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Chapter 1

Introduction

In many markets the firms' pricing decisions cannot be predicted. That is, firms randomly switch between low and high prices. Such behavior typically arises when firms have some pricing power over some consumers, but compete with other firms for other consumers. That is, in the equilibrium there is a tradeoff for firms between low prices, which increase the expected sales, and high prices, which increase the profit per expected sale. Hence, some prices can yield the same expected profits and consequently more prices can be optimal.

The pricing power of firms can have several origins. In this work, we want to concentrate on two important sources: information incompleteness of consumers and advertising.

In most consumer markets, we find a large number of competing retailers. Potential customers generally do not know the prices of firms in advance and have to visit the stores to collect prices. However, visiting all firms can be extremely costly. Therefore, (at least some) consumers may prefer to remain incompletely informed if these costs are substantially large. Hence, the prices observed by the consumers generally differ among them. Consequently, there are consumers who have only observed large prices and some who have observed lower prices. Moreover, if the consumers have different costs of acquiring the price information, the number of observed prices differs among them. Therefore, firms cannot exploit the consumers who have only observed large prices by charging a large price without risking to lose those who have observed lower prices and thus random pricing can occur. In Chapter 2 we analyze a model in which consumers are ex-ante uninformed about the prices set by the firms. Acquiring the price information is costly for consumers. In contrast to the existing literature, we allow firms to offer goods of different quality and to be different in costs. We analyze an equilibrium in which both firms have positive profits and use set prices randomly. Moreover, we want to clarify if this

equilibrium can also exist if one firm has a (cost) advantage. That is, we will find condition for which it is optimal for the firm with the advantage to charge prices so high that the competitor is not driven out of the market. Furthermore, there is the possibility that a high quality good is offered for a lower price than a low quality good. We find conditions for this to be possible.

Firms spend a lot of their budgets on advertising. This action increases the revenues of the firms by either increasing the (expected) sales or the price of the sales (or both). Through the spread of the internet and through the evolution of the so called gatekeepers, it has become easier (and cheaper) for firms to offer their products to a larger number of potential consumers. That is, gatekeepers create virtual market places in one country or in a group of countries. Firms can pay a fee to the gatekeeper to advertise their product and its price on the website operated by gatekeepers (or in other words, they pay an entry fee to sell their product on a virtual market place). More precisely, firms advertise to sell to more consumers. The potential customers who use the website have access to price lists and purchase the good which gives them the highest utility. Therefore, there is price competition for the consumers on the virtual market place. However, not all customers use the gatekeeper's website. Thus, they buy the good in local (physical) stores. Since the number of firms is limited in the local market or the costs of visiting a store is large, the firms have some pricing power over the non-users. Consequently, random pricing can occur. In Chapter 3, we analyze two local markets with one firm in each local market. However, the firms can compete for a fraction of consumers through advertising their prices on a gatekeeper's website. The existing literature has ignored that the firms which offer their products on these website cannot be treated as symmetric, since there are a lot of different firms selling on the virtual market places, e.g. large retailers chains and small shops with few employees. This raises the further questions. If one firm has a disadvantage, the gatekeeper might be able to eliminate that firm from the virtual market by charging high fees. This eliminates price competition too. Moreover, in our model, one good is socially preferred and the gatekeeper's presence can increase the consumption of this good and thus increase welfare. To analyze this, we establish an equilibrium in which firms are allowed to be asymmetric. We analyze the reaction of the agents on changes in a measure for asymmetry in the market. Moreover, we compare the expected profits of the gatekeeper and the welfare for different measures of asymmetry.

Another aim of advertising is to change the valuations of the receivers. More precisely, a firm designs its advertising in such a way that the receivers' willingness to pay for the firm's good increases relative to the goods of the competitors. In other words, firms advertise to gain pricing power. However, not all consumers receive the same advertisements or are equally affected by each

advertisement they receive. Consequently, consumers are heterogeneous in valuations after having received the advertisements. Thus, firms may not exploit their pricing power but use random pricing instead. In Chapter 4, we set up a model in which firms advertise to increase consumers' valuation for their goods. Moreover, firms are assumed to be price setters. We will establish and compare equilibria in four different scenarios. Two of those scenarios are extreme cases. Firms choose both strategic dimensions, their advertising intensities and their prices, noncooperatively or choose both actions to maximize industry profits. In the other two scenarios firms behave semicollusive. That is, they either choose their advertising intensities cooperatively and compete in prices or they set prices cooperatively and choose their advertising intensities noncooperatively. First, we analyze the model when advertising has no spillover effects. That is, we compare the equilibrium strategies, the expected profits and the welfare on the equilibrium paths. In a second step, we generalize the model by introducing spillovers of advertising. That is, a receiver of an advertisement is likely to change her willingness to pay not for the advertising firm's good only, but for all goods in the market. We analyze the effect of the spillovers on strategies, profits and welfare. More precisely, we examine whether cooperation is always welfare deterring or whether there exist ranges of spillovers in which semicollusion should be enforced by antitrust authorities.

All proofs can be found in the appendices of the chapters.

Chapter 2

Vertically Differentiated Goods and Sequential Consumer Search

2.1 Introduction

Most of the consumer search literature is concentrated on search for homogenous goods. They find equilibria in which firms randomize their prices over an interval in order that uninformed consumers and competing firms cannot predict it. Consequently, the analyzed markets exhibit price dispersion for a homogenous goods (see for example Stahl (1989)).

In the majority of these studies, markets are analyzed in which there exists a homogenous good of a known quality and the consumers search for prices. However, in many markets there are a lot of horizontally or vertically differentiated products. One only has to consider markets for electronic gadgets. Here we find a large number of different designs for a certain products as well as we find products, which differ in the number of functions, in their power, velocity, etc. When consumers start to search for products, which satisfy certain desires, they might not be sure which quality they eventually will buy. Therefore, incorporating differentiated products into consumer search theory can shed light on features which cannot be analyzed by models assuming a homogenous product.

A motivating observation can easily be made by studying price offers for vertically differentiated goods on websites which compare prices, e.g. www.idealo.co.uk, www.geizhals.de, etc. On these websites retailers quote their prices and users can costlessly access these price lists. It is not difficult to find products which price ranges overlap even if they are vertically differentiated, e.g. processors.

Moreover, one can actually find vertically differentiated processors of the same manufacturer, which have overlapping price supports.¹ If we assume that all consumers prefer later products, we can immediately conclude that it is strictly dominated for a consumer to buy the older good at the highest posted price, if the supports are overlapping and there is perfect information. Hence, setting this price can only be optimal if a firm offers the good to uninformed consumer, too. Therefore, incomplete information is a necessary condition for overlapping price distribution to be present.

Most studies in the consumer search literature concentrate on price search. That is, the consumer knows the quality the different shops offer. They are only uncertain about the price quoted by the firms. In this paper we want to adopt this assumption. Hence, we assume that the customers can distinguish the firms by their quality. Motivating this, one should consider, that the consumers have previous experience in the market. That is, they know that a certain firm always offers higher quality products and the other one offers low-quality products. It is much more obvious if some firms in the market are reputed to be discounters offering no-name products and others are reputed to be traditional retailers offering brands (assuming that the firms producing the brands are more experienced in making certain products and can therefore be expected to offer a higher quality).

The interaction of competing substitutes raises new questions to consumer search theory. Is there an equilibrium where all substitutes make positive sales, even if consumers have homogenous preferences and one good has a relative advantage (i.e. one firm is more profitable and hence the good is the socially desirable good)? How does the pricing strategy of the firms in this equilibrium look like? Does there exist an equilibrium in mixed pricing strategies? Is it possible that a substitute which is preferable over all other goods is offered for a lower price than its competing goods? That is, are the supports of the pricing strategies overlapping?

In an empirical study, Wildenbeest (2009) derives a pricing rule for firms offering different vertically differentiated goods to consumers who are involved in simultaneous search. He shows that firms with fixed qualities randomize their prices over different supports if consumers know which firm offers which quality. A necessary condition for the firms to have positive profits is that the average prices reflect the product difference. However, a shortcoming of Wildenbeest's model (2009) is that the maximum attainable profits per unit sold is constant

¹Take for example shops offering Intel processors on the price-search engine www.idealo.de on the 18. August 2009. The Intel Core 2 Quad Q9400 Tray's was offered in a price range was from 153.95 Euro to 202.94 Euro. However, the better Intel Core 2 Quad 9550 Tray's price range is from 185.86 Euro to 287.08 Euro. Hence, the prices ranges overlap in the interval 185.86 Euro and 202.94 Euro.

over firms, i.e. there are no relative advantages. This gives rise to the existence of a symmetric equilibrium in utility levels. Furthermore, Wildenbeest (2009) depicts the results of a simulation in which the price distributions for different qualities overlap. However, this is only a byproduct of the paper and not analyzed. In contrast to Wildenbeest (2009), our model allows for different profitability of the firms and it offers an intuitive explanations for the overlapping supports.

In an earlier empirical paper, Hortag su and Syverson (2004) derive optimal strategies for firms offering vertically differentiated products to homogenous consumers who search sequentially. In contrast to Wildenbeest (2009) and our model, the consumers do not know which firm offers which quality and the optimal pricing rules of the firms are in pure strategies.

In another study, Anderson and Renault (1999) examine a horizontally differentiated market with sequential search. They found that prices decrease with the degree of product differentiation. The reason for this is, that the consumers intensify search as the product's difference becomes larger if the heterogeneity of tastes of the consumers is large enough. Consequently, the competition between firms increases. However, we do not model how the firms choose the degree of differentiation. In the equilibrium of our model the firms set prices such that the visiting consumers buy the product and do not continue search. Hence, search does not intensify with the degree of differentiation, which is basically a result of homogeneity of the consumers utility function.

We set up a model, in which homogenous consumers search sequentially for prices of two vertically differentiated products. Each of the two goods is offered by one firm. We assume that the consumers know the quality offered by each shop ex-ante. First, the two firms simultaneously set their prices. Then, the consumers choose which shop they visit first. After the consumers have visited the shop and have observed the price of the product, the individual decides whether to continue search. First, we refer to a well-known benchmark case in which all customers are perfectly informed about the prices charged. Hence, the firms are involved in Bertrand-competition. We will show, that there exist pure-strategy equilibria. Furthermore, at most one firm can make positive profits. If consumers are incompletely informed, we show the existence of a particular equilibrium in mixed-strategies. In this equilibrium, the firms draw prices randomly from an interval according to a stochastic rule. In this equilibrium, each firm attracts a fraction of the non-shoppers. This equilibrium can exist even if one firm has a large advantage. A necessary condition is that search costs of the consumers are sufficiently large, given that consumers have positive expected payoffs. Interestingly, in this mixed-strategy equilibrium the price distributions overlap if search costs are sufficiently large and if the

products are not too differentiated. Furthermore, the pure-strategy equilibrium persists under incomplete information if the search costs are sufficiently small and the fraction of informed consumers as well as the product differentiation is sufficiently large.

The remainder of this chapter is as follows: Section 2 sets up the model. Section 3 analyzes a reference case with complete information. Section 4 is the heart of the paper and analyzes the sequential search model. We establish the existence of a mixed-strategy equilibrium. Furthermore, we show that such an equilibrium can arise even if the relative advantage is large. Moreover, we find condition under which the supports of the pricing strategies overlap. Ultimately, we establish an alternative equilibrium which exists, even if the other equilibrium fails to exist. Section 5 summarizes the results and concludes. The proofs can be found in Appendix A.

2.2 The Model

Assume two firms $i = H, L$ and a unit mass of risk-neutral consumers with unit demand. Let firm H to have constant marginal costs, c_H , and firm L to have constant marginal costs, c_L , with $c_L \leq c_H$. The firms are assumed to set prices simultaneously, $p_i \in [c_i, \infty)$, such that they maximize their expected profits. Let $F_i(\cdot)$ denote the c.d.f. of the price distribution for the firm offering good i . Moreover, we denote the lower and upper bound by $\underline{p}_i$ and $\bar{p}_i$, respectively.

Consumers are assumed to buy one good in the market or leave the market without consuming. Moreover, they are homogenous in preferences and prefer good H over good L . Thus, we have vertically differentiation in the market. Hence, the utility of consuming good i is given by

$$U(i) = V_i - p_i, \quad (2.1)$$

whereas V_i denotes the utility gained by consuming good i with $V_L \leq V_H$ ² and p_i is the price paid for this good. The utility of not buying anything is assumed to be zero.

Assume that the consumers are not fully informed about the firms. To get information about a firm's quotation, the consumers have to visit the firm. This search for prices is costly and the costs will be denoted by s . This can be understood as the costs of fuel or public transport to get to a shop plus the opportunity costs of time. Furthermore, as Stahl (1989), we assume that only a fraction $1 - \lambda$ of consumers, to which we will refer as non-shoppers, has search

²Since it could be the case, that $c_H = c_L$ and $V_H = V_L$, the homogenous goods case is contained in the analysis, too.

costs $s > 0$. Analogous to the literature, we assume that visiting the first shop is costless.³ However, the non-shoppers have to decide which shop to visit first. Let μ denote the fraction of non-shoppers who visit the low-quality firm first. For simplicity assume that there is perfect recall and that the non-shoppers can costlessly return to previously visited firms. The other fraction λ of consumers, to which we will refer as shoppers, like shopping or is able to use price search engines and these consumers are assumed to have no search costs.

To keep the problem interesting, we assume that firms can profitably sell to non-shoppers even if they actually search. That is, $V_H - c_H - s > 0$ and $V_L - c_L - s > 0$ hold.

2.3 Complete Information

In this section, we derive a reference case in which both prices and qualities are observable for firms and consumers. Hence under these assumption, the firms and the consumers are completely informed. Three cases can arise:

Proposition 2.1. *There exists a pure-strategy equilibrium under complete information. That is,*

(i) *if $V_H - c_H > V_L - c_L$, then $p_H = c_L + V_H - V_L$, $p_L = c_L$, $\mu = 0$ and all shoppers buy at the high-quality firm,*

(ii) *if $V_H - c_H < V_L - c_L$, then $p_L = c_H - (V_H - V_L)$, $p_H = c_H$, $\mu = 1$ and all shoppers buy at the low-quality firm,*

(iii) *if $V_H - c_H = V_L - c_L$, then $p_L = c_L$ and $p_H = c_H$ for any μ and any decision of the shoppers.*

In the complete information case, the firm which offers the best utility-price pair sells to all consumers whereas the other firm sells no units. Hence, at least one firm has an incentive to lower the price given the price is larger than the marginal costs. Consequently, at least one firm charges a price equal to its marginal costs. The results stated in Proposition 2.1 suggests that it is reasonable to have the following definition.

Definition 2.1. *One firm has a relative advantage if $V_H - c_H \neq V_L - c_L$.*

Intuitively, the maximum attainable profits per unit sold are larger for the firm with the relative advantage than those of the competitor.

³This assumption is commonly made in the consumer search literature. Assuming that the first search is costly complicates the analysis and does not give more insights.

In a related article of Singh and Vives (1984), who assume a duopoly in which the firms are involved in Bertrand competition, both firms are able to sell for prices larger than marginal costs. However, this is not possible under our assumptions, since we supposed unit demand, meaning that the costumer buys either the low-quality good or the high-quality good and not a combination of both. In Singh and Vives (1984) the customers have concave utility functions and combine the consumption of both goods, even if they are substitutes.

2.4 Incomplete Information

We consider a situation in which the prices quoted by the firms are private information and can only be observed by visiting the firms. The qualities offered by the firms are assumed to be publicly known, as well as the valuations and production costs.

The objective of this section is to establish the existence of an equilibrium in which both firms have positive expected profits. However, such an equilibrium cannot be in pure pricing strategies, because deterministic strategies make the consumers completely informed about the prices charged and hence the reference case arises. Hence, in such an equilibrium there must be necessarily at least one firm which mixes its prices such that price uncertainty prevails.

2.4.1 Which Firm Should Be Visited First?

In any equilibrium, the non-shoppers have to decide from which firm they receive their costless price quotation. If the expected payoffs of visiting one firm is larger than the expected payoffs of visiting the other firm, μ is a corner solution and all the non-shoppers have a pure strategy. However, if the non-shoppers are indifferent between visiting the high-quality firm and visiting the low-quality firm, then interior solutions, $\mu \in (0, 1)$, can arise too. If so, the consumers randomize their actions. If the rule is stochastic, the expected utility of visiting the low-quality firm first necessarily equals the expected utility of visiting the high-quality firm first.

2.4.2 The Reservation Price

Let the reservation price r_L be defined as the price at which a non-shopper who has visited the low-quality firm first is indifferent between buying the low-quality good and continuing search. The individual can continue search by visiting the high-quality firm. However, the costumer buys at the high-quality firm only if the utility of buying there is larger than returning (costlessly) to the low-quality

firm and buying for price r_L . Hence, the reservation price must fulfill

$$V_L - r_L = V_H - E[p_H | V_L - r_L \leq V_H - p_H] - s. \quad (2.2)$$

Analogously, we can define r_H as the price at which an individual who has visited the high-quality firm first is indifferent between buying the high-quality good and continuing search. That is,

$$V_H - r_H = V_L - E[p_L | V_H - r_H \leq V_L - p_L] - s. \quad (2.3)$$

2.4.3 The Expected Profits

The expected profits of a firm consist of two components, namely sales to the shoppers and sales to the non-shoppers. The expected profits for the firm offering the low quality good are given by

$$\Pi_L(p, F_H(\cdot)) = (p - c_L) \cdot [\lambda(1 - F_H(p + V_H - V_L)) + \mu(1 - \lambda)] \quad (2.4)$$

if $\bar{p}_L \leq r_L$ and the expected profits of the firm offering the high quality good are given by

$$\Pi_H(p, F_L(\cdot)) = (p - c_H) \cdot [\lambda(1 - F_L(p - (V_H - V_L))) + (1 - \mu)(1 - \lambda)] \quad (2.5)$$

if $\bar{p}_H \leq r_H$. The expected profits of the low-quality firm given in expression (2.4), represents the expected profits of firm L if it sets its price equal to p , given that the pricing strategy of the high-quality firm is F_H . The first term in the squared brackets represents the expected sales to shoppers, whereas the second term represents the expected amount of sales to non-shoppers. Note, that $(1 - F_H(p + V_H - V_L))$ represents the probability that the high-quality firm sets the price higher than $p + V_H - V_L$, which implies that the low-quality firm sells to all shoppers if it sets price p . The expected profits of the high-quality firm given in expression (2.5) can be explained analogously.

2.4.4 Equilibrium

We want to establish an equilibrium in which both firms have positive expected profits.

Lemma 2.1. *In any equilibrium in which both firms have positive expected profits, $\bar{p}_L = \bar{p}_H - (V_H - V_L)$ and $\bar{p}_i = r_i$.*

This Lemma states that in the equilibrium we want to examine, the upper bounds of both supports are equal to the reservation prices. Note, that it also

implies that $r_H - r_L = V_H - V_L$.

Lemma 2.2. *In any equilibrium in which both firms sell to non-shoppers, non-shoppers will not search beyond the first firm.*

This means that all non-shopper stop after having visited the first firm.

From Lemma 2.1 the conditional expected price in (2.2) is simply given by the unconditional price expectation

$$\begin{aligned} E[p_H | V_L - r_L \leq V_H - p_H] &= E[p_H | V_L - \bar{p}_L \leq V_H - p_H] \\ &= E[p_H | V_L - \bar{p}_H + (V_H - V_L) \leq V_H - p_H] \\ &= E[p_H | p_H \leq \bar{p}_H] \\ &= E[p_H] \end{aligned}$$

Similarly, the conditional expected price in (2.3) is simply given by the unconditional price expectation

$$E[p_L | V_H - r_H \leq V_L - p_L] = E[p_L].$$

In any equilibrium it is crucial whether

$$E[V_L - p_L] \stackrel{\geq}{\leq} E[V_H - p_H], \quad (2.6)$$

because this determines from which shop the non-shoppers receive their costless price quotation. The following Lemma rules out inequality in (2.6).

Lemma 2.3. *In any equilibrium in which both firms have positive expected profits, no firm attracts all non-shoppers.*

Thus, we can draw our attention to equilibria in which both shops are equally attractive to the consumers ex-ante. That is,

$$E[V_L - p_L] = E[V_H - p_H] \Leftrightarrow E[p_H] - E[p_L] = V_H - V_L \quad (2.7)$$

holds. Hence, the equilibrium which we want to analyze necessarily has the property that the expected additional expenditures of consuming the high-quality good equals the utility gain of consuming the high-quality good instead of the low-quality good.

In the equilibrium we are going to analyze, all consumers visit at least one firm, since visiting the first firm is costless. Furthermore, non-shoppers will buy at the first firm visited, by Lemma 2.1, whereas the shoppers buy at the firm which offers the highest utility.

To ensure that both firms have positive expected profits, it suffices to verify, that the expected profits of charging the upper bound are positive. That is

$$\mu(1-\lambda)(\bar{p}_L - c_L) > 0 \text{ and} \quad (2.8)$$

$$(1-\mu)(1-\lambda)(\bar{p}_H - c_H) > 0. \quad (2.9)$$

To keep the notation simple, let $\Phi_L(\lambda, \mu)$ and $\Phi_H(\lambda, \mu)$ be defined as

$$0 \leq \Phi_L(\lambda, \mu) \equiv \int_0^1 \frac{1-x}{(1-x) + \frac{1-\lambda}{\lambda}\mu} dx \leq 1 \text{ and} \quad (2.10)$$

$$0 \leq \Phi_H(\lambda, \mu) \equiv \int_0^1 \frac{1-x}{(1-x) + (1-\mu)\frac{1-\lambda}{\lambda}} dx \leq 1. \quad (2.11)$$

The expression $\Phi_i(\lambda, \mu)$ can be seen as an inverse measure of firm i 's market power, which arises from the informational incompleteness. It decreases in the fraction of non-shoppers and in the fraction of non-shoppers who visit firm i first.

We can establish the following equilibrium.

Proposition 2.2.

1. If

$$-\frac{1 - \Phi_L(\lambda, 1)}{\Phi_L(\lambda, 1)} < \frac{V_H - V_L - (c_H - c_L)}{s} < \frac{1 - \Phi_H(\lambda, 0)}{\Phi_H(\lambda, 0)}$$

then μ has an interior solution given by

$$\frac{1 - \Phi_H(\lambda, \mu)}{\Phi_H(\lambda, \mu)} - \frac{1 - \Phi_L(\lambda, \mu)}{\Phi_L(\lambda, \mu)} = \frac{V_H - V_L - (c_H - c_L)}{s} \quad (2.12)$$

and both firm's expected profits are strictly positive.

2. If μ has an interior solution, μ^* , then a mixed-strategy equilibrium exists, in which the atomless price distribution of the low-quality firm is given by

$$F_L(p_L) = 1 - \left(\frac{\frac{s}{\Phi_H(\lambda, \mu^*)} - p_L - (V_H - V_L) + c_H}{p_L + V_H - V_L - c_H} \frac{(1 - \mu^*)(1 - \lambda)}{\lambda} \right) \quad (2.13)$$

on the support

$$\left[\frac{s}{\Phi_H(\lambda, \mu^*)} \cdot \frac{(1 - \mu^*)\frac{1-\lambda}{\lambda}}{1 + (1 - \mu^*)\frac{1-\lambda}{\lambda}} - (V_H - V_L) + c_H, c_L + \frac{s}{\Phi_L(\lambda, \mu^*)} \right]$$

and the atomless price distribution of the high-quality firm is given by

$$F_H(p_H) = 1 - \left(\frac{\frac{s}{\Phi_L(\lambda, \mu^*)} - p_H + (V_H - V_L) + c_L}{p_H - (V_H - V_L) - c_L} \frac{\mu^*(1 - \lambda)}{\lambda} \right) \quad (2.14)$$

on the support

$$\left[\frac{s}{\Phi_L(\lambda, \mu^*)} \cdot \frac{\frac{1-\lambda}{\lambda} \mu^*}{1 + \frac{1-\lambda}{\lambda} \mu^*} + V_H - V_L + c_L, c_H + \frac{s}{\Phi_H(\lambda, \mu^*)} \right].$$

2.4.5 A Special Case: No Relative Advantages

For illustrative reasons, we first want to deal with the special case of the equilibrium described in Proposition 2.2. That is the case in which

$$V_H - c_H = V_L - c_L.$$

holds. The right-hand side of equation (2.12) is zero. Hence, we have

$$\frac{1 - \Phi_H(\lambda, \mu^*)}{\Phi_H(\lambda, \mu^*)} = \frac{1 - \Phi_L(\lambda, \mu^*)}{\Phi_L(\lambda, \mu^*)}.$$

This implies that $\Phi_H(\lambda, \mu^*) = \Phi_L(\lambda, \mu^*) \Leftrightarrow \mu = 0.5$, meaning that each firm attracts half of the non-shoppers in equilibrium.

Since $V_H - V_L - c_H = -c_L$ and $\Phi_H(\lambda, \frac{1}{2}) = \Phi_L(\lambda, \frac{1}{2})$ the equilibrium distribution of the low-quality firm simplifies to

$$F_L(p_L) = 1 - \left(\frac{c_L + \frac{s}{\Phi_L(\lambda, \frac{1}{2})} - p_L}{2(p_L - c_L)} \frac{(1 - \lambda)}{\lambda} \right)$$

and since $V_H - V_L + c_L = c_H$, the distribution of the high-quality firm simplifies to

$$F_H(p_H) = 1 - \left(\frac{c_H + \frac{s}{\Phi_H(\lambda, \frac{1}{2})} - p_H}{2(p_H - c_H)} \frac{(1 - \lambda)}{\lambda} \right).$$

Note that $\bar{p}_i = c_i + \frac{s}{\Phi_i(\lambda, \frac{1}{2})}$. These distribution functions are that derived by Janssen et al. (2005) (with $c_i \geq 0$ for $i = 1, 2$).

Since $\mu = \frac{1}{2}$ holds, a mixed-strategy equilibrium always exists if there is no relative advantage.

Corollary 2.1. *If no firm has a relative advantage, both firms have the same expected profits in the equilibrium established in Proposition 2.2.*

2.4.6 The Generic Case: Relative Advantages

Now, we want to consider the generic case, in which one firm has a relative advantage, that is

$$V_H - c_H \neq V_L - c_L.$$

The Role of μ and Positive Profits for Both Firms

Since μ is endogenously determined by equation (2.12), it is in general a function of $V_H - c_H - (V_L - c_L)$, s and λ . Therefore, we can write $\mu^*(V_H - c_H - (V_L - c_L), s, \lambda)$. Since the non-shoppers cannot costlessly observe the realization of prices at both shops, each of them has to decide which shop to visit first. However, in the equilibrium of Proposition 2.2, the difference of the expected prices is exactly $V_H - V_L$. This holds if the non-shoppers choose their first shop randomly following μ^* .

Actually, the role of the decision which firm to visit first is similar to the decision to participate in Janssen et al. (2005). In the equilibrium, the firms set prices such that the consumer is indifferent between participating and staying out of the market. However, in this model the consumer has the possibility to participate in two different markets. The non-shoppers use a strategy μ such that the expected payoffs in both markets are equal ex-ante.

In the equilibrium in which $E[p_H] - E[p_L] = V_H - V_L$ holds, it follows that $r_H - r_L = V_H - V_L$ (by Lemma 2.1 and the equilibrium equations for the reservation prices). The reservation prices are much easier to analyze than the expected prices. It can be seen that r_L increases in μ whereas r_H decreases in μ .

μ is decreasing in its first argument. This means, that in the equilibrium of Proposition 2.2, the larger the relative advantage of the high-quality firm, the larger the fraction of non-shoppers, which visits the high-quality firm. To keep $r_H - r_L$ and therefore $E[p_H] - E[p_L]$ on level $V_H - V_L$, μ has to increase. Intuitively, the high-quality firm needs an incentive to set higher prices. Hence, in equilibrium a larger fraction of non-shoppers visits the high-quality firm first. We can verify, that if the high-quality firm has a relative advantage, $V_H - c_H - (V_L - c_L) > 0$, the right-hand side of the equation is positive, implying that $\frac{1}{\Phi_H(\lambda, \mu)} > \frac{1}{\Phi_L(\lambda, \mu)} \Leftrightarrow \Phi_H < \Phi_L \Leftrightarrow \mu < \frac{1}{2}$. This means that the larger fraction of non-shoppers visits the high-quality firm first. On the other hand if the low-quality firm has a relative advantage, meaning $V_H - c_H - (V_L - c_L) < 0$, then $\mu > 0.5$. Thus, the larger fraction of non-shoppers visits to the low-quality firm first.

The search costs have exactly the opposite impact on μ as the relative advantage. Intuitively, in the equilibrium an increase in the search costs is equivalent

to an increase of profits per non-shopper. Hence with larger search costs, a smaller fraction of non-shoppers is needed to offset the incentive of the firm with the relative advantage to lower its prices. Therefore, increasing search costs drive μ towards $\frac{1}{2}$. (This can easily be seen by the reservation prices, which both depend positively on s , whereas the firm reservation price increases stronger for the firm with the relative advantage.) Positive search costs are a necessary condition for the mixed-strategy equilibrium to exist. Otherwise $[V_H - V_L - (c_H - c_L)]/s$ can never be in the bounded interval given in (1.) of Proposition 2.2.

An increase in λ means there are fewer non-shoppers present, and therefore a larger fraction of the non-shoppers is needed to offset the incentive of the firm with the relative advantage to lower its price (both reservation prices are decreasing in λ , however, it decreases faster for the firm with the relative advantage). Hence, μ approaches the closest corner solution as λ increases (which is one if the low-quality firm has a relative advantage and zero otherwise).

Consequently, since μ must have an interior solution such that both firms have positive expected profits, the equilibrium in Proposition 2.2 fails for large fractions of shoppers, λ , small search costs, s , and large relative advantages.

Relative Advantages, Search Costs and the Mixed-Strategy Equilibrium

Now we define $\underline{s}$ as the lower bound of the open interval of search costs, for which a mixed-strategy equilibrium exists.

Proposition 2.3. *The function $\underline{s}$ is given by*

$$\underline{s}(V_H - V_L - c_H + c_L, \lambda) = \begin{cases} \frac{V_H - V_L - c_H + c_L}{\frac{1}{\Phi_H(\lambda, 0)} - 1} & \text{if } V_H - V_L - c_H + c_L > 0 \\ 0 & \text{if } V_H - V_L - c_H + c_L = 0 \\ \frac{V_H - V_L - c_H + c_L}{1 - \frac{1}{\Phi_L(\lambda, 1)}} & \text{if } V_H - V_L - c_H + c_L < 0 \end{cases} \quad (2.15)$$

and increasing in the relative advantage.

The lower bound of the open interval of search costs for which a mixed-strategy equilibrium exists is an increasing function of the relative advantage. Intuitively, if one firm's relative advantage increases, then the fraction of non-shoppers attracted by this firm must increase such that $E[p_H] - E[p_L] = V_H - V_L$ is fulfilled. On the one hand, μ approaches the corner solution if the relative advantage increases, on the other hand the rents which can be extracted from any non-shopper increases in the search costs s . Hence, larger search costs are necessary if the relative advantage increases to prevent μ from becoming a

corner solution, which would cause the equilibrium to collapse,

Furthermore, it can easily be verified, that we can find search costs large enough for any relative advantage such that both firms have positive expected profits.

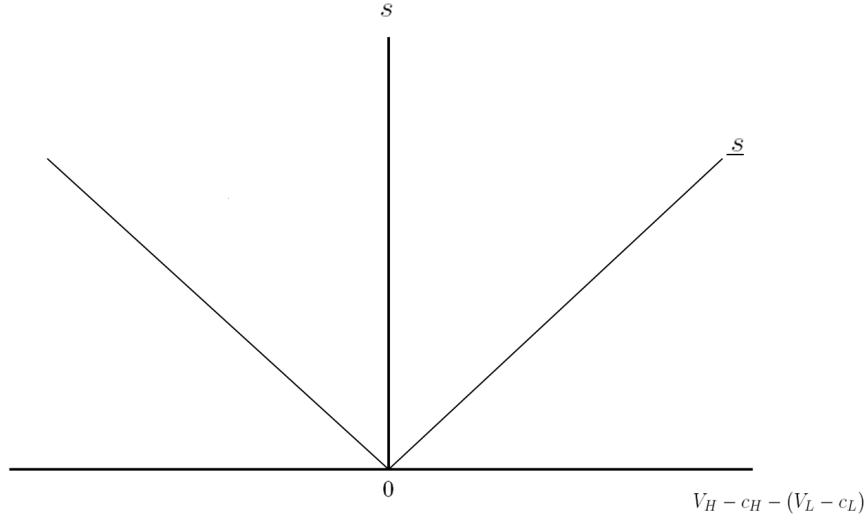


Figure 2.1: The lower bound of the open interval of search costs for which a mixed-strategy equilibrium exists $\underline{s}(V_H - V_L - c_H + c_L, \lambda)$.

In Figure 2.1 we depicted a coordinate system for all possible combinations of relative advantages and search costs. On the x-axis we find $V_H - c_H - (V_L - c_L)$ and the lower bound for which a mixed-strategy equilibrium exists is on the y-axis. Hence, on the right-hand side to the origin, the high-quality firm has a relative advantage, whereas on the left-hand side of the origin, the low-quality firm has a relative advantage. Farther from the origin means, that the relative advantage increases. All combinations strictly above the function $\underline{s}$ are combinations of relative advantages and search costs for which a mixed strategy equilibrium exists. For all the combinations below the function, the firm with the relative advantage charges prices so low, such that the competitor would have negative expected profits and hence the mixed-strategy equilibrium breaks down.

The combinations on the x-axis represent full-information ($s = 0$). For all this coordinates the equilibrium fails to exist. However, if s is large enough the equilibrium exists.

Furthermore, for all combinations on the y-axis no firm has a relative advantages. The mixed-strategy equilibrium exists for any $s > 0$. These equi-

libria are exactly those analyzed in the typically consumer search models with homogenous goods and symmetric firms. However, the graph displays that the mixed-strategy equilibrium persists even if the firms are asymmetric in the sense of relative advantages.

Corollary 2.2. *If any firm has a relative advantage, then the firm with the relative advantage has larger expected profits in the equilibrium described in Proposition 2.2.*

2.4.7 Overlapping Price Distributions

Proposition 2.4. *In the mixed-strategy equilibrium, the supports of the pricing distributions overlap if s is large or $V_H - V_L$ is sufficiently small. For homogenous goods the supports always overlap.*

This Proposition states that the pricing distributions in the mixed-strategy equilibrium overlap if the qualities of the goods are sufficiently similar and if search costs are sufficiently large. High search costs lead the firms to charge high prices (large upper bounds), however if the products difference is not too large, the expected prices do not differ much as we know from condition (2.7). This means that under these circumstances there is a positive probability that the low-quality product is sold for a higher price than the high-quality product.

Note, that if the goods are homogenous, $V_H = V_L$, the supports of the price distribution always overlap in the mixed-strategy equilibrium.⁴ This is the typical result we know for example from Stahl (1989).

2.4.8 Other Equilibria

First we note, that the equilibrium strategies in the complete information case do not necessarily form an equilibrium under incomplete information.

Under complete information there can exist an equilibrium in which both firms have zero profits. That is, both firms charge prices equal to their marginal costs and at least one of them sells a positive amount. The next proposition states that this equilibrium (without a relative advantage) does not persist under incomplete information for every λ and every $s > 0$.

Proposition 2.5. *In any equilibrium, at least one firm has positive expected profits.*

The reason is, that a firm which attracts non-shoppers has an incentive to increase the price, since this fraction of consumers do not observe the deviation

⁴However, the mixed-strategy equilibrium might not exist in the case if $c_H - c_L$ is too large.

and cannot costlessly switch to the other firm. Hence, the deviating firm has strictly positive profits.

However, also the other equilibria may fail to exist. A complete information equilibrium vanishes if additional profits from selling to non-shoppers can compensate for the loss of the shoppers. To be more precise, the firm which attracts non-shoppers increases the price by s . This makes the non-shoppers indifferent between continuing search and buying immediately. The additional profits of the deviating firm i are $(1 - \lambda)s$, whereas it loses $\lambda(c_{j \neq i} + V_i - V_{j \neq i} - c_i)$. Therefore, we have the next result.

Proposition 2.6. *If firm i has a relative advantage and*

$$\frac{1 - \lambda}{\lambda} < \frac{V_i - V_{j \neq i} - c_i + c_{j \neq i}}{s} \quad (2.16)$$

holds, the pure-strategy (complete-information) equilibrium is an equilibrium.

Hence, large search costs, a small fraction of informed consumers, λ , and a small relative advantage, causes the equilibrium to collapse.⁵

This raises the question, which kind of equilibria exists if the complete information equilibria does not exist.

The following proposition states that the only equilibria which might exist are those, in which both firms have positive expected profits.

Proposition 2.7. *If none of the complete information equilibria exist, there does not exist an equilibrium in which exactly one firm has positive expected profits.*

Hence, the only alternative equilibria which can exist if the complete information equilibrium fails are those each shop has positive expected profits and hence the consumers have ex-ante the same expected utility at each shop (by Lemma 2.3). Moreover, if the search costs are low and the fraction of shoppers are large, the complete information equilibrium can persist.

2.5 Conclusion

We have set up a model in which two firms compete in prices for consumers. One of the firms offers a low-quality product and the other one offers a high-quality product. The quality offered by the firms is commonly known and the firms can be distinguished by their quality.

In the situation in which all consumers are perfectly informed about the prices charged, we have established pure-strategy equilibria in which at least

⁵A proof with symmetric firms is presented in Nermuth (1982), pp. 85. With asymmetric firms, the idea is similar.

one firm has zero profits. Only the firm with the relative advantage is able to have positive profits.

However, in the interesting case, a fraction of the consumers (the non-shoppers) is involved in costly search. This means that this fraction does not know the realization of prices ex-ante. We have established the existence of a mixed-strategy equilibrium in which firms set prices according to a distribution. Moreover, each shop sells to a fraction of non-shoppers. Hence, both firms have positive profits. Such an equilibrium can exist even if relative advantages are large.

A necessary condition for the equilibrium to exist, is that search costs are strictly positive. Furthermore, the larger the search costs the larger is the set of disadvantaged firms which can survive.

Additionally, in the mixed-strategy equilibrium the realization of the price of the low-quality product can be larger than the realization of the price of the high-quality product. The reason for this is that non-shoppers will not visit the other firm if the expected utility gain from continued search is small. That is, high search costs and a low quality difference.

Finally, we have shown that the equilibrium found under complete information also exists under incomplete information if the fraction of shoppers as well as the product difference is large and the search costs are small. However, if it does not exist, the only class of equilibria which can exist, are equilibria in which both firms have positive profits and a share of the non-shoppers.

2.A Appendix: Proofs

2.A.1 Proof of Proposition 2.1

Proof. Consider there is a relative advantage for the high-quality firm, that is $V_H - c_H - (V_L - c_L) > 0$, the equilibrium strategies $p_L = c_L$ and $p_H = c_L + V_H - V_L$ and all consumers visit the high-quality firm. The low-quality firm has no incentive to deviate to higher prices, since it would not attract consumers. If the high-quality firm would deviate to higher prices, it would lose all consumers, since $V_L - c_L > V_H - p_H$ for $p_H > c_L + V_H - V_L$. Obviously, the high-quality firm has no incentive to lower its price, since it cannot attract additional consumers.

The same arguments hold if $V_H - c_H - (V_L - c_L) < 0$.

Consider now $V_H - c_H = V_L - c_L$ and the equilibrium strategies $p_L = c_L$ and $p_H = c_H$. Charging larger prices lead to the loss of all consumers one firm attracts. Thus, there is no incentive to deviate. $\square$

2.A.2 Proof of Lemma 2.1

Proof. Consider now, that $\bar{p}_L < \bar{p}_H - (V_H - V_L)$. However, then prices in the interval $(\bar{p}_L + V_H - V_L, \bar{p}_H)$ are not optimal for the high-quality firm, since increasing them does not lead to a loss of consumers. Hence, $F_H(\cdot)$ is constant on the interval $(\bar{p}_L + V_H - V_L, p_H)$. However, increasing $\bar{p}_L$ yields more profits per sold unit without losing expected consumers. Hence $\bar{p}_L < \bar{p}_H - (V_H - V_L)$ cannot be optimal for the low-quality firm.

Showing that $\bar{p}_L > \bar{p}_H - (V_H - V_L)$ cannot hold is analogous.

However, if the high-quality firm charges the upper-bound $\bar{p}_H$, the probability of attracting shoppers is $1 - F_L(\bar{p}_L) = 0$. Hence, as long as $\bar{p}_H < r_H$ the firm can increase its upper bound without losing consumers. $\square$

2.A.3 Proof of Lemma 2.2

Proof. From Lemma 2.1 we know that $\bar{p}_H = \bar{p}_L + V_H - V_L$. We want to show now that $\bar{p}_H > r_H$ cannot hold. If this would be the case, then the high-quality firm does not attract any consumers by charging the upper bound. The reason for this is that it attracts no shoppers, $1 - F_L(\bar{p}_L) = 0$. However, also the non-shoppers would leave and return with probability $1 - F_L(\bar{p}_L) = 0$. Thus, charging $\bar{p}_H > r_H$ yields zero profits.

It is analogous to show that $\bar{p}_L > r_L$ cannot hold. $\square$

2.A.4 Proof of Lemma 2.3

Proof. Suppose that all non-shoppers visit the high-quality firm first. Then the low-quality firm must attract shoppers with positive probability to have positive expected profits. By Lemma 2.1, $\bar{p}_L = \bar{p}_H - (V_H - V_L)$ must hold. However, then the low-quality firm attracts no shoppers by charging the upper bound, since $1 - F_L(\bar{p}_L) = 0$. Thus, there cannot be an equilibrium in which one firm attracts all non-shoppers and both firms have positive profits.

The case in which the low-quality firm attracts all non-shoppers is analogous. $\square$

2.A.5 Proof of Proposition 2.2

Proof. First, we want to show the second point of the proposition. Therefore, we suppose that $\mu \in (0, 1)$.

In any equilibrium, $F_L(p_L)$ and $F_H(p_H)$ must satisfy

$$\Pi_L(p, F_H(\cdot)) = \Pi_L(\bar{p}, F_H(\bar{p}_L + V_H - V_L))$$

for all prices p of the low quality firm which are optimal. Moreover, we must have

$$\Pi_H(p, F_L(\cdot)) = \Pi_H(\bar{p}_H, F_L(\bar{p}_H - (V_H - V_L)))$$

for all optimal prices p of the high quality firm.

By Lemma 2.1 $\bar{p}_i = r_i$ for $i = H, L$ and $\bar{p}_H = \bar{p}_L + (V_H - V_L)$. Since the distributions are assumed to have no mass points, a firm charging price $\bar{p}_i$ only sells to the non-shoppers. If the firm uses a mixed pricing strategy, any price charged in equilibrium must yield the same profits. Thus, any price charged by firm L in equilibrium fulfills

$$(p - c_L) \cdot [\lambda(1 - F_H(p + V_H - V_L)) + \mu(1 - \lambda)] = (\bar{p}_L - c_L)\mu(1 - \lambda) \quad (2A.1)$$

and any price charged by firm H fulfills

$$(p - c_H) \cdot [\lambda(1 - F_L(p - (V_H - V_L))) + (1 - \mu)(1 - \lambda)] = (\bar{p}_H - c_H)(1 - \mu)(1 - \lambda). \quad (2A.2)$$

The pricing strategies are given by the following expressions:

$$F_L(p - (V_H - V_L)) = 1 - \left(\frac{\bar{p}_H - p}{p - c_H} \frac{(1 - \mu)(1 - \lambda)}{\lambda} \right),$$

$$F_H(p + V_H - V_L) = 1 - \left(\frac{\bar{p}_L - p}{p - c_L} \frac{\mu(1 - \lambda)}{\lambda} \right).$$

Thus, the pricing strategies are given by

$$F_L(p) = 1 - \left(\frac{\bar{p}_H - p + V_H - V_L}{p + V_H - V_L - c_H} \frac{(1 - \mu)(1 - \lambda)}{\lambda} \right), \quad (2A.3)$$

$$F_H(p) = 1 - \left(\frac{\bar{p}_L - p + V_H - V_L}{p - (V_H - V_L) - c_L} \frac{\mu(1 - \lambda)}{\lambda} \right). \quad (2A.4)$$

The lower bounds of the distributions fulfill $F_i(\underline{p}) = 0$. It is then easy to derive the lower bound. They are given by

$$\underline{p}_L = \frac{(1 - \mu) \frac{1 - \lambda}{\lambda} \bar{p}_H + c_H}{1 + (1 - \mu) \frac{1 - \lambda}{\lambda}} - (V_H - V_L). \quad (2A.5)$$

$$\underline{p}_H = (\bar{p}_L - c_L) \frac{\frac{1 - \lambda}{\lambda} \mu}{1 + \frac{1 - \lambda}{\lambda} \mu} + V_H - V_L + c_L. \quad (2A.6)$$

Note, that the expressions for the lower bounds in (2A.5) and (2A.6) still depend on the upper bounds of the distributions. Thus, we have to find an expressions for r_H and r_L . For this purpose, we use the conditions (2.2) and (2.3) and

express the reservation prices, which are

$$\begin{aligned} r_L &= E[p_H] - (V_H - V_L) + s \text{ and} \\ r_H &= E[p_L] + (V_H - V_L) + s. \end{aligned}$$

Obviously, we first have to calculate the expectations of the prices, $E[p_i]$ for $i = H, L$. To do this, we change the variable and write $E[p_L] = \int_0^1 p_L dx$ and $E[p_H] = \int_0^1 p_H dx$. From the pricing strategies (2A.3) and (2A.4) we can express p_L and p_H and thus the expected prices are given by

$$\begin{aligned} E[p_H] &= \int_0^1 \left((\bar{p}_L - c_L) \frac{\frac{1-\lambda}{\lambda} \mu}{(1-x) + \frac{1-\lambda}{\lambda} \mu} + c_L + V_H - V_L \right) dx \\ &= \bar{p}_L \int_0^1 \frac{\frac{1-\lambda}{\lambda} \mu}{(1-x) + \frac{1-\lambda}{\lambda} \mu} dx \\ &\quad + c_L \int_0^1 \frac{1-x}{(1-x) + \frac{1-\lambda}{\lambda} \mu} dx + V_H - V_L \\ &= \bar{p}_L [1 - \Phi_L(\lambda, \mu)] + c_L \Phi_L(\lambda, \mu) + V_H - V_L \\ E[p_L] &= \left((\bar{p}_H - c_H) \frac{\frac{1-\lambda}{\lambda} \mu}{(1-x) + \frac{1-\lambda}{\lambda} \mu} + c_H - (V_H - V_L) \right) dx \\ &= \bar{p}_H \int_0^1 \frac{(1-\mu) \frac{1-\lambda}{\lambda}}{(1-x) + (1-\mu) \frac{1-\lambda}{\lambda}} dx \\ &\quad + c_H \int_0^1 \frac{1-x}{(1-x) + (1-\mu) \frac{1-\lambda}{\lambda}} dx - (V_H - V_L) \\ &= \bar{p}_H [1 - \Phi_H(\lambda, \mu)] + c_H \Phi_H(\lambda, \mu) - (V_H - V_L) \end{aligned}$$

Since $\bar{p}_i = r_i$ we obtain

$$\begin{aligned} r_L &= r_L [1 - \Phi_L(\lambda, \mu)] + c_L \Phi_L(\lambda, \mu) + s \Leftrightarrow \\ r_L &= c_L + \frac{s}{\Phi_L(\lambda, \mu)} \geq c_L + s \end{aligned} \tag{2A.7}$$

with Φ_L defined in (2.10) and

$$\begin{aligned} r_H &= r_H [1 - \Phi_H(\lambda, \mu)] + c_H \Phi_H(\lambda, \mu) + s \Leftrightarrow \\ r_H &= c_H + \frac{s}{\Phi_H(\lambda, \mu)} \geq c_H + s \end{aligned} \tag{2A.8}$$

with Φ_H defined in (2.11).

Finally, we insert the expression for the reservation prices into the lower bounds and into the pricing distributions to obtain expression which only depend on exogenous parameters.

Next, we want to verify that point one of the proposition is true. To do this we insert the upper bounds of the distribution into inequality (2.8) and (2.9) to ensure that both firms make positive profits. After some simple algebraic operations we obtain

$$\begin{aligned} s \cdot \mu(1 - \lambda) &> 0 \text{ and} \\ s \cdot (1 - \mu)(1 - \lambda) &> 0. \end{aligned}$$

These two inequalities hold if and only if μ has an interior solution, since $\lambda \in (0, 1)$ and $s > 0$. The fraction μ is endogenously determined by $E[p_H] - E[p_L] = V_H - V_L$. The difference of the expected prices is

$$\begin{aligned} E[p_H] - E[p_L] &= \bar{p}_L [1 - \Phi_L(\lambda, \mu)] + c_L \Phi_L(\lambda, \mu) + V_H - V_L \\ &\quad - \bar{p}_H [1 - \Phi_H(\lambda, \mu)] - c_H \Phi_H(\lambda, \mu) + V_H - V_L \\ &= \left(c_L + \frac{s}{\Phi_L(\lambda, \mu)} \right) [1 - \Phi_L(\lambda, \mu)] + c_L \Phi_L(\lambda, \mu) \\ &\quad - \left(c_H + \frac{s}{\Phi_H(\lambda, \mu)} \right) [1 - \Phi_H(\lambda, \mu)] - c_H \Phi_H(\lambda, \mu) \\ &\quad + 2(V_H - V_L) \\ &= c_L + \frac{s}{\Phi_L(\lambda, \mu)} [1 - \Phi_L(\lambda, \mu)] \\ &\quad - c_H - \frac{s}{\Phi_H(\lambda, \mu)} [1 - \Phi_H(\lambda, \mu)] + 2(V_H - V_L) \\ &= c_L + \frac{s}{\Phi_L(\lambda, \mu)} - c_H - \frac{s}{\Phi_H(\lambda, \mu)} + 2(V_H - V_L). \end{aligned}$$

And hence the solution for μ is implicitly given by

$$\frac{1}{\Phi_H(\lambda, \mu)} - \frac{1}{\Phi_L(\lambda, \mu)} = \frac{V_H - V_L - (c_H - c_L)}{s} \quad (2A.9)$$

Since $\Phi_H(\lambda, \mu)$ is increasing and $\Phi_L(\lambda, \mu)$ decreases in μ , the left hand side is strictly decreasing in μ for $\mu \in (0, 1)$. Therefore, μ is uniquely defined by (2A.9) as long as it is in the interval $(0, 1)$ and depends on $V_H - V_L - (c_H - c_L)$, λ and s . Hence, $\mu > 0$ if

$$\frac{1}{\Phi_H(\lambda, 0)} - \frac{1}{\Phi_L(\lambda, 0)} = \frac{1}{\Phi_H(\lambda, 0)} - 1 = \frac{1 - \Phi(\lambda, 0)}{\Phi(\lambda, 0)} > \frac{V_H - V_L - (c_H - c_L)}{s}$$

and $\mu < 1$ if

$$\frac{1}{\Phi_H(\lambda, 0)} - \frac{1}{\Phi_L(\lambda, 0)} = 1 - \frac{1}{\Phi_L(\lambda, 1)} = -\frac{1 - \Phi_L(\lambda, 1)}{\Phi_L(\lambda, 1)} < \frac{V_H - V_L - (c_H - c_L)}{s} \quad (2A.10)$$

holds. Therefore, we can conclude that point one of the proposition is true. $\square$

2.A.6 Proof of Corollary 2.1

Proof. If the expected profits are equal, the following holds:

$$\begin{aligned} \left(c_H + \frac{s}{\Phi_H(\lambda, \mu)} - c_H\right) \mu(1 - \lambda) &= \left(c_L + \frac{s}{\Phi_L(\lambda, \mu)} - c_L\right) \mu(1 - \lambda) \\ \Phi_L(\lambda, \mu) &= \Phi_H(\lambda, \mu) \end{aligned}$$

This holds only if $\mu = 0.5$. This is exactly the case when no firm has a relative advantage. $\square$

2.A.7 Proof of Proposition 2.3

Proof. The equilibrium of Proposition 2.2 exists if $\mu \in (0, 1)$.

Consider the high-quality firm has a relative advantage, that is $V_H - c_H - (V_L - c_L) > 0$. Then $\mu \in (0, 0.5)$ for an interior solution. Since μ is increasing in s , the largest value of s for which the mixed strategy equilibrium does not exist can be calculated from condition (2.12) and is given by

$$\bar{s} = \frac{V_H - c_H - (V_L - c_L)}{\frac{1}{\Phi_H(\lambda, 0)} - \frac{1}{\Phi_L(\lambda, 0)}}.$$

It is easy to see that it is an increasing function of $V_H - c_H - (V_L - c_L)$.

If $V_H - c_H - (V_L - c_L) < 0$, then $\mu \in (0.5, 1)$ for the interior solutions and μ is decreasing in s . Hence, the largest value of s for which the mixed strategy equilibrium does not exist can be calculated from condition (2.12) and is defined by

$$\bar{s} = \frac{V_H - c_H - (V_L - c_L)}{\frac{1}{\Phi_H(\lambda, 1)} - \frac{1}{\Phi_L(\lambda, 1)}}.$$

It is a decreasing function of $V_H - c_H - (V_L - c_L)$.

Hence the function

$$\bar{s}(V_H - c_H - (V_L - c_L)) = \begin{cases} \frac{V_H - c_H - (V_L - c_L)}{\frac{1}{\Phi_H(\lambda, 0)} - 1} & \text{if } V_H - c_H - (V_L - c_L) > 0 \\ 0 & \text{if } V_H - c_H - (V_L - c_L) = 0 \\ \frac{V_H - c_H - (V_L - c_L)}{1 - \frac{1}{\Phi_L(\lambda, 1)}} & \text{if } V_H - c_H - (V_L - c_L) < 0 \end{cases}$$

is increasing in the relative advantage, $|V_H - c_H - (V_L - c_L)|$, with a constant slope. Note, that the mixed strategy equilibrium does not exist for $s < \bar{s}(V_H - c_H - (V_L - c_L))$ $\square$

2.A.8 Proof of Corollary 2.2

Proof. Consider the case in which $V_H - c_H - (V_L - c_L) > 0$. Then the expected profits for any price charged in the equilibrium are given by $(c_H + \frac{s}{\Phi_H(\lambda, \mu)} - c_H)\mu(1 - \lambda)$ and $(c_L + \frac{s}{\Phi_L(\lambda, \mu)} - c_L)\mu(1 - \lambda)$ for the high-quality and low-quality firm, respectively. Thus, we have

$$\begin{aligned} \left(c_H + \frac{s}{\Phi_H(\lambda, \mu)} - c_H\right)\mu(1 - \lambda) &> \left(c_L + \frac{s}{\Phi_L(\lambda, \mu)} - c_L\right)\mu(1 - \lambda) \\ \Phi_L(\lambda, \mu) &> \Phi_H(\lambda, \mu), \end{aligned}$$

This inequality holds if $\mu < 0.5$, which is the case if the high-quality firm has a relative advantage.

The case in which the low quality firm has the relative advantage, $V_H - c_H - (V_L - c_L) < 0$, is analogous. $\square$

2.A.9 Proof of Proposition 2.4

Proof. The price distributions are overlapping if the largest price of the low-quality firm is larger than the lowest price of the high-quality firm.

$$\begin{aligned} c_L + \frac{s}{\Phi_L(\lambda, \mu)} &> \frac{s}{\Phi_L(\lambda, \mu)} \cdot \frac{\frac{1-\lambda}{\lambda}\mu}{1 + \frac{1-\lambda}{\lambda}\mu} + V_H - V_L + c_L \Leftrightarrow \\ \frac{s}{V_H - V_L} &> \left(1 + \frac{1-\lambda}{\lambda}\mu\right) \Phi_L(\lambda, \mu). \end{aligned} \quad (2A.11)$$

However, the right-hand side still depends on V_H , V_L and μ through μ . Fix now, V_H and V_L such that the low-quality good has a relative advantage, $V_H - V_L - c_H + c_L < 0$. Then $\mu \in (0.5, 1)$ and is decreasing in s . This means that,

$$\lim_{s \rightarrow \infty} \left(1 + \frac{1-\lambda}{\lambda}\mu\right) \Phi_L(\lambda, \mu) \rightarrow \left(1 + \frac{1-\lambda}{2\lambda}\right) \int_0^1 \frac{1-x}{1-x + \frac{1-\lambda}{2\lambda}} < \infty$$

Hence, we can find an s large enough, such that the inequality (2A.11) holds, since $\lim_{s \rightarrow \infty} \frac{s}{V_H - V_L} \rightarrow \infty$. This fraction tends faster to infinity if $V_H - V_L$ is small and hence the price distributions overlap for sufficiently large s and sufficiently low $V_H - V_L$. The case in which the high-quality firm has a relative advantage can be derived analogously. $\square$

2.A.10 Proof of Proposition 2.5

Proof. Suppose both firms have zero expected profits. This means, that the firms either charge prices lower than their marginal costs or so large that the buyers do not buy.

There is no equilibrium in which both firms charge prices strictly lower than their marginal costs, since at least one firm attracts consumers and has an incentive to increase the price to avoid losses. Thus, a firm which attracts consumers must charge the marginal costs. However, also the non-shoppers buy at such a price. Thus, the firm which attracts non-shoppers has an incentive to raise the price, since the non-shoppers cannot costlessly switch the firm.

If both firms charge prices $\hat{p}_i$ for $i = 1, 2$ such that no consumer buys, it is obvious that both firms have an incentive to decrease the price to attract consumers, since there exists an $\epsilon > 0$ such that $V_i - c_i - (\hat{p} - \epsilon) > 0$.

If one firm charges a price not larger than the marginal costs whereas the other charges prices such that no consumers buy, the first firm attracts all the consumers. However, then it can deviate to a price above marginal costs without loosing any consumers. $\square$

2.A.11 Proof of Proposition 2.6

Proof. In the text. $\square$

2.A.12 Proof of Proposition 2.7

Proof. Assume throughout the proof, that the high-quality firm has a relative advantage, $V_H - c_H > V_L - c_L$. The case in which the low-quality firm has a relative advantage is analogous.

In the equilibrium, the firm with the relative advantage is the firm which has positive expected profits. Assume that the high-quality firm has zero profits, whereas the low-quality firm has positive profits. The low-quality firm uses a strategy $F_L(p)$ with $p \in [\underline{p}_L, \bar{p}_L]$. However, then the high-quality firm can set the price equal to $\underline{p}_L + V_H - V_L$. Since $\underline{p}_L$ yields strictly positive expected profits, $\underline{p}_L > c_L$, and $V_H - V_L + c_L > c_H$, also $V_H - V_L + \underline{p}_L > c_H$ holds. Thus, the high-quality firm can attract all shoppers by undercutting $\underline{p}_L - (V_H - V_L) > c_H$ and thus can have strictly positive profits.

Now we show, that the firm with the relative advantage cannot be the firm with the only firm which has positive expected profits. Assume that the high-quality firm has positive profits, whereas the low-quality firm has zero profits. Then it must be the case that $\underline{p}_L + V_H - V_L \geq \bar{p}_H$ or $\underline{p}_L = c_L$. Assume $\underline{p}_L + V_H - V_L > \bar{p}_H$, then $\bar{p}_H$ can not be optimal, since the high-quality firm can increase the upper bound without loosing consumers. Moreover, if $\underline{p}_L + V_H - V_L = \bar{p}_H$ holds in equilibrium, then $\underline{p}_H < \bar{p}_H$ can not be optimal, since the high-quality firm can increase $\underline{p}_H$ without loosing consumers. Furthermore, also a pure strategy cannot be optimal for firm H . If firm L uses a mixed strategy, i.e. $\underline{p}_L < \bar{p}_L$. Then the low quality firm attracts shoppers at $\underline{p}_L$, but not at

any other price it charges. Thus, any price larger than $\underline{p}$ cannot be optimal. If both firms use pure-strategies then the only prices which form an equilibrium are $p_L = c_L$ and $p_H = c_L + V_H - V_L$ if all consumers visit the high quality firm. However, this is the complete information which is excluded by hypothesis. $\square$

Chapter 3

On the Profitability of Gatekeepers: The Role of Asymmetric Firms

3.1 Introduction

Gatekeepers in the internet like price comparison websites have gained much popularity in the last decade. Typically, sellers can choose to pay a fee to the gatekeepers to list their prices for certain products on these websites. Consumers usually have free access to the price lists. Now, a huge variety of products are listed on these websites. Moreover, there exists a large number of specialized price comparison sites which concentrate on only one category of products such as books or CDs. However, it is not clear why we find gatekeepers in some markets, but not in others. This question has not been answered in the existing literature. Moreover, all existing models assume markets for homogenous goods with firms having identical costs e.g. Baye and Morgan (2001). This paper introduces asymmetric firms into the framework of Baye and Morgan (2001) and examines the impact of asymmetry on the gatekeeper's expected profits. Moreover, it takes into account that gatekeepers do not only offer lists of prices for certain particular products, they also offer lists of products in certain categories.¹ That is, consumers also have to decide which product in the list has the best utility and price. This gives rise to consider vertically differentiated goods instead of homogenous goods. Moreover, a large number of different firms advertise their prices on gatekeepers' websites. On one hand, one finds small

¹For instance, a search for an USB-stick on a gatekeeper's site will provide the consumer with a list of USB-sticks with different capacities from different manufacturers.

retailers having only one shop, on the other hand there are also retailer chains. However, it is difficult to imagine that these different firms have the same cost structure. Consequently, this gives rise to consider differences in costs. This paper examines which markets offer the highest incentive for the gatekeeper to enter. For this purpose we adopt a simplified version of the Baye and Morgan model (2001), however, we introduce firms with non-identical products and non-identical costs, such that they have a different profitability.

We show that the firm which has an advantage over the competitor (in the sense of profitability) advertises more aggressively and captures the larger expected profits. Interestingly, the gatekeeper benefits from this asymmetry. That is, it prefers markets with large asymmetries in the sense of advantages over those with small ones. The rationale for this finding is that the gatekeeper increases the advertising fee when the advantage rises. This lets the advertising propensities drift apart and thus lowers price competition. This results into larger returns per sale of the firms, which can be extracted by the gatekeeper. Moreover, if the advantage is very large, the gatekeeper sets an advertising fee such that it is only profitable for the firm with the larger profitability to remain in the market. Furthermore, a more efficient allocation of goods can be achieved through the existence of the gatekeeper, in particular the existence of the gatekeeper can enhance the social welfare if the advantage is large and the fix costs of the gatekeeper are sufficiently small.² The entrance of a gatekeeper in markets in which firm's profitabilities are similar is detrimental to social welfare. Thus a social planner should prohibit entrance.

We consider two local markets. In each of the local markets there is one firm. We allow the firms to offer vertically differentiated goods and to have different marginal costs. There is a unit mass of consumers with unit demand and homogenous preferences. In each of the local markets there is one half of the consumers. Moreover, in each of the markets there is the same fraction non-users who can only buy at their local market. The other fraction of these consumers first observe the prices and the quality listed at a gatekeeper's website (which is a clearinghouse in the sense of Varian (1980)). This gatekeeper creates a virtual market place which allows the firms to offer their goods to a fraction of consumers in the other market. That is, without its existence, all consumers can only buy at their local markets. The gatekeeper sets an advertising fee for both firms at which the firms can advertise their prices at the gatekeeper's clearinghouse. The firms decide simultaneously if they want to advertise their price at the gatekeeper's website and which price to set.

²In this model half of the non-loyal consumers receive the socially inefficient good if there is no gatekeeper. However, if there is a gatekeeper and the advantage is large enough, this fraction of consumers purchase the socially efficient good. The gatekeeper enhances social welfare if this gain of a more efficient allocation exceeds the fix costs of the gatekeeper.

The most related article is that of Baye and Morgan (2001) in which a gatekeeper faces a market consisting of identical firms which compete in a homogeneous goods market. The gatekeeper sets an advertising fee and then the firms decide whether to advertise their prices or to offer it only in a local market with a small number of consumers. They show that several equilibria might arise. In the most interesting equilibria the firms use mixed pricing strategies. However, the results rely strongly on the assumption of identical firms and homogeneous products.

Arnold, Li, Saliba and Zhang (2008) have adopted the Baye and Morgan framework and introduced asymmetric market sizes, in the sense that each firm has a different number of loyal (local) consumers. They found that the firm with the smaller loyal consumer segment advertises more aggressively and prices less competitively than the firm with the larger loyal market. Moreover, they found a subgame in which the firm with the fewer loyal customers advertises with probability one. Other models as Narasimhan (1988) and Kocas and Kiyak (2006) study this asymmetry without modelling the gatekeeper and hence do not endogenize the advertising fee.

Schlag (2000) considers a model in which consumers do not know the prices charged by the firms in the market. To become informed about the price charged by a firm, consumers have to visit the firm, which is assumed to be costly. Moreover, the gatekeeper in the market knows all prices in the market and can sell this information to consumers. Schlag (2000) finds that a monopolistic gatekeeper (or colluding gatekeepers) induce maximal price dispersion³. Moreover, buying the information at the gatekeeper is very attractive for consumers if the number of competing firms is large. The reason is that firms either charge the competitive price or the monopoly price when the number of firms approaches infinity.

Baye and Morgan (2009) assume a gatekeeper model in which firms can brand their product. More precisely, they can spend resources in the first stage to increase the number of loyal customers who do not use the gatekeeper's website to compare prices. In particular, they show that the total fraction of loyal consumers is independent of the gatekeeper's advertising fee in the derived symmetric equilibrium. Consequently, the optimal advertising fee in their model is equivalent to the one in models in which branding is exogenous and hence the total number of loyal consumers.

Furthermore, Robert and Stahl (1993) as well as Janssen and Non (2009) study the relationship of costly search and informative price advertising in sequential search models. In Robert and Stahl (1993) a finite number of symmetric

³Price dispersion is measured by the normalized difference of the average price and the minimum price in the market of the symmetric pricing distribution of the firms.

firms compete in prices for a unit mass of consumers who are ex-ante uninformed. Any firm chooses the fraction of consumers which become informed about the posted price. The equilibrium converges to perfect competition if the advertising costs vanish. Moreover, advertising can decrease if the search costs become small and hence prices can remain above the marginal costs. Janssen and Non (2009) assume an additional stage in which firms first decide whether to sell the good or not. When they decide to sell the good, they can inform the consumers about the fact that they sell the good and about the price charged. Thus, the consumers are willing to pay a larger price at an advertising firm instead of searching for a non-advertising firm selling the good. Since advertising lowers the expected search costs, advertised prices are larger than non-advertised prices.

The remainder of this chapter proceeds as follows. In section 2 we set up our variant of the Baye and Morgan-model which is very similar to that of Arnold et al (2008). In section 3 we present the expected profits of the firms and derive the optimal behavior of them in any subgame. Section 4 presents the optimization problem of the gatekeeper and we derive the optimal advertising fee. In section 5 we establish the subgame perfect equilibrium and do some comparative statics. The final section concludes. All proofs can be found in Appendix B.

3.2 The Model

We assume two local markets. In each market there is one risk-neutral firm $i = H, L$. Firm H sells a high quality good and firm L sells a low quality good. We assume the firms to have different marginal costs, which we will denote by c_H and c_L and assume that $c_L \leq c_H$. Moreover, there is a unit mass of risk-neutral consumers with homogenous preferences and unit demand. That is, their reservation price for good H is given by V_H and their reservation price for good L is given by V_L with $V_L \leq V_H$ and $V_i > c_i$ for $i = H, L$. Moreover, we assume two types of consumers. In each local market there is a fraction of λ with $\lambda < \frac{1}{2}$ of non-users (or loyal consumers) who do not use the gatekeeper's website.⁴ They only buy at their local firm provided that it yields a positive utility. Otherwise they leave the market without buying. Therefore, we will refer to them as *loyals*. The other $\frac{1}{2} - \lambda$ consumers in each market first observe the prices quoted at the information gatekeeper's website. That is, if there are price quotations at the gatekeeper's website, the non-loyal consumers buy the advertised good which consumption yields the higher utility, provided that the utility is nonnegative. If both firms are listed and the consumption of the good

⁴The asymmetric case in which one market has a larger fraction of non-users with symmetric firms is analyzed by Arnold et al. (2008).

yields the same utility, the non-loyals buy at each store with probability one half. If no firm is listed, the non-loyals buy at their local firms, provided that the consumption yields a positive utility.⁵ The advertising fee, which is charged to firms who want to quote their price at the gatekeeper's site, is denoted by Φ . The gatekeeper is assumed to set the same advertising fee for both firms.⁶ Furthermore, the gatekeeper has no variable costs and the average fixed costs per consumer are given by C_G . Moreover, the advertising probability of firm i is denoted by α_i for $i = 1, 2$. Moreover, the firms are assumed to condition their price setting behavior on their advertising choice. Therefore, the cumulative distribution function of the advertised pricing strategy of the firms is given by $F_i(p) = \text{Prob}(p_i \leq p)$. The support's lower bound is denoted by $\underline{p}_i$ and the upper bound by $\bar{p}_i$ for $i = 1, 2$.⁷

The sequence of action is as follows. First, the gatekeeper sets the advertising fee. Then the firms observe the advertising fee and make simultaneously their advertising and pricing decisions. Finally, the consumers make their purchasing decisions.

Our strategy is to solve this problem by backward induction. Hence, we first assume that the advertising fee is fixed and solve the problem of the firms.

3.3 Subgames

In this section we first present the firm's expected profits from non-advertising and define a firm's advantage. Moreover, we define the expected profits from advertising and derive conditions under which only one firm advertises in equilibrium. Finally, we will derive the optimal behavior of the firms if it is optimal for both firms to advertise.

3.3.1 Expected Profits From Non-Advertising

The first lemma states that a firm which does not advertise, charges its valuation V_i .

⁵This assumption reproduces the consumer behavior of Baye and Morgan (2001). More precisely, one could assume, analogous to Baye and Morgan (2001), that all consumers have small costs $s > 0$ to visit the local shop. Then, the assumed consumer behavior arises in the equilibrium. The firms charge price $V_i - s$ for $i = H, L$ when they do not advertise. This only extends the analysis without leading to further insights.

⁶This assumption is made analogous to the relevant literature of Baye and Morgan (2001) and Arnold et al. (2008). Furthermore, it seems plausible in our setting that the gatekeeper has an incentive to discriminate the firms. However, you find an enormous amount of firms which advertise their products on price comparison website. Thus, it is implausible that gatekeepers charge different prices to all those firms. Therefore, we refrain from modelling price discrimination.

⁷We do not introduce a distribution function for the firms when they do not advertise, since it will be degenerated in the equilibrium.

Lemma 3.1. *In any equilibrium, firm i charges the monopoly price V_i if it does not advertise.*

The fraction of attracted consumers does not depend on the price of a non-advertising firm. Hence, charging lower prices than V_i cannot belong to an optimal strategy of firm i . Therefore, the expected profits, when firm i does not advertise, are given by

$$\Pi_i^N = (V_i - c_i) \left[\lambda + (1 - \alpha_j) \left(\frac{1}{2} - \lambda \right) \right] \quad (3.1)$$

for $i \neq j$ and $i = H, L$. Firm i has $V_i - c_i$ profits per sale and sells to its loyal consumers, λ . Additionally, it sells to the non-loyal shoppers in the market if the competitor, firm j , does not advertise.

As a benchmark, consider there is no gatekeeper. Hence, none of the firms has the possibility to attract all non-loyal customers and their expected profits reduce to

$$\Pi_i^N = \frac{V_i - c_i}{2}.$$

These profits do not have to be identical for $i = H, L$. Firm i has larger expected profits if $V_i - c_i \geq V_j - c_j$. Therefore, we have the following reasonable definition.

Definition 3.1. *Firm i has an advantage if $V_j - c_j < V_i - c_i$ for $i \neq j$ and the advantage is given by $V_i - c_i - (V_j - c_j)$.*

Moreover, we reduce our analysis to the case in which the high quality firm has a advantage, since the other case is the mirror image and does not yield additional insights.

Assumption 3.1. *The high quality firm has a advantage, that is $V_L - c_L < V_H - c_H$ holds.*

Moreover, we will consider changes in the advantage. For this purpose we will keep $V_L - c_L$ fixed and only consider changes in the profitability of the high quality firm.

Note, that none of the firms advertise if Φ fulfills $(V_H - c_H)(1 - \lambda) - \Phi < \frac{1}{2}(V_H - c_H)$, that is $\alpha_i = 0$ for $i = H, L$. This inequality can be rewritten to $\Phi > (V_H - c_H) \left(\frac{1}{2} - \lambda \right)$. Moreover, we can find a set of parameters for which only one firm advertises.

3.3.2 One Advertising Firm

Consider firm H advertises. The maximum additional profits firm L can achieve from advertising are given by $(V_L - c_L)(1 - 2\lambda)$. That is, it attracts all non-loyal

consumers instead of none. Thus, the low quality firm has no strict incentive to advertise as long as $\Phi \geq (V_L - c_L)(1 - 2\lambda)$. Clearly, $\Phi \leq (V_H - c_H)(\frac{1}{2} - \lambda)$ must hold, too. Therefore, we have

Proposition 3.1. *If $(V_L - c_L)(1 - 2\lambda) \leq \Phi \leq (V_H - c_H)(\frac{1}{2} - \lambda)$, then the high quality firm advertises price V_H with certainty and the low quality firm does not advertise and sets its price equal to V_L .*

For this equilibrium to be achievable, the above interval for Φ must be nonempty. Therefore, we necessarily must have $V_H - c_H \geq 2(V_L - c_L)$. Thus, the high quality firm must have a sufficiently large advantage. In these subgames the high quality firm advertises with certainty and can quote the monopoly price V_H , since the low quality firm abstains from advertising with certainty. Moreover, the low quality firm charges the monopoly price V_L by Lemma 3.1. The high quality firm has larger profits than the low quality firm. That is, the difference is given by $(V_H - c_H)(1 - \lambda) - \Phi - (V_L - c_L)\lambda$, which can be rearranged to $[V_H - c_H - (V_L - c_L)]\lambda + (V_H - c_H)(1 - 2\lambda) - \Phi \geq 0$. It can be easily seen that the difference is strictly increasing in the advantage.

3.3.3 Two Advertising Firms

In this subsection, we want to derive the optimal behavior when both firms advertise. That is, we assume throughout the analysis of this subsection that the gatekeeper's advertising fee is small enough to fulfill $\Phi < \min\{(V_L - c_L)(1 - 2\lambda), (V_H - c_H)(\frac{1}{2} - \lambda)\}$. First we derive some preliminary results which describes the firm's optimal behavior in any equilibrium.

The next result states that both firms advertise with positive probability.

Lemma 3.2. *If $\Phi < \min\{(V_L - c_L)(1 - 2\lambda), (V_H - c_H)(\frac{1}{2} - \lambda)\}$ holds, both firms advertise with positive probability in any equilibrium. That is $\alpha_i > 0$ for $i = H, L$.*

The net gain from advertising is positive at least for the high quality firm. Hence, at least the high quality firm advertises. However, if the high quality firm would be the only firm which advertises it would set the monopoly price. In this case also the low quality firm has an incentive to advertise a price slightly lower V_L to attract all non-loyal consumers, since $\Phi < (V_L - c_L)(1 - 2\lambda)$. Thus, there cannot be an equilibrium in which one firm does not advertise.

Furthermore, there is at least one firm which mixes between advertising and not advertising its price.

Lemma 3.3. *If $\Phi < \min\{(V_L - c_L)(1 - 2\lambda), (V_H - c_H)(\frac{1}{2} - \lambda)\}$ holds, at most one firm advertises with probability one in any equilibrium. That is $\alpha_i < 1$ for at least one i , $i = 1, 2$.*

If both firms advertise with certainty, they find themselves in a Bertrand-competition for the non-loyal consumers. Thus, the only possible candidate for an equilibrium is that at least one firm charges a price equal to its marginal costs. However, advertising leads to negative profits for this firm for any $\Phi > 0$. Therefore, it has an incentive to deviate from advertising, since abstaining from advertising and selling only to the loyal customers yields strictly positive profits.

Now, we want to analyze the advertised pricing distribution which the firms use when they advertise.

Lemma 3.4. *If $\Phi < \min\{(V_L - c_L)(1 - 2\lambda), (V_H - c_H)(\frac{1}{2} - \lambda)\}$ holds, the firms use mixed pricing strategies in any equilibrium. That is $F_i(\cdot)$ is not degenerated for $i = 1, 2$.*

A high deterministic pricing strategy of one firm gives the competing firm an incentive to advertise and undercut this price. A low deterministic price gives the competing firm an incentive to abstain completely from advertising (this contradicts Lemma 3.2). Thus, it becomes optimal for the firm to increase the price. Therefore, there can only be an equilibrium in mixed pricing strategies.

In any equilibrium the difference of the lower bounds is equal to the difference of the consumers' valuations for the goods, $V_H - V_L$.

Lemma 3.5. *If $\Phi < \min\{(V_L - c_L)(1 - 2\lambda), (V_H - c_H)(\frac{1}{2} - \lambda)\}$ holds, the lower bounds are set such that $\underline{p}_H - \underline{p}_L = V_H - V_L$ holds in any equilibrium.*

If this condition would not be fulfilled, then one firm could increase the lower bound of its advertised pricing strategy without risking to losing consumers. Hence, it increases the expected profits per sale without diminishing the expected sales. Although, this Lemma looks innocent, it plays a crucial role when we derive the equilibria.

The next two lemmata state that advertised pricing distributions have no mass points and are connected below the monopoly price.

Lemma 3.6. *If $\Phi < \min\{(V_L - c_L)(1 - 2\lambda), (V_H - c_H)(\frac{1}{2} - \lambda)\}$ holds, the advertised pricing distribution does not have a mass point on the interval $[\underline{p}_i, V_i)$ for $i = 1, 2$ in any equilibrium.*

Consider firm H would have a mass point, p^m , fulfilling $\underline{p} \leq p^m < V_H$. Then we can find an $\epsilon > 0$, such that firm L does not charge prices in an interval above p^m , $[p^m - (V_H - V_L), p^m - (V_H - V_L) + \epsilon]$, since undercutting the mass point of firm H increases the probability of attracting the non-loyal consumers sufficiently to raise the expected profits by a discrete amount. However, then firm H could shift its mass point to larger prices without losing consumers. Therefore, there cannot be mass points below the monopoly price in the equilibrium.

Lemma 3.7. *If $\Phi < \min\{(V_L - c_L)(1 - 2\lambda), (V_H - c_H)(\frac{1}{2} - \lambda)\}$ holds, the advertised pricing distribution is connected on the interval $[\underline{p}_i, V_i]$ for $i = 1, 2$ in any equilibrium.*

The optimal response of firm L to a strategy of firm H which puts zero probability weight on an interval (p_1, p_2) of prices is to put zero probability on the interval $(p_1 - (V_H - V_L), p_2 - (V_H - V_L))$. Consequently, if firm H charges a price p_1 it has the same number of expected consumers as charging prices from the interval (p_1, p_2) and can therefore not be optimal.

The next Lemma shows that at most one firm has an atom at its monopoly price if it advertises.

Lemma 3.8. *If $\Phi < \min\{(V_L - c_L)(1 - 2\lambda), (V_H - c_H)(\frac{1}{2} - \lambda)\}$ and firm i advertises V_i with strict positive probability, firm j does not advertise V_j in any equilibrium.*

If one firm has an atom at its monopoly price, the other firm has an incentive to undercut the atom slightly such that the profits per sale are decreased slightly, but the expected number of consumers are increased by a discrete amount.

Moreover, it follows immediately from Lemma 3.6 and 3.8 that either both firms advertised pricing strategies do not have mass points or firm j advertises V_j with positive probability and only firm i 's pricing strategy has no mass point. The supports of both firms' pricing strategies have the same length, but that of the low quality firm is shifted down by $V_H - V_L$.

Thus, the expected profits if firm L advertises a price $p \leq V_L$ are given by

$$\Pi_L^A = (p - c_L) [\lambda + (1 - \alpha_H F_H(p + V_H - V_L))(1 - 2\lambda)] - \Phi. \quad (3.2)$$

Firm L attracts its loyal consumers, L , with certainty and additionally attracts the non-loyal consumers with probability $1 - \alpha_H F_H(p + V_H - V_L)$. The term $\alpha_H F_H(p + V_H - V_L)$ is the probability that firm H advertises a $\hat{p}_H$ fulfilling $\hat{p} < p + V_H - V_L$ at which all non-loyal consumers prefer to buy the high quality good, since $V_H - \hat{p}_H > V_L - p$. Therefore, $1 - \alpha_H F_H(p + V_H - V_L)$ is the joint probability of all states in which firm L attracts the non-loyals if it advertises price p .

Analogously, the expected profits if firm H advertises a price $p \leq V_H$ are given by

$$\Pi_H^A = (p - c_H) [\lambda + (1 - \alpha_L F_L(p - (V_H - V_L)))(1 - 2\lambda)] - \Phi. \quad (3.3)$$

By Lemma 3.2 and 3.3 either both firms advertise with positive probability strictly less than one or one firm advertises with certainty and the competitor

advertises with positive probability strictly less than one. To figure out which equilibrium of these two types occur, we define $\tilde{p}_i$ to be the lowest price firm i is willing to advertise in equilibrium. That is, the maximum possible profits $\tilde{p}_i$ can yield must be equal to the profits of non-advertising. Thus, $\tilde{p}$ is defined by the following condition

$$(\tilde{p}_i - c_i)(1 - \lambda) - \Phi = (V_i - c_i) \left[\lambda + (1 - \alpha_j) \left(\frac{1}{2} - \lambda \right) \right] \quad (3.4)$$

for $i \neq j$ and $i = 1, 2$. If we express $\tilde{p}_i$ explicitly, we obtain

$$\tilde{p}_i = \frac{(V_i - c_i)[\lambda + (1 - \alpha_j) \left(\frac{1}{2} - \lambda \right)] + \Phi}{1 - \lambda} + c_i. \quad (3.5)$$

By Lemma 3.2 and 3.3 at least one firm is indifferent between advertising and non-advertising. Moreover, by 3.6 one firm's lower bound must be equal to $\tilde{p}$. We can now verify that $\alpha_L < 1$ holds. If this would not be the case, then $\tilde{p}_L > \tilde{p}_H - (V_H - V_L)$ holds and $\underline{p}_H = \tilde{p}_H$, since firm $\alpha_H < 1$ must hold. However, firm H can increase the lower bound without losing consumers and thus firm H has smaller profits if it abstains from advertising than if it advertises. Therefore, $\alpha_H = 1$, which contradicts Lemma 3.3. Therefore, we have the following proposition.

Proposition 3.2. *If $\Phi < \min\{(V_L - c_L)(1 - 2\lambda), (V_H - c_H) \left(\frac{1}{2} - \lambda \right)\}$ holds, the low quality firm advertises with probability strictly less than one, $\alpha_L < 1$.*

This proposition restricts the number of possible equilibria. Either both firms advertise with probability less than one or the high quality firm advertises with probability one and the low quality firm advertises with probability less than one. All cases will be discussed in the following subsections.

Both Firms Use Mixed Advertising Strategies

The following proposition establishes an equilibrium in which both firms advertise with probability strictly less than one.

Proposition 3.3. *If*

$$[V_H - c_H - (V_L - c_L)](1 - 2\lambda) \leq \Phi < \min \left\{ (V_L - c_L)(1 - 2\lambda), (V_H - c_H) \left(\frac{1}{2} - \lambda \right) \right\} \quad (3.6)$$

holds, there exists a unique equilibrium. The firms advertise with probabilities

$$\alpha_L = 1 - \frac{2\Phi}{(V_H - c_H)(1 - 2\lambda)} \quad (3.7)$$

$$\alpha_H = 2 \left[\frac{(V_H - c_H)(1 - 2\lambda) - \Phi}{(V_L - c_L)(1 - 2\lambda)} \right] - 1. \quad (3.8)$$

If the firms advertise, they use the pricing strategies

$$F_L(p) = \frac{1}{1 - \frac{2\Phi}{(1-2\lambda)(V_H - c_H)}} \left(1 - \frac{(V_L - p)\lambda + 2\Phi}{(p + V_H - V_L - c_H)(1 - 2\lambda)} \right) \quad (3.9)$$

$$F_H(p) = \frac{1}{2 \left[\frac{(V_H - c_H)(1 - 2\lambda) - \Phi}{(V_L - c_L)(1 - 2\lambda)} \right] - 1} \cdot \left(1 - \frac{(V_H - p)\lambda - [V_H - c_H - (V_L - c_L)](1 - 2\lambda) + 2\Phi}{(p - (V_H - V_L) - c_L)(1 - 2\lambda)} \right) \quad (3.10)$$

on the supports

$$\left[V_L - \frac{(V_H - c_H)(1 - 2\lambda) - 2\Phi}{1 - \lambda}, V_L \right] \text{ and } \left[V_H - \frac{(V_H - c_H)(1 - 2\lambda) - 2\Phi}{1 - \lambda}, V_H \right],$$

respectively. If the firms do not advertise they set their price equal to V_L and V_H , respectively.

The left hand side on the condition given in (3.6) ensures that none of the firms advertise with certainty. The right hand side ensures that it is optimal for both firms to advertise. Since this interval must not be empty for the equilibrium to exist, the advantage must be sufficiently small. This is exactly the case if $V_H - c_H < 2(V_L - c_L)$. Thus, it is only achievable by the gatekeeper if and only if the equilibrium in Proposition 3.1 is not achievable.

The equilibrium in Proposition 3.3 includes the equilibrium derived in Baye and Morgan (2001) for the special case with two firms. Generically, in this equilibrium the firms advertise with different probabilities if $V_H - c_H > V_L - c_L$. Moreover, the advertising propensities have the following property.

Proposition 3.4. *In the equilibrium in Proposition 3.3, the high quality firm advertises with larger probability.*

Intuitively, the maximal gain from advertising is larger for the high quality firm, since $(V_H - c_H)(1 - 2\lambda) - \Phi \geq (V_L - c_L)(1 - 2\lambda) - \Phi$. Hence, the high quality firm adopts a more aggressive advertising strategy than its competitor. Furthermore, it is remarkable that both firms advertise more aggressively if $V_H - c_H$ increases. That is, firm H 's maximum profits from advertising $(V_H - c_H)(1 - 2\lambda)$ increase and hence, firm H advertises with a larger probability. However, this decreases the expected profits of abstaining from advertising for

firm L and hence, it advertises with larger probability too. Surprisingly, the effect of $V_L - c_L$ is different. It only decreases the advertising propensity of the high quality firm. We conclude that the total expected demand for advertising, $\alpha_L + \alpha_H$, increases as the advantage becomes larger.

Proposition 3.5. *In the equilibrium in Proposition 3.3, the expected demand for advertising increases in the advantage.*

Finally, we can compare the expected profits. We conclude that firm H 's expected profits are not smaller than those of the competitor and that the difference in expected profits increases if the advantage increases.

Proposition 3.6. *In the equilibrium established in Proposition 3.3, (i) $\Pi_H - \Pi_L \geq 0$ holds; and (ii) the difference is increasing in the advantage.*

In this equilibrium, the high quality firm advertises its monopoly price with positive probability.⁸ Moreover, the mass point vanishes if the advantage decreases, $V_H - c_H - (V_L - c_L) \rightarrow 0$. Furthermore, the advertising probabilities approach each other as the advantage decreases.

However, these subgames are not reached if the gatekeeper sets an advertising fee lower than $\Phi < [V_H - V_L - (c_H - c_L)](1 - 2\lambda)$. We establish these subgames in the next subsection.

One Pure Advertising Strategy

We now want to establish an equilibrium for low advertising fees, i.e. $\Phi < [V_H - c_H - (V_L - c_L)](1 - 2\lambda)$. In this equilibrium the high quality firm advertises with certainty.

Proposition 3.7. *If*

$$0 < \Phi \leq [V_H - c_H - (V_L - c_L)](1 - 2\lambda) < \min \left\{ (V_L - c_L)(1 - 2\lambda), (V_H - c_H) \left(\frac{1}{2} - \lambda \right) \right\} \quad (3.11)$$

holds, then there exists a unique equilibrium in which the low quality firm advertises with probability

$$\alpha_L = \frac{(V_L - c_L)(1 - 2\lambda) - \Phi}{(V_H - c_H)(1 - 2\lambda)} \quad (3.12)$$

⁸Given that the high quality firm advertises, the monopoly price V_H is charged with probability $1 - \frac{1}{\alpha_H} \left(\alpha_H - \frac{V_H - c_H - (V_L - c_L)}{V_L - c_L} \right) > 0$.

and the high quality firm advertises with certainty. If the low quality firm advertises it uses the pricing distribution

$$F_L(p) = \frac{1}{\alpha_L} \left(1 - \frac{(V_L - p)\lambda + [V_H - c_H - (V_L - c_L)](1 - 2\lambda) + \Phi}{(p + V_H - V_L - c_H)(1 - 2\lambda)} \right) \quad (3.13)$$

on the support

$$\left[\frac{V_L\lambda + c_L(1 - 2\lambda) + \Phi}{1 - \lambda}, V_L \right]$$

and charges V_L if it does not advertise. The high quality firm sets prices according to the pricing distribution

$$F_H(p) = 1 - \frac{(V_H - p)\lambda + \Phi}{[p - (V_H - V_L) - c_L](1 - 2\lambda)}. \quad (3.14)$$

on the support

$$\left[V_H - \frac{(V_L - c_L)(1 - 2\lambda) - \Phi}{1 - \lambda}, V_H \right].$$

Necessarily, $V_H - c_H > V_L - c_L$ must hold for the interval for Φ to be nonempty, which is fulfilled by assumption. Moreover, Φ must not exceed $\Phi < \min\{(V_L - c_L)(1 - 2\lambda), (V_H - c_H)(\frac{1}{2} - \lambda)\}$. This equilibrium exists for all values of Φ which have not been discussed above. Therefore, we have derived the unique equilibrium strategies of the firms for any possible values of Φ .

In the subgames of Propositions 3.7, the low quality firm's advertising probability decreases in the advantage. Hence, the expected total demand for advertising increases if the advantage vanishes.

Proposition 3.8. *In the equilibrium established in Proposition 3.7, the expected demand for advertising decreases in the advantage.*

The reason why the reaction of the demand is different than in the previous equilibrium is that the high quality firm cannot increase the expected demand for advertising anymore. Only the low quality firm reacts to changes. It is interesting that for the two different types of subgames in which both firms advertise, there are two types of regimes. If the advertising fee is small enough such that the firms react as described in Proposition 3.7, the expected demand decreases in the advantage of the high quality firm. However, if the advertising fee is large enough such that the firms behave as in Proposition 3.3, then the expected demand is increasing.

Also in this regime, the high quality firm has the larger expected profits and the difference increases in the advantage.

Proposition 3.9. *In the equilibrium established in Proposition 3.7, (i) $\Pi_H -$*

$\Pi_L > 0$ holds; and (ii) the difference is increasing in the advantage.

Also in these subgames, the high quality firm advertises the monopoly price with positive probability.⁹

3.4 The Gatekeeper's Problem

Now, we can analyze the gatekeeper's optimization problem. From the analysis above we know that the gatekeeper can be in one of the two different scenarios. If $V_H - c_H < 2(V_L - c_L)$ holds, then the subgames in which only one firm advertises are not achievable. Thus, the gatekeeper can only set prices such that both firms advertise.¹⁰ If $V_H - c_H > 2(V_L - c_L)$ holds, then the subgames in which both firms advertise with probability less than one are not achievable. Thus, the gatekeeper can only set prices such that the high quality firm advertises with certainty.

The gatekeeper sets Φ so as to maximize the expected profits. These are given by

$$\Pi_G(\Phi) = (\alpha_L + \alpha_H) \cdot \Phi - C_G. \quad (3.15)$$

It cannot be optimal for the gatekeeper to set an advertising fee $\Phi > (V_H - c_H) \left(\frac{1}{2} - \lambda\right)$, since the demand would be equal to zero and hence $\Pi_G(\Phi) = -C_G$. Thus, we only have to consider the range $\Phi \in [0, (V_H - c_H) \left(\frac{1}{2} - \lambda\right)]$. The next proposition establishes the optimal strategy.

Proposition 3.10. *The gatekeeper sets the advertising fee according to*

$$\Phi^* = \begin{cases} \frac{(V_H - c_H)^2(1-2\lambda)}{2(V_H - c_H + V_L - c_L)} & \text{if } V_H - c_H \in [V_L - c_L, (V_L - c_L)\sqrt{2}] \\ [V_H - c_H - (V_L - c_L)](1-2\lambda) & \text{if } V_H - c_H \in [\sqrt{2}(V_L - c_L), 2(V_L - c_L)] \\ (V_H - c_H) \left(\frac{1}{2} - \lambda\right) & \text{otherwise.} \end{cases} \quad (3.16)$$

Note, that the gatekeeper's strategy is continuous in the parameters. Additionally, it is interesting that the gatekeeper induces the low quality firm to abstain from advertising as soon as it is possible. Moreover, we can conclude that the gatekeeper's advertising fee increases in the advantage of the high quality firm.

Proposition 3.11. *The advertising fee is increasing in (i) the advantage and (ii) the number of non-loyals.*

⁹The monopoly price V_H is charged with probability $\frac{\Phi}{(V_L - c_L)(1-2\lambda)} > 0$.

¹⁰Of course, the gatekeeper could also set a price such that no firm advertises. Obviously, this will not happen in the equilibrium.

Clearly, if the number of non-loyal consumers increase, both firms are willing to pay more for advertising. Thus, the gatekeeper increases its price. The rationale for the advertising fee being increasing in the advantage, will become clearer later in the analysis.

3.5 The Equilibrium

First, we establish the subgame perfect equilibrium just depending on the exogenous variables. Then we analyze the effects of the exogenous parameters on the expected profits of the gatekeeper. In addition, we will discuss the effects of the fraction of loyal consumers on the outcome.

3.5.1 The Subgame Perfect Equilibrium

To establish the subgame perfect equilibrium, we have to insert the optimal advertising fees of the gatekeeper into the equilibria in Proposition 3.3 and 3.7 for the respective parameters. From this we get immediately the following result.

Proposition 3.12. *If $V_H - c_H \in [V_L - c_L, (V_L - c_L)\sqrt{2})$ then there is a unique subgame perfect equilibrium in which the gatekeeper sets an advertising fee*

$$\Phi^* = \frac{(V_H - c_H)^2(1 - 2L)}{2(V_H - c_H + V_L - c_L)}$$

and the firms use the advertising and pricing strategies presented in Proposition 3.3.

If $V_H - c_H \in [(V_L - c_L)\sqrt{2}, 2(V_L - c_L))$, then there is a unique subgame perfect equilibrium in which the gatekeeper sets an advertising fee

$$\Phi^* = [V_H - c_H - (V_L - c_L)](1 - 2\lambda)$$

and the firms use the advertising and pricing strategy presented in Proposition 3.7.

If $V_H - c_H \in [2(V_L - c_L), \infty)$, then there is a unique subgame perfect equilibrium in which the gatekeeper sets an advertising fee

$$\Phi^* = (V_H - c_H) \left(\frac{1}{2} - \lambda \right)$$

and the high quality firm advertises with certainty and the low quality firms abstains from advertising. The low quality firm charges V_L and the high quality firm V_H .

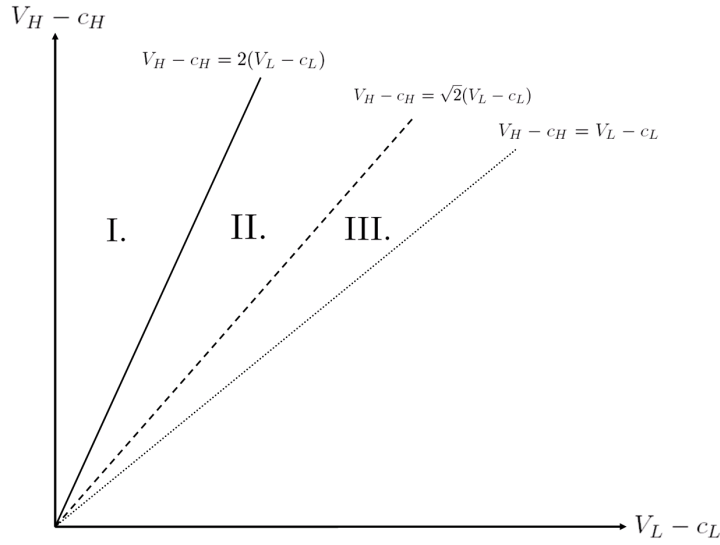


Figure 3.1: Different Areas of the Equilibrium

Thus, we have a unique subgame perfect equilibrium for all possible combinations of $V_H - c_H$ and $V_L - c_L$. Figure 3.1 depicts all combinations. On the 45-degree-line are all combination for which there is no advantage. In region I only the high quality firms advertises. In region II and III both firms advertise with positive probability. Whereas in the high quality firm advertises with certainty and the low quality firm advertises with positive probability strictly less than one in region II. In region III both firms advertises with probability less than one. The combinations below the 45-degree-line represent all combinations in which the low quality firm have a advantage, which are excluded by assumption.¹¹

3.5.2 Comparative Statics

Now, we want to analyze how the strategy of the gatekeeper influences the advertising behavior of the two firms. Moreover, we want to examine how expected profits of the gatekeeper respond to changes in the profitability of firms. Moreover, we do some welfare analysis.

The Expected Profits of the Gatekeeper

In the equilibrium, the gatekeeper raises the advertising fee if $V_H - c_H$ increases. However, it does not say something about the advertising behavior of firms.

¹¹If the low quality would have a advantage, then the figure is similar. It is simply mirrored around the 45-degree-line.

Proposition 3.13. *In the equilibrium, α_H is increasing in the advantage if $V_H - c_H < (V_L - c_L)\sqrt{2}$ and constant otherwise; α_L is decreasing in the advantage if $V_H - c_H < 2(V_L - c_L)$ and constant otherwise.*

From Proposition 3.11 we know that the gatekeeper raises the advertising fee when the advantage increases. Moreover, Proposition 3.13 states that the gatekeeper does it in such a way that the two advertising intensities drift apart given that the low quality firm is advertising with positive probability.

The next proposition establishes the central result of this paper. It states that the gatekeeper prefers markets with larger advantages.

Proposition 3.14. *In the equilibrium, the expected profits of the gatekeeper are increasing in the advantage.*

This suggests that an intense price competition which is induced by large advertising intensities of both firms is not beneficial for the gatekeeper. If $V_H - c_H - (V_L - c_L)$ is small, both firms advertise with similar probabilities. However, if the advantage increases the gatekeeper increases the advertising fee. This induces the advertising intensities to drift apart and price competition decreases. Thus, the sales to the non-loyal consumers are predominantly made by the high quality firm. If $V_H - c_H$ exceeds $2(V_L - c_L)$ then the gatekeeper charges a price which induces the low quality firm to abstain from advertising and all the sales to the non-loyal consumers are made by the high quality firm.

It seems to be natural, that the gatekeeper can benefit from the fact that a firm in the market becomes more profitable, that is $V_H - c_H + V_L - c_L$ increases, since its willingness to pay for advertising increases. The next proposition establishes this.

Proposition 3.15. *In the equilibrium, the expected profits are increasing in the aggregate profitability of the firms, when the advantage is kept constant.*

Therefore, the gatekeeper benefits if the aggregate profitability increases given that the advantage does not increase. However, the gatekeeper's expected profits do not generally increase in the aggregate profitability. The next proposition establishes that the gatekeepers profits can decrease in the advantage.

Proposition 3.16. *In the equilibrium, the expected profits are decreasing in $V_L - c_L$ if $V_H - c_H < 2(V_L - c_L)$ and constant otherwise.*

This underlines the role of the advantage. Even if the aggregate profitability increases, the gatekeeper does not benefit from it. The rationale for this is that price competition becomes more intensive if the firm L becomes more profitable (given that the low quality firm advertises with positive probability). Thus firms are less willing to pay for the advertisement.

Furthermore, the expected profits of the gatekeeper are increasing in the fraction of non-loyal consumers.

Proposition 3.17. *The expected profits of the gatekeeper increase in the fraction of non-loyal consumers.*

The equilibrium advertising propensities are decreasing in the advantage. However, the gatekeeper's advertising fee decreases in the fraction of loyal consumers. Hence, the expected profits of the gatekeeper increase if the fraction of non-loyal customers increase. Moreover, the gatekeeper's expected profits approach $-C_G$ as $\lambda \rightarrow \frac{1}{2}$. Thus, the gatekeeper is not active if λ is large.

Social Welfare

Advertising in this model allocates non-loyal consumers to the firm which offers the largest utility. Hence, non-loyal consumers do profit from advertising as well as loyal consumers, since they free ride on the information of the non-loyal consumers. In our model only the existence of the gatekeeper makes it possible for the firms to compete for non-loyal consumers and hence transfers profits from firms to the gatekeeper and to the consumers. Thus, firms are worse off with the gatekeeper and consumers better off. Therefore, the existence of the gatekeeper redistribution of the profits. Moreover, since $V_H - c_H > V_L - c_L$ holds, good H is the social efficient good. The existence of the gatekeeper makes it possible that more consumers buy good H instead of good L and thus may enhance welfare. However, the existence causes fix costs of C_G . Therefore, the existence of a gatekeeper is detrimental to welfare if the advantage is small. However, if $V_H - c_H \geq 2(V_L - c_L)$, then all the non-loyal in the market L consume good H instead of good L and thus the existence of the gatekeeper is efficient iff

$$[V_H - c_H - (V_L - c_L)] \left(\frac{1}{2} - \lambda \right) - C_G \geq (V_L - c_L) \left(\frac{1}{2} - \lambda \right) - C_G \geq 0.$$

Thus, the existence of the gatekeeper enhances the social welfare if the advantage is large enough.

3.6 Conclusion

We considered a duopoly market in which the firms are allowed to differ in their profitability. More precisely, we allowed the firms to offer vertically differentiated goods and to be different in costs. We analyzed the case when the high quality firm has an advantage. That is, it has a larger profitability. Moreover, each firm is a monopolist in a local market. However, through the existence

of the gatekeeper the firms can compete for a part of the consumers who have access to the gatekeeper's website.

We have shown, that there exists a unique subgame perfect equilibrium in which the firm with the advantage advertises more aggressively than its competitor. Moreover, in those subgames in which both firms advertise, there is a regime switch. If the advertising fee is small, the demand for advertising decreases in the advantage. If the advertising fee is large, then the demand for advertising is increasing in the advantage. Furthermore, the expected profits of the high quality firm is larger than those of her competitor and the difference increase if the advantage increases.

If the advantage of the high quality firm is small, then the advertising fee set by the gatekeeper is such that both firms advertise with positive probability less than one. For a moderate advantage the gatekeeper sets the advertising fee such that the high quality firm advertises with certainty. Moreover, for large advantages, the gatekeeper sets prices such that the low quality firm does not advertise. Only the high quality firm advertises with certainty. The gatekeeper benefits from a large advantage, since it can set the fee such that the advertising propensities drift apart and thus price competition becomes less intensive. Therefore, the firm's are more willing to pay for advertising. Thus, the gatekeeper can extract larger rents from the firms. Therefore, the expected profits of the gatekeeper are increasing in the advantage. Moreover, given any advantage, the firms are more willing to pay for advertisements if their profitability increases. This can be exploited by the gatekeeper. However, aggregate profitability of the firms is not in general advantageous for the gatekeeper. An increase in the aggregate profitability can decrease the gatekeeper's expected profits if the advantage decreases, since price competition intensifies. In addition, the expected profits increase in the fraction of consumers who can be reached via advertising (non-loyal consumers).

Moreover, if the advantage is large, the existence of the gatekeeper increases the social welfare, since all non-loyal consumers purchase the socially efficient good. Thus, this increase in efficiency exceeds the fix costs of the gatekeeper.

This indicates that gatekeepers are more likely to occur in markets in which firms have large differences in profitabilities. Thus, in the market with symmetric firms analyzed by Baye and Morgan (2001), it is most unlikely to find gatekeepers. Moreover, a social planner should prohibit gatekeepers to enter symmetric markets in our model.

3.A Proofs

3.A.1 Proof of Lemma 3.1

Proof. Assume both firms do not advertise their prices. Then both firms attract one half of the consumers. Since firm i knows the consumers reservation price V_i at which the consumers are willing to purchase the good, firm i does not have an incentive to charge a lower price.

Now, we want to show that lower prices than V_H cannot be optimal for firm H if firm H does not advertise. Assume it is optimal. Then firm H attracts half of the consumers if the competitor does not advertise independently of the price charged. Hence any price smaller than V_H cannot belong to an optimal strategy. However, even if the other firm advertises, the fraction of consumer attracted by firm H does only depend on the price advertised by firm L . If firm L advertises a price larger than V_L , then H attracts one half of the consumers. If firm L advertises a price not larger than V_L , then the firm H attracts only a fraction λ . Thus the number of consumers attracted by firm H is independent of the price charged by firm H . Therefore, firm H has no incentive to charge prices lower than V_H .

Showing that firm L charges V_L if it does not advertise, is analogous. $\square$

3.A.2 Proof of Proposition 3.1

Proof. If firm L has an incentive to advertise, also firm H has an incentive to advertise, since

$$(V_H - c_H) \left(\frac{1}{2} - \lambda \right) - \Phi > (V_L - c_L) \left(\frac{1}{2} - \lambda \right) - \Phi \geq 0$$

holds. Thus, the only equilibrium in which one firm advertises is one in which firm H advertises. Furthermore, in such an equilibrium firm H does not advertise prices lower than V_H . Moreover, in the equilibrium Φ must be such that firm L does not advertise. Thus, Φ must satisfy

$$(V_L - c_L) (1 - 2\lambda) - \Phi \leq 0.$$

The rest follows immediately. $\square$

3.A.3 Proof of Lemma 3.2

Proof. Note that advertising is generally profitable for both firms, that is $(V_i - c_i) \left(\frac{1}{2} - \lambda \right) > \Phi$ holds. Hence, it can not be optimal for both firms to abstain from advertising. Assume now, firm i advertises with probability zero and firm

j advertises with positive probability. Therefore, firm j knows, that it attracts the maximal consumers it can get, $(1 - \lambda)$, if it charges prices not larger V_j . Hence, the only price it advertises is V_j . However, there exists an $\epsilon > 0$ such that $(V_i - \epsilon - c_i)(1 - 2\lambda) - \Phi > (V_i - c_i)(\frac{1}{2} - \lambda) - \Phi > 0$. Thus, there is no equilibrium in which one firm does not advertise at all. $\square$

3.A.4 Proof of Lemma 3.3

Proof. Assume that both firms advertise with probability one. First, we note that the upper bound of the price range advertised by both firms must be such that $\bar{p}_H = \bar{p}_L + (V_H - V_L)$. If this would not be the case then one firm would never attract consumers if it charges the upper bound.

Consider now, that firm i charges the upper bound of the price distribution of firm $\bar{p}_i$. If firm j does not have a mass point at $\bar{p}_j$, then it attracts $(1 - \lambda)$ consumers with probability $1 - F_j(\bar{p}_j) = 0$. Therefore, firm i would strictly prefer to abstain from advertising instead of advertising price $\bar{p}_i$. If firm j has a mass point at $\bar{p}_j$, then firm i 's expected profits from charging $\bar{p}_i$ are given by $(\bar{p}_i - c_i)[1 - F_j(\bar{p}_j)]\{\lambda + \frac{1-2\lambda}{2}\} - \Phi$. However, there exists an $\epsilon > 0$ such that

$$(\bar{p}_i - \epsilon - c_i)\{\lambda + [1 - F_j(\bar{p}_j - \epsilon)](1 - 2\lambda)\} - \Phi > (\bar{p}_i - c_i)\{\lambda + [1 - F_j(\bar{p}_j)]\left(\frac{1}{2} - \lambda\right)\} - \Phi.$$

Therefore, it cannot be optimal for firm i to charge the upper bound $\bar{p}_i$. Consequently, there does not exist an equilibrium in which both firm advertise with certainty. $\square$

3.A.5 Proof of Lemma 3.4

Proof. By Lemma 3.2, both firms advertise in the equilibrium if $\Phi < \min\{(V_L - c_L)(1 - 2\lambda), (V_H - c_H)(\frac{1}{2} - \lambda)\}$.

First, we want to show that at least one firm uses a mixed pricing strategy when it advertises. By contradiction, assume that both firms use deterministic pricing strategies given that they advertise, which we denote as $\hat{p}_i$ for $i = 1, 2$. $\hat{p}_L < \hat{p}_H - V_H - V_L$ cannot hold, since the low quality firm could charge a price $\hat{p}_L + \epsilon$ for $\epsilon > 0$ small enough that $\hat{p}_L + \epsilon < \hat{p}_H - V_H - V_L$ holds, which still attracts all non-loyals when it advertises. A similar argument holds for $\hat{p}_L > \hat{p}_H - V_H - V_L$. If $\hat{p}_L = \hat{p}_H - V_H - V_L$ holds, then both firms attracts half of the non-loyals. Thus, both firms have an incentive to decrease the price by a small amount to attract all non-loyals. Therefore, at least one firm use a mixed pricing strategy in the equilibrium.

Suppose that the high quality firm uses a deterministic pricing strategy $\hat{p}_H$ when she advertises and the low quality firm uses a mixed pricing strategy $F_L(\cdot)$

when she advertises. Then consider $\epsilon > 0$ and two prices $\hat{p}_H - (V_H - V_L) + \epsilon$ and $\hat{p}_H - (V_H - V_L) - \epsilon$. The difference of the expected profits of these two prices is

$$[\hat{p} - (V_H - V_L) - \epsilon - c_L](1 - \lambda) - [\hat{p} - (V_H - V_L) + \epsilon - c_L] \cdot [\lambda + (1 - \alpha_H)(1 - 2\lambda)] > 0$$

for ϵ small enough. Therefore, for some ϵ prices in the interval $[\hat{p}_H - (V_H - V_L), \hat{p}_H - (V_H - V_L) + \epsilon]$ cannot belong to the support of the optimal pricing distribution of firm L . Hence, firm H can increase its deterministic price $\hat{p}_H$ without loosing consumers. This contradicts optimality. Moreover, $\hat{p}_H = V_H$ cannot hold, then the low quality firm does not advertise price V_L , since there exist an $\epsilon > 0$ such that $(V_L - \epsilon - c_L)(1 - 2\lambda) - \Phi > 0$.

Similarly, it is not optimal for firm L to use a deterministic pricing strategy when firm H uses a mixed advertised pricing strategy. $\square$

3.A.6 Proof of Lemma 3.5

Proof. Assume that the difference of the lower bounds is not equal to the difference of the products' qualities, $V_H - V_L$. First, we assume $\underline{p}_H - \underline{p}_L < V_H - V_L$. Then, firm H attracts all shoppers if it advertises a price $\underline{p}_H$. That is, the profits are given by $(\underline{p}_H - c_H)(1 - \lambda) - \Phi$. However, then there exists an $\epsilon > 0$ sufficiently small such that

$$(\underline{p}_H - c_H)(1 - \lambda) - \Phi < (\underline{p}_H + \epsilon - c_H)(1 - \lambda) - \Phi,$$

since $V_H - \underline{p}_H > V_L - \underline{p}_L$.

Similarly, we can show that $\underline{p}_H - \underline{p}_L > V_H - V_L$ cannot be optimal. Hence, in any equilibrium we must have $\underline{p}_H - \underline{p}_L = V_H - V_L$. $\square$

3.A.7 Proof of Lemma 3.6

Proof. Suppose firm H has a mass point in the interval $[\underline{p}_H, V_H)$. First, we suppose that the mass point of firm H is at $\underline{p}_H$. Then firm L 's expected profits of charging the lower bound $\underline{p}_L$ are given by $(\underline{p}_L - c_L)\{\alpha_H F_H(\underline{p}_H)^{\frac{1}{2}} + [1 - \alpha_H F_H(\underline{p}_H)](1 - \lambda)\}$, by Lemma 3.5. Consequently, there exists an $\epsilon > 0$ such that

$$\begin{aligned} (\underline{p}_L - \epsilon - c_L)(1 - \lambda) &> (\underline{p}_L - c_L) \left\{ \alpha_H F_H(\underline{p}_H)^{\frac{1}{2}} + [1 - \alpha_H F_H(\underline{p}_H)](1 - \lambda) \right\} \\ &> (\underline{p}_L + \epsilon - c_L) \{ \alpha_H F_H(\underline{p}_H + \epsilon) \lambda + [1 - \alpha_H F_H(\underline{p}_H + \epsilon)](1 - \lambda) \}. \end{aligned}$$

Thus, the interval $[\underline{p}_L, \underline{p}_L + \epsilon)$ cannot belong to the support of firm L . Hence, firm H can increase $\underline{p}_H$ without losing consumers. This contradicts optimality. Therefore, there cannot be a mass point at $\underline{p}_H$.

Suppose that firm H has a mass point at $p^m \in (\underline{p}_H, V_H)$. Consider a small $\epsilon > 0$. For ϵ small enough, the prices $p^m - (V_H - V_L) + \epsilon$ and $p^m - (V_H - V_L) - \epsilon$ the expected profits fulfill

$$\begin{aligned} & [p^m - (V_H - V_L) - \epsilon - c_L] \{F_H(p^m - (V_H - V_L) - \epsilon)\lambda \\ & \quad + [1 - F_H(p^m - (V_H - V_L) - \epsilon)](1 - \lambda)\} \\ & - [p^m - (V_H - V_L) + \epsilon - c_L] \{F_H(p^m - (V_H - V_L) + \epsilon)\lambda \\ & \quad + [1 - F_H(p^m - (V_H - V_L) + \epsilon)](1 - \lambda)\} > 0, \end{aligned}$$

since firm H has a mass point at p^m . Therefore, for some $\epsilon > 0$, prices in the interval $[p^m - (V_H - V_L), p^m - (V_H - V_L) + \epsilon]$ cannot belong to the support of the optimal pricing distribution of firm L . Hence, firm H can increase its mass point without losing consumers. This increases the expected profits of firm L , which contradicts optimality.

The proof that firm L has no mass point in the interval $[\underline{p}_L, V_L)$ is analogous. $\square$

3.A.8 Proof of Lemma 3.7

Proof. By Lemma 3.6, both pricing distributions do not have mass points on $[\underline{p}_i, V_i)$ for $i = H, L$. Suppose firm H 's equilibrium strategy distribution is not connected from (p_1, p_2) . Then it is not optimal for firm L to charge prices in the interval $(p_1 - (V_H - V_L), p_2 - (V_H - V_L))$, since all prices in the interval yield the same number of expected consumers. However, it cannot be optimal for firm H to charge a price p_1 , since it can charge prices in the interval $p \in (p_1, p_2)$ without losing consumers, but increase the profit per sale to $(p - c_H)$. This contradicts optimality. Hence, the support of the equilibrium distribution of firm H must be connected. Similar arguments hold for firm L . $\square$

3.A.9 Proof of Lemma 3.8

Proof. Suppose both firms have a mass point at V_i for $i = H, L$. Because of the implied discontinuity of $F_i(\cdot)$, there exists an $\epsilon > 0$ such that the following

holds:

$$(V_j - \epsilon - c_j) \{ \alpha_i F_i(V_i - \epsilon) \lambda + [1 - \alpha_i F_i(V_i)](1 - \lambda) \} \\ - (V_j - c_j) \left\{ \alpha_i F_i(V_i) \lambda + [1 - \alpha_i F_i(V_i)] \frac{1}{2} \right\} > 0.$$

Hence, if firm i has an atom at V_i , it is not optimal for firm j to charge V_j . $\square$

3.A.10 Proof of Proposition 3.2

Proof. The high quality firm has an advantage, that is $V_H - c_H - (V_L - c_L) > 0$. By contradiction we assume that $\alpha_L = 1$. Thus, it follows by Lemma 3.3, that $\alpha_H < 1$. Moreover, we have $\tilde{p}_L > \tilde{p}_H - (V_H - V_L)$. That is,

$$\frac{(V_L - c_L) [\lambda + (1 - \alpha_H) (\frac{1}{2} - \lambda)] + \Phi}{1 - \lambda} + c_L > \frac{(V_H - c_H) \lambda + \Phi}{1 - \lambda} + c_H - (V_H - V_L)$$

holds. This can be seen more easily if we simplify the inequality to

$$(V_L - c_L) \left[\lambda + (1 - \alpha_H) \left(\frac{1}{2} - \lambda \right) \right] + [V_H - c_H - (V_L - c_L)](1 - 2\lambda) > 0.$$

This holds since we assumed $V_H - c_H - (V_L - c_L) > 0$. However, by Lemma 3.5 and $\tilde{p}_L > \tilde{p}_H - (V_H - V_L)$, we have $\underline{p}_L > \tilde{p}_H - (V_H - V_L)$. However, then the high quality firm can advertise a price $\underline{p}_L + V_H - V_L$ which attracts a fraction of $1 - \lambda$ of the consumers. Hence, the profits are $\Pi_H^A(\underline{p}_L + V_H - V_L) = (\underline{p}_L + V_H - V_L - c_H)(1 - \lambda) - \Phi > (V_H - c_H)\lambda = \Pi_H^N(V_H)$. However, then a mixed advertising strategy $\alpha_H < 1$ cannot be optimal since the high quality firm strictly prefers to advertise. Which contradicts $\alpha_H < 1$. $\square$

3.A.11 Proof of Proposition 3.3

Proof. Suppose that $\alpha_i < 1$ for $i = 1, 2$. Hence, advertising and not advertising V_H yield the same expected profits for firm H . Therefore, we have

$$(V_H - c_H) \left[\lambda + (1 - \alpha_L) \left(\frac{1}{2} - \lambda \right) \right] = (V_H - c_H) [\lambda + (1 - \alpha_L)(1 - 2\lambda)] - \Phi \quad (3A.1)$$

and hence we obtain the unique equilibrium advertising propensity of firm L

$$\alpha_L = 1 - \frac{2\Phi}{(1 - 2\lambda)(V_H - c_H)}.$$

Moreover, the difference of the lowest prices which the firms are willing to advertise must be equal to $V_H - V_L$. That is, the following holds

$$\begin{aligned} & \frac{(V_H - c_H)[\lambda + (1 - \alpha_L)(\frac{1}{2} - \lambda)] + \Phi}{1 - \lambda} + c_H \\ &= \frac{(V_L - c_L)[\lambda + (1 - \alpha_H)(\frac{1}{2} - \lambda)] + \Phi}{1 - \lambda} + c_L + V_H - V_L. \end{aligned} \quad (3A.2)$$

We can insert α_L and solve for α_H . We obtain the unique advertising propensity of firm H

$$\alpha_H = 2 \left[\frac{(V_H - c_H)(1 - 2\lambda) - \Phi}{(V_L - c_L)(1 - 2\lambda)} \right] - 1.$$

Any advertised price charged by firm L yields the same expected profits as abstaining from advertising. That is

$$\begin{aligned} & (V_L - c_L) \left[\lambda + (1 - \alpha_H) \frac{1 - 2\lambda}{2} \right] \\ &= (p - c_L) [\lambda + (1 - \alpha_H F_H(p + V_H - V_L))(1 - 2\lambda)] - \Phi. \end{aligned} \quad (3A.3)$$

holds. Expressing $F_H(p + V_H - V_L)$ yields the unique advertised pricing distribution

$$F_H(p + V_H - V_L) = \frac{1}{\alpha_H} \left(1 - \frac{(V_L - p)\lambda + (V_L - c_L)(1 - \alpha_H) \frac{1 - 2\lambda}{2} + \Phi}{(p - c_L)(1 - 2\lambda)} \right)$$

and hence we receive the equilibrium distribution

$$F_H(p) = \frac{1}{\alpha_H} \left(1 - \frac{(V_H - p)\lambda + (V_L - c_L)(1 - \alpha_H) \frac{1 - 2\lambda}{2} + \Phi}{(p - (V_H - V_L) - c_L)(1 - 2\lambda)} \right),$$

which is the same as

$$F_H(p) = \frac{1}{\alpha_H} \left(1 - \frac{(V_H - p)\lambda - [V_H - c_H - (V_L - c_L)](1 - 2\lambda) + 2\Phi}{(p - (V_H - V_L) - c_L)(1 - 2\lambda)} \right).$$

The lower bound fulfills $F_H(\underline{p}_H) = 0$ and hence we get

$$\underline{p}_H = V_H - \frac{(V_H - c_H)(1 - 2\lambda) - 2\Phi}{1 - \lambda}.$$

The c.d.f. at the upper bound becomes $F_H(V_H) = \frac{1}{\alpha_H} \left(\alpha_H - \frac{V_H - c_H - (V_L - c_L)}{V_L - c_L} \right)$ and is smaller one, since $V_H - c_H - (V_L - c_L) > 0$ holds.

Moreover, any price charged by firm H yields the same profits as not ad-

vertising. Thus, we have

$$(V_H - c_H) \left[\lambda + (1 - \alpha_L) \left(\frac{1}{2} - \lambda \right) \right] = (p - c_H) \{ \lambda + [1 - \alpha_L F_L(p - (V_H - V_L))](1 - 2\lambda) \} - \Phi. \quad (3A.4)$$

Hence, we have the unique advertised pricing strategy of firm L

$$F_L(p) = \frac{1}{1 - \frac{2\Phi}{(1-2\lambda)(V_H - c_H)}} \left(1 - \frac{(V_L - p)\lambda + 2\Phi}{(p + V_H - V_L - c_H)(1 - 2\lambda)} \right).$$

It is easy to see that $F_L(V_L) = 1$. Moreover, for the lower bound $F_L(p_L) = 0$, hence we can calculate

$$p_L = V_L - \frac{(V_H - c_H)(1 - 2\lambda) - 2\Phi}{1 - \lambda}.$$

Finally, we have to verify when $0 < \alpha_L < 1$ and $0 < \alpha_H \leq 1$ holds. That is, $0 < \Phi < \frac{(V_H - c_H)(1 - 2\lambda)}{2}$ and $[V_H - c_H - (V_L - c_L)](1 - 2\lambda) \leq \Phi < \frac{(V_H - c_H)(1 - 2\lambda)}{2}$.

Since equations (3A.1), (3A.2), (3A.3) and (3A.4) must hold in this equilibrium, α_i and $F_i(\cdot)$ are uniquely defined for $i = 1, 2$ and thus the equilibrium is unique. $\square$

3.A.12 Proof of Proposition 3.4

Proof. We want to show that $\alpha_H - \alpha_L \geq 0$. Thus, we have

$$\begin{aligned} \alpha_H - \alpha_L &= 2 \frac{(V_H - c_H)(1 - 2\lambda) - \phi}{(V_L - c_L)(1 - 2\lambda)} - 1 - \left(1 - \frac{2\phi}{(V_H - c_H)(1 - 2\lambda)} \right) \\ &= 2 \left(\frac{V_H - c_H}{V_L - c_L} - \frac{[V_H - c_H - (V_L - c_L)]}{(V_H - c_H)(V_L - c_L)(1 - 2\lambda)} \right) \\ &= 2 \frac{[V_H - c_H - (V_L - c_L)] \cdot [(V_H - c_H)(1 - 2\lambda) - \phi]}{(V_H - c_H)(V_L - c_L)(1 - 2\lambda)} \geq 0. \end{aligned} \quad (3A.5)$$

The inequality follows from the fact, that for the equilibrium in Proposition 3.3 $\phi < (V_H - c_H)(1 - 2\lambda)$. $\square$

3.A.13 Proof of Proposition 3.5

Proof. The expected amount of advertising is given by

$$\alpha_L + \alpha_H = 2 - 2 \left[1 - \frac{(V_H - c_H)(1 - 2\lambda) - \Phi}{(V_L - c_L)(1 - 2\lambda)} \right] - \frac{2\Phi}{(V_H - c_H)(1 - 2\lambda)}$$

which derivative with respect to $V_H - c_H$ is given by

$$\frac{\partial(\alpha_L + \alpha_H)}{\partial(V_H - c_H)} = \frac{(1 - 2\lambda)}{(1 - 2\lambda)(V_L - c_L)} + \frac{2\Phi}{(V_H - c_H)^2(1 - 2\lambda)} > 0$$

and the derivative with respect to $V_L - c_L$ is given by

$$\frac{\partial(\alpha_L + \alpha_H)}{\partial(V_L - c_L)} = -\frac{(V_H - c_H)(1 - 2\lambda) - \Phi}{(V_L - c_L)^2(1 - 2\lambda)} < 0.$$

These two derivatives establish the result. $\square$

3.A.14 Proof of Proposition 3.6

Proof. Since the low quality firm uses a mixed strategy, the expected profits of the low quality firm are simply given by the expected profits if it does not advertise. That is

$$\begin{aligned} \Pi_L = \Pi_L^N &= (V_L - c_L) \left[2 - 2 \frac{(V_H - c_H)(1 - 2\lambda) - \Phi}{(V_L - c_L)(1 - 2\lambda)} \left(\frac{1}{2} - \lambda \right) \right] \\ &= (V_L - c_L)\lambda - [V_H - c_H - (V_L - c_L)](1 - \lambda) + \Phi \end{aligned}$$

and the profits of the high quality firm are given by

$$\Pi_H = \Pi_H^N = (V_H - c_H) \left[\lambda + \frac{2\Phi}{(V_H - c_H)(1 - 2\lambda)} \left(\frac{1}{2} - \lambda \right) \right] = (V_H - c_H)\lambda + \Phi.$$

Hence, the difference is given by

$$\Pi_H - \Pi_L = [V_H - c_H - (V_L - c_L)](1 - \lambda) > 0.$$

Obviously it increases as the advantage increases. $\square$

3.A.15 Proof of Proposition 3.7

Proof. Suppose $\alpha_H = 1$. By Lemma 3.3 $\alpha_L < 1$. Thus, we must have that $\tilde{p}_H - (V_H - V_L) \leq \tilde{p}_L$ holds, which is

$$\frac{(V_H - c_H) \left[\lambda + (1 - \alpha_L) \left(\frac{1}{2} - \lambda \right) \right] + \Phi}{1 - \lambda} + c_H - (V_H - V_L) \leq \frac{(V_L - c_L)\lambda + \Phi}{1 - \lambda} + c_L.$$

This can be reduced to

$$-[V_H - c_H - 2(V_L - c_L)](1 - 2\lambda) \leq (V_H - c_H)\alpha_L(1 - 2\lambda).$$

Any price charged by firm L fulfills

$$(p - c_L) [\lambda + (1 - F_H(p + V_H - V_L))(1 - 2\lambda)] - \Phi = (V_L - c_L)\lambda. \quad (3A.6)$$

From this condition we can express the unique advertised pricing strategy, which is

$$F_H(p + V_H - V_L) = 1 - \frac{(V_L - p)\lambda + \Phi}{(p - c_L)(1 - 2\lambda)}.$$

Hence we have

$$F_H(p) = 1 - \frac{(V_H - p)\lambda + \Phi}{(p - (V_H - V_L) - c_L)(1 - 2\lambda)}.$$

Since the lower bound fulfills $F_H(\underline{p}_H) = 0$, we obtain

$$\underline{p}_H = V_H - \frac{(V_L - c_L)(1 - 2\lambda) - \Phi}{1 - \lambda}$$

For the upper bound, V_H , we have $F_H(V_H) = 1 - \frac{\Phi}{(V_L - c_L)(1 - 2\lambda)} < 1$, since $\Phi > 0$ holds by assumption.

If firm H charges the lower bound $\underline{p}_H$ it must have the same expected profits as any other price charged, which is

$$(\underline{p}_H - c_H)(1 - \lambda) - \Phi = (p - c_H) [\lambda + (1 - \alpha_L F_L(p - (V_H - V_L)))(1 - 2\lambda)] - \Phi. \quad (3A.7)$$

Hence, we can express the unique advertised pricing strategy

$$F_L(p - (V_H - V_L)) = \frac{1}{\alpha_L} \left(1 - \frac{(\underline{p}_H - c_H)(1 - \lambda) - (p - c_H)\lambda}{(p - c_H)(1 - 2\lambda)} \right)$$

and then have

$$F_L(p) = \frac{1}{\alpha_L} \left(1 - \frac{(\underline{p}_H - c_H)(1 - \lambda) - (p + V_H - V_L - c_H)\lambda}{(p + V_H - V_L - c_H)(1 - 2\lambda)} \right)$$

and by inserting for $\underline{p}_H$ we have

$$F_L(p) = \frac{1}{\alpha_L} \left(1 - \frac{(V_L - p)\lambda + [V_H - c_H - (V_L - c_L)](1 - 2\lambda) + \Phi}{(p + V_H - V_L - c_H)(1 - 2\lambda)} \right)$$

and the lower bound is given by

$$\underline{p}_L = \frac{V_L\lambda + c_L(1 - 2\lambda) + \Phi}{1 - \lambda}.$$

Furthermore, charging the lower bound must yield the same expected profits

as charging the upper bound. Hence, we have

$$(\underline{p}_H - c_H)(1 - \lambda) - \Phi = (V_H - c_H)[\lambda + (1 - \alpha_L)(1 - 2\lambda)] - \Phi. \quad (3A.8)$$

By inserting for $\underline{p}_H$ and solving for α_L , we obtain

$$\alpha_L = \frac{(V_L - c_L)(1 - 2\lambda) - \Phi}{(V_H - c_H)(1 - 2\lambda)}.$$

Since equations (3A.6), (3A.7) and (3A.8) must hold, the strategies are uniquely defined and thus the equilibrium is unique. $\square$

3.A.16 Proof of Proposition 3.8

Proof. The expected demand for advertising is given by

$$\alpha_L + \alpha_H = \frac{(V_L - c_L)(1 - 2\lambda) - \Phi}{(V_H - c_H)(1 - 2\lambda)} + 1.$$

It can easily be seen that $\frac{\partial(\alpha_L + \alpha_H)}{\partial(V_H - c_H)} < 0$. This establishes the result. $\square$

3.A.17 Proof of Proposition 3.9

Proof. Since the low quality firm mixes between advertising and not advertising, the expected profits are given by the expected profits of non-advertising, which is simply $\Pi_L = \Pi_L^N = (V_L - c_L)\lambda$. The low quality firm advertises V_L with probability zero. Hence, the expected profits of the high quality firm are given by

$$\begin{aligned} \Pi_H &= \Pi_H^A = (V_H - c_H) \left[\lambda + \left(1 - \frac{(V_L - c_L)(1 - 2\lambda) - \Phi}{(V_H - c_H)(1 - 2\lambda)} \right) (1 - 2\lambda) \right] - \Phi \\ &= (V_H - c_H)\lambda + [V_H - c_H - (V_L - c_L)](1 - 2\lambda). \end{aligned}$$

Therefore, the difference of the expected profits is given by

$$\Pi_H - \Pi_L = [V_H - c_H - (V_L - c_L)](1 - \lambda) > 0,$$

Obviously, the difference is increasing in the advantage. $\square$

3.A.18 Proof of Proposition 3.10

Our strategy is to restrict the advertising fee such that it can only be chosen in the valid interval for each of the three different categories of subgames. Afterwards, we will combine these constrained choices of the advertising fee. We

start with the subgames in which only one firm advertises.

The Subgames with One Advertising Firm

If only the high quality firm advertises, then the only possible equilibrium candidate is the one in which the gatekeeper extracts all rents from advertising. Hence, we can establish the following equilibrium.

Lemma 3.9. *If $V_H - c_H \geq 2(V_L - c_L)$ holds, then the gatekeeper sets the advertising fee*

$$\Phi^* = (V_H - c_H) \left(\frac{1}{2} - \lambda \right).$$

Proof. Assume that the gatekeeper's advertising fee is restricted to the interval $(V_L - c_L)(1 - 2\lambda) \leq \Phi \leq (V_H - c_H) \left(\frac{1}{2} - \lambda \right)$ and that it is nonempty. Moreover, assume that the firms use the strategies described in Proposition 3.1. By contradiction assume that the gatekeeper sets a price $\Phi < (V_H - c_H) \left(\frac{1}{2} - \lambda \right)$. Then the expected demand is one. However, we can find an $\epsilon > 0$ small enough for which

$$\Pi_G(\Phi + \epsilon) = \Phi + \epsilon - C_G > \Phi - C_G = \Pi_G(\Phi).$$

Hence, $\Phi < (V_H - c_H) \left(\frac{1}{2} - \lambda \right)$ cannot be optimal.

However, these subgames are only achievable if $(V_L - c_L)(1 - 2\lambda) \leq (V_H - c_H) \left(\frac{1}{2} - \lambda \right)$, which is the case if $V_H - c_H \geq 2(V_L - c_L)$. $\square$

The Subgames of Proposition 3.3

Now, we consider that the two firms behave as in Proposition 3.3. That is, we restrict Φ to be in the interval described in condition (3.6). We can establish the following.

Lemma 3.10. *If $V_H - c_H < 2(V_L - c_L)$ holds, then the gatekeeper uses the strategy*

$$\Phi^* = \begin{cases} \frac{(V_H - c_H)^2(1 - 2\lambda)}{2(V_H - c_H + V_L - c_L)} & \text{if } V_H - c_H \leq (V_L - c_L)\sqrt{2} \\ [V_H - c_H - (V_L - c_L)](1 - 2\lambda) & \text{otherwise.} \end{cases} \quad (3A.9)$$

Proof. Assume that the firms use the equilibrium strategies described in Proposition 3.3. This results into the following maximization problem of the gatekeeper.

$$\max_{\Phi \in [(V_H - c_H - (V_L - c_L))(1 - 2\lambda), \min\{(V_L - c_L)(1 - 2\lambda), (V_H - c_H)(1 - 2\lambda)/2\}]} \Pi_G(\Phi),$$

whereas

$$\Pi_G(\Phi) = \left\{ 2 \frac{(V_H - c_H)(1 - 2\lambda) - \Phi}{(V_L - c_L)(1 - 2\lambda)} - \frac{2\Phi}{(V_H - c_H)(1 - 2\lambda)} \right\} \Phi - C_G$$

Taking the first derivative with respect to Φ and setting it equal to zero yields

$$\frac{\partial \Pi_G(\Phi)}{\partial \Phi} = 2 + 2 \frac{V_H - c_H - (V_L - c_L)}{V_L - c_L} - \frac{8\Phi}{(V_L - c_L)(1 - 2\lambda)} = 0$$

and after solving for Φ we receive

$$\Phi^* = \frac{(V_H - c_H)^2(1 - 2\lambda)}{2(V_H - c_H + V_L - c_L)},$$

which is the maximizer since $\frac{\partial^2 \Pi_G(\Phi)}{\partial \Phi^2} < 0$. However, we have to check when this solution lies in the appropriate range for the equilibrium in Proposition 3.3. That is

$$[V_H - c_H - (V_L - c_L)](1 - 2\lambda) \leq \frac{(V_H - c_H)^2(1 - 2\lambda)}{2(V_H - c_H + V_L - c_L)}$$

which can be represented by

$$V_H - c_H \leq (V_L - c_L)\sqrt{2}.$$

For this equilibrium to exist, the interval for Φ for which this equilibrium exist must not be nonempty. That is,

$$[V_H - c_H - (V_L - c_L)](1 - 2\lambda) < \min \left\{ (V_L - c_L)(1 - 2\lambda), (V_H - c_H) \left(\frac{1}{2} - \lambda \right) \right\},$$

which results into $V_H - c_H < 2(V_L - c_L)$.

Hence, the gatekeeper's strategy is given by

$$\Phi^* = \begin{cases} \frac{(V_H - c_H)^2(1 - 2\lambda)}{2(V_H - c_H + V_L - c_L)} & \text{if } V_H - c_H \leq (V_L - c_L)\sqrt{2} \\ [V_H - c_H - (V_L - c_L)](1 - 2\lambda). & \text{otherwise} \end{cases}$$

□

The Subgames of Proposition 3.7

Now we consider that both firms use the strategies as described in the equilibrium in Proposition 3.7. That is we restrict the possible advertising fee to the set $\Phi \in (0, [V_H - c_H - (V_L - c_L)](1 - 2\lambda)]$. Therefore, we can establish the following result.

Lemma 3.11. *If $2(V_L - c_L) > V_H - c_H$ holds, then the gatekeeper sets the advertising fee equal to*

$$\Phi^* = [V_H - c_H - (V_L - c_L)](1 - 2\lambda). \quad (3A.10)$$

Proof. If $V_H - c_H - (V_L - c_L) > 0$, then the equilibrium in Proposition 3.7 can only exist if the high quality firm advertises with certainty. That is, the gatekeeper's maximization problem becomes

$$\max_{\Phi \in (0, [V_H - c_H - (V_L - c_L)](1 - 2\lambda)]} \Pi_G(\Phi) = \left(1 + \frac{(V_L - c_L)(1 - 2\lambda) - \Phi}{(V_H - c_H)(1 - 2\lambda)}\right) \Phi - C_G.$$

Differentiating with respect to Φ and setting the derivative equal to zero yields

$$\frac{\partial \Pi_G(\Phi)}{\partial \Phi} = 1 + \frac{V_L - c_L}{V_H - c_H} - \frac{2\Phi}{(V_H - c_H)(1 - 2\lambda)} = 0,$$

which implies that the maximizer (by the fact that $\frac{\partial^2 \Pi_G(\Phi)}{\partial \Phi^2} < 0$) is given by $\Phi = (V_H - c_H + V_L - c_L) \frac{1 - 2\lambda}{2}$. However, this maximizer does not fulfill $(V_L - c_L)(1 - 2\lambda) > \Phi$. Consequently, we must have a corner solution, which is

$$\Phi^* = [V_H - c_H - (V_L - c_L)](1 - 2\lambda).$$

It fulfills the assumption $\Phi^* < \min\{(V_L - c_L)(1 - 2\lambda), (V_H - c_H) \left(\frac{1}{2} - \lambda\right)\}$ if $V_H - c_H < 2(V_L - c_L)$. $\square$

We can combine these results to derive the complete strategy of the gatekeeper.

Proof. We want to combine Lemma 3.9, 3.10 and 3.11. Suppose $V_H - c_H < 2(V_L - c_L)$, then either the strategy of 3.10 and 3.11 can be optimal. Hence, the only optimal strategy for the interval $V_H - c_H \in ((V_L - c_L)\sqrt{2}, 2(V_L - c_L))$ is

$$\Phi^* = [V_H - c_H - (V_L - c_L)](1 - 2\lambda).$$

Moreover, if $V_H - c_H \leq \sqrt{2}(V_L - c_L)$ then $[V_H - c_H - (V_L - c_L)](1 - 2\lambda) \leq \frac{(V_H - c_H)^2(1 - 2\lambda)}{2(V_H - c_H + V_L - c_L)}$. That is,

$$\Phi^* = \frac{(V_H - c_H)^2(1 - 2\lambda)}{2(V_H - c_H + V_L - c_L)}$$

is an interior solution and optimal for $V_H - c_H \in (V_L - c_L, (V_L - c_L)\sqrt{2})$. Finally,

if $V_H - c_H > 2(V_L - c_L)$, then the only advertising fee which can be optimal is

$$\Phi^*(V_H - c_H) \left(\frac{1}{2} - \lambda \right).$$

□

3.A.19 Proof of Proposition 3.11

Proof. The advertising fee of the gatekeeper is $\Phi^* = \frac{(V_H - c_H)(1 - 2\lambda)}{2\left(1 + \frac{V_L - c_L}{V_H - c_H}\right)}$ if $V_H - c_H < \sqrt{2}(V_L - c_L)$ holds. It is easy to see that Φ^* is increasing in $V_H - c_H$. Obviously, it is also increases when $\Phi \geq \sqrt{2}(V_L - c_L)$.

Moreover, all prices for the three different areas of $V_H - c_H$ depend negatively on the number of loyal consumers. Thus, the price is increasing in the number of non-loyal consumers. □

3.A.20 Proof of Proposition 3.13

Proof. If $V_H - c_H \in [V_L - c_L, (V_L - c_L)\sqrt{2})$, then the equilibrium advertising intensities are given by

$$\begin{aligned} \alpha_H &= \frac{V_H - c_H}{V_L - c_L} - \frac{V_L - c_L}{V_H - c_H + V_L - c_L} \text{ and} \\ \alpha_L &= \frac{V_L - c_L}{V_H - c_H + V_L - c_L}. \end{aligned}$$

The first derivatives with respect to $V_H - c_H$ are given by

$$\begin{aligned} \frac{\partial \alpha_H}{\partial V_H - c_H} &= \frac{1}{V_L - c_L} + \frac{V_L - c_L}{(V_H - c_H + V_L - c_L)^2} > 0 \text{ and} \\ \frac{\partial \alpha_L}{\partial V_H - c_H} &= -\frac{V_L - c_L}{(V_H - c_H + V_L - c_L)^2} < 0. \end{aligned}$$

If $V_H - c_H \in [(V_L - c_L)\sqrt{2}, 2(V_L - c_L))$, then the equilibrium advertising intensities are given by $\alpha_H = 1$ and

$$\alpha_L = 2 \frac{V_L - c_L}{V_H - c_H} - 1.$$

Obviously the advertising intensity α_H is constant in $V_H - c_H$ and the first derivative of α_L with respect to $V_H - c_H$ is

$$\frac{\partial \alpha_L}{\partial V_H - c_H} = -2 \frac{V_L - c_L}{(V_H - c_H)^2} < 0$$

Moreover, if $V_H - c_H \geq 2(V_L - c_L)$ holds, $\alpha_H = 1$ and $\alpha_L = 0$ and are obviously constant in $V_H - c_H$. $\square$

3.A.21 Proof of Proposition 3.14

Proof. Suppose that $V_H - c_H \in (V_L - c_L, (V_L - c_L)\sqrt{2})$, then the expected profits of the gatekeeper are given by

$$\Pi_G = \frac{V_H - c_H}{V_L - c_L} \frac{(V_H - c_H)^2(1 - 2\lambda)}{2(V_H - c_H + V_L - c_L)} - C_G = \frac{(V_H - c_H)^2(1 - 2\lambda)}{2(V_L - c_L + \frac{(V_L - c_L)^2}{V_H - c_H})} - C_G. \quad (3A.11)$$

The last expression is easy to analyze. The nominator is increasing in $V_H - c_H$ and the denominator is decreasing in $V_H - c_H$, therefore the expected profits increase in $V_H - c_H$.

Assume that $V_H - c_H \in [(V_L - c_L)\sqrt{2}, 2(V_L - c_L))$, then the profits of the gatekeeper are given by

$$\begin{aligned} \Pi_G &= 2 \frac{V_L - c_L}{V_H - c_H} [V_H - c_H - (V_L - c_L)](1 - 2\lambda) - C_G \\ &= 2 \left[(V_L - c_L) - \frac{(V_L - c_L)^2}{V_H - c_H} \right] (1 - 2\lambda) - C_G. \end{aligned} \quad (3A.12)$$

We take the derivative with respect to $V_H - c_H$ and obtain $\frac{\partial \Pi_G}{\partial (V_H - c_H)} = 4(1 - 2\lambda) \frac{(V_L - c_L)^2}{(V_H - c_H)^2} > 0$. Hence, the expected profits are strictly increasing in the advantage.

If $V_H - c_H \geq 2(V_L - c_L)$, then the expected profits of the gatekeeper are given by $\Pi_G = (V_H - c_H) \left(\frac{1}{2} - \lambda \right) - C_G$. Obviously, they are increasing in $V_H - c_H$. $\square$

3.A.22 Proof of Proposition 3.15

Proof. Suppose that $V_H - c_H \in (V_L - c_L, (V_L - c_L)\sqrt{2})$, then the expected profits are given by expression (3A.11). This can be rewritten to

$$\Pi_G = \frac{1 - 2\lambda}{2} \frac{(V_H - c_H)^3}{(V_L - c_L)[V_H - c_H - (V_L - c_L)]} - C_G.$$

Since the fraction $\frac{(V_H - c_H)^3}{V_L - c_L}$ is increasing when $V_H - c_H$ and $V_L - c_L$ increase at the same absolute amount, Π_G is increasing in the aggregate profitability if the advantage is kept constant.

Suppose now, that $V_H - c_H \in [(V_L - c_L)\sqrt{2}, 2(V_L - c_L))$, then the expected profits of the gatekeeper are given by expression (3A.12). The expected profits

can be rewritten to

$$\Pi_G = 2(1 - 2\lambda) \frac{V_L - c_L}{V_H - c_H} [V_H - c_H - (V_L - c_L)] - C_G.$$

Since $\frac{V_L - c_L}{V_H - c_H}$ is increasing when $V_H - c_H$ and $V_L - c_L$ are increased by the same amount, when $V_H - c_H - (V_L - c_L)$ is kept constant, Π_G is increasing when the profitability increases and the advantage is kept constant.

Finally, if $V_H - c_H \geq 2(V_L - c_L)$, the expected profits $\Pi_G = (V_H - c_H) \left(\frac{1}{2} - \lambda\right) - C_G$ are increasing when $V_H - c_H$ and $V_L - c_L$ are increased by the same absolute amount. $\square$

3.A.23 Proof of Proposition 3.16

Proof. Suppose that $V_H - c_H \in [V_L - c_L, (V_L - c_L)\sqrt{2})$, then the expected profits are given by expression (3A.11). It can easily be seen that the denominator increases in $V_L - c_L$ and thus the whole expression decreases in it.

Suppose now, that $V_H - c_H \in [(V_L - c_L)\sqrt{2}, 2(V_L - c_L))$, then the expected profits of the gatekeeper are given by expression (3A.12). The first derivative with respect to $V_L - c_L$ yields $\frac{\partial \Pi_G}{\partial (V_L - c_L)} = 2(1 - 2L)(1 - 2\frac{V_L - c_L}{V_H - c_H})$ which is smaller zero for $V_H - c_H < 2(V_L - c_L)$.

Finally, if $V_H - c_H \geq 2(V_L - c_L)$, the expected profits $\Pi_G = (V_H - c_H) \left(\frac{1}{2} - L\right) - C_G$ are constant in $V_L - c_L$. $\square$

3.A.24 Proof of Proposition 3.17

Proof. If one inserts the optimal advertising fee into the equilibrium propensities, one can find out that the equilibrium propensities are independent from λ . If $V_H - c_H \in [V_L - c_L, \sqrt{2}(V_L - c_L))$ the propensities are

$$\begin{aligned} \alpha_L &= \frac{V_L - c_L}{V_H - c_H + V_L - c_L} < \frac{1}{2} \\ \alpha_H &= \frac{V_H - c_H}{V_L - c_L} - \frac{V_L - c_L}{V_H - c_H + V_L - c_L} < 1 \end{aligned}$$

and if $V_H - c_H \in [\sqrt{2}(V_L - c_L), 2(V_L - c_L))$, then there is a unique subgame perfect equilibrium in which

$$\begin{aligned} \alpha_L &= 2 \frac{V_L - c_L}{V_H - c_H} - 1 < 1 \\ \alpha_H &= 1 \end{aligned}$$

and constant otherwise. However, the gatekeeper's advertising fee is decreasing in λ . The result follows immediately. $\square$

Chapter 4

Persuasive Advertising with Spillovers and Semicollusion

4.1 Introduction

The effects of collusive behavior has been well understood if firms have a one-dimensional action space, e.g. price or quantity. However, in many markets firms have to decide about more than one variable, such as capacities, R&D and in particular advertising. Thus, there is the opportunity for competing firms to cooperate in one dimension and to compete in the other. The so called semicollusion is of particular interest for firms if cooperation in the use of one strategic variable can easily be monitored by antitrust authorities, but not the cooperation in other variables. In the growing literature about semicollusion, persuasive advertising has widely been ignored.¹ However, a large fraction of advertisements does not explicitly provide information such as quality or prices and are therefore likely to be persuasive. In contrast to informative advertising, persuasive advertising generally leads to higher pricing power of advertising firms and consequently to larger prices (see for example Bagwell (2005)). On the one hand noncooperative behavior of the firms may lead to costly advertising wars and advertising levels may thus be inefficiently high. On the other hand persuasive advertising creates values for consumers and firms may choose inefficiently low advertising intensities if the effectiveness of advertising to create pricing power is too low. This raises the question whether social welfare could be increased if firms cooperate. Moreover, it is natural that firms which use persuasive advertising are no price takers. Thus, they have multi-dimensional

¹To best of my knowledge, the only paper analyzing a semicollusive outcome for a market with persuasive advertising is provided by Aluf and Shy (2001).

strategy spaces and semicollusion becomes a possible way of cooperation. That is, they collude in (at least) one dimension and compete in the other. In this paper we examine two possible types of semicollusion. That is, either the firms cooperate in prices, but set advertising levels noncooperatively or they cooperate in advertising² and set the price noncooperatively. We want to study under which condition semicollusion can be welfare enhancing when the firms can choose the level of persuasive advertising and the price they set.

We assume a duopoly model with complete information in which firms advertise to change preferences of the consumers. More precisely, consumers, who have received no advertisement of a good, have zero willingness to pay for that good. All consumers who have received an advertisement for a particular good have the same positive valuation for it. Thus, even if the consumers are assumed to have homogenous preferences ex-ante (without advertisements), the consumers receive different advertisements and are thus heterogenous ex post. Since some consumers only receive one advertisement and thus are willing to pay a positive price only for the good of a particular firm, advertising creates pricing power. Moreover, we analyze the four possible settings. In the competitive setting, both firms do not cooperate and decide about advertising and prices noncooperatively. In two other settings firms semicollude. That is, they collude to set the prices (advertising intensities) which maximize industry profits whereas the advertising intensity (price) is set noncooperatively. This is called price semicollusion (advertising semicollusion). Moreover, we examine a setting in which firms set advertising intensities and prices cooperatively. That is, they fully collude. In our model, the optimal strategies of a full collusion are welfare maximizing.³ As a benchmark case, we first assume that advertising is a strong tool. That is, a consumer has a high valuation for a firm's good only if she has received an advertisement of this firm. In a more general case, we allow advertisements to have spillovers. More precisely, even though a consumer has only received one advertisement, there is a positive probability that the consumer also becomes persuaded for both goods. This means, that some consumers change the valuation for a firm's good, even though it has perceived an advertisement from the competitor only. For analytical convenience we assume extreme spillovers, which change the valuation of the consumers for both goods to the same valuation.

²An example for such a advertising cooperation can be found in Austria. Several firms in this cooperation serve the same market, like Ariel and Persil which both serve the laundry detergent market. Companies joining this cooperation can be found on <http://www.achten-sie-auf-die-marke.at>.

³This is due to consumers having unit demand. Therefore, the price splits up the revenues in consumers' rents and profits. Since in a full collusion the firms set prices such that the whole rent is captured by the firms, the collusion chooses welfare maximizing advertising intensities.

We derive symmetric subgame-perfect equilibria of this model.⁴ Surprisingly, in the benchmark case without spillovers, the main results are equivalent to that derived by Simbanegavi (2009) in which advertising is informative. That is, firms profit from any cooperation⁵ and semicollusion is always welfare deterring and advertising semicollusion generates a lower welfare than price semicollusion if advertising costs are sufficiently low. These findings contradict those of Aluf and Shy (2001). In their model of persuasive advertising, advertising semicollusion is welfare enhancing (compared to the noncooperative equilibrium). A reason for this difference is due to the difference in the underlying advertising technology. Aluf and Shy assume comparison advertising in which advertising is a tool to lower the valuation for the competitor's good. Interestingly, if firms behave noncooperatively in our model, they choose the welfare maximizing advertising intensity.

In our model spillovers of advertising weaken the ability of firms to acquire pricing power. Thus, the firms' equilibrium advertising intensities are decreasing in the level of spillovers. One consequence is that larger spillovers can prevent firms to overinvest in advertising. More precisely, price semicollusion is more profitable and more efficient for intermediate spillovers. Furthermore, price semicollusion can create the maximum welfare and exceed the welfare generated by noncooperation and advertising semicollusion for spillovers large enough. Therefore, our model suggests that antitrust authorities should abstain from enforcing noncooperative behavior for large spillovers. They should motivate price cooperation instead. Moreover, full competition generates a larger welfare than advertising semicollusion for any level of spillovers. Thus, our model suggests that authorities should heavily monitor advertising activities of firms such that cooperation in advertising can be prevented.

When there are no spillovers, our results are consistent with the findings of Simbanegavi (2009). Simbanegavi (2009) assumes a duopoly version of the Grossman and Shapiro (1984) model with informative advertising. He finds that semicollusion is always detrimental to welfare, but advertising is more harmful than price semicollusion when advertising costs are low enough. Firms do always benefit from semicollusion.

The only work which examines semicollusion of persuasive advertising is that of Aluf and Shy (2001). Advertising here lowers the utility of the consumers for the competitor's good and creates heterogeneity among consumers. Firms advertise more aggressively when they semicollude on prices. Moreover, the equilibrium prices and expected profits are larger.

⁴Actually, we also show below that there exists a model with informative advertising in which the same strategies form the equilibria.

⁵Fershtman and Gandal (1994) show that this is not necessarily the case when firms semicollude.

Nakata (2006) studies informative advertising with spillovers in a dynamic framework. In this model spillovers lead to larger prices and profits.

Brod and Shivakumar (1999) consider a model in which firms choose their R&D investments and production plans. They allow R&D to have spillovers and show that an optimal antitrust policy strongly depends on the spillovers, since for some values of the spillovers cooperation in production increases welfare and decreases it for others.

The remainder of this section is organized as follows: In section 2 we briefly present the model. In section 3 we assume that there are no spillovers and briefly provide three equilibria, one in which firms behave noncooperatively and those two in which firms either semicollude on price or on advertising. In section 4 we generalize the model of the previous section and allow for spillovers. Again we derive symmetric equilibria for all cases. Section 5 concludes. The proofs can be found in Appendix C.

4.2 The Model

We assume two firms ($i=1,2$) and a unit mass of consumers with unit demand. The firms are assumed to set prices. We allow firms to have mixed pricing strategies and denote the random variable by P_i and its realization by p_i for $i = 1, 2$. Let $F_i(p_i) = \text{Prob}(P_i < p_i)$ denote the c.d.f. of the pricing strategy of firm i with lower bound $\underline{p}_i$ and upper bound $\bar{p}_i$. We assume all consumers to gain no utility from consuming the good. Moreover, consumers are assumed to be completely informed about the prices charged. However, the firms can advertise to persuade the consumers to buy the good. More precisely, all receivers of an advertisement of firm i change their preferences for good i to V . The consumers, who have not perceived the advertisement of firm i , do not change their preferences for good i . Hence, all consumers who have only received an advertisement from one firm have different valuations for the two goods and consumers have therefore heterogeneous preferences ex post.

The firms choose their advertising intensities $0 \leq \phi_1 \leq 1$ and $0 \leq \phi_2 \leq 1$, which are assumed to be stochastically independent. Then the market looks as follows: a fraction of $(1 - \phi_1)(1 - \phi_2)$ does not receive any advertisement; the fraction $\phi_1(1 - \phi_2)$ does only receive an advertisement from firm 1; the fraction $(1 - \phi_1)\phi_2$ does only receive an advertisement from firm 2; and the fraction $\phi_1\phi_2$ receives both advertisements. The costs of advertising are assumed to be convex and are given by the function $A(\phi_i) = a\frac{\phi_i^2}{2}$ for $i = 1, 2$. We restrict the parameters to $a > \frac{V}{2}$ to ensure that firms choose advertising intensities less than one in each of the equilibria considered. Moreover, we evaluate the welfare. Since valuations for the goods are different ex-ante and ex-post (before

and after advertising), we have to decide which valuations we choose for the welfare comparison. Using ex-ante valuations would never lead to a positive consumer surplus. Moreover, it becomes negative if at least a positive mass of consumers buys the good for a price larger zero. To avoid negative measures for the consumer surplus we use ex-post valuation for welfare comparison. Hence, we define the welfare as the sum of the consumer rents and the industry profits using ex-post valuations of the consumers.

4.2.1 Sequence of Action

We suppose a three stage game. In the first stage the firms simultaneously decide about their advertising intensity. In the second stage the firms observe the advertising intensity of their competitor and simultaneously decide about the price they charge for the good. In the last stage the consumer receive their advertisements, observe the prices and make their purchasing decisions. Then, the firms receive their profits.

4.2.2 Alternative Model: Informative Advertising

The model we are going to analyze can also be replaced by a model of incomplete information and informative advertising. That is, we have to assume that all consumers have a valuation V for both goods, but do not have any information about the exact locations of the firms. Suppose that it is too expensive for the consumers to do undirected search. However, directed search is costless. Hence, without becoming informed about the exact location of a firm, they are not willing to search for it. Therefore, the firms have to inform the consumers about their location. We assume that firms' advertising informs the consumers about their existence. A consumer who receives an advertisement knows about the existence of the firm and which price it charges. It can easily be verified that this model will have the same equilibria.

4.3 The Benchmark Case: No Spillovers

We want to establish symmetric subgame perfect equilibria for the three different cases.

We solve this game by backward induction. The purchasing decisions is trivial. All consumers who have not received an advertisement, do not buy any good for any price $p > 0$. The consumers who have received an advertisement from only one firm, buy at this firm if the price is less than the valuation, V . The consumers who have received both advertisements compare the prices of both firms and buy at the cheaper one. Provided that $F_i(\cdot)$ is continuous for

$i = 1, 2$, the expected profits of firm i by setting price p and advertising intensity ϕ_i are given by

$$\Pi_i(p, \phi_i) = p \{ \phi_i(1 - \phi_j) + \phi_i \phi_j [1 - F_j(p)] \} - a \frac{\phi_i^2}{2} \text{ for } i = 1, 2 \text{ given } p \leq V. \quad (4.1)$$

The expected sales consists of two parts: the expected number of consumers who have received only an advertisement from firm i and the expected sales to consumers who have received advertisements from both firms. However, those who have received both advertisements buy only at firm i if the price charged by firm i is lower than that of firm j , which happens with probability $[1 - F_j(p)]$.

Suppose both firms use pricing distribution without mass points. Since we want to establish a symmetric equilibrium, we know that $F_1 = F_2 \equiv F$. Then we can establish the following Lemma.

Lemma 4.1. *In any symmetric equilibrium, in which both firms use continuous pricing distributions, $\bar{p} = V$ holds.*

In any symmetric equilibrium, a firm i charging the upper bound, only sells to consumers who have received only received an advertisement from firm i only. Thus, it is optimal for the firms to exploit all those consumers by setting the price equal to the consumers' valuation.

4.3.1 Noncooperative Equilibrium

We now treat the advertising choices ϕ_1 and ϕ_2 as fixed and let the firms choose their prices optimally. Intuitively, in a symmetric equilibrium in which firms use atomless pricing strategies, a firm which charges the upper bound $\bar{p}$, sells to consumers, who have received both advertisements, with probability zero. Given that the pricing distributions do not have mass points, the expected profits of firm i are fixed by

$$\Pi_i(p, \phi_i) = p \{ \phi_i(1 - \phi_j) + \phi_i \phi_j [1 - F_j(p)] \} - a \frac{\phi_i^2}{2} = V \phi_i(1 - \phi_j) - a \frac{\phi_i^2}{2} \quad (4.2)$$

for $i \neq j$ and $i = 1, 2$.

We can establish the following symmetric equilibrium.

Proposition 4.1. *There exists a symmetric equilibrium with $\phi_1 = \phi_2 \equiv \phi^*$ in which the firms choose an advertising intensity*

$$\phi^* = \frac{V}{V + a} \quad (4.3)$$

and set prices according to the pricing strategy

$$F_i^*(p) = 1 - \frac{V-p}{p} \cdot \frac{1-\phi_i}{\phi_i}. \quad (4.4)$$

on the support $[(1-\phi_i)V, V]$ for $i = 1, 2$. On the equilibrium path the expected profits are given by

$$\Pi_i = \frac{a}{2} \cdot \left(\frac{V}{V+a} \right)^2 \quad (4.5)$$

for $i = 1, 2$ and the welfare is given by

$$W = \frac{V^2}{a+V}. \quad (4.6)$$

The expected profits of the derived equilibrium are increasing in V and in a . The pricing distribution on the equilibrium path are given by

$$F_i(p) = 1 - \frac{V-p}{p} \cdot \frac{a}{V}$$

and is decreasing in a . Moreover, the upper bound is increasing in a . Thus, price competition decreases in a , which enhances the expected profits of the firms.

Both firms choose a smaller advertising intensity if it advertising becomes more expensive. Thus, there are nearly no consumers who have received two advertisements if a is large. Consequently, there is only little price competition in the second stage. On the contrary, if a is close to zero, both firms choose a large advertising intensity and nearly all consumers receive both advertisements, which leads to an intensive price competition and thus lowers expected profits.

In the equilibrium, all consumers who have received an advertisement buy the good, since prices do not exceed V . Hence, the total welfare is simply given by the total number of informed consumers multiplied by V deducted by the total costs for advertising. The total welfare is increasing in the valuation V and decreasing in the costs of advertising a .

4.3.2 Equilibrium with Semicollusion on Price

We now want to derive a symmetric equilibrium when firms collude on prices but set their advertising intensities noncooperatively. When firms collude on the price, they set the monopoly price, which is given by V . Thus, they abstain from competing about the consumers who have received both advertisements to extract the complete rents of any receiver. Assuming that indifferent consumers

choose each firm with probability of $\frac{1}{2}$, their expected profits from advertising become

$$\Pi_i^{PC}(\phi_i) = \left[\phi_i(1 - \phi_j) + \frac{\phi_i\phi_j}{2} \right] V - a\frac{\phi_i^2}{2} \quad (4.7)$$

for $i \neq j$ and $i = 1, 2$.

We can derive the following symmetric equilibrium.

Proposition 4.2. *If firms semicollude on prices, there exists a symmetric equilibrium with $\phi_1 = \phi_2 \equiv \phi^{PC}$ in which both firms set price $p^{PC} = V$ and choose an advertising intensity equal to*

$$\phi^{PC} = \frac{2V}{V + 2a}. \quad (4.8)$$

On the equilibrium path the expected profits are given by

$$\Pi_i^{PC} = \frac{2aV^2}{(V + 2a)^2}. \quad (4.9)$$

for $i = 1, 2$ and the welfare is given by

$$W^{PC} = \frac{4aV^2}{(V + 2a)^2}. \quad (4.10)$$

Since all consumers have zero rents, the welfare in this equilibrium is simply the aggregate of the expected profits of the two firms. As expected, the expected profits and welfare are increasing in the valuation, V , and decreasing in the advertising costs. We compare this equilibrium with the noncooperative equilibrium of the previous subsection below.

4.3.3 Equilibrium with Semicollusion on Advertising

In this subsection, we suppose that firms set prices non-cooperatively, but advertising intensities are chosen to maximize expected industry profits.

The expected profits from noncooperative price setting are given in function (4.2). Thus, the expected industry profits in a symmetric equilibrium with $\phi_1 = \phi_2 \equiv \phi$ are given by

$$\begin{aligned} \Pi^{AC} &= \Pi_1(\phi_1) + \Pi_2(\phi_2) = V\phi_1(1 - \phi_2) - a\frac{\phi_1^2}{2} + V\phi_2(1 - \phi_1) - a\frac{\phi_2^2}{2} \\ &= 2V\phi - (2V + a)\phi^2. \end{aligned} \quad (4.11)$$

We can now derive the symmetric equilibrium with semicollusion on advertising.

Proposition 4.3. *If firms semicollude on advertising, there exists a symmetric equilibrium with $\phi_1 = \phi_2 \equiv \phi^{AC}$ in which firms choose advertising intensity*

$$\phi^{AC} = \frac{V}{2V + a} \quad (4.12)$$

and charge prices according to the atomless pricing strategy given by

$$F_i^{AC}(p) = 1 - \frac{V - p}{p} \cdot \frac{1 - \phi_i}{\phi_i} \quad (4.13)$$

on the support $[(1 - \phi_i)V, a]$ and $i = 1, 2$. On the equilibrium path the expected profits are given by

$$\Pi_i^{AC} = \frac{V^2}{2(2V + a)}. \quad (4.14)$$

for $i = 1, 2$ and the welfare is given by

$$W = (3V + a) \left(\frac{V}{2V + a} \right)^2. \quad (4.15)$$

In contrast to Simbanegavi (2009), in which expected profits are increasing in a when firms semicollude on advertising, the expected profits are here decreasing in a . Simbanegavi (2009) decomposes the effect of an increase in a into a direct effect, which means that the firm have higher costs, and into a strategic effect, which means that the firms will lower their advertising intensity if a increases. In contrast to Simbanegavi (2009), the strategic effect does not outweigh the direct effect and thus the firms have lower profits. Moreover, the pricing distribution on the equilibrium path is given by

$$F_i = 1 - \frac{V - p}{p} \cdot \frac{V + a}{V}$$

for $i = 1, 2$. Thus, larger a leads to larger prices and smaller consumer rents. Therefore, welfare is decreasing in a . Clearly, it is increasing in V .

4.3.4 Full Collusion

We now want to derive the equilibrium when firms fully collude. That is, they cooperate in advertising and pricing. Clearly, the firms set the monopoly price V . Thus, in a symmetric equilibrium with $\phi_1 = \phi_2 \equiv \phi$ the industry profits are given by

$$\Pi^{FC} = 2 \cdot \left\{ V \left[\phi(1 - \phi) + \frac{\phi^2}{2} \right] - \frac{a}{2} \phi^2 \right\} = V(2\phi - \phi^2) - a\phi^2 \quad (4.16)$$

And thus, we have the following equilibrium.

Proposition 4.4. *If firms cooperate in advertising and pricing, there exists a symmetric equilibrium in which firms set price $p^{FC} = V$ and choose an advertising intensity*

$$\phi^{FC} = \frac{V}{V+a}. \quad (4.17)$$

On the equilibrium path the profits are given by

$$\Pi_i^{FC} = \frac{V^2}{2(V+a)} \quad (4.18)$$

and welfare is given by

$$W^{FC} = \frac{V^2}{V+a}. \quad (4.19)$$

Interestingly, the firms set the same advertising intensity as when they do not cooperate. Thus, the resulting welfare is the same. However, there is no consumer rent in this equilibrium.

4.3.5 Comparison of the Equilibria

First we want to compare the equilibrium advertising intensities in the different equilibria.

Proposition 4.5. *The firms' advertising intensities in the different equilibria can be ranked as follows: $\phi^{PC} > \phi^* = \phi^{FC} > \phi^{AC}$.*

It is remarkable that firms choose the same advertising intensity if they do not cooperate than under full cooperation. That is, even though the firms gain market power through advertising, they choose the welfare maximizing advertising intensity.⁶ The rationale for the first inequality is that the expected profits from any additional attracted consumer are higher under price collusion than if firms behave noncooperatively. Thus, the firms have a larger incentive to attract additional consumers via advertising. However, if firms collude on advertising, they collude on low advertising levels to avoid an intensive price competition in the second stage. Hence, they forgo additional consumers to achieve larger (expected) prices in the second stage. This is established in the next Proposition, where P^* , P^{PC} , P^{AC} and P^{FC} denote the random equilibrium price in the noncooperative equilibrium, the equilibrium with price semicollusion, the equilibrium with advertising semicollusion and the full collusion equilibrium, respectively.

⁶That is, the firms are led by the invisible hand.

Proposition 4.6. *The expected prices on the equilibrium path of the different equilibria can be ranked as follows: $E[P^*] \leq E[P^{AC}] \leq E[P^{PC}] = E[P^{FC}] = V$.*

Clearly, the firms charge the highest price if they collude on prices, since they charge the monopoly price (reservation price). Since firms restrict advertising under advertising collusion to ease price competition, the prices with semicollusion on advertising is larger than if they behave completely noncooperatively.

However, even if the firms charge the largest price and sell most under price semicollusion it does not imply that it is the most profitable one for the firms, since advertising costs are convex. The next results examines this.

Proposition 4.7. *We can rank the expected profits on the equilibrium path as follows: $\Pi_i^{FC} > \Pi_i^{PC} > \Pi_i^{AC} > \Pi_i^*$.*

As expected, full collusion is most profitable for the firms. Moreover, semicollusion is always preferred by the firms to noncooperation since they can reduce price competition either directly or indirectly through advertising. The reason why the advertising costs must be sufficiently large for price semicollusion to be more profitable than advertising semicollusion is that firms advertise too much under price semicollusion relative to full collusion. Large advertising costs reduce the advertising levels and hence make the collusion more efficient (see also Simbanegavi (2009) for this effect).

Proposition 4.8. *Noncooperation and full cooperation lead to the largest welfare. Semicollusion is always detrimental to welfare. Moreover, the welfare is larger if firms semicollude on pricing than on advertising if V is sufficiently large and a is sufficiently small and vice versa.*

Since firms choose the same advertising intensity if they do not cooperate than when they fully collude, the welfare is the same in both scenarios and it is the maximum welfare. However, under full collusion there is no consumer rent. Moreover, the firm's advertising intensity is larger under price collusion than in the non-cooperative (or the full collusion) outcome. The fraction of consumers that are additionally reached under price semicollusion do not compensate for the increase in advertisement cost. The opposite is the case if firms semicollude on advertising. The reduction of advertising costs does not exceed the loss of consumers. Moreover, if the advertising costs are low (or if valuations are large), the loss of welfare, caused by the large advertising costs if firms semicollude on price, is lower than the loss of consumers caused by the reduction in advertising if firms semicollude on advertising.

4.3.6 Summary

The main results of the model are equivalent to the findings with informative advertising by Simbanegavi (2009). These are the following: (i) welfare is maximized if firms fully collude and if they behave noncooperatively; (ii) semicollusion is detrimental to welfare; (iii) if $V > a$ holds, the welfare loss is larger if firms semicollude on advertising; (iv) the firms' benefit from cooperation and the profits are the largest if the fully collude and price semicollusion is more profitable than advertising semicollusion; (v) the advertising intensities are largest if firms semicollude on price and lowest if firms semicollude on advertising. An explanation for these parallels with the findings of Simbanegavi (2009) is that this model can also be interpreted as a model with informative advertising.

In the model of Aluf and Shy (2001) in which the firms' advertising is persuasive the advertising intensities increase if firms semicollude on advertising. The difference of this behavior follows from the difference of the advertising technology. In our model, the valuation for the good is binary. In Aluf and Shy (2001) the valuation has continuous values and can be changed farther more if a firm spends more on advertising. Consequently, in Aluf and Shy (2001) the firms gain more market power over some consumers and can charge higher prices. Thus, firms increase the advertising intensity to weaken price competition. In our model, price competition increases in the advertising intensities for any $\phi > 0$ as can be easily seen by the equilibrium pricing strategies. That is, the firms find it more interesting to compete for the consumers who have perceived advertisements of both firms. Thus, firms are willing to cooperate to reduce the advertising intensities with the aim of weakening the price competition.

4.4 Advertising with Spillovers

Until now, we have assumed that a firm can persuade consumers only through its own advertising activity. In the model above, any advertising of a firm persuades the receivers only of the firm's good, but does not change the preference for the competitor's good. However, it is plausible that receivers of advertisement do not become persuaded from the product of one particular firm but for all products in one particular market. Therefore, we relax this assumption and allow a fraction of the receivers of a firm's advertisement to change the preferences for the competitor's good too. That is, any consumer who has only received the advertisement of firm i has valuation V only for good i with probability $0 \leq 1 - \eta \leq 1$ and has valuation V for both goods with probability η for $i = 1, 2$. Therefore, $0 \leq \eta \leq 1$ measures the level of the spillovers. Note, that this is an extreme form of spillovers. A consumer who has received only one

advertisement may have the same high valuation for both goods. Moreover, the spillovers do not have any direct welfare effect. If a consumer is once persuaded by one firm, it does not add anything to welfare if the consumer has also high valuation for the other good, since we assumed unit demand. However, there are indirect welfare effects, since spillovers change the equilibrium strategies.

Provided that $F_j(\cdot)$ is continuous, the expected profits of firm i become

$$\begin{aligned} \Pi_i(p, \phi_i) = & p\{\phi_i(1 - \phi_j)(1 - \eta) \\ & + [\phi_i\phi_j + [\phi_i(1 - \phi_j) + (1 - \phi_i)\phi_j]\eta] \cdot [1 - F_j(p)]\} - a\frac{\phi_i^2}{2} \end{aligned} \quad (4.20)$$

for $i \neq j$ and $i = 1, 2$ given $p \leq V$. The first term of the expected profits represents the expected revenues. The expected sales consist of two parts. The first term is the expected fraction of consumers who exclusively have valuation V for good i . These consumers have received one advertisement only (which is from firm i). This fraction of consumers buys the good at any price provided it does not exceed V . The second part represents the expected consumers who do price comparison. One part of these consumers has only received one advertisement, but are also willing to buy the other good. More precisely, a fraction η of those who have only received an advertisement of one firm. The other part of these consumers has received both advertisements and compare prices (analogous to the previous chapter).

We restrict parameters to $a > \frac{V(1-\eta)}{2}$. This ensures the existence of a symmetric equilibrium in the case of price semicollusion.

4.4.1 Noncooperative Equilibrium

Given the advertising intensities ϕ_1, ϕ_2 , all prices possibly charged in equilibrium must yield the same expected profits. In the symmetric equilibrium in which firms use continuous pricing distributions, we can apply Lemma 4.1, which holds for the case with spillovers as well. Thus, we obtain for any optimal price of firm i , that the expected profits are given by

$$\Pi_i^+(p, \phi_i) = V[\phi_i(1 - \phi_j)(1 - \eta)] - a\frac{\phi_i^2}{2}. \quad (4.21)$$

for $i \neq j$ and $i = 1, 2$. We are now able to establish the equilibrium if firms do not cooperate.

Proposition 4.9. *There exists a noncooperative symmetric equilibrium with*

$\phi_1 = \phi_2 \equiv \phi^+$, in which the firms choose advertising intensities

$$\phi^+ = (1 - \eta) \frac{V}{(1 - \eta)V + a} \quad (4.22)$$

and use the atomless pricing strategy

$$F_i^+(p) = 1 - \frac{(V - p)\phi_i(1 - \phi_j)(1 - \eta)}{p\{\phi_i\phi_j + [\phi_i(1 - \phi_j) + (1 - \phi_i)\phi_j]\eta\}} \quad (4.23)$$

on the support $\left[\frac{(1-\eta)\phi_i(1-\phi_j)}{\phi_i+(1-\phi_i)\phi_j\eta}V, V\right]$ for $i \neq j$ and $i = 1, 2$. On the equilibrium path the expected profits are given by

$$\Pi_i^+ = \frac{a}{2} \left(\frac{(1 - \eta)V}{(1 - \eta)V + a} \right)^2 \quad (4.24)$$

for $i = 1, 2$ and the welfare is

$$W^+ = (1 - \eta)V^2 \frac{(1 - \eta)V + (1 + \eta)a}{[(1 - \eta)V + a]^2}. \quad (4.25)$$

Both are decreasing in the spillovers.

Given any advertising intensity of the firms, the expected fraction of price insensitive consumers of firm i are given by $\phi_i(1 - \phi_j)(1 - \eta)$ and thus decreasing in the spillovers. That is, advertising is a less powerful tool to acquire pricing power when spillovers are large and hence price competition in the second stage is larger. That is, $F^+(p)$ is increasing in η and the lower bound is decreasing in η for any given ϕ_i and ϕ_j . Formally, we have

$$\frac{\partial^2 \Pi_i^+(p, \phi_i)}{\partial \phi_i \partial \eta} = -V(1 - \phi_j) < 0.$$

Therefore, the firms equilibrium advertising intensities are decreasing in the spillovers.

Since the spillovers lead to a more intensive price competition, the expected profits are decreasing in the spillovers. Moreover, the reduction of the advertising intensities due to larger spillovers is welfare deterring. Furthermore, the expected profits and the welfare converge to zero, if $\eta \rightarrow 1$. That is, the firms do not advertise if $\eta = 1$. Intuitively, advertising becomes a useless instrument to acquire pricing power, since all receivers do price comparison and thus the Bertrand outcome arises for any combination of ϕ_i and ϕ_j .

This is in sharp contrast to the findings of Nakata (2006) who find expected profits to be increasing in the spillovers. In Nakata's model, a more advantageous spillover effect (for the firms) is assumed. More precisely, the spillovers

from the competitor's advertising are a perfect substitute for a firm's advertising investment.⁷ In our model, this is not the case, since all consumers who are additionally persuaded via the spillover effect compare prices. Not all of these consumers compare prices in Nakata's model.

4.4.2 Equilibrium with Semicollusion on Price

If the two firms collude on the price but choose their advertising strategy noncooperatively, the equilibrium collusion price is $p^{+PC} = V$. Assuming that every firm attracts one half of the consumers who are persuaded of both goods, the expected profits of firm i become

$$\Pi_i^{+PC}(\phi_i) = V \left\{ \phi_i(1 - \phi_j)(1 - \eta) + \frac{\phi_i\phi_j + [\phi_i(1 - \phi_j) + (1 - \phi_i)\phi_j]\eta}{2} \right\} - a\frac{\phi_i^2}{2} \quad (4.26)$$

for $i \neq j$ and $i = 1, 2$. Hence, we can establish the following equilibrium.

Proposition 4.10. *There exists a symmetric equilibrium with $\phi_1 = \phi_2 \equiv \phi^{+PC}$ in which the firms collude on prices and choose advertising noncooperatively. The equilibrium price is $p^{+PC} = V$ and the equilibrium advertising intensities are given by*

$$\phi^{+PC} = (2 - \eta) \frac{V}{V + 2a}. \quad (4.27)$$

On the equilibrium path the expected profits are

$$\Pi_i^{+PC} = \frac{(2 - \eta)V^2}{2(V + 2a)^2} [\eta V + (2 + \eta)a] \quad (4.28)$$

for $i = 1, 2$ and the welfare is given by

$$W^{+PC} = \frac{(2 - \eta)V^2}{(V + 2a)^2} [\eta V + (2 + \eta)a]. \quad (4.29)$$

Both are increasing in the spillovers if $\eta \leq \frac{V}{V+a}$ holds and decreasing otherwise.

Thus, also in the price semicollusion the optimal advertising intensities are decreasing in the spillovers. This effect is due to the externality caused by advertising of the competing firm. For any given η , advertising is large if advertising costs are low or the valuation, V , is large. If there are no spillovers, $\eta = 0$, then each of the firms attracts the consumers who have positive valuation for their good only and half of those who have positive valuation for both⁸. If the

⁷Nakata (2006) assumes linear costs and a concave advertising technology, which is $\phi_i = \min\{\sqrt{aq^i + bq^{-i}}, 1\}$, where q^i is firm i 's advertising investment and q^{-i} are the advertising investments of all other firms. Thus, $b > 0$ measures the spillover effect.

⁸Those consumers have actually received both advertisements.

spillovers increase, two effects lower the incentive of advertising. A larger η lowers the fraction of consumers who have positive valuation for one good only, which is $\phi_i(1 - \phi_j)(1 - \eta)$. The other $\phi_i(1 - \phi_j)\eta$ are shared equally among the firms. The second effect is caused by the competitor. Given any strategy of firm j , a fraction $\phi_j\eta$ of the consumers has already changed its valuation for good i to V . Thus, it is of no value for firm i to address these consumers. Thus, the competitor causes a positive externality. Hence, the fraction of consumers who are worth to approach with the advertisement reduces to $[1 - \phi_j\eta]$. Formally, we have

$$\frac{\partial^2 \Pi^{+PC}(\phi_i)}{\partial \phi_i \partial \eta} = -\frac{V}{2} < 0.$$

The expected profits of both firms are positive for any spillovers. The welfare is given by the industry profits, since the firms exploit the consumers completely. For small spillovers, the welfare increases in the spillovers. To give a rationale for this, consider no spillovers, $\eta = 0$. Then, the firms advertise more aggressively under price collusion than they do when they do not cooperate and hence the advertising intensity exceeds the social optimal level (see Proposition 4.5 and 4.8). Moreover, firm's advertising intensities are decreasing in the spillovers when firms semicollude on prices. This indicates that the advertising intensities are inefficiently high when spillovers are low, but increasing spillovers lead to more efficiency when $\eta < \frac{V}{V+a}$. In contrast to the noncooperative case, the expected profits and welfare do not converge to zero when $\eta \rightarrow 1$. The rationale is that price cooperation prevents the firm from Bertrand-competition and thus advertising is still a beneficial instrument to acquire additional customers even though they are shared equally among the firms.

4.4.3 Equilibrium with Semicollusion on Advertising

We can again apply Lemma 4.1, since it holds for the case with spillovers as well. Thus, the pricing subgames are the as in the noncooperative scenario. Moreover, for continuous $F_i(\cdot)$ for $i = 1, 2$, the expected profits can also be represented by expression (4.21). We can establish the following equilibrium.

Proposition 4.11. *There exists a symmetric equilibrium with $\phi_1 = \phi_2 \equiv \phi^{+AC}$ in which firms semicollude on advertising, in which firms set advertising intensity*

$$\phi^{+AC} = \frac{(1 - \eta)V}{2(1 - \eta)V + a} \quad (4.30)$$

and use the pricing strategy

$$F_i^+(p) = 1 - \frac{(V - p)\phi_i(1 - \phi_j)(1 - \eta)}{p\{\phi_i\phi_j + [\phi_i(1 - \phi_j) + (1 - \phi_i)\phi_j]\eta\}} \quad (4.31)$$

on the support $\left[\frac{(1-\eta)\phi_i(1-\phi_j)}{\phi_i+(1-\phi_i)\phi_j\eta} V, V \right]$ for $i \neq j$ and $i = 1, 2$. On the equilibrium path, the expected profits are given by

$$\Pi_i^{+AC} = \frac{(1-\eta)^2 V^2}{2[2(1-\eta)V + a]}. \quad (4.32)$$

for $i = 1, 2$ and the welfare is given by

$$W^{+AC} = (1-\eta)V^2 \frac{3(1-\eta)V + (1+\eta)a}{[2(1-\eta)V + a]^2} \quad (4.33)$$

Both are decreasing in the spillovers.

The equilibrium advertising intensity decreases in the spillovers when firms semicollude on advertising. The reasons are the same as in the noncooperative case. Since the profit functions are equivalent with the noncooperative case, we also have $\frac{\partial^2 \Pi_i(p, \phi_i)}{\partial \phi_i \partial \eta} = -V(1 - \phi_j) < 0$. Since larger spillovers lead to a larger fraction of consumers who are persuaded from both goods, larger spillovers lead to a stronger price competition in the second stage. Consequently, the industry reduces the advertising levels to compensate for this. Moreover, larger spillovers lead to smaller expected profits. Since advertising intensities decrease if spillovers increase, fewer consumers receive advertisements. This increases the inefficiency due to low advertising. Moreover, similar to the noncooperative case, the expected profits and welfare converge to zero when $\eta \rightarrow 1$, because of the same intuition.

4.4.4 Full Collusion

If firms cooperate in pricing and advertising, then they set price equal to $p^{+FC} = V$ and thus the industry profit function becomes

$$\Pi^{+FC} = V(2\phi - \phi^2) - a\phi^2.$$

This function is independent of the spillovers η and equivalent to the profit function if there are no spillovers, Π_i^{FC} , given in equation (4.16). Moreover, the industry profits represent the welfare. Thus, the equilibrium with spillovers is completely described by Proposition 4.4. That is, the full collusion completely internalizes the external effect caused by the spillovers.

4.4.5 Firms' Behavior and Expected Profits

The firms reduce their advertising intensities when spillovers become larger. The intuition is that advertising becomes a less effective instrument to acquire

market power. The next proposition establishes the ranking of the advertising intensities.

Proposition 4.12. *The advertising intensities fulfill $\phi^{+AC} < \phi^+ < \phi^{FC} \leq \phi^{+PC}$ if $0 < \eta \leq \frac{V}{V+a}$ and $\phi^{+AC} \leq \phi^+ < \phi^{+PC} < \phi^{FC}$ if $\frac{V}{V+a} < \eta \leq 1$.*

This proposition establishes that the ranking of the advertising intensities in which firms compete in at least one strategic dimension does not change. It is remarkable, that the reduction of the advertising intensities under price semicollusion is so large, that the firms choose too large advertising intensities for low spillovers, the efficient advertising intensity if $\eta = \frac{V}{V+a}$ and too low intensities when spillovers are large.

It can easily be seen that the expected profits under full competition, (4.24), and advertising semicollusion, (4.32), converge to zero when η approaches one, whereas the expected profits under price semicollusion in (4.28) remain positive. However, the next proposition establishes that the ranking of the expected profits is independent from the level of spillovers.

Proposition 4.13. *The expected profits can be ranked as follows: $\Pi_i^+ \leq \Pi_i^{+AC} < \Pi_i^{+PC} \leq \Pi_i^{FC}$*

If there are no spillovers, firms advertise too much in the price semicollusion. However, as firms decrease their advertising intensity when spillovers increase they achieve larger expected profits (and reach the maximum level at $\eta = \frac{V}{V+a}$ at which $\Pi^{FC} = \Pi^{PC}$). For larger spillovers the profits of the price semicollusion decrease. Nevertheless, price semicollusion is more profitable for the firms than advertising semicollusion for any level of spillovers. As expected, the firms prefer any cooperation to the noncooperative scenario. Only if $\eta = 1$ $\phi^{+AC} = \phi^+ = 0$ and thus profits are zero in these two scenarios.

4.4.6 Welfare Analysis and Policy Implications

If advertising has no spillovers, semicollusion is detrimental to welfare and should therefore be monitored and prohibited by antitrust authorities (see the analysis above for persuasive advertising or Simbanegavi (2009) for informative advertising). However, with spillovers this does not necessarily persist, since firms might be able to internalize the external effects from spillovers. We now compare the three scenarios in which firms do not fully collude. First, one can note that price collusion approaches the maximum welfare if $\eta \rightarrow \frac{V}{V+a}$. Thus, $W^+(\eta) - W^{+PC}(\eta)$ and $W^{+AC}(\eta) - W^{+PC}(\eta)$ decrease in the spillovers if the spillovers are small and are negative at $\eta = \frac{V}{V+a}$. Furthermore, since the advertising intensities ϕ^+ and ϕ^{+AC} are inefficiently low and $\phi^+ > \phi^{+AC}$ holds for

all levels of spillovers, $W^+ > W^{+AC}$ must hold. That is, we can establish the following result.

Proposition 4.14. *The welfare is (i) larger under noncooperative behavior than if firms collude on advertising; (ii) larger under price semicollusion than under noncooperation when spillovers are large enough and vice versa; (iii) larger under price semicollusion than under advertising semicollusion when spillovers are large enough and vice versa.*

Thus, price semicollusion is preferable to the other two scenarios when spillovers are large enough. Without spillovers, the advertising intensity is too large when firms semicollude on the price. However, the spillovers lower the incentive to advertise. This lowers the equilibrium intensities in all scenarios. Therefore, the price semicollusion becomes more efficient and for $\eta = \frac{V}{V+a}$ it becomes welfare maximizing, whereas the advertising intensities in the other two scenarios create less welfare for increasing spillovers. Moreover, if spillovers are large enough such that $\eta > VV + a$ we have $\phi^{+AC} \leq \phi^+ < \phi^{+PC} < \phi^{FC} = \frac{V}{V+a}$. The result follows from the fact that the welfare is increasing in the advertising intensity for $\phi < \frac{V}{V+a}$.

This implies in our specific model that antitrust authorities should not enforce noncooperative behavior if there are large spillovers in advertising. Our analysis suggests that authorities should motivate price cooperation. Moreover, our analysis has shown that cooperation in advertising only yields a lower welfare than noncooperative behavior for all levels of spillovers and should therefore be intensively be monitored.

4.5 Conclusion

We have shown that the equilibrium behavior of the firms depend strongly on the underlying spillovers. Moreover, spillovers are decisive to determine the optimal antitrust policy. In our specific model with unit demand, a complete collusion maximizes the welfare and can internalize the external effect caused by the spillovers. It is remarkable that the firms choose the welfare maximizing advertising intensities also if they do not cooperate when there are no spillovers.

If the spillovers increase, advertising becomes a less powerful instrument to acquire pricing power since. Thus, firms lower their advertising intensities. That is, increasing spillovers can ease advertising wars, but can enforce underinvestments in advertising. In the extreme case in which spillovers are extremely large, advertising becomes a worthless tool for firms when there is price competition. Thus, they completely abstain from advertising. Therefore, spillovers lead to lower expected profits and lower welfare when firms choose too low advertising

intensities (compared with the efficient choice under full collusion). That is the case when firms do not cooperate and when they semicollude on advertising. However, the consequences when firms semicollude on the price are contrary. Since the advertising level is too large when there are no spillovers (and hence too costly), the reduction of the advertising levels due to spillovers makes the collusion more profitable and more efficient. Furthermore both approach their maximum when spillovers are low. For large spillovers the advertising intensities are under the efficient level and thus spillovers reduce welfare and expected profits.

The most striking result is that for large enough spillovers price semicollusion yields a larger welfare than the other two scenarios with competition. Thus, our model implies that antitrust authorities should consider spillovers to determine their optimal policy. In particular, it should motivate price cooperation for large spillovers. Moreover, price semicollusion is the most profitable cooperation for firms. Nevertheless, firms generically choose Pareto inefficient advertising levels. Advertising semicollusion creates less welfare than noncooperation for all levels of spillovers. Thus, our model implies that advertising choices of firms should be heavily monitored by the authorities so that cooperation in advertising can be avoided.

4.A Proofs

4.A.1 Proof of Lemma 4.1

Proof. Let F be the equilibrium advertising distribution with upper bound $\bar{p}$ and suppose it has no mass points. Moreover, let ϕ be the equilibrium advertising intensity. It is obvious that prices above V cannot be optimal, since they lead to zero sales. Hence, we only have to show, that $\bar{p} < V$ is suboptimal. Suppose that $\bar{p} < V$. The expected profits of the firms are given by $\bar{p}(1-\phi)(\phi) - a\frac{\phi}{2}$, since by charging $\bar{p}$ firm i only sells to consumers who have received an advertisement only from firm i . However, there exists an $\epsilon > 0$ small enough such that $\bar{p} + \epsilon \leq V$ such that $(\bar{p} + \epsilon)(1-\phi)(\phi) - a\frac{\phi}{2} > \bar{p}(1-\phi)(\phi) - a\frac{\phi}{2}$. Hence, $\bar{p} < V$ cannot be optimal and the only remaining candidate is $\bar{p} = V$. $\square$

4.A.2 Proof of Proposition 4.1

Proof. From condition 4.2 we can express the optimal pricing strategy, which is

$$F_j(p) = 1 - \frac{V - p}{p} \cdot \frac{1 - \phi_j}{\phi_j}.$$

For the lower bound $F_j(\underline{p}) = 0$ must hold. Hence, we obtain the lower bound

$$\underline{p}_j = V(1 - \phi_j).$$

The expected profits are given by

$$\Pi_i(\phi_i) = V\phi_i(1 - \phi_j) - a\frac{\phi_i^2}{2}$$

for $i = 1, 2$. The first order condition is

$$\frac{\partial \Pi(\phi_i)}{\partial \phi_i} = V(1 - \phi_j) - a\phi_i = 0$$

which gives the best response of firm i

$$\phi_i^*(\phi_j) = \frac{V}{a}(1 - \phi_j) \text{ for } i = 1, 2.$$

Since we want to establish a symmetric equilibrium, we have $\phi_i = \phi_j \equiv \phi$. We can insert this into the best response function and solve for ϕ . This yields the optimal advertising intensity

$$\phi^* = \frac{V}{V + a}.$$

Inserting $\phi^* = \phi_i = \phi_j$ into the expected profits function above, yields the equilibrium profits

$$\Pi_i = \frac{a}{2} \cdot \left(\frac{V}{V + a} \right)^2$$

for $i = 1, 2$. Moreover, the welfare is given by the expected number of receivers times their valuation subtracted by the total costs of advertising. This is

$$W = [2 \cdot \phi^*(1 - \phi^*) + (\phi^*)^2]V - 2 \cdot a \frac{(\phi^*)^2}{2} = \phi^*[2V - (a + V)\phi^*] = \frac{V^2}{a + V}.$$

□

4.A.3 Proof of Proposition 4.2

Proof. The first-order condition is then

$$\frac{\partial \Pi_i(\phi_i)}{\partial \phi_i} = \left[1 - \phi_j + \frac{\phi_j}{2} \right] V - a\phi_i = 0$$

and hence we obtain the best-response

$$\phi_i^{PC}(\phi_j) = \frac{V}{a} \left(1 - \frac{\phi_j}{2} \right)$$

of firm i on firm j 's advertising intensity for $i \neq j$ and $i = 1, 2$. Hence, the symmetric advertising intensity is given by

$$\phi^{PC} = \frac{2V}{V + 2a}.$$

The expected profits on the equilibrium path are given by

$$\begin{aligned} \Pi_i^{PC} &= \left[\phi^{PC} - \frac{(\phi^{PC})^2}{2} \right] V - a \frac{(\phi^{PC})^2}{2} \\ &= \frac{2V^2}{V + 2a} - 2(V + a) \left(\frac{V}{V + 2a} \right)^2 \\ &= \frac{2aV^2}{(V + 2a)^2} \end{aligned}$$

for $i = 1, 2$ and the welfare are given by the industry profits, since consumers are completely exploited. That is,

$$W^{PC} = 2 \cdot \Pi_i^{PC} = \frac{4aV^2}{(V + 2a)^2}.$$

□

4.A.4 Proof of Proposition 4.3

Proof. The first order condition of the expected industry profits in 4.11 is given by

$$\frac{\partial \Pi^{AC}}{\partial \phi} = 2V - 2\phi(2V + a) = 0.$$

Hence, the optimal advertising intensity is

$$\phi^{AC} = \frac{V}{2V + a}$$

The expected profits on the equilibrium path are given by

$$\Pi_i^{AC} = V\phi^{AC}(1 - \phi^{AC}) - \frac{a}{2} (\phi^{AC})^2 = V^2 \frac{V + \frac{a}{2}}{(2V + a)^2} = \frac{V^2}{2(2V + a)}$$

for $i = 1, 2$ and the welfare is given by

$$\begin{aligned} W^{AC} &= [2\phi^{AC} - (\phi^{AC})^2] V - 2a \frac{(\phi^{AC})^2}{2} \\ &= \phi^{AC} [2V - (V + a)\phi^{AC}] \\ &= (3V + a) \left(\frac{V}{2V + a} \right)^2. \end{aligned}$$

□

4.A.5 Proof of Proposition 4.4

Proof. It is obvious that firms extract all rents from consumers if they cooperate in prices by setting price equal to V . Then the firms choose their advertising intensity to maximize industry profits. The first-order condition is given by

$$\frac{\partial \Pi^{FC}}{\partial \phi} = 2V - 2V\phi - 2a\phi = 0$$

and thus the optimal advertising fee is

$$\phi^{FC} = \frac{V}{V + a}.$$

Therefore, the expected profits on the equilibrium path are given by

$$\Pi_i^{FC} = V \left(\phi - \frac{\phi^2}{2} \right) - \frac{a}{2} \phi^2 = \frac{V^2}{2(V + a)}$$

and welfare is the sum of the industry profits. □

4.A.6 Proof of Proposition 4.5

Proof. We have to show that $\Phi^{PC} \geq \Phi^*$. This is the case if $\frac{2V}{V+2a} \geq \frac{V}{V+a}$. After multiplying by $(V + a)(V + 2a)$ we receive $2V^2 + 2aV > V^2 + 2aV$, which holds by the assumption that $V > 0$.

To establish the result, it is sufficient to show that the advertising intensity under advertising collusion is smaller than the advertising intensity in the non-cooperative outcome. That is, $\Phi^* > \Phi^{AC}$ which holds if $\frac{V}{V+a} > \frac{V}{2V+a}$. Obviously, this is the case for all $V > 0$. □

4.A.7 Proof of Proposition 4.6

Proof. To show that $\int_0^1 p dF^{AC}(p) \geq \int_0^1 p dF(p)$, it suffices to show that $p^{AC} \geq p$ and that $F^{AC}(\cdot)$ dominates $F(\cdot)$. For the lower bounds $\frac{a}{V+a}V \leq \frac{V+a}{2V+a}V$ must

hold. By dividing through V and multiplying both sides by $(V + a)(2V + a)$ we receive $a(2V + a) \leq (V + a)^2$. This holds if $V^2 \geq 0$, which is the case by assumption. To show that $F^{AC}(\cdot)$ is dominated by $F(\cdot)$ we have to show that

$$1 - \frac{V - p}{p} \cdot \frac{a}{V} \geq 1 - \frac{V - p}{p} \cdot \left(1 + \frac{a}{V}\right)$$

holds. By subtracting one and dividing by $-\frac{V-p}{p}$ we receive $\frac{a}{V} \leq 1 + \frac{a}{V}$.

Moreover, both expected prices cannot exceed the monopoly price V since this presents the upper bound of both distributions. $\square$

4.A.8 Proof of Proposition 4.7

Proof. First, we show that on the equilibrium paths $\Pi_i < \Pi_i^{AC}$, which is equivalent to $\frac{a}{2} \cdot \left(\frac{V}{V+a}\right)^2 < \frac{V^2}{2(2V+a)} \Rightarrow a(2V + a) < V^2 + a(2V + a)$. This holds since $V > 0$.

Second, we show that on the equilibrium paths $\Pi_i^{PC} > \Pi_i^{AC}$ holds, which is equivalent to $2a \left(\frac{V}{V+2a}\right)^2 > \frac{V^2}{2(2V+a)} \Rightarrow 8aV + 4a^2 > V^2 + 4aV + 4a^2 \Rightarrow 4aV > V^2$. Since a is bounded below, $a > \frac{V}{2}$, we have the following

$$4aV > 2V^2 > V^2$$

Thus, the inequality is satisfied and price semicollusion is always more profitable than advertising semicollusion.

Moreover, we must have that full collusion is most profitable. Thus, we show that $\frac{V^2}{2(V+a)} > \frac{2aV^2}{(V+2a)^2}$. Multiplying by the denominators and dividing by V^2 , we receive $(V + 2a)^2 > 4a(V + a)$. It can easily be verified that this inequality holds for all $V > 0$. $\square$

4.A.9 Proof of Proposition 4.8

Proof. First, we want to show that $W^* = W^{FC} > W^{PC}$ holds. That is $\frac{V^2}{V+a} > \frac{4aV^2}{(V+2a)^2} \Rightarrow (V + 2a)^2 > 4a(V + a)$ which holds for any $V > 0$.

Second, we want to show that the welfare is larger when firms do not cooperate (or fully cooperate) than when they semicollude on advertising. That is, $W^* = W^{FC} > W^{AC}(\phi^{AC})$ must hold, which can be rewritten as $\frac{V^2}{a+V} > (3V + a) \left(\frac{V}{2V+a}\right)^2 \Rightarrow (2V + a)^2 > (3V + a)(a + V)$. After multiplication and cancellations one receives $V^2 > 0$, which holds by assumption.

Additionally, we have to show when $W^{PC} > W^{AC}$ holds. Inserting the optimal solution one receives $(3V + a) \left(\frac{V}{V+2a}\right)^2 > \frac{4aV^2}{(V+2a)^2} \Rightarrow (3V + a)(V + 2a) > 4a(2V + a)^2$. After multiplication and cancellation we receive $3V^2(V - a) > 0$,

which is larger zero for $V - a > 0$. Since a is bounded below by $\frac{V}{2}$, we have $V - a < V - \frac{V}{2} = \frac{V}{2} > 0$. Thus, $W^{PC} > W^{AC}$ holds if V is sufficiently large and a is sufficiently small and vice versa. $\square$

4.A.10 Proof of Proposition 4.9

Proof. Any price p charged in equilibrium must yield the same profits as the upper bound V . Thus, we have the following

$$\begin{aligned} V[\phi_i(1 - \phi_j)(1 - \eta)] - a \frac{\phi_i^2}{2} \\ = p \{ \phi_i(1 - \phi_j)(1 - \eta) + [\phi_i\phi_j + [\phi_i(1 - \phi_j) + (1 - \phi_i)\phi_j]\eta \} \cdot [1 - F_j(p)] \} - a \frac{\phi_i^2}{2} \end{aligned}$$

for $i \neq j$, $i = 1, 2$ and $p \leq V$. As above we can express from equation (4.21) the pricing strategy

$$F_i(p) = 1 - \frac{(V - p)\phi_i(1 - \phi_j)(1 - \eta)}{p[\phi_i\phi_j + [\phi_i(1 - \phi_j) + (1 - \phi_i)\phi_j]\eta]}$$

for $i \neq j$ and $i = 1, 2$.

The lower bound $\underline{p}$ fulfills $F(\underline{p}) = 0$ and is therefore given by

$$\underline{p}_i = \frac{V(1 - \eta)\phi_i(1 - \phi_j)}{\phi_i + (1 - \phi_i)\phi_j\eta}$$

which completely describes the optimal pricing strategy. We now have to derive the optimal advertising intensities. That is, we differentiate the expected profits with respect to ϕ_i and set it equal to zero, which is

$$\frac{\partial \Pi_i(p, \phi_i)}{\phi_i} = V(1 - \phi_j)(1 - \eta) - a\phi_i = 0.$$

Hence, the optimal response is given by

$$\phi_i = \frac{V(1 - \phi_j)(1 - \eta)}{a}.$$

For the symmetric equilibrium we look for a solution at which $\phi_i = \phi_j$. Thus, we obtain

$$\phi^+ = (1 - \eta) \frac{V}{V(1 - \eta) + a},$$

which is less than one. The expected profits are given by

$$\begin{aligned}\Pi_i^+(\phi^+) &= [\phi^+(1 - \phi^+)(1 - \eta)]V - a \frac{(\phi^+)^2}{2} \\ &= (1 - \eta)V\phi^+ - \left\{ \left[(1 - \eta)V + \frac{a}{2} \right] (\phi^+)^2 \right\} \\ &= \frac{a}{2} \left(\frac{(1 - \eta)V}{(1 - \eta)V + a} \right)^2 \geq 0.\end{aligned}$$

The welfare is given by the expected number of receivers multiplied by their valuation and subtracted by the total advertising costs.

$$\begin{aligned}W^+ &= V \left[2\phi^+ - (\phi^+)^2 \right] - 2 \cdot a \frac{(\phi^+)^2}{2} \\ &= 2V \frac{(1 - \eta)^2 V^2 + (1 - \eta)aV}{[(1 - \eta)V + a]^2} - (V + a) \frac{(1 - \eta)^2 V^2}{[(1 - \eta)V + a]^2} \\ &= (1 - \eta)V^2 \frac{(1 - \eta)V + (1 + \eta)a}{[(1 - \eta)V + a]^2}.\end{aligned}$$

We still have to show that the expected profits and the welfare are decreasing in η . For this purpose one can rewrite the expression for the expected profits to

$$\Pi_i^+ = \frac{a}{2} \left(\frac{V}{V + \frac{a}{1 - \eta}} \right)^2.$$

It is now obvious that $\frac{\partial \Pi_i^+}{\partial \eta} < 0$. The welfare is given by

$$W^+ = 2V\phi^+ - (V + a)(\phi^+)^2$$

and it is increasing in ϕ for $\phi < \frac{V}{V+a}$. Since $\phi^+ < \frac{V}{V+a}$ and ϕ^+ is decreasing in η , the welfare is decreasing in η . $\square$

4.A.11 Proof of Proposition 4.10

Proof. Firm i chooses ϕ_i such that it maximizes the expected profits $\Pi_i^{+PC}(\phi_i)$. The first-order condition is given by

$$\frac{\partial \Pi_i(\phi_i)}{\partial \phi_i} = V \left[(1 - \phi_j)(1 - \eta) + \frac{\phi_j + (1 - 2\phi_j)\eta}{2} \right] - a\phi_i = 0.$$

Therefore, the best-response function is given by

$$\phi_i^{+PC} = \frac{V}{2a} (2 - \phi_j - \eta).$$

The optimal choice in a symmetric equilibrium becomes

$$\phi^{+PC} = (2 - \eta) \frac{V}{V + 2a}.$$

The expected profits in the symmetric equilibrium are given by

$$\Pi_i^{+PC} = V(1 - \eta)\phi^{+PC} - \frac{V + a}{2} (\phi^{+PC})^2 = V^2 \frac{2 - \eta}{2(V + 2a)^2} [V\eta + a(2 + \eta)]$$

and are nonnegative. The welfare is equal to the industry profits and are given by

$$W^{+PC} = \frac{(2 - \eta)V^2}{(V + 2a)^2} [V\eta + a(2 + \eta)].$$

We have to show that W^{+PC} is increasing η for $\eta < \frac{V}{V+a}$. The first derivative is given by

$$\frac{\partial W^{+PC}}{\partial \eta} = 2V^2 [V(1 - \eta) - a\eta],$$

which is nonnegative if $V(1 - \eta) - a\eta \geq 0$. Rearranging this yields $\eta \leq \frac{V}{V+a}$. It is easy to see that the same inequality must hold for Π_i^{+PC} being increasing in η (since $\Pi_i^{+PC} = \frac{W^{+PC}}{2}$). $\square$

4.A.12 Proof of Proposition 4.11

Proof. The optimal pricing strategy is given in (4.23) on the support

$$\left[\frac{(1 - \eta)\phi_i(1 - \phi_j)}{\phi_i + (1 - \phi_i)\phi_j\eta} V, V \right].$$

However, the advertising decisions are chosen as to maximize expected industry profits, which are given by

$$\Pi^{AC} = 2V[\phi(1 - \phi)(1 - \eta)] - a\phi^2$$

for $p \leq V$. Taking the derivative with respect to ϕ and setting it equal to zero yields

$$\frac{\partial \Pi^+}{\partial \phi} = 2V(1 - \eta) - 4\phi V(1 - \eta) - 2a\phi = 0.$$

Hence, the optimal advertising strategy is

$$\phi^{+AC} = \frac{V(1 - \eta)}{2V(1 - \eta) + a}.$$

The expected profits are given by

$$\begin{aligned}\Pi_i^{+AC} &= V[\phi^{+AC}(1 - \phi^{+AC})(1 - \eta)] - \frac{a}{2} (\phi^{+AC})^2 \\ &= V(1 - \eta)\phi^{+AC} - \left(V(1 - \eta) + \frac{a}{2}\right) (\phi^{+AC})^2 \\ &= \frac{[V(1 - \eta)]^2}{2[2V(1 - \eta) + a]^2}\end{aligned}$$

for $i \neq j$ and $i = 1, 2$ and are nonnegative for all values of a , V and η . The welfare is given by

$$\begin{aligned}W^{+AC} &= 2V \frac{V(1 - \eta)}{2V(1 - \eta) + a} - (V + a) \left[\frac{V(1 - \eta)}{2V(1 - \eta) + a} \right]^2 \\ &= V^2(1 - \eta) \frac{3V(1 - \eta) + a(1 + \eta)}{(2V(1 - \eta) + a)^2}.\end{aligned}$$

We have to show that the expected profits Π_i^{+AC} and the welfare are increasing in $(1 - \eta)$.

$$\begin{aligned}\frac{\partial \Pi_i^{+AC}}{\partial \eta} &= V^2(1 - \eta) \frac{[-3V(1 - \eta) - a][2V(1 - \eta) + a] + 4V^2(1 - \mu)^2 + 2aV(1 - \eta)}{[2V(1 - \eta) + a]^3} \\ &= -V^2(1 - \eta) \frac{-2V^2(1 - \eta)^2 - 3aV(1 - \eta) - a^2}{[2V(1 - \eta) + a]^3} \leq 0\end{aligned}$$

The inequality follows from the parameters being larger zero. We know from the proof of Proposition 4.9, that the expected welfare is increasing in the equilibrium ϕ^{AC} for $\phi < \frac{V}{V+a}$. Since ϕ^{AC} decreases in η and we have $\phi^{AC} < \frac{V}{V+a}$, W^{AC} decreases in ϕ . $\square$

4.A.13 Proof of Proposition 4.12

Proof. Since the advertising intensities ϕ^+ and ϕ^{+AC} are strictly decreasing in η and by Proposition 4.5 we have $\phi^{AC} < \phi^* = \phi^{FC}$, we can immediately conclude that $\phi^{AC} < \phi^{FC}$ and $\phi^* < \phi^{FC}$. Moreover, we have to show that $\phi^{+PC} = (2 - \eta) \frac{V}{V+2a} > \frac{V}{V+2a} = \phi^{+FC}$ for $\eta > \frac{V}{V+a}$. This can be rearranged to $(2 - \eta)(V + a) > V + 2a$. After multiplication and cancellation we receive $\eta > \frac{V}{V+a}$. Showing the opposite is straight forward.

Additionally, we have to show that $\phi^{PC} > \phi^+$. That is $(2 - \eta) \frac{V}{V+2a} > (1 - \eta) \frac{V}{V(1 - \eta) + a}$. By multiplying by the denominators and dividing by V , we receive $(2 - \eta)[V(1 - \eta) + a] > (1 - \eta)(V + 2a)$. After cancellation we obtain $(1 - \eta)^2 V + \eta a > 0$

Moreover, we have to show that $\phi^+ > \phi^{+AC}$. That is, $(1 - \eta) \frac{V}{V(1 - \eta) + a} > \frac{V(1 - \eta)}{2V(1 - \eta) + a} \Rightarrow (1 - \eta)[2V(1 - \eta) + a] > (1 - \eta)[V(1 - \eta) + a]$, which is fulfilled

for any $V > 0$. It follows immediately that $\phi^{+PC} > \phi^{+AC}$. $\square$

4.A.14 Proof of Proposition 4.13

Proof. The welfare

$$W = 2V\phi - (V + a)\phi^2$$

is increasing in ϕ for $\phi < \frac{V}{V+a}$ and decreasing in ϕ for $\phi > \frac{V}{V+a}$. Under full collusion and under price collusion the profits are equal to half of the welfare. Since $\frac{V}{V+a} = \phi^{FC}$ for all η and $\phi^{+PC} = \frac{V}{V+a}$ only at $\eta = \frac{V}{V+a}$, we have $\Pi^{+PC} \leq \Pi^{+FC}$.

We have already shown in Proposition 4.11 that the profits are larger under price collusion than under advertising collusion when there are no spillovers. If $\eta < \frac{V}{V+a}$, the expected profits under price semicollusion are increasing and they are decreasing when firms semicollude on advertising. Thus, $\Pi^{+PC} > \Pi^{+AC}$ for $\eta < \frac{V}{V+a}$. It remains to be shown that the inequality also holds for $\eta > \frac{V}{V+a}$. Under price semicollusion there are no consumer rents and the profits of a firm is equal to half of the welfare. Moreover, welfare is increasing in $\phi < \frac{V}{V+a}$ and $\phi^{+AC} < \phi^{+PC} < \frac{V}{V+a}$ when $\eta > \frac{V}{V+a}$. Thus, it follows that $\Pi^{+AC} < \Pi^{+PC}$.

We now show that $\Pi^+ \leq \Pi^{+AC}$, which is equivalent to $\frac{a}{2} \left(\frac{(1-\eta)V}{(1-\eta)V+a} \right)^2 \leq \frac{[(1-\eta)V]^2}{2[2(1-\eta)V+a]}$. Dividing by the nominator and multiplying by both denominators yields $a[2(1-\eta)V+a] \leq [(1-\eta)V+a]^2$. Thus, it remains the inequality $(1-\eta)^2V^2 \geq 0$ with equality only if $\eta = 1$. $\square$

4.A.15 Proof of Proposition 4.14

Proof. Result (i) follows from the fact that $\phi^{+AC} \leq \phi^+ < \frac{V}{V+a}$ and by the fact that the welfare is continuously increasing in ϕ for $\phi < \frac{V}{V+a}$.

Result (ii) follows from the fact that W^+ is decreasing and W^{+PC} is increasing in η for $\eta < \frac{V}{V+a}$ and that W^{+PC} approaches the maximum welfare. For $\eta > \frac{V}{V+a}$, we have $\phi^+ < \phi^{+PC} < \frac{V}{V+a}$ and the rest follows from the welfare being continuously increasing in ϕ for $\phi < \frac{V}{V+a}$.

The welfare is increasing in η under price semicollusion if $\eta < \frac{V}{V+a}$. Moreover, it is equal to the maximum welfare when $\eta = \frac{V}{V+a}$. By the fact that welfare decreases continuously under advertising semicollusion, there exist a value $\hat{\eta}$ such that $W^{+PC} > W^{+AC}$ for $\eta \in (\hat{\eta}, \frac{V}{V+a}]$. It is left to show that $W^{+PC} > W^{+AC}$ for $\eta > \frac{V}{V+a}$. The welfare is increasing in ϕ for $\phi < \frac{V}{V+a}$. The result follows from $\phi^{+AC} < \phi^{+PC} < \frac{V}{V+a}$ for $\eta > \frac{V}{V+a}$. $\square$

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Abstract

We set up models in which firms use random pricing strategies in the equilibria. In the first model consumers are heterogenous in their costs of acquiring price information. We show that there exists an equilibrium in which both firms have positive profits when the costs of acquiring information are sufficiently large for some consumers. This equilibrium can exist even if one firm has a large advantage over its competitor. Moreover, there is a positive probability that a low quality good is offered for a larger price than good of higher quality when product differentiation is small and the consumer's costs of information are large.

In the second model, each firm has monopoly power over a local market. However, they have the possibility to compete about a fraction of consumers in the competitor's market by advertising the product on price comparison website. We show that the operator of the website (gatekeeper) benefits from firms having asymmetric profitability. That is, the gatekeeper increases the advertising fee when the advantage in the market increases. This lets the advertising intensities drift apart and thus lowers price competition. Therefore, the gatekeeper can extract larger rents from the firms.

In the third model, firms advertise to increase the valuation of consumers and to gain market power. We show that semicollusion is welfare deterring and that the full competition maximizes welfare when there are no spillovers of advertising. Moreover, when spillovers are large enough price semicollusion yields a larger welfare than advertising semicollusion or full competition. Thus, antitrust policy should depend on the underlying spillovers and should allow for price cooperation if spillovers are large.

Zusammenfassung

Wir bilden Modelle in denen Firmen in den Gleichgewichten ihre Preise zufällig wählen. Im ersten Modell haben Konsumenten ex-ante keine Information über die Preise die von den Firmen im Markt gesetzt wurden. Für die Konsumenten ist es kostspielig die Preisinformation zu erlangen. Wir zeigen die Existenz eines Gleichgewichts, in dem die Firmen positive Profite haben. Dieses existiert auch, wenn eine Firma einen kompetitiven Vorteil gegenüber, wenn die Informationskosten der Konsumenten hoch. Darüber hinaus kann ein schlechteres Gut zu einem höheren Preis angeboten werden als ein Besseres, wenn die Differenzierung der Güter klein ist und die Informationskosten hoch sind.

Im zweiten Modell hat jede Firma Monopolmacht in einem lokalen Markt. Durch die Existenz einer Preisvergleich-Website können die beiden Firmen jedoch konkurrieren. Das heißt, der Betreiber der Website (Gatekeeper) setzt eine Werbegebühr für die Firmen und die User der Website können kostenlos auf die Preislisten zugreifen. Der Gatekeeper erhöht die Werbegebühr wenn sich der kompetitive Vorteil einer Firma vergrößert. Dadurch erhält die Firma mit dem kompetitiven Vorteil eine größere Dominanz auf der Website des Gatekeepers, was verringert der Preiswettkampf verliert an Intensivität. Dadurch kann der Gatekeeper einen größeren Anteil der erhöhten Umsätze der dominierenden Firma extrahieren.

Im dritten Modell bewerben Firmen ihr Produkt um die Zahlungswilligkeit der Konsumenten zu erhöhen und dadurch Marktmacht zu generieren. Wir zeigen, dass Semikollusion den Wohlstand verringert und dass eine volle Kollusion den Wohlstand maximiert, wenn Werbung keine externen Effekte hat. Sind die externen Effekte jedoch groß genug, dann ist erhöht Preissemikollusion den Wohlstand. Daher ist die Größe der externen Effekte ausschlaggebend für eine optimale Kartellgesetzgebung. So sollte Werbungssemikollusion immer verhindert werden. Jedoch sollte eine Preissemikollusion gefördert werden, wenn die externen Effekte groß genug sind.

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