DIPLOMARBEIT

Titel der Diplomarbeit

"The multi-item, single-machine scheduling problem with stochastic demands: a comparison of heuristics with discrete and continuous time simulation"

Verfasserin

Elisabeth Brandl

angestrebter akademischer Grad

Magistra der Sozial- und Wirtschaftswissenschaften

(Mag. rer. soc. oec)

Wien, im Dezember 2010

Studienkennzahl lt. Studienblatt:	A 157
Studienrichtung lt. Studienblatt:	Internationale Betriebswirtschaftslehre
Betreuer:	Prof. Dr. Stefan Minner

Abstract

This paper presents a study of a variant of the stochastic economic lot scheduling problem in which a single production facility has to produce different items to meet random stationary demand. Demand that cannot directly be satisfied from inventory is lost. Raw materials are always available and a production change requires setup cost and setup time that do not depend on sequence. Six heuristics are simulated in discrete time and continuous time environment. Each time a decision was made whether to continue production, to initiate a changeover or to idle the machine based on the current state of the system which is defined by the current setup of the machine and the inventory level for all items. The objective is to minimize total costs while satisfying customer demands as good as possible. The heuristics are compared in order to get to know if there is one that outperforms all others.

Zusammenfassung

Die vorliegende Diplomarbeit präsentiert eine Studie von einer Variante des „stochastic economic lot scheduling“-Problems, in dem eine einzige Maschine mehrere Produkte produziert, um eine variable Nachfrage zu erfüllen. Jede Nachfrage, die nicht direkt vom Lager befriedigt werden kann, ist verloren. Rohmaterial ist immer verfügbar und ein Produktionswechsel benötigt Zeit und verursacht Kosten, wobei beide nicht von der Abfolge der Produkte abhängig sind, um die Maschine neu zu rüsten. Sechs Heuristiken werden in zwei verschiedenen Umgebungen simuliert. Im ersten Fall werde täglich, im zweiten wird bei jeder Veränderung des Systems eine Entscheidung gefällt. Jedes Mal wird - basierend auf dem aktuellen Status des Systems – entschieden, ob die Produktion weitergeführt wird, ob ein Produktionswechsel initiiert werden soll oder ob die Produktion pausieren soll. Der aktuelle Status des Systems zeigt, für welches Produkt die Maschine gerüstet ist und wie hoch die Lagerstände aller Produkte sind. Das Ziel ist es, die Gesamtkosten zu minimieren, wobei die Kundennachfrage bestmöglich befriedigt werden soll. Die Heuristiken werden miteinander verglichen, um herauszufinden, ob es ein Verfahren gibt, das besser funktioniert als alle anderen.

Table of contents

List of figures

List of tables

Abbreviation index

BP	Basic period approach
CC	Common cycle approach
DCL	Dynamic cycle length heuristic
EBP	Extended basic period approach
EDCL	Enhanced dynamic cycle length heuristic
ELSP	Economic lot scheduling problem
EMQ	Economic manufacturing quantity
EOQ	Economic order quantity
SCLSP	Stochastic capacitated lot scheduling problem
SELSP	Stochastic economic lot scheduling problem
e.g.	Exempli gratia (Latin), for example
eq.	Equation
et al.	Et alii (Latin), and others
i.e.	Id est (Latin), that is

Index of mathematical notation

a_i	Ratio of run length of item i in comparison to all other items
A	Setup cost (€/time unit)
A_i	Setup cost for item i (€/time unit)
b_i	Power-of-two multiplier
c	Setup time (time units)
c_i	Setup time for item i (time units)
cl	Remaining time for item setup (time units)
C	Total cost (€/time unit)
C^*	Total cost (€/time unit)
C_i	Total cost of item i (€/time unit)
$C_i(k_i, T)$	Total cost of item i depending on k_i and T (€/time unit)
$C_i(T_i)$	Total cost of item i depending on T_i (€/time unit)
d	Demand rate (units/time unit)
d_i	Demand rate for item i (units/time unit)
e	Minimum duration of a production run (units)
$F_m(\tau)$	Minimum cost of producing remaining $n - m$ items in time τ (€/time unit)
h	Inventory holding cost (€/unit & time unit)
h_i	Inventory holding cost for item i (€/unit & time unit)
I_i	Inventory level of item i (units)
k_i	Production frequency of item i
k_i^-	Nearest lower integer of T_i/T
k_i^+	Nearest higher integer of T_i/T
K	Number of basic periods
L	Lead-time (time units)
M and M'	Set of items
n	Number of items considered
o_i	Processing time for item i (time units)
p	Production rate (units/time unit)

p_i	Production rate for item i (units/time unit)
P	Review period (time units)
q_i	Order quantity for item i (units)
q^*	Order quantity (units)
Q	Order quantity (units)
r_i	Production time for item i (time units)
R	Reorder point (unit)
RO_i	Runout time of item i (time units)
s	Reorder point (unit)
s_i	Reorder point of item i (units)
ss_i	Safety stock level of item i (units)
sh_i	Shortage cost of item i (unit/time unit)
S	Maximum inventory level (units)
S_i	Maximum inventory level of item i (units)
t	Time used for production of item 1 to $m-1$ (time units)
T	Basic period or fundamental cycle (time units)
T_i	Cycle time for item i (time units)
T_i^o	Operational cycle time for item i (time units)
T^*	Cycle time (time units)
TS_j	Total slack of item j following item i
W	Common cycle time (time units)
w	Number of production cycles per year
x	Number of assigned working hours
y	Safety factor
Z	Parameter for confidence level
α	Reduction factor for operational cycle time (units)
α_{min}	Minimum reduction factor for operational cycle time (units)
ρ	Total utilization of machine (per cent)
ρ_i	Utilization of machine for item i (per cent)
σ_i	Forecasted standard deviation of the demand error
τ	Residual capacity (time units)

1 Introduction

The multi-item, single-machine scheduling problem is a common issue in inventory management and therefore the core concept of inventory management will be introduced. Afterwards the scheduling problem is defined, practical applications of the multi-item, single-machine problem are shown and the chapter ends with an outline of the paper.

1.1 Problem introduction

As discussed in Brander (2005), a lot of people deal with inventories on a daily basis without ever noticing it. Especially after going to the supermarket we have to store the food in the refrigerator and the other items around the house. If we do not keep our inventories in mind we could run out of space or buy the same things that are already at home. It could also happen that we find some food at home that is out of date because it was stored for too long. To avoid such situations everyone intuitively tries to manage its inventories.

Let us go back to the supermarket and assume we need some toilet paper. In these days there is a promotion for toilet paper, where it is possible to buy three packages and get one for free. The person that is out of toilet paper has to decide whether to take one package to the normal price or three packages to the promotional price. If the person assumes that there is enough space for storing at home, the promotion might be successful and three packages would be bought. This shows that the decision about how much inventories we store at home does not only imply the available storage space but also the cost saving that can be made by purchasing a higher quantity.

When companies are concerned with the inventory management problem they face different costs. These costs can be classified into four broad categories as shown in Ghiani, Laporte and Musmanno (2004, p. 121-122): procurement costs, inventory holding costs, shortage costs and obsolescence costs. Procurements costs are costs associated with the sourcing of goods. These costs may include reorder costs, for processing an order if the goods are bought or for setting up the production process if the goods are manufactured within the firm, purchasing or manufacturing costs, transportation costs and the costs for handling the goods. Inventory holding costs are associated with the storage of material for a period of time and include capital costs and warehousing costs. The capital or opportunity costs represent the return on investment the factory could have realized if the money had been invested in a more profitable economic activity. The warehousing costs are the costs for operating a warehouse or the rent that is paid for storing the goods in an outsourced warehouse. When operating a warehouse these costs include space and equipment costs, personal wages, insurance on inventories, maintenance costs, energy costs and taxes. Shortage costs are paid when customer orders cannot be satisfied. These costs are difficult to evaluate as no one knows about the negative effect the shortage could have on future sales. One differentiates between lost sales costs and back order costs. When an item is difficult to replace, the shortage often results in a delayed sale. A common item is easily purchased from a competitor and therefore the sale would be lost. Obsolescence costs arise when stored items lose some of their value over time. This happens, for example, when food deteriorates, clothing comes out of fashion or a new generation of electrical equipment is invented.

Keeping inventories can be very expensive. Ghiani et al. (2004, p. 121) show that the annual costs for holding inventories can be 30% of the value of materials stored or even more, but a lack of inventories could have serious negative effects. For instance, production facilities need the required raw materials to be able to produce their goods. Stores need enough finished goods to satisfy customer demand. Farmers need seeds for their fields otherwise they will have no crops to harvest and their animals need food to grow. According to Axsäter (2006, p. 2) the main reasons for inventories are economies of scale and uncertainties. “Economies of scale mean that we need to order in batches” (Axsäter, 2006, p. 2). Uncertainty in demand and supply creates a need for safety stock. Ghiani et al. (2004, p. 7) report several reasons for inventories like an improvement of service level, a reduction of overall logistics costs, the wish of coping with randomness in customer demand and lead time, the possibility of making seasonal items available throughout the year, the speculation on price patterns and the chance to overcome inefficiencies in managing the logistics system. When the fixed costs to setup a machine are high, the company may find it more convenient to produce more items at once than to produce small orders frequently. Inventories help to satisfy even unexpected peaks of demand and can relax when deliveries are delayed. Raw materials whose price varies greatly during the year can be purchased when prices are low, stored and used for production when needed. For Stock and Lambert (2001, p. 228) inventories serve the following five purposes within a firm. It enables to achieve economies of scale, it can balance supply and demand, it permits specialization in manufacturing, it provides protection against uncertainties in demand and production cycle and it acts as a buffer between critical interfaces within the supply chain.

All these positive and negative effects of inventories show how important it is to manage inventories carefully. Inventories occur on different positions of a supply chain and can be seen as supplies of goods like raw materials, components, semi-finished goods and finished goods, waiting to be manufactured, transported or used (Ghiani et al. 2004, p. 6-7). According to Brander (2005) every inventory lies between two activities or processes. The supply process adds new units to the inventory and the demand process takes items or materials from the inventory.

The inventories can also be categorized according to the way they are created. Krajewski and Ritzman (1995, p. 416-418) identify four types of inventories in this context: cycle, safety, anticipation and pipeline inventories. Cycle inventory results from batch production instead of processing one unit at a time and is the part of total inventory that varies immediately with the lot size. Safety stock is held by a company in order to protect against uncertainties in demand, lead time and supply. Anticipation stock is used to build up inventory to be able to satisfy future demand that is expected to be higher than available production capacity, e.g. when facing seasonal demand. The pipeline inventory represents the items that move from point to point in the material flow system. Silver, Pyke and Peterson (1998, p. 30-31) define two additional types, the congestion stock and the decoupling stock. The congestion stock occurs when multiple items share the same production equipment, especially when setup times are significant. The items compete for the limited capacity of the machine and inventories are built up as they have to wait for the equipment to become available for the next production. The decoupling stock is used in situations with many production levels through a supply chain to be able to separate the decision making at the different levels.

The objective of inventory management is to determine stock levels such that total operating costs are minimized while customer service requirements are satisfied (Ghiani et al., 2004, p. 7). For Stock and Lambert (2001, p. 21) the inventory management involves a compromise for the level of inventory held to achieve high customer service levels with the cost of holding inventory. Successful inventory control contains the determination of the level of inventory that is necessary to achieve the desired level of customer service while the cost of performing other logistics activities has to be considered. Axsäter (2006, p. 1-2) states that inventories cannot be decoupled from other functions and therefore the aim of inventory management is to balance conflicting goals. One goal is to keep inventory levels down to increase cash availability for other targets. The purchasing manager, the production manager and the marketing manager have other goals. The purchasing manager can get volume discounts when ordering in large batches. The production manager can avoid time consuming setups only if there are enough inventories that allow for long production runs. Additionally, he prefers to have a large raw material inventory to avoid that the production line must be shut down due to a lack of materials. The marketing manager wants to have high inventories of finished goods to be able to quickly respond to customer demands. Segerstedt (1999) means that, with production and inventory control, profitability is achieved by the difficult balancing of resource utilization (high), capital and inventory investments (low) and market services (high). These objectives may be inconsistent in some aspects but cooperate in others depending on the situation. For example, high utilization of resources may imply long delivery times resulting in a low service level but a large inventory can create high customer service. These objectives must be balanced in such a way that they coincide with other aggregate objectives of the company (Brander, 2005). If a good solution is developed the company makes one step further in achieving the mission of logistics, which is getting the right goods or services in the right quantity and the right quality at the right time to the right place at the right costs (Ballou, 2004, p. 6).

An inventory management system can be designed, so that the position of the inventory is monitored continuously or at certain time intervals. In the continuous review system an order is triggered as soon as the inventory is below a certain level. In the period review, the inventory position is only considered at certain time intervals that are generally constant. Both alternatives have advantages and disadvantages. The continuous review system will reduce the needed safety stock as the item will be ordered exactly at the reorder point and the safety stock should guard against demand during the lead-time L. The lead-time L is the time from ordering decision until the item is available on stock. It is the production time of the item or the arrival time if the item is produced from an external supplier plus the time that is needed to fulfill the order. The safety stock that is needed to satisfy demand in periodic review system will be higher as an order can only be triggered in P time units. P is the review period, i.e. the time interval between reviews. So the safety stock has to be high enough to satisfy demand in time $P + L$. The periodic review has advantages when coordinating orders for items with high demand even if modern technology has reduced the costs for inspections. Items with low demand could be reviewed continuously as there would only be a moderate increase in cost. It should be noted that periodic review with short review periods P are very similar to the continuous review. (Axsäter, 2006, p. 47)

The two most common ordering policies connected with inventory management are the (R, Q) policy and the (s, S) policy. When using the (R, Q) policy, one needs to determine the reorder quantity R and the order quantity Q. A new production has to start when the inventory reaches R and the amount that is produced is determined by Q. The (s, S) policy is similar to the (R, Q) policy where s stands for the reorder point and S represents a maximum inventory level. When the inventory reaches s or falls below the item is ordered up to the maximum inventory level S. The only difference to the (R, Q) policy is that the (s, S) policy does not order in given batch quantities. If the inventory level reaches the reorder point exactly when the product is ordered, the policies have the same outcome. But if the inventory falls below the reorder point the (R, Q) policy would not be able to reach the maximum inventory level while the (s, S) policy produces a higher quantity of the item and has no problem to reach the maximum inventory level. So the (s, S) policy is more important in periodic review systems. (Axsäter, 2006, p. 48-49)

Axsäter (2006) classifies three branches of inventory management problems corresponding to the problem that is studied. The first branch is for a single machine where items can be handled independently. For this problem one of the most well-known results in inventory management may be used: the economic order quantity (EOQ). The EOQ originates from Harris (1913) and assumes that each item is produced at its own machine and therefore there is no need for scheduling. The second branch of inventory management problems considers coordinated replenishments of two types. The first reason for the coordinated replenishment is to get a good utilization of the machine especially when a group of items is produced on the same machine. The other reason is where a group of items should be ordered at the same time to get discounts or to reduce transportation costs. This is called the joint replenishment. The third branch of problems considers multi-level inventory systems where several inventories are connected to each other, for instance in systems with a central warehouse. The research in this paper concentrates on the second branch and considers the first type of coordinated replenishment, namely the multi-item, single-machine system. (Brander, 2005)

1.2 Definition of the problem

Many companies produce various items in one facility. Typically, it is often more economical to invest in one high-speed machine that is capable of producing many items than to invest in many dedicated machines (Dobson, 1987; Gallego and Roundy, 1992). This machine can be the so-called bottleneck; that is one stage of a facility that generates most of the added value and utilizes the most expensive equipment (Winands, Adan and van Houtum, 2005). The scheduling decisions of the plant are generally dominated by this bottleneck meaning that the upstream and downstream production plans are derived from the plan inserted at the bottleneck. Typically such high-speed machines have limited capacity and only one item can be produced at a time requiring a setup each time production is changed (Gallego and Roundy, 1992).

Brander (2005) indicates the following characteristics as dominant characteristics of a multi-item, single-machine system:

- A single machine
- Multiple items
- One item produced at a time
- Limited capacity but enough to satisfy total demand
- Setup time between the processing of different items
- Possibly different cost structures and machine utilization

Winands et al. (2005) states that many variants of the multi-item, single-machine scheduling problem exist and classify them by three characteristics:

- *Presence or absence of setup times and/or costs.* The most important consequence of setups on the production plan is that the items need to be manufactured in batches for saving costly capacity that would be used for setups.
- *Customized or standardized items.* When facing customized items the production works in a make-to-order fashion where production only starts when an order has arrived. In case of standardized items one can work with a make-to-stock policy, because these items do not have to be produced to customer specifications. Obviously standardized items are easier to plan as the manufacturer can decide when to produce the item and in what quality.
- *Stochastic or deterministic environment*. A deterministic environment allows us to schedule production according to a cyclic production plan. For companies facing stochastic environment such a cyclic schedule will not be adequate anymore.

Within this categorization the present paper considers the production of multiple standardized items with the presence of setup times and setup costs in a stochastic environment. This is the so-called stochastic economic lot scheduling problem (SELSP). A production plan, which describes for each possible state of the system whether to continue current production, to initiate a changeover or to idle the machine, is needed. The objective is to minimize total costs, that is, the sum of setup costs and inventory holding costs, while satisfying customer demands as good as possible. (Winands et al., 2005)

1.3 Practical applications

The multi-item, single-machine problem can be used for many different practical applications and is found in many industries. Bomberger (1966) presents a 10-item problem for a metal stamping facility producing a number of different stampings on the same press line. This 10-item problem is widely used in the literature to test many different methods and algorithms. For Dobson (1987) the practical applications range from the production of toner for copiers to plastic wrap for potato chips. Gascon, Leachman and Lefranç ois (1994) show a packaging line of a large consumer products manufacturer where products such as soft drinks, bleach, soaps or detergents, produced in bulk, are bottled or boxed into retail sizes. For Sox, Jackson, Bowman and Muckstadt (1999) any production process with significant changeovers between items benefits from an effective scheduling system. They include bottling, paper production, molding and stamping operations. Winands et al. (2005) find the multi-item, single-machine scheduling problem in glass and paper production, injection molding, metal stamping as well as in semi-continuous chemical processes and in bulk production of consumer products such as detergents and beers. They point out some special applications like a laminate manufacturer, a glass-containers manufacturing company, a producer of plastic bumpers for cars and an aerospace component supplier. Liberopoulos, Pandelis and Hatzikonstantinou (2009) use the differentiation of Sox et al. (1999) to distinguish between discrete-parts manufacturing and continuous-processing manufacturing. The discrete-parts manufacturing that is characterized by individual items which are clearly distinguishable is often found in industries of computer and electronic products, electrical or transport equipment, machinery, wood, furniture products and so on. The process industries operate on material that is continuously flowing like petroleum and coal products, metallurgical products, food and beverage, paper products. In a typical process industry the production facility operates continuously, at a constant rate, and the different items are really variants of the same production family that differ in one or more attributes, such as quality, color, grade, weight, size, thickness.

All examples shown above consider a forward flow of goods in which raw materials are procured and items are produced at one or more machines, stored and sold to the customers (Brander, 2005). In many cases there is also an opposite flow back from the customer to the producer, mostly due to re-use activities. According to Thierry, Salomon, van Nunen and van Wassenhove (1999) returned items can be resold directly, recovered, or disposed. There are five recovering options that are ordered according to the required degree of disassembly: repair, refurbishing, remanufacturing, cannibalization and recycling. The re-use activities have become more and more important in recent years, mainly due to legislation and customer expectations (Brander, 2005). The multi-item single-machine systems can also be found in the reverse flow, e.g. for toner cartridges or beer bottles.

1.4 Outline of the paper

The remaining paper is organized as follows: Chapter 2 presents an overview of the research on the single-machine scheduling problem including the single-item, multi-items with deterministic environment and multi-items with stochastic demand. The single-item, single-machine problem is the basis for the multi-item case, as well as the deterministic environment is the basis for the stochastic research. Therefore different models, facing these problems, are introduced and applied to a small numerical example for easier understanding. The third chapter presents the heuristics that are used for the simulation study and the (s, S) policy. In chapter 4 the case study is outlined and it is shown how the heuristics are used for the simulations. Additionally the steps of the simulation model are explained for discrete time and continuous time review systems. Chapter 5 shows the impact of the modification that improves the sensitivity of the heuristic on the DCL and the EDCL heuristic and exhibits the results of the simulation study. The last chapter presents the conclusions that can be drawn from the study.

2 Literature review

The following chapter presents an overview of the research concerning the multi-item, single-machine scheduling problem. At the beginning the single-item, single-machine problem is discussed as it is used as a basis for the multi-item problem. Then research on the multi-item problem with deterministic demands will be covered and the chapter ends with presenting multi-item scheduling problems with stochastic demands.

2.1 Single-item, single-machine problem

At the beginning of the research for scheduling of economic lot sizes it is assumed that one machine produces one item at a constant and known rate. The demand for that item is also supposed to be constant and known. Whenever the machine starts production from idle condition a time is needed to do a setup. Each time the setup is accomplished, setup costs have to be paid. The inventory that is held on stock is charged by inventory holding costs on the basis of average inventory. (Maxwell, 1964)

In 1913 Ford W. Harris develops the economic order quantity (EOQ) model that explains the relationship between setup cost, inventory holding cost and average demand for an item on a single machine. Harris (1913) states that it is generally understood what set-up costs mean on small orders so managers would produce in large quantities to keep down these costs. In doing so, large deliveries would arrive in the storage resulting in a large stock. The optimal quantity that best balance these two cost components is calculated by using the following notations:

d Demand rate (units/time unit)

p Production rate (units/time unit)

h Inventory holding cost (€/unit & time unit)

A Setup cost (€/unit)

c Setup time (time units/unit)

Let q^* be the optimal quantity of items that are produced in an order. This optimal quantity is also called the optimal lot size.

$$q^* = \sqrt{\frac{2Ad}{h\left(1-\frac{d}{p}\right)}} \tag{1}$$

The cost C^* that is involved with the optimal lot size is calculated as:

$$C^* = \frac{Ad}{q^*} + \frac{hq\left(1-\frac{d}{p}\right)}{2} \ . \tag{2}$$

Whenever a parameter used for the calculation of the optimal lot size changes, the optimal lot size will change as well (Maxwell, 1964). When the setup cost, the setup time or the demand rate increase the lot size will increase as well while it will decrease when inventory holding costs or facility idle time decrease.

Maxwell (1964) shows that the analysis of the EOQ is incomplete if a positive setup time is required to start the machine. The time that is used for the setup may impose time requirements which would exceed the available time. In a time period T^*, the time that is available for setup is $(1 - d/p)\,T^*$. The amount of setups in time T^* is dT^*/q^*. The lot size must satisfy the following inequality since the setup time required cannot exceed the available time

$$q^* \geq \frac{dc}{1-\frac{d}{p}} \ . \tag{3}$$

This means that the complete solution of the single-item, single-machine problem looks like:

$$q^* = \max\left(\sqrt{\frac{2Ad}{h\left(1-\frac{d}{p}\right)}}, \quad \frac{dc}{1-\frac{d}{p}}\right). \tag{4}$$

To illustrate the importance of the EOQ model, let us consider a production facility. 250 items can be produced each day and a demand of 50 items per day has to be satisfied. Starting a production takes 0.5 days, generating setup costs of 20 €. Inventory holding costs are 0.04 € per item and day. To get the optimal lot size equation (4) is applied and gives an optimal lot size of 250 items according to eq. (1). The total costs of 8 € are calculated by using eq. (2). Deviating for the optimal quantity by 50 units less or more would increase the total costs by 25 or 16.6 per cent respectively. Therefore we decide to produce 250 items each period the machine is operating. To be able to satisfy the demand of 50 units per day the machine has to produce every five days. This cycle time is calculated by dividing the optimal quantity through the demand rate. The cycle time indicates for how long time the optimal order quantity will cover demand.

If we want to schedule this production we need to get some more information. We already know that we have to start a production run every fifth day. This production run has a setup time of 0.5 days. If we start the setup at time 0, the machine is ready for production at time 0.5. The time the facility is used to produce the order is called the processing time and is calculated by dividing the lot size with production rate. The processing will take one day as the lot size is as high as the daily production rate. If we start processing at time 0.5 the production will be finished at time 1.5. So 1.5 days is the total time the order needs to be finished as it is the processing time plus the setup time that is needed to start production.

At time 1.5 the machine is set idle and the demand is satisfied from inventory. At this time 200 items will be on inventory because the demand for one day had already been satisfied. The production cycle of 5 indicates that the next order should enter at time 5.0. The production is started after the 0.5 days setup, producing for one day, and the next order will arrive at time 10.0 with a production start at 10.5. As long as the parameters for the EOQ do not change, the scheduling will continue until the production of the item is totally stopped. Figure 1 shows the inventory of the product that result for the scheduling plan.

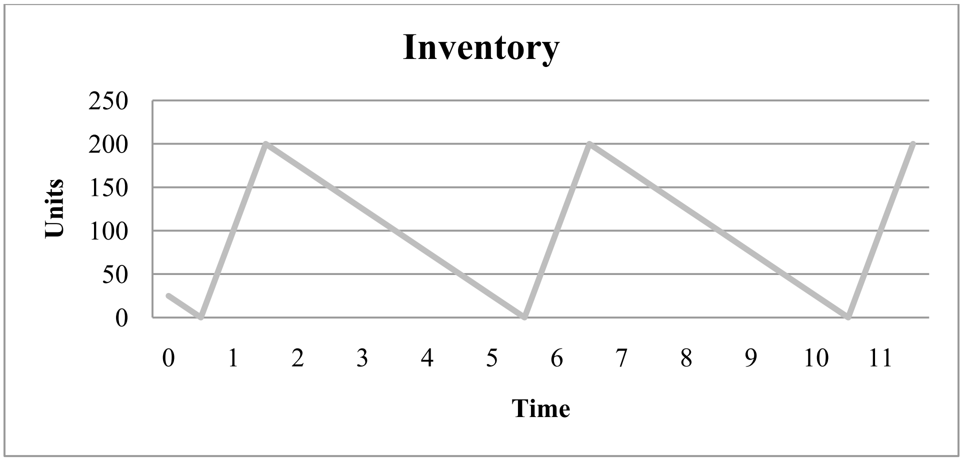

Figure 1 Inventory of single item

The economic order quantity gets optimal lot sizes for the single-item assumption but it cannot be applied optimally when more than one product is produced on the same machine. This should be logical as an assumption of the single-item analysis is that the entire production time is available for one product. The question is, if an application of the EOQ in a multi-item setting could generate feasible solutions, even if they are non-optimal. When regarding the feasibility, the question of scheduling conflicts arises. Can we sequence the lot sizes in a way that the required time intervals of production do not overlap? (Maxwell, 1964)

Let us consider the production facility again. The facility can now produce three items, with item A being the same item as before. The notations are the same as in the single product case with the supplement of subscript i, used to differentiate between the different items, $i = 1, \dots, n$. The characteristics of the three item production facility are shown in table 1.

Item	d_i	p_i	h_i	c_i	A_i
A	50	250	0.04	0.5	20
B	10	100	0.75	0.3	55
C	25	75	1	0.1	33

Table 1 Item characteristics

To be able to calculate the EOQ quantity we assume that each product is produced at its own machine, resulting in a need of three machines. The calculations are done as before, where the optimal quantity q_i for each item is achieved through equation (4). This lot size is used to determine all other values. The results are shown in table 2, where the first row represents the values that have been achieved before.

Item	q_i	T_i	τ_i	r_i	C_i
A	250	5	1	1.5	8.00
B	40	4	0.40	0.70	27.25
C	50	2	0.66	0.76	33.17

Table 2 Order quantities, times and total costs of EOQ

Producing the items on three machines, item A should be produced in a time interval of five time units, item B has a cycle time of four and item C should be produced every second period. If we want to produce these items together on one machine it is important that the machine utilization is smaller than one (Axsäter, 2006, p. 158). The machine utilization is the ratio of the time the machine is busy without setup times. Since we need some time for the setup of the items production cannot take all of the available time. The production capacity of one machine is potentially adequate as the facility will have a utilization of 63 per cent (50/250 + 10/100 + 25/75 = 0.63).

Knowing the production plan of item A, as shown in figure 1, we would start production of item B immediately after the completion of production of item A. This would imply that setup of item B begins at time 1.5 and production could start at time 1.8. With a cycle time of 4 time units, the second setup should begin at time 5.5 with production starting at time 5.8. This is not possible as the machine is busy with item A being produced from time 5.5 until time 6.5. The resulting schedule blockage can be seen in figure 2, where the light grey fields represent the setup times and the dark grey fields represent production times.

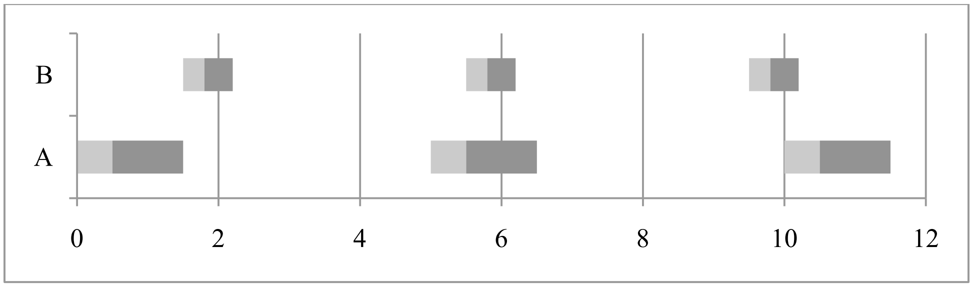

Figure 2 Scheduling item A and B

Figure 2 shows that there is another scheduling blockage at time 10.0. To avoid a scheduling blockage the production plan of item B must be re-scheduled.

The production of item B actually starts immediately after the production of item A. So, it is possible to postpone the production of item B. The second production of item A ends at time 6.5, so the second setup for item B could start at time 6.5. It would mean that the first setup should start at time 2.5. This would be possible, but the third setup at time 10.5 is impossible because at that time item A is produced. The machine is busy until time unit 11.5. Therefore we could try to postpone the production of item B by one more time unit. The first setup would start at time 3.5 and the machine would be busy until 4.2. If item B is finished at time 4.2 there should have been a previous production of item B that is finished at time 0.2 since the cycle time is 4 time units. This is not possible since the facility is busy producing item A from time 0 to time 1.5. So, finding a schedule for item A and B such that scheduling blockages are avoided is not possible when using the order quantities calculated from the EOQ formula. We should not forget that the production of item C has not yet been considered. A production cycle of 2 time units with a total production time of 0.76 is not possible to schedule when there is another product that has a total production time of 1.5. This shows that scheduling becomes more complicated when the amount of items considered increase.

Salveson (1956) addresses the question of feasibility and applicability of the economic order quantity for multi-items as follows: “The fundamental defect in the conventional formulations is that the formulae are designed for minimizing each part’s manufacturing cost independently. The formulae do not treat of the circumstances where two or more parts are produced by, or share, a common production facility. Of course, relatively few industrial enterprises manufacture only one part; those that do certainly must manufacture that part continuously so that they have no use for economic lot size formulae. Thus, the economic lot size formulae developed to date are either of no use where they are technically applicable or they are not technically applicable where they may be useful. The desired method when annual production quantity is fixed and when the parts are produced intermittently is to minimize total costs for all parts simultaneously."

Salveson (1956) and many more researchers have discussed the reasons for the difficulties of the EOQ in order to develop an alternative approach to the economic lot size concept. In the year 1978, Elmaghraby published a review of the economic lot scheduling problem (ELSP). He could find no less than 25 contributions concerning that problem as it has occupied researchers for a period of 25 years. The ELSP is the deterministic version of the SELSP and is easier to solve as demand and production rates are supposed to be constant and known in advance.

2.2 Multi-item, single-machine problem with deterministic demand

Elmaghraby (1978) makes a differentiation between the economic order quantity (EOQ) and the economic manufacturing quantity (EMQ). He states that the ELSP arises from the desire to accommodate the cyclical production patterns that are based on EMQ calculations for individual items on a single production facility while the EOQ deals with the impact of cash flow considerations. This is the reason why the economic manufacturing quantity (EMQ) is used from now on, instead of the EOQ.

According to Elmaghraby (1978) the developed approaches divide themselves into two broad categories:

I. Analytical approaches that achieve the optimum for a restricted version of the original problem.

II. Heuristic approaches that achieve solutions of the original problem at a high quality.

In some sense, each category presents a penalty that is paid as a price for deviating from the true optimum. The category I approaches solve a slightly different problem to its optimum while the category II approaches work with the original problem never knowing if a better solution exists.

The following approaches work with the same assumptions as Harris (1913). For each item i, $i = 1, 2, \ldots, n$, demand rate, d_i, is assumed to be constant and known deterministically and production rate, p_i, is assumed to be fixed with no learning effects and to be higher than demand. The setup time, c_i, is independent of sequence and occurs at each set up as well as the setup cost, A_i. Holding costs, h_i, have to be paid for stored items and backlogging is not allowed. (Elmaghraby, 1978)

With this data the following parameters can be derived: the utilization of the machine, ρ_i, of item i, $\rho_i = d_i/p_i$; and the total utilization of the machine, ρ, that has to be smaller than one, $\rho = \sum_i \rho_i \leq 1$. It is common to use cyclic schedules when different items are coordinated for replenishment (Axsäter, 2006, p. 149). So the (unknown) cycle time for item i, T_i, is used to calculated the remaining parameters as the processing time, o_i, for item i, $o_i = \rho_i T_i$; the total production time, r_i, for each item, $r_i = c_i + o_i$; the lot size, q_i, $q_i = d_i T_i = p_i o_i$. (Elmaghraby, 1978)

The solution that is achieved by regarding the items independently is a lower bound on the optimal value of any feasible solution (Elmaghraby, 1978). If any other cycle then T_i^* is adopted, $T_i \neq T_i^*$, the new costs are given by

$$C_i(T_i) = \frac{C_i^*}{2} * \left(\frac{T_i}{T_i^*} + \frac{T_i^*}{T_i}\right) \quad . \tag{5}$$

Taking the example of the company that produces three items as presented in table 1 we get a lower bound of 68.42 €. However we know that a feasible scheduling cannot be found with the cycles resulting from the EMQ.

According to Elmaghraby (1978) the most elementary approach to the ELSP is to a priori guarantee feasibility by inserting some constraints on the cycle times and then to optimize the individual cycle durations subject to the inserted constraint. Solutions obtained in this way are feasible and optimal over their sets of solutions. Including the publication of Elmaghraby (1978) three procedures have been developed with this approach in mind. The common cycle approach, where all cycle times are equal, the basic period approach, where cycle times are an integer multiple of a basic period and the extended basic period approach, where basic periods only have to be long enough to cover the average setup and production times for all items.

2.2.1 Common cycle approach

Maxwell (1964) states that earlier works assumed that the production works on a strict cycle or pure rotation basis, meaning that between the times of production of one product every other product is produced exactly once. According to Elmaghraby (1978) this common cycle (CC) approach was developed by Fred Hanssmann in the year 1962. Denote the common cycle length by W where $W_1 = W_2 = \cdots = W_n = W$. The lot size of each product is $q_i = d_i W$, since the inventory on stock has to last for W time units. The cost minimization under the assumption of a common cycle time is

$$C = \sum_{i=1}^{n} \frac{A_i}{W} + \frac{1}{2} \sum_{i=1}^{n} h_i d_i W \left(1 - \frac{d_i}{p_i}\right) . \qquad (6)$$

The optimal cycle time is obtained by differentiating the total cost expression with respect to the common cycle length W, setting the resultant expression equal to zero and solving for W:

$$W = \sqrt{2 \sum_{i=1}^{n} A_i \Bigg/ \sum_{i=1}^{n} h_i d_i \left(1 - \frac{d_i}{p_i}\right)} . \qquad (7)$$

As in the single item problem, the restraints due to the production saturation ($\rho = \sum_i \rho_i \leq 1$) and due to the changeover frequency saturation have to be satisfied (Maxwell, 1964). The latter constraint is a lower bound for the cycle time where:

$$W \geq \sum_{i=1}^{n} c_i \bigg/ 1 - \sum_{i=1}^{n} \frac{d_i}{p_i} . \tag{8}$$

So, the optimal solution for W is obtained by

$$W = \max\left(\sqrt{2 \sum_{i=1}^{n} A_i \bigg/ \sum_{i=1}^{n} h_i d_i \left(1 - \frac{d_i}{p_i}\right)} , \quad \sum_{i=1}^{n} c_i \bigg/ 1 - \sum_{i=1}^{n} \frac{d_i}{p_i} \right) . \tag{9}$$

Applying the common cycle length approach to the three item facility from table 1, cycle times of 2.94 and 2.45 time units are obtained from eq. (9). The largest value will be selected as optimum; the smallest value shows the minimum cycle length that satisfies the restraints due to changeover frequency. The resulting order quantities for cycle time 2.94 and the relevant costs are shown in table 3.

W = 2.94				
Item	q_i	τ_i	c_i	r_i
A	147	0.59	0.5	1.09
B	29	0.29	0.3	0.59
C	73	0.98	0.1	1.08
				2.76

Table 3 Order quantities and times of CC

The solution is feasible as the total time needed for production is less than the common cycle time and each item is produced exactly once, so there will be no problem of sequencing and scheduling. The total costs would be 73.51 €. These costs signify the upper bound for the optimal solution as the additional constraint, that all cycle times have to be equal, was enforced. (Axsäter, 2006, p. 159)

2.2.2 Basic period approach

The basic period (BP) approach, invented by Bomberger (1966), extends the common cycle approach. Different items are allowed to have different production cycles as long as each cycle time is an integer multiple of a basic period T, such that:

$$T_i = k_i T \ , \tag{10}$$

where k_i represents the integer multiplier of item i. The basic period is a time interval that is long enough to accommodate the production of all the items, which implies:

$$\sum_{i=1}^{n} r_i = \sum_{i=1}^{n} \left(c_i + \frac{d_i}{p_i} k_i T\right) \leq T \ . \tag{11}$$

This condition guarantees that solutions are feasible but the restriction may lead to suboptimal costs. For a given value of the basic period the analytical approaches normally use dynamic programming, as Bomberger (1966), or integer nonlinear programming models, as Davis (1990), to solve for the multipliers and the basic period in which each item is produced. (Brander, 2005)

Bomberger (1966) suppose for items $1, 2, \dots, m-1$ to have the multipliers $k_{1,} k_2, \dots, k_{m-1}$ and the consumption of t time units for setup and production. The remaining $n-m$ items can be produced with the residual capacity of τ, $\tau = T - t$, at costs of $F_m(\tau)$. The formulation of average total cost of item i looks like:

$$C_i(k_i, T) = \frac{A_i}{k_i T} + \frac{1}{2} h_i d_i \left(1 - \frac{d_i}{p_i}\right) k_i T \ . \tag{12}$$

The minimum cost of producing the remaining item $(m, \dots, n)$ in the remaining time τ, can be written as:

$$F_m(\tau) = \min_{k_m} \left[C_m(k_m, T) + F_{m+1}\left(\tau - \frac{k_m d_m T}{p_m} - c_m\right)\right] \ , \tag{13}$$

where $1 \leq k_m \leq \frac{(\tau - c_m) p_m}{d_m T}$; k_m is integer; and $F_{n+1}(.) = 0$. The multiplier k_m is determined by solving the function for the minimum cost beginning with $m = n$ and varying the residual capacity τ as $\tau = \Delta T, 2\Delta T, \dots, T$. (Brander, 2005)

The concept of the basic period approach had too tight restrictions for researchers like Stankard and Gupta (1969), Doll and Whybark (1973), Haessler and Hongue (1976) and Elmaghraby (1978). They relaxed the restriction that, if a product is not scheduled for production during some production cycle, no product can be produced instead. (Brander, 2005)

For Elmaghraby (1978), the heuristic approaches seemed to cluster around two poles: approaches that accept the concept of a basic period and those which reject it. Elmaghraby (1978) deals with procedures that belong to the first category like those published from Madigan (1968), Stankard and Gupta (1969), Doll and Whybark (1973) and Goyal (1973a). Rogers (1958) stood almost alone in the latter category, until Delporte and Thomas (1976) demonstrated that even if the basic period were accepted, the multipliers need not to be the same for each run of an item during a cycle to achieve feasibility. The Doll and Whybark (1973) procedure that iteratively determines the production frequencies and the basic period will be presented in the following.

2.2.3 Doll and Whybark heuristic

Doll and Whybark (1973) criticize that most of the procedures depend on judgment to define the desirable frequencies of production. They invent an iterative procedure to determine the target cycle for item i, T_i, directly. This target cycle, T_i, is a multiple of a basic period or target fundamental cycle, T, and a positive integer, k_i, where $T_i = k_i T$. k_i is an integer multiple as this facilitates the construction of feasible schedules and integer multiples have produced lower cost solutions to non-integer multiples. The objective of the procedure is to detect values of T and k_i that minimize the sum of setup costs and inventory holding costs for each item, that is

$$Min \sum_{i=1}^{n} C_i = \sum_{i=1}^{n} \left[\frac{A_i}{k_i T} + \frac{h_i d_i \left(1 - \frac{d_i}{p_i}\right) k_i T}{2} \right] . \qquad (14)$$

The iterative procedure works as follows (Doll and Whybark, 1973): At the beginning, the initial fundamental cycle length, T, and the positive integer multiplier, k_i, are calculated, and then these initial values are improved by iteration. The values of T_i are obtained for each product treated independently by applying the EMQ formula:

$$T_i = \sqrt{\frac{2A_i}{h_i d_i \left(1 - \frac{d_i}{p_i}\right)}} \quad . \tag{15}$$

The smallest T_i is selected as the initial fundamental cycle, that is $T = \min\{T_i\}$. The next step determines quotas between the optimal cycle time of each item and the fundamental cycle time by rounding to the next lower and higher integer:

$$k_i^- \leq \frac{T_i}{T} \leq k_i^+ \ . \tag{16}$$

New estimates of k_i are determined that best satisfy the cost function, $C_i(k_i, T)$, from equation (12). The cost penalty incurred by using $k_i^- T$ and $k_i^+ T$ as the production cycle for item i is evaluated and the new multiplier k_i are chosen by:

$$k_i = k_i^- \qquad \text{for} \qquad C_i(k_i^-) \leq C_i(k_i^+) \ ,$$

$$k_i = k_i^+ \qquad \text{for} \qquad C_i(k_i^+) \leq C_i(k_i^-) \ . \tag{17}$$

These estimations of k_i are used to recompute T by differentiating the cost expression:

$$T = \sqrt{\sum_{i=1}^{n} \left(\frac{A_i}{k_i}\right) \Bigg/ \sum_{i=1}^{n} \frac{h_i d_i k_i \left(1 - \frac{d_i}{p_i}\right)}{2}} \quad . \tag{18}$$

The current basic period, T, from eq. (18) is used to calculate new k_i^- and k_i^+ values from eq. (16). Another fundamental cycle, T, can be determined from eq. (18) with the new k_i from eq. (17). The end of the procedure is reached when identical values of k_i are produced from two consecutive iterations.

The iterative procedure will be illustrated for the facility with three items presented in table 1. The calculation of the optimal individual cycle time for each item from eq. (15) results in cycle time of five, four and two time units for A, B and C respectively as shown in table 2. The smallest T_i is selected as initial fundamental cycle time so $T = 2$. The integer multiples are calculated from eq. (16) and are used for the calculation of the cost using eq. (12). According to eq. (17) the smallest cost gives the multiplier to calculate the new basic period, T. Table 4 presents the results of these calculations:

Item	T_i	k_i^-	k_i^+	$C_i(k_i^-,T)$	$C_i(k_i^+,T)$	k_i
A	5	2	3	8.21	8.13	3
B	4	2	3	27.25	29.36	2
C	2	1	2	33.17	41.46	1

Table 4 Solution of Doll and Whybark

In the first iteration the smallest costs are resulted from the rounded up multiple three for item A, and from the rounded down multiple two and one for item B and C respectively. These values are taken to calculate the new fundamental cycle resulting in a T of 1.96 time units. Using the new fundamental cycle, the calculated multiples do not change, so the procedure can stop.

The final fundamental cycle is 1.96 time units and the items are produced every third cycle, every second cycle and in each cycle for item A, B and C respectively. This solution would generate total costs of 68.54 €, but unfortunately the solution is not feasible. Item A has a total production time of 1.68 time units and item C has a production time of 0.75 time units. As item C has to be produced in each cycle it is not possible to schedule item A within the fundamental cycle of 1.96 time units. The iterative procedure breaks down at this point.

Elmaghraby (1978) points out two serious drawbacks in the Doll and Whybark procedure. First, the procedure gives no guidance on how to proceed if one decides that the set of parameters (k_i, T) is infeasible. "The haphazard modification of the k_i's (such as the arbitrary rules of multipliers that are "power of 2") detracts from the apparent elegance of the first few steps of the iterative scheme" (Elmaghraby, 1987). The second problem is that if a modification for the multipliers is achieved to escape from infeasibility, the Doll and Whybark procedure may lead to a behavior in which convergence will never be achieved.

2.2.4 Extended basic period approach

The extended basic period (EBP) approach, published by Elmaghraby (1978), works with two basic periods. The items are loaded on the two periods simultaneously and the basic period has to be large enough to accommodate these simultaneous loadings. A feasible solution can be found when sets of items, M and M', are defined such that:

$$\sum_{i \in M} r_i \leq T \ ,$$
$$\sum_{i \in M'} r_i \leq T \ . \tag{19}$$

All basic periods are classified as even or odd. If the periods between runs, k_i, takes on an odd value, product i is registered as an element of both sets and will be run in all even and odd numbered periods. If k_i takes on an even value, product i is registered as an element of one set, either M or M' and will be run in either even or odd numbered periods but not in both. (Haessler, 1979)

Haessler (1979) presents an improved extended basic period procedure that starts by finding the cycle time, T_i, for each item considered individually, as

$$T_i = max\left(\sqrt{\frac{2A_i}{h_i d_i \left(1 - \frac{d_i}{p_i}\right)}}, \frac{c_i}{1 - \frac{d_i}{p_i}}\right) \ . \tag{20}$$

The smallest cycle time is selected as the initial estimate of the basic period, T, and values of b_i are achieved by

$$b_i \leq \frac{T_i}{T} \leq \ 2b_i \ , \tag{21}$$

where b_i is determined from the set {1, 2, 4, 8, … }. The target cycle, T_i, is set equal to $b_i T$ and $2b_i T$ to calculate the cost for each cycle time using eq. (12). The multiplier k_i is set to b_i or $2b_i$, depending on which presents the lower costs, equivalently to eq. (17).

A new basic period is calculated using the current values of k_i as:

$$T = max\left(\sqrt{\sum_{i=1}^{n}\left(\frac{A_i}{k_i}\right)\Big/\sum_{i=1}^{n}\frac{h_i d_i k_i\left(1-\frac{d_i}{p_i}\right)}{2}},\quad \sum_{i=1}^{n}\frac{c_i}{k_i}\Big/1-\sum_{i=1}^{n}\frac{d_i}{p_i}\right). \tag{22}$$

With the current basic period the process is repeated. New values of k_i are determine using eq. (21) and a new basic period T is calculated from eq. (22). The process terminates when consecutive iterations produce identical values. Thereafter the feasibility of the solution is investigated.

It is first tested if the total cost for the current k_i and T values can be reduced by simultaneously changing these values for each product considered individually. The total cost is given by eq. (14). Next, a feasible schedule is generated by sequentially assigning items to K basic periods. $K = \max_i(k_i)$, meaning that the highest k_i indicates how many basic periods are to be used. The items are sort based on the values of k_i from low to high. Those items with the same values of k_i are additionally sorted based on their total production time r_i from high to low, where

$$r_i = c_i + \frac{d_i k_i T}{p_i}. \tag{23}$$

The solution is generated by sequentially assigning the items starting with the lowest values of k_i to the first basic period with sufficient time available. If a feasible assignment is obtained the procedure terminates. If not, it continuous with systematically increasing the basic period time until a feasible solution is found for the current values of k_i. Use eq. (12) to compute the associated costs and save the solution, if it is the first feasible that is found or if the solution is better than the best feasible solution found so far. For each product that has a multiplier greater than one ($k_i \geq 1$), calculate the lower bound of cost using eq. (12) and eq. (22) if the value of k_i is halved. The product with the lowest lower bound is selected. If the lower bound is above the cost of the best solution found so far, the procedure is stopped. Otherwise, the value of k_i is halved for the selected product and generation of a feasible solution is started again.

The procedure must terminate with a feasible solution as

$$T \geq \sum_{i=1}^{n} \frac{c_i}{k_i} \Big/ 1 - \sum_{i=1}^{n} \frac{d_i}{p_i} \tag{24}$$

provides a condition for a feasible solution when all the k_i equal one. This would result in the same solution as achieved from the common cycle approach. (Haessler, 1979)

To illustrate the heuristic procedure it is applied on the three item example from table 1. At the beginning, the individual cycle times are calculated. The results are shown in table 2. The smallest cycle time is selected to be the initial basic period, $T = 2$. Equation (21) is used to determine the power-of-two multipliers and the related costs are calculated from eq. (12). This iteration is shown in table 5.

Item	T_i	b_i	$2b_i$	$C_i(b_i,T)$	$C_i(2b_i,T)$	k_i
A	5	2	4	8.21	8.88	2
B	4	2	4	27.25	33.77	2
C	2	1	2	33.17	41.46	1

Table 5 Initial solution of Haessler

The frequency, k_i, is set to the multiplier that achieves the lower costs, therefore the frequency is set to two for item A and B and for item C it is set to one. These values are used to determine a new basic period from eq. (22) resulting in $T = 2.06$. The second iteration use the actual basic period and achieves identical values on k_i. So the procedure is stopped.

The basic period is 2.06 time units and item A and B are produced every second period and item C is produced every period. This solution would cost 68.59 €, but it is not sure if it is feasible. At the beginning, it is tested if the total cost for the current solution can be reduced by simultaneously changing k_i and T for each product considered individually using eq. (22) and (12). No cost reduction is found, so the values of the solution are used to generate a feasible solution by sequentially assigning the items to the two basic periods. As discussed earlier this is done by sorting the items from low to high k_i and from high to low r_i, where r_i is calculated from eq. (23). The sorting of the three items is shown in table 6.

Item	k_i	r_i
C	1	0.79
A	2	1.32
B	2	0.71

Table 6 Sorting of solution of Haessler

Product C is assigned to both basic period as k_i is one. Item A is then assigned to the first basic period that has sufficient time available. This is not possible as the production of item A and C would require 2.11 time units and the basic period is 2.06 time units. So the basic period time is systematically increased until a feasible solution is found for the current values of k_i. This solution is found at a basic period of 2.25 time units and generates costs of 68.87 €. As this is the first feasible solution it is saved. In the following, for each product that has a multiplier higher than one, that is product A and B, a lower bound of cost is computed using eq. (22) and eq. (12) with the value of k_i being halved. Item A generates the lowest lower bound at costs of 71.51 €. As this lower bound is above the cost of the best solution found (71.51 € > 68.87 €) the procedure terminates. The values of the final solution are shown in table 7.

T = 2.25			
Item	k_i	r_i	C_i
C	1	0.85	33.42
A	2	1.4	8.04
B	2	0.75	27.41

Table 7 Final solution of Haessler

The items A and C are produced in the first basic period and B and C are produced in the second basic period. This solution is only 0.45 € above the lower bound from the EMQ but 4.64 € below the upper bound from the CC approach.

2.3 Multi-item, single-machine problem with stochastic demand

The previous section considered problems with deterministic demand, which is assumed by most of the approaches to the ELSP (Brander, 2005). However, in practice the demand rate for goods and services may not always be constant but vary greatly from one day to another. This stochastic demand increases the complexity of the problem in such a way that the deterministic version is investigated extensively within the literature whereas the stochastic problem is not as investigated (Sox et al., 1999). In the last years, a large number of papers have appeared on the SELSP which are surveyed by Winands et al. (2005). Vergin and Lee (1978) were one of the first researchers dealing with the problem of stochastic demand. They observed: “The literature is almost completely void of not only the development of analytical models, but even of discussion of the problem. A thorough review of the production scheduling and inventory management journals and books would almost suggest that the scheduling problem does not exist. Yet the multiple product single machine system is quite common in industry and demand is inevitably stochastic.”

When the question arises if deterministic models can be applied in stochastic environments researchers have different opinions. Maxwell (1964) means that one can make a strong argument for using the economic lot scheduling model under conditions of stochastic demand to obtain order quantities, followed by an analysis of stochastic behavior to obtain reorder points. However, Vergin and Lee (1978) point out that it is difficult in practice to establish and follow a specific schedule in the face of stochastic demand. They suggest that it may be more useful to utilize a dynamic scheduling rule that can react to disruptions like machine breakdowns, special rush orders and random deviations as well as shifts in demand pattern, than to seek to follow a rigid schedule. Graves (1980) states that deterministic results do not seem to be particularly helpful when solving a stochastic problem. The reason can be found in the complexity of the problem as any solution procedure has to simultaneously consider lot-sizing and capacity allocation decisions.

In 1978, Vergin and Lee could find two approaches to the stochastic environment but both were useless for the practical problem faced by schedulers because of their simplification assumptions. One approach, published by Dzielinski and Manne (1961), was limited to systems with items having identical customer demand, identical manufacturing times and a limit of one unit demand. The other approach, published by Goyal (1973b), assumed total production time equals zero, which implies a machine with infinite capacity. Vergin and Lee (1978) construct six different scheduling rules for stochastic demand that are based on the idea that production of an item should continue until either the inventory of some other item runs out or the inventory of the item being currently produced builds up to some proportion of the total inventory on hand or reaches its pre-specified maximum level. A simulation study showed that these rules always outperform approaches for deterministic ELSP models.

Graves (1980) analyzed a one-item problem as a Markov decision formulation. This analysis is used for the multi-item problem, where a composite product is introduced as a new notion to account for product interdependency due to the sharing of production capacity. This heuristic is compared to a minimum and maximum level policy where an item is produced up to its maximum level, unless some other item reaches its reorder point, at which the production is switched to the other product. Graves' heuristic is seen as an interesting approach that is limited by the substantial computations it requires. Tests showed effective heuristics but these tests were limited to problems in which all items have identical cost structures in order to reduce the amount of computation.

Since the beginning of the research on the SELSP by Vergin and Lee (1978) and Graves (1980) a lot of papers can be found in the literature. In 1999, Sox et al. review the research on the stochastic lot scheduling problem. In their survey, the SELSP is referred to the continuous time planning with infinite planning horizon and stationary demand. The discrete time planning with finite planning horizon and possibly non-stationary demand is called the stochastic capacitated lot size problem (SCLSP). The discrete time models are applicable for production planning of finished goods inventories or for systems in which demand is processed on a periodic basis while the continuous time models are appropriate for applications with relatively low inventory such as the production control of work-in-process inventory.

In 2005, Winands et al. published a new survey paper about the SELSP with a different classification of the literature, where the problem is seen by the manager of the production department. The critical elements of a production plan are the following two: production sequence and lot sizing policy. The sequence of production can be fixed or dynamic. In a fixed production sequence, a pre-defined order and frequency for the production of the items exists and makes it easy to implement the strategy. For such fixed production strategies an additional classification whether a fixed cycle length is used or not can be made. Working with a fixed cycle length has the advantage that one knows exactly when the production plan is restarted. Concerning the lot sizing it can be distinguished between global lot sizing and local lot sizing. In global lot sizing policies, lot sizing decisions depend on the complete state of the system, i.e. stock levels of all the individual items and the state of the machine, while local lot sizing policies decisions only depend on the stock level of the product currently set-up. The advantage of the global strategy is that it is able to react to changes in the system at each point in time, while the local strategy produces until the pre-specified end of the current production is achieved. (Winands et al., 2005)

While the fixed production sequence in combination with fixed cycle length has achieved little attention in the literature, the fixed production sequence with dynamic cycle length has achieved a lot of attention in the literature. According to Winands et al. (2005) only three papers are published in the case of fixed cycle length, one that works with global lot sizing policy and two papers that work with local lot sizing. One of the latter mentions some drawbacks of the fixed cycle strategy where the most important one is that pooling effects cannot be obtained. Representative works based on dynamic cycle scheduling with global lot sizing are Leachman and Gascon (1988), Gallego (1990, 1994), Bourland and Yano (1994), Fransoo, Sridharan and Bertrand (1995), Markowitz, Reiman and Wein (2000), Markowitz and Wein (2001).

Leachman and Gascon (1988) develop the dynamic cycle lengths (DCL) heuristic that is based on the concept of run-out times and total slack. The run-out time is the expected time until an item runs out of stock and the total slack is defined as the expected run-out time apart from setup time and production time. If the total slack is negative, lot sizes are reduced in such way that a schedule with positive total slack can be generated. Leachman, Xiong, Gascon and Park (1991) improve the calculations of production time with an expression that is more accurate than in Leachman and Gascon (1988).

In 1994, Gascon, Leachman and Lefranç ois make an extensive simulation study on the performance of six different heuristic with three production environments and conclude that the performance of the invented heuristics is satisfactory as long as the load is not extremely high. It turns out that two heuristics can be recommended, depending on the pursued objective and the production environment, and that heuristics invented for stochastic demands outperforms simple heuristics such as the EMQ rule. The Vergin and Lee heuristic (Vergin and Lee, 1978) was best at maintaining high service levels while the enhanced dynamic cycle lengths (EDCL) heuristic (Leachman *et al.* 1991) was best at minimizing total costs while maintaining fairly high service levels. Gascon, Leachman and Lefranç ois (1995) publish an improvement on the EDCL that outperforms all other heuristics for both pursued objectives and all three production environments tested.

Fransoo, Sridharan and Bertrand (1995) study a model for the SELSP under comparable assumptions to those of Leachman and Gascon (1988) and Leachman et al. (1991). They also show that the performance of the DCL decreases significantly if the production load increases, therefore an alternative heuristic was developed. This heuristic is able to keep the cycle lengths stable, and with a simulation study it is shown that the stable cycle length outperforms the DCL when demand is close to or exceeds production rate. Fransoo et al. (1995) explain the result with the problem that occurs when a product is expected to run out of stock in the upcoming cycle. The DCL heuristic will decrease the cycle length and, with this decision, the relative set-up frequency increases, so the capacity that is available for production decreases. Consequently, it becomes more complicated to fulfill demand in the future.

When working with local lot sizing policy in the context of fixed sequence strategies using dynamic cycle length, the papers of Federgruen and Katalan (1996a, 1996b, 1998) represent seminal papers. Three extensions to the papers by Federgruen and Katalan have appeared in the literature: one developed by Krieg and Kuhn (2002, 2004) and one by Winands, Adan and van Houtum (2004) and one by Grasman, Olsen and Birge (2008). All these strategies are some of the very few policies allowing for an analytical evaluation and optimization. Additional research in this field was published by

Anupindi and Tayur (1998), Vaughan (2003) and Wagner and Smits (2004). Anupindi and Tayur (1998) develop a simulation-based procedure to obtain the optimal base-stock levels for product focused measures as well as for order focused measures. They conclude that product focused measures such as costs and service levels based on individual items cannot be used as substitute for order focused measures like the order response time. (Winands et al., 2005)

For Liberopoulos et al. (2009) the literature on dynamic sequencing approaches is quite scarce, mainly because of the difficulty of obtaining an analytical solution even for problems of small size and the computational challenge of numerically solving problems of realistic size. They regard Zipkin (1986) as an indicative example of a dynamic sequencing approach that uses a local lot sizing policy. Winands et al. (2005) also state the publications of Altiok and Shiue (1994, 1995 and 2000) in the local lot sizing policy and the works of Karmarkar and Yoo (1994), Qiu and Loulou (1995) and Sox and Muckstadt (1997) in the global lot sizing policy.

3 Heuristics and scheduling policy

This chapter describes the four stochastic heuristics that are used for the comparison of heuristics; the Vergin and Lee heuristic, the lookahead heuristic, the dynamic cycle length heuristic and the enhanced dynamic cycle length heuristic. Additionally the values of the (s, S) policy will be specified.

3.1 Vergin and Lee heuristic

Vergin and Lee (1978) present six different decision rules for the stochastic demand case in order to develop a schedule that will minimize the sum of setup, carrying and shortage costs for all items. The first two rules, derived under conditions of deterministic demands, are included because they are often suggested for use, even though the scheduling may lead to excessive costs of carrying safety stock or excessive shortage costs. Rule number three was developed by Magee and Boodman (1967, p. 151). They suggest that it is more economical to set total inventory sufficient to provide for cyclic fluctuation in total demand for all items than to monitor the actual production operation to keep the inventory balanced among several items. The remaining three rules were invented by Vergin and Lee (1978) as improvements to rule number three.

Rule number one is the classical cyclical lot sizing model for multiple items. It computes the number of cycles per year, w, to produce each item in order to minimize the sum of setup plus carrying costs; shortage costs are ignored.

$$w = \sqrt{\sum_{i=1}^{n} d_i h_i \left(1 - \frac{d_i}{p_i}\right) \Big/ 2 \sum_{i=1}^{n} A_i} \tag{25}$$

The concept is the same as in the common cycle approach and the cycle length W is achieved by $1/w$. The second rule follows the same approach as above but incorporates a shortage factor in the cost equation, resulting in the following cycle length:

$$W = \sqrt{2\sum_{i=1}^{n} A_i \Big/ \sum_{i=1}^{n} h_i d_i \left(1-\frac{d_i}{p_i}\right)} \ * \ \sqrt{\sum_{i=1}^{n} h_i + \sum_{i=1}^{n} sh_i \Big/ \sum_{i=1}^{n} s_i} \ , \tag{26}$$

where sh_i is a shortage cost per item i backordered or lost.

Rule number three derives a series of ratios for each product that are based on the normal run length of each product compared with the other items produced on the machine:

$$a_i = 2d_i\left(1-\frac{d_i}{p_i}\right) \Big/ \sum_{i=1}^{n} d_i\left(1-\frac{d_i}{p_i}\right). \tag{27}$$

These ratios, a_i, can be used to monitor the cycling of items in the following way. The production of an item continues until one of the following two conditions results: (a) the inventory of some other items runs out of stock; or (b) the inventory of the item i currently produced builds up to the proportion a_i of the total inventory on hand. If current production is stopped because of condition (a), the production is shifted to the product that has run out. If production is stopped because of condition (b), the production is shifted to the item that has the lowest ratio of inventory on hand to usage. This rule helps to distribute the effect of variations in demand among all items. (Vergin and Lee, 1978)

Rule four establishes a maximum inventory level that should prevent excess production. In addition to the shown condition (b), the accumulating inventory of the item currently produced is compared to its maximum inventory level. If the current inventory is greater than or equal to the maximum level, production is shifted to the item with the lowest ratio of inventory on hand to usage. If the inventory level of this item is also greater than or equal to its maximum inventory level, no production will occur for the next review period. Rule number five concerns problems where backorders are allowed. Condition (a) is adjusted to include a search for items with backorders. If no item is backordered, it is searched for items with zero inventory. If any exists, this item will be produced, otherwise the procedure will continue as described in rule four. The last rule verifies if any item has less than a fixed minimum number of days usage on hand, if there are no items with backorders or zero inventory. If any exists this item will be produced, otherwise the rules described above will be applied. The last rule is invented to eliminate a lot of very short production runs. (Vergin and Lee, 1978)

As used by Vergin and Lee, a (s, S) policy, where an item i would be produced until either its inventory level reaches its maximum inventory level S_i or some other item j, $j \neq i$, reaches its reorder point s_j or lower, seems appealing for the problem of multi-item, single-machine scheduling with stochastic demands. Gascon (1988) shows how the target parameters s and S can be computed and these computations will be explained in the following.

3.2 The (s, S) policy

Harris (1913) uses the manufacturing interval to find the smallest quantity the inventory can fall before an order has to be made. Gascon (1988) presents a fairly simple way to achieve reasonable values of s_i and S_i for each item i, since the computational effort to find the best (s_i, S_i) pair would be intolerable.

The reorder point, s_i, for items facing probabilistic demands is defined as:

$$s_i = ss_i + c_i d_i , \quad (28)$$

where ss_i is the safety stock level of item i and $c_i d_i$ represents the expected demand during the setup time. The safety stock level ss_i is defined as:

$$ss_i = y\, \sigma_i \sqrt{c_i} , \quad (29)$$

where the safety factor y is multiplied with the standard deviation of forecast errors over the changeover time. The safety factor y can easily be determined in a single-item environment but under assumption of the multiple items it is quite difficult to relate a value of y with any service measure. Especially if the inventories of the items are not well balanced, it might happen that two or more items will reach their reorder point at the same time, making it hard to avoid stock out situations unless safety stocks are relatively high. To keep it simple, y is set empirically to provide corresponding service measures, as required by the user. (Gascon, 1988)

When the reorder point s_i is found, it is time to decide how much of the items should be produced. In the multi-item environment with stochastic demand it is quite difficult, if not impossible, to determine the optimal production quantity of an item. Therefore a target maximum inventory level, S_i, for each item i is specified, that indicates whenever the production of the item is finished. As the change of production could be induced by another item falling below the reorder point, the maximum inventory level will sometimes not be acquired. (Gascon, 1988) The maximum inventory level, S_i, is defined as

$$S_i = ss_i + \frac{T_i d_i (p_i - d_{i)}}{p_i} \ , \tag{30}$$

where the target cycle, T_i, can be taken from every heuristic that achieves reasonable results.

3.3 Lookahead heuristic

The lookahead heuristic, devised by Gascon (1988), works with a dynamic type of the (s, S) policy, where production tries to follow an ideal schedule provided by these parameters in order to minimize costs, with the possibility to reevaluate the parameters whenever necessary to avoid stock-out situations. This should help to overcome problems in which on one day the inventory level of each item is slightly above its reorder point and the next day the inventory levels of two or more items are below their reorder points. As only one item can be produced at a time, all other inventory levels would continue to fall, making stock-out situations unavoidable. Other problematic situations can occur when it is attractive to shut down the machine for a period of time because of overcapacity. Especially when facing uneven demands it can happen that low demand at the beginning of a period would lead to very little production in this time and high demand at the end of the period might not be able to be satisfied as not enough capacity would be left. Therefore, a lookahead function evaluates, if it is safe or not, in terms of possible future stock-out situations, to shut down the production facility for one time period.

The boolean lookahead function has one input parameter, x, that represents the amount of working hours assigned to production or changeover of an item on the considered day. $lookahead(x)$ is used to decide if a down time will be taken or if one or two shifts or even overtime should be used on a given day. To determine for how long the inventory will be able to satisfy future demand, Gascon (1988) defines a run-out time of item i, RO_i. The run-out time is the expected number of periods until the inventory level of the item will reach its reorder point. That is

$$RO_i = \frac{I_i - ss_i}{d_i} - ch_i \, , \tag{31}$$

where I_i represents the current inventory level of item i and where

$$ch_i = \begin{cases} cl & \text{if setup of the current item to be produced has already been initiated ,} \\ c_i & \text{otherwise .} \end{cases}$$

When the setup, for the item to be produced, is finished $cl = 0$; otherwise cl shows the remaining time that is needed to complete the setup of the item. The run-out times help in supervising which item should be next set up for production and give an estimate of when this setup will need to be done as $RO_i = 0$ is equivalent to $I_i = s_i$.

The considered items are numbered such that item 1 indicates the next or current item to be produced (Gascon, 1988). If $I_1 \geq S_1$ the production of item 1 is finished and the items are re-designated such that the item with the lowest run-out time becomes item number 1 and

$$RO_2 \leq RO_3 \leq \ldots \leq RO_n \, . \tag{32}$$

The lookahead function works as follows: At first, lookahead checks if there is enough time, with all available shifts and no overtime, for the currently produced item to reach its maximum inventory level without any other item running out of stock. This is done by assuming that the item with the lowest run-out time will be the next item to be produced and that its run-out time shows the time left for producing the current item. Next, lookahead goes through a simple simulation of the scheduling for the next 20 days and tests if the inventory level of each item is above its reorder point in every period. The scheduling is simulated by continuous production of the current item until another item runs out. If this happens the allocation of the production hours are switched to the other product. This process continues until the end of the 20 days period is reached. If the current item can reach its maximum inventory level and no item runs out of stock within the 20 days period the lookahead function returns the value true. True indicates that the use of x hours today should not generate any scheduling problems; otherwise $lookahead(x)$ would return the value false. If $x = 0$ and lookahead returns true, a down time can be taken, as it means that no production hours for that day are required to satisfy the two conditions. (Gascon, 1988)

The lookahead heuristic assumes a daily decision process with normal working time of one shift, and the possibility of two shifts and overtime. A down period can always be taken and indicates a shutdown of the production line for one shift. At the beginning of the day the run-out time of all items and the maximum inventory level of the item currently produced are calculated with updated values of the system like inventory levels, demands and machine setup. Next, it is decided which item should be produced if production will take place:

- If production of the current item is finished ($I_1 > S_1$) or if another item will reach its reorder point before the next decision will be made ($RO_2 < 1$), the item with the lowest run-out time will be set up for production. This assumption indicates that the item with the lowest run-out time is not the item currently being produced ($RO_2 < RO_1$).
- Otherwise the production continues with the currently set up product.

Afterwards it is decided if production should take place and if so, how many shifts are needed. If an item is in the middle of a setup, or if a setup was completed in the previous period without any production afterwards, the item is produced anyway. This insures that one item will be produced for at least one shift even if two items are running out of stock at the same time. The idea is to avoid situations in which production would switch back and forth between two items without any production in between and more time would be spent on setting up the items rather than actually producing them.

- If it is assumed that the item currently produced has enough inventory to meet the actual demand without reaching the reorder point ($RO_1 \geq 1$) and if it is assumed that some other item will run out ($RO_2 < 1$), production of the item currently produced is continued until it becomes absolutely necessary to produce the item with the lowest inventory on hand for use. Production will be changed to that product for the rest of the day.

- If it is estimated that a down time would be possible as $lookahead(0) = true$, the facility is shut down for the period.
- If it is estimated that, with one shift of 8 hours, the current item will reach its maximum level before any other items runs out of stock and the demand over the next 20 working days will be met without any item running out, $lookahead(8) = true$ and so, the current item will be produced for one shift. If $lookahead(8) = false$, the item will be produced for two shifts, if two shifts are available.
- If overtime can be taken and if $lookahead(8\ or\ 16) = false$, overtime hours are added to the production time of the current item produced, one hour at a time, until the conditions in the lookahead function are satisfied or until the limit on overtime hours is reached.

This simple simulation allows taking the demand over the next 20 working days, roughly a month, into account and to take the right decision now, considering what is coming next. (Gascon, 1988)

3.4 Dynamic cycle lengths heuristic

In developing the dynamic cycle length (DCL) heuristic Leachman and Gascon (1988) found the concept of rotation cycles appealing to practitioners but the nature of real-world systems precludes the direct use of solutions to the classical ELSP. So, Leachman and Gascon propose a modified solution and explain the essence of these modifications as follows: If it is estimated that there will be enough time to replenish each item after this item's inventory falls to its reorder point and before the next item's inventory falls to its reorder point they say that there exists positive slack between each consecutive pair of items. If it is estimated that there will be insufficient time to replenish one item before the other item runs out of stock they speak of negative slack between these items. If the total slack of an item, that is the sum of slacks until that item is produced, is negative, the system is out of balance.

To avoid stock-out situations the production run of one item could be cut short. Unfortunately, such an action would simply shift the negative slack to fall between another pair of items, leading to infeasibility in a following cycle and perhaps ultimately leading to stock-out situations anyway. As an alternative, one could reduce the production runs of all items. This alternative is used for the dynamic cycle length heuristic, where the cycle lengths of all items should be proportionately reduced from target lengths, just enough to eliminate negative total slack. These adjustments restore the balance in the production system and enable to maintain the rotation cycles. (Leachman and Gascon, 1988)

Figure 3 and 4 will illustrate the essence of the modifications. The values are chooses arbitrarily. The bars in light gray represent the inventory on hand to use that is measured in terms of estimated run-out times, RO_i, and the dark gray bars represent the duration that is required to replenish a rotation cycle quantity of each item, r_i. As measured in eq. (32), the items are numbered in such a way that item 1 is the item being currently produced and the other items are numbered in increasing order of estimated inventory run-out times.

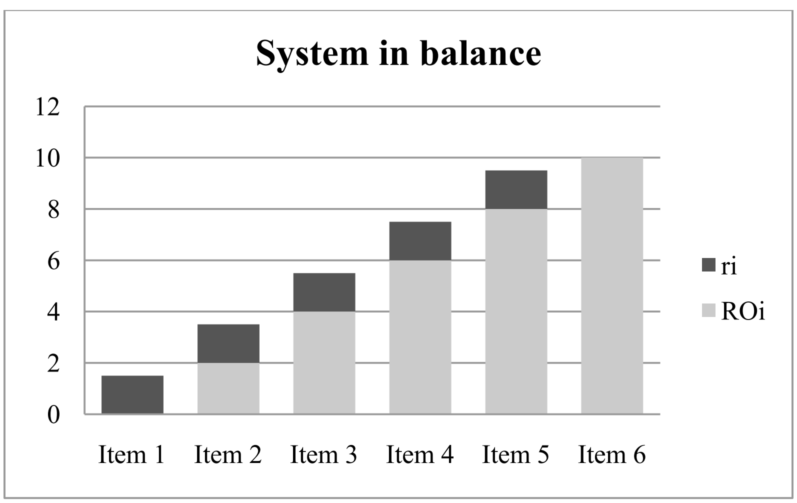

Figure 3 Production system in balance

Figure 3 illustrates a production system with positive slack between all items, so no modification is needed. The values are chosen arbitrarily, table 8 shows the values for the system in balance and table 9 shows the values for the system that is out of balance.

	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6
r_i	1.5	1.5	1.5	1.5	1.5	
RO_i	0	2	4	6	8	10

Table 8 Values for production system in balance

	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6
r_i	1.5	1.5	2.75	1.5	3	
RO_i	0	2	4	6	8	10

Table 9 Values for production system out of balance

Figure 4 displays a case in which it is estimated that there will be insufficient time to replenish item 3 before item 4 runs out of stock as well as to replenish item 5 before item 6 runs out of stock.

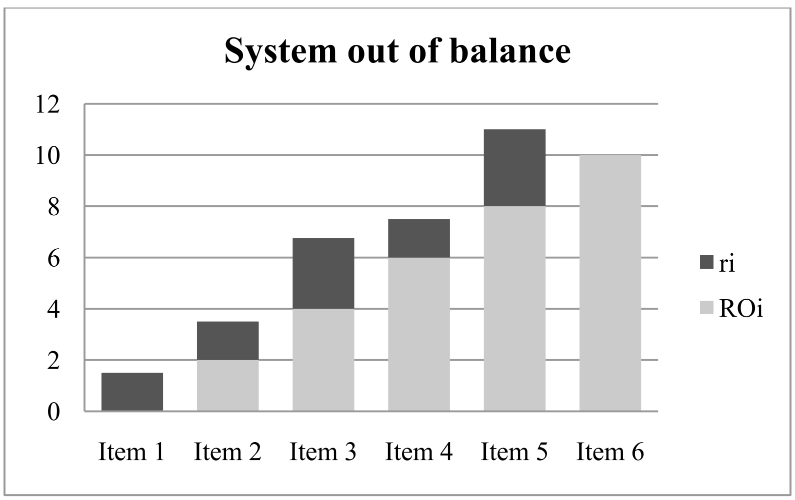

Figure 4 Production system out of balance

The negative slacks could bring the production system out of balance. To get to know if this really happens, we need to calculate the total slack that is the sum of slacks between item 1 and 2, 2 and 3, 3 and 4 and 4 and 5 and between 5 and 6. As two positive slacks exist, it would be possible to advance the start time of the replenishment of the items planned before item 4 to mitigate the negative slack. The total slack of item 4 is nonnegative meaning that the cycle quantities of each item may be feasibly produced by carrying out production runs one right after another. Even if we continue production without any idle time to diminish the negative slack of item 6 the cycle quantities of the six items cannot be fully produced without stock-outs as the total slack of item 6 is negative. A modification following the concept of the DCL heuristic would be needed to restore the balance of the scheduling.

The dynamic cycle length heuristic starts with a calculation of estimated target cycles T_i for item i. Leachman and Gascon (1988) mean that any method that provides good solutions in achieving the target cycles could be used. At the beginning of each decision process the heuristic compares the estimated inventory run-out time of each item with the total duration to complete target-cycle production runs of all items preceding that item in the order of rotation of the items. If the replenishment of an item is forced to begin at its run-out time, the spacing of the run-out times must be such that

$$RO_i - RO_{i-1} \geq c_{i-1} + \frac{T_{i-1} d_{i-1}}{p_{i-1}} \qquad i = 2, \dots, n \tag{33}$$

to allow for production of the target cycle quantities. The right hand side of the inequality represents the time to complete the batch of item $(i-1)$. Note that c_1 in eq. (33) is replaced by

$$c_1 = cl - \frac{I_1 - ss_1 - d_1 cl}{p_1 - d_1} \tag{34}$$

if setup of the machine to produce item 1 has already been initiated. This is due to the fact that the time to raise the inventory of item 1 to its maximum inventory level depends on the current inventory level if $cl = 0$, or the level that will be achieved after the completion of the setup if $cl \neq 0$. (Leachman and Gascon, 1988)

If the inequality for equation (33) is strictly satisfied item i has a positive slack, if the inequality is not satisfied, it has a negative slack (Leachman and Gascon, 1988). As discussed above, positive slack for an item i can be used to offset negative slack of a following item j by simply advancing the production of items $i, i+1, \dots, j-1$ in order to gain more time to produce item j. For this purpose the total slack of item j, defined as TS_j, is the sum of the differences between left and right sides of equation (33). If the total slack is insufficient, the target cycles of all items are proportionately reduced to operational cycles T_i^o for item i.

$$T_i^o = \alpha T_i \,, \tag{35}$$

where the reduction factor, α, is given by (Gascon et al., 1994):

$$\alpha = \min\left\{1, \min_{i=2,\dots,n}\left\{RO_i - \sum_{j=1}^{i-1} c_j \Bigg/ \sum_{j=1}^{i-1} \frac{d_j T_j}{P_j}\right\}\right\}. \tag{36}$$

The first term in (36) guarantees that the operational cycle does not exceed the target cycle, the second term compares the target cycle to cycle lengths allowed by the estimated run-out times (Gascon et al., 1994). Mathematically, it is possible to get a reduction factor, α, unacceptably small or even negative, especially if a shortage is inevitable. In such a case, the scheduling policy can no longer be based on the modifications of target cycles to get a balanced production system. Instead, a minimum duration for a production run is specified by e and the minimum of α, α_{min}, is calculated as (Leachman and Gascon, 1988):

$$\alpha_{min} = \frac{e\,p_1}{T_1 d_1}\,. \tag{37}$$

The revised definition of α is

$$\alpha = \max\left\{\alpha_{min}\;, \min\left\{1, \min_{i=2,\dots,n}\left\{RO_i - \sum_{j=1}^{i-1} c_j \Bigg/ \sum_{j=1}^{i-1} \frac{d_j T_j}{P_j}\right\}\right\}\right\}. \tag{38}$$

The maximum inventory level of item i, S_i, defined by (30) can be recomputed based on the operational cycles, T_i^o, calculated with the revised reduction factor, α. The new maximum inventory levels make it possible that $I_1 \geq S_1$. This indicates that the production of item 1 is completed and the numbering of the items must be revised so that the item with the lowest run-out time becomes item number 1 and the remaining items are numbered in order of increasing run-out times. (Leachman and Gascon, 1988)

However, if the total slack for each item is large enough, the DCL heuristic calls for leaving the machine idle for the next period (Gascon et al, 1994), i.e. $RO_1 \geq 1$, and

$$RO_i \geq 1 + \sum_{j=1}^{i-1} c_j + \sum_{j=1}^{i-1} \frac{k_j T d_j}{p_j} \qquad\qquad i = 2, 3, \dots, n\,. \tag{39}$$

If the conditions for taking a down time are not satisfied, the scheduling policy calls for production of item 1 until the inventory rises to the maximum inventory level or the end of the period is reached. If the inventory reaches S_1 before the end of the period, the policy calls for changeover and production of the item with the lowest run-out time until the end of the period. (Leachman and Gascon, 1988)

"The control adjustments of the heuristic are based on estimated run-out times of item inventories which are calculated from forecasts of uncertain demands" (Leachman and Gascon, 1988). To involve the fact that the uncertainty in forecast accuracy increases over time the basic heuristic is modified. The uncertainty of the demand forecasts includes the fact that the realized value of the run-out time could be higher or lower than planned, especially for large values of RO_i. A rough estimate of a confidence interval for the run-out time of item i, assuming normality of estimated forecast errors, can be expressed as:

$$RO_i \pm Z \frac{\sigma_i}{d_i} \sqrt{RO_i - 1} \ , \tag{40}$$

where Z is chosen according to the desired confidence level. This notation of the confidence interval is used in the decision process of the heuristic for determining the operational cycles and for deciding of whether or not to take a down time.

In the first instance it is not necessarily true that target cycles are infeasible if the total slack, $TS_j < 0$ for some large item j. Because of the uncertainty of demand forecasts it may happen that one day the run-out time seem to be too small and the next day everything might be back to normal. As there are a number of opportunities to reduce the operational cycle before RO_j falls to zero, the heuristic should be relatively insensitive to negative slack caused by items with large run-out times. Therefore RO_i is changed as follows:

$$RO_i^+ = \begin{cases} RO_i & if\ RO_i \leq 1\,, \\ RO_i + Z \dfrac{\sigma_i}{d_i} \sqrt{RO_i - 1} & otherwise\,, \end{cases} \tag{41}$$

and equation (38) is replaced by

$$\alpha = \max\left\{\alpha_{min}\,, \min\left\{1, \min_{i=2,\ldots,n}\left\{RO_i^+ - \sum_{j=1}^{i-1} c_j \middle/ \sum_{j=1}^{i-1} \frac{d_j T_j}{P_j}\right\}\right\}\right\}. \tag{42}$$

In the second instance, it is possible to shut down the production facility for a period if the run-out times are great enough. Because of the uncertainty in forecast accuracy it could happen that we make a down time now, only to realize a number of periods later that, because of the low forecasts, we lack capacity to meet demand. In this case it is suggested to be more conservative, meaning that a down time could only be taken if $RO_1^- \geq 1$ and

$$RO_i^- \geq 1 + \sum_{j=1}^{i-1} c_j + \sum_{j=1}^{i-1} \frac{k_j T d_j}{p_j} \qquad i = 2, 3, \dots, n \ , \tag{43}$$

where

$$RO_i^- = \begin{cases} RO_i & if\ RO_i \leq 1\ , \\ RO_i - Z\ \dfrac{\sigma_i}{d_i} \sqrt{RO_i - 1} & otherwise\ . \end{cases} \tag{44}$$

Leachman and Gascon (1988) indicate that a value of $Z = 0.7$ results in low costs for most of the tested cases but a higher value might be preferable when the machine usage rate is very high. Gascon et al. (1994) set the value of Z to zero as they could not find a single value of Z that performed well in all their different testing environments. Gascon, Leachman and Lefrancois (1995) show that the value of zero has an important impact on the results of the EDCL heuristic, that will be described below. They introduce a modification that allows for positive values of Z, but instead of running the heuristic with a single value of the confidence level parameter, the heuristic is applied many times to the data with different values of Z and returns the best solution. Z may vary between 0 and 2 with increments of 0.1. The results achieved through the use of the varying values of Z are significantly better than those obtained with the original heuristic but unfortunately, no particular pattern in the values could be detected with each simulation. However, a distribution of the best values of Z for each simulation shows that 0 provides the best solution in most of the time.

A further modification is proposed by Leachman et al. (1988) to account for the time to complete a second replenishment of the current item if that is anticipated occurring before run-out of some other item. If some item has a target cycle $k_i T$ with a multiplier k_i

being greater than one, there will be incidents when the current item is projected to become due for a second replenishment before the time of some other items with a large run-out time is projected to become due for replenishment. Therefore the estimates of the total slack of the items $j = 2, 3, \dots, n$ should be modified to account for the time to complete another replenishment of the current item, if that is anticipated to occur before run-out of item j. The modification estimates the run-out time of the current item following actual replenishment and adds an item so that the current item is item number 1 as well as some item $i \in \{2, 3, \dots, n+1\}$. The second value of i is chosen in such a way that the next run-out time of the current item is ordered in with the sorted run-out times of all other items so that:

$$RO_2 \leq RO_3 \leq \dots \leq RO_{n+1} . \quad (45)$$

3.5 Enhanced dynamic cycle lengths heuristic

Leachman, Xiong, Gascon and Park publish 1991 an improvement to the dynamic cycle lengths heuristic that was later called the enhanced dynamic cycle length (EDCL) heuristic. The EDCL expresses the duration to complete production runs of preceding items more accurately. The equations for the dynamic cycle length heuristic are based on the assumption of production of full cycle quantities for each of the items preceding item i, but with continuous production activity a production run may be initiated when the item inventory level is well above its safety stock level. In such a case the time, required to reach the maximum inventory level of item i, will be less than required for a full production run. The estimates of production time are refined by estimating the inventory level of each item at the time of its replenishment and therefore the procedure for calculating the reduction factor, α, is revised.

The EDCL heuristic works with the modification, introduced for the DCL heuristic, to account for a second replenishment of the current item if that is anticipated occurring before run-out of some other item. Additionally, the calculation for the run-out times involves the fact of forecast uncertainties.

So, the revised α is defined as

$$\alpha = \max\left\{\alpha_{min}, \min\left\{1, \min_{i=1,\dots,n}\left\{\frac{RO_{i+1}^{+} - B_i}{A_i}\right\}\right\}\right\} \ , \tag{46}$$

where α_{min} and RO_{i+1}^{+} are calculated according to eq. (37) and (41) respectively and where

$$A_i = \sum_{j=1}^{i}\left[\prod\nolimits_{k=j+1}^{i}\left(1 + \frac{d_k}{p_k - d_k}\right)\right]\frac{T_j d_j}{p_j} \tag{47}$$

and

$$B_i = \sum_{j=1}^{i}\left[\prod\nolimits_{k=j+1}^{i}\left(1 + \frac{d_k}{p_k - d_k}\right)\right]\left[c_j - \frac{I_j - c_j d_j - ss_j}{p_j - d_j}\right]. \tag{48}$$

$$\prod\nolimits_{k=j+1}^{i}\left(1 + \frac{d_k}{p_k - d_k}\right) \equiv 1 \tag{49}$$

as defined by Leachman et al. (1991).

According to Gascon et al. (1994) the condition for taking a down time are modified similarly, therefore a down time is taken if $RO_1^{-} \geq 1$ and

$$RO_i^{-} \geq 1 + B_i + A_i \qquad i = 1, 2, \dots, n \ , \tag{50}$$

where RO_i^{-} is calculated as defined in eq. (44), A_i and B_i as shown above through eq. (47) and (48).

The revised reduction factor, α, is used to calculate the operational cycles according to eq. (35), and with that the maximum inventory levels are achieved through eq. (30). The scheduling policy is the same as in the DCL heuristic. If production takes place, the item currently produced is produced up to its maximum inventory level or until the end of the period. If the maximum inventory level is reached before the end of the period, the item with the lowest run-out time is produced. If the maximum inventory level is reached due to the reduced maximum inventory level achieved with the operational cycle times, the ordering of the items is redone and α and all following parameters are recalculated. (Leachman et al., 1991)

4 Method

The simulation study published by Gascon et al. (1994) compares the EMQ and the Doll and Whybark heuristic with the Vergin and Lee, the lookahead, the DCL and the EDCL heuristic under the assumption of discrete time review systems with a period of one day. While the EMQ and the Doll and Whybark heuristic could not cope with the complexity of the problem, the EDCL heuristic seemed the best choice for three different production environments. The Vergin and Lee heuristic achieved very good service levels but at very high costs and Gascon et al. (1994) state that “the Vergin and Lee heuristic, in our belief, is not recommendable since it relies mainly on ‘filling up the warehouse’ to provide high service levels.” What would happen if a continuous time review system is used for the six heuristics? Will the heuristics perform in the same way or could a change in the review system change the output quality of the heuristics? Such questions make it interesting to work with the same six heuristics.

In doing so, the author specified an own empirical study that is based to the case study used by Gascon et al. (1994). Their formulation of the scheduling problem reflects characteristics of a packaging line of a large consumer products facility. These characteristics are typical for many companies in process industries therefore it seemed useful to rely on them. Some adaptations are made in order to be able to test the differences between the continuous and the discrete time system. The initial inventory levels are changed as it seemed to be too easy to work with initial inventories that cover demand for the first week.

4.1 Characteristics of the study

Simulations, where a single machine, that can produce only one item at a time, has to produce 6 items, are run on a 5-month horizon assuming on average 22 working days per month. Production can be done during one regular shift without any possibility of using overtime. Shutting down the production facility is possible. In the discrete time version this means that the down time takes at least one day while the down time in the continuous time simulation may only last until the next decision is made. When production is changed from one item to another, it takes half a day to clean and adjust the machine before production of the other item can be initiated. Such a machine setup affords 750 € per setup for each item. Due to the duration of the setup, a minimum production length of 1 day is imposed to avoid a series of successive production changes with little or no production. This minimum production length means that at least one production rate has to be produced or that the first decision after a setup can be made 1.5 days later. Inventory holding costs are assumed to be 0.00411 € per unit and day for each item on stock, indicating that it costs 1.5 € to hold one item of inventory for one year.

Item demands are stationary stochastic and generated randomly under the assumption that forecast errors follow normal distribution with a standard deviation equal to half of the demand rates. Customer demands has to be satisfied immediately from inventory on stock, otherwise it will be lost. The demand rates reflect three levels of machine usage, ρ, a low level of 30%, a medium level of 60% and a high usage level of 85%. As in Gascon et al. (1994) no tests above these levels had been performed because stock-out situations are more difficult to avoid with higher usage levels, especially when the setup time is high. The relative usage of the machine by each item, ρ_i/ρ, is specified as 25% for item 1, 10% for item 2 and 3 each, 15% for item 4, 35% for item 5 and 5 % for item 6 and remains the same for each machine usage level tested. In the discrete time simulation the demand is generated for the whole day and has to be satisfied before this day's delivery arrives. In the continuous time model the daily demand occur throughout the day as well as the delivery arrives whenever the item is finished. The tests are run on two different initial inventory levels in order to test how the heuristics handle low initial inventories and medium initial inventories. The details of demand and production data as well as the data about initial inventories can be found in table 8.

		Item 1	Item 2	Item 3	Item 4	Item 5	Item 6
Daily production rate		1000	2000	5000	4000	3000	2500
Daily demand forecast	85 % machine usage	212.5	170	425	510	892.5	106.25
	60 % machine usage	150	120	300	360	630	75
	30% machine usage	75	60	150	180	315	37.5
Initial inventory	Low	2000	2000	2500	4000	6000	2500
	Medium	3500	5000	7500	5500	6500	4000

Table 10 Item characteristics for case study

The heuristics are simulated under discrete time and continuous time review systems for 110 days. At the end of each simulation the setup costs and the holding costs represent the total costs needed to schedule production according to the heuristic. The service level indicates the fraction of customers that can be satisfied immediately from stock on hand. The aim is to minimize the total costs while maintaining, if possible, a 100% customer service level. All simulation variations are repeated 10 times using different seeds for the random generator to get good results.

4.2 Implementation of heuristics

The (s, S) policy, as described in section 3.2, is used to work with the data. All heuristics are based on the purpose of trying to produce a certain quantity of an item, but departing from this target production quantity, whenever needed, to avoid two items running out of stock at the same time. To be able to calculate the reorder point one needs to know the safety stock level. The safety stock level ss_i, depends on the safety factor y and that is set to 3 as in Gascon et al. (1994). The resulting safety stock levels and reorder points for a machine usage of 85% are calculated with eq. (29) and (28) and the results are shown in table 9.

	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6
ss_i	226	181	451	541	947	113
s_i	332	266	664	796	1393	166

Table 11 Safety stock levels and reorder points for 85% machine usage

The maximum inventory level, S_i, is calculated by using the target cycle time, T_i. Various methods to achieve target cycle times, T_i, exist, with EMQ as simplest by regarding the items as manufactured independently. Gascon (1988) and Leachman and Gascon (1988) use the Doll and Whybark heuristic to achieve target cycle times. The heuristics are used as described above, but the Doll and Whybark heuristic is added by an extra feasibility check at the end of the procedure. When the process terminates it is ensured that the changeover frequency saturation can be satisfied by using eq. (24). If the fundamental cycle, T, provided by the Doll and Whybark heuristic violates eq. (24), T is reset to be equal to the right-hand side of the feasibility condition. The target cycle times of the EMQ and the modified Doll and Whybark heuristic, and the related maximum inventory levels for a machine usage of 85%, calculated with eq. (30), are shown in table 10.

	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6
			EMQ			
T_i	46.70	48.44	30.64	28.64	24.13	59.90
S_i	8041	7715	12365	13285	16074	6207
			Doll and Whybark			
T_i	53.33	53.33	26.66	26.66	26.66	53.33
S_i	9150	8476	10820	12406	17665	5538

Table 12 Values of $\boldsymbol{T_i}$ and $\boldsymbol{S_i}$ for EMQ and Doll and Whybark for 85% machine usage

Table 10 indicates that shared capacity leads to change in cycle times and with it in maximum inventory levels. When item 1 reaches its maximum inventory in the EMQ heuristic, it is more than one production rate below the maximum inventory level of the Doll and Whybark heuristic. The opposite case is true for item 3, 4 and 6. While the production continues in the EMQ quantity, it would already been stopped by the Doll and Whybark procedure. The impact of the coordinated capacity allocation will be determined later.

The heuristics that are included in the study work with the two parameters achieved so far, s_i and S_i, together with the ordering of run-out times, as explained by eq. (32). When deciding about the item that should be produced or the need to shut down the machine, the inventory of all items is used to determine the run-out times and for the current item it is checked if the inventory is equal or higher than the maximum inventory level. A decision about a down time lasts until the next decision is made as well as a decision about production. This means that it can happen that the inventory level of the current item is one unit below the maximum inventory level and all other items have enough inventories. In such a case the production will continue until the next decision is made. So, for the discrete time planning, the maximum inventory level cannot signify the maximum storage space but indicates a produce-up-to-level.

The EMQ heuristic and the Doll and Whybark heuristic rely on the maximum inventory level achieved by the EMQ procedure and the modified Doll and Whybark procedure shown in table 12. The production of the current item continues if the current inventory level is smaller than its maximum inventory level. If it is higher, it is verified if all inventory run-out times are high enough to be able to satisfy the upcoming demand. If inventories are high enough, the machine is shut down, if not or if the inventory level of some item falls to its reorder point, the machine is set up for the item with the lowest run-out time.

The Vergin and Lee heuristic does not indicate how to specify the maximum inventory levels of each item, so the values of the EMQ procedure are used. Backordering is not allowed therefore the heuristic is implemented with the following three conditions. The first condition is met when the inventory of the current item rises to a proportion a_1 of the total inventory,

$$I_1 \geq a_1 \sum_{i=1}^{n} I_i \tag{51}$$

where a_1 is calculated according to equation (27). The second condition is met when the inventory of the current item rises to its maximum inventory level and the third condition is met when the inventory of another item falls to its reorder point. When any of these conditions occurs, the production is changed to the item with the lowest run-out time unless that item's inventory is at its maximum inventory level. In such a case, the machine would be shut down.

The lookahead heuristic works with the maximum inventory levels achieved by the modified Doll and Whybark procedure and is performed in such a way that the lookahead function is only used to decide about taking a down time or not. As there is no possibility of overtime or a second shift and the production is only changed due to decisions, there is no need for other evaluations than $lookahead(0)$. As long as the inventory of the current item is lower than its maximum inventory level, production of that item is continued. If the maximum inventory level is reached or if the inventory of some other item will not be able to satisfy the upcoming demand, the item with the lowest run-out time will be produced. If $lookahead(0) = true$, the current item will reach its maximum level before any other item runs out of stock and the demand over the next 20 working days will be met without any item running out, so a down time can be taken, no matter what was decided before.

The DCL heuristic and the EDCL heuristic also work with the modified Doll and Whybark procedure. For these heuristics the fundamental cycle is important. If the target cycles achieved from the fundamental cycle result in infeasible schedules, the cycles are modified to operational cycles according to eq. (35) with α values according to eq. (42) and eq. (46) respectively. The value of e, to calculate α_{min}, is set to one as the minimum production time is one day. The operational cycles are used to compute new maximum inventory level and the decisions depend on the new values. If the current item's inventory is below the maximum inventory level the production continues. If the inventory of the current item is above the maximum inventory level, the items are reordered according to the run-out times such that the item with the lowest run-out time becomes the current item and that item will be produced. The sorting of items for the DCL is done by eq. (32), for the EDCL by eq. (45). With the new current item the operational cycles are recalculated and the decision is made whether or not to leave the machine idle. That is done when $RO_1^- \geq 1$ and eq. (43) for the DCL or eq. (50) for the EDCL heuristic are satisfied.

4.3 The review systems

As shown in section 1.1, an inventory management system can be designed, so that the position of the inventory is monitored continuously or at certain time intervals. This section will show how the decisions of the review systems are obtained and how scheduling works with these decisions.

When working with a periodic review of one day, the simulation starts with an update of the inventory level as the items that are produced the day before arrive. The customer demand is generated for the day and has to be satisfied for available inventory. Unsatisfied demand will be lost. The inventory costs for items that are still on stock are derived and the current inventory on hand to use is taken to calculate the run-out times. A decision is made that indicates which item should be produced or if the machine should stay idle. The decision is used to make the production plan for that day. If production of the current item continues, the item is produced for the whole day. If production switches to another item it has to be guaranteed that the minimum production of one day is accomplished. If the current item has already been produced for one day, the setup for the new item can be done, setup costs are updated and the new item is produced for the rest of the day. If the currently set up item was only produced for half a day, the production of that item has to be finished at the beginning of the day. The rest of the day is used to setup the new item, setup costs are updated. If the item that is set up was not produced at all, as setup was done at the end of the last day, the currently set up item is produced for the whole day. When it is decided to set the machine idle, the minimum production of one day has to be considered as well. If the minimum production was done, the machine is immediately set idle. If the item was only produced for half a day, the production is finished at the beginning of the day and afterwards the machine is set idle. If the current item was not produced at all, it is produced for the whole day.

In the continuous time simulation, a decision may take place whenever a customer arrives or when an item is delivered. The customer arrival occurs throughout the whole day and as soon as setup is finished and production starts, the items are delivered after their production time. Whenever a decision the change production is made, the next decision is made 1.5 days later. That is due to the minimum production time of 1 day and the duration of the setup as the setup takes 0.5 days. Within that time, production is not allowed to change, and a decision has no use in the continuous time environment. As soon as the

1.5 days have passed, a decision is made and the production plan can follow this decision. The modeling steps shown for the discrete time review are also used for the continuous time simulation but not in such a strict manner. Let us suppose that we are in the middle of a down time as the maximum inventory level was reached for the last produced item and no other item is in danger of running out of stock. The machine is set idle and customer demands are satisfied from inventory. Whenever a customer demand is satisfied the new inventory level is achieved and the holding costs are calculated as the inventory is decreasing and is less expensive in future. The down time will be stopped when an item reaches its reorder point. At this time the machine will be set up for that item and the item will be produced for at least one day. Whenever an item is finished it will be delivered and the inventory of the item currently produced will increase. During the whole time customers arrive and the inventories of the demanded items are used. So, each time inventory changes, the holding costs are calculated and the current inventory levels are used for the decision. If some item is running out of stock, nothing can be done within the 1.5 days period but afterwards production can change. If no item is running out of stock the production continues until the maximum inventory level is reached or the machine is set idle.

The main difference of the discrete time scheduling and the continuous time planning is that with the latter it is possible to produce the item for any possible time greater than 1.5 days. The discrete time scheduling decides each day about the desired production but production can be changed only after 1.5 days. What this could mean is explained as follows: If a decision is made at a time when production of the current item was produced only for half of the production rate, production has to continue for at least half a period. When management decides to continue the current production, this production is done for the entire period. When management decides to change production, the production of the current item continues for half a period and afterwards the changeover for the new item is done. The new item can be produced on the following day. When management decides on the following day, the production plan for that day is already fixed. A decision for another new item would mean that the changeover for this item would start one day after the decision and then the item would be produced for one day. The need for such various changes of production occurs only when two or more items are running out of stock. In times when production is balanced, changeovers are not used that often and these problems will not occur.

To illustrate another advantage of the continuous time review system, let us assume the production of two items, where item a is currently produced and item b has a reorder point of 100, a maximum inventory level of 1000 and a production rate of 200. Assuming a reorder point of item b of 100 units, the continuous planning can produce the current item as long as the inventory of the item b falls to 100. Especially if the inventory is at 102 units, nothing will happen as well as with an inventory of 101 units, but if it reaches 100 units, the production will change. When the inventory reaches the reorder point, the run-out time will be zero. The discrete time planning will change the production as soon as the run-out time will be smaller than one, which indicates that the inventory on stock cannot satisfy the upcoming demand without running out of stock. The discrete time planning will change production before it is known what the upcoming demand will be. So the continuous counterpart can plan more precisely. The scheduling decision does not only depend on reorder points but also on maximum inventory levels. Assume, the production has changed to item b, as the reorder point was reached. In the continuous time scheduling with no other item running out we can produce the item until we reach the maximum inventory level of 1000. If we have an inventory of 998 units we continue production, as well as with an inventory of 999 units. As soon as we reach the inventory level of 1000 we stop production of item b. With the discrete time scheduling we will also continue production at an inventory level of 999 units and stop at its maximum level of 1000 but if we decide at a level of 999 units to continue production we will produce 200 units. Depending on the demand for that item the inventory can reach a level of 1199 units that is much higher than the maximum level.

The six heuristics are simulated in both review systems to find out if a more precise scheduling, as done with continuous time review systems, is preferable over the periodic review even if the implementation of such systems affords higher costs.

5 Results

In this chapter the outcome of the simulations are presented and compared to the outcome of the test made by Gascon et al. (1994). Before the six heuristics are compared for the discrete time and the continuous time review system, the modifications of the DCL heuristic and the EDCL heuristic, that incorporates the confidence level of the forecast error of demand, are shown to be able to decide if Z should be zero or not.

5.1 Results for the usage of Z

As described in the DCL heuristic, different positions exist about the modification, to incorporate a confidence level parameter, to improve the sensitivity of the heuristic. In the paper of Leachman et al. (1991) a value of $Z = 0.7$ is used in most cases while a value of $Z = 1$ is suggested for very high machine usage. Gascon et al. (1994) omit the usage of this modification since no value could be found that performed well in all the different testing environments. To get to know if the modification of the DCL and the EDCL could improve the outcome of the simulations for this study, the problems were one time solved with $Z = 0$ and another time with $Z = 1$ in the 85% usage case and $Z = 0.7$ in the 60 % usage case and in the 30 % usage case.

When the problem starts with low initial inventory, $Z = 1$ improves the service levels and the total costs for both review systems when regarding the 85 % machine utilization case. Table 13 shows a service level of 100 % achieved when Z equals zero as well as when it equals 0.7 when machine utilization is moderate, meaning that the utilization is less than 85 %, and planning done on discrete time. In the case of $Z = 0.7$ the total costs would be higher, indicating that a value of zero for Z is better. For the continuous time planning with moderate machine usage and low initial inventory the original schedule could not achieve full service while the modified schedule could achieve full service at lower total costs for the 60% usage case and slightly higher costs on the 30 % usage case. When initial inventories are low a value of Z higher than zero can improve service levels in both planning systems and in the continuous time planning the total costs are also improved. In the discrete time planning the cost increase in the moderate machine usage cases outperform the cost decrease in the high utilization case and therefore the total costs would be higher with $Z \neq 0$.

	DISCRETE TIME				CONTINUOUS TIME			
	Service level (%)		Total cost (€)		Service level (%)		Total cost (€)	
	Z = 0	Z = 1/0.7	Z = 0	Z = 1/0.7	Z = 0	Z = 1/0.7	Z = 0	Z = 1/0.7
Low initial inventory								
85 % machine usage	94.83	96.62	41889	40602	88.11	90.21	51976	51049
60% machine usage	100	100	33709	35100	99.89	100	40163	39593
30% machine usage	100	100	21309	21940	99.93	100	23659	23691

Table 13 Solution of DCL heuristic with low initial inventory using Z = 0 or Z = 1/ 0.7

When scheduling is started with medium initial inventory levels the usage of a positive value of *Z* decreases the service levels in four out of six cases and increases the total costs when machine usage is moderate in both review systems as can be seen in table 14. Only the costs at 85 % machine utilization are improved but this improvement is not big enough to overcome the negative effects in the other cases.

	DISCRETE TIME				CONTINUOUS TIME			
	Service level (%)		Total cost (€)		Service level (%)		Total cost (€)	
	Z = 0	Z = 1/0.7	Z = 0	Z = 1/0.7	Z = 0	Z = 1/0.7	Z = 0	Z = 1/0.7
Medium initial inventory								
85 % machine usage	99.99	99.98	35180	33361	100	99.97	40681	37622
60% machine usage	100	100	33270	35128	99.95	99.88	39215	41989
30% machine usage	100	100	19650	21249	99.68	99.61	22553	23124

Table 14 Solution of DCL heuristic with medium initial inventory using Z = 0 or 1/ 0.7

Gascon et al. (1995) suggest improving the EDCL heuristic by efficiently utilizing the confidence level parameter for adapting the sensitivity of the heuristic. With the improvement they show that the EDCL heuristic can outperform all other heuristics in the regarded production environments and for both objectives, the service level and the total costs. The see the effect of the suggested improvement table 15 show the results for low initial inventory level and table 16 for medium initial inventory level.

	DISCRETE TIME				CONTINUOUS TIME			
	Service level (%)		Total cost (€)		Service level (%)		Total cost (€)	
	Z = 0	Z = 1/0.7	Z = 0	Z = 1/0.7	Z = 0	Z = 1/0.7	Z = 0	Z = 1/0.7
Low initial inventory								
85 % machine usage	94.43	94.44	41468	42773	90.04	90.26	52016	50965
60% machine usage	100	100	30624	31450	99.67	99.91	32121	31863
30% machine usage	100	100	21135	21145	99.84	100	21024	21388

Table 15 Solution of EDCL heuristic with low initial inventory using Z = 0 or Z = 1/ 0.7

For the discrete time planning with low initial inventory level the service level could only be slightly increased while the relevant costs had a larger increase. In the continuous time planning, the modified simulation could achieve full service for the 30 % machine usage case and could increase the other service levels as well, while the total costs are decreased for all but the 30 % usage case. When scheduling starts with medium initial inventory levels the discrete time planning achieves full service when Z equals zero as well as when Z is positive. The total costs are smaller with positive Z when machine utilization is high. As soon as it is moderate, the smaller total costs result from $Z = 0$. The continuous time planning gets poor results for low machine usage when $Z = 0$. When confidence levels are used to adapt the scheduling, service levels are increased to 99.89 % for low machine usage and to full service for medium machine usage. In the first case the total costs could be reduced while in the latter case they are increased. The high machine usage case achieves full service in both cases but lower total costs when sensitivity of the heuristic is adapted.

	DISCRETE TIME				CONTINUOUS TIME			
	Service level (%)		Total cost (€)		Service level (%)		Total cost (€)	
	Z = 0	Z = 1/0.7	Z = 0	Z = 1/0.7	Z = 0	Z = 1/0.7	Z = 0	Z = 1/0.7
Medium initial inventory								
85 % machine usage	100	100	34003	33779	100	100	35916	35354
60% machine usage	100	100	28859	29249	99.91	100	29667	30351
30% machine usage	100	100	19447	19791	99.48	99.89	21290	20615

Table 16 Solution of EDCL heuristic with medium initial inventory using Z = 0 or 1/ 0.7

All the presented results should help for the decision whether or not to set the value of Z to zero. In regarding the DCL heuristic the benefits of $Z = 1 \ or \ 0.7$ depend on the initial inventory levels. As long as initial inventory is small, a positive Z can increase service levels and when scheduling becomes more difficult the positive Z will also improve total costs. As soon as one faces medium initial inventory the usage of a positive Z is outperformed by Z equals zero. For the EDCL heuristic the benefits of $Z = 1 \ or \ 0.7$ can be found when planning with continuous time reviews, where all service levels could be improved and total costs are improved in four out of six cases. When planning with periodic reviews the positive value of Z affords higher costs in almost all cases and can slightly increase service level in only one case.

A general solution for both heuristics and all variations cannot be the best choice as the positive values of Z can increase the performance of the heuristic in some cases and in some the performance is decreased. So, it is decided the value of Z will be zero for the DCL heuristic with medium initial inventory and for the EDCL heuristic with discrete time planning. For the DCL heuristic with low initial inventory and for the EDCL heuristic with continuous time review, the value of Z is set to one for the high machine utilization case and to 0.7 for the moderate machine utilization cases. In the comparison with the other heuristics, the modified heuristics are presented as DCL* and EDCL*.

5.2 Results for discrete time planning

The stationary demand tests of Gascon et al. (1994) achieve 100 % service in the 30 % machine usage case for all heuristics except the lookahead heuristic. For the 60 % machine usage case, half of the heuristics achieve full service while the other half has service levels greater than 99.92 per cent. In these two cases the total costs could be used to differentiate the performance of the heuristics while the 85 % usage case showed a wider range of achieved service levels. It is shown that the heuristics, developed for the stochastic demand case, outperform the simple deterministic approaches as the latter achieve less than 70 % service level while the stochastic approaches all reach service levels of more than 88 % and the Vergin and Lee heuristic reached full service.

The case study presented in this paper shows different results. Even with little initial inventory all heuristics could achieve a service level higher than 80 % in all different test runs. The test that can be best used to compare the outcome with the results of Gascon et al. (1994) is the discrete time simulation with medium initial inventory levels, as this is the higher initial inventory. The average simulation results are shown in table 17, where service level is indicated as satisfied demand in per cent of total demand and all costs are shown in €. The same holds for table 18, 19 and 20. The 30 % machine usage case achieves full service in all stochastic heuristics and 99.99 % in the two deterministic heuristics. While the results from Gascon et al. (1994) show much higher costs for the deterministic heuristics in comparison to the DCL, the EDCL and the lookahead heuristic the results from table 17 indicate that the deterministic heuristics would solve the problem at the lowest costs. In both studies the Vergin and Lee heuristic achieves full service and the costs are more than doubled in comparison to the cheapest solution. When regarding the more difficult scheduling of high demand in the 85 % machine usage case, table 17 displays that only the EDCL heuristic could avoid stock-out situations and all other stochastic heuristics achieve more than 99 % service level while the deterministic heuristics achieve less than 94% service and have the highest costs. In the scenario of discrete time planning with medium initial inventories the EDCL heuristic is the best as is achieves 100 % service level in all three usage cases and the smallest costs in the 85% case and smallest costs after the deterministic heuristics in the two other cases. The EMQ and the Doll and Whybark heuristic could achieve smaller costs as they have fewer machine setups in the 30 % and the 60 % case and less inventory on stock in the latter case. This might be the reason why they do not achieve full service level in both cases. The Vergin and Lee heuristic avoids stock-out situations at very high costs and cannot achieve full service at the 85 % case. In all three scenarios this heuristic has the highest inventory holding costs and in cases where demand is not more than 60 % of capacity, the setup costs are doubled in comparison to the average of the other setup costs. Interestingly the total costs decrease in the 85 % case in comparison to the 60 % machine usage case. The high inventories make it possible that the machine can produce the items for a longer time in comparison to the other heuristics. The Vergin and Lee heuristic has the lowest setup cost in this case and this leads to the second lowest total cost.

DISCRETE TIME							
Medium initial inventory							
Machine usage rate		EMQ	Doll & Whybark	Vergin & Lee	Lookahead	DCL	EDCL
85%	Service level	93.86	93.68	99.99	99.03	99.99	100
	Total cost	44898	45271	34988	41038	35180	34003
	Holding c.	7473	7171	16913	11563	14180	14878
	Setup cost	37425	38100	18075	29475	21000	19125
60%	Service level	99.91	99.83	100	99.94	100	100
	Total cost	25929	26284	51628	33153	33270	28859
	Holding c.	12504	12634	21703	11253	13095	14609
	Setup cost	13425	13650	29925	21900	20175	14250
30%	Service level	99.99	99.99	100	100	100	100
	Total cost	17808	17831	37330	21503	19650	19447
	Holding c.	10608	10106	17830	8078	8550	9697
	Setup cost	7200	7725	19500	13425	11100	9750

Table 17 Solutions of discrete time simulation with medium initial inventory

When initial inventory levels are low, it is more difficult to reach full service. In nearly all cases the total costs have increased in comparison to the medium initial inventories, as can be seen in table 18. The only heuristic that has lower total costs with low initial inventory is the Vergin and Lee heuristic. It has lower holding costs for the 60 % machine utilization because it is harder to build up such high inventory levels as before and lower setup cost as the production must not be changed that often because inventory reaches the maximum inventory level. All other heuristics have higher setup costs as the inventory on hand could not build up as good as before and it is needed to change production more often in order to avoid stock-out situations. This is indicated by the lower holding cost that can be seen for all heuristics except the modified DCL heuristic in case of 60% machine usage and the EDCL heuristic for 30 % utilization. These two heuristics are the only heuristics that can achieve full service with moderate machine utilization while the other heuristics all have a service level higher than 99.6 %.

DISCRETE TIME							
Low initial inventory							
Machine usage rate		EMQ	Doll & Whybark	Vergin & Lee	Lookahead	DCL*	EDCL
85%	Service level	83.90	83.23	90.08	87.38	96.62	94.43
	Total cost	55214	55528	51248	55512	40602	41468
	Holding c.	3464	3328	5198	4512	6927	6443
	Setup cost	51750	52200	46050	51000	33675	35025
60%	Service level	99.61	99.80	99.99	99.91	100	100
	Total cost	28763	28919	47278	33368	35100	30624
	Holding c.	11363	11894	19753	11018	13725	14349
	Setup cost	17400	17025	27525	22350	21375	16275
30%	Service level	99.97	99.96	100	99.98	100	100
	Total cost	18962	18672	38398	23393	21940	21135
	Holding c.	10112	9747	17473	7868	8515	10035
	Setup cost	8850	8925	20925	15525	13425	11100

Table 18 Solutions of discrete time simulation with low initial inventory

For the 85 % machine utilization case starting with low initial inventories results in service problems. None of the heuristics can reach a service level higher than 97 %. The high setup costs and low holding costs show that it is not possible to build up enough inventory to satisfy the customer. The minimum production time of 1.5 days, where setup time is included, allows for at most 73 setups within the 110 days. The EMQ, the Doll and Whybark and the lookahead heuristic have setup costs higher than € 50,000. That means that they setup the machine at least 67 times during the simulation. A lot of time is wasted to set up the machine and so no less than 12 % of the customers have to leave unsatisfied. The EDCL heuristic can satisfy around 94.43 % of the customers demand as it does not change the machine that often. Setup costs around € 35,000 show that roughly 46 setups are initiated and so more time is used to produce the items and the holding costs are much higher than for the other heuristics. The modified DCL heuristic outperforms all other heuristics in this case as 96.62 % of the customers are satisfied and it has the smallest total costs at 40,602 €. It saves one setup in comparison to the EDCL heuristic and with it the average inventory level is increased, resulting in higher holding costs.

5.3 Results for continuous time planning

When the simulations are run for the continuous time review system the results for the low initial inventory case are worse than for the discrete time planning in all but two cases, as can be seen in table 19. The Vergin and Lee and the lookahead heuristic are the only heuristics that benefit from the continuous time planning. Both heuristics can increase the service level in case of high utilization with the result that the Vergin and Lee has the highest service level and the lookahead heuristic has the second highest level in the high machine utilization case. They also have the lowest costs as setups are not than as often as in the remaining heuristics. The Vergin and Lee can also increase the service level in case of 60 % machine utilization, where the continuous review system makes it possible to achieve full service. The outcome for the deterministic heuristics is nearly the same as before. In case of 30 % machine utilization, the EMQ and the Doll and Whybark heuristic achieve the lowest costs but do not reach full service and when utilization is high they create the highest costs and lowest service levels. The modified DCL and the modified EDCL heuristic that could avoid too many setups for the discrete time system and therefore achieve good service levels, increase their setup rate to around 61 setups and with that loose service level of more than 6 and 4 %, respectively .

CONTINUOUS TIME							
Low initial inventory							
Machine usage rate		EMQ	Doll & Whybark	Vergin & Lee	Lookahead	DCL*	EDCL*
85%	Service level	83.15	81.96	93.27	91.03	90.21	90.26
	Total cost	55872	56113	48678	34625	51049	50965
	Holding c.	3447	3313	6753	14075	4624	4915
	Setup cost	52425	52800	41925	20550	46425	46050
60%	Service level	99.69	99.11	100	96.81	100	99.91
	Total cost	30103	31139	53377	37237	39593	31863
	Holding c.	9703	9164	19102	12112	12593	13488
	Setup cost	20400	21975	34275	25125	27000	18375
30%	Service level	99.92	99.90	100	99.51	100	100
	Total cost	18434	18627	40301	23308	23691	21388
	Holding c.	8009	8052	16901	7483	7116	8788
	Setup cost	10425	10575	23400	15825	16575	12600

Table 19 Solutions of continuous time simulation with low initial inventory

The last results shown are the result for the medium initial inventory levels at continuous time planning. As table 20 indicates, full service is most achieved in the high machine utilization case. The results present higher total costs, in comparison to the discrete time planning with the same initial inventory levels for all heuristics, except the lookahead heuristic for 85 % machine usage where service level decrease by 5.7 % and the deterministic heuristics for 30 % machine usage. The Vergin and Lee heuristic achieve full service for all machine usage cases but the full service costs 10 % more in the medium and the high usage cases and 5 % more for low machine usage. The modified DCL achieves full service only in the high utilization case at a cost increase in comparison to the discrete time planning of 15.6 %. The modified EDCL heuristic can achieve full service in the high and medium machine usage case at a cost increase of 4 % and 5 %., respectively. For the low machine usage case the deterministic heuristics outperform the lookahead, the DCL and the

EDCL heuristic in service levels and total costs as production changes are done infrequently. Customers demand can be satisfied in more than 99.9 per cent of their arrivals. The Vergin and Lee heuristic achieves full service for this case, but total costs are more than doubled in comparison to the deterministic heuristics.

CONTINUOUS TIME							
Medium initial inventory							
Machine usage rate		EMQ	Doll & Whybark	Vergin & Lee	Lookahead	DCL	EDCL*
85%	Service level	92.61	92.18	100	93.31	100	100
	Total cost	47534	47003	38703	36010	40681	35354
	Holding c.	6434	6278	16728	14110	13156	15479
	Setup cost	41100	40725	21975	21900	27525	19875
60%	Service level	99.83	99.76	100	99.19	99.95	100
	Total cost	27393	27002	57274	41311	39215	30351
	Holding c.	10293	10352	20899	12361	12065	13851
	Setup cost	17100	16650	36375	28950	27150	16500
30%	Service level	99.97	99.93	100	98.40	99.68	99.89
	Total cost	17034	17528	39392	22193	22553	20615
	Holding c.	9234	8978	17567	8318	7478	8840
	Setup cost	7800	8550	21825	13875	15075	11775

Table 20 Solutions of continuous time simulation with medium initial inventory

6 Conclusion

In this paper, six different heuristics that solve the multi-item, single-machine scheduling problem with stochastic demands are compared in four different production environments. On the one hand the initial inventory level is either low or medium and on the other hand the review system is either a discrete time review or a continuous time review.

The EMQ and the Doll and Whybark heuristic are included in the study to point out that simple heuristics are not capable of working with the complexity of the stochastic demand problem. Even if it is not possible to reach full service these simple heuristics minimize the setup costs in case of low machine utilization and achieve the lowest total cost. This good result is only seen at low utilization. As soon as the machine usage increases the simple rules cannot cope adequately with the problem. In all four environments tested with a 30 % machine usage rate, the EMQ works better than the Doll and Whybark heuristic.

The lookahead heuristic as stochastic heuristic provides slightly better results than the deterministic heuristics. It can detect situations where two or more items are running out of stock at the same time but the heuristic does not suggest the right decision to avoid such situations. In case of continuous time planning the deterministic heuristics outperform the lookahead heuristic as long as machine utilization is moderate. The lookahead heuristic benefits best from its more elaborate procedure if machine utilization is high.

The Vergin and Lee heuristic achieves high service levels at high costs. Only when machine utilization is high the heuristic results in goods costs in combination with high service levels. At low machine usage the inventory level is doubled in comparison to the other heuristics and the machine is often setup. The continuous time case with low initial inventory is one case where the Vergin and Lee heuristic performs best but the high costs for the other solution makes it impossible to recommend this heuristic.

The EDCL heuristic, original and modified, outperforms the DCL heuristic as expected. Only in two times the modified DCL works better, once when initial inventory is low and planning is done with a periodic review and 85% machine usage rate and when planning is done continuously and machine utilization is 60 % the DCL achieves full service at higher costs. Whenever the EDCL achieves full service it affords the lowest cost in comparison to other heuristic that achieve full service and it achieves full service as often as the Vergin and Lee heuristic.

When decisions are made with the discrete time review system, it is decided 110 times and scheduling is performed according to these decisions. The continuous time review system decides whenever an item arrives on stock or a customer demand arrives. This increases the amount of decisions and the effort that is needed to work with this planning system. The simulation results show on average higher costs and lower service levels with continuous time planning. When working with the underlying six heuristics the discrete time review system outperforms the continuous time system. The simulations are calculated within seconds and the more conservative planning results in higher service levels and lower total costs. In discrete time planning a production is changed due to the fact that the inventory will not be able to satisfy the upcoming demand. In this case the setup is done earlier as in the continuous time case and in the case of production change because the maximum inventory level is reached, the discrete time method achieves higher inventory levels as the production is continued for one day as long as the maximum inventory level is not reached. This means that the inventory can be up to 50 per cent higher than the maximum inventory level would indicate. The higher inventory helps to avoid stock-out situations.

When scheduling is done with periodic review the EDCL heuristic outperform all other heuristics in all but one case when. When initial inventory is low and machine usage is high the scheduling should be planned according to the modified DCL heuristic otherwise it is best to use the original EDCL heuristic. If a company prefers to save money in trade off service level the EMQ heuristic provides the lowest cost when machine usage rate is 30 % but cannot satisfy all demand.

7 Bibliography

Altiok, T., Shiue, G.A. (1995), Single-Stage, Multi-Product Production/Inventory Systems With Backorders, *IIE Transactions*, 26(2), 52-61.

Altiok, T., Shiue, G.A. (1995), Single-Stage, Multi-Product Production/Inventory Systems With Lost Sales, *Naval Research Logistics*, 42(6), 889-913.

Altiok, T., Shiue, G.A. (2000), Pull-type manufacturing systems with multiple product types, *IIE Transactions*, 32(2), 115-124.

Anupindi, R., Tayur, S. (1998), Managing stochastic multiproduct systems: Model, measures, and analysis, *Operations Research*, 46(3S), 98-111.

Axsäter, S. (2006), *Inventory Control*, 2nd ed., Springer, New York.

Ballou, R.H. (2004), *Business Logistics / Supply Chain Management*, 5th ed., Pearson Prenctice Hall, Upper Saddle River.

Bomberger, E.E. (1966), A Dynamic Programming Approach to a Lot Size Scheduling Problem, *Management Science*, 12(11), 778-784.

Bourland, K.E., Yano, C.A. (1994), The Strategic Use of Capacity Slack in the Economic Lot Scheduling Problem with Random Demand, *Management Science*, 40(12), 1690-1704.

Brander, P. (2005), Inventory Control and Scheduling Problems in a Single-Machine Multi-Item System, *Doctoral thesis, Luleå University of Technology, Sweden.*

Davis, S.G. (1990), Scheduling Economic Lot Size Production Runs, *Management Science*, 36(8), 985-998.

Delporte, C.M., Thomas, L.J. (1977), Lot Sizing and Sequencing for N Products on One Facility, *Management Science*, 23(10), 1070-1079.

Dzielinski, B.P., Manne, A.S. (1961), Simulation of a Hypothetical Multi-Item Production and Inventory System, *Journal of Industrial Engineering*, 12(6), 417-421.

Dobson, G. (1987), The Economic Lot-Scheduling Problem: Achieving Feasibility Using Time-Varying Lot Sizes, *Operations Research*, 35(5), 764-771.

Doll, C.L., Whybark, D.C. (1973), An Iterative Procedure for the Single-Machine Multi-Product Lot Scheduling Problem, *Management Science*, 20(1), 50-55.

Elmaghraby, S.E. (1978), The Economic Lot Scheduling Problem (ELSP): Review and Extensions, *Management Science*, 24(6), 587-598.

Federgruen, A., Katalan, Z. (1996a), The Stochastic Economic Lot Scheduling Problem: Cyclic Base-Stock Policies with Idle Times, *Management Science*, 42(6), 783-796.

Federgruen, A., Katalan, Z. (1996b), Customer Waiting-Time Distributions under Base-Stock Policies in Single-Facility Multi-Item Production Systems, *Naval Research Logistics*, 43(4), 533-548.

Federgruen, A., Katalan, Z. (1998), Determining production schedules under base-stock policies in single facility multi-item production systems, *Operations Research*, 46(6), 883-898.

Fransoo, J.C., Sridharan, V., Bertrand, J.W.M., (1995), A hierarchical approach for capacity coordination in multiple products single-machine production systems with stationary stochastic demands, *European Journal of Operations Research*, 86(1), 57-72.

Gallego, G. (1990), Scheduling the Production of Several Items with Random Demands in a Single Facility, *Management Science*, 36(12), 1579-1592.

Gallego, G. (1994), When Is a Base Stock Policy Optimal in Recovering Disrupted Cyclic Schedules?, *Naval Research Logistics*, 41(3), 317-333.

Gallego, G., Roundy, R. (1992), The Economic Lot Scheduling Problem with Finite Backorder Costs, *Naval Research Logistics*, 39(5), 729-739.

Gascon, A. (1988), The Lookahead Heuristic for Multi-Item Single Machine Production Scheduling with Dynamic, Stochastic Demand, *INFOR*, 26(2), 114-126.

Gascon, A., Leachman, R.C., Lefranç ois, P. (1994), Multi-item, single-machine scheduling problem with stochastic demands: a comparison of heuristics, *International Journal of Production Research,* 32(3), 583-596.

Gascon, A., Leachman, R.C., Lefranç ois, P. (1995), A note on improving the performance of the enhanced dynamic cycle lengths heuristic for the multi-item, single machine scheduling problem, *International Journal of Production Research*, 33 (3), 869-873.

Ghiani, G., Laporte, G., Musmanno, R. (2004), *Introduction to Logistics Systems Planning and Control*, John Wiley & Sons, New York.

Goyal, S.K. (1973a), Scheduling a Multi-Product Single Machine System, *Operational Research Quarterly*, 24(2), 261-269.

Goyal, S.K. (1973b), Lot Size Scheduling on a Single Machine for Stochastic Demand, *Management Science*, 19(11), 1322-1325.

Grasman, S.E., Olsen, T.L., Birge, J.R. (2008), Setting basestock levels in multiproduct systems with setups and random yield, IIE Transactions, 40(12), 1158-1170.

Graves, S.C. (1980), The Multi-Product Production Cycling Problem, *AIIE TRANSACTIONS*, 12(3), 233-240.

Haessler, R.W. (1979), An Improved Extended Basic Period Procedure for Scheduling the Economic Lot Scheduling Problem, *AIIE TRANSACTIONS*, 11(4), 336-340.

Haessler, R.W., Hogue, S.L. (1976), A Note on the Single-Machine Multi-Product Lot Scheduling Problem, *Management Science*, 22(8), 909-912.

Harris, F.W. (1913), How Many Parts to Make at Once, *Factory, The Magazine of Management*, 10 (2), 135-136, 152.

Karmarkar, U.S., Yoo, J. (1994), The stochastic dynamic product cycling problem, *European Journal of Operational Research*, 73(2), 360-373.

Krajewski, L.J., Ritzman L.P. (1990), *Operations Management: strategy and analysis*, 2nd ed., Addison-Wesley, Reading.

Krieg, G.N., Kuhn, H. (2002), A decomposition method for multi-product kanban systems with setup times and lost sales, *IIE Transactions*, 34(7), 613-625.

Krieg, G.N., Kuhn, H. (2004), Analysis of Multi-Product Kanban Systems with State-Dependent Setups and Lost Sales, *Annals of Operations Research*, 125, 141-166.

Leachman, R.C., Gascon, A. (1988), A Heuristic Scheduling Policy for Multi-Item, Single-Machine Production Systems with Time-Varying, Stochastic Demands, *Management Science*, 34(3), 377-390.

Leachman. R.C., Xiong, Z.K., Gascon, A., Park, K. (1991), Note: An Improvement to the Dynamic Cycle Lengths Heuristic for Scheduling the Multi-Item, Single-Machine, *Management Science*, 37(9), 1201-1205.

Liberopoulos, G., Pandelis, D.G., Hatzikonstantinou, O. (2009), The Stochastic Economic Lot Sizing Problem for Continuous Multi-Grade Production, *Working paper, University of Thessaly, Greece.*

Madigan, J.C. (1968), Scheduling a Multi-Product Single Machine System for an Infinite Planning Period, *Management Science*, 14(11), 713-719.

Magee, J.F., Boodman, D.M. (1967), *Production Planning and Inventory Control*, 2nd ed., McGraw-Hill, New York.

Markowitz, D.M., Reiman, M.I., Wein, L.M., (2000), The Stochastic Economic Lot Scheduling Problem: Heavy Traffic Analysis of Dynamic Cyclic Policies, *Operations Research*, 48(1), 136-145.

Markowitz, D.M., Wein, L.M., (2001), Heavy Traffic Analysis of Dynamic Cyclic Policies: a Unified Treatment of the Single Machine Scheduling Problem, *Operations Research*, 49(2), 246-270.

Maxwell, W.L. (1964), The Scheduling of Economic Lot Sizes, *Naval Research Logistics Quarterly*, 11(2), 89-124.

Qiu, J., Loulou, R. (1995), Multiproduct Production/Inventory Control Under Random Demands, *IEEE Transactions on Automatic Control*, 40(2), 350-356.

Rogers, J. (1958), A Computational Approach to the Economic Lot Scheduling Problem, *Management Science*, 4(3), 264-291.

Salveson, M.E. (1956), A Problem in Optimal Machine Loading, *Management Science*, 2(3), 232-260.

Segerstedt, A. (1999), Escape from the unnecessary – some guidelines for production management, *Production Planning & Control*, 10(2), 194-199.

Silver, E.A., Pyke, D.F., Peterson, R. (1998), *Inventory Management and Production Planning and Scheduling*, 3rd ed., Wiley, New York.

Sox, C. R., Muckstadt, J.A. (1997), Optimization- based planning for the stochastic lot scheduling problem, *IIE Transactions*, 29(5), 349-357.

Sox, C.R., Jackson, P.L., Bowman, A., Muckstadt, J.A. (1999), A review of the stochastic lot scheduling problem, *International Journal of Production Economics*, 62, 181-200.

Stankard Jr., M.F., Gupta, S.K. (1969), A Note on Bomberger's Approach to Lot Size Scheduling: Heuristic Proposed, *Management Science*, 15(7), 449-452.

Stock, J.R., Lambert, D.M. (2001), *Strategic Logistics Management*, 4th ed., McGraw-Hill, New York.

Thierry, M., Salomon, M., van Nunen, J., van Wassenhove, L. (1995) Strategic Issues in Product Recovery Management, *California Management Review*, 37(2), 114-135.

Vaughan, T.S. (2003), The effect of correlated demand on the cyclical scheduling system, *International Journal of Production Research*, 41(9), 2091-2106.

Vergin, R.C., Lee, T.N. (1978), Scheduling Rules for the Multiple Product Single Machine System with Stochastic Demand, *INFOR,* 16(1), 64-73.

Wagner, M., Smits, S.R. (2004), A local search algorithm for the optimization of the stochastic economic lot scheduling problem, International Journal of Production Economics, 90(3), 391-402.

Winands, E.M.M., Adan, I.J.B.F., van Houtum, G.J. (2004), A Quantity-Limited Base-Stock Policy in a Stochastic Two-Item Single-Capacity Production System, *Working paper, Beta Research School for Operations Management and Logistics, Eindhoven, Netherlands.*

Winands, E.M.M., Adan, I.J.B.F., van Houtum, G.J. (2005), The Stochastic Economic Lot Scheduling Problem: A Survey, *Working paper, Beta Research School for Operations Management and Logistics, Eindhoven, Netherlands.*

Zipkin, P.H. (1986), Models for Design and Control of Stochastic, Multi-Item Batch Production Systems, *Operations Research*, 34(1), 91-104.

Appendix A: Lebenslauf

Name:	Elisabeth Brandl
Geburtsdatum:	27. Februar 1983
Geburtsort:	Wels
Nationalität:	Österreich

Ausbildung

2002/10 – 2010/11	Internationale Betriebswirtschaftslehre, Universität Wien Spezialisierungen: Produktionsmanagement, Supply Chain Management
2006/09 – 2007/01	Auslandssemester School of Economics and Management, University Lund, Schweden
2002/06/12	Matura HBLA für wirtschaftliche Berufe, Linz Landwiedstrasse

Berufserfahrung

2009/03 – 2009/08	Studienassistentin Universität Wien, Institut Logistics and Supply Chain Management
2008/08 – 2008/11	Area Sales Assistant Starlinger Viscotech, Linz

Besondere Kenntnisse

Sprachkenntnisse:	Englisch	verhandlungssicher
	Französisch	in Wort und Schrift
	Schwedisch	in Wort und Schrift

EDV-Kenntnisse: MS Office, Adonis, AnyLogic, Xpress, SAP R/3, SPA (Schichtplanassistent), C++.