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1. INTRODUCTION

In a globalized economy where products, raw-materials and semi-finished goods are shipped around the world, the logistical system is playing a more and more important role in any company. Especially if we consider the share of the GDP which is represented by the overall cost of logistics, this role becomes more apparent. The Council of Supply Management Professionals estimates that currently 21 % of the Chinese GDP consists of logistical costs, although these numbers are quite high if compared to other countries. In India the percentage is about 13 %, the European average is around 11 % and in the US logistics make up about 8 % of the GDP.¹

With this importance in mind it is no surprise that companies place heavy emphasis on logistical problems and decisions. Two of these decisions will form the central point of this thesis: the facility-location-problem and the vehicle-routing-problem (VRP). Both of these problems have been thoroughly studied for the last decades, but through the difficulty of each problem they have been studied separately. Nonetheless in the last two decades more and more research has been published on a combination of these problems, called the Location Routing Problem (LRP). The idea behind this combination is quite simple: almost every solution can be improved if the status of an important corner pillar is changed from fixed to variable, meaning if we include the routing-problem into the location-decision a better solution can be found compared to the solution found looking at two separate problems.

The main-objective of this thesis is the implementation of a new iterative heuristic solution procedure for a real life LRP. The problem we are looking at is the location of distribution centres in Eastern Austria for an Austrian logistical company. The aim of this work is to present an adaptive solution-procedure for this problem which generates reasonable results in a short time span. Since we are dealing with a real-life problem the main emphasis of this heuristic lies on speed and simplicity. In the first part of this heuristic we will use linear programming to generate a p-median-starting

¹ The Economist, A Survey of Logistics – Cargo Cults, 17th of June 2006

solution, while a simple Nearest Neighbour heuristic will be applied for the change of depots between the iterations. The second part of the heuristic deals with the Multi-Depot-Vehicle Routing Problem with Time-Windows (MDVRPTW). The initial tours will be constructed by a Savings-Algorithm and later-on improved using an adapted r-opt-heuristic.

The rest of the work is organized as follows: Chapter 2 will continue with a presentation of the Location-Routing Problem and a linear program for it. This chapter will also shed some light on the characteristics of the LRP and its related problems. In the third Chapter we will present the real-life problem at our hands and a classification of it. Chapter 4 will describe the algorithms used in this heuristic, whereas the implementation will be shown in Chapter 5. This will be followed by a discussion of the results and the managerial implications in Chapter 6.

2. THE LOCATION-ROUTING PROBLEM

2.1 DEFINITION AND CHARACTERISTICS

In order to consider a problem in its details we have to define the problem first. Here we encounter our first difficulty since there is no generally accepted definition for the LRP, as there is for other classical problems like the Travelling Salesman Problem. On the other hand the definitions offered by the literature do circle around a common denominator. For example, Tuzun and Burke describe the problem this way: “A feasible set of potential facility sites and locations and expected demands of each customer are given. Each customer is to be assigned to a facility which will supply its demand. The shipments of customer demand are carried by vehicles which are dispatched from the facilities, and operate on routes that include multiple customers. [...] The LRP is to determine the location of the facilities and the vehicle routes from the facilities to the customers to minimize the sum of the location and distribution costs such that the vehicle capacities are not exceeded.”² Wu et al. give a similar but somewhat shorter definition. According to them the problem at hand is “to find the optimal number and locations of the DCs, simultaneously with the vehicle schedules and distribution routes so as to minimize the total system costs.”³

The common feature of these definitions and descriptions is the fact that they actually divide the LRP into two sub-problems. For example Lin and Kwok suggest that the LRP consists of a “facility location problem (FLP) and the vehicle routing problem (VRP).”⁴ An approach which slightly differs from this point of view can be found in Nagy and Salhi. According to them the objective is to solve “a facility location problem (the ‘master problem’), but in order to achieve this we simultaneously need to solve a vehicle-routing problem (the ‘sub problem’).”⁵

The basic principle behind all these ideas is that the overall cost of a complete logistical system can be lowered if the system is designed as a whole and not piece by piece. So we want to decrease the future routing cost by including the routing

² See Tuzun, D., Burke, L., 1999 , p. 88

³ See Wu, T., Low, C., Bai, J., 2001, p.1393

⁴ See Lin, C. K. Y., Kwok, R.C.W., 2006, p. 1834

⁵ See Nagy, G., Salhi S., 2007, p. 650

decisions into the location decision. The next step would be to formulate this problem as a linear program, but in order to do this and to increase our understanding we should take a closer look at the two underlying problems. First we will consider the facility-location problem and afterwards the MDVRP.

2.2 THE FACILITY - LOCATION PROBLEM

The literature considers the problem of locating facilities for almost a century. It was introduced by Weber in 1909. The problem he considered (which is now labelled “Weber-Problem”) was the location of a single warehouse in order to minimize the sum of the distances between this facility and all its customers⁶. But we cannot say that there is just one location problem, it is actually an umbrella term for a wide variety of different problems. Nonetheless we want to give a general definition for this research-area. “A location problem is a spatial resource allocation problem. In the general location paradigm, one or more facilities (“servers”) serve a spatially distributed set of demands (“customers”). [...] The objective is to locate facilities (and perhaps allocate customers to servers) to optimize an explicit or implicit spatially dependent objective.”⁷ In this area we can find problems like the Warehouse-Location Problem, where the objective is to locate one or more warehouses in order to minimize the distances to the customers but with keeping the cost-aspect of these facilities in mind, so it is actually an advancement of the Weber-Problem. Other examples for location-problems are the p-median-problem (which will be discussed later in detail) and the p-centre-problem, which has the objective to locate p-facilities in order to minimize the maximum-distance any customer has to travel to reach his closest facility. Another sub-category is the allocation of public emergency facilities like ambulances or fire departments which combine the objectives of short distances and coverage. In this category we can find the maximal covering location problem where you have to locate again p facilities in order to maximize the percentage of customers which have to travel less than a given distance

⁶ See Brandeau, M. L., Chiu, S. S., 1989

⁷ See Brandeau, M. L., Chiu, S. S., 1989, p. 646

D to reach an open facility. The Location Set Covering problem is almost the same. The main difference is the introduction of an additional restriction that every customer has to lie within a given distance from any opened facility. Of course this means that the maximization of the coverage is no longer the objective. The aim is now to minimize the number of facilities that have to be opened. A more complicated sub-group is the allocation of obnoxious facilities like a waste-disposal facility or a nuclear power plant. Here the difficulty is to compromise between two contradicting objectives. On the one hand such a facility should be located as far away from any city in order to minimize the negative influences such facilities tend to have on the population. On the other hand such facilities are highly interlinked to any population centres so a larger distance would cause an increase in transportation costs which is undesirable as well.

2.3 THE MULTI-DEPOT VEHICLE-ROUTING PROBLEM

Since most real-world logistical networks consist of more than one depot, the step of including multiple depots into the VRP brings it closer to praxis. Like in the VRP we have a graph $G = (V_A, E)$ where E is a set of edges and V_A is a set of vertices, which is again subdivided in V_C representing the customers and V_D representing the depots. Each customer has an associated and non-negative demand d_i as well as a service time δ_i . Additionally every edge has a specified travelling time c_{ij} . In order to fulfil the demand we have a set of k vehicles, which are not yet assigned to any depot. All the vehicles have a maximum-capacity Q and cannot drive longer than time L . The objective of the MDVRP is to construct “a set of vehicle routes in such a way that: (1) each route starts and ends at the same depot, (2) each customer is visited exactly once by a vehicle, (3) the total demand of each routes does not exceed the vehicle capacity Q , (4) the total duration of each route (including travel and service time) does not exceed a preset limit L and (5) the total routing cost is minimized.”⁸

⁸ See Renaud, J., Laporte, G., Boctor, F. F., 1996, p. 229

Now we want to present a LP formulation for the MDVRP. As a starting point we are using the classical three-index VRP-formulation and adapt it to the multi-depot case. We are using the notation for the parameters as described above and simply add two binary decision variables:

$$x_{ijk} \begin{cases} 1 & \text{If the edge between the points } i \text{ and } j \text{ is traversed by the vehicle } k, \\ 0 & \text{Otherwise,} \end{cases}$$

$$y_{ik} \begin{cases} 1 & \text{If the node } i \text{ is visited by the vehicle } k \\ 0 & \text{Otherwise,} \end{cases}$$

$$\min \sum_{i \in V_A} \sum_{j \in V_A} \sum_{k \in K} x_{ijk} * c_{ij} \quad (1)$$

Subject to

$$\sum_{k \in K} y_{ik} = 1 \quad \forall \quad i \in V_C \quad (2)$$

$$\sum_{i \in V_D} y_{ik} = 1 \quad \forall \quad k \in K \quad (3)$$

$$\sum_{k \in K} \sum_{i \in V_D} y_{ik} \leq K \quad (4)$$

$$\sum_{i \in V_C} d_i * y_{ik} \leq Q, \quad \forall \quad k \in K \quad (5)$$

$$\sum_{i \in V_A} \sum_{j \in V_A} c_{ij} * x_{ijk} + \sum_{h \in V_C} \delta_h * y_{hk} \leq L, \quad \forall \quad k \in K \quad (6)$$

$$\sum_{i \in V_A} x_{ijk} = y_{jk} \quad \forall \quad j \in V_A, k \in K \quad (7)$$

$$\sum_{j \in V_A} x_{ijk} = y_{ik} \quad \forall \quad i \in V_A, k \in K \quad (8)$$

$$\sum_{i \in V_C} \sum_{j \in V_C} x_{ijk} \leq |S| - 1, \forall S \subseteq V_C, |S| \geq 2, k \in K \quad (9)$$

$$\sum_{j \in V_C} x_{ijk} = \sum_{j \in V_C} x_{jik} \quad \forall \quad i \in V_D, k \in K \quad (10)$$

As stated before the objective function (1) is to minimize the total distance travelled by all vehicles. While the second equation specifies that every customer is assigned to only one vehicle and so will be visited exactly once, the third constraint assigns every vehicle to exactly one depot. Here we see the ambiguity of the binary variable y_{ik} : as long as i is a customer, the variable defines by which vehicle this customer is visited. If the same i is a depot, it specifies to which depot this vehicle is attached. With this in mind it is easy to see that equation (4) simply sets an upper limit for the number of vehicles used in the problem. While the next constraint (5) makes sure that the capacity of no vehicle exceeds the pre-defined limit Q , equation (6) ensures the compliance of the tour-length limit L . The next two equations are the so-called in-bound / out-bound degree constraints. The inbound degree - constraint (7) states that every node is entered as often as it is visited, while the outbound degree-constraint (8) states that every node is left as often as it is visited. Next we introduce the sub-tour-elimination constraint as proposed by Dantzig, Fulkerson and Johnson⁹.

⁹ See Dantzig, G., Fulkerson, R., Johnson, S., 1954

Here we consider any subset (excluding the depot) with at least two nodes of a tour. If there are fewer connections within the subset than there are nodes the constraint is fulfilled. If this is not the case we have a sub-tour with no connection to a depot and so the solution would be infeasible. Finally the last constraint ensures that every vehicle has to start and end its tour at the same depot, by considering the sum of all traversed edges leading away from a depot for any vehicle. If a specified vehicle is leaving a specific depot it also has to return to this depot. For this we consider the sum of all traversed edges leading back to the depot. If these two sums would be unequal, a vehicle would end its tour at another point than its starting-depot.

2.4 PRINCIPLE CRITICISM

Although the idea of combined location-routing has been around for more than two decades it received far less attention than other location or routing problems during this period. The reasons behind this fact are three basic points of criticism¹⁰, which we will now evaluate for our problem. The first criticism is that not every location decision is connected to a routing problem. Especially if we consider the location of ambulance-vehicles, hospitals, airports or networks like wastewater systems or metro lines, routing would be a useless criterion. But here we are dealing with the location of a distribution centre which has the main purpose of harbouring trucks which deliver goods to assigned customers. So we can ignore this point in our considerations. The next argument against combined Location-Routing considerations is the different level of those two decisions. While the location decision is a strategic one, the routing decisions appear even on a daily basis and so belong to the tactical level. Despite this discrepancy various studies have proven that the combination of facility-locating and vehicle-routing can lower the distribution-costs, if we consider the costs over a longer time horizon.¹¹ The third problematic point is the difficulty of this problem. The facility-location problem as well as the VRP is np-hard, so a combination of those two problems will be unlikely to create

¹⁰ See Nagy, G., Salhi S., 2007

¹¹ See Salhi, S., Nagy, G., 1999

good results within reasonable time for a real-life instance. We hope to prove these last two points invalid for our problem when we compare the results obtained by our heuristic to the current situation. Additionally the difficulty of a problem should actually not be a point of criticism, especially if we consider the large progression made in the sectors of IT and OR. This difficulty should rather be an incentive for further studies. But this leads us actually away from the topic at hand.

3. CLASSIFICATION AND PRESENTATION OF THE PROBLEM

3.1 MODELS FOR CLASSIFICATION

The literature offers a wide variety of classification-models for the LRP (Madsen, 1981; Laporte, 1988; List et al., 1991; Berman, 1995) but in this thesis we will only look at two classifications as well as a short comparison of those two. While the first article has a very comprehensive and sophisticated classification including all important studies done prior to 1998, the second model, although less elaborate is dealing with the more recent literature, focusing on the publications of the last decade. The first one is the classification by Min, Jayaraman and Srivastava presented in 1998¹². The second one has been described by Nagi and Salhi in 2007¹³. The first classification divides all LRPs according to 11 criteria. The first question is if we are dealing with a single staged or a two staged problem, whereas in the first case we only have to deal with outbound routes, the second case includes inbound routes (pickup and delivery problem). The next distinction is made between stochastic and deterministic LRPs. In a deterministic LRP all parameters like demand and supply are fixed while the stochastic one considers those parameters to be random. Further on they differentiate between single-depot and multi-depot as well as single-vehicle or multiple-vehicle problems, whether the facilities and / or the vehicles are uncapacitated or capacitated. Most of the LRPs are dealing with just secondary facilities like transshipment-points and intermediaries, other LRPs also take primary facilities like manufacturing plants into account. Criterion 8 considers if we are dealing with a static one-period problem or a dynamic problem with multiple periods. Another three sub-classes of LRPs are created by using the different types of time-windows, depending whether they are soft, hard or non-existing. The last two criteria incorporate the differentiation between single objective and multiple

¹² See Min, H., Jayaraman, V., Srivastava, R., 1998

¹³ See Nagi, G., Salhi, S., 2007

objective problems as well as the question whether the problem is based on hypothetical or real-life data. Additionally the authors offer a second taxonomy which is not based on the characteristics of the problem but based on the features of the solution method. This leaves us with two big groups: exact algorithms and heuristics, which both have four subgroups. The exact algorithm can be a (1) direct tree search /branch and bound, (2) dynamic programming, (3) integer programming or (4) non-linear programming. On the other side heuristics can be classified as (1) location-allocation- first, route-second, (2) route-first, location-allocation-second, (3) savings/insertion or (4) improvement / exchange. Apparently this taxonomy is problematic since a solution procedure can fit in more than one subgroup. For example the procedure presented in this thesis uses the principle of location-allocation-first route-second as well as a savings-heuristic and an improvement-algorithm.

Nagy and Salhi are not giving two different classifications but merge both aspects (nature of the problem and nature of solution procedure) into one classification with four key aspects and five additional criteria. The first key aspect is the hierarchical structure of the problem, which means that we have either facilities that solely serve customers (so-called secondary facilities) or are also take production and manufacturing facilities (so-called primary facilities) into account. The next two criteria for differentiation are the type of data used in the problem, meaning if stochastic or deterministic data is used, and the planning horizon, which can be single-period or multi-period. The last key-aspect is the differentiation whether a heuristic solution method or an exact algorithm is used. The other five criteria are only used to clarify a problem but not to group the problems because otherwise the different groups would apparently become too small. The fact that Min et al. had ten years earlier considerably less literature and even more criteria but had no problems with the size of their groups should not throw us off the track. The first criterion is the objective function, where we can see single objective and multiple objective functions. Another consideration is the underlying space of the problem which can be a network, discrete space or continuous space. Furthermore they differentiate between single-depot and multi-depot problems. Another point is the number and type of vehicles used in the routing, whereas the type is referring to a heterogeneous or a homogeneous fleet of vehicles. The last criterion is the route-structure. This

point refers to the different attributes of routing-problems, for example arc-routing and node-routing, multiple usage of vehicles or the combination of pickup and delivery.

In the table 1 we are going to compare these two classification-models. The model from Min et al. will be presented in the same order as before, the model of Nagy and Salhi will be rearrange so that comparable points are in the same line.

	Min et al. (1998)	Nagy and Salhi (2006)
I.	Hierarchical level	Route Structure (IX.)
II.	Nature of demand / supply	Type of input-data (II.)
III.	Number of facilities	Single or Multi-Depot (VII.)
IV.	Size of vehicle fleet	Number and type of vehicles (VIII.)
V.	Vehicle capacity	
VI.	Facility capacity	
VII.	Facility layer	Hierarchical Structure (I.)
VIII.	Planning horizon	Planning period (III.)
IX.	Time windows	
X.	Objective function	Objective function (V.)
XI.	Types of model-data	
		Solution-space (VI.)
	Seperate Classification-model	Solution-Methode (IV.)

Table 1: Comparison of Min et al. Vs. Nagy and Salhi

As one would anticipate most of the points can be found in both classifications, although some points are named different. Especially the points “hierarchical level” and “hierarchical structure” can be misleading. While Min et al. use hierarchical level for the differentiation between delivery and pickup & delivery (which is a sub-point in the route structure of the second model), the hierarchical structure in the Nagy and Salhi-classification is equal to the facility layer of the first model. Two points are completely missing in the second model, which are facility capacities and the differentiation between real-life and hypothetical data. Additionally two more points are not directly mentioned in the second model but the time-windows might be a part of the route-structure and the vehicle-capacities could fall under the type of vehicles, although the authors state nothing to support this opinion. The considerations concerning the solution space on the other hand only appear in the second model. Although the solution method is not mentioned in the first model, it is presented as a separate possibility to classify works concerning LRPs.

3.2 DESCRIPTION OF THE PROBLEM – INSTANCE

The starting-point of our problem is the distribution-network of an Austrian logistical company. They currently operate two depots in and around Vienna which are responsible for the customers located in Vienna, Burgenland and Lower Austria. The data we have obtained consists of the location, demand and time-window of all customers during a two-week period. Within these 10 days we have to perform 12.429 deliveries to 6.349 customers. As well as the number of deliveries during this period varies from customer to customer it also varies from day to day. To fulfil those deliveries we have a homogenous fleet of 160 trucks with a maximum load of 9000kg. Each vehicle can only be used once a day and must not exceed a 9 hour working time per day.

Aside from the two locations that are currently operated we researched 48 new ones in order to get 50 possible locations for our heuristic. Neither the price of the land or the availability has been of any concern during this search. The first step was to consider the location of other mayor international logistical companies like DHL, GLS, and Schenker and the location of their depots. Also some Austrian competitors

were considered. Through this we found 15 suitable locations. The next step was to look for business parks and industrial clusters because of their high demand for transportation, where we located additional 30 suitable candidates. The last three locations were found after a graphical analysis of the depots distribution compared to allocation of the customers. Since most of these facilities are hypothetical and we were not able to obtain data from the existing depots concerning operation and / or fixed costs of these facilities, the single objective is to minimize the total travelling time of all vehicles within the planning horizon. Due to this lack of information we did not consider any capacity-ceiling for the depots, but we introduced a minimum level for them. No depot should be opened unless at least 10 trucks are located at this depot. The number of vehicles is fixed and so not object to our optimization either, only the allocation of those trucks is considered in the heuristic. A list with all locations can be found in Appendix C.

If we reconsider the classification of Min et al. we can classify our problem as a single staged deterministic LRP with multiple uncapacitated depots and multiple capacitated vehicles. Further on we are considering secondary facilities over a multi-period planning horizon. We have strict time-windows for each customer, a single objective function and are using real-world data.

According to Nagy and Salhi we have a LRP considering secondary facilities based on deterministic data over a multi-period planning horizon which is solved by a heuristic procedure. Furthermore we have a single-objective problem on a discrete space with multiple depots and a homogenous fleet of 160 trucks. Concerning the route-structure we have a node-routing problem without multiple usage of trucks doing only deliveries and no pickups.

With these explanations in mind we want to present a linear program for the LRP adapted to our situation.

We have again a graph $G = (V_A, E)$ where E is a set of edges and V_A is a set of vertices, which is again subdivided in V_C representing the customers and V_D representing the depots. Additionally we have a set K representing all vehicles.

We introduce the following binary decision variables:

$$X_{ijk} \quad \left\{ \begin{array}{ll} 1 & \text{If the edge between the points } i \text{ and } j \text{ is traversed by the vehicle } k, \\ 0 & \text{Otherwise.} \end{array} \right.$$

$$y_{ik} \quad \left\{ \begin{array}{ll} 1 & \text{If the node } i \text{ is visited by the vehicle } k, \\ 0 & \text{Otherwise.} \end{array} \right.$$

$$Z_i \quad \left\{ \begin{array}{ll} 1 & \text{If a depot is opened at location } i, \\ 0 & \text{Otherwise.} \end{array} \right.$$

We use the following fixed values

Name	Explanation	In our example
P	Maximum Number of Facilities that can be opened	16
Q	Maximum capacity of any vehicle	9000 kg
$M_{\max}$	Maximum Number of vehicles available	160
$M_{\min}$	Minimum number of vehicles assigned to one depot so that it can be opened (min-flow-condition)	10
L	Maximum route-length of one vehicle	540 min
δ	Service time at any customer	15 min

Finally the used parameters:

f_h	Cost to open a facility at location h
d_i	Demand of customer i
c_{ij}	Cost of travelling between vertices i and j

$$\min \sum_{i \in V_A} \sum_{j \in V_A} \sum_{k \in K} x_{ijk} * c_{ij} + \sum_{h \in V_D} f_h * z_h \quad (11)$$

$$\sum_{k \in K} y_{ik} = 1 \quad \forall i \in V_C \quad (12)$$

$$\sum_{i \in V_D} y_{ik} = 1 \quad \forall k \in K \quad (13)$$

$$\sum_{k \in K} \sum_{i \in V_D} y_{ik} \leq M_{\max} \quad (14)$$

$$\sum_{k \in K} y_{ik} \geq z_i * M_{\min}, \forall i \in V_D \quad (15)$$

$$\sum_{h \in V_D} z_h \leq P \quad (16)$$

$$\sum_{i \in V_C} d_i * y_{ik} \leq Q, \quad \forall k \in K \quad (17)$$

$$\sum_{i \in V_A} \sum_{j \in V_A} c_{ij} * x_{ijk} + \sum_{h \in V_C} \delta_h * y_{hk} \leq L, \quad \forall k \in K \quad (18)$$

$$\sum_{i \in V_A} x_{ijk} = y_{jk} \quad \forall j \in V_A, k \in K \quad (19)$$

$$\sum_{j \in V_A} x_{ijk} = y_{ik} \quad \forall i \in V_A, k \in K \quad (20)$$

$$\sum_{j \in V_C} x_{ijk} = \sum_{j \in V_C} x_{jik} \quad \forall i \in V_D, k \in K \quad (21)$$

$$y_{ik} \leq z_i \quad \forall i \in V_D, k \in K \quad (22)$$

$$\sum_{i \in V_C} \sum_{j \in V_C} x_{ijk} \leq |S| - 1, \quad \forall S \subseteq V_C, |S| \geq 2, k \in K \quad (23)$$

Most of the constraints are directly taken from MDVRP model that was presented earlier, so we will only explain those not already used above. We expanded the objective function (11) in order to include also the fixed facility costs. While equation (14) sets an upper limit for the number of vehicles at our disposal, equation (15) sets a lower bound. No depot can be opened unless at least 10 vehicles are located there, in order to have a minimum flow at each depot. The next constraint (16) is responsible for the maximum number of depots that can be opened. The last difference between the MDVRP – LP and the LRP – LP is equation (22), which ensures that a vehicle is only assigned to a depot which is open.

4. ALGORITHMS

4.1 BASIC DEFINITIONS

Before we take a closer look at the different algorithms that are going to be used later on we should describe the two groups of solving procedures. On the one hand we have exact algorithms which solve a problem to its optimality, like the simplex-algorithm. Through their complexity the exact algorithms are mostly restricted to more simple problems or smaller instances. On the other hand we have so-called heuristics, which function on a trial and error base. These methods are normally faster than an exact algorithm but the disadvantage is that the result is most often just an approximation of the optimal solution, so the optimality is not guaranteed. Since we are dealing with a large scale problem an exact algorithm could only be applied if we are dealing with a simple problem. Since Karp has proven that both the VRP and the facility location problem are np-hard¹⁴, we can conclude that the LRP must be np-hard as well. The combination of the difficulty of the LRP and the scale of our problem is the main reason why we decided to apply a heuristic procedure.

4.2 THE SAVINGS – ALGORITHM

This algorithm was developed by Clark and Wright in 1964¹⁵ for the basic VRP, but the idea can be adapted quite easily for additional attributes like in our case multiple depots and time windows. It was designed to minimize the total distance of all routes, other objectives (like fleet-size) were not considered. The idea is the following: in the beginning every node is connected to a depot. Then a so-called savings-matrix will be computed.

¹⁴ See Karp, R., 1972

¹⁵ See Clarke, G., Wright, J. W., 1964

These savings are calculated by the following equation:

$$S_{ij} = C_{io} + C_{oj} - C_{ij} \quad (24)$$

Whereas i and j are customers and 0 is the corresponding depot. So the saving has two components. First the distance from customer i back to the depot plus the one from the depot to customer j . The second part is the distance between these two customers. So you save the first part, because you do not have to go back to depot, but you add the second part, which is the new connection between the customers. If we are considering a symmetric VRP, the saving cannot be negative since no side in a triangle can be larger than the sum of the other two sides. However, this assumption does not hold true if we are dealing with an asymmetric case like ours.

After the savings-matrix is computed we start with the highest saving and try to link the two customers. The next step is to check whether the new route is feasible. Afterwards this saving is erased and the next savings is considered until the savings-matrix is empty. The authors proposed that any algorithm suitable for a TSP could improve this final solution if applied to the individual routes.

4.3 THE P-MEDIAN-PROBLEM

This problem was first described by Hakimi in 1964¹⁶. We consider a network with a set of possible locations and a number of customer points with or without associated demand. Additionally we have a $m \times n$ -matrix which contains the distances between any location and any customer. While m is representing the number of possible locations, n is standing for the number of customers within the problem. The aim of this problem is to locate p -facilities among the possible locations in order to minimize the sum of the weighted distance between any customer and its nearest opened facility. The weight ω of these customers can either be their associated

¹⁶ See Hakimi, S. L., 1964

demand or they can be equally weighted. In the beginning we actually used three different weights in order to see the problem from different angles and improve the quality of the starting solution. In the first round we were using equal weights for every customer. The idea is that the frequency of the deliveries for any customer as well as the demands is only known for a short time period so the results in the long-run should improve if we are using equal weights. In the second round we added a weight which is equal to the times the customer has to be visited within our planning horizon. The main argument is that a customer, which is visited every day, should have more influence on the location of a depot than a customer which needs a delivery only every ten days. The last round used weights equal to the overall demand during the planning horizon. The advantage of these weights is that a few customers with small deliveries would have a similar influence like one customer with a rather large delivery. Nonetheless we dropped the results of the last round. The reason for this can be explained with a closer look at the capacities of the different vehicles. On average a vehicle is using 25 – 35 % of its capacity and there are relatively few vehicles using more than 65% of their maximum capacity. So we can concentrate on the first two ideas: using equal weights and using the number of deliveries as weights. Now we want to present the mathematical formulation of the p-median problem, which slightly differs from the classical formulation:

$$\min \sum_{j=0}^n \omega_j * \sum_{i=0}^m \sum_{j=0}^n d_{ij} * x_{ij} \quad (25)$$

Subject to

$$\sum_{i=0}^m y_i = p \quad (26)$$

$$\sum_{i=0}^m x_{ij} = 1, \quad \forall j \in N \quad (27)$$

$$\sum_{j=0}^n x_{ij} \geq y_i * F_{\min}, \quad \forall i \in M \quad (28)$$

$$x_{ij} \leq y_i, \quad \forall i \in M, j \in N \quad (29)$$

Whereas

$$x_{ij} = \begin{cases} 1 & \text{If customer } i \text{ is assigned to facility at location } j, \\ 0 & \text{Otherwise,} \end{cases}$$

$$y_j = \begin{cases} 1 & \text{If a facility is opened at location } j, \\ 0 & \text{Otherwise,} \end{cases}$$

The objective function (25) is to minimize the weighted product of the distance and assigning-variable x_{ij} . Equation (26) states that p facilities are opened, whereas equation (27) specifies that every customer is assigned to exactly one facility. The new feature of this formulation is represented by equation (28), which simply counts the number of customers assigned to a facility and ensures that this number is larger than a pre-defined minimum-flow. This constraint should ensure that a facility is not opened for just a small cluster of customers at the edge of the map, which would decrease the efficiency of the whole system. The final constraint (29) makes sure that a customer can only be assigned to facility which is open.

Although the facility location problem is np-hard, the p-median problem is a simpler case so for our small instance it can be solved up to optimality.

The results for the p-median-problem were obtained using the Xpress-Solver (Version 1.17.12). The detailed results can be found in Appendix A.

4.4 THE R-OPT-HEURISTIC

A simple way of classifying heuristics which solve a VRP is to divide them using the fact whether they need a starting-solution or not. The Savings-Algorithm does not need such a solution, that's why it belongs to the so-called construction heuristics. The other class consists of improvement-heuristics, which try to enhance an existing solution. One of these improvement-solutions is the r-opt-heuristic. The groundwork for this heuristic was laid by Croes¹⁷ and Lin¹⁸. The algorithm used in our case bases on the work of Lin and Kernighan¹⁹. The idea is that in any tour which is no optimal a set of k elements is not in the right order so in every step r-edges of the route are disconnected and then the remaining r pieces are reconnected to generate a new route. After this is done the tour-length is recalculated and compared to the existing length. If an improvement is found and the new route is still feasible, there are two ways to proceed. One way is called "first improvement", because as soon an improvement is found, this change is realized and the heuristic starts again at the beginning. On the other side is the "best-improvement"-variant, which means that every possibility is checked and the one with the highest improvement is realized, then the heuristic goes to the beginning as well. Both variants end, if no more improvements can be found.

¹⁷ See Croes, A., 1958

¹⁸ See Lin, S., 1965

¹⁹ See Lin, S., Kernighan, B. W., 1973

5. IMPLEMENTATION

This solution procedure begins by generating a starting solution using linear programming. By implementing a p-median-formulation as described in chapter 4 we generate 16 starting-solutions for all numbers of depots between 1 and 16. The upper limit of 16 is given by the fact that we need to allocate at least 10 vehicles to any depot and only 160 vehicles are available. As described in the previous chapter we were using two different formulations to generate our starting solutions. We tested the algorithm using both kind of starting solutions. The impact of this is discussed in the next chapter.

The next step is the allocation of customers to the depots determined by the starting solution, in order to split the problem into a series of simpler VRPs as proposed by Crevier et al²⁰. The primary variant of allocation is to assign each customer to its nearest depot, which is applied as the general method. The problem is the spatial and timely distribution of the customers in combination with the number of vehicles available. So we introduced a secondary allocation method, which divides the customers equally upon the different depots. In each step we let all depots cover their closest customer which is not yet assigned to a depot. This of course leads to a worse allocation and so it is only applied if after 15 iterations no feasible solution has been found.

The third part of our solution-procedure consists of the construction of multi-stop routes. For this problem we are using the Savings-algorithm. In each step we try to realize the biggest saving in the list. If the resulting tour is feasible in all 3 aspects (capacity, tour-length and time-windows), the new tour is saved. This particular saving will be deleted afterwards. In order to further improve the solution we are using first a 2-opt-heuristic and then a 3-opt-heuristic. In both cases the best-improvement-variant has been implemented. The reason for this is that we obtained better results with only slight deteriorations in respect to the computational time as with the first-improvement-variant. We also tried to apply an inter-route-move-

²⁰ See Crevier, B., Cordeau, J. F., Laporte, G., 2007

heuristic. The idea was to improve the overall solution by trying to take a single customer, remove it from his original route and insert him in any tour which lies within a predefined distance. The heuristic brought only small improvements while using substantial amounts of computational time, so we decided to drop this part.

The allocation, the tour-building and the improvement are redone for all of the ten days. As soon as one day is completed, the number of vehicles assigned to each depot is updated. At the beginning every depot gets 10 vehicles. After all tours have been constructed and improved, the algorithm checks if there are enough vehicles at every depot in order to service all routes. If this is not the case the number is raised up to the necessary level, otherwise the number stays the same. If at any point we exceed the maximum number of vehicles we declare this to be an infeasible solution. When this is the case we create a new set of depots. If we have reached the end of the 10th day and the solution is feasible, all routes of all days are summed up. This sum is compared with the current best solution, which is updated if we found a further improvement.

The next step is to create a new set of opened depots. For this we randomly choose one of the depots of the current best solution and close it. The successor of this depot is its nearest neighbour which is not on the tabu-list and still closed. Every candidate has its own tabu-list, consisting of the locations already implemented for this candidate. This should prevent a solution from cycling around a specific point. If the last switch of depots brought an improvement, the tabu-list will be erased for every depot, except the one that has been changed to cause the improvement. The reason for this is to reduce computational time by eliminating the possibility of recalculating older solutions, without eliminating possibilities for further improvement. If there has been no improvement for a specified number of iterations we try to rearrange the whole set of depots. This number of iterations is connected to the number of open depots, whereas we simply multiply the number of depots with 5 to get a limit for our search. For the relocation we are using all non-tabu neighbours of the depot which lie within a given distance in order to exclude the ones too far away and randomly choose one of these candidates. This is redone for every depot. The idea is that if we were not able to find any improvement in the neighbourhood, a random relocation might help to escape from a local optimum. After the generation of a new set of depots we start with the first day again.

The algorithm ends if there is no improvement for a certain number of iterations or the maximum number of iterations is reached. Both numbers are connected to the number of depots we are looking at due to the fact that the number of possibilities increases with the number of depots we are locating. The maximum number of iterations is 10 times the number of depots we have to locate, whereas this is not the case for the first two instances where we raise to the maximum number to 25. The second limit is 7 times the number of depots, whereas we have a lower bound of 20 and an upper bound of 60 iterations. If the number of iterations which produce feasible but inferior solutions reaches the second limit, we can conclude that we are stuck in a local optimum and skip the further search.

After the algorithm has solved the problem for a specific instance, the best solution with the opened depots and the corresponding tours for all days as well as the allocation of vehicles is printed. The next step is to raise the number of depots that are going to be opened and the algorithm starts again with the consideration of the corresponding p-median-solution.

6. RESULTS AND IMPLICATIONS

6.1 COMPUTATIONAL RESULTS

While the p-median-problem was solved by using the Xpress-Solver, the rest of solution-procedure is coded in C++. All experiments are run using a P IV / 3200 processor with 3072 MB RAM. All computations using a float point precision without rounding.

The following table shows the objective values, which equals the sum of all distances driven during the observed time span but without the service times, which cannot be optimized and are therefore excluded from the objective values.

Number of depots	Final Solution	Starting Solution	Computational time
1	44578	44578	02:47:45
2	40516	40539	03:52:55
3	38231	38428	02:11:44
4	37309	37309	02:50:53
5	35694	35749	02:07:38
6	35213	35840	02:00:59
7	34935	34935	01:39:45
8	34099	34099	03:09:42
9	54899	58352	02:18:50
10	55342	57608	02:05:58
11	54130	56004	02:13:29
12	49900	53723	03:17:34
13	54144	58112	02:04:08
14	50359	55532	02:56:15
15	53334	53857	01:52:45
16	54373	54596	04:06:19

Table 2: Primary and Final Results as well as computational times for all Instances

All results in table 2 are given in minutes. The values are presented for the starting and final solutions of our heuristic with different numbers of depots. The detailed solutions with the opened depots can be found in Appendix B. As we mentioned in Chapter 4 we obtained two types of starting solutions, whereas in the first type the number of deliveries is used as a weight for each customer. In the second type the customer are weighted equally. We were testing our solution procedure with both types and as a result we can conclude that the first type of starting solutions leads to inferior solutions, that is why we dropped these starting points and are only using and discussing the results obtained by applying the second type.

If we take a closer look at the solutions for the different number of depots, two attributes attract our attention immediately. The first point is the high degree of similarity between the final solutions found by our algorithm and the solutions presented by the p-median-problem, which is especially high during the beginning and becomes less important for the larger instances. Although simple straight line distances have been declared inefficient for the location of distribution centres²¹, it works at least partially in our case. But we have to keep three basic features in mind before drawing conclusions from this observation. The first point is that our depots are quite evenly distributed throughout the map whereas the customers are mainly concentrated in Vienna, which limits our possibilities. We also have to remember that we have only 50 possible locations at our disposal, which seemed to be a quite large number compared to the two existing depots but it was apparently not enough for our computational experiments. The third point is that most of the p-median-solutions (12 out of 16) have shown room for improvement. From this we can draw the conclusion that these solutions are effective starting points but inefficient means to support a real location decision.

The second important attribute is the development of the objective function as the number of facilities increases. We can observe a steady decrease in the overall travel time for every supplemental depot until the addition of a 9th depot. This addition raises the objective value by 61 % compared to the solution with 8 depots, which also provides the smallest sum at all. This development is of course confusing in the beginning since logic dictates that every additional depot has to lower the value of

²¹ See Perl, J., Daskin, M.S., 1985

the objective function, because every new depot lowers the distance to a subset of customers. If this is not the case, the customer would not be served from this new depot. But here we have the core of the problem. We have stated in the beginning that at least 10 vehicles have to be assigned to every depot and only 160 vehicles are available, which includes that a subset of customers has to be served from this new depot even if this deteriorates our overall travel time. Additionally above a certain number of depots the allocation of customers has to switch from the nearest-allocation-method to the even-allocation-method in order to fulfil those two restrictions. This equalisation of the consumers-allocation apparently creates by far inferior solutions. As the reader might assume this inferior allocation-procedure is applied for the first time during the case with 9 depots and has to be used from then on in order to create feasible solutions.

Another problematic point is the unequal distribution of customers during these ten days. Although this is difficult to describe, the p-median-solutions calculated for the single days might paint a clearer picture. These solutions for the first five instances are with the other solutions in Appendix A. These may also lead to infeasible solutions since a depot which is heavily used on one day might consume the trucks necessary at other depots at another day.

6.2 MANAGERIAL IMPLICATIONS

As described in Chien²² there are actually four main cost-aspects that need to be considered in order to make a competent location decision. The first aspect is the fixed cost for opening a depot at any given location, while the second one is a variable factor connecting the depot cost with the cargo-flow at this location. The third part is a fixed cost for the use of a vehicle and the last one should take the distance driven by any vehicle into account. Through our lack of data our solution is solely based on the minimization of the last aspect.

Nonetheless we want to do a detailed comparison between the existing depots and the two depots determined by our heuristic. In the table 3 we present the objective values for each day as well as the total again in minutes without the service time. The last column shows the discrepancy in percent between the two values whereas the existing depots represent the basis for this calculation.

	Existing depots	New depots	Discrepancy
Day 1	3991.53	3704.81	- 7.18 %
Day 2	4422.62	3959.92	- 10,46 %
Day 3	4465.89	4007.6	- 10,26 %
Day 4	4763.33	4364.74	- 8,37 %
Day 5	4501.7	4078.24	- 9,41 %
Day 6	2715.34	2528.06	- 6,90 %
Day 7	3794.5	3542	- 6,65 %
Day 8	3521.99	3264.48	- 7,31 %
Day 9	5914.09	5417.5	- 8,40 %
Day 10	6440.28	5648.28	- 12,30 %
Total	44531.27	40515.63	- 9,02 %

Table 3: Comparison of the current depots and our solutions

²² See Chien, W. T., 1993

This time the results paint a very clear picture. The new depots would improve the total travel-time between 6,65 % and 12,30 %, while the total solution shows an improvement of 9,02 %. Although this is only a sample and does not represent the whole set of customers served by this company, it clearly indicates that our solution could decrease transportation costs in this case. This point is very important if we reconsider the criticism against the LRP. We already dropped point one but still have the other points which are the different level of decision and the difficulty of the problem. Since we were able to improve the current solution for every day we can say that both points can be omitted since there is no ground to argument like this in our case.

Another important point would be the decision how many depots should be opened. Here the heuristic provides us only little insights which are only helpful to exclude some scenarios, for example all scenarios with more than 8 depots. On the other hand this leaves us still with 8 possibilities, which all have decreasing transportation costs but also would imply increasing fixed costs, but we have no possibility to compare these two values. But if we assume that we take the current situation as a basis for the new depots, we can also presume that the management would limit the number of depots to four, because it is very unlikely that the management would approve a number of depots that exceeds the current one by more than 100 %. So the possible scenarios would be limited to four or less opened depots. If we take a look at the decrease in travel time in percent realized by an additional depot, we get the following results presented in table 4.

Number of Depots	Total Travel Time	Reduction in %
1	44578	
2	40516	- 9,11 %
3	38206	- 5,70 %
4	37309	- 2,35 %

Table 4: Development in percent for cases one till four

This table shows again the steady decrease of travel time in the beginning. The second observation is that the cost saving is reducing itself in every step. So the most

promising reduction can be seen by the addition of a second depot. Since this would probably mean the least change in fixed costs and a simultaneous significant reduction in transportation costs this is our primary suggestion. The secondary suggestion is the reduction to one depot. Our solution for 1-depot-case has only slightly higher transportation costs than the current situation but would on the other side decrease the fixed facility costs. On the other hand an additional advantage for the 2-depot solution would be the increase in flexibility. Since we have no clues about the future distribution of customers as well as their numbers and their demand, this flexibility could prove quite useful for future route-constructions. That is why the solution for the 2-depot-case seems to be the most reasonable one found by the heuristic. A third option would be a middle way. One existing depot lies within proximity to one of the depots proposed by our algorithm. So this option would include keeping the first depot and relocating the second depot from the south of Vienna to the north, where it would apparently serve a better purpose. But we have to admit that a competent location decision cannot be made unless further data concerning the cost of vehicles and facilities is available.

7. CONCLUSION

This thesis has presented a detailed view onto the Location – Routing Problem and shed some light on the main components of it. This detailed view also explains why the LRP can quite easily incorporate different types of routing-problems and is not only restricted to vehicle routing but could also be expanded to arc-routing problems. Of course we also turned our attention to the critic against the LRP, but we have clearly shown that the arguments are quite inadequate in this case.

Based on the p-median-problem and a mathematical formulation for the MDVRP we constructed a linear program for the LRP, which includes all attributes of the problem instance underlying this thesis.

But the main work of this thesis was to create an effective solution procedure for the LRP. The main emphasis of this heuristic is to create reasonable results within a short time. Additionally we wanted to ensure that is quite simple to incorporate other restrictions and constraints beside the ones presented in this work. We have also shown that the p-median-problem is able to generate very good starting solutions for our heuristic. Another point is the route-construction, where we rely on the powerful combination of the savings-algorithm and the r-opt-heuristic. This team has shown that it can generate very good solutions and can be quite easily adapted to a number of restrictions.

We also want to mention the three weak spots of our heuristic, where we see the most potential for further improvement. The first point is the inter-route-exchange. Although the heuristic applied here has been inefficient for our instance, inter-route swaps or moves could of course show further improvement. The second point is the allocation of customers. Since the nearest-allocation-method delivers reasonable results it is probably a good starting point, but it has shown its deficiencies for higher instances. On the other hand the even-allocation-method has provided us apparently with inferior allocations. So the development of a new allocation method could improve the solutions as well. But since both of these points fall into the route-construction area we believe that they have little influence in the final location decision.

A point which could heavily influence the location decision is the heuristic for relocating the depots. The simple Nearest-Neighbour-Heuristic is fast but very inefficient. To use a more sophisticated heuristic could improve the speed as well as the quality of the solution significantly.

In order to find a reasonable and efficient location scheme we would need further data in order to assess and adjust the variable and fixed cost parameters linked to the facilities and the vehicles. Without this data the location decision is merely based on assumptions like it is the case here. Additionally an expansion of the set of possible locations, especially around the centre of the map where most customers are concentrated could cause further improvement.

APPENDIX A – SOLUTIONS FOR THE P-MEDIAN-PROBLEM

DAILY SOLUTIONS

	P = 1	P = 2	P = 3	P = 4	P = 5
Day 1	32	1/50	1/32/50	1/17/32/50	1/32/35/44/50
Day 2	36	1/46	1/33/49	1/16/36/50	1/16/37/49/50
Day 3	43	2/45	1/25/50	2/17/35/50	1/15/25/43/50
Day 4	42	1/50	1/30/50	1/14/31/50	1/14/31/39/50
Day 5	2	2/50	2/17/50	1/8/27/50	1/2/17/34/50
Day 6	34	6/50	1/29/50	2/13/35/50	1/9/20/35/50
Day 7	40	4/47	2/25/49	1/17/33/50	1/6/17/33/50
Day 8	18	2/43	2/22/43	1/18/42/50	1/18/30/42/50
Day 9	41	1/50	1/21/50	1/19/33/50	1/8/21/33/50
Day 10	27	1/50	1/21/50	1/11/27/50	1/11/27/37/50

SOLUTIONS WITH EQUAL WEIGHTS

P = 1	34
P = 2	7 / 24
P = 3	7 / 24 / 30
P = 4	24 / 25 / 34 / 40
P = 5	24 / 28 / 30 / 34 / 44
P = 6	24 / 28 / 30 / 34 / 40 / 44
P = 7	19 / 28 / 30 / 34 / 40 / 44 / 48
P = 8	2 / 13 / 19 / 28 / 34 / 40 / 44 / 48
P = 9	2 / 13 / 14 / 19 / 23 / 28 / 34 / 40 / 44
P = 10	2 / 13 / 14 / 23 / 24 / 28 / 34 / 40 / 42 / 44
P = 11	13 / 14 / 23 / 24 / 28 / 29 / 30 / 34 / 40 / 42 / 44
P = 12	13 / 14 / 23 / 24 / 28 / 29 / 30 / 34 / 40 / 41 / 42 / 44
P = 13	13 / 23 / 24 / 28 / 29 / 30 / 34 / 40 / 41 / 42 / 43 / 44 / 48
P = 14	2 / 13 / 14 / 18 / 23 / 24 / 28 / 29 / 34 / 40 / 41 / 42 / 43 / 44
P = 15	2 / 7 / 13 / 14 / 16 / 18 / 23 / 24 / 28 / 34 / 40 / 41 / 42 / 43 / 44
P = 16	2 / 7 / 13 / 14 / 16 / 18 / 23 / 24 / 28 / 34 / 40 / 41 / 42 / 43 / 44 / 49

SOLUTIONS WITH WEIGHTS EQUAL TO NUMBER OF DELIVERIES

P = 1	29
P = 2	1 / 50
P = 3	1 / 25 / 50
P = 4	1 / 24 / 48 / 50
P = 5	1 / 7 / 37 / 44 / 50
P = 6	1 / 2 / 6 / 8 / 10 / 45
P = 7	6 / 8 / 11 / 36 / 39 / 44 / 46
P = 8	1 / 2 / 15 / 30 / 40 / 48 / 49 / 50
P = 9	6 / 8 / 15 / 17 / 39 / 42 / 45 / 46 / 48
P = 10	1 / 4 / 8 / 15 / 21 / 27 / 33 / 39 / 45 / 50
P = 11	1 / 4 / 8 / 15 / 20 / 26 / 33 / 39 / 42 / 45 / 50
P = 12	1 / 4 / 8 / 11 / 15 / 20 / 24 / 29 / 37 / 42 / 45 / 50
P = 13	1 / 4 / 8 / 11 / 15 / 20 / 25 / 29 / 34 / 39 / 42 / 45 / 50
P = 14	1 / 4 / 8 / 11 / 15 / 20 / 25 / 29 / 34 / 39 / 42 / 45 / 48 / 50
P = 15	1 / 4 / 8 / 11 / 15 / 20 / 22 / 26 / 29 / 34 / 39 / 42 / 45 / 48 / 50
P = 16	1 / 4 / 6 / 8 / 11 / 15 / 20 / 25 / 29 / 33 / 36 / 39 / 42 / 45 / 48 / 50

APPENDIX B – FINAL SOLUTIONS

P = 1	34
P = 2	7 / 19
P = 3	24 / 30 / 31
P = 4	24 / 25 / 34 / 40
P = 5	24 / 28 / 30 / 34 / 46
P = 6	1 / 24 / 26 / 29 / 30 / 34
P = 7	19 / 28 / 30 / 34 / 40 / 44 / 48
P = 8	2 / 13 / 19 / 28 / 34 / 40 / 44 / 48
P = 9	2 / 13 / 14 / 19 / 23 / 25 / 28 / 34 / 44
P = 10	14 / 18 / 23 / 24 / 30 / 34 / 40 / 41 / 42 / 44
P = 11	9 / 11 / 13 / 14 / 23 / 24 / 28 / 34 / 40 / 42 / 44
P = 12	9 / 13 / 26 / 29 / 34 / 35 / 36 / 40 / 41 / 42 / 44 / 48
P = 13	9 / 11 / 13 / 19 / 31 / 33 / 34 / 40 / 41 / 43 / 44 / 48 / 50
P = 14	2 / 13 / 19 / 21 / 23 / 25 / 26 / 29 / 34 / 36 / 40 / 43 / 44 / 48

P = 15	2 / 7 / 8 / 13 / 14 / 16 / 18 / 23 / 24 / 28 / 36 / 40 / 42 / 43 / 44
P = 16	7 / 13 / 16 / 18 / 23 / 24 / 28 / 30 / 34 / 40 / 41 / 42 / 43 / 44 / 48 / 49

APPENDIX C – LOCATIONS

Location 01	Gewerbeparkstraße 35 1210 Wien 632430,36 / 432133,36
Location 02	Donaufelder Straße 101 1210 Wien, 629298,36 / 432768,06
Location 03	Bleibtreustraße 2 1110 Wien 631621,47 / 424440,04
Location 04	Freudenauer Hafenstraße 20-22 1020 Wien 633285,84 / 424677,27
Location 05	Laxenburgerstraße 365 1230 Wien 625055,81 / 419734,99
Location 06	Taborstraße 95 1200 Wien 626776,76 / 429719,20
Location 07	Weyringergasse 36 1040 Wien 626021,1 / 425408,25
Location 08	Clemens-Holzmeister-Straße 4-6 Wien 1100 624061,67 / 422978,36

Location 09	Giefinggasse 2 1210 Wien 629559,39 / 434550,57
Location 10	Freudenauer Hafenstraße 12 1020 Wien 633310,99 / 424676,5
Location 11	Litfaßstraße 8 1030 Wien 628610.64 / 425306.76
Location 12	Wiener Strasse 26 2326 Maria Lanzendorf 627744 / 416626
Location 13	Lastenstraße 1 2020 Hollabrunn 602122,81 / 466110,73
Location 14	Marktstraße 3 7000 Eisenstadt 639383,91 / 386017,07
Location 15	Speditionsstraße, Terminal 5 1300 Flughafen Wien-Schwechat 639828,08 / 418749,57
Location 16	Am Concorde Park 1/83/30 2320 Schwechat 635117,42 / 420133,60

Location 17	Handelsstraße 12 3130 Herzogenburg 574643,11 / 432323,98
Location 18	Handelsstraße 12 2100 Leobendorf 620856,15 / 445704,18
Location 19	Blätterstraße 1 2751 Steinabrückl 614696,97 / 388674,96
Location 20	GLS-Europa-Straße 1 3133 Traismauer 577180,26 / 443341,76
Location 21	Industriezentrum NÖ-Süd, Straße 3/6 A-2355 Wiener Neudorf 623137,76 / 412481,16
Location 22	Vösendorfer Südring 15 2334 Vösendorf 622198,89 / 415988,03
Location 23	Landhausboulevard 29-30 3109 St. Pölten 570984,39 / 425189,81
Location 24	Viktor-Kaplan-Straße 2 A-2700 Wiener Neustadt 618454,53 / 386136,19

Location 25	Donau-City-Str 1 1220 Wien 628609,31 / 430602,47
Location 26	Brunhildengasse 1 1150 Wien 622409,27 / 426905,77
Location 27	Fernkorngasse 10 1100 Wien 625328,28 / 424434,34
Location 28	Lerchenfelder Gürtel 43 1160 Wien 623286,39 / 427601,36
Location 29	Simmeringer Hauptstraße 24 1110 Wien 628448,52 / 424946,96
Location 30	Ignaz-Köck-Straße 10 1210 Wien 627647,64 / 434453,08
Location 31	Phorusgasse 8 1040 Wien 625234,92 / 425570,52
Location 32	Trabrennstr. 5 1020 Wien 628776,75 / 428258,2

Location 33	Viaduktstrasse 20 2353 Guntramsdorf 624584,83 / 411952,19
Location 34	1230 Wien Richard-Strauss-Straße 1 623875,46 / 420896,11
Location 35	Heinrich-Schneidmadl-Str 15 3100 St. Pölten 570887,73 / 426197,75
Location 36	Am Kanal 8-10 2352 Gumpoldskirchen 621209,8 / 409223,27
Location 37	Pottendorferstraße 15 1120 Wien 622885,01 / 423369,42
Location 38	Fabriksgelände 1 7011 Siegendorf 639619,85 / 379585,77
Location 39	Rottwiese 62 7350 Oberpullendorf 640207,32 / 348326,05
Location 40	Bahnstraße 30 2130 Mistelbach 638534,84 / 467945,82

Location 41	Brusattiplatz 14 2500 Baden 616166,58 / 404983,61
Location 42	Fabriksgasse 26 2620 Neunkirchen 606484,94 / 372961,43
Location 43	Werner-von-Siemens-Str 1 7343 Neutal 633236,41 / 353440,97
Location 44	Ludwig-Boltzmann-Str 2 7100 Neusiedl am See 661753,98 / 402817,19
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ABSTRACT

In a globalized economy the logistical system is an important part of every business. The two main pillars of these systems are facility location decision and the tour-planning. Instead of looking here at two different problems, the Location-Routing Problem (LRP) sees them as interlinked. We have obtained a real-life problem instance, where we have to locate the depots of an Austrian Logistical Company in Eastern Austria. The approach proposed by us is an iterative heuristic solution procedure, which is using p-median-solutions as a starting point, a Savings-Algorithm combined with a r-opt-heuristic for tour-planning and an adapted Nearest-Neighbour-Heuristic for the relocation of depots. The main emphasis during the development was to create a fast and relatively simply heuristic which provides reasonable results. The objective of this thesis is to implement this solution procedure in C++ and present the results. In order to provide the necessary background information this thesis will give some introductive definitions as well as explanations concerning the underlying problems and algorithms that are used during the implementation. Using these basic attributes and definitions we derived a linear program for the LRP which is adapted to our situation. Additionally we are going to present two models of categorizations for the Location-Routing Problem in order to be able to classify the instance we want to solve.

ZUSAMMENFASSUNG

In einer globalisierten Wirtschaft spielt das Logistiknetzwerk eine elementare Rolle. Die zwei Hauptsäulen dieses System ist die Standortauswahl und die Tourenplanung. Anstatt wie bisher diese beiden Probleme getrennt zu betrachten, das Location-Routing Problem (LRP) sieht sie als ein zusammenhängendes Problem. Wir haben eine reale Problemistanz, wo unsere Aufgabe darin besteht, die Depots eines österreichischen Logistikdienstleisters zu planen. Die Vorgehensweise die von uns verfolgt wird ist ein iterativer und heuristischer Lösungsansatz, welcher p-median-Lösungen als Startpunkt nutzen wird, einen Savings-Algorithmus in Kombination mit einer r-opt-Heuristik zur Tourenplanung verwenden und eine angepasste Nearest-Neighbour-Heuristik zur Verschiebung der Standorte einsetzen wird. Der Schwerpunkt während der Entwicklung war die Schaffung einer schnellen und einfachen Heuristik, welche akzeptable Ergebnisse liefert. Das Ziel dieser Arbeit ist die Implementierung dieser Lösungsmethode in C++ und die Präsentation der erzielten Ergebnisse. Um dem Leser auch die nötigen Hintergrundinformationen zu diesem Thema zur liefern werden wir auch einige einführende Defintionen, sowie Erklärungen für der verwendeten Subprobleme und Algorithmen, die während der Implementierung verwendet werden. Mittels dieser grundlegenden Definitionen und Eigenschaften kreierten wir ein lineares Programm für das LRP, welches auf unsere Situation zugeschnitten ist. Außerdem werden wir noch zwei Klassifizierungsmodelle für das Location-Routing Problem vorstellen um in der Lage zu sein unsere Problemistanz entsprechen einzuordnen.

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