G}^*}{\bar{C}_0^w}$$

$$\hat{\bar{C}} - \hat{\bar{C}}^* = \bar{r} \left(\frac{1}{1-n} \right) \frac{d\bar{F}}{\bar{C}_0^w} - \frac{d\bar{G}}{\bar{C}_0^w} + \frac{d\bar{G}^*}{\bar{C}_0^w} + \hat{y} - \hat{y}^* - (\hat{P} - \hat{P}^* + \hat{p}^*(f) - \hat{p}(h)), \text{ note that}$$

$$\hat{y} - \hat{y}^* = \theta \left(\hat{P} - \hat{p}(h) - \hat{P}^* + \hat{p}^*(f) \right), \text{ hence:}$$

$$\hat{\bar{C}} - \hat{\bar{C}}^* = \bar{r} \left(\frac{1}{1-n} \right) \frac{d\bar{F}}{\bar{C}_0^w} - \frac{d\bar{G}}{\bar{C}_0^w} + \frac{d\bar{G}^*}{\bar{C}_0^w} + \frac{\theta(\hat{y} - \hat{y}^*)}{\theta} - \frac{\hat{y} - \hat{y}^*}{\theta}$$

$$\hat{\bar{C}} - \hat{\bar{C}}^* = \bar{r} \left(\frac{1}{1-n} \right) \frac{d\bar{F}}{\bar{C}_0^w} - \frac{d\bar{G}}{\bar{C}_0^w} + \frac{d\bar{G}^*}{\bar{C}_0^w} + \frac{\theta-1}{\theta} (\hat{y} - \hat{y}^*), \text{ note that } \hat{y} - \hat{y}^* = -\frac{\theta}{\theta+1} (\hat{\bar{C}} - \hat{\bar{C}}^*)$$

$$\hat{\bar{C}} - \hat{\bar{C}}^* = \bar{r} \left(\frac{1}{1-n} \right) \frac{d\bar{F}}{\bar{C}_0^w} - \frac{d\bar{G}}{\bar{C}_0^w} + \frac{d\bar{G}^*}{\bar{C}_0^w} - \frac{\theta-1}{\theta} \frac{\theta}{\theta+1} (\hat{\bar{C}} - \hat{\bar{C}}^*)$$

$$\left(1 + \frac{\theta-1}{\theta+1} \right) (\hat{\bar{C}} - \hat{\bar{C}}^*) = \bar{r} \left(\frac{1}{1-n} \right) \frac{d\bar{F}}{\bar{C}_0^w} - \frac{d\bar{G}}{\bar{C}_0^w} + \frac{d\bar{G}^*}{\bar{C}_0^w}$$

$$\frac{2\theta}{\theta+1} (\hat{\bar{C}} - \hat{\bar{C}}^*) = \bar{r} \left(\frac{1}{1-n} \right) \frac{d\bar{F}}{\bar{C}_0^w} - \frac{d\bar{G}}{\bar{C}_0^w} + \frac{d\bar{G}^*}{\bar{C}_0^w}$$

$$\hat{\bar{C}} - \hat{\bar{C}}^* = \left(\frac{\theta+1}{2\theta} \right) \left(\frac{1}{1-n} \right) \bar{r} \frac{d\bar{F}}{\bar{C}_0^w} - \left(\frac{\theta+1}{2\theta} \right) \frac{d\bar{G}}{\bar{C}_0^w} + \left(\frac{\theta+1}{2\theta} \right) \frac{d\bar{G}^*}{\bar{C}_0^w}$$

Now multiply equation (6) by n and equation (7) by $1-n$, then add up:

$$n(\theta+1)\hat{y} = n \left(-\theta\hat{\bar{C}} + \hat{\bar{C}}^w + \frac{d\bar{G}^w}{\bar{C}_0^w} \right)$$

$$(1-n)(\theta+1)\hat{y}^* = (1-n) \left(-\theta\hat{\bar{C}}^* + \hat{\bar{C}}^w + \frac{d\bar{G}^w}{\bar{C}_0^w} \right)$$

$$(\theta+1)(n\hat{y} + (1-n)\hat{y}^*) = -\theta(n\hat{\bar{C}} + (1-n)\hat{\bar{C}}^*) + \hat{\bar{C}}^w + \frac{d\bar{G}^w}{\bar{C}_0^w}$$

Do the same procedure with equations:

$$\hat{y} = \theta \left(\hat{P} - \hat{p}(h) \right) + \hat{\bar{C}}^w + \frac{d\bar{G}^w}{\bar{C}_0^w} \text{ and } \hat{y}^* = \theta \left(\hat{P}^* - \hat{p}^*(f) \right) + \hat{\bar{C}}^w + \frac{d\bar{G}^w}{\bar{C}_0^w} :$$

$$n\hat{y} + (1-n)\hat{y}^* = n\theta\left(\hat{P} - \hat{p}(h)\right) + (1-n)\theta\left(\hat{P}^* - \hat{p}^*(f)\right) + \hat{C}^w + \frac{d\bar{G}^w}{\bar{C}_0^w}, \text{ recall that}$$

$$\hat{P}_t = n\hat{p}_t(h) + (1-n)(\hat{E}_t + \hat{p}_t^*(f)) \text{ and } \hat{P}_t^* = n(\hat{p}_t(h) - \hat{E}_t) + (1-n)\hat{p}_t^*(f)$$

$$n\hat{y} + (1-n)\hat{y}^* = \theta\left[n\left((n-1)\hat{p}_t(h) + (1-n)(\hat{E}_t + \hat{p}_t^*(f))\right) + (1-n)\left(n(\hat{p}_t(h) - \hat{E}_t) + (-n)\hat{p}_t^*(f)\right)\right] + \hat{C}^w + \frac{d\bar{G}^w}{\bar{C}_0^w}$$

After canceling out:

$$n\hat{y} + (1-n)\hat{y}^* = \hat{C}^w + \frac{d\bar{G}^w}{\bar{C}_0^w}$$

combining with the result obtain just before this one, $(\theta+1)\hat{y}^w = (1-\theta)\hat{C}^w + \frac{d\bar{G}^w}{\bar{C}_0^w}$ and

$$\hat{y}^w = \hat{C}^w + \frac{d\bar{G}^w}{\bar{C}_0^w}, \text{ where } \hat{y}^w = n\hat{y} + (1-n)\hat{y}^* \text{ gives :}$$

$$(\theta+1)\left(\hat{C}^w + \frac{d\bar{G}^w}{\bar{C}_0^w}\right) = (1-\theta)\hat{C}^w + \frac{d\bar{G}^w}{\bar{C}_0^w} \Rightarrow 2\theta\hat{C}^w = -\theta\frac{d\bar{G}^w}{\bar{C}_0^w}$$

$$\hat{C}^w = -\frac{1}{2}\frac{d\bar{G}^w}{\bar{C}_0^w}$$

$$\text{The same for } \hat{y}^w \text{ gives : } \hat{y}^w = \frac{1}{2}\frac{d\bar{G}^w}{\bar{C}_0^w}$$

Noticed that $nX + (1-n)X^* = X^w$, where X is equal to $\hat{C}$ or $\hat{y}$, can be rewritten in two ways: $X = X^w + (1-n)(X - X^*)$ or $X^* = X^w - n(X - X^*)$. Then:

$$\hat{C} = -\frac{1}{2}\frac{d\bar{G}^w}{\bar{C}_0^w} + (1-n)\left[\left(\frac{\theta+1}{2\theta}\right)\left(\frac{1}{1-n}\right)\frac{\bar{r}d\bar{F}}{\bar{C}_0^w} - \left(\frac{\theta+1}{2\theta}\right)\frac{d\bar{G}}{\bar{C}_0^w} + \left(\frac{\theta+1}{2\theta}\right)\frac{d\bar{G}^*}{\bar{C}_0^w}\right], \text{ since}$$

$d\bar{G}^w = nd\bar{G} + (1-n)d\bar{G}^*$ this could be simplified to:

$$\hat{C} = \left(\frac{\theta+1}{2\theta}\right)\frac{\bar{r}d\bar{F}}{\bar{C}_0^w} - \left(\frac{(1-n)(\theta+1)}{2\theta} + \frac{n}{2}\right)\frac{d\bar{G}}{\bar{C}_0^w} + \left(\frac{(1-n)(\theta+1)}{2\theta} - \frac{1-n}{2}\right)\frac{d\bar{G}^*}{\bar{C}_0^w}$$

$$\hat{C} = \left(\frac{\theta+1}{2\theta}\right)\frac{\bar{r}d\bar{F}}{\bar{C}_0^w} - \left(\frac{\theta+1-n}{2\theta}\right)\frac{d\bar{G}}{\bar{C}_0^w} + \left(\frac{1-n}{2\theta}\right)\frac{d\bar{G}^*}{\bar{C}_0^w}$$

For foreign:

$$\begin{aligned}\hat{\bar{C}}^* &= -\frac{1}{2} \frac{d\bar{G}^w}{\bar{C}_0^w} - n \left[\left(\frac{\theta+1}{2\theta} \right) \left(\frac{1}{1-n} \right) \bar{r} d\bar{F} - \left(\frac{\theta+1}{2\theta} \right) \frac{d\bar{G}}{\bar{C}_0^w} + \left(\frac{\theta+1}{2\theta} \right) \frac{d\bar{G}^*}{\bar{C}_0^w} \right] \\ \hat{\bar{C}}^* &= -\left(\frac{n}{1-n} \right) \left(\frac{\theta+1}{2\theta} \right) \frac{\bar{r} d\bar{F}}{\bar{C}_0^w} + \left(\frac{n(\theta+1)}{2\theta} - \frac{n}{2} \right) \frac{d\bar{G}}{\bar{C}_0^w} - \left(\frac{n(\theta+1)}{2\theta} + \frac{1-n}{2} \right) \frac{d\bar{G}^*}{\bar{C}_0^w} \\ \hat{\bar{C}}^* &= -\left(\frac{n}{1-n} \right) \left(\frac{\theta+1}{2\theta} \right) \frac{\bar{r} d\bar{F}}{\bar{C}_0^w} + \left(\frac{n}{2\theta} \right) \frac{d\bar{G}}{\bar{C}_0^w} - \left(\frac{\theta+n}{2\theta} \right) \frac{d\bar{G}^*}{\bar{C}_0^w}\end{aligned}$$

To obtain the solution for output, use equations (6) and (7) for a steady state.

Equation (6) $(\theta+1)\hat{y} = -\theta\hat{\bar{C}} + \hat{\bar{C}}^w + \frac{dG^w}{\bar{C}_0^w}$, can be rewritten by using $\hat{\bar{C}}^w = -\frac{1}{2} \frac{d\bar{G}^w}{\bar{C}_0^w}$ as:

$$\begin{aligned}\hat{y} &= -\frac{\theta}{\theta+1} \hat{\bar{C}} - \frac{1}{\theta+1} \left(\frac{1}{2} \frac{d\bar{G}^w}{\bar{C}_0^w} - \frac{dG_t^w}{\bar{C}_0^w} \right) \\ \hat{y} &= -\frac{\theta}{1+\theta} \hat{\bar{C}} + \left(\frac{1}{2(1+\theta)} \right) \frac{d\bar{G}^w}{\bar{C}_0^w}\end{aligned}$$

The same procedure from equation (7): $(\theta+1)\hat{y}^* = -\theta\hat{\bar{C}}^* + \bar{G}^w + \frac{dG^w}{\bar{C}_0^w}$ gives:

$$\hat{y}^* = -\frac{\theta}{\theta+1} \hat{\bar{C}}^* + \left(\frac{1}{2(1+\theta)} \right) \frac{d\bar{G}^w}{\bar{C}_0^w}$$

The solution for terms of trade can be obtained by recalling that:

$$\begin{aligned}\hat{y} - \hat{y}^* &= \theta \left(\hat{\bar{E}} - \hat{p}(h) + \hat{p}^*(f) \right) \quad \text{and} \quad \hat{y} - \hat{y}^* = -\frac{\theta}{\theta+1} \left(\hat{\bar{C}} - \hat{\bar{C}}^* \right). \quad \text{The equations can be} \\ \text{transformed to:} \quad \hat{p}(h) - \hat{p}^*(f) - \hat{\bar{E}} &= -\frac{1}{\theta} \left(\hat{y} - \hat{y}^* \right) = \frac{1}{1+\theta} \left(\hat{\bar{C}} - \hat{\bar{C}}^* \right)\end{aligned}$$

A.11. Linearizing the current account:

$F_t - F_{t-1} = r_{t-1} F_{t-1} + \frac{p_t(h)y_t}{P_t} - C_t - G_t$, recall that $\bar{F}_0 = \bar{F}_0^* = 0$ and that F_{t-1} denote the

stock of bond held by a home resident at the beginning of period t. Since the system is log-linearized around the zero steady state, the change from this state is examined.

Hence, F_{t-1} denote the bonds held in the baseline steady state, i.e. $F_{t-1} = \bar{F}_0 = 0$.

Then the equation is:

$F_t = \frac{p_t(h)y_t}{P_t} - C_t - G_t$, rearrange and take the natural logarithm of both sides:

$\ln \frac{p_t(h)y_t}{P_t} = \ln(F_t + C_t + G_t)$, totally differentiating gives:

$$\frac{dp_t(h)}{\bar{p}_0(h)} + \frac{dy_t}{\bar{y}_0} - \frac{dP_t}{\bar{P}_0} = \frac{dF_t}{\bar{F}_0 + \bar{C}_0 + \bar{G}_0} + \frac{dC_t}{\bar{F}_0 + \bar{C}_0 + \bar{G}_0} + \frac{dG_t}{\bar{F}_0 + \bar{C}_0 + \bar{G}_0}, \text{ since } \bar{G}_0 = \bar{G}_0^* = 0 \text{ and}$$

$$\bar{C}_0 = \bar{C}_0^* = \bar{y}_0 = \bar{y}_0^* = \bar{C}_0^w :$$

$$\frac{dp_t(h)}{\bar{y}_0} + \hat{y}_t - \hat{P}_t = \frac{dF_t}{\bar{C}_0^w} + \hat{C}_t + \frac{dG_t}{\bar{C}_0^w}, \text{ however here the short run is considered, then}$$

$$\hat{P} = (1-n)\hat{E} \text{ and } dp(h) = 0 :$$

$$\frac{dF_t}{\bar{C}_0^w} = \hat{y} - \hat{C} - (1-n)\hat{E} - \frac{dG}{\bar{C}_0^w}, \text{ notice that the assets at the end of period } t \text{ are the steady}$$

state levels, since the economy has returned to the steady state after one period:

$$\frac{d\bar{F}}{\bar{C}_0^w} = \hat{y} - \hat{C} - (1-n)\hat{E} - \frac{dG}{\bar{C}_0^w}$$

Equivalently for foreign, remember that $nF + (1-n)F^* = 0$:

$$\frac{d\bar{F}^*}{\bar{C}_0^w} = \hat{y}^* - \hat{C}^* + n\hat{E} - \frac{dG^*}{\bar{C}_0^w} = -\left(\frac{n}{1-n}\right)\frac{d\bar{F}}{\bar{C}_0^w}$$

A.12. Solving the short-run linearized model:

First solve for the difference between home and foreign variables. Subtracting (19) from (18) gives:

$$\hat{\bar{C}} - \hat{\bar{C}}^* = (1-\beta)\hat{r} + \hat{C} - (1-\beta)\hat{r} - \hat{C}^* = \hat{C} - \hat{C}^*$$

Subtracting (21) from (20) gives:

$$\hat{M} - \hat{P} - \hat{M}^* + \hat{P}^* = \frac{1}{\varepsilon}\hat{C} - \frac{\beta}{\varepsilon}\left(\hat{r} + \frac{\hat{\bar{P}} - \hat{P}}{1-\beta}\right) - \frac{1}{\varepsilon}\hat{C}^* + \frac{\beta}{\varepsilon}\left(\hat{r} + \frac{\hat{\bar{P}}^* - \hat{P}^*}{1-\beta}\right)$$

$$\hat{M} - \hat{M}^* - (1-n)\hat{E} - n\hat{E} = \frac{1}{\varepsilon}(\hat{C} - \hat{C}^*) - \frac{\beta}{\varepsilon(1-\beta)}(\hat{\bar{P}} - \hat{P} - \hat{\bar{P}}^* + \hat{P}^*), \text{ recall the purchasing}$$

parity equation $\hat{E} = \hat{P} - \hat{P}^*$, which holds always:

$$(\hat{M} - \hat{M}^*) - \hat{E} = \frac{1}{\varepsilon}(\hat{C} - \hat{C}^*) - \frac{\beta}{\varepsilon(1-\beta)}(\hat{\hat{E}} - \hat{E})$$

Consider the same equation one period later, when all variables are the steady state variables, alternatively the equation can be derived after observing that in the long-run

$$\hat{P}_t = \hat{M}_t - \frac{1}{\varepsilon}\hat{C}_t \text{ and } \hat{P}_t^* = \hat{M}_t^* - \frac{1}{\varepsilon}\hat{C}_t^* \text{ must hold:}$$

$$\hat{\hat{E}} = (\hat{\hat{M}} - \hat{\hat{M}}^*) - \frac{1}{\varepsilon}(\hat{\hat{C}} - \hat{\hat{C}}^*)$$

Substituting this into the equation for the difference in money holdings:

$$(\hat{M} - \hat{M}^*) - \hat{E} = \frac{1}{\varepsilon}(\hat{C} - \hat{C}^*) - \frac{\beta}{\varepsilon(1-\beta)}\left((\hat{\hat{M}} - \hat{\hat{M}}^*) - \frac{1}{\varepsilon}(\hat{\hat{C}} - \hat{\hat{C}}^*) - \hat{E}\right), \quad \text{note that}$$

$$(\hat{M} - \hat{M}^*) = (\hat{\hat{M}} - \hat{\hat{M}}^*) \text{ as the monetary shock is permanent, and that it was shown that}$$

$$(\hat{C} - \hat{C}^*) = (\hat{\hat{C}} - \hat{\hat{C}}^*), \text{ then:}$$

$$-\left(1 + \frac{\beta}{\varepsilon(1-\beta)}\right)\hat{E} = \left(1 + \frac{\beta}{\varepsilon(1-\beta)}\right)\frac{1}{\varepsilon}(\hat{C} - \hat{C}^*) - \left(1 + \frac{\beta}{\varepsilon(1-\beta)}\right)(\hat{M} - \hat{M}^*)$$

$$\hat{E} = (\hat{M} - \hat{M}^*) - \frac{1}{\varepsilon}(\hat{C} - \hat{C}^*)$$

Subtracting (23) from (22) gives:

$$\frac{d\bar{F}}{\bar{C}_0^w} + \left(\frac{n}{1-n}\right)\frac{d\bar{F}}{\bar{C}_0^w} = \hat{y} - \hat{C} - (1-n)\hat{E} - \hat{y}^* + \hat{C}^* - n\hat{E}$$

$$\left(\frac{1}{1-n}\right)\frac{d\bar{F}}{\bar{C}_0^w} = (\hat{y} - \hat{y}^*) - (\hat{C} - \hat{C}^*) - \hat{E}$$

$$\frac{d\bar{F}}{\bar{C}_0^w} = (1-n)\left[(\hat{y} - \hat{y}^*) - (\hat{C} - \hat{C}^*) - \hat{E}\right]$$

Then subtract (17) from (16):

$$\hat{y} - \hat{y}^* = \theta(1-n)\hat{E} + \theta n\hat{E} + \hat{C}^w - \hat{C}^w$$

$$\hat{y} - \hat{y}^* = \theta\hat{E}, \text{ plugging in this into the above equation for the current account:}$$

$$\frac{d\bar{F}}{\bar{C}_0^w} = (1-n)\left[(\theta-1)\hat{E} - (\hat{C} - \hat{C}^*)\right]$$

Now consider the solution for long-run steady state difference between consumptions, which was obtained in appendix A.10, shortly before the two separate equations for home and foreign consumption were obtained, (alternatively just subtract the two solutions):

$$\hat{\bar{C}} - \hat{\bar{C}}^* = \left(\frac{\theta+1}{2\theta} \right) \left(\frac{1}{1-n} \right) \frac{\bar{r}d\bar{F}}{\bar{C}_0^w} - \left(\frac{\theta+1}{2\theta} \right) \frac{d\bar{G}}{\bar{C}_0^w} + \left(\frac{\theta+1}{2\theta} \right) \frac{d\bar{G}^*}{\bar{C}_0^w} \quad \text{eliminate the fiscal policy and}$$

express for the current account:

$$\frac{d\bar{F}}{\bar{C}_0^w} = \left(\frac{2\theta}{\bar{r}(\theta+1)} \right) (1-n) (\hat{\bar{C}} - \hat{\bar{C}}^*), \quad \text{unite the two equations for the current account:}$$

$$(1-n) \left[(\theta-1)\hat{E} - (\hat{\bar{C}} - \hat{\bar{C}}^*) \right] = \left(\frac{2\theta}{\bar{r}(\theta+1)} \right) (1-n) (\hat{\bar{C}} - \hat{\bar{C}}^*)$$

$$(\theta-1)\hat{E} = \left(\frac{2\theta + \bar{r}(\theta+1)}{\bar{r}(\theta+1)} \right) (\hat{\bar{C}} - \hat{\bar{C}}^*)$$

$$\hat{E} = \left(\frac{2\theta + \bar{r}(\theta+1)}{\bar{r}(\theta^2-1)} \right) (\hat{\bar{C}} - \hat{\bar{C}}^*)$$

Combining this result with $\hat{E} = (\hat{M} - \hat{M}^*) - \frac{1}{\varepsilon} (\hat{\bar{C}} - \hat{\bar{C}}^*)$ gives:

$$\hat{E} = \left(\frac{2\theta + \bar{r}(\theta+1)}{\bar{r}(\theta^2-1)} \right) \varepsilon \left[(\hat{M} - \hat{M}^*) - \hat{E} \right]$$

$$\left(\frac{\varepsilon [2\theta + \bar{r}(\theta+1)]}{\bar{r}(\theta^2-1)} + 1 \right) \hat{E} = \left(\frac{\varepsilon [2\theta + \bar{r}(\theta+1)]}{\bar{r}(\theta^2-1)} \right) (\hat{M} - \hat{M}^*)$$

$$\hat{E} = \left(\frac{\varepsilon [2\theta + \bar{r}(\theta+1)]}{\varepsilon [2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2-1)} \right) (\hat{M} - \hat{M}^*)$$

Then plugging in this into $(\hat{\bar{C}} - \hat{\bar{C}}^*) = \varepsilon \left[(\hat{M} - \hat{M}^*) - \hat{E} \right]$:

$$(\hat{\bar{C}} - \hat{\bar{C}}^*) = \varepsilon \left[(\hat{M} - \hat{M}^*) - \left(\frac{\varepsilon [2\theta + \bar{r}(\theta+1)]}{\varepsilon [2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2-1)} \right) (\hat{M} - \hat{M}^*) \right]$$

$$(\hat{\bar{C}} - \hat{\bar{C}}^*) = \varepsilon \left(\frac{\varepsilon [2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2-1) - \varepsilon [2\theta + \bar{r}(\theta+1)]}{\varepsilon [2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2-1)} \right) (\hat{M} - \hat{M}^*)$$

$$(\hat{\bar{C}} - \hat{\bar{C}}^*) = \left(\frac{\varepsilon \bar{r}(\theta^2-1)}{\varepsilon [2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2-1)} \right) (\hat{M} - \hat{M}^*)$$

To solve for the current account, use $\frac{d\bar{F}}{\bar{C}_0^w} = \left(\frac{2\theta}{\bar{r}(\theta+1)} \right) (1-n) \left(\hat{\bar{C}} - \hat{\bar{C}}^* \right)$, and substitute the

difference in consumption with the result obtained above:

$$\frac{d\bar{F}}{\bar{C}_0^w} = \left(\frac{2\theta}{\bar{r}(\theta+1)} \right) (1-n) \left(\frac{\varepsilon \bar{r}(\theta^2-1)}{\varepsilon [2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2-1)} \right) (\hat{M} - \hat{M}^*)$$

$$\frac{d\bar{F}}{\bar{C}_0^w} = \left(\frac{2\theta(1-n)\varepsilon(\theta-1)}{\varepsilon [2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2-1)} \right) (\hat{M} - \hat{M}^*)$$

After obtaining the current account, all the long-term variables can be easily found:

Consumption:

$$\hat{\bar{C}} = \left(\frac{\theta+1}{2\theta} \right) \bar{r} \frac{d\bar{F}}{\bar{C}_0^w} - \left(\frac{\theta+1-n}{2\theta} \right) \frac{d\bar{G}}{\bar{C}_0^w} + \left(\frac{1-n}{2\theta} \right) \frac{d\bar{G}^*}{\bar{C}_0^w} \Rightarrow \hat{\bar{C}} = \left(\frac{\bar{r}(\theta+1)}{2\theta} \right) \frac{d\bar{F}}{\bar{C}_0^w}$$

$$\hat{\bar{C}} = \left(\frac{\bar{r}(\theta+1)}{2\theta} \right) \left(\frac{2\theta(1-n)\varepsilon(\theta-1)}{\varepsilon [2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2-1)} \right) (\hat{M} - \hat{M}^*)$$

$$\hat{\bar{C}} = \left(\frac{\bar{r}(1-n)\varepsilon(\theta^2-1)}{\varepsilon [2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2-1)} \right) (\hat{M} - \hat{M}^*)$$

$$\text{For foreign: } \hat{\bar{C}}^* = - \left(\frac{n\bar{r}\varepsilon(\theta^2-1)}{\varepsilon [2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2-1)} \right) (\hat{M} - \hat{M}^*)$$

Output:

$$\hat{y} = - \frac{\theta}{1+\theta} \hat{\bar{C}} + \left(\frac{1}{2(1+\theta)} \right) \frac{d\bar{G}^w}{\bar{C}_0^w} \Rightarrow \hat{y} = - \frac{\theta}{1+\theta} \hat{\bar{C}}$$

$$\hat{y} = - \left(\frac{\theta \bar{r}(1-n)\varepsilon(\theta-1)}{\varepsilon [2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2-1)} \right) (\hat{M} - \hat{M}^*)$$

$$\text{For foreign: } \hat{y}^* = \left(\frac{\theta n \bar{r} \varepsilon (\theta-1)}{\varepsilon [2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2-1)} \right) (\hat{M} - \hat{M}^*)$$

Terms of trade:

$$\hat{p}(h) - \hat{p}^*(f) - \hat{E} = \frac{1}{1+\theta} (\hat{\bar{C}} - \hat{\bar{C}}^*)$$

$$\hat{p}(h) - \hat{p}^*(f) - \hat{E} = \left(\frac{\varepsilon \bar{r}(\theta - 1)}{\varepsilon [2\theta + \bar{r}(\theta + 1)] + \bar{r}(\theta^2 - 1)} \right) (\hat{M} - \hat{M}^*)$$

To obtain the short-run variables, however, one must first solve for world aggregates:

Multiply the home Euler equation (18) by n and the foreign Euler equation (19) by $(1 - n)$ and add them up:

$$n\hat{C} + (1 - n)\hat{C}^* = n(1 - \beta)\hat{r} + n\hat{C} + (1 - n)(1 - \beta)\hat{r} + (1 - n)\hat{C}^*$$

$$\hat{C}^w = (1 - \beta)\hat{r} + \hat{C}^w$$

Recall that $\hat{C}^w = -\frac{1}{2} \frac{d\bar{G}^w}{\bar{C}_0^w}$. Thus, here $\hat{C}^w = 0$. Then:

$$\hat{C}^w = -(1 - \beta)\hat{r}$$

Now, multiply the home money demand equation (20) by n and the foreign money demand equation (21) by $(1 - n)$ and add them up:

$$n(\hat{M} - \hat{P}) + (1 - n)(\hat{M}^* - \hat{P}^*) = n \left[\frac{1}{\varepsilon} \hat{C} - \frac{\beta}{\varepsilon} \left(\hat{r} + \frac{\hat{P} - \hat{P}}{1 - \beta} \right) \right] + (1 - n) \left[\frac{1}{\varepsilon} \hat{C}^* - \frac{\beta}{\varepsilon} \left(\hat{r} + \frac{\hat{P}^* - \hat{P}^*}{1 - \beta} \right) \right]$$

Equations (14) and (15) give, respectively, $\hat{P} = (1 - n)\hat{E}$ and $\hat{P}^* = -n\hat{E}$, and let

$$\hat{M}^w = n\hat{M} + (1 - n)\hat{M}^*:$$

$$\hat{M}^w - n(1 - n)\hat{E} + n(1 - n)\hat{E} = \frac{1}{\varepsilon} \hat{C}^w - \frac{\beta}{\varepsilon} \hat{r} - \frac{\beta}{\varepsilon(1 - \beta)} (n\hat{P} - n\hat{P} + (1 - n)\hat{P}^* - (1 - n)\hat{P}^*)$$

$$\hat{M}^w = \frac{1}{\varepsilon} \hat{C}^w - \frac{\beta}{\varepsilon} \hat{r} - \frac{\beta}{\varepsilon(1 - \beta)} (n\hat{P} + (1 - n)\hat{P}^*)$$

Remember that $\hat{P} = \hat{M} - \frac{1}{\varepsilon} \hat{C}$ and $\hat{P}^* = \hat{M}^* - \frac{1}{\varepsilon} \hat{C}^*$:

$$\hat{M}^w = \frac{1}{\varepsilon} \hat{C}^w - \frac{\beta}{\varepsilon} \hat{r} - \frac{\beta}{\varepsilon(1 - \beta)} \left(\hat{M}^w - \frac{1}{\varepsilon} \hat{C}^w \right), \text{ but } \hat{C}^w = 0:$$

$$\hat{M}^w = \frac{1}{\varepsilon} \hat{C}^w - \frac{\beta}{\varepsilon} \hat{r} - \frac{\beta}{\varepsilon(1 - \beta)} \hat{M}^w$$

Combining this with $\hat{C}^w = -(1 - \beta)\hat{r}$ gives:

$$\hat{M}^w = -\frac{1}{\varepsilon} (1 - \beta)\hat{r} - \frac{\beta}{\varepsilon} \hat{r} - \frac{\beta}{\varepsilon(1 - \beta)} \hat{M}^w$$

$$\hat{M}^w + \frac{\beta}{\varepsilon(1-\beta)} \hat{M}^w = -\frac{\hat{r}}{\varepsilon}$$

$$\hat{r} = -\left(\varepsilon + \frac{\beta}{(1-\beta)}\right) \hat{M}^w$$

Alternatively combining the same equations gives:

$$\hat{M}^w = \frac{1}{\varepsilon} \hat{C}^w + \frac{\beta}{\varepsilon(1-\beta)} \frac{\hat{C}^w}{(1-\beta)} - \frac{\beta}{\varepsilon(1-\beta)} \hat{M}^w$$

$$\frac{\varepsilon(1-\beta) + \beta}{\varepsilon(1-\beta)} \hat{M}^w = \frac{(1-\beta) + \beta}{\varepsilon(1-\beta)} \hat{C}^w$$

$$(\varepsilon(1-\beta) + \beta) \hat{M}^w = \hat{C}^w$$

Furthermore, equation (3), global goods market equilibrium always holds:

$$\hat{C}^w = n(\hat{y} + \hat{p}(h) - \hat{P}) + (1-n)(\hat{y}^* + \hat{p}^*(f) - \hat{P}^*) - \frac{dG^w}{\bar{C}_0^w} \text{ and here it simplifies to:}$$

$$\hat{C}^w = n\hat{y} + (1-n)\hat{y}^* = \hat{y}^w$$

Lastly, the model can now be solved for short-run variables:

Noticed again that $nX + (1-n)X^* = X^w$, where X is equal to $\hat{C}$ or $\hat{y}_t$, can be rewritten in two ways: $X = X^w + (1-n)(X - X^*)$ or $X^* = X^w - n(X - X^*)$. Then:

$$\hat{y} = \hat{y}^w + (1-n)(\hat{y} - \hat{y}^*), \text{ as } \hat{y} - \hat{y}^* = \theta \hat{E}$$

$$\hat{y} = (\varepsilon(1-\beta) + \beta) \hat{M}^w + (1-n)\theta \hat{E}, \text{ recall that}$$

$$\hat{E} = \left(\frac{\varepsilon[2\theta + \bar{r}(\theta+1)]}{\varepsilon[2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2 - 1)} \right) (\hat{M} - \hat{M}^*)$$

$$\begin{aligned} \hat{y} &= \left[n(\varepsilon(1-\beta) + \beta) + (1-n)\theta \left(\frac{\varepsilon[2\theta + \bar{r}(\theta+1)]}{\varepsilon[2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2 - 1)} \right) \right] \hat{M} + \\ &+ \left[(1-n)(\varepsilon(1-\beta) + \beta) - (1-n)\theta \left(\frac{\varepsilon[2\theta + \bar{r}(\theta+1)]}{\varepsilon[2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2 - 1)} \right) \right] \hat{M}^* \end{aligned}$$

$$\text{For foreign: } \hat{y}^* = \left[n(\varepsilon(1-\beta) + \beta) - n\theta \left(\frac{\varepsilon[2\theta + \bar{r}(\theta+1)]}{\varepsilon[2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2 - 1)} \right) \right] \hat{M} +$$

$$+ \left[(1-n)(\varepsilon(1-\beta) + \beta) + n\theta \left(\frac{\varepsilon[2\theta + \bar{r}(\theta+1)]}{\varepsilon[2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2 - 1)} \right) \right] \hat{M}^*$$

For consumption:

$$\hat{C} = \hat{C}^w + (1-n)(\hat{C} - \hat{C}^*)$$

$$\hat{C} = (\varepsilon(1-\beta) + \beta) \hat{M}^w + (1-n)(\hat{C} - \hat{C}^*), \text{ as}$$

$$(\hat{C} - \hat{C}^*) = \left(\frac{\varepsilon \bar{r}(\theta^2 - 1)}{\varepsilon[2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2 - 1)} \right) (\hat{M} - \hat{M}^*) \text{ it follows that:}$$

$$\hat{C} = (\varepsilon(1-\beta) + \beta) \hat{M}^w + (1-n) \left(\frac{\varepsilon \bar{r}(\theta^2 - 1)}{\varepsilon[2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2 - 1)} \right) (\hat{M} - \hat{M}^*)$$

$$\text{For foreign: } \hat{C}^* = (\varepsilon(1-\beta) + \beta) \hat{M}^w - n \left(\frac{\varepsilon \bar{r}(\theta^2 - 1)}{\varepsilon[2\theta + \bar{r}(\theta+1)] + \bar{r}(\theta^2 - 1)} \right) (\hat{M} - \hat{M}^*)$$

Special case $\varepsilon = 1$:

$$\text{Then } 2\theta + \bar{r}(\theta+1) + \bar{r}(\theta-1)(\theta+1) = \theta(2 + (\theta+1)\bar{r})$$

$$\hat{y} = \left[n + (1-n) \left(\frac{2\theta + \bar{r}(\theta+1)}{2 + (\theta+1)\bar{r}} \right) \right] \hat{M} + (1-n) \left[1 - \left(\frac{2\theta + \bar{r}(\theta+1)}{2 + (\theta+1)\bar{r}} \right) \right] \hat{M}^*$$

$$\hat{y} = \left[\left(\frac{2n + n(\theta+1)\bar{r} + (1-n)(2\theta + \bar{r}(\theta+1))}{2 + (\theta+1)\bar{r}} \right) \right] \hat{M} + (1-n) \frac{2 + (\theta+1)\bar{r} - 2\theta - \bar{r}(\theta+1)}{2 + (\theta+1)\bar{r}} \hat{M}^*$$

$$\hat{y} = \frac{2(n + \theta(1-n)) + (\theta+1)\bar{r}}{2 + (\theta+1)\bar{r}} \hat{M} + \frac{2(1-n)(1-\theta)}{2 + (\theta+1)\bar{r}} \hat{M}^*$$

Mathematical Appendix B

B.1. Optimal allocation between home and foreign goods:

Note first that:

$$\text{Plugging in } c_t(z) = \left(\frac{p_t(z)}{P_t^H} \right)^{-\mu} C_t^H \text{ into total expenditure for home goods } \int_0^1 p_t(z) c_t(z) dz$$

gives:

$$\int_0^1 p_t(z) \left(\frac{p_t(z)}{P_t^H} \right)^{-\mu} C_t^H dz = C_t^H \frac{1}{(P_t^H)^{-\mu}} \int_0^1 p_t(z)^{1-\mu} dz = C_t^H \frac{(P_t^H)^{1-\mu}}{(P_t^H)^{-\mu}} = C_t^H P_t^H, \text{ since}$$

$$P_t^H \equiv \left(\int_0^1 p_t(z)^{1-\mu} dz \right)^{1/(1-\mu)}$$

Analogously:

$$\int_0^1 p_t^*(z) c_t^*(z) dz = P_t^* C_t^*$$

$$\int_0^1 P_t^* C_t^*(z) d* = P_t^F C_t^F$$

Let the individual has fixed income equal to I and let him/her gain utility from higher

values of $C_t \equiv \left[(1-\alpha)^{1/\eta} (C_t^H)^{(\eta-1)/\eta} + \alpha^{1/\eta} (C_t^F)^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}$, then the agent

maximizes $C_t \equiv \left[(1-\alpha)^{1/\eta} (C_t^H)^{(\eta-1)/\eta} + \alpha^{1/\eta} (C_t^F)^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}$, subject to his/her budget

constraint $P_t^H C_t^H + P_t^F C_t^F = I$. Take the Lagrangian:

$$L = \left[(1-\alpha)^{1/\eta} (C_t^H)^{(\eta-1)/\eta} + \alpha^{1/\eta} (C_t^F)^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)} - \lambda (P_t^H C_t^H + P_t^F C_t^F - I)$$

The first order conditions are:

$$\frac{\partial L}{\partial C_t^H} = \frac{\eta}{\eta-1} \left[(1-\alpha)^{1/\eta} (C_t^H)^{(\eta-1)/\eta} + \alpha^{1/\eta} (C_t^F)^{(\eta-1)/\eta} \right]^{1/(\eta-1)} \frac{\eta-1}{\eta} (1-\alpha)^{1/\eta} (C_t^H)^{-1/\eta} - \lambda P_t^H = 0$$

$$\frac{\partial L}{\partial C_t^F} = \frac{\eta}{\eta-1} \left[(1-\alpha)^{1/\eta} (C_t^H)^{(\eta-1)/\eta} + \alpha^{1/\eta} (C_t^F)^{(\eta-1)/\eta} \right]^{1/(\eta-1)} \frac{\eta-1}{\eta} \alpha^{1/\eta} (C_t^F)^{-1/\eta} - \lambda P_t^F = 0$$

Rearrange and divide the two FOCs:

$$\left(\frac{1-\alpha}{\alpha} \right)^{1/\eta} \left(\frac{C_t^H}{C_t^F} \right)^{-1/\eta} = \frac{P_t^H}{P_t^F}, \text{ take both sides to the power } \eta$$

$$\left(\frac{C_t^H}{C_t^F} \right)^{-1} = \left(\frac{1-\alpha}{\alpha} \right)^{-1} \left(\frac{P_t^H}{P_t^F} \right)^{\eta} \Rightarrow C_t^H = \frac{1-\alpha}{\alpha} \left(\frac{P_t^H}{P_t^F} \right)^{-\eta} C_t^F$$

Plug in this into the budget constraint gives:

$$P_t^H \frac{1-\alpha}{\alpha} \left(\frac{P_t^H}{P_t^F} \right)^{-\eta} C_t^F + P_t^F C_t^F = I, \text{ divide both sides by } P_t^F, \text{ and use the definition of } P_t^F.$$

$$\frac{\frac{1-\alpha}{\alpha} (P_t^H)^{1-\eta} (P_t^F)^{\eta} C_t^F + P_t^F C_t^F}{\left[(1-\alpha) (P_t^H)^{1-\eta} + \alpha (P_t^F)^{1-\eta} \right]^{1/(1-\eta)}} = \frac{I}{P_t^F}, \text{ multiply both sides by } \alpha (P_t^F)^{-\eta} :$$

$$\frac{(1-\alpha)(P_t^H)^{1-\eta} + \alpha(P_t^F)^{1-\eta}}{\left[(1-\alpha)(P_t^H)^{1-\eta} + \alpha(P_t^F)^{1-\eta}\right]^{1/(1-\eta)}} C_t^F = \alpha(P_t^F)^{-\eta} C_t, \text{ canceling out gives:}$$

$$\left[(1-\alpha)(P_t^H)^{1-\eta} + \alpha(P_t^F)^{1-\eta}\right]^{-\eta/(1-\eta)} C_t^F = \alpha(P_t^F)^{-\eta} C_t.$$

$$\text{Note that } \left[(1-\alpha)(P_t^H)^{1-\eta} + \alpha(P_t^F)^{1-\eta}\right]^{-\eta/(1-\eta)} = \left[\left[(1-\alpha)(P_t^H)^{1-\eta} + \alpha(P_t^F)^{1-\eta}\right]^{1/(1-\eta)}\right]^{-\eta}$$

$$(P_t)^{-\eta} C_t^F = \alpha(P_t^F)^{-\eta} C_t$$

$$C_t^F = \alpha \left(\frac{P_t^F}{P_t} \right)^{-\eta} C_t$$

By plugging in into the budget constraint not for C_t^H but for C_t^F , the analogous result for home goods consumption is obtained:

$$C_t^H = (1-\alpha) \left(\frac{P_t^H}{P_t} \right)^{-\eta} C_t$$

To show that $P_t^H C_t^H + P_t^F C_t^F = P_t C_t$, just plug in the results for C_t^H and C_t^F just obtained in the left-hand side:

$$P_t^H C_t^H + P_t^F C_t^F = P_t^H (1-\alpha) \left(\frac{P_t^H}{P_t} \right)^{-\eta} C_t + P_t^F \alpha \left(\frac{P_t^F}{P_t} \right)^{-\eta} C_t$$

$$P_t^H C_t^H + P_t^F C_t^F = \left[(1-\alpha)(P_t^H)^{1-\eta} + \alpha(P_t^F)^{1-\eta} \right] \frac{1}{P_t^{-\eta}} C_t = P_t^{1-\eta} \frac{1}{P_t^{-\eta}} C_t = P_t C_t$$

B.2. The agent's decision problem:

$$\max U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right]$$

$$\text{subject to } P_t C_t + E_t [Q_{t,t+1} D_{t+1}] = D_t + W_t N_t + T_t$$

The individual decides how much to consume - C_t , how much to work - N_t , and what payoff D_{t+1} , he/she wants to sent aside for the following period.

Expressing explicitly for C_t the budget constraint gives:

$$C_t = \frac{D_t + W_t N_t + T_t - E_t [Q_{t,t+1} D_{t+1}]}{P_t}, \text{ plug this into the utility function:}$$

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(\frac{D_t + W_t N_t + T_t - E_t [Q_{t,t+1} D_{t+1}]}{P_t} \right)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right] \text{ and maximize the unconstrained}$$

problem:

FOCs:

$$\frac{\partial U}{\partial N_t} = \beta^t \frac{W_t}{P_t} C_t^{-\sigma} - \beta^t N_t^{\varphi} = 0 \quad \Rightarrow \quad C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t}$$

$$\frac{\partial U}{\partial D_{t+1}} = -\beta^t C_t^{-\sigma} Q_{t,t+1}(h) \frac{1}{P_t} + \beta^{t+1} \frac{1}{P_{t+1}(h)} C_{t+1}^{-\sigma}(h) = 0, \text{ where } h=1, 2, \dots, H$$

Note that the second first-order condition must hold for all possible states of nature (the notation here is a bit sloppy.). And that $P_{t+1}(h)$ and $C_{t+1}(h)$ are conditional on the state of

nature. Written differently, using $Q_{t,t+1} = \frac{V_{t,t+1}(h)}{\xi_{t,t+1}(h)}$, this is:

$$\frac{V_{t,t+1}(h)}{P_t} C_t^{-\sigma} = \xi_{t,t+1}(h) \beta C_{t+1}^{-\sigma}(h) \frac{1}{P_{t+1}(h)} \quad \text{The left-hand side gives the utility loss from}$$

purchasing the Arrow-Debreu security due to forgone consumption in period t and the right-hand side gives the expected utility gain from additional consumption due to the revenue from the security in period $t+1$.

Rearranging the second first-order condition gives:

$$Q_{t,t+1}(h) = \beta \frac{P_t}{P_{t+1}(h)} \frac{C_{t+1}^{-\sigma}(h)}{C_t^{-\sigma}}, \text{ take conditional expectations on both sides:}$$

$$E_t [Q_{t,t+1}] = E_t \left[\beta \frac{P_t}{P_{t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \right]$$

$$\beta \frac{1}{E_t [Q_{t,t+1}]} E_t \left[\frac{P_t}{P_{t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \right] = 1$$

B.3. Log-linearizing (approximating) the FOCs:

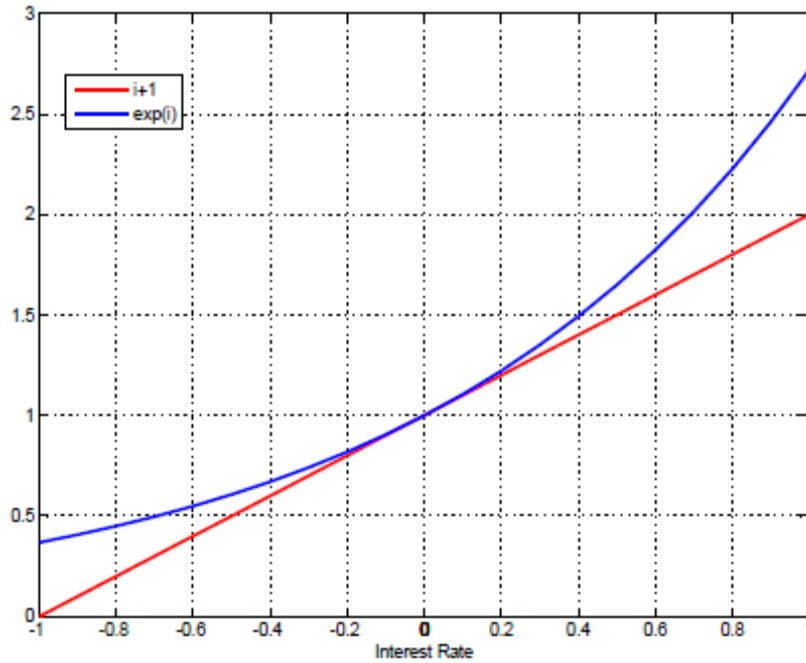
$C_t^\sigma N_t^\varphi = \frac{W_t}{P_t}$, take natural logarithm of both sides:

$$\ln(C_t^\sigma N_t^\varphi) = \ln \frac{W_t}{P_t} \Rightarrow \sigma \ln C_t + \varphi \ln N_t = \ln W_t - \ln P_t$$

$$\sigma c_t + \varphi n_t = w_t - p_t$$

$$\beta(1+i_t)E_t \left[\frac{P_t}{P_{t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \right] = 1$$

Notice that for small i_t , $(1+i_t)$ can be approximated by e^{i_t} . Formally, the first-order Taylor approximation of e^{i_t} around zero is given by $e^{i_t} \approx e^0 + e^0(i_t - 0) = 1 + i_t$, but one can best see this from the graph:



For $i_t \in [-0.2, 0.2]$ there is hardly a difference, and such high interest rates 0.2, i.e. 20%, (especially riskless) are not common. Then:

$$\beta(1+i_t)E_t \left[\frac{P_t}{P_{t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \right] \approx \beta e^{i_t} E_t \left[\frac{P_t}{P_{t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \right]$$

$\beta e^{i_t} E_t \left[\frac{P_t}{P_{t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \right] = 1$, take natural logarithm of both sides and interchange

expectations and logarithms:

$$\ln \beta + i_t + \ln P_t - E_t [\ln P_{t+1}] - \sigma E_t [\ln C_{t+1}] + \sigma \ln C_t = 0$$

$$\sigma c_t = \sigma E_t [c_{t+1}] + E_t [p_{t+1}] - p_t - \ln \beta - i_t$$

$$c_t = E_t [c_{t+1}] - \frac{1}{\sigma} [i_t - E_t (\pi_{t+1}) - \rho], \text{ where } \rho \equiv -\ln \beta$$

B.4. Some other Log-linearizations:

Before log-linearizing, note that, as shown in the previous appendix for the interest rate, $e^X \approx 1 + X$, when X is small (X can here stand for any variable). Therefore, after taking natural logarithms of both sides: $X \approx \ln(1 + X)$. Then the sequence following of equalities and approximated equalities holds:

$$\hat{X}_t = \frac{dX_t}{X} \approx \frac{X_t - X}{X} \approx \ln \left(1 + \frac{X_t - X}{X} \right) = \ln \frac{X_t}{X} = \ln X_t - \ln X$$

where $\hat{X}_t$ is, as defined before, percentage change from some steady state X . In this model, it is worked with the logarithms of the variables, not with the percentage changes, as in Obstfeld and Rogoff's model. Therefore, when $\frac{dX_t}{X}$ is reached, it would be interchanged with $\ln X_t - \ln X$, not with $\hat{X}_t$.

Terms of trade:

$$S_t = \frac{P_t^F}{P_t^H} = \left[\int_0^1 (S_t^*)^{1-\gamma} d^* \right]^{1/(1-\gamma)}$$

the first equality can be linearized by only taking logarithms of both sides:

$$\ln S_t = \ln \frac{P_t^F}{P_t^H} \Rightarrow s_t = p_t^F - p_t^H$$

take natural logarithms of both sides for the second equality as well:

$$\ln S_t = \ln \left[\int_0^1 (S_t^*)^{1-\gamma} d^* \right]^{1/(1-\gamma)}$$

$$\ln S_t = \frac{1}{1-\gamma} \ln \left[\int_0^1 (S_t^*)^{1-\gamma} d^* \right], \text{ now totally differentiate:}$$

$$\frac{dS_t}{S_0} = \frac{1}{1-\gamma} \int_0^1 \left(\frac{(1-\gamma)(S_0^*)^{-\gamma}}{\int_0^1 (S_0^*)^{1-\gamma} d^*} dS_t^* \right) d^*. \quad \text{It might, at first, seem strange that the total}$$

differentiation leads to expression like this one, but this is exactly the continuous time counterpart of the operation that was made many times in this paper in discrete time. The differentiation on the right-hand side is done with respect to S_t^* for all * . If instead the integral in the problem there would have been a sum, then the expression would have

looked like: $\left(\frac{\partial F}{\partial S_t^{*1}} \bigg|_{t=0} \right) dS_t^{*1} + \left(\frac{\partial F}{\partial S_t^{*2}} \bigg|_{t=0} \right) dS_t^{*2} + \dots$, a sum over all the differentials for the

different * . In continuous time this summation is done by integrating, as there is a unit interval of differentials. Hence, the expression:

$$\int_0^1 \left(\frac{(1-\gamma)(S_0^*)^{-\gamma}}{\int_0^1 (S_0^*)^{1-\gamma} d^*} dS_t^* \right) d^*, \text{ is nothing but the sum over all the different differentials.}$$

Now, note that $S_0^* = S_0 = 1$. Then:

$$\frac{dS_t}{S_0} = \int_0^1 dS_t^* d^*$$

$$\ln S_t - \ln S_0 = \int_0^1 (\ln S_t^* - \ln S_0) d^*$$

$$s_t = \int_0^1 s_t^* d^*$$

Consumer Price Index:

$$P_t \equiv \left[(1-\alpha)(P_t^H)^{1-\eta} + \alpha(P_t^F)^{1-\eta} \right]^{1/(1-\eta)}$$

$$\ln P_t = \frac{1}{1-\eta} \ln \left[(1-\alpha)(P_t^H)^{1-\eta} + \alpha(P_t^F)^{1-\eta} \right]$$

$$\frac{dP_t}{P_0} = \frac{(1-\alpha)(P_0^H)^{-\eta}}{(1-\alpha)(P_0^H)^{1-\eta} + \alpha(P_0^F)^{1-\eta}} dP_t^H + \frac{\alpha(P_0^F)^{-\eta}}{(1-\alpha)(P_0^H)^{1-\eta} + \alpha(P_0^F)^{1-\eta}} dP_t^F,$$

note that $P_0^H = P_0^F = P_0$

$$\frac{dP_t}{P_0} = (1-\alpha) \frac{(P_0)^{-\eta}}{(P_0)^{-\eta}} \frac{dP_t^H}{P_0} + \alpha \frac{(P_0)^{-\eta}}{(P_0)^{-\eta}} \frac{dP_t^F}{P_0}$$

$$\ln P_t - \ln P_0 = (1-\alpha)(\ln P_t^H - \ln P_0) + \alpha(\ln P_t^F - \ln P_0)$$

$$\ln P_t = (1-\alpha) \ln P_t^H + \alpha \ln P_t^F$$

$$p_t = (1-\alpha)p_t^H + \alpha p_t^F$$

Price index for foreign goods:

$$P_t^F = \left(\int_0^1 (P_t^*)^{1-\gamma} d* \right)^{1/(1-\gamma)}, \text{ plugging in } P_t^* = E_t^* P_{t,*}^* \text{ gives:}$$

$$P_t^F = \left(\int_0^1 (E_t^* P_{t,*}^*)^{1-\gamma} d* \right)^{1/(1-\gamma)}, \text{ plugging in } P_{t,*}^* \equiv \left(\int_0^1 (p_{t,*}^*(z))^{1-\mu} dz \right)^{1/(1-\mu)} \text{ gives:}$$

$$P_t^F = \left[\int_0^1 \left[E_t^* \left(\int_0^1 (p_{t,*}^*(z))^{1-\mu} dz \right)^{1/(1-\mu)} \right]^{1-\gamma} d* \right]^{1/(1-\gamma)}, \text{ take the natural logarithm of both sides:}$$

$$\ln P_t^F = \frac{1}{1-\gamma} \ln \left[\int_0^1 \left[E_t^* \left(\int_0^1 (p_{t,*}^*(z))^{1-\mu} dz \right)^{1/(1-\mu)} \right]^{1-\gamma} d* \right], \text{ totally differentiate:}$$

$$\begin{aligned} \frac{dP_t^F}{P_0^F} = & \int_0^1 \frac{\left(\int_0^1 (p_{0,*}^*(z))^{1-\mu} dz \right)^{(1-\gamma)/(1-\mu)} (E_0^*)^{-\gamma} dE_t^*}{\left[\int_0^1 \left[E_0^* \left(\int_0^1 (p_{0,*}^*(z))^{1-\mu} dz \right)^{1/(1-\mu)} \right]^{1-\gamma} d* \right]} d* + \\ & + \int_0^1 \frac{(E_0^*)^{1-\gamma} \left[\left(\int_0^1 (p_{0,*}^*(z))^{1-\mu} dz \right)^{1/(1-\mu)} \right]^{-\gamma} \frac{1}{1-\mu} \left(\int_0^1 (p_{0,*}^*(z))^{1-\mu} dz \right)^{\mu/(1-\mu)} \left[\int_0^1 (1-\mu) (p_{0,*}^*(z))^{-\mu} dp_{t,*}^*(z) dz \right]}{\left[\int_0^1 \left[E_0^* \left(\int_0^1 (p_{0,*}^*(z))^{1-\mu} dz \right)^{1/(1-\mu)} \right]^{1-\gamma} d* \right]} d* \end{aligned}$$

Note that in the case for $p_{t,*}^*(z)$ the summation as explain before is done twice, over z

and over $*$:

As in the steady state all the countries are the same, it holds that $E_0^* = 1$, and further more

$p_{0,*}^*(z)$ is the same for all $z, * \in [0,1]$ (meaning it can be factor out the integrals). Then:

$$\frac{dP_t^F}{P_0^F} = \int_0^1 \frac{(p_{0,*}^*(z))^{1-\gamma}}{(p_{0,*}^*(z))^{1-\gamma}} d* + \int_0^1 \frac{(p_{0,*}^*(z))^{-\gamma} (p_{0,*}^*(z))^\mu (p_{0,*}^*(z))^{-\mu} \left[\int_0^1 (dp_{t,*}^*(z)) dz \right]}{(p_{0,*}^*(z))^{1-\gamma}} d*$$

$$\frac{dP_t^F}{P_0^F} = \int_0^1 (dE_t^*) d* + \int_0^1 \left[\int_0^1 \left(\frac{dp_{t,*}^*(z)}{p_{0,*}^*(z)} \right) dz \right] d*, \text{ remember } E_0^* = 1, \text{ hence } dE_t^* = \frac{dE_t^*}{E_0^*}$$

$$\ln P_t^F - \ln P_0^F = \int_0^1 (\ln E_t^* - \ln E_0^*) d* + \int_0^1 \left[\int_0^1 (\ln p_{t,*}^*(z) - \ln p_{0,*}^*(z)) dz \right] d*$$

$$\ln P_t^F - \ln P_0^F = \int_0^1 (\ln E_t^*) d* + \int_0^1 \left[\int_0^1 (\ln p_{t,*}^*(z)) dz \right] d* - \ln E_0^* P_{0,*}^*, \text{ since } E_0^* = 1 \text{ and}$$

$$P_{0,*}^* = \left(\int_0^1 (p_{0,*}^*(z))^{1-\mu} dz \right)^{1/(1-\mu)} = p_{0,*}^*(z). \text{ Furthermore, } P_t^* = E_t^* P_{t,*}^* \text{ always, and since all the}$$

$$P_0^* \text{ are the same } P_0^F = \left(\int_0^1 (P_0^*)^{1-\gamma} d* \right)^{1/(1-\gamma)} = P_0^*. \text{ Then:}$$

$$p_t^F = \int_0^1 (\ln E_t^*) d* + \int_0^1 \left[\int_0^1 (\ln p_{t,*}^*(z)) dz \right] d*, \text{ where } p_t^F \equiv \ln P_t^F$$

$$p_t^F = \int_0^1 e_t^* d* + \int_0^1 (p_{t,*}^*) d*, \text{ where } e_t^* \equiv \ln E_t^* \text{ and } p_{t,*}^* \equiv \int_0^1 (\ln p_{t,*}^*(z)) dz$$

$$p_t^F = e_t + p_t^w, \text{ where } e_t \equiv \int_0^1 e_t^* d* \text{ and } p_t^w \equiv \int_0^1 (p_{t,*}^*) d*$$

B.5. International asset markets integration:

$$Q_{t,t+1} = \beta \frac{P_{t,*}}{P_{t+1,*}} \frac{C_{t+1,*}^{-\sigma}}{C_{t,*}^{-\sigma}} \frac{E_{t,*}}{E_{t+1,*}} \text{ is the optimality condition for the foreign country.}$$

$$Q_{t,t+1} = \beta \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \text{ is the optimality condition for Home.}$$

$$\beta \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} = \beta \frac{P_{t,*}}{P_{t+1,*}} \frac{C_{t+1,*}^{-\sigma}}{C_{t,*}^{-\sigma}} \frac{E_{t,*}}{E_{t+1,*}}, \text{ the bilateral real exchange rate is defined as } B_t^* \equiv \frac{E_t^* P_{t,*}}{P_t}:$$

$$\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} = \frac{C_{t+1,*}^{-\sigma} B_t^*}{C_{t,*}^{-\sigma} B_{t+1}^*} \Rightarrow \frac{C_t}{C_{t+1}} = \frac{C_{t,*}}{C_{t+1,*}} \frac{(B_t^*)^{1/\sigma}}{(B_{t+1}^*)^{1/\sigma}}$$

$$C_{t+1} = \frac{C_t}{C_{t,*} (B_t^*)^{1/\sigma}} C_{t+1,*} (B_{t+1}^*)^{1/\sigma}$$

Shift the equation one period in the past.

$$C_t = \frac{C_{t-1}}{C_{t-1,*} (B_{t-1}^*)^{1/\sigma}} C_{t,*} (B_t^*)^{1/\sigma}$$

Combining the two results gives:

$$C_{t+1} = \frac{\frac{C_{t-1}}{C_{t-1,*} (B_{t-1}^*)^{1/\sigma}} C_{t,*} (B_t^*)^{1/\sigma}}{C_{t,*} (B_t^*)^{1/\sigma}} C_{t+1,*} (B_{t+1}^*)^{1/\sigma}$$

$$C_{t+1} = \frac{C_{t-1}}{C_{t-1,*} (B_{t-1}^*)^{1/\sigma}} C_{t+1,*} (B_{t+1}^*)^{1/\sigma}, \text{ compare to } C_{t+1} = \frac{C_t}{C_{t,*} (B_t^*)^{1/\sigma}} C_{t+1,*} (B_{t+1}^*)^{1/\sigma}$$

One can go on with the shifting until the initial states are reached.

$$C_t = \frac{C_0}{C_{0,*} (B_0^*)^{1/\sigma}} C_{t,*} (B_t^*)^{1/\sigma}. \text{ Note that here the variables with the subscript 0 hold for any}$$

initial conditions, not only for the steady state. However, without loss of generality one can assume identical initial condition – no net foreign assets and identical ex-ante

environment, i.e. the steady state conditions. Let $\vartheta^* = \frac{C_0}{C_{0,*} (B_0^*)^{1/\sigma}}$. Hence ϑ^* is a

constant depending on the initial conditions and:

$$C_t = \vartheta^* C_{t,*} (B_t^*)^{1/\sigma}$$

With identical initial condition it must be that $C_0 = C_{0,*}$ and $B_0^* = 1$, then $\vartheta^* = 1$ (as in the steady state).

B.6. Log-linearizing the index of aggregate domestic output:

$$Y_t \equiv \left(\int_0^1 y_t(z)^{(\mu-1)/\mu} dz \right)^{\mu/(\mu-1)}, \text{ take natural logarithm of both sides:}$$

$\ln Y_t = \frac{\mu}{\mu-1} \ln \int_0^1 y_t(z)^{(\mu-1)/\mu} dz$, totally differentiate:

$$\frac{dY_t}{Y_0} = \frac{\int_0^1 y_0(z)^{-1/\mu} dy_t(z)}{\int_0^1 y_0(z)^{(\mu-1)/\mu} dz} dz$$

$$dY_t = \frac{Y_0 y_0(z)^{-1/\mu}}{y_0(z)^{(\mu-1)/\mu}} \int_0^1 dy_t(z) dz$$

$$\ln Y_t - \ln Y_0 = \int_0^1 (\ln y_t(z) - \ln y_0(z)) dz$$

$$y_t = \int_0^1 \ln y_t(z) dz , \text{ use the fact that } y_t(z) = A_t N_t(z)$$

$$y_t = \int_0^1 \ln A_t N_t(z) dz$$

$$y_t = a_t + \int_0^1 \ln N_t(z) dz , \text{ now log-linearize } N_t = \int_0^1 N_t(z) dz \text{ around the steady state:}$$

$$\ln N_t = \ln \int_0^1 N_t(z) dz , \text{ totally differentiate:}$$

$$\frac{dN_t}{N_0} = \frac{\int_0^1 \frac{dN_t(z)}{N_0(z)} dz}{\int_0^1 N_0(z) dz} , \text{ since } N_0(z) \text{ is the same for all } z \text{ producers in the symmetric}$$

equilibrium:

$$\frac{dN_t}{N_0} = \frac{\int_0^1 \frac{dN_t(z)}{N_0(z)} dz}{\int_0^1 N_0(z) dz} \Rightarrow \ln N_t - \ln N_0 = \int_0^1 (\ln N_t(z) - \ln N_0(z)) dz$$

$$n_t = \int_0^1 (\ln N_t(z)) dz , \text{ plugging in this into } y_t = a_t + \int_0^1 \ln N_t(z) dz \text{ gives:}$$

$$y_t = a_t + n_t$$

B.7. Optimal Pricing:

Let $\tilde{P}_t^H(z)$ be the new price set by the firm, producer of z , which is adjusting its price in period t . Note that $p_{t+k}^H(z) = \tilde{p}_t^H(z)$ with probability ϖ^k for $k = 0, 1, 2, \dots$. As all firms that are able to re-optimize in period t will set the same price, the indexes z are dropped.

Then the firm sets its price in period t in such a way that the current value of future profits is maximized, conditional on this price being still effective, subject to supply equals demand. That is:

$$\max \sum_{k=0}^{\infty} \varpi^k E_t \left[Q_{t,t+k} Y_{t+k} \left(\tilde{P}_t^H - MC_{t+k}^n \right) \right]$$

subject to $Y_{t+k} = \left(\frac{\tilde{P}_t^H}{P_{t+k}^H} \right)^{-\mu} \left(C_{t+k}^H + \int_0^1 C_{t+k,*}^H d* \right)$. Note that the demand function previously

derived was demand for home goods from Home. The term $\left(\frac{\tilde{P}_t^H}{P_{t+k}^H} \right)^{-\mu} \int_0^1 C_{t+k,*}^H d*$ captures the demand for home goods from foreign countries.

$MC_{t+k}^n \equiv \frac{(1-\tau)W_t}{A_t}$ is the nominal marginal cost.

Plugging in the budget constraint into the objective function gives:

$$\begin{aligned} & \max \sum_{k=0}^{\infty} \varpi^k E_t \left[Q_{t,t+k} \left(\frac{\tilde{P}_t^H}{P_{t+k}^H} \right)^{-\mu} \left(C_{t+k}^H + \int_0^1 C_{t+k,*}^H d* \right) \left(\tilde{P}_t^H - MC_{t+k}^n \right) \right] \\ & \max \sum_{k=0}^{\infty} \varpi^k E_t \left[Q_{t,t+k} \left(\frac{1}{P_{t+k}^H} \right)^{-\mu} \left(C_{t+k}^H + \int_0^1 C_{t+k,*}^H d* \right) \left(\left(\tilde{P}_t^H \right)^{1-\mu} - \left(\tilde{P}_t^H \right)^{-\mu} MC_{t+k}^n \right) \right] \end{aligned}$$

Then the FOC is:

$$\begin{aligned} & \sum_{k=0}^{\infty} \varpi^k E_t \left\{ \left[Q_{t,t+k} \left(\frac{1}{P_{t+k}^H} \right)^{-\mu} \left(C_{t+k}^H + \int_0^1 C_{t+k,*}^H d* \right) \right] \left[(1-\mu) \left(\tilde{P}_t^H \right)^{-\mu} + \mu \left(\tilde{P}_t^H \right)^{-\mu-1} MC_{t+k}^n \right] \right\} = 0 \\ & \sum_{k=0}^{\infty} \varpi^k E_t \left\{ \left[Q_{t,t+k} \left(\frac{\tilde{P}_t^H}{P_{t+k}^H} \right)^{-\mu} \left(C_{t+k}^H + \int_0^1 C_{t+k,*}^H d* \right) \right] \left[(1-\mu) + \mu \left(\tilde{P}_t^H \right)^{-1} MC_{t+k}^n \right] \right\} = 0, \quad \text{multiply} \end{aligned}$$

both sides by $-\tilde{P}_t^H$:

$$\sum_{k=0}^{\infty} \varpi^k E_t \left\{ \left[Q_{t,t+k} \left(\frac{\tilde{P}_t^H}{P_{t+k}^H} \right)^{-\mu} \left(C_{t+k}^H + \int_0^1 C_{t+k,*}^H d* \right) \right] \left[(\mu-1) \tilde{P}_t^H - \mu MC_{t+k}^n \right] \right\} = 0, \quad \text{now divide}$$

both sides by $\mu-1$:

$$\sum_{k=0}^{\infty} \varpi^k E_t \left\{ \left[Q_{t,t+k} \left(\frac{\tilde{P}_t^H}{P_{t+k}^H} \right)^{-\mu} \left(C_{t+k}^H + \int_0^1 C_{t+k,*}^H d* \right) \right] \left[\tilde{P}_t^H - \frac{\mu}{\mu-1} MC_{t+k}^n \right] \right\} = 0, \text{ plug in back } Y_{t+k}$$

from the budget constraint:

$$\sum_{k=0}^{\infty} \varpi^k E_t \left\{ \left[Q_{t,t+k} Y_{t+k} \right] \left[\tilde{P}_t^H - \frac{\mu}{\mu-1} MC_{t+k}^n \right] \right\} = 0$$

Recall that $Q_{t,t+1} = \beta \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}$. One can shift this expression by k periods to obtain:

$$Q_{t,t+k} = \beta^k \frac{P_t}{P_{t+k}} \frac{C_{t+k}^{-\sigma}}{C_t^{-\sigma}}. \text{ Plug this into the budget constraint:}$$

$$\sum_{k=0}^{\infty} \varpi^k E_t \left\{ \beta^k \frac{P_t}{P_{t+k}} \frac{C_{t+k}^{-\sigma}}{C_t^{-\sigma}} Y_{t+k} \left[\tilde{P}_t^H - \frac{\mu}{\mu-1} MC_{t+k}^n \right] \right\} = 0, \text{ multiply both sides by } \frac{C_t^{-\sigma}}{P_t}$$

$$\sum_{k=0}^{\infty} (\varpi\beta)^k E_t \left\{ (P_{t+k})^{-1} C_{t+k}^{-\sigma} Y_{t+k} \left[\tilde{P}_t^H - \frac{\mu}{\mu-1} MC_{t+k}^n \right] \right\} = 0$$

Or alternatively one can write this equation as:

$$\sum_{k=0}^{\infty} (\varpi\beta)^k E_t \left\{ C_{t+k}^{-\sigma} Y_{t+k} \frac{P_{t-1}^H}{P_{t+k}^H} \left[\frac{\tilde{P}_t^H}{P_{t-1}^H} - \frac{\mu}{\mu-1} \Pi_{t-1,t+k}^H MC_{t+k}^n \right] \right\} = 0$$

where $\Pi_{t-1,t+k}^H \equiv \frac{P_{t+k}^H}{P_{t-1}^H}$ and $MC_{t+k} \equiv \frac{MC_{t+k}^n}{P_{t+k}^H}$, rearrange:

$$\sum_{k=0}^{\infty} (\varpi\beta)^k E_t \left\{ C_{t+k}^{-\sigma} Y_{t+k} \frac{P_{t-1}^H}{P_{t+k}^H} \frac{\tilde{P}_t^H}{P_{t-1}^H} \right\} - \sum_{k=0}^{\infty} (\varpi\beta)^k E_t \left\{ C_{t+k}^{-\sigma} Y_{t+k} \frac{P_{t-1}^H}{P_{t+k}^H} \frac{\mu}{\mu-1} \Pi_{t-1,t+k}^H MC_{t+k} \right\} = 0,$$

rearrange again and take the natural logarithm of both sides:

$$\ln \sum_{k=0}^{\infty} (\varpi\beta)^k E_t \left\{ C_{t+k}^{-\sigma} Y_{t+k} \frac{\tilde{P}_t^H}{P_{t+k}^H} \right\} = \ln \sum_{k=0}^{\infty} (\varpi\beta)^k E_t \left\{ C_{t+k}^{-\sigma} Y_{t+k} \frac{P_{t-1}^H}{P_{t+k}^H} \frac{\mu}{\mu-1} \Pi_{t-1,t+k}^H MC_{t+k} \right\}$$

Now this expression can be linearized around a zero inflation steady state. Note that in

zero inflation steady state: $\Pi_{t-1,t+k}^H = 1$, $\frac{\tilde{P}_t^H}{P_{t-1}^H} = 1$, $MC_{t+k} = MC_t = MC$, $Y_{t+k} = Y_t = Y$,

$C_{t+k} = C_t = C$. It is as in the subscript 0 steady state, but with additional assumption that there are no changes in prices.

Linearizing around the steady state

$$\ln \sum_{k=0}^{\infty} (\varpi\beta)^k E_t \left\{ C_{t+k}^{-\sigma} Y_{t+k} \frac{\tilde{P}_t^H}{P_{t+k}} \right\} = \ln \sum_{k=0}^{\infty} (\varpi\beta)^k E_t \left\{ C_{t+k}^{-\sigma} Y_{t+k} \frac{P_{t-1}^H}{P_{t+k}} \frac{\mu}{\mu-1} \Pi_{t-1,t+k}^H MC_{t+k} \right\} \quad \text{is an}$$

extremely long and tedious process. In this appendix, the total differentiation is done with respect to only two variables, but they represent the main cases and all the others are equivalent to them.

The easiest case is with respect to variables that do not depend on k , such as $\tilde{P}_t^H$ and P_{t-1}^H . If the total differentiation is done with respect to $\tilde{P}_t^H$, it yields:

$$\left[\frac{\sum_{k=0}^{\infty} (\varpi\beta)^k E_0 \left\{ C_0^{-\sigma} Y_0 \frac{1}{P_0} \right\}}{\sum_{k=0}^{\infty} (\varpi\beta)^k E_0 \left\{ C_0^{-\sigma} Y_0 \frac{P_0^H}{P_0} \frac{\tilde{P}_0^H}{P_0^H} \right\}} \right] d\tilde{P}_t^H = \frac{d\tilde{P}_t^H}{\tilde{P}_0^H} = \ln \tilde{P}_t^H - \ln \tilde{P}_0^H$$

With respect to P_{t-1}^H it will yield $\ln P_{t-1}^H - \ln P_0^H$, the two second terms will cancel out.

The second case is for variables that depend on k , such as MC_{t+k} , $\Pi_{t-1,t+k}^H$, Y_{t+k} , C_{t+k} and the prices. If the total differentiation is done with respect to MC_{t+k} , it yields:

$$\sum_{k=0}^{\infty} E_t \frac{(\varpi\beta)^k E_t \left\{ C_0^{-\sigma} Y_0 \frac{P_0^H}{P_0} \frac{\mu}{\mu-1} \Pi_{0,0}^H \right\}}{\sum_{k=0}^{\infty} (\varpi\beta)^k E_t \left\{ C_0^{-\sigma} Y_0 \frac{P_0^H}{P_0} \frac{\mu}{\mu-1} \Pi_{0,0}^H MC_0 \right\}} dMC_{t+k}, \text{ the sum can be explained similarly}$$

to the explanation of why there is an integral in appendix B.4. Note that most of the variables cancel out, leaving:

$$\begin{aligned} \sum_{k=0}^{\infty} E_t \frac{(\varpi\beta)^k dMC_{t+k} MC_0^{-1}}{\sum_{k=0}^{\infty} (\varpi\beta)^k} &= \sum_{k=0}^{\infty} \frac{(\varpi\beta)^k E_t (dMC_{t+k} MC_0^{-1})}{\frac{1}{1-\varpi\beta}} = \\ &= (1-\varpi\beta) \sum_{k=0}^{\infty} (\varpi\beta)^k E_t (\ln MC_{t+k} - \ln MC_0). \end{aligned}$$

Same is done with respect to $\Pi_{t-1,t+k}^H$ and the prices. One can see that the terms obtained for Y_{t+k} and C_{t+k} cancel out.

The end result of this procedure is:

$$\tilde{p}_t^H = p_{t-1}^H + \sum_{k=0}^{\infty} (\varpi\beta)^k E_t(\pi_{t+k}^H) + (1-\varpi\beta) \sum_{k=0}^{\infty} (\varpi\beta)^k E_t(mc_{t+k} - mc_0)$$

This could be written in a more compact way. Just subtract the expectation in period t for the new price in period $t+1$ times $\varpi\beta$, i.e.:

Subtract

$$\varpi\beta E_t(\tilde{p}_{t+1}^H) = \varpi\beta p_t^H + \varpi\beta \sum_{k=0}^{\infty} (\varpi\beta)^k E_{t+1}(\pi_{t+1+k}^H) + \varpi\beta(1-\varpi\beta) \sum_{k=0}^{\infty} (\varpi\beta)^k E_{t+1}(mc_{t+k+1} - mc_0)$$

from:

$$\tilde{p}_t^H = p_{t-1}^H + \sum_{k=0}^{\infty} (\varpi\beta)^k E_t(\pi_{t+k}^H) + (1-\varpi\beta) \sum_{k=0}^{\infty} (\varpi\beta)^k E_t(mc_{t+k} - mc_0). \text{ Then:}$$

$$\tilde{p}_t^H - \varpi\beta E_t(\tilde{p}_{t+1}^H) = p_{t-1}^H - \varpi\beta p_t^H + E_t(\pi_t^H) + (1-\varpi\beta)(mc_t - mc_0)$$

$$\tilde{p}_t^H - p_{t-1}^H = \varpi\beta E_t(\tilde{p}_{t+1}^H) - \varpi\beta p_t^H + \pi_t^H + (1-\varpi\beta)(mc_t - mc_0)$$

$$\tilde{p}_t^H - p_{t-1}^H = \varpi\beta E_t(\tilde{p}_{t+1}^H - p_t^H) + \pi_t^H + (1-\varpi\beta)(mc_t - mc_0)$$

To proceed further, note that under the zero inflation steady state, as the firms never re-optimize (change prices), hence for how long prices remain fixed is irrelevant, the firm optimizes in every period (actually, all periods are the same) the problem (notice that in order for them not to re-optimize, they must not have an incentive to change prices, hence the solution must be identical to the one under flexible prices – that is also the reason why the indexes t are kept, since it is the general case when prices are flexible):

$$\max Y_t(p_t^H(z) - MC_t^n)$$

$$\text{subject to } Y_t = \left(\frac{p_t^H(z)}{P_t^H} \right)^{-\mu} \left(C_t^H + \int_0^1 C_{t,*}^H d* \right)$$

here the good index z is briefly introduced again, because there might be confusion otherwise. Once firms optimize, they will all set the same price, and basically $p_t^H(z) = P_t^H$ will hold. However, firms do not know that and they think they do not have an effect on the overall price index, hence they maximize as P_t^H is given.

Plugging the budget constraint into the objective function gives:

$$\max \left(\frac{p_t^H(z)}{P_t^H} \right)^{-\mu} \left(C_t^H + \int_0^1 C_{t,*}^H d* \right) (p_t^H(z) - MC_t^n)$$

FOC:

$$\left(\frac{1}{P_t^H} \right)^{-\mu} \left(C_t^H + \int_0^1 C_{t,*}^H d* \right) \left((1-\mu)(p_t^H(z))^{-\mu} + \mu(p_t^H(z))^{-\mu-1} MC_t^n \right) = 0, \text{ multiplying by}$$

$$\frac{-(p_t^H(z))^\mu}{\left(\frac{1}{P_t^H} \right)^{-\mu} \left(C_t^H + \int_0^1 C_{t,*}^H d* \right)} \text{ both sides gives:}$$

$$(\mu-1) - \mu(p_t^H(z))^{-1} MC_t^n$$

$$\frac{MC_t^n}{p_t^H(z)} = MC_t = \frac{\mu-1}{\mu} = \left(\frac{\mu}{\mu-1} \right)^{-1}, \text{ again note that all producers would set the same price,}$$

hence one can use $p_t^H(z)$ as deflator. Take natural logarithms of both sides:

$$mc_t^n - p_t^H = mc_t = -\ln \left(\frac{\mu}{\mu-1} \right) = -\hbar, \text{ which is the value of log real marginal cost under}$$

the zero inflation steady state, i.e. mc_0 . Note that because under this steady state the stickiness of the prices is irrelevant, this is actually the log real marginal cost that would be obtained under flexible prices.

$$\text{Plugging the last result into: } \check{p}_t^H - p_{t-1}^H = \varpi\beta E_t(\check{p}_{t+1}^H - p_t^H) + \pi_t^H + (1-\varpi\beta)(mc_t - mc_0)$$

$$\text{gives: } \check{p}_t^H - p_{t-1}^H = \varpi\beta E_t(\check{p}_{t+1}^H - p_t^H) + \pi_t^H + (1-\varpi\beta)(mc_t^n - p_t^H + \hbar)$$

$$\check{p}_t^H = p_{t-1}^H - \varpi\beta p_t^H + \varpi\beta E_t(\check{p}_{t+1}^H) + p_t^H - p_{t-1}^H - (1-\varpi\beta)p_t^H + (1-\varpi\beta)(mc_t^n + \hbar)$$

$$\check{p}_t^H = \varpi\beta E_t(\check{p}_{t+1}^H) + (1-\varpi\beta)(mc_t^n + \hbar), \text{ shift the same with one period in the future:}$$

$$\check{p}_{t+1}^H = \varpi\beta E_{t+1}(\check{p}_{t+2}^H) + (1-\varpi\beta)(mc_{t+1}^n + \hbar), \text{ plug into the first one.}$$

$$\check{p}_t^H = \varpi\beta E_t(\varpi\beta E_t(\check{p}_{t+2}^H) + (1-\varpi\beta)(mc_{t+1}^n + \hbar)) + (1-\varpi\beta)(mc_t^n + \hbar)$$

$$\check{p}_t^H = (\varpi\beta)^2 E_t(\check{p}_{t+2}^H) + \varpi\beta(1-\varpi\beta)\hbar + \hbar + \varpi\beta(1-\varpi\beta)(mc_{t+1}^n) + (1-\varpi\beta)mc_t^n$$

If one continues with the same procedure, the end result looks like:

$$\check{p}_t^H = \sum_{k=0}^{\infty} (\varpi\beta)^k (1-\varpi\beta)\hbar + \sum_{k=0}^{\infty} (\varpi\beta)^k (1-\varpi\beta)(mc_{t+k}^n)$$

$$\check{p}_t^H = \frac{1}{1-\varpi\beta} (1-\varpi\beta)\hbar + (1-\varpi\beta) \sum_{k=0}^{\infty} (\varpi\beta)^k (mc_{t+k}^n)$$

$$\check{p}_t^H = \hbar + (1-\varpi\beta) \sum_{k=0}^{\infty} (\varpi\beta)^k (mc_{t+k}^n), \text{ which is the equation in the text.}$$

For further uses it is convenient to derive one more equation

Notice that the price index for home goods can be defined as:

$$P_t^H \equiv \left[\varpi (P_{t-1}^H)^{1-\mu} + (1-\varpi) (\check{P}_t^H)^{1-\mu} \right]^{1/(1-\mu)}, \text{ which is quite straightforward to interpret as}$$

$(1-\varpi)$ of firms adopt new prices and the remaining keep their old prices.

Log-linearizing this around the zero inflation steady state:

$$\ln P_t^H = \frac{1}{1-\mu} \ln \left[\varpi (P_{t-1}^H)^{1-\mu} + (1-\varpi) (\check{P}_t^H)^{1-\mu} \right], \text{ totally differentiate:}$$

$$\frac{dP_t^H}{P_0^H} = \varpi \frac{(P_0^H)^{-\mu}}{\varpi (P_0^H)^{1-\mu} + (1-\varpi) (\check{P}_0^H)^{1-\mu}} dP_{t-1}^H + (1-\varpi) \frac{(\check{P}_0^H)^{-\mu}}{\varpi (P_0^H)^{1-\mu} + (1-\varpi) (\check{P}_0^H)^{1-\mu}} d\check{P}_t^H$$

$$\frac{dP_t^H}{P_0^H} = \varpi \frac{dP_{t-1}^H}{P_0^H} + (1-\varpi) \frac{d\check{P}_t^H}{\check{P}_0^H}, \text{ since in the steady state } P_0^H = \check{P}_0^H = P_{t-1}^H = P_t^H$$

$$\ln P_t^H - \ln P_0^H = \varpi \ln P_{t-1}^H - \varpi \ln P_0^H + (1-\varpi) \ln \check{P}_t^H - (1-\varpi) \ln \check{P}_0^H$$

$$p_t^H - p_{t-1}^H = (\varpi - 1) p_{t-1}^H + (1-\varpi) \check{p}_t^H$$

$$\pi_t^H = (1-\varpi)(\check{p}_t^H - p_{t-1}^H)$$

$$\check{p}_t^H - p_{t-1}^H = \frac{\pi_t^H}{(1-\varpi)}, \text{ plug this into the preciously derived equation:}$$

$$\check{p}_t^H - p_{t-1}^H = \varpi\beta E_t (\check{p}_{t+1}^H - p_t^H) + \pi_t^H + (1-\varpi\beta)(mc_t + \hbar)$$

$$\frac{\pi_t^H}{(1-\varpi)} - \pi_t^H = \varpi\beta E_t \left(\frac{\pi_{t+1}^H}{(1-\varpi)} \right) + (1-\varpi\beta)(mc_t + \hbar)$$

$$\pi_t^H = \frac{(1-\varpi)}{\varpi} \frac{\varpi\beta}{(1-\varpi)} E_t (\pi_{t+1}^H) + \frac{(1-\varpi)}{\varpi} (1-\varpi\beta)(mc_t + \hbar)$$

$$\pi_t^H = \beta E_t(\pi_{t+1}^H) + \lambda(mc_t + \hbar), \text{ where } \lambda \equiv \frac{(1-\varpi)}{\varpi}(1-\varpi\beta) \text{ (not a Lagrangian multiplier)}$$

B.8. Aggregating the equilibrium condition:

$$\text{Plugging } y_t(z) = \left(\frac{p_t(z)}{P_t^H} \right)^{-\mu} \left[(1-\alpha) \left(\frac{P_t^H}{P_t} \right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_t^H}{E_t^* P_{t,*}^F} \right)^{-\gamma} \left(\frac{P_{t,*}^F}{P_{t,*}} \right)^{-\eta} C_{t,*} d* \right] \text{ into}$$

$$Y_t = \left(\int_0^1 y_t(z)^{(\mu-1)/\mu} dz \right)^{\mu/(\mu-1)} \text{ leads to:}$$

$$Y_t = \left\{ \int_0^1 \left\{ \left(\frac{p_t(z)}{P_t^H} \right)^{-\mu} \left[(1-\alpha) \left(\frac{P_t^H}{P_t} \right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_t^H}{E_t^* P_{t,*}^F} \right)^{-\gamma} \left(\frac{P_{t,*}^F}{P_{t,*}} \right)^{-\eta} C_{t,*} d* \right] \right\}^{(\mu-1)/\mu} dz \right\}^{\mu/(\mu-1)}$$

$$Y_t = \left\{ \left[(1-\alpha) \left(\frac{P_t^H}{P_t} \right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_t^H}{E_t^* P_{t,*}^F} \right)^{-\gamma} \left(\frac{P_{t,*}^F}{P_{t,*}} \right)^{-\eta} C_{t,*} d* \right]^{(\mu-1)/\mu} \left(\frac{1}{P_t^H} \right)^{1-\mu} \int_0^1 [p_t(z)]^{1-\mu} dz \right\}^{\mu/(\mu-1)}$$

$$\text{Recall that } P_t^H \equiv \left(\int_0^1 p_t(z)^{1-\mu} dz \right)^{1/(1-\mu)}$$

$$Y_t = \left\{ \left[(1-\alpha) \left(\frac{P_t^H}{P_t} \right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_t^H}{E_t^* P_{t,*}^F} \right)^{-\gamma} \left(\frac{P_{t,*}^F}{P_{t,*}} \right)^{-\eta} C_{t,*} d* \right] \left(\frac{1}{P_t^H} \right)^{-\mu} (P_t^H)^{-\mu} \right\}$$

$$Y_t = (1-\alpha) \left(\frac{P_t^H}{P_t} \right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_t^H}{E_t^* P_{t,*}^F} \right)^{-\gamma} \left(\frac{P_{t,*}^F}{P_{t,*}} \right)^{-\eta} C_{t,*} d*$$

$$Y_t = \left(\frac{P_t^H}{P_t} \right)^{-\eta} \left[(1-\alpha) C_t + \alpha \int_0^1 \left(\frac{P_t^H}{P_t} \right)^{\eta} \left(\frac{E_t^* P_{t,*}^F}{P_t^H} \right)^{\gamma} \left(\frac{P_{t,*}}{P_{t,*}^F} \right)^{\eta} C_{t,*} d* \right]$$

$$Y_t = \left(\frac{P_t^H}{P_t} \right)^{-\eta} \left[(1-\alpha) C_t + \alpha \int_0^1 \left(\frac{E_t^* P_{t,*}^F}{P_t^H} \right)^{\gamma-\eta} \left(\frac{E_t^* P_{t,*}}{P_t} \right)^{\eta} C_{t,*} d* \right]$$

$$\text{Recall that } B_t^* \equiv \frac{E_t^* P_{t,*}}{P_t} \text{ and } S_t^* = \frac{P_t^*}{P_t^H}. \text{ Furthermore, } S_t = \frac{P_t^F}{P_t^H}, \text{ which looked from the}$$

$$\text{perspective of a foreign country } * \text{ becomes } S_{t,*} = \frac{P_t^F}{P_{t,*}^*}. \text{ Now notice that } S_{t,*} S_t^* = \frac{P_{t,*}^F}{P_{t,*}^*} \frac{P_t^*}{P_t^H}$$

and $\frac{E_t^* P_{t,*}^F}{P_t^H} = \frac{P_t^F}{P_t^H} \frac{P_t^*}{P_{t,*}^*}$, since $E_t^* = \frac{P_t^*}{P_{t,*}^*}$. Therefore, $S_{t,*} S_t^* = \frac{E_t^* P_{t,*}^F}{P_t^H}$, and the equilibrium relation becomes:

$$Y_t = \left(\frac{P_t^H}{P_t} \right)^{-\eta} \left[(1-\alpha) C_t + \alpha \int_0^1 (S_{t,*} S_t^*)^{\gamma-\eta} (B_t^*)^\eta C_{t,*} d* \right]$$

It was previously shown in this paper that $C_t = \vartheta^* C_{t,*} (B_t^*)^{1/\sigma}$, where ϑ^* is a constant determined by the initial condition. Furthermore, without loss of generality one can assume identical initial condition and then $\vartheta^* = 1$. The aggregate output equilibrium condition is given by:

$$Y_t = \left(\frac{P_t^H}{P_t} \right)^{-\eta} C_t \left[(1-\alpha) + \alpha \int_0^1 (S_{t,*} S_t^*)^{\gamma-\eta} (B_t^*)^{\eta-1/\sigma} d* \right]$$

B.9. The steady state:

Let all countries be symmetric. Furthermore, assume a constant productivity level equal to A.

Global goods equilibrium in the steady state is:

$$Y_0 = \left(\frac{P_0^H}{P_0} \right)^{-\eta} C_0 \left[(1-\alpha) + \alpha \int_0^1 (S_{0,*} S_0^*)^{\gamma-\eta} (B_0^*)^{\eta-1/\sigma} d* \right]$$

Since all countries are symmetric, it must be that all S_0^* and B_0^* are the same. In fact,

notice that the effective terms of trade are given by $S_t = \frac{P_t^F}{P_t^H} = \left[\int_0^1 (S_t^*)^{1-\gamma} d* \right]^{1/(1-\gamma)}$, but

since S_0^* are all the same, it follows that $S_0 = S_0^*$, and from their definitions $S_t^* \equiv \frac{P_t^*}{P_t^H}$ and

$S_t \equiv \frac{P_t^F}{P_t^H}$, it must hold that $P_0^* = P_0^F$ at the steady state.

The price index is given by $P_t \equiv \left[(1-\alpha) (P_t^H)^{1-\eta} + \alpha (P_t^F)^{1-\eta} \right]^{1/(1-\eta)}$, or alternatively:

$$\frac{P_t}{P_t^H} = \left[(1-\alpha) + \alpha \left(\frac{P_t^F}{P_t^H} \right)^{1-\eta} \right]^{1/(1-\eta)}, \text{ this holds always, including at the steady state. Then}$$

$$\frac{P_0}{P_0^H} = \left[(1-\alpha) + \alpha (S_0)^{1-\eta} \right]^{1/(1-\eta)}.$$

Let $B_0 = B_0^*$ as it is the same for all $*$. Notice that $B_0 = \frac{E_0^* P_{0,*}}{P_0} = \frac{E_0^* P_0^F}{P_0} = \frac{P_0^F}{P_0}$, as from

$$P_t^* = E_t^* P_{t,*}^* \Rightarrow P_t^F = E_t^* P_{t,*}^F.$$

$$B_0 = \frac{P_0^F}{P_0} = \frac{\frac{P_0^F}{P_0^H}}{\frac{P_0^H}{P_0}} = \frac{S_0}{\left[(1-\alpha) + \alpha (S_0)^{1-\eta} \right]^{1/(1-\eta)}}, \text{ a function of } S_0.$$

From the integration of the financial markets the relationship $C_t = C_{t,*} (B_t^*)^{1/\sigma}$ (again identical initial conditions are assumed). In the steady state that is $C_0 = C_{0,*} (B_0^*)^{1/\sigma}$.

Integrating over all $*$ gives:

$$C_0 = \int_0^1 C_{0,*} (B_0)^{1/\sigma} = (B_0)^{1/\sigma} C_0^w$$

And lastly, remember that when the terms of trade are aggregated over the whole world, they cancel out. Then the equation

$$Y_0 = \left(\frac{P_0^H}{P_0} \right)^{-\eta} C_0 \left[(1-\alpha) + \alpha \int_0^1 (S_{0,*} S_0^*)^{\gamma-\eta} (B_0^*)^{\eta-1/\sigma} d* \right] \text{ can be rewritten as:}$$

$$\begin{aligned} Y_0 &= \left[(1-\alpha) + \alpha (S_0)^{1-\eta} \right]^{\eta/(1-\eta)} \left(\frac{S_0}{\left[(1-\alpha) + \alpha (S_0)^{1-\eta} \right]^{1/(1-\eta)}} \right)^{1/\sigma} C_0^w \left[(1-\alpha) + \alpha (S_0)^{\gamma-\eta} (B_0)^{\eta-1/\sigma} \right] \\ Y_0 &= \left[(1-\alpha) + \alpha (S_0)^{1-\eta} \right]^{(\eta-1/\sigma)/(1-\eta)} (S_0)^{1/\sigma} \left[(1-\alpha) + \alpha (S_0)^{\gamma-\eta} \left(\frac{S_0}{\left[(1-\alpha) + \alpha (S_0)^{1-\eta} \right]^{1/(1-\eta)}} \right)^{\eta-1/\sigma} \right] C_0^w \\ Y_0 &= \left[(1-\alpha) (S_0)^{1/\sigma} \left[(1-\alpha) + \alpha (S_0)^{1-\eta} \right]^{(\eta-1/\sigma)/(1-\eta)} + \alpha (S_0)^\gamma \right] C_0^w \end{aligned}$$

World goods market clearing condition implies $C_0^w = Y_0^w$, then:

$$Y_0 = \left[(1-\alpha) (S_0)^{1/\sigma} \left[(1-\alpha) + \alpha (S_0)^{1-\eta} \right]^{(\eta-1/\sigma)/(1-\eta)} + \alpha (S_0)^\gamma \right] Y_0^w$$

The other equilibrium condition is obtained from the labor market clearing.

$C_0^\sigma N_0^\varphi = \frac{W_0}{P_0}$, substituting from the production function gives:

$$C_0^\sigma \left(\frac{Y_0}{A} \right)^\varphi = \frac{W_0}{P_0}$$

Recall that real marginal cost was defined as $MC_0 = \frac{(1-\tau)W_0}{A P_0^H}$ and moreover that it was

calculated in appendix B.7. that $MC_0 = \frac{\mu-1}{\mu}$. Then $W_0 = \frac{\mu-1}{\mu(1-\tau)} A P_0^H$. Hence:

$$C_0^\sigma \left(\frac{Y_0}{A} \right)^\varphi = A \frac{\mu-1}{(1-\tau)\mu} \frac{P_0^H}{P_0}. \text{ Use the facts obtained for } C_0 \text{ and } \frac{P_0^H}{P_0}:$$

$$B_0 (Y_0^w)^\sigma \left(\frac{Y_0}{A} \right)^\varphi = A \frac{\mu-1}{(1-\tau)\mu} \frac{1}{\left[(1-\alpha) + \alpha (S_0)^{1-\eta} \right]^{1/(1-\eta)}}$$

$$\frac{S_0}{\left[(1-\alpha) + \alpha (S_0)^{1-\eta} \right]^{1/(1-\eta)}} (Y_0^w)^\sigma \left(\frac{Y_0}{A} \right)^\varphi = A \frac{\mu-1}{(1-\tau)\mu} \frac{1}{\left[(1-\alpha) + \alpha (S_0)^{1-\eta} \right]^{1/(1-\eta)}}$$

$$S_0 (Y_0^w)^\sigma \left(\frac{Y_0}{A} \right)^\varphi = A \frac{\mu-1}{(1-\tau)\mu}$$

$$(Y_0)^\varphi = A^{1+\varphi} \frac{\mu-1}{(1-\tau)\mu S_0 (Y_0^w)^\sigma}$$

$$Y_0 = A^{(1+\varphi)/\varphi} \left(\frac{\mu-1}{(1-\tau)\mu S_0 (Y_0^w)^\sigma} \right)^{1/\varphi}$$

There is a system of two equations in two variables Y_0 and S_0 , conditional on A and Y_0^w

$$Y_0 = \left[(1-\alpha)(S_0)^{1/\sigma} \left[(1-\alpha) + \alpha (S_0)^{1-\eta} \right]^{(\eta-1/\sigma)/(1-\eta)} + \alpha (S_0)^\gamma \right] Y_0^w$$

$$Y_0 = A^{(1+\varphi)/\varphi} \left(\frac{\mu-1}{(1-\tau)\mu S_0 (Y_0^w)^\sigma} \right)^{1/\varphi}$$

Notice that the first function is strictly increasing in S_0 , while the second is strictly decreasing ($\mu > 1$), both are always in the positive numbers. Therefore, these two curves

meet only once, and there is a unique solution. If it is obtained, the problem is solved in its entirety.

Make guess of $Y_0 = Y_0^w = A^{(1+\varphi)/(\varphi+\sigma)} \left(\frac{\mu-1}{(1-\tau)\mu} \right)^{1/(\varphi+\sigma)}$ and $S_0 = 1$

From the first equation, one can see that $S_0 = 1$ satisfies the condition:

$$Y_0 = \left[(1-\alpha)(1)^{1/\sigma} \left[(1-\alpha) + \alpha(1)^{1-\eta} \right]^{(\eta-1/\sigma)/(1-\eta)} + \alpha(1)^\gamma \right] Y_0^w = Y_0^w$$

Check if the values satisfy the second condition:

$$A^{(1+\varphi)/(\varphi+\sigma)} \left(\frac{\mu-1}{(1-\tau)\mu} \right)^{1/(\varphi+\sigma)} = A^{(1+\varphi)/\varphi} \left(\frac{\mu-1}{(1-\tau)\mu S_0 \left(A^{(1+\varphi)/(\varphi+\sigma)} \left(\frac{\mu-1}{(1-\tau)\mu} \right)^{\frac{1}{\varphi+\sigma}} \right)^\sigma} \right)^{1/\varphi}$$

$$A^{(1+\varphi)/(\varphi+\sigma)} A^{-(1+\varphi)/\varphi} A^{\sigma(1+\varphi)/[(\varphi+\sigma)\varphi]} = (S_0)^{-1/\varphi} \left(\frac{\mu-1}{(1-\tau)\mu} \right)^{-1/(\varphi+\sigma)} \left(\frac{\mu-1}{(1-\tau)\mu} \right)^{[1-\sigma/(\varphi+\sigma)]\varphi}$$

The powers of the left-hand side are:

$$\frac{1+\varphi}{\varphi+\sigma} - \frac{1+\varphi}{\varphi} + \frac{\sigma(1+\varphi)}{(\varphi+\sigma)\varphi} = \frac{\varphi+\varphi^2-\varphi-\varphi^2-\sigma-\sigma\varphi+\sigma+\sigma\varphi}{(\varphi+\sigma)\varphi} = 0$$

The powers of the right-hand side (excluding the one of the S_0):

$$\frac{-1}{\varphi+\sigma} + \frac{1-\frac{\sigma}{\varphi+\sigma}}{\varphi} = \frac{-1}{\varphi+\sigma} + \frac{\frac{\varphi}{\varphi+\sigma}}{\varphi} = \frac{-1}{\varphi+\sigma} + \frac{1}{\varphi+\sigma} = 0$$

$$\text{Then: } 1 = (S_0)^{-1/\varphi} \Rightarrow S_0 = 1$$

Therefore, this is the unique solution of the system.

If $S_0 = 1$, it follows that $S_0^* = 1$ for all $*$. $B_0 = \frac{S_0}{\left[(1-\alpha) + \alpha(S_0)^{1-\eta} \right]^{1/(1-\eta)}} = 1$ Therefore, it

must be also that $E_0^* = 1$. Furthermore, $C_0 = (B_0)^{1/\sigma} C_0^w = C_0^w$.

B.10. Log-linearizing equilibrium relation in home goods market:

$$Y_t = \left(\frac{P_t^H}{P_t} \right)^{-\eta} C_t \left[(1-\alpha) + \alpha \int_0^1 (S_{t,*} S_t^*)^{\gamma-\eta} (B_t^*)^{\eta-1/\sigma} d^* \right]$$

Again use the fact that: $\frac{P_t}{P_t^H} = \left[(1-\alpha) + \alpha \left(\frac{P_t^F}{P_t^H} \right)^{1-\eta} \right]^{1/(1-\eta)} = \left[(1-\alpha) + \alpha (S_t)^{1-\eta} \right]^{1/(1-\eta)}$

$$Y_t = \left[(1-\alpha) + \alpha (S_t)^{1-\eta} \right]^{\eta/(1-\eta)} C_t \left[(1-\alpha) + \alpha \int_0^1 (S_{t,*} S_t^*)^{\gamma-\eta} (B_t^*)^{\eta-1/\sigma} d^* \right]$$

$$\ln Y_t = \frac{\eta}{1-\eta} \ln \left[(1-\alpha) + \alpha (S_t)^{1-\eta} \right] + \ln C_t + \ln \left[(1-\alpha) + \alpha \int_0^1 (S_{t,*} S_t^*)^{\gamma-\eta} (B_t^*)^{\eta-1/\sigma} d^* \right]$$

Totally differentiate:

$$\begin{aligned} \frac{dY_t}{Y_0} &= \frac{\eta \alpha (S_0)^{-\eta}}{(1-\alpha) + \alpha (S_0)^{1-\eta}} dS_t + \frac{dC_t}{C_0} + \int_0^1 \frac{(\gamma-\eta) \alpha (B_0^*)^{\eta-1/\sigma} (S_0^*)^{\gamma-\eta} (S_{0,*})^{\gamma-\eta-1} dS_{t,*}}{(1-\alpha) + \alpha \int_0^1 (S_{0,*} S_0^*)^{\gamma-\eta} (B_0^*)^{\eta-1/\sigma} d^*} d^* + \\ &+ \int_0^1 \frac{\alpha (\gamma-\eta) (B_0^*)^{\eta-1/\sigma} (S_0^*)^{\gamma-\eta-1} (S_{0,*})^{\gamma-\eta} dS_t^*}{(1-\alpha) + \alpha \int_0^1 (S_{0,*} S_0^*)^{\gamma-\eta} (B_0^*)^{\eta-1/\sigma} d^*} d^* + \int_0^1 \frac{\alpha (\eta - \frac{1}{\sigma}) (B_0^*)^{\eta-1-1/\sigma} (S_t^*)^{\gamma-\eta-1} (S_{0,*})^{\gamma-\eta} dB_t^*}{(1-\alpha) + \alpha \int_0^1 (S_{0,*} S_0^*)^{\gamma-\eta} (B_0^*)^{\eta-1/\sigma} d^*} d^* \\ y_t - \ln Y_0 &= \alpha \eta dS_t + c_t - \ln C_0 + \alpha (\gamma-\eta) \int_0^1 dS_{t,*} d^* + \alpha (\gamma-\eta) \int_0^1 dS_t^* d^* + \alpha (\eta - \frac{1}{\sigma}) \int_0^1 dB_t^* d^* \end{aligned}$$

$$y_t = c_t + \alpha \eta s_t + \alpha (\gamma-\eta) \int_0^1 s_{t,*} d^* + \alpha (\gamma-\eta) \int_0^1 s_t^* d^* + \alpha (\eta - \frac{1}{\sigma}) \int_0^1 b_t^* d^*. \text{ Note that } \int_0^1 s_{t,*} d^* = 0,$$

since when terms of trade are aggregated on a world basis, they cancel out (if they are not in logarithmic form they cancel out to 1, but here $s_{t,*}$ are logarithms, hence it is zero.)

$$y_t = c_t + \alpha \eta s_t + \alpha (\gamma-\eta) s_t + \alpha (\eta - \frac{1}{\sigma}) b_t$$

$$y_t = c_t + \alpha \gamma s_t + \alpha (\eta - \frac{1}{\sigma}) b_t. \text{ Recall that } b_t = (1-\alpha) s_t :$$

$$y_t = c_t + \alpha \gamma s_t + \alpha (\eta - \frac{1}{\sigma}) (1-\alpha) s_t$$

$$y_t = c_t + \frac{\alpha}{\sigma} (\sigma \gamma + (\sigma \eta - 1)(1-\alpha)) s_t = c_t + \frac{\alpha \omega}{\sigma} s_t$$

where $\omega \equiv \sigma \gamma + (\sigma \eta - 1)(1-\alpha)$

B.11. Modifying the Euler equation:

The linearized Euler equation (27) is given by:

$$c_t = E_t [c_{t+1}] - \frac{1}{\sigma} [i_t - E_t (\pi_{t+1}) - \rho]$$

Expressing c_t and c_{t+1} from $y_t = c_t + \frac{\alpha\omega}{\sigma} s_t$ and $y_{t+1} = c_{t+1} + \frac{\alpha\omega}{\sigma} s_{t+1}$, and plugging into the Euler equation gives:

$$y_t - \frac{\alpha\omega}{\sigma} s_t = E_t \left[y_{t+1} - \frac{\alpha\omega}{\sigma} s_{t+1} \right] - \frac{1}{\sigma} [i_t - E_t (\pi_{t+1}) - \rho]$$

$$y_t = E_t [y_{t+1}] - \frac{1}{\sigma} [i_t - E_t (\pi_{t+1}) - \rho] - \frac{\alpha\omega}{\sigma} E_t [\Delta s_{t+1}]$$

Recall that $\pi_t = \pi_t^H + \alpha \Delta s_t$

$$y_t = E_t [y_{t+1}] + \frac{\alpha}{\sigma} E_t [\Delta s_{t+1}] - \frac{1}{\sigma} [i_t - E_t (\pi_{t+1}^H) - \rho] - \frac{\alpha\omega}{\sigma} E_t [\Delta s_{t+1}]$$

$$y_t = E_t [y_{t+1}] - \frac{1}{\sigma} [i_t - E_t (\pi_{t+1}^H) - \rho] - \frac{\alpha(\omega-1)}{\sigma} E_t [\Delta s_{t+1}]$$

Combining $y_t = y_t^w + \frac{1}{\sigma_\alpha} s_t$ and $y_{t+1} = y_{t+1}^w + \frac{1}{\sigma_\alpha} s_{t+1}$ yields $\Delta s_{t+1} = \sigma_\alpha (y_{t+1} - y_t) - \sigma_\alpha \Delta y_{t+1}^w$

$$y_t = E_t [y_{t+1}] - \frac{1}{\sigma} [i_t - E_t (\pi_{t+1}^H) - \rho] - \frac{\alpha(\omega-1)}{\sigma} E_t [\sigma_\alpha (y_{t+1} - y_t) - \sigma_\alpha \Delta y_{t+1}^w]$$

$$y_t - \frac{\alpha(\omega-1)\sigma_\alpha}{\sigma} y_t = E_t [y_{t+1}] - \frac{\alpha(\omega-1)\sigma_\alpha}{\sigma} E_t [y_{t+1}] - \frac{1}{\sigma} [i_t - E_t (\pi_{t+1}^H) - \rho] + \frac{\alpha(\omega-1)\sigma_\alpha}{\sigma} E_t [\Delta y_{t+1}^w]$$

$$\frac{\sigma - \alpha(\omega-1)\sigma_\alpha}{\sigma} y_t = \frac{\sigma - \alpha(\omega-1)\sigma_\alpha}{\sigma} E_t [y_{t+1}] - \frac{1}{\sigma} [i_t - E_t (\pi_{t+1}^H) - \rho] + \frac{\alpha(\omega-1)\sigma_\alpha}{\sigma} E_t [\Delta y_{t+1}^w]$$

$$y_t = E_t [y_{t+1}] - \frac{1}{\sigma - \alpha(\omega-1)\sigma_\alpha} [i_t - E_t (\pi_{t+1}^H) - \rho] + \frac{\alpha(\omega-1)\sigma_\alpha}{\sigma - \alpha(\omega-1)\sigma_\alpha} E_t [\Delta y_{t+1}^w]$$

Note that:

$$\sigma - \alpha(\omega-1)\sigma_\alpha = \sigma - \frac{\sigma\alpha(\omega-1)}{1-\alpha+\alpha\omega} = \frac{\sigma - \sigma\alpha + \sigma\alpha\omega - \sigma\alpha\omega + \sigma\alpha}{1-\alpha+\alpha\omega} = \frac{\sigma}{1-\alpha+\alpha\omega} = \sigma_\alpha$$

$$\frac{\alpha(\omega-1)\sigma_\alpha}{\sigma - \alpha(\omega-1)\sigma_\alpha} = \frac{\alpha(\omega-1)\sigma_\alpha}{\sigma - \alpha(\omega-1)\frac{\sigma}{1-\alpha+\alpha\omega}} = \frac{\alpha(\omega-1)\sigma_\alpha}{\frac{\sigma - \sigma\alpha + \sigma\alpha\omega - \sigma\alpha\omega + \sigma\alpha}{1-\alpha+\alpha\omega}} = \frac{\alpha(\omega-1)\sigma_\alpha}{\sigma_\alpha} = \alpha(\omega-1)$$

Then:

$$y_t = E_t[y_{t+1}] - \frac{1}{\sigma_\alpha} \left[i_t - E_t(\pi_{t+1}^H) - \rho \right] + \alpha(\omega - 1)E_t[\Delta y_{t+1}^w]$$

$$y_t = E_t[y_{t+1}] - \frac{1}{\sigma_\alpha} \left[i_t - E_t(\pi_{t+1}^H) - \rho \right] + \alpha\Theta E_t[\Delta y_{t+1}^w]$$

where $\Theta = (\sigma\gamma - 1) + (\sigma\eta - 1)(1 - \alpha) = \omega - 1$

B.12. Formulating the model in terms of the output gap:

The natural output level can be determined from:

$$-\bar{h} = -v + (\sigma - \sigma_\alpha)y_t^w + (\varphi + \sigma_\alpha)y_t^n - (1 + \varphi)a_t$$

$$-(\varphi + \sigma_\alpha)y_t^n = \bar{h} - v + (\sigma - \sigma_\alpha)y_t^w - (1 + \varphi)a_t$$

$$y_t^n = \frac{v - \bar{h}}{\varphi + \sigma_\alpha} + \frac{\sigma_\alpha - \sigma}{\varphi + \sigma_\alpha} y_t^w + \frac{1 + \varphi}{\varphi + \sigma_\alpha} a_t$$

Let $\Omega \equiv \frac{v - \bar{h}}{\varphi + \sigma_\alpha}$; $\Gamma \equiv \frac{1 + \varphi}{\varphi + \sigma_\alpha}$; $\Psi \equiv -\frac{\Theta\sigma_\alpha}{\varphi + \sigma_\alpha}$. Then:

$$y_t^n = \Omega + \alpha\Psi y_t^w + \Gamma a_t$$

Notice that: $\sigma_\alpha - \sigma = \frac{\sigma}{1 - \alpha + \alpha\omega} - \sigma = \frac{\alpha\sigma - \alpha\omega\sigma}{1 - \alpha + \alpha\omega} = -\alpha \frac{(\omega - 1)\sigma}{1 - \alpha + \alpha\omega} = -\alpha\Theta\sigma_\alpha$

Furthermore, equation (29) is: $mc_t = -v + (\sigma - \sigma_\alpha)y_t^w + (\varphi + \sigma_\alpha)y_t - (1 + \varphi)a_t$ and,

correspondingly, $-\bar{h} = -v + (\sigma - \sigma_\alpha)y_t^w + (\varphi + \sigma_\alpha)y_t^n - (1 + \varphi)a_t$ when prices are flexible.

Combining these two results with the identity $mc_t + \bar{h} = mc_t - (-\bar{h})$ yields:

$$mc_t + \bar{h} = -v + (\sigma - \sigma_\alpha)y_t^w + (\varphi + \sigma_\alpha)y_t - (1 + \varphi)a_t + v - (\sigma - \sigma_\alpha)y_t^w - (\varphi + \sigma_\alpha)y_t^n + (1 + \varphi)a_t$$

$$mc_t + \bar{h} = (\varphi + \sigma_\alpha)x_t$$

Plug this into equation (28), $\pi_t^H = \beta E_t(\pi_{t+1}^H) + \lambda(mc_t + \bar{h})$:

$$\pi_t^H = \beta E_t(\pi_{t+1}^H) + \lambda(\varphi + \sigma_\alpha)x_t$$

$$\pi_t^H = \beta E_t(\pi_{t+1}^H) + \kappa_\alpha x_t, \quad \text{where } \kappa_\alpha \equiv \lambda(\varphi + \sigma_\alpha)$$

To derive the IS-type relation in terms of the output gap start from

$$y_t = E_t[y_{t+1}] - \frac{1}{\sigma_\alpha} \left[i_t - E_t(\pi_{t+1}^H) - \rho \right] + \alpha\Theta E_t[\Delta y_{t+1}^w]$$

$$x_t + y_t^n = E_t \left[x_{t+1} + y_{t+1}^n \right] - \frac{1}{\sigma_\alpha} \left[i_t - E_t \left(\pi_{t+1}^H \right) - \rho \right] + \alpha \Theta E_t \left[\Delta y_{t+1}^w \right], \text{ as } y_t = x_t + y_t^n$$

Recall that $y_t^n = \Omega + \alpha \Psi y_t^w + \Gamma a_t$. Then

$$E_t \left[y_{t+1}^n \right] = E_t \left[\Omega + \alpha \Psi y_{t+1}^w + \Gamma a_{t+1} \right] = \Omega + \alpha \Psi E_t \left[y_{t+1}^w \right] + E_t \left[\Gamma a_{t+1} \right]$$

Since $a_t = \rho_a a_{t-1} + \varepsilon_t$ with $E_t \left[\varepsilon_t \right] = 0$, it must be that:

$$E_t \left[y_{t+1}^n \right] = \Omega + \alpha \Psi E_t \left[y_{t+1}^w \right] + \Gamma \rho_a a_t$$

Using these facts the equation can be modified to:

$$x_t = E_t \left[x_{t+1} \right] - \Omega - \alpha \Psi y_t^w - \Gamma a_t + \Omega + \alpha \Psi E_t \left[y_{t+1}^w \right] + \Gamma \rho_a a_t - \frac{1}{\sigma_\alpha} \left[i_t - E_t \left(\pi_{t+1}^H \right) - \rho \right] + \alpha \Theta E_t \left[\Delta y_{t+1}^w \right]$$

$$x_t = E_t \left[x_{t+1} \right] - \Gamma (1 - \rho_a) a_t - \frac{1}{\sigma_\alpha} \left[i_t - E_t \left(\pi_{t+1}^H \right) - \rho \right] + \alpha (\Theta + \Psi) E_t \left[\Delta y_{t+1}^w \right]$$

$$x_t = E_t \left[x_{t+1} \right] - \frac{1}{\sigma_\alpha} \left[i_t - E_t \left(\pi_{t+1}^H \right) - \rho + \sigma_\alpha \Gamma (1 - \rho_a) a_t - \alpha \sigma_\alpha (\Theta + \Psi) E_t \left[\Delta y_{t+1}^w \right] \right]$$

$$x_t = E_t \left[x_{t+1} \right] - \frac{1}{\sigma_\alpha} \left[i_t - E_t \left(\pi_{t+1}^H \right) - r^n \right]$$

$$\text{where } r^n \equiv \rho - \sigma_\alpha \Gamma (1 - \rho_a) a_t + \alpha (\Theta + \Psi) E_t \left[\Delta y_{t+1}^w \right]$$

B.13. Solving the model:

The model is given by:

$$\pi_t^H = \beta E_t \left(\pi_{t+1}^H \right) + \kappa_\alpha x_t$$

$$x_t = E_t \left(x_{t+1} \right) - \frac{1}{\sigma_\alpha} \left[i_t - E_t \left(\pi_{t+1}^H \right) - r^n \right]$$

$$i_t = \rho + h_\pi \pi_{t+1}^H + h_x x_t + \delta_t$$

Use the last equation to eliminate the interest rate:

$$x_t = E_t \left(x_{t+1} \right) - \frac{1}{\sigma_\alpha} \left[\rho + h_\pi \pi_t^H + h_x x_t + \delta_t - E_t \left(\pi_{t+1}^H \right) - r^n \right]$$

$$\left(1 + \frac{h_x}{\sigma_\alpha} \right) x_t = E_t \left(x_{t+1} \right) - \frac{1}{\sigma_\alpha} \left[\rho + h_\pi \pi_t^H + \delta_t - E_t \left(\pi_{t+1}^H \right) - r^n \right]$$

$$x_t = \frac{\sigma_\alpha}{\sigma_\alpha + h_x} E_t(x_{t+1}) - \frac{1}{\sigma_\alpha + h_x} \left[\rho + h_\pi \pi_t^H + \delta_t - E_t(\pi_{t+1}^H) - r^n \right]$$

Plug this into the Phillips curve:

$$\begin{aligned} \pi_t^H &= \beta E_t(\pi_{t+1}^H) + \kappa_\alpha \left[\frac{\sigma_\alpha}{\sigma_\alpha + h_x} E_t(x_{t+1}) - \frac{1}{\sigma_\alpha + h_x} \left[\rho + h_\pi \pi_t^H + \delta_t - E_t(\pi_{t+1}^H) - r^n \right] \right] \\ \left(1 + \frac{\kappa_\alpha h_\pi}{\sigma_\alpha + h_x} \right) \pi_t^H &= \left(\beta + \frac{\kappa_\alpha}{\sigma_\alpha + h_x} \right) E_t(\pi_{t+1}^H) + \frac{\kappa_\alpha \sigma_\alpha}{\sigma_\alpha + h_x} E_t(x_{t+1}) - \frac{\kappa_\alpha}{\sigma_\alpha + h_x} (\rho + \delta_t - r^n) \\ \pi_t^H &= \frac{\beta(\sigma_\alpha + h_x) + \kappa_\alpha}{\sigma_\alpha + h_x + \kappa_\alpha h_\pi} E_t(\pi_{t+1}^H) + \frac{\kappa_\alpha \sigma_\alpha}{\sigma_\alpha + h_x + \kappa_\alpha h_\pi} E_t(x_{t+1}) - \frac{\kappa_\alpha}{\sigma_\alpha + h_x + \kappa_\alpha h_\pi} (\rho + \delta_t - r^n) \\ \pi_t^H &= \Xi_\alpha \left[(\beta(\sigma_\alpha + h_x) + \kappa_\alpha) E_t(\pi_{t+1}^H) + \kappa_\alpha \sigma_\alpha E_t(x_{t+1}) - \kappa_\alpha (\rho + \delta_t - r^n) \right] \end{aligned}$$

$$\text{where } \Xi_\alpha \equiv \frac{1}{\sigma_\alpha + h_x + \kappa_\alpha h_\pi}.$$

Now plug this into the IS relation:

$$\begin{aligned} x_t &= \frac{\sigma_\alpha}{\sigma_\alpha + h_x} E_t(x_{t+1}) - \\ &- \frac{1}{\sigma_\alpha + h_x} \left[\rho + h_\pi \Xi_\alpha \left[(\beta(\sigma_\alpha + h_x) + \kappa_\alpha) E_t(\pi_{t+1}^H) + \kappa_\alpha \sigma_\alpha E_t(x_{t+1}) - \kappa_\alpha (\rho + \delta_t - r^n) \right] + \delta_t - E_t(\pi_{t+1}^H) - r^n \right] \\ x_t &= \left(\frac{\sigma_\alpha}{\sigma_\alpha + h_x} - \frac{h_\pi \Xi_\alpha \kappa_\alpha \sigma_\alpha}{\sigma_\alpha + h_x} \right) E_t(x_{t+1}) - \frac{h_\pi \Xi_\alpha (\beta(\sigma_\alpha + h_x) + \kappa_\alpha) - 1}{\sigma_\alpha + h_x} E_t(\pi_{t+1}^H) + \frac{h_\pi \Xi_\alpha \kappa_\alpha - 1}{\sigma_\alpha + h_x} (\rho + \delta_t - r^n) \end{aligned}$$

Notice that:

$$\begin{aligned} \frac{h_\pi \Xi_\alpha \kappa_\alpha - 1}{\sigma_\alpha + h_x} &= \frac{\frac{h_\pi \kappa_\alpha}{\sigma_\alpha + h_x + \kappa_\alpha h_\pi} - 1}{\sigma_\alpha + h_x} = \frac{\frac{\sigma_\alpha + h_x}{\sigma_\alpha + h_x + \kappa_\alpha h_\pi} - 1}{\sigma_\alpha + h_x} = -\Xi_\alpha \quad \text{Hence:} \\ \frac{(1 - h_\pi \Xi_\alpha \kappa_\alpha) \sigma_\alpha}{\sigma_\alpha + h_x} &= \sigma_\alpha \Xi_\alpha \end{aligned}$$

Furthermore:

$$\frac{h_\pi \Xi_\alpha (\beta(\sigma_\alpha + h_x) + \kappa_\alpha) - 1}{\sigma_\alpha + h_x} = \frac{h_\pi \Xi_\alpha \beta \sigma_\alpha + h_\pi \Xi_\alpha \beta h_x + h_\pi \Xi_\alpha \kappa_\alpha - 1}{\sigma_\alpha + h_x} =$$

$$\frac{h_\pi \Xi_\alpha \beta \sigma_\alpha + h_\pi \Xi_\alpha \beta h_x}{\sigma_\alpha + h_x} - \Xi_\alpha = \Xi_\alpha (\beta h_\pi - 1). \text{ Then:}$$

$$x_t = \Xi_\alpha \left[\sigma_\alpha E_t(x_{t+1}) + (1 - \beta h_\pi) E_t(\pi_{t+1}^H) - (\rho + \delta_t - r^n) \right]$$

Putting this into matrix form:

$$\begin{pmatrix} x_t \\ \pi_t^H \end{pmatrix} = A_\alpha \begin{pmatrix} E_t(x_{t+1}) \\ E_t(\pi_{t+1}^H) \end{pmatrix} + b_\alpha (r^n - \rho - \delta_t)$$

$$\text{where } A_\alpha \equiv \Xi_\alpha \begin{pmatrix} \sigma_\alpha & 1 - \beta h_\pi \\ \sigma_\alpha \kappa_\alpha & \kappa_\alpha + \beta(\sigma_\alpha + h_x) \end{pmatrix} \quad b_\alpha \equiv \Xi_\alpha \begin{pmatrix} 1 \\ \kappa_\alpha \end{pmatrix}$$

Solving the system of equations:

Guess a solution of a form $x_t = G_{x\delta} \delta_t$ and $\pi_t^H = G_{\pi\delta} \delta_t$

Plug this into the system, and note that $r^n = \rho$,

$$E_t(\pi_{t+1}^H) = E_t(G_{\pi\delta} \delta_{t+1}) = G_{\pi\delta} \rho_\delta \delta_t + E_t(G_{\pi\delta} \varepsilon^\delta) = G_{\pi\delta} \rho_\delta \delta_t \text{ and hence } E_t(x_{t+1}) = G_{x\delta} \rho_\delta \delta_t:$$

The system:

$$\pi_t^H = \beta E_t(\pi_{t+1}^H) + \kappa_\alpha x_t$$

$$x_t = E_t(x_{t+1}) - \frac{1}{\sigma_\alpha} \left[\rho + h_\pi \pi_t^H + h_x x_t + \delta_t - E_t(\pi_{t+1}^H) - r^n \right]$$

becomes:

$$G_{\pi\delta} \delta_t = \beta G_{\pi\delta} \rho_\delta \delta_t + \kappa_\alpha G_{x\delta} \delta_t$$

$$G_{x\delta} \delta_t = G_{x\delta} \rho_\delta \delta_t - \frac{1}{\sigma_\alpha} [h_\pi G_{\pi\delta} \delta_t + h_x G_{x\delta} \delta_t + \delta_t - G_{\pi\delta} \rho_\delta \delta_t]$$

Cancel the δ_t and rearrange the first equation:

$$(1 - \beta \rho_\delta) G_{\pi\delta} = \kappa_\alpha G_{x\delta} \Rightarrow G_{x\delta} = \frac{1 - \beta \rho_\delta}{\kappa_\alpha} G_{\pi\delta} \text{ Plug this into the second:}$$

$$\frac{1 - \beta \rho_\delta}{\kappa_\alpha} G_{\pi\delta} = \frac{1 - \beta \rho_\delta}{\kappa_\alpha} G_{\pi\delta} \rho_\delta - \frac{1}{\sigma_\alpha} \left[h_\pi G_{\pi\delta} + h_x \frac{1 - \beta \rho_\delta}{\kappa_\alpha} G_{\pi\delta} + 1 - G_{\pi\delta} \rho_\delta \right]$$

$$\frac{1}{\sigma_\alpha} = \left(\frac{1 - \beta \rho_\delta}{\kappa_\alpha} \rho_\delta - \frac{1 - \beta \rho_\delta}{\kappa_\alpha} - \frac{h_\pi}{\sigma_\alpha} - h_x \frac{1}{\sigma_\alpha} \frac{1 - \beta \rho_\delta}{\kappa_\alpha} + \frac{1}{\sigma_\alpha} \rho_\delta \right) G_{\pi\delta}$$

$$1 = \frac{\sigma_\alpha(1-\beta\rho_\delta)(\rho_\delta-1) - h_\pi\kappa_\alpha - h_x(1-\beta\rho_\delta) + \rho_\delta\kappa_\alpha}{\kappa_\alpha} G_{\pi\delta}$$

$$G_{\pi\delta} = \frac{\kappa_\alpha}{(1-\beta\rho_\delta)[\sigma_\alpha(\rho_\delta-1) - h_x] - (h_\pi - \rho_\delta)\kappa_\alpha}$$

$$G_{\pi\delta} = \frac{-\kappa_\alpha}{(1-\beta\rho_\delta)[\sigma_\alpha(1-\rho_\delta) + h_x] + (h_\pi - \rho_\delta)\kappa_\alpha}$$

$$G_{\pi\delta} = -\kappa_\alpha H_\delta, \text{ where } H_\delta \equiv \frac{1}{(1-\beta\rho_\delta)[\sigma_\alpha(1-\rho_\delta) + h_x] + (h_\pi - \rho_\delta)\kappa_\alpha}$$

$$\text{Then: } G_{x\delta} = \frac{1-\beta\rho_\delta}{\kappa_\alpha} G_{\pi\delta} = \frac{1-\beta\rho_\delta}{\kappa_\alpha} (-\kappa_\alpha H_\delta) = -(1-\beta\rho_\delta) H_\delta$$

The solutions are given by:

$$x_t = -(1-\beta\rho_\delta) H_\delta \delta_t$$

$$\pi_t^H = -\kappa_\alpha H_\delta \delta_t$$

Note that $H_\delta \equiv \frac{1}{(1-\beta\rho_\delta)[\sigma_\alpha(1-\rho_\delta) + h_x] + (h_\pi - \rho_\delta)\kappa_\alpha} > 0$. If $(h_\pi - \rho_\delta)\kappa_\alpha > 0$ then it

obviously holds. More interestingly, if $(h_\pi - \rho_\delta)\kappa_\alpha < 0$, one can open the brackets on the left side and take out the term $(1-\beta\rho_\delta)h_x$, which is obviously positive. Note that $(1-\beta\rho_\delta)h_x > (1-\beta)h_x$ and, furthermore, the inequality that ensures unique solutions is $\kappa_\alpha(h_\pi-1) + (1-\beta)h_x > 0$. Notice that if $(h_\pi - \rho_\delta)\kappa_\alpha$ is negative, then $\kappa_\alpha(h_\pi-1)$ is even “more” negative, but the inequality still holds. Hence, $(1-\beta\rho_\delta)h_x + (h_\pi - \rho_\delta)\kappa_\alpha > 0$ and therefore $H_\delta > 0$.

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Abstracts in English and German:

This thesis aims at reviewing and comparing in a systematic manner some of the most prominent New-Keynesian models concerning an open economy: Obstfeld and Rogoff (1995) and Gali and Monacelli (2005). It examines the settings of the models, the concluding findings, and how differences in the settings have affected these findings. The paper investigates the applicability of the different models to diverse economic situations.

In dieser Masterarbeit werden zwei von den zentralsten und bekanntesten Ansätze aus der Klasse der New-Keynesian Modelle für offene Volkswirtschaften hergeleitet und vorgestellt: Obstfeld und Rogoff (1995) und Gali und Monacelli (2005). Es werden insbesondere die Unterschiede beider Modelle einander gegenüber gestellt und welche Unterschiede sich hinsichtlich ihrer Schlussfolgerungen daraus ergeben. Zuletzt wird die Anwendbarkeit beider Modelle auf diverse ökonomische Szenarien evaluiert.

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