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Decoherence and Entanglement of two qubit systems

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One particle meets
another one and they both
change their strangeness.

Time runs reverse.
A little secret slips away
from the world.

There is a bang.
A formula is fulfilled.
Champagne corks.

Contents

1 Introduction

In the first chapter we will briefly sketch the most important basics for describing a quantum system. In 1900 when Max Planck found a description for the black body radiation - since that time a hard nut to crack for mathematics - he revealed the top of an iceberg of a fundamental change in the physical paradigm. The iceberg itself has been growing in the water for ages and every now and then physicists were pushed on phenomena that could not be understood in the world view of classical physics. Since its foundation in 1900 quantum mechanics has been growing fast to a big and successful new science paradigm.

To get into the formalism of quantum physics it is essential to introduce the wave-function ψ which is the heart of quantum mechanics as it is the vehicle to describe a quantum state. Besides we will study the concept of entanglement which is the main feature of a quantum mechanical system. Why is that? Entanglement allows correlations of two systems that can no longer be understood as long as we stay in the paradigm of classical physics and stick strictly to a narrow plain down-to-earth understanding of the world. Taking the principle of entanglement seriously claims to overcome "closed"-minded views. In this sense the following chapter will be a challenge.

Figure 1: **Cartoon of Victoria Scott**

2 Description of a quantum state

2.1 Foundation of quantum mechanics

Quantum mechanics is a general theory. The aim is to cover descriptions of subatomic level going to the scale of galaxies and in between the region of our daily macro-world. Whereas historically quantum mechanics focused on the very nano-scale as this was the scale where quantum descriptions would work best, today the interest is attracted by larger scales like centimeters and kilometers.

Discreteness, *Diffraction* and *Coherence* are the three major headings for Leslie E. Ballentine [3] to illustrate phenomenological the essence of quantum theory.

The foundation of quantum mechanics took place in the year 1900. At this time the German physicist Max Planck could explain the black body radiation with the use of light quanta (specific amounts of energy) of the electromagnetic radiation field. This ansatz of discreteness of light was name-giving for quantum theory. For his research Planck received the Nobel prize in 1918.

The first experimental proof for discrete atomic energy levels was given by the German physicists James Franck and Gustav Ludwig Hertz in 1914 [36]. In their experiment they used electrons that were accelerated from a cathode through Hg-gas by means of an adjustable potential applied between the anode and the cathode. In contrast to the classical prediction the current as a function of the voltage does not increase monotonically, but rather displays a series of peaks at multiples of 4,9 eV, which is the required energy to excite an Hg-atom. The sequence of peaks can be explained like follows: When the voltage is sufficient for an electron to achieve a kinetic energy of 4,9 eV, it is able to excite an Hg-atom while losing itself kinetic energy. If the kinetic energy of an electron is more than twice times 4,9 eV it is able to excite two Hg-atoms before it reaches the cathode etc.

Diffraction and *interference patterns* are phenomena that are very similar to waves. In the history of physics there has been a long debate about whether light could be understood as a particle or as wave. The English scientist Isaac Newton who was the intellectual father of gravitation theory and classical mechanics developed in the 17th century a particle theory of light. In a celebrated series of experiments he studied the spectra of sunlight that passes through a prism. He concluded that white light as it is coming from the sun consists of rays of different colours that are refracted in the prism with different angles. With the assumption that light consists of particles he could also bring along his classical theory of reflection of light that would work like classical mechanic reflection of mass points.

His contemporary, the Dutch scientist Christiaan Huygens, had a completely opposite opinion of light: Together with the French physicist Augustin-Jean Fresnel he had founded the so-called *Huygens Fresnel principle* which is a wave-theory of light. It assumes that each point of an advancing wave front is itself the source of a new train of waves. But as Newton had more authority among the scientific community his particle-theory of light was much more accepted.

A u-turn in this respect brought an experiment of the English scientist Thomas Young -

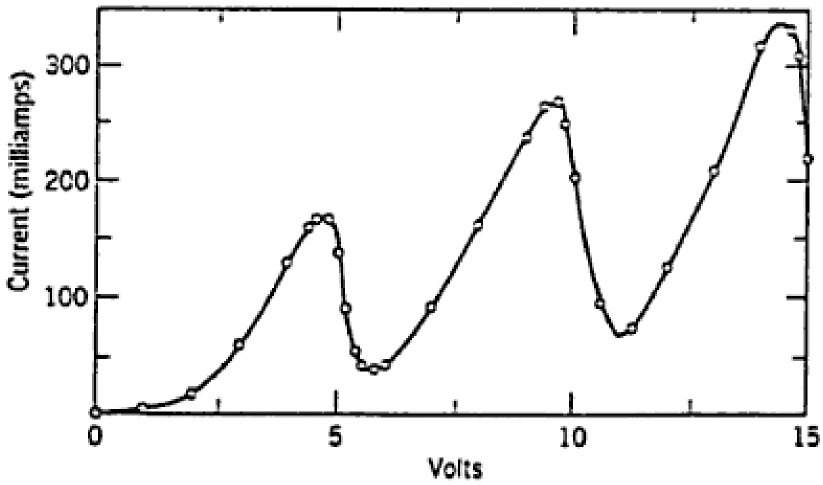

Figure 2: **Current through a tube of Hg gas versus applied voltage, from the data of Frank and Hertz [36]**, Reprinted from Ballentine [3]

the double-slit experiment - as it demonstrates the inseparability of the wave and particle nature of light. Young published his observations in a paper in 1803 [72]. The setup for this experiment is quite easy and works as follows: A beam of light passes trough a thin plate with two parallel narrow slits. Behind the double-slit light is detected on a screen and here the double nature of light appears: The wave character of light causes that the two coherent wave fronts from the slit interfere with each other and an interference pattern can be found. But as light is always found in small units, namely the so-called light particles *photons*, also the particle nature of light is visible in this experiment.

As other fundamental experiments for quantum physics the photo-electric effect and the Compton-effect are known. Both can only be explained with the particle- and wave-nature of light. For his explanation of the photo-electric effect in 1905 [27] Albert Einstein received the Nobel prize in 1922. So Einstein played an important part in the foundations of quantum theory on the one side, on the other - as we will see later - he also became one of its hardest critics.

Following a theoretical conjunction by French physicist Louis de Broglie in 1924 [22] (for which he received the Nobel prize in 1929) the American physicists Clinton Davisson and Lester Germer demonstrated the diffraction in the reflection of electrons from the surface of a nickel-crystal in 1927 [24]. This was the hour of birth for so-called *matter waves*, which means that quantum theory was extended now also for massive particles and not light only. Therefore Davisson received the Nobel prize in 1937.

This leads to the third major key words of quantum mechanics according to Ballentine - *coherence*. What is revolutionary about coherence in quantum theory? Ballentine: "In classical optics, coherence refers to the condition of phase stability that is necessary for interference to be observable. In quantum theory the concept of coherence also refers to phase stability, but it is generalized beyond any analogy with wave motion. In general, a coherent superposition of quantum states may have properties that are qualitatively different from a mixture of the properties of the component states. The possibility of combining quantum states in coherent superpositions that are qualitatively different from their components is perhaps the most distinctive feature of quantum mechanics." [3] What this means in mathematical terms shall be discussed in the following.

2.2 Basic Concepts

2.2.1 Schrödinger equation and the wavefunction

Going back to the Viennese physicist Erwin Schrödinger [62] a quantum system can be described by a vector state ψ living in a Hilbert-space $\mathcal{H}$. ψ is also called the *wavefunction* of a system. Indeed, just some years before Schrödinger published his wave-like description of quantum mechanics, the physicists Werner Heisenberg and Niels Bohr had formulated their mechanics via matrices. Still, when Schrödinger published his famous series of articles in 1929 "Quantisierung als Eigenwertproblem" in Annalen der Physik [62] in which he showed the equivalence of his wave-like formulation of quantum mechanics and the matrice-approach and also came forward with the important *Schrödinger-equation*, his formulation was instantaneously accepted as the standard formulation of quantum mechanics, as it is much easier to deal with than with the matrices after Heisenberg and Bohr.

For his fundamental approaches to quantum physics Erwin Schrödinger was awarded the Nobel Prize for Physics together with the French physicist Paul Dirac in 1933. Still, the way how Schrödinger found his equation resembles a fairy story of fortuity: We have already heard about Louis de Broglie and his suggestion to adapt wave-characters to particles with mass as well. De Broglie himself was member of a noble French family so when he sent in his dissertation with the bold conjunction the French community was unsure how to deal with it: On the one hand they thought Broglie's thesis was nonsense, on the other - due to his social standing they did not dare to tell him. So the thesis was sent to Albert Einstein to give his opinion on it. Surprisingly Einstein thought that de Broglie's idea was brilliant. Through Einstein, Schrödinger gained knowledge. In a letter to Einstein on November, 3rd 1925 Schrödinger wrote: "*mit größtem Interesse habe ich vor einigen Tagen die geistvollen Théses von Louis de Broglie gelesen.*"[1][51]

So Schrödinger was highly impressed but he also had his problems with the concept of particle-waves. In a letter to Alfred Landé on November, 16th, 1925 he wrote:

> "*Ganz besonders freut mich Ihre Mitteilung, dass Ihre Arbeit ein "Zurück zur Wellentheorie" sein sollte. Auch ich neige sehr dazu. Ich habe mich dieser Tage stark mit Louis de Broglies geistvollen Théses beschäftigt. Ist außerordentlich anregend, hat aber doch noch sehr große Härten. Ich habe vergebens versucht, mir von der Phasenwelle des Elektrons auf der Kepplerbahn ein Bild zu machen.*"[2][51]

[1] "With greatest interest I have read the brilliant thesis of Louis de Broglie some days ago."

[2] "Especially about your announcement that your work shall be a "back to wavetheory". I agree with

The Dutch physicist and chemist Peter Debye (Nobel prize for chemistry in 1936) who was - just like Schrödinger - professor in Zurich at that time asked him to give a talk on matter-waves in Zurich. Schrödinger did so. In his talk he was confronted with the suggestion from the audience to give a wave-like equation for this character. And indeed in his following cure-stay at Arosa around Christmas 1925 and New Year's Eve 1926 he achieved the break-through and discovered the wave-equation for matter-waves - the so-called *Schrödinger equation*.

As a starting point Schrödinger used the classical wave equation for plane waves $\psi(t, x)$:

$$\psi(t, x) = \psi_0 e^{i(kx \pm \omega t)} \tag{1}$$

Differentiated and multiplied with the factor $i\hbar$ and using the quantum properties for energy $E = \hbar\omega$ and momentum $p = \hbar\ k$ makes:

$$i\hbar \frac{\partial}{\partial t}\psi(t, x) = \hbar\omega\psi = E\psi \tag{2}$$

$$-i\hbar\nabla\psi(t, x) = \hbar k\psi = p\psi$$

$$-\hbar^2 \Delta\psi(t, x) = (\hbar k)^2\psi = p^2\psi \tag{3}$$

If we then consider the nonrelativistic energy-momentum-relation with time-independent potential $V = V(x)$

$$E = \frac{p^2}{2m} + V(x) \tag{4}$$

we easily arrive at the *time-dependent Schrödinger equation*:

$$i\hbar\psi(t, x) = (-\frac{\hbar^2}{2m}\Delta + V(x))\psi(t, x) = H\psi(t, x) \tag{5}$$

where $H = -\frac{\hbar^2}{2m}\Delta + V(x)$ is called the *Hamilton-operator* of the system.

Note:

With the relativistic energy-momentum-relation

$$E^2 - p^2c^2 = m^2c^4 \tag{6}$$

the so-called *Klein-Gordon-equation* is obtained:

$$\left[\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m^2c^2}{\hbar^2}\right]\psi(x, t) = 0 \tag{7}$$

Interestingly although this equations holds the correct energy-momentum relation in practice it turns out that it is only valid for particles without spin. Therefore in general

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that too. In the last days I have dealt intensively with Louis de Broglie's brilliant thesis. It is extremly simulating, but still has great severities. Without avail I tried to imagine the phasewave of the electron on a Keppler-path."

(for particles with spin) still the Schrödinger-equation is used although it is not consistent for the relativistic case.

So we see the discovery of Schrödinger's equation shows a deep context between quantum mechanics and classical mechanics, thus the two theories seem to be much more related to each other than physicists were thinking in the first years of development of quantum theory.

But how can we interpret this Schrödingian wavefunction ψ? Due to Max Born $|\psi|^2$ corresponds to the amplitude of the probability to find a particle at a certain time. But ψ itself is not a physical property that can be measured in nature but it represents just a physical concept to describe the statistics of experimental outcomes properly.

The fact that the wavefunction does not correspond to a physical measure leads to the question how to interpret the formalism. We will focus later on this question in chapter 6. For the beginning we will just give a sketch that tries to illustrate the quality of information provided by the wavefunction ψ. The poster below was composed together with Reinhold A. Bertlmann in 2009 and was shown at the exhibition "Chaos" at G.A.S - station, Berlin Tempelherrengasse 22 from Oct. 7th 2009 to Jan. 27th 2010.

2.2.2 The density matrix

The state vector ψ contains all information about a quantum system. But in many cases detail-information of a system are not known, for instance when a system interacts with its environment [67]. So if we want to describe a quantum system that is not isolated from its environment we have to replace the description by the state vector by a new concept - this will be the *density matrix*-notation.

The definition of the density matrix is motivated by the structure of an expectation value in quantum mechanics, which is given as follows: Let the observed quantum system be in state ψ. We consider the observable A in state ψ. Its expectation value is given by the tensor product:

$$\langle A \rangle = \langle \psi | A | \psi \rangle \tag{8}$$

Here we use Dirac's bra and ket notation that is very common in quantum mechanics. In this notation the vectors in any linear vector space V are called "ket"-vectors and denoted in brackets like this: $|.\rangle$. Whereas the linear functionals in the dual space $V's$ are called "bra"'s. They are the adjoint operators and therefore they are written in mirrored brackets: $\langle .|$.

Analogously to the expectation value we define the *density operator* ρ (for pure states):

$$\rho = |\psi\rangle\langle\psi| \tag{9}$$

ρ is governed by the following properties (only of interest for mathematically advanced readers):

- ρ is positive:
$$\rho \geq 0 \tag{10}$$

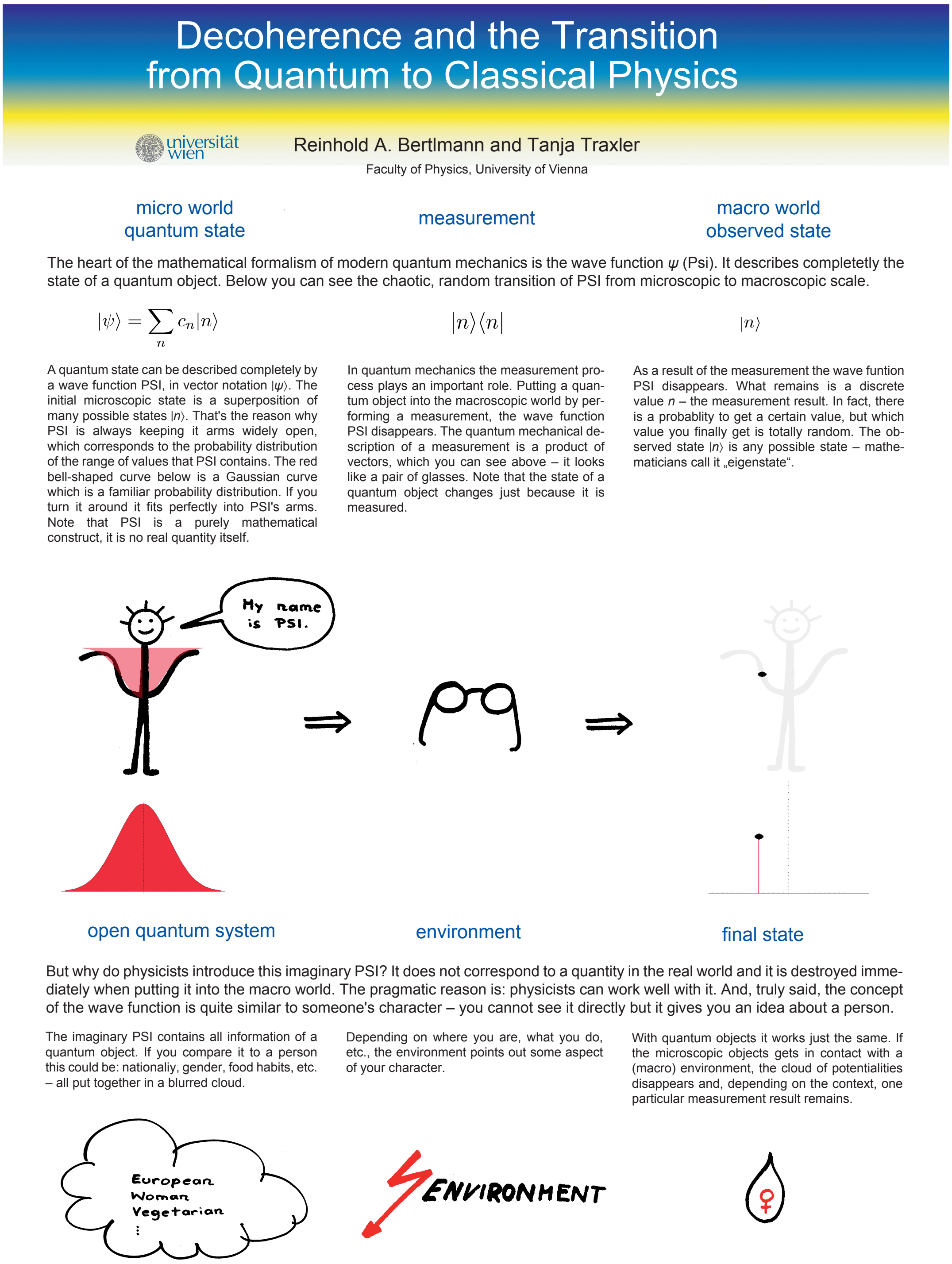

Figure 3: **Decoherence and the Transition from Quantum to Classical Physics**, R. A. Bertlmann, T. Traxler, 2009

By saying: "ρ is positive", we mean, that the eigen-values of ρ are always bigger than or equal to 0. Differently expressed: for all φ it is true that:

$$\langle\varphi|\rho|\varphi\rangle = \langle\varphi|\psi\rangle\langle\psi|\varphi\rangle = |\langle\varphi|\psi\rangle|^2 \geq 0 \tag{11}$$

- ρ is self-adjoint:

$$\rho = \rho^\dagger \tag{12}$$

Proof:

Commonly the adjoint $D^\dagger$ of an operator $D = |\varphi\rangle\langle\psi|$ is defined by $D^\dagger = (|\varphi\rangle\langle\psi|)^\dagger = |\psi\rangle\langle\varphi|$, which gives for ρ: $\rho^\dagger = |\psi\rangle\langle\psi|^\dagger = |\psi\rangle\langle\psi| = \rho$.

- Trace of ρ is 1:

$$\mathrm{tr}\rho = 1 \tag{13}$$

The common definition of the trace of an operator D is: $\mathrm{tr}D = \sum_n \langle n|D|n\rangle$ where $\{|n\rangle\}$ is an arbitrary complete orthogonal basis. With this definition we can calculate the trace of ρ:

$$\mathrm{tr}\rho = \sum_n \langle n|\rho|n\rangle = \sum_n \langle n|\psi\rangle\langle\psi|n\rangle = \sum_n \langle\psi|n\rangle\langle n|\psi\rangle = \langle\psi|\psi\rangle = 1$$

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$$\rho^2 = \rho \tag{14}$$

Proof:

$$\rho^2 = |\psi\rangle\langle\psi|\psi\rangle\langle\psi| = |\psi\rangle\langle\psi| = \rho \tag{15}$$

We can now rewrite the expectation value of an observable A like the following:

$$\langle A\rangle = \mathrm{tr}\rho A \tag{16}$$

which coincides with the definition: $\langle A\rangle = \langle\psi|A|\psi\rangle$.

Proof:

$$\mathrm{tr}\rho A = \sum_n \langle n|\psi\rangle\langle\psi|A|n\rangle = \sum_n \langle\psi|A|\underbrace{n\rangle\langle n}|\psi\rangle = \langle\psi|A|\psi\rangle = \langle A\rangle \tag{17}$$

Having found some general features of a density matrix ρ of pure states we will now classify between pure states and mixed states by certain features of the density matrix ρ. In the following we always consider an ensemble of objects, in contrast to isolated single quantum states.

2.3 Pure states and mixed states

In the following we will give a short overview about the mathematical features of pure and mixed states therefore this chapter addresses mainly to mathematical advanced readers.

- **Pure States:** As we have seen above pure quantum states can be described properly by a state vector $|\psi\rangle$ living in a Hilbert space $\mathcal{H}$. If we choose a basis $|n_i\rangle$ of the Hilbert space, every $|\psi\rangle$ can be written in the linear combination of these $|n_i\rangle$'s:

$$|\psi\rangle = \sum_{i=1}^{d} c_i |n_i\rangle \tag{18}$$

where the coefficients c_i are complex numbers and d denotes the dimension of the Hilbert space. To normalize the state we claim

$$\langle\psi|\psi\rangle = 1 \tag{19}$$

which goes hand in hand with the demand that the sum over all probabilities is equal to 1, therefore we have to claim for the coefficients:

$$\sum_{i=1}^{d} |c_i|^2 = 1 \tag{20}$$

Also if we consider more than one object while still the state of the composite system remains pure. This is the case if all considered objects are in one and the same state. To proof probability-predictions in the experiment, we have to consider an ensemble of objects with the same preparation.

Example:

Let us consider the state $|\psi\rangle = \sum_n c_n|n\rangle$ with the transition-coefficient $c_n = \langle n|\psi\rangle$, where $A|n\rangle = a_n|n\rangle$. Then

$$\langle A\rangle_\psi = \sum_n |c_n|^2 a_n = \sum_n a_n \frac{N_n}{N} \tag{21}$$

where $|c_n|^2$ is the probability of the transition and N_n the number, how often eigenvalue a_n was measured and N is the ensemble number. Then the density matrix is characterized by:

$$\rho = |\psi\rangle\langle\psi| \tag{22}$$

which fulfills the following properties:

$$\langle A\rangle = \mathrm{tr}\rho A\ , \quad \rho^\dagger = \rho\ , \quad \rho \geq 0\ , \quad \rho^2 = \rho\ , \quad \mathrm{tr}\rho = 1\ , \quad \mathrm{tr}\rho^2 = 1 \tag{23}$$

The time-evolution of pure states is governed by the time-dependent **Schrödinger equation**:

$$\frac{\partial}{\partial t}|\psi(t)\rangle = -iH(t)|\psi(t)\rangle \tag{24}$$

where $H(t)$ is the Hamilton-operator of the system and $\hbar$ is set to 1.

- **Mixed States:** If we consider a quantum system that has a certain probability p_i to be in the state $|\psi_i\rangle$ we call it a mixed state. Mixed states cannot be described just by a state vector $|\psi\rangle$. Instead we use the formulation of the *density matrix*, which we have introduced above [9] for pure states. For mixed states the density operator is defined:

$$\rho = \sum_i p_i|\psi_i\rangle\langle\psi_i| = \sum_i p_i\rho_i \tag{25}$$

where $\sum_i p_i = 1$ and the probabilities $p_i \geq 1$.

This more general case is very important for quantum statistics. In this case not all systems (objects) are in the same state. We consider N objects, let N_i objects of them be in state $|\psi_i\rangle$. The probability p_i that any object of the ensemble is in state $|\psi_i\rangle$ is given by

$$p_i = \frac{N_i}{N} \tag{26}$$

where N_i is the number of objects in state $|\psi_i\rangle$ and N is the total ensemble number. Then the expectation value of A is given by:

$$\langle A\rangle = \sum_i p_i\langle\psi_i|A|\psi_i\rangle \tag{27}$$

with the following properties:

$$\langle A\rangle = \mathrm{tr}\rho A \ , \ \rho^\dagger = \rho \ , \ \rho \geq 0 \ , \ \rho^2 \neq \rho \tag{28}$$

$$\mathrm{tr}\rho^2 < 1 \tag{29}$$

Proofs:

$\mathrm{tr}\rho A = \sum_{n,i} p_i\langle n|\psi_i\rangle\langle\psi_i|A|n\rangle = \sum_i p_i \sum_n\langle\psi_i|A|n\rangle\langle n|\psi_i\rangle = \sum_i p_i\langle\psi_i|A|\psi_i\rangle = \langle A\rangle$

$\rho^2 = \sum_i\sum_j p_ip_j|\psi_i\rangle\langle\psi_i|\psi_j\rangle\langle\psi_j| = \sum_i p_ip_i|\psi_i\rangle\langle\psi_i| \neq \rho$

$\mathrm{tr}\rho^2 = \sum_n\langle n|\sum_i\sum_j p_ip_j|\psi_i\rangle\langle\psi_i|\psi_j\rangle\langle\psi_j|n\rangle = \sum_i\sum_j\sum_n p_ip_j\langle\psi_i|\psi_j\rangle\langle\psi_j|n\rangle\langle n|\psi_i\rangle =$
$= \sum_i\sum_j p_ip_j|\langle\psi_i|\psi_j\rangle|^2 < \sum_i p_i\sum_j p_j = 1$

for all $|\varphi\rangle$: $\langle\varphi|\rho|\varphi\rangle = \sum_i p_i\langle\varphi|\psi_i\rangle\langle\psi_i|\varphi\rangle = \sum_i p_i|\langle\varphi|\psi_i\rangle|^2 \geq 0$

As the last property of (29) differs from the case above (23) where we have considered pure states we can associate $\delta = \mathrm{tr}\rho^2$ as a *measure of mixedness*.

But what is the physical difference between pure and mixed states?

pure state: A pure state is a coherent superposition of states, e.g. $|\uparrow\rangle, |\downarrow\rangle$. Off-diagonal-elements do exist, they contain the phase information and are responsible for coherence.

mixed state: A mixed state is an incoherent superposition of states, e.g. $|\uparrow\rangle, |\downarrow\rangle$. In this case off-diagonal-elements do not exist, so the phase information is lost, at least partially. It is lost totally for totally mixed state where $|\overrightarrow{a}| = 0$.

Comparison:

The density matrix of the totally mixed state has the following structure:

$$\rho_{\text{mix}} = \frac{1}{2}(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|) = \frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = \frac{1}{2}\mathbb{1} \tag{30}$$

The density matrix of the pure state with $\theta = 90$, $\varphi = 0$ is given by:

$$\rho_{\text{pure}} = \begin{pmatrix}\cos^2\frac{\theta}{2} & \frac{1}{2}\,\sin\theta e^{-i\varphi}\\ \frac{1}{2}\,\sin\theta e^{i\varphi} & \sin^2\frac{\theta}{2}\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} \tag{31}$$

Now we will consider the expectation value of an operator, so to say the measurement outcome. If we consider the spin along the z-axis, we can see that we can find no difference between the mixed and the pure state:

$$\langle\sigma_z\rangle_{\text{mix}} = \text{tr}\rho_{\text{mix}}\sigma_z = \text{tr}\frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} = 0 \tag{32}$$

$$\langle\sigma_z\rangle_{\text{pure}} = \text{tr}\rho_{\text{pure}}\sigma_z = \text{tr}\frac{1}{2}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} = 0 \tag{33}$$

In both cases 50% of the spins are oriented along $\uparrow$ and 50% along $\downarrow$. Now we choose projections on a definite spin, therefore we define the following projection operators:

$$P_{\uparrow} = |\uparrow\rangle\langle\uparrow| = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix} \tag{34}$$

$$P_{\downarrow} = |\downarrow\rangle\langle\downarrow| = \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}$$

$$P_{+} \equiv \rho(\theta = 90, \varphi = 0) = \frac{1}{2}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} \tag{35}$$

$$P_{-} \equiv \rho(\theta = 90, \varphi = 180) = \frac{1}{2}\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$$

Consequently, for the expectation values we obtain:

$$\langle P_\uparrow \rangle_{\text{mix}} = \text{tr}\ \rho_{\text{mix}} \text{P}_\uparrow = \text{tr} \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \frac{1}{2} \tag{36}$$

$$\langle P_\uparrow \rangle_{\text{pure}} = \text{tr}\ \rho_{\text{pure}} \text{P}_\uparrow = \text{tr} \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1}{2}$$

$$\langle P_\downarrow \rangle_{\text{mix}} = \text{tr} \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1}{2} \tag{37}$$

$$\langle P_\downarrow \rangle_{\text{pure}} = \text{tr} \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1}{2}$$

We see: There is no difference up to now between the mixed and the pure states. But if we choose the spin measurement along the x-axis we get:

$$\langle P_+ \rangle_{\text{mix}} = \text{tr}\ \rho_{\text{mix}} \text{P}_+ = \text{tr} \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \frac{1}{4} \text{tr} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \frac{1}{2} \tag{38}$$

$$\langle P_- \rangle_{\text{mix}} = \text{tr}\ \rho_{\text{mix}} \text{P}_- = \text{tr} \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \frac{1}{2} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = \frac{1}{4} \text{tr} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = \frac{1}{2}$$

Whereas:

$$\langle P_+ \rangle_{\text{pure}} = \text{tr}\ \rho_{\text{pure}} \text{P}_+ = \text{tr} \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \frac{1}{2} \text{tr} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = 1 \tag{39}$$

$$\langle P_- \rangle_{\text{pure}} = \text{tr}\ \rho_{\text{pure}} \text{P}_- = \text{tr} \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \frac{1}{2} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = \frac{1}{4} \text{tr} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = 0$$

Resumée

For **pure states** it is characteristic that there exists a maximal test such that the outcome occurs with 100%. But for **mixed states** such a test is not possible.

2.4 Time evolution

The time-evolution of the density matrix is given by the *von Neumann equation*. To derive it [67] we start with the Schrödinger-equation, which has been introduced in chapter (2.2):

$$i\hbar \frac{\partial}{\partial t} |\psi_i\rangle = H |\psi_i\rangle \tag{40}$$

If $H = H^\dagger$ is hermitian, the adjoint equation is given by:

$$-i\hbar \frac{\partial}{\partial t} \langle\psi_i| = \langle\psi_i| H \tag{41}$$

We apply this equations for the density matrix ρ and get:

$$i\hbar\frac{\partial}{\partial t}\rho = i\hbar\sum_i p_i\left(|\dot{\psi_i}\rangle\langle\psi_i| + |\psi_i\rangle\langle\dot{\psi_i}|\right) = \sum_i p_i\left(H|\psi_i\rangle\langle\psi_i| - |\psi_i\rangle\langle\psi_i|H\right) = H\rho - \rho H \tag{42}$$

Von Neumann-equation:

$$i\hbar\frac{\partial}{\partial t}\rho = [H, \rho] \tag{43}$$

Classical analogy: The von Neumann equation is analogous to the Liouville equation in classical statistical mechanics.

Liouville equation:

$$\frac{\partial}{\partial t}\rho = \{H, \rho\} \tag{44}$$

with the *Poisson brackets*:

$$\{\ ,\ \} = \frac{\partial}{\partial q}\frac{\partial}{\partial p} - \frac{\partial}{\partial p}\frac{\partial}{\partial q} \tag{45}$$

Here ρ is the classical density distribution in two variables p, q: $\rho = \rho(p, q)$.

The general equation of motion in statistical mechanics is described by the *Liouville*-equation:

$$\frac{\partial}{\partial t}\rho(t) = \mathcal{L}\rho(t) \tag{46}$$

where $\mathcal{L}$ is the so-called *Liouville*-operator. It has the formal solution:

$$\rho(t) = Te^{[\int_{t_0}^{t}\mathcal{L}(t')dt']}\rho(t_0) \tag{47}$$

The symbolization of this classical analogy is known as **Dirac rule**:

$$\{,\} \to -\frac{i}{\hbar}[,] \tag{48}$$

Here we can nicely see: The density matrix formalism introduced by John von Neumann and by Lev Landau is suitable to extend the tools of classical statistical mechanics to the quantum domain.

From the Schrödinger equation we also get the unitary *time-shift operator*:

$$U(t, t_0) = e^{-\frac{i}{\hbar}H(t-t_0)} \tag{49}$$

The density-shifts are given by:

$$\rho(t) = U(t, t_0)\rho(t_0)U^\dagger(t, t_0) \tag{50}$$

Proposition 1. *trρ^2 is time independent!*

This means that pure states remain pure and mixed states remain mixed (as long as they are isolated for any time).

Proof:

$$\text{tr } \rho^2(t) = \text{tr } U\rho(t_0)U^\dagger U\rho(t_0)U^\dagger = \text{tr } \rho^2(t_0)UU^\dagger = \text{tr } \rho^2(t_0)$$

2.5 Heisenberg's uncertainty relation

Can the world be observed arbitrary precisely as we want to? The thinking school of completely oberservability and determinism is known as "Laplace's demon" going back to a cite by the physicist Pierre-Simon Laplace [49] around 1795:

> "*We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes.*"

Before Heisenberg in fact it has been a matter of personal epistemological preferences to follow the ansatz of the complete describability of the world on the one hand or the limitations of knowledge on the other. In 1927 Werner Heisenberg [41] formulated in the course of quantum mechanics his concept of a principal uncertainty, which appears while measuring two complementary measures like momentum and place or energy and time of a quantum object at the same time.

Heisenberg's derivation of the uncertainty relation is an example for intuition in natural science as such. Heisenberg introduced some special case of a microscope - the so-called *Heisenberg's microscope* - but with this proposal he didn't bring the problem to the point. However, in the end he found the right relation.

Heisenberg's original sketch which he gave in 1930 [42] can be found below.

The idea is to detect the position of an electron via the scattering of light and to picture it on a screen. Therefore the electron shall move in such a distance from the apparatus that the rays scattered from it in the form of a cone with the angle ϵ. Then the uncertainty of the measurement of the x-coordinate of the electron which arises just from the laws of optics and not the efficiency of the microscope will be:

$$\Delta x = \frac{\lambda}{\sin\epsilon} \tag{51}$$

where λ is the wave-length of the light. To detect the electron at least one photon must travel from the electron through the microscope and this is the point where the

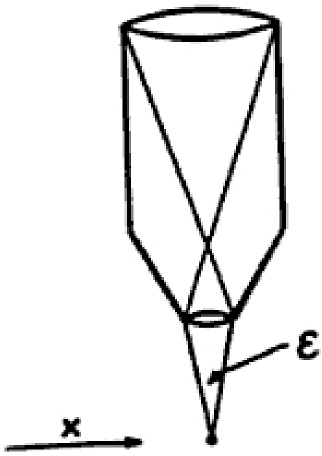


Figure 4: **Heisenberg's sketch of Heisenberg's microscope**, A gedankenexperiment by Werner Heisenberg to illustrate the uncertainty relation. Source: [42]

Compton effect comes into account: From the photon the electron gets a Compton recoil of the order of the magnitude h/λ - but its direction cannot be known precisely since the scattering of the photon is unknown within the bundle of rays. Thus for the uncertainty of the momentum of the electron in direction x we obtain:

$$\Delta p_x = \frac{h}{\lambda}\text{sin}\epsilon \tag{52}$$

By inserting the momentum uncertainty (52) to the uncertainty of the position given by equation (51) it follows the *uncertainty relation*:

$$\Delta x \Delta p_x = h \tag{53}$$

It can be shown that this uncertainty can be reduced by the factor 2π. Thus for the limit we get:

$$\Delta x \Delta p_x \geq \frac{\hbar}{2} \tag{54}$$

But in general the *Uncertainty Relation* shows that position and momentum of a particle cannot be measured arbitrary precisely at the same time. An analogous inequality can be found for energy and time measurements. For his approach Werner Heisenberg was awarded the Nobel price in 1932.

In his book "The physical principles of the quantum theory" [42], in which Heisenberg introduces Heisenberg's microscope, raises and shoots down two questions to overcome the uncertainty barrier.

Firstly Heisenberg suggests to move the microscope to reduce the width of the light-cone. But this method provokes the question of the position of the microscope - which is also ruled by the uncertainty relation. Therefore the uncertainty is not reduced but just shifted.

Secondly he proposes to measure the electron and a fixed scale simultaneously through the moving microscope. Heisenberg: "This seems to afford an escape from the uncertainty principle." But then the observation requires at least two photons - one to detect the electron and one to measure the scale and the measurement of the recoil is no longer sufficient to determine the direction of the light. Heisenberg: "And so on *ad infinitum.*"

Another argument that is sometimes used to overcome the uncertainty principle is the following: If in a first measurement the momentum of a particle is measured with maximal preciseness and a precisely determined time later the position is measured maximally precisely then the position from the first measurement or the momentum of the second measurement point could be calculated easily. The problem of this ansatz is that since we deal with quantum objects, here one cannot assume that the momentum of a propagating particle stays precisely the same over a given period of time. The exact quantity of momentum is just determined in the measurement point and for no other point later. Secondly, since momentum and energy are closely related in this case, time cannot be measured precisely at the same time with momentum.

Resumee: Since yet no violation of the uncertainty principle has been found therefore it states a fundamental principle of quantum physics. It puts Bohr's ideas of complementary measures like position and momentum or energy and time into a new light and overcomes epidemiological deterministic world views. Werner Heisenberg's pupil Fritjof Capra interprets:

> "*Die fundamentale Bedeutung des Unsicherheitsprinzips liegt darin, dass es die Grenzen unserer klassischen Begriffe in präzise mathematische Form bringt... Unsere klassischen Begriffe, die aus unserer gewöhnlichen makroskopischen Erfahrungen stammen, sind für die Beschreibung dieser Welt nicht ausreichend. Zuerst einmal ist der Begriff von einer selbstständigen physikalischen Einheit, eines Teilchens, eine Idealisierung ohne fundamentale Bedeutung. Es kann nur durch seine Beziehungen zum Ganzen definiert werden, und diese sind statistischer Natur, mehr Wahrscheinlichkeiten als Sicherheiten. Wenn wir die Eigenschaften einer solchen Einheit mit klassichen Begriffen wie Aufenthaltsort, Energie etc. beschreiben, stellen wir fest, dass es Begriffspaare gibt, die zusammenhängen und die gleichzeitig nicht genau definiert werden können.*"[1] [23]

[1] "The fundamental meaning of the uncertainty principle is that it brings the borders of our classical terms into a precise mathematical shape... Our classical terms that come from our common macroscopic experiences are not sufficient for the description of this world. Firstly the term of a stand-alone physical unity, a particle, an ideal without fundamental meaning. It can only be defined through its relations to a whole and those relations are of statistical nature, more probabilities than securities. If we describe the properties of such a unity with classical terms like position, energy etc. we find out that there are pairs of terms that belong together and that cannot be defined exactly simultaneously."

2.6 Entanglement

Another fundamental principle of quantum physics that stands in complete contrast to classical physics will be discussed in the following chapter: entanglement.

In 1935 Erwin Schrödinger published another famous series of articles "Die gegenwärtige Situation in der Quantenmechanik" [63] that would play an important role for quantum mechanics - this time not concerning the basic concepts of the formulation (as he did in 1929) but more concerning a deeper understanding of the new qualities of quantum physics.

Just in the same year Einstein, Podolsky and Rosen [28] who felt critical about the quantum formalism, constructed a gedankenexperiment for a quantum system of two distant particles to demonstrate: *Quantum mechanics is incomplete.* Although the physicists around Einstein wanted to reject quantum mechanics, indeed their paper inspired Schrödinger, as he said, to get an idea about a major feature of quantum physics which is even today one of the most fundamental keys to use it in quantum information like quantum teleportation, quantum cryptography, quantum computing, et cetera - this is what Schrödinger called *entangled states* or in his German phrasing *Verschränkte Zustände.*

Mathematically entanglement is defined via separability:

Definition 1. Separability
A state ρ is called separable if it can be written as the convex combination of product states like:

$$\rho = \sum_i p_i \rho_i^{(A)} \otimes \rho_i^{(B)} \tag{55}$$

where $\rho_i^{(A)}$ and $\rho_i^{(B)}$ are the density matrices of the subsystems and the probabilities satisfy $p_i \geq 1$ and $\sum_i p_i = 1$.

With this entanglement can be defined due to Werner [70]:

Definition 2. Entanglement *A quantum state ρ is called entangled if it is not separable.*

For many years there were hot discussions about entanglement and connected with that about the completeness of the quantum mechanical description. Whereas Einstein called entanglement "spukhafte Fernwirkung" ("spooky action at distance") [30], for Schrödinger it was *the essence* of quantum mechanics. Just 30 years later (when both, Schrödinger and Einstein had already died) the feature of entanglement was given also physical value when the Irish physicist John Stewart Bell [4] formulated his famous *Bell's inequalities* with which he could bring the since then purely philosophical debate to an experimental point.

Bell's inequalities

In 1964 Bell showed that the statistical correlations between the measurement outcomes of suitably chosen different quantities on the two systems are inconsistent with an inequality derived from Einstein's separability and locality assumptions, which is called

Bell's inequality. It implies an upper limit on the strength of correlations for any theory obeying "local realism". According to the inequality, quantum mechanical predictions can lead to correlations stronger than this limit, leading to results that are experimentally distinguishable from the results of a broad class of local hidden-variable theories. And the consequences of this statement are revolutionary - even as much as the foundation of quantum mechanics itself, thinks Alan Aspect: "I think it is not an exaggeration to say that the realization of the importance of entanglement and the clarification of the quantum description of single objects have been the root for a *second quantum revolution*, and that John Bell was its prophet", he wrote in the introduction to Bell's book "Speakable and unspeakable in quantum mechanics" [5] in 2003.

So in contrast to the discussions before Bell, the *Bell's inequalities* are experimentally testable. Indeed it could be shown, that quantum mechanics is complete and non-local. The first ones to do this experimental proof was the group around the French physicist Alain Aspect in 1982 [2].

And what can entanglement be used for? Quantum entanglement in general means that the objects that make up the system are linked in a way that the quantum state of one constituent of the system can no longer be described adequately without full mention of its counterparts. So to say there exist some kind of *quantum correlations*, in contrast to *classical correlations*. Even if two particles of an entangled state are physically separated by a great distance, they behave in some respects as a single entity.

The usefulness of entanglement emerges because it allows us to overcome the constraint of so called *Local Operations and Classical Communication* LOCC [57]. The LOCC constraint is a restriction that has both technological and fundamental motivations and arises in many physical settings involving quantum communication across distances.

Historical Remark:

Although the discussion about entanglement goes on now for about 80 years, there is still a lot of interest on the topic and it seems like the interest on entanglement increases year after year. In the following I want to give two examples to illustrate this growth of interest.

Example 1 (numerical): At the "Euro Science Open Forum" 2010 the Austrian quantum physicist Anton Zeilinger was invited to give a talk in the huge auditorium about the foundations of quantum physics [33]. One of the main slides Zeilinger showed to his audience was a graph of the citations of the paper of Einstein, Podolski and Rosen [28] per year. Summing over the citations Zeilinger found out that they emerge nearly exponentially per year: Whereas in the 30ies when the paper was published it was nearly not cited at all, in the 60ies there was an increase of interest when Bell published his inequalities and now in the age of the development of quantum information technology there are so many citations as have never been before.

Example 2 (social): In Schrödinger's publication which was already mentioned above [63] he introduced a famous gedankenexperiment known as *Schrödinger's cat.* The basic idea of this experiment is, that a cat is put into a black box together with an atom of a radioactive substance and a flask of poison which is released when the atom decays. After some time there is a certain probability that the particle has decayed and that therefore the cat is dead. Before we have a look into the box, the cat is in entangled state of "dead" and "alive".

Since Schrödinger published his gedankenexperiment many scientific and popular books and articles mentioned Schrödinger's cat or used the phrasing for a catchy title. A notable use of the metaphor I found in the editorial of the week-end-edition of the Austrian newspaper "Der Standard" on July 24th, 2010. Notable in the sense that description of the concept of Schrödinger's cat was given place the whole head paragraph (see 2.6), although the topic of the article was the at that time published stress-test for banks and had nothing to do with quantum physics.

Verdorbener Stresstest-Brei

Zu viele Köche waren beteiligt – Europa braucht eine starke Bankenaufsicht

Günther Oswald

Auf den österreichischen Physiker Erwin Schrödinger geht ein interessantes Gedankenexperiment zurück, bekannt unter dem Titel „Schrödingers Katze". Es geht dabei um die Wahrscheinlichkeit verschiedener Wirklichkeiten. Die Überlegung lautet: Steckt man eine Katze gemeinsam mit einem instabilen Atomkern in eine undurchsichtige Kiste, stirbt das Tier innerhalb einer Stunde mit einer Wahrscheinlichkeit von 50 Prozent. Schaut man nicht in die Kiste, weiß man also nicht, ob die Katze bereits tot ist oder noch lebt. Gewissheit bietet nur ein Blick in die Kiste. Schrödinger wollte damit zwar die Unvollständigkeit der Quantenmechanik demonstrieren, das Gedankenspiel passt aber auch sehr gut zur aktuellen Debatte über die Banken-Stresstests.

Am Freitag wurde uns von den europäischen Bankenaufsehern erklärt, warum das Bankensystem ganz gut dasteht, also am Leben ist. Der Gewiss-

Und das birgt bekannterweise die Gefahr des Verderbens. Die eigentlichen Prüfer von der Vereinigung der europäischen Bankaufseher (CEBS) konnten nicht in Ruhe und unabhängig arbeiten. Die Zentralbanken und Finanzminister durften ebenso mitmischen wie die geprüften Banken selbst. Die Vergleichbarkeit der Ergebnisse ist laut Experten nur sehr eingeschränkt gegeben. Schuld daran sind unterschiedliche Regeln in den teilnehmenden Ländern für die Kapitalquoten und die Bilanzierung.

Der Versuch der Politik, Ruhe in die Märkte zu bekommen, ist durchaus begrüßenswert. Angesichts der Börsenreaktion in den letzten Tagen ist das offenbar auch teilweise gelungen – Banktitel zählten zu den Gewinnern.

Einen nachhaltigen Vertrauensbeweis darf man sich von den Stresstests aber nicht erwarten. Dafür waren die Ergebnisse zu vorhersehbar. Der peinliche und chaotische Ablauf der Tests hat dafür einmal mehr gezeigt, wie dringend nötig eine starke und unabhängige europäische Bankenaufsicht ist. Die Verhandlungen zwischen EU-Parlament und Kommission müssen dringend zu einem positiven Ende gebracht werden.

KOPF DES TAGES

Teamplayerin auf psychologischer Mörderjagd

Nein, „Profilerin" will sie eigentlich nicht genannt werden, meint Christina Rieder, wenn man sie nach ihrer Arbeit fragt.

Die funktionieren nach standardisierten Abläufen, die im gesamten deutschsprachigen Raum gleich sind. Zu welchen Ergebnissen

Figure 5: **Facsimile of the Editorial of "Der Standard" on July 24th, 2010**

2.7 Entanglement measures

As we have seen, in principle it is well defined whether a state is separable or entangled, but in practice it turns out that given a general state it might be difficult to check if it is separable, because one would have to try all possible ways to factorize it.

The development of the *Bell inequalities* can be viewed as an early attempt to quantify the amount of entanglement of a state. The more the inequality is violated, the more the state is entangled. Even now there is no general theory for the quantification of entanglement but a huge field of research has emerged around the theory of *entanglement measures*.

Just for pure states there exists something like an *entanglement measure* which is the *von Neumann entropy*. For mixed states one general entanglement measure has not yet been found and a lot of different measures try to cover as much of the Hilbert space as possible. We will just pick out one of them - the *Concurrence* - (only valid for qubits) because we want to work with it later. Note: The introduction of this measures will be of course rather technical.

2.7.1 Von Neumann entropy

Let be a given state ρ. Then the **von Neumann entropy** is defined by

$$S(\rho) = -\text{tr } \rho \text{ log}\rho \tag{56}$$

Note:

Here log = ln, whereas for qubits it's better to use $\log_2 x = \frac{\ln x}{\ln 2}$.
The trace of the operator (density matrix) is defined via its eigenvalues. Thus for the entropy we get:

$$S(\rho) = -\sum_i \lambda_i \text{ log}\lambda_i \tag{57}$$

For a totally mixed state we have:

$$\rho_{\text{mix}} = \frac{1}{d}\mathbb{1}_{\text{d}} \tag{58}$$

$$S(\rho_{\text{mix}}) = -\text{tr } \rho_{\text{mix}} \text{ log}\rho_{\text{mix}} = -\text{tr}\frac{1}{d}\mathbb{1} \text{ log}\frac{1}{d}\mathbb{1} = -\sum_i \frac{1}{d}\lambda_i \text{ log}\frac{\lambda_i}{d} = \log d \tag{59}$$

The entropy can be normalized:

$$0 \leq S(\rho) \leq 1 \tag{60}$$

Examples:

- **pure state**: $|\alpha\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + e^{i\alpha}|\downarrow\rangle$
  Here the density matrix is given by:

$$\rho_\alpha = |\alpha\rangle\langle\alpha| = \frac{1}{2}\begin{pmatrix}1\\ e^{i\alpha}\end{pmatrix}\begin{pmatrix}1 & e^{-i\alpha}\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1 & e^{-i\alpha}\\ e^{i\alpha} & 1\end{pmatrix} \tag{61}$$

We calculate the eigenvalues:

$$\begin{vmatrix} 1-\lambda & e^{-i\alpha} \\ e^{i\alpha} & 1-\lambda \end{vmatrix} = 0 \;\Rightarrow\; (1-\lambda)^2 \Longrightarrow \lambda_1 = 2, \lambda_2 = 0 \tag{62}$$

$$\Downarrow$$

$$\rho_\alpha^{\text{diag}} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{63}$$

Thus for the von Neumann entropy we get:

$$S(\rho_\alpha) = -\text{tr}\rho_\alpha^{\text{diag}} \; \log\rho_\alpha^{\text{diag}} = -1 \; \log 1 - 0 \; \log 0 = 0 \tag{64}$$

So we can distinguish between the following cases:
i) $0 < S(\rho) \leq 1$ mixed state
ii) $S(\rho) = 1$ maximal mixed
iii) $S(\rho) = 0$ pure state

- **mixed state**: The density matrix is given in the spectral decomposition:

$$\rho = \sum_i p_i |\varphi_i\rangle\langle\varphi_i| \tag{65}$$

where $p_i \geq 0$ and $\sum_i p_i = 1$. Then the entropy becomes:

$$S(\rho) = -\text{tr}\rho \; \log\rho = -\text{tr} \sum_i p_i |\varphi_i\rangle\langle\varphi_i| \; \log \sum_j p_j |\varphi_j\rangle\langle\varphi_j| = \tag{66}$$

$$= -\text{tr} \sum_i p_i \sum_j \sum_k c_k (p_j)^k |\varphi_i\rangle\langle\varphi_i|\varphi_j\rangle\langle\varphi_j| ... = -\sum_i p_i \; \log p_i \equiv H(\{p_i\})$$

where $H(\{p_i\})$ denotes the *Shannon Information Entropy* of a classical probability distribution $\{p_i\}$ for random numbers i.

Resumée:

A statistical mixture is achieved by mixing pure states with weights p_i. Then the von Neumann entropy expresses the uncertainty - the lack of knowledge (partial information) - about the realization of a particular state in the mixture.

2.7.2 Concurrence

In the following we will introduce the *Concurrence*, which is a proper measurement for entanglement but which is only valid for qubits. Originally it was introduced by Bennett, DiVincenzo, Smolin and Wootters for pure states in 1996 [6] but it was labeled as "Concurrence" just one year later in a paper by Wootters and Hill [45] and in the same publication is was already generalized for mixed states too.

- **Concurrence for pure states**

  Every pure state can be re-expressed in the - by Wootters and Hill so-called "magic basis" [6] of the four maximally entangled Bell states:

$$
\begin{aligned}
|e_1\rangle = |\phi^+\rangle &= \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B) \\
|e_2\rangle = i|\phi^-\rangle &= \frac{i}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B - |1\rangle_A \otimes |1\rangle_B) \\
|e_3\rangle = i|\psi^+\rangle &= \frac{i}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B + |1\rangle_A \otimes |0\rangle_B) \\
|e_4\rangle = |\psi^-\rangle &= \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B)
\end{aligned} \tag{67}
$$

  In the basis of the Bell-states, the pure state $|\psi\rangle$ has the following structure:

$$|\psi\rangle = \sum_{j=1}^{4} \alpha_j |e_j\rangle \tag{68}$$

  Based on that Bennett, DiVincenzo, Smolin and Wootters introduced the *Concurrence* so to speak in passing on the way to a description of the amount of entanglement E via the binary entropy-function $H(x) = -x\log_2 x - (1-x)\log_2(1-x)$ as follows:

$$E = H[\frac{1}{2}(1 + \sqrt{1 - C^2})] \tag{69}$$

  where C denotes the Concurrence defined as:

$$C(|\psi\rangle) = |\sum_j \alpha_j^2| \tag{70}$$

  The functions Entanglement E and the Concurrence C are related closely: they both range from 0 to 1 (0 for separable and 1 for maximally entangled states) and as E grows monotonic with C the Concurrence itself can be regarded as a measurement for entanglement:

$$E(\psi) = \epsilon(C(\psi)) \tag{71}$$

  Bennett et al. note that if only one of the α_j's is sufficiently large enough in magnitude then the other α_j's will not have enough weight to make C equal to 0 and thus the state will have some entanglement. And this makes sense because if one particular completely entangled state is sufficiently strongly represented in $|\phi\rangle$ then $|\phi\rangle$ must have some entanglement.

- **Concurrence for mixed states**

  A mixed state can be represented by the density matrix ρ. With the use of the Sigma Pauli matrix $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ we can perform a so-called spin-flip operation like follows:

$$\widetilde{\rho} = (\sigma_2 \otimes \sigma_2)\rho^*(\sigma_2 \otimes \sigma_2) \tag{72}$$

  The operation to obtain a definition of the Concurrence for mixed states also was proposed by Wootters in a paper in 1997, which was published in 1998 [71]. Here the complex conjugate of the density matrix ρ^* is taken in the basis $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$. Then the Concurrence of mixed states is given by:

$$C = \max(0, 2\lambda_{max} - \mathrm{Tr}R) \tag{73}$$

  where λ_{max} is the largest eigenvalue and $R(\rho)$ represents the entanglement in matrix form like follows:

$$R(\rho) = \sqrt{\sqrt{\rho}\rho^*\sqrt{\rho}} \tag{74}$$

  Differently expressed we can re-write the Concurrence:

$$C(\rho) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4) \tag{75}$$

  where the λ_i's denote the eigenvalues of the matrix $R^2 := \rho \times \widetilde{\rho}$. If we want to give the concurrence explicitly for all density matrices of the more general form

$$\rho(t) = \begin{pmatrix} a(t) & 0 & 0 & f(t) \\ 0 & b(t) & 0 & 0 \\ 0 & 0 & c(t) & 0 \\ f(t) & 0 & 0 & d(t) \end{pmatrix} \tag{76}$$

  In this general form the Concurrence is then given by:

$$C(\rho(t)) = 2\max(0, |f(t)| - \sqrt{b(t) \times c(t)}) \tag{77}$$

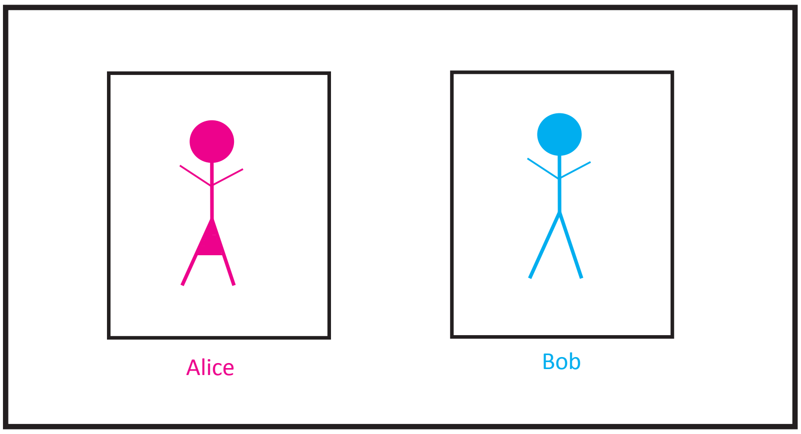
2.8 Composite quantum systems

2.8.1 Mathematical background for composite systems

Before we will have a look at some examples of bipartite quantum states at first we will introduce some mathematical aspects of systems of more than one particle. The composite quantum system consists of subsystems, or 2 atoms, or 2 particles, or 2 degrees of freedom of the same object, for example the spin-path of a neutron, etc. Interestingly, in quantum information theory this 2 characters are always called Alice and Bob - female and male like a polarity. An illustration for the composite system can be found below.

For a combined system

$$AB = A + B \tag{78}$$



described in the Hilbert-space-notation we get a tensor product of the subspaces:

$$\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B \tag{79}$$

If the state vectors in the subsystems are $\{|\varphi_i\rangle^A \ \epsilon \ \mathcal{H}_A\}$ and $\{|\varphi_j\rangle^B \ \epsilon \ \mathcal{H}_B\}$ then the space vector for the combined (composite) system is given by:

$$|\psi\rangle = \sum_{i,j} c_{ij} |\varphi_i\rangle^A \otimes |\varphi_j\rangle^B \tag{80}$$

Note:

$|\varphi_i\rangle^A \otimes |\varphi_j\rangle^B$ forms a basis in the tensorspace with the dimension:

$$\dim\mathcal{H}_{AB} = \dim\mathcal{H}_A \cdot \dim\mathcal{H}_B \tag{81}$$

We will now consider two operators that are acting in Hilbert-space, namely operator A acting in $\mathcal{H}_A$ and operator B acting in $\mathcal{H}_B$. We introduce the norm of an operator:

$$\|A\|_2^2 = \mathrm{tr} A^\dagger A < \infty \tag{82}$$

and the scalar product:

$$(A_1, A_2) = \mathrm{tr} A_1^\dagger A_2 \tag{83}$$

The Hilbert-Schmidt operators form a Hilbert space, the so-called *Hilbert-Schmidt-space*. The tensor product of an operator can be defined via the action on vectors:

$$(A \otimes B)(|\varphi_i\rangle^A \otimes |\varphi_j\rangle^B) \equiv A|\varphi_i\rangle^A \otimes B|\varphi_j\rangle^B \tag{84}$$

Any operator acting on $\mathcal{H}_{AB}$ is expressible by a linear combination of tensor products:

$$O = \sum_i a_i A_i \otimes B_i \tag{85}$$

In particular, observables of the subsystems A and B can be written:

$$A \otimes \mathbb{1}_B \tag{86}$$

$$\mathbb{1}_A \otimes B$$

where $\mathbb{1}_A$ and $\mathbb{1}_B$ are the identities in the subsystems. We now will consider the density matrix of the composite system, which is an operator action on $\mathcal{H}_{AB}$. If the subsystems are uncorrelated, the density matrix of the composite system is given by the density operators of the subsystems:

$$\rho^{AB} \equiv \rho = \rho^A \otimes \rho^B \tag{87}$$

where ρ^A is acting on $\mathcal{H}_A$ and ρ^B is acting on $\mathcal{H}_B$. The expectation value of the tensor product of operators factorizes:

$$\langle A \otimes B\rangle = \mathrm{tr} A \otimes B\rho = \mathrm{tr}(A \otimes B)(\rho^A \otimes \rho^B) = \mathrm{tr}[A\rho^A \otimes B\rho^B] =$$

$$= \mathrm{tr}_A A\rho^A \cdot \mathrm{tr}_B B\rho^B = \langle A\rangle \cdot \langle B\rangle \tag{88}$$

where tr_A and tr_B denote the partial traces over the subsystems. If an operator on the total space $\mathcal{H}_{AB}$ is given by:

$$O = |a_1\rangle\langle a_2| \otimes |b_1\rangle\langle b_2| \tag{89}$$

with the vectors $|a_i\rangle\ \epsilon\ \mathcal{H}_A$ and $|b_i\rangle\ \epsilon\ \mathcal{H}_B$, then the partial trace over the subsystem B is defined by:

$$\mathrm{tr}_B O = |a_1\rangle\langle a_2|\mathrm{tr}|\mathrm{b}_1\rangle\langle \mathrm{b}_2| = \langle \mathrm{b}_2|\mathrm{b}_1\rangle|\mathrm{a}_1\rangle\langle \mathrm{a}_2|\ \epsilon\ \mathcal{H}_\mathrm{A} \tag{90}$$

Thus, the reduced density matrices are defined by:

$$\rho^A = \mathrm{tr}_B\rho\ \epsilon\ \mathcal{H}_A \text{ (describes state on system A)} \tag{91}$$

$$\rho^B = \mathrm{tr}_A\rho\ \epsilon\ \mathcal{H}_B \text{ (describes state on system B)}$$

This definition will be intuitively clear when we consider the product state

$$\rho = \sigma \otimes \tau \tag{92}$$

where $\sigma\ \epsilon\ \mathcal{H}_A$ and $\tau\ \epsilon\ \mathcal{H}_B$. Then we gain:

$$\mathrm{tr}\rho^A = \mathrm{tr}_B\sigma \otimes \tau = \sigma \tag{93}$$

$$\mathrm{tr}\rho^B = \mathrm{tr}_A \sigma \otimes \tau = \tau$$

The reduced density matrix ρ^A completely describes the statistical properties of all observables of the subsystem A:

$$\langle A \rangle_\rho = \mathrm{tr} A\rho = \mathrm{tr}(A \otimes \mathbb{1}_B)\rho = \mathrm{tr}_A A \rho^A = \langle A \rangle_{\rho A} \tag{94}$$

Example:

As an example let us consider qubits. The states of the subsystems are given by:

$$|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \ , |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{95}$$

Thus for the composite system we get:

$$|\uparrow\rangle \otimes |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \tag{96}$$

Let's consider the operators:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \ , \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{97}$$

For the composite system we obtain:

$$\sigma_x \otimes \sigma_y = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} \tag{98}$$

Schmidt decomposition theorem (only for pure states)

For any state vector $|\psi\rangle \ \epsilon \ \mathcal{H}_A \otimes \mathcal{H}_B$ there exist orthonormal bases - the *Schmidt*-bases

$$\{|\chi_i\rangle^A \epsilon \mathcal{H}_A\} \text{ and } \{|\chi_i\rangle^B \epsilon \mathcal{H}_B\} \tag{99}$$

such that

$$|\psi\rangle = \sum_i c_i |\chi_i\rangle^A \otimes |\chi_i\rangle^B \tag{100}$$

with the Schmidt coefficients c_i. From the normalization $\langle\psi|\psi\rangle = 1$ we get:

$$\sum_i |c_i|^2 = 1 \tag{101}$$

Proof of the *Schmidt decomposition theorem*:

Without loss of generality we can suppose that $\dim\mathcal{H}_A = \dim\mathcal{H}_B$. Then the coefficient matrix in general decomposition is given by:

$$C = (c_{ij}) \text{ is square matrix} \tag{102}$$

$$\Downarrow$$

$$|\psi\rangle = \sum_{i,j} c_{ij} |\varphi_i\rangle^A \otimes |\varphi_j\rangle^B \tag{103}$$

We now use the *singular value decomposition theorem* of matrices:

$$C = UC^{\text{diag}}V \tag{104}$$

where U and V are unitary matrices and C^{diag} is a diagonal matrix with non-negative eigen-values. In components we can write:

$$c_{ij} = u_{ik} c_{kk}^{\text{diag}} v_{kj} \tag{105}$$

Then for the state vector $|\psi\rangle$ we get:

$$|\psi\rangle = \sum_{i,j} c_{ij} |\varphi_i\rangle^A \otimes |\varphi_j\rangle^B = \sum_{ijk} u_{ik} c_k v_{kj} |\varphi_i\rangle^A \otimes |\varphi_j\rangle^B = \sum_k c_k |\chi_k\rangle^A \otimes |\chi_k\rangle^B \text{ q.e.d.} \tag{106}$$

Remark:

Note that the Schmidt basis can always be chosen such that the Schmidt-coefficients $c_i \geq 0$ are real and non-negative.

Definition

The *Schmidt number* N_S is defined by the number of Schmidt coefficients $c_i > 0$. N_S is invariant under unitary transformations U^A and U^B on the subspaces $\mathcal{H}_A$ and $\mathcal{H}_B$. N_S is uniquely defined for a space-vector $|\psi\rangle$ (it does not depend on a particular Schmidt basis).

A state $|\psi\rangle$ is called *product state* if it can be written as a tensor product

$$|\psi\rangle = |\varphi\rangle^A \otimes |\varphi\rangle^B \tag{107}$$

A state $|\psi\rangle \;\epsilon\; \mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$ is called *entangled* if it cannot be written as a tensor product. From the Schmidt decomposition theorem follows:

$$|\psi\rangle \text{ is entangled if } N_S > 1 \tag{108}$$

$$|\psi\rangle \text{ is a product state if } N_S = 1$$

$$|\psi\rangle \text{ is maximal entangled if all Schmidt coefficients are equal } |c_i| = |c| \tag{109}$$

We will now consider the density matrix of a composite quantum system in a pure state.

Lemma 1. *If a system is in a pure state $\rho = |\psi\rangle\langle\psi|$ then the reduced density matrices $\rho^A = \mathrm{tr}_B\rho$ and $\rho^B = \mathrm{tr}_A\rho$ have same eigenvalues.*

Proof:

$$\rho^A = \mathrm{tr}_B|\psi\rangle\langle\psi| = \mathrm{tr}_B\left[\sum_i c_i|\chi_i\rangle^A \otimes |\chi_i\rangle^B \left(\sum_j c_j^{*\ A}\langle\chi_j| \otimes^B \langle\chi_j|\right)\right] =$$

$$= \mathrm{tr}_B\left[\sum_{ij} c_j c_j^* |\chi_j\rangle\langle\chi_j|^A \otimes |\chi_i\rangle\langle\chi_j|^B\right] = \sum_i |c_i|^2|\chi_i\rangle\langle\chi_i|^A \tag{110}$$

Analogously:

$$\rho^B = \mathrm{tr}_A|\psi\rangle\langle\psi| = \sum_i |c_i|^2|\chi_i\rangle\langle\chi_i|^B \text{ q.e.d.} \tag{111}$$

Remark:

Generally subsystems are in mixed states when the composite system is in a pure state. If a composite state ρ is maximally entangled, then the reduced densities ρ^A and $\rho^B \sim \mathbb{1}$ are maximally mixed. A composite state ρ is a product state if ρ^A and ρ^B are in pure states.

2.8.2 Some examples for bipartite states

Bell states

Bell states are named after John S. Bell as they show up in the *Bell inequalities* of which we have already heard in section (2.6). They have already been introduced in section (2.7) of two quantum bits (qubits):

$$|\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B \pm |1\rangle_A \otimes |0\rangle_B) \tag{112}$$

$$|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B \pm |1\rangle_A \otimes |1\rangle_B) \tag{113}$$

Therefore the density matrices are given by:

$$|\psi^\pm\rangle\langle\psi^\pm| = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & \pm1 & 0 \\ 0 & \pm1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{114}$$

$$|\phi^\pm\rangle\langle\phi^\pm| = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & \pm1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \pm1 & 0 & 0 & 1 \end{pmatrix} \tag{115}$$

Obviously the measure of *purity* given by:

$$P = \text{Tr}\rho^2 = 1 \quad \text{for pure states}$$
$$P = \text{Tr}\rho^2 < 1 \quad \text{for mixed states} \tag{116}$$

for the Bell states equals to 1. Also the concurrence, a measurement for entanglement, introduced in section(2.7) is 1. Therefore the Bell states are all pure and maximally entangled.

Werner state

Introduced by the German physicist Reinhard F. Werner in 1989 [70] is a mixture of the maximally mixed state and a Bell state:

$$\rho_W = \frac{1-\alpha}{4}\mathbb{1}_4 + \alpha|\psi^-\rangle\langle\psi^-| =$$
$$= \frac{1}{4}\begin{pmatrix} 1-\alpha & 0 & 0 & 0 \\ 0 & 1+\alpha & -2\alpha & 0 \\ 0 & -2\alpha & 1+\alpha & 0 \\ 0 & 0 & 0 & 1-\alpha \end{pmatrix} \tag{117}$$

with $0 \leq \alpha \leq 1$. If we calculate the purity we achieve that:

$$P_W(\alpha) = \frac{1+3\alpha^2}{4} \tag{118}$$

which means that the Werner state is only pure for $\alpha = 1$. In any other case it is a mixture of the maximally mixed state and a Bell state. To calculate the concurrence we firstly need the eigenvalues of ρ_W. With a simple calculation we get:

$$\lambda_1 = 1-\alpha, \quad \lambda_2 = 1-\alpha, \quad \lambda_3 = 1-\alpha, \quad \lambda_2 = 1+3\alpha \tag{119}$$

Obviously the concurrence as defined above (75) becomes for the Werner state to:

$$C_W(\alpha) = \lambda_4 - \lambda_1 - \lambda_2 - \lambda_3 = \frac{1}{2}(3\alpha - 1) \tag{120}$$

This means that the Werner state gets seperable for $\alpha \leq \frac{1}{3}$ and it is entangled for $\alpha > \frac{1}{3}$ with the concurrence above (120).

Note: If the dimensions of the subsystems are higher than 2 it is more usal to call the state *isotopic*. The diction *Werner state* is just used for 2-dimensional subspaces.

3 Decoherence and open quantum systems

In reality a quantum mechanical system is never perfectly isolated from its environment. The resulting interactions cause that a quantum system might lose its coherence - which is a major quantum feature as we have seen in chapter (2.1). An illustration for that can be found below.

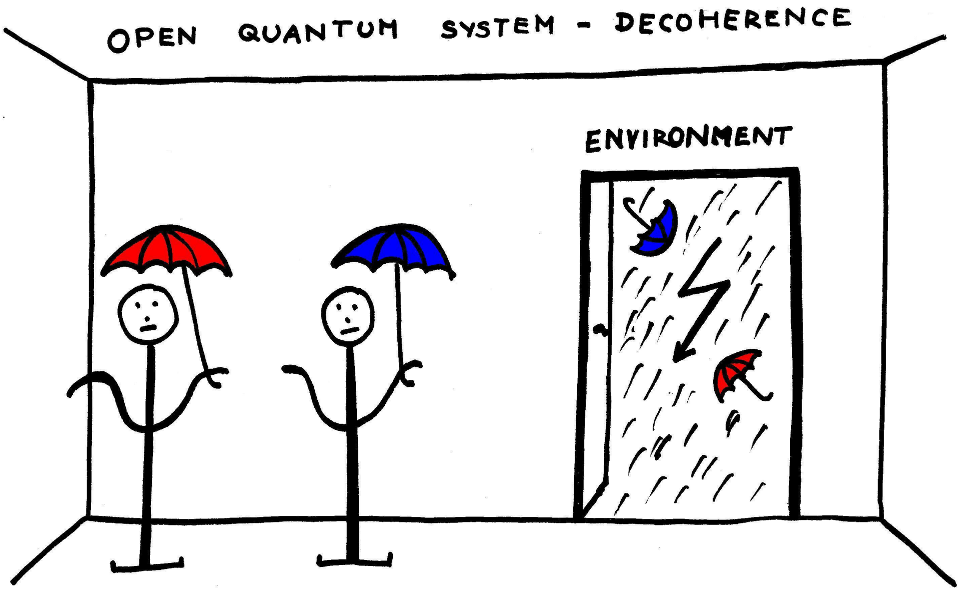


Figure 6: **Decoherence Cartoon**: when the entangled quantum system Alice and Bob interacts with its environment quantum correlations might get lost, (c) Tanja Traxler, 2009

In the study of decoherence we do not give a general formula for the loss of coherence depending on the interactions - indeed in many cases the detailed interactions and their acting is not known exactly. But what we know - because we can measure it - is the behavior (in mathematical terms: the composite Hamilton-operator) of the composite system consisting of a small quantum system and its environment.

Decoherence processes play a practical role for example in quantum computation. Until now they are *the* big barrier on the way to construct a quantum computer.

But decoherence also plays a crucial role as far as epistemological aspects are concerned: As we will see in chapter (6) it is of great importance to clarify the questions why the macro world does not fulfill quantum principles. If quantum mechanics wants to be a general theory it should be independent from scales. And if it is not it has to give a precise explanation why.

3.1 Open quantum system - dynamics

In the following we want to introduce the mathematical formalism to study quantum decoherence. This chapter is mainly based on references [67], [20], [38] and [61].

Therefore we will consider a quantum system S coupled to an environment E via interactions. E acts like a reservoir with infinite degrees of freedom, for example like a heat bath (in case of thermal equilibrium).

To study the dynamics of a quantum system in interaction with its environment (that is what we call an "*open quantum system*") we have to consider the total system $S + E$, because the dynamics of subsystem S is determined by the dynamics of the total system. The total Hilbertspace is constructed by the tensor product:

$$\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_E \tag{121}$$

The total Hamilton-operator has the following structure:

$$H(t) = H_S \otimes \mathbb{1}_E + \mathbb{1}_S \otimes H_E + H_I(t) \tag{122}$$

where H_S is the Hamiltonian of the open system, H_E is the Hamiltionian of the environment and $H_I(t)$ denotes the interaction between system and environment.

All observables refer to a subsystem S of the form:

$$A \otimes \mathbb{1}_E \quad \text{with} \quad A \,\epsilon\, \mathcal{H}_S \tag{123}$$

The density matrix of the system ρ^S - which is of central interest for us - we obtain by tracing over the environment E:

$$\rho^S = \mathrm{tr}_E \rho \tag{124}$$

The expectation value of A is represented by:

$$\langle A \rangle = \mathrm{tr}_S \rho^S A \tag{125}$$

The total system $S + E$ is closed and therefore follows a unitary time-evolution, which is determined by the operator:

$$U(t, t_0) = T e^{-i \int_{t_0}^{t} H(t) dt} \tag{126}$$

Thus, we can write the evolution of the density matrix of the reduced system in the following way:

$$\rho^S(t) = \mathrm{tr}_E U(t, t_0) \rho(t_0) U^\dagger(t, t_0) \tag{127}$$

For a closed system we know that: $\frac{\partial}{\partial t}\rho(t) = -i[H(t), \rho(t)]$, $(\hbar = 1)$. Thus, with tracing over the environment we get the equation of motion for the density matrix of the system:

$$\frac{\partial}{\partial t}\rho^S(t) = -i\mathrm{tr}_E[H(t), \rho(t)] \tag{128}$$

An example for the open quantum system description is an atom (system) in an external electromagnetical field (environment).

3.2 Dynamical map and operator sum representation

Let us suppose that at $t = 0$ the system and the environment are uncorrelated. Therefore, the density operator can be described by a tensor product of ρ^S and ρ^E:

$$\rho(0) = \rho^S(0) \otimes \rho^E \tag{129}$$

From the chapter above we already know the time-evolution of the reduced system:

$$\rho^S(0) \ \rightarrow \ \rho^S(t) = V(t)\rho^S(0) \equiv \mathrm{tr}_E U(t,0)\rho^S(0) \otimes \rho^E U^t(t,0) \tag{130}$$

where $V(t)$: $\mathcal{H}_S \rightarrow \mathcal{H}_E$ is called *dynamical map*. In the following we will show that a dynamical map $V(t)$ can be completely characterized by operators acting on $\mathcal{H}_S$. Therefore let us consider the spectral decomposition of the density matrix of the environment ρ^E:

$$\rho^E = \sum_k p_k |\phi_k\rangle\langle\phi_k| \tag{131}$$

where $0 \leq p_k \leq 1$ and $\sum p_k = 1$.

We can visualize the evolution in the following diagram:

$$\rho(0) = \rho^S(0) \otimes \rho^E \ \rightarrow^{U(t,0)}_{\text{unitary time-evolution}} \quad \rho(t) = U(t,0)\rho^S(0) \otimes \rho^E U^\dagger(t,0) \tag{132}$$

$$\downarrow \ \mathrm{tr}_E$$

$$\rho^S(0) \ \rightarrow^{V(t)}_{\text{dynamical map}} \quad \rho^S(t) = V(t)\rho^S(0)$$

For the overall density matrix ρ of $S + E$ we can choose that

- at beginning $t = 0$ the density matrix is given as a product state:

$$\rho(0) = \rho^S(0) \otimes \rho^E \tag{133}$$

- at $t = 0$ the environment represents a pure state - this is experimentally achievable and simplifies our discussions:

$$\rho^E = |\phi_0\rangle\langle\phi_0| \tag{134}$$

Then we consider the unitary time-evolution of the total system $S + E$ and study its effects on the system by tracing over E.

$$\rho_{\text{tot}} \ \rightarrow \ U\rho^S(0) \otimes \rho^E U^\dagger$$

$$\Downarrow$$

$$\rho^S = \mathrm{tr}_E \rho \ \rightarrow \ \sum_k \langle\phi_k|U\rho^S(0) \otimes |\phi_0\rangle\langle\phi_0|U^\dagger|\phi_k\rangle$$

where $\{|\phi_k\rangle\}$ denote the complete states of the environment.

We introduce operators on $\mathcal{H}_S$ - the so-called *Kraus-operators*:

$$W_k =: \langle\phi_k|U|\phi_0\rangle \tag{135}$$

$$W_k^\dagger =: \langle\phi_0|U^\dagger|\phi_k\rangle$$

with the property

$$\sum_k W_k^\dagger W_k = \sum_k \langle\phi_0|U^\dagger|\phi_k\rangle\langle\phi_k|U|\phi_0\rangle = \langle\phi_0|U^\dagger U|\phi_0\rangle = \langle\phi_0|\mathbb{1}_{S+E}|\phi_0\rangle = \mathbb{1}_S \tag{136}$$

Then we find for the dynamical map - the time evolution of the density matrix of the system - a representation in terms of a sum of *Kraus-operators*:

$$\rho^S(0) \;\rightarrow\; \sum_k W_k \rho^S(0) W_k^\dagger = V[\rho^S(0)] = \rho^S(t) \tag{137}$$

Properties of the dynamical map $V(t)$:

- The dynamical map $V(t)$ is trace conserving.

$$tr_S \rho^S(t) = tr_S V[\rho^S] = tr_S \sum_k W_k \rho^S(0) W_k^\dagger = tr_S \rho^S(0) \tag{138}$$

- $V(t)$ is a convex linear map.

$$V(t)\sum_i p_i\rho_i = \sum_i p_i V(t)\rho_i \;,\; \sum_i p_i = 1 \;\rightarrow\; \text{convex sum} \tag{139}$$

- The dynamical map $V(t)$ is completely positive.

$$V(t) \otimes 1_n \geq 0 \;\; \text{on } \mathcal{H}_S \otimes \mathbb{C}^n \tag{140}$$

Remark:

Let be a map $V(t)[\rho] \geq 0$ for all $\rho \geq 0$ and for all $t \geq 0$ on a finite dimensional complex Hilbert space. Then the map V is *completely positive* if the extention

$$V_n(t) = V(t) \otimes \mathbb{1}_n \tag{141}$$

defined on $\mathcal{H} \otimes \mathbb{C}^n$ for all n is positive

$$V_n(t)[\rho \otimes \omega] = V(t)[\rho] \otimes \omega \geq 0 \tag{142}$$

for all $\rho \,\epsilon\, \mathcal{H}$ and for all $\omega \,\epsilon\, \mathbb{C}^n$.

Theorem 1. *$V(t)$ is completely positive $\Leftrightarrow$ $V(t) \otimes V(t) \geq 0$ is positive.*

This theorem is important for entangled systems, a counter example to complete positivity is the partial transposition.

Let us now assume that the characteristic time scale of the environment is much smaller than the characteristic time scale of the system $\tau_E \ll \tau_S$, so to say, that the memory effects of the system about the environment are negligible (classical "Markov process"). The characteristic time scales are determined by some correlation functions proportional to $e^{-\frac{t}{\tau_E}}$ in case of the environment and $e^{-\frac{t}{\tau_S}}$ in case of the system.

Then the dynamical map V forms a semigroup:

$$V(t_1)V(t_2) = V(t_1 + t_2) \text{ where } t_1, t_2 \geq 0 \tag{143}$$

We construct a generator of the semigroup:

$$V(t) = e^{\mathcal{L}t} \tag{144}$$

$$\Downarrow$$

$$\rho^S(t) = V(t)\rho^S(0) = e^{\mathcal{L}t}\rho^S(0) \tag{145}$$

and find a so-called *master equation*:

$$\frac{\partial}{\partial t}\rho^S(t) = \mathcal{L}\rho^S(t) \tag{146}$$

in analogy to the classical *Liouville*-equation discussed in the beginning.

Resumée:

For the dynamical map of the density matrix there exists an operator decomposition. In the open quantum system formulation we consider a quantum system S in interaction with its environment E. We can assume that at the beginning $t = 0$:

- The density matrix of $S + E$ is represented by a product state:

$$\rho(0) = \rho^S(0) \otimes \rho^E \tag{147}$$

- The density matrix of the environment is a pure state:

$$\rho^E = |\phi_0\rangle\langle\phi_0| \tag{148}$$

  where $\{|\phi_k\rangle\}$ form a completely orthogonal system.

Visualized in a diagram:

$$\begin{array}{ccc} \rho(0) & \rightarrow & U\rho^S(0) \otimes \rho^E U^\dagger \\ & & \downarrow tr_E \end{array} \tag{149}$$

$$\rho^S(0) \;\rightarrow\; \sum_k \langle\phi_k|U(t)\rho^S(0) \otimes |\phi_0\rangle\langle\phi_0|U(t)^\dagger|\phi_k\rangle \tag{150}$$

With the *Kraus-operator* W_k

$$W_k =: \langle\phi_k|U(t)|\phi_0\rangle \tag{151}$$

the master-equation can finally be written as:

$$\rho^S(t) = \sum_k W_k(t)\rho^S(0)W_k^\dagger(t) = V(t)[\rho^S(0)] \tag{152}$$

The dynamical map $V(t)$ fulfills the following properties:

- trace conserving
- convex linear
- completely positive map

3.3 Measurement process

Experiments with quantum objects have shown that interference, for example of partial waves, disappears when the property characterizing these partial waves is measured [73]. Such partial waves may describe the passage through different slits of an interference device, or the two beams of a Stern – Gerlach device. Heinz Dieter Zeh states: "This loss of coherence is indeed required by mere logic once measurements are assumed to lead to definite results. In this case, the frequencies of events on the detection screen measured in coincidence with a *certain* passage can be counted separately, and thus have to be added to define the total probabilities. It is therefore a *plausible* experience that the interference disappears also when the passage is 'measured' without registration of a definite result. The latter may be *assumed* to have become a 'classical fact' as soon as the measurement has irreversibly 'occurred'. A quantum phenomenon may thus 'become a phenomenon' *without* being observed (in contrast to this early formulation of Bohr, which is in accordance with Heisenberg's idealistic statement about a trajectory coming into being by its observation, while Bohr later spoke of objective irreversible events occurring in the counter)." [73]

We see, the formula "Decoherence is caused by measurements" is quite too simple. Interference also disappears when a system is just somehow observed or even just by effects of noise. Still, the measurement process can be seen as a role model for a decoherence processes. In quantum mechanics the process of observation has not been clearly defined by now, but there are some types of mathematical definitions of different measurement processes that shall be discussed in the following.

von Neumann measurement, projective measurement

Let us consider the observable $A = \sum_n a_n P_n$ with the eigen-values a_n and the projection-operator $P_n = |n\rangle\langle n|$, $P_n^2 = P_n$, $\sum_n P_n = 1$ with the eigen-equation: $A|n\rangle = a_n|n\rangle$. The expectation value of A is given by:

$$\langle A\rangle = \mathrm{tr}\rho^S A = \sum p_n a_n \tag{153}$$

where $p_n = tr\rho^S P_n$ is the probability for an eigen-value a_n.

The *von Neumann-measurement* or *projective measurement* looks like the following:

$$\rho^S \;\rightarrow\; \sum_n p_n|n\rangle\langle n| = \sum_n P_n\rho^S P_n^\dagger \tag{154}$$

For measurement of this type the *Kraus-operator* is identical to the projection operator.

$$W_k \equiv P_n \tag{155}$$

Positive Operator Value Measurements POVM

We again consider the total system $S+E$ with some interaction $S \leftrightarrow E$. We define an unitary operator U such that at the same time:

- apply operator M_n on system S: $|\psi\rangle \rightarrow M_n|\psi\rangle$ with $|\psi\rangle$, $M_n|\psi\rangle\ \epsilon\ \mathcal{H}_S$
- state of environment changes $|e_0\rangle \rightarrow |e_n\rangle$, where $\{|e_n\rangle\}\ \epsilon\ \mathcal{H}_E$:

So the operation can be written as:

$$U(|\psi\rangle \otimes |e_0\rangle) = \sum_n M_n|\psi\rangle \otimes |e_n\rangle \tag{156}$$

where M_n has to fulfill the following property (normalization):

$$1 = \langle e_0|\langle\psi|U^\dagger U|\psi\rangle|e_o\rangle = \sum_{m,m'}\langle e_{m'}|\langle\psi|M_{m'}^\dagger M_m|\psi\rangle|e_m\rangle = \sum_m\langle\psi|M_m^\dagger M_m|\psi\rangle \tag{157}$$

$$\Downarrow$$

$$\sum_m M_m^\dagger M_m = 1 \tag{158}$$

In the method of the *Positive Operator Valued Measurements POVM* we measure the state of the environment by an operator B:

$$B = \mathbb{1}_S \otimes \sum_n b_n|e_n\rangle\langle e_n| = \sum_n b_n P_n^E \tag{159}$$

The expectation value of B is given by:

$$\langle B\rangle = \mathrm{tr}\rho_{SE}B = \mathrm{tr}U|\psi\rangle|e_0\rangle\langle e_0|\langle\psi|U^\dagger \mathbb{1}_S \otimes \sum_n b_n|e_n\rangle\langle e_n| =$$

$$[\text{using}: \quad \mathrm{tr}|\psi\rangle\langle\varphi| = \langle\varphi|\psi\rangle]$$

$$= \langle e_0|\langle\psi|U^\dagger \mathbb{1}_S \otimes \sum_n b_n|e_n\rangle\langle e_n|U|\psi\rangle|e_0\rangle = \sum_n p_n b_n \tag{160}$$

where p_n denotes the probability to get the measurement-result b_n.

$$p_n = \langle e_0|\langle\psi|U^\dagger \mathbb{1}_S \otimes |e_n\rangle\langle e_n|U|\psi|e_0\rangle =$$

$$= \sum_{m,m'} \langle e_{m'}|\langle\psi|M^\dagger_{m'} \mathbb{1}_S \otimes |e_n\rangle\langle e_n|M_m|\psi\rangle|e_m\rangle = \langle\psi|M_n^\dagger M_n|\psi\rangle \tag{161}$$

The effect of the measurement process is the following:

$$\rho_0^{SE} \;\rightarrow\; \sum_n p_n|\psi_n^{SE}\rangle\langle\psi_n^{SE}| \tag{162}$$

$$\downarrow \mathrm{tr}_E$$

$$\rho^S = |\psi\rangle\langle\psi| \;\rightarrow\; \mathrm{tr}_E \sum_n M_n|\psi\rangle|e_n\rangle\langle e_n|\langle\psi|M_n^\dagger = \sum_n M_n\rho^S M_n^\dagger \tag{163}$$

For this type of measurement the *Kraus-operator* is identical to M_n:

$$W_n \equiv M_n \tag{164}$$

3.4 Quantum channels - dynamical maps

Let us now consider a quantum operation with a spin- $\frac{1}{2}$ particle. Alice transmits such a particle to Bob:

$$\text{Alice} \;\rightsquigarrow\; \text{Bob} \tag{165}$$

There exists a noise caused by the interaction of the particle with the environment. In the following we will discuss different processes of this intereaction, the so-called *quantum channels* and the corresponding dynamical maps.

Depolarising channel

The first quantum channel we consider is the *depolarising channel.* This is a process with contributions from a total mixture:

$$\rho \;\rightarrow\; p\frac{1}{2}\mathbb{1} + (1-p)\rho \tag{166}$$

where p is the probability for an error and $(1-p)$ is the probability that the initial qubit remains O.K.

The dynamical map of this channel is given by:

$$V[\rho] = \frac{p}{2}\mathbb{1} + (1-p)\rho \tag{167}$$

To find out how the *Kraus-operators* look like we use the following lemma:

Lemma 2.

$$\frac{1}{2}(\mathbb{1}\rho\mathbb{1} + \overrightarrow{\sigma}\rho\overrightarrow{\sigma}) = \mathbb{1} \tag{168}$$

In general we can decompose the density matrix for qubits as:

$$\rho = \frac{1}{2}(1 + \overrightarrow{a} \cdot \overrightarrow{\sigma}) \tag{169}$$

where $\overrightarrow{a}$ is the Bloch-vector. Inserting Lemma 3 we get for the dynamical map:

$$V[\rho] = \frac{p}{2}\frac{1}{2}(\mathbb{1}\rho\mathbb{1} + \overrightarrow{\sigma}\rho\overrightarrow{\sigma}) + (1-p)\mathbb{1}\rho\mathbb{1} = \tag{170}$$

$$= \frac{p}{4}\overrightarrow{\sigma}\rho\overrightarrow{\sigma} + (\mathbb{1} - \frac{3p}{4})\mathbb{1}\rho\mathbb{1} = \frac{p'}{3}\overrightarrow{\sigma}\rho\overrightarrow{\sigma} + (1-p')\mathbb{1}\rho\mathbb{1}$$

where $p' = \frac{3p}{4}$. Thus the *Kraus-Operators* are given by:

$$W_0 = \sqrt{1-p'}\mathbb{1} \tag{171}$$

$$W_i = \sqrt{\frac{p'}{4}}\sigma_i$$

The Bloch-sphere shrinks by the factor $(1-p)$.

Bit-flip channel

The *Bit-flip channel* describes the process where spins are flipped:

$$|\uparrow\rangle \ \rightarrow \ |\downarrow\rangle \text{ analogous: } |0\rangle \ \rightarrow \ |1\rangle$$

$$|\downarrow\rangle \ \rightarrow \ |\uparrow\rangle \text{ analogous: } |1\rangle \ \rightarrow \ |0\rangle \tag{172}$$

Thus we have for the dynamical map of this process:

$$V[\rho] = p\ \sigma_x\rho\sigma_x + (1-p)\rho \tag{173}$$

The *Kraus-operators* are given by:

$$W_0 = \sqrt{p}\mathbb{1} \tag{174}$$

$$W_1 = \sqrt{p}\sigma_x$$

For this case the Bloch-sphere is invariant in x. In y, z it is shrinking by the factor $(1-2p)$.

Phase-flip channel

The *Phase-flip channel* describes the process where the spin obtains phases:

$$|\uparrow\rangle \ \rightarrow \ |\uparrow\rangle$$

$$| \downarrow\rangle \;\rightarrow\; -| \downarrow\rangle \tag{175}$$

Then the dynamical map for this process can be expressed by:

$$V[\rho] = p\ \sigma_z \rho\ \sigma_z + (1-p)\rho \tag{176}$$

with the *Kraus-operators*:

$$W_0 = \sqrt{1-p}\mathbb{1} \tag{177}$$
$$W_1 = \sqrt{p}\ \sigma_z$$

For this case the Bloch-sphere is invariant in z. In x, y it is shrinking by the factor $(1-2p)$.

Bit-flip-phase channel

The *Phase-flip channel* describes the process where the spin obtains phases:

$$| \uparrow\rangle \;\rightarrow\; i| \downarrow\rangle$$
$$| \downarrow\rangle \;\rightarrow\; -i| \uparrow\rangle \tag{178}$$

The dynamical map for this process is given by:

$$V[\rho] = p\ \sigma_y \rho\ \sigma_y + (1-p)\rho \tag{179}$$

with the *Kraus-operators*:

$$W_0 = \sqrt{1-p}\mathbb{1} \tag{180}$$
$$W_1 = \sqrt{p}\ \sigma_y$$

For this case the Bloch-sphere is invariant in y. In z, x it is shrinking by the factor $(1-2p)$.

Amplitude damping channel

The *amplitude damping channel* describes the process where the spin decays $| \downarrow\rangle \rightarrow | \uparrow\rangle$ via emission of a photon:

$$\rho_\downarrow = | \downarrow\rangle\langle\downarrow | \;\rightarrow\; | \uparrow\rangle\langle\uparrow | = \rho_\uparrow \tag{181}$$

Explicitly:

$$\sigma_+\ \rho_\downarrow\ \sigma_- = | \uparrow\rangle\langle\downarrow | \downarrow\rangle\langle\downarrow | \downarrow\rangle\langle\uparrow | = | \uparrow\rangle\langle\uparrow | = \rho_\uparrow \tag{182}$$

with $\sigma_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ and $\sigma_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$. Thus, one *Kraus-Operator* will be:

$$W_1 = \sqrt{p}\ \sigma_+ \tag{183}$$

From the normalization follows the other *Kraus-operator*:

$$\sum_i W_i^\dagger W_i = 1 \tag{184}$$

$$p\begin{pmatrix}0 & 0\\1 & 0\end{pmatrix}\begin{pmatrix}0 & 1\\0 & 0\end{pmatrix}+\begin{pmatrix}a & 0\\0 & b\end{pmatrix}=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$$

$$\Downarrow$$

$$a=1\ ,\ b=1-p$$

$$\Downarrow$$

$$W_0=\begin{pmatrix}1 & 0\\0 & \sqrt{1-p}\end{pmatrix} \tag{185}$$

For the dynamical map we finally have:

$$V[\rho_\downarrow]=\begin{pmatrix}1 & 0\\0 & \sqrt{1-p}\end{pmatrix}\rho_\downarrow\begin{pmatrix}1 & 0\\0 & \sqrt{1-p}\end{pmatrix}+p\begin{pmatrix}0 & 1\\0 & 0\end{pmatrix}\rho_\downarrow\begin{pmatrix}0 & 0\\1 & 0\end{pmatrix}=$$

$$=p\rho_\uparrow+(1-p)\rho_\downarrow \tag{186}$$

3.5 Masterequation

In the following we will construct the most general form of the Liouville equation for a finite dimensional complex Hilbert-space $\mathcal{H}_S$ with $dim\mathcal{H}_S = N^2$. We will construct out of *Kraus-operators* an equation which contains the Hamilton-operator (plus remaining operators). This method goes back to Göran Lindblad [50] and to Gorini-Kossakowski-Sudarshan [39] and was first published in 1976. It is the most general type for a markovian master equation.

The master equation a la Lindblad is given by:

$$\frac{d}{dt}\rho^S(t)=-\frac{i}{\hbar}[H,\rho^S(t)]-D[\rho^S(t)] \tag{187}$$

where ρ^S is the density matrix of the system and $D[\rho^S]$ is the so-called *dissipator*:

$$D[\rho^S]=\frac{1}{2}\sum_{k=1}^{N^2-1}\lambda_k\left(A_k^\dagger A_k\rho^S+\rho^S A_k^\dagger A_k-2A_k\rho^S A_k^\dagger\right) \tag{188}$$

The dissipator can be rewritten:

$$D[\rho^S]=\frac{1}{2}\sum_k\lambda_k([A_k^\dagger,A_k\rho^S]+[\rho^S A_k^\dagger,A_k]) \tag{189}$$

with the Lindblad-operators A_k and the (positive) decoherence constants $\lambda_k \geq 0$ which are a quantitative measure for decoherence.

Remark:
Here we assume a weak coupling limit between the system and the environment:

$$H\equiv H_{S+E}=H_S+H_E+H_{int} \tag{190}$$

For H_E, $H_{int} \to 0 \Rightarrow H = H_S$.

Proof of the Lindblad-master-equation:

We consider the dynamical map for the time-evolution of the density matrix of the system:

$$\rho^S \ \to \ V[\rho^S] = \sum_k W_k \rho^S W_k^\dagger \tag{191}$$

with the Kraus-operator:

$$W_k = \langle \phi_k | U | \phi_0 \rangle \tag{192}$$

with the unitary operator $U = e^{-\frac{i}{\hbar}Ht}$ and the property $\sum_k W_k^\dagger W_k = 1$. As we know, the dynamical map fulfills the following properties:

- trace conserving
- convex linear
- completely positive

About the time evolution we make the following assumptions:

- The characteristic time scale of the system δt is much smaller than the lifetime of the system τ_S:
$$\delta t \ll \tau_S \tag{193}$$
- The environment should "forget" about the system, this is a so-called *Markov process*:
$$\tau_E \ll \delta t \tag{194}$$

For the proof we start from the dynamical map under the assumptions made above:

$$\rho^S(\delta t) = V[\rho^S(0)] = \sum_k W_k \rho^S(0) W_k^\dagger = \rho^S(0) + \mathcal{O}(\delta t) \tag{195}$$

We see: First Kraus-operator $\sim 1_S + \mathcal{O}(\delta t)$, all further Kraus-operators $\sim \mathcal{O}(\delta t)$. Under these conditions we construct:

$$W_0 = \mathbb{1}_S + \left(K - \frac{i}{\hbar} H \right) \delta t \tag{196}$$

$$W_k = A_k \sqrt{\delta t}$$

where K and H are hermitian operators and A_k is the Lindblad operator. From the normalization we get:

$$\sum_k W_k^\dagger W_k = \mathbb{1}_S + \left(2K + \sum_k A_k^\dagger A_k \right) \delta t + \mathcal{O}(\delta t^2) \tag{197}$$

$$\Downarrow$$

$$K = -\frac{1}{2}\sum A_k^\dagger A_k \tag{198}$$

Thus, we find for the time evolution of the system S:

$$\rho^S(\delta t) = W_0 \rho^S W_0^\dagger + \sum W_k \rho^S(0) W_k^\dagger =$$

$$= \left(\mathbb{1}_S + \left(K - \frac{i}{\hbar}H\right)\delta t\right)\rho^S(0)\left(\mathbb{1}_S + \left(K + \frac{i}{\hbar}H\right)\delta t\right) + \delta t \sum_k A_k \rho^S(0) A_k^\dagger =$$

$$= \rho^S(0) + \delta t\{-\frac{i}{\hbar}\left[H, \rho^S(0)\right] - \frac{1}{2}\left(\sum A_k^\dagger A_k \rho^S(0) + \rho^S(0) A_k^\dagger A_k - 2A_k \rho^S(0) A_k^\dagger\right)\} \tag{199}$$

$$\Downarrow$$

$$\lim_{\delta t \to 0} \frac{\rho^S(\delta t) - \rho^S(0)}{\delta t} = \frac{d}{dt}\rho^S(t)|_{t=0} = -\frac{i}{\hbar}[H, \rho^S(t)]|_{t=0} - D[\rho^S(t)]|_{t=0} \tag{200}$$

Note:
Here we have derived Eq. (200) at $t = 0$ but it holds for any time and we have rescaled $A_k \to \sqrt{\lambda_k} A_k$.

Remarks:

- $\exists$ 1 Kraus-operator $\Leftrightarrow \nexists$ Lindblad operator. $\Rightarrow$ For $\lambda_k = 0$ there is no interaction. For this case H is the Hamiltonian of the system: $H = H_S$ and in the limit of weak coupling we also have $H \to H_S$ since $H_{int} \to 0$.

- The Hamiltonian is not unique, the master equation is invariant under the operation:

$$A_k \to A_k + a_k \mathbb{1}_S$$
$$H \to H + \frac{1}{2i}\sum_k (a_k^* A_k - a_k A_k^\dagger) + b\mathbb{1}_S$$

  Furthermore, the dissipator is invariant under unitary transformations

$$A_k \to U A_k$$

  where $UU^\dagger = \mathbb{1}$.

- The right hand side of the equation is linear functional in ρ^S:

$$\frac{d}{dt}\rho^S(t) = \mathcal{L}[\rho^S] \tag{201}$$

  formally:

$$\rho^S(t) = Te^{\int_0^t \mathcal{L}(t)dt}\rho^S(0) = e^{\mathcal{L}t}\rho^S(0) \text{ where } \mathcal{L} \text{ is constant} \tag{202}$$

$$\rho^S(t) = V(t)\rho^S(0) \to V(t) = e^{\mathcal{L}t}$$

To finalize this chapter on an introduction of decoherence I want to give a cartoon by Wojciech H. Zurek, one of the leading physicists to develop the theory of decoherence. The illustration addresses to the fact that decoherence processes are first of all mainly a matter of scales and that the study of decoherence goes hand in hand with epistemic questions about our perception of "reality".

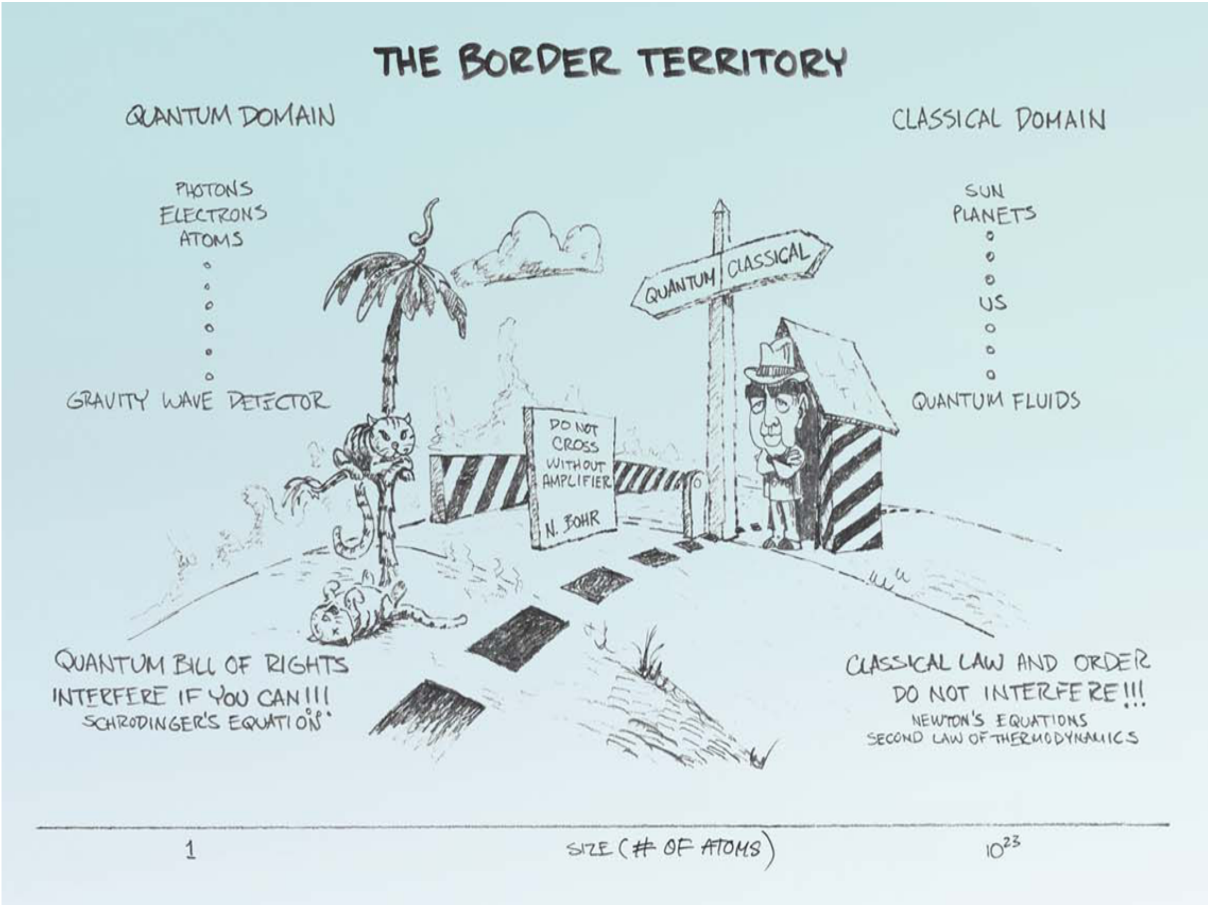


Figure 7: **Decoherence Cartoon by Wojciech H. Zurek**, Source: [74]

4 Decoherence with entangled kaonic qubits

4.1 Kaonic Qubits

Neutral K-mesons (Kaons) are fundamental particles that consist of two quarks, namely of a down-quark d and - what marks them in contrast to other fundamental particles - a strange-quark s. In the following we will write K^0 for a Kaon with quarkcontent $(d\bar{s})$ and the anti-particle we denote by $\overline{K^0}$ with $(\bar{d}s)$. The mass of a Kaon is about $M_K = 497MeV$. The behaviour of Kaons is ruled by the following quantum principles:

- **superposition**: As long as a Kaon is not measured, it may exist in a superposition of a particle and an anti-particle. Why this is possible is explained by the following principle:

- **oscillation**: Experiments have shown that the quarkcontent of a Kaon has the ability to oscillate. This means that $(d\bar{s})$ can change to $(\bar{d}s)$ and vice versa. Therefore as long as a Kaon is not measured it exists in a superposition of particle and anti-particle.

- **decay** The lifetime of Kaon is not endless. For example the positively charged Kaon K^+ has a medium lifetime of 1,2380 $\cdot 10^{-8}$ s. The decay of a K^+ is illustrated schematically below.

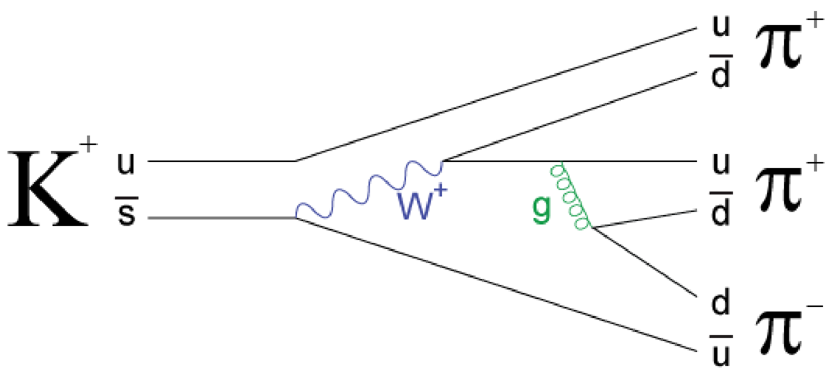


Figure 8: **Decay of a positively charged Kaon** K^+, Source: wikipedia.org

- **quasi spin** To describe the strangeness quantum number of Kaons in analogy to the spin quantum number, the concept of a so-called *quasi spin* is introduced. As we will see in the following chapter, a $|K^0|$ with spin $s = -1$ can be referred to a spin-up $|\uparrow\rangle$.

- **regeneration** As a result of oscillation and superposition another quantum principle determines the behavior of Kaon - this is regeneration. For instance let us consider a beam of neutral Kaons. When the beam decays in flight, two states appear - the short-lived state K_S decays first and leaves a beam of pure long-lived Kaons K_L. If this beam passes through matter, then the particle K^0 and anti-particle $\overline{K^0}$ interact differently with the nuclei therefore quantum coherence between these particles is lost. The emerging beam then contains different linear superpositions of the K^0 and $\overline{K^0}$, which means differently expressed, that various mixtures of K_Ss and K_Ls appear. So to say: K_S can be regenerated if a Kaon beam passes through matter.

In the following we will discuss this principles in detail.

4.1.1 Quantum states of K-mesons

Quantum-mechanically we can describe Kaons in the following way, we characterize them by quantum numbers. The eigen-equations of the strangeness-quantum-number are given by:

$$S|K^0\rangle = +|K^0\rangle \ , \ S|\overline{K^0}\rangle = -|\overline{K^0}\rangle \tag{203}$$

with strangeness operator S and strangeness eigen-values $+$ and $-$.

Parity:

$$P|K^0\rangle = -|K^0\rangle \tag{204}$$

Charge conjugation:

$$C|K^0\rangle = |\overline{K^0}\rangle \tag{205}$$

Charge conjugation - Parity:

$$CP|K^0\rangle = -|\overline{K^0}\rangle \ , \ CP|\overline{K^0}\rangle = -|K^0\rangle \tag{206}$$

We now construct eigen-states to the CP-operator:

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K^0}\rangle\right) \ , \ |K_2^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K^0}\rangle\right) \tag{207}$$

with the eigen-equations

$$CP|K_1^0\rangle = +|K_1^0\rangle \ , \ CP|K_2^0\rangle = -|K_2^0\rangle \tag{208}$$

From experiments we know, that Kaons decay with different decay times and the physical states we call K_S (short-lived-state) and K_L (long-lived-state). K_S decays into two pions with a decay-time of $\Gamma_S^{-1} = \tau_S = 10^{-10}s$. K_L decays into three pions with a decay-time of $\Gamma_L^{-1} = \tau_L = 5\cdot 10^{-8}s$. These states differ slightly in mass $\Delta m = m_L - m_S = 3,49\cdot 10^{-6}eV$. We also know that CP is violated due to weak-interactions with a probability of $|\varepsilon| \approx 10^{-3}$. Thus, we can write the states $|K_S\rangle$ and $|K_L\rangle$ as superposition of $|K_1^0\rangle$ and $|K_2^0\rangle$:

$$|K_S\rangle = \frac{1}{\sqrt{2}}\left(|K_1^0\rangle + \varepsilon|K_2^0\rangle\right) \ , \ |K_L\rangle = \frac{1}{\sqrt{2}}\left(\varepsilon|K_1^0\rangle + |K_2^0\rangle\right) \tag{209}$$

$$|K_S\rangle = \frac{1}{N}\left(p|K^0\rangle - q|\overline{K^0}\rangle\right) \ , \ |K_L\rangle = \frac{1}{N}\left(p|K^0\rangle + q|\overline{K^0}\rangle\right) \tag{210}$$

where $p = 1 + \varepsilon$ and $q = 1 - \varepsilon$ and $N = \sqrt{|p|^2 + |q|^2}$. The complex quantity ε is called *CP violating parameter*.

4.1.2 Strangeness oscillation

The decay is given by the non-hermitian effective Hamiltonian H:

$$H = M - \frac{i}{2}\Gamma \tag{211}$$

where M and Γ are hermitian operators. M corresponds to the mass and Γ is the decay-matrix.

The eigenequations of the effective Hamiltonian are satisfied by the states $|K_S\rangle$ and $|K_L\rangle$:

$$H|K_{S,L}\rangle = \lambda_{S,L}|K_{S,L}\rangle \tag{212}$$

with the energyeigen-values $\lambda_{S,L} = m_{S,L} - \frac{i}{2}\Gamma_{S,L}$ where $\Gamma_{S,L}$ is the width of the states.

From the Schrödinger equation we get the *Wigner-Weisskopf*-approximation:

$$|K_S(t)\rangle = e^{-i\lambda_S t}|K_S\rangle = e^{-\frac{\Gamma_S}{2}t}e^{-im_S t}|K_S\rangle \tag{213}$$

$$|K_L(t)\rangle = e^{-i\lambda_L t}|K_L\rangle = e^{-\frac{\Gamma_L}{2}t}e^{-im_L t}|K_L\rangle$$

Since $|K^0\rangle = \frac{N}{2p}(|K_S\rangle + |K_L\rangle)$ and $|\overline{K^0}\rangle = \frac{N}{2q}(-|K_S\rangle + |K_L\rangle)$ we have for the time-evolution of the strangeness-states:

$$|K^0(t)\rangle = g_+(t)|K^0\rangle + \frac{q}{p}g_-(t)|\overline{K^0}\rangle \tag{214}$$

$$|\overline{K^0}(t)\rangle = \frac{p}{q}g_-(t)|K^0\rangle + g_+(t)|\overline{K^0}\rangle$$

with $g_{+,-}(t) = \frac{1}{2}[\pm e^{-i\lambda_S t} + e^{-i\lambda_L t}]$. Suppose that a K^0-beam is produced at $t = 0$ then there occur transitions from $|K^0\rangle$ to $|\overline{K^0}\rangle$ with the following transitions-probabilities:

$$|\langle K^0|K^0(t)\rangle|^2 = g_+(t)g_+^*(t) = \frac{1}{4}[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-\Gamma t}\cos(\Delta m t)] \tag{215}$$

$$|\langle\overline{K^0}|\overline{K^0(t)}\rangle|^2 = g_+(t)g_+^*(t) \tag{216}$$

$$|\langle K^0|\overline{K^0(t)}\rangle|^2 = \frac{|p|^2}{|q|^2}g_-(t)g_-^*(t) = \frac{1}{4}\frac{|p|^2}{|q|^2}[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-\Gamma t}\cos(\Delta m t)] \tag{217}$$

$$|\langle\overline{K^0}|K^0(t)\rangle|^2 = \frac{|q|^2}{|p|^2}g_-(t)g_-^*(t) \tag{218}$$

where $\Delta m = m_L - m_S$ and $\Gamma = \frac{1}{2}(\Gamma_L + \Gamma_S)$.

4.1.3 Quasi-Spin of Kaon - Photon analogy

We can introduce the quasi-spin (strangeness) of a Kaon in analogy to the spin of a particle or to the polarization of a photon:

$$|K^0\rangle \quad \leftrightarrow \quad |\uparrow\rangle \quad \leftrightarrow \quad |V\rangle \tag{219}$$

$$\begin{aligned} |\overline{K^0}\rangle \quad &\leftrightarrow \quad |\downarrow\rangle \quad \leftrightarrow \quad |H\rangle \\ |K_S\rangle \quad &\leftrightarrow \quad |\rightarrow\rangle \quad \leftrightarrow \quad |L\rangle \\ |K_L\rangle \quad &\leftrightarrow \quad |\leftarrow\rangle \quad \leftrightarrow \quad |R\rangle \end{aligned} \tag{220}$$

Attention:

$$\langle K_S|K_L\rangle = \frac{2Re\varepsilon}{1+|\varepsilon|^2} \doteq 2Re\varepsilon \tag{221}$$

Whereas:

$$\langle L|R\rangle = 0 \tag{222}$$

where $|L\rangle = \frac{1}{\sqrt{2}}(|V\rangle - i|H\rangle)$ and $|R\rangle = \frac{1}{\sqrt{2}}(|V\rangle + i|H\rangle)$. Then we can describe the Kaon-features with the Pauli-matrices and we can decompose the Hamilton-operator in the following way:

$$H = a\mathbb{1} + \vec{b}\,\vec{\sigma} \tag{223}$$

In comparison with the effective Hamilton operator $H = M - \frac{i}{2}\Gamma$ we get:

$$b_1 = b\ \cos\alpha\ ,\ b_2 = b\ \sin\alpha \tag{224}$$

$$b_3 = 0 \quad \text{because of CPT-invariance}$$

$$a = (\lambda_L + \lambda_S)\frac{1}{2}\ ,\ b = (\lambda_L - \lambda_S)\frac{1}{2} \tag{225}$$

Because of the CP-violation the angle α corresponds to the parameter ε from chapter 4.1.1 via the following relation:

$$e^{i\alpha} = \frac{1-\varepsilon}{1+\varepsilon} \tag{226}$$

If we insert these relations we obtain for the Hamiltonian:

$$H = a\mathbb{1} + b\sigma_1 + 2i\varepsilon b\sigma_2 \tag{227}$$

4.1.4 Decoherence of entangled Kaons

Now let us describe and measure possible decoherence of entangled Kaons. The model we will discuss we developed by Reinhold A. Bertlmann, Walter Grimus and Beatrix C. Hiesmayr [8]. Decoherence provides some information on the quality of the entangled state. Experimentally a Bell-state is produced:

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}\left(|e_1\rangle - |e_2\rangle\right) \tag{228}$$

with the following notation:

$$|e_1\rangle = |K_S\rangle_l \otimes |K_L\rangle_r \ , \ |e_2\rangle = |K_L\rangle_l \otimes |K_S\rangle_r \tag{229}$$

where the indices l and r denote the left-moving and the right-moving particle and we have chosen the eigenstates of the Hamiltonian. Thus, the density matrix is described by:

$$\rho^- = |\psi^-\rangle\langle\psi^-| = \frac{1}{2}\left(|e_1\rangle\langle e_1| + |e_2\rangle\langle e_2| - |e_1\rangle\langle e_2| - |e_2\rangle\langle e_1|\right) \tag{230}$$

Possible decoherence arises from the interaction of the quantum system with its environment. To study decoherence we therefore consider the master equation

$$\frac{d}{dt}\rho = -iH\rho + i\rho H^\dagger - D[\rho] \tag{231}$$

In the first model we want to discuss the *Lindblad-operators* A_j act like projectors. This is reasonable as we have seen in chapter 3.3 measurement processes can be described via projectors. And as we will see at the end of this chapter in comparison to the experiment the ansatz works quite well.

$$A_j = \sqrt{\lambda} P_j \tag{232}$$

with $j = 1, 2$ and the projectors $P_j = |e_j\rangle\langle e_j|$. The operators P_j project onto the eigenstates of the 2-particle Hamiltonian $H = H_l \otimes \mathbb{1}_r + \mathbb{1}_l \otimes H_r$. The solution of the master equation provides the time dependence of the density matrix:

$$\rho(t) = \frac{1}{2} e^{-\Gamma t}\left(|e_1\rangle\langle e_1| + |e_2\rangle\langle e_2| - e^{-\lambda t}\left(|e_1\rangle\langle e_2| + |e_2\rangle\langle e_1|\right)\right) \tag{233}$$

where λ is the decoherence parameter. Decoherence arises through the factor $e^{-\lambda t}$ in the off-diagonal elements. It means that for $t > 0$ the density matrix $\rho(t)$ is not pure any more but mixed.

Experimentally kaons are produced in particle colliders, e.g. at e^+e^--collider DAΦNE, Frascati or at $p\bar{p}$-collider LEAR, CERN. The Kaons produced in such an experiment are entangled and detected with respect to their strangeness.

For the actual experiment let us consider the case:

$$K^0 \text{ will be measured at the left hand side at time } t_l$$

$$\overline{K^0} \text{ will be measured at the right hand side at time } t_r$$

and $t_l \geq t_r$. Then the probability of such a measurement is calculated by

$$P(K^0, t_l; \overline{K^0}, t_r) = \mathrm{tr}_l[\mathbb{1}_r \otimes |\overline{K^0}\rangle\langle\overline{K^0}|_l \rho(t_l)]\}\mathrm{tr}_r[\mathbb{1}_l \otimes |\overline{K^0}\rangle\langle\overline{K^0}|_r \rho(t_r)]\} \tag{234}$$

Analogously can be calculated the case K^0 left and K^0 right. The result for the probabilities is:

$$P_\lambda(K^0, t_l; \overline{K^0}, t_r) = P_\lambda(\overline{K^0}, t_l; K^0, t_r) = \tag{235}$$

$$= \frac{1}{8}\left(e^{-\Gamma_S t_l - \Gamma_L t_r} + e^{-\Gamma_L t_l - \Gamma_S t_r} + e^{-\lambda t_r} 2\cos(\Delta m \Delta t) \cdot e^{-\Gamma(t_l+t_r)}\right)$$

$$P_\lambda(K^0, t_l; K^0, t_r) = P_\lambda(\overline{K^0}, t_l; \overline{K^0}, t_r) = \tag{236}$$
$$= \frac{1}{8}\left(e^{-\Gamma_S t_l - \Gamma_L t_r} + e^{-\Gamma_L t_l - \Gamma_S t_r} - e^{-\lambda t_r} 2\cos(\Delta m \Delta t) \cdot e^{-\Gamma(t_l+t_r)}\right)$$

with $\Delta t = t_l - t_r$. Note that at equal times $t_l = t_r = t$ the like-strangeness probabilities

$$P_\lambda(K^0, t; K^0, t) = P_\lambda(\overline{K^0}, t; \overline{K^0}, t) = \frac{1}{4} e^{-2\Gamma t}(1 - e^{-\lambda t}) \tag{237}$$

do not vanish, in contrast to the pure quantum mechanical EPR-correlations. The interesting quantity is the *asymmetry of probabilities*; it is directly sensitive to the interference term and can be measured experimentally. For pure quantum mechanics we have

$$A^{QM}(\Delta t) =$$
$$= \frac{P(K^0, t_l; \overline{K^0}, t_r) + P(\overline{K^0}, t_l; K^0, t_r) - P(K^0, t_l; K^0, t_r) - P(\overline{K^0}, t_l; \overline{K^0}, t_r)}{P(K^0, t_l; \overline{K^0}, t_r) + P(\overline{K^0}, t_l; K^0, t_r) + P(K^0, t_l; K^0, t_r) + P(\overline{K^0}, t_l; \overline{K^0}, t_r)} =$$
$$= \frac{\cos(\Delta m \Delta t)}{\cosh(\frac{1}{2}\Delta\Gamma \Delta t)} \tag{238}$$

with $\Delta\Gamma = \Gamma_L - \Gamma_S$, and for our decoherence model we find, by inserting the probabilities (235), (236),

$$A^\lambda(t_l, t_r) = \frac{\cos\Delta m \Delta t}{\cosh(\frac{1}{2}\Delta\Gamma\Delta t)} e^{-\lambda \min\{t_l, t_r\}} = A^{QM}(\Delta t) e^{-\lambda \min\{t_l, t_r\}} \tag{239}$$

Thus, the decoherence effect, simply given by the factor $e^{-\lambda \min\{t_l, t_r\}}$, depends only on the time of the first measured kaon, in our case: $\min\{t_l, t_r\} = t_r$.

Experiment

Now we compare our model with the results of the CPLEAR experiment at CERN where $K^0\overline{K^0}$ pairs are produced in the $p\overline{p}$-collider: $p\overline{p} \to K^0\overline{K^0}$. These pairs are predominantly in an antisymmetric state with quantum numbers $J^{PC} = 1^-$ and the strangeness of the kaons is detected via strong interactions in surrounding absorbers (made of copper and carbon). The experimental set-up has two configurations. In configuration $C(0)$ both kaons propagate 2 cm, they have nearly equal proper times ($t_r \approx t_l$) when they are measured by the absorbers. This fulfills the condition for an EPR-type experiment. In configuration $C(5)$ one kaon propagates 2 cm and the other kaon 7 cm, thus, the flight-path difference is 5 cm on average, corresponding to a proper time difference $|t_r - t_l| \approx 1.2\tau_S$.

Fitting the decoherence parameter λ by comparing the asymmetry with the experimental data we find, when averaging over both configurations, the following bounds on λ:

$$\overline{\lambda} = (1.84^{+2.50}_{-2.17}) \cdot 10^{-12}\text{MeV} \quad \text{and} \quad \overline{\Lambda} = \frac{\overline{\lambda}}{\Gamma_S} = 0.25^{+0.34}_{-0.32} \tag{240}$$

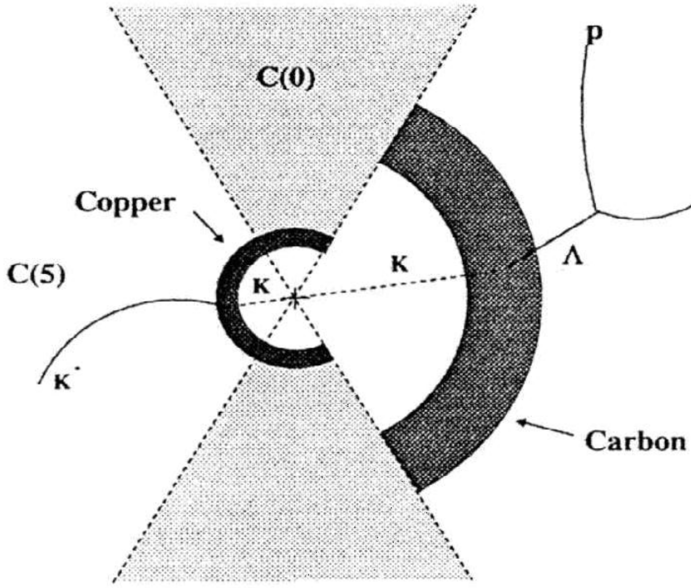


Figure 9: **Set-up of the CPLEAR-Experiment**, Source: reprinted from [13]

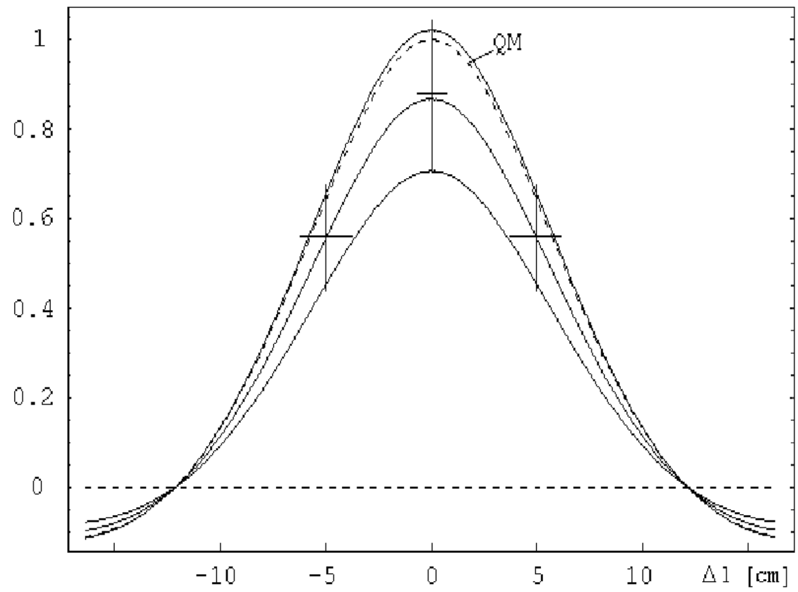


Figure 10: **Data from the CPLEAR-Experiment**, The asymmetry as a function of the distance of the kaons. Source: reprinted from [13]

The results are certainly compatible with quantum mechanics ($\lambda = 0$), nevertheless, the experimental data allow an upper bound $\overline{\lambda}_{\mathrm{up}} = 4.34 \cot 10^{-12}$ MeV for possible decoherence in the entangled $K^0\overline{K^0}$ system. But this decoherence is small $\Lambda = 0.25 + 0.34 < 1$.

Summary:

From the data we see the Kaons are still entangled although they extended over a macroscopic distance of about 7 cm. They form a quantum system of massive particles with a mass of about 1 GeV.

4.2 Projection-operator-model

Let us consider the time-evolution of the density matrix ρ of neutral K-mesons. We describe this decay via the non-hermitian Hamilton-operator

$$H = M - \frac{i}{2}\Gamma \tag{241}$$

where M is the mass and Γ the width of the particle. With this Hamilton-operator we get for the Schrödinger-equation:

$$\begin{aligned} H|K_S\rangle &= \lambda_S|K_S\rangle \\ H|K_L\rangle &= \lambda_L|K_L\rangle \end{aligned} \tag{242}$$

with the complex eigenvalues

$$\lambda_{S,L} = m_{S,L} - \frac{i}{2}\Gamma_{S,L} \tag{243}$$

The time-dependent density matrix $\rho(t)$ is given by:

$$\rho(t) = \sum_{i,j} \rho_{ij}(t)|i\rangle\langle j| \tag{244}$$

where $\rho_{ij} = \langle i|\rho(t)|j\rangle$ and $i, j = S, L$. By applying the von Neumann equation

$$\frac{d\rho(t)}{dt} = -\frac{i}{\hbar}\left(H\rho - \rho H^\dagger\right) \tag{245}$$

we gather for the time-evolution of the density matrix:

$$\dot{\rho}_{SS}(t) = -\frac{i}{\hbar}\left(\lambda_S - \lambda_S^*\right)\ \rho_{SS}(t) \tag{246}$$

$$\dot{\rho}_{LL}(t) = -\frac{i}{\hbar}\left(\lambda_L - \lambda_L^*\right)\ \rho_{LL}(t)$$

With $\lambda_S - \lambda_S^* = -i\Gamma_S$ the differential equation of the diagonal elements is:

$$\dot{\rho}_{SS}(t) = -\frac{1}{\hbar}\Gamma_S\rho_{SS}(t) \tag{247}$$

$$\dot{\rho}_{LL}(t) = -\frac{1}{\hbar}\Gamma_L \rho_{LL}(t)$$

Whereas the differential equation of the off-diagonal elements is:

$$\dot{\rho}_{SL}(t) = -\frac{1}{\hbar}\left(\Gamma + i\Delta m\right)\rho_{SL}(t) \tag{248}$$

where $\lambda_S - \lambda_L^* = m_S - m_L - \frac{i}{2}(\Gamma_S + \Gamma_L) = \Delta m - i\Gamma$ with $\Delta m = m_S - m_L$ and $\Gamma = \frac{\Gamma_S + \Gamma_L}{2}$ As a result for the solution we get the so-called **Wigner-Weisskopf**-approximation:

$$\rho_{SS}(t) = e^{-\frac{1}{\hbar}\Gamma_S t}\rho_{SS}(t) \tag{249}$$

$$\rho_{LL}(t) = e^{-\frac{1}{\hbar}\Gamma_L t}\rho_{LL}(t)$$

$$\rho_{SL}(t) = e^{-\frac{1}{\hbar}\Gamma t} e^{-\frac{i}{\hbar}\Delta m t}\rho_{SL}(t)$$

Example:

As an example let us now consider the time evolution of a spin-$\frac{1}{2}$ particle in an external magnetic field. For this case the Hamilton-operator is given by:

$$H = -\overrightarrow{\mu} \cdot \overrightarrow{B} \tag{250}$$

where $\overrightarrow{B}$ is a constant field parallel to the z-axis and $\overrightarrow{\mu}$ is the magnetic dipole $\overrightarrow{\mu} = g\mu\overrightarrow{s}$ with spin $\overrightarrow{s} = \frac{\hbar}{2}\overrightarrow{\sigma}$, $\overrightarrow{\sigma}$ is the Pauli-sigma-matrix. For electrons the gyromagnetic ratio $g \approx 2$. Bohr's Magneton is given by: $\mu_B = \frac{e\hbar}{2mc}$, we define $\gamma = g \cdot \mu_B$. Therefore for the Hamilton-operator we obtain:

$$H = -\frac{\gamma B}{2} \cdot \sigma_z \tag{251}$$

The solution for the density matrix is given by the von Neumann equation

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho] \tag{252}$$

As a solution we get:

$$\dot{\rho_{10}} = \dot{\rho_{01}} = 0$$

$$\dot{\rho_{00}} = \frac{i}{\hbar}\gamma B \rho_{00}$$

$$\dot{\rho_{11}} = \frac{i}{\hbar}\gamma B \rho_{11} \tag{253}$$

This means that:

$$\rho_{10}(t) = \rho_{10}(0) = const.$$

$$\rho_{01}(t) = \rho_{01}(0) = const. \tag{254}$$

$$\rho_{11}(t) = \rho_{11}(0) = const.$$

$$\rho_{00}(t) = e^{\frac{i}{\hbar}\gamma B t}\rho_{00}(0) = e^{i\omega t}\rho_{00}(0) \tag{255}$$

$$\rho_{11}(t) = e^{-\frac{i}{\hbar}\gamma B t}\rho_{11}(0) = e^{-i\omega t}\rho_{11}(0)$$

Like Ehrenfest's Theorem tells us, we can see that for the expectation value $\langle\overrightarrow{\sigma}\rangle = \overrightarrow{a}$ we get a classical vector, the so-called **Bloch-vector**.

4.3 Shift-operator-model

In the chapter above we have constructed the dissipator which is operational responsible for decoherence via projection operators. This is intuitively quite reasonable as projection operators are also the simplest mathematical construction for describing a measurement process. Mathematically, what a projection operator does (which is also name-giving for it) is to project a given state on its eigen-state.

In the following we want to develop another model for decoherence. This time not based on projection operators but on shift operators. Very literally this model can be associated to the notion, that decoherence causes a shift of a state.

For the model bases on shift operators we firstly define the shift-operators s_{ij} as follows:

$$s_{ij} := |e_i\rangle\langle e_j| \tag{256}$$

where the e_k are the basis vectors of the Hilbert space and the indices i and j run from 1 to 4 and $i \neq j$. To evolve different decoherence modes we need a generalized notation of the basis vectors e_k to be able to perform rotations easily as follows:

$$e_1 := \begin{pmatrix} \cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2} \\ \cos\frac{\theta_1}{2}\sin\frac{\theta_2}{2}e^{i\phi_2} \\ \sin\frac{\theta_1}{2}\cos\frac{\theta_2}{2}e^{i\phi_1} \\ \sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2}e^{i(\phi_1+\phi_2)} \end{pmatrix}, \quad e_2 := \begin{pmatrix} -\cos\frac{\theta_1}{2}\sin\frac{\theta_2}{2} \\ \cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2}e^{i\phi_2} \\ -\sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2}e^{i\phi_1} \\ \sin\frac{\theta_1}{2}\cos\frac{\theta_2}{2}e^{i(\phi_1+\phi_2)} \end{pmatrix} \tag{257}$$

$$e_3 := \begin{pmatrix} -\sin\frac{\theta_1}{2}\cos\frac{\theta_2}{2} \\ -\sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2}e^{i\phi_2} \\ \cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2}e^{i\phi_1} \\ \cos\frac{\theta_1}{2}\sin\frac{\theta_2}{2}e^{i(\phi_1+\phi_2)} \end{pmatrix}, \quad e_4 := \begin{pmatrix} \sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2} \\ -\sin\frac{\theta_1}{2}\cos\frac{\theta_2}{2}e^{i\phi_2} \\ -\cos\frac{\theta_1}{2}\sin\frac{\theta_2}{2}e^{i\phi_1} \\ \cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2}e^{i(\phi_1+\phi_2)} \end{pmatrix}$$

For example let us consider s_{12} for decoherence mode A ($\theta_1 = 0$, $\theta_2 = 0$, $\phi_1 = 0$, $\phi_2 = 0$):

$$s_{12} = |e_1\rangle\langle e_2| = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{258}$$

With the following formula (188) we can calculate the dissipator for shift operators as follows:

$$D[\rho^S]_{shift} = \frac{1}{2}(s_{12}^\dagger s_{12}\rho(t) - \rho(t)s_{12}^\dagger s_{12} - 2s_{12}\rho(t)s_{12}^\dagger + s_{13}^\dagger s_{13}\rho(t) - \rho(t)s_{13}^\dagger s_{13} -$$

$$-2s_{13}\rho(t)s_{13}^\dagger + s_{14}^\dagger s_{14}\rho(t) - \rho(t)s_{14}^\dagger s_{14} - 2s_{14}\rho(t)s_{14}^\dagger + s_{21}^\dagger s_{21}\rho(t) - \rho(t)s_{21}^\dagger s_{21} -$$

$$-2s_{21}\rho(t)s_{21}^\dagger + s_{23}^\dagger s_{23}\rho(t) - \rho(t)s_{23}^\dagger s_{23} - 2s_{23}\rho(t)s_{23}^\dagger + s_{24}^\dagger s_{24}\rho(t) - \rho(t)s_{24}^\dagger s_{24} -$$

$$-2s_{24}\rho(t)s_{24}^\dagger + s_{31}^\dagger s_{31}\rho(t) - \rho(t)s_{31}^\dagger s_{31} - 2s_{31}\rho(t)s_{31}^\dagger + s_{32}^\dagger s_{32}\rho(t) - \rho(t)s_{32}^\dagger s_{32} -$$

$$-2s_{32}\rho(t)s_{32}^\dagger + s_{34}^\dagger s_{34}\rho(t) - \rho(t)s_{34}^\dagger s_{34} - 2s_{34}\rho(t)s_{34}^\dagger + s_{41}^\dagger s_{41}\rho(t) - \rho(t)s_{41}^\dagger s_{41} -$$

$$-2s_{41}\rho(t)s_{41}^{\dagger}+s_{42}^{\dagger}s_{42}\rho(t)-\rho(t)s_{42}^{\dagger}s_{42}-2s_{42}\rho(t)s_{42}^{\dagger}+s_{43}^{\dagger}s_{43}\rho(t)-\rho(t)s_{43}^{\dagger}s_{43}-2s_{43}\rho(t)s_{43}^{\dagger}) \tag{259}$$

Mode A

As we can see from definition (256) the explicit structure of the shift-operators depends on the chosen basis vectors. Setting the angles of the basis system to $\theta_1 = 0$, $\theta_2 = 0$, $\phi_1 = 0$ and $\phi_2 = 0$ we achieve the familiar $|\uparrow\rangle$-$|\downarrow\rangle$-basis, which evolves the so-called *decoherence mode A*. For the dissipator we get:

$$D[\rho^S]_{shiftA} = \tag{260}$$

$$= \begin{pmatrix} -\sum_{i=2,3,4}\rho_{ii}(t) & 0 & 0 & 0 \\ 0 & -\sum_{i=1,3,4}\rho_{ii}(t) & 0 & 0 \\ 0 & 0 & -\sum_{i=1,2,4}\rho_{ii}(t) & 0 \\ 0 & 0 & 0 & -\sum_{i=1,2,3}\rho_{ii}(t) \end{pmatrix}$$

Mode B

Setting the angles to $\theta_1 = \frac{\pi}{2}$, $\theta_2 = 0$, $\phi_1 = 0$, $\phi_2 = 0$ *decoherence mode B* is achieved, which corresponds to a rotated (45°) $|\uparrow\rangle$-$|\downarrow\rangle$-basis. It corresponds to the basis vectors:

$$e_1 = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}, \quad e_2 = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}, \quad e_3 = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}, \quad e_4 = \begin{pmatrix} 0 \\ -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \tag{261}$$

The dissipator is then given by:

$$D[\rho^S]_{shiftB} = \frac{1}{2}\begin{pmatrix} d_{11} & 0 & \rho_{13}(t)+\rho_{31}(t) & 0 \\ 0 & d_{22} & 0 & \rho_{24}(t)+\rho_{42}(t) \\ \rho_{13}(t)+\rho_{31}(t) & 0 & d_{33} & 0 \\ 0 & \rho_{24}(t)+\rho_{42}(t) & 0 & d_{44} \end{pmatrix} \tag{262}$$

where $d_{11} = d_{33} = -\rho_{11}(t) - 2\rho_{22}(t) - \rho_{33}(t) - 2\rho_{44}(t)$ and $d_{22} = d_{44} = -2\rho_{11}(t) - \rho_{22}(t) - 2\rho_{33}(t) - \rho_{44}(t)$.

Using shift-operators the system of coupled differential equations for the time-evolution of the density matrix gets rather complicated as can bee seen. In case of starting with a Bell singlet state no analytical solution could be found. But of course the problem could be solved numerically.

5 Decoherence with neutrons

5.1 Complete positivity

Always when a quantum system is not isolated but weakly coupled to an environment, we call it - as we have already heard - an *open quantum system*. Typically we deal with a small well-known system coupled to an environment which is much bigger and whose details are unknown. Due to the interaction, the small quantum system might lose its characteristic quantum coherence properties.

The time-evolution of the density matrix of an open quantum system is determined by a so-called *quantum master equation* (187):

$$\frac{d}{dt}\rho^S(t) = -\frac{i}{\hbar}[H, \rho^S(t)] - D[\rho^S(t)] \tag{263}$$

which has already been introduced in chapter 3.5. As we have already learned the term $D[\rho^S(t)]$ is the so-called *dissipator*, which occurs if the considered system is not perfectly isolated but interacts weakly with its environment. Even thought in general the nature of such an interaction is unknown, but in the theory of decoherence we can find a suitable characterization for the Hamiltonian of the system. The time evolution of the density matrix ρ^S are maps which are compatible with the probabilistic interpretation of quantum mechanics as they preserve positivity . Moreover it turns out that the much stronger property of *complete positivity* holds for those maps which essentially ensures that tensor products of such maps remain positive. Although any theory of decoherence seems to be governed by complete positivity, until now no general proof could be brought up for that.

Complete positivity is definded as follows:

Proposition 2. *Let $\gamma_t[\rho] \equiv \rho(t)$ with $\rho(0) = \rho$ be a positive map on a finite complex Hilbert space $\mathcal{H}$, i.c. $\rho(t) \geq 0$ for all $t \geq 0$. Then γ_t is called completely positive if the extension $\gamma_{n;t} = \gamma_t \otimes \mathbb{1}_n$ defined on $\mathcal{H} \otimes \mathbb{C}^n$ is positive, i.e. $\gamma_{n;t}[\rho] \geq 0$ for all f, for all n=1,2,... and all density matrices on the space $\mathcal{H} \otimes \mathbb{C}^n$.*

Physically this condition is reasonable because the extension $\gamma_{n;t} = \gamma_t \otimes \mathbb{1}_n$ can be seen as an operator that acts locally on one side (Alice) without influencing the other one (Bob). This means that complete positivity signifies nothing more than that physical systems remain physical even if just a single part of a composite system is observed.

5.2 Experimental check of complete positivity with neutrons

5.2.1 Introduction to the model to check complete positivity

In the following we will discuss an experimental proposal by Reinhold A. Bertlmann and Walter Grimus to test complete positivity experimentally. Complete positivity is a theoretical feature that any model of decoherence contains. In fact, what it says is, that a physical system remains physical, also under decoherence - so there is no reason not to expect this requirement. However, not yet any experiment could proof that the feature of complete positivity holds for any process of decoherence.

At the beginning of the proposal we consider once more the quantum master equation after Lindblad (187):

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] - D[\rho] = G[\rho] \tag{264}$$

Here ρ denotes a 2×2-matrix. Via the Bloch-notation we can associate it to a 3-dimensional vector $\overrightarrow{\rho}$ like follows:

$$\rho = \frac{1}{2}(\mathbb{1} + \overrightarrow{\rho} \cdot \overrightarrow{\sigma}) \tag{265}$$

As every 2×2-matrix can be decomposed via the Pauli-matrices, we can analogously rewrite the Hamilton-operator H:

$$H = \frac{1}{2}a(\mathbb{1} + \overrightarrow{h} \cdot \overrightarrow{\sigma}) \tag{266}$$

To reformulate the Master Equation in times of the density-vector $\overrightarrow{\rho}$ we use a trick: We firstly multiply a factor, make the calculation to let it vanish again in the end:

$$\frac{1}{2}\overrightarrow{\sigma}\dot{\overrightarrow{\rho}} = -\frac{i}{\hbar}\frac{1}{4}\left[(\mathbb{1} + \overrightarrow{\rho} \cdot \overrightarrow{\sigma}), (\mathbb{1} + \overrightarrow{h} \cdot \overrightarrow{\sigma})\right] - D\left[\frac{1}{2}(\mathbb{1} + \overrightarrow{\rho} \cdot \overrightarrow{\sigma})\right] = \tag{267}$$

$$= -\frac{i}{\hbar}\frac{1}{4}(\overrightarrow{\rho} \cdot \overrightarrow{\sigma}\overrightarrow{h} \cdot \overrightarrow{\sigma}\overrightarrow{\rho} \cdot \overrightarrow{\sigma}) - D\left[\frac{1}{2}(\mathbb{1} + \overrightarrow{\rho}\overrightarrow{\sigma})\right] = -\frac{i \cdot i}{\hbar}\frac{1}{2}\overrightarrow{\sigma} \cdot (\overrightarrow{\rho} \times \overrightarrow{h}) - D\left[\frac{1}{2}(\mathbb{1} + \overrightarrow{\rho}\overrightarrow{\sigma})\right]$$

The second term can be reexpressed like the following:

$$D\left[\frac{1}{2}(\mathbb{1} + \overrightarrow{\rho}\overrightarrow{\sigma})\right] = \frac{1}{2}\overrightarrow{\sigma} \cdot L\overrightarrow{\rho} = D[\rho] \tag{268}$$

Analogously and choosing $a = 1$:

$$H = \frac{1}{2}\overrightarrow{\sigma} \cdot \overrightarrow{h} \tag{269}$$

Therefore from equation (267) we can cross out the factor $\frac{1}{2}\overrightarrow{\sigma}$ and get:

$$\dot{\overrightarrow{\rho}} = Q\overrightarrow{\rho} \tag{270}$$

where:

$$Q\overrightarrow{\rho} = \overrightarrow{\rho} \times \overrightarrow{h} - L\overrightarrow{\rho} \tag{271}$$

Here the first part $\overrightarrow{\rho} \times \overrightarrow{h}$ denotes the Hamiltonian part, while the second part $L\overrightarrow{\rho}$ describes decoherence. After equation (270) for the time-evolution of $\overrightarrow{\rho}$ we get:

$$\overrightarrow{\rho}(t) = e^{Qt}\overrightarrow{\rho(0)} \tag{272}$$

Therefore we finally obtain the Master equation in the following reexpression:

$$\rho(t) = \frac{1}{2}(\mathbb{1} + \overrightarrow{\rho}(t) \cdot \overrightarrow{\sigma}) \tag{273}$$

Note: The Bloch-vector $\overrightarrow{\rho}$ satisfies the inequality:

$$|\overrightarrow{\rho}(t)| \leq 1 \tag{274}$$

5.2.2 Rotational invariance

Assumption: Let us consider photons where the appearing decoherence is invariant under the direction of propagation. So to say: The photon is polarized transversely. Therefore the decoherence is invariant under any angle ϑ (see figure below). The rotation of the

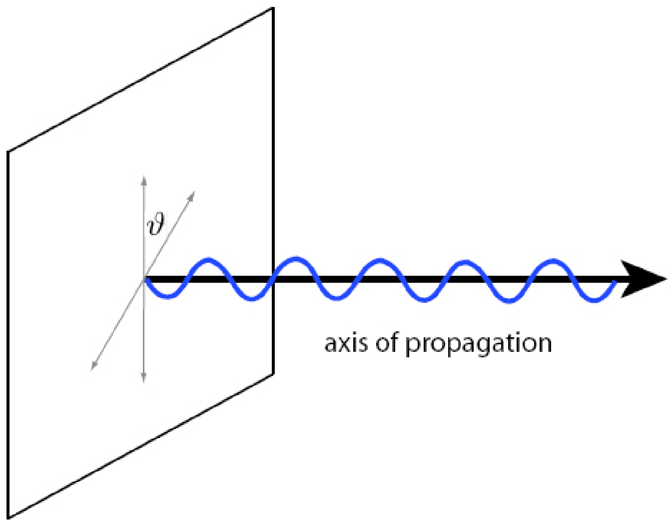


Figure 11: **Propagation of the polarized particle**, Decoherence is invariant under the direction of propagation under any angle ϑ.

polarization state of a photon along the propagation axis is given by the rotation matrix:

$$R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{275}$$

where θ is the angle of rotation. To express the demand of rotational invariance of the quantum state as sketched in figure 5.2.2 in mathematical terms, we consider the density matrix ρ: If $\rho(t)$ is a solution of the master equation (187) then also the rotation of $\rho(t)$: $R(\theta)\rho(t)R(\theta)^T$ has to be a solution of (187) for any rotation angle θ. Thus we may require:

$$G[R(\theta)\rho(t)R(\theta)^T] = R(\theta)G[\rho(t)]R(\theta)^T \tag{276}$$

for any θ. Performing the rotation we arrive at:

$$R(\theta)\rho(t)R(\theta)^T = \frac{1}{2}[\mathbb{1} + (\mathcal{R}(\theta)\overrightarrow{\rho}) \cdot \overrightarrow{\sigma}] \tag{277}$$

with the transposed 3-dimensional rotation matrix:

$$\mathcal{R}(\theta) = \begin{pmatrix} \cos 2\theta & 0 & \sin 2\theta \\ 0 & 1 & 0 \\ -\sin 2\theta & 0 & \cos 2\theta \end{pmatrix} \tag{278}$$

With (277) we may reexpress our demand (276) as:

$$\mathcal{R}(\theta)Q\overrightarrow{\rho} = Q\mathcal{R}(\theta)\overrightarrow{\rho} \tag{279}$$

for all θ's and any $\overrightarrow{\rho}$. So to say $\mathcal{R}$ commutes with Q and we arrive a the theorem:

Theorem 2. *Consider $\dot{\rho} = G[\rho]$ or equivalently $\dot{\overrightarrow{\rho}} = Q\overrightarrow{\rho}$. If $G[\rho]$ is rotational invariant then the matrix Q has the form:*

$$Q = \begin{pmatrix} -\lambda & 0 & -\omega \\ 0 & -\lambda' & 0 \\ \omega & 0 & -\lambda \end{pmatrix} \tag{280}$$

Note: We require non-negative constants λ and λ' in order to have physical density matrices $\rho(t) \geq 0$ for any $t \geq 0$.

5.2.3 Time evolution and complete positivity

Factorizing in the requirement of rotational invariance with the matrix Q from equation (280) we can decompose the time evolution of $\overrightarrow{\rho}$ in the following way:

$$\overrightarrow{\rho}(t) = e^{tQ}\overrightarrow{\rho} = \Omega(t)\overrightarrow{\rho}_d(t) \tag{281}$$

where

$$\Omega(t) = \begin{pmatrix} \cos\omega t & 0 & -\sin\omega t \\ 0 & 1 & 0 \\ \sin\omega t & 0 & \cos\omega t \end{pmatrix} \qquad \text{and} \qquad \overrightarrow{\rho}_d(t) = \begin{pmatrix} e^{-\lambda t}\rho_1 \\ e^{-\lambda' t}\rho_2 \\ e^{-\lambda t}\rho_3 \end{pmatrix} \tag{282}$$

This means that the time evolution of the density matrix (281) can be split into two parts: the decoherence part is given by $\overrightarrow{\rho}_d$ and the rotational part by $\Omega(t)$. For $\lambda > 0$ and $\lambda' > 0$ we arrive at the limit $t \longrightarrow \infty$ at the maximally mixed state $\lim_{t\to\infty}\rho(t) = \frac{1}{2}\mathbb{1}$ as desired.

It is possible to decompose Q into a Hamiltonian part and a decoherence part as follows:

$$Q\overrightarrow{\rho} = \overrightarrow{h} \times \overrightarrow{\rho} - L\overrightarrow{\rho} \tag{283}$$

where the first term is the Hamiltonian part with $\overrightarrow{h}$ related to H via:

$$H = \frac{1}{2}\overrightarrow{h} \cdot \overrightarrow{\sigma} \tag{284}$$

and the second term is the decoherence part where L is related to $D[\rho]$:

$$D[\rho] = \frac{1}{2}\overrightarrow{\sigma} \cdot L\overrightarrow{\rho} \tag{285}$$

With the parameterization of Q from (280) $\overrightarrow{h}$ and L are explicitly given by:

$$\overrightarrow{h} = -\omega \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \text{and} \qquad L = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda' & 0 \\ 0 & 0 & \lambda \end{pmatrix} \tag{286}$$

Obviously the Hamilton operator is then simply given as:

$$H = -\frac{1}{2}\omega\sigma_2 \tag{287}$$

So to say, the eigen-states of the given Hamiltonian are the eigen-states of the σ_2-Pauli-matrix - and those are well known: the helicities $|\pm\rangle$:

$$H|\pm\rangle = \mp\frac{1}{2}\omega|\pm\rangle \qquad \text{with} \qquad |\pm\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ \pm i\end{pmatrix} \tag{288}$$

To find a check-able criterion for complete positivity we now consider a very useful theorem which was introduced by Bertlmann and Grimus and has not been published yet:

Theorem 3. *The time evolution γ_t of a 2×2 density matrix ρ via*

$$\overrightarrow{\rho} \longrightarrow^t \overrightarrow{\rho}(t) = e^{tQ}\overrightarrow{\rho} \tag{289}$$

with $Q(\omega, \lambda, \lambda')$ given by equation (280) is completely positive iff

$$\lambda' \leq 2\lambda \tag{290}$$

Proof: From L given by equation (286) a matrix M can be defined by:

$$L = \begin{pmatrix}\lambda & 0 & 0\\ 0 & \lambda' & 0\\ 0 & 0 & \lambda\end{pmatrix} = \frac{1}{2}(\mathrm{tr}M\mathbb{1}_3 - M) \tag{291}$$

Bertlmann and Grimus have shown [11] that for complete positivity of the time evolution of the density matrix it has to been shown that:
(i) $M_{\alpha\alpha} \geq 0 \quad \forall\alpha = 1,2,3$
(ii) $M_{\alpha\alpha}M_{\beta\beta} \geq M_{\alpha\beta}^2 \quad \forall\alpha \neq \beta$
(iii) $\det M \geq 0$
The definition of M (291) with the given matrix L (286) fixes M to:

$$M = \begin{pmatrix}\lambda' & 0 & 0\\ 0 & 2\lambda - \lambda' & 0\\ 0 & 0 & \lambda'\end{pmatrix} \tag{292}$$

Proof:

$$\begin{aligned} L = \frac{1}{2}(\mathrm{tr}M\mathbb{1}_3 - M) = \frac{1}{2}\left(\mathrm{tr}\begin{pmatrix}\lambda' & 0 & 0\\ 0 & 2\lambda - \lambda' & 0\\ 0 & 0 & \lambda'\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} - \begin{pmatrix}\lambda' & 0 & 0\\ 0 & 2\lambda - \lambda' & 0\\ 0 & 0 & \lambda'\end{pmatrix}\right) = \\ = \frac{1}{2}\left(\begin{pmatrix}\lambda' + 2\lambda & 0 & 0\\ 0 & \lambda' + 2\lambda & 0\\ 0 & 0 & \lambda' + 2\lambda\end{pmatrix} - \begin{pmatrix}\lambda' & 0 & 0\\ 0 & 2\lambda - \lambda' & 0\\ 0 & 0 & \lambda'\end{pmatrix}\right) = \begin{pmatrix}\lambda & 0 & 0\\ 0 & \lambda' & 0\\ 0 & 0 & \lambda\end{pmatrix} = L \end{aligned} \tag{293}$$

(i) If $M_{\alpha\alpha} \geq 0 \quad \forall\alpha = 1,2,3$ we can immediately conclude that $\lambda' \leq 2\lambda$, then
(ii) is fulfilled as well (since M is diagonal).
(iii) is fulfilled since requirement (i) holds. *Q.E.D.*

5.2.4 Experimental observables

As we have already seen in chapter 3.5 for a completely positive time evolution of the density matrix the dissipator of the master equation has the following structure (188):

$$D[\rho^S] = \frac{1}{2}\sum_{k=1}^{N^2-1} \lambda_k \left(A_k^\dagger A_k \rho^S + \rho^S A_k^\dagger A_k - 2A_k \rho^S A_k^\dagger\right) = \frac{1}{2}\sum_{j=1}^{r} [A_j, [A_j, \rho]] \tag{294}$$

In chapter 4 we have already discussed some types of Lindblad operators A_j. In the case we are considering now it is useful to find operators corresponding to the matrix L. Therefore we will choose three matrices:

$$A_1 = \frac{1}{2}\sqrt{\lambda'}\sigma_1 \qquad A_2 = \frac{1}{2}\sqrt{2\lambda - \lambda'}\sigma_2 \qquad A_3 = \frac{1}{2}\sqrt{\lambda'}\sigma_3 \tag{295}$$

Note that the requirement of complete positivity of theorem 3 appears just in the second Lindblad operator A_2.

In the proposed experimental check, we are following observables for the measurement of the photon polarization. We choose the following projection operators:

$$\mathcal{P}_T(\alpha) = |V\rangle\langle V| = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} = \begin{pmatrix} \cos^2\alpha & \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha & \sin^2\alpha \end{pmatrix} \tag{296}$$

$\mathcal{P}_T$ measures the transverse polarization with azimuth angle α. As a second observable we choose $\mathcal{P}_\pm$ which measures positive (+) oder negative (-) helicity:

$$\mathcal{P}_\pm = |\pm\rangle\langle\pm| = \frac{1}{2}\begin{pmatrix} 1 & \mp i \\ \pm i & 1 \end{pmatrix} \tag{297}$$

With the general definition of the expectation value of an observable (16) we calculate the expectation values:

$$\langle \mathcal{P}_T(\alpha)\rangle_\rho = \mathrm{tr}(\rho \mathcal{P}_T(\alpha)) = \frac{1}{2}\{1 + \rho_1 \sin 2\alpha + \rho_3 \cos 2\alpha\} \tag{298}$$

$$\langle \mathcal{P}_\pm\rangle_\rho = \mathrm{tr}(\rho \mathcal{P}_\pm) = \frac{1}{2}\{1 \pm \rho_2\} \tag{299}$$

With the time evolution (281) for the density matrix ρ which was introduced before we get the time evolution of the observables:

$$\langle \mathcal{P}_T(\alpha)\rangle_{\rho(t)} = \frac{1}{2}\{1 + e^{-\lambda t}[\rho_1 \sin(\omega t + 2\alpha) + \rho_3 \cos(\omega t + 2\alpha)]\} \tag{300}$$

$$\langle \mathcal{P}_\pm\rangle_{\rho(t)} = \frac{1}{2}\{1 \pm \rho_2 e^{-\lambda' t}\} \tag{301}$$

As we see: We have done a tricky choice since the decoherence parameters λ and λ' are separated now: the transverse polarization which is governed by $\mathcal{P}_T$ depends on λ whereas the circular polarization of the helicities is governed by λ'.

With the chosen Lindblad operators we will now construct the dissipators. The operator $\mathcal{P}_T(\delta)$ generates decoherence with respect to an azimuthal angle δ by the corresponding dissipator:

$$D_\delta[\rho] = \lambda[\mathcal{P}_T(\delta)\rho\mathcal{P}_T^\perp(\delta) + \mathcal{P}_T^\perp(\delta)\rho\mathcal{P}_T(\delta)] \tag{302}$$

In their first conjunction Bertlmann and Grimus suggested to choose for the coefficient 2λ. But from experiments we know, that the coefficient λ is said to be a more reasonable value.

To achieve rotational invariance with respect to the propagation direction as sketched in the figure of chapter 5.2.2 it is necessary to average over δ:

$$D_T[\rho] = \frac{1}{2\pi}\int_0^{2\pi} d\delta D_\delta[\rho] \tag{303}$$

The calculation gives:

$$\begin{aligned} D_T[\rho] &= \frac{1}{2}\lambda(\rho_1\sigma_1 + 2\rho_2\sigma_2 + \rho_3\sigma_3) = \\ &= \frac{1}{2}\overrightarrow{\sigma} \cdot L\overrightarrow{\rho} \quad \text{with} \quad L = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \end{aligned} \tag{304}$$

If we compare this matrix to the matrix L given by equation (286) and take theorem 3 into account we see: here the decoherence parameter $\lambda' = \lambda$ corresponds to the half of the spectrum allowed by complete positivity.

Let us now consider the dissipator for the helicities:

$$D_H[\rho] = \lambda[\mathcal{P}_+\rho\mathcal{P}_- + \mathcal{P}_-\rho\mathcal{P}_+] \tag{305}$$

The calculation gives:

$$\begin{aligned} D_H[\rho] &= \frac{1}{2}\lambda(\rho_1\sigma_1 + \rho_3\sigma_3) = \\ &= \frac{1}{2}\overrightarrow{\sigma} \cdot L\overrightarrow{\rho} \quad \text{with} \quad L = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \lambda \end{pmatrix} \end{aligned} \tag{306}$$

The comparison with (286) and theorem 3 shows that $\lambda' = 0$ is the minimum. So all together we cover just the half spectrum from $\lambda' = 0$ to $\lambda' = \lambda$. This is why in the following I want to make a suggestion for another experimental setup aiming to cover the whole spectrum allowed through complete positivity.

5.2.5 Test with neutrons

Let us now consider particles with spin-$\frac{1}{2}$ instead of photons; the formalism can be used analogously. To check theorem 3 experimentally we have to find a procedure for a setup in which the barriers for λ' which are 0 and 2λ can be reached (or violated).

Noise model

From experimentalists we know that λ' which corresponds to the longitudinal relaxation-time is in general smaller than λ, which corresponds to the transverse relaxation. We note:

$$\lambda' < \lambda \qquad \text{homogeneous} \quad \text{case} \tag{307}$$

But it is possible to scale these parameters if the measurement is performed in an extern magnetic field. In detail we suggest two different experimental setups for spin-measurements. Always λ is measured transverse in the z-x plane and λ' is measured longitudinal along the y-axis. In setup (a) an external magnetic noise field is put along the y-axis. In setup (b) an external magnetic field acts in the x-z-plane. The sketch below illustrates these two setups.

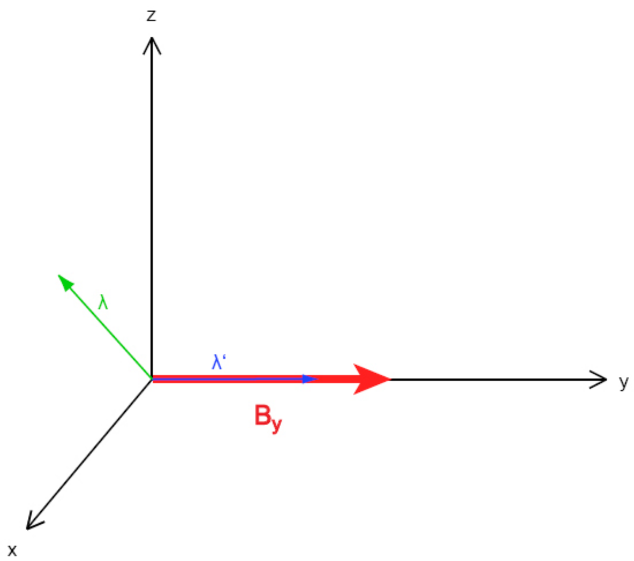


Figure 12: **Setup (a) for spin measurement**, The external magnetic field is acting along the y-axis.

To get the time-evolution of the density matrix in these setups we start again with the quantum master equation (3.5). In this description the noise through the external magnetic field is represented by the Lindblad operator, which will be labeled as Γ here:

$$\frac{\partial}{\partial t}\rho = -i[H,\rho] + \Gamma\rho\Gamma^\dagger - \frac{1}{2}(\Gamma^\dagger\Gamma\rho + \rho\Gamma^\dagger\Gamma) \tag{308}$$

Here ρ is a 2×2 matrix: $\rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix}$. We chose the Hamiltonian to lie along the y-axis: $H \sim \sigma_y$ and in the first step we will look at the case of a magnetic field along

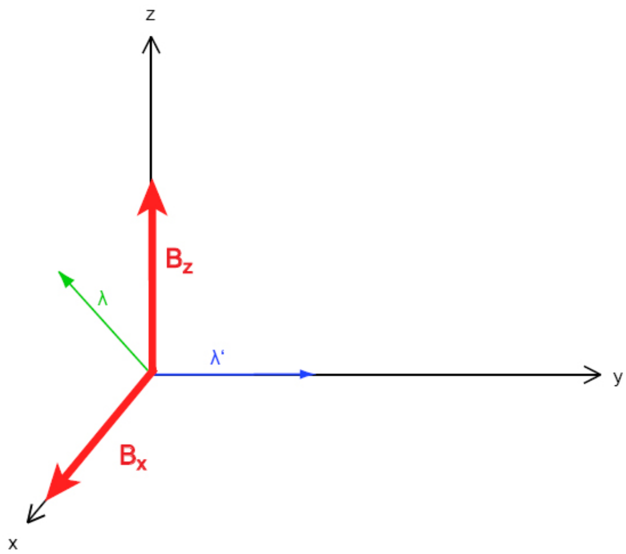


Figure 13: **Setup (b) for spin measurement**, The external magnetic field is acting in the x-z-plane.

the z-axis: $B(\Gamma) \sim \sigma_z$. Explicitly the Hamiltonian H and the Lindblad operator for the magnetic field Γ_z are given by:

$$H = -\frac{\omega}{2}\sigma_y \qquad \text{and} \qquad \Gamma_z = \sqrt{\frac{\Lambda_z}{2}}\sigma_z \tag{309}$$

Thus for the Hamiltonian part of the given master equation (308) we get:

$$-i[H,\rho] = -\frac{i\omega}{2}\left[\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}\rho_{00} & \rho_{01}\\ \rho_{10} & \rho_{11}\end{pmatrix} - \begin{pmatrix}\rho_{00} & \rho_{01}\\ \rho_{10} & \rho_{11}\end{pmatrix}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\right] =$$

$$= -\frac{i\omega}{2}\left[\begin{pmatrix}-i\rho_{01} & -i\rho_{11}\\ i\rho_{00} & i\rho_{01}\end{pmatrix} - \begin{pmatrix}i\rho_{01} & -i\rho_{00}\\ i\rho_{11} & -i\rho_{10}\end{pmatrix}\right] = \frac{\omega}{2}\begin{pmatrix}-(\rho_{01}+\rho_{10}) & \rho_{00}-\rho_{11}\\ \rho_{00}-\rho_{11} & \rho_{01}+\rho_{10}\end{pmatrix} \tag{310}$$

For the dissipator the calculation gives:

$$D[\rho] = \Gamma_z\rho\Gamma_z^\dagger - \frac{1}{2}(\Gamma_z^\dagger\Gamma_z\rho + \rho\Gamma_z^\dagger\Gamma_z) =$$

$$= \frac{\Lambda_z}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\rho_{00} & \rho_{01}\\ \rho_{10} & \rho_{11}\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} -$$

$$-\frac{\Lambda_z}{4}\left[\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\rho_{00} & \rho_{01}\\ \rho_{10} & \rho_{11}\end{pmatrix} + \begin{pmatrix}\rho_{00} & \rho_{01}\\ \rho_{10} & \rho_{11}\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\right] =$$

$$= \frac{\Lambda_z}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\rho_{00} & -\rho_{01}\\ -\rho_{10} & \rho_{11}\end{pmatrix} -$$

$$-\frac{\Lambda_z}{4}\left[\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\rho_{00} & \rho_{01}\\ -\rho_{10} & -\rho_{11}\end{pmatrix}+\begin{pmatrix}\rho_{00} & \rho_{01}\\ \rho_{10} & \rho_{11}\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\right]=$$

$$=\frac{\Lambda_z}{2}\begin{pmatrix}\rho_{00} & -\rho_{01}\\ -\rho_{10} & \rho_{11}\end{pmatrix}-\frac{\Lambda_z}{4}\left[\begin{pmatrix}\rho_{00} & \rho_{01}\\ \rho_{10} & \rho_{11}\end{pmatrix}+\begin{pmatrix}\rho_{00} & \rho_{01}\\ \rho_{10} & \rho_{11}\end{pmatrix}\right]\Longrightarrow$$

$$D[\rho]=\Lambda_z\begin{pmatrix}0 & -\rho_{01}\\ -\rho_{10} & 0\end{pmatrix} \tag{311}$$

Thus explicitly we get for the master equation:

$$\begin{pmatrix}\dot{\rho_{00}} & \dot{\rho_{01}}\\ \dot{\rho_{10}} & \dot{\rho_{11}}\end{pmatrix}=\frac{\omega}{2}\begin{pmatrix}-(\rho_{01}+\rho_{10}) & \rho_{00}-\rho_{11}\\ \rho_{00}-\rho_{11} & \rho_{01}+\rho_{10}\end{pmatrix}+\Lambda_z\begin{pmatrix}0 & -\rho_{01}\\ -\rho_{10} & 0\end{pmatrix} \tag{312}$$

In components we get the following set of differential equations:

$$\dot{\rho_{00}}=-\frac{\omega}{2}(\rho_{01}-\rho_{10}) \tag{313}$$

$$\dot{\rho_{01}}=\frac{\omega}{2}(\rho_{00}-\rho_{11})-\Lambda_z\rho_{01} \tag{314}$$

$$\dot{\rho_{10}}=\frac{\omega}{2}(\rho_{00}-\rho_{11})-\Lambda_z\rho_{10} \tag{315}$$

$$\dot{\rho_{11}}=\frac{\omega}{2}(\rho_{01}-\rho_{10}) \tag{316}$$

A short calculation beside gives us a feeling of the approximate structure of the time evolution. Therefore we consider the component $\dot{\rho_{01}}$ (314) and form the second derivative and insert then the relations (313) and (316):

$$\ddot{\rho_{01}}=\frac{\omega}{2}(\dot{\rho_{00}}-\dot{\rho_{11}})-\Lambda_z\dot{\rho_{01}}=$$

$$=\frac{\omega}{2}(-\frac{\omega}{2}(\rho_{01}-\rho_{10})-\frac{\omega}{2}(\rho_{01}-\rho_{10}))-\Lambda_z\dot{\rho_{01}}=$$

$$=-\frac{\omega^2}{2}(\rho_{01}-\rho_{10})-\Lambda_z\dot{\rho_{01}} \tag{317}$$

From this we may follow:

$$\rho_{01}\sim e^{\pm i\omega t}e^{-\Lambda_z t} \tag{318}$$

Thus the approximate structure of the time evolution looks like follows:

$$D[\rho]=\Lambda_z\begin{pmatrix}0 & e^{i\omega t}e^{-\Lambda_z t}\\ e^{-i\omega t}e^{-\Lambda_z t} & 0\end{pmatrix} \tag{319}$$

But now we want to take an explicit look at the dissipator. In particular we want to find an expression that enables us conclusions for our decoherence-parameters λ and λ'. Therefore we re-express the dissipator with use of the sigma-matrices $\sigma_+ := \frac{1}{2}(\sigma_1+i\sigma_2)$ and $\sigma_i := \frac{1}{2}(\sigma_1-i\sigma_2)$:

$$D[\rho]=\Lambda_z\begin{pmatrix}0 & -\rho_{01}\\ -\rho_{10} & 0\end{pmatrix}=-\Lambda_z(\rho_{01}\sigma_+ +\rho_{10}\sigma_-)=$$

$$= -\frac{\Lambda_z}{2}(\rho_{01}(\sigma_1 + i\sigma_2) + \rho_{10}(\sigma_1 + i\sigma_2)) =$$

$$= -\frac{\Lambda_z}{2}((\rho_{01} + \rho_{10})\sigma_1 + i(\rho_{01} - \rho_{10})\sigma_2) =$$

$$= -\frac{\Lambda_z}{2}(\rho_1\sigma_1 + \rho_2\sigma_2) \Longrightarrow$$

$$D[\rho] = -\frac{1}{2}\overrightarrow{\sigma} L \overrightarrow{\rho} \qquad \text{with} \qquad L = \begin{pmatrix} \Lambda_z & 0 & 0 \\ 0 & \Lambda_z & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{320}$$

The same calculation can be done for the case that the magnetic noise field acts along the x-axis - therefore this time we label the decoherence-parameter Λ_x. The calculation gives for the dissipator:

$$D[\rho] = -\frac{1}{2}\overrightarrow{\sigma} L \overrightarrow{\rho} \qquad \text{with} \qquad L = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Lambda_x & 0 \\ 0 & 0 & \Lambda_x \end{pmatrix} \tag{321}$$

Analogously for a magnetic field along the y-axis we get:

$$D[\rho] = -\frac{1}{2}\overrightarrow{\sigma} L \overrightarrow{\rho} \qquad \text{with} \qquad L = \begin{pmatrix} \Lambda_y & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Lambda_y \end{pmatrix} \tag{322}$$

If we now come back to our two setups and consider in the first place setup (a) with the magnetic field along the y-axis (322) and compare it to our model above of the dissipator of the helicity states (306) we see that we can associate setup (a) with the longitudinal measurement so that:

$$\Lambda_y \equiv \lambda \qquad \text{and} \qquad \lambda' = 0 \tag{323}$$

The comparison with the theorem 3 for complete positivity shows that here $\lambda' = 0$ is the minimum.

Whereas setup (b) with magnetic fields in the x-z-plane can be associated with the transverse measurement. If we put together a field in x-direction (321) and in z-direction (320) the comparison with the dissipator for transverse polarization (304) and the complete positivity theorem 3 gives:

$$\Lambda_x = \Lambda_z \equiv \lambda \qquad \text{and} \qquad \lambda' = 2\lambda \tag{324}$$

So we finally found a method to cover in principle the whole spectrum of complete positivity between $\lambda' = 0$ and $\lambda' = 2\lambda$, which is desirable. But of course it would be crucial to find a setup which allows also values $\lambda' > 2\lambda$ so that complete positivity can be tested experimentally.

6 Interpretation of the Quantum Mechanical Formalism

6.1 Introduction on the relation between physics and philosophy

Born through Max Planck's discovery of the Planck's constant in 1900, over the last 111 years quantum physics turned out not to be only an interesting, inspiring and also shocking field for physicist themselves, but also for biologists, chemists, social scientists, philosophers, artists and many people in other fields. Especially the relationship between quantum physics and philosophy is very interesting and shall therefore be stressed in this chapter.

As we will see in the following, in the beginning of the 20th century the relationship between philosophy and physics has emerged in two directions: On the one hand many philosophers got interested in the quantum sphere. On the other hand many physicists themselves moved towards philosophy to catch up with problems they where faced doing their job in physics.

In his book "Die Philosophie der Physiker" [59] the German professor for philosophy Erhard Scheibe cites a talk of the physicist Arnold Sommerfeld in 1948 to illustrate the new dynamics quantum physics brought into philosophy:

> *"Im 19. Jahrhundert war das Verhältnis zwischen Philosophie und Physik gespannt. Zuerst dominierte die Philosophie und wollte der Physik den Weg vorschreiben. Später waren die Physiker mißtrauisch geworden, sie lehnten jede Philosophie ab.*
>
> *Im 20. Jahrhundert änderte sich das Verhältnis grundlegend. Gleich zu Beginn im Jahre 1900 entdeckte Planck das Wirkungsquantum. Damit gab er der Philosophie die härteste Nuss zu knacken, mit der sie noch lange zu tun haben wird. Der entscheidende Schritt zu einer philosophisch vertieften Physik [wurde] von Einstein im Jahre 1905 getan.*
>
> *Seit Einstein gibt es keine Entfreumdung mehr zwischen Physikern und Philosophen. Die Physiker sind zu Philosophen geworden, und die Philosphen hüten sich, mit der Physik in Konflikt zu geraten."* [1]

In his book Scheibe also gives another example to illustrate the lively relationship between physics and philosophy in the early 20th century, another time he cites Sommerfeld around the same year - but this time the estimation comes from the corner of social sciences:

[1] In the 19th century the relationship between philosophy and physics was tense. Firstly philosophy dominated and wanted to rule the way for physics. Later the physicist became suspicious and refused all kind of philosophy.

In the 20th century the relationship changed radically. Already in the very beginning in the year 1900 Planck found the Planck's constant. Therewith he gave philosophy a hard nut to crack with which it will be dealing for long. The first crucial step towards a philosophically accentuated physics was done by Einstein in 1905.

Since Einstein there is no longer an estrangement between physicist and philosophers. The physicist have become philosophers and the philosophers beware of not to get in conflict with physics.

> *"Adolf von Harnack sagte einmal, wie mir berichtet wurde, im Sprechzimmer der Berliner Universität: Man klagt darüber, dass unsere Generation keine Philosophen habe. Mit Unrecht: Die Philosophen sitzen jetzt nur in der anderen Fakultät, sie heißen* Planck *und* Einstein."[2]

Following Scheibe this citation also shows that the relationship between philosophy and physics got closer but it did't really change for the better in terms of a growth of intellectual exchange. Much more the physicists got philosophers themselves - far away from philosophical faculties. This is what Scheibe calls "revolutionary" [59]. As the reason for that he sees, that physicists found themselves confronted with the fact that the new physical theories - Einstein's relativity and quantum mechanics - were standing in opposition to the basics of classical physics. In such a situation there is no way out except that the experts tackle the problems. This was also Einstein's opinion:

> *"Oft und gewiss nicht ohne Berechtigung ist gesagt worden, dass der Naturwissenschaftler ein schlechter Philosoph sei. Warum sollte es also nicht auch für den Physiker das Richtige sein, das Philosophieren dem Philosophen zu überlassen? In einer Zeit, in welcher die Physiker über ein festes, nicht angezweifeltes System von Fundamentalbegriffen und Fundamentalgesetzen zu verfügen glaubten, mag dies wohl so gewesen sein, nicht aber in einer Zeit, in welcher das ganze Fundament der Physik problematisch geworden ist, wie gegenwärtig. In solcher Zeit des durch die Erfahrung erzwungenen Suchens nach einer neuen solideren Basis kann der Physiker die kritische Betrachtung der Grundlagen nicht einfach der Philosophie überlassen, weil nur er selber am besten weiß und fühlt, wo ihn der Schuh drückt; auf der Suche nach einem neuen Fundament muss er sich über die Berechtigung beziehungsweise Notwendigkeit der von ihm benutzten Begriffe nach Kräften klarzuwerden versuchen."* [3] [31]

So there seems to be no way out that with the rise of the theory of relativity and quantum mechanics physicists didn't move towards philosophers but they became philosophers themselves in a non-academic, "revolutionary" (Scheibe) sense. So should the rise of new problems natural scientists were faced with through modern physics bring along the change to overcome the long known and lamented canon between the sciences? Like in

[2] Adolf von Harnack once said, how I was told, in his parlour office at the university of Berlin: One complains that in our generation there are no philosophers. This is wrong: The philosophers now just sit in another faculty and their names are Planck and Einstein.

[3] It has often been said, and certainly not without justification, that the man of science is a poor philosopher. Why then should it not be the right thing for the physicist to let the philosopher do the philosophizing? Such might indeed be the right thing to do a time when the physicist believes he has at his disposal a rigid system of fundamental laws which are so well established that waves of doubt can't reach them; but it cannot be right at a time when the very foundations of physics itself have become problematic as they are now. At a time like the present, when experience forces us to seek a newer and more solid foundation, the physicist cannot simply surrender to the philosopher the critical contemplation of theoretical foundations; for he himself knows best and feels more surely where the shoe pinches. In looking for an new foundation, he must try to make clear in his own mind just how far the concepts which he uses are justified, and are necessities.

many cases the answer to this question will depend on whom you ask. But the idea, that physics and philosophy have been merging into each other more and more since the upcoming of modern physics out of need for a lively exchange has to be seen critical. Paraphrasing Carl Friedrich von Weizsäcker there was no exchange since philosophizing physicists were philosophers of their own invented philosophy. In 1958 the German physicist and philosopher von Weizsäcker brought it to the point:

> *"Es ist ein empirisches Faktum, dass fast alle führenden theoretischen Physiker unserer Zeit philosophieren. Es ist ein zweites empirisches Faktum, dass ihre Philosophie im allgemeinen weitgehend ihre eigene Erfindung ist und sich mit den überlieferten Meinungen der Philosophen manchmal schlecht zusammenreimt. Beide empirischen Tatsachen scheinen mir aus einer sachlichen Notwendigkeit hervorgegangen zu sein, nämlich daraus, dass die moderne Physik ohne Philosophie nicht adäquat verstanden werden kann und dass es eine Philosophie, die dieses adäquate Verständnis liefern könnte, bis heute noch nicht gibt."* [4][68]

Also the Viennese physicist Wolfgang Pauli makes no secret about his distance to traditional philosophy, in a very personal statement in 1961 he committed:

> *"Zur Orientierung der Philosophen möchte ich von vornherein klarstellen, daß ich nicht zu einer der philosophischen Schulen gehöre, deren Namen mit einer Art von 'Ismus' enden. Darüber hinaus bin ich sehr dagegen, irgendeine spezielle physikalische Theorie, wie die Relativitätstheorie oder die Quanten- oder Wellenmechanik, unter einen dieser 'Ismen' zu bringen, obwohl dies von Zeit zu Zeit sogar von Physikern so gemacht worden ist."*[5] [55]

What we can read between these lines is, that the history of the relationship between physicists and philosophers is not only characterized by attraction, but also by ignorance. Also a citation by É. H. Gilson nicely illustrates this ambivalent relation of affinity and disaffirmation:

> *Nothing can be compared to the ignorance of modern philosophers concerning natural science accept for the ignorance modern scientist concerning philosophy.*

In the beginning of the 20th century when through the scientific approach many questions of philosophical interpretations arose, every physicist had to find his individual place

[4]It is an empirical fact, that virtually all leading physicists of the present philosophize. And it is a second that their philosophy is mainly their own invention and sometimes does not fit together with the traditional opinions of the philosophers. These two empirical facts seem to me to have developed out of a necessity: namely that modern physics cannot be understood adequately without philosophy and yet there does not exist a philosophy that could give a properly understanding.

[5]For the orientation of the philosophers I want to make clear that I do not belong to one of the philosophical schools whose names end with any kind of 'ismus'. Apart from that I reject that any special physical theory like relativity or quantum mechanics can put below any kind of those 'ismen' although even some physicists have done so from time to time.

in the stress field of philosophical approach or ignorance. Albert Einstein of course played a key role in this respect as he was the one to bring up many questions with his foundation of theory of relativity. And just as in his scientific life he became to hold an outstanding position as well what concerns the metaphysical components of physics. Einstein's position to philosophy was outstanding in this respect that he saw a unity of physics and philosophy through a religious, spiritual believe [31]:

> *"Ich glaube, dass jeder wahre Theoretiker ein gezähmter Metaphysiker ist... Der gezähmte Metaphysiker glaubt, dass die Totalität der sinnlichen Erfahrung auf der Grundlage eines Begriffssystems von großer Einfachheit 'verstanden' werden kann. Der Skeptiker wird sagen, daß dies ein 'Wunderglaube' sei. Das ist er allerdings, aber es ist ein Wunderglaube, der sich in einem erstaunlichen Maße bewährt hat in der Entwicklung der Wissenschaft."* [6]

Also in the introduction to the description of his theory of relativity Einstein wrote:

> *"Die Wichtigkeit des Ich im Weltbilde deucht mir ein Maßstab, an dem man Glaubendslehren, philosophische Systeme, künstlerische und wissenschaftliche Weltauffassungen aufreihen kann, wie Perlen auf einer Schnur...*
>
> *Das naturwissenschaftliche Denken steht an dem Ende jener Reihe, dort, wo das Ich... nur noch eine unbedeutende Rolle spielt, und jeder Fortschritt in den Begriffsbildungen der Physik, Astronomie, Chemie bedeutet eine Annäherung an das Ziel der Ausschaltung des Ich."*[7]

The following collage is an attempt to illustrate Einstein's position; from left to right: The humbleless believe in a so-called "Wunderglaube" involves the bold turning away from academic physics and philosophy and leads to deeper insights in science and is therefore an approach to the elimination of the self.

As already mentioned, Einstein's position was outstanding in this respect. Now and then the mainstream among physicists was to take the pragmatic stand which means not to think too much about these issues and concentrate more on purely physical problems. In the 1930ies Max Planck made the honest commitment [37]:

> *"...wer sucht, der muss etwas als vorhanden annehmen, nach dem er sucht. Dieses Etwas ist das Reale im metaphysischen Sinn...[ich halte]...das metaphysische Reale für die unerläßliche Voraussetzung der Naturwissenschaft und*

[6] I believe that every true theorist is a kind of tamed metaphysicist, no matter how pure a 'positivist' he may fancy himself. The metaphysicist believes that the logically simple is also the real. The tamed metaphysicist believes that not all that is logically simple is embodied in experienced reality, but that the totality of all sensory experiencec can be 'comprehended' on the basis of a conceptual system built on premises of great simplicity. The skeptic will say that this is a 'miracle creed'. Admittedly so, but it is a miracle creed which has been born out to an amazing extent by the development of science.

[7] The importance of the ego in the world-picture seems to me a measure according to which we may order confessions of faith, philosophic systems, world-views rooted in art and science, like pearls on a string...

Natural science is situated at the end of this series, at the point where the ego, the subject, plays only an insignificant part; every advance in the mouldings of the concepts of physics, astronomy, and chemistry denotes a further step towards the goal of excluding the ego.

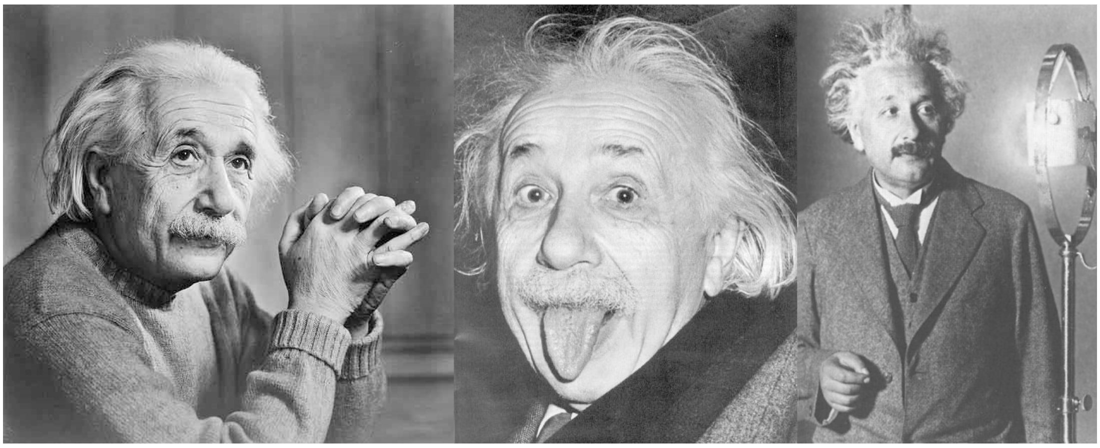


Figure 14: **Collage of photographs of Albert Einstein, (c) 2010**, Sources: eatourbrains.com, tillrathke.twoday.net, instein.com

den Glauben daran für die Wurzel des wissenschaftlichen Erkenntnistreibens, ja ich wage die Behauptung, dass jeder echte Naturforscher die Existenz einer realen Welt in dem geschilderten Sinn als eine Selbstverständlichkeit betrachtet, über die er nicht einmal gern kritisch nachdenkt, weil jeder Zweifel daran ihn nur von seiner Arbeit ablenken würde."[8]

6.2 Copenhagen interpretation

It is not the question whether a theory is too weird,
but whether it is weird enough.

Niels Bohr

With quantum physics the philosophical efforts have emerged ever since, up to now there are fare more than thousand philosophical publications by physicists [59]. Thus it can be seen as a stand-alone region in philosophy. In progress of time this contributions are spread since 1900 but a certain zenith can be found at the middle of the 20th century [59].

From the very first moment one interpretation of the - at that time new-born - quantum mechanical formalism was leading the debate, called after its place of origin, the *Copenhagen interpretation.* It is generally assumed, that it is going back to the intense discussions between the year 1925 and 1927 at the Institute of theoretical physics at Copenhagen, which was a hot spot for physics at that time. The three intellectual fathers of the Copenhagen interpretation can be seen as one of the opinion leaders of the

[8]... the one who searches has to accept something to be given for that he is looking. This something is the reality in the metaphysical sense... I repute the metaphysical reality for the essential condition of natural science and the believe in it for the root of scientific rush to knowledge, yes I make the assertion that every real natural scientist takes the existence of a real world for sure and he does not even like to think about it critically because every doubt about it would divert him from his work.

new area of quantum physics: Niels Bohr, Werner Heisenberg and Wolfgang Pauli. Still the Copenhagen interpretation has found its opponents from the very beginning, among them physicists that were not less famous and influential: Albert Einstein, Max Planck, Erwin Schrödinger and others.

"However, there has never been complete agreement about the actual meaning, or even definition, of this interpretation even among its main contributors. In fact, the Copenhagen interpretation has remained until today an amalgamation of different views," states Claus Kiefer in 2002 [48] - and it mirrors the incompatibility of Bohr's and Heisenberg's views on quantum mechanics.

In his early ages Heisenberg wanted to get rid of all intuitive concepts and therefore to base quantum theory solely on observable quantities. But his attitude changed drastically with the paper in which he introduced the uncertainty relations [41] in 1927. Since that his standpoint changed to: It is the theory that decides what can be observed. As Kiefer notices [48] Heisenberg's radical turn around can be "clearly understood as a reaction on the advent of Schrödinger's wave mechanics which, in particular due to its intuiveness, became soon very popular among physicists. In fact, the word *anschaulich* (intuitive) is contained in the title of Heisenberg's paper."

And this is very interesting if we just remember that Heisenberg - in the first place - felt deeply sceptic about Schrödinger's wave approach to quantum mechanics. That this simply formulation could contain all information that Heisenberg turned over in his complicated matrice calculations was just unbelievable to him.

Bohr's approach to the Copenhagen Interpretation was rather different, for him the core was the complementarity between particles and waves. In 1927 Bohr gave the first summary of his interpretation in his famous lecture in Como [15] in which he also introduced the notion of complementarity. Later Bohr extended the principle to non-physical themes and it became a central concept of his own philosophy. Complementarity means that a quantum object is neither a particle nor a wave. But for our intuition we have to use both pictures of course - and this is what Heisenberg as a mathematical physicist did not like about the complementarity principle. Kiefer: "He [Heisenberg] preferred to use one coherent set of concepts, rather than two incompatible ones. In fact, it was known by then that particle and wave language can be converted into each other and are transcended into the consistent formalism of quantum theory."[48]

Later Heisenberg wrote [43]:

> *"Licht und Materie sind einheitliche physikalische Phänomene, ihre scheinbare Doppelnatur liegt an der wesentlichen Unzulänglichkeit unserer Sprache. [...] Will man trotzdem von der Mathematik zur anschaulichen Beschreibung der Vorgänge übergehen, so muss man sich mit unvollständigen Analogien begnügen, wie sie uns Wellen- und Partikelbild bieten."*[1]

[1] Light and matter are unique physical phenomena, their apparent double nature is due to the essential inadequacy of our language. [...] If one nevertheless wants to procede from the mathematics to the intuitive description of the phenomena, we have to restrict ourselves to incomplete analogies as they are offered by the wave and particle pictures.

According to the Copenhagen interpretation, the act of measurement causes the "collapse" of the wave function of a state to the value defined by the measurement. So to say the interpretation says that the wave function involves the various probabilities that a given event will proceed to certain different outcomes. But when one or another of those more- or less-likely outcomes becomes manifest the other possible values vanish. For example, if a particle passes through a double slit apparatus there are various probabilities for where on the detection screen that individual particle will hit. But once it has hit, there is no longer any probability that it will hit somewhere else.

The interpretation says that in principle it is not possible to predict which value will be measured - there is just a probability distribution of possible outcomes but this is not caused by the incompleteness of the theory (as Einstein argued) but by the nature of reality. This standpoint was experimentally proven by the Bell-experiment by Aspect [2] already mentioned in chapter 2.2.

Another key feature of the Copenhagen interpretation is that it rejects to load up the quantum mechanical formalism - i.e. the wave function - with physical reality. Instead, the formalism is just seen as a tool that works in the prediction of probability distributions of measurement outcomes.

This involves another characteristic of the interpretation: the unity of quantum phenomena which means that the quantum system is influenced by the measurement process. In Bohr's wording [17]:

> *"The main difference between the study of phenomena in classical physics and in quantum physics is that in the first one the interaction between the studied objects and the measurement instrument can be ignored, whereas in the second one the interactions forms an incorporate component of the phenomena."*

6.3 Critics of the Copenhagen interpretation

6.3.1 Einstein's Critic

To understand Einstein's critic on the Copenhagen Interpretation it is essential to have a close look at Einstein's definition of reality. Einstein's attitude was that a complete description of a physical system is equivalent to the complete description of the state - and here comes in Einstein's understatement of the notion of reality, Einstein said:

> *"Die Physik ist eine Bemühung, das Seiende als etwas Begriffliches zu erfassen, was unabhängig vom Wahrgenommen-Werden gedacht wird. In diesem Sinne spricht man vom 'Physikalisch-Realen'. In der Vor-Quantenphysik war kein Zweifel, wie dies zu verstehen sei."* [1]

Or even more drastically Einstein claimed [29]:

[1]Physics is the effort to perceive that which exists as something conceptual, which can be thought independent from observing it. In this sense one speaks of the 'physical-real'. In pre-quantum physics there was no doubt how this could be understood.

> *"... die Begriffe der Physik beziehen sich auf eine reale Außenwelt, d.h., es sind Ideen von Dingen gesetzt, die eine von den wahrnehmenden Subjekten unabhängige 'reale Existenz' beanspruchen."* [2]

Especially between Einstein and Bohr a big debate was going on for decades - partly via face-to-face conversations, partly via letters. In a letter that Einstein wrote to Bohr in 1924 there is one passage that got quite popular as it brings the conflict clearly to the point. Einstein wrote:

> *"Zu einem Verzicht auf die strenge Kausalität möchte ich mich nicht treiben lassen, bevor man sich nicht noch ganz anders dagegen gewehrt hat als bisher. Der Gedanke, dass ein einem Strahl ausgesetztes Elektron* aus freiem Entschluss *den Augenblick und die Richtung wählt, in der es fortspringen will, ist mir unerträglich. Wenn schon, dann möchte ich lieber Schuster oder gar Angestellter in einer Spielbank sein als Physiker."* [3]

Two years later Einstein was a little bit more cautious. In another famous passage in a letter from Einstein to Bohr it says:

> *"Die Quantenmechanik ist sehr Achtung gebietend. Aber eine innere Stimme sagt mir, dass das doch nicht der wahre Jakob ist. Die Theorie liefert uns viel, aber dem Geheimnis des Alten bringt sie uns kaum näher. Jedenfalls bin ich überzeugt, dass* der *nicht würfelt."*[4]

6.3.2 Schrödinger's Position

Although Einstein and Bohr had completely opposite opinions about the interpretations of the "measurement problem" - but just the view of Schrödinger makes clear how close their interpretations were lying together namely in this sense that both of them thought the "measurement problem" to be the heart of the epidemic crises in which modern physics left the physicists behind. That they had completely opposite opinions how to solve this problem, is another story, which was told above. Also Schrödinger was aware of this crisis but he didn't think the "measurement problem" to be part of it (this is also the reason, why I put "measurement problem" under quotation marks).

For Schrödinger the epidemic crisis of the 1920ies was hard and deep. In 1955 he wrote [64]:

[2]"... the notations of physics apply to a real outside world, this means ideas of the things are assumed that claim a 'real existence' independent from the observing subjects."

[3]I don't want to be driven to the abandonment of strict causality before one didn't defense against it completely different than until now. The believe that an electron that is exposed to a beam takes the *free decision* at what time and in which direction it wants to jump off, is unbearable to me. If this is that case I would rather be a cobbler, or even an employee in a gaming-house rather than a physicist.

[4]One has to bring a lot of respect to quantum physics. But an inner voice tells me that this is not the real McCoy. The theory provides a lot, but it brings us little closer to the secrets of the old man. At least I am sure that *he* does not play dice.

"Worauf es mir ankommt, ist dieses: die moderne Entwicklung, die wirklich zu verstehen ihre Urheber noch weit entfernt sind, war ein Einbruch in die verhältnismäßig einfache Theorie der Physik, die gegen Ende des 19. Jahrhunderts recht gut umrissen schien. Dieser Einbruch hat in gewissem Sinne alles umgeworfen, was auf den Grundmauern errichtet war, die im 17. Jahrhundert...gelegt worden waren. Ja, die Grundmauern selbst beben."[5]

As Scheibe points out [59] Schrödinger's opinion was - just as his Nobel-partner Dirac's - that there could be no talk, that the attempts of Bohr or Einstein enabled to overcome this crisis. And in this sense is also be understood his comparison of quantum jumps with the epicycles of Ptolemeus' astronomy as well as that the supposed influence of the observer in the quantum measurement process was for him "just a very overestimated temporary aspect without deeper meaning". [59]

Therefore the deep crisis, Schrödinger was aware of, couldn't be solved by the Copenhagen interpretation. To understand his main philosophical problem one has to understand the following: His world view was the one of an idealistic monoism. That these fundamentally separated individual views could still find together in some sense - by the way especially in natural sciences - and come to the point to live in the same world in the end - this experience was *the* great unsolved mystery for him, his mayor philosophical problem. [59], [40]

As Scheibe notes, Schrödinger didn't accept the common explanation for this problem, to suppose a real outer world - he thought this interpretation was naive and not enough for a solution. And he thought that it was just as metaphysical and mystical as his own explanation: Inspired by Spinoza, Schopenhauer and the Indian *Vedanta* Schrödinger believed in a worldspirit, of which the individuals are just aspects and its unity are the community of experience which had to be explained. In "Geist und Materie" [66] Schrödinger wrote:

"Ich wage zu glauben, dass man beide Paradoxa lösen wird..., indem man dem Bau unsrer westlichen Naturwissenschaft die östliche Identitätslehre einverleibt. Bewusstsein gibt es seiner Natur nach nur in der Einzahl. Ich möchte sagen: die Gesamtzahl aller 'Bewusstheiten' ist immer bloß 'eins'. Ich wage, den Geist unzerstörbar zu nennen, denn er hat sein eigenes und besonderes Zeitmaß; nämlich er ist jederzeit jetzt. *Für ihn gibt es in Wahrheit weder früher noch später, sondern nur ein Jetzt, in das die Erinnerungen und die Erwartungen eingeschlossen sind."* [6]

[5] For me the important thing is the following: the modern development, which its founders are far away to understand, was the break down of the relatively simple theory of physics, which seemed to be quite complete at the end of the 19th century. This break down has stroke down in some sense everything which was built on the foundation walls, which were made up in the 17th century. Indeed, the foundation walls themselves tremble.

[6] I dare to beliebe that one can solve both paradoxes... by merging eastern identity doctrin with the building of our western natural sciences. Consciousness exists naturally just in singular. I want to say: the completeness of all 'consciousnesses' is always just 'one'. I dare to call the spirit indestructible because it has its own and special time-measure; namely it is always *now.* For it in reality there is no earlier or later, but just a now in which the memories and the prospects are melted.

Whereas the nature we experience with our senses is just illusion, is just *maya*. About this world, this universe invented and constructed by our senses but not as real as the underlying worldspirit Schrödinger said: "For myself is all that *maya*, even though very lawful and interesting *maya*." And this quote is a pointer to the inner tension with which Schrödinger was confronted: to feel close to the old Indian knowledge on the one hand and to be part of (and in Schrödinger's part also a very fundamental, mayor contributing part) the western culture and among it part of the innerst circle of the scientific community on the other hand, which focuses mainly on issues that his inner voice would call illusion, *maya*. In his last work in 1958 Schrödinger expressed this tension:

> *"Wir fühlen das Verlangen nach einer vollständigen Beschreibung der materiellen Welt in Raum und Zeit, und wir betrachten es als keineswegs erwiesen, dass dieses Ziel nicht erreicht werden könnte."*[7]

To stand this tension Schrödinger applied himself to extensive studies on the features of the physical world view. As Scheibe explains this scientific world view bases on two requirements, namely *comprehensibility* of the natural events and the requirement of *objectiveness*. For Schrödinger the crisis of modern physics was that it violated both of these requirements.

This is what I want to call the paradox of the paradox of *Schrödinger's cat*. Although he didn't think the "measurement problem" to be of fundamental meaning because of the reasons that we discussed above (keyword: world spirit) and therefore neither could share sympathy with the *Copenhagen interpretation* nor with Einstein's critics on Copenhagen, nevertheless Schrödinger was the first to catch the fundamental importance of the entangled particles that Einstein came up with in the EPR-paper [28]. Einstein himself didn't even label his concept of entanglement, he just invented it, but just Schrödinger understood it, catched it up, named it and with his gedanken-experiment of *Schrödinger's cat* - which was introduced in chapter 2.6 - made it comprehensible for the community. The paradox of the *Schrödinger's cat*-paradox is, that this illustrative picture is widely known in the community and as we have seen also far outside and found its way into completely other fields although the intellectual father of this macroscopic adoption of the quantum measurement problem supposed it to be of minor interested - and this out of philosophical reasons. But still Schrödinger's highly interesting world view isn't even known in the physical community, while the catchy picture of what his arguments defeated got celebrated.

6.4 Many worlds theory

As we have seen, the Copenhagen Interpretation places a boundary between quantum and classical. The existence of this boundary is a postulate but it is not visible just like the cartoon above (7) in chapter 3.5 suggests.

[7] We feel the desire for a complete description of the material world in space and time and we do not consider it as proven, that this goal might not be reachable.

At least on the first glance the *Many Worlds Interpretation* (or more precisely, the *Many Universes Interpretation*) claims to do away this boundary. Developed in the 1950s by Hugh Everett III [34] with encouragement from John Archibald Wheeler in this interpretation the entire universe is described by quantum theory. Superpositions evolve according to the Schrödinger equation but each time a suitable interaction takes place between any two quantum systems, the wave function of the universe splits, developing more "branches". Every possible measurement outcome is materialized - but in different universes that cannot communicate with each other. An illustration for that gives the figure below.

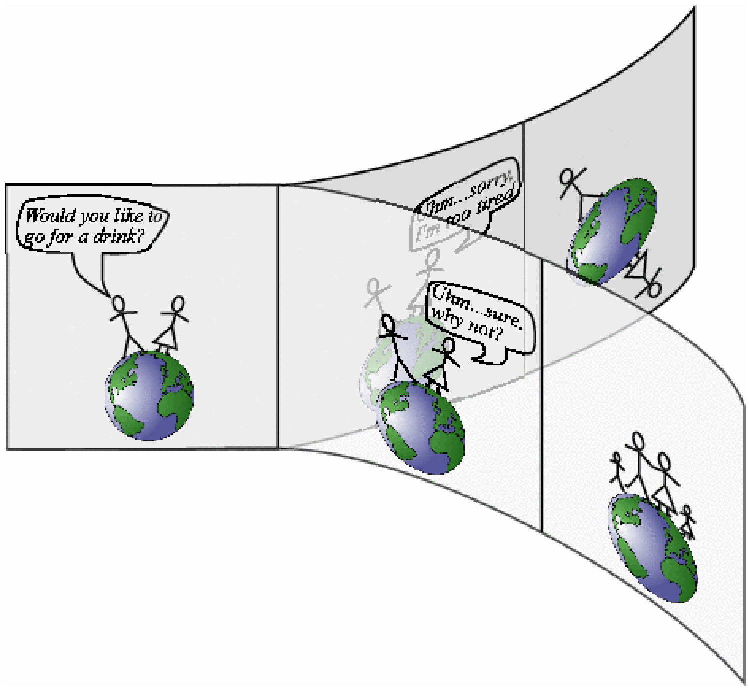


Figure 15: **Cartoon: Many Worlds Interpretation**, (c) Max Tegmark. Source: http://space.mit.edu/home/tegmark/crazy.html

Initially, Everett's work went almost unnoticed. But over a decade later, around 1970, Bryce DeWitt and Neill Graham managed to upgrade its status from "virtually unknown" to "very controversial" [74].

For example - rather diplomatic - John Bell called the Everett interpretation a "Bohm interpretation without trajectories". In fact, Everett assumed just as Bohm that the wave function is part of reality and that there is never any collapse. Therefore, after a measurement, all components corresponding to the different outcomes are equally present. It is claimed that the probablility interpretation of quantum theory can be derived from the formalism (which is, however, a contentious issue).

Whereas von Weizsäcker called the interpretation with great optimism: "... die einzige, die nicht hinter das schon von der Quantentheorie erreichte Verständnis zurück-, sondern vorwärts über es hinausstrebt."[1] [69]

Almost negative - and therefore matching the general tone in the community about the *Many Worlds Interpretation* is Zurek's judgement: "At first glance, the Many Worlds and Copenhagen Interpretations have little in common. The Copenhagen Interpretation demands an a priori 'classical domain' with a border that enforces a classical 'embargo' by letting through just one potential outcome. The Many Worlds Interpretation aims to abolish the need for the border altogether. Every potential outcome is accommodated by the ever-proliferating branches of the wave function of the Universe."[74] So far, so good, but as Zurek continues: "The similarity between the difficulties faced by these two viewpoints becomes apparent, nevertheless, when we ask the obvious question, 'Why do I, the observer, perceive only one of the outcomes?' Quantum theory, with its freedom to rotate bases in Hilbert space, does not even clearly define which states of the Universe correspond to the 'branches.' Yet, our perception of a reality with alternatives (not a coherent superposition of alternatives) demands an explanation of when, where, and how it is decided what the observer actually records. Considered in this context, the Many Worlds Interpretation in its original version does not really abolish the border but pushes it all the way to the boundary between the physical Universe and consciousness." With which he comes to the excoriating judgement: "Needless to say, this is a very uncomfortable place to do physics."

6.5 Theory of decoherence

But what is the loophole to escape from the measurement problem and the interpretation of the wavefunction ψ which shows up in Schrödinger's equation? Until now a general solution has not been found, but very promising in this epistemic context seems the ansatz of decoherence theory. As the key for the explanation from quantum to classical Zurek states: "Macroscopic systems are never isolated from their environments." [74] And therefore - as his college Heinz Dieter Zeh emphasized they should not be expected to follow Schrödinger's equation, which is applicable only to a closed system. Zurek continues:

> *"As a result, systems usually regarded as classical suffer (or benefit) from the natural loss of quantum coherence, which 'leaks out' into the environment. The resulting 'decoherence' cannot be ignored when one addresses the problem of the reduction of the quantum mechanical wave packet: Decoherence imposes, in effect, the required 'embargo' on the potential outcomes by allowing the observer to maintain records of alternatives but to be aware of only one of the branches."* [74]

So to say, in physical reality systems are never completely isolated, for example a flying particle will collide with air molecules or magnetism or thermal radiation will influence the particle. As a result, quantum coherence *leaks out* (as Zurek says) and with it entanglement. When quantum coherence is lost, the particle's state gets entangled with

[1] ...the only one that does not fall back behind the understanding already achieved by quantum theory but which strives forward and even beyond.

the state of the environment - but this *leaked out* entanglement cannot - at the moment - be measured somehow or used for quantum operatations, it is unfoundably hidden in the huge mixture of states the environment provides. And it is this moment, where the particle stops to behave due to quantum superposition but as a part of a classical ensemble of states. This is why the founders of decoherence theory argue that their theory is an elegant solution of the measurement problem, which comes out without a non-causal collapse of the wave-function nor with an infinity number of parallel universes.

Another notable point is the following: As we have seen, decoherence is the theory that describes the transition from quantum mechanical micro systems to classical macro systems. One of the reasons why there hasn't been found a fusion between the theory of quantum mechanics and relativity is because in relativity time plays a major role as the three space coordinates to describe the trajectory of a mass point, in relativity are enlarged via a fourth, time-like, coordinate. In contrast to that time doesn't play a key role in quantum physics. Of course there are time evolutions for quantum states (described, as we have seen in chapter 2, by the *Schrödinger equation* or respectively the *von Neumann equation*). But for instance if we describe an entangled system time doesn't even enter as a parameter, which is also the major reason why Einstein called entanglement *spooky action at distance.* He meant *spooky* in the sense, that correlations exist aside of time (and even away from long distances). Without decoherence the entanglement exists endlessly and if one particle is measured information about the other is transferred instantaneously.

Interestingly the theory of decoherence brings in time into the formalism of quantum mechanics namely via the decoherence-time. It so to say brings the "spooky", "magic" quantum regime back to the earth and our daily intuition - where time plays a key role.

Critics

Critics argue that the theory of decoherence does nothing more than to replace the mystical Copenhagian collapse of the wave function by the decoherence process, but does not touch the underlying question namely why interactions destroy superposition and entanglement. As we have seen (remember chapters 3, 4 and 5) in mathematical, technical terms the decoherence process has been much more enlightened with a dozen of spotlights: Mathematical constructions and physical models just like the master equation, the formalism of open quantum systems and decoherence models do bring light in the way of passing through the borders of quantum physics. In comparison the collapse of the wavefuncion is a real black-box-model. But from a philosophical epidemic point of view, one has to admit, that the theory of decoherence does not explain too much. Why decoherence happens and where and when exactly might be cast in physical formulas. Maybe the answer to these questions is hidden safely in the amalgation of dissipators, decoherence times and Lindblad operators. Or maybe it is not in there at all.

It seems that the interpretations of the quantum mechanical formalism behave just as Marcel Proust said his book to be: like glasses. One can try it and check if it fits and if one can see something with it which was invisible without it. If not, give it away and try another one. [25]

7 Quantum, the universe and everything

> *"Wahrscheinlich darf man ganz allgemein sagen, dass sich in der Geschichte des menschlichen Denkens oft die fruchtbarsten Entwicklungen dort ergeben haben, wo zwei verschiedene Arten des Denkens sich getroffen haben. Diese verschiedenen Arten des Denken mögen ihre Wurzeln in verschiedenen Gebieten der menschlichen Kultur haben oder in verschiedenen Zeiten, in verschiedenen kulturellen Umgebungen oder verschiedenen religiösen Traditionen. Wenn sie sich nur wirklich treffen, d. h., wenn sie wenigstens so weit zueinander in Beziehung treten, dass eine echte Wechselwirkung stattfindet, dann kann man darauf hoffen, dass neue und interessante Entwicklungen folgen. (Werner Heisenberg)"*[1]

Finally I want to end this thesis with a chapter that tries to reveal interactions of quantum physics with other fields. This is a huge claim and itself more than enough content for several thesises and years of research and contemplation. But for now I just want to give some flashlights to awake an idea to those who are interested to the broad diversity of fruits one may grab in the attempt to leave the narrow thinking culture of explanations and tiptoe on the path to revelation.

To do so one does not need a permission, not even as a physicist. And still I have the feeling, I want to give some impulses. Heisenberg's quote above can be read as a letter of motivation to surmount the quantum sphere to put in frame questions of the nature of the universe beyond the quantum physical explanation.

The borders of knowledge of quantum physics are known since its foundation and were studied closely by the founders themselves. As already mentioned, Erwin Schrödinger was outstanding in this respect. Robert Pogue Harrison, professor for literature at Stanford University, describes:

> *"Große Forscher und große Denker zeigen uns die Wirklichkeit, wie wir sie niemals vorher wahrgenommen haben, oder sie decken Wahrheiten auf, zu denen es vorher keinen Zugang gab...Doch diese nahen Verwandten - Forscher und Denken - werden von zwei grundlegend verschiedenen Leidenschaften angetrieben: der Leidenschaft, etwas erklären zu wollen, einerseits und der Leidenschaft, etwas offenbaren zu wollen, andererseits. Schrödinger war eine Ausnahmeerscheinung, weil er unter dem Einfluss dieser beiden Leidenschaften stand...*
>
> *Jeder Denker ist letzten Endes eine Art Mystiker, während jeder Wissenschaftler letzten Endes eine Art Detektiv ist. Schrödinger war ein Detektiv, der den Hinweisen bis an die Grenze des Blickfeldes der Wissenschaft folgte und*

[1] It is probably true quite generally that in the history of human thinking the most fruitful developments frequently take place at those points where two different lines of thought meet. These lines may have their roots in quite different parts of human culture, indifferent times, or different cultural environments or different religious traditions: hence is they actually meet, that is, if they are at least so much related to each other that a real interaction can take place, then one may hope that new and interesting development may occur.

dann über diese Grenze hinaus in das Mysterium der geistigen Wirklichkeit schaute, die mit der Wirklichkeit der Materie eng verwoben und doch so ganz anders ist als sie." [40] [2]

This last chapter of my thesis does not try to give a complete overview about the interactions of quantum physics with other fields. It is more meant as an attempt of deterritorialization and reterritorialization, as a *vanishing line* inspired by the ideas of Deleuze and Guattari [25]:

> *"Ein Buch hat weder Objekt noch Subjekt, es ist aus den verschiedensten Materialien gemacht, aus ganz unterschiedlichen Daten und Geschwindigkeiten. Sobald man das Buch einem Subjekt zuschreibt, vernachlässigt man die Arbeit der Materialien und die Äußerlichkeit ihrer Beziehungen. Man fabriziert einen lieben Gott der geologischen Bewegungen. Wie überall, so gibt es auch in einem Buch Linien der Artikulation oder Segmentierung, Schichten und Territorialitäten; aber auch Fluchtlinien, Bewegungen der Deterritorialisierung und Entschichtung."* [25][3]

Deleuze was a philosopher that grabbed to fruits as they grow for his philosophy, following vanishing lines. In "Rhizom" the method is described with the metaphor of the "pink panther":

> *"Der rosa Panther ahmt nichts nach, er reproduziert nichts, er malt die Welt in seiner Farbe, rosa auf rosa, das ist sein Welt-Werden: er wird selbst unsichtbar und asignifikant, macht seinen Bruch, treibt seine Fluchtlinie und seine "aparallele Evolution" auf die Spitze."* [25] [4]

7.1 Theories and pictures

> *Soweit das Ohr, so weit das Auge reicht;*
> *Du findest nur Bekanntes, das Ihm gleicht.*

[2] Great researchers and great thinkers show us reality like we never observed it before, they reveal truths to which there was no access before...But these close relatives - researchers and thinkers - are pushed by two fundamentally different passions: the passion to explain something on the one side and the passion to reveal something on the other side. Schrödinger was an exception because he was influenced by both of these passions...

Finally every thinker is a mystic, while every scientist is a kind of detective in the end. Schrödinger was a detective who followed hints to the border of science and then looked beyond this border into the mystery of mental truth, which is woven into the truth of matter, tightly but still so very different from it.

[3] A book has neither object nor subject, it is made from different data and velocities. As soon as one attributes a book to a subject, one neglects the work of the materials and their external relations. One fabricates a good god of geologic movements. Like everywhere, also in a book there are lines of articulations or segmentation, layers and territories; but also vanishing lines, movements of deterritorialization and de-layerness.

[4] The pink panther does not copy, he does not reproduce, he paints the world in his colour, pink on pink, this is his world-becoming: he himself becomes invisible and asignificant, he makes a break and carries his vanishing lines and his "in-parallel evolution" to the top.

Und deines Geistes höchster Feuerflug
Hat schon am Gleichnis, hat am Bild genug.

Wolfgang Goethe

Title and motto of this subchapter was taken from Scheibe [59]. As this thesis deals mainly with the topic of decoherence we will now have a closer look at the meanings of theories and pictures in the sphere of quantum decoherence.

Firstly we want to have a close look at the wording of decoherence. So to say the following two paragraphs try a tiny *quantization* of the measures, this time not in mathematical terms (this was done enough by now) but of the theory, namely the words and notations. For the first we will just observe this one notation: decoherence. As we have discussed above the interaction of the system with its environment causes the loss of entanglement because quantum coherence *leaks out*. This was name-giving for *de-coherence*. Since modern physics we know that the many-layeredness comes in when systems are coherent: Coherency is the condition that particles can act via *spooky actions at distance*, that they can form super-positions and carry entanglement. When coherence leaks out, these quantum properties cease to exist and the systems behave purely classical, predictable, straight-forward mechanical.

The interesting aspect in this wording is, that apart from physics these notations are used completely oppositely. "Are you already perfectly coherent?" Gilles Deleuze and Félix Guattari ask in "Rhizom" [25] and although they don't give a definition for coherence it is obvious what they mean in this context. Since before "blocks of wood bound on their legs" in the "stream they swim" locate the area where this question is posed, it is just a stone's throw away from the association that coherence here calls egalizing synchronization where no way beside the predictable mainstream is possible. By the way in this sense the word *coherence* is also used in daily, intuitive use. Of course this meaning is slightly different from the quantum physical connotation.

So far to the notation of *decoherence*. In the following I want to stress one conceptual remark because it will lead us over to a more general field of parallels between quantum physics and other disciplines. This remark will concern the role and meaning of time. As already mentioned, decoherence theory brings in time into the quantum regime as a major protagonist. In this sense, decoherence embodies timelikeness and timelessness at the same time. Timelikeness, because decoherence quantifies the period of time by giving an explicit decoherence-time until which the entanglement between two systems is conserved. Timelessness because the concept of entanglement is still a timeless one and as we already know, decoherence does not destroy the entanglement in general, it just enlarges the area of entanglement: Typically before decoherence emerges, two systems are entangled, but decoherence interactions with the environment cause that the systems get also entangled with their environment. Everything is entangled with everything. But this type of entanglement is not accessible to quantum operations because therefore perfectly isolated and sharply defined correlations of entanglement are necessary. Therefore this second part of the story, timelessness is not interesting for physicists any longer but it opens a big gate of inspiring parallels to other fields.

7.2 Quantum theory and eastern mythology

> *"Die Naturwissenschaftler kennen die Zweige des Baumes des Wissens, aber nicht seine Wurzel. Die Mystiker kennen die Wurzel des Baumes des Wissens, aber nicht seine Zweige.*
>
> *Die Naturwissenschaft ist nicht auf die Mystik angewiesen und die Mystik nicht auf die Naturwissenschaft - doch die Menschheit kann auf keine der beiden verzichten."* [1] (Fritjof Capra) [23]

One aspect of the magic of quantum physics is that it is able to explain counter-intuitive phenomena with logical-intuitive tools namely precise mathematics. Hand in hand goes that as soon as you start to interpret the quantum mechanical formalism or to adapt it to other fields you will find out that in some sense the mathematics of quantum theory is more than it is. This aspect we will reveal in the following.

With his book "Das Tao der Physik" [23] the Viennese physicist Fritjof Capra who lives in Berkeley now, was one of the very first in 1975 to reveal systematically the parallels between modern physics and old eastern mythology. Capra himself had a long period of academic education (he was also a student of Werner Heisenberg) and research as a particle physicist behind him when he started to get interest in the parallels of modern physics and eastern mythology. Capra studied eastern philosophers long-since, but - as he tells - it was a day at the sea when he started to literally feel the wide-ranging interconnections between his two focuses of interest:

> *"A couple of years ago I had a wonderful experience, after which I started the way that led me to write this book. One afternoon at late summer I was sitting at the sea and saw like the waves were rolling and I felt the rhythm of my breath when I suddenly became conscious that my surrounding was part of a galactic cosmic dance."* [23]

And moreover suddenly Capra had the enlightenment that his intellectual understanding of the world based on physical theories was just one and the same as his emotional and corporal feeling:

> *"When I was sitting at this beach my former experiments came to life. I really "saw" energy in cascades coming from outer space and how its particles were rhythmically produced and destroyed. I "saw" the atoms and those of my body as a part of this cosmic energy-dance; I felt the rhythm and I "heard" its sound and in this moment I knew that this was the dance of Shiva, the God of dancers, who is admired by the Hindu."*[23]

To illustrate this feeling, Capra brings a photo-collage in his book which can be found below. Capra's work inspired me to do a series of photo collages myself as well, that address to the same feeling Capra describes above. These series can be found at the end of this thesis.

[1] Scientists understand the branches of the Tao, but not its roots. Mystics understand its roots, but not its branches. Science does not need mysticism and mysticism does not need science - but man needs both.

Figure 16: **Dance of Shiva**, A photo-collage for the motivation of Capra's studies, source: [23]

Although the parallels between modern physics and mythology have been studied for a couple of decades now, in the daily physical discourse they do not appear. Almost unknown is also, that several of the intellectual fathers of quantum physics have already been aware of these parallels, which shall be illustrated to a couple of quotations in the following:

> *"Die allgemeinen Vorstellungen über die menschliche Erkenntnis..., wie sie durch die Entdeckungen der Atomphysik anschaulich werden, sind nicht ganz fremd oder unerhört. Sogar in unserer eigenen Kultur haben sie ihre Geschichte, und im buddhistischen oder hinduistischen Denken nehmen sie einen noch bedeutenderen Platz ein. Sie setzen Beispiele für, bestätigen und verfeinern die alte Weisheit."* [2] *(Julius Robert Oppenheimer)* [53]

> *"Um zur Lehre der Atomtheorie eine Parallele zu finden müssen wir uns den erkenntnistheoretsichen Problemen zuwenden, mit denen sich bereits Denker wie Buddha und Lao-tzu auseinandersetzten, wenn wir einen Ausgleich schaffen wollen zwischen unserer Position als Zuschauer und Akteure im großen Drama des Daseins."* [3] *(Niels Bohr)* [16]

> *"Zum Beispiel könnte der große wissenschaftliceh Beitrag in der theoretischen Physik, der seit dem letzten Krieg von Japan geleistet worden ist, als Anzeichen für gewisse Beziehungen zwischen den überlieferten Ideen des Fernen Ostens und der philosophischen Substanz der Quantentheorie angesehen werden."* [4] *(Werner Heisenberg)* [44]

Capra also notes that when physics today brings us to a mainly mystical point then it - in some sense - comes back to its origin about 2500 years ago at the age of the old Greeks. At that time the notation "physics" was derived from the word "physis", which denotes the "groundreason" of things, therefore physics was meant to be the science to explore the basic origin of things. In those days Aristophanes himself thought that the questions of human soul and the thinking about the completeness of God were more vulnerable than the research in the material world. (This is also the reason why Erwin Schrödinger went back until Ancient Greece in his profound studies of the underlying principle of

[2]The general notions about human understanding... which are illustrated by discoveries in atomic physics are not in the nature of things wholly unfamiliar, wholly unheard of or new. Even in our own culture they have a history, and in Buddhist and Hindu thought a more considerable and central place. What we shall find (in modern physics) is an exemplification, an encouragement, and a refinement of old wisdom.

[3]For a parallel to the lesson of atomic theory regarding the limited applicability of such customary idealizations, we must in fact turn to quite other branches of science, such as psychology, or even to that kind of epistemological problems with which already thinkers like Buddha and Lao Tzu have been confronted, when trying to harmonize our position as spectators and actors in the great drama of existence.

[4]The great scientific contribution in theoretical physics that has come from Japan since the last war may be an indication of a certain relationship between philosophical ideas in the tradition of the Far East and the philosophical substance of quantum theory.

phenomena. [64]) But before its final return to the nearly holistic world view of today, physics took a long way that lead very far away from mythological understanding as well. For example in the paradigm of a mechanical view of the world that had ruled physics for centuries, physicists were mainly interested to differentiate their observations of nature as much as possible - and this paradigm has not been completely overcome until today because it is still part of the scientific practice to split unsolved problems as much as possible rather than to think them together and find a synthesis between the mysteries.

As Capra mentions, in Buddhist philosophy our inclination to divide the observed world into different things and to feel ourselves as isolated egos, is regarded as an illusion, caused by the measuring, categorizing mentality of our mind. Buddhists call it "Avidya" and regard is as a disturbed state of our mind that should be overcome:

> *"Wenn der Geist gestört ist, wird die Vielfalt der Dinge produziert, aber wenn der Geist beruhigt wird, verschwindet die Vielfalt der Dinge."* [5]

The scientific paradigm of splitting-practice caused that among artist, philosophers, intellectuals, and physics (among other sciences), is seen as an unimaginative, narrow-minded discipline. On the other hand mythology has in our society the smell of washed-out, mysterious and highly non-scientific charlatanry. So just like in the 1970ies when Capra wrote his first book about this issue and also today both, physics and mythology are confronted - for opposite reasons - with a not completely unclouded image - and this was also one of the pivotal points for Capra to work in the interaction field of physics and mythology: "It (his book) tries to uncover, that physics goes far beyond pure technique and that the way - or the Tao - of physics can be a way with heart, a way to spiritual insights and self-realization." [23] To bring the both disciplines to a synthesis he tries to enrich the acceptance for both of them.

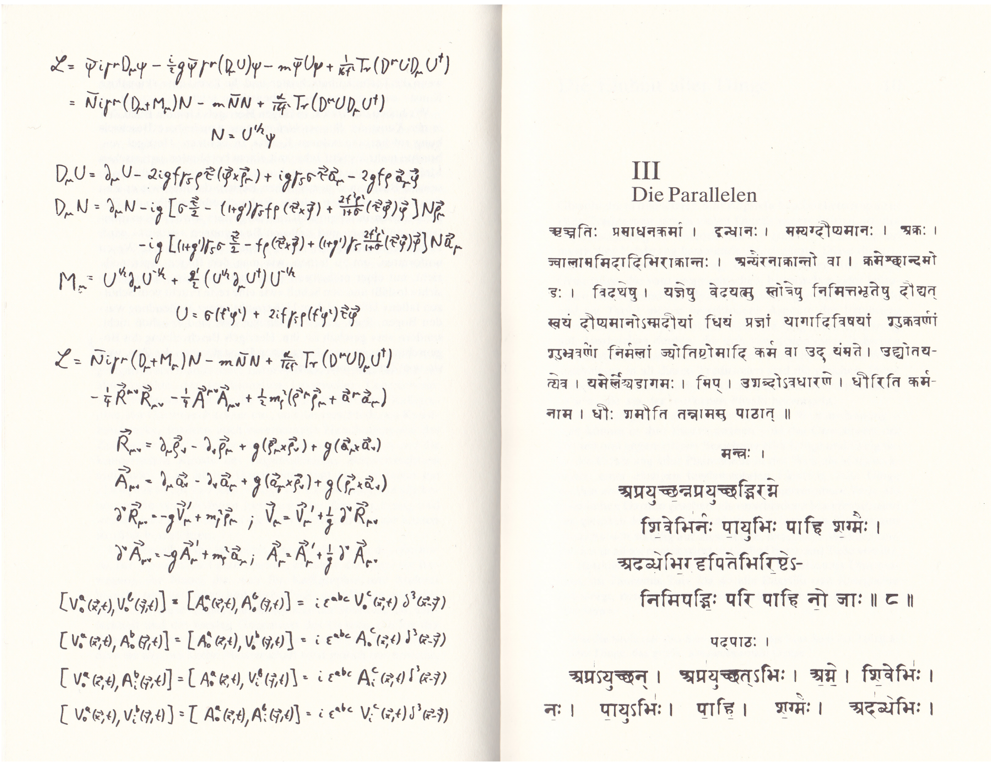
Apart from that Capra notices, that the developments of modern physics have broken with the long lasting requirement of scientific objectivity. In a surrounding of uncertainty relations and entanglement, the border between subject and object disappears - the measurement process influences the system and the environment of which the subject is part of, becomes entangled with the object of observation. Consequently Capra claims that scientists are not just intellectually responsible for their research but also morally. He drastically formulates that the way of modern physics provides two directions: the one to Buddha and the one to the (atomic) bomb. But of course this point is discussible. Firstly because one can never know in which technological applications fundamental research might lead. Secondly as for example the story of Oppenheimer's life shows sometimes there is a small path between morally honorable motivations and condemnable ones.

In the following we will not focuss on the moral implications but on the parallels between the world view of quantum theory and eastern mythology itselves.

On the very surface of these parallels swim the aesthetic of formal notations. Hopefully the main part of this thesis has given an idea of the beauty of the mathematical formalism

[5] When the mind is disturbed, the diversity of things is produced, but when the mind rests assured, the diversity of things disappears.

of quantum mechanics. The aesthetic of the eastern characters reveals itself. The collage below [23] aims to illustrate obvious similarities.



Below this formal surface a series of structural parallels can be found. Of very fundamental quality is the idea of matter - according to Capra [23] we will call this correspondence the *unity of matter*. As we have seen, in quantum physics (in contrast to classical physics), the fundamental particles of matter and the connected fundamental forces and phenomena are linked together, they are in relation with each other and nothing in the universe is completely isolated, but can just be described correctly as a part of a huge hole (this is what decoherence theory is all about). Heisenberg describes this unity as follows:

> *"Die Welt erscheint in dieser Weise (in der modernen Physik, Anm.) als ein kompliziertes Gewebe von Vorgängen, in dem sehr verschiedenartige Verknüpfungen sich abwechseln, sich überschneiden und zusammenwirken und in dieser Weise schließlich die Struktur des ganzen Gewebes bestimmen."* [6] *(Werner Heisenberg)*

A look into the Hindu Upanishads reveals an analogous idea of matter: *Brahman*, the last truth is understood as the core of entity of all things. Manifested in the human soul it is called "atman". In the Upanishads it says:

[6] The world thus appears as a complicate tissue of events, in which connections of different kinds alternate or overlap or combine and thereby determine the texture of the whole.

"Das, was der feinste Stoff ist, ist die Seele der ganzen Welt. Das ist das Wahre. Das ist Atman, das bist du." [7] *(Upanishads)*

And also in Hinduism the metaphor of "woven texture" is used parallel to Heisenberg's description of the nature of matter:

"Ihr kennt diesen, in den Himmel, Erde und Luftraum zusammen mit allen Hauchen verwebt sind, als den einen Atman." [8] *(Upanishads)*

As a second parallel apart from the *unity of matter* I want to mention the *wisdom in contradictions.* As we have seen, in contrast to classical physics, quantum physics is full of contradictions, its core is literally based on contradictions: the wave-particle-duality, the uncertainty relations, complementary, superposition are just some of its manifestations. Because of the observation of these counter-intuitive phenomena, physicists were forced to break with the purely logical paradigm of classical physics and thereby with a main column of western philosophy as well. As we know, now contradictions are integrated parts of the world view of modern physics. But this does not mean, that there doesn't remain a kind of astonishment, of a feeling of a certain strangeness. Robert Julius Oppenheimer describes it as follows:

"Wenn wir zum Beispiel fragen, ob die Position des Elektrons die gleiche bleibt, müssen wir 'nein' sagen; wenn wir fragen, ob die Position des Elektrons sich mit der Zeit ändert, müssen wir 'nein' sagen; wenn wir fragen, ob es in Bewegung ist, müssen wir 'nein' sagen." [9]

Also in eastern philosophy logical contradictions are an integrated part on the way to wisdom. The most illustrative example for that are the "koans" in Zen: to transfer knowledge Zen monks pose their pupils apparently senseless riddles to "solve" them. In the integration of the contradictions knowledge is earned.

Another look into the Upanishads shows great similarities to Oppenheimer's description of the position of the electron:

"Es bewegt sich. Es bewegt sich nicht.

Es ist weit, und es ist nahe.

Es ist in all diesem,

und es ist außerhalb von all diesem."[10]

[7] That what is the finest material, is the soul of the hole world. It is the truth. This is atman, this is you.

[8] You know this one, in which heaven, earth and air are woven together with all breaths, as the one atman.

[9] If we ask, for instance, whether the position of the electron remains the same, we must say 'no'; if we ask whether the electron's position changes with time, we must say 'no'; if we ask whether the electron is at rest, we must say 'no'; if we ask whether it is in motion, we must say 'no'.

[10] It moves. It doesn't move.
It is far and it is close.
It is in all of this,
and it is outside of all of this.

In Taoism the manifestations of nature are explained by the interaction of the antipoles Yin and Yang, which is conceptual of similar quality like the complementary in Heisenberg's uncertainty relation: as we have seen in chapter 2.5 position and momentum or respectively time and energy can not be measured at the same time with complete preciseness. If the uncertainty of the position is very small, the momentum is poorly defined and vice verse. A quotation by Lao-tzu about Yin and Yang just seems to describe this situation perfectly:

> *"Hat Yang seinen Gipfel erreicht, zieht es sich zugunsten des Yin zurück;*
> *hat Yin seinen Gipfel erreicht, zieht es sich zugunsten des Yang zurück."* [11]

Niels Bohr was aware of these connections and felt very close to Taoism. When he was ennobled in the 1940ies and was looking for an emblem that would fit to him, he didn't choose something physical, but as you can see below, the Yin-Yang-symbol in combination with his motto "contraria sunt complementa".

7.3 Quantum physics and literature

Fundamental postulate of quantum art ;-)

> *The most beautiful thing we can experience is the*
> *mysterious. It is the source of all true art and all*
> *science. He to whom this emotion is a stranger, who*
> *can no longer pause to wonder and stand rapt in*
> *awe, is as good as dead: his eyes are closed. (Albert Einstein)*

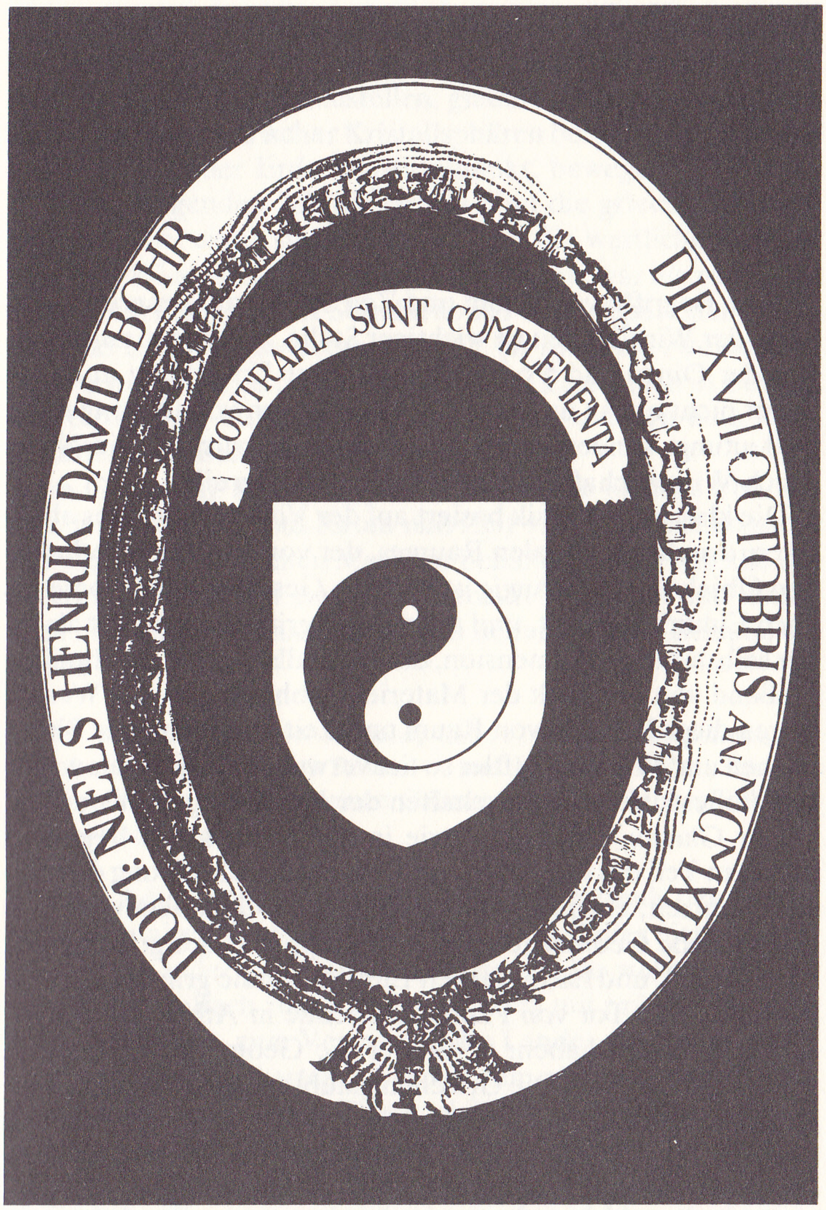
The quotation above can be seen as a fundamental postulate of the relations between quantum physics and art, a fundamental postulate with a twinkle in one's eye, because the quotation was of course not meant to be a fundamental postulate for quantum art by Einstein himself.

In this sense the following chapter is supposed to be read with eyes opened and not throw the narrow glasses of scientific completeness and objectiveness. Not that anything will be cheated in the following, but the connections between quantum physics and literature will be figured out more with personal impressions and individual snapshots than with analytical studies - not for arbitrariness but as it is more suitable for the subject.

Connections between art and science have been going on ever since, but the rise of the quantum paradigm in particular has brought the relations between art and physics to a new sphere, a sphere on which the connections are more tight than before, as no longer art was not just influenced by physics as far as contents are concerned but also in the conceptual structure and vice versa.

To give an idea how quantum physics may find its way into art, I will show some spotlights in the interacting field of quantum physics and literature.

[11] If Yang has reached its top, it withdraws in favor of Yin; if Yin has reached its top, it withdraws in favor of Yang.



Niels Bohrs Wappen
Aus dem Gedenkbuch Niels Bohr, *hrsg. von S. Rozental (North-Holland Publishing Company, Amsterdam 1967)*

Figure 17: **Emblem of Niels Bohr**, reprinted from [23]

On the 100th anniversary of quantum theory in 2000 H. Joachim Schlichting wrote [60] with reference to Elisabeth Emter [32] who has done a huge study on influences of quantum theory on the German literature:

> *"Obwohl die Kluft zwischen den zwei Kulturen seit C.P. Snow immer wieder beschworen wird, lassen sich in der zeitgenössischen Literatur vielfältige Bezüge zur Physik feststellen, insbesondere zu den konzeptuellen Umwälzungen, die mit der Etablierung der Quantenmechanik verbunden sind."* [1]

But quantum mechanics did not find into literature from the very first moment when it was established. According to Schlichting just after the second world war with the fall-down of the atomic bomb on Hiroshima and Nagasaki people - and among them also writers - realized that the progress in modern physics was not isolated from their daily life but - in opposite - dramatically influenced their environment. Therefore modern physics could no longer be thought isolated from cultural or social life. Schlichting argues:

> *"Die Physik war spätestens nachdem mit Galilei das Sichtbarkeitspostulat des Aristotelismus überwunden wurde und sich die physikalische Sehweise vom Common Sense zu emanzipieren begann, zum Gegenstand der literarischen Auseinandersetzung geworden. Aber erst nachdem mit der Atombombe die technische Umsetzung physikalischer Forschungsergebnisse zu einer realen Bedrohung für die Menschheit wurde, rückte die literarische Verarbeitung der Physik als naturwissenschaftliche Leitdisziplin in das Zentrum der öffentlichen Diskussion."* [2][60]

As examples for this processing Schlichting mentions "The life of Galilei" by Bertold Brecht, "The physicists" by Friedrich Dürrenmatt and "In the issue J. R. Oppenheimer" by Heiner Kipphardt.

Of special interest in the Viennese context and what the foundations of quantum physics are concerned is the opus magnum "Der Mann ohne Eigenschaften"[3] of the Austrian writer Robert Musil, at which we will have a looks in the following.

[1] "Although the gap between the two cultures is invoked since C.P. Snow again and again, manifold relations between contemporary literature and physics can be found, especially in the conceptual revolutions that are related to the establishment of quantum mechanics."

[2] At least since with Galilei the visablity-postulate of Aristoteles was overcome and physical views became independent from common sense, physics became part of the literary reception. But only after the technical implementation of physical research results became a real threat for mankind through the atomic bomb, the literary processing of physics moved to the center of public discussion.

[3] "The man without qualities"

The "probability-sense" of Robert Musil

As we have already seen in chapter 2 the principle of superposition is one of the key issues of quantum mechanics. It allows that a quantum particle is in several states at the same time. Until it is measured, it is not determined in which one, there exists only a certain probability distribution to find it in a certain state. One of the first writers who used this probabilistic view of the world was the Viennese writer Robert Musil in his huge unfinished novel "Der Mann ohne Eigenschaften" [52]. According to Schlichting [60] it is not sure whether Musil used the probalistic interpretation of quantum theory to adapt it to social life on purpose or incidentally due to parallel developments in culture. As Musil was mathematician himself and Vienna a center of physical related discussions at that time the first version is very likely especially because it is known that Musil followed the discussions about quantum physics with great interest as can be seen in his diary notes.

One of the main aims of the novel is to deconstruct reality into many different probabilities of the state of the world. Following this logic there is not just one reality but an ensemble of possible states. Hand in hand with this ensemble-interpretation of the world goes the so-called probability-sense which describes the ability to see one possible decision out of many ones. Robert Musil:

> *"Ein mögliches Erlebnis oder eine mögliche Wahrheit sind nicht gleich wirklichem Erlebnis und wirklicher Wahrheit weniger dem Werte des Wirklichseins, sondern sie haben, wenigstens nach Ansicht ihrer Anhänger, etwas sehr Göttliches in sich, ein Feuer, einen Flug, einen Bauwillen und bewussten Utopismus, der die Wirklichkeit nicht scheut, wohl aber als Aufgabe und Erfindung behandelt"* [3] [52]

But Musil is an utopian and follows a higher vision than just a probabilistic view of the world: By pushing away the - by causal thinking caused - "realitysense" he targets the "probabilitysense" as a coup against the bonds of the "realityorientation" and enables utopistic thinking. Musil:

> *"Utopien bedeuten ungefähr so viel wie Möglichkeiten; darin, dass eine Möglichkeit nicht Wirklichkeit ist, drückt sich nichts anderes aus, als dass die Umstände, mit denen sie gegenwärtig verflochten ist, sie daran hindern, denn andernfalls wäre sie ja nur eine Unmöglichkeit; löst man sie nur aus ihrer Bindung und gewährt ihr Entwicklung, so entsteht die Utopie."* [4] [52]

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[3] possible experience or truth is not the same as an actual experience or truth minus its "reality value" but has - according to its partisans, at least - something quite divine about it, a fire, a soaring, a readiness to build and a conscious utopianism that does not shrink from reality but sees it as a project, something yet to be invented. After all, the earth is not that old, and was apparently never so ready as now to give birth to its full potential.

[4] Utopias mean nearly as much as probabilities; in the sense that a probability is not reality, which expresses nothing else than that the circumstances with which it is currently woven, detain it, because otherwise it would just be an impossibility; if one just detaches it from its bonds and enables its development, then utopia arises.

Heisenberg and the relation of pain and desire

Before I jump from Vienna around 1900 to New York around 2000 I want to take a short stopover at postmodern literature of the 1970ies. With his most praised novel "Gravity's Rainbow" [58] Thomas Pynchon is rated among the most important postmodern writes and he is regularly cited as a contender of the Nobel prize of literature. When the Austrian writer Elfriede Jelinek received the Nobel prize in 2004 she said, she could not get it, as Pynchon doesn't have it. Three years of her life she spent to translate "Gravity's Rainbow" into German and often she emphasizes that she deeply admires Pynchon.

In "Gravity's Rainbow" Pynchon deals with many topics, characters and plots. Specially interest he shows to science and technology and their influences to the living of human beings and scientific metaphors are used to describe emotional occurrences, like in the following scene where the preciseness of Heisenberg's uncertainty relation is used to picture the crosscurrenting desires of drug use:

> *"Results have not been encouraging. We seem up against a dilemma built into Nature, much like the Heisenberg situation. There is nearly complete parallelism between analgesia and addiction. The more pain it takes away, the more we desire it. It appears we can't have one property without the other, any more than a particle physicist can specify position without suffering an uncertainty as to the particle's velocity. "*

Places and Nothing Places of Jonathan Safran Foer

A recent novel that shows parallels to quantum physics is "Extremely loud and incredibly close" by Jonathan Safran Foer. Although the author seems rather physics-affine (the main protagonist of the novel receives letters of Stephen Hawking every once in a while), it is very likely that here the parallels to quantum physics appear without purpose out of subconsciousness.

As an example I want to pick out one spot of the novel. It describes the estrangement of an old couple through changes in their flat:

> *"Only a few months into our marriage, we started to mark off areas in the apartment as 'Nothing Places', in which one could be assured of complete privacy, we agreed that we never would look at the marked-off zones, that they would be nonexistent territories in the apartment in which one could temporarily cease to exist, the first was in the bedroom, by the foot of the bed, we marked it off with red tape on the carpet, and it was just large enough to stand in, it was a good place to disappear, we knew it was there but we never looked at it."*

This touching description shows similarities to the concept of matter and anti-matter in quantum physics. And just as black holes tend to soak up everything around them, also the Foer's "Nothing Places" emerge more and more:

"It worked so well that we decided to create a Nothing Place in the living room, it seemed necessary, because there are times when one needs to disappear while in the living room, and sometimes one simply wants to disappear, we made this zone slightly larger so that one of us could lie down in it, it was a rule that you never could look at that rectangle of space, it didn't exist, and when you were in it, neither did you, for a while that was enough, but only for a while... There came a point, a year or two ago, when our apartment was more Nothing than Something...It wasn't until last night, our last night together, that the inevitable question finally arose, I told her, 'Something,' by covering her face with my hands and then lifting them like a marriage veil. 'We must be.' But I knew, in the most protected part of my heart, the truth."

Here Foer plays with the pulsing poles of life and death. Indeed, in quantum physics matter and anti-matter can be seen as basically the same thing but with different sign. Especially interesting in this respect and also what concerns a quantum-like probability-sense, is the place, when the same story as above is told again, half of the book later, told by the woman this time. In this context, the reader has to know that the man talking above does not want to have a baby but the woman does, and when she finally gets pregnant, the only places in which she feels safe are the "Nothing Places". What becomes a symbol for nothing and death for him, is in her view the place for life as such:

"I tried to wait to tell him until it was too late to do anything about it. It was the ultimate secret. Life. I kept it safe inside me...I wore loose shirts. I sat with pillows on my lap. I was naked only in the Nothing Places."

8 Conclusions and Outlook

It was the focus of this thesis to study decoherence and entanglement of qubit-systems. Based on a general introduction of the concepts of quantum mechanics, in the main part of the thesis open quantum systems were studied, which are quantum systems that interact with their environment. Without a precise knowledge of the occurring interactions, a Master-equation was derived which gives the complete time-evolution of the quantum system.

For the two-qubit-system of entangled Kaons two different kinds of ansatz for the Lindblad-operators of the Master-equation were discussed. Firstly the well known and closely studied ansatz with projection operators. Secondly a promising ansatz with shift operators which opens the gate for further studies. For some situations this ansatz might give a better description of the experimental phenomena: While projection-operators just kick the quantum state to its eigenstate, the ansatz with shift-operators would describe a slight shift of the state.

The decoherence processes of another two-qubit-system was studied closely, namely of neutrons. The mathematical requirement of Complete Positivity which occurs in every decoherence theory was brought to an experimental situation where it could be tested. Basing on a conjunction by R. A. Bertlmann and W. Grimus the idea is to test an inequality that is equivalent to the requirement of Complete Positivity and which involves the decoherence parameters in transverse and in longitudinal direction. With two different experimental setups of combined magnetic fields, measurements could be performed that cover the hole spectrum allowed by Complete Positivity. The crucial point - a set-up to check or violate Complete Positivity - needs further investigations. Since there hasn't been done an experimental test of complete positivity yet, this topic is very promising.

In general, the study of decoherence is currently of great interest, as it is the great restriction on the way of quantum technology - from quantum computing to quantum teleportation - wherever quantum operations are performed, decoherence governs the limits.

Leaving behind the mathematical formalism, this thesis also concerns the philosophical interpretations of the formalism of quantum mechanics. As we have seen, the concepts of decoherence and entanglement are of special interest in this respect. Also this field is open to new ideas as there hasn't been an agreement on one interpretation that can describe each aspect of quantum physics properly yet.

9 Deutsche Kurzfassung

Die vorliegende Diplomarbeit behandelt die quantenphysikalischen Konzepte *Dekohärenz* und *Verschränkung*. Basierend auf einer heuristischen Motivation dieser grundlegenden Phänomene der Quantenmechanik, wird im Hauptteil der Arbeit ein mathematischer Formalismus entworfen, der es erlaubt, sogenannte *offene Quantensysteme*, also Teilsysteme, die in Austausch mit ihrer Umgebung stehen, zu studieren. Ohne genauere Kenntnis der stattfindenden Wechselwirkungen ist es möglich, eine *Master*-Gleichung aufzustellen, die die vollständige Zeitentwicklung des Systems angibt.

Bei der Einführung in diese grundlegenden Konzepte werden sowohl quantenphysikalische Phänomene beleuchtet, deren Entdeckung vor 100 Jahren einen Paradigmenwechsel in der Physik erzwangen und damit verbunden grundlegende Änderungen des physikalischen Weltbilds anregte (Superposition, Unschärfe-Relation, Verschränkung, etc.), wie auch Konzepte, die gegenwärtig im Fokus des wissenschaftlichen Diskurses in der Physik stehen (Dekohärenz, Verschrän-kungs-Maße, etc.).

Am Beispiel eines Zwei-Qubit-Systems (verschränkte K-Mesonen) werden verschiedene Ansätze skizziert, die auftretenden Dekohärenzprozesse zu beschreiben, die sich vor allem auf unterschiedliche Ansätze, der in die Master-Gleichung eingehenden *Lindblad*-Operatoren beziehen. Dabei wird neben dem herkömmlichen Ansatz von Projektions-Operatoren ein Ansatz mit Verschiebungs-Operatoren vorgeschlagen.

Besondere Aufmerksamkeit wird der - für alle Dekohärenz-Theorien zutreffenden - Eigenschaft der *Vollständigen Positivität* geschenkt. Im Speziellen wird ein experimentelles Setup skizziert, in dem diese mathematische Eigenschaft am Beispiel von Neutronen (ebenfalls ein Zwei-Qubit-System) überprüft werden könnte. Dabei wird Bezug genommen auf eine von R. A. Bertlmann und W. Grimus vorgeschlagene Ungleichung, die im Falle Vollständiger Positivität erfüllt sein muss, und durch zwei Setups unterschiedlich kombinierter Magnetfelder mit den Dekohärenzparametern verschiedener Richtungen experimentell verifiziert werden kann. So konnte eine experimentelle Situation gefunden werden, in der das gesamte Spektrum für Vollständige Positivität realisiert werden kann - wesentlich allerdings wäre ein Setup, das auch "verbotene" Bereiche zulässt und somit die Vollständige Positivität experimentell testbar macht.

Um die mathematischen Betrachtungen zu ergänzen, wird ein Blick auf die erkenntnistheoretischen Interpretationen des quantenmechanischen Formalismus gewagt. Dabei werden besonders die Deutungsweisen von Niels Bohr, Werner Heisenberg, Albert Einstein und Erwin Schrödinger genauer betrachtet und zueinander in Beziehung gesetzt.

Den Abschluss der Arbeit bilden grenzüberschreitende Blitzlichter in die Wechselwirkungen und Parallelen der Quantentheorie mit anderen Denksystemen: von der traditionellen östlichen Mythologie bis in die postmoderne Literatur.

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11 Curriculum vitae

C.V.

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