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1 Introduction

The reasons why demand for capital guarantees rises are: the risk of losing substantial amounts of money in the short and long run, low interest rates since 2000, and aging of the population.

For a static pure stock investment, the long term shortfall-risk cannot be eliminated completely. Time diversification can reduce the risk of stocks because the downside risk can be partly compensated for by the upside yield in the long run. This is easier done in the long run than in the short run. The cumulative shortfall probability decreases with time. According to the results of Mauer and Schlag (2002), the cumulative shortfall probability for a 12-month investment is 48.09%, and for an accumulation period of 20 years it is still 2.72%. Even for very long time horizons, the shortfall expectation remains at a substantial level¹.

Secondly, since the end of the last century, interest rates have decreased dramatically worldwide. The life insurance companies who provide a minimum interest rate guarantee suffered from the fall in interest rates and even went bankrupt. One such company was Nissan Mutual Life, a large Japanese life insurer with about 1.2 million customers and capital assets of about 17 Billion USD. Since the end of the 1990s the interest rate on postal savings was almost zero in Japan (*Figure I-3*). The continuous low interest phase in Japan caused a state of affairs where the reinvestment of the investment funds yielded returns below the actual guaranteed interests of the insurance technical obligation. After a loss of approximate 2.5 Billion USD the company had to declare bankruptcy in 1997². The interest rates in Europe showed in *Figure I-1* clearly decreased between 2003 and 2005. The short-term interest rates in USA fell even below one percent per annum between 2003 and 2005 (*Figure I-2*).

¹ See Mauer and Schlag (2002)

² See Grosen and Jørgensen (1999)

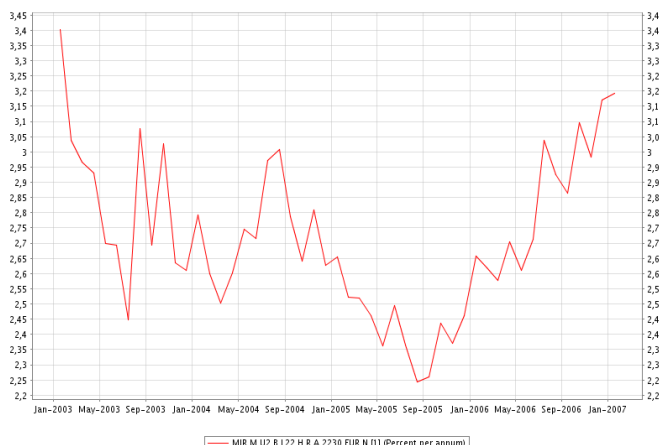


Figure 1-1 Interest rate from 2003 to 2007 in Europe

Source: ECB: MFI interest rate statistics

SHORT-TERM INTEREST RATES Weekly Average

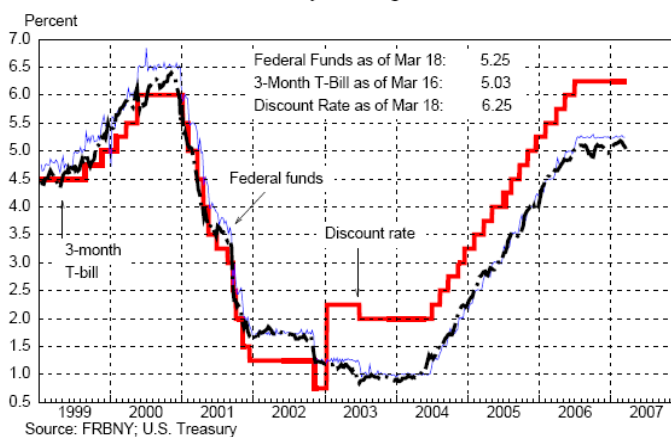


Figure 1-2 : Short-term interest rates in USA

Source: FRBNY; U.S. Treasury



Figure 1-3 Interest rate in Japan

Source: <http://www.investmentu.com/IUEL/2005/20050613.html>

Thirdly, according to the demographic development, the proportion of old people is on the rise. More and more countries started pension reforms in which national pensions were reduced by pension laws. Private pensions were started to compensate for the reduced

national pensions. The demand of additional forms of pensions increased. Since 1999, pension products which provided stronger capital market-orientation gained in significance. The debate over the stability of the public pension system caused an increased demand for public funds, funds and index-linked life insurances as well as pension funds. The pension law required a capital guarantee of these alternative pension products. A pure equity-linked life insurance is in reality not an insurance policy at all, but an investment programme, in which the insurance company invests the premia less expenses in an investment portfolio, and at expiration pays the policyholder the market value of the investment portfolio. Such policies provide for benefits which depend upon the performance of a reference portfolio subject to a minimum guaranteed benefit³.

Therefore investors also prefer safer capital redemption from their investments and the demand for more secure investments grew substantially. During the period of the decline in stock prices and interest rates in 1973-74, a new product arose – portfolio insurance⁴. Two major forms of this portfolio insurance were introduced in 1976 by Leland, Brennan and Schwartz. Leland's portfolio insurance was equivalent to a put option on an entire portfolio, while Brennan and Schwartz applied a call option on the equity-linked life insurance policy. Portfolio insurance plays a very important role in policies with guarantees. A simple portfolio insurance strategy ensures that the value of the insured portfolio, at some specified date, will not fall below a specified level⁵.

A capital guarantee product is an investment in which the investor receives at least the nominal value of his invested capital at the time of maturity. There are different options for providing a capital guarantee. According to the frequencies of payment with capital guarantees there are maturity and multi-period guarantees. A maturity guaranteed portfolio will be only paid back at maturity while a multi-period capital guarantee will be paid back in each period dependent on the policy terms. According to the guaranteed amount there are rate of return guarantees and benefit guarantees. Another example of a capital guarantee is the minimum interest payment provided by some life insurance companies. The minimum interest rate guarantee is based on the savings premium.

³ See Brennan and Schwartz (1976)

⁴ See Leland and Rubinstein (1988)

⁵ See Brennan and Schwartz (1988)

The characteristics of most capital guarantees are firstly that it is not possible to withdraw the capital before the date of maturity, otherwise capital guarantees are failed as promised. The investment horizon is usually between three to five years or even longer. Secondly, it is possible to earn a higher rate of return from other diversified portfolios which do not require that the invested capital will be frozen over a period of time. A portfolio with capital guarantees could be less advantageous if it yields a lesser return than the prevailing interest rates. Thirdly, capital guarantees could be structured in different ways. For example, some plans offer more potential growth if the index rises, but no guaranteed minimum growth. Fourthly, different types of capital guaranteed products are subject to different kinds of tax. Some products, for example, are held within a life insurance, so the gains are subject to income tax. Others like equity-linked life insurances are structured as shares, so capital gains tax is payable on the gain. Some products like Austria Individual Pension Account are under certain conditions totally tax-free. Most capital guarantees consist of stocks, bonds and options. So the returns depend on the various proportions of these components in a capital guarantee. The greater the investment in stocks, the higher the risk is. Hence the portfolio manager needs to choose proper stock markets for investors with different risk attitudes. Finally, most capital guarantees require fixed management fees or asset-based fees.

In contrast to the earlier capital guarantees, all the new capital guarantee products provide 100 percent repayment of the capitals. Some providers even offer a minimum rate of return and a minimum interest respectively. The older constructions guaranteed partly only between 40 to 90 percent of the capital.

The portfolio management applies portfolio insurance strategies to provide capital guarantees. Portfolio insurance strategies can be divided into option-based portfolio insurance and non-option based portfolio insurance. Option-based portfolio insurance (OBPI) is the application of call or put option derived by Leland (1980), Brennan and Schwartz (1976). A non-option based portfolio insurance like constant proportional portfolio insurance is a dynamic portfolio insurance and could be synthesized by a dynamic replicating portfolio invested in a risk-free asset (for instance, T-bills) and in the risky asset⁶. One problem of a simple portfolio insurance strategy (for instance, OPBI) is

⁶ See Bertrand and Prigent (2001)

that it is strongly time-dependent because in many circumstances, the specification of the precise date on which the insurance is to be effective, is arbitrary because institutional investment portfolios typically have no predetermined final date⁷. Therefore a time-invariant portfolio insurance strategy is derived by Brennan and Schwartz (1988). Another method which is more easily implemented was published by Estep, Kritzman (1988).

Section two will cover four types of capital guarantees, maturity, multi-period, rate of return and benefit capital guarantees. Section three describes guarantee funds and certificates to provide a capital guarantee. The hedging strategies for providing capital guarantees referred to as portfolio insurances are found in section four where the shortfall expectation and guarantee costs will also be considered. The effects of the payoffs and the shortfall expectations of hedging strategies, guarantees costs, management fees and the risk shifting problem of capital guarantees will be the topics of section five.

⁷ See Brennan and Schwartz (1988)

2 Forms of capital guarantees

A capital guarantee can be divided into different forms according to two main criteria. One is the frequency of redemption, the other is the guaranteed amount based on different levels of the capital. Based on the frequency of redemption, there are maturity and multi-period guarantees. Of these two guarantees, the former is paid only once at maturity and the latter is contractually dependent. Under this category the initial capital would be normally 100 percent guaranteed before tax and management fees.

According to the other criterion, the guaranteed amount based on different levels of the capital, most of the paid capital would be guaranteed. One form of this kind of capital guarantees is the interest rate guarantee, which guarantees only the yearly interest rate payment of the savings premium during the contract period. Other forms are rate of return and benefit capital guarantees. A rate of return guarantee provides a certain percentage rate of return on the initial capital no matter how the initial capital performs. A benefit capital guarantee is independent on the capital paid. The redemption would be a fixed amount determined by the contract.

2.1 Maturity and multi-period guarantee

According to the frequency of the payments, the capital guarantee can be divided into maturity guarantees and multi-period guarantees. Maturity guarantees promise the capital redeemed to the investor only on the date of maturity. The time of maturity can be either fixed in the contract or some special events like death, a particular age, or retirement with respect to mortality. If the policyholder sells or quits the contract before the date of maturity, he would not receive the guaranteed capital promised by the policy providers. This type of guarantee is pioneered by Brennan and Schwartz (1976) for the equity-linked life insurance policies with an asset value guarantee. Such policies provide benefits which depend upon the performance of a reference portfolio subject to a minimum guaranteed

benefit⁸. Returns above the guarantee at some time before maturity offset shortfalls at other times⁹. They are particularly valuable in case of downward markets¹⁰.

The multi-period guarantee is provided at the end of each period such as quarterly, annually, or multi-yearly. Hence, excess returns in one sub-period cannot be used to finance shortfalls in other sub-periods. Such guaranteed products appear in insurance policies, guaranteed investment contracts, and some pension plans¹¹. An example of multi-period guarantee is the UK insurance policies. The UK insurance policies declare at each sub-period a fraction of the surplus as reversionary bonus which is guaranteed. The remaining surplus is managed as an investment reserve, and is returned to customers as terminal bonus if it is positive at maturity or upon death. The bonus is contractually determined as a fraction of the portfolio excess return above the guaranteed rate during each sub-period. The guaranteed rate is also contractually specified¹².

2.2 Minimum interest rate guarantee

The minimum interest rate guarantee offers the investor an average minimum interest payment. This type of guarantee is widely used in the life insurance industry. For instance, the classical Austria capital insurance guarantees a minimum interest payment only based on the savings premium but not on the contributions. Savings premiums are equal to the total premium less four percent insurance tax, management fees and risk premium. The risk premium is deducted as long as mortality risk is involved. Only this savings premium will be provided a minimum interest payment¹³. In some countries the funded pension schemes, operating on an actuarial reserve basis, are also required by law to provide a minimum interest rate guarantee which ensures a minimum growth rate of the individual pension saver's reserves. This growth rate is guaranteed as an average over a long time interval. Such an insurance policy or a pension plan equipped with a minimum interest rate guarantee provides the buyer with a useful guarantee. The seller is issuing a put option enabling the buyer to receive a minimum guaranteed rate of return in cases where the return on the underlying investment falls short of this guaranteed rate of return. On the

⁸ See Brennan and Schwartz (1976)

⁹ See Lechance and Mitchell (2002)

¹⁰ See Fischer (1998)

¹¹ See Hansen and Miltersen (2001)

¹² See Consiglio et. al (2001)

other hand the buyer receives the return of the underlying investment whenever its return exceeds this minimum¹⁴. In Germany the BaFin (Bundesaufsichtsbehörde für das Finanzwesen) determines that for capital linked life insurance or pension insurance the minimum interest rate agreed by contract may not exceed 2.75 percent per annual since 01.01.2004¹⁵.

2.3 Benefit guarantee

A benefit guarantee gives investors certain benefits periodically or once at expiration of the contract, at maturity, or prior to death at least a minimum annuity or amount, irrespective of their account's actual investment performance¹⁶. A periodical guaranteed benefit could be limited by an upper amount¹⁷.

A benefit payable at once at maturity is a function of the value of some specified portfolios of common stocks: the insured or reference portfolio. This program, offered to investors in specified mutual funds, provides a benefit payable at maturity which is equal to the greater of zero and the difference between a guaranteed amount and the value at maturity of the proceeds yielded by a specified program of investment in the mutual fund. The investor who purchases such a contract is assured that his total return from the mutual fund investment and the insurance contract will not be less than the guaranteed amount.

In a capital guarantee contract with mortality the date of expiration is uncertain and the benefit payable at contract expiration, whether at maturity or prior to death, will be equal to the greater of the value of some reference portfolio of common stock and a guaranteed amount.

A variable life insurance contract is a whole life contract under which the insured receives a benefit at death which is proportional to the value of a reference portfolio of common stocks.¹⁸

¹³ See Ellmaier and Engel (2003)

¹⁴ See Jensen and Sorensen (2000)

¹⁵ <http://www.versicherungsnetz.de/Onlinelexikon/Mindestverzinsung.html>

¹⁶ See Lechance and Mitchel (2002)

¹⁷ See Fischer (1998)

¹⁸ See Brennan and Solanki (1981)

2.4 Rate of return guarantee

Under a rate of return guarantee, the policyholders would be entitled to receive the payments equal to their total contributions from the portfolio management plus some rate of return. One variant on this theme is a “principal guarantee” which is equivalent to guaranteeing a nominal rate of return of zero percent. Policyholders receive at least the initial investment at maturity. The other variant is a “real principal guarantee,” under which participants would be guaranteed their total contributions adjusted according to an inflation index¹⁹. This is especially important for a defined contribution pension plan. Continuously the insured can receive a principal guarantee plus some minimum rate of return which is greater than zero.

A rate of return guarantee can be fixed or relative. A minimum fixed rate of return guarantee is that the insurer guarantees a minimum rate of return of a fixed percentage. Thus the fund that earns less than this fixed percentage during a given year would require an insurer transfer to make up the difference. The other one is a minimum relative rate of return guarantee that is a function of the average annual real rate of return earned by all funds²⁰.

The greatest difference between the benefit and the rate of return guarantee is that the rate of return guarantee is a percentage rate of return based on the total investment or contributions while the benefit guarantee is a fixed amount.

¹⁹ See Lachance and Mitchel (2002)

²⁰ See Pennacchi (1999)

3 How to provide a capital guarantee

Most of the life insurance policies, the pension and the pension funds as well as the mutual funds provide capital guarantees. A capital guarantee can be provided by either an underlying with a long put or a fixed long zero bond with a long call.

A contract with risky (reference) assets imposes the full investment risk on the policyholder, a risk he may well be willing to take under conditions of uncertain inflation if he regards equities as providing a hedge against inflation. So the sellers of these contracts typically provide for different capital guarantees, payable depending on the contractible terms, thus relieving policyholders a part of the investment risk. The insurance companies are in fact selling an investment guarantee or insurance policy in addition to the straightforward investment plan²¹. So the seller is issuing a put option enabling the buyer to receive a minimum capital guarantee in cases where the value of the underlying investment falls short of this guaranteed amount. On the other hand the buyer receives the value of the underlying investment whenever its value exceeds this minimum²². The investor gives up some returns on the investment in order to get the minimum capital guarantee.

For a simple maturity guarantee, the portfolio management provides the investors, at maturity, a guaranteed value, V_T , which results from the greater of the final portfolio value, S_T , and the sum of his total contributions (the nominal capital), X , which can be seen as the capital guarantee and must be repaid to the investor at maturity. In detail, the guaranteed value, V_T , is equal to the nominal value of the capital, X , if the final portfolio value, S_T , is lower than the nominal value of the capital. Or the guaranteed value, V_T , is equal to the final portfolio value, S_T , if the final portfolio value is larger than the nominal value of the capital, X . In other words, the investor receives at least the nominal value of the capital at maturity.

²¹ See Brennan and Schwartz (1976)

²² See Jensen and Sorensen (2000)

$$V_T = \begin{cases} X & \text{if } S_T \leq X \\ S_T & \text{if } S_T > X \end{cases}$$

This portfolio with a capital guarantee can be structured either by buying a put option on the initial guaranteed capital or a call option on the reference portfolio. The former is called guarantee funds. The latter is called guarantee certificates²³. The strike prices of a put and a call are equal to the capital guarantee.

3.1 Guarantee funds

A guarantee fund is to hold the reference portfolio, the dotted blue line in *Figure 3-1*, and to buy a put option, the green line, on the portfolio with a strike price equal to the initial portfolio value X . The red line is the capital guarantee which is never below the strike price, X , but can provide a portfolio value larger than X .

$$V_T = \underbrace{S_T}_{\text{Long Underlying}} + \underbrace{\begin{cases} X - S_T & \text{if } S_T \leq X \\ 0 & \text{if } S_T \geq X \end{cases}}_{\text{Long Put}}^{24}$$

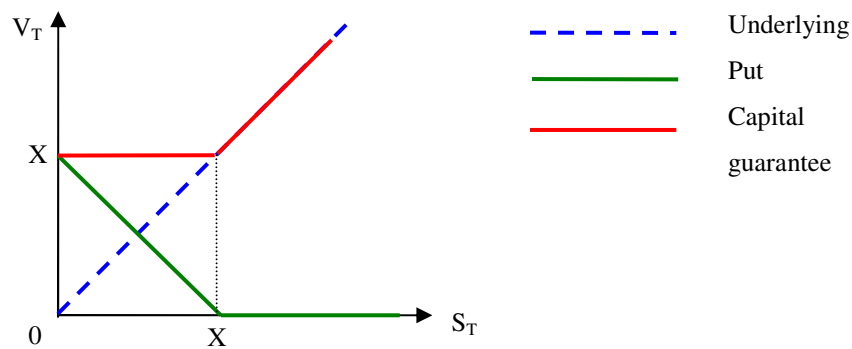


Figure 3-1 Capital guarantee with a put option

Suppose all wealth will be invested in a reference portfolio of one hundred percentage stocks. The initial wealth is equal to the strike price of the put option, X , and V_T is the end value of the reference portfolio. So the end-of-period value, V_T , is given by the greater of the value of the end value of the portfolio, S_T , and the strike price of the put option, X .

²³ See Raiffeisenblatt Heft (2/2005)

²⁴ See Fischer (2003)

$$V_T = \max\{S_T, X\}$$

If an investor buys a put option on the reference portfolio with a strike price of X which is equal to the guaranteed amount, the put option will be exercised at the end of period only if the final value of the portfolio, S_T , is smaller than the strike price, X , otherwise it will not be exercised, i.e. a put option is exercised and the total reference portfolio will be ensured by a put option only if the reference portfolio suffers a loss.

$$P_T = \max\{X - S_T, 0\}$$

A buy-and-hold strategy allows the investor to hold the reference portfolio until the end of the period. The value of the total portfolio at expiration is the end value of reference portfolio, S_T , plus the put option, P_T :

$$\begin{aligned} S_T + P_T &= S_T + \max\{X - S_T, 0\} \\ \text{If } S_T < X &= S_T + X - S_T \\ &= X \\ \text{OR} &= S_T + 0 \\ \text{If } S_T > X &= S_T \\ \rightarrow S_T + P_T &= \max\{S_T, X\} \\ &= V_T = X \quad \text{if } S_T < X \\ &= S_T \quad \text{if } S_T > X \end{aligned}$$

This implies that the value of the total portfolio is equal to either the strike price of the put option, X , if the end value of reference portfolio is smaller than the strike price of the put option, X , or the end value of reference portfolio without exercising the put option if the end value of the reference portfolio is larger than the strike price of the put option. This is how capital guarantee works and therefore buying a put option can ensure the total portfolio. In return, the investor must pay some premiums in order to get the capital back at the end of the period. The premiums are seen as the costs of a capital guarantee: this premium is equal to the price of a put option on the reference portfolio²⁵, i.e. the guarantee is seen as an option on a reference portfolio.

A guarantee fund is a variant of an investment fund which is regulated by investment fund law. A fund manager actively administers this fund which is invested in stocks and/or

bonds. This fund guarantees the investor the redemption of his initial capital minus the asset-based fee and management fee. Hence the guaranteed amount is slightly under 100 percent.

Compared to the guarantee fund, a guarantee certificate is a defined and fixed portfolio with derivatives but without management fees. Therefore the guarantee certificates follow a “buy-holding” strategy as trading strategy for the guarantee funds. Guarantee funds have a higher transparency and their portfolio is easier for the investor to understand than guarantee certificates. These funds cannot be bought at any time because after expiration of the subscription period they will be closed through a fixed maturity. A guarantee fund can be guaranteed only at maturity but not during the period. Other than this, a guarantee fund works similarly as guarantee certificates²⁶.

3.2 Guarantee certificates

An investor purchases a guarantee certificate by holding a zero bond or cash, the dotted blue line, with a nominal value of X , plus buying a call option, the green line, on the reference portfolio with a strike price equal to the initial portfolio value X . Again, the capital guarantee, the red line, offers a minimum payment of X and a possible higher gain.

$$V_T = \underbrace{X}_{\text{Long Zero Bond}} + \underbrace{\begin{cases} 0 & \text{if } S_T \leq X \\ S_T - X & \text{if } S_T > X \end{cases}}_{\text{Long Call}}^{27}$$

²⁵ See Brennan and Schwartz (1976)

²⁶ See Axer Partnerschaft (2006)

²⁷ See Fischer (2003)

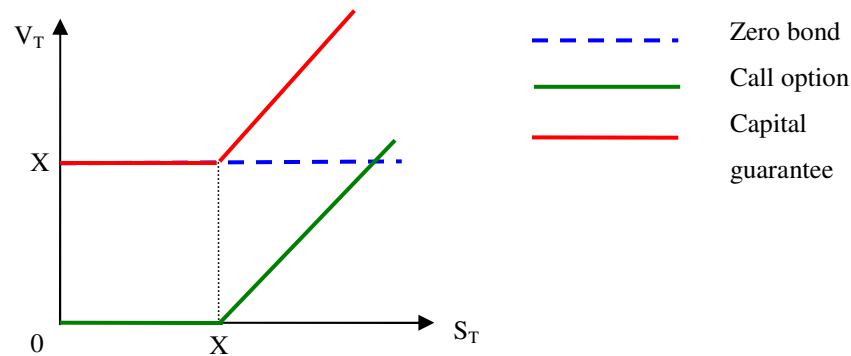


Figure 3-2 Capital guarantee with a call option

A call option will be exercised if the end value of the portfolio is larger than the initial value of the portfolio which is equal to the strike price of the call option. The value of the call option at maturity is

$$C_T = \max\{S_T - X, 0\}.$$

The value of portfolio at the end of the period is the initial wealth plus a call option:

$$\begin{aligned}
 X + C_T &= X + \max\{S_T - X, 0\} \\
 \text{If } S_T > X &= X + S_T - X \\
 &= S_T \\
 \text{OR} &= X + 0 \\
 \text{If } S_T < X &= X \\
 \\
 \rightarrow X + C_T &= \max\{S_T, X\} \\
 &= V_T = X \quad \text{if } S_T < X \\
 &= S_T \quad \text{if } S_T > X
 \end{aligned}$$

In this case, the call option ensures the total portfolio at maturity. The result is the same as in the case of a put option: the end value of the portfolio is at least the initial wealth (which is equal to the strike price of the call option) or the end value of the reference portfolio.

A large amount of a guarantee certificate will often be invested in a (zero) bond where the promised nominal redemption is ensured through regular interest payments. The remaining guarantee certificate will mostly be invested in stock options at stock markets like Dow Jones, DAX, EuroStoxx 50, etc. So a guarantee certificate is often a complicated structure. It leads to less transparency and is more difficult for the investor to understand.

The initial price, which moves mostly near the actual index price, will be agreed upon in the contract. At maturity the investor receives at least the initial price if the actual price is below the initial price or a possible distribution of the difference between the market price and initial price if the market price is higher than the initial price. This possible profit comes from the risky part (stock) of the certificate. The amount of profit depends on the level of the guarantee, the interest rates and the development of the stock price. The higher the level of the guarantee, the more the investment lies in the bond. In a case of more than one hundred percent guarantee the issuer assures the investor at least a minimum interest rate guarantee with the result that this certificate will be more conservative and yield a lower return. Guarantee certificates can be bought during maturity.²⁸

²⁸ See Axer Partnerschaft (2006)

4 Hedging strategies

In the following sub-sections we will discuss two main categories of strategies to ensure capital guarantees: the static and dynamic portfolio insurances which were popular in the 1980's. Static portfolio insurance is fixed at the time of purchase and is held until maturity. In contrast, dynamic portfolio insurance will change from time to time during the investment window. Shortfall expectations test whether the hedging strategies offer a capital guarantee. Shortfall expectations measure the sum of losses weighted by their loss probabilities. This measurement requires the shortfall probability and mean excess loss. In the case of capital guarantees, shortfall expectations should be equal to zero at expiration. Then we consider the guarantee costs for each strategy according to Mauer and Schlag (2000).

4.1 Portfolio insurance

4.1.1 An overview of portfolio insurances

Various literatures classify types of portfolio insurances differently. But the most common accepted classification is shown in *Table 4.1*. Here are the general descriptions, the advantages and the disadvantages of these four portfolio insurances.

Hedging strategy				
		Description	Advantages	Disadvantages
Static	Stop – Loss	• Single security hedging; • By reaching or falling below a certain price the security will be sold at the best price in the market.	• Simple handling; • Flexible structure possibility.	• Price gap risk; • Possible scenario of re-entry is necessary; • At many values, high expenditure of time; • Individual risk threshold.
	Protective put	• Total portfolio hedging; • Simultaneous purchase of security and hedging put,	• Costs and minimum value during maturity of the put are known.	• Costs of the put; • Maturity problem; • Possible that no suitable puts are available; • Valuation of option price theory (Basic price does not match the favored price.)
Dynamic	Synthetic	• Total portfolio hedging; • Combination of selling risky assets and purchase of riskless assets in a certain relationship, application of a synthetic put; • Regular changes of compositions dependent on appropriate influence factors.	• Inexpensive • Flexible, specific configuration as requested.	• Possible high transaction costs; • Very complex computation; • After end of the planning period possible that new investment necessary; • Valuation of option price theory.
	CPPI	• Constant Proportion Portfolio Insurance; • Total portfolio hedging; • Combination of selling risky assets and purchase riskless assets in a certain relationship • Regular changes of compositions dependent on appropriate influence factors.	• Flexible and simple handling.	• Possible high transaction costs; • Individual risk threshold.

Table 4-1: Different types of hedging strategies

Source: Urbatsch, Nagler: Technische Wertpapieranalyse

4.1.2 Stop-Loss strategy

Just as the name implies, the security paper would be sold at the next trading time point when it reaches or falls below a settled price limit. This strategy, though very simple and generally executed with no extra cost by brokers, has the drawback that the investor

will not participate in any further price rise above the limit, unless the asset is bought again²⁹.

Generally there are two ways to liquidate the security: either automatically or manually. For an automatic liquidation an order which will be filled after the presetting is taken directly in the trading system of the stock market. If an investor sells the security at a certain time, this action is denoted as manual liquidation³⁰.

The most common problem of stop-loss strategy is the risk of an explicit downward price gap. It is impossible for an investor to react on a strong price movement if he does not watch the price continuously. He will lose more on his purchase price because of the price gap which is shown in *Figure 4-1*.



Figure 4-1 Downward price gap

Source: Urbatsch, Nagler: Technische Wertpapieranalyse

4.1.3 Option based portfolio insurances (OBPI)

Option based portfolio insurances consist of buying an option on the whole portfolio and fixed at the beginning of the contract. This kind of strategy is called static strategy; once it is set up, it does not require any further adjustment until the end of the investment horizon³¹. It is also called one-period portfolio because the investor buys the contract and holds it until maturity. A put option is used in the downward market. Such a put option is called a protective put which protects the value of an entire portfolio from loss. This portfolio mostly provides a maturity guarantee which is described in Section 2.1.

²⁹ See Laurent (2003)

³⁰ See Urbatsch and Nagler (2001)

³¹ See Laurent (2003)

Over a very short time period, Black and Scholes showed that a call option could be perfectly hedged by a negative stock position³² which means a lower stock price than the basic price. Leland announced in 1976 a concept of portfolio insurance which was equivalent to a put option – not on a stock, but on the entire portfolio. So portfolio insurances are based on the Black-Scholes option price theory. Portfolio insurances enable an investor to avoid losses, but capture gains, at the cost of a fixed “premium”³³. This is also the main idea of a capital guarantee. Mostly the portfolio insurances are suitable for such capital guarantee contracts which avoid the value of reference portfolio at maturity below the guaranteed amount and simultaneously reduce the downside risk. Brennan and Schwartz (1976) derived an equilibrium value of an equity-linked life insurance policy with an asset value guarantee. This method could be implemented for both single period and multi-period contracts.

$C(S_t, T-t, X_T)$ is the value at time t of a call option to purchase the reference portfolio at time T for the exercise price, X_T . The value of the call option for the single premium contract for which there are no contributions to the reference portfolio after the initiation of the contract is given by³⁴

$$C(S_t, T-t, X_T) = S_t N(d_1) - X_T e^{-r(T-t)} N(d_2) \text{ where}$$

$$d_1 = \frac{\ln(S_t / X_T) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

$N(d)$ is the cumulated normal distribution. S_t represents the value of the reference portfolio at time t . X_T denotes the value of the capital guarantee at maturity and strike price of the option. r is the riskless interest rate. σ means the implied volatility.

At expiration of the contract, $t=T$, the equilibrium value of call option is

³² See Leland and Rubinstein (1988)

³³ See Leland and Rubinstein (1988)

³⁴ See Brennan and Schwartz (1976)

$$C(S_T, 0, X_T) = \max\{S_T - X_T, 0\}.$$

The value of the put option at the time t is equal to the sum of the call option and guaranteed capital minus the reference portfolio at the time t , $P_t = C(S_t, T - t, X_T) + X_T e^{-r(T-t)} - S_t$. That is the amount of the premium paid to the portfolio management.

4.1.3.1 Put-Call parity

A variant of OBPI is the put-call parity. Feldstein, Rangelova (2000) combined a put option and a call option as a “collar” in order to reduce the risk of the benefit guarantee. The costs of the put are partly financed by a short call. Namely, the premium paid is compensated by selling a call option.

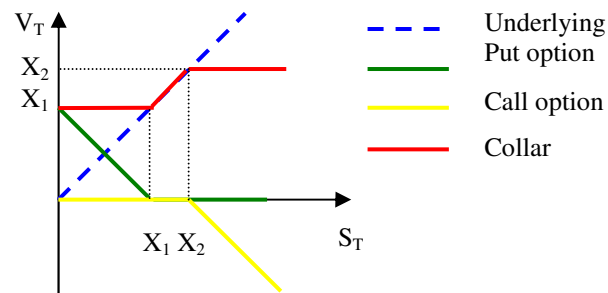


Figure 4-2 Collar with a long put and a short call

Source: Author's computation

This collar consists of a set of the long put and the short call. A collar has two strike prices. One strike price of a long put is equal to the guaranteed capital. The other strike price of a short call is solved through the put option price. The strike price of a call is higher than that of a put. If an investor buys a put option on the whole portfolio, he can sell a call option to compensate this put payment. As a result, the whole portfolio is ensured at no extra cost. The price of a put is calculated through the famous Black-Scholes option price theory given by maturity, initial portfolio value, strike price, implied volatility and risk-free rate.

After the price of a put option is solved, the price of a call is equal to the price of a put. The strike price of a call is also calculated through Black-Scholes option price theory, given by maturity, initial portfolio value, price of call, implied volatility and risk-free rate.

These calculations imply that an individual investing in a portfolio with a capital guarantee can apply a collar to avoid additional costs so that the insured portfolio will not be less than the capital guarantee and can exceed a certain rate of return of the initial wealth. One advantage of this collar is the saved hedging costs. The problem is the low possibility of exercising the call option. If the strike price of a call is too high, it is hardly possible to exercise it and then the call option is less valuable. As a result, the payoff of a collar is the same as of one put option but without any extra costs.

4.1.4 Constant proportion portfolio insurances (CPPI)³⁵

Constant proportion portfolio insurance dynamically changes the proportions invested in the stock and a riskless asset. As compared to static strategies, dynamic strategies once implemented must continuously be revised due to price changes of financial instruments³⁶. It is important to note that the dynamic trading strategy, which creates insured portfolio values, requires a higher investment in the reference portfolio as its value rises, and higher amounts in cash as its value falls³⁷. The portfolio value depends on the proportional value of stocks and bonds. CPPI trades between a risky asset and a 'riskless' asset. CPPI-Strategy is attractive because of its flexibility and simple manageability. It is not necessary to implement CPPI-Strategy through complex mathematic calculation or acknowledge of option price theory. Instead, the risk is based on a multiplier. The multiplier is in general larger than one. A large multiplier means that the investor would invest more in stocks, and vice versa.

The following show the elements of this method.

³⁵ See Jürgen (1999)

³⁶ See Laurent (2003)

³⁷ See Leland (1980)

F_t	Floor of the portfolio at time t
C_t	Cushion at the time t : difference between portfolio value and floor
E_t	Exposure at the time t : Stock value which is $F_t \leq E_t \leq V_{p,t}$
S_t	Value of stock at the time t
$r_{s,t}$	Rate of return of the stock at the time t
$r_{b,t}$	Rate of return of the bond at the time t
$V_{b,t}$	Value of bond at the time t
$V_{p,t}$	Value of portfolio at the time t
r	Constant, continuous interest rate of bond
V_T	Minimum Value of the portfolio at maturity
m	Multiplier, $m > 1$

The floor, F_t , is the minimum value which the portfolio value cannot exceed at any time. It can be understood as the present value of the guaranteed minimum value of the capital, V_T . It is also obvious that the initial value of the floor is less than the initial portfolio value. One exception is if the floor is higher than the capital guarantee in the short run, the present value at time $t=0$ could exceed the initial portfolio value.

$$F_t = V_T e^{-r(T-t)} \quad (1)$$

The cushion, C_t , is the difference between the portfolio value, $V_{p,t}$ and floor, F_t . It determines the minimum value invested in the stock. The cushion may not be negative. Hence, the cushion is equal to zero if the portfolio value is smaller than the floor.

$$C_t = \max\{0, V_{p,t} - F_t\} \quad (2)$$

An exposure, E_t , multiplied by the multiplier shows the amount invested in the stock. It depends on the value of the multiplier, m . A higher multiplier means a larger amount of portfolio invested in stocks. In the absence of short selling, the exposure cannot exceed the value of the portfolio. Otherwise, the exposure is equal to the actual value of the portfolio.

$$E_t = \min\{m * C_t, V_{p,t}\} \quad (3)$$

The value of the portfolio $V_{p,t}$ at any time t in the period $[0, T]$ is the value of the stock at the time, $t-1$, multiplied by the rate of return of the stock at the time t , plus the value of

the bond at the time, $t-1$, multiplied by the rate of return of the bond at the time t :

$$V_{p,t} = S_{t-1} * (1 + r_{s,t}) + B_{t-1} * (1 + r_{b,t}) .$$

In the next example we put a stock and a zero bond into a portfolio. For the stock we use ADIDAS and its price is from November 2005 to September 2006. The initial price of the stock and the bond is €89.9 and €97.3 respectively. The end price of the stock and the bond is €23.9 and €92.6 respectively.

In the following process a portfolio with a stock and a bond applying CPPI will be compared to a portfolio with 100% ADIDAS stock. Here we ignore the volume of the stock purchased, but only its value weighted in the portfolio value is considered.

In this example one buys the ADIDAS stock on 11.09.2005 at €89.9 and the bond at €97.3 per share. The investor has an initial wealth of €10000. The multiplier is 1.5. The value of the portfolio is 100 percent ensured which means the floor, F , is €10000. At the beginning of the contract the initial value of the cushion is the difference between the initial wealth and the present value of the floor. Then the value of the stock invested in the portfolio is €196.0528. The rest will be invested in the bond which is equal to €9803.947. In *Figure 4-3*, the red line shows the value of the portfolio with CPPI. At maturity the final wealth with CPPI is €10206.54, which consists of €103.2714 invested in the stock and €10103.27 in the bond. The yellow line shows the price development of 100% stock. In contrast, the value of the portfolio with 100 percent stock decreased from initial €10000 to €2658.509 finally because the stock price fell from €89.9 per share to around €25 per share. The highest return of stock is nearly 20% and is higher than the return of CPPI. The value of the portfolio with CPPI is always close to the actual value of the floor. In total, CPPI has well hedged the value of the portfolio.

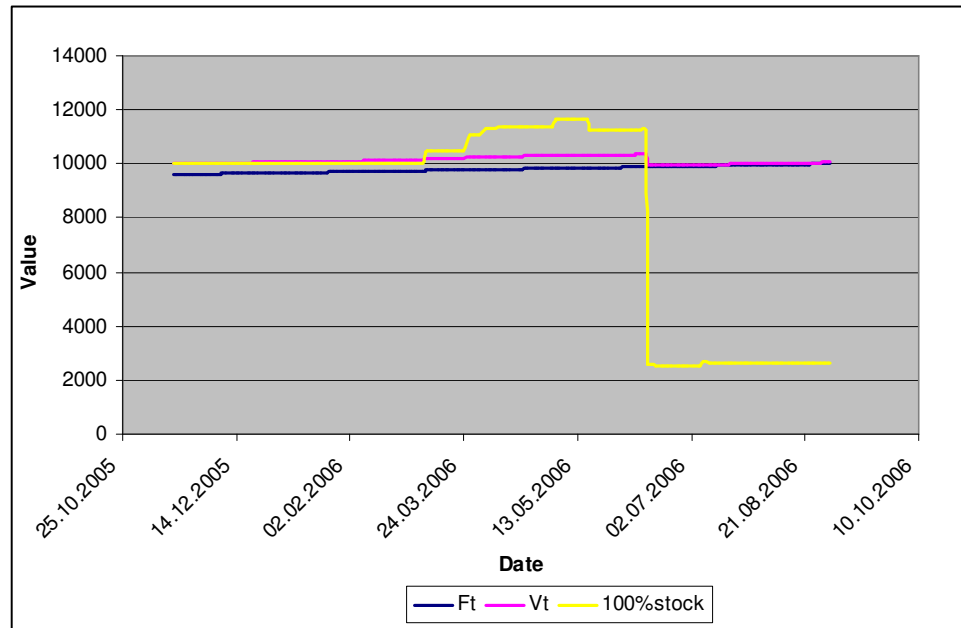


Figure 4-3 Value of the portfolio with and without CPPI, $m=1,5$

Source: Author's computation

The multiplier must be reasonably chosen in practice. For instance, it is related to the actual stock and the bond prices and the risk attitude of the investor. In this case, the stock price decreases dramatically from €89.9 at the beginning to €23.9. So at the end the portfolio consists most of the bond. The value of the portfolio could be totally different with different multipliers. Only a suitable multiplier can maximize the value of the portfolio.

4.1.5 Time irrelevant portfolio protection (TIPP) ³⁸

Under ideal conditions, a simple portfolio insurance strategy (maturity capital guarantee) ensures that the value of the insured portfolio, at a specified date, will not fall below a specified level. However, there are at least two difficulties with this simple type of portfolio insurance. First, under almost all circumstances, a simple portfolio insurance strategy is inconsistent with expected-utility maximization. Second, in many circumstances, the specification of the precise date on which the insurance is to be effective is arbitrary because institutional investment portfolios typically have no predetermined final date. Moreover, the specification of the effective date of the insurance induces an investment strategy that is strongly time-dependent. Given the indefinite

³⁸ See Estep and Kritzman (1988)

horizons of most institutional investment portfolios, it is therefore of interest to consider the class of investment strategies under which the fraction of wealth allocated to risky assets is independent of time³⁹.

A time invariant portfolio protection (TIPP) gives the fund sponsor a means to get protection that matches true goals, without introducing arbitrary considerations that depend on the starting or ending date of the program, without abstruse formulas, and without excessive trading⁴⁰. The total process of TIPP is explained in these six steps:

1. Calculate the total wealth at the time t , V_t (stocks and bonds)
2. Get the floor at time t , F_t , which is equal to the percentage rate, F , multiplied by the total wealth, V_t , at the time t .
3. If the new floor F_t in step 2 is greater than the previous floor F_{t-1} , it becomes the new floor; otherwise, keep the old floor.
4. Calculate the cushion, C_t , at the time t , which is the difference between the total wealth, V_t , and the floor, F_t .
5. Multiply the cushion, C_t , with the multiplier, m , in order to get the amount invested in the stock.
6. The rest of total wealth will be invested in the bond, B_t , if there is still some wealth left.

In TIPP the only difference to CPPI is the way to set the floor F_t . The floor in TIPP depends on the total wealth but not time in CPPI. The floor of TIPP increases while total wealth increases and holds if total wealth decreases. So the floor of TIPP today is always equal to or larger than that yesterday, but the floor of CPPI increases with the riskless interest rate and time.

³⁹ See Brennan and Schwartz (1988)

⁴⁰ See Estep and Kritzman (1988)

4.1.6 Comparison of OPBI, CPPI and TIPP⁴¹

OBPI combines a put option or a synthesized put option computed from the Black-Scholes option price theory on a reference portfolio in order to ensure the value of the initial portfolio. A synthesized put option enables an investor to change the weight between the stocks and the bonds and to liquidate the portfolio. This is also a trading strategy which produces the same results as buying a put option on the reference portfolio. But OBPI is also time-dependent and has normally a maturity of one year. As the investor approaches the expiration date, the portfolio will be heavily weighted toward bills, if its value is close to the floor value or towards stocks if its value significantly exceeds the floor value. The strategy's dependence on a particular expiration date has resulted in completely arbitrary trading. The other problem of OBPI is that there is no gain guarantee, especially by a collar. The call option limits the value of the portfolio from exceeding a certain level.

CPPI is less sensitive to arbitrary results caused by timing. But it is deficient in its ability to protect a portfolio from declines that start from a higher value than the portfolio's initial value. The floor grows at the risk-free interest rate. If the portfolio's value rises faster than the floor, after a while there is no meaningful protection. On the other hand, if the floor is allowed to grow at the risk-free rate while the stock prices are declining, the portfolio will soon be allocated entirely to bills, so that there is no meaningful participation in any subsequent market rise. So CPPI does not give a floor relevant to the market. CPPI does not take into account wealth generated by market increases during the insured period. In truth, when prices are rising the investor is always concerned about protecting today's fund value, not the value at some arbitrary date in the past. CPPI protects effectively when a decline starts at the same time the strategy is initialized. In practice, OBPI is sensitive both to the starting and ending dates of the put while CPPI is sensitive only to the starting date since CPPI could have no specified expiration.

On the contrary, TIPP is not sensitive to the starting date and expiration date. If the portfolio goes up, protection is in effect restarted daily; if the portfolio goes down, the decline will always begin from the floor that is most relevant to the fund sponsor. Risk exposure changes smoothly with changes in wealth, in accordance with a plausible theory

⁴¹ See Estep and Kritzman (1988)

of investor utility. No forecasts of market volatility or market actions are required, nor are any forecasts implicitly assumed in the practical operation of the process. The only difference between these two strategies is the floor. With CPPI the floor is defined as an absolute value that grows with time at the risk-free rate. With TIPP the floor is defined as a percentage of the portfolio's highest value up to a given moment.

4.1.7 Who needs portfolio insurances?⁴²

Portfolio insurances are one of the most popular hedging strategies using options or synthetic options, futures and derivatives in the money and capital market for one or more periods. It could be costly for an investor who aims to get his capital back. Under this goal the portfolio manager should buy portfolio insurance for the following two kinds of clients (investors):

1. Investors who have average expectations, but whose risk tolerance increases more rapidly with wealth than the average investor, will wish to obtain portfolio insurance. This type of investors might include pension or endowment funds which must exceed a minimum value at maturity or periodically, but can accept reasonable risks. "Safety-first" investors would find portfolio insurance attractive.
2. Investors who have average risk tolerance, but whose expectations of returns are more optimistic than average, will wish to obtain portfolio insurance. They would buy well-diversified funds to achieve an excess return⁴³ by superior stock selection.

In practice, two problems with portfolio insurance implementations can arise. One is the uncertainty of the interest rate. It is usually not significant and has little effect on the outcome of the trading strategy and further adjustments can be ignored. The other problem is the uncertainty of the volatility of the underlying portfolio. Both can be reduced by

⁴² See Leland (1980)

⁴³ Excess return is the difference between the portfolio return and the risk free interest rate

combining static positions in options with dynamic trading. Investor buys and holds the option until maturity (static strategy) while he trades the underlying (dynamic strategy)⁴⁴.

4.2 Shortfall expectation⁴⁵

The shortfall expectation measures the sum of losses weighted by their probabilities. This measurement requires the shortfall probability and mean excess loss. It answers how often a loss might occur and how big such a loss might be. In the next section, a Monte Carlo Simulation of ten years with 1000 runs for each year will be generated.

The shortfall probability of a maturity guarantee is measured by the cumulative return of the portfolio at maturity, R , and the capital guaranteed return. It is given by

$$SP = \text{Prob}(R < X)$$

where X is the capital guaranteed rate of return. For a principal guarantee described in Section 2.4, X is equal to zero. This answers the possibility that the rate of the portfolio return at maturity is lower than the capital guaranteed return.

To know how large such a loss might be, the mean excess loss (MEL) at maturity is calculated by

$$MEL = E[X - R | R < X]$$

where $E[.]$ means the expected average excess loss at maturity. This measures the average difference between the actual return and guaranteed return if the actual return is lower than the guaranteed return.

Then the shortfall expectation (SE) is measured by the shortfall probability multiplied by the mean excess loss.

$$SE = SP * MEL$$

⁴⁴ See <http://www.in-the-money.com/presentation/sld109.htm>

⁴⁵ See Mauer and Schlag (2002)

If the provider transfers the shortfall risk to an insurance company, the shortfall expectation could be seen as an important element of an appropriate premium⁴⁶.

4.3 Guarantee costs

For OBPI, the costs of funding the capital guarantee are the premiums paid by the investor. The premium is the price of the put option. For CPPI and TIPP, the portfolio management should fulfill the possible gap of the guaranteed capital and the actual value of the portfolio at maturity for a maturity capital guarantee. This gap could be seen as the capital charges paid for the portfolio management. The investors could pay a premium for the capital charges. The capital charges could be measured by the shortfall expectation which means the possible loss.

The hedging option is also problematic. In general an option has a maturity of one year. After the maturity, the investor must purchase new options and reinitialize the portfolio. Thus using options in the portfolio in the long run could be very costly. Problems of put options are first the costs for a long-term investment and how to finance this fund if one must pay for all the options at the beginning of the contract. Second, whether there is such an option with such a long time maturity. If not, buying option(s) annually (roll-over options) could be necessary and very costly in regards to the risks of prices and liquidity. Third, whether it is worth while for the institutions to hold a put option. The option seller must make a decision whether he can cover these liabilities at the end of maturity. Finally, it could be very complicated if the guarantor manages the accounts of investors separately. Then the guarantor must hold an appropriate put with different strike prices and maturity for each investor.

⁴⁶ See Mauer and Schlag (2002)

5 Effects on capital guarantees

In this section, we focus on a maturity guarantee generated by Monte Carlo simulation. The payoff and the shortfall expectation of option-based portfolio insurance, constant proportional portfolio insurance and time irrelevant portfolio insurance will be analyzed. The guarantee costs of OBPI are measured by the value of the put option. The guarantee costs of CPPI and TIPP are not considered since this capital charge is not paid by the investor directly but could be calculated in the form of management fees which are not considered here.

In the 1970's, academic papers put forward a simple framework for capital guarantees with call or put options. The insured portfolio payoff is a convex function of the reference portfolio's terminal value. Convexity implies greater protection from loss at lower values of the reference portfolio. There is also a reversible relationship between convexity and the insured reference portfolio. This means that a portfolio consisting of the reference portfolio plus put options will always provide a convex payoff function. Conversely, a convex payoff function can always be generated by holding the reference portfolio and cash, plus a suitable portfolio of put options on the reference portfolio⁴⁷. In this section, the simulations are dependent on the following parameter specifications.

Parameters	Value
Mean of the stock $E(r_s)$	9,3603% p.a.
Mean of the bond $E(r_b)$	4,86% p.a.
SD of the stock $\sigma(r_s)$	22,1396% p.a.
SD of the bond $\sigma(r_b)$	2,9833% p.a.
Correlation of the stock and the bond $\rho_{s,b}$	-0,17861627
Round, n , for each year	1000
Initial value V_0	100
Frequency of payment	Single period
Strike price X	80/100/110
Maturity T	1/2/3
Risk free rate r_f	3%/4%/5% p.a.
Multiplier m	≥ 1
SD of the stock according to simulation σ_s	5,34%/12,18%/30% p.a.
Floor f TIPP	0,8/1/1,1

⁴⁷ See Leland (1980)

We try to examine how the payoffs of these three strategies change if one of the parameters changes while the others stay constant. The payoff of OBPI is a function of the strike price of an option, the time horizon, the interest rate and the standard deviation of the portfolio. Suppose in the portfolio of OBPI there are only risky assets (stocks), riskless assets (bonds) and options. The strike price can be seen as the guaranteed value of the portfolio.

$$\text{Payoff}_{\text{OBPI}}(X, T, r, \sigma) \quad \text{equ. 1}$$

The payoff of CPPI is measured by a function of the floor, the time horizon, the interest rate and the multiplier.

$$\text{Payoff}_{\text{CPPI}}(f, T, r, m) \quad \text{equ. 2}$$

The payoff of TIPP is a function of only the floor and the multiplier since it is time invariant.

$$\text{Payoff}_{\text{TIPP}}(f, m) \quad \text{equ. 3}$$

From the above three equations, there are five effects of the capital guarantee on the payoff of the portfolio: the protective level for all three strategies, the time horizon, the interest rate, the risk for OBPI and CPPI as well as the multipliers for CPPI and TIPP. We start with a pure stock portfolio. Then a mixed portfolio of the stock and the bond will be introduced later.

The costs of the put option would be lower if the volatility of reference portfolio is lower, but at the other side a lower volatility brings also a lower return.

Management fees are paid at the end of the investment horizon, T .

The present value of short puts, P_0 , is equal to the initial wealth minus initial portfolio value, $V_0 - V_{p, 0}$.

After analyzing the effects of the capital guarantee on the payoffs, the shortfall expectation of each strategy is calculated in three steps: the shortfall probability (SP), the mean excess return (MEL) and the shortfall expectation (SE) described in *Section 4.2*. The shortfall expectation of a pure stock portfolio without capital guarantee is compared to a portfolio with capital guarantee. The shortfall expectation with capital guarantee should be zero at maturity.

5.1 Monte Carlo simulation

The valuation method used here is Monte Carlo (MC) simulation. A MC simulation is used to generate the full distribution of the payoffs, the shortfall expectation and a possible future price development since historical results do not represent the same situation in the future. The asset price generation using Monte Carlo simulation follows two assumptions:

1. Suppose the stock and the bond prices are normally distributed and according to a geometric Brownian motion:

$$dS_t = \alpha S_t dt + \sigma_S S_t dz_t \text{ and } dB_t = \beta B_t dt + \sigma_B B_t dz_t$$

where S_t , B_t denotes the price of the stock and the bond at time t respectively, dS_t , dB_t its infinitesimal price change, α and β , their per unit time means of the stock and the bond respectively, σ_S and σ_B , their per unit time standard deviations of the stock and the bond respectively, and dz_t , the increment of a Wiener process. Mean and volatility are constant. The only risk resource is the uncertain stock and bond price.

2. The weekly mean and the standard deviation refer to the DAX from 11.1990 – 12.2006 and REX from 1.2000 – 12.2006. The correlation of DAX and REX are calculated from 1.2000 – 12.2006 since REX started from 1. 2000.

A MC simulation was used and implemented in Excel according to the next four steps⁴⁸:

⁴⁸ See Lecture notes of Michael Halling

1. Generate for each asset 1000 random variables ε that are standard normally distributed (Mean=0 and Sigma=1) for each year.
2. Use Cholesky decomposition to transform these independent variables into correlated random variables.
3. Use these realizations for ε in the following equation to calculate 1000 realizations of asset price S for each year.

$$S_t = S_{t-\Delta t} * e^{(\mu - \frac{\sigma^2}{2}) * \Delta t + \sigma * \sqrt{\Delta t} * \varepsilon} \quad \text{where} \quad \Delta t = 1; \varepsilon \sim N(0,1); S \sim (\mu, \sigma) \quad \text{and}$$

$$B_t = B_{t-\Delta t} * e^{(\mu - \frac{\sigma^2}{2}) * \Delta t + \sigma * \sqrt{\Delta t} * \varepsilon} \quad \text{where} \quad \Delta t = 1; \varepsilon \sim N(0,1); B \sim (\mu, \sigma)$$

4. Evaluate the portfolio for every S and B respectively.

Assume the correlation matrix of two assets is $\rho = \begin{pmatrix} 1 & \varpi \\ \varpi & 1 \end{pmatrix}$. ω is the given correlation of these two assets. Then we have to find another matrix, H , the “square root” of ρ , such that $\rho = H * H^T$. The Cholesky Decomposition is $\begin{pmatrix} 1 & \varpi \\ \varpi & 1 \end{pmatrix} = \begin{pmatrix} a_{11} & 0 \\ a_{12} & a_{22} \end{pmatrix} * \begin{pmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{pmatrix}$. Solving this system of linear equations yields $\begin{pmatrix} a_{11} & 0 \\ a_{12} & a_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \varpi & \sqrt{1-\varpi^2} \end{pmatrix}$. Then we get the correlated standard normal distributed random variables $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \varpi & \sqrt{1-\varpi^2} \end{pmatrix} * \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix} = \begin{pmatrix} \varepsilon_1 \\ \varpi * \varepsilon_1 + \sqrt{1-\varpi^2} * \varepsilon_2 \end{pmatrix}$ ⁴⁹.

Stock and bond prices are simulated under the Cholesky framework so that both are calculated as two correlated assets. Each time period will be simulated by 1000 runs. The portfolio consists of one stock and one bond. The initial time unit is one year. The initial value of the portfolio is 100 Euro. According to the historical data from DAX and REX, the mean of the stock is set at 9.3603% per annum and of the bond at 4.860% per annum.

⁴⁹ See Lecture notes of Michael Halling

The standard deviations are 22.1396% and 2.9833% per annum for the stock and the bond respectively. Surprisingly, the correlation between the stock and the bond is negative during 1.2000-8.2007. *Figure 5-1* shows the 20 weeks moving average return of DAX and REX between 1.2000-8.2007. This could be due to the adverse price development. Especially, since the stock price fell between 2000 and 2002 then boomed again since 2003.

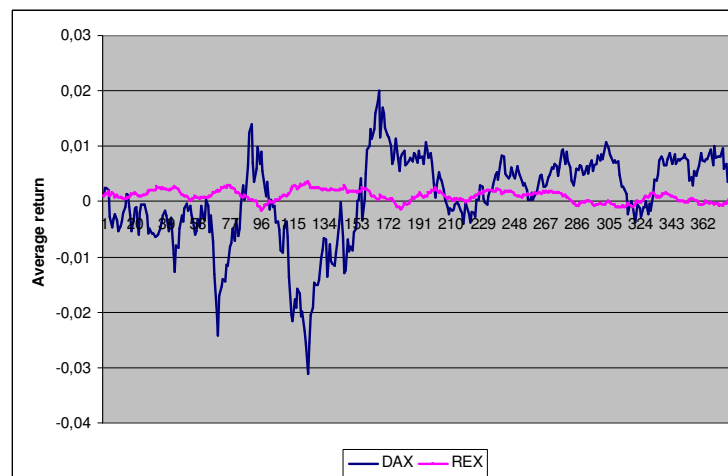


Figure 5-1 20-weeks average moving return of DAX and REX between 1.2000-8.2007

5.2 OBPI

5.2.1 Payoffs of capital guarantees

5.2.1.1 Effects of the protection level

At first we set three different strike prices of 80, 100 and 110 respectively. A low strike price of the option means a reduction of the guaranteed level. The price of the put option can be seen as the guarantee premium.

	X=80	X=100	X=110
Premium	1,22	7,29	12,88
Average payoff	109,30	109,38	111,85
Net aver. payoff	108,08	102,09	98,97

Table 5-1 Premium, average payoff and net average payoff with $T=1$, $r=0,03$ p.a.

Table 5-1 clearly demonstrates that when the protection level arrives at 80%, the premium is only 1.22 per annum. But its yearly average payoff is 109.30, i.e. 9.3% rate of

return per annum. Therefore the yearly net payoff is 108.08, also achieves 8.08% rate of return per annum. In comparison, the premiums are much higher with a protection level of 100% and 110% than with a protection level of 80%. The investor must pay seven times more premiums for a high protection level of 100% and twelve times more for a protection level of 110% than for a protection level of 80%. A higher than 100 percent capital guarantee could bring an average payoff higher than an 80 percent capital guarantee. But at the same time, the annual mean income of a higher capital guarantee does not increase proportionally with the guarantee level. The yearly mean payoff of 100% and 110% capital guarantee is 109.38 and 111.85 respectively. Then the net yearly mean income (after premium) of a 100% protection and a 110% protection is only 102.09 and 98.97 respectively. These results are even lower than the yearly mean payoff of 80% protection.

As the level of the guarantee increases, an increasing fraction of initial wealth must be allocated to the put option. Two effects of an increase in the level of the capital guarantee are the benefit of having a higher level of guaranteed payoff and the cost of paying for a higher level of guaranteed payoff. The second term must dominate the first term. The effect of increasing the level of the guarantee must be negative, because by increasing guarantee the set of feasible terminal payoffs of the restricted portfolio shrinks⁵⁰. The higher the minimum guarantee that the individual chooses, the greater the potential higher returns he must forego.

In *Figure 5-1* the payoff distributions of 80 percent and 100 percent protection level are presented very similarly. But the 80 percent protection level can not eliminate completely the payoffs under the initial capital. 0.3% random variables are lower than 100 at maturity of one year. All the random payoffs of a 100 percent protection level are larger than the initial capital, 100. But only around 35% random payoffs reach 110. Almost 60 percent of the random variables of 110 percent protection level reach a payoff of 110. It is difficult to achieve the capital guarantee with a lower protection level in one year. In the next section we will see the payoffs distribution in short and long run.

⁵⁰ See Jensen and Sorensen (2000)

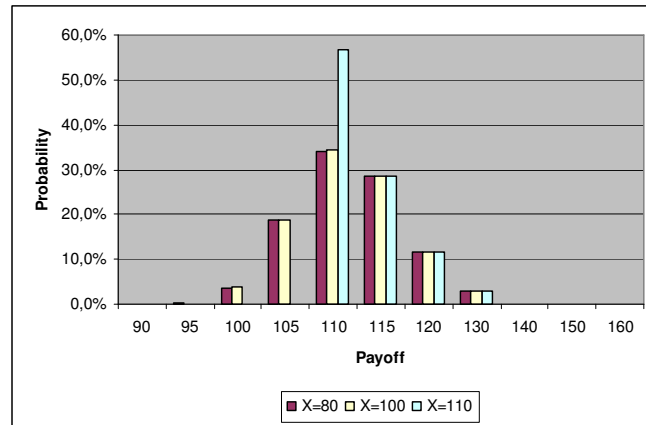


Figure 5-1 Distribution of payoff with different strike prices X, T=1, r=0,03 p.a.

5.2.1.2 Effects of the time horizon

In section 5.2.1.1, the effects of the protection level was shown. For OBPI the payoffs and costs increase with the protection level. These results are analyzed based on a maturity of one year. In this section a portfolio with a maturity of three and ten years using MC simulation is generated. Then we can see how the time horizon plays a role in the short and long term.

In general, the payoffs increase with the years and the time horizon. In *Table 5-2*, a three-year investment of OBPI has an average payoff of 158.26 at maturity compared to 109.38 of a one year investment. Its final payoff is about 20 percent more than that of one year maturity. The payoff of a three-year maturity in the first year is also higher than that of a one-year maturity. The payoff of a three-year maturity in the later years (122.07) is higher than in the early years (115.93), extending the portfolio insurance horizon is an indirect way of reducing the degree of protection, and thereby reducing the cost⁵¹.

Maturity	Year 1	Year 2	Year 3
1	109,38		
3	115,93	122,07	158,26

Table 5-2 Average payoff of OBPI with T=1, 3, r=0, 03 p.a., X=100

If an investor has unusual circumstances which require a short protection horizon, this discussion may be noted. For example, if a pension executive has a Board which expects positive returns each year, regardless of market movements, then a one-year insurance strategy may be the only prudent course. On the other hand, if the time horizon of the

⁵¹ See Arnott (1988)

underlying portfolio is long term, one-year insurance may help the investor to achieve a very costly objective (namely, positive returns each year), which is irrelevant for the long-term needs of the portfolio⁵².

The time horizon plays a key role in the determination of what is risky and what is not. A risk reduction strategy appropriate for a longer horizon may be quite dangerous if applied to an investment problem with a short horizon, and vice versa. When the horizon covers a short period of time such as one year, the strategy is typically to match these current liabilities with current assets; for example, cash. For an intermediate horizon, risk minimization involves ensuring against interest rate changes through the use of fixed income investments. But the bonds alone cannot cover all the risk of a portfolio. Only equities, with their automatic participation in gains, can adequately ensure against long-term risks⁵³. But some papers argue that the stock price risk cannot be eliminated even in the long run. A second long-term risk is that the company may not complete its promised payments, a risk that can be reduced by specific diversification against company risk⁵⁴.

5.2.1.3 Effects of the interest rates

For a long period, interest rates remained low until 2006. In 2007 interest rates increased globally to a height of around four percent per annum. This change brings a very popular topic for finance worldwide. Do interest rates have a significant influence on the capital guarantee? The results are shown in the following discussion.

In *Table 5-3* we can see that the payoff of OBPI decreases if the risk-free rate drops, then the implicit return available in the hedged portion of the portfolio similarly drops⁵⁵. A higher interest rate lowers the present value of the guarantee and increases the present value of a call. But at the end the payoffs decrease, also the present values of the put.

Payoff	r=0,03 p.a.	r=0,04 p.a.	r=0,05 p.a.
OBPI	132,09	131,79	131,52

Table 5-3 Average payoffs of OBPI with r=0, 03, 0, 04, 0, 05 p.a. , T=3, X=100

⁵² See Arnott (1988)

⁵³ See Wagner (1988)

⁵⁴ See Wagner (1988)

⁵⁵ See Arnott (1988)

5.2.1.4 Effects of the risks

In this sub-section we will firstly analyse the relationship between capital guarantees and two types of risks, systematic and unsystematic respectively. So we change the systematic risk (historical rate of return of DAX) to 15%, 22% and 30% p.a. respectively. From the *Table 5-4* we can see that the unsystematic risk of the stock are very close to the risk of the guaranteed portfolio consisting of 100% stock, when the systematic risk changes. This finding with respect to the market risk suggests that portfolio insurance is likely to have small effects on the fluctuation of the stock prices, at least in perfect markets in which the activities of portfolio insurers are fully anticipated⁵⁶.

The other finding from *Table 5-4* is the relationship between the payoffs and the risks. The payoffs increase with the systematic and unsystematic risks.

Sys. risk	Unsys. risk	σ_{OBPI}	Payoff _{OBPI}	Weight of the stock	Weight of the bond
15%	8%	9%	128,84	100%	0%
22%	12%	13%	132,09		
30%	17%	16%	134,78		

Table 5-4 Risks and average payoff with $X=100$, $r=0$, 03 p.a., $T=3$

While the systematic risk is hardly influenced through the portfolio insurance, the unsystematic risk could be partly reduced by diversification. Now the bond will be weighted in the portfolio with a proportion of 50% and 100% respectively. *Table 5-5* shows the results that the risks of the portfolio decrease with the average weights of stocks.

OBPI

σ_{OBPI}	Payoff	avr.w s	avr. w b
9%	128,84	100%	0%
5%	119,32	50%	50%
0%	110,24	0%	100%

Table 5-5 Payoffs of OBPI with a systematic risk of 15% p.a. and an unsystematic risk of 8% p.a.

5.2.2 Shortfall expectations

Now we use the initial parameters shown in the following table but with a maturity of ten years to test the shortfall probability, the mean expected loss and the shortfall expectation of three strategies, OBPI, CPPI and TIPP at the expiration.

⁵⁶ See Arnott (1988)

Maturity	10	year
Strike price	100	€
Floor	100	€
interest rate	0,03	p.a.
multiplier	3	
sys risk	22,1396%	p.a.

Through the comparison of the portfolio value with and without capital guarantee, we will see whether these hedge strategies provide capital guarantees and at which costs. At first, let us check the shortfall expectation of OBPI.

Figure 5-2 shows the shortfall probability of 100% stock portfolio without capital guarantee at maturity of ten years. About 0.90% of 1000 random variables are below the capital guarantee, 100. So the loss probability could not be ruled out even with a maturity of ten years. It could be much more severe if the investment horizon is one year. In order to see how much would be lost, the loss of each random variable which is below 100 is calculated. Then the means loss is also derived, 0.096. Finally, the shortfall expectation of 100% stock is equal to around 0.086% which means a loss of 0.086% of the portfolio with a probability of 0.90% at maturity.

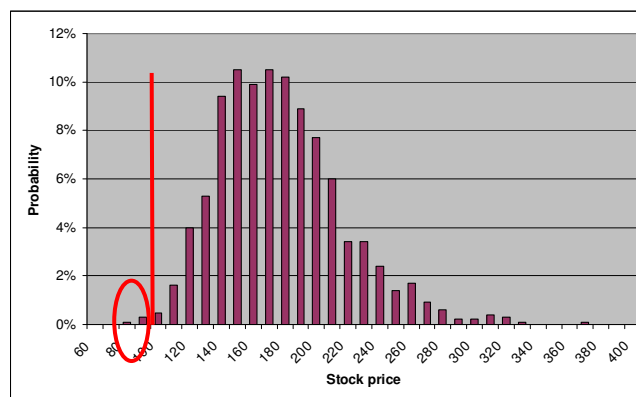
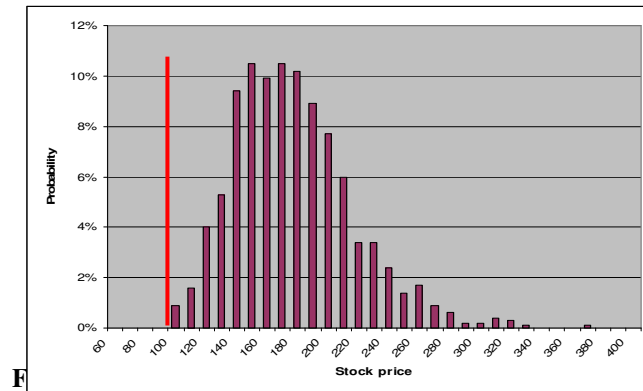


Figure 5-2 Shortfall probability of 100 % stock without capital guarantee at maturity

Then the shortfall probability of OBPI is zero at maturity but at the costs of a premium payment. The value of the premium depends on the value of the put option which is determined by the stock price, the protection level, the implied volatility, the interest rate and the maturity. Then the shortfall expectation is zero. Both have the similar distributions besides there are no shortfall probabilities for OBPI.



5.3 CPPI

5.3.1 Payoffs of capital guarantees

5.3.1.1 Effects of the floors

The average payoffs of OBPI increase with the protection level. On the contrary, for the synthesized strategies, CPPI decreases when the floor increases. In *Table 5-6* the payoffs of CPPI with a lower floor are more profitable than with a higher protection level at maturity of one year. A higher initial floor raises the actual floor so that the cushion declines. Then less wealth will be invested in the stock. As a result, the value of the portfolio decreases because the rate of return of the bond is less than that of the stock.

X	$V_{t, \text{cpbi}}$
80	107,35
100	103,61

Table 5-6 Average payoff of CPPI with $T=1$, $m=5$, $r=0.03$ p.a.

5.3.1.2 Effects of the time horizon

In order to see the time effect in the long run, a simulation of ten-year portfolio is added. In *Table 5-7* the payoff with a maturity of three years is much higher than with a maturity of one year if the multiplier is 5. But the average payoff with a ten-year maturity is even below the capital guarantee, 100, with a multiplier of 5. But the average payoff with a ten-year maturity is 114.91 if the multiplier is 1. CPPI could provide a capital guarantee only with a suitable combination of the floor and the multiplier.

Maturity	CPPI
1	103,61
3	117,86
10	91,23

Table 5-7 Average payoff of CPPI with $T=3$, $r=0,03$ p.a., $F=100$, $m=5$

The results of this ten-year simulation in *Table 5-8* show that the average values of the portfolio at maturity are almost equal to the values of the floor and increase with the floor. But a floor below one cannot offer a capital guarantee. In contrast, the values of a three-year portfolio decrease if the floors increase.

$V_{t, \text{cpbi}}$	$F=80$	$F=100$	$F=120$
$T=3$	142,59	117,86	106,78
$T=10$	80,00	100,00	120,00

Table 5-8 Average payoff of CPPI at maturity with different floors with $r=0,03$ p.a., $m=5$

In total, CPPI is suitable for the short run investment if the floor is low. A long run investment could be attractive if the floor is higher than one.

5.3.1.3 Effects of the interest rates

An increase of the interest rates leads to different changes of payoffs with different floors in the short and long run. In the short run, a lower floor gives the stock more space so that it is possible to get a higher payoff which is shown in *Figure 5-4*. With a maturity of three years, a floor equals 0.7 leads to the highest payoff compared to the other floors. But the payoffs tend to shrink as interest rates increase. In contrast, the payoff of a floor equal to 1.1 is lowest but it tends to increase with interest rates. In a word, the payoffs of lower floors are higher than of higher floors. A low floor leads to a higher payoff than a high floor. But the payoff of a low floor tends to shrink. In contrast, the payoff of a high floor tends to grow.

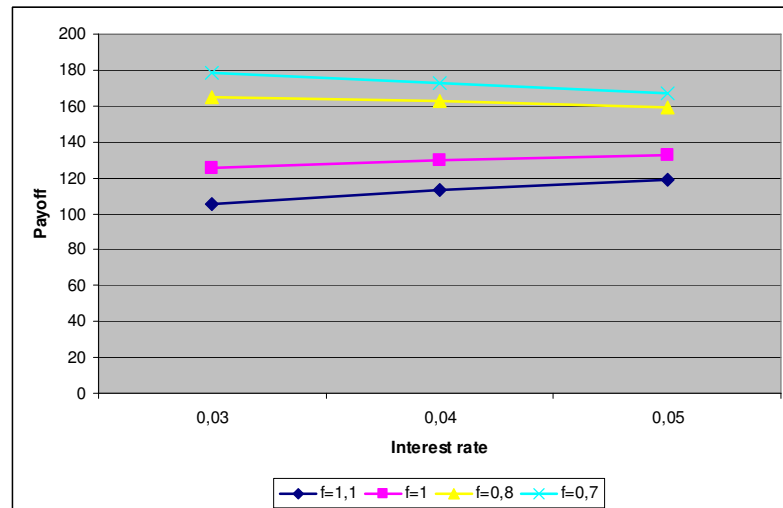


Figure 5-4 Average payoff of CPPI with different interest rate, T=3, m=5

The payoff in the short run is different from that in the long run. In the long run the payoff with a higher floor is always higher than that with a lower floor as shown in *Figure 5-5*. The payoff has a negative relationship with the interest rate during the periods. But at maturity, the payoffs with different interest rates are almost the same. Marginal long term payoffs are the same as the short term. In a word, a portfolio in the short run should have a lower floor when the interest rates increase, but in the long run a higher floor.

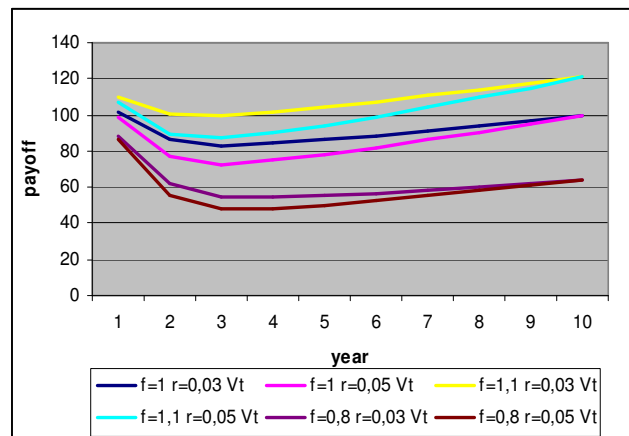


Figure 5-5 Payoff of CPPI with different interest rate and floor with T=10, m=5

The increase of interest rates is favourable to OBPI, but not to CPPI especially in the long run. It is more complicated to construct a capital guaranteed portfolio with CPPI in the long run. Since TIPP is time irrelevant, changes of interest rates do not influence the value of the floor, nor of the total payoffs.

5.3.1.4 Effects of the risks

Table 5-9 shows the results of CPPI in terms of the change of the risks. In comparison to *Table 5-4*, the risks and the payoffs of CPPI are lower than of OBPI with the same systematic and unsystematic risks. The average weights of the stock of CPPI are between 59% and 75% in comparison to 100% of OBPI. Obviously, the proportion of the stock decreases as the volatilities increase. This means that CPPI tends to be more conservative if the risks increase. As a result, the payoffs of CPPI decrease while the payoffs of OBPI increase with the risks.

Sys. volatility	Unsys. volatility	σ_{CPPI}	Payoff _{CPPI}	Avr. weight of the stock	Avr. weight of the bond
15%	8%	5%	119,60	75%	25%
22%	12%	6%	117,86	69%	31%
30%	17%	7%	113,18	59%	41%

Table 5-9 Average payoff with different risks with T=3, F=100, m=5, r=0,03 p.a.

5.3.1.5 Effects of the multipliers

The level of the multiplier measures the proportion of the stock in the portfolio as well as the risk tolerance of the investor. A higher multiplier means that the investor would like to invest more in stocks and therefore he is a risk taker. In other words, he has a higher risk tolerance. In this section, we will check the effect of the multiplier on the payoffs under a capital guarantee.

At first, let us have a look at the payoffs of CPPI with different multipliers. Then we check the effects of the multipliers on the payoffs. *Figure 5-6* shows the convex payoffs of CPPI. The payoffs are more convex as multipliers increase. The deep blue line, the red line and the yellow line show the payoffs of CPPI with multipliers of 1, 3 and 5 respectively. Curves with different multipliers intersect one another. It is not clear that one payoff of the multiplier dominates the others. The payoffs range from around 100 to 180. CPPI performs better for large fluctuations of the market while OPBI performs better in moderate bullish markets⁵⁷. The higher the multipliers, the higher the payoffs are.

⁵⁷ See Bertrand and Prigent (2002)

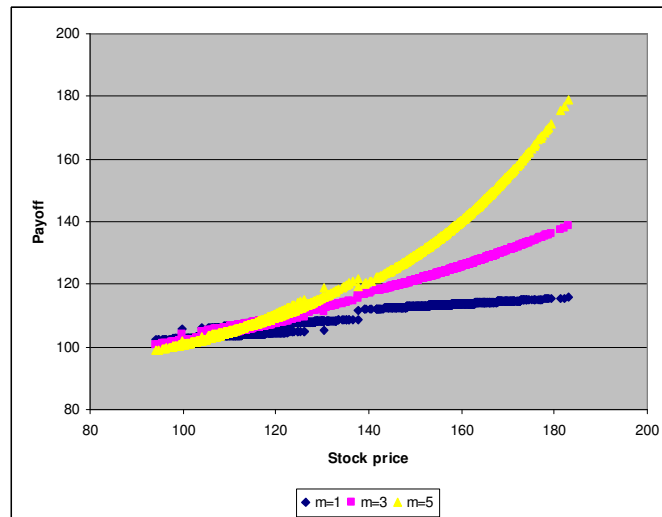


Figure 5-6 Payoffs of CPPI with $T=3$, $r=0,03$ p.a., $F=100$

For CPPI in the long term, *Table 5-10* shows that the multiplier cannot influence the payoffs which exceed the capital guarantee with a floor lower than 80 whatever the multipliers are, because the discounted floor is too low to ensure the minimum value of the portfolio. For a floor between 90 and 100, the payoffs of CPPI exceed the capital guarantee if the multipliers are close to 1. In the case of a floor beyond 110, the average payoffs of CPPI are always larger than the capital guarantee whatever the multipliers are, because the floor is high enough to ensure the value of the portfolio.

Payoff	F=80	F=90	F=100	F=110	F=120
m=1	98,90	106,03	114,91	125,55	137,94
m=5	61,68	75,58	91,23	108,64	127,80

Table 5-10 Average payoff of CPPI with different floor, multiplier, $T=10$ and $r=0,03$ p.a.

To see the changes in payoffs, the composition of the portfolio is calculated in *Table 5-11*. It is clear that the weight of the bond is always larger than 57.7% and increases with the floors and multipliers. More than 90 percent of the portfolio consists of the bond if the floor is 1.2.

	f	0,8	0,9	1	1,1	1,2
m=1	Avr. W stock	42,3%	32,4%	23,4%	15,4%	8,4%
	Avr. W bond	57,7%	67,6%	76,6%	84,6%	91,6%
m=5	Avr. W stock	14,3%	11,6%	9,2%	6,8%	4,4%
	Avr. W bond	85,7%	88,4%	90,8%	93,2%	95,6%

Table 5-11 Average weight of stock and bond with different floor and multiplier, $T=10$, $r=0,03$ p.a.

Now in the short run, despite the average payoffs, the average weights of assets have the same discipline as in the long run, namely, the higher the floor, the higher the proportion of bonds. The reason for the decreasing payoffs if the floors increase is that the initial floor is close to the initial portfolio value. Therefore the portfolio value depends mostly on the value of the stock if the floor is lower than 1. The payoffs could be much higher if the multipliers are high. Then there is only a bond in the portfolio if the initial value of the floor is beyond the initial portfolio value. Then the payoffs of different multipliers lead to the same portfolio value finally. The payoff with a combination of a low floor and a high multiplier leads to the highest average payoff of 148.04 shown in *Table 5-12*. The payoffs decrease with the floors and the multipliers. So in the short run, the investment tends to be less conservative.

Payoff	f=0,8	f=1	f=1,1
m=1	111,98	108,06	106,78
m=5	148,04	119,60	106,78

Table 5-12 Average payoff of CPPI with different floor and multiplier, T=3, r=0,03 p.a.

In the short run, the portfolio volatility, the average payoffs and the average weights of stock decrease with the multipliers.

CPPI				
m	σ	Payoff	avr.w s	avr. w b
5	5,19%	119,60	78,60%	21,40%
3	2,97%	112,91	41,20%	58,80%
1	1,78%	108,06	10,13%	89,87%

Table 5-13 Volatility, average payoffs and weights of assets of CPPI with T=3, f=1, r=0,03 p.a., sys. Vola.=15%, unsys. Vola=8%

To sum up, the portfolio with CPPI could be conservative in the long run and riskier in the short run.

5.3.2 Shortfall expectations

Since CPPI could provide a capital guarantee only with a suitable combination of the floor and multiplier, the shortfall probability of CPPI is also equal to zero at maturity of ten years if the multiplier is 1.5. More than 60% of the 1000 random variables range from 130 to 150 shown in *Figure 5-7*. The mean expected loss and the shortfall expectations are also zero.

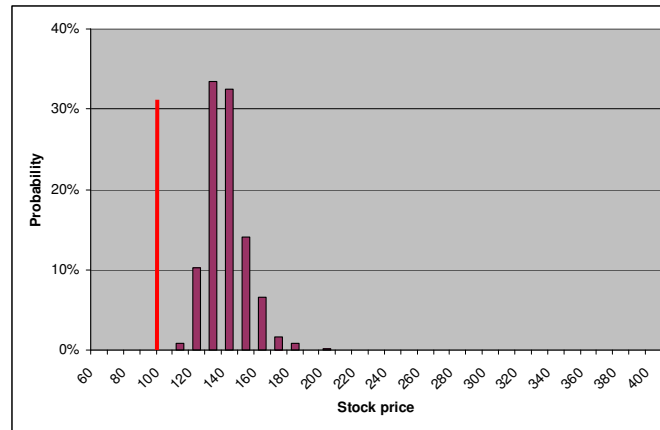


Figure 5-7 Shortfall probability of CPPI at maturity with $T=10$, $r=0,03$ p.a., $m=1,5$, $F=100$

5.4 TIPP

5.4.1 Payoffs of capital guarantees

The payoffs of TIPP are even easier than CPPI since it is time invariant and interest rate independent and does not need forecasts of risk either. The only relevant elements are the floors and the multipliers.

5.4.1.1 Effects of the floors

The value of the floors has the same effect on the payoffs of the TIPP as CPPI. Namely, the portfolio value decreases if the floor increases shown in *Table 5-14*.

F	V_{TIPP}
80	109,32
100	105,04

Table 5-14 Average payoff of TIPP with $T=1$, $m=5$, $r=0.03$ p.a.

For a portfolio with a maturity of one year, 60% of random values reach a portfolio value between 105 and 110, 40% between 100 and 105.

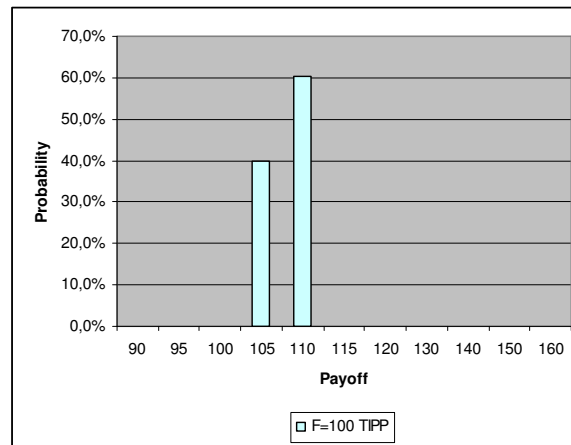


Figure 5-8 Distribution of payoff of TIPP with different floors with $T=1$, $r=0,03$ p.a., $m=5$, $F=100$

5.4.1.2 Effects of the multipliers

If the floor exceeds one, the initial cushion is zero. This leads to no stock investment in the portfolio. The entire portfolio consists of bonds only. Then multipliers can not affect payoffs, loss probabilities and weights of assets. The distribution of the payoffs is shown in *Figure 5-9*. The payoffs seem to be waved if the floor goes beyond 1. Since the bond is the only element of the portfolio, it is quite natural that the payoffs range from 104.8 to 105.15. The average payoff of ten-year maturity is 104.96 if the floor is 1, multiplier is 5.

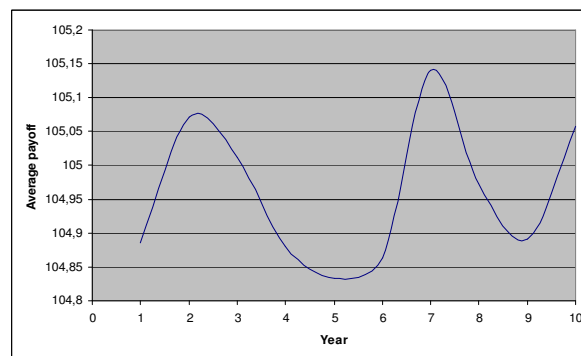


Figure 5-9 Average payoff of each year with $f=1$, $m=5$, $T=10$

In contrast, a low floor of 0.8 gives the stock more cushion so that the stocks could join the portfolio payoffs. But because the floor may only increase but not decrease, the weight of the stock in the later years is almost zero. As a result, the payoffs do not increase dramatically like CPPI. From *Figure 5-10* we see that the average payoffs range from 100 to 112. The average payoff in this case is 109.02 if the floor is 0.8, multiplier is 5 and maturity is 10.

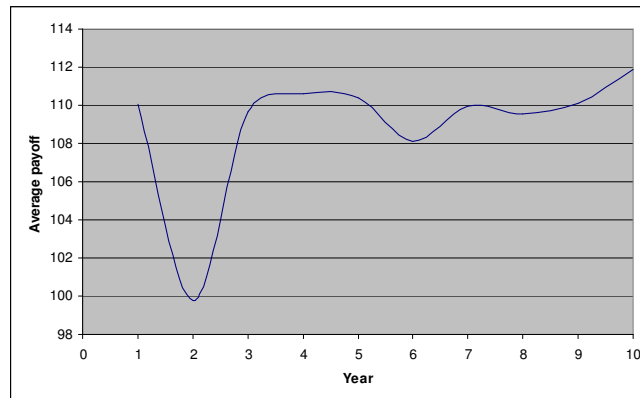


Figure 5-10 Average payoff of each year with $f=0.8$, $m=5$, $T=10$

5.4.2 Shortfall expectation

The shortfall probability is zero for TIPP at maturity. There is a possibility of 40% to reach an end value of the portfolio between 100 and 110 and 60% between 110 and 120 with no extra costs. Hence, there is no shortfall expectation either.

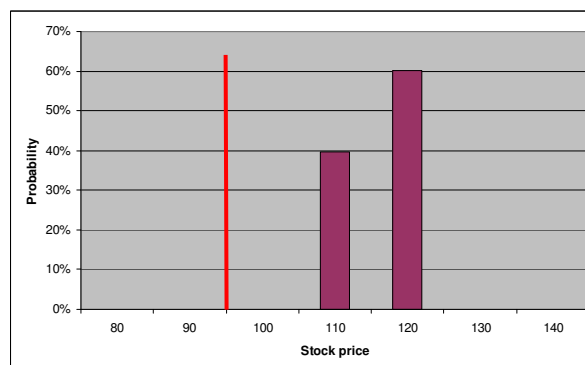


Figure 5-11 Shortfall probability of TIPP at maturity

5.5 Costs of guarantee

5.5.1 Nature of the downside risk⁵⁸

Lachance and Mitchell 2002 argued that popular belief regarding the nature of the downside risk tends to downplay the value of such guarantees. For instance, it is often recommended that investors with long investment horizons hold a larger proportion of

⁵⁸ See Lachance and Mitchell (2002)

stocks in their portfolios. This view is grounded in the argument that stocks are less risky in the long run or, putting it another way, that investors have more time to recoup their losses with longer investment horizons. Historically, stocks have outperformed bonds over long investment horizons, so the belief is that this trend will repeat in the future, resulting in costless guarantees. Empirical evidence shows that the standard deviation did not decrease but increase. These data illustrate the so-called “equity premium” — this is, because stocks are seen by the market as more volatile and hence riskier than bonds, purchasers of stocks require an additional risk premium or return in order to hold them.

The annualized stock returns become less volatile over time, but the opposite is true for compounded stock returns. Applied to the guarantee context, these findings imply that the volatility of capital guarantee portfolio should increase over time, because this volatility is affected by compounded, rather than annual, returns. Consequently, guarantee volatility rises over time, rather than being diversified away over longer investment periods.

5.2.2 Costs of guarantees

The costs of guarantees can be very important and may vary significantly with the protection level, the time horizon, the interest rate, the volatility and the guarantee design. In Section 5.2.1.1 *Table 5-1* shows that the premium of OBPI increases from 1.22 to 7.29 while the protection level increases from 80 to 100. The premium increases almost five times but the guarantee level increases only 25%. The cost of insurance rises more than proportionally to the amount insured⁵⁹. So a protection level over 100% can be much more costly than a protection level slightly below 100%.

Figure 5-12 shows the guarantee costs of different weighting of the stock and the bond. The blue curve is the costs of the pure stock portfolio in forty years. The red curve and yellow curve represent the costs of the fifty-to-fifty stock-bond portfolio and the pure bond portfolio respectively. The short-term premium increment can be associated with the extending investment maturity. In the long run all premiums increase then decrease after the tenth year for pure stock portfolio, seventh years for half of the stock and the bond portfolio and zero for pure bond portfolio.

⁵⁹ See Brennan and Schwartz (1989)

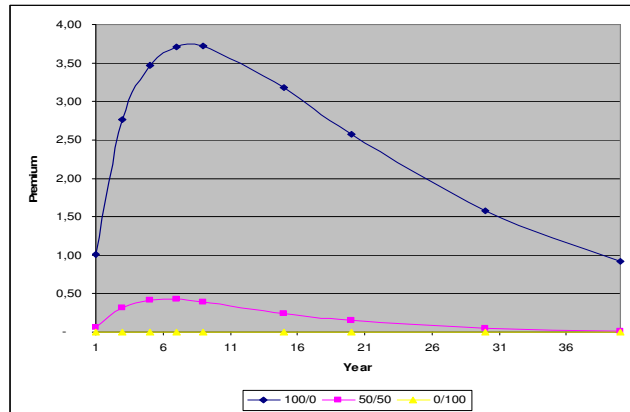


Figure 5-12 Premium with $X=80$, $r=0.05$ p.a.

A lower protection level of portfolio can reduce the costs of the portfolio insurance. This causes the risky assets to reach a higher return. Even so, this relationship between riskless rate and costs of portfolio insurance is often overlooked⁶⁰. It is less costly if the interest rate increases. Premiums decrease from 10.64 with a 3 percentage annual interest rate to 8.23 with a 5 percentage annual interest rate for a pure stock portfolio (Table 5-15), because a put option is further “out-of-the-money” at high-riskless rates than in an environment with low interest rates. Out-of-the-money options cost less than “at-the-money” options⁶¹. Disadvantages suffered due to the high costs in the past could be reduced by the increasing interest.

Put p.a.	$r=0,03$	$r=0,05$
100/0	10,64	8,23
50/50	4,68	2,92
0/100	0,08	0,00

Table 5-15 Present value of a put with different interest rates, $T=3$, $X=100$

The costs of the guarantee rise with the risk of the investments, and decreases as the difference between the real interest rate and the minimum rate of return widens. Companies typically offer guarantees that reduce individuals’ exposure to investment risks⁶².

Volatility	Premium		
Systematik	100/0	50/50	0/100
15%	4,20	1,29	0,00
22%	8,23	2,92	0,00
30%	12,88	5,01	0,00

Table 5-16 Premium with different systematic volatility and weights of assets, $T=3$, $r=0,05$ p.a., $X=100$

⁶⁰ See Arnott (1988)

⁶¹ See Arnott (1988)

⁶² See Pennacchi (1999)

But the greater the volatility of risky assets, the greater the cost of portfolio insurance⁶³. In *Table 5-16* the premium increases as market risk increases and decreases as less risky assets are weighted in the portfolio. One must pay much more premium for a pure stock portfolio than a mixed and a pure bond portfolio in both cases of a higher and lower market risk. All pure bond portfolios in any cases have almost zero premiums, however it is risky. This implies that giving a capital guarantee a choice over investment mix could be costly, in that they might boost the guarantee value by selecting a riskier investment portfolio. It is dangerous that a company provides an investor a guarantee without any restriction on portfolio mix. When guarantees are either very likely or very unlikely to be exercised, their costs are less sensitive to the portfolio allocation⁶⁴.

Hence it gives portfolio management an incentive to invest as riskless as possible, because the value of the put in this case is minimized so that the present value of management fees are not affected. Therefore the expected return of portfolio will lie near the riskless interest rate. However, this optimal assessment strategy for the management contradicts the objective of the investors, who expect as high a return as possible.

One more kind of costs of capital guarantees is the opportunity costs to a representative investor of switching from the expected utility maximizing investment strategy to the conservative portfolio insurance strategy, if total opportunity cost is equal to opportunity cost of investor minus opportunity cost of insurer. The fact that this opportunity cost rises as the fraction of wealth insured increases suggests that the higher the proportion of investors is following insurance strategies, the greater the disincentive for new investors to join them. The increase in the opportunity costs is the result of both an increase in the opportunity cost for the optimizing investor and a decrease of the opportunity cost of the insurer. This means that if the investor insures more of his portfolio, his opportunity costs are higher. Conversely, the less of the portfolio is insured, the lesser the opportunity costs of the portfolio manager⁶⁵.

Table 5-17 shows a summary of the shortfall probabilities, the mean expected losses, the shortfall expectations and the guarantee costs at expiration with a maturity of ten years.

⁶³ See Arnott (1988)

⁶⁴ See Lachance and Mitchell (2002)

⁶⁵ See Brennan and Schwartz (1989)

The guarantee costs of a pure stock using OBPI are highest of three hedging strategies, 12.91. CPPI and TIPP are costless with a proper combination of floors and multipliers.

	SP	MEL	SE	Costs
100%stock	0,90%	0,0956	0,086%	-
OBPI	0	0	0	12,91
CPPI	0	0	0%	0
TIPP	0	0	0%	0

Table 5-17 Shortfall probability, mean expected loss, shortfall expectation and costs at maturity

5.5.3 How to control costs⁶⁶

If the portfolio manager acts in the interest of his clients, he can control some costs while he may not be able to control others. The manager could control the following costs: the fraction of insured portfolio, the return of the floor, the risk of the insured underlying portfolio and the investment horizon covered by the insurance strategy. Costs which the manager cannot control are: the riskless interest rate, the expected equity risk premium and the volatility of the assets. The manager could control the transaction costs partially through reducing the frequency of trading assets.

Decreasing the protection level of portfolio could decrease the costs. The return of the floor reflects the risk tolerance of the investor. If the investor has a relative large tolerance, he can afford more losses and lower floor return. The long-term costs of the insurance are lower if the floor is low. The manager could choose part of the portfolio insured in order to reduce the costs of the portfolio insurance. Extending a longer investment horizon could also lead to lower costs.

5.6 Management fees

For a minimum rate of return, the higher the minimum rate of return guarantee, the more the company has to claim to be able to afford the contract. If the manager receives a percentage management fee of the minimum guarantee or actual return, whichever amount is larger. The investor gets a fraction, α , of the bonus reserve⁶⁷. The rate of payment fee required by the company might be marginally increasing in α for small values of the

⁶⁶ See Arnott (1988)

⁶⁷ Bonus reserve: the annual return of the customer's investment in the reference portfolio (positive or negative)

minimum rate of return guarantee. For small values of the minimum rate of return guarantee there is a greater possibility for investor to distribute more than guarantee, and hence the size of the bonus reserve will more sensitive to the chosen α . The larger the value of α , the more of the excess bonus⁶⁸ is distributed and the larger the probability of ending up with a negative bonus reserve. For higher α s the customer starts getting more than the minimum rate of return guarantee and, at the same time, the probability that the bonus reserve will be negative is increased, hence the company must have a higher payment for the contract, i.e., a higher management fee⁶⁹. For small values of α there is a high probability that the customer's account only grows at the minimum rate of return guarantee. Hence, the company receives almost all the surplus. However, the contract has to be fair since there is no distribution of extra funds to the customer.

In Austria for an individual private account of which is invested at least 40% of total wealth in Austria stock market, a capital guarantee improves only 0.0386% rate of return before guarantee, national premium and management fee. Management fees reduce the return with national premium by 1.92%. Then the final rate of after guarantee, national premium and management fees is left only 4.7274% which is less than the rate of return without capital guarantee⁷⁰.

The convex payoffs imply the risk-taking behaviour of the manager if he invests in reference portfolio like stocks. Such portfolio can be out of the control of the manager. Buying put option is aimed at ensuring this part of the reference portfolio.

The fund manager collects a fraction of the final wealth in the delegated portfolio when it is high and is required to make up the difference between the actual wealth and the promised guaranteed wealth when the portfolio wealth is low. The manager can get more management fee if the final wealth is higher. This provides incentives for the manager to maximize the portfolio wealth. Then manager will not take any risk at all⁷¹. It is also true for OBPI. Since the put option price is increasing in volatility, the present value of the payoff to the fund manager is decreasing.

⁶⁸ Excess bonus: bonus above the optimal buffer level which is a buffer ratio above a certain level.

⁶⁹ See Hansen and Miltersen (1999)

⁷⁰ See Fischer (2003)

⁷¹ See Liu and Wu (2005)

5.7 Risk shifting

Investors desire a higher return in addition to their minimum wealth. But the hedged portfolio consists most of the bond at maturity. Investing in bonds could satisfy this minimum wealth but not achieve a higher end wealth. So there is a conflict between investors and portfolio managers.

In a mutual fund the manager will shift to more risky assets to gamble for a better performance if the portfolio value exceeds the initial value shortly before its maturity date. In a contract with capital guarantee the manager would invest most of the wealth in riskless or risk poor assets. The relative laws are introduced to restrict the proportion of risky or riskless assets in order to reduce this conflict.

Under the restriction of capital guarantees, the proportion of the stock shifts to the bond. In the short run, the weight of bond decreases with time but mostly larger than 50% in average. In the long run, a portfolio of CPPI without loss at maturity tends to invest from 60% wealth in the bond at the beginning to 80% in average at maturity.

Nibert 2002 tested the model uncertainty of OBPI and CPPI in order to know how certain these two strategies can ensure the capital guarantees with different risk resources. The classical OBPI strategies shown in Section 4.1.3 can duplicate the desired option under geometric Brownian motion only, namely, only if the stock price is the unique risk resource. It fails to ensure a guaranteed minimum wealth if more than one risk type exist. Either fails the roll-over portfolio insurance even if one buys an option on each kind of the risks separately to hedge against a situation in which all the risks are present at the same time. But its duplication portfolio does not work when a maximum number of two risks are present at the same time. CPPI will be able to defend guaranteed income streams if each kind of risk is considered separately. It can cope with model uncertainty because cushion (wealth minus the floor discounted at the riskfree rate if the wealth level is lower than the minimum guaranteed wealth) of all portfolio weights coincides. Nevertheless, CPPI is influenced by model uncertainty because it calls for a portfolio strategy that is too conservative in particular under geometric Brownian motion.

He showed a solution to the portfolio problem that crucially hinges on the assumption that model uncertainty is taken into account by adding an explicit preference for models' similarity to the objective function of the decision problem, a so-called preference for robustness. As a result, only one trivial portfolio strategy is able to defend minimum investment goals, namely invest in the riskless assets the amount guaranteed discounted at the riskfree rate. For OBPI, the only portfolio strategy that is capable of defending guaranteed minimum wealth irrespective of the capital market environment reads: invest in the riskless asset guaranteed wealth discounted at the riskfree rate. It is more complicated for CPPI. The portfolio tends to be non-trivial if the stock price is the unique risk resource, but there is no pronounced worst case scenario, in that all portfolio weights have the same structure. But then the portfolio tends to be trivial if there are more risk resources considered: it invests in the riskless asset times guaranteed income per period discounted at the riskfree rate⁷².

⁷² See Nietert (2002)

6 Conclusions

To sum up, issuing a capital guarantee should refer to the investment time horizon, the risk attitude of the clients and different strategies. For OBPI, the payoffs are higher if the portfolio management provides a higher than 100% capital guarantee level and a long term investment plan, takes advantage of the higher interest rates and chooses relative less risky assets. But to reduce the costs of the capital guarantee, portfolio management should choose a lower than 100% protection level. A higher interest rate and less risky assets could also reduce the costs of the capital guarantee.

CPPI could provide a capital guarantee at maturity mainly conditional on the multipliers, the floors and time. In the short run, portfolio management should choose a lower floor, an investment time horizon of more than one year, higher multipliers and more risky assets. Despite of the high multipliers and proportions of the risky asset, the loss probabilities are very low. But it seems to be more conservative in the long run. The floors tend to be higher and the multipliers are lower in order to reach the capital guarantee. As a result, the total portfolio consists most of the bonds. This conservative portfolio leads to a low loss probability. Its losses are much less than a pure stock portfolio. But the losses could be eliminated through extra costs which are much less than the premium of OBPI.

Since TIPP is time irrelevant, the portfolio management could only consider the floors and the multipliers. The payoffs and shortfall expectations of TIPP are not sensitive to the multipliers and the floors. At maturity, the portfolio consists only of the bond however the multipliers and floors are. The payoffs with a floor below one are higher than of a floor beyond one. The payoffs are lowest in compare to the pure stock, OBPI and CPPI. But TIPP could always ensure a maturity capital guarantee with no extra.

A capital guarantee contract with a put option issued by the portfolio management to the individual restricts the investors to maximize their utilities. Taking the inability to short this saving and other institutional restrictions into account the individual may actually face a restriction on the feasible set of portfolio choices, hence be better off without such guarantees. The only possible response to a more binding constraint is to switch from risky

investments into risk-free positions in the bond market. But in cases where the institutional saving constitutes a major part of the savings of individuals, and where this saving in part or in full may be required by law or somehow have a mandatory character, this can actually be a binding constraint on the overall asset allocation problem⁷³. A put option involves a short position in the underlying asset, the effect of a capital guarantee is to limit the investment in the otherwise optimal risky portfolio. Since OBPI and CPPI could not ensure the capital guarantee with different risky sources, the only solution of this model uncertainty is to invest the total portfolio in riskless asset. In a delegated portfolio the manager would maximize his total income inclusive of management fee by investing only in riskless assets and bonds. In an optimal portfolio he invests all in bonds.

There is a conflict between investors and managers because investors want to have a higher rate of return in addition to the guaranteed rate of return, while managers, on the other hand, wish to minimize the volatility to ensure the capital guarantee and to maximize his management fees. As a result investors get only a lower rate of return at maturity by paying premium and management fees to the manager. The manager harvests the total management fees by investing in riskless assets.

⁷³ See Jensen and Sorensen (2000)

7 Abstract

The reasons why demand for capital guarantees rises are: the risk of losing substantial amounts of money in the short and long run, low interest rates since 2000, and aging of the population. Section two will cover the four types of capital guarantees, maturity, multi-period, rate of return and benefit capital guarantees. Section three describes guarantee funds and certificates to provide a capital guarantee. The hedging strategies for providing capital guarantees refer to portfolio insurances in section four which the shortfall expectation and guarantee costs using Monte Carlo Simulation will also be considered. Effects of payoffs and shortfall expectations of hedging strategies, guarantees costs, management fees and the risk shifting problem of capital guarantees will be the topics of section five.

Die Gründe für den Anstieg an der Nachfrage der Kapitalgarantien begründen sich im Risiko kurz- und mittelfristig investiertes Geld zu verlieren, den niedrigen Zinsen seit 2000 und den Altern der Bevölkerung. Der Abschnitt Zwei wird vier Arten der Kapitalgarantie, Kapitalgarantie an der Fälligkeit, multi-periodische Kapitalgarantie, die Renditekapitalgarantie und Benefizkapitalgarantie abdecken. Der Abschnitt Drei beschreibt Garantiefonds und Garantiezertifikate um eine Kapitalgarantie anzubieten. Die Wertabsicherungsstrategien beziehen sich auf die Portfolioversicherungen in dem Abschnitt Vier, wobei der erwarteter Ausfall und die Garantiekosten mit der Monte Carlo Simulation auch berücksichtigt werden. Die Auswirkungen der Auszahlungen und des erwarteten Ausfällen von den Wertabsicherungsstrategien, der Kapitalkosten, der Verwaltungsgebühren und der Verschiebung des Risikos sind die Themen des Abschnitt Fünf.

8 References

Clarke Arnott. Patterns of Reward for Portfolio Insurance. Portfolio Insurance: A guide of dynamic strategies, edited by Don Luskin. 128-137, 1988.

Axer Partnerschaft 03.04.2006.

Philippe Bertrand, Jean-luc Prigent. Portfolio Insurance Strategies: OPBI versus CPPI. University of CERGY Working paper No. 2001-30, 12.2001.

M. J. Brennan, E. S. Schwartz. The Pricing of Equity-linked Life Insurance Policies with an Asset Value Guarantee. Journal of Financial Economics 3, 195-213, 1976.

M. J. Brennan and E. S. Schwartz. Time-Invariant Portfolio Insurance Strategies. The Journal of Finance 43 (2), 283-299, 6.1988.

M. J. Brennan and R. Solanki. Optimal portfolio insurance. The Journal of Financial and Quantitative Analysis 16(3), 279-300, 9.1981.

A. Consiglio, F. Cocco, and S.A. Zenios. Asset and liability modeling for participating policies with guarantees. Journal of Risk and Insurance, 2002.

Wolfgang Ellmaier, Arno Engel. Pensionsvorsorge & Vermögensaufbau : [Leitfaden zur privaten Vorsorge ; Tipps zu allen Veranlagungsformen], Verl. Österreich, 2003.

Tony Estep, Mark Kritzman. TIPP: Insurance without complexity. The Journal of Portfolio Management 38-42, Summer 1988.

Edwin O. Fischer 2003. Die prämienbegünstigte Zukunftsvorsorge in Österreich. Zeitschrift für die gesamte Versicherungswissenschaft 93, 161-184 2003.

Klaus P. Fischer. A Discrete Martingale Model of Pension Fund Guarantees in Colombia: Pricing and market effects. CREFA Working Paper No 98-02, 2.1998.

Hendrik Garz, Stefan Günther, Cyrus Morabadi. Portfolio-Management: Theorie und Anwendung. Frankfurt am Main: Bankakad.-Verl., 2002.

Anders Grosena, Peter Løchte Jørgensen. Fair valuation of life insurance liabilities: The impact of interest rate guarantees, surrender options, and bonus policies. *Mathematics and Economics* 26, 37–57, 2000.

Michael Halling. Lecture notes of Value Based Bank Management. 2006.

M. Hansen and K.R. Miltersen. Minimum Rate of Return Guarantees: The Danish Case. *Scandinavian Actuarial Journal* 4, 280-318, 2002.

Bjarne Astrup Jensen, Carsten Sørensen. Paying for minimum interest rate guarantees: Who should compensate who? CIES Working Papers No. 2000-1, 1.2000.

Bär Jürgen. Optionsbewertung und Absicherungsstrategien. Lang. 1999.

Konsument 1/2003, 12/2004.

Andrea Laurent. Derivatives and the asset allocation decision. St. Gallen, Univ., Diss., 2003.

Lechance, Mitchell 2002. Understanding individual account guarantees, NBER working paper series, 2002.

Hayne E. Leland. Who should buy portfolio insurance? *Journal of Finance* 35, 581-594, 1980.

Hayne E. Leland and Mark Rubinstein. The Evolution of Portfolio Insurance. *Portfolio insurance: a guide to dynamic hedging*, edited by Don Luskin, 3-10, 1988.

Jun Liu, Youchang Wu. Concave Incentives and Delegated Portfolio Choice. Working Paper, 8.2005.

Raimond Mauer, Christian Schlag. Money-Back Guarantees in Individual Pension Accounts: Evidence from the German Pension Reform. to be published in „The Pension Challenge: Risk Transfers and Retirement Income Security”, Eds. Olivia S., 6.2002.

Bernhard Nietert. Model Uncertainty and Portfolio Insurance. OR Spectrum 25 (3), 295-316, 8.2003.

George G. Pennacchi. The value of guarantees on pension fund returns. The Journal of Risk and Insurance 66 (2), 219-237, 6.1999.

Kent Smetters. Controlling the cost of minimum benefit guarantees in public pension conversions. NBER Working paper Series 8732, 2002.

Manfred Steiner, Christoph Bruns. Wertpapiermanagement. Stuttgart: Schäffer-Poeschel, 1995.

Urbatsch, Nagler. Technische Wertpapieranalyse - Grundlagen, Technische Wertpapieranalyse, Stop-Loss-Orders –Teil 3. Fachbereich Wirtschaftswissenschaften, Hochschule Mittweida (FH) · University of Applied Sciences, 2001.

Wayne H Wagner. The many dimensions of risk. The journal of portfolio management, Winter 1988.

<http://www.in-the-money.com/presentation/sld>

http://xbcapital.com/research/alt_ubs_strat.pdf